Throughput Optimization in Multi-hop Wireless Networks with Random Access

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
Abstract

This research investigates cross-layer design in multi-hop wireless networks with random access. Due to the complexity of the problem, we study cross-layer design with a simple slotted ALOHA medium access control (MAC) protocol without considering any network dynamics. Firstly, we study the optimal joint configuration of routing and MAC parameters in slotted ALOHA based wireless networks under a signal to interference plus noise ratio based physical interference model. We formulate a joint routing and MAC (JRM) optimization problem under a saturation assumption to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters. The JRM optimization problem is a complex non-convex problem. We solve it by an iterated optimal search (IOS) technique and validate our model via simulation. Via numerical and simulation results, we show that JRM design provides a significant throughput gain over a default configuration in a slotted ALOHA based wireless network.

Next, we study the optimal joint configuration of routing, MAC, and network coding in wireless mesh networks using an XOR-like network coding without opportunistic listening. We reformulate the JRM optimization problem to include the simple network coding and obtain a more complex non-convex problem. Similar to the JRM problem, we solve it by the IOS technique and validate our model via simulation. Numerical and simulation results for different networks illustrate that (i) the
jointly optimized configuration provides a remarkable throughput gain with respect to a default configuration in a slotted ALOHA system with network coding and (ii) the throughput gain obtained by the simple network coding is significant, especially at low transmission power, i.e., the gain obtained by jointly optimizing routing, MAC, and network coding is significant even when compared to an optimized network without network coding. We then show that, in a mesh network, a significant fraction of the throughput gain for network coding can be obtained by limiting network coding to nodes directly adjacent to the gateway.

Next, we propose simple heuristics to configure slotted ALOHA based wireless networks without and with network coding. These heuristics are extensively evaluated via simulation and found to be very efficient. We also formulate problems to jointly configure not only the routing and MAC parameters but also the transmission rate parameters in multi-rate slotted ALOHA systems without and with network coding. We compare the performance of multi-rate and single rate systems via numerical results.

We model the energy consumption in terms of slotted ALOHA system parameters. We found out that the energy consumption for various cross-layer systems, i.e., single rate and multi-rate slotted ALOHA systems without and with network coding, are very close.
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Finally, I am grateful to the great Almighty for everything good in my life. All praises are for him.
Dedication

To my beloved parents.
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<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CSMA/CA</td>
<td>Carrier Sense Multiple Access with Collision Avoidance</td>
</tr>
<tr>
<td>DFT</td>
<td>Delayed First Transmission</td>
</tr>
<tr>
<td>DSDV</td>
<td>Destination-sequenced Distance Vector</td>
</tr>
<tr>
<td>DSR</td>
<td>Dynamic Source Routing</td>
</tr>
<tr>
<td>ERC</td>
<td>Expected Resource Consumption</td>
</tr>
<tr>
<td>ETT</td>
<td>Expected Transmission Time</td>
</tr>
<tr>
<td>ETX</td>
<td>Expected Transmission Count</td>
</tr>
<tr>
<td>ICT</td>
<td>Information &amp; Communications Technology</td>
</tr>
<tr>
<td>ILS</td>
<td>Iterated Local Search</td>
</tr>
<tr>
<td>IOS</td>
<td>Iterated Optimal Search</td>
</tr>
<tr>
<td>JRM</td>
<td>Joint Routing and MAC</td>
</tr>
<tr>
<td>JRM-BiNC</td>
<td>Joint Routing, MAC, and Bidirectional Network Coding</td>
</tr>
<tr>
<td>JRM-BiNC-SN</td>
<td>Joint Routing, MAC, and Bidirectional Network Coding at the Selected Nodes</td>
</tr>
<tr>
<td>JRM-NC</td>
<td>Joint Routing, MAC, and Network Coding</td>
</tr>
<tr>
<td>JRM-NC-RA</td>
<td>Joint Routing, MAC, Network Coding, and Rate Allocation</td>
</tr>
<tr>
<td>JRM-RA</td>
<td>Joint Routing, MAC, and Rate Allocation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>MAC</td>
<td>Medium Access Control</td>
</tr>
<tr>
<td>NC</td>
<td>Network Coding</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to Interference Plus Noise Ratio</td>
</tr>
<tr>
<td>WiFi</td>
<td>Wireless Fidelity</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
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List of Symbols

\( c_{fl} \)  Effective rate of flow \( f \) on link \( l \)
\( \mathbf{c} \)  Effective rate vector
\( c_{f}^{r_{l}} \)  Effective rate of flow \( f \) on link \( l \) at rate \( r \)
\( d_{o} \)  Near-field crossover distance
\( d_{n_{1}n_{2}} \)  Distance between nodes \( n_{1} \) and \( n_{2} \)
\( \mathcal{F} \)  Set of data flows
\( f^{s} \)  Source of flow \( f \)
\( f^{d} \)  Destination of flow \( f \)
\( \overline{f} \)  Bi-directional counterpart flow of flow \( f \)
\( G_{n_{1}n_{2}} \)  Normalized channel gain between nodes \( n_{1} \) and \( n_{2} \)
\( L \)  Number of directed links
\( \mathcal{L} \)  Set of directed links
\( \mathcal{L}_{o}^{n} \)  Set of outgoing links of node \( n \in \mathcal{N} \)
\( \mathcal{L}_{i}^{n} \)  Set of incoming links of node \( n \in \mathcal{N} \)
\( l_{o} \)  Originating node of link \( l \in \mathcal{L} \)
\( l_{d} \)  Receiver node of link \( l \in \mathcal{L} \)
\( M_{n} \)  Number of bidirectional flow pairs relayed by node \( n \)
$N$ Number of nodes
$N_a$ Number of active nodes
$\mathcal{N}$ Set of nodes in the network
$\mathcal{N}_l$ Set of nodes in the network except the transmitter of link $l$
$\mathcal{N}_T$ Set of permitted nodes to perform network coding
$N_o$ Received background noise power
$P_t$ Transmission power
$P_K$ Packet loss probability
$p^*_l$ Probability of successful transmission on link $l$
$p^s$ Vector for the probabilities of successful transmission
$p^s_l(r)$ Successful transmission probability on link $l$ at transmission rate $r$
$q_{f,l}$ Probability of selecting flow $f$ on link $l$ by the transmitter of link $l$
$q$ Flow and link selection probability vector
$q^*_l(r)$ Probability of selecting flow $f$ on link $l$ at rate $r$ by the transmitter of link $l$
$q^{WNC}_{f,l}$ Probability of selecting flow $f$ on link $l$ by the transmitter of link $l$
to transmit without network coding
$q^{WNC}_{f,l}(r)$ Probability of selecting flow $f$ on link $l$ by the transmitter of link $l$
to transmit without network coding with rate $r$
$q^{NC}_{f_i,l_i,f_j,l_j}$ Probability of selecting flow $f_i$ on link $l_i$ and $f_j$ on link $l_j$
by the transmitter of links $l_i$ and $l_j$ to transmit with network coding
$q^{NC}_{f_i,l_i,f_j,l_j}(r)$ Probability of selecting flow $f_i$ on link $l_i$ and $f_j$ on link $l_j$ by
the transmitter of links $l_i$ and $l_j$ to transmit with network coding with rate $r$
$R$ Number of available modulation and coding schemes at each node
$\mathcal{R}$ Set of transmission rates
\( \mathcal{R}(l) \) Set of feasible transmission rates on link \( l \)

\( \mathcal{R}(l_i, l_j) \) Set of feasible transmission rates on both links \( l_i \) and \( l_j \)

\( S_l \) Set of states \( \sigma_l \) for which a transmission on link \( l \) is successful

\( S'_l \) Set of states \( \sigma_l \) for which a transmission on link \( l \) is successful at rate \( r \)

\( w_f \) Weight of flow \( f \)

\( w \) Traffic rate ratio of a downlink flow to an uplink flow

\( y_n \) Traffic (or effective traffic) carried by node \( n \)

\( \eta \) Path loss exponent

\( \gamma \) Required SINR for successful packet reception

\( \gamma(r) \) Required SINR for successful packet reception at rate \( r \)

\( \lambda \) Max-min flow rate

\( \lambda_f \) Rate of flow \( f \in \mathcal{F} \)

\( \pi_n \) Attempt probability of node \( n \)

\( \pi \) Attempt probability vector

\( \pi_0 \) Unknown multiplying vector

\( \pi_I \) Probability that the system is idle in a slot

\( \rho \) Queue utilization factor

\( \sigma_l \) Set of active nodes in a time slot except the transmitter of link \( l \)

\( \tau_{f,l} \) Transmission probability of flow \( f \) on link \( l \) in a time slot

\( \tau'_{f,l} \) Transmission probability of flow \( f \) on link \( l \) at rate \( r \)

\( \tau \) Transmission probability vector

\( \tau_{f,l}^{WNC} \) Transmission probability of flow \( f \) on link \( l \) without network coding

\( \tau_{f,l}^{WNC}(r) \) Transmission probability of flow \( f \) on link \( l \) without network coding at rate \( r \)

\( \tau_{f_i,l_i,f_j,l_j}^{NC} \) Transmission probability of flow \( f_i \) on link \( l_i \) and \( f_j \) on link \( l_j \)
with network coding

\[ \tau_{f_i,l_i,f_j,l_j}^{NC}(r) \]
Transmission probability of flow \( f_i \) on link \( l_i \) and \( f_j \) on link \( l_j \) with network coding at rate \( r \)

\( \tau^{WNC} \)
Transmission probability vector for the transmissions without network coding

\( \tau^{NC} \)
Transmission probability vector for the transmissions with network coding
Chapter 1

Introduction

The number of users and their demands for new applications and bandwidth in wireless networks are increasing day by day. To meet the demand of the users in the future, the coverage area of wireless networks and their throughputs have to be increased as much as possible. Multi-hop networking has emerged as a promising technology for future wireless networks to increase coverage area and to meet the demand of the users. Traditionally, network functionalities of wired networks are performed in network protocol stack by several independent layers. Each layer is designed to perform separate functions, and the interaction between two adjacent layers is performed through a well-defined interface. In wireless networks, the functionalities of the different protocol layers impact each other significantly. To take advantage of these interactions, as opposed to suffer from them, a cross-layer design has to be performed. In this chapter, we provide an overview on multi-hop wireless networks, throughput optimization, and cross-layer design and present the motivation of this research and contributions.
1.1 Multi-hop Wireless Networks

Multi-hop wireless networks have evolved into two classes: mobile ad hoc networks and fixed wireless networks. A mobile ad hoc network is an infrastructure less and temporary network consisting of a set of self-organized and self-managed mobile nodes. Vehicular ad hoc networks, mobile sensor networks, emergency response networks, and military networks are typical examples of such networks [1], [2]. Mobile ad hoc networking is promising for safety driving, emergency rescue and relief, and conferences.

Protocol design and management in fixed wireless networks are easier as compared to those of the mobile ad hoc networks. The static nature of the node location in fixed wireless networks provides advantages in network management, protocol design, spectral efficiency, and link reliability. Wireless mesh networks are promising fixed multi-hop wireless networks for future Internet services [3]-[5]. A wireless mesh network consists of gateways, mesh routers, and mesh clients [3], [4] as shown in Fig.1.1. Mesh routers and gateways are fixed and form a mesh backbone network to provide broadband access to the clients or other networks such as cellular networks and wireless local area networks (WLANs). Clients (static or mobile) are connected directly or through other networks to the routers or gateways of the mesh backbone network to access the Internet, while the Internet is connected to the gateways of the mesh backbone network through wireline or separate wireless links. Besides providing access to the Internet, the mesh backbone network also can provide client-to-client communication facility.
1.1. MULTI-HOP WIRELESS NETWORKS

Figure 1.1: Typical wireless mesh network topology
1.2 Cross-layer Design and Throughput Optimization

Although the worldwide success of the Internet is partly due to the simplicity and robustness of its layered network architecture, this architecture developed for wired networks is not flexible enough for multi-hop wireless networks. In a multi-hop wired network, the capacities of the links are usually fixed and independent of each other. As a result, the traditional layered architecture does not impose too much penalty in wired network. But the phenomenon in multi-hop wireless networks is different due to the inherent broadcast nature of wireless transmission. Links and their performance are no longer independent of each other since a receiver not only receives the desired signal from the target transmitter but also the signals from all the other nodes transmitting simultaneously which is known as interference. In a wireless network, the performance of the links is strongly dependent on the interactions of the different layers due to interference. Since the performance of the links is dependent on the interactions of the different layers, cross-layer design provides an opportunity to optimize a certain performance measure by jointly tuning the parameters at the different layers, which cannot be done with the strict layer architecture. As a consequence, cross-layer approaches have been proposed to enhance the adaptability and performance in wireless networks [6]-[9].

Throughput is a critical performance metric in wireless networks and so is fairness. There is a trade-off between the total throughput of a network and fairness among the flows. In general, fairness among the flows is severely degraded when the total throughput of a network is maximized. In this research, we will focus on maximizing
1.3 Motivation

Based on the link layer protocol, multi-hop wireless networks can be classified into two distinct classes: random access networks and scheduled access networks. Although the throughput performance of a scheduled access network is in general much better than that of a random access network, scheduled access protocols are complex, usually centralized, and difficult to implement. On the other hand, random access protocols are distributed, robust, and easy to implement. Both access classes have attracted lot of attention from the wireless research community due to their advantages and features. In our research, we consider a single channel random access network. In a random access network, nodes access the channel according to their access parame-

\[1\text{Although } \text{max-min throughput provides better service to the worst flows, it may reduce the overall network throughput.}\]
ters and collisions may happen due to lack of knowledge of the other transmissions. In a certain period, the numbers of packet transmissions, collisions, and successful transmissions of a node are determined mainly by the access parameters of the nodes in the network. Thus, the achievable rates of the links are adjustable by setting the access parameters. In a wireless network, routing affects the load carried by each link and hence, the minimum throughput of the flows in the network can be improved significantly via configuring the node access parameters according to the traffic loads in the different links.

End-to-end throughputs of the flows not only depend on the configuration of the node access parameters but also the routing of the flows. If the routes of the flows are not chosen properly, considering the impact of interference, then end-to-end throughputs of the flows may be poor [11]. Given node access parameters, each flow has an optimal route\(^2\), and the optimal route of each flow may change with node access parameters since the rates of the links change with the access parameters. On the other hand, for given routes of the flows, each node should have an optimal access parameter and the optimal access parameter of each node changes with the routes of the flows. Thus, routing of the flows and channel access of the nodes significantly interact with each other and jointly determine the throughputs of the flows. If the routes of the flows and the access parameters of the nodes are determined separately, optimal performance may not be achieved.

Due to a high degree of interaction between the network layer and link layer, cross-layer design with routing and scheduling is addressed in many research works [9], [11]-[13]. Configuring a wireless network based on random access is much more difficult,

\(^2\)Based on the network topology, a flow could have more than one optimal routes.
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and one might be tempted to simply use a so-called default configuration comprised, for example in the case of slotted ALOHA [14], of a minimum hop routing and an equal attempt probability at all nodes. Depending on the network topology, the minimum throughput of the flows in a network with default routing and access parameters may be significantly lower than that with jointly optimal routing and node access parameters. While it is expected that joint configuration of routing and access parameters of a random access network can provide a significant throughput gain over a default configuration, there is no clear indication so far on how much improvement can be achieved by joint routing and MAC (JRM) design and how to jointly configure the parameters. Our first focus in this research is to study the joint configuration of routing and MAC parameters in random access based multi-hop wireless networks and quantify the throughput improvement by joint design with respect to a default design.

Usually nodes in wireless networks are capable to use different modulation and coding schemes. Each modulation and coding scheme is characterized by a physical transmission rate and a signal to interference plus noise ratio (SINR) threshold. If a transmitter transmits a packet with a higher rate, then the received SINR must be higher for the receiver in order to decode the packet successfully. It means that a larger number of nodes close to the receiver must not transmit during the transmission, i.e., there is a trade-off between transmission rate and spatial frequency reuse. Thus, it is important to know what transmission rate should the nodes choose if they are able to use only one transmission rate. Further, one would expect that, if all the nodes in a network are able to use multiple transmission rates and the routing, MAC, and transmission rate parameters are chosen optimally, the network throughput will
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improve significantly with respect to the throughput obtained by JRM design with optimal single rate. Next focus of our research is to provide insight about the optimal rate allocation in single rate random access networks and quantify the throughput improvement given by a multi-rate system over a single rate system.

Network coding has emerged as a promising technique both in wireline and wireless networks [15], [16] to improve throughput performance. Wireless networks suffer from interference due to the inherent broadcast nature of the wireless medium. Network coding is an important method that turns this apparent broadcast limitation of wireless communication into an advantage for better throughput performance. Network coding has been used in many contexts in wireless networks, including (i) end-to-end multicasting [17], (ii) end-to-end unicasting [18], [19], (iii) at the link layer [20], [22], [25], [58], and (iv) physical layer transmission [24]. The existing works in (i)-(ii) and (iv) are mainly theoretical. Link layer network coding is studied theoretically in [25] for unicast applications, and COPE bridges the gap between theory and practice and provides an operational protocol for general unicast traffic [22]. Due to the simplicity and practicality of link layer network coding, this technique has attracted a lot of attention from the wireless research community.

In a wireless network, (link layer) network coding opportunities significantly depend on the routing and MAC parameters and the interactions between the two layers. It is expected that network coding opportunities as well as throughput performance can be improved significantly by joint configuration of routing and MAC parameters. However, how to jointly configure the routing and MAC parameters when network coding is enabled is unknown. Our next focus of this research is to study the JRM-NC problem in single rate multi-hop wireless networks and provide insights on (i)
throughput gains obtained by the joint design over a default design, and (ii) throughput gains obtained by network coding. Further, it is important to quantify the gain obtained by a multi-rate system over a single rate system when network coding is enabled.

The total energy consumption by the communication, computing, and networking devices and the relevant global carbon dioxide (CO2) emission are increasing day by day due to dramatic increase of the use of these devices. Currently, the Information & Communications Technology (ICT) infrastructure consumes 3% of the world-wide energy and causes about 2% of the world-wide CO2 emissions [26]. It has been reported by Ericsson that the total energy cost of the mobile systems is approximately half of the total operating expenses [27]. While the researchers in the networking community are proposing more and more new techniques, e.g., network coding, to improve throughput performance of the networks, it is not clear what is the impact of these techniques on energy consumption. If a new technique brings throughput gain at the price of a high energy consumption, then the technique will not be a good solution for telecommunication operators as well as for the global environment. Hence, the next step of this research is to study energy consumption in different cross-layer systems (i.e., single rate as well multi-rate cross-layer system without and with network coding) and to provide some useful insights about energy consumption.
1.4 Thesis Overview

1.4.1 Cross-layer Design without Network Coding

Firstly, we focus on the JRM design problem in random access networks without network coding. To quantify the throughput improvement by the JRM design with respect to a default design, the off-line joint configuration problem is addressed under the assumption that there is no dynamics in traffic, network topology, and wireless channel, i.e., the number of nodes and their positions as well as the number of flows and their source and destination pairs are fixed and the channel variation with time is negligible.

In a single channel wireless network, during a transmission between a transmitter and a receiver, the interference received by the receiver is the additive interference from all the other simultaneous transmissions. As a consequence, it is essential to use a proper interference model when configuring the wireless network. The physical interference model based on SINR is the more realistic interference model for wireless networks [28]. The simple interference models such as primary interference model, protocol model, and capture threshold model can provide misleading insights about the optimal configuration of routing and MAC parameters as well as throughput improvements [28]. Thus, in this research, we consider the SINR based physical interference model to account interference at the receivers.

Throughput optimization problem of any network is a link rate constraint optimization problem [10], [11]. For popular but complex MAC protocols such as the IEEE 802.11 based carrier sense multiple access with collision avoidance (CSMA/CA) MAC protocol [29]-[31], modeling the effective link rate in terms of MAC parame-
1.4. THESIS OVERVIEW

ters under a realistic interference model is an open research issue for a multi-hop wireless network. The fundamental random access protocol, slotted ALOHA, was first proposed in 1970 by Abramson [14]. It has similar contention characteristics to CSMA/CA in a WLAN [32]. Due to its simplicity of operation and analytical formulation, the protocol is still attractive for understanding the contention in random access networks. In this research, we consider a simple slotted ALOHA MAC protocol instead of a complex MAC protocol for link layer operation. The objective is to provide insights on (i) the interaction of the routing layer and MAC layer and (ii) throughput gains obtained by a joint configuration over a default configuration.

The primary challenges in formulating the JRM problem are to define a JRM based slotted ALOHA system and model the effective rates of the flows in the different links under the physical interference model. We define a JRM based slotted ALOHA system using probabilistic routing and MAC strategies. We model the effective rate of a flow on a given link using the concept of conflict free set of the nodes under a saturation assumption. The link rate model is found to be very complex (the computational complexity exponentially increases with the number of nodes in the network) and is a non-linear and non-convex function of routing and MAC parameters. Using link rate constraints and the other necessary constraints, we formulate a JRM optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters. This optimization problem turns out to be a non-linear and non-convex problem.

The next challenge of this research is to solve the computationally complex non-linear and non-convex optimization problem. We choose to solve it by an iterated optimal search (IOS) technique [33] (which is an iterated local search (ILS) tech-
1.4. THESIS OVERVIEW

...technique [34]) and focus on small to medium size networks\(^3\). We use MINOS 5.51 [35] to compute the local maxima at each iteration of the IOS algorithm.

Before analyzing the results obtained by the JRM optimization model, it is necessary to validate the optimization model because of the saturation assumption. We validate the configurations of routing and MAC parameters obtained via our optimization model by simulation. We show that, if we use the routing and access parameters calculated by the model in a simulated network, the network can handle the max-min throughput obtained by the model and that any larger value for the throughput will make the network unstable.

We try to understand how the optimal routing and MAC parameters differ from a default configuration of those parameters and how they are related to network topology and traffic flows. In the default slotted ALOHA system, each node attempts to transmit in a time-slot with equal probability \(1/N_a\), where \(N_a\) is the number of active nodes in the system, and each flow chooses, among all the min-hop paths, the one with the shortest distance (the sum of the physical distance of each link of the path). From 10 different scenarios (by varying the number of flows and choosing the source and destination pairs randomly, i.e., ad hoc like networks) in two 10-node networks, we obtain that an optimal configuration has the following characteristics, at least in all the scenarios that we studied: (i) single path routing is optimal, (ii) most of the flows choose a path with high link quality instead of a minimum hop path, and (iii) the attempt probabilities of the nodes differ from each other significantly, where a node carrying high traffic and suffering high interference has a high attempt probability.

\(^3\)In fact, the number of nodes in a small as well as medium size network depends on the type of the wireless network. We refer 9-node to 16-node networks as the small to medium size network in the context of wireless mesh networks.
To determine the performance gain obtained by the JRM configuration over the default configuration, we determine the max-min throughput of the flows for each scenario with the default configuration by simulation. We found out that the max-min throughput performance gain when using JRM is between 9.07% to 181.73% depending on the scenario.

Next, we study max-min throughput performance of the JRM and default configurations in two 16-node mesh networks (one is grid topology and the another is random). Each network consists of a single gateway and all flows are either destined for the gateway or generated by the gateway. In this case, it is natural to refer to an uplink flow if it is destined for the gateway and a downlink flow if it is generated by the gateway. We consider that each node has a downlink flow as well as an uplink flow. We determine the optimal max-min throughput by solving the JRM optimization problem in the two 16-node mesh networks using the IOS technique. The max-min throughput with the default configuration is determined by simulation. We found out that 80% to 450% throughput gain is achievable with joint design.

The computational complexity of the JRM optimization problem limits us to solve it only for a small to medium size network and hence, for a large network, we propose a simple heuristic to configure the routing and MAC parameters. To investigate how the simple heuristic performs, the max-min throughput in the two 16-node mesh networks is determined for heuristic configuration by simulation. The max-min throughput obtained by the heuristic is significantly higher when compared to the max-min throughput obtained by the default configuration and compares well to the optimal max-min throughput. We also compare the throughput performance of the heuristic and default designs for a 30-node random network. The heuristic is
found to be very effective for this larger network.

Since there is a trade-off between physical transmission rate and spatial frequency reuse, we try to understand what is the best physical transmission rate if all the nodes are able to use only one transmission rate from a set of available transmission rates. We determine the optimal max-min throughput in the two 16-node mesh networks by solving the JRM problem with different physical transmission rates. We found out that the higher the physical transmission rate, the higher the throughput given that route of each flow is available for using the higher transmission rate.

Further, to study the joint routing, MAC, and transmission rate allocation (JRM-RA) problem in multi-rate slotted ALOHA systems, we formulate a JRM-RA optimization problem by reformulating the JRM optimization problem. We determine the optimal max-min throughput in the two 16-node mesh networks by solving the JRM-RA problem using the IOS technique. We found out that the throughput improvements by a multi-rate system over a single rate system with the optimal rate (for the case of two normalized rates 1 and 2) is negligible for the grid network, while they depend on the node transmit power and the available transmission rates for the random network. The throughput improvement is at most 16% for the random network only when the node transmit power is not sufficient to route all the flows at the highest available physical transmission rate, negligible otherwise.

1.4.2 Cross-layer Design with Network Coding

Due to its simplicity and practicality, we consider a link layer network coding to increase throughput performance in slotted ALOHA based wireless networks. COPE [22] proposed an implementation of link layer network coding with opportunistic listening.
But it is too complex to analyze link layer network coding with opportunistic listening for a wireless network and optimize the network parameters. In this research, we consider link layer network coding without any opportunistic listening to simplify network operation as well as theoretical analysis. In the absence of opportunistic listening, network coding opportunity at a node involves XOR-ing exactly two packets and these packets must enter through a pair of incoming links and leave through an opposite pair of outgoing links.

We consider a single rate slotted ALOHA system where this simple network coding is enabled. We extend the link rate model for the system with network coding under the saturation assumption. However, we model network coding constraints to ensure that a node cannot do more network coding than available packets allow. Using link rate constraints, network coding constraints, and the other necessary constraints, we then formulate a JRM-NC optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters in a single rate slotted ALOHA system. Similar to the JRM optimization problem, this optimization problem is non-linear and non-convex, but the computational complexity, the number of variables, and the number of constraints in this problem increase significantly compared to the JRM optimization problem.

We focus only on wireless mesh networks to show results with network coding, although the JRM-NC optimization problem is formulated for a general network. Due to the complexity of the JRM-NC optimization problem, we solve it for two 9-node mesh networks by the IOS technique and validate the optimization model by simulation.

To reduce the computational complexity, we reformulate the JRM-NC problem
by restricting network coding to bi-directional flows\(^4\) (called as bi-directional network coding). To compare the throughput performance of full network coding and bi-directional network coding, the optimal max-min throughputs of the two 9-node mesh networks are determined by solving the optimization problem with bi-directional network coding. Interestingly, we found out that only a small amount (less than 1\%) of throughput is lost if bi-directional network coding is used instead of full network coding under the assumption that each uplink flow (resp. downlink flow) has the same weight. Hence, we use bi-directional network coding instead of full network coding to study medium size networks (i.e., 16-node mesh networks).

To determine the throughput improvements by joint configuration, we define a default slotted ALOHA system with network coding as a benchmark. We determine the max-min throughput of the two 16-node mesh networks with default configuration by simulation. The optimal max-min throughput of these networks are determined numerically by using the IOS technique. Similar to the throughput results in slotted ALOHA systems without network coding, we found out that the joint design provides superior performance to the default design in slotted ALOHA systems with network coding. We can achieve 100\% – 450\% throughput gain with the jointly optimized configuration in the 16-node mesh networks.

Next, we determine the throughput improvements for enabling network coding in slotted ALOHA systems, i.e., throughput gain by jointly configuring routing, MAC, and network coding with respect to the JRM design without network coding. We found out that at low transmission power, roughly 30\% – 50\% throughput gain is

\(^4\)Two flows \(f_i\) and \(f_j\) are called bidirectional if \(f_i^s = f_j^d\) and \(f_i^d = f_j^s\), where the source and destination of flow \(f\) are denoted by \(f^s\) and \(f^d\).
achievable for the 16-node mesh networks. At higher transmission power, network coding becomes less attractive because there are more and more single hop paths to the gateway. One may expect that the throughput gain for network coding (i.e., network coding opportunities) is less if there is a rate imbalance between the downlink and uplink flows. Surprisingly, we found out that the typical imbalance between downlink and uplink flows in wireless mesh networks plays in favor of network coding due to retransmissions.

Further, we reformulate the JRM-NC optimization problem when network coding (bi-directional) is only employed at a subset of nodes. The optimal max-min throughput in the two 16-node mesh networks are determined by limiting network coding operation to the nodes directly adjacent to the gateway. Interestingly, we found out that a large part of the throughput improvement for network coding can be obtained by limiting network coding to the nodes directly adjacent to the gateway.

Due to the computational complexity of the optimization problems, for a larger slotted ALOHA network with network coding, we propose a simple heuristic to configure the routing and MAC parameters. We determine the max-min throughput in the two 16-node mesh networks with the heuristic configuration by simulation. We found out that this simple heuristic is also very effective, i.e., max-min throughput obtained by the heuristic compares well to the optimal max-min throughput and is significantly higher with respect to the max-min throughput obtained by the default design. We also compare the throughput performance of the heuristic and default designs for the same 30-node random network. The heuristic is found to be very efficient for this larger network.

To investigate whether the obtained insights on multi-rate slotted ALOHA sys-
tems without network coding remain the same when network coding is enabled, we formulate a joint routing, MAC, network coding, and rate allocation (JRM-NC-RA) optimization problem for a multi-rate system with network coding. We determine the optimal max-min throughput in the two 16-node mesh networks with single rate (for all the other available rates) and multiple rates when network coding is enabled. We found that the insights on transmission rate dimension remain the same even when network coding is enabled in the system.

We summarize the research works overviewed in sub-sections 1.4.1-1.4.2 using the diagram shown in Fig. 1.2.
1.4.3 Energy Consumptions

To determine the energy consumption in the different proposed cross-layer systems, we model the energy consumption for slotted ALOHA systems as a function of the system parameters. We calculate the energy consumption with the optimal configuration obtained by each of the joint designs, i.e., joint design in single rate as well as multi-rate system without and with network coding. We found out that the energy consumption for all the cross-layer systems are very close. Thus, the throughput improvement by enabling network coding as well as a multi-rate technique can be obtained without a significant impact in term of energy consumption.

1.5 Contributions

It is well known that cross-layer design enhances throughput performance in multi-hop wireless networks. However, it is a very difficult problem to deal with, especially in random access based multi-hop wireless networks. This is due to the difficulty of modeling link rates in terms of system parameters, and formulating and solving a joint optimization problem. To the best of our knowledge, there have been limited study on the cross-layer design among the routing layer, MAC layer, and physical layer in random access based multi-hop wireless networks. In this research, we study the optimal joint configuration of routing, MAC, and transmission rate parameters in slotted ALOHA based multi-hop wireless networks without and with network coding and provide various useful insights. Our contributions are as follows:

- We model the effective link rate for a slotted ALOHA system with the SINR based physical interference model via the concept of conflict free set of nodes.
1.5. CONTRIBUTIONS

- We formulate the JRM optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters in slotted ALOHA based wireless networks. We extend this problem to JRM-NC problem. These problems turn out to be very large non-linear and non-convex optimization problems.

- We solve the optimization problems numerically by using the IOS technique.

- Via numerical and simulation results, we quantify the performance gains obtained by jointly optimizing configuration of routing and MAC parameters over a default configuration in slotted ALOHA systems without and with network coding.

- Via numerical results, we also quantify the performance gains obtained by jointly optimizing routing, MAC, and network coding over a joint design without network coding in mesh networks, i.e., gains obtained for enabling network coding, and provide several interesting engineering insights on network coding opportunities.

- We propose simple heuristics to configure slotted ALOHA-based wireless mesh networks without and with network coding. We show via simulation that the max-min throughputs obtained by the heuristics are significantly higher than the max-min throughputs obtained by default designs and compare well with the optimal max-min throughputs.

- We also formulate problems to jointly configure routing, MAC, and transmission rate parameters in multi-rate slotted ALOHA systems without and with network
coding and compare the performance of multi-rate and single rate systems.

- We model the energy consumption in terms of system parameters for slotted ALOHA systems. We show that the amount of energy consumptions for all the cross-layer systems (i.e., single rate as well as multi-rate system without and with network coding) are very close.

1.6 Outline

The rest of the thesis is organized as follows. Chapter 2 presents a literature review. In Chapter 3, we study cross-layer design in slotted ALOHA based wireless networks without network coding. We present the formulation of the JRM optimization problem and the IOS solution technique. We describe the simple heuristic to configure routing and MAC parameters in slotted ALOHA systems. We present the reformulation of the JRM problem to include optimal rate allocation in multi-rate slotted ALOHA systems. We provide numerical and simulation results for various network scenarios. In Chapter 4, we study cross-layer design in slotted ALOHA based wireless networks with network coding. We present the reformulation of the optimization problems and heuristic design to include network coding. We provide numerical and simulation results for various scenarios of wireless mesh networks. We study the energy consumption in different cross-layer based slotted ALOHA systems. We summarize the thesis and discuss some future work in Chapter 5.
Chapter 2

Literature Review

In this research, we mainly focus on the joint configuration of routing and MAC parameters in slotted ALOHA based wireless networks to maximize the minimum throughput of the flows. Since network coding is a promising technique to improve throughput performance in wireless networks, cross-layer design between the network layer and MAC layer is studied for slotted ALOHA systems without and with network coding. We also study the jointly optimized configuration of routing, MAC, and transmission rate parameters in multi-rate slotted ALOHA systems without and with network coding. Further, we study the energy consumptions in the proposed cross-layer systems. We categorize the related work into three sections: (i) cross-layer design without network coding, (ii) cross-layer design with network coding, and (iii) energy consumption.

We use the IOS technique to solve the cross-layer optimization problems, which is similar to the ILS technique [34]. The ILS technique is also described in this chapter.
2.1 Cross-layer Design without Network Coding

The performance of the protocols in the different layers in wireless networks is strongly dependent on each other since radio transmissions are susceptible to interference. As a result, the traditional layered architecture is very inefficient for wireless networks even when the protocols in the different layers are designed carefully. In the last decades, the layering architecture based networking approach has been criticized in the context of wireless networks in many papers [6]-[8] and many cross-layer designs have been proposed to improve different performance measures.

Usually cross-layer design approaches are classified into two categories: loosely coupled and tightly coupled. In the loosely coupled cross-layer design, researchers attempted to improve performance of networks by exchanging information and setting the parameters among the different layers [37]. Most of the loosely coupled cross-layer design approaches focus on configuring the parameters of one protocol layer heuristically or by solving an optimization problem using the information of the other layers. On the other hand, in the tightly coupled cross-layer design, parameters in different layers are optimized altogether as one optimization problem to optimize a certain performance measure. The tightly coupled cross-layer design is in general more complex and difficult to implement than the loosely coupled cross-layer design, but it provides optimal performance by taking full advantage of interaction among the layers. Although the loosely coupled cross-layer design is simple and easy to implement, unfortunately, this approach does not usually provide enough benefits since it cannot account for the complete interaction among the layers [38].

Most researches on tightly coupled cross-layer design for random access based
multi-hop wireless networks focus on the transport layer and MAC layer. Cross-layer based congestion control and MAC in slotted ALOHA networks have been addressed in [47], [48] using a protocol interference model. In [47], the authors propose two distributed joint rate control and MAC algorithms, a dual based algorithm and a primal based algorithm, to provide proportional fairness among the flows by controlling the attempt probabilities of the nodes. In [48], the authors extend the work of [47] with the objective to maximize the network utility$^1$. Recently, the authors in [49], [50] propose joint congestion control and MAC algorithms using a general interference model to maximize network utility in CSMA based multi-hop wireless networks. In [49], the authors consider a CSMA protocol with ideal carrier sensing, i.e., carrier sensing time is zero and hence, back-off time is continuous. Rates of the sources and MAC parameters (i.e., back-off time of the links) are adapted distributively based on the queue length of the links to optimize network utility. In [50], the same authors consider a realistic carrier sensing mechanism, i.e., back-off time is discrete for the links, and distributively optimize the network utility by controlling source rates and payload sizes (MAC parameters) based on the queue length of the links. In [51], the authors consider a slotted like CSMA protocol and propose a distributed algorithm similar to [49], [50]. In [47]-[51], the authors do not consider the routing as well as rate allocation problem into their cross-layer design problems. They investigate the joint congestion control and MAC problem and provide distributed algorithms by solving the joint congestion control and MAC problems. In this thesis, we study a

$^1$In the literature, the objective functions $\sum_{f \in \mathcal{F}} \lambda_f (1 - \zeta)/(1 - \zeta), \zeta \neq 1, \zeta \geq 0$, and $\sum_{f \in \mathcal{F}} \log(\lambda_f), \zeta = 1$, are considered as network utility functions for data flows, where $\mathcal{F}$ is the set of data flows, $\lambda_f$ is the rate of flow $f \in \mathcal{F}$, and $\zeta$ is a parameter that represents the fairness among the flows. Note that $\zeta \to \infty$ provides max-min throughput of the flows and the objective function $\sum_{f \in \mathcal{F}} \log(\lambda_f)$ provides proportional fairness among the flows.
cross-layer design problem between the routing and MAC layers (also physical layer) in slotted ALOHA networks and focus on an off-line static configuration of the network parameters. Tightly coupled joint design of routing and scheduling (as opposed to random access MAC) is addressed in many papers (For example see [9], [11]-[13]).

A large number of cross-layer design studies in random access networks are based on the loosely coupled approach [39]-[46]. Since early 1990’s, researchers have tried to address the problem of JRM for multi-hop ALOHA wireless networks [39], [40], although the term “cross-layer design” was not familiar to the researchers at that time. In [39], a nonlinear joint optimization problem is formulated using a simple interference model and solved by decoupling the routing and the MAC problems. For the routing problem, a heuristic is used to find the minimum hop path with low interference and then the MAC problem is solved by an iterative numerical method. In [40], the problem is solved by forcing the attempt probabilities to be fixed and equal for all nodes. This transforms the original problem into a linear program which can be easily solved. In both papers, the authors have decoupled the MAC and routing problems to get some workable solution. In [41]-[43], cross-layer between the network layer and MAC layer is addressed by designing different routing metrics. The purpose of these routing metrics are to determine the optimal route of a newly arriving session or an existing session after a route failure by computing the metric value of different paths based on MAC layer information. Cross-layer design between the network layer and MAC layer based on routing metric improves throughput performance by exploiting the MAC layer information and is easy to implement distributively. However, it cannot achieve the optimal throughput performance since routes of the existing sessions and MAC parameters are not adapted to the routing impact of a new arriving or
2.1. CROSS-LAYER DESIGN WITHOUT NETWORK CODING

failure session. In [44], a cross-layer design between the network layer and MAC layer is proposed to find out a stable route in mobile ad hoc networks based on the stability and life time information of the links. In [45], [46], cross-layer designs between the transport layer and MAC layer are proposed to improve throughput performance in IEEE 802.11 based multi-hop wireless networks.

In Chapter 3, we study cross-layer design in slotted ALOHA based wireless networks to maximize the minimum throughput of the flows. We formulate the JRM optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of the routing and MAC parameters in a slotted ALOHA based wireless network. The researches in [39], [40] are the most relevant works to the JRM problem of this thesis. In [39], [40], the authors model the JRM optimization problems using the protocol model for wireless interference and solve their problems by decoupling the routing and MAC problems. We formulate the JRM optimization problem under the physical interference model. To model the JRM optimization problem, we derive the expression of the rate of a flow on a given link under the physical interference model. The link rate model is found to be very complex (the computational complexity exponentially increases with the number of nodes in the network). We provide a methodology to reduce the computation complexity of link rates. Unlike decoupling the routing and MAC problems, we solve the joint problem using the IOS technique.

We also propose a simple heuristic to configure the routing and MAC parameters in a slotted ALOHA-based wireless network to maximize the minimum throughput of the flows. In [41]-[43], the authors address only the routing problem in CSMA/CA based wireless networks by designing different routing metrics based on heuristic. Since the
purposes of these routing metrics are not to maximize the minimum throughput of the flows, it is not clear how they will perform for max-min throughput. For simplicity, our heuristic for routing is the same as the default routing, i.e., it is based on min-hops. The heuristic for MAC is based on the insights of the optimal configuration of MAC parameters obtained by solving the JRM optimization problem. We compare the performance of our heuristic with the optimal design. The heuristic is found to be very effective.

We formulate the JRM-RA optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of the routing, MAC, and transmission rate parameters in a multi-rate slotted ALOHA based wireless network. To the best of our knowledge, JRM-RA problem in random access based wireless networks is not addressed so far in any paper.

2.2 Cross-layer Design with Network Coding

Since the pioneering work on network coding for multicast applications on wireline networks [15], a large body of work has explored network coding for multicast as well as unicast applications on wireline and wireless networks [17]-[19], [21], [23]. These works investigate end-to-end network coding which is complex and very difficult to implement. In [25], Wu et al. introduce a simple link layer network coding, i.e., XOR-type network coding, for unicast applications. Ho et al. study the construction of XOR coding between a pair of flows in wireless networks with multiple unicast flows [52]. COPE [22] provides an operational protocol for XOR-type network coding with opportunistic listening in CSMA/CA networks for general unicast traf-
A testbed deployment has shown that COPE can significantly increases network throughput. In [36], the authors show that in a multi-rate system the opportunistic (greedy) scheduling with COPE-type network coding may not satisfy a throughput requirement that can be achieved by scheduling without network coding. Due to the operational complexity and energy consumption of opportunistic listening, they suggest to study COPE without opportunistic listening. They propose an optimal adaptive network coding scheme joining with scheduling to take the advantage of network coding. They also propose a new network coding scheme XOR-Sym for bi-directional flows which requires decoding of XOR packets only at the destination node. They show that the throughput performance of XOR-Sym is similar to COPE without opportunistic listening but the operational complexity of XOR-Sym is very low. Interestingly, our optimization formulation for bi-directional network coding fits well with the XOR-Sym network coding scheme. The studies in [22], [25], [36], [52] focus on the construction of different network coding schemes. The throughput performance of any network coding scheme in a wireless network significantly depends on the configuration of the network parameters. However, these studies do not focus on this issue. We consider the XOR-type network coding scheme introduced in [25] in a slotted ALOHA based wireless network and focus on the configuration of the network parameters to maximize the minimum throughput of the flows.

In [53], the authors study joint link adaptation and network coding, and show that link rate adaptation has a significant impact on network coding opportunities due to retransmissions. Recently, the throughput performance of a two-hop relay network (i.e., three-node network) with network coding is studied in [54]-[56]. In [54], the authors consider a slotted ALOHA MAC protocol and show that the transmission
2.2. CROSS-LAYER DESIGN WITH NETWORK CODING

probability of the relay node is a design parameter that is crucial to maximize the throughput. In [55], the same authors consider a CSMA MAC protocol instead of slotted ALOHA and provide similar insight. In [56], the authors consider a CSMA MAC protocol and show that long processing times for network coding reduce throughput significantly. The works in [54]-[56] demonstrate that network coding opportunities significantly depend on the configuration of MAC parameters, although these works are limited to a two-hop relay network.

In [22], the authors studied network coding by using a dynamic source routing (DSR) protocol under the expected transmission time (ETT) routing metric and the default MAC parameters of 802.11 wireless cards. BEND, a more opportunistic link layer network coding scheme than COPE, is proposed in [58] and is studied using a destination-sequenced distance vector (DSDV) routing protocol under the same MAC protocol. In BEND, network coded packets can be constructed from 2 or more non-network coded packets. Furthermore, in BEND, XORed packets that are constructed from a greater number of non-network coded packets use a smaller contention window in order to increase the efficiency of the medium access.

In [57], the expected resource consumption (ERC) routing metric is proposed to determine a route for a given flow that has good network coding opportunities. The efficiency of the ERC routing metric is evaluated via simulation under the default IEEE 802.11 MAC layer and compared to the expected transmission count (ETX) routing metric.

In Chapter 4, we study cross-layer design in slotted ALOHA based wireless networks with network coding. While it is clear that network coding opportunities in a wireless network significantly depend on the routing, MAC, and transmission rate
parameters [53]-[57], the existing study [22], [57], [58] do not explicitly consider the interaction between network parameters and network coding (i.e., do not formulate and solve a joint problem). We formulate the novel JRM-NC and JRM-NC-RA optimization frameworks to study cross-layer design in slotted ALOHA systems with network coding. We solve the joint problems using the IOS technique and provide various engineering insights. We also provide a simple heuristic to configure the routing and MAC parameters in a slotted ALOHA based wireless mesh network with network coding.

With respect to conflict free scheduled networks (as opposed to random access MAC), network coding has been studied in [59]-[61]. In [59], the authors study joint routing, scheduling, and network coding under a simplistic interference model and provide bounds on throughput. In [60], [61], the authors study joint congestion control, scheduling, and bi-directional network coding.

2.3 Energy Consumption

In the past, the research on energy consumption was limited to the energy constrained wireless networks. Recently, energy consumption is becoming a focus not only for those networks but also for all the other wireless networks for environmental friendly future networking. In [62]-[65], the authors study energy consumption in single hop wireless networks. In [62], the authors model the energy efficiency of MAC schemes whose operations can be described by finite state-space Markov chains. Energy efficiencies of different versions of a hybrid protocol are compared using slotted ALOHA and reservation concepts. Chen et al. [63] model the energy consumption for
some MAC protocols, including the IEEE 802.11 under a simple back-off mechanism and demonstrate that energy consumption can be reduced by reducing contention on the channel. In [64], the authors present an energy consumption model for an IEEE 802.11 WLAN with a practical back-off mechanism and demonstrate that the transmit mode of a node has marginal impact on the overall energy consumption, while other modes (receive, idle, etc.) are responsible for most of the energy consumption. The energy consumption in the IEEE 802.11 protocol is studied in [65] by measuring energy consumptions for sending, receiving, and discarding data packets. It is concluded that the energy consumption associated with receiving data is not negligible when compared to the transmission energy. The energy consumption models used in the research works mentioned above are not expressed as a function of slotted ALOHA system parameters. As a result, we use the insights of the reasons of energy consumption, i.e., energy is consumed not only to send packets but also to receive packets, and model the energy consumption for slotted ALOHA systems.

In [66]-[69], the authors study energy consumption in multi-hop wireless networks. In [66], the authors study the trade-offs between transmission range (i.e., transmission power), average energy consumption, and the achievable throughput in a slotted ALOHA system. It is found that increasing the throughput significantly by means of transmitter power control requires only a very moderate increase in energy consumption and transmission range. In [67], the authors study joint routing, scheduling, and link adaptation to minimize energy consumptions in wireless networks for a given rate of the flows. It is demonstrated that when only the transmission energy is considered, multi-hop routing saves energy but single-hop transmission may be more efficient than a multi-hop routing scheme when the circuit processing energy is included, i.e., circuit
processing energy has a significant impact on the routing of the flows. However, link rate adaptation can reduce energy consumption significantly. In [68], [69], the authors study joint routing, scheduling, and power control to minimize power consumptions in wireless networks for a given rate of the flows. It is found that there is a trade-off between energy consumption and the throughput and delay performance in a wireless network. In this research, we propose various cross-layer systems. We found out that throughput performance improves when enabling network coding as well as rate adaptation technique. The above studies do not provide any insights whether the energy consumption will increase when enabling these techniques. We determine the energy consumption for the optimal configurations of the network parameters, and compare the energy consumption of the proposed cross-layer systems.

2.4 Iterated Local Search Technique

The cross-layer optimization problems solved in this thesis are non-convex and non-linear. ILS is a simple, robust, and highly effective technique to solve a non-convex problem [34]. This technique iteratively applies the local optimal solutions of the previous iterations to perturb the current search point and find out a new local optimum. To describe the ILS algorithm, denote the vector of variables of a given optimization problem by $x$. Initially, the \textit{GenerateInitialSolution} procedure generates initial values of the variables, $x^I$, for the optimization problem and the \textit{LocalSearch} procedure finds out the local optimal solution $x_0$ using $x^I$. The iterative procedure described below starts with $x_0$ and continues until a given termination condition satisfies. In each iteration of the iterative procedure, three procedures are performed to generate
2.5. SUMMARY

We have reviewed the related work on cross-layer design in multi-hop wireless networks. There have been limited study on the cross-layer design among the routing layer, MAC layer, and physical layer in wireless networks without and with network coding. The existing studies on energy consumption in wireless networks are reviewed. However, these studies do not provide any insights on whether the energy consumption will increase when enabling network coding and multi-rate techniques. We also describe the ILS technique as background of the IOS technique used in this
research to solve several non-convex optimization problems.
Chapter 3

Cross-layer Design without Network Coding

In this chapter, we mainly study the joint configuration of routing and MAC parameters to maximize the minimum throughput of the flows in slotted ALOHA based wireless networks. We define a JRM based single channel slotted ALOHA system. We model the effective rate of a flow on a given link based on the concept of conflict free set of the nodes under the physical interference model and saturation assumption. We formulate the JRM optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters. The JRM optimization problem is a complex non-convex problem. We present the IOS technique to solve the optimization problem numerically. We validate the configurations of routing and MAC parameters obtained via our optimization model by simulation to justify the saturation assumption.

Due to its computational complexity, the optimal configuration for a large network
3.1 Joint Routing and Medium Access Control

is intractable, and thus one needs to develop heuristics to configure slotted ALOHA-based wireless networks. We present a simple heuristic to configure the routing and MAC parameters in a slotted ALOHA-based wireless network.

We provide numerical and simulation results for various network scenarios and demonstrate that a proper configuration (i.e., jointly optimized or heuristic) of routing and MAC parameters yields significant improvement in throughput performance in wireless networks using slotted ALOHA with respect to a default configuration.

We also extend the JRM optimization framework to maximize the minimum throughput in multi-rate slotted ALOHA systems by jointly optimizing routing, MAC, and transmission rate. We compare the throughput performance of multi-rate and single rate systems using numerical results.

3.1 Joint Routing and Medium Access Control

3.1.1 System Model

Network Topology and Flows: We consider a wireless network consisting of \( N \) stationary nodes with known locations using the same transmission power \( P_t \). The set of nodes is denoted by \( \mathcal{N} \). Let \( \mathcal{L} \) be the set of directed links in the network and \( L = |\mathcal{L}| \). Clearly the set of links depends on \( P_t \) (see later). A directed link \( l \in \mathcal{L} \) is represented as \((l^o, l^d)\), where \( l^o \) and \( l^d \) are the originating and destination nodes of the link. We denote the sets of links coming into and going out of node \( n \) by \( \mathcal{L}_n^I \) and \( \mathcal{L}_n^O \). There are \( F \) data flows in the network, belonging to set \( \mathcal{F} \). A data flow \( f \) is characterized by its source \( f^s \) and its destination \( f^d \). The rate for flow \( f \) is constant
and denoted by $\lambda_f$. Denote the weight $w_f$ for flow $f \in \mathcal{F}$ such that

$$\frac{\lambda_f}{w_f} = \lambda \quad \forall f \in \mathcal{F} \tag{3.1}$$

where $\lambda$ is the common base throughput for all the flows. We want to maximize $\lambda$.

**Channel and Interference Models:** The channel gain of a link is assumed to be time invariant [33], [70]. The channel gain between nodes $n_1$ and $n_2$, $G_{n_1n_2}$, is given by $(d_{n_1n_2}/d_0)^{-\eta}$, where $d_{n_1n_2}$ is the distance between nodes $n_1$ and $n_2$, $d_0$ is a reference distance in the far field of the transmit antenna, and $\eta$ is the path loss exponent. We assume that all the nodes use the same modulation and coding scheme characterized by a unit rate and an SINR threshold $\gamma$. A directed link between $n_1$ and $n_2$ exists if the signal-to-noise ratio (SNR) for the link is greater than $\gamma$, i.e.,

$$\frac{G_{n_1n_2}P_t}{N_0} \geq \gamma \tag{3.2}$$

where $N_0$ is the received background noise power. Time is slotted and the size of a packet is fixed and corresponds to the duration of one time slot. A packet sent by $n_1$ in a given time slot is considered to be successfully received by the receiver $n_2$ if the received SINR is higher than $\gamma$. Thus, a packet transmission from $n_1$ to $n_2$ is successful if

$$\frac{G_{n_1n_2}P_t}{N_0 + \sum_{n' \neq n_1} G_{n'n_2}P_t Y_{n'}} \geq \gamma \tag{3.3}$$

where $Y_{n'}$ is a binary variable being equal to 1 if node $n'$ transmits in the given slot and 0 otherwise.

**Medium Access Control:** We consider a slotted ALOHA MAC protocol, where
3.1. JOINT ROUTING AND MEDIUM ACCESS CONTROL

The nodes in the network are synchronized and probabilistically access the channel in each time-slot. Denote $\pi_n$ to be the probability that node $n$ tries to access the channel in a given slot, i.e., the attempt probability, and the corresponding probability vector $\pi = [\pi_1, \pi_2, \ldots, \pi_N]$. For medium access, at each slot, node $n$ first generates a Bernoulli variable with probability $\pi_n$. If the result is 1, then it performs the routing operation as follows to transmit a packet. If the result is zero, it keeps silent.

**Routing:** Given that node $n$ does try to access the channel, the routing decision is to determine which packet to send and whom to send it to. We consider a probabilistic routing strategy to select a packet (i.e., flow) and the receiver (i.e., link) of the packet. The routing operation is described by the following random variables. Given that node $n$ does try to access the channel, we denote the conditional probability that it will select a packet of flow $f$ to transmit on link $l \in L_n$ by $q_{f,l}$ with the condition

$$\sum_{f \in \mathcal{F}, l \in L_n} q_{f,l} = 1. \quad (3.4)$$

The collection of $q_{f,l}$ variables is represented by the flow selection probability vector $q$.

We assume that each node maintains a separate infinite queue for each flow. Further, we assume that if a node attempts to transmit in a time slot and selects a flow, a packet of the flow will be available at that node. This is what we call the saturation assumption. We will explain more about this assumption after formulating the optimization problem and we will validate this assumption by simulation.

**Retransmission Strategy:** We assume that a transmitter knows immediately at the end of the current slot whether its transmission is successful or not. We consider a
delayed first transmission (DFT) retransmission policy, where the transmitting node keeps a copy of the packet in the queue that it is transmitting. This copy is deleted if the transmission is successful; otherwise it is retransmitted when the transmitter selects that flow again.

3.1.2 Effective Link Rate

In the absence of interference, two nodes can communicate at a nominal rate, $C$, determined by physical layer parameters. We normalize the physical transmission rate to $C = 1$. The presence of other nodes and the MAC policy will reduce the normalize rate to a lower value because of collisions and retransmissions. This is referred to as the effective link rate.

Let $\tau_{f,l}$ be the probability that a packet of flow $f$ will be transmitted on link $l$ in a given time slot. It is given by

$$\tau_{f,l} = \pi_n q_{f,l} \quad \forall n \in \mathcal{N}, \forall f \in \mathcal{F}, \forall l \in \mathcal{L}_n^O. \quad (3.5)$$

The collection of $\tau_{f,l}$ is called the transmission probability matrix, denoted by $\tau$. Because nodes are able to know immediately whether a collision has occurred, the effective rate of flow $f$ on link $l$, $c_{f,l}$, can be expressed as

$$c_{f,l} = \tau_{f,l} p_l^s \quad (3.6)$$

where $p_l^s$ is the probability that a packet can be transmitted successfully on link $l$, i.e., that the SINR at $l_d$ will be greater than the threshold $\gamma$. The main difficulty of
the link rate model is the calculation of $p^s_l$. We denote the effective link rate matrix by $c$.

**Computation of $p^s_l$:** Let $N_l$ be the set of nodes except the transmitter of link $l$, i.e., $N_l = \mathcal{N} \setminus l^0$. We denote a state of $N_l$ in a time slot by $\sigma_l$, where $\sigma_l \subset N_l$ is the set of active nodes in the time slot. Because each node decides whether or not it will transmit independently of all the other nodes, the probability $P\{\sigma_l\}$ that the system is in state $\sigma_l$ in a time slot is given by

$$P\{\sigma_l\} = \prod_{i \in \sigma_l} \pi_i \prod_{j \in N_l \setminus \sigma_l} (1 - \pi_j).$$  \hspace{1cm} (3.7)

A transmission on link $l$ is successful for a state $\sigma_l$ depending on the received SINR at the receiver. Let $S_l$ be the set of states for which the transmission on link $l$ is successful and the number of successful states, $|S_l| = K_l$. It is clear that $P\{\sigma_i^l \cap \sigma_j^l\} = 0$ for any two successful states $i$ and $j$ as the system cannot be in two states simultaneously, and hence the successful transmission probability $p^s_l$ is given by

$$p^s_l = P\left\{ \bigcup_{\sigma_l \in S_l} \sigma_l \right\}$$
$$= \sum_{\sigma_l \in S_l} P\{\sigma_l\}$$
$$= \sum_{\sigma_l \in S_l} \prod_{i \in \sigma_l} \pi_i \prod_{j \in N_l \setminus \sigma_l} (1 - \pi_j).$$  \hspace{1cm} (3.8)

The calculation of the successful transmission probability for a given link $l$ is then made up of two parts. The first one is the enumeration of all the successful states $S_l$. This depends on the parameters of the physical layer and on the position of the nodes,
but does not depend on the \( \pi \) variables. The second step is the evaluation of the polynomial in \( \pi \) given by (3.8). This calculation has to be done whenever the values of \( \pi \) change, for instance during an iterative optimization procedure. The complexity in determining all the successful states is \( 2^{(N-1)} \). This complexity can be reduced significantly by using a suitable enumeration technique [9], [71]. The computational complexity of \( p^s_l \) in (3.8) depends on the number of nodes \( N \) and the number of successful states \( K_l \), where \( K_l \) is given by the network topology and physical layer parameters. The computational complexity can be further reduced significantly by applying the following proposition.

**Proposition 3.1.1** If \( \sigma^1_l \) and \( \sigma^2_l \) are two successful states of the set of nodes \( N_l \) such that \( \sigma^1_l \cup \{n\} = \sigma^2_l \), then

\[
P\{\sigma^1_l\} + P\{\sigma^2_l\} = \prod_{i \in \sigma^1_l} \pi_i \prod_{j \in N_l \setminus \sigma^1_l} (1 - \pi_j) \tag{3.9}
\]

where \( N'_l = N_l \setminus \{n\} \).

**Proof:** Using (3.7), we have

\[
P\{\sigma^2_l\} = \prod_{i \in \sigma^1_l \cup \{n\}} \pi_i \prod_{j \in N_l \setminus (\sigma^1_l \cup \{n\})} (1 - \pi_j)
= \frac{\pi_n}{1 - \pi_n} \prod_{i \in \sigma^1_l} \pi_i \prod_{j \in N_l \setminus \sigma^1_l} (1 - \pi_j)
= \frac{\pi_n}{1 - \pi_n} P\{\sigma^1_l\}. \tag{3.10}
\]
Thus, from (3.10) we get

$$P\{\sigma^1_i\} + P\{\sigma^2_i\} = \frac{P\{\sigma^1_i\}}{1 - \pi_n}. \quad (3.11)$$

Using (3.7) in (3.11), (3.9) can be obtained. □

This proposition means that if two successful states satisfy the condition, they can be combined into one successful state, and hence \( \mathcal{N}_i \) can be replaced by set \( \mathcal{N}'_i \) for the combined state. Since a successful state is made by adding a node to another successful state, eventually, this proposition will reduce the computational complexity significantly.
3.1.3 Joint Routing and MAC Optimization Problem

The JRM optimization problem to maximize the minimum throughput of the flows is given by

\[
\max_{\tau, \pi, \lambda} \lambda \quad \text{subject to} \quad 0 \leq \lambda, c \quad \text{and} \quad 0 \leq \tau, \pi \leq 1.
\]

The objective function in (3.12) ensures that the minimum throughput of the flows is maximized. The flow conservation constraints in (3.13) guarantee that the outgoing and incoming traffic of a flow are equal at each intermediate node, that the outgoing traffic of a flow is equal to the source rate at the source node, and that the incoming traffic of a flow is equal to the source rate at the destination node. This ensures that intermediate nodes cannot create flows. The link rate constraints in (3.14) ensure that the traffic rate on a link is not larger than the link rate for each flow. The equality constraints in (3.15) relate the attempt probabilities to the transmission probabilities.
Equations (3.16) and (3.17) are the bounds on the variables.

Now, we explain the saturation assumption, i.e., we assume that a packet of a flow is always available at a node if it selects the flow in a time slot. Since flow conservation constraints ensure that intermediate nodes cannot create flows, due to the link rate constraints, the solution of the JRM optimization problem will provide us such configuration of the MAC and routing parameters so that an intermediate node will attempt to transmit a flow only if it relays the flow and the arrival rate of the flow at the intermediate node will be exactly equal to the service rate of the flow.

The JRM optimization problem in (3.12)-(3.17) is a non-linear optimization problem because the constraints in (3.14) have a strong non-linear dependence on the \( \pi \) variables. Furthermore, constraints in (3.14) are not convex since both sides of the constraints turn out as posynomials [72]. Thus, finding a global optimal solution is a challenge. Note that the solution of the JRM determines the optimal \( \pi^*, \tau^* \) and \( \lambda^* \). The optimal values for \( q^* \) can be determined from \( \pi^* \) and \( \tau^* \) using (3.5).

### 3.2 Solution Technique

Since the JRM optimization problem (3.12)-(3.17) is non-linear and non-convex, computing a global optimum is difficult if not impossible for large networks. Branch and bound [73], [74], simulated annealing [75], and ILS [34] techniques are well known to solve non-convex optimization problems. We choose to solve the JRM optimization problem by the IOS technique which is an ILS technique [34] described in Section 2.4 of Chapter 2. However, we will only be able to obtain solution for small to medium size networks.
3.2.1 Iterated Optimal Search Algorithm

For a given problem, the IOS algorithm finds a sequence of local maxima by starting from different initial values at each iteration. The main feature of this method is that the initial values of a local search are chosen using the best solution of the previous iterations. Denote by $M$ the total number of iterations of the algorithm. Further, let $\mathbf{x}$ be the vector of variables of the optimization problem and $\mathbf{x}_m$ be the initial values of the variables for the $m^{th}$ iteration. At each iteration, we use MINOS 5.51 [35] to compute the local maxima. The initial values of variables for the first iteration, $\mathbf{x}_1$, are taken from a reasonable range of the variables. At the start of the $m^{th}$ iteration, $1 < m \leq M$, $\mathbf{x}_m$ is computed by $\mathbf{x}_m = \mathbf{x}_m^B + \mathbf{x}_m^p$, where $\mathbf{x}_m^B$ is the best solution among the first $m - 1$ iterations and $\mathbf{x}_m^p$ is a perturbation vector given by $\mathbf{x}_m^p = \alpha_m \odot \mathbf{x}_1$, where $\odot$ is an element-wise multiplication operator and each element of the vector $\alpha_m$ is chosen independently from a uniform distribution on $[-a, a]$. At the end of the $M^{th}$ iteration, this algorithm selects the best local optimal solution.

3.2.2 Determining the Optimal Solution

To determine the optimal solution of a given problem, we run the IOS algorithm with several different initial vectors $\mathbf{x}_1$ and three values of $a$ for each initial $\mathbf{x}_1$, and we then select the best solution. In our study, we selected $M = 30$ and the 3 values of $a$ to be 0.25, 0.5, and 1.
### 3.3 Model Validation

The *effective rate* model of a flow on a given link that we use to compute the optimal max-min throughput and the routing and the MAC configuration is based on the assumption that the queues of source and the relaying nodes of the flow are saturated. This is not always the case in practice so that it is important to validate this saturation assumption. This can be done by simulating a network configured with the optimal parameters calculated by the IOS algorithm and increasing the rate $\lambda$ of each flow $f$ (assuming $w_f = 1 \ \forall f \in \mathcal{F}$) until instability is seen (see [33]). If we obtain by simulation that $\forall \lambda < \lambda'$ the system is stable and for $\lambda \geq \lambda'$ the system is unstable, then if $\lambda^* \approx \lambda'$, we have validated our model. For the simulation, $q^*$ is calculated from (3.5) using the optimal configuration $\tau^*$ and $\pi^*$.

### 3.3.1 Network and Algorithm Parameters

We use two 10-node random networks (Rand10A and Rand10B) with different flow sets, yielding 10 different scenarios. Two sets of different physical layer parameters for the two 10-node networks are given in Table 3.1. The two networks are shown in Fig. 3.1 (a) and Fig. 3.2 (a) with only the odd numbered directed links for clarity. The directed links in the opposite direction have the following even numbers. The

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Network Rand10A</th>
<th>Network Rand10B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission power (dBm)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SINR threshold (dB)</td>
<td>15</td>
<td>6.4</td>
</tr>
<tr>
<td>Noise power (dBm)</td>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>Path-loss exponent</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Far-field crossover distance (m)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3.1: Physical layer parameters
3.3. MODEL VALIDATION

| Flow set | $|\mathcal{F}|$ | S-D pairs (Rand10A) | S-D pairs (Rand10B) |
|----------|-----------------|---------------------|---------------------|
| 1        | 2               | $(6, 4), (8, 9)$    | $(1, 5), (7, 6)$    |
| 2        | 3               | $(3, 4), (8, 5), (6, 10)$ | $(7, 5), (9, 6), (6, 5)$ |
| 3        | 4               | $(4, 6), (8, 9), (7, 4), (9, 2)$ | $(4, 1), (1, 5), (5, 6), (6, 9)$ |
| 4        | 5               | $(5, 2), (6, 4), (9, 8), (10, 7), (3, 9)$ | $(9, 5), (1, 6), (6, 5), (5, 1), (7, 6)$ |
| 5        | 9               | $\{(i, 9) : i = 1 \ldots 10, i \neq 9\}$ | $\{(i, 6) : i = 1 \ldots 10, i \neq 6\}$ |

links are determined using (3.2). A scenario is characterized by the network (either network Rand10A or Rand10B) and a flow set (i.e., set of source and destinations pairs). The weight of each flow is assumed to be equal, i.e. $w_f = 1 \quad \forall f \in \mathcal{F}$. The different scenarios are specified in Table 3.2, where $S-D$ represents source and destination.

3.3.2 Simulator Setup

The average rates of the sources are all set to equal values and their traffic is assumed to be Poisson. The node decision to transmit or not and the selection of which flow to transmit on which link are implemented in the simulation as described in the system model. When the source rate is low, a node may not always have a packet of the selected flow to transmit. In that case, the node does not transmit.

Each node maintains a separate queue for each flow with a buffer of size 1000 packets. In the simulator, the number of packets in a queue is increased by one if a new packet arrives, decreased by one if a transmission is successful, and kept unchanged if a transmission is unsuccessful. Since a separate queue is maintained for
each flow, this strategy is equivalent to the DFT retransmission strategy mentioned in sub-section 3.1.1 on page 40. The simulation is done using C++.

3.3.3 Determining the Max-min Throughput of a Network Configuration

For a particular source rate, the packet loss probability of each queue is estimated from the ratio of the number of loss packets and the number of packets that arrived at the queue over a window of $1 \times 10^8$ slots after a network loading time of $10^6$ slots. The total simulation time is then $1.01 \times 10^8$ slots. To determine the max-min throughput with a small error, the source rate is increased from a starting value $\lambda_0$ by small increments of 0.0001 till the system becomes unstable. The system stability is checked at each step using the statistical test described in Appendix.

3.3.4 Numerical vs. Simulation Results

The comparison between our numerical results and the simulation results is summarized in Table 3.3. The column labeled “Numerical” contains the maximum throughput computed by the JRM algorithm. The column labeled “Simulation” contains the maximum and minimum values of the largest stable throughput obtained over 10 simulation runs. The difference between the numerical and simulation results is less than 1% in most of the cases and the maximum difference is found to be 4.25%. Based on this, we can consider that the model has been validated.
3.4. OPTIMAL VS. DEFAULT CONFIGURATION

Table 3.3: Numerical versus simulation max-min throughput

<table>
<thead>
<tr>
<th>Network</th>
<th>Flow Set</th>
<th>Numerical</th>
<th>Simulation</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rand10A</td>
<td>1</td>
<td>0.1227</td>
<td>0.1247−0.1249</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1107</td>
<td>0.1112−0.1115</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0493</td>
<td>0.0494−0.0495</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0359</td>
<td>0.0359</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.0247</td>
<td>0.0247</td>
<td>0</td>
</tr>
<tr>
<td>Rand10B</td>
<td>1</td>
<td>0.1546</td>
<td>0.1543−0.1547</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0875</td>
<td>0.0877−0.0879</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0659</td>
<td>0.0686−0.0687</td>
<td>4.25</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0427</td>
<td>0.0430−0.0431</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.0293</td>
<td>0.0294</td>
<td>0.34</td>
</tr>
</tbody>
</table>

3.4  Optimal vs. Default Configuration

In this section, we define a default configuration for a slotted ALOHA system. We compare the optimal configuration of the routing and MAC parameters with the default configuration of those parameters for each scenario of the two 10-node networks. We want to understand how the optimal routing and MAC parameters differ from a default configuration of those parameters and how they are related to network topology and traffic flows.

3.4.1 Default Configuration

We assume that each flow uses a single path with min-hop routing. For each flow we choose, among all the min-hop paths, the one with the shortest distance (the sum of the physical distances of each link of the path) since the quality of a link depends on the distance between the transmitter and the receiver. If the number of shortest distance min-hop paths is more than one, e.g. in a network with grid topology, the path yielding the maximum total traffic load is chosen to reduce collisions by...
3.5. ADVANTAGES OF JOINT CONFIGURATION

decreasing traffic in the competing nodes.

The default configuration uses the same attempt probability at all nodes. We set the node attempt probability to $1/N_a$. If the routing is given, $N_a$, the number of active nodes in the network, can be computed easily. If a node decides to transmit, it will select a flow from all the flows that it will transmit with equal probability.

3.4.2 Comparison

We show in Figures 3.1 (b)–(f) and 3.2 (b)–(f) the optimal routing of each flow with solid lines and the optimal attempt probability of each node for the 10 scenarios. The computed max-min throughput of the flows is given at the bottom of each figure. In each figure, we also present the routing for the default configuration, indicated by dotted lines. We can see that the optimal attempt probabilities are very different from those of the default configuration and that, in most cases, minimum hop routing is not optimal. In particular, a node carrying high traffic and suffering high interference has a high attempt probability. From the optimal routing, we also note that most of the flows choose a path with high link quality. It is also interesting that, for all the scenarios, the optimal route of each flow is a single path. It means that splitting a flow to balance the load in a network does not seem to be a good solution for a random access network, since it increases collisions by increasing traffic in the competing nodes. A similar phenomenon is also observed in [71].
Figure 3.1: Network Rand10A: optimal routing (solid lines) and MAC configurations and min-hop routing (dotted lines)
Figure 3.2: Network Rand10B: optimal routing (solid lines) and MAC configurations and min-hop routing (dotted lines)
3.5 Advantages of Joint Configuration

The max-min throughput for the default configuration of each scenario of the Rand10A and Rand10B networks is determined by simulation\(^1\) and compared to the max-min throughput obtained by the JRM design. The max-min throughput for the JRM and default configurations are shown in Table 3.4, where \(\lambda_{JRM}\) and \(\lambda_D\) are the max-min throughput achieved for the JRM and default configurations. We calculate throughput gains by the JRM design over the default design by

\[
% \text{Throughput gain} = \frac{\lambda_{JRM} - \lambda_D}{\lambda_D} \times 100. \tag{3.18}
\]

It is seen that the performance gain varies significantly from one scenario to another. The relative throughput gain ranges from 9.07% to 181.73%.

We also study the max-min throughput performance of the JRM and default designs for two 16-node mesh networks (Grid16 and Rand16) as shown in Fig. 3.3,\(^1\)

\(^1\)We select the minimum throughput obtained over 10 simulation runs.
where the gateway node is shown by a rectangle in each figure. The total number of flows in each network is set to be \(2(N - 1)\), where \(N - 1\) flows are the uplink flows to the gateway and the other \(N - 1\) flows are the downlink flows from the gateway. We assume that the weight of each uplink flow is 1 and the weight for each downlink flow is \(w\), i.e., the traffic rate ratio of a downlink flow to an uplink flow is \(w\). Except for the transmit power, the physical layer parameters are taken as described in Table 3.1 for network Rand10B. The max-min node throughput (i.e., \(w\lambda + \lambda\)) achieved for the JRM and default designs are shown in Fig. 3.4. It is seen that max-min node throughput increases with transmission power for both designs and it is very sensitive to the transmission power for the JRM design but not for the default one. Further, max-min node throughput with \(w = 2\) is higher than with \(w = 1\) for the JRM configuration. We attribute this to the fact that the downlink links of a node have higher successful transmission probabilities than its uplink links due to congestion as traffic increases for the nodes close to the gateway and the gateway node itself generates a large amount of traffic. Because of this fact and since the relative downlink traffic increases with \(w\) with respect to the uplink traffic and the JRM design optimizes the routing and access parameters of the nodes according to traffic, per node throughput increase with \(w\). But it is the reverse for the default configuration. We attribute this to the fact that the bottleneck is at the gateway for the default configuration since it has to transmit significantly higher traffic than the other nodes but it has the same access parameter than the others. Since the relative traffic at the gateway increases with \(w\) with respect to the traffic at the other nodes, max-min node throughput decreases with increasing \(w\). We show the throughput gains obtained by the JRM design over the default design in Fig. 3.5 for
both networks. The results indicate that 80-300% throughput gain can be achieved with joint design for the equal weighting case, i.e., \( w = 1 \). The throughput gains with \( w = 2 \) is higher than with \( w = 1 \) and hence the maximum throughput gain increases up to 450%.

## 3.6 Heuristic Configuration

In this section, we describe a simple heuristic to configure routing and MAC parameters in slotted ALOHA based wireless networks that allows us to configure a large size network. Then by comparing the performance of the heuristic with the performance of the JRM and default designs, we show how a simple heuristic can bring significant benefits. Before defining precisely this heuristic, we need to better understand what parameters are to be configured. Clearly, we need to define a per flow routing strategy that will be used to fill up the forwarding table in each node and a way to set the attempt probabilities \( \pi_n \). In addition, we also need to describe how, if a node decides
Figure 3.4: Max-min node throughput in the two 16-node networks: Left: Grid16; Right: Rand16.

Figure 3.5: Throughput gain of the JRM design over the default design in the two 16-node networks: Left: Grid16; Right: Rand16.
to transmit, it will select a flow. All these parameters must be explicitly configured.

### 3.6.1 Routing

While it is clear that min-hop routing can be sub-optimal, its simplicity makes it a good candidate for a heuristic. We consider that each flow chooses a min-hop path as in Section 3.4.1 for the default configuration.

### 3.6.2 Medium Access Control

In order to get a better understanding on how the optimal attempt probabilities $\pi_n$’s are related to the input parameters, we have studied them using the optimal routing configuration of joint design. The value of $\pi_n$ clearly should depend on the traffic carried by node $n$ as well as the traffic carried by the other nodes (which are function of routing). We conjecture that a good approximation would be of the type:

$$
\pi_n = \frac{y_n}{\sum_{n' \in N} y_{n'}} \pi_0
$$

where $y_n$ is the amount of traffic transmitted by node $n$ and $\pi_0$ is an unknown factor depending on network topology. From the optimal routing configuration, the heuristic values of $\pi_n$’s are determined using (3.19). The optimal values of $\pi_n$’s and the heuristic values are shown in Fig. 3.6 for the two 16-node networks (the X-axis represents the node index), where the node index for the gateway is 16. These results are surprisingly good. Since it is not clear what is a suitable value of $\pi_0$, we decided to use $\pi_0 = 1$ such that $\sum_n \pi_n = 1$. 
3.6.3 Flow(s) and Link(s) Selection

Once a node has decided to transmit, it has to determine which flow it will transmit. Since a single route has been selected per flow, the link on which the selected packet will be transmitted is known. Thus, after a decision of transmission, a node needs only to select a flow. In our heuristic, a node probabilistically selects a carried flow to transmit. We consider that node $n$ will select the carried flow $f$ with probability $\frac{\lambda_f}{y_n}$ when it decides to transmit.

3.6.4 Performance of the Heuristic

We determine the max-min node throughput in the two 16-node mesh networks by simulation with the heuristic configuration. The max-min node throughput for the JRM, default, and heuristic designs are shown in Fig. 3.7. Clearly, the heuristic design provides a significantly higher throughput when compared to the default design and
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comparisons well with the JRM design.

We also study the performance of the heuristic in a 30-node random (Rand30) network as shown in Fig. 3.8, where the gateway node is shown by a rectangle. The total number of flows is set to be $2(N - 1)$, where $N - 1$ flows are the uplink flows to the gateway and the other $N - 1$ flows are the downlink flows from the gateway. Except for the transmit power, the physical layer parameters are taken as described in Table 3.1 for network Rand10B. The max-min node throughput for the default and heuristic designs are shown in Fig. 3.9. Clearly, the heuristic is very effective for this larger network.

3.7 Joint Routing, MAC, and Rate Adaptation

3.7.1 System Model

We assume that all the nodes are able to use $R$ modulation and coding schemes characterized by the set of physical transmission rates $\mathcal{R} = \{r_1, r_2, \cdots, r_R\}$. The minimum SINR for the physical transmission rate $r \in \mathcal{R}$ is $\gamma(r)$. Network topology, node transmit power, and traffic model are assumed to be the same as those defined in the single rate system, where $L$ is the set of directed links with the minimum physical transmission rate $r_{\text{min}} = \min_{r \in \mathcal{R}} r$. Note that a directed link between nodes $n_1$ and $n_2$ exists with physical transmission rate $r$ (i.e., transmission rate $r$ is feasible between nodes $n_1$ and $n_2$) if the SNR for the link is greater than $\gamma(r)$, i.e.,

$$\frac{G_{n_1n_2}P_i}{N_0} \geq \gamma(r).$$

(3.20)
Figure 3.7: Comparison of max-min node throughput among the JRM, heuristic, and default designs: Top-left: Grid16, $w = 1$; Top-right: Grid16, $w = 2$; Bottom-left: Rand16, $w = 1$; Bottom-right: Rand16, $w = 2$. 
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Figure 3.8: The 30-node random network.

Figure 3.9: Comparison of max-min node throughput between the heuristic and default designs in the Rand30 network.
Denote $\mathcal{R}(l) \subseteq \mathcal{R}$ to be the set of feasible rates on link $l$. We assume that each node can adjust the size of a packet according to the transmission rate such that the transmission time of the packet is equal to the duration of one time slot. In a given time slot, a packet sent by $n_1$ with physical transmission rate $r$ is considered to be successfully received by receiver $n_2$ if the received SINR is higher than $\gamma(r)$, i.e., a packet transmission from node $n_1$ to $n_2$ with physical transmission rate $r$ is successful if

$$ \frac{G_{n_1n_2}P_t}{N_0 + \sum_{n' \neq n_1} G_{n'n_2}P_tY_{n'}} \geq \gamma(r). \quad (3.21) $$

The medium access strategy of each node is assumed to be the same as before, i.e., each node $n$ attempts to access the channel in a time slot with probability $\pi_n$. Since each node is capable to use all the modulation and coding schemes, it requires to re-define the routing operation as follows to include rate adaptation while the queue management and retransmission strategies also remain the same as those described in the single rate system. For routing operation, we assume that node $n$ selects a packet of flow $f$ to transmit on link $l \in \mathcal{L}_n^O$ with physical transmission rate $r \in \mathcal{R}(l)$ with probability $q_{f,l}^r$ such that

$$ \sum_{f \in \mathcal{F}, l \in \mathcal{L}_n^O, r \in \mathcal{R}(l)} q_{f,l}^r = 1 \quad (3.22) $$

given that it does try to access the channel.
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3.7.2 Problem Formulation

Let $\tau_{f,l}^r$ be the probability that a packet of flow $f$ will be transmitted on link $l$ in a given time slot with transmission rate $r \in \mathcal{R}(l)$. Thus,

$$
\tau_{f,l}^r = \pi_n q_{f,l}^r \quad \forall n \in \mathcal{N}, \forall f \in \mathcal{F}, \forall l \in \mathcal{L}_n^O, \forall r \in \mathcal{R}(l).
$$

(3.23)

The effective rate of flow $f$ on link $l$ is then given by

$$
c_{f,l} = \sum_{r \in \mathcal{R}(l)} r \tau_{f,l}^r \hat{p}_l^s(r)
$$

(3.24)

where $p_l^s(r)$ is the probability that a packet can be transmitted successfully on link $l$ with transmission rate $r$, i.e., that the SINR at $l_d$ will be greater than the threshold $\gamma(r)$.

Let $S_r^l$ be the set of states of the nodes in $\mathcal{N}_l$ for which a transmission on link $l$ is successful with transmission rate $r$. Thus, similar to (3.8), $p_l^s(r)$ can be calculated as

$$
p_l^s(r) = \sum_{\sigma_l \in S_r^l} \prod_{i \in \sigma_l} \pi_i \prod_{j \in \mathcal{N}_l \setminus \sigma_l} (1 - \pi_j).
$$

(3.25)

Similar to the JRM optimization problem (3.12)-(3.17), the JRM-RA problem can
be expressed as

\[
\max_{\tau, \pi, c} \lambda
\]

\[
\sum_{l \in \mathcal{L}_h^O} c_{f,l} - \sum_{l \in \mathcal{L}_h} c_{f,l} = \begin{cases} w_f \lambda & \text{if } n = f^s \\ -w_f \lambda & \text{if } n = f^d \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \tag{3.27}
\]

\[
c_{f,l} = \sum_{r \in \mathcal{R}(l)} r^{\tau_{f,l}} \left( \sum_{\sigma_i \in \mathcal{S}^f_l} \prod_{i \in \sigma_i} \pi_i \prod_{j \in \mathcal{N} \setminus \sigma_i} (1 - \pi_j) \right) \quad \forall f \in \mathcal{F}, \forall l \in \mathcal{L} \tag{3.28}
\]

\[
\pi_n = \sum_{f \in \mathcal{F}, l \in \mathcal{L}_h^O, r \in \mathcal{R}(l)} \tau_{f,l} \quad \forall n \in \mathcal{N} \tag{3.29}
\]

\[
0 \leq \lambda, c \tag{3.30}
\]

\[
0 \leq \tau, \pi \leq 1. \tag{3.31}
\]

This problem is similar to the JRM problem, but the complexity in computing the link rates and the attempt probabilities increases $R$ times.

### 3.7.3 Multi-Rate vs. Single Rate

To compare the performance of multi-rate and single rate slotted ALOHA systems, we determine the optimal max-min throughput of the two 16-node mesh networks by solving the JRM-RA optimization problem using the IOS technique. We consider that each node uses two modulation and coding schemes characterized by rates 1 and 2 and SINR thresholds 6.4 dB and 9.4 dB. Previously, we have solved the JRM optimization problem for the two 16-node networks with transmission rate 1. Hence,
we also solve the JRM optimization problem with transmission rate 2. The optimal max-min throughput of the JRM and JRM-RA designs in the two 16-node networks are shown in Fig. 3.10. Clearly, higher throughput is achievable using a higher physical transmission rate in a single rate system when the network is connected at this higher transmission rate. The throughput improvement for using multiple rates is negligible for the 16-node grid network. In the case of 16-node random network, it depend on the transmit power and the available rates. The results show that the throughput gain is at most 16% for the random network only when the node transmit power is not sufficient to route all the flows with the highest available physical transmission rate, negligible otherwise.

3.8 Conclusion

We have studied the optimal joint configuration of routing and MAC parameters to maximize the minimum throughput of the flows in slotted ALOHA-based wireless networks. Via an extensive simulation campaign, we demonstrate that cross-layer design of routing and MAC yields remarkable improvement in throughput performance in wireless networks using slotted ALOHA when compared to a default design. We also provide a simple heuristic to configure routing and MAC parameters in slotted ALOHA based wireless networks and demonstrate that throughput performance can be improved significantly by a heuristic configuration of the access parameters based on the traffic load of the node.

Further, we have studied the optimal joint configuration of routing, MAC, and transmission rate parameters in multi-rate wireless networks. We found that the
3.8. CONCLUSION

Figure 3.10: Comparison of max-min node throughput between single rate and multirate systems: Top-left: Grid16, \( w = 1 \); Top-right: Grid16, \( w = 2 \); Bottom-left: Rand16, \( w = 1 \); Bottom-right: Rand16, \( w = 2 \).
throughput improvement for using multiple rates is negligible in a grid network while it depends on the node transmit power and the available rates in a random network. A moderate amount of throughput gain is achievable in random networks only when the node transmit power is not sufficient to route all the flows with all the available physical transmission rates, throughput gain is negligible otherwise.
Chapter 4

Cross-layer Design with Network Coding

In Chapter 3, we show that the jointly optimized routing and MAC configuration provides a significant throughput gain over a default configuration in a slotted ALOHA based wireless network without network coding. In a wireless network, network coding opportunities significantly depend on the routing and MAC configuration and on their interactions. To achieve a high throughput a JRM-NC design is necessary in a wireless network with network coding. In this chapter, we study the joint configuration of routing and MAC parameters to maximize the minimum throughput of the flows in slotted ALOHA based wireless networks when a simple network coding is enabled. We define a slotted ALOHA system in which the simple network coding is enabled. The operations of a slotted ALOHA system change when enabling the simple network coding and hence, we require to re-model the effective link rate for the system with network coding. We re-model the effective link rate under the physical interference
model and saturation assumption. Only replacing the link rate constraints to the JRM optimization problem is not sufficient to formulate the JRM-NC optimization problem. We require to model the network coding constraints to ensure that a node cannot do more network coding than allowed with the available packets. We model the network coding constraints, re-formulate some constraints to reduce the computation complexity, and formulate the JRM-NC optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters. We also formulate the problems when we restrict network coding to bidirectional flows and when network coding is only employed at a subset of nodes. Similar to the JRM optimization problem, these optimization problems are non-convex and very complex. We solve these optimization problems numerically by using the IOS technique and validate our model by simulation. We provide numerical and simulation results for various mesh network scenarios and quantify the throughput gains obtained by the JRM-NC design over a default design with network coding. We also quantify the throughput improvements given by the simple network coding. We present a simple heuristic to configure slotted ALOHA based wireless mesh networks when network coding is enabled. The heuristic is extensively evaluated via simulation and found to be very efficient.

We also extend the JRM-NC formulation for a multi-rate slotted ALOHA system with network coding. Using numerical results, we compare the throughput performance of multi-rate and single rate slotted ALOHA systems with network coding.

We study energy consumption for the different proposed cross-layer systems. We model the energy consumption in terms of the slotted ALOHA system parameters. We determine the energy consumption for various network scenarios and compare the
energy consumption of the different proposed cross-layer systems.

## 4.1 Joint Routing, MAC, and Network Coding

### 4.1.1 System Model

We consider link layer network coding without opportunistic listening. Thus, network coding at a node involves XOR-ing exactly two packets and these packets must enter through a pair of incoming links and leave through an opposite pair of outgoing links [22]. In Fig 4.1, for example, assume that node \( a \) (resp. \( c \)) needs to send packets of flow \( f_1 \) (resp. \( f_2 \)) to node \( c \) (resp. \( a \)) through the intermediate node \( b \). If two packets from two flows are available at node \( b \), it can transmit both packets simultaneously by XOR operation and hence, node \( a \) (resp. \( c \)) can decode the packet of flow \( f_2 \) (resp. \( f_1 \)) from the XOR packet if it receives the XOR packet successfully.

Now, we define a slotted ALOHA system to include this simple link layer network coding. Network topology, flows, and MAC operation of the system are considered to be the same as defined in the single rate slotted ALOHA system without network coding. We also consider a similar physical layer model, i.e., a packet – single or
XORed\(^1\) – sent by transmitter \(n_1\) in a given time slot is considered to be successfully received by receiver \(n_2\) if the received SINR is higher than \(\gamma\). The main differences of the slotted ALOHA systems without and with network coding are in routing and queue maintenance operations as described in the following. Given that node \(n\) does try to access the channel, we denote the conditional probability that (i) it will select packets of flows \(f_i\) and \(f_j\), \(f_i \neq f_j\), to transmit on links \(l_i \in \mathcal{L}_n^O\) and \(l_j \in \mathcal{L}_n^O\), resp., \(l_i \neq l_j\), using network coding by \(q_{f_i,l_i,f_j,l_j}^{NC}\), (ii) it will select a packet of flow \(f_i\) to transmit on link \(l_i \in \mathcal{L}_n^O\) without network coding by \(q_{f_i,l_i}^{WNC}\). These probabilities are related by the following equation:

\[
\sum_{f_i \in \mathcal{F}_n, l_i \in \mathcal{L}_n^O, f_j \in \mathcal{F}_n, l_j \in \mathcal{L}_n^O, f_i \neq f_j, l_j < l_i} q_{f_i,l_i,f_j,l_j}^{NC} + \sum_{f_i \in \mathcal{F}_n, l_i \in \mathcal{L}_n^O} q_{f_i,l_i}^{WNC} = 1. \tag{4.1}
\]

We assume that each node maintains a separate infinite buffer for each flow and each node maintains for each packet the information on which incoming link it was received. We assume that if a node attempts to transmit in a time slot and selects one or two flows, packet(s) of the selected flow(s) is available at the node so that it can transmit. This is the saturation assumption for a slotted ALOHA system with network coding.

### 4.1.2 Effective Link Rate

Let \(\tau_{f_i,l_i,f_j,l_j}^{NC}\) be the probability that packets of flows \(f_i\) and \(f_j\) will be transmitted using network coding on links \(l_i\) and \(l_j\), resp., in a given time-slot and \(\tau_{f_i,l_i}^{WNC}\) be the probability that a packet of flow \(f_i\) will be transmitted on link \(l_i\) without network

\(^1\)The network coded packet is called an XORed packet.
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coding in a given time slot. Note that $\tau_{f_i,l_i,f_j,l_j}^{NC} = \tau_{f_j,l_j,f_i,l_i}^{NC}$. The collection of $\tau_{f_i,l_i,f_j,l_j}^{NC}$'s and $\tau_{f_i,l_i}^{WNC}$'s variables are denoted by $\mathbf{\tau}^{NC}$ and $\mathbf{\tau}^{WNC}$, respectively. We assume that the links have some (arbitrary) ordering, i.e., given any two distinct links $l_1 \neq l_2$, we have that either $l_1 < l_2$ or $l_2 < l_1$. To keep the number of variables to a minimum and having ordered the links, we denote the collection of $\tau_{f_i,l_i,f_j,l_j}^{NC}$'s by $\mathbf{\tau}^{NC} = \{\tau_{f_i,l_i,f_j,l_j}^{NC} : f_i \in F, f_j \in F, f_i \neq f_j, l_i \in L, l_j \in L, l_j < l_i\}$. Thus,

$$\tau_{f_i,l_i}^{WNC} = \pi_n q_{f_i,l_i}^{WNC} \quad \forall n \in N, \forall f_i \in F, \forall l_i \in L_n^O$$  \hspace{1cm} (4.2)

$$\tau_{f_i,l_i,f_j,l_j}^{NC} = \pi_n q_{f_i,l_i,f_j,l_j}^{NC} \quad \forall n \in N, \forall f_i,f_j \in F, \forall l_i,l_j \in L_n^O, f_i \neq f_j,l_j < l_i$$ \hspace{1cm} (4.3)

and

$$\pi_n = \sum_{f_i,f_j \in F, l_i,l_j \in L_n^O, f_i \neq f_j, l_j < l_i} \tau_{f_i,l_i,f_j,l_j}^{NC} + \sum_{f_i \in F, l_i \in L_n^O} \tau_{f_i,l_i}^{WNC}.$$ \hspace{1cm} (4.4)

We assume that a transmitter knows immediately (i.e., at the end of the current slot) whether or not its packet (single or XORed) is successfully received by the intended receiver(s). For an XORed transmission $a \oplus b$, if the receiver intending to decode $b$ from $a \oplus b$ cannot receive $a \oplus b$ successfully while the other receiver can, then the transmitter will retransmit only packet $b$, not packet $a$, using network coding or without network coding later. We assume that each node maintains an infinite buffer to store all the successfully transmitted packets such that they can be used to decode
the necessary packets from the received XORRed packets.

For the transmissions associated with the transmission probabilities $\tau_{\mathcal{F},f_i,l_i,f_j,l_j}^{NC}$, with $l_i, l_j \in \mathcal{L}$, $l_j < l_i$, $f_i \neq f_j$, the effective rate of flow $f_i$ on link $l_i$, $c_{f_i,l_i,f_j,l_j}^{NC}(f_i,l_i)$, is given by

$$c_{f_i,l_i,f_j,l_j}^{NC}(f_i,l_i) = \tau_{\mathcal{F},f_i,l_i,f_j,l_j}^{NC} p_i^l$$

(4.5)

and the effective rate of flow $f_j$ on link $l_j$, $c_{f_i,l_i,f_j,l_j}^{NC}(f_j,l_j)$, is given by

$$c_{f_i,l_i,f_j,l_j}^{NC}(f_j,l_j) = \tau_{\mathcal{F},f_i,l_i,f_j,l_j}^{NC} p_j^l.$$  

(4.6)

For the transmissions associated with the transmission probabilities $\tau_{\mathcal{F},f_i,l_i}^{WN}$, the effective rate of flow $f_i$ on link $l_i$ is given by

$$c_{f_i,l_i}^{WN} = \tau_{\mathcal{F},f_i,l_i}^{WN} p_i^l.$$  

(4.7)

The effective rate of flow $f_i$ on link $l_i$ for the system with network coding can be written as

$$c_{f_i,l_i} = c_{f_i,l_i}^{WN} + \sum_{f_j \notin \mathcal{F}, f_j \neq f_i \in \mathcal{L}_p^{\text{out}}, l_j < l_i} c_{f_i,l_i,f_j,l_j}^{NC}(f_i,l_i) + \sum_{f_j \notin \mathcal{F}, f_j \neq f_i \in \mathcal{L}_p^{\text{out}}, l_j > l_i} c_{f_j,l_j,f_i,l_i}^{NC}(f_i,l_i)$$

$$= (\tau_{\mathcal{F},f_i,l_i}^{WN} + \sum_{f_j \notin \mathcal{F}, f_j \neq f_i \in \mathcal{L}_p^{\text{out}}, l_j < l_i} \tau_{\mathcal{F},f_i,l_i,f_j,l_j}^{NC} + \sum_{f_j \notin \mathcal{F}, f_j \neq f_i \in \mathcal{L}_p^{\text{out}}, l_j > l_i} \tau_{\mathcal{F},f_j,l_j,f_i,l_i}^{NC}) p_i^l.$$  

(4.8)
4.1.3 Joint Routing, MAC, and Network Coding Optimization Problem

We now formulate the JRM-NC optimization problem. In Section 4.1.2, we derive the expression of the effective rate of a flow on a given link by combining the rates achieved by both types of transmissions under the saturation assumption. Similar to the JRM optimization problem (3.12)-(3.17), we will use this expression to model the link rate constraints. In the JRM optimization problem, we use flow conservation constraints and link rate constraints to guarantee that an intermediate node cannot create a flow and the arrival rate of a flow is equal to the service rate of the flow at an intermediate node. Unfortunately, these constraints are not sufficient to forbid a node to do more network coding than allowed with the available packets. To ensure that a node cannot do more network coding than allowed, we add network coding constraints to the optimization problem described in the following. Since the packets in an XORed transmission must enter through a pair of incoming links and leave through an opposite pair of outgoing links, considering only the transmission probability \( \tau_{NC}^{f_i,l_i,f_j,l_j} \), we require that the effective rates of flow \( f_i \) on link \( l_i \), \( c_{NC}^{f_i,l_i,f_j,l_j}(f_i, l_i) \), and flow \( f_j \) on link \( l_j \), \( c_{NC}^{f_i,l_i,f_j,l_j}(f_i, l_i) \), are restricted by the rates of flow \( f_i \) on link \( \overline{l}_j \), \( c_{f_i,\overline{l}_j} \), and flow \( f_j \) on link \( \overline{l}_i \), \( c_{f_j,\overline{l}_i} \), resp., where the opposite link of \( l \) is denoted by \( \overline{l} \), i.e., \( l^o = \overline{l}^d \) and \( l^d = \overline{l}^o \). If node \( n \) attempts to transmit only with the transmission probability \( \tau_{NC}^{f_i,l_i,f_j,l_j} \), then the network coding constraints for node \( n \) can be written as

\[
\begin{align*}
    c_{NC}^{f_i,l_i,f_j,l_j}(f_i, l_i) & \leq c_{f_i,\overline{l}_j} \\
    c_{NC}^{f_i,l_i,f_j,l_j}(f_j, l_j) & \leq c_{f_j,\overline{l}_i}.
\end{align*}
\]
4.1. JOINT ROUTING, MAC, AND NETWORK CODING

Note that the effective rate of flow $f_i$ on link $l_i$ (flow $f_j$ on link $l_j$) and the arrival rate $c_{f_i,l_i}$ ($c_{f_j,l_j}$) depend on each other, due to the common transmission probability $\tau_{f_i,l_i,f_j,l_j}^{NC}$ (see (4.5) and (4.6)). Thus, if we can derive a network coding constraint for the arrival flow $f_i$ at node $n$ through the incoming link $l_j$, the constraint for any arrival flow at node $n$ through any incoming link can be written in a similar way.

The arrival packets of flow $f_i$ at node $n$ through the incoming link $l_j$ are transmitted with transmission probabilities $\{\tau_{f_i,l_i,f_j,l_j}^{NC} : f_j \in F, l_i \in L_n^O, f_i \neq f_j, l_j < l_i\}$ and the total effective rate of flow $f_i$ achieved by all of these transmission probabilities is restricted by the flow rate $c_{f_i,l_j}$. Thus, for $n \in N$, $f_i \in F$, $l_j \in L_n^O$, the network coding constraint can be written as

$$\sum_{f_j \in F, f_i \neq f_j, l_i \in L_n^O, l_j < l_i} c_{f_i,l_i,f_j,l_j}^{NC}(f_i,l_i) + \sum_{f_j \in F, f_i \neq f_j, l_i \in L_n^O, l_j > l_i} c_{f_j,l_j,f_i,l_i}^{NC}(f_i,l_i) \leq c_{f_i,l_j}$$

(4.11)

where the left hand side represents the total effective rate of flow $f_i$ on all the outgoing links (except $l_j$) of node $n$ for network coding with the traffic of the other flows on link $l_j$.

To compute $\pi_n$ from (4.4), the number of additive terms is $O(F^2L^2)$ which is very high and this will limit the size of the network that we can handle. To reduce this
complexity, we rewrite $\pi_n$ as follows

$$
\pi_n = \frac{1}{2} \sum_{f_i \in F, l_i \in L_n^o} \sum_{j \in F, l_j \in L_n^o} \tau_{NC}^{l_i,l_j} + \sum_{f_i \in F, l_i \in L_n^o} \tau_{WN C}^{l_i}.
$$

(4.12)

Using (4.8) in (4.12), we have

$$
\pi_n = \frac{1}{2} \sum_{f_i \in F, l_i \in L_n^o} \frac{c_{f_i,l_i}}{p_{l_i}^s} + \frac{1}{2} \sum_{f_i \in F, l_i \in L_n^o} \tau_{WN C}^{l_i}.
$$

(4.13)

Thus, using (4.13), the number of additive terms in the computation of $\pi_n$ is reduced from $O(F^2L^2)$ to $O(FL)$.

Let $p^s$ be the vector for successful transmission probabilities on the links and let $\epsilon$ be a very small positive constant. We formulate the JRM-NC optimization problem as shown in (4.14)-(4.22). The JRM-NC optimization problem is similar to the JRM optimization problem (3.12)-(3.17). Hence, we include network coding constraints in (4.17) to ensure that a node cannot do network coding more often than what packet arrivals allow. We also include boundary constraints in (4.22) for $p^s_l$ variables. We use $\epsilon$ as a lower bound of $p^s_l$ since the constraints in (4.18) become infeasible at $p^s_l = 0$ and for a practical network usually $p^s_l > 0, \forall l \in L$. Thus, we do not consider the case where $p^s_l = 0$ for any link $l \in L$. Similar to the JRM optimization problem, the JRM-NC optimization problem is non-linear and non-convex due to the non-linear and non-convex constraints in (4.16)–(4.19), but the computational complexity
4.2. MODEL VALIDATION

\[
\max_{\tau^{NC}, \tau^{WN}, \pi, \mathbf{p}, \mathbf{c}} \lambda \\
\sum_{l \in L_n^O} c_{f_i,l_i} - \sum_{l \in L_n^I} c_{f_i,l_i} = \begin{cases} 
  w_f \lambda & \text{if } n = f^s \\
  -w_f \lambda & \text{if } n = f^d \\
  0 & \text{otherwise}
\end{cases} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \tag{4.15}
\]

\[
c_{f_i,l_i} = (\tau^{WNC}_{f_i,l_i} + \sum_{f_j \in \mathcal{F}, l_j \in L_n^O, l_i \neq f_j, l_j < l_i} \tau^{NC}_{f_i,l_i, f_j,l_j} + \sum_{f_j \in \mathcal{F}, l_j \in L_n^O, l_i \neq f_j, l_j > l_i} \tau^{NC}_{f_i,l_i, f_j,l_j}) p^*_l
\]

\[
\forall n \in \mathcal{N}, \forall f_i \in \mathcal{F}, \forall l_i \in L_n^O
\]

\[
\left( \sum_{f_j \in \mathcal{F}, l_j \neq f_i, l_i \in L_n^O, l_j < l_i} \tau^{NC}_{f_i,l_i, f_j,l_j} p^*_l + \sum_{f_j \in \mathcal{F}, l_j \neq f_i, l_i \in L_n^O, l_j > l_i} \tau^{NC}_{f_i,l_i, f_j,l_j} p^*_l \right) \leq c_{f_i,l_i} \tag{4.16}
\]

\[
\pi_n = \frac{1}{2} \sum_{f_i \in \mathcal{F}, l_i \in L_n^O} \frac{c_{f_i,l_i}}{p^*_l(\pi)} + \frac{1}{2} \sum_{f_i \in \mathcal{F}, l_i \in L_n^O} \tau^{WNC}_{f_i,l_i} \quad \forall n \in \mathcal{N} \tag{4.17}
\]

\[
p^*_l = \sum_{\sigma_i \in \pi} \prod_{i \in \sigma_i} \pi_i \prod_{j \in \mathcal{N} \setminus \sigma_i} (1 - \pi_j) \quad \forall l \in \mathcal{L} \tag{4.18}
\]

\[
0 \leq \lambda, \mathbf{c} \tag{4.20}
\]

\[
0 \leq \tau^{NC}, \tau^{WN}, \pi \leq 1 \tag{4.21}
\]

\[
\epsilon \leq \mathbf{p}^* \leq 1 \tag{4.22}
\]

significantly increases in this problem.

4.2 Model validation

To validate the JRM-NC optimization model, we use a simulation technique similar to the one described in Section 3.3 of Chapter 3.
4.2. MODEL VALIDATION

4.2.1 Networks and Algorithm Parameters

Due to its complexity, solving the JRM-NC problem is difficult if not impossible for a medium to large size network. At this point, we use two 9-node mesh networks (Grid9 and Rand9) to validate the JRM-NC model. The two 9-node networks are shown in Fig. 4.2, where the gateway node is shown by a rectangle in each figure. The total number of flows in each network is set to be $2(N - 1)$, where $N - 1$ flows are the uplink flows to the gateway and the other $N - 1$ flows are the downlink flows from the gateway. Except for the transmit power, the physical layer parameters are taken as described in Table 3.1 for network Rand10B.

4.2.2 Simulator Setup

The average rates of all the sources of the uplink flows are set to the same equal value (say, $\lambda$), the average rates at the gateway for all the downlink flows are set to $w\lambda$, and the traffic arrivals are assumed to be Poisson. The node decision to transmit or not and the selection of which flow(s) on which link(s) to transmit are implemented in the simulation as described in the problem formulation. Each node maintains a separate queue for each flow with a buffer of size 1000 packets. In a queue, the incoming link on which a packet arrived is stored. When the source rate is low, a node may not always have a packet(s) of the selected flow(s) to transmit and hence, the node does not transmit anything (or if only one packet is available when network coding is attempted then the packet is sent without network coding). The simulation is done using C++.
4.2. MODEL VALIDATION

Table 4.1: Numerical versus simulation max-min throughput.

<table>
<thead>
<tr>
<th>Net.</th>
<th>$P_t$ (dBm)</th>
<th>$w$</th>
<th>Num.</th>
<th>Simu.</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid9</td>
<td>-35</td>
<td>1</td>
<td>0.01587</td>
<td>0.0158</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.00979</td>
<td>0.0097</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>-33</td>
<td>1</td>
<td>0.02109</td>
<td>0.0212</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.01445</td>
<td>0.0144-0.0145</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>-31</td>
<td>1</td>
<td>0.02230</td>
<td>0.0223</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.01582</td>
<td>0.0158</td>
<td>0.13</td>
</tr>
<tr>
<td>Rand9</td>
<td>-38</td>
<td>1</td>
<td>0.01790</td>
<td>0.0178</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.01166</td>
<td>0.0116</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>-36</td>
<td>1</td>
<td>0.02081</td>
<td>0.0207-0.0208</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.01557</td>
<td>0.0154-0.0155</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>-34</td>
<td>1</td>
<td>0.02222</td>
<td>0.0222-0.0223</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.01604</td>
<td>0.0160</td>
<td>0.25</td>
</tr>
</tbody>
</table>

4.2.3 Numerical vs. Simulation Results

For a network scenario, the stable maximum throughput is determined using the technique described in sub-section 3.3.3. The comparison between the numerical results and simulation results is summarized in Table 4.1 for different transmit powers and different values of $w$. The column labeled “Num.” contains the max-min throughput computed by the JRM-NC algorithm. The column labeled “Simu.” contains the maximum and minimum values of the largest stable throughput obtained over 10 simulation runs using the optimal routing and MAC configurations obtained by the IOS technique. The largest difference between the numerical and simulation results is found to be 1.09%. Based on this, we can consider that our saturation assumption has been validated.
4.3 Joint Routing, MAC, and Bidirectional Network Coding

The JRM-NC optimization problem (4.14)-(4.22) is a complex non-linear and non-convex problem. To reduce the complexity, one way is to restrict network coding to bidirectional flows. In the following, we focus on wireless mesh networks even if all our formulations and solution techniques are not limited to this kind of network.

4.3.1 Problem Formulation

We define a bi-directional network coding model as follows. Denote $f_i \in F$ to be the corresponding uplink (downlink) flow of the downlink (uplink) flow $f_i \in F$. Nodes are allowed to do network coding only between $f_i \in F$ and $\overline{f}_i \in F$. Let $\pi_n$ be the probability that node $n$ will try to access the channel in a given slot. Given that node $n$ does try to access the channel, we then denote the conditional probability that (i) it will select packets of flows $f_i \in F$ and $\overline{f}_i \in F$ to transmit on links $l_i \in L_n^O$ and $l_j \in L_n^O$, resp., $l_j < l_i$, using network coding by $q_{f_i,l_i}^{NC}$, (ii) it will select a packet of flow $f_i \in F$ to transmit on link $l_i \in L_n^O$ without network coding by $q_{f_i,l_i}^{WC}$ with the condition

$$
\sum_{f_i \in F, l_i \in L_n^O, l_j \in L_n^O, l_j < l_i} q_{f_i,l_i}^{NC} + \sum_{f_i \in F, l_i \in L_n^O} q_{f_i,l_i}^{WC} = 1. \tag{4.23}
$$

One can formulate the joint routing, MAC, and bi-directional network coding (JRM-BiNC) optimization problem, by replacing $f_j \in F$ with $\overline{f}_i$ in all the constraints in the original problem formulation (4.14)-(4.22).
4.3. JOINT ROUTING, MAC, AND BIDIRECTIONAL NETWORK CODING

4.3.2 Bi-directional Network Coding vs. Full Network Coding

To compare the throughput performance of the JRM-BiNC (i.e., bi-directional network coding) and JRM-NC (i.e., full network coding\(^2\)) designs, we determine the optimal max-min throughput in the Grid9 and Rand9 networks for different transmission power levels by using the IOS technique. We compute the percentage throughput difference between the two cases as

\[
\% \text{ Diff.} = \frac{\lambda_{JRM-NC} - \lambda_{JRM-BiNC}}{\lambda_{JRM-NC}} \times 100 \tag{4.24}
\]

where \(\lambda_{JRM-NC}\) is the max-min node throughput (i.e., the max-min throughput of the combined uplink and downlink flows) for the JRM-NC design and \(\lambda_{JRM-BiNC}\) is the max-min node throughput achieved when allowing only bi-directional network coding between any two flows is possible.

---

\(^2\)Network coding between any two flows is possible.
### 4.4 Advantage of Joint Configuration

To investigate the advantages of joint configuration, as a benchmark in the following, we define a default slotted ALOHA system when network coding is enabled. Since we restrict ourselves to bidirectional network coding, a node has two types of flows, the
4.4. ADVANTAGE OF JOINT CONFIGURATION

‘local’ ones (i.e., the one it generates and the one it receives) and the ‘relayed’ ones (the number of relayed flows depends on the routing). We assume that each flow uses a single path with min-hop routing, and to take full advantage of network coding, the paths of corresponding downlink $f_i$ and uplink $\bar{f}_i$ flows are the same (with the links directed in the opposite direction) and a node always attempts to network code a relayed flow with its bidirectional counterpart. Thus, only the paths of the uplink flows need to be determined. In the default configuration, for each uplink flow, a min-hop path is chosen as in sub-section 3.4.1 of Chapter 3 and the attempt probability of each node is chosen to be $1/N$. From the routing decision, each node knows the “local flow” and bi-directional flow pairs that it will transmit. The gateway transmits only $N-1$ “local” flows (i.e., downlink flows) while the other nodes can transmit one “local” (i.e., own generated) as well as bi-directional flows. Let $M_n$ be the number of bidirectional flow pairs that node $n \in \mathcal{N} \setminus \{g\}$ relays, where the gateway is denoted by $g$. Once node $n \in \mathcal{N} \setminus \{g\}$ has decided to transmit, it selects either one of the bi-directional flow pairs that it relays or its own generated flow with equal probability $\frac{1}{M_n+1}$. The gateway selects the downlink flows with equal probability $1/(N-1)$.

We determine the optimal max-min throughput in the two 16-node networks (Grid16 and Rand16) by solving the JRM-BiNC optimization problem using the IOS technique. With the default configurations, the max-min throughput in the two 16-node networks are determined by simulation. The physical layer parameters are considered to be the same as in the single rate system without network coding. In Fig. 4.3, we show the max-min throughput performance for the joint and default designs for the systems with and without network coding. Clearly, a joint design with network coding yields a higher throughput especially at low transmission power.
At higher transmission power, network coding becomes less attractive because there are more and more single hop paths to the gateway. The throughput improvement for enabling network coding is less with the default configuration than that with the joint configuration. The default design with network coding provides much worse performance than the joint designs and the max-min node throughput for the default designs is not very sensitive to transmission power. The percentage throughput gains obtained by the joint design with network coding over the default design with network coding are shown in Fig. 4.4 for the two 16-node networks. The results show that remarkable throughput gains (100% to 450%) can be achieved by jointly optimizing the configuration.

### 4.5 Advantages of Network Coding

Now, we study the throughput gains achieved by the simple network coding. In Fig. 4.5, we present the percentage throughput gain obtained by the JRM-BiNC design with respect to the JRM design for the two 16-node networks. Note that the percentage throughput gains are computed by

\[
\% \text{ Throughput gain} = \frac{\lambda_{\text{JRM-BiNC}} - \lambda_{\text{JRM}}}{\lambda_{\text{JRM}}} \times 100 \tag{4.25}
\]

where \( \lambda_{\text{JRM}} \) is the max-min node throughput achieved by the JRM design. The results show that, at low transmission power, network coding can provide a significant throughput gain. Specifically, at low transmission power, roughly 30% – 50% throughput gain can be achieved.
Figure 4.3: Comparison of max-min node throughput among the joint and default designs: Top-left: Grid16, $w = 1$; Top-right: Grid16, $w = 2$; Bottom-left: Rand16, $w = 1$; Bottom-right: Rand16, $w = 2$. 

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4.5. ADVANTAGES OF NETWORK CODING

Figure 4.4: Throughput gain of the joint design with network coding over the default design with network coding: Left: Grid16; Right: Rand16.

Figure 4.5: Throughput gain of the JRM-BiNC design with respect to the JRM design: Left: Grid16; Right: Rand16.
Interestingly, except at very low transmission power, the throughput gain for a downlink/uplink ratio of $w = 2$ is higher than for a ratio of $w = 1$. We attribute this to the fact that, in a network coding pair, the downlink link has a higher successful transmission probability than the uplink link due to congestion as traffic increases for the nodes close to the gateway and the gateway node itself generates a large amount of traffic. Although the traffic rate is balanced on a network coding link pair at $w = 1$, differences in the successful transmission probabilities on the two links for a network coded packet creates an imbalance in offered traffic on network coded link pairs due to retransmissions, and hence the number of network coding opportunities is significantly reduced. On the other hand, at $w = 2$, there is traffic imbalance on a network coded link pair. However, due to a high retransmission rate on the lower traffic uplink link and a low retransmission rate on the higher traffic downlink link, offered traffic on a network coded link pair is in fact more balanced. As a result, there are more network coding opportunities at $w = 2$ than at $w = 1$ and a higher throughput gain is obtained at $w = 2$.

Except at low transmission power with $w = 2$, the throughput gain decreases with increasing transmission power. We attribute this to the fact that the number of routing hops decreases and thus the number of opportunities to perform network coding is reduced as well. In the low transmission power regime with a ratio $w = 2$, the offered load between the uplink and downlink links of a network coded link pair becomes more balanced with increasing transmission power since the successful transmission probability in the downlink component becomes greater than that of the uplink component as transmission power is increased. As a result, for a ratio $w = 2$ and in the very low transmission power regime, the throughput gain increases with
transmission power.

In Fig. 4.6, given a transmission power value, the throughput gains for different values of $w$ are presented for the two networks. Although the value of $w$ at which the highest throughput gain is obtained differs from one network to another, from all the networks we have studied, we found that the value of $w$ at which the highest throughput gain is obtained is typically in the range 1 to 2.5. Since typical values of $w$ for Internet traffic are around 2, these results show that the typical imbalance of downlink and uplink traffic rates will increase network coding opportunities.

### 4.6 Limiting Network Coding at a Few Nodes

In this section, we provide the problem formulation of joint design where (bi-directional) network coding is enabled only at a few nodes and then we compare the performance of this simple design with the JRM and JRM-BiNC designs.
4.6. LIMITING NETWORK CODING AT A FEW NODES

4.6.1 Problem Formulation

Denote $\mathcal{N}_T \subset \mathcal{N}$ to be the subset of nodes which are permitted to do (bi-directional) network coding. Let $\pi_n$ be the probability that node $n$ will try to access the channel in a given slot. Given that node $n \in \mathcal{N} \setminus \mathcal{N}_T$ does try to access the channel, it will select a packet of flow $f$ to transmit on link $l \in \mathcal{L}_n^O$ without network coding by $q_{f,l}^{WNC}$ with the condition

$$\sum_{l \in \mathcal{L}_n^O, f \in \mathcal{F}} q_{f,l}^{WNC} = 1.$$

(4.26)

Similar to (4.2) and (4.7), $\forall n \in \mathcal{N} \setminus \mathcal{N}_T, \forall f \in \mathcal{F}, \forall l \in \mathcal{L}_n^O$, we have

$$\tau_{f,l}^{WNC} = \pi_n q_{f,l}^{WNC}$$

(4.27)

and

$$c_{f,l} = \tau_{f,l}^{WNC} p_l^s$$

(4.28)

where

$$\sum_{l \in \mathcal{L}_n^O, f \in \mathcal{F}} \tau_{f,l}^{WNC} = \pi_n.$$

(4.29)

Given that node $n \in \mathcal{N}_T$ does try to access the channel, it will select flow (flows) and link (links) as described in Section 4.3. For each $n \in \mathcal{N}_T$, the expressions for the attempt probability and the effective link rate of flow $f_i$ on link $l_i \in \mathcal{L}_n^O$, and the network coding constraints can be obtained from the original problem formulation of JRM-NC design by replacing $f_j \in \mathcal{F}$ with $\overline{f}_i$. Thus, the joint optimization problem
4.7. HEURISTIC CONFIGURATION

allowing bi-directional network coding only to a subset of nodes (JRM-BiNC-SN)
can be formulated from the JRM-NC optimization problem (4.14)-(4.22) by setting
appropriate expressions of effective link rate and attempt probability according to the
category of each node \( n \). Note that network coding constraints are not required for
the non network coding nodes.

4.6.2 Performance Comparison

We determine the optimal max-min throughput in the two 16-node mesh networks by
limiting network coding operation to the nodes directly adjacent to the gateway by
using the IOS technique. The max-min node throughput for the JRM, JRM-BiNC,
and JRM-BiNC-SN designs are shown in Fig. 4.7. For the JRM-BiNC-SN design,
the number of nodes directly adjacent to the gateway is shown as a label for each
point. Clearly, a large part of the throughput improvement for network coding can
be obtained by limiting network coding to the nodes directly adjacent to the gateway.

4.7 Heuristic Configuration

In this section, we describe a simple heuristic to configure the routing and MAC
parameters in a slotted ALOHA based wireless mesh network with bi-directional
network coding. By comparing the performance of our heuristic not only to the
optimal solution but also to the benchmark, we illustrate the effectiveness of the
heuristic design.
Figure 4.7: Throughput performance by limiting network coding to the nodes directly adjacent to the gateway: Top-left: Grid16, $w = 1$; Top-right: Grid16, $w = 2$; Bottom-left: Rand16, $w = 1$; Bottom-right: Rand16, $w = 2$.  

4.7. HEURISTIC CONFIGURATION

4.7.1 Routing

Routing of the flows is the same as the default one.

4.7.2 Medium Access Control

In sub-section 3.6.2 of Chapter 3, we investigate that, for each node \( n \), the optimal attempt probability \( \pi_n \) of a slotted ALOHA system without network coding is strongly related to the traffic carried by node \( n \) as well as the traffic carried by the other nodes. We provide a heuristic solution to calculate the attempt probabilities of the nodes based on the transmitted traffic by the nodes. For a system with network coding, we want to use a similar model by computing the effective traffic (described below) carried by the nodes.

Once routes of the flows have been selected, it is possible to calculate the amount of traffic transmitted by each node assuming that each uplink flow has a throughput \( \lambda \) (and each downlink flow has a throughput \( w\lambda \)). Clearly, the amount of traffic transmitted by node \( n \in \mathcal{N} \setminus \{g\} \) is \( M_n(w\lambda + \lambda) + \lambda \), where \( w\lambda + \lambda \) is the total rate of each bi-directional flow pair and \( \lambda \) is the rate of its own generated flow. On the other hand, the gateway does not have any opportunity to network code since it does not relay any flow and it has to transmit all the downlink flows without network coding. The amount of traffic transmitted by the gateway is \( (N - 1)w\lambda \). Since node \( n \in \mathcal{N} \setminus \{g\} \) is able to do network coding on each bi-directional flow pair that it relays, it could transmit all the uplink relaying traffic \( M_n\lambda \) by network coding with the downlink relaying traffic \( M_nw\lambda \) since \( w \geq 1 \). Thus, effectively, it needs medium access to transmit an amount of traffic \( M_nw\lambda + \lambda \). Denote the effective amount of
traffic for which node $n \in \mathcal{N} \setminus \{g\}$ needs medium access by $y_n$, i.e.,

$$y_n = M_n w \lambda + \lambda. \quad (4.30)$$

Since the gateway transmits all the traffic without any network coding, we set

$$y_g = (N - 1) w \lambda. \quad (4.31)$$

Then in our heuristic, the attempt probability of node $n$ is calculated as

$$\pi_n = \frac{y_n}{\sum_{n' \in \mathcal{N}} y_{n'}'}. \quad (4.32)$$

To investigate how accurate our heuristic is in configuring $\pi_n$ parameters, we compute the optimal routing and $\pi_n$’s configurations of the two 16-node networks for the JRM-BiNC design and then we calculate the heuristic $\pi_n$’s using the formula in (4.32). The optimal and heuristic values of $\pi_n$’s are shown in Fig. 4.8 for the two 16-node mesh networks. The heuristic attempt probabilities follow the trends of the optimal solution for the both networks. The heuristic attempt probabilities of the Rand16 are quite accurate.

### 4.7.3 Flow(s) and Link(s) Selection

Once a node has decided to transmit, it has to determine which packet(s) of which flow(s) it will transmit. Since a single route has been selected per flow, the link(s) on which the selected packet(s) will be transmitted is known. We assume that each node probabilistically selects its local flow to transmit without network coding or a
4.7. HEURISTIC CONFIGURATION

Figure 4.8: Optimal and heuristic attempt probabilities of the nodes at \( w = 1 \) for the case with bi-directional network coding: Top: Grid16, \( P_t = -33 \) dBm; Bottom: Rand16, \( P_t = -34 \) dBm.

bidirectional (relayed) flow to transmit with network coding. Node \( n \in N \setminus \{g\} \) selects one of the bidirectional flow pairs that it relays with probability \( \frac{w}{M_n w + 1} \) and its own generated flow with probability \( \frac{1}{M_n w + 1} \) as the effective amount of traffic for which it needs medium access is \((M_n w + 1) \lambda\), the effective amount of traffic of a bidirectional flow pair is \( w \lambda\), and the effective amount of traffic of its own flow is \( \lambda\). The gateway selects the downlink flows with equal probability \( 1/(N - 1) \).

4.7.4 Performance of the Heuristic

We determine the max-min node throughput in the two 16-node mesh networks and 30-node mesh network by simulation with the heuristic configuration. Max-min node throughputs obtained by the joint, heuristic, and default designs for the two 16-node networks with network coding are shown in Fig. 4.9. The max-min throughput obtained by the heuristic compares well to the optimal max-min throughput and
is significantly higher when compared to the max-min throughput obtained by the default configuration. Max-min node throughputs obtained by the heuristic and default designs for the Rand30 network are shown in Fig. 4.10. Clearly, the heuristic is very effective for the Rand30 mesh network.

4.8 Joint Routing, MAC, Network Coding, and Rate Adaptation

4.8.1 System Model

Network topology, flows, physical layer model, and MAC operation of the system are considered to be the same as defined in the multi-rate slotted ALOHA system without network coding in sub-section 3.7.1 of Chapter 3. Queue maintenance and retransmission strategies are assumed to be the same as in the single rate slotted ALOHA system with network coding in sub-section 4.1.1. The main differences are in the routing operation described below. We assume that network coding between two packets can only be performed with the same modulation and coding scheme such that network coding operations remain simple and practical. Denote $\mathcal{R}(l_i, l_j)$ to be the set of common available rates in links $l_i$ and $l_j$, i.e., $\mathcal{R}(l_i, l_j) = \mathcal{R}(l_i) \cap \mathcal{R}(l_j)$. Given that node $n$ does try to access the channel, we denote the conditional probability that (i) it will select packets of flows $f_i$ and $f_j$, $f_i \neq f_j$, to transmit on links $l_i \in \mathcal{L}_n^O$ and $l_j \in \mathcal{L}_n^O$, resp., $l_i \neq l_j$, using network coding with transmission rate $r \in \mathcal{R}(l_i, l_j)$ by $q_{f_i,l_i,f_j,l_j}^{NC}(r)$, (ii) it will select a packet of flow $f_i$ to transmit on link $l_i \in \mathcal{L}_n^O$ without network coding with transmission rate $r \in \mathcal{R}(l_i)$ by $q_{f_i,l_i}^{WNC}(r)$. These probabilities are
Figure 4.9: Comparison of max-min node throughput among the joint, heuristic, and default designs with network coding: Top-left: Grid16, \( w = 1 \); Top-right: Grid16, \( w = 2 \); Bottom-left: Rand16, \( w = 1 \); Bottom-right: Rand16, \( w = 2 \).
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Figure 4.10: Comparison of max-min node throughput between the heuristic and default designs with network coding in the Rand30 network.

related by

\[
\sum_{f_i \in F, l_i \in L_i^O, f_j \in F, l_j \in L_j^O, f_i \neq f_j, l_i < l_i, r \in R(l_i, l_j)} q^{NC}_{f_i,l_i,f_j,l_j}(r) + \sum_{f_i \in F, l_i \in L_i^O, r \in R(l_i)} q^{WNC}_{f_i,l_i}(r) = 1.
\]

(4.33)

4.8.2 Effective Link Rate

Let \(\tau^{NC}_{f_i,l_i,f_j,l_j}(r)\) be the probability that packets of flows \(f_i\) and \(f_j\) will be transmitted using network coding on links \(l_i\) and \(l_j\), resp., in a given time-slot with transmission rate \(r \in R(l_i, l_j)\) and \(\tau^{WNC}_{f_i,l_i}(r)\) be the probability that a packet of flow \(f_i\) will be transmitted on link \(l_i\) without network coding in a given time slot with transmission rate \(r \in R(l_i)\). We denote the collection of \(\tau^{NC}_{f_i,l_i,f_j,l_j}\)'s by \(\boldsymbol{\tau}^{NC} = \{\tau^{NC}_{f_i,l_i,f_j,l_j}(r) : f_i \in\)

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\( \mathcal{F}, f_j \in \mathcal{F}, f_i \neq f_j, l_i \in \mathcal{L}, l_j \in \mathcal{L}, l_j < l_i, r \in \mathcal{R}(l,l_j) \). Thus,

\[
\tau_{f_i,l_i}^{\text{WNC}}(r) = \pi_n q_{f_i,l_i}^{\text{WNC}}(r) \quad \forall n \in \mathcal{N}, \forall f_i \in \mathcal{F}, \forall l_i \in \mathcal{L}_n, \forall r \in \mathcal{R}(l_i) \quad (4.34)
\]

and

\[
\tau_{f_i,l_i,f_j,l_j}^{\text{NC}}(r) = \pi_n q_{f_i,l_i,f_j,l_j}^{\text{NC}}(r) \forall n \in \mathcal{N}, \forall f_i, f_j \in \mathcal{F}, \forall l_i, l_j \in \mathcal{L}_n, f_i \neq f_j, l_j < l_i, \forall r \in \mathcal{R}(l_i,l_j) \quad (4.35)
\]

and

\[
\pi_n = \sum_{f_i, f_j \in \mathcal{F}, f_i \neq f_j \in \mathcal{L}_n, f_i \neq f_j, l_j < l_i, r \in \mathcal{R}(l_i,l_j)} \tau_{f_i,l_i,f_j,l_j}^{\text{NC}}(r) + \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}_n, r \in \mathcal{R}(l_i)} \tau_{f_i,l_i}^{\text{WNC}}(r). \quad (4.36)
\]

Denote \( c_{f_i,l_i}^r \) to be the effective rate of flow \( f_i \) on link \( l_i \) with the physical rate \( r \in \mathcal{R}(l_i) \). Similar to (4.8), \( c_{f_i,l_i}^r \) can be written as

\[
c_{f_i,l_i}^r = \left[ \tau_{f_i,l_i}^{\text{WNC}}(r) + \sum_{f_j \in \mathcal{F}, f_j \neq f_i \in \mathcal{L}_n(l_i), l_j < l_i} \tau_{f_i,l_i,f_j,l_j}^{\text{NC}}(r) + \sum_{f_j \in \mathcal{F}, f_j \neq f_i \in \mathcal{L}_n(l_i), l_j > l_i} \tau_{f_j,l_j,f_i,l_i}^{\text{NC}}(r) \right] r p_{l_i}^s(r)
\]

\[\quad (4.37)\]

where \( \mathcal{L}_n^O(r) \) is the set of feasible links going out from node \( n \) at the transmission rate \( r \).
The effective rate of flow $f_i$ on link $l_i$ is then given by

$$c_{f_i,l_i} = \sum_{r \in R(l_i)} c_{f_i,l_i}^r.$$  \hfill (4.38)

### 4.8.3 Problem Formulation

First, we reformulate the network coding constraints for a multi-rate system similar to the formulation of network coding constraints in a single rate system. The arrival packets of flow $f_i$ at node $n$ through the incoming link $l_j$ are transmitted with the transmission probabilities \( \{ \tau_{NC}^{f_i,l_i,f_j,l_j}(r) : f_j \in \mathcal{F}, l_i \in \mathcal{L}_n^O, f_i \neq f_j, l_j < l_i, r \in R(l_i,l_j) \} \) and the total effective rate of flow $f_i$ achieved by all of these transmission probabilities is restricted by flow rate $c_{f_i,l_j}$. Thus, for a multi-rate system the network coding constraint for $n \in \mathcal{N}$, $f_i \in \mathcal{F}$, $l_j \in \mathcal{L}_n^O$ can be written as

$$\sum_{f_j \in \mathcal{F}, f_i \neq f_j, l_i \in \mathcal{L}_n^O, l_j < l_i, r \in R(l_i,l_j)} \tau_{NC}^{f_i,l_i,f_j,l_j}(r) r p_{l_i}^s(r) + \sum_{f_j \in \mathcal{F}, f_i \neq f_j, l_i \in \mathcal{L}_n^O, l_j > l_i, r \in R(l_i,l_j)} \tau_{NC}^{f_j,l_j,f_i,l_i}(r) r p_{l_i}^s(r) \leq c_{f_i,l_j} \hfill (4.39)$$

where the left hand side represents the total effective rate of flow $f_i$ on all the outgoing links (except $l_j$) of node $n$ for network coding with the traffic of the other flows on link $l_j$. 

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Next, to reduce the computational complexity of $\pi_n$, we rewrite (4.36) as follows.

\[
\pi_n = \frac{1}{2} \left( \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}_n^O, r \in \mathcal{R}(l_i)} \sum_{f_j \in \mathcal{F}, l_j \in \mathcal{L}_n^O(r), f_i \neq f_j, l_j < l_i} \tau_{NC}^{f_i,l_i,f_j,l_j}(r) \right) + \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}_n^O, r \in \mathcal{R}(l_i)} \tau_{WNC}^{f_i,l_i}(r)
\]

Using (4.37) in (4.40), we have

\[
\pi_n = \frac{1}{2} \left( \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}_n^O, r \in \mathcal{R}(l_i)} \frac{c_{f_i,l_i}}{p_{l_i}(r)} \right) + \frac{1}{2} \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}_n^O, r \in \mathcal{R}(l_i)} \tau_{WNC}^{f_i,l_i}(r).
\]

Now, we formulate the JRM-NC-RA optimization problem as shown in (4.42)-(4.50). Clearly, the complexity in computing each of the link rate, attempt probability, and network coding constraints in the optimization problem increases by $R$ times when including multiple rates in the system. Since we limit ourselves to bi-directional network coding, we use the optimization problem (4.42)-(4.50) by replacing $f_j \in \mathcal{F}$ with $\overline{f}_i$ in all the constraints for bi-directional network coding.

### 4.8.4 Multi-Rate vs. Single Rate

To compare the performance of multi-rate and single rate slotted ALOHA systems with network coding, we determine the optimal max-min throughput in the two 16-node mesh networks by solving the JRM-BiNC-RA optimization problem using the IOS technique. Physical transmission rates and the corresponding SINR thresholds are set to be the same as in multi-rate slotted ALOHA systems without network.
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\[
\max_{\tau^{NC}, \tau^{WNC}, \pi, \mathbf{p}^s, \mathbf{c}} \lambda
\]

\[
\sum_{l \in \mathcal{L}^O_n} c_{f,l} \leq \sum_{l \in \mathcal{L}^O_n} c_{f,l} = \begin{cases} \omega_f \lambda & \text{if } n = f^s \\ -\omega_f \lambda & \text{if } n = f^d \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \tag{4.42}
\]

\[
c_{f_i,l_i} = \sum_{r \in \mathcal{R}(l_i)} \left( \tau^{WNC}_{f_i,l_i}(r) + \sum_{f_j \in \mathcal{F}, l_j \in \mathcal{L}^O_n, f_j \neq f_i, l_j < l_i} \tau^{NC}_{f_i,l_i,f_j,l_j}(r) \right) \quad \forall n \in \mathcal{N}, \forall f_i \in \mathcal{F}, \forall l_i \in \mathcal{L}^O_n \tag{4.43}
\]

\[
\sum_{f_j \in \mathcal{F}, l_j \in \mathcal{L}^O_n, f_j \neq f_i, l_j > l_i} \sum_{r \in \mathcal{R}(l_i), f_j \in \mathcal{F}, l_j \in \mathcal{L}^O_n, l_j > l_i} \tau^{NC}_{f_i,l_i,f_j,l_j}(r) \leq c_{f_i,l_i} \quad \forall n \in \mathcal{N}, \forall f_i \in \mathcal{F}, \forall l_i \in \mathcal{L}^O_n \tag{4.44}
\]

\[
\pi_n = \frac{1}{2} \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}^O_n} c_{f_i,l_i} \frac{\pi_i}{r p_i^s(r)} + \frac{1}{2} \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}^O_n} \tau^{WNC}_{f_i,l_i}(r) \quad \forall n \in \mathcal{N} \tag{4.45}
\]

\[
p_i^s(r) = \sum_{\sigma_i \in \mathcal{S}^l \setminus \sigma_i} \prod_{j \in \mathcal{N} \setminus \sigma_i} (1 - \pi_j) \quad \forall l \in \mathcal{L}, \forall r \in \mathcal{R}(l) \tag{4.46}
\]

\[
0 \leq \lambda, \mathbf{c} \tag{4.47}
\]

\[
0 \leq \tau^{NC}, \tau^{WNC}, \pi \leq 1 \tag{4.48}
\]

\[
\epsilon \leq \mathbf{p}^s \leq 1 \tag{4.49}
\]
coding in sub-section 3.7.3 of Chapter 3. We also solve the JRM-BiNC optimization problem for these two networks with physical transmission rate 2. The max-min throughput of the JRM-BiNC and JRM-BiNC-RA designs for the two 16-node networks are shown in Fig. 4.11. Clearly, the insights on rate allocation that we obtained in slotted ALOHA systems without network coding remain the same even when network coding is enabled in the system.

4.9 Energy Consumption

In this section, we model the energy consumption in terms of slotted ALOHA system parameters. We compare the energy consumption of the different proposed cross-layer systems.

4.9.1 Energy Consumption Model

**Assumptions:** We assume that, at the beginning of each time slot, a node is capable of detecting whether any transmission is occurring in the network if it does not transmit, and hence, it will try to decode the received signal if any transmission occurs. Each node switches to a sleep mode if the medium is idle in a time slot. Note that a high transmit power level is required to transmit a packet, a medium power level is required to decode a received signal, and a low power is required to stay in a sleep mode [62]-[65]. We assume that the processing power for network coding of two pack-

---

3This power is required to process the received signal to decode a packet, not the received signal power, and it is required for each non-transmitting node when any transmission occurs in the network.
Figure 4.11: Comparison of max-min node throughput between single rate and multi-rate systems: Top-left: Grid16, \( w = 1 \); Top-right: Grid16, \( w = 2 \); Bottom-left: Rand16, \( w = 1 \); Bottom-right: Rand16, \( w = 2 \).
ets is negligible compared to the transmission power\(^4\) [78]. The required power to decode one of the packets from an XOR packet is negligible compared to the required power to receive the XOR packet\(^5\). Further, we assume that the processing power does not depend on modulation and coding scheme\(^6\). Thus, the energy consumption model for single rate as well as multi-rate slotted ALOHA systems without and with network coding remains the same.

**Model:** As mentioned earlier, a node consumes one of the three energies in a time slot. Thus, to compute energy consumption, for each node \(n\), we need to calculate the probabilities of different level of energy consumptions in a time slot. The probability that node \(n\) transmits in a slot is \(\pi_n\). The probability that the medium is idle in a time slot is

\[
\pi_I = \prod_{n \in \mathcal{N}} (1 - \pi_n).
\]

(4.51)

The probability that node \(n\) does not transmit in a time slot but there is at least one transmission in the network is given by

\[
\pi_B^{(n)} = (1 - \pi_n) \left( 1 - \prod_{i \neq n} (1 - \pi_i) \right)
= (1 - \pi_n) - \pi_I.
\]

(4.52)

---

\(^4\)One XOR combination of two packets of size 1000 byte results in the energy consumption of 191 nJ.

\(^5\)To decode one of the packets from an XOR packet requires one XOR operation. Thus, this assumption is reasonable.

\(^6\)In [65], authors measure the currents for 2 Mbps and 11 Mbps rates in the transmit and receive modes in an IEEE 802.11 card. The currents for both data rates are very close for each mode.
Denote $P_{tx}$ to be the required power at a transmitter for processing and transmitting a data packet. The required power to decode a received signal in a time slot is denoted by $P_{rx}$. Denote $P_I$ to be the required power to stay in sleep mode. Thus, the total energy consumption per unit time in the system is then given by

$$E_t = \sum_{n \in \mathcal{N}} (P_{tx}\pi_n + P_{rx}\pi_B^{(n)} + P_I\pi_I)$$  \hspace{1cm} (4.53)

Using (4.52) in (4.53), we get

$$E_t = \sum_{n \in \mathcal{N}} (P_{tx}\pi_n + P_{rx}(1 - \pi_n) - P_{rx}\pi_I + P_I\pi_I).$$  \hspace{1cm} (4.54)

Denote

$$\Delta = \sum_{n \in \mathcal{N}} \pi_n.$$  \hspace{1cm} (4.55)

The total energy consumption per unit time is then given as

$$E_t = P_{rx}\left(\Delta\left(\frac{P_{tx}}{P_{rx}} - 1\right) + N(1 - \pi_I)\right) + N\pi_IP_I$$  \hspace{1cm} (4.56)

Assuming, $P_{tx} = P_t + P_{rx}$, i.e., the required energies for modulation and demodulation are equal, and $P_I = 0$, we have

$$E_t = \Delta P_t + N(1 - \pi_I)P_{rx}.$$  \hspace{1cm} (4.57)
4.9.2 Energy Consumptions in Different Cross-layer Systems

Using (4.57), we calculate the energy consumption in the Grid16 and Rand16 mesh networks for the JRM, JRM-RA, JRM-BiNC, and JRM-BiNC-RA designs. The value of $P_{rx}$ is assumed to be equal to the minimum transmission power $P_{\text{min}}$ of network. For the Grid16 and Rand16 mesh networks $P_{\text{min}}$’s are -36.2573 dBm and -36.2509 dBm, respectively. The energy consumption of all the cross-layer systems are shown in Fig. 4.12. It is seen that, in most of the cases, the energy consumption of all the cross-layer systems are very close. Thus, the throughput improvements for enabling network coding as well as rate adaptation technique are almost free in term of energy consumption.

4.10 Conclusion

In this chapter, we study the problem of throughput-optimal configuration of routing and MAC parameters in wireless mesh networks with network coding. We formulate optimization problems, solve them by using the IOS technique, and validate our models by simulation. Via extensive numerical and simulation results, we demonstrate that (i) joint configuration of routing and MAC parameters provides a remarkable throughput gain over a default configuration in slotted ALOHA based wireless networks with network coding, (ii) throughput improvement for enabling simple network coding is significant, especially at low transmit power, when the routing and MAC parameters are jointly optimized with network coding, (iii) the typical rate imbalance between downlink and uplink flows in wireless mesh networks plays in favor of network coding, (iv) a large part of the throughput gain of network coding can be
Figure 4.12: Comparison of energy consumptions among different cross-layer systems: Top-left: Grid16, \( w = 1 \); Top-right: Grid16, \( w = 2 \); Bottom-left: Rand16, \( w = 1 \); Bottom-right: Rand16, \( w = 2 \).
obtained by limiting network coding at nodes directly adjacent to the gateway.

We propose a simple heuristic to configure routing and MAC parameters in slotted ALOHA based mesh networks with network coding. We show that our heuristic design compares well with the joint design and provides a significantly higher throughput than a default design.

We also formulate an optimization problem to optimize not only the MAC and routing parameters but also the transmission rate parameters in multirate systems with network coding. We found that using multiple rates a moderate amount of throughput gain is achievable in a network with random topology only when the node transmit power is not sufficient to route all the flows at each available physical transmission rate in case of single rate system, throughput gain is negligible otherwise.

We study the energy consumptions for various cross-layer systems. We show that the amount of energy consumptions for all the cross-layer systems (i.e., single rate as well as multi-rate system without and with network coding) are very close.
Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this research, we investigate cross-layer design in multi-hop wireless networks with random access. Tightly coupled cross-layer design with a practical MAC protocol (i.e., CSMA/CA) is a very difficult problem due to the difficulty of modeling link rates in multi-hop networks. Due to the difficulty of the problem, a simple slotted ALOHA MAC protocol is chosen for link layer operation, which has a similar contention behavior to the practical MAC protocol CSMA/CA in WLANs [32]. The objective is to provide insights on the cross-layer design in random access based multi-hop wireless networks, especially (i) the interaction among the lower layers and (ii) throughput gains obtained by a joint configuration over a default configuration.

We study the cross-layer design between the routing layer and MAC layer. We formulate the JRM optimization problem to maximize the minimum throughput of the flows by jointly optimizing the configuration of routing and MAC parameters.
We then extend the formulation to include a simple network coding, namely XOR without opportunistic listening. The optimization problems are found to be very complex, non-linear, and non-convex. We use the IOS technique to solve the problems. However, we are only able to solve the problem for small to medium size networks. Via extensive numerical and simulation results, we demonstrate that joint design improves throughput significantly with respect to a default design in slotted ALOHA based wireless networks without and with network coding. We also show that at low transmit power, a simple XOR network coding without opportunistic listening can yield non negligible throughput gains. The most surprising finding is may be that the typical imbalance between downlink and uplink flow rates in wireless mesh networks increases network coding opportunities. We also found that, in mesh networks, a large part of the throughput improvement for network coding can be obtained by limiting network coding to the nodes directly adjacent to the gateway.

Due to the computational complexity, solving the joint optimization problems for a large network is impossible and hence, we propose a simple heuristic. We found that the optimal configuration of the attempt probabilities of the nodes is highly related to the traffic load of the nodes. We propose simple heuristic based on a min-hop routing and the traffic load of nodes to configure routing and MAC parameters in large networks without and with network coding. We found that heuristic configuration of the transmission probabilities based on the traffic load of the nodes over performs the default configuration and compares well with the optimal design.

We extend the optimization problems for multi-rate systems without and with network coding and compare the performance of multi-rate and single rate systems. We found that the throughput improvements for using multiple rates depend on network
topology, node transmit power, and the available rates. The throughput improvement in a grid network is negligible, but a moderate amount of throughput gain is achievable in a random topology only when the node transmit power is not sufficient to route all the flows at each available rate if a single rate system is considered.

5.2 Future Work

In this research, we have done a preliminary study on cross-layer design in multi-hop wireless networks with random access and provided various useful insights. However, significant challenging issues remain to be resolved for real class applications and real networks. Our work can be extended in several directions. The heuristic for routing of the flows that we propose for a large slotted ALOHA network is not good enough since it is based on min-hops. There are many routing metrics in the literature to improve throughput performance in multi-hop wireless networks [41]-[43]. It would be interesting to study the interaction of these routing metrics with our heuristic of MAC.

We address the JRM-NC problem without opportunistic listening. The performance gain achieved by network coding with opportunistic listening is higher than that without it. But it is not clear how much gain can be achieved by enabling opportunistic listening. Thus, it would be interesting to study the JRM-NC problem with opportunistic listening and quantify the throughput gain given by opportunistic listening.

Successive interference cancellation [76] and superposition coding [77] are promising physical layer techniques to improve throughput in wireless networks by taking
advantage from interference. It would be interesting to address the joint configuration problem to include these physical layer techniques and provide some insights about the performance gain given by these techniques.

The efficiency of a slotted ALOHA MAC protocol is in general worse than the practical CSMA/CA MAC protocol. Thus, the most exciting future work would be to use the formulations and insights of this research for CSMA/CA MAC based wireless networks. For physical reason, time in CSMA/CA MAC protocol is divided into mini-slots and nodes in a network access the channel in a min-slot according to their access parameters (i.e. the minimum contention windows) by using a binary exponential back-off mechanism [29]-[31]. The access rate of a node in a mini-slot is related to its minimum contention window. It would be interesting to study CSMA/CA MAC based multi-hop wireless networks by configuring the node minimum contention windows based on the traffic load of nodes under a min-hop as well as the other well known routing protocols and compare their performance with a default configuration.

For multi-hop wireless networks with dynamic traffic, dynamically configuring the routing and MAC parameters in a distributed fashion would be required. Researches on dynamic routing consider the impact of interference partially by designing various routing metrics [41]-[43]. However, it is not known how to dynamically configure the MAC parameters according to the traffic distribution in a distributed fashion. Thus, it is important to design a distributed protocol to dynamically configure the routing and MAC parameters using the routing and MAC heuristics.
Appendix A

A Statistical Test of Stability

A.0.1 Methodology

The max-min throughput of a network is the maximum traffic rate that can be injected in each source such that the network will be stable. We consider that a network is stable if all its queues are stable. The problem is then to estimate whether a queue is stable for a given load. This is a complex problem for which we do not have a rigorous solution. Instead, we use a simple statistical test that can be justified as follows.

The test is based on the behavior of M/M/1/K queues (note that the same argument can be done using M/D/1/K queues). Recall that the loss probability $P_K$ in an M/M/1/K is given by

$$P_K = \left(\frac{1 - \rho}{1 - \rho^{K+1}}\right) \rho^K$$  \hspace{1cm} (A.1)

with queue utilization factor $\rho$. When $K$ is large, if $\rho < 1$, we have

$$P_K \simeq (1 - \rho)\rho^K$$  \hspace{1cm} (A.2)
which is the standard formula for the M/M/1/∞ queue. This value will go to zero rather quickly as $K$ gets large, so that the loss probability is very small unless $\rho$ is very close to 1. If $\rho > 1$, we get for a large $K$

$$P_K \simeq (\rho - 1) \frac{\rho^K}{\rho^{K+1}} = \frac{\rho - 1}{\rho}$$

which is a pure fluid model. If $\rho = 1$, we get

$$P_K = \frac{1}{K + 1}.$$  \hspace{1cm} (A.4)

In other words, the buffer loss probability is a very powerful test for the stability of a queue. It gets close to 0 very quickly when $\rho < 1$ and increases reasonably fast when $\rho > 1$, as can be seen from part (a) of Fig. A.1 for $K = 1000$.

To determine the stability of a network for a particular source rate, we consider that the buffer size of each queue is $K$ instead of infinity, and assume that the system is unstable if $P_K$ of any queue exceeds $1/(K + 1)$. Increasing the source rate from a low value in several steps and checking the stability of each queue at each step by simulation, the maximum source rate yielding stability of all queues can be determined for a given network configuration.

### A.0.2 Validation of the Test

Although the queues of a multi-hop slotted ALOHA network are not $M/M/1/K$, we assume that its packet loss behavior should be similar if the buffer size is set to a large value. We have verified this assumption as follows. The packet loss probabilities of
all the queues for different source rates are plotted in Figures A.1 (b) and (c) for the default (default configurations are described in Section 3.4) and JRM configurations of flow set 5 of network Rand10A. We see that in all cases $P_K$ does increase from zero to a high value very quickly when the rate reaches a certain threshold similar to $M/M/1/1000$ queue as shown in Fig A.1 (a), in the present case, within about 1% of the max-min throughput. Based on this, we can use the test with reasonable confidence that the error in estimating the maximum rate is not much more than 1%.
Bibliography


[70] V. Erceg, L. Greenstein, Y. Tjandra, S. Parkoff, A. Gupta, B. Kulic, A. Julius, and A. Bianchi, “An empirically-based path loss model for wireless channels in


