Hierarchical Graph Models for Conflict Resolution

by

Shawei He

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Systems Design Engineering

Waterloo, Ontario, Canada, 2015

Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contributions

I declare that the original ideas, theories, and case studies in this thesis were proposed and developed by myself alone. My two PhD supervisors, Professor Keith W. Hipel and Professor D. Marc Kilgour, provided guidance and suggestions for refining my ideas as well as improving the organization and presentation of this thesis. Chapters 3 and 4 are based on two published journal articles, of which I am the first author. These papers, listed below, are also shown in the References and referred to at appropriate locations in my thesis.

He, S., Kilgour, D. M., Hipel, K. W., and Bashar, M. A. (2013). A Basic Hierarchical Graph Model for Conflict Resolution with Application to Water Diversion Conflicts in China. INFOR: Information Systems and Operational Research, 51(3), 103-119.

He, S., Hipel, K. W., and Kilgour, D. M. (2014). Water diversion conflicts in China: A hierarchical perspective. Water Resources Management, 28(7), 1823-1837.

Abstract

The hierarchical graph model is developed for representing strategic conflicts having a hierarchical structure. More specifically, in a hierarchical graph model, one or more decision makers (DMs) at a higher level are involved in lower level or local disputes, such as when a central government is participating in separate disputes with different provincial governments. These newly defined hierarchical models constitute significant expansions of the Graph Model for Conflict Resolution (GMCR) methodology. Moreover, relationships are developed between preference and stability in local conflicts and those in the higher level conflict. To test and refine the various hierarchical definitions, the new approach is applied to three real world conflicts: controversies over water diversions in China, sales competition of aircraft between Airbus and Boeing, and the dispute between the USA and China over greenhouse gas emissions. DMs are provided with possible resolutions and guidance for courses of actions to follow, which can be beneficial in a hierarchical conflict.

GMCR is a conflict analysis methodology that possesses a flexible structure which allows it to be applied to a wide range of real world disputes. Hierarchy in a graph model is defined in this thesis for describing the structure of a complex conflict that includes smaller interrelated conflicts. In a hierarchical graph model, DMs are classified as common decision makers (CDMs) and local decision makers (LDMs). They can initiate different types of moves, and have interrelated preferences.

Among the different hierarchical graph models, the basic hierarchical graph model has the simplest structure, containing three DMs: a CDM appearing in both of the two smaller conflicts, plus one LDM in each of the two smaller conflicts. A duo hierarchical graph model has two CDMs and two local conflicts. Each LDM, together with both CDMs, appears

in one local conflict. A general hierarchical model may include any number of smaller or local conflicts, and any number of CDMs and LDMs.

Stability results in a graph model reflect different solution concepts or stability definitions. Because of the connections between local conflicts, stability in the hierarchical model can be partially obtained from the stability calculations in local models. One component of this thesis is the investigation of the stability interrelationships between stabilities in a hierarchical graph and those in the local conflicts.

Hierarchical graph models can be applied to various practical conflicts. Water diversion conflicts in China, caused by the implementation of the South-North Water Diversion Project, are an excellent example. The duo hierarchical graph model can model a competition between Airbus and Boeing, regarding the marketing of wide-body and narrow-body aircraft in the Asia Pacific region. Conflicts between China and the USA over compliance with a bilateral greenhouse gas emissions agreement are modelled by a hierarchal graph model with a more complicated structure. The disputes take place between the two national governments, both of which also face domestic opposition and concerns.

Acknowledgements

Picture myself in the forest of Robert Frost; the pursuit of a doctoral degree means, for many, a road not taken. A long and winding road straggles in the woods, with the destination hidden in the mist. The bird of truth, whistling over my head, disappears suddenly when I am about to find her. Confusion, anxiety, and uncertainty – call them history. My profound gratitude goes to my two supervisors, Professor Keith W. Hipel and Professor D. Marc Kilgour, who made light shine out of darkness, explored my unknown potential, and above all, made the impossible possible. Dr. Hipel's passion for research has always motivated me to seek, to try, and to discover. Dr. Kilgour's extensive knowledge and academic acuteness have often inspired me to turn ideas into fruitful research. Without them, the past four years would not have been such a rewarding journey.

I am grateful to Dr. M. Abul Bashar who provided expert guidance on improving various mathematical definitions and theorems. I appreciate the help received from Dr. Yacov Haimes, Dr. Stacey Scott, the External Examiner, Dr. Monica Emelko, and Dr. Kevin Li, who served as the examiners of my PhD Thesis dissertation. I also thank Dr. Haiyan Xu and Dr. Christophe Sibertin Blanc, who earlier made intellectually stimulating comments and suggestions on my research. Special thanks should be delivered to Conrad Hipel who carefully proofread my thesis.

Additionally, I am grateful for the scholarship provided by Chinese Scholarship Council (CSC) over the entire course of my entire PhD program as well as funding received from my supervisors, NSERC Discovery Grants. My sincere gratitude also goes to the amiable Canadians, who, though mostly strangers to me, generously tolerated my ignorance and clumsiness, and ardently helped me get settled in Canada. My four-year study in Canada was a memorable experience for me. Forever will I cherish it!

Most importantly, I would like to thank my dearest wife, Kun Liu, and my parents. I could not have completed my PhD program without their endless encouragement, unconditional support, and abundant love.

Dedication

I dedicate this dissertation to my beloved wife, Kun, and my parents: Professor Jiang-sheng He and Longning Li.

Table of Contents

List of Figures

List of Tables

List of Symbols

Symbols in Chapter 2

G	Graph Model
i, N	Decision maker (DM), set of all DMs in G
s, S	State, set of states
A_i	Set of moves for DM i
$R_i(s)$	Reachable list for i from state s
$R_i^+(s)$	Unilateral improvement (UI) list for i from s
$\Pi_H(s, s')$	Set of DMs in H whose UMs are in the legal sequence from s to s'
$\Omega_{ij}(s)$	The j^{th} preference statement for i at s
$\Psi_{ij}(s)$	Score of the j^{th} preference statement for i at s
$J_i(s, s')$	Reachable matrix for i from s to s'
$J_i^+(s, s')$	UI matrix for i from s to s'
M_H	Joint movement matrix for a coalition of DMs H
M_H^+	UI matrix for a coalition of DMs H

Symbols in Chapter 4

$G^{(1)}$, $G^{(2)}$	Local graph models in hierarchical graph model G
CDM, LDM_1, LDM_2	A common decision maker (CDM) and two local decision makers (LDMs) in G
AC, AL_1	Set of moves for CDM in G, set of moves for LDM_1 in G
$S^{(1)}$	Set of states in local graph model $G^{(1)}$
$s = (s^{(1)}, s^{(2)})$	State s in hierarchical graph model G, consisting of two component states, $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$
$\succsim_C$, $\succsim_{L_1}$	Preference relations for CDM, LDM_1 in G
$\succsim_C^{(1)}$, $\succsim_{L_1}^{(1)}$	Preference relations for CDM, LDM_1 in $G^{(1)}$

Symbols in Chapter 5

J_C, J_{L_1}	Reachable matrix for CDM, LDM_1, in G
$J_C^{(1)}$, $J_{L_1}^{(1)}$	Reachable matrix for CDM, LDM_1, in $G^{(1)}$
M_{N-L_1}	Joint movement matrix for all DMs except LDM_1 in G
$w^{(k)}$	Weight of local graph model $G^{(k)}$ for CDM
$\lvert S^{(1)} \rvert = m$	Number of states in $S^{(1)}$ is m
$\boldsymbol{e_{q^{(1)}}}$	m-dimension 0-1 vector with ${q^{(1)}}^{th}$ entry being 1 and others 0, where $q^{(1)} \in S^{(1)}$

Symbols in Chapter 7

N_C, $N_L^{(k)}$	Set of CDMs in G, set of LDMs, in general hierarchical graph model G
$LDM_l^{(k)}$, or short for l_k	LDM l in local graph model $G^{(k)}$
$G^{(\tau_1)}, ..., G^{(\tau_K)}$	Local graph models $G^{(1)}, ..., G^{(K)}$ in the order from τ_1 to τ_K

$w_{C_i}^{(k)}$	Weight of local graph model $G^{(k)}$ for CDM_i

Chapter 1

Introduction

1.1 Hierarchical Conflicts

Strategic conflict refers to competitive or opposing actions that reflect the inconsistent interests and objectives of human beings (Hipel, 2002). Conflicts are ubiquitous in every human society, ranging from divergence of opinion among individuals to fierce antagonism between social groups. A strategic conflict is an interaction among decision makers (DMs), who can affect the courses of the conflict by their independent actions, in accordance with their preferences (Kilgour and Hipel, 2005).

Strategic conflicts often take place when DMs attempt to seize natural and social resources. Because of the scarcity of natural resources and the significant impacts of human activities on the natural environment, environmental issues have become an origin of conflicts in the modern world. An environmental conflict involves a clash of interests between individuals or social groups seeking profits by exploiting natural resources, and stakeholders whose well-being may be at risk. Major environmental conflicts include disputes over water usage among nations or parties, the negotiations on reducing greenhouse gas emissions among countries, and the connection between the economic growth and the deteriorating air quality in the developing world.

The struggle over social resources, such as wealth and power, is also at the root of many conflicts. In business competition, enterprises contest over selling goods to gain more profits or market share. The terminology of business competition has been well defined in economics (Stigler, 1988; Blaug, 2008; Fleisher and Bensoussan, 2003): it can be regarded as conflict when competitors pursue the same resources, such as market share, and interact with each other. For example, in a duopoly market, each company uses business strategies to grab more market share and thereby put its opponent at a disadvantage.

In the real world, a dispute often includes smaller conflicts that are connected spatially or logically. These connected conflicts are called hierarchical conflicts. Failure to perceive these connections may lead to inaccurate predictions about the outcome of the conflict and irrational solutions for DMs. For example, a large-scale construction project initiated by a national government can evoke disputes during the implementation at different locations. The government cannot properly resolve these disputes without considering the conflicts at all locations.

A historical example is the global rivalry between Britain and France during the Seven Years' War. The military contest which occurred in three major theatres, comprising the European continent, North America, and India, ended in a victory for the British and their allies. With its strong navy, Britain carried out several successful military operations in overseas colonies, while the strategy for France with its overwhelming army was to concentrate on continental Europe, hoping that losses overseas could be traded for victories in Europe through treaty negotiation. Victory for Britain was the result of its vision over all continents, and the effective deployment of troops in all theatres.

The word “hierarchy” originated from the Greek word “hierarchia”, meaning “rule of a high priest” or “leader of sacred rites” (Liddell et al., 1940). Hierarchy refers to the arrangement of a particular set of ranks or levels (Dawkins, 1976; Simon, 1991). The hierarchical structure of conflicts has been widely discussed within the Game Theory paradigm. Hierarchical games denote interrelated games with different ranks or multiple levels. Weights and thresholds may be assigned to define the seniority of players in a hierarchical game (Beimel et al., 2008). The weight structure in hierarchical games was investigated by Gvozdeva et al. (2013). The hierarchical game was used for allocating resources among players (Gilles, 2010; Beimel et al., 2008; Farras and Padro, 2010), while the solutions to this game are complex. Markov models have been utilized to analyze hierarchical games within the game of tennis. The mathematical models describe players as attempting to win in tennis by optimizing their available energy (Gale, 1971; George, 1973; Gillman, 1985; Walker and Wooders, 2001; Brimberg et al., 2004). The results obtained for tennis have also been applied to other real world conflicts, such as defence strategy, which can be modelled by a hierarchical scoring system (Epstein, 2012). Theoretical contributions to hierarchical scoring include calculations of the parameters in a hierarchical model using probabilistic functions (Morris, 1977).

Stackelberg games also constitute hierarchical conflicts, because players are divided into a leader and several followers (Von Stackelberg, 1934; Simaan and Cruz, 1973). Different from other models in game theory, players in a Stackelberg game move sequentially and have asymmetric information about the game.

This brief literature review shows that models have been constructed to analyze hierarchical conflicts. However, these models require a large amount of input information, thereby forfeiting flexibility and simplicity. For example, in game theory, utility values that describe the preferences of DMs, and threshold values to define levels in a hierarchical game, are needed. The Markov model used to analyze tennis games requires computations using probabilities. However, these probabilities are difficult to calibrate (Lichtenstein et al., 1982). Therefore, a formal methodology to model hierarchical conflict using a flexible structure and simpler model input is required to provide better predictions and resolutions (Howard, 1971; Kilgour and Hipel, 2005).

1.2 Classical Approaches of Conflict Analysis

The ubiquitousness of strategic conflicts in the real world and the importance of conflict support the call for comprehensive methodologies to understand conflicts and produce positive resolutions for DMs to take reasonable and beneficial actions (Kilgour and Hipel, 2005). The conflict methods can also help mediators propose resolutions.

Various approaches have been developed to analyze decision behaviour. The study of decision making processes of human beings is a psychological science (Marshall, 1920). Among the descriptive approaches used to model decision making behaviour in reality is Prospect Theory, developed by Kahneman and Tversky (1979), which states that the decisions of mankind are based on relative gains and losses, and that losses are more significant than gains. In Satisficing Theory (Simon, 1978; 1990), a decision maker will search for a reasonable outcome that reflects the information the decision makers have, the constraints of time and other resources, and the decision makers' cognitive limitations (Simon, 1991).

The normative approaches to analyze decision process were developed to investigate how a better decision should be made. Analysis on human conflicts is a multidisciplinary domain of research, combining knowledge in economics, sociology, philosophy, and mathematics (Hipel, 2009). Among various conflict analysis methods in different disciplines, game theory is a formal methodology to investigate strategic conflicts using mathematical tools. Not until the mid 1940s did game theory rapidly develop. The book written by Von Neumann and Morgenstern (1944), called Theory of Games and Economic Behavior, is widely considered as the start of modern game theory. In this book, the rationality of players is discussed and their choices are formally analyzed (Kilgour and Hipel, 2005). Decision makers in game theory are assumed rational, because it is important for policy makers or mediators to know how a better outcome can be achieved. Besides, although individuals may not be rational, the actions for large organizations are close to rational. It is strategically important to know what an organization should do for its managers or policy makers. To describe rationality in a game, Nash (1950) defined a solution concept of a non-cooperative game, called Nash equilibrium, to describe a situation for two DMs, at which both of them cannot make a better choice.

The game theoretic models can be classified as being quantitive or non-quantitive. The genealogy of these models is depicted in Fig. 1.1. In quantitive models, preference relations for DMs are expressed using utility values (Von Neumann and Morgenstern, 1944) to represent a DM's gains or losses in making a choice. Strategies for a DM can be classified as pure strategies, which mean definite actions for the DM, and mixed strategies, denoted by the probabilistic mixture of actions. In comparison, non-quantitive models were designed for use with relative preferences, in which a DM may prefer one state to another or consider the two states to be equally preferred. Normal and extensive form models are two representative quantitive models. In a normal form model, each DM can only move once by changing his or her choices. Extensive games were developed to describe the sequence of moves for DMs. Cooperative games, which describe conflicts among DMs in allocating limited resources, can also be analyzed using quantitive approaches. In quantitive models, the use of utility values has drawbacks because they are usually hard to measure. The determination of utility is usually intuitive and lacks statistics to validate the value. In addition, the probabilities underlying mixed strategies may be difficult to estimate in practice.

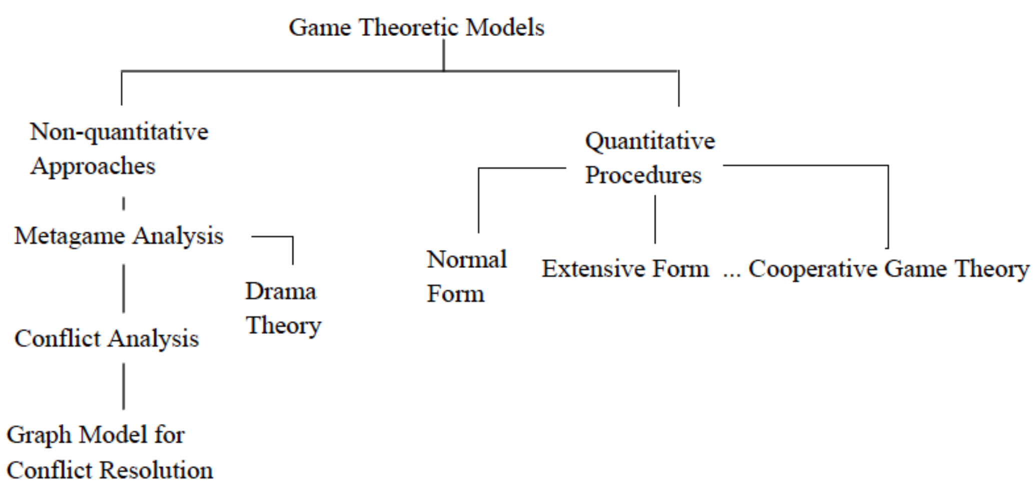

Figure 1.1: Genealogy of conflict analysis methodologies (Hipel and Fang, 2005)

Non-quantitive models, in comparison, can overcome the aforementioned drawbacks.

Howard developed the Metagame Analysis methodology (1971) to mitigate these shortcomings. In a metagame, DMs can move in any order and at any time. Relative preference for a DM is represented by a comparison of each pair of possible outcomes. An outcome, formally called a state, can be more preferred, less preferred, or equal in preference. Drama theory is an extension of metagame theory by considering emotions for DMs (Howard, 1999). In a drama theoretic model, emotional interactions among DMs and possible scenarios are investigated.

Conflict Analysis, devised by Fraser and Hipel (1984), is an important extension of the metagame methodology by introducing sequential stability (SEQ) and symmetric stability (SMR), as additional new solution concepts for capturing human behaviour under conflict (Kilgour and Hipel, 2005). In general, a DM will consider a state to be stable if other DMs can somehow block possible improvements. Stability analyses can be carried out by investigating the stability of each feasible state for each DM under a given solution concept. An equilibrium is a state that is stable under a given solution concept for all DMs. Conflict analysis can be used with the option form proposed by Howard (1971) and stability calculation can be conveniently carried out using the tableau form proposed by Fraser and Hipel (1979; 1984)

The Graph Model for Conflict Resolution (GMCR) (Fang et al., 1993), placed at the bottom of Fig. 1.1, further improved Conflict Analysis. It possesses a flexible theoretical structure based on graph theory and can produce meaningful analytical results (Kilgour and Hipel, 2005). A directed graph for each DM keeps track of the moves in one step that the DM has from each state. Its unique advantages over other conflict methodologies are that it can handle irreversible moves, model common moves, and provide a flexible framework for calculating stability results. It aims to obtain strategic insight into practical conflicts occurring either currently or in the past. The solution concepts defined in the graph model are Nash stability (R) (Nash, 1950; 1951), sequential stability (SEQ) (Fraser and Hipel, 1979; 1984), general metarationality (GMR) (Howard, 1971), symmetric metarationality (SMR) (Howard, 1971), and limited move stability (Zagare, 1984; Brams and Wittman, 1981; Fang et al., 1993). Furthermore, a decision support system, called GMCR II, was designed to facilitate the calculation of stabilities in real world conflicts (Fang et al., 2003a; b). It is also a useful platform for carrying out in-depth analyses, such as status quo analysis (Li et al., 2005) and sensitivity analysis. Other studies on GMCR methodologies and applications include coalition analysis (Kilgour et al., 2001; Inohara and Hipel, 2008a;b), preference uncertainty (Li et al., 2004), strength of preference (Hamouda et al., 2004; Xu et al., 2010), fuzzy preference (Bashar et al., 2014), and the matrix representation of a conflict (Xu et al., 2009a). A detailed introduction to GMCR methodologies is provided in Chapter 2.

1.3 Research Objectives

The objective of this research is to develop theoretical models within the GMCR paradigm to analyze strategic conflicts having a hierarchical structure. A hierarchical graph model is constructed to describe a hierarchical conflict, in which each subconflict is modelled by a regular graph model, called a local graph model. DMs, states, moves for DMs, and their preference relations are defined with respect to hierarchical conflicts as follows. In a hierarchical conflict, some DMs appear in all subconflicts while others only take part in one of them. States denote the possible outcomes of a conflict. The states in a hierarchical conflict model are said to be the combination of all possible outcomes in local graph models. The moves in a hierarchical conflict represent a collection of possible actions for DMs in subconflicts. Preference relations for DMs in hierarchical conflicts will be defined in this thesis. Because of the unique structure of hierarchical graph model, the preferences for DMs in a hierarchical conflict cannot be fully determined by the preference information in all subconflicts. This feature of the hierarchical graph model will be explained in Section 4.2.2. Different patterns of preference structure will be designed by ordering the subconflicts according to importance.

Stabilities refer to the stable states at which, according to the graph model, DMs may stop depending on their different perceptions of sanctions. For some DMs in a hierarchical graph model, an improvement does not necessarily imply improvements in all local models. Hence, the stabilities in a hierarchical conflict cannot be completely predicted from the stabilities in subconflicts. Therefore, the interrelationships between stabilities in the hierarchical model and local models deserve formal investigation. Theorems will be formulated and proven to offer insights into the conditions under which a state in a hierarchical graph model is stable under a given solution concept.

As a hierarchical graph model usually has numerous states, the exhaustive method of calculating stabilities becomes time-consuming. Thus, different algorithms for calculating stabilities in a hierarchical model will be designed in the thesis. A possible way is to use the stability interrelationships. Theorems describing the interrelationship will be proposed in order to determine the stabilities in a hierarchical model. The decision support system GMCR II will be utilized to calculate stabilities in a hierarchical model, provided the preference relations for DMs are easy to input. The matrix representation approach (Xu et al, 2009a) will also be used for carrying out stability calculations. Corresponding matrices should be defined to represent moves and preferences in a hierarchical model.

One of the most important objectives in this research is to apply the hierarchical graph model to real world conflicts. Environmental conflicts, such as water diversion conflicts in China and greenhouse gas treaty disputes between China and the USA, are analyzed. The hierarchical graph model is also used to study the sales competition between Airbus and Boeing in different markets. In each case study, the advantages of the new methodology are demonstrated by comparing the stabilities in the hierarchical model with those obtained separately in local models.

1.4 Outline of the Thesis

The rest of the thesis is organized into eight chapters. A detailed introduction to GMCR methodology is provided in Chapter 2. A lead-in example, a water diversion conflict in China, is given in Chapter 3.

A hierarchical graph model with the simplest structure, called basic hierarchical graph model, is introduced in Chapters 4 and 5. Preferences for DMs in the two chapters are defined in different ways, referred to as lexicographic preference in Chapter 4 and weighted preference in Chapter 5, respectively. In both chapters, water diversion conflicts in China are reinvestigated using the hierarchical graph model paradigm. The approaches of calculating stabilities of the real world conflicts in the two chapters are developed based on different preference structures. The efficiencies of the two approaches in calculating stabilities are compared.

Another hierarchical graph model with a more complex structure, called the duo hierarchical graph model, is proposed in Chapter 6. This model is applied to a sales competition between Airbus and Boeing in the narrow and wide body jet markets.

The general expression of the hierarchical graph model, called the general hierarchical graph model, is introduced in Chapter 7. The disputes between China and USA on adherence to the greenhouse emissions treaty are studied using this model. The conclusion of this thesis and possible extensions of the hierarchical graph model are introduced in Chapter 8. The structure of the thesis is depicted in Fig. 1.2.

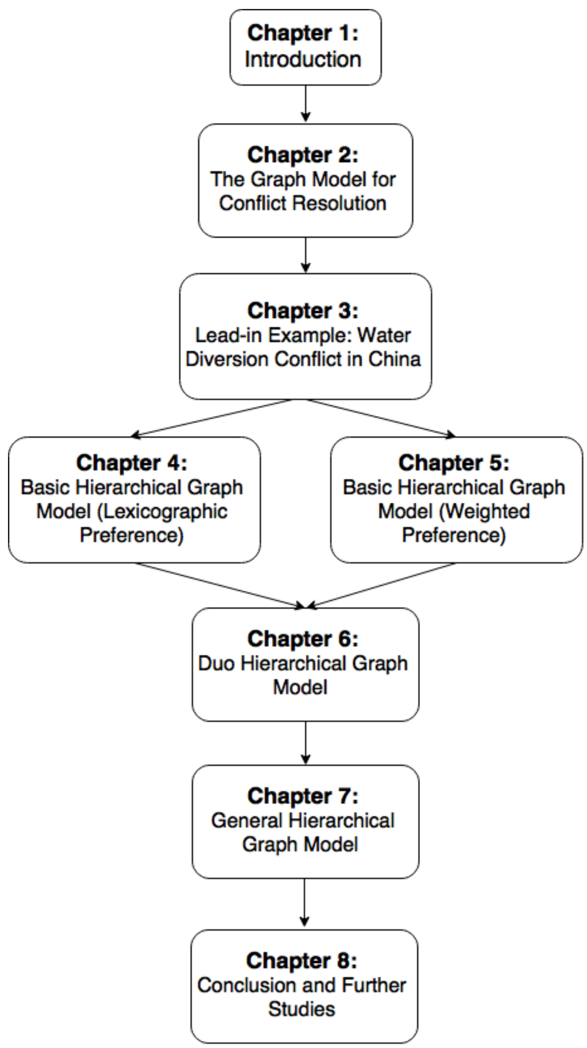

Figure 1.2: Structure of thesis

Chapter 2

Graph Model for Conflict Resolution

2.1 Introduction

The Graph Model for Conflict Resolution (GMCR) constitutes a systems thinking methodology used for modeling and analyzing strategic conflicts. GMCR is used to represent decision makers (DMs), states, possible moves, and preference relations in a strategic conflict (Kilgour et al., 1987). DMs are defined as stakeholders who can affect the outcome of a conflict. Usually, a state is defined as a combination of possible choices, called options, selected by all DMs. Only states which are feasible could occur in the real world and are analyzed. A DM can move between two states by changing his or her option. The options for other DMs remain fixed.

A DM has his or her relative preferences over the feasible states in a conflict. The preferences of a specific DM can be represented by either a pairwise comparison between two states or a list of all the states from most to least preferred, where ties are allowed for the case of transitive preferences. Solution concepts, or stability definitions, are utilized to describe different types of human behaviour under conflict in a graph model, including Nash (R) stability, sequential stability (SEQ), general metarationality (GMR), and symmetric metarationality (SMR). States are identified under each stability definition for each DM, which reflect his or her possible actions based on individual interests. Equilibria are states which are stable for all DMs for a specific solution concept. These equilibria are the outcomes or possible resolutions of a conflict that are likely to happen (Fang et al., 1993).

In-depth analyses of a conflict under the graph model paradigm include coalition analysis, status quo analysis, and sensitivity analysis. In a coalition, two or more DMs believe that their cooperation is more advantageous than individual moves. Status quo analysis is used to indicate the evolution of a conflict from the starting state to the actual outcome. A sensitivity analysis can be executed to test the robustness of the results in a conflict by examining how these results are affected by changing model input.

2.2 Formal Definition of Graph Model

A graph model is defined as a set of directed graphs to represent a strategic conflict. The nodes in a graph represent possible status of the conflict, i.e. states. The directed arcs connecting two states are called moves for a focal DM. The structure of a graph model is defined as follows:

Assuming that N denotes the set of DMs and S represents the set of feasible states, suppose that D_i is DM i's directed graph and A_i is the set of directed arcs in D_i, a graph model is defined as $\{D_i = S, A_i : i \in N\}$. The ordered pair of states (s, s') for $s, s' \in S$

denotes a directed arc starting from its tail s to the head s'. Hence, the arcs for DM i are the Cartesian product of two sets of states $A_i \subseteq S \times S$. Note that the moves may or may not be reversible. The preference for DM i is defined as the ordinal relation between each pair of states, i.e. more preferred ($\succ_i$), less preferred ($\prec_i$), or equally preferred ($\sim_i$). The preference relation in a graph model can be collectively denoted as $\succsim$. Thus, a graph model G can be defined as a set including N, S, A, and $\succsim$, by $G = \{N, S, A, \succsim\}$.

The reachable list is the set of all possible moves, called unilateral movements (UMs), for a DM and does not depend on its preferences. Suppose $s, s' \in S$, the reachable list for i from state s is defined as

$$R_i(s) = \{s' \in S : (s, s') \in A_i\}.$$

The unilateral improvement (UI) list is a reachable list containing all accessible states from a starting state which are more preferred by the focal DM. Suppose $s, s' \in S$, the UI list for i from state s is defined as

$$R_i^+(s) = \{s' \in S : (s, s') \in A_i, s' \succ_i s\}.$$

2.3 Joint Movement and Joint Unilateral Improvement

When making a move, a DM may wish to consider possible responses by other DMs in a coalition, say H. In a graph model, unilateral moves for H are defined as follows:

Definition 2.1 (Joint Unilateral Movement in G): Let $s \in S$ and $H \subset N$, $H \neq \phi$. A unilateral move by H is a member of $R_H(s) \subseteq S$, defined inductively beginning with $i \in H$, then for all $s' \in R_i(s)$, $s' \in R_H(s)$ and $\Pi_H(s,s') = \{i\}$. Now if $s' \in R_H(s)$, $j \in H$, $s'' \in R_j(s')$, and $s'' \neq s$, then provided $\Pi_H(s,s') \neq \{j\}$, $s'' \in R_H(s)$ and $\Pi_H(s,s'') = \{j\}$.

Coalition H represents a group of DMs with no less than one DM. $R_H(s)$ denotes the set of all states that can be accessed from s through any legal sequence of unilateral moves by some or all of the DMs in H. Note that in a legal sequence of unilateral moves, no DM can move twice consecutively. The state s' represents any state reached from s by DMs in H. The set $\Pi_H(s,s')$ contains all the DMs in H whose UMs are in the legal sequence from s to s'.

This definition constructs $R_H(s)$ by expanding the set of DMs who can move from s to s', which is initially assumed to be empty. States reachable from s are identified and included in $R_H(s)$. Furthermore, for $j \in H$, reachable states from s' are identified and added to $R_H(s)$. The process stops when no further state can be included in $R_H(s)$. Since $R_H(s) \subseteq S$ and S is finite, the process is completed in a finite number of steps.

A legal sequence of UIs for a group of DMs can be defined similarly. In the definition below, $R_H^+(s)$ denotes the set of states that can be accessed by a legal sequence of UIs, by some or all DMs in H, starting at state s. If $s' \in R_i^+(s)$, then $\Pi_H^+(s,s')$ in the following definition means the set of all DMs whose UIs are in the legal sequences from state s to state s'.

The joint UIs in a graph model are defined analogously.

Definition 2.2 (Joint Unilateral Improvement in G): Let $s \in S$ and $H \in N, H \neq \phi$. A unilateral improvement by H is a member of $R_H^+(s) \subseteq S$, defined inductively beginning with $i \in H$. For all $s' \in R_i^+(s)$, one has $s' \in R_H^+(s)$ and $\Pi_H^+(s, s') = \{i\}$. Now if $s' \in R_H^+(s)$, $j \in H$, $s'' \in R_j^+(s')$, $s'' \neq s$, and $\Pi_H^+(s, s') \neq \{j\}$, then $s'' \in R_H^+(s)$ and $\Pi_H^+(s, s') = \{j\}$.

Definition 2.2 differs from Definition 2.1 in that the moves are required to be UIs. Each state reached by a DM is strictly preferred to the starting state. Accordingly, $\Pi_H^+(s, s')$ represents all DMs in H whose UMs are UIs from state s to state s'.

2.4 Option Form and Option Prioritization

Option form proposed by Howard (1971) has been widely used to represent strategic conflicts. A graph model can be expressed in option form by listing all feasible states as the selection of options for all DMs. Each state can be written as a list of "Y"s and "N"s. A "Y" for an option means this option is selected and an "N" means the opposite. For example, supposing a graph model contains a total of 2 options, the states in the graph model can be written as (YY), (YN), (NY), (NN). Among them, state (YN) indicates an outcome at which the first option is selected and the second is not.

Option prioritization is an effective approach to represent preferences for a DM by investigating its selection of options at each state. In this approach, the preference of a DM is denoted by a list of preference statements ordered from the most preferred for the DM at the top of a table to the least preferred at the bottom. A preference statement for a DM can be expressed by option numbers connected by logical symbols, including negation ("NOT" or "−"), conjunction ("AND" or "&"), disjunction ("OR" or "|"), and conditions ("IF" and "IFF") (Fang et al., 2003a). In a graph model, the preference statements for DM i can be denoted as Ω_{ij}, where $0 < j \leq h$ and h denotes the number of statements.

Each preference statement can be either true (T) or false (F) at each particular state. A score is assigned to each state for a focal DM to represent its truth values when the statements are applied. Specifically, the score of state $s \in S$ for i can be written as $\Psi_i(s)$, where

$$\Psi_{ij}(s) = \begin{cases} 2^{h-j} & \Omega_{ij}(s) = T \\ 0 & otherwise \end{cases}$$

$$\Psi_i(s) = \sum_{j=1}^{h} \Psi_{ij}(s).$$

Then, the preference relation for i between any two states $s, s' \in S$ can be determined by comparing their scores $\Psi_i(s)$ and $\Psi_i(s')$.

2.5 Matrix Representation of Graph Model

The matrix representation method, proposed by Xu et al. (2007; 2009a), can be used for effectively describing states, moves, and preferences in a graph model. This approach is powerful in determining stabilities for DMs, as the calculation is performed by matrix computation using a decision support system, MRSC, designed by Xu et al (2009a).

The reachable list and UI list for a DM are denoted by a reachable matrix and UI matrix, respectively. The reachable list for DM i, denoted as $R_i(s) = \{s' \in S, (s, s') \in A_i\}$, can be defined by an $m \times m$ 0-1 matrix J_i with the entry $J_i(s, s')$ in the s^{th} row and the s'^{th} column written as:

$$J_i(s,s') = \begin{cases} 1 & if\ (s,s') \in A_i \\ 0 & otherwise \end{cases}$$

J_i is called a reachable matrix for DM i. The preference matrix P_i^+ for DM i represents the preference relation between any pair of states, which can be written as

$$P_i^+(s,s') = \begin{cases} 1 & if\ s' \succ_i s \\ 0 & if\ s' \precsim_i s \end{cases}$$

Note that $s' \precsim_i s$ means s' is less or equally preferred to s for DM i. The symbol $\precsim$ contains two relations between s' and s, which are less preferred ($\prec$) and equally preferred ($\sim$).

The unilateral improvement (UI) list from a state $s \in S$ for DM $i \in N$ is marked as $R_i^+(s) = \{s' \in R_i(s) : s' \succ_i s\}$. The UI matrix for DM i can be denoted as the Hadamard Product of J_i and P_i^+:

$$J_i^+ = J_i \circ P_i^+ \tag{2.1}$$

where "$\circ$" denotes the Hadamard Product, with $J_i^+(s,s') = J_i(s,s') \cdot P_i^+(s,s')$.

In a graph model, the joint movement and improvement for DMs $H \subseteq N$ can be noted as M_H and M_H^+, for $H \neq \phi$. The expression of M_H and M_H^+ can be shown below:

$$M_H = \bigvee_{t=1}^{\delta} \bigvee_{i \in H} M_i^{<t>} \tag{2.2}$$

and

$$M_H^+ = \bigvee_{t=1}^{\delta'} \bigvee_{i \in H} M_i^{<t,+>} \tag{2.3}$$

where

$$M_i^{<t>} = sign(J_i \cdot (\bigvee_{i' \in H-i} M_{i'}^{<t-1>})) \tag{2.4}$$

with $M_i^{(1)} = J_i$ and

$$M_i^{<t,+>} = sign(J_i^+ \cdot (\bigvee_{i' \in H-i} M_{i'}^{<t-1,+>})) \tag{2.5}$$

with $M_i^{<1,+>} = J_i^+$.

The *sign* function for $m \times n$ matrix A is defined as

$$sign(A) = \begin{pmatrix} sign[A(1,1)] & ... & sign[A(1,n)] \\ \vdots & \ddots & \vdots \\ sign[A(m,1)] & ... & sign[A(m,n)] \end{pmatrix} \tag{2.6}$$

where $A(m,n)$ is the entry in m^{th} row and n^{th} column, with

$$sign[A(m,n)] = \begin{cases} 1 & \text{if } A(m,n) > 0 \\ 0 & \text{if } A(m,n) = 0 \\ -1 & \text{if } A(m,n) < 0. \end{cases}$$

The operator "$\bigvee$" is defined as:

if M_1, M_2, ..., M_l are all $h \times h$ matrices, then $\bigvee_{i=1}^{l} M_i$ is the $h \times h$ matrix with (s, s') entry

$$ind\left(\sum_{i=1}^{l} M_i(s,s')\right)$$

when

$$ind(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases} \tag{2.7}$$

In equations (2.2) and (2.3), δ and δ' represent the number of iterations required to form the joint movement and improvement, respectively. δ and δ' have been proven to be less than the number of UM and UI arcs $|\bigcup_{i\in N} A_i|$ and $|\bigcup_{i\in N} A_i^+|$ for all DMs, respectively, i.e. $\delta \leq |\bigcup_{i\in N} A_i|$ and $\delta' \leq |\bigcup_{i\in N} A_i^+|$ (Xu et al., 2009a).

Equations (2.4) and (2.5) denote the UM and UI matrices for a coalition of DMs H, which are the aggregations of UMs and UIs for each DM in the coalition. In Equation (2.4), $M_i^{<1>} = J_i$ means the joint moves made in one step by DM i. Following this move, other DMs in H, marked as $H - i$, move in sequence, while each of them cannot move consecutively. For example, the two-step joint moves starting from DM i can be written as

$$M_i^{<2>} = sign(J_i \cdot \bigvee_{i' \in H-i} M_{i'}^{<1>}) \tag{2.8}$$

where $M_{i'}^{<1>} = J_{i'}$. Equation (2.5) has analogous notation to Equation (2.4).

2.6 Solution Concepts

GMCR methodology is a powerful tool for predicting the outcomes of a conflict. The outcomes that are more likely to happen are denoted by stable states in a graph model. These states are stable under different solution concepts, reflecting different foresights of DMs and their perception of risks. The four types of solution concepts are Nash stability (R), sequential stability (SEQ), general metarationality (GMR), and symmetric metarationality (SMR). Detailed definitions for these solution concepts are given below.

(1) Nash Stability (R) (Nash 1950, 1951): A state is Nash Stable or Rational for a DM if and only if there is no UI by the DM from this state to another. Nash stability can be mathematically defined as: $s \in S$ is Nash stable for DM $i \in N$ iff $R_i^+(s) = \phi$.

(2) Unstable (u): A state is unstable for a DM if the DM has at least one UI which cannot be credibly blocked by other DMs.

(3) Sequential Stability (SEQ) (Kilgour, 1985; Fang et al., 1993): A state is Sequentially Stable for a DM when all of the DMs' UIs are sanctioned by subsequent UIs by other DMs. At an SEQ state, a DM's UI will be blocked by other DMs' UIs. The definition can be written as: $s \in S$ is SEQ for DM $i \in N$ iff for every state $s' \in R_i^+(s)$, there exists at least $s'' \in R_{N-i}^+(s')$, such that $s'' \precsim_i s$, where $R_{N-i}^+(s')$ is the set of UIs for all DMs except i at state s'.

(4) In General Metarationality (GMR) (Howard 1971), all of a DM's UIs are sanctioned by subsequent unilateral moves by other DMs. GMR describes a DM who perceives his possible moves conservatively by ignoring his own possible counteractions while considering all potential sanctions to his move.

GMR differs from SEQ in that a focal DM is afraid of being sanctioned by other DMs' UMs. The definition is written as: $s \in S$ is GMR for DM $i \in N$ iff for every state $s' \in R_i^+(s)$, there exists at least $s'' \in R_{N-i}(s')$, such that $s'' \precsim_i s$, where $R_{N-i}(s')$ is the set of UMs for all DMs except i at state s'.

(5) At an Symmetric Metarational (SMR) (Howard 1971) state, a DM considers not only its own possible moves and the reaction of the opponents, but also its own counteractions.

The DM has a horizon of three moves, while GMR reflects its perception of two moves.

The formal definition is provided as: $s \in S$ is SMR for DM $i \in N$ iff for every state $s' \in R_i^+(s)$, there exists at least $s'' \in R_{N-i}(s')$, such that $s'' \precsim_i s$, and $s^* \precsim_i s$ for all $s^* \in R_i(s'')$.

Note that a stable state under one solution concept may also be stable under other solution concepts. A Nash stable state is also SEQ, GMR, and SMR, because the focal DM cannot move away from this state. A SEQ or SMR state is also GMR, because there must exist a sanctioning UM at an SEQ or SMR state. The interrelationship among these solution concepts, shown in Fig. 2.1, has been discussed by Kilgour and Hipel (2010). For a focal DM, a Nash state is also SEQ, GMR, and SMR. A Nash, SEQ, and SMR state is also GMR for the focal DM.

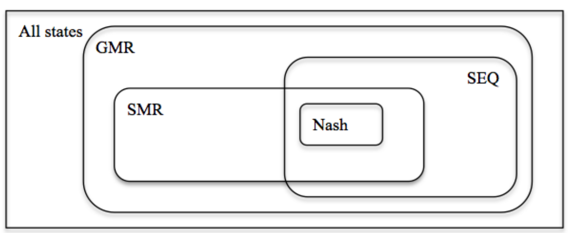

Figure 2.1: Interrelationship of four solution concepts

For each DM, a state can be stable under different solution concepts. A stable state under one solution concept for all DMs is an equilibrium and can be regarded as a possible resolution for the conflict. A Nash or SEQ stable state for all DMs is a strong equilibrium (SE), which can reflect an actual outcome that occurs in reality. A GMR or SMR state for all DMs constitutes a weak equilibrium (WE), indicating an outcome which is less likely to happen.

2.7 Decision Support System: GMCR II

The decision support system for GMCR, GMCR II, is a user-friendly computer package that encodes conflict modelling and can be used for calculating stabilities for DMs. Apart from presenting stability results for either individual DM or overall DMs, GMCR II can also be used to perform coalition analysis and status quo analysis.

The structure of GMCR II is shown in Fig. 2.2. GMCR II is divided into modelling subsystem, analysis engine, and output interpretation subsystem. DMs, options, feasible states, and preferences are the input in the modelling subsystem. Stability analysis, coalition analysis, and status quo analysis can be carried out in the analysis engine. Individual stability and equilibria are demonstrated in the output interpretation subsystem (Fang et al., 2003a; b).

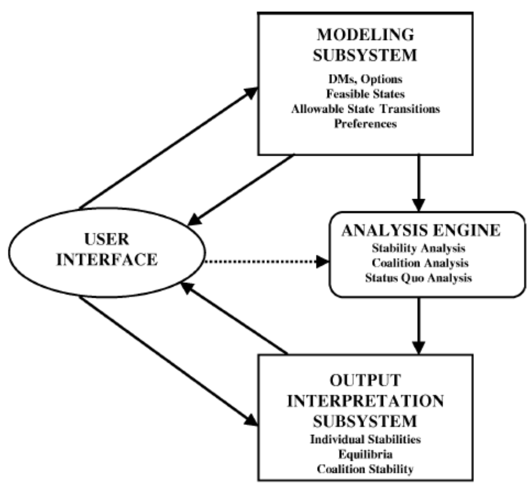

Figure 2.2: Structure of GMCR II (Fang et al., 2003a)

Chapter 3

A Lead-in Example: Water Diversion Conflict in China

3.1 Introduction

A real world example regarding water diversion conflicts in China is given in this chapter to show the hierarchical structure of these conflicts and the need for developing new methodologies. Chapter 3 is based on a journal article written by He et al. (2014).

To address China's large population and uneven distribution of water storage, the South-North Water Diversion Project (SNWDP) aims at transferring water from the Yangtze River (Changjiang) Basin to the North China Plain. This project is being implemented over three different routes, referred to as the Eastern, Central, and Western routes, each of which is giving rise to strategic conflicts. In this Chapter, the Graph Model for Conflict Resolution (GMCR) is used to systematically investigate these conflicts and obtain strategic insights into them. The Chinese Central Government is involved in each conflict, together with local decision makers, making the entire conflict hierarchical. The conflict is analyzed both as an overall graph model and as three local conflicts, and the resulting equilibria are compared. The Central Government's preferences, which can be fully elaborated in the single or overall model, account for the differences in equilibria between the overall model and the three local models. The Central Government,which utilizes various strategies in planning, modifying, and building these routes, may be able to control the overall project as it wishes, thereby reaching separate agreements with the relevant decision makers along each route.

China is one of the world's largest countries and has abundant water resources. China's total renewable fresh water is about 2804 km^3 and ranked the fifth in the world, following Brazil, Russia, United States, and Canada. However, water availability per capita in China amounts to only 1/5 of the world average (World Factbook, 2014). In the North China Plain, severe droughts have been reported recently (United Nations News Report, 2012). Human activities and crops in rural areas have been dramatically affected. Meanwhile, water demand is rocketing as a result of fast urbanization. As more of the population moves to cities from rural areas, household demands, as well as demands in agricultural and industrial sectors, grow year by year (Larson, 2012). Moreover, waste and inefficient usage also contribute to the water scarcity. A 2009 World Bank report reveals that in China only 45 percent of the water withdrawn for agriculture actually reaches the crops (Cho, 2011). To tackle the aforementioned issues, new approaches are required to meet the water demands and reasonably manage water resources in the North China Plain.

Water resources management has been studied in the area such as water pricing, water allocation, and water monitoring and assessment. In studies on water pricing, the transaction costs (Zhang et al., 2013) and efficiency of urban water use (Hung and Chie, 2013; Garcia-Valinas, 2005; Harris, 2011) have been analyzed. An area-based pricing method was introduced by Dono et al. (2012). With respect to water allocation, an Integrated-Water Resource Management (IWRM) model for river basin water allocations has been put forward (Silva-Hidalgo et al., 2009). Cooperative game theory has been applied to transboundary water allocations (Sechi et al., 2013; Abed-Elmdoust and Kerachian, 2012; Kucukmehmetoglu, 2009). To monitor and evaluate the impacts of climate change and human activities on water resources, a Soil and Water Assessment Tool (SWAT) model has been used to model water flows in the Xixian Watershed (Shi et al., 2011). The impact of urbanization in the Ganga basin has also been revealed and measures to better utilize water in this region have been proposed (Misra, 2011).

3.2 The South-North Water Diversion Projects in China

Water diversion projects are effective in easing water shortages and have been adopted in many countries. Various studies can be found on interbasin water diversions. The interbasin water transfer project in Brazil was compared with two international cases by de Andrade et al. (2011): the Colorado-Big Tompson Project in the US and an Australian water diversion project. Their studies revealed the environmental, political, and economic complexities associated with these large water diversion projects. Another water diversion project in Brazil was examined for its suitability to supply the metropolitan area of Sao Paulo, using a dynamic systems simulation model (Gonzalez et al., 2011). When possible water diversions from the North American Great Lakes were analyzed within a game theory framework (Becker and Easter, 1995), states and provinces along the lakes would divert water even it is unnecessary, because they would lose more by not diverting water. Their decisions result in a Tragedy of the Commons.

The last decade has witnessed increasingly serious water shortages in North China due to industrialization and population growth, while South China has suffered from frequent flooding. The increasing water demand in the North China plain calls for more water supplies. A possible solution is to divert water from regions with abundant water resources, so the water rights are transferred from users in one basin to another. The theory of water rights defines the total amount of available water and rules to divide it among users (Speed, 2009a; Xie, 2008). Water rights are enforced, and tradable in markets (Productivity Commission, 2003). In China, rights-based water allocation systems have not yet been developed (WET, 2006). China's 2002 water law marks a great effort toward establishing "an initial water distribution system" and "a water trading system" (State Council, 2006). These systems are designed to transfer water within a given river basin (Speed, 2009b). The Yellow River basin in the North China plain is already short of water, and it is planned to transfer water to it from other river basins. This is currently feasible, as water is defined in the constitution as public property shared by the people (Speed, 2009a). Thus, water transfer projects in China are implemented by the Central Government.

The South-North Water Diversion Project (SNWDP), depicted in Fig. 3.1, was proposed to ease the water scarcity in the North China Plain. This huge project includes three main water transport routes: Eastern, Central, and Western. The eastern route is being constructed following the Grand Canal, starting from Jiangdu, a town along the lower Yangtze River, to the city of Tianjin. Along the eastern route are the most industrialized and highly populated areas of China. Tunnels carry the diverted water beneath the Yellow River (Huang He). This route can meet industrial and urban demands in the eastern part of the North China Plain, and provide water for agricultural use if necessary. The central route starts from Danjiangkou Reservoir, China's largest artificial freshwater lake, located in Hubei Province. The water is to be diverted from the reservoir to Beijing via canals and tunnels. The water will flow through Henan, Hebei, and Beijing, where it will be used for urban and industrial purposes. During and after the construction of the Danjiangkou Reservoir, large populated areas will be inundated, forcing residents to relocate. Tunnels will divert the water across the Yellow River near Zhengzhou, the capital of Henan Province. The western route is planned to connect the Yangtze River and the Yellow River in the Tibetan Plateau. Water shortages in the Yellow River can be alleviated by transferring water from the Yangtze River. This project is controversial on account of some unresolved technical difficulties in construction. Moreover, as the headwaters of many international rivers flow from this region, the project may affect water usage in neighboring countries. This megaproject could have significant impacts on local societies and their environment. Provincial governments, residents, and other stakeholders are involved and their interests and rights may be dramatically affected. Conflicts are arising along all three routes. On the eastern route, there are disputes over water usage between Beijing and eastern provinces along the Yangtze and Huai Rivers, such as Jiangsu and Anhui provinces. Frequent droughts have also been reported in recent years in some eastern provinces (Buckley, 2011), intensifying the existing disputes. It is challenging for the Central Government to satisfy all users while maintaining the profitability of this project (Chen et al., 2013).

Figure 3.1: The location of the South-North Water Diversion Project (Source: francistopia.edublogs.org, 2011)

On the central route, many residents must be relocated, resulting in dissent from the residents due to inadequate compensation. However, local governments may seek profits through corruption while executing these policies, particularly in the distribution of funds for compensation. They may even block the residents petitions from reaching the Central Government. Nevertheless, corruption is risky for local governments, since they will be harshly punished if the corruption is revealed to the public (McCormack, 2001; Gleick, 2009).

The western route will affect water usage in neighbouring countries, as water will be diverted from some international rivers in the Tibetan Plateau. A large number of dams and reservoirs have been built in the upstream of the Yangtze River, significantly affecting water flows in the Tibetan Plateau (Zhang et al., 2012). Opposition has arisen in these countries, where there have been calls for negotiations (Holslag, 2011).

Many studies have focused on issues and conflicts related to the SNWDP in China. The risk of water shortages caused by this project in areas around the Danjiangkou Reservoir has been determined and evaluated (Gu et al., 2012). Monthly flow data in the reservoir were simulated for optimized operation at minimum risk; a case study in the Yellow River Basin provides approaches to allocate water resources taking into account a flexible limit to water storage (Shao et al., 2009). Accompanied by the approaches put forward to optimize water usage, SNWDP is believed to be effective in solving the water crisis in the Yellow River Basin.

During the implementation of the project, conflicts have taken place among various stakeholders. For the Central Government of China, ignorance of these conflicts is risky because it may increase the dissatisfaction of other stakeholders. It is important to systematically investigate these conflicts to obtain strategic resolutions.

3.3 Graph Model of Water Diversion Conflict in China

The hierarchical water diversion conflict in China is modelled using GMCR. This hierarchal conflict contains two subconflicts. DMs and their options are identified. Feasible states are obtained and ranked according to the preferences of each DM.

3.3.1 Decision Makers

DMs in the overall water diversion conflict are defined with respect to the three routes: Chinese Central Government (CG) and eastern provincial governments (EPGs) on the eastern route; Chinese Central Government (CG), central provincial governments (CPGs), and local residents (LRs) on the central route; Chinese Central Government (CG) and neighboring countries (NCs) on the western route. CG, which is involved with all three routes, refers to the officials in the Central Political Bureau and their subordinates. It also includes affiliated ministries such as the Ministry of Water Resources and the National Development and Reform Commission.

The acronym EPGs refers to the governments in the eastern provinces including Jiangsu, Anhui, and Jiangxi. LRs refer to the residents who are forced to relocate due to the construction of the central route occupying the land on which they reside. CPGs represent the local governments in and below provincial level on the central route in charge of relocating residents and granting compensation, such as the governments in the municipal and county levels in Hubei and Henan Provinces. NCs are the countries bordering southwestern China,

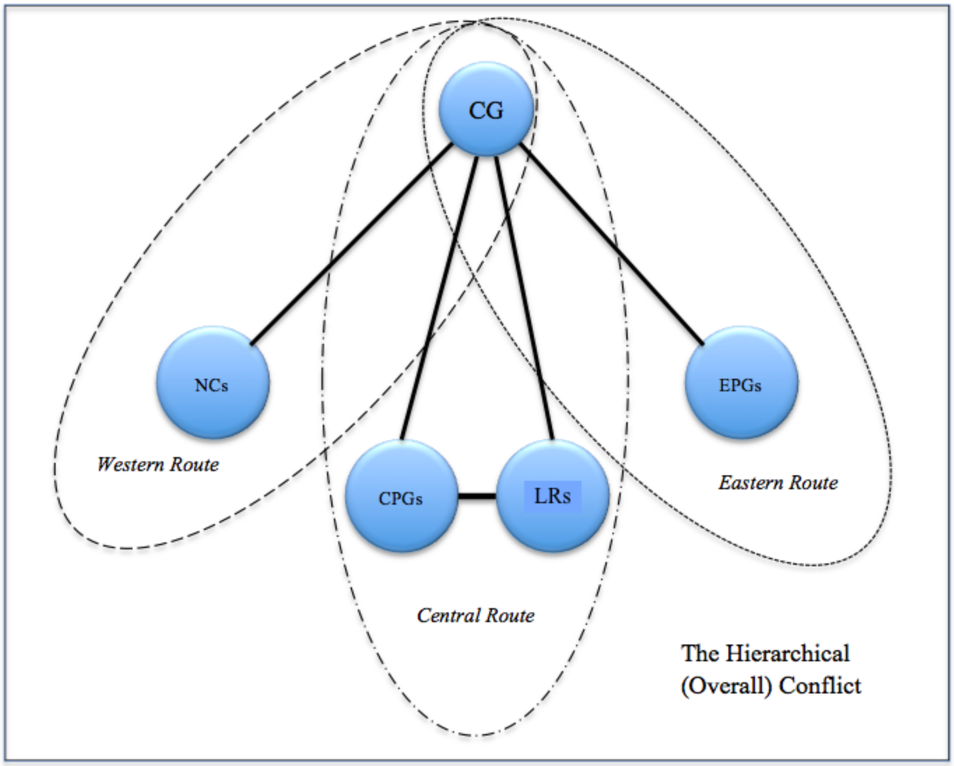

Figure 3.2: The structure of the overall water diversion conflict in China

whose water usage is affected by the construction of the western route. In this chapter, an overall hierarchical conflict consisting of conflicts pertaining to three routes is analyzed, where CG is the only DM involved in all three routes.

As shown in Fig. 3.2, the hierarchical water conflict consists of three routes. A black line connecting two DMs represents an interaction between the two. CG is the only DM who belongs to all three routes and interacts with all other DMs. There is also an interaction between CPGs and LRs within the central route.

3.3.2 Options

Options are possible actions that DMs expect to take in a conflict. In this conflict, CG wishes to initiate all of the scheduled projects on the three routes if no opposition occurs. However, it will face opposition from other DMs on the three routes. CG has two options for each route. On the eastern route, it can either initiate the full projects or implement modifications, such as changing the water prices and controlling the amount of water to be transferred. EPGs can either agree with CG's initiatives or oppose them. On the central route, CG can similarly either initiate the full projects or grant further compensation to appease the residents. The local residents can comply with CG's policies or refuse to move. They will not concede to CG's plans unless their living conditions are maintained and job opportunities are secured by CG. On paper, CPGs should fully execute CG's policies regarding the relocated residents. However, they are likely to misappropriate the funds used for compensation and conceal the local residents' oppositions from being known by CG. On the western route, CG can either execute the original plan or modify the plan by cancelling some dams and reservoirs that will affect the water flow in neighboring countries. NCs can agree with CG's original plan or protest. The DMs and their options are listed in Table 3.1. Each option is expressed with a number to facilitate further calculation. As shown in Table 3.1, there are 14 options in total.

3.3.3 The Status Quo State

The status quo state (SQ) is the initial situation at the time for which a conflict study is executed. In the overall conflict, the status quo state is listed as the column of Ys and Ns in Table 3.1. An option marked with Y represents that it is taken in this state, while

Table 3.1: Options of DMs and Status Quo State

DMs	Option Numbers	Options	Status Quo
China Central Government (CG)	1	Fully initiate the eastern route	Y
	2	Modify the eastern route	N
	3	Fully initiate the central route	Y
	4	Modify the project and change compensation policies	N
	5	Original plan on the western route	Y
	6	Modified western plan	N
Eastern Provincial Governments (EPGs)	7	Agree with CG's initiative	N
	8	Oppose CG	Y
Central Provincial Governments (CPGs)	9	Fully initiate CG's policies	N
	10	Initiate CG's policies in a corrupt and arbitrary manner	Y
Local Residents (LRs)	11	Comply with CG's policies	N
	12	Refuse to move	Y
Neighboring Countries (NCs)	13	Agree with CG regarding the western route	N
	14	Protest and pressure CG	Y

an option with N means the opposite. Initially, CG intends to implement the full projects on all of the three routes, which results in opposition from EPGs, LRs, and NCs in each route. CPGs are likely to engage in corruption if there is no possible sanction from other DMs.

3.3.4 Removing Infeasible States

As each option can be either selected or rejected, there are 2^{14} states that are mathematically possible. However, many of the combinations are logically infeasible. For example, CG must choose exactly one option from the two for each route. Another infeasibility, called option dependence, is caused by the importance of the three routes perceived by

CG. Option dependence reflects CG's discrepant view regarding the three routes and its strategy of initiating each route in a preferred order. Specifically, CG considers the eastern route as the most important route and the western route the least important. Therefore, the full proposed central route cannot be adopted by CG until the full project on the eastern route is carried out. Likewise, CG will not initiate the full western route until both the full eastern and central routes are implemented. Thus, the modified eastern route with either full central or full western are infeasible. Similarly, states with modified central or eastern route and full western should also be removed. After removing these infeasible states, there are 64 feasible states left for further analysis. The different relative importances of each route, reflected by option dependence, results in analytical results that are different from findings that would result from assuming the three routes are equally important. This difference is discussed later by comparing the hierarchical conflict with the three separate conflicts.

3.3.5 Ranking of Feasible States

For each DM, the feasible states are ranked according to the preferences of the DM using the option prioritization method (Kilgour and Hipel, 2010). Preference statements written in terms of option numbers are listed from the most preferred at the top to the least preferred at the bottom, as shown in the right column of Table 3.2 for CG.

For example, as the eastern route is the most important for CG, it would construct the full project (option 1) regardless of oppositions from EPGs. Following this, it expects agreement from EPGs on its decision (1 IF 7). If EPGs oppose the full route, CG will consider modifying the project (2 IF 8). CG would consider the central route after the eastern project has been implemented. It prefers to modify the project on account of strong opposition from LRs (4). It would also like to see satisfaction from LRs (11). If there is no significant opposition from LRs, CG would fully initiate the central project (3 IF 11). CG also expects its compensation policies to be well executed by CPGs (option 9). The western route is the last to be considered. CG's first option is to change the western project if the neighbouring countries protest (6 IF 14). Otherwise, it will initiate the original plan if NCs acquiesce (5 IF 13). Note that the order of the three routes reflects option dependence among the three routes, which forms a basis for removing certain infeasible states.

EPGs would agree on the eastern project if it is modified (2 then 7 IF 2). Otherwise, they will pressure CG to modify (8 IF 1). CPGs will not fully execute CG's policies until being pressured by other DMs (10 then 9). In particular, they will execute the entirety of CG's policies if LRs oppose (9 IF 12). CPGs also wish to see CG change the policies (i.e. increase compensation) so that they can confiscate more funds (4). LRs will be content if CG changes policies (11 IF 4). They will refuse to move if CG insists on the original central plan (12 IF 3). Moreover, LRs expect the compensation executed by CPG to be transparent (9). They will move if properly compensated (11 IF 9). On the western route, NCs would firstly protest (14 IF 5) if CG insists on the original plan. They can be satisfied if the western plan is modified (13 IF 6).

Table 3.2: Preference Statements for all DMs in Overall Conflict

Descriptions	Preference Statements
Chinese Central Government (CG)	
CG wishes to fully initiate the eastern route.	1
CG initiates the full eastern route if EPGs agree with CG.	1 IF 7
CG will modify the eastern route if EPGs oppose it.	2 IF 8
CG changes the compensation policies on the central route.	4
CG wishes to see contentment on the part of LRs.	11
CG initiates the full central route if LRs are satisfied.	3 IF 11
CPGs fully initiate CG's policies.	9
CG will change the western plan if NCs oppose it.	6 IF 14
CG will initiate the original western plan if NCs agree to it.	5 IF 13
Eastern Provincial Governments (EPGs)	
EPGs expect CG to modify the eastern route.	2
EPGs will agree with CG if it modifies.	7 IF 2
EPGs will oppose the eastern route if CG fully initiates it.	8 IF 1
Central Provincial Governments (CPGs)	
CPGs initiate CG's policies with corruption.	10
CPGs fully initiate CG's policies.	9
CPGs fully initiate CG's policies if LRs refuse to move.	9 IF 12
CPGs wish that CG changes compensation policies.	4
Local Residents (LRs)	
LRs will be happy if CG changes compensation policies.	11 IF 4
LRs will refuse to move if CG initiates the full central route.	12 IF 3
LRs expect CPGs to fully initiate CG's policies.	9
LRs will accept CG's policies if CPGs are not corrupted.	11 IF 9
Neighbouring Countries (NCs)	
NCs will protest and pressure CG if it initiates the original western plan.	14 IF 5
NCs will be satisfied with a modified western plan.	13 IF 6

3.4 Stability Analysis of the Hierarchical Water Diversion Conflict in China

A detailed stability analysis of the hierarchical conflict is conducted using GMCR II. Stabilities are examined for each DM and the equilibrium states are discovered. These analytical results are interpreted with respect to their strategic significance in the overall dispute.

3.4.1 Stability Calculation of the Hierarchical Conflict

All feasible states are examined for stabilities for each DM, using GMCR II. An outcome with some types of stabilities for all DMs is considered an equilibrium or resolution. As indicated previously, an SE is more likely to be an actual outcome; thus, it can be considered as a possible resolution. As shown in Table 3.3, state 15 is an SE in this conflict because it is Nash rational for all DMs. This equilibrium is also SEQ, GMR and SMR for all DMs. State 47 is a WE; it is only GMR stable for all DMs. The options taken by all DMs in the two equilibria are indicated in Table 3.3.

State 15 is the actual outcome that has been officially reported (South-North Water Diversion, 2011), in which CG initiates the full eastern route while modifying the central and western projects. Its decision results in disagreement from EPGs and the corruption of CPGs (Duggan, 2013). LRs accept CG's decision since the compensation scheme has been modified. NCs also agree with CG's modified western plan. The official news reveals that the eastern project based on the original schedule nears completion. As the most developed region in China, the Yangtze Delta needs an increasing amount of water to

Table 3.3: The Equilibria with Options of all DMs in the Stability Analysis

DMs	Options	State 15 (SE)	State 47 (WE)
CG	1. Full Eastern	Y	Y
	2. Modified Eastern	N	N
	3. Full Central	N	N
	4. Modify Compensation	Y	Y
	5. Original Western	N	N
	6. Modified Western	Y	Y
EPG	7. Agree with CG	N	N
	8. Oppose the Eastern	Y	Y
CPG	9. Full Implementation	N	N
	10. Corruption	Y	Y
LRs	11. Accept Compensation	Y	Y
	12. Refuse to move	N	N
NCs	13. Agree with CG	Y	N
	14. Protest	N	Y

support its fast-growing economy. Opposition still persists due to recent seasonal droughts in the Yangtze River and water pollution caused by the diversion.

3.4.2 Stability Analyses of the Separate Conflicts for the Three Routes

The conflict on each route can also be modelled separately. In calculating the equilibria for each route, the preference statements in the three separate conflicts are the same as those in the overall conflict, as described in Table 3.2. In the separate modelling, CG considers each route equally; thus, option dependence across different subconflicts does not exist. Note that only strong equilibria (SEs) in separate conflicts are analyzed.

The DMs, options, Status Quo (SQ) states, and equiibria for the three single conflicts are listed in Table 3.4. Note that the option numbers are consistent with those in Table 3.1, while the state numbers are only assigned within each single conflict. For example, state 3 in the single eastern and western conflicts refers to different states. All equilibria in Table 3.4 are SEs. There is one SE in each of the eastern and central routes. States 2 and 4 are two SEs on the western route.

3.5 Comparison of the Three Separate Conflicts with the Overall Dispute

The stability results in the single conflicts are compared with those in the hierarchical conflict. To make the results comparable, a combination of equilibrium states for the three

Table 3.4: The DMs, Options, SQ states, and Equilibria in the Three Single Conflicts

Single Eastern Conflict		SQ State	Equilibrium	
DMs	Options	State 3	State 3	
China Central Government (CG)	1. Fully initiate the eastern route	Y	Y	
	2. Modify the eastern route	N	N	
Eastern Provincial Governments (EPGs)	7. Agree with CG's initiative	N	N	
	8. Oppose CG	Y	Y	
Single Central Conflict		SQ State	Equilibrium	
DMs	Options	State 7	State 4	
China Central Government (CG)	3. Full Central Route	Y	N	
	4. Modify and Change Compensation	N	Y	
Central Provincial Governments (CPGs)	9. Full Implementation	N	N	
	10. Corruption	Y	Y	
Local Residents (LRs)	11. Accept Compensation	N	Y	
	12. Protest	Y	N	
Single Western Conflict		SQ State	Equilibria	
DMs	Options	State 3	State 2	State 4
Central Government	5. Fully Initiate the Western Project	Y	N	N
	6. Modify the Western Plan	N	Y	Y
Neighbouring Countries	13. Agree with CG on the Western Route	N	Y	N
	14. Protest and Pressure CG	Y	N	Y

routes are obtained, as shown in Fig. 3.3. Each block on the left contains the SE(s) with DMs' options in the corresponding single conflict. Option numbers for each route are displayed on the top of each block, followed by options in the SE(s) listed below. For example, in the Eastern block located at the top left in Fig. 3.3, the numbers "1. 2. 7. 8." refer to the options for CG and EPGs in the eastern conflict. Then "(Y N N Y)" means that options 1 and 8 are selected while options 2 and 7 are not taken. Therefore, two combined equilibria are generated on the right by including the selection of options for all DMs in the three blocks. The combined equilibria are identical to states 15 and 47 in the hierarchical conflict, which are SE and WE, respectively. State 15 is an SE in the hierarchical conflict which is composed of SEs in the single conflicts. Also constituted by SEs in the single conflicts, state 47 is a WE in the hierarchical conflict. State 15 is underlined in Fig. 3.3 to be distinguished from state 47.

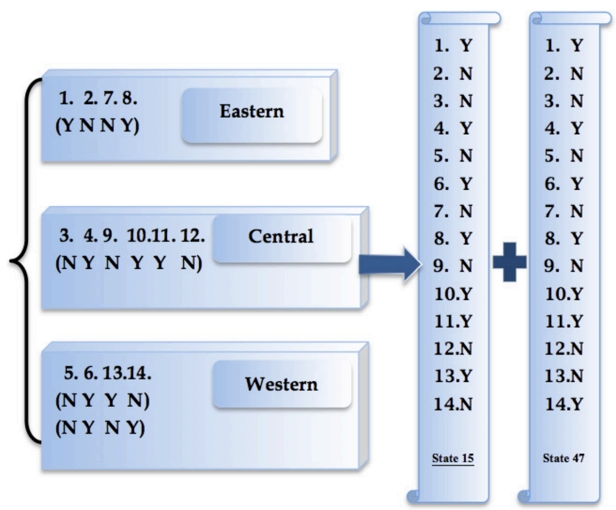

Figure 3.3: Combination of the equilibria for the single conflicts

CG's preferences can explain the differences in equilibria. For example, state 4 is SEQ for NCs in the western conflict, which means NCs fear the possible sanctions from CG. State 4 in the western conflict is contained in state 47 in the overall conflict. Because the western route is of least importance for CG in the hierarchical conflict, CG will not improve on the western route at the cost of a disimprovement on more important central route. Thus, in the overall conflict, state 4 as a component state in the western subconflict will not be sanctioned by CG, because CG overviews the conflicts at all locations. Thus, NCs in state 47 cannot be sanctioned by CG's UI. This can explain why state 47 is a WE in the hierarchical conflict consisting of SEs in the subconflicts.

3.6 Summary

The conflicts in SNWDP are analyzed in a hierarchical structure with CG's different priorities over the three routes. GMCR II is used to obtain the comprehensive analytical results. The SE (state 15) echoes the actual outcome and provides insights for DMs. This result is compared with the two SEs, states 15 and 47, in Fig. 3.3, which are a combination of the equilibria calculated in the single conflicts.

According to the findings, the modified western project may still result in opposition from NCs, if the western route is treated equally with the other routes by CG. However, CG could fully initiate the eastern project while easing the opposition to the central and western routes by having different priorities towards the three routes. This also implies that NCs could protest if the western route is simultaneously started with the other routes. This result explains why the western project should not be initiated until completion of projects on the other routes. It is rational for CG to first construct the original eastern project regardless of opposition from EPGs. Modifications to the central route can be implemented as the second step. The western project should be suspended. Therefore, CG may wish to strategically, rather than evenly, carry out a plan and distribute resources on the three routes, such as time, capital, and human resources, to improve its prospects of achieving a better outcome that will meet the desires of the various stakeholders as much as possible.

Other DMs can also learn from the analytical results. As the opposition cannot change CG's plan on the eastern route, EPGs may wish to change their preferences or options to increase their impact on the project. On the central route, LRs can expect a favourable outcome as CG may change the project and increase the compensation. The only concern comes from CPGs, who can still confiscate compensation without the knowledge of CG. For NCs, the situation on the western route is acceptable as long as CG does not initiate the western project immediately. As state 47 is GMR for NCs, their protests, although currently not highly preferred, can be a possible solution in the future. In general, a better outcome on the western route can be achieved if CG deals with all the conflicts hierarchically.

The hierarchical structure of water diversion conflicts in China has been demonstrated in this chapter. To study similar conflicts in the real world with a hierarchical structure, different types of hierarchical graph models are developed within the graph model paradigm in the following chapters. Applications of these models are also provided.

Chapter 4

Basic Hierarchical Graph Model with Lexicographic Preference

4.1 Introduction

A hierarchical graph model with the simplest structure, called the basic hierarchical graph model with lexicographic preference, is constructed in this chapter. The basic hierarchical graph model combines two component graph models. The theoretical framework of the combined model is constructed. A simple approach to determine preferences in the hierarchical graph model, called lexicographic preference, is developed in this chapter. Theorems are developed to relate stable states in the hierarchical model to stable states in the local graph models. These theorems can be employed to calculate stabilities in the hierarchical model by using stabilities in subconflicts. This methodology is applied to the water diversion conflict in China, the same example that was used in Chapter 3, but with only two subconflicts. The analytical results show how DMs can obtain strategic resolutions for the entire conflict. Chapter 4 is revised based on the journal article written by He et al (2013).

4.2 Formal Definitions

Recall that a graph model for a strategic conflict consists of a finite set of DMs, N; a finite set of feasible states, S; and, for each DM $i \in N$, a preference relation on S. As well, a directed arc connecting two states for each DM i, given as $A_i \subseteq S \times S$, is required. The nodes of each DM's graph are the states of the model, and possible moves for a given DM are represented by directed arcs.

4.2.1 Framework of Basic Hierarchical Graph Model

A hierarchical graph model for a strategic conflict contains smaller graph models, called local graph models. These smaller graph models feature one or more common DMs (CDMs) who appear in each of the local graphs. DMs appearing in only one local graph are called local DMs (LDMs).

The basic hierarchical graph model contains two smaller graph models with only one CDM. Besides CDM, each local graph has one LDM. The hierarchical graph model, G, has local graphs $G^{(1)}$ and $G^{(2)}$, with LDMs, LDM_1 and LDM_2, respectively. The interactions of the three DMs are shown in Fig. 4.1. CDM has interactions with both LDMs, separately with each in its local graph. No interaction is assumed between the two LDMs. States in G are specified using two entries, representing states in $G^{(1)}$ and $G^{(2)}$. CDM can move either

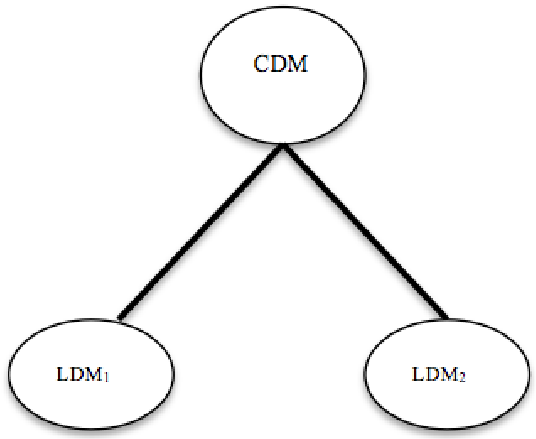

Figure 4.1: Interactions of DMs in the basic hierarchical graph model

locally within $G^{(1)}$ or $G^{(2)}$, or in both local graphs simultaneously. The basic hierarchical graph model is defined as follows.

Definition 4.1 (Basic Hierarchical Graph Model): There are three DMs, consisting of CDM, LDM_1, and LDM_2, and two local graph models,

$$G^{(1)} = \langle \{CDM, LDM_1\}, S^{(1)}, \{AC^{(1)}, AL^{(1)}\}, \{\succsim_C^{(1)}, \succsim_{L_1}^{(1)}\}\rangle$$

where $AC^{(1)} \subseteq S^{(1)} \times S^{(1)}$, $AL^{(1)} \subseteq S^{(1)} \times S^{(1)}$, and $\succsim_C^{(1)}$ and $\succsim_{L_1}^{(1)}$ are preference relations on $S^{(1)}$ for CDM and LDM_1 respectively.

$$G^{(2)} = \langle \{CDM, LDM_2\}, S^{(2)}, \{AC^{(2)}, AL^{(2)}\}, \{\succsim_C^{(2)}, \succsim_{L_2}^{(2)}\}\rangle$$

where $AC^{(2)} \subseteq S^{(2)} \times S^{(2)}$, $AL^{(2)} \subseteq S^{(2)} \times S^{(2)}$, and $\succsim_C^{(2)}$ and $\succsim_{L_2}^{(2)}$ are preference relations on $S^{(2)}$.

Then the graph model $G = \langle \{CDM, LDM_1, LDM_2\}, S = S^{(1)} \times S^{(2)}, \{AC, AL_1, AL_2\}, \{\succsim_C, \succsim_{L_1}, \succsim_{L_2}\}\rangle$ is a *basic hierarchical graph model based on* $G^{(1)}$ *and* $G^{(2)}$, provided that the following conditions hold:

(C_1) For $s_a^{(k)}, s_b^{(k)} \in S^{(k)}$ $(k = 1, 2)$ and $(s_a^{(1)}, s_a^{(2)}), (s_b^{(1)}, s_b^{(2)}) \in S$, $AC \subseteq S \times S$ is defined by $\Big((s_a^{(1)}, s_a^{(2)}), (s_b^{(1)}, s_b^{(2)})\Big) \in AC$ iff either $(s_a^{(1)}, s_b^{(1)}) \in AC^{(1)}$ and $s_a^{(2)} = s_b^{(2)}$, or $(s_a^{(2)}, s_b^{(2)}) \in AC^{(2)}$ and $s_a^{(1)} = s_b^{(1)}$.

(C_{21}) $AL_1 \subseteq S \times S$ is defined by $\Big((s_a^{(1)}, s_a^{(2)}), (s_b^{(1)}, s_b^{(2)})\Big) \in AL_1$ iff $(s_a^{(1)}, s_b^{(1)}) \in AL_1^{(1)}$ and $s_a^{(2)} = s_b^{(2)}$

(C_{22}) $AL_2 \subseteq S \times S$ is defined by $\Big((s_a^{(1)}, s_a^{(2)}), (s_b^{(1)}, s_b^{(2)})\Big) \in AL_2$ iff $(s_a^{(2)}, s_b^{(2)}) \in AL_2^{(2)}$ and $s_a^{(1)} = s_b^{(1)}$

(C_{31}) The preference relation $\succsim_{L_1}$ is defined on S by $(s_a^{(1)}, s_a^{(2)}) \succsim_{L_1} (s_b^{(1)}, s_b^{(2)})$ iff $s_a^{(1)} \succsim_{L_1}^{(1)} s_b^{(1)}$

(C_{32}) The preference relation $\succsim_{L_2}$ is defined on S by $(s_a^{(1)}, s_a^{(2)}) \succsim_{L_2} (s_b^{(1)}, s_b^{(2)})$ iff $s_a^{(2)} \succsim_{L_2}^{(2)} s_b^{(2)}$

(C_{41}) The preference relation $\succsim_C$ on S is related to the preference relation $\succsim_C^{(1)}$ on $S^{(1)}$ as follows:

$$\text{If } s_a^{(1)}, s_b^{(1)} \in S^{(1)} \text{ and } s_a^{(1)} \succsim_C^{(1)} s_b^{(1)}, \text{ then } (s_a^{(1)}, s_0^{(2)}) \succsim_C (s_b^{(1)}, s_0^{(2)}) \text{ for all } s_0^{(2)} \in S^{(2)}$$

(C_{42}) The preference relation $\succsim_C$ on S is related to the preference relation $\succsim_C^{(2)}$ on $S^{(2)}$ as follows:

$$\text{If } s_a^{(2)}, s_b^{(2)} \in S^{(2)} \text{ and } s_a^{(2)} \succsim_C^{(2)} s_b^{(2)}, \text{ then } (s_0^{(1)}, s_a^{(2)}) \succsim_C (s_0^{(1)}, s_b^{(2)}) \text{ for all } s_0^{(1)} \in S^{(1)}$$

Definition 4.1 describes the structure of the basic hierarchical graph model, G, which constitutes two local graph models ($G^{(1)}$ and $G^{(2)}$) sharing a common DM, called CDM. Other DMs are local decision makers (LDMs) and participate only in one local graph. The set of states in G is the Cartesian Product of the sets of states in the two local models.

CDM can participate in either or both of the two local models, while a LDM can move only within one local model.

Note: If $G^{(1)}$ and $G^{(2)}$ are given, then all components of G are determined by Definition 4.1 except for CDM's preference relation $\succsim_C$. This relation must satisfy (C_{41}) and (C_{42}); these conditions restrict $\succsim_C$ but do not determine it. In general, basic hierarchical graph models based on $G^{(1)}$ and $G^{(2)}$ differ only in $\succsim_C$.

In Definition 4.1, $\{CDM, LDM_1\}$ is the set of DMs and $S^{(1)}$ is the set of states in $G^{(1)}$. $AC^{(1)}$ is the set of moves in one step controlled by CDM in $G^{(1)}$, while $AL^{(1)}$ is the set of moves under the control of LDM_1 in $G^{(1)}$. Similarly, AC represents the set of moves by CDM in G and AL_1 the set of moves by LDM_1 in G. The preference relations in G for CDM and LDM_1 are expressed by $\succsim_C$ and $\succsim_{L_1}$, respectively.

In particular, CDM's preference between two states in the basic hierarchical model is determined by C_{41} and C_{42} if, and only if, the states are identical in one local graph model. Thus, more than one basic hierarchical graph model based on $G^{(1)}$ and $G^{(2)}$ can be found satisfying the conditions listed above.

4.2.2 Lexicographic Preference

An illustration of CDM's preferences in a basic hierarchical graph model based on $G^{(1)}$ and $G^{(2)}$ is given next. Suppose that there are three states in $G^{(1)}$: $S^{(1)} = \{a, b, c\}$ and two states in $G^{(2)}$: $S^{(2)} = \{A, B\}$, and write $S = S^{(1)} \times S^{(2)} = \{aA, aB, bA, bB, cA, cB\}$. The preference relations of CDM in $G^{(1)}$ and $G^{(2)}$ are defined as $\succsim_C^{(1)} = a \succ b \succ c$ and $\succsim_C^{(2)} = A \succ B$. Accordingly, the preference relation of CDM in G must satisfy

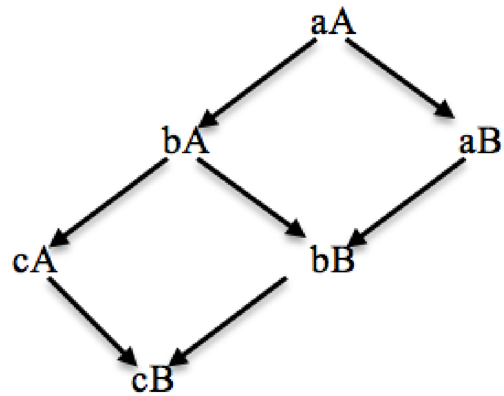

Figure 4.2: Preference tree of a basic hierarchical graph model for the CDM

(1) $aA \succ bA \succ cA$

(2) $aB \succ bB \succ cB$

(3) $aA \succ aB$

(4) $bA \succ bB$

(5) $cA \succ cB$

The above five relations are generated according to conditions C_{41} and C_{42}. A preference tree that describes CDM's preferences on all states in G is shown in Fig. 4.2. A directed arrow points to a state that is less preferred than the starting state by CDM. Preferences between states without connection are not determined.

According to Fig. 4.2, there are 5 possible rankings of states in G:

(i) $aA \succ bA \succ cA \succ aB \succ bB \succ cB$

(ii) $aA \succ bA \succ aB \succ cA \succ bB \succ cB$

(iii) $aA \succ bA \succ aB \succ bB \succ cA \succ cB$

(iv) $aA \succ aB \succ bA \succ cA \succ bB \succ cB$

(v) $aA \succ aB \succ bA \succ bB \succ cA \succ cB$

Note that state aA must be CDM's most preferred state and state cB the least preferred. The second-best state for CDM can be either bA or aB. Ranking (i) demonstrates a possible ranking in which $cA \succ aB$, but $cA \prec aB$ in rankings (ii) to (v).

To further determine preference relations on all states in G for CDM, additional rules for comparing states in G can be introduced. A possible way to rank all states in G is called Lexicographic Ordering, which depends on the importance of local graphs.

Definition 4.2 (Lexicographic Order): Given two sets A and B with a_1, $a_2 \in A$ and b_1, $b_2 \in B$, the lexicographical order on the Cartesian product $A \times B$ is defined as $(a_1, b_1) < (a_2, b_2)$ if and only if

(1) $a_1 < a_2$

(2) $a_1 = a_2$ and $b_1 < b_2$

The lexicographic order provides an efficient way of comparing two states in G. To establish a general rule for comparison, pairwise importance between two local graphs is introduced. For two local graphs $G^{(1)}$ and $G^{(2)}$, the CDM's attitude on which local graph is more important is called local graph importance. CDM always prefers to achieve a better outcome in the more important local graph, regardless of the situation in the other local graph. Between two combined states with equally preferred components in the more important local graph, the CDM prefers the combined state that has the more preferred component in the less important local graph. Thus, $G^{(1)} > G^{(2)}$ and $G^{(1)} < G^{(2)}$ respectively denote that the CDM considers $G^{(1)}$ more important and less important than

$G^{(2)}$. The pairwise importance between the two local graphs is defined as follows.

Definition 4.3 (Pairwise Importance): Let $G^{(1)}$ and $G^{(2)}$ denote two local graphs in G. Then $G^{(1)} > G^{(2)}$ denotes that CDM considers $G^{(1)}$ more important than $G^{(2)}$, and $G^{(1)} < G^{(2)}$ indicates the reverse.

Based on the pairwise importance between the two local graphs, preference relations in G for CDM are completely determined. For two states in G, $s_1 = (s_1^{(1)}, s_1^{(2)}) \in S$ and $s_2 = (s_2^{(1)}, s_2^{(2)}) \in S$, CDM's preferences between s_1 and s_2 are defined as follows:

If $G^{(1)} > G^{(2)}$ for CDM,

(a) $s_1 \succ_C s_2$, if either $s_1^{(1)} \succ_C^{(1)} s_2^{(1)}$ or $(s_1^{(1)} \sim_C^{(1)} s_2^{(1)}$ and $s_1^{(2)} \succ_C^{(2)} s_2^{(2)})$;

(b) $s_1 \sim_C s_2$, if $s_1^{(1)} \sim_C^{(1)} s_2^{(1)}$ and $s_1^{(2)} \sim_C^{(2)} s_2^{(2)}$; and

(c) $s_1 \prec_C s_2$, if either $s_1^{(1)} \prec_C^{(1)} s_2^{(1)}$ or $(s_1^{(1)} \sim_C^{(1)} s_2^{(1)}$ and $s_1^{(2)} \prec_C^{(2)} s_2^{(2)})$.

CDM's preference in G is similar if $G^{(1)} < G^{(2)}$.

Pairwise importance acts as extra information to determine the preference ranking of all states in the basic hierarchical model. Among the five possible rankings of states demonstrated from (i) to (v), (i) is concluded by assuming $G^{(1)} < G^{(2)}$ and (v) by considering $G^{(1)} > G^{(2)}$. If $G^{(1)} < G^{(2)}$, then $cA \succ_C aB$ due to $A \succ_C^{(2)} B$ even though $c \prec_C^{(1)} a$. Therefore, a strict local graph importance relation corresponds to a unique ranking of states. The remaining three rankings from (ii) to (iv) suggest approximately equal importance of the two local graphs for CDM.

4.3 Moves and Improvements

Movements and preferences of DMs are defined within the framework of the hierarchical graph model. Only CDM can participate in both local graph models, while LDMs' moves can only affect their own subconflict. The preferences of LDMs in their local graphs, $\succsim_{L_1}^{(1)}$ and $\succsim_{L_2}^{(2)}$, are essentially the same as their preferences in the hierarchical graph model, $\succsim_{L_1}$ and $\succsim_{L_2}$. However, CDM's preference relations between states in the hierarchical graph are constrained by conditions C_{41} and C_{42} shown in Definition 4.1, and completely determined if there is an importance relation between two local graphs.

As introduced in Chapter 2, the reachable list is the set of all possible moves for a DM and does not depend on its preferences. For CDM, possible moves from $s = (s^{(1)}, s^{(2)}) \in S$ can occur from either $s^{(1)}$ or $s^{(2)}$. In comparison, LDM_1 can only move within its own local graph, i.e. from $s^{(1)} \in S^{(1)}$.

Definition 4.4 (Reachable List): Suppose $s_1 = (s_1^{(1)}, s_1^{(2)})$, $s_2 = (s_2^{(1)}, s_2^{(2)}) \in S$. The reachable list for CDM from state s_1 is defined as

$$R_C(s_1) = \{s_2 \in S : (s_1, s_2) \in AC\}.$$

Note that $(s_1, s_2) \in AC$ if and only if either $(s_1^{(1)}, s_2^{(1)}) \in AC^{(1)}$ and $s_1^{(2)} = s_2^{(2)}$ or $(s_1^{(2)}, s_2^{(2)}) \in AC^{(2)}$ and $s_1^{(1)} = s_2^{(1)}$ or $(s_1^{(1)}, s_2^{(1)}) \in AC^{(1)}$ and $(s_1^{(2)}, s_2^{(2)}) \in AC^{(2)}$. The reachable list for LDM_1 is defined as

$$R_{L_1}(s_1) = \{s_2 \in S : (s_1, s_2) \in AL_1\}$$

Note that $(s_1, s_2) \in AL_1$ if and only if $(s_1^{(1)}, s_2^{(1)}) \in AL_1^{(1)}$ and $s_1^{(2)} = s_2^{(2)}$. The reachable list for LDM_2 is defined similarly.

Note that $(s_1, s_2) \in AL_2$ if and only if $(s_1^{(2)}, s_2^{(2)}) \in AL_2^{(2)}$ and $s_1^{(1)} = s_2^{(1)}$.

4.4 Interrelationships of Stabilities between Basic Hierarchical Graph Model and Local Graph Models

Relative relationships between stabilities in the basic hierarchical graph model and stabilities in the local graph model are now investigated. Theorems following the definitions for each stability type are proposed to reveal the connection between the overall stability and the individual stabilities in smaller graphs. Four stabilities, Nash, sequential stability (SEQ), general metarationality (GMR), and symmetric metarationality (SMR), are discussed. Representative proofs of some theorems can be seen in Appendices.

The four types of stabilities reflect different foresights for DMs (Kilgour and Hipel, 2010). Nash rationality has the lowest foresight, as the focal DM perceives only displacements of one step. It is the strongest stability. Sequential stability (SEQ) and general metarationality (GMR) have a foresight of two steps. In sequential stability (SEQ), the focal DM considers only unilateral improvements by other DMs as responses to his initial move.

In general metarationality (GMR), the focal DM expects that other DMs will sanction its UIs, even if the only available sanctions are disimprovements. Compared with GMR, symmetric metarationality (SMR) extends the foresight of the focal DM one step further. The focal DM takes into account not only sanctions from other DMs, but also its own counteractions.

In a graph model, a state is an equilibrium if it is stable for all DMs. For example, an SEQ equilibriumm is a state that is SEQ stable for all DMs. Equilibria in a graph model suggest possible outcomes of the conflict or resolutions for DMs as courses of action to follow.

The interrelationship of these four stabilities is shown in Fig. 4.3. For example, if a combined state is Nash stable, it is also SEQ, GMR, and SMR. A state with any other form of stability is also GMR (Kilgour and Hipel, 2010).

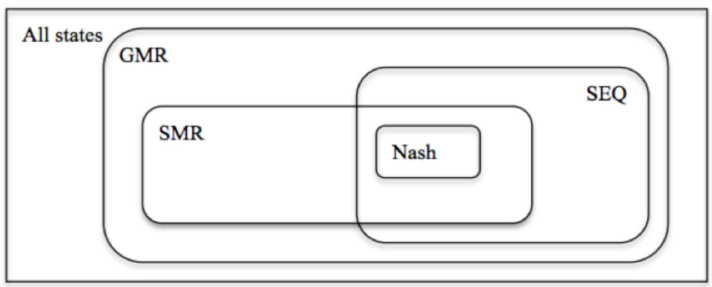

Figure 4.3: The interrelationship of four stabilities

4.4.1 Nash (R)

The standard definition for Nash stability in the graph model is given by Nash (1951):

Definition 4.5: Let $i \in N$ and $s_1 \in S$. State s_1 is Nash stable or rational (R) for DM i if and only if $R_i^+(s_1) = \phi$.

Thus, a state $s = (s^{(1)}, s^{(2)})$ in the hierarchical model is Nash stable for a DM if and only if no UI can be found for the DM from this state. For CDM, this means no UI can be found for CDM from either $s^{(1)}$ in $G^{(1)}$ or $s^{(2)}$ in $G^{(2)}$, while for LDM_k there can be no UI within $G^{(k)}$ from $s^{(k)}$ $(k = 1, 2)$.

Theorems to determine the Nash stable states are discussed for CDM and LDMs separately. To determine Nash stability for a DM, every state must be examined for whether there is a UI. A UM for CDM in the hierarchical model may lead to a less preferred state if this UM includes a local UM to a less preferred state within one local graph. For an LDM, an analysis is focused only on his or her own local graph. Note that R_C^+ represents the list of UIs for the CDM in the hierarchical model and $R_C^{(1)+}$ the list of UIs in $G^{(1)}$. $R_{L_1}^+$ denotes the list of UIs for LDM_1 and R_L^+ the list of UIs by the coalition of LDM_1 and LDM_2.

Theorem 4.1 (Nash for CDM in G): If $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are Nash stable for CDM in $G^{(1)}$ and $G^{(2)}$, respectively, then $(s^{(1)}, s^{(2)})$ is Nash stable for CDM in G. $\square$

Theorem 4.2 (Nash for LDMs in G): If $s^{(1)} \in S^{(1)}$ is Nash stable for LDM_1 in $G^{(1)}$, then $(s^{(1)}, s^{(2)})$ is Nash stable for LDM_1 in G for any $s^{(2)} \in S^{(2)}$. $\square$

All Nash stable states for LDM_2 can be determined analogously by Theorem 4.2. Theorems 4.1 and 4.2 can be used for determining Nash stability for all DMs in G if the Nash stable states in each local graph are known.

4.4.2 Sequential Stability (SEQ)

Sequential Stability is analyzed for the CDM and LDMs separately. For CDM, a state in the basic hierarchical model is sequentially stable if and only if sanctions by LDMs can be found for each possible UI for CDM. Theorems are proposed to identify the SEQ states in the basic hierarchical model. A combined state is sequentially stable if its component states are both sequentially stable in the local graph, as demonstrated in Theorem 4.3. If the combined state is sequentially stable for CDM, the stabilities of its components are presented in Theorem 4.4 for $G^{(1)} > G^{(2)}$. The SEQ states for LDMs are determined in Theorem 4.6. Note that the importance relations between two local graphs for CDM are strict, and are denoted as $G^{(1)} > G^{(2)}$ and $G^{(1)} < G^{(2)}$.

Definition 4.6: Recall that $R_C^+(s)$ is the set of UIs from state $s = (s^{(1)}, s^{(2)}) \in S$ for CDM and that $R_L^+(q)$ is the set of UIs from state $q \in S$ for the LDMs as a coalition. State s is sequentially stable for CDM if and only if, for every state $q \in R_C^+(s)$, there exists at least one state $r \in R_L^+(q)$ such that $r \precsim_C s$.

Definition 4.7: Recall that $R_{L_1}^+(s)$ is the set of UIs for LDM_1 from state $s \in S$ and that $R_C^+(q)$ is the set of UIs from state $q \in S$ for the CDM. The state s is SEQ for LDM_1 if and only if for every state $q \in R_{L_1}^+(s)$, there exists at least one state $r \in R_C^+(q)$ such that $r \precsim_{L_1} s$.

Theorem 4.3 (SEQ for CDM): Suppose that $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are sequentially stable for CDM in $G^{(1)}$ and $G^{(2)}$, respectively. Then $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G.

Theorem 4.3 indicates that a combined state in G is SEQ for CDM if both of its component states are SEQ in the corresponding local graphs. This theorem does not require a particular importance relation between the two local graphs. In Theorem 4.3, sequential stability for both $s^{(1)}$ and $s^{(2)}$ in the component model is a sufficient condition to imply the sequential stability for the combined state $(s^{(1)}, s^{(2)})$.Reversely, if $(s^{(1)}, s^{(2)})$ is sequentially stable for CDM, the stabilities of its components are investigated in Theorems 4.4 and 4.5. Strict sequential stability for CDM, which occurs in the two theorems, is defined in Remark 1. At a strict SEQ state, CDM is only sanctioned by less preferred states.

Remark 4.1 (Strict SEQ): Recall that $R_C^+(s)$ is the set of UIs from state $s = (s^{(1)}, s^{(2)}) \in S$ for CDM and that $R_L^+(q)$ is the set of UIs from state $q \in S$ for the LDMs as a coalition. State s is strictly sequentially stable for CDM iff, for every state $q \in R_C^+(s)$, there exists at least one state $r \in R_L^+(q)$ such that $r \prec_C s$.

Theorem 4.4: Assume $G^{(1)} > G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G, iff either

(1) both $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are SEQ for CDM, or

(2) $s^{(1)} \in S^{(1)}$ is strictly sequentially stable for CDM in $G^{(1)}$ and, for all $q^{(1)} \in R_C^{(1)+,=}(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{(1)+}(q^{(1)})$ such that $r^{(1)} \prec_C^{(1)} s^{(1)}$, where $R_C^{(1)+,=}(s^{(1)})$ is the set of UMs for CDM from $s^{(1)}$ which are no less preferred to $s^{(1)}$. $\square$

A small example makes situation (2) of Theorem 4.4 easy to understand. Suppose the sets of states in $G^{(1)}$ and $G^{(2)}$ are written as $S^{(1)} = \{1, 2, 3, 4\}$ and $S^{(2)} = \{5, 6, 7, 8\}$, respectively. Let $R_C^{(1)+}(1) = 3$; $R_{L_1}^{(1)+}(3) = 2$ with $2 \prec_C^{(1)} 1$; $R_{L_1}^{(1)+}(1) = 4$ with $4 \prec_C^{(1)} 1$, and $R_C^{(2)+}(5) = 6$; $R_{L_2}^{(2)+}(6) = 7$ with $7 \succ_C^{(2)} 5$. We investigate whether state $(1, 5) \in S$ is SEQ for CDM in G, where $S = S^{(1)} \times S^{(2)}$. According to situation (2) of Theorem 4.4, $s^{(1)} = 1$; $s^{(2)} = 5$, then, $q^{(1)}$ can be state 1 or 5.

It can be concluded that state 1 is SEQ for CDM in $G^{(1)}$, while state 5 is not SEQ for CDM in $G^{(2)}$. All UIs for CDM from state (1, 5) are

(a) (3, 5)

(b) (1, 6)

(c) (3, 6)

State (1, 5) is SEQ for CDM in G iff at least one sanction can be found on each of (a), (b), and (c). For (a), there exists $R^+_{L_1}(3,5) = (2,5)$, such that $(2,5) \succ_C (1,5)$. For (b), there exists $R^+_{L_1}(1,6) = (4,6)$, such that $(4,6) \succ_C (1,5)$ because $G^{(1)} > G^{(2)}$. For (c), there exists $R^+_{L_1}(3,6) = (2,6)$, such that $(2,6) \succ_C (1,5)$ because $G^{(1)} > G^{(2)}$. Then, at least a sanction can be found for each of (a), (b), and (c). Thus, one can conclude state (1, 5) is SEQ for CDM in G.

Similarly, Theorem 4.5 is proposed by assuming $G^{(1)} < G^{(2)}$.

Theorem 4.5: Assume $G^{(1)} < G^{(2)}$, if $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G, iff either

(1) both $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are SEQ for CDM, or

(2) $s^{(2)} \in S^{(2)}$ is strictly SEQ for the CDM in $G^{(2)}$ and for all $q^{(2)} \in R_C^{(2)+,=}(s^{(2)})$ and $q^{(2)} = s^{(2)}$, there exists $r^{(2)} \in R_{L_2}^{(2)+}(q^{(2)})$ such that $r^{(2)} \prec_C^{(2)} s^{(2)}$ $\square$

The proof of this theorem is analogous to Theorem 4.4. According to Theorems 4.4 and 4.5, $(s^{(1)}, s^{(2)})$ can be sequentially stable for CDM in G if the component state in the more important local graph is sequentially stable for CDM. If both component states are sequentially stable in the corresponding local graph for CDM, the combined state is SEQ, as indicated in Theorem 4.3. If the component state in the less important local graph is not sequentially stable, $(s^{(1)}, s^{(2)})$ is SEQ for CDM provided the conditions in situation (2) of Theorem 4.4 and 4.5 are satisfied.

The sequential stability for LDMs in the hierarchical graph can be determined by Theorem 4.6.

Theorem 4.6 (SEQ for LDM_1 in G): A state $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for LDM_1 in G iff either

(1) $s^{(1)} \in S^{(1)}$ is sequentially stable for LDM_1 in $G^{(1)}$, or

(2) for every $q^{(1)} \in R_{L_1}^{(1)+}(s^{(1)})$, there exists $(r^{(1)}, r^{(2)}) \in R_{\{C,L_2\}}^{+}(q^{(1)}, s^{(2)})$, such that $r^{(1)} \precsim_{L_1}^{(1)} s^{(1)}$.

□

The SEQ for LDM_2 is analogous.

4.4.3 General Metarationality (GMR)

General metarationality (GMR) is a weaker stability than sequential stability (SEQ) in that a focal DM considers not only UIs by the opponents as possible sanctions but also their UMs. The focal DM is afraid of being sanctioned by other DMs' UMs even if these UMs are less preferred for them. Similar to sequential stability, general metarational states in G for the CDM and LDMs are identified by Theorems 4.7, 4.8, and 4.9.

Definition 4.8: Recall that $R_C^+(s)$ is the set of UIs from state $s = (s^{(1)}, s^{(2)}) \in S$ for CDM and $R_L(q)$ is the set of UMs from state $q \in S$ for LDMs as a coalition. State s is GMR for CDM if and only if, for every state $q \in R_C^+(s)$, there exists at least one state $r \in R_L(q)$ such that $r \precsim_C s$.

Definition 4.9: Recall that $R_{L_1}^+(s)$ is the set of UIs for LDM_1 from state $s \in S$ and $R_C(q)$ is the set of UMs from state $q \in S$ for the CDM. State s is GMR for LDM_1 if and only if for every state $q \in R_{L_1}^+(s)$, there exists at least one state $r \in R_C(q)$ such that $r \precsim_{L_1} s$.

The GMR stability for LDM_2 can be analogously defined.

Similar to Theorem 4.3, GMR can be concluded for CDM in the combined state if both component states are GMR in the local graphs, as shown in Theorem 4.7.

Theorem 4.7 (GMR for CDM): Suppose that $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are GMR for CDM in $G^{(1)}$ and $G^{(2)}$, respectively. Then $(s^{(1)}, s^{(2)})$ is GMR for CDM in G. □

Besides the case mentioned in Theorem 4.7, $(s^{(1)}, s^{(2)})$ can be GMR in other possibilities. The stabilities of component states in a combined GMR state for CDM are discussed in Theorems 4.8 and 4.9. Strict GMR that appears in the two theorems is defined in Remark 4.2.

Remark 4.2 (Strict GMR): Recall that $R_C^+(s)$ is the set of UIs from state $s = (s^{(1)}, s^{(2)}) \in S$ for CDM and $R_L(q)$ is the set of UMs from state $q \in S$ for LDMs as a coalition. State s is GMR for CDM if and only if, for every state $q \in R_C^+(s)$, there exists at least one state $r \in R_L(q)$ such that $r \prec_C s$.

Theorem 4.8 Assume $G^{(1)} > G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is GMR for CDM in G iff either

(1) both $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are GMR for CDM, or

(2) $s^{(1)} \in S^{(1)}$ is strictly GMR for the CDM in $G^{(1)}$ and, for all $q^{(1)} \in R_C^{(1)+,=}(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $r^{(1)} \prec_C^{(1)} s^{(1)}$. □

Theorem 4.9 is similarly proposed by assuming $G^{(1)} < G^{(2)}$.

Theorem 4.9 Assume $G^{(1)} < G^{(2)}$, if $(s^{(1)}, s^{(2)})$ is GMR for CDM in G, iff either

(1) both $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are GMR for CDM, or

(2) $s^{(2)} \in S^{(2)}$ is strictly GMR for the CDM in $G^{(2)}$ and for all $q^{(2)} \in R_C^{(2)+,=}(s^{(2)})$ and $q^{(2)} = s^{(2)}$, there exists $r^{(2)} \in R_{L_2}^{(2)}(q^{(2)})$ such that $r^{(2)} \prec_C^{(2)} s^{(2)}$ □

The proof of this theorem is analogous to Theorem 4.8.

Similar to sequential stability, $(s^{(1)}, s^{(2)})$ can be GMR for CDM in G if the component state in the more important local graph is GMR. According to case (2) in Theorems 4.7 and

4.8, additional sanctions should exist so that $(s^{(1)}, s^{(2)})$ is GMR for CDM if the component state in the less important local graph is not GMR for CDM.

The general metarationality for LDMs can be examined by Theorem 4.10 indicated below. Only component states in the LDM's own local graph need to be investigated.

Theorem 4.10 (GMR for LDM_1 in G): Suppose $s^{(1)} \in S^{(1)}$ is a state in $G^{(1)}$. A state $(s^{(1)}, s^{(2)}) \in S$ is GMR for LDM_1 in G if $s^{(1)}$ is GMR for LDM_1 in $G^{(1)}$ and $s^{(2)} \in S^{(2)}$ is any state in $G^{(2)}$. $\square$

The general metarationality for LDM_2 can be determined analogously.

According to Theorem 4.10, general metarationality for a given LDM can be identified if the component state is GMR for the LDM in its own local graph.

4.4.4 Symmetric Metarationality (SMR)

Symmetric metarationality is a more restrictive stability reflecting the vision of the focal DM one step further than general metarationality. In the basic hierarchical model, the combined states are SMR for CDM if every type of his or her UIs is sanctioned by an LDM's UM and also by CDM's subsequent UM. $(s^{(1)}, s^{(2)})$ is investigated for symmetric metarationality for CDM based on the relationship between SMR and GMR, as depicted in Fig. 4.3.

Definition 4.10 Recall that $R_C^+(s)$ is the set of UIs from state $s = (s^{(1)}, s^{(2)}) \in S$ for CDM and $R_L(q)$ is the set of UMs from state $q \in S$ for LDMs as a coalition. State s is SMR for CDM if and only if, for every state $q \in R_C^+(s)$, there exists at least $r \in R_L(q)$ such that $r \precsim_C s$ and $t \precsim_C s$ for all $t \in R_C(r)$.

Definition 4.11 Recall that $R^+_{L_1}(s)$ is the set of UIs for LDM_1 from state $s \in S$ and $R_C(q)$ is the set of UMs from state $q \in S$ for CDM. State s is SMR for LDM_1 if and only if for every state $q \in R^+_{L_1}(s)$, there exists at least $r \in R_C(q)$ such that $r \precsim_{L_1} s$ and $t \precsim_{L_1} s$ for all $t \in R_{L_1}(r)$.

If both component states are SMR for CDM in the corresponding local graph, the symmetric metarationality for CDM is discussed in Theorem 4.11.

Theorem 4.11 (SMR for CDM in G): Suppose that $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are states in $G^{(1)}$ and $G^{(2)}$, respectively. Assume that $s^{(1)}$ is SMR for CDM in $G^{(1)}$ and $s^{(2)}$ is SMR for CDM in $G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G. $\square$

One can determine whether $(s^{(1)}, s^{(2)})$ is SMR for CDM in G using Theorem 4.12 for $G^{(1)} > G^{(2)}$ and Theorem 4.13 for $G^{(1)} < G^{(2)}$. Strict SMR that occurs in the two theorems is defined in Remark 4.3.

Remark 4.3 (Strict SMR): Recall that $R^+_C(s)$ is the set of UIs from state $s = (s^{(1)}, s^{(2)}) \in S$ for CDM and $R_L(q)$ is the set of UMs from state $q \in S$ for LDMs as a coalition. State s is SMR for CDM iff, for every state $q \in R^+_C(s)$, there exist at least $r \in R_L(q)$ such that $r \prec_C s$ and $t \prec_C s$ for all $t \in R_C(q)$.

Theorem 4.12: Assume $G^{(1)} > G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G, iff either

(1) $s^{(1)}$ is SMR for CDM in $G^{(1)}$ and $s^{(2)}$ is SMR for CDM in $G^{(2)}$, or

(2) $s^{(1)}$ is strict SMR for CDM in $G^{(1)}$ and, for all for all $q^{(1)} \in R^{(1)+,=}_C(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exist $r^{(1)} \in R^{(1)}_{L_1}(q^{(1)})$ such that $r^{(1)} \prec^{(1)}_C s^{(1)}$ and $t^{(1)} \prec^{(1)}_C s^{(1)}$ for all $t^{(1)} \in R^{(1)}_C(r^{(1)})$. $\square$

According to Theorem 4.12, $(s^{(1)}, s^{(2)})$ can be SMR for CDM in G if it contains a component state which is SMR in the more important local graph. The additional condition is required for $s^{(1)}$ so that $(s^{(1)}, s^{(2)})$ is SMR. Similarly, if $(s^{(1)}, s^{(2)})$ is SMR for CDM and assume $G^{(1)} < G^{(2)}$, the stabilities of the component states are discussed in Theorem 4.13.

Theorem 4.13: Assume $G^{(1)} < G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G, iff either

(1) $s^{(1)}$ is SMR for CDM in $G^{(1)}$ and $s^{(2)}$ is SMR for CDM in $G^{(2)}$, or

(2) $s^{(2)}$ is strict SMR for CDM in $G^{(2)}$ and, for all $q^{(2)} \in R_C^{(2)+,=}(s^{(2)})$ and $q^{(2)} = s^{(2)}$, there exists $r^{(2)} \in R_{L_2}^{(2)}(q^{(2)})$ such that $r^{(2)} \prec_C^{(2)} s^{(2)}$ and $t^{(2)} \prec_C^{(2)} s^{(2)}$ for all $t^{(2)} \in R_C^{(2)}(r^{(2)})$.
□

The SMR states for a LDM can be examined by Theorem 4.14 indicated below. Only states in the local graph where the LDM participates need to be investigated.

Theorem 4.14 (SMR for LDM_1 in G): Suppose $s^{(1)} \in S^{(1)}$ is a state in $G^{(1)}$. A state $(s^{(1)}, s^{(2)}) \in S$ is SMR for LDM_1 in G if $s^{(1)}$ is SMR for LDM_1 in $G^{(1)}$ and $s^{(2)} \in S^{(2)}$ is any state in $G^{(2)}$. □

The symmetric metarationality for LDM_2 can be determined analogously.

According to Theorem 4.14, symmetric metarationality for a given LDM can be identified if the component state is SMR for the LDM in its own local graph.

4.5 Water Diversion Conflicts in China

The basic hierarchical graph model with lexicographic preference is applied to the water diversion conflicts in China, the same example that was investigated in Chapter 3. In

Chapter 3, the subconflicts at three locations were investigated. In this section, the water diversion conflicts have been simplified to contain two subconflicts.

As the eastern project is already complete, only the subconflicts at the central and the western locations are discussed in this section. Moreover, the example in this section is analyzed using the basic hierarchical graph model developed in this Chapter.

The preferences in the hierarchical conflict and subconflicts in Chapter 3 have been determined using the option prioritization method. In comparison, the preferences in the hierarchical conflict are not required in calculating the stability results. To calculate these stability results, only the preferences in subconflicts and the relative importance of local graphs are required. Despite different assumptions and modelling approaches, some comparisons can still be made between the case study in Chapter 3 and this section.

As the eastern route is already complete, the conflicts along the remaining central and western routes need to be properly handled before the implementation of the projects. Thus, the two subconflicts are modelled in this case study. At each location, China Central Government (CG) disputes with one DM: local residents (LRs) in the central subconflict, and neighbouring countries (NCs) in the western subconflict.

At the central location, local residents (LRs) are forced to relocate to make way for the construction, which ignites their opposition against CG. The western route aims at transferring water from the upper Yangtze River, situated on the Tibet Plateau, to the Yellow River Basin. Neighboring countries (NCs) such as India and Bangladesh are protesting against this plan since their water usage will be greatly affected once the project is constructed.

4.5.1 Conflict Modeling

In this hierarchical conflict, disputes on each route are represented by a local graph. CG is the CDM in the overall conflict, who has one option in each local graph: to fully initiate the central project and to resume the western project. LRs and NCs are LDMs on the central and western route, respectively. LRs and NCs could either agree or disagree. Note that the disagreement is expressed by not selecting the corresponding option, which is opposite to "agree".

Table 4.1: DMs, Their Options, and States in the Central Conflict

DM	Option				
CG	1) Full Central	Y	Y	N	N
LRs	3) Agree	Y	N	Y	N
		1	2	3	4

Table 4.2: DMs, Their Options, and States in the Western Conflict

DM	Option				
CG	2) Resume Western	Y	Y	N	N
NCs	4) Consent	Y	N	Y	N
		5	6	7	8

These two local graphs in option form are illustrated in Tables 4.1 and 4.2. The option form of the overall conflict is shown in Table 4.3. Note that the states in the overall conflict

Table 4.3: DMs, Their Options, and States in the Overall Conflict

CG	1) Full Central	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
	2) Resume Western	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
LRs	3) Agree	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
NCs	4) Consent	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
	State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		(4,8)	(2,8)	(4,6)	(2,6)	(3,8)	(1,8)	(3,6)	(1,6)	(4,7)	(2,7)	(4,5)	(2,5)	(3,7)	(1,7)	(3,5)	(1,5)

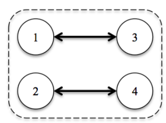

Figure 4.4: UMs for CG in the central conflict

can also be expressed in two dimensions. For example, state 1 in the overall conflict can also be represented as state (4, 8), in which the first entry is state 4 in the central conflict and the second entry is state 8 in the western conflict.

The unilateral moves for CG in the two individual conflicts and the overall conflict are shown in Figs. 4.4, 4.5, and 4.6. In the central conflict, CG can move between state 1 and 3 by changing its option selection on the central route. Similarly, it can also move between

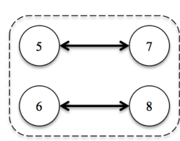

Figure 4.5: UMs for CG in the western conflict

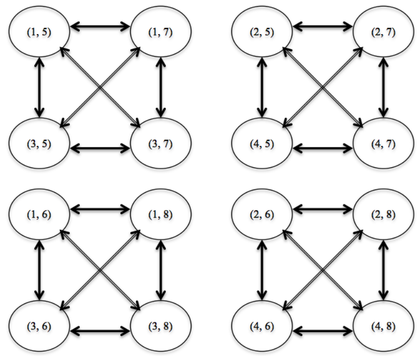

Figure 4.6: UMs for CG in the overall conflict

state 2 and 4. The UMs for CG in the overall conflict can take place on both entries of a state. For example, in Fig. 4.6, CG can either move between state (1, 5) and (1, 7) by switching its option on the western route, or between (1, 5) and (3, 5) on the central route. In addition, it can change options on both central and western routes, resulting in moves between (1, 5) and (3, 7). States in the overall conflict are connected by CG's UMs, as depicted in Fig. 4.7. Note that these UMs are reversible.

The stability analyses are carried out in each local graph separately. The preference for each DM is given by the ranking of states from the most preferred on the left to the least preferred on the right. In the central conflict, states are ranked as "$1 \succ 2 \succ 3 \succ 4$" for CG. The preferences for LRs are "$3 \succ 4 \succ 2 \succ 1$". In the western conflict, the preferences for CG and NCs are "$5 \succ 7 \succ 6 \succ 8$" and "$7 \succ 8 \succ 6 \succ 5$", respectively. CG's preferences indicate that it favours the full central plan and reaching an agreement with LRs on the central route. In the western disputes, CG wishes to see the consent from NCs. LRs and NCs have the same preferences in each local graph. They would not stop protesting until CG changes the original plan on each route. The results of four types of stabilities, Nash (R), sequential stability (SEQ), general metarationality (GMR), and symmetric metarationality (SMR) in the central and western conflicts are presented in Tables 4.4 and 4.5, respectively. According to the interrelationship among the four stabilities in Fig. 4.3, a Nash rational state is also sequentially stable. For each DM, the SEQ states are marked in bold if they are also Nash stable. For example in Table 4.4, states 2 and 3 are Nash stable for LRs. They are automatically sequential stable and are thus written in bold. Similarly, some GMR and SMR states are also marked in bold in Tables 4.4 and 4.5.

Table 4.4: Stability Results in the Central Conflict

	Nash	SEQ	GMR	SMR
CG	1, 2	**1, 2**	**1, 2**	**1, 2**
LRs	2, 3	**2**, **3**, 4	**2, 3, 4**	**2, 3, 4**

Table 4.5: Stability Results in the Western Conflict

	Nash	SEQ	GMR	SMR
CG	5, 6	**5**, **6**, 7	**5, 6, 7**	**5, 6, 7**
NCs	6, 7	**6**, **7**, 8	**6, 7, 8**	**6, 7, 8**

The central and western conflicts can be represented by two local graphs $G^{(1)}$ and $G^{(2)}$. The stabilities for the overall conflict are obtained by assuming that CG considers the central route more important than the western, namely $G^{(1)} > G^{(2)}$. Assume $s^{(1)}, s^{(2)} \in S$

are two states in G, where $s_a = (s_a^{(1)}, s_a^{(2)})$ and $s_b = (s_b^{(1)}, s_b^{(2)})$. Since $G^{(1)} > G^{(2)}$ for CG, CG's preferences are determined as follows:

(1) $s_a \succ_{CG} s_b$ iff either $s_a^{(1)} \succ_{CG}^{(1)} s_b^{(1)}$ or ($s_a^{(1)} \sim_{CG}^{(1)} s_b^{(1)}$ and $s_a^{(2)} \succ_{CG}^{(2)} s_b^{(2)}$);

(2) $s_a \sim_{CG} s_b$ iff $s_a^{(1)} \sim_{CG}^{(1)} s_b^{(1)}$ and $s_a^{(2)} \sim_{CG}^{(2)} s_b^{(2)}$; and

(3) $s_a \prec_{CG} s_b$ iff $s_a^{(1)} \prec_{CG}^{(1)} s_b^{(1)}$ or ($s_a^{(1)} \sim_{CG}^{(1)} s_b^{(1)}$ and $s_a^{(2)} \prec_{CG}^{(2)} s_b^{(2)}$)

As a result, all the states in G can be ranked as $(1,-) \succ_{CG} (2,-) \succ_{CG} (3,-) \succ_{CG} (4,-)$, where "-" denotes any state in the western conflict.

4.5.2 Stability Analysis

Four types of stabilities, Nash (R), sequential Stability (SEQ), general metarationality (GMR), and symmetric metarationality (SMR), are identified by the theorems demonstrated in Chapter 4.4. Acronyms of these theorems are used in the analysis. For example, Th 4.3 is short for Theorem 4.3 in Table 4.6. Stabilities in the hierarchical graph are identified using corresponding theorems and illustrated by Tables 4.6, 4.7, and 4.8.

(1) Nash (R)

As indicated in Th 4.1, the combined states in the hierarchical graph are Nash stable for CDM if both components are Nash stable in the corresponding local graphs. According to Tables 4.4 and 4.5, the Nash states for CG are states (1, 5), (1, 6), (2, 5), and (2, 6). The Nash states are (2, -) and (3, -) for LRs, and (-, 6) and (-, 7) for NCs. The "-" denotes any state in the corresponding entries. The Nash stable state for all DMs is (2, 6).

(2) Sequential Stability (SEQ)

Sequential stability is investigated for CG by Ths 4.3 and 4.4. The analysis is demonstrated in Table 4.6. As $G^{(1)} > G^{(2)}$, the combined states can be SEQ for CDM in G only if $s^{(1)}$ is SEQ for CDM in $G^{(1)}$. These possibilities are denoted as (s, s) and (s, $\overline{s}$), where s represents the SEQ states for CDM in the corresponding subconflict and $\overline{s}$ the states that are not SEQ.

In (s, s), the set of component states in $G^{(1)}$ and $G^{(2)}$ are $\{1, 2\}$ and $\{5, 6, 7\}$, respectively. They are called the candidate sets. Note that Nash states are also included in "s", according to the interrelationship between Nash and SEQ depicted in Fig. 4.3. Each state in the candidate sets is tested for eligibility of being a component SEQ states in the hierarchical graph, based on corresponding theorems. A "$\surd$" is marked behind each state if it is qualified. Otherwise, an "$\times$" is shown behind to denote the opposite. A combined state in the hierarchical graph is sequentially stable if both of its components are marked with "$\surd$"s. The last column in Table 4.6 shows the theorems used for this identification.

In situation (s, s), all states in the candidate sets are eligible to constitute combined sequentially stable states in G for CG. Nash rational states are distinguished with "r"s in Table 4.6. Thus, states (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7) are SEQ for CG in G. In (s, $\overline{s}$), states 1 and 2 are candidate states in $G^{(1)}$ and state 8 in $G^{(2)}$, according to case (2) in Th 4.4. In $G^{(1)}$, only state 1 is eligible because there exists $r^{(1)} = \{2\}$ for $s^{(1)} = \{1\}$, $r^{(1)} \in R^{(1)+}_{\{LRs, NCs\}}(q^{(1)})$, and $q^{(1)} \in R^{(1)+}_{CG}(s^{(1)})$. Thus, state (1, 8) is also SEQ for CG in G. The SEQ states for CG in the overall conflict are states (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7), and (1, 8). According to Th 4.6, states (2, -), (3, -), and (4, -) are SEQ for LRs and (-, 6), (-, 7), and (-, 8) for NCs.

(3) General Metarationality (GMR)

Table 4.6: Identification of SEQ States in the Overall Conflict for CG

Situations	Candidate Sets	Eligibilities of $s^{(1)}$ and $s^{(1)}$	SEQ States	Theorems Used
$(s,\ s)$	$G^{(1)}: \{1, 2\}$	$\{1\}$: r $\surd$	(1, 5), (1, 6)	Th 4.3
		$\{2\}$: r $\surd$	(2, 5), (2, 6)	
	$G^{(2)}: \{5, 6, 7\}$	$\{5\}$: r $\surd$	(1, 7), (2, 7)	
		$\{6\}$: r $\surd$		
		$\{7\}$: $\surd$		
$(s,\ \overline{s})$	$G^{(1)}: \{1, 2\}$	$\{1\}$: exist $R^{(1)+}_{LRs}(1) = \{2\} \prec^{(1)}_{CG} \{1\}\surd$	(1, 8)	Th 4.4
		$\{2\}$: $R^{(2)+}_{LRs}(2) = \phi\ \times$		
	$G^{(2)}: \{8\}$	$\{8\}$: $\surd$		

Similar to SEQ, general metarationality (GMR) in the overall conflict is identified by theorems from 4.7 to 4.10. A combined state could be GMR for CDM in G only if its component in $G^{(1)}$ is GMR for CDM. Thus, situations (g, g) and $(g, \overline{g})$ are discussed, for which "g" represents general metarationality and "$\overline{g}$" the opposite. As shown in Table 4.7, state 1 in $G^{(1)}$ and state 8 in $G^{(2)}$ qualify for GMR according to Th 4.8. Thus, state (1, 8) is GMR for CDM in the overall conflict. The GMR states for CG in the overall conflict are states (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7), and (1, 8). States (2, -), (3, -), and (4, -) are also GMR for LRs and (-, 6), (-, 7), and (-, 8) are GMR for NCs, according to Th 4.10.

(4) Symmetric Metarationality (SMR)

Table 4.7: Identification of GMR States in the Overall Conflict for CG

Situations	Candidate Sets	Eligibilities of $s^{(1)}$ and $s^{(2)}$	GMR States	Theorems Used
$(g,\ g)$	$G^{(1)}:\{1,2\}$	$\{1\}$: $\surd$	(1, 5), (1, 6)	Th 4.7
		$\{2\}$: $\surd$	(2, 5), (2, 6)	
	$G^{(2)}:\{5,6,7\}$	$\{5\}$: $\surd$	(1, 7), (2, 7)	
		$\{6\}$: $\surd$		
		$\{7\}$: $\surd$		
$(g,\ \overline{g})$	$G^{(1)}:\{1,2\}$	$\{1\}$: exist $R^{(1)}_{LRs}(1)=\{2\}\prec^{(1)}_{C}\{1\}\surd$	(1, 8)	Th 4.8
		$\{2\}$: $R_{LRs}(2)=\phi\ \times$		
	$G^{(2)}:\{8\}$	$\{8\}$: $\surd$		

According to Fig. 4.3, SMR states for CDM in G are also GMR for CDM. Hence, the GMR states for CG in G are identified for symmetric metarationality. According to Th 4.11, states (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7) are SMR for CDM in G. According to Th 4.12, state (1, 8) is investigated for symmetric metarationality for CDM in G. State 1 is tested for whether $r^{(1)}$ and $t^{(1)}$ exist, for $t^{(1)}\in R^{(1)}_{CG}(r^{(1)})$, according to case (2) in Th 4.12. As demonstrated in Table 4.8, for $s^{(1)}=\{1\}$, there exists $r^{(1)}=\{2\}$, which is a sanction for CG. In addition, $R^{(1)}_{CG}(2)=\{4\}$ and state 4 is less preferred for CDM. Hence, state (1, 8) is SMR for CG in G. Note that $(\sigma,\ \sigma)$ in Table 4.8 means the combined states with two SMR component states, while in $(\sigma,\ \overline{\sigma})$, the second component is not SMR for CG in $G^{(2)}$.

Table 4.8: Identification of SMR States in the Overall Conflict for CG

Situations	Candidate Sets	Eligibilities of $s^{(1)}$ and $s^{(2)}$	SMR States	Theorems Used
(σ, σ)	$G^{(1)}: \{1, 2\}$	$\{1\}$: $\surd$	(1, 5), (1, 6)	Th 4.11
		$\{2\}$: $\surd$	(2, 5), (2, 6)	
	$G^{(2)}: \{5, 6, 7\}$	$\{5\}$: $\surd$	(1, 7), (2, 7)	
		$\{6\}$: $\surd$		
		$\{7\}$: $\surd$		
$(\sigma, \overline{\sigma})$	$G^{(1)}: \{1, 2\}$	$\{1\}$: exist $R^{(1)}_{LRs}(1) = \{2\} \prec^{(1)}_{C} \{1\}$	(1, 8)	Th 4.12
		and $\{4\} \prec^{(1)}_{CG} \{1\}$ for $R^{(1)}_{CG}(2) = \{4\}$ $\surd$		
	$G^{(2)}: \{8\}$	$\{8\}$: $\surd$		

As a result, the symmetric metarational states in the overall conflict for CG are (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7), and (1, 8). States (2, -), (3, -), and (4, -) are symmetric metarational for LRs and (-, 6), (-, 7), and (-, 8) for NCs, according to Theorem 4.14.

4.5.3 Outcome Interpretation

The stability results of four types of stabilities for all DMs in the overall conflict are listed in Table 4.9. The SEQ, GMR, and SMR states for CG are the same. The rational state for all DMs is (2, 6). States (2, 6) and (2, 7) are SEQ, GMR, and SMR for all DMs. Among them, state (2, 7) is the status quo state, which indicates the start of the conflict. State (2, 6) can be reached from the starting state by DMs' UIs. Therefore, it could theoretically occur.

Table 4.9: Stability Results in the Overall Conflict

	Nash	SEQ	GMR	SMR
CG	(1, 5), (1, 6)	(1, 5), (1, 6), (2, 5)	(1, 5), (1, 6), (2, 5)	(1, 5), (1, 6), (2, 5)
	(2, 5), (2, 6)	(2, 6), (1, 7), (2, 7)	(2, 6), (1, 7), (2, 7)	(2, 6), (1, 7), (2, 7)
		(1, 8)	(1, 8)	(1, 8)
LRs	(2, -), (3, -)	(2, -), (3, -), (4, -)	(2, -), (3, -), (4, -)	(2, -), (3, -), (4, -)
NCs	(-, 6), (-, 7)	(-, 6), (-, 7), (-, 8)	(-, 6), (-, 7), (-, 8)	(-, 6), (-, 7), (-, 8)
Overall	(2, 6)	(2, 6), (2, 7)	(2, 6), (2, 7)	(2, 6), (2, 7)

State (2, 7), which is sequentially stable for all DMs, is historically recorded (The-Economic-Times, 2012; The-Hindu, 2013). CG can be assertive in initiating the central project while compromising with NCs on the western route. The counteractions from LRs cannot stop the implementation of the central project. This state is stable for all DMs due to their fears of potential sanctions. LRs cannot move to a better outcome from state 2 according to Table 4.4 and state 7 is the best outcome for NCs indicated in Table 4.5. CG will not move away from state 7 to 5 although the latter one is more preferred, because state 7 is sequentially stable. Although state (2, 6) can be reached from (2, 7), it could not occur unless CG is short-sighted. If CG is tempted to take advantage of its UI to state 5, NCs will in response levy a sanction by protesting. This will result in state 6 on the western route, which is the first equilibrium (2, 6). No DM can improve from this state on the western route and this state is less preferred by both CG and NCs. This justifies CG's current decision of suspending the western project. Therefore, the only possible resolution for all DMs is state (2, 7), in which CG fully initiates the central project and suspends the western project. The rational strategy for LRs is to oppose, while it is wise for NCs to consent with CG.

4.5.4 Comparison of the Two Case Studies in Chapter 3 and Section 4.5

The water diversion conflicts in China discussed in Chapter 3 have been analyzed using the basic hierarchical graph model methodology in this section. There are several differences between the case studies in Chapter 3 and Section 4.5. Firstly, the hierarchical conflict in Chapter 3 contains subconflicts at three locations, while only the central and western subconflicts are considered in Section 4.5. Secondly, the water diversion conflicts in Chapter 3 are investigated using classical GMCR methodology. In Section 4.5, the hierarchical conflicts are modelled by a basic hierarchical graph. The stability results are concluded by stabilities in subconflicts. Moreover, the central conflict in Chapter 3 contains three DMs, while only two DMs, CG and LRs, are included in Section 4.5.

The stability results obtained by the two modelling approaches are also different. In Chapter 3, the equilibria in the hierarchical conflict indicate that CG will insist on building the eastern project while compromising at the central and western locations. This outcome is caused by CG's priority on the eastern project. Although relative importance of local graphs is not defined in Chapter 3, the preference statements regarding the eastern project are placed on the top of Table 3.2, indicating that the eastern project is the most important for CG. In Section 4.5, as the eastern project is complete, CG considers the central project more important, which results in the equilibria in Section 4.5. The equilibrium state, state (2, 7), suggests that CG will initiate the central project and suspend the western project. Therefore, the lead-in example in Chapter 3 is a complete version of the water diversion conflicts by assuming the eastern project is the most important for CG. The change of CG's priority from the eastern to the central location results in different stability results. The conflicts at three locations in Chapter 3 can be considered as the conflicts at an earlier stage, while the conflicts modelled in Section 4.5 take place after the eastern project is complete. The central government then transfers its attention from the eastern to the central project.

4.6 Summary

A basic hierarchical graph model for conflict resolution is constructed to handle complex conflicts containing two subconflicts with a common decision maker in both subconflicts. Preference relations in this hierarchical model are defined in lexicographic order. The relation between the stabilities in the basic hierarchical graph model and stabilities in the local graph models is investigated. Theorems for each type of stability are proposed to reveal this relation. The stabilities in the basic hierarchical model can be obtained using these theorems and stability results in the local graph models.

The resolution obtained in the water diversion conflicts in China indicate that the Chinese Central Government should consider both smaller conflicts and prioritize the central route to achieve a desired outcome. As illustrated in this example, the identification of stability results does not require calculation executed on each state in the basic hierarchical graph. Therefore, the simplicity of the calculation in the basic hierarchical model can be preserved when more than one conflict in this model is investigated.

Chapter 5

Basic Hierarchical Graph Model in Matrix Form with Weighted Preference

5.1 Introduction

In this chapter, a basic hierarchical graph model with weighted preference is developed. Weighted preference is a preference structure that is more general than the lexicographic preference. The basic hierarchical graph model is represented by matrices to facilitate the calculation for stability results. Theorems reveal the relationship between the stability results in matrix form in the hierarchical graph and in each local graph. Algorithms are designed to capitalize on these relationships in the calculation of stability. The same example as in Chapters 3 and 4, water diversion conflicts in China, is investigated in this chapter to show the effectiveness of the newly designed algorithms and the connection between the new methodology in this chapter and the basic hierarchical model in Chapter 4. The weighted hierarchical graph model improves the modeling of hierarchical conflicts by providing more flexibility in describing the preference of CDM.

5.2 Formal Definition

As defined in Chapter 4, a hierarchical graph model for a strategic conflict contains smaller graph models, called local graph models. These local models feature one or more common DMs (CDMs) who appear in each of the local graphs. Local DMs (LDMs) appear only in one local graph. A basic hierarchical graph model has been constructed in Definition 4.1. A basic hierarchical graph model contains two smaller graph models with only one CDM.

5.2.1 Weighted Preference

If all the features of the two local graphs are known, the preference relations for LDMs in the basic hierarchical graph model can be determined. A new preference structure, called weighted preference, is constructed in this chapter. Different from the lexicographic preference, the relative importance of local graphs for CDM is represented by weight. Weighted preference is a more general way of describing preferences in basic hierarchical graph model.

The preference structure for CDM can be obtained as follows:

Suppose two states $s_a, s_b \in S$ in a basic hierarchical graph model G consisting of $G^{(1)}$

and $G^{(2)}$, where $s_a = (s_a^{(1)}, s_a^{(2)})$ and $s_b = (s_b^{(1)}, s_b^{(2)})$. The preferences for CDM in each local graph can be depicted by the option prioritization method (Fang et al., 2003a;b).

Let $\{\Omega_1^{(k)}, \Omega_2^{(k)}, ..., \Omega_{h_k}^{(k)}\}$ be the set of preference statements in $G^{(k)}$, $k = 1, 2$. In local graph $G^{(k)}$ ($k = 1, 2$), a score $\Psi_{j_k}^{(k)}(s_a^{(k)})$ is assigned to state $s_a^{(k)}$ according to its true values when the statements are applied (Peng et al., 1997), $0 < j_k < h_k$. Then,

$$\Psi_{j_k}^{(k)}(s_a^{(k)}) = \begin{cases} 2^{h_k - j_k} & if\ \Omega_{j_k}^{(k)}(s_a^{(k)}) = T \\ 0 & \text{otherwise} \end{cases} \tag{5.1}$$

and

$$\Psi^{(k)}(s_a^{(k)}) = \sum_{j_k=1}^{h_k} \Psi_{j_k}^{(k)}(s_a^{(k)}) \tag{5.2}$$

The importance of each local graph for CDM is denoted by a weight $w^{(k)}$ ($w^{(k)} > 0$). Thus, the score for state s_a in G is defined as:

$$\Psi(s_a) = \Psi^{(1)}(s_a^{(1)})w^{(1)} + \Psi^{(2)}(s_a^{(2)})w^{(2)} \tag{5.3}$$

The score on s_b for CDM can be similarly obtained as $\Psi(s_b)$. Then, s_a and s_b can be compared:

$s_a \succ s_b$ if $\Psi(s_a) > \Psi(s_b)$;

$s_a \prec s_b$ if $\Psi(s_a) < \Psi(s_b)$;

$s_a \sim s_b$ if $\Psi(s_a) = \Psi(s_b)$.

5.2.2 Connection with Lexicographic Preference

The connection between the weighted preference and lexicographic preference is investigated. Lexicographic preference is a special case of weighted preference. Their connections are indicated in Theorem 5.1.

Theorem 5.1: Suppose that $\Omega_C^{(k)} = \{\Omega_{C1}^{(k)}, ..., \Omega_{Cj_k}^{(k)}, ..., \Omega_{Ch_k}^{(k)}\}$ $(k = 1, 2;\ j_k = 1, ..., h_k)$ is the set of preference statements for CDM in $G^{(k)}$, and $\Omega_C = \{\Omega_{C1}, ..., \Omega_{Cj}, ..., \Omega_{Ch}\}$ $(j = 1, ..., h)$ is the set of preference statements for CDM in G, with $\Omega_{Cj} = \Omega_{Cj_k}^{(k)}$, and state $s = (s^{(1)}, s^{(2)}) \in S$ for $s^{(k)} \in S^{(k)}$, for the weighted preference,

$$\Psi(s) = \Psi^{(1)}(s^{(1)})w^{(1)} + \Psi^{(2)}(s^{(2)})w^{(2)} \tag{5.4}$$

Then, lexicographic preference is a special case of weighted preference for

(1) If $w^{(1)} > w^{(2)}$,

$w^{(1)} = 2^{h_2}$ and $w^{(2)} = 1$;

(2) If $w^{(1)} < w^{(2)}$,

$w^{(2)} = 2^{h_1}$ and $w^{(1)} = 1$. $\square$

Note that $w^{(1)} \neq w^{(2)}$, because equal importance is not considered in the lexicographic preference. The weight $w^{(k)}$ can be further normalized as $|w^{(k)}|$, where $|w^{(k)}| = \frac{w^{(k)}}{w^{(1)}+w^{(2)}}$ and $|w^{(1)}| + |w^{(2)}| = 1$. For $s_a, s_b \in S$, the relation between $\Psi_C(s_a)$ and $\Psi_C(s_b)$ does not change when substituting $w^{(k)}$ with $|w^{(k)}|$. Thus, to facilitate the calculation for stabilities, simply let

$$\Psi(s) = \Psi^{(1)}(s^{(1)})|w^{(1)}| + \Psi^{(2)}(s^{(2)})|w^{(2)}|. \tag{5.5}$$

5.2.3 Reachable Matrix

Matrix representation is an effective way of describing graph models and calculating stability results (Xu et al., 2007; 2009a). The weighted basic hierarchical graph model is represented by matrices to facilitate the calculation for stabilities. The reachable matrix is constructed to denote the reachable list for a given DM in the hierarchical model. The reachable matrix for CDM in the hierarchical model is a Tensor product of the reachable matrices in the two local graphs.

Theorem 5.2: Suppose $J_C^{(1)}$ is the $m \times m$ reachable matrix for CDM in $G^{(1)}$ and $J_C^{(2)}$ the $n \times n$ reachable matrix for CDM in $G^{(2)}$, I_n is an identity matrix of n scale, then the $mn \times mn$ hierarchical reachable matrix for CDM J_C in G is written as:

$J_C = J_C^{(1)} \bigotimes_R J_C^{(2)}$

$$= \begin{pmatrix} J_C^{(1)}(1,1) \otimes_r J_C^{(2)} & \dots & J_C^{(1)}(1,m) \otimes_r J_C^{(2)} \\ \vdots & \ddots & \vdots \\ J_C^{(1)}(m,1) \otimes_r J_C^{(2)} & \dots & J_C^{(1)}(m,m) \otimes_r J_C^{(2)} \end{pmatrix} \tag{5.6}$$

where $J_C^{(1)}(s^{(1)}, q^{(1)})$ is an entry in $J_C^{(1)}$ $(s^{(1)}, q^{(1)} = 1, \dots, m)$ and

$J_C^{(1)}(s^{(1)}, q^{(1)}) \otimes_r J_C^{(2)}$

$$= \begin{cases} J_C^{(2)} & s^{(1)} = q^{(1)} \\ J_C^{(1)}(s^{(1)}, q^{(1)})(I_n + J_C^{(2)}) & s^{(1)} \neq q^{(1)} \end{cases} \tag{5.7}$$

□

The reachable matrix for an LDM in the hierarchical graph is the Kronecker Product of the reachable matrix in the local graph and an identity matrix.

Theorem 5.3: Suppose that states $s, q \in S$ are two states in G where $s = (s^{(1)}, s^{(2)})$ and $q = (q^{(1)}, q^{(2)})$ $(s^{(1)}, q^{(1)} = 1, ..., m;\ s^{(2)}, q^{(2)} = 1, ..., n)$. Let $J_{L_1}^{(1)}$ denote the $m \times m$ reachable matrix for LDM_1 in $G^{(1)}$ and $J_{L_2}^{(2)}$ the $n \times n$ reachable matrix for LDM_2 in $G^{(2)}$, I_m and I_n represent identity matrices of m and n scales, respectively, and $\bigotimes$ mean the Kronecker Product of two matrices. Then the $mn \times mn$ hierarchical reachable matrices J_{L_1} and J_{L_2} for LDM_1 and LDM_2 are expressed as

$$J_{L_1} = J_{L_1}^{(1)} \bigotimes I_n = \begin{pmatrix} J_{L_1}^{(1)}(1,1)I_n & \dots & J_{L_1}^{(1)}(1,m)I_n \\ \vdots & \ddots & \vdots \\ J_{L_1}^{(1)}(m,1)I_n & \dots & J_{L_1}^{(1)}(m,m)I_n \end{pmatrix} \tag{5.8}$$

$$J_{L_2} = I_m \bigotimes J_{L_2}^{(2)} = \begin{pmatrix} J_{L_2}^{(2)} & & \\ & \ddots & \\ & & J_{L_2}^{(2)} \end{pmatrix} \tag{5.9}$$

□

The proof of Theorem 5.3 is analogous to Theorem 5.2.

5.2.4 UI Matrix

The UIs for DMs are also expressed in matrix form. In particular, suppose $s, q \in S$ for $s = (s^{(1)}, s^{(2)})$ and $(q^{(1)}, q^{(2)})$. The entries for CDM in the UI matrix J_C^+ can be written as

$$J_C^+(s,q) = \begin{cases} 1 & s \prec_C q \\ 0 & \text{other.} \end{cases} \tag{5.10}$$

The entries for LDMs in their UI matrices J_{L_1} and J_{L_2} can be expressed as

$$J_{L_1}(s,q) = \begin{cases} 1 & s^{(1)} \prec_{L_1}^{(1)} q^{(1)} \text{ and } s^{(2)} = q^{(2)} \\ 0 & \text{other} \end{cases}$$

and

$$J_{L_2}(s,q) = \begin{cases} 1 & s^{(2)} \prec_{L_2}^{(2)} q^{(2)} \text{ and } s^{(1)} = q^{(1)} \\ 0 & \text{other} \end{cases} \tag{5.11}$$

5.2.5 Joint Movement and Improvement Matrices

The joint movement and improvement matrices have been constructed by Xu et al. (2009a) to represent joint movements and joint improvements. The joint movement and improvement matrices have been introduced in Chapter 2. For example, the joint movement and improvement for LDMs in the hierarchical model can be expressed as:

$$M_L = \bigvee_{t=1}^{\delta} \bigvee_{L} M_L^{<t>} \tag{5.12}$$

where $L = \{L_1, L_2\}$ and

$$M_L^+ = \bigvee_{t=1}^{\delta'} \bigvee_L M_L^{<t,+>} \tag{5.13}$$

where

$$\begin{cases} M_{L_1}^{<t>} = sign(J_{L_1} \cdot M_{L_2}^{<t-1>}) \\ M_{L_2}^{<t>} = sign(J_{L_2} \cdot M_{L_1}^{<t-1>}) \end{cases}$$

and

$$\begin{cases} M_{L_1}^{<t,+>} = sign(J_{L_1} \cdot M_{L_2}^{<t-1,+>}) \\ M_{L_2}^{<t,+>} = sign(J_{L_2} \cdot M_{L_1}^{<t-1,+>}) \end{cases}$$

The joint movement and improvement matrices for CDM and LDM_1 and for CDM and LDM_2 can be obtained analogously.

5.3 Stability Definitions

The matrix form of stability definitions in a basic hierarchical graph model can be rewritten based on the definitions proposed by Xu et al (2009a). Note that E is a matrix with the same size as J_C^+ and all of its entries are 1.

(1) Nash Stability (R)

In a basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s \in S$ is Nash stable for CDM iff $e_s^T \cdot J_C^+ = 0^T$, where T denotes the transpose of matrix. The state s is Nash stable for LDM_1 iff $e_s^T \cdot J_{L_1} = 0^T$.

(2) Sequential Stability (SEQ)

A state $s \in S$ in G is sequentially stable for CDM iff $M_C^{SEQ}(s,s) = 0$, where $M_C^{SEQ} = J_C^+ \cdot \{E - sign[M_L^+ \cdot (P_C^{-,=})^T]\}$, and M_L^+ is the joint improvement matrix by LDMs.

State s is sequentially stable for LDM_1 iff $M_{L_1}^{SEQ}(s,s) = 0$, where $M_{L_1}^{SEQ} = J_{L_1} \cdot \{E - sign[M_{N-L_1}^+ \cdot (P_C^{-,=})^T]\}$ and $M_{N-L_1}^+$ is the joint improvement matrix by CDM and LDM_2.

(3) General Metarationality (GMR)

A state $s \in S$ in G is general metarational for CDM iff $M_C^{GMR}(s,s) = 0$, where $M_C^{GMR} = J_C^+ \cdot \{E - sign[M_L \cdot (P_C^{-,=})^T]\}$, and M_L is the joint movement matrix by LDMs.

State s is general metarational for LDM_1 iff $M_{L_1}^{GMR}(s,s) = 0$, where $M_{L_1}^{GMR} = J_{L_1} \cdot \{E - sign[(M_{N-L_1} \cdot (P_C^{-,=})^T]\}$ and M_{N-L_1} is the joint movement matrix by CDM and LDM_2.

(4) Symmetric Metarationality (SMR)

A state $s \in S$ in G is symmetric metarational for CDM iff $M_C^{SMR}(s,s) = 0$, where $M_C^{SMR} = J_C^+ \cdot [E - sign(M_L \cdot W_C)]$, and $W_C = (P_C^{-,=})^T \circ [E - sign(J_C \cdot (P_C^+)^T)]$ and $\circ$ is the Hadamard Product of the two matrices.

State s is symmetric metarational for LDM_1 iff $M_{L_1}^{SMR}(s,s) = 0$, where $M_{L_1}^{SMR} = J_{L_1}^+ \cdot [E - sign(M_{N-L_1} \cdot W_{L_1})]$, and $W_{L_1} = (P_{L_1}^{-,=})^T \circ [E - sign(J_{L_1} \cdot (P_{L_1})^T)]$.

5.4 Interrelationship Between Stabilities in the Hierarchical Graph and the Local Graphs

The four types of solution concepts for DMs in matrix form in the hierarchical model are linked with those in each local model. The interrelationship between the stabilities in the hierarchical model and the local models is investigated in theorems. Different from the theorems in Chapter 4, they are presented and proven in matrix form. Representative proofs of some theorems are provided in Appendices. The algorithms for calculating stabilities in the hierarchical graph model are designed based on these theorems. Note that the vectors appeared in this thesis are written in bold.

5.4.1 Nash Stability (R)

Theorem 5.4: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is Nash stable for CDM in G iff $s^{(k)}$ $(k = 1, 2)$ is Nash stable for CDM in $G^{(k)}$. □

Theorem 5.5 (Nash for LDM): In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is Nash rational for LDM_1 in G iff $s^{(1)}$ is Nash rational for LDM_1 in $G^{(1)}$. □

Theorem 5.5 indicates that a state in a hierarchical graph model is Nash stable for a LDM if and only if the component state is Nash stable for the LDM in the local graph in which it participates.

5.4.2 Sequential Stability (SEQ)

Theorem 5.6: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G if $s^{(k)}$ ($k = 1, 2$) is sequentially stable for CDM in $G^{(k)}$. □

Theorem 5.7: Suppose G is a weighted basic hierarchical graph model consisting of $G^{(1)}$ and $G^{(2)}$, if a state $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G, then

(1) $s^{(k)}$ is sequentially stable for CDM in $G^{(k)}$ for both $k = 1$ and 2, or

(2) when $s^{(k)}$ is not sequentially stable for CDM in $G^{(k)}$ ($k = 1$ or 2, but not both), there exists $r = (r^{(1)}, r^{(2)}) \in S$, such that $\Psi(r) \leq \Psi(s)$, where state $r \in S$ corresponds to the entries in vector $\boldsymbol{r}$:

$\boldsymbol{r} = \boldsymbol{r^{(1)}} \times \boldsymbol{r^{(2)}} - \boldsymbol{e_{q^{(1)}}} \times \boldsymbol{e_{q^{(2)}}}$, for $\times$ symbolizes Cartesian Product;

$\boldsymbol{r^{(1)}} = (J_{L_1}^{(1)+})^T \cdot \boldsymbol{e_{q^{(1)}}} + \boldsymbol{e_{q^{(1)}}}$ and $\boldsymbol{r^{(2)}} = (J_{L_2}^{(2)+})^T \cdot \boldsymbol{e_{q^{(2)}}} + \boldsymbol{e_{q^{(2)}}}$ for all $(q^{(1)}, q^{(2)}) \in R_C^+(s)$;

$\boldsymbol{e_{q^{(k)}}}$ is a 0-1 vector with the ${q^{(k)}}^{th}$ entry being 1 and others 0. □

State $r^{(1)}$ can be either a UI for LDM_1 from $q^{(1)}$, referred to as entries in $(J_{L_1}^{(1)+})^T \cdot \boldsymbol{e_{q^{(1)}}}$, or just $q^{(1)}$, written as entries in $\boldsymbol{e_{q^{(2)}}}$.

A small example makes Theorem 5.7 easier to understand. Let the sets of state in $G^{(1)}$ and $G^{(2)}$ written as $S^{(1)} = \{1, 2, 3, 4\}$ and $S^{(2)} = \{5, 6, 7, 8\}$, respectively. Suppose

$$J_C^{(1)+} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \quad J_C^{(2)+} = \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

and

$$J_{L_1}^{(1)+} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \quad J_{L_2}^{(2)+} = \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

We analyze when state 1 is not SEQ for CDM in $G^{(1)}$, whether state $(1,5)$ is SEQ for CDM in G.

As can be seen from $J_C^{(1)+}$ and $J_C^{(2)+}$, $(q^{(1)}, q^{(2)})$ can be written as

(a) $(q^{(1)}, q^{(2)}) = (2, 6)$;

(b) $(q^{(1)}, q^{(2)}) = (2, 5)$;

(c) $(q^{(1)}, q^{(2)}) = (1, 6)$, where $(q^{(1)}, q^{(2)}) \in R^+(s^{(1)}, s^{(2)})$, $s^{(1)} \in S^{(1)}$, and $s^{(2)} \in S^{(2)}$.

Then $q^{(1)} = 1$ or 2; $q^{(2)} = 5$ or 6. Thus, $\boldsymbol{e_{q^{(1)}}} = (1 \quad 0 \quad 0 \quad 0)^T$ or $(0 \quad 1 \quad 0 \quad 0)^T$; $\boldsymbol{e_{q^{(2)}}} = (1 \quad 0 \quad 0 \quad 0)^T$ or $(0 \quad 1 \quad 0 \quad 0)^T$.

For (a), $\boldsymbol{e_{q^{(1)}}} = (0 \quad 1 \quad 0 \quad 0)^T$ and $\boldsymbol{e_{q^{(2)}}} = (0 \quad 1 \quad 0 \quad 0)^T$. Note that state 6 is actually the second entry of $\boldsymbol{e_{q^{(2)}}}$.

One can calculate $(J_{L_1}^{(1)+})^T \cdot \boldsymbol{e_{q^{(1)}}} = (0 \quad 0 \quad 1 \quad 0)^T$, which is the transpose of the second row of $J_{L_1}^{(1)+}$. Then, $\boldsymbol{r^{(1)}} = (0 \quad 1 \quad 1 \quad 0)^T$. Analogously, $\boldsymbol{r^{(2)}} = (0 \quad 1 \quad 0 \quad 1)^T$. Then, the non-zero entries in $\boldsymbol{r}$ correspond to states (2, 8), (3, 6), (3, 8). Thus, state (1, 5) is SEQ for CDM if it is more preferred to at least one of the three states.

Analogously for (b) and (c), state (1, 5) can also be SEQ for CDM in G.

Sequential stability for CDM in G is also affected by the weights of local graphs. The relation between the weights and sequential stability for CDM is indicated by Corollary 5.1.

Corollary 5.1: State $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G iff $|w^{(k)}| \in (\alpha, \beta)$ for either $\alpha = 0$ or $\beta = 1$. □

According to the corollary, a state is SEQ for CDM in G if and only if each weight w_k (k = 1,2) ranges from either side of the interval (0, 1), i.e., the range of $|w^{(k)}|$ should be either (0, β) or (α, 1).

The stabilities for LDMs are investigated in Theorem 5.8.

Theorem 5.8: Suppose there exists a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, and the number of states in $G^{(1)}$ is $|S^{(1)}| = m$ and $|S^{(2)}| = n$ in $G^{(2)}$. If a state $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for LDM_1 in G, then

(1) $s^{(1)}$ is sequentially stable for LDM_1 in $G^{(1)}$ or

(2) $s^{(1)}$ is general metarational for LDM_1 in $G^{(1)}$,

$\boldsymbol{e}_{\boldsymbol{s^{(2)}}}^T \cdot J_C^{(2)+} \neq 0$ and $\boldsymbol{e}_{\boldsymbol{s^{(2)}}}^T \cdot M_{\{C,L_2\}}^{(2)+} \neq 0$

where $M_{\{C,L_2\}}^{(2)+}$ is the joint improvement matrix for CDM and LDM_2 in $G^{(2)}$ and $\boldsymbol{e}_{\boldsymbol{s^{(2)}}}^T$ is the vector with ${s^{(2)}}^{th}$ element 1 and others 0, and

$\exists r \in S$, such that $\Psi(r) > \Psi(q)$ for CDM and the r^{th} element in 0-1 vector $\boldsymbol{r}$ is 1, where $\boldsymbol{r} = \left(\boldsymbol{e}^T_{\boldsymbol{q^{(1)}}} \cdot J^{(1)}_C\right) \times \left(\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}} \cdot M^{(2)+}_{\{C,L_2\}}\right) \neq 0 \quad \forall q^{(1)} \in R^{(1)+}_{L_1}(s^{(1)})$, and $q = (q^{(1)}, s^{(2)})$. □

5.4.3 General Metarationality (GMR)

Theorem 5.9: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is general metarational for CDM in G if $s^{(k)}$ ($k = 1$, 2) is general metarational for CDM in $G^{(k)}$. □

Theorem 5.10: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, if a state $s = (s^{(1)}, s^{(2)}) \in S$ is general metarational for CDM in G, then

(1) $s^{(k)}$ is general metarational for CDM in $G^{(k)}$ for both $k = 1$ and 2, or

(2) if $s^{(k)}$ is not general metarational for CDM in $G^{(k)}$ ($k = 1$ or 2, but not both), there exists $r = (r^{(1)}, r^{(2)}) \in S$, such that $\Psi(r) \leq \Psi(s)$, where state $r \in S$ corresponds to entries in vector $\boldsymbol{r}$:

$\boldsymbol{r} = \boldsymbol{r^{(1)}} \times \boldsymbol{r^{(2)}} - \boldsymbol{e}_{\boldsymbol{q^{(1)}}} \times \boldsymbol{e}_{\boldsymbol{q^{(2)}}}$;

$\boldsymbol{r^{(1)}} = (J^{(1)}_{L_1})^T \cdot \boldsymbol{e}_{\boldsymbol{q^{(1)}}} + \boldsymbol{e}_{\boldsymbol{q^{(1)}}}$ and $\boldsymbol{r^{(2)}} = (J^{(2)}_{L_2})^T \cdot \boldsymbol{e}_{\boldsymbol{q^{(2)}}} + \boldsymbol{e}_{\boldsymbol{q^{(2)}}}$ for all $(q^{(1)}, q^{(2)}) \in R^+_C(s)$;

$\boldsymbol{e}_{\boldsymbol{q^{(k)}}}$ is a 0-1 vector with the ${q^{(k)}}^{th}$ entry being 1 and others 0. □

Theorem 5.11: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is general metarational for LDM_1 in G, iff $s^{(1)} \in S^{(1)}$ is general metarational for LDM_1 in $G^{(1)}$. □

As suggested in Theorem 5.11, a state is GMR for an LDM in G if and only if the component state in its local graph is GMR.

5.4.4 Sequential Metarationality (SMR)

Theorem 5.12 (SMR): In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is symmetric metarational for CDM in G if $s^{(k)}$ ($k =$ 1, 2) is symmetric metarational for CDM in $G^{(k)}$. $\square$

Theorem 5.13: Suppose G is a weighted basic hierarchical graph model consisting of $G^{(1)}$ and $G^{(2)}$, if a state $s = (s^{(1)}, s^{(2)}) \in S$ is symmetric metarational for CDM in G, then

(1) $s^{(k)}$ is symmetric metarational for CDM in $G^{(k)}$ for both $k = 1$ and 2, or

(2) if $s^{(k)}$ is not symmetric metarational for CDM in $G^{(k)}$ ($k =$ 1, 2, but not both),

there exists $r = (r^{(1)}, r^{(2)}) \in S$, such that $\Psi(r) \leq \Psi(s)$ and $\Psi(t) \leq \Psi(s)$ for all $t = (t^{(1)}, t^{(2)}) \in S$, where r, t correspond to vectors $\boldsymbol{r}$ and $\boldsymbol{t}$, respectively:

$\boldsymbol{t} = \boldsymbol{t^{(1)}} \times \boldsymbol{t^{(2)}} - \boldsymbol{e_{r^{(1)}}} \times \boldsymbol{e_{r^{(2)}}}$;

$\boldsymbol{t^{(1)}} = (J_C^{(1)})^T \cdot \boldsymbol{e_{r^{(1)}}} + \boldsymbol{e_{r^{(1)}}}$ and $\boldsymbol{t^{(2)}} = (J_C^{(2)})^T \cdot \boldsymbol{e_{r^{(2)}}} + \boldsymbol{e_{r^{(2)}}}$ for $\boldsymbol{e_{r^{(k)}}}$ ($k = 1, 2$) represents a 0-1 vector with ${r^{(k)}}^{th}$ entry being 1 and others 0;

$\boldsymbol{r} = \boldsymbol{r^{(1)}} \times \boldsymbol{r^{(2)}} - \boldsymbol{e_{q^{(1)}}} \times \boldsymbol{e_{q^{(2)}}}$;

$\boldsymbol{r^{(1)}} = (J_{L_1}^{(1)})^T \cdot \boldsymbol{e_{q^{(1)}}} + \boldsymbol{e_{q^{(1)}}}$ and $\boldsymbol{r^{(2)}} = (J_{L_2}^{(2)})^T \cdot \boldsymbol{e_{q^{(2)}}} + \boldsymbol{e_{q^{(2)}}}$ for all $(q^{(1)}, q^{(2)}) \in R_C^+(s)$. $\square$

Note that $\boldsymbol{e_{r^{(k)}}}$ is different from $\boldsymbol{r^{(k)}}$. The entries in $\boldsymbol{e_{r^{(1)}}}$ refer to state $r^{(1)}$, which is the component of $(r^{(1)}, r^{(2)})$. Vector $\boldsymbol{r^{(1)}}$ contains not only state $r^{(1)}$, but also state $q^{(1)}$. Thus, $\boldsymbol{r^{(1)}}$ can be written as $\boldsymbol{r^{(1)}} = \boldsymbol{e_{r^{(1)}}} + \boldsymbol{e_{q^{(1)}}}$. In other words, $\boldsymbol{e_{r^{(1)}}} = (J_{L_1}^{(1)})^T \cdot \boldsymbol{e_{q^{(1)}}}$.

Theorem 5.14: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is symmetric metarational for LDM_1 in G iff $s^{(1)} \in S^{(1)}$ is symmetric metarational for LDM_1 in $G^{(1)}$. $\square$

Theorems 5.12, 5.13, and 5.14 are analogous to the Theorems 5.9, 5.10, and 5.11, respectively.

5.5 Algorithms for Calculating Stability

The algorithms for calculating stability results in the hierarchical model are designed according to the theorems demonstrated in Section 5.4. Each algorithm is described in steps.

5.5.1 Nash Stability (R)

According to Theorem 5.4, a state $s = (s^{(1)}, s^{(2)}) \in S$ is Nash stable for CDM in G if and only if $s^{(k)} \in S^{(k)}$ is Nash stable for CDM in $G^{(k)}$ ($k = 1, 2$). As demonstrated in Theorem 5.5, state s is Nash stable for LDM_k in $G^{(k)}$ if and only if $s^{(k)}$ is Nash stable for LDM_k in $G^{(k)}$.

5.5.2 Sequential Stability (SEQ)

The sequential stable (SEQ) states in the hierarchical graph model can be calculated according to Theorems 5.6 and 5.7. The algorithm for this calculation for CDM is shown in Fig. 5.1. The detailed steps for the calculation are shown as follows.

Step 1: For $(s^{(1)}, s^{(2)}) \in S$, if $s^{(k)}$ is SEQ for CDM for both $k = 1$ and 2,

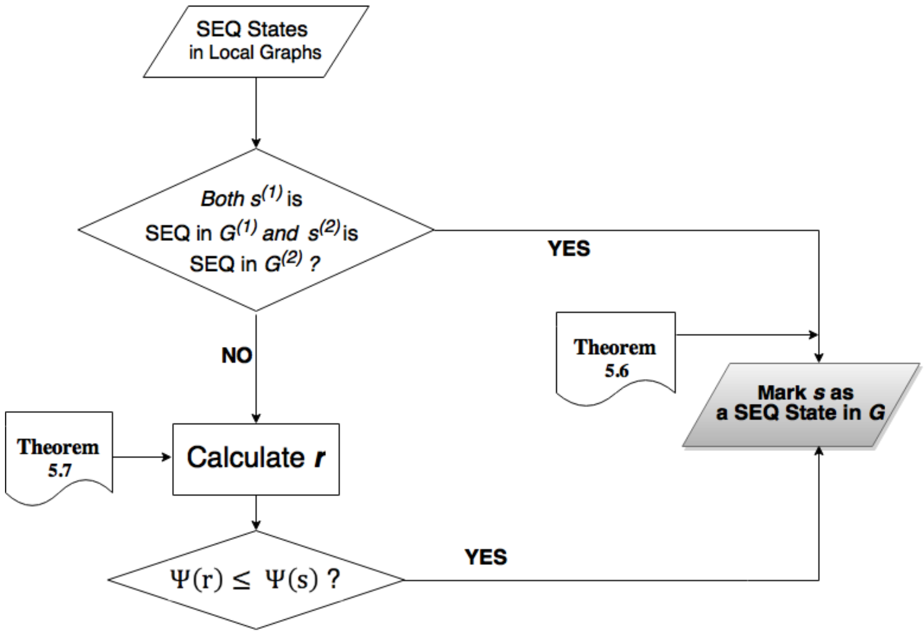

Figure 5.1: Algorithm to determine SEQ states for CDM in G

then $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G according to Theorem 5.6; otherwise go to step 2.

Step 2: Calculate $\boldsymbol{r}$; if the non-zero element r in $\boldsymbol{r}$ satisfies $\Psi(r) \leq \Psi(s)$,

then $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G according to Theorem 5.7.

The algorithm to determine sequential stability (SEQ) for LDM_1 in the hierarchical model can be depicted in Fig. 5.2. The procedure for this calculation is demonstrated as:

Step 1: For $(s^{(1)}, s^{(2)}) \in S$, if $s^{(1)}$ is GMR for LDM_1 for $G^{(1)}$,

then $(s^{(1)}, s^{(2)})$ is SEQ for LDM_1 in G according to Theorem 5.8; otherwise go to step 2.

Step 2: If $s^{(1)}$ is GMR for LDM_1 in $G^{(1)}$ and $s^{(2)}$ satisfies $\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}} \cdot J_C^{(2)+} \neq 0$ and $\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}} \cdot M^{(2)+}_{\{C,L_2\}} \neq 0$ according to Theorem 5.8,

then go to step 3.

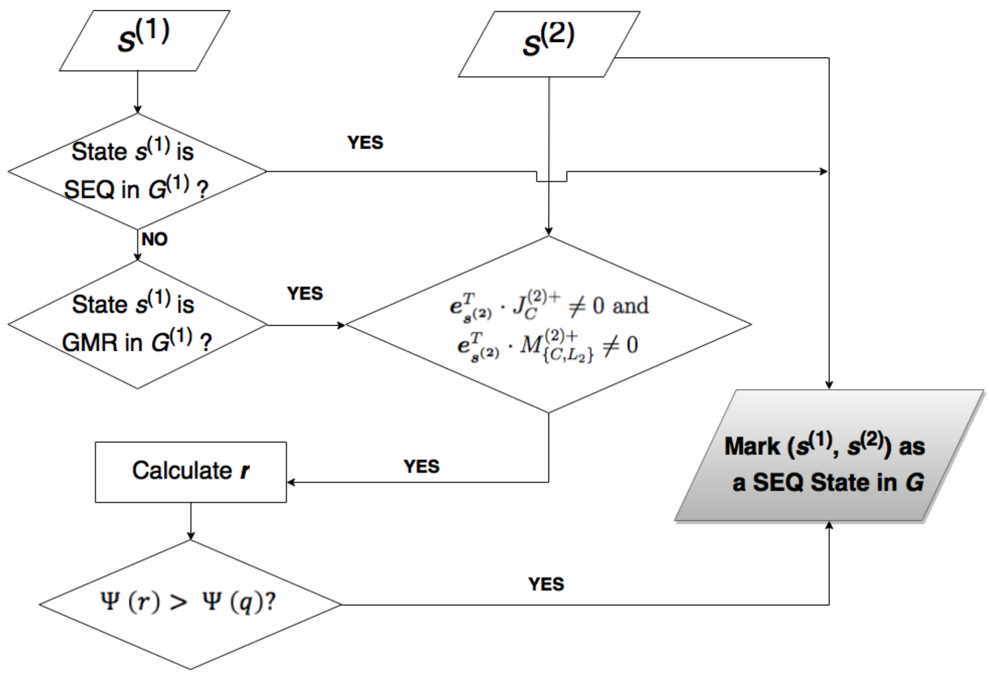

Figure 5.2: Algorithm to determine SEQ states for LDM_1 in G

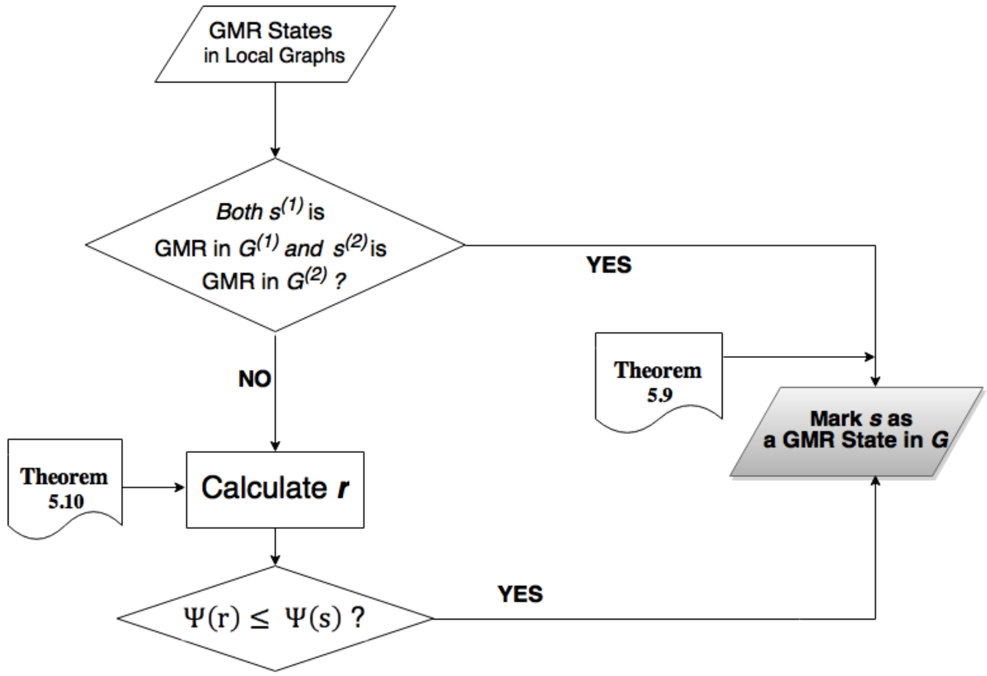

Figure 5.3: Algorithm to determine GMR states for CDM in G

Step 3: Calculate $\boldsymbol{r}$; if there exists non-zero element r in $\boldsymbol{r}$ such that $\Psi(r) > \Psi(q)$, then $(s^{(1)}, s^{(2)})$ is SEQ for LDM_1 in G.

5.5.3 General Metarationality (GMR)

The GMR states for CDM in the hierarchical model can be determined analogously to calculating SEQ for CDM. The algorithm is shown in Fig. 5.3.

According to Theorem 5.11, $(s^{(1)}, s^{(2)})$ is GMR for LDM_k in G if and only if $s^{(k)}$ is GMR for LDM_k in $G^{(k)}$.

5.5.4 Symmetric Metarationality (SMR)

SMR states for CDM in the hierarchical model can be determined according to Theorems 5.12 and 5.13. As depicted in Fig. 5.4, the calculation is described in the following steps.

Step 1: For $(s^{(1)}, s^{(2)}) \in S$, if $s^{(k)}$ is SMR for CDM for both $k = 1$ and 2,

then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G according to Theorem 5.12; otherwise go to step 2.

Step 2: Calculate $\boldsymbol{r}$, if $\Psi(r) \leq \Psi(s)$,

then go to step 3.

Step 3: Calculate $\boldsymbol{t}$; if $\Psi(t) \leq \Psi(s)$ for all $t \in R_C^+(r)$,

then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G according to Theorem 5.13.

According to Theorem 5.14, the SMR states for LDMs can be calculated analogously to obtaining GMR states for LDMs.

5.6 Reinvestigation of Water Diversion Conflicts in China

The water diversion conflicts in China are reinvestigated to show the effectiveness of the basic hierarchical graph model using weighted preference in calculating stabilities. The DMs, options, and preferences in each subconflict are the same as those in Chapter 4. The preferences in the overall conflict are expressed in the weighted structure. The calculation for stabilities in the overall conflict is carried out using matrices.

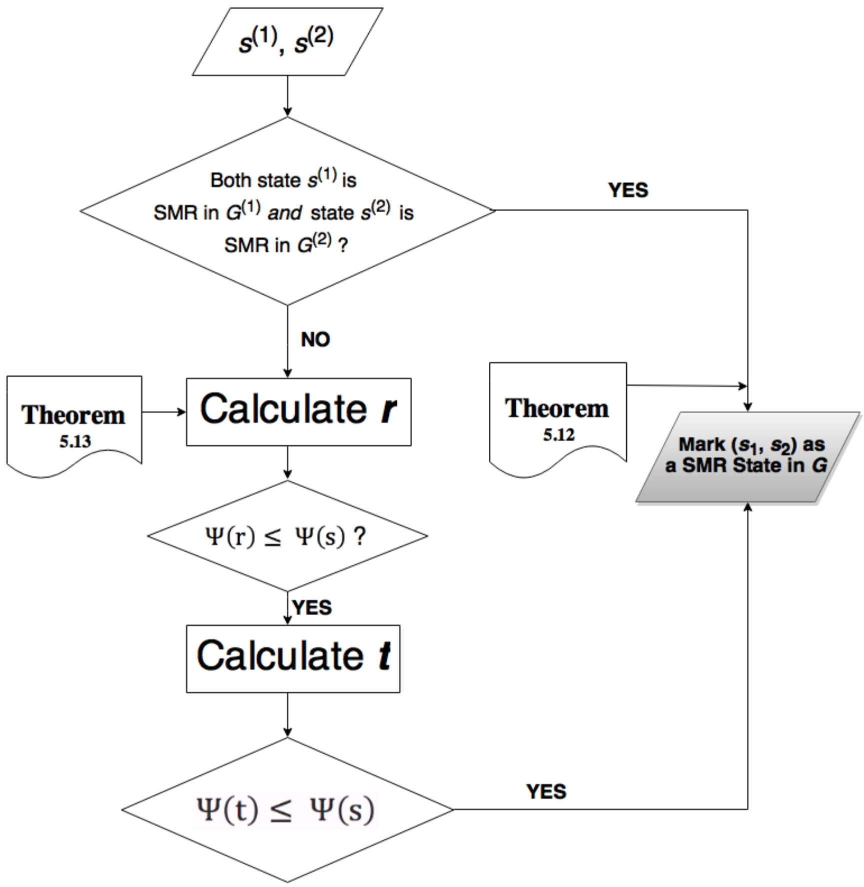

Figure 5.4: Algorithm to determine SMR states for CDM in G

The DMs, options, and states in the water diversion conflict have been shown in Tables 4.1, 4.2, and 4.3.

The preferences for DMs in the local graphs are consistent with those in Section 4.5. These preferences are represented here using the option prioritization method, listed in Table 5.1. In $G^{(1)}$, preference statements for CG are written in the left column. Option 1) is the first statement for CG, which means that the original construction plan at the central location is the most important issue for CG. The next preference statement, option 3), is written below option 1), which denotes that the agreement from LRs has the second importance for CG. According to Equation (5.1), the scores for the two statements are 2^{2-1} and 2^{2-2}, respectively. As a state in G can be written as a selection of options, each state is investigated for whether the option selection matches a given preference statement. In the third column, a true value is assigned at each state for each preference statement. For example, state 2 in G can be expressed as “YN”. Because option 1) is selected and option 3) is not, the truth value of state 2 is “T” on option 1) and “F” on option 3). According to Equation (5.1), the score for each state is aggregated. For state 2, the score is $1 \cdot 2^1 + 0 \cdot 2^0 = 2$. Thus, the scores for states 1, 2, 3, and 4 are 3, 2, 1, 0. The four states can be ranked according to the scores from state 1, the most preferred, to state 4, the least preferred. This ranking is also consistent with the preferences for CG expressed by the UI matrix. Note that all state numbers are in bold. The preferences for CG in $G^{(2)}$ can be analyzed analogously.

The UI matrices for DMs in the local graphs are shown as $J_{CG}^{(1)+}$, $J_{CG}^{(2)+}$, $J_{L_1}^{(1)+}$, and $J_{L_2}^{(2)+}$. Note that all moves are assumed reversible for each DM.

$$J_{CG}^{(1)+} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 \end{array} \quad J_{CG}^{(2)+} = \begin{array}{c|cccc} & 5 & 6 & 7 & 8 \\ \hline 5 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 & 0 \\ 8 & 0 & 1 & 0 & 0 \end{array}$$

$$J_{L_1}^{(1)+} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array} \quad J_{L_2}^{(2)+} = \begin{array}{c|cccc} & 5 & 6 & 7 & 8 \\ \hline 5 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 & 0 \end{array}$$

The stability results in the local graphs can be seen in Tables 4.4 and 4.5. The stability results in the hierarchical conflict are calculated using the algorithms introduced in Section 5.5. Taking SEQ for CDM as an example, as shown in Tables 4.4 and 4.5, states 1 and 2 are SEQ for CG in $G^{(1)}$ and states 5, 6, and 7 are SEQ for CG in $G^{(2)}$. According to Theorem 5.6, states (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), and (2, 7) are SEQ for CG in the hierarchical conflict.

To determine the rest of the SEQ states for CDM in G, states 3 and 4 in $G^{(1)}$ and state 8 in $G^{(2)}$ are investigated according to Fig. 5.1. For example, let $s^{(1)} = 1$ and $s^{(2)} = 8$; then $q^{(1)} = 1$ and $q^{(2)} = 6$, which can be written as $\boldsymbol{e_{q^{(1)}}} = (1 \quad 0 \quad 0 \quad 0)^T$ and $\boldsymbol{e_{q^{(2)}}} = (0 \quad 1 \quad 0 \quad 0)^T$. Then, $\boldsymbol{r^{(1)}} = (1 \quad 1 \quad 0 \quad 0)^T$, and $\boldsymbol{r^{(2)}} = (0 \quad 1 \quad 0 \quad 1)^T$. In $\boldsymbol{r}$, the non-zero elements correspond to states (1, 8), (2, 6), and (2, 8). Since CG considers the central subconflict more important, the weights for the two subconflicts are assumed as $|w^{(1)}| = 0.9$ and $|w^{(2)}| = 0.1$. According to Equation (5.5) and Table 5.1, $\Psi(s) = 3 \cdot 0.9 + 0 \cdot 0.1 = 2.7$. When $r = (2, 6)$, $\Psi(r) = 2 \cdot 0.9 + 1 \cdot 0.1 = 1.9$. Thus, $\Psi(r) < \Psi(s)$ for $s = (1, 8)$ and $r = (2, 6)$. Then state (1, 8) is an SEQ state for CG in G. Other stable states in G can be determined analogously. The stability results calculated in this chapter are the same as those in Table 4.9.

The weighted preference is more flexible for expressing CDM's assessment on the importance of each local graph. The stability results for CG can change according to different value of weights assigned to the local graphs. To carry out further analysis, these stability results with a complete range of $|w^{(1)}|$ in the water diversion conflicts are investigated.

According to Theorem 5.4, Nash states for CG are not affected by the weights for the local graphs. The SEQ and GMR states for CG with different normalized weights are shown in Fig. 5.5 and Fig. 5.6. The relationship between SMR and $|w^{(1)}|$ for CG are the same with that between GMR and $|w^{(1)}|$ shown in Fig. 5.6.

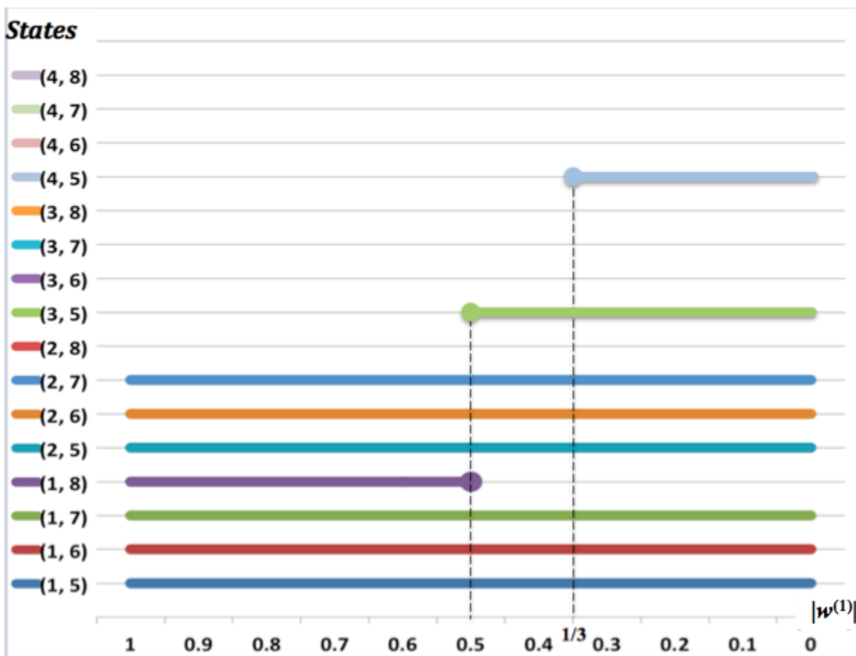

Figure 5.5: SEQ states for CDM in the hierarchical graph changed with normalized weights

Table 5.1: Preferences for CG in Two Subconflicts Obtained by Option Prioritization Method

$G^{(1)}$					
Preference Statements	Score of Each Statement	States			
		1	**2**	**3**	**4**
1)	2^1	T	T	F	F
3)	2^0	T	F	T	F
	Score of Each State	3	2	1	0
	Ranking of States	**1**	**2**	**3**	**4**
$G^{(2)}$					
Preference Statements	Score of Each Statement	States			
		5	**6**	**7**	**8**
4)	2^1	T	F	T	F
2)	2^0	T	T	F	F
	Score of Each State	3	1	2	0
	Ranking of States	**5**	**7**	**6**	**8**

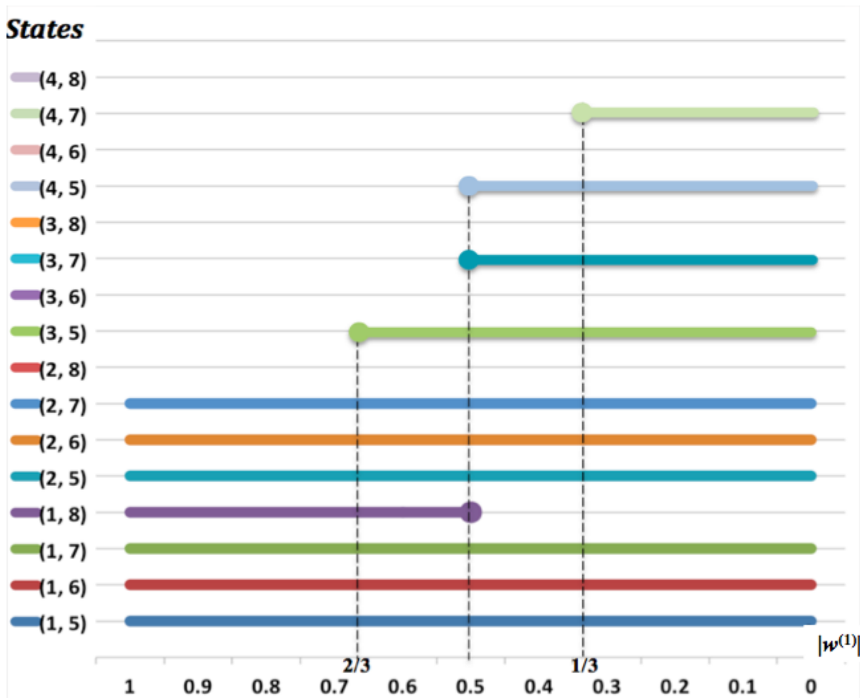

Figure 5.6: GMR and SMR states for CDM in the hierarchical graph changed with normalized weights

In Fig. 5.6, the horizontal axis denotes the value of $|w^{(1)}|$ ranging from 1 to 0. The 16 states in the overall conflict are listed in the vertical axis. Each bar represents the range of $|w^{(1)}|$ within which the corresponding state is SEQ for CDM. For example, state (4, 5) is SEQ for CDM when $|w^{(1)}| \in (0, 1/3]$. Note that the left end of the bar is round and the right end is flat, since $|w^{(1)}|$ can equal 1/3 but never reach 0.

According to Fig. 5.6, states (1, 5), (1, 6), (1, 7), (2, 5), (2, 6), (2, 7) are sequentially stable for CG regardless of the change of $|w^{(1)}|$. State (1, 8) is SEQ for CG when $|w^{(1)}|$ is no less than 0.5. States (3, 5) and (4, 5) are SEQ for CG if $|w^{(1)}|$ is no larger than 0.5 and 1/3, respectively. Taking state (1, 8) as an example, CG's UI is sanctioned by a LRs' UI if

CG considers the central project more important, which means $|w^{(1)}| \in [0.5, 1)$. Thus, the counteraction from LRs will be highly valued. As $|w^{(1)}|$ is below 0.5, the central project will be less important. This countermove from LRs would fail to sanction CG when CG has a view of both central and western projects.

The sequentially stable states for LDMs in the hierarchical conflict are the same when the weights for local graphs are changed. For *LRs*, states (2, -), (3, -), (4, -) are SEQ since the corresponding component states 2, 3, and 4 are SEQ. Although the states can be SEQ for *REs* when the corresponding component states are not SEQ, no SEQ state is found among states (1, -) in this example.

The results calculated in the weighted hierarchical graph model indicate resolution for DMs in this conflict. Equilibrium (2, 7) suggests that the project at the central location can be successfully constructed despite opposition from LRs. CG will suspend the construction plan at the western project to appease NCs. For CG, the central projects should be constructed first. Although the dissatisfaction from LRs cannot affect the course of the construction, more action plans are advised to better accommodate LRs after relocation. As the western project is likely to cause transboundary disputes, CG should fully negotiate with NCs before resuming the project. As LRs do not have much impact in this conflict, they can still express their concerns by communicating with local governments. According to Figs. 5 and 6, the results of this conflict may change if CG adjusts the importance of each local graph. Therefore, CG can be provided with a wide range of resolutions depending on which outcome it desires to achieve.

5.7 Comparison of Weighted Hierarchical Graph Model and Former Methodologies

The advantage of the weighted hierarchical graph model is explained by comparing it with the basic hierarchical model with lexicographical preferences in Chapter 4. The weighted hierarchical graph model has a more flexible preference structure compared with the basic hierarchical graph model with lexicographic preferences. The lexicographic preference structure is a crisp approach of determining preference relations for CDM. A local graph can be either more important, less important, or equally important than the other local graph. In the weighted preference structure, the relative importance between the two local graphs is expressed by numbers between 0 and 1. Thus, such importance can be described more flexibly.

The calculation for stabilities in the weighted hierarchical graph model is also more effective than the former calculation method. The stabilities in the basic hierarchical model with lexicographic preferences are determined by theorems describing the interrelationship between stabilities in the hierarchical model and local models. In comparison, the stabilities in the weighted hierarchical model are obtained by matrix computation based on the algorithms. As demonstrated in the case study, the new calculation approach is more simplified and easier to implement.

The stabilities in the weighted hierarchical graph model are more sensitive to the relative importance of local graphs, compared with the stabilities in the hierarchical graph model with lexicographic preferences. As depicted in Fig. 5.6, state (3, 5) is not GMR for CG if $|w^{(1)}|$ is greater than 2/3. However, in the hierarchical model with lexicographic preferences, this state is still GMR for CG when it considers $G^{(1)}$ more important, which is interpreted as $|w^{(1)}| \in (0.5, 1)$ in the weighted hierarchical model.

5.8 Summary

In this chapter, a basic hierarchical graph model has been proposed with a weighted preference structure. The UMs and UIs in this model have been expressed in matrices. Algorithms have been constructed for calculating stability results in the hierarchical model based on the theorems which link stability definitions of the hierarchical graph with the local graphs. The water diversion conflicts in China have been reinvestigated to demonstrate the effectiveness of the new basic hierarchical model in representing hierarchical conflicts and calculating stabilities.

The hierarchical graph model can effectively solve strategic conflicts that contain several related subconflicts. Instead of focusing on one single conflict, DMs can have a comprehensive understanding of all related conflicts. The structure of the hierarchical graph model can be extended. The following two chapters demonstrate different types of hierarchical graph models: a model containing two CDMs, called a duo hierarchical graph model, and a general hierarchical graph model containing any number of CDMs.

Chapter 6

Duo Hierarchical Graph Model

6.1 Introduction

A duo hierarchical graph model for conflict resolution containing two CDMs and two LDMs is presented in this chapter. The preferences for DMs in the new model are constructed in lexicographic order. A fast approach for calculating stabilities is developed based on the option prioritization method. The duo hierarchical graph model is applied to a sales competition between Airbus and Boeing in the Asia Pacific region. The two manufacturers compete in both wide and narrow body models for purchase by airlines. Two types of Asian airlines with different operating strategies plan to purchase new aircraft to profit from the booming air travel in this region. The wide and narrow body markets constitute two subcompetitions and are modelled using the hierarchical graph model. The stability results indicate how the aircraft manufacturers and their customers may act strategically in the process of reaching a resolution. Thus, this model can provide decision makers with a

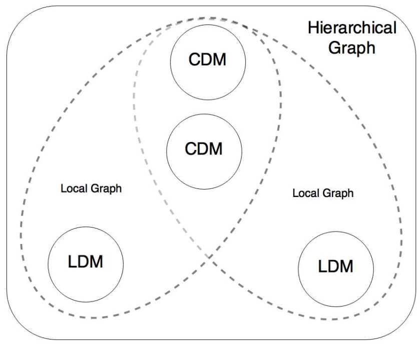

Figure 6.1: Structure of duo hierarchical graph model

comprehensive understanding of the dynamics of this competition, and guidance for taking beneficial actions.

6.2 Formal Definitions

6.2.1 Definition of Duo Hierarchical Graph Model

A duo hierarchical graph model consists of two local graphs. Two CDMs appear in both local graphs. Each local graph contains an LDM. The structure of the duo hierarchical graph model is depicted in Fig. 6.1.

The formal definition of the duo hierarchical graph model is provided below.

Definition 6.1 (A Hierarchical Graph Model with two CDMs): Suppose there are four DMs, consisting of CDM_1, CDM_2, LDM_1, and LDM_2. The two local graph models can be written as

$G^{(1)} = \langle \{CDM_1, CDM_2, LDM_1\}, S^{(1)}, \{AC_1^{(1)}, AC_2^{(1)}, AL_1^{(1)}\},$
$\{\succsim_{C_1}^{(1)}, \succsim_{C_2}^{(1)}, \succsim_{L_1}^{(1)}\}\rangle$

$G^{(2)} = \langle \{CDM_1, CDM_2, LDM_2\}, S^{(2)}, \{AC_1^{(2)}, AC_2^{(2)}, AL_2^{(2)}\},$
$\{\succsim_{C_1}^{(2)}, \succsim_{C_2}^{(2)}, \succsim_{L_2}^{(2)}\}\rangle$

where $S^{(k)}$ denotes the set of states in $G^{(k)}$ $(k = 1, 2)$; $AC_1^{(k)}, AC_2^{(k)}, AL_1^{(k)}, AL_2^{(k)} \subseteq S^{(k)} \times S^{(k)}$ means the sets of arcs between all pairs of states in $S^{(k)}$ for CDM_1, CDM_2, and LDM_k, respectively; $\succsim_{C_1}^{(k)}$, $\succsim_{C_2}^{(k)}$, $\succsim_{l_k}^{(k)}$ are preference relations on $S^{(k)}$ for the respective DMs, where, for instance, $\succsim_{C_1}^{(k)}$ represents the following relations between $s_1^{(k)}$ and $s_2^{(k)}$ for $s_1^{(k)}, s_2^{(k)} \in S^{(k)}$:

(1) CDM_1 prefers $s_1^{(k)}$ to $s_2^{(k)}$ in $G^{(k)}$, i.e. $s_1^{(k)} \succ_{C_1}^{(k)} s_2^{(k)}$;

(2) CDM_1 prefers $s_2^{(k)}$ to $s_1^{(k)}$ in $G^{(k)}$, i.e. $s_1^{(k)} \prec_{C_1}^{(k)} s_2^{(k)}$;

(3) $s_1^{(k)}$ is equally preferred to $s_2^{(k)}$ for CDM_1 in $G^{(k)}$, i.e. $s_1^{(k)} \sim_{C_1}^{(k)} s_2^{(k)}$;

(4) $s_1^{(k)}$ is no less preferred to $s_2^{(k)}$ for CDM_1 in $G^{(k)}$, i.e. $s_1^{(k)} \succsim_{C_1}^{(k)} s_2^{(k)}$;

(5) $s_2^{(k)}$ is no less preferred to $s_1^{(k)}$ for CDM_1 in $G^{(k)}$, i.e. $s_1^{(k)} \precsim_{C_1}^{(k)} s_2^{(k)}$.

Then the graph model:

$G = \langle \{CDM_1, CDM_2, LDM_1, LDM_2\}, S = S^{(1)} \times S^{(2)}, \{AC_1, AC_2, AL_1, AL_2\}, \{\succsim_{C_1}, \succsim_{C_2}$
$, \succsim_{L_1}, \succsim_{L_2}\}\rangle$

is a hierarchical graph model based on $G^{(1)}$ and $G^{(2)}$, where $AC_1, AC_2, AL_1, AL_2 \subseteq S \times S$ are sets of arcs in S for CDM_1, CDM_2, LDM_1, and LDM_2, respectively, and $\succsim_{C_1}$, $\succsim_{C_2}$, $\succsim_{L_1}$, $\succsim_{L_2}$ are the preference relations for the respective DMs in G.

Several properties can be concluded from Definition 6.1.

(i) For $s_1^{(k)}, s_2^{(k)} \in S^{(k)}$ $(k = 1, 2)$ and $(s_1^{(1)}, s_1^{(1)}), (s_2^{(1)}, s_2^{(2)}) \in S$, $AC_1 \subseteq S \times S$ is defined by $((s_1^{(1)}, s_1^{(2)}), (s_2^{(1)}, s_2^{(2)})) \in AC_1$ iff either $(s_1^{(1)}, s_2^{(1)}) \in AC_1^{(1)}$ and $s_1^{(2)} = s_2^{(2)}$, or $s_1^{(1)} = s_2^{(1)}$ and $(s_1^{(2)}, s_2^{(2)}) \in AC_1^{(2)}$, or $(s_1^{(1)}, s_2^{(1)}) \in AC_1^{(1)}$ and $(s_1^{(2)}, s_2^{(2)}) \in AC_1^{(2)}$.

(ii) $AL_1 \subseteq S \times S$ is defined by $((s_1^{(1)}, s_1^{(2)}), (s_2^{(1)}, s_2^{(2)})) \in AL_1$ iff $(s_1^{(1)}, s_2^{(1)}) \in AL_1^{(1)}$ and $s_1^{(2)} = s_2^{(2)}$

(iii) $AL_2 \subseteq S \times S$ is defined by $((s_1^{(1)}, s_1^{(2)}), (s_2^{(1)}, s_2^{(2)})) \in AL_2$ iff $(s_1^{(2)}, s_2^{(2)}) \in AL_2^{(2)}$ and $s_1^{(1)} = s_2^{(1)}$

(iv) The preference relation $\succsim_{L_1}$ is defined on S by $(s_1^{(1)}, s_1^{(2)}) \succsim_{L_1} (s_2^{(1)}, s_2^{(2)})$ iff $s_1^{(1)} \succsim_{L_1}^{(1)} s_2^{(1)}$

(v) The preference relation $\succsim_{L_2}$ is defined on S by $(s_1^{(1)}, s_1^{(2)}) \succsim_{L_1} (s_2^{(1)}, s_2^{(2)})$ iff $s_1^{(2)} \succsim_{L_2}^{(2)} s_2^{(2)}$

(vi) The preference relation $\succsim_{C_1}$ on S is related to the preference relation $\succsim_{C_1}^{(1)}$ on $S^{(1)}$ as follows:

if $s_1^{(1)}, s_2^{(1)} \in S^{(1)}$ and $s_1^{(1)} \succsim_{C_1}^{(1)} s_2^{(1)}$, then $(s_1^{(1)}, s_0^{(2)}) \succsim_{C_1} (s_2^{(1)}, s_0^{(2)})$ for all $s_0^{(2)} \in S^{(2)}$

(vii) The preference relation $\succsim_{C_1}$ on S is related to the preference relation $\succsim_{C_1}^{(2)}$ on $S^{(2)}$ as follows:

if $s_1^{(2)}, s_2^{(2)} \in S^{(2)}$ and $s_1^{(2)} \succsim_{C_1}^{(2)} s_2^{(2)}$, then $(s_0^{(1)}, s_1^{(2)}) \succsim_{C_1} (s_0^{(1)}, s_2^{(2)})$ for all $s_0^{(1)} \in S^{(1)}$.

6.2.2 Lexicographic Preference Structure

In this chapter, the preferences in local graphs are represented in the option prioritization method. The preferences in the duo hierarchical model are constructed by the preferences in the local graphs. As demonstrated in Chapters 4 and 5, weighted preference is a general way of expressing preferences in the hierarchical graph model, while lexicographic preference is more efficient in calculating stabilities. Thus, the preferences in the duo hierarchical graph model are presented in lexicographic order.

Recall that in the option prioritization approach, a preference statement for a DM can be expressed by option numbers connected by logical symbols, including negation ("NOT" or "−"), conjunction ("AND" or "&"), disjunction ("OR" or "|"), and conditions ("IF" and "IFF"). Preference statements are ordered from the most preferred for a given DM at the top of a table to the least preferred at the bottom. As an example, the set of preference statements for CDM_i ($i = 1,\ 2$) in G can be marked as $\{\Omega_{C_i1},\ \Omega_{C_i2},\ ...,\ \Omega_{C_ij},\ ...,\ \Omega_{C_ih}\}$, where $0 < j \leq h$ and h denotes the number of statements.

The preference structure in lexicographical order for CDM_i in G can be expressed by the preference statements in $G^{(1)}$ and $G^{(2)}$, as demonstrated in Theorem 6.1.

Theorem 6.1: In a hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, suppose $G^{(1)}$ is more important than $G^{(2)}$ for CDM_i $(i = 1, 2)$. Let $\{\Omega^{(k)}_{C_i1},\ \Omega^{(k)}_{C_i2},\ ...,\ \Omega^{(k)}_{C_ih_k}\}$ be the set of preference statements in $G^{(k)}$ ($k = 1,\ 2$). Then the set of preference statements for CDM_i in G can be written as $\{\Omega^{(1)}_{C_i1},\ \Omega^{(1)}_{C_i2},\ ...,\ \Omega^{(1)}_{C_ih_1},\ \Omega^{(2)}_{C_i1},\ \Omega^{(2)}_{C_i2},\ ...,\ \Omega^{(2)}_{C_ih_2}\}$. $\square$

The proof of theorem 6.1 is given in the Appendices. Theorem 6.1 can be used to calculate stabilities in the duo hierarchical graph model. The preference statements for CDM in G can be determined based on the preference statements in the local graphs. The stabilities are then calculated using GMCR II, the decision support system introduced in Chapter 2.

6.3 Sales Competition between Airbus and Boeing

6.3.1 Background Introduction

Airbus and Boeing, the two largest commercial aircraft manufacturers, have forecasted a huge demand for both wide body and narrow body planes (Airbus, 2014; Boeing 2014). The former refers to the double-aisle aircraft having a capacity of 200 to 850 passengers, while the latter can carry passengers of approximately 200 with a single-aisle layout (Doganis, 2002).

Eager to grab a greater share of the Asia Pacific market, both aircraft makers have launched a vigorous sales campaign in this region by promoting their most competitive models. Airbus and Boeing wish to market their new generation models as well as advocate their best selling planes. The European Aerospace Consortium wishes to grab deals from some major Asian airlines for its two-deck jumbo aircraft A380. The existing long haul model A330 can also be an alternative (Hepher, 2013a). Boeing competes with Airbus in the wide body market in the Asia Pacific by promoting the B787 Dreamliner, a next generation model with better performance and higher efficiency. Confronted with a delay in delivering the B787s, Boeing also plans to promote the existing B777 model to compensate for this disadvantage (Forbes, 2013). Potential buyers in the Asia Pacific region include flag carriers such as Cathy Pacific, Japan Airlines, Korean Air, and Singapore Airlines.

In the narrow body aircraft market, the low budget carriers in this region, such as Air Asia, Spring Airlines, and Tigerair, provide flexible air travel by operating services between non-hub cities in the region. As their business grows, these airlines can upgrade and expand their fleets by purchasing new planes. To meet these demands, Airbus has started to develop the A320neo model, an upgraded version of the existing A320 (Hepher, 2013b). Meanwhile, the upgrade for B737, called B737MAX, is also underway (Boeing, 2011).

6.3.2 Decision Makers and Their Options

The competition between Airbus and Boeing in the wide and narrow body aircraft market can be modelled using a hierarchical graph, represented as G. The competition in each market is represented by a local graph: the wide body competition is denoted by $G^{(1)}$ and narrow body by $G^{(2)}$. Airbus (A) and Boeing (B) are two CDMs in the hierarchical graph model and are labelled as CDM_1 and CDM_2, respectively. The two types of airline carriers are the LDMs, each of which appears in one local graph. In the wide body market, customer X, denoted simply as X, is a flagship carrier airline in the region. Customer Z (Z) represents a low cost airline in southeast Asia, which operates some short haul flights between major cities in this region. Several assumptions are part of the model:

(1) X plans to purchase one type of wide body jet from either A or B.

(2) X will not choose two models from the same manufacturer. Negotiation with one manufacturer can only result in ordering one of its aircraft types.

(3) Z can choose from A or B, but not both, because the manufacturer of Z's fleet should be consistent to reduce costs. As with (2), Z will consider only one model from the selected manufacturer.

The hierarchical structure of the market competition is shown in Fig. 6.2. The options for each DM are shown in Table 6.1. To simplify the modelling process, there are 7 options

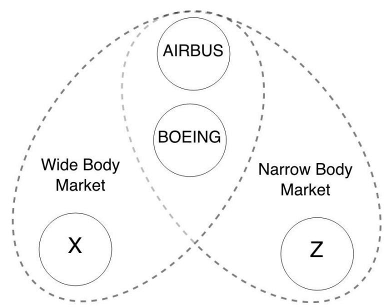

Figure 6.2: Hierarchical structure of the overall competition

for all DMs shown in Table 6.1. Each option is assigned a number. The negation of each option for A and B is written after the slash in the same line with the given option.

In the wide body market, Airbus can either promote the new jumbo jet A380 to X or persuade X to buy the A330, which is written as option 1 on the right in Table 6.1. The negation of option 1, which is to promote A330, is written after the slash in option 1. In the narrow body market, A can advertise the existing A320 (option 2). The other option for A in selling the narrow body aircraft is to promote A320neo, which is the negation of option 2. The A320neo is designed with new technology that can improve the performance and save operating costs (Airbus, 2014). Despite the previous issue of delivery delay, Boeing still attempts to sell the B787 to X, or attract the carrier to buy B777s. The two alternatives are combined as option 3. Analogously, B can either advertise the B737, or

Table 6.1: DMs and Their Options in the Overall Competition

DMs:	Options
Airbus (A)	1. Promote A380/ Promote A330
	2. Promote A320/ Promote A320neo
Boeing (B)	3. Promote B787/ Promote B777
	4. Promote B737/ Promote B737MAX
Customer X (X)	5. Choose A
	6. Choose B
Customer Z (Z)	7. Choose A/ Choose B

market B737MAX to Z (option 4).

As the buyer in the wide body market, X can choose one aircraft model from either A or B. Hence, options 5 and 6 represent the selection of A and B for X, respectively. Note that X can also purchase two aircraft models, one from each manufacturer. In this case, both options 5 and 6 are selected by X. However, as a budget airline carrier, Z could buy aircraft from only one manufacturer. Thus, option 7 means the selection of A by Z, while the contract with B is denoted by the negation of this option.

6.3.3 States in the Sales Competition

The 7 options in the model indicate 2^7 possible states in total. Some infeasible states are removed because they are illogical in the context of the market competition. X should select at least one option, either option 5 or 6, since this carrier will definitely purchase one model from at least one manufacturer. In the narrow body market, Z will only select one model from one manufacturer considering the budget and the smaller scale of the company. After eliminating the infeasible states, 96 states are left for analysis.

The DMs and their options in the two local models are given in Tables 6.2 and 6.3. As shown in Tables 6.2 and 6.3, states in the two local models are assigned with decimal numbers. The states are numbered from 1 to 12 in the wide body market competition and from 13 to 20 in the narrow body competition. According to Definition 6.1, the states in the overall competition are the Cartesian Product of the states in the two local models. Hence, each state in the hierarchical model is expressed by a two-dimension number. Each dimension denotes a component state in the local model. For example, state (1, 14) in the market competition is composed of state 1 and state 14 in the wide and narrow body competitions, respectively.

6.3.4 Preferences of Decision Makers

Preference relations for DMs are determined using the option prioritization method. In this approach, preference statements are written in option numbers from the most preferred at the top to the least preferred at the bottom. The preference statements in each local model are listed in Tables 6.4 and Tables 6.5. The preference statements in the hierarchical model

Table 6.2: DMs and Their Options in the Wide Body Competition

DMs	Options	States											
A	1. A380/ A330	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
B	3. B787/ B777	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
X	5. Choose A	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
	6. Choose B	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
	State Number	1	2	3	4	5	6	7	8	9	10	11	12

Table 6.3: DMs and Their Options in the Narrow Body Competition

DMs	Options	States							
A	2. A320/A320neo	N	Y	N	Y	N	Y	N	Y
B	4. B737/B737MAX	N	N	Y	Y	N	N	Y	Y
Z	7. Choose A	Y	Y	Y	Y	N	N	N	N
	State Number	13	14	15	16	17	18	19	20

for CDMs are combined according to Theorem 6.1. Because Airbus has a slight advantage in the narrow body market due to a shorter delivery period of the A320neo, the European manufacturer considers the campaign of selling the models to the low budget carrier more important. In comparison, the American company is more interested in selling planes to the wide body customer due to the strong competitiveness of B787s and B777s. Thus, the narrow body market is more important for Airbus and the wide body market more important for Boeing in the hierarchical model, i.e. $G^{(1)} < G^{(2)}$ for Airbus and $G^{(1)} > G^{(2)}$ for Boeing.

The preference statements for CDMs in the overall competition are demonstrated in Table 6.6. As shown in Table 6.6, the first preference statement for Airbus is written as (-2) & 7, which means the purchase of the A320neo by Z is the most desired scenario for the European company. The selling of the existing A320 model, denoted as option 7, would also be acceptable for A. The next preference statement, 1 & 5, reflects the expectation of A of sealing the deal with X who only chooses the A380 model. According to the last three statements, 1, 5, and -6, the European company does not wish to see the successful deal between its rival and X. This outcome is not the worst for A as long as X also buys A380s.

As Boeing values the wide body market more than the narrow body one, the American manufacturer wishes to sign a deal with X on any type of model (option 6). If X plans to sign a deal with Airbus, B will advocate the B777 ((-3) IF 5). In the following preference statement, 3 IFF 6, B prefers to promote the B787 model to X if and only if the American company has gained the interest from X. The deal between A and X is not favoured by B due to the rivalry between the two manufacturers (-5). In the narrow body market, Boeing

Table 6.4: Preference Statement for A in the Wide and Narrow Body Competition

Wide Body Market	Narrow Body Market
1 & 5	(-2) & 7
1	7
5	
-6	

prefers to promote the B737 model regardless of the reaction of Z (4). The order from Z will be a preferred outcome. However, as the B737MAX will not be available in the near future, the current version of this model, the B737, would be more competitive (4 & -7).

6.3.5 The Uncertainty of Preferences for LDMs

In reality, the selection of commercial aircraft for airlines is often affected by various factors, such as retail prices, operation costs, and the relationship with the two manufacturers. The selection of these factors has been carried out using decision making approaches (Dozic and Kalic, 2014; Flouris, 2010; Ozdemir et al., 2011). Some representative variables for selecting aircraft for an airline company include retail price, fuel cost per seat per nautical mile, and the number of seats. In Table 6.7, these variables are listed for each type of aircraft mentioned in this case study. An ideal aircraft model for the airline companies should be cheaper in retail price, more fuel efficient, and have a larger seating capacity. As

Table 6.5: Preference Statement for B in the Wide and Narrow Body Competition

Wide Body Market	Narrow Body Market
6	4
(-3) IF 5	4 & (-7)
3 IFF 6	
-5	

Table 6.6: Preference Statements for DMs in the Overall Competition

A	B
(-2) & 7	6
7	(-3) IF 5
1 & 5	3 IFF 6
1	-5
5	4
-6	4 & -7

can be seen from Table 6.7, none of the models is predominately better than others under these criteria in either the wide or narrow body category. In the real world scenario, airline companies need to weight these variables before making reasonable decisions. In some cases, these decisions could also be affected by other factors that are hard to predict or measure, such as the relationship with the two manufacturers and the results of negotiation. Thus, it is difficult to determine the preferences for the two LDMs affected the aforementioned factors.

The relationships between the various factors and the preferences for the LDMs are demonstrated in Fig. 6.3. These factors are considered as inputs, and the preferences are outputs. The mechanism regarding how these factors affect the preferences is unknown, and thus denoted by a black box. Although this mechanism is hard to investigate, some preference patterns can be concluded: the preference for an LDM is either "pro-Airbus" or "pro-Boeing". The two preference patterns for each LDM will generate four different preferences in the overall competition, demonstrated in Table 6.8. Stabilities in the overall competition are calculated under the four preferences. The similarities in these stabilities are investigated. Common stable states can be used to predict the outcome of the overall competition, or suggest possible resolutions for DMs.

For airline X, the "pro-Airbus" preference can be represented by the following preference statements, from most to least important: 5 IF 1, 6 IFF -3, and -3, which mean that A380 is the ideal model for X and B777 can be the second choice. The pro-Boeing preference can be written in reverse: 6 IFF -3, -3, and 5 IF 1. For airline Z, the "pro-Airbus" preference is written from most to least important as: 7, -7. The "pro-Boeing" preference is denoted in reverse as: -7, 7. The 4 preference patterns for all DMs are marked as I, II, III, and IV

Table 6.7: Variables for Selecting Commercial Aircraft (Axlegeeks, 2015; Leeham, 2015)

Wide Body			
X	Retail Price (Million USD)	Fuel Cost Per Seat Per Nautical Miles	Typical Seating
A380	318	11.38 ¢	525
A330	245	8.19 ¢	295
B787	157	11.68 ¢	210
B777	281	9.93 ¢	365
Narrow Body			
Z	Retail Price (Million USD)	Fuel Cost Per Seat Per Nautical Miles	Typical Seating
A320	70	7.76 ¢	150
A320neo	103	20% less fuel per seat than A320	195
B738	79	8.17 ¢	162
B737MAX 8	104	14% less than B738	162

in Table 6.8. Pattern I indicates the situation in which both X and Z are "pro-Airbus". Pattern II differs from pattern I in that Z is "pro-Boeing" rather than "pro-Airbus". In pattern III, X is "pro-Boeing" and Z is "pro-Airbus". Pattern IV is the situation in which both X and Z are "pro-Boeing". Note that the preferences for A and B are not listed because they are the same in all of the 4 patterns.

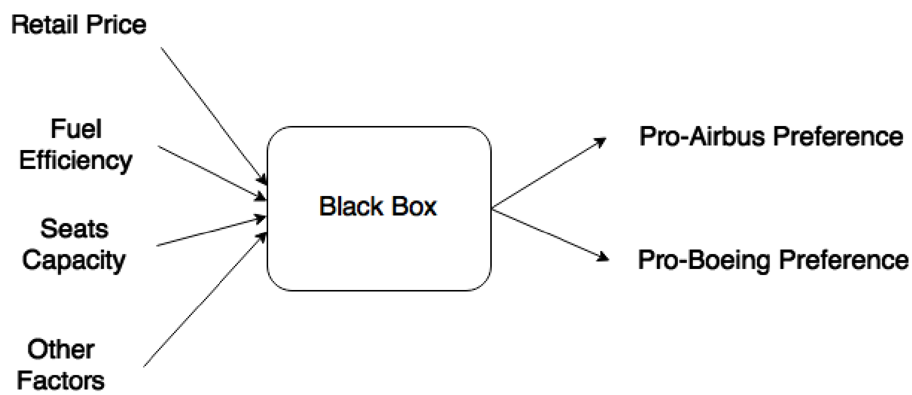

Figure 6.3: Generation of preference patterns for customers

6.3.6 Stability Calculation

The stability results in the market competition are calculated using GMCR II. Four types of solution concepts, Nash stability (R), sequential stability (SEQ), general metarationality (GMR), and symmetric metaratonality (SMR), are investigated for each preference pattern.

The stable states for each DM under four solution concepts in the overall competition are listed in Table 6.9. The state numbers are listed in the first and second columns. The state numbers in the two columns are identical, but represented in different forms. The state numbers in the second column are denoted by two entries. Each entry represents a state number shown in Tables 6.2 and 6.3. The 96 states in total in the overall competition are also numbered from 1 to 96 in the first column. Nash, SEQ, GMR, and SMR stabilities are denoted as "r", "s", "g", and "σ", respectively. These stabilities are investigated for each DM and all DMs under 4 different preferences, referred to as I, II, III, and IV. At each state, the stabilities for each DM are marked. A column entitled "Ovr" demonstrates the stabilities for all DMs, i.e. equilibria, under a preference pattern. The states with the same

Table 6.8: Preference Patterns in the Overall Competition

	I	II	III	IV
X:	5 IF 1	5 IF 1	6 IFF -3	6 IFF -3
	6 IFF -3	6 IFF -3	-3	-3
	-3	-3	5 IF 1	5 IF 1
Z:	7	-7	7	-7
	-7	7	-7	7

equilibria under all preference patterns, called common equilibria, are marked in bold.

Table 6.9: Stabilities for DMs in the Overall Competition in Different Preference Patterns

State Num.	State Num.	I					II					III					IV				
		A	B	X	Z	Ovr	A	B	X	Z	Ovr	A	B	X	Z	Ovr	A	B	X	Z	Ovr
1	(1,17)		s,σ	g				s,σ	g	r			s,σ	g				$s\ ,\sigma$	g	r	
2	(2,17)	r	s,σ	g			r	s,σ	g	r	g	r	s,σ	g			r	s,σ	g	r	g
3	(1,18)		s,σ	g	r			s,σ	g	r			s,σ	g	r			s,σ	g	r	
4	**(2,18)**	r	s,σ	g	r	g	r	s,σ	g	r	g	r	s,σ	g	r	g	r	s,σ	g	r	g
5	(3,17)			r					r	r				r					r	r	
6	(4,17)	r		r			r		r	r		r		r			r		r	r	
7	(3,18)			r	r				r	r				r	r				r	r	
8	(4,18)	r		r	r		r		r	r		r		r	r		r		r	r	
9	(1,19)		r	g				r	g				r	g				r	g		
10	(2,19)	r	r	g			r	r	g			r	r	g			r	r	g		
11	(1,20)		r	g	s,σ			r	g	s			r	g	s,σ			r	g	s	
12	**(2,20)**	r	r	g	s,σ	g	r	r	g	s	g	r	r	g	s,σ	g	r	r	g	s	g
13	(3,19)			r					r					r					r		
14	(4,19)	r		r			r		r			r		r			r		r		
15	(3,20)			r	s,σ				r	s				r	s,σ				r	s	
16	(4,20)	r		r	s,σ		r		r	s		r		r	s,σ		r		r	s	
17	(5,17)		s,σ	r				s,σ	r	r			s,σ	r				s,σ	r	r	
18	(6,17)	r	s,σ				r	s,σ		r		r	s,σ	σ			r	s,σ	σ	r	σ
19	(5,18)		s,σ	r	r			s,σ	r	r			s,σ	r	r			s,σ	r	r	
20	(6,18)	r	s,σ		r		r	s,σ		r		r	s,σ	σ	r	σ	r	s,σ	σ	r	σ
21	(7,17)		$s,\ \sigma$					s,σ		r			s,σ					s,σ		r	
22	(8,17)	r	s,σ				r	s,σ		r		r	s,σ				r	s,σ		r	

23	(7,18)		s, σ		r			s,σ		r			s,σ		r			s,σ		r	
24	(8,18)	r	s,σ		r		r	s,σ		r		r	s,σ		r		r	s,σ		r	
25	(5,19)		s,σ	r				s,σ	r				s,σ	r				s,σ	r		
26	(6,19)	r	s,σ				r	s,σ				r	s,σ	σ			r	s,σ	σ		
27	(5,20)		s,σ	r	s,σ			s,σ	r	s			s,σ	r	s,σ			s,σ	r	s	
28	(6,20)	r	s,σ		s,σ		r	s,σ		s		r	s,σ	σ	s,σ	σ	r	s,σ	σ	s	g
29	(7,19)		r					r					r					r			
30	(8,19)	r	r				r	r				r	r				r	r			
31	(7,20)		r		s,σ			r		s			r		s,σ			r		s	
32	(8,20)	r	r		s,σ		r	r		s		r	r		s,σ		r	r		s	
33	(9,17)			r					r	r				r					r	r	
34	(10,17)	r		r			r		r	r		r		r			r		r	r	
35	(9,18)			r	r				r	r				r	r				r	r	
36	(10,18)	r		r	r		r		r	r		r		r	r		r		r	r	
37	(11,17)									r										r	
38	(12,17)	r					r			r		r					r			r	
39	(11,18)				r					r					r					r	
40	(12,18)	r			r		r			r		r			r		r			r	
41	(9,19)		r	r				r	r				r	r				r	r		
42	(10,19)	r	r	r			r	r	r			r	r	r			r	r	r		
43	(9,20)		r	r	σ			r	r	g			r	r	σ			r	r	g	
44	(10,20)	r	r	r	σ	σ	r	r	r	g	g	r	r	r	σ	σ	r	r	r	g	g
45	(11,19)																				
46	(12,19)	r					r					r					r				
47	(11,20)			g	s,σ					s					s,σ					s	
48	(12,20)	r			s,σ		r			s		r			s,σ		r			s	
49	**(1,13)**	σ	s,σ	g	r	g	s,σ	s,σ	g	s	g	σ	s,σ	g	r	g	s,σ	s,σ	g	s	g
50	**(2,13)**	r	s,σ	g	r	g	r	s,σ	g	s	g	r	s,σ	g	r	g	r	s,σ	g	s	g
51	**(1,14)**	σ	s,σ	g	s	g	s,σ	s,σ	g	s	g	σ	s,σ	g	s	g	s,σ	s,σ	g	s	g
52	**(2,14)**	σ	s,σ	g	s	g	s,σ	s,σ	g	s	g	σ	s,σ	g	s	g	s,σ	s,σ	g	s	g
53	(3,13)	σ		r	r		s,σ		r	s		σ		r	r		s,σ		r	s	
54	(4,13)	r		r	r		r		r	s		r		r	r		r		r	s	
55	(3,14)	σ		r	s		s,σ		r	s		σ		r	s		s,σ		r	s	
56	(4,14)	σ		r	s		s,σ		r	s		σ		r	s		s,σ		r	s	
57	**(1,15)**	σ	r	g	r	g	σ	r	g	r	g	σ	r	g	r	g	s,σ	r	g	r	g
58	**(2,15)**	r	r	g	r	g	r	r	g	r	g	r	r	g	r	g	r	r	g	r	g
59	**(1,16)**	σ	r	g	r	g	σ	r	g	r	g	σ	r	g	r	g	s,σ	r	g	r	g
60	**(2,16)**	σ	r	g	r	g	σ	r	g	r	g	σ	r	g	r	g	s,σ	r	g	r	g
61	(3,15)	σ		r	r		s,σ		r	r		σ		r	r		s,σ		r	r	
62	(4,15)	r		r	r		r		r	r		r		r	r		r		r	r	
63	(3,16)	σ		r	r		s,σ		r	r		σ		r	r		s,σ		r	r	
64	(4,16)	σ		r	r		s,σ		r	r		σ		r	r		s,σ		r	r	
65	(5,13)	σ	s,σ	r	r	σ	s,σ	s,σ	r	s	s	σ	s,σ	r	r	σ	s,σ	s,σ	r	s	s
66	(6,13)	r	s,σ		r		r	s,σ		s		r	s,σ	σ	r	σ	r	s,σ	σ	s	g
67	(5,14)	σ	s,σ	r	s	g	s,σ	s,σ	r	s	s	σ	s,σ	r	s	g	s,σ	s,σ	r	s	s
68	(6,14)	σ	s,σ		s		s,σ	s,σ		s		σ	s,σ	σ	s	g	s,σ	s,σ	σ	s	g
69	(7,13)	σ	s,σ		r		s,σ	s,σ		s		σ	s,σ		r		s,σ	s,σ		s	
70	(8,13)	r	s,σ		r		r	s,σ		s		r	s,σ		r		r	s,σ		s	

71	(7,14)	σ	s,σ		s		s,σ	s,σ		s		σ	s,σ		s		s,σ	s,σ		s	
72	(8,14)	σ	s,σ		s		s,σ	s,σ		s		σ	s,σ		s		s,σ	s,σ		s	
73	(5,15)	σ	s,σ	r	r	σ	s,σ	s,σ	r	r	s,σ	σ	s,σ	r	r	σ	s,σ	s,σ	r	r	s,σ
74	(6,15)	r	s,σ		r		r	s,σ		r		r	s,σ	σ	r	σ	r	s,σ	σ	r	σ
75	(5,16)	σ	s,σ	r	r	σ	s,σ	s,σ	r	r	s,σ	σ	s,σ	r	r	σ	s,σ	s,σ	r	r	s,σ
76	(6,16)	σ	s,σ		r		s,σ	s,σ		r		σ	s,σ	σ	r	σ	s,σ	s,σ	σ	r	σ
77	(7,15)	σ	r		r		s,σ	r		r		σ	r		r		s,σ	r		r	
78	(8,15)	r	r		r		r	r		r		r	r		r		r	r		r	
79	(7,16)	σ	r		r		s,σ	r		r		σ	r		r		s,σ	r		r	
80	(8,16)	σ	r		r		s,σ	r		r		σ	r		r		s,σ	r		r	
81	(9,13)	σ		r	r		s,σ		r	g		σ		r	r		s,σ		r	s	
82	(10,13)	r		r	r		r		r	g		r		r	r		r		r	s	
83	(9,14)	σ		r	s		s,σ		r	g		σ		r	s		s,σ		r	s	
84	(10,14)	σ		r	s		s,σ		r	g		σ		r	s		s,σ		r	s	
85	(11,13)	σ			r		s,σ			g		σ			r		s,σ			s	
86	(12,13)	r			r		r			g		r			r		r			s	
87	(11,14)	σ			s		s,σ			g		σ			s		s,σ			s	
88	(12,14)	σ			s		s,σ			g		σ			s		s,σ			s	
89	**(9,15)**	σ	r	r	r	σ	σ	r	r	r	σ	σ	r	r	r	σ	σ	r	r	r	σ
90	**(10,15)**	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
91	**(9,16)**	σ	r	r	r	σ	σ	r	r	r	σ	σ	r	r	r	σ	σ	r	r	r	σ
92	**(10,16)**	σ	r	r	r	σ	σ	r	r	r	σ	σ	r	r	r	σ	σ	r	r	r	σ
93	(11,15)	σ			r		s,σ			r		σ			r		s,σ			r	
94	(12,15)	r			r		r			r		r			r		r			r	
95	(11,16)	σ			r		s,σ			r		σ			r		s,σ			r	
96	(12,16)	σ			r		s,σ			r		σ			r		s,σ			r	

According to Table 6.9, state (10, 15), also written as state 90, is the strong equilibrium in the market competition because it is Nash stable for all DMs. Among the common equilibria, there are 10 GMR and 3 SMR equilibria. No state is sequentially stable for all DMs under all preference patterns. State (10, 15) indicates a possible outcome that is most likely to happen because each DM has no incentive to move away from this state. The GMR and SMR equilibria suggest the outcomes that are also possible but are less likely to happen than state (10, 15).

The equilibria indicate possible outcomes for DMs in the market competition. At the Nash equilibrium, X signs deals with both A and B by purchasing A380s and B777s.

Customer Z selects A320neo as its next generation fleet. Airbus and Boeing have reached a draw in the wide body campaign, while the European Consortium has gained the upper hand in the narrow body battle. As a large airline carrier with a sufficient budget, Customer X can expand their fleets with both A380s and B777s. The two-deck jumbo jets will be used for carrying more passengers along transcontinental routes, while the B777s can supplement the existing fleets. This outcome has been validated by press release (Airbus, 2012; Planespotters, 2015), as Singapore Airlines intends to purchase both types of aircraft. The advantage of Airbus in the narrow body market results from the availability of the newly introduced A320neo. In comparison, B737MAX is also a next generation model to compete with A320neo. However, this model is still under development. The long delivery time of B737MAX and the high fuel efficiency of the A320neo drives the narrow body customer to select the next generation Airbus model. This outcome echoes the reality that Air Asia, a budget airline company, ordered A320neo aircraft (Sepang, 2014).

Other equilibria can also be considered as possible resolutions for DMs. For instance, a GMR equilibrium, state (2, 16), indicates the selection of A380s by X and the order of A320s by Z. No DM will move away from this state because each DM will sanction UIs for other DMs at the cost of disadvantaging itself. For example, Airbus will not promote the A320neo model, because it fears Z's subsequent move which is the selection of the American aircraft, although this move is unfavorable for Z.

It is worth noting that the difference in the LDMs' preference, reflected by the 4 preference patterns, does not affect the strong equilibria, and some of the other equilibria. In other words, state (10, 15) is the outcome most likely to happen regardless of whether each LDM is "pro-Airbus" or "pro-Boeing".

6.3.7 Comparison with the Results in Separate Competitions

Stability results in the overall competition are compared with the stable states in each local model to investigate their interrelations. The stable states for each DM and the equilibria in each local model are listed in Tables 6.10, 6.11, and 6.12. In each table, the stabilities for an LDM being "Pro-Airbus" and "Pro-Boeing" are both listed. The bolded states indicate the common equilibria in both tables. Note that in Table 6.12, the equilibria are listed under the solution concepts in "narrow sense", which means the interrelationships among the solution concepts are not considered. For example, the Nash equilibrium, state 10, in the wide body market is also SEQ, GMR, and SMR for all DMs, but it is only listed as the Nash equilibrium in Table 6.12.

The common Nash equilibrium (10, 15) in Table 6.9 is constituted by the common Nash equilibria in the local models: state 10 in the wide body competition and state 15 in the narrow body competition. However, the equilibria in the local models cannot predict all the equilibria in the hierarchical model. For example, a GMR equilibrium in the hierarchical model may contain a component state that is not a GMR equilibrium in a local model. At a common GMR equilibrium, state (2, 18), state 18 is not a GMR equilibrium in the narrow body competition under any preference pattern.

Moreover, a stable state for a given DM in the hierarchical model may also contain an unstable component for the focal DM in a local model. Taking Boeing as an example, state (2, 13) in Table 6.9 is both SEQ and SMR for the American manufacturer under all preference patterns. However, state 13 is not a stable component for B under any preference pattern in the narrow body competition, according to Table 6.11.

The UIs and possible sanctions for a given DM in the overall competition are analyzed under a particular preference pattern. For example, under pattern I, in which both X and Z are "pro-Airbus", state (7, 13) indicates the situation where the B787 beats the A330 in the wide body competition and Z intends to choose the A320neo. To regain Z's interest, Boeing plans to change its strategy by advocating the existing B737 model. As a subsequent UI, X can purchase the A330 instead of the B787. Although B can benefit in the narrow body market by reaching state (7, 15) from state (7, 13), the subsequent UI by X to state (3, 15) is less desired by the American manufacturer. As B considers the wide body market more important, it does not wish to benefit in the narrow body market at the cost of reaching a worse situation in its more valued market.

Still under pattern I, as a component in state (7, 13), state 13 is unstable for B in the narrow body model. The American company will move away from this state because it cannot recognize the sanction levied by X if it considers the two subcompetitions separate. The CDM will misjudge the situation in the two competitions due to ignorance of this sanction. Thus, the stability results in the hierarchical graph model can reflect the situations in the overall competition more precisely than those in the local models.

6.4 Summary

The sales competition between Airbus and Boeing in the wide and narrow body aircraft markets is investigated by employing the Hierarchical Graph Model for Conflict Resolution. The theoretical structure of this model containing two CDMs is defined such that the overall competition is modelled using the hierarchical graph model. The preference structure for CDMs in lexicographical order was constructed using the option prioritization method.

Table 6.10: Stability Results in the Wide Body Competition

	Pro-Airbus				Pro-Boeing			
	A	B	X	Equilibria	A	B	X	Equilibria
1		r	g	g	s,σ	r	g	g
2	r	r	g	g	r	r	g	g
3	σ		r		s,σ		r	
4	r		r		r		r	
5	r	s,σ	r	s,σ	r	s,σ	r	s,σ
6	r	s,σ			r	s,σ	σ	σ
7	r	r			r	r		
8	r	r			r	r		
9	σ	r	r	σ	σ	r	r	σ
10	r	r	r	r	r	r	r	r
11	σ	σ			s,σ	σ		
12	r	σ			r	σ		

Table 6.11: Stability Results in the Narrow Body Competition

	Pro-Airbus				Pro-Boeing			
	A	B	Z	Overall	A	B	Z	Overall
13	r		r		r		s	
14	σ		s		s,σ		s	
15	r	r	r	r	r	r	r	r
16	σ	r	r	σ	σ	r	r	σ
17	r				r		r	
18	r		r		r		r	
19	r	r			r	r		
20	r	r	σ	σ	r	r	g	g

Table 6.12: Equilibria in the Two Local Models

Equilibria	Wide Body	Narrow Body
Nash	10	15
SEQ	5	
GMR	1, 2	
SMR	9	16, 20

Furthermore, stability results calculated using the hierarchical model were compared with the results obtained by modelling separate competitions. The difference in stability results suggests that CDMs can have an enhanced understanding of the overall competition when it is analyzed using the hierarchical graph model methodology.

In the commercial aircraft competition, the duo hierarchical graph model can provide strategic insights and advice for DMs, without the requirement of various quantitative data such as retail price, operation costs, and other factors affecting DMs' actions. Instead, these data have been reflected by DMs' preferences, or preference patterns when some part of the preference relations are uncertain. It is worth noting that time is not considered in modelling the market competition. The hierarchical graph model only indicates what DMs should do by giving DMs' options and preferences. The resolutions for DMs can either be consistent with the reality or suggest a future outcome. Thus, these resolutions are more valuable if considered as a guidance of actions to follow when making strategic decisions.

The hierarchical graph model methodology can be further extended to having more DMs and more complex structures. A general hierarchical graph model, containing any number of DMs and local graph models, will be introduced in Chapter 7.

Chapter 7

General Hierarchical Graph Model

7.1 Introduction

In this chapter, a general hierarchical graph model for conflict resolution has been proposed. A general hierarchical graph model contains any number of CDMs, who supervise all the related conflicts, and LDMs, who only participate in one conflict. Different preference structures in the general hierarchical graph model are discussed. Theorems are provided to investigate the relationship between the stabilities in the hierarchical model and the stabilities in the component graph models. The hierarchical graph mode is applied to greenhouse gas emissions disputes between the USA and China. The stability results suggest how the two countries can obtain strategic resolutions for the bilateral disputes.

7.2 Formal Definitions

Recall that a graph model is defined as a 4-tuple set, written as $\{N, S, A, \succsim\}$, where N, S, A, and $\succsim$ represent DMs, states, arcs of moves, and preference relations, respectively.

A general hierarchical graph model consists of several smaller graph models, called local graphs. There are two types of DMs in a general hierarchical graph model. The DMs who participate in all local graphs are called common decision makers (CDMs). Other DMs that only appear in one local graph are defined as local decision makers (LDMs).

7.2.1 Definition of General Hierarchical Graph Model

A formal definition of a general hierarchical graph model is given below.

Definition 7.1 Suppose K graph models are written as

$$G^{(k)} = \{N_C, N_L^{(k)}, S^{(k)}, AC^{(k)}, AL^{(k)}, \succsim_C^{(k)}, \succsim_L^{(k)}\},$$

for $k = 1, ..., K$.

Let N_C denote the set of CDMs; $N_L^{(k)}$ the set of LDMs in $G^{(k)}$; $S^{(k)}$ the set of states in $G^{(k)}$; $AC^{(k)}$ and $AL^{(k)}$ the sets of arcs for CDMs and LDMs, respectively in $G^{(k)}$; $\succsim_C^{(k)}$ and $\succsim_L^{(k)}$ the preference relations for CDMs and LDMs, respectively, in $G^{(k)}$.

Then, the graph model

$G = \{N_C, N_L, AC, AL, S, \succsim_C, \succsim_L\}$ is called a *general hierarchical graph model*, where

$$N_L = \bigcup_{k=1}^{K} N_L^{(k)};$$

$$\bigcup_{k=1}^{K} AC^{(k)} \subseteq AC;$$

$$AL = \bigcup_{k=1}^{K} AL^{(k)};$$

$$S = S^{(1)} ... \times S^{(K)},$$

denoting the set of states in G is a Cartesian Product of the sets of states in local graphs. State $s \in S$ can be written as $s = (s^{(1)}, ..., s^{(K)})$, where $s^{(K)} \in S^{(K)}$.

The moves and preferences for DMs in the general hierarchical model satisfy the following properties.

(i) For $s_a^{(k)}, s_b^{(k)} \in S^{(k)}$ and $s_a = (s_a^{(1)}, ..., s_a^{(K)}) \in S, s_b = (s_b^{(1)}, ..., s_b^{(K)}) \in S$, $AC \subseteq S \times S$ is defined by $(s_a, s_b) \in AC$ iff there exists $k = 1, ..., K$ for $(s_a^{(k)}, s_b^{(k)}) \in AC^{(k)}$ and $s_a^{(k')} = s_b^{(k')}$ for all $k' = 1, ..., K$ except k.

(ii) Moves for $LDM_l^{(k)} \in N_L^{(k)}$ in G, short for $l_k \in N_L^{(k)}$, $AL_{l_k} \subseteq S \times S$, is defined by $(s_a, s_b) \in AL_{l_k}$ iff $(s_a^{(k)}, s_b^{(k)}) \in AL_l^{(k)}$ and $s_a^{(k')} = s_b^{(k')}$ for all $k' = 1, ..., K$ except k.

(iii) The preference relation for l_k, $\succsim_{l_k}$, in G is defined by $s_a \succsim_{l_k} s_b$ iff $s_a^{(k)} \succsim_{l_k}^{(k)} s_b^{(k)}$.

(iv) The preference relation ($\succsim_C$) for a CDM, $CDM_i \in N_C$, in G is related to the preference relation $\succsim_C^{(k)}$ as follows:

If $s_a^{(k)}, s_b^{(k)} \in S^{(k)}$ and $s_a^{(k)} \succsim_{C_i}^{(k)} s_b^{(k)}$, then $s_a \succsim_{C_i} s_b$ for all $k' = 1, ..., K$ except k, such that $s_a^{(k')} = s_b^{(k')}$.

Property (iv) can be used to determine the relative preferences for CDMs for some states in G, but not all of them. In Property (iv), suppose $s_a^{(k)} \succsim_{C_i}^{(k)} s_b^{(k)}$, the relation between states s_a and s_b for CDM_i is unknown when $s_a^{(k')} \prec_{C_i}^{(k')} s_b^{(k')}$ for some $k' \neq k$. Hence, to determine the preference relation between states s_a and s_b for CDMs in G, extra information is required.

The structure of the general hierarchical graph model is depicted in Fig. 7.1. The nodes in Fig. 7.1 represent CDMs and LDMs. The boundary of a local graph is expressed by a dashed circle. An ellipsis ("...") between two DMs in a local graph model means the number of DMs in the local graph can vary. An ellipsis between two dashed circle denotes that there can be any number of local graphs in the general hierarchical graph model.

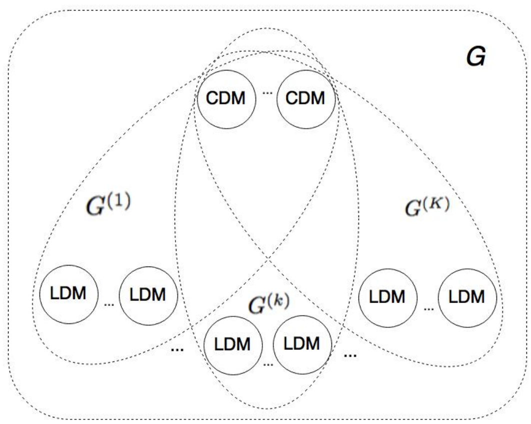

Figure 7.1: Structure of general hierarchical graph model

7.3 Preference Structures based on Option Prioritization

Due to the incompleteness of preference information for CDMs in G, relative importance of local graphs is defined for each CDM. The relative importance reflects the focal CDM's perception of the importance of each local graph. The importance relations between two local graphs are defined in Definition 7.2.

Definition 7.2: (Pairwise Importance) In a hierarchical graph model G, the relative importance between two local graphs, $G^{(k)}$ and $G^{(k')}$ $(k, k' = 1, ..., K)$, for $CDM_i \in N_C$ is indicated by the relation $>_i$ (strictly more important than) as $G^{(k)} >_i G^{(k')}$.

Note that weakly ordered importance, which means "no less important than", is not considered for the sake of model simplicity.

If all local graphs in G can be ordered as $G^{(\tau_1)} >_i G^{(\tau_2)} ... >_i G^{(\tau_K)}$ for $\tau_k = 1, 2, ..., K$, then G is said to be transitive for CDM_i. Note that $G^{(\tau_1)}, ..., G^{(\tau_K)}$ is identical to $G^{(1)}, ..., G^{(K)}$, except that they are ordered by the relative importance for CDM_i. The transitive importance is defined in Definition 7.3.

Definition 7.3: (Transitive Importance) In the general hierarchical graph model G, a transitive ordering of the local graphs by importance for CDM_i $(CDM_i \in N_C)$ can be written as $G^{(\tau_1)} >_i G^{(\tau_2)} ... >_i G^{(\tau_K)}$, where $>_i$ is the importance relation between local graphs satisfying:

$G^{(\tau_{k-1})} >_i G^{(\tau_k)}$ and $G^{(\tau_k)} >_i G^{(\tau_{k+1})}$ implies $G^{(\tau_{k-1})} >_i G^{(\tau_{k+1})}$.

The preferences for DMs in G are determined using the option prioritization method, which has been demonstrated in Chapters 5 and 6. Weighted preference and lexicographic preference in the general hierarchical graph model are defined below.

Definition 7.4: (Weighted Preference) Let $\Omega^{(k)}_{C_i}$ $(k = 1, ..., K)$ be the set of preference statements for CDM_i in $G^{(k)}$, and $\Omega^{(k)}_{C_i j_k} \in \Omega^{(k)}_{C_i}$ for $0 < j_k < h_k$. Suppose a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ where $s^{(K)} \in S^{(K)}$. A score $\Psi^{(k)}_{C_i}(s^{(k)})$ is assigned to state $s^{(k)}$ for CDM_i.

Then,

$$\Psi_{C_i}^{(k)}(s_k) = \sum_{j_k=1}^{h_k} sgn\left(\Omega_{C_i j_k}^{(k)}(s_k)\right) 2^{h_k - j_k} \tag{7.1}$$

where

$$sgn\left(\Omega_{C_i j_k}^{(k)}(s_k)\right) = \begin{cases} 1 & \Omega_{C_i j_k}^{(k)}(s_k) = T \\ 0 & \text{otherwise} \end{cases}$$

The relative importance of $G^{(k)}$ for CDM_i is denoted by a weight $w_{C_i}^{(k)}$ ($w_{C_i}^{(k)} > 0$). Then $G^{(k)} >_i G^{(k')}$ indicates $w_{C_i}^{(k)} > w_{C_i}^{(k')}$ for $k' = 1, ...K$ and $k \neq k'$.

Thus, the aggregated score for state $s \in S$ in G is defined as:

$$\Psi_{C_i}(s) = \sum_{k=1}^{K} w_{C_i}^{(k)} \Psi_{C_i}^{(k)}(s_k) \tag{7.2}$$

Equation (7.2) is a general expression of $\Psi_{C_i}(s)$ as an aggregation of scores for the component states, $\Psi_{C_i}^{(k)}(s^{(k)})$ $(k = 1, ..., K)$, and the weights for local graphs, $w_{C_i}^{(k)}$.

Definition 7.5 (Lexicographic Preference) Suppose $G^{(k)} >_{C_i} G^{(k')}$ for $k, k' = 1, ..., K$, then the lexicographic preference for CDM_i is defined as

(1) Suppose $s = (s^{(1)}..., s^{(K)}) \in S$ and $t = (t^{(1)}..., t^{(K)}) \in S$ for $s^{(k^*)} = t^{(k^*)}$ ($k^* = 1, 2, ..., K$ except k and k'). If $s^{(k)} \succ_{C_i}^{(k)} t^{(k)}$, then $s \succ_{C_i} t$

(2) If $s^{(k)} \sim_{C_i}^{(k)} t^{(k)}$, then

1) $s \succ_{C_i} t$ if $s^{(k')} \succ_{C_i}^{(k')} t^{(k')}$;

2) $s \sim_{C_i} t$ if $s^{(k')} \sim_{C_i}^{(k')} t^{(k')}$;

3) $s \prec_{C_i} t$ if $s^{(k')} \prec_{C_i}^{(k')} t^{(k')}$.

The relation between weighted preference and lexicographic preference is investigated in Theorem 7.1.

Theorem 7.1: Suppose in a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, $w_{C_i}^{(k_1)}$ and $w_{C_i}^{(k_2)}$ denote the weight of $G^{(k_1)}$ and $G^{(k_2)}$ for CDM_i $(k_1, k_2 = 1, 2, ..., K)$ and $G^{(k_1)} >_{C_i} G^{(k_2)}$. Let the preference statements in $G^{(k_1)}$ and $G^{(k_2)}$ be $\Omega_{C_i j_{k_1}}^{(k_1)} \in \Omega_{C_i}^{(k_1)}$ and $\Omega_{C_i j_{k_2}}^{(k_1)} \in \Omega_{C_i}^{(k_1)}$, for $0 < j_{k_1} < h_{k_1}$ and $0 < j_{k_2} < h_{k_2}$. Then, the weighted preference is a lexicographic preference when $\frac{w_{C_i}^{(k_1)}}{w_{C_i}^{(k_2)}} > 2^{h_{k_2}} - 1$. □

Theorem 7.1 presents the condition for the weighted preference to become a lexicographic preference. For a pair of local graphs $G^{(k_1)}$ and $G^{(k_2)}$, the ratio of their weights for a CDM, $\frac{w_{C_i}^{(k_1)}}{w_{C_i}^{(k_2)}}$, is related with the number of its preference statements in $G^{(k_2)}$.

The relation between CDM_i's preference statements in G, $\Omega_{C_i j}$ for $0 < j < h$, and its preference statements in local graphs, $\Omega_{C_i j_k}^{(k)}$ for $0 < j_k < h_k$, is discussed in Theorem 7.2.

Theorem 7.2: (Preference statements) In a hierarchical graph model G consisting of $G^{(1)}$ ..., $G^{(K)}$, let $\Omega_{C_i j_k}^{(k)} \in \Omega_{C_i}^{(k)}$ $(k = 1, 2, ..., K;\ j_k = 1, 2, ..., h_k)$ be the preference statements for $CDM_i \in N_C^{(k)}$ in $G^{(k)}$, and $\Omega_{C_i j} \in \Omega_{C_i}$ $(j = 1, 2, ..., h)$ be the set of preference statements for CDM_i in G, with $\Omega_{C_i j} = \Omega_{C_i j_k}^{(k)}$. If CDM_i's preference is lexicographic in G with $G^{(\tau_1)} ... >_{C_i} G^{(\tau_K)}$ $(\tau_k = 1, ..., K)$, then

$$\Omega_{C_i} = \bigcup_{k=\tau_1}^{\tau_K} \Omega_{C_i}^{(k)} \tag{7.3}$$

and $\Omega_{C_i j} \in \Omega_{C_i}$ can be written in order as:

$\Omega_{C_i} = \{\Omega^{(\tau_1)}_{C_i1}, ..., \Omega^{(\tau_1)}_{C_ih_1}, \Omega^{(\tau_2)}_{C_i1} ..., \Omega^{(\tau_2)}_{C_ih_2}, ..., \Omega^{(\tau_K)}_{C_i1}, ..., \Omega^{(\tau_K)}_{C_ih_K}\}$ for $h = h_1... + h_K$.

The proof of Theorem 7.2 is analogous to Theorem 6.1. Corollary 7.1 is given to discuss the weight $w^{(k)}_{C_i}$ in the two types of preference.

Corollary 7.1: Recall that $\Omega^{(k)}_{C_ij_k} \in \Omega^{(k)}_{C_i}$ ($k = 1, 2, ..., K$; $j_k = 1, ..., h_k$) represents the preference statements for $C_i \in N^{(k)}_C$ in $G^{(k)}$, and $\Omega_{C_ij} \in \Omega_{C_i}$ ($j = 1, ..., h$) represents the set of preference statements for CDM_i in G, with $\Omega_{C_ij} = \Omega^{(k)}_{C_ij_k}$, and state $s = (s^{(1)}, ..., s^{(K)}) \in S$ for $s^{(k)} \in S^{(k)}$,

(1) For the weighted preference,

$$\Psi_{C_i}(s) = \sum_{k=1}^{K} \Psi^{(k)}_{C_i}(s^{(k)}) w^{(k)}_{C_i} \tag{7.4}$$

where

$$\Psi^{(k)}_{C_i}(s^{(k)}) = \sum_{j_k=1}^{h_k} \Psi^{(k)}_{C_ij_k}(s^{(k)}).$$

and

$$\Psi^{(k)}_{C_ij_k}(s^{(k)}) = \begin{cases} 2^{h_k-j_k} & \Omega^{(k)}_{C_ij_k}(s^{(k)}) = T \\ 0 & otherwise \end{cases}$$

(2) For the lexicographic preference, Equation (7.4) exists, plus

$$w^{(k)}_{C_i} = 2^{h_{k+1}+...+h_K} \tag{7.5}$$

and

$$h = \sum_{k=1}^{K} h_k \tag{7.6}$$

□

As indicated in Corollary 7.1, the lexicographic preference is a special case of the weighted preference when Equations (7.5) and (7.6) are satisfied.

Preferences for a CDM in G can be determined in Theorem 7.2. Thus, to simplify the calculation for stabilities in G, the preferences for all DMs in G are assumed to be lexicographic in the case study as mentioned in Section 7.5.

7.4 Interrelationships of Stabilities between General Hierarchical Graph Model and Local Graph Models

As demonstrated in Section 4.2, stabilities in the general hierarchical graph model cannot be totally predicted from the stabilities in each local graph. However, several connections can be concluded under each solution concept. Theorems are proposed to demonstrate the interrelation between stabilities in G and those in each local graph. Representative proofs for some theorems are provided. Proofs for other theorems are omitted because they are analogous.

7.4.1 Nash (R)

Theorem 7.3: A state $s = (s^{(1)}, ..., s^{(K)}) \in S$ in general hierarchical graph model G is Nash stable for $CDM_i \in N_C$ if for all $k = 1, ..., K$, $s^{(k)} \in S^{(k)}$ is Nash stable for CDM_i in $G^{(k)}$. $\square$

According to Theorem 7.3, a state in the general hierarchical graph model is Nash stable for a CDM if all of its component states are Nash stable in the corresponding local graphs. Note that state s is Nash stable for CDM_i in G does not imply that $s^{(k)} \in S^{(k)}$ is Nash in $G^{(k)}$ for all $k = 1, ..., K$, but for at least one of them.

Theorem 7.4: A state $s = (s^{(1)}, ..., s^{(K)}) \in S$ in general hierarchical graph model G is Nash stable for $l_k \in N_L^{(k)}$ iff $s^{(k)} \in S^{(k)}$ is Nash stable for l_k in $G^{(k)}$. $\square$

Theorem 7.4 indicates that Nash stability for an LDM in the general hierarchical graph model is only determined by the Nash stability in the LDM's local graph.

7.4.2 Sequential Stability (SEQ)

Theorem 7.5: A state $s = (s^{(1)}, ..., s^{(K)}) \in S$ in general hierarchical graph model G is sequentially stable for CDM_i if for all $k = 1, ..., K$, $s^{(k)} \in S^{(k)}$ is sequentially stable for CDM_i in $G^{(k)}$. $\square$

Theorem 7.5 is similar to Theorem 7.3: a state in the general hierarchical graph model is SEQ for a CDM if all of its component states are SEQ in the corresponding local graphs.

Theorem 7.6 (SEQ for CDM): Suppose G is a general hierarchical graph model consisting of $G^{(k)}$ for $k = 1, ..., K$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is sequentially stable for CDM_i in G, then

(1) either $s^{(k)}$ is sequentially stable for CDM_i in $G^{(k)}$ for every $k = 1, ..., K$ or

(2) if $s^{(k)}$ is not sequentially stable for CDM_i in $G^{(k)}$ for some $k = 1, ..., K$, but not all, there exists state $r \in R^{+}_{N-C_i}(q)$ for every $q \in R^{+}_{C_i}(s)$, such that $r \precsim_{C_i} s$, where $R^{+}_{N-C_i}(q)$ is the set of UIs from state q by all DMs in G except CDM_i. $\square$

Situation (1) in Theorem 7.6 is identical to Theorem 7.5. Situation (2) in Theorem 7.6 indicates that a state in the general hierarchical graph model can be SEQ for the focal CDM if some component states are not SEQ in the corresponding local graphs.

Theorem 7.7 (SEQ for LDMs): In a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is sequentially stable for $l_k \in N_L^{(k)}$ in G, then

(1) either $s^{(k)}$ is SEQ for l_k in $G^{(k)}$, or

(2) $s^{(k)}$ is GMR for l_k in $G^{(k)}$ and there exist $r^{(k')} \in R_{N^{(k')}}(s^{(k')})$ for $k' = 1, ..., K$ except k, such that $r = (r^{(1)}, ..., r^{(k)}, ..., r^{(K)}) \in R^{+}_{N-l_k}(s)$ and $r \precsim_{l_k} s$, where $R_{N^{(k')}}$ denotes the set of joint UMs by all DMs in $G^{(k')}$ and $R^{+}_{N-l_k}$ the set of UIs by all DMs in G except l_k. $\square$

A state in the general hierarchical graph model is SEQ for an LDM if the component state describing the LDM's actions is SEQ in its local graph. If the component state is not SEQ for the LDM, the conditions indicated in situation (2) in Theorem 7.7 should be satisfied.

7.4.3 General Metarationality (GMR)

Theorem 7.8 (GMR for CDM): In a general hierarchical graph model G consisting of $G^{(1)}, G^{(2)}, ..., G^{(k)}, ..., G^{(K)}$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is general metarational for CDM_i in G, and $q = (q^{(1)}, ..., q^{(K)}) \in R^{+}_{C_i}(s)$, then

(1) either $s^{(k)}$ is general metarational for CDM_i in $G^{(k)}$ for every $k = 1, 2, ..., K$ or

(2) if $s^{(k)}$ is not GMR for CDM_i in $G^{(k)}$ ($k = 1, 2, ..., K$, but not all), there exists $r \in R_{N-C_i}(q)$ for every $q \in R^+_{C_i}(s)$, such that $r = (r^{(1)}, ..., r^{(K)}) \precsim_{C_i} s$. □

Theorem 7.8 is analogous to Theorem 7.6. The difference between the two theorems is that in Theorem 8, the UMs from DMs in G other than the focal CDM are considered.

Theorem 7.9 (GMR for LDM): In a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is general metarational for l_k in G if and only if $s^{(k)}$ is general metarational for l_k in $G^{(k)}$. □

Theorem 7.9 is simpler than Theorem 7.7 in that, to determine GMR states for l_k in G, only the GMR states in $G^{(k)}$ for l_k need to be considered.

7.4.4 Symmetric Metarationality (SMR)

Theorem 7.10 (SMR for CDM): In a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is symmetric metarational for CDM_i in G, and $q = (q^{(1)}, ..., q^{(K)}) \in R^+_{C_i}(s)$, then

(1) either $s^{(k)}$ is symmetric metarational for CDM_i in $G^{(k)}$ for every $k = 1, ..., K$ or

(2) if $s^{(k)}$ is GMR but not SMR for CDM_i in $G^{(k)}$ ($k = 1, ..., K$, but not all), for every $q \in R^+_{C_i}(s)$, there exists $t \in R_{C_i}(r)$, such that $t = (t^{(1)}, ..., t^{(K)}) \precsim_{C_i} s$, where $r = (r^{(1)}, ..., r^{(K)}) \precsim_{C_i} s$ and $r \in R_{N-C_i}(q)$. □

Theorem 7.10 indicates that a state in the hierarchical graph model is SMR for a CDM if every component state in the local graph is SMR. A state in the hierarchical graph model can be SMR for the focal CDM if some component states are GMR but not SMR in local graphs.

Theorem 7.11 (SMR for LDM): In a general hierarchical graph model G consisting of $G^{(1)},...,G^{(K)}$, a state $s = (s^{(1)},...,s^{(K)}) \in S$ is symmetric metarational for l_k in G if and only if $s^{(k)}$ is symmetric metarational for l_k in $G^{(k)}$. $\square$

7.5 Steps for Calculating Stabilities

The steps for calculating stabilities in the general hierarchical graph model are given for CDMs and LDMs. To model hierarchical conflicts, each subconflict is considered as a local graph. DMs and their options in each local graph are identified. Preference statements for DMs in a local graph are provided.

For CDMs, the preference statements in G can be determined according to Theorem 7.2. For LDMs, the preference statements in local graphs are the same as those in the hierarchical graph. The calculation for stabilities under different solution concepts is carried out using the decision support system GMCR II. Stable states for each DM and equilibria, defined as the stable states for all DMs, can be obtained. A stable state indicates a status in the real world conflict, at which a focal DM will stop under a particular solution concept. Stable states for all DMs, also called equilibria, indicate outcomes that are more likely to happen than others. The steps for stability calculation are listed below.

Step 1: Divide G into $G^{(1)},...,G^{(K)}$;

Step 2: Identify CDMs and LDMs ($N_C^{(k)}$ and $N_L^{(k)}$), options, and preference statements $\Omega_{C_i}^{(k)}$ in each $G^{(k)}$ ($k = 1,...,K$);

Step 3: For CDMs: determine preference statements Ω_{C_i} in G according to Theorem 7.2; For LDMs: preference statements for l_k in G and $G^{(k)}$ are the same: $\Omega_{l_k}^{(k)} = \Omega_{l_k}$;

Step 4: Calculate stabilities in G using GMCR II.

7.6 Greenhouse Gas Emissions Disputes between China and USA

7.6.1 Conflict Background

The general hierarchical graph model is applied to disputes between USA and China over adhering to the bilateral climate change agreement. On November 11th, 2014, the two superpowers reached a deal to curb carbon emissions in the next few decades (Goldenberg et al., 2014). According to the treaty, China committed to limiting carbon emissions by 2030 and increase the percentage of clean energy use to 20%. Meanwhile, the US agreed to emit 26% to 28% less than the 2005 carbon levels by 2025.

This agreement has shown the determination of the two nations in reducing carbon emissions by cooperation. However, challenges and disputes take place in both nations. In the US Congress, the Republicans threatened to block this deal, because many of the representatives have connections with traditional energy industries. They believed that this agreement would result in fewer jobs and higher energy prices. Although the White House claimed that the abatement target for the US is achievable under the existing environmental laws, experts have pointed out that these goals are hard to meet without new legislation (Levi, 2014). However, as the Republicans have held the majority of seats in Congress, the new climate laws are likely to be blocked by them. Furthermore, the emissions targets need a few decades to be achieved. Because the actions for the next administration are unknown, the future of this agreement is in doubt.

In China, large efforts should be made by the government to reach the abatement goal. As China still relies heavily on traditional energy, the energy industry needs to be reformed by stricter environmental laws. For fear of losing profits, stakeholders such as state-owned energy companies would actively oppose these laws by lobbying the government using their political influence.

Despite the fact that the two countries have successfully cooperated in handling many global issues in the 21^{st} century, they may still be suspicious about whether their counterpart would fully adhere to the climate change agreement, due to the unpleasant memory of decades of antagonism in the Cold War. The Republicans attacked the current treaty by accusing it of being a mere strategy of buying time on the part of China (Yuhas, 2014). Dubious sentiment is also widespread in China. Many consider the agreement to be a US conspiracy to undermine social and economic development in China.

7.6.2 Conflict Modeling

The disputes regarding the climate change agreement are analyzed using the general hierarchical graph model to predict likely outcomes and provide each DM with possible resolutions. The disputes among DMs in the US and China are represented by two local graph models, called US graph and China graph. The Chinese government and the White House are the two CDMs, marked as CG and WH, respectively. In the US graph, denoted as $G^{(1)}$, the Republicans (GOP) and the Democrats (USS) are two LDMs. In the China graph, denoted as $G^{(2)}$, state owned energy companies (ECs) is the only LDM. The DMs and their options in the climate change conflict are shown in Table 7.1.

In Table 7.1, each option is marked with a number followed by a half parenthesis. WH has two options in the climate change conflict. To reach the abatement goals, it will propose new environmental laws and expect the approval of these laws in Congress, which is denoted as option 1). Note that an option in Table 7.1 represents two possible actions for a focal DM. The negation of option 1) means that WH does not propose any environmental law. WH will also decide whether to comply with the climate agreement, marked as option 2). The option for the Republicans to block the approval of these laws is represented as option 3). Option 4) denotes the support of the Democrats for the current Obama administration. The Chinese government (CG) will reform the energy industry and force ECs to comply if they pose any opposition (option 5)). CG will also consider whether to adhere to the climate agreement (option 6)). Option 7) indicates the compliance of ECs to CG's reform instructions.

The climate change conflict can be divided into two component conflicts, in the US and China, respectively. The DMs and their options in two local graph models are listed in Tables 7.2 and 7.3. The two CDMs are marked in bold.

The preferences for DMs in each local graph model are expressed using the option prioritization method. In Tables 7.4 and 7.5, the preference statements for WH and CG are listed. Taking Table 7.4 as an example, the first statement for WH in the US subconflict is option 6), which means WH expects CG to adhere to the agreement. The next statement, written as 1 & -3, indicates that WH plans to propose new environmental laws that will not be blocked by GOP. To follow up, WH does not wish to see any opposing actions from

Table 7.1: DMs and Options in the Overall Conflict

DMs	Options
The White House (WH)	1) Propose extra environmental laws
	2) Comply with the climate agreement
Republicans (GOP)	3) Block the laws in Congress
Democrats (USS)	4) Support the WH
Chinese Government (CG)	5) Pressure ECs to reform the energy industry
	6) Comply with the climate agreement
State Owned Energy Companies (ECs)	7) Follow the reform instructions

Table 7.2: DMs and Options in the US Subconflict

DMs	Options										
WH	1) New Laws	N	Y	Y	Y	Y	N	Y	Y	Y	Y
GOP	3) Block	N	N	Y	N	Y	N	N	Y	N	Y
USS	4) Support	N	N	N	Y	Y	N	N	N	Y	Y
CG	6) Comply	N	N	N	N	N	Y	Y	Y	Y	Y
	State Number	1	2	3	4	5	6	7	8	9	10

Table 7.3: DMs and Options in the China Subconflict

DMs	Options						
WH	2) Comply	N	N	N	Y	Y	Y
CG	5) Pressure ECs	Y	N	Y	Y	N	Y
ECs	7) Follow CG	N	Y	Y	N	Y	Y
	State Number	11	12	13	14	15	16

GOP (-3). In particular, the support from USS is desired ((-3) & 4). If GOP blocks the passing of new laws, WH still expects support from USS (4). In Table 7.4, the statements for WH in the China subconflict are analogously listed in the right column. The preference statements for the three LDMs in the subconficts, GOP, USS, and ECs, are shown in Table 7.6.

The preferences for WH and CG in the overall conflict are obtained by combining preference statements in the two subconflicts according to Theorem 7.1. Because WH considers the US subconflict more important, the preference statements in the US subconflict are listed before the statements in the China subconflict, shown in Table 7.7. The preference statements for CG in the overall conflict are described in Table 7.8. As CG regards the China subconflict as more important, the preference statements for CG in the China subconflict are placed first. The preferences for LDMs in the overall conflict can be represented with the same statements in the corresponding subconflicts.

Table 7.4: Preference Statements for WH in Each Subconflict

US Subconflict	China Subconflict
6	5
1 & -3	7
-3	2 IF 5
(-3) & 4	-(2&-5)
4	

Table 7.5: Preference Statements for CG in Each Subconflict

US Subconflict	China Subconflict
1	2
-3	(-5) & 7
4	7
6 IF 1	5 IF -7
-(6 & -1)	

Table 7.6: Preference Statements for GOP, USS, and ECs

GOP	USS	ECs
-1	4	-5
3 IF 1	-3	-7
-4		(-7) IF 5

Table 7.7: Preference Statements for WH in the Overall Conflict

The Overall Conflict
6
1 & -3
-3
(-3) & 4
4
5
7
2 IF 5
-(2&-5)

Table 7.8: Preference Statements for CG in the Overall Conflict

The Overall Conflict
2
(-5) & 7
7
5 IF -7
1
-3
4
6 IF 1
-(6 IF -1)

7.6.3 Removal of Infeasible States

The four options in the US subconflict ($G^{(1)}$) and three options in the China subconflict ($G^{(2)}$) result in 2^4 and 2^3 states in the respective local graphs. Some states are removed because they are logically or preferentially infeasible in practice. The infeasibility in each local graph can also be described by preference statements. In the US subconflict, it is logically impossible for GOP to block (option 3) when WH does not propose any new environmental legislation (-1), expressed as "(-1) & 3". Additionally, the support from USS (option 4) cannot happen when no new law is proposed by WH, marked as "(-1) & 4". In the China subconflict, it is preferentially impossible for ECs to oppose CG when it does not exert pressure. Thus, "(-5) & -7" is infeasible. By removing these infeasible states, there are 10 states in $G^{(1)}$ and 6 states in $G^{(2)}$. The states in $G^{(1)}$ are numbered from state 1 to state 10, and the states in $G^{(2)}$ are marked from state 11 to state 16, shown in Tables 7.2 and 7.3, respectively.

7.6.4 Stability Analysis

The stabilities for DMs in the overall conflict are calculated using GMCR II. This decision support system provides an effective way of stability calculation using the option prioritization method. DMs and their options in G are the important inputs in the "Decision Maker and Options" Panel, shown in Fig. 7.2. Preference statements for each DM are inputed in the "Option Prioritization" Panel in Fig. 7.3. By clicking the "Analysis" button listed on the top of Fig. 7.2, stabilities can be calculated. Fig. 7.4 is a screen to show some equilibria in G under different solution concepts. For example, state 3, which can be also written as state (1, 14), is GMR, SEQ, and SMR equilibria in G.

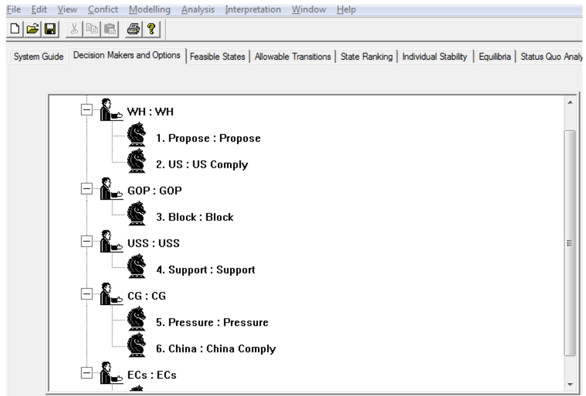

Figure 7.2: Decision makers and options panel in GMCR II

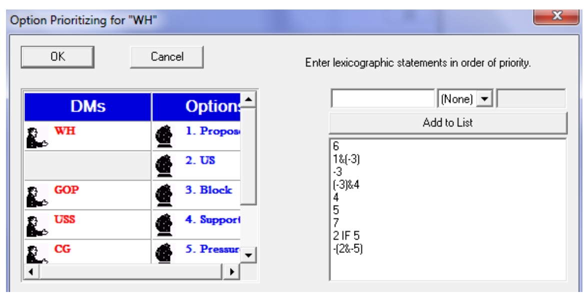

Figure 7.3: Option prioritization panel in GMCR II

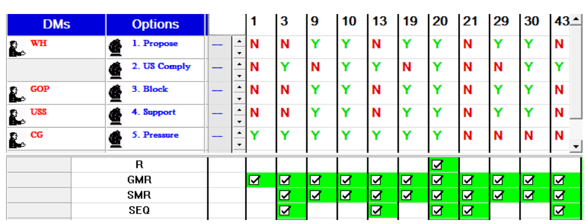

Figure 7.4: Stability results panel in GMCR II

The stability results for individual DMs and all DMs are contained in Table 7.9. By excluding infeasible states, the 60 states in the overall conflict are numbered from state 1 to 60 in the 9th column of Table 7.9. Each state is also expressed as a pair of numbers contained by parentheses, shown in the 10th column in the same table. Each number represents a state number in the corresponding local graph. For example, state 3 can also be written as state (1, 14), in which the first entry denotes state 1 in the US subconflict and the second entry represents state 14 in the China subconflict. The selection of options for DMs in each state is also presented. Four types of solution concepts, Nash stability (R), sequential stability (SEQ), general metarationality (GMR), and symmetric metarationality (SMR), are investigated for each DM and all DMs. They are abbreviated as "r", "s", "g", and "δ", respectively, to save space in Table 7.9. The stabilities for all DMs, listed in the last column with "Overall" on top, indicate the equilibria of the conflict. States 20 and 49 are two Nash equilibria. SEQ equilibria include states 3, 13, 21, and 43, all of which are also SMR equilibria. Other SMR equilibria are states 9, 10, 19, 29, 30, and 50. State 1 is the only GMR equilibrium.

The two Nash equilibria, states 20 and 49, indicate outcomes that are likely to happen. At state 20, which can be written as state (10, 14), both governments will adhere to the agreement. GOP would block WH's proposal of new environmental laws that are supported by USS. The pressure from CG on reforming the energy industry would be resisted by ECs. Another Nash equilibrium, state 49 (state (10, 12)), suggests that WH cannot reach the abatement goals. ECs will follow CG's instruction to reform even when CG does not exert pressure. Although the two equilibria can both indicate possible outcomes theoretically, further investigation is needed to determine which one would actually evolve from the starting state of the conflict.

Table 7.9: The Overall Conflict in Option Form and Stabilities

DMs	WH		GOP	USS	CG		ECs			WH	GOP	USS	CG	ECs	Overall
Option Num.	1)	2)	3)	4)	5)	6)	7)	State Num.	State Num.						

Options	N	N	N	N	Y	N	N	1	(1,11)	g	r	r	r	r	g
	Y	N	N	N	Y	N	N	2	(2,11)	s, δ			s, δ	r	
	N	Y	N	N	Y	N	N	3	(1,14)	s, δ	r	r	r	r	s, δ
	Y	Y	N	N	Y	N	N	4	(2,14)	r			s, δ	r	
	Y	N	Y	N	Y	N	N	5	(3,11)	δ	r		δ	r	
	Y	Y	Y	N	Y	N	N	6	(3,14)	r	r		δ	r	
	Y	N	N	Y	Y	N	N	7	(4,11)	s, δ		r	s, δ	r	
	Y	Y	N	Y	Y	N	N	8	(4,14)	r		r	s, δ	r	
	Y	N	Y	Y	Y	N	N	9	(5,11)	δ	r	r	δ	r	δ
	Y	Y	Y	Y	Y	N	N	10	(5,14)	r	r	r	δ	r	δ
	N	N	N	N	Y	Y	N	11	(6,11)	s, δ	r	r		r	
	Y	N	N	N	Y	Y	N	12	(7,11)	s, δ			r	r	
	N	Y	N	N	Y	Y	N	13	(6,14)	s, δ	r	r	s, δ	r	s, δ
	Y	Y	N	N	Y	Y	N	14	(7,14)	r			r	r	
	Y	N	Y	N	Y	Y	N	15	(8,11)	δ	r		r	r	
	Y	Y	Y	N	Y	Y	N	16	(8,14)	r	r		r	r	
	Y	N	N	Y	Y	Y	N	17	(9,11)	s, δ		r	r	r	
	Y	Y	N	Y	Y	Y	N	18	(9,14)	r		r	r	r	
	Y	N	Y	Y	Y	Y	N	19	(10,11)	δ	r	r	r	r	δ
	Y	Y	Y	Y	Y	Y	N	20	(10,14)	r	r	r	r	r	r
	N	N	N	N	N	N	Y	21	(1,12)	s, δ	r	r	r	r	s, δ
	Y	N	N	N	N	N	Y	22	(2,12)	r			s, δ	r	
	N	Y	N	N	N	N	Y	23	(1,15)		r	r	r	r	
	Y	Y	N	N	N	N	Y	24	(2,15)	s, δ			s, δ	r	
	Y	N	Y	N	N	N	Y	25	(3,12)	r	r		δ	r	
	Y	Y	Y	N	N	N	Y	26	(3,15)		r		s, δ	r	
	Y	N	N	Y	N	N	Y	27	(4,12)	r		r	s, δ	r	
	Y	Y	N	Y	N	N	Y	28	(4,15)	s, δ		r	s, δ	r	
	Y	N	Y	Y	N	N	Y	29	(5,12)	r	r	r	δ	r	δ
	Y	Y	Y	Y	N	N	Y	30	(5,15)	δ	r	r	s, δ	r	δ
	N	N	N	N	Y	N	Y	31	(1,13)	s	r	r			
	Y	N	N	N	Y	N	Y	32	(2,13)	s, δ					
	N	Y	N	N	Y	N	Y	33	(1,16)	s, δ	r	r	s, δ		
	Y	Y	N	N	Y	N	Y	34	(2,16)	r			s, δ		
	Y	N	Y	N	Y	N	Y	35	(3,13)	s, δ	r				
	Y	Y	Y	N	Y	N	Y	36	(3,16)	r	r		s, δ		
	Y	N	N	Y	Y	N	Y	37	(4,13)	s, δ		r			
	Y	Y	N	Y	Y	N	Y	38	(4,16)	r		r	s, δ		
	Y	N	Y	Y	Y	N	Y	39	(5,13)	s, δ	r	r			
	Y	Y	Y	Y	Y	N	Y	40	(5,16)	r	r	r	s, δ		
	N	N	N	N	N	Y	Y	41	(6,12)	s, δ	r	r		r	
	Y	N	N	N	N	Y	Y	42	(7,12)	r			r	r	
	N	Y	N	N	N	Y	Y	43	(6,15)	s, δ	r	r	s, δ	r	s, δ
	Y	Y	N	N	N	Y	Y	44	(7,15)	s, δ			r	r	
	Y	N	Y	N	N	Y	Y	45	(8,12)	r	r		r	r	
	Y	Y	Y	N	N	Y	Y	46	(8,15)	δ	r		r	r	
	Y	N	N	Y	N	Y	Y	47	(9,12)	r		r	r	r	
	Y	Y	N	Y	N	Y	Y	48	(9,15)	s, δ		r	r	r	

	Y	N	Y	Y	N	Y	Y	49	(10,12)	r	r	r	r	r	r
	Y	Y	Y	Y	N	Y	Y	50	(10,15)	δ	r	r	r	r	δ
	N	N	N	N	Y	Y	Y	51	(6,13)	s, δ	r	r			
	Y	N	N	N	Y	Y	Y	52	(7,13)	s, δ					
	N	Y	N	N	Y	Y	Y	53	(6,16)	s, δ	r	r	s, δ		
	Y	Y	N	N	Y	Y	Y	54	(7,16)	r			s, δ		
	Y	N	Y	N	Y	Y	Y	55	(8,13)	s, δ	r				
	Y	Y	Y	N	Y	Y	Y	56	(8,16)	r	r		s, δ		
	Y	N	N	Y	Y	Y	Y	57	(9,13)	s, δ		r			
	Y	Y	N	Y	Y	Y	Y	58	(9,16)	r		r	s, δ		
	Y	N	Y	Y	Y	Y	Y	59	(10,13)	s, δ	r	r			
	Y	Y	Y	Y	Y	Y	Y	60	(10,16)	r	r	r	s, δ		

7.6.5 Evolution of the Conflict

The evolution of the climate change conflict from the status quo to possible equilibria is investigated to predict the actual outcome in reality. At the current status of the conflict, WH has not proposed new laws in Congress. Thus, no action has been made from GOP and USS. As the agreement was recently signed, both WH and CG show willingness to adhere to it. CG plans to pressure ECs on reforming the energy industry. However, they have voiced some discontent. Hence, the status quo corresponds to state (6, 14), which is also state 13. The evolution of the overall conflict from the status quo on the left to the equilibrium on the right is demonstrated as a range of states connected by UIs made by DMs.

As can be seen in Table 7.10, the overall conflict evolves from the status quo to state (7, 14) when CG proposes new environmental legislation. The subsequent UI from GOP to block the laws results in state (8, 14). State (10, 14), which is a Nash equilibrium, can be accessed from state (8, 14) by USS's UI. At state (10, 14), no DM has any UI. Thus, the overall conflict will stay at state (10, 14), which indicates the outcome that is most likely to happen.

7.6.6 Implications for Decision Makers

The stability results in the hierarchical model indicate possible resolutions for DMs. The Nash equilibrium that can be reached from the status quo suggests that both WH and CG will adhere to the agreement. For WH, the proposal of new environmental laws will be opposed by GOP. To achieve the abatement goals, WH may wish to persuade Congress to pass these laws. Alternatively, it should find other methods to abate the CO_2 emissions under the current legislation framework, such as by supporting the development of new energy and encouraging an energy-saving lifestyle.

As CG abides by the bilateral agreement, it will reform the current energy industry to reach the abatement goals. The China-US agreement is also an opportunity for CG to improve the efficiency of China's energy usage. A possible way for CG to accomplish this is to put forward stricter environmental legislations. The Nash equilibrium, state (10, 14), indicates that ECs are unwilling to support CG's initiative. CG needs to impose political and economic pressure on these energy companies, such as stricter penalties for violating environmental laws, and replacing the board of directors in these companies, because CG has direct control of these state-owned companies.

Table 7.10: Evolution of the Conflict

DMs	Options	SQ	(7, 14)	(8, 14)	(10, 14)
WH	1. Propose New Laws	N $\longrightarrow$	Y	Y	Y
	2. Comply	Y	Y	Y	Y
GOP	3. Block	N	N $\longrightarrow$	Y	Y
USS	4. Support	N	N	N $\longrightarrow$	Y
CG	5. Pressure ECs	Y	Y	Y	Y
	6. Comply	Y	Y	Y	Y
ECs	7. Follow CG	N	N	N	N

7.6.7 Comparison of Stability Results

The stability results in the general hierarchical graph model are compared with the results in the separate graph models. The stabilities in each local graph model are listed in Table 7.11. Compared with Table 7.9, each component of a Nash stable state for a CDM in G is also Nash stable in the corresponding local graph. A Nash state for an LDM in G contains the Nash component in the subconflict it participates in. At an SEQ state for a CDM in G, one of the two component states can be a non-SEQ state. Thus, the SEQ states for the focal CDM in the local graph model cannot predict all the SEQ states in G. For example, state (6, 15) is SEQ for WH in G. The component in $G^{(1)}$, state 6, is SEQ for WH. However, the other component, state 15, is not SEQ for WH in $G^{(2)}$. The UIs for WH at state (6, 15) are demonstrated in Table 7.12. At this state, WH can improve in either state 6 or state 15, or both, resulting in three UIs, states (6, 12), (7, 12), and (7, 15). Note that the sanctions on these UIs are imposed on the component state in $G^{(1)}$. Thus, these UIs can be sanctioned even though state 15 is not an SEQ state. Similar findings can also be concluded for GMR and SMR states for CDMs in G. For LDMs, a GMR or SMR state in G contains the GMR or SMR component in the subconflict it participates in.

State (6, 15) indicates the situation in which both WH and CG adhere to the greenhouse gas agreement. WH does not plan to propose new environmental laws. Thus, GOP does not show any opposition, and USS does not need to show any support. ECs are willing to follow CG's directive to reform without being pressured by CG. This state is SEQ for WH in G, even though a component state, state 15, is not SEQ for WH in $G^{(2)}$. Facing the difficulty of achieving the abatement goals using current environmental laws, WH would break the bilateral agreement. This UI results in state (6, 12). Subsequently, CG would also break the agreement. The resulting state, state (1, 12), is less preferred by WH. Comparing state (1, 12) with the starting state, state (6, 15), state 1 is less preferred to state 6 for WH in $G^{(1)}$, and state 12 is more preferred to state 15 in $G^{(2)}$. Since the US subconflict ($G^{(1)}$) is more important for WH, state (1, 12) is less preferred to state (6, 15). The unilateral breach of the greenhouse gas emissions agreement will result in a disadvantageous situation for both WH and CG. Thus, WH would not move away from state (6, 15). For WH, the sanction from CG cannot be foreseen by considering the two subconflicts separately. In the US subconflict, WH would break the agreement if it only considers the situation within the country. The narrow vision of WH will give rise to a less desired outcome in reality. Thus, the general hierarchical graph model can provide WH with a comprehensive understanding of the dynamics of the overall conflict, and guide WH

Table 7.11: Stability Results in the Two Local Graphs

	$G^{(1)}$						G_2			
States	WH	GOP	USS	CG	Overall	States	WH	CG	ECs	Overall
1	s, g, δ	r, s, g, δ	r, s, g, δ	r, s, g, δ	s, g, δ	11	g, δ	r, s, g, δ	r, s, g, δ	g, δ
2	r, s, g, δ			s, g, δ		12	r, s, g, δ	r, s, g, δ	r, s, g, δ	r, s, g, δ
3	r, s, g, δ	r, s, g, δ		g, δ		13	s, g, δ			
4	r, s, g, δ		r, s, g, δ	s, g, δ		14	r, s, g, δ	r, s, g, δ	r, s, g, δ	r, s, g, δ
5	r, s, g, δ	r, s, g, δ	r, s, g, δ	g, δ	g, δ	15		r, s, g, δ	r, s, g, δ	
6	s, g, δ	r, s, g, δ	r, s, g, δ			16	r, s, g, δ	s, g, δ		
7	r, s, g, δ			r, s, g, δ						
8	r, s, g, δ	r, s, g, δ		r, s, g, δ						
9	r, s, g, δ		r, s, g, δ	r, s, g, δ						
10	r, s, g, δ	r, s, g, δ	r, s, g, δ	r, s, g, δ	r, s, g, δ					

in taking beneficial actions.

7.7 Summary

The general hierarchical graph model for conflict resolution has been developed to model hierarchical conflicts having more DMs compared with former hierarchical graph models presented in previous chapters. The preference structure for the general hierarchical graph model has been defined based on the option prioritization method. Two types of preferences, weighted preference and lexicographic preference, are proposed. As a simplified preference structure, lexicographic preference can be used to effectively rank states in the hierarchical graph model and facilitate the calculation of stabilities. Moreover, the interrelation between stabilities in the general hierarchical graph model and those in its local

Table 7.12: UIs for WH at SEQ State (6, 15)

Focal State		UIs		Sanctioning UIs
(6, 15)	$\xrightarrow{WH^+}$	(6, 12)	$\xrightarrow{CG^+}$	(1, 12)
	$\xrightarrow{WH^+}$	(7, 12)	$\xrightarrow{CG^+}$	(8, 12)
	$\xrightarrow{WH^+}$	(7, 15)	$\xrightarrow{GOP^+}$	(8, 15)

graphs has been demonstrated. As an application, the bilateral greenhouse gas disputes between USA and China are investigated. A detailed modelling and calculation process have been provided, as well as implications for DMs and in-depth results analysis. The general hierarchical graph model has been demonstrated to provide DMs with guidance for how to act in any interrelated conflict in the real world.

Further studies can be carried out within the hierarchical graph model methodology. A preference structure to incorporate uncertainties deserves further research. As the general hierarchical graph model usually has a complex structure, more efficient algorithms to implement calculations of stabilities need to be developed. Furthermore, as current hierarchical graph models have only two levels, a multiple level model should be constructed to solve more complex hierarchical conflicts in the real world, such as climate change negotiations among countries and between different levels of governments.

Chapter 8

Conclusions and Further Study

Hierarchical graph models are developed to solve interrelated conflicts in the real world. After investigating water diversion conflicts in China, a typical hierarchical conflicts in the real world, the basic hierarchical graph model is developed using lexicographic preference and weighted preference. Different ways of representing the model and approaches of calculating stabilities have been provided.

More complex hierarchical graph models, the duo hierarchical graph model and the general hierarchical graph model, have defined using lexicographic preference. Algorithms of calculating stability results have been designed. The duo hierarchical graph model has been applied to the market competition between Airbus and Boeing. The greenhouse gas emissions disputes have been studied using the general hierarchical graph model.

Compared with classical graph model methodologies, hierarchical graph model has the following advantages. Firstly, the novel methodology can reveal the interrelation among conflicts that are connected logically or geographically. Secondly, the resolutions in a hierarchical graph model can provide DMs with a more comprehensive understanding of the interrelated conflicts and guidance of taking reasonable actions. In comparison, the resolutions in a local model may be reasonable for DMs within the corresponding subconflict, but less favourable in the overall conflicts. In addition, hierarchical graph model methodology can be used to efficiently determine stability results using the theorems developed from Chapter 4 to 7. Calculations for stability results can be realized by GMCR II, a decision support system that has been designed for classical graph models.

8.1 Major Contributions

Interrelated conflicts are ubiquitous in the real world. The most important contribution of this thesis is the establishment of hierarchical graph models to analyze interrelated conflicts. Based on the graph model for conflict resolution paradigm, hierarchical graph models have been developed in this thesis, and are proposed to describe the decision process of DMs in certain forms of interrelated conflict.

Two types of preference structure, lexicographic preference and weighted preference, have also been developed for hierarchical graph models. Preference information in hierarchical graph models is not completely determined by preferences in the local graphs. The proposed preference structures are crucial to provide the complete preference information in hierarchical graph models.

Furthermore, theorems have been developed to elucidate the relationship between stabilities in hierarchical graph models and stabilities in local graph models. These theorems can be used to efficiently conclude stabilities in hierarchical graph models using the stability information in local models. As indicated by the theorems, a stable states in a hierarchical model may contain a component state which is unstable in local models. These theorems have indicated the conditions on which this hierarchical state is stable.

Different methods of calculating stabilities in a hierarchical graph model have been provided in this thesis. In the basic hierarchical graph model with lexicographic preference, each state in the hierarchical model is investigated for stabilities using the theorems. In the basic hierarchical graph model with weighted preference, algorithms are designed for calculating stabilities in matrix form. In the duo and general hierarchical graph models, the stabilities have been calculated when the preferences are expressed using the option prioritization method. The decision support system, GMCR II, has been used to facilitate the stability calculations. The lexicographic preference is simple and efficient in calculating stability results, while weighted preference is more formal and precise in describing the relative importance for a CDM.

The three real world applications in this thesis have demonstrated the usefulness of the hierarchical graph model methodology in understanding hierarchical conflicts. In the water diversion conflicts, Chinese Government is well-advised to implement its construction plan in different locations sequentially. In the sales competition between Airbus and Boeing, both aircraft manufacturers can adjust their marketing strategies in the wide and narrow body markets. In the greenhouse emissions disputes, the two national governments are given suggestions on how to deal with their domestic oppositions while adhering to the bilateral agreement.

Different from previous GMCR methodologies, which are used to investigate simple conflicts, hierarchical graph model methodologies can provide DMs with a comprehensive understanding of the interrelated components of a hierarchical conflict. With new insights, DMs can obtain strategic resolutions in hierarchical disputes and reach win-win situations.

8.2 Assumptions and Limitations of Hierarchical Graph Models

The hierarchical graph models developed in this thesis are based on certain assumptions that are reviewed here. First, some general assumptions regarding the use of the graph model methodology apply to hierarchical graph models. All decision makers (DMs) in a graph model are aware of the options and preferences of all other DMs. All DMs can move in any order whatsoever and may not decide to move at all. DMs mainly care about the final state; intermediate states do not affect their preferences. Second, there are some specific assumptions for the hierarchical graph model methodology. More specifically, in a hierarchical graph model, a CDM can move across local models, while a LDM can only move within its own local model. If LDMs can move in many local models, the hierarchical graph model must be considered as a classic graph model. DMs who can move in some local models, but not all, are not considered in this thesis, but may be investigated in the future.

The hierarchical graph model methodologies are used to provide strategic resolutions for DMs in solving interrelated conflicts. Some limitations of the model are now clarified. Graph model methodologies are used to analyze the strategic aspects of a conflict by considering only the basic elements of the conflict, such as decision makers, options, moves, and preferences. The output of the model consists of possible resolutions of the conflict at the strategic level. Some specific details of the conflict may not be considered such as how the types of moves made may affect preference. Furthermore, stability results in a graph model can be used for many purposes, including to predict the outcome of the conflict. When more than one equilibrium may occur, the equilibrium that is closest to the status quo is considered as the outcome most likely to happen. Other equilibria can suggest possible scenarios when DMs have different foresights and perception of risks. In addition, the stability results for the hierarchical graph model do not reflect any lapse of time within the conflict. Any change of model inputs with time, such as options, preferences, and DMs, is not considered in this thesis, but deserves further study.

8.3 Further Study

The variety of the interrelated conflicts in the real world calls for further development of hierarchical graph model methodologies. As current models feature the hierarchical structure in only two levels, new hierarchical graph models consisting of multiple levels can be developed. DMs at a higher level can be considered as common decision makers (CDMs), compared with DMs at a lower level, and this structure may be repeated at other levels.

Uncertainty is an important ingredient in interactive strategic decision making. A well-developed conflict analysis methodology should take account of uncertainties in the interaction among DMs. To consider uncertainty in hierarchical graph models, the preferences for DMs can be expressed using fuzzy (Bashar et al., 2014), grey (Kuang et al., 2015), or probabilistic relations, adopting suitable stability definitions. These uncertainty theories already exist, but are yet to be incorporated into the hierarchical graph model methodology.

The definition of preferences in hierarchical graph models can be extended. For a DM, one unilateral improvement (UI) can improve its well being or interests in a conflict more significantly than another UI. Thus, the focal DM discriminates amongst the UIs in the conflict. Xu et al. (2009b) have studied multiple levels of preferences, referred to as strength of preference by Hamouda et al. (2004), in a classical graph model. Thus, this new preference structure in a hierarchical graph model should be investigated.

The time frame in hierarchical graph model can also be studied. The time frame can influence the decision making process, as DMs, options, and preferences may change with the lapse of time. Under the graph model paradigm, Yasser et al. (2013) have used "robustness" to measure the sustainability of a resolution in a given conflict. As a conflict may evolve over time, some equilibria may change with time, while others are stable. In a hierarchical graph model, the evolution of a hierarchical conflict should be investigated. Furthermore, an optimization algorithm can be developed to determine the relative importance of local models for a CDM at different periods of a conflict, in order to achieve a more desired outcome for the CDM.

The concept of coalitions in graph model was first studied by Hipel and Meister (1994). Coalitions in hierarchical graph models can also be analyzed. As LDMs usually have limited influence in that they participate in only one local graph, the situation in which they act collectively for a common purpose should be analyzed. Stability results containing such coalitions can be compared with the stability results without any coalition. In addition, attitudes (Walker et al., 2012; Inohara et al., 2007) and emotion (Obeidi et al., 2005) have been incorporated into the graph model paradigm, and may be adapted for employment within the hierarchical graph model methodology.

References

Abed-Elmdoust, A. and Kerachian, R. (2012). Water resources allocation using a cooperative game with fuzzy payoffs and fuzzy coalitions. *Water Resources Management*, 26(13):3961–3976.

Airbus. (2014) Global market forecast 2014-2033. http://www.airbus.com. Accessed 29-Jan-2015.

Airbus (2012). Singapore Airlines to Order More A380s and A350 XWBs. http://www.airbus.com. Accessed 5-Oct-2015.

Bashar, M. A., Hipel, K. W., Kilgour, D. M., and Obeidi, A. (2014). Coalition fuzzy stability analysis in the graph model for conflict resolution. *Journal of Intelligent and Fuzzy Systems*, 29(2):593–607.

Becker, N. and Easter, K. W. (1995). Water diversions in the Great Lakes basin analyzed in a game theory framework. *Water Resources Management*, 9(3):221–242.

Beimel, A., Tassa, T., and Weinreb, E. (2008). Characterizing ideal weighted threshold secret sharing. *SIAM Journal on Discrete Mathematics*, 22(1):360–397.

Blaug, M. (2008). Invisible hand. *The New Palgrave Dictionary of Economics*, pages 564–566.

Boeing. (2011) Boeing introduces 737 MAX With launch of new aircraft family. http://boeing.mediaroom.com. Accessed 29-Jan-2015.

Boeing. (2014) Global market outlook/ Asia Pacific 2014-2033. http://www.boeing.com/boeing/commercial. Accessed 29-Jan-2015.

Brams, S. J. and Wittman, D. (1981). Nonmyopic equilibria in 2× 2 games. *Conflict Management and Peace Science*, (6):39–62.

Brimberg, J., Mladenovic, N., and Salhi, S. (2004). The multi-source weber problem with constant opening cost. *Journal of the Operational Research Society*, 55(6):640–646.

Buckley, C. (2011) China drought raises questions about climate change. Reuters. http://online.barrons.com. Accessed 3-June-2011.

Chen, Z., Wang, H., and Qi, X. (2013). Pricing and water resource allocation scheme for the South-to-North Water Diversion Project in China. *Water Resources Management*, 27(5):1457–1472.

Cho, R. (2011) How China is dealing with its water crisis. State of the planet. http://blogs.ei.columbia.edu. Accessed 5-May-2011.

Compare Airplanes (2015). Airbus vs Boeing. http://planes.axlegeeks.com. Accessed 30-Jul-2015.

Dawkins, R. (1976). *Hierarchical Organization: A Candidate Principle for Ethology.* Cambridge U Press.

de Andrade, J. G., Barbosa, P. S., Souza, L. C., and Makino, D. L. (2011). Interbasin water transfers: the Brazilian experience and international case comparisons. *Water Resources Management*, 25(8):1915–1934.

Doganis, R. (2002). *Flying off Course: The Economics of International Airlines*. Psychology Press.

Dono, G., Giraldo, L., and Severini, S. (2012). The cost of irrigation water delivery: an attempt to reconcile the concepts of cost and efficiency. *Water Resources Management*, 26(7):1865–1877.

Dožić, S. and Kalić, M. (2014). An AHP Approach to Aircraft Selection Process. *Transportation Research Procedia*, 3:165–174.

Duggan, J. (2013). China's mega water diversion project begins testing. http://www.theguardian.com. Accessed 5-Oct-2015.

Epstein, R. A. (2012). *The Theory of Gambling and Statistical Logic*. Academic Press.

Fang, L., Hipel, K. W., and Kilgour, D. M. (1993). *Interactive Decision Making: the Graph Model for Conflict Resolution*, volume 3. John Wiley & Sons.

Fang, L., Hipel, K. W., Kilgour, D. M., and Peng, X. (2003a). A decision support system for interactive decision making-Part I: model formulation. *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on*, 33(1):42–55.

Fang, L., Hipel, K. W., Kilgour, D. M., and Peng, X. J. (2003b). A decision support system for interactive decision making-Part II: analysis and output interpretation. *Sys-*

tems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on, 33(1):56–66.

Farras, O. and Padró, C. (2010). Ideal hierarchical secret sharing schemes. In *Theory of Cryptography*, pages 219–236. Springer.

Fleisher, C. S. and Bensoussan, B. E. (2003). *Strategic and Competitive Analysis: Methods and Techniques for Analyzing Business Competition*. Prentice Hall Upper Saddle River, NJ.

Flouris, T. (2010). Rationalizing aircraft performance dynamic modeling in airline fleet planning decisions. *Enterprise Risk Management*, 2(1):39–67.

Forbes. (2013) Boeing looks set to beat Airbus in orders and deliveries In 2014. http://www.forbes.com. Accessed 29-Jan-2015.

Fraser, N. M. and Hipel, K. W. (1979). Solving complex conflicts. *Systems, Man and Cybernetics, IEEE Transactions on*, 9(12):805–816.

Fraser, N. M. and Hipel, K. W. (1984). *Conflict Analysis: Models and Resolutions, Series Volume 11*. North-Holland.

Gale, D. (1971). Optimal strategy for serving in tennis. *Mathematics Magazine*, 44(4):197–199.

García-Valiñas, M. (2005). Efficiency and equity in natural resources pricing: a proposal for urban water distribution service. *Environmental and Resource Economics*, 32(2):183–204.

George, S. (1973). Optimal strategy in tennis: a simple probabilistic model. *Applied Statistics*, 22(1):97–104.

Gilles, R. P. (2010). *The Cooperative Game Theory of Networks and Hierarchies*, volume 44. Springer Science & Business Media.

Gillman, L. (1985). Missing more serves may win more points. *Mathematics Magazine*, 58(4):222–224.

Gleick, P. H. (2009). China and Water. *The World's Water 2008–2009: The Biennial Report on Freshwater Resources*, pages 79–100.

Goldenberg, S., Taylor, L., and Branigan, T. US-China climate deal boosts global talks but Republicans vow to resist. http://www.theguardian.com. Accessed 30-Mar-2015.

González, A., Sánchez-Román, R. M., Folegatti, Marcos, V., and Zolin, C. A. (2011). *Water Resources Management*, 25(13):3371–3386.

Gu, W., Shao, D., and Jiang, Y. (2012). Risk evaluation of water shortage in source area of middle route project for South-to-North water transfer in China. *Water Resources Management*, 26(12):3479–3493.

Gvozdeva, T., Hameed, A., and Slinko, A. (2013). Weightedness and structural characterization of hierarchical simple games. *Mathematical Social Sciences*, 65(3):181–189.

Hamouda, L., Kilgour, D. M., and Hipel, K. W. (2004). Strength of preference in the graph model for conflict resolution. *Group Decision and Negotiation*, 13(5):449–462.

Harris, E. (2011). The impact of institutional path dependence on water market efficiency in Victoria, Australia. *Water resources management*, 25(15):4069–4080.

He, S., Hipel, K. W., and Kilgour, D. M. (2014). Water diversion conflicts in China: A hierarchical perspective. *Water Resources Management*, 28(7):1823–1837.

He, S., Kilgour, D. M., Hipel, K. W., and Bashar, M. A. (2013). A Basic Hierarchical Graph Model for Conflict Resolution with application to water diversion conflicts in China. *INFOR: Information Systems and Operational Research*, 51(3):103–119.

Hepher, T. (2013a) Boeing wide-body jet battle triggers warnings of oversupply. http://www.reuters.com. Accessed 29-Jan-2015.

Hepher, T. (2013b) Boeing and Airbus are neck-and-neck heading into the end of 2014. http:// www.businessinsider.com. Accessed 29-Jan-2015.

Hipel, K. (2009). *Conflict Resolution Vol. I and II.* Eolss Publishers Company Limited.

Hipel, K. W. (2002). Conflict resolution: Theme overview paper in conflict resolution. *Encyclopedia of Life Support Systems (EOLSS). Oxford, UK: EOLSS Publishers*, 2:50–52.

Hipel, K. W. and Fang, L. (2005). Multiple participant decision making in societal and technological systems. In *Systems and Human Science, for Safety, Security, and Dependability: Selected Papers of the 1st International Symposium SSR2003, Osaka, Japan, November 2003*, pages 3–29. Elsevier.

Hipel, K. W. and Meister, D. B. (1994). Conflict-analysis methodology for modelling coalition in multilateral negotiations. *Information and Decision Technologies*, 19(2):85–103.

Holslag, J. (2011). Assessing the Sino-Indian water dispute. *Journal of International Affairs*, 64(2):19–35.

Howard, N. (1971). *Paradoxes of Rationality: Theory of Metagames and Political Behavior*, volume 1. MIT press.

Howard, N. (1999). Confrontation analysis: How to win operations other than war. Technical report, DTIC Document.

Hung, M. and Chie, B. (2013). Residential water use: efficiency, affordability, and price elasticity. *Water Resources Management*, 27(1):275–291.

Inohara, T. and Hipel, K. W. (2008a). Coalition analysis in the graph model for conflict resolution. *Systems Engineering*, 11(4):343–359.

Inohara, T. and Hipel, K. W. (2008b). Interrelationships among noncooperative and coalition stability concepts. *Journal of Systems Science and Systems Engineering*, 17(1):1–29.

Inohara, T., Hipel, K. W., and Walker, S. (2007). Conflict analysis approaches for investigating attitudes and misperceptions in the War of 1812. *Journal of Systems Science and Systems Engineering*, 16(2):181–201.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 47(2):263–291.

Kilgour, D. and Hipel, K. (2005). The graph model for conflict resolution: Past, present, and future. *Group Decision and Negotiation*, 14(6):441–460.

Kilgour, D. M. (1985). Anticipation and stability in two-person noncooperative games. *Dynamic Models of International Conflict*, pages 26–51.

Kilgour, D. M. and Hipel, K. W. (2010). Conflict analysis methods: The graph model for conflict resolution. In *Handbook of Group Decision and Negotiation*, pages 203–222. Springer.

Kilgour, D. M., Hipel, K. W., and Fang, L. (1987). The graph model for conflicts. *Automatica*, 23(1):41–55.

Kilgour, D. M., Hipel, K. W., Fang, L., and Peng, X. J. (2001). Coalition analysis in group decision support. *Group Decision and Negotiation*, 10(2):159–175.

Kuang, H., Bashar, M. A., Hipel, K. W., and Kilgour, D. M. (2015). Grey-based preference in a graph model for conflict resolution with multiple decision makers. 45(9):1254–1267.

Kucukmehmetoglu, M. (2009). A game theoretic approach to assess the impacts of major investments on transboundary water resources: the case of the Euphrates and Tigris. *Water Resources Management*, 23(15):3069–3099.

Larson, C. (2012) Growing shortages of water threaten China's development. http://e360.yale.edu. Accessed 23-Dec-2012.

Leeham (2015). 747-8 vs A380 costs: airlines weigh in. http://leehamnews.com. Accessed 30-Jul-2015.

Levi, M. What the big U.S.-China climate announcement means. http://blogs.cfr.org/levi. (Accessed 30-Mar-2015).

Li, K. W., Hipel, K. W., Kilgour, D. M., and Fang, L. (2004). Preference uncertainty in the graph model for conflict resolution. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 34(4):507–520.

Li, K. W., Hipel, K. W., Kilgour, D. M., and Noakes, D. (2005). Integrating uncertain preferences into status quo analysis with applications to an environmental conflict. *Group Decision and Negotiation*, 14(6):461–479.

Lichtenstein, S., Fischhoff, B., and Phillips, L. (1982). *Calibration of Probabilities: The State of the Art to 1980. D. Kahneman, P. Slovic, and A. Tverski (Eds.) Judgement under Uncertainty: Heuristics and Biases.* New York, Cambridge University Press.

Liddell, H. G., Scott, R., and Jones, H. S. (1940). A Greek-English Lexicon/compiled by Henry George Liddell and Robert Scott.

Marshall, A. (1920). *Principles of Economics, 8th Edition.* Macmillan, London.

Matbouli, Y. T., Hipel, K. W., and Kilgour, D. M. (2013). Robustness of equilibrium in conflict eesolution. *Proceedings of Group Decision and Negotiation-GDN*, pages 52–53.

McCormack, G. (2001). Water margins: Competing paradigms in China. *Critical Asian Studies*, 33(1):5–30.

Misra, A. K. (2011). Impact of urbanization on the hydrology of Ganga Basin (India). *Water Resources Management*, 25(2):705–719.

Morris, C. (1977). The most important points in tennis. *Optimal Strategies in Sports*, 5:131–140.

Nash, J. F. (1950). Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences. USA*, 36(1):48–49.

Nash, J. F. (1951). Non-cooperative games. *Annals of Mathematics*, 54(2):286–295.

Obeidi, A., Hipel, K. W., and Kilgour, D. M. (2005). The role of emotions in envisioning outcomes in conflict analysis. *Group Decision and Negotiation*, 14(6):481–500.

Ozdemir, Y., Basligil, H., and Karaca, M. (2011). Aircraft selection using Analytic Network Process: a case for Turkish Airlines. *Assessment*, 8:9.

Planespotters.net (2015). B777 Production List. https://www.planespotters.net. Accessed 5-Oct-2015.

Productivity Commission (2003). Water rights arrangements in Australia and overseas. Commission Research Paper, Melbourne: Productivity Commission.

Sechi, G. M., Zucca, R., and Zuddas, P. (2013). Water costs allocation in complex systems using a cooperative game theory approach. *Water Resources Management*, 27(6):1781–1796.

Sepang (2014). AirAsia X places firm order for 55 A330neo. http://www.airasia.com. Accessed 5-Oct-2015.

Shao, W., Yang, D., Hu, H., and Sanbongi, K. (2009). Water resources allocation considering the water use flexible limit to water shortage: a case study in the Yellow River Basin of China. *Water Resources Management*, 23(5):869–880.

Shi, P., Chen, C., Srinivasan, R., Zhang, X., Cai, T., Fang, X., Qu, S., Chen, X., and Li, Q. (2011). Evaluating the swat model for hydrological modeling in the Xixian watershed and a comparison with the XAJ model. *Water Resources Management*, 25(10):2595–2612.

Silva-Hidalgo, H., Martín-Domínguez, I. R., Alarcón-Herrera, M. T., and Granados-Olivas,

A. (2009). Mathematical modelling for the integrated management of water resources in hydrological basins. *Water Resources Management*, 23(4):721–730.

Simaan, M. and Cruz Jr., J. B. (1973). On the Stackelberg strategy in non zero-sum games. *Journal of Optimization Theory and Applications*, 11(5):533–555.

Simon, H. A. (1978). Rationality as process and as product of thought. *The American Economic Review*, 68(2):1–16.

Simon, H. A. (1990). *Reason in Human Affairs*. Stanford University Press.

Simon, H. A. (1991). *The Architecture of Complexity*. Springer.

South-North Water Diversion Official Site (2011). Massive water diversion project put on fast track. http://www.nsbd.gov.cn. Accessed 29-Nov-2011.

Speed, R. (2009a). A comparison of water rights systems in China and Australia. *Water Resources Development*, 25(2):389–405.

Speed, R. (2009b). Transferring and trading water rights in the People's Republic of China. *Water Resources Development*, 25(2):269–281.

State Council (2006). National Economic and Social Development Plan for Eleventh-Five Year Period, 2006-10. Available at: http://english.gov.cn/special. Accessed 12-Oct-2015.

Stigler, G. J. (1988). Palgrave's dictionary of economics. *Journal of Economic Literature*, pages 1729–1736.

The Economic Times (2012). China wants to deepen strategic cooperation with India. http://articles.economictimes.indiatimes.com/2012-03-30/news/31261180_1_brics-summit-bilateral-ties-hu-jintao/. Accessed 30-Mar-2012.

The Hindu (2013). Borderwater disputes in focus as Khurshid heads for China-How to handle thorny issues is the crux of talks. www.thehindu.com. Accessed 12-Sep-2013.

United Nations News Report (2012). Drought alert has been issued by FAO for the three-month insufficient precipitation in North China Plain (in Chinese). United Nations. http:// www.un.org. Accessed 23-Dec-2012.

Von Neumann, J. and Morgenstern, O. (2007). *Theory of Games and Economic Behavior*. Princeton University Press.

Von Stackelberg, H. (1934). *Marktform und Gleichgewicht*. J. Springer.

Walker, M. and Wooders, J. (2001). Minimax play at Wimbledon. *American Economic Review*, 91(5):1521–1538.

Walker, S. B., Hipel, K. W., and Inohara, T. (2012). Attitudes and preferences: Approaches to representing decision maker desires. *Applied Mathematics and Computation*, 218(12):6637–6647.

WET (2006). Water entitlements and trading project (WET Phase 1) final report November 2006 [in English and Chinese] (Beijing: Ministry of Water Resources, People's Republic of China and Canberra: Department of Agriculture, Fisheries and Forestry, Australian Government). Available at:

http://www.environment.gov.au/water/action/international/wet1.html. Accessed 12-Oct-2015.

World Factbook (2014). Total renewable water resources. Central Intelligence Agency. https://www.cia. gov. Accessed 10-Jan-2014.

Xie, J. (2008). *Addressing China's Water Scarcity: A Synthesis of Recommendations for Selected Water Resource Management Issues*. Washington, DC: World Bank Publications.

Xu, H., Hipel, K. W., and Kilgour, D. M. (2009a). Matrix representation of solution concepts in multiple-decision-maker graph models. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 39(1):96–108.

Xu, H., Hipel, K. W., and Kilgour, D. M. (2009b). Multiple levels of preference in interactive strategic decisions. *Discrete Applied Mathematics*, 157(15):3300–3313.

Xu, H., Hipel, K. W., Kilgour, D. M., and Chen, Y. (2010). Combining strength and uncertainty for preferences in the graph model for conflict resolution with multiple decision makers. *Theory and Decision*, 69(4):497–521.

Xu, H., Kilgour, D. M., and Hipel, K. W. (2007). Matrix representation of solution concepts in graph models for two decision-makers with preference uncertainty. *Dynamics of Continuous, Discrete and Impulsive Systems*, 14(S1):703–737.

Yuhas, A. How the Republican-led Congress could kill the climate change deal. http://www.theguardian.com. (Accessed 30-Mar-2015).

Zagare, F. C. (1984). Limited-move equilibria in 2× 2 games. *Theory and Decision*, 16(1):1–19.

Zhang, L., Jia, S., Leung, C., and Guo, L. (2013). An analysis on the transaction costs of water markets under DPA and UPA auctions. *Water Resources Management*, 27(2):475–484.

Zhang, N., He, H., Zhang, S., Jiang, X., Xia, Z., and Huang, F. (2012). Influence of reservoir operation in the upper reaches of the Yangtze River (China) on the inflow and outflow regime of the TGR-based on the improved SWAT model. *Water Resources Management*, 26(3):691–705.

APPENDICES

Appendix A

Proofs for Theorems

A.1 Proof for Theorem 4.1

Theorem 4.1 (Nash for CDM in G): If $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are Nash stable for CDM in $G^{(1)}$ and $G^{(2)}$, respectively, then $(s^{(1)}, s^{(2)})$ is Nash stable for CDM in G.

Proof. Because $s^{(1)}$ is Nash stable for CDM in $G^{(1)}$, then $s^{(1)} \succsim_C^{(1)} q^{(1)}$ for all $q^{(1)} \in R_C^{(1)}(s^{(1)})$. Similarly, $s^{(2)} \succsim_C^{(2)} q^{(2)}$ for all $q^{(2)} \in R_C^{(2)}(s^{(2)})$. Suppose $(s^{(1)}, s^{(2)})$ is not Nash stable for CDM in G, then there exists $(r^{(1)}, r^{(2)}) \in S$ such that $(r^{(1)}, r^{(2)}) \in R_C(s^{(1)}, s^{(2)})$ and $(r^{(1)}, r^{(2)}) \succ_C (s^{(1)}, s^{(2)})$.

Assume that $r^{(1)} \precsim_C^{(1)} s^{(1)}$ and $r^{(2)} \precsim_C^{(2)} s^{(2)}$. Then $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. This contradicts $(r^{(1)}, r^{(2)}) \succ_C (s^{(1)}, s^{(2)})$. Therefore, either $r^{(1)} \succ_C^{(1)} s^{(1)}$ or $r^{(2)} \succ_C^{(2)} s^{(2)}$. Hence, either $s^{(1)}$ is not Nash stable for CDM in $G^{(1)}$ or $s^{(2)}$ is not Nash stable for CDM in $G^{(2)}$. By contradiction, we conclude that $(s^{(1)}, s^{(2)})$ is Nash stable for CDM in G. □

A.2 Proof for Theorem 4.3

Theorem 4.3 (SEQ for CDM): Suppose that $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are sequentially stable for CDM in $G^{(1)}$ and $G^{(2)}$, respectively. Then $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G.

Proof. Let $(q^{(1)}, q^{(2)})$ be a UI from $(s^{(1)}, s^{(2)})$ for CDM in $G^{(1)}$, i.e. $(q^{(1)}, q^{(2)}) \in R_C^{+}(s^{(1)}, s^{(2)})$. Then $(q^{(1)}, q^{(2)}) \succ_C (s^{(1)}, s^{(2)})$.

As before, either $q^{(1)} \succ_C^{(1)} s^{(1)}$ or $q^{(2)} \succ_C^{(2)} s^{(2)}$. Assume $q^{(1)} \succ_C^{(1)} s^{(1)}$, then $q^{(1)} \in R_C^{(1)+}(s^{(1)})$. As $s^{(1)}$ is SEQ for CDM in $G^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{+}(q^{(1)})$ such that $s^{(1)} \succsim_C^{(1)} r^{(1)}$. Similarly, if $r^{(2)} \succ_C^{(2)} s^{(2)}$, there exists $r^{(2)} \in R_{L_2}^{(2)+}(q^{(2)})$ such that $s^{(2)} \succsim_C^{(2)} q^{(2)}$.

(1) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \precsim_C^{(2)} s^{(2)}$, $(r^{(1)}, q^{(2)}) \in R_L(q^{(1)}, q^{(2)})$. Then $(r^{(1)}, q^{(2)}) \in R_L^{+}(q^{(1)}, q^{(2)})$ and $(r^{(1)}, q^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Thus, $(r^{(1)}, q^{(2)})$ is a sanction and the UI for CDM from $(s^{(1)}, s^{(2)})$ to $(q^{(1)}, q^{(2)})$ is sanctioned.

(2) Similarly, if $q^{(1)} \precsim_C^{(1)} s^{(1)}$ and $s^{(2)} \succ_C^{(2)} s^{(2)}$, there exists $(q^{(1)}, r^{(2)}) \in R_L^{+}(q^{(1)}, q^{(2)})$ such that $(q^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Thus, $(q^{(1)}, r^{(2)})$ is a sanction to the UI for CDM from $(s^{(1)}, s^{(2)})$ to $(q^{(1)}, q^{(2)})$.

(3) Assume that $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, then there exists $(r^{(1)}, r^{(1)}) \in R_L^{+}(q^{(1)}, q^{(2)})$ such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Thus, $(r^{(1)}, r^{(2)})$ is a sanction.

Overall, a sanction can be found for every UI for CDM from $(s^{(1)}, s^{(2)})$ to $(q^{(1)}, q^{(2)})$. Therefore, $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G. □

A.3 Proof for Theorem 4.4

Theorem 4.4: Assume $G^{(1)} > G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G, iff either

(1) both $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are SEQ for CDM, or

(2) $s^{(1)} \in S^{(1)}$ is strictly sequentially stable for CDM in $G^{(1)}$ and, for all $q^{(1)} \in R_C^{(1)+,=}(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{(1)+}(q^{(1)})$ such that $r^{(1)} \prec_C^{(1)} s^{(1)}$, where $R_C^{(1)+,=}(s^{(1)})$ is the set of UMs for CDM from $s^{(1)}$ which are no less preferred to $s^{(1)}$.

Proof. Assume that $(s^{(1)}, s^{(2)})$ is SEQ for CDM in G. Then, if $(q^{(1)}, q^{(2)}) \in R_C^+(s^{(1)}, s^{(2)})$, there exists $(r^{(1)}, r^{(2)}) \in R_L^+(q^{(1)}, q^{(2)})$ such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)}$. Note that $(q^{(1)}, q^{(2)}) \succ_C (s^{(1)}, s^{(2)})$ implies either $q^{(1)} \succ_C^{(1)} s^{(1)}$ or $q^{(2)} \succ_C^{(2)} s^{(2)}$.

As both $s^{(1)}$ and $s^{(2)}$ can be either SEQ or not, there are four possibilities:

(i) If both $s^{(1)}$ and $s^{(2)}$ are SEQ for CDM in $G^{(1)}$ and $G^{(2)}$, respectively, then $(s^{(1)}, s^{(2)})$ is SEQ by Theorem 4.3. This is case (1) of the Theorem.

(ii) Suppose $s^{(1)} \in S^{(1)}$ is SEQ for CDM in $G^{(1)}$ while $s^{(2)} \in S^{(2)}$ is not SEQ for CDM in $G^{(2)}$. We investigate whether every UI from $(s^{(1)}, s^{(2)})$ is sanctioned.

Assume $q^{(1)} \succ_C^{(1)} s^{(1)}$. Then there exists $r^{(1)} \in R_{L_1}^{(1)+}(q^{(1)})$ such that $r^{(1)} \precsim_C^{(1)} s^{(1)}$. Because $s^{(2)}$ is not SEQ for CDM in $G^{(2)}$, there must be a UI $q^{(2)} \in R_C^{(2)+}(s^{(2)})$ such that $r^{(2)} \succ_C^{(2)} s^{(2)}$ for all $r^{(2)} \in R_{L_2}^{(2)+}(q^{(2)})$.

a) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \precsim_C^{(2)} s^{(2)}$, then $(r^{(1)}, q^{(2)}) \in R_L^+(q^{(1)}, q^{(2)})$ and $(r^{(1)}, q^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Therefore, $(r^{(1)}, q^{(2)})$ is a sanction, so $(s^{(1)}, s^{(2)})$ is SEQ.

b) If $q^{(1)} \precsim_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, then since $G^{(1)} > G^{(2)}$ and $(q^{(1)}, q^{(2)}) \succ_C (s^{(1)}, s^{(2)})$, $q^{(1)} \sim_C^{(1)} s^{(1)}$ or $q^{(1)} = s^{(1)}$. In G, the sanction against $(q^{(1)}, q^{(2)})$ by LDMs can be

$(r^{(1)}, r^{(2)}) \in R_L^+(q^{(1)}, q^{(2)})$ when either $r^{(1)} = q^{(1)}$ or $r^{(2)} = q^{(2)}$, but not both. Note that $r^{(2)} \succ_C^{(2)} s^{(2)}$, so $(r^{(1)}, r^{(2)})$ is a sanction, i.e. $(r^{(1)}, r^{(2)}) \prec_C (s^{(1)}, s^{(2)})$, only if $r^{(1)} \prec_C^{(1)} s^{(1)}$.

c) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, the sanction against $(q^{(1)}, q^{(2)})$ by the LDMs satisfies $(r^{(1)}, r^{(2)}) \prec_C (s^{(1)}, s^{(2)})$ iff $r^{(1)} \prec_C^{(1)} s^{(1)}$. In particular, $s^{(1)}$ is strict SEQ for CDM in $G^{(1)}$.

Together, (a), (b), and (c) prove case (2) of the Theorem. To complete the proof, we show that if (1) and (2) fail, then $(s^{(1)}, s^{(2)})$ is not SEQ for CDM in G.

(iii) Suppose that $s^{(1)} \in S^{(1)}$ is not SEQ for CDM in $G^{(1)}$ while $s^{(2)} \in S^{(2)}$ is sequentially stable for CDM in $G^{(2)}$.

(iv) $s^{(1)}$ is not SEQ for CDM in $G^{(1)}$ and $s^{(2)}$ is not SEQ for CDM in $G^{(2)}$.

(iii) and (iv) can be proved together. $(s^{(1)}, s^{(2)})$ cannot be SEQ if one of its UIs cannot be sanctioned.

Assume $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$. As before, every UI from $(q^{(1)}, q^{(2)})$ for LDMs can be written as $(r^{(1)}, r^{(2)})$. As $s^{(1)}$ is not SEQ for CDM in $G^{(1)}$, $r^{(1)} \succ_C^{(1)} s^{(1)}$ for all $r^{(1)} \in R_{L_1}^{(1)+}(s^{(1)})$. Since $G^{(1)} > G^{(2)}$, $(r^{(1)}, r^{(2)}) \succ_C (s^{(1)}, s^{(2)})$ is alway true. Thus, no sanction is found in (iii) and (iv) and $(s^{(1)}, s^{(2)})$ cannot be SEQ for CDM in G in cases other than (1) and (2) in the Theorem. □

A.4 Proof for Theorem 4.6

Theorem 4.6 (SEQ for LDM_k in G): A state $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for LDM_1 in G iff either

(1) $s^{(1)} \in S^{(1)}$ is sequentially stable for LDM_1 in $G^{(1)}$, or

(2) for every $q^{(1)} \in R_{L_1}^{(1)+}(s^{(1)})$, there exists $(r^{(1)}, r^{(2)}) \in R_{\{C,L_2\}}^{+}(q^{(1)}, s^{(2)})$, such that $r^{(1)} \precsim_{L_1}^{(1)} s^{(1)}$.

Proof. (1) If $s^{(1)} \in S^{(1)}$ is SEQ in $G^{(1)}$, then for every $q^{(1)} \in R_{L_1}^{(1)+}(s^{(1)})$, there exists $r^{(1)} \in R_C^{(1)+}(q^{(1)})$, such that $r^{(1)} \precsim_{L_1}^{(1)} s^{(1)}$. For LDM_1, for every UI from $(s^{(1)}, s^{(2)})$ in G, i.e. $(q^{(1)}, s^{(2)}) \in R_{L_1}^{+}(s^{(1)}, s^{(2)})$, there exists $(r^{(1)}, s^{(2)}) \in R_C^{+}(q^{(1)}, s^{(2)})$, such that $(r^{(1)}, s^{(2)}) \precsim_{L_1} (s^{(1)}, s^{(2)})$. Thus, $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for LDM_1 in G.

(2) If $s^{(1)} \in S^{(1)}$ is not SEQ in $G^{(1)}$, then $r^{(1)} \succ_{L_1}^{(1)} s^{(1)}$ for every $q^{(1)} \in R_{L_1}^{(1)+}(s^{(1)})$ and $r^{(1)} \in R_C^{(1)+}(q^{(1)})$. If $G^{(1)} > G^{(2)}$ for CDM, for every $(r^{(1)}, r^{(2)}) \in R_{\{C,L_2\}}^{+}(q^{(1)}, s^{(2)})$, there must satisfy: $(r^{(1)}, r^{(2)}) \precsim_{L_1} (s^{(1)}, s^{(2)})$, which can be simplified as $r^{(1)} \precsim_{L_1}^{(1)} s^{(1)}$.

□

A.5 Proof for Theorem 4.7

Theorem 4.7 (GMR for CDM): Suppose that $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are GMR for CDM in $G^{(1)}$ and $G^{(2)}$, respectively. Then $(s^{(1)}, s^{(2)})$ is GMR for CDM in G.

Proof. Let $(q^{(1)}, q^{(2)})$ be a UI from $(s^{(1)}, s^{(2)})$ for CDM in G, i.e. $(q^{(1)}, q^{(2)}) \in R_C^{+}(s^{(1)}, s^{(2)})$. Then $(q^{(1)}, q^{(2)}) \succ_C (s^{(1)}, s^{(2)})$.

As before, either $q^{(1)} \succ_C^{(1)} s^{(1)}$ or $q^{(2)} \succ_C^{(2)} s^{(2)}$. Assume $q^{(1)} \succ_C^{(1)} s^{(1)}$, then $q^{(1)} \in R_C^{(1)+}(s^{(1)})$. As $s^{(1)}$ is GMR for CDM in $G^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $s^{(1)} \succsim_C^{(1)} r^{(1)}$. Similarly, if $q^{(2)} \succ_C^{(2)} s^{(2)}$, there exists $r^{(2)} \in R_{L_2}^{(2)}(q^{(2)})$ such that $s^{(2)} \succsim_C^{(2)} r^{(2)}$.

(1) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \precsim_C^{(2)} s^{(2)}$, $(r^{(1)}, q^{(2)}) \in R_L(q^{(1)}, q^{(2)})$. Then $(r^{(1)}, q^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Thus, $(r^{(1)}, q^{(2)})$ is a sanction and the UI for CDM from $(s^{(1)}, s^{(2)})$ to $(q^{(1)}, q^{(2)})$ is sanctioned.

(2) Similarly, if $q^{(1)} \precsim_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, there exists $(q^{(1)}, r^{(2)}) \in R_L(q^{(1)}, q^{(2)})$ such that $(q^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Thus, $(q^{(1)}, r^{(2)})$ is a sanction to the UI for CDM from $(s^{(1)}, s^{(2)})$ to $(q^{(1)}, q^{(2)})$.

(3) Assume that $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$. Then there exists $(r^{(1)}, r^{(2)}) \in R_L(q^{(1)}, q^{(2)})$ such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Thus, $(r^{(1)}, r^{(2)})$ is a sanction.

Overall, a sanctioning UM can be found on every UI for CDM from $(s^{(1)}, s^{(2)})$ to $(q^{(1)}, q^{(2)})$. Therefore, $(s^{(1)}, s^{(2)})$ is GMR for CDM in G. □

A.6 Proof for Theorem 4.8

Theorem 4.8 Assume $G^{(1)} > G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is GMR for CDM in G iff either

(1) both $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are GMR for CDM, or

(2) $s^{(1)} \in S^{(1)}$ is strictly GMR for the CDM in $G^{(1)}$ and, for all $q^{(1)} \in R_C^{(1)+,=}(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $r^{(1)} \prec_C^{(1)} s^{(1)}$.

Proof. Assume that $(s^{(1)}, s^{(2)})$ is GMR for CDM in G. Then if $(q^{(1)}, q^{(2)}) \in R_C^{+}(s^{(1)}, s^{(2)})$, there exists $(r^{(1)}, r^{(2)}) \in R_L(q^{(1)}, q^{(2)})$ such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Note that $(q^{(1)}, q^{(2)}) \succ_C (s^{(1)}, s^{(2)})$ implies either $q^{(1)} \succ_C^{(1)} s^{(1)}$ or $q^{(2)} \succ_C^{(2)} s^{(2)}$.

As both $s^{(1)}$ and $s^{(2)}$ can be either GMR or not, there are four possibilities:

(i) If both $s^{(1)}$ and $s^{(2)}$ are GMR, then $(s^{(1)}, s^{(2)})$ is GMR by Theorem 4.7. This is case (1) of the Theorem.

(ii) Suppose $s^{(1)} \in S^{(1)}$ is GMR for the CDM in $G^{(1)}$ while $s^{(2)} \in S^{(2)}$ is not GMR for the CDM in $G^{(2)}$. We investigate whether every UI from $(s^{(1)}, s^{(2)})$ is sanctioned by an LDMs' subsequent UM.

Assume $q^{(1)} \succ_C^{(1)} s^{(1)}$. Then there exists $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $r^{(1)} \precsim_C^{(1)} s^{(1)}$. Because $s^{(2)}$ is not GMR for CDM in $G^{(2)}$, there must be a UI $q^{(2)} \in R_C^{(2)+}(s^{(2)})$ such that $r^{(2)} \succ_C^{(2)} s^{(2)}$ for all $r^{(2)} \in R_{L_2}^{(2)}(q^{(2)})$.

a) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $s^{(2)} \precsim_C^{(2)} s^{(2)}$, then $(r^{(1)}, q^{(2)}) \in R_L(q^{(1)}, q^{(2)})$ and $(r^{(1)}, q^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$. Therefore, $(r^{(1)}, q^{(2)})$ is a sanction, so $(s^{(1)}, s^{(2)})$ is GMR.

b) If $q^{(1)} \precsim_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, then since $G^{(1)} > G^{(2)}$ and $(q^{(1)}, q^{(1)}) \succ_C (s^{(1)}, s^{(2)})$, $q^{(1)} \sim_C^{(1)} s^{(1)}$ or $q^{(1)} = s^{(1)}$. The sanction against $(q^{(1)}, q^{(2)})$ by LDMs can be $(r^{(1)}, r^{(2)}) \in R_L(q^{(1)}, q^{(2)})$ when either $r^{(1)} = q^{(1)}$ or $r^{(2)} = q^{(2)}$, but not both. Note that $r^{(2)} \succ_C^{(2)} s^{(2)}$, so $(r^{(1)}, r^{(2)})$ is a sanction, i.e. $(r^{(1)}, r^{(2)}) \prec_C (s^{(1)}, s^{(2)})$, only if $r^{(1)} \prec_C^{(1)} s^{(1)}$.

c) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, the sanction against $(q^{(1)}, q^{(2)})$ by the LDMs satisfies $(r^{(1)}, r^{(2)}) \prec_C (s^{(1)}, s^{(2)})$ iff $r^{(1)} \prec_C^{(1)} s^{(1)}$. In particular, $s^{(1)}$ is strict GMR for CDM in $G^{(1)}$.

Together, (a), (b), and (c) prove case (2) of the Theorem. To complete the proof, we show that if (1) and (2) fail, then $(s^{(1)}, s^{(2)})$ is not GMR for CDM in G.

(iii) Suppose that $s^{(1)} \in S^{(1)}$ is not GMR for the CDM in $G^{(1)}$ while $s^{(2)} \in S^{(2)}$ is GMR for the CDM in $G^{(2)}$.

(iv) $s^{(1)}$ is not GMR for CDM in $G^{(1)}$ and $s^{(2)}$ is not GMR for CDM in $G^{(2)}$.

(iii) and (iv) can be proved together. $(s^{(1)}, s^{(2)})$ cannot be GMR for CDM if one of its

UIs cannot be sanctioned.

Assume $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$. As before, every UM from $(q^{(1)}, q^{(2)})$ for LDMs can be written as $(r^{(1)}, r^{(2)})$. As $s^{(1)}$ is not GMR for CDM in $G^{(1)}$, $r^{(1)} \succ_C^{(1)} s^{(1)}$ for all $r^{(1)} \in R_{L_1}^{(1)}(s^{(1)})$. Since $G^{(1)} > G^{(2)}$, $(r^{(1)}, r^{(2)}) \succ_C (s^{(1)}, s^{(2)})$ is always true. Thus, no sanction is found in (iii) and (iv) and $(s^{(1)}, s^{(2)})$ cannot be GMR for CDM in G in cases other than (1) and (2) in the Theorem. □

A.7 Proof for Theorem 4.11

Theorem 4.11 (SMR for CDM in G): Suppose that $s^{(1)} \in S^{(1)}$ and $s^{(2)} \in S^{(2)}$ are states in $G^{(1)}$ and $G^{(2)}$, respectively. Assume that $s^{(1)}$ is SMR for CDM in $G^{(1)}$ and $s^{(2)}$ is SMR for CDM in $G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G.

Proof. Because $s^{(1)}$ and $s^{(2)}$ are SMR for CDM in $G^{(1)}$ and $G^{(2)}$ respectively, they are also GMR in $G^{(1)}$ and $G^{(2)}$, respectively. According to Theorem 4.7, $(s^{(1)}, s^{(2)})$ is then GMR for CDM in G. Thus, if $(q^{(1)}, q^{(1)}) \in R_C^{+}(s^{(1)}, s^{(2)})$ for $q^{(k)} \in R_C^{(k)}(s^{(k)})$ $(k = 1,\ 2)$, then there exists $(r^{(1)}, r^{(2)}) \in R_L(q^{(1)}, q^{(2)})$, such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$.

As $s^{(k)}$ is SMR for CDM in $G^{(k)}$ $(k = 1,\ 2)$, then if $q^{(k)} \in R_C^{(k)+}(s^{(k)})$, there exists at least $r^{(k)} \in R_{l_k}^{(k)}(q^{(k)})$ such that $r^{(k)} \precsim_C^{(k)} s^{(k)}$, and $t^{(k)} \precsim_C^{(k)} s^{(k)}$ for all $t^{(k)} \in R_C^{(k)}(r^{(k)})$. Therefore, there exists $(t^{(1)}, t^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$ for all $(t^{(1)}, t^{(2)}) \in R_C(r^{(1)}, r^{(2)})$. Hence, $(s^{(1)}, s^{(2)})$ is SMR for CDM in G. □

A.8 Proof for Theorem 4.12

Theorem 4.12: Assume $G^{(1)} > G^{(2)}$. Then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G, iff either

(1) $s^{(1)}$ is SMR for CDM in $G^{(1)}$ and $s^{(2)}$ is SMR for CDM in $G^{(2)}$, or

(2) $s^{(1)}$ is strict SMR for CDM in $G^{(1)}$ and, for all for all $q^{(1)} \in R_C^{(1)+,=}(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exist $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $r^{(1)} \prec_C^{(1)} s^{(1)}$ and $t^{(1)} \prec_C^{(1)} s^{(1)}$ for all $t^{(1)} \in R_C^{(1)}(r^{(1)})$.

Proof. Assume that $(s^{(1)}, s^{(2)})$ is SMR for CDM in G. Then if $(q^{(1)}, q^{(2)}) \in R_C^{+}(s^{(1)}, s^{(2)})$ for $q^{(k)} \in R_C^{(k)}(s^{(k)})$ $(k = 1, 2)$, there exists $(r^{(1)}, r^{(2)}) \in R_L(q^{(1)}, q^{(2)})$ such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$, and $(t^{(1)}, t^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$ for all $(t^{(1)}, t^{(2)}) \in R_C(r^{(1)}, r^{(2)})$. Note that $(q^{(1)}, q^{(2)}) \succ_C (s^{(1)}, s^{(2)})$ implies either $q^{(1)} \succ_C^{(1)} s^{(1)}$ or $q^{(2)} \succ_C^{(2)} s^{(2)}$.

As $(s^{(1)}, s^{(2)})$ must be GMR for CDM in G if $(s^{(1)}, s^{(2)})$ is SMR, Theorems 4.11 and 4.12 are used in the following proof.

As both $s^{(1)}$ and $s^{(2)}$ can be either SMR or not, there are three possibilities:

(i) If both $s^{(1)}$ and $s^{(2)}$ are SMR, then $(s^{(1)}, s^{(2)})$ is SMR for CDM in G by Theorem 4.11. This corresponds to case (1) of the theorem.

(ii) If $s^{(1)}$ is SMR for CDM in $G^{(1)}$ while $s^{(2)}$ is not SMR for CDM in $G^{(2)}$, we have proven $(s^{(1)}, s^{(2)})$ can be GMR for CDM by Theorem 4.12. To further determine GMR, we investigate whether $(t^{(1)}, t^{(2)})$ is also a sanction for CDM based on part (ii) in the proof of Theorem 4.12.

Because $s^{(1)}$ is SMR for CDM in $G^{(1)}$, then if $q^{(1)} \in R_C^{(1)+}(s^{(1)})$, there exists $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $r^{(1)} \precsim_C^{(1)} s^{(1)}$ and $t^{(1)} \precsim_C^{(1)} s^{(1)}$ for all $t^{(1)} \in R_C^{(1)}(r^{(1)})$.

a) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \precsim_C^{(2)} s^{(2)}$, the sanction by LDM_1 is $(r^{(1)}, q^{(2)})$. We investigate all UMs for CDM from $(r^{(1)}, q^{(2)})$, which can be written as $(t^{(1)}, r^{(2)})$ with the move on $G^{(1)}$, $(r^{(1)}, t_0^{(2)})$ on $G^{(2)}$ for $t_0^{(2)} \in R_C^{(2)}(q^{(2)})$, and $(t^{(1)}, t_0^{(2)})$ on both $G^{(1)}$ and $G^{(2)}$. They are sanctions for CDM if $r^{(1)} \prec_C^{(1)} s^{(1)}$ and $t^{(1)} \prec_C^{(1)} s^{(1)}$, which means $s^{(1)}$ must be strict SMR for CDM in $G^{(1)}$.

b) If $q^{(1)} \sim_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, the sanction by $LDMs$ is $(r^{(1)}, r^{(2)})$ if $r^{(1)} \prec_C^{(1)} s^{(1)}$. We investigate all UMs for CDM from $(r^{(1)}, r^{(2)})$, which can be written as $(t^{(1)}, r^{(2)})$ with the move on $G^{(1)}$, $(r^{(1)}, t^{(2)})$ on $G^{(2)}$, and $(t^{(1)}, t^{(2)})$ on both $G^{(1)}$ and $G^{(2)}$. Because $r^{(1)} \prec_C^{(1)} s^{(1)}$, $(r^{(1)}, t^{(2)})$ is a sanction for CDM. The other UMs, $(t^{(1)}, q^{(2)})$ and $(t^{(1)}, t^{(2)})$, are sanctions for CDM if $t^{(1)} \prec_C^{(1)} r^{(1)}$ for all $t^{(1)} \in R_C^{(1)}(r^{(1)})$.

c) If $q^{(1)} \succ_C^{(1)} s^{(1)}$ and $q^{(2)} \succ_C^{(2)} s^{(2)}$, the sanction by $LDMs$ is $(r^{(1)}, r^{(2)})$ if $s^{(1)}$ is strict GMR for CDM in $G^{(1)}$. All UMs for CDM from $(r^{(1)}, r^{(2)})$ can be written as $(t^{(1)}, q^{(2)})$ with the move on $G^{(1)}$, $(r^{(1)}, t^{(2)})$ on $G^{(2)}$, and $(t^{(1)}, t^{(2)})$ on both $G^{(1)}$ and $G^{(2)}$. As $t^{(1)} \prec_C^{(1)} s^{(1)}$ and $r^{(1)} \prec_C^{(1)} s^{(1)}$, all of the UMs are sanctions for CDM.

Overall for (ii), $(s^{(1)}, s^{(2)})$ is SMR for CDM in G if $s^{(1)}$ is strict SMR and for all for all $q^{(1)} \in R_C^{(1)+,=}(s^{(1)})$ and $q^{(1)} = s^{(1)}$, there exists $r^{(1)} \in R_{L_1}^{(1)}(q^{(1)})$ such that $r^{(1)} \prec_C^{(1)} s^{(1)}$ and $t^{(1)} \prec_C^{(1)} r^{(1)}$ for all $t^{(1)} \in R_C^{(1)}(r^{(1)})$.

The case (2) in the Theorem is thus proved. To complete the proof, we show that if (i) and (ii) fail, then $(s^{(1)}, s^{(2)})$ is not SMR for CDM in G.

(iii) If $s^{(1)} \in S^{(1)}$ is GMR for CDM in $G^{(1)}$, then if $q^{(1)} \in R_C^{(1)+}(s^{(1)})$, there exists $r^{(1)} \in$

$R^{(1)}_{L_1}(q^{(1)})$ such that $r^{(1)} \precsim^{(1)}_C s^{(1)}$ and $t^{(1)} \succ^{(1)}_C s^{(1)}$ for some $t^{(1)} \in R^{(1)}_C(r^{(1)})$.

If $q^{(1)} \succ^{(1)}_C s^{(1)}$ and $q^{(2)} \precsim^{(2)}_C s^{(2)}$, same as a) in case (ii), the sanction by LDM_1 is $(r^{(1)}, q^{(2)})$. All UMs for CDM from $(r^{(1)}, q^{(2)})$ can be written as $(t^{(1)}, r^{(2)})$ with the move on $G^{(1)}$, $(r^{(1)}, t^{(2)}_0)$ on $G^{(2)}$, and $(t^{(1)}, t^{(2)}_0)$ on both $G^{(1)}$ and $G^{(2)}$. As $t^{(1)} \succ^{(1)}_C s^{(1)}$, $(t^{(1)}, r^{(2)}) \succ_C (s^{(1)}, s^{(2)})$. Thus, CDM's UM $(t^{(1)}, r^{(2)}) \in R_C(r^{(1)}, q^{(2)})$ fails to be sanctioned by other DMs.

Thus, $(s^{(1)}, s^{(2)})$ is not SMR for CDM in G for case (iii). Overall, SMR states for CDM in G only exist in cases (i) and (ii). Therefore, this theorem is proved.

□

A.9 Proof for Theorem 5.1

Theorem 5.1: Suppose that $\Omega^{(k)}_C = \{\Omega^{(k)}_{C1}, ..., \Omega^{(k)}_{Cj_k}, ..., \Omega^{(k)}_{Ch_k}\}$ $(k = 1, 2;\ j_k = 1, ..., h_k)$ is the set of preference statements for CDM in $G^{(k)}$, and $\Omega_C = \{\Omega_{C1}, ..., \Omega_{Cj}, ..., \Omega_{Ch}\}$ $(j = 1, ..., h)$ is the set of preference statements for CDM in G, with $\Omega_{Cj} = \Omega^{(k)}_{Cj_k}$, and state $s = (s^{(1)}, s^{(2)}) \in S$ for $s^{(k)} \in S^{(k)}$, for the weighted preference,

$$\Psi(s) = \Psi^{(1)}(s^{(1)})w^{(1)} + \Psi^{(2)}(s^{(2)})w^{(2)} \tag{A.1}$$

Then, lexicographic preference is a special case of weighted preference for

(1) If $w^{(1)} > w^{(2)}$,

$w^{(1)} = 2^{h_2}$ and $w^{(2)} = 1$;

(2) If $w^{(1)} < w^{(2)}$,

$w^{(2)} = 2^{h_1}$ and $w^{(1)} = 1$.

Proof. According to Equation (5.1),

$$\Psi_{j_k}^{(k)}(s^{(k)}) = \begin{cases} 2^{h_k - j_k} & if\ \Omega_{j_k}^{(k)}(s^{(k)}) = T \\ 0 & \text{otherwise} \end{cases} \tag{A.2}$$

and

$$\Psi^{(k)}(s^{(k)}) = \sum_{j_k=1}^{h_k} \Psi_{j_k}^{(k)}(s^{(k)}) \tag{A.3}$$

Also,

$$\Psi_j(s) = \begin{cases} 2^{h-j} & if\ \Omega_j(s) = T \\ 0 & \text{otherwise} \end{cases} \tag{A.4}$$

and

$$\Psi(s) = \sum_{j=1}^{h} \Psi_j(s) \tag{A.5}$$

Suppose $w^{(1)} > w^{(2)}$, then

$$\sum_{j=1}^{h} \Psi_j(s) = \sum_{j_1=1}^{h_1} \Psi_{j_1}^{(1)}(s^{(1)}) w^{(1)} + \sum_{j_2=1}^{h_2} \Psi_{j_2}^{(2)}(s^{(2)}) w^{(2)} \tag{A.6}$$

where $h = h_1 + h_2$.

Let $j' = j_1$ and $j'' = j_2$, for $j', j'' = 1, ..., h$, because $w^{(1)} > w^{(2)}$, it can be easily concluded that $j' < j''$. Thus,

$$\Psi_j(s) = \begin{cases} \Psi_{j_1}^{(1)}(s^{(1)})2^{h_2} & j \leq h_1 \\ \Psi_{j_2}^{(2)}(s^{(2)}) & j > h_1 \end{cases} \tag{A.7}$$

Thus, $w^{(1)} = 2^{h_2}$ and $w^{(2)} = 1$.

□

A.10 Proof for Theorem 5.2

Theorem 5.2: Suppose $J_C^{(1)}$ is the $m \times m$ reachable matrix for CDM in $G^{(1)}$ and $J_C^{(2)}$ the $n \times n$ reachable matrix for CDM in $G^{(2)}$, I_n is an identity matrix of n scale, then the $mn \times mn$ hierarchical reachable matrix for CDM J_C in G is written as:

$J_C = J_C^{(1)} \bigotimes_R J_C^{(2)}$

$$= \begin{pmatrix} J_C^{(1)}(1,1) \otimes_r J_C^{(2)} & ... & J_C^{(1)}(1,m) \otimes_r J_C^{(2)} \\ \vdots & \ddots & \vdots \\ J_C^{(1)}(m,1) \otimes_r J_C^{(2)} & ... & J_C^{(1)}(m,m) \otimes_r J_C^{(2)} \end{pmatrix} \tag{A.8}$$

where $J_C^{(1)}(s^{(1)}, q^{(1)})$ is an entry in $J_C^{(1)}$ $(s^{(1)}, q^{(1)} = 1, ..., m)$ and

$J_C^{(1)}(s^{(1)}, q^{(1)}) \otimes_r J_C^{(2)}$

$$= \begin{cases} J_C^{(2)} & s^{(1)} = q^{(1)} \\ J_C^{(1)}(s^{(1)}, q^{(1)})(I_n + J_C^{(2)}) & s^{(1)} \neq q^{(1)} \end{cases} \tag{A.9}$$

Proof. Suppose that states $s, q \in S$ are two states in G where $s = (s^{(1)}, s^{(2)})$ and $q = (q^{(1)}, q^{(2)})$ ($s^{(1)}, q^{(1)} = 1, ..., m$; $s^{(2)}, q^{(2)} = 1, ..., n$). The reachable matrix J_C for CDM in G can be defined as:

$$J_C(s,q) = \begin{cases} 1 & if (s,q) \in AC \\ 0 & otherwise \end{cases} \tag{A.10}$$

Note that AC is the set of moves for CDM in G according to Definition 4.1. $(s, q) \in AC$ means $(s^{(1)}, q^{(1)}) \in AC^{(1)}$ or $(s^{(2)}, q^{(2)}) \in AC^{(2)}$, where $AC^{(1)}$ and $AC^{(2)}$ are the sets of moves for CDM in $G^{(1)}$ and $G^{(2)}$, respectively.

(1) If $s^{(1)} = q^{(1)}$, then $J_C(s, q) = J_C^{(2)}(s^{(2)}, q^{(2)})$. For each pair of $s^{(1)}$ and $s^{(2)}$ ($s^{(1)}, q^{(1)}$ $= 1, 2, ..., m$), the entries in J_C can be considered as a block. In each block, $J_C(s, q)$ can be written as $J_C^{(2)}$. Thus, $J_C^{(1)}(s^{(1)}, q^{(1)}) \otimes_r J_C^{(2)} = J_C^{(2)}$ in the theorem can be proven.

(2) If $s^{(1)} \neq q^{(1)}$, then

2.1) if $s^{(2)} = q^{(2)}$, then $J_C(s, q) = J_C^{(1)}(s^{(1)}, q^{(1)})$

2.2) if $s^{(2)} \neq q^{(2)}$, then $J_C(s, q) = J_C^{(1)}(s^{(1)}, q^{(1)}) J_C^{(2)}(s^{(2)}, q^{(2)})$

In an $n \times n$ identity matrix I_n, the entries can be written as

$$\theta_{s^{(2)},q^{(2)}} = \begin{cases} 1 & s^{(2)} = q^{(2)} \\ 0 & s^{(2)} \neq q^{(2)} \end{cases} \tag{A.11}$$

Also recall that $J_C^{(2)}(s^{(2)}, q^{(2)}) = 0$ if $s^{(2)} = q^{(2)}$.

Thus, 2.1) and 2.2) can be combined as $J_C(s, q) = J_C^{(1)}(s^{(1)}, q^{(1)})[\theta_{s^{(2)},q^{(2)}} + J_C^{(2)}(s^{(2)}, q^{(2)})]$.

Hence, $J_C(s, q)$ can be denoted as $J_C^{(1)}(s^{(1)}, q^{(1)})[I_n + J_C^{(2)}]$ when $s^{(1)} \neq q^{(1)}$.

Thus, Theorem 5.2 is proven.

□

A.11 Proof for Theorem 5.4

Theorem 5.4: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is Nash stable for CDM in G iff $s^{(k)}$ ($k = 1, 2$) is Nash stable for CDM in $G^{(k)}$.

Proof. Suppose $s, q \in S$ for $s = (s^{(1)}, s^{(2)})$ and $q = (q^{(1)}, q^{(2)})$. The UI from s to q in J_C^+ can be written as $J_C^+(s,q) = J_C(s,q)P_C^+(s,q)$, where $J_C^+(s,q)$ denotes the entry of J_C^+ in the s^{th} row and q^{th} column.

(I) If $s^{(k)}$ is Nash stable for CDM in $G^{(k)}$,

then

$$J_C^{(k)+}(s^{(k)}, q'^{(k)}) = 0 \quad \forall q'^{(k)} \in S^{(k)} \tag{A.12}$$

for $k =$ both 1 and 2.

Suppose $s = (s^{(1)}, s^{(2)})$ is not Nash stable for CDM in G. Then

$$J_C^+(s, q^*) \neq 0 \quad \exists q^* \in S. \tag{A.13}$$

Equation (A.13) implies that

$$J_C(s, q^*) \neq 0 \tag{A.14}$$

and

$$P_C^{+}(s, q^*) \neq 0 \tag{A.15}$$

As defined by Xu et al. (2009a),

$$P_C^{+}(s, q^*) = sign\left(\frac{\left|\Psi(q^*) - \Psi(s)\right| + \Psi(q^*) - \Psi(s)}{2}\right) \tag{A.16}$$

where

$$\Psi(q^*) - \Psi(s) = [\Psi^{(1)}(q^{*(1)}) - \Psi^{(1)}(s^{(1)})]w^{(1)} + [\Psi^{(2)}(q^{*(2)}) - \Psi^{(2)}(s^{(2)})]w^{(2)} \tag{A.17}$$

according to Equation (A.1).

Assume that $P_C^{(k)+}(s^{(k)}, q'^{(k)}) = 0$ for both $k = 1$ and 2. Then, $P_C^{+}(s, q^*) = 0$ according to Equation (A.16). This contradicts Equation (A.15). Therefore, we conclude that state s is Nash stable for CDM in G.

(II) If $s = (s^{(1)}, s^{(2)})$ is Nash stable for CDM in G, we prove $s^{(1)}$ and $s^{(2)}$ are Nash stable for CDM in $G^{(1)}$ and $G^{(2)}$, respectively.

Because $J_C^{+}(s, q^*) = 0$ for any $q^* = (q^{*(1)}, q^{*(2)}) \in S$ ($q^{*(1)} = 1, 2, ...m$ and $q^{*(2)} = 1, 2, ...n$), and

$$J_C^{+}(s, q^*) = J_C^{(1)}(s^{(1)}, q^{*(1)}) J_C^{(2)}(s^{(2)}, q^{*(2)}) P_C^{+}(s, q^*) \tag{A.18}$$

if $s^{(k)}$ is not Nash stable for CDM in $G^{(k)}$, then there exists $q^{(k)} \in S$, such that $J_C^{(k)}(s^{(k)}, q^{(k)}) \neq 0$, which contradicts $J_C^{+}(s, q^*) = 0$ according to Equation (A.18). Thus, $s^{(k)}$ is Nash stable for CDM in $G^{(k)}$.

Overall, the proof for (I) and (II) is completed. □

A.12 Proof for Theorem 5.6

Theorem 5.6: In a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, a state $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G if $s^{(k)}$ $(k = 1, 2)$ is sequentially stable for CDM in $G^{(k)}$.

Proof. According to the stability definition in Section 5.3, if state $s^{(k)}$ $(k = 1, 2)$ is sequentially stable for CDM in $G^{(k)}$, then $M_C^{(1)SEQ}(s^{(1)}, s^{(1)}) = 0$ and $M_C^{(2)SEQ}(s^{(2)}, s^{(2)}) = 0$.

The SEQ matrix for CDM in G can be written as $M_C^{SEQ} = J_C^{+} \cdot [E - sign(M_L^{+} \cdot (P_C^{-,=})^T)]$, where M_L^{+} is the joint UI matrix by LDM_1 and LDM_2.

$M_C^{SEQ}(s, s) = 0$ implies that

for any $q \in R_C^{+}(s)$, there exists $r \in R_L^{+}(q)$ such that

$$M_L^{+}(q, r) P_C^{-,=}(s, r) \neq 0 \tag{A.19}$$

$M_C^{(1)SEQ}(s^{(1)}, s^{(1)}) = 0$ and $M_C^{(2)SEQ}(s^{(2)}, s^{(2)}) = 0$ imply that

$\exists r^{(1)} \in R_{L_1}^{(1)+}(q^{(1)})$, such that

$$J_{L_1}^{(1)+}(q^{(1)}, r^{(1)}) P_C^{(1)-,=}(s^{(1)}, r^{(1)}) \neq 0 \tag{A.20}$$

and

$\exists r^{(2)} \in R_{L_2}^{(2)+}(q^{(2)})$, such that

$$J_{L_2}^{(2)+}(q^{(2)}, r^{(2)}) P_C^{(2)-,=}(s^{(2)}, r^{(2)}) \neq 0 \tag{A.21}$$

Equations (A.20) and (A.21) indicate that

$$J_{l_k}^{(k)+}(q^{(k)}, r^{(k)}) \neq 0$$

and

$$P_C^{(k)-,=}(s^{(k)}, r^{(k)}) \neq 0 \tag{A.22}$$

for $k = 1$, 2.

Thus, there exists $r = (r^{(1)}, r^{(2)}) \in R_L^+(q)$ such that $P_C^{-,=}(s, r) \neq 0$.

Equation (A.19) can be written as

$$M_L^+(q, r) \, P_C^{-,=}(s, r) \neq 0 \tag{A.23}$$

According to equation (5.13), $M_L^+(q, r)$ can be written as

$$M_L^+(q, r) = \bigvee_{t=2}^{\delta'} (J_{L_1}(q, r) + J_{L_2}(q, r) + M_{L_1}^{<t,+>}(q, r) + M_{L_2}^{<t,+>}(q, r)) \tag{A.24}$$

where obviously $\delta' > 2$ and

$$\begin{cases} M_{L_1}^{<t,+>}(q,r) = sign(J_{L_1}(q,r)M_{L_2}^{<t-1,+>}(q,r)) \\ M_{L_2}^{<t,+>}(q,r) = sign(J_{L_2}(q,r)M_{L_1}^{<t-1,+>}(q,r)) \end{cases}$$

There exists $J_{L_1}(q,r) = J_{L_1}^{(1)+}(q^{(1)}, r^{(1)})$ when $q^{(2)} = r^{(2)}$ and $J_{L_2}(q,r) = J_{L_2}^{(2)+}(q^{(2)}, r^{(2)})$ when $q^{(1)} = r^{(1)}$. Then, there exist $J_{L_1}(q,r) \neq 0$ and $J_{L_2}(q,r) \neq 0$ such that $M_L^+(q,r) \neq 0$.

Thus, $\left(\boldsymbol{e_q}^T \cdot M_L^+\right) \cdot (\boldsymbol{e_s}^T \cdot P_C^{-,=})^T \neq 0$, and therefore $M_C^{SEQ}(s,s) = 0$. Note that $\boldsymbol{e_q}^T$ is the transpose of a 0-1 vector with the q^{th} entry being 1 and others 0. □

A.13 Proof for Theorem 5.7

Theorem 5.7: Suppose G is a weighted basic hierarchical graph model consisting of $G^{(1)}$ and $G^{(2)}$, if a state $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G, then

(1) $s^{(k)}$ is sequentially stable for CDM in $G^{(k)}$ for both $k = 1$ and 2, or

(2) when $s^{(k)}$ is not sequentially stable for CDM in $G^{(k)}$ ($k = 1$ or 2, but not both), there exists $r = (r^{(1)}, r^{(2)}) \in S$, such that $\Psi(r) \leq \Psi(s)$, where state $r \in S$ corresponds to the entries in vector $\boldsymbol{r}$:

$\boldsymbol{r} = \boldsymbol{r^{(1)}} \times \boldsymbol{r^{(2)}} - \boldsymbol{e_{q^{(1)}}} \times \boldsymbol{e_{q^{(2)}}}$, for $\times$ symbolizes Cartesian Product;

$\boldsymbol{r^{(1)}} = (J_{L_1}^{(1)+})^T \cdot \boldsymbol{e_{q^{(1)}}} + \boldsymbol{e_{q^{(1)}}}$ and $\boldsymbol{r^{(2)}} = (J_{L_2}^{(2)+})^T \cdot \boldsymbol{e_{q^{(2)}}} + \boldsymbol{e_{q^{(2)}}}$ for all $(q^{(1)}, q^{(2)}) \in R_C^+(s)$;

$\boldsymbol{e_{q^{(k)}}}$ is a 0-1 vector with the ${q^{(k)}}^{th}$ entry being 1 and others 0.

Proof. If $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G, there exists $(r^{(1)}, r^{(2)}) \in$

$R_L^+(q^{(1)}, q^{(2)})$ for every $(q^{(1)}, q^{(2)}) \in R_C^+(s^{(1)}, s^{(2)})$, such that $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$, where $R_L^+(\cdot)$ represents the set of joint UIs for LDM_1 and LDM_2.

Situation (1) has been proven in Theorem 5.4.

In situation (2), because $s^{(1)}$ is not SEQ for CDM in $G^{(1)}$, then $r^{(1)} \succ_C^{(1)} s^{(1)}$. State $r^{(1)}$ can be either a UI for LDM_1 from $q^{(1)}$, referred to as the non-zero entry in $(J_{L_1}^{(1)+})^T \cdot \boldsymbol{e_{q^{(1)}}}$, or just $q^{(1)}$, corresponding to the non-zero entry in $\boldsymbol{e_{q^{(1)}}}$. Thus, the vector to represent $r^{(k)}$ can be written as $\boldsymbol{r^{(k)}} = (J_{L_k}^{(k)+})^T \cdot \boldsymbol{e_{q^{(k)}}} + \boldsymbol{e_{q^{(k)}}}$ $(k = 1, 2)$.

In addition, $(r^{(1)}, r^{(2)}) \precsim_C (s^{(1)}, s^{(2)})$ can also be written as $\Psi(r) \leq \Psi(s)$, where $r = (r^{(1)}, r^{(2)}) \in S$. □

A.14 Proof for Corollary 5.1

Corollary 5.1: State $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G iff $|w^{(k)}| \in (\alpha, \beta)$ for either $\alpha = 0$ or $\beta = 1$.

Proof. This corollary can be proven based on Theorem 5.7.

(1) According to Theorem 5.7, if $s^{(1)}$ and $s^{(2)}$ are SEQ for CDM in $G^{(1)}$ and $G^{(2)}$, respectively, then $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G regardless of the value of $|w^{(k)}|$. Hence, $|w^{(k)}| \in (0, 1)$.

(2) If neither $s^{(1)}$ nor $s^{(1)}$ is SEQ for CDM, $(s^{(1)}, s^{(2)}) \in S$ cannot be sequentially stable for CDM in G regardless of the value of $|w^{(k)}|$.

(3) According to Theorem 5.7, if $s^{(1)}$ is SEQ for CDM in $G^{(1)}$ and $s^{(2)}$ is not SEQ for CDM in $G^{(2)}$, $(s^{(1)}, s^{(2)}) \in S$ is sequentially stable for CDM in G iff $P_C^{-,=}(s, r) \neq 0$.

This means that there exists $r = (r^{(1)}, r^{(2)}) \in R_L^+(q)$, such that

$$[\Psi^{(1)}(s^{(1)}) - \Psi^{(1)}(r^{(1)})]|w^{(1)}| + [\Psi^{(2)}(s^{(2)}) - \Psi^{(2)}(r^{(2)})]|w^{(2)}| \geq 0 \tag{A.25}$$

Note that $\Psi^{(1)}(s^{(1)}) - \Psi^{(1)}(r^{(1)}) \geq 0$ and $\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)}) > 0$.

This expression can also be written as

$$|w^{(1)}| \geq \frac{[\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)})]}{[\Psi^{(1)}(s^{(1)}) - \Psi^{(1)}(r^{(1)})] + [\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)})]} \tag{A.26}$$

Thus,

$$\alpha = \frac{[\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)})]}{[\Psi^{(1)}(s^{(1)}) - \Psi^{(1)}(r^{(1)})] + [\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)})]} \tag{A.27}$$

and $\beta = 1$.

Analogously, if $s^{(1)}$ is not SEQ for CDM in $G^{(1)}$ and $s^{(2)}$ is SEQ for CDM in $G^{(2)}$, for $|w^{(1)}|$, $\alpha = 0$ and

$$\beta = \frac{[\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)})]}{[\Psi^{(1)}(s^{(1)}) - \Psi^{(1)}(r^{(1)})] + [\Psi^{(2)}(r^{(2)}) - \Psi^{(2)}(s^{(2)})]}. \tag{A.28}$$

Therefore, this corollary has been proven.

□

A.15 Proof for Theorem 5.8

Theorem 5.8: Suppose there exists a weighted basic hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, and the number of states in $G^{(1)}$ is $|S^{(1)}| = m$ and $|S^{(2)}| = n$ in $G^{(2)}$. If a state $s = (s^{(1)}, s^{(2)}) \in S$ is sequentially stable for LDM_1 in G, then

(1) $s^{(1)}$ is sequentially stable for LDM_1 in $G^{(1)}$ or

(2) $s^{(1)}$ is general metarational for LDM_1 in $G^{(1)}$,

$\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}} \cdot J_C^{(2)+} \neq 0$ and $\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}} \cdot M^{(2)+}_{\{C,L_2\}} \neq 0$

where $M^{(2)+}_{\{C,L_2\}}$ is the joint improvement matrix for CDM and LDM_2 in $G^{(2)}$ and $\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}}$ is the vector with $s^{(2)}th$ element 1 and others 0, and

$\exists r \in S$, such that $\Psi(r) > \Psi(q)$ for CDM and the r^{th} element in 0-1 vector $\boldsymbol{r}$ is 1,

where $\boldsymbol{r} = \left(\boldsymbol{e}^T_{\boldsymbol{q^{(1)}}} \cdot J_C^{(1)}\right) \times \left(\boldsymbol{e}^T_{\boldsymbol{s^{(2)}}} \cdot M^{(2)+}_{\{C,L_2\}}\right) \neq 0 \quad \forall q^{(1)} \in R^{(1)+}_{L_1}(s^{(1)})$, and $q = (q^{(1)}, s^{(2)})$.

Proof. If state $s^{(1)}$ is sequentially stable for LDM_1 in $G^{(1)}$, then $M^{SEQ}_{L_1}(s^{(1)}, s^{(1)}) = 0$. This implies that

for any $q^{(1)} \in R^{(1)+}_{L_1}(s^{(1)})$, there exists $r^{(1)} \in R^{(1)+}_C(q^{(1)})$, such that

$$J_C^{(1)+}(q^{(1)}, r^{(1)}) P^{-,=}_{L_1}(s^{(1)}, r^{(1)}) \neq 0. \tag{A.29}$$

State $s = (s^{(1)}, s^{(2)})$ is SEQ for LDM_1 in G iff,

for any $q = (q^{(1)}, q^{(2)}) \in R^+_L(s)$, there exists $r = (r^{(1)}, r^{(2)}) \in R^+_{N-L_1}(q)$, such that

$$M^+_{C,L_2}(q, r) P^{-,=}_{L_1}(s, r) \neq 0 \tag{A.30}$$

where $M^{+}_{C,L_2}(q,r)$ is the joint UI matrix by CDM and LDM_2 in G .

Because $J^{(1)+}_{C}(q^{(1)}, r^{(1)}) \neq 0$, then $\exists r = (r^{(1)}, r^{(2)}) \in R^{+}_{C,L_2}(q)$, such that $J^{+}_{C}(q,r) \neq 0$. Hence, $M^{+}_{C,L_2}(q,r) \neq 0$. Because $P^{-,=}_{L_1}(s^{(1)}, r^{(1)}) \neq 0$, then $r^{(1)} \precsim_{L_1} s^{(1)}$. One can conclude that $(r^{(1)}, r^{(2)}) \precsim_{L_1} (s^{(1)}, s^{(2)})$, $\forall r^{(2)}, s^{(2)} \in S^{(2)}$. This means that $P^{-,=}_{L_1}(s,r) \neq 0$.

Therefore, for any $q = (q^{(1)}, q^{(2)}) \in R^{+}_{L}(s)$, there exists $r = (r^{(1)}, r^{(2)}) \in R^{+}_{C,L_2}(q)$, such that $M^{+}_{C,L_2}(q,r)P^{-,=}_{L_1}(s,r) \neq 0$. State $s = (s^{(1)}, s^{(2)})$ is SEQ for LDM_1 in G for any $s^{(2)} \in S^{(2)}$, if $s^{(1)}$ is SEQ for LDM_1 in $G^{(1)}$.

□

A.16 Proof for Theorem 6.1

Theorem 6.1: In a hierarchical graph model G consisting of $G^{(1)}$ and $G^{(2)}$, suppose $G^{(1)}$ is more important than $G^{(2)}$ for CDM_i $(i = 1, 2)$. Let $\{\Omega^{(k)}_{C_i1}, \Omega^{(k)}_{C_i2}, ..., \Omega^{(k)}_{C_ih_k}\}$ be the set of preference statements in $G^{(k)}$ $(k = 1, 2)$. Then the set of preference statements for CDM_i in G can be written as $\{\Omega^{(1)}_{C_i1}, \Omega^{(1)}_{C_i2}, ..., \Omega^{(1)}_{C_ih_1}, \Omega^{(2)}_{C_i1}, \Omega^{(2)}_{C_i2}, ..., \Omega^{(2)}_{C_ih_2}\}$.

Proof. Let $\Omega_{C_ij} = \Omega^{(k)}_{C_ij_k}$ for $\Omega_{C_i} = \{\Omega_{C_i1}, ..., \Omega_{C_ij}, ..., \Omega_{C_ih}\}$ $(j = 1, 2, ..., h)$ and $\Omega^{(k)}_{C_i} = \{\Omega^{(k)}_{C_i1}, ..., \Omega^{(k)}_{C_ij_k}, ..., \Omega^{(k)}_{C_ih_k}\}$ $(k = 1, 2; j_k = 1, 2, ..., h_k)$. Suppose $G^{(1)} >_i G^{(2)}$, and suppose $s^{(1)} \succ^{(1)}_{C_i} t^{(1)}$, and $s^{(2)} \prec^{(2)}_{C_i} t^{(2)}$ where $s = (s^{(1)}, s^{(2)}) \in S$ and $t = (t^{(1)}, t^{(2)}) \in S$. Theorem 6.1 is equivalent to proving:

For any $\Omega_{C_ij_a} = \Omega^{(1)}_{C_ij_1}$ and $\Omega_{C_ij_b} = \Omega^{(2)}_{C_ij_2}$, $j_a < j_b$ always holds, where $j_a, j_b = 1, ..., h$ and $j_k = 1, ..., h_k$ $(k = 1, 2)$.

(1) Suppose $h_1, h_2 = 1$, $s^{(1)} \succ^{(1)}_{C_i} t^{(1)}$ implies

$\Omega_{C_i1}^{(1)}(s^{(1)}) = T$ and $\Omega_{C_i1}^{(1)}(t^{(1)}) = F$;

and $s^{(2)} \prec_{C_i}^{(2)} t^{(2)}$ implies

$\Omega_{C_i1}^{(2)}(s^{(2)}) = F$ and $\Omega_{C_i1}^{(2)}(t^{(2)}) = T$.

Because $s \succ_{C_i} t$, then $j_a < j_b$ for $\Omega_{C_ij_a} = \Omega_{C_i1}^{(1)}$ and $\Omega_{C_ij_b} = \Omega_{C_i1}^{(2)}$.

(2) Suppose $h_1 = 2, h_2 = 1$. Then the set of preference statements for CDM_i in G is written as $\{\Omega_{C_i1}^{(1)}, \Omega_{C_i2}^{(1)}, \Omega_{C_i1}^{(2)}\}$.

Because $\Omega_{C_i1}^{(2)}(s^{(2)}) = F$ and $\Omega_{C_i1}^{(2)}(t^{(2)}) = T$, then $\Omega_{C_ij_b}(s) = F$ and $\Omega_{C_ij_b}(t) = T$. Because $s \succ_{C_i} t$, then $j_a < j_b$ for $j_a = 1, 2$.

Inductively, $j_a < j_b$ holds for $j_a = 1, ...h_1$ and $h_2 = 1$. Analogously, $j_a < j_b$ holds for $j_a = 1, ...h_1$ and $j_b = 1, ..., h_2$.

□

A.17 Proof for Theorem 7.1

Theorem 7.1: Suppose in a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, $w_{C_i}^{(k_1)}$ and $w_{C_i}^{(k_2)}$ denote the weight of $G^{(k_1)}$ and $G^{(k_2)}$ for CDM_i $(k_1, k_2 = 1, 2, ..., K)$ and $G^{(k_1)} >_{C_i} G^{(k_2)}$. Let the preference statements in $G^{(k_1)}$ and $G^{(k_2)}$ be $\Omega_{C_ij_{k_1}}^{(k_1)} \in \Omega_{C_i}^{(k_1)}$ and $\Omega_{C_ij_{k_2}}^{(k_1)} \in \Omega_{C_i}^{(k_1)}$, for $0 < j_{k_1} < h_{k_1}$ and $0 < j_{k_2} < h_{k_2}$. Then, the weighted preference is a lexicographic preference when $\frac{w_{C_i}^{(k_1)}}{w_{C_i}^{(k_2)}} > 2^{h_{k_2}} - 1$.

Proof. Suppose $s = (s^{(1)}, ..., s^{(k_1)}, ..., s^{(k_2)}, ..., s^{(K)}) \in S$ and $s' = (s^{(1)}, ..., s'^{(k_1)}, ..., s'^{(k_2)}, ..., s^{(K)}) \in S$ $(k_1, k_2 = 1, 2, ..., K)$ for $s^{(k_1)}, s'^{(k_1)} \in S^{(k_1)}$ and $s^{(k_2)}, s'^{(k_2)} \in S^{(k_2)}$, where $s^{(k_1)} \succ_{C_i}^{(k_1)} s'^{(k_1)}$, $s^{(k_2)} \prec_{C_i}^{(k_2)} s'^{(k_2)}$. The aggregated score for s and s' can be expressed as

$$\Psi_{C_i}(s) = \sum_{k=1}^{K} \Psi_{C_i}^{(k)}(s^{(k)}) w_{C_i}^{(k)} \tag{A.31}$$

$$\Psi_{C_i}(s') = \sum_{k=1}^{K} \Psi_{C_i}^{(k)}(s'^{(k)}) w_{C_i}^{(k)} \tag{A.32}$$

Thus,

$$\begin{aligned} \Psi_{C_i}(s) - \Psi_{C_i}(s') = \left(\Psi_{C_i}^{(k_1)}(s^{(k_1)}) - \Psi_{C_i}^{(k_1)}(s'^{(k_1)})\right) w_{C_i}^{(k_2)} + \\ \left(\Psi_{C_i}^{(k_2)}(s^{(k_2)}) - \Psi_{C_i}^{(k_2)}(s'^{(k_2)})\right) w_{C_i}^{(k_2)} \end{aligned} \tag{A.33}$$

Suppose $\Delta\Psi_{C_i}^{(k_1)} = \Psi_{C_i}^{(k_1)}(s^{(k_1)}) - \Psi_{C_i}^{(k_1)}(s'^{(k_1)})$ and $\Delta\Psi_{C_i}^{(k_2)} = \Psi_{C_i}^{(k_2)}(s'^{(k_2)}) - \Psi_{C_i}^{(k_2)}(s^{(k_2)})$, then

$$\Psi_{C_i}(s) - \Psi_{C_i}(s') \geq \left(\Delta\Psi_{C_i}^{(k_1)}\right)_{min} w_{C_i}^{(k_1)} - \left(\Delta\Psi_{C_i}^{(k_2)}\right)_{max} w_{C_i}^{(k_2)} \tag{A.34}$$

where there exist

$\left(\Delta\Psi_{C_i}^{(k_1)}\right)_{min} = 1$, and $\left(\Delta\Psi_{C_i}^{(k_2)}\right)_{max} = 2^{h_{k_2}-1} + ...2^1 + 2^0 = 2^{h_{k_2}} - 1$.

Then, $\Psi_{C_i}(s) - \Psi_{C_i}(s') > 0$ indicates that $\left(\Delta\Psi_{C_i}^{(k_1)}\right)_{min} w_{C_i}^{(k_1)} - \left(\Delta\Psi_{C_i}^{(k_2)}\right)_{max} w_{C_i}^{(k_2)} > 0$, namely,

$$\frac{w_{C_i}^{(k_1)}}{w_{C_i}^{(k_2)}} > 2^{h_{k_2}} - 1 \tag{A.35}$$

□

A.18 Proof for Corollary 7.1

Corollary 7.1: Recall that $\Omega^{(k)}_{C_i j_k} \in \Omega^{(k)}_{C_i}$ ($k = 1, 2, ..., K$; $j_k = 1, ..., h_k$) represents the preference statements for $CDM_i \in N^{(k)}_C$ in $G^{(k)}$, and $\Omega_{C_i j} \in \Omega_{C_i}$ ($j = 1, ..., h$) represents the set of preference statements for CDM_i in G, with $\Omega_{C_i j} = \Omega^{(k)}_{C_i j_k}$, and state $s = (s^{(1)}, ..., s^{(K)}) \in S$ for $s^{(k)} \in S^{(k)}$,

(1) For the weighted preference,

$$\Psi_{C_i}(s) = \sum_{k=1}^{K} \Psi^{(k)}_{C_i}(s^{(k)}) w^{(k)}_{C_i} \tag{A.36}$$

where

$$\Psi^{(k)}_{C_i}(s^{(k)}) = \sum_{j_k=1}^{h_k} \Psi^{(k)}_{C_i j_k}(s^{(k)}).$$

and

$$\Psi^{(k)}_{C_i j_k}(s^{(k)}) = \begin{cases} 2^{h_k - j_k} & \Omega^{(k)}_{C_i j_k}(s^{(k)}) = T \\ 0 & otherwise \end{cases}$$

(2) For the lexicographic preference, Equation (A.36) exists, plus

$$w^{(k)}_{C_i} = 2^{h_{k+1} + ... + h_K} \tag{A.37}$$

and

$$h = \sum_{k=1}^{K} h_k \tag{A.38}$$

Proof. Situation (1) can be proved by Definition 2. In Situation (2), Equation (A.38) can be proved by Theorem 2. Also, according to Theorem 2, Equation (A.36) can be written as

$$\sum_{j=1}^{h} 2^{h-j} sgn(\Omega_{Cij}) = w_{C_i}^{(1)} \sum_{j_1=1}^{h_1} 2^{h_1-j_1} sgn(\Omega_{Cij_1}^{(1)}) + ...$$

$$+ w_{C_i}^{(K)} \sum_{j_K=1}^{h_K} 2^{h_K-j_K} sgn(\Omega_{Cij_K}^{(K)}) \tag{A.39}$$

where

$$w_{C_i}^{(1)} = 2^{h-h_1}$$

$$\vdots$$

$$w_{C_i}^{(k)} = 2^{h-(h_1+...+h_k)}$$

$$\vdots$$

$$w_{C_i}^{(K)} = 2^{h-h} = 1$$

□

A.19 Proof for Theorem 7.3

Theorem 7.3: A state $s = (s^{(1)}, ..., s^{(K)}) \in S$ in general hierarchical graph model G is Nash stable for $CDM_i \in N_C$ if for all $k = 1, ..., K$, $s^{(k)} \in S^{(k)}$ is Nash stable for CDM_i in $G^{(k)}$.

Proof. If for all $k = 1, 2, ..., K$, state $s^{(k)} \in S^{(k)}$ is Nash stable for CDM_i in $G^{(k)}$, then $R_{C_i}^{(k)+}(s^{(k)}) = \phi$. Thus $q \precsim_{C_i} s$ for any $q = (q^{(1)}, ..., q^{(K)}) \in R_{C_i}(s)$. □

A.20 Proof for Theorem 7.5

Theorem 7.5: A state $s = (s^{(1)}, ..., s^{(K)}) \in S$ in general hierarchical graph model G is sequentially stable for CDM_i if for all $k = 1, ..., K$, $s^{(k)} \in S^{(k)}$ is sequentially stable for CDM_i in $G^{(k)}$. □

Proof. If for all $k = 1, ..., K$, state $s^{(k)} \in S^{(k)}$ is SEQ for CDM_i in $G^{(k)}$, there exists $r^{(k)} \in R_{C_i}^{(k)+}(q^{(k)})$ for every $q^{(k)} \in R_{C_i}^{(k)+}(s^{(k)})$, such that $r^{(k)} \precsim_{C_i}^{(k)} s^{(k)}$.

If $q = (q^{(1)}, ..., q^{(K)}) \in R_{C_i}^{+}(s)$ with $q^{(k)} \in R_{C_i}^{(k)+}(s^{(k)})$ for every $k = 1, 2, ..., K$, then there exists $r = (r^{(1)}, ..., r^{(K)}) \in R_{N-C_i}^{+}(q)$ such that $r \precsim_{C_i} s$, where $R_{N-C_i}^{+}(q)$ is the set of UIs for all DMs in G except CDM_i at state q.

If $q = (q^{(1)}, ..., q^{(k)}) \in R_{C_i}^{+}(s)$ with $q^{(k)} \in R_{C_i}^{(k)+}(s^{(k)})$ for some $k = 1, 2, ..., K$, then there exists $r' = (q^{(1)}, ..., r^{(k)}, ..., q^{(K)}) \in R_{N-C_i}^{+}(q)$, such that $r' \precsim_{C_i} s$.

□

A.21 Proof for Theorem 7.6

Theorem 7.6 (SEQ for CDM): Suppose G is a general hierarchical graph model consisting of $G^{(k)}$ for $k = 1, ..., K$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is sequentially stable for CDM_i in G, then

(1) either $s^{(k)}$ is sequentially stable for CDM_i in $G^{(k)}$ for every $k = 1, ..., K$ or

(2) if $s^{(k)}$ is not sequentially stable for CDM_i in $G^{(k)}$ for some $k = 1, ..., K$, but not all, there exists state $r \in R^+_{N-C_i}(q)$ for every $q \in R^+_{C_i}(s)$, such that $r \precsim_{C_i} s$, where $R^+_{N-C_i}(q)$ is the set of UIs from state q by all DMs in G except CDM_i.

Proof. Situation (1) has been proven in Theorem 7.5.

In situation (2), if $s^{(k)}$ is not sequentially stable for CDM_i in $G^{(k)}$, then $r^{(k)} \succ^{(k)}_{C_i} s^{(k)}$ for any $r^{(k)} \in R^+_{N-C_i}(q^{(k)})$ and $q^{(k)} \in R^+_{C_i}(s^{(k)})$. Then, state s is SEQ for CDM_i iff there exists state $r \in R^+_{N-C_i}(q)$ for every $q \in R^+_{C_i}(s)$, such that $r \precsim_{C_i} s$. □

A.22 Proof for Theorem 7.7

Theorem 7.7 (SEQ for LDMs): In a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is sequentially stable for $l_k \in N^{(k)}_L$ in G, then

(1) either $s^{(k)}$ is SEQ for l_k in $G^{(k)}$, or

(2) $s^{(k)}$ is GMR for l_k in $G^{(k)}$ and there exist $r^{(k')} \in R_{N^{(k')}}(s^{(k')})$ for $k' = 1, ..., K$ except k, such that $r = (r^{(1)}, ..., r^{(k)}, ..., r^{(K)}) \in R^+_{N-l_k}(s)$ and $r \precsim_{l_k} s$, where $R_{N^{(k')}}$ denotes the set of joint UMs by all DMs in $G^{(k')}$ and $R^+_{N-l_k}$ the set of UIs by all DMs in G except l_k. □

Proof. If $s^{(k)} \in S^{(k)}$ is SEQ for l_k in $G^{(k)}$, then there exists $r^{(k)} \in R^{(k)+}_{N^{(k)}-l_k}(q^{(k)})$, such that $r^{(k)} \precsim^{(k)}_{l_k} s^{(k)}$ for every $q^{(k)} \in R^{(k)+}_{l_k}(s^{(k)})$. $R^{(k)+}_{N^{(k)}-l_k}$ is the set of UIs for all DMs in $G^{(k)}$ except l_k. Then, for $s = (s^{(1)}, ..., s^{(k)}, ..., s^{(K)}) \in S$, there must exist $r = (q^{(1)}, ..., r^{(k)}, ..., q^{(K)}) \in R^{+}_{N-l_k}(q)$, such that $r \precsim_{l_k} s$ for every $q = (q^{(1)}, ..., q^{(k)}, ..., q^{(K)}) \in R^{+}_{l_k}(s)$. Thus, situation (1) in the theorem applies in this case.

If $s^{(k)} \in S^{(k)}$ is not SEQ but GMR for l_k in $G^{(k)}$, then there exists $r^{(k)} \in R^{(k)}_{N^{(k)}-l_k}(q^{(k)})$, such that $r^{(k)} \precsim^{(k)}_{l_k} s^{(k)}$ for every $q^{(k)} \in R^{(k)+}_{l_k}(s^{(k)})$. To be an SEQ state for l_k in G, state $s = (s^{(1)}, ..., s^{(k)}, ..., s^{(K)}) \in S$ must satisfy:

there exists $r^{(k')} \in R_{N^{(k')}}(s^{(k')})$ for $k' = 1, ..., K$ except k, such that $r = (r^{(1)}, ..., r^{(k)}, ..., r^{(K)}) \in R^{+}_{N-l_k}(s)$ and $r \precsim_{l_k} s$.

Then, for every $q = (q^{(1)}, ..., q^{(k)}, ..., q^{(K)}) \in R^{+}_{l_k}(s)$ where $q^{(k)} \in R^{(k)+}_{l_k}(s^{(k)})$ and $q^{(k')} = s^{(k')}$, there exists $r \precsim_{l_k} s$. Therefore, situation (2) applies in this case. □

A.23 Proof for Theorem 7.9

Theorem 7.9 (GMR for LDM): In a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is general metarational for l_k in G if and only if $s^{(k)}$ is general metarational for l_k in $G^{(k)}$.

Proof. If $s^{(k)}$ is GMR for l_k in $G^{(k)}$ ($l_k \in N^{(k)}_L$), it can be easily proved that s is GMR for l_k in G.

If $s^{(k)}$ is not GMR for l_k in $G^{(k)}$ ($l_k \in N^{(k)}_L$), then $r^{(k)} \succ^{(k)}_{l_k} s^{(k)}$ always exists for every $r^{(k)} \in R^{(k)}_{N^{(k)}-l_k}(q^{(k)})$ and $q^{(k)} \in R^{(k)+}_{l_k}(s^{(k)})$, where $R^{(k)}_{N^{(k)}-l_k}(q^{(k)})$ is the set of joint UMs from

$q^{(k)}$ made by all DMs in $G^{(k)}$ except l_k.

For any $r' = (r'^{(1)}, ..., r'^{(K)}) \in S$ where $r'^{(k')} \in S^{(k')}$ $(k' = 1, 2, ..., K$ except $k)$, $r' \succ_{l_k} s$ always stands. Thus, s is not GMR for l_k in G. $\square$

A.24 Proof for Theorem 7.10

Theorem 7.10 (SMR for CDM): In a general hierarchical graph model G consisting of $G^{(1)}, ..., G^{(K)}$, if a state $s = (s^{(1)}, ..., s^{(K)}) \in S$ is symmetric metarational for CDM_i in G, and $q = (q^{(1)}, ..., q^{(K)}) \in R^+_{C_i}(s)$, then

(1) either $s^{(k)}$ is symmetric metarational for CDM_i in $G^{(k)}$ for every $k = 1, ..., K$ or

(2) if $s^{(k)}$ is GMR but not SMR for CDM_i in $G^{(k)}$ ($k = 1, ..., K$, but not all), for every $q \in R^+_{C_i}(s)$, there exists $t \in R_{C_i}(r)$, such that $t = (t^{(1)}, ..., t^{(K)}) \precsim_{C_i} s$, where $r = (r^{(1)}, ..., r^{(K)}) \precsim_{C_i} s$ and $r \in R_{N-C_i}(q)$.

Proof. (1) Because $s^{(k)}$ is symmetric metarational for CDM_i in $G^{(k)}$ for every $k = 1, ..., K$, for every $q^{(k)} \in R^{(k)+}_{C_i}(s^{(k)})$, there exists $r^{(k)} \in R^{(k)}_{N^{(k)}-C_i}(q^{(k)})$, such that $r^{(k)} \precsim^{(k)}_{C_i} s^{(k)}$ and $t^{(k)} \in R^{(k)}_{C_i}(r^{(k)})$ for all $t^{(k)} \in R^{(k)}_{C_i}(r^{(k)})$.

Because $s^{(k)}$ is symmetric metarational for CDM_i in $G^{(k)}$, $s^{(k)}$ is also GMR for CDM_i in every $G^{(k)}$. Then, s is GMR for CDM_i in G. This implies that there exists $r = (r^{(1)}, ..., r^{(K)}) \in R_{N-C_i}(q)$ for $q \in R^+_{C_i}(s)$, such that $r \precsim_{C_i} s$. In every $G^{(k)}$, $t^{(k)} \precsim^{(k)}_{C_i} s^{(k)}$ for all $t^{(k)} \in R^{(k)}_{C_i}(r^{(k)})$. Thus, $t = (t^{(1)}, ..., t^{(K)}) \precsim_{C_i} s$ for all $t \in R_{C_i}(r)$.

Therefore, for every $q = (q^{(1)}, ..., q^{(K)}) \in R^+_{C_i}(s)$ where $q^{(k)} \in R^{(k)+}_{C_i}(s^{(k)})$ and $q^{(k')} \in R^{(k')}_{C_i}(s^{(k')})$ $(k' = 1, ..., K$ except $k)$, there exist $(t^{(1)}, ..., t^{(K)}) \precsim_{C_i} s$ and $r = (r^{(1)}, ..., r^{(K)}) \precsim_{C_i}$

s for $t^{(k')} \in R_{C_i}^{(k')}(r^{(k')})$ and $r^{(k')} \in R_{N^{(k')}}^{(k')}(q^{(k')})$.

(2) If $\exists k = 1, ..., K$, for $s^{(k)}$ is not symmetric metarational for CDM_i in $G^{(k)}$, a SMR state $s \in S$ for CDM_i must be GMR in G. Then, for every $q = (q^{(1)}, ..., q^{(K)}) \in R_{C_i}^{+}(s)$ where $q^{(k)} \in R_{C_i}^{(k)}(s^{(k)})$, there exists $r = (r^{(1)}, ..., r^{(K)}) \precsim_{C_i} s$.

Because, $t^{(k)} \succ_{C_i}^{(k)} s^{(k)}$, then there must exist $t = (t^{(1)}, ..., t^{(K)}) \in R_{C_i}(r)$, such that $(t^{(1)}, ..., t^{(K)}) \precsim_{C_i} s$. $\square$