The effect of P2P marketplaces on retailing in the presence of mismatch risk

by

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Author’s Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Consumers frequently face mismatch risk as goods they purchase may be deemed inappropriate or below expectations. Due to this risk, consumers may avoid purchasing such goods and consequently hurt retailers. Can the emergence of peer-to-peer (P2P) marketplaces benefit retailers? On the one hand, P2P marketplaces can mitigate some of this risk by allowing consumers to trade mismatched goods. On the other hand, P2P marketplaces impose a threat on retailers as they compete with them over consumers. We develop a two-period model that highlights the effects introduced by P2P marketplaces.

We show that a P2P marketplace benefits both the retailer and consumers when the wholesale price is sufficiently high and hurts them both when the wholesale price is low. The introduction of a P2P marketplace can relieve consumers from the mismatch risk and induces the retailer to post a higher price. However, when the wholesale price is low, the platform manages to extract most, or all, of the consumers surplus and directly hurts consumers, and eventually the retailer who experiences lower sales in both periods. With a high wholesale price the P2P marketplace is limited in its ability of extracting consumer surplus, which increases the retailer sales and benefits both the retailer and consumers. We further observe that social welfare is generally higher unless the wholesale price is relatively low.

Keywords: mismatch risk, P2P marketplace, retailing strategy, backward induction.
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Chapter 1

Introduction

When consumers purchase goods, they frequently face risky decisions. After purchasing a product, consumers may find that products do not meet their expectation and their utility is diminished (Grewal et al., 1994; Teo and Yeong, 2003). This kind of concern can inhibit potential consumers from buying products, and hence harm the retailer. This phenomenon is particularly true for new goods as firms may intentionally limit the amount of information released to consumers to induce more profits (Chu and Zhang, 2011). With the development of the Internet and e-business (Bakos, 2001), consumers can trade with each other on peer-to-peer (P2P) marketplaces like eBay, which offers a channel for consumers to dispose of those products they do not want any more.

Obviously, to some extent, the emergence of P2P marketplaces relieves consumers’ concern about product mismatch risk, especially for those forward-looking consumers who take future payoff into consideration. As a result, more products will be sold when they are just released, but in the future, new products will also face competition from used goods sold on the P2P marketplaces. On the one hand, retailers can take advantage of the P2P platform to lower consumers’ risk about mismatch. On the other hand, they need to compete with the P2P platform. This leads us to explore several important research questions. Can, and under what conditions, P2P marketplaces benefit or harm retailers in the presence of mismatch risk? Additionally, how are consumers affected due to the emergence of P2P marketplaces? Namely, can consumers materialize the benefits presented by P2P marketplaces, or are these benefits neutralized by the operators of these marketplaces and/or retailers? Lastly, does a P2P marketplace induce a net social welfare gain? Specifically, is this a “bigger slice or a larger pie” type of argument? Namely, a marketplace will emerge if it can generate positive profit. Is this P2P profit merely a redistribution of wealth or generation of new welfare?
To study the implications of a P2P marketplace on the retailer, consumers and society, in this paper, we consider a two-period setting in which a single retailer sells new products to consumers. We start with a benchmark case in which there is no P2P marketplace. The retailer sets optimal selling prices (quantities) over the two periods to maximize his total profit. This analysis reveals the decisions made by the retailer as a function of the wholesale price and the magnitude of mismatch risk faced by consumers. Since consumers are forward-looking, they take into consideration the possibility that they will find the product to mismatch their expectations after purchase.

We then incorporate an electronic P2P platform into the setting. The platform allows consumers to salvage the mismatched products by selling them in the second period. The platform generates revenue by charging consumers a fixed transaction fee, and the forward-looking consumers now take into account the money they can recover by selling through the platform when they consider buying the product. Importantly, we assume that in the second period the uncertainty about the product’s valuation is eliminated and consumers who face the buying decision in the second period face no mismatch risk. Both retailer and platform operator optimize their decisions to maximize their respective profits. We derive the equilibrium results in each of the scenarios by backward induction.

The analysis gives rise to several important insights. Our primary finding is that when a P2P platform is involved, the retailer and consumers will benefit if the wholesale price is sufficiently high, and vice versa. The intuition behind is that a P2P platform will induce the retailers to increase the price of new products and may stimulate first period demand by eliminating product mismatch risk, however, it also imposes competition that impairs the retailer’s second period profit. When the wholesale price is low, the ratio of transaction fee and used goods selling price is high, which means the platform extracts all or most of the surplus form its consumers and hence reduces first period demand. As a result, the retailer will not gain enough to compensate the loss from the competition brought by the platform because of low first period demand. For the consumers, although they benefit from an additional marketplace in the second period, the P2P platform will not compensate their loss in the first period due to a higher retail price and relatively low salvage value. Only when the wholesale price is sufficiently high, will the platform sufficiently compensate consumers who may experience a product mismatch, and hence the benefit will outweigh the potential harm of having a platform, making the retailer and consumers better off. Interestingly, from society’s perspective, a P2P platform can harm total welfare when the wholesale price is sufficiently low.

The rest of the paper is organized as follows. §2 presents a review of related literature. §3 describes the model framework. The benchmark case and the setting that accounts for the presence of a platform are analyzed in §4. In §5, we further discuss the implications of
the platform on the retailer and the consumers. Lastly, §6 concludes the paper.
Chapter 2

Literature Review

The related literature can be grouped into two main streams. One stream is related to the emerging e-business and its marketplaces, while the other stream is related to retailers’ pricing strategies. We review each of these streams and other related papers below.

The first stream studies the impact of the rising e-marketplace on traditional retailing industry. There are two main types of e-market intermediaries, one is usually referred to as Business-to-Business/Business-to-Consumer (B2B/B2C) merchant who acquires goods from suppliers and resells them to buyers, and the other is usually referred to as P2P platform which allows its “affiliated” sellers to sell directly to its “affiliated” buyers. Hagiu (2007) discusses several trade-offs between operating as a B2B or as a P2P, and has revealed conditions under which the latter is preferred over the former.

The literature on B2B/B2C platforms has found many inspiring results. Choi et al. (2004) argue that the introduction of the e-marketplace can be beneficial to the supply chain and the amount of improvement depends on the fixed operational cost and the demand size of the e-marketplace. Bernstein et al. (2008) note that when traditional retailers use the Internet as a new selling channel in an oligopoly setting, the equilibrium does not necessarily imply higher profits for the firms: in some cases, rather, it emerges as a strategic necessity, and consumers are generally better off with clicks-and-mortar retailers who incorporate Internet sales. Ghose et al. (2005) build a duopoly model to examine conditions under which it is optimal for suppliers to operate in electronic used goods markets in addition to brick-and-mortar channels, explaining why these markets may not always be detrimental for them. They also highlight the strategic role that used goods commission set by the retailer plays in determining profits for suppliers. Under a potential revenue-sharing mechanism, Ryan et al. (2013) explore the interaction between a retailer
only selling his products online and an e-marketplace like Amazon, addressing questions such as whether the retailer should choose to contract with the online marketplace firm to sell through the marketplace system, and if so, at what price; on the other hand, whether the marketplace firm should sell a competing product, how to price that competing product, and how to design a contract with the participating retailer.

As many other papers, we focus on one intermediary type: peer-to-peer, or P2P. Whereas B2B/B2C platforms set their own prices for their products, P2P platforms give rise to market clearance mechanisms. Yin et al. (2009) consider the interaction between two behaviorally distinct used goods markets—retail and P2P—for a durable product. They argue that compared to a no used goods market scenario, the presence of either retail or P2P used goods market can be either beneficial or harmful for both channel members; while in the presence of a retail used market, the addition of a P2P market is usually beneficial for the retailer, but not when used products are valued highly by consumers. Gumus et al. (2013) explore the role of consumer valuation for used products in shaping the incentive of having a P2P used goods market. They show that higher consumer valuation of used goods in the P2P market increases the manufacturers’ incentive to offer a returns policy contract to the retailer. From empirical aspect, Ghose et al. (2006) study the elasticity of new product demand with respect to used product prices and the resulting changes in new and used product sales and overall surplus. Their analysis suggests that used books are poor substitutes for new books for most of Amazon’s consumers. Also, the increase in book readership from Amazon’s used book marketplace increases consumer surplus, decreases publisher welfare, and increases Amazon’s profits, which leads to an increase in total society welfare from the introduction of used book markets at Amazon.com. Most of the results in this stream are based on the assumption of a frictionless market, however, using e-platforms in reality is not always free. Similar to Mantin et al. (2013), our model incorporates a platform as a decision maker who charges a fixed transaction fee to its consumers, and explores how the platform’s decision affects the retailer in such a market setting.

The second related stream of research explores pricing strategies in the presence of product quality risk; that is, when consumers may face a product mismatch. Hsiao and Chen (2012) note that quality risk has an impact on a retailer’s return policy and pricing strategy by investigating the interplay in a two-segment market setting distinguishing consumers between having low valuation and high valuation. Gu and Liu (2013) argue that better matching between consumers and products may hurt the retailer’s profit, in which case the retailer cuts its own sales commissions and blocks manufacturer SPIFF (Sales Person Incentive Funding Formula) programs so as to suppress retail sales advisory. In e-business, retailers and consumers face not only product quality risk, but also mismatch
risk due to information asymmetry. We believe mismatch risk plays an important role in consumer behavior, but it is seldom considered in supply chain studies, hence we address the role of mismatch risk in the interaction between a platform and a retailer in our model.

There are several other related streams addressing issues in supply chain management based on a monopoly model setting. Bulow (1982), Huang et al. (2001), Gilbert et al. (2012) explore trade-offs between selling and renting in traditional retail businesses using a monopoly model. Altug (2012), Chen and Bell (2012) study one monopolistic retailer’s optimal return policy. Li and Zhang (2013) focus on exploring sellers’ preorder strategies. Inspired by these papers, we incorporate deterministic demand and forward-looking consumers into our two-period monopoly model.
Chapter 3

Modeling Framework

In this chapter, we describe our model setup and the methods to solve the model. We first consider a benchmark scenario, where a single profit-maximizing retailer sells new products to consumers over two periods in the absence of a platform. Similar to Yin et al. (2009), a new set of consumers shows up in each period and their valuation \( v \) is uniformly distributed between 0 and 1. Consumers face a mismatch risk: due to a variety of issues, not entirely modeled in this paper, such as lack of information, consumers may find the good to be unfit for their purposes. Consumers who are assumed to be forward looking, take this mismatch risk into account. Specifically, as in Hsiao and Chen (2012), after some preliminary use, with some probability \( \alpha \in [0, 1] \), consumers find the product to be mismatched and hence has no utility to them.\(^1\) We use \( \alpha_i \in \{0, 1\} \) to indicate consumer \( i \)'s mismatch realization, where \( \alpha_i = 1 \) implies the product is a mismatch and hence the realized valuation of the product is 0, and \( \alpha_i = 0 \) indicates a match and hence the entire valuation is materialized. We also let \( \alpha \in [0, 1] \) to represent the fraction of consumers having \( \alpha_i = 1 \). That is, \( \alpha \) is the expected mismatch risk faced by consumers. Since consumers are risk-neutral (as are all other agents in our setting) and forward looking, they purchase the good only if their expected utility, \( (1 - \alpha) v - p \), where \( p \) denotes the price, is positive.

The sequence of events in the benchmark setting is depicted in Figure 3.1. In period 1, the retailer orders an amount, \( q_{1n}^{BR} \), of new products from a wholesaler at a cost \( w \), and sets the first period’s selling price \( p_{1n}^{BR} \). We assume \( w < \bar{w} \equiv 1 - \alpha \); otherwise, the retailer will make negative profit in expectation, thus this is an individual rationality constraint and

\(^1\)We assume that the goods, since they have already been used by consumers, cannot be returned for a refund. In our follow-up research we also consider returns as a retailer’s strategy to combat P2P marketplaces.
holds for all scenarios. $w$ can be endogenized and becomes a decision variable. In that case the results follow through, only that the implications would be at the supply chain level rather than at the retailer level. In our setting, prices and quantities are equivalent: once the quantity is determined, the price is set to sell out. Intuitively, due to the deterministic nature of the model, since $w > 0$, the retailer will not purchase more units than it will sell. After $p_{1n}^{BR}$ has been announced, consumers buy products based on their expected utility. At the end of period 1, individual mismatch, $\alpha_i$, is realized and all consumers with mismatched products ($\alpha q_{1n}^{BR}$) get a negative utility, $-p_{1n}^{BR}$.

In period 2, a new set of consumers shows up. The retailer orders an amount, $q_{2n}^{BR}$, of new products from a wholesaler at a fixed price, $w$, and sets the second period’s selling price $p_{2n}^{BR}$. Then, as before, consumers buy products based on their expected utilities.

To capture the effect of a profit-maximizing P2P platform, which allows used goods transactions between consumers, on the retailer and the consumers, we consider a variation to the benchmark scenario. In this augmented scenario, the retailer sells new products in both periods, while used products are traded through the platform only in the second period. The used goods sold on the platform are all mismatched goods from the first period.

The stackelberg game sequence of events in the presence of a platform, which is shown in Figure 3.1, is as follows. Following Mantin et al. (2013), in the first stage, the platform commits to a fixed transaction fee $t$, the amount consumers are charged for using the platform. For example, Amazon’s fees are posted online and are rarely changed. Then, in the first period, the retailer orders an amount, $q_{1n}^{PR}$, of new products from a wholesaler at a cost $w$ per unit. This amount corresponds to the first period’s selling price $p_{1n}^{PR}$. Then consumers buy products based on their expected utilities. At the end of period 1, individual $\alpha_i$s are realized and all consumers with mismatched products ($\alpha q_{1n}^{PR}$) wait to sell their products through the P2P on-line market. Used products traded through the P2P market are devalued by a factor $\delta \in [0,1]$. That is, the expected benefit of these products is $(1 - \alpha)\delta v$.

In period 2, a new set of consumers shows up. The retailer sells $q_{2n}^{PR}$ new products, whereas $q_{2p}^{PP} = \alpha q_{1n}^{PR}$ used products are offered through the platform. Since used products on the P2P platform are sold to clear the market, the corresponding prices of new and used products may be determined, and are denoted as $p_{2n}^{PR}$ and $p_{2p}^{PP}$ respectively.

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2We abstract away from the issue of strategic inventories and accordingly retailers have no reason to store excess inventory. For more on strategic inventories, refer to Anand et al. (2008), Hartwig et al. (2012).
Next, we solve the model using backward induction for each of the two scenarios described earlier. We start with the benchmark case (§4), and then proceed with the platform setting (§5). Throughout the analysis, we use the following notation. Let the first superscript denote the scenarios: B for the benchmark scenario, P for the platform scenario. Let the second superscript denote the players: R for the retailer, and P for the platform. Let the first subscript denote the periods: 1 for the first, and 2 for the second. Finally, let the second subscript denote the types of products: n for the new products, and p for the used products on the platform. Not all notations will have superscripts and subscripts present, however, the meaning, given an order of these superscripts and subscripts, remains the same. For example, $p_{1n}^{BR}$ denotes the retailing price of a new product in the first period in the benchmark scenario (see also Appendix A for the complete notation).

Figure 3.1: Timeline of events: events that occur only in the presence of the platform are with dashed arrows; non-decision events are in italic.
Chapter 4

Model Analysis

We analyze the benchmark scenario and further the scenario incorporating a platform in this chapter.

4.1 Benchmark scenario: no P2P platform

In this section, we explore the retailer’s equilibrium decisions and resulting profit in the benchmark scenario without a P2P platform. In period 1, the expected utility from buying a product is a consumer’s expected valuation of the product minus its selling price, that is, \( U^B_1 = (1 - \alpha)v - p^BR_1 \). The consumer indifferent between buying and not buying a product in period 1 has a valuation of \( \bar{v}^B_1 \equiv p^BR_1 n / (1 - \alpha) \). Then the quantity of products sold in period 1 is given by \( q^BR_1 n = 1 - \bar{v}^B_1 = 1 - p^BR_1 n / (1 - \alpha) \), which follows from the assumption that consumers’ valuations are uniformly distributed between 0 and 1. Similarly, in period 2, we have \( q^BR_2 n = 1 - p^BR_2 n / (1 - \alpha) \).

Having derived the demand functions, we can express the retailer’s profit functions. In period 2, the profit of the retailer is given by \( \Pi^BR_2 = (p^BR_2 n - w)q^BR_2 n = ((1 - \alpha)(1 - q^BR_2 n) - w)q^BR_2 n \), which is concave in \( q^BR_2 n \). Maximizing \( \Pi^BR_2 \) with respect to \( q^BR_2 n \), we find that \( q^BR^*_2 n = (1 - \alpha - w)/(2(1 - \alpha)) \), \( p^BR^*_2 n = (1 - \alpha + w)/2 \) and consequently \( \Pi^BR^*_2 n = (1 - \alpha - w)^2/(4(1 - \alpha)) \). Evidently, in this benchmark setting, the two periods are independent of each other. In period 1, the profit of the retailer is given by \( \Pi^BR_1 = (p^BR_1 n - w)q^BR_1 n \). Therefore, the total profit of the retailer is \( \Pi^BR = \Pi^BR_1 + \Pi^BR_2 = (1 - \alpha)(1 - q^BR_1 n - w)q^BR_1 n + (1 - \alpha - w)^2/(4(1 - \alpha)) \), which is concave in \( q^BR_1 n \). Maximizing \( \Pi^BR \) with respect to \( q^BR_1 n \), we find that \( q^BR^*_1 n = q^BR^*_2 n = (1 - \alpha - w)/(2(1 - \alpha)) \), \( p^BR^*_1 n = p^BR^*_2 n = (1 - \alpha + w)/2 \) and
\[ \Pi^{BR*} = (1 - \alpha - w)^2/(2(1 - \alpha)) \]. Recall that we assume \( w < 1 - \alpha \), which guarantees strictly positive quantities. The profit is decreasing in \( w \), and decreasing in \( \alpha \). That is, higher wholesale price and higher mismatch risk will harm the retailer’s profit.

In this benchmark scenario, there is no channel for consumers to trade their mismatched products; hence they receive negative utility when a product mismatch occurs. This risk is taken into account by consumers of which are all forward-looking in both the first and second periods. In the next section we explore if a P2P platform can remedy the mismatch issue.

### 4.2 A P2P platform scenario

In the presence of a platform, a consumer has the option of selling the product in the second period at the market clearing price, \( p_{2p}^{PP} \), while paying a cost \( t^{PP} \) to the platform for using this service. Thus, in period 1, the expected utility from buying a product is a consumer’s expected valuation of a product minus its selling price, plus his expected payoff from selling the product through the platform, that is, \( U_1^P = (1 - \alpha)v - p_{1n}^{PR} + \alpha(p_{2p}^{PP} - t^{PP}) \). A consumer is indifferent between buying and not buying a product in period 1 when his valuation of the product is \( \bar{v}_1^P = (1 - \alpha)v - p_{1n}^{PR} + \alpha(p_{2p}^{PP} - t^{PP}) \). The indifference valuation implies that the quantity of products sold in period 1 is \( q_{1n}^{PR} = 1 - \bar{v}_1^P = 1 - (p_{1n}^{PR} - \alpha(p_{2p}^{PP} - t^{PP}))/\alpha \). In period 2, the expected utility from buying a new product is \( U_2^P = (1 - \alpha)v - p_{2n}^{PR} \), whereas the expected utility of buying a used product from the platform is \( U_p^P = (1 - \alpha)\delta v - p_{2p}^{PP} \). A consumer is indifferent between buying a new product and a used product in period 2 if his valuation of the used product is \( \bar{v}_2^P = (p_{2n}^{PR} - p_{2p}^{PP})/\delta(1 - \alpha) \), implying \( q_{2n}^{PR} = 1 - \bar{v}_2^P = 1 - (p_{2n}^{PR} - p_{2p}^{PP})/(1 - \delta)(1 - \alpha) \). A consumer is indifferent between buying and not buying a used product from the platform when his valuation of the used product is \( \bar{v}_p^P = p_{2p}^{PP}/\delta(1 - \alpha) \), implying \( q_{2p}^{PP} = \bar{v}_p^P - \bar{v}_p^P = (p_{2n}^{PR} - p_{2p}^{PP})/(1 - \delta)(1 - \alpha) - p_{2p}^{PP}/\delta(1 - \alpha) \). Having derived the demand functions, we solve the game via backward induction.

We solve the Stackelberg game backwards by first maximizing the retailer’s total profit in the two periods with respect to \( q_{1n}^{PR} \) and \( q_{2n}^{PR} \). As the model is solved backwards, we find a threshold of first period quantity due to nonnegative constraint on \( q_{2n}^{PR} \), which leads to two subgames: low \( q_{1n}^{PR} \) (such that \( q_{2n}^{PR} > 0 \)) and high \( q_{1n}^{PR} \) (such that \( q_{2n}^{PR} = 0 \)). Further, by maximizing the platform’s profit with respect to \( t^{PP} \), subject to \( 0 \leq t^{PP} \leq p_{2p}^{PP} \) under low \( q_{1n}^{PR} \) case, we have two subgames: low \( t^{PP} \) (such that \( q_{1n}^{PR} > 0 \)), and high \( t^{PP} \) (such that
depending on the value of $w$; consumers who use the platform to sell their mismatched products. Let $w_1 \equiv 1 - \alpha - \frac{\alpha \delta (1 - \alpha)(2 + \alpha^2 \delta - \alpha^2 \delta^2)}{2 - 3 \alpha^2 \delta^2 + 3 \alpha^3 \delta - \alpha^3 \delta^2 + \alpha^4 \delta^3}; w_2 \equiv 1 - \alpha - \frac{\alpha^2 \delta^2 (1 - \alpha)}{2 + 2 \alpha^3 \delta - \alpha^3 \delta^2 - \alpha^4 \delta^3};$ and $w_3 \equiv 1 - \alpha - \frac{\alpha^2 \delta^2 (1 - \alpha)}{2 + 2 \alpha^3 \delta - \alpha^3 \delta}$. We have the following Lemma. All proofs are provided in Appendix B.

**Lemma 1.** In equilibrium, the platform sets the transaction fee, $t^{PP}$, as follows:

1. If the wholesale price is sufficiently low, $w \leq w_1$, then the platform transaction fee is equal to the used product’s selling price ($t^{PP*} = p^{PP}_{2p}$) to extract all benefits from its consumers;

2. If $w_1 < w \leq w_2$, then the transaction fee is lower than the used product’s selling price ($t^{PP*} < p^{PP}_{2p}$) to induce more consumers to purchase in the first period;

3. If $w_2 < w \leq w_3$, then the platform sets the transaction fee to price the retailer out of the second period (i.e., $q^{PR}_{2n} = 0$);

4. If the wholesale price is sufficiently high, $w > w_3$, the platform sets the transaction fee as though it were a monopolist in the second period.

Formally,

$$t^{PP*} = \begin{cases} \frac{\delta(w(1+\alpha-\alpha\delta)+(1-\alpha)(1-\alpha+\alpha\delta+\alpha^2\delta^2-\alpha^2\delta))}{1-\alpha-w+\alpha\delta w} & \text{if } w \leq w_1; \\ \frac{2\alpha}{w(1-\alpha+\alpha^2\delta)-(1-\alpha)(1-\alpha\delta+\alpha^2\delta^2)} & \text{if } w_1 < w \leq w_2; \\ \frac{1-\alpha-w+\alpha\delta(1-\alpha)}{2\alpha} & \text{if } w_2 < w \leq w_3; \\ & \text{if } w > w_3. \end{cases}$$

The corresponding retailer’s decisions are given by:

$$\{q^{PP*}_{1n}, q^{PP*}_{2n}\} = \begin{cases} \{\frac{(1-\alpha-w)(2-\alpha\delta)}{2(1-\alpha)(2+\alpha^2\delta^2-\alpha^2\delta^2)}, \frac{(1-\alpha-w)(2+\alpha^2\delta^2-2\alpha\delta)}{(1-\alpha)(2+\alpha^2\delta^2-2\alpha\delta)}\} & \text{if } w \leq w_1; \\ \frac{1-\alpha-w+\alpha\delta(1-\alpha)}{2\alpha(1-\alpha)}, 0 & \text{if } w_1 < w \leq w_2; \\ \{\frac{(1-\alpha-w)(1+\alpha^2\delta-\alpha^2\delta^2)}{4(1-\alpha)(1+\alpha^2\delta-\alpha^2\delta^2)}, \frac{4(1-\alpha)(1+\alpha^2\delta-\alpha^2\delta^2)}{4(1-\alpha)(2+\alpha^2\delta^2-\alpha^2\delta^2)-(1-\alpha)\alpha^2\delta^2}\} & \text{if } w_2 < w \leq w_3; \\ \frac{1-\alpha-w+\alpha\delta(1-\alpha)}{(1-\alpha)(1+\alpha^2\delta)}, 0 & \text{if } w > w_3. \end{cases}$$
The optimal transaction fee and retailer’s corresponding decisions are demonstrated in Figure 4.1. From the platform’s perspective, his profit is affected by the transaction fee and the demand of new products in the first period, which in turn, determine the number of used goods available in the second period. Therefore, the platform is trading-off between setting a high transaction fee, referred to as the transaction fee effect, and having higher first period demand, referred to as the demand effect, because a higher transaction fee decreases the demand, and vice versa. Therefore, the optimal transaction fee is a result of the interaction between the transaction fee effect and the demand effect.

Lemma 1 shows that when the wholesale price is sufficiently low, the platform can set the transaction fee as high as possible, equal to the selling price of the used goods. In other words, the transaction fee effect dominates the demand effect, and the platform has the power to extract the entire surplus from consumers who use its service. As the wholesale price increases, the transaction fee effect decreases while the demand effect increases, as seen in segment 2 in Figure 4.1a, in which case the platform’s power is weakened. In order to stimulate first period demand, the platform must reduce its transaction fee so as to compensate consumers that may experience a mismatched product. As the wholesale
price continues to increase, the platform will induce the retailer to abandon his business in the second period, as seen in segment 3 in Figure 4.1b. Therefore, the used goods on the platform are the only goods sold in the market during the second period. The platform may extract more of its consumers’ surplus, but not enough to extract the entire surplus because the threat of the retailer selling new products in the second period. With the wholesale price sufficiently high, \( w \geq w_3 \), this threat is eliminated and the first period demand is very low due to the high retail price, consequently the platform needs to induce demand in the first period by reducing the transaction fee as \( w \) further increases.
Chapter 5

The impact of a P2P platform

In this chapter we investigate the net effect of a P2P platform on retailers and consumers. Consumers may sell mismatched products through the platform, however they must pay a transaction fee to the platform operator. On the one hand, the mismatch risk can be somewhat mitigated, as they can sell those unwanted products; while, on the other hand, the retailer faces competition in the second period. Which of these effects dominates? More specifically, how does the entrance of a P2P platform affect the retailer in terms of retail price and profit? Is the retailer better off or worse off? How is the consumers’ surplus affected? These questions are all explored in this section.

We have the following theorem.

**Theorem 1.** The introduction of a P2P platform

(1) reduces the expected profit of the retailer when the wholesale price is sufficiently low, i.e., \( w < \tilde{w} \equiv 1 - \alpha - \frac{\alpha(1-\alpha)(1-\alpha+2\sqrt{1+\alpha^2 \delta - \alpha^2 \delta^2})}{3+2\alpha \delta + 4\alpha^2 \delta - 5\alpha^2 \delta^2} \in [w_1, w_2] \);

(2) increases the expected profit of the retailer when the wholesale price is sufficiently high, i.e., \( w \geq \tilde{w} \).
Figure 5.1: Wholesale price thresholds for retailer and consumers, $\delta = 0.9$

Theorem 1 is demonstrated in Figure 5.1. From the retailer’s perspective, the emergence of a P2P platform induces several effects. In the second period, the platform exposes the retailer to competition from used products, which results in lower selling prices as well as lower demand for new products (see Figure 5.2b), and hence lower profit in the second period (see Figure 5.3a). At the same time, because the platform mitigates some of the mismatch risk, the retailer can increase the selling price of new products in the first period. However, higher selling price also has a negative effect on the first period demand. As the wholesale price increases, $w \geq w_1$, the platform lowers the transaction fee to increase consumers’ expected payoff to stimulate first period demand (the slope of $q_{in}^{PR}$ is less steeper in segment 2 than that in segment 1 in Figure 5.2a). When the platform’s demand stimulation effect dominates the negative effect of higher selling price, it results in greater first period profit compared to the benchmark scenario (see segment 2 in Figure 5.3a). As the wholesale price increases ($w > \tilde{w}$), the gain in the first period from benefit from the platform exceeds the loss in the second period from the platform competition, which leads to a net positive effect of the platform (see segment 2 in Figure 5.3b). In other words, the advantage of a P2P platform is that it increases the price of new products and may stimulate first period demand, but the disadvantage is that the competition impairs the retailer’s profit. Only when the wholesale price is sufficiently high, will the benefit compensate the potential harm of having a platform, making the retailer better off.
Figure 5.2: Comparisons of retailer’s optimal decision variables between Benchmark and Platform scenarios, $\alpha = 0.8$, $\delta = 0.9$
Figure 5.3: Comparisons of retailer's optimal profit between Benchmark and Platform scenarios: retailer’s profit, $\alpha = 0.8$, $\delta = 0.9$

Proposition 1. $\frac{\bar{w} - \tilde{w}}{\bar{w}}$ is increasing in $\alpha$ for all $\alpha \in [0, 1]$.

Proposition 1 shows that the relative range (with respect to the range of feasible $w$) of the retailer being better off with the platform is increasing in $\alpha$. In other words, with a given wholesale price, $w$, higher mismatch risk leads to higher likelihood of platform benefiting the retailer. If the retailer could not prohibit the platform from entering the market, he may want the products to have a mismatch risk as high as possible, $\alpha = 1 - w$, which gives her the highest probability, 1, of benefiting from the introduction of a P2P platform. However, higher $\alpha$ also reduces her profit in both scenarios, which means if the retailer wants to negotiate with her wholesaler to increase the product mismatch risk to give her a higher chance of benefiting from potential entrance of a P2P platform, he will actually lose profit because of a higher product mismatch risk.

We also have the following insight regarding consumer surplus, which is the cumulative integration of consumers’ net utility over the two periods. That is, the monetary gain obtained by consumers for purchasing a product at a price less than the highest price that they would be willing to pay.

Theorem 2. The introduction of a P2P platform
(1) decreases consumer surplus when the wholesale price is sufficiently low, \( w < \tilde{w} \);
(2) increases consumer surplus when the wholesale price is sufficiently high, \( w \geq \tilde{w} \).

This is an important insight. Although the perceived knowledge is that a P2P platform always benefit consumers as it offers a channel to trade used goods and further pressure retailers to lower prices, we find that this is not always true. In the presence of mismatch risk, a platform allows consumers to recover some of the loss from the mismatch. However, a platform can be detrimental to consumers’ welfare, when the wholesale price is low. For example, \( w < w_1 \), the platform sets the transaction fee equal to the selling price of the online products, and consequently consumers do not gain anything by trading through the platform. Even worse, they suffer from a higher price in the first period, which results in a lower consumer surplus. When the transaction fee is lower than the selling price of the used products on the platform, then consumers strictly gain from trading, yet the higher first period price erodes these gains. It is only when the wholesale price is sufficiently high, \( w > \tilde{w} \), that consumers gain more from the presence of a platform as compared to the benchmark scenario (see Figure 5.4).
Lastly we consider the net effect on social welfare. Trivially, when the wholesale price is high, a platform is welfare-imposing, as both the retailer and consumers are better off and the platform makes positive profit. However, the two previous results reveal the erosion of the retailer’s and the consumers’ welfare in the presence of a platform when the wholesale price is low. Is it merely a gravitation of welfare from these two agents—the retailer and the consumers—to the new agent—the platform? We have the following insight.

**Observation 1.** The introduction of a P2P platform

1. reduces total welfare when the wholesale price is sufficiently low, $w < \hat{w}$;

2. increases total welfare when the wholesale price is sufficiently high, $w \geq \hat{w}$,

where $\hat{w}$ is the threshold value that solves $SW^P - SW^B \equiv \Pi^{PP} + \Pi^{PR} + CS^P - (\Pi^{BR} + CS^B) = 0$.

![Figure 5.5: Wholesale price thresholds for social welfare, $\delta = 0.6$](image)

Figure 5.5: Wholesale price thresholds for social welfare, $\delta = 0.6$
This observation indicates that a P2P platform is not always beneficial to society and is shown in Figure 5.5. Specifically, when the wholesale price is sufficiently low, the emergence of a P2P platform has a detrimental effect on social welfare. Figure 5.6, plots of the welfare with a platform, which is $\Pi^{PP} + \Pi^{PR} + CS^{P}$, and the welfare without a platform, which is $\Pi^{BR} + CS^{B}$, further illustrates this insight. When a product’s wholesale price is sufficiently low, $w < \hat{w}$, both the retailer and the consumers suffer from the presence of a P2P platform ($\Delta \Pi^{R} \equiv \Pi^{PR} - \Pi^{BR} < 0, \Delta CS^{P} \equiv CS^{P} - CS^{B} < 0$), and the profit obtained by the platform ($\Delta \Pi^{R} \equiv \Pi^{PR} - 0 > 0$) does not compensate for the loss of welfare of the retailer and the consumers. Hence, social welfare diminishes as the platform enters the market.

Figure 5.6: Social welfare components, $\alpha = 0.8, \delta = 0.9$
Chapter 6

Conclusion

The development of P2P platforms has changed traditional retailing. Our paper fills in the academic literature gap by incorporating a P2P platform as a decision maker and taking product mismatch risk into consideration at the same time. We show in this paper that when product mismatch risk is present, a P2P platform can mitigate some of the risk by allowing consumers to trade their used goods, however, at the same time, used goods on the platform also compete with new goods from retailers. Specifically, our model shows that a P2P platform can benefit both the retailer and consumers when the wholesale price is sufficiently high. The intuition is that a P2P platform will increase the price of new products and stimulates first period demand, but the competition it imposes will impair the retailer’s second period profit. Only when the wholesale price is sufficiently high, will the benefit compensate the potential harm of having a platform, making the retailer better off. In terms of social welfare, a P2P platform can harm society, all agents in our model, when the wholesale price is sufficiently low.

The results in this paper are based on a monopolistic model. In practice, there could be multiple retailers offering identical or similar products and various P2P platforms can co-exist. Accordingly, fiercer competition may arise along more complicated interactions among market agents. A possible extension is to extend the present model to involve two or more retailers and platforms. For instance, considering the competition between Microsoft and Sony in the video-game industry, or Best Buy and Future Shop, it could be better presented by a duopolistic or oligopolistic setting. However, we believe our core insights still apply in such setting, and our work is a necessary fist step in analyzing these, more complex, settings.

In addition, the wholesale price is exogenously given in our model for simplification.
One possible extension is to incorporate a supplier as a decision maker, who maximizes his profit over the wholesale price, which may change the wholesale price threshold in this paper to a cost threshold related to the supplier. Also, because the supplier moves before the retailer in the Stackelberg game, the resulting model may have two thresholds when characterizing the benefits from the presence of a P2P platform, one is relative to the supplier and the other is relative to the retailer, which may lead to a revenue-sharing contract to benefit both the supplier and the retailer as a whole.

Last, but not least, one may consider the retailer applying platform-mitigation strategies in order to improve his profit, such as buying out the platform, offering refunds, revenue-sharing with the platform and so on. If any strategy is effective in increasing retailer profitability is an open question and needs to be explored further. For example, as shown in this paper, the platform can mitigate some of the mismatch risk, but also impose competition. If the retailer buys out the platform and operates it by himself, he may reduce some of the competition by suppressing first period demand, or go the other way by stimulating the first period demand to earn more transaction fees. Either way, it may help the retailer eliminate the threat of a P2P platform and boost his profit.
References


APPENDICES
Appendix A

Notations

\( \alpha \), probability of consumers observing a mismatch product, \( \alpha \in [0, 1] \).

\( \alpha_i \), individual realization of mismatch risk, \( \alpha_i \in \{0, 1\} \).

\( \delta \), discount factor for used goods on the P2P platform.

\( w \), wholesale price.

\( v \), consumers valuation, \( v \in U[0, 1] \).

\( U_B^1, U_B^2 \), consumers’ expected utility from buying a new product in period 1 and 2 in benchmark scenario, respectively.

\( U_P^1, U_P^2 \), consumers’ expected utility from buying a new product in period 1 and 2 in platform scenario, respectively.

\( U_P^p \), consumers’ expected utility from buying a used product in period 2 in platform scenario.

\( p_{BR}^{1n}, p_{BR}^{2n} \), selling price of new products in period 1 and 2 in benchmark scenario, respectively.

\( q_{BR}^{1n}, q_{BR}^{2n} \), demand of new products in period 1 and 2 in benchmark scenario, respectively.

\( p_{PR}^{1n}, p_{PR}^{2n} \), selling price of new products in period 1 and 2 in platform scenario, respectively.

\( q_{PR}^{1n}, q_{PR}^{2n} \), demand of new products in period 1 and 2 in platform scenario, respectively.

\( p_{PP}^2 \), selling price of used products in period 2 in platform scenario.

\( q_{PP}^2 \), demand of used products in period 2 in platform scenario.

\( t^{PP} \), platform transaction fee in platform scenario.
\( \Pi_1^{BR}, \Pi_2^{BR}, \) retailer’s first and second period profit, respectively, in benchmark scenario.

\( \Pi^{BR}, \) retailer’s total profit in benchmark scenario.

\( \Pi_1^{PR}, \Pi_2^{PR}, \) retailer’s first and second period profit, respectively, in platform scenario.

\( \Pi^{PR}, \) retailer’s total profit in platform scenario.

\( CS^B, \) consumer surplus in benchmark scenario.

\( CS^P, \) consumer surplus in platform scenario.
Appendix B

Proofs

B.1 Proof of Lemma 1

Using backward induction, we first solve for the retailer’s decisions in the two periods, \( q_{1n}^{PR}, q_{2n}^{PR} \), followed by the platform’s charge, \( t^{PP} \). Specifically, the retailer sets \( (q_{1n}^{PR}, q_{2n}^{PR}) \) to maximize his total profit

\[
\max_{q_{1n}^{PR},q_{2n}^{PR}} \Pi^{PR} = (p_{1n}^{PR} - w)q_{1n}^{PR} + (p_{2n}^{PR} - w)q_{2n}^{PR} \quad \text{(B.1)}
\]

subject to

\[
q_{2n}^{PR} \geq 0, \quad q_{1n}^{PR} \geq 0, \quad \text{(B.2)-(B.3)}
\]

where \( p_{1n}^{PR} = (1+\alpha \delta)(1-\alpha)q_{1n}^{PR} - \alpha \delta(1-\alpha)q_{2n}^{PR} + \alpha t^{PP} \) and \( p_{2n}^{PR} = (1-\alpha)(1-\alpha \delta q_{1n}^{PR} - q_{2n}^{PR}) \).

The platform sets \( t^{PP} \) to maximize its profit

\[
\max_{t^{PP}} \Pi^{PP} = \alpha q_{1n}^{PR} t^{PP} \quad \text{(B.4)}
\]

subject to

\[
t^{PP} \geq 0, \quad t^{PP} \leq p_{2p}^{PP} \quad \text{(B.5)-(B.6)}
\]

where \( p_{2p}^{PP} = \delta(1-\alpha)(1-\alpha q_{1n}^{PR} - q_{2n}^{PR}) \).

\( \Pi^{PR} \) is jointly concave in \( (q_{1n}^{PR}, q_{2n}^{PR}) \). Thus there exists an unconstrained unique global optimal \( (q_{1n}^{PR*}, q_{2n}^{PR*}) \). Instead of solving problems (B.1)-(B.3) and (B.4)-(B.6), we simplify the retailer problem as problems (B.1)-(B.2) by including (B.3) into the platform problem.
This means that for the retailer problem, we only have two cases, depending on whether (B.2) is binding.

Solving problem (B.1)–(B.2) for \( q_{PR}^{*} \) for a given \( q_{1n}^{PR} \) gives:

\[
q_{2n}^{PR} = [((1 - \alpha)(1 - 2\alpha \delta q_{1n}^{PR}) - w)/(2(1 - \alpha))]^{+}.\]

Specifically, there exists a threshold \( q_{1n}^{PR} \equiv (1 - \alpha - w)/(2\alpha \delta(1 - \alpha)) \), such that if \( q_{1n}^{PR} < \tilde{q}_{1n}^{PR} \), then \( q_{2n}^{PR} = ((1 - \alpha)(1 - 2\alpha \delta q_{1n}^{PR}) - w)/(2(1 - \alpha)) > 0 \); otherwise, if \( q_{1n}^{PR} \geq \tilde{q}_{1n}^{PR} \), then \( q_{2n}^{PR} = 0 \). The platform problem depends on whether constraint (B.2) is binding or not. We have two cases.

Case 1: (B.2) is not binding. In this case, \( q_{1n}^{PR} < \tilde{q}_{1n}^{PR} \) and \( q_{2n}^{PR} = ((1 - \alpha)(1 - \alpha \delta q_{1n}^{PR}) - w)/(2(1 - \alpha)) > 0 \). Solving problem (B.1)–(B.2) gives us the unconstrained optimal \( q_{1n}^{PR} = (1 - \alpha - w + \alpha \delta w - \alpha t_{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta - \alpha^2 \delta^2)) \). Accordingly, we rewrite the platform problem as:

\[
\max_{t_{PP}} \Pi_{PP} = \alpha q_{1n}^{PR} t_{PP} \] (B.7)

subject to

\[
i_{PP} \geq 0, \] (B.8)

\[
t_{PP} \leq p_{2n}^{PP}, \] (B.9)

\[
q_{1n}^{PR} \geq 0, \] (B.10)

\[
q_{1n}^{PR} < \tilde{q}_{1n}^{PR}, \] (B.11)

where \( q_{1n}^{PR} = (1 - \alpha - w + \alpha \delta w - \alpha t_{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta - \alpha^2 \delta^2)) \), which is the unconstrained optimal solution to (B.1)–(B.2).

Rearranging (B.10) reveals an important threshold: \( \hat{t}_{PP} \equiv (1 - \alpha - w + \alpha \delta w)/\alpha \). If \( t_{PP} < \hat{t}_{PP} \), then \( q_{1n}^{PR} = (1 - \alpha - w + \alpha \delta w - \alpha t_{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta - \alpha^2 \delta^2)) > 0 \); otherwise, if \( t_{PP} \geq \hat{t}_{PP} \), \( q_{1n}^{PR} = 0 \). Thus for the modified platform problem, we have two subcases, depending on whether (B.10) is binding or not.

Case 1.1: (B.10) is not binding. In this case, \( q_{1n}^{PR} = (1 - \alpha - w + \alpha \delta w - \alpha t_{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta - \alpha^2 \delta^2)) > 0 \). Constraint (B.11) leads to \( t_{PP} > t_{1n}^{PP} \equiv (w(1 - \alpha \delta + \alpha^2 \delta) - (1 - \alpha)(1 - \alpha \delta + \alpha^2 \delta - \alpha^2 \delta^2))/(\alpha^2 \delta) \). Constraint (B.9) leads to \( t_{PP} \leq t_{2n}^{PP} \equiv \delta((1 - \alpha)(1 + \alpha \delta + \alpha^2 \delta - \alpha - \alpha^2 \delta^2) + w(1 + \alpha - \alpha \delta))/(2 + \alpha^2 \delta - \alpha^2 \delta^2) \). Then we can rewrite the platform problem as:

\[
\max_{t_{PP}} \Pi_{PP} = \frac{\alpha t_{PP} (1 - \alpha - w + \alpha \delta w - \alpha t_{PP})}{2(1 - \alpha)(1 + \alpha^2 \delta - \alpha^2 \delta^2)} \] (B.12)

subject to

\[
i_{PP} \geq 0, \] (B.13)

\[
t_{PP} > t_{1n}^{PP}, \] (B.14)

\[
t_{PP} \leq t_{2n}^{PP}. \] (B.15)
(B.12) is concave in $t^{PP}$. The unconstrained optimal $t^{PP*} = (1 - \alpha - w + \alpha \delta w)/(2\alpha) \geq 0$, so (B.13) is always satisfied. Given this value of $t^{PP}$, solving (B.15) gives us a threshold of $w$: $w_1 \equiv 1 - \alpha - \alpha \delta(1 - \alpha)(2 + \alpha^2 \delta - \alpha^3 \delta^2)/(2 - 3\alpha^2 \delta^2 + 3\alpha^2 \delta - \alpha^3 \delta^2 + \alpha^3 \delta^3)$, such that if $w \leq w_1$, then the unconstrained optimal $t^{PP*} \geq t^{PP}_2$; if $w > w_1$, then the unconstrained optimal $t^{PP*} < t^{PP}_2$. The platform problem has two subcases depending on whether (B.15) is binding or not.

Case 1.1a: (B.15) is binding. In this case, $w \leq w_1$ and $t^{PP*} = t^{PP} = \delta((1 - \alpha)(1 + \alpha \delta + \alpha^2 \delta - \alpha - \alpha^2 \delta^2) + w(1 + \alpha - \alpha \delta))/(2 + \alpha^2 \delta - \alpha^2 \delta^2 - \alpha \delta)$. Since $w < 1 - \alpha$, $t^{PP}_2 > t^{PP}_1$. Therefore (B.14) is always satisfied.

Case 1.1b: (B.15) is not binding. In this case, $w > w_1$ and $t^{PP*} = (1 - \alpha - w + \alpha \delta w)/(2\alpha)$. Rewriting constraint (B.14) leads to $w < w_2 \equiv 1 - \alpha - \alpha^2 \delta(1 - \alpha)/(2 + 2\alpha^2 \delta - \alpha^2 \delta^2 - \alpha \delta)$. Since $w_2 > w_1$, only when $w < w_2$ is case 1.1b feasible.

Case 1.2: (B.10) is binding. In this case, $q_{1n}^{PP*} = 0$, which means there is no business for both the retailer and the platform. So this subcase is a dominated strategy for the platform.

Combining cases 1.1 and 1.2, we conclude that the platform prefers case 1.1, which is a dominant strategy.

Case 2: (B.2) is binding. In this case, $q_{1n}^{PP} \geq \tilde{q}_{1n}^{PP}$ and $q_{2n}^{PP*} = 0$. Solving problem (B.1)-(B.2) gives us the unconstrained optimal $q_{1n}^{PP*} = ((1 - \alpha)(1 + \alpha \delta) - w - \alpha t^{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta)) \geq \tilde{q}_{1n}^{PP} > 0$. Therefore, (B.3) is always satisfied. Thus, $q_{1n}^{PP} = ((1 - \alpha)(1 + \alpha \delta) - w - \alpha t^{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta))$. Accordingly, we can rewrite the platform problem as:

$$\max_{t^{PP}} \quad \Pi^{PP} = \alpha q_{1n}^{PP} t^{PP}$$  \hspace{1cm} (B.16)

subject to

$$t^{PP} \geq 0, $$ \hspace{1cm} (B.17)

$$t^{PP} \leq p_{2p}^{PP}, $$ \hspace{1cm} (B.18)

$$q_{1n}^{PP} \geq \tilde{q}_{1n}^{PP}, $$ \hspace{1cm} (B.19)

where $q_{1n}^{PP} = ((1 - \alpha)(1 + \alpha \delta) - w - \alpha t^{PP})/(2(1 - \alpha)(1 + \alpha^2 \delta))$, which is the unconstrained optimal solution to (B.1)-(B.2).

Rewriting constraint (B.19) leads to $t^{PP} \leq t^{PP}_1$. Rearranging constraint (B.18) leads to $t^{PP} \leq t^{PP}_3 \equiv \delta(\alpha w + (1 - \alpha)(2 + \delta \alpha^2 - \alpha))/(2 + \delta \alpha^2)$, and since $w < 1 - \alpha$, then $t^{PP}_1 \leq t^{PP}_3$, so (B.19) is stronger than (B.18), which means as long as (B.19) is satisfied, (B.18) is
always satisfied. Then we can rewrite the platform problem as:

\[
\max_{t^{PP}} \Pi^{PP} = \frac{\alpha t^{PP}(1-\alpha)(1+\alpha \delta) - w - \alpha t^{PP}}{2(1-\alpha)(1+\alpha^2 \delta)}
\] (B.20)

subject to

\[
t^{PP} \geq 0, \quad (B.21)
\]
\[
t^{PP} \leq t^{PP}_1, \quad (B.22)
\]

(B.20) is concave in \(t^{PP}\). The unconstrained optimal \(t^{PP*} = ((1-\alpha)(1+\alpha \delta) - w)/(2\alpha) > 0\). Given this value of \(t^{PP}\), solving (B.22) reveals one threshold: \(w_3 \equiv 1 - \alpha - (1-\alpha)\alpha^2 \delta^2 / (2 + 2\alpha^2 \delta - \alpha \delta)\), such that if \(w \leq w_3\), then the unconstrained optimal \(t^{PP*} \geq t^{PP}_1\); otherwise if \(w > w_3\), then the unconstrained optimal \(t^{PP*} < t^{PP}_1\). Depending on whether (B.22) is binding or not, we have two subcases.

Case 2a: (B.22) is binding. In this case, \(w \leq w_3\) and \(t^{PP*} = t^{PP}_1 = ((1-\alpha)(1+\alpha \delta) - w)/(2\alpha) > 0\). However, only when \(w \geq w_4 \equiv 1 - \alpha - (1-\alpha)\alpha^2 \delta^2 / (1 + \alpha^2 \delta - \alpha \delta)\), \(t^{PP}_1 \geq 0\). Since \(w_4 < w_3\), only when \(w_4 \leq w \leq w_3\) is case 2a feasible.

Case 2b: (B.22) is not binding. In this case, \(w > w_3\) and \(t^{PP*} = ((1-\alpha)(1+\alpha \delta) - w)/(2\alpha) < t^{PP}_1\). (B.21) is always satisfied.

Combining cases 1 and 2, we conclude that under case 1, when \(w \leq w_1\), the platform prefers subcase 1.1a; when \(w_1 < w < w_2\), the platform prefers subcase 1.1b. Under case 2, when \(w_4 \leq w \leq w_3\), the platform prefers subcase 2a; when \(w > w_3\), the platform prefers subcase 2b.

Since \(w_1 < w_4 < w_2\), when \(w < w_4\), the retailer chooses case 1; when \(w_4 \leq w < w_2\), the retailer needs to compare his profit in case 1 and 2; when \(w \geq w_2\), the retailer chooses case 2. Further we find that when \(w_4 \leq w < w_2\), the retailer’s profit in case 1 is greater than that in case 2. Therefore, in general, when \(w < w_2\), the retailer prefers case 1; otherwise, he prefers case 2.

\[\square\]

### B.2 Proof of Theorem 1

Denote \(\Delta\) as the retailer’s profit difference between benchmark scenario and platform scenario, that is, \(\Delta = \Pi^{BR*} - \Pi^{PR*}\). We then consider \(\Delta\) in each of the ranges defined earlier in Lemma 1.

When \(w < w_1\), \(\Delta\) is convex in \(w\) since

\[
\frac{d^2 \Delta}{dw^2} = \frac{\alpha \delta (4 - 3\alpha \delta - 4\alpha^2 \delta^2)}{2(1-\alpha)(1+\alpha^2 \delta^2)^2} \geq 0.
\]

Also, when \(d\Delta/dw = 0\), minimum \(\Delta = 0\) is achieved at \(w = 1 - \alpha \geq w_1\). So \(\Delta\) is
When $w_1 \leq w < w_2$, $\Delta$ is convex in $w$ since $\frac{d\Delta}{d^2w} = \frac{3+2\alpha\delta+4\alpha^2\delta-5\alpha^2\delta^2}{8(1-\alpha)(1+\alpha\delta-\alpha^2\delta^2)} \geq 0$. Also when $\Delta = 0$, $w = \bar{w}$ or $\bar{w}'$, where $\bar{w} = 1 - \alpha - \frac{\alpha\delta(1-\alpha)(1-\alpha\delta+2\sqrt{1+\alpha\delta-\alpha^2\delta^2})}{3+2\alpha\delta+4\alpha^2\delta-5\alpha^2\delta^2}$, $\bar{w}' = 1 - \alpha + \frac{\alpha\delta(1-\alpha)(1-\alpha\delta-2\sqrt{1+\alpha\delta-\alpha^2\delta^2})}{3+2\alpha\delta+4\alpha^2\delta-5\alpha^2\delta^2}$. Therefore, $\bar{w}'$ is infeasible because of violation of individual rationality ($w \leq 1 - \alpha$). Hence, if $w \leq \bar{w}$, $\Delta \geq 0$; otherwise, $\Delta < 0$.

When $w_2 \leq w < w_3$, $\Delta$ is concave in $w$ since $\frac{d\Delta}{d^2w} = \frac{2\alpha^2\delta^2-\alpha^2\delta-1}{2(1-\alpha)(1+\alpha\delta-\alpha^2\delta^2)} \leq 0$. Also, when $d\Delta/dw = 0$, maximum $\Delta = 0$ is achieved at $w = 1 - \alpha \geq w_3$. So $\Delta$ is concavely increasing when $w_2 \leq w < w_3$ and $\Delta \big|_{w=w_3} = \frac{(1-\alpha)(2\alpha^2\delta^2-\alpha^2\delta-1)}{4(2+2\alpha^2-\alpha\delta)^2} < 0$ for any $0 \leq \alpha < 1$.

When $w \geq w_3$, $\Delta$ is convex in $w$ since $\frac{d\Delta}{d^2w} = \frac{7+8\alpha\delta}{8(1-\alpha)(1+\alpha\delta)} \geq 0$. Also, when $\Delta = 0$, $w = 1 - \alpha - \frac{\alpha\delta(1-\alpha)(2\sqrt{2+2\alpha^2\delta-1})}{7+8\alpha\delta} \leq w_3$, or $1 - \alpha + \frac{\alpha\delta(1-\alpha)(2\sqrt{2+2\alpha^2\delta-1})}{7+8\alpha\delta} \geq 1 - \alpha$. Therefore, $\Delta < 0$ when $w_3 \leq w \leq 1 - \alpha$.

### B.3 Proof of Proposition 1

Denote $r$ as the likelihood of platform benefiting the retailer, that is $r = \frac{\bar{w}-\bar{w}}{\bar{w}} = \frac{\alpha\delta(1-\alpha\delta+2\sqrt{1+\alpha^2\delta(1-\delta)})}{3+2\alpha\delta+4\alpha^2\delta-5\alpha^2\delta^2}$.

\[
\frac{dr}{d\alpha} = \frac{\delta((3+3\alpha^2\delta^2-6\alpha\delta-4\alpha^2\delta)\sqrt{1+\alpha^2\delta(1-\delta)}+4\alpha^3\delta^2+4\alpha^2\delta-4\alpha^3\delta^3+6-2\alpha^2\delta^2)}{(3+2\alpha\delta+4\alpha^2\delta-5\alpha^2\delta^2)^2}.
\]

Note that $\delta(1-\delta) = \frac{1}{4} - (\delta - \frac{1}{2})^2 \leq \frac{1}{4}$ for all $\delta$ between 0 and 1 and $\alpha \leq 1$; hence $1 \leq \sqrt{1 + \alpha^2\delta(1-\delta)} \leq \frac{\sqrt{6}}{2} < \frac{6}{5}$.

Therefore, $(3+3\alpha^2\delta^2-6\alpha\delta-4\alpha^2\delta)\sqrt{1+\alpha^2\delta(1-\delta)}+4\alpha^3\delta^2+4\alpha^2\delta-4\alpha^3\delta^3+6-2\alpha^2\delta^2 > 3+3\alpha^2\delta^2-\frac{3\alpha^2\delta^2}{5} - 2\alpha^2\delta^2 + 4\alpha^2\delta^2+4\alpha^2\delta-4\alpha^3\delta^3+6-2\alpha^2\delta^2 = 9 - \frac{4\alpha^3\delta(9+\alpha)}{5} + \alpha^2\delta^2+4\alpha^3\delta^2(1-\delta) > 9 - 8\alpha\delta + \alpha^2\delta^2+4\alpha^3\delta^2(1-\delta) > 0$ for all $\alpha$ and $\delta$ between 0 and 1. Hence, $\frac{dr}{d\alpha} > 0$. 

### B.4 Proof of Theorem 2

Denote $\Delta$ as the consumer surplus’s difference between benchmark scenario and platform scenario, that is, $\Delta = CS_B - CS_P$, where $CS_B = \int_{v_1}^{1} U_1^B dv + \int_{v_2}^{1} U_2^B dv = \frac{1}{4}(1 - \alpha -
\[
\frac{2w-2aw-w^2}{1-\alpha}, \quad C\mathcal{S}P = \int_{\bar{v}^1}^{v} U_1^P \, dv + \int_{\bar{v}^2}^{v} U_2^P \, dv + \int_{\bar{v}^3}^{v} U_3^P \, dv = \left\{
\begin{array}{ll}
(1+\alpha^2)\delta(2-\alpha^2)(1-\alpha-w)^2+(1-\alpha-w)^2(2-2\alpha+\alpha^2\delta)(2+\alpha^2\delta-2\alpha^2\delta^2) \\
8(1-\alpha)(2+\alpha^2\delta-\alpha^2\delta^2)^2 \\
(1-\alpha-\alpha^2\delta-\alpha^2\delta^2)^2 \\
(1-\alpha^2w)^2 \\
32(1-\alpha)(1+\alpha^2\delta-\alpha^2\delta^2)^2 \\
\end{array}\right.
\]
if \( w \leq w_1; \)
if \( w_1 < w \leq w_2; \)
if \( w_2 < w \leq w_3; \)
if \( w > w_3. \)

When \( w < w_1, \) \( \Delta \) is convex in \( w \) since \( \frac{d\Delta^2}{dw} = \frac{\alpha\delta(4-\alpha^2-3\alpha^2\delta^2+\alpha^2\delta+2\alpha^2\delta^3+4\alpha^2\delta-4\alpha^2\delta^2)}{4(1-\alpha)(2+\alpha^2\delta-\alpha^2\delta^2)^2} \geq 0. \)
Also, when \( d\Delta/dw = 0, \) minimum \( \Delta = 0 \) is achieved at \( w = 1-\alpha \geq w_1. \) So \( \Delta \) is convexly decreasing when \( w < w_1 \) and \( \Delta \big|_{w=w_1} = \frac{\alpha\delta(4-\alpha^2-3\alpha^2\delta^2+\alpha^2\delta+2\alpha^2\delta^3+4\alpha^2\delta-4\alpha^2\delta^2)}{8(2+\alpha^2\delta-\alpha^2\delta^2-\alpha^2\delta^3)^2} > 0 \) for all \( 0 \leq \alpha < 1. \)

When \( w_1 \leq w < w_2, \) \( \Delta \) is convex in \( w \) since \( \frac{d\Delta^2}{dw} = \frac{3+2\alpha^2+4\alpha^2\delta-5\alpha^2\delta^2}{16(1-\alpha)(1+\alpha^2\delta-\alpha^2\delta^2)} \geq 0. \) Also when \( \Delta = 0, \) \( w = \bar{w} \) or \( \tilde{w}, \) where \( \bar{w} = 1-\alpha - \frac{\alpha\delta(1-\alpha)(1-\alpha^2+2\sqrt{1+\alpha^2\delta-\alpha^2\delta^2})}{3+2\alpha^2+4\alpha^2\delta-5\alpha^2\delta^2} \) \( (w_1 \leq \bar{w} \leq 1-\alpha \leq \tilde{w}'). \) Therefore, \( \tilde{w}' \) is infeasible because of violation of individual rationality \( (w_1 \leq \alpha). \) Hence, if \( w \leq \bar{w}, \) \( \Delta \geq 0; \) otherwise, \( \Delta < 0. \)

When \( w_2 \leq w < w_3, \) \( \Delta \) is concave in \( w \) since \( \frac{d\Delta^2}{dw} = \frac{2\alpha^2\delta^2-\alpha^2\delta-1}{4\alpha^2\delta(1-\alpha)} \leq 0. \) Also, when \( d\Delta/dw = 0, \) maximum \( \Delta = 0 \) is achieved at \( w = 1-\alpha \geq w_3. \) So \( \Delta \) is concavely increasing when \( w_2 \leq w < w_3 \) and \( \Delta \big|_{w=w_3} = \frac{(1-\alpha)(2\alpha^2\delta^2-\alpha^2\delta-1)}{8(2+2\alpha^2-\alpha\delta)^2} < 0 \) for any \( 0 \leq \alpha < 1. \)

When \( w \geq w_3, \) \( \Delta \) is convex in \( w \) since \( \frac{d\Delta^2}{dw} = \frac{7+8\alpha^2\delta}{16(1-\alpha)(1-\alpha^2)} \geq 0. \) Also, when \( \Delta = 0, \)
\[
w = 1-\alpha - \frac{\alpha\delta(1-\alpha)(2\sqrt{2+2\alpha^2\delta^2})}{7+8\alpha^2\delta} \leq w_3, \text{ or } 1-\alpha + \frac{\alpha\delta(1-\alpha)(2\sqrt{2+2\alpha^2\delta^2}-1)}{7+8\alpha^2\delta} \geq 1-\alpha.
\]
Therefore, \( \Delta < 0 \) when \( w_3 \leq w \leq 1-\alpha. \)