Toward a unified global regulatory capital framework for life insurers

by

Ishmael Sharara

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Actuarial Science

Waterloo, Ontario, Canada, 2010

© Ishmael Sharara 2010
I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
Abstract

In many regions of the world, the solvency regulation of insurers is becoming more principle-based and market oriented. However, the exact forms of the solvency standards that are emerging in individual jurisdictions are not entirely consistent. A common risk and capital framework can level the global playing field and possibly reduce the cost of capital for insurers. In the thesis, a conceptual framework for measuring the insolvency risk of life insurance companies will be proposed. The two main advantages of the proposed solvency framework are that it addresses the issue of incentives in the calibration of the capital requirements and it also provides an associated decomposition of the insurer’s insolvency risk by term. The proposed term structure of insolvency risk is an efficient risk summary that should be readily accessible to both regulators and policyholders. Given the inherent complexity of the long-term guarantees and options of typical life insurance policies, the term structure of insolvency risk is able to provide stakeholders with more complete information than that provided by a single number that relates to a specific period. The capital standards for life insurers that are currently existing or have been proposed in Canada, U.S., and in the EU are then reviewed within the risk and capital measurement framework of the proposed standard to identify potential shortcomings.
Acknowledgements

I would like to thank my supervisors, Prof Mary Hardy and Prof David Saunders, for the help and support that they provided to me during a very challenging time. I also want to acknowledge the other members of the PhD committee: Thanks to Prof Rob Brown for making those long trips to attend the defenses! Prof Hyuntae Kim for challenging me to look deeper into my work and Prof James Thompson for the interesting read, it was worthwhile! I also want to thank Dr. Allen Brender for making himself available and for the helpful comments that he provided.

Last but not least, I want to say a special thanks to Mary Lou Dufton who helped me in numerous ways throughout the course of my program.
Dedication

Ecclesiastes 9:11

I returned, and saw under the sun, that the race [is] not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill; but time and chance happeneth to them all.

I dedicate this thesis to God and my young family. Thank you all for being very patient and fair with me.
# Table of Contents

List of Tables .................................................. x

List of Figures .................................................. xi

1 Introduction .................................................. 1

1.1 Background ................................................. 1

1.1.1 Overview ................................................ 1

1.1.2 U.S. Risk Based Capital .............................. 3

1.1.3 Canadian MCCSR ........................................ 4

1.1.4 Solvency II ............................................... 5

1.2 Thesis motivation .......................................... 8

1.2.1 Why regulate life insurers? ........................... 8

1.2.2 The measurement problem of ruin theory .......... 10

1.3 Accomplishments of thesis ............................. 18

1.4 Outline of thesis .......................................... 21

2 A conceptual framework for a unifying global capital standard .... 22

2.1 Introduction ............................................... 22

2.2 An asset-liability model for a life insurer ................ 23
<table>
<thead>
<tr>
<th>2.2.1 Model definition</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.2 Market value of assets and liabilities</td>
<td>24</td>
</tr>
<tr>
<td>2.2.3 Solvency capital rules</td>
<td>26</td>
</tr>
<tr>
<td>2.2.4 Term structure of insolvency risk</td>
<td>27</td>
</tr>
<tr>
<td>2.2.5 Framework for policyholder-oriented risk management incentives</td>
<td>29</td>
</tr>
<tr>
<td>2.3 Outline of the proposed regulatory capital framework</td>
<td>30</td>
</tr>
<tr>
<td>2.3.1 GFAs for asset and liability valuation</td>
<td>32</td>
</tr>
<tr>
<td>2.3.2 GFAs for regulatory capital</td>
<td>37</td>
</tr>
</tbody>
</table>

### 3 A review of the MCCSR, US RBC and Solvency II formulas

#### 3.1 Overview

<table>
<thead>
<tr>
<th>3.2 A benchmark review of the standard formulas</th>
<th>53</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.1 Definition of terms</td>
<td>53</td>
</tr>
<tr>
<td>3.2.2 Market-valuation based balance sheet</td>
<td>55</td>
</tr>
<tr>
<td>3.2.3 Capital requirements</td>
<td>62</td>
</tr>
<tr>
<td>3.3 Chapter conclusion</td>
<td>68</td>
</tr>
</tbody>
</table>

### 4 Proposal of an ALM risk measurement framework

#### 4.1 Overview

<table>
<thead>
<tr>
<th>4.1.1 Motivation</th>
<th>69</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.2 Outline of chapter</td>
<td>71</td>
</tr>
<tr>
<td>4.2 Model insurer</td>
<td>72</td>
</tr>
<tr>
<td>4.2.1 Model insurance portfolio</td>
<td>72</td>
</tr>
<tr>
<td>4.2.2 Assumed investment strategies of the model insurer</td>
<td>73</td>
</tr>
<tr>
<td>4.3 Definitions of supervisory target capital</td>
<td>76</td>
</tr>
<tr>
<td>4.4 A numerical comparison of the proposed ALM risk capital requirements</td>
<td>80</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Sample distribution of supervisory target capital</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Insolvency risk profile implied by supervisory target capital</td>
</tr>
<tr>
<td>4.4.3</td>
<td>What risk is being measured?</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Term structure of ALM risk</td>
</tr>
<tr>
<td>4.4.5</td>
<td>An incentive-based review of proposed capital standards</td>
</tr>
<tr>
<td>4.5</td>
<td>The impact of model risk on relative risk and capital assessments</td>
</tr>
<tr>
<td>4.6</td>
<td>Chapter conclusion</td>
</tr>
<tr>
<td>5</td>
<td><strong>Policy recommendations and future research</strong></td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>5.2</td>
<td>Market discipline</td>
</tr>
<tr>
<td>5.3</td>
<td>Proposed Canadian and Solvency II capital standards</td>
</tr>
<tr>
<td>5.4</td>
<td>US PBA capital standard</td>
</tr>
<tr>
<td>5.5</td>
<td>Other potential applications of thesis results</td>
</tr>
<tr>
<td>5.6</td>
<td>Conclusion and future research</td>
</tr>
<tr>
<td>Appendices</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td><strong>Data and formulas for Chapter 3 calculations</strong></td>
</tr>
<tr>
<td>A.1</td>
<td>Model portfolio and valuation assumptions</td>
</tr>
<tr>
<td>A.2</td>
<td>MCCSR formula</td>
</tr>
<tr>
<td>A.3</td>
<td>U.S. RBC formula</td>
</tr>
<tr>
<td>A.4</td>
<td>Solvency II standard formula</td>
</tr>
<tr>
<td>B</td>
<td><strong>Assumptions for Chapter 4 calculations</strong></td>
</tr>
<tr>
<td>B.1</td>
<td>Mortality table assumption</td>
</tr>
<tr>
<td>B.2</td>
<td>Initial interest rates</td>
</tr>
</tbody>
</table>
# List of Tables

4.1 Fisher-Weil duration and convexity risk measures ............... 75  
4.2 A summary of the proposed capital requirements ............... 77  
4.3 Initial balance sheet of the model insurer ..................... 82  
4.4 Correlations of scenario total balance sheet requirements .......... 91  
4.5 Required capital for alternative investment strategies ............ 96  
4.6 Incentive structure of proposed capital standards ............... 108  

A.1 U.S. dollar swap rates from December 31, 2000 to December 31, 2008 126  
A.2 Solvency valuation assumptions on December 31, 2008 ............ 127  
A.3 MCCSR factors for C-3 risk .................................. 130  
A.4 U.S. RBC factors for C-2 risk ................................ 131  

B.1 1986-92 Canadian CIA basic male, ultimate, non-smoker, nearest age 139  
B.2 Interest rates on September 30, 2009 .......................... 140
List of Figures

3.1 Total balance sheet requirements ........................................ 57
3.2 Free capital of the model insurer ......................................... 58

4.1 Scenario Total Balance Sheet Requirements .......................... 81
4.2 Insolvency risk profile implied by supervisory target capital ....... 84
4.3 Incremental capital charge for balance sheet solvency .............. 86
4.4 Risky scenario set for U.S. PBA ........................................ 88
4.5 Risky scenario set for Solvency II .................................... 89
4.6 Measured risk exposure of investment strategies .................... 94
4.7 Measured risk exposure of investment strategies (cont’d) ......... 95
4.8 Tail-dependencies of alternative measures of (long-bond) ALM risk . 100
4.9 Tail-dependencies of alternative measures of (short-bond) ALM risk . 101
4.10 Marginal risk contributions of the long-bond strategy by period ... 104
4.11 Marginal risk contributions of the short-bond strategy by period ... 105
4.12 Reinvestment risk of the short bond strategy ......................... 111

B.1 C3 Phase III scenarios: sample key rates ............................ 143
Chapter 1

Introduction

1.1 Background

1.1.1 Overview

The subject matter pertaining to the solvency regulation of financial institutions has become very topical in recent years. The changes to the solvency regulation of financial institutions that are occurring globally are in response to factors such as the need to level the global playing field, the increase in complexity of products, the globalization of insurance and financial capital markets, and the need to prevent regulatory arbitrage across sectors of the financial industry. Additionally, the recent global financial crisis has proven to be a significant catalyst for advancing the crucial debate on the regulation of financial institutions, especially those deemed to be systemically important.

The Basel Committee on Banking Supervision (BCBS) which effectively sets global prudential bank standards issued the first Basel capital accord in 1988 (BIS
(1988)). To date, there have been two further iterations of the original accord (BIS (2006, 2010)) to address perceived shortcomings in the global bank capital rules. The International Association of Insurance Supervisors (IAIS) and International Actuarial Association (IAA) have done considerable work on developing a global standard for insurer solvency assessment (e.g. Sandstrom (2006); IAIS (2002, 2005, 2007); IAA (2004)). The International Accounting Standards Board (IASB) and U.S. Financial Accounting Standards Board (FASB) are collaborating to develop a single global standard for insurance contracts. This collaboration has recently resulted in the issuance of an exposure draft of the proposed International Financial Reporting Standard (IFRS) for insurance in July 2010. The main elements of the proposed insurance accounting standard are summarized in IASB (2010). In contrast to the accounting model that was described in the previously issued discussion paper (IASB (2007)), the exposure draft values insurance contracts using a fulfilment cost method rather than exit value. Insurance liabilities that are outlined in the exposure draft include deferred amounts of profit that result from a calibration to premium at inception. The measured contract liability is therefore not economic. The final IFRS for insurance is not expected to be issued until June 2011. The importance of having harmonized insurance accounting bases for public financial and regulatory reports is noted in IAA (2004).

Individual jurisdictions have also undertaken projects to modernize their solvency standards for insurers. Brief descriptions of the developments that are occurring in the U.S., Canada and EU are provided next.
1.1.2 U.S. Risk Based Capital

Equation (1.1) shows the U.S. RBC formula for life insurers, as described in NAIC (2008).

\[ \rho(S_{US}) = \rho(X_0) + \rho(X_{4a}) \]
\[ + (\sqrt{\rho(X_{1cs}) + \rho(X_{3c})})^2 + \rho(X_2)^2 + \rho(X_{3b})^2 + \rho(X_{4b})^2 + (\rho(X_{1o}) + \rho(X_{3a}))^2 \]

where

• \( \rho(S_{US}) \) is the Total-Risk Based Capital After Covariance

• \( \rho(X_0) \): Asset risk-affiliates

• \( \rho(X_{1cs}) \): Unaffiliated common stock and affiliated noninsurance common stock components

• \( \rho(X_{1o}) \): Asset risk-other (excluding common stock)

• \( \rho(X_2) \): Insurance risk

• \( \rho(X_{3a}) \): Interest rate risk

• \( \rho(X_{3b}) \): Health credit risk

• \( \rho(X_{3c}) \): Market risk (variable products)

• \( \rho(X_{4a}) \): Business risk-premium and liability components

• \( \rho(X_{4b}) \): Business risk-health administrative risk
As illustrated in Equation (1.2), capital for a given risk $X_i$ is generally calculated as the product of a factor $f_i$ and a measure of exposure $E_i$ that is derived from amounts in the audited financial statements. However, there are exceptions.

$$\rho(X_i) = f_i \times E_i$$ (1.2)

The C-3 capital amounts for certain interest sensitive and variable annuity products are determined using stochastic simulation (e.g. AAA (2002) and AAA (2003)). The outcome of the C-3 Phase 3 (AAA (2008a)) project was a proposed calculation methodology for determining the C-3 risk of all life insurance products using stochastic simulation. A principle-based reserving methodology for all life insurance and annuity products has also been finalized.

### 1.1.3 Canadian MCCSR

The formula for the Canadian MCCSR is shown in Equation (1.3) (OSFI (2010b)).

$$\rho(S_{CAN}) = \rho(X_1) + \rho(X_2) + \rho(X_3) + \rho(X_4) + \rho(X_5)$$ (1.3)

where

- $\rho(S_{CAN})$ is the Canadian MCCSR
- $\rho(X_1)$ is asset default or C-1 risk
- $\rho(X_2)$ is insurance risk which includes mortality, morbidity and lapse risks
- $\rho(X_3)$ is the change in interest rate environment or C-3 risk
- $\rho(X_4)$ is segregated funds (variable annuities in the US) risk i.e. the risk of loss arising from guarantees embedded in segregated funds
• $\rho(X_5)$ is foreign exchange risk

Capital amounts for individual risks are simply summed to get the total amount. There is therefore no recognition of diversification among the risk classes that underly the MCCCSR formula since the risks are effectively assumed to be perfectly correlated. As in the case of U.S. RBC, capital for individual risks is generally determined as the product of a factor and a measure of exposure that is obtained from the annual financial statement. However, the amount of capital with respect to segregated funds risk can be determined using internal models (OSFI (2010b)).

All public insurers in Canada have started to report under IFRS as of January 1, 2011. With the expected finalization of Phase 2 of the insurance accounting project in June 2011, preparations are underway for the adoption of the new standard. Since regulatory reports are based on public financial reports in Canada, the adoption of IFRS requires that changes be made to the current MCCCSR framework in anticipation of the new insurance accounting standard. There are also other reasons why the Office of the Superintendent of Financial Institutions (OSFI) has been reviewing its capital adequacy framework for life insurers. Among these are the increase in actuarial and risk management expertise allowing more sophisticated methods to be used and the need to consider changes that are currently occurring in other jurisdictions (JCOAA (2008a)). Background information on the proposed internal model framework is summarized in Table 4.2 of Chapter 4. For more detailed information, refer to JCOAA (2008a,b); MAC (2007), for example.

1.1.4 Solvency II

Solvency II is the new solvency standard that will apply to insurers that operate in the European Union. A comprehensive history of the evolution of Solvency II is provided
in Sandstrom (2006). Solvency I, the predecessor to Solvency II, relies on measures of risk that are not sufficiently risk-sensitive because they are essentially volume-based measures (Eling et al. (2007)). An example of a volume-based measure is the determination of capital as the product of the written premium and a fixed percentage factor. In this case, the amount of capital simply depends on the written premium and does not differentiate between risk profiles. Another shortcoming of Solvency I is that it did not consider asset risks (Linder and Ronkainen (2004)). The current design and calibration of Solvency II can be reviewed in the supporting documentation for the recently conducted fifth quantitative impact study (CEIOPS (2010)). The design of Solvency II is modeled after the three pillar structure of the Basel II capital accord for banks (BIS (2006)).

The three pillars are:

1. Quantitative capital requirements (Pillar 1)
2. Supervisory review (Pillar 2)
3. Market discipline (Pillar 3)

A combination of scenarios, stress tests and factors are used to calculate the capital amount for individual risk categories (i.e. $\rho(X_i)$ for $i=1..N$) under the Solvency II standard formula. The scenarios, stress tests and factors are calibrated to a 0.5% probability of ruin over a one-year period. For example, Equation (1.4) shows the calculation of the capital charge for mortality risk $\rho(X_{mort})$.

$$\rho(X_{mort}) = \sum_l (\Delta NAV|mortality)$$

where $l$ denotes each policy subject to mortality risk, $mortality$ is a permanent increase of 15% (based on QIS 5 calibration) applied to mortality rates at every age
and $\Delta NAV$ is the change in the insurer’s net asset value (i.e. assets minus liabilities) given the mortality shock.

The capital amounts for individual risks that have been determined under the Solvency II standard formula are typically aggregated using Equation (1.10).

\[
\rho(S_{EU}) = \sqrt{\sum_i \sum_j \phi(i,j) \cdot \rho(X_i) \cdot \rho(X_j)} \tag{1.5}
\]

where

- $\rho(X_i)$ is the solvency capital requirement for risk $i$.
- $\phi(i,j)$ denotes the (tail) correlation between risks $i$ and $j$.

A given insurer can calculate its required capital amount using a standard formula, partial internal model or fully internal model. Under the internal model approach, an insurer can use an internal model to determine the capital requirement subject to specific calibration standards in addition to a “use test”. The second pillar of the Solvency II framework is the means by which the regulator promotes good governance and risk management practices of the insurer. Under the third pillar, the insurance supervisor encourages the provision of timely and relevant information on the insurer’s operations to market participants so that they can monitor the insurer effectively. The design of Solvency II is also very similar to that of the Swiss Solvency Test (SST) (see Sandstrom (2006)). In particular, they both determine capital by applying an appropriately calibrated shock to a risk factor and measuring the impact on the net assets of the insurer using a market-valuation balance sheet. The notable differences between the SST and Solvency II include the SST’s use of scenarios to model abnormal losses.
1.2 Thesis motivation

In many regions of the world, the solvency regulation of insurers is becoming more principle-based and market oriented. However, the exact nature of the solvency standards that are emerging in individual jurisdictions are not entirely consistent, as has been suggested above in the case of the U.S. RBC, MCCSR and Solvency II. A common risk and capital framework can level the global playing field and possibly reduce the cost of capital for insurers. As has been noted in the introduction, the IAIS and IAA have done a lot of work in developing a global framework for insurer solvency assessment. For example, see IAA (2004); IAIS (2002, 2005, 2007). To date, they have outlined principles for a global capital framework and issued standards and guidelines to promote the goal of convergence in insurer solvency assessment among their members.

The primary goal of this thesis is to propose an insolvency risk and capital measurement framework that can be used as benchmark standard for life insurers. The arguments that will be used to justify the proposed framework will go beyond the typical considerations of actuarial ruin-theory. As such, the proposed risk and capital measurement framework should be relatively more robust.

1.2.1 Why regulate life insurers?

Unlike in banking, the failure of an insurer would generally not result in a contagion effect across the industry. According to Cummins et al. (1993), solvency regulation (for insurers) should be designed to duplicate as closely as possible the outcome of a competitive market in which all parties have access to all relevant information. In particular, solvency regulation should address the agency problem that
is created by the information asymmetry between the firm owners and policyholders/debtholders. This is necessary since policyholders are “...dispersed and insufficiently informed; none of them (individually) has enough incentive to spend time, energy, and/or financial resources in monitoring the management of her insurance company” (Plantine and Rochet (2007)). Accordingly, Plantine and Rochet (2007) suggest a “banker model” of insurance regulation in which the regulator’s role is limited to the effective representation of the policyholders in the corporate governance structure of the insurer. Consistent with the banker model of regulation, Doff (2008) also states that “..the focus of an insurance supervision framework should be to decrease information asymmetries and to align incentives for policyholders and the insurance company”.

The following factors tend to make life insurers especially prone to insolvency risk:

1. Insurers generally have ample liquidity even in times of financial distress since they typically receive premiums well in advance of having to pay claims to policyholders. The liquidity risk profile of life insurers also differs from that of banks since life insurance and annuity contracts tend to be long-term, and penalties and/or other fees are usually assessed for early withdrawal or cancelation of the policies. The availability of ample liquidity means an insurer is able to continue in business even if it is losing money as long as its management is able to conceal current losses in the income statement by understating reserves for example. The result is that troubled insurers will usually come to the attention of the regulators and the market when the likely losses for policyholders are severe (Plantine and Rochet (2007)).

2. Life insurance products generally have long-term guarantees and options that are difficult to assess in terms of risk. The notable collapse of Equitable Life in
the UK was partly due to inadequately priced options in the pension annuity portfolio (e.g. IAA (2004)). In Eling (2010), insurance business model and product complexity are suggested as possible reasons why market participants are not able to effectively monitor the insurer’s risk taking.

Based on the preceding discussion, solvency regulation is essentially required since the acquisition of information (e.g. assessment of willingness and ability to pay claims of insurer) by policyholders is costly (e.g. Eling et al. (2007); Cummins et al. (1993)). As a means to mitigate the information asymmetry between the firm owners and policyholders, a very accessible and efficient risk summary for life insurers will be proposed in the thesis.

The disadvantages of stringent solvency regulation include unintended consequences such as unnecessarily high insurance prices and the squeezing out of small insurers from the market (e.g. Eling et al. (2007); Cummins et al. (1993)).

1.2.2 The measurement problem of ruin theory

In general, the notion that risk can be measured accurately can incentivise management to engage in risky behaviour that is not properly accounted for under the capital standard either because it is too difficult to assess, or because of model error.

Ruin-theory based capital requirements are determined such that they limit the probability of insurer failure to some very low level that is acceptable to the supervisor. For an introduction and overview of ruin-theory, see Klugman et al. (1998). The existing and proposed capital standards for life insurers all tend to be based on ruin-theory (e.g. MCCSR, US RBC, and Solvency II). However, there are significant implementation challenges associated with a ruin-theoretic capital adequacy framework. Plantine and Rochet (2007) discuss several practical and conceptual lim-
itations of ruin-theory. From a theoretical standpoint, the failure of ruin-theory to account for the response of the market to capital requirements is a particularly notable limitation that they cited. They also note that the results of several studies (e.g. Cummins et al. (1995)) suggest that RBC type formulas are not good predictors of insurer failure. The computational challenges of ruin theory will be further discussed in the next sections.

In IAA (2004), the International Actuarial Association (IAA) endorsed the three pillar structure of Basel II for insurance solvency supervision. The three pillar structure was discussed in Section 1.1 in the context of Solvency II. Within the three pillar solvency regulation framework, the pillar 1 capital requirements are viewed as a buffer to protect against residual risks that were not adequately dealt with under pillars 2 and 3. For this reason, capital requirements are termed the “last line of defence” in IAA (2004). Accordingly, we can describe the measurement of pillar 1 capital requirements within a three pillar framework by Equation (1.6).

\[ \rho_{p1}(X) = \rho(X) - \Delta \rho_{p2}(X) - \Delta \rho_{p3}(X) \]  

where \( \rho(X) \) is the appropriate capital amount in the absence of any pillar 2 and pillar 3 risk mitigating effects i.e. \( \Delta \rho_{p2}(X) \) and \( \Delta \rho_{p3}(X) \) respectively. \( \rho_{p1}(X) \) is the pillar 1 capital requirement with respect to residual risks only i.e. after considering the risk mitigating effects of pillar 2 and pillar 3.

Given the qualitative and/or subjective nature of pillars 2 and 3, the measurement problem of ruin theory can only be compounded within a three pillar regime.

Eling (2010) defines market discipline in the context of insurance as “the ability of customers, investors, and intermediaries to monitor and influence the management of insurance companies”. The results in Eling (2010) suggest that the extent
of market discipline in life insurance is very limited. One reason for the ineffectiveness of market-discipline is due to the complexity of the products that are offered by life insurers which makes them very difficult to understand by stakeholders. Additionally, from the viewpoint of prudential regulation, it is worthwhile to ask to what extent the market-discipline that can be expected to exist in a given insurance market is “policyholder-oriented”. That is, does the market-discipline properly reflect the regulator’s goal of protecting policyholders? The significant impediments to policyholder-oriented market discipline that exist in life insurance imply that the effectiveness of the third pillar is probably very limited. For reasons already mentioned, it is likely that the ability of policyholder discipline to be effective will remain hampered even in the presence of increased disclosures by life insurers.

Pillar 2 is the supervisory effort in promoting good corporate governance and risk management practices (e.g. stress testing, asset-liability management) by the insurer. The inclusion of corporate governance and risk management in a solvency regulation framework was a key recommendation of the “Sharma report” (Sharma (2002)). Under pillar 2, larger insurers are likely to be using economic capital models and sophisticated ERM systems to manage their risk. Generally speaking, insurers using internal models to determine regulatory capital would be required to ensure consistency with their economic capital models. The recent global financial crisis showed that the sophisticated ERM systems that had been built by the banks were inadequate. The supervisor needs to be confident that the models being used are appropriate from the legislated viewpoint of safeguarding the policyholders’ interest. The verification problem of internal economic capital models led Plantine and Rochet (2007) to not advocate their use in the determination of regulatory capital. Eling et al. (2007) cite the possibility of “model arbitrage” as another disadvantage of using internal models.
for regulatory capital purposes. In fact, based on their overall analysis, including review of available literature on solvency, Eling et al. (2007) conclude that risk-based capital models should only be used as guidelines rather than strict requirements since they have limited predictive utility.

Further, the extent to which the regulator is able to provide effective monitoring in the design and application of the insurer’s complex risk management models is questionable at best. The global financial crisis revealed that even rating agencies, who are paid to provide similar monitoring, were not able to properly assess the risk of securities in which financial institutions had an interest. Given that the problem of insurer insolvency is really about bad (and or dishonest) management in many situations, and the information asymmetry that generally exists between regulators and the company’s management, the potential rewards of the regulator’s effort under pillar 2 can be limited.

In conclusion, the accurate application of Equation (1.6) can be seen to be an extremely challenging endeavour.

**Quantitative capital requirements (Pillar 1)**

To further our understanding of the challenges involved in the implementation of a ruin-theory based capital framework, let’s consider the following example. Assume that a given insurer is faced with individual risk exposures $X_i$ for $i=1,\ldots,N$. For example, $i$ could be either of market, credit, insurance or operational risks. Assume further that each

$$X_i = \mu_i (1 + Z_i \nu_i) \quad (1.7)$$

where $\mu_i$ is the mean, $\nu_i = \frac{\sigma_i}{\mu_i}$ is the coefficient of variation of $X_i$ and $Z_i$ is the standardized random variable corresponding to $X_i$. The total risk exposure of the
insurer is then given by

\[ S = X_1 + X_2 + \ldots + X_N = \sum_{i=1}^{N} \mu_i (1 + Z_i \nu_i) \]  

(1.8)

To define a capital requirement based on the given insurer’s risk profile, an appropriate risk measure must be specified.

A risk measure is defined as any mapping from a random variable to the real number line, Jorion (2005). The purpose of a risk measure is to summarise the entire risk distribution \( X \) by one number \( \rho(X) \). Examples of popular risk measures include the standard deviation principle, Value at Risk (VaR) and Conditional Tail Expectation (Tail VaR). Desirable properties of a risk measure for capital adequacy will be explored in Chapter 2. The overall capital requirement for the given insurer can be expressed as:

\[ \rho(S) = \mu_S (1 + \kappa_S \nu_S) \]  

(1.9)

where \( \mu_S = \sum_{i=1}^{N} \mu_i, \sigma_S = \sigma(\sum_{i=1}^{N} X_i), \nu_S = \frac{\sigma_S}{\mu_S} \) is the coefficient of variation of \( S \) and \( \kappa_S \) is a parameter that depends on the compound distribution \( S \) and the solvency standard. For example, if the regulatory solvency standard is calibrated to a 1% probability of ruin and \( S \) is normally distributed, then \( \kappa_S = \Phi^{-1}(0.99) \) where \( \Phi^{-1} \) is the inverse standard normal cumulative distribution function.

**Standardized solvency models**

The U.S. RBC, Canadian MCCSR, and the Solvency II standard formulas have been discussed in previous sections. Under the Canadian MCCSR or U.S. RBC formula, capital for a given risk \( X_i \) is generally calculated as the product of a factor and a measure of exposure that is derived from audited financial statements as shown
in Equation (1.2). For example, the MCCSR capital buffer for the default risk of
$100 million of AA-rated bonds is calculated by substituting $E_i = $100 million and
$f_i = 0.5\%$ (i.e. the 2010 MCCSR factor). By definition, the calibration of a stan-
dardized solvency model (e.g. factors $f_i$ in Equation (1.2)) is conservative since it
is meant to cover the tail-risk profile of insurers that operate in a given market or
industry. In a heterogenous insurance market, the conservative calibration of the stan-
dard model implies that it will not accurately portray the economic risk exposures
of a given insurer, especially one that is on the lower-end of the risk-profile spec-
trum. Additionally, volume-based risk measures such as those derived, for example,
by multiplying premium volume by a fixed factor provide misaligned incentives for
prudent risk management. If inappropriate incentives are to be avoided, an insurer’s
regulatory capital should not deviate significantly from its economic capital.

The U.S. RBC, Canadian MCCSR and Solvency II standard formulas use dif-
f erent aggregation techniques to approximate the overall capital requirement of the
insurer $\rho(S)$ from the capital requirements of the individual risk categories ($\rho(X_i)$
for $i=1..N$). The U.S. RBC capital amounts are aggregated using Equation (1.1) (see
NAIC (2008)). Equation (1.1) effectively assumes correlations of either 0 or 1 between
categories of risk. On the other hand, the capital requirements for individual risk
categories under the Canadian MCCSR are aggregated using Equation (1.3) (OSFI
(2008)). By simply summing up the capital requirements for individual risks, the
Canadian MCCSR effectively assumes that the risks $X_i$ are perfectly correlated. The
prescribed correlation matrix approach that is used under the Solvency II standard
formula to combine risks at each aggregation level is summarized by Equation (1.10)
(for example, see CEIOPS (2010)). The calibration of the correlations $\phi(i, j)$ is such
that they produce an overall capital requirement $\rho(S_{EU})$ at the 99.5% confidence level
over a one year period. Equation (1.10) can be shown to produce correct results in the case of linear correlations only when the underlying distributions $X_i$ are elliptical, e.g. multivariate normal. However, many loss distributions in insurance are skewed, as are credit, market and operational risks.

$$\rho(S_{EU}) = \sqrt{\sum_i \sum_j \phi(i, j) \cdot \rho(X_i) \cdot \rho(X_j)}$$ (1.10)

where

- $\rho(X_i)$ is the solvency capital requirement for risk $i$.
- $\phi(i, j)$ denotes the (tail) correlation between risks $i$ and $j$.

The US RBC, MCCSR and the Solvency II standard formulas use different approaches to combine risks. This is a reflection of the inherent difficulty in modelling dependency.

**Internal solvency models**

The solvency regulation of insurers in many jurisdictions is moving toward more principle-based regimes for reasons cited in the introductory paragraph. In particular, insurers are for the first time being given the opportunity to use internal models to determine regulatory capital, generally subject to specific calibration standards and a “use test”. The idea of using an institution’s internal model for determining regulatory capital was introduced in a 1996 market-risk amendment to the Basel II accord (BIS (1996)). An internal model framework allows a better alignment of a financial institution’s regulatory capital with economic capital by giving the
institution more control over the definition and calibration of the parameters of the regulatory capital model. In Hardy (1993) a compelling argument for the use of stochastic modeling in life insurer solvency assessment was presented. In the analysis that was conducted, both relative and absolute solvency risk assessments of several life offices with different risk profiles were shown to be potentially misleading when they are conducted using deterministic scenarios rather than stochastic scenarios.

For example, an insurer might use the method of copulas to model the dependencies among the risk categories $X_i$ rather than a simpler approach underlying a given solvency standard.

**Definition 1. Embrechts et al. (2001)**

An $N$-dimensional copula is a function $C$ with domain $[0,1]^N$ such that

1. $C$ is grounded and $n$-increasing.

2. $C$ has margins $C_k$, $k=1,\ldots,N$, which satisfy $C_k(u) = u$ for all $u$ in $[0,1]$.

Copulas are a useful alternative to linear correlation in dependency modeling since they can be chosen and calibrated to more accurately capture the assumed joint tail-behavior of the risk factors, which could be very different from their behaviour under normal situations. Sklar’s Theorem (see Embrechts et al. (2001)) as restated below demonstrates the usefulness of the copula concept in risk management.

**Theorem 1. (Sklar’s Theorem)** Let $H$ be an $N$-dimensional distribution function with margins $F_1,\ldots,F_N$. Then there exists an $N$-copula $C$ such that for all $x$ in $R^N$, $H(x_1,\ldots,x_N) = C(F_1(x_1),\ldots,F_N(x_N))$.

If $F_1,\ldots,F_N$ are all continuous, then $C$ is unique; otherwise $C$ is uniquely determined on $Ran F_1,\ldots,Ran F_N$. Conversely, if $C$ is an $N$-copula and $F_1,\ldots,F_N$ are
distribution functions, then the function $H$ defined above is an $N$-dimensional distribution function with margins $F_1, \ldots, F_N$.

Using the technique of copulas, the joint distribution of the component risks $H(X_1, \ldots, X_N)$ which is required to calculate $\rho(S)$, is modeled by separately specifying the distribution of the marginals ($F_i$, $i=1,\ldots,N$) and the dependence structure, as represented by the copula $C$. An example of a copula that can be used to derive multivariate distributions from the modeled univariate distributions $F_i$, $i=1,\ldots,N$ is the $N$-dimensional Gaussian copula $C_\theta$ given by

$$C_\theta(F_1(x_1), \ldots, F_N(x_N)) = \Phi_\theta(\Phi^{-1}(F_1(x_1)), \ldots, \Phi^{-1}(F_N(x_N)))$$

where $\Phi_\theta$ is the multivariate standard normal distribution with correlation matrix $\theta$ and $\Phi^{-1}$ is the inverse of $\Phi$, the standard normal distribution function.

There are other risk management tools that can afford the insurer an opportunity to enhance the accuracy of the regulatory capital calculations performed using a fully-internal model. For example, Extreme Value Theory (EVT) can be very useful in approximating the tail-losses that are important for calculating regulatory capital. McNeil et al. (2005) contains a comprehensive discourse on EVT and its applications.

### 1.3 Accomplishments of thesis

The practical implementation challenges of a ruin-theory based capital framework have been discussed in the preceding sections. Plantine and Rochet (2007) considered the shortcomings of ruin theory significant enough to abandon the theory altogether in favor of an alternative theory that is based on corporate finance arguments. Starting with the perfect capital market assumptions of Modigliani and Miller (1958) and
then introducing agency risk into the analysis, they conclude that the potential role of capital in a solvency regulatory framework is in its use as an incentive device for aligning the interests of shareholders and policyholders. The impact of capital on shareholder risk-taking behavior is then likened to the effect of a deductible on an insurance policyholder.

In this thesis, a different approach is taken from that of Plantine and Rochet (2007). The framework for insurer capital requirements that is developed in the thesis combines both the incentive and ruin-theoretic roles of capital, rather than completely discarding ruin-theory altogether. The two key insights of Plantine and Rochet (2007) that are retained in the proposed framework are the following:

1. Capital can be used as incentive device to mitigate moral hazard risk, as explained by the deductible analogy.

2. The role of the solvency/prudential regulator is limited to the aggressive representation of the policyholders in the management of the insurer since policyholders typically do not have representation in the corporate governance structure of insurers.

The problem with using pillar 2 as the main instrument of solvency supervision is that it is qualitative and very subjective by nature. It is hard to legislate “desired” behaviour. However, it is possible to promote sound policyholder-oriented risk management policies by the insurer through the use of carefully designed and calibrated incentives. In the recommended framework, capital has a dual role. It is a buffer against residual risks and also a robust mechanism of incentives for sound risk behaviour by shareholders. The incentive effect reinforces the pillar 2 supervisory effort. It can be argued that the more important of the two roles of capital is the one in which
it is regarded as a system of incentives. Given the important and powerful role of incentives in driving risk management behaviour, a capital framework that considers incentives as an input rather than an output of the model should be superior.

The main accomplishments of the thesis are the following:

1. A unified framework for analyzing insurer insolvency risk and capital is proposed. Additionally, a term structure decomposition of the insurer’s insolvency risk is proposed to assist in the insolvency risk analysis. By providing an additional (time) dimension to insolvency risk measurement, the term structure decomposition provides a more complete picture of the insurer’s overall insolvency risk compared to traditional capital standards that only provide a single number to summarize the insurer’s total exposure. For example, a term structure decomposition of an insurer’s insolvency risk might reveal that a one-year based capital requirement is grossly misleading as a measure of the insurer’s time-dependent risk exposure.

2. The unified capital framework is also applied to the measurement of ALM risk for life insurers. The proposed ALM risk and capital framework is “policyholder-oriented”, defined in this thesis to mean capital requirements or corresponding incentives that are completely aligned with the overall goal of prudential regulation.

3. The unified capital framework and its associated term structure decomposition can be applied to enhance the effectiveness of all three-pillars of the solvency regulation framework.

4. The term structure decomposition of insolvency or ALM risk can be used in other applications as well. For example, it can be used to provide more complete
risk information to the stakeholders of an insurer in public financial reports. It can also be potentially used in portfolio optimization and economic capital calculations.

1.4 Outline of thesis

The context of the current thesis engagement has been provided in this chapter. In Chapter 2, the proposed benchmark global capital framework will be outlined. A method to decompose the capital or risk that has been calculated within the framework of the benchmark standard will also be presented in this chapter. In Chapter 3, the effectiveness of the existing Canadian MCCSR and US RBC standards, and the Solvency II standard formula, will be compared against the benchmark capital standard that is proposed in Chapter 2. In particular, special attention will be paid to the incentives that are provided under these standardized capital frameworks to assess whether they are “policyholder-oriented”. The unified global framework for measuring ALM risk and capital is presented in 4. Finally, Chapter 5 concludes by offering policy recommendations regarding the Solvency II, US PBA, and Canadian capital standards. Suggestions for future research are also listed in that final chapter.
Chapter 2

A conceptual framework for a unifying global capital standard

2.1 Introduction

In this chapter, a conceptual framework for a unifying global standard will be proposed. The two main advantages of the proposed solvency framework are that it addresses the issue of incentives in the calibration of the capital requirements and it also provides an associated decomposition of the insurer’s insolvency risk by term. When the incentive effect of capital is considered, pillar 1 of the three pillar solvency framework is no longer just a buffer to absorb residual risks. Rather, the incentives that are created by the capital requirements can facilitate the qualitative and subjective efforts of the supervisor under the remaining pillars. According to Cummins et al. (1993), solvency regulation should be designed to duplicate as closely as possible the outcome of a competitive market in which all parties have access to all relevant information. The proposed term structure of insolvency risk is an efficient summary of the
insurer’s risk information that should be readily accessible to all market participants, including regulators and policyholders. Given the inherent complexity of the long-term guarantees and options of typical life insurance policies, the term structure of insolvency risk is able to provide stakeholders with more complete information than that provided by a single number that relates to a specific period.

A solvency model for a life insurer will now be presented in the context of asset-liability mismatch risk as a means to introduce notation and solvency concepts that will be used in the remainder of the thesis.

2.2 An asset-liability model for a life insurer

2.2.1 Model definition

The insurance model is a multi-period discrete time model. The solvency risk measurements are based on monte-carlo simulations of many sample paths of the relevant risk factors (e.g. equity prices, interest rates or commodity futures prices) over the time horizon T. Denote each simulated sample path by $\omega \in \Omega$. Suppose that $(\Omega, F, P)$ is a probability space with filtration $\{F_t; t=1,..,T\}$. $P$ is a real-world or physical probability measure. Let $X(\omega)$ denote the value taken by the random variable $X$ in the state of the world $\omega$.

We define the following adapted stochastic processes:

- $MVA_t$ is the market value of assets of the insurer at time $t$.
- $MVL_t$ is the market value of insurer’s liabilities at time $t$.
- $ALR_t = MVA_t/MVL_t$ is the assets to liability ratio.
• \( \Theta = \{ \Theta_t, t=1,...,T \} \) denotes an investment, risk management or trading strategy. Each \( \Theta_t \) is a random vector showing the amount of each security invested in the asset portfolio at time \( t \).

\[
\text{LCF}(\omega) = \{ \text{LC}_1(\omega), \text{LC}_2(\omega), ... \} \quad \text{and} \quad \text{ACF}(\omega) = \{ \text{AC}_1(\omega), \text{AC}_2(\omega), ... \}
\]
are the liability and asset cash flows along the path \( \omega \).

### 2.2.2 Market value of assets and liabilities

In this section, the meaning of market value of insurance liabilities will be explained. Generally speaking, financial economics principles should be the basis for calculating the market value of insurance liabilities. Panjer et al. (1998) contains illustrative applications of financial economics to insurance and pension valuation. Babbel et al. (2002) examine the specific issues, such as the treatment of default and liquidity risk, that arise in the application of financial economics principles to the valuation of illiquid life insurance liabilities. In Black and Scholes (1973); Merton (1974), a holding in corporate debt can be treated as a combined position in risk-free debt and a short put. Babbel et al. (2002) illustrate the pricing of a bullet GIC and whole life insurance product using a similar decomposition technique that disaggregates the insurance liability into a risk-free amount that measures the insurer’s indebtedness (“Treasury” or “defeasance” value) and a put option amount that reflects default risk. Using the notation of the previous section, Equation (2.1) shows the calculation of this Treasury value for a whole life insurance policy that is exposed to interest-sensitive surrenders. At \( t=0 \), \( \text{TVL}_0 \) is the Treasury value of the liabilities, \( \text{E}_r \) represents risk-neutral expectation with respect to risk-free interest rates \( r \) and \( \text{E}_{m|r} \) is the conditional expectation with respect to mortality risk given interest rates. If mortality risk is orthogonal to interest rate risk, mortality can be treated
as deterministic in Equation (2.1) (i.e. in the evaluation of the inner expectation $E_{m|r}$). A surrender function that summarizes the relationship between interest rates and surrenders can be postulated and used as the basis for determining the interest-sensitive component cash flows of $LC_t$ for $t=1,2,\ldots,T$. To price in the ‘default’ put option $POVL_0$, option-adjusted spreads in respect of non-completely orthogonal and non-diversifiable (e.g. systematic mortality) risks are applied along the interest rate paths. Equation (2.2) then shows that the market value of liabilities $MVL_0$ is the defeasance value minus the value of the put option.

$$TVL_0 = E_r(E_{m|r}(\sum_{t=1}^{T} LC_t e^{-r_t \cdot t}))$$

$$MVL_0 = TVL_0 - POVL_0$$

The valuation of insurance liabilities in this manner is consistent with the market-based valuations of interest-rate derivative securities which require the specification and calibration of an arbitrage-free model for the short rate $r$. For example, Hull and White (1993) compare different approaches to developing arbitrage-free term structure models and describe a numerical procedure for constructing a variety of single-factor models.

A comprehensive review of the pricing, reserving and capital assessment techniques of equity-linked life insurance contracts from both the actuarial and dynamic hedging perspectives can be found in Hardy (2003). Option-pricing techniques for valuing insurance are also illustrated in Boyle and Hardy (1997, 2007). Bader and Gold (2003) support the application of financial economics principles to the valuation of defined pension plans for regulatory solvency and public reporting purposes. In particular, they argue against the use of book valuation methods, and the traditional actuarial-
ial practice of using a valuation discount rate that reflects the assumed long-term
investment strategy of the pension plan.

2.2.3 Solvency capital rules

The following definitions of solvency have been used in the actuarial literature.

1. An insurer is solvent at time $t$ if $\text{MVA}_t \geq \text{L}_t$. Define this as “point-in-time”
   (PIT) solvency.

2. If at time $t$, $\text{MVA}(\omega)_{t+1} \geq \text{MVL}(\omega)_{t+1}$, the insurer is solvent under scenario $\omega$.
   Define this as “short-term solvency” (STS). The regulatory capital framework
   that was proposed in IAA (2004) is based on the STS perspective. Similarly,
   the Solvency II (CEIOPS (2010)), Swiss Solvency Test (Sandstrom (2006)), and
   the proposed Canadian capital requirements (OSFI (2010a); MAC (2007)) for
   life insurers are also calibrated in this setting.

3. If $\text{MVA}(\omega)_{t} \geq \text{MVL}(\omega)_{t}$ for $t=0,1,...,n$ for $n=2,...,T-1$ where $T$ is the maturity of
   liability cash flows, then the insurer meets the “n-year balance sheet solvency”
   (XnYR) condition under scenario $\omega$. The short-term solvency (STS) definition
   would be equivalent to an X1YR rule but has been separately considered due
   to its relevance as indicated in the preceding bullet.

4. We define long-term balance sheet solvency (LTBS) as the situation when
   $\text{MVA}_t \geq \text{MVL}_t$ for $t=0,1,...,T$, where $T$ is the maturity of the liability cash
   flows. The determination of principle-based U.S. RBC that is described in
   AAA (2008a, 2002, 2003) is based on the LTBS formulation albeit using statutory
   rather than market values of assets and liabilities.
5. Let \( MVA_{t+1} = MVA_t (1 + R_t) - LCF_{t+1} \), where \( R_t \) is the random \( t \)-period investment return. Define \( \delta_t = MVA_{t+1} - MVA_t \). The insurer is solvent in the cash flow sense (i.e. “cash-flow solvency” (CFS)) if \( MVA_0 + \sum_{t=0}^{T-1} \delta_t \geq 0 \). The Canadian Asset Liability Method (CALM) (ASB (Canada) (2009)) and U.S. principle-based reserve method are applications of the cash flow solvency concept.

### 2.2.4 Term structure of insolvency risk

It is useful to consider a method to map out the insolvency risk profile of the insurer through time \( t \). The insolvency risk term structure would provide similar information to that provided by a mortality table for a cohort of insureds. Alternatively, it can be likened to an implied volatility term structure derived from option prices of different maturities.

Equation (2.3) shows the recursive calculation of the insurer’s assets \( MVA_t(\omega) \) for \( t=0,1,\ldots,T-1 \); given projected liability cashflows \( \{LC_1(\omega), LC_2(\omega), LC_3(\omega), \ldots\} \) under scenario path \( \omega \), where \( R_t(\omega) \) is the investment return and \( MVA_0(\omega) \) is the amount used as the starting assets in the projection.

\[
MVA_{t+1}(\omega) = MVA_t(\omega)(1 + R_t(\omega)) - LCF_{t+1}(\omega)
\]  

(2.3)

The projected liability values \( L_t(\omega) \) for \( t=0,1,\ldots,T \) are determined using the applicable valuation basis.

If we require \( MVA_t \geq L_t \) for \( t=0,1,\ldots,n \), for solvency, we can define the aggregate loss random variable \( S_n \) for a given solvency assessment horizon \( n=1,2,\ldots,T \) using Equation (2.5), where \( T \) is the maturity of the liability cashflows.
\[ S_n = \min_{\Delta \in \mathbb{R}} \{ \text{MVA}_0 = (L_0 + \Delta) | \text{ALR}_t \geq 1; t = 0, 1, \ldots, n \} \quad (2.4) \]
\[ = X_1 + X_2 + \ldots + X_n \quad (2.5) \]

where \( X_t \) for \( t=1,\ldots,n \) is the marginal loss random variable corresponding to time period \( t \). Equation (2.6) expresses \( X_t \) in terms of \( S_t \) (assume \( S_0 = 0 \)).

\[ X_t = S_t - S_{t-1} \quad (2.6) \]
\[ = \min_{\Delta \in \mathbb{R}} \{ \text{MVA}_0 = (S_{t-1} + \Delta) | \text{ALR}_t \geq 1 \} \quad (2.7) \]

The interpretation of \( X_t(\omega) \) is that it is the non-negative amount of top-up required to the assets at time \( t=0 \) so that \( \text{ALR}_t(\omega) \geq 1 \), given that the insurer was solvent under that scenario path \( \omega \) to the beginning of the period (i.e. \( \text{ALR}_i(\omega) \geq 1 \) for all \( i=0,1,\ldots,t-1 \)).

Let \( \Omega_{\text{tail}} = \{ \omega \in \Omega | S_T(\omega) > \text{VaR}_\alpha(S_T) \} \) for \( \alpha \) in \([0,1]\). Now taking expectations of the quantities in Equation (2.5) for \( n=T \), we obtain the insolvency risk decomposition in Equation (2.8).

\[ E(S_T|\Omega_{\text{tail}}) = E(X_1|\Omega_{\text{tail}}) + E(X_2|\Omega_{\text{tail}}) + \ldots + E(X_T|\Omega_{\text{tail}}) \quad (2.8) \]

The insolvency term structure expressed by Equation (2.8) is a useful construct since it provides an attribution of the insurer’s default risk (as measured by \( \text{TVaR}_\alpha(S_T) \)) to future periods. It will be used to explain important results in the remainder of the thesis. Tail-value at risk (TVaR) has been widely used in capital allocation problems similar to its use in Equation (2.8). For example, Panjer (2001) uses it for allocating capital to the business lines of a financial conglomerate.
2.2.5 Framework for policyholder-oriented risk management incentives

In this section, notation for explaining the role of policyholder-oriented incentives in the design and calibration of the proposed model for capital requirements will be introduced.

Define \( d(X, Y) \) to be some measure of distance between the multivariate random vectors \( X = (X_1, X_2, \ldots, X_T) \) and \( Y = (Y_1, Y_2, \ldots, Y_T) \), with means \( \mu_X, \mu_Y \) and covariance matrices \( \Sigma_X \) and \( \Sigma_Y \). Assume that \( Y \) is an objective that has to be achieved, and that \( X \) represents the random possible outcomes of a given process that has been designed to achieve those goals. Then \( d(X, Y) \) can been interpreted as a measure of the risk of the process in achieving the objective. In the present context, we can assume that \( Y \) represents the liability cash flows of the insurer, that is, \( Y = \{ LC_1, LC_2, LC_3, \ldots \} \). In our proposed framework, as discussed in previous sections, the sole objective of the prudential regulator is to safeguard these policyholder obligations. If \( X \) represents the corresponding cash flows from the insurer’s assets given a particular operational, investment or business strategy, we have the interpretation that \( d(X, Y) \) is a measure of the riskiness of the strategy, or of the insurer’s insolvency risk, in general. We can make the same argument using the market values of assets and liabilities instead of cash flows.

In order to promote the alignment of the interests of the shareholders with the regulatory objective, the capital requirements can be defined in manner that creates appropriate shareholder/managerial incentives for sound policyholder-oriented risk management (PORM). To better articulate the role of capital as an incentive device in
solvency regulation, Equation 2.9 defines an “incentive function” on given regulatory capital requirements $\rho(.)$.

$$I(J, K) = \rho(J) - \rho(K)$$  \hfill (2.9)

$I(J, K)$ in Equation (2.9) provides a measure of the monetary incentives or reward for changing the underlying strategy from $J$ to $K$. If $J$ is a riskier strategy than $K$, in terms of the regulatory objective, we should have $I(J, K) \geq 0$. If $K$ corresponds to an insurer strategy that perfectly replicates the liability cash flows, then

$$I(J, K) = \rho(J) - \rho(K) = \rho(J)$$ \hfill (2.10)

since $\rho(K) = 0$. That is, the monetary incentive for adopting the minimum-risk strategy $K$ is the full amount of regulatory capital associated with the current strategy $J$. In the special case when the strategy $K$ is the benchmark strategy (i.e. exact proxy for regulatory objective), we refer to $I(J, K)$ as the PORM incentives of strategy $J$. They are policyholder-oriented since they represent the “reward” for moving down the risk spectrum toward the regulatory objective.

### 2.3 Outline of the proposed regulatory capital framework

The proposed framework for a unified global capital standard for life insurers is presented in the following sections. The proposed Global Framework Attributes (GFAs) will be discussed under two broad categories of a regulatory capital framework: (1) valuation of assets and liabilities (2) required capital. As has been previously noted,
the IAIS has done related work. To date, they have published principles, standards and guidelines for a solvency assessment framework that they hope will be adopted by each of the IAIS member states. In that regard, some of the GFAs presented will be largely consistent with the IAIS principles, outlined in IAIS (2002, 2005, 2007), for example. In Cummins et al. (1993), seven specific objectives of risk-based capital are provided. One of these objectives is that the risk-based capital requirements should provide “incentives” for insurers to reduce insolvency risk.

The distinguishing feature of the work in this thesis is the great emphasis it places on having appropriate “policyholder-oriented risk management (PORM) incentives” within a pillar 1 capital framework for reasons previously cited in the thesis. As discussed in previous sections, the empirical evidence available (e.g. Sharma (2002)) suggests that failure in corporate governance and risk management is frequently the cause of insurer insolvency, rather than inadequate capital per se. The proposed GFAs are therefore generally based on the following two important insights:

1. The overriding goal of prudential supervision is limited to the aggressive representation of an insurer’s existing policyholders in its corporate governance structure. In the proposed framework, the prudential regulator does not need to consider the interests of other stakeholders such as employees and shareholders of the insurer, similar to the approach of Plantine and Rochet (2007). The problem of poor corporate governance and risk management that is generally associated with failed insurers can be expected to be especially pronounced in a principle-based environment for solvency regulation.

2. The single objective of all three pillars of the solvency framework is to promote policyholder-oriented corporate governance and risk management by the insurer’s shareholders/management in a coherent and harmonized manner. In
particular, pillar 1 capital requirements should be structured to provide appropriate shareholder/management behavioral incentives to support the second and third pillars.

2.3.1 GFAs for asset and liability valuation

The GFAs for the valuation of assets and liabilities are largely consistent with the IAIS principles stated in IAIS (2002, 2005, 2007).

GFA 1

(1.1) The solvency assessment of all assets and liabilities should be based on fundamental economic values

Fundamental economic values require the use of realistic assumptions and methods to value assets and liabilities. They do not include arbitrary levels of conservatism. A solvency framework benefits from the use of economic values since they are more objective, transparent and relevant, compared to alternative valuation systems. For example, the book valuation of assets and the formulaic-approach to life insurance reserves under the current U.S. NAIC standard (see Lombardi (2006)) do not represent assessments of fundamental economic value.

Using an equity-indexed annuity portfolio as an example, Wallace (2006) demonstrates that the underlying risk of the portfolio is only properly reflected when economic values (in this example, also market values) for assets and liabilities are used. When historical accounting or other U.S. GAAP-based valuations are used, the resulting measured risk can be dangerously misleading.

Further, for the policyholder-oriented incentives defined in Equation (2.9) to have their intended effect, they must be structured in the context of economic
valuations.

(1.2) The valuation of assets and liabilities should be calibrated to the market, as far as possible.

When the market for an asset or liability is active, transparent and liquid, its market value should be used as the basis for measuring fundamental economic value. If there is no ready market for the asset or liability, the estimate of fundamental economic value should be based on market-inputs that have been derived from similar or other instruments, provided that this is reasonable. The added transparency and objectivity of market values allows market participants and the regulator to make more meaningful assessments of the insurer’s financial position. Therefore, the solvency monitoring of the insurer that is performed under pillars 2 and 3 is enhanced when market values are used. The book valuation of assets and the formula-based life insurance reserves under the U.S. NAIC system are objective, but they do not properly reflect the risk of the underlying cash flows. Other advantages of market-based valuation include the following:

- As stated previously, a major root cause for insurer solvency failures is due to incompetent or dishonest management. For example, in the situation when an insurer’s management consistently understates reserves or under-prices policies, the insolvency risk profile of the insurer takes the character of a ponzi scheme with an ultimate ruin or default probability of 1. The objectivity and transparency of market-based valuations helps to mitigate such operational risks.

- A mark-to-market solvency framework would also be advantageous since
the use of fair value or market value in public accounting has generally increased. The desirability of a harmonized solvency and public reporting standard is noted in IAA (2004).

Three categories of fair-value measurements are defined under the U.S. FASB Statement No. 157, Fair Value Measurements (i.e. US GAAP). In order of reliability, they are: quoted market prices in active markets (level 1), mark-to-model prices (level 2), and the unobservable inputs category that uses (subjective) estimates and assumptions in the valuation (level 3). The corresponding International Accounting Standard, IAS 39, is very similar.

The convergence of financial reporting standards to IFRS globally, as noted in Chapter 1, lends credence to the use of a mark-to-market valuation framework in the solvency assessment of an insurer. As described in the exposure draft of the IFRS for insurance IASB (2010), the measured insurance contract liability does not reflect a fair or “exit” value estimate. However, the sum of the first two components (i.e. best estimate liability plus risk adjustment, from the insurer’s perspective) has a strong resemblance to fair or market value making it easier to reconcile the two amounts.

- A mark-to-market (MtM) paradigm for all the assets and liabilities of the insurer ensures consistency in their valuation for solvency assessment. As suggested in IAA (2004), an inconsistent valuation of assets and liabilities would create hidden surplus or deficit.

- Alternative statutory solvency asset valuation approaches such as historical cost, amortized cost and the equity method do not provide current
estimates of value that are risk-sensitive. The disadvantages of using val-
uations that are not market-based will be elaborated later in Chapter 3, in the context of the statutory valuation practices that are in current use in Canada and the U.S.. The results of Chapter 3 demonstrate the inap-
propriate incentives that result when non market-based or risk-insensitive methods are used to value the insurer’s assets and liabilities.

As described in IAA (2004), the collapse of the Equitable Life Assurance Society in the U.K. was partly a result of inadequately priced guarantees and options in a portion of their pension portfolio. If market or option pricing theory-based methods had been used to explicitly reflect the value of these options and guarantees in a transparent manner, it is possible that the collapse of a such a major insurer could have been prevented.

Notwithstanding the significant risk management benefits of market valuation, there are also significant implementation challenges. Examples of the challenges include:

- Most insurance liabilities are illiquid and will not have a readily available quoted market price, the most reliable estimate of fair value (Level 1) under US GAAP, as described above. In the terminology of US FAS 157, many insurance liabilities would fall under the Level 3 type of fair value, which is the more subjective and least reliable of the three categories of fair valuation. Under Solvency II, the challenge of determining an “implied” or “extrapolated” market value of insurance liabilities was addressed by prescribing the cost of capital methodology (e.g. CEIOPS (2010)) for measuring the liability risk margin. As can be expected, a lot of simplifying
assumptions were required to make this approach viable within a principle-based solvency framework. For example, a fixed cost of capital rate is assumed for all insurers, life and P&C insurers alike. Towers Perrin (2004) discuss the practical implementation challenges of a fair value approach in the context of P&C insurers.

- As observed in the recent global financial crisis, liquidity can suddenly dry up for some markets under stressed conditions. In this case, estimates of fundamental value are made using the less reliable level-3 type of valuation, for example.

- Procyclical capital requirements are those that tend to exacerbate prevailing market conditions. Risk-sensitive market valuations are more prone to procyclicality, in comparison to book valuation methods for example.

(1.3) The calculation of the policyholder liability should provide separate estimates for the best estimate liability and solvency margin

Separate estimates of the best estimate liability and solvency margin enable effective absolute and relative assessments of insurer financial strength. Knowledge of the best estimate liability would allow stakeholders of the insurer to make informed judgements of the actual levels of capitalization. Hirst et al. (2007) also note how market participants feel more confident about the quality of an earnings forecast when information on the source of the earnings by line item is also included in the disclosure. Therefore, the effectiveness of market discipline can be potentially enhanced when market participants receive disaggregated risk information on the liabilities since they are more confident to act on that information.
2.3.2 GFAs for regulatory capital

GFA 2

(2.1) The global capital framework must be principle-based

(2.2) The regulatory capital requirements should be calibrated to an overall enterprise level of statistical confidence

The capital requirement should be principle-based to accommodate the varied insurer risk profiles. The calibration of capital requirements should be at the enterprise level to incentivise integrated risk management by the insurer.

(2.3) The determination of regulatory capital should be assessed within the context of an integrated asset-liability model

The interactions between both sides of the insurer’s balance sheet need to be properly modeled for accurate measurement of the insurer’s net risk exposures and corresponding capital requirements.

(2.4) All material risk exposures should be dealt with under the solvency framework, whether quantitatively or qualitatively

Material insurance underwriting, market, credit, operational and other risks should be included in the solvency assessment framework if inappropriate behavioral incentives are to be avoided.

(2.5) Risk dependencies, diversification and concentration

A capital requirement should be set reflecting the net risk exposure of the insurer at the enterprise level. Therefore, dependencies among risks should be considered to the extent possible.

In IAA (2004), the WP states that the solvency assessment method should
recognise the impact of dependencies, diversification, concentration.

(2.6) The use of internal models should be allowed subject to appropriate policyholder-oriented constraints

The constraints that are to be applied to internal models should reflect the long and cash-flow nature of the insurance obligations.

GFA 3

(3.1) The minimum capital requirements should anticipate the development of all material risks and cash flows over the full term of the existing liabilities

In general, life insurance is a long-term cash flow business. Accordingly, insurers use pricing models with a sufficiently long horizon to accurately measure the underlying risk-return profile of the insured portfolios. If the horizon is not long enough, inappropriate risk-return decisions will be made. The long-horizon is important for profitability assessment since there is generally no ready secondary market to sell the liabilities. In this regard, policy obligations can be rightly characterised as “sell and hold” liabilities. To match the “sell and hold” insurance liabilities, insurers have traditionally used buy and hold strategies for their investment portfolios. In other words, insurers are more accurately categorised as investors rather than traders. Consequently, it would be inappropriate to view the risk arising from the investment operations of a typical insurance enterprise as trading risk (i.e. short-term) rather than investment risk. Many regulatory capital systems for insurers around the world are currently calibrated using a solvency assessment horizon that is much longer than one year. This includes those of Canada and the United States as will be seen
In Chapter 3.

In IAA (2004), the WP suggests that a reasonable period for solvency assessment is about one-year. In the context of market risk, the combination of a one-year assessment horizon and market-based valuation of assets and liabilities will incentivise short-term risk-taking behavior by insurers. In most situations, an insurer will have well established and documented investment policies and procedures that can be reliably used for modeling future cash flows. For example, cash-flow modeling of the insurer’s investment strategy and liabilities is the basis for determining the liability margin for investment risk under the Canadian Asset Liability Method (CALM) that will be described in Chapter 3. Given a buy-and-hold investment philosophy, the limited usefulness of short-term market value changes in assessing the long-term solvency of an insurance portfolio can be evidenced by the following current accounting practices for insurers:

1. Book valuation methods for hold-to-maturity (HTM) assets have been in widespread use globally to this point - a reflection that cash flows tend to be more important in assessing long-term profitability and solvency strength in the business of insurance, rather than short-term MtM gains/losses.

2. Historically, the smoothing of MtM asset and liability gains and losses that has characterised the balance sheet evaluation of insurers around the world supports the need for a risk assessment period that is more than one-year. For example, the Asset Valuation Reserve (AVR) and Interest Maintenance Reserve (IMR) are mechanisms that are employed under the U.S. statutory system to smooth the impact of asset gains on statutory surplus.
3. The underlying risk assessment horizons that have been historically used to calibrate minimum capital requirements for ALM risk have typically been longer than one-year. For example, in Canada, the CALM that is described in Chapter 3 calibrates investment risk margins using an assessment period that is over the full term of the liabilities, similar to the principle-based approach that has now been proposed in the United States (e.g. AAA (2008a)). Another example is the calibration of the bond default factors under the U.S. RBC formula for life insurance to a 10-year period.

4. The current accounting practice for cash flow hedges under FAS 133—Accounting for Derivative Instruments and Hedging Activities \(^1\) supports the view that short-term MtM changes should not be given undue weight when the reference instrument that is being valued is being held for its cash flow value (similar to a HTM asset) rather than being held for trading or as a fair value hedge.

Finally, risks that were previously thought to be easily hedgeable might become unhedgeable under stressed market conditions. A one-year market-value based risk measurement would therefore not adequately measure the extent of the insurer’s exposure to an investment strategy that utilizes currently liquid securities that could potentially become very illiquid under stressed conditions.

(3.2) The minimum capital requirements must be optimally risk-sensitive from the policyholders’ perspective

We define optimally risk-sensitive capital requirements to be those that meet

\(^1\) Under FAS 133, fair value changes associated with derivative instruments that are classified as cash-flow hedges are recorded in AOCI and are not included in the P&L.
the following conditions:

1. The underlying risk measure must be policyholder-oriented in the sense that it measures the tail-risk of the insurer defaulting on the payment of insurance benefits to policyholders. This meaning of risk is appropriate since the sole objective of prudential regulation should be to secure the existing policyholder obligations.

2. The underlying risk measure must give due recognition to measurement error. Further, it must achieve an “optimal” balance in the tradeoff between measurement error and the solvency assessment horizon.

We also distinguish three types of measurement error risk that can contaminate a risk measure for capital adequacy:

Type A: Specification and calibration error for the asset valuation model. Let’s denote this error by $\varepsilon_A$.

Type B: Specification and calibration error for the liability valuation model. Let’s denote this error by $\varepsilon_L$.

Type C: Asset and liability cashflow model specification and calibration error. Denote this by $\varepsilon_{CF}$.

The discussions in Section 2.2.2 and 2.3.1 highlight the difficulty involved in calculating an accurate market value of insurance liabilities. Consequently, we would expect the error term $\varepsilon_L$ to be somewhat large. On the other hand, the size of $\varepsilon_A$ would depend on the nature of the insurer’s portfolio. If it is invested in private equity, hedge funds, OTC derivatives and other illiquid investments, $\varepsilon_A$ can be expected to be significantly large. Conversely, investment in publicly traded securities that are quoted on a major exchange implies a smaller $\varepsilon_A$. 

41
Consider, for example, a capital requirement \( \text{VaR}_\alpha(\Delta S) \) calculated at the \( \alpha \% \) confidence level of the change in net assets or surplus (\( \Delta S \)) over a one-year period. Equation (2.11) highlights the impact of the asset and liability valuation errors on the measured capital requirement. In this equation, \( A', L' \) and \( S' \) are the theoretically correct valuations of assets, liabilities and surplus given assumed changes in risk factors \( \Delta \) over the one-year period. Given that the scenario model for the underlying risk factors is also a potentially significant source of measurement error, Equation (2.12) shows the measured capital amount in terms of the “true” unobserved surplus amount \( S'' \) and the three categories of measurement error.

\[
\text{VaR}_\alpha(\Delta S) = \text{VaR}_\alpha(\Delta A - \Delta L) \\
= \text{VaR}_\alpha(\Delta A' - \Delta L' + \Delta \varepsilon_A - \Delta \varepsilon_L) \\
= \text{VaR}_\alpha(\Delta S' + \Delta \varepsilon_A - \Delta \varepsilon_L)
\]

\[
\text{VaR}_\alpha(\Delta S) = \text{VaR}_\alpha(\Delta S'' + \Delta \varepsilon_{CF} + \Delta \varepsilon_A - \Delta \varepsilon_L)
\]

If the proportion of the measured capital that is due to the error terms (i.e. \( \Delta \varepsilon_{CF}, \Delta \varepsilon_A \) and \( \Delta \varepsilon_L \) in Equation (2.12)) is significant for a given capital requirement, the regulatory capital framework will not be effective and might have unintended consequences. The dependence of the capital requirement on the character of the asset and liability portfolios should be noted in the context of the error terms. An optimally risk-sensitive solvency framework should minimize the distorting impact of measurement error on the overall required capital amount and on the system of incentives described by Equation (2.9).
A regulatory capital framework that is optimally risk-sensitive has the following advantages:

1. It provides appropriate incentives for insurers to implement sound policyholder-oriented risk management policies and practices.

2. The deployment of capital in the insurance sector is not discouraged since the capital requirements properly reflect the solvency regulatory objective, and are not inappropriately burdensome.

(3.3) The minimum capital requirement framework should be cognisant of the economic incentives that it creates. To promote sound business practice and reduce insolvency risk, the economic incentives created by the minimum capital requirements should be aligned with the economic fundamentals of insurance business.

Capital can be an expensive resource. If the minimum capital requirements for a given set of mutually exclusive investment or risk management strategies, for example, are inappropriately ordered given the insurer’s liabilities, unwarranted biases can be created against or for a given strategy. Insurers may also judge the adequacy of their economic capital requirements using the minimum capital requirements as a benchmark. For example, a 400% ratio of available capital to minimum requirements might be considered to be excessive, regardless of the differences in the underlying assumptions and methodologies of the economic and regulatory capital amounts. This will be especially problematic where the time horizons used in calibrating the capital requirements are different.

(3.4) The proposed solvency capital rule

Consistent with GFAs 3.1, 3.2 and GFA 1, and using the notation of Section
2.2.4, the aggregate loss random variable $S_T$ that is associated with the pillar 1 capital requirements under the proposed solvency framework is described by Equation (2.13).

$$S_T = \min_{\Delta \in \mathbb{R}} \{ A_0 = (L_0 + \Delta) | \text{ALR}_t \geq 1; t = 0, 1, ..., T \} \quad (2.13)$$

If Tail-VaR or Conditional Tail Expectation (CTE) is the risk measure that is used for determining capital requirements, Equation (2.8) then gives a useful decomposition of the insolvency risk exposure of the insurer by time horizon, as was discussed in Section 2.2.4.

A given capital requirement $\rho_\alpha(S_T)$ that is defined on the aggregate loss random variable $S_T$ at the $\alpha$-confidence level can also be decomposed into cash-flow and market-valuation based components of insurer insolvency risk as described by Equation (2.14).

$$\rho_\alpha(S_T) = \rho_\alpha(\text{CFS}) + \rho_\alpha(\text{ALR}) \quad (2.14)$$

The cash-flow solvency (CFS) condition that was discussed in Section 2.2.3 directly measures the risk of actual non-payment of the contractual benefits to policyholders (similar to the risk exposure of ponzi scheme members). An incremental capital charge $\rho_\alpha(\text{ALR})$ to that based on cash-flow default risk considerations alone $\rho_\alpha(\text{CFS})$ is then obtained by imposing a marginal constraint on the market-based leverage ratio of the insurer ($\text{MVA}_t \geq \text{MVL}_t$ for $t=0,1,..,T$). The incremental capital charge is based on the insight that an investment scenario that otherwise generates sufficient returns for the insurer to be able to discharge all policyholder obligations as they fall due may still be risky if it achieves those
investment results at the expense of increased volatility in the market-valuation based leverage ratio. Hence $\rho_a(\text{ALR})$ will increase with the volatility of the market leverage ratio that is not explained by cash-flow volatility.

The imposition of a floor on the market-value based leverage ratio of the insurance portfolio $(\text{ALR}_t \geq 1$ for $t=0,1,\ldots,T)$ enhances the security of the policyholder obligations for two reasons. A purely cash-flow based capital requirement (i.e. without the constraint on the market-based solvency/leverage ratio) would significantly depend on the subjective modeling assumptions of the insurer. The constraint on the market-based leverage ratio should increase the relative objectivity of the capital requirements. In Chapter 3, the ill-incentives that result from subjective valuation assumptions are illustrated in the context of the current Canadian MCCSR formula (OSFI (2008)) which relies on subjective CALM liability estimates that are cash-flow based.

The market leverage-ratio constraint is also relevant in the context of the expected IFRS for insurance (IASB (2010)). The first two components of the insurance contract liability that are described in the exposure draft (i.e. best estimate liability and risk adjustment) should be easier to reconcile with a market-value based measurement.

The imposition of a marginal market-based LTBS condition on a cash flow-based capital requirement therefore appears to provide an adequate basis for calibrating policyholder-oriented capital requirements.

Based on the preceding discussion, we now define policyholder-oriented capital requirements to be those that correspond to a system of risk management incentives, as defined by Equation (2.9), that are consistent with the benchmark
risk management incentives defined on the capital requirements specified by Equation (2.14).

**GFA 4**

The capital requirements should be based on a coherent measure of risk

A risk measure is any mapping from a random variable to the real number line (Jorion (2005)).

In Wirch and Hardy (1999) it is shown that many risk measures can be expressed as an expected value of the loss random variable $X$ given some probability distortion function or change of measure.

A distorted probability $\Gamma$ is defined on a $\sigma$-algebra $\Omega$ as $\Gamma(A) = g[P(X \in A)]$, where $A \in \Omega$, and $g:[0,1] \rightarrow [0,1]$ is an increasing function with $g(0)=0$ and $g(1)=1$, and $P$ is a probability measure on $\Omega$.

Given that $F_X$ is the distribution function of $X$, a non-negative random variable, and $S_X = 1 - F_X$ is the survival distribution function, then

$$E_G[X] = \int_0^\infty g[P(X > x)]dx = \int_0^\infty g[S_X(x)]dx$$

is a risk measure.

If $g$ is a concave function, the risk measure $E_G[X]$ will be “coherent”. A coherent risk measure is a desirable basis for capital requirements since it satisfies some consistency criteria, referred to as axioms in Artzner et al. (1999).

**Definition 2.** A risk measure that satisfies the following axioms is called coherent:
• **Monotonicity**: If $X_1 \leq X_2$, then $\rho(X_1) \leq \rho(X_2)$

If one risk is always smaller than another, the capital required to support it should be correspondingly smaller.

• **Translation Invariance**: For $k > 0$, $\rho(X + k) = \rho(X) + k$

The capital for a certain risk is the amount of the certain risk.

• **Positive Homogeneity**: For $b > 0$, $\rho(bX) = b\rho(X)$

The capital for the same risk but in a different measurement unit is simply the capital amount in one unit multiplied by the unit conversion factor e.g. different units of currency.

• **Subadditivity**: For random losses $X_1$ and $X_2$, $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

The overall capital required to support two risks should not be greater than the sum of their individual capital amounts.

Two popular risk measures that are widely used in the financial industry are Value-at-Risk (VaR) and Conditional Tail Expectation (CTE). A brief description of each measure is provided below.

**Value-at-Risk (VaR)**

VaR is the maximum loss over a target horizon such that there is a low, pre-specified probability that the actual loss will be larger (see Jorion (2005)). VaR is not coherent since it does not satisfy the subadditivity axiom in general.

Given some confidence level $\alpha$, the VaR of the portfolio at the confidence level $\alpha$ is given by the smallest number $x$ such that the probability that the loss $X$ exceeds $x$ is not larger than $(1 - \alpha)$.

\[
\text{VaR}_\alpha = \inf \{x \in \mathbb{R} : P(X \leq x) \geq \alpha\}
\]  

(2.15)
Disadvantages of VaR include the following:

1. It is not coherent since it does not satisfy the subadditivity axiom.

2. It does not consider losses in the tail of the distribution. Therefore very different loss distributions can have the same VaR computed at some confidence level.

**Conditional Tail Expectation (CTE)**

An example of a risk measure that is coherent is the Conditional Tail Expectation (CTE). It is the expected value of the loss given that it exceeds VaR. For a continuous random variable $X$, the $\alpha$-level CTE can be defined as:

$$CTE_\alpha = E(X|X > \text{VaR}_\alpha)$$

$$= \frac{\int_{\text{VaR}_\alpha}^\infty x \cdot f(x) \, dx}{\int_{\text{VaR}_\alpha}^\infty f(x) \, dx}$$

**GFA 5**

The capital adequacy framework should have a minimum capital floor which is objective and cannot be easily gamed or manipulated by insurers.

In an internal model-based regulatory capital environment, it is important to impose an objective floor on the internal model-based minimum capital requirements. The necessity of this safety measure is supported by the Basel III amendments (BIS (2010)) which resulted in the adoption of a leverage ratio for banks.
GFA 6

There should be no regulatory capital assessment on assets representing free capital

The imposition of capital requirements on these free assets discourages insurers from maintaining more capital than is absolutely necessary in the insurer (IAA (2004)). Therefore, the assessment of capital on free assets should be avoided to the extent that the free assets do not include assets that can potentially turn into a liability at a future date, such as a swap or forward contract.

GFA 7

The capital framework should include a stabilizing adjustment to counter systemic risk

Market-based capital requirements would have the tendency to exacerbate market cycles i.e. they are procyclical. In the interest of promoting financial market stability, it is desirable for the cyclical effect of market-based risk measurements to be muted to the extent possible.
Chapter 3

A benchmark review of the MCCSR, US RBC and Solvency II formulas

3.1 Overview

In this chapter, the Canadian Minimum Continuing Capital and Surplus Requirements (MCCSR) (OSFI (2008)), the U.S. Risk Based Capital (RBC) (NAIC (2008)) and the Solvency II (CEIOPS (2010)) standard formulas for life insurance will be evaluated against the global capital framework that was proposed in Chapter 2. An outline of the three standard formulas was provided in Chapter 1. In that same chapter it was mentioned that the solvency regulation of insurers in the U.S., Canada and the EU is currently undergoing major review. One common trend among the changes that are occurring in these jurisdictions is the move toward approaches that are more principle-based. Similarly to Basel II (and now Basel III), insurers will be allowed
to use internal models to determine their regulatory capital needs under the evolving standards. However, the extent and manner of use will vary under each standard. For example, the use of internal models under Solvency II is generally motivated as a means to lower capital requirements from those determined under the standard formula. In contrast, capital requirements in the U.S. that have been determined using internal models tend to be subject to a floor that is based on a standard approach (Vaughan (2009)). In either case, however, standardized approaches will continue to serve as relevant benchmarks for their internal model counterparts. Therefore, for this reason as well, it is important to consider what are the optimal design and calibration features of a standard capital formula.

The main purpose of the analysis that is conducted in this chapter is to illustrate the application of the proposed global capital framework as a benchmark against which alternative capital standards can be evaluated. Specifically, the advantages offered by the following characteristics of the proposed global capital framework will be further discussed with the aid of numerical illustrations:


2. A calibration horizon for solvency capital that is based on the term of the liabilities.

3. The incentives that are offered under a capital standard must be policyholder-oriented, as defined in the previous chapter. If capital requirements are badly structured, unintended consequences can result (Cummins et al. (1993)). Kim and Santomero (1988) show how badly structured capital requirements can incentivise banks to undertake activities that are specifically designed to exploit the inconsistencies in the capital requirements.
4. The required capital levels must be efficient, that is, there must not be any redundancies or deficiencies in the capital amount. Redundancies or deficiencies can cause a misallocation of capital.

5. Explicit calibration of capital requirements to an enterprise probability of ruin.

The analysis in this chapter is primarily concerned with the design aspects of each standard formula. It is not concerned with the matter of the actual calibration of parameters of a given standard formula. In any case, the appropriate calibration of the parameters is dependent on the insurance market so it is difficult to directly draw comparisons across the standard formulas.

Descriptions and comparisons of the major insurance solvency systems in the world can be found in Sandstrom (2006). A comprehensive comparison of the US RBC and Solvency II systems was also conducted by Vaughan (2009). The author identified features of Solvency II that are potentially problematic. Some of these features are: (1) The use of a one-year horizon to measure market risk; (2) The danger from promoting the adoption of internal models as a means to reduce capital requirements; (3) The absence of investment restrictions; and (4) The associated challenges of implementing a capital standard that is based on market valuations. Plantine and Rochet (2007) proposed the use of simple verifiable ratios based on public statements rather than the more complicated US RBC type formulas that seem to perform poorly in predicting insolvency.

A standardized approach to determining capital is limited in terms of accuracy since it is based on conservative “one-size-fits-all” assumptions that do not necessarily reflect the actual risk exposure of any given insurer. The primary beneficiaries under a standardized risk-based capital framework are those insurers with the riskiest profiles. The numerical examples that are presented in the chapter have been deliberately
tailored to highlight certain unique features of each standard formula. The illustrative
term life insurance portfolios and assumed investment strategies are therefore not
necessarily representative of the typical insurer. They have been simply designed
to facilitate a review of the general features of the three standard formulas, and to
highlight their shortcomings when the underlying risk profile is arbitrary rather than
average.

3.2  A benchmark review of the standard formulas

3.2.1 Definition of terms

Let $MVA$ and $MVL(\text{i}_c, \text{l}_c, \text{m}_c)$ be the market values of the assets and liabilities of the
model insurer respectively, where $\text{i}_c$, $\text{l}_c$ and $\text{m}_c$ are the current interest rates, lapse
rates and mortality rates for determining the liability. And let $\text{SA}_i$ and $\text{SL}_j(\text{i}_c, \text{l}_c, \text{m}_c)$
be the statutory values of the same assets and liabilities in accordance with the
standard formula $j=$CAN or EU. We will use the notation $\text{SL}_{\text{us}}(\text{i}_p, \text{l}_p, \text{m}_p)$ where $\text{i}_p,$
$l_p$ and $m_p$ are the prescribed interest rates, lapse rates and mortality rates based on
the policy issue date. The major risk categories that underly the MCCSR, US Life
RBC, and the Solvency II standard formulas are indicated in Equations (1.1), (1.3)
and (1.10).

The supervisory target capital $\text{STC}_j$ amount for each standard formula $j$ is de-
defined as the minimum amount of capital required by an insurer to avoid supervisory
intervention.

Specifically, supervisory target capital should be interpreted as follows:

- Canada: Regulatory capital corresponding to an MCCSR ratio of 150%
• United States: Company Action Level RBC (i.e. 200% of the ACL RBC)

• European Union: Solvency Capital Requirement

The statutory liability valuations \( SL_j \) are defined below:

• In Canada, the Canadian Asset Liability Method (CALM) is used to determine reserves. The amount of CALM reserves \( SL_{can}(i_c, l_c, m_c) \) is determined using the cash-flow solvency rule that was explained in Section 2.2.3. It is the carrying value of starting assets in a cash-flow projection such that the terminal surplus is zero (ASB (Canada) (2009)).

• In the United States, formula-based reserves \( SL_{us}(i_p, l_p, m_p) \) are determined using a prescribed modified net premium method (Commissioners Reserve Valuation Method) and prescribed valuation assumptions that are based on the policy issue date (Lombardi (2006)).

• Equation (3.1) defines the technical provisions (TP) or market-consistent value of liabilities \( SL_{eu}(i_c, l_c, m_c) \) under Solvency II as the sum of the best-estimate liability (BEL) and a market value margin (MVM) which is estimated using a cost of capital approach (e.g. CEIOPS (2010)).

\[
SL_{eu}(i_c, l_c, m_c) = BEL + MVM
\]  

(3.1)

where the best estimate liability is defined as the probability-weighted average of discounted cash-flows at risk-free rates. The MVM is calculated as the present value of the future costs of capital requirements discounted at risk-free rates. The insurance portfolio is assumed to be run-off when determining the MVM (CEIOPS (2010)).
The definitions of the total balance sheet requirement $TBSR_j$ and free capital $FC_j$, as they are used in this thesis, are given by Equation (3.2).

$$TBSR_j = SL_j + STC_j \quad (3.2)$$

$$FC_j = MVA - TBSR_j \quad (3.3)$$

Free capital is the actual dollar amount of capital that can be used to pay shareholder dividends, repurchase shares, or embark on business expansion projects at the discretion of the shareholders. As such, the amount of free capital can be directly compared across jurisdictions regardless of the differences in statutory accounting practices.

The following analysis does not consider the different definitions of available capital under the three jurisdictions. The categorization of the different elements in the capital structure of a typical insurance enterprise, and the corresponding tier-specific regulatory capital requirements, are an important aspect of any solvency system but are outside the scope of this thesis.

### 3.2.2 Market-valuation based balance sheet

In this section, the risk-management benefits of a market-value based balance sheet will be illustrated using a simple example of a model term life insurer. The assumptions that are required to obtain illustrative results of capital are identified in the next section.
Assumptions for illustrative calculations

In this section, the assumptions that were required to obtain illustrative capital calculations for each of the US RBC, MCCSR and Solvency II standard formulas are listed. They are categorized into three groups: (1) model insurance portfolio; (2) liability valuation assumptions; and (3) asset portfolio. The assumptions for the model insurance portfolio and liability valuation are listed in Appendix A. The assets of the model insurer are assumed to be invested as follows: Reserve assets are assumed to be invested in risk-free debt securities which are assumed to be perfectly matched with liability cash-flows determined using the maximum permissible valuation margins in accordance with accepted Canadian actuarial practice. Surplus assets are assumed to be invested in a 60 - 40% combination of 30 and 10-year risk-free zero coupon securities. As mentioned at the beginning of this chapter, the motivation for this portfolio choice is precisely because it is an unusual strategy. A standard formula that is able to reasonably capture the significant tail-risk of such an arbitrary portfolio should be regarded positively.

Advantages of market valuation

Figures 3.1 and 3.2 show graphs of the required total balance sheet capital and free capital amounts, respectively, for the hypothetical term life insurance portfolio that have been determined in accordance with the requirements of the three jurisdictions. The amounts shown are all expressed as percentages of the best estimate liability (BEL).

The following two assumptions can be considered to enable a comparison of the total balance sheet requirement that has been calculated using amortized cost under the US standard with the market-value equivalent amounts determined under the
Figure 3.1: Total balance sheet requirements

Solvency II and Canadian standards:

1. We can assume the market and amortized cost values of the insurer’s assets are
Figure 3.2: Free capital of the model insurer

equal at the solvency assessment date.

In the graphs presented in this section, the amounts labeled “U.S. - 4.75%”
are directly comparable to the corresponding amounts under Canadian and EU Solvency II rules, subject to the assumption of equal market and amortized cost values. This might be the case if, prior to December 31, 2008, the insurer had been investing in and rolling over one-year Treasury bills upon maturity. Given that risky investment strategy, the AVR and IMR would have been nil on December 31, 2008, when the portfolio is rebalanced to the assumed matched strategy.

2. The valuation rate for calculating formula-based reserves can be adjusted to reflect current interest rates. The amounts labeled in the graphs as “U.S. - 2.7%” are comparable to the corresponding Canadian and Solvency II amounts in the sense that they have been calculated using the same reference interest rates. The 2.7% rate corresponds to the 10-year swap rate in Table A.1 of the appendix.

The risk management benefits of basing a solvency assessment on market values are illustrated in Figures 3.1 and 3.2:

1. To demonstrate the risk management benefits of a market valuation balance sheet, we first need an economic measure of liabilities. Let us assume that the Solvency II liability is a good proxy for this economic liability i.e. \( MVL(i_c, l_c, m_c) \approx SL_{eu}(i_c, l_c, m_c) \). Figure 3.1 then implies that the financial position of the insurer (i.e. net assets) is grossly overstated when the solvency liabilities are determined using the U.S. formula-based approach (based on \( i_p = 4.75\% \)). In fact, \( TBSR_{us}(i_p = .0475, l_p = 0, m_p) \leq MVL(i_c, l_c, m_c) \). The main reason for the significantly lower capital requirements under the U.S. rules is the relatively high statutory (lagged) interest rate of 4.75% that was used to determine liabil-
ities. The fixed interest rate of 4.75% is sufficiently high to offset the embedded conservatism in the prescribed mortality and lapse assumptions that are used in calculating U.S. formula-based reserves. As a result, the U.S. formula-based reserves cause an understatement of the insurer’s overall default risk with respect to policyholder obligations.

2. We can also assess the adequacy of the US reserve from the going-concern perspective. In the situation depicted in Figure 3.1, the U.S. statutory reserves might reasonably be inferred to be insufficient to assure the regulator that the insurer will be able to meet all policyholder benefits, both as a going-concern (i.e. $SL_{us}(i_p = .0475, l_p = 0, m_p) \leq SL_{can}(i_c, l_c, m_c)$) and in the event of immediate bankruptcy (using the Solvency II current exit value as a crude proxy for liquidation value). Therefore the overall U.S. statutory capital requirements are deficient based on economic considerations in this particular instance. This increased risk to policyholders will therefore go unnoticed.

3. Additionally, the locked-in nature of U.S. valuation assumptions by policy issue-year can result in different regulatory capital requirements for two life insurance policies that represent an identical residual obligation to the insurer. For example, single premium policies that are identical in every respect as of a given point in time, with the exception of the policy issue date. The current age of the insured, remaining coverage period and other underwriting characteristics are identical. That is, $SL_{[x+t]}(i_p, l_p, m_p) \neq SL_{[x+t]}(i_{p'}, l_{p'}, m_{p'}).$

4. The difference between the economic and statutory liabilities $\rho(i_{m'})$ of a given policy is described by Equation (3.4). Equation (3.4) shows that at any given time the redundancy or deficiency of U.S. formula reserves depends on the
relationship between the prescribed and current valuation assumptions.

\[
\rho(im') = MVL(i_c, l_c, m_c) - SL_{us}(i_p, l_p, m_p) \tag{3.4}
\]

For example, when interest rates are falling (i.e. \(i_p - i_c \geq 0\)) there will be a bias towards undervaluing liabilities (i.e. \(\rho(im') \geq 0\)). The opposite effect would be expected when interest rates are increasing (i.e. \(i_c - i_p \geq 0\)). For a given insurer at any given moment, the aggregate deficiency or redundancy of reserves with respect to interest rates will depend on the history of interest rates as well as the volume of new business that was issued in each previous year. If interest rates are assumed to be cyclical, the net redundancy or deficiency could be expected to cancel out over time for a mature insurer. However, since interest rates can have prolonged periods in which they are either trending up or down, the assumption that the deficiencies and redundancies will cancel is not guaranteed.

5. When the statutory valuation interest rates under the U.S. framework are much lower than market rates, resulting in conservative reserves, insurers in financial difficulty may be incentivized to arbitrage the regulatory capital requirements through reinsurance, securitization or other innovative capital market transactions. An economic valuation of assets and liabilities that uses current assumptions may be expected to mitigate the possibility of regulatory arbitrage by insurers.

6. The MCCSR amounts in Figures 3.1 and 3.2 are based on the most-conservative valuation assumptions in accordance with generally accepted Canadian actuarial practice ASB (Canada) (2009). The MCCSR amounts would be lower if
more aggressive assumptions had been used, resulting in more free capital for shareholders. Therefore, there appears to be an incentive for shareholders to understate reserves when policy liabilities are subjective.

Since the determination of the Canadian GAAP liability requires the subjective input of the actuary, it is possible to have multiple balance sheet measurements of the same liability. Subjective liabilities also make it difficult to obtain robust comparisons of financial strength across insurers.

7. Figure 3.2 shows that the amount of the insurer’s free capital for the U.S.-4.75% case is significantly greater than the amounts for the other standard formulas. The real danger of misclassifying capital or liabilities as free capital is that the money may be permanently withdrawn by shareholders.

3.2.3 Capital requirements

In this section, the appropriateness of the standard formula capital requirements is determined using the proposed global capital framework as the benchmark.

Calibration horizon and cash-flow versus market valuation perspectives

Solvency II uses a calibration period of one year (CEIOPS (2010)). Equation (A.13) in the appendix illustrates the calculation of credit spread risk under Solvency II. The need to determine capital for spread risk under Solvency II is because risk is defined under that standard in terms of changes in market values over a one-year period. The calibration of the formula is such that the resulting capital requirement corresponds to VaR 99.5% of the market-value losses due to spread risk over a one-year period. In contrast, the calibration of the credit default risk factors under the
MCCSR and US RBC is based on cash-flow modeling over longer periods. For example, the bond default factors in NAIC (2008) were based on cash-flow modeling of an actual bond portfolio over a period of 10 years. The calculation of the corresponding capital charges for default risk under the US RBC and MCCSR formulas is based on Equations (A.7) and (A.1) in the appendix. For reasons that were supported in the previous chapter, the proposed global capital framework requires a calibration period that is commensurate with the term of the liabilities. In that regard, the MCCSR and US RBC are more compatible with the proposed capital framework.

Consistency in asset and liability valuation

It was a fundamental insight of Reddington (1952) that the assets and liabilities of an insurer should be treated in an equal manner, since they both represent series of cash-flows. If the assets and liabilities are not consistently treated, it would be hard to interpret the changes in the net asset value of the insurer.

As described in Section 3.2.1, the formula-based reserves $SL_{us}(i_p, l_p, m_p)$ that underly the US RBC framework are based on a prescribed modified net premium valuation method and prescribed assumptions that vary by policy issue date. For example, the valuation interest rate for a policy that was issued 10 years ago would depend on the level of interest rates that existed at that time. On the other hand, the underlying interest rates that are used to value assets are most likely different. Consequently, the statutory valuation of assets and liabilities in the U.S. is inconsistent.

In Canada, policy liabilities $SL_{can}(i_c, l_c, m_c)$ are based on the book value of supporting assets such that the terminal surplus in a cash-flow projection of the assets and liabilities is zero. The scenario-wise cash flow projection requires subjective assumptions such as the anticipated portfolio credit loss rates, return premiums on
risky investments and the exercise of borrower and issuer options. Section 2340 of ASB (Canada) (2009) provides guidance on the derivation of the valuation assumptions. The CALM is “cash-flow consistent” in the sense that it establishes the equivalence of assets and liabilities on the basis of projected, albeit subjective, cash-flows.

As previously explained in Section 3.2.2, when policy liabilities are subjective, there is an incentive for shareholders to understate reserves since this would increase free capital. The use of subjective investment premiums under the CALM, for example, might incentivize insurers to undertake risky investment strategies. Due to the vulnerability of the MCCSR to this kind of exploitation, strong governance is required to minimize the discrepancy between the insurer’s actual investment strategy and the one that is assumed in calculating liabilities. There is also a need to vet the assumed investment premiums in the liability valuation model. An economic or market-based valuation of liabilities would eliminate such perverse incentives by de-linking the liability valuation from the assets backing the liabilities.

**Total balance sheet approach**

Generally speaking, capital requirements under Solvency II are calculated by applying shocks to risk factors and assessing their impact on the insurer’s net asset position over a one-year period. For example, Equation A.14 in the appendix provides the calculation of capital with respect to interest rate risk. The nature of the calculation implies that an insurer with a greater amount of surplus will be assessed more capital than an otherwise identical insurer with lower capitalization.

The factor-based approaches in Canada and the U.S. determine risk-based capital for interest rate risk (C-3) as the product of a prescribed factor and statutory reserves as described by Equations A.6 and A.8, respectively. The factor-based interest
rate risk calculation assumes that the insurer’s assets and liabilities are reasonably matched (see NAIC (2008)) and therefore does not need to explicitly consider the assets. The Solvency II total balance sheet approach considers the impact of interest rate risk on the actual net asset (assets-liabilities) position of the insurer and is therefore more risk-sensitive. From that perspective, the Solvency II total balance sheet approach is more consistent with the proposed global capital framework. In terms of liability valuation, the CALM is a total balance sheet approach that considers the interaction of asset and liability cash flows in setting reserves, including liability margins for interest rate or investment risk.

**Optimally risk-sensitive capital requirements**

In the previous chapter, the concept of optimally-risk sensitive capital requirements was introduced to highlight the importance of reducing bias and other measurement errors associated with a given capital standard.

The exposure base for determining US RBC for underwriting risks is the net amount at risk given by Equation (3.5).

\[
\text{Net amount at risk} = \text{Total inforce amount} - \text{Formula reserves} \quad (3.5)
\]

\[
= \text{Total inforce amount} - SL_{us}(i_p, t_p, m_p)
\]

To determine capital, decreasing percentages are progressively applied to higher level tranches (see Table A.4 in the appendix) to reflect the diminishing risk of more diversified insurance portfolios. Given formula-based reserves, the resulting net amount at risk is not particularly risk-sensitive. In contrast, underwriting risk margins under the Canadian and EU standard formula frameworks are generally based on changes
in the reserve requirements due to a prescribed change in the underlying risk factors, and are therefore more risk sensitive and portfolio-specific. For example, the calculation of mortality parameter risk under the Solvency II formula is provided by Equation (A.10). In the fifth quantitative impact study of Solvency II (i.e. QIS 5), the mortality shock which should be applied to the balance sheet has been changed from 10% to 15%. In Canada, mortality parameter risk is included in the CALM liability, with the remaining two components of mortality risk (i.e. volatility and catastrophic risks) being included in capital requirements (OSFI (2010b)).

**Categories of risk**

A capital framework should include all material risk exposures. Equation (1.1) shows that the US RBC for life insurance formula does not include catastrophe and operational risks. However, the U.S. RBC formula includes a charge for general business risk (C4) which is not included in the Solvency II or MCCSR formulas. In Canada, an arbitrary loading of 50% is applied to capital requirements that have been determined in accordance with Equation (1.3) to account for qualitative risk exposures.

The Solvency II standard formula does not include a margin for mortality volatility or process risk which is explicitly included in the MCCSR formula. The significance of this component can be expected to rise with increased skewness in the distribution of life insurance amounts and with decreasing size of the portfolio.

**Diversification credits**

The calibration of the dependencies among risks is primarily a qualitative exercise. The difficulty in measuring tail dependencies was discussed in Chapter 1. From the point of view of incentives, Equation (2.9), implies that there will be no incentive
to diversify if perfect correlation is assumed among risk types. On the other hand, significant diversification incentives will encourage insurers to bulk up, and perhaps squeeze out smaller players out of the market. The appropriate amount of incentives to provide for diversification will be determined by the specifics of the insurance market, among other considerations.

Equation (1.10) suggests the potential for significant diversification credits under Solvency II. The assumed correlation structure of risks under Solvency II describes the joint behaviour of the risks under a financially adverse scenario of events. Specifically, the tail correlations are calibrated such that they produce a solvency capital requirement at the enterprise level that has an associated ruin probability of 0.5% over a one year period. In general, the diversification incentives that are provided under the U.S. RBC formula (described by Equation (1.1)) are more modest than the amounts under Solvency II. Equation (1.3) shows that there are no diversification credits under Canada’s MCCSR formula since the total capital requirement is simply the sum of its component requirements.

**Enterprise-level ruin probability**

The solvency capital requirement under Solvency II is calculated to achieve a 99.5% confidence level over a one-year period at the enterprise level. In contrast, the total solvency buffer (i.e. TBSR\textsubscript{i} - BEL, for i=CAN, US ) under the Canadian and US regimes has no explicit confidence level attached to it. If there is no specific targeted security level, then it is plausible to expect that such systems will result in significant levels of redundancies and/or deficiencies that cause capital to be misallocated.
3.3 Chapter conclusion

For reasons described in Chapter 2, and based on the analysis of this chapter, an economic or market-based valuation framework that meets the conditions stipulated in the global framework attributes (GFAs) appears to be a reasonable unifying basis for a global capital framework. A long-term, cash-flow based economic valuation framework is preferable from a risk management perspective since it provides a relatively more objective, transparent and consistent solvency benchmark for determining appropriate capital requirements. A single global capital framework would also tend to mitigate the extent of arbitrage that is possible by international insurers.
Chapter 4

Proposal of an asset-liability mismatch risk measurement framework

4.1 Overview

4.1.1 Motivation

The global capital framework that was proposed in Chapter 2 will be applied to the specific measurement of asset-liability mismatch (ALM) risk in this chapter. The proposed ALM risk measurement approaches under the U.S. Principle-Based Approach (US PBA) (AAA (2008a)), Future Internal Model Approach (FIMA) in Canada (JCOAA (2008a,b); MAC (2007)) and Solvency II (e.g. CEIOPS (2010)), will be critically compared against the incentive-based capital framework that was proposed in Chapter 2. The main features of the (internal-model) regulatory capital
standards that have been proposed in the U.S., Canada and the EU are summarized in Table 4.2. The proposed capital standard in Canada is preliminary and has not been finalized.

As the complexity of the features that are embedded in insurance contracts has increased, so has the sophistication of the risk management programs that are used to hedge those features. As part of the asset-liability management programs of insurers worldwide, the embedded derivatives in insurance contracts are frequently hedged using portfolios of interest rate and credit default swaps, options, futures and other capital market instruments. To promote innovation and prudent risk management by the insurer, the regulatory capital framework must properly reflect the risk mitigating impact of these strategies on the insurer’s overall risk profile. In the context of Equation 2.12, all three measurement errors ($\Delta \varepsilon_{CF}$, $\Delta \varepsilon_A$ and $\Delta \varepsilon_L$) can be expected to be large when a significant proportion of the insurer’s portfolio consists of over-the-counter (OTC) and other derivatives that are suitable for matching complex insurance liabilities. A comparison of alternative capital standards should therefore consider the potential exposure to measurement error of each standard. Additionally, the incentive effect of the ALM risk capital requirements should be considered for the same reasons that were outlined in previous chapters. For example, it was argued in Chapter 2 under GFA 3 that a solvency capital framework should be compatible with the generally long-term cash-flow nature of insurance business. In this regard, a capital standard that is calibrated to short-term market movements can inappropriately incentivise insurers to take a myopic view of their risk exposures.

The primary goals of this chapter are twofold:

1. To apply the global capital framework that was proposed in Chapter 2 to the specific measurement of the ALM risk of life insurers. The proposed ALM risk
capital standards in the U.S., Canada and the EU will also be reviewed against this proposed benchmark standard to highlight their deficiencies.

2. The appropriateness of the incentive structures that are implied by the proposed ALM risk capital requirements in the U.S., Canada and the EU will also be examined using the framework for policyholder-oriented incentives that was outlined in Chapter 2. Doff (2008) reviewed the extent to which Solvency II satisfied the seven objectives of risk-based capital proposed by Cummins et al. (1993). He concluded that Solvency II generally satisfies the criteria. A different conclusion is reached in this chapter regarding the appropriateness of the incentives of the Solvency II framework (i.e. the first objective in Cummins et al. (1993)) with respect to the measurement of ALM risk.

4.1.2 Outline of chapter

The remainder of this chapter is structured as follows. The assumptions of the model insurer for which the monte-carlo simulation experiments that underly the results of this chapter were performed are provided in Section 4.2. The definitions of supervisory target capital that are used in this chapter are provided in Section 4.3. In Section 4.4, alternative ALM risk capital standards including the proposed US PBA, Canadian and Solvency II standards, will be critically reviewed against the benchmark standard that was proposed in Chapter 2. The term structure of insolvency risk decomposition of the proposed ALM risk measurement framework will be used to reconcile observed differences among the alternative capital standards. In Section 4.5, the potential impact of model error on relative ALM risk and capital assessments will be discussed. The main conclusions of this chapter are outlined in Section 4.6.
4.2 Model insurer

4.2.1 Model insurance portfolio

The monte-carlo analysis conducted in this chapter is based on a model life insurance company that issues only 20-year term life insurance. The 20-year term product is being used as a proxy for long-dated insurance products. It is assumed that the model insurer has been in business for the last 20 years and has been growing at a rate of 5% per year (number of policies) during that period. The number of policies issued in the most recent period is assumed to be 10,000. Therefore, the number of policies issued in a given previous year is $10,000 \times 1.05^{-t}$, where $t$ is the number of years preceding the most recent period.

- The original term of the insurance policies is 20 years
- The premiums are level and the product is not renewable
- The face amount of each policy is $500,000
- The annual premium is $1.80 per $1,000 face amount
- Expenses are 5% of the gross premium for all renewal years
- Issue age: 35
- All death benefits are assumed to be paid at the end of the year of death, while premiums and expenses are paid at the beginning of the year

The cashflows of the term life insurance portfolio are projected assuming a lapse rate of 4% per year, and mortality in accordance with Table B.1 in the appendix.
The monte-carlo simulation also assumes that there will be no further issuance of new policies in future periods so that the insurance portfolio will be managed on a run-off basis.

### 4.2.2 Assumed investment strategies of the model insurer

In this chapter, reference will be made to six alternative investment strategies ($\Theta_i$ for $i=1,\ldots,6$) of the model insurance company: the “bullet”, “barbell”, “laddered”, “short bond”, “long bond” and “duration matching” portfolio strategies. The first five of these strategies are more or less static (i.e. buy and hold) in the sense that no portfolio rebalancing is assumed to occur other than the reinvestment or divestment of net cash flows (investment income plus par redemptions less insurance cashflows) to the insurer. Reinvestment cash flows are used to purchase (hypothetical) bonds of a maturity which corresponds to the residual maturity of the portfolio holdings at the time, while existing bond holdings are sold as needed to fund net cash outflows to the policyholders.

The details of the portfolio strategies are as follows:

1. **Bullet strategy** ($\Theta_1$): The initial portfolio of this strategy consists entirely of hypothetical 14-year risk-free (annual) coupon bonds. After 14 years when the bond matures, the proceeds are reinvested in 5 year bonds. As needed, bonds are sold to fund insurance policyholder benefits.

2. **Barbell strategy** ($\Theta_2$): The initial portfolio consists of 1-year (23.9%) and 20-year (76.1%) risk-free coupon bonds.

3. **Laddered strategy** ($\Theta_3$): The initial portfolio is invested in risk-free coupon bonds of the following maturities, with the proportion corresponding to each
maturity indicated in parentheses: 1-year (5%), 3-year (5%), 5-year (10%), 7-year (15%), 10-year (10%), 15-year (10%), 20-year (20%) and 30-year (25%).

4. **Short bond strategy** ($\Theta_4$): Under this strategy, the entire portfolio is invested in 5-year risk-free (annual) coupon bonds. At every five-year anniversary thereafter, when the remaining bonds have matured, the portfolio is rolled over into new five year bonds.

5. **Long bond strategy** ($\Theta_5$): The initial portfolio consists entirely of 30-year risk-free (annual) coupon bonds.

6. **Duration matching strategy** ($\Theta_6$): Under the duration matching strategy, the portfolio is rebalanced annually to match the duration of the liabilities. At any given time, the minimum bond maturity that has a duration that is greater or equal to that of the liabilities under the prevailing interest rate environment is determined. If the duration of the determined bond maturity is greater than the liability duration, an exact duration match is achieved by a proportionate investment in one-year bonds, with the remainder of the portfolio being allocated to the determined bond maturity. The initial portfolio under this strategy consists of 1-year (2.7%) and 14-year (97.3%) default-free bonds.

In the monte-carlo simulation study, all bonds are assumed to be purchased at par or face value. The interest rates that were used as the starting point for the simulation are shown in the appendix in Table B.2. The coupon rates for the bonds purchased on the start date of the simulation correspond to the par yield column in Table B.2. Beyond the start date, the C-3 Phase III interest rate generator was used to obtain 10,000 stochastic scenarios. The C-3 Phase III generator has been recommended by the Life Reserve Working Group (LRWG) of the American Academy of Actuaries.
Table 4.1 summarizes the Fisher-Weil duration and convexity risk metrics of the initial bond holdings of the investment strategies on the start date of the simulation. The Fisher-Weil duration and convexity risk measures of the term-life insurance portfolio are also included for comparison.

The investment strategies were selected to roughly match the duration of the term-life insurance portfolio initially. Over the long-term, the investment strategies are expected to exhibit divergent risk characteristics. An internal model that underlies a regulatory capital framework should be able to distinguish these divergent risk profiles over time and appropriate amounts of capital that are commensurate with the measured risk exposures. Other reasons for selecting the investment strategies described in this section are as follows:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Duration</th>
<th>Convexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullet</td>
<td>11.2</td>
<td>144.1</td>
</tr>
<tr>
<td>Barbell</td>
<td>10.9</td>
<td>186.3</td>
</tr>
<tr>
<td>Laddered</td>
<td>10.9</td>
<td>189.9</td>
</tr>
<tr>
<td>Short bond</td>
<td>4.8</td>
<td>23.4</td>
</tr>
<tr>
<td>Long bond</td>
<td>17.8</td>
<td>429.4</td>
</tr>
<tr>
<td>Duration matching</td>
<td>10.9</td>
<td>140.3</td>
</tr>
<tr>
<td>Term-insurance</td>
<td>10.9</td>
<td>140.4</td>
</tr>
</tbody>
</table>

Table 4.1: Fisher-Weil duration and convexity risk measures

(AAA) for calculating U.S. PBA reserves for all life insurance products. Additional information on the C-3 Phase III generator is available in the appendix and is also detailed in the Economic Scenario Work Group Report (AAA (2008b)).
1. The short and long bond strategies can be considered to be proxies for high risk investment or risk management strategies of an insurer. From a supervisory standpoint, it is especially important for an internal model to be able to accurately capture the increased insolvency tail-risk of these strategies through commensurately increased capital requirements. Given typically long-term insurance liabilities, the short bond strategy can also be viewed as a representative depiction of the very real-life situation that exists in many insurance markets where there are no investable assets of sufficient duration to match that of the liabilities. In those insurance markets where this is the case, the reinvestment risk is intrinsic to the insurance market. On the other hand, the long bond strategy creates an asset-liability mismatch exposure to price risk, akin to the situation that is faced by banking institutions that borrow short and lend long when the yield curve is upward sloping.

2. The duration matching strategy is a risk management strategy that has been selected to assess the relative impact of hedging ALM risk on capital requirements under the different internal model-based capital frameworks.

### 4.3 Definitions of supervisory target capital

In this section, the measurement of ALM risk under the proposed Canadian, U.S. and Solvency II regulatory capital frameworks will be reviewed. A summary of the important features of these proposed capital frameworks is provided in Table 4.2.

The total capital requirements under each proposed capital standard will be compared using the Total Balance Sheet Requirement (TBSR). The Scenario Total Balance Sheet Requirement quantity (i.e. ALM risk loss random variable) will be first
<table>
<thead>
<tr>
<th>Liability valuation</th>
<th>U.S.</th>
<th>Canada</th>
<th>EU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuarial Valuation Approach (AVA) (AVA is the statutory value of assets needed to pay benefits at a CTE X% confidence level)</td>
<td>IFRS GAAP Liability (BEL + IFRS MVM) (details to be finalised with issuance of IFRS Insurance Contracts Phase II)</td>
<td>Market Value Liability (BEL+SH MVM) (i.e. Solvency II transfer value)</td>
<td></td>
</tr>
<tr>
<td>Asset valuation</td>
<td>Statutory (combination of book, equity, historical cost, fair value)</td>
<td>IFRS Fair Value</td>
<td>Market Value</td>
</tr>
<tr>
<td>Solvency assessment basis</td>
<td>Total asset requirement</td>
<td>Total asset requirement = best estimate liability (BEL) + capital buffer (but capital ratios will continue to be used)</td>
<td>Total asset requirement= BEL + MVM + SCR</td>
</tr>
<tr>
<td>Risk metric</td>
<td>CTE</td>
<td>CTE</td>
<td>VAR</td>
</tr>
<tr>
<td>Solvency assessment horizon</td>
<td>Lifetime of liability</td>
<td>1 year</td>
<td>1 year</td>
</tr>
<tr>
<td>Confidence level</td>
<td>90%</td>
<td>99%</td>
<td>99.5%</td>
</tr>
<tr>
<td>Investments</td>
<td>Quantitative limits and restrictions</td>
<td>To be determined</td>
<td>No restrictions</td>
</tr>
<tr>
<td>Internal models</td>
<td>only C3-risk for now, and by product (model segment); various floors for capital and reserves</td>
<td>By risk, and not expected to be implemented before 2014; floor based on standard formula</td>
<td>Enterprise-wide model (option to use partial internal model); only floor is the MCR</td>
</tr>
<tr>
<td>Risk mitigation</td>
<td>Can be reflected in the cash flow model over the liability term</td>
<td>Only instruments held on valuation date can be recognised</td>
<td>Can be reflected over the 1-year horizon</td>
</tr>
</tbody>
</table>

Table 4.2: A summary of the proposed capital requirements
defined for each of the proposed regulatory capital regimes (STBSRᵢ where i=U.S., CAN or EU).

Under the proposed Canadian market-risk framework, the STBSRᵢ^{CAN}(ω|Θ) is the value of assets required to pay policyholder benefits within a 1-year period and provide for an adequate reserve, called the terminal provision, at the end of the one-year period. The STBSRᵢ^{CAN}(ω|Θ) is a conditional amount that is calculated with respect to given scenario ω of changes in the relevant market risk factors over the one-year solvency assessment period and an assumed investment strategy Θ. In our analysis, the Solvency II best estimate liability (BEL) has been substituted for the terminal provision at the one-year solvency assessment horizon date.

Making use of the notation that was introduced in Section 2.2, Equation (4.1) shows the calculation of the STBSRᵢ^{CAN}(ω|Θ) and TBSRᵢ^{CAN}(Θ) amounts.

STBSRᵢ^{CAN}(ω|Θ) = \inf\{Δ ∈ \mathbb{R}^+: MVA₁ > MVL₁|MVA₀ = BEL₀ + Δ, Θ, ω ∈ Ω\}

TBSRᵢ^{CAN}(Θ) = CTE_{0.99}(STBSRᵢ^{CAN}(ω|Θ))

where MVA₀ is the starting amount of assets that is used in the simulation and Θ is the underlying investment strategy in risk-free bonds as described in Section 4.2.2. It can be seen that the STBSRᵢ^{CAN} loss-random variable is based on the short-term solvency (STS) or X1YR capital rule of Section 2.2.3. As shown in Table 4.2, TBSRᵢ^{CAN} should be calibrated using a CTE-99% level.¹

The STBSRᵢ^{US}(ω|Θ) is the exact amount of assets that is needed at the valuation date to pay all policyholder benefits as they fall due, given a scenario path ω of the

¹As described in JCOAA (2008a), the solvency buffer will be calibrated so that a company can withstand adverse conditions over a one year time horizon with a very high degree of confidence and have enough assets to sell or run off the business after the year.
relevant market risk factors and assuming an investment strategy \( \Theta \).

The \( \text{STBSR}_{\text{US}}(\omega|\Theta) \) and \( \text{TBSR}_{\text{US}}(\Theta) \) amounts were calculated in accordance with Equation (4.2).

\[
\text{STBSR}_{\text{US}}(\omega|\Theta) = \inf\{\Delta \in \mathbb{R}^+: \text{MVA}_k \geq 0; k = 0, 1, \ldots, T|\text{MVA}_0 = \Delta, \Theta, \omega \in \Omega\}
\]

(4.2)

\[
\text{TBSR}_{\text{US}}(\Theta) = \text{CTE}_{0.9}(\text{STBSR}_{\text{US}}(\omega|\Theta))
\]

where \( T \) is the term of the policy obligations, and \( \Delta \) is the starting amount of assets (i.e. at \( t=0 \)) that is used in the simulation for a given interest rate scenario path \( \omega \) and investment strategy \( \Theta \). The \( \text{STBSR}_{\text{US}} \) loss-random variable has been defined in Equation (4.2) using the cash-flow solvency concept of Section 2.2.3.

As defined in Equation (4.2), the \( \text{TBSR}_{\text{US}}(\Theta) \) or Total Asset Requirement (TAR) amount ignores the potential impact of the Deterministic Reserve calculation or any other floor amount since it is not within the scope of the thesis.

Table 4.2 shows that the solvency capital requirement (i.e. \( \text{SCR}_{\text{EU}} \)) for market risk under Solvency II is the 99.5% Value-at-Risk of the change in the net assets of the insurer over the following year due to a change in interest rates. We define \( \Delta S(\omega|\Theta) \) to be the change in the insurer’s net asset position over the following year due to a given change in interest rates represented by \( \omega \) and assuming an investment strategy \( \Theta \).

The \( \text{TBSR}_{\text{EU}} \) has been calculated as shown in Equation 4.3.

\[
\text{TBSR}_{\text{EU}}(\Theta) = \text{TP} + \text{VaR}_{0.995}(\Delta S(\omega|\Theta))
\]

(4.3)

\[
\Delta S(\omega|\Theta) = (\text{MVA}_1 - \text{MVL}_1) - (\text{MVA}_0 - \text{MVL}_0)|(\omega, \Theta)
\]
where TP corresponds to the Solvency II technical provisions that were defined in Section 3.2.1. The Solvency II best-estimate liability (BEL) has been substituted for the TP amount since the focus of this chapter is on the measurement of ALM risk.

4.4 A numerical comparison of the proposed ALM risk capital requirements

In this section, we will take a closer look at some of the more fundamental aspects of the proposed regulatory capital standards for ALM risk. The analysis presented is based on monte-carlo simulation results of the model insurance company that was described in Section 4.2. The model insurer will be assumed to invest its assets in accordance with the long-bond strategy in cases where it is not necessary to reflect the impact of varying the investment strategy. In a risk-based solvency supervision framework, the long-bond strategy should command the greatest attention of the regulator since it is the most unmatched strategy.

4.4.1 Sample distribution of supervisory target capital

Figure 4.1 shows the distribution of the scenario-specific total balance sheet requirement (STBSR) for each of the three proposed capital adequacy regimes. The model insurer is assumed to commit its entire asset portfolio to the long-bond investment strategy that has been described in Section 4.2.

The assumed initial market-consistent values of the insurer’s assets and liabilities are shown in Table 4.3.

The distributions of STBSR_{EU} and STBSR_{CAN} have been left-truncated at the
Figure 4.1: Scenario Total Balance Sheet Requirements
amount of the market-consistent technical provisions thus generating a probability mass-point at $215,599,000. Under the proposed Canadian and Solvency II frameworks, it seems reasonable that a solvent insurer should have an amount of assets that is at least sufficient to sell or runoff the business on the solvency assessment date.

In the case of the long-bond strategy, Figure 4.1 shows that the U.S. PBA total capital amount of $258 million is the highest of the three regimes. The Canadian and Solvency II total capital amounts are each $221 million when rounded to the nearest 1 million dollars. In other words, for the same ALM risk exposure, the U.S. PBA requires an additional $37 million of capital, which is 17.0% of the liabilities. This example clearly highlights the extent of regulatory capital arbitrage that is possible given the different perspectives of risk under the three regimes.

Equations 4.1 and 4.3 show that there are two main sources of difference between the Canadian and Solvency II capital requirements:

1. The proposed Canadian framework utilizes the Conditional Tail Expectation as the risk measure while the Value-at-Risk metric is the basis of the target capital requirement under Solvency II.

2. The Solvency II capital requirement includes an incremental capital charge on free assets i.e. those assets over and above the target requirement. Equation 4.1 shows that the proposed Canadian standard does not include this incremental

<table>
<thead>
<tr>
<th>Market value of assets</th>
<th>266,004,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best estimate of liabilities</td>
<td>215,599,000</td>
</tr>
<tr>
<td>Net Assets</td>
<td>50,405,000</td>
</tr>
</tbody>
</table>

Table 4.3: Initial balance sheet of the model insurer
capital charge.

In the example given, the two differences between the Canadian and Solvency II capital calculations offset each other producing roughly identical target capital amounts.

The superimposed normal distributions in Figure 4.1 suggest that the insurer’s ALM risk exposure is skewed when determined under each of the three capital adequacy regimes given the long-bond investment strategy.

4.4.2 Insolvency risk profiles implied by supervisory target capital amounts of proposed standards

The supervisory target capital amounts or TBSRs that were determined in the previous section will be used to “bootstrap” an implied solvency risk profile of the model insurer that corresponds to the TBSR of each capital regime. Figure 4.2 shows the time variation of the asset-to-economic liability (AL) ratios of the model insurance company for different amounts of starting assets.

The four cases correspond to different assumed amounts of starting assets of the model insurer: (1) best estimate liability (BEL) of the insurer’s obligations (2) TBSR_{CAN} (3) TBSR_{US}; and (4) TBSR_{EU}. The one-year solvency assessment period that underlies the calibration of the target capital requirements under the Canadian and Solvency II regimes results in an implied long-term solvency profile of the model insurer that is very concerning as shown in Figure 4.2. The capital requirements under the proposed Canadian and Solvency II capital regimes can leave the insurer exposed to significant insolvency risk in future periods. The long-term calibration perspective of the U.S. PBA capital requirements is evident in Figure 4.2. At any
Figure 4.2: Insolvency risk profile implied by supervisory target capital
given time horizon, the insolvency probability is less than 5% when the amount of initial assets is set equal to the total balance sheet requirement under the U.S. framework. However, it is important to note that there is no explicit requirement for the insurer to remain solvent on a market-consistent balance sheet basis during the risk assessment period (i.e. $\text{MVA}_t \geq \text{MVL}_t \quad \forall t=1,..,T$) under the U.S. PBA for capital. As shown in Equation 4.2, a solvent scenario under the U.S. PBA is simply one in which the insurer is able to discharge all policyholder benefit obligations as they fall due.

Equation (2.13) in Section 2.3.2 suggests that the solvency-risk profiles corresponding to capital amounts that have been determined using the capital framework that has been proposed in the thesis will be superior to those based on the proposed Solvency II, Canadian and U.S. capital standards.

We can calculate the marginal dollar cost of imposing the leverage constraint on the cash-flow based risk measure under the U.S. PBA (i.e. the amount corresponding to $\rho_\alpha(\text{ALR})$ in Equation (2.14)). This incremental capital charge can be viewed as the value of a “floor” on the solvency ratio with a strike value of 1.

Figure 4.3 shows the empirical density function of the scenario total balance sheet requirement under the U.S. principle-based approach, with and without the marginal constraint that the insurer has to maintain (economic) balance sheet solvency for the entire duration of the insurance liabilities.

Figure 4.3 shows that the imposition of the leverage constraint skews the distribution of the STBSR$_{US}$ to the right. In this particular example of the model insurer, the capital add-on for the balance sheet solvency constraint is $15.2$ million or about 40% of the total capital requirement. The significance of this amount implies that capital requirements that are determined within the U.S. PBA framework can still
Figure 4.3: Incremental capital charge for balance sheet solvency
leave the insurer greatly exposed to balance sheet insolvency risk over the term of the liabilities.

4.4.3 What risk is being measured?

The importance of having a minimum capital requirement that reflects the long-term cash flow nature of life insurance was underscored by GFA 3 of the global capital framework that was proposed in Chapter 2. In particular, GFA 3.2 requires the regulatory capital framework to be based on a risk measure that directly considers the policyholder cash flows as the primary objective to be met by the insurer.

To assess the extent to which the proposed standards reflect regulatory or policyholder oriented risk measurements, Figures 4.4 and 4.5 show plots of the model insurer’s asset to liability ratios (ALRt in Section 2.2) that correspond to the ten worst-case interest rate scenarios under the proposed U.S. and Solvency II capital standards respectively.

The interest rate scenarios corresponding to the ten largest STBSRs under each framework are considered to be the “worst”. As before, the model insurer is assumed to follow the risky long-bond investment strategy (which should therefore be of more interest to the prudential regulator). In addition to the adverse interest rate scenario paths, Figure 4.4 also shows the term-liability cash flow profile. The following important observations can be made from these graphs:

- The interest rate scenarios identified as adverse under each framework are very different. The worst case scenarios under the U.S. cash flow perspective are those where interest rates generally increase sharply over the term of the liability cash flows. The sharp increase in interest rates leads to severely depressed asset market prices at the times when assets need to be sold to fund net term liability
Figure 4.4: Risky scenario set for U.S. PBA
Figure 4.5: Risky scenario set for Solvency II
cash flows. Under the one-year market valuation risk perspective, an adverse scenario path is one characterised by a sharp rise in interest rates over a one-year horizon. As the model insurer is long-duration and convexity risks (based on the long-bond strategy), the sharp rise in interest rates over the one-year horizon leads to a significant loss of economic value on a total balance sheet basis. The one-year measurement horizon for ALM risk that underlies Solvency II and the proposed Canada framework implies that the progression of interest rates and the investment strategy beyond the one-year horizon will not be accounted for under those regimes. The apparent lack of a common trend beyond the one-year horizon among the ten worst scenarios in Figure 4.5 is consistent with this fact.

- Under each adverse interest rate scenario path in Figure 4.4, the sharp increase in interest rates causes the model insurer to become technically insolvent (i.e. \( \text{ALR}_t < 1 \)) at some future time point. The earliest predicted time of insolvency or ruin is about five years in the future. However, the asset-liability ratios of the model insurer in Figure 4.5 show a very different picture. Figure 4.5 shows that the \( \text{ALR}_t \) ratios of the model insurer are greater than one at all durations and for all scenarios. The insurer is therefore never really technically insolvent under any of these “adverse” interest scenarios. This observation leads one to question the ability of the one-year, market-valuation based risk measurement framework to adequately assess ALM risk from a policyholder-oriented or regulatory perspective. The correlation matrix of the scenario total balance sheet requirement amounts (STBSR) in Figure 4.1 is shown in Table 4.4 to further demonstrate the apparent lack of dependence between the U.S. PBA and the one-year based Solvency II and proposed Canadian capital requirements.

Given long-term liabilities, the low correlation between the U.S. and the Sol-
<table>
<thead>
<tr>
<th></th>
<th>CAN</th>
<th>US</th>
<th>EU</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAN</td>
<td>1.00</td>
<td>0.03</td>
<td>0.9</td>
</tr>
<tr>
<td>US</td>
<td>0.03</td>
<td>1.00</td>
<td>0.04</td>
</tr>
<tr>
<td>EU</td>
<td>0.9</td>
<td>0.04</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4.4: Correlations of scenario total balance sheet requirements

Solvency II (and Canadian) risk measures suggests that the minimum capital regimes are not targeting the same risk measurement objective.

- The actual ALM risk exposure under the U.S. PBA approach depends on the interaction of at least three factors: (1) insurer’s investment strategy (2) insurance liability cash flows; and (3) evolution of interest rates.

The one-year solvency assessment horizon for ALM risk that underlies the proposed Canadian and Solvency II minimum capital regimes appears to be more appropriate for assessing risk from a short-term profit oriented shareholder’s perspective. The one-year assessment horizon appears to be less suited as a basis for establishing regulatory minimum capital requirements that aim to guarantee the short and long-term security of policy obligations at a high level of statistical confidence.

### 4.4.4 Term structure of ALM risk

In this section, alternative capital standards will be assessed against the benchmark ALM risk measurement standard that was proposed in Section 2.3.2 under GFA 3.4 of the global regulatory capital framework to assess the degree to which they are policyholder-oriented. It was argued in Chapter 2 that the measurement objective underlying the proposed benchmark capital standard was appropriate (i.e.
policyholder-oriented) given the goal of prudential regulation.

The relevant risky scenario set of the proposed benchmark capital standard \( \Omega''(\alpha, \theta) \) given a confidence-level \( \alpha \) and investment or risk management strategy \( \theta \) is formally defined in Equation (4.4).

\[
\Omega''(\alpha, \theta) = \{\omega : S_T(\omega, \theta) \geq \text{VaR}_\alpha(S_T(\omega, \theta))\} \tag{4.4}
\]

where \( S_T \) is the aggregate loss-random variable that underlies the benchmark capital standard that is defined in Equation (2.13).

The results of Section 4.4.3 showed that the worst-case or risky scenario set for some confidence-level \( \alpha \), i.e. \( \Omega'_i(\alpha) = \{\omega : \text{STBSR}_i(\omega) \geq \text{VaR}_\alpha(\text{STBSR}_i) ; i = \text{CAN, US, EU} \} \), at least depends on the length of the calibration period that underlies the capital standard. Based on the analysis of that section, the proposed Solvency II and Canadian capital standards were shown to be not policyholder-oriented. That is, \( \Omega'_{\text{CAN}}(\alpha, \theta) \) and \( \Omega'_{\text{EU}}(\alpha, \theta) \) were shown to be significantly different from \( \Omega''(\alpha, \theta) \).

It is worthwhile to consider whether capital requirements can be policyholder-oriented when the underlying risk measure is based on a calibration horizon that is less than the full-term of the liabilities. To investigate that possibility, capital amounts \( \text{TBSR}_{XnYR} \) corresponding to the XnYR solvency capital rules that were introduced in Section 2.2.3 are defined here in Equation (4.5). For notational convenience, we denote the benchmark capital standard by \( X20YR \). This notation is appropriate since in the case of the term life insurance portfolio that was described in Section 4.2, the full-term of the liability cash flows is 20 years.
\[
\text{STBSR}_{X_n Y R}(\omega|\Theta) = \inf \{ x \in \mathbb{R} : \text{ALR}_t \geq 1 \quad \forall t = 1, \ldots, n | \Theta, \omega \in \Omega \} \quad (4.5)
\]

\[
\text{TBSR}_{X_n Y R}(\Theta) = \text{CTE}_\alpha(\text{STBSR}_{X_n Y R}(\omega|\Theta))
\]

where \( n \) is the term of the risk measure and \( \alpha = 0.9 \) is the desired confidence-level. For example, the \( \text{STBSR}_{X_5 Y R} \) is the amount needed to discharge policyholder obligations and also maintain the market-value based leverage-ratio \((\text{ALR}_t = \text{MVA}_t / \text{MVL}_t; \quad t=1,2,\ldots,5)\) above 1 over a five-year calibration horizon. As explained before, the proposed Canadian capital standard is equivalent to a X1YR solvency capital rule.

In Figures 4.6 and 4.7, risk profiles of the six investment strategies that were described in Section 4.2 (i.e. laddered (LADD), bullet (BULL), barbell (BARB), long bond (LB), short bond (SB) and duration matching (MATCH)) are provided as a basis for a comparison of the alternative capital standards.

In particular, the box plots represent the (sampling) distributions of the scenario total balance sheet requirement amount for each given capital standard. Given the apparent lack of correlation that was observed between the proposed one-year based capital standards (i.e. in Canada and the EU) and the longer-term U.S. PBA standard, in Sections 4.4.3 and 4.4.4, it is not surprising that the corresponding risk profiles of the investment strategies are significantly different. The capital requirements for each investment strategy that have been determined in accordance with the proposed capital standards are summarized in Table 4.5. The capital amounts are shown as percentages of the Solvency II best-estimate liability.

A preliminary assessment of the degree to which a given capital standard is policyholder-oriented, in terms of whether it is measuring the same underlying tail-risk as the benchmark X20YR standard, can be made by observing the following from Figures 4.6, 4.7 and Table 4.5:
Figure 4.6: Measured risk exposure of investment strategies
Figure 4.7: Measured risk exposure of investment strategies (cont’d)
The relative risk of the short-bond (SB) strategy appears to decrease with an increase in the length of the calibration period underlying the capital standard. Given the historically low level of interest rates that prevailed on the start date of the simulation, as shown in Table B.2, the likelihood of interest rates falling further over the long-term is expected to be small due to the mean reversion property of interest rates. However, the probability of interest rates falling even lower in the near future could potentially be significant. The result is a sharp difference in the relative risk of the short-bond strategy as perceived under the proposed Canadian and U.S. PBA capital standards, for example. The SB strategy is considered the riskiest investment strategy under the proposed Canadian capital standard. However, the box-plots corresponding to the longer-term U.S. PBA capital standard show that the downside risk of the short-bond strategy is significantly limited while the upside potential is considerable.

Table 4.5 shows that the capital requirements for the long bond strategy under the EU framework are almost double those of the short bond strategy. However, the capital requirements for these two strategies are almost the same under the Canadian framework, with those of the short bond strategy being slightly

<table>
<thead>
<tr>
<th></th>
<th>CAN</th>
<th>US</th>
<th>EU</th>
<th>X20YR</th>
<th>X15YR</th>
<th>X10YR</th>
<th>X5YR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laddered</td>
<td>0.005</td>
<td>0.087</td>
<td>0.001</td>
<td>0.105</td>
<td>0.102</td>
<td>0.071</td>
<td>0.021</td>
</tr>
<tr>
<td>Bullet</td>
<td>0.003</td>
<td>0.021</td>
<td>0.001</td>
<td>0.026</td>
<td>0.026</td>
<td>0.021</td>
<td>0.016</td>
</tr>
<tr>
<td>Barbell</td>
<td>0.010</td>
<td>0.080</td>
<td>0.005</td>
<td>0.112</td>
<td>0.111</td>
<td>0.097</td>
<td>0.050</td>
</tr>
<tr>
<td>Long bond</td>
<td>0.023</td>
<td>0.196</td>
<td>0.025</td>
<td>0.258</td>
<td>0.253</td>
<td>0.210</td>
<td>0.110</td>
</tr>
<tr>
<td>Short bond</td>
<td>0.024</td>
<td>0.152</td>
<td>0.013</td>
<td>0.162</td>
<td>0.162</td>
<td>0.161</td>
<td>0.149</td>
</tr>
<tr>
<td>Matching</td>
<td>0.003</td>
<td>-0.006</td>
<td>0.001</td>
<td>0.014</td>
<td>0.014</td>
<td>0.013</td>
<td>0.011</td>
</tr>
</tbody>
</table>
greater. The primary reason for this difference is the assessment of incremental capital on free assets under Solvency II as was explained in Section 4.4.1. The distorting impact of this aspect of Solvency II in risk assessment is therefore evident.

- The relative risk ranking of the laddered, barbell and bullet strategies under the U.S., Canadian and EU frameworks is also of interest. The duration and convexity risk measures of the initial portfolios of these strategies were summarised in Table 4.1. Under the Solvency II capital standard, the risk of the laddered and bullet strategies is the same, while that of the barbell is about five times that of either the bullet or laddered strategies. Under the proposed Canadian capital standard, the risk of the laddered strategy is about 67% greater than that of the bullet strategy, while the risk of the barbell strategy is more than 300% that of the bullet strategy. Of the three strategies, the Canadian and EU standards are consistent in their ranking of the barbell strategy as the riskiest. This is in contrast to the risk assessment under the U.S. PBA standard which ranks the laddered portfolio strategy as having the greatest risk exposure to interest rate risk among the three investment strategies, with the barbell strategy ranking second.

- As evidenced by the capital requirements in Table 4.1, the longer horizon X-measurement methods (i.e. 10, 15, and 20 year methods) are more or less consistent with the U.S. framework in their relative risk rankings of the investment portfolio strategies. The minor exception is the relative ranking of the laddered and barbell strategies which is reversed. However, when the actual order of magnitude of the difference in capital requirements is taken into account, this
inconsistency does not appear to be very material. On the other hand, the only consistency between the rankings that are based on the five-year horizon X5YR method and the U.S. rankings is the ranking of the bullet and duration matching strategies as the second least and least risky strategies respectively.

- With the exception of the duration matching strategy, the capital requirements for all other investment strategies determined under the U.S. capital framework appear to be significantly more conservative than the corresponding requirements under the Canadian and Solvency II regimes. The capital requirement for the duration matching strategy under the U.S. framework is -0.6% of the best estimate liability since the total balance sheet requirement under that strategy is less than the amount of the Solvency II best estimate liability. This is possible since we have not imposed the Solvency II best-estimate liability floor on TBSR_{US} in Equation 4.2.

- The significantly different character of the risk exposures underlying the six investment strategies in Figures 4.6 and 4.7 highlights the challenge of implementing a standard factor-based capital formula that is able to reasonably capture the essential features of each unique exposure, as was explained in Chapter 1.

To further establish whether capital requirements under a given standard are policyholder-oriented, the tail-dependency of a given alternative capital standard with the benchmark standard can be analyzed. In Figure 4.8, the 100 worst-case scenarios of the long-bond strategy in terms of the benchmark capital standard are plotted against corresponding outcomes of each given capital standard for those same scenarios. That is, given $\omega \in \Omega''(\alpha = 0.01, \theta = \text{LB})$, $F_1$ is the empirical distribution of $\text{STBSR}_{X20YR}(\theta = \text{LB})$, $F_i$ is the empirical distribution of $\text{STBSR}_i(\theta = \text{LB})$ for a given
capital standard $i$, the tail-dependency plots in Figure 4.8 are plots of points of the form $(x,y)$, where $x=F_1(STBSR_{X20YR}(\omega, \theta = \text{LB}))$ and $y= F_i(STBSR_i(\omega, \theta = \text{LB}))$.

A plot which shows a strong linear relationship (note that the line $x=y$ is indicated by a black dotted line in each plot) such as that for the X15YR method suggests that the given capital standard is policyholder-oriented. The tail-dependency plots of the short-bond investment strategy are also shown in Figure 4.9 for comparison.

An analysis of Figures 4.8 and 4.9 leads to the following conclusions:

- Figures 4.8 and 4.9 show that a significant number of interest rate scenarios that are classified as adverse (i.e. the worst 1%) under the benchmark capital standard are actually considered the least risky from the perspective of the proposed Canadian and EU capital standards since they correspond to the minimum target capital requirement $^2$ under those standards. Similarly to previous results, the proposed Canadian and EU capital adequacy standards are therefore not policyholder-oriented.

- The graphs also suggest that the tail-dependency between an alternative capital standard and the benchmark standard increases with the calibration horizon underlying the alternative standard. In the case of the long-bond investment strategy, for example, Figure 4.4 shows that the use of a time period such as 5-years to determine regulatory capital would result in the exclusion of the continued adverse development of interest scenarios and the investment strategy beyond the 5-year period. For each given adverse scenario in Figure 4.4, interest rates continue to increase beyond the 5-year calibration period, resulting in more assets being sold at even more depressed levels to fund net payouts.

$^2$The minimum target capital requirement under the Canadian and EU standards correspond to a probability mass point at the amount of the best estimate liability at time $t = 0$. 99
Figure 4.8: Tail-dependencies of alternative measures of (long-bond) ALM risk
Figure 4.9: Tail-dependencies of alternative measures of (short-bond) ALM risk
insurance policyholders. Consequently, a 5-year calibration horizon is too short to adequately capture the complete tail-risk profile of the long-bond strategy. The X5YR-based capital requirement is therefore not policyholder-oriented in the case of the long bond strategy because of its low tail-risk correlation with the benchmark capital standard which reflects all material risk over the term of the liabilities. However, Figure 4.8 suggests that a 15-year period might be adequate. Conversely, the tail-dependency plots of the short-bond strategy in Figure 4.9 show that a 5-year horizon is likely adequate to completely describe the long-term risk profile of this strategy. The same figure suggests that a 10-year calibration period will result in an extremely good proxy for the benchmark capital standard. Relative to the long-bond strategy, the risk of the short-bond strategy appears to be front-loaded, therefore allowing shorter calibration periods to be used.

• In Section 2.2.4, a method to decompose the ALM risk exposure of a given investment or risk management strategy by future time period, as measured under the benchmark capital standard, was suggested by Equation 2.8. In order to better understand the tail-dependency plots in Figures 4.8 and 4.9, we can make use of the term structure decomposition of the ALM risk exposures of the long and short-bond investment strategies. The ALM risk term structures corresponding to these strategies are respectively depicted in Figures 4.10 and 4.11. An analysis of Figure 4.10 shows that only 2% of the long-bond strategy’s risk is due to exposure in the first period. The cumulative percentages of the long-bond’s total ALM risk exposure that can be explained at the end of the fifth, tenth, and fifteenth years are respectively 26%, 74% and 98%. Given this risk decomposition, it is relatively easy to explain the observed tail-correlations

102
in Figure 4.8. For example, the observed low tail-correlations of the proposed Canadian and Solvency II capital standards with the proposed benchmark capital standard are due to the fact the one-year based standards are only capturing about 2\% of the total ALM risk exposure of the long-bond strategy. The term structure decomposition of the ALM risk exposure of the short-bond strategy that is shown in Figure 4.11 reveals a different picture. By the end of the fifth year, about 90\% of the total ALM risk has already been explained. This observation confirms the earlier conclusion that was reached in the examination of the tail-dependency plots that the X5YR capital standard was probably adequate or policyholder-oriented in terms of analyzing the risk of the short-bond strategy. The combined use of the benchmark capital standard that has been proposed in this thesis and the associated term structure decomposition of capital that is described by Equation 2.8, thus appears to be an immensely powerful and more complete tool for describing the insurer’s ALM risk exposure given an underlying investment or risk management strategy.

In conclusion, the analysis of this section has shown us that a capital standard need not use a calibration horizon that extends to the full-term of the liabilities in order to be considered policyholder-oriented, in the sense that was defined in Chapter 2. A shorter calibration horizon is possible if the term structure decomposition of the benchmark ALM risk exposure indicates that a significant percentage of the risk is explained within that measurement period.
Figure 4.10: Marginal risk contributions of the long-bond strategy by period
Figure 4.11: Marginal risk contributions of the short-bond strategy by period
4.4.5 An incentive-based review of proposed capital standards

In the three-pillar solvency regulation framework, it has been argued throughout this thesis that the pillar 1 capital requirements should be designed and calibrated such that the resulting incentive structure of the capital requirements is appropriate from a prudential regulation perspective. Studies by Kim and Santomero (1988); Rochet (1992) conclude that poorly designed capital requirements can create incentives for regulatory arbitrage by banks. Accordingly, GFA 3.3 requires a capital framework to be cognisant of the economic incentives that it creates.

The behavioral incentives that are implied by a given capital standard will be analyzed in this section using notation that was introduced in Section 2.2.5. The corresponding incentives for each of the proposed Canadian FIMA, US PBA, Solvency II and benchmark capital standards are shown in Table 4.6. For a given capital standard \( l \), each element of the incentive matrix corresponds to the quantity \( \Gamma_l(\theta_i, \theta_j) \) which is defined by Equation (4.6). It measures the capital savings in percentage terms when the insurer changes its investment strategy from \( \theta_i \) to \( \theta_j \).

\[
\Gamma(\theta_i, \theta_j) = \frac{I(\theta_i, \theta_j)}{I(\theta_i)} \tag{4.6}
\]

where \( I(\theta_i, \theta_j) \) is as defined in Equation (2.9). That is, \( I(\theta_i, \theta_j) \) is the dollar amount of capital savings when the insurer changes its investment strategy from \( \theta_i \) to \( \theta_j \), where \( i \) and \( j \) are the row and column indicators of the incentive matrix. \( I(\theta_i) \) is simply the amount of regulatory capital for investment strategy \( \theta_i \) that is indicated in Table 4.5.

As previously defined, a system of incentives will considered to be policyholder-oriented if it is largely consistent with the incentive structure of the benchmark capital
standard (i.e. $\Gamma_{BM}(\theta_i, \theta_j)$). Policyholder-oriented incentives are designed to encourage insurer behaviour that is consistent with the overall goal of prudential supervision, which is to ensure the complete and timely discharge of existing policyholder obligations. Accordingly, if the incentive matrix of a given capital standard is inconsistent with the benchmark incentives, we can conclude that the insurer’s shareholders/management are being encouraged to engage in risk-taking behavior that is not completely aligned with the overall objective of prudential regulation.

An examination of Table 4.6 shows that the incentive structures implied by the proposed capital standards, with the exception of the proposed US PBA, are significantly different from those of the benchmark standard. Therefore, the proposed Solvency II and Canadian capital standards do not necessarily result in incentives that encourage prudent risk management from a policyholder or regulatory perspective.

The percentage capital savings when an insurer changes from a long-bond ($\theta_5$) to a short-bond investment strategy ($\theta_4$) are ordered as follows: $(\Gamma_{EU} = 0.5) > (\Gamma_{BM} = 0.4) > (\Gamma_{US} = 0.2) > (\Gamma_{CAN} = 0.0)$. The Solvency II capital standard therefore offers the greatest incentive for this portfolio change while the proposed Canadian standard does not offer any incentive at all. This sharp difference between the one-year based Canadian and Solvency II capital standards is an example of the potential distortion in risk assessment that results from the determination of capital on free assets under Solvency II. From the viewpoint of prudential regulation, the change to a short bond strategy is a favorable move that should be encouraged (since $\Gamma_{BM} = 0.4$). In terms of all the capital standards in Table 4.6, the matching strategy ($\theta_6$) is considered the least risky. Consequently, an insurer would expect non-negative capital savings if it were to adopt this derisking strategy. The degree to which the
## Table 4.6: Incentive structure of proposed capital standards

<table>
<thead>
<tr>
<th></th>
<th>Laddered</th>
<th>Bullet</th>
<th>Barbell</th>
<th>Long bond</th>
<th>Short bond</th>
<th>Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Canada FIMA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laddered</td>
<td>0.005</td>
<td>0</td>
<td>0.4</td>
<td>-1</td>
<td>-3.6</td>
<td>-3.8</td>
</tr>
<tr>
<td>Bullet</td>
<td>0.003</td>
<td>-0.7</td>
<td>0</td>
<td>-2.3</td>
<td>-6.7</td>
<td>-7</td>
</tr>
<tr>
<td>Barbell</td>
<td>0.01</td>
<td>0.5</td>
<td>0.7</td>
<td>0</td>
<td>-1.3</td>
<td>-1.4</td>
</tr>
<tr>
<td>Long bond</td>
<td>0.023</td>
<td>0.8</td>
<td>0.9</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Short bond</td>
<td>0.024</td>
<td>0.8</td>
<td>0.9</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Matching</td>
<td>0.003</td>
<td>-0.7</td>
<td>0</td>
<td>-2.3</td>
<td>-6.7</td>
<td>-7</td>
</tr>
<tr>
<td><strong>Solvency II</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laddered</td>
<td>0.001</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>-24</td>
<td>-12</td>
</tr>
<tr>
<td>Bullet</td>
<td>0.001</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>-24</td>
<td>-12</td>
</tr>
<tr>
<td>Barbell</td>
<td>0.005</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
<td>-4</td>
<td>-1.6</td>
</tr>
<tr>
<td>Long bond</td>
<td>0.025</td>
<td>1</td>
<td>1</td>
<td>0.8</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Short bond</td>
<td>0.013</td>
<td>0.9</td>
<td>0.9</td>
<td>0.6</td>
<td>-0.9</td>
<td>0</td>
</tr>
<tr>
<td>Matching</td>
<td>0.001</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>-24</td>
<td>-12</td>
</tr>
<tr>
<td><strong>US PBA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laddered</td>
<td>0.087</td>
<td>0</td>
<td>0.8</td>
<td>0.1</td>
<td>-1.3</td>
<td>-0.7</td>
</tr>
<tr>
<td>Bullet</td>
<td>0.021</td>
<td>-3.1</td>
<td>0</td>
<td>-2.8</td>
<td>-8.3</td>
<td>-6.2</td>
</tr>
<tr>
<td>Barbell</td>
<td>0.08</td>
<td>-0.1</td>
<td>0.7</td>
<td>0</td>
<td>-1.5</td>
<td>-0.9</td>
</tr>
<tr>
<td>Long bond</td>
<td>0.196</td>
<td>0.6</td>
<td>0.9</td>
<td>0.6</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Short bond</td>
<td>0.152</td>
<td>0.4</td>
<td>0.9</td>
<td>0.5</td>
<td>-0.3</td>
<td>0</td>
</tr>
<tr>
<td>Matching</td>
<td>-0.006</td>
<td>-15.5</td>
<td>-4.5</td>
<td>-14.3</td>
<td>-33.7</td>
<td>-26.3</td>
</tr>
<tr>
<td><strong>Benchmark Capital Standard</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laddered</td>
<td>0.105</td>
<td>0</td>
<td>0.8</td>
<td>-0.1</td>
<td>-1.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>Bullet</td>
<td>0.026</td>
<td>-3</td>
<td>0</td>
<td>-3.3</td>
<td>-8.9</td>
<td>-5.2</td>
</tr>
<tr>
<td>Barbell</td>
<td>0.112</td>
<td>0.1</td>
<td>0.8</td>
<td>0</td>
<td>-1.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>Long bond</td>
<td>0.258</td>
<td>0.6</td>
<td>0.9</td>
<td>0.6</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>Short bond</td>
<td>0.162</td>
<td>0.4</td>
<td>0.8</td>
<td>0.3</td>
<td>-0.6</td>
<td>0</td>
</tr>
<tr>
<td>Matching</td>
<td>0.014</td>
<td>-6.5</td>
<td>-0.9</td>
<td>-7</td>
<td>-17.4</td>
<td>-10.6</td>
</tr>
</tbody>
</table>
matching strategy is incentivized under each capital standard follows the general pattern: $\Gamma_{US} > \Gamma_{BM} > \Gamma_{CAN} > \Gamma_{EU}$.

Lastly, as general rule, capital standards that are based on a short-term calibration horizon can be expected to encourage a short-term perspective to risk assessment by the shareholders of the insurer.

4.5 The impact of model risk on relative risk and capital assessments

In Chapter 2, GFA 3.2 of the global capital framework requires the capital requirements to be optimally risk-sensitive from the policyholder’s perspective, as defined in Section 2.3.2 of that chapter. In other words, the capital requirements must satisfy the following two conditions:

1. They must be policyholder-oriented as discussed in the preceding sections.

2. They should be structured such that they are not prone to significant measurement error that can distort risk measurements.

Three types of measurement error ($\varepsilon_A, \varepsilon_L$ and $\varepsilon_{CF}$) that should be considered in the design and calibration of the capital framework were discussed in Section 2.3.2. It was noted in that section that the relative magnitude of the errors depended on the nature of the asset and liability portfolios. In particular, all three errors can be expected to be large when the asset and liability portfolios contain complex guarantees and embedded options that are difficult to model and value.

The implementation of the general framework for assessing the market value of insurance liabilities that was outlined Section 2.2.2 is challenging. The valuation error
for typically illiquid insurance liabilities $\varepsilon_L$ can therefore be potentially significant and this should be a consideration in the final design and calibration of the corresponding regulatory capital requirements.

The potential impact of the Type C (scenario generator) model error on the determination of US PBA capital requirements will be illustrated in the case of the short bond strategy in this section. Under the short bond strategy, the entire portfolio is invested in five-year risk-free coupon bonds as described in Section 4.2. Whenever the bonds in the portfolio mature, the proceeds are rolled over into new five-year bonds. The uncertainty of interest rates at the times when new bonds have to be purchased is the primary source of the investment risk of the short bond strategy.

Reinvestment risk is an important consideration in the many insurance markets where the universe of available investment instruments is not of sufficient duration to match that of the liabilities. Zero-coupon swaps or notes, long-dated cross currency swaps and inverse floaters are among the tools that can be used to hedge reinvestment risk in such markets provided the infrastructure to use such hedging techniques is sufficiently developed. Swaption strategies can also be used to allow the insurer the opportunity to capitalize on favorable movements in interest rates.

The top graph in Figure 4.12 shows the risky ('worst case') interest rate scenario set for the short bond strategy in terms of the U.S. PBA capital framework.

As can be identified in Figure 4.12, the risky scenario set is characterized by low interest (reinvestment) rates at time horizons of 5, 10 and 15 years, when the insurer’s investment portfolio is rolled over into new five-year bonds.

It was noted in Figures 4.6 and 4.7 that the relative risk of the short bond strategy appeared to be much greater when a shorter horizon was used to measure the risk. Figure 4.12 reflects the expectation that is embedded in the 10,000 interest rate
Figure 4.12: Reinvestment risk of the short bond strategy
scenarios that were used in the simulation that five-year interest rates will tend to increase with time. Further, given that the starting interest rates in the simulation were very low and because of the mean reversion property of interest rates, Figure 4.12 shows that the downside risk in terms of interest rates falling further is significantly limited over the long-haul. However, it is likely that interest rates will fall further in the short-term. These underlying “views” on the short and long-term paths of future interest rates that are embedded in the scenario generator model largely contribute to the conflicting relative risk assessments of the short bond strategy versus the long bond or barbell strategy (as shown in Figures 4.6 and 4.7), for example. In particular, given the view that interest rates are much more likely to rise than fall over the long-term, Figure 4.6 suggests that the downside risk of the short bond strategy is limited while the upside potential is significant. The determination of capital and assessment of risk under the U.S. cash-flow based capital standard is therefore potentially exposed to significant model error. To the extent that the scenario generator model has been misspecified, the corresponding risk management decisions and solvency capital calculations will be misleading. For example, if the likelihood of reinvestment rates being lower than current rates over the long-haul is actually much higher than that which is embedded in the 10,000 C-3 Phase III scenarios that were used in the risk calculations, the relative risk of the short bond strategy would be much higher than determined in Section 4.4.5.

The potential benefit of imposing a market-based leverage constraint on a cash-flow based capital requirement (e.g. US PBA capital framework) in terms of reducing the impact of the subjectivity of modeling assumptions was motivated in Section 2.3.2 under GFA 3.4.
4.6 Chapter conclusion

A general framework for assessing the ALM-risk exposure of a life insurer has been provided in this chapter. The recommended approach for the measurement of ALM risk can serve as a unifying framework for the principle-based ALM-risk capital standards that have been proposed in the U.S., Canada, and the EU.

In Section 4.4.4, alternative capital standards were compared against the benchmark standard that has been proposed in the thesis to determine the extent to which they are ‘policyholder-oriented’. As defined in Chapter 2, capital requirements are policyholder-oriented if the measurement objective of the underlying risk measure reflects the ultimate goal of prudential supervision, the complete and timely discharge of all existing policyholder obligations. If the calibration period that underlies a given capital standard is such that a significant proportion of the overall ALM risk exposure of the insurer is explained by exposure in that period, then the resulting capital requirement adequately reflects the policyholders’ exposure to the default risk of the insurer. In the analysis of Section 2.2.4, the benchmark capital standard and the ALM risk decomposition technique that is described in Section 2.2.4 were shown to be invaluable tools for making this determination.

Throughout the thesis, it has been stressed that the incentive effect of capital requirements should be a primary consideration in the design and calibration of a regulatory capital framework. For example, the decision of what, if any, incentives should be provided through capital requirements for insurers to bulk up or diversify is an important one. As noted in Chapter 3, the Canadian MCCSR formula does not offer any credits for diversifying among its component risk categories. On the other hand, as noted in that chapter, the incentives for diversification under the Solvency II standard formula appear to be very significant. The overall consideration
in determining the appropriateness of incentives offered under a capital framework depends on whether their application serves to promote the overall goal of prudential supervision. In Section 4.4.5, a suggestive framework for analyzing and calibrating the incentives that underly a capital standard for life insurers was presented. Essentially, the incentive structure that is implied by a given life insurer capital standard must be largely consistent with the incentive structure of the benchmark capital standard. Such an incentive scheme is considered to be policyholder-oriented in the sense that has been defined in the thesis. The design of the proposed Solvency II and Canadian capital standards implies an incentive structure for ALM risk that is not policyholder-oriented.

Finally, the design of a capital framework should consider the three sources of measurement error that could distort the underlying risk measures. They are errors in asset valuation, liability valuation, and in the cash-flow model of the assets and liabilities (which includes sampling error). For example, measurements of capital that are based on a short-term calibration horizon such as one year (e.g. proposed Solvency II and Canadian capital standards) are particularly prone to errors in asset and liability valuation. Longer-horizon based capital standards such as the US PBA and the proposed benchmark capital standard would be more prone to errors in cash-flow modeling than they are to valuation risk. Policyholder-oriented capital requirements that minimize the incidence of measurement error risk are considered to be ‘optimally’ risk sensitive. An optimally risk-sensitive regulatory capital framework reflects the overall goal of prudential supervision in an optimal fashion, giving due consideration to the associated challenges of implementation.

In Section 4.4.3, the apparent low correlation between the ALM-risk measurements that underly the U.S. and the proposed Canadian and EU capital standards was
illustrated in the case of a twenty-year life insurance product. The one-year calibration horizon of the Solvency II and Canadian frameworks was shown to be inadequate for describing the ALM-risk profile of long-term strategies. An analysis of the ALM-risk term structure of the long-bond strategy validated this conclusion since only 2% of the strategy’s overall long-term risk could be explained by exposure in the first year. Additionally, the inadequacy of the one-year based capital standards in measuring long-term risk exposures implies that they cannot function as effective early warning indicators of financially distressed insurers to insurance supervisors.

The global financial crisis of 2007-2009 highlighted the pitfalls of a short-term oriented approach to the appraisal of enterprise risk that was caused, among other factors, by poorly structured managerial incentives. The troubled financial institutions that were at the epicenter of the crisis were widely criticized for making short-term business decisions that did not give due regard to long-term consequences. In the context of life insurance, the work of this chapter can be applied to determine the appropriate calibration horizon for a regulatory capital standard so that the corresponding incentive structure is aligned with the single goal of prudential regulation, the minimization of losses to policyholders due to insurer insolvency.
Chapter 5

Policy recommendations, applications of thesis results and future research

5.1 Introduction

As discussed in Chapter 1, the solvency regulation standards that are emerging in different jurisdictions can be generally organized under three pillars – quantitative capital requirements, supervisory review and market discipline. The use of a three-pillar solvency regulation mechanism has the potential problem that is described by Eling (2010) as follows, “Combining two approaches always runs the risk that a little of everything will be done, but not enough of anything (just like the “stuck in the middle” problem described by Porter (1980))”.

In Chapter 4, significant differences were identified among the proposed Canadian, US PBA and Solvency II capital standards with respect to the measurement of ALM
risk. In Chapter 2, a benchmark framework for measuring the insolvency risk of a life insurer was proposed. This framework was then applied to the measurement of ALM risk in Chapter 4 and was suggested as a possible basis for achieving global convergence in solvency standards.

The goal of this chapter is to suggest changes that can be made to the proposed Canadian, Solvency II and US PBA capital standards, as described in this thesis, in order to bring about convergence to the proposed global benchmark standard.

5.2 Market discipline

The primary reason for subjecting insurers to solvency regulation that was provided in Chapter 1 is based on the fact that the policyholders of an insurer are typically not able to effectively monitor and influence the insurer in their own capacity to protect their interest. In that same chapter, significant impediments that may exist to render ineffective the operation of market-discipline were provided. Information asymmetry and costly information are cited in Eling (2010) as possible impediments to the operation of market discipline. Life insurance and annuity contracts typically contain complex long-term guarantees and options that are extremely difficult to analyze. This makes it difficult for market participants, including policyholders and intermediaries such as rating agencies and brokers, to provide effective monitoring of the insurer’s risk portfolio. Consequently, market participants generally have to rely on the integrity of the insurer’s management for accurate disclosure of the insurer’s risk positions. Given that dishonest or incompetent management is frequently the reason cited for insurer failure, the ability of market participants to effectively monitor the insurer’s insolvency risk is at best questionable.
The effectiveness of market-discipline can be enhanced if the insurance policyholders and other market participants are furnished with relevant and timely information that they are able to process and act upon. Eling (2010) notes that for market discipline to work, the information that is provided to market participants must be “timely, standardized, consistent and transparent”. The term structure of insolvency risk decomposition that was proposed and illustrated in Chapters 2 and 4, respectively, can be provided to market participants as a pillar-three disclosure. The benefits of including the term structure of insolvency risk in the pillar-three disclosures are:

1. It is an efficient risk summary that is particularly suited for dissemination to market participants. It can be provided at the portfolio, business line or enterprise-wide levels depending on the particular application.

2. It is easy to understand. The amount of capital allocated to a given period is based on the marginal risk contribution of that period to the insurer’s overall insolvency risk over the term of the liabilities.

3. It provides more complete information than any single number. For example, it allows ALM strategies with risk profiles that are ‘front-loaded’ to be distinguished from those that are “back-loaded” (similar to a ponzi-scheme). Hirst et al. (2007) show that market participants find earnings forecasts by management more credible when information on the component earnings by source (i.e. income statement line items) is included in the disclosure along with the combined earnings forecast. Accordingly, we can expect that market participants will be more likely to act on the disaggregated risk information that is provided by the term structure decomposition of insolvency or ALM risk since they will find it to be more credible and informative than a single number.
4. It provides a context for assessing the appropriateness or relevance of shorter-term based risk measures. For example, a short-term risk measure would be judged to be inadequate for assessing the risk exposure of an insurer strategy that is back-loaded since it effectively cuts off the tail of the risk profile.

In summary, providing the term structure decomposition of the insurer’s insolvency risk to market participants should enhance the effectiveness of pillar-three of the solvency regulation framework. This applies whether the underlying pillar-one capital requirements are based on the U.S. PBA, proposed Canadian or Solvency II standards. Increased transparency in the insurer’s risk portfolio should also have the benefit of reducing its cost of capital.

5.3 Proposed Canadian and Solvency II capital standards

As noted in Chapter 4, the proposed Canadian and Solvency II capital standards are not policyholder oriented. Accordingly, the incentives that are provided under these (pillar 1) standards are not always consistent with the overall objective of prudential regulation as discussed in Section 4.4.5 of the previous chapter. To that end, the other pillars can be potentially used to negate the impact of the misaligned incentives.

In order of preference, the following recommendations are provided to address the potential shortcomings of the one-year based capital standards:

1. The pillar 1 capital requirements for ALM risk should be redesigned so that they become policyholder-oriented. It is understood that this will be a difficult
task to undertake at this time given the considerable progress that has been made toward adopting Solvency II in its current form.

2. The insurer’s term structure of insolvency risk should be used as primary tool for the conduct of the supervisory review pillar of the solvency regulation framework. As a quantitative summary measure of the insurer’s overall risk exposure with respect to existing business, it should add a lot of value to the primarily qualitative exercises conducted under this pillar.

3. As explained in the previous section, market participants should be furnished with information on the insurer’s term structure of risk exposures.

5.4 US PBA capital standard

The US PBA capital framework fared better than the proposed Solvency II and Canadian capital standards in the analysis that was conducted in Chapter 4. The absence of the market-based leverage-ratio constraint in the US PBA capital standard is the only source of difference with the benchmark standard. The risk-management benefits of the proposed market-based leverage constraint were explained in Section 2.3.2 under GFA 3.4. Accordingly, application of this leverage constraint should be an improvement of the proposed US PBA capital standard for all life insurance and annuity products.

In Table 4.5, the capital requirement for the matching strategy was found to be negative when the market-value measure of the liability was used. Accordingly, imposing a balance-sheet constraint that is based on market values would make the US PBA capital standard more compatible with a market valuation balance-sheet. The first two components of the insurance contract liability that are described in the
exposure draft of the IFRS insurance standard (IASB (2010)) should be reasonably correlated with a market-value based measurement. The leverage constraint would therefore also make the US PBA capital standard more consistent with the proposed IFRS for insurance, which is important due to the global reach of the international reporting standards.

### 5.5 Other potential applications of thesis results

Other potential applications of the results in this thesis include the following:

1. The disclosures that have been recommended in the previous section can also be used in public financial reports.

2. The term structure decomposition can be used in portfolio selection in the case where assets are being managed against a liability benchmark.

3. The risk decomposition technique can also be applied in the relative assessment of economic capital and risk given alternative insurer strategies.

### 5.6 Conclusion and future research

The potential applications of the benchmark capital standard and its associated term structure decomposition have been reviewed in this chapter. A possible pathway of achieving convergence in the proposed US PBA, Canadian and Solvency II capital standards has also been offered.

Opportunities for future research include the following:
1. Eling (2010) suggests there is a strong need of research to enhance our currently limited understanding of market discipline as it applies to insurance. From a solvency regulation standpoint, customer or insurance product-driven market discipline should be the primary focus of these further investigations.

2. The specific analysis of the proposed U.S. PBA, Canadian and Solvency II capital standards that was presented in Chapter 4 can be enhanced by including the following elements in the comparison:

   - Applicable minimum capital floors. For example, the Minimum Capital Requirement (MCR) under Solvency II, the Deterministic Reserve (DR) under the US PBA, etc.

   - A numerical analysis of dynamic aspects of each capital regime. For example, further research to evaluate the extent of the increase in the volatility of the capital requirements under the proposed Solvency II and Canadian standards, and to anticipate the impact of the increased volatility on the insurer’s ALM risk practices.

3. The ALM risk measurement framework that has been proposed in this thesis implies that the term structure decomposition of the insurer’s risk exposure will vary with the nature of the insurer’s product portfolio and investment/risk management. Research can be undertaken to develop guidelines for determining the appropriate calibration period for specific insurance products and common investment strategies.

In conclusion, the unified framework for analyzing insolvency risk and associated capital requirements that has been presented in this thesis should be a valuable con-
tribution to the development of a global regulatory capital framework for life insurers.
Appendices
Appendix A

Data and formulas for Chapter 3 calculations

A.1 Model portfolio and valuation assumptions

Model insurance portfolio

- 1,000,000 identical policies issued to males aged 35
- 30-year level, non-renewal term life product
- $500,000 face amount per policy
- Premium rate of $2 per $1,000 face amount

Interest rates

The interest rates that were used in the calculations are shown in Table A.1.
Solvency valuation assumptions

The solvency liability valuation assumptions are shown in Table A.2.

A.2 MCCCSR formula

The amounts of required and available capital under the Canadian MCCCSR formula are defined in OSFI (2008). A general overview of the MCCCSR formula was presented in Section 1.1. Equation (1.3) shows that the total MCCSR is simply the sum of the component requirements.

The actual MCCCSR-based formulas for asset default (C-1), mortality, lapse, and change in interest environment risks that were used to calculate the results of Section 3.2 are presented below.
<table>
<thead>
<tr>
<th>Assumption</th>
<th>US NAIC</th>
<th>Canada</th>
<th>Solvency II</th>
</tr>
</thead>
<tbody>
<tr>
<td>mortality</td>
<td>CSO 80 ultimate, non-smoker, male</td>
<td>Makeham(^1) + CIA High Margin(^2)</td>
<td>Makeham(^1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A=0.0007,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B=0.00005,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>c=1.08</td>
</tr>
<tr>
<td>lapse</td>
<td>nil</td>
<td>3.2%</td>
<td>4%</td>
</tr>
<tr>
<td>expense</td>
<td>nil</td>
<td>renewal: 5% of gross premium</td>
<td>renewal: 5% of gross premium</td>
</tr>
<tr>
<td>discount rates</td>
<td>based on policy issue year (4.75%)</td>
<td>expected portfolio return (based on current risk-free rates)</td>
<td>current risk-free rates</td>
</tr>
</tbody>
</table>

\(^1\) The force of mortality at age \(x\) in accordance with the Makeham law of mortality is given by \(\mu_x = A + Bc^x\)

\(^2\) The maximum margin permitted under the CIA standards of practice

Table A.2: Solvency valuation assumptions on December 31, 2008

**Asset default risk (C-1)**

MCCSR capital for default risk \(\rho(X_1)\) is based on Equation (A.1).

\[
\rho(X_1) = \text{MCCSR default risk factor} \times \text{statement value of assets} \quad \text{(A.1)}
\]

For example, OSFI (2008) gives an asset default factor of 1% for A-rated corporate bonds.
Mortality risk

Equation (A.2) gives overall mortality risk $\rho(X_{2,m})$ as the sum of the volatility and catastrophic risk components, which are respectively defined by Equation (A.3) and Equation (A.4).

$$\rho(X_{2,m}) = \text{volatility risk component} + \text{catastrophic risk component}$$  \hspace{1cm} (A.2)

with

$$\text{volatility risk component} = 2.5 \times A \times B \times E/F$$  \hspace{1cm} (A.3)

where:

- $A$ is the standard deviation of the projected death claims in the year immediately following the valuation date.

It is calculated by the following formula:

$$A = \sqrt{\sum_{\text{all policies}} q(1-q)b^2}$$

where $q$ is the valuation mortality and $b$ is the death benefit for the policy.

- $B$ is defined as follows:

$$B = \max(1, \ln D)$$

where $D$ is the Macaulay Duration of the projected net death claims for the term-life insurance portfolio at a discount rate of 5%.

- $E$ is the total net amount at risk for the term-life portfolio.
- F is the total face amount

\[
\text{catastrophic risk capital} = \alpha \times C \times E/F
\]  

(A.4)

where:

- \( \alpha = 10\% \)
- C is next year’s expected death claims
- E is the total net amount at risk
- F is the total face amount of insurance

**Lapse risk**

MCCSR capital for lapse risk \( \rho(X_{2,t}) \) is based on Equation (A.5).

\[
\rho(X_{2,t}) = \text{CALM reserve}_A - \text{CALM reserve}_B
\]  

(A.5)

where:

- \( \text{CALM reserve}_A \) is the CALM reserve for the term insurance portfolio that has been calculated using appropriate valuation assumptions.
- \( \text{CALM reserve}_B \) is the CALM reserve that has been recalculated using a more conservative lapse assumption i.e. an additional 15% margin to that already assumed in the valuation. For example if a 10% margin had already been assumed in the valuation, \( \text{Reserve}_B \) will be based on a lapse margin assumption of 10% + 15% = 25%.
Changes in interest rate environment (C-3) risk

The MCCSR capital for change in interest rate environment risk $\rho(X_3)$ is given by Equation (A.6). It depends on the type of insurance product as well as the remaining guarantee period under the contract.

$$\rho(X_3) = \text{MCCSR interest risk factor} \times \text{CALM reserve amount} \quad (A.6)$$

In OSFI (2008), the C-3 RBC factors varied by the remaining guarantee period at the valuation date as shown in table A.3. The factors in the table were halved for the hypothetical term-life product of Section 3.2 since there are no guaranteed cash surrender values.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Guarantee Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>Less than 5 years</td>
</tr>
<tr>
<td>0.02</td>
<td>Less than 10 years, greater than or equal to 5 years</td>
</tr>
<tr>
<td>0.03</td>
<td>Greater than 10 years</td>
</tr>
</tbody>
</table>

Table A.3: MCCSR factors for C-3 risk

A.3 U.S. RBC formula

A general overview of the determination and aggregation of the component U.S. RBC amounts was provided in Section 1.1. For more specific details on the US RBC formula, see NAIC (2008).

The calculation of the asset default, insurance, interest and business risk capital charges that were used to calculate the results of Section 3.2 are presented below.
Asset default risk (C-1)

US RBC capital for default risk $\rho(X_1)$ is based on Equation (A.7).

$$\rho(X_1) = \text{US RBC default risk factor} \times \text{statement value of assets} \quad (A.7)$$

Insurance risk (C-2)

Life RBC factors are applied to each tranche of net amount at risk in accordance with table A.4.

<table>
<thead>
<tr>
<th>Net Amount at Risk</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>First 500 Million</td>
<td>0.0023</td>
</tr>
<tr>
<td>Next 4,500 Million</td>
<td>0.0015</td>
</tr>
<tr>
<td>Next 20,000 Million</td>
<td>0.0012</td>
</tr>
<tr>
<td>Over 25,000 Million</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

Table A.4: U.S. RBC factors for C-2 risk

Interest rate risk (C-3)

The C-3 RBC factor is based on the type of insurance product including the presence or absence of liberal withdrawal provisions which would expose the insurer to greater risk from interest rate fluctuations. For those products that do not fall under the
scope of C-3 Phase I and C-3 Phase II, the C-3 risk capital requirements $\rho(X_3)$ are generally determined by Equation (A.8).

$$\rho(X_3) = \text{US RBC factor for C-3 risk} \times \text{formula-based reserves} \quad (A.8)$$

The pre-tax factor of 0.0115 which is applicable to the Low-Risk Category products was used to determine the interest rate RBC for the term-life insurance portfolio example in Section 3.2.

**Business risk (C-4)**

Regulatory capital for exposure to general business risk, $\rho(X_4)$, is determined by Equation (A.9).

$$\rho(X_4) = \text{US RBC factor for C-4 risk} \times \text{premium income} \quad (A.9)$$

The pre-tax C-4 factor of 3.08% was used to determine the business risk capital charge for the term-life insurance portfolio example in Section 3.2.

**A.4 Solvency II standard formula**

A general overview of the Solvency II framework was provided in Section 1.1.

The formulas for mortality, lapse, credit spread, interest rate and operational risks that were used to calculate the results of Section 3.2 are stated below.
Life underwriting: mortality risk

The capital charge for mortality risk, $\rho(X_{mort})$, was based on Equation (A.10).

$$\rho(X_{mort}) = \sum_i (\Delta NAV|mortshock)$$  \hspace{1cm} (A.10)

where $i$ denotes each policy subject to mortality risk. The remaining terms are:

- $\Delta NAV$ is the change in net asset value given the mortality shock
- $mortshock$ is a permanent increase of 15% applied to mortality rates at every age

Life underwriting: catastrophic risk

The capital charge for exposure to catastrophic mortality risk, $\rho(X_{cat})$, was based on Equation (A.11).

$$\rho(X_{cat}) = \Delta NAV|catshock = \sum_i \text{Net Amount at Risk}_i \times 0.0015$$  \hspace{1cm} (A.11)

The terms in the formula are:

- subscript $i$ denotes summation over each policy subject to mortality risk i.e. all term life policies at the valuation date
- $\Delta NAV$ is the change in net asset value given the catastrophic mortality shock
- $catshock$ is an absolute increase in mortality rates of 1.5 per thousand

133
Life underwriting: lapse risk

The formula for the capital charge for lapse risk, $\rho(X_t)$, is calculated on a policy by policy basis using Equation (A.12).

$$\rho(X_t) = \max(Lapse_{\text{down}}; Lapse_{\text{up}}; Lapse_{\text{mass}})$$  \hspace{1cm} (A.12)

where:

- $Lapse_{\text{down}}$ is the capital charge for a permanent decrease in lapse rates
  $$Lapse_{\text{down}} = \sum_i (\Delta NAV|lapseshock_{\text{down}})$$
  where $lapseshock_{\text{down}}$ is a permanent decrease in lapse rates of 50% at all policy durations where a lapse event would incur a payout that is smaller than the best estimate liability at that duration i.e. negative surrender strain

- $Lapse_{\text{up}}$ is the capital charge for a permanent increase in lapse rates
  $$Lapse_{\text{up}} = \sum_i (\Delta NAV|lapseshock_{\text{up}})$$
  where $lapseshock_{\text{up}}$ is a permanent increase in lapse rates of 50% at all policy durations where a lapse event would incur a payout that is greater than the best estimate liability at that duration i.e. positive surrender strain

- $Lapse_{\text{mass}}$ is the risk capital charge for a mass lapse event. It is calculated as 30% of the sum of the surrender strains for policies where the surrender strain is positive.
Market: credit spread risk

The capital charge for the spread risk of bonds, $\rho(X_{sp})$, was calculated using Equation (A.13).

$$\rho(X_{sp}) = \sum_i MV_i \cdot \text{dur}_i \cdot F_{up}(\text{rating}_i)$$  \hspace{1cm} (A.13)

where:

- $MV_i$ is the market value of each bond $i$ subject to credit spread risk.
- $\text{dur}_i$ is the duration of bond exposure $i$.
- $F_{up}(\text{rating}_i)$ is a function of the credit rating class of the bond that is calibrated to a 99.5% VaR metric for credit spread related market value losses.

<table>
<thead>
<tr>
<th>Rating</th>
<th>$F_{up}$</th>
<th>Duration floor</th>
<th>Duration cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.9%</td>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>AA</td>
<td>1.1%</td>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>A</td>
<td>1.4%</td>
<td>1</td>
<td>23</td>
</tr>
<tr>
<td>BBB</td>
<td>2.5%</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>BB</td>
<td>4.5%</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>B or lower</td>
<td>7.5%</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Unrated</td>
<td>3.0%</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>

Market: Interest rate risk

The capital charge for the interest rate risk due to the mismatch of asset and liability cashflows $\rho(X_{int})$ was calculated using Equation (A.14).
\[
\rho(X_{int}) = \max(Mkt_{int}^{up}, Mkt_{int}^{down})
\]  
\hspace{1cm} (A.14)

where:

\[
Mkt_{int}^{up} = \Delta NAV|upshock
\]

is the change in the Net Asset Value due to a prescribed upward shock to interest rates. \(Mkt_{int}^{down} = \Delta NAV|down\) is the change in net assets due to a prescribed downward shock to interest rates. The interest rate shocks are calibrated to VaR 99.5% over a one-year period.

### Operational risk

The current-year capital charge for operational risk, \(\rho(X_{op})\), was based on Equation (A.15).

\[
\rho(X_{op}) = \min(0.3 \times \text{BSCR}; \max(0.03 \times \text{EP}; 0.003 \times \text{TP}))
\]  
\hspace{1cm} (A.15)

where:

- BSCR is the Basic Solvency Capital Requirement i.e. the sum of all the risk charges calculated above including diversification credits, and before adjustments for the risk reduction arising from future profit sharing and deferred taxes (which we have ignored).

- EP is the total earned premium (gross of reinsurance, if any) for the term life portfolio in the year following the valuation date.
• TP are the Technical Provisions on the valuation date. Best estimate provisions are required to be used in the computation to avoid circularity (since the cost of capital margin in TP includes a charge for operational risk).
Appendix B

Assumptions for Chapter 4 calculations

B.1 Mortality table assumption

The mortality rates that were used to perform the calculations in Chapter 4 are given in Table B.1 of this appendix.

B.2 Initial interest rates

The interest rates on the starting date of the monte-carlo simulation experiments of Chapter 4 are given in Table B.2 of this appendix.
<table>
<thead>
<tr>
<th>Age</th>
<th>q(x)</th>
<th>Age</th>
<th>q(x)</th>
<th>Age</th>
<th>q(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>0.00109</td>
<td>56</td>
<td>0.00491</td>
<td>81</td>
<td>0.07543</td>
</tr>
<tr>
<td>32</td>
<td>0.00114</td>
<td>57</td>
<td>0.00542</td>
<td>82</td>
<td>0.08332</td>
</tr>
<tr>
<td>33</td>
<td>0.00118</td>
<td>58</td>
<td>0.006</td>
<td>83</td>
<td>0.09187</td>
</tr>
<tr>
<td>34</td>
<td>0.00119</td>
<td>59</td>
<td>0.00666</td>
<td>84</td>
<td>0.10112</td>
</tr>
<tr>
<td>35</td>
<td>0.00118</td>
<td>60</td>
<td>0.0074</td>
<td>85</td>
<td>0.11113</td>
</tr>
<tr>
<td>36</td>
<td>0.00116</td>
<td>61</td>
<td>0.00824</td>
<td>86</td>
<td>0.12191</td>
</tr>
<tr>
<td>37</td>
<td>0.00115</td>
<td>62</td>
<td>0.00919</td>
<td>87</td>
<td>0.13354</td>
</tr>
<tr>
<td>38</td>
<td>0.00115</td>
<td>63</td>
<td>0.01026</td>
<td>88</td>
<td>0.14605</td>
</tr>
<tr>
<td>39</td>
<td>0.00118</td>
<td>64</td>
<td>0.01147</td>
<td>89</td>
<td>0.15949</td>
</tr>
<tr>
<td>40</td>
<td>0.00122</td>
<td>65</td>
<td>0.01284</td>
<td>90</td>
<td>0.17393</td>
</tr>
<tr>
<td>41</td>
<td>0.00131</td>
<td>66</td>
<td>0.01439</td>
<td>91</td>
<td>0.18941</td>
</tr>
<tr>
<td>42</td>
<td>0.0014</td>
<td>67</td>
<td>0.01614</td>
<td>92</td>
<td>0.20598</td>
</tr>
<tr>
<td>43</td>
<td>0.00152</td>
<td>68</td>
<td>0.0181</td>
<td>93</td>
<td>0.22371</td>
</tr>
<tr>
<td>44</td>
<td>0.00164</td>
<td>69</td>
<td>0.02031</td>
<td>94</td>
<td>0.24263</td>
</tr>
<tr>
<td>45</td>
<td>0.00178</td>
<td>70</td>
<td>0.02279</td>
<td>95</td>
<td>0.26281</td>
</tr>
<tr>
<td>46</td>
<td>0.00194</td>
<td>71</td>
<td>0.02555</td>
<td>96</td>
<td>0.28427</td>
</tr>
<tr>
<td>47</td>
<td>0.00212</td>
<td>72</td>
<td>0.02864</td>
<td>97</td>
<td>0.30704</td>
</tr>
<tr>
<td>48</td>
<td>0.00231</td>
<td>73</td>
<td>0.03207</td>
<td>98</td>
<td>0.33115</td>
</tr>
<tr>
<td>49</td>
<td>0.00253</td>
<td>74</td>
<td>0.03587</td>
<td>99</td>
<td>0.35658</td>
</tr>
<tr>
<td>50</td>
<td>0.00277</td>
<td>75</td>
<td>0.04007</td>
<td>100</td>
<td>0.38543</td>
</tr>
<tr>
<td>51</td>
<td>0.00304</td>
<td>76</td>
<td>0.04471</td>
<td>101</td>
<td>0.42814</td>
</tr>
<tr>
<td>52</td>
<td>0.00334</td>
<td>77</td>
<td>0.0498</td>
<td>102</td>
<td>0.49724</td>
</tr>
<tr>
<td>53</td>
<td>0.00367</td>
<td>78</td>
<td>0.05539</td>
<td>103</td>
<td>0.60527</td>
</tr>
<tr>
<td>54</td>
<td>0.00403</td>
<td>79</td>
<td>0.0615</td>
<td>104</td>
<td>0.76478</td>
</tr>
<tr>
<td>55</td>
<td>0.00445</td>
<td>80</td>
<td>0.06817</td>
<td>105</td>
<td>1</td>
</tr>
</tbody>
</table>

Table B.1: 1986-92 Canadian CIA basic male, ultimate, non-smoker, nearest age
<table>
<thead>
<tr>
<th>Term (years)</th>
<th>Spot yields</th>
<th>Par yields</th>
<th>Term (years)</th>
<th>Spot yields</th>
<th>Par yields</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.004000</td>
<td>0.0040</td>
<td>16</td>
<td>0.037360</td>
<td>0.0357</td>
</tr>
<tr>
<td>2</td>
<td>0.009500</td>
<td>0.0095</td>
<td>17</td>
<td>0.038070</td>
<td>0.0363</td>
</tr>
<tr>
<td>3</td>
<td>0.014500</td>
<td>0.0144</td>
<td>18</td>
<td>0.038780</td>
<td>0.0368</td>
</tr>
<tr>
<td>4</td>
<td>0.018800</td>
<td>0.0186</td>
<td>19</td>
<td>0.039490</td>
<td>0.0373</td>
</tr>
<tr>
<td>5</td>
<td>0.023100</td>
<td>0.0227</td>
<td>20</td>
<td>0.040200</td>
<td>0.0379</td>
</tr>
<tr>
<td>6</td>
<td>0.026200</td>
<td>0.0256</td>
<td>21</td>
<td>0.040210</td>
<td>0.0379</td>
</tr>
<tr>
<td>7</td>
<td>0.029300</td>
<td>0.0285</td>
<td>22</td>
<td>0.040220</td>
<td>0.0380</td>
</tr>
<tr>
<td>8</td>
<td>0.030567</td>
<td>0.0297</td>
<td>23</td>
<td>0.040230</td>
<td>0.0381</td>
</tr>
<tr>
<td>9</td>
<td>0.031833</td>
<td>0.0308</td>
<td>24</td>
<td>0.040240</td>
<td>0.0381</td>
</tr>
<tr>
<td>10</td>
<td>0.033100</td>
<td>0.0320</td>
<td>25</td>
<td>0.040250</td>
<td>0.0382</td>
</tr>
<tr>
<td>11</td>
<td>0.033810</td>
<td>0.0326</td>
<td>26</td>
<td>0.040260</td>
<td>0.0382</td>
</tr>
<tr>
<td>12</td>
<td>0.034520</td>
<td>0.0333</td>
<td>27</td>
<td>0.040270</td>
<td>0.0383</td>
</tr>
<tr>
<td>13</td>
<td>0.035230</td>
<td>0.0339</td>
<td>28</td>
<td>0.040280</td>
<td>0.0383</td>
</tr>
<tr>
<td>14</td>
<td>0.035940</td>
<td>0.0345</td>
<td>29</td>
<td>0.040290</td>
<td>0.0384</td>
</tr>
<tr>
<td>15</td>
<td>0.036650</td>
<td>0.0351</td>
<td>30</td>
<td>0.040300</td>
<td>0.0384</td>
</tr>
</tbody>
</table>

Table B.2: Interest rates on September 30, 2009
B.3 The C-3 phase III interest rate generator: model specification

The following is a description of the Stochastic Log Volatility (SLV) Model. The SLV model simulates the following three (3) correlated stochastic processes in discrete monthly time:

1. \( i_t \): The natural logarithm of the long maturity interest rate
2. \( \alpha_t \): The nominal spread between the long and short maturity rates
3. \( \nu_t \): The natural logarithm of the volatility of the long maturity rate process

The SLV discrete time equations are:

\[
\begin{align*}
\log_{11r_t} &= \max(\lambda_U, \min(\lambda_L, (1 - \beta_1) \cdot i_{t-1} + \beta_1 \cdot \log \tau_1 + \psi(2\tau_t - \alpha_{t-1}))) + 1 \sigma_1 \cdot Z_t \tag{B.1} \\
\alpha_t &= (1 - \beta_2) \cdot \alpha_{t-1} + \beta_2 \cdot \tau_t + \phi(1i_{t-1} - \log \tau_1) + 1 \sigma_2 \cdot Z_t \cdot (1r_{t-1})^\theta \tag{B.2} \\
\nu_t &= (1 - \beta_3) \cdot \nu_{t-1} + \beta_3 \cdot \log \tau_3 + \sigma_3 \cdot \nu_t \tag{B.3}
\end{align*}
\]

where

- \( \log_{11r_t} \) = \( \log 11r_t \)
- \( \lambda_U = \log 11r_{\text{max}} \)
- \( \lambda_L = \log 11r_{\text{min}} \)
- \( 2r_t = \exp 1i_t - \alpha_t \)
- If \( 2r_t < 2r_{\text{min}} \), then \( 2r_t = \kappa \cdot 1r_t \cdot \alpha_t \)
- \( \sigma_t = \exp(\nu_t) \)
- \( 1Z_t, 2Z_t \) and \( 3Z_t \sim N(0, 1) \) with constant correlation matrix \( \rho \)
B.4 Simulated sample key interest rates from the C3 Phase III generator

The sample distributions of the 1, 7, 10 and 30 year spot interest rates based on the 10,000 C3 Phase III generator scenarios that were used in the analysis of Chapter 4 are summarized in Figure B.1 of this appendix.
Figure B.1: C3 Phase III scenarios: sample key rates
Bibliography


Eling, M. (2010). What do we know about market discipline in insurance?. University of Ulm, Germany.


OSFI (2010b). Minimum Continuing Capital and Surplus Requirements (MCCSR) for Life Insurance Companies.


Towers Perrin (2004). How Might the Presentation of Liabilities at Fair Value Have Affected the Reported Results of U.S. Property and Casualty Insurers?
