Dynamic pricing under demand uncertainty in the presence of strategic consumers

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

We study the effect of strategic consumer behavior on pricing, inventory decisions, and inventory release policies of a monopoly retailer selling a single product over two periods facing uncertain demand. We consider the following three-stage two-period dynamic pricing game. In the first stage the retailer sets his inventory level and inventory release policy; in the second stage the retailer faces uncertain demand that consists of both myopic and strategic consumers. The former type of consumers purchase the good if their valuations exceed the posted price, while the latter type of consumers consider future realizations of prices, and hence their future surplus, before deciding when to purchase the good; in the third stage, the retailer releases its remaining inventory according to the release policy chosen in the first stage.

Game theory is employed to model strategic decisions in this setting. Each of the strategies available to the players in this setting (the consumers and the retailer) are solved backward to yield the subgame perfect Nash equilibrium, which allows us to derive the equilibrium pricing policies.

This work provides three primary contributions to the fields of dynamic pricing and revenue management. First, if, in the third stage, inventory is released to clear the market, then the presence of strategic consumers may be beneficial for the retailer. Second, we find the optimal inventory release strategy when retailers have capacity limitation. Lastly, we numerically demonstrate the retailer's optimal decisions of both inventory level and the inventory release strategy. We find that market clearance mechanism and intermediate supply strategy may emerge as the retailers optimal choice.

Acknowledgements

It has been my good fortune to have the advice and guidance of many talented people, whose knowledge and skills have enhanced this thesis in many ways. I would like to thank my supervisor, Dr. Benny Mantin, for his insightful guidance in advising me with this thesis. Without his support and encouragement, my contribution to this thesis would not have been possible. Secondly, I would like to thank Dr. Stanko Dimitrov and Dr. Qi-Ming He for their careful reviewing and constructive criticism of this thesis.

Dedication

I would like to thank my loving family and my friends. This work would not have been possible without their continuous support, both financially and emotionally. They have been tremendously caring and understanding throughout all these years. It was a long haul, but it is finally done.

Table of Contents

List of Figures

Chapter 1

Introduction

1.1 Background

In recent years, there has been an increasing interest in research on dynamic pricing and revenue management. Numerous papers have studied revenue management, or markdown pricing; e.g., Lazear (1986), Gallego and Van Ryzin (1994), Feng and Gallego (1995). The structure of a markdown mechanism influences buyer behavior, and in turn, the seller's profits. Lazear (1986) employs a two step markdown mechanism. The latter two study demand in a Poisson process with known intensity where they show that prices are not necessarily decreasing.

The mainstream literature on dynamic pricing has focused, thus far, on managing clearance prices: finding the optimal timing and magnitude of the discount price along the selling horizon (see, e.g., Lazear 1986, Feng and Gallego 2000, Aviv and Pazgal 2008, Zhang and Cooper 2008). As research has been maturing in the field of dynamic pricing (DP) and revenue management (RM) employing passive demand, researchers have shifted their focus on to modeling strategic consumer behavior. The general result is that the presence of these consumers is detrimental to firms (Besanko and Winston 1990, Aviv and Pazgal 2008). Besanko and Winston (1990) demonstrate that the retailer's profit decreases as a result of mistakenly treating forward-looking consumers as myopic. It has been suggested that strategic consumer behavior suppresses the benefits of segmentation under medium to high values of heterogeneity and modest rates of decline in valuations. Also the seller cannot effectively avoid the adverse impact of strategic behavior even under low levels of initial inventory (Aviv and Pazgal 2008).

Recently, some researchers have tried to show how to mitigate strategic consumers' behavior (Liu and van Ryzin 2008, Cachon and Swinney 2009, Levin et al. 2009, Aviv, Levin and Nediak 2010). Liu and van Ryzin (2008) demonstrate that rationing can be a profitable strategy to influence the strategic consumers' behavior. Cachon and Swinney (2007) study the additional value of quick response to mitigate the negative consequences of strategic purchase behavior of customers. Levin et al. (2009) demonstrate that the initial capacity can be used together with the appropriate pricing policy to effectively reduce the impact of strategic consumer behavior, when the initial capacity is a decision variable. Aviv, Levin and Nediak (2010) study whether price matching of either internal or external type can lead to a decrease in strategic waiting by the consumers. Our approach is to consider a clearance sales mechanism, similar to the market clearance mechanism employed in Lee and Whang (2002), Cachon and Kők (2007), but with pricing decisions instead of studying the news vendor model. Lee and Whang (2002) investigate the impacts of a secondary market where retailers can buy and sell excess inventories. Cachon and Kők (2007) use the clearance sales mechanism to determine the salvage value in the newsvendor setting.

1.2 Problem of Interest

We study the effect of strategic consumer behavior on pricing, inventory decisions and inventory release policies of a monopoly retailer selling a single product over two periods under uncertain demand. Under this premise, the retailer offers opportunity to consumers that price in period 2 could be lower than the posted price in period 1; while, at the same time, consumers face the risk that the price may increase in period 2. We are interested in the benefits that such a mechanism can offer the firm, which can be a better segmentation of consumers to price discriminatively and/or a significant mitigation of the uncertain demand to capture more profit. Moreover, we seek to find whether the clearance sales strategy is always the optimal choice in the context of other inventory release strategies. Our primary goal in this paper is to address the optimal inventory release strategy when capacity is know and, ultimately, when the retailer can set the optimal inventory at the beginning of the selling horizon under uncertain demand.

In our two period selling horizon setting, we assume that a monopoly retailer satisfies first period demand[1] but considers the following two inventory release policies for the second

[1]This is not a binding constraint under most circumstances, and we find it to be binding in a limited interval where the entire inventory is depleted in period 1. Therefore, the results are consistent with the assumption. Liu and van Ryzin (2008) adopted a similar assumption that demand in the first period is satisfied.

period: Clearance Sale strategy (CS): all leftover inventory is released to the market and the market determines the price; Dynamic Pricing strategy (DP): the leftover inventory is released to maximize the second period profit by ignoring the inventory constraints. We study the Clearance Sales and Dynamic Pricing strategies separately. The latter policy gives rise to another instance, which we refer to as Intermediate Supply (IS), which occurs if the inventory level is at some intermediate level. Specifically, under Intermediate Supply strategy, if demand is low, the leftover inventory is released to maximize the profit and if demand is high, the entire leftover inventory is released.

In the model, a fraction of consumers are strategic and time their purchases to maximize their own expected surplus. Consumers may respond strategically to a retailer's pricing decisions, while a myopic consumer acts impulsively and purchases the good if his immediate surplus is positive. Therefore, there is a need to incorporate consumer responses in the decision-making process. Similar to Cachon and Swinney (2009), we assume that while myopic consumers visit the store in the first period only, strategic consumers may return in the second period, and that these strategic consumers possess a discounting factor associated with their patience level (Levin et al. 2009) or with their reduced discounted surplus obtained due to the delayed purchase. The number of consumers that arrive in the first period could be either high or low. Though this information is common knowledge, the strategic consumers, when they arrive at the store in the first period, do not know the realization of demand until the end of the first period.

Our work addresses the following research questions: (i) Given an inventory level, which inventory release policy should the retailer choose under uncertain demand? (ii) If the retailer can set both the inventory level and the inventory release policy, which inventory release policy should the retailer choose under uncertain demand? (iii) Is the retailer always better off with lower strategic consumers' patience levels? (iv) Is the retailer always better off if fewer strategic consumers exist in the market?

1.3 Overview of Results and Organization of the Thesis

By employing Clearance Sale strategy, the price that the retailer charges in the second period may be higher, or lower, than the price posted in the first period. Specifically, if the demand in the first period is high, more of the inventory is depleted and with lower remaining inventory, the second period price could be higher than the first period price. Strategic consumers, when they decide on their purchase timing, should account for the possibility of the second period price being higher, as this price could very well exceed their valuations (and they may end up not purchasing at all).

Each of the policies is solved backward to yield the subgame perfect Nash equilibrium, which allows us to derive the equilibrium pricing policies. We find that for certain parameters' range, the retailer may price to skim high-valuation consumers of both consumer types (myopic and strategic) in the first period; otherwise, the retailer will price to skim only high-valuation myopic consumers in the first period, while deferring all strategic consumers to the second period. Quite trivially, in the Dynamic Pricing strategy, we also find that the prices charged in period 2 are less than the selling price in period 1 under a certain parameter range.

The results of this study indicate that, given the inventory level, a retailer's profit function is strictly concave in the inventory quantity stocked under feasible region for both Clearance Sales (CS) strategy and Intermediate Supply (IS) strategy, which guarantees an optimal stocking decision under both Clearance Sales and Intermediate Supply strategies. Thus, the retailer's profit function is piecewised concave in Clearance Sales and Intermediate Supply strategies, and linear in Dynamic Pricing strategy. When the retailer can set both the inventory level and inventory release policy, it appears that the retailer sets a rather low inventory level. By employing a low inventory level, the retailer can charge high prices in the first period, and still retain strategic consumer demand in the second period.

Consistent with some previous studies, we also find that more myopic consumers existing in the market is beneficial for the retailer, but only when the number is above a certain threshold. We find that optimal stocking and corresponding profit may decrease as the proportion of myopic consumers increases under both Clearance Sales and Dynamic Pricing strategies. Indeed, with fewer strategic consumers, the retailer's potential profit from the second period diminishes (recall that only strategic consumers may delay their purchase to the second period). The retailer cares less about the strategic consumers and seeks to defer them all to the second period, while focusing attention on first period demand stemming from myopic consumers. Yet, the retailer stocks a lower inventory level and sets a higher first period price (partially to divert strategic consumers to the second period). Ultimately, this results in less profit for the retailer. Also, we further prove that, most of the time, the retailer is better off with less patient strategic consumers under a simplifying condition (high demand and low demand occur with equal probabilities, and the magnitude of low demand is half of that of high demand).

In the Dynamic Pricing strategy, the analysis reveals that the choice of inventory level determines the corresponding inventory release in the second period. If the inventory level is high, leftover inventory is released optimally in the second period; if the inventory level is intermediate, then Intermediate Supply strategy follows, and eventually, if inventory is low, Clearance Sales emerges. The choice of stocking level depends on the optimal profit in each of these outcomes. This strong relationship between the inventory release strategy and inventory level is demonstrated, which solves our first research question.

Lastly, we compare the profit functions under the different inventory release strategies in order to characterize the conditions under which each policy is preferred by the retailer when he can also determine the stocking level. Since it is very difficult to compare analytically the profit functions under the three inventory release strategies, we resort to numerical examples. Our numerical analysis suggests that the retailer is always better off with Intermediate Supply strategy than with Dynamic Pricing strategy, and the retailer is mostly better off employing an Intermediate Supply strategy if he can make decisions about both inventory level and an inventory release strategy. A few examples also show that the possibilities of a Clearance Sales strategy dominate sometimes when inventory costs and/or consumers' patience levels are high.

The remainder of the paper is organized as follows. Section 2 lays out the theoretical dimensions of the research and looks at the related literature. The modeling framework and assumptions are set up in Section 3 and the preliminary analysis is conducted in Section 4. Section 5 studies the Clearance Sale strategy. Section 6 analyzes Dynamic Pricing and Intermediate Supply strategies and integrates the different policies to reveal the Optimal Release strategy. Section 7 summarizes.

Chapter 2

Literature Review

A considerable amount of work has been conducted in the area of profit management and dynamic pricing (e.g., reviews by Bitran and Caldentey 2003, Elmaghraby and Keskinocak 2003, Talluri and van Ryzin 2004, Chan et al. 2004, and Shen and Su 2007). For example, in their book, Talluri and van Ryzin (2004) provide an extensive review of revenue management with seat inventory and capacity-planning problems. However, this review focuses almost exclusively on myopic consumers, and strategic consumers are considered only if the firm is not capacity constrained. They state that "some industries use price-based RM (retailing), whereas others use quantity-based RM (airlines).

Even in the same industry, firms may use a mixture of price- and quantity-based RM. For instance, many of the RM practices of the new low-cost airlines more closely resemble dynamic pricing than the quantity-based RM of the traditional carriers" (p. 176); Shen and Su (2007) review previous models of customer behavior in the revenue management and auction literature; Elmaghraby and Keskinocak (2003) focus on dynamic pricing in the presence of inventory considerations, where they pointed out that the increased availability of demand data, the ease of changing prices as a result of new technologies and the availability of decision-support tools for analyzing demand data are the three factors which contributed to this phenomenon. In our work, a mixture of price- and quantity-based Profit Management is studied uner demand uncertainty.

At the passive demand research has been maturing in the field of dynamic pricing (DP) and revenue management (RM), researchers shift their focus to modeling strategic consumer behavior. And two types of behaviors are analyzed together, which are myopic and strategic behaviors. The general result is that the presence of this type of consumers is detrimental to firms (Besanko and Winston 1990, Aviv and Pazgal 2008). Modeling strategic consumers' behavior can be traced back to Coase (1972). Gallego and van Ryzin (1994) characterized the optimal pricing policy in the presence of strategic consumers. Elmaghraby et al. (2002) analyze the optimal design of a markdown pricing mechanism in the presence of strategic consumers. Zhou, Fan and Cho (2009) focus on the optimal purchasing strategy of a single strategic consumer, and they numerically find that strategic behavior may benefit the retailer. Aviv and Pazgal (2008) study the optimal pricing of a finite quantity of a fashion-like seasonal good in the presence of forward-looking (strategic) customers. They consider two classes of pricing strategies in their paper: contingent and announced fixed-discount. We demonstrated that the behavior may actually benefit retailers.

Some contributes have tried to show how to mitigate such behavior (Liu and van Ryzin 2008, Levin et al. 2009, Aviv, Levin and Nediak 2010). Our approach is to consider the clearance sales mechanism, similar to the market clearance mechanisms demonstrated in Lee and Whang (2002) and Cachon and Kők (2007). In their works, the clearance mechanism was used to determine the salvage value in the newsvendor model, but the researchers make no price decisions. Lee and Whang (2002) consider two interdependent effects which are a quantity effect (sales by the manufacturer) and an allocation effect. Cachon and Kők (2007) highlight the importance of understanding how a model can interact with its own inputs.

Continuous updating of prices over time is not a practical pricing policy in the view of Gallego and van Ryzin (1994) and Bitran and Mondschein (1997). While periodic pricing policies are employed by Lazear (1986) and Elmaghraby et al. (2002) where prices are updated at fixed time intervals, Gallego and van Ryzin (1994) set the price to the "optimal" fixed price to maximize the retailer's profit. In addition, clearance pricing has been studied extensively in the operations literature. Here we do not attempt to provide a complete review. Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003) provide surveys in this field. Zhang and Copper (2008) study the effect of strategic consumer behavior on pricing and rationing decisions of a firm selling a single product over two periods. We model a retailer charge a single price in each period, with two price options in period 2 depending on the demand level.

Numerous studies have attempted to explain the markdown mechanism (e.g., Lazear 1986, Smith and Achabal 1998, Gupta et al. 2006, Zhang and Cooper 2008). Even though they all refer to the inventory release policy as a clearance sale, their approach is not a market clearance mechanism as employed in this work. Lazear (1986) and Smith and Achabal (1998) try to move merchandizes at a price significantly lower than its original price in their studies. In the studies of Gupta et al. (2006) and Zhang and Cooper' s study (2008), the seller may limit the availability of the product in the clearance period. In our model, the retailer may only charge a lower price in latter period of the selling period if the demand level is low.

This paper shows more about that prices may in fact increase, due, e.g., to limited supply, and the choice of inventory release policy of retailers. Elmaghraby and Keskinocak (2003) have also considered whether prices are allowed to increase over time depending on the underlying modeling assumptions: "current research suggests that prices either decrease over time (Lazear 1986) or prices move both up and down (Bitran and Mondschein 1997, Feng and Gallego 1995, Gallego and van Ryzin 1994)." Possible explanations for the rise in price are the stochastic arrival of consumers or the poisson arrivals of consumers under continuous time and the fact that high valuation consumers arrive later during the selling season. Although some research has been carried out on this topic, no single study exists which adequately covers the possibility that limited supply might be one of the reasons for price rises over time.

Several attempts have been made to study the pricing decisions using different consumer arrival processes. A deterministic model is studied by Elmaghraby et al. (2002) where all consumers arrive at the market at the beginning of the selling season with known valuations. Lazear (1986) uses a simplified model of the stochastic demand. Gallego and van Ryzin (1994) formulate the problem using intensity control (Bremaud 1980) and study the optimal dynamic pricing strategy using different demand functions, including exponential arrival of families, general linear function and compound poisson function. Feng and Gallego (1995) and Aviv and Pazgal (2008) model the demand as a homogenous (time-invariant) poisson process. Su (2007) model the demand function as the continuous deterministic arrival of consumers.

Demand is one of the most important elements that influences pricing decisions, a fact that also explains reactions to price changes and other factors. The variety of products in the market has significantly risen, while the product life cycles have become shorter in the last decades. It is even complicated for the retailer to make inventory decisions in advance if demand is uncertain. Most studies in the field of dynamic pricing have focussed only on considering initial inventory decisions as exogenously determined. The following literature consider initial inventory decision as an object decision the retailer must make. Cachon and Swinney (2009) study the pricing and stocking decisions by a monopolist facing myopic consumers, bargain-hunters, and strategic consumers. Liu and van Ryzin (2008) focus on capacity rationing to induce early purchases. Smith and Achabal (1998) and Mantrala and Rao (2001) study initial inventory decisions as well as markdown pricing decisions before the selling season in the absence of strategic consumers. We consider the optimal inventory release strategy with and without setting the initial inventory decision as an object decision.

There may be many reasons for consumers to delay their purchase decisions. Most of the research to date has tended to focus on the possibility of lower prices rather than the uncertain demand. Fay and Xie (2008) define a unique type of product or service offering, termed probabilistic goods, and they analyze the probabilistic selling strategy under capacity constraints and demand uncertainty. They model a seller offering two-component products with unknown consumers' preference. Xie and Shugan (2001) consider a two-period model where N consumers arrive in each period. They modify the demand uncertainty based on the uncertain valuation of consumer based on consumption states. Similarly, Swinney (2010) addresses the practice of matching supply and demand in the presence of strategic consumers when product value is uncertain. Consumers do not know their private valuation for the product before the selling season in his study. This kind of uncertainty is resolved by time alone, and each consumer exogenously learns his value of the product at a random time during the selling season. In our work, consumers may delay their purchase due to the possible lower price resulting from low demand level.

Even though a considerable amount of literature has been published in the area of revenue management and dynamic pricing, most studies in this field have focussed only on a single inventory release strategy, without considering other strategies. Therefore, our research mainly differs from past research in several aspects. First, a mixture of price- and quantity-based Profit Management is studied under demand uncertainty. Second, the combination decision of the inventory level and inventory release strategy is demonstrated.

Chapter 3

Model Setup

We consider a three-stage game: in the first stage, inventory decisions are made by a monopoly retailer, whereas in the second and third stages, the retailer faces consumers. Specifically, in the first stage, the retailer stocks K units of an item, which will be available for sale during the two-period sale, at a inventory cost c per unit. At this stage, the retailer also chooses the inventory release policy, which will be discussed later on. We assume that the inventory can not be replenished during the selling horizon and that the retailer has pricing flexibility. We assume that N_i consumers arrive simultaneously at the beginning of the sale season, in the spirit of early papers by Stokey (1979) and Besanko and Winston (1990), and each buys at most one unit of the product.

In the first stage a monopolistic retailer chooses the inventory release policy based on the parameters' range and/or sets its inventory level; in the second stage the retailer faces uncertain demand that consists of both myopic and strategic consumers. The former type of consumers purchase the good if their valuations exceed the posted price, while the latter type of consumers consider future realization of prices, and hence their future surplus, before deciding when to purchase the good. Namely, strategic consumers time their purchases to maximize expected surplus, in that they may decide to postpone their purchases if they believe that a later purchase may bring a higher expected surplus than what they can gain from an early purchase in period 1; in the third stage, after demand uncertainty is resolved in the first period, the retailer releases its remaining inventory according to the release policy stated in stage one. Figure 3.1 shows the timeline of this dynamic pricing game.

Each consumer has an individual maximum willingness to pay for the product Throughout the paper, we use the term valuation, denoted by v, to refer to each consumer's maximum willingness to pay. At the beginning of the selling season, consumers are certain about their valuations of the product, v, which are independently drawn from a uniform distribution between 0 and 1. The valuations of consumers are fixed, and all consumers are risk neutral. Strategic consumers are rational and solve the problem faced that retailer, hence they can develop expectations about the future price which will be consistent with realized outcome.

The consumers do not only have different valuations, but they are also heterogeneous along the dimension of behavior type (i.e., strategic or myopic). In our model, a fraction α of the consumers are myopic, and the remaining consumers, $1 - \alpha$, exhibit strategic behavior. We assume that a consumer's myopic or strategic behavior is independent of his valuation in period 1. Myopic consumers behave impulsively, and make purchases as long as their valuations are higher than the selling price posted in period 1, R_1. Myopic

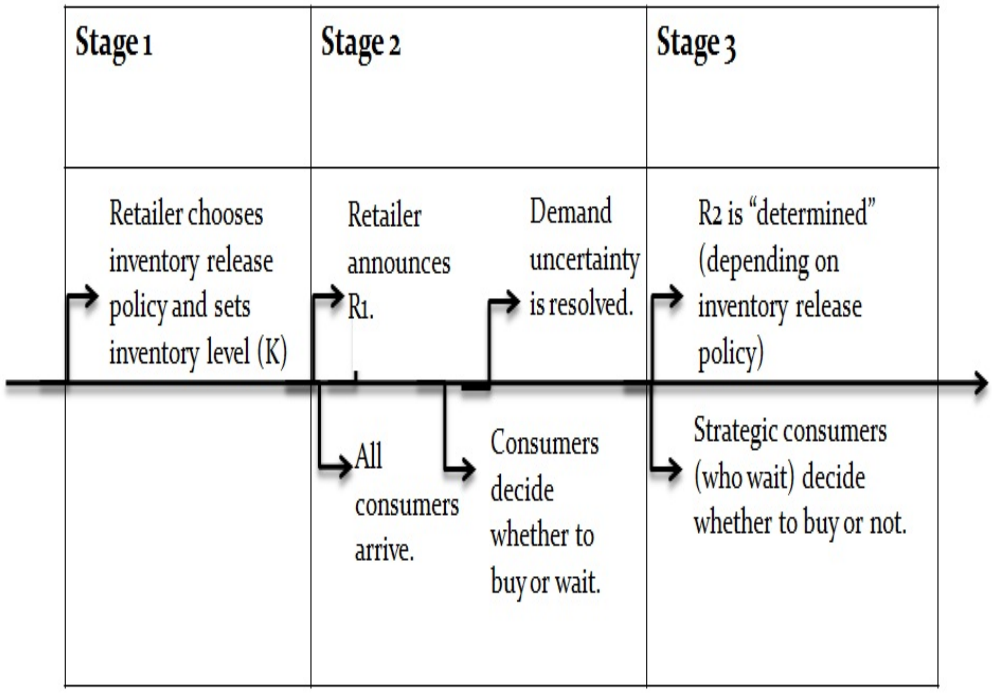

Figure 3.1: Timeline of the three-stage two-period dynamic pricing game.

consumer behavior allows the retailer to ignore any detrimental effects of future price cuts on current consumer purchases. Similar to Cachon and Swinney (2009), we assume that only strategic consumers may return in period 2, and the difference is that we ignore the third type proposed in their paper, bargain hunter, who only pursue the discounting product offered in period 2. A discounting factor referred to as patience level, δ, is applied to consumers' surplus if they choose to wait and purchase in the second period. Namely, while their surplus in period 1 is, $v - R_1$, their discounted surplus obtained in period 2 is $\delta(v-R_2)$. Similar to Cachon and Swinney (2009), we assume that only strategic consumers may return in the second period, except that only two behavior types of consumers exist in the market without the bargain hunter type in their paper. Additionally, consumer composition is stationary over time.

In the first period, all consumers arrive, and N refers the number of consumers. In the second period, a clear or an optimal release in the market mechanism takes place. When demand arrives, it could be N_H (that is, high demand) with probability p or N_L with probability $1 - p$, which are denoted as N_i, $i \in \{H, L\}$. Since not all consumers have the capability to purchase, the actual sales for the product depend upon the price of the product. In this setting, without loss of generality, we normalize N_H to 1 and clearly the capacity K is less or equal to one. Assuming that the actual demand can be satisfied based on the initial inventory decision and let D_{it} $(\leq K)$ denote the realized demand under state i, $i \in \{H, L\}$, in period $t, t \in \{1, 2\}$. Demand for the product in period 1 depends upon the price of the product. As illustrated by Elmaghraby and Keskinocak (2003), it is important for the retailer to capture the information from consumers' side to charge the appropriate price, like current customer values of the product and the future demand. We assume that there is no opportunity for inventory replenishment during the selling horizon.

$$N_i = \begin{cases} N_H & w.p. \quad p \\ N_L & w.p. \quad 1 - p \end{cases} \tag{3.1}$$

During the first part of the selling season, the price R_1 is posted by the retailer, and during the second phase of the season, one of the two distinct prices will be offered depending on the realized demand in the first period. Specifically, R_{H2} is offered when N_H consumers arrive at the beginning of the sale season with probability p, and R_{L2} is offered when N_L consumers arrive at the beginning of the sale season with probability $(1-p)$. At the end of the selling season, leftover units have zero value. The retailer's objective is to set the prices to maximize the expected profit collected during the sale horizon. The retailer can manipulate the demand by setting selling prices and the retailer ensures that the inventory level can satisfy the demand, both of which imply that the consumers do not need to worry about the risk of stock out as all demand can be met. When visiting the store, consumers must choose either to buy the product at the current price R_1 or to wait for the later price $R_{i2}, i \in \{H, L\}$. The prices should not exceed one since consumers' valuation is between 0 and 1.

The optimal inventory release strategy is referred as Optimal Release strategy (OR). According to the unsold product amount at the end of the selling season, we study the scenarios listed below:

(1) Clearance Sale strategy (CS): After the first-period demand is realized, all leftover inventory is released to the market and the market determines the price.

(2) Dynamic Pricing strategy (DP): After the first-period demand is realized, the left-over inventory is released to maximize profit ignoring the capacity constraints.

The latter policy gives rise to another instance, to which we refer as Intermediate Supply strategy (IS). In this strategy, the leftover inventory is released to maximize the profit under low demand and all left over inventory is depleted under high demand, after the first-period demand is realized.

We focus the analysis on the first two scenarios. Our model is characterized by a set of parameters $\{K, p, \alpha, \delta, c, N_L, N_H\}$, assumed to be known to the retailer and all consumers. We assume that the retailer has great credibility and that all consumers believe the retailer's announcement. Additionally, each consumer has private information about his own valuation, v. The consumers know the initial inventory quantity, K, but they have to predict the period 2 selling price before they make their purchase decisions. In those regards, the game between the retailer and the consumers follows the Stackelberg model, with the retailer being the leader and the consumers being the followers.

Chapter 4

Preliminary Analysis

4.1 Consumer's Purchasing Behavior

In this section, we study the consumers' purchasing decisions. Myopic consumers behave impulsively, and purchase the product if their valuations are higher than the selling price in period 1, R_1. We focus the analysis on strategic consumers below.

A strategic consumer's optimal purchase decision is based on a threshold valuation, V. Specifically, a strategic consumer will purchase an available unit immediately during period 1 if his valuation exceeds V. Otherwise, the consumer will revisit the store in period 2 and purchase an available unit if his base valuation is higher than R_2 (R_{H2} or R_{L2}). Based on the prices in the second period, R_{L2} and R_{H2}, we can segment the strategic consumers into three groups: those who can buy the product in period 2 if $R_2 = R_{H2}$, those who can buy the product in period 2 if $R_2 = R_{L2}$, and those who can not buy in period 2 at either price. Based on the above groupings, two critical valuations emerge as relevant for our analysis: V_H and V_L. The former is relevant to consumers with $v > R_{H2}$, those consumers who may purchase the product in the second period regardless of the demand state; the latter threshold valuation, V_L,is relevant for consumers whose valuations are less than R_{H2} and, hence, they do not purchase the product in the second period if demand is in the high demand state. When a consumer's valuation is less than R_{L2}, he can not make any purchases when either prices is offered, and the threshold valuation is 1 for this type of consumers since they wait during all the selling season and make no purchase. Specifically, we have

$$V = \begin{cases} V_H & v \in [R_{H2}, 1] \\ V_L & v \in [R_{L2}, R_{H2}) \\ 1 & v \in [0, R_{L2}) \end{cases} \tag{4.1}$$

Consumers observe the selling price R_1 and supply quantity in period 1. Specifically, consumers make immediate purchase only when the surplus from immediate purchase (SIP) exceeds the expected surplus from waiting (ESW), which is equal to $(1-p)\delta(v - R_{L2}) + p\delta(v - R_{H2})$, for consumers whose valuations are in $[R_{H2}, 1]$. The surplus of an immediate purchase is $v - R_1$. Consumers make an immediate purchase when both (a) $SIP \geq \max\{0, ESW\}$. By solving $SIP = ESW$, we get the critical value V_H. That is, strategic consumers with $v \geq V_H$ purchase the product in period 1.

Strategic consumers whose valuations are in $[R_{L2}, R_{H2}]$, do not buy in period 2 if the demand is the high state. Hence, for these consumers, $ESW_L = (1-p)\delta(v - R_{L2})$, and V_L

is solved from $SIP = ESW_L$. We assume that $R_{L2} < R_1$ for now (and we will later prove that this assumption holds true in Lemma 7); hence, when $v < R_{L2}$, strategic consumers do not purchase in the first period.

The threshold functions are given by

$$V_H \equiv \frac{R_1 - \delta p R_{H2} - \delta(1-p)R_{L2}}{1-\delta}; V_L \equiv \frac{R_1 - \delta R_{L2}(1-p)}{1-\delta(1-p)}. \tag{4.2}$$

To determine the sales of the product in each period, we need to know the relationship between R_{H2}, V_H and V_L.

By solving $V_H = R_{H2}$ for δ, we have $\delta = \delta_1$

$$\delta_1 \equiv \frac{R_{H2} - R_1}{(R_{H2} - R_{L2})(1-p)}, \tag{4.3}$$

and by solving $V_L = R_{H2}$ for δ, we have $\delta = \delta_2 \equiv \frac{R_{H2}-R_1}{(R_{H2}-R_{L2})(1-p)}$. Based on the assumption of $R_{H2} \geqslant R_1$, $R_{H2} \geqslant R_{L2}$ and $0 \leqslant p \leqslant 1$, we know that δ_1 is positive. The functions of R_{H2}, V_H, and V_L join at the same intersection, δ_1.

Lemma 1 *If $\delta < \delta_1$, then $V_H < V_L < R_{H2}$; otherwise, $V_H \geq V_L \geq R_{H2}$.*

Proof. Consider the difference

$V_H - V_L$

$= \frac{R_1-\delta p R_{H2}-\delta(1-p)R_{L2}}{1-\delta} - \frac{R_1-\delta R_{L2}(1-p)}{1-\delta(1-p)}$

$= -\frac{\delta p[(R_{H2}-R_{L2})(p-1)\delta+(R_{H2}-R_1)]}{(1-\delta)(1-\delta+\delta p)}$

$= \frac{\delta p(R_{H2}-R_{L2})(1-p)}{(1-\delta)(1-\delta+\delta p)}[\delta - \frac{R_{H2}-R_1}{(R_{H2}-R_{L2})(1-p)}]$

$= \frac{(R_{H2}-R_{L2})(1-p)}{1-\delta(1-p)}[\delta - \delta_1] > 0$

Similarly, $V_L - R_{H2}$

$= \frac{R_1-\delta R_{L2}(1-p)}{1-\delta(1-p)} - R_{H2}$

$= \frac{(R_{H2}-R_{L2})(1-p)\delta+(R_1-R_{H2})}{1-\delta(1-p)}$

$= \frac{(R_{H2}-R_{L2})(1-p)}{1-\delta(1-p)}[\delta - \frac{R_{H2}-R_1}{(R_{H2}-R_{L2})(1-p)}]$

$= \frac{(R_{H2}-R_{L2})(1-p)}{1-\delta(1-p)}[\delta - \delta_1] > 0$

By routine calculations, if $\delta > \delta_1$, then $V_H - V_L > 0$ and $V_L - R_{H2} > 0$, implying $V_H > V_L > R_{H2}$. Similarly, $V_H < V_L < R_{H2}$ if $\delta < \delta_1$. ■

Figure 4.1 illustrates the relationship between R_{H2}, V_H, and V_L assuming $R_1 = 0.5, R_{L2} = 0.2, R_{H2} = 0.65$, which implies $\delta_1 = 0.66$. When δ is less then δ_1, both threshold valuations are smaller than the selling price in period 2 under high demand. When δ is larger then δ_1, both threshold valuations are larger than R_{H2}, and V_H is larger than V_L. The basic models of Skim Both and Skim Myopic cases are only feasible in the region of $[\delta_1, 1]$. The graph also shows that there has been a steady increase in the threshold valuations as patience level increases. Intuitively, as strategic consumers become more patient, expected surpluses from waiting increases and the threshold valuation increases. Therefore, less strategic consumers purchase in period 1 and more of them wait for the purchase in period 2.

Strategic consumers who wait for the second period purchase a product only if their

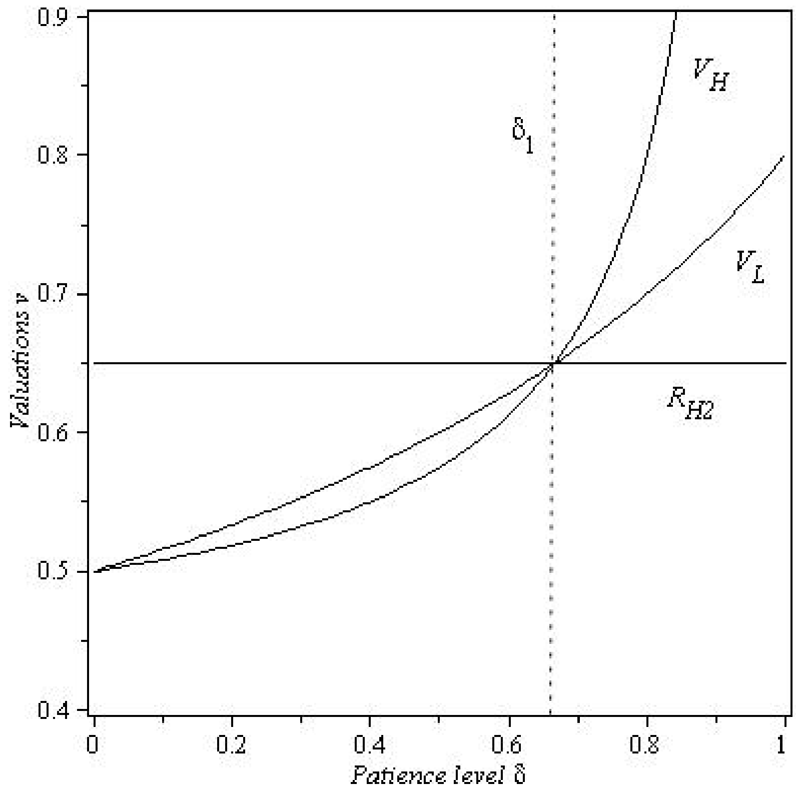

Figure 4.1: An example of the relationship between the threshold valuation functions and selling price in period 2 under high demand for case $R_1 = 0.5, R_{L2} = 0.2, R_{H2} = 0.65$ and $\delta_1 = 0.66$.

valuations are higher than the selling price in period 2 ($\geq R_{L2}$). In accordance with Lemma 1, when $\delta < \delta_1$, $V_H < V_L < R_{H2}$, $(1 - V_L)$ is the fraction of strategic consumers who purchase the product in period 1. The remaining strategic consumers may buy only when the firm charges R_{L2} in period 2. Under this condition, the critical valuation V_H and the price R_{H2} do not have any influence on strategic consumer' purchase timing decision. So the firm should not charge a price in period 2 that is higher than in period 1 under low demand. So the threshold valuation V_L is employed when $\delta < \delta_1$.

We note, however, that when $\delta > \delta_1$, $V_L > R_{H2}$, the critical value could not be calculated by equation $SIP = ESW_L$ anymore due to consumers' ability to purchase the product at R_{H2}. So $(1 - V_H)$ is the percentage of strategic consumers who purchase the product in period 1, and the remain strategic consumers wait for period 2 if their base valuations belong to $[R_{H2}, V_H]$. Therefore, when Clearance Sales strategy is employed,

$$V = \begin{cases} V_H & \delta \geq \delta_1 \\ V_L & \delta < \delta_1 \end{cases}$$

When Dynamic Pricing and Intermediate Supply strategies are employed,

$$V = V_H.$$

4.2 Pricing Policies

In this setting, without loss of generality, we normalize p and N_L to $\frac{1}{2}$ and assume c is less than $\frac{1}{2}$. Retailers may practise one of two specific prices policies, depending on the parameter interval: prices may be set to skim both types (myopic and strategic) of high valuation consumers in period 1; or set to skim only high valuation myopic consumers in period 1, deferring all strategic consumers to period 2. Following Mantin and Granot (2010), we refer to these pricing policies as Skim Both (SB) pricing and Skim Myopic (SM) pricing, respectively. Specifically, we have

Skim Both (SB): price to skim high-valuation consumers of both types in the first period, implying $V < 1$.

Skim Myopic (SM): price to skim only high-valuation myopic consumers in the first period. That is, none of the strategic consumers purchases in the first period; they all wait for the second period; i.e. $V = 1$.

The retailer must switch from case Skim Both to case SM if V equals 1, but he may choose to switch at an earlier stage to impose waiting on strategic consumers. We need to derive the switching point based on the other parameters. Hence, we derive δ_3, which is obtained by solving $V = 1$ for δ. That is

$$\delta_3 \equiv \frac{2(1 - R_1)}{2 - R_{H2} - R_{L2}} \tag{4.4}$$

Therefore, when δ exceeds δ_3, the retailer must switch to the Skim Myopic case. The consistency for the profit functions of Skim Both and SM will be shown later in this section.

Pictorially, the strategic consumers' type space (valuation) is divided into four regions, as shown in Figure 4.2, which also provides an insight for the behavior of strategic consumers under Clearance Sales strategy. Strategic consumers with valuations below R_{L2} never purchase as their willingness to purchase is too low. For those consumers with valuations between R_{L2} and R_1, the only profitable option is to wait and purchase when the retailer sells the product for the price of R_{L2}. Consumers with valuations above R_1 act strategically according to the threshold function V, such that a strategic consumer waits strategically only if his or her valuation satisfies $R_1 < v \leq V$. Those with valuation $v > V$ are the buy-now strategic consumers who tend to avoid the waiting cost. Notice that part (a) in figure 4.2 is only feasible for Skim Both up to the point where V reaches 1.

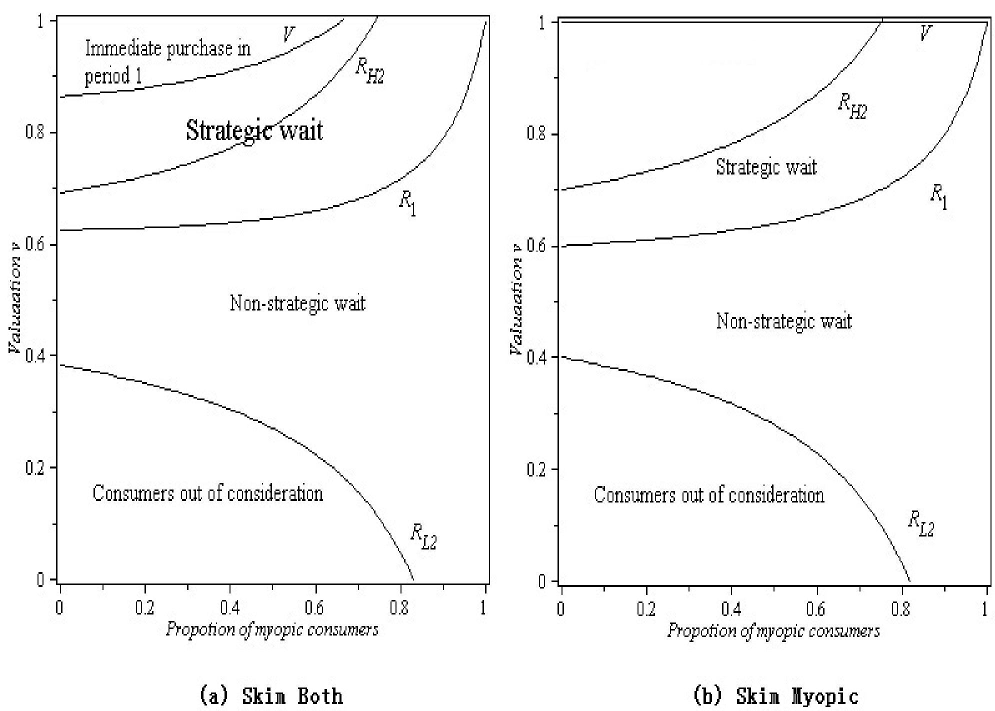

Figure 4.2: An example of a purchasing strategy of the strategic consumers under the Clearance Sale strategy for the case $\delta = 0.73, c = 0.1$. Note that the Skim Both case is only feasible when $V < 1$ in panel (a).

Chapter 5

Clearance Sales Strategy

In this scenario, all leftover inventory is released into the market in the second period. The retailer sets the price, R_1, in the first period whereas the selling price in the second period, $R_{i2}, i \in \{H, L\}$, is determined by the market. Having capacity of K units implies that at most K units of the product will be sold during the selling season. Specifically, D_{i1} units are sold in the first period, and the remaining units, at most $K-$ D_{i1} products are sold in the second period. That is $D_{i2} \leq K - D_{i1}$. Based on the relationship between D_{i2} and $K - D_{i1}$ and when the entire inventory is sold out, we further separate the analysis of Clearance Sales strategy into three scenarios.

(a) If $D_{i2} = K - D_{i1}$ $\forall i$ and $i \in \{H, L\}$: inventory is depleted completely by the end of the selling season.

(b)[1] If $D_{L2} = K - D_{L1}$ and $D_{H1} = K$: inventory is depleted completely at the end of

[1]Case (b) is a special case of case (a), we analyze case (b) separately to capture its specialities.

period 1 under high demand.

(c) If $D_{i2} < K - D_{i1}$ $\forall i$ and $i \in \{H, L\}$: some leftover inventory remains (when demand is low) by the end of the selling season.

5.1 Case a: Inventory Depleted Completely over Two-period Sales

Based on the information in Figure 4.2 above, we can derive the demand function of this product under both low and high demands. Let D_{i1}^{ϕ} denote the realized demand under state i, $i \in \{H, L\}$ in period $t, t \in \{1, 2\}$, and $\phi = SB$, SM; note that if $\phi = SM$, then $V = 1$. In this setting, the realized demand in period 1 at price R_1 is the number of myopic consumers with valuations above R_1 plus the number of strategic consumers with valuations above V in the Skim Both case, and only the number of myopic consumers with valuations above R_1 in the Skim Myopic case. Specifically,

$$D_{i1}^{\phi} \equiv \alpha N_i(1 - R_1) + (1 - \alpha)N_i(1 - V) = \alpha N_i(V_H - R_1) + N_i(1 - V)$$

The realized demand in period 2 at price R_{i2} is the number of strategic consumers with valuations below V but above R_{i2} in Skim Both case, and the number of strategic consumers with valuations below 1 but above R_{i2} in Skim Myopic case.

$$D_{i2}^{\phi} \equiv (1 - \alpha)N_i(V - R_{i2})$$

Let Π^{SB} and Π^{SM} denote the retailer's profits under Skim Both and Skim Myopic pricing, respectively. The profit function under low demand is given by $\Pi_L = R_1 D_{L1} +$

$R_{L2}D_{L2} - cK$, and under high demand is given by $\Pi_H = R_1 D_{H1} + R_{H2}D_{H2} - cK$. Thus, the retailer's total expected profit is $\Pi = p\Pi_H + (1-p)\Pi_L$.

5.1.1 Equilibrium Pricing Policies and Model Analysis

The model is solved backward to yield the subgame perfect Nash equilibrium, which allows us to derive the equilibrium pricing policies. For both pricing strategies, we identify the subgame-perfect Nash equilibrium, and show that given the retailer's strategy, the equilibrium in the consumer subgame is unique.

Let R_{it}^{ϕ} denote the selling price under state i, $i \in \{H, L\}$ in period $t, t \in \{1, 2\}$, and $\phi = SB$, SM. Recalling our assumption that the retailer employs Clearance Sales strategy, the selling prices in period 2, R_{i2}, are determined by the demand (the strategic consumers who wait for the second period), and the left-over inventory (i.e. $K - D_{i1}, i \in \{H, L\}$). Thus,

$(R_{H2}^{\phi}, R_{L2}^{\phi}) = (\frac{-K-\alpha R_1+1}{1-\alpha}, \frac{-2K-\alpha R_1+1}{1-\alpha})$, $\phi \in \{SB, SM\}$

The first period price is obtained by solving the first-order condition, which is to maximize the profit. For the first-order conditions, an optimal solution is

$R_1^{SB} = \frac{2}{3}\alpha K\delta - \frac{2}{3}K\alpha + 1 - \frac{3}{4}K\delta - \frac{2}{3}K$, if *Skim Both* is employed and

$R_1^{SM} = 1 - \frac{4}{3}K$, if SM is employed.

Recall that the precondition of the Clearance Sale scenario is that the consumers' threshold valuation, V, should be higher than the selling price in period 2 under high demand; δ should be larger than δ_1 to satisfy this condition (Lemma 1). Substituting the price expressions in the expression of δ_1 yields

$$\delta_1 = \frac{4(1-2\alpha)}{3-8\alpha}. \tag{5.1}$$

Similarly, we get

$$\delta_3 = \frac{8(1-\alpha)}{9-8\alpha}. \tag{5.2}$$

Notice that $\delta_1 = \delta_3$ when $\alpha = \frac{3}{4}$.

When $\alpha \in [0, 0.5]$, δ_1 equals 0 since the valuation of δ_1 over the regular interval of δ, thus the basic Skim Both and Skim Myopic models are always feasible in this interval of α. When $\alpha \in (0.5, 0.75]$, δ_1 is between 0 and 1, and the basic Skim Both and Skim Myopic models are only feasible when δ is larger than δ_1. We assume that the retailer does not deplete all inventory in period 1 in the basic Skim Both and Skim Myopic models. Therefore, we solve the value of α, where the retailer sell out everything in period 1. Quite surprisingly, we find out that the retailer depletes all inventory if the proportion of myopic consumer is equals to or over $\frac{3}{4}$, and $\alpha = \frac{3}{4}$ is also where $\delta_1 = \delta_3$. As a result of this fact, retailer switch from Skim Myopic to sell everything in period 1 when $\alpha = \frac{3}{4}$. Therefore, δ_1 equals 1 when $\alpha \in (0.75, 1]$.

From Figure 5.1, the value of δ_1 can be observed.

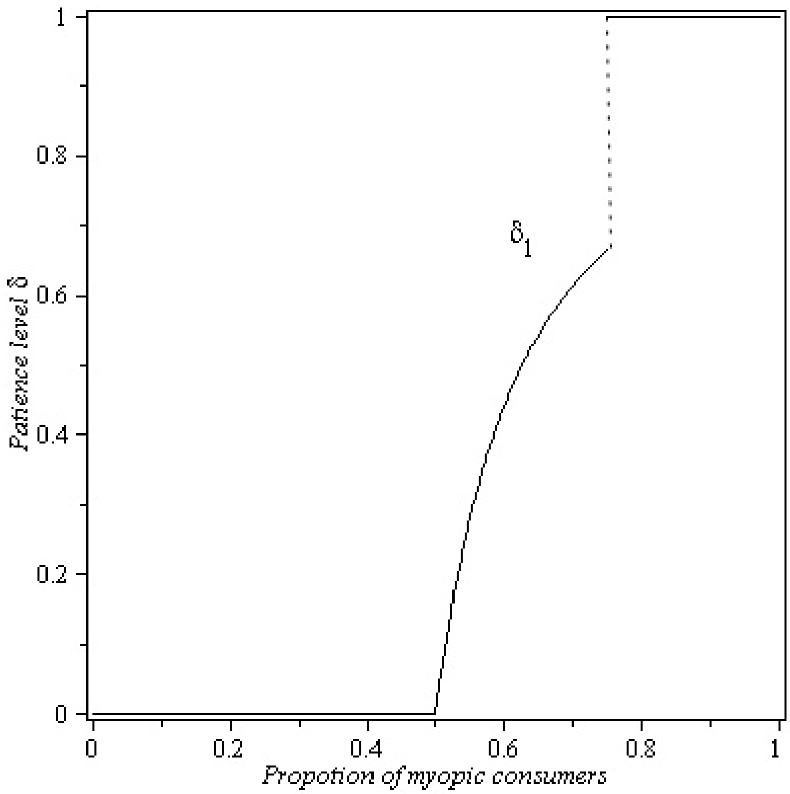

Figure 5.1: The value of δ_1. When $\delta > \delta_1$, the basic CS is employed by the retailer; when $\delta < \delta_1$, the retailer employs AH.

$$\delta_1 \in \begin{cases} 0 & \text{if } \alpha \in [0, 0.5] \\ (0,1) & \text{if } \alpha \in (0.5, 0.75] \\ 1 & \text{if } \alpha \in (0.75, 1] \end{cases} \tag{5.3}$$

Since we are more interested in the cases where $V > R_{H2} > R_1$, we focus the analysis on the feasible interval of α and δ. δ_1 equals zero in the region of $\alpha \in [0, 0.5]$, which implies that the threshold valuation is always larger than the selling price in period 2 in this interval. So we always have $V > R_{H2} > R_1$ when $\alpha \in (0, 0.5]$. Basic skim both and skim myopic cases can be applied in this interval. When $\alpha \in (0.5, 1)$, the threshold valuation is only larger than the selling price in period 2 if $\delta > \delta_1$. This may happen under high demand and if the entire inventory is sold during the first period. A subsection named sell all under high demand is given under this section.

In the case where all products are sold in period 1 under high demand, the selling prices in period 2 are irrelevant anymore since no product is available in period 2. This case is named AH, which means the retailer sells all units under high demand in period 1. Intuitively, we know that all the sales occur in period-1 with a relatively high price if the proportion of myopic consumers is relatively high in the market. If the proportion of myopic consumers is over half, the retailer will not have any inventory left in period 2 under high demand under most of the conditions, Specifically, the retailer depletes all inventory if the proportion of myopic consumers equals or exceeds 75%.

Based on Lemma 1 and Figure 5.1, the retailer employ the basic Skim Both and Skim Myopic models when consumers' patience lever is less than δ_1, and the retailer switches from basic Skim Both and Skim Myopic to case AH when $\delta \geq \delta_1$ since all inventory is depleted in period 1 of the selling season. Moreover, the retailer employs basic Skim Myopic instead of basic Skim Both if consumers' patience level equals or exceeds δ_3. The separation of the cases is given in Figure 5.2.

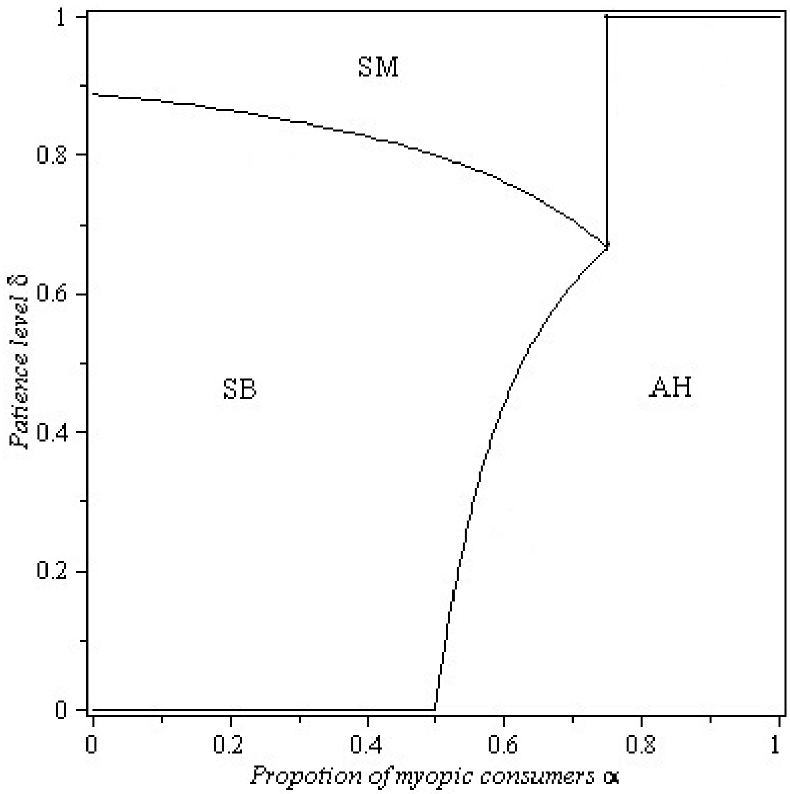

Figure 5.2: The case separation based on α and δ when supply equals demand, where SB=Skim Both; SM=Skim Myopic and AH=sell All under High demand in period1. Note that the downsloping line is δ_3; the stepwise increasing curve is δ_1.

5.1.2 Basic Skim Both and Skim Myopic

In the basic Skim Both and Skim Myopic cases, the retailer has some left-over inventory in period 2 after the sales in period 1. And both R_{L2} and R_{H2} exist and are strictly positive ($0 < R_{L2},\ R_{H2} < 1$). Later in Section 4.2, binding constraints are imposed. Naturally, a retailer prefers Skim Both over Skim Myopic whenever $\Pi^{SB} \geq \Pi^{SM}$. The retailer is indifferent between Skim Both and Skim Myopic at δ_3, where δ_3 is the value of δ solved by ($V = 1$), and $\delta = \delta_3$, when we solve $\Pi^{SB} = \Pi^{SM}$ in δ, finding that further proves the consistency of this model. A retailer employs Skim Both when δ is less than δ_3; otherwise, the retailer is forced to switch to Skim Myopic pricing due to the strategic consumers' behavior.

There are a few conditions that need to be satisfied in this model. First of all, we need to satisfy the assumption of $R_1 < R_{H2}$, so δ_4 is solved by $R_1 = R_{H2}$ for δ. Hence, R_{H2} is larger than R_1 When $\delta > \delta_4$.

$$\delta_4 \equiv \frac{4(1-2\alpha)}{9-8\alpha} = \frac{8(1-\alpha)-4}{9-8\alpha} < \delta_3 \tag{5.4}$$

Note that $\delta_4 < \delta_3$. Also we need to satisfy the positivity of R_{L2}. Since R_{L2} is decreasing in α, we employ α_1 solved from $R_{L2} = 0$ for α. This condition will be released after the inventory level is decided.

$$\alpha_1 = \begin{cases} \frac{12-8K-9K\delta-\sqrt{832K^2-576K-624K^2\delta+144+168K\delta+81K^2\delta^2}}{16K(1-\delta)} & \text{if } \delta_1 < \delta < \delta_3 \\ \frac{3(1+2c)}{4(1+c)} & \text{if } \delta \geq \delta_3 \end{cases} \tag{5.5}$$

Substitute the prices in the profit functions. The corresponding profit functions are: $\Pi^{SB} = \frac{KA_1}{192(1-\delta)(1-\alpha)}$, and $\Pi^{SM} = \frac{K(-6\alpha+8K\alpha-9K+6-6c+6c\alpha)}{6(1-\alpha)}$.

where $A_1 = K(8\alpha-9)^2\delta^2 + (-192+192\alpha+16K\alpha-128K\alpha^2+192c-192c\alpha+144K)\delta + 92-224K-192\alpha+128K\alpha-192c+64K\alpha^2+192c\alpha$

Proposition 2 *Consider the Clearance Sales strategy in Case (a) where $\delta > \delta_1$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, if $\alpha < \min\{\alpha_1, \frac{3}{4}\}$, if $\delta_1 < \delta < \delta_3$, then Skim Both is employed;*

otherwise, if $\delta > \delta_3$, then Skim Myopic is employed. If $\alpha \geq \frac{3}{4}$, then the retailers always employ Skim Myopic policy.

Proof. Recall that $\delta > \delta_1$ ensures $V_H > V_L > R_{H2}$, and recall that $\delta_1 = \frac{4(1-2\alpha)}{3-8\alpha}$. The retailer switches from Skim Both to Skim Myopic at $\delta = \delta_3 = \frac{8(1-\alpha)}{9-8\alpha}$.

There is not a certain relationship between δ_1 and δ_3. Note that $\delta_1 = \delta_3$ when $\alpha = \frac{3}{4}$. If $\alpha < \frac{3}{4}$, then $\delta_1 < \delta_3$, both Skim Both and Skim Myopic cases exist in the market. If $\alpha \geq \frac{3}{4}$, then $\delta_1 \geq \delta_3$, and only the Skim Myopic case is feasible.

We also need to guarantee the positivity of R_{L2} by limiting $\alpha < \min\{\alpha, \frac{3}{4}\}$. Figure 5.2 shows that if $\delta < \delta_1$, the retailer deplete all inventory in period 1 and AH is employed. Further details about case AH will be given in Section 4.2. ■

The profit function is quadratic in inventory level (K), and we need to find out if it is convex or concave in K. If the profit function is convex in K, the corner solution(s) within the feasible region will be employed, and if the profit function is concave in K, the optimal solution within the feasible region will be chosen.

Proposition 3 *Consider the Clearance Sales strategy in Case (a) where $\delta > \delta_1$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the retailer's profit function is strictly concave in the inventory quantity stocked, thus guaranteeing an optimal stocking decision.*

Proof. Under Skim Both pricing, $\frac{\partial^2 \Pi^{SB}}{\partial K^2} = \frac{(8\alpha-9)^2\delta^2+(144+16\alpha-128\alpha^2)\delta-224+128\alpha+64\alpha^2}{96(1-\alpha)(1-\delta)}$. The denominator is clearly positive. The numerator can be written as $f_1 = a\delta^2+b\delta+c$, where f_1 is convex in δ. $\frac{\partial^2 f_1}{\partial \delta^2} = 2(8\alpha-9)^2 > 0$ and the discriminant of f_1 is $\Delta_{f_1} = 1152(8\alpha-9)^2 \geq 0$.

Hence, f_1 has two real number roots which are $-\frac{4(2+3\sqrt{2}+2\alpha)}{9-8\alpha}$ and $-\frac{4(2-3\sqrt{2}-2\alpha)}{9-8\alpha}$, where the former is negative and the latter is positive. Let δ_5 denote the positive root. That is $\delta_5 \equiv -\frac{4(2-3\sqrt{2}-2\alpha)}{9-8\alpha}$. Since $\delta_5 - \delta_3 = \frac{4(-4+3\sqrt{2})}{9-8\alpha} > 0$, δ_5 is larger than δ_3.

Since the retailer employs Skim Both in the region of $[\delta_1, \delta_3]$, $\frac{\partial^2 \Pi^{SB}}{\partial K^2}$ is negative in the region of $\delta \in [0, \delta_3]$, which means that Π^{SB} is strictly concave in the inventory quantity stocked.

Under Skim Myopic pricing, $\frac{\partial^2 \Pi^{SM}}{\partial K^2} = -\frac{9-8\alpha}{3(1-\alpha)} < 0$. Hence, Π^{SM} is strictly concave in the inventory quantity stocked. ■

The corresponding optimal stocking decision is

$$K = \begin{cases} \frac{96(1-c)(1-\delta)(1-\alpha)}{-(8\alpha-9)^2\delta^2-(144+16\alpha-128\alpha^2)\delta+224-128\alpha-64\alpha^2} & \text{if } \delta_1 < \delta < \delta_3 \\ \frac{3(1-c)(1-\alpha)}{9-8\alpha} & \text{if } \delta \geq \delta_3 \end{cases}$$

Substituting the K in the profit function:

$$\Pi^{SB} = \frac{48(1-c)^2(1-\delta)(1-\alpha)}{-(8\alpha-9)^2\delta^2-(144+16\alpha-128\alpha^2)\delta+224-128\alpha-64\alpha^2},$$

$$\Pi^{SM} = \frac{3(1-\alpha)(1-c)^2}{2(9-8\alpha)},$$

and the corresponding strategic consumer critical valuation is

$$V = \begin{cases} \frac{64c(\delta-1)^2\alpha^2-8(\delta-1)(9\delta+9c\delta+16)\alpha+72\delta-160+81\delta^2+72c\delta-64c}{-224+144\delta+16\alpha\delta+128\alpha-128\alpha^2\delta+64\alpha^2\delta^2-144\alpha\delta^2+81\delta^2+64\alpha^2} & \text{if } \delta_1 < \delta < \delta \\ 1 & \text{if } \delta \geq \delta_3 \end{cases}$$

The proposition above implies that there is always an optimal inventory level decision, and the retailer will obtain more profit if he employs a relatively low stocking decision to satisfy part of the demand or a relatively high stocking decision to satisfy all demand. With a high δ, the consumers are very patient and wait for the product discounts. Let's take luxury goods as a example. If the retailer is selling luxury goods, a relatively low stocking decision would be employed to stimulate the demand at a high selling price in both periods. If the product is non-seasonal or non-perishable, the retailer sets a relatively high stocking decision to sell as many units as possible at a lower rate.

Myopic consumers purchase the product if their valuations are higher than the first period price. Intuitively, as the percentage of myopic consumers in the market increases, more strategic consumers purchase in the second period instead of the first period due to retailer's pricing strategy. Recall that only the strategic consumers with valuations between V and one purchase goods in the first period. As this critical valuation, V, increases, more strategic consumers wait and purchase in the second period. We seek to characterize the change in the critical valuation as more myopic consumers exist in the market.

$\frac{\partial V}{\partial \alpha} = \frac{8(1-\delta)(1-c)f_2}{(f_3)^2}$, where $f_2 = -64(9\delta+16)(\delta-1)^2\alpha^2 + \alpha 16(\delta-1)(81\delta^2+72\delta-160) - 729\delta^3 + 2016\delta - 1024$ and f_3 is some function of α and δ, which we suppress since it is clear that the denominator is positive. As $(1-c)$ and $(1-\delta)$ are positive, so the relationship between critical valuation and α is purely dependent on B_1. B_1 can be written as $f_2 = a\alpha^2 + b\alpha + c$. Since the discriminant of function f_2 is $\Delta_{f_2} = 73728(32-9\delta^2)(\delta-1)^2 \geq 0$, function f_2 has two real number roots: $\frac{-81\delta^2-72\delta+160\pm 12\sqrt{64-18\delta^2}}{8(9\delta+16)(1-\delta)}$. $\frac{\partial^2 f_2}{\partial \alpha^2} = -(128(9\delta+16))(\delta-1)^2 < 0$, so function B_1 is concave in α.

Since one root is larger than 1 and the other root is less than one, we denote the smaller root as α_2: $\alpha_2 \equiv \frac{-81\delta^2-72\delta+160-12\sqrt{64-18\delta^2}}{8(9\delta+16)(1-\delta)}$, where$\alpha_2$ is between 0 and 1 when $\delta \in [0, 0.578]$, and α_2 is less than 0 if $\delta \in [0.578, \delta_3]$. Note that if $\delta \geq \delta_3$, Skim Myopic is employed and $V = 1$.

Proposition 4 *Consider the Clearance Sales strategy in Case (a) where* $\delta > \delta_1$*, and assume* $p = \frac{1}{2}$ *and* $N_L = \frac{1}{2}$*. If* $\delta \in [0, 0.578]$*, then if*

$\alpha < \alpha_2 = \frac{-81\delta^2 - 72\delta + 160 - 12\sqrt{64 - 18\delta^2}}{8(9\delta+16)(1-\delta)}$*,* V *is decreasing in* α*; otherwise, if* $\alpha > \alpha_2$*, then* V *is increasing in* α*. If* $\delta \in [0.578, \delta_3]$*, then* V *is decreasing in* α*; If* $\delta \geq \delta_3$*, then* V *is independent of* α*.*

Proof. When $\delta \in [0, 0.578]$, α_2 is between 0 and 1, so f_2 is negative in the range of $[0, \alpha_2]$, and positive in the range of $[\alpha_2, 1]$.

Note that B_1 is concave in α since $\frac{\partial^2 B_1}{\partial \alpha^2} < 0$, so $\frac{\partial V}{\partial \alpha}$ is negative in the range of $[0, \alpha_2]$, and positive in the range of $[\alpha_2, 1]$. Therefore, V decreases in α when $\alpha \in [0, \alpha_2]$ and increases in α when $\alpha \in [\alpha_2, 1]$.

When $\delta \in [0.578, \delta_3]$, α_2 is less than 0, so $\frac{\partial V}{\partial \alpha}$is always positive, which means that V increases in α. ■

Based on the proposition above, above an intermediate high level of patience, δ, the threshold valuation increases as more myopic consumers exist in the market. Even through fewer strategic consumers are in the market, but the proportion of them who choose to wait and purchase in period 2 increases because their patience levels are comparatively high. In contrast to the the high patience level, the threshold valuation declines first then increase gently as the proposition of myopic consumer increases. Intuitively, we know that a retailer's pricing strategy results in the changes in the threshold valuation of strategic consumers.

In Figure 5.3, two examples are given to illustrate the pricing and threshold valuation in the proportion of myopic consumers. When consumers' patience levels are relatively low, the threshold valuation decreases gently in α when α is small; then the threshold valuation increases when α is sufficiently high in α. As shown in Figure 5.3, the selling price in period 2 under high demand increases in α, and the selling price in period 1 decreases first then increases in α, which explains the trend of threshold valuation above an intermediate high level of patience level.

The retailer tries to manipulate strategic consumers' behavior by changing selling prices. Based on the threshold valuation, more strategic consumers purchase in period 1 as R_1 decreases, and more of them decide to wait for period 2 as R_1 increases, as shown in panel (a) of Figure 5.3. As shown in panel (b), the graph shows that there is a steep rise in the selling price in period 2 under high demand with a slight change in R_1, and the selling price in period 2 under low demand is relatively higher than what we have in panel (a). Therefore, the threshold valuation increases steadily as more myopic consumers exist in the market under a high patience level.

The left panel of Figure 5.3, feasible exists only when α is sufficiently small, since V needs to be larger than R_{H2}. However, this condition only holds if δ is larger than δ_1 and the valuation of δ_1 is given in Figure 5.1, and $\delta_1 = \frac{4(1-2\alpha)}{3-8\alpha}$. The infeasible region further verifies the correctness of Lemma 1. Since R_{H2} is larger than V in the infeasible region in the left part of Figure 5.3, the basic Clearance Sales strategy models do not work in this region. As illustrated in Figure 5.2, the retailer sells everything in period 1 under high demand, a scenario that fits this region. More details are given later in a section 4.13.

Quite surprisingly, numerical analysis suggests that optimal stocking and corresponding profit decrease as the proportion of myopic consumers increases, which implies that

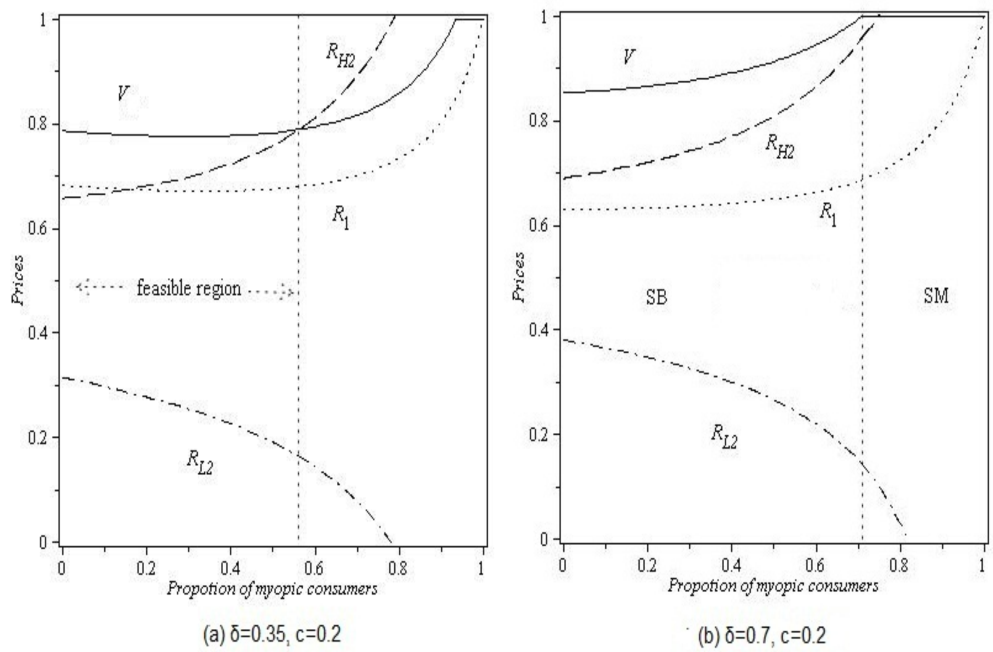

Figure 5.3: Examples of prices and threshold valuations under Clearance Sales strategy, and basic Skim Both and Skim Myopic cases are employed. Note that the feasible region in panel (a) is for basic SB and the left region is feasible for AH.

strategic consumer behavior may actually benefit the retailer. This finding counters common intuition that strategic customers who consider all available purchase choices hurt retailer's profit. Su (2007) identified finding as the result of scarcity and the heterogeneity of consumers. The threat of stock-outs discourages some strategic consumers from waiting, which may also increase their willingness to pay. In our model, the valuations of consumers are fixed, and all consumers are risk neutral. We can, however, consider the scarcity in a different way. As more myopic consumers exist in the market, fewer products are left for period 2, which further increases the selling price in period 2 so that fewer strategic consumers have the ability to purchase. The retailer then cares less about strategic consumers and defers more of them to period 2 by increasing R_1 and decreasing R_{L2} as α increases. With fewer strategic consumers, the retailer's potential profit from both period 1 and period 2 diminishes (recall that only strategic consumers may delay their purchase to the second period). Another possible explanation for this behavior is that the retailer will not lose the strategic consumers immediately if he charges a very high price in period 1, and extra profit may be obtained from strategically waiting consumers if a lower price is charged in the second period.

As the selling price in period 1 increases in α, fewer myopic consumers are capable of purchasing in period 1 since their valuations may be lower than the selling price. However, considering the sales lost from strategic consumers, the retailer stocks less as more myopic consumers exist in the market. Therefore, these variations may result in a corresponding decrease of profit in α. Cachon and Swinney (2009) also find that a firm stocks less with strategic customers than without them in the uncertain demand case. More interestingly, they study the additional value of quick response to mitigate the negative consequences of strategic purchase behavior of customers.This finding has important implications for developing the dynamic pricing theory since strategic consumers' behavior may actually benefit the retailer. The described trends of inventory level and profit functions are shown in Figure 5.4 using the same parameter values as in Figure 5.3.

Proposition 5 *Consider the Clearance Sales strategy in Case (a) where $\delta_1 < \delta < \delta_3$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as myopic consumers are more numerous in the market, (i) the capacity function K is strictly decreasing in α, which implies that as myopic consumers are more numerous in the market, the retailer chooses a lower capacity; (ii) the retailer*

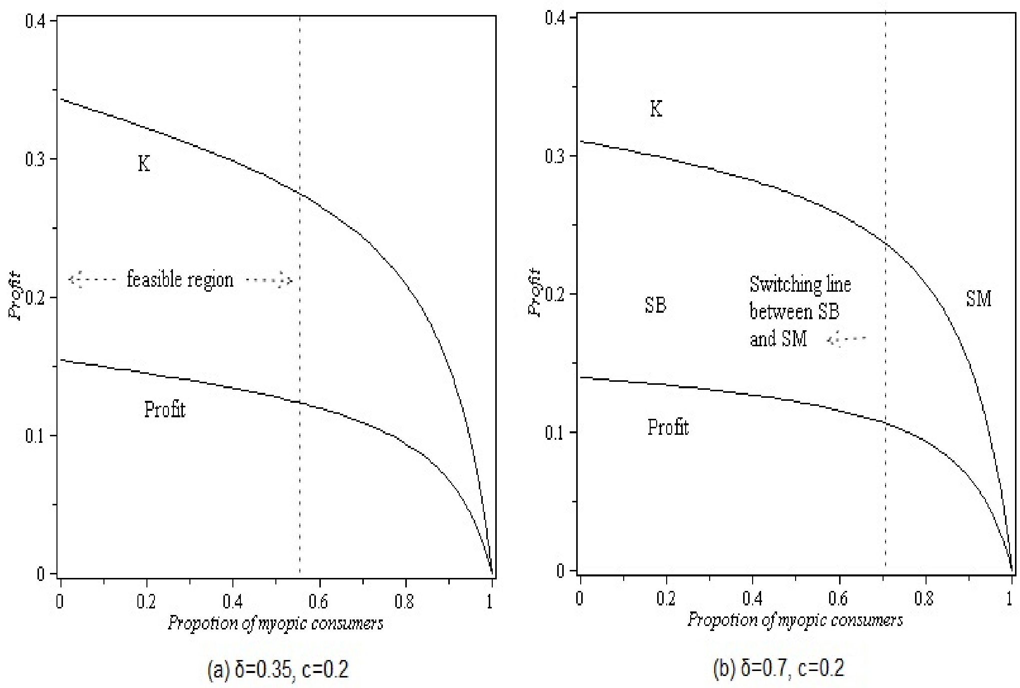

Figure 5.4: Two examples of profits and corresponding inventory level under Basic Skim Both and Skim Myopic cases. Note that the profit decreases to 0 when α increases to 1. Later we will see that this is not the case in equilibrium.

gets less profit in basic Skim Both and Skim Myopic cases.

Proof. In Skim Both, $\frac{\partial K}{\partial \alpha} = -\frac{96(1-c)(1-\delta)f_3}{(f_4)^2}$, where f_4 is some function of α and δ, and $f_3 = 64(\delta-1)^2\alpha^2 - 128(\delta-1)^2\alpha - 160\delta + 63\delta^2 + 96$.

To show $\frac{\partial K}{\partial \alpha} < 0$, we need to proof $f_3 > 0$. Since $\frac{\partial^2 C_1}{\partial \alpha^2} = 128(\delta-1)^2 \geq 0$, f_3 is convex in α.

The discriminate of function f_3 is $\Delta_{f_3} = 256(\delta^2+32\delta-32)(\delta-1)^2$. Because $(\delta^2+32\delta-32)$ is negative in the region of $\delta \in [0, \delta_3]$. Recall that $\delta_3 = \frac{8(1-\alpha)}{9-8\alpha}$ which is the switching point from Skim Both to Skim Myopic. So $\Delta_{f_3} < 0$.

Since the discriminant of function f_3 is less than zero, function f_3 does not have any real roots. So function f_3 is positive in the region of $\delta \in [0, \delta_3]$. Hence, The capacity function K is strictly decreasing in α.

Similarly, we can prove that the profit function is strictly decreasing in α.

$\frac{\partial \Pi^{SB}}{\partial \alpha} = \frac{48(1-c)^2(1-\delta)f_6}{(f_5)^2}$, where f_5 is some function of α and δ, and $f_6 = -64(\delta-1)^2\alpha^2 - 128(\delta-1)^2\alpha - 96 - 63\delta^2 + 160\delta$. Since $\frac{\partial^2 C_4}{\partial \alpha^2} = -128(\delta-1)^2 \leqslant 0$, f_6 is concave.

Since the discriminant of function f_6 is $\Delta_{f_6} = 256(\delta^2 + 32\delta - 32)(\delta-1)^2 < 0$, function f_6 does not have any real roots. So f_6 is negative.

Since$\frac{\partial \Pi^{SB}}{\partial \alpha} < 0$, the profit function Π_{SB} is strictly decreasing in α.

In Skim Myopic,$\frac{\partial K}{\partial \alpha} = -\frac{3(1-c)}{(8\alpha-9)^2} < 0$, and $\frac{\partial \Pi^{SM}}{\partial \alpha} = -\frac{3(1-c)^2}{2(9-8\alpha)^2} < 0$ ∎

Many previous studies have addressed the fact that a retailer may profit less if consumers are more patient. This finding is very straightforward as we consider patience level as a waiting cost. A high patience level implies a low waiting cost, which means the surplus of purchasing in period 2 is still very high for strategic consumers in our model. As the patience level increases, the threshold valuation decreases, and more strategic consumers would like to wait and pursue a better deal in period 2. If the retailer charges R_{H2} under high demand, some of the waiting strategic consumers would not purchase, which resulting in lost sales. If the retailer charges R_{L2} under low demand, the potential profit for period 2 also diminishes since the waiting strategic consumers pay less to obtain the product. Thus, a retailer gets less profit if consumer patience level increases.

Proposition 6 *Consider the Clearance Sales strategy in Case (a) where* $\delta_1 < \delta < \delta_3$*,* *assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as strategic consumers are more patient, that is, as δ increases, the retailer's profit decreases.*

Proof. In Skim Both, $\frac{\partial \Pi^{SB}}{\partial \delta} = \frac{48(1-c)^2(1-\alpha)f_8}{(f_7)^2}$, where f_7 is some function of α and δ, and $f_8 = -(8\alpha\delta - 9\delta + 8 - 8\alpha)(8\alpha\delta - 9\delta - 8\alpha + 10)$.

$\frac{\partial^2 f_8}{\partial \delta^2} = -2(9-8\alpha)^2 \leqslant 0$, f_8 is concave in δ. Since the discriminant of function f_8 is larger than 0, function f_8 has two real roots: $\frac{2(5-4\alpha)}{9-8\alpha}$ and $\frac{8(1-\alpha)}{9-8\alpha}$.

Since $\frac{2(5-4\alpha)}{9-8\alpha} > 0$ and $\frac{8(1-\alpha)}{9-8\alpha} \in [0,1]$, function A is negative in the region of $\delta \in [0, \frac{8(1-\alpha)}{9-8\alpha}]$ and positive in the region of $\delta \in [\frac{8(1-\alpha)}{9-8\alpha}, 1]$. Recall that $\delta_3 = \frac{8(1-\alpha)}{9-8\alpha}$ which is the switching point from Skim Both to Skim Myopic. So Skim Both case is only applicable in the region of $\delta \in [0, \frac{8(1-\alpha)}{9-8\alpha}]$. Therefore, $\frac{\partial \Pi^{SB}}{\partial \alpha} < 0$ in Skim Both case. ■

Now we plug the value of K to obtain the optimal prices.

$$R_1 = \frac{-160+152\delta-64c+128\alpha-120\alpha\delta-8\alpha\delta^2-8c\delta+9\delta^2+136c\alpha\delta-136\alpha\delta^2c-128\alpha^2\delta c+64\alpha^2\delta^2c+64c\alpha^2+72c\delta^2}{-224+144\delta+16\alpha\delta+128\alpha-128\alpha^2\delta+64\alpha^2\delta^2-144\alpha\delta^2+81\delta^2+64\alpha^2}$$

$$R_{H2} = \frac{-128\alpha^2\delta c+64\alpha^2\delta^2c+64c\alpha^2+8\alpha\delta+64\alpha-72\alpha\delta^2+8c\alpha\delta-72\alpha\delta^2c+64c\alpha+48\delta+81\delta^2-128+96c\delta-96c}{-224+144\delta+16\alpha\delta+128\alpha-128\alpha^2\delta+64\alpha^2\delta^2-144\alpha\delta^2+81\delta^2+64\alpha^2}$$

$$R_{L2} = \frac{-128\alpha^2\delta c+64\alpha^2\delta^2c+64c\alpha^2+8\alpha\delta+64\alpha-72\alpha\delta^2+8c\alpha\delta-72\alpha\delta^2c+64c\alpha-48\delta+81\delta^2-32+192c\delta-192c}{-224+144\delta+16\alpha\delta+128\alpha-128\alpha^2\delta+64\alpha^2\delta^2-144\alpha\delta^2+81\delta^2+64\alpha^2}$$

Usually, the selling price in period 2 is lower than the selling price in period 1 as the retailer adopts the markdown mechanism (Lazear 1986, Smith and Achabal 1998, Gupta et al. 2006, Zhang and Cooper 2008). Under the condition of uncertain demand, the price that the retailer charges in the second period may be higher, or lower, than the price posted in the first period. Specifically, if the demand in the first period is high, more of the inventory is depleted, and with less inventory, the second period price can be higher than the first period price. If the demand in the first period is low, all leftover inventory is released in period 2. The retailer has to charge a lower price in period 2 than in period 1 to sell as much stock. The findings of the current study are consistent with those of Bitran and Mondschein (1997), Feng and Gallego (1995), Gallego and van Ryzin (1994), who found that prices may go either up or down over time. Possible explanations for the rise in price are the stochastic arrival of consumers or the poisson arrivals of consumers under continuous time and the fact that high valuation consumers arrive later during the selling season.

Proposition 7 *Consider the Clearance Sales strategy in Case (a) where $\delta > \delta_1$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the selling price in period 1 is always higher than the selling price in period 2 under low demand.*

Proof. Under Skim Both case, $R_1 - R_{L2} = \frac{(1-\delta)(1-c)f_{10}}{f_9}$, where $f_{10} = 8(\delta-1)\alpha - 9\delta + 16$, and f_{10} is a liner function which decreases in α. Recall that when $\alpha < \min\{\alpha_1\ , \frac{3}{4}\}$, retailers employ Skim Both if the discounting factor of consumers is between δ_1 and δ_3. Since $\alpha \in [0, \frac{3}{4})$, function f_{10} is between $[10-3\delta, 16-9\delta]$, function f_{10} is positive. $f_9 = (-64\alpha^2+144\alpha-81)\delta^2+(-144-16\alpha+128\alpha^2)\delta+224-128\alpha-64\alpha^2$. Since $\frac{\partial^2 f_9}{\partial \delta^2} = -2(8\alpha-9)^2$, f_9 is concave in δ. The roots of f_9 are $\frac{4(-2-2\alpha\pm 3\sqrt{2})}{9-8\alpha}$. Since one root is less than 0 and the other root is larger than $\frac{3}{4}$, function f_9 is positive when $\alpha \in [0, \frac{3}{4})$. Thus, $R_1 > R_{L2}$ for Skim Both cases.

Under Skim Myopic case, $R_1 - R_{L2} = \frac{2(1-c)}{9-8\alpha} > 0$ ■

The equilibrium prices for both cases are illustrated in Figure 4.2. As seen in Figure 4.2, for fixed values of δ, as α increases, the retailer increases the first period price as well as the second period under high demand and decreases the second period under low demand for both Skim Both and Skim Myopic pricing. Compared to the price curves under Skim Both, the retailer try to increase the first period price and decrease the second period price steeply in Skim Myopic. The retailer tries to encourage strategic consumers to make purchases in the second period. This behavior coincides with the proposition above. With fewer strategic consumers, the retailer's potential profit from the second period diminishes. Retailer cares less about strategic consumers and seeks to defer them all to the second period, while focusing attention on first period demand stemming from myopic consumers. Yet, the retailer stocks a lower inventory level and sets a higher first period price (partially to divert strategic consumers to the second period). Ultimately, this behavior results in less profit for the retailer.

In this section, so far, we have found that the second period selling price in low demand scenario, R_{L2}, decreases in α. Thus, as more myopic consumers exist in the market, R_{L2}, keeps decreasing until it reach 0. Under low demand, the retailer employs a very low selling price in period 2 to sell all the left-over inventory. As the proportion of myopic consumers increases in the market, more myopic consumers purchase products in period 1, and more strategic consumers wait to purchase in period 2 as a result of the low selling price R_{L2}. The analysis conducted so far is not applicable any more if the selling price in period 2 is less than 0. Cases in which the selling price in period 2 equals 0 as discussed in the next section (4.2).

The single most striking observation to emerge from the comparison is that profit function decreases as the number of myopic consumers increases. As mentioned, this finding corroborates the ideas of Su (2007), who identified this as the result of scarcity and heterogeneity of consumers. As more myopic consumers exist in the market, fewer products are left for period 2, which further increases the selling price in period 2 so that fewer strategic consumers have the ability to purchase. The retailer at this point cares less about strategic consumers and defers more of them to period 2 by increasing R_1 and decreasing R_{L2} as α increases. Thus, with fewer strategic consumers, the retailer's potential profit from both periods 1 and 2 diminishes (recall that only strategic consumers may delay their purchase to the second period). Another possible explanation for this finding is that strategic waiting consumers may result in extra sales if a lower price is charged in the second period.

5.2 Case b: Inventory Depleted Completely in period 1 under High Demand

Recall that if either $\alpha \leq 0.5$ or $\delta > \delta_1$ and $\alpha > 0.5$, then $V_H > V_L > R_{H2}$. We analyzed this condition in the Clearance Sales strategy case. Next, we investigate what happens if δ is less than δ_1. When $\delta < \delta_1$ and $\alpha > 0.5$, $V_H < V_L < R_{H2}$. The selling price under high demand in period 2, R_{H2}, is irrelevant at this point since the entire inventory is sold in period 1. Therefore, the demand under high demand in period 1, D_{H1}, is equal to the inventory level, K, and the selling season ends right after period 1. This strategy is good for the retailer to shorten the shelf time and save the inventory cost, which may result in higher profit compared to the based case illustrated in Section 4.1.

Based on the analysis in section 4.12, the combining valuations of α and δ results in this situation. It happens when the proportion of myopic consumers is between 50% and 75% and the consumers' patience level is relatively low or the proportion of myopic consumers exceeds 75%. Since the patience level is very low for strategic consumers, and few strategic consumers can benefit from the waiting and purchasing in period 2, the retailer can charge a relatively low price for the product in period 1 to induce early purchases in period 1.

Since there is no second period if demand is high, V_H is not relevant, and strategic consumers consider only V_L, $V = V_L = \frac{R_1 - \delta R_{L2}(1-p)}{1-\delta(1-p)}$. Even though the sales under high demand is set to be equal to inventory level (K), we do not know if the retailer sells everything under low demand. Hence, we still need to explore whether inventory is completely depleted during the selling season or not.

As before, we get the corresponding demand functions under Skim Both. To distinguish this special case from the basic ones, we call it SB-AH.

Under Low demand:

$D_{L1}^{\text{SB-AH}} = \alpha N_L(1-R_1) + (1-\alpha)N_L(1-V) = \alpha N_L(V-R_1) + N_L(1-V)$ and $D_{L2}^{\text{SB-AH}} = (1-\alpha)N_L(V-R_{L2})$.

Under high demand:

$D_{H1}^{\text{SB-AH}} = K$ and $D_{H2}^{SB-AH} = 0$.

The profit function under low demand is given by $\Pi_L^{SB-AH} = R_1 D_{L1} + R_{L2} D_{L2} - cK$, and under high demand is $\Pi_H^{\text{SB-AH}} = R_1 D_{H1} - cK$. Thus, the retailer's total expected profit is $\Pi^{\text{SB-AH}} = p \cdot \Pi_H + (1-p) \cdot \Pi_L$.

Recalling that we assume that the retailer employs Clearance Sales strategy, the selling price in period 2, R_{L2}, is determined by the demand and the leftover inventory (i.e., $K = D_{L1} + D_{L2}$). Specifically, $R_{L2} = \frac{-2K-\alpha R_1+1}{1-\alpha}$

Since the inventory is sold completely in period 1 if demand is high, the selling price in period 2, R_1, is also determined by the demand and the leftover inventory (i.e., $K = D_{H1}$). Thus, $R_1 = -K - \frac{1}{2}K\delta + 1$.

The model is solved backward to yield the subgame perfect Nash equilibrium, which allows us to derive the equilibrium pricing policies. We identify subgame-perfect Nash equilibrium and show that, given the retailer's strategy, the equilibrium in the consumer subgame is unique.

The profit function is by:

$\Pi^{\text{SB-AH}} = \frac{K(-10K-3K\delta+8+8K\alpha+4\alpha K\delta-8\alpha-8c+8c\alpha)}{8(1-\alpha)}$.

Based on the profit function above, the profit function is concave in K, there is always an optimal inventory level exist.

Proposition 8 *Consider the Clearance Sales strategy in Case SB-AH where $\delta \leq \delta_1$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, where SB-AH is employed, the retailer's profit function is strictly concave in the inventory quantity stocked.*

Proof. In SB-AH case, $\frac{\partial^2 \Pi^{\text{SB-AH}}}{\partial K^2} = -\frac{(8+4\delta)\alpha-10-3\delta}{4(1-\alpha)}$. The denominator is positive since $\alpha \in (0.5, 1)$. And the numerator is strictly increasing in α; when $\alpha = 0.5$, the numerator $= -6 - \delta < 0$, and when $\alpha = 1$, the numerator$= -2 + \delta < 0$. Thus, the numerator is negative, $\frac{\partial^2 \Pi^{\text{SB-AH}}}{\partial K^2} < 0$. So $\frac{\partial^2 \Pi^{\text{SB-AH}}}{\partial K^2}$ is negative and the profit function is strictly concave in K. $\blacksquare$

The corresponding optimal stocking decision is $K = K^{\text{SB-AH}} \equiv \frac{4(1-c))(1-\alpha)}{10+3\delta-8\alpha-4\alpha\delta}$.

Substituting the K in the profit function: $\Pi^{\text{SB-AH}} = \frac{2(c-1)^2(1-\alpha)}{10+3\delta-8\alpha-4\alpha\delta}$, and the corresponding prices and threshold valuation are

$(R_1, R_{L2}) = (\frac{-4c+4c\alpha-6+4\alpha-2c\delta+2c\alpha\delta-\delta+2\alpha\delta}{-10-3\delta+8\alpha+4\alpha\delta}, \frac{-2-3\delta+4\alpha+2\alpha\delta-8c+4c\alpha+2c\alpha\delta}{-10-3\delta+8\alpha+4\alpha\delta})$;

$V = \frac{-3\delta+2\alpha\delta+2c\alpha\delta-4c+4c\alpha-6+4\alpha}{-10-3\delta+8\alpha+4\alpha\delta}$.

Proposition 9 *Consider the Clearance Sales strategy where $\delta \leq \delta_1$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as myopic consumers are more numerous in the market, (i) the capacity function K is strictly decreasing in α, which implies that as myopic consumers are more numerous in the market, the retailer chooses a lower capacity; (ii) the retailer gets less profit in case SB-AH.*

Proof. Follows immediately from $\frac{\partial \Pi^{\text{SB-AH}}}{\partial \alpha} = -\frac{2(1-c)^2(2-\delta)}{(-10-3\delta+8\alpha+4\alpha\delta)^2} < 0$
and $\frac{\partial K^{\text{SB-AH}}}{\partial \alpha} = -\frac{4(1-c)(2-\delta)}{(-10-3\delta+8\alpha+4\alpha\delta)^2} < 0$ ■

However, the retailer cannot always take the optimal stocking decision for K since the retailer may switch to Skim Myopic, or R_{L2} may reach zero as α increases; and either of them may happen first. The next step is to analyze the switching points under this case where all products are sold in period 1 under high demand.

When $R_{L2} = 0$, $K^{\text{SB-AH-RL}} \equiv \frac{2(1-\alpha)}{-4+2\alpha+\alpha\delta}$. In this case, the inventory decision is equal to the true demand so that the retailer does not have any leftovers, and it is denoted as SB-AH-RL for skim both case and SM-AH-RL for skim myopic case. The inventory decision should be equal to the smaller of $K^{\text{SB-AH}}$ and $K^{\text{SB-AH-RL}}$ for feasibility. Let α_3 denote the value of α which solves $K^{\text{SB-AH}} = K^{\text{SB-AH-RL}}$, and $\alpha_3 \equiv \frac{2+3\delta+8c}{2(2+\delta+2c+c\delta)} > 0.5$. $K^{\text{SB-AH}}$ is less than $K^{\text{SB-AH-RL}}$ when $\alpha < \alpha_3$, so α_3 is the value of α for the switching point of $R_{L2} = 0$.

Let α_4 denote the value of α which solves $V = 1$, and $\alpha_4 = \frac{2}{2+\delta} > 0.5$. We need to compare α_3 and α_4 to learn the switching sequence.

$\alpha_3 - \alpha_4 = \frac{3\delta-2+4c}{2(2+\delta)(1+c)}$ can be either positive or negative, so there is no fixed relationship between α_3 and α_4. When $\alpha_3 < \alpha_4$, the retailer switches from SB-AH to SB-AH-RL then SM-AH-RL as α increases. When $\alpha_3 > \alpha_4$, V reaches 1 before R_{L2} becomes 0, and the retailer switches from SB-AH to Skim myopic when he sells everything under high demand in period 1, which is denoted as SM-AH; then he switches from SM-AH back to SM-AH-RL.

Let δ_a denote the value of δ which solves $\alpha_3 = \alpha_4$, and $\delta_a \equiv \frac{2}{3} - \frac{4}{3}c$. The switching sequence is listed below as α increases.

$$\begin{cases} \delta \leq \delta_a & \text{SB-AH} \rightarrow \text{SB-AH-RL} \rightarrow \text{SM-AH-RL} \\ \delta > \delta_a & \text{SB-AH} \rightarrow \text{SM-AH} \rightarrow \text{SM-AH-RL} \end{cases}$$

There is a significant difference between the two conditions, which is demonstrated in more in detail in the next two subsections.

5.2.1 The Switching Sequence When $\delta \leq \delta_a$

Case SB-AH-RL

When $\delta \leq \delta_a$, $K = K^{\text{SB-AH-RL}}$ if $\alpha > \alpha_3$. So all demand functions are the same as in SB-AH. The only difference between SB-AH and SB-AH-RL is the inventory decision.

Substituting K in the profit function, $\Pi^{\text{SB-AH-RL}} = \frac{(1-\alpha)(6-3\delta-16c+8c\alpha+4c\alpha\delta)}{2(-4+2\alpha+\alpha\delta)^2}$.

The corresponding prices functions and threshold valuation function are $(R_1, R_{L2}) = (\frac{2-\delta}{4-2\alpha-\alpha\delta}, 0)$ and $V = \frac{2}{4-2\alpha-\alpha\delta}$.

Solving $V = 1$ in α, we have $\alpha = \frac{2}{2+\delta} = \alpha_4$, so when $\alpha > \alpha_4$, the retailer switches from SB-AH-RL to SM-AH-RL.

Proposition 10 *Consider the Clearance Sales strategy where* $\delta \leq \min\{\delta_1,\ \delta_a\}$ *and* $\alpha_3 \leq \alpha \leq \alpha_4$*, assuming* $p = \frac{1}{2}$ *and* $N_L = \frac{1}{2}$*, as more myopic consumers exist in the market, (i) the capacity function* K *is strictly decreasing in* α*, which implies that as more myopic consumers exist in the market, the retailer chooses a lower capacity; (ii) the retailer gets less profit in SB-AH-RL case.*

Proof. $\frac{\partial K^{\text{SB-AH-RL}}}{\partial \alpha} = -\frac{2(2-\delta)}{(-4+2\alpha+\alpha\delta)^2} < 0$

$\frac{\partial \Pi^{\text{SB-AH-RL}}}{\partial \alpha} = -\frac{(2-\delta)f_{11}}{2(4-2\alpha-\alpha\delta)^3}$, where $f_{11} = (3\delta + 4c\delta + 6 + 8c)\alpha - 6\delta - 16c$.

The denominator is positive since $4 > 2\alpha - \alpha\delta$.

f_{11} is increasing in α, and solve f_{11} in α, $\alpha_i \equiv \frac{2(3\delta+8c)}{3\delta+4c\delta+6+8c}$. Recall that SB-AH-RL can be implemented only if $\alpha > \alpha_3$. Since $\alpha_3 - \alpha_5 = \frac{3(2-\delta)}{2(2+\delta)(3+4c)(1+c)} > 0$, $\alpha_3 > \alpha_5$. Therefore, f_{11} is positive when $\alpha > \alpha_3$, so $\frac{\partial \Pi_{SB_{RL}}}{\partial \alpha} < 0$. ∎

Case SM-AH-RL

Since the retailer switches from SB-AH-RL to SM-AH-RL, the inventory decision is still $R_{L2} = 0$, and only myopic consumers decide to purchase in period 1, so the threshold valuation (V) is 1. The profits function remains the same and the updated demand functions are listed below.

Under low demand:

$D_{L1}^{\text{SM-AH-RL}} = \alpha N_L(1 - R_1)$ and $D_{L2}^{\text{SM-AH-RL}} = (1 - \alpha)N_L(1 - R_{L2})$.

Under high demand:

$D_{H1}^{\text{SM-AH-RL}} = K_2$ and $D_{H2}^{\text{SM-AH-RL}} = 0$.

As before, the selling price in period 2, R_{L2}, is determined by the demand and the leftover inventory (i.e., $K = D_{L1} + D_{L2}$).

$R_{L2} = \frac{-2K-\alpha R_1+1}{1-\alpha}$

Since the entire inventory is sold completely in period 1 if demand is high, the selling price in period 2, R_1, is also determined by the demand and the leftover inventory (i.e., $K = D_{H1}$). Thus, $R_1 = \frac{\alpha-K}{\alpha}$.

The profit function is given by:

$\Pi^{\text{SM-AH-RL}} = \frac{(-3K+2K\alpha+4\alpha-4\alpha^2-4c\alpha+4c\alpha^2)K}{4(1-\alpha)\alpha}$.

Recall that all products are sold during the selling season and $R_{L2} = 0$. So $K^{\text{SM-AH-RL}} \equiv 1 - \alpha$, which is solved from $R_{L2} = 0$.

Substituting K in the profit function, $\Pi^{\text{SM-AH-RL}} = \frac{(1-\alpha)(-3+6\alpha-4c\alpha)}{4\alpha}$. And the corresponding prices are $(R_1, R_{L2}) = (\frac{-1+2\alpha}{\alpha}, 0)$.

Proposition 11 *Consider the Clearance Sales strategy where $\delta \leq \min\{\delta_1,\ \delta_a\}$ and $\alpha > \alpha_4$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as more myopic consumers exist in the market, (i) the capacity function K is strictly decreasing in α, which implies that as more myopic consumers exist in the market, the retailer chooses a lower capacity; (ii) the retailer gets less profit in SM-AH-RL case.*

Proof. $\frac{\partial K^{\text{SM-AH-RL}}}{\partial \alpha} = -1 < 0$

$\frac{\partial \Pi^{\text{SM-AH-RL}}}{\partial \alpha} = \frac{f_{12}}{4\alpha^2}$, where $f_{12} = -6\alpha^2 + 3 + 4c\alpha^2$, and f_{12} is concave in α.

Solve f_{12} in α, and we get $\alpha = \pm\frac{3}{\sqrt{18-12c}}$. So f_{12} is positive when $\alpha \in [0, \frac{3}{\sqrt{18-12c}}]$, and negative when $\alpha \in (\frac{3}{\sqrt{18-12c}}, 1]$.

Recall that SM-AH-RL can be implemented only if $\alpha > \alpha_4$ and $\delta > \delta_a$. Recall that $\alpha_4 = \frac{2}{2+\delta}$ and $\delta_a = \frac{2}{3} - \frac{4}{3}c$, so $\alpha > \frac{3}{4-2c}$

Since $4 - 2c < \sqrt{18 - 12c}$, so $\frac{3}{4-2c} > \frac{3}{\sqrt{18-12c}}$. Thus, A is always negative when $\alpha \in (\frac{3}{4-2c}, 1]$. Therefore,$\frac{\partial \Pi_{SB_{RL}}}{\partial \alpha} < 0$. ∎

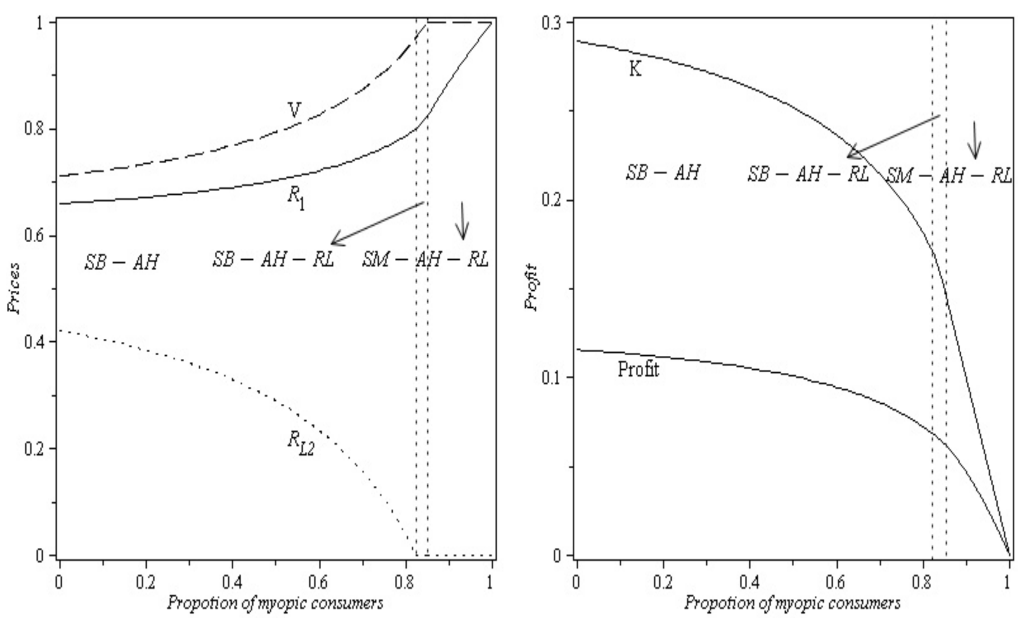

Figure 5.5: Optimal pricing policy and inventory level under Clearance Sale assuming AH is employed; $\delta = 0.35, c = 0.2$. Note that the profit decreases to 0 as α increases to 1, which is unlikely to be optimal.

When c equals to 0.1, $\delta_a = \frac{2}{3} - \frac{4}{3}c \mid_{c=0.1} = 0.4$. If the patience level is evaluated as 0.35

which is lower than δ_a, the selling price in period 2 under low demand decreases to zero before the threshold valuation reaches one. Therefore, the retailer employs SB-AH when α is between 0 and α_3; SB-AH-RL is adopted when α falls into the interval of $[\alpha_3, \alpha_4]$; SM-AH-RL is employed when α is larger than α_4. The results obtained from the preliminary analysis of case AH are shown in Figure 5.5. As can be seen from the Figure 5.5 below, only the selling prices under low demand is given for period 2 since all products are sold in period 1 under high demand.

There are clear trends of decreasing inventory level and profit function as the proportion of myopic consumer increases in Figure 5.5. The observed correlation between α and inventory level, K, might be explained in this way. Since all products are sold in period 1 under high demand, strategic consumers only have two options left: purchase in period 1 or purchase in period 2 only when demand is low. More strategic consumers will consequently consider purchasing in period 1. However, the consumer's patience level is at an intermediate low level. Intuitively, the threshold valuation is lower than the one in basic Skim Both and Skim Myopic cases. As more myopic consumers exist in the market, more products will be sold in period-1, leading to even fewer units left for period 2. Since the demand in period 1 is increasing, the selling price in period 1 is relatively high and increasing in α, which limits the demand in period 1. The total demand therefore decreases as α increases.

Even though the selling price in period 1 is higher as α increases, the extra incomes on products sold in period 1 is not going to compensate for sales lost because of the quantity of multi-units of strategic consumers. It is possible to hypothesis that these conditions cause the declining profit function trend when myopic consumers are more numerous in the market.

5.2.2 The Switching Sequence When $\delta > \delta_a$

SM-AH

When $\delta > \delta_a$, we switch from SB-AH directly to SM-AH while the selling price in period 2 under low demand, R_{L2}, is strictly positive. The switching point between SB-AH and SM-AH is solved from $\Pi^{\text{SB-AH}} = \Pi^{\text{SM-AH}}$, which is happen to be the same one as the one solved rom $V^{\text{SB-AH}} = 1$. So the switching point in α is α_4.

The demand functions are the same as in case SB-AH, but substituting V as 1. The profit function under low demand is given by $\Pi_L^{\text{SM-AH}} = R_1 D_{L1} + R_{L2} D_{L2} - cK$, and under high demand is $\Pi_H^{\text{SM-AH}} = R_1 D_{H1} - cK$. Thus, the retailer's total expected profit is $\Pi^{\text{SM-AH}} = p\Pi_H + (1-p)\Pi_L$.

As the process done in case SB-AH, we get $R_{L2} = \frac{-2K-\alpha R_1+1}{1-\alpha}$ and $R_1 = \frac{\alpha - K}{\alpha}$. The profit function is $\Pi^{\text{SM-AH}} = \frac{(-3K+2K\alpha+4\alpha-4\alpha^2-4c\alpha+4c\alpha^2)K}{4(1-\alpha)\alpha}$. Based on the profit function above, the profit function is concave in K, so there is always an optimal inventory level exist.

Proposition 12 *Consider the Clearance Sales strategy where $\delta_a < \delta < \delta_1$ and $\alpha > 0.5$, , assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the retailer's profit function is strictly concave in the inventory quantity stocked, which guarantees an optimal stocking decision in SM-AH case.*

Proof. $\frac{\partial^2 \Pi^{\text{SM-AH}}}{\partial K^2} = -\frac{3-2\alpha}{2(1-\alpha)\alpha} < 0.$

So $\frac{\partial^2 \Pi^{\text{SM-AH}}}{\partial K^2}$ is negative and the profit function is strictly concave in K. ■

The corresponding optimal stocking decision is $K^{\text{SM-AH}} \equiv \frac{2(1-c)(1-\alpha)\alpha}{3-2\alpha}$.

Substituting K in the profit function, $\Pi^{\text{SM-AH}} = \frac{\alpha(1-\alpha)(c-1)^2}{3-2c}$, and the corresponding prices and threshold valuation are $(R_1, R_{L2}) = (\frac{1+2c-2c\alpha}{3-2\alpha}, \frac{-4\alpha+2c\alpha+3}{3-2\alpha})$ and $V = 1$.

Proposition 13 *Consider the Clearance Sales strategy where $\delta_a < \delta < \delta_1$ and $\alpha > 0.5$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as more myopic consumers exist in the market, (i) the capacity function K is strictly decreasing in α, which implies that as more myopic consumers exist in the market, the retailer chooses a lower capacity; (ii) the retailer gets less profit in the SM-AH case.*

Proof. $\frac{\partial K^{\text{SM-AH}}}{\partial \alpha} = \frac{2(1-c)(2\alpha^2-6\alpha+3)}{(-3+2\alpha)^2} < 0.$

$\frac{\partial \Pi^{\text{SM-AH}}}{\partial \alpha} = \frac{f_{13}(1-c)^2}{(3-2\alpha)^2}$, where $f_{13} = (2\alpha^2 - 6\alpha + 3)$, and f_{13} is convex in α. Solve f_{13} in α, and we get $\alpha \equiv (2.366, 0.6340)$. Recall that the precondition of SM-AH is $\alpha > \alpha_4 = \frac{2}{2+\delta}$ and $\alpha_4 > \frac{2}{3}$. So f_{13} is negative when $\alpha \in [\alpha_4, 1]$. So $\frac{\partial \Pi^{\text{SM-AH}}}{\partial \alpha} < 0$. ■

When $\delta > \delta_a$, the switching point between SM-AH and SM-AH-RL is derived from the of selling price function in period 2 under low demand in case SM-AH. By solving $R_{L2} = 0$ in α, we have $\alpha_5 \equiv \frac{3}{2(2-c)}$.

When c is equal to 0.1, δ_a is 0.4. If the patience level is evaluated as 0.7 which is larger than δ_a, the threshold valuation reaches 1 before the selling price in period2 under low demand decreases to 0. The retailer employs SB-AH when α is between 0 and α_4; SM-AH is adopted when α falls into the interval of $[\alpha_4, \alpha_5]$; SM-AH-RL is employed when α is larger than α_5. The results obtained from the preliminary analysis of case AH are shown in Figure 5.6. Similar to Figure 5.5, only selling price under low demand is given for period 2 in Figure 5.6 since all products are sold in period 1 under high demand.

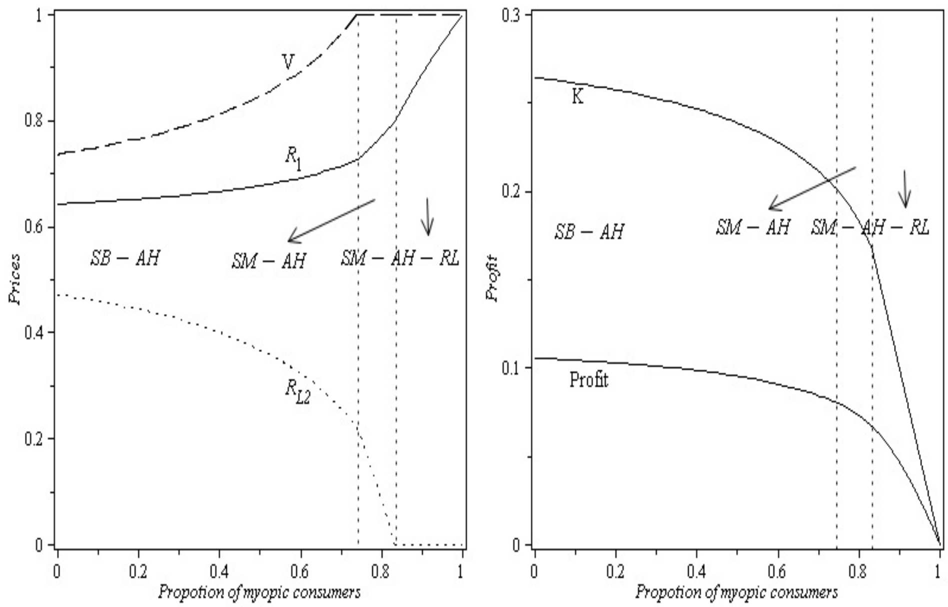

Figure 5.6: Optimal pricing policy and inventory level under Clearance Sale assuming AH is employed; $\delta = 0.7, c = 0.2$.

There are also clear trends of decreasing inventory level and profit function as the proportion of myopic consumer increases in Figure 5.6. The observed correlation between α and inventory level, K, follow the same arguments as in the previous segment of switches since they are both under case AH.

Next, we state the relationship between the patience level and retailer's profit function. Intuitively, we know that the profit should decrease if strategic consumers are more patient since they have a higher chance to wait and purchase in period 2, but still satisfy with their purchases.

Proposition 14 *Consider the Clearance Sales strategy in Case (b) where $\delta_a < \delta < \delta_1$ and $\alpha > 0.5$, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as strategic consumers are more patient in case*

AH, (i) the retailer either gets less profit under Skim Both cases; (ii) the retailer's profit is irrelevant in in δ under Skim Myopic cases.

Proof. In SB-AH,$\frac{\partial \Pi^{SB-AH}}{\partial \delta} = -\frac{2(c-1)^2(3-4\alpha)(1-\alpha)}{(-10-3\delta+8\alpha+4\alpha\delta)^2} < 0$.

In SB-AH-RL, $\frac{\partial \Pi^{SB-AH-RL}}{\partial \delta} = \frac{(1-\alpha)f_{14}}{2(4-2\alpha-\alpha\delta)^3}$, and the denominator is positive since $4 > 2\alpha - \alpha\delta$.

$f_{14} = -12 + (8c + 4c\delta)\alpha^2 + (18 - 3\delta - 16c)\alpha$, solve f_{14} in α, the roots are $\frac{-18+3\delta+16c\pm\sqrt{324-108\delta-192c+9\delta^2+288c\delta+256c^2}}{8(2+\delta)c}$. One root is negative and the other is between 0 and 1. Let's demote the positive root.

That is $\alpha_6 = \frac{-18+3\delta+16c+\sqrt{324-108\delta-192c+9\delta^2+288c\delta+256c^2}}{8(2+\delta)c}$. Recall that the precondition of SB_{RL} is $\alpha > \alpha_3$. Since $\alpha_3 - \alpha_6 > 0$, $\alpha_3 > \alpha_6$. Therefore, f_{14} is negative when $\alpha > \alpha_3$. So $\frac{\partial \Pi^{SB-AH-RL}}{\partial \delta} < 0$.

In SM-AH-RL,$\frac{\partial \Pi^{SM-AH-RL}}{\partial \delta} = 0$. In SM-AH, $\frac{\partial \Pi^{SM-AH}}{\partial \delta} = 0$.

profit function is therefore irrelevant in δ in SM-AH-RL and SM-AH cases. ■

The analysis has so far focused on the cases where supply was completely depleted. We have noticed that this issue as the selling price in period 1, R_1, is equal to the consumer's maximum willingness to pay, no consumers purchase in the first period when all consumers are myopic. Also, the retailer charges a price as 0 in period 2. Therefore, the retailer ends up in getting zero profit when all consumers in the market are myopic. The model where supply exceeds demand fills in the gap to resolve the problem.

5.3 Case c: Possible Leftover Inventory after Clearance Sales

If $D_{i2} < K - D_{i1}$, supply exceeds demand and some inventory remains after the selling season. This case is named after supply exceeds demand, denoted as SD. The demand functions for both Skim Both and Skim Myopic cases are the same as in the basic Skim Both and Skim Myopic models in section 4.12.

Since supply exceeds demand when demand is low, the selling price in period 2 under low demand is zero if the retailer releases all the inventory.

Recalling that under Clearance Sales strategy, the selling price in period 2 under high demand, R_{h2}, is determined by the demand and the leftover inventory (i.e., $K = D_{h1}+D_{h2}$). We have $(R_{H2}^{\text{SB-SD}}, R_{L2}^{\text{SB-SD}}) = (\frac{-K-\alpha R_1+1}{1-\alpha}, 0)$ and $(R_{H2}^{\text{SB-SM}}, R_{L2}^{\text{SB-SM}}) = (\frac{-K-\alpha R_1+1}{1-\alpha}, 0)$.

The selling price in period 1 is obtained from the first-order condition, which is to maximize the profit. For the first-order conditions, an optimal solution is

$R_1^{\text{SB-SD}} = \frac{-\alpha\delta-2\alpha+7\alpha K\delta-4\alpha K-3\delta+10-3K\delta-4K}{2(-3\alpha\delta-2\alpha+\alpha^2\delta+6)}$ and $R_1^{\text{SB-SM}} = \frac{5-\alpha-4K}{2(3-\alpha)}$.

The profit functions are $\Pi^{\text{SB-SD}} = \frac{A_2}{32(1-\delta)(3\alpha\delta+2\alpha-\alpha^2\delta-6)}$ and

$\Pi^{\text{SM-SD}} = \frac{-\alpha^2+\alpha+16cK\alpha+24K-24K^2-48cK}{16(3-\alpha)}$, where $A_2 = -4+12\alpha K\delta-112K+4\alpha+16\alpha K+12\delta-12\alpha\delta+132K\delta-64cK\alpha-192cK\delta+80K^2+192cK-72K^2\delta-32cK\alpha\delta-32cK\alpha^2\delta^2+32cK\alpha^2\delta+96cK\alpha\delta^2-30\alpha K\delta^2-72\alpha K^2\delta+57\alpha K^2\delta^2-9\delta^2+9\alpha\delta^2-18K\delta^2-9K^2\delta^2+16K^2\alpha$

Based on the profit function above, the profit function is concave in K, so there is always an optimal inventory level exist.

Proposition 15 *Consider the Clearance Sales strategy, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the retailer's profit function is strictly concave in the inventory quantity stocked, which guarantees an optimal stocking decision under case SD.*

Proof. Under Skim Both pricing, $\frac{\partial^2 \Pi^{SB-SD}}{\partial K^2} = -\frac{57\alpha\delta^2 - 72\alpha\delta + 16\alpha - 9\delta^2 - 72\delta + 80}{16(3-\alpha)(1-\delta)(2-\alpha\delta)}$, where the denominator is positive.

The numerator can be written as $f_{15} = a\delta^2 + b\delta + c$. Since $\frac{\partial^2 f_{15}}{\partial \delta^2}$ is $114\alpha - 18 > 0$ when $\alpha > 0.2$, function f_{15} is convex. The discriminant of f_{15} is $\Delta_{f_{15}} = 384(7-4\alpha)(3-\alpha) > 0$, f_{15} has two real number roots, which are $\frac{4(9+9\alpha+\sqrt{126-114\alpha+24\alpha^2})}{3(-3+19\alpha)}$ and $\frac{4(9+9\alpha-\sqrt{126-114\alpha+24\alpha^2})}{3(-3+19\alpha)}$. The two roots are both larger than 1 when $\alpha > 0.2$. So function f_{15} is positive. Therefore, function $\frac{\partial^2 \Pi^{SB-SD}}{\partial K^2}$ is negative, which means that Π_{SB} is strictly concave in the inventory quantity stocked.

Under Skim Myopic pricing, $\frac{\partial^2 \Pi^{SM-SD}}{\partial K^2} = -\frac{3}{3(1-\alpha)} < 0$ ■

The corresponding optimal stocking decision is

$$K = \begin{cases} \frac{(16\alpha^2\delta^2 - 16\alpha^2\delta - 48\alpha\delta^2 + 16\alpha\delta + 32\alpha + 96\delta - 96)c + 15\alpha\delta^2 - 6\alpha\delta - 8\alpha + 9\delta^2 - 66\delta + 56}{57\alpha\delta^2 - 72\alpha\delta + 16\alpha - 9\delta^2 - 72\delta + 80} & \text{if } \delta < \delta_3 \\ \frac{1}{3}c\alpha - c + \frac{1}{2} & \text{otherwise.} \end{cases}$$

Substituting K in the profit function, we have

$$\Pi^{\text{SB-SD}} = \frac{(24\alpha\delta^2 - 8\alpha\delta + 8\alpha^2\delta + 48 - 48\delta - 8\alpha^2\delta^2 - 16\alpha)c^2 + (-56 - 9\delta^2 + 6\alpha\delta + 66\delta - 15\alpha\delta^2 + 8\alpha)c - 27\delta + 9\delta^2 + 18}{57\alpha\delta^2 - 72\alpha\delta + 16\alpha - 9\delta^2 - 72\delta + 80}$$

and $\Pi^{\text{SM-SD}} = -\frac{1}{6}c^2\alpha + \frac{1}{16}\alpha + \frac{1}{8} - \frac{1}{2}c + \frac{1}{2}c^2$,

and the corresponding strategic consumer critical valuation is

$$V = \begin{cases} \frac{36c\alpha\delta^2 - 9\delta^2 - 72c\alpha\delta - 36\delta - 24c\delta + 48 + 32c\alpha + 32c}{57\alpha\delta^2 - 72\alpha\delta + 16\alpha - 9\delta^2 - 72\delta + 80} & \text{if } \alpha < \alpha_x \\ 1 & \text{otherwise.} \end{cases}$$

α_x is solve from $\Pi^{\text{SB-SD}} = \Pi^{\text{SM-SD}}$ in α.

And $\alpha_7 \equiv \frac{4(6c\delta-9\delta-8c+8)}{36c\delta^2-57\delta^2-72c\delta+72\delta+32c-16}$

The model should satisfy the condition, which is that supply exceeds demand. We therefore need to test whether the inventory level, K, is larger or equal to the actual sales under low demand. By solving $D_{L1}^{SB-SD} + D_{L2}^{SB-SD} = K$ in α, we get $\alpha_8 \equiv \frac{A_3}{16c(11\delta^2-15\delta+4)}$.

$A_3 = 120c\delta^2 + 3\delta^2 - 24c\delta + 12\delta - 96c - 16 + (256 - 112128c^2\delta + 33792c^2 - 1024c - 384\delta - 8784c\delta^4 + 21504c\delta - 48512c\delta^2 + 36816c\delta^3 + 9\delta^4 + 14400c^2\delta^4 - 73344c^2\delta^3 + 137280c^2\delta^2 + 48\delta^2 + 72\delta^3)^{\frac{1}{2}}$

So case SB-SD is only feasible when the proportion of myopic consumers is larger than α_8. Similarly, we solve $D_{L1}^{SM} + D_{L2}^{SM} = K$ in α, and get $\alpha_9 \equiv \frac{12c}{3+8c}$. Hence, the case SM-SD is only feasible when the proportion of myopic consumer is larger than α_9.

Proposition 16 *Consider the Clearance Sales strategy where $\alpha > \alpha_8$ in Skim Both and $\alpha > \alpha_9$ in Skim Myopic, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as more myopic consumers exist in the market, (i) the capacity function K is strictly increasing in α, which implies that as more myopic consumers exist in the market, the retailer chooses a lower capacity; (ii) the retailer gets more profit in SD case.*

Proof. $\frac{\partial K^{SB-SD}}{\partial \alpha} = \frac{1}{16} - \frac{1}{6}c^2 > 0$ since $c \in [0, 0.5]$.

$\frac{\partial \Pi^{SB-SD}}{\partial \alpha} = \frac{(1-\delta)f_{16}}{(57\alpha\delta^2-72\alpha\delta+16\alpha-9\delta^2-72\delta+80)^2}$ where $f_{16} = (456c^2\delta^3 - 576c^2\delta^2 + 128c^2\delta)\alpha^2 + (-144c^2\delta^3 - 1152c^2\delta^2 + 1280c^2\delta)\alpha + 513\delta^3 + 2688c^2\delta - 648c\delta^3 + 216c^2\delta^3 - 3648c\delta + 2736c\delta^2 - 864c^2\delta^2 - 288 - 2048c^2 - 1674\delta^2 + 1440\delta + 1536c$

Since $\frac{\partial^2 f_{16}}{\partial\alpha^2} = 16c^2\delta(57\delta^2 - 72\delta + 16)$, $\frac{\partial^2 K}{\partial\alpha^2} > 0$ when $\delta \in [0, \frac{12}{19} - \frac{8}{57}\sqrt{6}]$ and $\frac{\partial^2 A}{\partial\alpha^2} < 0$ when $\delta \in (\frac{12}{19} - \frac{8}{57}\sqrt{6}, \frac{12}{19} + \frac{8}{57}\sqrt{6}]$.

Function f_{16} is convex when $\delta \in [0, \frac{12}{19} - \frac{8}{57}\sqrt{6}]$ and function f_{16} is concave when $\delta \in (\frac{12}{19} - \frac{8}{57}\sqrt{6}, \frac{12}{19} + \frac{8}{57}\sqrt{6}]$. The two roots of function f_{16} solved in α are both negative when $\delta \in [0, \frac{12}{19} - \frac{8}{57}\sqrt{6}]$. When $\delta \in (\frac{12}{19} - \frac{8}{57}\sqrt{6}, \frac{12}{19} + \frac{8}{57}\sqrt{6}]$, one root is positive and the other is negative. Thus, function f_{16} is always positive. And $\frac{\partial\Pi^{SB-SD}}{\partial\alpha} > 0$.

$\frac{\partial K^{SM-SD}}{\partial\alpha} = \frac{1}{16} - \frac{1}{6}c^2 > 0$ since $c \in [0, 0.5]$.

$\frac{\partial\Pi^{SM-SD}}{\partial\alpha} = \frac{c}{3} > 0$. ■

Figure 5.7 shows how prices, inventory levels and corresponding profits change as the proportion of myopic consumers increases under the Clearance Sale strategy where supply exceeds demand. In this figure, we can also notice that the threshold valuation is higher than the selling price in period 2 under high demand (R_{H2}). Note that $R_{H2} > R_1 > R_{L2}$ under SD. The graph shows that prices steady increases as the proportion of myopic consumer increases in the Skim Both cases, whereas the prices are irrelevant with α in the Skim Myopic case. The cause of this difference is the adoption of a different pricing policies. Recall that the retailer sells all inventory during the selling season if the demand level is high and sells the leftover in the price of 0 in period 2 under low demand level since supply exceeds demand in the second period.

The intuition for the increasing prices in α is straightforward. When more myopic consumers exist in the market, more consumer will purchase in period 1 which improve the competition between consumers for available inventory units. In Skim Myopic cases, all strategic consumers wait to purchase in period 2 and they either pay a higher price compared to period 1 or get the product for free. This situation happens when the proportion of myopic consumers is relatively high in the market, so when α is high enough, the retailer manipulates prices to defer all strategic consumers to period 2 and charges a constant price in this interval of α to capture the profit from strategic consumers.

Interestingly, some strategic consumers purchase in period 1 under Skim Both case even though they know that they have the chance to get the product in period 2 for free if the demand level is low. Those strategic consumers do so to avoid the risk of purchasing the product in a much higher price in period 2 under high demand, which is a logical and reasonable strategy.

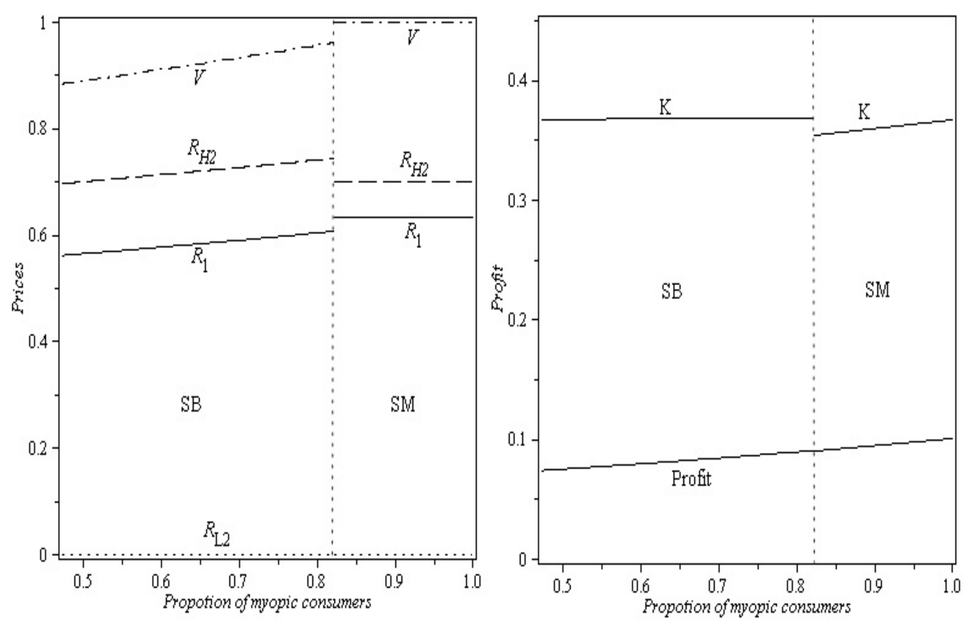

Figure 5.7: Optimal pricing policy and inventory level under Clearance Sale assuming SD is employed; $\delta = 0.6, c = 0.2$.

Compared with the analysis in basic Skim Both and Skim Myopic as well as AH, the analysis conducted in this section can let the retailer obtain more profit if the proportion of myopic consumers is high in the market. The proposition above proves that the profit function is strictly increasing in α. In Skim Both, the inventory stocked by the retailer barely changes as α increases, however, the prices is steady increasing as the myopic consumers are more numerous in the market. Therefore, the retailer obtains more profit as α increases in Skim Both case. By analyzing the Skim Myopic only, we know that the retailer will isolate myopic consumers from strategic consumers so that he can price differently for the two groups. Therefore, the retailer can capture the profit from both groups by pricing discrimination.

Proposition 17 *Consider the Clearance Sales strategy in Case (c) where $\alpha > \alpha_y$ in Skim Both and $\alpha > \alpha_Z$ in Skim Myopic, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, as strategic consumers are more patient when supply exceeds demand, (i) the retailer gets less profit under Skim Both case; (ii) the retailer's profit is irrelevant in δ under Skim Myopic case.*

Proof. In Skim Both,$\frac{\partial \Pi^{SB-SD}}{\partial \delta} = -\frac{(1-\alpha)f_{17}}{(f_{18})^2}$,where f_{18} is a function of α and δ , and $f_{17} = (20c\alpha\delta - 16c\alpha - 36 + 16c + 33\delta - 24c\delta)(6c\alpha\delta - 8c\alpha - 24 + 24c + 27\delta - 18c\delta)$.

Since$\frac{\partial^2 f_{17}}{\partial \alpha^2} = 2(20c\delta - 16c)(6c\delta - 8c) > 0$, f_{17} is convex. Since the discriminant of function f_{17} is larger than 0, function f_{17} has two same real roots which are $(\frac{36-16c-33\delta+24c\delta}{4(5\delta-4)c}, \frac{3(8-8c-9\delta+6c\delta)}{2(3\delta-4)c})$. Both roots are less than zero, so function f_{17} is positive. Therefore, $\frac{\partial \Pi^{SB-SD}}{\partial \delta} < 0$ in Skim Both case.

In Skim Myopic,$\frac{\partial \Pi^{SB-SD}}{\partial \delta} = 0$, so profit function is irrelevant in δ under Skim Myopic case. ■

How the profit changes as consumer's patience level increases is demonstrated in the proposition above. The retailer will lose profit if the patience level of strategic consumers increases under Skim Both cases.

5.4 Summary of Clearance Sale Strategy

Three parts of the Clearance Sale strategy have been analyzed separately in the previous sections. It would be hard to interpret the models and pricing policy separately. Thus, we need to combine the different elements of the Clearance Sale strategy to draw a full picture of it. However, with different combination of parameter valuations for α, δ, and c, caution must be applied, as different models may be feasible in the same range and the optimal one with higher profit should be chosen by the retailer. Since it is hard to analyze the model based on parameter ranges, numerical examples are give below.

The first example is given in $\delta = 0.35$ and $c = 0.2$ in Figure 5.8. As discussed above, we know that basic Skim Both and Skim Myopic, SB-AH→SB-AH-RL→SM-AH-RL under AH and SD models are feasible in this region of parameters. By comparing the profit functions of those models, the dominated ones are employed based on different intervals of α. As shown in Figure 5.8 when $\delta = 0.35$ and $c = 0.2$, the basic Skim Both model is adopted when less than 0.56 of consumers are myopic. When the proportion of myopic consumers is between 0.56 and 0.64, the retailer employs the model of SB-AH to sell everything in period 1 under high demand since over half consumers in the market are myopic and strategic consumer's patience level is relatively low. When the proportion of myopic consumers is sufficiently high (above 0.64 in this instance), the Skim Both model

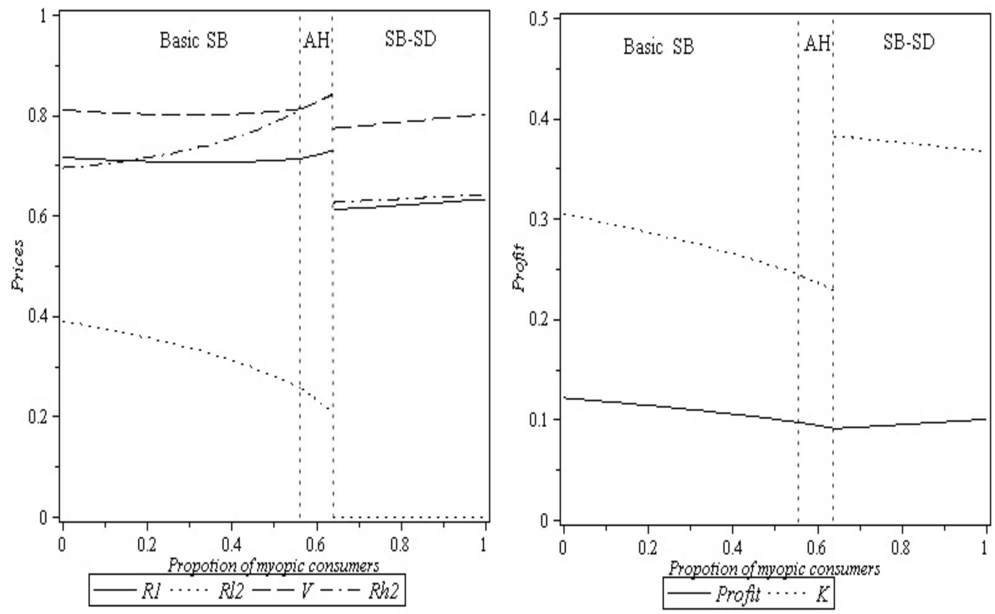

Figure 5.8: An example of combination of Clearance Sale strategy in the case of $\delta = 0.35, c = 0.2$.

under SD strategy will be adopted. The retailer stocks much more inventory compared to the basic Skim Both and SB-AH and charges a lower price in both periods to capture less profit from individual consumers, but sells much more stock.

Now, let us increase the strategic consumers' patience level to $\delta = 0.7$. This case is illustrated in Figure 5.9. Following analysis steps similar to those in the last example, we find that the best choice for the retailer when α is small is still basic Skim Both. However, the retailer should switch from basic Skim Both into Skim Myopic directly if α is larger than 0.7.

Numerically, the two combinations of choices for the retailer mostly dominate all parameter intervals. One may notice that the trends of inventory levels under sufficiently

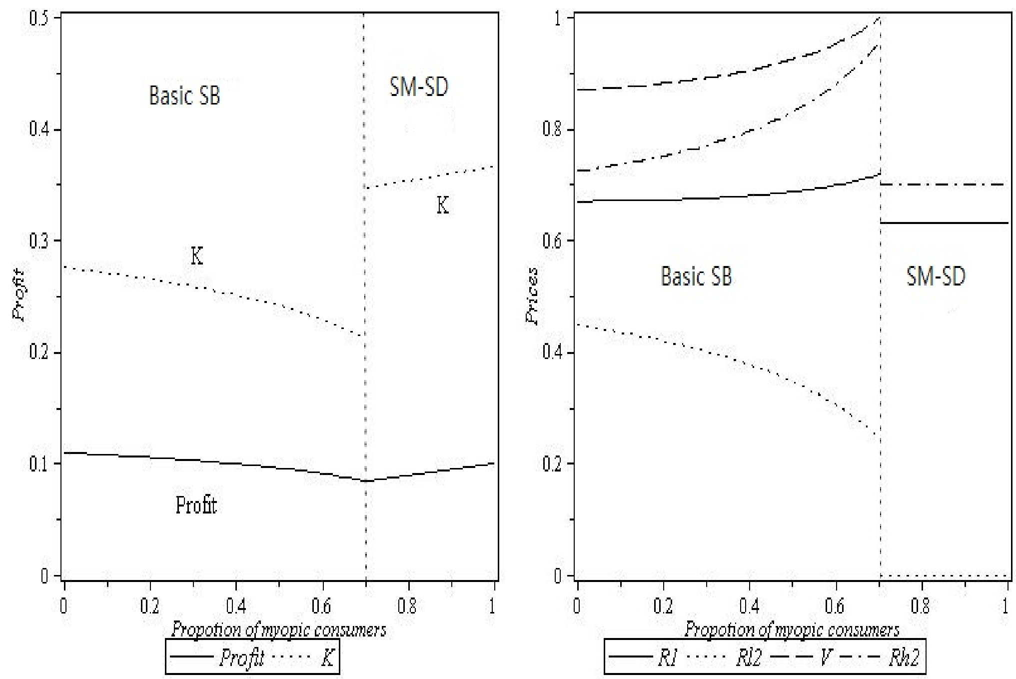

Figure 5.9: An example of combination of Clearance Sale strategy in the case of $\delta = 0.7, c = 0.2$.

high α are different in Figures 5.8 and 5.9. At intermediate low level of δ ($\delta = 0.35$), K decreases in α when supply exceeds demand. Moreover, the retailer uses SB-SD under this parameter level. As more myopic consumers exist in the market, fewer and fewer strategic consumers will wait for period 2 since delta is not high, and retailer keeps less inventory to save the inventory cost. And retailer can also charge a higher price within a limited inventory level to capture higher profit. However, when δ equals to 0.7, K is increasing in α because the retailer employs Skim Myopic. All strategic consumers wait for period 2. Even though sales number (realized demand) in period 2 is decreasing in α. But the sales number in the first period is significantly increasing in α. Also the prices are irrelevant with alpha in Skim Myopic. That is why retailer has an increasing K in α when patience level is high.

Consider the cases of basic SB/SM and AH under the Clearance Sales strategy, the retailer's profit function is strictly concave in the inventory quantity stocked. which guarantee an optimal inventory decision. Also the capacity function K is strictly decreasing in α, which implies that as myopic consumers are more numerous in the market, the retailer chooses a lower capacity. Also, as the proportion of myopic consumers increases in the market, the retailer gets less profit. As we stated before, this finding counters common intuition that strategic customers who consider all available purchase choices hurt retailer's profit. Su (2007) identified finding as the result of scarcity and the heterogeneity of consumers. In our model, with fewer strategic consumers, the retailer's potential profit from both period 1 and period 2 diminishes.

Another possible explanation for this behavior is that the retailer will not lose the strategic consumers immediately if he charges a very high price in period 1, and extra profit may be obtained from strategically waiting consumers if a lower price is charged in the second period. Considering the sales lost from strategic consumers, the retailer stocks less as more myopic consumers exist in the market. Therefore, these variations may result in a corresponding decrease of profit in α. Cachon and Swinney (2009) also find that a firm stocks less with strategic customers than without them in the uncertain demand case. This finding has important implications for developing the dynamic pricing theory since strategic consumers' behavior may actually benefit the retailer. We note that in the cases of basic SB/SM and AH, the retailer ends getting zero profit when all consumers are myopic. This seems unlikely the optimal case, which also leads us to the case SD.

In Skim Both case, the inventory stocked by the retailer barely changes as α increases, however, the prices is steady increasing as the myopic consumers are more numerous in the market. Therefore, the retailer obtains more profit as α increases in Skim Both case. By analyzing the Skim Myopic only, we know that the retailer will isolate myopic consumers from the strategic consumers so that he can price differently for those two groups. Therefore, the retailer can capture the profit from both part by pricing discrimination.

Case SB-SD is only feasible when the proportion of myopic consumer is larger than α_8. Similarly, the case SM-SD is only feasible when the proportion of myopic consumer is larger than α_9. Therefore, the retailer only consider the basic SB/SM and AH when α is relatively small. Moreover, if basic SB/SM or AH models is employed, the retailer obtains zero profit when all consumers are myopic. The retailer has to adopt SD to profit. Then we know a switch must occur for a certain value of α, hence up to this value of α, retailer's profit decreases in α, thereafter his profit increases in α.

As strategic consumers are more patient in cases of basic Skim Both and Skim Myopic, the retailer's profit strictly decreases. However, as strategic consumers are more patient in cases of AH and SD, the retailer gets less profit under Skim Both cases and the retailer's profit is irrelevant in in δ under Skim Myopic cases. The retailer isolates myopic consumers from the strategic consumers in Skim Myopic so that he can price differently for those two groups. The retailer ignores strategic consumers' patience level when he makes the pricing decisions. Hence, the profit function is irrelevant with δ.

Chapter 6

Optimal Release Strategy

In the previous section, the Clearance Sales strategy was studied in isolation. In this section, we assume that Clearance Sales strategy is one available inventory release strategy among other options. To find out the optimal release strategy, we study the Dynamic Pricing strategy and Intermediate Supply strategy in this section. Of the three strategies, the prevailing optimal release strategy is the one that maximizes the retailer's profit.

6.1 Dynamic Pricing Strategy

In this setting, the retailer ignores capacity constraints during the second period of the selling horizon and determines an original inventory ,K, that can satisfies the demand under both N_H and N_L. However, after the first period demand is realized, the left-over inventory is released optimally to maximize the profit. The retailer would like to keep a proportion of the left-over inventory rather than sell items at a much lower price. In this section, we conduct the analysis for both Skim Both and Skim Myopic cases, but we find, as we show later, that Skim Myopic is not feasible in this setting.

The realized demand functions are the same as the ones in the setting of Clearance Sales strategy. We calculate the selling prices in period 2 differently here than we do for the Clearance Sales strategy. The model is also solved backwards to yield subgame perfect Nash equilibrium, which allows us to derive equilibrium pricing policies. Since the demand in period 1 is already realized, the selling prices in period 2 are determined to maximize the profit in period 2 only. We have $R_{H2} = R_{L2} = \frac{2R_1}{4-\delta} < R_1$.

Then the selling price in period 1 is obtained from the first-order condition of the profit function: $R_1 = \frac{(\delta-4)^2(\delta-1)}{2(\alpha\delta^3-6\alpha\delta^2+10\alpha\delta-4\alpha-3\delta^2+14\delta-12)}$.

And $\Pi^{\text{SB-DP}} = \frac{(16c\alpha\delta^3-96c\alpha\delta^2+160c\alpha\delta-64c\alpha-48c\delta^2+224c\delta-192c)K-3\delta^3+27\delta^2-72\delta+48}{16(-\alpha\delta^3+6\alpha\delta^2-10\alpha\delta+4\alpha+3\delta^2-14\delta+12)}$.

Since $\frac{\partial\Pi^{\text{SB-DP}}}{\partial K} = -c < 0$, the profit strictly decreases in K. In the Dynamic Pricing strategy setting, the retailer needs to have enough leftover inventory in period 2 to satisfy the demand under both high demand, D_{H2}, and low demand, D_{L2}, as he releases inventory to maximize his profit in period 2. Thus, the retailer needs to choose a relatively high inventory level to satisfy this condition. Therefore, the retailer needs to choose the sales under high demand to ensure the availability of Dynamic Pricing strategy. The inventory level is given below:

$K = D_{H1} + D_{H2} = \frac{-5\alpha\delta^2+6\alpha\delta+\alpha\delta^3-4\delta^2+18\delta-16}{2(\alpha\delta^3-6\alpha\delta^2+10\alpha\delta-4\alpha-3\delta^2+14\delta-12)}$

Substitute K in the profit function:

$\Pi^{\text{SB-DP}} = \frac{-8c\alpha\delta^3+40c\alpha\delta^2-48c\alpha\delta+3\delta^3+32c\delta^2-27\delta^2-144c\delta+72\delta+128c-48}{16(\alpha\delta^3-6\alpha\delta^2+10\alpha\delta-4\alpha-3\delta^2+14\delta-12)}$, and the corresponding strategic consumer critical valuation is

$V^{\text{SB-DP}} = \frac{(4-3\delta)(4-\delta)}{2(-\alpha\delta^3+6\alpha\delta^2-10\alpha\delta+4\alpha+3\delta^2-14\delta+12)}$ and

$R_{H2} = R_{L2} = \frac{(4-\delta))(1-\delta)}{-\alpha\delta^3+6\alpha\delta^2-10\alpha\delta+4\alpha+3\delta^2-14\delta+12}$.

In the Skim Myopic case, we get the optimal prices of $\frac{1}{2}$ for both periods, and the corresponding cut-off valuation V is equal to $\frac{1}{2}$, which violates the condition for Skim Myopic to hold $V = 1$. This situation implies that the case of Skim Myopic is not realizable under this setting, so we only have case Skim Both in the Dynamic Pricing strategy setting.

Since the second period price is always lower than the first period price, more strategic consumers are motivated to wait and purchase in period 2. So the retailer would get less profit than in the Clearance Sales strategy setting. Also, as the patience level increases, more strategic consumers wait for period 2, to maximize their expected surpluses.

Proposition 18 *When Dynamic Pricing strategy is employed, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the retailer's profit is decreasing in strategic consumers' patience, δ.*

Proof. $\frac{\partial \Pi^{\text{SB-DP}}}{\partial \delta} = -\frac{(1-\alpha)f_{20}}{16 f_{19}^2}$, where f_{19} is some function of α and δ, and where $f_{20} = 8(\delta^2 - 4\delta + 6)(\delta - 2)^2 c\alpha + (-16\delta^2 + 64)c + 3(\delta - 4)(3\delta^3 - 16\delta^2 + 26\delta - 16)$.

Function f_{20} is strictly increasing in α since $8(\delta^2 - 4\delta + 6)(\delta - 2)^2 c > 0$. Therefore, we estimate f_{20} at $\alpha = 0$ to show $f_{20} \mid_{\alpha=0} > 0$, which guarantees f_{20} is always positive, and hence $\frac{\partial \Pi_{SB}}{\partial \delta} < 0$.

$f_{20} \mid_{\alpha=0} = (-16\delta^2 + 64)c + 3(\delta - 4)(3\delta^3 - 16\delta^2 + 26\delta - 16)$ is strictly increasing in c, so we further evaluate f_{20} at c. $f_{20} \mid_{c=0} = 3(\delta - 4)(3\delta^3 - 16\delta^2 + 26\delta - 16) > 0$. Thus, $f_{20} > 0$ for $\alpha \geq 0$ and $c \geq 0$ implying as required. ■

Under Clearance Sales strategy, we found that Π may increase in the proportion of strategic consumers. Does the same hold true under Dynamic Pricing?

Proposition 19 *When Dynamic Pricing strategy is employed, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, if $c < \frac{3}{8}$ and $\delta < -\frac{4}{3}c + 2 - \frac{1}{3}\sqrt{16c^2 + 18}$, then the retailer's profit is decreasing in α; otherwise, the retailer's profit is increasing in α.*

Proof. $\frac{\partial \Pi^{\text{SB-DP}}}{\partial \alpha} = \frac{(1-\delta)(2-\delta)(\delta-4)^2 f_{22}}{16 f_{21}^2}$, where f_{21} is some function of α and δ, and $f_{22} = 3\delta^2 + 8c\delta - 12\delta - 16c + 6$. Since $\frac{\partial^2 f_{22}}{\partial \delta^2} = -6$, f_{22} is concave in δ and function f_{22} has two distinct real roots, which are $f_{22} = -\frac{4}{3}c + 2 \pm \frac{1}{3}\sqrt{16c^2 + 18}$. Note $-\frac{4}{3}c + 2 + \frac{1}{3}\sqrt{16c^2 + 18} > 1$ and $0 < -\frac{4}{3}c + 2 - \frac{1}{3}\sqrt{16c^2 + 18} < 1$.

Set $\delta_5 \equiv -\frac{4}{3}c + 2 - \frac{1}{3}\sqrt{16c^2 + 18}$, and δ_5 is decreasing in c. Note $\delta_5 \leq 0$ if $c \geq \frac{3}{8}$. Function f_{22} is negative in the region of $[0, \delta_5]$, and positive in the region of $[\delta_5, 1]$ if $c < \frac{3}{8}$. Therefore, $\frac{\partial \Pi^{\text{SB-DP}}}{\partial \alpha}$ is negative $c < \frac{3}{8}$ and $\delta < -\frac{4}{3}c + 2 - \frac{1}{3}\sqrt{16c^2 + 18}$; otherwise, $\frac{\partial \Pi^{\text{SB-DP}}}{\partial \alpha}$ is positive. ■

The proposition above states that the profit is decreasing in α when the patience level is relatively low and the unit cost is not too high. As more myopic consumers exist in the market, more consumers purchase in period 1 by paying a higher price compared to the price in period 2. We also need to consider the effect of patience level, δ. If the strategic consumers have a low patience level, less strategic consumers choose to wait for period 2 and pay a higher price in period 1. As α increases, only a very small portion of strategic consumers choose to wait. In addition, the retailer decreases R_1 to induce early purchase by strategic consumers. Therefore, profit decreases due the decreasing prices in both periods.

If the patience level is high, more strategic consumers wait to purchase in the second period since the purchasing cost is also lower than in period 1. The retailer expects outcomes before the selling season, so he sets the selling price in period 1 lower (but still higher than the selling prices in period 2) to encourage strategic consumers to purchase in period 1. But the retailer obtains lower profit in this condition compared to when consumers' patience level is low overall since myopic consumers also pay a low price in period 1. Two examples are given in Figure 6.1 to demonstrate the results of the previous discussion. The differences are shown by employing same value of c, but different values of δ. In panel (a), the prices, threshold valuation and profit decrease in α when δ is low; while in panel (b), all of them increases in α when δ is high. Also, we notice that the threshold valuation under high patience level is much higher than the one under low patience level.

6.2 Intermediate Supply Strategy

After first period demand is realized, the leftover inventory is released to maximize the profit under low demand, and all leftover inventory is sold under high demand. The realized demand functions are the same as the ones in the setting of Clearance Sales strategy. We find the selling prices in period 2 differently under low demand than we do in the Clearance Sales strategy. The model is also solved backwards to yield the subgame perfect Nash equilibrium, which allows us to derive the equilibrium pricing policies. Since the demand in period 1 is already realized, the selling price in period 2 under low demand is determined to maximize the profit. Then $R_{H2} = \frac{1-\alpha R_1 - K}{1-\alpha}$ and $R_{L2} = \frac{-\delta+\alpha\delta R_1+K\delta+2R_1-2\alpha R_1}{2(1-\alpha)(2-\delta)}$. The selling price in period 1 is solved backward by taking the first-order condition of the

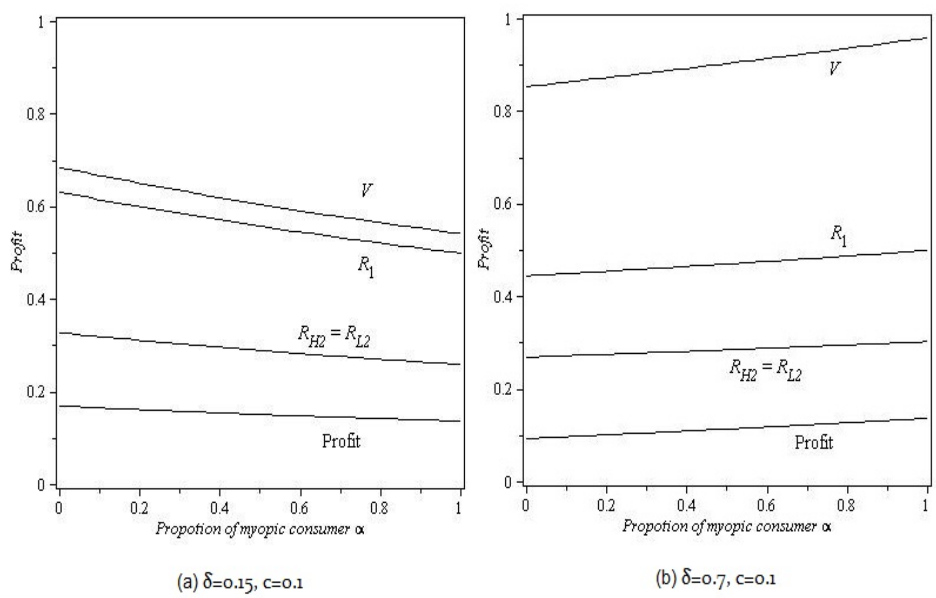

Figure 6.1: Two examples of profits and correponding prices and threshold valuations under Optimal Release strategy.

profit function, we have $R_1 = \frac{(-9\delta^2+12\alpha\delta^2-2\delta-30\alpha\delta+16\alpha+16)K-3\delta^2-6\alpha\delta+38\delta-40+8\alpha}{3\alpha^2\delta^2-6\alpha\delta^2-8\alpha^2\delta+4\alpha\delta+36\delta+4\alpha^2-44+8\alpha}$. And $\Pi^{IS} = \frac{f_{23}}{32(1-\delta)(2-\delta)((3\delta^2+4-8\delta)\alpha^2+(-6\delta^2+4\delta+8)\alpha+36\delta-44)}$, where $f_{23} = (1152 - 2496\delta - 144\delta^3 + 384\alpha + 1568\delta^2 - 840\alpha\delta^3 + 1648\alpha\delta^2 + 153\alpha\delta^4 - 1344\alpha\delta - 81\delta^4)K^2 + (-96c\alpha^2\delta^4 + 4864c\delta^2 - 176\alpha\delta^2 + 3744\delta + 192c\alpha\delta^4 - 18\alpha\delta^4 - 1152c\delta^3 + 544c\alpha^2\delta^3 + 96\alpha\delta^3 - 54\delta^4 - 704c\alpha\delta^3 - 256\alpha^2c - 6528c\delta + 2816c - 3040\delta^2 + 512c\alpha\delta - 1536 + 96\alpha\delta + 512c\alpha\delta^2 + 888\delta^3 - 1088c\alpha^2\delta^2 - 512c\alpha + 896\alpha^2c\delta)K - 192 + 9\alpha\delta^4 + 192\alpha - 400\delta^2 + 120\delta^3 - 120\alpha\delta^3 - 480\alpha\delta + 480\delta - 9\delta^4 + 400\alpha\delta^2$. The corresponding threshold valuation is

$V = \frac{((15K\delta^3-62K\delta^2+2\delta^2+16\delta+80K\delta-32K-3\delta^3-16)\alpha+42\delta^2-18K\delta^2+80-120\delta+48K\delta-32K)(3\delta-4)}{4(1-\delta)(2-\delta)(3\alpha^2\delta^2-6\alpha\delta^2+36\delta+4\alpha\delta-8\alpha^2\delta+4\alpha^2-44+8\alpha)}$.

Based on the profit function above, the profit is quadratic in K.

Proposition 20 *When SB-IS is employed, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the retailer's profit function is strictly concave in the inventory quantity stocked, which guarantees an optimal stocking decision for the market clearing strategy.*

Proof. Under Skim Both pricing, $\frac{\partial^2 \Pi^{IS}}{\partial K^2} = \frac{f_{24}}{16(1-\alpha)(1-\delta)f_{25}}$, where $f_{25} = -6\alpha\delta^2 + 3\alpha^2\delta^2 + 36\delta - 8\alpha^2\delta + 4\alpha\delta - 44 + 4\alpha^2 + 8\alpha$.

The numerator $f_{24} = (153\delta^4 - 840\delta^3 + 1648\delta^2 - 1344\delta + 384)\alpha - 81\delta^4 - 144\delta^3 + 1568\delta^2 - 2496\delta + 1152$. Since function f_{24} is in a linear relationship in α, function f_{24} is in the range of $[f_{24|\alpha=0},\ f_{24|\alpha=1}]$.

$f_{24|\alpha=0} = -81\delta^4 - 144\delta^3 + 1568\delta^2 - 2496\delta + 1152 > 0$ and $f_{24|\alpha=1} = 24(2-\delta))(1-\delta)(3\delta^2 - 32\delta + 32) > 0$, since both are positive, f_{24} is positive as well.

Now consider f_{25}, which concave in δ (since $\frac{\partial^2 f_{25}}{\partial \delta^2} = -6\alpha(2-\alpha) < 0$). Since the discriminant of function f_{25} is $\Delta_{f_{25}} = 16(\alpha-3)(\alpha^3 - \alpha^2 + 7\alpha - 27) \geq 0$, function f_{25} has two real number roots, which are $\frac{(2\alpha-2\alpha^2+9\pm\sqrt{10\alpha^2-4\alpha^3-48\alpha+\alpha^4+81})}{3\alpha(2-\alpha)} > 1$. Thus, function f_{25} is negative. And $\frac{\partial^2\Pi^{IS}}{\partial K^2} < 0$. ■

6.2.1 Optimal capacity is employed for Skim Both under Intermediate Supply Strategy

The optimal stocking decision under IS, when Skim Both is employed, is

$K^{\text{SB-IS}} \equiv \frac{f_{26}}{(384+1648\delta^2+153\delta^4-1344\delta-840\delta^3)\alpha+1152-2496\delta-144\delta^3+1568\delta^2-81\delta^4}$, where

$f_{26} = (-448c\delta + 544c\delta^2 + 128c + 48c\delta^4 - 272c\delta^3)\alpha^2 + (352c\delta^3 - 48\delta^3 - 48\delta + 88\delta^2 + 9\delta^4 - 256c\delta^2 + 256c - 96c\delta^4 - 256c\delta)\alpha - 444\delta^3 - 1872\delta + 27\delta^4 + 3264c\delta + 576c\delta^3 + 1520\delta^2 + 768 - 2432c\delta^2 - 1408c$

Substituting K in the profit function and threshold function, we have

$\Pi^{\text{SB-IS}} = \frac{f_{27}}{(153\delta^4-840\delta^3-1344\delta+384+1648\delta^2)\alpha+1568\delta^2-81\delta^4-2496\delta-144\delta^3+1152}$, where

$f_{27} = -(16(\delta-1))(-2+\delta)(3\alpha^2\delta^2 - 6\alpha\delta^2 + 36\delta - 8\alpha^2\delta + 4\alpha\delta + 4\alpha^2 - 44 + 8\alpha)c^2 + (96\alpha\delta^3 - 18\alpha\delta^4 + 96\alpha\delta - 1536 - 3040\delta^2 + 888\delta^3 - 54\delta^4 + 3744\delta - 176\alpha\delta^2)c + 576 + 1206\delta^2 - 369\delta^3 - 1440\delta + 27\delta^4$

And $V^{\text{SB-IS}} = \frac{(-4+3\delta)(-27\delta^3+60c\alpha\delta^3-248c\alpha\delta^2-24\delta^2-72c\delta^2+192c\delta+240\delta+320c\alpha\delta-128c\alpha-128c-192)}{153\alpha\delta^4-81\delta^4-840\alpha\delta^3-144\delta^3+1568\delta^2+1648\alpha\delta^2-2496\delta-1344\alpha\delta+384\alpha+1152}$.

Prices functions are given by:

$R_1 = \frac{4(\delta-1)(-2+\delta)(9\delta^2+48c\alpha\delta^2-36c\delta^2-120c\alpha\delta-96\delta-8c\delta+64c\alpha+64c+96)}{153\alpha\delta^4-81\delta^4-840\alpha\delta^3-144\delta^3+1568\delta^2+1648\alpha\delta^2-2496\delta-1344\alpha\delta+384\alpha+1152}$

$R_{L2} = \frac{2(\delta-1)(-27\delta^3+60c\alpha\delta^3-248c\alpha\delta^2-24\delta^2-72c\delta^2+192c\delta+240\delta+320c\alpha\delta-128c\alpha-128c-192)}{153\alpha\delta^4-81\delta^4-840\alpha\delta^3-144\delta^3+1568\delta^2+1648\alpha\delta^2-2496\delta-1344\alpha\delta+384\alpha+1152}$

$R_{H2} = \frac{4(\delta-1)(-2+\delta)(-27\delta^2+60c\alpha\delta^2-152c\alpha\delta-6\delta-144c\delta+48+80c\alpha+176c)}{153\alpha\delta^4-81\delta^4-840\alpha\delta^3-144\delta^3+1568\delta^2+1648\alpha\delta^2-2496\delta-1344\alpha\delta+384\alpha+1152}$

Under Skim Both, the threshold valuation should be less than 1. However, the threshold valuation may exceed one if the optimal inventory level is employed under a relatively high patience level, and the corresponding case is not a Skim Both case anymore since all strategic consumers wait to make their purchases in period 2.

6.2.2 Skim Myopic under Intermediate Supply Strategy

The realized demand functions and the methods of calculating prices are the same as the ones in the setting of Skim Both case in the Intermediate Supply scenario. The only change is to replace the threshold valuation with 1. We have $R_{H2} = \frac{1-\alpha R_1 - K}{1-\alpha}$, $R_{L2} = \frac{1}{2}$. The selling price in period 1 is solved backward by taking the first-order condition of the profit function, we have $R_1 = \frac{5-\alpha-4K}{2(2-\alpha)}$. And $\Pi^{\text{SM-IS}} = \frac{16cK\alpha - 3\alpha + 24K - 48cK + 3 - 24K^2}{16(3-\alpha)}$; $V^{\text{SM-IS}} = \frac{-2\alpha + 2\alpha\delta + 6K\delta - 8K + 10 - 9\delta}{4(1-\delta)(3-\alpha)}$.

Based on the profit function above, the profit is quadratic in K.

Proposition 21 *When SM-IS is employed, assuming $p = \frac{1}{2}$ and $N_L = \frac{1}{2}$, the retailer's profit function is strictly concave in the inventory quantity stocked, which guarantees an optimal stocking decision for the market clearing strategy.*

Proof. $\frac{\partial^2 \Pi^{\text{SM-IS}}}{\partial K^2} = -\frac{3}{3-\alpha} < 0$ ■

The corresponding optimal stocking decision is $K^{\text{SM2-IS}} \equiv \frac{1}{3}c\alpha + \frac{1}{2} - c$.

Since we are solving the model under a Skim Myopic case, the threshold valuation should be larger than or equal to 1, but the threshold valuation is actually less than one when the optimal inventory decision is employed within a low interval of patience level (δ). If the optimal inventory level is employed under a relatively how patience level, then the corresponding case is not a Skim Myopic case anymore since some strategic consumers purchase in period 1. A different inventory decision is adopted to ensure the validity of Skim Myopic cases, which is solved from $V^{SM} = \frac{-2\alpha+2\alpha\delta+6K\delta-8K+10-9\delta}{4(1-\delta)(3-\alpha)} = 1$ in K. The inventory level is denoted as K^{SM1}. And $K^{\text{SM1-IS}} \equiv \frac{2\alpha-2\alpha\delta-2+3\delta}{2(4-3\delta)}$.

The switching point between these two cases is solved from $K^{\text{SM1-IS}} = K^{\text{SM2-IS}}$ in δ, which is denoted as δ_c, $\delta_c \equiv \frac{3-4c}{2(1-c)}$. Later, we further solve $\Pi^{\text{SM1-IS}} = \Pi^{\text{SM2-IS}}$ to derive the switching point for consistency.

The smaller one between $K^{\text{SM1-IS}}$and $K^{\text{SM2-IS}}$ is chosen as the inventory decision. When $\delta \leq \delta_c$, $K^{\text{SM2-IS}} > K^{\text{SM1-IS}}$. So the inventory decision for $V = 1$ should be chosen when $\delta \leq \delta_c$; and the optimal inventory decision should be employed when $\delta > \delta_c$. Therefore, when $\delta \leq \delta_c$, $K = K^{\text{SM1-IS}} = \frac{2\alpha-2\alpha\delta-2+3\delta}{2(4-3\delta)}$.

Substituting K in the profit function, we have

$\Pi^{\text{SM1-IS}} = \frac{-(8(-4+3\delta))(-2\alpha+2\alpha\delta+2-3\delta)c-48\alpha\delta+24\alpha\delta^2+24\alpha-45\delta^2-24+72\delta}{16(4-3\delta)^2}$.

And $(R_1, R_{L2}, R_{H2}) = (\frac{8-7\delta}{2(4-3\delta)}, \frac{1}{2}, \frac{10-9\delta}{2(4-3\delta)})$

When $\delta > \delta_c$, $K = K^{\text{SM2-IS}} = \frac{1}{3}c\alpha + \frac{1}{2} - c$. Substituting K in the profit function, we have $\Pi^{\text{SM2-IS}} = -\frac{1}{6}c^2\alpha - \frac{1}{2}c + \frac{3}{16} + \frac{1}{2}c^2$. And $(R_1, R_{L2}, R_{H2}) = (\frac{1}{2} + \frac{2}{3}c, \frac{1}{2}, c + \frac{1}{2})$.

Figure 6.2 shows how the retailer's profit changes as the consumer's patience level increases when the retailer employs the Intermediate Supply strategy. When consumers' patience level is low, the retailer gets less profit than if strategic consumers are more patient. This result is similar to what we have in the Clearance Sales and Dynamic Pricing strategies. Interestingly, when patience levels are sufficiently high, retailers may actually benefit from increased consumer patience. This result may be explained by the fact that the Skim Myopic pricing policy is employed when consumers' patience level is sufficiently high. Intuitively, the retailer encourages all strategic consumers to buy in period 2 by manipulating the selling prices in periods 1 and 2. When strategic consumers' patience level is high, these consumers prefer to wait for period 2, which is also the retailer's purpose. Since retailer is able to time consumers' purchase based on their types, the retailer can price discriminatively to capture higher profit.

This finding has important implications for developing inventory release strategies and pricing policies in revenue management. The retailer can take advantage of strategic consumers' behavior when they take into accounts all available information to make their purchasing decisions. The market is becoming more transparent, especially with the increased popularity of electronic tickets and on-line payment (Xie and Shugan 2001).

Compared to the Clearance Sale strategy and Dynamic Pricing strategy, Intermediate Supply strategy is far too complicated to analyze. On the other hand, little research has been found that provides models using Intermediate Supply strategy. It can, therefore, be assumed that those are the reasons why this strategy is not employed widely.

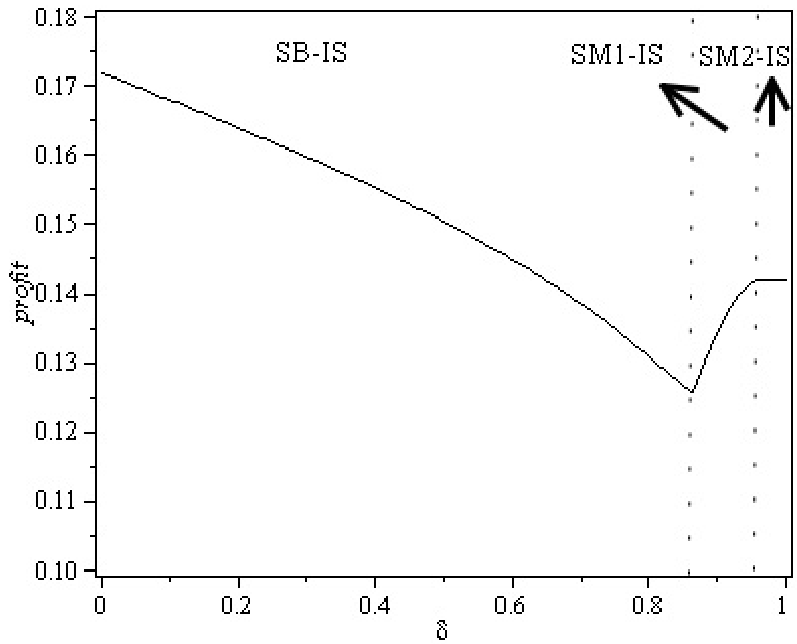

Figure 6.2: Retailer's profit after adoption of an Intermediate Supply strategy; $c = 0.1, \alpha = 0.3$.

6.3 Integrating Clearance Sales, Intermediate Supply and Dynamic Pricing strategies

We have analyzed each inventory release strategy separately. Now let's identify the optimal inventory release policy when the retailer faces capacity limitation. Since many cases need to be compared and different cases are feasible under different parameter ranges, so the results of this research question is analyzed numerically.

One example, the case of $c = 0.2, \alpha = 0.5$ and $\delta = 0.7$ is given in Figure-6.3. Firstly, at least one of the inventory release strategies is feasible. Then we find out the one that maximize the retailer's profit as well as the switching points. Figure-6.3 is drawn after all the redundancies are cleared, with only the optimal one left. We notice that, for a low capacity level, a Clearance Sales strategy is employed. When the retailer has an intermediate level of capacity, he employs the Intermediate Supply strategy, and the Dynamic Pricing strategy is used if the inventory level is high.

This result is reasonable since it is beneficial for the retailer to sell every inventory unit when the capacity level is low compared to the demand; the consumer quantity may cause the retailer to charge higher prices in both periods than when using the Intermediate Supply strategy. When the capacity level is appropriate for the retailer, he or she can sell everything under high demand and optimally release the inventory under low demand. Intermediate Supply strategy is able to let the retailer satisfy the demand under high demand level and save a lot of inventory cost under low demand. It is obvious that Dynamic Pricing strategy is chosen when the capacity level is comparatively high. Therefore, the retailer may price strategically to maximize his profit. However, inventory cost is a great consideration for the retailer in our model setting. We hypothesize that Dynamic Pricing is less likely to be employed by the retailer since it may generate the least profit. This hypothesis is numerically demonstrated in Figure 6.3, and we also notice that retailer will not benefit from the market at all if his inventory level is sufficiently high.

Most of the numerical examples studied show that a retailer's decision to adopt Clearance Sales, Intermediate Supply and Dynamic Pricing strategies, should depend on whether the inventory level is low, medium, or high, respectively. We also find examples where only Clearance Sales and Intermediate Supply strategies are chosen since Intermediate Supply and Dynamic Pricing strategies are both feasible when inventory level is high, but the retailer is better off if the Intermediate Supply strategy is chosen. Figure 6.4 provides an

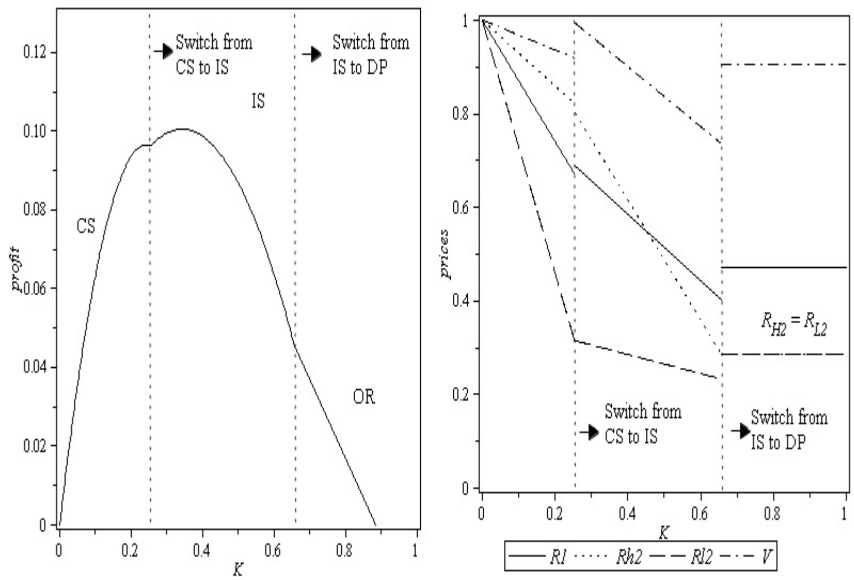

Figure 6.3: Profit functions of three inventory release strategies with undecided inventory decision (K) in the case of $c = 0.2, \alpha = 0.5$ and $\delta = 0.7$.

example for the case where $c = 0.3, \alpha = 0.5$ and $\delta = 0.7$, where only Clearance Sales and IS may be chosen.

In Figure 6.3, maximum profit is obtained by the retailer if Intermediate Supply strategy is chosen in the case of $c = 0.2, \alpha = 0.5$ and $\delta = 0.7$. Thus, the retailer can set, K, then he employs IS strategy and sets the optimal inventory level, $K = 0.35$. In contrast to this case, Figure 6.4 shows a different result where Clearance Sales strategy dominates IS if the optimal inventory level can be set by the retailer. To further analyze the validity of this result, we further release one parameter of α to figure out the optimal inventory release strategy.

To identify the optimal inventory release policy with no capacity limitation, the optimal inventory levels for each strategy is employed. The corresponding optimal inventory levels

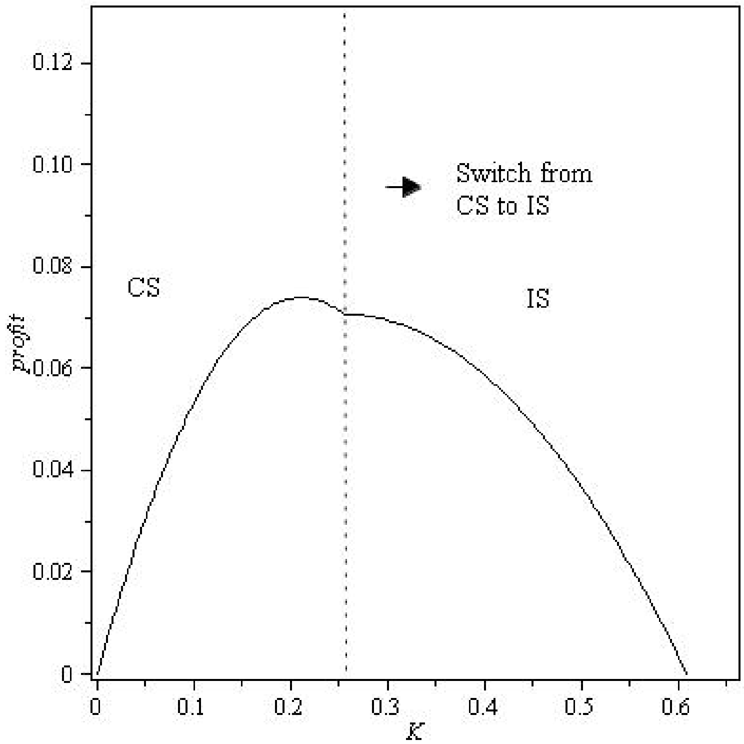

Figure 6.4: Profit functions of two inventory release strategies with undecided inventory decision (K) in the case of $c = 0.3, \alpha = 0.5$ and $\delta = 0.7$.

are highlighted in each section. One example of three inventory release strategies using optimal inventory level for the case of $c = 0.1$ and $\delta = 0.35$ is given in panel (a) of Figure 6.5. The graph shows that Intermediate Supply strategy always dominates when $c = 0.1$ and $\delta = 0.35$, and the profit function of IS gradually decreases in proportion to myopic consumers, a finding not interpreted under the IS model. This result further supports our statements that a strategic consumer's behavior may benefit the retailer.

Using the same valuation of parameters as in the examples given in IS and Clearance Sales strategies, we have a graph showing a different result compared to panel (a) in Figure

6.5. Panel (e) in Figure 6.5 shows the profit functions of three strategies by employing the optimal inventory level in the case of $c = 0.2$ and $\delta = 0.5$. From the graph, we notice that the retailer can get higher profit by adopting a Clearance Sales strategy when the proportion of myopic consumers in the market is less than 0.2. On the other hand, the retailer chooses an Intermediate Supply strategy if α is sufficiently high (above 0.2).

To show the result in a better way, Figure 6.5 shows nine examples using different valuations, inventory costs and patience levels to display the profit functions possible using the three inventory release strategies. We notice that in all instances the profit decreases in α and as inventory cost increases. Most of the time, the Intermediate Supply strategy is the best choice for the retailer under a low inventory cost condition and/or relatively low consumers' patience levels. Clearance Sales strategy emerges as a dominant when inventory costs and consumers' patience levels are high.

In general, it seems that the retailer is mostly better off if he chooses Intermediate Supply strategy as the inventory release strategy with no capacity limitation. Under some circumstances, retailer may also get higher profit if Clearance Sales strategy is employed depending the parameters' ranges.

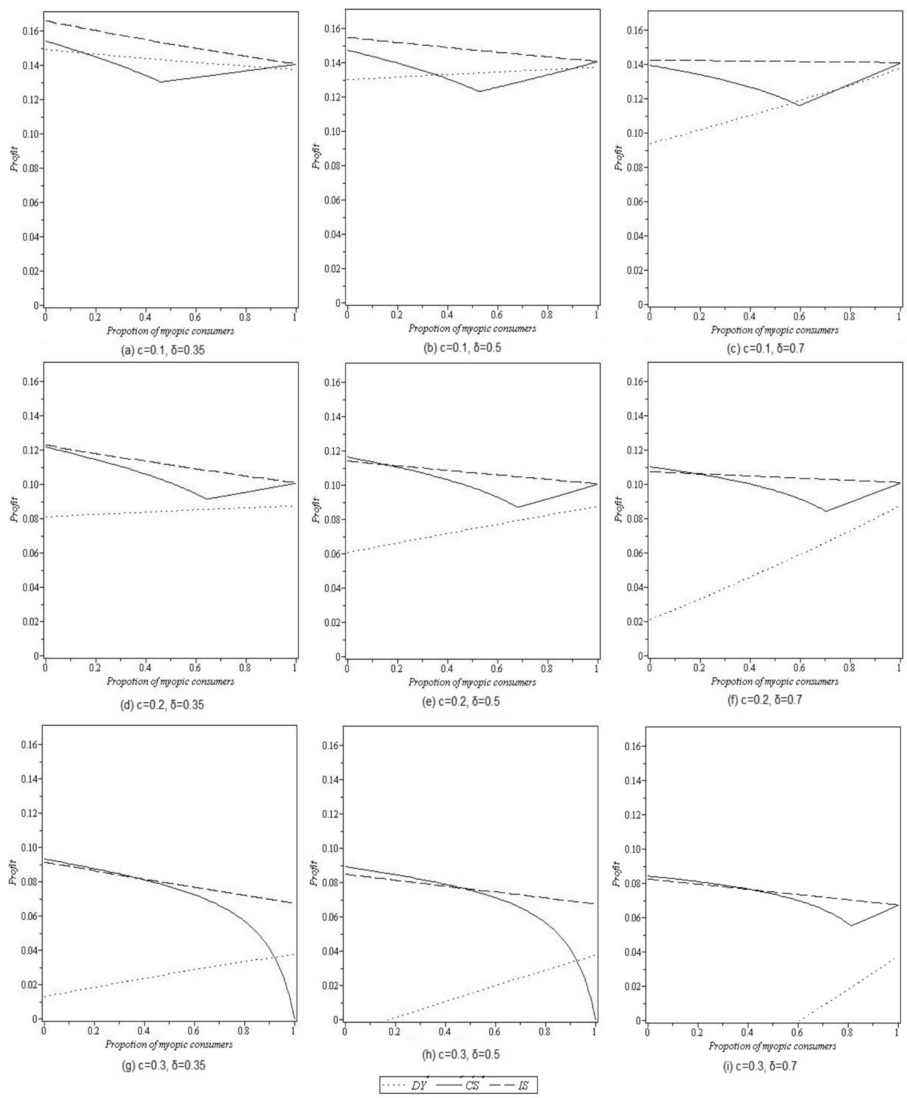

Figure 6.5: Nine examples are given using different valuations, inventory costs and patience levels to display the profit functions by using the three inventory release strategies.

Chapter 7

Summary and Managerial Insights

Our work sheds some insight into pricing strategies under uncertain demand in the presence of strategic consumers. In our model, a monopoly retailer sells a single kind of product over a two-period selling horizon faced with uncertain demand. We analyze three inventory release strategies and the corresponding pricing policies with limited capacity level. We perform the analysis by normalizing the high demand level to 1 and the low demand level to $\frac{1}{2}$, each occurring with equal probability.

With a low inventory level, a Clearance Sale strategy, the case where all leftover inventory is released to the market in the third stage, is employed, and the uncertainty with respect to the consumer quantity may cause the retailer to charge higher price under high demand and a lower price under low demand in period 2. The same holds true if Intermediate Supply strategy is adopted for an intermediate inventory level. Dynamic Pricing strategy shall be used if the inventory level is high.

Based on the results from analyzing these three inventory release strategies, we prove that the existence of strategic consumers may actually benefit the retailer. A retailer may also obtain extra profit if strategic consumers have high levels of patience employing Intermediate Supply strategy. This combination of findings provides some support for the conceptual premise that strategic consumers' behavior may actually benefit the retailer if the retailer can clearly understand it.

Consider the Clearance Sales and Intermediate Supply strategies, the retailer's profit function is strictly concave in the inventory quantity stocked, which guarantee an optimal inventory decision. Interestingly, the inventory level decreases as more myopic consumers exist in the market in Clearance Sales strategy, which implies that as myopic consumers are more numerous in the market, the retailer chooses a lower capacity. As strategic consumers are more patient in cases of basic Skim Both and Skim Myopic, the retailer's profit strictly decreases. However, as strategic consumers are more patient, the retailer gets less profit under Skim Both cases and the retailer's profit is irrelevant with patience level under Skim Myopic cases when all inventory units are sold in period 1 under high demand level or some leftovers exist after the selling season. The retailer isolates myopic consumers from the strategic consumers in Skim Myopic so that he can price differently for those two groups. The retailer ignores strategic consumers' patience level when he makes the pricing decisions.

We find that a retailer is mostly better off employing an Intermediate Supply strategy if he can make decisions about both inventory level and an inventory release strategy. A few examples also show that the Clearance Sale strategy emerges as the optimal inventory release strategy when inventory costs and/or consumers' patience levels are high. This thesis has given an account of and the reasons for the widespread use of Clearance Sale strategy as the inventory release strategy. Our study also confirms previous findings and contributes additional evidence that suggests retailers should employ Intermediate Supply strategy as the inventory release policy under uncertain demand when inventory costs and/or consumers' patience levels are low.

This thesis can stimulate further research. For examples, we have assumed that consumers are risk neutral. However, this assumption can be relaxed and explored the corresponding effects when consumers are, for instance, risk averse. The risk averse consumers are reluctant to purchase in period 2 with an uncertain payoff rather than purchasing in period 1 with a more certain, but possibly lower, expected payoff. If strategic consumers are risk averse, less strategic consumers wait and purchase in the second period. Retailers may charge a higher price in period 1 to increase their profits. As myopic consumers are more numerous in the market, the retailer's profit may increase as well.

Another direction related to the inventory release strategy we have studied. As numerical results indicate, a retailer is better off employing an Intermediate Supply strategy if he can make decisions about both inventory level and an inventory release strategy when inventory costs and/or consumers' patience levels are low. Future research question can focus the Intermediate Supply strategy's attributes since it may be adopted widely under uncertain demand markets. Since Intermediate Supply strategy is derived from Clearance Sales strategy and Dynamic Pricing strategy, it is reasonable to assume that they share some attributes. A good example in case is that the retailer's profit may decrease as the consumer's patience level increase.

Appendices

Definitions

Inputs:

v: consumer's valuation, which is assumed to be uniformly distributed between 0 and 1.

δ: patience level of strategic consumers.

α: fraction of myopic consumers in the market; $1-\alpha$ is the fraction of strategic consumers in the market.

c: inventory cost per unit.

p: probability of high level of demand.

N_i: number of consumers arriving at the beginning of the selling season under state i, $i \in \{H, L\}$, where H stand for High and L stands for Low.

We have $N_i = \begin{cases} N_H & w.p.\ p \\ N_L & w.p.\ 1-p \end{cases}$

D_{it} ($\leq K$): the realized demand under state i, $i \in \{H, L\}$, in period $t, t \in \{1, 2\}$.

Decisions:

K: inventory level.

R_1: posted selling price in period 1.

R_{i2}: selling price in period 2 under state i, $i \in \{H, L\}$.

Pricing Policies:

(1) Skim Both (SB): price to skim high-valuation consumers of both types in the first period, implying $V < 1$.

(2) Skim Myopic (SM): price to skim only high-valuation myopic consumers in the first period. That is, none of the strategic consumers purchases in the first period; they all wait for the second period; i.e. $V = 1$.

Inventory Release Strategies:

(1) Clearance Sale strategy (CS): After the first-period demand is realized, all leftover inventory is released to the market and the market determines the price.

(2) Dynamic Pricing strategy (DP): After the first-period demand is realized, the left-over inventory is released to maximize profit ignoring the capacity constraints.

(3) Intermediate Supply strategy (IS): After the first-period demand is realized, the leftover inventory is released to maximize the profit under low demand and all left over inventory is depleted under high demand, after the first-period demand is realized.

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