Sizing & Allocation of PV Units in Distribution Systems

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

The thesis focuses on allocation and sizing of PV units targeting the minimization of total cost of electricity purchase from the grid taking in consideration the capital cost of the units. The PV sizing and allocation problem is formulated as a mixed integer non linear programming (MINLP) problem where the objective is to supply local loads and if excess generation is available, the PV system would sell electricity back to the grid. The study is performed on the 13 bus radial feeder. The allocation and sizing problem of PV units is performed following two approaches. In the first approach, the problem is studied under demand-supply balance; while in the second approach, the problem is investigated under AC power balance.

Since claims about PV units' payback time exist in practice, this thesis considers a different strategy based on which it would calculate the PV units payback time. It considers the capacity factor of PV units that represents a percentage of PV output power depending on the availability of solar radiation. The proposed problem formulation in this work can become a good tool for both utility and customers. For utilities, the model proposed can provide an insight on the price of electricity that should be paid for green PV energy. From a customer's perspective, the proposed model can provide the customer with a more accurate estimate of the PV payback time since the model takes into account the variability in PV as well as the fact that not all PV generation will be exported to the grid at a given moment. It would set the prices at which the customer would sell electricity to the grid at certain age of PV units and would investigate the PV operation period at which the system would consider their availability to be an advantage at the current PV electricity selling price as available in the market.

Finally, the model presented in the study can be adapted to fit any region in the world taking into account two major factors, the electricity market price in the area and the capacity factor of PV units.

Acknowledgements

I praise and glorify Allah who creates a vicegerent on earth and who knows what I do not know. Thanks goes to Allah giving me strength and support during the master degree journey.

I am heartily thankful to my home country United Arab Emirates and its rulers for their continuous encouragement and support. My deepest gratitude goes to Sheikh Mohammed Bin Zayed Al Nahayan for inspiration.

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I would like to express my sincere gratitude to my supervisors Professor Mehrdad Kazerani and Professor Hatem Zeineldien for their endless encouragement and invaluable guidance throughout this journey. My appreciation is also extended to Professor Magdy Salama for his encouragement.

I have been lucky enough to be raised in a loved family. Life without my parents would not be the same. I would like to dedicate flowers: The first flower is a red rose and goes for my mother; the second flower is a purple flower and is dedicated to my father. Gladiolus flowers are dedicated to my beloved brother Abdulla and my uncle Hussein, yellow roses to my sisters and brothers and a lily to me. Since flowers cannot be green, I would like to dedicate a white flower to my brother Faisal who passed away before my graduation.

My deepest thanks go to my best friend in Waterloo, Zainab Meraj for her care and support. I cannot imagine how life would be without her friendship.

To my colleagues, I would like to thank them for their advice and continuous encouragement.

Lastly, my regards go to all of those who supported me in any respect during the completion of my thesis.

Dedication

To His Highness Sheikh Mohammed Bin Zayed Al-Nahayan

With my greatest gratitude

Table of Contents

List of Figures

List of Tables

Chapter 1
Introduction

Distribution Generation (DG) and Renewable Energy Sources (RES) have attracted the attention of different countries in the world. The term distributed generation can be defined as the local generation of electricity that can be associated with heat generation in a cogeneration system. Different terminologies can be used to refer to distributed generation. These are on site generation, dispersed generation, embedded generation, decentralized generation or decentralized energy.

The distribution generation technology is developing very fast in many countries as its resources provide clean energy. Abu Dhabi City located in UAE has a bold vision to transform itself to a global leader in the field of sustainable energy. Masdar city is a global cooperative platform initiated by the city of Abu Dhabi in order to determine solutions for energy security, climate change and sustainable human development. It has been chosen as the headquarters for International Renewable Energy Agency.

Solar cells become an attractive option of renewable energy sources when considering the climate of the country. This thesis targets the selection of optimal sizes and locations of the PV units in the city by minimizing the total cost of electricity purchase from the grid plus the PV capital cost minus the electricity sold to the grid, but since the city is under establishment, the study is performed on a 13 bus radial feeder that is originally adapted from IEEE 13 bus feeder but with certain modifications made to the system.

The domain of this thesis is in the field of modeling and analysis. The optimization problem is solved using mixed integer non linear programming. The problem is solved taking into account power balance constraints, total power loss, voltage limits, transmission line limits, PV unit’s capacity and its initial cost limits.

1.1 Thesis Objective

There are two important factors that determine the payback time of PV units which are the PV capital cost, the electricity price as well as the PV capacity factor. Thus, the main objective of this thesis is twofold:

1. To determine the payback time of PV units taking into account PV output variation.
2. To determine the utility electricity price at which the PV payback time would be reasonable.

1.2 Thesis Outline

The proposed work is interesting as it is a suggestion for a real application on a city under establishment. The thesis is organized as follows:

Chapter 2 provides the reader with an overview of distribution generation and renewable energy resources. It presents the advantages of distribution generation.

Chapter 3 introduces photovoltics that is the selected distributed generation class to be used and shows the availability of solar energy over the world. It presents Abu Dhabi vision toward sustainable renewable energy reflected into Masdar city. Moreover, it describes the photovoltaic system and the materials used to manufacture solar cells.

Chapter 4 starts with introducing the objective of the thesis that is to determine the optimal size and location of PV units in different distribution systems under demand-supply balance and AC power balance. It presents the software used "GAMS" as a simulation tool for the studies. It also presents the data to be integrated with the system under study to achieve the target of the thesis and considers the first type of study which is to allocate the PV units and size them under demand-supply balance.
It discusses the optimization problem and the models needed.

Chapter 5 is focused on finding the optimal size and location of PV units under demand-supply balance. It presents the optimization problem formulation for this type of study and discusses the determined results. Moreover, the recommendation is also presented for this study.

Chapter 6 considers the problem of sizing and allocation of PV units under AC power balance and provides the reader with the results and recommendation.

Chapter 7 summarizes the project and suggests future work.

Chapter 2

Distribution Generation & Renewable Energy Sources

The distribution generation technology is developing very fast in many countries as its renewable resources provide clean energy. This chapter defines distribution generation. It presents its advantages and provides the reader with an overview of renewable energy sources.

2.1 Definition

Distribution Generation (DG) can be defined as electricity generation at small scale to satisfy the demands close to the load being supplied at distribution level voltage [1],[2]. There is no common agreed definition on DG. The installation and operation of electric power generation units that are connected to the network on the customer side of meter are recognized as distribution generation [3]. CIRED identified five factors behind the increased interest in DG. These are reduced gaseous emissions, completion policy deregulation, variety of energy sources, the efficiency of energy or logical use of energy and the requirement of national power [4]. CIGRE added to these factors the following: presence of modular plant for generation, locations for smaller generators that are to be found easily, the length of the construction period that is to be short and the capital costs of smaller plants that should be low. Generation may be sited closer to load resulting in a reduction in the transmission costs [5].

Building new transmission lines in an energy plant would be expensive. This problem can be avoided through the investment in DG. Since there are a variety of energy sources and a reliable grid is targeted, DG is considered as an attractive option. DG is considered to be a flexible technology as it is capable of meeting peak demands of power. Such flexibility is coupled with load profile, cost, reliability and availability of energy as power supply might be needed to be uninterrupted. The environment is a major concern as carbon emission results in pollution. Such factors define the reasons behind the increased interest in DG based on the definition of International Energy Agency (IEA) [2], [6], [7].

2.2 National Eras in the Development of Electricity Generation

Central station plants have been in use to generate electric power as a matter of economics of scale [8]. The first main power plant was opened by George Westinghouse in Niagra Falls in 1895 by using alternating current [9].

Some of electricity customers mostly in the industrial field decided to run their own power generator considering the economical perspective. Besides that, facilities such as hospitals and telecommunication sectors use their own power generators during power outage. These power generators were under the control of the customers rather than the utilities, which made it advantageous in overall as customers are supplied with their demands especially when operating away from the grid rather than purchasing electricity from the local electricity provider or when it is not possible to supply the customers operating away from it. Under the first circumstance, it is possible to expand the electricity network as the electricity to be generated to supply such customers is not utilized and it can be redirected to the network to be invested [10].

The utility has decided to switch from the economics of scale to what is called mass production [11]. In 1970s and 1980s, the system of electricity in some countries was developed to be hybrid including both centralized and distributed generation units as environmental concerns started to arise because of the availability of natural gas for power generation [12]. Based on [9], 2% of the energy in the U.S was involved in producing electricity in 1920; while today the percentage is much more above that.

2.3 Distribution Generation Resources

Electricity can be generated at large scale or can be distributed. DG resources can be classified into two categories, combined heat and power resources and renewable energy resources. The first class can be defined as the generation of both electricity and heat through the use of power generator [13]. The renewable energy sources are these natural sources of energy that are considered to be over lasting [1]. These sources include hydro-power, biomass, wind, solar, geothermal, wave and tidal energy and biodegradable waste sources. It is important to note that such resources belonging to the two described classes are considered as DG under certain characteristics determined by the scale of generation, location and application. Such resources are classified in Table 2-1 to distinguish them from large scale generation of electricity and heat.

Table 2-1 Classification of RES

Generation	Combined heat &power	Characteristic	Renewable energy source	Characteristic
Large Scale Generation.	Large district heating >50MW	Scale	Large hydro >10MW	Scale
	Large industrial combined heat and power >50MW	Application & scale	Offshore wind	Location
			Co-firing biomass in coal power plant	Application
			Geothermal energy	Scale
DG	Medium district heating	Scale	Medium & small hydro	Scale
	Medium industrial combined heat and power	Scale & application	Onshore wind	Location
	Commercial combined heat and power	Application	Tidal energy	Scale
	Micro combined heat and power	Scale	Biomass and waste incineration/gasification	Scale & application
			Solar energy	Scale

2.4 Distribution generation Capacity

The capacity of DG is based on the capacity of the distribution system to which the DG is connected and the voltage level of such system [14]. Table 2-2 shows the most common DG capacity rating.

Table 2-2 DG capacity

Class	**Capacity rating of DG**
Micro	1W-5KW
Small	5KW-5MW
Medium	5MW-50MW
Large	50MW-300MW

The energy supplied by DG is either consumed within the distribution system or fed back to the transmission system if the energy produced is greater than what the load in a distribution system requires [14].

2.5 Advantages of DG and Regulations

Distribution generation is a term that includes a variety of technologies, including many renewable technologies, combined heat and power plants , back-up and peak load systems. These technologies provide many advantages including new market opportunities and improved competitiveness in the industrial sector [6].

In the future, DGs are expected to transform distribution networks [15]. Figure 2-1 represents some of the advantages of the DG and renewable energy sources. This section will briefly describe the advantages offered by DG and RES.

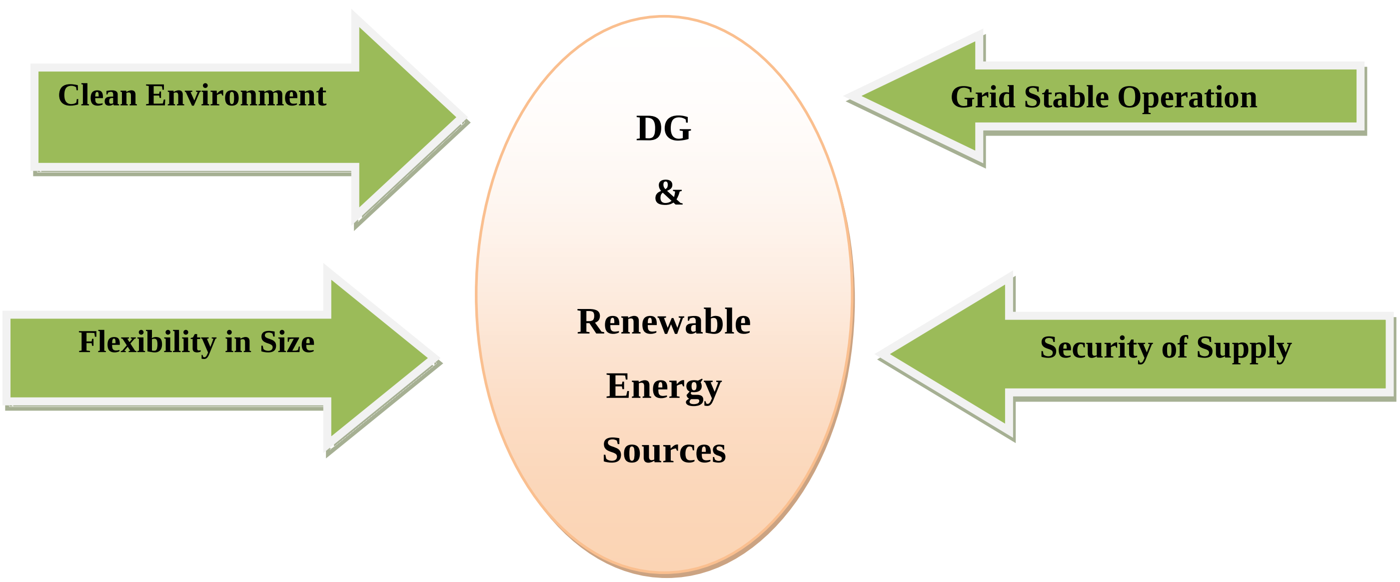

Figure 2-1 DG and RES advantages

2.5.1 Electricity Market Liberalization & Flexibility Factor

DG permits the players in the electricity sector to respond in a flexible manner to the changes in the conditions of the market. The flexibility of the technologies of DG results from their small sizes and the short construction lead times when compared to the large power plants that are centralized [12].

Since DG technologies are flexible in their size and operation, and they can be expanded easily, they play a major role in reacting to electricity price fluctuations. For instance, heat applications in Europe drive the need for DG in the market; while in US, the volatility of price drives the demand for DG [12]. Distributed generation had been developed for the reason of improving the overall fuel efficiency of the power plant.

The demand for DGs is controlled by price volatility. DGs can either operate continuously or for specific periods within the day. The schedule of operation of DG technologies is dependent on the demand of electricity and thermal energy. Furthermore, fuel prices and utility rates contribute to the operation of DGs. When a DG is selected to operate continuously, it would operate for 8760 hours per year in order to supply a continuous power to the demand. This time excludes, the time needed for its maintenance. On the other hand, if the DG is chosen to be operating for part of the day, then it is responsible for the supply of an intermediate power. Intermediate power can be defined as the power generated during the schedule of operation of DG. To decide on the operating schedule of the DG, the difference between the cost of electricity generation and electricity purchasing from the utility should be determined. If the cost of generation is less than the cost of purchasing by the utility, then DG is to operate; otherwise it will not operate [14].

Based on option value theory, [16] and [17] recommend the operation of flexible power plants during the peak periods as this is to be more profitable when compared to the conventional evaluations' option.

A long term market offer for DG at small scale in U.S and worldwide is projected by Gas Research Institute (GRI). Certain issues resembled in the restructuring of the utility would result in uncertainty in the market that would provide a limit on the market penetration. The projection made by GRI for U.S expects the power to be 27GW with a capital equipment purchase of $10 billion by the year 2015 as shown in Figure 2-2 [18].

Figure 2-2 DG market projection for U.S. [18]

2.5.2 Power Quality

Power quality is a term that is composed of two other qualities that are the current and the voltage. Voltage quality is the deviation of the voltage signal from the ideal waveform that is a single frequency sine wave characterized with an amplitude and frequency that are both constant. Similarly, current quality can be defined as the deviation of the current signal from the ideal waveform that is a single frequency since wave characterized with amplitude and frequency that are constant. Voltage quality is the deviation of the voltage signal from the ideal waveform [19].

DG serves as an alternative for a better quality power supply as better electricity is needed. The output power of DG can be DC like in fuel cells, photovoltaic cells and batteries or AC such as the power supplied from micro-turbine and combustion engines [14].

2.5.3 DGs Advantage to the Grid

DGs are to maintain a stable operation of the grid rather to supply directly the customers. Based on the demand of the operator of the grid, DGs generate energy [12]. The transmission grid is characterized by high voltage for high flow of power. Its operation in Europe is at voltage level that is greater than 110KV. Such high voltage for transmission results in a reduction in the losses of the grid. The distribution grid can be classified based on the voltage level in Europe into three levels. The first level is a high voltage level distribution grid with a voltage range between 60KV to 110KV. The second class is a medium voltage level distribution grid with a voltage level varying between 10KV to 50KV. The third class is a low voltage level distribution grid with a voltage ranging between 240V to40V. Figure 2-3 presents a schematic diagram of an average European electricity grid and connection levels for DG and RES voltage levels [20]. It is important to keep in mind that these levels vary from one country to another.

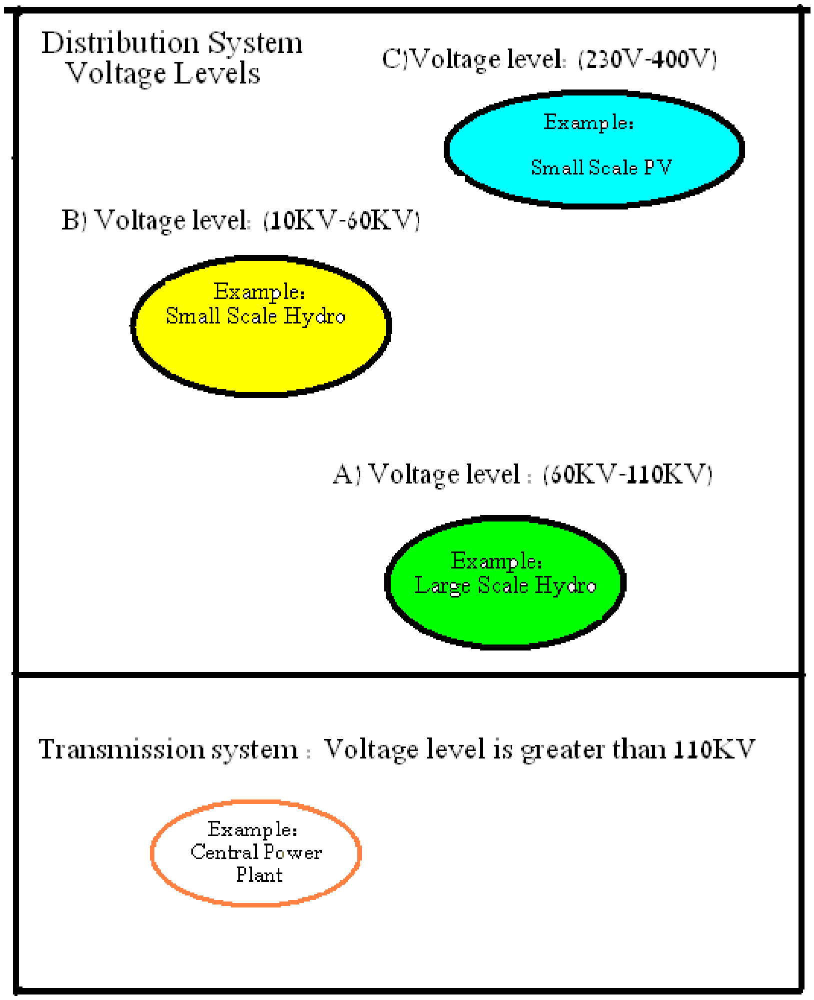

Figure 2-3 Examples on average European electricity grid voltage level at which DG is connected

[20]

2.5.4 DGs & Environment

Environmental issues started rising as a matter of the power industry. Burning fossil fuels to generate energy, results in many pollutants and carbon dioxide. DG can be used to provide clean energy.

Combined heat and power generation can be used for applications where heat and electricity are both in demand rather than using an external boiler to deliver heat and purchasing electricity from the grid. The DG market is partially driven by the availability of more efficient, more cost-effective distribution technologies. CHP conserves energy by 10% to 30% based on the size and consequently the efficiency of cogeneration units. Moreover, through the installation of DG units, it is possible to use cheap fuel [12]. Table 2-3 and Figure 2-4 provide an example on a comparison case between two distribution generators different energy sources both DGs and centralized generation in terms of efficiency and emission [21]. Such comparison is made on the basis of lb/MWh. A higher efficient system is recognized for producing less pollution per MWh [21].

Table 2-3 Comparison between DGs in terms of efficiency and emission of CO_2 [21]

Comparison	**Uncontrolled gas fired lean burn IC engine**	**SCR controlled diesel generator**	**Solid oxide fuel cell**	**Phosphoric acid fuel cell**	**Micro-turbine**	**Gas turbine (small)**
CO_2 (LB/MWh)	1099	1537	867	937	1477	1329
Efficiency (Btu/KWh)	9402	9646	7420	8324	12641	11374

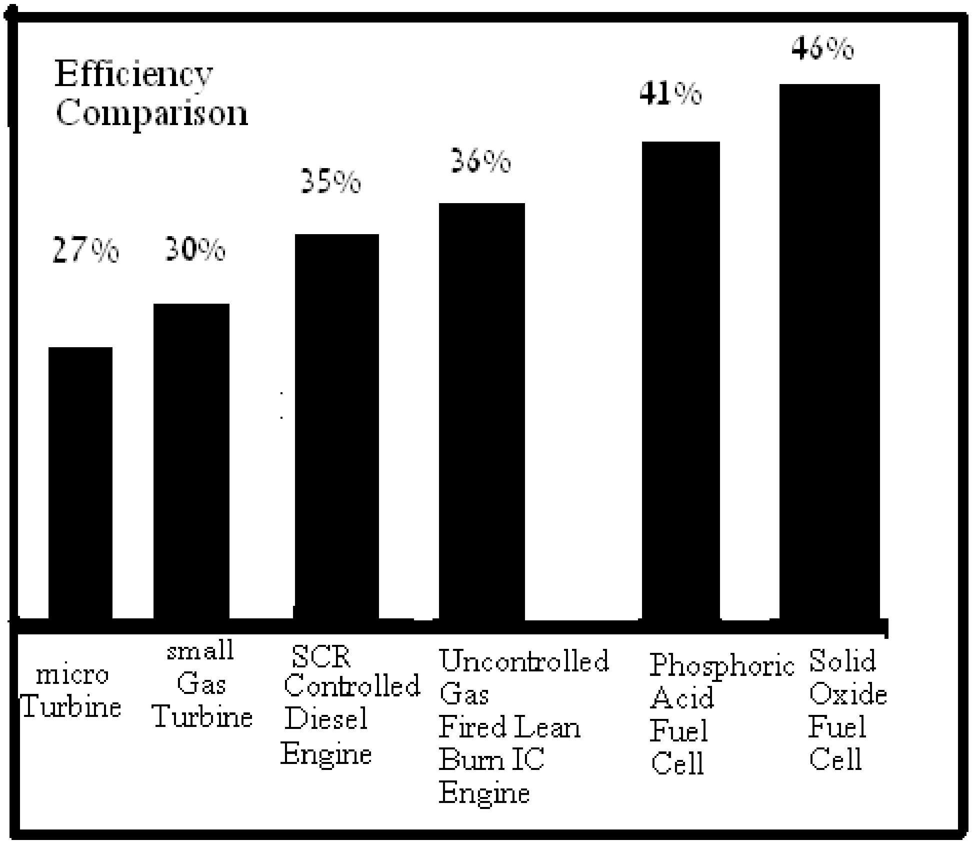

Figure 2-4 Efficiency comparison of different energy sources [21]

2.5.5 Energy Security

The security of the supply is dependent on the ability to have sufficient generation, secure grid connections, interconnections and mix generation. Energy security can be regionally developed as the sites of distributed generation are selected considering the following factors [22]:

- Emissions
- Energy usage
- Grid utilization

Since the demand and the use of natural gas as a primary source of energy increases, the security of energy decreases. On the other hand, there is a case in which the security of energy is enhanced as efficiency of the fuel is high and its consumption is low. This case is CHP in which both heat and power are generated. The increased penetration of both renewable energy sources and DGs with a high efficiency in terms of energy results in a secure supply through a reduction in the energy imports and the establishment of a diverse portfolio of energy [12],[23].

2.5.6 Points to Consider while Dealing with DGs

There are certain points to be considered while dealing with DGs such as operating frequency of the system, voltage profile, reactive power, power conditioning and system protection. Frequency deviation is encountered when DG is connected to the system and this deviation should be kept as low as possible to achieve the right performance of the system by careful planning of DG installation. DG can cause instability problems on voltage profile as a matter of bi-directional power flow of the current that makes it difficult to tune protection schemes. When DGs at small and medium sizes are used, they may not produce reactive power as they use asynchronous generators. This problem can be solved by having a DG unit with a power electronic interface for certain reactive power production. In terms of power conditioning, PV and fuel cells as examples of DGs are capable of DC production. As a result, a DC/AC inverter is needed to connect them to the grid which results in harmonics [12]. Specific control techniques can be applied to control the injection of these harmonics [14].

The increase in the rate of electricity as well as the rates of standby and backup in areas that are served by utility, result in off grid application which is the most economical mean for electricity production for remote areas with high load factors [24].

In off grid applications, the site is disconnected from the electric grid. In such applications, DG units are responsible for continuous generation with backup capabilities onsite. In addition, usually utilities assess DG operators with capacity charges that are dependent on the size of the DG system. Utility charges different rates for both energy and demand when the DG system is down [24].

2.5.7 Cost

From an investment point of view, it is most likely to be much easier to locate RES and DG compared to large central power plant. Furthermore, the time needed to have DG units on the site is shorter than what will be spent in constructing in a power plant.

The capital cost for installing a DG is high when compared to large central plants. The capital cost differs from one DG technology to another. For example, the capital cost for combustion turbine is $1292.24 US/ KW; while it is $25848 US/KW for fuel cells [2].

DGs can reduce the transmission and distribution losses as well as the transmission and distribution costs. As the customer size is smaller, the sharing price of the transmission and distribution in the electricity cost is larger. This is greater than 40% for households [2]. Using DG results in a reduction in the losses of the grid by 6.8% and this will contribute to 10% to 15% saving of the cost [2], [25].

As a matter of the vital role that renewable energy plays in the reduction of CO_2, such reduction in gaseous emission is reflected into cost. Moreover, Jobs have been created as a matter of the investments carried out in the field of renewable energy.

Chapter 3
Photovoltaic System

Photovoltaic (PV) system can be defined as an energy system that converts the solar energy into electricity through the use of semiconductor materials compromising PV cells. The PV cells are connected together in different combinations that can be series, parallel or both to form PV arrays. They might be integrated with storage banks to save the excess generated energy or to act as a battery back up to supply electricity when needed. PV systems can be classified into two classes based on the application they are used for. The first class is the grid connected PV system; while the second class is the stand alone PV system. The following sections will provide a brief description of the materials used to design PV and will describe the differences between the two PV systems [26]. Moreover, it presents the different classes of storage system and the selected storage system to be involved in this thesis.

3.1 Motivation to Use PV

The economy of United Arab Emirates had been dependent on crude oil and gas exports as the main source of national income. Its reserve of crude oil is 9.5%; while the reserve of natural gas is 3.5% [27].

United Arab Emirates is located in region where solar energy is widely available, which makes solar energy an attractive option to be invested to raise the national income while maintain sustainability and clean environment. The sustainability is defined by the integration of economy, society and environment Abu Dhabi city, the capital of United Arab Emirates, occupies 80% of the country land with 30% of the total population. The city sets a long term vision to be not reliant on fossil fuel and to maintain a safe environment [27].

When selecting a renewable energy resource, certain requirements should be studied in advance. These are limits in the efficiency, size of the plant, structural restrictions that is considered to be essential in case of PV such that the availability of suitable areas or competitive uses is studied, and reliability of energy supply, and space requirements. When considering solar energy, it can be noted that the natural availability of continental solar irradiation is extraordinary huge as represented by the largest cube in Figure 3-1 [28].

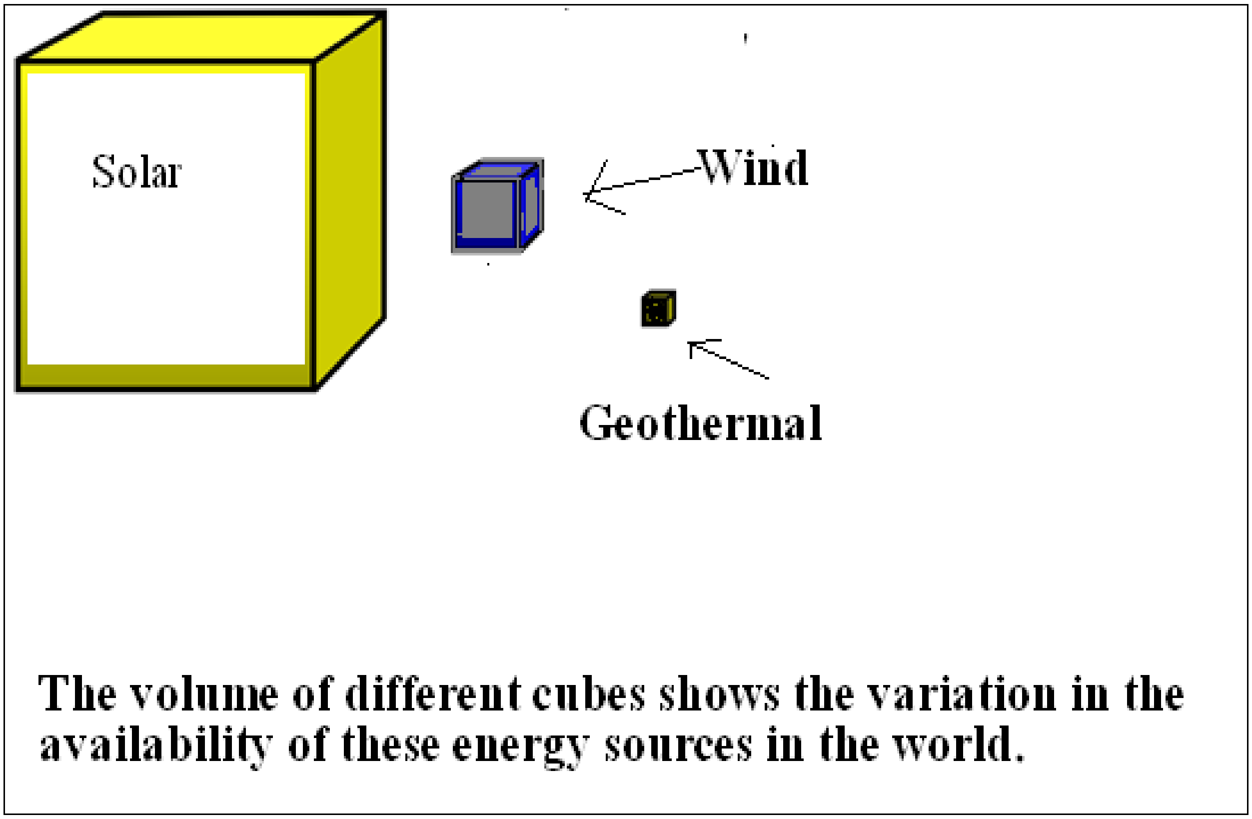

Figure 3-1 Natural availability of continental solar irradiation [28]

The world wide has been attracted by solar technologies as solar energy is sustainable. Masdar initiative at Abu Dhabi is adopting solar technologies in United Arab Emirates. Photovoltaics and concentrating solar power projects are developing in Abu Dhabi such that Masdar is provided with broad coverage of the solar sector [29].

3.2 PV History

The history of photovoltaic (PV) started with the discovery of its effect in 1839. The French physicist Alexandre Becquerel discovered this effect as light was converted to electricity. His experiment considered the use of metal electrodes and electrolyte concluding that conductance increases with insulation. This discovery was followed by two discoveries in 1873 and 1876 when Willoughby Smith discovered the PV effect in selenium and he and his student, William Adams, found about the PV effect when illuminating junction between selenium and platinum existed. In the following year, the first solar cell was implemented. Table 3-1 presents the expansion of PV research after the discovery of the first solar cell [30].

Table 3-1 History of PV

[30]

Year	**Author**	**Discovery**
1904	Albert Einstein	Theoretical explanation of the photovoltaic effect
1916	Robert Millikan's	Practical proof of the t Theoretical explanation of the photovoltaic effect carried out by Albert Einstein
1918	Jan Czochralski	The first silicon solar cells
1951	Dr. Dan Trivich of Wayne State University	Theoretical calculation on solar cell efficiency with different materials
1953	Dr. Dan Trivich of Wayne State University	Theoretical calculation on solar spectrum wavelengths
1954	RCA Laboratories	CdS photovoltaic effect
1955	Hoffman Electronics- Division	The first sun-powered automobile

1961	A United Nation's conference on solar energy application in developing countries	The first photovoltaic conferences
1962	Bell Laboratories	The first commercial telecommunications satellite Telstar. The photovoltaic system peak power for satellite power supply was 14 W
1963	Sharp Corporation	The first solar modules
1969	Roger Corporation	The first bigger company
1972	Solar Power Corporation	The first application of photovoltaic technologies on Earth
1976	NASA LeRC	The first photovoltaic systems for the third world rural areas
1977	NASA LeRC	The first photovoltaic applications for supply of technologically sophisticated devices on Earth
1980	ARCO Solar	Large standalone systems installations
1983	Solarex Corporation	Solar cars
1984	NASA LeRC	The first amorphous solar module
1985	University of New South Wales in Australia	High efficient silicon solar cells and thin film solar module
1997	General Motors Sunracer	Solar car races - a new challenge for research labs
1989	Solarex & ARCO	Third world projects and new production capacities
1990 -1999	ECD, SERI, NREL,BPSI &NREL	Large photovoltaic companies co-operation
2000	Germany & Japan(Sharp & Kyocera)	Renewable energy and the Stock exchange
2002– 2003	Germany	Large photovoltaic plants

3.3 PV Cell

PV cells can be defined as large area semiconductor diodes converting light into electricity as a matter of the photovoltaic effect. An electric field is established at the p-n junction, resulting in the separation of the charge carriers, holes and electrons. As shown in Figure 3-2, once the semiconductor material is strike by sun light with sufficient energy that is the source of excitation, the electrons are released. When a load is connected to the PV cell, a direct current flows. The energy of photons comes in quanta. The energy of each quantum is dependent on the lights' wavelength [31].

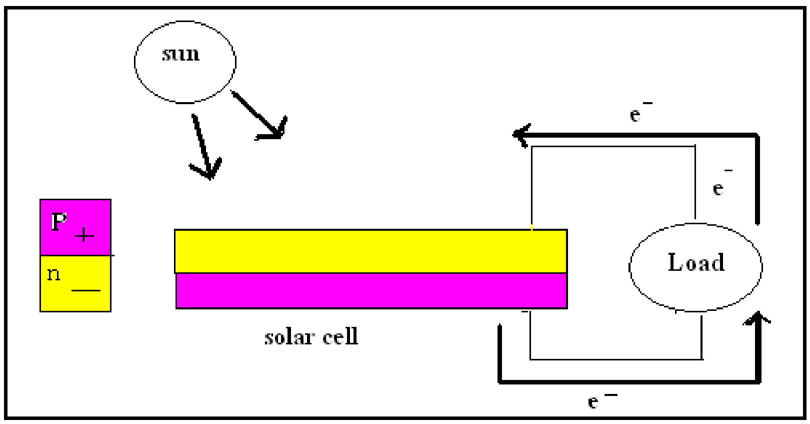

Figure 3-2 PV Cell

3.4 PV Materials

A variety of semiconductor materials are available to manufacture PV cells. Today, silicon is considered to be the most important element to be considered when manufacturing PV. Based on the chemical structure of the silicon, PV can be produced in three forms:

3.4.1 Mono-Crystalline Silicon

Mono-crystalline is made of a single and continuous crystal lattice structure that is characterized with no defects. This category of silicon is expensive and from the manufacture point of view, it is associated with a complicated manufacture process. If PV is to be manufactured from mono-crystalline Silicon, the conversion efficiency will be high [32].

3.4.2 Poly-Crystalline Silicon

Polycrystalline Silicon is composed from many silicon crystals that are small in size. This category of silicon has been used in the MOSFET and CMOS industry since a long time because of its conducting characteristic. Polycrystalline silicon is recognized for showing greater stability when exposed to electric field and light induced stress. Polycrystalline silicon has the advantage of being simpler and cheaper comparing to mono-crystalline silicon; however, its grain boundaries between the crystalline in silicon cell results in a lower efficiency [32].

3.4.3 Amorphous Silicon

Amorphous silicon has lower efficiency compared to mono-crystalline and poly-crystalline silicon. It is characterized to have a low cost and low efficiency [33].

3.4.4 Industrial Perspective toward PV Material

The main differences between the different classes of PV materials in terms of efficiency, cost and power per area are presented in Table 3.2 [33].

Table 3-2PV Material Types

[33]

Solar module material type	Efficiency of solar module	Cost of solar module	Power Area of solar module
A) Mono - crystalline solar module	Varies between 10% to 13%	High cost	High power area
B) Poly - crystalline solar module	Varies between 9% to 13%	Moderate cost	Moderate power area
C)Amorphous solar module	Varies between 6% to 8%	Low cost	Low power area

From the industrial point of view presented in an article in renewable energy access in [34], the industry should focus on utilizing multi-crystalline silicon rather than mono-crystalline silicon even though the latter shows better performance because of the lower cost of the first class of materials. The production cost of mono-crystalline silicon is less than the multi-crystalline one which would be reflected into saving from the economical perspective.

3.5 PV Configuration

Every PV system is composed of photovoltaic units and modules. The module is implemented through a series and parallel combination of solar cells as shown in Figure 3-3 and Figure 3-4. The series connection of PV modules results in a higher voltage while their parallel connection increases the current [38]. The most commercial crystalline modules are made of 36 or 72 solar cells. Once Solar cells are connected, they are placed on tedlar film and covered by tempered glass on their top [30].

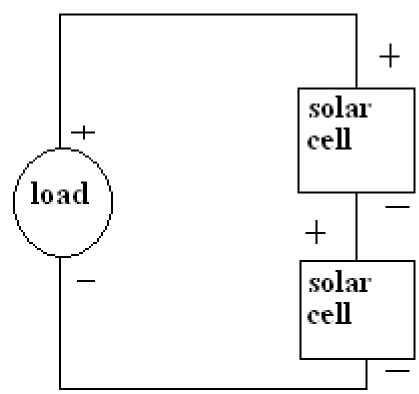

Figure 3-3 Solar cells in series connection

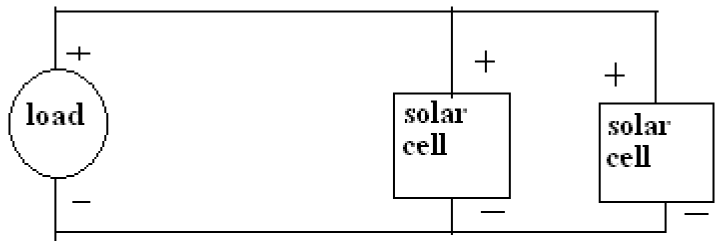

Figure 3-4 Solar cells in parallel connection

Grouping of PV modules results in a PV unit and grouping of PV units results in PV array as shown in Figure 3-5. An array can be achieved by connecting one to thousands of modules together. The output power of an array would vary from few watts to tens of mega watts based on the output power of each module and the number of integrated modules. The output of the array is DC and can be used to supply a load. On the other hand, for a PV array to be connected to the utility grid, the DC output should be converted first to AC through a DC/AC converter. Most converters have an efficiency of 90% [35].

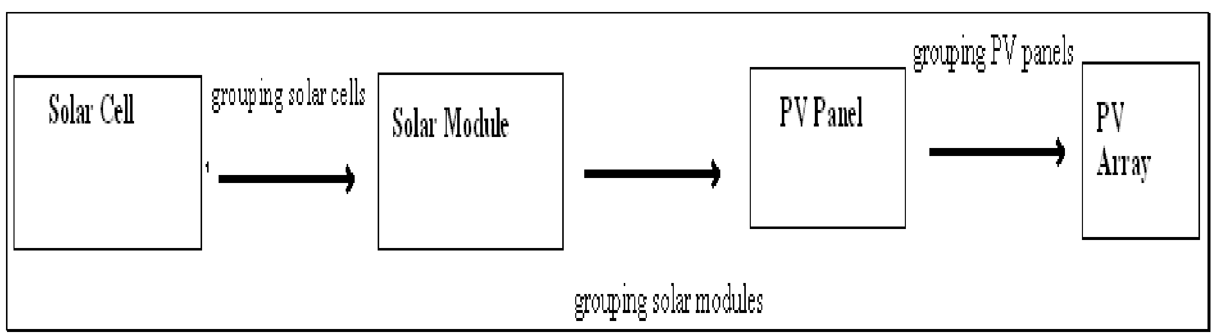

Figure 3-5 PV system

PV system might be integrated with storage banks to save the excess generated energy or to act as a battery back up to supply electricity when needed

3.6 PV System Classes

PV systems can be classified into two classes based on the application they are used for. The first class is the grid connected PV system; while the second class is the stand alone PV system. The following section will describe the differences between the two systems [26].

3.6.1 Stand Alone PV System

Stand alone PV system is capable of supplying the load with power when they are off grid connected. This might take place when there is a fault in the distribution system. Such system is often integrated with back up batteries storing solar energy during the day time to be used at night or when needed [26].

From the economical point of view, this system could be used to supply remote areas as it becomes an attractive option when considering how cost effective it is when comparing this cost to the cost of connecting the load with other utility line extensions [36],[37],[38].

Various research have been carried out to optimize the energy systems. For example, [39] proposes a model to optimize the PV array and storage bank for a stand-alone hybrid wind/PV system taking into account the long term hourly solar insulation level data and the peak load demand data for the selected site. The study presents the number of PV arrays to be used; however, it does not show the influence of such system on the hybrid system cost. Later on, the authors in [39] expanded their study to include the cost of the PV modules and storage bank [40].

3.6.2 Grid Connected PV System

Grid connected PV system is recognized as the latest technology of PV systems. This system is capable to supplement the electricity supplied by the utility company. When the energy level generated by a grid connected PV system is greater than the load level of the customer, the difference in energy can be transferred to the utility. As a matter, the meter of the customer will be turned backward and that would be reflected on the total cost of electricity to be paid by the customer. On the other hand, when the condition that the energy generated from PVs is not enough to meet the demands of the customer, electricity will be purchased from the utility company [26].

The problem with the use of a grid connected PV system is its cost. This problem can be solved in two different aspects. The first solution might be the use of new PV units that are made of less costly materials known for their high energy conversion efficiency. In [41] a set of industrial guidelines for the proper design of industrial PV plants were presented. These guidelines are based on the use of power electronic conditioning systems and simulation aided design tools. Since following this option would require the replacement of the existing PV units with newer one, another alternative could be in needed. The cost of grid connected PV system can be made less if one or more of the following approaches is targeted [42]:

- Maximizing the production of energy from PV units in all operating conditions.
- Minimizing system losses of the distribution system.
- Utilizing grid connected PV system to achieve both the generation of electrical energy and providing auxiliary services such as power quality control and load demand control
- Integrating different renewable energy resources in the distribution system (Hybrid systems).

3.7 Solar Radiation Characteristics

Solar radiance and solar insulation are the main characteristics of solar radiation. The solar radiance can be defined as the instantaneous power density measured in KW/m^2. It is dependable on two main factors which are the location and the local weather. Its measurements include the global and the direct radiation during the day. Pyranometer, pyrheliometer or sunshine recorder are the devices that can be used to make the measurements [43].

The second characteristic of solar radiation is solar insulation that can be defined as the total amount of solar energy received at a certain point during certain period of time. It is measured in KWh/m^2 day. It can be measured from cloud cover data taken from existing satellite images, data during the period at which the sun is shine, global isoflux contours for a complete year, quarter of a year or certain month, solar insulation at certain location averaged over a year, a month or a day, typical mean year data at certain location or calculations for solar radiation [43].

3.8 Previous Studies on optimal Size and Location of RES

In [44], an iterative scheme to find the mix of wind-PV system with a storage system is presented. The optimal size is determined based on the calculated values of life cycle unit cost of power generation or relative excess power generated or unutilized energy probability for a certain deficiency of power supply probability. The authors in [45] apply genetic algorithm to allocate DG in order to reduce losses and improve voltage profile. In [46], the excess capacity of a system composed on PV, wind, hydro and diesel is optimized. Strategic placement of distribution generation capacity is described in [47]. In [48], a heuristic approach is presented to optimize the investment by determining the optimal site and size of DG under the assumption that the DG size is a multiple of a provided capacity. In [49], a method to determine the optimal location and size of PV grid connected systems in distribution systems is presented. The problem was formulated as multi-objective function involving strategies to evaluate both the technical impact associated with improving the stability of the voltage of the feeder and the economical effect associated with the increase of loading limits is used.

Particle swarm optimization algorithm is proposed in [50] in order to allocate three type of DG by minimizing the losses in the system. The location and size of DG is randomly generated. The particle keeps moving from its recent position considering the distance from its local point with a velocity until reaching the global point. The results prove that particle swarm optimization would lead to better results when it is compared to heuristic search technique [50].

Genetic algorithm and tabu search are applied in a new technique to determine the optimal location of dispersed generation is distribution systems. In this algorithm, losses in distribution systems have been reduced when compared to losses in genetic algorithm [51].

The allocation and sizing problem of DG is considered in [52] to minimize system losses. The algorithm followed is tabu search and system losses are determined when DGs are allocated in the system. Numerical simulation has been conducted in order to check the validity of the algorithm.

The optimization problem that targets the allocation and sizing of the distribution generators in distribution system presented in [53] is focused on minimizing system losses taking into consideration, the variability of the load with respect to voltage and frequency. The algorithm followed is genetic algorithm. The loads in the study are considered to be fixed and varying. It has been found that the location of distributed generator is independent on the load model, but the objective function and the size of distributed generator affects by the load model. The variation in the frequency affected the losses of the system and the size of distributed generator. The voltage has improved in the system as a matter of the presence of distributed generator.

A combination of genetic algorithm and simulated annealing for optimal DG allocation in distribution networks is considered in [54]. The problem focuses on minimizing system losses at fixed number of DGs and specific total capacity. This study shows the effectiveness of the proposed mixed algorithm when compared to SGA.

The optimal location of DG that is operating at optimal power factor is determined in [55]. The study follows an analytical method in order to find the location of DG and it places the DG at busses with highest suitability index. The study recommends operating DG at 0.8 unit factor as in order to achieve a better performance of the system. This improvement is in terms of system losses reduction and voltage profile improvement.

In [56], the optimal location and of distribution generators is determined applying bee colony optimization that is a member of swarm intelligence, while considering a multiple objective function that is to minimize the real power loss and violation function of contingency analysis. The major constraints are the power generation limits and power balance. This study proves that less simulation time can be reached applying bee colony optimization when compared to genetic algorithm, search tabu and simulated annealing.

In [57], the allocation and sizing of distributed generators are achieved by focusing on minimizing the distribution generators cost and maximizing the reliability at the same time. A major constraint in the optimization problem is the active power balance between the distributed generators and the load during the isolation time. Load shedding has been assumed in this study such that the active power of distribution generators is in one zone separated from the fault in the system by sectionalizers is less than total active power of the loads in that zone. Loads will be shed one by one applying the priority maintaining the active power balance. Such consideration of load shedding would result in a smaller reliability index if compared to the case that loads are shed when the power of distribution generators becomes less than the total power of the loads.

The optimal allocation of DG is determined based on a cost/worth analysis. The study takes into consideration both technical and economical factors which involve energy loss, load point, reliability indices, cost of DG cost and DG's portability. The optimization problem focuses on maximizing the benefit to cost ratio of DG application. It has been found in this study that the number of DG displacement to decrease when the benefit to cost ratio increases [58].

In [59], optimal power flow and genetic algorithm are considered in order to allocate certain number of DGs in distribution network. The use of such combination would allow dynamic network operators to search network for the optimal locations that permits strategic placement of small number of DGs through a large number of potential combinations. The software implementing the suggested algorithm is Matlab incorporating some features used in MATPOWER.

Genetic algorithm is applied in [60], in order to optimally allocate and size DGs by minimizing the location charges for active power at the busses in the distribution system. This is achieved by using various voltage dependent static load models. Nodal pricing and per unit location charges that are involved in short term operation of transmission systems can be applied also in distribution systems. The simulation is carried out on radial feeder and networked system involving one DG and many of them. A major constraint in the optimization problem is that the voltage should not be violated at all the busses. The results of this study show that the location of DG does not change irrespective of different load models; however, the size of DG is affected by the models of load. As the load exponent increases, there is decay in the objective function until it reaches the minimum. Further increase in the load exponent would lead the objective function to increase. In networked systems, the influence of the load exponent is found to be marginal when compared to radial distribution. This is due to small variation in the voltage in such systems in networked systems. If radial distribution system is considered, then the allocation of many DGs in the system would results decrease the objective function compared to distribution system involving only one DG. The availability of more than one DG in the network would improve the objective function by reducing the average location charges at the busses without violating the voltage constraints.

There is no common agreement on the payback time of PV units. The payback time of PV units can be defined in terms of energy and cost. If the payback time is defined in terms of energy, it would mean the time needed to recover the inserted energy to manufacture PV units such that the PV units can output the corresponding power [61]. The other definition is based on cost and that is associated with the recovery of PV cost through selling energy to the grid. The Feed in Tariff is the price at which the PV output power would be sold at to the grid. This price is set by Ontario's government in such a way to incentivize consumers to use PV units. By doing so, the government can achieve two main objectives which are increased energy efficiency and increased renewable energy penetration. The difference between the optimal payback time calculated in the work and the payback life time presented in [61] and [62] is that this thesis targets the supplement of system demands from the power generated from PV units such that the electricity purchase from the grid is minimized taking into consideration the capital cost of PV units. Moreover, it adds to previous research by considering the variability of solar radiation over the 24 hours of the year. This is implemented by defining a capacity factor which is the ratio of PV output power to the PV rated capacity.

From previous research, it can be noted that most of the research targeted the allocation and sizing of distributed generators based on system losses minimization. Some of them performed cost/worth analysis. Different optimization methods had been followed. Comparing to previous studies, this work is focused on optimal allocation and size of certain distributed generation units that are PV units. The work considers a 13 bus radial distribution feeder. Two approaches have been followed in order to allocate and size PV units in the systems. These are the demand-supply balance without feeder power flow representation and the AC power balance with the representation of feeder power flow. The problem in the first approach is modeled as a mixed integer linear programming problem; while in the second approach it is modeled as a mixed integer non linear programming problem. This study determines the location and the size of PV units by minimizing the total cost of electricity purchase from the grid plus the capital cost of PV units minus the electricity cost of selling energy from PV units back to the grid. This study differs from previous studies in that it considers the variability of the load being supplied, the capacity factor of PV units and the requirement to supply the demand from PV units before selling any extra generated electricity from the PV units to the grid. The work determines the location and size of PV units in the following way:

1. It determines the objective function behavior, the minimum total cost of electricity purchase from the grid plus the cost of PV units minus the cost of electricity generated from PV and sold back to the grid, over the expected operation period of PV units and finds the point at which it would be suitable to allocate and size PV units in the system.
2. It finds the best electricity selling price of PV units for a selected operation period of PV units

based on which the PV units will be allocated and sized in the distribution system.
3. It investigates the effect of the variation of the capital cost of PV units on the allocation of PV units in the system.

It is important to emphasize that the study assumes an average market price and neglects the discount rate that could be a point to be investigated in the future to improve the study.

Chapter 4
Sizing & Allocation of PV units in Distribution systems Optimization Problem Definition

4.1 Objectives

The thesis targets the optimal allocation and sizing of different PV units whose maximum capacities are selected based on the number of busses in the distribution system selected. The location and the size of PV units are determined by focusing on minimizing the total cost of electricity purchase from the grid plus the capital cost of PV units net the cost of selling electricity generated from PV units back to the grid. In this thesis, the system selected on which the problem is to be studied is13 bus feeder that has been originally adapted from IEEE 13 bus radial feeder but after applying certain modifications. The optimization problem formulated is tested by implementing two studies which include:

- Demand-supply balance without feeder power flow representation.
- AC power balance with the representation of feeder power flow.

The second objective is to investigate the impact of variation in the cost of selling electricity from the PV to the grid on the PV location and total cost.

4.2 Optimal Power Flow

During the last decade, optimal power flow literature has seen dramatic rise focusing on two points that are the solution methodologies and the areas of application. Optimal power flow was defined in 1960s as an extension of the conventional economic load dispatch problem to find the optimal setting for control variables satisfying power system constraints. The optimal power flow solution is considered to be more accurate that the economic load dispatch solution. The optimal power flow problem can have different objective functions depending on the nature of the problem selected. For example, the objective can be minimizing transmission loss such as in reactive power planning area, or can be minimizing the generation shift. The optimal power flow problem can involve different control variables and system constraints depending on the requirement of the problem. These control variables can involve [63]:

- Active power generation
- Reactive power generation
- Switched capacitor setting
- Active power of the load
- Reactive power of the load
- Transformer tap setting

4.3Type of Optimization Problem to be solved

The problem of allocating and sizing PV units is formulated as mixed integer linear programming (MILP) under demand-supply balance without power flow representation and as a mixed integer non linear programming problem (MINLP) under AC power balance with the representation of feeder power flow. In a linear programming problem, the objective function and the constraints are linear. An NLP problem can be defined as the process of finding a solution to a mathematical system that is composed of equalities and inequalities recognized as constraints over unknown variables that are real with an objective function to be maximized or minimized. For NLP problems, either the objective function or the constraint is nonlinear. In mixed integer programming some or all the variables are integers.

4.4 Selected Software

GAMS software is a general algebraic modeling system that has been chosen to be the simulation tool for this work. This software is recognized for being a high level modeling system for optimization problems. The software is associated with many solvers. CPLEX is used to solve the MIP problem; while SBB is chosen to be the solver for the MINLP problem [64].

4.5 Optimization Problem

The optimization problem to be solved is made of an objective function subject to certain constraints. The objective function, the decision variable and the data needed to optimally locate and size PV units are summarized in Figure4-1.

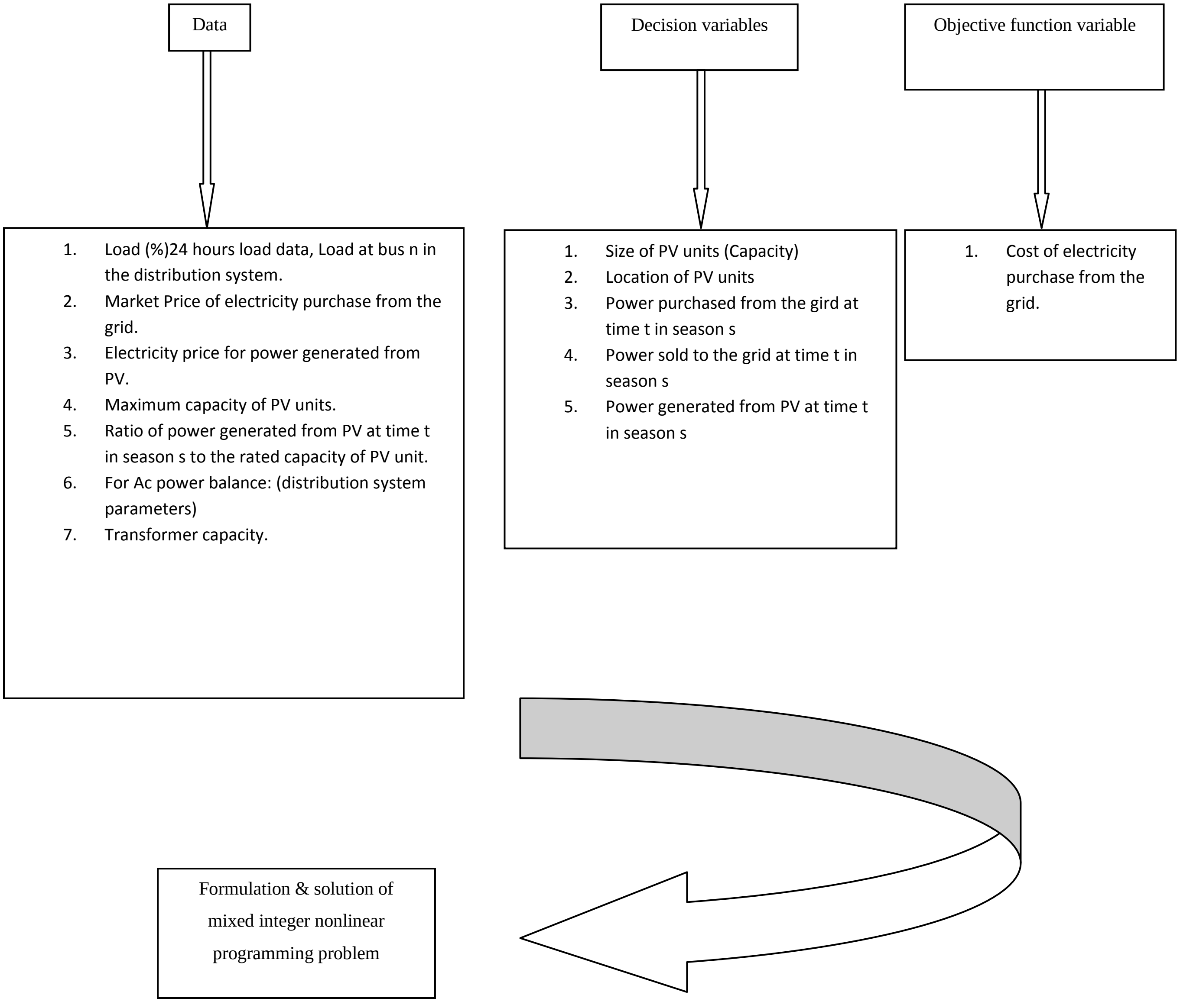

Figure 4-1 Formulation of the optimization problem

4.6 Estimated Market Price

The on peak and off peak hourly Ontario energy electricity prices are used to formulate the market price model for the demand-supply balance without feeder power flow representation case study. The on peak prices occur between 8am to 11pm; while the off peak prices occur between 12am to 7am [65]In this work, it is assumed that these prices do not change over the weekend and holidays. These prices are considered to be the cost of electricity purchase from the grid. The average of the on peak and off peak prices over the four seasons of the year has been determined as shown in Table 4-1. These prices are considered as an average and not to vary over years ahead. Moreover, this work did not take into consideration the variation of $ value over the years. In other words, the value of the $ today is assumed to be the same in the future. It also does not consider any discount rate that can be a point to consider in the future to further improve the study,

Table 4-1 Estimated market price

year 2009	**$/MWh**	**$/MWh**
Season	**on peak**	**off peak**
Fall	35.67	25.54
Winter	44.56	31.67
Spring	30.95	16.39
Summer	28.47	16.39

For the AC power balance with feeder power flow representation case study, the price at which the system will be purchasing electricity from the grid is considered to be fixed and it is taken as an average between the minimum off peak price and the maximum on peak price that is almost $30.48/MWh. If the estimated market price is considered to follow the estimated market price under demand-supply balance without feeder power flow representation, this will lead to a longer time when simulating the problem using GAMS software. The estimated market price under AC power balance with feeder power flow representation is assumed to be fixed to avoid complexity.

The power generated from PV units is sold to the grid at a cost of $420/MWh based on [66]. One of the goals in this project is to study the possibility of changing this cost such that the total cost of electricity purchased from the grid is minimized.

The capacity cost per MW installed of PV was found by finding the linear equation that relates the cost in $ to the capacity of PV unit from BP company [67]. The relationship is plotted in Figure 4-3and the linear factor is found to be $4000/KW.

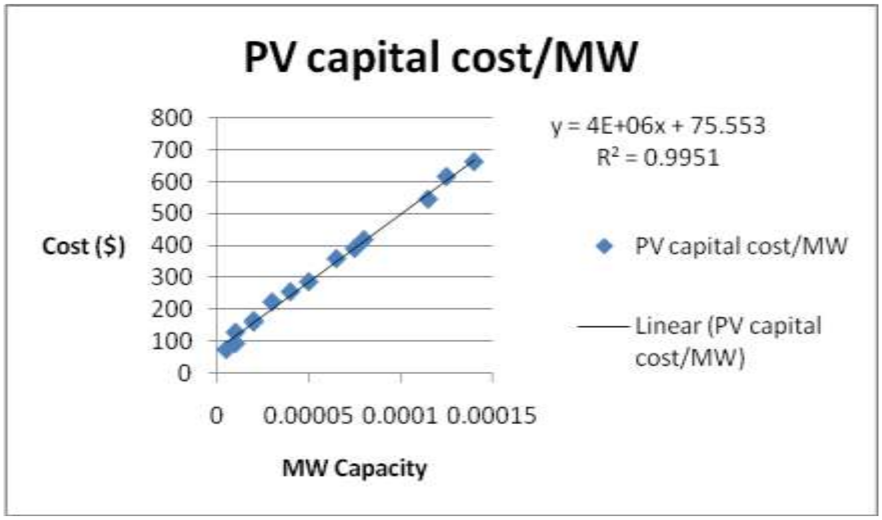

Figure 4-2 $/W installed of PV units.

4.7 PV Output Power

The output power of PV to the rated capacity over 24 hours of the day is to be determined. With the availability of global irradiance measurements in a typical day in different seasons, the output power can be found applying equation (0-1):

$$PV\ otput\ \ power = A\ \lambda\ \eta$$ **(4-1)** [68]

where

A: Area of PV unit

λ: Irradiance (W/m^2)

η : Efficiency

η is the resultant from different efficiencies as shown in equation (4-2):

$$\eta = \eta_{rated}\ \eta_{dust}\ \eta_{mismatch}\ \eta_{DC\ \ loss}\ \eta_{MPPT}\ \eta_{inv}$$ **(4-2)** [69]

Where

η_{rated} : Rated efficiency of PV module (10.77%).

η_{dust} : 1 - the fractional power loss due to dust and debris on the PV array (96%).

$\eta_{mismatch}$: 1- the fractional power loss due to module parameter mismatch (95%).

$\eta_{DC\ loss}$: 1 - the DC-side I^2R losses (98%).

η_{MPPT} : 1 - the power loss due to DC current ripple and "algorithm error" caused by the switching converter which performs the maximum power point tracking function (95%).

η_{inv} : DC/AC inverter efficiency (97%)

The data for solar irradiance is obtained from the Solar Radiation Research Laboratory [70]. Since the available capacity of PV, in this work, is 1MW, the area of the PV unit can be found as shown in Table 4-2.

Table 4-2 Area of IMW PV

Area of 50W PV module [71]	839mm*537mm =0.450543m^2
Area of 1MW	(1MW/50W)*Area of 50W PV module= 9010.86m^2

The ratio between the PV output power and its rated capacity (1MW) in different seasons ($a_{t,s}$) has been calculated applying (4-4) as shown in Table 4-3:

$a_{t,s}$ = PV output power / Rated capacity of PV **(4-3)**

This variation is plotted in Figure 4-4. It can be concluded from this table that the maximum radiation duration is in the summer season followed by spring, fall and winter.

Table 4-3 $a_{t,s}$ in fall, winter, spring and summer

Time (hour)	Fall	Winter	Spring	Summer	Time (hour)	Fall	Winter	Spring	Summer
1	0	0	0	0	13	0.344921	0.174347	0.525306	0.634744
2	0	0	0	0	14	0.420396	0.181895	0.509456	0.665689
3	0	0	0	0	15	0.310202	0.118496	0.539646	0.612102
4	0	0	0	0	16	0.227934	0.057361	0.323788	0.533608
5	0	0	0	0	17	0.10491	0.023397	0.252086	0.412093
6	0	0	0	0	18	0.018869	0.003774	0.12076	0.2702
7	0	0	0.007547	0.027171	19	0	0	0.018869	0.123779
8	0.006793	0	0.053587	0.135855	20	0	0	0	0.018869
9	0.081513	0.016604	0.07472	0.282276	21	0	0	0	0
10	0.186423	0.10491	0.24001	0.420396	22	0	0	0	0
11	0.289824	0.205292	0.440019	0.537382	23	0	0	0	0
12	0.365299	0.218877	0.42115	0.624178	24	0	0	0	0

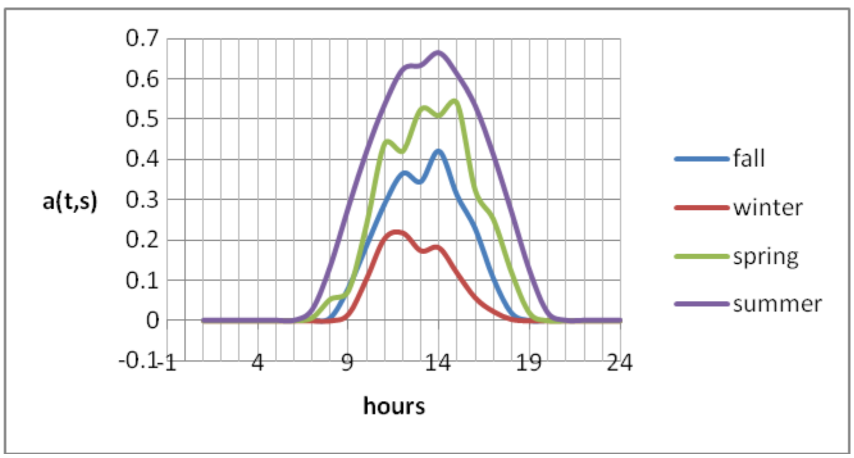

Figure 4-3 $a_{t,s}$ variation in fall, winter, spring and summer

4.8 Chapter 4 Summary

This chapter presented the type of optimization problem to be solved and the selected simulation tool. The optimization problem under the demand-supply balance without feeder power flow representation is considered in Chapter 5; while the optimization problem under the AC power flow with the representation of feeder power flow is considered in Chapter 6. The estimated market price of electricity, the PV unit capital cost per MW installed and the capacity factor of PV units presented in this chapter are used as the input data to the optimization problems presented in the next chapters.

Chapter 5

Optimal Sizing & Location of PV Units under Demand-Supply Balance without Feeder Power Flow Representation

This chapter focuses on selecting the optimum size and location of PV units under demand-supply balance without feeder power flow representation. The optimization problem formulation for this case is discussed.

5.1 Problem Definition & Optimization Problem Formulation

The problem is to locate and size PV units over that operation period of PV minimizing the total cost of electricity purchase from the grid considering the capital cost of PV and the possibility to sell extra electricity generation from PV units to the grid. The PV operation period is in years in which each year is made of four seasons (fall, winter, spring and summer) and each season is almost 91.25days. Twenty four hours days in different seasons of the year are selected as typical days in the year based on which the problem is formulated as the following:

5.1.1 Objective Function

The objective function is to minimize the total cost of electricity purchase from the grid plus the capital cost of PV units net the cost of selling electricity generated from PV units to the grid. This is formulated as shown in (5-1):

$$\min \quad Z = CC\sum_{n=2}^{k} C_n + 91.25L\sum_{s=1}^{4}\sum_{t=1}^{24} bP_{t,s}\, PP_{t,s} - 91.25L\ bS\sum_{s=1}^{4}\sum_{t=1}^{24} PS_{t,s} \qquad \textbf{(5-1)}$$

Where

n: Bus number excluding the slack bus (n_2, $n_{3,}$.....n_i).

k: Number of busses in the distribution system.

t: Time starting at 1am until 24.

s: Season from 1 to 4 (fall, winter, spring, summer).

CC: Capacity cost of PVs per MW installed.

C_n: Positive variable representing the capacity of the PVs connected on bus n (MW).

91.25: Number of days in a season.

L: Expected operation period of PV unit in years.

bP: Market price of electricity purchased from the grid ($/MWh).

$PP_{t,s}$: Positive variable representing power purchased from grid (MW) at time t in season s.

bS: Price of selling generated power from PV to the grid.

$PS_{t,s}$: Positive variable representing power generated from PV and sold to the grid (MW) at time t in season s.

The problem is subjected to two classes of constraints, equality and inequality constraints as the following:

5.1.2 Equality Constraints

1. PV output power at bus n at time t in season s ($PG_{n,t,s}$)

$$PG_{n,t,s} = a_{t,s} C_n \qquad \textbf{(5-2)}$$

Where $a_{t,s}$:Ratio between the PV output power at time t in season s and the rated capacity of the PV unit as determined in Chapter 5.

2. Demand-supply balance:

$$\sum_{n=1}^{k} Demand_{t,s} = PP_{t,s} - PS_{t,s} + \sum_{n=1}^{k} PG_{n,t,s} \quad \textbf{(5-3)}$$

Where

$Demand_{t,s}$: is the power consumed by the load at time t in season s.

5.1.3 Inequality Constraints

1. Electricity can be either purchased from the grid or sold to the grid:

$$XS_{t,s} + XB_{t,s} \le 1 \quad \textbf{(5-4)}$$

Where:

$$XS_{t,s} \text{ is a binary variable}: \begin{cases} 1: \text{if the system is selling power to grid at } t \text{ in season } s. \\ 0: \text{otherwise} \end{cases}$$

$$XB_{t,s} \text{ is a binary variable}: \begin{cases} 1: \text{if the system is purchasing power from the grid at } t \text{ in } s. \\ 0: \text{otherwise} \end{cases}$$

2. Electricity purchase condition at time *t* in season *s* is that the system can only purchase electricity from the grid if the binary variable is 1:

$$PP_{t,s} \le TC \;\; XB_{t,s} \quad \textbf{(5-5)}$$

Where:

TC: The main transformer capacity that is supplying power from the main substation.

3. Electricity sold from PV units condition at time t in season s is that the micro-grid can only sell electricity to the utility if the binary variable is 1:

$$PS_{t,s} \le TC \;\; XS_{t,s} \quad \textbf{(5-6)}$$

4. Capacity limit of PV unit to be connected to bus *n*:

$$C_n \leq 1MW \quad \textbf{(5-7)}$$

5. Total capacity of PV units:

$$\sum_n C_n \leq 10MW \quad \textbf{(5-8)}$$

5.2 13 bus Feeder Case Study

PV units are to be allocated and sized for 13 bus distribution system provided in Figure 5-1. The system presented here is a modified version of the IEEE 13 bus feeder. The regulator in the original system connecting buses 1 and 4 is neglected. Moreover, in the original system an intermediate node between node 9 and 10 is present. This node is connected to node 9 through a switch that has been removed in the system under study. Bus 13 has been added as the substation transformer is to connect bus 1 and bus 13. Capacitors are connected to buses 10 and 7 in the original system, but their presence here has been neglected. Furthermore, all the lines in the system are assumed to be three phase lines which is not the case in the original system. It is important to emphasize that any conductance available in the original system has been neglected and the resistance and the inductance of any line of the system have been assumed to follow the line data presented in Chapter 6. The main substation is located at bus 13 with a capacity of 5000KVA in the demand-supply balance without power flow feeder representation case study; while it has a value higher than that under AC power balance with the representation of feeder power flow. Spot loads are located at buses 6, 3, 2, 11, 9, 10 and 7 [72]. The distributed load is neglected. These modifications have been considered for simplicity reasons. Moreover, due to the nature of the problem being solved under the AC power balance with feeder power flow representation modeled as mixed integer non linear programming problem that is non convex, and since the accessible solvers for MINLP in the selected simulation tool, GAMS, can solve non convex problems with limited capacity, the problem is solved with SBB solver with the capability of solving convex problems. This might result in a local optimal solution that can be avoided with certain modification in the system such that the optimization problem formulation would provide a solution and infeasibility is avoided.

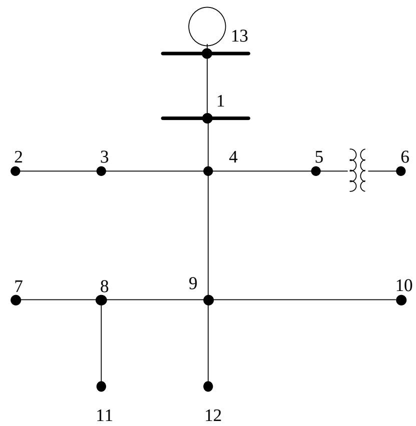

Figure 5-1 13 bus radial feeder after modifications to IEEE 13 bus feeder for simplicity reasons

5.2.1 Substation Transformer

Considering the grid to supply the local distribution company represented by the system below bus 1 through a transformer, the capacity of this transformer is to be determined. In general, the transformer rating is the substation capacity that is 5000KVA. This would mean as well that the substation can supply a maximum of 5000KW load [73]

5.2.2 Load

The actual demands of IEEE 13 bus system are provided in Table 5-1. Only spot loads of the system are calculated; while distributed loads are neglected in the analysis.

Table 5-1Peak demands

Bus	**Spot load (MW)**	**Spot load (MVAR)**
6	0.4	0.29
3	0.17	0.125
2	0.23	0.132
11	0.128	0.086
9	1.325	0.811
10	0.843	0.462
7	0.17	0.08
Total	3.266	1.986

A typical 24 hours load profile in different seasons (fall, winter, spring, summer) is provided in Table 5-2. The data has been assumed to follow a similar system and they are taken from [74] that has been and published in [75]. The data is expressed as a fraction of the peak load. It can be noted from this table that the maximum power demand takes place between 5pm and 7pm in winter, 1pm and 3pm in spring, and 7pm and 8pm in summer and Table 5-3 presents the load data of the system shown in Figure 5-1during the four terms of the year winter, spring, summer and fall. These are expressed as a fraction of the total peak load of the system. It can be noted from this table that the maximum power demand by the load occurs between (9am-10am) and (11am-12pm) in fall, (4pm-5pm) in winter, (9am-10am) and (11am-12pm) in spring and (11am-12pm) and (1pm-3pm) in summer.

The actual demand of the IEEE 13 bus system for 24 hours in different seasons of the year is to be calculated using the following equation:

$$Demand\,(i,t,s) = DP(i)\ X\ \%L(t,s) \quad \textbf{(5-9)}$$

Where

Demand(i,t,s): Actual demand of the system at bus i, time t and season s.

t : Time (certain hour, 12am-12pm & 12pm-12am) at which load is consumed.

s: Season of the year (fall, winter, spring, summer).

DP(i): Peak demand at bus i obtained from the distribution system spot load data.

%L(t,s): typical load profile of Ontario at time t and season s as a percentage of the peak demand.

Table 5-2 Typical 24 hours load profile in four seasons (%).

Taken from[74], [75]

Hour	Fall	Winter	Spring	Summer
12-1 am	0.3717	0.4757	0.3969	0.64
1—2	0.3658	0.4473	0.3906	0.6
2—3	0.354	0.426	0.378	0.58
3—4	0.3422	0.4189	0.3654	0.56
4—5	0.3481	0.4189	0.3717	0.56
5—6	0.3835	0.426	0.4095	0.58
6—7	0.4248	0.5254	0.4536	0.64
7—8	0.5015	0.6106	0.5355	0.76
8—9	0.5605	0.6745	0.5985	0.87
9—10	0.5841	0.6816	0.6237	0.95
10—11	0.59	0.6816	0.63	0.99
11-12pm	0.5841	0.6745	0.6237	1
12—1	0.5487	0.6745	0.5859	0.99
1—2	0.5428	0.6745	0.5796	1
2—3	0.531	0.6603	0.567	1
3—4	0.5192	0.6674	0.5544	0.97
4—5	0.531	0.7029	0.567	0.96
5—6	0.5428	0.71	0.5796	0.96
6—7	0.5664	0.71	0.6048	0.93
7—8	0.5782	0.6816	0.6174	0.92
8—9	0.5664	0.6461	0.6048	0.92
9—10	0.531	0.5893	0.567	0.93
10—11	0.472	0.5183	0.504	0.87
11-12am	0.413	0.4473	0.441	0.72

5.3 Results based on Demand-Supply Balance without Feeder Power Flow Representation for13 Bus Radial Feeder

The results for three types of study are presented in this section. In the first subsection, the price at which the PV units would sell electricity to the grid is considered to be fixed and an investigation on the corresponding PV operation period at which the 13 bus radial feeder would allocate PV units in the system or sell electricity to the grid is carried out. In the second subsection, the price at which the electricity would be sold from PV units to grid after meeting system demands is allowed to vary at fixed operation period of PV. The capital cost at both studies is set to be invariable. In the third subsection, the capital cost variation effect on the age at which the system would allocate PV units or sell electricity to the grid is considered.

5.3.1 Objective Function With Respect to PV Operation Period

The objective function is to be minimized in this work. This objective function is a function of the operation period of the PV units. The operation period of the PV units has been set to vary between 1 year and 30 years. The objective function corresponding to such variation was determined through the optimization problem. The results have been displayed in Figure 5-2.

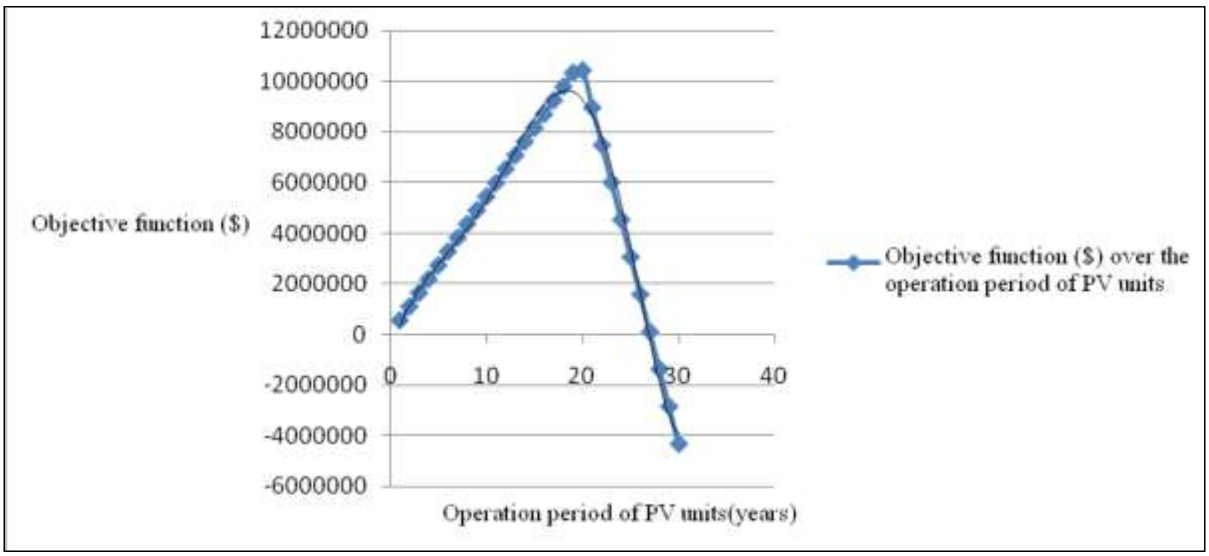

Figure 5-2 Electricity cost with respect to PV operation period.

It can be noted that for the set data used in the problem, the total cost of electricity purchased from the grid is a positive number and keeps increasing until a maximum point where the cost switches to decrease. When the operation period of PV units is considered to be 28years, it can be noted that the cost becomes negative. This can be explained as the following:

- When the operation period of PV was set to be between 1 and 19 years, no PV units were allocated in the13bus feeder. This is because the installation cost of PV/MW is very high and the operation of PV over these years would not result in any revenue. In other words, it is preferable to purchase electricity from the grid rather than supplying it from PV units since the optimization problem is a minimization problem targeting minimizing the total cost of electricity purchased from the grid and involving the initial cost of PV. If the PV units are to be operating in such system, 18 years would not be sufficient to pay for the capital cost of PV. That is the main reason for having a positive cost that is increasing over the years.

- When the operation period of PV was set to be 20 years, PV units were allocated and sized in the 13bus feeder, as the capacity of the PV unit is s function of the bus at which it is installed as shown in the objective function presented in equation (5-1). The location of the PV units is considered in the optimization problem formulation as the capacity of the PV unit at certain bus is involved in equation (5-2) and this equation is involved in the demand-supply balance equation (5-3). The cost of electricity purchased from the grid per year decreased in comparison to the cost at 19 years. This is because the PV units have been allocated and sized to minimize the total cost of electricity. With the allocation and sizing of PV units in the feeder, the load started to be supplied by the PV units and less MWh were to be purchased from the grid. This leads to a reduction in the cost compared to the previous case.
- For an operation period of 28 years, the PV units system is capable of paying back the capital cost of the PV which has been reflected back into the total cost such that the objective function become negative. However, the payback time seems to be high when compared to what is claimed in practice. This is due to two main factors:

  A) The PV capacity factor varies between 0 and a maximum value of 0.665689 in summer season. It is important to note that this factor would differ based on weather conditions and the area in which the study is performed.

  B) The optimization model is set such that the system could supply its demands from the power generated from PV units at first and then sell electricity to the grid in case of extra generation.

5.3.2 Allocation and Sizing of PV Units

Running the optimization problem using GAMS software for different PV operation periods, the optimal location and size of PV units are determined as shown in Table 5-3 and Table 5-4.

It can be seen from Table 5-3 that the system decided to allocate the maximum capacity of PV system at bus 12 when the PV unit capacity constraint is not involved in the optimization problem; however, when this constraint is introduced, the system will allocate PV units at different nodes in the 13 bus radial feeder such that not more than 1MW PV unit is connected to each node and the total capacity of the PV system is at maximum 10MW that is reached in this study.

Table 5-3 Allocation & sizing PV when total capacity of PV units is at most 10MW

PV operation period	Location	Size (MW)
1-19 years	-	0
20-30years	Bus 12	10

Table 5-4 Allocation & sizing PV when the capacity of one PV unit is at most 1MW and the total capacity of PV units is at most 10MW.

PV operation period	Location	Size (MW)
1-19 years	-	0
20-30years	Bus 1	1
	Bus 2	1
	Bus 3	1
	Bus 4	1
	Bus 5	1
	Bus 6	1
	Bus 7	1
	Bus 8	1
	Bus 9	1
	Bus 10	1

It can be seen from Table 5-4 that the capacities of all PV units allocated in the system for an operation period of 20-30 years are set at the maximum that is 1MW. The main reason behind this is that in order to minimize the objective function, the system would allocate PV in it to have extra generation and sell to the grid so that the objective function is minimized.

The electricity purchased yearly from the grid when PV units are allocated in the system is 10441.526MWh, while the electricity sold yearly to the grid is 4265.406MWh. The power sold to and the power purchased from the grid at time *t* in season *s* at 20 years of PV unit operation period are shown in Table 5-5 and Table 5-6. It can be seen from this table that the power generated from PV in winter is not sufficient to supply the demand, and in order to determine the optimal location and size of PV units provided the variability of the load in this season, it would be cheaper to purchase electricity from the grid in this season.

Table 5-5 Power sold to the grid over 24 hours in different seasons of the year

Time (hour)	**Fall (MW)**	**Winter (MW)**	**Spring (MW)**	**Summer (MW)**	**Time (hour)**	**Fall (MW)**	**Winter (MW)**	**Spring (MW)**	**Summer (MW)**
1	0	0	0	0	13	1.246	0	1.987	4.575
2	0	0	0	0	14	2.047	0	1.829	4.923
3	0	0	0	0	15	0.922	0	2.228	4.425
4	0	0	0	0	16	0	0	0.103	3.602
5	0	0	0	0	17	0	0	0	2.348
6	0	0	0	0	18	0	0	0	0.852
7	0	0	0	0	19	0	0	0	0
8	0	0	0	0	20	0	0	0	0
9	0	0	0	0.915	21	0	0	0	0
10	0	0	0	2.277	22	0	0	0	0
11	0.695	0.016	1.134	3.466	23	0	0	0	0
12	1.45	0.275	0.978	4.45	24	0	0	0	0

Table 5-6 Power purchased from the grid at time *t* in season *s* in typical days of the year

Time (hour)	Fall (MW)	Winter (MW)	Spring (MW)	Summer (MW)	Time (hour)	Fall (MW)	Winter (MW)	Spring (MW)	Summer (MW)
1	1.461	1.276	1.96	1.195	13	0	0.15	0	0
2	1.391	1.235	1.894	1.156	14	0	0.033	0	0
3	1.368	1.193	1.829	1.118	15	0	0.626	0	0
4	1.368	1.214	1.829	1.137	16	0.016	1.278	0	0
5	1.391	1.337	1.894	1.253	17	1.27	1.659	0.615	0
6	1.716	1.481	2.09	1.387	18	2.13	1.938	1.83	0
7	1.994	1.749	2.407	1.366	19	2.226	2.016	2.816	0.651
8	2.135	1.955	2.306	0.472	20	2.11	1.975	3.005	1.661
9	1.411	1.871	2.355	0	21	1.925	1.852	3.037	1.734
10	0.362	1.008	0.833	0	22	1.693	1.646	2.841	1.542
11	0	0	0	0	23	1.461	1.44	2.352	1.349
12	0	0	0	0	24	1.554	1.296	2.09	1.214

5.3.3 Capital Cost Reduction over Years under Demand-Supply Balance without Feeder Power Flow Representation

Since the capital cost of PV units is expected to reduce over year, the question arises: what would happen if the set price at which the utility is purchasing electricity from the system is set to be fixed at $420/MWh. Table 5-7 presents the simulation results obtained by reducing the PV capital cost. It can be seen that as the PV capital cost decreases, the operation period at which PV units are allocated in the system to supply its demand and then to sell extra generation to the grid decreases. The model presented could be a good tool, for utilities, to modify the PV selling price as PV capital cost decreases.

Table 5-7 The operation period of PV units at which the system would allocate PV units in it when the capital cost varies.

Capital cost ($/MW)	Operation period at which PV is allocated in the system	Objective function ($)
4000000	20	1.04E+07
3000000	15	7826383.343
2000000	10	5217588.895
1000000	5	2608794.448
500000	2.5	1304397.224

5.3.4 Recommendation

If the operation period of PV units used in the project is set to be 25years, the total cost of electricity purchase from the grid over these years is $7830740.275. The objective function in this case is $3043972.238. At this expected operation period of PV units and with this cost, PV units are allocated and sized for 13 bus radial feeder. In order for the system to achieve profit, the objective function should be negative. This can be reached by modifying the price of electricity for the PV units. Figure 5.3 presents the cost of electricity purchase from the grid for different costs of selling electricity from PV units to the grid over the 25 years operation period of PV units. From this figure, it is recommended that the price at which electricity is sold from PV to the grid is to be modified to almost $449/MWh because it would be profitable.

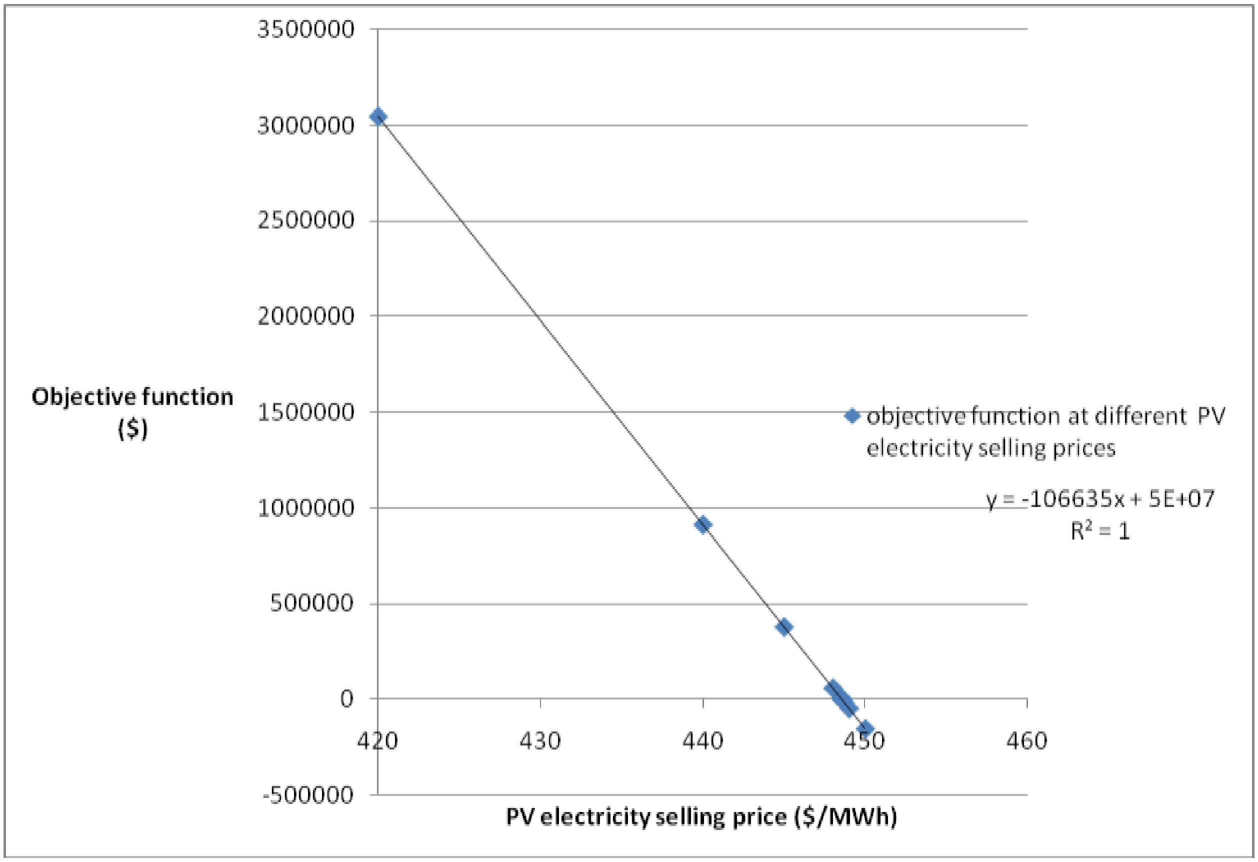

Figure 5-3 Study on variation on the price of electricity sold from PV to the grid.

In this work, two terms are defined. The first term is the location price and the second term is the selling price. The location price is the set PV electricity selling price at which the system will decide to allocate PV units based on the optimization problem without achieving any profit. The selling price is the price at which the objective function will become negative and the system will achieve profit. At this price, the PV units already exist in the system. These terms and the corresponding objective functions are determined for different PV units' operation period as shown in Table 5-8 considering the capital cost to be fixed and it is to be recovered over these years.

Table 5-8 Allocation and selling prices for different PV operation period.

Life	Allocation Price ($/MWh)	Selling Price ($/MWh)	Objective function at allocation price ($)	Objective function at selling price ($)
20	415	542.32337	1.09E+07	-0.11029967
25	322	448.5456709	-4.35E+06	-8.12E-04

5.4 Chapter 5 Summary

This chapter determined the location and the size of PV units under demand-supply balance without feeder power flow representation. It found the location and the size of PV units under different studies. It studied the effect of PV operation period on the objective function when the capital cost and the PV electricity selling price were set to be fixed. Moreover, found the best PV units' electricity selling price at a certain operation period when the capital cost of PV units was set to be fixed. Also, it assumed the capital cost of PV units to decay in the future and correspondingly it found the expected PV operation period at which PV units would be allocated in the system. In the next chapter, the problem would be investigated under AC power balance taking into consideration the feeder power flow representation.

Chapter 6

Optimal Sizing & Location of PV Units under AC Power Balance with Feeder Power Flow Representation

Real power systems are operated under AC power balance. The main objective of this section is to optimally allocate and size PV units while considering the optimal AC power balance. The objective of the optimal power balance is to minimize the total cost of electricity generation by optimally allocating and sizing PV units to increase the power generated from them in order to satisfy the demand and selling extra generation to the grid. In this section, the problem formulation is provided and the results are analyzed investigating the effect of PV operation period on the objective. Furthermore, it looks for the best PV electricity selling price under which the PV will be allocated in the 13 bus radial feeder system. Since the capital cost of PV units is expected to drop over the years and assuming that the utility is fixing the price at which it purchases electricity generated from PV units, the PV operation period at which the system will decide to allocate them in it and the PV operation period at which the system will be able to achieve profit are determined.

6.1 Problem Formulation

The objective function and constraints for the optimal allocation and sizing of PV units for 13 radial feeder that is provided in the following subsections:

6.1.1 Objective Function

The objective function is to minimize the total cost of electricity generation as shown below:

$$\min \quad \cos t = CC\sum_{i=1}^{M} C(i) + basexDxlife\sum_{s=1}^{4}\sum_{t=1}^{24} bP_{t,s}\ PP_{t,s} + basexDxlifexbs\sum_{s=1}^{4}\sum_{t=1}^{24} Ps_{t,s} \qquad \textbf{(6-1)}$$

Where

Cost : total cost of electricity generation from the substation located at bus 13 in case of 13 bus feeder.

CC: capital cost of PV per MW approximated to be $4X10^6$ as in section 4.6.

i: bus number (integer: *i*=1,…..,13.) for 13 bus system.

M: bus number 1 in case of 13 bus feeder.

$C(i)$: capacity of PV Unit in MW allocated at bus i.

base: since the power balance is run to be in per unit system, the power is multiplied back by the base unit that is 5000KVA such that the output of multiplication is in MW.

D: number of days in a season considering the year to be made of 365 days.

life: PV unit operation period.

s: seasons of the year (fall, winter, spring and summer).

t: hours of the day that is an integer number (t=1,….24).

$bP_{t,s}$:price of electricity purchased from the market at time t in season s. This price is considered to be fixed in the demand-supply balance case study; while it is set to be fixed to in the AC power balance case study for simplicity.

$PP_{t,s}$: power purchased from the grid at time t in season s to meet the hourly demands of the system in measured in per unit and it is a free variable.

bs: price of electricity generated from PV units and sold back to the grid.

$Ps_{t,s}$: power sold from PV units to the grid at time t in season s measured in per unit and it is a free variable.

6.1.2 Constraints

The optimization problem is subject to two classes of constraints that are the equality constraints and the inequality constraints.

1. ***Equality constraints***

These constraints include the parameters of power balance constraints, system losses, power transmitted from substation to bus 1 in the 13 bus system, energy purchased from grid and energy sold from PV units to the grid.

- Power balance Parameters constraints:

$$\delta_{i=M,t,s} = 0 \quad \textbf{(6-2)}$$

$$V_{i=M,t,s} = 1.04 \quad \textbf{(6-3)}$$

The upper and lower levels on the voltage at different busses at different times in different seasons are inequality constraints but they are set as equality constraints as programmed in GAMS software as in the following:

$$V_{i,t,s}(upper) = 1.05 \quad \textbf{(6-4)}$$

$$V_{i,t,s}(lower) = 0.8 \quad \textbf{(6-5)}$$

The main reason behind setting the lower limit on the voltage to 0.8, that is a low voltage value not preferable in practice, is to relax the constraint to avoid infeasibility. This would lead certain nodes in the 13 bus feeder to experience a low voltage at certain instances of time. The location of PV units might be at these nodes that might be considered a problem. A reason behind the voltage drop at certain nodes, to which a PV unit is connected with the setting of the lower limit on the voltage to be 0.8, is that the IEEE standards do not allow PV system to generate reactive power that can be a solution to the problem faced under this case. Other factors for experiencing low voltage at certain nodes in the system with the set lower limit on the voltage are the load factor and the capacity factor of the PV units. Moreover, the nodes experiencing low voltages at certain instances of time are the end of the line nodes. This problem can be solved by integrating capacitors or voltage regulators to the nodes with low voltage.

- Total system losses constraint:

$$\textbf{\textit{losses}}\textbf{=} \sum_{s=1}^{4} \sum_{t=1}^{24} \sum_{i=1}^{M} \sum_{j=1}^{M} 0.5\ G_{(i,j)}\ (V^{2}{}_{i,t,s} + V^{2}{}_{j,t,s} - 2\ V_{i,t,s}\ V_{j,t,s}\ \cos\ (\delta_{j,t,s} - \delta_{i,t,s}))$$

(6-6) [74]

- Active AC load flow constraint (modified from the original load flow equation [76] to include the load flow at time t in season s and to include the power generated from PV unit):

$$P_{i,t,s} + (a_{t,s}\ C(i)/base) - demand_{i,t,s} = \sum_{j=1}^{M} V_{i,t,s}\ V_{j,t,s}\ Y_{(i,j)}\ \cos\ (\theta_{(i,j)} + \delta_{j,t,s} - \delta_{i,t,s})$$

(6-7)

The voltages at different nodes play a role in determining the location of the PV unit in the system as the capacity of the PV unit at a certain node is a part of the active AC load flow constraint. Moreover, the role of other constraints should not be neglected as most of them involve the voltage at a certain bus that is a variable.

- Reactive AC load flow constraint at time t in season s (modified from the original load flow equation [76] to include the load flow at time t in season s):

$$Q_{i,t,s} - Qdemand_{i,t,s} = -\sum_{j=1}^{M} V_{i,t,s}\, V_{j,t,s}\, Y_{(i,j)} \sin\left(\theta_{(i,j)} + \delta_{j,t,s} - \delta_{i,t,s}\right) \tag{6-8}$$

- Power transmitted between substation (bus 13) and bus 1 at time *t* in season *s*:

$$Pt_{t,s} = V_{i=1,t,s}\, V_{j=13,t,s}\, \frac{1}{X_{i=1,j=13}} \sin(\delta_{i=13,t,s} - \delta_{j=1,t,s}) \tag{6-9}$$

- Upper and lower limits of power transmitted between substation (bus 13) and bus 1:

The following two constraints are inequality constraints but they are set as equality constraints considering the upper and lower levels as programmed in GAMS software:

$$Pt_{t,s}(upper) = 1 \tag{6-10}$$

$$Pt_{t,s}(lower) = -1 \tag{6-11}$$

- Power transmitted relationship with power purchased from the grid and power sold from PV units to the grid:

$$Pt_{t,s} = PP_{t,s} + Ps_{t,s} \tag{6-12}$$

2. Inequality Constraints

These constraints include the power balance limits constraints, constraints on the power purchased from the grid and constraints on the power sold from PV units to the grid as in the following:

- Power balance limits constraints:

$$P_{i,t,s} \leq P_{\max imum} \quad \textbf{(6-13)}$$

$$P_{i,t,s} \geq P_{\min imum} \quad \textbf{(6-14)}$$

$$Q_{i,t,s} \leq Q_{\max imum} \quad \textbf{(6-15)}$$

$$Q_{i,t,s} \geq Q_{\min imum} \quad \textbf{(6-16)}$$

- Power purchased from grid constraints:

$$PP_{t,s} \geq 0 \quad \textbf{(6-17)}$$

$$PP_{t,s} \leq Pt_{t,s}(upper)\ \ YY_{t,s} \quad \textbf{(6-18)}$$

- Power sold from PV units to the grid constraints:

$$Ps_{t,s} \leq 0 \quad \textbf{(6-19)}$$

$$Ps_{t,s} \geq Pt_{t,s}(lower)\ \ (1-YY_{t,s}) \quad \textbf{(6-20)}$$

Where

$\delta_{i,t,s}$: voltage angle at bus i at time t in season s.

$V_{i,t,s}$: voltage magnitude at bus i at time t in season s.

$\theta_{(i,j)}$: angle of the admittance that is in per unit between from node i to node j in the system.

$V_{i,t,s}(upper)$: upper limit on the voltage at bus i at time t in season s.

$V_{i,t,s}(lower)$: lower limit on the voltage at bus i at time t in season s.

$G_{(i,j)}$: conductance between two nodes i and j in the system.

$P_{i,t,s}$: active power at bus i at time t in season s.

$Q_{i,t,s}$: reactive power at bus i at time t in season s.

$P_{\max imum}$: maximum value of active power generated at a bus that is set to be 0 when no generator is connected to that bus.

$Q_{\max imum}$: maximum value of reactive power generated at certain bus that is set to be 0 when no generator is connected to that bus.

$P_{\min imum}$: minimum value of active power at certain bus that is set to be 0 when no generator is connected to that bus.

$Q_{\min imum}$: minimum value of reactive power at certain bus that is set to be 0 when no generator is connected to that bus.

$Pt_{t,s}$: Power transmitted from substation at bus 13 to bus 1 at time *t* in season *s*.

$Pt_{t,s}(upper)$: upper limit on power transmitted from substation at bus 13 to bus 1 at time *t* in season *s*.

$Pt_{t,s}(lower)$: lower limit on power transmitted from substation at bus 13 to bus 1 at time t in season *s*.

$a_{t,s}$: ratio between PV unit output power at time t in season s and the capacity of PV unit (PV power /1MW) as determined in Chapter 4.

$demand_{i,t,s}$: active power demand at bus *i* at time *t* in season *s* calculated as multiplication of the active peak demand at bus i by the percentage load variability over 24hours in 4 seasons.

$Qdemand_{i,t,s}$: reactive power demand at bus i at time t in season s calculated as multiplication of the reactive peak demand at bus *i* by the percentage load variability over 24hours in 4 seasons.

$Y_{i,j}$: admittance from node *i* to node *j* in per unit.

$YY_{t,s}$: binary variable (0 or 1) at time t in season s based on which either the power would be purchased from the grid or sold from PV units to the grid at time *t* in season *s*.

6.2 Systems Parameters

Since AC power balance is considered, the system line data and loads (spot loads) in PU for the 13 radial bus feeder is presented in Table 6-1 and Table 6-2:

Table 6-1 Line data for the 13bus radial feeder.

From	To	R (PU)	X(PU)	Charging	Length of the line (feet)
1	4	0.0369	0.1147	0	2000
2	3	0.0123	0.0197	0	300
3	4	0.0205	0.0328	0	500
4	5	0.0205	0.0328	0	500
4	9	0.0369	0.1147	0	2000
8	9	0.0130	0.0066	0	300
7	8	0.0130	0.0066	0	300
9	10	0.0216	0.0111	0	500
9	12	0.0185	0.0573	0	1000
8	11	0.0345	0.0177	0	800
1	13	0	0.0800	0	Transformer (0)
6	5	0.0011	0.0020	0	Transformer (0)

Table 6-2 Load data13 bus radial feeder

Bus number	Active Power (PU)	Reactive Power (PU)
1	0	0
2	0.046	0.0264
3	0.034	0.025
4	0	0
5	0	0
6	0.008	0.058
7	0.034	0.016
8	0	0
9	0.265	0.1622
10	0.1686	0.0924
11	0.0256	0.0172
12	0	0
13	0 in demand-supply balance 1 in AC power balance	0

6.3 Results of 13 bus Feeder Case Study under AC Power Balance

This section analyzes the results obtained for the optimization problem considering AC power balance under two cases of investigation. It studies the effect of expected operation period of PV unit on the total cost of electricity generation looking for the best operation period. Furthermore, it determines the appropriates selling price of electricity from PV units at which PV units would be allocated and sized in 13bus radial distribution system.

6.3.1 Expected PV Operation Period Effect on Total Cost of Electricity

The problem of sizing PV units under AC power balance targeting minimizing the total cost of electricity is considered starting with the original case to be the following:

- The capital cost of PV units is $4X10^6$/MW.
- Selling cost of electricity generated from PV units to the grid is \$420/MWh.

- PV units' operation period is set to 10 years.

Targeting the objective, the results for this case were as the following:

- There were no PV units allocated in the system under the set operation period.
- The total cost of electricity was \$5.52X10^6 for the 10 years of operation.
- The energy transmitted from the grid to the system is the energy purchased from the grid and it is 1.812285X10^{4}MWh yearly; while this case does not include any selling to the grid as there was no PV in the system.

If the operation period of PV is modified while fixing the capital cost and the set PV electricity selling price, the objective function would be increasing linearly until PV units are allocated in the system and that would take place around 33 years of PV operation period which does not take place in practice as PV operation period is expected to reach 25 years [77] as shown in Figure 6-1. This is represented by the point after which the curve starts decaying. This is due to the fact that the system will not be able to recover its capital cost over the expected PV operation period with the set PV electricity selling price considering AC power losses. The power purchased at time *t* in season *s* at 10 years of PV operation period is shown in Table 6-3.

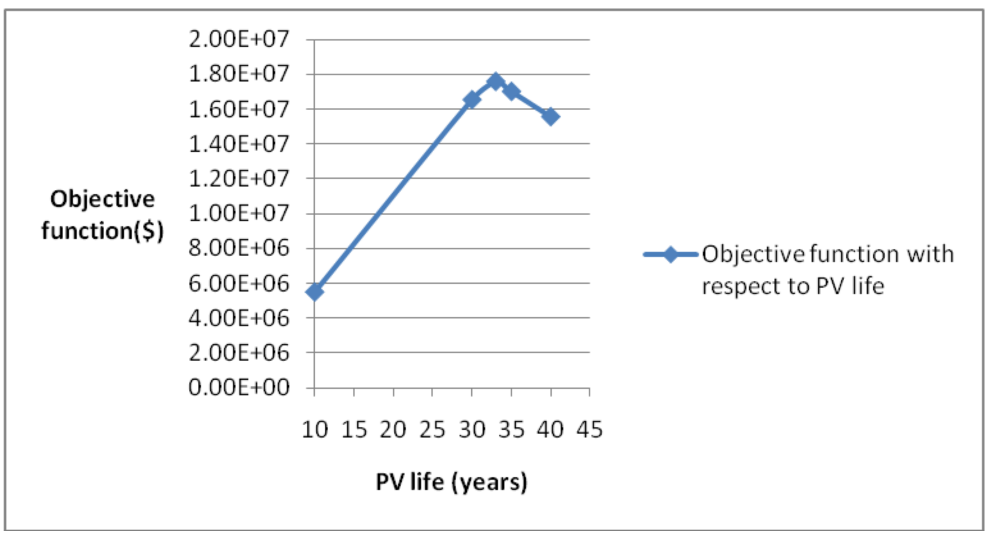

Figure 6-1: Objective function with respect to PV operation period

Table 6-3 Power purchased in per unit (PU) from grid when PV operation period is 10

Time Hour	Fall (PU)	Winter (PU)	Spring (PU)	Summer (PU)	Time Hour	Fall (PU)	Winter (PU)	Spring (PU)	Summer (PU)
1	0.299605	0.260692	0.406087	0.243769	13	0.459054	0.391707	0.701203	0.365892
2	0.28495	0.252087	0.391988	0.235737	14	0.448901	0.38285	0.701203	0.357648
3	0.280075	0.243497	0.377938	0.227717	15	0.453974	0.374012	0.677952	0.349419
4	0.280075	0.24779	0.377938	0.231726	16	0.479444	0.38285	0.670254	0.357648
5	0.28495	0.273628	0.391988	0.255842	17	0.48456	0.391707	0.670254	0.365892
6	0.353741	0.303948	0.434435	0.284126	18	0.48456	0.409478	0.647305	0.382429
7	0.413579	0.36079	0.520803	0.337105	19	0.464141	0.418393	0.639702	0.390722
8	0.459054	0.405028	0.602015	0.378289	20	0.438776	0.409478	0.639702	0.382429
9	0.464141	0.422858	0.66258	0.394875	21	0.398538	0.38285	0.647305	0.357648
10	0.464141	0.427329	0.693426	0.399032	22	0.348792	0.338842	0.602015	0.316656
11	0.459054	0.422858	0.701203	0.394875	23	0.299605	0.295265	0.491778	0.276028
12	0.459054	0.396142	0.693426	0.37002	24	0.319216	0.265001	0.434435	0.24779

The power transmitted at time *t* in season *s* at 33years of PV operation period is presented in Table 6-4. The positive values correspond to the power purchase from the grid to supply the system; while the negative values represent the power sold to the grid from the system at time *t* in season *s* due to extra generation from PV units.

Table 6-4 Power transmitted at time *t* in season *s* when PV operation period is 33 years

Time Hour	Fall (PU)	Winter (PU)	Spring (PU)	Summer (PU)
1	0.299605	0.260692	0.406087	0.243769
2	0.28495	0.252087	0.391988	0.235737
3	0.280075	0.243497	0.377938	0.227717
4	0.280075	0.24779	0.377938	0.231726
5	0.28495	0.273628	0.391988	0.255842
6	0.353741	0.303948	0.434435	0.284126
7	0.413579	0.36079	0.509769	0.298739
8	0.44924	0.405028	0.522785	0.187367
9	0.347297	0.399054	0.550851	0.002881
10	0.199979	0.278271	0.340435	-0.17759
11	0.053061	0.134417	0.069168	-0.33402
12	-0.04886	0.090261	0.088053	-0.4675
13	-0.02148	0.14714	-0.04593	-0.48459
14	-0.13168	0.128224	-0.02469	-0.531
15	0.020651	0.207302	-0.08517	-0.47074
16	0.157077	0.301526	0.201476	-0.363
17	0.334037	0.358366	0.301964	-0.1973
18	0.457209	0.404072	0.468614	0.007435
19	0.464141	0.418393	0.611404	0.216225
20	0.438776	0.409478	0.639702	0.355568
21	0.398538	0.38285	0.647305	0.357648
22	0.348792	0.338842	0.602015	0.316656
23	0.299605	0.295265	0.491778	0.276028
24	0.319216	0.265001	0.434435	0.24779

6.3.2 Study of Effect of PV Electricity Selling Price on Total Cost of Electricity

In this subsection, the PV electricity selling price was modified until allocation is accomplished and such modification was carried on when PV operation period is set to be 15 years. This is presented in Table 6-5. The capacities of PV units are displayed in Table 6-6 at age 33years.

Table 6-5 PV electricity selling price effect on objective function and PV availability in the system

Value $/MWh (set PV selling price)	Objective function ($)	Allocation	Selling
420	8.29E+06	No	No
700	8.29E+06	No	No
800	8.29E+06	No	No
800.5	1.60E+07	Yes	Yes
801	1.39E+07	Yes	Yes
804.9	1.39E+07	Yes	Yes
810	1.37E+07	Yes	Yes
830	1.33E+07	Yes	Yes
850	1.28E+07	Yes	Yes
900	1.17E+07	Yes	Yes

Table 6-6 PV units' capacities

Location	Size (MW)
2	0.61057810
3	0.02472857
6	0.65778886
7	0.95599374
8	1
9	1
10	1
11	0.52188562
12	1
Total capacity (MW)	6.77097488

The power purchased and powers sold over 24 hours in the four seasons of the year are available in Table 6-7 and Table 6-8, respectively. The total energy purchased per one year is$1.186063X10^4$MWh; while the total energy sold per year is $1.54374X10^3$MWh. Comparing the energy purchased from the grid when PV is allocated in the system to the energy purchased from the grid when there was no PV allocation, it can be noted that with PV units available in the system, less energy is purchased from the grid and that is reflected back on the cost such that less payments are made to the grid. It can be noted from Table 6-8 that there is no energy sold in the winter season. This is due to the low PV capacity factor in that season**.**

Table 6-7 Power purchased at time *t* in season *s* in per unit

Time Hour	Fall	Winter	Spring	Summer	Time Hour	Fall	Winter	Spring	Summer
1	0.299605	0.260692	0.406087	0.243769	13	0	0.14714	0	0
2	0.28495	0.252087	0.391988	0.235737	14	0	0.128224	0	0
3	0.280075	0.243497	0.377938	0.227717	15	0.020651	0.207302	0	0
4	0.280075	0.24779	0.377938	0.231726	16	0.157077	0.301526	0.201476	0
5	0.28495	0.273628	0.391988	0.255842	17	0.334037	0.358366	0.301964	0
6	0.353741	0.303948	0.434435	0.284126	18	0.457209	0.404072	0.468614	0.007435
7	0.413579	0.36079	0.509769	0.298739	19	0.464141	0.418393	0.611404	0.216225
8	0.44924	0.405028	0.522785	0.187367	20	0.438776	0.409478	0.639702	0.355568
9	0.347297	0.399054	0.550851	0.002881	21	0.398538	0.38285	0.647305	0.357648
10	0.199979	0.278271	0.340435	0	22	0.348792	0.338842	0.602015	0.316656
11	0.053061	0.134417	0.069168	0	23	0.299605	0.295265	0.491778	0.276028
12	0	0.090261	0.088053	0	24	0.319216	0.265001	0.434435	0.24779

Table 6-8 Power sold at time *t* in season *s* in per unit

Time Hour	Fall	Spring	Summer
10	0	0	0.17759
11	0	0	0.33402
12	0.04886	0	0.4675
13	0.02148	0.04593	0.48459
14	0.13168	0.02469	0.531
15	0	0.08517	0.47074
16	0	0	0.363
17	0	0	0.1973

6.3.3 Capital Cost Reduction over Years under AC Power Balance

Assuming the utility decided to fix the cost at which it purchases electricity from PV system, the operation periods at which the system would allocate PV in the system and sell electricity to the grid at the various capital costs expected to decay in the future as displayed in Table 6-9.

Table 6-9 PV operation period at which capital cost is recovered at fixed PV electricity selling price

Capital cost $/MW	Operation period at which PV is allocated in the system	Objective function at allocation operation period $
4000000	25	1.99E+07
3000000	14	1.62E+07
1000000	8	4.48E+06

It can be seen from this table that when the capital cost of PV units has reduced, the operation period at which the system would allocate PV units in it or would sell electricity to grid will reduce as well. Toward the future, the payback time would reach a reasonable value taking in consideration the PV capacity factor and meeting the system demands before any selling is made from PV units to the grid.

6.4 Chapter 6 Summary

This chapter presented the optimal location and size of PV units in 13 bus radial feeder under AC power balance with feeder power flow representation. It considered different studies. It finds the objective function variation with respect to the operation period of PV units. It selected the best PV unit's electricity selling price at which the system would allocate it in it taking into consideration certain operation period of PV unit and its fixed capital cost. Since the capital cost of PV units is expected to decay over the years, the work studied the effect of different capital cost on PV units' allocation in the system. In the next chapter, suggestions to further develop this work are presented.

Chapter 7
Conclusion & Future Work

In conclusion, the problem of optimal PV sizing and allocation in 13bus system has been considered. The problem was carried out under both demand-supply balance and AC power balance. The proposed work sets the basic optimization model suggested to be used for any distribution system taking into consideration the PV capacity factor and the capability of the system in supplying its loads first and then selling extra generated power to the grid. The optimization problem is formulated to minimize the total cost of electricity in the system involving the capital cost of PV units. The work differs from other previous works in that it determines the payback time of the PV units considering a more realistic case that involves the variation in the daily solar radiation reflected back into the system as a PV capacity factor rather than considering the full rating of the PV units. Moreover, the payback time is different from previous studies as it considers the supplement of the system demands from the units as a major objective before selling to the grid.

It has been found out that at fixed capital cost of PV units and at fixed PV electricity selling price to the grid (set by the utility), the total cost of electricity of the system that involves the capital cost of PV units would keep increasing over years until it reaches a certain age at which the cost would flip and starts decreasing. This would represent the point after which the system would find it optimal to allocate PV units such that the total cost of electricity of the system is reduced. Then the study goes for fixing the operation period of PV units and their capital costs, and determines the prices at which the system would be able to allocate PV units in the system and the prices at which it would achieve profit.. In addition, the operation period of PV units at which payback time is found to decrease as the capital cost of PV units is decreasing (if the utility decides to fix the price at which it purchases PV electricity from the system), is determined. Since the world is moving toward renewable clean energy, the utility would gain the advantage of meeting safe environment standards, and the customer would be willing to achieve profit.

This work can be further extended by taking into consideration the discount rate of the market price. It can also be applied on the original IEEE 13 bus radial feeder without any modifications such that all the capacitors and the regulator are involved without the assumption of all the lines to be three phase lines considering a different optimization simulation tool that can be used to solve non convex problems.

In the future, the approach can be applied on larger distribution systems. It can involve more classes of renewable energy sources such as wind or combinations of both PV and wind technologies.

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