Building complex number words:

How and when do children learn the meaning of multipliers.

by

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A thesis

presented to the University of Waterloo

in fulfillment of the

thesis requirement for the degree of

Master of Arts

in

Psychology

Waterloo, Ontario, Canada, 2013

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Number words or numerals are built using a compositional system, wherein a small number of words can be combined in multiple ways to represent many different numbers. Children not only have to learn the rules for combining numerals, but must also map certain combinations to specific arithmetic functions. One such combination involves a class of words called multipliers that are used in a multiplicative structure (e.g. “two hundred” maps to “two times one hundred”). How and when do children learn this mapping? There have been two contrasting theories of acquisition: (1) That the compositional rules themselves provide all the necessary tools in order to create the mapping (Hurford, 1975) or (2) the rules are learned by rote and children only make the mapping via explicit instruction and experience with real world objects (Fuson, 1990). To test these theories, 99 children between 4.5 and 6.5 years old were trained on a novel numeral phrase that either did (Experiment 1) or did not (Experiment 2) use a multiplier structure. With all other stimuli remaining the same, more children (43% vs. 10%) were able to determine the novel word was a multiplier when in the correct structure. Other possible avenues for learning this mapping, including being taught the place value system (Experiment 3) and experience counting (Experiment 4), did not fully explain why children did better with the correct syntax. Although the results of these experiments cannot entirely discount the theory put forth by Fuson, they do support Hurford’s theory that it is the rules themselves which allow children to map meaning onto complex numerals.

Acknowledgments

I would like to thank Dr. Mathieu Le Corre for his help and input with the project.

I would also like to thank Dr. Katherine White and Dr. Ori Friedman for agreeing to review this manuscript. Special thanks to Clark Amistad and Michelle Cariati for their assistance in collecting data.

Finally, thanks to my lab mates Amanda Pogue and Pierina Cheung for being exceptional friends and colleagues through all grad school has to offer.

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Introduction

In today's world of space travel, powerful computers and global markets, everyday conversations commonly involve talk about numbers in the billions or trillions. How do we create and understand words for such astronomically large numbers? We use a compositional system where words for distinct numbers are combined together to make words for new numbers. According to the only systematic linguistic study of numerical composition systems in human languages, numerical systems have two fundamental components: compositional rules for combining simple numerals (number words) into complex ones; and systematic mappings between particular combinations of numerals and addition or multiplication (Hurford, 1975). For example, in English, some combinations map onto addition (e.g., "twenty-three") and others map onto multiplication (e.g., "two hundred").

Together, compositional rules and systematic mappings between combinations and arithmetic operations provide numerical composition systems with great expressive power – they allow us to create easily interpretable expressions for an enormous range of numbers on the basis of a small number of words. For example, English speakers can easily create expressions for numbers in the trillions –numbers more than three times larger than the number of stars in our galaxy. Without a compositional system, one would need that many words to express all these numbers – a clear strain on cognitive resources. In contrast, a method of creating numbers that uses compositional rules allows for exponential growth in number with only minimal increased memorization of numerals. For example, in English, to count to one hundred a person must memorize only 28 words, the sequence one through nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, and hundred. Counting to one trillion, a number 10,000,000,000 times larger than 100 requires only 4 additional words!

What are the developmental origins of the adult number composition system? Specifically, what are its linguistic and conceptual building blocks? Do children acquire this system as a by-product of some other process, such as being explicitly taught how to use formal written numerical notation (i.e., Arabic digits and the place-value system)? Or are both the compositional rules and the systematic mappings to addition and multiplication available to children prior to explicit formal instruction?

This set of questions breaks down into three sub-questions, namely what are the developmental origins of (1) the compositional rules for combining numerals into complex numerals; (2) the representations of addition and multiplication; and (3) the systematic mappings between the compositional rules and addition or multiplication. The experiments presented here begin to address the third question by investigating the development of the mapping between complex numerals and multiplication. Specifically, I ask when children are first able to use the ordering of the numerals in two-word expressions to infer whether they are multiplicative. I also test several hypotheses concerning how children learn the mapping between numeral ordering and multiplication. But first, I turn to a review of existing literature on the nature of the adult numerical composition system, on the acquisition of its compositional rules, and on the development of the concepts required to represent multiplication.

The adult numerical composition system

In all languages, the compositional rules are based on two types of numerals: digits and multipliers (Hurford, 1975). Multipliers are words such as hundred, thousand, and million, and are named such because they use a structure based around multiplication. The simplest syntactic difference between them is that multipliers have some properties of nouns while digits do not. For example, in English, “hundred,” “thousand” and “million” can all be pluralized (e.g., “Millions watched the game”), but digits cannot (e.g., “Threes watched the game”). Moreover, just like English singular count nouns, multipliers must be preceded by a numeral or a determiner – e.g., “A/one million people watched the game” is grammatical but “Million people watched the game” is not. In contrast, digits can be used without any word preceding them (“Three people watched the game”).

The distinct status of multipliers and digits as different parts is supported by neurological evidence. McCloskey and colleagues (1986) showed that when English-speaking brain-damaged patients were asked to read large Arabic numerals aloud, patients would typically only make one naming error while saying the rest of the word correctly. The types of errors provide insight into how neurologically intact adults represent the structure of complex numerals. While multiplier substitutions (e.g. 543 said as five thousand forty-three) and decade substitutions (e.g. 43 said as sixty-three) were common, errors between decade terms and multipliers (e.g. 43 said as four hundred three) were not, suggesting that for English-speakers multipliers and decade terms are not considered as the same class of word. McCloskey concluded that adults assign numerals for digits, decades, or multipliers to different linguistic categories. Both the linguistic and neurological theories imply that adults store complex numerals as their component parts.

According to Hurford (1975), digit and multiplier numerals can combine in two ways. First, in a process akin to the combination of a determiner and a noun into a noun phrase (e.g. “a” + “dog”→ “a dog”), they can combine into numeral phrases (e.g. “one” + “hundred” → “one hundred”). The simplest numeral phrase consists of the combination of a digit with a multiplier – e.g. “two hundred.” More complex numeral phrases consist of combinations of numeral phrases with a multiplier – e.g. “two hundred thousand.” Second, numerals can combine via conjunction. The conjunction can be explicit as in “one hundred *and* one” or implicit as in “twenty-three.” Each type of combination maps onto a unique arithmetic operation. Conjunctions map onto addition (e.g. “twenty-three” means 20 + 3; numeral phrases map onto multiplication (e.g. “two hundred” means 2 x 100).

In English, the different types of combinations can be recognized by the ordering of their constituents. Numeral phrases always begin with a digit – whether they are simple (“one hundred”) or complex (“one hundred thousand million”). All conjunctions consist of a numeral phrase followed by a digit (e.g., “twenty-three”)[1], or by another numeral phrase (“one thousand one hundred”). But conjunctions never begin with a simple digit. Therefore, the ordering of the digit in combinations of English numerals provides a strong cue to meaning. That is, all combinations of two numerals that begin with a digit are numeral phrases --- i.e., complex numerals that map onto multiplication.

[1]Hurford analyses the English decade terms as numeral phrases.

Learning the compositional rules

How do children discover the compositional rules of their language's numeral system? There are two contrasting theories purposed by Fuson (1990) and Hurford (1975). Fuson's theory claims that children require explicit instruction of their language's compositional rules. Children must learn complex numerals by rote, one at a time, prior to decomposing them. For example, after learning "One hundred and one," children would still have to separately learn "One hundred and two."Hurford's theory is that children discover the rules of the system on their own. Hurford would predict both a sudden explosion of complex numerals in a child's language and that the more productive a numeral system – i.e., the fewer individual numerals required to make all complex ones – the easier it is to learn. Fuson would not predict either of these patterns.

The pattern of children's counting can provide insight into how children learn compositional rules. If children are learning by rote, with no decomposition or impression of the rules, then there should be no pattern to the highest number children can count. However, by using compositional rules to assist in creating complex numerals, children should be able to follow that rule for as long as they know the individual components of the complex numeral. Siegler & Robinson (1982) found that if children, who had received no explicit instruction in counting complex numerals, could count above 20 then their highest number would almost always end with nine (twenty-nine, thirty-nine, etc.), suggesting that they understood the within-decade pattern and were limited only by their knowledge of the next decade word. For the majority of children they had not simply memorized up to an arbitrary complex numeral, but rather were using the pattern of digits within each decade, supporting Hurford's theory of spontaneous discovery of the rules over Fuson's rote memorization.

The two theories can also be evaluated by examining children's performances across languages. While all human languages use a structure built around digits and multipliers, many languages have a variety of irregularities which mask the implicit pattern of numerals. If children are using rule based learning for complex numerals, then languages with few or no irregularities in their numerical rules should be easier to learn than those with numerous irregularities. While if children are using a rote memorization system, variations in rules should have no effect.

The first step toward learning the system of numeral composition in all languages is to memorize the digits. These are the backbone to building all other numbers: they are the smallest of the natural numbers and those which children typically learn first (Baroody et al., 1984; Wynn, 1990, 1992). Unlike larger numerals, the words one to nine have no relation to each other. Knowing the word *five* does not help you predict the word *six* and cannot itself be predicted by knowing the word *four*. While the specific words used are individual to each language, this is a property that all languages with a numerical system share. The list of digits must be memorized by rote. Children can very easily memorize this list of words, and by age two many children can recite the list, despite the fact that they are not aware of the numerical meanings of the words until months or years later (Fuson, 1988; Le Corre et al., 2006; Le Corre & Carey, 2007; Wynn, 1992).

Differences in languages emerge upon combining numerals. In English, the tens decade is different from all other decade structures, and does not follow the standard linguistic rules for either multiplication or addition. Rather than the digit appearing after the decade term, it is said first with the suffix –teen. It is very unlikely that children consider the teens by their separate parts; instead they treat them as single, indecomposable words (Miller & Zhu, 1991). Similarly, the decade terms are variations on the digits with the suffix –ty. While it is possible to predict the next decade term given the preceding one by knowing the list of digits, evidence from children's counting indicate that they do not make the connection to the structure of the decade word and its underlying meaning (Fuson, Richards, & Briars, 1982). The irregularities in English mask the implicit rules used to combine numerals. In comparison, many Asian languages including Korean, Japanese, and Chinese all employ an explicit system with no irregularities. In these languages the numbers following ten (十, *shi* in Chinese) are ten-one (十一, *shiyi*) and ten-two (十二, *shier)*, while the words from twenty and thirty are two-ten (二十, *ershi*), and three-ten (三十, *sanshi*), respectively. This system requires only ten words to count to ninety-nine. With no irregularities children can see the pattern of numerals more easily, allowing them to predict the next decade term.

How then does children's counting differ across these languages? One study by Miller and colleagues (1995) of 3-5-year-olds in both the US and China found that 3-year-olds in both countries could count roughly to 10. As the digits list in both languages involves the same properties of arbitrary words that must be memorized in order, these are expected results. However, while 4-year-old Chinese children could count to 40, English children could not count that high until they were 5 years old. This divergence in performance is consistent with rule-based learning but not with rote memorization, as the only explanation for the English-speakers' delay is the irregularity of the teens and decade numerals interfering with establishing the compositional rules, once again supporting Hurford's theory over Fuson's.

While more transparent languages allow for earlier rule learning, they also introduce children to the use of both addition and multiplication in numeral composition much earlier than in English. As previously discussed, since English-speaking children do not decompose decade words, they only need to perform addition to determine the quantity being named (e.g. twenty-five is twenty plus five). However, in the more transparent Asian languages the numerals are constructed using a combination of addition and multiplication. The Chinese word *sanshiyi* (三十一) consists of the sequence of words *three, ten and one.* The rules of their language require it to be interpreted as *three times ten plus one*. In this case the number three does not count individual objects but instead counts the number of "tens" represented in this number. Children speaking Chinese learn that tens are units that can be counted, whereas in English, because of the individual names for each decade, children are not exposed to this type of counting numbers until they start learning numeral forms for numbers in the hundreds.

The structure of the English hundreds is similar to the structure of the Asian languages' decade words. The number preceding the word "hundred" specifies how many times hundred is counted. Children speaking languages such as Chinese use a multiplier system for decade terms, and it appears that this structure gives them an advantage in learning up to one hundred more quickly than their English counterparts. However, given that English-speaking children have learned a very different system for numerals up to ninety-nine it is unknown how they will adapt to the introduction of multipliers to their numerals. As English-speaking children do not decompose the decade terms, but instead treat them as whole words they may be resistant to viewing the multiplier structure as an amalgamation of its component parts and may initially treat “one hundred” as a single indecomposable word. Another possibility is that children will recognize the more transparent structure and be willing to accept that one hundred consists of two separate numerals more easily than with the decade terms. In either case, it is only after decomposing multipliers that English children will be able to map the complex numerals to their multiplicative meaning.

Acquisition of the mappings between number word orderings and arithmetic operations

Once children decompose complex numerals into their constituents, they can begin to learn how different orderings of constituents map onto addition or multiplication. Since my experiments test knowledge of the mapping between numeral phrases and multiplication, my review focuses on the acquisition of that mapping only.

To our best knowledge, there has been no research on children’s ability to infer the meaning of digit + multiplier complexes based only on the syntax of these numerals. In order to correctly interpret a number such as “one hundred”, first children must recognize that "one hundred" is a numeral, then that both one and hundred are separate numerals in a common structure, and finally that they are in a multiplicative structure wherein one is a count of the number of hundreds. This final step could be particularly difficult for young children because of biases about possible referents for words. As will be discussed in the following section, that a word can refer to the property of a group of individuals rather than the individuals themselves, is difficult for children to understand.

Mapping numeral phrases to multiplication. Mapping numeral phrases to multiplication involves taking the digit to count instances of the number denoted by the multiplier. For example, in “two hundred,” the digit “two” counts instances of 100. Some evidence suggests that understanding that numerals can count numbers is a slow developmental process.

Children have a very strong discrete object bias - they have difficulty taking anything other than discrete spatiotemporal chunks as individuals (Spelke, 1994; Wynn, 1995). For example, if asked to count the number of objects in a display with a broken object, preschoolers are likely to count the individual pieces of the broken object (Shipley &Shepperson, 1990; Brooks, Pogue, &Barner, 2011). Similarly, when asked to count the number of kinds of objects (e.g. asking “how many kinds of animals are there?” when there are 2 chickens, 2 cows, and 2 pigs) children count all individual objects instead of the kinds (Shipley &Shepperson, 1990). This difficulty extends to words that explicitly refer to more than one object such as collective nouns. For example, when children who are familiar with the word “forest” are asked to identify "one forest" they choose a single tree or a single unfamiliar object over a group of many trees (Huntley-Fenner, 1995). This strong aversion to grouping terms could hinder children’s ability to understand the meaning of multipliers, by limiting words that can be counted only to individuals. If children cannot accept multipliers as words that refer to groups of objects, then they will not be able to decipher that the first numeral in multiplicative number phrases such as “two hundred” counts groups of objects instead of individual spatiotemporal chunks.

This problem begins to wane between 4 and 5 years of age. Bloom and Kelemen (1995) found that 3-year-olds would never accept novel words as having a collective meaning. However, 4- and 5-year-olds would say that a word represented a group of objects over a single object, but only when the syntax surrounding that word denoted a collective noun. The research presented above by Shipley and Shepperson (1990) and Huntley-Fenner (1995) also showed a shift in children’s interpretation of collections within the same age range. As 4 years old appears to be the youngest age that children begin to take words to refer to groups, the present studies will ask whether these children can recognize and interpret multipliers.

The Experiments

Numeral composition systems are made of three parts: compositional rules, representations of addition and multiplication, and mappings between numeral orderings and addition or multiplication. Previous studies suggest that sometime between 3 and 4 years of age, children acquire the first two parts. They discover the compositional rules of their language’s numeral system by learning to count, and they are able to map pieces of language onto concepts similar to the ones involved in multiplication -- they are able to take nouns to refer to collections, and they are able to take numerals to quantify over collections. Here we ask when children acquire the mapping between the compositional structure of complex numerals and their meaning. Specifically, we ask when English-speaking children recognize that two-word complex numerals that begin with a digit are numeral phrases that map onto multiplication.

Children's ability to map a complex numeral to the multiplicative structure was tested by creating a novel numeral phrase. This phrase consisted of a familiar digit followed by a novel word, "gobi" (Experiment 1). Children were given experience with the novel numeral phrase before being tested on what, if any, mappings they made between this complex numeral and multiplication. Additional experiments (Experiment 2a and 2b) examined whether the mapping children were making between the novel word "gobi" and multiplication was based on the unique word forms of complex numeral phrases. The second half of the thesis attempts to determine how children make the mapping between complex numerals in the form of "digit numeral" and multiplication. Two plausible avenues of acquisition were examined: the mapping is made as a by-product of learning the written place value system (Experiment 3); or children learn through experience counting into the hundreds (Experiment 4). These possibilities are discussed in the context of the contrasting theories of acquisition proposed by Hurford (1975) and Fuson (1990).

Experiment 1: Learning a novel numeral phrase, and mapping it onto multiplication

Method

Participants. Thirty 4.5- to 6.5-year-old children (15 girls, mean age = 5 years 4 months, range = 4 years 6 months to 6 years 5 months) from the Kitchener-Waterloo area participated in the study. Participants were recruited through their daycare or the local children's museum. All children spoke English as a first language.

Materials and procedure. All children were tested one-on-one in a private room with the experimenter. Some children had their parents present. Parents sat behind the child and were instructed not to assist the child in any way. Children sat in front of a laptop using Microsoft PowerPoint to display the testing images.

Familiarization and training. Children were told they were going to learn a new word: *Gobi*. This word was presented as being from another language. They were shown a number of slides of three objects that were labeled as "this is one gobi objects". They were also shown a slide with groups of two, three, and four objects and were told "This is one gobi objects (while the experimenter pointed to the group of three), and "This is not one gobi objects" (experimenter pointed to the group of two), and "This is not one gobi objects" (experimenter pointed to the group of four). Children were given further training via a two-alternative forced choice task with feedback. They were shown a boy and a girl, one of whom had three objects while the other had either four or two, and were asked "who has one gobi objects?"Children were corrected if they chose the wrong character and praised for picking the correct one. In order to succeed in these training trials, children only had to identify which character had three objects, not necessarily consider them as a single group of three. Children were presented with training trials until they answered correctly 4 times in a row, up to a maximum of 16 trials.

Forced choice generalization task*.* All thirty children passed training and moved onto the testing phase. Again using PowerPoint, children were shown the same boy and girl character but now asked "Who has two gobi objects?" This time one character had two groups of three, while the other had one of a variety of alternatives (Fig. 1). These alternatives were designed to test whether children had learned that the meaning of expressions in the form "D gobi (digit gobi)" is determined both by the number of groups and by the number of individuals within each group. To determine whether the learned meaning was formulated over groups, the target (two groups of three) was tested against two individuals. To determine whether it was formulated over groups of three, the target was tested against two groups of two. To determine whether children took *gobi* to mean three, the target was against a single group of three individuals. Finally, to ensure that children could not succeed by always choosing the larger group, the target was tested against four groups of three. On most trials, the target objects for two gobi were organized in two distinct perceptual groups of three objects. However, on some trials, they were presented in a single perceptual group of six objects. The latter trials allowed us to test whether children identified "D gobi" by counting perceptual groups (e.g., rows or triangles) or whether they genuinely counted groups of three. There were 24 forced choice trials, four of each type, with the correct answer being equally distributed between the two characters.

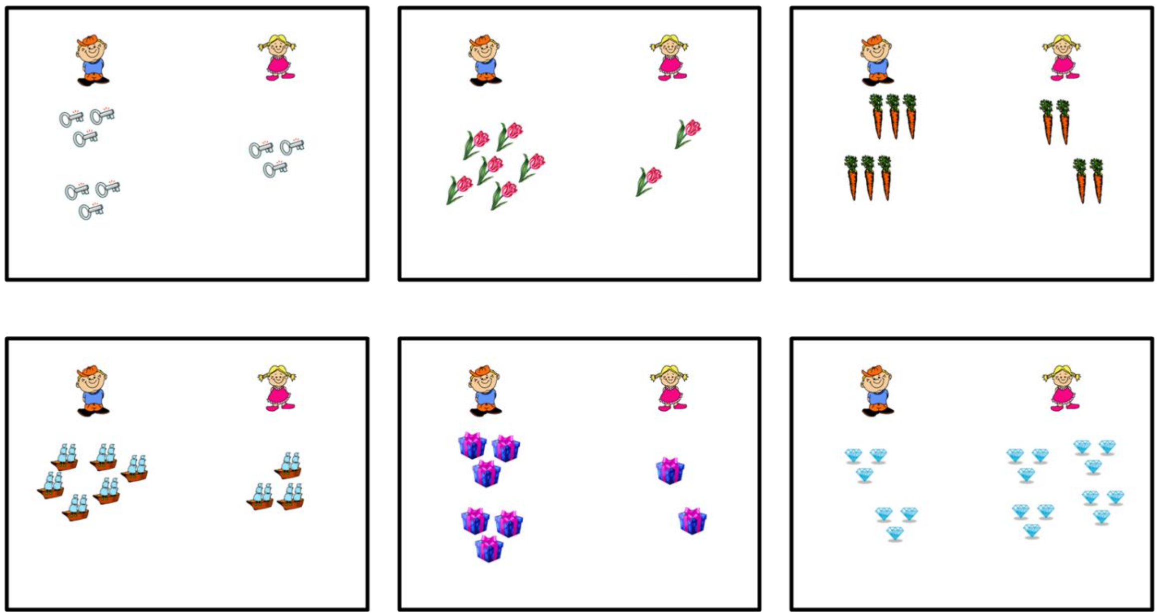

Figure 1. *Examples of each alternative in the Forced Choice Generalization task.*

Counting circles. In this task, participants were presented with four circles containing objects and were asked “How many circles have one gobi objects in them?” There were three trials: four circles with one group of three in each; three circles with one group of three in each and one circle with two objects; and three circles with one group of three in each and one circle with a single object.

How many gobi. Children were presented with six identical toy alligators and asked “How many gobi can you make from this many alligators?” The children were then allowed to manipulate the alligators before answering. This procedure was then repeated with five alligators.

***Give two gobi*.** The procedure for this task was based on Wynn’s (1990, 1992) Give-a-Number task. Children were presented with ten alligators and asked “Can you give me two gobi alligators?" Children were able to freely manipulate the alligators for as long as they needed. When they stopped manipulating objects, they were asked to confirm "Is that two gobi?" If they said no they were asked "Can you fix it and make it two gobi?"

Counting groups. The purpose of this task was to determine whether success on the novel numeral phrase tasks was determined by knowledge that is specific to complex numerals or whether it was determined by more general capacities, namely the capacity to construe individual objects as parts of groups and to count these groups; and the capacity to take words to refer to groups, as opposed to individual objects.

In this task, participants were shown a boy and a girl character, each of which was next to its own set of objects. Then they were asked "Who has two groups of three?" One character had the correct number; the other had one of the four alternatives described in the Forced Choice Generalization task: i.e., two individual objects, a single group of three, two groups of two, or four groups of three. There were two examples of each trial type for a total of eight trials. Each character had the correct set an equal number of times.

Two patterns of performance are possible: either children succeed on the counting groups task regardless of whether they succeed on the novel multiplicative numerals task; or children succeed on the counting groups task only if they also succeed on the multiplicative number task. The former pattern would suggest that success on the novel multiplicative numerals task reflects knowledge that is specific to numerals; the latter would suggest that it reflects the acquisition of the capacity to represent and refer to groups in general.

Results

All 30 children tested successfully passed the training phase[2]; 29 did so in 4 to 6 trials. Children's responses to the Forced Choice Generalization task created a bimodal distribution by falling into two distinct, significantly different groups (Fig.2). Thirteen children performed significantly above chance – i.e., they answered correctly on 17 or more trials, $t(13) = 12.71$, $p < 0.001$ ($M = 20.46$, $SD = 2.40$).The remaining 17 participants did not. Rather, they performed significantly *below* chance, answering correctly on 7 or fewer trials, $t(16) = 8.29$, $p < 0.001$ ($M = 6.41$, $SD = 2.78$). That is, the latter participants consistently chose the *smaller* of the two sets. Henceforth, the group of children who performed above chance will be referred to as "m-learners" (multiplier-learners); the others will be referred to as "non-learners."

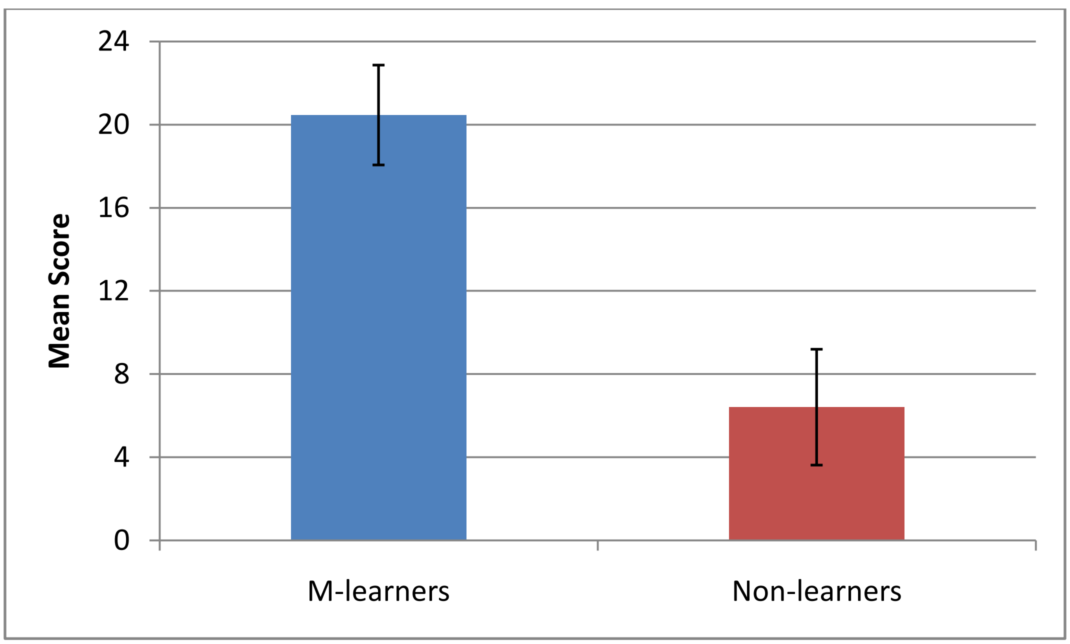

Figure 2. *Mean score in forced choice generalization task by learner status.*

[2]Age effects were tested for all tasks in Experiments 1 through 4. No age effects were significant (all *p's* > 0.09). Hence, age is not included in any of the following analyses.

The m-learners performed well above chance for 5 of the 6 trial types (all *p's* <0.001), marginally reaching significance only when two sets of three were compared to four groups of three (M = 2.77, SD = 1.48), $t(12) = 1.87$, $p = 0.043$. The non-learners consistently chose the incorrect answer in 5 of the 6 trials types, while choosing the correct answer in the four groups of three condition (M = 3.29, SD = 0.77, Table 1), again suggesting that they consistently chose the smaller number of objects. Although m-learners performed above chance in all trial types, m-learners performed better when presented with two visually distinct groups of three rather than a single group of six when judging against a single group of three, $t(13) = 2.22$, p = 0.036,but not when judging against two individuals, $t(13) = 1.32$, $p = 0.199$.

Table 1. *Average correct for m-learners and non-learners as a function of trial type.*

Trial Type		M-learners		Non-learners	
Competitor set	Correct set	Mean	SD	Mean	SD
Two individuals	Two groups of three	3.69	0.85	0.41	0.87
	One group of six	3.23	0.93	0.29	0.47
One group of three	Two groups of three	3.85	0.37	0.94	1.14
	One group of six	3.23	0.93	1.06	1.39
Two groups of four	Two groups of three	3.69	0.48	0.41	0.62
Four groups of three	Two groups of three	2.77	1.48	3.29	0.77

M-learner and non-learner groups were examined separately for each of the other tasks. This was done for the novel word tasks primarily for two reasons. First, this was to gather converging evidence as to the reliability of these two classifications. Second, to determine whether children were using perceptual cues, as some tasks, specifically the How Many Gobi and Give Two Gobi tasks, did not lend themselves to a perceptual strategy. In addition, comparing m-learners to non-learners in the Counting Groups task determines whether the successful mapping is specific to numeral phrases. A summary of m-learners' and non-learners' performance in each task can be seen in Table 2.

Table 2. *Average correct of m-learners and non-learners for each task in Experiment 1.*

		M-learners		Non-learners	
Task	Max Score	Mean	SD	Mean	SD
Counting Circles	3	2.15	1.07	0.71	0.92
How Many Gobi	2	1.54	0.78	0.23	0.44
Give Two Gobi	1	0.62	0.51	0.06	0.24
Counting Groups	8	8	0	6.53	1.33

In the Counting Circles task, children were given a score out of 3 for correctly identifying the number of circles that contained one gobi Xs on each trial. M-learners performed well (M = 2.15, SD = 1.06), with 7 out of 13 participants answering correctly on all 3 trials. Only one m-learner was unable to answer correctly on any trial. In contrast, non-learners were generally unable to correctly count the circles (M = 0.71, SD = 0.91), as only 1 of the 17 non-learners was able to correctly answer all 3 trials. The majority of non-learners were unable to give the correct answer on any trial (N = 9) or only on the first trial when presented with four circles with one gobi in each of them (N = 5). Overall, m-learners performed significantly better than non-learners, $t(28) = 3.99$, $p<0.001$.

In the How Many Gobi task, children's responses were recorded and they were given a score out of 2. Although children could say any number, all children used one of two strategies: correctly count the number of gobi; or count all individual alligators. When given six alligators and asked how many gobi they could make, they answered either two (correct) or six (incorrect). Similarly, when given five alligators children answered either one (correct) or five (incorrect). The majority (N = 9) of m-learners responded with the correct answer for both the six alligator trial and the five alligator trial; two counted gobi for one of the trials, and two others counted individuals for both trials. In sharp contrast, none of the non-learners answered correctly on both trials. Rather, the majority (N = 13) of non-learners counted individuals for both trials; the others counted gobi for one of two trials. . M-learners were significantly more likely to count gobi correctly than non-learners, χ^2 (2, N = 30) = 17.51, p< 0.001.

With the final task, Give Two Gobi, children gave their answer non-verbally by manipulating the toys. Children's answers were scored on whether they gave the correct number of alligators (six) as well as whether they formed groups or counted out individuals. Children's responses were grouped into three categories: two groups of three (correct); two or three individuals; or other. The other category included giving all individuals, giving unequal groups, or giving two equal groups of another number (e.g., two groups of four).The majority of m-learners (N = 8) correctly gave the experimenter two groups of three, while 3 children gave two or three individuals, and 2 children were in the other category. In contrast, only 1 non-learner gave the correct answer, while the majority (N=13) gave two or three individuals, and 3 children were in the "other" category. The distributions of response types for m-learners and non-learners were significantly different from each other, χ^2 (2, N = 30) = 11.57, p = 0.003.

Overall children's classification of m-learner or non-learner was consistent across tasks. Individually, 3 m-learners were successful in all three tasks (Counting Circles, How Many Gobi, and Give Two Gobi), with most m-learners (*N*=8) passing two of the tasks, and 3 m-learners passing only one. While all m-learners were successful on at least one task, 12out of17 non-learners passed none of the tasks, and the remaining 5 passed only one task. There was no pattern as to which task the m-learners failed or which task the non-learners passed.

Counting groups*.* M-learners performed at ceiling: all 13 answered 8 out of 8 correctly. Non-learners also performed above chance. Ten of the non-learners answered at least 7 out of 8 questions correctly. Overall the non-learner group had an average of 6.53 (SD = 1.33) correct answers, which is significantly more than predicted by chance, $t(16) = 7.85$, $p< 0.001$. This suggests that unsurprisingly, children acquire the capacity to represent and refer to numerically equal groups prior to learning to map numerals to the multiplicative structure; and the main difference between m-learners and non-learners was specific to their knowledge of multiplicative numerals.

Discussion

In Experiment 1, 4.5- to 6.5-year-olds were trained to pair the novel numeral phrase "one gobi" with sets of three, that is they were tested on their ability to decompose the novel complex numeral into its constituents and map each numeral onto a multiplicative structure in the form of D groups of M, where M referred to sets of three. Over 40% generalized to a numeral phrase that they had not been trained on, namely "two gobi." They were able to determine the meaning of "two gobi" despite never being told that it refers to sets of six (or two times three). The only plausible way that children could successfully generalize the phrase "one gobi" to "two gobi" is by decomposing the phrase and mapping it to a multiplicative meaning. If children thought that "one gobi" was a single word then "two gobi" would similarly be another single unrelated word and they would not be able to use their knowledge from training to assist them in answering the Forced Choice Generalization task.

Decomposition, while a necessary first step, was not enough. In order to succeed m-learners would have to map "one gobi" onto the multiplicative structure D groups of M. Unlike digits used alone, digits in a multiplier structure count *groups* of objects, as opposed to the individual objects themselves. For example, in "three cars," the "three" counts individual cars, but in "three hundred cars," it counts groups of one hundred. The primary difference between m-learners and non-learners was whether they took "one gobi" to refer to one group of three or to a number that counts individual objects (i.e. three). Several aspects of the performance of both m-learners and non-learners confirm this distinction.

M-learners' performance suggests that they attempted to break down "two gobi" into two equal groups. Although they were able to consistently choose the set of six in the Forced Choice Generalization task regardless of whether the sets were presented in two perceptual groups or not, they were more accurate when presented with two groups. Moreover, in the Give Two Gobi task, m-learners would give groups of objects when asked to give "two gobi": giving first one group of three and then a second group of three. Even those m-learners whose responses were in the "other" category still gave groups rather than individuals, even though the groups themselves were incorrect. Despite errors, m-learners had a strong inclination that the novel phrase “D gobi” referred to groups of objects, rather than individuals.

In contrast, non-learners’ performance suggests that they believed “one gobi” to refer to a numeral phrase that counts three instances of individual objects, explaining why they failed all of the test trials while still passing training. Non-learners could not be distinguished from m-learners during training trials, as with the exception of one non-learner, neither group required any more than six training trials to correctly choose “one gobi Xs” in four trials in a row. The key difference is that while both groups assigned a numerical meaning to “one gobi”, non-learners not only failed to choose the sets of six when asked for two gobi, but also could not count the circles that contained one gobi, or say how many gobi could be formed from a set of six or five. If non-learners have a numerical definition of “one gobi” during training, why then do they fail all tasks even when not asked to generalize to “two gobi”? The most probable explanation is that while m-learners took the complex numeral “D gobi” to count groups of three, non-learners believed that it counted three individual objects. The counting circles task required children to treat each set of three as a countable unit; however, if children were counting individuals then they would instead count either all circles regardless of what was in them or all objects within the circles, which is consistent with the pattern of non-learners’ answers. As well, when asked to report how many gobi could be made from sets of six or five, all but one non-learner counted all individual objects. Again, if non-learners assumed that “D gobi” was a numeral phrase that counted three individual objects rather than groups, this would explain their responses. Finally, only one non-learner created two groups of objects when asked to give “two gobi,” while the remaining non-learners gave either a subset of individuals or all individuals, once again indicating an individual based approach, rather than group. In sum, this pattern suggests that while both m-learners and non-learners assigned a numerical meaning to “one gobi” during training, the meanings they assigned were subtly but significantly different. As only m-learners took “one gobi” to mean one group of three, they were the only ones who could use what they had learned about “one gobi” to infer that “two gobi” referred to two groups of three.

In order to succeed, m-learners must decompose and interpret the novel phrase as one that refers to groups. However, the digit in the D groups of M structure does not refer to any group, but a group whose identity is determined by the number of objects it contains, and not by any perceptual features of the array. We know that m-learners used a specific number to determine the groups based on their performance on three tasks: Forced Choice Generalization; How Many Gobi, and Give Two Gobi. In all three of these tasks, objects were presented in various forms in order to diminish the possibility that m-learners could use any spatial cues. Despite these variations in presentation, m-learners consistently took “two gobi” to refer to two groups of three objects, suggesting that their definition of the novel complex numeral was of groups determined by the amount of objects they contained and not perceptual features of the arrays. In sum, the results suggest that sometime between 4.5 and 6.5 years old, some children are able to map the novel numerical phrase “D gobi” onto the multiplicative structure D groups of M.

While the results of Experiment 1 show that some children are able to create a mapping between “D gobi” and D groups of M, they do not demonstrate that this mapping is unique.

Specifically, they do not show that complex numerals of the form "digit multiplier" are the only ones that map onto the multiplicative structure D groups of M for children in the age range of 4.5 to 6.5 years old. From these results, a causal link from the structure "one gobi" to the meaning "one group of three" cannot be determined. In fact, it may be that the context of the training and generalization tasks cause some children to interpret any novel numeral phrase with a multiplicative structure, regardless of the actual form of the complex numeral itself.

Experiments 2a and 2b address this question by presenting children in the same age range with "gobi" alone (e.g., "This is gobi houses"; Experiment 2a), or with the reversed phrase "gobi one" (e.g., "This is gobi one houses"; Experiment 2b) in the context of the same training sets that were used in Experiment 1. Children are then asked to interpret the phrase "two gobi Xs" (Experiment 2a) or "gobi two Xs" (Experiment 2b) in the context of two of the generalization tasks used in Experiment 1. As performance was consistent in all tasks in Experiment 1, it is not necessary to conduct every task. Experiments 2a and 2b will use only two tasks: Forced Choice Generalization, as this task was able to create the reliable groups of m-learners and non-learners; and Give Two Gobi, as this gives children the fewest perceptual cues to produce a response and therefore is the most stringent test of children's interpretation of gobi.

Finding that children in Experiments 2a and 2b do not take "two gobi Xs" or "gobi two Xs" to refer to two groups of three objects would provide strong evidence that the children in Experiment 1 who mapped "one gobi" and "two gobi" onto the multiplicative structure did so because of a compositional structure that mirrors English. This, in turn, would show that, sometime between age 4.5 and 6.5, English-speaking children learn that only complex numerals of the form "digit multiplier" can be mapped to the multiplicative meaning D groups of M.

Experiment 2: Are m-learners truly using the word form to access the multiplicative meaning?

Experiment 2a: Do children map a simple novel numeral ("gobi") onto multiplication?

Method

Participants. Thirty 4.5- to 6.5-year-old children (17 girls, mean age = 5 years 4 months, range = 4 years 5 months to 6 years 6 months) from the Kitchener-Waterloo area participated in the study. Participants were recruited through their schools at the Waterloo Region District School Board. All children spoke English as a first language.

Materials and procedure. Participants were presented with the same familiarization and training pictures as in Experiment 1. The only differences were that (1) the familiarization sets were labeled as "gobi objects" instead of as "one gobi objects"; and (2) during the two-alternative forced choice training, children were asked to point to the set with "gobi objects" instead of "one gobi objects." Children who were able to correctly identify "gobi objects" in 4 consecutive training trials were then given the Forced Choice Generalization task and Give Two Gobi task exactly as in Experiment 1.

Results

All 30 children passed the training phase. Children required an average of 5.17 (*SD* = 1.12) trials in order to pass training, which is not significantly different from the children in Experiment 1, $t(58) = 1.42$, $p = 0.159$. As with Experiment 1, children performed either significantly above or below chance on the Forced Choice Generalization task (i.e., they were either m-learners or non-learners). Only 3 children had scores consistent with m-learners (*M* =

20.67, *SD* = 2.31), *t*(2) = 6.5, *p* = 0.022, while the remaining 27 were non-learners (*M* = 5.48, *SD* = 2.99), *t*(26) = 11.3209, *p*< 0.001.There were significantly fewer m-learners than in Experiment 1, χ^2 (1, *N* = 60) = 6.903, *p* = 0.008 (Fig.3). Two of the 3m-learners were able to correctly give two groups of three when asked for two gobi; the other m-learner and all non-learners gave two or three individuals (Table 3). This suggests that the m-learners in Experiment 1 mapped the novel complex numeral onto the multiplicative structure because the novel numeral was complex.

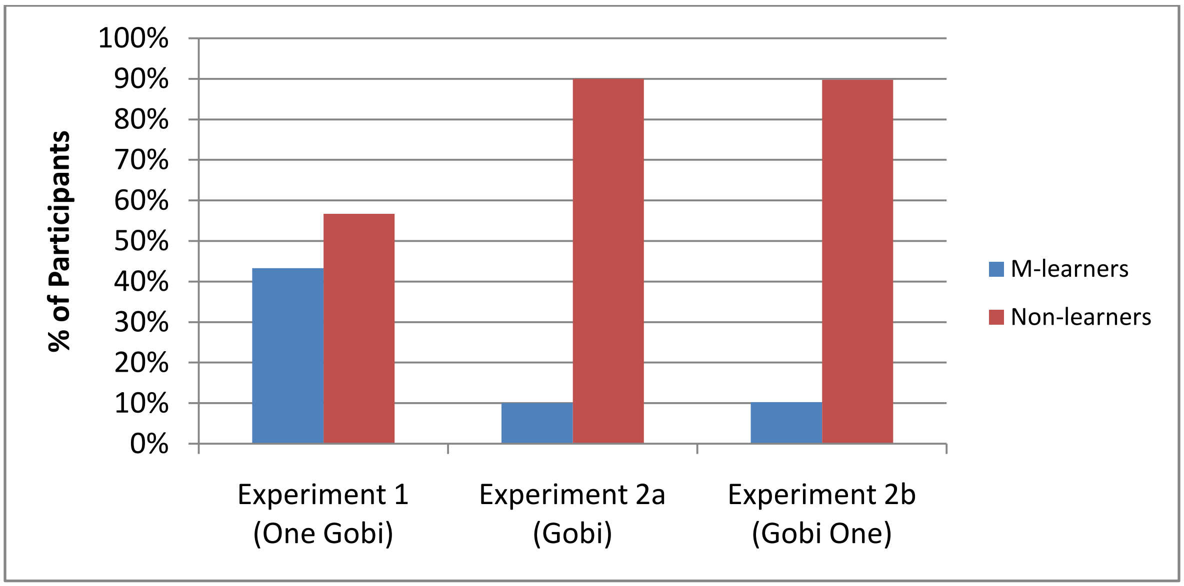

Figure 3. *Percentage of m-learners and non-learners as a function of complex numeral structure in Experiments 1 and 2.*

Experiment 2b: Did the order of the terms in "one gobi" determine whether children mapped it onto multiplication?

Method

Participants. Thirty-nine 4.5- to 6.5-year-old children (22 girls, mean age = 5 years 3 months, range = 4 years 5 months to 6 years 4 months) from the Kitchener-Waterloo area participated in the study. Participants were recruited through their schools at the Waterloo Region District School Board. All children spoke English as a first language.

Materials and procedure. Participants received the same familiarization, training pictures, Forced Choice Generalization, and Give Two Gobi tasks as in Experiment 1.As in Experiment 1, children were presented with a novel complex numeral. However, the order of the words in the novel complex numeral was reversed. The familiarization sets were labeled as "gobi one objects" instead of as "one gobi objects." During the two-alternative forced choice training, children were asked to point to the set with "gobi one objects" instead of "one gobi objects." Children were required to identify who had "gobi two objects" rather than "two gobi objects" during the Forced Choice Generalization task, and were asked to give "gobi two alligators" in the Give Two Gobi task.

Results

All 39 children passed the training phase; however, on average children required 6.93 (*SD* = 3.85) trials which is significantly more than in previous experiments, $t(58) = 2.99$, $p = 0.004$. Children's responses fell into the same classification as previous experiments. Four children were classified as m-learners (*M*=20.75, *SD* = 0.96), $t(3) = 18.28$, $p< 0.001$, while the remaining 35 were non-learners (*M* = 7.14, *SD* = 2.90), $t(34) = 9.90$, $p < 0.001$. This is significantly fewer m-learners than in Experiment 1, χ^2 (1, *N* = 69) = 8.29, $p = 0.004$, but not significantly more than in Experiment 2, χ^2 (1, *N* = 69) = 0.001, $p = 0.972$ (Fig 3). When asked for "gobi two objects", 2 of the m-learners gave six objects, while 1 gave two objects, and 1 gave four objects (classified as "other"). Thirty-two of the non-learners gave two or three individual objects; the remaining 3 gave all ten objects (Table 3).

Table 3. *Number of participants per response type by learner status for Experiments 1 and 2 in the Give two gobi task.*

		2 Groups of 3	2 or 3 Individuals	Other
M-learners	Experiment 1 ("Give two Gobi")	8	3	2
	Experiment 2a ("Give two Gobi")	2	1	0
	Experiment 2b ("Give Gobi Two")	2	1	1
Non-learners	Experiment 1 (" Give two Gobi")	1	13	3
	Experiment 2a ("Give two Gobi")	0	28	0
	Experiment 2b ("Give Gobi Two")	0	24	10

Discussion

As in Experiment 1, children aged between 4.5 and 6.5 years were presented with a novel word and were taught that the phrases containing the novel word referred to sets of three objects. However, unlike Experiment 1, the novel word was not part of a complex numeral of the form "digit numeral." Rather, it was either presented without a familiar digit preceding it (e.g. "These are gobi houses;" Experiment 2a) or it immediately preceded a familiar digit (e.g. "These are gobi one houses;" Experiment 2b). The children were then asked to identify and create sets of "two gobi Xs" (Experiment 2a) or of "gobi two Xs" (Experiment 2b). Despite identical test phases, the children in Experiment 2a were far less likely to take "two gobi Xs" to refer to two groups of three than the children in Experiment 1. Similarly, the number of children who took "gobi two Xs" to refer to two groups of three was significantly smaller than the number who took "two gobi Xs" to refer to two groups of three in Experiment 1. This strongly suggests that the compositional structure of the novel complex numeral "digit gobi" played a causal role in getting children in Experiment 1 to map onto the multiplicative structure.

The results of the Forced Choice Generalization task in Experiments 2a and 2b were very similar, with almost all children performing well below chance. This suggests that similar to the non-learners in Experiment 1, they had a specific definition of gobi as three, and were not simply guessing. The one significant difference was the number of training trials required for the "gobi one" condition. It is not very surprising that children required more trials to correctly identify gobi one, as it is comparatively uncommon structure in English compared to the other versions of the task. In "one gobi", gobi can be replaced in all sentences with "group of three" and they will remain correct, while in the "gobi" condition gobi can be replaced with "three" with no issue. However, for "gobi one", neither "three" nor "group of three" will yield a grammatically correct response, yet all visual evidence indicates that gobi refers to three objects. It is possible that children could assume that gobi means two and that gobi one is an additive structure similar to the decade terms (e.g. twenty-one), but based on their responses to the Give Two Gobi task that is unlikely, as none of the non-learners gave four (2+2) when asked for gobi two objects.

Despite overall fewer children being classified as m-learners, roughly 10% of children in both samples interpreted “two gobi” or “gobi two” as two groups of three. These data cannot determine exactly what strategies these children used to arrive at this interpretation, and it is possible that for a small number of children the mapping between the multiplicative structure and meaning is not unique. It is also possible that children in the “gobi” condition may have discarded or adapted their definition from training once presented with “two gobi” and began interpreting gobi using a multiplier structure. While these possibilities could also be used to explain some of the successes in Experiment 1, there still remains significantly more m-learners when given the correct structure which confirms that the structure of the novel complex numeral in Experiment 1 played a causal role in allowing children to access the multiplicative meaning.

Part II: How do children learn that complex numerals of the form "digit numeral" map onto the multiplicative structure?

The novel numeral experiments of Part I show that some children between 4.5 and 6.5 years old can identify numeral phrases and map them onto multiplication. How do children learn this mapping? There are two predominant views as to how this connection is made: it is innate or it is learned through explicit instruction.

Hurford (1975) argues that numerals are no different from other syntactic units, and the connection between these units and their meanings is innate and built into the syntax itself. With this view, Hurford would argue that all languages have multipliers and a "D M" syntactic structure that refers to "D groups of M". When children encounter this syntax in their language, they would recognize these numerals as multipliers with very little exposure and would immediately connect them with the grouping meaning with no extra step necessary.

On the opposite side of the spectrum, Fuson (1990) argues that the meaning of multipliers must be constructed through experience and instruction. On her view, children must encounter situations where multiunit collections are presented as single units which are then counted. These collections are physical objects in the world such as bundles of sticks, base-ten blocks, or lengths of string. Then through explicit instruction in how to count these groups of objects as a single unit, children connect this concept with the spoken and written system. In this view, while children learn how to say and write numbers by rote, they do not truly understand their meaning without sufficient real world experience.

It is currently unknown whether either of these two theories truly represents children's acquisition of multipliers. The following experiments attempt to address these theories while also examining two indirect avenues by which children could plausibly acquire the meaning of multipliers: place value and counting to hundreds.

Experiment 3: Do children acquire the mapping between numeral phrases and multiplication as a by-product of learning the meaning of the written place value system?

In a base-10 place value system, the relative position indicates the power of 10 that digit represents. For example, the 5 in 529 represents five times 100 because it is the third digit from the right. The rules of this system are explicitly taught to children; very few children spontaneously understand it (Kouba et al., 1988). The nature of instruction links this learning experience to the spoken numerals; e.g., to teach children that the written numeral “500” denotes the number 500 because the 5 refers to five groups of 100, teachers necessarily use the spoken numeral phrase “five hundred.” Therefore, while children are learning the written place-value system they simultaneously receive information that is relevant to mapping numeral phrases to multiplication. The present experiment asks whether children use information about the place value system to map numeral phrases to multiplication.

Method

Participants. The participants were the same as in Experiment 2a.

Materials and Procedure

Familiarization Children were shown a frog puppet and told that the frog loves candy. They were instructed to help the frog count his candy. They were shown a paper cut-out of a wrapped candy and told it was one candy. Then they were shown a paper cut-out of a bag and told that inside the bag were ten candies so they should count the bag as ten candies. Although children do not tend to dissect decade numerals, children are taught the place value system first with two digit numbers and it is likely they would be most familiar with those rather than three digit written numbers. To familiarize children with the procedure, they were shown a PowerPoint slide with two boxed single digit numbers, one on each side of the slide. They were told that one of those numbers represented how many candies the frog had, and were asked to point or say which one was correct. Children were given two training trials with single digit responses (1v2 and 3v5).

Test. On each trial, the experimenter presented the child with a set of bags and a set of individual candies. The number of bags and the number of individual candies varied between 2 and 5. On each trial, the experimenter told the child how many bags and how many individual candies she had given the frog; e.g."Froggy has 2 bags of ten and 3 single candies. How many candies is that?" The number of bags and candies were never the same. The two numbers on the screen were always the correct answer and its mirror-image. For example, on trials with two bags of ten and three individuals, the numbers shown were 23 and 32. Therefore, to select the correct answer, children had to understand that the leftmost digit in a written number corresponds to the number of bags – i.e., to multiples of 10. There were a total of eight trials.

Results

All children could correctly identify the Arabic numbers 1 and 3 as the appropriate number of candy in the familiarization trials. During testing children's overall performance was at chance (M = 4.63, SD = 1.88), with only 16.7% (N = 5) of children performing above chance according to a binomial distribution – that is, able to answer at least 7 out of 8 trials correctly,

$p < 0.035$. This proportion of children is significantly smaller than the proportion of m-learners in Experiment 1, χ^2 (1, N = 60) = 3.89, p = 0.02.

Discussion

While some children were able to correctly use the place value system, almost all children tested were not able to understand the underlying meaning of the digits in two-digit numbers. The proportion of m-learners in Experiment 1 was greater than the proportion of children who passed this task, implying that there are a number of children who failed to identify place value but who would still be m-learners. It is therefore unlikely that learning place value is what causes children to map numeral phrases onto multiplication.

This experiment requires children to have advanced knowledge of the place value system, i.e. not only how to read it but what each digit represents. It is possible that children create the mapping to multiplication based on information acquired during the learning of the place value system, but yet they still do not understand the system well enough to succeed on the task they were given. While this possibility cannot be ruled out based on this experiment, evidence from cross-cultural studies suggests that the mapping between numerals and multiplication likely occurs prior to learning the place value system. Studies of Japanese and Chinese children find that they learn the place value system faster and at a younger age than English-speaking children (Miura, Kim, Chang, & Okamoto, 1988; Stigler, Lee, & Stevenson, 1990).The spoken numerals are more transparent in Japanese and Chinese than in English, yet the place value system is identical in all three languages. Therefore, it is likely that for Asian children their advanced understanding of their spoken language is facilitating the learning of the place value system, rather than the reverse.

Experiment 4: Does experience using the numeral "hundred" provide information about the meaning of numeral phrases?

Another alternative to innate knowledge of the mapping between numeral phrases and multiplication is that children obtain information about the meaning of numeral phrases via experience using numeral phrases within their language. There are primarily two ways that children would be exposed to complex numerals: counting and simple arithmetic problems. Each way can contribute information to children's understanding. First, the count list from one multiple of a numeral phrase to the next involves the repetition of one cycle of all numerals that precede the first numeral of the numeral phrase. For example, in English, counting from a multiple of "hundred" to the next involves repeating the same cycle of "one" to "ninety-nine." This provides information that going from one multiple of a numeral phrase to the next always involves adding the same number. Second, children could obtain information about the mapping of numeral phrases to multiplication by performing simple arithmetic with real world objects or representations in word problems. For example, English-speakers could learn to map "two hundred" to two times one hundred by counting two hundred objects and noticing – either spontaneously or through explicit instruction -- that they counted two equal groups of one hundred objects.

In English, the first structurally transparent numeral phrases are the hundreds – i.e., one hundred, two hundred, etc… Therefore, to test whether English-speaking children obtain information about the mapping between numeral phrases and multiplication from counting, Experiment 4a asked whether the proportion of 4.5- to 6.5-year-olds who can count to the hundreds is at least as great as the proportion of m-learners in Experiment 1, while Experiment 4b asked whether a greater proportion of m-learners would be able to perform simple arithmetic using hundreds. If not, then children probably do not use information from experience using the numeral “hundred” to map numeral phrases to multiplication.

Experiment 4a: Does counting up to the hundreds contribute to learning a mapping between “*D gobi* Xs” and a multiplicative meaning?

Method

Participants. The participants were the same as in Experiment 2a.

Materials and Procedure. Participants were told they would be playing a counting game with the experimenter in which the experimenter would start counting and the child should continue from where they left off. The experimenter would count the first three numerals then stop and wait for the child to continue the count. If the child did not continue the experimenter would count the same three numerals again. The experimenter would start counting at 5, 35, 95, 195, 395, and 795. As the interest was in children’s ability to correctly use numeral phrases, the experimenter would stop the child once they had successfully crossed over at the numeral phrase (i.e. ... 198, 199, 200).If the child ever hesitated or said they did not know the next numeral, the experimenter would prompt them to try to remember what came next. If they still could not answer the task would end. Children’s highest successful count was recorded, such that if children could successfully count from 5, 35, and 95 but could not correctly continue from 195 their highest count was recorded as 100.

Results

Children's highest counts are reported in Figure 4. Only 10% (*N*=3) of children were able to count beyond one hundred. The majority of children either stopped at forty (*N*=7) or at one hundred (*N*=11), with the remaining children barely able or unable to perform counting-on at all (*N* = 9). The proportion of children who could count beyond one hundred is significantly smaller than the proportion of children who were m-learners in Experiment 1, $p = 0.007$. The same result is obtained if we include only those children who could perform the counting on task for at least some numerals, $p = 0.04$.

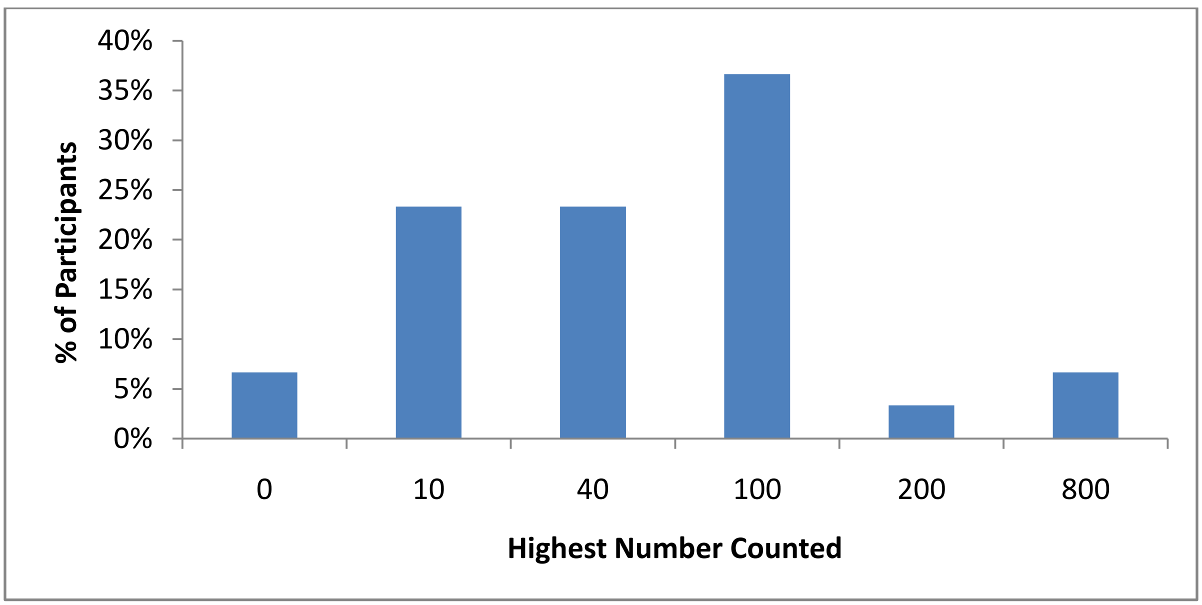

Figure 4. *Distribution of participants by highest number counted.*

Experiment 4b: Are m-learners better able to do simple arithmetic involving the numeral hundred than non-learners?

Method

Participants. The participants were the same as in Experiment 1.

Materials and Procedure. Children were shown a picture of a bag. They were told "This bag has one hundred candies in it. It will always have one hundred candies." To confirm they understood the premise, children were shown a boy character with one bag and asked how many candies he had. Once children were able to confirm that he had one hundred candies, they were shown a girl character with two bags. The experimenter said "This girl has two bags" and then asked "How many candies does she have?"

Results

Nine of the 13 m-learners from Experiment 1 (69%) correctly answered two hundred, while only 5 of the 17 non-learners from Experiment 1 (29%) answered correctly, a marginally significant difference, χ^2 (1, N = 30) = 3.23, p = 0.072. Children gave a wide range of answers from five to one million. Roughly half (47%) of children gave the correct answer (two hundred), with other common answers being one hundred (23%), one thousand (10%), and two thousand (7%).

Discussion

The results of the counting task suggest that of children between 4.5 and 6.5 years old, the proportion who can count up to one hundred is less than the proportion who have mapped numeral phrases to multiplication (Experiment 4a). This is true even if we only consider those children who can count on from a given numeral, suggesting that they are not using information provided by counting to map numeral phrases onto multiplication. However, the results of Experiment 4b indicate that the majority of children tested, and 69% of m-learners, are able to perform simple arithmetic in order to produce hundred in the proper form when asked directly. Together, these results suggest that some children, primarily m-learners, between 4.5 and 6.5 years old are able to correctly map the numeral phrase “one hundred” to multiplication before they can effectively count up to one hundred.

General Discussion

A set of four experiments tested whether children make the connection between numeral phrases and multiplication using nothing but their combinatorial structure. The results showed that while some children will succeed regardless of syntactic information, the phrase structure gives a significant boost to understanding. Children had no difficulty accepting a digit followed by a novel word as a numeral, and m-learners were able to determine it was a numeral phrase (with multiplier structure) based on limited experience with it. Further experiments were inspired by two major theories on the origins of the mapping between numeral phrases and multiplication. These experiments were designed to determine what tools or skills are needed before the connection between numeral phrases and multiplication is made. The results show that the ability to count groups is a prerequisite, yet not sufficient (Experiment 1); explicit instruction in the form of learning the place value system likely does not contribute (Experiment 3); and relatively limited experience is required in using multipliers while counting (Experiment 4). How then do these results support or refute the two theories under investigation?

Fuson's theory is that experience with explicit instruction and real world objects is required to link complex numerals to their meaning. This theory is not supported by the evidence presented. The first instance where children are typically explicitly taught to decompose complex spoken numerals is while learning the place value system (Fuson, 1990), yet by and large children in this age group do not understand that each written digit in a multi-digit number counts powers of ten. This suggests that children learn to decompose spoken numerals and to map numeral phrases to multiplication before they learn the equivalent in the written system. The other method Fuson puts forth is through experience in using numerals in a variety of physical contexts. If this were the case, we would expect age effects in our data. The age range tested is quite large, spanning the two years between 4.5 and 6.5 years old. If experience manipulating objects in the world was a primary factor in mapping syntax to meaning, we should see older children more likely to be m-learners than younger children, yet age was not a factor in determining whether a child was an m-learner or not. Instruction and experience do not appear to determine children's understanding of the mapping between syntax and meaning, indicating that Fuson's theory is not representative of how children map numeral phrases to multiplication.

In contrast, Hurford's theory is that the link between numeral phrases and multiplication is similar to other aspects of language learning. That is, children have expectations that their language will contain numeral phrases that represent multiplication, and need only learn the specific words their language uses. While these experiments cannot speak to whether this mapping is innate, they do mirror studies of general language learning. Of the children tested here, m-learners required very little experience with the novel complex numeral in order to determine that it referred to a multiplicative structure. This is similar to the novel word learning study by Brown (1957) wherein preschoolers were given the novel word "sib" in a verb, count noun, or mass noun structure. From this limited exposure they were able to make assumptions about the type of word "sib" was by matching it with an appropriate action, object, or substance. As with these novel numeral studies, the context and structure of the phrase was able to give the children the tools to map this new word to a meaning. The similarity between the results of Brown and those presented here are in line with Hurford's theory.

Learning numerals like multipliers does mirror general language learning in the way that Hurford proposes. However, the additional claim that this connection between the multiplier syntax and group meaning is innate is not entirely supported by the data. The Counting Groups task in Experiment 1 showed that most children, regardless of learner status, could correctly create groups by number. We know from previous research that assigning words that refer to groups is a difficult concept for children. It is a skill that must be learned, and as this data shows, is a skill that is mastered before learning the connection to the meaning of multipliers.

In summary, some children are able to learn about multipliers without ever being explicitly taught to do so and with limited experience, yet all are able to reorganize objects into groups before they make this connection. Neither of the previous theories account for this pattern of data, leaving room for alternative proposals.

Alternative Proposals

One possibility is that children may have an innate expectation that their language will contain multipliers, but until they have learned about object grouping, and specifically learned that words can refer to groups of individuals rather than the individuals themselves, they are not sensitive to this syntax. We know that adults separate numerals into units, decades, and multipliers. In this scenario, children may classify all numerals as the same until they have learned to use grouping words and only then begin to differentiate between different types of numerals.

A second theory is that the nature of English artificially delays learning multipliers. Children may have the capacity to make the connection between multipliers and their meaning immediately upon learning to count groups, yet as they need to learn the numeral phrase "one hundred" before they encounter the multiplier structure this skill does not manifest until children have progressed into using the numeral "hundred" in their own speech. The lack of transparency of the decade terms would not allow children to use this knowledge of counting groups to help them. If this were the case, Chinese- or Japanese-speaking children would be able to interpret a novel multiplier much earlier than English-speaking children, as those languages uses a transparent multiplier system immediately upon reaching ten.

There is no singular explanation for the data presented, and further studies are required to determine if either of the proposed alternatives explains how children learn the meaning of multipliers. All that can clearly be said from this series of experiments is that structure of the complex numeral itself provides an important role in the learning of numerals. The structure of numerals is not simply a tool to express quantity but acts as a representation of number itself, and studying how we learn the linguistic aspects of our number system allows us to better understand how we conceptualize number.

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