

Throughput Scaling Laws in Point-to-Multipoint Cognitive Networks

by

Nadia Jamal

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Simultaneous operation of different wireless applications in the same geographical region and the same frequency band gives rise to undesired interference issues. Since licensed (primary) applications have been granted priority access to the frequency spectrum, unlicensed (secondary) services should avoid imposing interference on the primary system. In other words, secondary system's activity in the same bands should be in a controlled fashion so that the primary system maintains its quality of service (QoS) requirements.

In this thesis, we consider collocated point-to-multipoint primary and secondary networks that have simultaneous access to the same frequency band. Particularly, we examine three different levels at which the two networks may coexist: pure interference, asymmetric co-existence, and symmetric co-existence levels.

At the pure interference level, both networks operate simultaneously regardless of their interference to each other. At the other two levels, at least one of the networks attempts to mitigate its interference to the other network by deactivating some of its users. Specifically, at the asymmetric co-existence level, the secondary network selectively deactivates its users based on knowledge of the interference and channel gains, whereas at the symmetric level, the primary network also schedules its users in the same way.

Our aim is to derive optimal sum-rates (i.e., throughputs) of both networks at each co-existence level as the number of users grows asymptotically and evaluate how the sum-rates scale with the network size. In order to find the asymptotic throughput results, we derive two propositions; one on the asymptotic behaviour of the largest order statistic and one on the asymptotic behaviour of the sum of lower order statistics.

As a baseline comparison, we calculate primary and secondary sum-rates for the time division (TD) channel sharing. Then, we compare the asymptotic secondary sum-rate in TD to that under simultaneous channel sharing, while ensuring the primary network maintains the same sum-rate in both cases.

Our results indicate that simultaneous channel sharing at both asymmetric and symmetric co-existence levels can outperform TD. Furthermore, this enhancement is achievable when user scheduling in uplink mode is based only on the interference gains to the opposite network and not on a network's own channel gains. In other words, the optimal secondary sum-rate is achievable by applying a scheduling strategy, referred to as the least interference strategy, for which only the knowledge of interference gains is required and can be performed in a distributed way.

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Chapter 1

Introduction

1.1 Cognitive Radio Networks

The frequency spectrum is a limited resource and yet should accommodate the increasing amount of wireless services and applications. Therefore, fixed spectrum assignment is traditionally used to allow simultaneous operation of different wireless applications in the same geographical region. Using this policy, government agencies (such as the Federal Communications Commission (FCC) in the United States) have licensed specified frequency bands (licensed bands) to the wireless services (licensed services or licensees) on a long term basis. Accordingly, a licensee has the exclusive right to access its allocated frequency band and is therefore protected from interference from other wireless services.

The licensed frequency bands may differ from one country to another. Radio band, television (TV) band, cellular band, satellite band, and the air traffic control band are examples of the licensed frequency bands. There is also another part of the frequency spectrum which is allocated to unlicensed users with the purpose of encouraging innovations, i.e., anybody can use these bands under specific rules (such as the maximum transmitted power per Hertz) without any need to purchase a license. As an example, systems such as Bluetooth, IEEE 802.11b/g/n (Wi-Fi) and cordless phones operate in 2.4 GHz unlicensed band.

According to [1], in the United States, significant portions of the frequency spectrum have been allocated to licensed applications by the FCC, while leaving only some tight bands available for unlicensed applications. In the FCC frequency allocation chart, [1], the unlicensed bands are the green bands which are referred to as the *amateur* bands.

As the popularity of wireless services increase and innovative systems appear, the demand for more bandwidths and data rates grows dramatically. More bandwidth is required for the operation of unlicensed systems, while the existing licensed applications also demand for higher data rates

and thus higher bandwidth.

On the other hand, measurements conducted by government agencies indicate that licensed bands are underutilized by their licensees both temporally and geographically. In other words, monitoring of the frequency spectrum has revealed that some licensed frequency bands are not occupied all the time and thus are not utilized efficiently. According to the FCC, temporal and geographical variations in the utilization of the assigned frequency spectrum range from 15% to 85%, which means that utilization of frequency spectrum strongly depends on time and place [2], [3].

In [4], the sporadic usage of frequency recourses in the frequency range 50MHz to 1GHz is depicted. The measurement was conducted in Lichtenau, Germany during one day in 2001. According to that study, the electromagnetic field strength changes from one frequency band to another. The frequency bands below 300MHz, which are allocated to analog audio and video broadcasts, are constantly used. There is a spectral peak in 900 MHz allocated to GSM (European 2G Global System for Mobile Communications). Furthermore, some wide ranges of frequency bands appear to be partially used which leads to spectral inefficiency.

Emergence of the idea of cognitive radio networks: Motivated by these measurements and observations, there has been recent interest in finding a new communication paradigm, rather than the current fixed allocation policy, that increases the spectral efficiency and enables optimal accommodation of the wireless services in the limited frequency spectrum. In this regard, dynamic or opportunistic spectrum access was first introduced by Mitola [5].

The basic idea of opportunistic spectrum access is to allow unlicensed users to communicate over and utilize the licensed bands in a controlled fashion so that the performance of the licensed services does not severely degrade and a quality of service (QoS) requirement maintains for the licensed users.

In such systems, unlicensed users must have the capability of acquiring knowledge on the wireless environment, for example the licensed system's activity, and consequently adapt their behaviour based on this knowledge. For this reason, a user with the mentioned capability is broadly referred to as a *cognitive radio* and a network that employs cognitive radios is called a *cognitive radio network*.

The idea of cognitive radio networks has evoked much interest in regulatory bodies (including the FCC) to provide new spectrum allocation policies in order to support cognitive radio [2], [3]. According to [3], a cognitive radio is defined as:

“Cognitive radio is a radio that can change its transmitter parameters based on interaction with the environment in which it operates. This interaction may involve active negotiation or communications with other spectrum users and/or passive sensing and decision making within the radio.”

The report [2] concludes that smart radio technologies can enable better access to spectrum and thus increase spectral efficiency. It also recommends the FCC to remove the regulatory barriers to the use of cognitive radios and their access to the licensed bands.

In this thesis, we refer to the licensed users (resp. network) as the primary users (resp. network) since they have priority access to the allocated bands, while the unlicensed users (resp. network) are referred to as secondary users (resp. network).

Thus far, three distinct approaches to cognitive radio have been proposed which are namely *interweave*, *overlay*, and *underlay* approaches [6]. Each approach focuses on enabling spectrum sharing among primary and secondary users and consequently increasing the spectral efficiency of the licensed bands. Hybrid approaches to cognitive radio are also studied in the literature. For example in [7], the authors combine the overlay and interweave approaches.

In the following, we overview the basic ideas and challenges regarding to the interweave, overlay, and underlay paradigms.

1.1.1 Interweave Approach

Most prior works in the area of cognitive radio networks focus on the interweave approach where the secondary users sense the entire licensed bands dynamically in order to detect idle frequency slots and thus exploit them for transmission. An example of this approach is the emerging IEEE 802.22 standard which is based on exploiting underutilized frequency slots in the TV band [8].

This approach is also called *interference avoidance* in the literature because the objective of the secondary system is to not interfere at all with the primary system's activity. Otherwise, sensing errors may lead to unwanted collisions which degrade performance of the primary system. Accordingly, the secondary users must have advanced capabilities to dynamically monitor the licensed bands, detect frequency gaps (also called spectrum holes or whitespaces), and opportunistically communicate over these gaps. In [9, Chapter 1], spectrum holes are formally defined as:

“A spectrum hole is a band of frequency assigned to the primary users, but at a particular time and specific geographical location, the band is not utilized by those users.”

The technical challenge in this approach is related to identifying and exploitation of the spectrum holes. In [10], these challenges are further classified as:

- *Spectrum sensing*; related to detection of the spectrum holes.
- *Spectrum management*; related to capturing the best spectrum hole that meets the cognitive user's communication requirements like QoS.
- *Spectrum sharing*; related to providing a fair spectrum scheduling method with other cognitive users.

- *Spectrum mobility*; related to immediately vacating the spectrum hole and exchanging it upon detection of the primary systems's activity.

Much of the research on sequential detection of spectrum holes [11], [12] is motivated by the interweave approach. Typically, the licensed spectrum occupancy is modeled by a Markov chain where the transition probabilities are derived based on the dynamics of the primary traffic and are irrespective of the secondary users' operations [11].

Matched filter detection, energy detection, cyclostationary detection, and wavelet detection are among the methods proposed for spectrum sensing [13], [14].

In [15], the author discusses the signal processing challenges such as spectrum hole detection, channel state estimation, transmitter power control, and dynamic spectrum management in detail.

In cases with multiple secondary users (multiuser cognitive networks), cooperative detection of the spectrum holes can help to improve performance and achieve efficient spectrum management. The cooperation can be either through a distributed or a centralized way. The interested reader is referred to [13] and [16]–[18] for an overview of cooperative spectrum sensing in interweave cognitive systems. The multiuser cooperative approach may require synchronization among the users and thus a more complex network design. In [19], the case of multiuser noncooperative (competitive) cognitive radio network is addressed as a game-theoretic problem.

Upon detection of the spectrum holes, the secondary users should exploit appropriate modulation schemes and coding formats. Orthogonal frequency division multiplexing (OFDM) is a flexible modulation scheme that the secondary system can apply to communicate over the licensed frequency gaps [4]. Consequently, to avoid interference to the adjacent licensed bands, more effective sidelobe suppression techniques are required which are discussed in [20] and [21].

Some prior literature also investigates a cross-layer approach to cognitive medium access control (MAC) which aims to integrate the physical layer and MAC layer [9, Chapter 10].

1.1.2 Overlay Approach

The overlay approach is based on the assumption that the secondary system has knowledge of the primary system's messages or possibly its codebooks. Thus, it can dedicate a part of its power to relay the primary's message and use the remaining power for its own communication. This power split can be adjusted so that the degradation in the primary's performance caused by the interference from the secondary system is compensated for by the primary's performance enhancement due to the assistance from the secondary system [6]. Therefore, the primary system's rate can remain unchanged. On the other hand, in order to mitigate the interference at the secondary receiver and provide positive secondary throughput, the secondary transmitter can apply dirty paper coding (DPC) [22], based on the knowledge of the primary's messages.

The simplest overlay network is a two-user interference channel, where one user has knowledge of the other user's transmission. These systems and their capacity results are further analyzed in [23]–[27].

1.1.3 Underlay Approach

In the underlay approach, which is our focus in this work, primary and secondary systems are allowed to simultaneously communicate over the same frequency band. In this case, unlike the overlay approach, none of the primary and secondary systems have knowledge of the other system's messages or codebooks. Thus, no DPC is applied and interference is treated as noise.

Due to the constraint on the interference power at the primary receiver, the capacity of the secondary system in the underlay approach can be derived under received power constraints at the primary receiver rather than transmitted power constraints at the secondary transmitter in different channel models (point-to-point AWGN¹ non-fading channels, point-to-point AWGN fading channels, Gaussian multiple access channels, etc).

In point-to-point AWGN non-fading channels, since the received power is a deterministically scaled version of the transmitted power, the capacity under the received power constraint and that under the transmitted power constraint are identical [28]. However, in fading environments, the secondary system can take advantage of the fading characteristic of the channel by transmitting at higher power levels whenever the channel between the secondary transmitter and the primary receiver is in a deep fade and achieve positive secondary throughput [29].

For the point-to-point AWGN fading channel considered in [29], capacity of the secondary system is derived under received power constraints at the primary receiver. Interestingly, it is shown that an increase in the variance of the fading distribution gives rise to a higher capacity and in some cases, this capacity result is significantly more than that under a transmitted power constraint at the secondary transmitter.

In [30], the authors consider underlay cognitive systems that exploit multiple transmit antennas at the cognitive transmitter, and investigate spatial multiplexing gains for the secondary system under interference constraints on the primary receivers.

1.2 Thesis Highlights and Objectives

In this thesis, we consider underlay operation of primary and secondary point-to-multipoint networks in an AWGN fading environment.

¹additive white Gaussian noise

When a point-to-multipoint network operates in downlink mode, we assume that its base station transmits to the user with the highest channel gain. Thus, the achievable throughput is equivalent to the capacity of a point-to-point AWGN fading channel derived by using Shannon’s theorem [31]. For the uplink transmission of a point-to-multipoint network, the maximum sum-rate of a multiple access channel is achievable using Gaussian codebooks at the transmitters [31].

For the two point-to-multipoint networks, each network’s interference to the the opposite network is regarded as noise and advanced techniques such as DPC is not performed. Since each network may operate in either of uplink or downlink modes, there are four possible uplink-downlink scenarios to be considered.

For channel sharing between the primary and secondary networks in uplink mode, we consider three different levels of co-existence based on utilization of the channel state information (CSI), i.e., the networks can apply the information on the channel and interference power gains to selectively activate or deactivate their users so that they can achieve a higher sum-rate and at the same time impose limited interference on the opposite network’s receiver. The levels of co-existence are further discussed in Chapter 4.

Our main interest is studying the tradeoffs between the sum-rate of each network and the number of users. Consequently, we measure the asymptotic sum-rate of the secondary network for each co-existence level while ensuring the primary network’s sum-rate is not reduced by more than a specified primary protection factor $0 < f \leq 1$.

As a base reference, we consider sum-rate results of time division (TD) channel sharing where the primary network employs the whole channel for a fraction $0 < f \leq 1$ of time, leaving a fraction $1 - f$ for the secondary network’s transmission. Then, in each of the uplink-downlink scenarios, we compare the asymptotic secondary sum-rate under simultaneous transmission to that in TD for the same value of f .

Sum-rates of primary and secondary networks in TD are in principle similar to those in channel sharing by means of orthogonal subchannels (frequency division channel sharing) and those in perfect opportunistic channel sharing (interweave approach with no sensing errors). In the former, f refers to the fraction of frequency band dedicated to the primary’s transmission, while in the latter, it is the fraction of licensed frequency band in which the primary is active.

We assume that channel power gains are continuous, independent and identically distributed (i.i.d.), and have a cumulative distribution function (CDF) with a specific behaviour in the very high gain regime referred to as the *exponential tail* property. The interference power gains are also assumed to be continuous, i.i.d. , and have any arbitrary CDF with a property corresponding to its order in the very low gain regime. These properties are presented in detail in Chapter 2.

The results obtained in this work indicate that significant asymptotic performance improvements can be gained over TD by selectively deactivating users in uplink mode.

Interestingly, the enhanced sum-rate results are achievable when uplink scheduling and deactivation of users is based only on the interference gains to the opposite network. Moreover, in uplink mode, user scheduling based on interference to the other network as well as gains to one's own network does not improve the results. Therefore, by only allowing activation of users that generate interference gains less than a certain threshold, the secondary network can achieve a significant throughput while the primary network is protected to the desired protection factor.

Our results in this thesis are partly presented in [32] where the channel and interference gains are Rayleigh distributed. As we shall illustrate later, the CDF of the power gains corresponding to the fading distributions such as Rayleigh, Rician, and Nakagami- m have both of the mentioned properties. Therefore, our general results in this paper can be applied to these three fading distributions.

Our results also verify that an increase in the variance of the Nakagami- m distribution (decrease in m) expands the range of f for which better sum-rates are achieved. In other words, the richer the scattering environment, the better the asymptotic sum-rates over TD.

1.3 Outline of the Thesis

The rest of this thesis is organized as follows: In Chapter 2, we introduce our system model, notations, and a definition on the asymptotic behaviour of a random sequence. Sum-rates of a single network in uplink and downlink modes, and a proposition on the asymptotic behaviour of the largest order statistic of a random sequence are presented in Chapter 3. We introduce the three co-existence levels and our uplink scheduling strategies in Chapter 4, and provide a proposition on the sum of lower order statistics of a random sequence. In Chapter 5 through Chapter 8, sum-rates of primary and secondary networks are derived in each of the four uplink-downlink scenarios. Then, in Chapter 9, we compare the secondary sum-rates in simultaneous transmission to that in TD and discuss about the distributed uplink scheduling. Some simulation results on the accuracy of the propositions are provided in Chapter 10. We finally conclude this work in Chapter 11 and discuss directions for further research.

Chapter 2

Preliminaries

2.1 System Model and Notations

Our system model consists of two collocated point-to-multipoint networks that share the same frequency band. We assume that the network with priority access to the band (the primary network) comprises a base station and n users. The channel power gains between each primary user and the primary base station are denoted by $G_i^p, i = 1, \dots, n$.

Likewise, the second network (the secondary network) consists of a base station and k users. Also, we denote by $G_j^s, j = 1, \dots, k$, the channel power gains between each of the secondary users and the secondary base station.

Each of the primary and secondary networks can operate in either uplink or downlink modes, resulting in four possible scenarios for their simultaneous activity, which are namely:

1. primary and secondary uplink (PUSU),
2. primary downlink and secondary uplink (PDSU),
3. primary uplink and secondary downlink (PUSD),
4. primary and secondary downlink (PDSD).

When a network is in uplink mode, its users transmit to the network's base station and in downlink mode, we assume that a base station transmits to its user which has the highest channel power gain. Therefore, at an instant, in either of the four scenarios mentioned above, only one entity in each network acts as a receiver. We thus denote by $G_j^{s \rightarrow p}$ the interference power gain between the j th secondary user and the primary receiver. Similarly, the interference power gain between the i th primary user and the secondary receiver is denoted by $G_i^{p \rightarrow s}$, with the special case that $i = 0$ (resp. $j = 0$) denotes the interference from the primary (resp. secondary) base station.

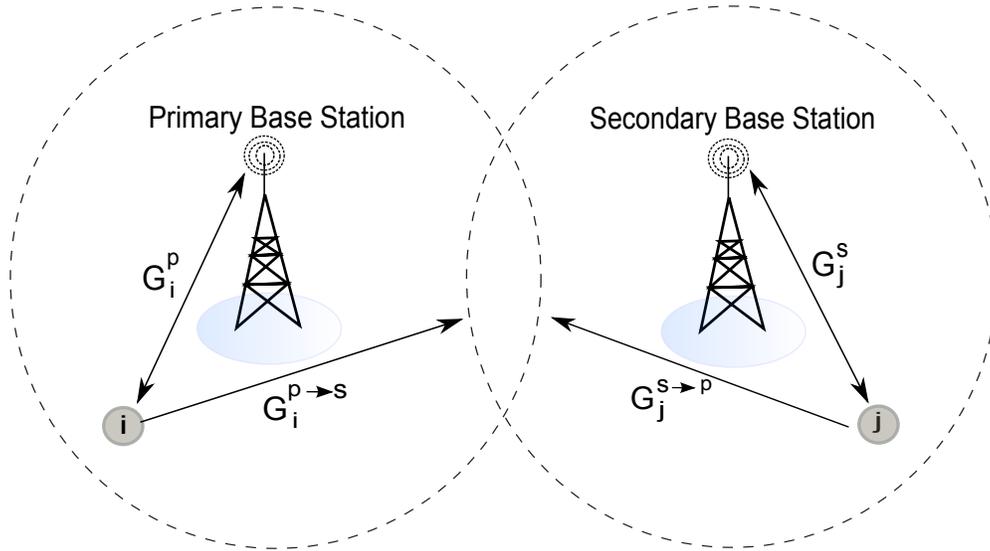


Figure 2.1: Channel power gain between the i th primary (resp. the j th secondary) user and primary (resp. secondary) base station. Interference power gain between the i th primary (resp. the j th secondary) user and secondary (resp. primary) receiver.

Fig. 2.1 illustrates the primary and secondary networks, channel power gains, and interference power gains. For simplicity, we only show one user in each network and its corresponding channel and interference power gains.

Throughout the thesis, we assume that in both networks each transmitter employs a complex Gaussian codebook with power P , and each receiver is subject to AWGN with power N .

2.2 Channel and Interference Power Gains

In wireless channels, if noise and interference are neglected, the variation in the received signal's power over time is caused by *large scale propagation effects* and *small scale propagation effects*. Shadowing and path loss contribute to the large scale propagation effects since they cause variations in the received signal power over relatively large distances, while small scale effects are characterized by multipath fading [33].

In this thesis, we only consider the small scale propagation effects in the received power. Hence, the channel power gains (G_i^p and G_j^s) and the interference power gains ($G_i^{p \rightarrow s}$ and $G_j^{s \rightarrow p}$) are characterized by multipath fading. The received power is thus the product of the transmitted power (which is equal to P) and the respective channel or interference power gain, and is represented by

a random variable.

Throughout the thesis, we assume that all the primary and secondary channel power gains are continuous, i.i.d. , and without loss of generality, we assume they have unit mean. We also assume that all the interference power gains are continuous, i.i.d. , and have unit mean. Furthermore, the interference power gains and the channel power gains are assumed to be independent.

Definition 2.1: Let X be a random variable with probability distribution function (pdf) $f_X(x)$. We say X has an exponential tail with parameter c if:

$$\lim_{x \rightarrow \infty} \frac{\ln f_X(x)}{x} = -c, \quad (2.1)$$

where c is a positive real number.

Definition 2.2: Let X be a continuous random variable with CDF $F_X(x)$. We say X has parameters $\lambda > 0$ and $\gamma > 0$ as $x \rightarrow 0$, if and only if there exist positive numbers δ and M such that

$$|F_X(x) - \lambda x^\gamma| \leq M |x^{\gamma+1}|, \quad (2.2)$$

for every $|x| < \delta$. We express this fact by

$$F_X(x) \approx \lambda x^\gamma + O(x^{\gamma+1}), \quad (2.3)$$

as $x \rightarrow 0$.

- Throughout the thesis, the channel power gains are considered to have the exponential tail property and the interference power gains are considered to have parameters λ and γ .

Now, we present some examples of the properties mentioned in definitions 2.1 and 2.2 where X is a random variable with $E[X^2] = 2$.

Rayleigh and Rician fading: Let X be a Rician distributed random variable with parameter K (K -factor) as the ratio of the line of sight component's power to the power of the multipath fading component [33], and $E[X^2] = 2$. Then, $G = X^2/2$ has unit mean and its pdf is [34]:

$$f_G(g) = (1 + K)e^{-K} \cdot e^{-(K+1)g} \cdot I_0\left(2\sqrt{K(K+1)g}\right), \quad (2.4)$$

for $g > 0$, where $I_0(\cdot)$ is a modified Bessel function. Then,

$$\begin{aligned} \lim_{g \rightarrow \infty} \frac{\ln f_G(g)}{g} &= \lim_{g \rightarrow \infty} \frac{\ln \left[(1 + K)e^{-K} \cdot e^{-(K+1)g} \cdot I_0\left(2\sqrt{K(K+1)g}\right) \right]}{g} \\ &= \lim_{g \rightarrow \infty} \frac{\ln(1 + K)e^{-K}}{g} - (K + 1) + \lim_{g \rightarrow \infty} \frac{\ln I_0\left(2\sqrt{K(K+1)g}\right)}{g}. \end{aligned} \quad (2.5)$$

Since for the Bessel function we have $I_0(\sqrt{g}) \approx \frac{e^{\sqrt{g}}}{\sqrt{2\pi\sqrt{g}}}$ when $g \rightarrow \infty$, then

$$\lim_{g \rightarrow \infty} \frac{\ln f_G(g)}{g} = -(K+1), \quad (2.6)$$

which implies that G has an exponential tail with parameter $c = (K+1)$.

On the other hand, when $g \rightarrow 0$, for the Bessel function we have

$$I_0\left(2\sqrt{K(K+1)g}\right) = 1 + \frac{4K(K+1)g}{2^2} + \dots \approx 1 + O(g), \quad (2.7)$$

and using Taylor series expansion we have

$$e^{-(K+1)g} = 1 - (K+1)g + \frac{(K+1)^2 g^2}{2} + \dots \approx 1 + O(g). \quad (2.8)$$

Note that in (2.7) and (2.8), similar to Definition 2.2, we use the notation $O(\cdot)$ to describe the limiting behaviour of the Bessel function and the exponential function respectively when g tends towards 0.

Substituting in (2.4), we obtain $f_G(g) \approx (1+K)e^{-K}[1+O(g)]$. Thus, the CDF $F_G(g) \approx (1+K)e^{-K}g + O(g^2)$ when $g \rightarrow 0$ and consequently for the Rician distribution with K -factor we obtain the parameters $\lambda = (1+K)e^{-K}$ and $\gamma = 1$.

Since Rician distribution reduces to Rayleigh distribution for $K = 0$, if X is Rayleigh distributed with unit mean then G has an exponential tail with parameter $c = 1$ and it has parameters $\gamma = 1$ and $\lambda = 1$.

Nakagami- m fading: Let X have a Nakagami- m distribution and $E[X^2] = 2$. Then $G = X^2/2$ has unit mean and its pdf is [35]:

$$f_G(g) = \frac{1}{\Gamma(m)} m^m g^{m-1} e^{-mg}, \quad (2.9)$$

for $g > 0$, where $m = \frac{E[G]^2}{\text{VAR}[G]}$. Then,

$$\lim_{g \rightarrow \infty} \frac{\ln f_G(g)}{g} = \lim_{g \rightarrow \infty} \frac{\ln \left[\frac{m^m}{\Gamma(m)} g^{m-1} e^{-mg} \right]}{g} = \lim_{g \rightarrow \infty} \frac{\ln \frac{m^m}{\Gamma(m)}}{g} + \lim_{g \rightarrow \infty} \frac{(m-1) \ln g}{g} - m = -m. \quad (2.10)$$

Thus, G has an exponential tail with parameter $c = m$.

On the other hand, when $g \rightarrow 0$ we have

$$e^{-mg} = 1 - mg + \frac{m^2 g^2}{2} + \dots \approx 1 + O(g).$$

Table 2.1: Parameters c , γ , and λ for Rayleigh, Rician, and Nakagami- m distributions.

	c	γ	λ
Rayleigh	1	1	1
Rician	$K + 1$	1	$(1 + K)e^{-K}$
Nakagami- m	m	m	$\frac{m^{m-1}}{\Gamma(m)}$

Then, $f_G(g) \approx \frac{m^m}{\Gamma(m)} (g^{m-1} + O(g^m))$ and the CDF $F_G(g) \approx \frac{m^{m-1}}{\Gamma(m)} g^m + O(g^{m+1})$ for $g \rightarrow 0$. Thus, for the Nakagami- m distribution we obtain $\lambda = \frac{m^{m-1}}{\Gamma(m)}$ and $\gamma = m$.

Table 2.1 summarizes the values of c , γ , and λ for Rayleigh, Rician, and Nakagami- m distributions.

2.3 Asymptotic Behaviour of a Random Sequence

In the following, we provide a definition on the asymptotic behaviour of a sequence of random variables which we will broadly use hereafter to derive the asymptotic sum-rate of each network in our system model.

Definition 2.3: Let X_1, X_2, \dots be a sequence of random variables whose mean may not converge. We say that the sequence X_n *concentrates* as the non-random sequence $a_n > 0$ if

$$\Pr[|X_n - a_n| \geq \varepsilon a_n] \rightarrow 0, \quad (2.11)$$

as $n \rightarrow \infty$, for all $\varepsilon > 0$. In other words,

$$\Pr\left[\left|\frac{X_n}{a_n} - 1\right| \geq \varepsilon\right] \rightarrow 0, \quad (2.12)$$

as $n \rightarrow \infty$, which implies X_n/a_n converges in probability to 1, [36]. We denote this fact by $X_n \sim a_n$.

Remark 2.1 We also abuse notation and write $X_n \sim 0$ when X_n converges in probability to zero.

Example 2.1: Let X_1, X_2, \dots, X_n be i.i.d. random variables with finite mean $E[X]$ and finite variance $\text{VAR}[X]$. Let $S_n = \sum_{i=1}^n X_i$. Therefore, we have $E[S_n] = nE[X]$ and $\text{VAR}[S_n] = n\text{VAR}[X]$. Then, using Chebychev inequality [36], we have

$$\Pr[|S_n - E[S_n]| \geq \varepsilon E[S_n]] \leq \frac{\text{VAR}[S_n]}{\varepsilon^2 E[S_n]^2} = \frac{\text{VAR}[X]}{\varepsilon^2 n E[X]}, \quad (2.13)$$

and thus

$$\Pr[|S_n - nE[X]| \geq \varepsilon nE[X]] \rightarrow 0, \quad (2.14)$$

as $n \rightarrow \infty$ for all $\varepsilon > 0$, which implies $S_n \sim nE[X]$.

Lemma 2.1: Given positive sequences a_n and b_n ,

a) if $X_n \sim a_n$ and $\lim_{n \rightarrow \infty} a_n/b_n = 1$, then $X_n \sim b_n$. We then write $X_n \sim a_n \sim b_n$.

b) if $X_n \sim a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$, then $X_n \sim 0$. We then write $X_n \sim a_n \sim 0$.

Proof of Lemma 2.1:

a) $\lim_{n \rightarrow \infty} a_n/b_n = 1$ implies that for every real number $\varepsilon_0 > 0$ there exists a natural number N such that for every $n > N$ we have $(1 - \varepsilon_0)b_n < a_n < (1 + \varepsilon_0)b_n$. On the other hand, since

$$\Pr[(1 - \varepsilon)a_n \leq X_n \leq (1 + \varepsilon)a_n] \rightarrow 1, \quad (2.15)$$

as $n \rightarrow \infty$ for every $0 < \varepsilon < 1$, then as $n \rightarrow \infty$, we have

$$\Pr[(1 - \varepsilon)(1 - \varepsilon_0)b_n \leq X_n \leq (1 + \varepsilon)(1 + \varepsilon_0)b_n] = \Pr[(1 - \delta)b_n \leq X_n \leq (1 + \delta)b_n] \rightarrow 1, \quad (2.16)$$

for every $\delta > 0$, which implies $X_n \sim b_n$.

b) $\lim_{n \rightarrow \infty} a_n = 0$ implies that for every real number $\varepsilon_0 > 0$ there exists a natural number N such that for every $n > N$ we have $-\varepsilon_0 < a_n < \varepsilon_0$. Then, as $n \rightarrow \infty$, we have

$$\Pr[-(1 - \varepsilon)\varepsilon_0 \leq X_n \leq (1 + \varepsilon)\varepsilon_0] = \Pr[-\delta b_n \leq X_n \leq \delta b_n] \rightarrow 1, \quad (2.17)$$

for every $\delta > 0$, which implies $X_n \sim 0$.

■

Remark 2.2: Let the random sequence X_n concentrate as $a_n > 0$ and we have $\lim_{n \rightarrow \infty} a_n = L$, where $L \geq 0$ is a real number. Then, using Lemma 2.1 we have $X_n \sim L$ and we can express this fact by

$$\text{plim}_{n \rightarrow \infty} X_n = L, \quad (2.18)$$

where “plim” denotes limit in probability.

Lemma 2.2: Let X_n and Y_n be two sequences of random variables where $X_n \overset{\sim}{\sim} x_n$ and $Y_n \overset{\sim}{\sim} y_n$ for strictly positive sequences x_n and y_n . Then,

$$\frac{X_n}{Y_n} \overset{\sim}{\sim} \frac{x_n}{y_n}. \quad (2.19)$$

Proof of Lemma 2.2: $X_n \overset{\sim}{\sim} x_n$ and $Y_n \overset{\sim}{\sim} y_n$ imply that

$$\Pr[(1 - \varepsilon)x_n \leq X_n \leq (1 + \varepsilon)x_n] \rightarrow 1, \quad (2.20)$$

and

$$\Pr[(1 - \varepsilon)y_n \leq Y_n \leq (1 + \varepsilon)y_n] \rightarrow 1, \quad (2.21)$$

as $n \rightarrow \infty$ for every $0 < \varepsilon < 1$. Therefore, we have

$$\Pr \left[\frac{(1 - \varepsilon)x_n}{(1 + \varepsilon)y_n} \leq \frac{X_n}{Y_n} \leq \frac{(1 + \varepsilon)x_n}{(1 - \varepsilon)y_n} \right] \rightarrow 1, \quad (2.22)$$

as $n \rightarrow \infty$ for every $\varepsilon > 0$, which implies $\frac{X_n}{Y_n} \overset{\sim}{\sim} \frac{x_n}{y_n}$.

■

Note that for the sequences in Lemma 2.2, if $\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = L$, where L is a nonnegative real number, then recalling Remark 2.2 we have

$$\text{plim}_{n \rightarrow \infty} \frac{X_n}{Y_n} = \lim_{n \rightarrow \infty} \frac{x_n}{y_n} = L. \quad (2.23)$$

Lemma 2.3: Let the real valued random sequence X_1, X_2, \dots concentrate as a_n , where $a_n > 1$ for all n . Then, $\ln X_n \overset{\sim}{\sim} \ln a_n$, where \ln is the natural logarithm, when

a) $a_n \rightarrow \infty$ as $n \rightarrow \infty$,

b) $\lim_{n \rightarrow \infty} a_n$ exists.

Proof of Lemma 2.3: $X_n \overset{\sim}{\sim} a_n$ implies that

$$\Pr[(1 - \varepsilon)a_n \leq X_n \leq (1 + \varepsilon)a_n] \rightarrow 1, \quad (2.24)$$

as $n \rightarrow \infty$ for all $0 < \varepsilon < 1$. Since the function $\ln x$, $x \in \mathbb{R}^+$, is one-to-one, then we have

$$\Pr[\ln((1 - \varepsilon)a_n) \leq \ln X_n \leq \ln((1 + \varepsilon)a_n)] \rightarrow 1, \quad (2.25)$$

as $n \rightarrow \infty$ for all $0 < \varepsilon < 1$. Consequently, we have

$$\Pr[\ln(1 - \varepsilon) + \ln a_n \leq \ln X_n \leq \ln(1 + \varepsilon) + \ln a_n] \rightarrow 1, \quad (2.26)$$

and since $a_n > 1$ for all n , then

$$\Pr\left[1 + \frac{\ln(1 - \varepsilon)}{\ln a_n} \leq \frac{\ln X_n}{\ln a_n} \leq 1 + \frac{\ln(1 + \varepsilon)}{\ln a_n}\right] \rightarrow 1, \quad (2.27)$$

as $n \rightarrow \infty$ for all $0 < \varepsilon < 1$, which implies

$$\Pr\left[1 - \delta \leq \frac{\ln X_n}{\ln a_n} \leq 1 + \delta\right] \rightarrow 1, \quad (2.28)$$

as $n \rightarrow \infty$ where $\delta \geq \max\left\{-\frac{\ln(1-\varepsilon)}{\ln a_n}, \frac{\ln(1+\varepsilon)}{\ln a_n}\right\}$.

a) In the case that $a_n \rightarrow \infty$ as $n \rightarrow \infty$, for every $\delta > 0$ we can find a sufficiently large $N \in \mathbb{N}$ such that $\delta \geq \max\left\{-\frac{\ln(1-\varepsilon)}{\ln a_N}, \frac{\ln(1+\varepsilon)}{\ln a_N}\right\}$. Therefore, (2.28) holds as $n \rightarrow \infty$ for every $\delta > 0$, which implies that $\ln X_n \sim \ln a_n$.

b) In the case that $\lim_{n \rightarrow \infty} a_n$ exists, for every $\delta > 0$ we can find a sufficiently small $0 < \varepsilon < 1$. Therefore, (2.28) holds as $n \rightarrow \infty$ for every $\delta > 0$, which implies that $\ln X_n \sim \ln a_n$.

■

Example 2.2: Recall Example 2.1. Then, using Lemma 2.3 we have

$$\ln\left(1 + \sum_{i=1}^n X_i\right) \sim \ln(1 + nE[X]). \quad (2.29)$$

Example 2.3 Recall Example 2.1. Then, using Lemma 2.1, Lemma 2.2, and Lemma 2.3, we have

$$\ln\left(1 + \frac{\sum_{i=1}^n X_i}{1 + \sum_{i=1}^n X_i}\right) \sim \ln\left(1 + \frac{nE[X]}{1 + nE[X]}\right) \sim \ln 2, \quad (2.30)$$

i.e.,

$$\text{plim}_{n \rightarrow \infty} \ln\left(1 + \frac{\sum_{i=1}^n X_i}{1 + \sum_{i=1}^n X_i}\right) = \ln 2. \quad (2.31)$$

Remark 2.3: It can be easily verified that Lemma 2.3 is also valid in case of the logarithm base-2 function. In other words, for the sequences in Lemma 2.3 we have $\log X_n \sim \log a_n$, where \log is logarithm base-2.

Chapter 3

Sum-rates of a Single Network

In this chapter, we derive asymptotic sum-rates of a single network in our system model when it operates in either uplink or downlink modes.

3.1 Sum-rate of a Single Network in Uplink Mode

When the users in a point-to-multipoint network employ Gaussian codebooks in uplink mode, the maximum sum-rate of a Gaussian multiple access channel is achievable according to [31].

For the primary network in our system model, the maximum sum-rate can be written as¹

$$R_{\Sigma} = \log \left(1 + \frac{P \sum_{i=1}^n G_i^p}{N} \right). \quad (3.1)$$

Since the i.i.d. channel power gains have unit mean, then following Example 2.1 we have

$$\sum_{i=1}^n G_i \sim nE[G] = n, \quad (3.2)$$

and thus

$$R_{\Sigma} \sim \log \left(1 + \frac{Pn}{N} \right) \sim \log n. \quad (3.3)$$

Similarly, for the secondary network with $k = n^{\alpha}$ ($\alpha > 0$) users, the sum-rate can be written as

$$R_{\Sigma} = \log \left(1 + \frac{P \sum_{j=1}^{n^{\alpha}} G_j^s}{N} \right) \sim \log \left(1 + \frac{Pn^{\alpha}}{N} \right) \sim \alpha \log n. \quad (3.4)$$

¹Throughout the thesis, log is logarithm base-2 and the sum-rates are expressed in bits/sec/Hz.

3.2 Sum-rate of a Single Network in Downlink Mode

According to [37], when a point-to-multipoint network operates in downlink mode, transmitting to the receiver with the strongest gain optimizes the system throughput. Following this scheme, the primary network's sum-rate in downlink mode can be written as

$$R_{\Sigma} = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N} \right). \quad (3.5)$$

Note that knowledge of the network's channel power gains is required for the base station's downlink operation.

3.3 Asymptotic Behaviour of the Largest Order Statistic

In this thesis, we denote the order statistics of a set of random variables X_1, X_2, \dots, X_n , by $X_{1:n}, \dots, X_{n:n}$, i.e., $X_{1:n} \leq X_{2:n} \leq \dots \leq X_{n:n}$. Consequently, $X_{n:n}$ denotes the largest order statistic.

In order to find the asymptotic behaviour of the maximum of channel power gains and thus the asymptotic sum-rate in downlink mode, we provide the following proposition.

Proposition 3.1: Let X_1, X_2, \dots, X_n be i.i.d. random variables with pdf $f_X(x)$ and an exponential tail with parameter $c > 0$. Define $X_{n:n} = \max_{i=1}^n X_i$. Then

$$X_{n:n} \sim \frac{\ln n}{c}. \quad (3.6)$$

Proof of Proposition 3.1: Let X_1, X_2, \dots, X_n be i.i.d. random variables with pdf $f_X(x)$. Since the random variables have an exponential tail with parameter $c > 0$, then $\lim_{x \rightarrow \infty} \frac{\ln f_X(x)}{x} = -c$. Thus, for every real $\varepsilon_0 > 0$, there exists a real $s_0 > 0$ such that $\left| \frac{\ln f_X(x)}{x} + c \right| \leq \varepsilon_0$ whenever $x > s_0$. Therefore, for $x > s_0$

$$e^{-(c+\varepsilon_0)x} \leq f_X(x) \leq e^{-(c-\varepsilon_0)x}. \quad (3.7)$$

Integrating (3.7) from $t > s_0$ to infinity, we obtain

$$1 - \frac{e^{-(c-\varepsilon_0)t}}{c - \varepsilon_0} \leq F_X(t) \leq 1 - \frac{e^{-(c+\varepsilon_0)t}}{c + \varepsilon_0}, \quad (3.8)$$

for every $t > s_0$, where $F_X(t)$ denotes the CDF of random variables X_1, X_2, \dots, X_n .

Define $X_{n:n} = \max_{i=1}^n X_i$. Then, we obtain the CDF of $X_{n:n}$ as $F_{X_{n:n}}(t) = [F_X(t)]^n$ [38]. Thus, for every $t > s_0$

$$\left[1 - \frac{e^{-(c-\varepsilon_0)t}}{c - \varepsilon_0} \right]^n \leq F_{X_{n:n}}(t) \leq \left[1 - \frac{e^{-(c+\varepsilon_0)t}}{c + \varepsilon_0} \right]^n. \quad (3.9)$$

Replacing $t = \left(\frac{1-\varepsilon_0}{c+\varepsilon_0} \ln n\right) > s_0$ and $t = \left(\frac{1+\varepsilon_0}{c-\varepsilon_0} \ln n\right) > s_0$ in (3.9), then we have

$$F_{X_{n:n}}\left(\frac{1-\varepsilon_0}{c+\varepsilon_0} \ln n\right) \leq \left[1 - \frac{1}{(c+\varepsilon_0)n^{1-\varepsilon_0}}\right]^n, \quad (3.10)$$

and

$$F_{X_{n:n}}\left(\frac{1+\varepsilon_0}{c-\varepsilon_0} \ln n\right) \geq \left[1 - \frac{1}{(c-\varepsilon_0)n^{1+\varepsilon_0}}\right]^n, \quad (3.11)$$

respectively.

Since

$$\lim_{n \rightarrow \infty} \left[1 - \frac{1}{(c+\varepsilon) \cdot n^{1-\varepsilon_0}}\right]^n = 0, \quad (3.12)$$

and

$$\lim_{n \rightarrow \infty} \left[1 - \frac{1}{(c-\varepsilon) \cdot n^{1+\varepsilon_0}}\right]^n = 1, \quad (3.13)$$

therefore

$$\Pr \left[\frac{1-\varepsilon_0}{c+\varepsilon_0} \ln n \leq X_{n:n} \leq \frac{1+\varepsilon_0}{c-\varepsilon_0} \ln n \right] \rightarrow 1, \quad (3.14)$$

as $n \rightarrow \infty$, and thus

$$\Pr \left[\left| X_{n:n} - \frac{\ln n}{c} \right| \leq \delta \frac{\ln n}{c} \right] \rightarrow 1, \quad (3.15)$$

for all $\delta > 0$, which implies $X_{n:n} \sim \frac{\ln n}{c}$.

■

Now, recall 3.5 and assume the channel power gains have an exponential tail with parameter $c > 0$, then the network's sum-rate in downlink mode concentrates as

$$R_\Sigma = \log \left(1 + \frac{P \cdot \frac{\ln n}{c}}{N} \right) \sim \log \log n. \quad (3.16)$$

Chapter 4

Simultaneous Transmission

In this chapter, we investigate channel sharing between the primary and secondary networks in our system model. For the two networks to share the same frequency band, different ways of channel sharing can be applied:

- *Time division (TD) channel sharing*: Using TD channel sharing, one network utilizes the whole frequency band for a fraction $0 < f \leq 1$ of time while the other network is silent. The other fraction of time $(1 - f)$ is dedicated to the other network's operation.
- *Frequency division (FD) channel sharing*: Using FD channel sharing, orthogonal frequency sub-bands can be assigned to each of the networks. Then, the networks can operate in their respective allocated sub-bands simultaneously. For example, one network can operate over a fraction f of the frequency band, while at the same time the other network operates in the other $1 - f$ fraction of the frequency band.
- *Simultaneous channel sharing*: In this case, both networks simultaneously utilize the whole frequency band and interference of each network to the opposite network is regarded as noise. Each network can schedule its users based on the CSI and thus mitigate its interference to the other network.

Note that channel sharing among the users in a single network (in the literature referred to as multiple access) is different from channel sharing between the two networks. In our model, the former can be performed for example by using code division multiple access (CDMA), whereas the latter can be done by either of the above schemes.

Table 4.1: Asymptotic primary and secondary sum-rates in TD.

Primary	Secondary	$R_{\Sigma, \text{TD}}^p(n, n^\alpha)$	$R_{\Sigma, \text{TD}}^s(n, n^\alpha)$
up	up	$f \log n$	$\alpha(1-f) \log n$
up	down	$f \log n$	$(1-f) \log \log n$
down	up	$f \log \log n$	$\alpha(1-f) \log n$
down	down	$f \log \log n$	$(1-f) \log \log n$

4.1 Baseline Comparison

In this part, we compute the asymptotic sum-rates of both primary and secondary networks in the four uplink-downlink scenarios when channel is shared via TD channel sharing.

The sum-rate results of TD channel sharing are equivalent to those derived by FD channel sharing in term of efficiency in spectral utilization (expressed in bits/sec/Hz). In this thesis, we consider TD channel sharing as a reference to compare the secondary sum-rates achieved by our proposed channel sharing (simultaneous channel sharing). We define the primary protection factor $0 < f \leq 1$ as the limit

$$f = \text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{TD}}^p(n, k)}{R_{\Sigma}(n)}, \quad (4.1)$$

i.e., it represents the fraction of time that the primary network utilizes the frequency band. Note that in (4.1), $R_{\Sigma}(n)$ refers to the the sum-rate of a single network with n users (primary network in our model), and $R_{\Sigma, \text{TD}}^p(n, k)$ denotes the sum-rate of the primary network in TD channel sharing, where n and k are the number of primary and secondary users, respectively.

Also, we denote by $R_{\Sigma, \text{TD}}^s(n, k)$ the sum-rate of the secondary network in TD channel sharing. If we take $k = n^\alpha$ ($\alpha > 0$) and let the channel power gains have an exponential tail with parameter $c > 0$, then using Proposition 3.1, we obtain asymptotic sum-rates listed in table 4.1 for the TD channel sharing.

We further refer to the limit

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{TD}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)}, \quad (4.2)$$

as the secondary throughput factor in TD which is verified to be equal to $1 - f$. Note that, in (4.2), $R_{\Sigma}(n^\alpha)$ refers to the the sum-rate of a single network with n^α users (secondary network in our model).

Unsurprisingly, if the primary network is not to be impacted at all (i.e., $f = 1$), then the secondary network is always silent.

4.2 Simultaneous Channel Sharing

With this scheme, despite TD and FD channel sharing, both networks operate simultaneously in the same frequency band and interfere with each other. By exploiting sufficient CSI, in uplink mode, each network can schedule (activate or deactivate) its users and hence control its interference to the other network while maximizing its own sum-rate.

4.2.1 CSI and Co-existence Levels

In *downlink mode*, CSI is the knowledge of a network's own channel power gains which is assumed to be available to the network's base station. When the network operates in downlink mode with such CSI, the highest sum-rate is obtained by transmitting to the user with the greatest channel power gain (the strongest user). Transmission to the strongest user is done regardless of the interference that the base station imposes on the opposite network's receiver.

In *uplink mode*, CSI refers to the knowledge of a network's own channel power gains as well as the knowledge of the interference power gains that its users impose on the other network's receiver. We assume that a central entity called the *network's scheduler* has knowledge of this information and applies the CSI to activate or deactivate users of the network so that the network obtains the highest sum-rate and also the interference to the other network is minimized.

Based on utilization of the CSI, three different levels of co-existence are considered for the uplink transmission:

- **Pure interference:** At this simplest level of co-existence, the primary and secondary networks operate independently. None of the networks exploit the CSI to schedule users. Therefore, all the primary and secondary users are active and interfere with each other.
- **Asymmetric co-existence:** At the asymmetric co-existence level, only the secondary network schedules its users. In other words, only the secondary network's scheduler selectively activates or deactivates secondary users. The primary network however, operates as usual.
- **Symmetric co-existence:** The third level is referred to as the symmetric co-existence level. In this case, the primary network cooperates with the secondary network, i.e., the primary scheduler applies its CSI and selectively schedules the primary users.

4.2.2 Scheduling Strategies in Uplink Mode

At asymmetric and symmetric co-existence levels, the primary and secondary users can be activated or deactivated based on two different scheduling strategies which are referred to as the *joint*

optimization, and the *least interference* strategies. These strategies are performed by the scheduler in each network.

- **Joint optimization:** Using the joint optimization strategy, the scheduler of a network can optimally deactivate the network's users based on the entire CSI which is the network's own channel power gains and the interference power gains between its users and the opposite network's receiver. The joint optimization scheduling can be performed in either of the networks such that the highest secondary sum-rate is achieved while the primary network is protected by some protection factor $0 < f \leq 1$.
- **Least interference:** Using the least interference scheduling method, activation or deactivation of users is based only on their interference power gains to the opposite network and not on the network's own channel power gains. In other words, the scheduler only considers the interference power gains rather than the network's own channel power gains.

For the pure interference case as well as each scheduling strategy at the symmetric or asymmetric co-existence levels, we are interested in maximizing the secondary throughput factor defined as

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)}, \quad (4.3)$$

subject to the constraint,

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^p(n, n^\alpha)}{R_{\Sigma}(n)} \geq f \quad (4.4)$$

for some primary protection factor $0 < f \leq 1$, where $R_{\Sigma, \text{Sim}}^p(n, n^\alpha)$ and $R_{\Sigma, \text{Sim}}^s(n, n^\alpha)$ represent primary and secondary sum-rates in simultaneous channel sharing, respectively.

The constraint (4.4) indicates that the primary sum-rate in simultaneous channel sharing cannot be less than that under TD channel sharing.

4.2.3 Asymptotic Sum of Lower Order Statistics

In order to find the asymptotic behaviour of the total interference imposed from one network on the opposite network for the least interference strategy, we have to find the asymptotic sum of lower order statistics. In this regard, we provide the following proposition which plays a key role in the sequel.

Proposition 4.1: Let X_1, \dots, X_n be continuous i.i.d. random variables with unit mean and CDF $F_X(x)$ with parameters λ and γ as $x \rightarrow 0$. Furthermore, let $f : \mathbb{N} \rightarrow \mathbb{N}$ be such that $f(n) \rightarrow \infty$ and $f(n)/n \rightarrow 0$ as $n \rightarrow \infty$. Then the sum of the $f(n)$ lowest order statistics

$$S_n = \sum_{i=1}^{f(n)} X_{i:n}, \quad (4.5)$$

concentrates as

$$S_n \sim \frac{n}{\lambda^{\frac{1}{\gamma}} \left(1 + \frac{1}{\gamma}\right)} \left(\frac{f(n)}{n}\right)^{1 + \frac{1}{\gamma}}. \quad (4.6)$$

In order to prove Proposition 4.1, we first present the following lemma.

Lemma 4.1: Let $g(x) \approx \lambda x^\gamma + O(x^{\gamma+1})$ as $x \rightarrow 0$, where λ and γ are positive real numbers. Then, we have $g^{-1}(x) \approx \left(\frac{x}{\lambda}\right)^{\frac{1}{\gamma}} + O(x^{\frac{2}{\gamma}})$, as $x \rightarrow 0$.

Proof of Lemma 4.1: Let

$$y = g(x) \approx \lambda x^\gamma + O(x^{\gamma+1}), \quad (4.7)$$

as $x \rightarrow 0$. Then

$$\left(\frac{y}{\lambda}\right)^{\frac{1}{\gamma}} \approx \left(\frac{\lambda x^\gamma + O(x^{\gamma+1})}{\lambda}\right)^{\frac{1}{\gamma}} \approx (x^\gamma(1 + O(x)))^{\frac{1}{\gamma}} = x(1 + O(x))^{\frac{1}{\gamma}}, \quad (4.8)$$

as $x \rightarrow 0$. Since by Taylor series expansion we have $(1 + O(x))^{\frac{1}{\gamma}} \approx 1 + O(x)$ for all $\gamma > 0$ as $x \rightarrow 0$, then

$$\left(\frac{y}{\lambda}\right)^{\frac{1}{\gamma}} \approx x + O(x^2), \quad (4.9)$$

as $x \rightarrow 0$, and consequently

$$x \approx \left(\frac{y}{\lambda}\right)^{\frac{1}{\gamma}} + O(x^2). \quad (4.10)$$

To find the error equivalent to $O(x^2)$ in (4.10) in terms of y , we need to find the degree $q > 0$ so that $y^q \approx (\lambda x^\gamma + O(x^{\gamma+1}))^q \approx O(x^2)$. Solving this, we find $q = \frac{2}{\gamma}$. Therefore, by substituting in (4.10) we obtain $x \approx \left(\frac{y}{\lambda}\right)^{\frac{1}{\gamma}} + O(y^{\frac{2}{\gamma}})$ as $y \rightarrow 0$.

■

Now, we provide the proof of Proposition 4.1.

Proof of Proposition 4.1: Let X_1, \dots, X_n be continuous i.i.d. random variables with CDF $F_X(x) \approx \lambda x^\gamma + O(x^{\gamma+1})$ when $x \rightarrow 0$. We denote their order statistics by $X_{1:n}, X_{2:n}, \dots, X_{n:n}$. Also, let U_1, \dots, U_n be i.i.d. uniformly distributed random variables defined in $[0, 1]$. Then, we have $X_{k:n} \stackrel{d}{=} F_X^{-1}(U_{k:n})$ [38], where $\stackrel{d}{=}$ denotes equality in distribution. Thus

$$S_n = \sum_{r=1}^{f(n)} X_{r:n} \stackrel{d}{=} \sum_{r=1}^{f(n)} F_X^{-1}(U_{r:n}). \quad (4.11)$$

In [38], the expected value and the variance of $U_{f(n):n}$ are derived as

$$\mathbb{E}[U_{f(n):n}] = \frac{f(n)}{n+1}, \quad (4.12)$$

and

$$\text{VAR}[U_{f(n):n}] = \frac{f(n)(n+1-f(n))}{(n+1)^2(n+2)}, \quad (4.13)$$

respectively.

Let $f(n) \rightarrow \infty$ and $f(n)/n \rightarrow 0$ as $n \rightarrow \infty$, then using Chebychev inequality we have

$$\Pr \left[|U_{f(n):n} - \mathbb{E}[U_{f(n):n}]| \geq \varepsilon \frac{f(n)}{n+1} \right] \leq \frac{\text{VAR}[U_{f(n):n}]}{\varepsilon^2 \mathbb{E}[U_{f(n):n}]^2} \leq \frac{1}{\varepsilon^2 f(n)}. \quad (4.14)$$

Thus, $U_{f(n):n} \sim \frac{f(n)}{n+1}$ and for all $\varepsilon > 0$, the event

$$S_n \leq \sum_{r=1}^n F_X^{-1}(U_{r:n}) \mathbf{1}_{\{U_{r:n} \leq \frac{f(n)}{n+1}(1+\varepsilon)\}} = \sum_{r=1}^n F_X^{-1}(U_r) \mathbf{1}_{\{U_r \leq \frac{f(n)}{n+1}(1+\varepsilon)\}}, \quad (4.15)$$

holds at least with probability $1 - \frac{1}{\varepsilon^2 f(n)}$ and the event

$$S_n \geq \sum_{r=1}^n F_X^{-1}(U_{r:n}) \mathbf{1}_{\{U_{r:n} \leq \frac{f(n)}{n+1}(1-\varepsilon)\}} = \sum_{r=1}^n F_X^{-1}(U_r) \mathbf{1}_{\{U_r \leq \frac{f(n)}{n+1}(1-\varepsilon)\}}, \quad (4.16)$$

also holds at least with probability $1 - \frac{1}{\varepsilon^2 f(n)}$.

Now, define

$$Y_r = F_X^{-1}(U_r) \mathbf{1}_{\{U_r \leq \frac{f(n)}{n+1}(1+\varepsilon)\}}, \quad (4.17)$$

and

$$W_r = F_X^{-1}(U_r) \mathbf{1}_{\{U_r \leq \frac{f(n)}{n+1}(1-\varepsilon)\}}. \quad (4.18)$$

Then we have

$$\Pr \left[\sum_{r=1}^n W_r \leq S_n \leq \sum_{r=1}^n Y_r \right] \rightarrow 1 \quad (4.19)$$

as $n \rightarrow \infty$.

On the other hand, we have

$$E[Y_r] = \int_0^{\frac{f(n)}{n+1}(1+\varepsilon)} F_X^{-1}(u_r) du_r \quad (4.20)$$

,

$$E[Y_r^2] = \int_0^{\frac{f(n)}{n+1}(1+\varepsilon)} [F_X^{-1}(u_r)]^2 du_r, \quad (4.21)$$

and

$$\text{VAR}[Y_r] = E[Y_r^2] - E[Y_r]^2. \quad (4.22)$$

By Lemma 4.1, $F_X^{-1}(x) \approx (\frac{x}{\lambda})^{\frac{1}{\gamma}} + O(x^{\frac{2}{\gamma}})$ as $x \rightarrow 0$. Substituting in (4.20) we obtain

$$E[Y_r] \approx a \left(\frac{f(n)}{n+1} \right)^{1+\frac{1}{\gamma}} + O \left(\left(\frac{f(n)}{n+1} \right)^{1+\frac{2}{\gamma}} \right), \quad (4.23)$$

as $n \rightarrow \infty$, where $a = \frac{(1+\varepsilon)^{1+\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}$. In addition,

$$\text{VAR}[Y_r] \approx b \left(\frac{f(n)}{n+1} \right)^{1+\frac{2}{\gamma}} + O \left(\left(\frac{f(n)}{n+1} \right)^{2+\frac{2}{\gamma}} \right), \quad (4.24)$$

as $n \rightarrow \infty$, where $b = \frac{(1+\varepsilon)^{1+\frac{2}{\gamma}}}{\lambda^{\frac{2}{\gamma}}(1+\frac{2}{\gamma})}$. Also, since

$$E \left[\sum_{r=1}^n Y_r \right] = nE[Y_r], \quad (4.25)$$

and

$$\text{VAR} \left[\sum_{r=1}^n Y_r \right] = n\text{VAR}[Y_r], \quad (4.26)$$

using Chebychev inequality, we have

$$\Pr \left[\left| \sum_{r=1}^n Y_r - n\mathbb{E}[Y_r] \right| \geq \delta n \mathbb{E}[Y_r] \right] \leq \frac{\text{VAR}[Y_r]}{n\delta^2 \mathbb{E}[Y_r]^2}, \quad (4.27)$$

for all $\delta > 0$. Since from (4.23) and (4.24) we have $\frac{\text{VAR}[Y_r]}{n\delta^2 \mathbb{E}[Y_r]^2} \rightarrow 0$ as $n \rightarrow \infty$, then for all $\varepsilon > 0$ we obtain

$$\sum_{r=1}^n Y_r \sim n \frac{(1+\varepsilon)^{1+\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}} \left(1+\frac{1}{\gamma}\right)} \left(\frac{f(n)}{n}\right)^{1+\frac{1}{\gamma}}. \quad (4.28)$$

Similarly, one can show that

$$\sum_{r=1}^n W_r \sim n \frac{(1-\varepsilon)^{1+\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}} \left(1+\frac{1}{\gamma}\right)} \left(\frac{f(n)}{n}\right)^{1+\frac{1}{\gamma}}, \quad (4.29)$$

and consequently from (4.19), (4.28), and (4.29), we obtain

$$S_n \sim \frac{n}{\lambda^{\frac{1}{\gamma}} \left(1+\frac{1}{\gamma}\right)} \left(\frac{f(n)}{n}\right)^{1+\frac{1}{\gamma}}. \quad (4.30)$$

■

In the next four chapters, we examine each of the four uplink-downlink scenarios and derive the asymptotic sum-rates at each co-existence level.

Chapter 5

Primary and Secondary Uplink

In this chapter, we derive the asymptotic primary and secondary sum-rates under simultaneous channel sharing when both networks operate in uplink mode. We examine the three distinct co-existence levels and at each level, we find the asymptotic result for the maximum secondary throughput factor (4.3), provided that the primary network is protected by the protection factor $0 < f \leq 1$, as shown in (4.4).

5.1 Pure Interference

We first consider the pure interference level at which all users are active in both networks. Therefore, having n and n^α ($\alpha > 0$) active primary and secondary users respectively, the sum-rates of the primary and secondary networks can be written as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \sum_{i=1}^n G_i^p}{N + P \sum_{j=1}^{n^\alpha} G_j^{s \rightarrow p}} \right), \quad (5.1)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{j=1}^{n^\alpha} G_j^s}{N + P \sum_{i=1}^n G_i^{p \rightarrow s}} \right), \quad (5.2)$$

respectively. For primary and secondary uplink scenario, $G_j^{s \rightarrow p}$ is the interference power gain from the j th secondary user to the primary base station and $G_i^{p \rightarrow s}$ is the interference power gain from the i th primary user to the secondary base station. Also, note that the interference power from one network to the opposite network is treated as noise, i.e., it is added to the Gaussian noise power N .

The primary sum-rate in (5.1) concentrates as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{Pn}{N + Pn^\alpha} \right) \sim [1 - \alpha]^+ \log n, \quad (5.3)$$

where $x^+ := \max(0, x)$, and we have

$$R_{\Sigma}(n) = \log \left(1 + \frac{P \sum_{i=1}^n G_i^p}{N} \right) \dot{\sim} \log \left(1 + \frac{Pn}{N} \right) \dot{\sim} \log n. \quad (5.4)$$

as the primary sum-rate when the secondary network is silent.

In order to protect the primary network by its protection factor f , (4.4) should be satisfied for any $0 < f \leq 1$. However, the primary protection constraint cannot be satisfied for any value $\alpha > 0$ when $f = 1$. For $0 < f < 1$, the primary network is protected when $1 - \alpha \geq f$ which results in

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) \dot{\sim} \log \left(1 + \frac{Pn^\alpha}{N + Pn} \right) \dot{\sim} [\alpha - 1]^+ \log n = 0. \quad (5.5)$$

Therefore, having

$$R_{\Sigma}(n^\alpha) = \log \left(1 + \frac{P \sum_{j=1}^{n^\alpha} G_j^s}{N} \right) \dot{\sim} \log \left(1 + \frac{Pn^\alpha}{N} \right) \dot{\sim} \log n^\alpha, \quad (5.6)$$

we obtain

$$\max_{0 < \alpha \leq 1-f} \left[\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)} \right] = 0. \quad (5.7)$$

for any $0 < f < 1$.

This negative result is not surprising, because the secondary network has no method to limit its interference to the primary network except by decreasing the number of active secondary users. Thus, at the pure interference level, no better secondary sum-rate can be achieved over TD.

5.2 Co-existence

Using CSI, the secondary and primary networks can sufficiently mitigate the interference to each other by deactivating some of their users. Therefore, a positive secondary throughput factor may be achieved while the primary network is protected. In the following, we examine the two scheduling strategies.

Joint optimization strategy: In this case, if the primary network activates n^β ($0 \leq \beta \leq 1$) users out of n and the secondary network activates $n^{\bar{\alpha}}$, users out of n^α ($0 \leq \bar{\alpha} \leq \alpha$), then the sum-rates of the primary and secondary networks can be written as follows

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^\beta} G_{i_\ell}^p}{N + P \sum_{\bar{\ell}=1}^{n^{\bar{\alpha}}} G_{j_{\bar{\ell}}}^{s \rightarrow p}} \right), \quad (5.8)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^{\bar{\alpha}}} G_{j_\ell}^s}{N + P \sum_{\ell=1}^{n^\beta} G_{i_\ell}^{p \rightarrow s}} \right). \quad (5.9)$$

The joint optimization is over $\bar{\alpha}$, β , and the sets $(j_1, j_2, \dots, j_{n^{\bar{\alpha}}})$ and $(i_1, i_2, \dots, i_{n^\beta})$, such that the primary is protected and the secondary throughput factor is maximized.

Least interference strategy: Following the least interference strategy, the primary network activates n^β users ($0 \leq \beta \leq 1$) out of n that result in the least interference power gains to the secondary base station and similarly, the secondary network activates $n^{\bar{\alpha}}$ users out of n^α ($0 \leq \bar{\alpha} \leq \alpha$) which generate the least interference power gains to the primary base station. Since the channel power gains and the interference power gains are independent, the sum-rates of primary and secondary networks can be written as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^\beta} G_{i_\ell}^p}{N + P \sum_{j=1}^{n^{\bar{\alpha}}} G_{j:n^\alpha}^{s \rightarrow p}} \right), \quad (5.10)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^{\bar{\alpha}}} G_{j_\ell}^s}{N + P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}} \right). \quad (5.11)$$

Note that $\sum_{j=1}^{n^{\bar{\alpha}}} G_{j:n^\alpha}^{s \rightarrow p}$ is the sum of the $n^{\bar{\alpha}}$ lowest interference power gains to the primary base station and $\sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}$ is the sum of the n^β lowest interference power gains to the secondary base station.

Furthermore, we have

$$R_\Sigma(n) = \log \left(1 + \frac{P \sum_{i=1}^{n^\beta} G_i^p}{N} \right) \sim \log \left(1 + \frac{P n^\beta}{N} \right) \sim \log n^\beta, \quad (5.12)$$

and

$$R_\Sigma(n^\alpha) = \log \left(1 + \frac{P \sum_{j=1}^{n^{\bar{\alpha}}} G_j^s}{N} \right) \sim \log \left(1 + \frac{P n^{\bar{\alpha}}}{N} \right) \sim \log n^{\bar{\alpha}}. \quad (5.13)$$

Given a scheduling strategy, we are interested in finding the maximum secondary throughput factor while ensuring the primary is protected by a protection factor at least equal to f .

Theorem 5.1: Let the i.i.d. channel power gains have unit mean and an exponential tail with parameter $c > 0$, also let the i.i.d. interference power gains have unit mean and parameters $\lambda > 0$

and $\gamma > 0$. Given the protection of the primary network and using the joint optimization strategy, an upper bound on the maximum achievable secondary throughput factor at the asymmetric co-existence level is $\left[\frac{\alpha-1-f\gamma}{\alpha(1+\gamma)}\right]^+$ for every $0 < f \leq 1$ and $\alpha > 0$. In case of symmetric co-existence, for $0 < f \leq \frac{1}{1+\gamma}$ we have the upper bound

$$\begin{cases} \left[\frac{\gamma}{\alpha(\gamma+1)^2} - \left(\frac{\gamma}{\gamma+1}\right)\frac{f}{\alpha} + \frac{1}{1+\gamma}\right]^+, & \alpha \geq \frac{1}{1+\gamma} - f \\ 1, & 0 < \alpha < \frac{1}{1+\gamma} - f, \end{cases} \quad (5.14)$$

and for $\frac{1}{1+\gamma} < f \leq 1$, we have the upper bound

$$\left[\frac{1}{\alpha\gamma} - \left(1 + \frac{1}{\gamma}\right)\frac{f}{\alpha} + \frac{1}{1+\gamma}\right]^+, \quad (5.15)$$

for $\alpha > 0$. Furthermore, these bounds are also achievable by using the least interference scheduling strategy.

Proof of Theorem 5.1: In the first part of the proof, we show that the above result is achievable when scheduling users is based only on the least interference power gains.

Since the interference power gains have parameters γ and λ , then using Proposition 4.1 for any $\alpha > 0$, we have

$$P \sum_{j=1}^{n^\alpha} G_{j:n^\alpha}^{s \rightarrow p} \sim P \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma})-\frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}, \quad (5.16)$$

and

$$P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s} \sim P \frac{n^{\beta(1+\frac{1}{\gamma})-\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}, \quad (5.17)$$

as the total interference power from the secondary network to the primary base station and the total interference power from the primary network to the secondary base station, respectively.

Therefore, following from (5.10), the sum-rate of the primary network concentrates as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{Pn^\beta}{N + P \cdot \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma})-\frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}} \right) \quad (5.18)$$

$$\sim \left(\beta - \left[\left(1 + \frac{1}{\gamma}\right)\bar{\alpha} - \frac{\alpha}{\gamma} \right]^+ \right)^+ \log n. \quad (5.19)$$

Likewise, following from (5.11), the sum-rate of the secondary network concentrates as

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) \sim \log \left(1 + \frac{Pn^{\bar{\alpha}}}{N + P \cdot \frac{n^{\beta(1+\frac{1}{\gamma}) - \frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}(1+\frac{1}{\gamma})}}} \right) \quad (5.20)$$

$$\sim \left(\bar{\alpha} - \left[\left(1 + \frac{1}{\gamma}\right)\beta - \frac{1}{\gamma} \right]^+ \right)^+ \log n. \quad (5.21)$$

Intuitively, in (5.18) (resp. in (5.20)), if $\bar{\alpha}(1 + \frac{1}{\gamma}) \leq \frac{\alpha}{\gamma}$ (resp. $\beta(1 + \frac{1}{\gamma}) \leq \frac{1}{\gamma}$), the total interference to the primary (resp. secondary) network concentrates to at most a bounded quantity, which suggests that the primary (resp. secondary) network would suffer no adverse effects asymptotically.

We now consider the two co-existence levels:

- **Asymmetric co-existence:** In this case, we have $\beta = 1$ and we must maximize $[\bar{\alpha} - 1]^+$ subject to the primary protection constraint

$$1 - \left[\left(1 + \frac{1}{\gamma}\right)\bar{\alpha} - \frac{\alpha}{\gamma} \right]^+ \geq f, \quad (5.22)$$

for every $0 < f \leq 1$, as well as $0 \leq \bar{\alpha} \leq \alpha$.

Solving the optimization problem when $\alpha > 1$, we find $\bar{\alpha} = \frac{\gamma - f\gamma + \alpha}{1 + \gamma}$ as an optimal value. In the case that $\alpha \leq 1$, the secondary sum-rate trivially concentrates as 0. Therefore, the maximum secondary throughput factor is

$$\max_{\bar{\alpha}} \left[\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)} \right] = \left[\frac{\alpha - 1 - f\gamma}{\alpha(1 + \gamma)} \right]^+, \quad (5.23)$$

for any $0 < f \leq 1$ and $\alpha > 0$.

- **Symmetric co-existence:** In this case, we have $0 \leq \beta < 1$ and we must maximize $\left[\bar{\alpha} - \left[\left(1 + \frac{1}{\gamma}\right)\beta - \frac{1}{\gamma} \right]^+ \right]^+$ subject to the primary protection constraint

$$\beta - \left[\left(1 + \frac{1}{\gamma}\right)\bar{\alpha} - \frac{\alpha}{\gamma} \right]^+ \geq f, \quad (5.24)$$

for every $0 < f \leq 1$, as well as $0 \leq \bar{\alpha} \leq \alpha$.

For $0 < f \leq \frac{1}{1+\gamma}$, the optimal solution is to select

$$\begin{cases} \beta = \frac{1}{1+\gamma}, \bar{\alpha} = \frac{\gamma(\beta-f)+\alpha}{1+\gamma}, & \alpha \geq \frac{1}{1+\gamma} - f \\ \beta = \frac{1}{1+\gamma}, \bar{\alpha} = \alpha, & 0 < \alpha < \frac{1}{1+\gamma} - f. \end{cases} \quad (5.25)$$

For $\frac{1}{1+\gamma} < f \leq 1$, the optimal solution is to select $\beta = f$ and $\bar{\alpha} = \frac{\alpha}{1+\gamma}$ when $\alpha > \frac{(1+\gamma)^2}{\gamma} f - \frac{1+\gamma}{\gamma}$. In the case that $\alpha \leq \frac{(1+\gamma)^2}{\gamma} f - \frac{1+\gamma}{\gamma}$, the secondary sum-rate trivially concentrates as 0.

Thus, for $0 < f \leq \frac{1}{1+\gamma}$ the maximum secondary throughput factor is

$$\begin{cases} \left[\frac{\gamma}{\alpha(\gamma+1)^2} - \left(\frac{\gamma}{\gamma+1}\right) \frac{f}{\bar{\alpha}} + \frac{1}{1+\gamma} \right]^+, & \alpha \geq \frac{1}{1+\gamma} - f \\ 1, & 0 < \alpha < \frac{1}{1+\gamma} - f, \end{cases} \quad (5.26)$$

and for $\frac{1}{1+\gamma} < f \leq 1$, the maximum secondary throughput factor is

$$\left[\frac{1}{\alpha\gamma} - \left(1 + \frac{1}{\gamma}\right) \frac{f}{\alpha} + \frac{1}{1+\gamma} \right]^+, \quad (5.27)$$

when $\alpha > 0$.

The final step of the proof of Theorem 5.1 is to show that using joint optimization we achieve the same sum-rate results. Scheduling users based on joint optimization, one can find upper bounds for the primary and secondary sum-rates as follows

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \leq R_{\Sigma, \text{UB, Sim}}^p(n, n^\alpha), \quad (5.28)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) \leq R_{\Sigma, \text{UB, Sim}}^s(n, n^\alpha), \quad (5.29)$$

where

$$R_{\Sigma, \text{UB, Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{Pn^\beta G_{n:n}^p}{N + P \sum_{j=1}^{n^{\bar{\alpha}}} G_{j:n^\alpha}^{s \rightarrow p}} \right), \quad (5.30)$$

and

$$R_{\Sigma, \text{UB, Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{Pn^{\bar{\alpha}} G_{n^\alpha:n^\alpha}^s}{N + P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}} \right). \quad (5.31)$$

These upper bounds are obtained by assuming that all activated users have the maximum channel power gains to their own network's base station and the least interference power gains to the opposite network's base station.

Since the channel power gains have exponential tail with parameter $c > 0$, we have

$$R_{\Sigma, \text{UB, Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{Pn^\beta \cdot \frac{\ln n}{c}}{N + P \cdot \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma})} n^{-\frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}(1+\frac{1}{\gamma})}}} \right) \sim \log \left(1 + \frac{Pn^\beta}{N + P \cdot \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma}) - \frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}(1+\frac{1}{\gamma})}}} \right), \quad (5.32)$$

and

$$R_{\Sigma, \text{UB, Sim}}^s(n, n^\alpha) \sim \log \left(1 + \frac{Pn^{\bar{\alpha}} \cdot \frac{\ln n^\alpha}{c}}{N + P \cdot \frac{n^{\beta(1+\frac{1}{\gamma})} n^{-\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}} \right) \sim \log \left(1 + \frac{Pn^{\bar{\alpha}}}{N + P \cdot \frac{n^{\beta(1+\frac{1}{\gamma})} n^{-\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}} \right). \quad (5.33)$$

But (5.32) and (5.33) are identical to (5.18) and (5.20) respectively, thus by finding

$$\max_{\bar{\alpha}, \beta} \left[\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{UB, Sim}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)} \right], \quad (5.34)$$

subject to

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{UB, Sim}}^p(n, n^\alpha)}{R_{\Sigma}(n)} \geq f, \quad (5.35)$$

for $0 < f \leq 1$, we obtain the upper bound for the maximum secondary throughput factor to be equal to the maximum secondary throughput factor when user scheduling is based on the least interference strategy. Therefore, by using the joint optimization strategy, the secondary network can not achieve a better sum-rate than that achieved by using the least interference strategy. ■

The results obtained in this chapter indicate that at asymmetric and symmetric co-existence levels, it is possible for the secondary network to provide positive asymptotic sum-rate while the primary network is still protected by factor f , whereas at the pure interference level, the secondary network is asymptotically unable to deliver a positive sum-rate.

Chapter 6

Primary Downlink and Secondary Uplink

In this chapter, we obtain throughput results for the secondary and primary networks, when the former is in uplink mode and the latter is in downlink mode. Since the primary network is operating in downlink mode, there is no symmetric co-existence to be considered. Therefore, in this scenario, we only examine pure interference and asymmetric co-existence levels.

In this scenario, the primary base station transmits to the primary user with the highest channel power gain.

6.1 Pure Interference

In this case, all n^α ($\alpha > 0$) secondary users are active. Therefore, the sum-rates of primary and secondary networks are

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N + P \sum_{j=1}^{n^\alpha} G_j^{s \rightarrow p}} \right), \quad (6.1)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{j=1}^{n^\alpha} G_j^s}{N + P G_0^{p \rightarrow s}} \right), \quad (6.2)$$

respectively.

Since the channel power gains have an exponential tail with parameter $c > 0$, then by Proposition 3.1 we have

$$\max_{i=1}^n G_i^p \sim \frac{\ln n}{c}. \quad (6.3)$$

Thus

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{P \cdot \frac{\ln n}{c}}{N + Pn^\alpha} \right) \sim 0, \quad (6.4)$$

for any $\alpha > 0$, whereas

$$R_\Sigma(n) = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N} \right) \sim \log \left(1 + \frac{P \cdot \frac{\ln n}{c}}{N} \right) \sim \log \log n. \quad (6.5)$$

Therefore, due to the excessive interference from the secondary network on the primary receiver, the primary protection constraint, (4.4), can not be satisfied for any value of $\alpha > 0$ and therefore the networks can not coexist at this level.

6.2 Asymmetric Co-existence

Joint optimization strategy: Using the joint optimization scheduling strategy in the secondary network with $n^{\bar{\alpha}}$ ($0 \leq \bar{\alpha} \leq \alpha$) active secondary users, sum-rates of the primary and secondary networks can be written as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N + P \sum_{\ell=1}^{n^{\bar{\alpha}}} G_{j_\ell}^{s \rightarrow p}} \right), \quad (6.6)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^{\bar{\alpha}}} G_{j_\ell}^s}{N + P G_0^{p \rightarrow s}} \right), \quad (6.7)$$

respectively.

To solve the above joint optimization problem, we should find optimal $\bar{\alpha}$ and the set $(j_1, j_2, \dots, j_{n^{\bar{\alpha}}})$ such that the primary network is protected and the secondary throughput factor is maximized.

Least interference strategy: Using the least interference strategy and letting the secondary network activate only $n^{\bar{\alpha}}$ users that generate the least interference power gains to the primary receiver, sum-rates of the primary and secondary networks can be written as follows

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N + P \sum_{j=1}^{n^{\bar{\alpha}}} G_{j:n^\alpha}^{s \rightarrow p}} \right), \quad (6.8)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^{\bar{\alpha}}} G_{j_\ell}^s}{N + P G_0^{p \rightarrow s}} \right). \quad (6.9)$$

Note that in order to derive the above sum-rates, we use the assumption that the channel power gains and the interference power gains are independent.

Furthermore, at the asymmetric co-existence level, we have

$$R_{\Sigma}(n^{\alpha}) = \log \left(1 + \frac{P \sum_{j=1}^{n^{\bar{\alpha}}} G_j^s}{N} \right) \sim \log \left(1 + \frac{P n^{\bar{\alpha}}}{N} \right) \sim \log n^{\bar{\alpha}}. \quad (6.10)$$

Given a scheduling strategy, we are interested in finding the maximum secondary throughput factor while ensuring the primary is protected by a protection factor at least equal to f .

Theorem 6.1: Let the i.i.d. channel power gains have unit mean and an exponential tail with parameter $c > 0$, also let the i.i.d. interference power gains have unit mean and parameters $\lambda > 0$ and $\gamma > 0$. At the asymmetric co-existence level, provided the protection of the primary network and using the joint optimization scheduling strategy, an upper bound on the maximum secondary throughput factor is $\frac{1}{1+\gamma}$ for any $0 < f \leq 1$ and $\alpha > 0$. Furthermore, this bound is also achievable by using the least interference scheduling strategy.

Proof of Theorem 6.1: First, we examine sum-rate results when user scheduling is based only on the least interference power gains. Since the interference power gains have parameters λ and γ , by Proposition 4.1 we have

$$\sum_{j=1}^{n^{\bar{\alpha}}} G_{j:n^{\bar{\alpha}}}^{s \rightarrow p} \sim \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma})-\frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}. \quad (6.11)$$

Since the channel power gains have an exponential tail with parameter $c > 0$, the primary sum-rate concentrates as

$$R_{\Sigma, \text{Sim}}^p(n, n^{\alpha}) \sim \log \left(1 + \frac{P \cdot \frac{\ln n}{c}}{N + P \cdot \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma})-\frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}} \right). \quad (6.12)$$

Also, since the channel power gains are i.i.d. , the secondary sum-rate concentrates as

$$R_{\Sigma, \text{Sim}}^s(n, n^{\alpha}) \sim \bar{\alpha} \log n. \quad (6.13)$$

To obtain the highest secondary throughput factor, we must maximize $\bar{\alpha}$ subject to the primary protection constraint (4.4).

In (6.12), when $\bar{\alpha} > \alpha/(1+\gamma)$, the sum-rate of the primary network concentrates as 0. On the other hand, in the case that $\bar{\alpha} \leq \alpha/(1+\gamma)$, (4.4) is satisfied for every $0 < f \leq 1$. Thus,

$\bar{\alpha} = \alpha/(1 + \gamma)$ is the optimal value and the maximum secondary throughput factor is then

$$\max_{\bar{\alpha}} \left[\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)} \right] = \frac{1}{1 + \gamma}, \quad (6.14)$$

for any $0 < f \leq 1$ and $\alpha > 0$. Hence, the first part of Theorem 6.1 is proven.

In what follows, we show that applying the joint optimization strategy, the sum-rate results are the same in the best case.

If users are scheduled based on the joint optimization strategy, we have

$$R_{\Sigma, \text{UB}, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N + P \sum_{j=1}^{n^\alpha} G_{j:n^\alpha}^{s \rightarrow p}} \right), \quad (6.15)$$

and

$$R_{\Sigma, \text{UB}, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P n^{\bar{\alpha}} G_{n^\alpha:n^\alpha}^s}{N + P G_0^{p \rightarrow s}} \right), \quad (6.16)$$

as the upper bounds for the primary and secondary sum-rates respectively. These upper bounds are derived by assuming that the secondary active users have the highest channel power gains to their own base station and they also have the least interference power gains to the primary receiver.

Using Proposition 3.1 and Proposition 4.1, we have

$$R_{\Sigma, \text{UB}, \text{Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{P \cdot \frac{\ln n}{c}}{N + P \cdot \frac{n^{\bar{\alpha}(1+\frac{1}{\gamma})} n^{-\frac{\alpha}{\gamma}}}{\lambda^{\frac{1}{\gamma}(1+\frac{1}{\gamma})}}} \right), \quad (6.17)$$

and

$$R_{\Sigma, \text{UB}, \text{Sim}}^s(n, n^\alpha) \sim \log \left(1 + \frac{P n^{\bar{\alpha}} \cdot \frac{\ln n^\alpha}{c}}{N + P G_0^{p \rightarrow s}} \right) \sim \bar{\alpha} \log n. \quad (6.18)$$

Since (6.17) and (6.18) are identical to (6.12) and (6.13) respectively, by maximizing (5.34) over $\bar{\alpha}$ subject to (5.35), we find that the maximum secondary throughput factor is upper bounded by that achieved with the least interference strategy. ■

Chapter 7

Primary Uplink and Secondary Downlink

In this chapter, we obtain sum-rates of the primary and secondary networks, when the primary network is in uplink mode and the secondary network is in downlink mode. Since the secondary network is in downlink mode, there is no asymmetric co-existence for this scenario to be considered. In this scenario, the secondary base station transmits to the secondary user with the highest channel power gain.

7.1 Pure Interference

In this case, all n primary users are active. The sum-rate of primary and secondary networks are then

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \sum_{i=1}^n G_i^p}{N + P G_0^{s \rightarrow p}} \right), \quad (7.1)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + P \sum_{i=1}^n G_i^{p \rightarrow s}} \right), \quad (7.2)$$

respectively.

We find

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{Pn}{N + P G_0^{s \rightarrow p}} \right) \sim \log n, \quad (7.3)$$

and

$$R_{\Sigma}(n) \sim \log \left(1 + \frac{Pn}{N} \right) \sim \log n. \quad (7.4)$$

Thus, (4.4) is satisfied for any $0 < f \leq 1$.

Since the channel power gains have a positive exponential tail c , using Proposition 3.1 we have

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) \sim \log \left(1 + \frac{P \cdot \frac{\ln n^\alpha}{c}}{N + Pn} \right) \sim 0, \quad (7.5)$$

for any $\alpha > 0$ and thus

$$\max_{\alpha > 0} \left[\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_\Sigma(n^\alpha)} \right] = 0, \quad (7.6)$$

for any $0 < f \leq 1$. Hence, the maximum secondary throughput factor is equal to zero in the pure interference case.

7.2 Symmetric Co-existence

Following the same line of development as in the first two scenarios (chapters 5 and 6), we can obtain primary and secondary sum-rates by using each of the scheduling strategies.

Joint optimization strategy: In this case, if the primary network activates n^β ($0 \leq \beta \leq 1$) users out of n , then the sum-rates of the primary and secondary networks can be written as follows

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^\beta} G_{i_\ell}^p}{N + P G_0^{s \rightarrow p}} \right), \quad (7.7)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + P \sum_{\ell=1}^{n^\beta} G_{i_\ell}^{p \rightarrow s}} \right). \quad (7.8)$$

The joint optimization is over β , and the set of vectors $(i_1, i_2, \dots, i_{n^\beta})$, such that the primary is protected and the secondary throughput factor is maximized.

Least interference strategy: Following the least interference strategy, the primary network activates n^β users ($0 \leq \beta \leq 1$) out of n that result in the least interference power gains to the secondary receiver. Since the channel power gains and the interference power gains are independent, the sum-rates of primary and secondary networks can be written as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \sum_{\ell=1}^{n^\beta} G_{i_\ell}^p}{N + P G_0^{s \rightarrow p}} \right), \quad (7.9)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}} \right). \quad (7.10)$$

At the symmetric co-existence level, we have

$$R_{\Sigma}(n) = \log \left(1 + \frac{P \sum_{i=1}^{n^\beta} G_i^p}{N} \right) \sim \log \left(1 + \frac{P n^\beta}{N} \right) \sim \log n^\beta. \quad (7.11)$$

We also have

$$R_{\Sigma}(n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N} \right) \sim \log \left(1 + \frac{P \cdot \frac{\ln n^\alpha}{c}}{N} \right) \sim \log \log n. \quad (7.12)$$

Given a scheduling strategy, we are interested in finding the maximum secondary throughput factor while ensuring the primary is protected by a protection factor at least equal to f .

Theorem 7.1: Let the i.i.d. channel power gains have unit mean and an exponential tail with parameter $c > 0$, also let the i.i.d. interference power gains have unit mean and parameters $\lambda > 0$ and $\gamma > 0$. For $\alpha > 0$, given the protection of the primary network and using the joint optimization strategy, an upper bound on the maximum secondary throughput factor is 1 for any $0 < f \leq \frac{1}{1+\gamma}$ and 0 for any $\frac{1}{1+\gamma} < f \leq 1$. Furthermore, this result is also achievable by using the least interference scheduling strategy.

Proof of Theorem 7.1: When scheduling users is based only on the least interference power gains, the primary sum-rate concentrates as

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \sim \beta \log n. \quad (7.13)$$

Using Proposition 4.1, we find

$$\sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s} \sim \frac{n^{\beta(1+\frac{1}{\gamma})-\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}, \quad (7.14)$$

and using Proposition 3.1, we have

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) \sim \log \left(1 + \frac{P \cdot \frac{\ln n^\alpha}{c}}{N + P \cdot \frac{n^{\beta(1+\frac{1}{\gamma})-\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}}(1+\frac{1}{\gamma})}} \right). \quad (7.15)$$

Choosing $\beta = f$, the primary network is protected and the secondary throughput factor is obtained as

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_{\Sigma}(n^\alpha)} = \begin{cases} 1 & 0 < f \leq \frac{1}{1+\gamma} \\ 0 & \frac{1}{1+\gamma} < f \leq 1, \end{cases}$$

for any $\alpha > 0$.

Using the joint optimization strategy, we have

$$R_{\Sigma, \text{UB}, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P n^\beta G_{n:n}^p}{N + P G_0^{s \rightarrow p}} \right), \quad (7.16)$$

and

$$R_{\Sigma, \text{UB}, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}} \right), \quad (7.17)$$

as the upper bounds for primary and secondary sum-rates respectively. We find the upper bounds to concentrate as

$$R_{\Sigma, \text{UB}, \text{Sim}}^p(n, n^\alpha) \sim \log \left(1 + \frac{P n^\beta \cdot \frac{\ln n}{c}}{N + P G_0^{s \rightarrow p}} \right) \sim \beta \log n, \quad (7.18)$$

and

$$R_{\Sigma, \text{UB}, \text{Sim}}^s(n, n^\alpha) \sim \log \left(1 + \frac{P \cdot \frac{\ln n^\alpha}{c}}{N + P \frac{n^{\beta(1+\frac{1}{\gamma})} n^{-\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}(1+\frac{1}{\gamma})}}} \right), \quad (7.19)$$

respectively.

Again, by solving the optimization problem (5.34) subject to (5.35), one can show that no better result is achievable by applying the joint optimization strategy. ■

Chapter 8

Primary and Secondary Downlink

As the primary and secondary networks both operate in downlink mode, there is no co-existence level to be considered. In this scenario, the primary base station transmits to the primary user with the strongest channel power gain and similarly, the secondary base station transmits to the secondary user with the highest channel power gain. Thus, the sum-rates of the primary and secondary networks are

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) = \log \left(1 + \frac{P \max_{i=1}^n G_i^p}{N + PG_0^{s \rightarrow p}} \right), \quad (8.1)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + PG_0^{p \rightarrow s}} \right), \quad (8.2)$$

respectively. Since the i.i.d. channel power gains have unit mean and an exponential tail with parameter $c > 0$, then using Proposition 3.1, we have

$$R_{\Sigma, \text{Sim}}^p(n, n^\alpha) \sim \log \log n, \quad (8.3)$$

and

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) \sim \log \log n. \quad (8.4)$$

Since

$$R_\Sigma(n) \sim \log \log n, \quad (8.5)$$

and

$$R_\Sigma(n^\alpha) \sim \log \log n, \quad (8.6)$$

therefore

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^p(n, n^\alpha)}{R_\Sigma(n)} = 1 \geq f, \quad (8.7)$$

for any $0 < f \leq 1$ and

$$\text{plim}_{n \rightarrow \infty} \frac{R_{\Sigma, \text{Sim}}^s(n, n^\alpha)}{R_\Sigma(n^\alpha)} = 1. \quad (8.8)$$

As a result, for any given $0 < f \leq 1$, the primary network is always protected, while the secondary achieves a positive throughput factor.

Note that in this scenario, there is no restriction on the interference power gains to be i.i.d. and have parameters γ and λ .

Chapter 9

Performance Evaluation

9.1 Distribution of the Channel and Interference Power Gains

The results obtained in the first three uplink-downlink scenarios are valid in case of channel power gains that are i.i.d. and have exponential tail property (such as Rayleigh, Rician, or Nakagami- m distributed power gains) and interference power gains that are i.i.d. and have parameters $\gamma > 0$ and $\lambda > 0$ (such as Rayleigh, Rician, or Nakagami- m distributed power gains). Furthermore, as mentioned earlier, in the last scenario only the exponential tail property is required for the channel power gains.

Our results are still valid for the following cases:

1. When the mean of the i.i.d. channel power gains or that of the i.i.d. interference power gains are not unity.
2. When the interference power gains from the primary network to the secondary network and that from the secondary network to the primary network are not identically distributed but have the property indicated in Definition 2.2 with different values of λ and the same value of γ .
3. When the primary and secondary channel power gains are not identically distributed but have the exponential tail property with different values of c .
4. When conditions 2 and 3 both hold at the same time.

In cases when 2 holds but the values of γ are also different, the sum-rate results can be derived in a similar way.

Table 9.1: Ranges of f with higher secondary sum-rates over TD in the PUSU scenario. For Nakagami- m interference gains, $\gamma = m$ and for Rayleigh and Rician interference gains, $\gamma = 1$.

Asymmetric	$\frac{1+\gamma\alpha}{\alpha-\gamma+\gamma\alpha} < f \leq 1$,	$\alpha > 1 + \gamma$
Symmetric	$\frac{\frac{\alpha\gamma^2}{\gamma+1}-1}{\gamma\alpha-1-\gamma} < f \leq 1$,	$\alpha > 1 + \gamma, \gamma \geq 1$
	$0 < f < \frac{\gamma\alpha-\frac{\gamma}{\gamma+1}}{\gamma\alpha+\alpha-\gamma}$,	$0 < \alpha < \frac{1}{1+\gamma}, \gamma \geq 1$

9.2 Comparison to TD

In this part, we compare the maximum secondary throughput factor achievable under simultaneous channel sharing (derived in chapters 5–8) to that achievable in TD channel sharing. As shown in Chapter 4, the asymptotic secondary throughput factor in TD is equal to $1 - f$ in all of the uplink-downlink scenarios.

9.2.1 Primary and Secondary Uplink

Asymmetric co-existence: At the asymmetric co-existence level, when $\alpha > 1 + \gamma$, for $\frac{1+\gamma\alpha}{\alpha-\gamma+\gamma\alpha} < f \leq 1$ simultaneous channel sharing outperforms TD.

Symmetric co-existence: In case of symmetric co-existence, when $\gamma \geq 1$ for

$$\begin{cases} \frac{\frac{\alpha\gamma^2}{\gamma+1}-1}{\gamma\alpha-1-\gamma} < f \leq 1, & \alpha > 1 + \gamma \\ 0 < f < \frac{\gamma\alpha-\frac{\gamma}{\gamma+1}}{\gamma\alpha+\alpha-\gamma}, & 0 < \alpha < \frac{1}{1+\gamma}, \end{cases} \quad (9.1)$$

the secondary network achieves a greater sum-rate compared to TD.

As shown in Chapter 2, we obtain $\gamma = 1$ when interference gains have Rayleigh or Rician distributions. Therefore, when $\alpha > 2$ symmetric co-existence outperforms TD for $1/2 < f \leq 1$, and when $0 < \alpha < 1/2$ symmetric co-existence outperforms TD for $0 < f < 1/2$. Furthermore, with asymmetric co-existence, secondary sum-rate is higher than that in TD for $\frac{1+\alpha}{2\alpha-1} < f \leq 1$ when $\alpha > 2$.

The tradeoff between the secondary throughput factor and the primary protection factor f is illustrated in Fig. 9.1 for $\gamma = 1$ and $\alpha = 4$. Surprisingly, when $f = 1$, for both symmetric and asymmetric levels, the secondary network can provide positive sum-rates.

Fig. 9.2 shows the secondary throughput factor versus f under simultaneous channel sharing when interference gains have Nakagami- m distribution with $m = 3/2$ and $\alpha = 4$. At the asymmetric co-existence level, for $14/17 < f \leq 1$, simultaneous channel sharing is better than TD and at the symmetric co-existence level, simultaneous channel sharing outperforms TD for $26/35 < f \leq 1$.

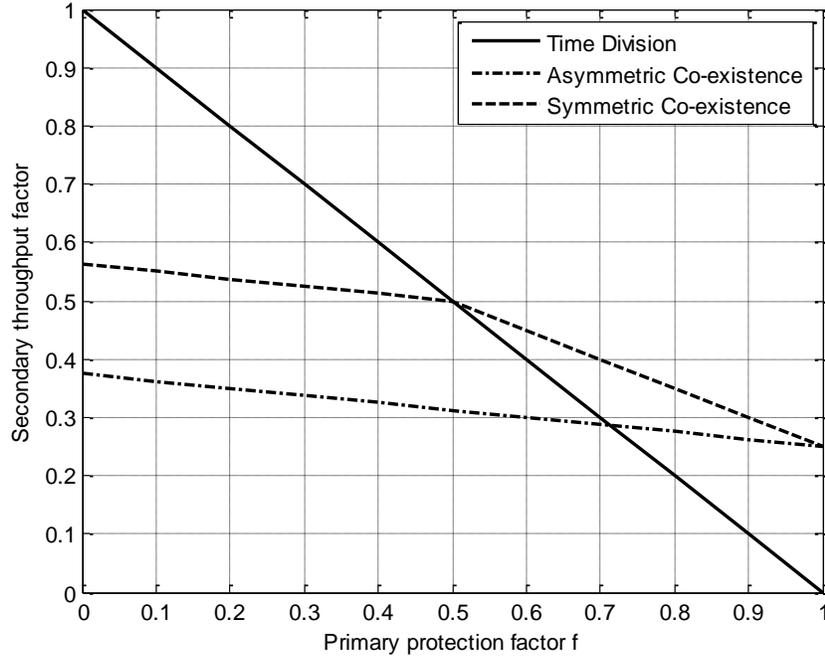


Figure 9.1: Asymptotic result for maximum secondary throughput factor versus primary protection factor f for $\alpha = 4$ and when both networks are in uplink mode. Rayleigh or Rician interference power gains are assumed. Symmetric (resp. asymmetric) co-existence is better than TD when $1/2 < f \leq 1$ (resp. $5/7 < f \leq 1$).

Table 9.1 summarizes the ranges of f for every $\gamma > 0$ in the asymmetric co-existence case and the ranges of f when $\gamma \geq 1$ in the symmetric co-existence. The results for the symmetric co-existence case can be applied to the Nakagami- m interference gains when $m \geq 1$. When $\gamma < 1$, higher asymptotic sum-rates are achieved over TD for some values of f at the symmetric co-existence level. In this case, by comparing the secondary throughput factor derived in Chapter 5 and the secondary throughput factor in TD, one can find the ranges of f for which simultaneous channel sharing outperforms TD.

9.2.2 Primary Downlink and Secondary Uplink

In this scenario, simultaneous channel sharing asymptotically outperforms TD for $\frac{\gamma}{1+\gamma} < f \leq 1$.

Particularly, in case of Rayleigh or Rician interference gains with $\gamma = 1$, for $1/2 < f \leq 1$, simultaneous channel sharing asymptotically outperforms TD.

When the interference gains have the Nakagami- m distribution with $\gamma = m$, for $\frac{m}{1+m} < f \leq 1$, simultaneous channel sharing results in a higher sum-rate for the secondary network compared

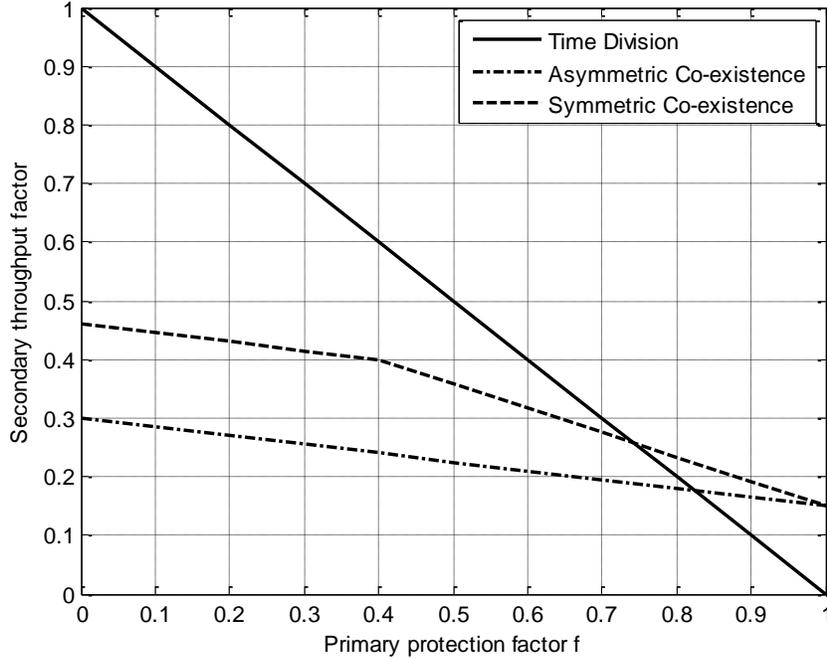


Figure 9.2: Coefficients of sum-rate scalings for secondary versus primary protection factor f for $\alpha = 4$, when interference gains have Nakagami- m distribution with $m = 3/2$ when both networks operate in uplink mode. Symmetric (resp. asymmetric) co-existence is better than TD when $26/35 < f \leq 1$ (resp. $14/17 < f \leq 1$).

to TD. Decreasing m results in expansion of the interval of f for which a better performance is achievable. Since m is reversely proportional to the variance of the Nakagami- m distribution, [35], an increase in the variance (the more scattering in the environment) improves the range of f for which simultaneous channel sharing outperforms TD.

9.2.3 Primary Uplink and Secondary Downlink

In simultaneous channel sharing, for $0 < f \leq \frac{1}{1+\gamma}$, the maximum secondary throughput factor is 1 which is greater than that in TD.

In case of Rayleigh and Rician interference power gains, for $0 < f \leq 1/2$, simultaneous channel sharing results in a higher secondary throughput factor than TD.

In case of Nakagami- m interference gains, for $0 < f \leq \frac{1}{1+m}$, we obtain better results over TD. Also, smaller m increases the interval of f for which a higher sum-rate is achieved.

Table 9.2: Ranges of f with higher secondary sum-rates over TD in the last three scenarios. For Nakagami- m interference gains, $\gamma = m$ and for Rayleigh and Rician interference gains, $\gamma = 1$.

Primary Network	Secondary Network	Range of f
down	up	$\frac{\gamma}{1+\gamma} \leq f \leq 1$
up	down	$0 < f \leq \frac{1}{1+\gamma}$
down	down	$0 < f \leq 1$

9.2.4 Primary and Secondary Downlink

In this scenario, under simultaneous channel sharing, the secondary network can achieve maximum secondary throughput factor of 1. Comparing to the secondary throughput factor in TD, which is $1 - f$, one can conclude that simultaneous channel sharing asymptotically outperforms TD for any $0 < f \leq 1$. Thus, when both networks operate in downlink mode, neither is asymptotically impeded by the other. This result is achievable regardless of the distribution of interference power gains and thus is valid for any distribution for the interference gains including, Rayleigh, Rician, and Nakagami- m .

Table 9.2 summarizes the ranges of protection factor f for which enhanced results over TD are achieved in the last three scenarios.

9.3 Distributed Uplink Scheduling

As demonstrated in Chapter 5 through Chapter 7 (the first three scenarios), the optimal secondary sum-rates are achievable using the least interference strategy in uplink transmission. Thus, the networks' schedulers do not require knowledge of the networks' own channel power gains in order to activate the users. Therefore, activation of users can be performed in a distributed (decentralized) way, i.e., it can be performed by a simple comparison between the interference power gain that each user imposes on the other network's receiver and a certain threshold followed by activating only the users whose interference power gains are less than the threshold. This distributed scheduling can be performed by each user rather than by a central entity like a scheduler.

Furthermore, in order to deploy the least interference scheduling strategy in uplink mode, only a user's own interference power gain to the opposite network's receiver is required which can be estimated and obtained.

9.3.1 Value of the Threshold

If we let $F_G(g)$ be the CDF of the interference power gains, from the proof of Proposition 4.1, using $F_G^{-1}(\frac{f(n)}{n+1})$ as a threshold gives the best achievable secondary sum-rates with high probability as $n \rightarrow \infty$. Therefore, $F_G^{-1}(n^{\beta-1})$ and $F_G^{-1}(n^{\bar{\alpha}-\alpha})$ can be used as thresholds for the primary and secondary networks, respectively. Since the optimal values of $\bar{\alpha}$ and β , that achieve the maximum secondary sum-rates in each uplink-downlink scenario are known, the thresholds can be derived for any large n .

Chapter 10

Simulation Results

Using Proposition 3.1 and Proposition 4.1, we have derived asymptotic sum-rates for the primary and secondary networks under simultaneous channel sharing. In this chapter, in order to demonstrate the accuracy of the propositions, we provide some simulation results.

Simulation setup: In the simulations, we assume that the channel and interference gains are i.i.d. and Rayleigh distributed with unit mean. Furthermore, the users are scheduled based on the least interference strategy. We also take $P = 10$ dB and $N = 0$ dB.

Primary and secondary uplink: When the primary and secondary networks are both in uplink mode, for $\beta = 0.5$, the sum-rate of the secondary network is

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{j=1}^{n^{\bar{\alpha}}} G_j^s}{N + P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}} \right) \quad (10.1)$$

$$\sim \log \left(1 + \frac{n^{\bar{\alpha}}}{\frac{N}{P} + \frac{1}{2}} \right) \quad (10.2)$$

$$\sim \bar{\alpha} \log n. \quad (10.3)$$

We call (10.2) an intermediate secondary sum-rate as compared to the asymptotic sum-rate in (10.3). The actual sum-rate, (10.1), and the intermediate sum-rate are plotted for $\alpha = 3$ and $\bar{\alpha} = 3/2$ in Fig. 10.1.

Primary uplink and secondary downlink: When the primary is in uplink and the secondary is in downlink mode, for $\beta = 0.5$, the sum-rate of the secondary network is

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + P \sum_{i=1}^{n^\beta} G_{i:n}^{p \rightarrow s}} \right) \quad (10.4)$$

$$\sim \log \left(1 + \frac{\ln n^\alpha}{\frac{N}{P} + \frac{1}{2}} \right). \quad (10.5)$$

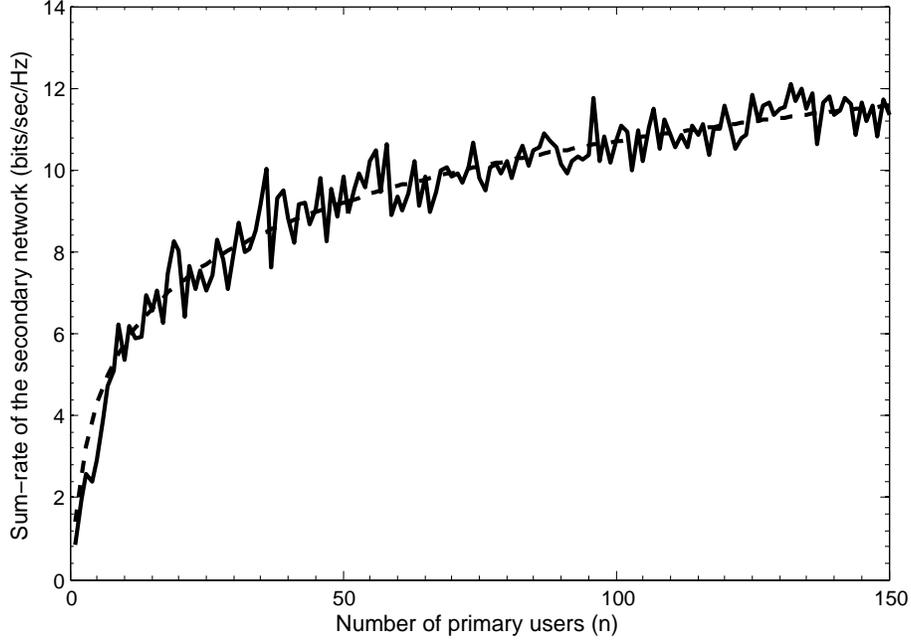


Figure 10.1: The actual (solid) and intermediate (dashed) sum-rates of the secondary network when primary and secondary are both in uplink mode.

The actual, (10.4), and the intermediate, (10.5), secondary sum-rates are plotted for $\alpha = 3$ in Fig. 10.

Primary downlink and secondary uplink: When the primary is in downlink and the secondary is in uplink mode, the sum-rate of the secondary network is

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \sum_{j=1}^{n^\alpha} G_j^s}{N + PG_0^{p \rightarrow s}} \right) \quad (10.6)$$

$$\dot{\sim} \log \left(1 + \frac{n^{\bar{\alpha}}}{\frac{N}{P} + 1} \right). \quad (10.7)$$

The actual, (10.6), and the intermediate, (10.7), sum-rates are plotted for $\alpha = 3$ and $\bar{\alpha} = 3/2$ in Fig. 10.3.

Primary and secondary downlink: When the primary and secondary networks are both in downlink mode, the sum-rate of the secondary network is

$$R_{\Sigma, \text{Sim}}^s(n, n^\alpha) = \log \left(1 + \frac{P \max_{j=1}^{n^\alpha} G_j^s}{N + PG_0^{p \rightarrow s}} \right) \quad (10.8)$$

$$\dot{\sim} \log \left(1 + \frac{\ln n^\alpha}{\frac{N}{P} + 1} \right). \quad (10.9)$$

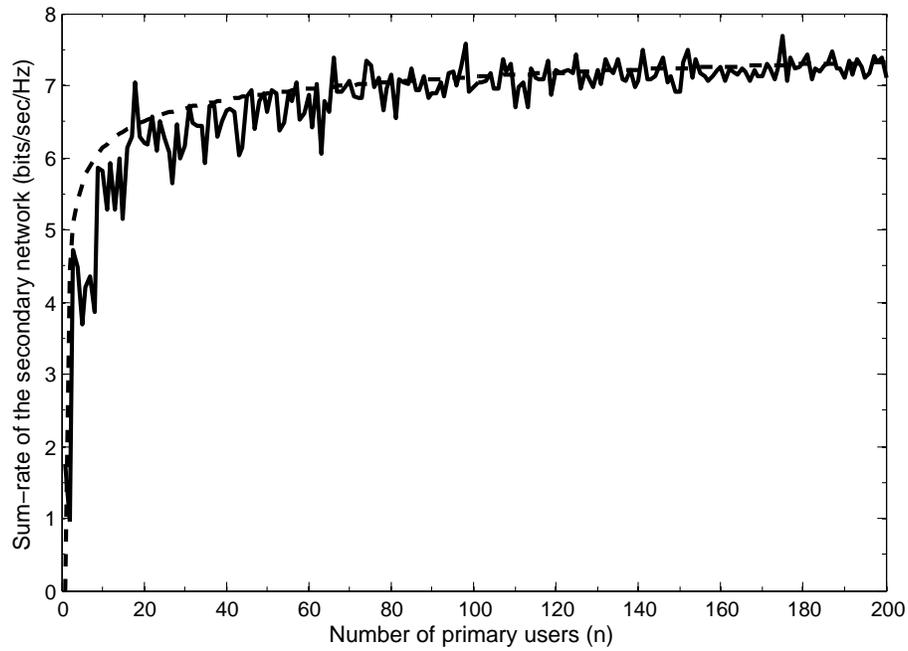


Figure 10.2: The actual (solid) and intermediate (dashed) sum-rate of secondary network when primary is in uplink and secondary is in downlink mode.

The actual, (10.8), and the intermediate, (10.9), sum-rates are plotted for $\alpha = 3$ in Fig. 10.4.

As the figures demonstrate, the intermediate sum-rates track the actual sum-rates well as n increases.

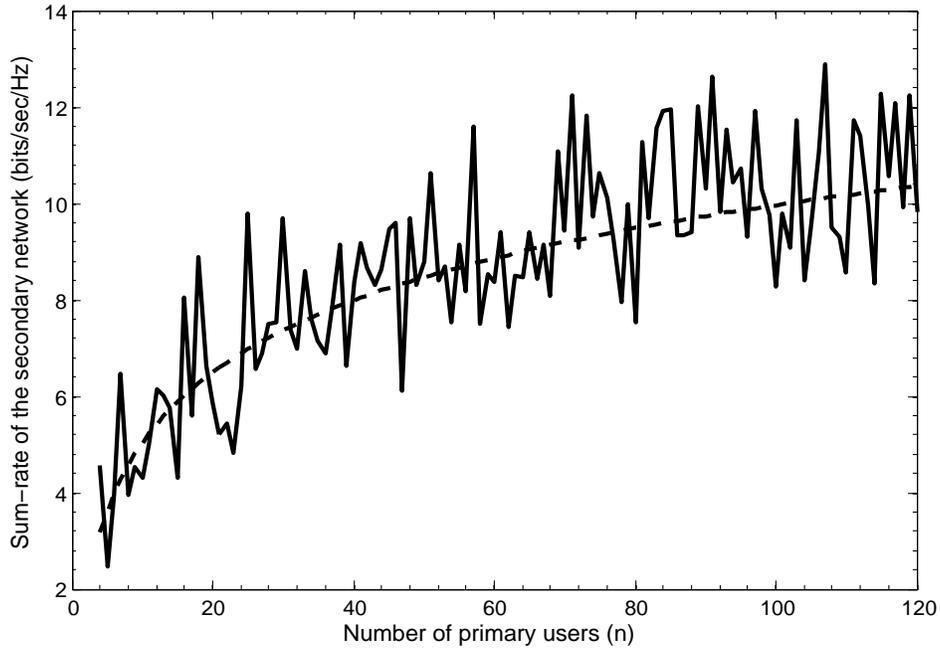


Figure 10.3: The actual (solid) and intermediate (dashed) sum-rate of secondary network when primary is in downlink and secondary is in uplink mode.

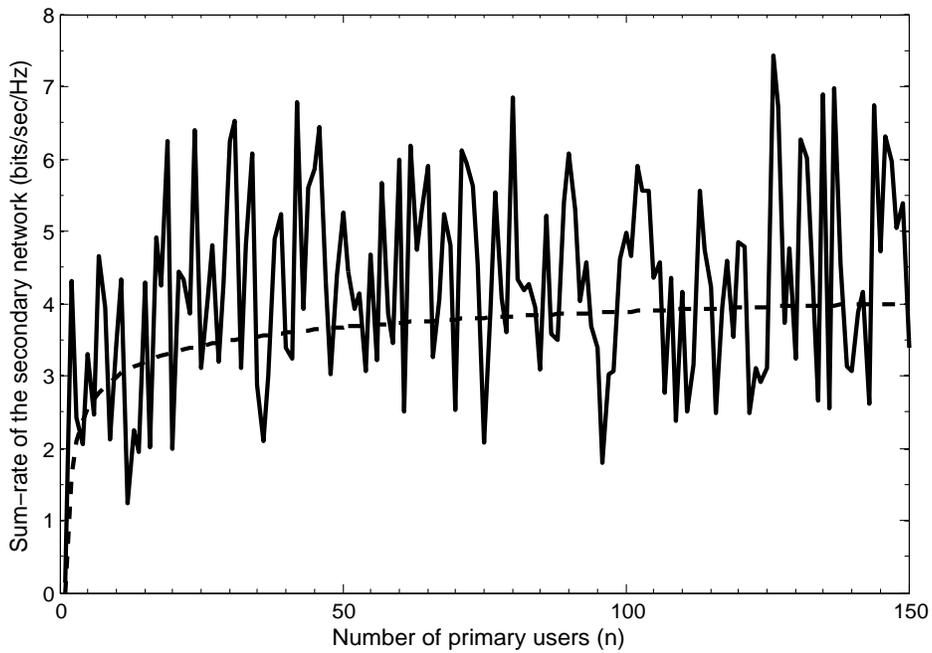


Figure 10.4: The actual (solid) and intermediate (dashed) sum-rate of secondary network when primary and secondary are both in downlink mode.

Chapter 11

Conclusions and Future Work

In this study, we have examined co-existence of primary and secondary point-to-multipoint underlay networks. The networks simultaneously share the same spectrum in a fading environment where the channel power gains have the exponential tail property and the interference power gains have another property which is presented in Chapter 2.

We have shown that at either of the symmetric or asymmetric co-existence levels, a higher asymptotic sum-rate is achievable for the secondary network than TD, while the primary is protected by factor f . These enhanced results can be obtained when user scheduling in uplink mode is based only on the least interference gains. In other words, no additional throughput enhancement is made by using the joint optimization strategy.

Specifically, some conclusions that we draw are that at suitable co-existence levels, when the channel power gains have exponential tail property

- If both networks operate in uplink mode and interference gains are Rayleigh or Rician distributed, in some specific cases, for $1/2 < f \leq 1$ underlay operation results in a better asymptotic sum-rate for the secondary network over TD and in some other cases, for $0 < f < 1/2$, underlay operation outperforms TD.
- If the primary network is in downlink and the secondary network is in uplink mode, in case of Rayleigh or Rician interference gains, for $1/2 < f \leq 1$, underlay is asymptotically preferred over TD. When the interference gains are Nakagami- m distributed, then for $\frac{m}{1+m} < f \leq 1$, simultaneous channel sharing outperforms TD.
- If the primary network is in uplink and the secondary is in downlink mode, in case of Rayleigh or Rician interference gains, for $0 < f \leq 1/2$, underlay is asymptotically preferred over TD. When the interference gains have Nakagami- m distribution, then for $0 < f \leq \frac{1}{1+m}$, simultaneous channel sharing outperforms TD.

- If both networks are in downlink mode, simultaneous channel sharing asymptotically outperforms TD for any $0 < f \leq 1$. In other words, both networks may operate simultaneously in the same band at no loss.

Also, an increase in the variance of the Nakagami- m distribution (increase in the scattering characteristic of the wireless environment) expands the range of f for which better sum-rates are achieved.

From a practical standpoint, in uplink transmission the scheduling can be performed in a distributed way and the secondary network still achieves the optimal sum-rate. In other words, each user can estimate its interference gain to the opposite network's receiver and then compare it with a certain threshold and remain active if its interference is less than the threshold. In downlink mode however, the base stations transmit to their respective network's strongest user.

This work serves as a first step in enhancement of the secondary network's sum-rate in the underlay approach. An extension to this work is to include path loss in the system model. In this case, channel and interference power gains will be products of the corresponding pathloss and the multipath fading gains. Another suitable extension is that instead of activating or deactivating users (transmit power set to P or zero respectively), we apply power control in each network and find optimal transmitted power allocations for users in a way that better secondary sum-rates are obtained.

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