Optimization Method for Inventory and Supply Chain Management

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

This thesis presents an optimization model which helps retailers to reduce product costs by taking advantage of parts commonality in manufacturing and production areas, when selling similar units with uncertainty in demands. The concept of component commonality can be often found in the assemble-to-order system, which is the foremost concept used by prominent manufacturing companies in the global market. The method developed uses genetic algorithm (GA) to solve real world optimization problems that contain integer values for parts and finished items, and uncertain information.

Numerical examples are solved using generated stochastic scenarios to show the impact of uncertainty on solutions. This impact is verified using two important criteria, Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS). The obtained solutions present significant monetary benefits for the manufacturer illustrating the importance of the model presented here for retailers.

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Finally special thanks to my dear parents, Manilachelvan Raju and Thilakavathy Manilachelvan; and my little sister Dhivetha Manilachelvan for their continuous support and best wishes.

Dedication

I dedicate my thesis to my grandfather Raju Thambusami, His words of wisdom and encouragement will always ring in my ears. I would like to express my deepest indebtedness through this dedication.

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Chapter 1

Introduction

Globalization of product manufacturing has new challenges. There are new strategic, tactical and operation plans to be made to maximize profits, follow new trade zones, and consider socio-environmental-economic factors. In addition, uncertainty cannot be avoided due to multiple countries and complex transportation needs, but optimization models for retailers manufacturing items considering risk from uncertainty in supply chain management is continuing to be rare as suggested by Mentzer et al. in [18].

The models for the use of manufacturers require optimization, which involves the selection of an appropriate or best element from some set of existing choices. We consider two types of models; deterministic model and stochastic model for comparison of the final results, in other words, comparing the final results of models without uncertainty and the model where uncertainty is explicitly considered with a risk factor. The role of uncertainty comes (i) in time to produce when several companies and different transportation facilities are

required in the manufacturing and selling of an item, (ii) prices, and (iii) demand for products.

To try to win in international markets, short manufacturing lead time is necessary; this can be achieved through component commonality and assemble-to-order strategies which are a part of the supply chain management system. There are various definitions for the term supply chain, which has been an area of research for more than a decade.

After examining the works of Mentzer et al. of [18] supply chain is defined as a set of three or more organizations openly involved in the upstream (supply) and downstream (distribution) movement of products, amenities and data from a source to a customer.

1.1 Problem Statement

The thesis focuses on analyzing the impact of the solutions obtained using the mathematical model for any manufacturer who manufactures items using common parts or components. For example consider Figure 1.1, which shows the different levels involved in manufacturing an item based on the two

important strategies used in this thesis; component commonality and assemble-to-order system.

Level 1 contains the parts supplies, which includes three different parts P1, P2 and P3. The term parts and components is used as synonyms over the entire thesis. Level 2 is the stage for assembling, where the assemble-to-order conception takes place. In Figure 1.1, the parts P1 and P2 are assembled together to produce item 1. Some of which are sold and some of which are sent to the next level to add part 3 to make another item. Level 4 is where the finished items are transported to the retailer or vendor for sale.

Levels 1, 2 and 4 shown in the figure need not be in one location, each entity can be in different regions. For instance the raw materials can be available in one region which might get transported to another region to do the assembling and finally transported to the destination to reach the customers. Thus there is a possibility for uncertainty in each of these levels while manufacturing an item.

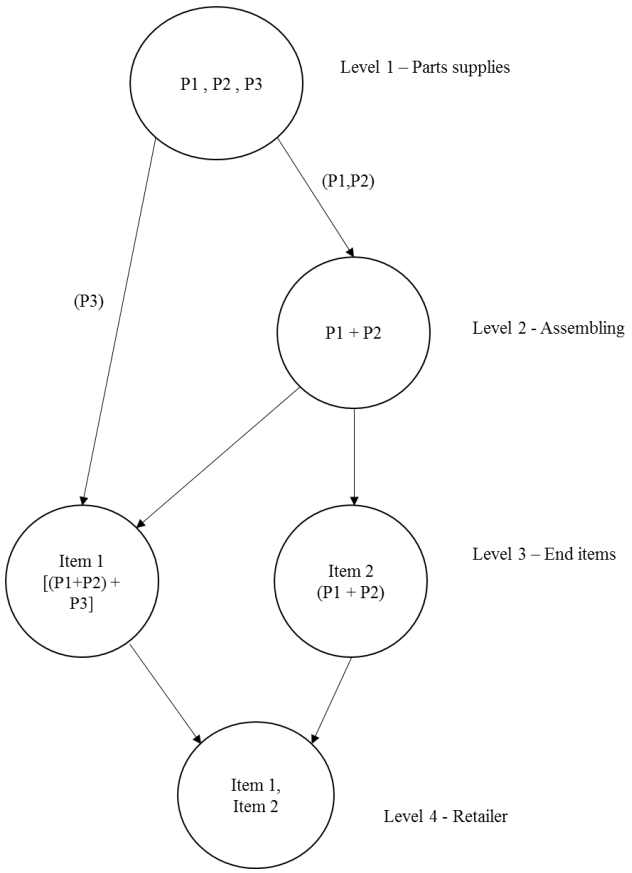

Figure 1.1: Problem statement description

The main objective of this study is to develop models that can be used by any original equipment manufacturer (OEM) to gain maximum profit by minimizing the equipment assembling cost while using the two strategies: component commonality and assemble-to-order systems.

1.2 Contribution

The main contribution of the thesis is a stochastic programming model that solves difficult integer variable problems using genetic algorithm. Demonstrating the significance of the model using Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS), both terms explained in detail later, is a novel contribution.

1.3 Content Organization

The thesis is composed of six chapters. The first chapter was an introduction giving an overview of the problem statement and a brief outline of the thesis contribution.

Chapter 2 deals with the literature review. It starts with the background summary of the research on the related works of supply chain management and contains a brief explanation about the two main streams focused in the thesis, component commonality and the assemble-to-order systems. This chapter also includes a short description about stochastic programming and global optimization methods used in the study.

Chapter 3 presents the mathematical model explanation, followed by Chapter 4 comprising the methodology used in the thesis. This methodology is explained using a numerical example in Chapter 5, which also deals with a case study of a real world problem in a new section under the same chapter.

The last chapter presents the summary and conclusion of the thesis work done and also contains the expected future works in the field.

Chapter 2
Literature Review

2.1 Supply Chain Management

Supply chain management is a growing and vibrant area of research in recent years. There are various definitions given by researchers and authors for the term supply chain management. The thesis of Mentzer et al. of [18] provides a good review of the definitions of supply chain and supply chain management.

Ultimately, a supply chain is a system of activities, resources or organizations involved in moving a product or service from seller to customer. Supply chain management may include all movements and loadings of raw supplies, work-in-process inventories, and completed goods from starting point to the point of consumption.

As per Lambert et al. [13], the eight procedures involved in supply chain management as classified by the Global Supply Chain Forum (GSCF) are "manufacturing flow management, order fulfilment, demand management, supplier relationship management, product development and commercialization, returns management, customer service management and finally customer

relationship management". Each process plays a vital role in the structure of supply chain.

All the above mentioned eight processes can be categorized under five entities as in Figure 2.1. The preliminary entity, suppliers, is the one who supplies the basic commodities and resources for a product, which is followed by the manufacturer who completes the work of manufacturing the desired product. The distributor follows the manufacturer with distributing the finished goods to various places by making the items reach the retailer, who in turn sells it to the customers, who are the final entity of the chain.

Figure 2.1: Supply chain management entities

Managing the supply chain is a complex task, from the point of origin to the point of consumption, it is a challenging work. In most of the developed industries there may be multiple elements in each entity. For instance, a leading manufacturing company like Hyundai or Honda would have more than one supplier who might deliver different needed merchandises to manufacture a car.

It is also possible that these suppliers can be from different parts of the world and similarly the distribution and sales of these cars happen in multiple countries all over the world.

Being an extensive field of research and implementation, there are different areas that are related to supply chain by using organizational theories. For instance, the organizational theories such as Materials Requirement Planning (MRP), Total Quality Management (TQM) play a vital role in managing an industry. Some of the problems addressed by supply chain management are distribution network configuration, inventory management, logistics, cash flow and transportation strategies.

In order to have an effective supply chain, Liberti [14] suggested three closely inter related elements in a supply chain management framework: the organization of supply chain, the supply chain business process and lastly, the supply chain management components.

The supply chain include the networks of members and the connection between the members, while the business process are the actions that yield an explicit productivity and significance to the consumer. Finally the management components are the managerial components such as planning and control, work

structure, product flow and other components that are related to the process of management in an organization.

2.2 Component Commonality

In order to achieve world–class manufacturing and to compete well in international markets, companies try to attain short manufacturing lead time; the lead time is predicted as the main factor for on-time delivery, quality and productivity, as suggested by Song and Yao [23]. All these elements influence the success of a company in the long run. The manufacturing lead time denotes the time between the beginning and completion of a production process.

The process that most of the leading manufacturers follow is parts propagation, which is an upsurge of the number of divergent part type that a product might require for manufacturing. Hence every time a new product is planned, new parts are designed and added on with the existing parts to launch a seemingly newer product for the consumers in the market.

The practice of following parts propagation in engineering terms is referred to as parts-commonality or component commonality. As proposed by Baker [2], parts commonality refers to a state where one component is common to more

than one end item. Many enterprises benefit from parts commonality as it streamlines the process of engineering design of products and it also leads to more economical manufacturing.

For example, consider Figure 2.2, where there are two products; Product A and Product B and the requirement of parts for each product is also shown. On comparing the products, it can be clearly seen that part 1 and part 4 are commonly used in the manufacturing of both A and B. Hence parts 1 and 4 are referred to as common components and the process of using such method is identified as component commonality.

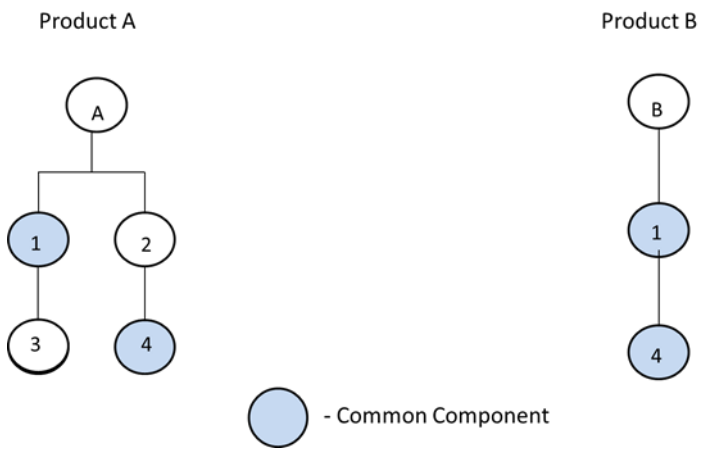

Figure 2.2: Component commonality

It can also be inferred from Figure 2.2, that parts 2 and 3 are used specifically for the manufacturing of product A and hence they are called product specific components. It has been suggested by previous studies that the introduction of component commonality in the shop floor has profitable effects on aspects such as planning and development, lower safety stock and order quantity improvement are some factors as suggested by Song and Yao in [23].

Some literature explain the effects of combining certain product specific components with common parts of an item to study the qualitative influences on components and common parts stock. For example, the Gerchak et al. of [7] proposed that the stock of product specific components constantly rises when other components are joined into common parts.

The set capacities for low level components are based on demand anticipation rather than on industry orders. If this expectancy turns out to be mistaken, strategies may have to be adjusted. The consequential scheduling, speed up and predictable inventory accumulation are expensive and time consuming and significantly delay the accomplishment of a smooth movement through the industrial system.

Therefore commonalities significantly reduce the costs of increased product lines and moderates the properties of product production and process difficulty. It decreases the cost of safety stock. Ultimately, taking commonality into consideration can decrease the inventory level of a product, condense the time for getting in market, and decline the set-up time, increase efficiency, and advance flexible manufacturing.

2.3 Assemble-to-order systems

In manufacturing units where many finished products are assembled from a set of regular components, the process of assemble-to-order (ATO) is followed. It is a method in which assembling of the components is done based on the order received from the customers. The subassemblies of parts are done based on predictions and estimations attained while the final assembly of components for a finished product is done at a later stage.

As suggested by Mula et al. in [20], the ATO system proves to be beneficial in conditions where the assembling time of products is substantially shorter than the manufacturing time of its respective parts and components. This method

also provides a positive difference in the holding cost, variation of product design and on-time delivery of products.

In industries it is also a known factor that customer satisfaction is the key result of the satisfaction gained through on-time delivery of the products leading to a company's growth. One of the main reasons that inhibits timely delivery of items is the shortage of components; which in turn brings a manufacturing capacity loss that brings down the rate of the industry growth in a global market. This problem of on-time delivery can be resolved by ATO system.

Another factor that influences the assembling of system is component commonality. Since all products do not need most of the features and every single product require only a subset of the design structures, the commonality of parts speeds up the assembling process.

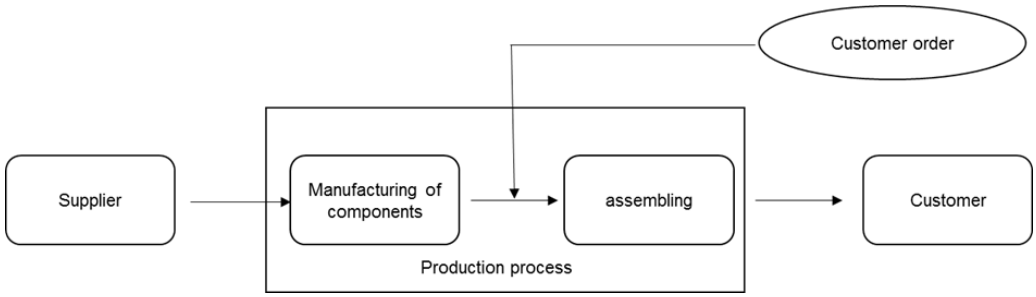

Figure 2.3: ATO process flow

Figure 2.3, illustrates a simple ATO process where the assembling of components is done based on a customer order, however manufacturing of components is done prior to the assembling stage. The manufacturing and assembling process are together referred to as the production process. As discussed by Mentzer et al. in [17], another important aspect that should be well thought out is that the inventory and the manufacturing capacity of items must be mutually monitored and managed at the same time.

The stock-out of one item might delay the delivery of different products since products use common parts for the manufacturing of an end item. Moreover demands are not considered to be independent of the items ordered, which will be described in the next chapter.

The ATO system also makes it possible for items to be assembled in a different country by transporting the components manufactured from multiple countries based on profitable prices. This provides an opportunity for industries to collaborate in the global market and furthermore provide a huge variety of products.

Also the ATO approach is a fusion of a make-to-stock approach, where end items are completely manufactured in advance; and the make-to-order strategy,

where products are contrived once the customer order is received. The ATO approach tries to take advantages of both approaches by making products reach customers more speedily while letting the product to be modified.

2.4 Uncertainty and Stochastic Programming

The Oduguwa et al. in [21] refer uncertainty as the difference between the total information required to complete a task and the total info already attained. Jing et al. in [9] classify uncertainty into two sets, environmental uncertainty and system uncertainty. The former includes uncertainties such as demand and supply which are beyond the production processes while the latter involves uncertainties inside the production processes like lead time.

There are various measures of uncertainty that have been suggested in previous studies. In a problem, numerous parameters can be considered uncertain, which are represented as random variables. For example, in any manufacturing unit the production and distribution costs depend on the fuel cost, which is meant to be random. Uncertain market conditions is a dynamic factor for future demands.

Oduguwa et al. in [21] suggest that one way of measuring uncertainty is by finding the variance of the output. In this thesis we consider the uncertainties in demand and supply, however, because of the stochastic programming model we use it is easy to extend it to cost uncertainties by simply including such uncertainties in the generated scenarios. Hereafter, we will consider only demand uncertainty in the proposed models.

Though various methods have been used to model uncertain quantities in optimization, stochastic programming is used for their flexibility and effectiveness in various fields. In a stochastic model, the uncertainty (randomness) is described by probability distributions. Uncertainty can be introduced in one or many points of the model, for example, objective function coefficients, right-hand side values of constraints, or coefficients in the constraint left-hand sides.

Stochastic programming models take advantage of the fact that probability distribution of the data are known or can be predicted from historic data analysis. The objective is to find some strategy that is feasible for almost all the conceivable data occurrences and maximizes or minimizes the expectancy of some function of the decisions considering the random variables. King et al. in

[11] explain that stochastic programming offers modelling and solution procedures that connect the supremacy of optimization to resolve models that are sensitive to uncertainty.

Linear programming (LP or linear optimization) is a mathematical method for determining the best outcome (of maximization or minimization) in a given mathematical model containing only linear relations. LP is used in this study, however, when risk is considered the model becomes nonlinear and is a nonlinear optimization problem.

The two stage stochastic optimization supports the dual purpose of obtaining an (approximately) optimal first stage solution, usually a small set of design variables in the first stage, appropriate to the optimization problem, then the second stage optimization is constrained by the solution of the first stage design variables and determines an optimal solution for the second stage for a given random scenario.

Hence in a two stage stochastic programming method, the decision maker takes some action in the first stage, and later a random event follows affecting the result of the first-stage decision. An alternative decision can then be made in the second stage that recompenses for any drawbacks that might have occurred

as a result of the first-stage decision. Two stage stochastic programming considers both these stages simultaneously allowing for any number of random scenarios and this method is adopted in the thesis of Brandimarte [3].

2.5 Global Optimization Methods

Global optimization is a division of mathematics and numerical analysis that deals with the optimization of a function or a set of functions and are especially suitable for discrete optimization and non-smooth optimization. It is recalled that supply chain optimization models have a large number of integer variables and therefore global optimization methods are useful for solving such problems. Usually, a set of bound and other common constraints also exist and the decision variables are optimized considering the constraints. There are various methods of global optimization which are broadly classified into the following sections: deterministic methods, stochastic methods, heuristic and metaheuristic methods.

The goal of global optimization is to discover the best solution of the modeled design problem, subject to various constraints. Smooth methods depending upon derivatives, such as quadratic programming, can give only locally optimal

solutions. Also, when there are integer variables finding the solution becomes extremely hard and global optimization methods are some of the best methods to solve such problems as suggested by Oduguwa et al. [21].

A small number of application areas of global optimization methods comprise malignancy treatment forecasting, chemical process modeling, data analysis, organization and conception, financial plus economic forecasting, ecological hazard controlling, manufacturing invention, laser apparatus plan, portfolio management, development mechanism, robot design and operations as suggested by Whitney [28].

The approaches that were initially used in global optimization were deterministic techniques, generally grounded according to the divide-and-conquer principle. The principle is built upon the impression that, as an alternative of trying to find a minimum by solving a set of equations by algebraic methods, making a sequence of estimated solutions for the problem which has been divided into smaller sub-problems.

Brandimarte [3] states one usual algorithm which symbolizes the divide-and-conquer principle is the Branch-and-Bound algorithm (BB). And the first

applications of BB to global optimization problems was in discrete problems such as the Travelling Salesman Problem (TSP).

Stochastic methods are centered on an element of random choice. Due to this, the chance of complete guarantee of achievement within a finite amount of computation is not possible. Stochastic optimization methods are optimization methods that create and use randomly generated candidate solutions. The stochastic optimization method that is used in this thesis is the genetic algorithm (GA), which comes under the type heuristic methods and is an evolutionary algorithm.

2.6 Genetic Algorithm

The evolutionary techniques heuristically imitate organic evolution: that is, selection, mutation, recombination, and reproduction. The selection may be guided by a fitness criteria. As explained by Whitney [28] an adaptive exploration process for generation of new populations is used.

Repetitions include an uncertain range that reduces the chance of having less significant solutions. The enduring group of candidates with advanced suitability rate are then recombined with further results by trading components

with another; they can furthermore be transformed by creating a small alteration to a candidate solution.

The recombination and mutation changes are applied successively; the goal is to produce fresh results that are influenced towards subsets. Several alternatives of this general strategy based on diverse evolution guidelines can be created. The different types of evolutionary search methods consist of methods that are targeted towards solving combinatorial problems, called as genetic algorithms (GA).

An implementation of a GA begins with a population of typically random chromosomes, which refers to a set of properties of each candidate result. These structures are then evaluated and the reproductive chances are assigned, in a way that those chromosomes which signify a better result to the target problem are given more opportunities to produce offspring than those chromosomes which give lower results. A good result is usually identified with respect to the current solution.

The two main elements of most GAs are problem encoding and the definition of evaluation function. Consider a constraint optimization problem, where a set

of variables need to be optimized either to maximize some objective such as profits or to minimize costs.

The only output is a value given back by an evaluation function signifying how good a specific combination of constraint setting solves the optimization problem. Constraint violations are added to the evaluation function as penalties.

GA is mostly used for nonlinear, non-smooth and integer programming problems. The main objective of GA in the thesis is to reduce the fitness function value, which is otherwise referred to as the penalty value of the model. The fitness function value is similar to the objective function, which gets modified in the model when solutions change. In later chapters the term penalty value is used in the graphs, which represents the fitness function value. An important idea in GA is that variables can be denoted by bit strings. This signifies that the variables are discretized that the basis of the discretization is some power of 2.

The evaluation function must also be comparatively quick to solve. This is normally true for any optimization method, but it may be a problem. Since GA works with a population of potential solutions usually large in number, the cost of evaluating this population could be very large. Moreover the population is

swapped entirely or partially on a generational basis. However, in our problem, described in detail later, the evaluation is quick and hence is not a serious problem.

2.7 Summary

In this chapter, a brief review of the key concepts such as component commonality, assemble-to-order and the genetic algorithm that will be eventually used have been presented. Combining these tools and using various criteria of impacts of uncertainty to demonstrate the proposed methodology will be the major contribution of this thesis.

The next chapter describes the mathematical model, which contains the explanation of deterministic and stochastic models. These models are used to find solutions for both the continuous variable problem and the integer variable problem.

Chapter 3
Mathematical Model

3.1 Model Description

In this chapter, the deterministic model for optimizing the total revenues of a retailer considering various manufacturing constraints is presented followed by the description of the extensions made to this model for stochastic demands.

A manufacturing process for the production of *n* end items with relevant processing times, component cost, the selling price, the machine capacity and the demand for each product are considered in this research. The overview of this problem was presented in Chapter 1. The average demand for each product is also taken into consideration for the deterministic model. The production of an item containing more than one part may require different types of machine to manufacture the parts based on the requirement such as thickness, length, type of material and etc. Hence all the end items or the finished products considered are made up of more than one component or part. The finished product may use mutual components or can be product specific based on the individual manufacturing requirement as in Figure 2.2. A deterministic model for the

optimal manufacturing of systems with common components (as described in Chapters 1 and 2) that will maximize the revenues of a retailer is first developed.

- **Indices**

i: Index of components for each item (*i* = 1,…., I)

j: Index of end item/ finished product (j = 1,…., J)

m: Index of machine types (m = 1,…., M)

t: Index of processing time (t = 1,……, T)

s: Index of the scenarios (s = 1,……,S)

3.1.1 Problem Assumptions

The assumptions of the problem are as follows:

1. We have items with i components to be manufactured.

2. The component cost includes the raw material costs.

3. The assembly is not a bottleneck.

4. Production of components is planned before knowing the demand, but final assembly of the finished products is delayed depending upon the customer order.

3.1.2 Parameters

T_{im}: Processing time for each component i at each machine type m.

L_m: Available capacity of each machine type m.

G_{ij}: Number of components i required for each end item j.

d_j: demand for each end item j.

$\overline{d_j}$: average demand for each end item j.

x_i: quantity of component i to be produced in the first stage before knowing the demand for the finished product j

y_j: produced quantity of the finished product j.

3.1.3 Costs and Decision Variables

C_i: C_i is unit production cost for each component i.

P_j: P_j is unit selling price for each end item j and quantity y_j is sold.

3.1.4 Objective Function for Deterministic Model

Maximization of revenue is done by:

$$\sum_{j=1}^{J} P_j y_j$$

Equation 3-A: Maximization of revenue

Minimization of component costs:

$$\sum_{i=1}^{I} C_i x_i$$

Equation 3-B: Minimization of component cost

3.1.5 Constraints

$$\sum_{i=1}^{I} T_{im} x_i \leq L_m \qquad m = 1,\ldots\ldots\ldots\ldots,M \qquad \textbf{(3 - C)}$$

$$\sum_{j=1}^{J} G_{lj} y_j \leq x_i \qquad i = 1,\ldots\ldots\ldots\ldots,I \qquad \textbf{(3 - D)}$$

$$y_j \leq \overline{d_j} \qquad j = 1,\ldots\ldots\ldots\ldots,J \qquad \textbf{(3 - E)}$$

$$x_i, y_j \geq 0$$

(3-C) denotes the constraint for the total processing time of each component to be less than the machine capacity. (3-D) states the unit requirement of the components for each end item (assembly constraint) and finally (3-E) indicates the average demand of each end items. Hence the deterministic model considered in this study is represented as follows:

$$max \quad -\sum_{i=1}^{I} C_i x_i + \sum_{j=1}^{J} P_j y_j$$

$$\sum_{i=1}^{I} T_{im} x_i \leq L_m \qquad m = 1,\ldots\ldots\ldots\ldots,M \qquad \textbf{(3 - C)}$$

$$\sum_{j=1}^{J} G_{ij} y_j \leq x_i \qquad i = 1,\ldots\ldots\ldots\ldots,I \qquad \textbf{(3 - D)}$$

$$y_j \leq \overline{d_j} \qquad j = 1,\ldots\ldots\ldots\ldots,J \qquad \textbf{(3 - E)}$$

$$x_i, y_j \geq 0$$

Model 3-1: Deterministic model

3.1.6 Objective Function for Stochastic Model

The equation for maximization of revenue is same as in Equation 3-A of the deterministic model. The stochastic model contains different demand scenarios for the end items, while in the deterministic model the average demand of the

end items is used. And due to the different scenarios in the stochastic model, there is uncertainty present in each scenario.

Maximization of revenue with respect to the uncertainty in each scenario modifies the Equation 3-B as maximizing the expected revenues:

$$\sum_{s=1}^{S} \pi^{s} \left(\sum_{j=1}^{J} P_j y_j \right)$$

Equation 3-F: Maximization of selling price with uncertainty

The probability of random scenario (scenario contains a single instant of all the random variables) is represented by π^s for each scenario s. For simplicity we show in the model only the random demand. The probability may come from either a uniform distribution or a non-uniform distribution.

For clarification of the term scenario, each scenario may refer to the following situation, for example consider Table 3.1, which shows two end items y1 and y2 with their respective demand values. For ease of explanation just two end items are considered.

y1	y2
100	50
60	30

Table 3.1: Demand of end items y1 and y2

In order to use the stochastic model, scenarios need to be formed. Table 3.2 shows the four possible demand scenarios for the end items y1 and y2.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
y1	100	100	60	60
y2	50	30	50	30

Table 3.2: Stochastic scenarios of end items y1 and y2

The constraints for the stochastic model contain the same parameters as the deterministic model but for each of the scenarios being considered. Hence the model is as follows:

$$max \; - \sum_{i=1}^{I} C_i x_i + \sum_{s=1}^{S} \pi^s \left(\sum_{j=1}^{J} P_j y_j^s \right)$$

s.t.

$$\sum_{i=1}^{I} T_{im} x_i \leq L_m \qquad m = 1,\ldots\ldots\ldots,M \qquad (3 - G)$$

$$\sum_{j=1}^{J} G_{ij} y_j^s \leq x_i \qquad i = 1,\ldots\ldots,I;\; s = 1,\ldots\ldots,S \qquad (3 - H)$$

$$y_j^s \leq d_j^s \qquad j = 1,\ldots\ldots,J\;;\; s = 1,\ldots\ldots,S \qquad (3 - I)$$

$$x_i, y_j \geq 0$$

Model 3-2: Stochastic Model

(3-G) is similar to constraint (3-C) of the deterministic model with no changes. (3-H) denotes the unit requirement of the component for the end items at each scenario *s*, while (3-I) represents the demand of the end items at each scenario. Hence the decision variables associated with assembly and sales are scenario dependent. We adopted the approach discussed by Brandimarte in [3] for this thesis by extending to real world problems which also require integer variables. The comparison of the solution from both the models and the use of integer values along with GA is experimented in the further chapters.

3.2 Summary

This chapter described the mathematical model for both the deterministic and stochastic type that is used to illustrate how these models can increase expected revenues of the retailer. The role of scenarios in stochastic models was also explained briefly. The following chapter explains the methodology implied with the defined models in detail.

Chapter 4
Methodology

The supply chain optimization problem considered in this thesis has the following problem; the component manufacturing has to start before knowing the demand for the finished product. However, the retailer can use the concept of component commonality to distribute and optimize the assembly of finished products to maximize revenues. For these two reasons, we proposed the two stage stochastic programming model in Chapter 3, where the first stage decision variables corresponded to the ordering of parts and the second stage variables corresponded to the assembling of finished products whose demands are uncertain. In this chapter, we propose some criteria to compare results of the proposed stochastic programming model with deterministic models commonly used today.

The commonly used deterministic model is executed first and the results are obtained, followed by the stochastic model is solved. Initially, linear programming, a continuous variable optimization method which is significantly faster to solve is used to develop and demonstrate the criteria for comparing

before the problems with discrete variables are solved. In order to compare both the solutions, the model that is the deterministic model, is repeatedly run with the generated scenarios (the same scenarios used in the stochastic model) and the expected cost weighted by their respective probabilities is evaluated. The result is used for defining Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS). Lastly, mean, variance, and coefficient of variation of the maximized revenues are used to compare the effect of the shape of probability distributions on the solution.

4.1 Criteria for Comparing Stochastic Solutions with Deterministic Solutions

This section presents the definition of two significant factors that determine the importance of modelling uncertainties in optimization models, namely, EVPI and VSS. EVPI is the measure in determining the cost significance of uncertainties in the optimization problem. This quantity represents the maximum amount one would be willing to pay in order to gain the perfect information as suggested by Galbraith in [5]. Perfect information refers to the

situation in which one has all the relevant information with which to make a decision and there are no uncertainties.

This value is calculated by finding the difference between the objective function value of the deterministic model which has perfect information and the stochastic model. The value thus obtained is assumed as the amount one would be willing to pay to gain the perfect information.

$$EVPI = \text{Weighted Objective Function Value of the Deterministic Model} - \text{Objective Function Value of the Stochastic Model}$$

Situations in which one cannot gather more data about the future and in order to know how well the deterministic model solutions perform compared to solutions from stochastic programs, a quantity called the value of stochastic solution is used. This value is calculated using the objective function value of stochastic model and the objective function value of the deterministic solution in stochastic situation.

$$VSS = \frac{\text{Objective function value of stochastic model} - \text{Objective function value of the deterministic solution in stochastic situation}}{\text{Objective function value of the deterministic solution in stochastic situation}}$$

The EVPI helps us understand the cost of uncertainty and VSS helps us understand the significance of using a stochastic programming model. The calculation of these values is explained further in this thesis.

Also the mean, variance and coefficient of variation of the total revenues are calculated. In order to understand the sensitivity of probability distribution shapes, we compare the results of a numerical example using both uniform and non-uniform distribution values.

The coefficient of variation, in general is defined as the standardised amount of scattering of random value knowing only its mean and standard deviation; in other words it represents the relative uncertainty in the model.

However in the real world the data consists of only integer decisions, in order to get suitable results for integer values, a global optimization method such as the genetic algorithm is used. When using this algorithm the results are run a few times to get a better variability in optimal solutions. The model is made to run for several generations in order to perceive the changes in the output perceived.

4.2 Taxonomy of Methodologies

Figure 4.1 shows a diagrammatic representation of the entire methodology process followed in the thesis.

To summarise the methodology discussed above consider Figure 4.1, which shows the diagrammatic representation of the methods applied. Both the deterministic model and the stochastic model may obtain either integer solution or continuous real number solution based on the type of data given. When the values are real numbers, with the attained solution, the EVPI and VSS value using both the models are calculated which is represented as A1 in the figure. The EVPI value is the amount that a retailer would be willing to pay to reduce the uncertainty while the VSS value is the value of proposed model for the retailer. These two amounts are calculated in dollars which is the unit of the objective function.

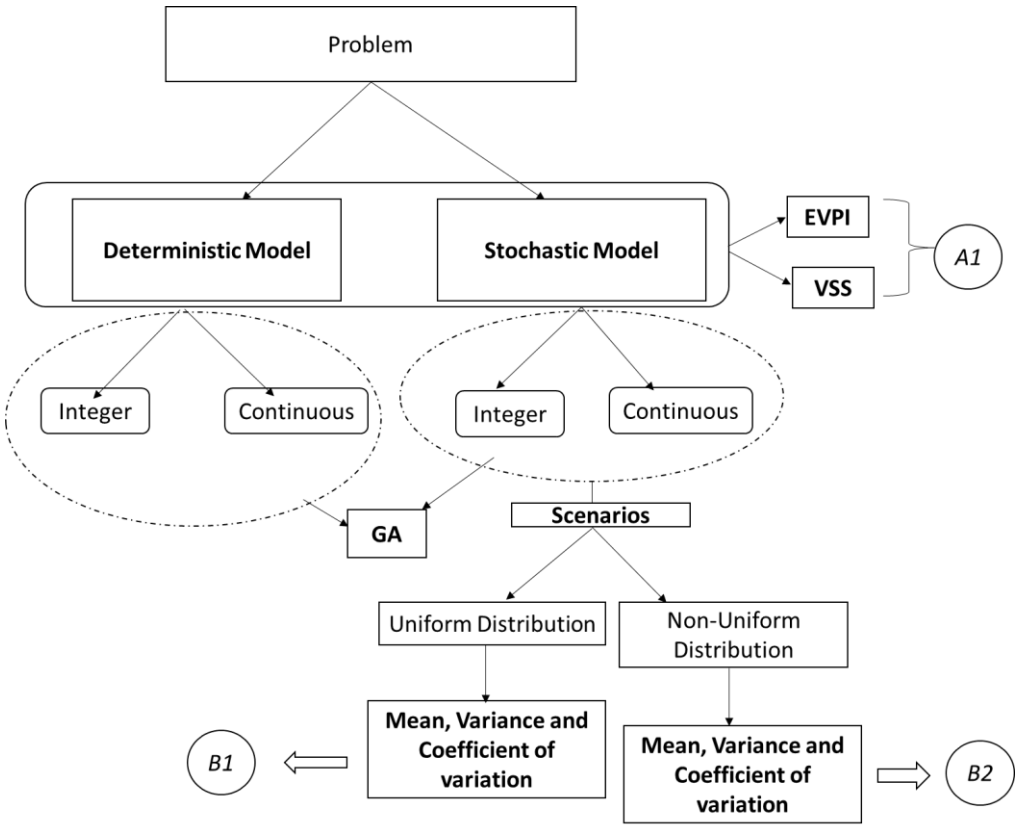

Figure 4.1: Methodology

In addition the stochastic model contains different demand scenarios with uncertainty present in each scenario and all scenarios are used in stochastic distribution. Also from the stochastic model based on the type of distribution (uncertainty) used, the mean, variance and the coefficient of variation values are calculated and compared to observe the importance of the distribution type, which is represented as B1 and B2 in the figure. The coefficient of variation

gives the uncertainty factor present in the model. The genetic algorithm is used to find the best solution for both the models to get integer values.

4.3 Summary

The methodology that is used in the thesis is discussed in this chapter and in the next chapter through a numerical example we use GA to calculate the best mean value and observe the difference in the results obtained. A case study is also included in the next chapter that deals with the real world data obtained for the purpose of this study.

Chapter 5
Numerical Analysis and a Case study

In Chapters 3 and 4, the methodology for solving the supply chain management optimization problem, when there is parts commonality, was presented while considering uncertainty in demand. In this chapter, a numerical example is considered first to demonstrate the various results for evaluating the methodology presented. Finally, results from applying the methodology in a real world case study is presented.

Consider that six end items, finished products, (y1, y2, ….., y6) are assembled using eight different component types (x1, x2, ……, x8) as shown in Table 5.1. From the data in the table, which corresponds to the matrix G in equation 3-H, it can be seen that components x1 and x2 are used as common components for all the six end items. While component 8 is used only in the manufacturing of item y6 which makes it a product-specific-component [23].

	x1	x2	x3	x4	x5	x6	x7	x8
y1	1	1	1	1	0	0	0	0
y2	1	1	1	1	1	0	0	0
y3	1	1	0	0	1	1	0	0
y4	1	1	0	0	1	1	0	0
y5	1	1	0	0	0	1	1	0
y6	1	1	0	0	0	1	1	1

G

Table 5.1: Component (x) requirement for end items (y)

We assume that three machine types (M1, M2, and M3) with the processing times matrix (T), equation 3-G, and the components cost (C), equation part 3-B, including raw material costs are given in Table 5.2. The last row gives the available capacity (L) of each machine type.

	M1	M2	M3	Unit Cost (C)
x1	1	3	2	20
x2	3	3	1	30
x3	2	3	1	10
x4	2	2	2	10
x5	1	1	2	10
x6	1	1	2	20
x7	2	0	1	30
x8	1	0	1	20
Capacity (L)	600	800	900	

Table 5.2: Processing time (t), component cost (c) & machine capacity (l)

5.1 Example 1 – Uniform Probability

Four equally likely demand scenarios are considered which is shown in Table 5.3 with the average demand for each finished product. In each column (either S1, S2, or S3 or S4), the demand for each finished product is presented. The entire scenario (column) is assumed to have equal probability of occurring.

	S1	S2	S3	S4	Average Demand	Price (P)
y1	100	50	120	60	82.5	80
y2	60	30	80	25	48.75	70
y3	40	70	55	90	63.75	90
y4	80	70	40	60	62.5	40
y5	70	40	90	50	62.5	60
y6	50	20	70	30	42.5	30

Table 5.3: Four equal demand scenarios with the selling price of the end items

Using Model 3-1 which comprises of the deterministic model and solving the model using the above data, the objective function value of the optimal deterministic solution obtained is $f = \$625$. It is to be noted that the average

demand values are used in the model and at the moment all decisions are real values. Solutions for integer decisions will be described later.

In in the stochastic model 3-2, the four demand scenarios in Table 5.3 are used along with equal probability. The four demand scenario values are implemented as constraints connecting the first stage and second stage variables in the model along with their probabilities in the objection function.

The optimal objective function value, which is the maximum total expected revenues, of the stochastic model is, $f = \$305$, which is comparatively less than the deterministic model solution which assumed perfect information. In order to compare the objective function values of both the models the EVPI and the VSS values are calculated.

Along with the objective function values, the results also contain the quantity of the components (x) and the amount of end items (y) as part of the output. The value of the end items obtained in the deterministic model plays a significant role for the calculation of VSS. The entire output values for the deterministic model are given in Table A 1 of the Appendix.

5.1.1 EVPI and VSS

The Expected Value of Perfect Information (EVPI) is given as follows:

Objective Function Value of Deterministic Model = $625

Objective Function Value of Stochastic Model = $305

EVPI = $625 – $305 = $320

In this example, the maximum amount the retailer would be willing to pay in order to reduce uncertainty and gain the perfect information would be *$320*.

The Value of Stochastic Solution (VSS) is given by the following calculation:

Objective Function Value of Stochastic Model = 305

Objective Function Value of the Deterministic Solution in Stochastic Situation = *258.98*

$$VSS = \frac{305 - 258.98}{258.98}$$

$$= 0.1776 = 17.76\%$$

From this example, the relative improvement from using the stochastic optimization model instead of solutions from a deterministic model is *17.76%*.

If the retailer estimates developing the stochastic model costs significantly less than the VSS, then the retailer may choose the stochastic model.

5.1.2 Coefficient of Variation of Demands for Finished Products

Consider all the scenarios from Table 5.3, for the given demand values and the scenarios the coefficient of variation is calculated as standard deviation/mean. It is assumed that each product has the same coefficient of variation which is about 0.40. The objective of finding the coefficient of variation is for the comparison of the results with a non-uniform distributed demands to show the importance of the effect of the distribution type on VSS.

These obtained values are used for the comparison of the results attained for a non-uniform probability in the later part of this chapter. Until now the solution determined are of real number values, to obtain integer solutions, GA is implemented with the same set of data.

5.1.3 Integer Solution using GA

In the real world, the decision variables x and y are integers. This integer optimization problem, using the deterministic model 3-1, is solved by genetic algorithm (GA). Because the GA uses statistical samples for integer solutions, it is run several times to get the best output. Each iteration gives a different best penalty value (objective function value minus penalties for violations). Final chosen solution is more or less close to the value obtained without GA (that is, as in Example 5.1 when considering continuous variables).

With the same data as in Example 1, GA is implemented to get integer solutions. In GA the fitness function value includes the objective function value including the penalty for violating the constraints. So taking the solutions from GA the actual objective function value is calculated. Taking the GA solution the actual objective function value is calculated. The GA solution chosen is shown below:

x	82	81	6	14	77	75	0	0	6	0	13	62	0	0

Table 5.4: GA solution used for the objective function calculation

Using the values of the above solution the actual objective function value was calculated to be $f = 465$. The comparison of the result for the deterministic model got without GA and using GA is shown in Table 5.5.

In a similar way, GA is implied for the stochastic model and the actual objective function value calculated was $f = 248$. By knowing the objective function value of both the deterministic and stochastic model, the EVPI and VSS values can be calculated, which results in $EVPI = \$220$ and $VSS = 12.21\%$. All the obtained results are lower when compared to the results that were obtained without GA which is to be expected because of the requirement for integer solutions.

	Deterministic Model Without GA	Deterministic Model with GA
Objective function value	625	465
Solutions		
Component Values		
x1	79.88	82
x2	79.88	81
x3	17.38	6
x4	17.38	14
x5	79.88	77
x6	62.50	75
x7	0	0
x8	0	0
End Items		
y1	0	6
y2	17.38	0
y3	0	13
y4	62.50	62
y5	0	0
y6	0	0

Table 5.5: Deterministic model solution comparison

5.2 Example 2 – Non-uniform probability

Now assume different probability values of 0.2, 0.6, 0.14 and 0.06 for each of the scenarios in Table 5.3, respectively. These distribution values are chosen to provide a non-uniform distribution. Using the same data as in Table 5.3, the coefficient of variation values is calculated as 0.45.

As explained earlier, the EVPI and VSS value for the non-uniform distribution values is also calculated in a similar way and is compared with the uniform distribution to see the difference between the values. The comparison between the distributions is shown in Table 5.7.

	Uniform Distribution	**Non-uniform Distribution**
Deterministic Solution	625	625
Stochastic Solution	305	612.99
EVPI	320	12.01
VSS	17.76%	33.53%

Table 5.6: Impact of uniform distribution versus non-uniform distribution on results

Table 5.7 shows the comparison between uniform probability and non-uniform probability. The EVPI and VSS value shows the impacts of using correct distributions.

	Uniform Distribution	**Non-Uniform Distribution**
Coefficient of Variation	0.40	0.45
VSS in %	17.76%	33.53%

Table 5.7: Importance of the effect of distribution type for the demand data

The coefficient of variation gives the uncertainty factor for comparison purposes. It can be seen that there is not a huge difference between the coefficients of variation, but when comparing the VSS value a bigger difference is found. This shows the importance of the distribution type used in the model.

5.3 Case Study

In order to experiment with real world data, we pursued real world data from different wireless company products. The products are high speed broadband wireless products, and are produced as numerous products based on the assemble-to-order approach with component commonality. The data was collected from published data in the internet (many sources and the data here are reflective of the true data but not actual data as prices and costs are kept confidential by various companies) in such a way that the products that were chosen had similar characteristics with common parts involved in their production.

Therefore, Table 5.10 and Table 5.11 contain the data that were made up based on some broadband wireless products that are active in market and which uses the strategies of ATO and component commonality in their manufacturing.

Table 5.9 gives the details of the components (x1,...., x12) used to produce end items (y1,…,y6). It can be seen that the table contains both the concepts of common components as well product specific components as discussed in the literature review. The product y4 in the table uses only x10 component, which makes it a product specific component.

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12
y1	1	1	1	1	1	1	1	1	1	1	1	1
y2	1	1	1	1	1	1	1	1	1	1	1	1
y3	1	1	0	1	1	1	1	1	1	0	0	0
y4	0	0	0	0	0	0	0	0	0	1	0	0
y5	1	1	1	1	1	1	1	1	1	0	0	0
y6	1	1	0	1	1	0	0	0	0	1	1	1

Table 5.8: Component requirement for end items

Table 5.10 gives the machine types, processing times, component costs and the available machine capacity.

	M1	M2	M3	M4	Unit Cost
x1	1	2	3	1	800
x2	2	1	2	3	600
x3	3	1	1	2	400
x4	1	3	2	1	700
x5	1	3	2	2	500
x6	2	2	1	3	900
x7	1	2	1	1	200
x8	3	1	3	2	800
x9	3	1	2	2	400
x10	2	3	2	1	100
x11	1	2	1	3	100
x12	2	1	1	3	200
Capacity	600	900	800	500	

Table 5.9: Processing time, component cost and machine capacity

And finally Table 5.11 gives the three scenario requirements, average demand and the selling price of each product.

	S1	S2	S3	Average Demand	Price
y1	60	120	60	80	1500
y2	80	40	90	70	1000
y3	35	125	80	80	1100
y4	100	55	70	75	1600
y5	50	70	45	55	1900
y6	65	95	80	80	1750

Table 5.10: Three equal demand scenarios with the selling price of the end items

Using Model 3-1 comprising the deterministic model, the objective function value (total revenues) of the deterministic solution is calculated, which is f = *$951500*. Then by using Model 3-2 containing the stochastic model, the stochastic solution is computed which is given by f = *$117380*. Using these values the EVPI and VSS are calculated. The Expected Value of Perfect Information (EVPI) is given as follows:

Objective Function Value of Deterministic Model = $951500

Objective Function Value of Stochastic Model = $117380

EVPI =$951500 - $117380 = $834120

Hence the maximum amount the manufacturer would be willing to pay in order to gain the perfect information would be *$834120*. This may suggest that it may be prohibitively expensive to try to obtain perfect information and hence using stochastic models may be the only practical possibility.

The Value of Stochastic Solution (VSS) is given by the following calculation:

Objective Function Value of Stochastic Model = 117380

Objective Function Value of the Deterministic Solution in Stochastic Situation } *= 73817*

$$VSS = \frac{117380 - 73817}{73817}$$

$$= 0.590 = 59\%$$

Thus the relative improvement from optimization models when uncertainty is considered explicitly is *59%*.

5.3.1 Solution using GA

As experimented with Example 1 previously, GA is used for the case study problem consisting of integer variables to examine the results. Using the outputs for the deterministic model with GA, the actual objective function value is calculated. Considering the least solution from the iterations, the calculated result is $f = 48500$.

Similarly the actual objective function value for the stochastic model with GA is calculated to be $f = 33916$. Knowing both the values of the deterministic and the stochastic solution, the EVPI can be calculated:

Objective Function Value of Deterministic Model = $48500

Objective Function Value of Stochastic Model = $33916

EVPI =$48500 - $33916 = $14584

When compared to the EVPI value obtained without GA, the above result f = *$14584* is lower. Thus it can be inferred that in integer solutions in this case study, the cost of uncertainty given by the EVPI value reduces.

5.4 Summary

This chapter presented a generated numerical example experimenting the methodology explained in the previous Chapter. The EVPI and VSS value calculations were done. A comparison between modelling the demand with uniform distribution and non-uniform distribution was also shown in the example. GA was used in the example for solving integer problems. Furthermore a case study was also done based on real data obtained.

Chapter 6
Summary & Conclusion

6.1 Summary

In this thesis, deterministic optimization models and stochastic optimization models for solving supply chain optimization problems when there are parts commonality and uncertainty in demand are presented. EVPI and VSS to demonstrate the usefulness of the stochastic models are proven.

In Chapter 2, a brief literature review of some essential topics such as supply chain management, component commonality and assemble to order system that is related to this thesis was done. The significance of component commonality is that it delivers a way to deal with high variability in finished product demands while retaining low variation in manufacturing tasks and to reduce lower costs. It also helps to reduce manufacturing lead time which could be one of the uncertainties. Increasing commonality advances material usability and decreases system complexity.

High commonality makes a better percentage of the product structure appropriate for repetitive manufacturing, which in turn helps in shortened

planning and scheduling time. This commonality reduces setup and holding costs, reduces lead time and risk throughout product development.

A short review about uncertainty and stochastic programming techniques was also done and a brief discussion on global optimization related to the thesis is reviewed was also done in Chapter 2.

A detailed description of the mathematical model comprising both the deterministic and stochastic model was explained in Chapter 3, followed by the explanation of the methodology in Chapter 4. The criteria, EVPI and VSS, to use to evaluate the impact of stochastic model when scenarios are introduced are also discussed in Chapter 4.

In Chapter 5, the methodology was illustrated using numerical examples and was applied to solve the optimization problem in a case study. The difference in impacts due to the shape of distributions was demonstrated using uniform distribution and non-uniform distribution distributions.

6.2 Conclusion

Further research is continuously being carried on component commonality and assemble-to-order strategy to produce finished products with reduced costs and

lead times. However the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS) when considering also the risk (or variance of the objective functions) are topics that can be explored further in the future.

The effect of risk pooling on component commonality can also be studied. Moreover extending and implementing stochastic programming to consider both tree and fan based generations can also be done in the future.

Appendix A

Deterministic Model Solution

Value of	Result
x1	79.88
x2	79.88
x3	17.38
x4	17.38
x5	79.88
x6	62.50
x7	0
x8	0
y1	0
y2	17.38
y3	0
y4	62.50
y5	0
y6	0
Objective Function	625

Table A 1: Example 1 deterministic model solution

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