Facilitating Brownfield Redevelopment Projects: Evaluation, Negotiation, and Policy

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

A risky project evaluation technique called the fuzzy real options analysis is developed to evaluate brownfield redevelopment projects. Other decision making techniques, such as multiple criteria analysis and conflict analysis, can be incorporated into fuzzy real options analysis to facilitate negotiations on brownfield redevelopment among decision makers (DMs). The value of managerial flexibility, which is important in negotiations and policy making for brownfield redevelopment, is overlooked when the traditional evaluation method, net present value (NPV), is employed. Findings of this thesis can be used to promote brownfield redevelopment, thereby helping to eliminate environmental threats and enhance regional sustainability.

A brownfield is an abandoned or underutilized property that contains, or may contain, pollutants, hazardous substances, or contaminants from previous usage, typically industrial activity [45]. Brownfields often occur when the local economy transits from industrial to service-oriented seeking more profit. Governments actively promote brownfield redevelopment to eliminate public health threats, help economic transition, and enhance sustainability. However, developers are reluctant to participate in brownfield redevelopment because they often regard these projects as unprofitable when using classic evaluation techniques. On the other hand, case studies show that brownfield redevelopment projects can be good business opportunities for developers. An improved evaluation method is developed in order to estimate the value of a brownfield more accurately.

The main reason that makes the difference between estimates and "actual" values lies in the failure of the deterministic project evaluation tool to price the value of uncertainty, which leads to efforts to enhance the decision making under uncertainty. Real options modelling, which extends the ability of option pricing models in real asset evaluation, is employed in risky project evaluation because of its capacity to handle uncertainties. However, brownfield redevelopment projects contain uncertain factors that have no market price, thus violating the assumption of option pricing models for which all risks have been reflected in the market. This problem, called private risk [106], is addressed by incorporating fuzzy numbers into real options in this thesis, which can be called fuzzy real options. Fuzzy real options are shown to generalize the original model to deal with additional kinds of uncertainties, making them more suitable for project evaluation.

A numerical technique based on hybrid variables is developed to price fuzzy real options.

We proposed an extension of Least Squares Monte-Carlo simulation (LSM) that produces numerical evaluations of options. A major advantage of this methodology lies in its ability to produce results regardless of whether or not an analytic solution exists. Tests show that the generalized LSM produces similar results to the analytic valuation of fuzzy real options, when this is possible.

To facilitate parameter estimation for the fuzzy real options model, another numerical method is proposed to represent the likelihood of contamination of a brownfield using fuzzy boundaries. Linguistic quantifiers and ordered weighted averaging (OWA) techniques are utilized to determine the likelihood of pollution at sample locations based on multiple environmental indicators, acting as a fuzzy deduction rule to calculate the triangle membership functions of the fuzzy parameters. Risk preferences of DMs are expressed as different "ORness" levels of OWA operators, which affect likelihood estimates. When the fuzzy boundaries of a brownfield are generated by interpolation of sample points, the parameters of fuzzy real options, drift rate and volatility, can be calculated as fuzzy numbers. Hence, this proposed method can act as an intermediary between DMs and the fuzzy real options models, making this model much easier to apply.

The values of DMs to a brownfield can be input to the graph model for conflict resolution (GMCR) to identify possible resolutions during brownfield redevelopment negotiation among all possible states, or combinations of DMs' choices. Major redevelopment policies are studied using a brownfield redevelopment case, Ralgreen Community in Kitchener, Ontario, Canada. The fuzzy preference framework [5] and probability-based comparison method to rank fuzzy variables [53] are employed to integrate fuzzy real options and GMCR. Insights into this conflict and general policy suggestions are provided.

A potential negotiation support system (NSS) implementing these numerical methods is discussed in the context of negotiating brownfield redevelopment projects. The NSS combines the computational modules, decision support system (DSS) prototypes, and geographic information systems (GIS), and message systems. A public-private partnership (PPP) will be enhanced through information sharing, scenario generation, and conflict analysis provided by the NSS, encouraging more efficient brownfield redevelopment and leading to greater regional sustainability.

The integrated usage of fuzzy real options, OWA, and GMCR takes advantage of fuzziness and randomness, making better evaluation technique available in a multiple DMs

negotiation setting. Decision techniques expand their range from decision analysis, multiple criteria analysis, to a game-theoretic approach, contributing to a big picture on decision making under uncertainty. When these methods are used to study brownfield redevelopment, we found that creating better business opportunities, such as allowing land use change to raise net income, are more important in determining equilibria than remediation cost refunding. Better redevelopment policies can be proposed to aid negotiations among stakeholders.

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Huaxin Sun for her patience, support, and understanding throughout these years of our marriage. My daughter, Haiting Wang, is probably the most important and special person in my life. Thanks to her for bringing me much joy and happiness during these challenging years.

Dedication

This is dedicated to Huaxin Sun, my beloved wife who has been kindly supporting my studies for a very long time.

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Acronyms

CAPM capital asset pricing model. 29

CPF correlation pricing formula. 21, 29

CSM conceptual site model. 58

DM decision maker. 2–4, 6, 24, 35, 38, 45, 48, 53, 55, 58, 60–62, 64, 68, 72–74, 76, 78–85, 87, 90, 93, 98, 99, 102, 103, 105, 110, 111, 114

DSS decision support system. 6, 14, 101–103, 105, 107, 113

DST Dempster-Shafer Theory. 20

ERASE environmental remediation and site enhancement. 11, 12

ESA environmental site assessment. 12, 55, 58

EU European Union. 12

GIS geographic information system. 4, 61, 64, 102, 113

GMCR graphic model for conflict resolution. 4, 6, 74, 76, 79, 80, 82, 84–87, 90, 92, 93, 98–100, 102, 107, 111, 113, 115

GML geographic markup language. 105

GMR general metarationality. 79, 82, 93

IDW inverse distance weighting. 61, 62, 67

IRR internal return rate. 1, 19

IVP integrated valuation procedure. 27, 28, 30, 51, 53, 55, 115

LSM least squares Monte-Carlo simulation. 6, 36, 38, 39, 41, 46, 48, 55, 107

MAD market asset disclaimer. 26, 27

MFC Microsoft foundation class. 107

MOE ministry of environment. 12, 55

NGO non-governmental organization. 9, 11, 12, 76

NPV net present value. 1, 3, 17, 19, 21, 22, 25, 48, 72, 101, 105, 113

NSS negotiation support system. 3, 4, 6, 61, 101, 102, 105, 107, 110, 113, 115

OWA ordered weighted averaging. 6, 34, 58–63, 72, 73, 102, 103, 111, 113

PPP public-private partnership. 1, 12

QP qualified person. 102

R & D research and development. 21

RSC record of site condition. 12, 55

SEQ Sequential rationality. 79, 82, 98

SMR symmetric metarationality. 79, 82, 99

SoS system of systems. 8, 115

SSRA site-specific risk assessment. 12, 99

UI unilateral improvement. 78, 81–83, 86, 87, 93, 98

UM unilateral movement. 78, 80, 81, 83, 86, 92

US the United States. 9, 12, 72

USEPA United States environmental protection agency. 9

VOID value of individual disapproval. 60–62

WPF Windows presentation foundation. 105, 107

Chapter 1

Introduction

A brownfield is an abandoned or underutilized property that is, or suspected to be, contaminated from previous usage, typically industrial activity [45]. Governments across the world have established programs to promote brownfield redevelopment in order to eliminate public health threats, help economic transition, and enhance sustainability. Local governments in Canada also attempt to address brownfield problems via public-private partnership (PPP) [88]. However, developers are generally reluctant to participate in brownfield redevelopment because they often regard these projects as unprofitable, leading to slow progress of redeveloping brownfield sites in Canada [26]. On the other hand, case studies show that brownfield redevelopment projects can be good business opportunities for developers, indicating that developers actually underestimate the value of brownfields.

The difference between the higher value of a brownfield revealed in case studies and the lower value estimated using the classic project evaluation method must be addressed by including the value of uncertainty. Demonstrating that brownfield redevelopment is a good opportunity for developers is critical to facilitate negotiation on redeveloping contaminated sites; a better project evaluation technique also helps governments to design more effective brownfield initiatives. Because a brownfield redevelopment project has highly uncertain payoffs, techniques of decision making under uncertainty are studied in this thesis.

1.1 Problem Statement

A developer normally employs net present value (NPV) or internal return rate (IRR) to evaluate a project [40]. But these techniques are all deterministic in nature. Because uncertainty plays a significant role in brownfield redevelopment, both NPV and IRR provide little assistance to developers searching for optimal decisions. Models capable of pricing uncertainty should be considered instead.

This research attempts to employ and customize the option pricing model for better evaluation of brownfield redevelopment projects. But as options become "real", the underlying uncertainties become more difficult to deal with. Some risks associated with real options are not priced in the market, challenging the validity of using option pricing models. Hence, volatilities in real options usually cannot be accurately estimated. These risks are usually referred to as private risk, which are prevalent in brownfield redevelopment projects. The private risk problem places a major obstacle on adopting the real options approach to evaluate brownfields. Fuzzy real options, initialized by Carlsson and Fuller [12], are employed to accommodate private risks in brownfield redevelopment. The proposed model is able to tackle private risks and preferences, making it more suitable for employment in risky project evaluations, such as the brownfields.

When the risky projects evaluation technique is developed, decision makers (DMs) have clearer ideas on the value of uncertainties. The way to integrate fuzzy real options into game-theoretic approach is also studied to support negotiation among multiple DMs. Conflict resolutions on brownfield redevelopment can be identified with the aid of multi-criteria analysis and conflict analysis. Suggestions on brownfield redevelopment policies could be provided to enhance regional sustainability.

1.2 Research Objectives

The overall goals can be divided into three objectives: to explore fuzzy real options analysis for risky project evaluation, to employ fuzzy real options to evaluate brownfields, and to integrate a game-theoretic approach with fuzzy real options in order to facilitate negotiations. These three objectives, which are discussed in the next three subsections, are investigated thoroughly within this thesis.

1.2.1 Risky Project Evaluation Technique

An option is the right, but not the obligation, to buy or sell a certain security at a specified price at some time in the future [4]. Because option pricing models quantify the value of uncertainties, this technique has been migrated to broader usage, such as strategy selection [4], risky project valuation [106] [98], and policy assessment [24]. The idea of employing an option pricing model to value real assets or investments with uncertainties is usually called real options analysis or real options modelling [4] [30].

However, risky projects sometimes contain uncertainties that cannot be reflected in markets, violating the basic assumption underlying option pricing models. This problem is normally called private risk [106] [79]. Because the remediation and redevelopment costs can be regarded as private risk, methods addressing private risk must be found before applying real options analysis to brownfield redevelopment evaluation. An objective of this thesis is to utilize fuzzy real options analysis to deal with private risk. If a fuzzy representation could reflect private risk, fuzzy real options could be used to evaluate risky projects in a much broader area, therefore, for contaminated site pricing.

1.2.2 Decision Support in Brownfield Redevelopment

Another objective of this research is to utilize the fuzzy real options approach developed in this thesis to evaluate brownfields. To accomplish this task, three problems must be addressed:

- *Numerical Method of Fuzzy Real Options:* Analytic-form solutions of fuzzy real options are difficult to identify and even impossible in some cases. Hence, to ensure that the fuzzy real options model can evaluate any brownfield, a numerical method of fuzzy real options is designed.

- *Risk Preference:* Because fuzzy variables are utilized to represent expert estimates, the risk preference of the expert must have an impact on the value of a brownfield. The way to consider risk preferences of DMs is therefore explored.

- *Parameter Estimation for the Brownfield Evaluation Model:* Fuzzy real options models are much more complex than classic evaluation methods such as NPV. Thus, an intuitive process to facilitate usage of fuzzy real options to evaluate brownfields is required. An algorithm linking experts' subjective estimates on a brownfield site to parameters of fuzzy real options is proposed, which is a key component in the negotiation support system (NSS) design.

1.2.3 Negotiation and Conflict Analysis

When the conflicting values of different DMs are identified with respect to a brownfield project, conflict resolution can be suggested to facilitate negotiation and policy making. A game-theoretic approach is incorporated into fuzzy real options, thereby extending single decision making to the case of mulitple DMs'. Accordingly, a fuzzy game approach is added to the decision making under uncertainty framework due to its compatibility with fuzzy real options. In this thesis, fuzzy real options can provide DMs' fuzzy payoffs. Negotiation results are suggested as some kind of equilibria, which in turn can generate strategic insights into brownfield redevelopment policy making in Section 5.4.

1.3 Main Findings

The relationships among concepts and research areas considered in this thesis are summarized in Figure 1.3. Briefly, fuzzy real options are employed to evaluate risky projects with private risks, in order to integrate uncertainty representation of fuzziness and randomness [118]. Tests carried out in Section 4.3 and 5.3 show that private risk slightly increases the value of a brownfield, which is similar to the effect of market risk.

To determine values of fuzzy real options, a numerical method of option pricing models called NSS is extended to accommodate fuzziness. A program implementing this method is developed firstly using Python, then in C with parallel capacity, which becomes the core module in the NSS design [121].

Multicriteria analysis is employed for parameter estimation in fuzzy real options. DMs' risk preferences are reflected as different values of parameters in the fuzzy real options model, then different values on brownfield sites. A geographic information system (GIS) module is added on top of fuzzy real options to facilitate the multicriteria analysis procedure for DMs to determine the likelihood of contamination of a brownfield [119].

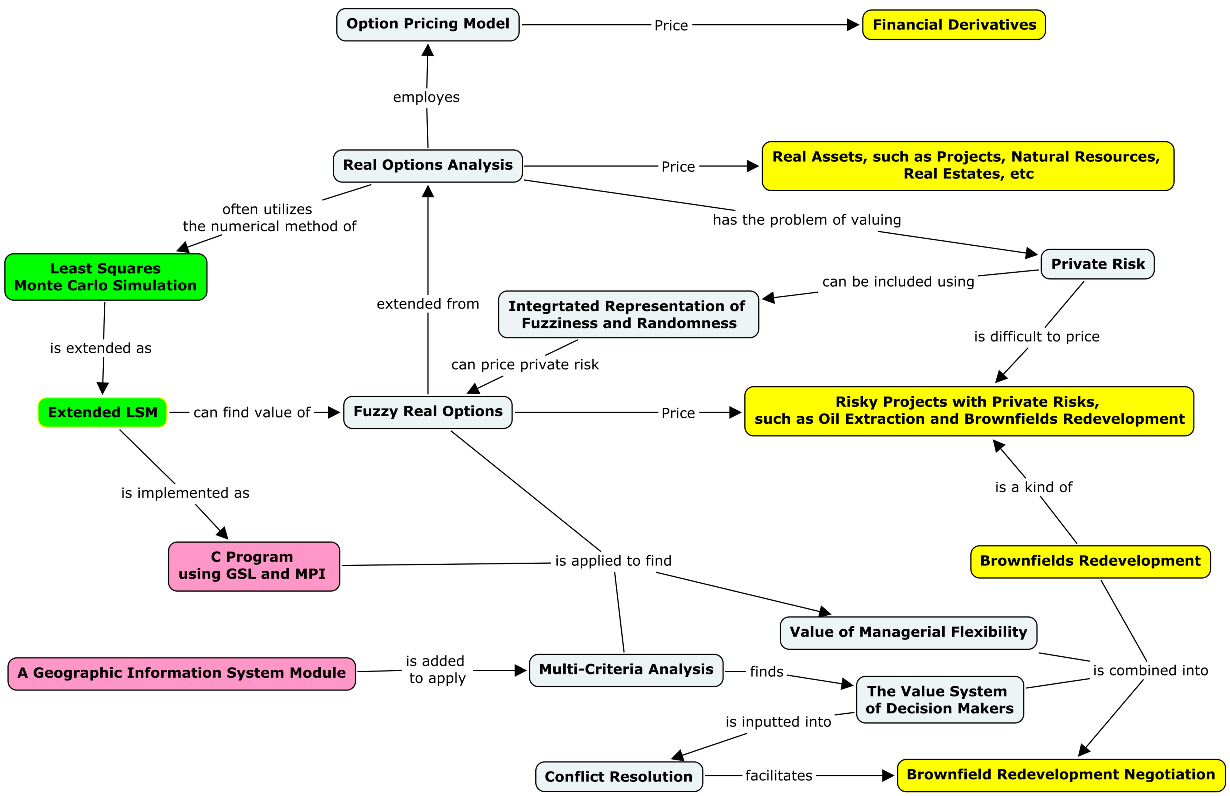


Figure 1.1: Conceptual Map of Research Areas in this Thesis

The case of the Ralgreen community redevelopment located in Kitchener, Ontario, Canada, is studied using these proposed methods. The values of DMs under various situations (policy settings) are identified. Results reveal that these fuzzy variable with trapezoid membership functions as brownfield values overlap, therefore are highly uncertain in preference comparison. In this case, fuzzy preference framework under the graphic model for conflict resolution (GMCR) is utilized to analyze brownfield redevelopment conflict [5]. Insights on redevelopment settings are summarized.

1.4 Thesis Structure

The structure of this thesis is illustrated in Figure 1.2. After the introduction, a background to brownfield redevelopment is provided, such as brownfield characteristics, redevelopment difficulties, and redevelopment initiatives in Chapter 2. Then, the fuzzy real options modelling is discussed in Chapter 3 as a better evaluation tool to address the underestimation problem in pricing brownfields.

Obstacles in applying fuzzy real options to evaluate brownfields are addressed in Chapter 4, which are numerical methods of computing fuzzy real options and their parameter estimates. Least squares Monte-Carlo simulation (LSM) is extended based on chance theory to calculate fuzzy real options. Ordered weighted averaging (OWA) is also added to identify the likelihood of contamination in a brownfield with linguistic quantifiers reflecting a DM's risk preference. Then, equations linking contamination likelihood to parameters in fuzzy real options for brownfields are proposed to evaluate brownfield sites.

A brownfield redevelopment conflict among stakeholders is analyzed using GMCR and fuzzy preference framework in Chapter 5. Fuzzy numbers with trapezoid membership functions derived from fuzzy real options are ranked under a fuzzy preference framework. The overlapping fuzzy numbers are converted into fuzzy degrees of preference. GMCR is used to determine equilibria with DMs' attitudes as the parameter of α-cut levels, which convert a fuzzy variable into an interval. The effectiveness of policy choices can be ranked by studying generated equilibria in various situations.

To facilitate negotiation in a brownfield redevelopment project, a NSS, which extends a couple of decision support system (DSS) prototypes, is designed and discussed in Chapter 6. Finally, conclusions and future work are summarized in Chapter 7.

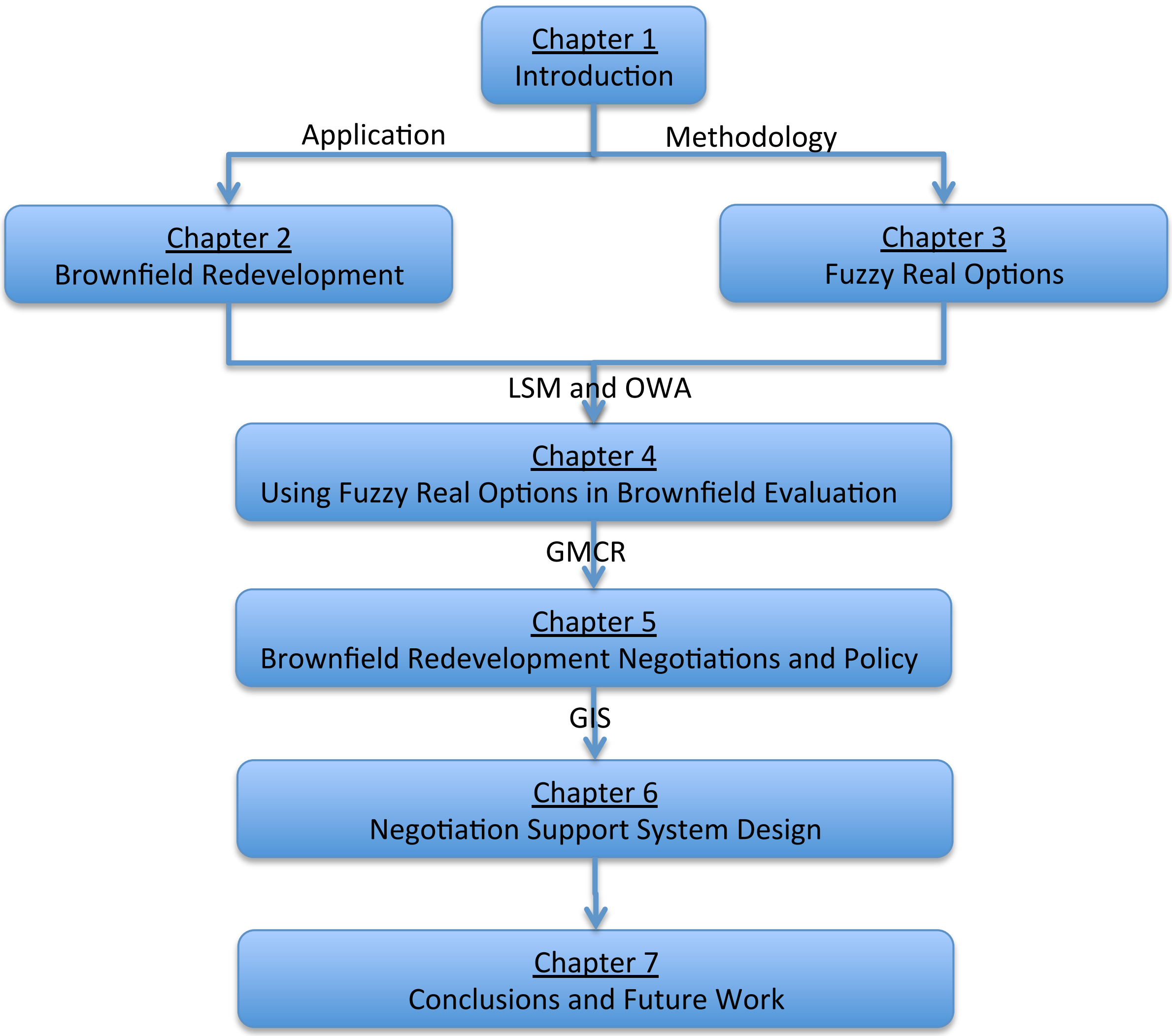


Figure 1.2: Thesis Structure

Chapter 2

Brownfield Redevelopment

2.1 Brownfield Redevelopment

A brownfield, the opposite of a greenfield, refers to a developed property that is abandoned or underutilized [45]. Brownfields usually occur when an industrialized region evolves into a service-oriented economy [111]. For instance, Hamilton, one of the major industrial cities in Ontario, Canada, is famous for its steel and chemical plants. However, many factories are relocating to developing countries, and these properties have been left as unproductive brownfields, as suspicion of contamination has prevented redevelopment.

Brownfields represent an unsustainable development pattern because existing infrastructure is wasted and greenfields are irreversibly developed for business or residential purposes. In addition, brownfields usually pose a threat to public health as the hazardous materials left in these properties may eventually leak into groundwater. Hence, leaving brownfields intact reduces the sustainability of cities.

On the other hand, redeveloping brownfields can revive the downtown areas of cities. Historically, many cities were developed around major plants; factories, residential areas, and community facilities constituted the urban core. Redeveloping brownfields reduces not only public health threats, but also unemployment. Therefore, brownfields are challenges to local governments, but also provide opportunities if redevelopment is properly conducted. Hence, many countries have launched their brownfield redevelopment programs to revitalize brownfield to a sustainable way.

But brownfield redevelopment is a typical system of systems (SoS) problem [55], as it involves various systems with complex interconnections, as shown in Figure 2.1. Brownfield redevelopment has many characteristics of an SoS, such as possessing high uncertainty, exhibiting nonlinear behaviour, and being interdisciplinary in nature [47]. Due to the complex interactions of soil-groundwater and societal systems, uncertainties in redevelopment costs, knowledge and technologies, and liabilities, are high and thereby play a critical role in preventing redevelopment [120]. These redevelopment projects are too risky to be undertaken by any single stakeholder. Uncertainties involved in brownfield redevelopment can be classified into the following categories:

- *Uncertainties due to limited knowledge of a brownfield*: Generally, knowledge and data about brownfields are limited. Identifying appropriate models, characteristics, and parameters can be costly and time-consuming;
- *Uncertainties originating from environmental systems*: Environmental systems have complex interactions, especially between groundwater and soil. Complex site-specific characteristics hinder remediation and redevelopment, because they imply highly uncertain remediation costs [32];
- *Uncertainties originating from societal systems*: Many stakeholders, with different interests, are involved in brownfield redevelopment. Complex conflicts and interactions create high levels of uncertainty in liabilities and cost sharing polices.

Hence, an effective and fair allocation system of liabilities, costs, and knowledge are required for financial resources and technical support in dealing with brownfields redevelopment across different levels of public sectors, non-governmental organizations (NGOs), and private entities. Coordinated efforts from all stakeholders are important in brownfield negotiation and redevelopment policy design.

2.2 Brownfield Redevelopment Situation in Canada

To discuss the situation of brownfield redevelopment in Canada, main factors in redevelopment policy design identified by United States environmental protection agency (USEPA)

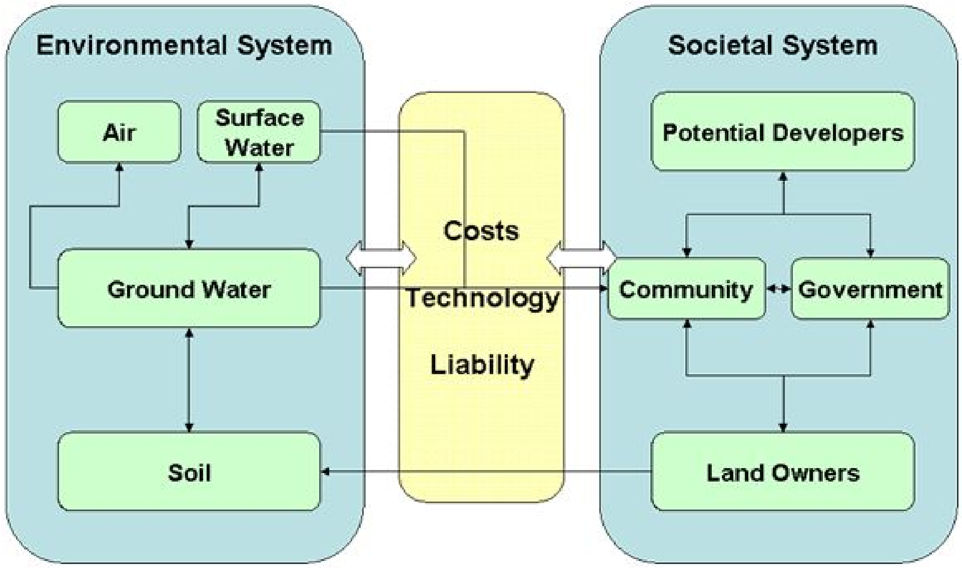


Figure 2.1: Systems Diagram of Brownfield Redevelopment

are summarized and classified them in the following based on lessons learned from pilot projects conducted in the United States (US) [112]:

- *Community Profiling:* Sustainable development is an evolutionary concept. Local sustainable development can only be realized when the community profile is clear, which means comprehensive surveys on environmental baseline, landscape situation, social and economic structure profile, labour knowledge and skills, and so on;

- *Comprehensive Community Planning:* Brownfield redevelopment should be integrated into a comprehensive community planning and not run as a sole program. A common vision and goals through public participation process are critical to the success of brownfield redevelopment;

- *Organizational Focus and Structure:* Unsuccessful brownfield redevelopment projects often originate from lack of government commitment and funding [27]. Brownfields redevelopment programs should be combined into the political and administrative workings of local government;

- *Site Identification and Characterization:* Adopting better site identification technology can reduce uncertainty and environmental risks, which have great impacts on remediation costs, marketing, and redevelopment process;

- *Risk Management and Restoration:* Uncertainty always leads to miscalculations and misunderstandings of redevelopment projects, ruining negotiating and wise decision-making for DMs. Rigid risk management balancing benefits with costs is one of the key factors of successful brownfield redevelopment projects;

- *Legal/Regulatory Issues:* Legislation for environmental remediation is important in clarifying stakeholders rights and liabilities. Liability exemption for brownfield redevelopment is helpful in sharing uncertainty and encouraging potential developers to participate in by ensuring them that they will not be responsible for environmental risks caused by former owners;

- *Site Marketing and Redevelopment*: Promoting the concept of brownfield redevelopment to the general public is critical in establishing positive image on redevelopment projects and correcting developers overestimation on redevelopment [27];

- *Technology Applications:* A state-of-the-art technology is always needed in improving site identification, hazardous removal rate, and associate costs cutting. Both hard and soft science and technologies are very useful;

- *Project Funding and Finance*: Providing fiscal basis to initiate programs and ensure their continued operations are important because market alone usually fails to handle environmental issues until better systems are established [117];

- *Environmental Justice:* Ensuring environmental equality and fair benefits and costs allocation in stakeholders are one of the most important goals in sustainable development. Realizing justice is the only way to gain local communities supportive to brownfield redevelopment.

Considering Canadian characteristics in legal and economic situations, above factors can be simplified into three categories: liability clarification and exemption, costs sharing and financial resources allocation, and technical supports. A conflict analysis on a brownfield case, called environmental remediation and site enhancement (ERASE) program in Hamilton, Ontario, Canada, is conducted to derive insights on redevelopment policy design, which supports the study using fuzzy real options [120].

Stakeholders in Canada generally complain about the complexity, uncertainty, and variability of regulatory systems supervising redevelopment issues [88]. Financial, human, and technological resources distribute in various entities, such as federal government, provincial government, local government, communities, NGOs, consulting companies, and so on. However, environmental laws (federal or provincial level) in Canada put entire liability for pollutions on brownfields landowners. This liability even includes personal liability for responsible individuals [22]. Plea for "innocent landowner defense' is invalid in Canada.

Potential developers are reluctant to enter and take over brownfields, fearing of being solely responsible for all environmental risks, including pollutants emit by former owners. Hence, when facing high uncertainty in brownfield redevelopment, an improved legislation system of various liabilities allocation is needed for risk sharing in different stakeholders. Regulatory liability exemption is especially in need for attracting new comers in brownfield redevelopment.

Another major obstacle in brownfield redevelopment lies in costs sharing. Unlike US and European Union (EU), Canada does not regard brownfield redevelopment as a serious problem [27]. Public funding for brownfield redevelopment is not enough because of the Canadain political structure. Environmental legislation is on the provincial level, restricting the power of the federal government other than coordinating related regulations in provincial governments [88]. Furthermore, provincial governments usually regard brownfield redevelopment as a low priority task, given that regional planning and infrastructure maintenance are main responsibilities of local governments. This fact forces stakeholders in Canada to find more innovative way of getting financial resources and sharing redevelopment benefits and costs.

Technical support, the assumption of adequate information on system is hardly to satisfy in brownfield redevelopment. In fact, information on brownfield is far from enough [111] [112]. Unified knowledge source accumulating knowledge in brownfield redevelopment is in high need [27]. Coordination efforts of collecting knowledge and founding specialized entities (NGOs and private consultancies) are underdeveloped up to now.

Learnt form the ERASE program, brownfield redevelopment can achieve great success if the local government is active in promoting brownfield redevelopment with sufficient technical support. A strong PPP can form and effectively clean brownfield sites. In response to the regal system regulating brownfields, Ontario passed Ontario Bill 56 and

Regulation 511/09 in support of redevelopment. The property development process is resigned considering environmental risks. Partial liability is exempted as long as a record of site condition (RSC) is submitted to ministry of environment (MOE). site-specific risk assessment (SSRA) is also permitted if the typical environmental site assessment (ESA) cannot be conducted.

Technical support and guidance is effective at reducing uncertainty in brownfield redevelopment. Staff with appropriate expertise can facilitate and promote the redevelopment process. In addition, specialized technical staff and marketing can reduce potential developers' misconceptions about brownfield redevelopment costs, which are usually lower than expected [26]. Hence, provision of technical support is an indispensable option to encourage brownfield redevelopment.

Given that the primary goal of private developers is profit, monetary compensation is supposed to stimulate developers to participate. There are two ways to reduce the costs of brownfield redevelopment: reduce taxes and supply funds. Tax reduction usually covers a portion of redevelopment costs. But even with this cost sharing, developers face uncertain costs. On the other hand, funds provision generally allows for full recovery of redevelopment costs [26].

Because basic legal system and technical support have been provided in Ontario, Canada, the focus will be decision technique of brownfield evaluation, more specifically, real options analysis that will be explained in the next chapter. A game-theoretic approach will be employed combined with fuzzy real options to study policy options mentioned above.

2.3 Literature on Brownfield Redevelopment Evaluation

The uncertainty on the pollutants' dissemination in the unsaturated layer, called the vodose zone, is systematically discussed in [32]. Sources of uncertainty in modelling vodose zone are also summarized. The redevelopment cost, which directly relates to the extent of contamination, can be represented as a stochastic process, which naturally leads researcher to think about using real options. Related studies have been made from various perspectives as explained below. However, a real options model that recognizes private risk in brownfield redevelopment has not been developed.

- *Real options on brownfield redevelopment*: Erzi [34] conducts a systematic study on brownfield redevelopment projects using real options approach, which is one of the major contributions in this area. However, the real options model derived by Dixit and Pindyck [30] does not take private risk into consideration and assumes only two options: the option to defer and the option to exit;

- *Contaminated property redevelopment evaluation*: Lentz and Tse [66] establish one of the first valuation formulae in contaminated property redevelopment using a real options approach. Their paper is a milestone in contamination remediation field. Private risk problem is considered as coefficients between underlying properties and derived securities and between costs and benefits. This approach not only requires subjective setting of the coefficient, but also results in complex and inflexible formulae;

- *Fuzzy real options on private risk*: As fuzzy real options are introduced [12], many applications adopt this approach if subjective uncertainty estimation is required. An effective DSS based on the fuzzy real options is built [110]. The fuzzy real options approach is applied to the brownfield redevelopment problem in order to overcome the private risk issue.

- *Real options on gas stations*: Yu [134] studied the contamination problem of gas stations when methane is added. Physical simulations are conducted in order to determine parameters in an extended numerical model for the vodose zone modelling. Although Yu's contribution was mainly on ground water modelling when cosolvent existis, she also proposed to use the real options analysis to determine a "fair" price of the environmental issuance premium.

Of the available real options models, the one proposed by Lentz and Tse is chosen for brownfield redevelopment. Pricing formulae of contaminated properties are derived using the option pricing approach and have the following advantages, making their work the basic case to extend toward fuzzy real option in this thesis:

- *Analytic framework*: Valuation formulae come from analytic framework. Hence, the result is simple and does not have the problem of overlapping portfolio as using the numerical framework, in which may repeatedly count some options' values [84]. For instance, the option to expand and the option of staging are overlapped since they both allow to expand the project at some time points. Therefore, adding both options using numerical methods will lead to higher-than-actual project value;

- *Reasonable options*: Unlike the options considered in the Dixit and Pindyck model [30] [34], which includes the option to exit, Lentz and Tse devise the option to remove and option to redevelop instead. This is more realistic because once the site is contaminated, property owners have the liability to conduct environmental remediation, which eliminates the option to abandon in the Dixit and Pindyck's model;

- *Consideration of private risk*: The Lentz and Tse's model explicitly considers the private risk problem via uncertainty coefficients between hedging portfolio and underlying cash flow, hedging portfolio and underlying redevelopment cost, and cash flow and redevelopment cost, which can also be fuzzy variables;

- *Simple implementation*: Given that their model is just a group of equations, it is easy for it to be implemented in the MatLab. Modification is also easier, facilitated by symbolic computation in MatLab.

In the Lentz and Tse's model, the value of a brownfield site is regarded as two Geometric Wiener Motions, which can be generally written as as $\frac{dS}{S} = \mu dt + \sigma dz$, where the parameter μ denotes the growth rate, the parameter σ the volatility, t the time, and z the Wiener process: the cash flow generated from this site without contamination (denoted x) and the redevelopment cost for this site (denoted R). To make private risk distinct from market risk, both are treated as partially hedged portfolios, a cash flow portfolio (denoted P) and a redevelopment cost portfolio (denoted K). Three coefficients are included: the coefficient of the cash flow and its hedged portfolio (ρ_{xP}), the coefficient of the redevelopment cost and its hedged portfolio (ρ_{RK}), and the coefficient of the cash flow and the redevelopment cost (ρ_{xR}). Formal introduction of the real options, which is the basis of Lentz and Tse's work, is in Section 3.2.

In addition, the model includes four coefficient parameters. The parameters φ_1, φ_2, and ϕ focus on cash flows. As cash flows from all states are proportional to the cash flow from the clean state, the cash flow generated under contamination is $\varphi_1 x$; the cash flow after removal to the clean flow $\varphi_2 x$; and cash flow after redevelopment is ϕx. The coefficients α_1 and α_2 denote the removal and restoration costs $\alpha_1 R$ and $\alpha_2 R$, which are assumed to be proportional to the total redevelopment cost R. Therefore, the cleanup cost C equals $(\alpha_1 + \alpha_2)R$. Furthermore, the riskless growth rate is denoted as r, which is a common notation in economic literature.

Lentz and Tse apply the contingent claim approach to analyze their model. Because the option to switch is a option to option, a multi-stage analysis is employed [66]. In other words, the value of the option to switch relies on the value of another option. Hence, the problem will be studied in a backward induction manner. The value of the underlying option is determined first by constructing a portfolio (denoted as H) holding the riskfree hedging position. Then the option to switch is identified on its basis.

To evaluate the contaminated properties, Lentz and Tse studies the value of a "clean" property first to determine whether to redevelop at all. If the value of the property to determine is denoted as V, the riskless portfolio can be expressed as $dH = dV - mdP - ndK + \varphi_2 xdt$, where m and n is unknown parameters while $\varphi_2 xdt$ is regarded as a kind of dividend. By doing this, the option to redevelop can be determined as Formula 2.1. Its critical value is reflected in $Z = \frac{x}{R}$. The parameters can be found in Formula 2.2 in [66], where g represents the risk-adjusted rate of the cashflow x and ω_K is the risk-adjusted rate plus the risk premium of the uncertain redevelopment R.

$$V = \begin{cases} \frac{\varphi_2 x}{r-g} + (\frac{(q-1)^{q-1}}{q^q})(\frac{\phi-\varphi_2}{r-g})^q \frac{x^q}{R^{q-1}}, & \text{if } Z \leq Z^*; \\ \frac{\phi x}{r-g} - R & \text{if } Z > Z^*; \end{cases} \tag{2.1}$$

If the optimal decision is not doing nothing, the redevelopment problem can be divided into two subproblems: to clean and redevelop sequentially or simultaneously. They are studied as separate cases. If the contaminated properties were to be cleaned and redeveloped sequentially, their values can be expressed as Formula 2.3 in [66], depending on the critical value of Y^*, the ratio of the clean cash flow (x) to the cleanup cost (C), as Formula 2.4 in [66]. If $Y > Y^*$, the removal action should be taken right now. Otherwise, the optimal executing time is in Formula 2.6 in [66]. After the cleanup action, redevelopment is better to be conducted when $Z > Z^*$ in Formula 2.6.

$$\begin{cases} \sigma^2 = \sigma_x^2 + \sigma_R^2 - 2\sigma_{xR} \\ \gamma = \omega_k - \mu_R \\ \delta = g - (\mu_R - \mu_K + r) \\ g = \mu_x - (\mu_P - r)\beta_x \\ \beta_x = \rho_{xP}\frac{\sigma_x}{\sigma_P} \\ \beta_R = \rho_{RK}\frac{\sigma_R}{\sigma_K} \\ \omega_K = r + (\mu_K - r)\beta_R \\ q = 0.5(\frac{\sigma^2-2\delta}{\sigma^2} + \sqrt{\frac{(2\delta-\sigma^2)^2+8\gamma\sigma^2}{\sigma^4}}) \end{cases} \tag{2.2}$$

$$V_1 = \begin{cases} \frac{\varphi_1 x}{r-g} + (\frac{(q-1)^{q-1}}{q^q})((\frac{1-\varphi_1}{r-g})^q(\frac{x}{C})^{q-1} + (\frac{\phi-1}{r-g})^q(\frac{x}{R})^{q-1})x, & \text{if } Y \leq Y^*; \\ \frac{x}{r-g} + (\frac{(q-1)^{q-1}}{q^q})(\frac{\phi-1}{r-g})^q(\frac{x}{R})^{q-1}x - C, & \text{if } Y > Y^*; \end{cases} \tag{2.3}$$

$$\begin{cases} Y^* = \frac{r-g}{1-\varphi_1}\frac{q}{q-1} \\ W^* = \frac{r-g}{\phi-\varphi_1}\frac{q}{q-1} \\ Z^* = \frac{r-g}{\phi-1}\frac{q}{q-1} \end{cases} \tag{2.4}$$

If the brownfield sites were to be cleaned and redeveloped simultaneously, their values can be expressed in Formula 2.5 in [66], depending on the critical value of W^* in Formula 2.4, the ratio of the clean cash flow (x) to the combined cost of removal and redevelopment as a joint action. If $W > W^*$, the removal action should be taken right now. Otherwise, the optimal executing time is in Formula 2.6.

$$V_2 = \begin{cases} \frac{\varphi_1 x}{r-g} + (\frac{\phi-\varphi_1}{r-g})^q(\frac{(q-1)^{q-1}}{q^q})\frac{x^q}{(\alpha_1 R+R)^{q-1}}, & \text{if } W \leq W^*; \\ \frac{\phi x}{r-g} - (\alpha_1 R + R), & \text{if } W > W^*; \end{cases} \tag{2.5}$$

$$\begin{cases} \tau_Y = \frac{lnY^*-lnY}{m_x-m_R} \\ \tau_W = \frac{lnW^*-lnW}{m_x-m_R} \\ \tau_Z = \frac{lnZ^*-lnZ}{m_x-m_R} \end{cases} \tag{2.6}$$

for $m_x > m_R$, where $m_x = \mu_x - 0.5\sigma_x^2$ and $m_R = \mu_R - 0.5\sigma_R^2$.

The final value of the brownfield site is the maximum of V_1 and V_2. When Y or W exceeds its corresponding critical value, the formula are very similar to NPV equations. Otherwise, we can find the formula are a summation of a NPV term with some additional option values. An optimized redevelopment strategy can also be formed based on where it locates in the decision region, since all critical values can be converted into x/R.

Chapter 3

Fuzzy Real Options

Profitability of a project is perhaps the most important factor affecting developer's decision on whether to undertake it. There are many techniques available to assess economic feasibility, such as NPV and IRR [40]. But these techniques are all deterministic in nature. When uncertainty plays a significant role, they provide little assistance to developers searching for optimal decisions.

Efforts using various uncertainty representations have been made to enhance the decision-making under uncertainty. In this chapter, uncertainty representations will be introduced first. Real options analysis, a popular approach based on stochastic calculus, is reviewed. Its obstacle to application, namely, private risk is explained. Then, an innovative solution called fuzzy real options is discussed, which addresses private risk using fuzzy presentations.

3.1 Uncertainty Representations

The uncertain future costs and income of a project can be modelled as stochastic processes, the main model used in real options analysis [30]. However, other representations of uncertainty might be helpful in extending real options analysis, which can be classified into two categories: those having underlying distribution, and those with an interval but no underlying distribution [46]. The first class relies on the distribution to quantify the degree of uncertainty. It can differ from classical probability theory by modifying the axioms, thereby allowing for different measurements of uncertainty [60] [61].

Fuzzy theory, developed by Zadeh [135], is one of the most frequently-used representations of uncertainty. It can serve as the basis of possibility theory [136]. Its major difference from probability theory lies in the principle that fuzzy measure of multiple events equals to the maximum rather than the sum [60] [73]. Fuzzy theory has had great success as a bridge between quantitative information and qualitative description, and is appropriate for expert estimates utilizing descriptive terms [137].

Dempster-Shafer Theory (DST) of evidence can be regarded as a generalization of subjective probability. It allocates probability to events, which are elements of the power set of A, rather than elements in the set A itself, making room for the storage of prior knowledge [60] [61]. For events with non-void intersections, DST measure is not additive [61].

When applying DST, two steps are modifications of subjective probability. One is that the distribution of probability on events is determined using a belief function, which is also called a basic probability assignment in DST. The other is that, since additivity does not hold, the evidence of the combination rule must be employed in calculations [61]. The main advantage of these modifications is that the imprecision of subjective estimation can be modelled as ignorance and vagueness [16]. DST has been used in many areas, such as health care and medicine [109], decision making and business [122], and artificial intelligence [100].

As opposed to distribution-based uncertainty representations, the min-max principle and info-gap theory can be utilized to produce boundaries on estimation when severe uncertainty affects the decision process [8]. Information gap theory defines a measure of robustness and aims at finding a safe boundary within a set of nested functions [7]. In fact, the distribution of uncertainty is not needed to calculate the envelope. Due to its reduced need for information, info-gap theory is widely used in engineering design and safety analysis [46]. Finally, a unique approach to handling uncertainty, for the case of preferences, which is non-probabilistic, non-fuzzy and non-info-gap, is provided by Li et al. [67].

Since different uncertainty representations have advantages in different application areas, some researchers have considered integrating them for more general purposes. Combining fuzziness and randomness has been carefully designed in combination with appropriate axioms and suitable implementation algorithm as a basis for further studies [61] [70]. This thesis utilizes these results and attempts to find a numerical method for hybrid processes.

3.2 Real Options Analysis

The cash flow of a project is usually classified into income and costs [40]. For a risky project, income or costs (or both) are highly unpredictable. For instance, a toll highway could have very different incomes, depending on whether or not people choose it as their main route. The costs of extracting oil vary greatly, determined by the geology of the oil site. Research and development (R & D) projects have both highly uncertain income and costs. The market for products cannot be accurately predicted. While the development costs, such as patents and human resources, are also largely unknown ahead of time. As a result, the values of these risky projects are uncertain. Accordingly, a single value calculated using NPV is inappropriate to help decision makers identify correct decisions.

To make better estimates of risky projects, various NPV based methods have been suggested. Correlation pricing formula (CPF) changes the riskless discount rate into a risk-adjusted one, so that the risk premium is included [77]. Expected NPV calculation combines several scenarios into one. Fuzzy NPV can help in this situation, but gives the final value as a fuzzy number [95]. Another way of evaluating risky projects is called real options, which is the one of the main focus of this thesis.

In this section, option pricing model, real option analysis, and numerical methods to solve options are explained in subsections in order. The process of applying real options analysis, its results, and advantages and disadvantages of real options modelling will be discussed to establish the basis of fuzzy real options.

3.2.1 Option Pricing Models

Option pricing models were used in the financial market to price derivatives, whose value depends on underlying securities. Black, Scholes and Merton proposed their option pricing frameworks (Formula 3.1 and 3.2) [9] [81], regarding the value of underlying asset as the Geometric Brownie Motion. The risk-neutral approach they employed is also widely adopted for deducing other option pricing models [52] [21].

$$Call = SN(d_1) - Xe^{-\mu T}N(d_2) \tag{3.1}$$

$$Put = Xe^{-\mu T}N(-d_2) - SN(-d_1) \tag{3.2}$$

where $d_1 = \frac{ln(S_0/X)+(\mu+\sigma^2/2)T}{\sigma\sqrt{T}}$*;* $d_2 = d_1 - \sigma\sqrt{T}$*;* S *is the current price of the stock;* X *is the exercise price of the option;* $Call$ *and* Put *denote respectively the European call and put option prices; and* $N(x)$ *is the cumulative distribution of the normal distribution.*

Option pricing models explicitly acknowledge uncertainty (Table 3.1). By adding the term σdz, they regard the assets underlying derivatives as stochastic processes, not deterministic ones, thus dealing with risks and uncertainties more realistically. The volatility reflects uncertainties that can be estimated using historical data and has a solid mathematical basis. The values of uncertainties are reflected in the option prices.

Table 3.1: Comparison of Option Models and NPV

	NPV	**Option models**
Value of underlying assets	$\frac{dS}{S} = \mu dt$	$\frac{dS}{S} = \mu dt + \sigma dz$
Value of derivative options	0	Black-Scholes

With option pricing models, the value of flexibility can be more accurately valued. For instance, if we are certain that Google stock price will be 700 dollars one year later from NPV, the ability to sell the stock of Google at 650 dollars after a year is worthless. However, if we admit that the probability of Google stock to fall behind 650 dollars exists (say 10%), the put option at \$650 will have some value, which can be determined by Black-Sholes equation.

3.2.2 Real Options

The idea of using option pricing model to price real assets is called real options analysis, or real options modelling [4]. The ability to evaluate managerial flexibility makes real options appropriate for employment in many applications. The relationship between the value of managerial flexibility and the level of uncertainty, which is important in risk management, decision making, and policy making, can be identified using real options analysis. For instance, real options analysis has been applied to supply chains [83], infrastructure development [29] [1] [103], energy management [28] [89], and environmental management [97] [24].

Since the value of a project is mainly determined by the market values of its products or consumed resources, real options analysis can be utilized to evaluate risky projects. In the literature, real options analysis has been applied to different risky projects, such as information technology [71], product development [38] [82], and oil extraction [105].

A project can be regarded as a cash flow and a portfolio of options reflecting managerial flexibilities, such as closing the project at any time, expanding project scale at specified times, and so on [21]. The following options may exist in different kinds of projects and situations, and can be evaluated using option formulae developed for the financial market [4] [98] [30] [21] [84].

- *The option to defer*: The option of waiting for the best time to start a project can be valued as an American call option or a Bermuda call option;
- *The option to expand*: The option of expanding the scale of a project can be valued as an American call option or a barrier option;
- *The option to contract*: The option of shrinking the scale of a project can be valued as an American put option;
- *The option to abandon*: The ability to quit a project can be valued as an American put option or a European put option;
- *The option of staging*: The ability to divide a project into several serial stages, with the option of abandoning it at the end of each stage ("option on option"), can be valued as a compound option, also known as a learning option;
- *The option to switch*: The flexibility to convert a project to another use can be valued as a switch option.
- *Options with multiple uncertainties*: When the underlying asset of an option has more than one uncertainty, it is called a rainbow option and requires a more complex pricing formula.

There are normally four steps in the process of valuing a risky project using the real options approach [4]. Real option modeling involves iterative design, implementation, testing, and modification:

1. *Frame the application*: Identify relevant aspects of the decision problem, including uncertainties, decision rules, and available information:

    - The decision problem: Start from a big picture of the application and identify the appropriate problem to study;
    - The uncertainties: Identify all sources of uncertainty and classify them into market uncertainties and private uncertainties (also called private risks), which can be treated differently;
    - The decision rule: Express decision rules in a simple mathematical form, such as the stochastic processes of the uncertainties and the optimization goal (profit maximization or risk minimization);
    - The available market information: Find data supporting parameter estimation and uncertainty calculation.

2. *Implement an option pricing model*: This step involves establishing all inputs and then calculating the result using analytic formulae or an option calculator, which is normally pre-installed with frequently used option equations, i.e. European option equations.

    - Establishing inputs: Obtain the current value of the underlying asset, plus the historical data needed for parameter estimation;
    - Calculating the result: A value can be determined using an option calculator with pre-built option valuation functions. Alternatively, formulae for options in a specific situation can be deduced analytically using the risk-neutral approach [30].

3. *Review the result*: Outputs from real options modelling should be reviewed not only to identify insights and conclusions, but also to assess the model and recalibrate it if necessary;

4. *Redesign if necessary*: If the quality of the model is not satisfactory, it should be improved iteratively.

The output of option valuation models usually includes valuations, critical values, and strategy regions to help a DM to make decisions [4]. These important products are explained next:

- *Evaluation*: The most important output is the value of the risky project, the original goal of the real options model;

- *Critical values*: In project valuation, there are usually formulae for calculating critical values, which are thresholds for deciding whether to undertake the project. Critical values thus play the same role as the NPV zero;

- *Strategy space*: The multi-dimensional strategy space is divided into regions, corresponding to which option is best to implement. This output is often optional.

3.2.3 Numerical Methods of Real Options

Numerical analysis focuses on determining the fair price of options with the aid of a computer. Although numerical results are approximate, numerical methods can be intuitive and convenient to use. Three main classes of numerical methods are binomial trees, Monte-Carlo simulations, and differential equations [52].

- *Binomial tree*: The binomial (and trinomial) trees simulate the up-and-down paths of the random walk. Its structure is similar to a decision tree, using nodes to represent events and linking them with probabilities. The two main steps are predicting future events with known parameters, and then calculating values of real options at the root node by finding the expected values backward from the bottom level to the top. In fact, the binomial tree approach is a kind of explicit numerical solutions for partial difference equations. But, because it is very popular among real options practitioners, it is made as a distinct type of solutions.

- *Monte-Carlo simulation*: Steps in applying Monte-Carlo simulation are similar to the binomial tree. The key difference from the binomial tree is that nodes on different paths have no linkages. The probability of going from one node to the other is unknown in Monte-Carlo simulation, while it is predetermined in the binomial tree approach.

- *Numerical differential equation solution*: This solution normally employs the numerical solution of partial differential equations. The whole space will be divided into grids. Boundary conditions are used to identify feasible regions. Then, forward or backward algorithm are utilized to find the final result.

Each numerical method has its own advantages and disadvantages. A binomial tree is intuitive and easy to implement. However, the complexity of a binomial tree algorithm has exponential growth as the number of random variables increases. Monte-Carlo simulation is also intuitive and easy to be parallelized, making it favoured by financial engineers. Nonetheless, it requires higher computer power than other methods if the problem is not too complex. A numerical differential equation method is more accurate and previously widely used. But this algorithm is more difficult to understand than the previous two, thereby preventing others from building more complex real options models.

3.3 Private Risk and Solutions

Risks considered in the option pricing models are market risks, assumed to satisfy the requirements given below. However, these assumptions may not be realistic for real assets. Many uncertainties cannot be matched by any basketed of market goods, and hence, are unable to be included in the option pricing models [79]. Risks violating these assumptions are referred as private risks [30] [106], which have to be considered in real options analysis.

- *Complete Market*: All risks can be hedged by a portfolio of options [86]. In other words, all risks are reflected in the market price. In some publications, it is also called market asset disclaimer (MAD) [106];

- *Arbitrage-free Market*: Unless a player in the market is willing to take some risk, there is no opportunity for profit [106]. In other words, there is no risk-free way of making profit;

- *Frictionless Market*: There are no barriers to trading, borrowing, shorting, or any associated transaction costs. Furthermore, the underlying assets are infinitely divisible [79].

These assumptions are generally realistic in the financial market, but these may not be the case for real options. In fact, it is unusual for project-specific uncertainties to be replicated as a market portfolio. The options modelling process must be customized to make the valuation framework flexible enough to fit real options.

While Dixit and Pindyck [30] linked the real options model with decision analysis implicitly, Smith and Nau [106] realized the similarity between the two in the presence of private risk, and conducted the first study on comparing and integrating them in order to handle private risk. This idea was further studied in [96], [10], and [101].

Given that private risk violates the MAD assumption, Smith suggests that there is no unique value of the project, but instead an interval of value, between the dominating and dominated replicating trading strategies [106] [33]. The dominating trading strategy is defined as a strategy that generates a cash flow that is always greater or equal to the project; while a dominated trading strategy is the strategy that always produce a cash flow smaller or equal to the project. Therefore, we are able to achieve the value interval of a project with private risk.

If "incompleteness" can be reduced to "partially complete" (Definition 3.3.1), the integrated valuation procedure (IVP) can be applied for better real options valuation [105]. The basic idea of IVP is to evaluate market-priced uncertainties using a real options model while pricing private risks using decision analysis.

Definition 3.3.1 *The market is partially complete if it satisfies [106]:*

- *Replicated security prices rely only on market states and can be expressed as* $s(t, \omega_t^m)$*;*
- *The market is complete in terms of treating market-priced uncertainties;*
- *Private events do not provide any information about future markets, implying that* ω_t^m *and* ω_{t-1}^p *are independent.*

where $s(t, \omega_t)$ *denotes a vector of all prices in the portfolio at time* t *in state* ω_t*; and* ω^m *and* ω^p *represent states of market and private information, respectively.*

When two preference restrictions, namely, the additive independence (Definition 3.3.2) and Δ-Property (Definition 3.3.3) are added, the decision analysis technique can be applied to solve the investment problem. While option pricing model is used to tackle the investment problem under a partially complete market. Then, based on the separation theorem [48], these two parts can be combined to solve the grand problem.

Definition 3.3.2 *A firm's preference is an additive independence one if it does not rely on the joint distributions of its net incomes at each period [106]:*

Definition 3.3.3 Δ*-property means that a firm's preference toward its net income at a specific time* t *has constant absolute risk aversion. If the firm has no preference between an uncertain cash flow* $\widetilde{x}(t)$ *and a deterministic one* $x(t)$*,* Δ*-property requires this firm to be indifferent between* $\widetilde{x}(t) + \Delta$ *and* $x(t) + \Delta$ *[106]:*

Smith's work established an important approach to the private risk problem with dynamic programming [116] [10] [80] [33] [105]. Given its numerical simplicity, this method is frequently used by real options practitioners. Mattar [80] extends IVP by differentiating unique and private risks, which were both previously regarded as private risks. Unique risk is private risk that can be diversified by holding various assets in the portfolio, and requires no risk premium. With this extension, Mattar allows the exercise price to be stochastic and to have a project-specific risk, making the valuation framework more flexible.

Monte-Carlo simulation is another option to handle the private risk problem, and is widely adopted by real options practitioners. Because of its intuitive approach and convenience of implementation, Monte-Carlo simulation is one of the default volatility estimation methods in major real option software, such as Crystal Ball [13] and @Risk [44]. The key idea lies in to represent the uncertainty in parameter estimation using subjective probability [44]. Sample points will be generated using Monte-Carlo simulation and then used to obtain a value distribution as outputs [84] [21]. Monte-Carlo simulation normally involves the following steps [21]:

1. *Define assumptions*: Specify the distributions of the uncertainties of each variable , i.e. uniform, normal, lognormal, beta, etc, by consulting experts;

2. *Set autocorrelations*: Set the autocorrelation parameter for each variable, which is important in time series analysis;

3. *Define forecast variable*: The forecast variable is the target of the estimation;

4. *Run the simulation*: Run Monte-Carlo simulation to construct present value event trees, so that the distribution of the output values can be obtained.

On the other hand, although the basis of Monte-Carlo simulation is similar to IVP, the assumptions of Monte-Carlo simulation is frequently not met in the brownfield context. An equivalent riskfree portfolio is difficult to construct because the MAD condition fails. Therefore, the contingent claim approach cannot be applied to obtain the value of a brownfield.

Besides dynamic programming and the Monte-Carlo methods used by financial engineers, economists focus on finding better market-priced counterparts for private risks. Historical data used to estimate the volatility of private risk can be classified as estimation using the market price of the underlying asset, the market price of compatible assets, the market price of the company's stock, and related industry indexes [44] [34]. This classification provides guidance to real options users on how to find the most appropriate market-priced target for private risk estimation.

The idea of finding similar market-priced items can be rationalized into CPF (Formula 3.3) [77]. CPF is very similar to the capital asset pricing model (CAPM) model. The only difference is that CPF does not require Y to be a market portfolio [86]. Another mathematical treatment is to add a correlation coefficient ρ_{ud} between the Brownian processes of derivatives dz_d and underlying asset dz_u (Formula 3.4) [66] [102]. Because the coefficient is added, it implies that a riskless portfolio can be constructed by holding diversified goods.

$$\overline{\mu_w} = \frac{1}{r}[\overline{w} - \beta_{wy}(\overline{y} - rS_y)] \tag{3.3}$$

where y is the normalized market asset that is the most similar to the project w to be priced; β_{wy} is the beta between w and y; S, μ and r is defined the same as in previous sections.

$$dz_d = \rho_{ud} dz_u + \sqrt{1 - \rho_{ud}^2} dz_{ud} \tag{3.4}$$

where the random variable dz_{ud} is assumed to be unsystematic with zero mean and variance dt of a normal distribution.

3.4 Fuzzy Real Options

Fuzzy real options were first introduced by Carlsson and Fuller [12], who attempted to identify optimal strategies using real options analysis with uncertain parameters. The possibility mean and variance were introduced in combination of real options analysis [31] [11]. Unlike the crisp parameters required in real options analysis, fuzzy real options allow parameters as fuzzy numbers.

This idea can be extended to tackle the private risk problem [121]. Similar to the IVP, random variables model market uncertainty, while fuzzy representations are used for private uncertainty. More specifically, the triangle fuzzy membership function is usually used to represent the three-point estimation of experts, which is widely applied in project management [93]. The probabilistic mean is calculated first using the Black-Scholes equations, and then the possibilistic mean is applied to obtain the final answer.

When the effects of private risk are not negligible and expert estimation is unavoidable, fuzzy real options may be more appropriate to use than (crisp) real options, even though the models are more complex. In areas such as environmental management, infrastructure development [79], and strategy selection [12], fuzzy real options are more convenient to employ, as they require fewer assumptions and include fewer correlated market factors.

Since fuzzy real options modelling incorporates fuzzy variables into stochastic processes, it integrates fuzziness and randomness. The advantage of this combination is that it meets practical needs in many applications, especially when descriptive expert knowledge, which can be fuzzy in nature, is used to calibrate the parameters of a stochastic process. In such cases, it is convenient to employ fuzziness and randomness at the same time.

Randomness is widely used in modelling natural and societal phenomena either because it is inherent or because specific detailed information is missing. In view of the complexity of the model, the latter situation is common. Even though unknown disturbances to a system may be very small, their overall impact is not negligible in many cases. Probability and stochastic processes are the prevalent modelling paradigm in fluid dynamics, the financial market, and thermodynamics. Fuzzy representation cannot replace randomness in these applications, given that these behaviours are countably additive, and thereby in conformity with the axioms of probability.

However, the parameters determining these processes, such as drift rate and volatility, may not be easy to estimate. In fact, selecting parameters is more art than science, and typically involves some subjective judgement, an area where fuzzy representation has demonstrated its effectiveness. Compared to the Bayesian subjective probability approach, fuzzy representation is more tolerant of the inaccuracy of human expression. Fuzzy parameter estimates are like likelihood estimates in that they are not normalized - their sum is not a constant. Additionally, the minimum and maximum operators used with fuzzy variables are cognitively simpler than addition. Because of these advantages, fuzziness has been widely applied to natural language processes and gained considerable success in this area.

Hence, combining fuzziness and randomness is natural in some real-world applications, and sometimes even seems inevitable. Related mathematical concepts such as a fuzzy random variable and a random fuzzy variable, and their applications, have been explored by authors in many papers, such as [70], [62], [63], [94], [133], and [23].

Fuzzy real options can utilize fuzziness and randomness in a complementary fashion when needed in evaluating risky projects. Its basis, the hybrid process, is further discussed in [133], [74], [70], [18], and [19]. Treatments differ in terms of the processes, the definitions of means, and the definitions of variances. But all of them generate very similar results for a basic case such as a fuzzy variable with a triangular membership function and a normally distributed random variable. In this thesis, Chance Theory and the hybrid process developed by Li and Liu [70] is employed, thereby taking advantage of their systematic study of different uncertainty representations [74] [70].

In Liu's theory [74], fuzziness is measured as credibility (Definition 3.4.1). Then the chance space is defined as the product of a credibility space and a probability space (Definition 3.4.2). The hybrid variable that integrates fuzziness and randomness is defined in Definition 3.4.3. Then, the chance measure is defined in Definition 3.4.4. It can be proved that the chance measure is subadditive (Theorem 3.4.5 [70]). Since people normally weigh facts differently, subadditivity is more appropriate to model human assessment of evidence [61].

Definition 3.4.1 *[74] Credibility (denoted as* Cr*) is a measure on a nonempty set* Θ*, which satisfies the following axioms:*

1. *Normality:* $\mathrm{Cr}\{A\} = 1$

2. *Monotonicity:* $\mathrm{Cr}\{A_1\} \le \mathrm{Cr}\{A_2\}$ *if* $A_1 \subset A_2$

3. *Self-duality:* $\mathrm{Cr}\{A_1\} + \mathrm{Cr}\{A_1^c\} = 1$*, where* A_1^c *is the complementary set of* A_1

4. *Maximality:* $\mathrm{Cr}\{\cup_i A_i\} = \sup_i \mathrm{Cr}\{A_i\}$ *for any events* $\{A_i\}$ *with* $\sup_i \mathrm{Cr}\{A_i\} < 0.5$*, where* sup *denotes the largest set among* A_i

where A *is a* σ*-algebra over* Θ *[124]. And* A_i *represents an event, which is essentially a subset of* A*.*

Definition 3.4.2 *Let* (Ω, P, Pr) *denote a probability space and* (Θ, A, Cr) *a credibility space. Then the product of these two spaces,* $(\Theta, A, \mathrm{Cr}) \times (\Omega, P, \mathrm{Pr})$*, is a chance space [70].*

In this definition, Ω *is a nonempty set;* P *is a* σ*-algebra over* Ω*;* Pr *is a probability measure. Similarly,* Θ *is a nonempty set;* A *is a* σ*-algebra over* Θ*; and* Cr *is a credibility measure. The tuple of these three elements defines a credibility space.*

Definition 3.4.3 *A hybrid variable is a measurable function from a chance space* $(\Theta, A, \mathrm{Cr}) \times (\Omega, P, \mathrm{Pr})$ *to the set of real numbers. For instance, for any Borel set* B *of real numbers, which is generated from open spaces (intervals) of real number sets, the set* $\{\Lambda \in B\} = \{(\theta, \omega) \in \Theta \times \Omega | \Lambda(\theta, \omega) \in B\}$ *is an event as* $\theta \in \Theta$ *and* $\omega \in \Omega$ *[70].*

Definition 3.4.4 *Given a hybrid space* $(\Theta, A, \mathrm{Cr}) \times (\Omega, P, \mathrm{Pr})$*, the chance measure of a hybrid event* Λ *is defined as [70]:*

$$\mathrm{Ch}(\Lambda) = \begin{cases} \sup\limits_{\theta \in \Theta}(\mathrm{Cr}\{\theta\} \wedge \mathrm{Pr}\{\Lambda(\theta)\}), \textit{if} \sup\limits_{\theta \in \Theta}(\mathrm{Cr}\{\theta\} \wedge \mathrm{Pr}\{\Theta(\theta)\}) < 0.5 \\ 1 - \sup\limits_{\theta \in \Theta}(\mathrm{Cr}\{\theta\} \wedge \mathrm{Pr}\{\Lambda^c(\theta)\}), \textit{if} \sup\limits_{\theta \in \Theta}(\mathrm{Cr}\{\theta\} \wedge \mathrm{Pr}\{\Theta(\theta)\}) \ge 0.5 \end{cases} \tag{3.5}$$

Theorem 3.4.5 *The chance measure is subadditive, which means that, for any events* Λ_1 *and* Λ_2*,* $\mathrm{Ch}(\Lambda_1 \cup \Lambda_2) \le \mathrm{Ch}(\Lambda_1) + \mathrm{Ch}(\Lambda_2)$ *[70].*

We often meet situations that a random variable contains parameters as fuzzy variables. In other words, the probability density of a random variable cannot be determined without fuzziness on its function parameters. The density of this random variable rv can be expressed as $\phi(rv; fv_1, fv_2, \cdots, fv_m)$, where ϕ is the density function on rv, with the number of m fuzzy parameters as $fv_1, fv_2, \ldots, fv_m$. Every fuzzy variable has its own membership function $\mu_i(fv_i)$ $(i = 1, 2, \ldots, m)$. In this case, the chance measure defined in Definition 3.4.4 can be more specific as follows Formula 3.6.

$$\mathrm{Ch}(\xi \in B) = \begin{cases} \sup\limits_{fv_1, fv_2, \ldots, fv_m} ((\min\limits_{1 \le i \le m} \frac{\mu_i(fv_i)}{2}) \wedge \int_B \phi(rv; fv_1, fv_2, \cdots, fv_m) drv), \\ \text{if} \sup\limits_{fv_1, fv_2, \ldots, fv_m} ((\min\limits_{1 \le i \le m} \frac{\mu_i(fv_i)}{2}) \wedge \int_B \phi(rv; fv_1, fv_2, \ldots, fv_m) dx) \le 0.5 \\ 1 - \sup\limits_{fv_1, fv_2, \ldots, fv_m} ((\min\limits_{1 \le i \le m} \frac{\mu_i(fv_i)}{2}) \wedge \int_{B^c} \phi(rv; fv_1, fv_2, \cdots, fv_m) dx), \\ \text{otherwise} \end{cases} \tag{3.6}$$

Among all types of hybrid variables, the normal distributed random variable with triangle formed fuzzy parameter may be the simplest while most useful ones. From the randomness perspective, the normal distribution is the most frequently used form with only two parameters. According to the central limit theorem, the sum of numerous small random disturbances will form the normal distribution regardless to their own distributions. On the other hand, the triangle-form membership function is easy to be applied to natural language process and three-point estimation, which are important in subjective parameter estimation. Therefore, this type of hybrid is employed in fuzzy real options modelling in this thesis.

The distribution of the hybrid variable is shown in Figure 3.1. This hybrid variable is a combination of a fuzzy variable and a random variable. It appears to follow the normal distribution function in some intervals, while becoming a straight line elsewhere. The hybrid variable is continuous, since both fuzzy variables and random variables are continuous, while its derivative is not.

When fuzzy real options are applied to evaluate brownfields, the same input data is adopted for the sake of result comparison. The private risk of redevelopment cost, reflected in the volatility rate of the redevelopment cost, σ_R, is deemed to be a fuzzy variable. The volatility of the redevelopment cost is difficult to estimate due to the complexity of the vodose zone modelling [32]. For instance, groundwater passes through peat (or cinders) at

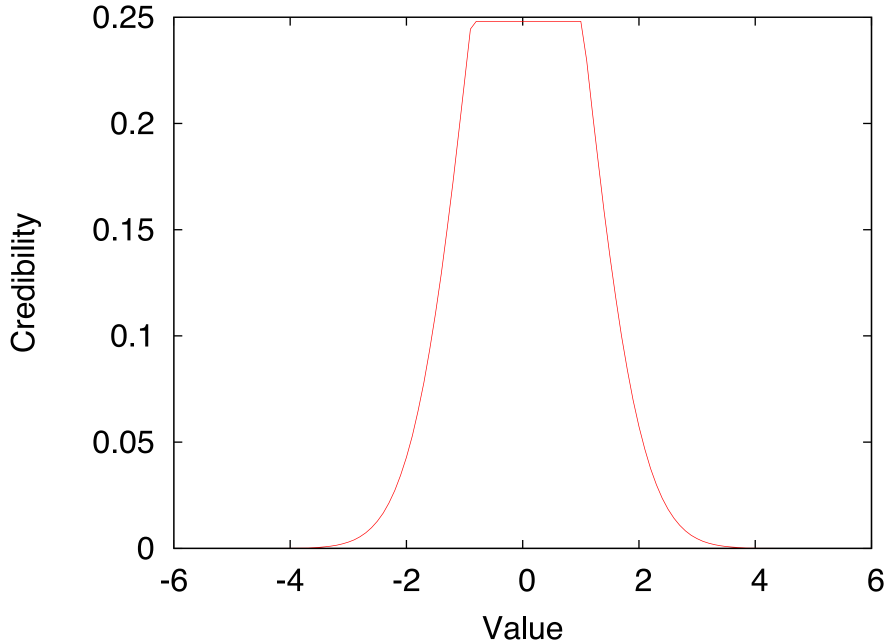


Figure 3.1: Chance Distribution of a Typical Hybrid Variable

the velocity of 177cm/day, which is hundreds of times the speed in the silt till (0.49cm/day) at the site of the Ralgreen Community in Kitchener, Ontario, Canada [54]. Moreover, redevelopment cost also depends on the residual rate of pollutants and excavation cost, which are hard to estimate using market data neither.

The redevelopment volatility is represented as a fuzzy variable with a triangle membership function, because project managers and experts usually use the three-point estimation method [93]. Based on the hydraulic conductivity, volume of contaminated soil, and elevations, we found the 20% volatility rate is valid and roughly realistic. Nonetheless, since there are only two wells drilled for sampling, a relatively large interval should be added. As the result, the fuzzy redevelopment volatility is inputted as (0.15, 0.2, 0.25). The three-point parameter estimation will be extended with OWA in Section 4.2, which is a kind of fuzzy deduction rules.

The main result indicates that the private risk of redevelopment volatility has impact on the value of brownfields. And because the output indicators are less than their corresponding critical values, the optimal decision is to wait and see, which helps to explain why developers are reluctant to undertake brownfield redevelopment projects.

Moreover, the critical values become fuzzy outputs since there is fuzzy input. These critical values can be converted into the ratio of x/R and shown in one figure as different decision regions in the strategy space. Fuzzy areas are calculated based on their fuzzy means and standard deviations, providing DMs an intuitive decision suggestion with the aid of the decision region chart.

Chapter 4

Using Fuzzy Real Options in Brownfield Evaluation

4.1 Extended Least-Squares Monte Carlo Simulation

Finding numerical valuations of fuzzy real options is challenging. Up to now, fuzzy real options have been used only when a closed-form solution is available [12] [118]. If the analytic form cannot be found, fuzzy real options are hard to evaluate. This thesis uses a Monte-Carlo simulation based algorithm to find numerical results for fuzzy real options, attempting to fill this gap.

In the following subsections, Monte-Carlo simulation and LSM are explained. Then the hybrid process and extended LSM algorithm are introduced [70]. The design of the associated tests is also discussed in connection with optimization of these algorithms.

4.1.1 Monte-Carlo Simulation and Real Options

Monte-Carlo simulation is one the major classes of numerical algorithms to price options. There are three major steps involved in Monte-Carlo valuation [3]:

1. *Generate sample paths*: Discretize the continuous sample paths and generate them based on specified random variables, such as Geometric Brownian Motion, using

$S(t + \Delta t) - S(t) = \mu S(t)\Delta t + \sigma S(t)\epsilon\sqrt{\Delta t}$, where S represents the present value, t represents time, ϵ is a standard normal variable distributed as $N(0, 1)$, μ represents the drift rate (or called growth rate), and σ represents the volatility rate.

2. *Apply backward induction to price the option for every path*: Calculate the maximum value on every path $V_i = max(V_{i,t})$, where $V_{i,t}$ stands for the value of the option at the time $t \in [0, T]$ on the i^{th} sample path and T is the expiration time. The maximum value can be obtained by recursively finding the higher value between the value of executing the option at the time t with the continuation value without execution at the time $t+1$ ($V_{i,t+1}$) when the option becomes exercisable at t. Backward induction means that calculation starts from the expiration time T, where $V_{i,T}$ is only determined by payoff and option execution cost, and backwards induction processes until $t = 0$.

3. *Estimate expectation on all paths as the value of the option*: Calculate the price of an option as the expectation of all sample paths, which is the mean of values on all paths $V = E[V_0] = \frac{1}{N}\sum_{i=1}^{N} V_{i,0}$, where N is the number of sample paths.

Monte-Carlo simulation is popular among financial engineers and real options practitioners, because of several advantages, described below (see [52] for details):

- *Stable converge speed*: The error rate of the Monte-Carlo simulation is $Error = O(Max[\Delta t, \frac{1}{\sqrt{N}}])$, where Δt is the time interval in discrete time values and N is the number of samples generated. Intuitively, both the number of samples (according to the Law of Large Numbers) and the number of time steps have impact on the accuracy. Hence, the larger one is the bottleneck. In other words, if the number of samples is small, therefore the bottleneck, it is impossible to achieve high precision no matter how small the Δt is.

- *Linear growth of algorithm complexity as the number of random variables increases*: Unlike the binomial tree approach, where complexity grows exponentially with the number of random variables, the complexity of Monte-Carlo simulation has only a linear increase. In real situations, Monte-Carlo simulation is particularly appropriate when the number of random variables is large.

- *Greater flexibility in parallel computing*: The Monte-Carlo simulation algorithm is easily parallelized and therefore benefits from multi-core development in the chip industry [17].

- *Expandability*: Monte-Carlo is easily combined with the Quasi-Monte Carlo random number generation algorithm, double sampling technique, and so on. All these techniques can greatly accelerate the performance [57].

The flexibility of scale expansion justifies the choice in this thesis of Monte-Carlo simulation as the basis for the numerical algorithm to price fuzzy real options. Once the proposed algorithm is demonstrated to be effective, it can quickly be applied to large-scale real applications. But when Monte-Carlo simulation is used to evaluate real options, there are some problems that must be addressed:

- *Early executable options*: Since real options are usually used to evaluate real assets, in most cases the execution time of real options is flexible. Asset owners have great power to deal with their own properties. The value of a sample path depends on when the option is executed, and therefore on the path. The normal backward algorithm in the Monte-Carlo Simulation must be modified in order to deal with this situation [76]. In addition, the analytic solution of the Black-Scholes equation is hard to find due to the unknown execution time [52].

- *Infinite time horizon*: Once an asset, real estate, is purchased, in theory ownership lasts forever. Many real options can be executed at any time with no expiration [30]. However, sample path generation in Monte-Carlo simulation cannot extend forever, as there must be an end in order to begin backward induction.

- *Critical values and expected execution time*: Unlike options in financial markets, DMs want to know not only the value of options, but also the optimal decisions and expected time to execute their options. The algorithms of finding these values must be part of the evaluation algorithm.

To evaluate early executable options using Monte-Carlo simulation, Longstaff and Schwartz [76] proposed the LSM, which has gained wide acceptance. The key point of LSM is to compare the expected payoff of continuation of the status quo against the payoff for immediate execution at every step. The expectation is calculated by utilizing information from all sample paths, and is estimated using a least squares method.

In carrying out the backward induction, the continuation values at time $t-1$ on all sample paths, which normally equal the expected values ($E[V_t]$) at the next step (t), are calculated using least-squares estimation. The payoffs at the next step (t) are converted to present value using $V_{t-1} = e^{-r\Delta t}V_t$. These values are collected in a vector denoted $\vec{Y}$. The known payoffs at time $t-1$ on all sample paths form a vector $\vec{X}$. Then, the following steps are conducted to find the expected payoffs at the next time t:

1. *Select functional form*: Various functional forms can be used in least squares estimation [64], such as $\vec{Y} = f(x) = a_0 + a_1\vec{X} + a_2\vec{X}^2 + a_3\vec{X}^3 + a_4\vec{X}^4 + a_5\vec{X}^5$.

2. *Conduct least squares estimation*: Use least squares estimation to estimate all of the coefficient parameters in the function by $E[Y|X] = \widehat{Y} = f(x)$ (for instance, $(a_0, a_1, a_2, a_3, a_4, a_5)$ in the above example).

3. *Calculate the expected continuation value*: The expected continuation value on every sample path is calculated as $E[Y = y|X = x] = \widehat{y} = f(X = x)$.

The LSM approach is elegant and powerful, extending the application of the Monte-Carlo method to almost all path-dependent options. However, least squares estimates of continuation values are proven valid within the realm of probability theory, and therefore limited to the stochastic case. Whether LSM can price fuzzy real options credibly is unknown, and should be tested [105]. As usual, the critical value and infinite horizon problems will be addressed in algorithms proposed below. All of these considerations are summarized in Figure 4.1, which are all included in the newly developed extended LSM introduced in the next subsection 4.1.2 .

4.1.2 Extending the LSM Algorithms

The Hybrid Process

Similar to stochastic processes, we define a hybrid process to be a sequence of real numbers retrieved from hybrid events indexed by an index set T [74]. A hybrid process is incrementally stationary if and only if for all t, $X_{t+1} - X_t$ is an independent and identical hybrid

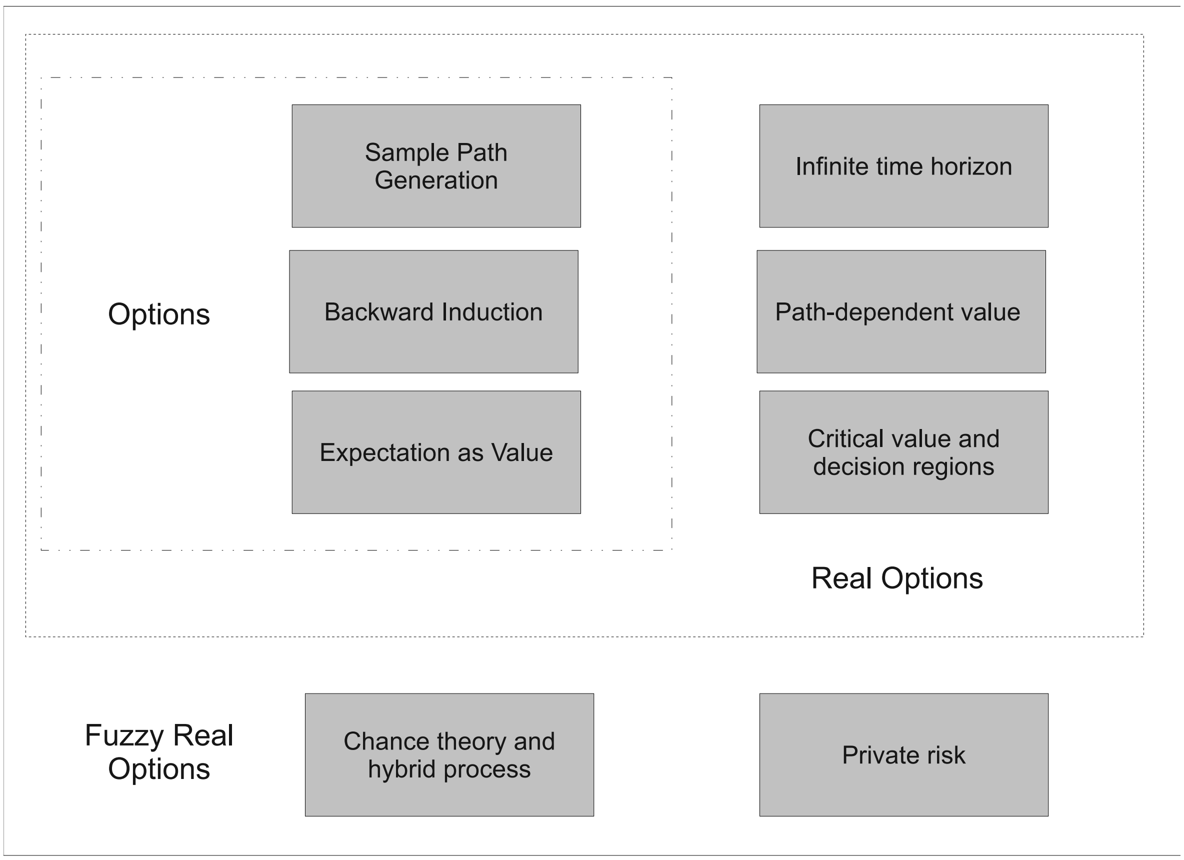


Figure 4.1: Conceptual Map of Considerations for Extending Option Pricing Models to Fuzzy Real Options

variable. Following the assumptions made in real options analysis, the hybrid processes used to value fuzzy real options are incrementally stationary [70].

The hybrid variable used as stationary increment in this thesis is a normally distributed random variable $N \sim (\mu, \sigma)$ with parameters (μ and σ) that are fuzzy variables with triangular membership functions. This hybrid variable is selected because it is analogous to the normal random variable that is the basis of Wiener process. When private risk is present in fuzzy real options, parameter estimates from experts are often expressed as fuzzy variables with triangle membership functions. Hence, a normally distributed random variable with triangle-form fuzzy parameters must be studied as the base case.

To generate sample paths for the hybrid process, two separate steps are required: first the fuzzy samples are generated, as these processes have different parameters; second the hybrid process is simulated by splitting it into sets of stochastic processes, each corresponding to possible values of the fuzzy parameters. Because all of the increments on every sample path come from the same hybrid variable, it is easy to repeatedly simulate the entire hybrid process. The algorithm is set out as Algorithm 1.

Algorithm 1 divides the hybrid process into two levels to deal with the fuzzy parameters and the stochastic process separately. This helps to cope with the difficulty of fuzzy variable simulation. Separating fuzziness and randomness also facilitates following steps, such as backward induction and calculation of expectation. Validation tests are conducted in each case study.

Least Squares Estimation of Conditional Expectation of Hybrid Variables

As mentioned above, LSM employs the least squares estimation to find the expected value of continuation with no action and compares it with the payoff of option execution at the present time. The least squares estimation can utilize a cross-section of information to find the expected value of continuation on every sample path. Hence, there is a question about the validity of the least squares estimation when a hybrid process is introduced.

This problem can be answered using levelled least-squares estimation. If least-squares estimation is conducted on a group of stochastic processes with the same fuzzy sample, LSM can be assumed valid, as the hybrid variable has been degraded back to a random variable. When the backward induction is complete, values with fuzzy membership degrees

Algorithm 1 Hybrid Process Sample Paths Generation Algorithm

Require: Parameters of normal distribution with fuzzy three-point estimates of μ and σ

Ensure: A $SamNum * Step$ matrix ($out[][]$) of all hybrid sample paths.

▷ Where $SamNum$ is the number of sample paths

▷ $CrNum$ is the number of fuzzy samples

▷ $PrNum$ is the number of random samples

▷ $Steps$ is the number of induction steps, which equals T/Δt

▷ Where $SamNum$ is the number of sample

$SamNum \leftarrow CrNum * PrNum$

▷ Fuzzy samples generation with (min, $most$, max) triangular membership functions

for $i = 1$ **to** $CrNum$ **do**

  $CrValue[i] \leftarrow Random.uniform(min, max)$

  if $CrValue[i] \leq most$ **then**

    $CrDegree[i] \leftarrow \frac{CrValue[i]-min}{2(most-min)}$

  else

    $CrDegree[i] \leftarrow \frac{CrValue[i]+max-2most}{2(max-most)}$

  end if

end for

for $i = 1$ **to** $CrNum$ **do**

  $sigma \leftarrow CrValue[i]$

  for $j = 1$ **to** $CrNum$ **do**

    $out[i * CrNum + j][1] \leftarrow IntialValue$

    for $k = 2$ **to** $Step$ **do**

      $out[i * CrNum + j][k] \leftarrow out[i * CrNum + j][k-1] * exp(mu * dT - 0.5 * sigma^2 * dT + sigma * \sqrt{dT} * Random.gauss(0, 1))$

    end for

  end for

end for

are obtained. Steps of finding the expected value will be applied later. The algorithm is listed as Algorithm 2.

In addition, the validity of the non-levelled least-squares estimation can be checked by comparing its result to the levelled one. Since the separated estimates do not utilize cross-sectioned information across different fuzzy parameters, they can be regarded as a test bench for the non-levelled algorithm.

Expected Value Estimation

The final vector calculated using Algorithm 2 is associated with a fuzzy membership degree for every sample path. Each option value at this last step has an associated fuzzy membership. The mean of this fuzzy vector cannot be calculated as the average of all samples. so the definition of the expected value of a fuzzy variable is employed to obtain a final value. The algorithm is based on [73].

Further, normalized fuzzy membership averaging can be used to compare results. After the fuzzy membership function is normalized, weighted averaging is employed to calculate the expected value. Because normalization makes the fuzzy membership degree to be additive, the mean obtained can also be used to explore the linkage between fuzziness and randomness. Results can be compared using previous cases, so that better candidates are identified for further study.

Critical Values and Optimal Strategies

Critical values divide the state space into decision regions with different optimal strategies. For an American put option, such as the ability to abandon a project at anytime, a critical value splits the space into two decision regions: keep the project running if the value of the state variable is higher than the critical value; and reject the project once the state variable becomes lower than the critical value. Execution of the option to abandon occurs when the state variable falls from keeping the project to rejecting it.

When finding critical values using Monte-Carlo simulation, infinite horizon is an important problem to overcome. Since Monte-Carlo simulation must have an end node to start backward induction, infinite horizon is treated in two parts: a long time to simulate

Algorithm 2 Hybrid Process Backward Induction Algorithm

Require: The $SamNum * Step$ payoff matrix of all hybrid sample paths

Ensure: The present value vector for all hybrid sample paths

▷ Valcont and ValExec denote the value of continuation and execution respectively.

```
for i = 1 to SamNum do
    if ValCont ≤ ValExec then
        Payoff[i][Step] ← ValExec
    else
        Payoff[i][Step] ← ValCont
    end if
end for
for i = 1 to CrNum do
    for j = Step − 1 to 1 do
        for k = 1 to CrNum do
            RealContVal[i*CrNum+k][j] ← Payoff[i*CrNum+k][j+1]*exp(−r*dT)
            Y[k] = RealContVal[i * CrNum + k][j]
            X[k] = Payoff[i * CrNum + k][j + 1]
            RealExeuVal[i * CrNum + k][j] ← (Payoff[i * CrNum + k][j + 1] + out[i * CrNum + k][j]) * exp(−r * dT)
        end for
        Coef ← LeastSquareEstimate(Y, X)
        for k = 1 to CrNum do
            ExptContVal[i * CrNum + k][j] ← Coef * X[k]
            if ExptContVal[i*CrNum+k][j] ≤ RealExeuVal[i*CrNum+k][j] then
                Payoff[i][Step] ← RealExeuVal[i * CrNum + k][j]
            else
                Payoff[i][Step] ← ExptContVal[i * CrNum + k][j]
            end if
        end for
    end for
end for
```

so that the value approximately equals the infinite one; and the expected project value after the last simulated nodes, which will be regarded as an added constant during the backward induction to make the simulation more accurate.

Stated as the continuous and smooth pasting conditions [30], critical values are the points where both the payoff and its derivative equal the value and its derivative of the option. Meeting critical values from the no-execution zone to another usually implies executing the option. To simplify the process of identifying critical values and expected execution time, this paper suggests a process utilizing properties of infinite horizons.

Since every node can be regarded as a start node with only one sample path, critical values can be identified at the last time where the continuous and smooth pasting conditions are satisfied in the backward process. The execution time at this node is recorded for the expected execution time calculation. In other words, critical values and execution time can be identified by adding some conditional statements to the backward induction algorithm.

When implementing a given algorithm, a parameter such as 10 years out of 200 years simulated time, is assumed to be the latest time for critical values and execution time identification. This is because nodes near the bottom cannot be regarded as starting nodes. Once the nodes are identified, the control parameter has to be traced back. This algorithm is proposed as Algorithm 3.

In Algorithm 3, tracing back to the control variable is conducted via the linkage between the control variable and its payoffs, which are stored in the price matrix and payoff matrix. With this mapping, one can always find the value of the control variable.

Algorithm 3 becomes more complex when applied to a hybrid process. As usual, the critical values and execution times will be identified first. But, for hybrid processes, sample paths have fuzzy memberships, so the final critical values and expected execution times are fuzzy variables as well.

4.1.3 Illustrative Applications using Extended LSM

Two illustrative applications will be introduced in the following sections for two main reasons: to test proposed algorithms by comparing their outputs with previous results, and to demonstrate what information fuzzy real options can provide to DMs. The first is a brownfield study, while the second is an oil development evaluation. Although oil

Algorithm 3 Algorithm for finding critical values and expected execution time

Require: The $SamNum{*}Step$ cash flow matrix (out) and the option value matrix (Payoff)

Ensure: Critical Values and Execution Times

```
Year ← TotalYear * 5%
Tol ← 0.01
for i = Step − 1 to 1 do
    for j = 1 to SamNum do
                    ▷ Comparison based on the continuous and smooth pasting conditions
        if i*δt < Year and out[i][j] − Payoff[i][j] ≤ Tol and out[i][j+1] − out[i][j] ≤ Tol then
            ExeTime[i] ← i * δt
            CriVal[i] ← P which leads to out[i][j]
        end if
    end for
end for
```

development is not a typical brownfield, it relates to brownfield problem due to its risk to groundwater, and therefore, the possibility of becoming a brownfield. Backgrounds, input information, result comparisons, and insights are explained later.

Brownfield Evaluation using Fuzzy Real Options

The same data of a brownfield example from Lentz and Tse, a classic real options study on brownfield evaluation, is used so that outputs of the extended LSM algorithm are comparable to results in literature. Numerical outputs and analytic result [66] are compared and analyzed. Interpretation of fuzzy outputs of the extended LSM is further discussed later. Studies are conducted as follows:

- *Least-square Estimation*: Two ways of applying least-square estimation are tested. One is to restrict least-square estimation within a group with the same fuzzy parameter, which is called levelled LSM in this paper. The other is to use least-square estimation in all sample paths, the same as the original without any consideration of fuzzy values, which is called non-levelled LSM.

- *Expected Value*: Identify appropriate method of calculating the expected value of the hybrid process, including fuzzy expected value definition, expected value as averages, and normalized averages as a quick way, which normalizes the resulted fuzzy variable at first, and then compute the expected value as the weighted average.

- *Critical Values*: Similar to the value of brownfields, algorithms to find critical values differentiating optimal decisions will be compared with results in the literature having the same parameters. Fuzzy critical values are plotted to check the effect of fuzziness.

When applying fuzzy real options to evaluate brownfields, income (x) and cost (R) are assumed to be uncertain, for which both follow normal distributions. Although uncertainty in incomes ($x \sim N(\mu_x, \sigma_x)$) is reflected in the market data, uncertainties regarding remediation and redevelopment costs are private risks. Therefore, redevelopment cost (R) is represented by a hybrid variable, $R \sim N(\mu_R, \sigma_R)$ with its variance as a triangular-form fuzzy variable ($\sigma_R \in (a_R, b_R, c_R)$ [118].

The above variable definitions for brownfield redevelopment fit Model IV of Liu [73], as both hybrid variables has a probability density function (normal distribution) with fuzzy parameters with triangle membership functions. Liu's Model IV is used in this case study [73].

In order to compare results, the brownfield model remains the same except for differences in private risk treatments. They are compared as follows [66] [121]:

- Brownfield modelling
  - There are two uncertain parts: payoffs and redevelopment costs. Payoffs are continuous cash flows, while redevelopment costs are one-time payments;
  - There are three stages in brownfield redevelopment: before remediation, after remediation, and after redevelopment. The payoffs are assumed to be proportional in these stages;
  - Redevelopment costs have two components: remediation and redevelopment;
  - There are three strategies a developer can choose: take no action; clean and redevelop brownfield sequentially; and clean and redevelop simultaneously.
- Private Risk

– Real options: Both payoffs and redevelopment costs are private risks, but correlated to two market-priced variables;

– Fuzzy real options: Income only has market risk, while uncertainty about redevelopment costs is private risk. The volatility of costs (σ) is typically estimated by experts using three-point estimation as a triangle-form fuzzy variable.

When parameters similar to those given in the literature are used (Table 4.1), similar results are obtained (Table 4.2). The value of the brownfield is found to be slightly higher than the output using NPV. The optimal decision, to wait and develop simultaneously later, is suggested. The expected waiting time is also provided. In addition to the NPV value of the property, DMs can find the value of the managerial flexibility of waiting for a better market environment and the way to realize it. Furthermore, because fuzzy variables are added, decision boundaries marked by critical values become fuzzy. The critical value using non-levelled LSM is plotted in Figure 4.2 as an example.

When comparing results from different approaches, the findings suggest that numerical methods can produce answers consistent with analytic methods. When the analytic form of fuzzy real options is difficult to derive, extended LSM can help to evaluate fuzzy real options. But, we can also see that the result of expected time and critical value from the non-levelled LSM is not satisfactory, indicating that the levelled LSM algorithm is preferred over the non-levelled one.

Furthermore, in terms of the estimator used in the least-square estimation, the fifth level of polynomial function is found to be appropriate because it achieves both satisfactory performance and good quality of estimates. The impact of different estimators does not significantly affect the results, but has various performance times. Polynomial function is selected due to its flexibility in curve fitting. As a higher level is included, the running time grows exponentially. The balance between satisfactory estimates and tolerable performance must be kept. A better discussion on selection of different types of estimators and associated parameters can be found in [64].

As for the methods to determine expected value (Table 4.3), we found that when the fuzzy interval approaches zero, the fuzzy expected value definition, average, and normalized average share similar results, suggesting that when the fuzzy variable becomes a crisp one, real options and fuzzy real options are identical. But, even though the fuzzy expected value definition is accurate in theory, the normalized average is suggested, given that the

Table 4.1: Input Data of a Brownfield [66]

Benefit	drift rate	$\mu_x = 2.5\%$
	volatility	$\sigma_x = 20\%$
	present annual income	$x_0 = \$ 300{,}000$
Cost	drift rate	$\mu_R = 7\ \%$
	present redevelopment cost	$R_0 = \$ 5{,}000{,}000$
Parameters	income rate with contamination	$\varphi_1 = 0.4$
	income rate after clean-up	$\varphi_2 = 1.0$
	income rate after redevelopment	$\phi = 2.0$
	clean-up cost rate	$\alpha_1 = 0.3$
	redevelopment cost rate	$\alpha_2 = 1.0$
	riskless interest rate	$r = 5\%$
Private Risk	real options	correlations $\rho_{xP} = 0.7$ and $\rho_{RK} = 0.8$
	fuzzy real options	Fuzzy σ_R with triangular distribution (0.15;0.2;0.25)

Note: The main data source comes from Lentz and Tse [66], whose parameters are explained in Section 2.3.

Table 4.2: Main Result Comparison between Analytic and Numerical Outputs

	Analytic	Levelled	Non-levelled
Value (Million Dollars)	17.838	16.896	17.122
Critical Value (x/R)	0.0823	0.0733	0.0747*
Current Ratio (x/R)	0.06		
Expected Time (Year)	10.5	9.18	6.63
Optimal Decision	Wait and develop simultaneously in future		

Note: Critical values and expected times are fuzzy variables. Crisp values in the table are expected value of these fuzzy variables. The fuzzy critical value retrieved from non-layered method is plotted in Figure 4.2 as an example.

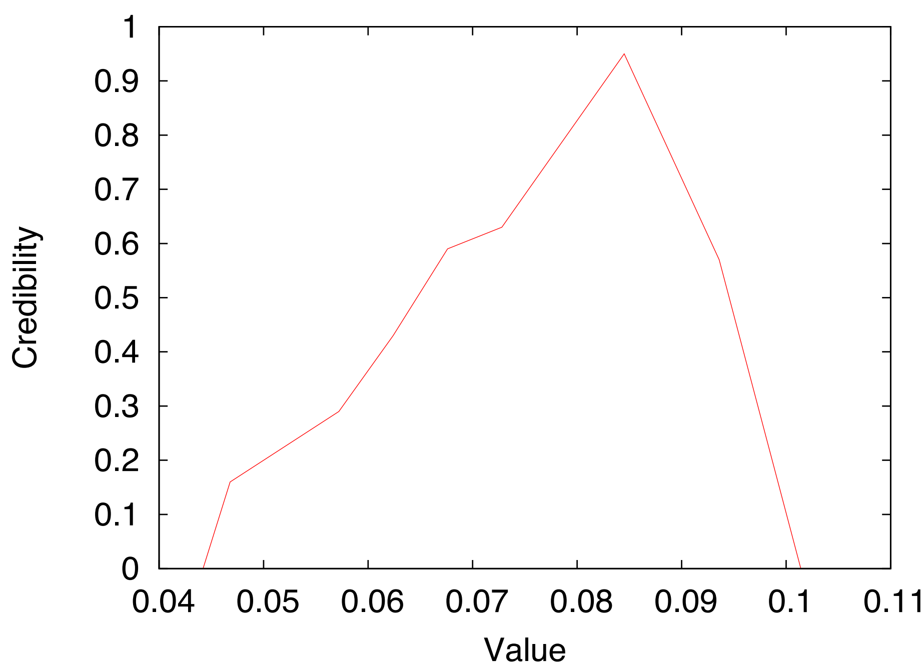


Figure 4.2: A Fuzzy Critical Value

Notes: The vertical measurement "Credibility" is essentially the fuzzy membership degree, while the horizontal measurement "value"is the ratio of the annual income to the redevelopment cost (x/R).

output of the fuzzy expected value definition is unstable when the sample number is small. Generating a large number of fuzzy samples requires too much computing resources. The normalized average is also superior to the average, as there is no distribution consideration in calculating the average. The result will be the same regardless of the fuzzy membership distribution if the average is examined. The normalized average was selected due to its consistency and precision. The appropriate sample number can be determined by observing the convergence rates of different sizes with a much shorter time parameter T. We found the sample size is better to be set between 100 and 200 in fuzzy real options modelling.

Table 4.3: Comparison of Methods to Calculate the Expected Value (Million Dollars)

	Sample Number			
	30 (T=200 years)		200 (T=15 years)	
	mean	interval	mean	interval
Mean	16.724	3.2	5.442	0.8
Unified	17.106	3.6	5.478	0.9
Fuzzy	18.092	18.7	5.650	0.9

Studying the effect of fuzzy uncertainty shows that the brownfield evaluation increases as fuzzy uncertainty increases (Figure 4.3). This phenomenon confirms a finding in literature [12] [118], indicating that the private risk has similar effect on the output values to market risks. The higher uncertain the future is, the larger value of managerial flexibility (or the adaptivity) becomes.

Oil Extraction using Different Private Risk Solutions

Oil extraction is a classical problem used in studies of real options modelling and decision analysis [43] [91] [41] [92]. One popular case is a study of oil extraction in Southwestern United States based on the original work of Smith and McCardle [105], whose data are derived from Gibson [41].

The purposes of this case study are to compare fuzzy real options with IVP, the fuzzy effect on the growth rate, and an initial study on how value changes as preference varies.

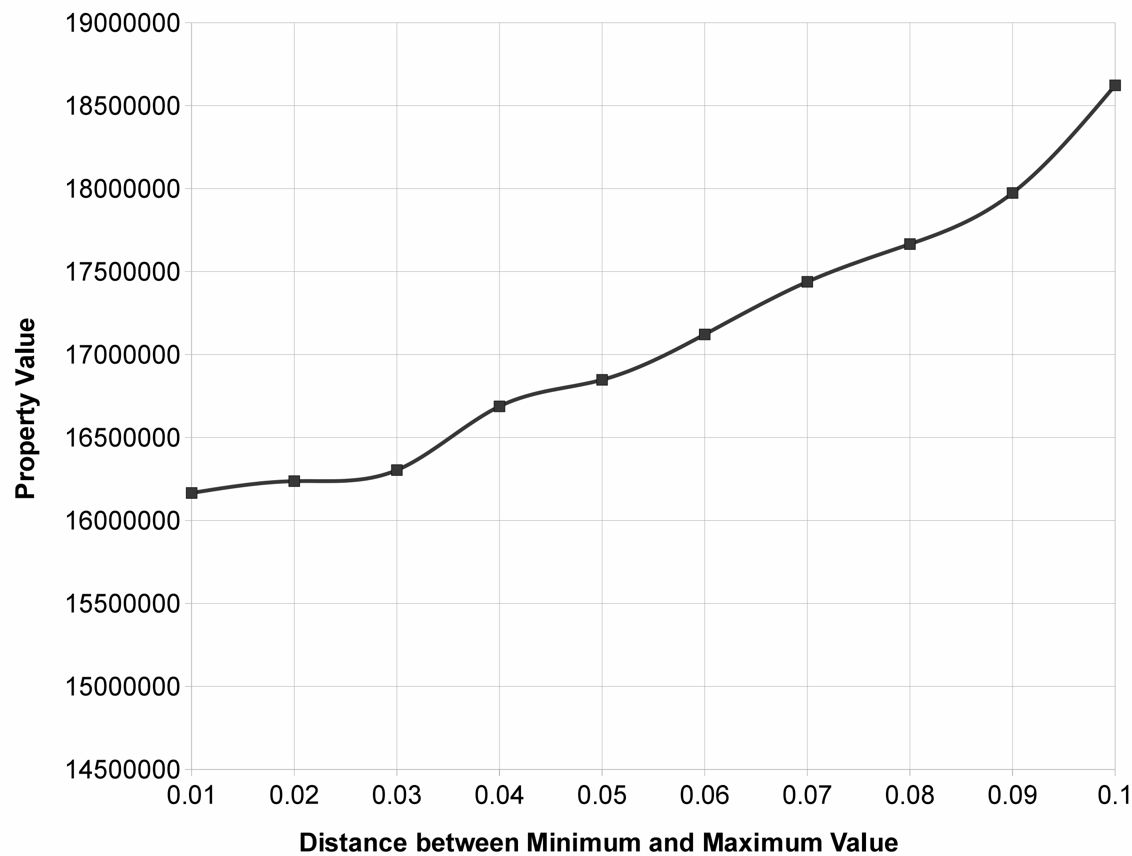


Figure 4.3: Brownfield Value as Fuzzy Interval Increases
Note: The property value is measured in dollars. The distance between minimum and maximum value refers to the three-point estimation on the volatility. In this diagram, the triangle is assumed to be symmetric representing the risk-neutral case. Therefore, the distance is actually measured between the most likely and the minimum (or maximum) values, which is a half of the distance between minimum and maximum value. The value shown in Table 4.2 is the point at 0.05 in this figure.

Table 4.5 shows the main results, including value and critical value using input data that are similar to Smith and McCardle's (Table 4.4).

Table 4.4: Data used in Evaluating Oil Properties [41]

Input Variables		Value
Riskless interest rate	r	0.5 %
Drift rate of oil price	μ_{price}	-8%
Volatility of oil price	σ_{price}	33%
Drift rate of oil production	$\mu_{production}$	-10%
Volatility of oil production	$\sigma_{production}$	2%
Present oil price	P_0	\$ 18.00
Present oil production per day	V_0	\$ 600
Annual operation cost	Op_C	\$ 255500.0
Tax rate	tax	74.192%
Decommissioning cost	abandon	\$ 350000.0

In addition, the value of fuzzy real options, risk neutral, and IVP cases can be compared as the initial value changes from 0 to 20 million dollars, which can be calculated out by $P_0 \times V_0 \times 365$. For the case shown in Table 4.4, the inivial value is around four million dollars, because $18 \times 600 \times 365 = 3942000$. Consistent with previous results, the uncertainty in fuzziness slightly increases the value of oil property compared to the risk neutral case. Risk tolerance requires some premium to compensate the private risk.

Figure 4.5 shows the consequences of using the skewness of the triangle membership function as an indicator of preference. It shows the effect of moving the most likely value from maximum to minimum. The closer the most likely value to the minimum, the lower the value it indicates. Note that the most pessimistic value is less than the risk neutral value. This phenomenon is similar to the IVP case when the risk tolerance is added. Overall, this analysis confirms that fuzzy real options are appropriate to represent private risk.

Hence, we find that fuzzy real options also provide DMs a way of reflecting their risk preference into the risky project evaluation. Estimates from experts contain their prefer-

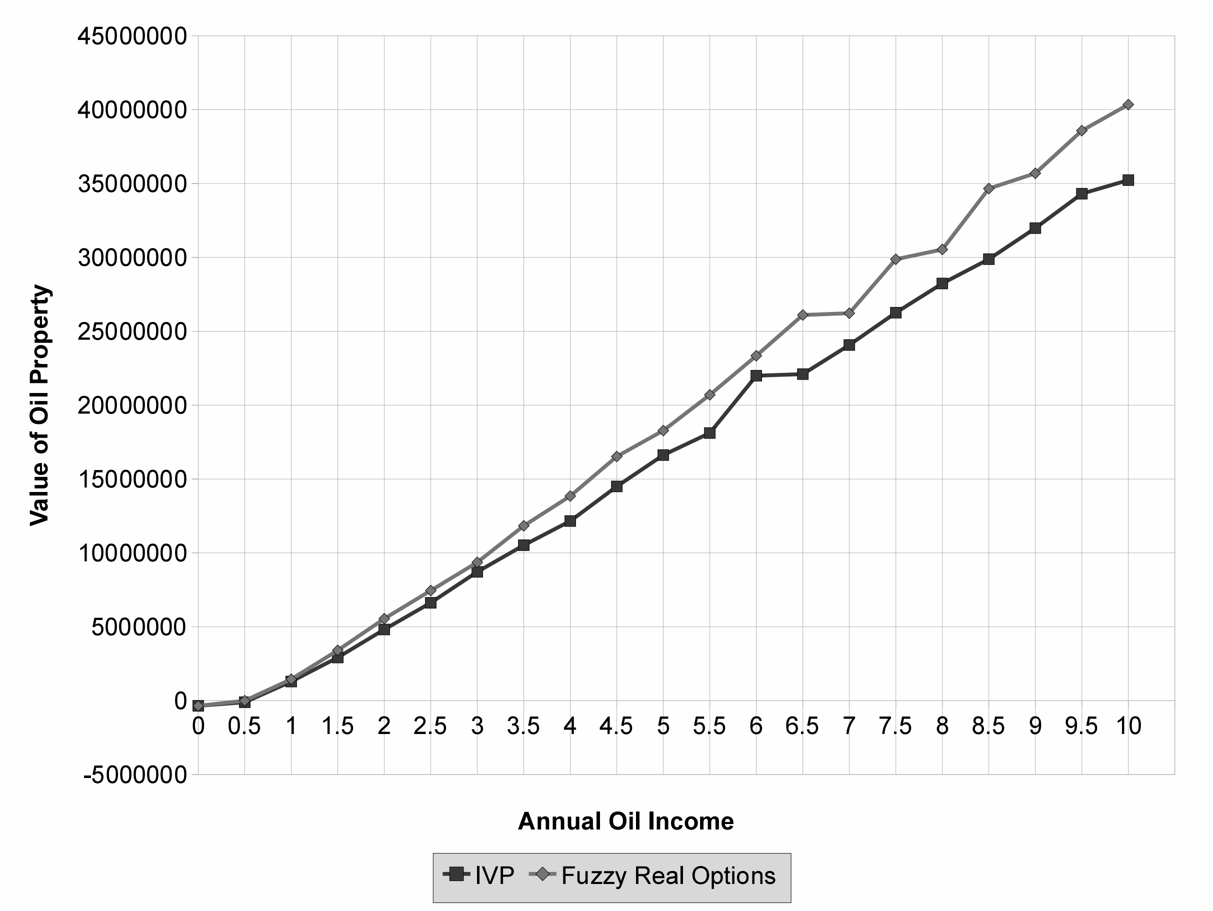


Figure 4.4: Value Comparison between IVP (risk neutral) and Fuzzy Real Options
Note: The property value is measured in dollars. The oil annual income is shown in million dollars. The data shown in Table 4.4 is around the point 4 million dollars, because $18 \times 600 \times 365 = 3942000$.

Table 4.5: Result Comparison for Oil Property

	Real Options	IVP	Fuzzy Real Options
	risk neutral	R = 10	$\mu_{production} = (-0.2; -0.1; 0.0)$
Value	12.096	11.923	12.231
Critical Value	267.383	263.122	NA
Optimal Strategy	Acquire immediately	Acquire immediately	Acquire immediately

Note: The critical value of fuzzy real options is omitted here because it is a fuzzy number. Except for the fuzzy drift rate, all other numbers shown in this table are measured in million dollars.

ences, and therefore affect the evaluation of the project. DMs can understand the impact of their risk preferences on their optimal decision by utilizing fuzzy real options.

In conclusion, extended LSM can generate numerical results for fuzzy real options modelling. The numerical and analytic results are approximately equal in cases where comparison is possible. Fuzzy estimation normally slightly increases the value of fuzzy real options, which is easily understood as the value of managerial flexibility in dealing with private risk. As the skewness of the fuzzy membership function changes, the value of fuzzy real options also changes. Compared to the IVP, extended LSM produces similar results and can be regarded as another option for representing preference toward private risks.

The numerical method studied in this thesis enables DMs to use fuzzy real options in more applications. The flexibility of fuzzy real options generalized to stochastic models can be utilized whenever DMs are unsure on certain parameters. Both market and private risk can be accommodated in fuzzy real options, and can be calculated even if no analytic solution can be found. This advantage can promote the usage of real options analysis in areas where private risk cannot be overlooked. DMs can learn the value of the managerial flexibility, the impact of their risk preferences, and the optimal decision regarding a risky project.

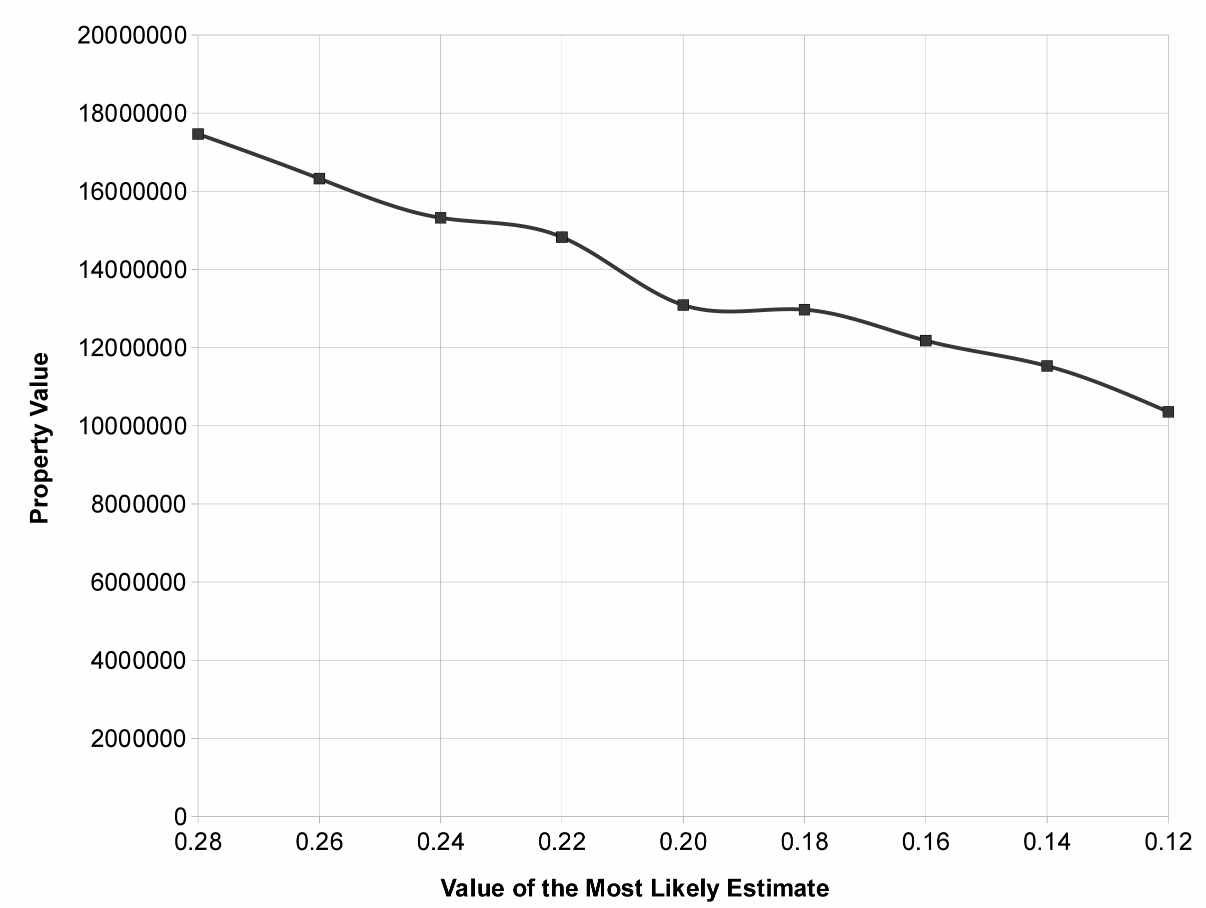


Figure 4.5: Value as Preference Changes

Note: The property value is measured in dollars. The minimum and maximum value of the three-point estimation on the volatility are 0.1 and 0.3 respectively. The horizontal axiom shows the value of the most likely point, changing from the maximum to the minimum.

4.2 OWA for Contamination Likelihood Estimation

4.2.1 Brownfield Redevelopment Process in Ontario, Canada

In Ontario, Canada, brownfield redevelopment contains two main stages: redevelopment and long-term monitoring according to the Ontario Regulation 511/09 [90]. The redevelopment process can be further divided into three phases. Phase I is ESA I. An expert investigates the brownfield site and uses his or her judgement to decide whether this site has been contaminated. If so, the scope of Phase II (ESA II) will include surveying, monitoring, and remediation; if not, a RSC will be submitted to the MOE. Then the site undergoes long-term monitoring. This process is illustrated in Figure 4.2.1.

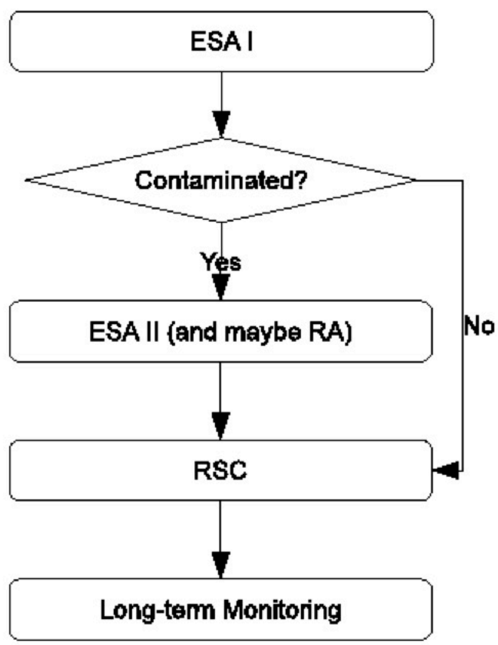


Figure 4.6: Process of Brownfield Redevelopment in Ontario, Canada

Human judgement plays a critical role in determining the likelihood that contaminants have affected the property; therefore, subjective uncertainty should be included in pollution estimates. A fuzzy boundary is an appropriate representation for dividing contaminated and clean regions, which can be determined using an OWA operator including judgements on different environmental indicators [115]. The proposed numerical method of calculating the fuzzy boundaries will be further discussed later.

A typical brownfield redevelopment negotiation process occurred in the brownfield redevelopment case in the Ralgreen Community, Kitchener, Ontario, Canada [54]. At first, the community observed the degradation of some environmental indicators, including odour in basements, sinking of garages, and killing vegetation. They complained to the City of Kitchener, since the landowner was bankrupt. After a year-long negotiation, an air photo showing a landfill site in 1950s under some properties in Ralgreen was found, providing strong evidence of pollution. Since all phenomena indicated that the community has been polluted, corresponding to ESA I, a land survey was conducted by a third-party engineering consulting company (similar to ESA II). A redevelopment plan was proposed and implemented after hazardous materials were detected.

Through this negotiation process, subjective judgements on the likelihood of contamination were critical. This is especially true in ESA I, when surveying and monitoring efforts were minimal. In addition, DMs may have different judgements based on the same evidence. For instance, residents in a community are more likely to believe their community is contaminated than a landowner. Multi-criteria aggregation with preference characterizes this process.

To facilitate the estimation of contamination likelihood, OWA can be employed to generate fuzzy boundaries around a brownfield, dividing contaminated and clean areas with degrees of fuzzy membership. In this case, OWA acts as the fuzzy deduction rule to determine the triangle membership functions of the parameters in the fuzzy real options. Unlike crisp boundaries, overlaps between polygons are allowed [115]. Fuzzy boundaries reflect the reality that the transition of a contaminated area to a non-contaminated area occurs gradually, rather than abruptly. The transition can be represented using fuzzy membership functions.

DMs and experts can mark their judgement at some sample spots on the conceptual site model (CSM) (Figure 4.7), which records the contamination information through the ESA processes, such as site-specific hydrogeology, site layout, and map of surrounding area in Ontario. Their descriptive estimates and preference are represented as fuzzy membership degrees. An OWA operator is applied to compute the likelihood on the spot, where preference is added via linguistic quantifiers [132]. The process is illustrated in Figure

4.8. Interpolating the pollution level on sample spots to the whole site generates the fuzzy boundaries of a brownfield, which is used to determine parameters of its fuzzy real options model.

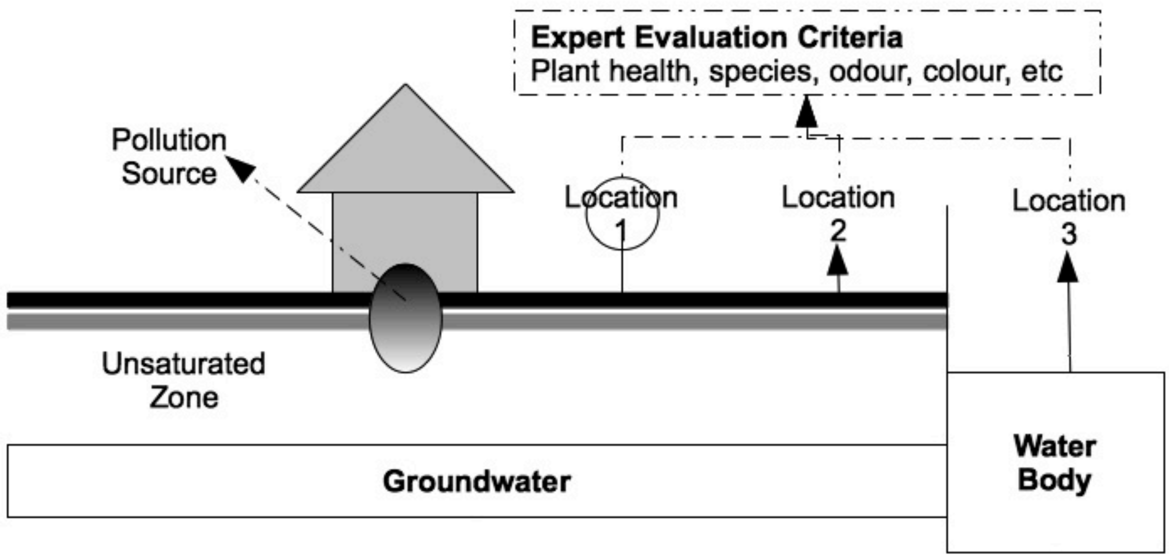


Figure 4.7: Conceptual Site Model of a Brownfield

4.2.2 Introduction to OWA

Formally, OWAs are defined in Definition 4.2.1. They assume a total weight of 1 ($\sum_{i=1}^{n} w_i = 1$), and produce expected value of the form $w_1 b_1 + w_2 b_2 + \ldots + w_n b_n$. OWAs are generalized T-norm and S-norm operators in fuzzy logic, and can be used to find membership degrees in a fuzzy set from multiple factors.

Definition 4.2.1 *An ordered weighted operator F is a mapping that converts the multi-dimensional unit interval into the one-dimensional unit interval: $F : I^n \rightarrow I$, where $I = [0, 1]$. F is determined by a vector with n elements, denoted $\overrightarrow{W} = [w_1, w_2, \ldots, w_n]$, and follows the rule [132]:*

$$F(a_1, a_2, \ldots, a_n) = w_1 b_1 + w_2 b_2 + \ldots + w_n b_n \tag{4.1}$$

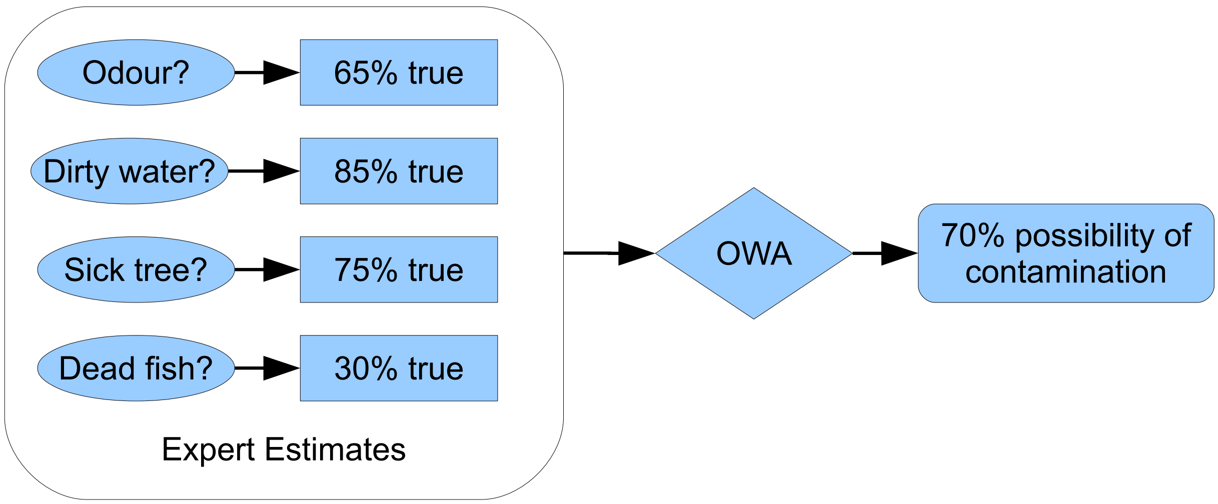


Figure 4.8: An Example of using OWA in Determining Pollution Likelihood

where $a_1, a_2, \ldots, a_n$ *are the multiple inputs with values between 0 and 1 and* $b_1, b_2, \ldots, b_n$ *is the ordered vector of these inputs from the maximum (*$b_1 = \max(a_1, a_2, \ldots, a_n)$*) to the minimum (*$b_n = \min(a_1, a_2, \ldots, a_n)$*). The elements in* $\overrightarrow{W}$ *satisfy two conditions:* $w_i \in [0, 1]$ *for all* $i = 1, 2, \ldots, n$ *and* $\sum_{i=1}^{n} w_i = 1$.

The "ORness" measurement of F is defined as $Orness = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$ [131]. There are three special cases: if $w_1 = 1$ and all other $w_i = 0$, F becomes the "OR" operator in fuzzy sets $F(a_1, a_2, \ldots, a_n) = max(a_1, a_2, \ldots, a_n)$; if $w_n = 1$ and all other $w_i = 0$, F turns into the "AND" operator in fuzzy sets $F(a_1, a_2, \ldots, a_n) = min(a_1, a_2, \ldots, a_n)$; if all $w_i = \frac{1}{n}$, the OWA is an average, the arithmetic mean.

There are many ways of determining OWA operators [129]. In the context of negotiation and expert knowledge representation, the equation of determining weights from the linguistic quantifier can be utilized, such as "all", "any", and "most". The template "Q criteria must be met" can be applied to identify a function Q. An OWA can be determined with Q according to $w_i = Q(\frac{1}{n}) - Q(\frac{i-1}{n})$ (for all $i = 1, 2, \ldots, n$). Preference inherent in linguistic quantifiers can be found using the indicator value of individual disapproval (VOID) $VOID = 1 - \int_0^1 Q(x)dx$ [132].

4.2.3 OWA and Fuzzy Boundaries

The proposed numerical method contains three main parts. First, OWA is used to aggregate DMs judgements on multiple environmental indicators. The three-point estimates are calculated with different preference parameters. While the minimum and maximum corresponds to the two extreme cases, the most likely contamination level will be determined by DMs risk preferences expressed in linguistic quantifiers.

Second, once the likelihood of pollution at sample points is determined, an interpolation method must be applied to compute the contamination level in the entire brownfield. Different interpolation techniques are discussed. The method called inverse distance weighting (IDW) is suggested due to its flexibility in dealing with discrete layer boundaries. Then the equations for calibrating parameters in fuzzy real options are derived.

Third, the likelihood of contamination will be visualized and stored as fuzzy objects in a GIS system, enabling iterative modification and negotiation. Ways to store such information in databases are introduced and implemented. All the three parts are further explained in the following parts.

OWA and Interpolation

Since brownfield redevelopment involves geographic information, spatial analysis and GIS are helpful in decision-making and negotiation. Techniques in GIS software to combine multiple geographic factors include spatial logic operators (i.e. union and intersection) and simple additive weighting [78]. OWA can be regarded as a generalization of both methods, so it is entirely appropriate for a spatial decision-making environment.

Because the condition of brownfields varies greatly, there are both generic and site-specific approaches to brownfield redevelopment [90]. Criteria in judging pollution level differ case-by-case. Furthermore, the number of criteria may be too great for DMs to keep them in perspective [87]. Thus, linguistic quantifiers are helpful in making criteria cognitively manageable [78]. Linguistic quantifiers can also be extended to processing descriptive assessments [137].

The function $Q = x^q$ is employed here to determine VOID and the associated OWA weights. DMs can express their risk preferences descriptively, or by changing the parameter

q. Using this function, VOID, q, and ORness level are linked, where $VOID = \frac{1}{q+1}$ [132]. This relationship will be shown graphically in the planned NSS, making DMs aware of the impact of changing their risk preference on the OWA weights generated. The steps involved in determining the likelihood of contamination are as follows:

1. Identify criteria used in judging the contamination level and select appropriate linguistic quantifiers expressing DMs risk preferences.

2. Let DMs express their judgements ranging from 0 to 1 at sample points.

3. Calculate the likelihood of contamination of the entire brownfield site by interpolation, which will be explained below.

4. Map the result and encourage iterative modification if appropriate.

The main interpolation methods are IDW and Kriging. In the brownfield redevelopment application, Kriging is inappropriate for the following reasons: Firstly, Kriging is based on stationary spatial stochastic processes with spatial correlations. Since fuzzy real options assume independent and identical distributed stochastic processes with exponential growth, Kriging fails to satisfy independence and no correlation [42]. Secondly, Kriging tends to generate continuous results. However, as geological layers are often discrete, Kriging is inappropriate when some crisp boundaries must be accounted for [2]. Furthermore, Kriging demands considerable computational power and is difficult to implement. Hence, IDW, with none of these drawbacks, is selected as the interpolation technique.

The idea of IDW is to utilize the distances between each sample point and the estimated positions as the main factor in determining weights (Formula 4.2), where u is the estimate value; v is the real value at the sample point; and d the the distance between two points.

$$u(x) = \sum_{k=0}^{N} \frac{v_k(x)}{\sum_{i=0}^{N} v_i(x)} v_k, \text{ where } v_k(x) = \frac{1}{d(x, x_k)^p} \tag{4.2}$$

The parameter p affecting weights is often greater than 1. Since gravity decreases as the square of distance, is a frequently-used value. Other possible values can be also tested in the future.

The combined utilization of OWA, VOID, and IDW enables DMs to make subjective judgements on the brownfield site. The fuzzy membership degree for each point of the site can be fed into equations to compute the parameters in fuzzy real options and displayed as a map for iterative modification and negotiation, as will be further discussed below.

Parameter Estimation

The OWA and interpolation stages produce three maps of the likelihood of contamination. Each of them shows a contamination area with fuzzy membership degrees, corresponding to minimum, most likely, and maximum scenarios. This information can be used to estimate the parameters of fuzzy real options.

Because remediation cost is assumed to satisfy $\frac{dS}{S} = \mu dt + \sigma dz$, there are three parameters to be estimated: the initial (or current) remediation cost S_0 , the drift rate (for cost) μ , and the volatility σ [119]. Initially, the contamination disseminates rapidly in the vodose zone but the rate of spread quickly decreases to nearly zero. Therefore, the contamination volume can be assumed to remain constant after a short initial period [134] [32]. Hence, given that remediation cost directly depends on the contamination volume, remediation cost and volatility must be fuzzy numbers, while the growth rate can be assumed to be crisp and independent of remediation volume. The rate of growth of the clean-up cost can be estimated based on market data.

In hydrogeological models, five main factors determine pollutant dissemination: advection, diffusion, dispersion, sorption, and biodegradation [32]. Parameters for these processes are difficult to calibrate. Uncertainties are represented by fuzzy boundaries around the brownfield site, which reflect the volatility.

The initial remediation cost must be proportional to the contaminated volume, which can be regarded as constant as long as pollutants have not entered the saturated zone [134]. This relationship can be expressed as Formula 4.3, where k is a coefficient and A^{high} denotes the area value with a high membership degree that is decided by the expert as an empirical parameter, which is similar to A^{low} later. Although the initial remediation might be functionally related to the area in a more complex manner than expressed in Formula 4.3, this relationship does not have a great impact on the final result and is easy to improve in future work. With three different risk preferences, a three-point estimate of initial cost can be determined.

$$S_0 = kA^{high} \tag{4.3}$$

The volatility will be computed based on the difference between A^{high} and A^{low}, the areas that are certainly polluted and clean, which is shown in Formula 4.4. This formula is derived from the variance equation for a triangular fuzzy variable, $x = (a, b, c)$, which is $\sigma_x = \frac{33\alpha^3+21\alpha^2+11\alpha\beta^2-\beta^3}{384\alpha}$, where $\alpha = (b-a) \vee (c-b)$ and $\beta = (b-a) \wedge (c-b)$ [73]. Because subjective estimates of pollution level are usually linear and fit the triangular form, the transition from contaminated to clean area is assumed to follow a right-angled triangle form. In this case, we can assert that $a = b$, leading to $\alpha = c - b = A^{low} - A^{high}$ and $\beta = 0$.

With Formula 4.3 and 4.4, the parameters needed to evaluate fuzzy real options can be calculated. Although the area value used might be replaced by a function of the area, it is believed that both equations are approximately correct due to the layered structure of the hydrogeology [134]. For now, the equations are simple to use and easy to modify if necessary.

$$\sigma = \sqrt{\frac{11}{128}(A^{low} - A^{high})} \tag{4.4}$$

Fuzzy Boundaries

The mapping capacity of GIS can be utilized to facilitate iterative multi-criteria analysis, which is helpful in negotiation. As Jankowski et al. [56] emphasized, exploratory decision analysis is critical in multi-criteria decision-making. Given that preference and subjective judgement are often expressed with intrinsic vagueness, the mechanism of allowing a DM to check the output and modify unsatisfactory inputs will likely be useful. Hence, fuzzy boundaries of a brownfield are proposed in order to employ mapping tools for aid.

As mentioned in the previous subsection, every location in a brownfield site has a fuzzy membership degree measuring likelihood of contamination. Unlike crisp logic used in GIS either within or outside a parcel, fuzzy boundaries of a brownfield are a challenging representation problem for a GIS system. The representation of fuzzy boundaries has been studied in the literature, such as [115] and [99].

Since representations of geographic features can be classified in vector-based or grid-based storage [138], efforts may be made in both directions. The representation of fuzzy boundaries in vector form is preferred in normal GIS applications because of its compact format and compatibility to structured query languages like structured query language (SQL) and relational database.

However, the vector form representation of fuzzy boundaries normally requires a known fuzzy membership function, which may be difficult to obtain. In addition, the performance of spatial operations, such as union, intersection, and buffering, is weaker than that of the grid form. In the brownfield redevelopment application, little information other than the fuzzy membership degree is required, so the advantages of vector form do not compensate for its performance burden. Hence, the grid-form of representation of fuzzy boundaries should be considered.

When more information is to be associated with locations, the vector form representation of fuzzy boundaries can be implemented. The work on fuzzy representations, fuzzy query language, and even a fuzzy relational data model for geographic information can be added to existing work toward a more integrated system that is capable of processing natural language [115] [75]. A membership degree will be associated with each tuple (record) in the database, just as for other mandatory geographic attributes [138].

4.3 A Case Study applying Extended LSM and OWA

In this thesis, the case of the Ralgreen Community redevelopment in Kitchener, Ontario, Canada is employed to illustrate how to apply proposed numerical methods. This case was selected due to the relatively rich set of available documents and to the long history of controversy concerning the contamination of the site, which is typical in brownfield redevelopment.

Background information on the Ralgreen Community redevelopment is introduced first. Subsequently, steps for determining the likelihood of contamination are shown. Results are discussed in comparison to the case documents.

4.3.1 Ralgreen Community Redevelopment

Until 1948, the Ralgreen property was farmland with a small pond. Then, with the owner's agreement, the City of Kitchener dumped garbage into the pond and surrounding area as land-fill. Some twenty years later, the property was developed into a residential community in 1968-69 [20]. On August 22, 1969, 65 and 67 Ralgreen Crescent were devastated by fire, caused by methane gas. During the subsequent investigation, three other semi-detached buildings, 64-66, 68-70, and 94-96 Ralgreen Crescent, and three houses, 1257, 1259, and 1261 Queens Boulevard were found to be in a potentially hazardous situation [20].

In response to this danger and a by-law, the Building By-law (Special Requirements on Filled Lands), was passed by the City of Kitchener in October 1969, requiring venting systems to be installed in all buildings in potential danger. Furthermore, garbage and organic materials were removed and replaced with compacted granular fill (HEATH, 1997). All properties passed a methane gas test on June 20, 1978, and were not listed as closed disposal sites by the Ontario Ministry of the Environment.

After 67 Ralgreen Crescent was sold around 1993, the possibility of contamination arose again. At the end of 1995, the homeowner reported a leaked sewage-like water in the basement. A high level of combustible gases was detected in April 1996. In the following year, underground monitoring was conducted and contamination was confirmed in the surrounding area, which roughly coincided with the original pond [54].

In 1999, a group of residents undertook legal action against the City of Kitchener. In the following year, an agreement was reached by the parties, under which the City of Kitchener purchased 15 properties in the former pond area and cleaned the land according to the MOE 1997 guidelines [54]. In the end, the Ralgreen Community was remediated and redeveloped based on the agreement.

The evidence of pollution found through this process can be classified into five categories: foundation settlement, interior methane gas levels, soil and groundwater quality, basement water leakage, and indoor air quality [20]. Each class contains several indicators, around 20 in total, ranging from garage tilting, leaking sewage-like water, and odour, to mould on the wall [54].

4.3.2 Main Steps in Pricing Brownfield using Proposed Method

The steps involved in determining the value of a brownfield based on subjective judgements are: identify the judgement criteria, assess the likelihood of contamination at sample points, derive the map of the pollution extent of the brownfield as fuzzy boundaries, estimate parameters in a fuzzy real options model of the brownfield, and calculate the value. The case of Ralgreen Community is used to demonstrate this process.

As mentioned in the previous subsection, five criteria were employed to measure the contamination level. The assessment is given in Table 4.6, summarized from the literature [20].

Table 4.6: Criteria and Assessment used in Contamination Judgement

Criteria	**Assessments**
Foundation settlement	Confirmed problem (100%); main structure settlement (80%); shear cracking (60%); attached garage settlement (40%); detached garage settlement (20%); and no problem (0%).
Interior methane gas levels	Methane gas level is at least 200 ppm (100%); 100-200 ppm (50%); greater than 0% (25%); and zero (0%).
Soil and groundwater quality	At least 5 contaminants (100%); 4 contaminants (80%); 3 contaminants (60%); 2 contaminants (40%); 1 contaminant (20%); none (0%).
Basement water leakage	Confirmed problem (100%); detected (measured) contaminants (70%); odour (40%); none (0%).
Indoor air quality	Confirmed problem (100%); detected (measured) contaminants (50%); none (0%).

The linguistic quantifiers used are "most", "average", and "few" for community residents, the non-partisan expert, and the City of Kitchener, respectively, where the parameter q is set as 10, 1, and 0.1. The weights of the OWA operator are listed in Table 4.7, ordered from largest (applies to maximum assessment) to smallest (applies to minimum assessment).

When a IDW system is applied to determine the likelihood of contamination in the Ralgreen community, five maps based on different linguistic quantifiers are generated as shown in Figure 4.9-13. In these figures, the rectangles denote individual properties, the lighter areas are more polluted and the darker shading indicates less pollution. Points spreading in the community are samples where judgements are made. In fact, the lightest

Table 4.7: OWA Weights of Different Linguistic Quantifiers

Linguistic quantifiers	**Parameter** q	**VOID**	**Weight 1**	**Weight 2**	**Weight 3**	**Weight 4**	**Weight 5**
Max	∞	0	1.0000	0.0000	0.0000	0.0000	0.0000
Most	10	0.0909	0.8926	0.1013	0.0059	0.0001	0.0000
Average	1	0.5	0.2000	0.2000	0.2000	0.2000	0.2000
Few	0.1	0.9091	0.0221	0.0277	0.0378	0.0611	0.8513
Min	0	1	0.0000	0.0000	0.0000	0.0000	1.0000

patch in this figure is the location of the former pond, which has the highest level of pollution as mentioned in various reports [20]. A 50% possibility of contamination is selected as the high threshold, and 10% as the low level.

For the areas exceeding the α^{50} and α^{10} thresholds, the parameters of the initial redevelopment cost and volatility are shown in Table 4.8. The excavation and refill cost is assumed to be $\$100/m^2$. The volatility coefficient is set to 10^6. Other parameters are listed in Table 4.9.

With the inputs shown in the above tables, the fuzzy real options model of brownfields generated the results in Table 4.10. We see that differing risk preferences among DMs generates different property values. But the differences are minor compared to the overall value. The less the likelihood of contamination, the less the lower redevelopment cost, and the higher the property value. Since current income/cost ratio exceeds the critical value in each scenario, all DMs tend to select the option to wait, with an expected time of 6 years.

4.4 Insights

As shown in the results, the proposed method can help determine the likelihood of contamination and the corresponding value of the brownfield. The results can be understood from the perspectives of fuzzy boundaries, property values, critical values, and expected time.

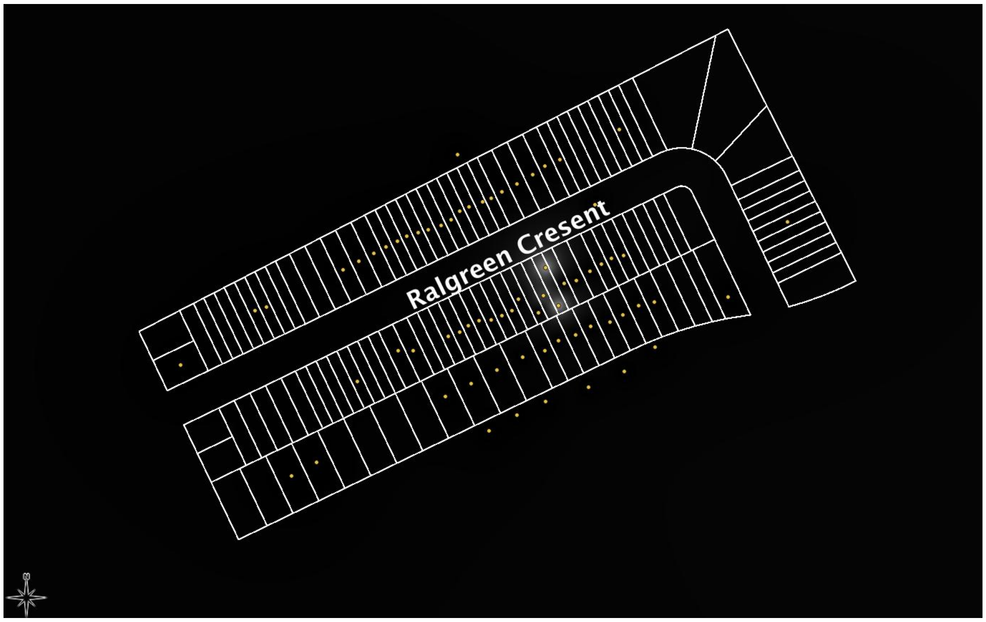


Figure 4.9: Fuzzy Boundaries of the Ralgreen Brownfield for the Min Likelihood

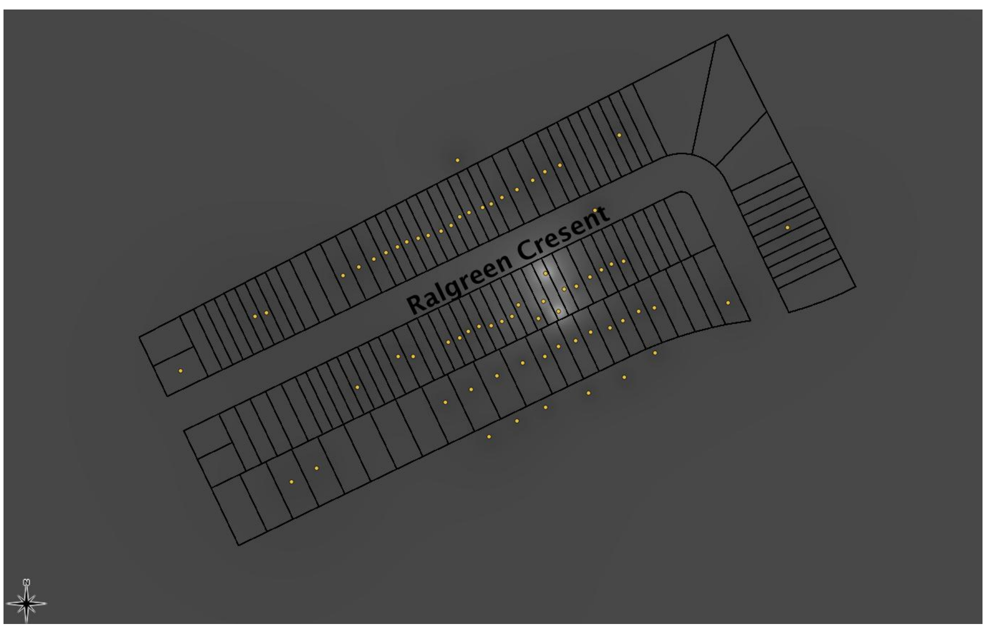


Figure 4.10: Fuzzy Boundaries of the Ralgreen Brownfield for the Likelihood based on “Few”

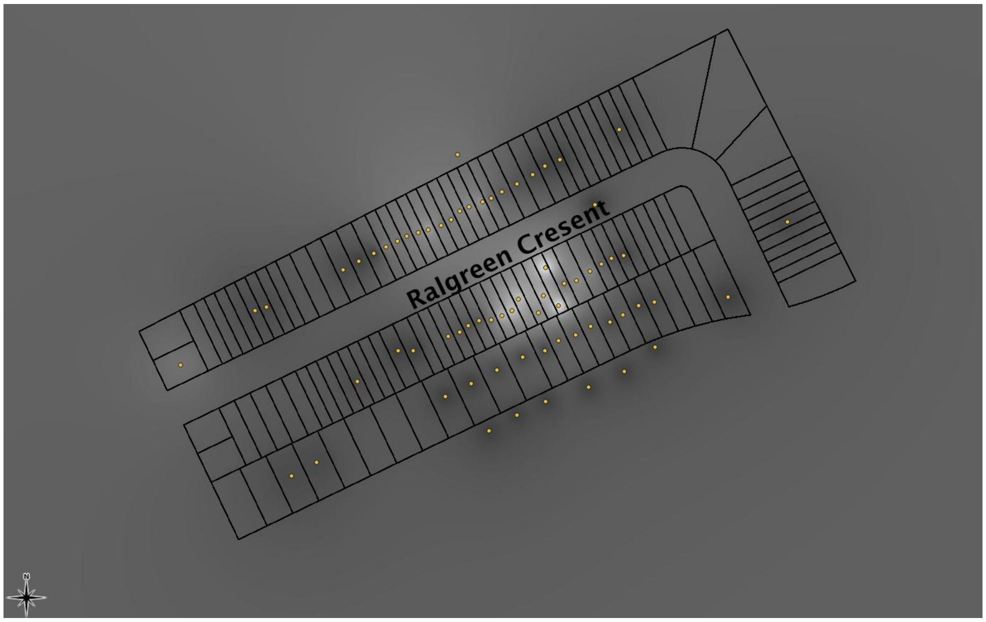


Figure 4.11: Fuzzy Boundaries of the Ralgreen Brownfield for the Average Likelihood

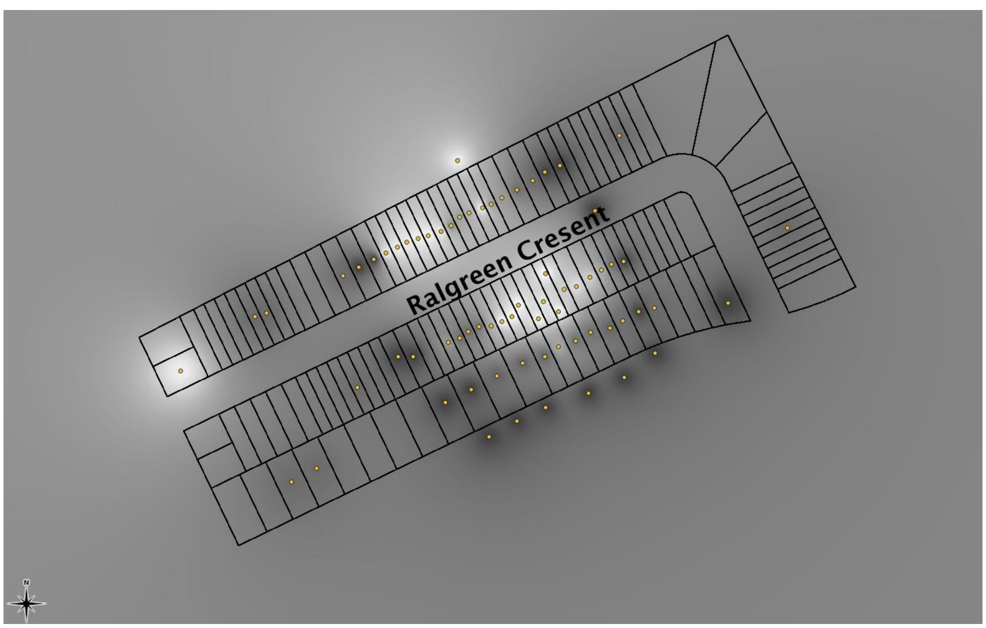


Figure 4.12: Fuzzy Boundaries of the Ralgreen Brownfield for the Likelihood based on "Most"

Table 4.8: Estimated Parameters derived from Pollution Judgements

Scenario	α^{50} **Area** (m^2)	α^{10} **Area** (m^2)	S_0 **($)**	**Volatility**
Max	39845.25	1137538.00	3984525	0.321790
Most	27575.00	1137311.00	2757500	0.325320
Average	1006.50	1115614.00	110650	0.326748
Few	37.75	3715.25	3775.00	0.001078
Min	0.00	1511.50	0.00	0.000443

Table 4.9: Parameters other than Fuzzy Inputs in Ralgreen Case Study

Parameter	**Value**
Riskless Rate (r)	5%
Payoff Drift Rate (μ_x)	2.5%
Payoff Volatility (σ_x)	0.2
Initial Annual Payoff (S_{0x})	216000

Table 4.10: Results of Fuzzy Real Options Model for Brownfields with OWA

	Property Value (Million $)	**Critical Value**	**Expected Waiting Time (Year)**
Few	6.813	0.1811	6.34
Average	6.607	0.1976	6.25
Most	6.440	0.1353	5.91

Note: The critical values have been “defuzzified” by calculating the fuzzy expected values.

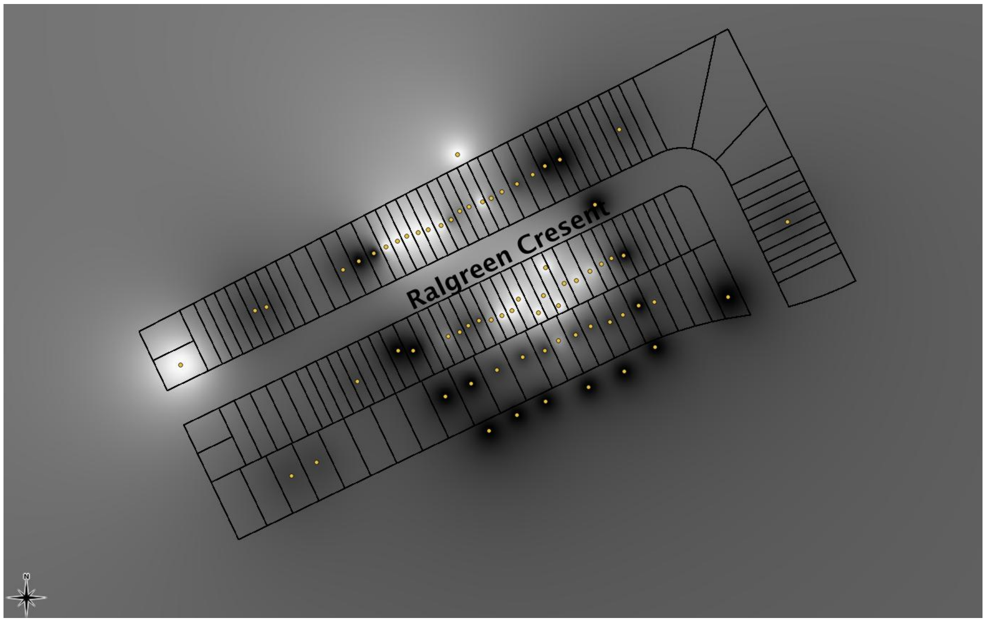


Figure 4.13: Fuzzy Boundaries of the Ralgreen Brownfield for the Max Likelihood

The fuzzy boundaries of the brownfield reflect the reality of contamination in literature [20] [54]. Depending on the linguistic quantifiers, the polluted areas of “max”, “most”, and “average” spread around the former pond, although the “average” quantifier is much less probable than the other two. With the linguistic quantifier “few”, the contaminated area is restricted to the two most problematic properties: Ralgreen 65 and 67. The “min” operator even suggests that there was no contamination in the Ralgreen Community. The conflict over pollution extent between residents and developers is clearly shown in these different scenarios.

Comparing the five OWA weight vectors and the scenarios they generated, it is observed that the “most” and “max” cases are similar, as are the “few” and “min” cases. Hence, the derived triangle-form fuzzy estimates are skewed, indicating that DMs have strong risk preference.

When fuzzy real options analysis is employed, we found minor differences (about 3%) in values for the brownfield. The major factor affecting values should be the estimates of the initial redevelopment cost. Volatility also has an impact on value, but it is not as influential.

Under all scenarios, critical values are slightly greater than the current x/R ratio. Hence, the optimal decision should be to wait. Otherwise the strategy should be to redevelop now, which generates identical value with NPV. In addition, given that the difference between the critical values and the current x/R ratio, the values generated using fuzzy real options analysis should be only a little higher than those of NPV. A similar result was found in a case study in US [34]. In particular, only a minor modification or compensation would change the developer's decision from wait and see to participate immediately.

All expected waiting times are around 6 years, which roughly equals the negotiation process from 1995 (contamination found) to 2001 (redevelopment complete). In contrast to the brownfield value, in which the DM became more optimistic about the pollution level, the expected waiting time increases when preference becomes less risk adverse. The reason is the volatility of redevelopment cost, which increases as pollution extent shrinks. Therefore, DMs tend to wait longer in anticipation of more business opportunities due to higher uncertainty. This result explains why developers are reluctant to redevelop, even though they understand the value of brownfields. On the other hand, it is unclear whether fuzzy real options can model the behaviour of community residents. Since community residents live in the contaminated properties, concerns for public health may be a more important factor than property value.

When the proposed method is applied to the Ralgreen Community case study, outputs reflect the reality of the negotiation process. Conflict among DMs is shown as different values for the same brownfield, using fuzzy real options and OWA. Even when brownfields have a high value, developers usually select the option to wait, seeking business opportunities that maximize the price. The fuzzy real options model of brownfield pricing thus explains DMs behaviour under different policy scenarios.

Chapter 5

Brownfield Redevelopment Negotiations and Policy

5.1 Conflict Analysis

Brownfield redevelopment is a situation of strategic conflict, where multiple independent DMs choose options, leading to one of many possible outcomes over which every DM has preferences [59]. DMs usually have conflicting value systems, and therefore different preferences. In brownfield redevelopment, developers prefer states when the government shares remediation costs, while the government usually has the opposite preference. Since all DMs have some power to change outcomes, each DM must consider not only his or her own preference, but also others' choices. Hence, the final output will be a compromise resolution that all parties can agree on.

However, methodologies to understand conflict decision-making and identify potential resolutions were not systemically developed until recent years. Two major approaches are non-cooperative game theory (quantitative) and conflict analysis (non-quantitative) (Figure 5.1). The game-theoretic approach has a large and well-developed body of theory [113]. But game-theoretic approaches impose some restrictions on strategic conflict, such as predefined order of moves and quantitative utility, and may suggest mixed strategies, which are difficult to convince decision-makers are optimal choices [59]. Thus, conflict analysis is selected to analyze brownfield redevelopment, due to its simplicity and flexibility.

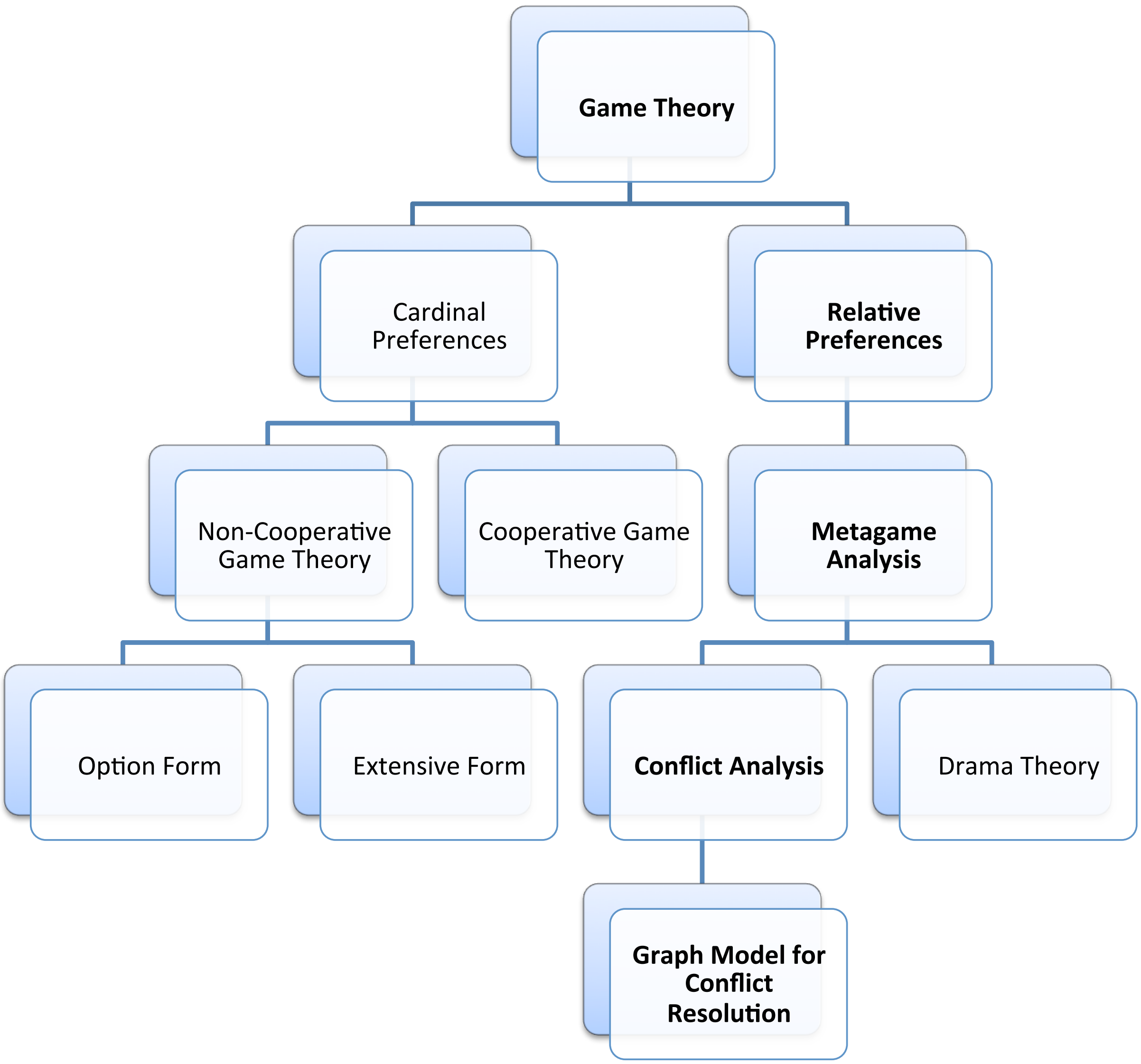


Figure 5.1: Genealogy of Conflict Analysis Models [35]

Conflict Analysis studies a strategic conflict or policy problem qualitatively from the perspective of actors' behaviour. It started from Metagame Analysis by Howard [49]. Fraser and Hipel [39] expanded metagame analysis into conflict analysis by introducing more equilibrium definitions to model human behaviour under conflict and computerized implementation of modelling and analytical techniques. Later, Fang, Hipel and Kilgour proposed the GMCR to enhance model flexibility and breadth [35]. The GMCR model is applied in this thesis to analyze brownfield conflict, because a graph model provides an intuitive representation of the potential moves and counter-moves of each DM and, establishes a solid foundation that makes many novel expansions possible.

A graph model for conflict resolution is normally implemented in two stages (Figure 5.2): a modelling stage through which all required information is input, and an analysis process, that conducts stability analysis, equilibrium identification, and so on. Following this structure, modelling components are introduced in the next subsection. Then different types of stabilities are defined. New extensions of GMCR are discussed, preparing for the combination of fuzzy real options analysis and the GMCR system.

5.1.1 Components in GMCR

At the modelling stage, the DMs are identified and the available options for each DM are determined. State are generated as combinations of options for all DMs. Then, infeasible states are removed from the model. Then, all that is required for a graph model analysis is knowledge of each DM's preference ranking over the feasible states, where ties are allowed [120]. The graph model components can be summarized as follows:

- *DMs:* A DM may be an individual or a group who makes a single decision. In brownfield redevelopment conflict, typical DMs may be stakeholders such as local government, federal governmental agencies, community, developer, land owner, and NGO. In previous research, a DM is also referred to as a player, actor, stakeholder, or participant. Mathematically, DMs can be denoted as a finite set $N = \{1, 2, ..., n\}$;
- *States:* States, or called outcomes, are generated from the combination of the options DMs choose with logical restrictions. For a conflict with q options, where q is the sum of DMs' options, a state can also be represented as a q-dimensional vector, which

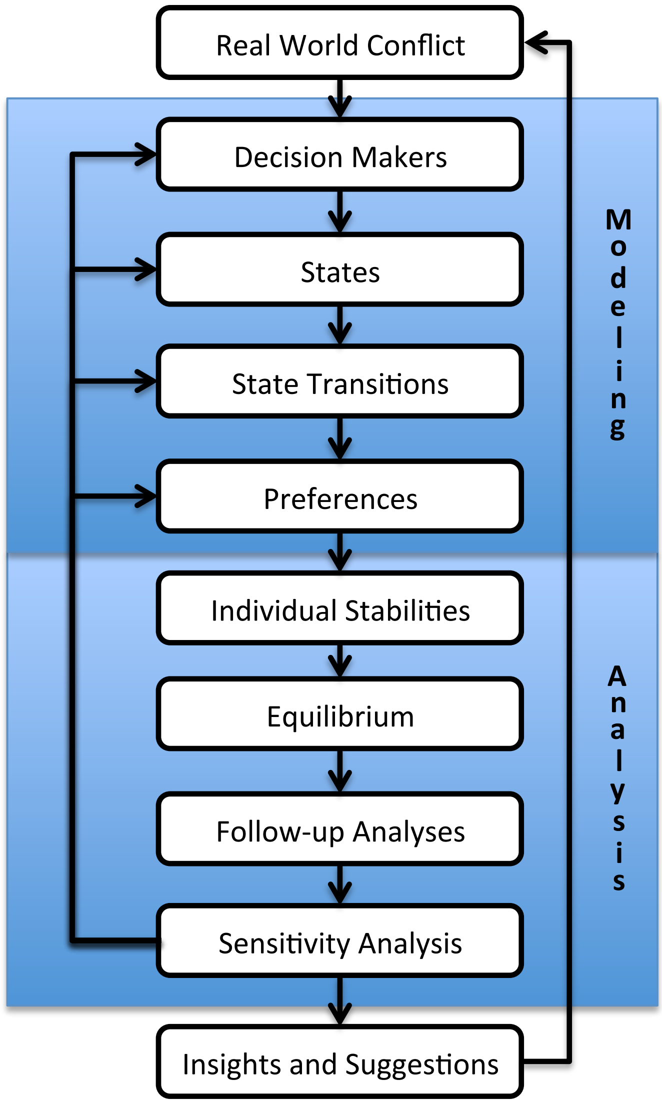


Figure 5.2: Procedure of applying GMCR [35]

"Y" at the i-th position means the i-th option is selected or "N" for not chosen. Therefore, a conflict with q options can have 2^q states at the most, which can be denoted as $S = \{s_1, s_2, \ldots, s_t\}$;

- *State Transitions:* State transition employs graph model to keep track of the possible evolution of a conflict. If a DM can cause a state transition on his or her own, then this transition is called a unilateral movement (UM) for that DM. Formally, for each DM $i \in N$, a directed graph $G_i = (S, A_i)$, where $A_i \subset S \times S$ is a set of oriented arcs representing all possible UMs by DM i;

- *Preferences:* For each DM_i $(i \in N)$, relative preferences are required, which can be obtained by pairly compared all possible states. There are various approaches in representing preferences. More specifically, the fuzzy preference is employed in this paper, which will be introduced in later section. But the most basic way to express preference is a pair of binary relations, $\succ_i$ and $\sim_i$ on S, where $s \succ_i q$ indicates DM_i strictly prefers state $s \in S$ to state $q \in S$, and $s \sim_i q$ means DM_i is indifferent between s and q. This fundamental representation is compatible with other extended approaches.

In addition to the components that requires inputs from modellers, various definitions of movements are important in analyzing a graph model. In particular, let $i \in N$ and $s \in S$. Then $R_i(s) = \{q \in S : (s, q) \in A_i\}$ is called DM_i's reachable list from state s, and represents all the states to which DM_i can move the conflict, in one step, starting at state s. Unilateral improvements (UIs) are UMs by a DM that result in a preferred target state. They are defined by $R_i^+(s) = \{q \in R_i(s) : q \succ_i s\}$, which is a subset of $R_i(s)$. If there are more than two DMs in a graph model, there may be joint movements and joint improvements. These concepts are similar to UMs and UIs, and may involve sequences of moves by different DMs in which a particular DM may move more than once, but not twice consecutively. These movement definitions are widely used in determining a state's stability for one DM.

5.1.2 Stability Analysis in GMCR

At the analysis stage, the stability of each state for each DM is calculated. A state is deemed to be stable for a given DM if it is not advantageous for the DM to unilaterally move away from it according to a specific stability definition of how DMs behave in a conflict situation. A state that is stable for all DMs, is a possible equilibrium or resolution to the dispute. Sensitivity analysis can then strengthen the interpretation of the modelling and analytical results.

In GMCR, a stability definition (or solution concept) is a set of rules for DMs to determine whether he or she should stay at a state or unilaterally move toward a more preferred one. A stability definition is therefore a model of a DM's strategic approach, or more generally of human behavior in strategic conflict. An equilibrium is a state that is stable for all DMs. Since all DMs have no incentive to move to another state, equilibria can be regarded as potential conflict resolutions [35].

The main stability definitions frequently used in GMCR contain Nash stability (Nash), general metarationality (GMR) stability, symmetric metarationality (SMR) stability, and Sequential rationality (SEQ) stability. Table 5.1 describes some features of these definitions that relate them to behaviour in conflicts. The main classification scheme is foresight, which refers to the number of moves a DM considers to determine stability, and disimprovement, which is defined as when any opponent is likely to move to a state less favoured by the focal DM [59]. More details about formal definitions, explanations, examples, and original references can be found in Fang et al. [35].

Table 5.1: Main Stability Definitions used in the Graph Model [59]

Stability	**Foresight**	**Disimprovement**	**Definition**
Nash	1	Never	The focal DM has no UIs.
GMR	2	Sanctions only	All of the focal DM's UIs are sanctioned by subsequent UMs by other DMs.
SMR	3	Sanctions only	All of the focal DM's UIs are sanctioned by subsequent UMs by other DMs, which cannot be sanctioned by any focal DM's UM.
SEQ	2	Never	All of the focal DM's UIs are sanctioned by subsequent UIs by other DMs

5.1.3 Some Extensions of GMCR

Main extensions that are useful in analyzing brownfield redevelopment are matrix representation of GMCR, coalition analysis, and status-quo analysis. Hence, they are introduced in in the following subsections.

Matrix Representation of GMCR

We now turn to the matrix representation of a graph model, a newly developed procedure that simplifies the analysis by allowing for convenient computing using software such as MatLab [126]. With the help of matrix representation of GMCR, all stability calculations can be expressed in program modules that are easy to extend, embed, and reuse. This paper employs graph models in matrix form to calculate equilibria [125].

It is a remarkable fact that all operations and solutions of the graph model for conflict resolution can be expressed using matrices and matrix operations in place of logical statements. All definitions in GMCR have corresponding concepts in matrix form [125], which are introduced according to the process of applying matrix representation of GMCR.

At first, two kinds of matrices are required as inputs: the UM matrices and preference matrices. They are defined as follows:

- *UM:* UMs are recorded in $m \times m$ 0-1 matrices, denoted as J_is, where $i \in N$ and $m = \mid S \mid$. If $R_i(s)$ represents DM$_i$'s reachable list of the state s, then J_i is defined as

$$J_i(s,q) = \begin{cases} 1 & \text{if } q \in R_i(s) \\ 0 & \text{otherwise.} \end{cases} \tag{5.1}$$

- *Preference Relation Matrices:* Several preference relation matrices (P_i^+, $P_i^=$, and P_i^-) can be defined as,among whom the most important is P_i^+ and can be extended by fuzzy preference later

$$P_i^+(s,q) = \begin{cases} 1 & \text{if } q \succ_i s, \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

$$P_i^=(s,q) = \begin{cases} 1 & \text{if } q \sim_i s, \\ 0 & \text{otherwise.} \end{cases} \tag{5.3}$$

$$P_i^-(s,q) = \begin{cases} 1 & \text{if } q \prec_i s, \\ 0 & \text{otherwise.} \end{cases} \tag{5.4}$$

When information is inputted, there is an intermediate step that focuses on calculating UI matrices. Related concepts and equations are listed as follows:

- *UI:* Similar to UM, the UI matrix, denoted as J_i^+, is defined as

$$J_i^+(s,q) = \begin{cases} 1 & \text{if } q \in R_i(s) \text{ and } q \succ_i s, \\ 0 & \text{otherwise.} \end{cases} \tag{5.5}$$

- *$n-$Step Movement Matrix:* For a $n-$DMs conflict, a DM should not only consider movements from all other individual DMs, but also their possible joint movements, which are reflected in some stability definitions. To do so, UM matrix is extended to $n-$Step movement matrix, which is defined as: For a DM_i ($i \in N$) and a series of steps $t = 1, 2, 3, \ldots$

$$M_i^{(t)}(s,q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \text{ in exactly } t \text{ legal UMs} \\ & \text{with first mover DM}i, \\ 0 & \text{otherwise.} \end{cases} \tag{5.6}$$

- *$n-$Step Improvement Matrix:* Similarly, UI matrix is extended to $n-$Step improvement matrix, which is defined as: For a DM_i ($i \in N$) and a series of steps $t = 1, 2, 3, \ldots$

$$M_i^{(t,+)}(s,q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \text{ in exactly } t \text{ legal UIs} \\ & \text{with first mover DM}i, \\ 0 & \text{otherwise.} \end{cases} \tag{5.7}$$

- *Two Important Operators:* There are two operators that should be introduced to simply formula. The first is Hadamard product $\circ$. For two $m \times m$ matrices M and G, $W = M \circ G$ means that each entry of W satisfies $W(s,q) = M(s,q) \cdot G(s,q)$. The other is $\bigvee$. $H = M \bigvee G$ is defined as

$$H(s,q) = \begin{cases} 1 & \text{if } M(s,q) + G(s,q) \neq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{5.8}$$

- *An important formula*: The relationship between UI, preference, and reachable list can be expressed as $J_i^+ = J_i \circ P_i^+$.

- *Joint Movement Matrix:* Joint movement matrix M_H is a 0-1 matrix as well, whose entry $M_H(s,q)$ will be 1 if and only if q is in the joint reachable list R_H, where $H \subseteq N$ and $H \neq \emptyset$. In a sense, it is the union of all reachable lists for every DM$_i$ and their joint reachable lists. M_H is easy to calculate under matrix representation based on the equation of $M_H = \bigvee_{t=1}^{\delta} \bigvee_{i \in H} M_i^t$, where δ determines the time of iteration, which can be set to a large number, normally, $n \times m \times m$.

- *Joint Improvement Matrix:* Similar to the joint movement matrix, joint improvement matrix M_H^+ is a 0-1 matrix, whose entry $M_H(s,q)$ will be 1 if and only if q is in the joint improvement list R_H^+, where $H \subseteq N$ and $H \neq \emptyset$. M_H^+ can be computed as $M_H = \bigvee_{t=1}^{\delta} \bigvee_{i \in H} M_i^{t,+}$.

When UIs and joint improvement matrices are calculated, the $m \times m$ stability matrices can be derived using some formula. If a diagonal element for a specific stability concept is zero, the corresponding state is a stability of that type for the DM. Therefore, if all DMs have stability on this state, it is an equilibrium [125]. These formula are as follows:

- *Nash Stability Matrix M_i^{Nash}:* Let $i \in N$. $M_i^{Nash} = J_i^+ \cdot E$.

- *SEQ Stability Matrix M_i^{SEQ}: $M_i^{SEQ} = J_i^+ \cdot [E - sign(M_{N-i}^+ \cdot (P_i^{-,=})^T)]$*, where the symbol $sign(M)$ is the sign function for the $m \times m$ matrix M, where each entry (s,q) equals 1 if $M(s,q)$ is positive, 0 if $M(s,q)$ is zero, and -1 if $M(s,q)$ is negative and $P_i^{-,=} = P_i^- \bigvee P_i^=$.

- *GMR Stability Matrix M_i^{GMR}: $M_i^{GMR} = J_i^+ \cdot [E - sign(M_{N-i} \cdot (P_i^{-,=})^T)]$.*

- *SMR Stability Matrix M_i^{SMR}: $M_i^{SMR} = J_i^+ \cdot [E - sign(M_{N-i}) \cdot W]$*, where $W = (P_i^{-,=})^T \circ [E - J_i \cdot (P_i^+)^T)]$

A program implementing above definitions and equations is developed in MatLab by Xu [125]. In this thesis, this program is modified to fit the case of fuzzy preference, which is listed in the Appendix C. The four kinds of stabilities mentioned above can be easily identified with the aid of this program.

Status Quo Analysis

Status quo analysis focus on the evolution of the conflict from the current state (situation) to possible stabilities, which extends GMCR to dynamic analysis [69]. Essentially, status quo analysis employs the directed graph in GMCR to keep track of moves and countermoves starting from the status quo and a table to store the reachability status of states from it. Applying status quo not only provides the track of how a conflict might evolute, but also is able to eliminate unreachable equilibria.

Depending on cases, four algorithms with different computation complexity and generality are developed, depending on properties of moves and preferences hold in a conflict. The lists of moves from status quo at most h moves, at exact h-th move, the states for DM_i to move from the current state will be initially constructed as empty. Then these lists will be iteratively updated as the number of moves increases until the termination condition is satisfied (basically, no move toward other states for all DMs) [69]. In case of the brownfield redevelopment conflict, only the general case can be applied as preferences are intransitive.

Because of the complex process to run through, the matrix representation is employed in this thesis due to its simplicity [128]. Xu et al. observed the similarity between the reachable list employed in status quo analysis and the joint movement (and improvement) matrix. They defined status-quo joint movement $M_i^{SQ^t}$ (and improvement $M_i^{SQ^{t,+}}$) matrix based on M_i^t (and $M_I^{t,+}$) as follows, where $V_i^t(s)$ denotes all states reachable from s in at most k legal UMs and $V_i^{t,+}(s)$ for UIs:

$$M_i^{SQ^t}(s,q) = \begin{cases} 1 & \text{if } q \in V_i^t(s) \\ 0 & \text{otherwise.} \end{cases} \tag{5.9}$$

$$M_i^{SQ^{t,+}}(s,q) = \begin{cases} 1 & \text{if } q \in V_i^{t,+}(s) \\ 0 & \text{otherwise.} \end{cases} \tag{5.10}$$

Then they proved that the relationship between $M_i^{SQ^k}$ and M_i^k for any integer $k > 1$ is $M_i^{SQ^t} = \bigvee_{t=1}^{k} M_i^t$, while the improvement linkage is $M_i^{SQ^{k,+}} = \bigvee_{t=1}^{k} M_i^{t,+}$ [128].

As a follow-up analysis, status quo analysis provides information on predicting the conflict evolution toward equilibrium. Possible paths will be drawn and discussed based on the case study.

5.2 Preference Ranking based on Values Generated from Fuzzy Real Options

Since fuzzy real options can be combined with multicriteria analysis to identify DM's values on a brownfield, it is natural to think how to utilize this result in a multiple DM setting. Intuitively, conflicting values lead to a compromise outcome. Hence, a kind of game-theoretic approach should be incorporated into the fuzzy real options model, so that suggestions on resolving the brownfield conflict can be provided.

In this section, literature on the combination of real options analysis and games is reviewed. The advantages of combining GMCR and fuzzy real options are explained. Then, the difficulty of integrating these two models, namely, representing preferences under uncertainty in GMCR, is discussed with several approaches. At the end of this section, the process of converting values from fuzzy real option into fuzzy preferences are explained.

5.2.1 Fuzzy Real Options and Game-Theoretic Approaches

Researches on real options and games have been conducted in area of infrastructure [103] [104], supply chain [25], and invesetment assessment [85]. Other than the attempt to link equilibria and Black-Sholes equation together [25], the integration of real options and games mainly starts from two approaches.

One approach is to embed real options into extensive form of games [103]. In the tree showing the game evolution, real options analysis is applied to obtain the value on each node under certain state. Essentially, the algorithm runs iteratively from the bottom to the top. In every iteration, the analytic formula of real options are applied to determine the value (or utility) under uncertainty. This method requires to strictly follow the order of movements due to the use of extensive form of games, .

The other is to apply game-theoretic approach under the real options framework [25]. Monte-Carlo simulation is usually employed to generate sample paths in order to evaluate real options. At every step on the sample paths, some kind of equilibrium formula is applied to take the interaction among DMs (or players) into consideration. From the real options analysis perspective, equilibira act as some payoff maximization condition in the path-dependant options. Obviously, this approach normally assumes that interaction among

DMs (or the game) occurs all the time while can reach some equilibrium immediately.

However, these quantitative game-theoretic approach seem inappropriate to be combined with fuzzy real options. The order of moves in a brownfield redevelopment conflict is not clearly specified, making extensive form of games difficult to apply. In addition, all above methods require some analytic form solution in order to reduce computational complexity. Unfortunately, given that fuzzy real options usually have to be evaluated using numerical method, it is impractical to execute the numerical method of fuzzy real options at every step in a loop. Furthermore, assuming the conflict can reach an equilibrium at any time seems unrealistic. In this case, GMCR is utilized as a game-theoretic approach due to its flexibility.

As a qualitative game-theoretic approach, GMCR has the advantage of accepting various preference information in addition to ordinal values. Since the output value of a fuzzy real options model is essentially fuzzy, representations of preferences under uncertainty in GMCR will be introduced first. The newly developed fuzzy preference framework will be discussed in detail. The process of converting outputs from fuzzy real options to fuzzy preference rankings will be proposed and applied to the brownfield redevelopment conflict as a case study.

5.2.2 Preferences under Uncertainty in GMCR

Efforts on representing preferences under uncertainty includes information-gap theory [6] , fuzzy multicriteria analysis and fuzzy move [68] , uncertain preference [67], and fuzzy preference framework [5]. The information-gap models are designed to deal with situations when a DM has severe preference-uncertainty in a conflict. Information-gap theory can be regarded as a sensitive analysis technique based on the set theory. The unit modification $\pi(s, p)$ and distance between different preference rankings is defined first. Then the upper and lower boundary with associated parameter α and β is determined based on set theory, which can be calculated iteratively [6] . Then the robustness of a conflict is defined and used as an indicator of the uncertainty of preferences associated with its impact on the stability [6].

In Li et al. [68] , fuzzy mulitcriteria analysis and fuzzy moves are proposed to analyze uncertain preference. Besides fuzzy moves that does not deal with preference, fuzzy multicriteria analysis considers a DM's attitude toward other DMs. In other words, a DM's utility is determined by not only his or her own payoff but also others', expressed as fuzzy membership degrees. The multiple objectives are aggregated with weights. Results indicate that, based on attitudes among DMs, equilibria can be cooperative or noncooperative. This conforms to people's intuition that personalities affect game's outcomes. However, the fuzzy multi-criteria analysis approach is difficult to apply when more than two DMs participate [107].

Another major work is uncertain preference, which is defined as another kind of relation than $\succ$, $\sim$, and $\succeq$ [67]. Depending on the strength of incentive to depart and sanction, four methods of dealing with uncertain information are proposed, leading to four times of original stability definitions. Equilibria derived with uncertain preference can be updated as new information on preference is input. The uncertain preference can also be represented under the matrix representation framework [127]. The main extension is including the uncertain preference matrix into the UM and UI matrices, which are extended and defined as UIUUM. Since the union of original preference set and uncertain preference set can be expressed as the add operation of UI (or UM) and uncertain preference matrix, the rest operations in the matrix representation are parallel to the original one mentioned in Section 5.1.3 [127].

Apart from above approaches, fuzzy preference framework for GMCR is proposed by Barshar et al. [5] based on fuzzy preference orderings [108]. Similar to the ordinal preference relations, the fuzzy preference ranking focuses on binary comparison, which usually satisfies reciprocity and max-min transitivity [108]. Fuzzy preference framework has many advantages over other approaches. As a binary relation framework, its measurement has a solid basis. In addition, fuzzy preference is also more compatible with GMCR, which does not require mixed strategy or uncertain moves. More importantly, fuzzy preference can be regarded as a extension of crisp state comparison. GMCR becomes more flexible under the fuzzy preference framework using the α-cut parameter without great changes on stability definitions [5]. The two contradictory features, simplicity and flexibility, are unified under this theory.

Since the fuzzy preference framework is employed in this thesis to combine fuzzy real options and GMCR, its definitions and properties are listed below [5] [15]. The method of converting fuzzy numbers into fuzzy preferences will be explained in Section 5.2.3, based on possibility theory.

- *Fuzzy Preference Relation:* A fuzzy relation on S is a membership function $\mu_R : S \times S \rightarrow [0,1]$, mapping any two states $s, q \in S$ into a fuzzy membership degree between 0 and 1. The preference degree of s over q is denoted as $\mu_R(s,q) = r_{sq}$, and satisfies $r_{sq} + r_{qs} = 1$, therefore $r_{ss} = 0.5$ [15] [130]. The matrix representation can be employed to define a $m \times m$ ($m =\mid S \mid$) fuzzy preference relation matrix, R, whose entries are r_{sq}.

- *Fuzzy Relative Strength of Preference:* Fuzzy relative strength of preference is defined based on fuzzy preference relation. For a DM_i ($i \in N$), the fuzzy relative strength of preference of state $s \in S$ over $q \in S$ is defined as $\alpha^i(s,q) = r_{sq} - r_{qs}$. It is clear that fuzzy relative strength of preference is just a linear transformation of fuzzy preference relation. The key difference is that the indifference value, 0 rather than 0.5, is easier to identify.

- *Fuzzy UI:* For a given α-cut level $\gamma \in [0,1]$, a fuzzy UI for a DM_i ($i \in N$) from state $s \in S$ is defined as any state q that satisfies $\alpha^i(q,s) > \gamma$. The α-cut parameter for DM_i, γ^i, is usually called the fuzzy satisficing threshold. We can see that when the fuzzy satisificing threshold is applied, the fuzzy preference relation matrix, R, will be converted to J_i^+. This property is critical in extending matrix representation program to accommodate the fuzzy preference framework.

Once above definitions are employed to use the fuzzy preference relation in GMCR, stability and equilibria definitions are very similar to the original ones. The key difference is the involvement of fuzzy satisficing threshold in determining fuzzy UI. One feature should be noted is that multiple fuzzy satificing thresholds must be considered, because by definition, the thresholds in stability definitions are for different DMs.

Fuzzy preference framework extends GMCR applications when uncertain utility or fuzzy multicriteria analysis are applied. Since fuzzy real options is a generalized model of real options, which can produce fuzzy numbers, the feasibility of integrating fuzzy real options and GMCR is ensured. A detailed conversion procedure will be explained in the next section using the probabilistic approach.

5.2.3 Ranking of Feasible States

Fuzzy numbers generated from fuzzy real options models are trapezoids, and therefore convex Type-I fuzzy sets. Huynh et al. [53] proposed a useful probability-based comparison method to rank fuzzy variables. For two fuzzy numbers, A and B, fuzzy preference degree, defined as $P(A \succeq B) = \int_0^1 P(A^\alpha \succeq B^\alpha)d\alpha$ [53] can be extended to a likelihood-based approach with a satisfaction function [65]. These ideas are consistent with fuzzy preference relations [14].

For the simple comparison of two intervals $X = (a, b)$ and $Y = (c, d)$, which can be regarded as α-cuts, the result is illustrated in Figure 5.3. The fuzzy preference degree of $X \succeq Y$ is the ratio of area below and to the right of the line of $x = y$ to the area of the entire rectangle. In other words, the fuzzy preference degree of $X \succeq Y$ is the probability that a randomly chosen point in the rectangle falls in the right-hand part.

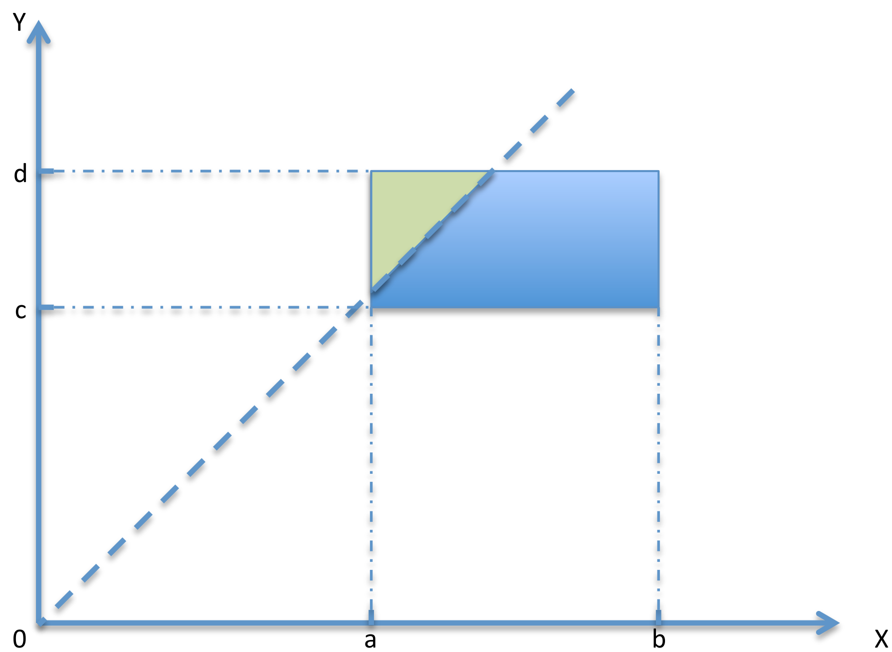


Figure 5.3: Geometric Interpretation of Two Intervals' Comparison [53]
Note: This plot diagram is illustrative. The axioms can be any measurement.

Following this geometric interpretation, we could regard an interval X as a fuzzy variable, whose fuzzy membership degree is 1 within a and b and 0 otherwise, etc, and then calculate the preference relation as the area under membership function. In other words, instead of integrating values horizontally based on α-cut levels, we can also conduct integration vertically as usual, formally as $P(X \succeq Y) = \int_{-\infty}^{\infty} P_X(x)[\int_{-\infty}^{x} P_Y(y)dy]dx$ [53].

When comparing two trapezoids $X = (a_1, a_2, a_3, a_4)$ and $Y = (b_1, b_2, b_3, b_4)$, the generalized procedure depends on the membership functions of a trapezoid and split into three cases. We denote the function in (a_1, a_2) interval as μ_A^X, function in (a_2, a_3) as μ_B^X, and μ_C^X for the case of (a_3, a_4). The notation is similar for Y, with a different super-script, $\mu_{A|B|C}^Y$. Then the following procedure can be applied based an illustrative example in Figure 5.4:

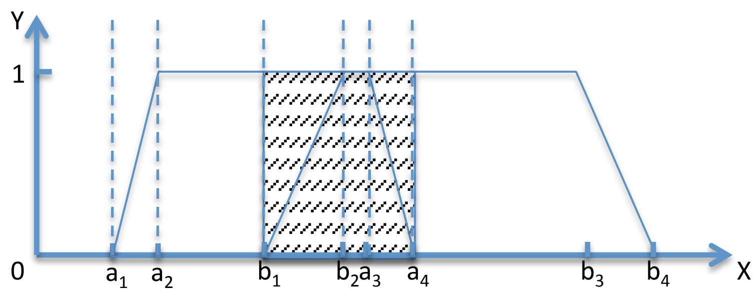


Figure 5.4: Trapezoid Comparison Diagram
Note: This plot diagram is illustrative. The axiom Y is the fuzzy membership degree, while the axiom X can be any measurement.

1. Identify the trapezoid to the left by comparing a_1 and b_1, say a_1 in Figure 5.4, which means that X is the "left" fuzzy variable in the illustrative example.

2. Determine the comparison region. Since the "left" fuzzy variable can not exceed the value of its right boundary, which is a_4 in the example, the likelihood for X of being greater than a_4 is 0. Therefore, we only need to calculate values in intervals between a_1 and a_4 in this case.

3. Divide the left trapezoid into two regions based on the left boundary of the other trapezoid, which is b_1 in the example. We are certain that $Y > X$ if the value of X is less than b_1, but not sure when these two intervals overlap (shaded area in Figure 5.4).

4. Rank the eight values in X and Y in order and form several intervals on which membership functions, such as μ_A^X, μ_B^X, μ_C^X, μ_A^Y, are to be integrated. If the interval falls in the certain area, only fuzzy membership function of one trapezoid is non-zero.

5. Integrate on these intervals (denoted $1, 2, \ldots$) until a_4 is met (there are five intervals (a_1, a_2), (a_2, b_1), (b_1, b_2), (b_2, a_3), and (a_3, a_4) in the example). Accumulate the fuzzy preference value based on the following equations:

   - For an interval i in the certain area, the contribution to $P(X \succeq Y)$ is $\frac{1}{S_X} \int_i \mu_i^X dx$, where S_X is the area of the trapezoid of X.
   - If the interval i falls in the uncertain area, then the contribution to $P(X \succeq Y)$ is $\frac{1}{S_X S_Y} \int_i \mu_i^X (S_Y - y\mu_i^Y) dxdy$, where S_Y is the area of the trapezoid for Y.

6. Add the contributions of all intervals identified in 5.

5.3 Case Study

5.3.1 Background and Models

The Ralgreen community case mentioned in Section 4.3 is employed for the further exploration from the policy making perspective. Different choices in policy design are tested in the multiple DMs environment. Equilibria derived under various settings are provided as possible conflict resolutions and discussed for policy design insights into brownfield redevelopment.

There are three DMs involved in the Ralgreen community conflict: community residents who suspected to be exposed to contamination, municipal government who is promoting brownfield redevelopment, and private developers who seek to business opportunities in redevelopment. Their options are listed in Table 5.2. Each option corresponds to the setting of a parameter, which is shown in Table 5.3. The values employed are generated in the fuzzy real options modelling of brownfield redevelopment with different parameter settings for all DMs in Section 4.3.

Since each state in the GMCR associates with a specific combination of parameter settings, DMs' values can be calculated out for each state. Sample values with fuzzy

Table 5.2: DMs and Their Options

Local government (DM_{GOV})		Main power to make policies
Wait	Wait Permission	Allow developers to delay action to wait for optimal situation
Redev	Landuse Change	Authorize the brownfield to be redeveloped for more profitable usage
Cost	Cost Sharing	Share a portion of the redevelopment cost
Community (DM_{COM})		Property owners who lilve in the community
Risk	Risk Sharing	Reduce risk sensitivity and permit SSRA
Developer (DM_{DEV})		Private developers who seek profit maximization
Dev	Development	Participate in brownfield redevelopment

Table 5.3: DMs' Options and Associated Parameter Settings

Option	**Choice**	**Parameter Setting**
Wait	Y	The option to wait is included.
	N	Only NPV is considered.
Redev	Y	Income after redevelopment increases twice.
	N	Income after redevelopment resumes to the clean state.
Cost	Y	Government shares 30% redevelopment cost.
	N	Developers pay the entire redevelopment cost.
Risk	Y	Community agrees to smaller contamination level with lower redevelopment cost.
	N	Developers must remediate contamination according to community's requirement.
Dev	Y	The brownfield value is higher than the status-quo value.
	N	The brownfield value is lower than the status-quo value.

membership degrees are generalized into the trapezoid-shape fuzzy numbers. Values of all states for every DM are plotted in Figure A.1, A.2, and A.3. A sample of trapezoid fuzzy numbers is illustrated in Figure 5.5. We can easily identify that these fuzzy outputs overlap a great deal, indicating that the preference ranking in GMCR is highly uncertain. This phenomena leads to the use of fuzzy preference relations for state ranking, so that GMCR can be applied.

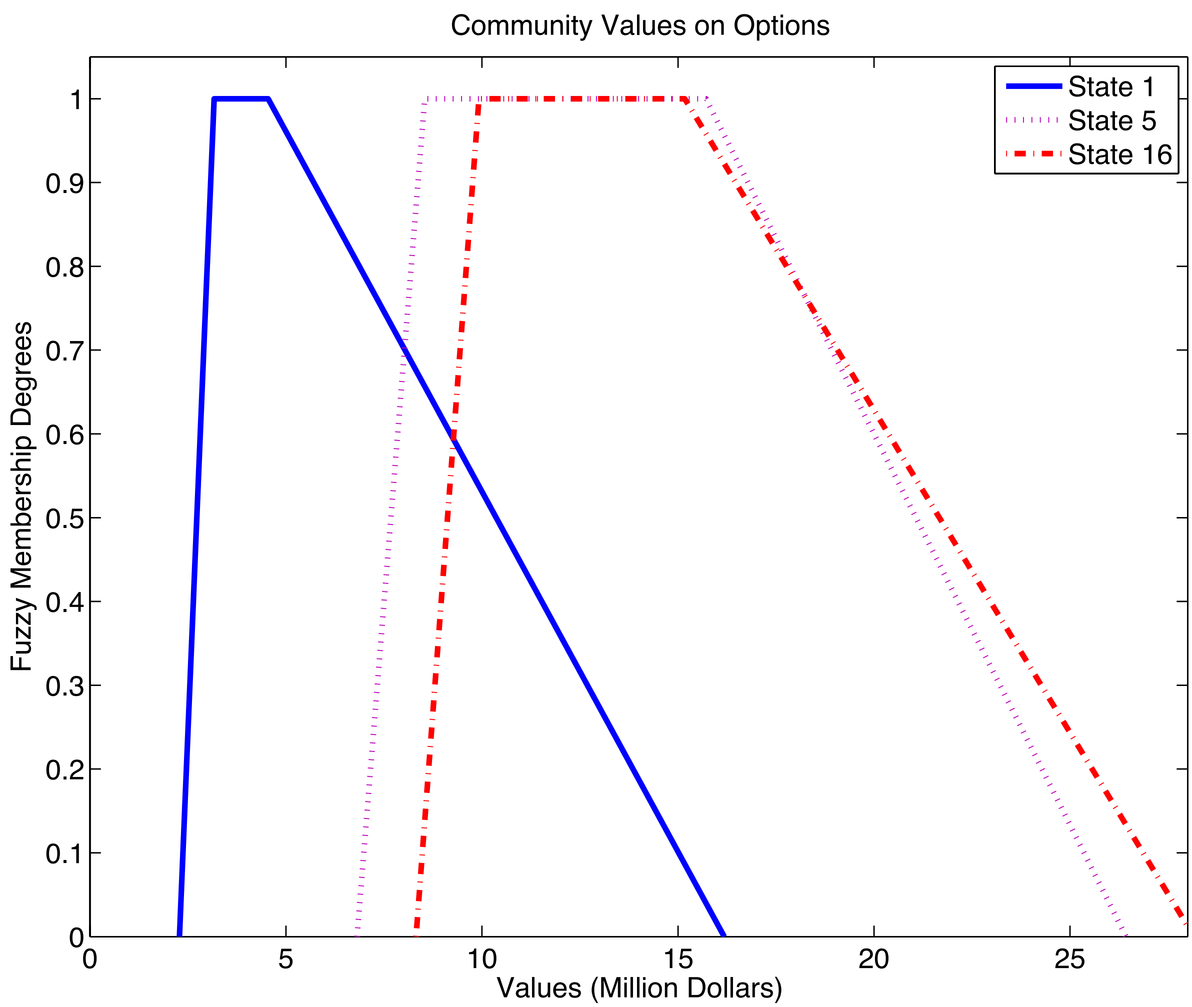


Figure 5.5: Sample Fuzzy Trapezoids Comparison

States in the GMCR for brownfield redevelopment is shown in Table 5.4. Other than the easy-to-understand states (1 - 16), state 17 is a combined state, which essentially means that the conflict will stay in stagnant if only no developer is willing to undertake this redevelopment project. This state can be regarded as status-quo with a certain threshold value that must be exceeded with some policies. In addition, state transition is simple in this GMCR (Figure 5.6). Since all UMs are bidirectional, the property of transitivity holds. All linked nodes in graphs are connected to each other, even though no direct link is drawn for the sake of simplicity.

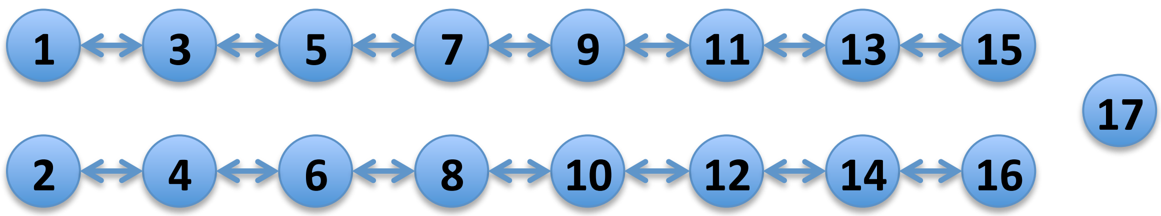
Table 5.4: The Feasible State List

Local Government (DM_{GOV})																	
Wait	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	-
Redev	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y	-
Cost	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	-
Community (DM_{COM})																	
Risk	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	-
Developer (DM_{DEV})																	
Dev	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N
No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**

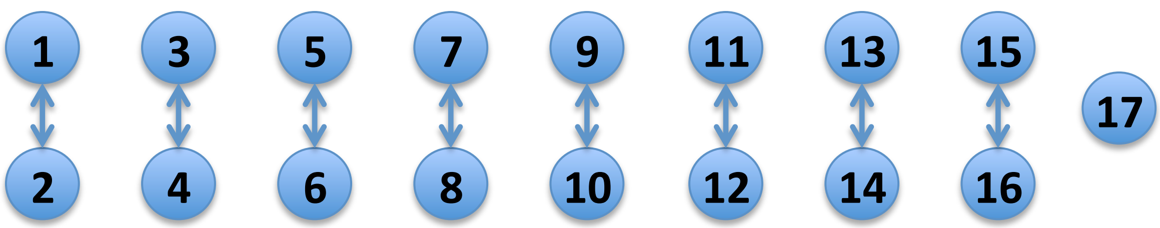
Trapezoid fuzzy values of all states for every DM in Figure 5.5 are paired compared, which produces three fuzzy preference matrices (Table 5.5, 5.6, and 5.7). From these matrices, we can find that transitivity does not hold for fuzzy preference matrices. Most values are only slightly greater than 0.5, confirming our observation that fuzzy numbers under different parameter settings are similar to each other. When α-cut levels of DMs are applied, fuzzy preference matrices can be converted into crisp ones and used to determine fuzzy UI lists.

5.3.2 Results

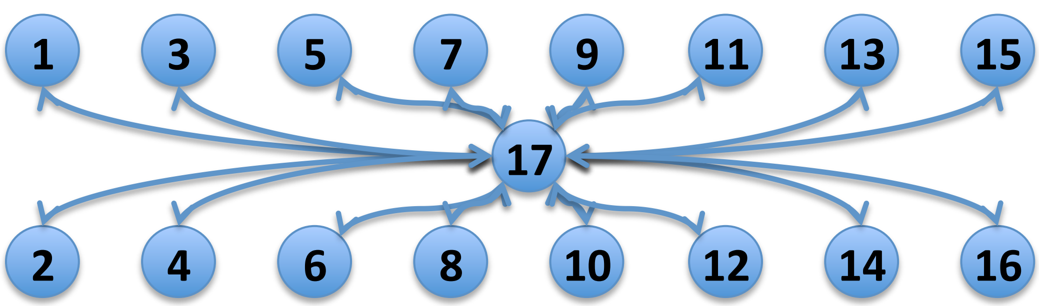
Since the fuzzy preference matrices and reachable lists have been prepared, algorithms of matrix representation of GMCR implemented in MatLab, are applied to determine stabilities and equilibiria. Equilibrium of various definitions under different α-cut level parameters are listed in Table 5.8. It is obvious that, because almost all states are stable for all DMs, GMR equilibrium provides little insights. This may be due to the highly



(a) Unilateral Movements of DM_{GOV}



(b) Unilateral Movements of DM_{COM}



(c) Unilateral Movements of DM_{DEV}

Figure 5.6: DMs' Unilateral Movements

Table 5.5: The Fuzzy Preference Matrix of DM_{GOV}

Matrix	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.5000	0.5000	0.5669	0.5669	0.0587	0.0587	0.0623	0.0623	0.3278	0.3278	0.3578	0.3578	0.0174	0.0174	0.0281	0.0281	0.4219
2	0.5000	0.5000	0.5669	0.5669	0.0587	0.0587	0.0623	0.0623	0.3278	0.3278	0.3578	0.3578	0.0174	0.0174	0.0281	0.0281	0.4219
3	0.4331	0.4331	0.5000	0.5000	0.0530	0.0530	0.0564	0.0564	0.2874	0.2874	0.3138	0.3138	0.0166	0.0166	0.0263	0.0263	0.3712
4	0.4331	0.4331	0.5000	0.5000	0.0530	0.0530	0.0564	0.0564	0.2874	0.2874	0.3138	0.3138	0.0166	0.0166	0.0263	0.0263	0.3712
5	0.9413	0.9413	0.9470	0.9470	0.5000	0.5000	0.5497	0.5497	0.9062	0.9062	0.9112	0.9112	0.3416	0.3416	0.5087	0.5087	0.9883
6	0.9413	0.9413	0.9470	0.9470	0.5000	0.5000	0.5497	0.5497	0.9062	0.9062	0.9112	0.9112	0.3416	0.3416	0.5087	0.5087	0.9883
7	0.9377	0.9377	0.9436	0.9436	0.4503	0.4503	0.5000	0.5000	0.8999	0.8999	0.9051	0.9051	0.3985	0.3985	0.4517	0.4517	0.9911
8	0.9377	0.9377	0.9436	0.9436	0.4503	0.4503	0.5000	0.5000	0.8999	0.8999	0.9051	0.9051	0.3985	0.3985	0.4517	0.4517	0.9911
9	0.6731	0.6731	0.7126	0.7126	0.0938	0.0938	0.1001	0.1001	0.5000	0.5000	0.5317	0.5317	0.0322	0.0322	0.0487	0.0487	0.6432
10	0.6731	0.6731	0.7126	0.7126	0.0938	0.0938	0.1001	0.1001	0.5000	0.5000	0.5317	0.5317	0.0322	0.0322	0.0487	0.0487	0.6432
11	0.6422	0.6422	0.6862	0.6862	0.0888	0.0888	0.0949	0.0949	0.4683	0.4683	0.5000	0.5000	0.0285	0.0285	0.0460	0.0460	0.6010
12	0.6422	0.6422	0.6862	0.6862	0.0888	0.0888	0.0949	0.0949	0.4683	0.4683	0.5000	0.5000	0.0285	0.0285	0.0460	0.0460	0.6010
13	0.9826	0.9826	0.9834	0.9834	0.6584	0.6584	0.6015	0.6015	0.9678	0.9678	0.9705	0.9705	0.5000	0.5000	0.5597	0.5597	1.0000
14	0.9826	0.9826	0.9834	0.9834	0.6584	0.6584	0.6015	0.6015	0.9678	0.9678	0.9705	0.9705	0.5000	0.5000	0.5597	0.5597	1.0000
15	0.9719	0.9719	0.9737	0.9737	0.4913	0.4913	0.5483	0.5483	0.9513	0.9513	0.9540	0.9540	0.4403	0.4403	0.5000	0.5000	1.0000
16	0.9719	0.9719	0.9737	0.9737	0.4913	0.4913	0.5483	0.5483	0.9513	0.9513	0.9540	0.9540	0.4403	0.4403	0.5000	0.5000	1.0000
17	0.5781	0.5781	0.6288	0.6288	0.0117	0.0117	0.0089	0.0089	0.3568	0.3568	0.3990	0.3990	0.0000	0.0000	0.0000	0.0000	0.5000

Table 5.6: The Fuzzy Preference Matrix of DM_{COM}

Matrix	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.5000	0.5654	0.4517	0.4970	0.0870	0.1110	0.0425	0.0700	0.3909	0.4128	0.3870	0.4068	0.0735	0.0703	0.0533	0.0622	0.2405
2	0.4346	0.5000	0.3774	0.4271	0.0352	0.0587	0.0038	0.0208	0.2909	0.3269	0.3006	0.3166	0.0238	0.0174	0.0103	0.0148	0.4249
3	0.5483	0.6226	0.5000	0.5520	0.0759	0.1040	0.0282	0.0573	0.4215	0.4493	0.4180	0.4411	0.0610	0.0555	0.0401	0.0487	0.2692
4	0.5030	0.5729	0.4480	0.5000	0.0479	0.0758	0.0075	0.0305	0.3671	0.3959	0.3631	0.3849	0.0340	0.0265	0.0166	0.0228	0.5058
5	0.9130	0.9648	0.9241	0.9521	0.5000	0.5328	0.4391	0.4798	0.9290	0.9415	0.9078	0.9367	0.4678	0.4868	0.4257	0.4685	0.9998
6	0.8890	0.9413	0.8960	0.9242	0.4672	0.5000	0.4102	0.4478	0.8923	0.9062	0.8731	0.9006	0.4366	0.4545	0.3968	0.4373	0.9883
7	0.9575	0.9962	0.9718	0.9925	0.5609	0.5898	0.5000	0.5416	0.9839	0.9906	0.9684	0.9919	0.5284	0.5554	0.4826	0.5306	1.0000
8	0.9300	0.9792	0.9427	0.9695	0.5202	0.5522	0.4584	0.5000	0.9520	0.9631	0.9313	0.9598	0.4867	0.5083	0.4418	0.4882	1.0000
9	0.6091	0.7091	0.5785	0.6329	0.0710	0.1077	0.0161	0.0480	0.5000	0.5362	0.4893	0.5225	0.0527	0.0437	0.0291	0.0379	0.6773
10	0.5872	0.6731	0.5507	0.6041	0.0585	0.0938	0.0094	0.0369	0.4638	0.5000	0.4571	0.4857	0.0413	0.0322	0.0203	0.0277	0.6432
11	0.6130	0.6994	0.5820	0.6369	0.0922	0.1269	0.0316	0.0687	0.5107	0.5429	0.5000	0.5322	0.0734	0.0659	0.0469	0.0570	0.6589
12	0.5932	0.6834	0.5589	0.6151	0.0633	0.0994	0.0081	0.0402	0.4775	0.5143	0.4678	0.5000	0.0447	0.0346	0.0209	0.0297	0.6551
13	0.9265	0.9762	0.9390	0.9660	0.5322	0.5634	0.4716	0.5133	0.9473	0.9587	0.9266	0.9553	0.5000	0.5198	0.4977	0.5020	1.0000
14	0.9297	0.9826	0.9445	0.9735	0.5132	0.5455	0.4446	0.4917	0.9563	0.9678	0.9341	0.9654	0.4802	0.5000	0.4605	0.4789	1.0000
15	0.9467	0.9897	0.9599	0.9834	0.5743	0.6032	0.5174	0.5582	0.9709	0.9797	0.9531	0.9791	0.5023	0.5395	0.5000	0.5476	1.0000
16	0.9378	0.9852	0.9513	0.9772	0.5315	0.5627	0.4694	0.5118	0.9621	0.9723	0.9430	0.9703	0.4980	0.5211	0.4524	0.5000	1.0000
17	0.7595	0.5751	0.7308	0.4942	0.0002	0.0117	0.0000	0.0000	0.3227	0.3568	0.3411	0.3449	0.0000	0.0000	0.0000	0.0000	0.5000

Table 5.7: The Fuzzy Preference Matrix of DM_{DEV}

Matrix	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.5000	0.4906	0.4583	0.4185	0.0567	0.0691	0.0283	0.0341	0.3473	0.3292	0.3178	0.3225	0.0421	0.0321	0.0242	0.0280	0.2005
2	0.5094	0.5000	0.4662	0.4227	0.0433	0.0587	0.0141	0.0208	0.3496	0.3269	0.3139	0.3166	0.0278	0.0174	0.0101	0.0148	0.4249
3	0.5417	0.5338	0.5000	0.4564	0.0555	0.0707	0.0241	0.0309	0.3811	0.3589	0.3460	0.3500	0.0391	0.0280	0.0201	0.0243	0.4527
4	0.5815	0.5773	0.5436	0.5000	0.0583	0.0758	0.0222	0.0305	0.4220	0.3959	0.3816	0.3849	0.0395	0.0265	0.0180	0.0228	0.5058
5	0.9433	0.9567	0.9445	0.9417	0.5000	0.4912	0.4468	0.4324	0.9454	0.9288	0.9248	0.9230	0.4711	0.4406	0.4391	0.4200	1.0000
6	0.9309	0.9413	0.9293	0.9242	0.5088	0.5000	0.4589	0.4478	0.9221	0.9062	0.9015	0.9006	0.4835	0.4546	0.4324	0.4373	0.9883
7	0.9717	0.9859	0.9759	0.9778	0.5532	0.5411	0.5000	0.4853	0.9830	0.9729	0.9710	0.9712	0.5306	0.4976	0.4692	0.4720	1.0000
8	0.9659	0.9792	0.9691	0.9695	0.5676	0.5522	0.5147	0.5000	0.9747	0.9631	0.9607	0.9598	0.5412	0.5083	0.4830	0.4882	1.0000
9	0.6527	0.6504	0.6189	0.5780	0.0546	0.0779	0.0170	0.0253	0.5000	0.4659	0.4459	0.4511	0.0341	0.0210	0.0134	0.0178	0.6110
10	0.6708	0.6731	0.6411	0.6041	0.0712	0.0938	0.0271	0.0369	0.5341	0.5000	0.4816	0.4857	0.0479	0.0220	0.0322	0.0277	0.6432
11	0.6822	0.6861	0.6540	0.6184	0.0752	0.0985	0.0290	0.0393	0.5541	0.5184	0.5000	0.5034	0.0509	0.0344	0.0235	0.0296	0.6667
12	0.6775	0.6834	0.6500	0.6151	0.0770	0.0994	0.0288	0.0402	0.5489	0.5143	0.4966	0.5000	0.0521	0.0346	0.0226	0.0297	0.6551
13	0.9579	0.9722	0.9609	0.9605	0.5289	0.5165	0.4694	0.4588	0.9659	0.9521	0.9491	0.9479	0.5000	0.4681	0.4414	0.4459	1.0000
14	0.9679	0.9826	0.9720	0.9735	0.5594	0.5454	0.5024	0.4917	0.9790	0.9678	0.9656	0.9654	0.5319	0.5000	0.4730	0.4789	1.0000
15	0.9758	0.9899	0.9799	0.9820	0.5609	0.5676	0.5308	0.5170	0.9866	0.9780	0.9765	0.9774	0.5586	0.5270	0.5000	0.5051	1.0000
16	0.9720	0.9852	0.9757	0.9772	0.5800	0.5627	0.5280	0.5118	0.9822	0.9723	0.9704	0.9703	0.5541	0.5211	0.4949	0.5000	1.0000
17	0.7995	0.5751	0.5473	0.4942	0.0000	0.0117	0.0000	0.0000	0.3890	0.3568	0.3333	0.3449	0.0000	0.0000	0.0000	0.0000	0.5000

uncertain preferences in the brownfield redevelopment conflict. Thus, GMR equilibrium are only provided for the first two settings and excluded in later computation.

If all DMs' α-cut levels are set to 0.5, no Nash equilibrium exists in this conflict. This is a major difference between quantitative game theory approach and GMCR. Since the states become discrete, Existence of Nash equilibrium is not ensured. However, SEQ equilibrium must exist based on Fraser-Hipel Theorem [39].

Table 5.8: Parameters and Equilibria

Equilibirum	γ_{GOV}	γ_{COM}	γ_{DEV}	**States**
Nash	0.5	0.5	0.5	ϕ
GMR				3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17
SMR				15
SEQ				14, 15, 16
Nash	0.55	0.55	0.55	14
GMR				2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17
SMR				14, 15
SEQ				14, 15
Nash	0.6	0.6	0.6	13, 14, 15, 16
GMR				unknown
SMR				13, 14, 15, 16
SEQ				13, 14, 15, 16
Nash	0.5	0.55	0.6	13, 14
GMR				unknown
SMR				13, 14, 15, 16
SEQ				13, 14
Nash	0.6	0.5	0.6	13
GMR				unknown
SMR				13, 14, 15, 16
SEQ				13

Seen from the Table 5.8, the number of equilibrium normally increases when higher α-cut levels are selected, which can be explained by the smaller UI set resulted from higher α-cut level, leading to more stable states. However, SEQ equilirium does not follow this thumb of rule. When α-cut levels of all DMs are raised to 0.55 from 0.5, state 16 becomes unstable and removed from equilibrium. By definition, unlike other definitions, SEQ involves UIs of other DMs. Such interactions make SEQ behaves differently.

5.4 Implications

The likely equilibria are clearly states 13, 14, 15, and 16. These states share the common policy options of allowing flexibility: developers can wait to develop, or redevelop for other usage. Flexibility of choosing the best time and changing land use are critical factors in promoting brownfield redevelopment. On the other hand, risk and cost sharing are not as important as expected. In other words, the real estate business component is more important than the land remediation component in brownfield redevelopment. This conclusion is similar to Erzi's work [34].

Furthermore, when land use change permits and wait-and-see flexibility in redevelopment are compared, changing land use is more effective in promoting brownfield redevelopment. Economic transition, which can attract more property income, is the most important factor in determining the success of a brownfield redevelopment project. In terms of the risk sharing and cost sharing options, risk sharing is more important. If the community has a more positive attitude toward the other two DMs and allows SSRA, the brownfield redevelopment is more likely to be successful [114]. Hence, the ranking of options in decreasing order of importance is the redevelopment option, the wait-and-see option, the risk-sharing option, and the cost-sharing option.

From the stability types perspective, sequential equilbrium seems to be the most appropriate, given that it considers not only a DM's own incentive to move, but also other DM's incentives as sanctions. SMR is also appropriate, although it is less sensitive to the change of α-cut levels among DMs. Nash equilibrium is not very meaningful, since there are few stable states. When fuzzy preference relations are highly uncertain, Nash equilibrium is not very helpful.

When we study on the relationship between α-cut levels and equlibria, we can find that the α-cut level can be interpreted as the DM's attitude toward change of brownfield value. In other words, a higher α-cut level means the DM is reluctant (or less incentive) to change. Unless this DM is "sure" the state transition is good, he or she prefers to stay at the status-quo. The GMCR model of brownfield redevelopment is more sensitive to the developer's α-cut level, which implies that persuading the developer to be more positive toward redevelopment is more important.

From the redevelopment policy perspective, it is interesting to find that cost sharing is not an effective as many people think. This result is not difficult to understand, since cost-sharing is a zero-sum process. If the developer's remediation cost is less, the local government pays more anyway. Thus, either local government or developers are reluctant to change to another state, leading to stability of both states that differ only in the option of cost sharing. In this case, we suggest that the local government focus on creating better business opportunities. Joint risk sharing is more useful in promoting brownfield redevelopment.

This case study of fuzzy preference and GMCR demonstrates the feasibility and usefulness of the fuzzy preference extension on GMCR. For instance, we find that, when the α-cut level of the community is changed from 0.6 to 0.5 with all other parameters fixed, state 14 is removed from the equilibria and only state 13 remains. This result is easy to understand given that the community becomes more sensitive to incentives, therefore less tolerant of fuzzy utility decrease. Fuzzy preference generalizes the application of GMCR in a simple but powerful way. More insights can be provided in this approach.

Beside factors considered in this GMCR model of brownfield redevelopment, future study can include developing environmental insurance as an improved way of cost sharing. Rather than sharing cost proportionally, insurance usually takes into account all unexpected remediation cost that exceeds a certain level. Insurance design can be discussed based on [34] [134]. In addition, the comparison of fuzzy numbers other than trapezoids must be extended in future work, which will be built on a better GMCR model.

Chapter 6

Negotiation Support System Design

6.1 NSS for Brownfield Negotiation

A NSS to facilitate brownfield negotiation is proposed in order to simplify the use of complex evaluation model. Compared to the usual DSS, an NSS aims to find optimal multi-party agreements, utilizing innovative models, workflows, and associated communication support, sometimes with a non-partisan mediator [58]. NSS can be regarded as a superset of DSS, implementing not only decision making techniques, but also additional communication subsystems for negotiation.

From the negotiation process perspective, NSS can be employed at the stages of preparation, position and interest assessment, and proposal [123]. In addition to a DSS component using models based on decision analysis, game theory, or economic theory, an NSS often includes an coordination module based on psychological and behavioural theory [72].

Negotiations are inevitable in promoting brownfield redevelopment. To facilitate negotiations among stakeholders of brownfield redevelopment, the following difficulties must be addressed:

- *Evaluation techniques:* Benefits and costs of brownfield redevelopment projects are highly unpredictable, making deterministic evaluation tools, such as NPV, inappropriate for pricing brownfields. A better pricing technique called fuzzy real options analysis can be employed in order to evaluate uncertainties involved in brownfield redevelopment [121]. The more accurate estimates generated using fuzzy real options provide a solid basis for negotiation.

- *Information sharing:* Another obstacle to negotiation is the limited information available on brownfield redevelopment, especially for site-specific conditions. Information sharing will be very helpful in building a positive environment for negotiation.

- *User friendly interface:* The utilization of the complex fuzzy real options model should be automated and concealed so that it will not be an obstacle for DMs. Hence, the proposed method will act as a bridge between the interface, which allows end users to mark their judgements on a brownfield map, and the fuzzy real options model for brownfield evaluation, which needs this information for parameter estimation.

In the context of the brownfield negotiation, OWA is appropriate to estimate the parameters of the fuzzy real options, and hence to determine the values of a brownfield to different DMs, which can be used as the basis of negotiation. Expert judgements of contamination likelihood and risk preferences of decision makers can be expressed graphically and reflected using fuzzy real options.

A non-partisan professional (qualified person (QP)) should be added to facilitate the brownfield negotiation. A QP can be invited to offer professional opinions on the likelihood of contamination, which will be used as the reference point of pollution likelihood. The QP can help in building trust and identifying solutions in the negotiation process.

Maps and fuzzy boundaries will be used to facilitate information sharing and communication based on a GIS module. DMs judgements can be illustrated using fuzzy boundaries for iterative assessment in order to obtain an accurate subjective estimation. The GMCR and report generation functions will be incorporated into the proposed NSS in the future.

6.2 Structure and Process Design

The structure of the proposed DSS illustrated in Figure 6.1 will also be the core module of a NSS. The DSS will be distributed across three locations: a server with powerful computational capacity where the core DSS component is installed, a server sharing geographic information publicly, and a mobile device with graphic interface to capture judgements on site.

- *Geographic Information Server:* This component provides public information to all decision makers, facilitating negotiation by information sharing. All contamination information will be updated here, avoiding information management issues, such as version control, accessibility, and backup.

- *Core Components for Brownfield Evaluation:* Because fuzzy real options models require computational power even in a parallel computing environment, the parameter estimation and option evaluation algorithms will be installed and utilized on a powerful server via web services. This mechanism reduces costs while increasing the availability of fuzzy real options models.

- *Mobile Device connected to the Internet:* A mobile device is easily portable, enabling DMs to record and modify their judgements on site or during the negotiation process anywhere. Exploratory decision making and negotiation will be facilitated through this arrangement.

Negotiation using the proposed DSS can follow a process briefly described below:

1. DMs take a mobile device to the brownfield, retrieving maps with appropriate geographic information from local government, and then mark their judgements at sample locations. OWA will be called from another server to combine multiple assessments and interpolate across the brownfield. The output will be fed back to the mobile device, adding the likelihood of contamination as a layer on the map. Decision makers can modify their estimates if they prefer. Final outputs will be stored on the public server.

2. Once judgements are fixed, the parameters for fuzzy real options will be computed, and the fuzzy real options model for brownfield evaluation will be called to determine the value of the site, critical values, and optimal decisions for DMs with different risk preferences.

3. Since the values of the brownfield for DMs have been determined, conflicts are now clear. Negotiation can be facilitated through equilibria found using conflict analysis

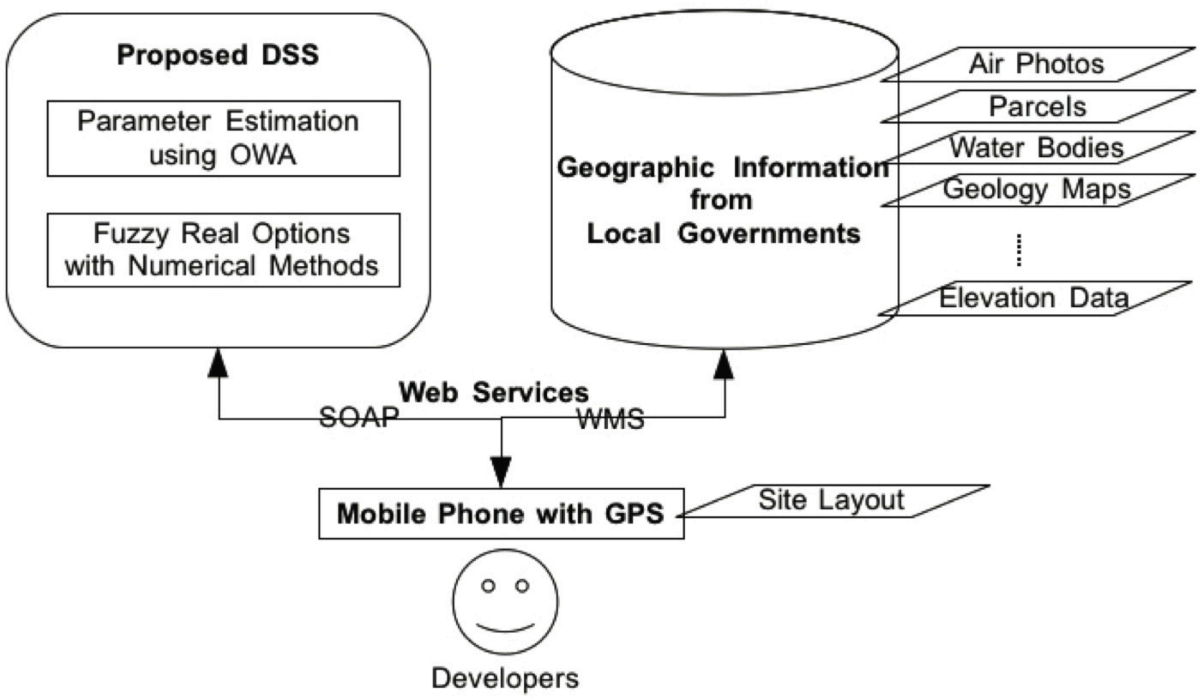


Figure 6.1: Structure of the Proposed DSS

methods. DMs can also compromise by adjusting their judgements, changing their attitudes, or adding more options.

Negotiation workflow can be optimized in the future and added as another module on top of the DSS. Better negotiation processes that encourage candid reporting of judgement may be added. Another possible improvement would be an additional component dealing with communication.

6.3 NSS Components and Prototypes

6.3.1 Fuzzy Real Options

A DSS prototype using fuzzy real options is designed for to convenient evaluation of brownfield projects. It has the following features [121] :

- *Ease of use*: All input parameters are easily input, including riskless rate, volatility, and fuzzy variables. Users can quickly understand the DSS;
- *Optimal operation suggestions*: Since the fuzzy real options approach has not been widely adopted by developers, giving values and associated operation suggestions can help to convince DMs that higher than NPV estimated brownfield values generated from fuzzy real options are achievable;
- *Strategy spaces shown graphically*: Plotting the strategy spaces and boundaries can clarify the project situation and give DMs intuition about how to choose options in order to attain the maximum value of a brownfield;

The system architecture of the prototype DSS is shown in Figure 6.2. Experts input parameters via the Windows presentation foundation (WPF) layer shown in Figure 6.3. Then, an event and process management module controls the work flow to convert all information to the MatLab format and directs the computation. Finally, the output is presented graphically via WPF.

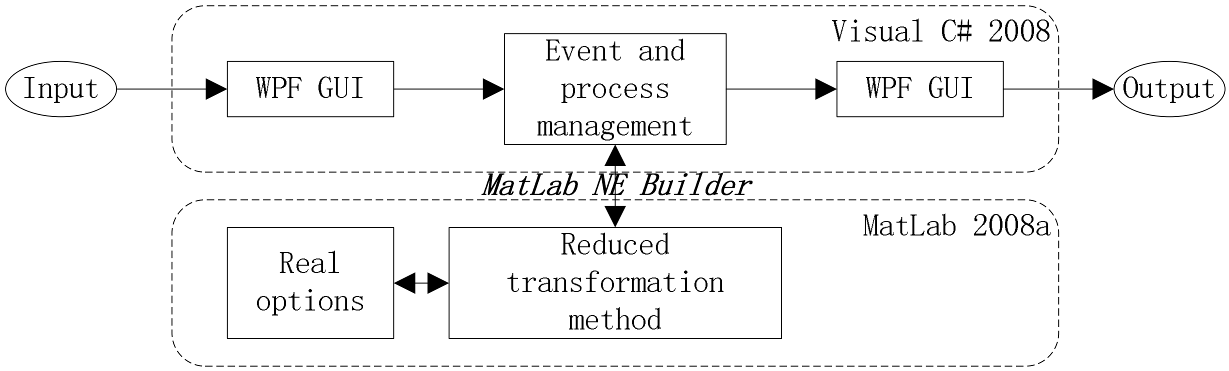


Figure 6.2: A DSS Prototype of Fuzzy Real Options

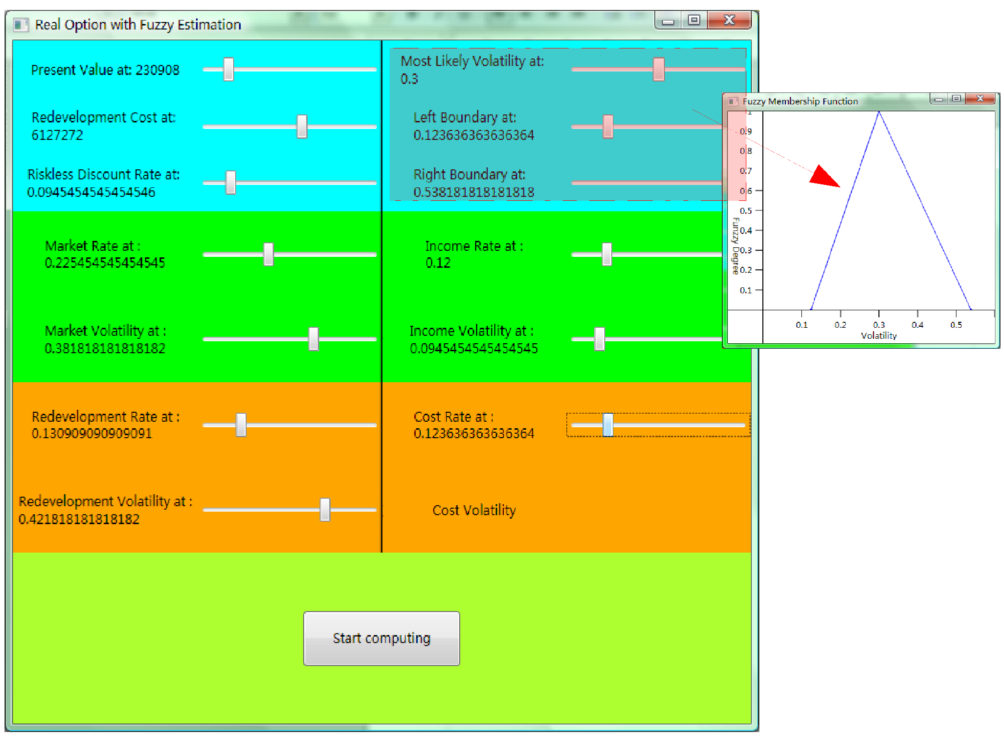


Figure 6.3: GUI for Input of the DSS

6.3.2 Fuzzy Boundaries and Numerical Methods

Two major extensions to the DSS prototype toward the proposed NSS are browser-server structure mentioned in Figure 6.1, which is critical in collaborative process integration, and the mapping capacity that can make judgements on the likelihood of contamination. As shown in Figure 6.4, DMs can add sample points on a base map, such as Google Maps, and then make multicriteria assessments. Local-government-owned data can also be added as layers on the map, providing more information. All information can be integrated under the geographic markup language (GML) standard.

When the "Submit" button is clicked, input information, including the subjective estimates on the map, will be sent to the server implemented in Geodjango. The generated fuzzy boundaries of a brownfield are then fed back as a layer overlaying the contaminated site. Geodjango is employed due to its backbone programming language, Python. The extended LSM algorithm is written in Python (Appendix B).

Python has the ability to integrate different components and act as "system glue". Information retrieved from clients can be input to the GeoStat module for interpolation via the R-Python interface. Once the parameters of the fuzzy real options model have been estimated, the parallelized C program under the multiple processing interface (MPI) is called by Geodjango via C-Python interface. Values generated by the fuzzy real options model are passed into the GMCR model, which can be easily rewritten in Python.

6.3.3 GMCR and Matrix Representation

A DSS is developed implementing GMCR in C++ and Microsoft foundation class (MFC) [36] [37]. With this system, users can follow the modelling process by switching among tabs (Figure 6.5) and obtain equilibria as suggestions. This DSS is mature and has demonstrated its effectiveness for years.

But because MFC is essentially a closed-form solution, it is difficult to accommodate this DSS as a module in the proposed NSS. Therefore, a MatLab program using the matrix representation of GMCR is utilized as a basis for multiple DM analysis (Appendix C). To ensure system integration, this program would be better rewritten in Python, which has a counterpart matrix system called SciPy.

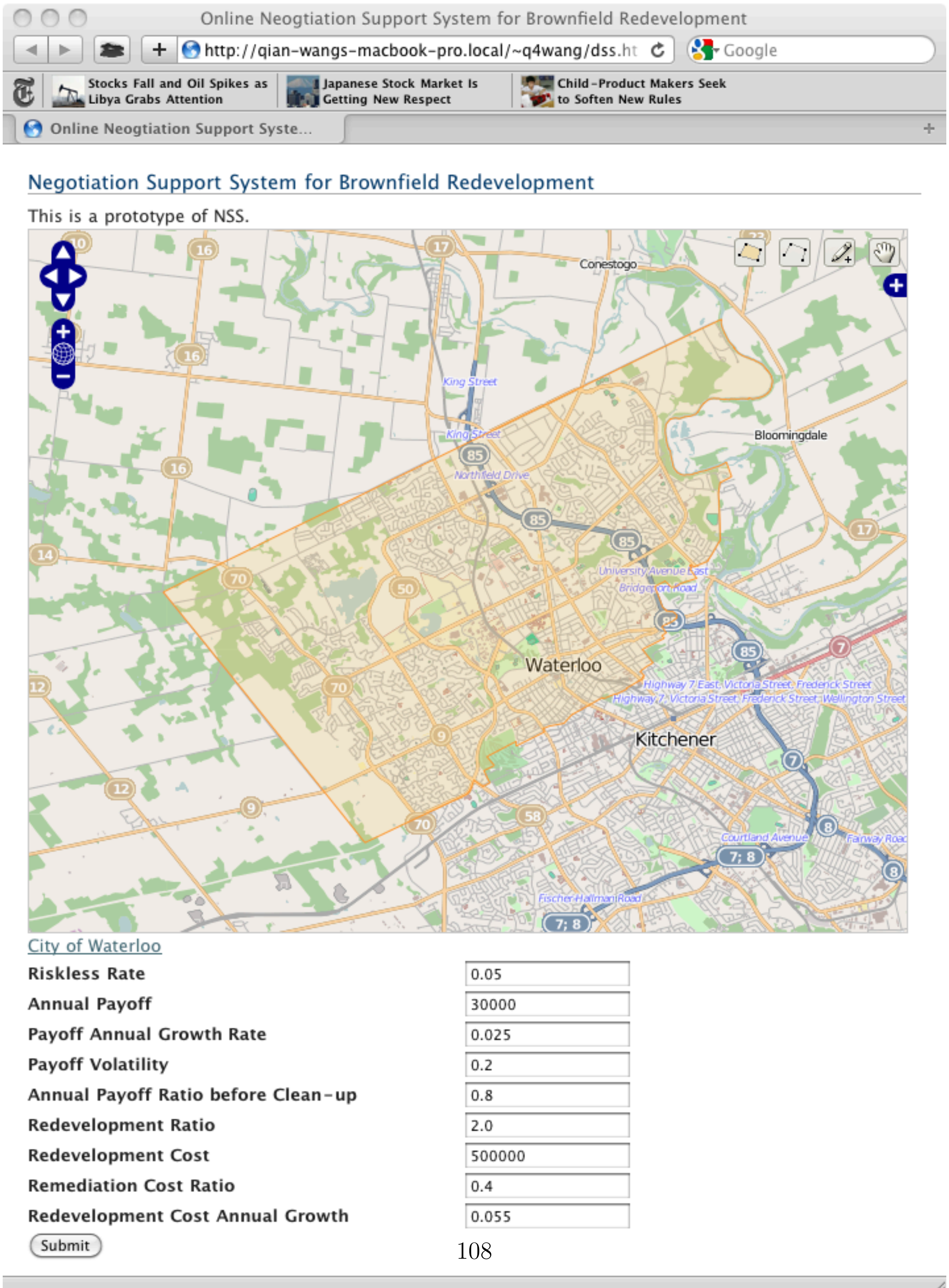


Figure 6.4: Client of the Proposed NSS

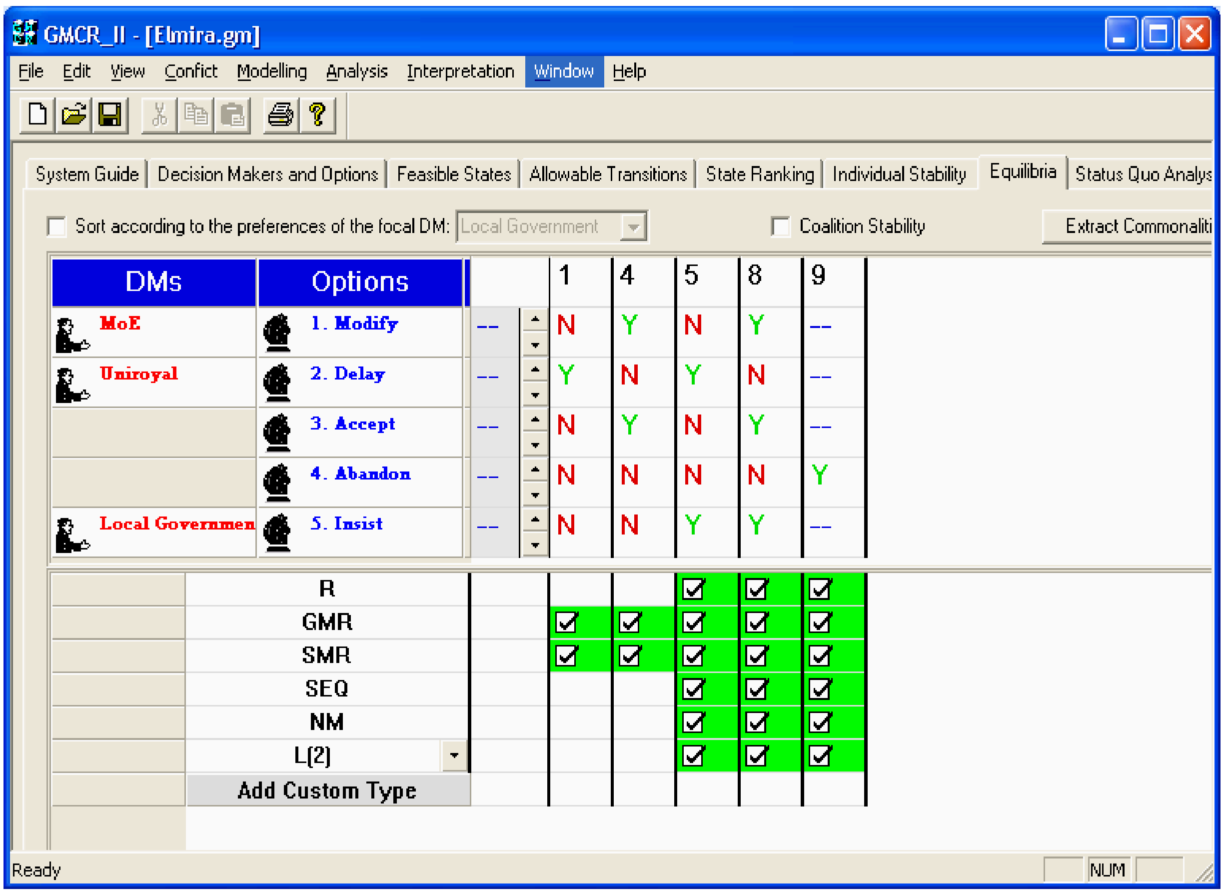


Figure 6.5: GMCR DSS

6.4 Summary

In summary, the proposed NSS will be based on browser-server architecture integrating different DSS prototypes. On the client side, AJAX, which is also compatible to WPF, will be used for user interface building. On the server side, workflow will be controlled using Python as a central language linking everything. Mapping capacity and location-based services (LBSs) are provided via Geodjango. The geostatistical module is called via an R-Python bridge, and then sends output to the fuzzy real options model in the C programming language.

Further development will be easy using Python, and customization will also be easy. This flexible structure balances the requirements of mobility, computation power, and compatibility. Handheld devices can be used providing web browsers are installed. Computing-intensive operations are allocated on the server grid, forming a cloud-computing environment. The usage of XML format enables data exchange across platforms.

All in all, the proposed NSS implements the process recommended earlier in this thesis. Equilibria under different settings provide DMs with insights into brownfield redevelopment conflicts. Effective collaboration can be enhanced using this NSS in design, and positive results are expected.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

This thesis follows the tasks outlined in the research proposal submitted in the comprehensive examination, which is shown in Table 7.1. Decision techniques are studied in order to facilitate brownfield negotiations. To assist a single DM's decision making, fuzzy real options and OWA are combined to calculate the value of a brownfield. Fuzzy boundaries and an online mapping module are employed to extend the fuzzy real options model and strengthen communication among DMs. Outputs are used as input into GMCR, which generates equilibria as suggestions for resolution of redevelopment conflicts. Main findings on decision-making under uncertainty, as well as brownfield negotiations, are summarized respectively in the following subsections.

7.1.1 Decision-making under Uncertainty

This thesis covers most decision-making techniques classified in Table 7.2. In other words, a line of methods to make decisions under uncertainty is extended, studied, and combined for synthesized application, such as brownfield redevelopment. Findings are listed below:

- *Fuzzy Real Options:* Fuzzy real options can be employed for risky project evaluation, regardless of the existence of analytic results or the types of risk. Compared to IVP,

Table 7.1: Comparison of Plan and Implementation

Tasks in Proposal	**Results in Thesis**
1. Fuzzy real options	
1.1. Numerical methods for fuzzy real options	Extended LSM based on chance theory
1.2. Uncertainty of fuzziness and randomness	
1.3. More computational intelligence methods	OWA and linguistics quantifier to represent risk preference
2. DSS extension with a GIS module	NSS design
	Fuzzy boundaries and geostatistics
3. Fuzzy real options and game-theoretic approaches	GMCR and fuzzy preference

Table 7.2: Classification of Decision Making Models [35]

		Objectives	
		One	Two or More
DMs	One	Decision Analysis Models	Multi-Criteria Decision Making
	Two or More	Team Theory	Conflict Analysis

the fuzzy real options approach is compatible with other artificial intelligence techniques, especially for linguistic quantifiers, because of the importance of descriptive knowledge in subjective estimation. In addition, exploration of fuzzy real options contributes to the integration of various uncertainty representations, aiding further research. More specifically, the fuzzy real options method enriches the application of possibility theory, which can be very useful for practitioners.

- *GMCR:* Newly developed extensions of GMCR are utilized to combine fuzzy real options and the original GMCR. The fuzzy preference framework demonstrates its advantages in representing state ranking under uncertainty, acting as a bridge between fuzzy real options and GMCR. The matrix representation of GMCR is convenient to implement and build with other modules [125]. We can see that GMCR and fuzzy real options are compatible with each other, making an integrated NSS possible.

- *NSS for brownfield negotiations:* An NSS for brownfield redevelopment is designed in this thesis. The fuzzy boundary representation is proposed with the aid of a distributed GIS module. The brownfield negotiation process, NSS architecture, and technical issues are discussed. The infrastructure supporting the proposed NSS, which has been tested via DSS prototypes, can be based on internet protocols. On the server side, Python modules and a parallel computer environment are also explored via SharcNet, the supercomputer grid in Ontario, Canada, and other servers. Hence, the feasibility of the proposed NSS is ensured.

7.1.2 Brownfield Redevelopment Negotiation

Because decision analysis methods have been improved for dealing with uncertainty, new results are found for these methods when addressing brownfield redevelopment cases. Fuzzy real options and GMCR provide the following results:

- *Impact of redevelopment cost:* The application of OWA confirms that the redevelopment cost is high and varies greatly case by case. However, the impact of uncertain cost on the brownfield value is not as high as people estimate. Therefore, brownfield projects are determined more by their real estate value than their remediation factors. This result partially explains why prosperous cities regenerate quickly leaving few brownfields, while others have to be abandoned entirely with no new investment.

- *Value of managerial flexibility:* The managerial flexibility adds some value to the project, which is roughly 5% - 10% higher than NPV estimates in brownfield case studies. But the flexible value should be higher than the appeared number, given that the redevelopment cost could dramatically rise in the future. The value of managerial flexibility is often higher than the value added when cost sharing is allowed. In other words, managerial flexibility is important in brownfield redevelopment.

- *Impact of risk preference:* The conflict among values of a brownfield, caused by risk preferences, is important, accounting for about 10% of the total value in the Ralgreen community case study. Nontheless, the difference is also not as high as one might imagine. When a non-partisan professional participates in the negotiation, one whom all stakeholders trust, redevelopment deals are much easier to make.

- *Issues in brownfield negotiation:* Because the differences between fuzzy values of a brownfield under several scenarios are not high, the DMs' attitudes are important in determining the equilibria, which constitute potential resolutions. The degree of tolerance of the community to contamination, as well as the eagerness of developers for taking advantage of business opportunities, are critical factors. Land use change is the most important factor in defining such opportunities.

- *Monte-Carlo simulation with fuzzy variables*: The impact of increasing the number of fuzzy samples on the result accuracy leads to an interesting question - what is the converge rate when fuzzy samples are generated in Monte-Carlo simulation. Although there has been no analytical work completed in this area yet, some tests can be conducted to obtain some empirical rules, which is very helpful in practise.

- *Implications of brownfield policy design:* In terms of identifying effective brownfield redevelopment policy, the creation of better economic opportunity is critical. Local governments need to design and implement innovative plans to attract developers. By contrast, cost sharing might not be effective in promoting redevelopment. A property tax cut after redevelopment is suggested over cost sharing due to its greater impact on future payoffs. In addition to these policies, employing non-partisan professionals and establishing risk-sharing policies are helpful.

7.2 Future Work

Although novel framework of decision making under uncertainty is proposed, there are many improvements that can be made. Some interesting ideas are as follows:

- *IVP and subjective probability:* Subjective probability is another good choice in dealing with private risks. The application of IVP can be further studied with the inclusion of Bayesian method, which can create a learning process gradually changing the estimate from subjective to objective. Transition between market and private risks would be a great discovery.

- *More types of real options:* Similar to the evolution of options in financial markets, more complex options will emerge when people realize the importance of real options and deliberately use them in contract negotiations. Timely value computation of options must be found in the future, this being very important to practitioners.

- *More types of processes:* The political uncertainty, such as regulation update, has not been included in the current model yet. But it can be added as a jump process or a fuzzy process. This will be another interesting topic for further research, which enhances the uncertainty study in SoS.

- *More game theoretic approaches:* Apart from GMCR, drama theory can be explored for possible combination with fuzzy real options [50] [51]. Also, quantitative games will also be tested in order to study the evolution of a conflict. Multi-agent simulation can be conducted in order to avoid the complexity of finding analytic results, while fully utilize computation power for complex system studies.

- *More flexible NSS design:* State-of-the-art information technology can be employed for developing a more flexible NSS design, namely, cloud computing. Cloud computing has a parallel structure, which can allocate computing resource effectively and accommodate various mobile devices. Better technologies should be used to develop improved NSS.

- *More application areas:* Because fuzzy real options and GMCR have demonstrated their advantages in studying environmental problems, more specifically, evaluating hydrogeolocial uncertainties, more applications can be executed. For instance, the oil sands development in Alberta, Canada, which requires large amounts of water resources and threatens the quality of ground water, can be studied within the research framework of this thesis. Better policy suggestions can be expected since decision-making under uncertainty is improved.

APPENDICES

Appendix A

Values

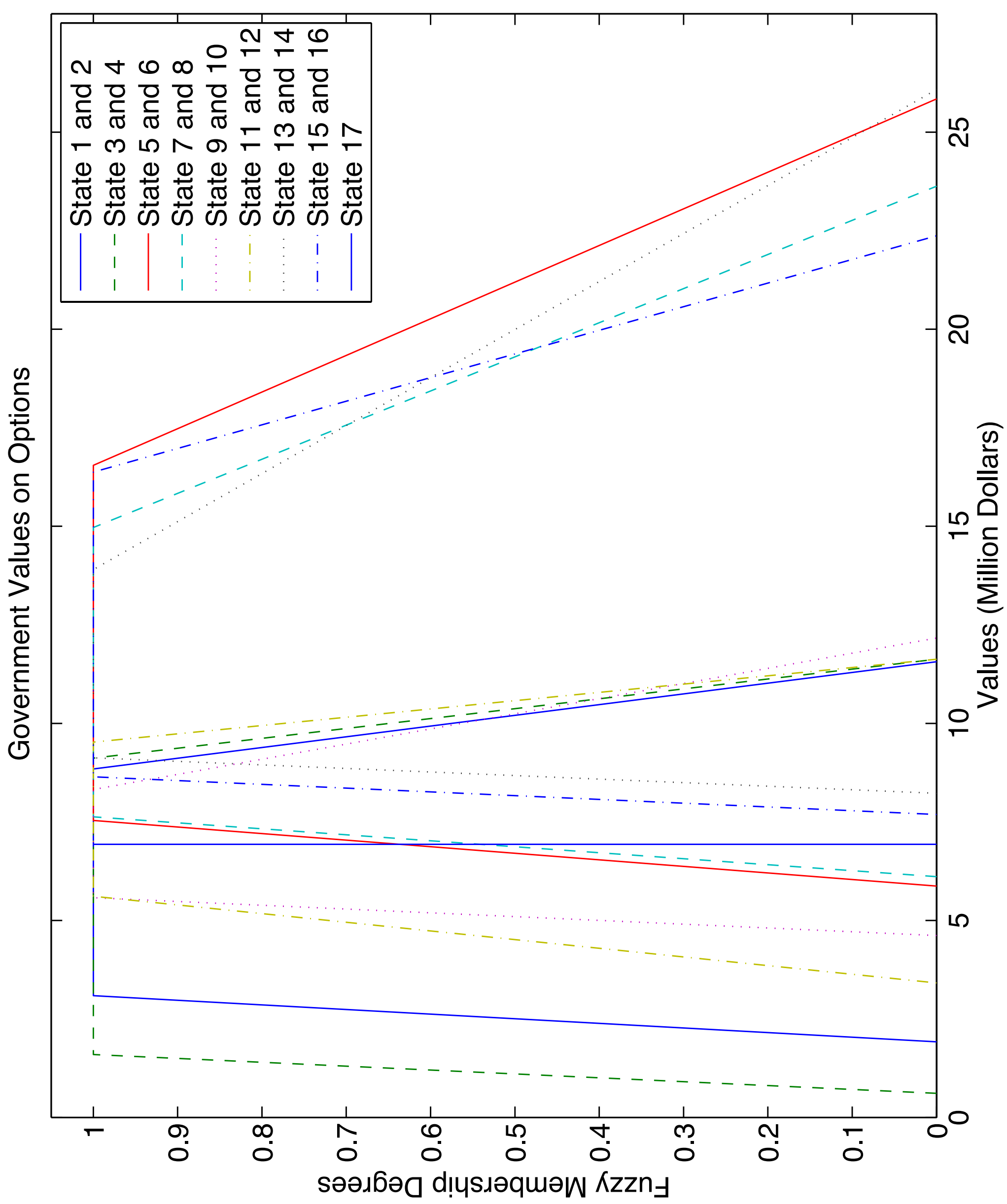


Figure A.1: DM_{GOV}'s Values on Options

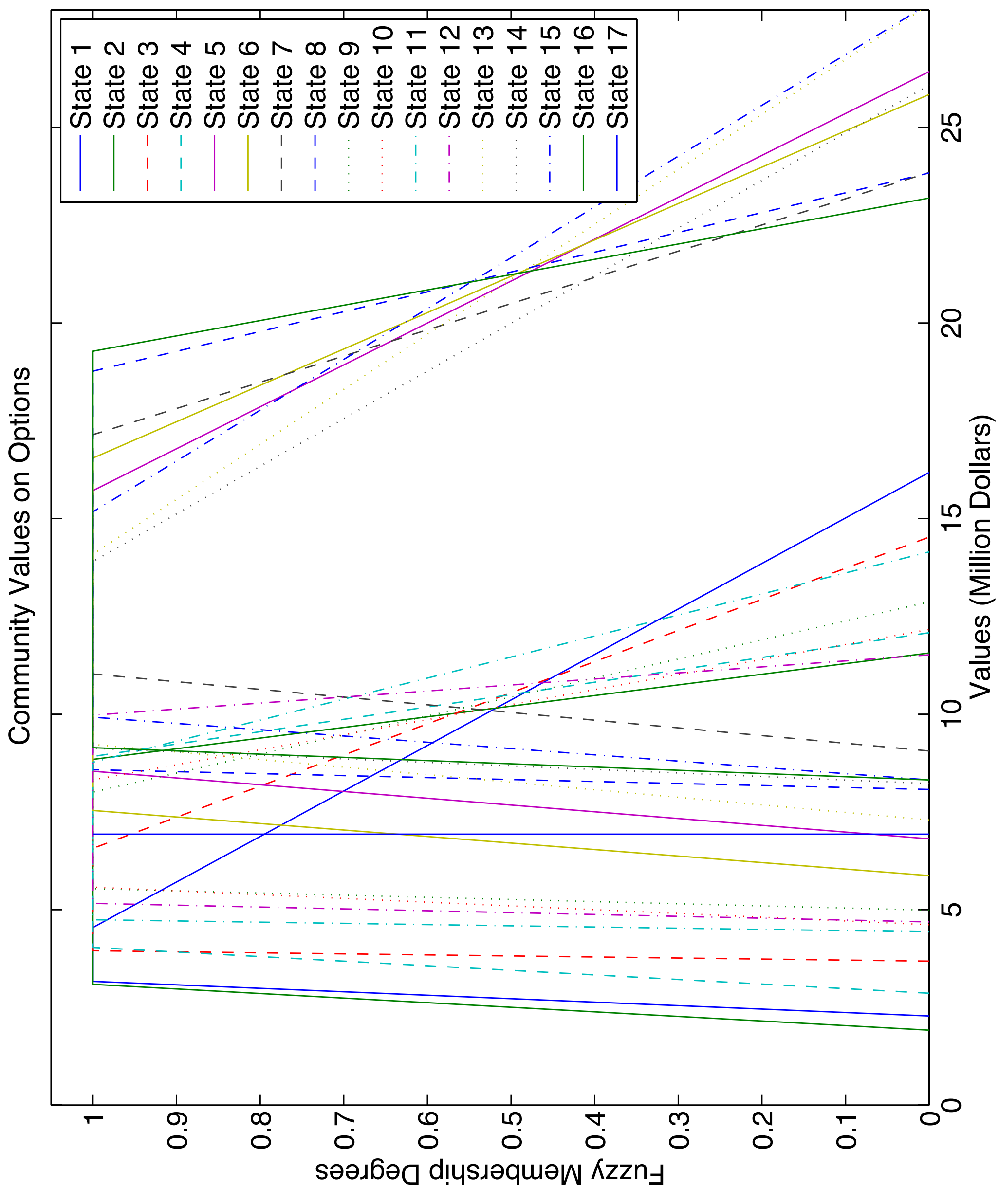


Figure A.2: DM_{COM}'s Values on Options

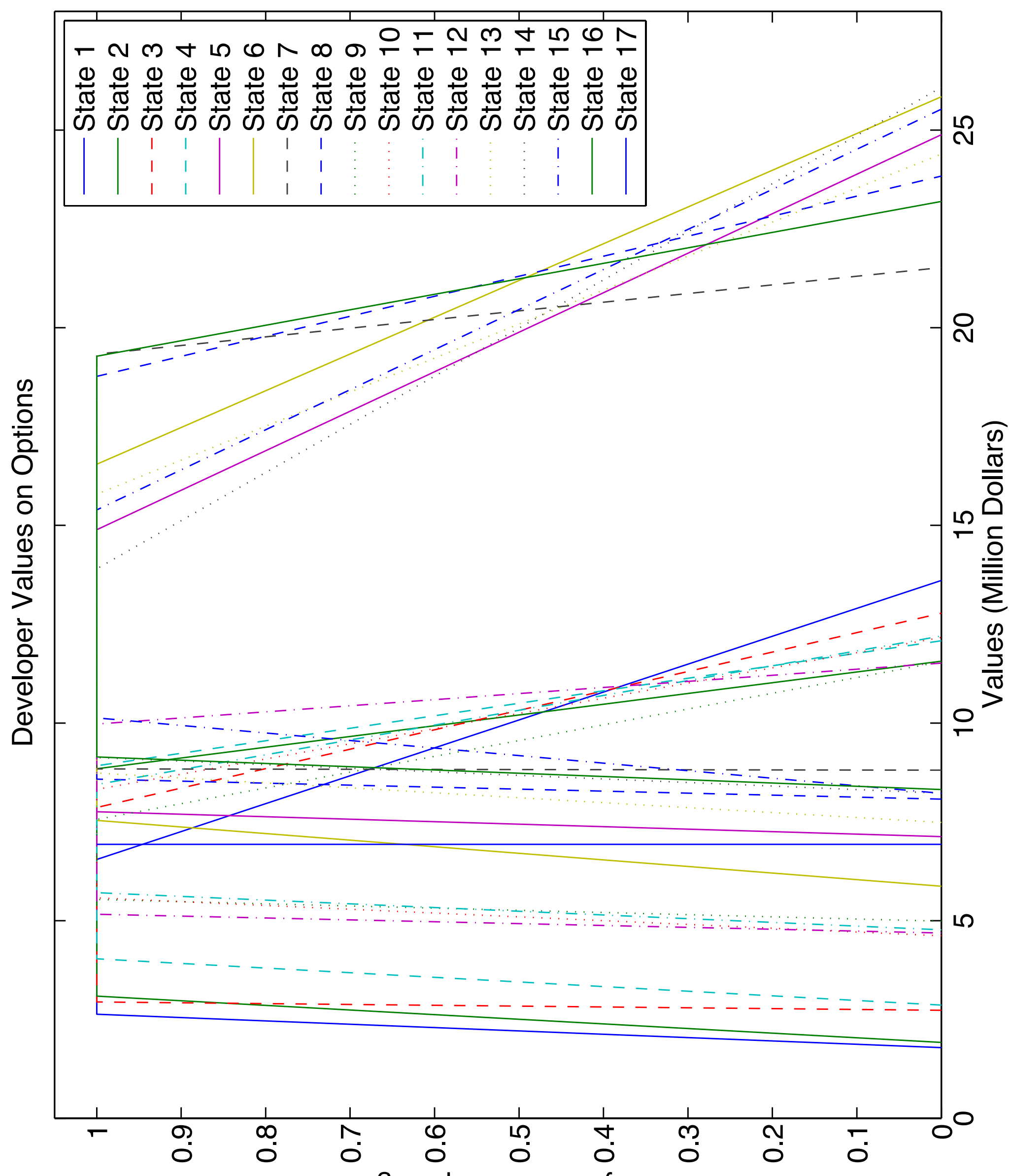


Figure A.3: DM_{DEV}'s Values on Options

Appendix B

Code of the Extended LSM Algorithm

```
from numpy import *
import random
import math
from math import sqrt
from Scientific.Functions.LeastSquares import leastSquaresFit
import cPickle, Gnuplot

Sam_num = 60 * 60; T = 200; dT = 0.25; Step_num = T / dT
Cr_num = sqrt(Sam_num)
abandon = 0; r = 0.05

# ----------------------------------------------------------------------------
# Cash Flow part
phi1 = 0.4; theta = 2.0;
alpha1 = 0.3

mu_in = 0.025; sigma_in = 0.2
mu_out = 0.055; low = 0.1; mode = 0.2; high = 0.3
```

```
dZ_in = array([random.gauss(0, 1) for i in xrange(Sam_num * Step_num)])
dZ_in.shape = (Sam_num, Step_num)
inFlow = zeros((Sam_num, Step_num))
Price = zeros((Sam_num, Step_num - 1))
V_P = zeros((Sam_num, Step_num - 1))

dZ_out = array([random.gauss(0, 1) for i in xrange(Sam_num * Step_num)])
dZ_out.shape = (Sam_num, Step_num)
outFlow = zeros((Sam_num, Step_num))

inFlow[:, 0] = 72341.067
outFlow[: , 0] = 5000000.0
Price[:, 0] = 300000.0  # will be the x-axis all the time

for i in xrange(Step_num-2):
    inFlow[:, i+1] = inFlow[:, i] * exp((mu_in - 0.5*sigma_in**2 - r)*dT \
     + sigma_in*sqrt(dT)*dZ_in[:, i])
    Price[:, i+1] = Price[:, i] * exp((mu_in - 0.5*sigma_in**2)*dT \
     + sigma_in*sqrt(dT)*dZ_in[:, i])
inFlow[:, Step_num-1] = inFlow[:, Step_num-2] / (r - mu_in) * exp(-r*dT)

# Credibility
Cr_sigma = array([random.uniform(low, high) for i in xrange(Cr_num)])
Cr_Cred = where(Cr_sigma < mode, (Cr_sigma - low) / ((mode - low)*2.0), \
(high + Cr_sigma - 2.0*mode) / (high - mode))

for i in xrange(Cr_num):
    sigma_out = Cr_sigma[i]
    for j in xrange(Step_num-1):
        outFlow[Cr_num*i:Cr_num*(i+1), j+1] = outFlow[Cr_num*i:Cr_num*(i+1), j] \
         * exp((mu_out - 0.5*sigma_out**2 - r)*dT \
         + sigma_out*sqrt(dT)*dZ_out[Cr_num*i:Cr_num*(i+1), j])
# ============================================================================
```

```
# -----------------------------------------------------------------------------
# Cashflow
net = zeros((Sam_num, Step_num))
net = inFlow
net_sum = zeros((Sam_num, Step_num))        # V(P)
option = zeros((Sam_num, Step_num))         # F(P)
Cost = outFlow * (1.0 + alpha1)

del inFlow; del dZ_in; del dZ_out; del outFlow

# Base function for least squares estimation
def basefunction(p, x):
    a1, a2, a3, a4, a5, b = p
    e = b + a1 * x + a2 * x**2 + a3 * x**3 + a4 * x**4 + a5 * x**5
    return e

# -----------------------------------------------------------------------------
# Backwards Valuation
net_sum[:, Step_num-1] = net[:, Step_num-1]
E = zeros((Sam_num, 1))
ValCont = net_sum[:, Step_num-1] * phi1
ValExec = net_sum[:, Step_num-1] * theta - Cost[:, Step_num-1]
option[:, Step_num-1] = where (ValCont > ValExec, ValCont, ValExec)
CriticalValue = zeros(Sam_num)
ExpTime = zeros(Sam_num)
ExpTime[:] = T

for i in range(Step_num, 1, -1):
    net_sum[:, i-2] = net[:, i-2] + net_sum[:, i-1]
    ValCont = net_sum[:, i-2] * phi1
    ValExec = net_sum[:, i-2] * theta - Cost[:, i-2]
    option[:, i-2] = where(ValCont > ValExec, ValCont, ValExec)
```

```
    CriticalValue = where (ValCont <= ValExec, Price[:, i-2] \
     / Cost[:, i-2], CriticalValue)
    ExpTime = where (ValCont <= ValExec, i*dT, ExpTime)
    for j in range(Cr_num):
        data = [(option[Cr_num*j+k, i-1], option[Cr_num*j+k, i-2]) for k \
         in range(Cr_num)]
        par = leastSquaresFit(basefunction, (0, 0, 0, 0, 0, 0), data)
        E[Cr_num*j:Cr_num*(j+1), 0] = basefunction(par[0], option[Cr_num*j: \
         Cr_num*(j+1), i-1]) # expected cashflow next step
        for l in range(Cr_num):
            option[Cr_num*j+l, i-2] = max(option[Cr_num*j+l, i-2], E[Cr_num*j+l])
    print "%.2f percent is finished" % ((Step_num - i) / Step_num * 100)
# ================================================================================

# --------------------------------------------------------------------------------
P = zeros((Cr_num, 1))
su1 = zeros((Cr_num, 1))
su2 = zeros((Cr_num, 1))
C = zeros((Cr_num, 1))
for i in range(Cr_num):
    C[i] = mean(option[Cr_num*i:Cr_num*(i+1), 0])

l = min(C[:, 0])
m = max(C[:, 0])
e = 0.0
N = 10000

for k in range(N):
    r = random.uniform(l, m)
    L = 0.0; temp = 0.0; sup1 =0.0; sup2 = 0.0;
    if r >=0:
        for i in range(Cr_num):
            if (C[i, 0] >= r) & (sup1 < Cr_Cred[i]):
```

```
                sup1 = Cr_Cred2[i]
            if (C[i, 0] < r) & (sup2 < Cr_Cred[i/Cr_num]):
                sup2 = Cr_Cred2[i]
        e = e + (sup1 + 1 - sup2) / 2.0
    else:
        for i in range(Cr_num):
            if (C[i, 0] <= r) & (sup1 < Cr_Cred2[i]):
                sup1 = Cr_Cred[i]
            if (C[i, 0] > r) & (sup2 < Cr_Cred2[i]):
                sup2 = Cr_Cred[i]
        e = e - (sup1 + 1 - sup2) / 2.0

average = max(l, 0.0) + min(m, 0.0) + e * (m-l) / N
print "The fuzzy mean is %.2f" % average

ttt = 0.0
for i in range(Sam_num):
ttt = ttt + Cr_Cred[i/Cr_num]*option[i, 0]

print "The united value is %.2f " % (ttt/sum(Cr_Cred)/Cr_num)
# ======================================================================

# ----------------------------------------------------------------------
su=zeros(Cr_num); n=zeros(Cr_num); TT = zeros(Cr_num)
for i in range(Cr_num) :
    for j in range(Cr_num) :
    if ExpTime[i*Cr_num+j] != min(ExpTime[i*Cr_num:(i+1)*Cr_num]) \
     and ExpTime[i*Cr_num+j] !=max(ExpTime[i*Cr_num:(i+1)*Cr_num]) :
    TT[i] = su[i] + ExpTime[i*Cr_num+j]
    n[i] = n[i] + 1
    su[i] = su[i] + CriticalValue[i*Cr_num+j]
    if n[i] == 0:
    n[i] = 1
```

```
for i in range(Cr_num) :
    su[i] = su[i] / Cr_num
    TT[i] = TT[i]/ n[i]
print "The rough time is %.2f " % (mean(TT))
# ====================================================================
```

Appendix C

Code for the Matrix Representation of GMCR with Fuzzy Preferences

```
% Qian Wang 2011 (c)
% Ralgreen Community
%_____________________________________
% Code modified based on Haiyan Xu (c) 2007

n = 3; % the number of decision maker
m = 17; % the number of state
delta = m * n;  % delta = |UA|

J = zeros(m,m,n);
Jp = zeros(m,m,n);
Pp = zeros(m,m,n);
Pne = zeros(m,m,n);
alpha = zeros(n, 1);

Mh = zeros(m,m,n);
Mhp = zeros(m,m,n);

M = zeros(m,m,n,n,delta);
```

```
Mp = zeros(m,m,n,n,delta);

Mnash = zeros(n,m);
Mgmr = zeros(m,m,n);
Msmr = zeros(m,m,n);
Mseq = zeros(m,m,n);

E = ones(m,m);

%__________________________________________
% Only need to input J and Pp
Pp(:, :, 1) = GOV';
Pp(:, :, 2) = COM';
Pp(:, :, 3) = DEV';
alpha(1) = 0.5;
alpha(2) = 0.5;
alpha(3) = 0.6;
J(:, :, 1) = Jgov';
J(:, :, 2) = Jcom';
J(:, :, 3) = Jdev';

for i = 1: n
    for j = 1 : m
        for k = 1: m
            if Pp(j, k, i) > alpha(i)
                Pp(j, k, i) = 1;
            else
                Pp(j, k, i) = 0;
            end
        end
    end
end
```

```
%_______________________________________________
% calculate Pne

for i = 1:n
    Pne(:,:,i) = ones(m,m) - Pp(:,:,i) - eye(m,m);
end

%_______________________________________________
% calculate Jp
for i = 1:n
    Jp(:,:,i) = J(:,:,i).*Pp(:,:,i);
end
Jp = sign(Jp);

%_______________________________________________
% calculate M, Mp
for i = 1:n
    for j = 1:n
        M(:,:,i,j,1) = J(:,:,j);     % initial
        Mp(:,:,i,j,1) = Jp(:,:,j);
    end
end

for k = 2:delta
    for i = 1:n
        for j = 1:n
            if j ~= i
                tempM = zeros(m,m);
                tempMp = zeros(m,m);

                for t = 1:n
                    if (t ~= i) && (t ~= j)
                        tempM = tempM + M(:,:,i,t,k-1);
```

```
                        tempMp = tempMp + Mp(:,:,i,t,k-1);
                    end
                end

                M(:,:,i,j,k) = sign(J(:,:,j)*tempM);
                Mp(:,:,i,j,k) = sign(Jp(:,:,j)*tempMp);
            end
        end
    end
end

%____________________________________________
% calculate Mh, Mhp
for i = 1:n
    for k = 1:delta

        for j = 1:n
            if j ~= i
                Mh(:,:,i) = Mh(:,:,i) + M(:,:,i,j,k);
                Mhp(:,:,i) = Mhp(:,:,i) + Mp(:,:,i,j,k);
            end
        end

    end
    Mh(:,:,i) = sign(Mh(:,:,i));
    Mhp(:,:,i) = sign(Mhp(:,:,i));
end


%___________________________________________
% stability

for i = 1:n
```

```
    e = ones(m,1);
    for j = 1:m
        es = zeros(m,1); es(j) = 1;
        Mnash(i,j) = es'*Jp(:,:,i)*e;     %Nash
    end

    Mgmr(:,:,i) = Jp(:,:,i)*(E - sign(Mh(:,:,i)*Pne(:,:,i)'));  %GMR
    Msmr(:,:,i) = Jp(:,:,i)*(E - sign(Mh(:,:,i)*(Pne(:,:,i)'.*(E - sign(J(:,:,i)* ..
     Pp(:,:,i)')))));  %SMR
    Mseq(:,:,i) = Jp(:,:,i)*(E - sign(Mhp(:,:,i)*Pne(:,:,i)'));  %SEQ
end

Mgmr = sign(Mgmr);
Msmr = sign(Msmr);
Mseq = sign(Mseq);

MgmrT = diag(Mgmr(:,:,1))+diag(Mgmr(:,:,2))+diag(Mgmr(:,:,3));
MsmrT = diag(Msmr(:,:,1))+diag(Msmr(:,:,2))+diag(Msmr(:,:,3));
MseqT = diag(Mseq(:,:,1))+diag(Mseq(:,:,2))+diag(Mseq(:,:,3));

%MgmrT',MsmrT',MseqT',

GMR = []; SMR = []; SEQ = [];

for i = 1:n
    GMR = [GMR; diag(Mgmr(:,:,i))'];
    SMR = [SMR; diag(Msmr(:,:,i))'];
    SEQ = [SEQ; diag(Mseq(:,:,i))'];
end

MH=zeros(m,m); MHp=zeros(m,m);
for i=1:n
    MH = MH + Mh(:,:,i);
```

```
    MHp = MHp + Mhp(:,:,i);
end

MH=sign(MH);
MHp=sign(MHp);

Mnash=Mnash
GMR = GMR;
SMR = SMR
SEQ = SEQ
```

References

[1] A.N. Ajah and P.M. Herder. Addressing flexibility during process and infrastructure systems conceptual design: real options perspective. In *Proceedings of IEEE International Conference on Systems, Man and Cybernetics*, volume 4, pages 3711–3716, October 2005. 23

[2] D.M. Allen, N. Schuurman, and Q. Zhang. Using fuzzy logic for modeling aquifer architecture. *Journal of Geographic Systems*, 9:289–310, 2007. 62

[3] D. Allenotor and R.K. Thulasiram. G-from: Grid resources pricing a fuzzy real option model. In *Proceedings of The IEEE International Conference on e-Science and Grid Computing*, pages 388–395, 2007. 36

[4] Martha Amram and Nalin Kulatilaka. *Real Options: Managing Strategic Investment in an Uncertain World*. Harvard Business School Press, 1999. 3, 22, 23, 24, 25

[5] Abul Bashar, Keith W. Hipel, and D. Marc Kilgour. Fuzzy preferences in a two-decision maker graph model. In *Proceedings of The IEEE International Conference on Sysytem, Man and Cybernetics*, pages 2964–2970, 2010. iv, 6, 85, 86

[6] Yacov Ben-Haim and Keith W. Hipel. The graph model for conflict resolution with information-gap uncertainty in preferences. *Applied Mathematics and Computation*, 126:319–340, 2002. 85

[7] Yakov Ben-Haim. Set-models of information-gap uncertainty: axioms and an inference scheme. *Journal of the Franklin Institute*, 336:1093–1117, 1999. 20

[8] Yakov Ben-Haim. *Info-Gap Theory: Decisions Under Severe Uncertainty*. Academic Press, 2nd edition, 2006. 20

[9] Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–659, 1973. 21

[10] Luiz E. Brandao and James S. Dyer. Decision analysis and real options: A discrete time approach to real option valuation. *Annals of Operations Research*, 135:21–39, 2005. 27, 28

[11] Christer Carlsson and Robert Fuller. On possibilistic mean value and variance of fuzzy numbers. *Fuzzy Sets and Systems*, 122:315–326, 2001. 30

[12] Christer Carlsson and Robert Fuller. A fuzzy approach to real option valuation. *Fuzzy Sets and Systems*, 13:297–312, 2003. 2, 14, 30, 36, 51

[13] John M. Charnes. *Financial Modeling with Crystal Ball and Excel*. John Wiley and Sons, 2007. 28

[14] Shyi-Ming Chen and Li-Wei Lee. Fuzzy decision-making based on likelihood-based comparison relations. *IEEE Transactions on Fuzzy Systems*, 18(3):613–628, June 2010. 88

[15] F. Chiclana, F. Herrera, and E. Herrera-Viedma. Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations. *Fuzzy Sets and Systems*, 122:277–291, 2001. 86, 87

[16] Kwai-Sang Chin, Jian-Bo Yang, Min Guo, and J.P.-K. Lam. An evidential-reasoning-interval-based method for new product design assessment. *IEEE Transactions on Engineering Management*, 56(1):142 –156, feb. 2009. 20

[17] A.R. Choudhury, A. King, S. Kumar, and Y. Sabharwal. Optimizations in financial engineering: the least-squares Monte Carlo method of Longstaff and Schwartz. In *Proceedings of the 2008 IEEE International Parallel and Distributed Processing Symposium*, pages 1701–1711, Piscataway, New Jersey, 2008. 38

[18] Mikael Collan. A new method for real option valuation using fuzzy numbers. Technical report, Institute for Advanced Management Systems Research, Ábo Akademi University, 2008. 31

[19] Mikael Collan, Robert Fullr, and Jzsef Mezei. A fuzzy pay-off method for real option valuation. In *2009 International Conference on Business Intelligence and Financial Engineering, BIFE 2009*, pages 165–169. IEEE Computer Society, 2009. 31

[20] HEATH Consultants. Summary of technical issues on the ralgreen subdivision. Technical report, HEATH, London, Ontario, Canada, 1997. 66, 67, 68, 72

[21] Tom Copeland and Vladimir Antikarov. *Real Options: A Practitioner's Guide*. Thomson, 2003. 21, 23, 28

[22] International Economic Development Council. International brownfields redevelopment. Technical report, IEDC, Washington DC, USA, 2005. 12

[23] Ines Couso and Didier Dubois. On the variability of the concept of variance for fuzzy random variables. *IEEE Transactions on Fuzzy Systems*, 17(5):1070–1080, 2009. 31

[24] Christopher R. Cunningham. Growth controls, real options, and land development. *The Review of Economics and Statistics*, 89(2):343–358, 2007. 3, 23

[25] Viswanath Cvsa and Stephen M. Gilbert. Strategic commitment versus postponement in a two-tier supply chain. *European Journal of Operational Research*, 141:526–543, 2002. 84

[26] Christopher A. De Sousa. *The Brownfield Problem in Urban Canada: Issues, Approaches, and Solutions*. Doctoral dissertation, University of Toronto, 2000. 1, 13

[27] Christopher A. De Sousa. Contaminated sites: The Canadian situation in an international context. *Journal of Environmental Management*, 62:131–154, 2001. 10, 11, 12

[28] M. Denton, A. Palmer, R. Masiello, and P. Skantze. Managing market risk in energy. *IEEE Transactions on Power Systems*, 18(2):494–502, May 2003. 23

[29] Yann d'Halluin, Peter A. Forsyth, and Kenneth R. Vetzal. Managing capacity for telecommunications networks under uncertainty. *IEEE Transactions on Networking*, 10(4):579–588, August 2002. 23

[30] Avinash K. Dixit and Robert S. Pindyck. *Investment under Uncertainty*. Princeton University Press, 1994. 3, 14, 15, 19, 23, 24, 26, 27, 38, 45

[31] Didier Dubois and Henri Prade. The mean value of a fuzzy number. *Fuzzy Sets and Systems*, 24:279–300, 1987. 30

[32] Dima S. El-Gamal. *Streamlined Approach to to Vadose Zone Modelling a Contribution to Brownfield Redevelopment*. PhD thesis, Department of Civil and Environmental Engineering in Wayne State University, 2005. 9, 13, 33, 63

[33] Hakan Erdogmus. Valuation of learning options in software development under private and market risk. *The Engineering Economist*, 47(3):308–353, 2002. 27, 28

[34] Ipek Erzi-Akcelik. *An Analysis of Uncertainty in Brownfield Redevelopment using Real Options*. PhD thesis, Department of Civil and Environmental Engineering, Carnegie Mellon University, May 2002. 14, 15, 29, 73, 99, 100

[35] Liping Fang, Keith W. Hipel, and D. Marc Kilgour. *Interactive decision making: The graph model for conflict resolution*. Wiley, New York, 1993. xiv, xvi, 75, 76, 77, 79, 112

[36] Liping Fang, Keith W. Hipel, D. Marc Kilgour, and John Peng. A decision support system for interactive decision making, part 1: Model formulation. *IEEE Transactions on Systems, Man and Cybernetics, Part C*, 33(1):42–55, 2003. 107

[37] Liping Fang, Keith W. Hipel, D. Marc Kilgour, and John Peng. Decision support system for interactive decision making, part 2: Analysis and output interpretation. *IEEE Transactions on Systems, Man and Cybernetics, Part C*, 33(1):56–66, 2003. 107

[38] David N. Ford and Durward K. Sobek II. Adapting real options to new product development by modeling the second toyota paradox. *IEEE Transactions on Engineering Management*, 52(2):175–185, 2005. 23

[39] Niall M. Fraser and Keith W. Hipel. *Conflict Analysis: Models and Resolutions*. North-Holland, New York, 1984. 76, 98

[40] Niall M. Fraser, Elizabeth M. Jewkes, Irwin Bernhardt, and May Tajima. *Global Engineering Economics: Financial Decision Making for Engineers*. Pearson Education Canada, 4th edition, 2008. 2, 19, 21

[41] R. Gibson and E. Schwartz. The valuation of long term oil-linked assests. In D. Lund and B Oksendal, editors, *Stochastic Models and Option Values*. Elsevier Science Publisher, Amsterdam, 1991. xiii, 51, 53

[42] Pierre Goovaerts. *Geostatistics for Natural Resources Evaluation*. Oxford University Press, New York, USA, 1997. 62

[43] C.J. Jr. Grayson. *Decisions under Uncertainty, Drilling Decisions by Oil and Gas Operators*. Harvard Business School, Boston, MA, USA, 1960. 51

[44] Hyun-Jin Han. *Estimating Project Volatility and Developing Decision Support System in Real Options Analysis*. PhD thesis, Auburn University, 2007. 28, 29

[45] Keith W. Hipel. Systems engineering approaches for brownfield redevelopment. Online at http://researchcommunity.iglooresearch.org/systemseng, Feburary 2010. iii, 1, 8

[46] Keith W. Hipel and Yacov Ben-Haim. Decision making in an uncertain world: Information-gap modeling in water resources management. *IEEE Transactions on Systems, Man and Cybernetics, Part C*, 29(4):506–517, 1999. 19, 20

[47] Keith W. Hipel, Mo M. Jamshidi, James M. Tien, and Chelsea C. White III. The future of systems, man and cybernetics: Application domains and research methods. *IEEE Transactions on Systems, Man, and Cybernetics, Part C*, 37(5):726–743, 2007. 9

[48] J. Hirshleifer. *Investment, Interest, and Capital*. Prentical Hall, 1970. 28

[49] Nigel Howard. *Paradoxes of Rationality: Theory of Metagames and Political Behaviour*. MIT Press, Cambridge, MA, 1971. 76

[50] Nigel Howard. Drama theory and its relation to game theory. part 1: Dramatic resolution vs. rational solution. *Group Decision Negotiation*, 3(2):187–206, 1994. 115

[51] Nigel Howard. Drama theory and its relation to game theory. part 2: Formal model of the resolution process. *Group Decision Negotiation*, 3(2):207–235, 1994. 115

[52] John C. Hull. *Options, Futures, and other Derivative Securities*. Prentice-Hall Press, 2nd edition, 1993. 21, 25, 37, 38

[53] Van-Nam Huynh, Yoshiteru Nakamori, and Jonathan Lawry. A probability-based approach to comparison of fuzzy numbers and applications to target-oriented decision making. *IEEE Transaction on Fuzzy Systems*, 16(2):371–387, April 2008. iv, xvi, 88, 89

[54] "Frontline Environmental Management Inc". Restroation work plan for the ralgreen community. Technical report, Frontline Environmental Management Inc., 2001. 34, 58, 66, 72

[55] Mo M. Jamshidi, editor. *System of Systems Engineering - Innovations for the 21st Century*. Wiley, New York, 2009. 9

[56] P. Jankowski, N. Andrienko, and G. Andrienko. Map-centered exploratory approach to multiple criteria spatial decision making. *Journal of Geographic Systems*, 15(2):101127, 2001. 64

[57] Corwin Joy, Phelim P. Boyle, and Ken Seng Tan. Quasi-monte carlo methods in numerical finance. *Management Science*, 42(6):926–938, 1996. 38

[58] Gregory E. Kersten and Hsiangchu Lai. Negotiation support and e-negotiation systems: an overview. *Group Decision Negotiation*, 16(6):553586, 2007. 101

[59] D. Marc Kilgour and Keith W. Hipel. *Handbook of Group Decision and Negotiation*, volume 4, chapter Conflict Analysis Methods: The Graph Model for Conflict Resolution, pages 203–222. Springer, 1st edition, 2010. xiii, 74, 79

[60] George J. Klir. Principles of uncertainty: What are they? why do we need them? *Fuzzy Sets and Systems*, 74(1):15–31, 1995. 19, 20

[61] George J. Klir and Richard M. Smith. On measuring uncertainty and uncertainty-based information: Recent developments. *Annals of Mathematics and Artificial Intelligence*, 32:5–33, 2001. 19, 20, 31

[62] Huibert Kwakernaak. Fuzzy random variablesi: definitions and theorems. *Information Sciences*, 15:1–29, 1978. 31

[63] Huibert Kwakernaak. Fuzzy random variablesii: Algorithms and examples for the discrete case. *Information Sciences*, 17:253–278, 1979. 31

[64] Samuel P. Lai, Tze L.and Wong. Valuation of American options via basis functions. *IEEE Transactions on Automatic Control*, 49(3):374–385, 2004. Lai, Tze Leung and Wong, Samuel Po-Shing. 39, 48

[65] H. Lee-Kwang and Jee-Hyong Lee. A method for ranking fuzzy numbers and its application to decision-making. 7(6):677–685, 1999. 88

[66] George H. Lentz and K.S. Maurice Tse. An option pricing approach to the valuation of real estate contaminated with hazardous materials. *Journal of Real Estate Finance and Economics*, 10:121–144, 1995. xiii, 14, 16, 17, 29, 46, 47, 49

[67] Kevin W. Li, Keith W. Hipel, D. Marc Kilgour, and Liping Fang. Preference uncertainty in the graph model for conflict resolution. *IEEE Transactions on Systems, Man, and Cybernetics, Part A*, 34(4):507–520, 2004. 20, 85, 86

[68] Kevin W. Li, Fakhreddine Karray, Keith W. Hipel, and D. Marc Kilgour. Fuzzy approaches to the game of chicken. *IEEE Transaction on Fuzzy Systems*, 9(4):608–623, August 2001. 85

[69] Kevin W. Li, D. Marc Kilgour, and Keith W. Hipel. Status quo analysis in the graph model for conflict resolution. *Journal of the Operational Research Society*, 56:699–707, 2005. 83

[70] Xiang Li and Baoding Liu. Chance measure for hybrid events with fuzziness and randomness. *Soft Computing*, 13(2):105–115, 2009. 20, 31, 32, 36, 41

[71] Xiaotong Li. Preemptive learning, competency traps, and information technology adoption: A real options analysis. *IEEE Transactions on Engineering Management*, 56(4):650–662, November 2009. 23

[72] Lai-Huat Lim and Izak Benbasat. A theoretical perspective of negotiation systems. *Journal of Management Information Systems*, 9(3):27–44, 1993. 101

[73] Baoding Liu. *Uncertainty Theory*. Springer-Verlag Berlin, 2nd edition, 2007. 20, 43, 47, 64

[74] Baoding Liu. Fuzzy process, hybrid process and uncertain process. *Journal of Uncertain Systems*, 2(1):3–16, 2008. 31, 39

[75] Paul A. Longley, Michael F. Goodchild, David J. Maguire, and David W. Rhind. *Geographic information systems and science*. John Wiley and Sons, 2nd edition, 2005. 65

[76] Francis A. Longstaff and Eduardo S. Schwartz. Valuing American options by simulation: A simple least-squares approach. *The Review of Financial Studies*, 14(1):113–147, Spring 2001. 38

[77] David G. Luenberger. A correlation pricing formula. *Journal of Economic Dynamics and Control*, 26:1113–1126, 2002. 21, 29

[78] Jacek Malczewski and Claus Rinner. Exploring multicriteria decision strategies in gis with linguistic quantifiers: A case study of residential quality evaluation. *Journal of Geographic Systems*, 7:249–268, 2005. 61

[79] Mahdi H. Mattar and Charles Y. J. Cheah. Valuing large engineering projects under uncertainty: private risk effects and real options. *Construction Management and Economics*, 24(8):847–860, Aug. 2006. 3, 26, 30

[80] Mahdi Haidar Mattar. *Private Risk*. PhD thesis, MIT, 2002. 28

[81] Robert C. Merton. Theory of rational optin pricing. *Bell Journal of Economics and Management Science*, 4:141–183, 1973. 21

[82] Bruno Miller and John-Paul Clarke. Strategic guidance in the development of new aircraft programs: A practical real options approach. *IEEE Transactions on Engineering Managemnt*, 55(4):566–578, November 2008. 23

[83] Xu Min-li and Jian Hui-yun. Application of real options to the risk management of supply chain. In *Proc. International Conference on Management Science and Engineering ICMSE 2007*, pages 722–727, 20–22 Aug. 2007. 23

[84] Johnathan Mum. *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*. Wiley Finance, 2002. 15, 23, 28

[85] Pauli Murto, Erkka Näsäkkälä, and Jussi Keppo. Timing of investments in oligopoly under uncertainty: A framework for numerical analysis. *European Journal of Operational Research*, 157:486–500, 2004. 84

[86] Francis P. Ng and Hans C. Bjornsson. Using real option and decision analysis to evaluate investments in the architecture, construction and engineering industry. *Construction Management and Economics*, 22:471–482, June 2004. 26, 29

[87] P. Nijkamp, H. Leitner, and N. Wrigley. *Measuring the Unmeasurable.* Martinus Nijhoff Publishers, 1985. 61

[88] NRTEE. Contaminated sites issues in canada. Technical report, Financial Services Task Force, Ottawa, Ontario, Canada, 1996. 1, 12

[89] Adekunle Richard Oduntan. *Real Options and Asset Valuation in Competitive Energy Markets.* PhD thesis, Department of Management Science, University of Waterloo, 2007. 23

[90] Government of Ontario. Ontario Regulation 511/09: Records of Site Condition. 57, 61

[91] James L. Paddock, Daniel R. Siegel, and James L. Smith. Option valuation of claims on real assets: the case of offshore petroleum leases. *Quarterly Journal of Economics*, 10:479508, 1988. 51

[92] E. Pickles and J.L. Smith. Petroleum property evaluation: a binomial lattice implementation of option pricing theory. *Energy Journal*, 14(2):1–26, 1993. 51

[93] Inc. Project Management Institute. *A Guide to the Project Management Body of Knowledge.* Project Management Institute,, 2004. 30, 34

[94] M.L. Puri and D.A. Ralescu. Fuzzy random variables. *Journal of Mathematical Analysis and Applications*, 114:409–422, 1986. 31

[95] Bogdan Rebiasz. Fuzziness and randomness in investment project risk appraisal. *Computers and Operations Research*, 34:199–210, 2007. 21

[96] Bert De Reyck, Zeger Degraeve, and Roger Vandenborre. Project options valuation with net present value and decision tree analysis. *European Journal of Operational Research*, 184:341–355, 2008. 27

[97] Joseph Sarkis and Maurry Tamarkin. A quadranomial real options model for evaluation of emissions trading and technology. volume 5997. SPIE, 2005. 23

[98] Malte Schneider, Mauricio Tejeda, Gabriel Dondiw, Florian Herzogw, Simon Keelw, and Hans Geering. Making real options work for practitioners: a generic model for valuing r&d projects. *R&D Management*, 38(1):85–106, 2008. 3, 23

[99] Markus Schneider. *Handbook of Research on Fuzzy Information Processing in Databases*, chapter Fuzzy Spatial Data Types for Spatial Uncertainty Management in Databases, pages 490–515. IGI Global, 2008. 64

[100] Glenn Shafer. Perspectives on the theory and practice of belief functions. *International Journal of ApproximateReasoning*, 4:323–362, 1990. 20

[101] Takashi Shibata. The impacts of uncertainties in a real options model under incomplete information. *European Journal of Operational Research*, 187:1368–1379, 2008. 27

[102] David C. Shimko. *Finance in Continuous Time: A Primer*. Kolb Publishing, 1992. 29

[103] Han T.J. Smit. Infrastructure investment as a real options game: The case of European airport expansion. *Financial Management;*, 32(4):27–57, 2003. 23, 84

[104] Han T.J. Smit and Lenos Trigeorgis. *Strategic Investment: Real Options and Games*. Princeton University Press, 2004. 84

[105] James E. Smith and Kevin F. McCardle. Valuing oil properties: Integrating option pricing and decision analysis approach. *Operations Research*, 46(2):198–217, 1998. 23, 27, 28, 39, 51

[106] James E. Smith and Robert F. Nau. Valuing risky projects: option pricing theory and decision analysis. *Management Science*, 41(5):795–816, May 1995. iii, 3, 26, 27, 28

[107] Qian Song and Abraham Kandel. A fuzzy approach to strategic games. *IEEE Transactions on Fuzzy Systems*, 7:634–642, December 1999. 86

[108] Tetsuzo Tanino. Fuzzy preference orderings in group decision making. *Fuzzy Sets and Systems*, 12:117–131, 1984. 86

[109] James M. Tien and Pascal J. Goldschmidt-Clermont. Health care: A complex service system. *Journal of Systems Science and Systems Engineering*, 18(3):257–282, September 2009. 20

[110] Ashutosh Tiwari, Kostas Vergidis, and Kevin Y. Kuo. Computer assisted decision making for new product introduction investments. *Computers in Industry*, 59:96–105, 2008. 14

[111] USEPA. The effects of environmental hazards and regulation on urban redevelopment. Technical Report EPA 06542-003-00, USEPA, Washington DC, USA, 1997. 8, 12

[112] USEPA. Characteristics of sustainable brownfields projects. Technical Report EPA500-R-98-001G, USEPA, 1998. 10, 12

[113] John von Neumann and Oskar Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, NJ, 1944. 74

[114] Sean B. Walker, Terry Boutilier, and Keith W. Hipel. Systems management study of a private brownfield renovation. *Journal of Urban Planning and Development*, 136(3):246–260, 2010. 99

[115] Fangju Wang and G. Brent Hall. Fuzzy representation of geographical boundaries in gis. *International Journal of Geographical Information Systems*, 10(5):573–590, 1996. 58, 64, 65

[116] Juite Wang and W.-L. Hwang. A fuzzy set approach for r&d portfolio selection using a real options valuation model. *Omega*, 35:247–257, 2007. 28

[117] Qian Wang. Assessment of the emission trading policy: A case study for the acid rain program in the United States. Master's thesis, University of Waterloo, 2004. 11

[118] Qian Wang, Keith W. Hipel, and D. Marc Kilgour. Fuzzy real options in brownfield redevelopment evaluation. *Journal of Applied Mathematics and Decision Sciences*, 2009:16, 2009. 4, 36, 47, 51

[119] Qian Wang, Keith W. Hipel, and D. Marc Kilgour. A numerical method of evaluating brownfields using fuzzy boundaries and fuzzy real options. In *Proceedings of The 2010 IEEE International Conference on Systems, Man and Cybernetics*, 2010. 4, 63

[120] Qian Wang, K.W. Hipel, and D.M. Kilgour. Conflict analysis in brownfield redevelopment: The ERASE program in Hamilton, Ontario. In *Proceedings of The IEEE International Conference on Sysytem, Man and Cybernetics*, pages 2913–2918, 2008. 9, 11, 76

[121] Qian Wang, K.W. Hipel, and D.M. Kilgour. Using fuzzy real options in a brownfield redevelopment decision support system. In *Proceedings of The IEEE International Conference on Sysytem, Man and Cybernetics*, pages 1545–1550, 2009. 4, 30, 47, 102, 105

[122] Yingming Wang, Jianbo Yang, and Kwaisang Chin. The evidential reasoning approach for multiple attribute decision analysis using interval belief degrees. *European Journal of Operational Research*, 175(1):35–66, 2006. 20

[123] Zhen Wang, John Lin, and Xiaojia Guo. Negotiator satisfaction in nss-facilitated negotiation. *Group Decision and Negotiation*, 19:279–300, 2010. 101

[124] Richard L. Wheeden and Antoni Zygmund. *Measure and Integral: An Introduction to Real Analysis*. CRC Press, 1977. 32

[125] Haiyan Xu, Keith W. Hipel, and D. Marc Kilgour. Matrix representation of solution concepts in multiple-decision-maker graph models. *IEEE Transactions on Systems, Man and Cybernetics - Part A: Systems and Humans*, 39(1):96–108, January 2009. 80, 82, 113

[126] Haiyan Xu, D. Marc Kilgour, and Keith W. Hipel. Matrix representation of solution concepts in graph models for two decision-makers with preference uncertainity. *Dynamics of Continuous, Discrete and Impulsive Systems, Supplement on Advances in Neural Networks - Theory and Applications*, 14:703–707, 2007. 80

[127] Haiyan Xu, D. Marc Kilgour, and Keith W. Hipel. Matrix representation of conflict resolution in multiple-decision-maker graph models with preference uncertainty. *Group Decision and Negotiation*, page Published Online, 2010. 86

[128] Haiyan Xu, Kevin W. Li, Keith W. Hipel, and D. Marc Kilgour. A matrix approach to status quo analysis in the graph model for conflict resolution. *Applied Mathematics and Computation*, 212(2):470480, 2009. 83

[129] Zeshui Xu. An overview of methods for determining owa weights. *International Journal of Intelligent Systems*, 20:843–865, 2005. 60

[130] Zeshui Xu. A survey of preference relations. *International Journal of General Systems*, 36:179–203, 2007. 87

[131] Ronald R. Yager. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man, and Cybernetics*, 18(1):183–190, 1988. 60

[132] Ronald R. Yager. Multi-agent negotiation using linguistically expressed mediation rules. *Group Decision and Negotiation*, 16(1):1–23, 2007. 58, 59, 60, 62

[133] Yuji Yoshida, Masami Yasuda, Jun-ichi Nakagami, and Masami Kurano. A new evaluation of mean value for fuzzy numbers and its application to American put option under uncertainty. *Fuzzy Sets and Systems*, 157:2614–2626, 2006. 31

[134] Soonyoung Yu. *An Optimal Framework of Investment Strategy in Brownfields Redevelopment by Integrating Site-Specific Hydrogeological and Financial Uncertainties.* PhD thesis, University of Waterloo, 2009. 14, 63, 64, 100

[135] Lotfi A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965. 20

[136] Lotfi A. Zadeh. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, 1:3–28, 1978. 20

[137] Lotfi A. Zadeh. Precisiated natural language. *Artificial Intelligence Magazine*, 25(3):74–91, 2004. 20, 61

[138] Michael Zeiler. *Modeling Our World*. ESRI Press, 2010. 64, 65