Sequential Decision Making Schemes in Inventory and Transportation Environments

by

Marc Coulombe

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Management Sciences

Waterloo, Ontario, Canada, 2012

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Many mathematical models exist for the simultaneous optimization of transportation and inventory functions. A simultaneous model, while giving the lowest total cost, may not be easily implemented in a firm with decentralized transportation and inventory departments. As such, this thesis studies sequential models, where the primary department is artificially given the authority to make some set of decisions prior to the decisions made by the secondary department. Some known formulations for simultaneous models are studied in an attempt to create a sequential process for the same environment. Finally, a generalized sequential approach is developed that can be applied to any transportation and inventory model with separable costs. The generalized approach allows for the full optimization of the primary departmental costs, and then sequentially allows the optimization of the secondary departmental costs subject to a maximum allowable increase in the costs of the primary department. The analysis of this sequential approach notably reveals that when the relative deviation from the optimal cost of each department is equal, a reasonable solution with respect to total cost is attained. This balance in relative deviation is defined as the fairness point solution. Differing cost scenarios are thus tested to determine the relationship between the cost ratio among departments and the performance of the fairness point solution. The fairness point solution provides an average deviation of total cost from the total optimal cost of less than 1% in four of the seven scenarios tested. Other sequential approaches are discussed and fairness with respect to these new approaches is considered.

Acknowledgements

I would like to take this opportunity to express my gratitude to my supervisor, Professor James Bookbinder, for his support in preparing this Master's thesis. I would also like to thank my readers, Professor Fuller and Professor Ozaltin, for their input.

Dedication

This thesis is dedicated to my family and friends.

Table of Contents

List of Figures

List of Tables

1. Introduction

Many mathematical models have been designed to optimize either transportation decisions or inventory decisions; a smaller subset of these models aim to simultaneously optimize both transportation and inventory decisions. Whether minimizing cost or maximizing potential profit, these simultaneous models necessarily assume the existence of a centralized decision making power, as some optimal decisions may, on the surface, seem unfavourable to one of the two parties. However, in reality, many firms implement a decentralized approach where the individual departments are solely concerned with optimizing their own objective function. These firms consider the transportation department and inventory department as separate profit or cost centres, despite the direct relationship both departments have on one another. In such a decentralized environment, it is possible that the decisions of one department can heavily affect the objective function of the other. As such, it may be pertinent to study the effect of one department's decisions on the other's objective function.

A possible way to study the effect of one party's decisions on the ability for the other party to optimize its own objective function is known in economics and game theory as a Stackelberg game. In such a game, one party is known as the leader (whose decisions are made known first) and the second party is known as the follower (whose decisions are made following those of the leader). In a Stackelberg game, the leader optimizes his own objective function by analyzing the response function of the follower. In this thesis, models with a leader and follower will be explored. These models don't necessarily fit the strict definition of a Stackelberg game, as the leader does not necessarily consider the response function of the follower. Therefore, we will refer to such models to as *sequential* models.

The study of sequential models of that type, representing the decisions of the transportation and inventory departments, could help to understand the types of relationships with regard to sources of power within an organization under various scenarios.

2. Literature Review

An extensive review of much of the literature concerning joint transportation and inventory problems was conducted by Schwarz et al. and presented in a working paper (Schwarz et al., 2004). The review categorizes the literature based on ten methods of classification: subject, horizon, review, vehicle, product, orders/demands, assignment, routing, allocation, and policy variables. The *subject* identifies the type of problem to be solved. Some commonly known subjects are the inventory routing problem, vehicle routing problem, and delivery dispatching problem. Next, the time *horizon* is categorized as finite or infinite and the *review* system is classified as periodic or continuous. The *vehicle* category is subcategorized by the number of vehicles (limited or unlimited) and by the capacity of vehicles (capacitated or uncapacitated).

The *product* category indicates the number of products (single or multiple), while the *orders/demands* category indicates whether orders or demands are considered to be deterministic or stochastic. The *assignment* category involves the assignment of retailers to vehicles, groups of retailers to vehicles, and various other assignments. The *routing* policy can be described as either static or dynamic policies, with static policies implying a route is established and is used repeatedly while dynamic policies allow for the routes to be updated and changed over time. Similarly, *allocation* of each vehicle's inventory to retailers can be grouped by static or dynamic policies. Finally, the *policy variables* indicate if the variables within the model are transportation policy variables, inventory policy variables, or a mixture of both (joint transportation-inventory policy variables).

Given the breadth of the review by Schwarz et al. (2004), the same classification system will be used throughout the literature review in this thesis. Also, due to the scale of the literature concerning joint inventory and transportation decisions, our review will first place a focus on those papers that consider multiple products. This review will then branch into some of the more relevant papers that consider one single product with joint transportation-inventory policy variables, as it can be seen that the methods proposed in this thesis can certainly be applied in those cases. One can remark that while considering models with multiple products, most of these models can be trivially reduced to a single product. It is also noteworthy that the number of products has little effect on most of the procedures developed in this thesis.

Adelman (2004) presents an inventory routing problem with an infinite horizon, periodic review, an unlimited number of vehicles of non-identical capacity, multiple products, stochastic product-specific demand, retailer selection (a set of retailers to be visited is determined each time before the vehicle leaves), static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Note that a routing policy is not applicable to this model. Adelman uses optimal shadow prices in linear programs to attribute potential cost values to current actions. Shipment itineraries are generated and transportation costs are then allocated. However, this is the extent to which this model behaves in a sequential manner.

Bassok & Ernst (1995) present a product distribution problem with an infinite horizon, periodic review, a limited number of vehicles of non-identical capacity, multiple products, stochastic retailer-specific and product-specific demand, vehicle assignment (retailers are assigned to vehicles), a static routing policy, dynamic allocation of vehicle inventory to retailers, and inventory policy variables. It is shown that the problem is separable into two sub-problems: a product allocation problem and a space allocation problem. The former problem is solved using a recursive formula while the latter is solved using a Monte-Carlo method. It is important to note that in their study, the transportation route is assumed to be known in advance. There is no mention in the paper regarding the split between transportation and inventory costs.

Bertazzi et al. (2000) analyze a shipment problem with an infinite horizon, periodic review, an unlimited number of vehicles of non-identical capacity, multiple products, deterministic product-specific demand, vehicle assignment (retailers are assigned to vehicles), a direct-delivery routing policy, static allocation of vehicle inventory to retailers, and inventory policy variables. Dominance rules are derived to help analyze potential solutions by further tightening the bounds on variables. These rules can be used with branch and bound algorithms to improve their efficiencies. Finally, heuristic approaches to solving the shipment problem are compared to EOQ-based algorithms. The branch and bound algorithm using the derived dominance rules performs significantly faster than the same branch and bound algorithm without the use of those rules. Depending on the number of available frequencies, the algorithm using dominance rules terminated in 0.18% to 1.4% of the time required for the algorithm without the dominance rules. The best heuristic of those tested performed well, with an average error below 0.34% for all the tested instances, and found the optimal solution for approximately 36% of those instances. There is no mention in the paper regarding the split between transportation and inventory costs.

Bertazzi et al. (2002) study an inventory routing problem with a finite horizon, periodic review, one single capacitated vehicle, multiple products, deterministic retailer-specific demand, retailer selection (a set of retailers to be visited is determined each time before the vehicle leaves), a dynamic routing policy, and transportation policy variables. Note that allocation is not applicable to this model. Using a heuristic approach, the method presented seeks to minimize total cost. The analysis provided by Bertazzi et al. (2002) is of particular interest, as the heuristic was used to explore the total costs, transportation costs, and inventory costs while varying the objective function between subsets of those costs. The maximum increase in total costs by using different objective functions was seen to be only 14%. Although there was no second optimization of costs afterwards, the approach used is somewhat similar to the approach in this thesis.

Fumero and Vercellis (1999) discuss a production and distribution planning problem with a finite horizon, periodic review, a limited number of vehicles of non-identical capacity, multiple products, deterministic retailer-specific and product-specific demand, a dynamic routing policy, static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Note that vehicle assignment is not considered in this model. The solution method proposed uses a relaxation to split the problem into separate lot-sizing and vehicle routing sub-problems. Although the paper separately considers production, inventory, distribution, and routing costs, there was little discussion of the interaction between such costs.

Qu et al. (1999) analyze an integrated inventory and transportation problem with an infinite horizon, periodic review, one single uncapacitated vehicle, multiple products, stochastic product-specific demand, retailer selection (a set of retailers to be visited is determined each time before the vehicle leaves), a dynamic routing policy, static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. In this paper, total cost is comprised of transportation costs (dispatching, stopover, and routing) as well as inventory costs (ordering, holding, and backlog). Separate methods are used for making inventory and routing decisions simultaneously. Namely, an independent transportation problem is solved as a sub-problem while the inventory problem is used as a master problem. Once again, the focus of that research is not the interaction of transportation cost and inventory cost.

Viswanathan and Mathur (1997) study an integrated inventory and vehicle routing problem with an infinite horizon, periodic review, an unlimited number of vehicles of identical capacity, multiple products, deterministic retailer-specific and product-specific demand, vehicle assignment (retailers are assigned to vehicles), static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Note that a routing policy is not considered in this model. The paper first proposes a heuristic to solve the uncapacitated-vehicle version of this problem, and proceeds to develop the heuristic to treat the capacitated case. Only total cost is considered for analysis while the interaction between transportation and inventory cost is not considered.

Cetinkaya and Lee (2000) present a vendor-managed inventory system in which scheduling of shipments and replenishment of stock is considered. The problem has an infinite horizon, periodic review, one single capacitated vehicle, a single product, stochastic independent and identically distributed demand for each retailer, static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Retailer and vehicle assignment as well as routing policies are not applicable. Long run average total cost is minimized using renewal theory. Expected total costs per replenishment cycle are broken into the following parts: the costs of inventory replenishment, delivery, carrying inventory, and customer waiting times. Although the total costs are presented separately, the solution method primarily uses the Hessian of the total cost function, and as such the separate costs are not further considered separately.

Chan and Simchi-Levi (1998) analyze a three-level distribution system with an infinite horizon, continuous review, an unlimited number of vehicles of non-identical capacity, a single product, constant deterministic retailer demand, grouped retailers, a static routing policy and static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Bounds on costs are analyzed in order to develop an efficient algorithm to solve the problem. Total transportation costs and inventory costs are not considered separately in any fashion.

Chan et al. (1998) discuss an inventory-routing problem for a distribution system with an infinite horizon, periodic review, an unlimited number of vehicles of non-identical capacity, a single product, deterministic retailer-specific demand, grouped retailers, a static routing policy, static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Several bounding methods are used on the objective function that is minimized, which represents "system-wide infinite-horizon costs", to develop an asymptotically-optimal efficient algorithm for the policies studied. This algorithm performs well for problems that were randomly generated. Long run average cost is considered for bounding and subsequent analysis and, as such, separate transportation and inventory costs are not analyzed.

Christiansen and Nygreen (1998) study a ship-routing problem with time windows, but with harbours rather than retailers. The problem has a finite horizon, a limited number of vehicles of non-identical capacity, a single product, harbour-specific deterministic demand, vehicle assignment (harbours are assigned to vehicles), a dynamic routing policy, static allocation of vehicle inventory to harbours, and joint transportation and inventory policy variables. A review policy is not applicable to this problem. The approach uses a Dantzig-Wolfe decomposition method and decomposes into sub-problems for each harbour and ship. While the costs of the sub-problems can be analyzed separately, the transportation and inventory costs are not analyzed separately.

Kleywegt et al., (2002) consider a direct-delivery, stochastic inventory routing problem with an infinite horizon, periodic review, a limited number of vehicles of non-identical capacity, a single product, stochastic retailer-specific demand, a direct-delivery routing policy, static allocation of vehicle inventory to retailers, and joint transportation and inventory policy variables. Vehicle assignment is not applicable in this model. The proposed solution uses Markov decision processes to estimate costs and to develop a method for choosing the optimal action. There is no focus in this study on the interaction of transportation costs and inventory costs.

The reader will have noted that, in most literature on joint transportation and inventory problems, the costs of each department are not considered separately. Therefore, the focus of this review now shifts to articles not included in the review by Schwarz et al.

(2004) that, in some way, do allow for the separation of the transportation and inventory departments.

Following the preceding review, the same authors detailed the interactions between routing and inventory in an attempt to represent the tradeoffs between those departments (Schwarz et al., 2006). It is assumed that a manager of such a distribution system, as presented in the paper, has been given the authority to make policy decisions for both inventory and transportation departments. The method starts with an optimal static transportation route, then determines the interactions between routing and inventory policy decisions, and then a heuristic routing policy determines the final routes.

Blumenfeld et al. (1985) examine tradeoffs not only between transportation and inventory but also production costs. A link is established between transportation and production costs, and the effect of both those costs on inventory costs is studied under many varied scenarios.

Burns et al. (1985) consider the tradeoffs between inventory and transportation cost by making simplifying assumptions. The tradeoffs are used to determine optimal values for parameters governing shipment size and the number of customers in a delivery region.

One way to treat departmental objectives separately is to use several objective functions, i.e. multi-objective optimization. Liao et al. (2011) describe an evolutionary approach for a vendor managed inventory problem. Although the transportation and inventory costs are not split apart (total cost, fill rate, and responsiveness act as three separate objective functions), the paper provides some detail as to the treatment of competing objective functions. While total cost is to be minimized, both fill rate and responsiveness are to be maximized, presenting a situation in which solutions obtained may not be optimal to any of the three individual objectives. The approach in the paper by Liao et al. is beyond the scope of this thesis, but the motivation can be seen to be similar in many respects.

Esparcia-Alcazar et al. (2009) considered both single and multiple objective approaches to the joint inventory and transportation problem. The goal was to determine if an approach using separate objective functions for inventory and transportation could provide an advantage over the approach using a single combined objective function. This research provides the most similar motivation to that of this thesis. In the cases studied, it was concluded that the approach using only the single objective function was superior to the alternative. Although the conclusion is not favourable for the multi-objective approach, the motivation provided by Esparcia-Alcazar et al. for considering the transportation and inventory objectives separately is largely the same as it is in this thesis. The objectives of minimizing both transportation and inventory costs can be seen to be contrary to each other and, thus, it is expected generally that decreasing the cost of one department will increase the cost of the other. The authors set out to determine if the results of the multi-objective approach are “better” than the single objective approach. As will be seen in our own work, “better” must be defined more thoroughly than simply the solution with the lower cost, especially when dealing with algorithms that solve the problems to optimality. There are many soft constraints and concepts, such as “fairness”, that cannot necessarily be directly included in the objective functions for each department. The determination in the paper by Esparcia-Alcazar et al. thus does not vitiate the approach taken in this thesis.

Nozick and Turnquist (2001) study the tradeoffs between facility costs, inventory costs, transportation costs, and finally customer responsiveness in solving for optimal locations of distribution centres. A weight parameter, W, is implemented and assigned to the objective of minimizing unmet demand. As such, when W is varied from small to large and the model is solved for each instance, a family of tradeoff solutions is identified. This family creates an efficient set or efficient frontier from which an appropriate solution can be chosen. The solutions on the efficient frontier vary from lowest cost to lowest unmet demand. The challenge for management when presented with this set of solutions is to choose a specific element that is acceptable and meets many of the soft constraints and other criteria that are not captured within the model.

Bhaskaran and Turnquist (1990) consider a multiple-facility location problem with several objective functions. It is shown that when optimizing over a single objective function, substantial deviation can occur with respect to other objectives, indicating a need for a multi-objective approach.

Aguezzoul and Ladet (2007) recognize the highly interrelated nature of transportation and inventory and their importance to total logistics cost in the area of supplier selection. The inherent multi-objective nature of supplier selection is discussed and is grouped into the main categories of linear weighting models, mathematical progrraming models, and statistical approaches. It was found that little literature existed showing the effect of transportation decisions on supplier selection. In order to consider multiple objectvies, Aguezzoul and Ladet normalize the data in a fashion similar to the normalization performed in this thesis. The absolute value of the relative deviation for each objective from its goal is employed as the performance measure for that objective. A weighted sum of these performance measures is used as the overall objective function. As is typical with weighted objective functions, a set of solutions corresponding to varied values of the weights can be presented to the decision maker, at which point a subjective choice may be made.

In a book by Zeleny (1982), entitled *Multi Criteria Decision Making*, an entire chapter is devoted to the concept of “compromise programming”. Compromise is defined there as “an effort to approach or emulate the ideal solution as closely as possible.” It is noted that the acceptability of a compromise can be easily measured with respect to an ideal solution. Such measurements could be the “closeness to the ideal” or “the remoteness from the anti-ideal.” Zeleny discusses a way to measure closeness to the ideal in a relative sense, similar to the method utilized in Section 3.3 of this thesis. The main problem with this technique is easily being able to define an ideal solution with respect to all necessary values of a given firm. In fact, Russell L. Ackoff is quoted in the chapter noting that “To the extent that OR's concept of optimality fails to take extrinsic value of ends, progress towards ideals, into account, it is seriously deficient." The chapter goes on to describe multi-criteria decisions in a group-decision-making environment. Typically a huge set of non-dominated solutions is generated by an optimization procedure, and the decision makers are responsible for filtering the available solutions so as to intelligently identify the most preferable solution in the reduced set. The author ends in noting that literature concerning compromise programming, evolving from the "notion of the displaced ideal", is relatively limited.

It can be remarked that taking into account all intrinsic and extrinsic value judgements in the objective function (or functions) of a mathematical model is very difficult if not impossible. Bentley and Wakefield (1997) discuss the concept of choosing acceptable solutions from a Pareto-optimal set using multi-objective genetic algorithms. It is once again observed that choosing an acceptable solution manually is often an arduous chore, due to the large size of the set of Pareto-optimal solutions. A method is discussed for using genetic algorithms to hone in on only acceptable solutions within the Pareto-optimal set. To do this, the authors provide the following definition: "A solution is an acceptable solution if it is Pareto-optimal and it is considered to be acceptable by a human." This definition shows, again, the problem of objectively defining the value judgements of the decision makers.

Lucas (2011) notes that, when considering a Pareto-optimal set, the decision criteria need to be prioritized, or preferences that incorporate the subjective nature of the environment need to be established. To do this while considering the full contextual situation, three methods can be considered. Firstly, the prioritization can be performed before the optimization procedure by developing a scalar fitness function, which is appropriate when preferences are well defined and objectives are changing. Secondly, it can be performed after the optimization procedure by selecting the solution from the full Pareto-optimal set, which is appropriate when objectives are well defined and preferences are changing. Lastly, prioritization can be done in an interactive manner which encourages convergence on a solution deemed to be acceptable by the decision maker. This last method is appropriate when both objectives and preferences are dynamic. In the models studied in this thesis, objectives are well defined and preferences may be dynamic, therefore it may be prudent to select the solution from the full Pareto-optimal set. That being said, the process that we present can be applied to a wide variety of problems. Therefore, it is important to keep this prioritization method in mind when applying the methods proposed here.

Luss (1999) discusses resource allocation problems in which scarce resources need to be distributed equitably among competing parties. In particular, Luss discusses performance functions for those parties, including a method termed the "weighted shortfall of the selected activity level from a specified target." The weighted shortfall method discussed draws some similarities to the methods proposed in this thesis. Equitable solutions are defined similar to pareto-optimal solutions, such that for an equitable solution "no performance function value can be improved without either violating a constraint or degrading an already equal or worse-off performance function value." A lexicographic minimax solution method is used to determine equitable solutions, but that method is beyond the scope of this thesis.

Kostreva et al. (2004), Ogryczak (2000), and Marsh and Schilling (1994) respectively discuss equity, equality, or inequality with regard to multi-criteria or multi-objective approaches. Each goes beyond the typical measures of efficiency or effectiveness to explore concepts of justice and fairness, or the perception of those attributes. Though defined for different problems, all propose objective measures that include "fairness", similar to those objectives that are proposed in this thesis.

Considering the lack of analysis on the effects of transportation costs and inventory costs individually on each other and thus on total cost, Chapter 3 will begin to explore possible models that consider one department sequentially after the other.

3. Sequential Model Development

In this chapter, several sequential models will be developed, starting with a simple model to better illustrate the sequential procedure, then moving to a more complex model with a generalized approach.

3.1 Simple Sequential Models

In this section, a simple simultaneous transportation and inventory model is presented and an attempt is made to alter this model into the form of a sequential model. A simultaneous model similar to the one here is developed in Bertazzi et al. (2000).

To begin, the assumptions, data, parameters, and variables of the simultaneous model are outlined below. Note that this is relatively simple, but even so, it is still a mixed integer linear programming formulation, that can be difficult to solve to optimality with reasonably sized input.

Model Assumptions:

- One origin (supplier)
- One destination (retailer)
- All shipments are dispatched with a fixed frequency
- Demand rate and production rate are constant and equal
- Trucks have a stated capacity and cost
- Unlimited trucks are available at any given time
- Products can be shipped in any fractional amount

Sets and Parameters in the Simultaneous Model

I Set of products to be shipped, with individual index i

J Set of predefined frequencies, with individual index j

f_j Represents the j^{th} frequency

r_i The rate of production and demand of product i

v_i Volume of product i

h_i Denotes the inventory holding cost of one unit of product i for one time unit

V Represents the capacity of each truck

C The fixed cost of one truck

Decision Variables in the Simultaneous Model

x_{ij} The fraction of product i which is shipped at frequency f_j

y_j The number of trucks that travel at frequency f_j

Simultaneous Model

Minimize:	$\sum_{i \in I} \sum_{j \in J} \frac{h_i r_i x_{ij}}{f_j} + \sum_{j \in J} C f_j y_j$		(1)
Subject to:	$\frac{1}{f_j} \sum_{i \in I} v_i r_i x_{ij} \leq V y_j$	$j \in J$	(2)
	$\sum_{j \in J} x_{ij} = 1$	$i \in I$	(3)
	$0 \leq x_{ij} \leq 1$	$i \in I, j \in J$	(4)
	$y_j \geq 0$, integer	$j \in J$	(5)

Now, to better comprehend the effects of the transportation department's decisions on the inventory costs, a sequential version of the preceding model is presented. Costs in the original objective function will be split apart by department. The inventory costs are used as the response function for that department, while the transportation costs are used as the objective function in the transportation problem.

So, if we consider the sequential version where the Transportation decisions (shipment frequencies) are decided first, we have the following:

<u>Transportation Problem</u>

Minimize:	$\sum_{j \in J} Cf_j y_j$	(Trans.)
Subject to:	Inventory Problem Feasibility	

Note that the transportation problem is solved by deciding which of the predefined frequencies in set J are to be made available to the inventory department. The transportation problem is solved knowing that the inventory decision will be made to minimize cost, using the Inventory Problem that follows. We remark that the inventory problem must be feasible, given the resulting output of the transportation problem. If the inventory problem is not feasible, then neither is the transportation problem.

Inventory Problem

Minimize:	$\sum_{i\in I}\sum_{j\in J}\frac{h_i r_i x_{ij}}{f_j}$		(Inv.) (Decision variables: x_{ij}, y_j)
Subject to:	$\frac{1}{f_j}\sum_{i\in I} v_i r_i x_{ij} \le V y_j$	$j \in J$	(2)
	$\sum_{j\in J} x_{ij} = 1$	$i \in I$	(3)
	$0 \le x_{ij} \le 1$	$i \in I, j \in J$	(4)
	$y_j \ge 0$, integer $j \in J$		(5)

To better illustrate the way in which the sequential version of the model is used, we consider a small example and analyze the results. This simple example may then be expanded to provide a better understanding of more complicated situations.

So, assume that there are only two available frequencies from which to choose: ship every day or ship every second day. Also, assume that there is only a single product to be shipped. Now, the transportation department has the choice: to make only one of the two frequencies available, or to make both available.

Clearly, the transportation department, while solely considering its own costs, has the motivation to allow only shipments with the lowest frequency (or largest period). In this way, a higher fraction of truck volume can be utilized and thus lower transportation costs achieved. Since just one of two frequencies is made available, the inventory decision (x_{ij}) becomes trivial, as all products must be shipped at that frequency.

This simple result also extends to cases where more than two frequencies are available and when more than one product needs to be shipped. The transportation department will always choose to make only the frequency with the largest period available. This result is clearly not an accurate representation of the actual behaviour in such an environment. To encourage the transportation department to make other frequencies available, one must extend this simple model to account for additional costs and requirements such as a maximum inventory position allowed, costs associated with service levels, and service-level constraints (when considering stochastic models with probabilistic demand). However, it may be more prudent to explore various other existing simultaneous models, rather than attempting to extend the simple model above.

Also, if we consider the other sequential version of the simple model, where now the inventory decisions are made first, we are left with a similar problem: decision variables for the inventory problem depend entirely on those of the transportation problem. For example, if the inventory department chooses its inventory positions (essentially by setting x_{ij} and y_j), then whenever y_j takes a positive value, the transportation department must make the corresponding frequency, f_j, available.

So as to study the effects of each decision on the other's costs using this style of approach, we must find or develop a simultaneous model with symmetry that can be split apart, as above, while maintaining the effectiveness of decision making by each party, whichever goes first in a sequential model. Given that such symmetry is obviously not always the case, an approach is considered next that allows an optimization procedure by one department, followed by a reoptimization procedure by the other.

3.2 Generalized Sequential Approach

To accomplish the approach just discussed, we consider a simultaneous model presented by Bertazzi et al. (2002), whose assumptions, data, parameters, and variables are now outlined. Although the model presented in that paper is not necessarily symmetrical between transportation and inventory department decisions, it is of particular interest, as the objective function is already broken down into three separable parts: transportation cost, retailer inventory cost, and supplier inventory cost. The method for separating costs may provide some insight into the separation of costs in the sequential approach.

Model Assumptions:

- One origin (supplier)
- Several retailers (destinations)
- Multiple periods
- Deterministic supply and demand
- Trucks have a given capacity and cost
- Shipments occur in discrete time instants
- Lower and upper levels of inventory for each product exists at each retailer
- If a retailer is visited, its inventory position is restored to the upper level
- A single product and single vehicle are studied (however, this can be relaxed later)

Data and Parameters in the Simultaneous Model

R Represents the set of retailers, where $R = \{1, 2, \ldots, n\}$

H The time horizon

t Denotes the current discrete time, where $t \in T = \{1, 2, \ldots, H\}$

r_{it} Amount of product sold at retailer i at time t

r_{0t} Represents the amount of product made available at the supplier at time t

L_i The minimum inventory level of retailer i

U_i The maximum inventory level of retailer i

C Truck capacity

c_{ij} Represents the transportation cost from node *i* to node *j*

h_i Unit inventory cost at node *i*

Decision Variables in the Simultaneous Model

x_{it} Amount of product dispatched to retailer *i* at time *t*

Note that $x_{it} = U_i - I_{it}$ if product is dispatched to retailer *i* at time *t*, and $x_{it} = 0$ otherwise.

$y_{it} \in \{0,1\}$ Denotes whether or not product is dispatched to retailer *i* at time *t*

$a_{ijt} \in \{0,1\}$ Indicates if arc *ij* is used in the delivery route for time period *t*

Decision Variables in the Simultaneous Model

I_{it} Level of inventory at retailer *i* at time *t*

B_t Level of inventory at the supplier at time *t*

ICR Total retailer inventory cost, where ICR = $\sum_{t \in T \cup \{H+1\}} h_i I_{it}$

ICS Total supplier inventory cost, where ICS = $\sum_{t \in T \cup \{H+1\}} h_0 B_t$

TC Total transportation cost, where TC = $\sum_{t \in T, i \neq j} a_{ijt} c_{ij}$

Simultaneous Model

Minimize:	$ICR + ICS + TC$		(6)	
Subject to:	$B_{t+1} = B_t + r_{0t} - \sum_{i \in R} x_{it}$		(7)	Supplier inventory constraints
	$I_{i,t+1} = I_{it} + x_{it} - r_{it}$		(8)	Retailer inventory constraints
	$\sum_{i \in R} x_{it} \leq C$	$t \in T$	(9)	Capacity constraints
	$\sum_{i \in R} x_{it} \leq B_t$	$t \in T$	(10)	Stockout constraints at the supplier
	$I_{it} \geq L_i$	$t \in T', i \in R$	(11)	Stockout constraints at the retailer
	$x_{it} = y_{it}\left(U_i - I_{it}\right)$	$t \in T', i \in R$	(12)	Order-up-to constraints
	y_{it} binary		(13)	
	$\sum_{i \neq j} a_{ijt} = y_{jt}$	$t \in T, j \in R$	(14)	TSP style constraints
	$\sum_{i \neq j} a_{ijt} = y_{it}$	$t \in T, i \in R$	(15)	TSP style constraints
	Sub-tour Elimination Constraints		(16)	TSP style constraints

Observe that (14), (15), and (16) form a travelling salesman problem (TSP) style sub-problem. Essentially, a travelling salesman problem is solved for each time period, only including those nodes that must have product delivered at that *t*. There are many ways to model and solve the TSP sub-problem; however that is not the focus of this study. As such, the classical TSP constraints are listed above. It is important to realize that as the number of retailers grows, then due to the TSP sub-problem, it may be necessary to consider other modeling methodologies or heuristics to be able to reach a solution in a reasonable computational time. In fact, Bertazzi et al. (2002) discuss this problem, and develop an appropriate heuristic for the simultaneous model presented above with 50 retailers and 30 periods.

Now, given that *ICR* and *ICS* are independent of the arcs used within the transportation routes, we see that this problem does not take the exact form of a Stackelberg game. However, it is possible to view the effects of sequential decision making if constraints are added to ensure minimal deviation from an initial decision. For example, if the transportation department solves the problem above (subject to all constraints 7 through 16) while only minimizing *TC* (with an optimal cost of TC_T^*), the inventory department can then resolve the problem while only minimizing *ICR* + *ICS* with an additional constraint that the new transportation cost must not exceed a certain threshold. The problem can be tested using various threshold constraints. For example: $TC \leq (1.1)(TC_T^*)$, $TC \leq (1.2)(TC_T^*)$, and $TC \leq (1.3)(TC_T^*)$; which correspond to a maximum increase in transportation cost of 10%, 20%, and 30%, respectively. We will refer to this method as the “incremental sequential model”.

Bertazzi et al. (2002) do perform the optimization of the above problem using the restricted transportation and inventory objective functions; however, there is no study of further optimization from the perspective of the other department.

Although adding these threshold constraints and reoptimizing with a different objective function does not strictly adhere to the form of a Stackelberg game, it may provide some valuable insights into the tradeoffs of cost in one department to the other. Once the second optimization has taken place, one can compare the percentage decrease in cost in one department to the percentage increase in the other, as well as comparing the absolute changes in cost. It may also help to determine if the decisions of one department dominate the decisions, and therefore costs, of the other department. Ideally, the results would allow an answer to the following questions: In an environment where the authority to decide first can be assigned to either the transportation or inventory department, under which conditions should one or the other department be allowed to decide first? Which sequence of decisions allows for a total cost closest to the overall simultaneously-minimized cost?

Under many of the methods discussed in this thesis, it is assumed that the optimal total cost is attainable using a mathematical model. This assumption necessitates that both departments are aware of the other's constraints and variables. It will further be assumed that both departments are greedy and seek to minimize only their own cost, even at the detriment of the other department.

To begin to answer the questions above, the model is represented in the mathematical programming language, AMPL. To increase the robustness of the programmed model, it is divided into three sections: the model file, the data file, and the run file. The model file contains a listing of sets, parameters, and variables. It also lists the objective functions and constraints that are used. However, as much of the data as possible is listed separately from the model file in the data file. Finally, the run file gives AMPL instructions on how to execute the optimizations and deals with inputs and outputs to the system. As AMPL is a modelling language (and not, strictly speaking, a language that performs the optimization), the AMPL files are used by CPLEX to perform the optimization. (All relevant AMPL files can be found in Appendix 1.) Note that the model file is an extended version of the basic travelling salesman model provided on the AMPL website. (Fourer, 2004)

Our model has travelling salesman style sub-problems on a subset of all nodes. To accommodate these sub-problems, the model includes two distinct sets of sub-tour elimination constraints and a set of adapted connectivity constraints. The first set, Elimination1, ensures that no subset of nodes contains a sub-tour. This is done by limiting the number of arcs with both endpoints in the subset to the size of the subset minus 1. The second set, Elimination2, is similar to the first; however, it ensures that the number of arcs with both endpoints in the subset is less than the number of visited nodes in that subset.

The set of connectivity constraints, called Tour, ensures that each visited node is connected to exactly two arcs that are employed in the network. It also ensures that if a node is not used, it is connected to no arcs that are used in the network. In combination with the set of connectivity constraints, Elimination1 and Elimination2 disallow any sub-tour within the network. Note that a unique realization of each of these sets of constraints exists for each period in the model.

Unfortunately, due to the nature of the Elimination1 and Elimination2 sets of constraints, there is an exponential number of such constraints. The tables below show the total number of variables and the total number of constraints, according to the length of the time horizon and the number of retailers. Cells containing “out of range” indicate that the size of the problem is too large for the student version of AMPL to load. Loading the model simply evaluates and enumerates all of the data and constraints. Tables 1 and 2, however, do not indicate the ability of AMPL to solve the model with the given input sizes. In order to solve an instance of the problem, AMPL must be able to first load the instance.

Table 1: Total Number of Variables by Time Horizon and Retailers

Time Horizon (Number of Periods)		Number of Retailers		
		5	10	15
	6	198	528	1,008
	10	330	880	1,680
	15	495	1,320	2,520
	20	660	1,760	out of range
	30	990	2,640	out of range

Table 2: Total Number of Constraints by Time Horizon and Retailers

Time Horizon (Number of Periods)		Number of Retailers		
		5	10	15
	6	428	11,822	469,280
	10	712	19,702	782,132
	15	1,067	29,552	1,173,200
	20	1,422	39,402	out of range
	30	2,132	59,102	out of range

As the size of the input (number of retailers and number of periods) is increased, the solution time for such a model will increase dramatically. Bertazzi et al. (2002) develop a heuristic for the model with 50 retailers and 30 periods. Using the full version of AMPL, this input is far out of range for the full optimization model to solve to optimality. To explore the exact change in costs due to a reoptimization by one department or the other, it may thus be necessary to develop a more efficient solution algorithm that can solve to optimality without considering all of the elimination constraints at once. It will likely also be necessary to consider smaller instances of the problem, even with a more efficient solution method, to compare *exact* optimal solutions.

One such efficient approach for solving the travelling salesman problem is known as the Dantzig, Fulkerson, and Johnson (DFJ) Cutting-Plane Method. It starts by choosing a subset of arcs in the network, and uses the reduced costs of those arcs to cleverly add additional arcs to the subset of arcs to be analyzed. In this way, the model can "ignore" arcs that would likely not be part of the optimal solution, thus decreasing the size of the problem significantly.

More precisely, the DFJ algorithm begins by only including a small subset of low-cost arcs from the original set of arcs and, iteratively, DFJ solves a relaxed version of the model and uses the maximum flow problem to find sub-tours within the solution. Once sub-tours are found, appropriate cuts are added to the model to eliminate the corresponding sub-tours until none remain. Once a valid tour is established, the reduced costs of each remaining unconsidered arc is employed to determine if including that arc could potentially provide a better solution. This process repeats until the optimal solution is found.

The method by DFJ can be extended to allow for the "periodic" problem, as we presented above. The DFJ-inspired algorithm for the periodic problem has been implemented in AMPL, and can be found in Appendix 2. Note that the files in Appendix 2 are extended and modified versions of the DFJ Cutting-Plane Method files referenced in Lee and Raffensperger (2006).

3.3 Incremental Sequential Approach

Despite developing the efficient optimization procedure above, that approach cannot compare to the efficiency of the heuristic method presented by Bertazzi et al. (2002). Sequential optimization analysis will therefore be performed with the DFJ-inspired algorithm on problems with a more reasonable size of input. Although results for the heuristic were based on 50 retailers and 30 periods, the exact method cannot be run on such a large instance. As such, a variety of different scenarios (varied number of retailers and periods) will be tested to help clarify the interaction of costs between departments. The goal of our analysis is to develop functions that represent the changes in cost (total cost and departmental costs) in terms of the allowable proportion of change in the cost of the department performing the reoptimization. To start, we will test three scenarios (of sizes which the DFJ-inspired algorithm can solve to optimality): 15 retailers and 15 periods; 15 retailers and 25 periods; as well as 20 retailers and 10 periods. For each scenario, data points are randomly generated to create 10 unique sets of parameters. For each of those sets, output is obtained as follows:

i. The problem is solved to optimality using total cost as the objective function.
ii. The problem is solved to optimality using inventory cost as the objective function.
iii. The problem is solved to optimality using transportation cost as the objective function.
iv. Starting with the solution to the optimal inventory-cost objective function, a transportation reoptimization occurs, allowing inventory cost to increase by 1 percent through to 100 percent. (This process terminates early if the optimal transportation cost is attained.) For each reoptimization, all costs (total, inventory, and transportation costs) are recorded.
v. Starting with the solution to the optimal transportation cost objective function, an inventory reoptimization occurs, allowing transportation cost to increase by 1 percent through to 100 percent. (Again, this process terminates early if the optimal inventory cost is attained.) For each reoptimization, all costs (total, inventory, and transportation costs) are recorded.

To make valid comparisons between any two sets of solutions, each of the costs output above are normalized by dividing that cost by the relevant optimal cost. For example, any particular transportation cost is reported as (Transportation Cost)/(Optimal

Transportation Cost). In this way, the data can be summarized in the form of an average normalized cost for the relevant maximum percentage increase in cost. The process just outlined may be more evident in Appendix 3, where a normalized data table is presented. Chapter 4 summarizes the results of our sequential process for the three differing scenarios.

4. Results of the Incremental Sequential Model

Many parameters of the model are randomly generated for each instance. Below, the ranges used for some of the parameters of the incremental sequential model are listed. It is important to note these ranges, as they may be varied later to test the validity of the results. (For a discussion of the parameters used within AMPL, see Appendix 2. For a more complete discussion of the randomly generated parameters, see Bertazzi et al. (2002), as the same parameter ranges are used there.) The retailers are placed on a grid with horizontal and vertical (x and y) coordinates obtained according to a discrete uniform distribution between 0 and 500. These coordinates are used to determine the cost of travelling between destinations, c_{ij}, and are calculated using the Euclidean distance.

The amount of product absorbed (sold or used up) at retailer i during time t, r_{it}, is constant over time and generated according to a discrete uniform distribution between 10 and 100. The minimum inventory level of retailer i, L_i, is generated from a discrete uniform distribution between 50 and 150, while the maximum inventory level of retailer i, U_i, is the minimum inventory plus an amount $r_i g_i$, where g_i is randomly selected from the set of divisors of the total number of periods. The capacity of the truck, C, is simply the sum of all demand for any time period. The unit inventory cost at node i, h_i, is randomly generated according to a uniform distribution between 0.6 and 1. Finally, the unit inventory cost at the supplier, h_0, is 0.3.

Figures 1 through 12 represent the change in transportation costs, inventory costs, and total costs given an allowable increase in cost for the one department as well as the portion of total cost for which each department is responsible.

The plots reveal some major insights into the relationships between transportation cost, inventory cost, and the total cost. (In what follows, by a *scenario* we shall mean a full set of results for the case of n retailers and H periods.) First of all, as we can see in the stacked plots for all three scenarios, the cost for the department responsible for reoptimization decreases as a percentage of total cost as the allowable increase in the primary departmental cost increases. This relationship is fully expected and acts as a validation of the results. It is also noteworthy that under all three scenarios, inventory cost represents the vast majority of total cost, at least 70% of that total, independent of the percentage of allowable increase in either departmental cost.

Figure 1: Normalized Costs – Trans. Reoptimization (20 Retailers, 10 Periods)

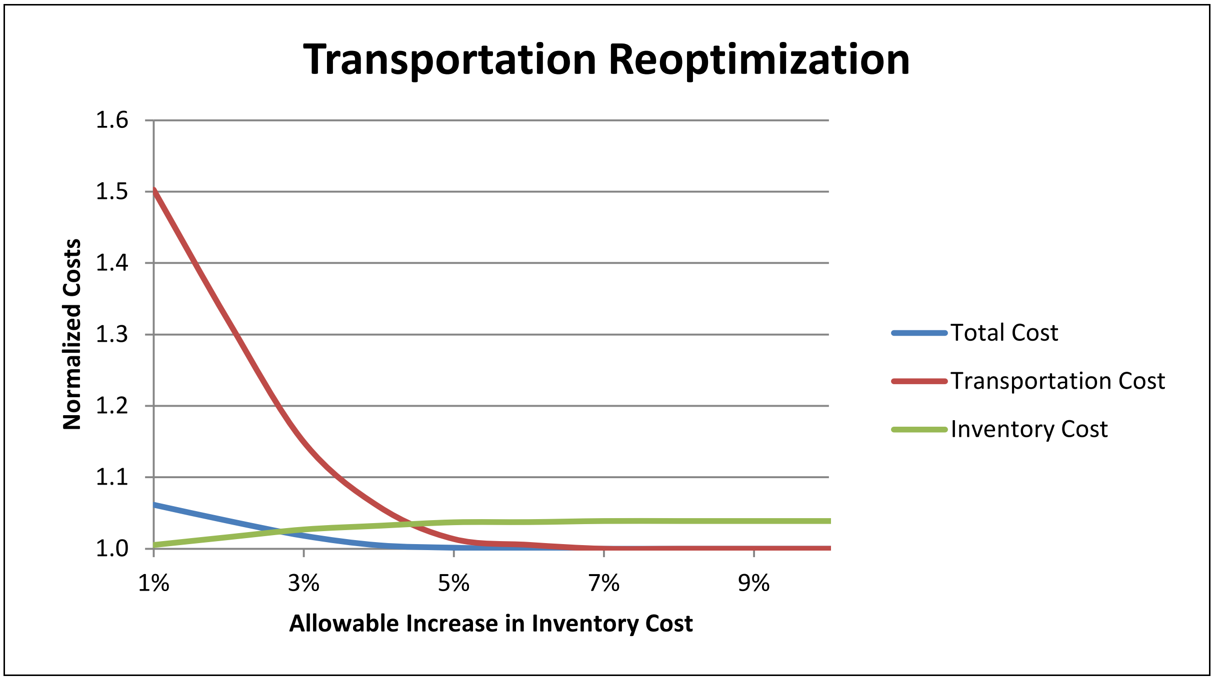


Figure 2: Dept. Cost Split – Trans. Reoptimization (20 Retailers, 10 Periods)

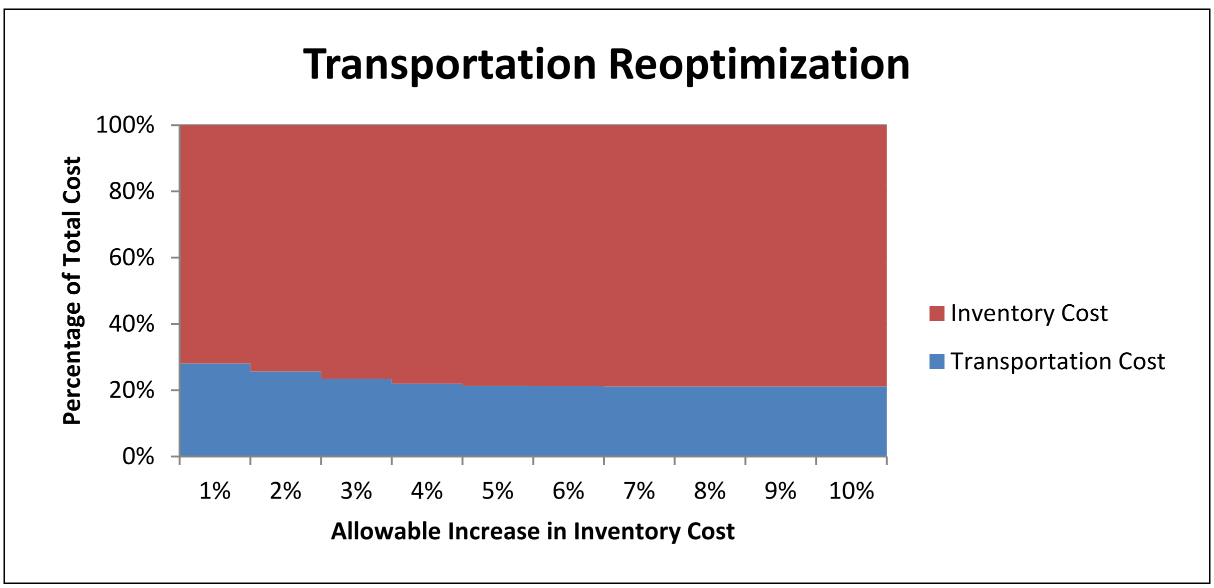


Figure 3: Normalized Costs - Inv. Reoptimization (20 Retailers, 10 Periods)

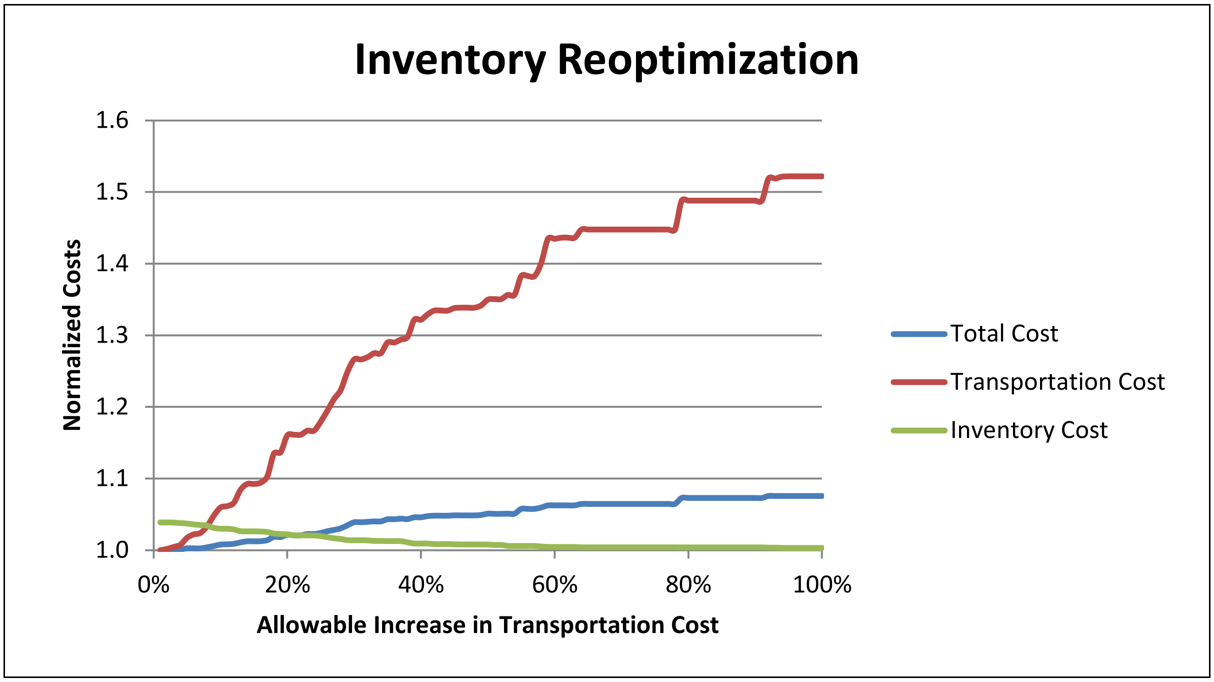


Figure 4: Dept. Cost Split - Inv. Reoptimization (20 Retailers, 10 Periods)

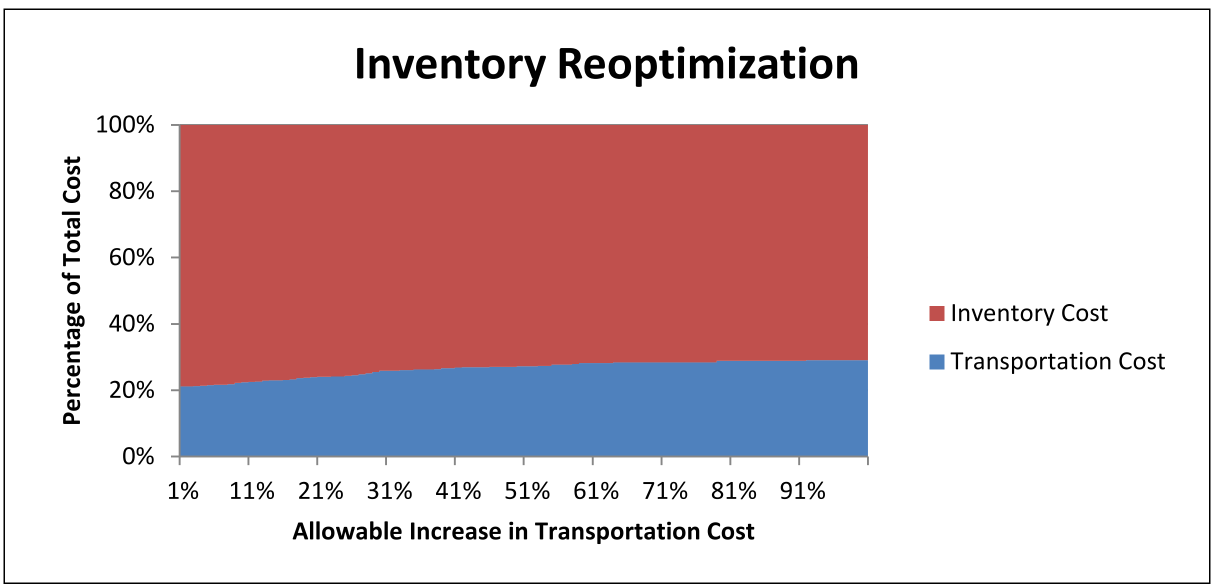


Figure 5: Normalized Costs - Trans. Reoptimization (15 Retailers, 25 Periods)

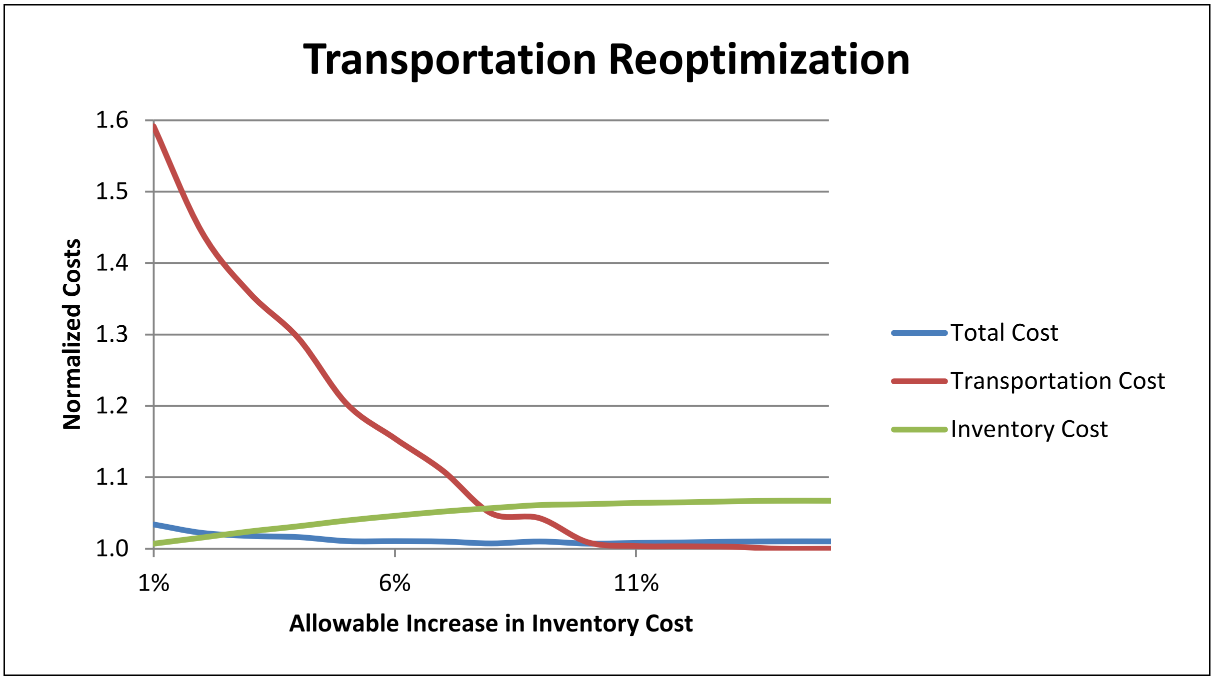


Figure 6: Dept. Cost Split - Trans. Reoptimization (15 Retailers, 25 Periods)

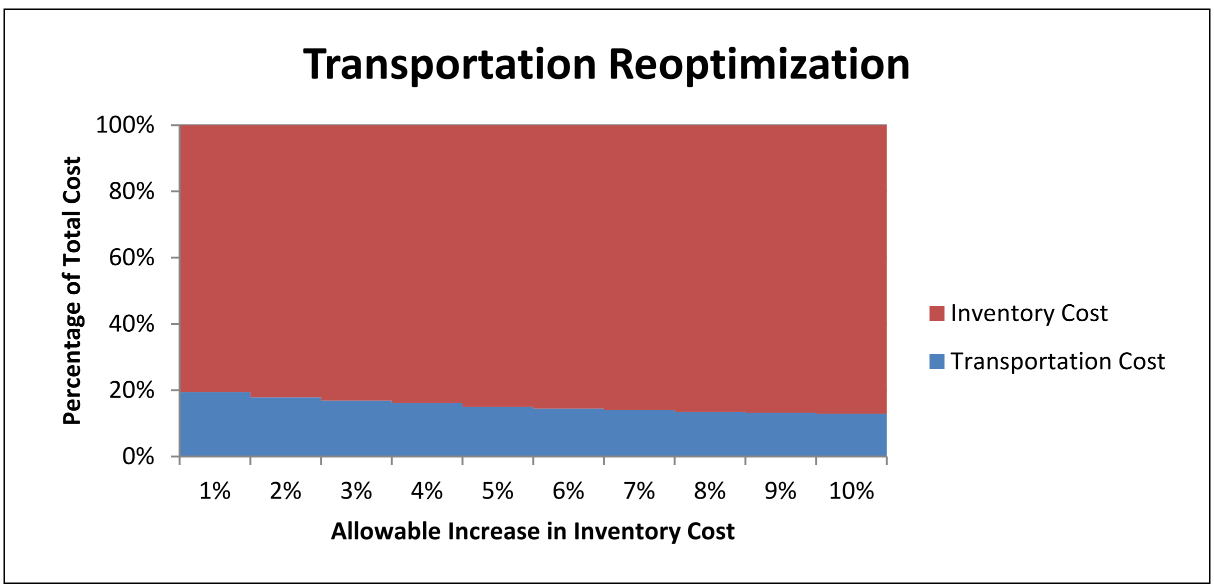


Figure 7: Normalized Costs - Inv. Reoptimization (15 Retailers, 25 Periods)

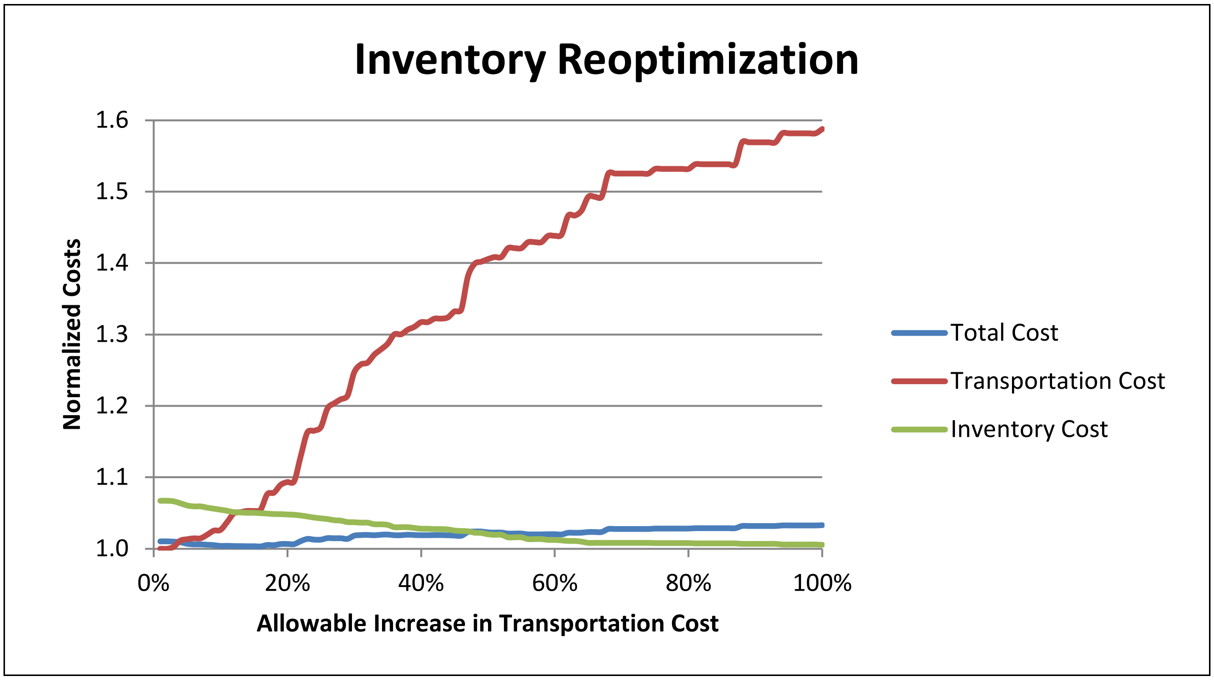


Figure 8: Dept. Cost Split - Inv. Reoptimization (15 Retailers, 25 Periods)

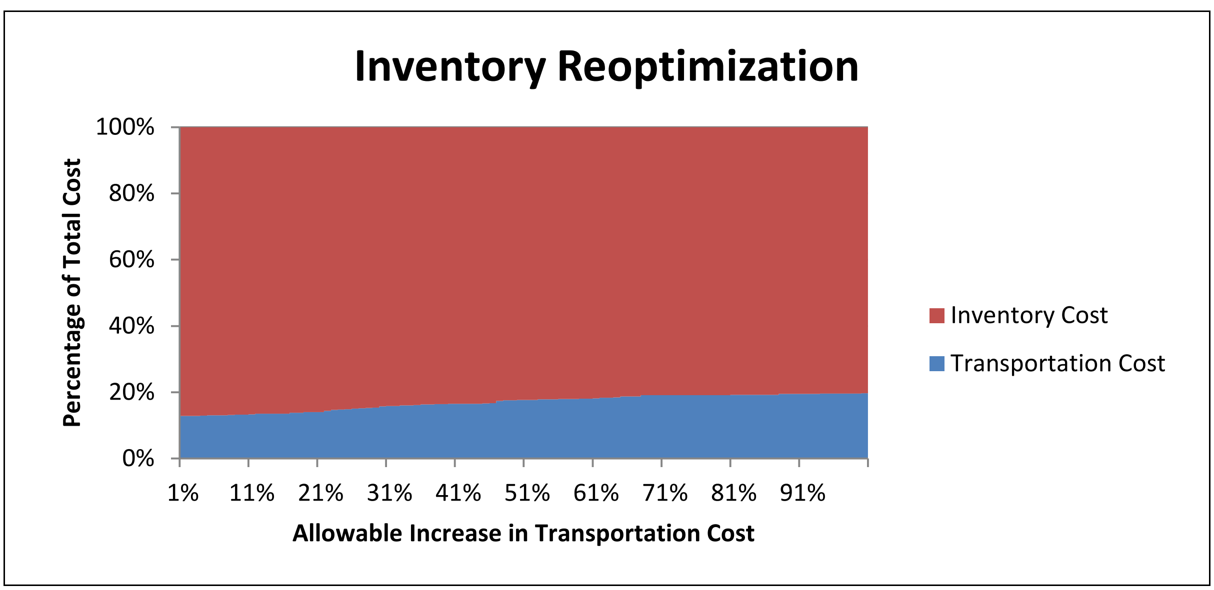


Figure 9: Normalized Costs - Trans. Reoptimization (15 Retailers, 15 Periods)

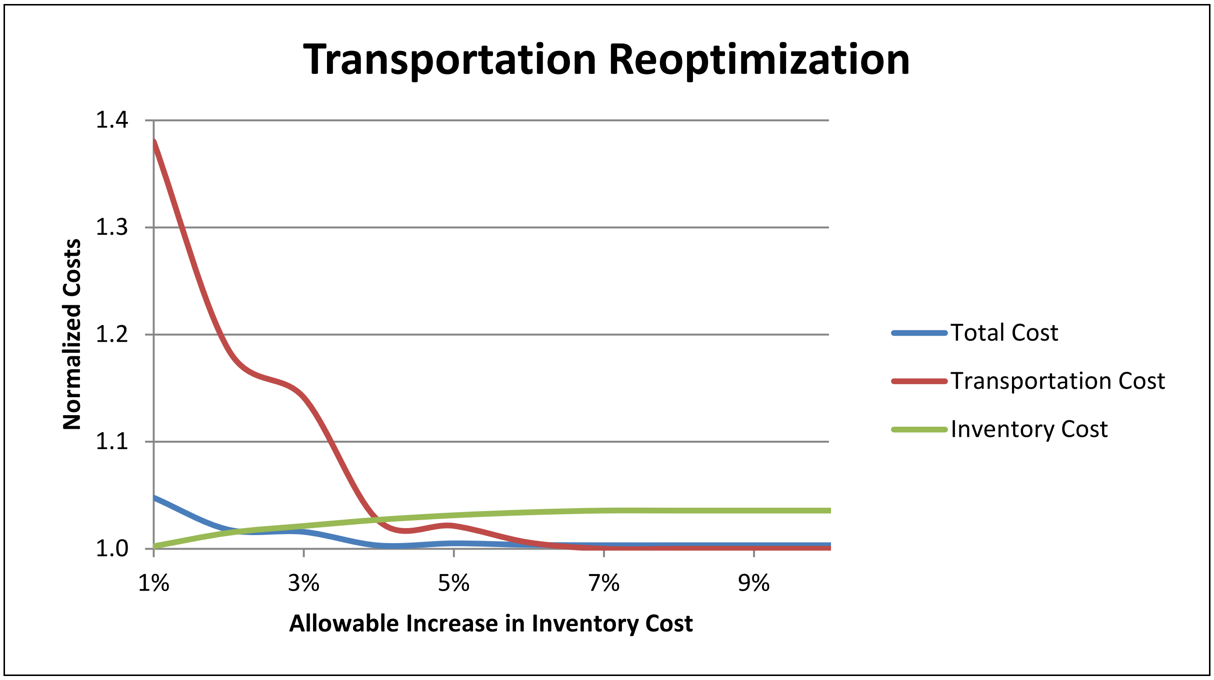


Figure 10: Dept. Cost Split - Trans. Reoptimization (15 Retailers, 15 Periods)

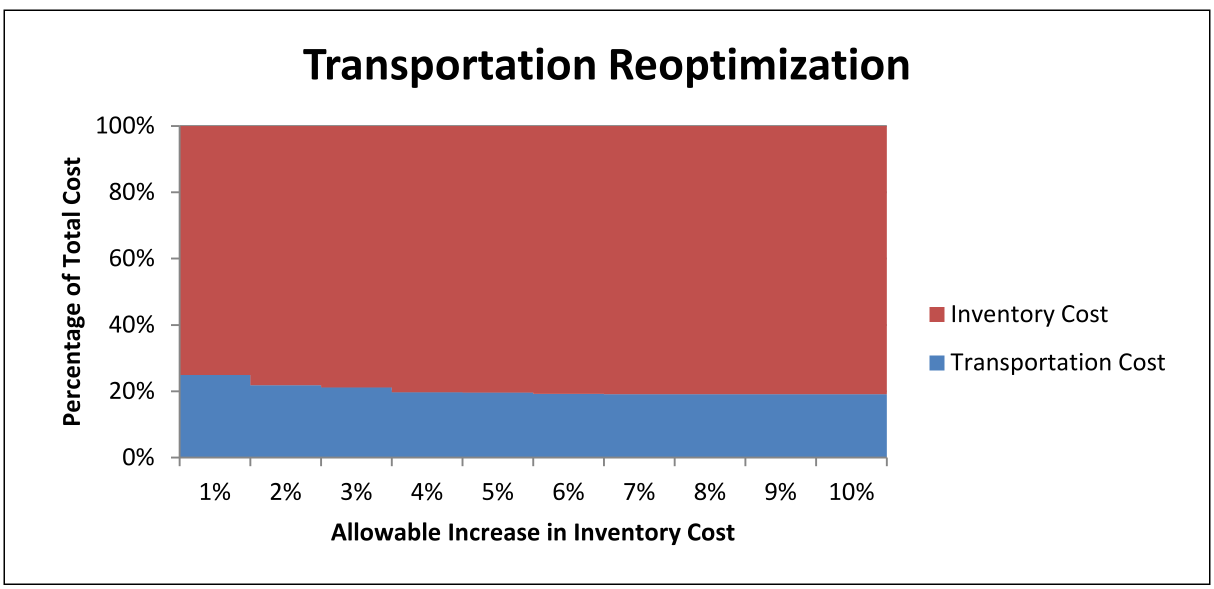


Figure 11: Normalized Costs - Inv. Reoptimization (15 Retailers, 15 Periods)

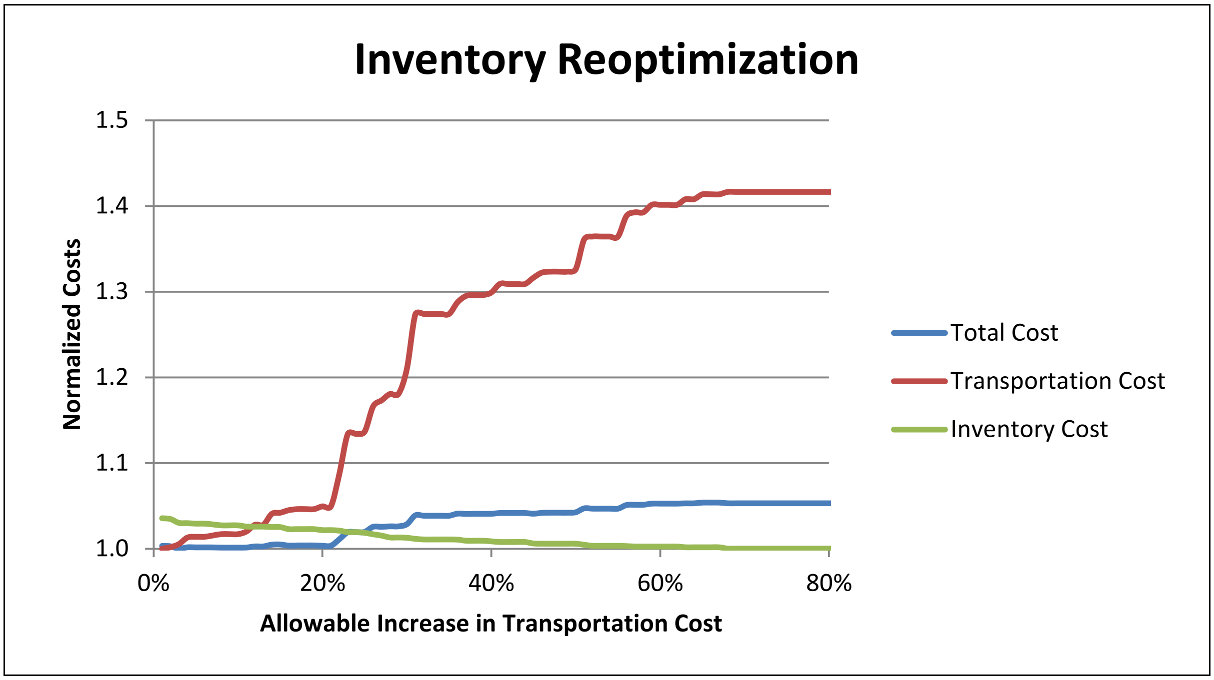
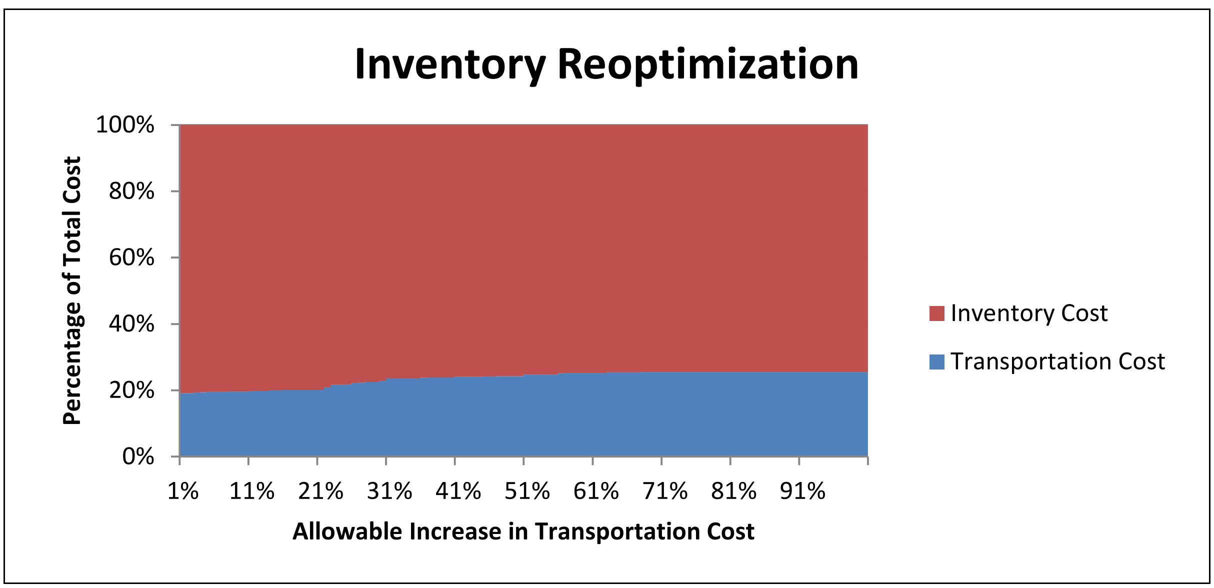


Figure 12: Dept. Cost Split - Inv. Reoptimization (15 Retailers, 15 Periods)



As a result of the large portion of cost represented by inventory cost, the transportation department is able to achieve optimality with a smaller (usually less than 10%) allowable increase in inventory cost. This is true as each allowable percentage increase represents a larger absolute allowable change in cost, due to the larger scale of inventory costs in comparison to transportation costs. This relationship also explains the seemingly large increase in transportation cost during the inventory reoptimization, as the inventory department requires a larger allowable percentage increase in transportation cost to achieve the optimal inventory cost.

We can see from the normalized cost plots that in all three scenarios, total cost generally increases when the inventory department is responsible for reoptimization, while total cost generally decreases when the transportation department is responsible for reoptimization. This trend is largely due to the structure of costs between departments. We have seen that inventory costs represent the vast majority of costs under all of these scenarios, and, as such, the optimal total cost solution is likely dominated by decisions that are beneficial to the inventory department. This trend, however, may not hold under varying cost structures that will be discussed in the next chapter.

Most interestingly, under all three scenarios and independent of the sequence of optimization, total cost is remarkably near its minimum when the normalized transportation cost intersects the normalized inventory cost. As a matter of fact, for the 15-retailer and 15-period test cases, when the difference between normalized transportation cost and normalized inventory cost is at its minimum, the total cost differs from the optimal total cost by at most 1.4% and on average is within 0.2% of optimality. Similarly for 15 retailers and 25 periods, the total cost differs from the optimal total cost by at most 3.1% and on average is within 0.9% of optimality. Finally for 20 retailers and 10 periods, the total cost differs from the optimal total cost by at most 1.8% and on average is within 0.4% of optimality.

The next chapter outlines several of the implications of the results obtained using the sequential approach, and pursues some of the recommendations based on those implications.

5. Implications of the Results

Chapter 5 discusses various implications derived from the results in the previous chapter. These implications and recommendations lead to additional scenarios with varied cost structures that are examined, as before.

5.1 Primary Decision Making Power

Given the results of the analysis in Chapter 4, several recommendations are logically drawn below. If, in a given organization, it is decided that communication between departments is too difficult or fragile to achieve and implement a simultaneous optimization model, a holistic approach to cost may be prudent. Under such an approach, the power to optimize one's own departmental cost first can be given to such a department, so as to achieve close to optimal total costs after the reoptimization process is complete. In the scenarios studied in this thesis and outlined above, allowing the inventory department to perform the primary optimization creates a situation in which the transportation department reoptimization brings total cost closer to the overall optimum. Under such a situation, the organization may find it prudent to set a maximum allowable increase in inventory cost.

In the cases provided above, a maximum allowable increase of 10% would provide the transportation department the opportunity to drastically decrease its cost, and would allow the possibility of achieving a total cost near optimality. However, this maximum-allowable-increase approach does not provide any additional motivation to the transportation department to choose a better *total-cost* solution over a better transportation-cost solution. The result of this reoptimization procedure would simply be the result of one optimization with the maximum allowable increase parameter set to 10%. There is no motivation for the department to even consider results with the allowable increase parameter below 10%.

A more appropriate benchmark may thus be to minimize the difference between normalized costs, while implementing a maximum allowable increase. As such, the inventory department is made aware ahead of time of an upper bound to their costs, while the organization maintains somewhat of a holistic approach – achieving close to optimal total costs in the end.

This intersection between normalized transportation and normalized inventory costs represents a point at which the departments have both made a compromise by deviating from their own optimal costs. Henceforth, this point will therefore be referred to as the “fairness point”. Although there are many ways to define a fair solution between departments, similar relative deviation from one’s own optimal cost can definitely be seen in this way.

The data used to generate Figures 1 through 12 was obtained using data generated from the same distributions for each randomly generated parameter. As such, the recommended application only necessarily applies to situations that closely resemble environments that are randomly generated in this way. However, to check whether the recommended approach yields satisfactory results for other scenarios, we can simply alter the ranges of some of the randomly generated parameters. In the previous scenarios, approximately 20% of the total cost is attributable to the transportation department. This may be an accurate representation of a manufacturer of small high-end electronics such as smartphones. Inventory costs are expected to be considerable, due to the high value of each device, while transportation costs are expected to be low, due to the phone’s small size. However, considering ever-increasing energy costs, transportation costs may eventually to play a larger role. The ranges for certain transportation cost parameters will thus be altered in the Section 5.2, so as to determine if the fairness point still yields a reasonable solution with respect to total cost.

5.2 Artificially Increased Transportation Costs

In this section, the same procedure as before is used to generate cases with significantly higher transportation costs. In this way, the recommended approach can be applied to scenarios to judge the efficacy of our decision rule. To artificially increase transportation costs, the ranges for vertical and horizontal coordinates have been doubled, and the overall transportation cost per unit distance has also been doubled. Only the 15-retailer 15-period scenario is tested, and the results of the increased costs follow. One notices immediately that the parameter changes bring the cost of transportation to be approximately equal to the cost of inventory. (A similar split between costs is yielded by the inventory reoptimization, hence the plot of that cost split is not shown.)

Figure 13: Dept. Cost Split: Trans. Reoptimization (15 Retailers, 15 Periods)

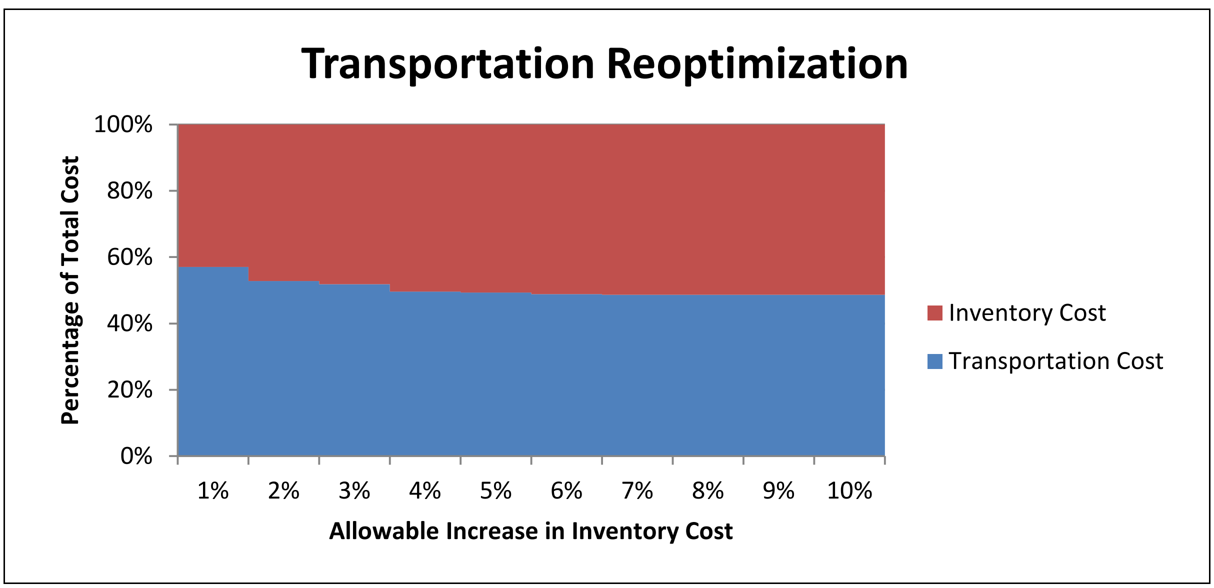


Figure 14: Normalized Costs - Trans. Reoptimization (15 Retailers, 15 Periods)

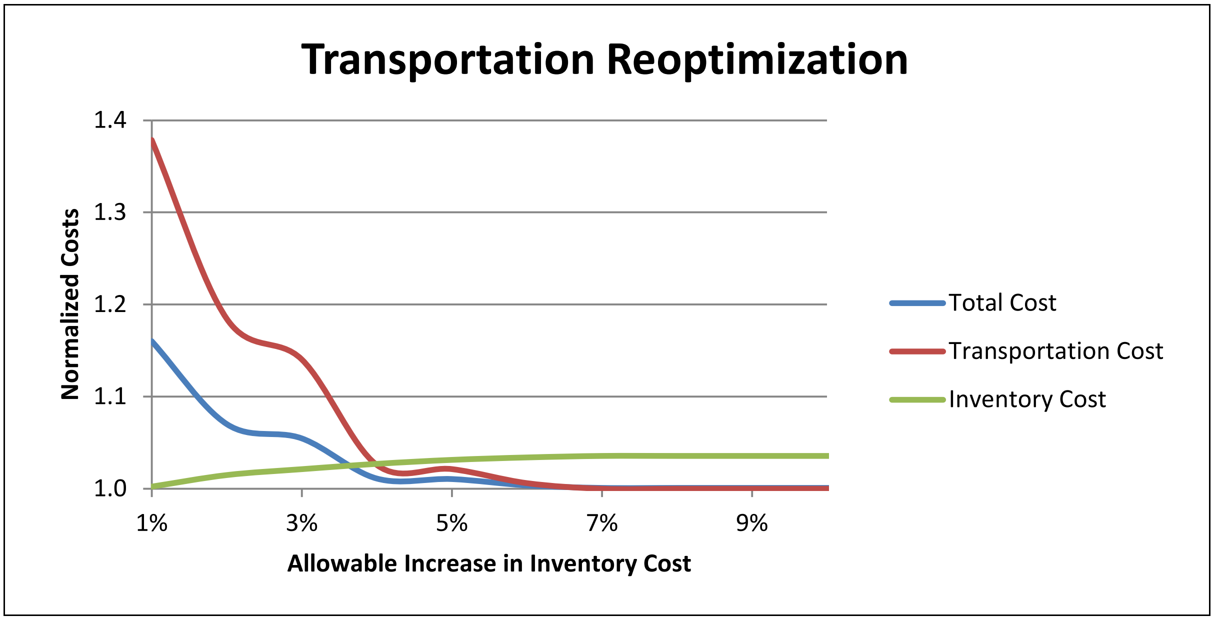
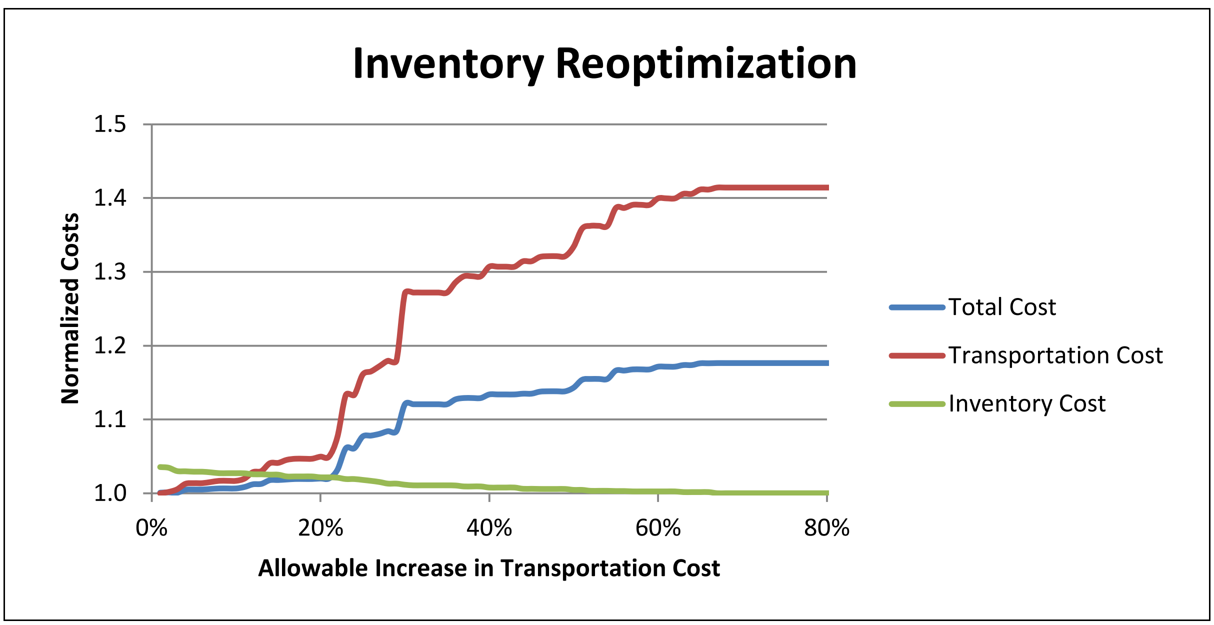


Figure 15: Normalized Costs - Inv. Reoptimization (15 Retailers, 15 Periods)



If the reoptimization process is stopped when the difference between normalized transportation cost and normalized inventory cost is minimized, on average the results are not quite as close to optimality as in the previous cases, but are still likely quite acceptable. For the transportation reoptimization, the process would stop on average with a total cost within 1.1% of optimality. For the inventory reoptimization, the process would stop on average with a total cost within 1.2% of optimality. Considering the individual cases of transportation and inventory reoptimization procedures, the total cost differs from the optimum by at most 3.5% and 2.8%, respectively.

To better understand the relationship between the fairness point and the total cost solution, several other scenarios will be explored and summarized in the following section.

5.3 Differing Cost Structures

As with the artificially-increased transportation cost structure above, the data in this section will be generated using 15 retailers and 15 periods. This should help to provide a baseline for fairness point performance across the various scenarios and various cost structures.

First of all, we have seen a ratio of average transportation to average inventory cost of approximately 1:4 and 1:1 in the previous 15 retailer and 15 period scenarios. Thus it is now prudent to consider a cost ratio of approximately 4:1. This will help to understand the behaviour of the fairness-point solution when implemented in companies with similar inventory and transportation policies, but with a largely varied cost structure. In an attempt to achieve the 4:1 cost ratio, the doubled transportation cost structure will be coupled with a decreased inventory cost structure, as in Bertazzi et al. (2002). The unit inventory cost at node i, h_i, will be randomly generated according to a uniform distribution between 0.1 and 0.5 instead of between 0.6 and 1. Also, the unit inventory cost at the supplier, h_0, will be reduced from 0.3 to 0.1.

The preceding cost structure yields approximately a 3:1 cost ratio. As such, the unit inventory cost at node i, h_i, will be further reduced and generated according to a uniform distribution between 0.05 and 0.3. Also, the unit inventory cost at the supplier, h_0, will be further reduced to 0.05. The results of this case, among others, can be seen in Tables 3 and 4.

Lastly a cost ratio of 1:2 and 2:1 should be considered as the midpoint scenarios between the 1:4, 1:1, and 4:1 scenarios that have already been explored. The cost structure will be modified using similar techniques to achieve the new cost ratios. The results of these instances can also be seen in Tables 3 and 4. Note that in these tables, the cost ratio always indicates the ratio of transportation costs to inventory costs.

Table 3: Summarized Fairness Point Performance (Department Optimum)

Retailers	Periods	Approximate Cost Ratio	Deviation from Department Optimum		
			Average	Maximum	Minimum
20	10	1:4	3.1%	8.3%	0.0%
15	25	1:4	4.7%	12.1%	0.0%
15	15	1:4	2.1%	6.2%	0.0%
15	15	1:2	4.6%	11.9%	0.0%
15	15	1:1	2.1%	6.3%	0.0%
15	15	2:1	5.0%	14.4%	0.0%
15	15	4:1	6.9%	14.0%	0.0%

Table 4: Summarized Fairness Point Performance (Total Optimum)

Retailers	Periods	Approximate Cost Ratio	Deviation from Total Optimum		
			Average	Maximum	Minimum
20	10	1:4	0.4%	1.8%	0.0%
15	25	1:4	0.9%	3.1%	0.0%
15	15	1:4	0.2%	1.4%	0.0%
15	15	1:2	1.4%	5.4%	0.0%
15	15	1:1	0.5%	3.6%	0.0%
15	15	2:1	2.5%	7.4%	0.0%
15	15	4:1	5.4%	11.9%	0.0%

One can immediately note that in all scenarios, the fairness point solution allowed for a minimum deviation from both department and total optimal cost of 0%. This indicates that the fairness point method is able to attain the overall total cost optimal solution (as well as a department cost optimal solution) in some cases for each of the scenarios studied. For many of those scenarios, the average deviation is less than 1% from the optimal total cost.

The worst performance by the fairness point method can be seen for the 4:1 cost ratio (with 15 retailers and 15 periods). This example experienced an average deviation of 5.4% at the fairness point from the total optimal cost.

Whether or not the performance indicated above is acceptable for a given firm is to be determined by the respective managers of the transportation and inventory departments. It can, however, be seen that for most of the cases studied, the fairness point method gives a reasonable solution with a cost that does not deviate wildly from the total optimal cost. Having noted the role of the department managers in determining acceptance of a given solution, there should likely be benchmarks set in accordance with the firm's overall goals. In this way, the managers will be able to determine if a given solution is acceptable. Using this type of method, management can set a total cost acceptability threshold – a maximum allowable deviation of the total cost of the fairness point solution from the optimal total cost. Table 5 shows the percentage of solutions that are considered unacceptable, given various thresholds between 1% and 10%.

Table 5: Acceptability of Fairness Point Solutions

Total Cost Acceptability Threshold	Percentage of Unacceptable Fairness Point Solutions
1%	34.3%
2%	23.6%
3%	20.0%
4%	15.7%
5%	13.6%
6%	9.3%
7%	7.1%
8%	4.3%
9%	4.2%
10%	3.6%

The total cost acceptability threshold indicated in Table 5 represents the maximum allowable deviation from the optimal total cost. That table summarizes all of the data points from every case discussed up to now.

Given a firm's particular threshold for total cost, especially for those thresholds close to 1%, many of the fairness point solutions would be discarded as unacceptable. Another method must therefore be used to develop a solution that is both fair and acceptable with respect to total cost. In the next chapter, other possible approaches to the problem are discussed. It may be possible to use one of those approaches when a solution is deemed unacceptable.

6. Other Possible Approaches

Section 6 discusses other approaches that consider the sequential nature of decisions in decentralized inventory and transportation environments.

6.1 Full Cost Absorption by Secondary Department

One might consider dropping the allowable-increase constraint, and simply making the secondary department responsible for absorbing any primary department cost above the primary department's optimal cost. However, if the secondary department is responsible for absorbing all of the additional primary cost, the solution reduces to the total cost solution, as we now show.

> Consider two departments: P (the primary department that optimizes its costs first) and S (the secondary department that optimizes its costs afterward). Let P_{cost} denote the value of the primary department's objective function while S_{cost}

denotes the value of the secondary department's objective function. Also, let T_{cost} denote the total cost. Finally, let P_{cost}*, S_{cost}*, and T_{cost}* represent the relevant optimal costs.

Note that we know in general $P_{cost}^* + S_{cost}^* \geq T_{cost}^*$, as the single P or S objective functions do not take advantage of the synergies between departments that are incorporated in the total cost solution.

Now, if P attains P_{cost}* and hands the solution off to S, S must now minimize its total relevant costs. The costs relevant to S are $S_{cost} + P_{cost} - P_{cost}^*$. However, P_{cost}* is fixed, and so can be removed from the objective function, leaving $S_{cost} + P_{cost}$. But, $T_{cost} = S_{cost} + P_{cost}$, so it is in S's best interest to minimize T_{cost}. Therefore, the solution reduces to the total cost solution, as noted above.

6.2 Partial Cost Absorption by Secondary Department

As opposed to making the secondary department responsible for absorbing any primary department cost above the primary department optimal cost, one can consider dropping the allowable increase constraint and making the secondary department responsible for absorbing a specified portion of any primary department cost above the primary department's optimal cost.

> Consider two departments: P and S (as above). Let P_{cost}, S_{cost}, T_{cost} and their respective optimal costs be defined as above. Let S be responsible for absorbing a portion Ω of P_{cost} above P_{cost}*.

> Now, if P attains P_{cost}* and hands the solution off to S, S must now minimize their total relevant costs. The costs relevant to S are $S_{cost} + \Omega(P_{cost} - P_{cost}*)$. However, P_{cost}* is fixed, and as such can be removed from the objective function, leaving $S_{cost} + \Omega(P_{cost})$. We know three solutions are available, giving costs P_{cost}*, S_{cost}*, and T_{cost}*. The costs for these solutions are as follows:
>
> 1. $S_{cost}* + \Omega(P_{cost})$
> 2. $S_{cost} + \Omega(P_{cost}*)$
> 3. $S_{cost} + \Omega(T_{cost}* - S_{cost})$
>
> However, it may also be possible for a solution to take the following form:
>
> 4. $S_{cost} + \Omega(P_{cost})$,
>
> where neither S_{cost} nor P_{cost} attain its own individual optimum at the total optimal solution.

It may therefore be prudent to study the relationship between this method and the fairness point method presented thoroughly in Chapters 3 and 4. The choice of procedure used to balance the need for fairness and for cost optimization depends strongly on the nature of the cost environment and the relationship between departments in an organization.

The method of partial cost absorption by the secondary department closely resembles weighted multi-criteria decision making models. The primary difference with the objective function listed above is that the form of the function does not necessarily fit into the standard additive or multiplicative forms. This is due to the fact that the “weights” (1 and Ω) attributed to the secondary and primary departmental costs do not necessarily sum to 1 and are not really “weights” in the traditional sense.

Using this type of model, one can vary Ω between 0 and 1 and compare the total cost to the optimal total cost. One may also compare the actual transportation and inventory costs, the costs actually incurred by each department, and the departmental optimal costs. Most importantly, determining the minimum value of Ω for each department that achieves the optimal total cost will give great insight into the significance of one department’s cost function on the others solution.

Note that the total cost of this process approaches the overall total optimal cost as Ω approaches 1, since then the objective function is equal to the total cost objective function. However, given the dependent nature of one department on the other for optimal decision making, it can be seen that (using the base 15-retailer, 15-period scenario) Ω need not be increased far from 0 to achieve the overall optimal total cost solution. As a matter of fact, considering the transportation department as the secondary department, Ω only needs to be increased to 5.5% on average to achieve the optimal total cost. Considering the inventory department in the same way, Ω only needs to be increased to 8.5% on average to achieve the optimal total cost. The maximum increase in Ω needed was 24% for transportation as the secondary department, and was 15% for inventory as the secondary department.

This approach somewhat fails to consider "fairness" between the departments, in contrast to the fairness-point approach discussed before. Hence, another approach to fair solutions will now be discussed.

6.3 Weighted Deviation from Optimal Costs

So as to consider fairness and cost in a consistent manner, a multi-objective approach that incorporates both into the objective function would be best. To optimize cost and fairness in one weighted objective function, the measures need to be expressed in the same consistent manner. Using cost in dollars is not appropriate, since fairness cannot necessarily be measured in that way and as such. It may then be prudent to develop unit-less measures for fairness and for cost.

One such way to consider cost without units is to express the deviation from the total optimum, as before, since ([total cost / optimal total cost] – 1) gives the percentage increase over optimal total cost. The fairness point was previously indicated by the minimization of the difference in relative deviation from each department's own optimal cost and, as such, the measure for fairness is now simply the sum of the relative deviations from each department's own optimal cost. Note that the optimal departmental costs here denote the best attainable costs for the individual departments while satisfying all constraints in the model, not the relevant cost in the optimal total cost solution. Therefore one approach to the new weighted objective function is of the following form:

$$w_1\left(\frac{\text{Total Cost}}{\text{Optimal Total Cost}}-1\right)+w_2\left(\frac{\text{Inventory Cost}}{\text{Optimal Inventory Cost}}-1\right)+w_3\left(\frac{\text{Transportation Cost}}{\text{Optimal Transportation Cost}}-1\right)$$

Since w_1, w_2, and w_3 are constant for any individual optimization, the objective function can take the following form:

$$w_1\left(\frac{\text{Total Cost}}{\text{Optimal Total Cost}}\right)+w_2\left(\frac{\text{Inventory Cost}}{\text{Optimal Inventory Cost}}\right)+w_3\left(\frac{\text{Transportation Cost}}{\text{Optimal Transportation Cost}}\right)$$

Given the motivation of fairness, it can easily be argued that the percentage increase over optimal inventory cost should be weighted the same as the percentage increase over optimal transportation cost. Thus, w_2 is equal to w_3 and the objective function simplifies to the following form:

$$w\left(\frac{\text{Total Cost}}{\text{Optimal Total Cost}}\right)+(1-w)\left(\frac{\text{Inventory Cost}}{\text{Optimal Inventory Cost}}+\frac{\text{Transportation Cost}}{\text{Optimal Transportation Cost}}\right)$$

Due to the similarity in the first weighted portion of the objective function (the total cost portion) to the second weighted portion (the departmental cost portion), for weights between 0.2 and 0.8 the minimization achieves within 2% of the overall optimal cost for all cases considered.

6.4 Difference in Deviation from Optimal Departmental Costs

Taking the results from the sequential analysis presented in Chapter 4, where the fairness point is defined as the intersection of the normalized inventory cost curve and the normalized transportation cost curve, one can define a new objective function to simply minimize the absolute deviation between the normalized costs for the two departments. An objective function of this type would take the following form:

$$\left| \frac{\text{Inventory Cost}}{\text{Optimal Inventory Cost}} - \frac{\text{Transportation Cost}}{\text{Optimal Transportation Cost}} \right|$$

However, given the integer programming formulation of the model used in this thesis (and many others used in theory and practice), the minimization of an absolute value typically becomes the following:

$$\text{Minimize:} \qquad M$$

$$\text{Subject to:}$$

$$M \geq \left(\frac{\text{Inventory Cost}}{\text{Optimal Inventory Cost}} - \frac{\text{Transportation Cost}}{\text{Optimal Transportation Cost}} \right)$$

$$M \geq \left(\frac{\text{Transportation Cost}}{\text{Optimal Transportation Cost}} - \frac{\text{Inventory Cost}}{\text{Optimal Inventory Cost}} \right)$$

One would expect the results of this objective function to yield similar results to the incremental approach proposed in Section 3. There are, however, advantages to using the objective function above. Primarily, only three optimizations need to be performed: to attain the optimal transportation cost, the optimal inventory cost, and then the minimal absolute difference in deviation. Secondly, the incremental approach is confined to a very granular set of solutions, whereas the minimal absolute difference in deviation approach can achieve solutions between the granular points for which the incremental approach solves to optimality.

6.5 Bounding Methods for Larger Instances

Given that many of the approaches discussed here require solving a complicated mathematical model multiple times, it may be prudent to consider optimization methods that are considerably more efficient than those discussed in this thesis. As our focus is on fair solutions for these environments, a spotlight was not placed on the efficiency of the solution algorithm used. As such, it is known that many considerably more efficient methods exist to solve significantly larger instances within a certain percentage of optimality. These fairness methods may be able to be implemented with more efficient heuristics if bounds are considered on the optimal departmental costs. If such methods are to be used with bounding on optimal costs, it may be necessary to alter the way that acceptability of solutions is defined. For example, if the best available inventory solution can be shown to be within 3% of the optimal cost for the inventory department, while the best available transportation solution can be shown to be within 2% of the optimal cost for the transportation department, one can simply update the best estimate of the departmental optima to $T^* = (T^1/1.03)$ and $I^* = (I^1/1.02)$. Note here that T^* and I^* represent the best estimate for the optimal transportation and inventory departments, respectively, while T^1 and I^1 represent the best available solutions for the same departments, respectively.

Using bounding methods, as above, can make fairness more accessible in organizations with environments with up to 50 retailers and 30 periods, for example by using the heuristic method developed in Bertazzi et al. (2002). Depending on the choice of fairness procedure, it may be prudent to develop new heuristics or simply enhance existing algorithms to efficiently solve these types of joint inventory and transportation problems.

The final chapter leaves the reader with some concluding remarks and some direction for possible further research.

7. Conclusions

This thesis considers sequential methods for solving joint transportation and inventory problems in a decentralized environment, in such a way that ensures some measure of fairness is observed. First of all, simple sequential models are discussed and then enhanced into a generalized form that is applicable in most joint transportation and inventory models. The use of the generalized sequential approach is studied under various scenarios for a given model. It is noted that a solution with a deviation from each departmental optimum that is approximately equal is typically a reasonable solution with respect to total cost. This solution was defined as the *fairness point solution*. In the seven cases that were tested, four had a total cost less than 1% over the optimal total cost on average. Similarly, six of the seven cases had a total cost over the optimal total cost on average by less than 2.5%.

The acceptability of fairness point solutions is expressed in terms of a total cost acceptability threshold, in which any solution beyond the threshold is deemed unacceptable. A total cost acceptability threshold of 6% leaves 9.3% of the fairness point solutions deemed unacceptable for all cases tested. Similarly, a threshold of 10% leaves only 3.6% of all fairness point solutions deemed unacceptable.

Other methods for fairness were discussed in Chapter 6. These included cost absorption methods, multi-objective approaches, and bounding techniques for heuristic algorithms.

Specific recommendations for which procedure is appropriate depend thoroughly on the type of logistics environment in which the model will be implemented. However, it can be noted that it is often possible to consider fairness between departments without sacrificing too much as far as total cost is concerned.

In order to study the effect of fairness measures more thoroughly, it is recommended that future research be focused primarily on the interaction of fairness with efficient heuristics for solving large instances of joint inventory and transportation models. Many real world logistics environments consist of huge numbers of “component parts”, such as retailers, vehicles, depots, cross docks, warehouses, and so on. As such, optimization methods with significantly increased efficiency must be used in order to achieve optimal or near optimal solutions.

Appendices

Appendix 1 – AMPL Code (Simple Approach)

AMPL Model File:

```
param n;
param num_periods;
param delta_trans;
param delta_inv;

set LOCATION ordered := 1 .. n by 1;
set TIME ordered := 1 .. num_periods by 1;
set divisor_num ordered;

param invcostint;
param invcostsup;
param posind;
param capmultiplier;
param num_divisors := card(divisor_num);
```

```
param divisors {divisor_num};
param time_to_consume {i in LOCATION} :=
     divisors[trunc(Uniform(1,num_divisors+1))];
param demand_ind {i in LOCATION} := trunc(Uniform(10,101));
param x_coord {i in LOCATION} := trunc(Uniform(0,501+posind*500));
param y_coord {i in LOCATION} := trunc(Uniform(0,501+posind*500));

set LINKS := {i in LOCATION, j in LOCATION: ord(i) < ord(j)};
set SEC ordered := 0 .. (2^n -1);

set POW {k in SEC} := {i in LOCATION: (k div 2**(ord(i)-1)) mod 2 = 1};

param T := card(TIME);
param demand {i in LOCATION, t in TIME} :=
     (if i=1 then sum {j in LOCATION: j>1} demand[j,t] else
     demand_ind[i]);
param L {i in LOCATION} := trunc(Uniform(50,151));

param U {i in LOCATION} := L[i] + demand_ind[i]*time_to_consume[i];

param start {i in LOCATION} := (if i=1 then sum{j in LOCATION: j>1}
     (U[j]-L[j]) else U[i] - demand_ind[i]);
param C:= sum{i in LOCATION: i>1} capmultiplier*demand_ind[i];
```

```
param cost {(i,j) in LINKS} :=
     trunc(sqrt((x_coord[i]-x_coord[j])^2+(y_coord[i]-y_coord[j])^2));

param h {i in LOCATION} := (if i=1 then invcostsup else
     (invcostint*Uniform(0.1,0.5) + (1 - invcostint)*Uniform(0.6,1)));

var x {LOCATION, TIME} >= 0;
var y {LOCATION, TIME} binary;
var z {LINKS, TIME} binary;
var I {LOCATION, TIME} >= 0;

minimize Total_Cost:
     sum {t in TIME, i in LOCATION} h[i]*I[i,t] + sum {(i,j) in LINKS, t
     in TIME} z[i,j,t]*cost[i,j];
minimize Trans_Cost:
     sum {(i,j) in LINKS, t in TIME} z[i,j,t]*cost[i,j];
minimize Inv_Cost:
     sum {t in TIME, i in LOCATION} h[i]*I[i,t];

subject to Inv_Start {i in LOCATION}: I[i,1]=start[i];
subject to Inv_Supplier {t in TIME: t>1}: I[1,t] = I[1,t-1] + demand[1,t-
     1] - sum{i in LOCATION: i>1} x[i,t-1];
subject to Inv_Retailer {i in LOCATION, t in TIME: i>1 and t>1}:
     I[i,t] = I[i,t-1] + x[i,t-1] - demand[i,t-1];
```

```
subject to Must_Visit_Supplier {t in TIME}:
     y[1,t]=0 ==> sum {i in LOCATION: i>1} y[i,t] = 0;
subject to Capacity {t in TIME}: sum {i in LOCATION} x[i,t] <= C;
subject to Sup_Stockout {t in TIME}: sum {i in LOCATION} x[i,t] <=
     I[1,t];
subject to Ret_Stockout {i in LOCATION, t in TIME}: I[i,t] >= L[i];
subject to Order {i in LOCATION, t in TIME: i>1}:
     y[i,t]=0 ==> x[i,t] = 0 else x[i,t] = U[i] - I[i,t];
subject to Tour {i in LOCATION, t in TIME}:
     sum {(i,j) in LINKS} z[i,j,t] + sum {(j,i) in LINKS} z[j,i,t] =
     2*y[i,t];
subject to Elimination1 {k in SEC, t in TIME:
     card(POW[k]) >= 3 and card(POW[k]) <= n/2}: sum {(i,j) in LINKS:
     (i in POW[k]) and (j in POW[k])} z[i,j,t] <= card(POW[k]) -1;
subject to Elimination2 {k in SEC, t in TIME:
     card(POW[k]) >= 3 and card(POW[k]) <= n/2}: sum {(i,j) in LINKS:
     (i in POW[k]) and (j in POW[k])} z[i,j,t] <= sum{i in LOCATION}
     y[i,t] -1;
subject to Limit_Trans_Cost: sum {(i,j) in LINKS, t in TIME}
     z[i,j,t]*cost[i,j] <= delta_trans;
subject to Limit_Inv_Cost: sum {t in TIME, i in LOCATION}
     h[i]*I[i,t] <= delta_inv;
```

AMPL Data File:

```
param n := 16;

param num_periods := 15;

param:      divisor_num:      divisors :=
      1           3
      2           5
      3           15;

param invcostint := 0;

param invcostsup := 0.3;

param posind := 0;

param capmultiplier := 2;

param delta_trans := Infinity;

param delta_inv := Infinity;
```

AMPL Run File:

```
option solver cplex;

option randseed 0;


model thesis.mod;

data thesis.dat;


objective Total_Cost;

#objective Trans_Cost;

#objective Inv_Cost;

solve;
```

```
display Total_Cost;

display Trans_Cost;

display Inv_Cost;
```

AMPL Parameters:

- n represents the number of retailers + 1, since the supplier is also counted
- num_periods represents the number of periods over which the model will be used
- delta_trans and delta_inv are used to indicate a maximum allowable change in transportation or inventory cost
- invcostint is used to determine which interval will be used for the inventory cost random number generation
- invcostsup is used to determine the inventory cost at the supplier
- posind is used to determine which interval will be used for the x and y coordinate random number generation
- capmultiplier is used to determine the capacity of the transportation vehicle
- num_divisors represents the number of divisors of the number of periods
- divisors represents the set of divisors of the number of periods
- time_to_consume[i] represents the number of periods for retailer i to consume goods
- demand_ind[i] represents the demand for each individual retailer

- x_coord[i] and y_coord[i] represent the coordinates for each location in the network
- T represents the number of periods or the time horizon
- demand represents the overall demand for retailer i during time period t
- demand[1,t] represents the supply at the supplier during time period t
- L[i] represents the lower bound on inventory for each retailer
- U[i] represents the upper bound on inventory for each retailer
- start[i] represents the starting inventory level at each location
- C represents the capacity of the transportation vehicle
- cost[i,j] represents the cost of using an arc between two locations in the network
- h[i] represents the holding cost of inventory for each location

Note that these parameters are calculated based on uniform random number generation, as in Bertazzi et al. (2002).

AMPL Sets:

- LOCATION represents the set of locations including all retailers and the supplier
- TIME represents the set of time periods
- divisor_num represents the set of divisors of the time horizon
- LINKS represents the arcs between locations in the network
- SEC represents the subtour elimination constraints
- POW represents the power set of all locations

AMPL Variables:

- x[i,t] represents the amount of product shipped to location i at time t
- y[i,t] is a binary indicator, indicating if product is shipped to location i at time t
- z[(i,j),t] is a binary indicator, indicating if an arc is used in the tour at time t
- I[i,t] represents the level of inventory at location i at time t

Appendix 2 – AMPL Code (DFJ Inspired Approach)

Note that the files included below are extended and modified versions of the DFJ Cutting-Plane Method files referenced in Lee and Raffensperger (2006).

AMPL Model File:

```
param n;
param num_periods;
param delta_trans;
param delta_inv;

set RUNS1 ordered;
param results {RUNS1};

set LOCATION ordered := 1 .. n by 1;
set TIME ordered := 1 .. num_periods by 1;
set divisor_num ordered;

param invcostint; #1 if using interval [0.1,0.5], 0 if using interval
[0.6,1]
param invcostsup; #0.3 or 0.8
param posind;     #0 if using interval [0,500], 1 if using interval
[0,1000]
```

```
param capmultiplier;    #1,2,or 3
param num_divisors := card(divisor_num);
param divisors {divisor_num};
param time_to_consume {i in LOCATION} :=
divisors[trunc(Uniform(1,num_divisors+1))];
param demand_ind {i in LOCATION} := trunc(Uniform(10,101));
param x_coord {i in LOCATION} := trunc(Uniform(0,501+posind*500));
param y_coord {i in LOCATION} := trunc(Uniform(0,501+posind*500));

set LINKS := {i in LOCATION, j in LOCATION: ord(i) < ord(j)};
      #represents arcs between locations

param reducedcost {LINKS};
set SUBLINKS dimension 2 default {};
param epsilon default 0.0001;
param maxcuts integer >= 0 default 10000;
param numcuts integer >= 0 default 0;
set SEC {1 .. numcuts} default {};
param T := card(TIME);  #time horizon

param demand {i in LOCATION, t in TIME} := (if i=1 then sum {j in
LOCATION: j>1} demand[j,t] else demand_ind[i]);
      #amount demanded at LOCATION i, at time t. Note LOCATION 1 is the
      supplier, so demand[1,t] represents supply
```

```
param L {i in LOCATION} := trunc(Uniform(50,151));
      #min inventory level of LOCATION i
param U {i in LOCATION} := L[i] + demand_ind[i]*time_to_consume[i];
      #max inventory level of LOCATION i
param start {i in LOCATION} := (if i=1 then sum{j in LOCATION: j>1}
(U[j]-L[j]) else U[i] - demand_ind[i]);
      #starting inventory levels
param C:= sum{i in LOCATION: i>1} capmultiplier*demand_ind[i];
      #capacity of the truck
param cost {(i,j) in LINKS} := trunc(sqrt((x_coord[i]-
x_coord[j])^2+(y_coord[i]-y_coord[j])^2));
      #transportation cost from LOCATION i to j
param h {i in LOCATION} := (if i=1 then invcostsup else
(invcostint*Uniform(0.1,0.5) + (1 - invcostint)*Uniform(0.6,1)));
      #unit inventory cost at LOCATION i


var x {LOCATION, TIME} >= 0;        #amount shipped to LOCATION i at time
t
var y {LOCATION, TIME} binary;      #1 if product is shipped to LOCATION
                                    i at time t, 0 otherwise
var z {SUBLINKS, TIME} binary;      #1 if a link is used in the tour at
time t
```

```
var I {LOCATION, TIME} >= 0;        #level of inventory at LOCATION i, at
                                    time t
                                    #note I[1,t] represents level of
                                    inventory at supplier at time t

minimize Total_Cost:    sum {t in TIME, i in LOCATION} h[i]*I[i,t] + sum
{(i,j) in SUBLINKS, t in TIME} z[i,j,t]*cost[i,j];
minimize Trans_Cost:    sum {(i,j) in SUBLINKS, t in TIME}
z[i,j,t]*cost[i,j];
minimize Inv_Cost:      sum {t in TIME, i in LOCATION} h[i]*I[i,t];


subject to Inv_Start {i in LOCATION}: I[i,1]=start[i];#Sets first
inventory level
subject to Inv_Supplier {t in TIME: t>1}: I[1,t] = I[1,t-1] + demand[1,t-
1] - sum{i in LOCATION: i>1} x[i,t-1];
subject to Inv_Retailer {i in LOCATION, t in TIME: i>1 and t>1}: I[i,t] =
I[i,t-1] + x[i,t-1] - demand[i,t-1];
subject to Must_Visit_Supplier {t in TIME}: y[1,t]=0 ==> sum {i in
LOCATION: i>1} y[i,t] = 0;
subject to Capacity {t in TIME}: sum {i in LOCATION} x[i,t] <= C;
subject to Sup_Stockout {t in TIME}: sum {i in LOCATION} x[i,t] <=
I[1,t];
subject to Ret_Stockout {i in LOCATION, t in TIME}: I[i,t] >= L[i];
```

```
subject to Order {i in LOCATION, t in TIME: i>1}: y[i,t]=0 ==> x[i,t] = 0
else x[i,t] = U[i] - I[i,t];

subject to Tour {i in LOCATION, t in TIME}:     sum {(i,j) in SUBLINKS}
z[i,j,t] + sum {(j,i) in SUBLINKS} z[j,i,t] = 2*y[i,t];
subject to Elimination1 {k in 1..numcuts, t in TIME: card(SEC[k]) >= 3
and card(SEC[k]) <= n/2}:     sum {(i,j) in SUBLINKS: (i in SEC[k]) and
(j in SEC[k])} z[i,j,t] <= card(SEC[k]) -1;
subject to Elimination2 {k in 1..numcuts, t in TIME: card(SEC[k]) >= 3
and card(SEC[k]) <= n/2}:     sum {(i,j) in SUBLINKS: (i in SEC[k]) and
(j in SEC[k])} z[i,j,t] <= sum{i in LOCATION} y[i,t] -1;

subject to Limit_Trans_Cost: sum {(i,j) in SUBLINKS, t in TIME}
z[i,j,t]*cost[i,j] <= delta_trans;
subject to Limit_Inv_Cost: sum {t in TIME, i in LOCATION} h[i]*I[i,t] <=
delta_inv;

###Max Flow Problem Finds Appropriate Cuts

param source symbolic in LOCATION default 1;
param sink symbolic in LOCATION, <> source;
param bound {SUBLINKS} default 1;
```

```
var MF {(i,j) in SUBLINKS} >= - bound[i,j], <= bound[i,j];

maximize Flow: sum {(source,j) in SUBLINKS} MF[source,j] - sum
{(j,source) in SUBLINKS} MF[j,source];

subject to Conserveflow {i in LOCATION diff {source, sink}}: sum {(j,i)
in SUBLINKS} MF[j,i] - sum {(i,j) in SUBLINKS} MF[i,j] = 0;
```

AMPL Data File:

```
param n := 16;

param num_periods :=    15    ;

param:      divisor_num:      divisors :=
      1           3
      2           5
      3           15;

param invcostint := 0;
param invcostsup := 0.3;
param posind := 0;
param capmultiplier := 2;
param delta_trans := Infinity;
param delta_inv := Infinity;
```

AMPL Run File:

```
option solver cplex;

option randseed 5;

option solver_msg 0;


model dfj.mod;

data dfj.dat;


problem TSPmodel: x, y, z, I, Inv_Cost, Inv_Start, Inv_Supplier,
Inv_Retailer, Must_Visit_Supplier, Capacity, Sup_Stockout, Ret_Stockout,
Order, Tour, Elimination1, Elimination2, Limit_Trans_Cost,
Limit_Inv_Cost;

let {(i,j) in SUBLINKS, t in TIME} z[i,j,t].relax := 1;


problem Findsubtour: MF, Flow, Conserveflow;


param cutdualprice;

param foundcut default 1;

param lowerbound default 0;

param number_of_newvars default 1;

param numpass default 0;

param upperbound default Infinity;


param longest default (sum {(i,j) in LINKS} cost[i,j])/card(LINKS);
```

```
param high default 100000000;
param low default 0;

param num_start = 80;

repeat
{     let SUBLINKS := {(i,j) in LINKS: cost[i,j] <= longest};
      display card(SUBLINKS);
      if (card(SUBLINKS) < num_start*1.05 and card(SUBLINKS) >
num_start*0.95) then break;
      else if card (SUBLINKS) > num_start then
      {     let high := longest; let longest := (low + longest)/2;
      }
      else
      {     let low := longest; let longest := (high + longest)/2
      }
}

repeat while number_of_newvars >= 1
{     let numpass := 0;
      repeat while numpass < 2
      {     let numpass := numpass + 1;
            let foundcut := 1;
```

```
            repeat while foundcut = 1
            {     let foundcut := 0;
                  if numpass = 2 then {let {(i,j) in SUBLINKS, t in TIME}
z[i,j,t].relax := 0;};

                  for {(i,j) in SUBLINKS} let bound[i,j] := 0;
                  problem TSPmodel;
                  solve TSPmodel;

                  display z;

                  for {t in TIME}
                  {
                        if foundcut = 1 then {break;};
                        let upperbound := Inv_Cost;
                        for {(i,j) in SUBLINKS} let bound[i,j] :=
z[i,j,t];

                        for {nn in LOCATION diff{source}: y[nn,t]=1}
                        {
                              if y[1,t] < epsilon then break;
                              let sink := nn;

                              problem Findsubtour;
```

```
                              solve Findsubtour;

                              if Flow < 2 - epsilon then
                              {     let numcuts := numcuts + 1;
                                    let SEC[numcuts] := {source} union {k
in LOCATION diff {source,sink}: Conserveflow[k].dual >= epsilon};
                                    let foundcut := 1;
                                    break;
                              };
                        };
                  };
            };

            if forall {(i,j) in SUBLINKS, t in TIME} (z[i,j,t] <= epsilon
or z[i,j,t] >= 1-epsilon) then {break;}
      };

      let {(i,j) in SUBLINKS, t in TIME} z[i,j,t].relax := 1;
      problem TSPmodel;
      solve TSPmodel;

      let number_of_newvars := 0;

      let lowerbound := Inv_Cost;
```

```
      for {(i,j) in LINKS: (i,j) not in SUBLINKS}
      {     let cutdualprice := sum {k in 1..numcuts, t in TIME:
card(SEC[k]) >= 3} (Elimination1[k,t].dual + Elimination2[k,t].dual) *
(if (i in SEC[k]) and (j in SEC[k]) then 1 else 0);

            let reducedcost[i,j] := cost[i,j] - cutdualprice - sum{t in
TIME} (Tour[i,t].dual + Tour[j,t].dual);

            if reducedcost[i,j] + lowerbound < upperbound then
            {     let SUBLINKS := SUBLINKS union {(i,j)};
                  let number_of_newvars := number_of_newvars + 1;
            }
      }
}
let {(i,j) in SUBLINKS, t in TIME} z[i,j,t].relax := 0;
option relax_integrality 0;
let SUBLINKS := LINKS;
problem TSPmodel;
solve TSPmodel;
display z;
display Total_Cost;
display Trans_Cost;
display Inv_Cost;
```

Appendix 3 – Output Data

Presented below are modified normalized data tables for first of the 15 retailer, 15 period scenarios studied. The table represents the percentage difference from optimality for all three objective functions: total cost, transportation cost, and inventory cost.

15 retailers and 15 periods

Allowable Increase	Transportation Reoptimization			Inventory Reoptimization		
	Total Cost	Transportation Cost	Inventory Cost	Total Cost	Transportation Cost	Inventory Cost
1%	5.65%	51.86%	0.18%	0.51%	0.00%	6.59%
2%	4.38%	39.98%	1.53%	0.51%	0.00%	6.59%
3%	1.86%	21.31%	2.96%	0.00%	2.69%	5.24%
4%	0.93%	15.90%	3.11%	0.00%	2.69%	5.24%
5%	1.76%	13.63%	4.77%	0.00%	2.69%	5.24%
6%	0.00%	2.69%	5.24%	0.00%	2.69%	5.24%
7%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
8%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
9%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
10%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
11%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
12%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
13%	0.51%	0.00%	6.59%	0.00%	2.69%	5.24%
14%	0.51%	0.00%	6.59%	1.76%	13.63%	4.77%
15%	0.51%	0.00%	6.59%	1.97%	14.97%	4.71%
16%	0.51%	0.00%	6.59%	0.93%	15.90%	3.11%
17%	0.51%	0.00%	6.59%	0.93%	15.90%	3.11%
18%	0.51%	0.00%	6.59%	0.93%	15.90%	3.11%
19%	0.51%	0.00%	6.59%	0.93%	15.90%	3.11%
20%	0.51%	0.00%	6.59%	0.93%	15.90%	3.11%
21%	0.51%	0.00%	6.59%	0.93%	15.90%	3.11%

Allowable Increase	Transportation Reoptimization			Inventory Reoptimization		
	Total Cost	Transportation Cost	Inventory Cost	Total Cost	Transportation Cost	Inventory Cost
22%	0.51%	0.00%	6.59%	1.86%	21.31%	2.96%
23%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
24%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
25%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
26%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
27%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
28%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
29%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
30%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
31%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
32%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
33%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
34%	0.51%	0.00%	6.59%	1.71%	22.39%	2.49%
35%	0.33%	0.00%	3.56%	3.85%	27.40%	1.09%
36%	0.33%	0.00%	3.56%	4.10%	28.76%	1.06%
37%	0.33%	0.00%	3.56%	4.06%	29.50%	0.94%
38%	0.33%	0.00%	3.56%	4.08%	29.60%	0.94%
39%	0.33%	0.00%	3.56%	4.08%	29.60%	0.94%
40%	0.33%	0.00%	3.56%	4.08%	29.90%	0.87%
41%	0.33%	0.00%	3.56%	4.17%	30.90%	0.79%
42%	0.33%	0.00%	3.56%	4.17%	30.90%	0.79%
43%	0.33%	0.00%	3.56%	4.17%	30.90%	0.79%
44%	0.33%	0.00%	3.56%	4.17%	30.90%	0.79%
45%	0.33%	0.00%	3.56%	4.09%	31.64%	0.62%
46%	0.33%	0.00%	3.56%	4.19%	32.22%	0.59%
47%	0.33%	0.00%	3.56%	4.21%	32.33%	0.59%
48%	0.33%	0.00%	3.56%	4.21%	32.33%	0.59%
49%	0.33%	0.00%	3.56%	4.21%	32.33%	0.59%
50%	0.33%	0.00%	3.56%	4.26%	32.63%	0.59%
51%	0.33%	0.00%	3.56%	4.72%	36.10%	0.47%
52%	0.33%	0.00%	3.56%	4.68%	36.43%	0.34%

Allowable Increase	Transportation Reoptimization			Inventory Reoptimization		
	Total Cost	Transportation Cost	Inventory Cost	Total Cost	Transportation Cost	Inventory Cost
53%	0.33%	0.00%	3.56%	4.68%	36.43%	0.34%
54%	0.33%	0.00%	3.56%	4.68%	36.43%	0.34%
55%	0.33%	0.00%	3.56%	4.68%	36.43%	0.34%
56%	0.33%	0.00%	3.56%	5.10%	38.82%	0.31%
57%	0.33%	0.00%	3.56%	5.13%	39.25%	0.26%
58%	0.33%	0.00%	3.56%	5.13%	39.25%	0.26%
59%	0.33%	0.00%	3.56%	5.27%	40.13%	0.26%
60%	0.33%	0.00%	3.56%	5.27%	40.13%	0.26%
61%	0.33%	0.00%	3.56%	5.27%	40.13%	0.26%
62%	0.33%	0.00%	3.56%	5.27%	40.13%	0.26%
63%	0.33%	0.00%	3.56%	5.30%	40.79%	0.17%
64%	0.33%	0.00%	3.56%	5.30%	40.79%	0.17%
65%	0.33%	0.00%	3.56%	5.39%	41.36%	0.17%
66%	0.33%	0.00%	3.56%	5.39%	41.36%	0.17%
67%	0.33%	0.00%	3.56%	5.39%	41.36%	0.17%
68%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
69%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
70%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
71%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
72%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
73%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
74%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
75%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
76%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
77%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
78%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
79%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
80%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
81%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
82%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
83%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%

Allowable Increase	Transportation Reoptimization			Inventory Reoptimization		
	Total Cost	Transportation Cost	Inventory Cost	Total Cost	Transportation Cost	Inventory Cost
84%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
85%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
86%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
87%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
88%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
89%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
90%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
91%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
92%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
93%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
94%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
95%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
96%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
97%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
98%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
99%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%
100%	0.33%	0.00%	3.56%	5.31%	41.64%	0.02%

References

Adelman, D., 2004. A Price-Directed Approach to Stochastic Inventory Routing. *Operations Research*, 52(4), pp.499-514.

Aguezzoul, A. & Ladet, P., 2007. A nonlinear multibojective approach for the supplier selection, integrating transportation policies. *Journal of Modelling in Management*, 2(2), pp.157-69.

Bassok, Y. & Ernst, R., 1995. Dynamic Allocations for Multi-Product Distribution. *Transportation Science*, 29(3), pp.256-66.

Bentley, P.J. & Wakefield, J.P., 1997. Finding Acceptable Solutions in the Pareto-Optimal Range using Multiobjective Genetic Algorithms. In *Soft Computing in Engineering Design and Manufacturing*. 1st ed. London: Springer. pp.231-40.

Bertazzi, L., Paletta, G. & Speranza, G., 2002. Deterministic Order-Up-To Level Policies in an Inventory Routing Problem. *Transportation Science*, 36(1), pp.119-32.

Bertazzi, L., Speranza, M.G. & Ukovich, W., 2000. Exact and Heuristic Solutions for a Shipment Problem with Given Frequencies. *Management Science*, 46(7), pp.973-88.

Bhaskaran, S. & Turnquist, M.A., 1990. Multiobjective Trasnportation Considerations in Multiple Facility Location. *Transportation Research Part A: General*, 24(2), pp.139-48.

Blumenfeld, D.E., Burns, L.D. & Diltz, J.D., 1985. Analyzing Trade-Offs Between Transportation, Inventory and Production Costs on Freight Networks. *Transportation Research Part B: Methodological*, 19(5), p.361–380.

Burns, L.D., Hall, R.W., Blumenfeld, D.E. & Daganzo, C.F., 1985. Distribution Strategies that Minimize Transportation and Inventory Costs. *Operations Research*, 33(3), pp.469-90.

Cetinkaya, S. & Lee, C.-Y., 2000. Stock Replenishment and Shipment Scheduling for Vendor-Managed Inventory Systems. *Management Science*, 46(2), pp.217-32.

Chan, L.M.A., Federgruen, A. & Simchi-Levi, D., 1998. Probabilistic Anlayses and Practical Algorithms for Inventory-Routing Models. *Operations Research*, 46(1), pp.96-106.

Chan, L.M.A. & Simchi-Levi, D., 1998. Probabilistic Analyses and Algorithms for Three-Leve Distribution Systems. *Management Science*, 44(11), pp.1562-76.

Christiansen, M. & Nygreen, B., 1998. A Method for Solving Ship Routing Problems with Inventory Constraints. *Annals of Operations Research*, 81, pp.357-78.

Esparcia-Alcazar, A., Merelo, J., Martinez-Garcia, A., Garcia-Sanchez, P., Alfaro-Cid, E., & Sharman, K., 2009. *Comparing Single and Multiobjective Evolutionary Approaches to the Inventory and Transportation Problem*. Draft Submitted to Evolutionary Computation. Ithaca, NY: Cornell University.

Fourer, R., 2004. *AMPL: Travelling Salesman Model*. [Online] Available at: www.ampl.com/FAQ/tsp.mod [Accessed 15 January 2012].

Fumero, F. & Vercellis, C., 1999. Synchronized Development of Production, Inventory, and Distribution Schedules. *Transportation Science,* 33(3), pp.330-40.

Kleywegt, A.J., Nori, V.S. & Savelsbergh, M.W.P., 2002. The Stochastic Inventory Routing Problem with Direct Deliveries. *Transportation Science*, 36(1), pp.94-118.

Kostreva, M.M., Ogryczakw, W. & Wierzbicki, A., 2004. Equitable Aggregations and Multiple Criteria Analysis. *European Journal of Operational Research*, 158(2), pp.362-77.

Lee, J. & Raffensperger, J.F., 2006. Using AMPL for teaching the TSP. *INFORMS Transactions on Education*, 7(1), pp.37-69.

Liao, S.-H., Hsieh, C.-L. & Lai, P.-J., 2011. An evolutionary approach for multi-objective optimization of the integrated location–inventory distribution network problem in vendor-managed inventory. *Expert Systems with Applications*, 38(6), pp.6768-76.

Lucas, C., 2011. *Practical Multiobjective Optimisation*. [Online] Available at: http://www.calresco.org/lucas/pmo.htm [Accessed 31 October 2012].

Luss, H., 1999. On Equitable Resource Allocation Problems: A Lexicographic Minimax Approach. *Operations Research*, 47(3), pp.361-78.

Marsh, M.T. & Schilling, D.A., 1994. Equity Measurement in Facility Location Analysis: A Review and Framework. *European Journal of Operational Research*, 74(1), pp.1-17.

Nozick, L.K. & Turnquist, M.A., 2001. Inventory, transportation, service quality and the location of distribution centers. *European Journal of Operational Research*, 129, pp.362-71.

Ogryczak, W., 2000. Inequality Measures and Equitable Approaches to Location Problems. *European Journal of Operational Research*, 122(2), pp.374-91.

Qu, W., Bookbinder, J.H. & Iyogun, P., 1999. An Integrated Inventory-Transportation System with Modified Periodic Policy for Multiple Products. *European Journal of Operational Research*, 115(2), pp.254-69.

Schwarz, L.B., Ward, J. & Zhai, X., 2004. *Joint Transportation-and-Inventory Problems in Supply Chains: A Review*. Working Paper. West Lafayette, IN: Purdue University (Krannert School of Management).

Schwarz, L.B., Ward, J.E. & Zhai, X., 2006. On the Interactions Between Routing and Inventory-Management Policies in a One-Warehouse N-Retailer Distribution System. *Manufacturing and Service Operations Management*, 8(3), pp.253-72.

Viswanathan, S. & Mathur, K., 1997. Integrating Routing and Inventory Decisions in One-Warehouse Multiretailer Multiproduct Distribution Systems. *Management Science*, 43(3), pp.294-312.

Zeleny, M., 1982. *Multiple Criteria Decision Making*. 1st ed. New York, NY: McGraw-Hill.