Probabilistic Assessment of Common Cause Failures in Nuclear Power Plants

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Common cause failures (CCF) are a significant contributor to risk in complex technological systems, such as nuclear power plants. Many probabilistic parametric models have been developed to quantify the systems subject to the CCF. Existing models include the beta factor model, the multiple Greek letter model, the basic parameter model, the alpha factor model and the binomial failure rate model. These models are often only capable of providing a point estimate, when there are limited CCF data available. Some recent studies have proposed a Bayesian approach to quantify the uncertainties in CCF modeling, but they are limited in addressing the uncertainty in the common failure factors only.

This thesis presents a multivariate Poisson model for CCF modeling, which combines the modeling of individual and common cause failures into one process. The key idea of the approach is that failures in a common cause component group of n components are decomposed into superposition of k $(> n)$ independent Poisson processes. Empirical Bayes method is utilized for simultaneously estimating the independent and common cause failure rates which are mutually exclusive. In addition, the conventional CCF parameters can be evaluated using the outcomes of the new approach. Moreover, the uncertainties in the CCF modeling can also be addressed in an integrated manner. The failure rate is estimated as the mean value of the posterior density function while the variance of the posterior represents the variation of the estimate. A MATLAB program of the Monte Carlo simulation was developed to check the behavior of the proposed multivariate Poisson (MVP) model. Superiority over the traditional CCF models has been illustrated.

Furthermore, due to the rarity of the CCF data observed at one nuclear power plant, data of the target plant alone are insufficient to produce reliable estimates of the failure rates. Data mapping has been developed to make use of the data from source plants of different sizes. In this thesis, data mapping is combined with EB approach to partially assimilate information from source plants and also respect the data of the target plant. Two case studies are presented using different database. The results are compared to the empirical values provided by the United States Nuclear Regulatory Commission (USNRC).

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To

My Family

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List of Abbreviations and Notations

AF	Alpha Factor
BF	Beta Factor
BFR	Binomial Failure Rate
BN	Bayesian Network
BP	Basic Parameter
CCCG	Common Cause Component Group
CCF	Common Cause Failure
CPF	Conditional Probability Function
CV	Coefficient of Variation
EB	Empirical Bayes
EDG	Emergency Diesel Generator
FO	Failure to Open
G-V-EB	Vaurio's EB with Grouping Method
HPP	Homogeneous Poisson Process
ID	Influence Diagram
JS	James-Stein
LER	Licencee Event Report
MBF	Multiple Beta Factor
MGL	Multiple Greek Letter
MOV	Motor-operated Valve
MP	Multivariate Poisson
MSE	Mean Squared Error
NUREG	Nuclear Regulatory
PDF	Probability Density Function
UPM	Unified Partial Method
USNRC	United States Nuclear Regulatory Commission

Chapter 1
Introduction

1.1 Background

A nuclear power plant is a kind of complex technological system that requires high level of safety in operation. Enormous and severe consequences that impact the public safety may be caused by the failure of the plant. In order to achieve the high reliability of the system, current nuclear power plants employ the redundancy design. Probabilistic risk assessment (PRA) is a methodology to analyze the risk associated with a complex technological system.

If a failure event cannot be expressed by a product of independent failures of the components, it is classified as the dependent failure. A special type of dependent failure is named as common cause failure (CCF) because the multi-component failure event is caused by a shared root cause. CCF is the reason why at least two components become unavailable simultaneously (Börcsök and Holub, 2008). There are four broad types of root causes: hardware equipment failure, human error during operation, environmental stress applied to components, and external events that causes environmental effects (Mosleh *et al*., 1989). Most of the common causes can be classified into one of the above. The CCF stemming from the first type root cause is also known as cascading failure or component-caused CCF. The rest can be simply considered as externally caused CCF (Vaurio, 2007). The NUREG series guides for the CCF analysis only take the latter type into consideration and independent failure is regarded to result from internal component failure mechanisms (Mosleh *et al*., 1989). In this thesis, CCF is externally caused if not specified.

Consider for example a *two-out-of-three* (or 2/3) system as shown by Figure 1.1, which is a common redundancy design in nuclear power plants. The *two-out-of-three* system fails if at least two of the three components are in the failure state. This kind of redundancy design improves overall system safety as the failures of an individual component do not cause a system failure. This type of failure is called independent failure, denoted by the component name with a subscript I such as A_I. The actual improvement in system safety can be easily evaluated, if the component failures are considered to be stochastically independent to each other.

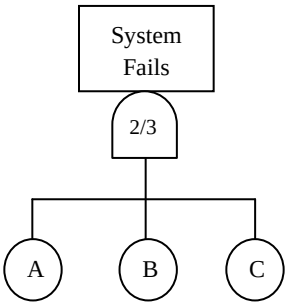

Figure 1.1:2-*out-of*-3 system without CCF

In this case, the Boolean expression of the system failure is given as

$$S = (A_I \cap B_I) \cup (A_I \cap C_I) \cup (B_I \cap C_I) \cup (A_I \cap B_I \cap C_I) \quad 1.1$$

However, based on the observations in redundant systems, one component does not only fail independently but also simultaneously with other components due to the same root cause within the same group. This type of failure is CCF as defined above. Let the capital letter C denote common cause failure and subscript denote the components involved in the failure event. Therefore, there exist many other failure scenarios for each component. For example, the failure of component A can be decomposed into four possible situations rather than the independent one.

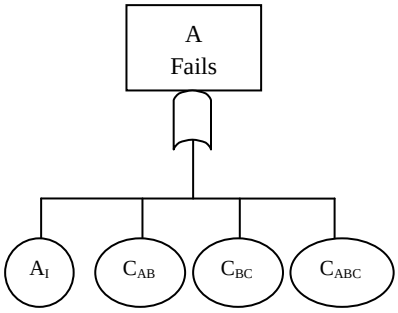

Figure 1.2: Failure scenarios of the component A

According to the analysis above, the system failure is expressed by Equation 1.2, rather than Equation 1.1 above.

$$S = (A_I \cap B_I) \cup (A_I \cap C_I) \cup (B_I \cap C_I) \cup (A_I \cap B_I \cap C_I) \cup C_{AB} \cup C_{BC} \cup C_{AC} \cup C_{ABC} \quad 1.2$$

However, the above expression is still arguably correct in practice because it is difficult to classify the multi-component events. For instance, when A and B fail simultaneously, one may not be able to

tell, if historical operation logs are unavailable, whether the event is a CCF event or an event that two independent failures happen to occur simultaneously. Therefore, it is necessary to propose a clear mutually exclusive classification system for the possible failure scenarios. Unfortunately the present literatures cannot meet such a requirement. Meanwhile, since the multiple independent failures do not frequently occur simultaneously, the multi-component failures such as C_{AB}, C_{BC}, C_{AC} and C_{ABC} are the main contributors to the system failure. Hence, it is important to take the multiple component failures into account and quantify them such that a more accurate and reliable failure rate can be given to the plant operators.

There have been several parametric models developed for the CCF analysis up to now such as the beta factor model, multiple Greek letter model and alpha factor model. These models are easy to understand and calculate, and hence widely utilized in North America. However, they are highly dependent on certain simplifying assumptions and rely on plentiful data collected from nuclear plants, assumptions which are unrealistic and data which are often unavailable. In the UK, another evaluation procedure called utilizes unified partial method (UPM) is adopted for CCF analysis. The UPM requires lots of expert judgments that many professionals do not feel comfortable to exercise. The field failure data may not be recorded in such specific and detailed manner as the UPM expects. It is thus desirable to develop a new modeling framework for CCF that is easy to understand and straightforward in parameter estimation.

In the probabilistic assessment of common cause failure, two types of problems are considered, i.e., failure to start and failure to run. The first one usually occurs in dormant/standby safety devices or systems whose functioning is often triggered by certain external events. The probability that a standby safety system fail to start when triggered is simply called the probability of failure to start. In contrast, the second type of failure occurs when an element or system becomes unavailable during operation. It is often quantified by failure rate. This thesis focuses on the estimation of failure rates of components in the nuclear systems.

1.2 An illustrative example

Consider a redundant system with three components, as shown in Figure 1.3. The failure of an i^{th} component is modeled as HPP with the failure rate λ_i .

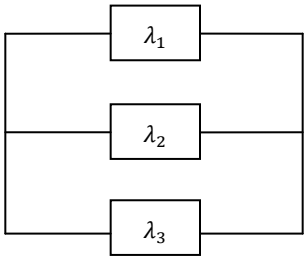

Figure 1.3: A parallel system with 3 components

And the probability of failure $P_i(T)$ and reliability $R_i(T)$ of the component i in the time interval T are

$$P_i(T) = 1 - e^{-\lambda_i T} \tag{1.3}$$

$$R_i(T) = e^{-\lambda_i T} \tag{1.4}$$

1.2.1 One-out-of-three system

Let's define a *one-out-of-three* system here, which means at least one component should be reliable in order to assure that the system is reliable. The system failure block diagram is shown below.

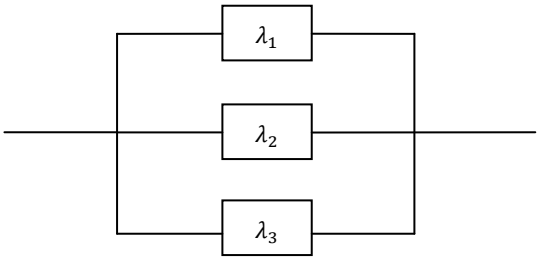

Figure 1.4: System failure block diagram given *one-out-of-three* configuration

This system only involves independent failures of the components. In order to fail the system, all the three components should be failed. Based on the above criteria, the probability of failure of the system is

$$P_s(T) = P_1 P_2 P_3 = \left(1 - e^{-\lambda_1 T}\right)\left(1 - e^{-\lambda_2 T}\right)\left(1 - e^{-\lambda_3 T}\right) \tag{1.5}$$

Since the symmetry assumption is often adopted in the CCG, all the components would be considered as similar/identical. In this case, their failure rates for each type are the same. Let $\lambda_{k/3}$ denote the component level rate of a specific *k*-component failure event, e.g., $\lambda_{1/3} = \lambda_1 = \lambda_2 = \lambda_3$.

And let $\Lambda_{k/3} = \binom{3}{k}\lambda_{k/3}$ denote the total failure rate of *k*-component failure event in the system, for example, $\Lambda_{1/3} = \lambda_1 + \lambda_2 + \lambda_3$. Then the above Equation 1.5 could be simplified as

$$P_s(T) = \left(1 - e^{-\lambda_{1/3}T}\right)^3 \tag{1.6}$$

Therefore, the reliability of the above system is

$$\begin{aligned} R_s(T) &= 1 - \left(1 - e^{-\lambda_{1/3}T}\right)^3 \\ &= 3e^{-\lambda_{1/3}T} - 3e^{-2\lambda_{1/3}T} + e^{-3\lambda_{1/3}T} \end{aligned} \tag{1.7}$$

However, due to the existence of CCF, the multi-component failures are not only caused by multiple independent failures, but also by CCFs in most cases. Therefore, the system failure block diagram is modified as shown below.

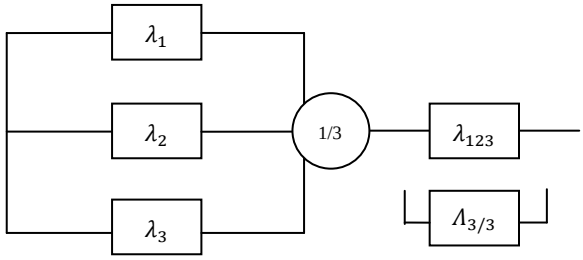

Figure 1.5: System failure block diagram given *one-out-of-three* configuration

Since four CCF terms have been added to the diagram in series, in order to ensure that the whole system is reliable, each part should be reliable. Hence, the new reliability of the whole system becomes

$$\begin{aligned} R_s'(T) &= R_s(T)\left(e^{-\lambda_{123}T}\right) \\ &= \left(3e^{-\lambda_{1/3}T} - 3e^{-2\lambda_{1/3}T} + e^{-3\lambda_{1/3}T}\right)\left(e^{-\lambda_{3/3}T}\right) \end{aligned} \tag{1.8}$$

1.2.2 Two-out-of-three system

Similarly, define a *two-out-of-three* system, which means at least two reliable components can assure the reliability of the system. Therefore, the cutsets for failing the system are shown in the figure below.

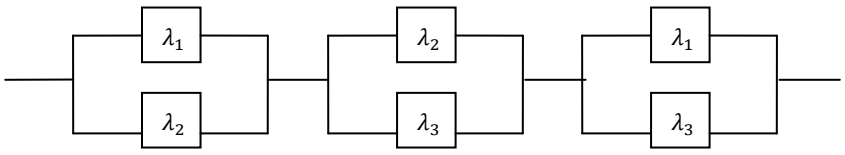

Figure 1.6: System failure block diagram given *two-out-of-three* configuration

According to the symmetry assumption, the reliability of the system without CCF is

$$R_s(T) = 3R_1^2P_1 + R_1^3 = 3e^{-2\lambda_{1/3}T}\left(1 - e^{-\lambda_{1/3}T}\right) + e^{-3\lambda_{1/3}T}$$
$$= 3e^{-2\lambda_{1/3}T} - 2e^{-3\lambda_{1/3}T} \qquad 1.9$$

Taking account of CCF, the system failure block diagram is modified as shown below.

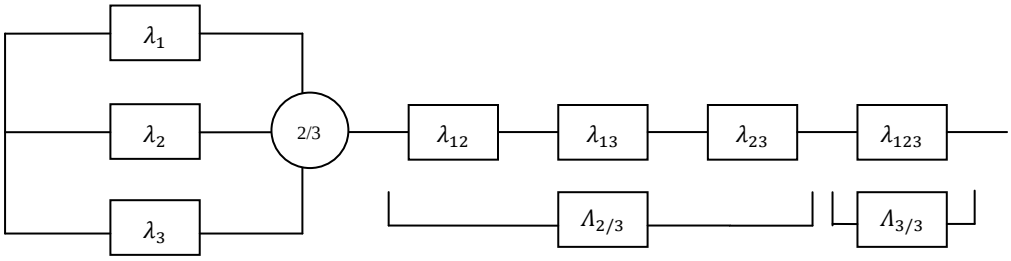

Figure 1.7: System failure block diagram given *two-out-of-three* configuration

Then the reliability of the system with CCF becomes

$$R_s'(T) = R_s(T)\left(e^{-\lambda_{12}T}\right)\left(e^{-\lambda_{13}T}\right)\left(e^{-\lambda_{23}T}\right)\left(e^{-\lambda_{123}T}\right)$$
$$= \left(3e^{-2\lambda_{1/3}T} - 2e^{-3\lambda_{1/3}T}\right)\left[e^{-(3\lambda_{2/3}+\lambda_{3/3})T}\right] \qquad 1.10$$

1.2.3 Three-out-of-three system

Again, assume a *three-out-of-three* system, meaning that all the component must be working to make sure the system is safe. In other words, any type of failure event will fail the whole system. Since it is difficult to calculate all the failure scenarios, one can compute the reliability of system without CCF as

$$R_s(T) = R_1^3 = e^{-3\lambda_{1/3}T} \tag{1.11}$$

Taking account of CCF, four CCF terms are added to the system failure block diagram. And the reliability becomes Equation 1.12.

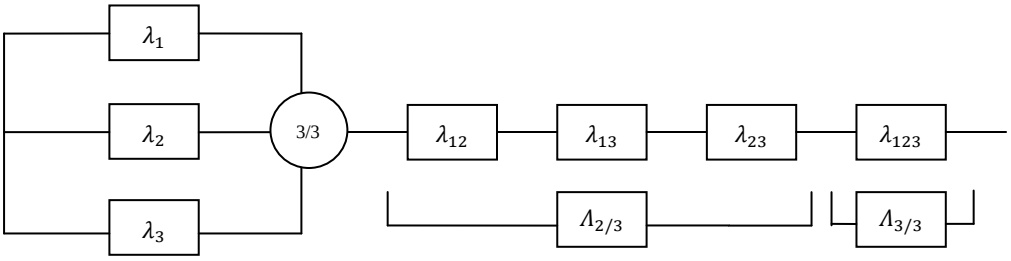

Figure 1.8: System failure block diagram given *three-out-of-three* configuration

$$\begin{aligned} R_s'(T) &= R_s(T)\left(e^{-\lambda_{12}T}\right)\left(e^{-\lambda_{13}T}\right)\left(e^{-\lambda_{23}T}\right)\left(e^{-\lambda_{123}T}\right) \\ &= \left(e^{-3\lambda_{1/3}T}\right)\left(e^{-(3\lambda_{2/3}+\lambda_{3/3})T}\right) = e^{-(3\lambda_{1/3}+3\lambda_{2/3}+\lambda_{3/3})T} \end{aligned} \tag{1.12}$$

1.2.4 Numerical example of 3-component system

Some failure data are collected from the nuclear power plants. The component is motor-operated valve (MOV) and failure mode is 'fail to open' (FO). Operation time is 216 months. Both the independent and common cause failures are listed in the table below.

Table 1.1: Failure rates in a 3-component system

k	1	2	3
$\lambda_{k/3}$	0.04321	0.001543	0.00463

Substituting the above failure rates into the Equations 1.7 to 1.12 and assuming further operation time is 100 (month), one can obtain the reliability of the *k-out-of-three* system in two different situations. The reliabilities of system for each type are calculated without/with CCF. Based on the results listed in the table below, it is obvious that without taking account of CCF, the system reliability is too much optimistic. This is why the CCF must be accurately evaluated.

Table 1.2: Reliability of a *k-out-of*-3 system

System type	Reliability of system	
k-out-of-3	Without CCF	With CCF
1-*out-of*-3	3.93×10^{-2}	2.48×10^{-2}
2-*out-of*-3	5.25×10^{-4}	2.08×10^{-4}
3-*out-of*-3	2.35×10^{-6}	9.29×10^{-7}

1.3 Motivations

Although various methods are available for the probabilistic risk assessment of the CCF, a few problems remain. First of all, the present parametric models being used are strictly dependent on some specific assumptions which simplify the computation and application but are unrealistic to some extent. For example, the beta factor model assumes only one type of common cause failure that all the components fail as long as a common cause occurs. This is practical and effective for two-component group while impractical in situations where the group size is more than two (Kumamoto and Henley, 1996). Moreover, all these traditional models presume that the components within the same group are identical. However, according to the various component manufactories, environments, maintenance procedures and other factors, it is better to consider them as different individuals. This treatment enables analysts to evaluate a specific component's behavior.

Second, failure data in nuclear power plants are very limited even though the nuclear industry has had more than 60 years of operation history since the construction of the first commercial nuclear reactor. Most of the existing CCF models introduced provide a point estimates of the failure rates of interest. However, it is not convincing to use these estimations to represent the real unknown failure rates of the components. From a statistical point of view, the point estimator is accurate only when the sample size is large enough. Moreover, there may be no failure of a specific scenario observed in a certain time interval. Despite the lack of historical failure, it is obviously inappropriate to think that the events will never happen in the future and a non-zero failure rate is required for the future prediction. Therefore, it is an urgent industrial need that an uncertainty interval be provided.

Third, to make up for the rarity of data in the target nuclear system, the analysts and researchers have adopted a straightforward method called data mapping in order to make use of data collected from source plants of different group sizes, the procedures of which have been presented in

NUREG/CR-4780. Based on the traditional data mapping, Kvam and Miller (2002) suggest that the operation times from source plants be modified and rates be calculated from the sum of failures over the sum of modified times. The inherent assumption is that these failures are produced from a single Poisson process and the uncertainty in each plant rate estimate can be minimized, which may be against the practical observations. In addition, to make the mapping up more accurate, the extra parameters are estimated using empirical alpha factors provided by NUREG/CR-5497. Also, the Bayesian method is recommended but not addressed in detail (Vaurio, 2007). Therefore, the commonly used data mapping can be further improved using some statistical methods so that higher effectiveness can be achieved.

1.4 Objectives

Regarding the drawbacks of existing models and the scarcity of data in nuclear power plants, it is essential to propose new solutions to solve the problems and to yield more reliable estimates for the failure rates of the components.

This thesis applies a multivariate Poisson (MVP) model for the CCF modeling which distinguishes the components within the common cause component group (CCCG). Failures observed in the CCCG are decomposed into the superposition of several independent homogeneous Poisson processes (HPP). Empirical Bayes method is adopted to improve the estimation accuracy achieves a lower sum of squared error (SSE). In this way, all the possible failure scenarios of a specific component can be quantified, and the factors of traditional parametric models, such as the alpha factors, are easy to be obtained using the outcomes of the MVP model. This approach makes use of other components' information within the same CCCG to resolve the limitation of data of a specific component within a nuclear power plant.

Besides, since the current data mapping approaches are arguably useful in one way or another, empirical Bayes method will be combined with the traditional data mapping, which makes it relatively more rational to assimilate information from source plants for the target plant. This can be seen as an action to make use of other plants of different sizes by the means of statistics.

1.5 Organization

This thesis is divided into six chapters including this introductory one. In Chapter 2, the current approaches for CCF modeling are reviewed. Chapter 3 introduces statistical methods that can be applied to the estimation of the failure rates in nuclear power plants. One of the methods proposed by

different academics is regarded as optimal after some numerical examples and will be adopted in the chapters later. In chapter 4, concept and algorithm of the MVP model are described in detail. Monte Carlo simulation is utilized to evaluate the performances of different methods of modeling CCF. Chapter 5 presents a comparison between the combination of conventional data mapping with statistical method and the existing mapping approaches. An illustrative case study is included to explain the effectiveness of the recommended combination. In the last chapter, the main outcomes of the thesis are concluded and the remaining challenges need further investigations are pinpointed. The numerical calculation and Monte Carlo simulation are implemented using Excel spreadsheets or MATLAB functions developed by the author.

Chapter 2
Literature Review

This chapter first reviews the existing literature on the probabilistic models for the CCF assessment. Since most of the models have drawbacks one way or another, it is necessary to be familiar with the concepts, advantages and disadvantages in order to propose superior solutions.

Furthermore, data mapping, as a common approach to make use of the data from source plants, is considered unsatisfactory for giving point estimates of failure rates with large uncertainties. Therefore, a comprehensive understanding of the theory is essential so that some improvement measures can be introduced in the Chapter 5.

2.1 Parametric common cause failure models

A variety of parametric CCF models have been developed in the past decades(Fleming, Mosleh and Deremer, 1986) such as the beta factor model, multiple Greek letter model, alpha factor model, just name a few. Some of them are easy to adopt and cover all the possible failure scenarios that happened in the nuclear industrial practice. However, an unavoidable fact is all of the models are highly dependent on their own assumptions, which is quite arguable regarding the reality. In the next sections, those methods will be introduced in details and given critical reviews respectively.

2.1.1 Beta factor (BF) model and its generalization

The BF model is one of the earliest for CCF modeling and originated from a report written by Fleming K. N. (1974). This method is presented to calculate the reliability of redundant system for the failure to start mode at first, and then extended to the failure to run mode.

The BF model assumes that all the components fail when common cause occurs in a group of n components (group size is n). There are only two types of failures meaning that the probability of failure (or failure rate) of one component is either λ_1 due to independent causes or λ_n due to common causes. The key part in the model is to introduce a fraction of failure β_s corresponding to CCF as defined in Equation 2.1. Then the total failure probability of a component λ_{total} consists of two parts as shown in Equation 2.2.

$$\beta = \frac{\lambda_n}{\lambda_n + \lambda_1} = \frac{N_n}{N_n + (N_1/n)} \qquad 2.1$$

$$\lambda_{total} = \lambda_n + \lambda_1 = \frac{N_n + (N_1/n)}{N} \quad 2.2$$

The data required in the BF model are:

N_1 = Number of independent failure events in the group.

N_n = Number of failure events that simultaneously involve n components in the group.

N = Number of total demands applied to the group.

For the case of estimating failure rates, rather than probability of failure, only the number N should be replaced by the total system operation time T.

The simplifying assumption of the BF model makes it easy to represent the failure consequences and to compute with the sole factor β defined. However, when the group size n is larger than 2, the assumption is obviously against the practical situations because the CCF is not necessarily involving all the components within the group. Besides, as a common defense application to prevent CCF, diverse components are often used in a CCCG. Then the number of independent failures of a particular component it is not the same as those of other components, i.e., $N_{1,1} \neq N_{1,2} \neq N_1/n$. And the data are not enough to support the point estimation. Sometimes the Bayesian theorem is recommended to solve the problem of lacking data while additional assumption of distribution is often brought in (Kumamoto and Henley, 1996).

By the time when the analysts began to assess the CCF, the database available from the industry such as the Licensee Event Report (LER) did not count independent and multiple failures clearly. The application of the BF model met the obstacle of poor quality of data when quantifying the β factor. Evans *et al*. (1983) presented a novel angle at which the dependent failure rate can be estimated. This method utilizes the same assumption of the BF model that a component fails either independently or simultaneously with all the other ones. The difference lies in the parameter definitions. The occurrence rate of common cause failure is regarded as a proportion of that of the independent failure by a factor $P_c(\lambda_n = P_c\lambda_1)$. Since the independent failure rate is relatively easy to evaluate from the database, the total failure rate and CCF rate can be calculated.

The proposed approach by Evans *et al*. gives an easy way to make use of the data of poor quality compared to the BF model. Unfortunately, the inherent limitations of the model's assumption such as lack of data, inaccuracy of point estimation, and overmuch assumption, still exist and the calculation result is conservative.

In order to extend the BF model to more general cases in which multiple ($1 < k < n$) components may fail at the same time. The multiple beta factor (MBF) model has been developed (Hokstad and Rausand, 2008). A set of factors β_i are defined, each of which means the probability that a specific train $i + 1$ fails given the trains 1 to i have failed. β_1 is the original parameter β for two trains. The probability of failure of a group containing m trains is calculated from the Equation 2.3 below.

$$Q_{koon} = C_{koon}\beta\lambda_{total} \tag{2.3}$$

The subscript $koon$ refers to a k-out-of-n configuration meaning that at least k trains failing at the same time will fails the whole system. Hokstad derives the expression of the key factor C_{koon} using the set of β_i values and gives some examples of small size groups for comparison. Since it is much more complicated to obtain C_{koon} when n is large (Hokstad, 2004), a simplification has been adopted, which assumes that $\beta_i = \beta_3 (i \geq 3)$. A base case is introduced as $\beta_2 = 0.3;\ \beta_i = 0.5$ when $i \geq 3$.

This MBF model generalizes the basic BF model to the groups of larger sizes and enables all the possible failures existing in the nuclear industry. The factor β_i can be seen as a portion of β_{j-1} which makes sense because from qualitative investigation, an event involving three trains' failure can certainly be regarded as one involving two trains' failure while a two-train failure may not necessarily affect a third train. Hence, the dependency between β_i and β_{i-1} ensures that the proportions of 2 to *n*-component failures do not vary greatly from each other.

However, since the configuration factors vary greatly given different beta values and these values are usually assumed according to the expert judgment regarding the past experience. It is critical to choose appropriate values in order to obtain a realistic prediction. This, unfortunately, is not included in the paper and is supposed to be paid more attention to in the further research. Furthermore, the assumption of conditional probability is robust to some extent and the simplification case lacks large quantities of practical data to support.

2.1.2 Multiple Greek letter (MGL) model

The MGL model is the most general extension of the BF model (Fleming and Kalinowski, 1983). A set of different parameters are defined according to the ratio of multi-component CCFs as shown in the followings. From these definitions below, it can be seen that the MGL model completely becomes the BF model for the 3-component group if $\gamma = 1$. More parameters can be assumed if necessary.

λ_{total} = Total failure rate for each component.

β = Conditional probability that a component's failure is shared by one or more additional components, given that the former component fails.

γ = Conditional probability that a component's failure is shared by two or more additional components, given that the former component fails together with one or more additional components.

In order to estimate the parameters, the data required are the same as those for the BF model. The equations are shown below.

$$\lambda_{total} = \frac{1}{nT}\sum_{i=1}^{n} iN_i \qquad 2.4$$

$$\beta = \sum_{i=2}^{n} iN_i \Big/ \sum_{i=1}^{n} iN_i \qquad 2.5$$

$$\gamma = \sum_{i=3}^{n} iN_i \Big/ \sum_{i=2}^{n} iN_i \qquad 2.6$$

The advantages of the MGL model lie in that it covers all the possible failures as the MBF model does and that the factors are directly estimated from the data rather than dependent on robust assumptions like the MBF model. The drawback is that the data may not be sufficient to ensure the accuracy of many factors especially when the group size is large. Moreover, a problem of over-parameterization occurs as the group size increases to a high level.

2.1.3 Basic parameter (BP) model

The BP model is either time-based (to calculate failure rates) or demand-based (to calculate probability of failure) according to Kumamoto and Henley (1996). The parameter λ_i is failure-to-operate rate for a particular group of $i(1 \leq i \leq n)$ components. Considering a three-component system for example, the rate of events involving exactly two components is thus $3\lambda_2$. From symmetry assumption that the probabilities of failures involving the same number of components are equal (e.g. $\lambda_{1,1} = \lambda_{1,2} = \lambda_{1,3}$), the independent and common cause failure rates depend only on the numbers of failures of different types. The required data are listed below and the formula of failure rate is given in Equation 2.7 (Fleming *et al*., 1986; Mosleh *et al*., 1989).

$$\lambda_i = \frac{N_i}{\binom{n}{i} T} \quad 2.7$$

T = Total operation time of the system.

N_i = Number of failure events involving exactly i components in failed states.

The basic parameter (BP) model is a straightforward approach to adopt with simple concepts. Since the components in various trains are tested differently and the testing scheme affects the CCF probabilities, it is necessary to investigate the formulas of CCF factors under different testing schemes. Hwang, M. J. and Kang, D. I. (2011) develops the calculation equations of CCF factors within a CCCG under the staggered testing scheme, non-staggered testing scheme and mixed testing scheme based on the BP model. It is a significant generalization of the BP model that makes the method more applicable in the CCF modeling.

2.1.4 Binomial failure rate (BFR) model

The binomial failure-rate (BFR) model is specialized from a general model proposed by Marshall and Olkin (1967). Due to the different causes, failures are divided into two types, namely independent failures and nonlethal shocks. The occurrence of the nonlethal shock follows the Poisson process with a rate μ. Each component has a constant failure probability of p under the shocks and the failure distribution is binomial (Vesely, 1977). Afterwards, Atwood developed the BFR model further and introduced lethal shock with rate ω. The basic parameters are shown below:

λ_I = Independent failure rate for each component.

μ = Occurrence rate for nonlethal shocks.

p = Probability of failure for each component under nonlethal shocks. $(q = 1 - p)$

ω = Occurrence rate of lethal shocks.

The relations hold between the BP and BRF models as follows:

$$\lambda_1 = \lambda_I + \mu p q^{n-1} \quad 2.8$$

$$\lambda_k = \mu p^k q^{n-k} \quad 2.9$$

$$\lambda_n = \mu p^n + \omega \quad 2.10$$

A distinguished feature of this model is that the total number of parameters remains constant regardless of the number of components. Each event is classified as lethal or nonlethal (including independent cause) shock (Kumamoto and Henley, 1996). However, μ cannot be estimated directly from the data recorded because nonlethal shocks do not necessarily cause visible failures. Only shocks that cause at least one component failure are counted. Hence, the rate of visible nonlethal shocks, μ_+, is regarded as a basic parameter instead of μ. Then the expected total number of failures caused by nonlethal shocks is

$$N_{non} = \mu Tnp = \frac{\mu_+ Tnp}{1-q^n} = \frac{N_+ np}{1-q^n} \tag{2.11}$$

Then the probability p can be calculated by solving Equation 2.11. The other parameters are easy to estimate following the equations below

$$\lambda_I = \frac{N_I}{nT} \tag{2.12}$$

$$\mu_+ = \frac{N_+}{T} \tag{2.13}$$

$$\omega = \frac{N_L}{T} \tag{2.14}$$

where

N_I = Number of single-component failures due to independent cause.

N_+ = Number of nonlethal shocks causing at least one component failure.

N_L = Number of lethal shocks.

This BFR model assumes a binomial distribution for the failure mechanism which describes the failure situations by the means of mathematics. Unfortunately, the practical situation is a black box that no one can predict. Even the components within a CCCG are not necessarily equally subjected to a nonlethal shock due to different location, internal structure, lifetime, manufactory, etc. Therefore the probability p may not be a constant for all the components.

2.1.5 General multiple failure model

General multiple failure model is based on the ratio of multi-component CCFs. Factors, $\alpha_{1/n}, \alpha_{2/n}, \ldots, \alpha_{n/n}$, correspond to CCF events of different orders. The alpha factors are easy to calculate and often used in practice. First, define some parameters as follows where $k = 1,2, \ldots, n$.

$N_{k/n}$ = Number of failure events involving k components in the n-component system.

T = Operation time of the n-component system.

$\lambda_{k/n}$ = Rate of CCF events failing specific k components in a n-component system.

$\Lambda_{k/n}$ = Total rate of CCF events failing exactly k components in a n-component system.

Λ_n = Total rate of failure events occurred in the system.

$\alpha_{k/n}$ = Alpha factor for k/n event, ratio of parameters.

Based upon the above definitions, the relations among the parameters are illustrated as follows.

$$\Lambda_{k/n} = \binom{n}{k} \lambda_{k/n} = \frac{N_{k/n}}{T} \tag{2.15}$$

$$\Lambda_n = \sum_{k=1}^{n} \binom{n}{k} \lambda_{k/n} = \sum_{k=1}^{n} \Lambda_{k/n} \tag{2.16}$$

$$\alpha_{k/n} = \frac{\Lambda_{k/n}}{\Lambda_n} = \frac{N_{k/n}}{\sum_{k=1}^{n} N_{k/n}} \tag{2.17}$$

From the definitions of the alpha factors, it can be seen that they are solely dependent on the ratios of multiple failures. All the possible failures can be evaluated using the parameters obtained from the data. If the database is not large enough, the parameter estimation is not likely to be reliable. The method of data mapping can be used to utilize data from other plants. This topic will be addressed in a later Section.

2.1.6 Markov model

All the models above are developed for non-repairable systems. As to the repairable systems, Markov transition diagrams are used with appropriate parameters: common cause occurrence rates $\lambda_1 \sim \lambda_n$ and repair rate μ (Platz, 1984).

Let P_i, a function of time, denote the probability of the state i. The probability can transit from state i to state j. A transfer matrix A consisting of $\lambda_1 \sim \lambda_n$ and μcan be formed, the element of which is a_{ij}.The probability of transiting to state j during the intervaldt is $a_{ij}dt$. Then differential equations for P_i can be written in the form of matrix. Then the probability P_i can be calculated by solving the differential equations.

$$P_i(t + \Delta t) = AP_i(t)dt \quad 2.18$$

This thesis does not concentrate on solving differential equations as the Markov model does, although it is widely applied to describe the dependencies among the CCFs in redundant systems.

2.1.7 Case study

Data regarding diesel generator (CCCG = 4) failures are summarized in Table 2.1 (Becker *et al.*, 2009). Observation time is 2.91E6 (the unit is unknown in the report). CCF models discussed previously will be applied to compare their performance. Since BF model does not consider *k* out *n* failures, it is not included in the case study. In addition, Markov model is not employed either because no further information is available indicating that the systems are repairable or the failures are time-related. First, α factors are calculated following Equation 2.15 to 2.17. The results are listed in the table below.

Table 2.1: Failure data of diesel generators in SKI Report 2009:07.

k	1	2	3	4
$N_{k/4}$	3	19	2	8
$\alpha_{k/4}$	0.09375	0.59375	0.0625	0.25

Second, the MGL, AF, BP, and BFR models are compared for estimating the *k-out-of-*4 failure rate. By following the formulas of each model, it is not difficult to obtain the component level and system level failure rates. Simplified expressions for the *k-out-of-*4 system level failure rates are able to obtain as well. The estimation of the system failure rates are listed below.

An interesting phenomenon shows up that the MGL, AF and BP models produce the same estimates. This result stems from the fact that parameters of MGL and AF models are defined by those of BP model, and that in order to model the CCF, the basic parameters λ_i still need to be

calculated even for the former two models. Besides, the BFR model generates a higher estimate for the 4-*out-of*-4 failure rate. That is because non-lethal shock also contributes to the lethal failure with a probability of p.

Table 2.2: Failure rates of *k-out-of*-4 events calculated by different models.

Model	$k = 1$	$k = 2$	$k = 3$	$k = 4$
MGL	1.10E-05	9.96E-06	3.44E-06	2.75E-06
AF	1.10E-05	9.96E-06	3.44E-06	2.75E-06
BP	1.10E-05	9.96E-06	3.44E-06	2.75E-06
BFR	1.09E-05	8.94E-06	6.61E-06	3.91E-06

2.2 Unified partial method (UPM) and influence diagram (ID) extension

Since the parametric models introduced above are not always accurate due to the over-simplifying assumption and the lack of available data respectively, the UK nuclear industry are currently adopting the unified partial method (UPM) which is based on the beta factor model and the cut-off methodology to assess the CCF in a novel way.

2.2.1 Unified partial method (UPM)

In the UPM common cause method (Zitrou and Bedford, 2003), the defenses against CCF are broken down into 8 sub-factors such as Operator Interaction, Redundancy, Analysis, etc. Each sub-factor has 5 levels of strength (A, B, C, D, E) accompanied by corresponding scores from low to high.

From the database, a beta factor for a specific system i can be calculated using the following equation,

$$\beta_i = \frac{N_{i\_e}}{N_{i\_c}} \tag{2.19}$$

where N_{i_e} and N_{i_c} are the numbers of common cause events and failed components respectively. Next, a linear regression model as shown in Equation is assumed to investigate the relationships between the beta factor and the scores (x_{ij}) corresponding to the eight sub-factor levels.

$$\beta_i = w_1 x_{i1} + w_2 x_{i2} + \cdots + w_8 x_{i8} \tag{2.20}$$

After the weights (w_i) have been determined, the dependencies of failure rate on the sub-factors are also revealed. And the failure rate of the system λ_i is assumed to be the sum of the eight scoresx_i.

The advantage of the UPM is its ease of implementation framework. The steps are clear to follow and the calculation is not difficult; a number of tables are enough to obtain the scores of the sub-factors for each system. However, the data required such as N_{i_e} and N_{i_c}, and x_i are not available at most times because not many countries are utilizing this method currently. The failure events and data may not be recorded in such a detail as the UPM expects and the scores have to be determined by the experts following some guides and past experience. Besides, any change of the design and age of the system are likely to require rescoring the system. A lot of qualitative analysis is required when analysts are assigning the scores. Moreover, the assumed linear relationship between failure rate and the scores needs further investigation. The levels of certain sub-factors may change the other factors' impacts on CCF probability according to Zitrou and Bedford (2003).

2.2.2 Influence diagram (ID) extension

The Bayesian network (BN) model is a directed acyclic graph in which the nodes represent random variables and the directed arrows always point from parents to children. All the conditional probability functions (CPFs) for the variables are written in such a conditional way: $f(x_i|pa(x_i))$ where $pa(x_i)$ means the parents of a variable x_i have already happened. It is necessary to choose an appropriate family of distribution for each CPF and assign values to the parameters. In this process, empirical data and/or expert judgments may be utilized (Langseth and Portinale, 2007).

An obvious advantage of the BN model lies in the flexibility of the formulation. The parent-child conditional structure enables analysts to add possible causes of failures into the model. What's more, since the number of probabilities is reduced significantly, the data or expert judgments required are reduced which are usually expensive and difficult to obtain. The difficulty of the method is that the analyst should get inputs from the real experts because of the complexity of the networks. All the possible dependencies of the variables should be considered in order to reflect the practical situations. This makes it rather difficult to construct a BN structure.

As an extension of the Bayesian network, influence diagram (ID) can be used to modify the UPM (Zitrou, Bedford and Walls, 2004). The ID contains different types of nodes (variables) such as decision node, uncertainty node and value node, which is more detailed compared to the BN. It has two major advantages in the CCF defense analysis. First, the ID graphically portrays the

dependencies among the defenses and the influences on the CCF probability. Second, the expert judgments can be taken into account together with the statistical data.

The assumption made in the model is that the CCF event occurs in an independent homogeneous Poisson process (HPP) with rate γ_i and hence the overall failure rate λ is the superposition of them. It is also assumed that the parameters of rates are uncertain variables rather than unknown constants.

There are two factors causing CCFs, (i.e., root cause and coupling mechanisms, three defense actions are proposed aimed at reducing the frequency of root causes, or against the coupling mechanisms, or both. And the domain variables are classified to three types related to different objectives, namely defenses, root causes and coupling factors. Three scenarios are illustrated: a), decision variable describes the defense against the occurrence of root cause; b), decision variable describes the defense against the coupling factor; and c), decision variable describes the defense against both of them.

Zitrou *et al*. (2007) further introduce the ID extension of the UPM afterwards. Two advantages are shown in the application of the ID, namely modeling non-linear dependencies amongst the defenses and taking account for expert judgments as an important source. First, functional interactions between any two defenses can be modeled by assuming parameters to represent impacts on a certain variable between different levels of the defenses. The relationships may be functionally independent, functionally dependent and threshold functionally dependent. Second, when constructing the ID network, every expert is expected to draw the relationships between defense, root cause and coupling mechanism variables individually. Then all the opinions are incorporated and disagreements can be discussed within the panel.

One of the most important advantages of the ID is the capability of representing various kinds of information (e.g., root cause, coupling factor, defense, rate, etc.) by different types of variables. Besides, the dependencies between various defenses can be taken into account. Moreover, expert judgments are valued as an important resource. However, different interpretations, personal experience and ambiguities in definitions of the defenses may lead to different understandings of the interactions. This will lead to several rounds of reflections so that the experts in the panel can reach an agreement in the end.

2.3 Traditional data mapping

Due to the rarity of failure data in nuclear power plants, it is essential to combine those from other component groups of various sizes. It is the data mapping that enables making use of the event data of a different size system. In practice, each event can be reinterpreted based on the components' status in the event such as complete failure, degraded, incipient, or working (Becker *et al*., 2007). The outcome of the reinterpretation is impact vector which is a set of probabilities of the possible failure scenarios which is the objectives of data mapping (Mosleh *et al*., 1989). Meanwhile, since the mapping process is purely mathematical, the utilization of the method is not only limited to the impact vector, but also applicable for many other kinds of data, such as number of failures, failure rates or even some CCF factors (Vaurio, 2007).

Suppose a system of two MOVs is to be assessed but available data exist from another system of three MOVs. Then the estimated failure rates or number of failures of the 2-MOV CCCG depend on which specific two out of the three MOVs in the source group are selected (Kvam and Miller, 2002). Therefore, the data mapping is a purely mathematical approach based on the symmetry assumptions of the components such as identical design, separation and maintenance (Vaurio, 2007).

Furthermore, since the size of source plant is not necessarily equal to the one of target plant, mapping can be specifically classified into two categories, mapping down and mapping up corresponding to whether the outer plant size is larger or smaller than the target. As to the plants of the same size, a simplifying assumption that they are identical contributes to easy computation and hence no more mapping is needed, unless convincing evidence shows that they are not identical in the performance. Since the thesis refers to externally caused CCF only, which is the majority of CCF at the present, the formulas of both the mappings are derived for the CCF of interest in the following.

2.3.1 Mapping down

The objectives of the mapping methods are the failure rates (it can also be number of failures) including both independent and common caused. As the same as before, they are denoted by $\lambda_{k/n}$ which mean the rate of failure events simultaneously involving specific k components within a CCCG of size n. The figure below illustrates the process of mapping failure rates from a system of size 2 to the rate of size 1 (Vaurio, 2007). Based on the previous definitions of component level and system level failure, we can come up with the relations $\Lambda_{2/2} = \lambda_{2/2}$, $\Lambda_{1/2} = 2\lambda_{1/2}$ and $\Lambda_{1/1} = \lambda_{1/1}$.The mapped rate of interest is calculated from Equation 2.21.

$$\Lambda_{1/1} = \frac{1}{\binom{2}{1}}\Lambda_{1/2} + \Lambda_{2/2} \tag{2.21}$$

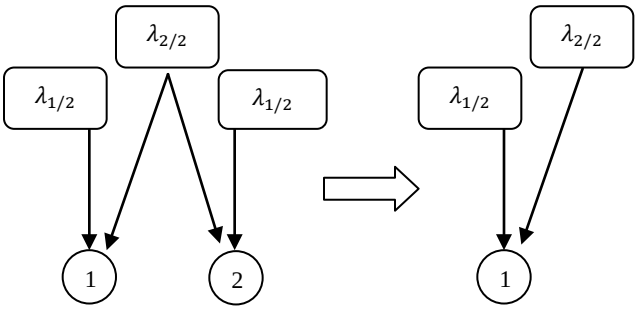

Figure 2.1: Mapping down CCF rates from *n* = 2 to 1.

When this method is applied to larger systems, for example, mapping from known CCF rates of size n to the rates of size $n-1$, the general equation is easy to find as shown in Equation 2.22. Its simplified expression is given in Equation 2.23.

$$\frac{1}{\binom{n-1}{k}}\Lambda_{k/n-1} = \frac{1}{\binom{n}{k}}\Lambda_{k/n} + \frac{1}{\binom{n}{k+1}}\Lambda_{k+1/n} \tag{2.22}$$

$$\Lambda_{k/n-1} = \frac{n-k}{n}\Lambda_{k/n} + \frac{k+1}{n}\Lambda_{k+1/n} \tag{2.23}$$

Sometimes the size distance is equal or larger than 2. By the means of substitution, it is easy to derive the desired formulas, e.g., mapping from n to $n-2$ using Equation 2.24.

$$\frac{1}{\binom{n-2}{k}}\Lambda_{k/n-2} = \frac{1}{\binom{n}{k}}\Lambda_{k/n} + \frac{2}{\binom{n}{k+1}}\Lambda_{k+1/n} + \frac{1}{\binom{n}{k+2}}\Lambda_{k+2/n} \tag{2.24}$$

Kvam and Miller (2002) derive a more generalized form of traditional mapping down formula without any substitutions in the calculation shown in Equation 2.25. NUREG/CR-4780 gives several mapping down formulas commonly used which are also listed in Table 2.3 below. They also verify the derived equations above.

$$\Lambda_{k/n'} = \sum_{r=1}^{n} C_{k,r}\Lambda_{r/n} \quad for\ (k = 1, \dots, n') \tag{2.25}$$

$$where \quad C_{k,r} = \frac{\binom{r}{k}\binom{n-r}{n'-k}}{\binom{n}{n'}} \quad (k \le r \le n - n' + k)$$

Table 2.3: Formulas for mapping down failure rates for systems of size 1 to 4.

		Size of system mapping to		
		3	2	1
Size of system mapping from	4	$\Lambda_{1/3} = \frac{3}{4}\Lambda_{1/4} + \frac{1}{2}\Lambda_{2/4}$ $\Lambda_{2/3} = \frac{1}{2}\Lambda_{2/4} + \frac{3}{4}\Lambda_{3/4}$ $\Lambda_{3/3} = \frac{1}{4}\Lambda_{3/4} + \Lambda_{4/4}$	$\Lambda_{1/2} = \frac{1}{2}\Lambda_{1/4} + \frac{2}{3}\Lambda_{2/4} + \frac{1}{2}\Lambda_{3/4}$ $\Lambda_{2/2} = \frac{1}{6}\Lambda_{2/4} + \frac{1}{2}\Lambda_{3/4} + \Lambda_{4/4}$	$\Lambda_{1/1} = \frac{1}{4}\Lambda_{1/4} + \frac{1}{2}\Lambda_{2/4} + \frac{3}{4}\Lambda_{3/4} + \Lambda_{4/4}$
	3		$\Lambda_{1/2} = \frac{2}{3}\Lambda_{1/3} + \frac{2}{3}\Lambda_{2/3}$ $\Lambda_{2/2} = \frac{1}{3}\Lambda_{2/3} + \Lambda_{3/3}$	$\Lambda_{1/1} = \frac{1}{3}\Lambda_{1/3} + \frac{2}{3}\Lambda_{2/3} + \Lambda_{3/3}$
	2			$\Lambda_{1/1} = \frac{1}{2}\Lambda_{1/2} + \Lambda_{2/2}$

2.3.2 Mapping up

According to the derivations, it is clear that mapping down is rather deterministic, which means that given a set of failure rates of a larger system, the ones of a smaller system are straightforward to be estimated by following the formulas presented above. However, when it comes to mapping up, the estimation of failure rates from a small system to a larger one is not deterministic and extra uncertainties are unavoidable.

Before mapping up, recall several concepts introduced in the BFR model, which are independent failures, nonlethal and lethal shocks. The first type refers to the failure event in which only one component is failed and no correlations with others are shown. Since the number of independent failure is simply proportional to the size of the CCCG, it is reasonable to map the failure rates from small groups to large groups solely based on their sizes. In contrast, the lethal shock occurs resulting

in all the n components being in the state of failure regardless the size of the system. In other words, a CCCG is regarded as a whole and the lethal shock then follows a constant occurrence rate when impacting each of the CCCGs. In addition to the above two, the nonlethal shock causes multiple, more than 1 and less than n, components to fail. Each component is likely to fail with a probability of ρ, given a nonlethal shock which has a constant occurrence rate impacting a CCCG. Therefore, when the analysts map the failure rates up from small to large size, the uncertainty lies in the estimation of the parameter ρ, the probability of failing one more hypothetical component attached to the current system. Table 2.4 reveals the commonly used formulas of mapping up when the size is from 1 to 4.

Table 2.4: Formulas for mapping up failure rates for systems of size 1 to 4.

		Size of system mapping to		
		2	3	4
Size of system mapping from	1	$\Lambda_{1/2} = 2(1-\rho)\Lambda_{1/1}$ $\Lambda_{2/2} = \rho\Lambda_{1/1}$	$\Lambda_{1/3} = 3(1-\rho)^2\Lambda_{1/1}$ $\Lambda_{2/3} = 3\rho(1-\rho)\Lambda_{1/1}$ $\Lambda_{3/3} = \rho^2\Lambda_{1/1}$	$\Lambda_{1/4} = 4(1-\rho)^3\Lambda_{1/1}$ $\Lambda_{2/4} = 6\rho(1-\rho)^2\Lambda_{1/1}$ $\Lambda_{3/4} = 4\rho^2(1-\rho)\Lambda_{1/1}$ $\Lambda_{4/4} = \rho^3\Lambda_{1/1}$
	2		$\Lambda_{1/3} = \frac{3}{2}(1-\rho)\Lambda_{1/2}$ $\Lambda_{2/3} = \rho\Lambda_{1/2} + (1-\rho)\Lambda_{2/2}$ $\Lambda_{3/3} = \rho\Lambda_{2/2}$	$\Lambda_{1/4} = 2(1-\rho)^2\Lambda_{1/2}$ $\Lambda_{2/4} = \frac{5}{2}\rho(1-\rho)\Lambda_{1/2} + (1-\rho)^2\Lambda_{2/2}$ $\Lambda_{3/4} = \rho^2\Lambda_{1/2} + 2\rho(1-\rho)\Lambda_{2/2}$ $\Lambda_{4/4} = \rho^2\Lambda_{2/2}$
	3			$\Lambda_{1/4} = \frac{4}{3}(1-\rho)\Lambda_{1/3}$ $\Lambda_{2/4} = \rho\Lambda_{1/3} + (1-\rho)\Lambda_{2/3}$ $\Lambda_{3/4} = \rho\Lambda_{2/3} + (1-\rho)\Lambda_{3/3}$ $\Lambda_{4/4} = \rho\Lambda_{3/3}$

The calculation process of mapping up is straightforward to implement. However, the parameter ρ is assumed for all the scenarios in all the system sizes, which is quite arguable regarding the practical situation. Furthermore, the transition of the assumption from the BFR model to mapping up computation is not necessarily reflecting the reality all the time. This assumption is robust and the inherent failure mechanism is complicated and never known to the analysts.

2.4 Newly developed data mapping

Based on the introduction of data mapping above, it can be seen that the tradition method has several drawbacks. First, the data of each resource group can be mapped to a set of rates of the target group size. But the newly mapped sets of failure rates are rather different from each other. This fact relates to the fact that the configurations and designs of diverse size systems are usually inconsistent. Hence the mapping process simply based on probability is arguable because of the defect of inherent assumptions. Second, since the data of the source plants are not necessarily enough, it is a promising attempt to assimilate each other's information by some sort of means, other than taking the average of them. Third, though this thesis does not focus on the component-caused CCF, it is still of interest to extend the data mapping from the external caused CCF to this type. In the following sections, some recently developed approaches will be addressed.

2.4.1 Data mapping proposed by Kvam and Miller

Kvam and Miller (2002) propose a novel angle from which the database can be enlarged by making use of other source plants in the data mapping. In the traditional method, the failure rates of the large system are transformed down to the smaller target one by modifying the rates directly according to the number of components in each system. In further detail, the times of operation are fixed to the practical values while the numbers of failures are mapped based on the ratio of source size to target size. In contrast, the proposed approach is to modify time intervals given that the numbers of failures are known and unchanged. This is an inverse process of the traditional one. The core assumption is that all the failures are recorded from a typical system following a homogeneous Poisson process (HPP) in different time intervals. Therefore, by dividing the summation of numbers of failures of different plants by the summation of their times, one can obtain a more reliable result with greatly enlarged time of operation compared to the one calculated from the individual plant itself.

An example is given accompanied to the description of the method (Kvam and Miller, 2002). Numbers of the different types of failures and the operation times for each size system are listed in Table 2.5. The failure rate type indicates the failure scenario k/n.

Table 2.5: Common cause failure data for EDGs

Size	Failure Type	Number	Time
2	1/2	17	75
	2/2	14	75
3	1/3	9	50
	2/3	5	50
	3/3	6	50
4	1/4	11	50
	2/4	10	50
	3/4	7	50
	4/4	6	50
5	1/5	2	10
	2/5	2	10
	3/5	1	10
	4/5	2	10
	5/5	1	10

Suppose the 2-component group to be the target and the rate under study is $\Lambda_{2/2}$ as indicated in the table. For each of the sizes, a mapped time is calculated by the means proposed by Kvam and Miller. The mapped result $\Lambda_{2/2}$ can be obtained by the total number of failure over total modified time which turns out to be 0.0610. The computation results are shown in Table 2.6. For comparison, the traditional mapping down is also utilized to generate the target rate $\Lambda_{2/2}$ for each size listed in the right column.

The new approach holds an original inverse view on the data from CCCGs of various sizes compared to the conventional mapping down process. The strong and robust assumption extends the time interval to a large magnitude and makes the result seem more reliable. However, the internal assumption may be fundamentally against the nature and hence leads the mapping result to be completely wrong. Mathematically, the four systems are likely to be following different HPPs because the traditional mapped $\Lambda_{2/2}$ for them are not identical (0.1867, 0.1533, 0.2233, 0.27). The typical HPP with a mean failure rate of 0.0610 is rare to generate four samples whose mean rates are

much larger than the true one. And as a matter of fact, it is irrational to believe that identical configuration is designed for each of the systems. This fact makes the assumption less trust worth. Anyway, if there is further convincing information to support typical HPP assumption, or the group sizes are close, then this method of mapping failure rates is still reasonable to apply.

Table 2.6: Transformed time on test for EDG failure data

Size	k/n	Number	Time (year)	Mapping Modifier	Modified Time (year)	Traditional Mapped $\Lambda_{2/2}$
2	1/2	17	75	0	0	
	2/2	14	75	1	75	0.1867
3	1/3	9	50	0	0	
	2/3	5	50	0.3333	150	
	3/3	6	50	1	50	0.1533
4	1/4	11	50	0	0	
	2/4	10	50	0.1667	300	
	3/4	7	50	0.5	100	
	4/4	6	50	1	50	0.2233
5	1/5	2	10	0	0	
	2/5	2	10	0.1	100	
	3/5	1	10	0.3	33.3333	
	4/5	2	10	0.6	16.6667	
	5/5	1	10	1	10	0.2700

2.4.2 Data mapping proposed by Vaurio

Vaurio (2007) derives the mapping up and down equations corresponding to both types of CCFs, external caused CCF and component-caused CCF, respectively. The results of external caused CCF are consistent with Table 2.3 and Table 2.4 which have been given by Mosleh *et al*. (1989).

Mapping up from $n-1$ to n involves a new parameter $\eta_{n-1,n}$. The value of this parameter is unknown but can be estimated from empirical alpha factors. The various empirical alpha factors for different types of components, failure modes and system sizes turn out to be different as well (Marshall, Rasmuson and Mosleh, 1998). Compared to the traditional mapping up process which utilizes a single extra parameter ρ, the introduction of $\eta_{n-1,n}$ is making more sense and does not completely rely on expert judgments as ρ does. Besides, Vaurio gives a suggestion about treating

plants of same size, Bayesian update. This, however, requires a sufficient database and is rather complicated, while a wiser choice is to consider them as identical ones. This treatment sums the failures and observation times respectively and hence makes the database larger and greatly simplifies the calculations (Vaurio, 2008). Moreover, based on the mapping equations of failure rates and the definitions of alpha factor and $\eta_{n-1,n}$, the alpha factors can also be mapped up/down amongst plants of different sizes. In the end, although it is unnecessary to apply Bayesian method to the systems of the same size, it is worth an attempt to extend the Bayesian update to the ones of different sizes. This will make use of other sources and at the same time respect the data of the target its own.

2.5 Summary

According to the reviews above, there are two aspects that need further investigation, proposing an advanced model to assess the reliability of a system of CCCG and borrowing data information from other size systems in order to enrich the database under study.

Most of the existing parametric models rely on strong assumptions one way or another. These assumptions are not always following the unknown nature of the failure. When the CCCG size becomes large, some models such as the MGL model have to estimate many parameters, which are faced with a problem of lacking enough data. The UPM and its ID extension are good applications to the CCF modeling but lots of reliable expert judgments are required to ensure the accuracy of the results. That makes it difficult for a less experienced analyst to adopt them in practice.

Since the CCF data are sparse, there is a demand to make good use of the data from other plants which are often of different sizes. The traditional data mapping generates a group of failure rates from the source plants of various sizes to the target plant. But there are no further instructions about how to merge these mapped rates from different sizes. Also, the assumed parameter for mapping up should not be a constant. The recently developed methods have some defects of inherent assumption (Kvam and Miller, 2002), or lack a clear solution to make use of the data from various plants (Vaurio, 2007). Hence, there need to be a new approach to solve the existing problems.

Chapter 3
Statistical Approaches for Failure Rate Estimation

Since the existing CCF models only generate point estimates for the failure rates and common cause factors, it is necessary to explore some superior approaches which give confidence intervals or even the distributions for the factors to be evaluated. Furthermore, assimilating data information from other components, systems or plants is an effective measure to make up the drawbacks of sparse CCF data. These requirements can be met by the means of statistical approaches. This chapter introduces the maximum likelihood estimate (MLE) of the failure rate, James-Stein estimator and its application to the failure rate estimation, and then the commonly utilized empirical Bayes method and the developments for the topic under study. Two numerical examples will be given to investigate the similarity and diversity of these methods and to choose the optimal one to be adopted in this thesis.

3.1 Maximum likelihood estimate (MLE)

The maximum likelihood estimate is one of the simplest methods in statistics. In the case of failure to run, the parameter needs to be estimated is failure rate, which is the ratio of number of failures and operation time. Failure to run event can be modeled by a homogeneous Poisson process (HPP). Some agreements are made: a) the occurrence of failure is independent identically distributed, b) repair time is negligible and c) the failure rate follows exponential distribution (Pandey and Jyrkama, 2012). According to Crowder *et al.*(1991), the MLE and its standard error are

$$\hat{\lambda}_i = \frac{N_i}{T_i} \tag{3.1}$$

$$\sigma(\hat{\lambda}_i) = \sqrt{\frac{N_i}{{T_i}^2}} \tag{3.2}$$

The above approach is defined as MLE in this thesis and estimates the failure rate of a single component. If more failure data are available from other components, there are many other statistical methods to make use of outer components' data to estimate the target one. James-Stein estimator and empirical Bayes method are two of them and will be introduced later in this chapter.

3.2 James-Stein estimators

3.2.1 Basic concepts

The James-Stein (JS) estimator was first proposed (James and Stein, 1961) to solve multivariate normal distribution problem. The estimator is written in terms of an overall average and the distance from the maximum likelihood estimate (MLE) to the average multiplied by corresponding fractions. By the means of mathematical transformation, the estimates of the different components, expressed in the form of $\lambda_i = B_i\mu + (1 - B_i)N_i/T_i$, are shrunk from the MLE $\hat{\lambda}_i = N_i/T_i$ towards an overall average μ.The effect of shrinkage is reflected by the factors B_i which are related to the variances of the normal random variables.

An example of baseball hit rate was used (Efron and Morris, 1975) to show the shrinking effect of the JS estimator. Numbers of hits and tries for 18 players early in 1970 season are listed in the second and third columns of Table 3.1 below. The JS estimator is utilized to predict each player's true hit rate which can be taken as the average throughout the whole season. It can be seen that the JS estimator always gives the estimation results between the overall average and the MLE. To illustrate more clearly, the calculation results are plotted in Figure 3.1.

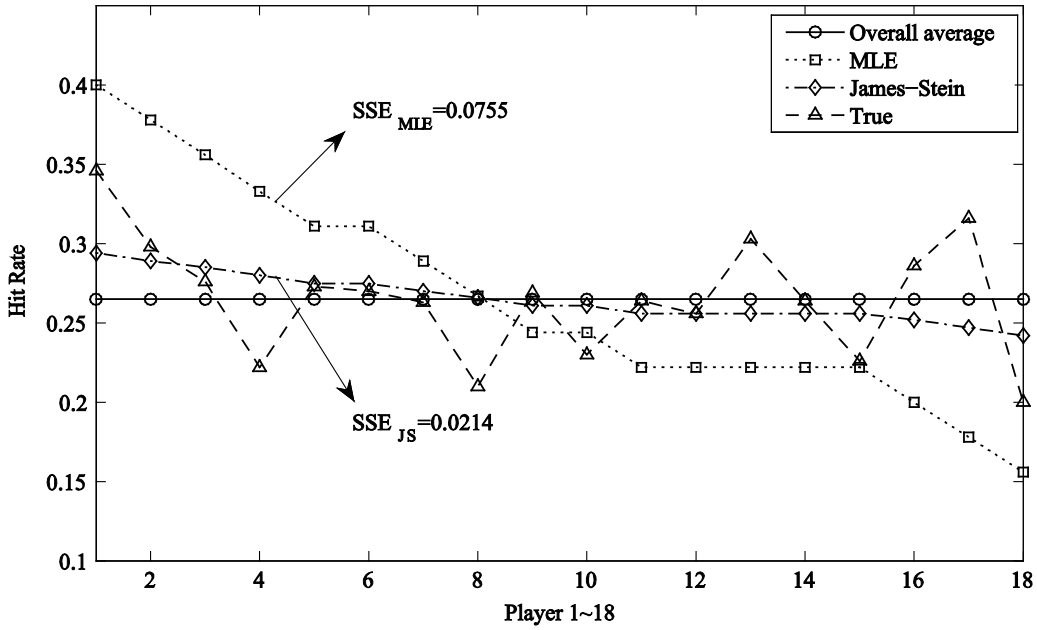

Figure 3.1: Example of baseball hit rate

James-Stein estimator dominates the MLE in terms of expected total squared error with a lower (or at least equal) risk than that of the MLE. This method is more accurate than the method of MLE because it always produces results between the overall mean and point estimates. It takes account of other components' data as a larger database instead of using its own. The sums of squared errors for the MLE and James-Stein estimator are 0.0755 and 0.0214 respectively, which validate the superior to the MLE.

Table 3.1: Example of baseball hit rate

Player	Hits	Tries	Overall average	MLE	James-Stein	True
1	18	45	0.265	0.4	0.294	0.346
2	17	45	0.265	0.378	0.289	0.298
3	16	45	0.265	0.356	0.285	0.276
4	15	45	0.265	0.333	0.28	0.222
5	14	45	0.265	0.311	0.275	0.273
6	14	45	0.265	0.311	0.275	0.27
7	13	45	0.265	0.289	0.27	0.263
8	12	45	0.265	0.267	0.266	0.21
9	11	45	0.265	0.244	0.261	0.269
10	11	45	0.265	0.244	0.261	0.23
11	10	45	0.265	0.222	0.256	0.264
12	10	45	0.265	0.222	0.256	0.256
13	10	45	0.265	0.222	0.256	0.303
14	10	45	0.265	0.222	0.256	0.264
15	10	45	0.265	0.222	0.256	0.226
16	9	45	0.265	0.2	0.252	0.286
17	8	45	0.265	0.178	0.247	0.316
18	7	45	0.265	0.156	0.242	0.2

3.2.2 Vaurio's James-Stein estimator

The James-Stein estimator is not only limited to normal or binomial variables, but also applicable to some other unknown distributed variables. Vaurio and Jänkälä (1992) have derived the JS estimator for failure rates based on Poisson data.

At first, the authors assume that the$n$ failure rates λ_i are independent realizations of a random variable λ with a probability density of $\pi(\lambda)$. The form of the distribution is not necessary to be

known. Afterwards, through the moment method, the expected mean and variance of the MLE for each of the components are derived in terms of population mean μ and variance σ^2as shown below.

$$E(\lambda_i) = \mu \tag{3.3}$$

$$Var(\lambda_i) = \frac{\mu}{T_i} + \sigma^2 \tag{3.4}$$

In order to better estimate the population mean and variance, the authors propose an unbiased weighted average estimator of μ, denoted by m.

$$m = \sum_{i=1}^{n} w_i \frac{N_i}{T_i} \tag{3.5}$$

The mean and variance of this estimator are then calculated as Equation 3.6 and 3.7.

$$E(m) = \mu \sum_{i=1}^{n} w_i \tag{3.6}$$

$$Var(m) = \sum_{i=1}^{n} {w_i}^2 \left(\sigma^2 + \frac{\mu}{T_i}\right) \tag{3.7}$$

By minimizing $Var(m)$ under the constraint of $\sum_{i=1}^{n} w_i = 1$, a set of optimal weights are obtained as shown in

$$w_{io} = \frac{1}{\mu/T_i + \sigma^2} \left(\sum_{j=1}^{n} \frac{1}{\mu/T_i + \sigma^2} \right)^{-1} \tag{3.8}$$

The population mean and variance in the equation above can be substituted by m_o and V_o.

$$m_o = \sum_{i=1}^{n} w_{io} \, N_i/T_i \tag{3.9}$$

$$V_o = \frac{1}{1 - \sum_{i=1}^{n} {w_{io}}^2} \left[S - m_o \sum_{i=1}^{n} \frac{w_{io}(1 - w_{io})}{T_i} \right] \tag{3.10}$$

$$S = \sum_{i=1}^{n} w_{io} (N_i/T_i - m_o)^2 \tag{3.11}$$

Following the iteration procedure from Equation 3.8 to 3.11, the optimal w_i, μ and σ^2 can be obtained until m_o, V_o andw_{io} converge.

The James-Stein estimator of the failure rate is written in the form of

$$\lambda_i = B_i\mu + (1 - B_i)N_i/T_i \tag{3.12}$$

By minimizing the expected mean squared error of the JS estimator for each of the n components, a set of corresponding shrinkage factors B_i are selected.

$$B_i = \frac{\mu}{\mu + \sigma^2 T_i} \tag{3.13}$$

Note that the authors claim that the optimal weights for calculating the population mean and variance are also optimal to minimize the mean squared error for each component. Detailed derivation is given in Appendix A.

An advantage of the JS estimator lies in the capability of achieving lower sum of squared errors (SSE) compared to the MLE. This fact means that the JS estimator is more accurate than the conventional point estimate and will be validated in the numerical example later in this chapter. In addition, this specific approach developed by Vaurio and Jänkälä still produces only an estimate of the failure rate. It is hard to estimate a confidence interval or distribution. A novel point of the JS for estimating failure rates lies in taking the view point of Bayesian statistics more or less, although the JS is regarded as Frequentist statistics at all the times. The difference is that failure rate is an outcome of a population distribution while the form of the distribution does not have to be specified. However, it would be better if one could give a distribution to visually illustrate the variation trend of the failure rates to be estimated. In order to achieve this goal, empirical Bayes would be an appropriate method.

3.3 Empirical Bayes method

Bayesian method is an effective approach to describe the variability of an objective and makes use of the specific data to generate a more reliable distribution named as posterior. In this section, some basic concepts of Bayesian and empirical Bayes (EB) will be introduced. Afterwards, two EB algorithms developed by different scholars are will be presented.

3.3.1 Basic concepts

In Bayesian Method, the unknown failure rate λ is assumed to be produced from a prior distribution $\pi(\lambda)$, which is usually determined according to past experience, expert judgment and so on. The likelihood function of an event for a given failure rate λ can be expressed as $p(x|\lambda)$. The purpose of the method is to calibrate the failure rate distribution with the help of gained data. In order

to achieve this, a marginal distribution density function which has nothing to do with the parameter λ is defined:

$$m(x) = \int p(x|\lambda)\pi(\lambda)d\lambda \tag{3.14}$$

Then the posterior distribution is the updated distribution based on the available data and calculated from the following equation:

$$p(\lambda|x) = \frac{h(x,\lambda)}{m(x)} = \frac{p(x|\lambda)\pi(\lambda)}{\int p(x|\lambda)\pi(\lambda)d\lambda} \tag{3.15}$$

Since the posterior has taken both of the prior judgment and sample data into account, it can be regarded as a calibration of the knowledge of the unknown parameter λ after obtaining the data. Hence, the posterior is believed to be more accurate and closer to the truth (Mao, 1999).

The choice of prior distribution is vitally important for the accuracy of posterior. The posterior distribution can be seen as a compromise between prior and data. If the prior is too far from the reality reflected by data, the posterior may probably have two peaks. If no information is available for prior, it is reasonable to choose a non-informative prior such as uniform prior. Then the posterior solely depends on the data. Moreover, selecting a prior distribution from a family which is conjugate to the likelihood function leads to a posterior distribution belonging to the same family of distribution. The conjugate prior contributes to a convenient calculation of the posterior (Carlin and Louis, 2000). Some common conjugate priors and corresponding likelihood distributions, either discrete or continuous, are listed in Table 3.2. In the reliability assessment field where Poisson data are collected, a gamma prior distribution is usually assumed. A broad consensus has been reached, upon which the EB algorithms developed by different researchers are based.

Table 3.2: Common conjugate prior and likelihood distributions

Likelihood Distribution	Conjugate Prior
Negative Binomial	Beta
Poisson	Gamma
Normal	Normal
Gamma	Gamma
Log-Normal	Gamma/Normal

Empirical Bayes (EB) method in general can be seen as an approximation to a fully Bayesian inference procedure of a hierarchical model in which the hyper-parameters of the prior distribution are estimated from the data per se (Carlin and Louis, 2000), rather than from other sources such as expert judgments. It works the best when there are several 'similar' inferential groups, which are somehow related to each other and nevertheless each has its own unknown feature. For example, in probabilistic risk analysis of nuclear power plants, failure rates of piping at different pipe size are combined together for statistical inference to improve the statistical power. This is different from a simple pooled analysis in which all pipe segments are assumed homogeneous. In EB, the unique nature of each pipe segment is respected while other components of variation are treated and modeled together to improve the inference efficiency.

The EB method has been used to estimate failure rates, although originally not for CCF. Consider a group of n components. For each of them, its failure is modeled by a homogeneous Poisson process with a failure rate which probably is different from that of the other components. Each component has experienced N_i failures in a time interval of length T_i where $i = 1, 2, \ldots, n$. The objective is to estimate an accurate failure rate λ_i for component i. For this purpose, the Bayesian estimation method assumes a gamma prior distribution for the failure rate as

$$\pi(\lambda_i) = \frac{\beta^{\alpha} {\lambda_i}^{\alpha-1} e^{-\beta\lambda_i}}{\Gamma(\alpha)} \quad , \qquad \lambda_i, \alpha, \beta > 0 \tag{3.16}$$

where α and β are the hyper-parameters. The gamma prior is adopted because it is flexible to fit different types of data and conjugate to the Poisson likelihood as

$$L[N_i|\lambda_i, T_i] = \frac{e^{-\lambda_i T_i}(\lambda_i T_i)^{N_i}}{N_i!} \tag{3.17}$$

Hence the corresponding posterior distribution of the failure rate is also a gamma distribution (Carlin and Louis, 2000):

$$p(\lambda_i|N_i, T_i) = \frac{(\beta + T_i)^{(\alpha+N_i)} {\lambda_i}^{\alpha+N_i-1} e^{-(\beta+T_i)\lambda_i}}{\Gamma(\alpha + N_i)} \quad , \qquad \lambda_i, \alpha, \beta > 0 \tag{3.18}$$

This property makes the computation of posterior rather straightforward: the hyper-parameters become $\alpha + N_i$ and $\beta + T_i$. The estimated failure rate can be expressed by the mean of the posterior distribution as a point estimate and the uncertainty is summarized by the variance of the posterior; they are,

$$Mean(\hat{\lambda}_i) = (\alpha + N_i)/(\beta + T_i) \quad 3.19$$

$$Var(\hat{\lambda}_i) = (\alpha + N_i)/(\beta + T_i)^2 \quad 3.20$$

So far, this approach is called fully Bayesian if the hyper-parameters α and β are determined through expert judgment or other procedures that rely on historical data or other information. The above procedure is called empirical Bayesian if the same data of the components are used to estimate the hyper-parameters.

3.3.2 Vaurio's EB method

Vaurio (1987) proposes a procedure to estimate the hyper-parameters using a moment matching method. More specifically, since the mean and variance of the prior are able to be expressed as α/β and α/β^2, if one can calculate them accurately, then α and β are obtained as well.

A set of normalized weight w_i are introduced into the procedure which sum up to one. A weighted average of the sample mean M_0 is calculated by the following equation:

$$M_0 = \sum_{i=1}^{n} w_i N_i/T_i \quad 3.21$$

which is an unbiased estimate of the true mean of prior distribution, denoted as M.

The variance of the sample mean is calculated as

$$V_0 = S + M_0/T^* \quad 3.22$$

where

$$S = 1/(1 - \sum_{i=1}^{n} {w_i}^2) \sum_{i=1}^{n} w_i (N_i/T_i - M_0)^2 \quad 3.23$$

$$T^* = \sum_{i=1}^{n} T_i - max\ (T_i) \quad 3.24$$

in order to avoid the problem caused by identical data that the component with largest observation time has the highest precision while the others do not.

Based on the method of moments, the author concludes that V_0 is a biased estimate of the true variance V and derives the variance of the estimated sample mean as follows.

$$Var(M_0) = V\sum_{i=1}^{n} {w_i}^2 + M\sum_{i=1}^{n} ({w_i}^2/T_i) \quad 3.25$$

The normalized weights are generated by minimizing the variance of sample mean as shown above. The formula is

$$w_i = u_i / \sum_{j=1}^{n} u_j \quad 3.26$$

where

$$u_i = T_i/(T_i + M_0/V_0) \quad 3.27$$

However, since the true values of M and V are never known in practice, one can use the estimates of them, M_0 and V_0, and iterate from Equation 3.21 to 3.27 with a set of initial weights as $w_i = 1/n$. When M_0, V_0 and w_i converge, the iteration can be terminated and the values of M_0 and V_0 can be seen as the real ones of M and V. With the converged prior mean and variance, calculation procedures of α and β have been given as follows according to Vaurio (1987).

$$\beta = M/V \quad 3.28$$

$$\alpha = M^2/V + 0.5\beta/T^* \quad 3.29$$

There is a similarity between the EB and JS estimators. First, transform the posterior mean of the EB estimator into the form of stein estimator.

$$\lambda_i = \frac{\alpha + N_i}{\beta + T_i} = \frac{\alpha}{\beta}\frac{\beta}{\beta + T_i} + \frac{N_i}{T_i}\frac{T_i}{\beta + T_i} \quad 3.30$$

According to the prior distribution, the mean and variance are expressed as $\mathrm{M} = \alpha/\beta$ and $\mathrm{V} = \alpha/\beta^2$. Then the hyper-parameter can be obtained in the form of the statistical characters as $\beta = \mathrm{M}/\mathrm{V}$. Substitute β into the equation above, one can come up with a new expression without hyper-parameters.

$$\lambda_i = M\frac{M/T_i}{M/T_i + V} + \frac{N_i}{T_i}\frac{V}{M/T_i + V} \quad 3.31$$

Based on the derivations given by Vaurio (1987), $\mathrm{M}/\mathrm{T_i}$ can be seen as the variance of likelihood distribution while V is the variance of prior distribution. Therefore, the EB estimator mean value can

be expressed in the form of a balance between the overall average and MLE with weights determined by the magnitudes of prior and likelihood's variances.

$$\lambda_i = M\frac{Var(Likelihood)}{Var(Likelihood)+Var(Prior)} + \frac{N_i}{T_i}\frac{Var(Prior)}{Var(Likelihood)+Var(Prior)} \qquad 3.32$$

Second, recall the formulas of the JS estimator given in 3.12 and 3.13. The estimator is also able to be written as a balance between overall average and MLE as shown in Equation 3.33 below. The fractions in this formula are also proportions taken by likelihood and prior in the total variance based on the derivations in the literature (Vaurio and Jänkälä, 1992) and they are exactly the same with those in the EB mean value expression.

$$\lambda_i = \mu\frac{\mu}{\mu+\sigma^2 T_i} + \frac{N_i}{T_i}\frac{\sigma^2 T_i}{\mu+\sigma^2 T_i} = \mu\frac{\mu/\mathrm{T_i}}{\mu/\mathrm{T_i}+\sigma^2} + \frac{N_i}{T_i}\frac{\sigma^2}{\mu/\mathrm{T_i}+\sigma^2} \qquad 3.33$$

As a conclusion, there is a close connection between the EB and JS estimators, although the JS estimator does not need to specify the exact form of prior distribution of the failure rates.

3.3.3 Quigley-Bedford-Walls (QBW) EB procedure

In order to estimate the rate of derailment, a statistical estimation procedure has been proposed by Quigley, Bedford and Walls (2007). The same HPP and gamma assumptions are made in this approach as those in Vaurio's EB estimator and result in a gamma posterior distribution. For convenience of presentation, it is called QBW procedure hereafter.

Quigley *et al*. treat the data of all the components equally as their numbers of failures and corresponding time intervals are summed up in the calculation procedure. Hyper-parameters, as given in Equation 3.34 are calculated through two middle terms U and W. The method of moment is used (Arnold, 1990) in the estimation process of these middle terms, Equation 3.35. The details are presented in the Appendix B.

$$\alpha = \frac{U^2}{W-U^2} \quad , \qquad \beta = \frac{U}{W-U^2} \qquad 3.34$$

$$U = \frac{\sum_{i=1}^{n} N_i}{\sum_{i=1}^{n} T_i} \quad , \qquad W = \frac{\sum_{i=1}^{n} {N_i}^2 - \sum_{i=1}^{n} N_i}{\sum_{i=1}^{n} {T_i}^2} \qquad 3.35$$

This paper proposes an empirical Bayes method to solve the problem existing in reliability analysis: lack of data. Normally an accurate evaluation cannot be guaranteed unless a lot of data are available.

With the help of this method, researchers can calculate a relatively more reliable estimation of rate of occurrence based on the limited available data. Since common cause failure (CCF) is also a rare event similar to derailment in the paper, it is worth trying to adopt the EB in the estimation of the failure rate by replacing the miles and numbers of derailment events by operation times and numbers of CCFs respectively.

Unlike the Vaurio's EB procedure, the newly developed one (QBW EB procedure) does not distinguish the components under study. As can be seen from the formulas, all the data collected are treated equally. In other words, the QBW EB method is more applicable for identical or similar components.

Since α and β must be positive, the relation $W > U^2$ is supposed to hold all the time. However, given certain industrial data, the requirement is not always satisfied, which means the applicability of QBW EB is somehow dependent on the data. To investigate the percentage of data that can be processed by QBW EB method, Monte Carlo simulation is utilized. Six components are assumed who have the same failure rate (0.001) and operation time (1000). The data are generated from the Poisson process. The percentage of data that fit the algorithm versus the number of trials is plotted in Figure 3.2 below. It can be seen that given large quantities of trials, the percentage finally stabilized at a rather low level.

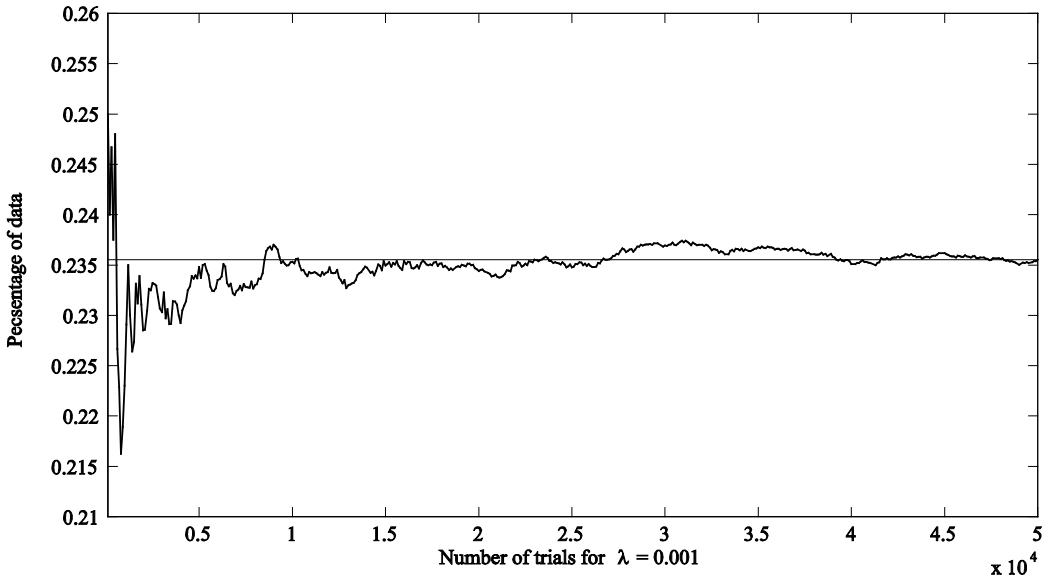

Figure 3.2: Percentage of data fitting QBW EB

Next, change the value of the assumed failure rates to check the relationship between the percentage and the magnitude of rate. Results have been plotted in Figure 3.3 below. If the failure rate is in the range of 0 to 0.01, the percentage of the data fitting QBW's model increases as the rate increase, whereas as the failure rate is larger than 0.01, the percentage is stabilized.

The above simulations are rather arbitrary because the failure rate, operation time, number of trials, number of components in the group are all randomly assumed. Further investigation is essential to reveal the inherent relationships between the percentage and the factors such as a) whether the percentage relates to the number of component in the group, b) the effect of components having different failure rates, and c) the impact of different operation times. However, the improvement of the method is not the focus of the thesis. QBW EB is merely an alternative to evaluate the failure rates of CCFs.

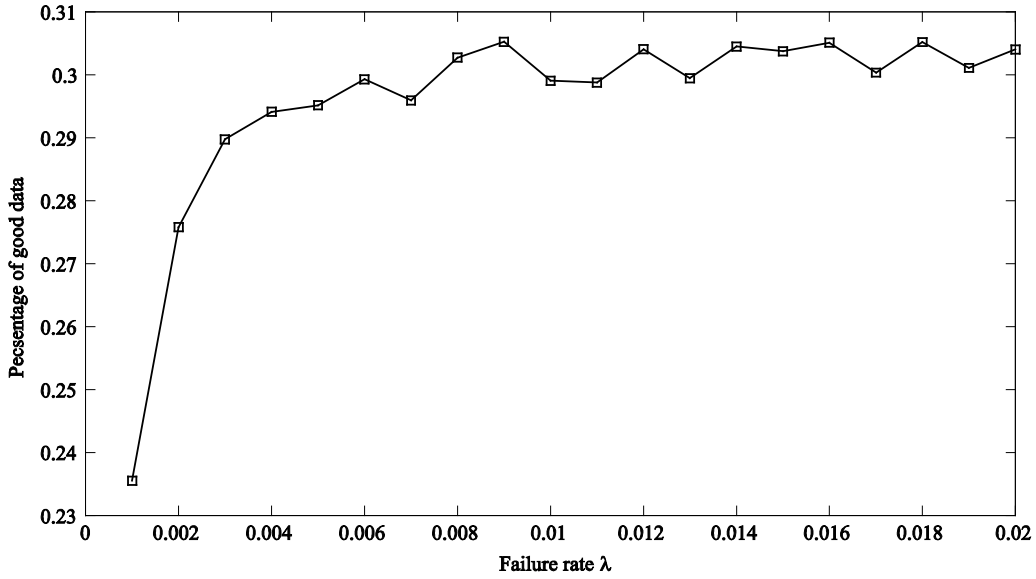

Figure 3.3: Percentage of data versus the value of failure rate.

3.3.4 Uncertainties of hyper-parameters

In practice, the industry prefers a confidence interval of the failure rate rather than just a point estimate. Given a pair of hyper-parameters, by either Vaurio's EB or QBW EB procedure, one can adopt different approaches to estimate the confidence interval.

First, with known α and β values, it is easy to utilize Monte Carlo simulation to generate large quantities of random variables following the specific gamma distribution. After ranking the variables, one can pick up the percentiles corresponding to the confidence level of interest. This method requires large amount of computation and is therefore time consuming if a highly precise interval is to be obtained.

In contrast, one can also utilize the inverse gamma function to analytically calculate the percentiles of interest. It can be proved that this analytical result is consistent with the Monte Carlo simulation result as long as the simulation time is huge enough. Both of the two methods will be utilized in the numerical example later.

However, the above confidence interval obtained is based on the presumption that the empirically calculated α and β are right the true parameters which obviously is not correct. In order to quantify the uncertainty of the failure rate estimation, it is reasonable to count in the uncertainties existing in the hyper-parameter estimation procedure.

3.4 Estimation of zero failure events

Sometimes, no observation is recorded during the time of the study. Then precise estimation may be difficult but a non-zero estimate is still desirable. Some approaches have been proposed to solve the problem of evaluating zero failure events.

Hanley and Lippman-Hand (1983) presents a rule of three for estimating a confidence interval of the zero failure event probability at 95% confidence level. This is a simple approximation based on binomial distribution. When doctors are interested in the unknown probability of cancer of each person (p), they choose a sample of n people, among which no one suffers from cancer. At a 95% confidence level, the zero event probability is

$$(1-p)^n = 0.05 \tag{3.36}$$

$$nln(1-p) = \ln(0.05) \tag{3.37}$$

According to Taylor's theorem, and the fact that natural log of 0.05 is approximately -3, Equation 3.37 can be transformed into the following form

$$n(-p) = -3 \tag{3.38}$$

Therefore, the probability is in a 95% confidence interval from 0 to $3/n$. This rule of three is accurate when the sample size is larger than 30. This means that even no observation appears, one can still get an estimate of the unknown probability at the confidence level of 95%. In the assessment of CCF of the type of failure to start in the nuclear power plants, it can be applied when no CCF is recorded in the total n demands.

As the failure rate is calculated as the ratio of the number of failures and the operation time, the MLE estimate would be zero in this case which is obviously unrealistic. When it comes to the failure rate estimation given zero failure events, the Chi-square distribution can be used to find the confidence intervals from 0 to an upper bound at a confidence interval $100(1-\alpha)$.

$$Pr\left\{\lambda \leq \frac{\hat{\lambda}\left(\chi^2_{2;100(1-\alpha)}\right)}{2}\right\} = 1-\alpha \tag{3.39}$$

$$\hat{\lambda} = \frac{1}{nT} \tag{3.40}$$

Therefore the upper $100(1-\alpha)$ confidence limit for the rate is given as follows.

$$\lambda_{100(1-\alpha)} = \frac{\chi^2_{2;100(1-\alpha)}}{2nT} \tag{3.41}$$

Since Chi-square distribution with a degree of freedom of 2 is equal to an exponential distribution with mean value of 2, after simplification, the upper bound limit is obtained as

$$\lambda_{100(1-\alpha)} = \frac{-ln(\alpha)}{nT} \tag{3.42}$$

where$n$ is the number of components that are tested. So far, the problem of estimating zero event failure rates has been solved using a simple method as well.

3.5 Numerical examples

In this section, two numerical examples will be analyzed by James-Stein, Vaurio's EB and Quigley *et al.*'s EB estimators in order to investigate the similarity and diversity of them. As a contrast, the MLE method which is introduced at the beginning of this chapter is also included such that the superiorities of the statistical methods to the point estimate can be demonstrated. Based on the calculation result, one can conclude each method's applicability and performance on certain data. The choice of approach is vital for the new CCF model is proposed in Chapter 4.

3.5.1 Example 1

The data (N_i,T_i), the number of failures in relative time of exposure of each component, are used in Vaurio's paper on James-Stein estimator (1992). As given in Table 3.3, the estimates are the mean values of the posterior distributions or MLE, and the standard deviations (S.D.) of the estimates are listed as well. Vaurio's EB method yields a single prior for all the seven estimates and a set of parameters is generated, as $\alpha = 0.9070$ and $\beta = 0.7485$. Similarly, QBW EB procedure produces a different prior for the estimates with $\alpha = 37.8144$ and $\beta = 62.6769$. It can be seen that the QBW EB hyper-parameters are much larger than those of Vaurio's procedure, which can explain the reason why the QBW posterior is less influenced by a specific component's data.

Table 3.3: MLE, JS, Vaurio's EB and QBW EB failure rate estimates for valves

i	N_i	T_i	MLE		James-Stein		Vaurio's EB		QBW EB	
			Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	31	236.9020	0.1309	0.0235	0.1355	0.0711	0.1343	0.0238	0.2297	0.0277
2	157	115.9440	1.3541	0.1081	1.3528	0.1014	1.3532	0.1077	1.0907	0.0781
3	30	36.8120	0.8150	0.1488	0.8255	0.1783	0.8229	0.1480	0.6816	0.0828
4	13	7.5970	1.7112	0.4746	1.6506	0.3734	1.6664	0.4469	0.7231	0.1014
5	7	5.4660	1.2806	0.4840	1.2683	0.4303	1.2723	0.4525	0.6577	0.0982
6	7	1.6890	4.1445	1.5665	3.0310	0.6652	3.2439	1.1536	0.6962	0.1040
7	0	1.1230	0.0000	0.0000	0.5749	0.7476	0.4846	0.5089	0.5927	0.0964
8	0	0.5520	0.0000	0.0000	0.7826	0.8723	0.6974	0.7323	0.5981	0.0973

In order to show the effects of different methods on the failure rate estimation, plot the mean values of all the components in Figure 3.4. First, as can be seen in the figure, all the three statistical approaches shrink the failure rates from the MLE to an unknown overall average. This is a novel and valuable feature that could solve the limitation of data to some extent in the nuclear power plant reliability assessment by borrowing data from similar components, systems or plants. Second, the JS and EB estimators developed by Vaurio show similar performances due to the estimation procedures.

Third, QBW EB estimator is less sensitive to specific data than the other two. All the components of interest are considered as though they are highly identical or even the same while in fact the point estimates distinguish from each other. This result can be foreseen from the estimation procedure when the data of different components are summed up without any unequal treatments. Last but not least, all the three approaches can produce a positive failure rate for the event that has never happened in the past. This is of great importance in the field of probabilistic safety assessment of rare events. Zero-data event can still be evaluated by assimilating information from other sources.

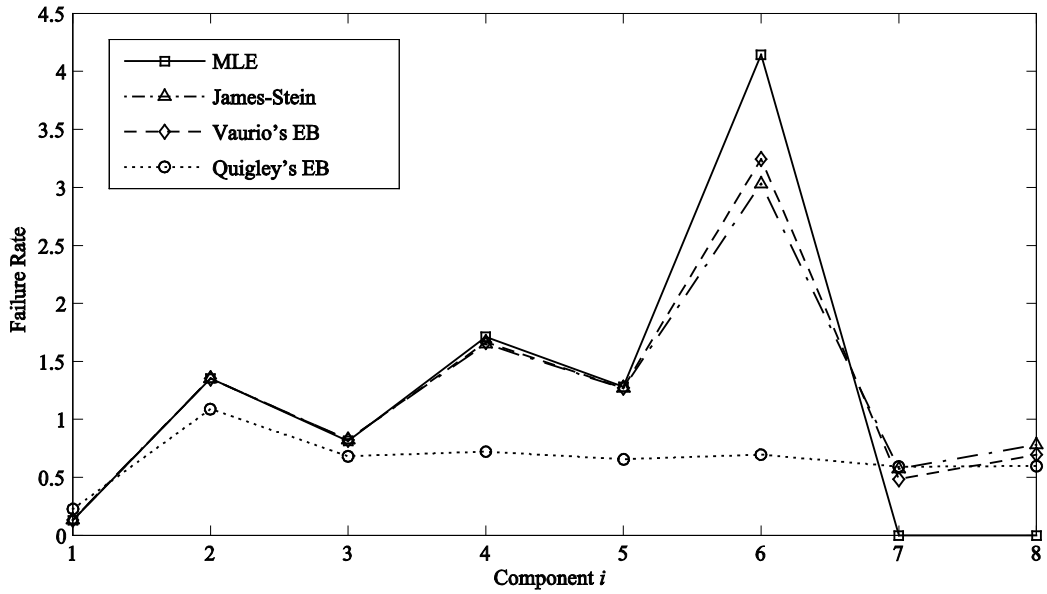

Figure 3.4: Failure rate estimates by different methods

Although the estimation results of failure rates are promising, it is still necessary to compare the standard deviations (S.D.) of different methods to check the variability. The S.D. of the MLE is derived from the variance of gamma distribution. Besides, the standard deviations of the two EB methods' posteriors are also plotted in the same figure below. It is clear that all the James-Stein, Vaurio's empirical Bayes and QBW EB methods are more accurate by achieving lower S.D. of the estimate. Besides, the MLE method gives zero estimates and standard deviations for the zero failure events (last two components). This is obviously incorrect as the observation times are not long enough to record failures.

Moreover, the uncertainties in the calculation of α and β are not taken into consideration. The authors take the estimates of α and β based on the data as the true ones as granted. But in fact, the

estimated parameters are not completely accurate. Last but not least, QBW EB procedure seems rather advanced because of the much lower standard deviations of its posterior distributions. This stems from the unique estimation procedure and relatively large α and β. One should also keep in mind that QBW EB procedure is more suitable for identical data.

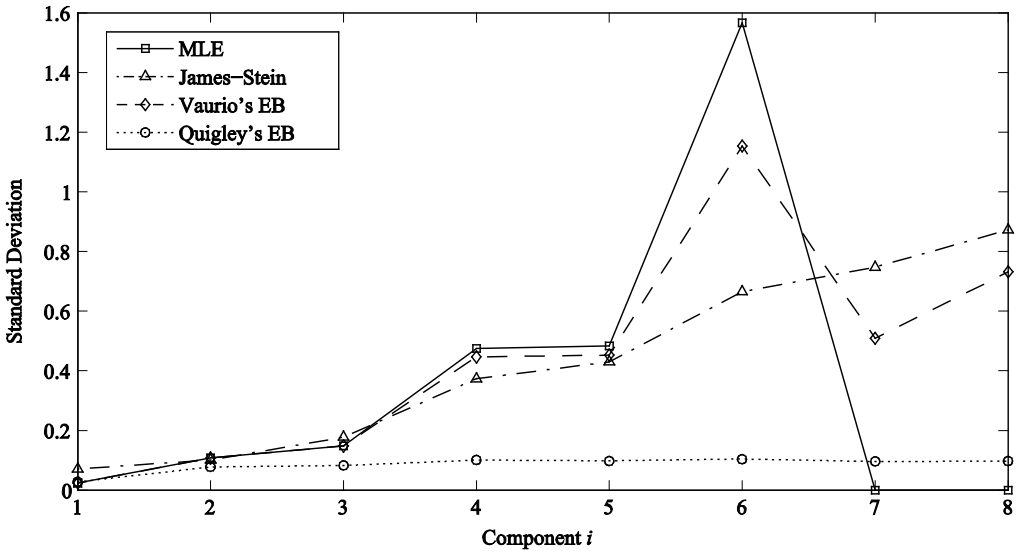

Figure 3.5: Standard deviations of the estimates by different methods

In reality, the probability density functions (PDFs) may be more welcomed by the industry compared to the point estimate. With the help of the PDF, one can visually be aware of the statistical characteristics of the estimate such as the scale parameter, shape parameter as well as the variability. The James-Stein estimator is not included as it is a Frequentist approach rather than a Bayesian method. Therefore, in this example, the posterior distributions of each component generated by different approaches will be plotted. Prior distributions are not included as they are the same for all the components. One should be aware that the posterior is generally narrower than prior meaning that the uncertainty is reduced after the Bayesian update.

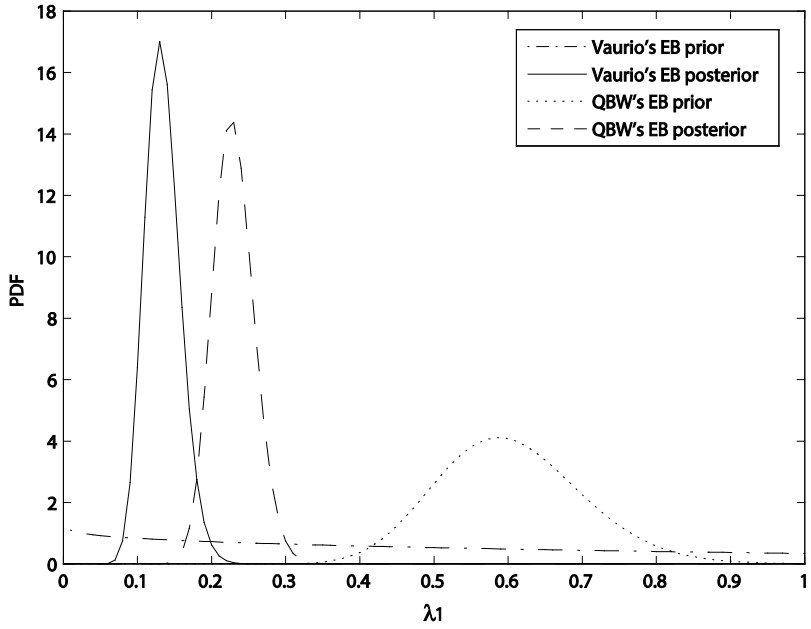

Figure 3.6: Posterior distributions of the failure rate of component 1

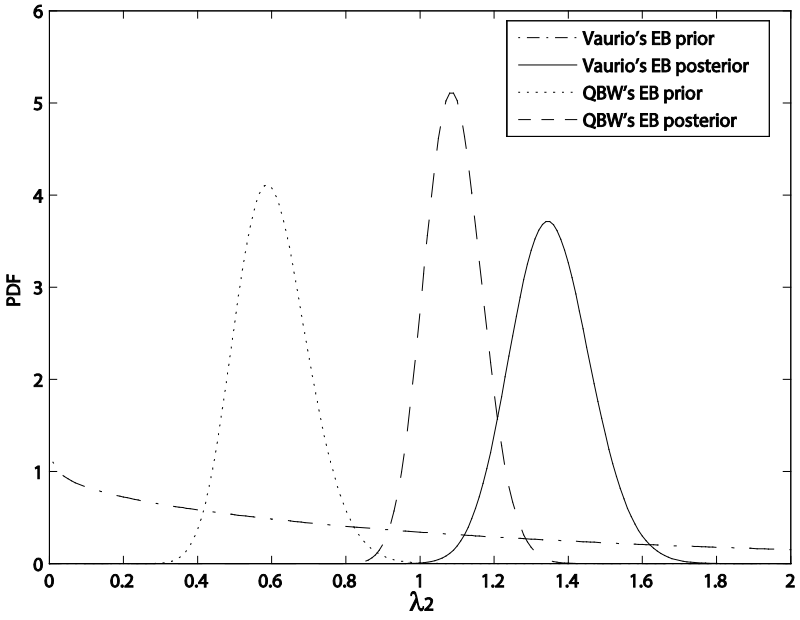

Figure 3.7: Posterior distributions of the failure rate of component 2

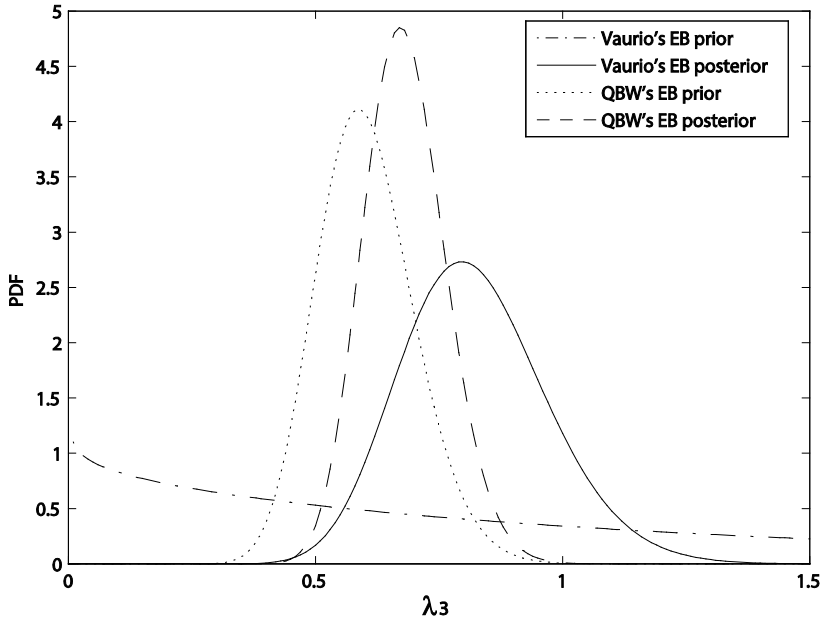

Figure 3.8: Posterior distributions of the failure rate of component 3

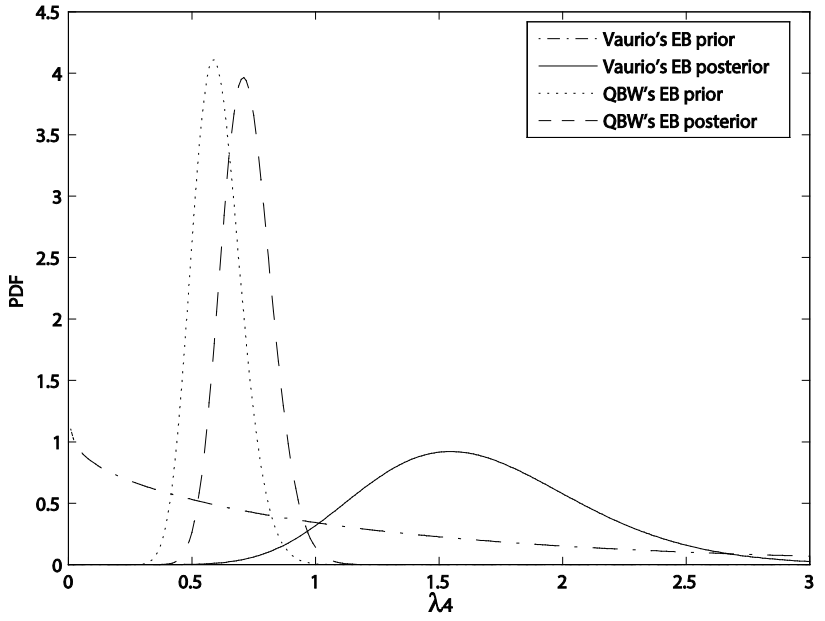

Figure 3.9: Posterior distributions of the failure rate of component 4

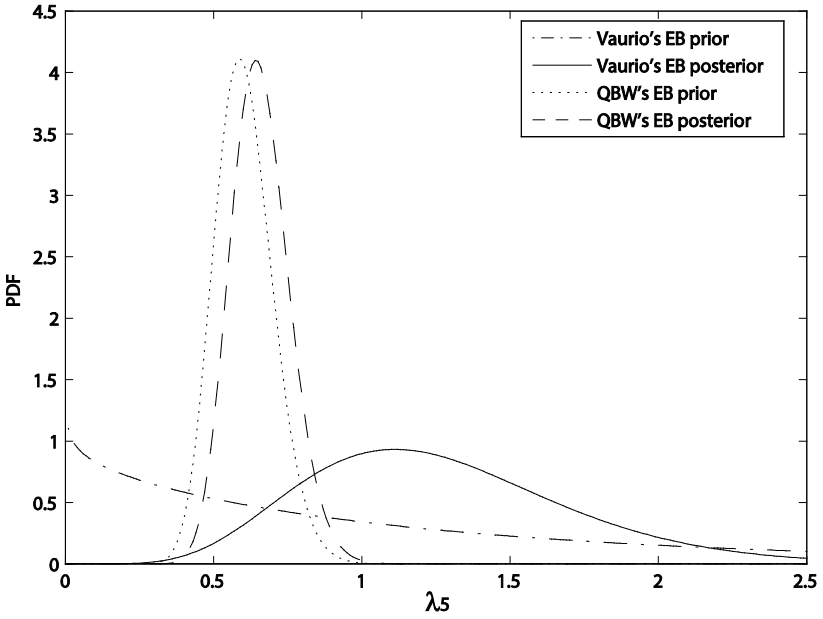

Figure 3.10: Posterior distributions of the failure rate of component 5

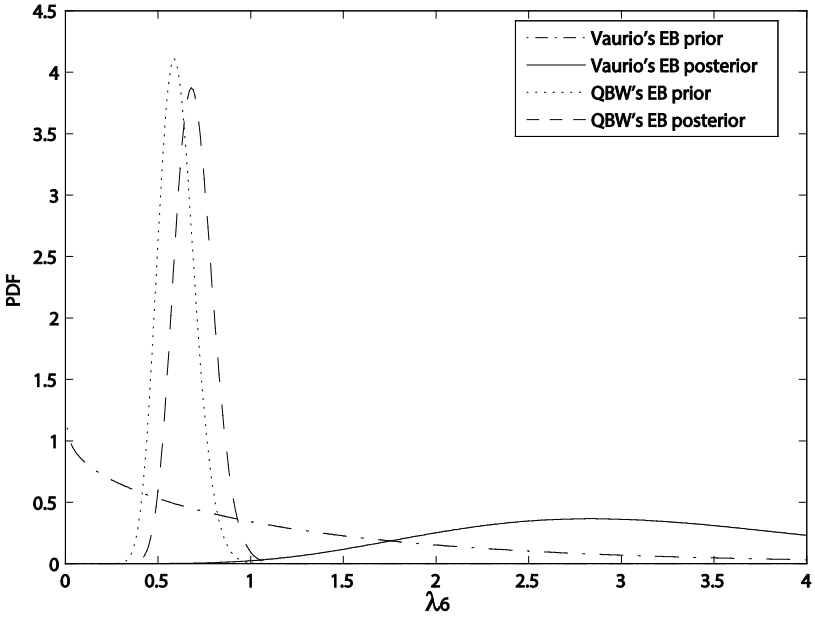

Figure 3.11: Posterior distributions of the failure rate of component 6

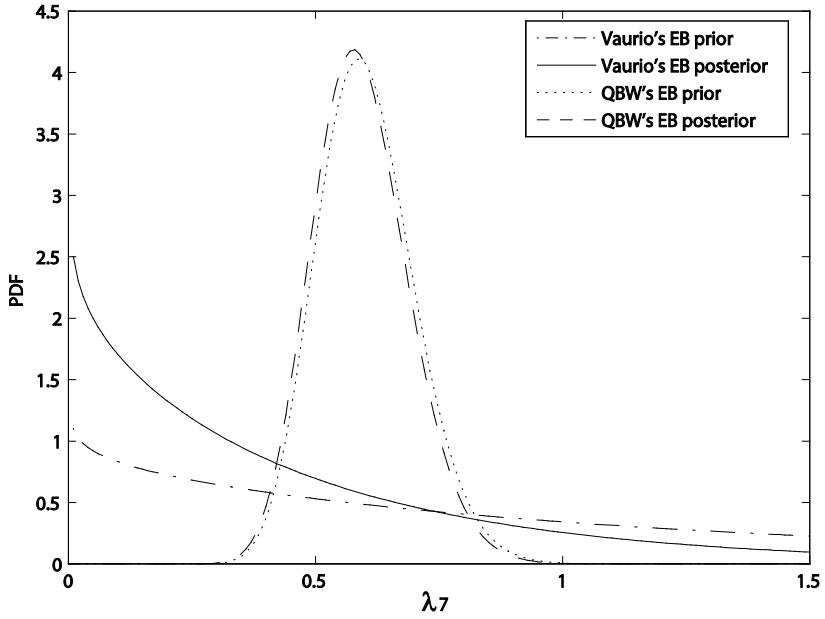

Figure 3.12: Posterior distributions of the failure rate of component 7

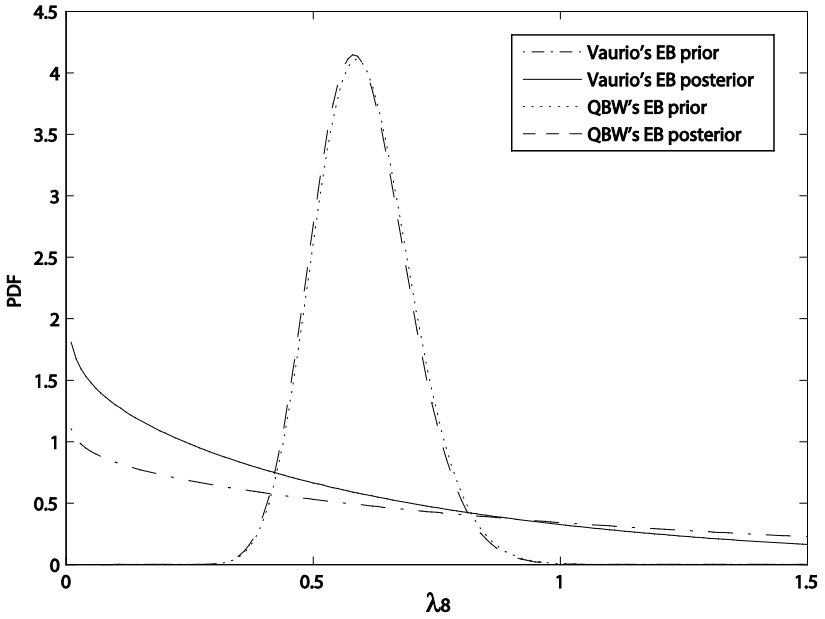

Figure 3.13: Posterior distributions of the failure rate of component 8

Some conclusions can be drawn from the above 8 PDF figures. First of all, according to the figures, Vaurio's EB estimator is more sensitive to specific component's data than QBW EB procedure. The posteriors in Figure 3.6 to Figure 3.10 almost coincide with the MLE results because the hyper-parameters are far smaller than a certain component's data. In contrast, QBW EB procedure is always stable in a narrow range no matter if the data are enough or not. A set of much larger hyper-parameters are generated from the estimation procedure and hence leads to the insensitivity to the individual data. Moreover, if there is indication showing that the components under study are highly identical, then QBW EB procedure may be a good choice to evaluate their failure rates. Last, according to the PDFs in Figure 3.12 and Figure 3.13, zero-failure events can be estimated by the means of EB. The EB posterior distributions still exist by assimilating information from other components, which shows the advantage of the means of EB.

If one is interested in the 90% confidence interval of the fourth component, for instance, the methods given in Section 3.2.4 are available to be used. By calculating the 5% and 95% percentiles, the confidence interval can be regarded as from the lower one to the higher. The results are listed in the table below. It is obvious that the two methods yield consistent result of confidence interval.

Table 3.4: The 90% confidence intervals of component 4 using Monte Carlo simulation and inverse gamma function

	5%	95%
Monte Carlo simulation 10^4 times	0.6079	1.5091
Monte Carlo simulation 10^5 times	0.6117	1.4916
Monte Carlo simulation 10^6 times	0.6107	1.4958
Inverse gamma function	0.6104	1.4951

3.5.2 Example 2

Example 2 involves a small sample of data used by Vaurio (1986). The data of the four components are recorded in pairs (N_i,T_i). The mean and variance/standard deviation of all the four estimators are listed in the table below.

Table 3.5: MLE, JS, Vaurio EB and Quigley EB results of Example 2

i	N_i	T_i	MLE		James-Stein		Vaurio's EB		QBW EB	
			Mean	S.D.	Mean	Var	Mean	S.D.	Mean	Var
1	4	1	4	2	0.0600	-31.3766	3.6892	1.3096	48.3333	2497.2222
2	3	1	3	1.7321	3.1541	-31.3766	3.2243	1.2243	-3.3333	-172.2222
3	2	1	2	1.4142	6.2482	-31.3766	2.7594	1.1326	-55.0000	-2841.6667
4	1	0.2	5	5	2.6882	-2.7773	3.6532	1.6443	2.6446	-3.3877

In the example, the JS estimator produces a negative overall variance that is certainly wrong. Hence it can be concluded that for small sample data, the JS estimator proposed by Vaurio may not be applicable. In addition, QBW EB procedure is not applicable either because of $W < U^2$ resulting in negative hyper-parameters and variances. Therefore, the only one in the three statistical approaches is Vaurio's EB estimator. According to the plot of MLE and posterior mean values of Vaurio's EB procedure in Figure 3.14, the shrinkage effect is still obvious for the small sample unlike the unrealistic results of the JS or QBW EB procedure estimators. The hyper-parameters are: $\alpha = 3.9359$ and $\beta = 1.1511$.

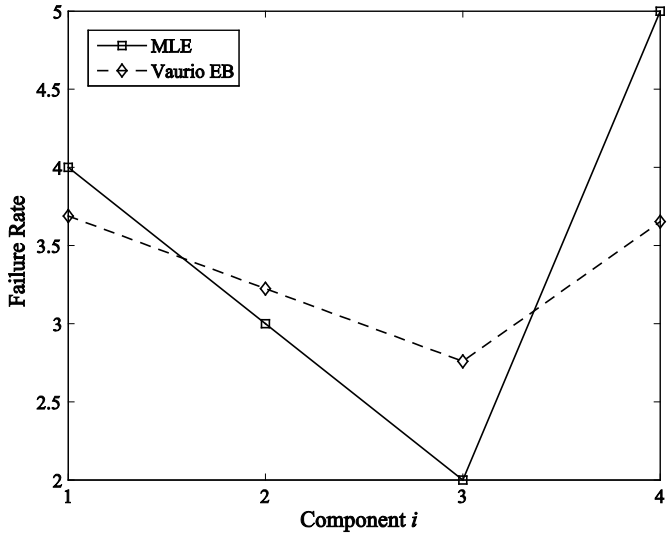

Figure 3.14: Failure rate estimates by the MLE and Vaurio's EB procedure

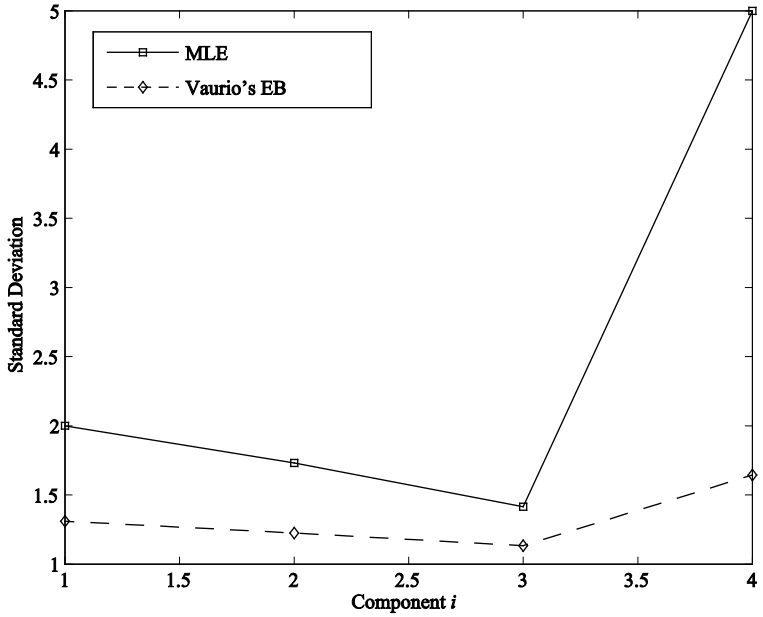

Figure 3.15: Standard deviations of the estimates by different methods

Second, as shown above, plot the standard deviations of the estimates of MLE and Vaurio's EB method. It can be seen that EB method generates much smaller standard deviations meaning that its failure rates are more accurate than the MLE.

In the next, the prior and posterior distributions of Vaurio's EB are plotted in the four figures below. For all the four components, the posterior distributions are higher and narrower than the priors as indicated below. The uncertainties of the estimates are therefore reduced by the means of Bayesian. Besides, the fourth component has a short operation time meaning large uncertainty and unreliability of the MLE estimate. With the help of Vaurio's EB method, one can still generat a posterior distribution for the failure rate regardless the limitation of the data of this specific component.

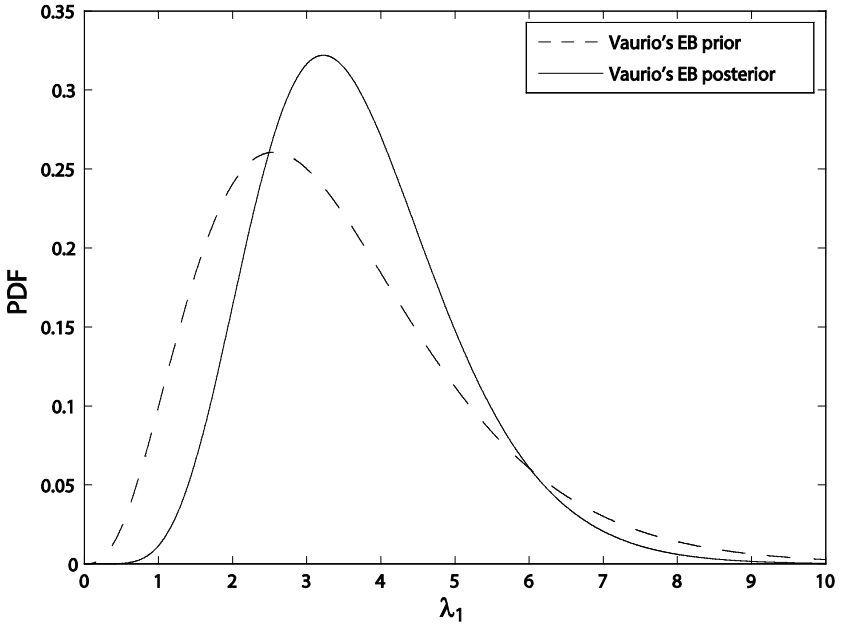

Figure 3.16: Prior and posterior distributions of component 1

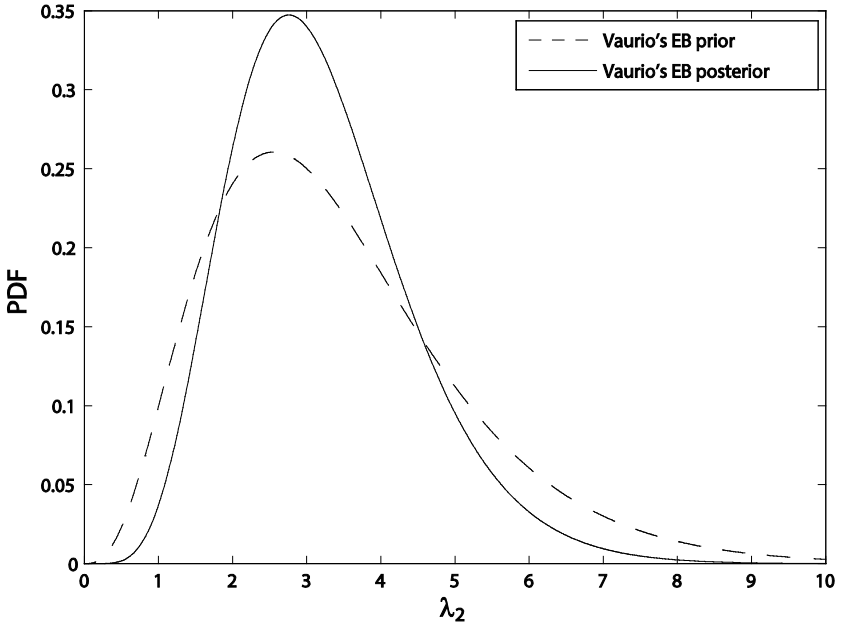

Figure 3.17: Prior and posterior distributions of component 2

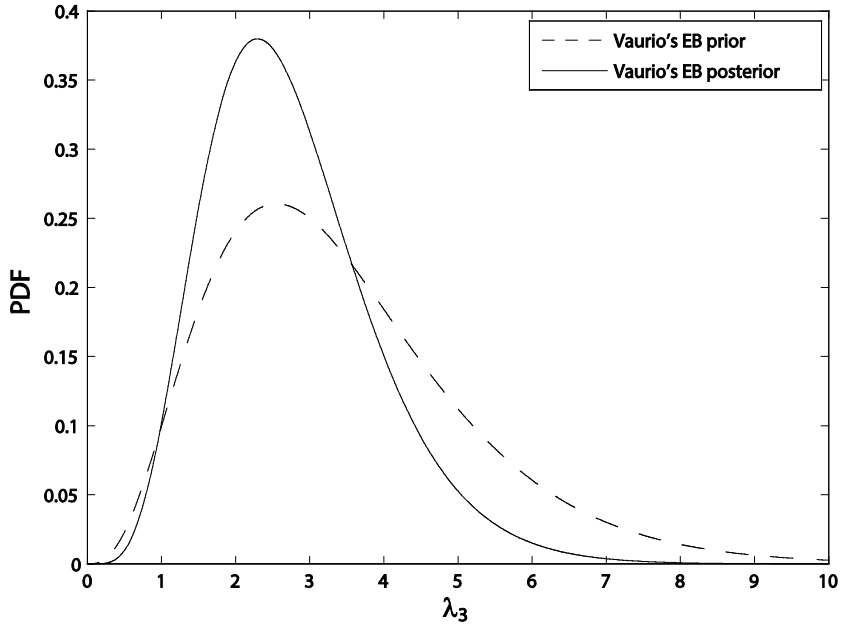

Figure 3.18: Prior and posterior distributions of component 3

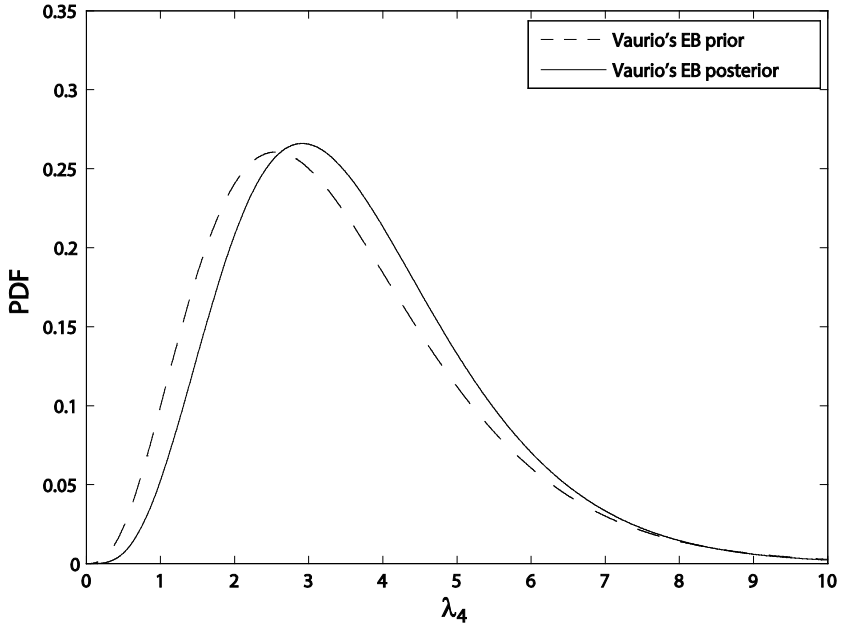

Figure 3.19: Prior and posterior distributions of component 4

3.6 Summary

This chapter has presented three statistical approaches theoretically and numerically other than the MLE. Generally, all the three methods are more reliable than the MLE by assimilating data information from other resources. The MLE is subject to large uncertainty especially when the data are sparse and the observation time if limited.

The JS method proposed by Vaurio is taking the view point of Frequentist statistics and dominates the point estimate all the time. However, according to the second example, inapplicability of the method arises when the component group is small which results from the algorithm. Also, this Frequentist approach only produces an estimate but the industry is more likely to expect a confidence interval or even a whole distribution for the objective of the estimation.

QBW EB estimator is able to provide a distribution based on the Bayesian statistics. It is especially suitable for identical components in a group because of the strong pooling of the data. Whereas, it cannot be used for small samples either because of the negative hyper-parameter α or β or both. The improvement of the algorithm needs further research.

In the end, Vaurio's EB estimator is regarded as an optimal one for the failure rate estimation in the assessment of rare events, for instance the common cause failure. Its broad applicability, remarkable shrinkage effect and capability of describing the estimate with distributions are quite superior to the others, not to mention the widely used point estimate. In the next chapter, a distinguished model for assessing the CCF will be proposed. Vaurio's EB will be utilized in the estimation procedure.

Chapter 4
Multivariate Poisson Model

4.1 Introduction of the multivariate Poisson (MVP) model

As discussed in the chapter before, some of the current CCF models are somehow dependent on strong assumptions such as the BF and BFR models. These assumptions are hardly following the unknown nature of the failure which makes the models less trustworthy. In contrast, other models like the MGL and BP models are solely based on the recorded numbers of failures in order to estimate the specific failure rates of interest. However, the sparseness of the data in practice reduces the accuracy associated with the point estimate estimated directly from the data. Given a limited database of a certain CCCG, it is of importance to propose a new method which does not rely on robust assumptions, respects practical data, and achieves high accuracy as well.

Karlis (2002) presents a multivariate Poisson model that can be extended to the CCF assessment and solves the above remaining problems nicely. Consider a 3-component common cause component group again. The failure scenarios of the CCCG can be decomposed into the following seven scenarios:$\{1, 2, 3, 12, 13, 23, 123\}$where the first three refer to the individual-component failures, and the remaining four elements to the CCFs. Accordingly, the problem has been decomposed into the estimation of a group of seven failure rates$[\lambda_1, \lambda_2, \lambda_3, \lambda_{12}, \lambda_{13}, \lambda_{23}, \lambda_{123}]$.

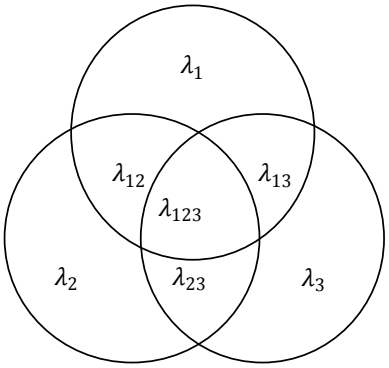

Figure 4.1: Multivariate Poisson model of CCCG of size 3

Due to the facts that the occurrence of failure events (both independent and dependent) can be modeled by a Poisson process, and that the superposition of Poisson processes is still a Poisson process, the total failure events involving a specific component, i.e., component 1, can be expressed as in Equation 4.5.

$$X_1 \sim Poisson(\lambda_1) \quad 4.1$$

$$X_{12} \sim Poisson(\lambda_{12}) \quad 4.2$$

$$X_{13} \sim Poisson(\lambda_{13}) \quad 4.3$$

$$X_{123} \sim Poisson(\lambda_{123}) \quad 4.4$$

$$X_1 + X_{12} + X_{13} + X_{123} \sim Poisson(\lambda_1 + \lambda_{12} + \lambda_{13} + \lambda_{123}) \quad 4.5$$

Therefore, all the failures that may occur in practice are able to be characterized by the mutually exclusive elements in the MVP model. This complete decomposition of the failure events eliminates the confusion in the conventional models, e.g., the alpha factor model, that whether a multi-component failure is supposed to be a single common cause event or a combination of multiple events involving fewer components.

4.2 Relation between the MVP and traditional parametric CCF models

Some parametric CCF models are available at present such as the beta factor model, multiple Greek letter model, alpha factor model, basic parameter model, as well as binomial failure rate model. The first difference is that the MVP model does not strongly rely on assumptions as those traditional ones, e.g., beta factor model and binomial failure rate model, do. It solely depends on the practical data collected during the operation time. Second, the MVP distinguishes all the possible failures and treats them as unique components. On the contrary, all the other conventional parametric models make a symmetry assumption, which supposes that failure events involving the same number of components are the same. For instance, in a three-component group, the independent failure rates of component 1, 2, 3 are equal and calculated as the total independent failure rate divided by three. So is the double-component failure rates treated. In other words, failure scenarios with the same multiplicity are pooled to be the same and diversity is ignored completely. In fact, however, components in the same CCCG are supposed to be diverse as a defense in order to prevent CCF. Third, there is confusion in the traditional parametric CCF models about whether a multi-component failure event is caused by a single CCF. By contrast, the MVP model eliminates the confusion by the exclusive failure rates defined in the framework.

Apart from the above differences, some connections between the MVP and traditional models are readily established. For instance, given the seven failure rates of the mutually exclusive events, one can calculate the total failure rate of a specific component, i.e., Component 1, as

$$\lambda_{Total-1} = \lambda_1 + \lambda_{12} + \lambda_{13} + \lambda_{123} \quad 4.6$$

Similarly, if one is interested to know the failure rate of the *k-out-of-n* system, it can be also easily calculated from the failure rates of the MVP model. For a 2-*out-of*-3 configuration, it is

$$\Lambda_{2oo3} = \lambda_{12} + \lambda_{13} + \lambda_{23} + \lambda_{123} \quad 4.7$$

Note that the possibilities of multiple failure events happening simultaneously have already been screened out according to the assumption of MVP model.

Furthermore, common cause factors such as the alpha factors, beta factor and multiple Greek letters defined by various models are the ones that are used to represent the plant's characteristics in practice. These can also be obtained based on their definition once the failure rates of the MVP are given. For instance, the alpha factors are calculated as

$$\alpha_{1/3} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_{12} + \lambda_{13} + \lambda_{23} + \lambda_{123}} \quad 4.8$$

$$\alpha_{2/3} = \frac{\lambda_{12} + \lambda_{13} + \lambda_{23}}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_{12} + \lambda_{13} + \lambda_{23} + \lambda_{123}} \quad 4.9$$

$$\alpha_{3/3} = \frac{\lambda_{123}}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_{12} + \lambda_{13} + \lambda_{23} + \lambda_{123}} \quad 4.10$$

4.3 Comparison of the estimation procedures

First, apply Vaurio's EB and QBW EB procedure to the group of seven failure rates respectively. Each method generates one prior which means that the seven rates are realizations of a single distribution. However, it is recommended that the data be grouped into some subgroups that are more homogeneous. Then apply EB method to each sub-group. For example, when predicting many students' grades, it is better to group them into three subgroups (good, medium, bad) based on their past grades in history and apply the EB method to each of the subgroups respectively (Shen, 2006). Therefore, Vaurio's procedure combined with grouping method (denoted as G-V-EB) is to be adopted as the third potential estimation approach. When applying G-V-EB method, the first three rates are

put in one sub-group while the remaining four are grouped into another according to their magnitudes and understandings of their natures: $\{(\lambda_1, \lambda_2, \lambda_3), (\lambda_{12}, \lambda_{13}, \lambda_{23}, \lambda_{123})\}$ .In this way, two prior distributions will be produced, one is for independent failures and the other is for CCFs, which seems more reasonable. Grouping is not adopted in QBW procedure because of its narrow applicability to sub-groups of small sizes. Included for the comparison is also the maximum likelihood estimate (MLE), which is introduced in Chapter 3.

Before recommending a procedure for estimating the hyper-parameters of the proposed MVP model, it is necessary to compare the performance of the four estimation procedures reviewed above. Monte Carlo simulation is conducted for the comparison. The exact values of seven failure ratesλ_i are listed in Table 4.1below. The operation time of the system is assumed to be 5000 months in the simulation. Failure data are randomly generated from the Poisson processes with the corresponding failure rates. By comparing the estimates of various methods with the true values, it is ready to conclude the best procedure. To achieve a reliable conclusion, a total number of 10,000 simulations are run.

Now that the required information for Monte Carlo simulation has been determined previously, the next step is to set some indicators to represent the effectiveness of various estimation methods namely the MLE, QBW-EB, V-EB and G-V-EB. Three indicators are adopted for the comparison.

4.3.1 Squared error (SE) of a specific failure rate

The first one is the squared error (SE) of a specific failure rate λ_i obtained from a certain method. For instance, four different estimates of λ_1 are generated whose distances to the true value 0.011 are either positive or negative and certainly not the same. The squared error is therefore utilized to represent the accuracy of the estimation.

$$SE(\hat{\lambda}_i) = (\hat{\lambda}_i - \lambda_i)^2 \qquad 4.11$$

Any estimation method of the four can be seen as superior provided that its SE of λ_1 is smaller than the others'. However, due to the fact that the generation of the failure data is subject to large uncertainty, especially when the operation time is not long enough, one time simulation is hence obviously insufficient to draw a conclusion. As a result, 10^4 simulations are run and the percentages of the simulations to yield the smallest SE for $\lambda_1$by the four approaches are calculated as shown in the first row of Table 4.1. All the four percentages in the same row sum up to one. By extending the

calculation of the squared error of λ_1 to all the failure rates λ_i, one can conclude that G-V-EB is much more accurate for the estimation of a specific rate than the other three approaches in most cases. The average errors of the seven failure rates yielded by G-V-EB are 9.3, 9.1, 10.5, 25.6, 26.1, 27.9, 31 percentages of the exact values respectively. This results from the uncertainty in the generation of Poisson failure data. If the operation time is longer, the error will become smaller.

Table 4.1: Calculation of the percentage of the smallest SE by each method

i	λ_i	Percentage of the smallest SE (%)			
	(/month)	MLE	QWB-EB	V-EB	G-V-EB
1	0.011	33.30	5.11	5.99	55.60
2	0.01	14.08	3.93	20.73	61.26
3	0.009	10.27	1.83	37.17	50.73
12	0.0011	3.52	14.01	29.72	52.75
13	0.001	20.14	0.39	35.94	43.53
23	0.0009	11.45	10.59	23.24	54.72
123	0.0008	30.28	1.21	29.66	38.85

4.3.2 Sum of squared errors (SSE) of the whole set of failure rates

Although G-V-EB is more accurate than the other three approaches, the superiority over the MLE for some of the failure rates is not quite convincing. That is because the EB method aims to improve the overall accuracy of all estimates rather than a certain one. Therefore, secondly, it is also essential to examine the overall precision of the set of seven failure rates, which can be measured by a sum of squared errors (SSE).

$$SSE = \sum_{i=1}^{123} \left(\hat{\lambda}_i - \lambda_i\right)^2 \text{ where } i \in \{1, 2, 3, 12, 13, 23, 123\} \quad 4.12$$

Similar to the above, a proportion of each estimation approach is to be calculated. For a random simulation, all the squared errors of different failure rates λ_i computed by a certain method, e.g., MLE, are added up, so are the ones obtained from the other three methods. By comparing the magnitudes of the SSEs for one simulation, one can come up with a more accurate method for estimating the whole set of failure rates. Unavoidably, however, uncertainty associated with a single simulation is likely to interfere with the conclusion. Large sample is required as well and the

percentage of each method that yields the smallest SSE is to be calculated, in order to get a more dependable result of the overall precision. The calculation results have been listed in Table 2 below. It can be seen that the proposed G-V-EB is remarkably superior to the others for estimating all the set of rates.

Table 4.2: Calculation of the smallest SSE by each method

Method	MLE	QBW-EB	V-EB	G-V-EB
Percentage (%)	5.66	1.31	12.37	80.66

4.3.3 Variations of the estimates by different methods

While numerical calculation has proved that the proposed Vaurio's EB combined with grouping method (G-V-EB) to be obviously the best among the alternatives, one may be interested in the variation of the estimates as well. In the next, thirdly, illustration of the variations of the above four estimation approaches is considered as well. A visual illustration is to plot their PDFs for each failure rate of the MVP model. The true failure rates and operation time remain the same as previously.

A random simulation is run with the exposure time of 5000 and the results are plotted in the figures below. The three different lines represent the posterior distributions of the three EB approaches. Moreover, to show the accuracy of the PDFs, the true value of each failure rate is marked by the vertical solid line in each figure.

First of all, the elements in the same group or sub-group interact with each other. Although it is hard to guarantee that all the posteriors are closer to the true values of the failure rates (vertical line), the effectiveness of EB measures is still visible in most cases. Besides, due to the strategy of putting the elements of similar natures or magnitudes into the same sub-group, the G-V-EB is generally more accurate than the other two, resulting in narrower distributions and the closer centers to the true values and.

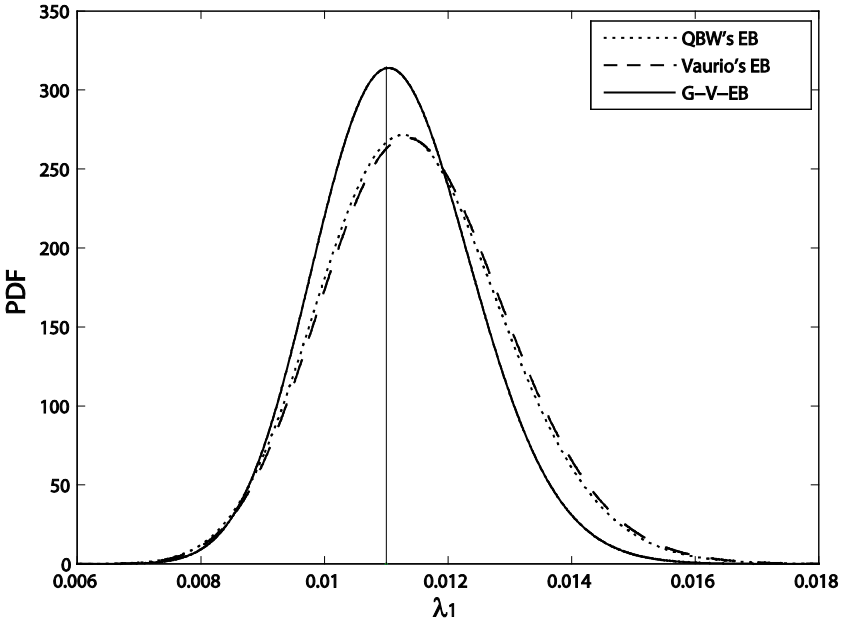

Figure 4.2: PDFs of λ_1 generated by different methods

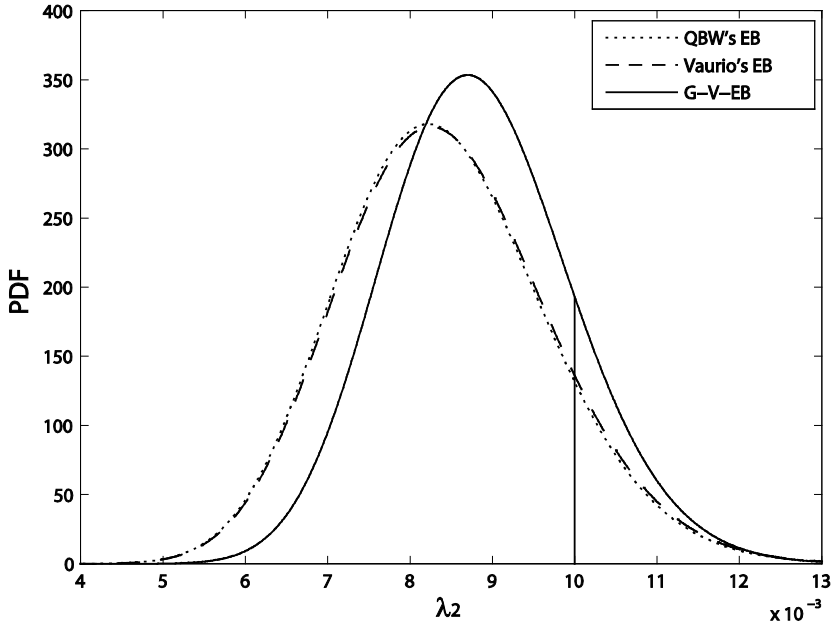

Figure 4.3: PDFs of λ_2 generated by different methods

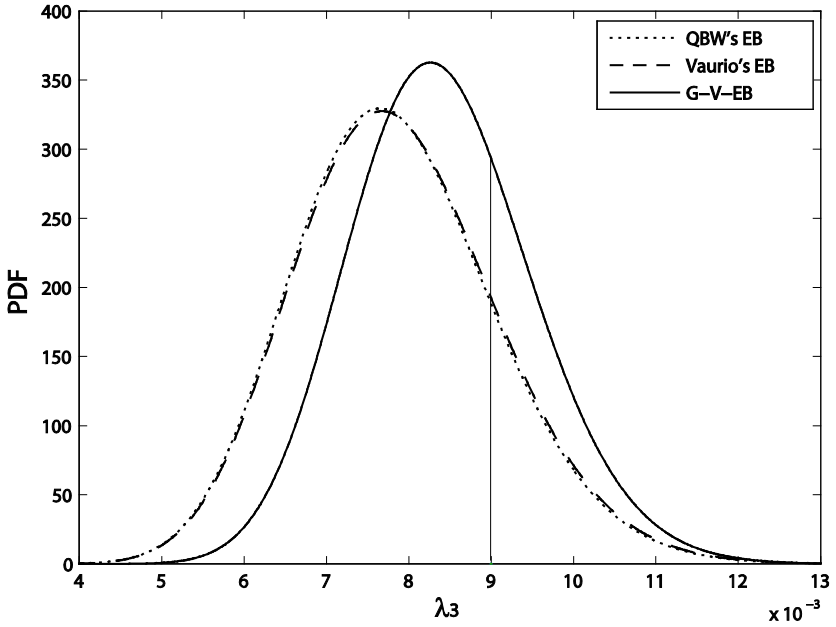

Figure 4.4: PDFs of λ_3 generated by different methods

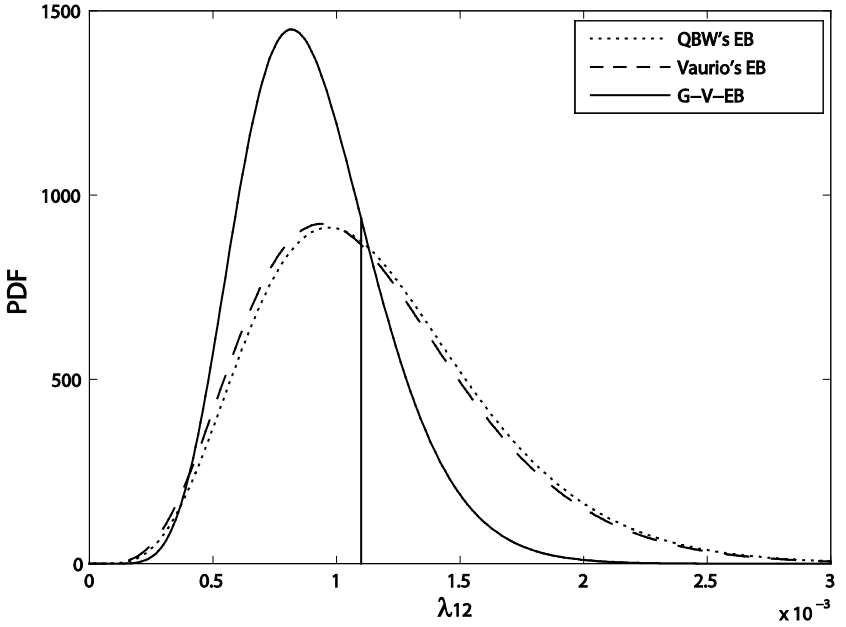

Figure 4.5: PDFs of λ_{12} generated by different methods

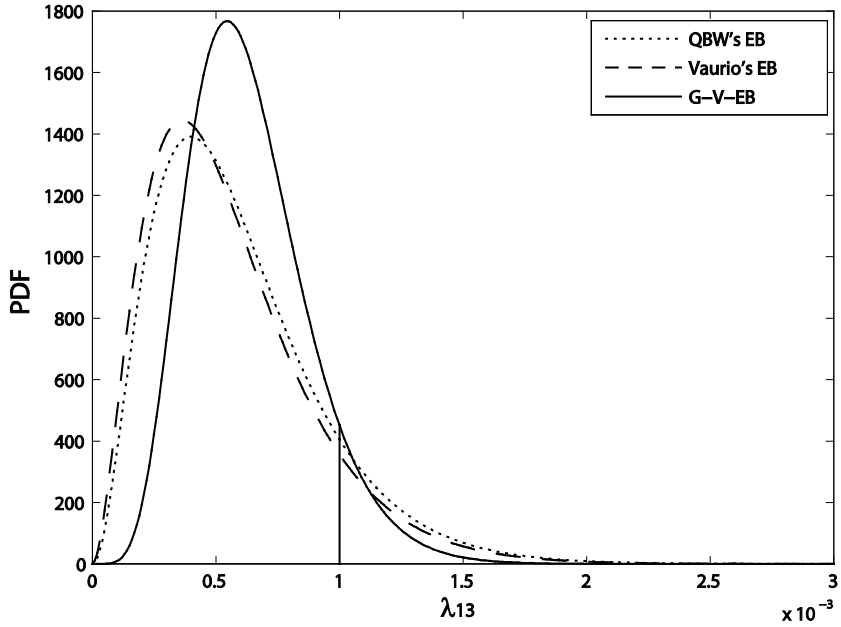

Figure 4.6: PDFs of λ_{13} generated by different methods

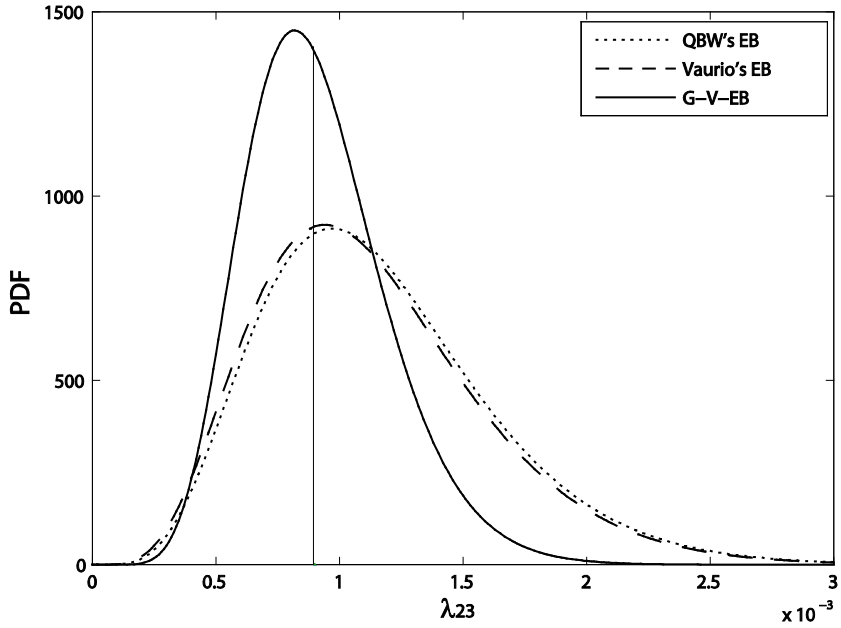

Figure 4.7: PDFs of λ_{23} generated by different methods

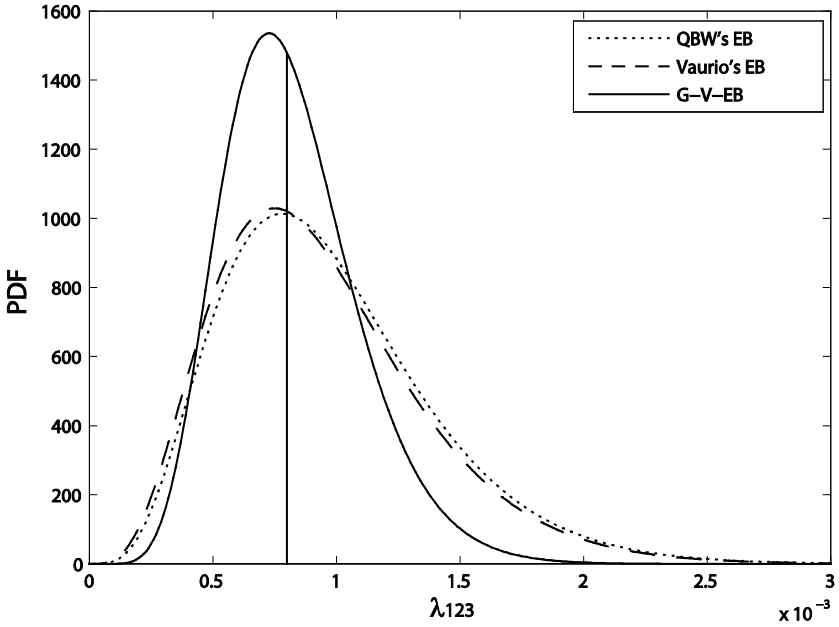

Figure 4.8: PDFs of λ_{123} generated by different methods

Apart from the visual illustration of the variation, one can also utilize the variance/standard deviation of the posterior distribution to measure it. The variance is easy to calculate by $(\alpha + N_i)/(\beta + T_i)^2$. Numerical values are not given here as one time simulation is subject to large uncertainty. But it can be assured that G-V-EB yields the smallest variance/standard deviation in most of the times.

Due to the higher accuracies in the estimation of one rate or the whole set of rates proved by statistics, as well as the lower variability illustrated by the density functions, the Grouped Vaurio's EB (G-V-EB) is considered optimal to be adopted in the MVP model.

The MVP model proposed at the beginning of this section contains a lot of parameters to be estimated. For a CCCG of size n, the number of failure rates is $2^n - 1$. For a three-component group, there are seven failure rates to be estimated. According to NUREG/CR-5497, the occurrence rate of independent failure is significantly higher than that of CCF while the CCFs do not obviously distinguish from each other. As a result, one can simply divide all the possible failures into two groups: one for the independent failures and the other for all the CCFs. From practical point of view, it is also reasonable to separate independent and common cause failures because of their distinct mechanisms, and two prior distributions are produced for both sub-groups.

Based on the discussion above, the MVP model transforms the quantification of CCF into the estimation of a number of mutually exclusive failure rates using the G-V-EB procedure. The outcomes are a set of posterior mean values and variances, if necessary, of the mutually exclusive failure rates. One can therefore obtain the rate of any possible failure event, such as the total failure rate of a specific component, and *k-out-of-n* system failure rate. Moreover, the parameters of the conventional parametric CCF models such as the alpha factors can be calculated based on these estimated failure rates as well. In the next section, the proposed MVP model will be compared to traditional parametric CCF models.

4.4 Comparison of the MVP and traditional CCF models

After choosing the appropriate estimation approach, the MVP model is necessarily to be compared to the traditional parametric models, for instance the MGL, AF, BFR models as presented in Chapter 2. Consider again the 3-component system utilized in the simulation above. Both independent and common cause failures are generated from the simulation procedure in the operation time of 5000.

As one of the earliest and simplest models, the beta factor (BF) model is not applicable here because the 2-component failure event always exists in the simulated database. This fact is against the assumption of the BF model. As a result, its extension the multiple-Greek letter (MGL) model is taken as a comparison with the MVP model. Besides, alpha factor model is one whose parameters are solely dependent on the data. It respects the practical observations a lot without extra robust assumptions while large uncertainties are unavoidable due to the rarity of the CCF data in practice. This kind of performance deserves to be a control group for the MVP model. Moreover, the binomial failure rate (BFR) is rather different from the others because of its unique assumptions about lethal and non-lethal shocks. The performance of BFR model is to be examined in the simulation as well. Furthermore, the true values of the estimates are able to be obtained as a benchmark since the failure rates have been presumed for the simulation already.

4.4.1 Calculation of the reliability of system

Two factors will be calculated within the Monte Carlo simulation. One is the reliability of *k-out-of-*3($k = 1, 2, 3$) system in an exposure time of 200 units. The calculations formulas have been derived in Chapter 1. It is of importance to check the overall performance of the proposed MVP model when applied in practice. All the three conventional parametric models and the true values mentioned above will be calculated likewise. The table below shows the results of one trial in the simulation. One can

easily conclude that the MVP model is more accurate than the traditional ones. In fact, after large quantities of simulation, this superiority retains.

Table 4.3: Reliability of a *k-out-of*-3 system using different models

Multiplicity *k*	1	2	3
True	3.05E-01	2.37E-02	1.16E-03
MGL	3.28E-01	2.39E-02	1.26E-03
AF	3.28E-01	2.39E-02	1.26E-03
BFR	2.59E-01	2.65E-02	1.30E-03
MVP	3.16E-01	2.39E-02	1.23E-03

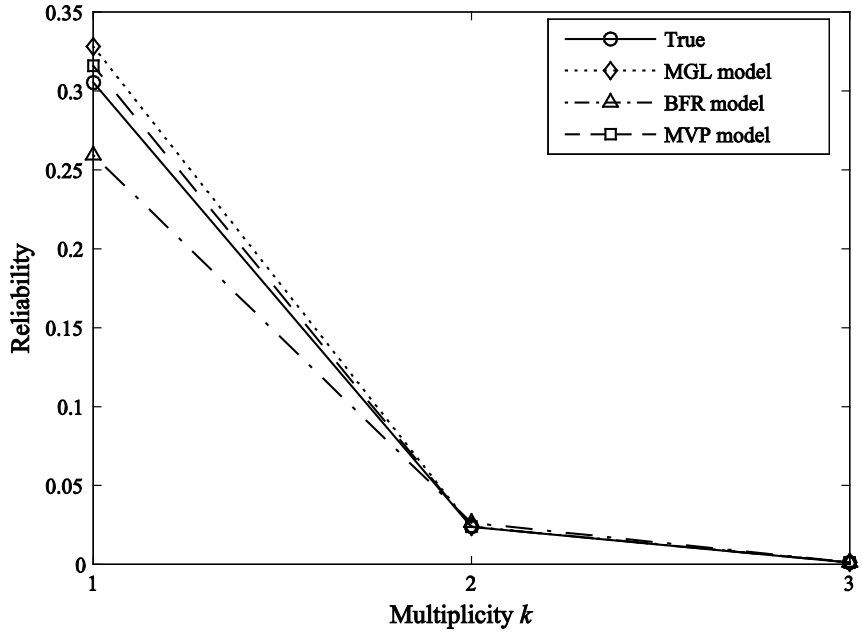

Figure 4.9: Reliability of a *k-out-of*-3 system using different models

In order to visually illustrate the difference, the failure rates are plotted in the same figure. The true reliability of the system is easy to obtain using the assumed values of the simulation as listed in Table 4.1. Since both MGL and AF models have their factors defined in terms of BP model parameters which depend merely on the data, their performances are the same in the example. As can be seen in the figure, the MVP model is much more accurate than the traditional models as the evaluated failure rates are closer to the true ones. Based on this phenomenon, one can conclude that in most cases, the newly developed MVP model is superior to those ones that are currently widely adopted.

4.4.2 Calculation of the CCF factors

Apart from the failure rate of a system, another aspect to be evaluated is accuracy of the traditional CCF factors estimation based on outcomes the MVP model. Since the alpha factor model is one of the most commonly used, take the alpha factors as example. Again, the true values of alpha factors are included for comparison. Details are included in the table below. To visually illustrate the difference, plot the alpha factors in the figure below and apply the logarithmic coordinates to y axis.

Table 4.4: Alpha factors using different models

	$\alpha_{1/3}$	$\alpha_{2/3}$	$\alpha_{3/3}$
True	8.88E-01	8.88E-02	2.37E-02
Alpha	8.68E-01	1.15E-01	1.65E-02
MVP	8.67E-01	1.09E-01	2.39E-02

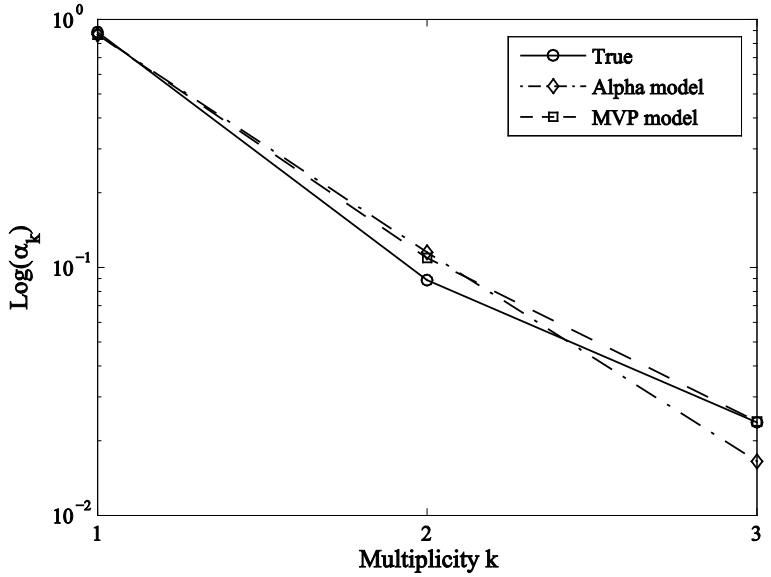

Figure 4.10: Alpha factor calculation using the AF and MVP models

According to the simulation results, it is apparent that in most cases the MVP model yields closer estimates of alpha factors than the alpha factor model does. The reason lies in the unique property of EB approach as discussed in Chapter 3. This characteristic solves the problem that the AF model is subject to large uncertainties in the parameter estimation when the data are sparse. Since all the CCF factors are proportions of different types of failure events (except the BFR parameters) and also able

to be obtained using failure rates instead of numbers of failures, one can conclude that the estimation of these factors is supposed to be more accurate with less uncertainty as long as the MVP model is adopted.

4.5 Summary

This chapter proposes a multivariate Poisson (MVP) model for CCF assessment. It also recommends the Vaurio's EB procedure combined with a grouping strategy (called the G-V-EB procedure) for the failure rate estimation. Compared to the conventional parametric CCF models, the MVP model is believed to be able to assimilate information from other components in the same sub-group, achieve higher accuracy, and get rid of strong assumptions. The main characteristic of the method is that every type of failure scenario is considered unique, which requires the failure data recorded in detail. In other words, when a failure event appears, the operation engineer should keep a record of which specific components are involved in the event. This newly proposed model is rather efficient for plant-specific estimation because all the data are observed within the target plant and no outer data source is introduced.

Two tests have been conducted to examine the performance of MVP model in the evaluation of the whole system and the estimation of traditional CCF factors. Obvious evidence has indicated that the MVP model is more reliable than the traditional ones in most cases because the calculation results using MVP model's outcomes are closer to the real values assumed.

However, there remain some drawbacks for the newly proposed MVP model. Since the practical operation team sometimes does not record the specific component combination of the CCFs, the requirement of data is hard to be satisfied. The failure events are simply included in the daily log with their times, root causes as well as component statuses. Sometimes the failures from source plants of the same size are merged together. Moreover, due to the sparseness of the CCF, the number of common cause failures is probably zero unless the operation time is long enough whereas the component may not be able to function if the time is too long. Once a component is replaced by a new one, the previous recorded failures of it are useless owing to the unique component assumption. This contradict will always remain with the MVP model.

Chapter 5
Data Mapping Combined with Empirical Bayes

5.1 Introduction of the combination of data mapping and empirical Bayes

Chapter 4 has introduced a multivariate Poisson (MVP) model for CCF modeling. It is an effective approach to implement plant-specific assessment, which means the data used are collected from the target plant of interest and the evaluation result is generated for specific components in the system. In other words, the MVP model is not able to make use of the data of source plants and reliable evaluation depends on large database of its own in a long operation time.

In contrast, there is a method called data mapping to assimilate information from source plants such that the limitation of data can be solved. Data mapping has been introduced in Chapter 2, as well as the developments by Vaurio *et al*. The traditional data mapping procedure generates a group of failure rates from the source plants of various sizes to the target plant. But there are no further instructions about how to merge these mapping results from different sizes. Kvam and Miller (2002) proposed a homogeneous Poisson process (HPP) for the mapping results which are assumed to be the failure data recorded from the target plant in different time intervals. This treatment, however, is rather robust not only based on the numerical magnitudes but also from the practical point of view. Hence, there need to be a new approach to solve the existing problems.

In this chapter, data mapping will be combined with empirical Bayes approach in order to make better use of the outcomes of the traditional data mapping and to result in a more reasonable conclusion. First of all, the failure data are processed following the data mapping formulas provided by Table 2.3 and Table 2.4 in Chapter 2. Detailed derivations are available in NUREG/CR-4780 Volume 1 and 2. At this stage, the data of source plants are mapped in to the size of the target plant as though various plants (including the target one) of the same size are available, i.e., N_{i_k} and T_i are already known for every multiplicity k in system i. Second, since these systems are not believed to be identical, the HPP assumption is quite arguable. As a result, for each multiplicity k, the number of failures N_k of the target system is able to be re-evaluated by the means of EB, partially assimilating from other systems and respecting its own data at the same time. After the implementation of EB approach, either the failure rates or the numbers of the target system are believed to be more reliable compared to the raw data alone. Third, with the data processed by data mapping and EB algorithm

one can easily estimate the failure rate of each multiplicity k and conventional CCF factors such as the alpha factors.

Note the component type is MOV and failure mode is failure to open (FO) in this chapter. There are various component statuses, namely working (W), complete failure (C), degraded (D) and incipient (I). It is common to utilize the impact vector to evaluate the possibility of every type of failure event in practice. However, for the ease of calculation, the thesis takes the statuses C, D and I as failure. Illustration of the combination of the data mapping and EB is the main purpose of the chapter. For higher accuracy, one can replace the data used below by impact vectors easily.

5.2 Case study 1 using a small motor-operated valve dataset

In this Section, we have used a sample of MOV data that represent typical operating experience in a nuclear plant, purely for illustration purposes. In nuclear power plants there are various sizes for the systems of MOVs, i.e., 2, 4, 8 and 16. The number of systems for each size varies. A simple assumption has been made that systems of the same size are identical and the failure data can be seen as collected from a typical system in different time intervals. Hence, it is reasonable to sum up the operation times and numbers of failures respectively for the typical system of a specific size. The operation time for all the plants are 18 years and the unit of time in the case study is month.

Let the 4-MOV system be the target system. The failure data of all the other three sizes have to be mapped upward/downward to $n = 4$. First, mapping down from $n = 8$ or 16 can be implemented by following Equation 2.25 or formulas listed in Table 2.3 in Chapter 2. The mapping down procedure is straightforward. Second, as to mapping up from $n = 2$ to 4, formulas are available in Table 2.4. What should be notable is that there is no further indication of whether the failures are caused by lethal or non-lethal shock. Some simplifying assumptions are made as follows: a) all the single-component failures are independent and b) all the multi-component failures are caused by non-lethal shocks. Therefore, an extra parameter ρ has to be introduced and also assumed as 0.2 according to NUREG/CR-4780. Following the mapping formulas provided above, one can come up with the results in the table below. The data of each different size are transformed to be of the target system size 4.

Table 5.1: Data after mapping process

System size	Number of failures			
n	$1/n$	$2/n$	$3/n$	$4/n$
2	36	0.6400	0.3200	0.0400
4	18	2	10	1
8	3.5714	0.2143	0	0
16	4.0456	0.4209	0.1099	0.0082
sum	61.6170	3.2752	10.4299	1.0482

Based on the data after mapping, there are two options about how to make use of them such as calculating CCF parameters. Take alpha factors as an example. Option 1: Assume a homogeneous Poisson process (HPP) for the whole database which means that the mapped data from source plants are collected from a typical system of size 4in four different operation times. In order to obtain the alpha factors, the numbers of all the k/n failures are to be summed up respectively (as has been done in the table above), and then calculate the proportions of them. Results are listed in the table below. One can refer to Equation 2.15 to 2.17.

Table 5.2: Alpha factors of 4-MOV system based on the assumption of HPP

Multiplicity k	1	2	3	4
$N_{k/n}$	61.6170	3.2752	10.4299	1.0482
$\alpha_{k/n}$	0.8068	0.0429	0.1366	0.0137

Option 2: Since the data of each size differ greatly from each other, it is not convincing to make an HPP assumption upon the whole database, considering both the numerical variations and the practical situations such as system size, environment, and maintenance. It is hence necessary to propose an approach which is capable of assimilating information from other sources and respecting the target system's data as well. Fortunately, empirical Bayes is the one that can meet the requirement.

In order to estimate alpha factors of the 4-MOV system accurately, precise evaluations of the numbers of failures for all multiplicities are necessary. Consider 1/4 failures of all the four sizes as an example. The number of failures after data mapping and the corresponding time are listed in the second and third columns of the table below, which are required by the empirical Bayes algorithm. As

QBW EB method has some limitations of inapplicability for small sample and strong pooling effect, the EB procedure proposed by Vaurio (1987) is utilized for the calculation here. The detailed calculation procedure has been listed in the table below. The outcome is the posterior mean value of the failure rate $\Lambda_{1/4}$ of the target system after Bayesian update which has been included in the fourth column.

Table 5.3: Calculation of the 1/4 failure rate by the means of EB

System size n	$N_{1/4}$	T_n	$\Lambda_{1/4}$
2	36	10584	3.44E-03
4	18	3672	4.49E-03
8	3.5714	1728	2.91E-03
16	4.0456	1080	3.71E-03

After applying the EB algorithm to all the multiplicities k/n respectively, the failure rates $\Lambda_{k/n}$ of the target system (of size 4) are believed to be closer to the inherent unknown rates. The results are shown in Table 5.4 below.

Table 5.4: Failure rates of all the multiplicities in different systems using EB approach

Size	$\Lambda_{k/n}$			
n	1/4	2/4	3/4	4/4
2	3.44E-03	1.13E-04	5.76E-05	3.23E-05
4	4.49E-03	4.58E-04	2.55E-03	2.26E-04
8	2.91E-03	2.48E-04	1.48E-04	9.14E-05
16	3.71E-03	3.50E-04	2.89E-04	1.11E-04

The alpha factors can be easily calculated by following Equation 2.17. All the calculation results are given in Table 5.5.

Table 5.5: Alpha factors of 4-MOV system using the EB approach

Multiplicity k	1	2	3	4
$\Lambda_{k/n}$	4.49E-03	4.58E-04	2.55E-03	2.26E-04
$\alpha_{k/n}$	0.5813	0.0594	0.3300	0.0293

The two options' results have been plotted in the figure below for comparison. An apparent difference has been shown visually in Figure 5.1 below. First, the CCF data show a higher possibility of 3/4 failure event than the other CCF, i.e., 2/4 and 4/4. Second, the HPP assumption yields a larger proportion of independent failures. This phenomenon may probably stem from the fact that independent failure is more likely to occur and the summation of independent failures takes a higher percentage than the data of any specific plant itself.

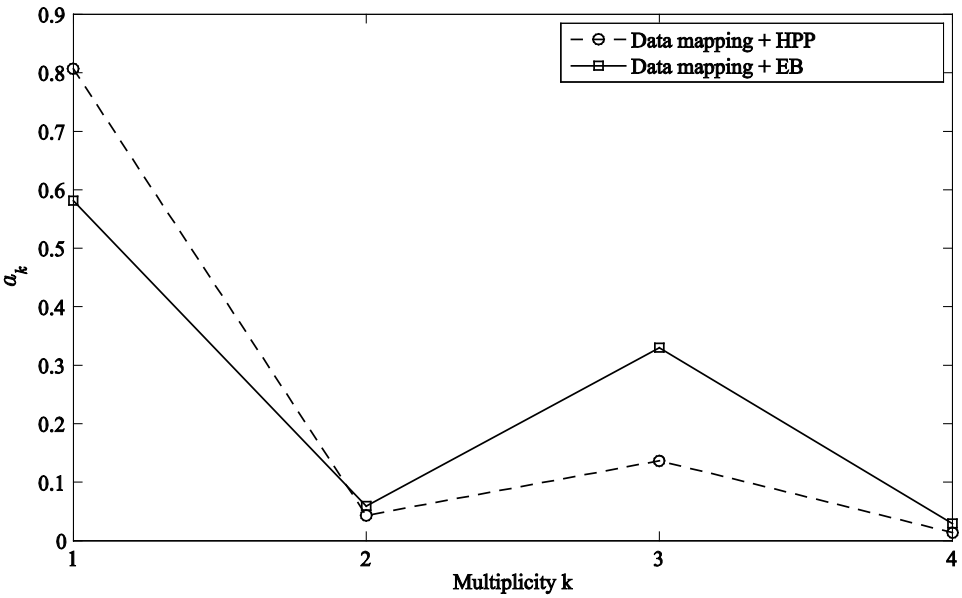

Figure 5.1: Comparison of alpha factors calculated by different approaches using database 1

5.3 Case study 2 using a large MOV dataset

As the same as section 5.2 above, the calculation of alpha factors will be repeated by constructing a larger dataset. The idea is to understand how different method works when sufficiently large data are available. The system sizes contain 2, 4, 8, 12, 16, and 24. Actual data are confidential.

By the means of data mapping, the failure data of all the different sizes can be transformed to be size 4, as if they are recorded in a typical 4-MOV system. Details have been given in Table 5.6.

Table 5.6: Data after mapping process: case study 2

System size	Number of failures			
n	$1/n$	$2/n$	$3/n$	$4/n$
2	154	5.1200	2.5600	0.3200
4	209	10	12	11
8	170.3571	2.3571	0.1429	0.0000
12	10.6909	1.0182	0.1939	0.0061
16	5.1670	0.5995	0.1170	0.0082
24	3.2464	0.0435	0.0000	0.0000
sum	552.4615	19.1383	15.0138	11.3343

Based on these data after mapping, one can easily apply the HPP assumption and EB algorithm again. By assuming HPP process, numbers of failures of all the four multiplicities are summed up respectively. Alpha factors are hence obtained as the same way in previous example.

Table 5.7: Alpha factors of 4-MOV system based on the assumption of HPP

Multiplicity k	1	2	3	4
$N_{k/n}$	552.4615	19.1383	15.0138	11.3343
$\alpha_{k/n}$	0.9239	0.0320	0.0251	0.0190

Similar to the above, apply EB approach to each multiplicity. One can gain the corresponding failure rate as follows.

Table 5.8: Failure rates of all the multiplicities in different systems using EB approach

Size	$\Lambda_{k/n}$			
n	1/4	2/4	3/4	4/4
2	1.74E-03	5.99E-05	3.24E-05	6.91E-06
4	2.92E-03	1.41E-04	1.58E-04	1.44E-04
8	1.91E-03	2.91E-05	7.42E-06	3.55E-06
12	1.42E-02	7.40E-04	9.03E-05	4.62E-05
16	3.18E-03	3.09E-04	7.22E-05	4.06E-05
24	6.85E-03	2.08E-04	6.94E-05	4.67E-05

Following Equation 2.17, it is easy to obtain the alpha factors with the outcomes of EB approach for the target system of size 4.

Table 5.9: Alpha factors of 4-MOV system applying EB approach

Multiplicity k	1	2	3	4
$\Lambda_{k/n}$	2.92E-03	1.41E-04	1.58E-04	1.44E-04
$\alpha_{k/n}$	0.8682	0.0420	0.0470	0.0429

In the next, plot the results of alpha factors using CCF data in the figure below. It can be concluded that the percentage taken by independent failure calculated by HPP method is still larger than that of EB method. But since the operation time is long enough for the size of 4, the difference is not as obvious as shown in Figure 5.1 above.

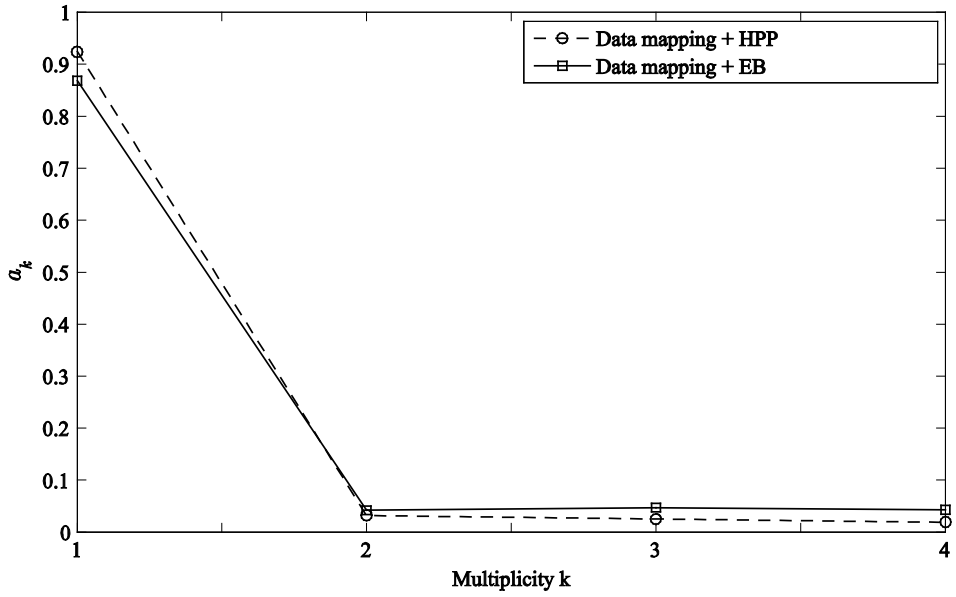

Figure 5.2: Comparison of alpha factors calculated by different approaches using database 2.

5.4 Summary

This Chapter presents two illustrative case studies in which various methods of CCF analysis are compared. It can be seen that the results using different data sets differ from each other. This may results from the different configurations, maintenances, environments and so on. Besides, the EB

algorithm yields higher CCF probability than HPP. The phenomenon may stem from the fact that independent failure is more likely to occur than CCF. Therefore, for the HPP method, summing up the independent failures (i.e., 1/4) among all sizes may probably lead to a higher proportion of independent failure than the data of a certain system its own, unless the operation time is long enough. Furthermore, it is not easy to judge which way is superior because the real value is never known in practice. However, if one is interested in plant-specific evaluation, the EB approach is supposed to be better.

Chapter 6
Conclusion and Challenges

6.1 Conclusion

First, this thesis has reviewed the current CCF modeling approaches, highlighting some of the disadvantages of existing methodologies such as the large uncertainty, reliance on overly simplified assumptions, and requirement for plentiful failure data in nuclear power plants. Second, a multivariate Poisson model incorporated with the empirical Bayes technique has been proposed for the CCF assessment. Third, Vaurio's EB method has been combined with traditional data mapping approach in order to assimilate information from source plants due to the limitation of failure data of the target plants. A comprehensive case study has been presented in Chapter 5. Chapter 3 and 4 are the main body of a paper accepted by 11th International Conference on Structural Safety & Reliability.

Since the Vaurio's empirical Bayes method has some distinctive properties such as broad applicability, higher accuracy and capability of quantifying the estimation uncertainty with posterior distributions, it has been adopted for estimating the failure rates in the framework of MVP model. According to the large sample of Monte Carlo simulation via MATLAB, some of the drawbacks of the current parametric models have been overcome by the newly proposed MVP model. Afterwards, two tests were conducted to examine the performance of MVP model in the evaluation of the whole system's reliability and the traditional CCF factors. Evidence has indicated that the MVP model is more reliable than the traditional ones in most cases because the calculation results using MVP model's outcomes are closer to the real values assumed in the simulation previously. Hence, it can be concluded that compared to the current parametric CCF models, the MVP model has better features.

In addition, the great features of EB method can be employed to make use of the failure data collected from source plants. The illustrative case studies in Chapter 5 has shown that the results of different methods can be quite different. Therefore, the combination of traditional data mapping and EB method is a useful way to perform plant-specific estimation. This method makes up for the limitation of data and generates specific evaluation results for the plant of interest.

6.2 Remaining challenges

Although great advantages of the MVP model have been proved, some drawbacks still remain. The MVP model only makes use of the data of the target plant. In addition, as the MVP model

distinguishes the components within the same CCCG, the shortage of available data becomes more obvious unless the observation time is long enough. Sometimes the data of each component may not be sufficient to perform EB estimation.

Apart from the above, the uncertainty of the EB estimation also exists in the evaluation of hyper-parameters which has not been taken into consideration. A robust assumption is made that the parameters α and β calculated by QBW or Vaurio's EB methods are assumed to be correct. However, it is necessary to take account of the uncertainty in the process of parameter estimation.

In the data mapping process in this thesis, a simplification assumption is made that all the component statuses of complete failure (C), degraded (D) and incipient (I) are considered as failure. However, in order to obtain a more realistic result, impact vectors should be considered in the analysis.

Appendix A

James-Stein estimator formula derivations

The James-Stein (JS) estimator proposed by Vaurio and Jänkälä (1992) takes another form other than EB in assimilating the information from others. The paper starts with the property of the MLE measured by the mean and variance of the estimator, namely $E[\hat{\lambda}_i]$ and $Var[\hat{\lambda}_i]$.To investigate this, the authors first evaluate the conditional and unconditional moments of the number of failure eventsN_i.

Givenλ_i, a realization of the unknown λ, the conditional moments of N_i can be found by routine as

$$E[N_i|\lambda_i] = \lambda_i T_i, \qquad Var[N_i|\lambda_i] = \lambda_i T_i$$

The unconditional moments of N_i can be found by taking integration overλ_i,

$$E[N_i] = \int E[N_i|\lambda_i]\pi(\lambda_i)d\lambda_i = T_i \int \lambda_i \pi(\lambda_i)d\lambda_i = T_i E[\lambda_i] = \mu T_i$$

The unconditional variance can be found by first evaluating the unconditional second original moment

$$E[N_i^2] = \int E[N_i^2|\lambda_i]\pi(\lambda_i)d\lambda_i = \int (\lambda_i T_i + \lambda_i^2 T_i^2)\pi(\lambda_i)d\lambda_i = T_i E[\lambda_i] + T_i^2 E[\lambda_i^2]$$
$$= \mu T_i + T_i^2(\mu^2 + \sigma^2)$$

Then,

$$Var[N_i] = E[N_i^2] - E^2[N_i] = \mu T_i + T_i^2\sigma^2$$

With these, the mean and variance of the MLE can be found as

$$E[\hat{\lambda}_i] = E\left[\frac{N_i}{T_i}\right] = \frac{E[N_i]}{T_i} = \mu$$

$$Var[\hat{\lambda}_i] = \frac{Var[N_i]}{T_i^2} = \frac{\mu}{T_i} + \sigma^2$$

The above two equations correspond to Eqs. (14) and (15) in the paper, respectively.

For the estimation of the population mean μ, the authors proposed an unbiased, minimum variance estimator as

$$m = \sum_{i=1}^{n} w_i \hat{\lambda}_i = \sum_{i=1}^{n} w_i \frac{N_i}{T_i}$$

It can be readily shown by using the above unconditional moments that

$$E[m] = \mu \sum_{i=1}^{n} w_i\,, \qquad Var[m] = \sum_{i=1}^{n} w_i^2 \left(\sigma^2 + \frac{\mu}{T_i}\right)$$

In order to obtain an unbiased estimate of μ, it is required that

$$\sum_{i=1}^{n} w_i = 1$$

Final determination of the weights w_i results from the solution to the constrained minimization problem with $Var[m]$ being the objective function and the above normalization equation being the constraint function.By using theLagrangian multiplier method, the constrained minimization problem can be modified to the following equivalent unconstrained minimization:

$$Minimize:\ f = Var[m] + 2\alpha\left(1 - \sum_{i=1}^{n} w_i\right)$$

where α is the Lagrangian multiplier. Taking derivative of f, one has

$$\frac{\partial f}{\partial w_i} = 2w_i\left(\sigma^2 + \frac{\mu}{T_i}\right) - 2\alpha = 0$$

for $i = 1, \dots, n$. This equation implies that w_i is inversely proportional to $(\sigma^2 + \mu/T_i)$, which is also $Var[\hat{\lambda}_i]$. Thus, the optimal weights for the estimation of μ is

$$w_{io} = \frac{\psi_i}{\sum_{i=1}^{k} \psi_i}$$

in which ψ_i is the precision[1] of $\hat{\lambda}_i$, the reciprocal of $Var[\hat{\lambda}_i]$.

With the optimal weights, the variance of the estimator for population mean is thus

$$Var[m_o] = \sum_{i=1}^{n} w_{io}^2\left(\sigma^2 + \frac{\mu}{T_i}\right) = \frac{1}{\sum_{i=1}^{n} \psi_i}$$

Finally the expected squared error of the JS estimator of the form $\bar{\lambda}_i = B_i m + (1 - B_i)\hat{\lambda}_i$ can be evaluated.

$$Q_i = E\left(\bar{\lambda}_i - \lambda_i\right)^2 = E\left(B_i m + (1 - B_i)\hat{\lambda}_i - \lambda_i\right)^2 = E\left(-B_i\left(\hat{\lambda}_i - m\right) + \left(\hat{\lambda}_i - \lambda_i\right)\right)^2$$

$$= B_i^2 E\left(\hat{\lambda}_i - m\right)^2 - 2B_i E\left(\left(\hat{\lambda}_i - m\right)\left(\hat{\lambda}_i - \lambda_i\right)\right) + E\left(\hat{\lambda}_i - \lambda_i\right)^2$$

Clearly, the third term is

$$E\left(\hat{\lambda}_i - \lambda_i\right)^2 = \int \left\{E\left(\hat{\lambda}_i - \lambda_i\right)^2 \middle| \lambda_i\right\} \pi(\lambda_i) d\lambda_i = \int Var(\hat{\lambda}_i|\lambda_i)\pi(\lambda_i)d\lambda_i = \frac{\mu}{T_i}$$

For the first term,

$$E\left(\hat{\lambda}_i - m\right)^2 = E\left(\left(\hat{\lambda}_i - \mu\right) - (m - \mu)\right)^2 = E\left(\hat{\lambda}_i - \mu\right)^2 - 2E\left(\hat{\lambda}_i - \mu\right)(m - \mu) + E(\mu - m)^2$$

[1] In Bayesian statistics, the precision is more often used than the variance.

in which $E\left(\hat{\lambda}_i - \mu\right)^2 = Var(\hat{\lambda}_i)$, $E(\mu - m)^2 = Var(m)$, and

$$E\left(\hat{\lambda}_i - \mu\right)(m - \mu) = E\left[\left(\hat{\lambda}_i - \mu\right)\left(w_i\hat{\lambda}_i - w_i\mu\right) + \left(\hat{\lambda}_i - \mu\right)\sum_{j\neq i} w_j\left(\hat{\lambda}_j - \mu\right)\right] = w_i Var(\hat{\lambda}_i)$$

Thus, the first term can be expressed as

$$E\left(\hat{\lambda}_i - m\right)^2 = (1 - 2w_i)Var\left(\hat{\lambda}_i\right) + Var(m)$$

For the second term,

$$E\left(\left(\hat{\lambda}_i - m\right)\left(\hat{\lambda}_i - \lambda_i\right)\right) = E\left[\left((1 - w_i)\left(\hat{\lambda}_i - \lambda_i\right) - \sum_{j\neq i} w_j\left(\hat{\lambda}_j - \lambda_i\right)\right)\left(\hat{\lambda}_i - \lambda_i\right)\right]$$

$$= (1 - w_i)E\left(\hat{\lambda}_i - \lambda_i\right)^2 = (1 - w_i)\frac{\mu}{T_i}$$

Putting these terms back, one thus has

$$Q_i = \frac{\mu}{T_i}[1 - 2B_i(1 - w_i)] + B_i^2\left[(1 - 2w_i)\left(\sigma^2 + \frac{\mu}{T_i}\right) + \sum_{j=1}^{n} w_j^2\left(\sigma^2 + \frac{\mu}{T_j}\right)\right]$$

$$= \frac{\mu}{T_i}[1 - 2B_i(1 - w_i)] + B_i^2(1 - w_i)^2\left(\sigma^2 + \frac{\mu}{T_i}\right) + B_i^2\sum_{j\neq i} w_j^2\left(\sigma^2 + \frac{\mu}{T_j}\right)$$

Finally, the expected sum of squared errors (SSE) is[2]

$$Q = \sum_{i=1}^{n} Q_i$$

[2] It is important to calculate the expected sum of square errors. Otherwise, the optimal weights for m would not be optimal for the JS estimator. In this case,

$$\frac{\partial Q_i}{\partial w_i} = 2B_i\frac{\mu}{T_i} - 2(1 - w_i)B_i^2\left(\sigma^2 + \frac{\mu}{T_i}\right) + 2\alpha = 0$$

Substituting the optimal shrinkage factor B_{io} into the equation yields

$$\alpha = (1 - w_i)B_i^2\left(\sigma^2 + \frac{\mu}{T_i}\right) - B_i\frac{\mu}{T_i} = w_i\frac{(\mu/T_i)^2}{\sigma^2 + \mu/T_i}$$

This is not a constant for all i, which is contrary to the property of a Langrangian multiplier.

To minimize Q under the normalization condition $\sum w_i = 1$, one can use the same Lagrangian multiplier approach that has been used earlier for m. Taking derivative of Q yields, for each $i = 1, \dots, n$,

$$\frac{\partial Q}{\partial w_i} = 2\sum_j B_j \frac{\mu}{T_j} - 2\sum_j B_j^2 \left(\sigma^2 + \frac{\mu}{T_j}\right) + 2w_i \left(\sigma^2 + \frac{\mu}{T_i}\right) \sum_j B_j^2 + 2\alpha$$

Also, we have

$$\frac{\partial Q_i}{\partial B_i} = -2(1 - w_i)\frac{\mu}{T_i} + 2B_i \left[(1 - 2w_i)\left(\sigma^2 + \frac{\mu}{T_i}\right) + \sum_{j=1}^{n} w_j^2 \left(\sigma^2 + \frac{\mu}{T_j}\right)\right] = 0$$

The authors claimed after equation (30) that Q_i is minimized by selecting the optimal weights $w_i = w_{io}$ which also minimize the variance of m. Based on this claim, the authors further derive the optimal shrinkage factor as

$$B_{i0} = \frac{\mu/T_i}{\sigma^2 + \mu/T_i}$$

which can be seen as the ratio of the sample variance to the total variance of the MLE.

Indeed, when $w_{io} = \frac{\psi_i}{\sum \psi_j} = \frac{\psi_i}{s}$ as shown above,

$$\left(\sigma^2 + \frac{\mu}{T_i}\right) = \frac{1}{\psi_i} = \frac{1}{w_i S}$$

Thus,

$$(1 - 2w_i)\left(\sigma^2 + \frac{\mu}{T_i}\right) + \sum_{j=1}^{n} w_j^2 \left(\sigma^2 + \frac{\mu}{T_j}\right) = \frac{1 - 2w_i}{w_i S} + \sum_{j=1}^{n} \frac{w_j^2}{w_j S} = \frac{1}{S}\left(\frac{1}{w_i} - 1\right)$$

Hence,

$$B_{io} = \frac{(1 - w_i)\,\mu/T_i}{(1/w_i - 1)/S} = \frac{\mu/T_i}{1/(S w_i)} = \frac{\mu/T_i}{\sigma^2 + \mu/T_i}$$

With the optimal shrinkage factors, the first derivative of Q with respect to w_i is expressed as

$$\frac{\partial Q}{\partial w_i} = 2\sum_j \left[B_{jo}\frac{\mu}{T_j} - B_{jo}^2 \left(\sigma^2 + \frac{\mu}{T_j}\right)\right] + 2w_i \left(\sigma^2 + \frac{\mu}{T_i}\right) \sum_j B_{jo}^2 + 2\alpha$$

Noting that

$$B_{jo}\frac{\mu}{t_j} - B_{jo}^2 \left(\sigma^2 + \frac{\mu}{T_j}\right) = 0$$

and $\sum_j B_{jo}^2 = \text{Constant}$, it is readily shown that the optimal w_{io} is the same optimal weight for m.

Finally, the minimized expected sum of square errors of the JS estimator is simply expressed as

$$Q_{io} = \sigma^2 B_{io} + \frac{\mu}{T_i} B_{io} w_{io}$$

Also note that Q_{io} is always smaller than

$$Var\left[\hat{\lambda}_i\right] = \frac{Var[N_i]}{T_i^2} = \frac{\mu}{T_i} + \sigma^2$$

Appendix B

Derivation of QBW's EB estimator

A gamma prior distribution is assumed as (1), which is conjugate to the likelihood function. Then the posterior is also a gamma distribution.

$$\pi(\lambda_i) = \frac{\beta^{\alpha}\lambda_i{}^{\alpha-1}e^{-\beta\lambda_i}}{\Gamma(\alpha)} \quad , \qquad \lambda_i, \alpha, \beta > 0 \tag{1}$$

Mean and variance of the prior are expressed by hyper-parameters.

$$\mu = \frac{\alpha}{\beta} \text{ and } \sigma^2 = \frac{\alpha}{\beta^2} \tag{2}$$

Due to the moment method, it is easy to obtain the following relations.

$$E(N_i|\lambda_i) = \frac{\alpha}{\beta}T_i \tag{3}$$

$$E\left(N_i^2|\lambda_i\right) = E\left(\lambda_i^2 T_i^2 + \lambda_i T_i\right) = E\left(\lambda_i^2\right)T_i^2 + \frac{\alpha}{\beta}T_i$$

$$= [Var(\lambda_i) + [E(\lambda_i)]^2]T_i^2 + \frac{\alpha}{\beta}T_i$$

$$= \left[\frac{\alpha}{\beta^2} + \left(\frac{\alpha}{\beta}\right)^2\right]T_i^2 + \frac{\alpha}{\beta}T_i \tag{4}$$

Let

$$U = \frac{\sum_{i=1}^{n} N_i}{\sum_{i=1}^{n} T_i} \text{and} \quad W = \frac{\sum_{i=1}^{n} N_i{}^2 - \sum_{i=1}^{n} N_i}{\sum_{i=1}^{n} T_i{}^2} \tag{5}$$

Then the expectation of *U* and *W* are

$$E(U) = \frac{\sum \frac{\alpha}{\beta}T_i}{\sum T_i} = \frac{\alpha}{\beta} \tag{6}$$

$$E(W) = \frac{\sum\left[\frac{\alpha}{\beta^2} + \left(\frac{\alpha}{\beta}\right)^2\right]T_i^2}{\sum T_i^2} = \frac{\alpha}{\beta^2} + \left(\frac{\alpha}{\beta}\right)^2 \tag{7}$$

From equation (6) and (7), the obtained in the following form.

$$\hat{\alpha} = \frac{U^2}{W - U^2} \text{ and } \hat{\beta} = \frac{U}{W - U^2} \tag{8}$$

And Quigley *et al*. have already proved the above are consistent estimators of the hyper-parameters α and β.

Finally, the posterior distribution is shown as (9). And the mean and variance of the posterior of each event can be calculated from (10).

$$\pi(\lambda_i | N_i, \alpha, \beta) = \frac{(\beta + T_i)^{\alpha + N_i} {\lambda_i}^{\alpha + N_i - 1} e^{-(\beta + T_i)\lambda_i}}{\Gamma(\alpha + N_i)} \tag{9}$$

$$M_i = \frac{(\alpha + N_i)}{(\beta + T_i)} \text{ and } V_i = \frac{(\alpha + N_i)}{(\beta + T_i)^2} \tag{10}$$

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