Fuzzy Preferences in the Graph Model for Conflict Resolution

by

Md. Abul Bashar

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Systems Design Engineering

Waterloo, Ontario, Canada, 2012

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Md. Abul Bashar

Abstract

A *F*uzzy Preference Framework for the *G*raph *M*odel for Conflict Resolution (FGM) is developed so that real-world conflicts in which decision makers (DMs) have uncertain preferences can be modeled and analyzed mathematically in order to gain strategic insights. The graph model methodology constitutes both a formal representation of a multiple participant-multiple objective decision problem and a set of analysis procedures that provide insights into them. Because crisp or definite preference is a special case of fuzzy preference, the new framework of the graph model can include—and integrate into the analysis—both certain and uncertain information about DMs' preferences. In this sense, the FGM is an important generalization of the existing graph model for conflict resolution.

One key contribution of this study is to extend the four basic graph model stability definitions to models with fuzzy preferences. Together, fuzzy Nash stability, fuzzy general metarationality, fuzzy symmetric metarationality, and fuzzy sequential stability provide a realistic description of human behavior under conflict in the face of uncertainty. A state is fuzzy stable for a DM if a move to any other state is not sufficiently likely to yield an outcome the DM prefers, where sufficiency is measured according to a fuzzy satisficing threshold that is characteristic of the DM. A fuzzy equilibrium, an outcome that is fuzzy stable for all DMs, therefore represents a possible resolution of the conflict. To demonstrate their applicability, the fuzzy stability definitions are applied to a generic two-DM sustainable development conflict, in which a developer plans to build or operate a project inspected by an environmental agency. This application identifies stable outcomes, and thus clarifies the necessary conditions for sustainability. The methodology is then applied to an actual dispute with more than two DMs concerning groundwater contamination that took place in Elmira, Ontario, Canada, again uncovering valuable strategic insights.

To investigate how DMs with fuzzy preferences can cooperate in a strategic conflict, coalition fuzzy stability concepts are developed within FGM. In particular, coalition fuzzy Nash stability, coalition fuzzy general metarationality, coalition fuzzy symmetric metarationality, and coalition fuzzy sequential stability are defined, for both a coalition and a single DM. These concepts constitute a natural generalization of the corresponding non-cooperative fuzzy preference-based definitions for Nash stability, general metarationality, symmetric metarationality, and sequential stability, respectively. As a follow-up analysis of the non-cooperative fuzzy stability results and to demonstrate their applicability, the coalition fuzzy stability definitions are applied to the aforementioned Elmira groundwater contamination conflict. These new concepts can be conveniently utilized in the study of practical problems in order to gain strategic insights and to compare conclusions derived from both cooperative and non-cooperative stability notions.

A fuzzy option prioritization technique is developed within the FGM so that uncertain preferences of DMs in strategic conflicts can be efficiently modeled as fuzzy preferences by using the fuzzy truth values they assign to preference statements about feasible states. The preference statements of a DM express desirable combinations of options or courses of action, and are listed in order of importance. A fuzzy truth value is a truth degree, expressed as a number between 0 and 1, capturing uncertainty in the truth of a preference statement at a feasible state. It is established that the output of a fuzzy preference formula, developed based on the fuzzy truth values of preference statements, is always a fuzzy preference relation. The fuzzy option prioritization methodology can also be employed when the truth values of preference statements at feasible states are formally based on Boolean logic, thereby generating a crisp preference over feasible states that is the same as would be found using the existing crisp option prioritization approach. Therefore, crisp option prioritization is a special case of fuzzy option prioritization. To demonstrate how this methodology can be used to represent fuzzy preferences in real-world problems, the new fuzzy option prioritization technique is applied to the Elmira aquifer contamination conflict. It is observed that the fuzzy preferences obtained by employing this technique are very close to those found using the rather complicated and tedious pairwise comparison approach.

Acknowledgements

First and foremost, I would like to express my deep gratitude to my supervisors, Professor Keith W. Hipel and Professor D. Marc Kilgour, for their continuous guidance, unreserved support, valuable insights, and encouragement. The time and effort, they spent throughout my PhD research, have contributed significantly to its success and completion, and will be forever appreciated. I am extremely grateful to them for their genuine guidance for my smooth transition from a background in pure mathematics to a specialization in engineering. Truly, all my gains in Engineering is owing to their fine assistance and support. It is almost impossible to express my appreciation to them in words.

I wish to convey my sincere thanks to Professor Ni-Bin Chang for being the External Examiner of my PhD thesis and for his suggestions to improve it. I am most thankful to my other PhD examination committee members, Professor Fakhri Karray, Dr. John Zelek, and Dr. Kevin Li, for their valuable time, useful comments from different perspectives, and prompt support whenever needed. Their helpful advice and suggestions have significantly enhanced my research.

My special thanks go to Sheila Hipel and Conrad Hipel for their help in improving the English of my writing whenever needed. I would also like to extend my gratitude to my friends and colleagues, among whom are Ye (Richard) Chen, Haiyan Xu, Qian Wang, Yi (Ginger) Ke, Michele Bristow, Yasser Matbouli, Rami Kinsara, Young-Jae Kim, and Hanbin (Eric) Kuang to name a few, for their enormous support and help during this journey.

I would like to express my sincere thanks to the the Natural Science and Engineering Research Council of Canada for providing me with full financial support throughout my

PhD studies from the NSERC Strategic Grant project entitled "Systems Engineering Approaches for Brownfield Redevelopment" as well as from the NSERC Discovery Grants of Professor Keith Hipel and Professor Marc Kilgour. I wish to express my tribute to the Department of Systems Design Engineering for providing me with a stimulating research environment. I also owe special thanks to Ms. Vicky Lawrence and Ms. Colleen Richardson of Systems Design Engineering for their assistance whenever requested.

I am greatly indebted to my parents, siblings, and relatives for their unchanging love, endless patience, and understanding. My deepest gratitude is for my wife, Pushpa, for her loving support, understanding, and encouragement, which were indispensable for the completion of my PhD studies. My endless love goes to my son, Tashfiq Aiman Bashar, and daughter, Tazmeen Aairah Bashar, who are too young to understand what their dad has been doing for all these years at university, sometimes, day and night, instead of being with them as they deserve. Thanks to them for their patience.

Dedication

To my son, Tashfiq Aiman Bashar, and daughter, Tazmeen Aairah Bashar.

Table of Contents

List of Tables

List of Figures

Acronyms

CCFIL	class coalitional fuzzy improvement list
CCI	class coalitional improvement
CCM	class coalitional move
CFE	coalition fuzzy equilibrium
CFGMR	coalition fuzzy general metarational
CFI	coalition fuzzy improvement
CFIL	coalition fuzzy improvement list
CFR	coalition fuzzy Nash stable or coalition fuzzy rational
CFSEQ	coalition fuzzy sequentially stable
CFSMR	coalition fuzzy symmetric metarational
CGMR	coalition general metarational
CR	coalition Nash stable or coalition rational
CSEQ	coalition sequentially stable
CSMR	coalition symmetric metarational
DEV	developers
DM	decision maker
ENV	environmental agencies
FE	fuzzy equilibrium
FGM	*F*uzzy Preference Framework for the *G*raph *M*odel for Conflict Resolution
FGMR	fuzzy general metarational
FR	fuzzy Nash stable or fuzzy rational

FRCP	fuzzy relative certainty of preference
FSEQ	fuzzy sequentially stable
FSMR	fuzzy symmetric metarational
FST	fuzzy satisficing threshold
FUI	fuzzy unilateral improvement
FUIL	fuzzy unilateral improvement list
GMCR	Graph Model for Conflict Resolution
GMCR II	decision support system of GMCR
GMR	general metarational
L	Local Government
M	The Ontario Ministry of the Environment
NDMA	N-nitroso demethylamine
$\underline{\text{P}}$	proactive
R	Nash stable or rational
$\underline{\text{R}}$	reactive
$\underline{\text{S}}$	sustainable development
SEQ	sequentially stable
SMR	symmetric metarational
U	Uniroyal Chemical Ltd.
$\underline{\text{U}}$	unsustainable development
UI	unilateral improvement
UIL	unilateral improvement list

Chapter 1

Introduction

Decision making, in the sense of choosing a course of action from available states or alternatives, is one of the most common activities in life. It ranges from simple everyday decisions to strategic decisions in war. To make decisions easier, a number of methodologies have been developed, including linear and non-linear optimization (Bartholomew-Biggs, 2008; Chang, 2010; Taha, 1971), multiple-criteria decision analysis (MCDA) (Chang, 2010; Figueira et al., 2005; Hipel et al., 1993b), game theory (von Neumann and Morgenstern, 1944), fuzzy decision making (Nakamura, 1986; De Wilde, 2004), and the Graph Model for Conflict Resolution (GMCR) (Fang et al., 1993; Kilgour et al., 1987). Depending on the number of decision makers (DMs) and objectives, decision making techniques are divided into four main categories: (i) single participant-single objective (such as most operations research models), (ii) single participant-multiple objective (such as MCDA methods), (iii) multiple participant-single objective (such as team theory), and (iv) multiple participant-multiple objective (such as GMCR) decision making.

Strategic conflict is a common phenomenon in multiple participant-multiple objective decision making situations, and is observed whenever humans interact through their decisions (Hipel, 2002, 2009a,b; Kilgour and Eden, 2010). For example, two or more individuals or groups may have (i) opposing objectives, as when a seller tries to get a high price while the buyer aims for a low price, or (ii) differing strategies, as when one political party wants to remove the current ruler through a peaceful protest while another would like a revolution. Other human activities that incorporate strategic conflict include bargaining settings, meetings, military actions, and peace-keeping activities (Kilgour and Hipel, 2005).

A number of formal methodologies have been developed to facilitate the analysis of strategic conflicts and to advise on possible resolutions. These methodologies, which include game theory (von Neumann and Morgenstern, 1944), metagame analysis (Howard, 1971), conflict analysis (Fraser and Hipel, 1984), drama theory (Howard, 1999), and GMCR (Kilgour et al., 1987; Fang et al., 1993), share many characteristics. They all provide means to represent and analyze conflict situations with at least two DMs, each of whom has multiple options and multiple objectives, which imply distinctive preferences over the outcomes.

Among these methodologies, conflict resolution researchers and practitioners praise GMCR because of its simplicity and flexibility (Kilgour and Hipel, 2005). Its advantages include its ability to model both irreversible and common moves. It provides a flexible framework for defining, comparing, and characterizing various stability concepts, and is easy to apply to real-world disputes. The GMCR methodology has been used for resolving conflicts, including many arising in engineering, such as water resources management, sustainable development, and environmental engineering (Hipel et al., 2001, 2008a,b; Hipel and Obeidi, 2005; Kilgour and Hipel, 2005). The methodology can also be applied to disputes arising in other areas, such as in social and political sciences, economics and business.

A graph model is a structure describing systematically the main characteristics of a conflict, which may be either current or historical. The major components of a graph model are the DMs, the possible states of the conflict, the movements among states that each DM controls, and each DM's preferences over the available states. Often, a DM's choices are represented as options or courses of action, any combination of which can be selected (Howard, 1971; Kilgour et al., 1987; Fang et al., 1993).

To apply the GMCR methodology, there are two steps: modeling and analysis. In the modeling step, feasible states and moves among them are usually constructed using options (Kilgour et al., 1987; Fang et al., 1993), although states can be defined in other ways. A feasible state is a feasible combination of options, selected or not selected, and a move is a change of options for a DM. In the analysis step, states are assessed for stability employing a number of stability definitions developed to account for the diversity of decision styles, including four basic stabilities: Nash stability or rationality (R) (Nash, 1950, 1951), general metarationality (GMR) (Howard, 1971), symmetric metarationality (SMR) (Howard, 1971), and sequential stability (SEQ) (Fraser and Hipel, 1984). Note that a state is stable for a DM if that DM would not choose to move away from it. A state that is stable for all DMs is called an equilibrium of the model; if it forms, it is predicted to persist.

To demonstrate how a real-world decision problem is formulated within the GMCR framework, consider the environmental conflict in Elmira (a small town in Southern Ontario, Canada) that began in late 1989 when the Ontario Ministry of the Environment (M) found that an underground aquifer was contaminated by a carcinogen. The main suspect was a chemical company in Elmira, Uniroyal Chemical Ltd. (U), which produced the same carcinogen as a by-product. M issued a Control Order demanding that U take necessary measures to remedy the contamination. However, U appealed the control order. The Local Government (L) was another DM of the dispute, as it attempted to represent local interests. These DMs had differing objectives; for example, M wanted to require U to rectify the contamination, while U wanted the control order lifted or at least modified. The conflict is modeled as a graph model, in which each DM has one or more options that it either selects or not. For instance, to attempt to reach a preferable outcome, U could *delay* the appeal process, *accept* the original control order, or *abandon* its Elmira operation. This interesting conflict will be examined at various locations in this thesis to demonstrate, test and refine the new ideas put forward.

1.1 Research Motivation

Preference information is crucial to the identification of states that are stable for a particular DM. Existing stability definitions for the GMCR are based only on relative preferences, expressed using the binary relations "is (strictly) preferred to" and "is indifferent to"; that is, preference input is assumed to be crisp (Fang et al., 1993). A limitation of the existing GMCR (here called the crisp GMCR) is that its associated stability definitions cannot accommodate uncertainty or vagueness in DMs' preferences, which is a major issue in many real-world multiple participant-multiple objective decision problems. DMs may be unclear or uncertain about the preferences between two states, perhaps reflecting cultural and educational backgrounds, personal habits, lack of information, and the inherent vagueness of human judgment. For example, in the Elmira conflict model, U may be in doubt about the desirability of abandoning its Elmira operation. More specifically, the preference of U for a state in which U delays the appeal process and L insists on the application of the original control order, relative to a state in which U abandons its operation, may be uncertain.

Among various formal approaches to modeling uncertainty, probability theory (Feller, 1968, 1971) and fuzzy logic (Zadeh, 1965, 1973) are two widely used platforms. These two concepts are different in meaning, and have both strengths and limitations with respect to their realms of applicability. Probability theory is a model of "randomness, and is effectively used in areas in which probabilistic models can be calibrated using available data sets for applications such as weather forecasting, simulating possible future events, trend analysis, quality control, and risk assessment. Fuzzy logic is based on "linguistic intelligence, and is intended to describe a system, event, or entity, for which quantitative data may be scarce or some of the information may be qualitative in nature. A key application area of fuzzy logic is control systems, as in antilock braking mechanisms, automatic washers, air conditioners, and subway trains. Note that certainty or uncertainty of preferences between two states or scenarios is characterized by a DM's choice and is not based on chance. A DM's choice is more likely to reflect linguistic intelligence, thereby making fuzzy logic a potential tool to model uncertain preferences within the framework of the graph model.

A number of questions arise. Which types of preference uncertainties are encountered in conflict models? Was there any attempt to use preference uncertainties in the stability calculations within the GMCR framework? If there is a development of the GMCR to handle preference uncertainties, how useful is it? Is there a suitable generalized preference structure by which various uncertain preference information can be modeled? Is it possible to develop the GMCR framework so that it can be used to calculate stability of states incorporating various types of preference uncertainties that may be encountered in real-world conflict models?

Only two attempts have been found to incorporate DMs' uncertain preferences into the

GMCR from different points of view. Li et al. (2004a) introduced a new preference structure for the graph model that includes uncertain or unknown preference in the comparison of two states. They considered the situation in which a DM, for the time being, might be uncertain about the preference between two states, but knew that with full information he or she would strictly prefer one state to the other, or be indifferent. The modified stability definitions for R, GMR, SMR, and SEQ were also introduced to accommodate this incomplete binary preference structure into the GMCR. Then, a partial stability analysis could be carried out, with a plan to modify (sharpen) it if more complete preference information became available later.

A fuzzy approach was developed in (Al-Mutairi et al., 2008) to model uncertainty in the preferences of DMs involved in a conflict. The authors divided the fuzzy domain of preferences into five regions with linguistic labels: much more preferred, more preferred, indifferent, less preferred, and much less preferred. Based on these divisions, and adapting the concepts of strong and weak stability proposed by Hamouda et al. (2004, 2006), they introduced an analogous strong and weak stability, and hence strong and weak equilibrium, to suggest possible resolutions of the conflict.

However, the above two approaches fail to accommodate uncertainty about preferences between two states in any general sense. For example, a DM may wish to express his or her preference as a degree or grade of the preference for one state over another. When a fuzzy truth value is assumed in the option prioritization, a novel preference modeling technique within the GMCR framework, as an assessment of the truth of a preference statement at a feasible state in the case of uncertainty, a fuzzy score (which is a real number calculated using fuzzy information, as demonstrated in Chapter 6 of this thesis) may be obtained for each state as a measure of preference. Sometimes a DM may be able to provide a crisp cardinal utility for some states but not others, to which he or she assigns a *fuzzy utility*, i.e., a utility in the form of a fuzzy number.

When information is lacking, a DM may wish to employ a fuzzy multi-criteria decision making technique by using fuzzy weights for the criteria according to which the states are being judged. A participating DM then finds a fuzzy weighted sum (which is a fuzzy number) for each state as a measure of preference. For all of these cases preference between two states is not crisp, but some fuzzy preference information is available. By pairwise comparisons of the fuzzy numbers or scores it may be possible to obtain, for each pair of states, a degree of preference for one state over another. The formal representation of these degrees of preference for one state over another would constitute a fuzzy preference relation.

In the representation of fuzzy preferences, the highest preference degree is 1.0, which implies definite preference (or, in other words, crisp preference), and the lowest degree is 0, which implies definite reverse preference. The degree 0.5 indicates that the states, which are being compared, are likely to be indifferent. The interpretation of other degrees in the unit interval, $[0, 1]$, follows accordingly. Hence, a crisp or certain preference relation is a special case of a fuzzy preference relation.

The objective of this research is to develop a new framework for the graph model that considers fuzzy preference instead of crisp preference as a basic input. The GMCR will then be applicable to decision problems having both certain and uncertain preference information.

1.2 Research Objectives

The main objective of this research is to develop a fuzzy preference methodology for the GMCR to broaden its applicability in strategic conflicts. The proposed comprehensive methodology aims to characterize, accommodate, and analyze potential human interactions in strategic conflict within the paradigm of the GMCR, including fuzzy preferences. The specific goals of this study are listed below.

1. To develop a Fuzzy Preference Framework for the GMCR (FGM) to incorporate uncertain preferences in conflict resolution.

    - Introduce the concept of fuzzy relative certainty of preference to make fuzzy preference useable in stability calculations carried out within the GMCR methodology.
    - Propose the idea of fuzzy satisficing threshold to take into account various DMs' satisficing behavior in strategic conflicts.
    - Introduce the definition of fuzzy unilateral improvement to identify states to which a DM wish to move, if such a move is permitted.
    - Propose definitions of the four basic fuzzy stabilities, specifically, fuzzy Nash stability or fuzzy rationality (FR), fuzzy general metarationality ($FGMR$), fuzzy symmetric metarationality ($FSMR$), and fuzzy sequential stability ($FSEQ$), as well as the definitions of the associated fuzzy equilibria, for two-DM graph models.
    - Extend the the same four fuzzy stability definitions as well as the definitions of the associated fuzzy equilibria for a two-DM graph model to an n-DM ($n > 2$)

case.

- Establish that the crisp graph model is a special case of the FGM.

2. To develop coalition fuzzy stability concepts as a follow-up analysis technique within the FGM.

    - Introduce the ideas of a coalition fuzzy improvement and a class coalitional fuzzy improvement.
    - Propose the definitions of coalition fuzzy stabilities, specifically the coalitional fuzzy Nash stability or coalition fuzzy rationality (*CFR*), coalitional fuzzy general metarationality (*CFGMR*), coalitional fuzzy symmetric metarationality (*CFSMR*), and coalition fuzzy sequential stability (*CFSEQ*), for a coalition.
    - Put forward the definitions of the *CFR*, *CFGMR*, *CFSMR*, and *CFSEQ*, for a DM, and then define the associated coalition fuzzy equilibria.

3. To develop the fuzzy option prioritization technique to model fuzzy preferences for DMs within the GMCR framework.

    - Assume fuzzy truth values of preference statements at feasible states to capture preference uncertainty.
    - Calculate a fuzzy score for each state as a measure of preference by using fuzzy truth values of preference statements at feasible states.
    - Propose a formula to compute a fuzzy preference degree for one state over another, thereby establishing a fuzzy preference relation over the set of feasible states.

4. To apply the fuzzy preference methodologies for the GMCR to real-world disputes.

- Apply the two-DM case of the FGM to the sustainable development conflict.
- Apply the n-DM ($n > 2$) case of the FGM to the Elmira groundwater contamination conflict.
- Carry out the coalition fuzzy stability analysis on the Elmira groundwater contamination conflict.
- Apply the fuzzy option prioritization technique to the Elmira groundwater contamination conflict for eliciting (crisp or fuzzy) preferences of the DMs.

1.3 Outline of the Thesis

The graph model fuzzy preference methodology is illustrated in Figure 1.1. As depicted in the figure, the general steps to apply this methodology to a real-world dispute are: (i) modeling, (ii) fuzzy stability analysis, and (iii) follow-up analysis (if needed). Each step is accomplished by employing one or more techniques. For example, in the modeling step, fuzzy option prioritization is employed to represent DMs crisp or fuzzy preferences.

The outline of the thesis is as follows. The present chapter mainly describes the motivation and objectives of this research. The existing GMCR methodology is reviewed in the first part of Chapter 2, briefly describing its modeling components and four basic stability concepts, while in the second part, the concept of a fuzzy preference relation and its main properties are discussed. The contributions of this PhD research are clustered into the rest of the chapters.

In Chapter 3, a fuzzy preference framework for a two-DM graph model is developed to introduce four basic fuzzy stability concepts and apply them to simple conflicts with

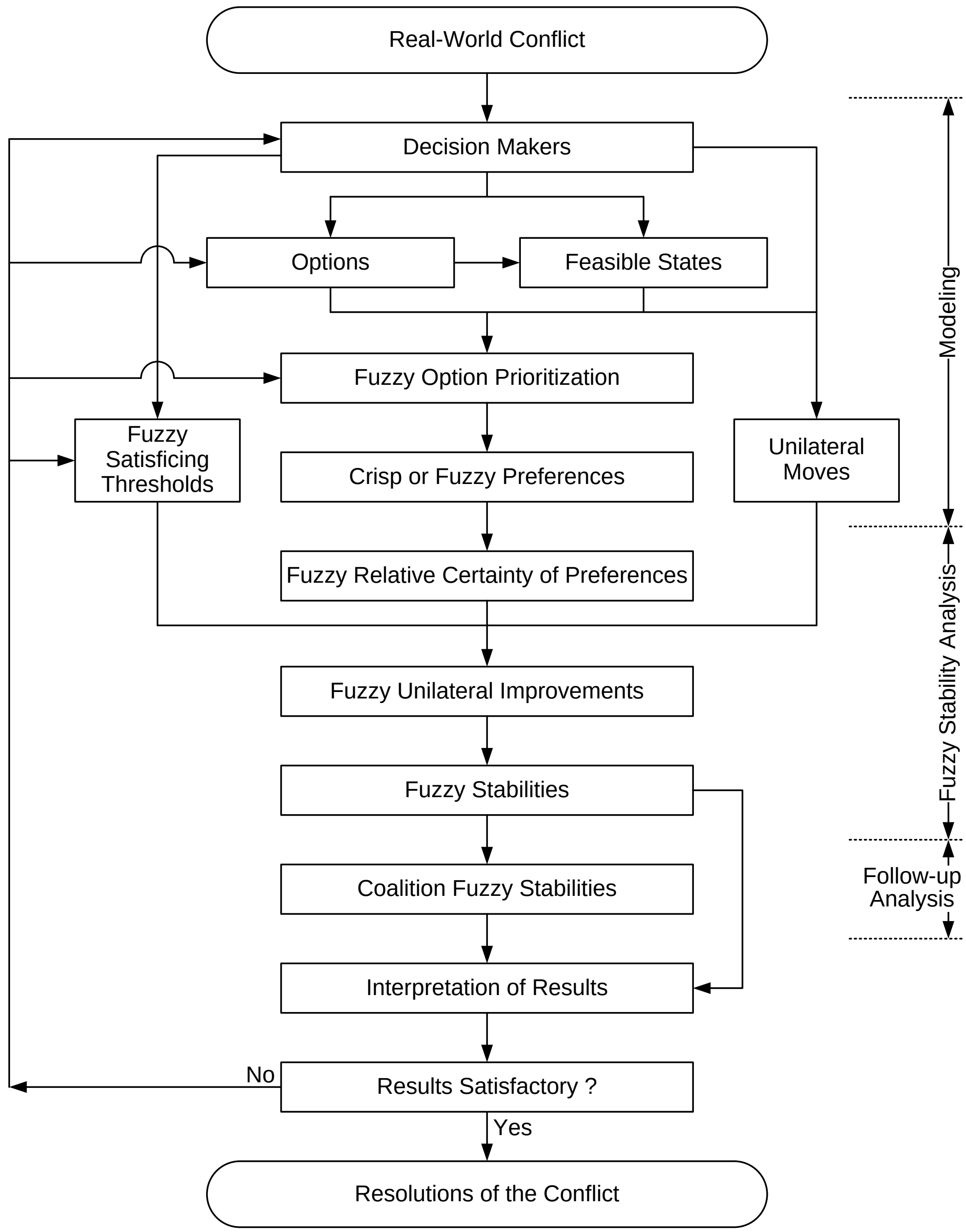

Figure 1.1: Fuzzy Preferences in the Graph Model for Conflict Resolution

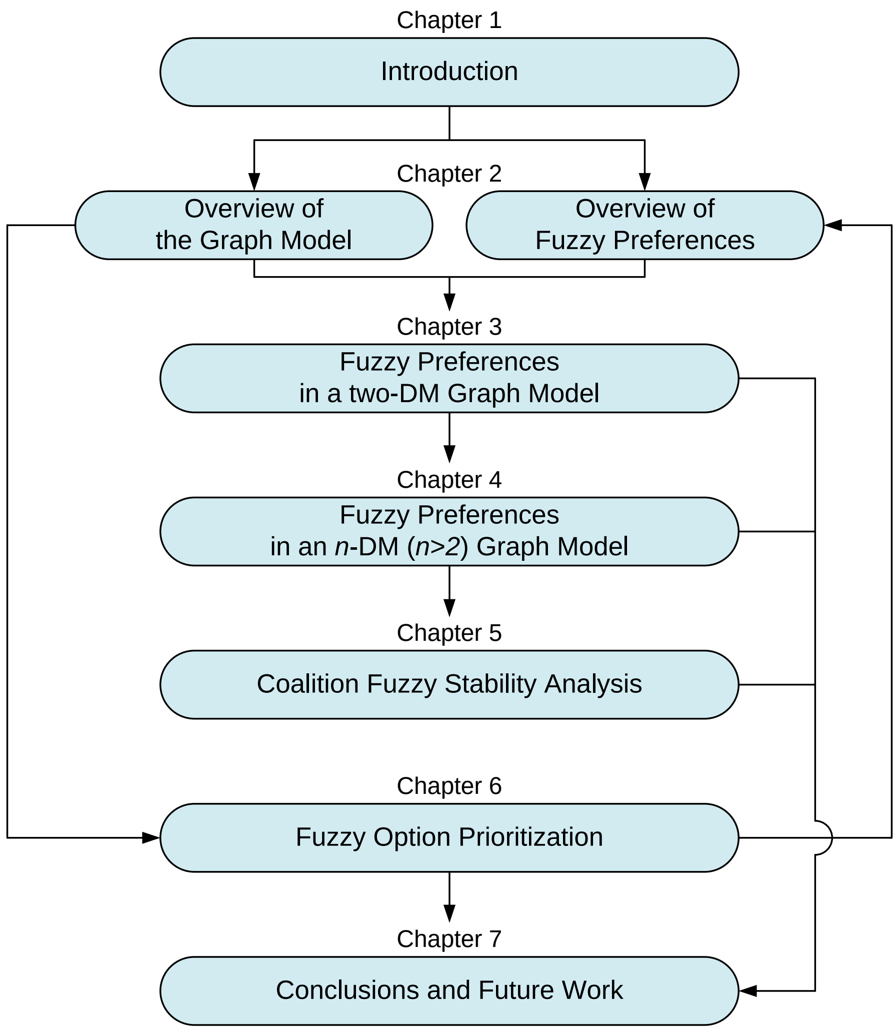

Figure 1.2: Thesis Outline

two DMs exhibiting uncertain preferences. Addressing the necessity of fuzzy stability definitions for a more general n-DM ($n > 2$) graph model, the fuzzy preference framework is then extended in Chapter 4 to accommodate graph models with any number of DMs, definitely generalizing the associated fuzzy stability definitions. To further analyze the individual level fuzzy stabilities introduced in Chapters 3 and 4, the coalition fuzzy stability concepts are developed in Chapter 5.

A fuzzy option prioritization methodology is formalized in Chapter 6 to facilitate the modeling of fuzzy preferences for DMs in strategic conflicts that feature uncertain preferences for DMs. The main contributions of this thesis are compiled in Section 7.1 while a number of directions for potential future research is listed in Section 7.2. The outline of the thesis is also summarized in Figure 1.2.

Chapter 2

Background and Literature Review

2.1 Introduction

Among the four categories of decision making techniques mentioned in Chapter 1, multiple participant-multiple objective decision making is the most complicated. This research is intended to develop an appropriate solution methodology for it, especially for the case in which the participants or DMs have uncertain preferences over the states or alternatives. The GMCR, a variant of classical game theory, is a novel methodology for modeling and analyzing disputes occurring in multiple participant-multiple objective decision problems. The key difference between the GMCR and classical game theory is that the preference structure of GMCR is binary, and not based on utility theory. Accordingly, the preference inputs for GMCR do not need to be transitive. Furthermore, in a Graph Model, the order in which DMs choose to move, or not to move, need not be specified in advance.

In this chapter, related literature on the crisp GMCR and fuzzy preferences are reviewed. More specifically, the components as well as the structure of a crisp graph model are described and the four basic crisp stability definitions are presented. Moreover, the literatures on a number of approaches for modeling uncertain preferences, including fuzzy preferences, are reviewed. Next, the notions of a fuzzy set, fuzzy number and fuzzy relation are presented as bases of a fuzzy preference. Then the concept of a fuzzy preference and its properties are introduced.

2.2 Literature Review of the Graph Model for Conflict Resolution

The GMCR is a methodology for modelling, analyzing, and understanding strategic conflicts, which is common in multiple participant-multiple objective decision makings. The main motivation for developing the GMCR was the demand for a comprehensive method to understand conflict decision-making and conflict resolution as existing methods were cumbersome and often failed to provide the needed analysis and advice (Kilgour and Hipel, 2005). The graph model is designed to be simple and flexible, as well as to have minimal information requirements. The original idea of the graph model was introduced by Kilgour, Hipel, and Fang in (Kilgour et al., 1987), while the first comprehensive representation was furnished by Fang, Hipel, and Kilgour as (Fang et al., 1993).

The application of the GMCR begins with the representation of a real-world conflict problem. By careful examination of the conflict, the DMs who have direct impact on the conflict and interest in its outcomes are identified. Taking the available options or courses of actions of these DMs into account, a set of feasible states are generated. Note that a state is a combination of options chosen by the participating DMs; the set of feasible states is a subset of all possible states, and can be thought of as the set of all feasible combinations of the options. A DM's possible moves among states are determined by allowable state transitions, and are identified by fixing all other DMs' options. Investigating historical data or information supplied directly by the DMs, the preference relation between any two feasible states (pairwise preferences, or relative preferences) for each DM are determined. After these steps have been completed, stability of each feasible state is investigated for various stability definitions including *R*, *GMR*, *SMR*, and *SEQ* for each DM. Using these stability results, states that are stable under a suitable stability definition for every participating DM are identified and interpreted as equilibrium states or possible resolutions of the conflict (Fang et al., 1993).

A stability concept prescribes what a DM can do when acting independently in a conflict, based on his or her own interests. To further develop insights into the conflict, a coalition analysis is carried out. A coalition consists of two or more DMs who may act as a group if they can do better together than individually. A recent development in the graph model is status quo analysis, which investigates equilibria that are more likely to occur in a conflict situation, to help analysts identify likely resolutions.

2.2.1 The Structure of the Graph Model for Conflict Resolution

A graph model of a conflict is represented mathematically by a set of DMs, a set of states, each DM's directed graph indicating movements controlled by the DM, and each DM's preference relation over the states. The nodes in the DMs' graphs are common, referred to as the feasible states, whereas the (directed) arcs of a DM's graph are the possible state-to-state moves controlled by that DM. Note that moves may or may not be reversible. As mentioned earlier, in a crisp graph model, DMs' preferences are given by binary relations on the set of feasible states (Fang et al., 1993; Kilgour et al., 1987).

In a graph model, each state or scenario is often defined as a combination of a number of options that reflect the participating DMs' strategies to achieve their objectives. In a model, available options are uniquely represented by $O_1, O_2, ...$. In a state, a particular option may or may not be selected by the DM controlling it; if the option is selected, it is given by a "Y" or "1" and if the option is not chosen, it is represented by an "N" or "0". Hence, a state is an ordered tuple of Ys and Ns or of 1s and 0s, usually written as a column in which the number of entries is the same as the total number of options in the model. Accordingly, if the number of options in a model is λ, then there are 2^λ mathematically possible states; however, only a portion of them may be feasible in practice because of various option constraints (Fang et al., 1993; Peng, 1999; Fang et al., 2003).

Note that in a graph model, there may be a group of formally "distinct" but practically "indistinguishable" states. Such a group is represented as one state using "–"s against appropriate options, indicating that it is the same whether Ys or Ns are chosen for those options. A state of this type is called a composite state (Fang et al., 1993; Peng, 1999; Fang et al., 2003).

Denote by $N = \{1, 2, ..., n\}$, the set of DMs, and by $S = \{s_1, s_2, ..., s_m\}$, $m > 1$, the set of feasible states. For $k \in N$, let $A_k \subseteq S \times S$ (Cartesian product of S with itself) represent the moves controlled by DM k, so that for $s_i, s_j \in S$, $(s_i, s_j) \in A_k$ if and only if DM k can cause the conflict to move (directly) from state s_i to state s_j. Then $D_k = (S, A_k)$ is DM k's directed graph. Also, DM k's preferences are recorded by a binary relation $\succsim_k$, with the interpretation that, for $s_i, s_j \in S$, $s_i \succsim_k s_j$ if and only if DM k prefers s_i to s_j ($s_i \succ s_j$), or is indifferent between them ($s_i \sim s_j$). With the notations given above, a Graph Model can be represented as

$$\langle N,\ S,\ \{(D_k, \succsim_k) : k \in N\}\rangle.$$

Note that the graph model methodology can handle both transitive and intransitive preferences over the feasible states.

2.2.2 Reachable Lists

The GMCR methodology uses DMs' unilateral improvement lists for its stability calculations. As a basis of the construction of a DM's unilateral improvement list from a given state, as well as to study the countermoves by the opponent(s), it is necessary to record all the states to which a DM can cause the conflict to move unilaterally from an initial state in one step.

2.2.2.1 Reachable List of a Decision Maker

A DM's reachable list from a specified starting state is a record of all the states that the DM can reach in one step. In a graph model, the states that are joined by an arc in A_k beginning at state s form the DM k's reachable list from state s. A formal definition is given below.

Definition 2.2.1. (Reachable List for a DM): The *reachable list* from a state $s \in S$ for DM k is

$$R_k(s) = \{s_i \in S\ :\ (s, s_i) \in A_k\}.$$

2.2.2.2 Reachable List of a Coalition

The reachable list provided by Definition 2.2.1 is the set of unilateral moves under the control of DM k. However, the GMCR methodology takes into account moves and countermoves in its stability calculations. When there are more than two DMs in a model, the countermoves are performed by more than one DM. Hence, the definition of unilateral moves by a group or coalition of DMs is needed.

Assume $n > 2$. Any set of DMs, $H \subseteq N$, is called a *coalition*. If $|H| > 0$, then the coalition H is non-empty. Throughout the thesis, each coalition $H \subseteq N$ is assumed to be non-empty. If $|H| \geq 2$, then the coalition H is non-trivial.

For $s \in S$, let $R_H(s) \subseteq S$ denote the set of all states reachable from s via a legal sequence of moves by some or all of the DMs in H. Note that a sequence of moves for a coalition H is called *legal* if no DM in H moves twice consecutively. For any $s_1 \in R_H(s)$, let $\Omega_H(s, s_1)$ denote the set of all last DMs in legal sequences from s to s_1. The reachable list by a coalition can now be defined formally. Note that the coalition $H \subseteq N$ with $|H| = 1$ is trivial in the sense that it is equivalent to a single DM, and is excluded from this definition. In fact, if $H = \{k\}$, then $R_H(s) = R_k(s)$.

Definition 2.2.2. (Reachable List for a Coalition): Let $s \in S$ and $H \subseteq N$, $|H| \geq 2$. Define the subset $R_H(s) \subseteq S$ inductively as follows:

(1) If $k \in H$ and $s_1 \in R_k(s)$, then $s_1 \in R_H(s)$ and $k \in \Omega_H(s, s_1)$;

(2) If $s_1 \in R_H(s)$, $k \in H$, $s_2 \in R_k(s_1)$, and $\Omega_H(s, s_1) \neq \{k\}$, then $s_2 \in R_H(s)$ and $k \in \Omega_H(s, s_2)$.

The set $R_H(s)$ is called the *reachable list* from s for the coalition H, and any member of $R_H(s)$ is called a *unilateral move* from s by the coalition H.

Note that, in Definition 2.2.2, the induction stops as soon as no new state (s_2) can be added to $R_H(s)$ and $|\Omega_H(s, s_1)|$ cannot be increased for any $s_1 \in R_H(s)$.

Below is an algorithm that implements this definition. The set $R_H(s, i)$ consists of the states achievable by coalition H in at most $i \geq 0$ legal moves, starting from state s. For $s_1 \in R_H(s, i), \Omega_H(s, s_1, i)$ denotes the set of all last DMs in legal sequences from s to s_1 with at most i moves.

(1) For $i = 0$, set $R_H(s, 0) = \{s\}$ and $\Omega_H(s, s_1, 0) = \emptyset$ for all $s_1 \in S$.

(2) Now find $R_H(s, i+1) \supseteq R_H(s, i)$ and $\Omega_H(s, s_1, i+1) \supseteq \Omega_H(s, s_1, i)$ for all $s_1 \in S$. Select any $s_2 \in S$ satisfying $s_2 \in R_k(s_1)$ for some $k \in H$ and some $s_1 \in R_H(s, i)$. Then, if $s_2 \notin R_H(s, i)$ and $\Omega_H(s, s_1, i) \neq \{k\}$, add s_2 to $R_H(s, i + 1)$ and k to $\Omega_H(s, s_2, i + 1)$. Also, if $s_2 \in R_H(s, i)$ and $\Omega_H(s, s_1, i) \neq \{k\}$ but $k \notin \Omega_H(s, s_2, i)$, add k to $\Omega_H(s, s_2, i + 1)$. Continue until $R_H(s, i + 1)$ and $\Omega_H(s, s_2, i + 1)$ cannot be further increased.

(3) If $R_H(s, i + 1) = R_H(s, i)$ and $\Omega_H(s, s_1, i + 1) = \Omega_H(s, s_1, i)$ for all $s_1 \in R_H(s, i)$, stop. Otherwise, increase i by 1 and repeat step (2).

Note that the algorithm stops as soon as $R_H(s, i + 1) = R_H(s, i)$ and, for all $s_1 \in R_H(s, i)$, $\Omega_H(s, s_1, i + 1) = \Omega_H(s, s_1, i)$. The corresponding value of i is the maximum length of any legal path for H from s.

2.2.3 Crisp Preferences

As mentioned earlier, each DM's preference information over feasible states or alternatives is an important input to the GMCR methodology. A crisp preference over feasible states, mathematically a crisp binary relation, reflects the certainty of preference between any two states. A crisp preference is often denoted by $\succsim$, and for DM k, it is given by $\succsim_k$. For any $s_i, s_j \in S$, $s_i \succsim_k s_j$ means that DM k finds s_i at least as preferable as s_j, and is stated as "s_i is preferred or indifferent to s_j." Therefore, $s_i \succsim_k s_j$ implies that DM k likes s_i better than s_j, or doesn't care whether s_i or s_j is chosen. In fact, the symbol "$\succ$" stands for strict preference and "$\sim$" for indifference. In summary, $s_i \succsim_k s_j$ indicates that DM k strictly prefers s_i to s_j ($s_i \succ s_j$), or is indifferent between them ($s_i \sim s_j$).

Given a strict crisp preference $\succ$ on S, $\prec$ is defined as follows.

Definition 2.2.3. For $s_i, s_j \in S$, $s_i \prec s_j$ if and only if $s_j \succ s_i$.

There are other representations of a crisp preference between two states s_i and s_j. One uses an index d_{ij} to distinguish three cases of preference or indifference (Garcia-Lapresta and Montero, 2006):

$$d_{ij} = \begin{cases} 1, & \text{if } s_i \text{ is preferred to } s_j \\ 0, & \text{if } s_i \text{ is indifferent to } s_j \\ -1, & \text{if } s_j \text{ is preferred to } s_i \end{cases}.$$

This index can be normalized to take values in the unit interval $[0, 1]$ as follows:

$$r_{ij} = \frac{d_{ij}+1}{2} = \begin{cases} 1, & \text{if } s_i \text{ is preferred to } s_j \\ 0.5, & \text{if } s_i \text{ is indifferent to } s_j \\ 0, & \text{if } s_j \text{ is preferred to } s_i \end{cases}. \tag{2.1}$$

2.2.4 Crisp Stabilities in a two-Decision Maker Graph Model

In the final part of the GMCR study, the main focus is on examination of the stability of states for a DM. From a stable state, the focal DM has no incentive to deviate in a sense determined by a particular stability definition. The crisp GMCR accounts for crisp stabilities, which are based on crisp preferences described in Subsection 2.2.3. To identify the states that are worthwhile for a DM to move to from a given state, the definition of unilateral improvements by a DM is provided in the following subsection.

2.2.4.1 Unilateral Improvements by a Decision Maker

Note that the GMCR methodology considers moves and countermoves by the opponent(s) in calculating various stabilities. In the case of a two-DM graph model, the focal DM, as well as the opponent, is a single DM. Hence, the definition of a unilateral improvement from a given state by a single DM is needed. A state is a unilateral improvement from an initial state $s \in S$ by a DM if the state is reachable from s by the DM in one step and is preferred to s. A formal definition is given below.

Definition 2.2.4. (Unilateral Improvement by a DM): Recall that $R_k(s)$ represents the set of reachable states from a given initial state $s \in S$ by DM $k \in N$. A state $s_i \in S$ is called a *unilateral improvement* (UI) from s by DM k if $s_i \in R_k(s)$ and $s_i \succ_k s$.

Definition 2.2.5. (Unilateral Improvement List by a DM): The collection of all UIs from a state s by DM k is called the *unilateral improvement list* (UIL) from s by DM k, denoted $R_k^+(s)$. Mathematically,

$$R_k^+(s) = \{s_i \in R_k(s) \ : \ s_i \succ_k s\}.$$

2.2.4.2 Crisp Stability Definitions in a two-Decision Maker Graph Model

As a basis for identifying the states from which a DM does not like to move away, the stability concepts within the GMCR framework are provided here. Note that different DMs may show different behavior patterns in responding to a strategic conflict. For example, they may have different levels of foresight for which some DMs look far ahead before making a decision, while others consider only immediate consequences. Furthermore, DMs may have different perspectives about the risks of moving. Some DMs may be ready to accept temporary dis-improvements in the expectation of achieving a better outcome in the end, while others may wish to avoid all dis-improvements. To capture these varied human behavior and decision techniques formally, a number of stability definitions have been introduced within the GMCR framework of which the four basic definitions are presented below. Note that these definitions are for a two-DM graph model. The stability definitions for a general n-DM ($n > 2$) graph model are provided in Susubsection 2.2.5.2.

Definition 2.2.6. (Nash Stability or Rationality): Let $k \in N$ and $s \in S$. State s is *Nash stable* or *rational* (R) for DM $k \in N$ if and only if $R_k^+(s) = \varnothing$.

As there are two DMs in the model, for the following definitions assume that $N = \{k, l\}$.

Definition 2.2.7. (General Metarationality): A state $s \in S$ is *general metarational*

(GMR) for DM k if and only if for every $s_1 \in R_k^+(s)$ there exists an $s_2 \in R_l(s_1)$ such that $s_2 \precsim_k s$.

Definition 2.2.8. (Symmetric Metarationality): A state $s \in S$ is *symmetric metarational* (SMR) for DM k if and only if for every $s_1 \in R_k^+(s)$ there exists an $s_2 \in R_l(s_1)$ such that $s_2 \precsim_k s$, and $s_3 \precsim_k s$ for all $s_3 \in R_k(s_2)$.

Definition 2.2.9. (Sequential Stability): A state $s \in S$ is *sequentially stable* (SEQ) for DM k if and only if for every $s_1 \in R_k^+(s)$ there exists an $s_2 \in R_l^+(s_1)$ such that $s_2 \precsim_k s$.

2.2.5 Crisp Stabilities in a Graph Model with More than Two Decision Makers

The stability definitions provided in Subsubsection 2.2.4.2 are specifically for a two-DM graph model where the opponent of the focal DM is a single DM. However, in a graph model with more than two DMs, the opponent of the focal DM is a group or coalition of two or more DMs. Hence, the stability definitions given in Subsubsection 2.2.4.2 will not work for a graph model that has more than two DMs. Accordingly, stability definitions for a general n-DM ($n > 2$) graph model are needed.

2.2.5.1 Unilateral Improvements by a Coalition

The UIL for a DM, given by Definition 2.2.5, is sufficient for defining stabilities for a two-DM graph model as represented in Subsubsection 2.2.4.2. However, to extend these stability concepts for a general n-DM ($n > 2$) graph model, one needs the definition of a UI by a coalition of DMs.

Definition 2.2.10. (Unilateral Improvement by a Coalition): Let $s \in S$ and $H \subseteq N$, $|H| \geq 2$. Define $R_H^+(s) \subseteq S$ inductively as follows:

(1) If $k \in H$ and $s_1 \in R_k^+(s)$, then $s_1 \in R_H^+(s)$ and $k \in \Omega_H^+(s, s_1)$, where $\Omega_H^+(s, s_1)$ represents the set of all last DMs in legal sequences from s to s_1;

(2) If $s_1 \in R_H^+(s)$, $k \in H$, $s_2 \in R_k^+(s_1)$, and $\Omega_H^+(s, s_1) \neq \{k\}$, then $s_2 \in R_H^+(s)$ and $k \in \Omega_H^+(s, s_2)$.

A *unilateral improvement* (UI) from s by the coalition H is any member of $R_H^+(s)$.

Note that the induction in Definition 2.2.10 stops as soon as no new state (s_2) can be added to $R_H^+(s)$, and $|\Omega_H^+(s, s_1)|$ cannot be increased for any $s_1 \in R_H^+(s)$.

2.2.5.2 Crisp Stability Definitions in an n-Decision Maker ($n > 2$) Graph Model

The definitions of *GMR*, *SMR*, and *SEQ* stabilities for an n-DM ($n > 2$) graph model are provided here. Note that *Nash* stability does not depend on the responses of the opponents. Therefore, the definition of *Nash* stability for an n-DM graph model is the same as for the two-DM case. In the following definitions, $N - k$ denotes the set of DMs other than k, or in other words, k's opponents.

Definition 2.2.11. (General Metarationality): A state $s \in S$ is *general metarational* (*GMR*) for DM $k \in N$ if and only if for every $s_1 \in R_k^+(s)$ there exists an $s_2 \in R_{N-k}(s_1)$ such that $s_2 \precsim_k s$.

Definition 2.2.12. (Symmetric Metarationality): A state $s \in S$ is *symmetric metarational* (*SMR*) for DM $k \in N$ if and only if for every $s_1 \in R_k^+(s)$ there exists an $s_2 \in R_{N-k}(s_1)$ such that $s_2 \precsim_k s$, and $s_3 \precsim_k s$ for all $s_3 \in R_k(s_2)$.

Definition 2.2.13. (Sequential Stability): A state $s \in S$ is *sequentially stable* (*SEQ*) for DM $k \in N$ if and only if for every $s_1 \in R_k^+(s)$ there exists an $s_2 \in R_{N-k}^+(s_1)$ such that $s_2 \precsim_k s$.

2.2.5.3 Crisp Equilibrium

The stability definitions, given in Subsubsections 2.2.4.2 and 2.2.5.2, characterize a single DM's unwillingness to deviate from a state. However, the GMCR methodology identifies a state as a potential resolution, from which no DM would like to move away, referred to as an equilibrium state. A formal definition is given below.

Definition 2.2.14. A state that is stable for all DMs under a specific stability definition is called an *equilibrium* under that definition.

2.2.6 Coalition Stability Analysis

The old saying "Many hands make light work" means that working together may make a task easier compared to working individually. In the same way, in decision making—more specifically, in multiple participant-multiple objective decision making—it is logical to raise the question whether there will be a better outcome when a group of DMs joins together to make a decision, even when the DMs have individual objectives that may be in conflict. Coalition formation is often found in various real-world multiple-participant decision situations. For instance, in a debate of the United Nations general assembly on a proposal to help nations affected by greenhouse gas emissions, like-minded countries may work together in support of amendments they prefer, or on the choice to accept or reject the proposal.

Kuhn et al. (1983) applied some simple concepts of coalition analysis to strategic conflicts within a crisp GMCR structure by introducing rules for formation of a coalition in a conflict, assuming that a coalition would last throughout the dispute. Kilgour et al. (2001) made the first general approach to developing coalition formation guidelines and formalizing the idea of coalition moves. They introduced coalition stability, parallel to the concept of individual (crisp) Nash stability. Later, Inohara and Hipel (2008a,b) developed coalition stability definitions parallel to individual (crisp) general metarationality, symmetric metarationality, and sequential stability, and also characterized general relationships among them. The coalition stability concepts reviewed in this subsection are due to (Kilgour et al., 2001; Inohara and Hipel, 2008a,b).

Below, $H \subseteq N$ represents a coalition of DMs in N, and $\mathcal{P}(N)$, the class of all coalitions of DMs in N.

Definition 2.2.15. (Coalition Improvement): A state $s_i \in S$ is a *coalition improvement* from a state $s \in S$ by a coalition $H \subseteq N$ if $s_i \in R_H(s)$ and $s_i \succ_k s$ for all $k \in H$. The *coalition improvement list* from s by the coalition H, denoted $R_H^{++}(s)$, is

$$R_H^{++}(s) = \{s_i \in S \ : \ s_i \in R_H(s) \text{ and } s_i \succ_k s \text{ for all } k \in H\}.$$

Definition 2.2.16. (Coalition Nash Stability or Coalition Rationality for a Coalition): Let $H \in \mathcal{P}(N)$ and $s \in S$. State s is *coalition Nash stable* or *coalition rational*

(CR) for coalition H if and only if $R_H^{++}(s) = \varnothing$.

Definition 2.2.17. (Coalition Nash Stability or coalition Rationality for a DM): Let $k \in N$ and $s \in S$. State s is *coalition Nash stable* or *coalition rational* (CR) for DM k if and only if s is CR for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

To define the coalitional versions of *GMR*, *SMR* and *SEQ*, a class coalitional move and class coalitional improvement by a class of coalitions of DM in N must first be defined.

Definition 2.2.18. (Class Coalitional Move): Let $s \in S$, and $\mathcal{C}$ be a class of coalitions of DMs in N, i.e., $\mathcal{C} \subseteq \mathcal{P}(N)$. The *class reachable list* or *set of class coalitional moves* from state s by class $\mathcal{C}$ is defined inductively as the set $R_{\mathcal{C}}(s)$ that satisfies the following two conditions:

(1) If $H \in \mathcal{C}$ and $s_1 \in R_H(s)$, then $s_1 \in R_{\mathcal{C}}(s)$;

(2) If $s_1 \in R_{\mathcal{C}}(s)$ and $H \in \mathcal{C}$, and $s_2 \in R_H(s_1)$, then $s_2 \in R_{\mathcal{C}}(s)$.

A *class coalitional move* (CCM) from s by the class $\mathcal{C}$ is any member of $R_{\mathcal{C}}(s)$. Note that, because of the definition of a coalitional unilateral move (Definition 2.2.2), no DM in any coalition in $\mathcal{C}$ may move twice consecutively in passing from s to any state in $R_{\mathcal{C}}(s)$.

Definition 2.2.19. (Class Coalitional Improvement): Let $s \in S$ and $\mathcal{C} \subseteq \mathcal{P}(N)$. The *class improvement list* or *class coalitional improvement list* from state s by class $\mathcal{C}$, denoted $R_{\mathcal{C}}^{++}(s)$, is defined inductively as follows:

(1) If $H \in \mathcal{C}$ and $s_1 \in R_H^{++}(s)$, then $s_1 \in R_{\mathcal{C}}^{++}(s)$;

(2) If $s_1 \in R_{\mathcal{C}}^{++}(s)$ and $H \in \mathcal{C}$, and $s_2 \in R_H^{++}(s_1)$, then $s_2 \in R_{\mathcal{C}}^{++}(s)$.

A *class coalitional improvement* (CCI) from s by the class $\mathcal{C}$ is any member of $R^{++}_{\mathcal{C}}(s)$. As in Definition 2.2.18, this definition ensures that no DM in any coalition in $\mathcal{C}$ may move twice consecutively.

The definitions of the coalitional forms of *GMR*, *SMR* and *SEQ* given by Definitions 2.2.11, 2.2.12 and 2.2.13, respectively, are now provided below.

Definition 2.2.20. (Coalition General Metarationality for a Coalition): For $H \in \mathcal{P}(N)$, state $s \in S$ is *coalition general metarational* (*CGMR*) for coalition H if and only if for every $s_1 \in R^{++}_H(s)$ there exists a CCM $s_2 \in R_{\mathcal{P}(N-H)}(s_1)$ such that $s_2 \precsim_k s$ for some $k \in H$.

Definition 2.2.21. (Coalition General Metarationality for a DM): For $k \in N$, state $s \in S$ is *coalition general metarational* (*CGMR*) for DM k if and only if s is *CGMR* for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

Definition 2.2.22. (Coalition Symmetric Metarationality for a Coalition): For $H \in \mathcal{P}(N)$, state $s \in S$ is *coalition symmetric metarational* (*CSMR*) for coalition H if and only if for every $s_1 \in R^{++}_H(s)$ there exists a CCM $s_2 \in R_{\mathcal{P}(N-H)}(s_1)$ such that $s_2 \precsim_k s$ for some $k \in H$, and for every $s_3 \in R_H(s_2)$, $s_3 \precsim_l s$ for some $l \in H$.

Definition 2.2.23. (Coalition Symmetric Metarationality for a DM): For $k \in N$, state $s \in S$ is *coalition symmetric metarational* (*CSMR*) for DM k if and only if s is *CSMR* for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

Definition 2.2.24. (Coalition Sequential Stability for a Coalition): For $H \in \mathcal{P}(N)$, state $s \in S$ is *coalition sequentially stable* (*CSEQ*) for coalition H if and only if for every $s_1 \in R^{++}_H(s)$ there exists a CCI $s_2 \in R^{++}_{\mathcal{P}(N-H)}(s_1)$ such that $s_2 \precsim_k s$ for some $k \in H$.

Definition 2.2.25. (Coalition Sequential Stability for a DM): For $k \in N$, state $s \in S$ is *coalition sequentially stable* ($CSEQ$) for DM k if and only if s is $CSEQ$ for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

Definition 2.2.26. (Coalition Equilibrium): A state $s \in S$ is a *coalition equilibrium* under a specific coalition stability concept if and only if s is coalition stable for each DM under that coalition stability notion. For instance, state s is coalition Nash equilibrium or CR equilibrium if and only if it is CR stable for each DM in N.

2.2.7 Crisp Option Prioritization

Each DM's preferences over feasible states, which are inputs to the analysis step of the GMCR methodology and GMCR II (Peng, 1999; Fang et al., 2003), a decision support software developed to implement the GMCR, are traditionally modeled by pairwise comparisons of states. However, it may be hard for a DM or an analyst to identify the preferred state from a pair by comparing them, especially when the model has a large number of options. Note that DMs' options, whose feasible selection constitutes a state, characterize a possible solution space of the problem under study. The greater the number of options contained in a model, the larger is the number of criteria needed to compare one state with another. Other techniques used to model preferences between two states include the preference tree (Fraser, 1993, 1994; Fraser and Hipel, 1988; Hipel and Meister, 1994), option weighting (Fang et al., 2003; Kilgour, 1997), and option prioritization (Peng et al., 1997; Peng, 1999; Fang et al., 2003) (now called, crisp option prioritization). Among these methodologies, crisp option prioritization, which is a generalization of the preference tree, is a very useful preference modeling technique in the GMCR. This technique overcomes the limitations that other methods have, and has been implemented in GMCR II.

In crisp option prioritization, each DM is asked to provide a priority ordered set of preference statements. Each preference statement takes a truth value, either “True” (T) or “False” (F), at each state. A preference statement is composed of options by using logical connectives. It can be non-conditional, conditional, or bi-conditional. A non-conditional preference statement is simple and is given as a combination of options relevant to that particular statement, joined by various connectives such as *negation* (“not”, “—”, or “¬”), *conjunction* (“and”, “&”, or “$\wedge$”), and *disjunction* (“or”, “|”, or “$\vee$”). The priority of operations in a preference statement is often controlled by round parentheses “(” and “)”. A conditional or bi-conditional preference statement consists of two non-conditional preference statements joined by a connective *implies* (or, *if-then*) (“IF”) or *if and only if* (“IFF”).

The truth value of a non-conditional preference statement at a state is straightforward. For example, if O_1 and O_2 represent two options, then the statement $O_1 \wedge O_2$ is true at a state if both O_1 and O_2 occur at that state, otherwise $O_1 \wedge O_2$ is false. However, the truth value of a conditional or bi-conditional preference statement at a state depends on the truth values of its component non-conditional statements. The truth values of these preference statements are determined according to the conditional or bi-conditional truth tables standardized in mathematical logic (Chiswell and Hodges, 2007).

For crisp option prioritization, each DM's preference statements, say Ω_1, Ω_2, ..., Ω_q, are listed in order of priority, which are often represented vertically from the most important to least. For $s \in S$, let $\Omega_t(s)$ $(1 \leq t \leq q)$ denote the truth value of the preference statement Ω_t at state s. A DM's crisp preference between two states is determined based on the truth values of the preference statements at those states in lexicographic ordering fashion. A state with a truth value “T” of a more important preference statement is preferred to a state having a truth value "F" of the same preference statement, or to a state with a truth value "T" or "F" of a less important preference statement. Specifically, a state $s_1 \in S$ is preferred to a state $s_2 \in S$ ($s_1 \neq s_2$) if and only if either $\Omega_1(s_1) = T$ and $\Omega_1(s_2) = F$, or there exists t, $1 < t \leq q$, such that

$$\begin{gathered} \Omega_1(s_1) = \Omega_1(s_2) \\ \Omega_2(s_1) = \Omega_2(s_2) \\ \vdots \\ \Omega_{t-1}(s_1) = \Omega_{t-1}(s_2), \\ \text{and } \Omega_t(s_1) = T \text{ and } \Omega_t(s_2) = F. \end{gathered} \tag{2.2}$$

If there is no such t, then either s_1 and s_2 are indifferent or s_2 is preferred to s_1. Note that it is a convention in GMCR II that the "–"s are considered as "N"s in determining the truth of a preference statement at a composite state (Peng et al., 1997; Peng, 1999; Fang et al., 2003).

An equivalent scheme that can result in the same ranking as in (2.2) is to assign a "score" $\Psi(s)$ to each feasible state $s \in S$ according to its truth values when the preference statements are applied. Assume that q is the total number of preference statements for a DM. Denote by $\Psi_t(s)$ the *incremental score* of state s for preference statement Ω_t, $1 \leq t \leq q$. Define

$$\Psi_t(s) = \begin{cases} \frac{1}{2^t}, & \text{if } \Omega_t(s) = T \\ 0, & \text{if } \Omega_t(s) = F \end{cases}$$

and

$$\Psi(s) = \sum_{t=1}^{q} \Psi_t(s). \tag{2.3}$$

Then the states are ranked according to their scores; a state with a higher score is preferred to a state with a lower score. More specifically, for $s_1, s_2 \in S$, $s_1 \succ s_2$ if and only if $\Psi(s_1) > \Psi(s_2)$. Furthermore, $s_1 \sim s_2$ if and only if $\Psi(s_1) = \Psi(s_2)$. This results in exactly the same ranking as that obtained from the lexicographic ordering. Note that, even though a cardinal score is involved, it only plays a temporary role in determining the ranking; it does not tell anything about the intensity of this ranking.

2.3 Fuzzy Preferences with Literature Review

Failing to order feasible states or alternatives with certainty is the main reason for studying preference uncertainty. Preference uncertainty is modelled qualitatively or quantitatively. Qualitatively, it is represented by linguistic labels, such as good, fair and poor (Herrera and Herrera-Viedma, 2000; Xu, 2004a); while quantitatively, it is given by numbers, such as degrees of preference (Orlovsky, 1978; Xu, 2007). Because of their importance in various decision making techniques, uncertain preference relations have been an active area of research and many variants have been developed over the last few decades.

Widely used uncertain preference relations include multiplicative preferences (Saaty, 1980; Herrera et al., 2001), incomplete multiplicative preferences (Harker, 1987; Nishizawa, 1997), interval multiplicative preferences (Islam et al., 1997; Xu, 2005a), incomplete interval multiplicative preferences (Xu, 2006), triangular fuzzy multiplicative preferences (Chang, 1996; Mikhailov, 2003), incomplete triangular fuzzy multiplicative preferences (Xu, 2006), the fuzzy preference relation (Orlovsky, 1978; Tanino, 1984, 1988; Chiclana et al., 2001; Xu, 2007), the incomplete fuzzy preference relation (Herrera-Viedma et al., 2007; Xu, 2005b), interval fuzzy preferences (Jiang, 2007; Xu, 2004b), incomplete interval fuzzy preferences (Xu, 2006), triangular fuzzy preferences (Xu, 2002), incomplete triangular fuzzy preferences

(Xu, 2006), linguistic preferences (Herrera and Herrera-Viedma, 2000; Xu, 2004a), and incomplete linguistic preferences (Alonso et al., 2009; Xu, 2005c). Among these preference relations, fuzzy preference relations are a convenient way of representing both certain and uncertain relative preferences between two states or alternatives. A fuzzy preference between two states is represented by a preference degree, which is interpreted as the grade of certainty of the preference for one state over the other.

2.3.1 Literature Review on Fuzzy Preferences

Zadeh (1965, 1973) developed the concepts of a fuzzy logic and fuzzy set as effective tools for mathematically modelling uncertainty or vagueness. Based on Zadeh's notion of fuzzy logic, Orlovsky (1978) proposed a fuzzy preference relation to generalize crisp preference in a decision making situation. He introduced and studied fuzzy preference and its properties, and the fuzzy set of non-dominated alternatives. He established that if the fuzzy preference relation in a fuzzy decision-making problem satisfies some topological properties, then the problem has "un-fuzzy" (crisp) non-dominated solutions.

Keeping in mind that fuzzy utilities could be a flexible way of representing utilities of states, Nakamura (1986) proposed a method to construct a fuzzy preference, given a set of fuzzy utilities, to allow rational decision making. Tanino (1984) discussed the use of fuzzy preference orderings in group decision making. He defined a fuzzy preference ordering as a fuzzy binary relation satisfying reciprocity and max-min transitivity, and developed group fuzzy preference orderings applicable when individual preferences are represented by utility functions, developing a method for group decision processes analogous to the extended contributive rule.

Chiclana et al. (1998) introduced a general multipurpose decision model that is able to handle problems with a range of preference information: preference orderings, utility functions, or fuzzy preference relations. First, the preference information is made uniform using fuzzy preference relations, and then selection processes are introduced based on the concept of fuzzy majority (Kacprzyk, 1986) and on ordered weighted averaging operators (Yager, 1988).

Chiclana et al. (2001) also carried out research on how to integrate multiplicative preference relations into fuzzy multipurpose decision models using preference orderings, utility functions, or fuzzy preference relations. Together with the work in (Chiclana et al., 1998), the authors provided a more flexible framework to manage different preference structures. This constituted a decision model that approximated real decision situations involving experts from different knowledge areas very well. Also, a number of other fuzzy preference structures and their connections in social choices were discussed in (Banerjee, 1994; Dutta, 1987; Richardson, 1998) and the references contained therein.

2.3.2 Fuzzy Sets, Fuzzy Numbers, and Fuzzy Relations

The concept of a fuzzy preference relation is derived from fuzzy sets, fuzzy numbers, and fuzzy relations. These three notions are briefly described below.

2.3.2.1 Fuzzy Sets

The notion of a fuzzy set was introduced by Zadeh (1965) to generalize the classical idea of a set, now called a *crisp set*. In classical set theory, membership of an element in a set is binary: an element either belongs to the set, or not. In contrast, fuzzy set theory allows the membership of an element to be described by any number in the unit interval, $I = [0, 1] = \{x : 0 \leq x \leq 1\}$, referred to as the degree or grade of membership. A formal definition is given below.

Definition 2.3.1. Let X denote a nonempty collection of objects. A *fuzzy set* in X is characterized by a *membership function*, $\delta : X \longrightarrow I$, where, for an $x \in X$, $\delta(x)$ is interpreted as the *degree* or *grade of membership* of x in the fuzzy set.

Example 2.3.2. The set of tall students in a class can be described as a fuzzy set. For instance, if students $1, 2, \ldots, 10$ are numbered in increasing order of height, one might have $\delta(x) = 0$ for $x = 1, \ldots, 5$, $\delta(6) = 0.4$, $\delta(7) = 0.6$, $\delta(8) = 0.9$, and $\delta(9) = \delta(10) = 1$.

Note that the closer is the value of $\delta(x)$ to 1, the higher is the grade of membership of x in the fuzzy set. A conventional, or crisp, set is a fuzzy set, in that the membership function is a $0 - 1$ function, assigning 1 to each element of the set and 0 to each element not in the set.

2.3.2.2 Fuzzy Numbers

A fuzzy number is a fuzzy set of a particular form defined on the set of real numbers, $\mathbb{R}$. Recall that, if $c, d \in \mathbb{R}$ satisfy $c \leq d$, then $[c, d] = \{x : c \leq x \leq d\}$ is a (closed) interval of real numbers. The definition of a fuzzy number follows (Goetschel and Voxman, 1986; Klir and Yuan, 1995):

Definition 2.3.3. A *fuzzy number* is a fuzzy set in $\mathbb{R}$ defined by a membership function $\delta : \mathbb{R} \longrightarrow I$ with the following properties:

- δ is upper semi-continuous;

- There is an interval $[c, d]$ such that $\delta(x) = 0$ for all $x \notin [c, d]$;
- There are real numbers a and b satisfying $c \leq a \leq b \leq d$ such that
  (i) $\delta(x) = 1$ for all $x \in [a, b]$;
  (ii) If $c \leq x_1 \leq x_2 \leq a$, then $\delta(x_1) \leq \delta(x_2)$;
  (iii) If $b \leq x_1 \leq x_2 \leq d$, then $\delta(x_1) \geq \delta(x_2)$.

Note that δ is upper semi-continuous if and only if $\{x \in \mathbb{R} : \delta(x) < \alpha\}$ is an open set in $\mathbb{R}$ for every $\alpha \in (0, 1]$.

Example 2.3.4. The set of all real numbers close to 3 can be thought of as a fuzzy number. For instance, one might choose $c = 2, a = b = 3$, and $d = 4$, with $\delta(x)$ linear on each interval.

2.3.2.3 Fuzzy Relations

Traditionally, a preference or crisp preference is characterized using a binary relation. In consequence, a fuzzy preference is defined using a fuzzy binary relation or simply a fuzzy relation. A classical or crisp relation indicates that an object $x \in X$ is either related to, or not related to, an object $y \in Y$; so, it is natural that a fuzzy relation assigns a degree or grade to the relation of x to y. A formal definition is provided below (Klir and Yuan, 1995).

Definition 2.3.5. Let X and Y denote nonempty collections of objects. A fuzzy relation from X to Y, denoted $\mathcal{R}$, is a fuzzy set in $X \times Y$ with membership function:

$$\mu_{\mathcal{R}} : X \times Y \longrightarrow [0, 1],$$

where $\mu_{\mathcal{R}}(x, y)$ represents the degree or grade of the relationship of $x \in X$ to $y \in Y$.

Note that the sets X and Y may or may not be identical. If $X = Y$, $\mathcal{R}$ is said to be a fuzzy relation on X. A fuzzy relation from X to Y is usually represented by a matrix in which the members of X are the row labels and the members of Y are the column labels. The entry in row x and column y represents the degree to which x is related to y.

Example 2.3.6. The relation "likes", between two students in a class, can be thought of as a fuzzy relation.

2.3.3 Fuzzy Preference and its Properties

A fuzzy preference is an important type of fuzzy binary relation. It represents preference between two states or alternatives as a preference degree for the first state over the second, and thus naturally includes both certain and uncertain preferences. A formal definition of a fuzzy preference relation is presented below (Tanino, 1984, 1988; Chiclana et al., 2001; Xu, 2007).

Definition 2.3.7. Let $S = \{s_1, s_2, \ldots, s_m\}$, $m > 1$, denote a set of states or alternatives. A *fuzzy preference* over S is a fuzzy relation on S, represented by a matrix $\mathcal{R} = (r_{ij})_{m\times m}$, with membership function $\mu_{\mathcal{R}} : S \times S \longrightarrow [0, 1]$, where $\mu_{\mathcal{R}}(s_i, s_j) = r_{ij}$, the degree of preference for s_i over s_j, satisfies

$$r_{ij} + r_{ji} = 1 \text{ and } r_{ii} = 0.5, \text{ for all } i, j = 1, 2, ..., m.$$

The condition $r_{ij} + r_{ji} = 1$ is referred to as the *additive reciprocity.*

One often writes $r = \mu_{\mathcal{R}}$, so $r(s_i, s_j) = \mu_{\mathcal{R}}(s_i, s_j) = r_{ij}$. Interpretations of the values of $r(s_i, s_j)$ follow:

(1) $r(s_i, s_j) = 1$ indicates that state s_i is definitely preferred to state s_j;

(2) $r(s_i, s_j) > 0.5$ implies that state s_i is likely to be preferred to state s_j; the larger $r(s_i, s_j)$, the more likely that s_i is preferred to s_j;

(3) $r(s_i, s_j) = 0.5$ means that state s_i is likely to be indifferent to state s_j, or that each state is equally likely to be preferred to the other;

(4) $r(s_i, s_j) < 0.5$ indicates that state s_j is likely to be preferred to state s_i; the smaller $r(s_i, s_j)$, the more likely that s_j is preferred to s_i;

(5) $r(s_i, s_j) = 0$ implies that state s_j is definitely preferred to state s_i.

Note that the amount of preference cannot be inferred from a degree of preference. The degree of preference of one state relative to another is the level of certainty that a DM prefers the first state to the second, and says nothing about how strong this preference may be. Thus, if s_i is definitely preferred to s_j, then it is certain that s_i is preferred to s_j, but there is no implication about how much more preferred is s_i than s_j. For $k \in N$, DM k's fuzzy preference is often denoted by $\mathcal{R}^k$.

Example 2.3.8. Tom has a cold and would like a hot drink. He prefers coffee from Tim Hortons cafes and tea from Williams cafes, and is indifferent between them. This means that Tom prefers coffee from Tim Hortons to coffee from Williams and tea from Williams to tea from Tim Hortons. However, he is indifferent between coffee from Tim Hortons and tea from Williams.

Tom's new roommate Dave brings Tom a tea from Tim Hortons and a coffee from Williams. Now Tom's preference for "tea" or "coffee" is unclear; he does not definitely prefer one to the other. In this case, Tom's preference can be described as a fuzzy preference relation.

If Tom is more likely to take tea, then Tom's preference can be represented as a fuzzy preference with $r^{\text{Tom}}(\mathrm{T_T}, \mathrm{C_W}) > 0.5$, where $r^{\text{Tom}}(\mathrm{C_W}, \mathrm{T_T})$ is defined by $r^{\text{Tom}}(\mathrm{C_W}, \mathrm{T_T}) = 1 - r^{\text{Tom}}(\mathrm{T_T}, \mathrm{C_W})$, in which $\mathrm{T_T}$ means tea from Tim Hortons and $\mathrm{C_W}$ means coffee from Williams. In particular, if $r^{\text{Tom}}(\mathrm{T_T}, \mathrm{C_W}) = 0.7$, then Tom's preference over $\{\mathrm{C_W}, \mathrm{T_T}\}$ can be represented by the following matrix.

$$\mathcal{R}^{\text{Tom}} = \begin{array}{c c} & \begin{array}{cc} \mathrm{C_W} & \mathrm{T_T} \end{array} \\ \begin{array}{c} \mathrm{C_W} \\ \mathrm{T_T} \end{array} & \begin{pmatrix} 0.5 & 0.3 \\ 0.7 & 0.5 \end{pmatrix} \end{array}.$$

Example 2.3.9. Example 2.3.8 can be expanded to represent Tom's preferences over "coffee" or "tea" from "Tim Hortons" or "Williams" and can be represented by the following matrix:

$$\mathcal{R} = \begin{array}{c c} & \begin{array}{cccc} \mathrm{C_T} & \mathrm{C_W} & \mathrm{T_T} & \mathrm{T_W} \end{array} \\ \begin{array}{c} \mathrm{C_T} \\ \mathrm{C_W} \\ \mathrm{T_T} \\ \mathrm{T_W} \end{array} & \begin{pmatrix} 0.5 & 1.0 & 0.8 & 0.5 \\ 0 & 0.5 & 0.3 & 0.1 \\ 0.2 & 0.7 & 0.5 & 0 \\ 0.5 & 0.9 & 1.0 & 0.5 \end{pmatrix} \end{array},$$

where $\mathrm{C_T}$, $\mathrm{C_W}$, $\mathrm{T_T}$, and $\mathrm{T_W}$ denote alternatives: coffee from Tim Hortons, coffee from Williams, tea from Tim Hortons, and tea from Williams, respectively.

Chapter 3

Fuzzy Preferences in a Two-Decision Maker Graph Model

3.1 Introduction

The first step in developing the FGM is to integrate fuzzy preferences into a two-DM graph model. Note that the simplest genuine conflict has two DMs, each of whom has two choices or strategies. To gain fundamental insights into what is actually occurring strategically and what can be done to obtain favorable resolutions, one can investigate such basic, or generic form, conflicts, including the Sustainable Development, Prisoner's Dilemma, and Chicken. For example, in sustainable development disputes, two major groups of agents involved are usually environmental agencies and developers. Environmental agencies are committed to monitoring development activities such that no component of a healthy environment is significantly degraded or destroyed. On the other hand, developers are engaged in activities intended to increase the quality of human life, and are often dominated by their business views of making profits. To study conflicts with two DMs, such as those mentioned above, with uncertain preferences, a fuzzy preference model for two DMs within the GMCR framework is proposed in the current chapter.

In addition to developing the concepts of fuzzy relative certainty of preference, fuzzy satisficing threshold, and fuzzy unilateral improvement as tools for incorporating fuzzy preferences in the GMCR, the four basic fuzzy stability definitions—fuzzy Nash stability, fuzzy general metarationality, fuzzy symmetric metarationality, and fuzzy sequential stability—are introduced here. These fuzzy stabilities are applied to an ongoing sustainable development dispute to demonstrate their applicability. The contributions of this chapter are partly due to the papers by Bashar et al. (2009a,b, 2010a, 2011).

3.2 Fuzzy Relative Certainty of Preference

A fuzzy preference captures preference uncertainty using numbers between 0 and 1, indicating pairwise preference degree to which one state is preferred over the other. Fuzzy preference can be thought of as an increasing function of preference degrees for which larger preference degree means more likely preferred. The maximum preference degree, 1.0, implies definite preference. When a preference degree is less than 1.0 (but greater than 0) for a DM, he or she perceives that either state of the pair may be preferable to the other, even if he "leans" toward one of the states. In particular, if $r(s_i, s_j) < 1$, then the DM does not definitely prefer state s_i to state s_j. Due to additive reciprocity, the number $r(s_j, s_i) = 1 - r(s_i, s_j)$ can be interpreted as the degree to which state s_i is not preferred over state s_j. Hence, the following definition describes the intensity of preference for a state (relative to another), which will be called the fuzzy relative certainty of preference of a DM. Recall that N represents the set of DMs and $S = \{s_1, s_2, ..., s_m\}$, $m > 1$, represents the set of feasible states.

Definition 3.2.1. Let $k \in N$, and for $s_i, s_j \in S$, let $r^k(s_i, s_j)$ denote the preference degree of state s_i over s_j for DM k. Then the k-th DM's *fuzzy relative certainty of preference* (FRCP) for state s_i over s_j, denoted $\alpha^k(s_i, s_j)$, is $\alpha^k(s_i, s_j) = r^k(s_i, s_j) - r^k(s_j, s_i)$.

The number $\alpha^k(s_i, s_j)$ measures the relative certainty of DM k's preference for state s_i over state s_j. It is clear from Definitions 2.3.7 and 3.2.1 that for any $k \in N$ and for all $i, j = 1, 2, ..., m$, $-1 \leq \alpha^k(s_i, s_j) \leq 1$. In particular,

(1) $\alpha^k(s_i, s_j) = 1$ indicates that DM k definitely prefers state s_i to state s_j;

(2) $\alpha^k(s_i, s_j) = 0$ means that DM k is equally likely to favor state s_i over state s_j, or to favor state s_j over state s_i;

(3) $\alpha^k(s_i, s_j) = -1$ indicates that DM k definitely prefers state s_j to state s_i.

Denoting $\alpha^k_{ij} = \alpha^k(s_i, s_j)$ for any $i, j = 1, 2, ..., m$, the k-th DM's FRCP over S can be represented by matrix $(\alpha^k_{ij})_{m \times m}$.

Example 3.2.2. Let the matrix

$$\mathcal{R}^{\mathrm{p}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0.5 & 1.0 & 0.8 & 0.5 \\ 0 & 0.5 & 0.3 & 0.1 \\ 0.2 & 0.7 & 0.5 & 0 \\ 0.5 & 0.9 & 1.0 & 0.5 \end{pmatrix} \end{array}$$

represent the (fuzzy) preference of a DM, p, over the set of states $S = \{s_1, s_2, s_3, s_4\}$. Then by employing Definition 3.2.1, p's FRCP over S can be represented by the matrix:

$$(\alpha_{ij}^{\mathrm{p}}) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0 & 1.0 & 0.6 & 0 \\ -1.0 & 0 & -0.4 & -0.8 \\ -0.6 & 0.4 & 0 & -1.0 \\ 0 & 0.8 & 1.0 & 0 \end{pmatrix} \end{array}.$$

Remark 3.2.3. The matrix representing FRCP is skew-symmetric in that, for all $k \in N$ and all $i, j = 1, 2, ..., m$, $\alpha^k(s_j, s_i) = -\alpha^k(s_i, s_j)$ and $\alpha^k(s_i, s_i) = 0$.

3.3 Fuzzy Satisficing Threshold

In analyzing a graph model, one important task is to determine whether a DM is better off to stay at a focal state or to move to some other states. A DM may wish to achieve a certain amount of confidence in identifying a better state. The fuzzy satisficing threshold of a DM describes his or her criterion to identify a state that is worthwhile. Note that different DMs may have different criteria in choosing states that benefit them. The fuzzy satisficing threshold of a DM is a number that characterizes the level of FRCP required for the DM to find an advantageous state. A formal definition is given below.

Definition 3.3.1. For $k \in N$, DM k would be willing to move from state $s \in S$ to state $s_i \in S$ if and only if $\alpha^k(s_i, s) \geq \gamma_k$, where γ_k is called the *fuzzy satisficing threshold* (FST) of DM k.

The FST is a behavioral parameter that represents the DM's criterion for deciding whether to take advantage of some possible moves. Because of Definition 3.2.1, it is reasonable to assume that an FST is positive and does not exceed 1, i.e., for all $k \in N$, $0 < \gamma_k \leq 1$. If for a DM $k \in N$, $\gamma_k = 1$, it means that DM k finds a state worthy only if his or her preference for the state over an initial state is crisp, since $\alpha^k(s_i, s) \geq \gamma_k = 1$ implies $r^k(s_i, s) - r^k(s, s_i) = 1$ indicating $r^k(s_i, s) = 1$ and $r^k(s, s_i) = 0$. The FST of a DM may be supplied by the DM himself or herself, determined by an analyst by interviewing the DM, or by other means such as reading background information about the DM.

3.4 Fuzzy Unilateral Improvements by a Decision Maker

Stability analysis within the GMCR depends fundamentally on which states a DM would move to, given that he or she could do so, starting at some given initial state. In a graph model with fuzzy preference, this choice must depend on the DM's FST as it characterizes the level of FRCP to be required to identify states that are worthy for the DM. A fuzzy unilateral improvement signals a DM's attractiveness to move. More specifically, a fuzzy unilateral improvement for a DM is a state that the DM could and would move to, in other words, a state in the DM's reachable list for which the FRCP over the initial state is not less than the DM's FST. Recall that $R_k(s)$ is the set of states reachable from a given state $s \in S$ for DM $k \in N$ and γ_k is the FST of DM k.

Definition 3.4.1. Let $s \in S$ and $k \in N$. A state $s_i \in R_k(s)$ is called a *fuzzy unilateral improvement* (FUI) from s by DM k if and only if $\alpha^k(s_i, s) \geq \gamma_k$.

Definition 3.4.2. The set of all FUIs from a state $s \in S$ for DM k is called the *fuzzy unilateral improvement list* (FUIL) from s by DM k, and is denoted $\widetilde{R}^{+}_{k,\gamma_k}(s)$.

To summarize Definition 3.4.2, $\widetilde{R}^{+}_{k,\gamma_k}(s) = \{s_i \in R_k(s) : \alpha^k(s_i, s) \geq \gamma_k\}$. For simplicity, one writes $\widetilde{R}^{+}_{k}(s) = \widetilde{R}^{+}_{k,\gamma_k}(s)$.

Example 3.4.3. Suppose that the matrix $\mathcal{R}^{\mathrm{p}}$ in Example 3.2.2 represents the (fuzzy) preference of a DM, p, over the set of states $S = \{s_1, s_2, s_3, s_4\}$ in a conflict model and that p's reachable list from state s_3 is $R_{\mathrm{p}}(s_3) = \{s_1, s_2, s_4\}$. Also suppose that $\gamma_{\mathrm{p}} = 0.5$, that is, the FST of p is 0.5. Then s_1 is an FUI from s_3 for p, since $s_1 \in R_{\mathrm{p}}(s_3)$ and $\alpha^{\mathrm{p}}(s_1, s_3) = 0.6 \geq 0.5$, the FST of p. Hence, one can find that the FUIL from s_3 for p is

$$\widetilde{R}^{+}_{\mathrm{p},0.5}(s_3) = \{s_1, s_4\}.$$

Likewise, if p had FST 0.7, then his or her FUIL from s_3 would be $\widetilde{R}^{+}_{\mathrm{p},0.7}(s_3) = \{s_4\}$.

Remark 3.4.4. When the FST of a DM is 1.0, the definitions of a DM's FUI and FUIL coincide with the definitions of a DM's (crisp) UI and (crisp) UIL, respectively.

3.5 Fuzzy Stabilities for a Two-Decision Maker Graph Model

The concept of fuzzy stability is incorporated into the GMCR to accommodate fuzzy preference, which is an effective tool in representing both certain and uncertain preference information. Note that in a graph model with two DMs, the opponent of a focal DM is always a single DM. In a strategic conflict, a DM never moves to a state that is not advantageous according to his or her FUIL. If there is an FUI from the current state, a DM may consider the opponent's countermove and also subsequent move before deciding on whether to take advantage of the immediate FUI. Like the crisp GMCR, the FGM takes into account these human behavior in identifying states that represent potential resolutions of a dispute. More specifically, four basic fuzzy stability definitions—fuzzy Nash stability, fuzzy general metarationality, fuzzy symmetric metarationality, and fuzzy sequential stability—are introduced to integrate various kinds of human behavior into graph model fuzzy stability calculations. As there are two DMs in the model, assume that $N = \{k, l\}$; accordingly, the FSTs will be denoted γ_k and γ_l.

Definition 3.5.1. (Fuzzy Nash Stability or Fuzzy Rationality): A state $s \in S$ is *fuzzy Nash stable*, or *fuzzy rational* (FR) for DM k if and only if

$$\widetilde{R}_k^+(s) = \varnothing.$$

Under FR stability, the focal DM is considered to take into account only of his or her FUIs when deciding whether to move from a given initial state, and ignores any possible responses by the opponent. Thus, state s is FR stable for DM k if and only if DM k has no FUIs from s.

Definition 3.5.2. (Fuzzy General Metarationality): A state $s \in S$ is *fuzzy general metarational* ($FGMR$) for DM k if and only if for every $s_1 \in \widetilde{R}_k^+(s)$ there exists an $s_2 \in R_l(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$.

For $FGMR$, the focal DM asks whether each of his or her FUIs could subsequently be sanctioned by the opponent, using one of the opponent's unilateral moves. Note that the

DM does not consider whether the opponent would be better off making this sanctioning move. If the focal DM has no FUIs from state s, then s is automatically *FGMR* stable in the sense that there is no FUI from s that cannot subsequently be sanctioned by the opponent using a unilateral move. In particular, *FR* stability implies *FGMR* stability.

Definition 3.5.3. (Fuzzy Symmetric Metarationality): A state $s \in S$ is *fuzzy symmetric metarational* (*FSMR*) for DM k if and only if for every $s_1 \in \widetilde{R}_k^+(s)$ there exists an $s_2 \in R_l(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$, and $\alpha^k(s_3, s) < \gamma_k$ for all $s_3 \in R_k(s_2)$.

In *FSMR* stability, the focal DM looks one more step ahead (in comparison to *FGMR* stability) when deciding whether to take advantage of an FUI. If there is a sanction by the opponent, the focal DM asks if he or she has a unilateral move that escapes the sanction. If the focal DM cannot escape the sanction, then the original state is *FSMR* stable. If the focal DM has no FUIs from the current state, then it is *FSMR* stable in the sense that there is no FUI from the initial state for which a sanction by the opponent can be escaped by the focal DM. In particular, *FR* stability implies *FSMR* stability.

Definition 3.5.4. (Fuzzy Sequential Stability): A state $s \in S$ is *fuzzy sequentially stable* (*FSEQ*) for DM k if and only if for every $s_1 \in \widetilde{R}_k^+(s)$ there exists an $s_2 \in \widetilde{R}_l^+(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$.

FSEQ stability is the same as *FGMR* stability except that the focal DM considers only sanctions of his or her FUIs that are "credible" in the sense that they are FUIs for the opponent. Note that the definition of *FSEQ* depends not only on the focal DM's FST, γ_k, but also on the opponent's FST, γ_l. If the focal DM has no FUIs from the initial state, then it is *FSEQ* stable in the sense that there is no FUI from the current state that cannot subsequently be sanctioned by the opponent using an FUI. In particular, *FR* stability implies *FSEQ* stability.

Definition 3.5.5. (Fuzzy Equilibrium): A state $s \in S$ that is fuzzy stable for both DMs k and l under a specific fuzzy stability definition is called a *fuzzy equilibrium* (*FE*) under that definition.

Note that DMs k and l may have different FSTs in identifying their own fuzzy stable states. Therefore, *FE* corresponding to all the fuzzy stability definitions above, even fuzzy Nash equilibrium, depend on both DMs' FSTs.

3.6 Application of Fuzzy Stabilities to the Sustainable Development Conflict

3.6.1 Sustainable Development and Related Issues

Development is crucial to the advancement of civilization. More specifically, economic development increases standards of living, and sustainable development, as described in (Brundtland Report, 1987), meets the needs of the present without compromising the ability of future generations to meet their own requirements. Although development is essential to fulfil human needs and to improve the quality of life, it must be based on the efficient and responsible use of human, economic and natural resources. Theoretically, development that does not cause significant damage to the planet is possible, but conflicting motivations make it difficult to achieve. One instance is the temptation to improve an economy at the cost of environmental protection, for example, by not treating industrial wastes nor enhancing industrial processes (Gore, 2006a,b; Hipel and Obeidi, 2005).

A wide variety of environmental disputes is taking place around the globe on an ongoing basis, including the continuing controversies surrounding the reduction of greenhouse gasses and the preservation of ecosystems (Hipel and Bernath Walker, 2010). The great sparrow campaign (also known as kill a sparrow campaign, and officially, the four pests campaign) between 1958 and 1960 in China is an example of an environmental disaster. Under this campaign, sparrows were killed by peasants to save their grain seeds. However, this action caused populations of harmful insects to balloon, leading to a major ecological imbalance (Shapiro, 2001). An example of a recent environmental disaster is the Gulf of Mexico Oil Spill caused by an explosion on a drilling rig off the coast of southeast Louisiana, USA on April 20, 2010 (The New York Times, 2010; Center for Biological Diversity, 2011). More than 200 million gallons of oil fouled the ocean and Gulf coastlines, spreading along more than 1,000 miles of shoreline, and causing the death or harm of more than 82,000 birds, about 6,000 sea turtles, nearly 26,000 marine mammals including dolphins, as well as an unknown but enormous number of fish and invertebrates (Center for Biological Diversity, 2011).

3.6.2 Application of Fuzzy Stabilities for a Two-Decision Maker Graph Model to the Sustainable Development Conflict

The Sustainable Development conflict (Hipel, 2002) is a 2×2 game having two DMs or players each of whom has two options or strategies (Kilgour and Fraser, 1988; Fraser and Kilgour, 1986; Rapoport et al., 1976). One DM represents the environmental agencies

(ENV) and the other potential developers (DEV). ENV consists of government officials, environmentalists, and/or community groups. The main task of ENV is to oversee development activities to ensure that they remain sustainable. This means that the development projects will not only be economically beneficial but also environmentally viable. DEV, on the other hand, is composed of individuals or business enterprises, whose aim is to initiate development projects that will be economically feasible. Generally speaking, DEV's major goal is to make profit. However, DEV often feels some sort of environmental responsibility; some of them may place environmental priorities higher than others.

Table 3.1: States in the Sustainable Development Conflict

		DEV: S	DEV: U
ENV	P	s_1	s_2
ENV	R	s_3	s_4

In summary, in monitoring development activities and their effects on the environment, ENV can be proactive ($\underline{\text{P}}$) or reactive ($\underline{\text{R}}$). On the other hand, based on the level of responsibility to the environment and society, DEV may practice sustainable development ($\underline{\text{S}}$) or unsustainable development ($\underline{\text{U}}$). The model is presented in Table 3.1 in which each cell represents one of the four possible states. For example, state s_1 indicates the situation in which ENV is proactive and DEV practices sustainable development. Figures 3.1 and 3.2 show how ENV and DEV can cause the conflict to move from one state to another. For example, ENV can move from state s_1 to state s_3 by changing its strategy from proactive to reactive, but cannot move from s_1 to s_2.

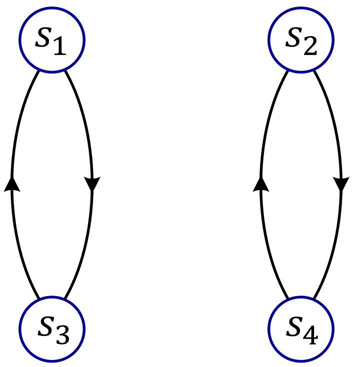

Figure 3.1: Possible Moves by Environmental Agencies (ENV)

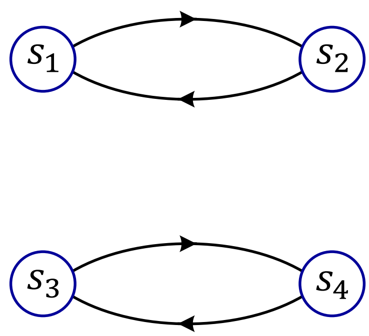

Figure 3.2: Possible Moves by Developers (DEV)

The Graph Model of the Sustainable Development conflict studied here is similar to the one investigated in (Hipel, 2002) and (Hipel and Bernath Walker, 2010), except that in this study, preference uncertainties between some states are considered for both ENV and DEV. For example, when DEV practices sustainable development, ENV may not have enough reason to definitely prefer state s_1 over state s_3, even though, by nature, ENV may want to be proactive rather than reactive. For DEV, when ENV is proactive, it may be unsure which of states s_1 and s_2 is better (even though it may want to choose unsustainable development instead of sustainable development) because it is unaware about ENV's plans in administering relevant environmental regulations. Taking these and other

Table 3.2: Fuzzy Preferences of Environmental Agencies (ENV) and Developers (DEV)

$$\mathcal{R}^{\mathrm{ENV}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0.5 & 1.0 & 0.75 & 1.0 \\ 0 & 0.5 & 0 & 1.0 \\ 0.25 & 1.0 & 0.5 & 1.0 \\ 0 & 0 & 0 & 0.5 \end{pmatrix} \end{array}$$

$$\mathcal{R}^{\mathrm{DEV}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0.5 & 0.25 & 0 & 0 \\ 0.75 & 0.5 & 0.7 & 0 \\ 1.0 & 0.3 & 0.5 & 0 \\ 1.0 & 1.0 & 1.0 & 0.5 \end{pmatrix} \end{array}$$

preference uncertainties into account, a typical fuzzy preference model for ENV and DEV is constructed, and is represented in Table 3.2 by matrices $\mathcal{R}^{\mathrm{ENV}}$ and $\mathcal{R}^{\mathrm{DEV}}$. In particular, the number 0.75 in $\mathcal{R}^{\mathrm{ENV}}$ represents ENV's preference degree of being proactive over reactive when DEV practices sustainable development. Using Definition 3.2.1, the FRCPs for ENV and DEV are calculated, and are represented by matrices α^{ENV} and α^{DEV} in Table 3.3.

A fuzzy stability analysis is carried out by applying the *FR*, *FGMR*, *FSMR*, and *FSEQ* stability definitions introduced in Section 3.5 to the sustainable development model. The results are presented in Table 3.4 in which ENV or DEV in a cell indicates that the state in the corresponding row is fuzzy stable for the indicated DM but not for the opponent while *FE* indicates that the state is a fuzzy equilibrium, under the indicated fuzzy stability definition. In this analysis, four sets of FSTs for ENV and DEV—(i) $\gamma_{\mathrm{ENV}} = 0.4$, $\gamma_{\mathrm{DEV}} = 0.3$; (ii) $\gamma_{\mathrm{ENV}} = 0.6$, $\gamma_{\mathrm{DEV}} = 0.3$; (iii) $\gamma_{\mathrm{ENV}} = 0.4$, $\gamma_{\mathrm{DEV}} = 0.6$; and (iv) $\gamma_{\mathrm{ENV}} = 0.6$,

Table 3.3: Fuzzy Relative Certainty of Preferences of Environmental Agencies (ENV) and Developers (DEV)

$$\alpha^{\mathrm{ENV}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0 & 1.0 & 0.5 & 1.0 \\ -1.0 & 0 & -1.0 & 1.0 \\ -0.5 & 1.0 & 0 & 1.0 \\ -1.0 & -1.0 & -1.0 & 0 \end{pmatrix} \end{array}$$

$$\alpha^{\mathrm{DEV}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0 & -0.5 & -1.0 & -1.0 \\ 0.5 & 0 & 0.4 & -1.0 \\ 1.0 & -0.4 & 0 & -1.0 \\ 1.0 & 1.0 & 1.0 & 0 \end{pmatrix} \end{array}$$

$\gamma_{\mathrm{DEV}} = 0.6$—are considered.

It can be seen from Table 3.4 that when satisficing criteria of both ENV and DEV are weak, that is, for smaller FSTs, state s_2 is the only *FE* under all four fuzzy stability definitions. This result is similar to the one found in (Hipel, 2002) for the case of typical developers who are not concerned about environmental impacts of their activities. State s_2, in which environmental agencies are proactive and developers practice unsustainable development, represents a reasonable resolution for these developers. An increase in the FST of ENV does not significantly change the fuzzy stability results. However, when the FST of DEV is increased from 0.3 to 0.6, state s_1 also becomes a *FE* under all four fuzzy stability definitions. Recall that state s_1 represents a circumstance in which environmental agencies are proactive and developers practice sustainable development. Note that s_1 is the outcome predicted in (Hipel, 2002) for the case of more environmentally responsible

Table 3.4: Fuzzy Stability Results of the Sustainable Development Conflict

FSTs	States	*FR*	*FGMR*	*FSMR*	*FSEQ*
$\gamma_{\mathrm{ENV}} = 0.4$ $\gamma_{\mathrm{DEV}} = 0.3$	s_1	ENV	ENV	ENV	ENV
	s_2	*FE*	*FE*	*FE*	*FE*
	s_3		ENV	ENV	ENV
	s_4	DEV	DEV	DEV	DEV
$\gamma_{\mathrm{ENV}} = 0.6$ $\gamma_{\mathrm{DEV}} = 0.3$	s_1	ENV	ENV	ENV	ENV
	s_2	*FE*	*FE*	*FE*	*FE*
	s_3	ENV	ENV	ENV	ENV
	s_4	DEV	DEV	DEV	DEV
$\gamma_{\mathrm{ENV}} = 0.4$ $\gamma_{\mathrm{DEV}} = 0.6$	s_1	*FE*	*FE*	*FE*	*FE*
	s_2	*FE*	*FE*	*FE*	*FE*
	s_3		*FE*	*FE*	DEV
	s_4	DEV	DEV	DEV	DEV
$\gamma_{\mathrm{ENV}} = 0.6$ $\gamma_{\mathrm{DEV}} = 0.6$	s_1	*FE*	*FE*	*FE*	*FE*
	s_2	*FE*	*FE*	*FE*	*FE*
	s_3	ENV	*FE*	*FE*	*FE*
	s_4	DEV	DEV	DEV	DEV

developers.

When the DEV's FST increases, that is, when developers do not see moves as improvements unless they are relatively certain to be better off, they may end up choosing to stay either at state s_1 or s_2. This indicates that when environmental agencies are proactive, developers do not have enough incentive to move away from either of these two states even though s_2 is likely to be preferred to state s_1. A move from s_1 is not sufficiently likely to satisfy developers' desire for improvement. This represents developers' "stickiness" in moving to a reachable state implying the likely indifference between states s_1 and s_2. In this case, state s_3, being an *FGMR*, *FSMR* and *FSEQ* equilibrium, is also a potential resolution if both ENV and DEV are farsighted in identifying the benefits of possible moves.

3.7 Summary

A new framework for the GMCR, the FGM, is developed to handle strategic conflicts in which DMs have fuzzy preferences over the feasible states. This makes it possible to use all forms of DMs' preference information—certain or uncertain—in a graph model. Within FGM, the four basic crisp graph model stability definitions, *R*, *GMR*, *SMR*, and *SEQ* for a two-DM graph model are redefined as *FR*, *FGMR*, *FSMR*, and *FSEQ*, respectively, and called fuzzy stabilities. The FST, a parameter, is introduced to take into account the interacting DMs' satisficing behavior and is incorporated into the fuzzy stability definitions.

When the fuzzy stability definitions developed in this chapter are applied to the well-known sustainable development conflict, the analysis provides new insights into the dispute. The predicted equilibria in two different cases of a previous study (developers being less or more environmentally responsible) are obtained from the same fuzzy preference model. The analysis also finds that developers' satisficing behavior has more impact on the solutions than the satisficing behavior of environmental agencies.

Chapter 4

Fuzzy Preferences in an n-Decision Maker ($n > 2$) Graph Model

4.1 Introduction

The fuzzy preference framework for the GMCR developed in Chapter 3 is specifically applicable to a conflict with only two DMs. However, the number of DMs in a real-world dispute is not limited to two; for example, the Elmira groundwater contamination dispute has three DMs. To study strategic conflicts with more than two DMs, at least one of whom has uncertain preferences over feasible states, a general fuzzy preference framework within the structure of the GMCR is developed in this chapter. More specifically, the concept of a fuzzy unilateral improvement (FUI) by a coalition of DMs is introduced and the four basic fuzzy stability definitions—*FR*, *FGMR*, *FSMR*, and *FSEQ*—for a two-DM case are extended to an n-DM ($n > 2$) graph model.

The fuzzy stability definitions given in Section 3.5 apply only to two-DM graph models. In such a model, the opponent of a focal DM is an individual. But in a graph model with more than two DMs, the "opponent" of a focal DM may not be an individual DM but could be a coalition of two or more DMs. Except for fuzzy Nash stability, the vital issue in fuzzy stability determination is whether any possible response by the opponent(s) would constitute a sanction. The fuzzy stability definitions in Section 3.5 apply the criteria provided by Definition 3.4.1 to identify FUIs from a given state by an individual DM.

Note that the *FSEQ* stability is defined based on countermoves by the opponent(s) that are credible in the sense that the opponent(s)' moves are taken into account only if they benefit the opponent(s). To facilitate the definition of *FSEQ* stability for a graph model with more than two DMs, the definition of an FUI by a coalition of DMs is given in the next section. Subsequently, fuzzy stability definitions are generalized in the context of an n-DM ($n > 2$) graph model, and are presented in Section 4.3. The fuzzy stabilities developed in Section 4.3 are then applied to the Elmira groundwater contamination dispute—a three-DM conflict—to demonstrate the applicability of the generalized FGM. Part of the research in this chapter is based upon the papers by Bashar et al. (2010b,c, 2012b); Hipel et al. (2011).

4.2 Fuzzy Unilateral Improvements by a Coalition

Definition 3.4.1 provides criteria to identify states that benefit a single DM who is willing to move to an advantageous state or to make a credible sanctioning move. This case is clear and straightforward. However, the identification of states that benefit a coalition of DMs is complicated and depend not only on the FUIs of the coalition members but also on their joint unilateral moves and countermoves. The next definition integrates the idea of a coalitional unilateral move, given by Definition 2.2.2, with the individual FUIs of Definition 3.4.1.

Definition 4.2.1. (Fuzzy Unilateral Improvements by a Coalition): Let $s \in S$ and $H \subseteq N$, $|H| \geq 2$. Let $H = \{1, 2, ..., p\}$ and define $\gamma_H = (\gamma_1, \gamma_2, ..., \gamma_p)$. Now, define the subset $\widetilde{R}^+_{H,\gamma_H}(s) \subseteq S$ inductively by the following:

(1) If $k \in H$ and $s_1 \in \widetilde{R}^+_k(s)$, then $s_1 \in \widetilde{R}^+_{H,\gamma_H}(s)$ and $k \in \widetilde{\Omega}^+_{H,\gamma_H}(s, s_1)$, where $\widetilde{\Omega}^+_{H,\gamma_H}(s, s_1)$ denotes the set of all last DMs in legal sequences from s to s_1;

(2) If $s_1 \in \widetilde{R}^+_{H,\gamma_H}(s)$, $k \in H$, $s_2 \in \widetilde{R}^+_k(s_1)$, and $\widetilde{\Omega}^+_{H,\gamma_H}(s, s_1) \neq \{k\}$, then $s_2 \in \widetilde{R}^+_{H,\gamma_H}(s)$ and $k \in \widetilde{\Omega}^+_{H,\gamma_H}(s, s_2)$.

A *fuzzy unilateral improvement* (FUI) from s by the coalition H is any member of $\widetilde{R}^+_{H,\gamma_H}(s)$.

Note that the induction stops in Definition 4.2.1 as soon as (i) $\widetilde{R}^+_{H,\gamma_H}(s)$ cannot be augmented by any new state, s_2, and (ii) $|\widetilde{\Omega}^+_{H,\gamma_H}(s, s_1)|$ cannot be increased beyond 1 for any $s_1 \in \widetilde{R}^+_{H,\gamma_H}(s)$. As in Definition 2.2.2, Definition 4.2.1 imposes the requirement that all sequences of moves be legal in the sense that no DM ever moves twice consecutively. Also note that Definition 4.2.1 depends essentially on the FSTs of all DMs in H, since a new state s_2 is added to $\widetilde{R}^+_{H,\gamma_H}(s)$ only if it belongs to $\widetilde{R}^+_k(s_1) = \widetilde{R}^+_{k,\gamma_k}(s_1)$ for a suitable s_1 and k. For simplicity, one writes $\widetilde{R}^+_H(s) = \widetilde{R}^+_{H,\gamma_H}(s)$. It is important to note that if $|H| = 1$, such as $H = \{k\}$, then $\widetilde{R}^+_H(s) = \widetilde{R}^+_k(s)$.

Remark 4.2.2. When $\gamma_k = 1.0$ for all $k \in H$, that is, when the FST of each DM in the coalition is 1, the definition of a coalition's FUI coincides with the definition of a coalition's (crisp) UI.

4.3 Fuzzy Stabilities for an n-Decision Maker ($n > 2$) Graph Model

Now that the definition of FUIs from a given initial state by a coalition of DMs in a graph model with fuzzy preferences is introduced, appropriate fuzzy stabilities for an n-DM ($n > 2$) graph model can be defined. Note that fuzzy Nash stability does not depend on the responses of the opponents, so the definition of fuzzy Nash stability for an n-DM graph model is unchanged from the two-DM case.

The definitions of the remaining three basic fuzzy stabilities for models with $n > 2$ DMs and fuzzy preference, namely fuzzy general metarationality, fuzzy symmetric metarationality, and fuzzy sequential stability, are now put forward. In these definitions, $N - k$ represents the coalition of all DMs other than k or, in other words, k's opponents. Thus, $R_{N-k}(s)$ and $\widetilde{R}^{+}_{N-k}(s)$ represent the unilateral moves and FUIs, respectively, from s by DM k's opponents.

Definition 4.3.1. (Fuzzy General Metarationality): A state $s \in S$ is *fuzzy general metarational* (*FGMR*) for DM $k \in N$ if and only if for every $s_1 \in \widetilde{R}^{+}_{k}(s)$ there exists an $s_2 \in R_{N-k}(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$.

In *FGMR* stability, the focal DM inquires whether each of his or her potential FUIs is sanctioned by the opponents using a coalitional unilateral move, even if this move hurts any of the opponents. If the focal DM has no FUI from the current state, the state is automatically *FGMR* stable in the sense that there is no FUI that cannot be sanctioned by the opponents using a coalitional unilateral move.

Definition 4.3.2. (Fuzzy Symmetric Metarationality): A state $s \in S$ is *fuzzy symmetric metarational* (*FSMR*) for DM $k \in N$ if and only if for every $s_1 \in \widetilde{R}_k^+(s)$ there exists an $s_2 \in R_{N-k}(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$, and $\alpha^k(s_3, s) < \gamma_k$ for all $s_3 \in R_k(s_2)$.

For *FSMR* stability, if any of the FUIs of the focal DM is sanctioned by a coalitional unilateral move of the opponents, the focal DM asks whether he or she can escape the sanction using a unilateral move. If the focal DM cannot escape the sanction, the current state is *FSMR* stable for the DM. For the situation where the focal DM has no FUI from the current state, the state is *FSMR* stable in the sense that there is no FUI from the initial state for which a sanction by the opponents (as a coalition) can be escaped by the focal DM.

Definition 4.3.3. (Fuzzy Sequential Stability): A state $s \in S$ is *fuzzy sequentially stable* (*FSEQ*) for DM $k \in N$ if and only if for every $s_1 \in \widetilde{R}_k^+(s)$ there exists an $s_2 \in \widetilde{R}_{N-k}^+(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$.

FSEQ stability is the same as *FGMR* stability except that while considering the sanction of each of his or her potential FUIs, the focal DM takes into account only credible sanctions (i.e., FUIs) by the opponents (as a coalition). If the focal DM has no FUI from the current state, the state is *FSEQ* stable in the sense that there is no FUI from the initial state that cannot be sanctioned by the opponents using a coalitional FUI. Note that *FSEQ* stability depends on all DMs' FSTs because $\widetilde{R}_{N-k}^+(s_1)$ appears in this definition.

From above, one can summarize that if there is no FUI from an initial state, the state is automatically *FGMR*, *FSMR*, and *FSEQ* stable. But by definition, if there is no FUI from a given state, the state is *FR* stable. Therefore, it can be concluded that *FR* stability implies *FGMR*, *FSMR*, and *FSEQ* stability.

Definition 4.3.4. (Fuzzy Unstable): A state is *fuzzy unstable* for a DM under a specific fuzzy stability definition if the state is not fuzzy stable for that DM under that definition.

Definition 4.3.5. (Fuzzy Equilibrium): A state $s \in S$ that is fuzzy stable for all DMs under a specific fuzzy stability definition is called a *fuzzy equilibrium* (FE) under that definition. In particular, state s is *fuzzy Nash equilibrium* if s is FR stable for all DMs in N. Note that the FE corresponding to all the fuzzy stability definitions above, even fuzzy Nash equilibrium, depend on all DMs' FSTs.

When the FST of each DM in a graph model is 1.0, the definitions of FR, $FGMR$, $FSMR$, $FSEQ$, and FE coincide with the definitions of R, GMR, SMR, SEQ, and crisp equilibrium, respectively. This follows the following theorem.

Theorem 4.3.6. The crisp GMCR is a special case of the FGM.

4.4 Application of Fuzzy Stabilities to the Elmira Groundwater Contamination Conflict

In this section, the fuzzy stability concept is applied to a real-world environmental conflict that took place in Elmira, a small town in Ontario, Canada. First, a brief background of the dispute is presented. Next, the dispute is modelled within the structure of the FGM followed by a fuzzy stability analysis.

4.4.1 Background of the Elmira Groundwater Contamination Conflict

Elmira, a town with about 10,000 residents renowned for its annual maple syrup festival, is located in an agricultural region of southwestern Ontario, Canada, roughly equally distant from three Great Lakes: Lake Ontario, Lake Erie, and Lake Huron. Domestic water supplies for the town are sourced mainly from an underground aquifer. In late 1989, the Ontario Ministry of the Environment (now the Ministry of Environment and Energy), labeled "M", found that the aquifer was contaminated by a carcinogen, N-nitroso demethylamine (NDMA), causing a major environmental crisis (Hipel et al., 1993a; Sanderson et al., 1995; Conestoga-Rovers and Associates, 1999).

NDMA is formed by the combination of nitrates and amines and belongs to the nitrosamine group of chemicals. It is highly water soluble and a potent carcinogen. NDMA was found in two well fields in Elmira at concentrations of 40 ppb and 1.5 ppb (parts per billion). Although at that time there were no official NDMA drinking water guidelines, the Ontario Environmental Appeal Board stated in 1992 that concentrations of NDMA in drinking water should not exceed 0.009 ppb. Suspicion fell on Uniroyal Chemical Ltd., called "U", which since 1942 had been operating a pesticide and rubber products plant in Elmira that had a history of environmental problems, and was associated with NDMA-producing processes (Sanderson et al., 1995; Conestoga-Rovers and Associates, 1999).

M issued a control order under the Environmental Protection Act of Ontario, requiring that U take immediate and expensive measures to rectify the contamination. In reply, U appealed the control order. The Local Government, labeled "L", consisting of the Regional Municipality of Waterloo and the Township of Woolwich, felt that it should take a position as the authority responsible for protecting local interests, and sought legal and technical advice from independent consultants.

4.4.2 A Graph Model of the Elmira Groundwater Contamination Conflict

The graph model of the Elmira conflict considered here is based on the situation in mid-1991 (Hipel et al., 1993a). At that time, the control order was still under appeal, and the situation had not changed for more than one year. The main DMs in the dispute, M, U and L, each had distinctive objectives. M aimed to carry out what it saw as its mandate as efficiently as possible, U wanted the control order lifted or at least modified, while L wanted to protect both its citizens and industrial base.

Table 4.1: Decision Makers, Options, and Feasible States for the Elmira Conflict

M									
O_1: Modify	N	Y	N	Y	N	Y	N	Y	–
U									
O_2: Delay	Y	Y	N	N	Y	Y	N	N	–
O_3: Accept	N	N	Y	Y	N	N	Y	Y	–
O_4: Abandon	N	N	N	N	N	N	N	N	Y
L									
O_5: Insist	N	N	N	N	Y	Y	Y	Y	–
States	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9

To secure a desirable outcome, each DM had one or more options or courses of action. The DMs, their main options, and feasible states are represented in Table 4.1. The "modify" option for M means that M could modify the original control order to make it more favorable to U. As are also listed in the first column of Table 4.1, U could *delay* the appeal process, *accept* the original control order as is, or simply *abandon* its operations in Elmira; and L could *insist* on the application of the original control order. In Table 4.1, states are defined by indicating options selected by the controlling DM with “Y” and options not selected with “N”; the symbol “–” means that the state is the same whether “Y” or “N” is chosen. Although there are 32 mathematically possible states, only 9 states were considered feasible due to various option constraints (Fang et al., 1993; Hipel et al., 1993a, 1999; Kilgour et al., 2001). For example, U can select only one option at a time out of the available three—delay, accept, and abandon.

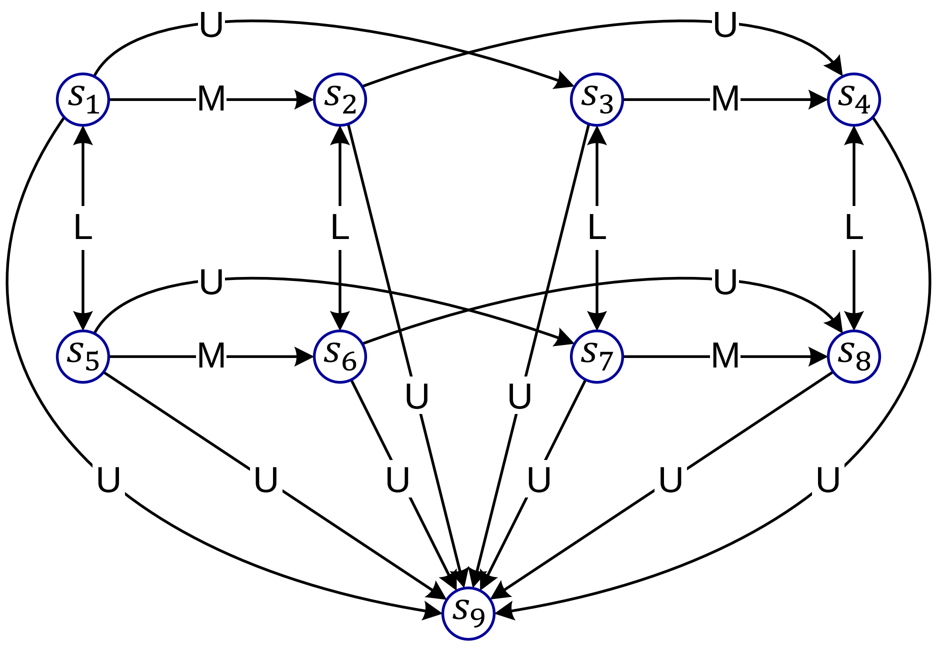

Figure 4.1: Unilateral Moves in Elmira Conflict

Figure 4.1, the integrated graph for the model of the Elmira conflict in Table 4.1, shows all unilateral moves. The nodes of the graph represent feasible states and the labels on the arcs indicate the controlling DM. The arrowhead(s) of an arc indicate the allowable move directions. Note that the model includes both reversible and irreversible moves; for

Table 4.2: Preference of the Ontario Ministry of the Environment (M) in the Elmira Conflict: Most to Least Preferred

s_7	s_3	s_4	s_8	s_5	s_1	s_2	s_6	s_9

example, the move between states s_1 and s_5 by L is reversible while the move from s_1 to s_3 by U is irreversible.

Note that M, a provincial authority that is responsible for environmental issues in the entire Province of Ontario, might not be as closely connected to the Elmira dispute as the more local DMs, U and L, and may therefore have more precisely defined preferences. Hence, the preference of M over the feasible states is assumed to be crisp and shown in Table 4.2, listed as the most preferred state on the left to the least preferred on the right, which is identical to (Hipel et al., 1993a, 1999; Kilgour et al., 2001).

But in the present study, possible preference uncertainty of U and L over some states is taken into account and its effects are assessed. For example, U may be in doubt about the desirability of abandoning its Elmira operation. More specifically, the preference of U for state s_5, where U delays the appeal process and L insists on the application of the original control order, over state s_9, where U abandons its operation, may be uncertain. Although L would like to insist on the application of the original control order, it may be uncertain about its preference when a control order (original or modified) is accepted by U. Thus, when M modifies the original control order and U accepts it, L may be unsure whether state s_8 (insist) or s_4 (not) is better. L may not find enough reason to definitely prefer state s_8 over s_4 as assumed in (Hipel et al., 1993a, 1999; Kilgour et al., 2001); rather, it may lean toward s_4 over s_8.

Taking these and other possible preference uncertainties of U and L into account, a

Table 4.3: Fuzzy Preferences of Uniroyal Chemical Ltd. (U) and Local Government (L) in the Elmira Conflict

$$
\mathcal{R}^{\mathrm{U}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{array} \\
\left(\begin{array}{ccccccccc}
0.5 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
0 & 0.5 & 0.85 & 0 & 0.4 & 0.7 & 0.9 & 0 & 0.2 \\
0 & 0.15 & 0.5 & 0 & 0 & 0 & 0.8 & 0 & 0.1 \\
0 & 1.0 & 1.0 & 0.5 & 1.0 & 1.0 & 1.0 & 1.0 & 0.9 \\
0 & 0.6 & 1.0 & 0 & 0.5 & 0.7 & 1.0 & 0.1 & 0.3 \\
0 & 0.3 & 1.0 & 0 & 0.3 & 0.5 & 1.0 & 0 & 0.2 \\
0 & 0.1 & 0.2 & 0 & 0 & 0 & 0.5 & 0 & 0.05 \\
0 & 1.0 & 1.0 & 0 & 0.9 & 1.0 & 1.0 & 0.5 & 0.7 \\
0 & 0.8 & 0.9 & 0.1 & 0.7 & 0.8 & 0.95 & 0.3 & 0.5
\end{array}\right)
\end{array}
$$

$$
\mathcal{R}^{\mathrm{L}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{array} \\
\left(\begin{array}{ccccccccc}
0.5 & 1.0 & 0 & 0.8 & 0 & 1.0 & 0 & 0.7 & 1.0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\
1.0 & 1.0 & 0.5 & 1.0 & 1.0 & 1.0 & 0.65 & 1.0 & 1.0 \\
0.2 & 1.0 & 0 & 0.5 & 0.25 & 1.0 & 0 & 0.6 & 1.0 \\
1.0 & 1.0 & 0 & 0.75 & 0.5 & 1.0 & 0 & 0.7 & 1.0 \\
0 & 1.0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 1.0 \\
1.0 & 1.0 & 0.35 & 1.0 & 1.0 & 1.0 & 0.5 & 1.0 & 1.0 \\
0.3 & 1.0 & 0 & 0.4 & 0.3 & 1.0 & 0 & 0.5 & 1.0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5
\end{array}\right)
\end{array}
$$

fuzzy preference model for U and L has been developed, as represented by matrices $\mathcal{R}^{\mathrm{U}}$ and $\mathcal{R}^{\mathrm{L}}$ in Table 6.6. For example, the number 0.7 in the 9-th row and 5-th column of $\mathcal{R}^{\mathrm{U}}$ represents the degree of preference of state s_9 over state s_5 for U, while the number 0.6 in the 4-th row and 8-th column of $\mathcal{R}^{\mathrm{L}}$ represents L's preference degree for state s_4 over state s_8. To demonstrate how the satisficing behavior of DMs within an FGM influences fuzzy stabilities, four sets of FSTs of the DMs are considered. The FSTs used in the analysis are: (i) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4, \gamma_{\mathrm{L}} = 0.2$; (ii) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4, \gamma_{\mathrm{L}} = 0.3$; (iii) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.6, \gamma_{\mathrm{L}} = 0.2$; and (iv) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.6, \gamma_{\mathrm{L}} = 0.3$. Note that in each case $\gamma_{\mathrm{M}} = 1.0$, since the preference of M is crisp.

4.4.3 Fuzzy Stability Analysis of the Elmira Groundwater Contamination Conflict

To carry out a fuzzy stability analysis of the Elmira conflict model described above means to apply the fuzzy stability definitions in order to identify states with high degrees of stability. The results are presented in Table 4.4, where a $\surd$ in a cell indicates that the state in the corresponding row is fuzzy stable for the indicated DM, or a fuzzy equilibrium, under the indicated fuzzy stability definition. Note that the fuzzy stabilities are calculated for each of the four sets of FSTs mentioned in Subsection 4.4.2.

As can be seen from Table 4.4, when weaker satisficing criteria for U and L, such as $\gamma_{\mathrm{U}} = 0.4$ and $\gamma_{\mathrm{L}} = 0.2$, are considered, the two predicted equilibria (states s_5 and s_8) of the analysis in (Hipel et al., 1993a, 1999; Kilgour et al., 2001) disappear for *FR* and *FSEQ* stability types. However, there is a new fuzzy equilibrium at state s_4. For stronger satisficing criteria for U and L, that is, for increased FSTs, states s_5 and s_8 join the fuzzy equilibrium list. For *FSEQ*, when moves and countermoves determined using DMs' FUIs become relevant, state s_1 is no longer fuzzy stable for L when the FST of U is increased. That is, increasing γ_{U} from 0.4 to 0.6 unblocks L's FUI from s_1 to s_5, causing state s_1 to be *FSEQ* unstable for L.

Table 4.4: Fuzzy Stability Results of the Elmira Groundwater Contamination Conflict

FSTs	States	*FR*				*FGMR*				*FSMR*				*FSEQ*			
		M	U	L	*FE*	M	U	L	*FE*	M	U	L	*FE*	M	U	L	*FE*
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.4$ $\gamma_{\mathrm{L}} = 0.2$	s_1	√	√			√	√	√	√	√	√	√	√	√	√	√	√
	s_2	√				√		√		√		√		√		√	
	s_3	√		√		√		√		√		√		√		√	
	s_4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_5	√		√		√		√		√		√		√		√	
	s_6	√		√		√		√		√		√		√		√	
	s_7	√				√		√		√		√		√		√	
	s_8	√	√			√	√	√	√	√	√	√	√	√	√		
	s_9	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.4$ $\gamma_{\mathrm{L}} = 0.3$	s_1	√	√			√	√	√	√	√	√	√	√	√	√	√	√
	s_2	√				√		√		√		√		√		√	
	s_3	√		√		√		√		√		√		√		√	
	s_4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_5	√		√		√		√		√		√		√		√	
	s_6	√		√		√		√		√		√		√		√	
	s_7	√				√		√		√		√		√		√	
	s_8	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_9	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.6$ $\gamma_{\mathrm{L}} = 0.2$	s_1	√	√			√	√	√	√	√	√	√	√	√	√		
	s_2	√				√		√		√		√		√		√	
	s_3	√		√		√		√		√		√		√		√	
	s_4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_5	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_6	√		√		√		√		√		√		√		√	
	s_7	√				√		√		√		√		√		√	
	s_8	√	√			√	√	√	√	√	√	√	√	√	√		
	s_9	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.6$ $\gamma_{\mathrm{L}} = 0.3$	s_1	√	√			√	√	√	√	√	√	√	√	√	√		
	s_2	√				√		√		√		√		√		√	
	s_3	√		√		√		√		√		√		√		√	
	s_4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_5	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_6	√		√		√		√		√		√		√		√	
	s_7	√				√		√		√		√		√		√	
	s_8	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_9	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

As is clear from Table 4.4, states s_4 and s_9 have a high degree of stability—they are FE (fuzzy stable for all DMs) under all four fuzzy stability definitions for each of the four sets of FSTs. The addition of state s_5 to the FE list results from the increase of U's FST from 0.4 to 0.6, while the inclusion of state s_8 as a FE results from the increase of L's FST from 0.2 to 0.3. It should be noted that state s_9, where U closes its operation in Elmira, is the least preferred for both M and L as can be seen from their preference representations. Moreover, as depicted in Figure 4.1, U alone controls the movement of the dispute to state s_9, which is not its most preferred state, and seems relatively unlikely to happen.

In state s_5, U delays the appeal process while L insists on application of the original control order. In this circumstance, the DMs are working to reach a reasonable (win-win) resolution, so it seems that, like the original analysis (Hipel et al., 1993a), state s_5 cannot be a likely outcome. State s_1, which is similar to state s_5 except that L does not insist on application of the original control order, also cannot be likely. Thus, the alternatives are state s_8, where M modifies the original control order and U accepts it as modified, despite the objection of L, and state s_4, which is the same as state s_8 except that L does not raise any objection.

Thus, if L would move to a reachable state that is relatively less favorable, that is, if its satisficing criterion is weak, the most reasonable resolution is state s_4. However, as L becomes stricter about making only more favorable moves, that is, if L's satisficing criterion is high, the outcome is either state s_4 or s_8. Note that L controls the movement between states s_4 and s_8. Therefore, which of these two equilibria is more likely depends greatly on how sensitive L is toward U's interests. Recall that the objective of L was not only to care for its citizens but also to safeguard its financial base. Furthermore, U controls threats from states s_4 and s_8 to abandon its Elmira operation. Thus, L has good reasons to be

Table 4.5: Comparison of the Stability Results between the Crisp GMCR and FGM Analyses of the Elmira Groundwater Contamination Conflict

States	Stability Results in the Previous Analysis (using the Crisp GMCR (Hipel et al., 1993a, 1999; Kilgour et al., 2001))	Stability Findings in the Present Analysis (using the FGM)
s_1	*GMR* and *SMR* equilibrium.	*FGMR*, *FSMR* and *FSEQ* equilibrium for each of the four sets of FSTs except that for larger FST of U, the state becomes *FSEQ* unstable for L.
s_4	*GMR* and *SMR* equilibrium.	*FE* under all four fuzzy stability definitions for each of the four sets of FSTs—**one of the recommended resolutions**.
s_5	Equilibrium under all four stability definitions.	*FE* under all four fuzzy stability definitions for only larger FST of U.
s_8	Equilibrium under all four stability definitions—**recommended resolution**.	*FE* under all four fuzzy stability definitions for only larger FST of L. For smaller FST of L, it is only an *FGMR* and *FSMR* equilibrium—**one of the recommended resolutions**.
s_9	Equilibrium under all four stability definitions.	*FE* under all four fuzzy stability definitions for each of the four sets of FSTs—**relatively unlikely to happen**.

conciliatory to U. If this is the case, then the most likely resolution is state s_4; otherwise, state s_8 would be most likely. The role of L in this model is definitely a new insight into the conflict; earlier analyses (e.g., (Hipel et al., 1993a)) concluded that, despite its efforts, L had essentially no effect on the outcome. A comparison of these results with the findings from the previous crisp graph model analysis of the Elmira groundwater contamination conflict (Hipel et al., 1993a, 1999; Kilgour et al., 2001) is presented in Table 4.5.

4.5 Summary

A FGM is developed for the general situation of the n-DM ($n > 2$) graph model to incorporate preference uncertainty into conflict decision making. Within this framework, three basic fuzzy stability definitions—*FGMR*, *FSMR*, and *FSEQ*—are introduced. Since *FR* stability does not depend on the responses by the opponents, the *FR* stability definition for an n-DM model is the same as for the two-DM case. Moreover, when there are two DMs in a model ($|N| = 2$), the fuzzy stabilities given by Definitions 4.3.1, 4.3.2 and 4.3.3 coincide with the Definitions 3.5.2, 3.5.3 and 3.5.4, respectively, since in this case the coalition $H = N - k \subset N$ of the opponents of the focal DM k is trivial. In particular, if $H = N - k = \{l\}$, then $R_{N-k}(s) = R_l(s)$ and $\widetilde{R}^+_{N-k}(s) = \widetilde{R}^+_l(s)$. Hence, it follows that the fuzzy stability definitions provided in Section 4.3 constitute a set of fuzzy stability criteria that can be applied to a graph model with any number of DMs.

When applied to the Elmira groundwater contamination conflict, the fuzzy stability analysis leads to some different predictions from the original analysis and provides new insights. The outcomes can be interpreted not only as predictions, but also as answers to "What-If?" questions. In particular, the final outcome depends on how much L cares about U's interests. If L is strict in pursuing its own objectives, the final outcome may be s_8, which does not add any value compared to the other possible outcome, s_4, but creates a gap between these two DMs. On the other hand, if L is more sympathetic to U's interests, the outcome may be s_4.

Chapter 5

Coalition Fuzzy Stability Analysis

5.1 Introduction

The fuzzy stability definitions introduced in Sections 3.5 and 4.3 are based solely on the non-cooperative behavior of DMs; that is, each DM identifies his or her favorable states based only on his or her own interests. However, in reality, people may cooperate with each other to see if they can do even better compared to what they can achieve individually.

As mentioned in the Subsection 2.2.6, a coalitional form of stability definitions for Nash, *GMR*, *SMR*, and *SEQ* within the (crisp) graph model structure are developed in (Kilgour et al., 2001; Inohara and Hipel, 2008a,b). In this chapter, necessary tools, such as coalition fuzzy improvements and class coalitional fuzzy improvements, are introduced to facilitate the coalitional form of fuzzy stability analysis. Next, the four basic coalitional fuzzy stability definitions, such as coalition fuzzy Nash stability, coalition fuzzy general metarationality, coalition fuzzy symmetric metarationality, and coalition fuzzy sequential stability are developed.

The coalition fuzzy stability definitions introduced in this chapter are applied to a model of the groundwater contamination dispute in Elmira, described in Section 4.4, demonstrating how these new concepts can be conveniently applied to practical problems in order to identify a likely outcome. This application also reveals a comparison between cooperative and non-cooperative form of fuzzy stabilities, and provides valuable strategic insights. The research in this chapter was initiated in the paper by Bashar et al. (2012a).

5.2 Fuzzy Improvements by Coalitions

In the fuzzy stability definitions given in Sections 3.5 and 4.3, the focal DM is a single DM; therefore, to determine states that are advantageous for the DM to move to, one needs to find the FUI list for each individual DM. However, the coalition fuzzy stabilities are primarily defined for a coalition of DMs. Note that the concept of a coalition fuzzy stability is an extension of the notion of a fuzzy stability for an individual DM. Thus, the coalition fuzzy stabilities will be characterized based on whether a coalition is better off to stay at the current state or to move to a reachable state. Hence, one needs to identify states that benefit the members of the coalition.

Recall that $H \subseteq N$ represents a coalition of DMs in N, and $\mathcal{P}(N)$, the class of all coalitions of DMs in N. Also, recall that if $H = \{1, 2, ..., p\}$ then $\gamma_H = (\gamma_1, \gamma_2, ..., \gamma_p)$.

Definition 5.2.1. (Coalition Fuzzy Improvement): A state $s_i \in S$ is a *coalition fuzzy improvement* (CFI) from a state $s \in S$ by a coalition $H \subseteq N$ if $s_i \in R_H(s)$ and $\alpha^k(s_i, s) \geq \gamma_k$ for all $k \in H$.

Definition 5.2.2. (Coalition Fuzzy Improvement List): The *coalition fuzzy improvement list* (CFIL) from state s by coalition H is the collection of all CFIs from s by the coalition H, denoted $\widetilde{R}^{++}_{H,\gamma_H}(s)$.

In summary,

$$s_i \in \widetilde{R}^{++}_{H,\gamma_H}(s) \text{ if and only if } s_i \in R_H(s) \text{ and } \alpha^k(s_i, s) \geq \gamma_k \text{ for all } k \in H.$$

For simplicity, one writes $\widetilde{R}^{++}_{H}(s) = \widetilde{R}^{++}_{H,\gamma_H}(s)$.

For coalitional form of *FR* stability, one needs to identify states from which a coalition does not have an incentive to move to another state. The criteria given by Definitions 5.2.1 and 5.2.2 are sufficient to find these states. However, to define coalitional forms of *FGMR*, *FSMR* and *FSEQ* stabilities, one needs to consider possible sanctions by the coalitions of the opponent DMs. Accordingly, the concept of class coalitional fuzzy improvements is provided below. Note that the notion of class coalitional moves is given by Definition 2.2.18.

Definition 5.2.3. (Class Coalitional Fuzzy Improvement): Let $s \in S$ and $\mathcal{C} \subseteq \mathcal{P}(N)$. Let $\bigcup_{H \in \mathcal{C}} H = \{1, 2, ..., \eta\}$ and define $\gamma_{\mathcal{C}} = (\gamma_1, \gamma_2, ..., \gamma_\eta)$. The *class fuzzy improvement list* or *class coalitional fuzzy improvement list* (CCFIL) from state s by class $\mathcal{C}$, denoted $\widetilde{R}^{++}_{\mathcal{C},\gamma_{\mathcal{C}}}(s)$, is defined inductively as follows:

(1) If $H \in \mathcal{C}$ and $s_1 \in \widetilde{R}^{++}_{H}(s)$, then $s_1 \in \widetilde{R}^{++}_{\mathcal{C},\gamma_{\mathcal{C}}}(s)$.

(2) If $s_1 \in \widetilde{R}^{++}_{\mathcal{C},\gamma_{\mathcal{C}}}(s)$ and $H \in \mathcal{C}$, and $s_2 \in \widetilde{R}^{++}_{H}(s_1)$, then $s_2 \in \widetilde{R}^{++}_{\mathcal{C},\gamma_{\mathcal{C}}}(s)$.

A *class coalitional fuzzy improvement* (CCFI) from s by the class $\mathcal{C}$ is any member of

$\widetilde{R}^{++}_{\mathcal{C},\gamma_{\mathcal{C}}}(s)$. As in Definition 2.2.18, this definition ensures that no DM in any coalition in $\mathcal{C}$ may move twice consecutively. For simplicity, one writes $\widetilde{R}^{++}_{\mathcal{C}}(s) = \widetilde{R}^{++}_{\mathcal{C},\gamma_{\mathcal{C}}}(s)$.

5.3 Coalition Fuzzy Stabilities

Definition 5.3.1. (Coalition Fuzzy Nash Stability or Coalition Fuzzy Rationality for a Coalition): Let H be a coalition of DMs in N and $s \in S$. State s is *coalition fuzzy Nash stable* or *coalition fuzzy rational* (*CFR*) for coalition H if and only if $\widetilde{R}^{++}_{H}(s) = \varnothing$.

It is clear from this definition that state s is a *CFR* for a coalition if and only if there is no state that is a CFI from s by the coalition. Note that *CFR* stability is a natural generalization of the *FR* stability given in Definition 3.5.1.

Definition 5.3.2. (Coalition Fuzzy Nash Stability or Coalition Fuzzy Rationality for a DM): Let $k \in N$ and $s \in S$. State s is *coalition fuzzy Nash stable* or *coalition fuzzy rational* (*CFR*) for DM k if and only if s is *CFR* for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

In the following definitions, $\mathcal{P}(N - H)$ represents the class of coalitions of DMs in N other than those in H.

Definition 5.3.3. (Coalition Fuzzy General Metarationality for a Coalition): Let $H \in \mathcal{P}(N)$ and $s \in S$. State s is *coalition fuzzy general metarational* (*CFGMR*) for coalition H if and only if for every $s_1 \in \widetilde{R}^{++}_{H}(s)$ there exists a CCM $s_2 \in R_{\mathcal{P}(N-H)}(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$ for some $k \in H$.

Under *CFGMR*, each of the initial coalition H's CFIs is sanctioned by a subsequent coalitional move by some or all of the sanctioning coalitions such that no DM in any sanctioning coalition may move twice consecutively. Keep in mind that, as in the case of *CFR* stability, *CFGMR* stability generalizes the concept of *FGMR* stability provided in Definition 4.3.1.

Definition 5.3.4. (Coalition Fuzzy General Metarationality for a DM): Let $k \in N$ and $s \in S$. State s is *coalition fuzzy general metarational* (*CFGMR*) for DM k if and only if s is *CFGMR* for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

Definition 5.3.5. (Coalition Fuzzy Symmetric Metarationality for a Coalition): Let $H \in \mathcal{P}(N)$ and $s \in S$. State s is *coalition fuzzy symmetric metarational* (*CFSMR*) for coalition H if and only if for every $s_1 \in \widetilde{R}_H^{++}(s)$ there exists a CCM $s_2 \in R_{\mathcal{P}(N-H)}(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$ for some $k \in H$, and for every $s_3 \in R_H(s_2)$, $\alpha^l(s_3, s) < \gamma_l$ for some $l \in H$.

CFSMR describes the stability of states by looking one more step ahead compared to the *CFGMR* by checking whether the initial coalition H can escape sanctions, if any, caused by the subsequent coalitional moves against each of H's CFIs. If H cannot escape the sanction using a coalitional move, the state is *CFSMR* stable. As before, the *CFSMR* stability constitutes an intuitive extension of the *FSMR* stability furnished in Definition 4.3.2.

Definition 5.3.6. (Coalition Fuzzy Symmetric Metarationality for a DM): Let $k \in N$ and $s \in S$. State s is *coalition fuzzy symmetric metarational* (*CFSMR*) for DM k if and only if s is *CFSMR* for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

Definition 5.3.7. (Coalition Fuzzy Sequential Stability for a Coalition): Let $H \in \mathcal{P}(N)$ and $s \in S$. State s is *coalition fuzzy sequentially stable* (*CFSEQ*) for coalition H

if and only if for every $s_1 \in \widetilde{R}^{++}_{H}(s)$ there exists a CCFI $s_2 \in \widetilde{R}^{++}_{\mathcal{P}(N-H)}(s_1)$ such that $\alpha^k(s_2, s) < \gamma_k$ for some $k \in H$.

CFSEQ is the same as *CFGMR* except that while considering sanctions of the initial coalition H's CFIs, H takes into account only subsequent coalitional fuzzy improvements rather than coalitional moves. Like other coalition fuzzy stabilities, *CFSEQ* stability is a natural generalization of the *FSEQ* stability given in Definition 4.3.3.

Definition 5.3.8. (Coalition Fuzzy Sequential Stability for a DM): Let $k \in N$ and $s \in S$. State s is *coalition fuzzy sequentially stable* (*CFSEQ*) for DM k if and only if s is *CFSEQ* for all coalitions $H \in \mathcal{P}(N)$ such that $k \in H$.

Definition 5.3.9. (Coalition Fuzzy Equilibrium): A state $s \in S$ is a *coalition fuzzy equilibrium* (*CFE*) under a specific coalition fuzzy stability concept if and only if s is coalition fuzzy stable for all DMs under that coalition fuzzy stability notion. For instance, state s is coalition fuzzy Nash equilibrium or *CFR* equilibrium if and only if it is *CFR* stable for all DMs in N.

Remark 5.3.10. If the FST of each DM in N is 1.0, the definitions of coalition fuzzy stabilities and associated coalition fuzzy equilibrium developed in this section coincide with the definitions of the corresponding coalition stabilities and associated coalition equilibrium provided in Subsection 2.2.6.

The following theorem is due to Remark 5.3.10.

Theorem 5.3.11. Coalition fuzzy stability concepts of the FGM are generalizations of (crisp) coalition stability notions of the GMCR.

5.4 Application of Coalition Fuzzy Stabilities to the Elmira Groundwater Contamination Conflict

To clearly understand the coalition fuzzy stability concepts as well as the associated definitions, and to demonstrate how they can be applied to practical decision problems, the methodology is applied to an actual dispute over the contamination of the groundwater aquifer supplying the town of Elmira, Canada, which is described in detail in Section 4.4. Note that a crisp graph model coalition analysis of the Elmira conflict was initiated by Kilgour et al. (2001) while an extensive coalition analysis was done by Inohara and Hipel (2008a).

To study the impacts of DMs' satisficing behavior in the coalition fuzzy stability analysis of the Elmira dispute, four sets of FSTs—(i) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4, \gamma_{\mathrm{L}} = 0.2$; (ii) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4, \gamma_{\mathrm{L}} = 0.3$; (iii) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.6, \gamma_{\mathrm{L}} = 0.2$; and (iv) $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.6, \gamma_{\mathrm{L}} = 0.3$—are considered, which are the same as in Section 4.4. The reason for choosing the same set of FSTs as in Section 4.4 is to compare the outcomes of the present analysis with the results obtained in Section 4.4 using non-cooperative fuzzy stabilities.

Recall that the set of DMs in the conflict is $N = \{\mathrm{M, U, L}\}$ and the set of feasible states, $S = \{s_1, s_2, ..., s_9\}$. Hence, all possible coalitions of DMs are {M}, {U}, {L}, {M, U}, {M, L}, {U, L}, and {M, U, L}. For simplicity, these coalitions are written as M, U, L, MU, ML, UL, and MUL, respectively; hence $\mathcal{P}(N) = \{\mathrm{M, U, L, MU, ML, UL, MUL}\}$. The reachable lists for all coalitions of DMs are calculated by using Definition 2.2.2, and are presented in Table 5.1. Next, for each coalition $H \in \mathcal{P}(N)$, Definitions 5.2.2 and 5.2.3 are employed to calculate CFILs $\widetilde{R}^{++}_H(s)$ and CCFILs $\widetilde{R}^{++}_{\mathcal{P}(N-H)}(s)$, respectively, from each state s for each of the four sets of FSTs. To save space in the thesis, $\widetilde{R}^{++}_H(s)$ and $\widetilde{R}^{++}_{\mathcal{P}(N-H)}(s)$ are

Table 5.1: Reachable Lists of Coalitions

States (s)	Reachable Lists $R_H(s)$ by Coalitions $H \subseteq N$						
	M	U	L	MU	ML	UL	MUL
s_1	s_2	s_3, s_9	s_5	s_2, s_3, s_4, s_9	s_2, s_5, s_6	s_3, s_5, s_7, s_9	$s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9$
s_2		s_4, s_9	s_6	s_4, s_9	s_6	s_4, s_6, s_8, s_9	s_4, s_6, s_8, s_9
s_3	s_4	s_9	s_7	s_4, s_9	s_4, s_7, s_8	s_7, s_9	s_4, s_7, s_8, s_9
s_4		s_9	s_8	s_9	s_8	s_8, s_9	s_8, s_9
s_5	s_6	s_7, s_9	s_1	s_6, s_7, s_8, s_9	s_1, s_2, s_6	s_1, s_3, s_7, s_9	$s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9$
s_6		s_8, s_9	s_2	s_8, s_9	s_2	s_2, s_4, s_8, s_9	s_2, s_4, s_8, s_9
s_7	s_8	s_9	s_3	s_8, s_9	s_3, s_4, s_8	s_3, s_9	s_3, s_4, s_8, s_9
s_8		s_9	s_4	s_9	s_4	s_4, s_9	s_4, s_9
s_9							

presented in Tables 5.2 and 5.3, respectively, for only the FSTs $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4$, and $\gamma_{\mathrm{L}} = 0.2$. In particular, one can see from Table 5.2 that state s_8 is a CFI of coalition MU from state s_5, since $s_8 \in R_{\mathrm{MU}}(s_5)$, and $\alpha^{\mathrm{M}}(s_8, s_5) = 1.0 - 0 = 1.0 = \gamma_{\mathrm{M}}$ and $\alpha^{\mathrm{U}}(s_8, s_5) = 0.9 - 0.1 = 0.8 > 0.4 = \gamma_{\mathrm{U}}$. From Table 5.3, state s_9 is a CCFI from state s_1 by the class of coalitions of the opponents of M (that is, of the class of coalitions {U, L, UL}), since s_9 is a CFI from state s_5 by U and s_5 is a CFI from state s_1 by L.

Although four different coalition fuzzy stability definitions are introduced to include varied human behavior under conflict within the FGM, it is reasonable to assume that *CFR* and *CFSEQ* stability represent a majority of the DMs' behavioral patterns, since DMs levying sanctions will not hurt themselves when sanctioning by moving to a less preferred state. Hence, a coalition fuzzy stability analysis for *CFR* and *CFSEQ* are presented here for the dispute over the groundwater contamination in Elmira. Note from Definitions 5.3.2, 5.3.4, 5.3.6, and 5.3.8 that the coalition fuzzy stability for an individual DM depends on the coalition fuzzy stability results for all possible coalitions in which that DM is a member.

Table 5.2: Coalition Fuzzy Improvement Lists for Fuzzy Satisficing Thresholds (FSTs): $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4, \gamma_{\mathrm{L}} = 0.2$

States (s)	Coalition Fuzzy Improvement Lists $\widetilde{R}_H^{++}(s)$ for Coalitions $H \subseteq N$						
	M	U	L	MU	ML	UL	MUL
s_1			s_5		s_5		
s_2		s_4, s_9	s_6	s_4		s_4, s_8	s_4, s_8
s_3		s_9					
s_4							
s_5		s_9		s_8			
s_6		s_8, s_9		s_8		s_4, s_8	s_4, s_8
s_7		s_9	s_3			s_3	
s_8			s_4		s_4	s_4	s_4
s_9							

Table 5.3: Class Coalitional Fuzzy Improvement Lists for Fuzzy Satisficing Thresholds (FSTs): $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4, \gamma_{\mathrm{L}} = 0.2$

States (s)	Class Coalitional Fuzzy Improvement Lists $\widetilde{R}_{\mathcal{P}(N-H)}^{++}(s)$ for Coalitions $H \subseteq N$						
	M	U	L	MU	ML	UL	MUL
s_1	s_5, s_9	s_5		s_5			
s_2	s_4, s_6, s_8, s_9	s_6	s_4, s_9	s_6	s_4, s_9		
s_3	s_9		s_9		s_9		
s_4							
s_5	s_9		s_8, s_9		s_9		
s_6	s_4, s_8, s_9		s_8, s_9		s_8, s_9		
s_7	s_3, s_9	s_3	s_9	s_3	s_9		
s_8	s_4	s_4		s_4			
s_9							

In particular, in the Elmira conflict, a state is *CFR* stable for DM M if it is *CFR* stable for coalitions M, MU, ML, and MUL. The *CFR* and *CFSEQ* stability results for DM M for FSTs $\gamma_M = 1.0, \gamma_U = 0.4, \text{and} \gamma_L = 0.2$ are presented in Table 5.4. One can notice from Table 5.4 that state s_4 is *CFR* for DM M since it is *CFR* for each of the four coalitions

M, MU, ML and MUL, while state s_5 is not *CFR* for DM M because it is not *CFR* for coalition MU, although it is for the other three coalitions M, ML and MUL.

Table 5.4: Coalition Fuzzy Nash (*CFR*) and Sequential (*CFSEQ*) Stability Results for the Ontario Ministry of the Environment (M) for Fuzzy Satisficing Thresholds (FSTs): $\gamma_{\text{M}} = 1.0, \gamma_{\text{U}} = 0.4, \gamma_{\text{L}} = 0.2$

States (s)	*CFR* for Coalition $H \subseteq N$				*CFR* for DM M	*CFSEQ* for Coalition $H \subseteq N$				*CFSEQ* for DM M
	M	MU	ML	MUL		M	MU	ML	MUL	
s_1	√	√		√		√	√	√	√	√
s_2	√		√			√		√		
s_3	√	√	√	√	√	√	√	√	√	√
s_4	√	√	√	√	√	√	√	√	√	√
s_5	√		√	√		√		√	√	
s_6	√		√			√		√		
s_7	√	√	√	√	√	√	√	√	√	√
s_8	√	√				√	√			
s_9	√	√	√	√	√	√	√	√	√	√

The overall stability results, that is, the *CFR* and *CFSEQ* stability findings for M, U and L as well as the corresponding *CFE* for all four sets of FSTs, are presented in Table 5.5. It can be seen from Table 5.5 that the most consistent *CFE* are states s_4 and s_9, as they appear to be *CFE* under both *CFR* and *CFSEQ* stability concepts for all four sets of FSTs. If L requires more certainty of improvements, that is, if L wants to gain more in identifying its FUIs, then state s_8 joins the *CFE* list. Notice that for smaller FST of U, state s_1 is not *CFE* under *CFR* but it is under *CFSEQ*. The reason for this is that there is a CFI s_5 from state s_1 by each of the coalitions L and ML that is sanctioned by the subsequent CCFI(s) of the class of coalitions of the opponents of each of L and ML. In particular, the CFI s_5 of coalition ML from s_1 is sanctioned by the subsequent CCFI s_9 of the class {U} of coalitions of the opponent(s) of ML, making state s_1 to be *CFSEQ* stable for ML. However, for increased FST of U, CFI s_5 from s_1 by coalition ML becomes unsanctioned, so that s_1 can no longer be *CFE* under *CFSEQ*.

Now, these findings can be compared with the non-cooperative form of fuzzy stability results of the Elmira conflict found in Section 4.4 (given in Table 4.4). For this purpose, Table 5.6 is reproduced from Table 4.4, representing only *FR* and *FSEQ* stability results. From Tables 5.5 and 5.6, one can find that, the non-cooperative form of *FE* state s_5 is no longer a *FE* when DMs coordinate their moves. This is because the coalition MU has a CFI s_8 from s_5 that cannot be sanctioned by any subsequent CCFI of the class of coalitions of the opponents of MU. Hence, state s_5 is not *CFR* or *CFSEQ* stable for any of the DMs M and U. This means that M and U can find a better outcome than s_5 if they form a coalition; accordingly, s_5 is not a good choice as a resolution. Thus, the coalition fuzzy stability analysis can help narrow down the list of possible resolution(s).

Although there are no other differences between the *FE* and *CFE* results, there is a substantial amount of difference in individual level fuzzy stability findings between the non-cooperative and coalitional fuzzy stability concepts. For example, when considering the non-cooperative form, M does not envision an FUI from any of the states, thereby making each feasible state *FR* as well as *FSEQ* stable. However, if it decides to cooperate with others, some states become fuzzy stable, but not all. For instance, for FSTs $\gamma_{\mathrm{M}} = 1.0, \gamma_{\mathrm{U}} = 0.4$, and $\gamma_{\mathrm{L}} = 0.2$, states $s_1, s_3, s_4, s_7, \text{and} s_9$ are *CFSEQ* stable for M. This means that M can join in some coalitions for each of which there are some CFIs that cannot be sanctioned by any subsequent CCFIs of the class of coalitions of the opponents of the initial coalition. Hence, M now has a shorter list of states from which it cannot do any better. This fact is very important to come up with a suitable resolution. Even if this does not change the equilibrium list at this time, the DM may reconsider its preferences to see

Table 5.5: Coalition Fuzzy Stability Results of the Elmira Conflict

FSTs	States (s)	*CFR*				*CFSEQ*			
		M	U	L	*CFE*	M	U	L	*CFE*
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.4$ $\gamma_{\mathrm{L}} = 0.2$	s_1		√			√	√	√	√
	s_2								
	s_3	√		√		√		√	
	s_4	√	√	√	√	√	√	√	√
	s_5			√				√	
	s_6								
	s_7	√				√			
	s_8								
	s_9	√	√	√	√	√	√	√	√
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.4$ $\gamma_{\mathrm{L}} = 0.3$	s_1		√			√	√	√	√
	s_2								
	s_3	√		√		√		√	
	s_4	√	√	√	√	√	√	√	√
	s_5			√				√	
	s_6								
	s_7	√				√			
	s_8	√	√	√	√	√	√	√	√
	s_9	√	√	√	√	√	√	√	√
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.6$ $\gamma_{\mathrm{L}} = 0.2$	s_1		√				√		
	s_2								
	s_3	√		√		√		√	
	s_4	√	√	√	√	√	√	√	√
	s_5			√				√	
	s_6								
	s_7	√				√			
	s_8								
	s_9	√	√	√	√	√	√	√	√
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.6$ $\gamma_{\mathrm{L}} = 0.3$	s_1		√				√		
	s_2								
	s_3	√		√		√		√	
	s_4	√	√	√	√	√	√	√	√
	s_5			√				√	
	s_6								
	s_7	√				√			
	s_8	√	√	√	√	√	√	√	√
	s_9	√	√	√	√	√	√	√	√

Table 5.6: Non-cooperative Form of Fuzzy Nash (FR) and Sequential ($FSEQ$) Stability Results of the Elmira Conflict (reproduced from Table 4.4)

FSTs	States (s)	FR				$FSEQ$			
		M	U	L	FE	M	U	L	FE
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.4$ $\gamma_{\mathrm{L}} = 0.2$	s_1	$\surd$	$\surd$			$\surd$	$\surd$	$\surd$	$\surd$
	s_2	$\surd$				$\surd$		$\surd$	
	s_3	$\surd$		$\surd$		$\surd$		$\surd$	
	s_4	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_5	$\surd$		$\surd$		$\surd$		$\surd$	
	s_6	$\surd$		$\surd$		$\surd$		$\surd$	
	s_7	$\surd$				$\surd$		$\surd$	
	s_8	$\surd$	$\surd$			$\surd$	$\surd$		
	s_9	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.4$ $\gamma_{\mathrm{L}} = 0.3$	s_1	$\surd$	$\surd$			$\surd$	$\surd$	$\surd$	$\surd$
	s_2	$\surd$				$\surd$		$\surd$	
	s_3	$\surd$		$\surd$		$\surd$		$\surd$	
	s_4	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_5	$\surd$		$\surd$		$\surd$		$\surd$	
	s_6	$\surd$		$\surd$		$\surd$		$\surd$	
	s_7	$\surd$				$\surd$		$\surd$	
	s_8	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_9	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.6$ $\gamma_{\mathrm{L}} = 0.2$	s_1	$\surd$	$\surd$			$\surd$	$\surd$		
	s_2	$\surd$				$\surd$		$\surd$	
	s_3	$\surd$		$\surd$		$\surd$		$\surd$	
	s_4	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_5	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_6	$\surd$		$\surd$		$\surd$		$\surd$	
	s_7	$\surd$				$\surd$		$\surd$	
	s_8	$\surd$	$\surd$			$\surd$	$\surd$		
	s_9	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\gamma_{\mathrm{M}} = 1.0$ $\gamma_{\mathrm{U}} = 0.6$ $\gamma_{\mathrm{L}} = 0.3$	s_1	$\surd$	$\surd$			$\surd$	$\surd$		
	s_2	$\surd$				$\surd$		$\surd$	
	s_3	$\surd$		$\surd$		$\surd$		$\surd$	
	s_4	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_5	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_6	$\surd$		$\surd$		$\surd$		$\surd$	
	s_7	$\surd$				$\surd$		$\surd$	
	s_8	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	s_9	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$

how outcomes are influenced.

Table 5.7: Evolution from the Status Quo State to the Coalition Fuzzy Equilibrium (*CFE*), s_4, in the Elmira Conflict (when $\gamma_{\mathrm{L}} = 0.2$)

	Status Quo		Intermediate Cooperative Moves				*CFE*
M							
1. Modify	N		N	$\longrightarrow$	Y		Y
U							
2. Delay	Y		Y	$\longrightarrow$	N		N
3. Accept	N		N	$\longrightarrow$	Y		Y
4. Abandon	N		N		N		N
L							
5. Insist	N	$\longrightarrow$	Y		Y	$\longrightarrow$	N
States	s_1		s_5		s_8		s_4

Table 5.8: Evolution from the Status Quo State to the Coalition Fuzzy Equilibrium (*CFE*), s_8, in the Elmira Conflict (when $\gamma_{\mathrm{L}} = 0.3$)

	Status Quo		Intermediate Cooperative Moves		*CFE*
M					
1. Modify	N		N	$\longrightarrow$	Y
U					
2. Delay	Y		Y	$\longrightarrow$	N
3. Accept	N		N	$\longrightarrow$	Y
4. Abandon	N		N		N
L					
5. Insist	N	$\longrightarrow$	Y		Y
States	s_1		s_5		s_8

From Table 5.5, one can see that any of states s_4, s_8 and s_9 can be a final resolution. From Table 5.2, states s_4 and s_8 can only be CFIs if all three DMs join in a coalition. Because the final outcome should to be a state that is favorable to all DMs, state s_9 cannot be a suitable choice. If the status quo state is considered to be s_1, L can take the conflict from s_1 to s_5, which is an FUI for L. Then, the coalition MU can take the conflict from state s_5 to s_8, which is a coalitional fuzzy improvement. If the FST of L is small such that a move is not highly restricted, it can join the coalition MU to make the final move from state s_8 to s_4, which is a CFI for the coalition MUL, and hence, the final outcome will be s_4. If L is too strict to make a move, then s_8 will remain as the final outcome. The possible evolutions of this conflict from the status quo state s_1 to the *CFE* states s_4 and s_8 are exhibited in Tables 5.7 and 5.8, respectively.

5.5 Summary

The coalition fuzzy stability definitions for Nash, general metarational, symmetric metarational, and sequential stability concepts of the GMCR are developed so that they constitute a natural generalization of the individual level *FR*, *FGMR*, *FSMR* and *FSEQ* stabilities. By employing these definitions, each state can be investigated for not only how preferable it is for an individual DM, but also how desirable it is for the DM as a potential coalition member. Specifically, coalition fuzzy stability for a DM identifies states from which neither the DM himself or herself, nor any of the coalitions that he or she can join, would like to move away. A DM can first assess how well he or she can do by acting on his own and then ascertain whether he can fare even better by cooperating within a coalition in the face of high uncertainty.

When applied to the Elmira groundwater contamination conflict, the methodology identifies some states that were fuzzy stable for M with respect to the non-cooperative fuzzy stability definitions developed in Chapters 3 and 4, but fail to be coalition fuzzy stable for DM M. Hence, the coalition fuzzy stability analysis may narrow down the list of individual-level fuzzy stabilities, thereby providing the analyst with valuable strategic insights into the conflict under study. Furthermore, the possible evolution of a conflict from a status quo state to a final outcome can be conveniently explained using CFILs. Therefore, as an analysis tool to augment non-cooperative fuzzy stabilities, coalition fuzzy stability analysis constitutes an important addition to the FGM.

Chapter 6

Fuzzy Option Prioritization

6.1 Introduction

In any decision making technique, DMs' preference information, expressed implicitly or explicitly, is an essential component. Preference information may be given in various forms, for instance as utilities (as in classical game theory (von Neumann and Morgenstern, 1944)), as fuzzy utilities (as in fuzzy decision making (Nakamura, 1986; De Wilde, 2004)), via crisp option prioritization (as in the crisp GMCR (Hipel et al., 1997; Peng et al., 1997; Peng, 1999; Hipel et al., 2001; Fang et al., 2003)), or simply as pairwise relative preferences over the feasible states or scenarios (as in the GMCR (Kilgour et al., 1987; Fang et al., 1993)). In whatever form a DM's preference information is provided, the objective is always to represent a preference relation over the feasible states or alternatives.

Crisp option prioritization is a methodology to model a DM's preference over feasible states within a graph model structure using his or her priority list of combinations of courses of action, referred to as preference statements (Peng et al., 1997; Peng, 1999; Fang et al., 2003). To be more specific, a DM's preference statements are composed of options using logical connectives such as "and", "or", and "if–then", and listed from the most important to least. The option prioritization methodology takes into account the truth value ("true" or "false") of each preference statement at each feasible state. Note that in calculating a DM's preferences, a more important preference statement dominates a less important one, and the truthfulness of a preference statement dominates its falsity. For example, in the Elmira model, the most important preference statement of L is "U does not close its operations in Elmira"; so any state or scenario in which U continues its operations in Elmira is preferred by L to any state in which U closes its operations.

Note that the FGM developed in Chapters 3 and 4, extending the crisp GMCR, as well as the coalition fuzzy stabilities introduced in Chapter 5, take into account a DM's fuzzy preferences. But, like crisp preferences, modeling fuzzy preferences by pairwise comparison of states is difficult, and even impractical, for larger problems. Until now, there is no single procedure to model a DM's fuzzy preference within a graph model framework. In this chapter, a fuzzy version of the crisp graph model option prioritization methodology, called fuzzy option prioritization, is developed by assuming fuzzy truth values of preference statements at feasible states to model DMs' fuzzy preferences for use in the analysis step of the FGM. The original research carried out in this chapter is the extension of the paper by Bashar et al. (2012c).

6.2 Fuzzy Truth Values and Fuzzy Scores

Preference uncertainty is an important issue in modeling and analyzing a real-world decision problem. When preference uncertainty is incurred in a multiple participant-multiple objective decision problem, preference statements, as modeled for the crisp option prioritization technique, developed in (Peng et al., 1997; Peng, 1999; Fang et al., 2003) and described in Subsection 2.2.7, may not be assessed to be precisely "true" or "false" at some feasible states. Note that an earlier version of crisp option prioritization was first introduced by Fraser and Hipel (1988), discussed by Fraser (1993, 1994), and then generalized for use within GMCR by Peng et al. (1997); Peng (1999); Hipel et al. (1997); Fang et al. (2003).

A limitation of preference modeling by crisp option prioritization technique is that it assesses a preference statement based only on whether it occurs (i.e., "true") or not (i.e., "false") at a state, and does not consider other courses of action that are present in that state but not included in that particular preference statement. These courses of action may influence the truth value of the preference statement at that state. For example, in the Elmira dispute, L may consider the preference statement "insisting on the application of the original control order" in modeling its preferences. It may be reasonable to restrict the truth value of this preference statement to either "true" or "false" at states in which U is delaying the appeal process. However, at a state in which U accepts a control order (original or modified), the truth value of this preference statement may not be precisely "true" or "false". Accordingly, to make the crisp option prioritization a more useful preference elicitation technique, a more flexible and realistic truth value of a preference statement is needed.

In the circumstances described above, it may be reasonable to represent the truth value of a preference statement at a state by a numerical value taken from the closed unit interval $[0, 1]$, referred to as a *truth degree* or *degree of truth*, called a *fuzzy truth value*. A truth degree of 1 for a preference statement at a state indicates that the preference statement is true, while a truth degree of 0 implies that the preference statement is false. Note that fuzzy truth value is the main concept behind fuzzy logic and fuzzy set, introduced by Zadeh (1965), that has a wide variety of applications in engineering, decision sciences, and other areas (Bellman and Zadeh, 1970; Bojadziev and Bojadziev, 2007; Ross, 2010).

A lower truth degree of a preference statement at a particular state indicates less suitability, while a higher truth degree implies more suitability of the statement in the circumstances of the state. If $\sigma_t(s) = \sigma(\Omega_t, s)$ denotes the fuzzy truth value of a preference statement Ω_t at a given state $s \in S$, then $\sigma_t(s) = 0$ (that is, the truth degree of the preference statement Ω_t at state s is 0) means that the statement Ω_t does not make any sense at s, which is equivalent to the fact that Ω_t is "false" at s, that is, $\Omega_t(s) = F$. Likewise, $\sigma_t(s) = 1$ is equivalent to $\Omega_t(s) = T$.

Recall from Subsection 2.2.7 that $\Omega_1, \Omega_2, ..., \Omega_q$ represent a DM's preference statements listed in order of importance from most to least. For $1 \leq t \leq q$, let $\widetilde{\Psi}_t(s)$ denote the *fuzzy incremental score* of a state $s \in S$ for preference statement Ω_t, defined by

$$\widetilde{\Psi}_t(s) = \frac{1}{2^t}\sigma_t(s). \tag{6.1}$$

Now, let $\widetilde{\Psi}(s)$ denote the *fuzzy score* of a state $s \in S$. Then define

$$\widetilde{\Psi}(s) = \sum_{t=1}^{q} \widetilde{\Psi}_t(s). \tag{6.2}$$

In (6.2), the fuzzy truth values of preference statements at the feasible states, as assumed for preference uncertainty, are taken into account to calculate fuzzy scores for the states. The fuzzy scores of the feasible states will be used in a formula, developed in the following section, to calculate a preference degree for each pair of states.

6.3 Fuzzy Preference Elicitation

The crisp option prioritization, representing a crisp preference ordering of feasible states for a DM lexicographically or by using the scores calculated by employing the Equation 2.3, is a standard ranking methodology if there is no preference uncertainty and the states are assessed according to the preference statements of the DM using Boolean or classical logic. However, because preference uncertainty is the main justification for the assumption of fuzzy truth values in the assessment of preference statements at feasible states, the crisp preference ordering of states using the fuzzy scores as given by (6.2) is not enough to capture the vagueness in preferences. Rather, the cardinal values of the fuzzy scores should be used to identify the preference intensity between two states. A function $r : S \times S \longrightarrow [0, 1]$ is defined below to express this preference information.

$$r(s_i, s_j) = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{\widetilde{\Psi}(s_i)-\widetilde{\Psi}(s_j)}{\max\{|\Psi(s_i)-\Psi(s_j)|,|\widetilde{\Psi}(s_i)-\widetilde{\Psi}(s_j)|\}}, & \text{if } \widetilde{\Psi}(s_i) \neq \widetilde{\Psi}(s_j) \\ 0.5, & \text{otherwise,} \end{cases} \tag{6.3}$$

where for an $s \in S$, $\Psi(s)$ and $\widetilde{\Psi}(s)$ are defined by (2.3) and (6.2), respectively. The following theorem establishes that the function defined by (6.3) represents a fuzzy preference relation over the set of feasible states, S.

Theorem 6.3.1. The fuzzy relation $\mathcal{R} = (r_{ij})_{m\times m}$ on S, with membership function

$$\mu_{\mathcal{R}} : S \times S \longrightarrow [0,1]$$

defined by $\mu_{\mathcal{R}}(s_i, s_j) = r_{ij} = r(s_i, s_j)$, where $r(s_i, s_j)$ is given by (6.3), is a fuzzy preference relation on S.

Proof: First we show that for any $s_i, s_j \in S$, $\mu_{\mathcal{R}}(s_i, s_j) = r_{ij} = r(s_i, s_j) \in [0,1]$.

Let $s_i, s_j \in S$. If $\widetilde{\Psi}(s_i) = \widetilde{\Psi}(s_j)$, then by (6.3), $r(s_i, s_j) = 0.5$. If $\widetilde{\Psi}(s_i) \neq \widetilde{\Psi}(s_j)$, then from the fact that

$$\mid \widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j) \mid \leq \max\{\mid \Psi(s_i) - \Psi(s_j) \mid, \mid \widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j) \mid\},$$

we obtain

$$-1 \leq \frac{\widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j)}{\max\{\mid \Psi(s_i) - \Psi(s_j) \mid, \mid \widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j) \mid\}} \leq 1,$$

and hence,

$$0 \leq \frac{1}{2} + \frac{1}{2}\frac{\widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j)}{\max\{\mid \Psi(s_i) - \Psi(s_j) \mid, \mid \widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j) \mid\}} \leq 1,$$

that is, $0 \leq r(s_i, s_j) \leq 1$. Accordingly, $\mu_{\mathcal{R}}(s_i, s_j) = r_{ij} = r(s_i, s_j) \in [0,1]$, for all $s_i, s_j \in S$.

Next we show that $\mu_{\mathcal{R}}(s_i, s_j) + \mu_{\mathcal{R}}(s_j, s_i) = 1$, for any $s_i, s_j \in S$. For $s_i, s_j \in S$ for which $\widetilde{\Psi}(s_i) = \widetilde{\Psi}(s_j)$, the above identity is obvious, since in this case $r(s_i, s_j) = r(s_j, s_i) = 0.5$, and hence, $\mu_{\mathcal{R}}(s_i, s_j) = \mu_{\mathcal{R}}(s_j, s_i) = 0.5$. For $s_i, s_j \in S$ satisfying $\widetilde{\Psi}(s_i) \neq \widetilde{\Psi}(s_j)$, we obtain that

$$\begin{aligned}
\mu_{\mathcal{R}}(s_i, s_j) + \mu_{\mathcal{R}}(s_j, s_i) &= r(s_i, s_j) + r(s_j, s_i) \\
&= \tfrac{1}{2} + \tfrac{1}{2}\frac{\widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j)}{\max\{|\Psi(s_i) - \Psi(s_j)|, |\widetilde{\Psi}(s_i) - \widetilde{\Psi}(s_j)|\}} + \\
&\quad \tfrac{1}{2} + \tfrac{1}{2}\frac{\widetilde{\Psi}(s_j) - \widetilde{\Psi}(s_i)}{\max\{|\Psi(s_j) - \Psi(s_i)|, |\widetilde{\Psi}(s_j) - \widetilde{\Psi}(s_i)|\}} \\
&= 1.
\end{aligned}$$

Finally, $\mu_{\mathcal{R}}(s_i, s_i) = r_{ii} = r(s_i, s_i) = 0.5$, from (6.3). Hence, $\mathcal{R} = (r_{ij})_{m \times m}$ is a fuzzy preference relation on S. □

Theorem 6.3.2. Crisp option prioritization is a special case of fuzzy option prioritization. Specifically, if the truth value of each preference statement at each feasible state is based on Boolean logic, then preferences over feasible states obtained by employing fuzzy option prioritization are crisp and are the same as would be found by using crisp option prioritization.

Proof: Assume that the truth value of each preference statement Ω_t, $t = 1, 2, ..., q$, at each state in S is based on Boolean logic. Then for any $s \in S$,

$$\sigma_t(s) = \begin{cases} 1, & \text{if } \Omega_t(s) = T \\ 0, & \text{if } \Omega_t(s) = F \end{cases}.$$

Hence, $\widetilde{\Psi}_t(s) = \frac{1}{2^t}\sigma_t(s) = \Psi_t(s)$. Accordingly,

$$\widetilde{\Psi}(s) = \sum_{t=1}^{q} \widetilde{\Psi}_t(s) = \sum_{t=1}^{q} \Psi_t(s) = \Psi(s).$$

For $s_i, s_j \in S$, by crisp option prioritization, $s_i \succ s_j$, or $s_i \sim s_j$, or $s_i \prec s_j$ if and only if $\Psi(s_i) > \Psi(s_j)$, or $\Psi(s_i) = \Psi(s_j)$, or $\Psi(s_i) < \Psi(s_j)$, respectively. Now, if $\Psi(s_i) > \Psi(s_j)$, then since $\widetilde{\Psi}(s_i) = \Psi(s_i)$ and $\widetilde{\Psi}(s_j) = \Psi(s_j)$, we can find from (6.3) that

$$\begin{aligned} r(s_i, s_j) &= \tfrac{1}{2} + \tfrac{1}{2}\tfrac{\widetilde{\Psi}(s_i)-\widetilde{\Psi}(s_j)}{\max\{|\Psi(s_i)-\Psi(s_j)|,|\widetilde{\Psi}(s_i)-\widetilde{\Psi}(s_j)|\}} \\ &= \tfrac{1}{2} + \tfrac{1}{2}\tfrac{\Psi(s_i)-\Psi(s_j)}{\max\{|\Psi(s_i)-\Psi(s_j)|,|\Psi(s_i)-\Psi(s_j)|\}} \\ &= \tfrac{1}{2} + \tfrac{1}{2} = 1, \end{aligned}$$

which is equivalent to $s_i \succ s_j$.

If $\Psi(s_i) = \Psi(s_j)$, then by (6.3), $r(s_i, s_j) = 0.5$, which is equivalent to $s_i \sim s_j$. Finally, if $\Psi(s_i) < \Psi(s_j)$, then by (6.3),

$$\begin{aligned} r(s_i, s_j) &= \tfrac{1}{2} + \tfrac{1}{2}\tfrac{\Psi(s_i)-\Psi(s_j)}{\max\{|\Psi(s_i)-\Psi(s_j)|,|\Psi(s_i)-\Psi(s_j)|\}} \\ &= \tfrac{1}{2} - \tfrac{1}{2} = 0, \end{aligned}$$

which is equivalent to $s_i \prec s_j$. This completes the proof. □

6.4 Application of Fuzzy Option Prioritization to the Elmira Groundwater Contamination Conflict

To demonstrate how the fuzzy option prioritization methodology can be employed to model fuzzy preferences in a real-world multiple participant-multiple objective decision problem, it is applied to the Elmira groundwater contamination conflict described in Section 4.4. Recall from Section 4.4 that in Elmira dispute, the uncertain preferences of U and L were modeled as fuzzy preferences by complicated and time consuming pairwise comparison of states, represented in Table 6.6 by matrices $\mathcal{R}^{\mathrm{U}}$ and $\mathcal{R}^{\mathrm{L}}$, respectively. In this section, the uncertain preferences of U and L are modeled as fuzzy preferences by using the systematic

Table 6.1: Lexicographic Preference Statements and Interpretations for the Ontario Ministry of the Environment (M) in the Elmira Conflict

Preference Statement	Interpretation
$-O_4$	Concerned about the provincial economy, M does not want U to abandon its operations in Elmira.
O_3	M wants U to accept a control order, original or modified.
$-O_2$	M does not want the procedure delayed.
$-O_1$	M prefers that the original control order not be modified.
O_5 IFF $-O_1$	M would like L to support the original control order if and only if it does not modify the order.

fuzzy option prioritization technique developed in this chapter. The methodology also generates a crisp preference for M as expected.

Peng (1999) developed a set of preference statements for each of the DMs, M, U and L, of the Elmira dispute to apply the crisp option prioritization for eliciting preferences. In this study, the preference statements of M, U and L are considered to be the same as in (Peng, 1999), which are presented in Tables 6.1, 6.2 and 6.3, respectively. In these tables, preference statements are listed vertically from most to least important. Recall that crisp option prioritization (Fang et al., 2003; Peng et al., 1997; Peng, 1999) is a methodology that is used to order feasible states lexicographically based on truth values, "true" or "false", of preference statements at each state.

Unlike Peng's assumption in (Peng, 1999), the truth values of some of the preference statements of U and L at some states may not be concluded as exactly "true" or "false" because of the presence of some specific combinations of courses of actions in those states. For Example, according to Boolean logic, the truth value of U's preference statement Ω_2 ($-O_4$, meaning that U prefers not to abandon its operations in Elmira) is "true" at state

Table 6.2: Lexicographic Preference Statements and Interpretations for Uniroyal Chemical Ltd. (U) in the Elmira Conflict

Preference Statement	Interpretation
O_3 IFF O_1	U would accept the control order if and only if it is modified.
$-O_4$	U does not want to abandon its operations in Elmira.
$-O_5$	U does not want L to insist on the application of the original control order.
O_2 IFF $-O_5$	U would like to delay the procedure if and only if L's attitude is softened.

Table 6.3: Lexicographic Preference Statements and Interpretations for Local Government (L) in the Elmira Conflict

Preference Statement	Interpretation
$-O_4$	Concerned about the negative impacts on local economy, L does not want U to abandon its operations in Elmira.
$-O_1$	L prefers that the original control order not be modified.
O_3 IF $-O_1$	L wants U to accept the control order if it is not modified.
O_5 IF O_1	L would insistently ask for the original control order if M plans to modify it.
$-O_2$	L does not want the procedure delayed.
O_5	L insists on the application of the original control order.

s_3 in which U accepts the original control order. However, when U has to accept the original control order, it may not prefer to choose "not abandon" with 100% truth (that is, a degree of truth 1), even if it would rather not abandon; instead, it may prefer to choose "not abandon" with some degree of truth between 0 and 1. A similar argument may be given for L when judging truthfulness of the preference statement Ω_6 (O_5, meaning that L tends to insist on the application of the original control order in any circumstances) at state s_4, where a modified control order is accepted by U without pressure from L. Specifically, if M modifies the original control order and U accepts it, as described by state s_4, L may take into account U's possible threat of abandoning its operations in Elmira, and prefer to assign a non-zero truth degree to Ω_6 at s_4, rather than a truth value of "false" (equivalent to a truth degree of 0) in accordance with Boolean logic.

The above circumstances necessitate the consideration of fuzzy truth values of some preference statements at some states to calibrate preferences of U and L in the Elmira conflict. By taking these and similar situations into account, fuzzy truth values are assigned to the preference statements of U and L at each feasible state, which are presented in Table 6.4. Recall that a truth degree of 1 indicates the absolute truth of a preference statement (which is equivalent to the truth value "true"), while a truth degree of 0 means the absolute falsity (equivalent to the truth value "false"). As is also explained in Section 4.4, M is a provincial authority that looks after the environmental issues of the entire Province of Ontario. Its interest regarding the Elmira conflict may not be as closely connected to the dispute as the more local DMs, U and L, and may therefore be considered to have precisely defined preferences. Hence, by examining the preference statements listed in Table 6.1, one can ascertain that the truth value of each of these statements at each feasible state is Boolean logic-based, "true" or "false", which is also presented in Table 6.4.

In Tables 6.1, 6.2 and 6.3, there are a total of 5, 4 and 6 preference statements for M, U and L, respectively. Since there is exactly one truth degree for one preference statement at a given state, M, U and L have 5, 4 and 6 truth degrees, respectively, at each state. The first, second and third columns of Table 6.4 list these truth degrees as 5-tuples, 4-tuples and 6-tuples, respectively, in which the truth degrees appear in the decreasing order of importance of the preference statements. That is, the first entry of a 4-tuple represents the truth degree of the most important preference statement of U at a state, while the last entry is the truth degree of the least important preference statement. For example, in the 4-tuple $(0, 0.7, 1, 0)$ in the third row and third column of Table 6.4, the first entry 0 is the truth degree of the most important preference statement "O_3 IFF O_1" of U at state s_3, the second entry 0.7 is the truth degree of the second most important preference statement "$-O_4$", and so on.

Table 6.4: Fuzzy Truth Values of the Preference Statements of the Ontario Ministry of the Environment (M), Uniroyal Chemical Ltd. (U), and Local Government (L) in the Elmira Conflict

State (s)	Fuzzy Truth Values of Preference Statements (Most Important to Least) at State s		
	M	U	L
s_1	(1, 0, 0, 1, 0)	(1, 1, 1, 1)	(1, 1, 0, 1, 0, 0)
s_2	(1, 0, 0, 0, 1)	(0.25, 1, 1, 1)	(1, 0, 1, 0, 0, 0)
s_3	(1, 1, 1, 1, 0)	(0, 0.7, 1, 0)	(1, 1, 1, 1, 1, 0.5)
s_4	(1, 1, 1, 0, 1)	(1, 1, 1, 0)	(1, 0.2, 1, 0.7, 1, 0.7)
s_5	(1, 0, 0, 1, 1)	(1, 0.65, 0, 0)	(1, 1, 0, 1, 0, 1)
s_6	(1, 0, 0, 0, 0)	(0.25, 1, 0.4, 0.5)	(1, 0, 1, 1, 0, 1)
s_7	(1, 1, 1, 1, 1)	(0, 0.7, 0, 1)	(1, 1, 1, 0.9, 1, 0.5)
s_8	(1, 1, 1, 0, 0)	(1, 1, 0, 1)	(1, 0.2, 1, 0.5, 1, 0.5)
s_9	(0, 0, 1, 1, 0)	(1, 0.35, 1, 0)	(0, 1, 0, 1, 1, 0)

Now one employs Equation 6.2 to calculate a fuzzy score for each state in S for M, U and L. Next, these fuzzy scores are used in (6.3) to find the fuzzy preference degrees for M, U and L. From the results, it is clear that the preferences of M are crisp, represented by Table 6.5 from most preferred on the left to least preferred on the right, and are the same as found in (Peng, 1999), because the truth values of the preference statements of M at feasible states are assumed to be Boolean. This same preferences of M are also considered in the fuzzy stability analysis of the Elmira dispute in Section 4.4 as well as in the coalition fuzzy stability analysis in Section 5.4. However, the application of Equation 6.3 generates

Table 6.5: Preferences of the Ontario Ministry of the Environment (M) in the Elmira Conflict: Most to Least Preferred

s_7	s_3	s_4	s_8	s_5	s_1	s_2	s_6	s_9

Table 6.6: Fuzzy Preferences of Uniroyal Chemical Ltd. (U) and Local Government (L) in the Elmira Conflict

$$\underline{\mathcal{R}}^{\mathrm{U}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{array} \\ \begin{pmatrix} 0.5 & 0.875 & 1.0 & 1.0 & 1.0 & 0.85 & 1.0 & 1.0 & 0.86 \\ 0.125 & 0.5 & 1.0 & 0.1429 & 0.34 & 0.7833 & 1.0 & 0.1667 & 0.1 \\ 0 & 0 & 0.5 & 0 & 0.0167 & 0 & 1.0 & 0 & 0 \\ 0 & 0.8571 & 1.0 & 0.5 & 1.0 & 0.835 & 1.0 & 1.0 & 0.825 \\ 0 & 0.66 & 0.9833 & 0 & 0.5 & 0.7063 & 0.9857 & 0 & 0.3 \\ 0.15 & 0.2167 & 1.0 & 0.165 & 0.2938 & 0.5 & 1.0 & 0.1833 & 0.1583 \\ 0 & 0 & 0 & 0 & 0.0143 & 0 & 0.5 & 0 & 0 \\ 0 & 0.8333 & 1.0 & 0 & 1.0 & 0.8167 & 1.0 & 0.5 & 0.7667 \\ 0.14 & 0.9 & 1.0 & 0.175 & 0.7 & 0.8417 & 1.0 & 0.2333 & 0.5 \end{pmatrix} \end{array}$$

$$\underline{\mathcal{R}}^{\mathrm{L}} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{array} \\ \begin{pmatrix} 0.5 & 1.0 & 0 & 0.665 & 0 & 1.0 & 0.0409 & 0.93 & 1.0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.0196 & 0 & 1.0 \\ 1.0 & 1.0 & 0.5 & 0.845 & 1.0 & 1.0 & 0.7 & 0.9933 & 1.0 \\ 0.335 & 1.0 & 0.155 & 0.5 & 0.3045 & 1.0 & 0.181 & 0.6 & 1.0 \\ 1.0 & 1.0 & 0 & 0.6955 & 0.5 & 1.0 & 0.045 & 0.9417 & 1.0 \\ 0 & 1.0 & 0 & 0 & 0 & 0.5 & 0.025 & 0 & 1.0 \\ 0.9591 & 0.9804 & 0.3 & 0.819 & 0.955 & 0.975 & 0.5 & 0.95 & 0.989 \\ 0.07 & 1.0 & 0.0067 & 0.4 & 0.0583 & 1.0 & 0.05 & 0.5 & 1.0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.011 & 0 & 0.5 \end{pmatrix} \end{array}$$

fuzzy preferences for U and L as preference degrees for one state over another, which are represented by matrices $\underline{\mathcal{R}}_{\mathrm{U}}$ and $\underline{\mathcal{R}}_{\mathrm{L}}$ in Table 6.6.

As is also mentioned earlier, fuzzy preferences of U and L are modeled in Section 4.4 as matrices $\mathcal{R}_{\mathrm{U}}$ and $\mathcal{R}_{\mathrm{L}}$ by pairwise comparison of feasible states for use in the fuzzy stability analysis of the Elmira dispute. One can verify that the fuzzy preferences $\underline{\mathcal{R}}_{\mathrm{U}}$ and $\underline{\mathcal{R}}_{\mathrm{L}}$, as obtained here, are very close to $\mathcal{R}_{\mathrm{U}}$ and $\mathcal{R}_{\mathrm{L}}$, respectively. In particular, with the same sets of FSTs, as considered in Section 4.4, fuzzy stability results are also the same. As the objective of this chapter is to develop an efficient technique to model fuzzy preferences of DMs involved in a dispute and not to make an analysis, the details of the fuzzy stability results are not shown here.

It may be mentioned that it was hard to model the fuzzy preferences of U and L in Section 4.4 by comparing the states pairwise. However, the fuzzy option prioritization methodology developed in this chapter can do this job without difficulty, and the fuzzy preference outputs are very close to those obtained by a pairwise comparison of states. It follows that the new methodology can utilize human judgements on preference statements at feasible states efficiently and generate a fuzzy preference that is reasonably close to the one obtained by systematic but tedious pairwise comparison of states. Since the truth values of preference statements of M at feasible states are all based on Boolean logic, the methodology provides a crisp preference ordering of states for M, exactly the same as in Peng (1999) from crisp option prioritization.

6.5 Summary

The fuzzy option prioritization methodology is developed to aid the modeling of uncertain preferences as fuzzy preferences within the framework of the GMCR. Equation 6.1 is introduced to represent fuzzy incremental score of a state for a preference statement while (6.2) gives the overall fuzzy score of a state for a DM, both capturing preference uncertainty via fuzzy truth values of the DM's preference statements at feasible states. The fuzzy scores of states are then utilized to define a fuzzy relation in (6.3), which is established later as a fuzzy preference relation. It is also proved that the fuzzy option prioritization methodology generalizes the existing crisp option prioritization technique in the sense that the crisp option prioritization is a special case of the fuzzy option prioritization.

When applied to the Elmira groundwater contamination conflict, the methodology not only models fuzzy preferences for the DMs efficiently so that they are close to those that were obtained by detailed human pairwise comparison of states, but also accomplishes it without difficulty. Of course, fuzzy preferences may be modeled by pairwise comparison of states for small problems. But, for larger conflicts, modeling fuzzy preferences by pairwise comparison of states is unrealistic and may be impossible. However, fuzzy option prioritization can be applied to a conflict model of any size to model DMs' fuzzy preferences. The methodology is based on simple calculations and can be easily implemented using a small computer program.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

The key to the applicability of the GMCR is that it contains both modeling and analysis components. Many real-world conflict decision problems exhibit preference uncertainty, but until now there was no suitable methodology to model and analyze them. This thesis has remedied this problem.

Together with the non-cooperative form of fuzzy stabilities, the concepts of coalitional form of fuzzy stabilities are put forward to provide follow-up analysis of a dispute. Addressing the difficulties of modeling fuzzy preferences from uncertain preference information, the fuzzy option prioritization methodology is developed within the framework of the GMCR to efficiently model fuzzy preferences of DMs for use in the graph model fuzzy stability analysis. The main contributions of this study are summarized below:

1. A fuzzy preference framework for a two-DM graph model is developed to introduce

fuzzy stability concepts and apply them to simple conflicts with two DMs who have uncertain preferences over feasible states. Specifically,

- The concept of an FRCP is introduced to characterize preference intensity between two feasible states in the presence of preference uncertainty (Section 3.2).
- A parameter called "FST" is introduced to take into account the interacting DMs' satisficing behavior in strategic conflicts, and is incorporated into the graph model fuzzy stability definitions (Section 3.3).
- To identify states to which a DM would be willing to move to, the concept of an FUI for a DM is put forward (Section 3.4).
- The four basic crisp stability definitions for a two-DM graph model—*R*, *GMR*, *SMR*, and *SEQ*—are generalized as *FR*, *FGMR*, *FSMR*, and *FSEQ* to facilitate the (fuzzy) stability analysis of a two-DM graph model with uncertain preference information (Section 3.5).
- The fuzzy stability definitions for a two-DM graph model are illustrated using the sustainable development conflict. It is found that the developers' satisficing behavior greatly influences the fuzzy stability results, which is quite reasonable because developers' decisions in development activities are volatile and mostly context-dependent. Environmental agencies' roles in choosing how strictly to enforce environmental regulations may guide developers' satisficing behavior. If developers want to grab every last penny, environmental security will remain under threat and environmental disasters can never be completely avoided (Section 3.6).

2. The fuzzy preference framework for the two-DM graph model is extended to ac-

commodate graph models with any number of DMs, generalizing the fuzzy stability definitions. More specifically,

- The concept of an FUI by a group or coalition of DMs is introduced for use in the fuzzy stabilities (Section 4.2).
- *FGMR*, *FSMR*, and *FSEQ* stability definitions for a two-DM graph model are extended for a general n-DM ($n \geq 2$) graph model. In these definitions, fuzzy stabilities for a DM depend on the responses (credible or not) of the coalition of the remaining DMs, rather than the single opponent DM of the two-DM case. Since *FR* stability does not depend on the responses by the opponent(s), the *FR* stability definition for a general n-DM graph model remains the same as in the two-DM case (Section 4.3).
- A fuzzy stability analysis is carried out on the Elmira groundwater contamination conflict by employing the fuzzy stability definitions for an n-DM graph model. This analysis predicts a new strong equilibrium (state s_4), indicated in Table 4.5, that is a possible resolution under all four fuzzy stability definitions. The fuzzy stability results tableau provided in Table 4.4 demonstrates how a DM's satisficing criteria can affect the final outcome (Section 4.4).
- The FGM is shown to be a more general approach to decision making under conflict compared to the crisp graph model, as it can handle both certain and uncertain (fuzzy) preferences. Hence, the FGM constitutes an extension of the crisp graph model that permits the modeling and analysis of more realistic multiple participant-multiple objective decision problems. In particular, by setting the FST of each DM to 1.0, FGM becomes the crisp GMCR (Remarks 3.4.4, 4.2.2, and Theorem 4.3.6).

- Like the crisp GMCR in which the associated preferences can be transitive or nontransitive, the FGM is developed such that any transitive or nontransitive fuzzy preferences can be utilized in the fuzzy stability calculations (Chapters 3 and 4).

3. The coalition fuzzy stability concept is developed as a follow-up analysis technique within the FGM, intending to further analyze the individual level fuzzy stabilities of a conflict model with uncertain preference information. In particular,

   - The concepts of CFI and CCFI are introduced as tools for facilitating the coalition fuzzy stability definitions (Section 5.2).
   - The coalition fuzzy stability definitions for *R*, *GMR*, *SMR*, and *SEQ* stability concepts of the GMCR are developed so that they constitute a natural generalization of the individual level *FR*, *FGMR*, *FSMR* and *FSEQ* stabilities (Section 5.3).
   - Coalition fuzzy stability definitions can be applied to a crisp graph model by assigning an FST of 1.0 to each DM, thereby making them more general coalition analysis tools within the graph model structure. Accordingly, the four coalition fuzzy stability definitions—*CFR*, *CFGMR*, *CFSMR*, and *CFSEQ*—form a strong solution methodology for strategic conflicts with both certain and uncertain preference information. Although the implementation of these stabilities is not straightforward, a suitable decision support system could bring this capability to the fingertips of the DM and the analyst (Section 5.3).
   - A coalition fuzzy stability definition for a DM identifies states from which neither the DM himself or herself, nor any of the coalitions that he or she can

join, would like to move away. These characteristics are justified regarding a particular DM, M, when the coalition fuzzy stability definitions are applied to the Elmira groundwater contamination conflict. Specifically, some states fail to be coalition fuzzy stable for M that were fuzzy stable with respect to non-cooperative fuzzy stability definitions. Accordingly, the coalition fuzzy stability analysis may narrow down the list of individual-level fuzzy stabilities, thereby providing the analyst with valuable strategic insights into the conflict under study (Section 5.4).

- It can also be concluded from the application that the possible evolution of a conflict from a status quo state to a final outcome can be conveniently explained using CFILs. Therefore, as an analysis tool to augment individual-level fuzzy stabilities, coalition fuzzy stability analysis constitutes an important addition to the FGM (Section 5.4).

4. The fuzzy option prioritization methodology is developed within the FGM structure to facilitate the modeling of fuzzy preferences for DMs involved in a strategic conflict with uncertain preference information. Specifically,

    - Fuzzy option prioritization is the first formal methodology to model a fuzzy preference within the GMCR framework in order to deal with uncertain preferences. This technique offers flexibility to DMs or analysts who can assume the intensity of truth of a preference statement at a feasible state to be any number between 0 and 1, referred to as a fuzzy truth value (Chapter 6).

    - A fuzzy preference relation over the set of feasible states is constructed by taking into account the fuzzy truth values of preference statements at feasible states

(Sections 6.2 and 6.3).

- The fuzzy option prioritization methodology generalizes the existing crisp option prioritization technique in the sense that crisp option prioritization is a special case of fuzzy option prioritization (Theorem 6.3.2).
- When applied to the Elmira groundwater contamination conflict, the methodology models fuzzy preferences for the DMs efficiently so that they are close to those obtained by a complicated human assessment based on pairwise comparison of states (Section 6.4).
- For larger problems, modeling fuzzy preferences by pairwise comparison of states is unrealistic and may be impossible. However, the fuzzy option prioritization methodology can be applied to a dispute of any size without difficulty (Chapter 6).
- Since the FGM is developed in this research to study multiple participant-multiple objective decision problems by carrying out fuzzy stability analysis and coalition fuzzy stability analysis based on DMs' fuzzy preferences, the introduction of the fuzzy option prioritization methodology, an efficient tool to model fuzzy preferences, will make the FGM more useful (Chapters 3, 4, 5, and 6).

7.2 Future Work

The FGM methodology developed in this PhD thesis is a complete fuzzy preference approach for both modeling and analyzing real-world multiple participant-multiple objective decision problems with certain or uncertain preference information for DMs. However, fuzzy stability, introduced in this thesis, is a new concept and may therefore be integrated with recent developments and initiatives within the framework of the GMCR. A number of directions for potential future research is listed below.

- *Matrix Representations of the Fuzzy Stability Definitions*: A recent addition to the GMCR is a matrix representation of the graph model solution concepts for easy computer coding and manipulation (Xu et al., 2009a,b, 2011). This idea may be adapted to FGM to represent fuzzy stability definitions.

- *Fuzzy Stabilities with Transitive Fuzzy Preferences*: The fuzzy stability definitions proposed in this thesis are based on fuzzy preferences that are not restricted by any transitivity property. However, various transitivity properties may be imposed on fuzzy preferences to study their implications for fuzzy stability.

- *Fuzzy Status Quo Analysis*: Status quo analysis technique (Li et al., 2004b, 2005a,b) was developed within the crisp GMCR to inspect whether a potential resolution (i.e., an equilibrium state) is attainable from the status quo state, and to analyze how DMs may act and interact to direct a conflict to that attainable resolution. A fuzzy version of the status quo analysis technique may be developed within the FGM to keep track of the evolution of a conflict from the status quo state to a *FE*.

- *Fuzzy Stability Definitions for Other Stability Types*: The fuzzy stability definitions introduced in this study are *FR*, *FGMR*, *FSMR*, and *FSEQ*. However, fuzzy stability definitions for other graph model stability concepts, such as *limited move* and *non-myopic* stabilities, may be defined by imposing appropriate transitivity property on DMs' fuzzy preferences.

- *Use of Other Techniques to Model Fuzzy Preferences*: Fuzzy option prioritization, developed in this thesis, is the only methodology to formally model fuzzy preferences for DMs with uncertain preference information in a graph model, based on fuzzy truth values of preference statements at feasible states. Other techniques may be developed to represent DMs' fuzzy preferences by taking into account other uncertain information about preferences, such as fuzzy utilities, fuzzy option weighting, and fuzzy multi-criteria decision making.

- *Use of Other Types of Fuzzy Preferences*: There may be a fuzzy preference relation in which the additive reciprocity condition is not met; that is, there may be a certain degree for a pair of states to which a DM does not prefer one state of the pair over the other. Techniques may be developed within the GMCR framework to handle this type of fuzzy preference relation.

- *Decision Support System for* FGM: Calculating various graph model fuzzy stabilities by hand is tedious even for a small model. Therefore, the design of a suitable decision support system for the FGM may be an important future project.

- *Applications to Challenging Real-World Conflicts*: FGM can be applied to many challenging real-world disputes to gain strategic insights.

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