

Foundations of Deduction's Pedigree: A Non-Inferential Account

by

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Abstract

In this thesis I discuss the problems associated with the epistemological task of arriving at basic logical knowledge. This is knowledge that the primitive rules of inference we use in deductive reasoning are correct. Knowledge of correctness, like all knowledge, is available to us either as the product of inference, or it is available non-inferentially. Success in the campaign to justify the correctness of these rules is mired by opposing views on how to do this properly. Inferential justifications of rules of inference, which are based on reasons, lead to regressive or circular results. Non-inferential justifications, based on something other than reasons, at first do not seem to fare any better: without a basis for these justifications, they appear arbitrary and unfounded.

The works of Boghossian and Dummett who argue for an inferentialist approach, and Hale who supports non-inferentialism are carefully examined in this thesis. I conclude by finding superiority in Hale's suggestion that a particular set of basic logical constants are indispensable to deductive reasoning. I suggest that we endorse a principle which states that rules are not premises, and are therefore to be excluded from expression as statements in a deductive argument. I argue that the quality of being indispensable is sufficient for a *basic* rule of deduction to be countenanced as default-justified, and therefore need not be expressed in argument. By a rule's evading expression in argument, it avoids circular reasoning in deductive arguments about its own correctness.

Another important outcome that emerges from my research is the finding that non-inferential knowledge is ontologically prior to the inferential sort. This is because plausible inferential knowledge of basic logical constants shall always be justified by circular reasoning that already assumes the correctness of the rule to be vindicated. This initial assumption is tantamount to non-inferential knowledge, and therefore this latter is more primitive—in fact the only primitive—species of basic logical knowledge.

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Chapter 1

Basic Logical Knowledge and its Problems

1.1 Introduction

Every thing we know is based on some other thing that we know, or so the saying goes. If this were true, then there should be some knowledge with origins that are inaccessible to us, and so this adage might never be confirmed or disproven. The issue of finding a basis for knowledge is of primary concern to the logician and epistemologist, because knowing how we possess the knowledge we do, if such sort of knowing is possible, gives us confidence to pursue new knowledge that we may regard as soundly derived. It also provides us with some reassurance that our ‘webs of belief’ are consistent, viz., that the corporate body of individual beliefs we each possess more-or-less agree with each other as a conglomerate whole. In the case of logic, my concern in this project, finding a basis for our knowledge that a given logical language permits the inference only of sentences that are consistent with each other and valid in that system involves demonstrating that its most fundamental elements are correct. If we may be persuaded of this, then warrant exists to reason according to the principles of a particular logic with confidence.

To establish the correctness of these principles, we shall likely have to proceed in a fashion that is at odds with the supposition of the adage above, or else we shall never arrive at what is first, basic, and certain. The following example from fictional literature suggests a strategy we might apply to overcome the obstacle presented by the adage. It suggests an epistemological attitude towards knowledge that I shall favour and defend in this thesis: Baron Münchhausen recounts the story of his falling into quicksand. To avoid certain drowning, he hoists himself to safety by a firm tug of his own ponytail. This amusing tale is analogous to the problem that concerns me where, in the case of knowledge of logic, some approaches to justification like the one championed by the adage succumb to a regress and fail to arrive at a basis—at what lies first in a chain of reasoning. But Münchhausen’s tale portends an alternative; the ‘bootstrapping’ that he performs stands as a place to

start *that begins in itself*. The solution to his problem comes from nothing external to his situation—such as a low-lying vine or wooden plank to facilitate his escape.

In similar spirit, this thesis shall defend the idea that knowledge of the correctness of the rules governing basic logical constants is knowledge that is non-inferential in status. Furthermore, I shall argue that any plausible inferential strategy proposed to justify that correctness shall depend on non-inferential knowledge. The latter therefore stands prior to what has been derived inferentially, and this renders non-basic any inferentially-derived knowledge of correctness. I shall presently deliver a characteristic of basic logical knowledge, followed by an account of the two ways available to us for arriving at knowledge: by inference or by non-inferential means.

A rule of inference in deductive logic is a metalinguistic expression. It governs operations on—or defines the function of—a logical operator, which is a syntactic element of the formal system of deductive logic. Rules of inference that define an operator come in pairs; one rule articulating how to introduce the operator in a chain of reasoning, another telling us how to eliminate that operator from the chain. Rules of inference act as functions taking us from premises to conclusions, and the rules of inference admitted in deductive logic are universally valid. *Basic logical knowledge* (or BLK) is knowledge we purportedly have about rules of inference; knowledge that a rule is correct. Important paradoxes emerge from attempts to arrive at this knowledge; the substance of this thesis is largely a reckoning of whether this knowledge is possible and if so, how.

An inferential justification of a claim about a rule of inference, for instance the claim that the basic rule *modus ponens* (hereafter MPP) is correct achieves its objective by means of inference; it uses deductive reasoning to corroborate that claim. The virtue of an inferential justification in the eyes of a reasoner who assesses an argument for validity is that an inferential argument presents us with some sort of substantiation in support of our conclusion. I anticipate one point of view an inferentialist might hold about deductive arguments such that in an important sense it seems like the preferred approach to justification—for it appears to answer the question *how are we justified?* in a rather effective way, viz., via deductively valid argument. In this thesis deductively valid arguments shall be regarded as rigorous ones because deductive logic allows us to establish, for any given conclusion, whether it follows from premises by means of a systematic proof-checking procedure based on accepted rules of inference or axioms. Similar rigour is attributable to the domain of mathematics. Kreisel for instance describes the sort of rigour that commonly serves as the touchstone of mathematical proofs: rigour may be thought of in the ‘Turing sense’ with respect to decidability.¹

The non-inferential account of knowledge takes a different approach by grounding a justification of our knowledge that a rule of inference is correct in a stipulation, a proffered definition, a stated intuition, or something along those lines.

¹See his paper *Informal Rigour and Completeness Proofs*, International Colloquium in the Philosophy of Science. Amsterdam: North-Holland Press, 1967. Pg. 138.

Many philosophical views exist about these forms of non-inferential knowledge, and what is common to all of them is the view that they are basic in themselves. Some (radical) non-inferentialists hold that these rules are not in need of any justification, or perhaps that they are default-justified. Other non-inferentialists such as Hale argue more reasonably that non-inferential knowledge *of a specific kind* is indispensable to the practice of deduction, and therefore that our non-inferential knowledge about basic rules is justified because of this indispensability. Hale's view is the one I defend in this thesis. The inferentialist repudiates the basis on which rests non-inferential knowledge, and this creates an epistemological tension between deductive and rigorous knowledge on one hand, and intuitive notions on the other.

This project fundamentally involves a rejection of the attitude expressed by the adage at the beginning of this section. It argues for the primacy and superiority of non-inferential justification of the correctness of the most basic logical constants on the basis that (a.) we can effectively bestow basic logical rules and operators with meaning when their introduction and elimination rules are in harmony. (b.) That a particular subset of basic rules or operators with fixed meaning should not be doubted because those rules are indispensable to the deductive reasoning process. (c.) That those rules that are impossible to doubt participate in a minimal kit of rules (viz., basic rules) that we cannot help but use in reasoning. (d.) Because we cannot help but use them in reasoning they are essential to logic and deductive reasoning, and are correct. (e.) Finally, because the basic rules enjoy this status as indispensable, essential, and implicit in the reasoning process they need not, and should not figure as premises in deductive reasoning. Items (a.) through (d.) are ideas belonging to Hale which I expound and endorse. (e.) is one of my hypotheses, and the following is my own contribution to this enquiry: The upshot of a defense of non-inferentialism is that in the case of an argument that seeks to justify a basic rule of inference, we shall have license to suppose the correctness of that rule without a deductively valid proof of its correctness. A further result emerging from the status of a basic rule's implicit nature is that the reasoner is absolved from obligation to have that rule figure assertorically in reasoning. The theory establishes a purported distinction between rules and premises in deductive reasoning. By omitting explicit expression of a particular rule in an argument, we avoid circularity when reasoning about the correctness of that rule and thereby arrive at a plausible justification for our application of the rule—viz., for its correctness. The most original argument I advance in my thesis is one that I have not come across anywhere in the literature: it says that non-inferential knowledge of logical rules can be shown to be ontologically prior to knowledge arrived at inferentially. In fact, inferential knowledge *depends* on non-inferential knowledge, and this in turn renders inferential knowledge non-basic.

Most people familiar with deductive logic understand notions such as validity and soundness of arguments. While these notions are discussed at great length in this thesis, it is important to acknowledge my principal concern here—for the correctness of the rules of inference. Validity and soundness are not called into question - I am not concerned to know, for example, whether validity is the right sort of criterion to respect in order for our conclusions to follow necessarily from

premises. We enter into an enquiry about the correctness of basic rules because, as we will see, their origin and epistemological status is still undecided, evidenced by the opposing views held by inferentialists and non-inferentialists on the matter.

This thesis discusses problems nested in the epistemology of justification—the study of how belief is justified. Its philosophical literature generally countenances three main underlying theories: foundationalism, coherentism, and infinitism. Infinitism holds that the structure of justificatory reasons is infinite and non-repeating. It is an appealing theory because it can provide an acceptable account of rational belief, viz., beliefs held on the basis of adequate reasons.² But most theorists reject infinitism as absurd because under this theory, the evidential ancestry of any statement is infinite. This is particularly troublesome to a deductive justification of a conclusion. Foundationalism is typified by Descartes' famous statement '*I think therefore I am*'. He believed that he could build his edifice of knowledge on that fact; a fact about which he was undoubtedly certain; a fact that is 'default-justified', or 'self-justifying'. The fact served as his foundation—as a place to begin. Opponents of a foundationalist theory of justification argue that it is unacceptable because it countenances an arbitrary reason as its base—arbitrary in a similar way to the status of non-inferential knowledge I described above. In an important sense we cannot call a foundationalist belief rational because no further reasons exist as justification for it. Coherentism sees justification as holistic. Instead of there being a chain of reasons justifying a statement, a statement is justified because it is included in and coheres with a set of statements that are accepted. Minimally, the statement must be consistent with the other statements of the set. Objections to coherentism include the argument that it leads to circularity. Furthermore, if we examine the set of accepted statements just described, no justification of the set of statements necessarily exists, and may have been accepted in foundationalist spirit. In this sense coherentism is disguised as foundationalism. Just as I have shown the correlation between foundationalism and non-inferentialism several sentences above, so too do I loosely suggest a synergy between coherentism and inferentialism.

A defense of my thesis shall unfold in the following manner. Chapter 1 introduces the concept of basic logical knowledge and the practice of deductive inference. It presents a sceptical case from Lewis Carroll challenging the possibility of justifying our knowledge that the rule MPP is correct. Chapter 2 surveys responses to this challenge offered by Belnap, Boghossian, Dummett, Engels, Hale, and Prior. In all cases these responses include profound and insightful discussion about how the basic rules figure in logic and reasoning. Chapter 3 will conclude the thesis by defending the claims (a.) through (e.), as well as by defending my own findings about the primacy of non-inferential knowledge, in support of my preference for a non-inferential strategy in any justification of the correctness of a select number of basic logical constants.

²This explanation of infinitism comes from Klein in his (1999).

1.2 Rules and Regress Explained

What qualifies as a rule of inference? Since we are concerned with logic in this enquiry, we discuss forms of reasoning rather than particular arguments, forms such as $\lceil \alpha \rightarrow \beta, \alpha, \therefore \beta \rceil$. The form expressed, which instantiates conditional-elimination or MPP, is universally valid: it exemplifies the standard or universal form of a deductive argument that applies MPP in order to arrive at its conclusion, and because any contentful argument, viz., one that contains at least one premise leading to a conclusion about something in a particular domain of discourse, can be translated into this (or some similar) logical representation, we call it universal. But this is not to make an argument by enumerative induction for the validity of MPP. Enumerative induction fails to give a rigorous justification because it is impossible to test every (past, present, and future) instantiation of MPP for validity. Must we qualify the universality of MPP's logical applicability such that it is universal in the context of deductive or classical logic but perhaps not in some other contexts? Well, we don't want to.³ This is one of several important issues on the block in any attempt to justify basic rules of inference. The universality, necessity, and *a priority* of MPP that we seek to demonstrate should tell us that MPP is truth-preserving, in all instances, in any model, and showing that MPP has these properties constitutes some way towards success to the epistemologist attempting to justify basic rules of inference.

Rules of inference must be necessary. Necessity emerges from one of the characteristics of validity: it obtains when the premises of an argument are true, and the conclusion is validly inferred from the premises by means of a correct rule of inference. It's impossible for that conclusion to be false in any case where the premises are true. That was a brief characteristic of necessity as it pertains to deductive arguments. But we are also concerned with what it means for *a rule of inference* to be necessary, and necessity comes in two forms, *absolute* and *relative*. The sort of necessity we wish to assign to rules of inference in order for them to be warranted in deduction is *absolute* necessity. The difference between the two is that relative necessities are only applicable in specific or restricted contexts. A useful example might be mathematical induction; while Frege took it to be a logical principle, it is nowadays not reckoned as such because while it holds for the natural numbers and the many domains for which suitable isomorphisms to the natural numbers can be defined, there are many domains in which it does not hold, including, for instance, the reals but also even some weak arithmetics. Hence the principle of mathematical induction is merely a relative necessity. In the present case, absolutely necessary rules of inference are those for which there is no possibility that (*a.*) they are inapplicable or not warranted in the domain of discourse, and (*b.*) do not preserve the validity of arguments. We may cautiously call certain rules of inference such as MPP absolutely necessary and accept the impossibility that they are doubtable. We

³Rules of inference also exist that are valid in some contexts but not in others, for example the inference rule $\lceil \neg\neg \alpha \rightarrow \alpha \rceil$ (viz., the rule of double-negation) is valid in classical logic, but not in intuitionistic logic.

also accept the indefeasibility of certain other types of knowledge such as knowledge of the axioms of arithmetic. However we must acknowledge the case of Euclidean geometry, knowledge and understanding of those axioms having been held to be tantamount to logical knowledge and therefore by extension necessary for a very long time. This geometry showed us that even seemingly necessary knowledge is defeasible. So acknowledging this possible obstacle, we nevertheless want to talk about MPP as a rule of inference that is absolutely necessary, universal as described above, and finally *a priori*. *A priori* characterises the way in which a reasoner arrives at a conclusion, or arrives at knowledge. To be realised *a priori* means that a reasoner surpasses the character of experience to deduce his conclusion. We then want to present the right sort of reasons that demonstrate basic rules of inference as being in possession of each of these properties.

Without giving much thought to the matter, the rule of inference MPP seems like one of those rules we need not question. The obvious nature of a deductive argument that eliminates its conditional by applying MPP thus; $\lceil \alpha \rightarrow \beta, \alpha, \therefore \beta \rceil$, seems like an argument that employs a rule that may be unreflectively countenanced as correct. However, as we unpack this simple rule of inference and attempt to justify our warrant for applying it, we run into the obstacles of regress and circularity. The work of the philosophers I discuss in Chapter 2 is concerned with finding solutions to and ways-around these obstacles.

The background to these problems circulate around epistemological and metaphysical stances towards the right method of justification—of basic rules of inference. Lewis Carroll’s puzzle presented in his paper *What the Tortoise Said to Achilles*⁴ demonstrates that *knowledge* of logic is insufficient to warrant our use of basic rules of inference. Warrant must therefore be grounded in something other than our knowledge of logical laws if it is groundable at all. Otherwise, a warrant grounded in such knowledge may lead to regress. Patrice Philie presents this regress in an extremely clear way in his paper *Carroll’s Regress and the Epistemology of Logic*:⁵

First we consider an argument employing MPP:

- (1) If Socrates is a man, then Socrates is mortal.
- (2) Socrates is a man.

Therefore,

- (C) Socrates is Mortal.

We must know two things about the argument; first, that the premises themselves are warranted, and second, that the argument possesses a valid logical form. We must therefore be in possession of (viz., we must *know*) the following principle;

⁴Carroll, (1895).

⁵Philie, (2006), 185.

(**W**) If an argument displays the logical form of MPP, then it is valid.

Principle (**W**) is therefore the warrant that moves us from premises to conclusion. But how does it do this? A reasoner in this case must know two things; first he must know (**W**), and second that the argument above is an instance of MPP.

So what one must know can be summarized as follows:

- (a) (**W**): If an argument displays the logical form of MPP, then it is valid.
- (b) The argument above ((1), (2), (C)) is an instance of MPP.
- (c) Therefore, the conclusion (C)—that Socrates is mortal—can be drawn.

The problem is immediate: the step from (a) and (b) to (c) is itself inferential and possesses the MPP form. In order to have warrant for drawing conclusion (c), the reasoner would need to possess an additional piece of knowledge, namely:

- (i) (**W**): If an argument displays the logical form of MPP, then it is valid.
- (ii) The argument above ((a), (b), (c)) is an instance of MPP.
- (iii) Therefore, the conclusion (c)—that the conclusion (C) can be drawn—can be drawn.

This can go on indefinitely and constitutes an infinite regress of warrants. The challenge to the epistemologist and philosopher of logic, then, is to establish a cogent apparatus for justifying our entitlement to reason according to (this) basic rule of inference. I want to ask about an argument that contains as conclusion the very claim that MPP is itself a correct rule of inference. Intuitively, we seem to *know* that MPP preserves truth, yet the way we know this is not so obvious, as evidenced by the regress argument above. I seek the *knowledge* that MPP is a correct rule of inference and is truth-preserving in all cases, delivered via rigorous justification if possible. Alternatively, I seek some sort of vindication for the *intuition* that MPP is correct. We know that an argument with true premises that instantiates MPP is a valid one. We now seek an answer to a *how?* question that stands one step prior to the application of a rule of inference, a question that asks after justification of MPP itself which, if answered, can give us the knowledge about MPP that we seek, viz., the substantiated knowledge that it is unexceptionally truth-preserving, delivered via the force of an argument in a deductive proof, or via some other convincing sanction of our intuition. The sort of knowledge sought is knowledge about logic rather than knowledge gained by using it. Paradoxically however, it seems as though we might be required to use logic in order to gain knowledge about it.

The problem of justifying our knowledge that MPP is a truth-preserving rule of inference seems to lie in the fact that MPP is construed as *basic*. A principle of inference is basic logical knowledge according to Hale if ‘. . . the explanation of how we have it makes no appeal to the soundness of any principle of inference.’⁶ In other words, our knowledge that MPP is truth preserving does not rest on our knowledge of any other rules of inference that allow us to arrive at this knowledge. Under Hale’s definition, we therefore might be inclined to believe that the justification of our knowledge that MPP is truth-preserving cannot be the product of some piece of reasoning, for reasoning employs rules of inference, thereby violating Hale’s requirement. What other approaches to justification of knowledge exist then that might allow us to justify the knowledge about MPP that we seek to vindicate?

This is basic logical knowledge (BLK)—knowledge not only of correct forms of inference, but also knowledge of the warrant that blesses these forms as correct. The apparatus we endorse for the task must naturally avoid the problem of regress just explained, and it should not come to a sceptical conclusion about this sort of knowledge. If it succeeds in its task, then the account we ultimately adopt shall come about as the product of either an inferential piece of reasoning or a determination of non-inferential composition. These are the two epistemological approaches we employ to arrive at knowledge, and any proffered solution to my enquiry will be grounded in one of these two strategies.

1.3 The Usefulness of Deduction and a Slice of its History

Philosophers have disputed the usefulness of deductive inference in the past, and my interest for this section is the difference in attitudes maintained by Kant and Frege towards deduction. The brunt of the difference pertains to disagreement over the fruitfulness of analytic judgments. Briefly, an analytic judgment occurs according to Kant, when some predicate B is judged to be covertly contained in the concept A.⁷ The statement ‘All bodies are extended’ is an analytic one for Kant because the concept ‘body’ possesses extension as an essential property.⁸ Conversely, the statement ‘All bodies are heavy’ is not analytic, but rather synthetic, for ‘weight’, according to Kant, is not essential to (and thus not covertly contained in) the concept ‘body’. A synthetic judgement is one such that predicate B lies outside the concept A. For Kant therefore, analytic judgments clarify and synthetic ones amplify. Though both sorts of judgments can be inferred *a priori* at least for some propositions. Deductive inferences that arrive at conclusions of an analytic nature are less interesting for Kant because he held that inferred conclusions are simply reiterations of that already contained in the premises.

⁶Hale (2002), pg. 280.

⁷Kant (1998), B10.

⁸The apriority of this statement is also apparent because we do not need to appeal to intuition or experience in order to establish the truth of the statement (on Kant’s account).

Frege's understanding of analyticity differed from Kant's. An analytic statement for Frege is one that follows from the fundamental laws of logic. He holds that we can establish these fundamental laws in the following manner: He distinguishes the content of a judgement (such as the concept 'extension' being contained in the concept 'body') from *the justification for making the judgement*.⁹ The latter is '... a judgement about the ultimate ground upon which rests the justification for holding it to be true.'¹⁰ We attain the ground by commencing from deductive inferences and working backward.¹¹ This process demands that a proof for each justificatory step be demonstrated, and if it is possible to arrive at primitive truths (which are general logical laws and definitions according to him) in this reverse-forensic manner, then the truth is analytic. So we have just seen how Frege thought 'the ground' could be established, *viz.*, by deducing the fundamental laws of logic and nominating accompanying definitions. Now to arrive at analytic truths, we do not use the reverse-forensic process just described, rather, we apply the basic rules and definitions to arrive at particular analytic judgments. The following quote from *The Foundations* supports this method and sheds some light on how analyticity is to be preserved:

Often we need several definitions for the proof of some proposition, which consequently is not contained in any one of them alone, yet does follow purely logically from all of them together.¹²

With an understanding of what it means for a statement to be analytic according to Frege we can now explain why he held some analytic propositions to be fruitful—that is, why they *extend* our knowledge rather than merely clarify it as Kant believed. Frege distinguishes between the terms 'function' and 'argument'. A function is a statement containing a constant component '... which represents the totality of the relations.'¹³ An argument signifies the object that stands in those relations. In other words, the argument is replaceable by a variable. Therefore, in the proposition 'David has reached the age of majority,' 'David' is the argument, and 'reaching the age of majority' is the function. Dummett¹⁴ observes that functions arise in one of two ways according to Frege's ontology. The first is by the identification of a pattern (what I have called 'form' in my discussion of arguments in the previous sections) that stays constant regardless of what argument is inserted. The other way in which a function can be distilled, applicable to such concepts as shape, number, and direction, is by abstraction of concepts from complex predicates which are then transformable into definitions by way of simple predicates. The example below consisting of (1.), (2.), and (3.) will clarify what this means. Frege believed that the sense of a complex predicate (*viz.*, what the complex *means*) could not

⁹Frege (1953), §3.

¹⁰*ibid.*

¹¹This process is discussed in Frege (1997), pg. 309

¹²Frege (1953), §88.

¹³Frege (1967), §9.

¹⁴Dummett (1991), pg. 39 - 40.

be entirely derived from its components, which is why he endorsed a strategy that dissects a proposition into components rather than build it up from them. This strategy agrees with the ‘backwards method’ of Frege’s later writings briefly mentioned in the preceding paragraph. This is also our first clue that provides insight into Frege’s ontology, where, by supposing that a complex designates something ‘richer’ than the sum of its constituent components is capable of expressing, we see that the complex extends the content articulated by its own constituent components. By means of example Dummett explains Frege’s reasons for preferring dissection over the ‘building up’ of a concept;¹⁵

- (1.) Consider the proposition ‘**Either** Jupiter is larger than Neptune **and** Neptune is larger than Mars, **or** Mars is larger than Neptune **and** Neptune is larger than Jupiter.’
- (2.) We extract the predicate ‘Either Jupiter is larger than x and x is larger than Mars, or Mars is larger than x and x is larger than Jupiter’
- (3.) We thus attain the concept ‘intermediate in size between Jupiter and Mars.’

But if we are to regard the logical connectives ‘and’ and ‘or’ as primitive (or ‘basic’ in the terminology of this project), then they cannot be explained in terms of how they function in a complex predicate. They must be explained rather in terms of how they function when acting as main connectives. Complex predicates are therefore only to be understood such that they are extractable from sentences such as (1.). Such an extraction is exemplified by (2.), and (3.) is a new simple predicate by definition.

‘...its [viz., the complex predicate’s] sense may be seen as being given as a function carrying the sense of the name Neptune [or Venus, or some other body] on the thought expressed by [(1.)] ... We can regard it as such a function only because we *already* understand the complete propositions; it is in grasping *their* contents that we directly advert to the meanings of the connectives ‘or’ and ‘and.’”¹⁶

Dissection of a proposition is therefore counter to the process of ‘building one up,’ where in the latter, we incite its constituent components. We don’t need to acknowledge the pattern of the complex predicate (exemplified by (2.)), instead, all that is required is to be able to understand the subsentences of the complex and the meaning of ‘and’ in order to arrive at our definition (3.).

Dissection is therefore justly described as a process of concept-formation: it reveals something new, one pattern among many discernible in the proposition and shared with it by others, but not, in general, intrinsic to a grasp of its content.¹⁷

¹⁵ *ibid.*, pg. 40 - 41.

¹⁶ *ibid.*, pg. 40. Dummett’s *emphases*, my clarification and adaptations in square braces.

¹⁷ *ibid.*, pg. 41.

The ‘richness’ I adverted to earlier is what our understandings bestow upon a complex predicate. It is an aspect of the content *over and above* that offered by its constituent components. When we acknowledge a complex predicate, we exceed the content offered by the components of that predicate (or the content offered by the premises in the case of a whole argument), and in this way what has been derived extends our knowledge.

Why discuss analyticity in a study of deduction? Having surveyed two opposing views about the fabric of deduction, one being Frege’s claim that deductions leading to analytic judgments advance our knowledge, the other view belonging to Kant who claims that analytic judgments do not, we should want to confront Kant’s charge. One way in which Frege interprets Kant’s ontology is by means of analogy. Frege describes the Kantian view as seeing deductive inferences follow from premises ‘as the beams are contained in a house.’ Whereas Frege sees them as ‘plants are contained in their seeds.’¹⁸ My view is that complex predicate (1.) above is not the only one that can be dissected to arrive at definition (3.). There are probably an infinite number of complex predicates that can be so dissected to arrive at that exact definition, and conversely, we should be able to dissect (1.) and arrive at some definition other than (3.) as well. In the present case, the choice to dissect (1.) *in the particular way expressed* in order to arrive at (3.) constitutes something extra—something over and above the content expressed in (1.); ‘here, we are not simply taking out of the box again what we have just put into it.’¹⁹ Frege therefore regards both form/pattern recognition and dissection as fruitful, and the consequences so derived advance our knowledge.

On this much Kant and Frege were in agreement: deductively valid arguments deliver a new item of knowledge—an unobserved conclusion—that was purportedly unobvious until a rule of inference was applied. They also agreed that *a priori* was a quality that a deductive argument must possess. A conclusion must follow necessarily from the premise(s) of an argument, and the argument must be universal. The condition of universality as it relates to the *a priori* is as follows: where the conclusion of an argument is deduced beyond the character of our experience, the universality of the way we come about that deductive inference takes the form of argument (3.) above. We are permitted to countenance it as universal because in talking about its form (once again, exemplified by (3.)), we have established that every instantiation of its form is valid, rather than pointing to a subset of particular instances as evidence for its validity. Divorcing the notion of particular instantiation (*viz.*, experience) from any circumscribed argument form is essential to claiming the right to countenance that argument form as universal.

I conclude this section with emphasis on the notion that deduction is useful. Frege’s arguments, accompanied by Dummett’s analysis give strong reasons for endorsing the fruitfulness of deduction. If we were to go with Kant on this matter and suppose that only the synthetic is capable of bearing fruit, then we would

¹⁸Frege (1953), §88.

¹⁹*ibid.*

have to possess an unreasonable amount of background knowledge. For consider: If the statement ‘*all bachelors are unmarried men*’ is an analytic one for Kant and tells us nothing new, then we would have to know ALL the synonyms for ‘unmarried man’ that also make the statement analytic. For Kant’s story to work, the thesauruses of our minds would have to be unabridged. We would then be able to say that synonymies are unfruitful because we would already possess knowledge of all concepts. To put the case in other terms, we would have to know the meanings of all words in our language in order for us to already possess all the knowledge that is being deduced. Knowledge of such synonymies is something we just don’t have. In light of persuasive arguments for the fruitfulness of deduction, we should therefore acknowledge that it is a valuable practice and that every effort to understand its foundations, most notably a defense of the correctness of its basic rules of inference, should be pursued.

1.4 The Problems Facing BLK

I have thus far defined BLK as the sort of knowledge we might possess about rules of inference. The knowledge that interests me is about the correctness of the basic rule MPP. In order for us to have this knowledge, we either (*a.*) suppose it to be knowledge in one of the non-inferential forms previously mentioned, or (*b.*) we attempt to demonstrate via inferential justification that the inference rule is correct. Both of these accounts are problematic however, and in this section I explain why.

If we attempt to provide an inferential justification for our entitlement to reason according to MPP, then we must offer up an argument to that effect. An argument, as we have seen, employs inference to arrive at its conclusion. But an argument that defends the validity of a basic rule of inference must make use of some rule of inference in order to arrive at that conclusion. The basic rule we are attempting to defend would cease to be basic by virtue of being justified by something (*viz.*, another rule) more basic than it. This leads to an infinitistic view of justification. Carroll’s message presented in the introduction is similar but not identical. Carroll does not argue for a regressive chain of increasingly basic rules, rather, he argues that a regressive series of applications of MPP ensue from any attempt to justify it. This will be discussed to greater depth in sections §1.5 and §2.1. As I will show in those sections, there is an additional complication in the case of a justification for our warrant to reason according to MPP, because the only rule available to us for that justification is MPP itself. This will highlight the problem of circularity: the use of a rule of inference in an argument that defends the validity of that very same rule.

A non-inferential account of a justification for our entitlement to reason according to MPP does not rely on inference. Instead, it finds its grounding in stipulations that are informed by, *inter alia*, beliefs or intuitions. This account, some argue, can lead directly to regress; if our entitlement to reason according to MPP is grounded in a stipulation (for example), what justification exists then for our entitlement to

countenance that stipulation as correct? The next level in a regress never answers this question in a satisfactory way, always requiring some sort of ‘reinforcement’ one level beyond. But that is not even the most important shortcoming of the non-inferential account. Without a deductive demonstration showing *how* we come to adopt the stipulation as correct, our endorsement of that stipulation is less than rigorous; its correctness is not demonstrable by some formal method governed by rules. A stipulation tells us *that* something is the case, but not how it comes to be so.

The final problem in need of attention pertains to the possibility of justification altogether. Upon evaluation of the problems facing inferential and non-inferential accounts of our entitlement to reason according to a rule of inference, a stalwart sceptic would dismiss both citing their problematic natures just discussed as his reason. Instead he would defend the thesis that no satisfactory justification exists. Is this a reasonable position? How would we have to change our attitudes about deductive logic if we were to accept the position that the basic rules of inference we endorse are unjustifiable? The sceptic introduces a legitimate challenge to the possibility of the sort of justification we seek. However, accepting the sceptical thesis is tantamount to defeat: it is difficult to accept that no proof exists for the correctness of our logics and basic epistemology, for, if we cannot justify basic logical rules, how may we declare with certainty that our science of deductive reasoning is sound? Scepticism is a thesis that we should refuse to accept. Once again, intuitively, we feel as though we cannot be wrong about the soundness of deductive logic. The sceptic’s thesis is self-undermining and therefore indeterminate, for, in order to support his thesis that MPP is unjustifiable, an argument employing MPP must be presented, which commits the sceptic to its correctness. In this paradoxical case, the sceptic’s endorsement of correctness is inevitable because it would be unreasonable for him to apply a rule of inference in an argument that he deemed *unjustifiable* or even worse, *incorrect*. If the sceptical thesis is vindicated, then so will the thesis that logical norms (such as the necessity of MPP) do not exist. Without them we no longer have a science of deductive reasoning—of logic. Assessment of the validity of arguments thus becomes a bankrupt practice.

1.5 Carroll

Lewis Carroll’s 1895 paper *What the Tortoise Said to Achilles* shows that any application of MPP is inferentially justified only by appeal to an additional application of that same rule. The result not only leads to infinite regress, but also highlights a difficulty present in *any* attempt to justify a rule of logic using that same logical framework. What follows is a truer iteration of the Tortoisean regress argument. Despite presenting a variation of it in §1.2, a rendition of its original form is appropriate here because of its historical importance, and also because I will refer to specific aspects of the argument throughout this thesis.

Carroll’s story begins with Achilles and the Tortoise discussing their race. The

Tortoise piques Achilles's interest by boasting of being able to demonstrate a sort of race that ultimately has no end. The proverbial race, of course, is explained in such a way as to mimic the path a logician might take in her justification of our entitlement to apply a rule of inference. The Tortoise presents a Euclidean-inspired argument about a triangle that applies MPP as a rule of inference:

- (A) Things that are equal to the same are equal to each other.
- (B) The two sides of this triangle are things that are equal to the same.

Therefore,

- (Z) The two sides of this triangle are equal to each other.

Achilles initially believes that the argument arrives at its conclusion after only two steps (two premises), but the Tortoise holds otherwise; she distinguishes between thinkers who might hold one of several attitudes towards an argument taking that form:

1. The thinker who believes in the *logical necessity* of the truth of Z following from true premises A and B in a valid argument.
2. The thinker who regards the hypothetical *sequence* as valid, viz., someone who endorses the validity of an argument employing the MPP rule of inference, but who does not yet accept the truth of A or B .
3. The thinker who accepts the truth of the premises A and B , but does not accept the hypothetical $\ulcorner (A \wedge B) \rightarrow Z \urcorner$.

Carroll wants to make an argument for the case of thinker (3.), and his Tortoise defies Achilles to make her accept Z . Achilles responds by entreating the Tortoise to accept statement (C):

- (C) If A and B are true, Z must be true.

But (C) is another hypothetical; we are therefore left with a new argument:

- (A) Things that are equal to the same are equal to each other.
- (B) The two sides of this triangle are things that are equal to the same.
- (C) If A and B are true, Z must be true.

Therefore,

- (Z) The two sides of this triangle are equal to each other.

But the Tortoise steadfastly resists acceptance of Z —she refuses to accept the consequent of the hypothetical, whereby Achilles argues that if the Tortoise accepts A and B and C , then she must, by logical necessity, accept Z . However this last argument by Achilles involves another hypothetical of the form,

(D) If A and B and C are true, Z must be true.

So long as the Tortoise fails or refuses to acknowledge the truth of Z which, being unconvinced of the correctness of MPP, she is under no logical obligation to accept, justification for Z must appeal to a regressive series of hypotheticals. The next, and obvious step in this series would be:

(E) If A and B and C and D are true, Z must be true.

This argument persists *ad infinitum*. Pascal Engel²⁰ proposes several morals that we might draw from Carroll's paper. One moral that is relevant to this enquiry is that a paradox arises when we countenance a rule of inference, in this case MPP, as a premise. If we fail to see the distinction between premises and rules (and subsequently treat the latter as having the same status or force as the former), then it is conceivable that someone—an inferentialist—might never be able to grasp the necessity of conclusion Z following from premises A and B . The same applies for the necessity existing between A , B , C , and conclusion Z , and so on. If we always preserve the distinction between rules, premises, and conditionals, then the regress cannot start because the regress of hypotheticals simply does not arise. In this case a non-inferential justification for our warrant to use MPP would be required, a justification that does not call upon an argument in order to demonstrate validity.

The idea behind the infinitely regressive nature of an inferential justification for MPP is therefore that any axiom, rule, or principle we adopt and hold to be valid is always subject to the sceptical challenge that asks us to justify the decision we have made to regard the rule as correct. This challenge will always ask after a level of justification higher than the one at which we are presently working. In this way the regress conceivably never ends, or results in circularity.

1.6 Agenda for Resolution of the Problem of Warrant

Several problems confront attempts to justify our knowledge of basic rules. The Carroll story shows how infinite regress ensues from an inferential justification of MPP. Regress is unsatisfactory because—roughly—we never arrive at some grounded principle whose correctness we can be certain of. That principle is important; upon it we could demonstrate our entitlement to reason according to a given rule of

²⁰Pascal Engel (2007), *Dummett, Achilles, and the Tortoise*, Open Court, 728 - 745.

inference, and subsequently effect valid deductive inferences culminating in conclusions that follow necessarily from their premises. This last sentence describes the objective of the inferentialist approach.

Further scrutiny of inferential justification also reveals that circularity ensues from any attempted justification of MPP by deductive argument. This is because a deductive argument that concludes that MPP is correct must use MPP in its reasoning. If we wish to vindicate an inferential approach to the justification of our entitlement to use basic rules of inference in deductive reasoning, we shall have to find a strategy that does not succumb to regress or circularity. Michael Dummett proposes a solution available to the inferentialist in the form of *pragmatic circularities* to be discussed in detail in the following chapter. This proposal improves the prospects for the inferentialist campaign, and figures in numerous solutions proposed by other philosophers, including that of Boghossian also to be discussed.

Non-inferentialism countenances justifications for rules of inference that are not themselves arguments. These sorts of justifications, as we have seen, include stipulative definition, sense experience or even an intuition. They seem less satisfactory as grounding principles however because of the lack of support available to corroborate them as correct, support such as a formal argument. Sense experience is not inter-subjective, and therefore it is impossible to codify a justification for it in universal fashion. Intuition is the very practice we seek to explain (*viz.*, our intuition to countenance certain rules of inference as correct), and therefore appealing to intuition in order to justify it leads to circularity. And stipulative definition without justification is arbitrary, regardless of how effective and plausible the definition is. How then do we satisfy someone seeking greater explanatory power in a justification than the solution proffered by the non-inferentialist? It might seem as though an inferential justification, one that is a product of reasoning, better satisfies the criteria for justificatory rigour. Consider the non-inferential stipulation: ‘MPP is a correct and truth-preserving rule of inference,’ and let us attempt to compose an argument in its defense;

P1. If a rule of inference is correct and truth-preserving, then it facilitates the inference of a valid argument.

P2. MPP is a correct and truth-preserving rule of inference.

Therefore,

C. MPP yields valid arguments

This simple argument seems to show that the non-inferentialist’s supposition that MPP is correct may be even more deeply justified by argument—by reason—in an inferential context. The ultimate goal of any justificatory account of rules of inference would be the presentation of a justification in a chain of reasoning that terminates. We see that the non-inferentialist is satisfied with justification that begins and terminates at once in definition, intuition, or sense-experience, but by

the most recent argument above, the inferentialist account seems to be capable of justifying MPP more ‘deeply’. Now what would a terminated justification look like? Should it be presented in inferentialist or non-inferentialist fashion, and what are its properties? Intuitively, we should seek a justification for which complete justificatory rigour exists, if possible. Ultimately, this ends up being the yet unachieved objective of all philosophers mentioned in this thesis, not to mention my own.

Rules of inference lie, more-or-less, at the beginning of an evidential chain. Again, what strategy do we have at our disposal that guarantees that MPP is a correct rule of inference? What, in other words, would an argument consist in, that has as conclusion the very claim that MPP is a correct rule of inference? The next chapter will present and evaluate several proposed solutions to the challenge of justifying our entitlement to reason according to basic rules of inference.

Chapter 2

Treatment of the Problems

2.1 Tonk

Carroll's Tortoise lays a large obstacle in the inferentialist's path. Achilles' attempt to justify a rule of inference by argument is hampered by the fact that he cannot do so without the consequence of regress in that argument. The alternatives to justification by argument, described above, may seem less palatable in light of our interest in justificatory rigour, but this non-inferentialist strategy of justification does carry with it results that are worthy of consideration. By regarding the rules governing deductive validity as *analytically correct*, and at the same time holding that those rules are conventions that emerge based on the language(s) we use, we improve the prospect and plausibility of logical rules being arbitrary; 'arbitrariness' is effectively what a stipulation connotes.

A *statement* is analytic when its truth can be determined solely by the meanings of the terms contained in that statement. Quine's example *No unmarried men are married* is the archetypical analytic statement in philosophical literature. It is analytic because no matter what the non-logical elements of the sentence represent, we can determine the truth of a sentence by what its logical structure forces upon us. Quine's example can be converted into the universal statement *No non- x^s are x* , which is a tautology—a logical truth. An analytically valid *inference* is one that arrives at its conclusion by respecting prescribed meanings of the rules of inference and operators involved. These meanings are expressible in terms of introduction and elimination rules which explain when and how we are permitted to apply the logical term in deductive inference. An analytically valid inference is simply an inference that has arrived at its conclusion by respecting the meanings of the logical terms, that is, by conducting inference in a manner sanctioned by rules telling us how a logical term may be used in inference. In *The Runabout Inference Ticket*, Arthur Prior asks whether the notion of an analytically valid inference is corrupt, based on his demonstration that '...any statement whatever may be inferred, in an analytically valid way, from any other.'¹

¹Prior (1960), pg. 38.

He observes that in inference involving conjunction, the validity of that inference is buttressed ‘solely [in] the meaning of the word *and*’. He demonstrates by applying its elimination rule to the following inference:

Grass is green and the sky is blue;

Therefore,

Grass is green.

Furthermore, Prior holds that we may deliver a *complete* (or all-encompassing) account of the meaning of the word ‘and’ by enumerating that operator’s introduction and elimination rules, thereby specifying the role this operator plays in our inferences:

- i. from any pair of statements P and Q, we can infer the statement P-and-Q.
- ii. from any conjunctive statement P-and-Q we can infer P.
- iii. from any conjunctive statement P-and-Q we can infer Q.

Prior then points out that based on this account of analytic validity, we can generate all sorts of absurdities in an analytically valid way; for example, the statement ‘2 and 2 are 4, therefore 2 and 2 are 5.’ He accomplishes this with the following argument:

(P1.) 2 and 2 are 4.

(P2.) 2 and 2 are 4 *tonk* 2 and 2 are 5.

Therefore,

(C.) 2 and 2 are 5.

Tonk, being a hitherto unknown operator, takes its *complete* definition in similar fashion to the definition of ‘and’ above using the following introduction and elimination rules:

- i. from any statement P we can infer any statement formed by joining P to any statement Q by *tonk* (which compound statement we hereafter describe as ‘the statement P-tonk-Q’).

- ii. from any ‘contonktive’ statement P-tonk-Q we can infer the contained statement Q.²

Prior sarcastically characterises this new logical construction as *convenient*, promising ‘to banish *falsche Spitzfindigkeit* from Logic for ever.’ It is convenient and banishes *false niceties* from logic because this operator as defined admits the inference of any statement from any another in an analytically valid way. Naturally, we should not take this message to mean that Prior subscribes to the thesis that analytically valid statements are corrupt. He acknowledges that as response to his introduction of tonk, ‘more enlightened views will surely prevail at last...’³ We find such an enlightened view in Belnap’s discussion of conservative extensions in the next section.

So it would seem as though some analytically valid arguments are semantically legitimate, this last term being characterised by arguments employing operators actually used in English and extra-logical contexts such as the connective ‘and’. Some other operators however are illegitimate, tonk being our prime example, where we find no such term in everyday parlance. If we are to preserve and vindicate the notion of analytic validity, we should have to (a.) explain it in such a way that it permits of all inferences we hold to be legitimate (such as those inferred from conjunction-elimination), while at the same time excludes all inferences that have no semantic relevance or value (such as the inference of ‘2 and 2 are 5’ from ‘2 and 2 are 4’); and (b.) present analytic validity in a way that does not enable it to generate inconsistencies. Furthermore, analytic validity in the more formal sense ensures that all statements expressible in a system of logic are, programmatically, permitted by the rules and axioms, and are valid. There is something about the nature of analytic validity that is not captured in the account articulated by Prior, something that erroneously vindicates illegitimate operators such as tonk. If we can identify what that is and incorporate it into a more restrictive account of analytic validity, perhaps the notion may be preserved. Analytic validity is a goal worth pursuing because if it can be made stable, viz., if it can satisfy our requirements for semantic legitimacy and avoid inconsistency as mentioned in the previous paragraph, then the inferentialist can point to the quality of being analytically valid as warrant for his entitlement to reason according to a rule of inference. The introduction and elimination rules could conceivably lie at the onset of the inference pedigree, and a possible Carroll-style regress might not arise in an analytically valid scenario.

2.2 Tonk gets *Belnapped*

Let us recharacterise the most salient point of Prior’s argument. Tonk’s introduction and elimination rules are expressible thus;

²Prior (1960), pg. 39.

³*ibid.*

$A \vdash A\text{-tonk-B}$ (*tonk-introduction*);

$A\text{-tonk-B} \vdash A$ (*tonk-elimination*);

$A\text{-tonk-B} \vdash B$ (*tonk-elimination*).⁴

By the transitivity of deducibility in a logical framework that recognises tonk as an operator, the logical form $A \vdash B$ follows. However $A \vdash B$ does not express a universal truth, for if A is true and B is false, then B does not follow from A . The tonk framework allows us to make this contingent inference, and when B is actually false, we have a counterexample derived in an analytically valid way. The crux of Prior’s point therefore argues that operators characterised in an analytically valid way permit the valid derivation of falsehoods from true premises. But Belnap shows his point to be mistaken, as I shall presently explain.

Belnap observes two options for defining connectives. The first option is to define them in terms of the role they play in inference and with respect to deducibility. The second option is to deliver some conception of what the connective means ‘independently of the role it plays as premiss and as conclusion’.⁵ This second option might see, for example, the creation of simple truth tables to establish the independent meaning of the given connective. Yet despite the problems caused to the first option by tonk, Belnap nevertheless wishes to preserve that approach to defining connectives. But how is this to be achieved without generating *tonktive* results? ‘How are we to make good the claim that there is no connective such as tonk though there is a connective such as *and*...?’⁶ What is required is a characterisation of a connective in terms of the role it plays in inference such that when that connective is used in argument, only analytically valid inferences that lead to sound conclusions are the result. Furthermore, when the connective is used in deductive inference according to the rules governing its meaning, the conclusion inferred must follow necessarily from the premise or premises of that argument.

The solution, explains Belnap, is to acknowledge that connectives that are defined using the first option, viz., in terms of the role they play in inference, are not defined or conjured up from nothing—*ab initio*, to use his words—but rather are defined in terms of an *antecedently given context of deducibility*. In the recharacterisation of Prior’s argument at the beginning of this section, we see that Prior has helped himself to one of the properties of deducibility—that of transitivity. That is how he is able to validly deduce the argument form $A \vdash B$ using tonk.⁷ But nowhere in Prior’s characterisation of tonk are the rules for deducibility characterised; they are *presupposed*, and constitute the antecedent context just mentioned.

⁴Note that *tonk*’s introduction rule is coextensive with disjunction-introduction, and its elimination rule with conjunction-elimination.

⁵Belnap (1962), pg. 130.

⁶Belnap (1962), pg. 131.

⁷Transitivity functions largely like the cut rule in propositional logic, where the cut rule states that If $\Phi \vdash_0 \delta_i$ for each $i = 1, 2, \dots, k$ and $\Psi \cup \delta_1, \delta_2, \dots, \delta_k \vdash_0 \alpha$ then $\Phi \cup \Psi \vdash_0 \alpha$.

When we deliver a formal characterisation of deducibility however, we see that introduction of the tonk rules to that characterisation (*viz.*, adding tonk as an *extension* to deducibility) renders the newly conjoined system inconsistent. Belnap chooses Gentzen’s characterisation⁸ of deducibility (hereafter \mathfrak{D}) to demonstrate his point. This characterisation is a sequent calculus, one among several used to demonstrate proofs that apply certain styles of formal inference. In this case, \mathfrak{D} was conceived (by Gentzen, who named it ‘LK’ for *Logischer Kalkül*) to study natural deduction. All and only sequents that correspond to universally valid arguments are provable in \mathfrak{D} , and it, in Belnap’s words, ‘completely determine[s] the context’ of deducibility. The difference between sequents and statements is that sequents are better suited to express *forms* of reasoning, whereas statements, as have been used in Chapter 1, are employed to express *particular* premises and conclusions, *viz.*, non-universal content.⁹ A sequent such as $\Gamma \vdash \Sigma$ in \mathfrak{D} expresses the intuitive notion that if every formula contained in Γ is true, then at least one formula in Σ shall be true as well. This also means that Σ is *provable* in \mathfrak{D} . There were numerous attempts to characterise natural deduction in the 1900^s. Common to all these attempts was the aim to minimise or eliminate the number of axioms used to characterise deduction, attempting instead to formally model reasoning as it ‘naturally’ occurs.

Table of Belnap’s Presentation of G. Gentzen’s Characterisation of Deducibility (\mathfrak{D})

<i>Axiom.</i>	$A \vdash A$
<i>Rules.</i>	
<i>Weakening:</i>	from $A_1, \dots, A_n \vdash C$ to infer $A_1, \dots, A_n, B \vdash C$.
<i>Permutation:</i>	from $A_1, \dots, A_i, A_{i+1}, \dots, A_n \vdash B$ to infer $A_1, \dots, A_{i+1}, A_i, \dots, A_n \vdash B$.
<i>Contraction:</i>	from $A_1, \dots, A_n, A_n \vdash B$ to infer $A_1, \dots, A_n \vdash B$.
<i>Transitivity:</i>	from $A_1, \dots, A_m \vdash B$ and $C_1, \dots, C_n, B \vdash D$ to infer $A_1, \dots, A_m, C_1, \dots, C_n \vdash D$.

While \mathfrak{D} permits the sequent $A \vdash A$, it does not enable us to derive $A \vdash B$. However, as we have seen, the addition of the tonk rules to \mathfrak{D} makes that sequent derivable. Since \mathfrak{D} already outputs all and only the universally valid sequents expressible in that language, and since $A \vdash B$ is not one of those sequents, $A \vdash B$

⁸summarized by Belnap in his (1962), pg. 132.

⁹Other important differences between statements and sequents exist as well. For instance, the order in which formulas occur in a sequent may have bearing on the resultant sequent inferred. Additionally, the possibility of a formula being repeated in a sequent exists as well. This is in contrast to sets where the order of set members is insignificant, and multiple occurrences of members is prohibited.

is inconsistent with \mathfrak{D} . This problem is the result of the tonk rules not being a *conservative extension* of \mathfrak{D} .

An extension of a system (or language) is characterised by the introduction of operators or new rules and/or axioms to that system, comprising an articulation of how the sequents deduced by way of those extensions may be proved.¹⁰ A system might be extended so as to allow it to deduce sequents that it was unable to previously, or to allow the system to deduce more thriftily that which it was already able to in more cumbersome fashion. A *conservative* extension has the following essential property: every new sequent provable by means of the extension must include instances of the new operator introduced by that extension. When these conditions are transported to contexts involving particular arguments, as in Chapter 1, we realise the result that a conclusion follows from premises not including the new operator only if *the conclusion* contains the new operator. The reason we require a sequent to contain a newly introduced operator is that if it was already possible to prove some sequent without it, then it was never necessary to introduce the new operator in the first place. It is illuminating to note as well that were the characterisation of deducibility to augment the tonk rules instead of vice-versa, then that *would* be a conservative extension because standard deducibility is (much) more restrictive in terms of what can be proved than the tonk rules, in fact, anything follows from a rule that permits the sequent $A \vdash B$. When the tonk rules extend the formal characterisation of deducibility, the augmented system makes the sequent $A \vdash B$ provable, violating the conservativeness requirement: the sequent $A \vdash B$, deduced by means of tonk-elimination, does not contain tonk, and was not expressible by the original characterisation of deducibility alone.

Belnap realised several items of importance in his reply to Prior. He describes a concise criterion for deciding whether or not a newly proposed operator shall be problematic in the framework of natural deduction. One aspect of that criterion requires that a newly introduced operator be verified against an antecedent context of deducibility exemplified by (but not limited to) the Gentzen characterisation above. The other aspect involves the conservativeness requirement: it is impermissible to admit an operator to a system of natural deduction if that operator makes possible the deduction of sequents that do not contain that operator and which are not provable in the unextended system. Belnap showed how tonk violates this criterion by permitting the deduction of a non-conservative, inconsistent sequent through the application of the rule of transitivity. Belnap's suggestion is that taking these points into account rescues the supposition that correct inferences such as MPP are analytically valid and truth-preserving. He has shown that tonk **does not** corrupt the notion of analytic validity where logical rules and operators are defined in terms of the role they play in inference.

So might we regard analytic validity as capable of adequately characterising BLK in a way that satisfies all of Hale's criteria mentioned in §1.2? Not yet, one

¹⁰In the case of tonk for example, the rule 'for any P, P-tonk-Q', explains how to use/introduce the rules governing the tonk operator.

problem still plagues analytic validity: If a logical operator is implicitly characterised by prescribing its introduction and elimination rules, thereby delivering its meaning, once those operators are part of a deductive context then there is nothing stopping a reasoner from using the very defining rule to vindicate the defined operator. Suppose we were to adopt the introduction and elimination rules for tonk presented at the beginning of this subsection. We could then put forward the following argument:

(P1.) $9 \times 7 = 63$,

(P2.) $9 \times 7 = 63$ tonk ‘tonk’ is a valid operator, (by tonk-introduction)

therefore,

(C.) ‘Tonk’ is a valid operator. (by tonk-elimination)

This argument is circular and certainly appears to lead to a problematic conclusion. It argues for the validity of tonk while using that very operator in its reasoning. We know that tonk is an unsound rule of inference, and that this argument is unsound, but it is no thanks to our test for validity that we know this. For in a situation in which tonk is part of the deductive context, this argument is valid. Were a similar argument put forth to demonstrate the correctness of MPP, it would, unlike tonk, result in a conclusion of which we approve and regard as sound. But what, then, distinguishes these two valid cases, each of which involves its own justification via a rule (namely, itself) which is supposed to be part of the pre-established deductive context? The lesson suggests that we cannot rely solely on analytic validity—mere acceptance of introduction and elimination rules—as the strategy for establishing the correctness of those rules. There are two reasons for this; (*a*) because analytic validity does not prevent the vindication of a rule by means of a circular argument. And (*b*), it allows us to find validity in some patently bad rules of inference.

As reasoners who admit MPP to deductive logic, we occlude tonk from deduction because of the unfavourable results that emerge when we apply it. But again this is no thanks to our test for validity. Furthermore, the disapproval we have for tonk is informed by our acceptance of MPP—the latter is already part of our established logical context, and so when we evaluate those unfavourable results that ensue from an application of tonk, we find the rationale behind an adoption of tonk to be broken-backed. But suppose some reasoner accepts tonk in the same way MPP is accepted. He might view the choice to accept MPP just as erroneous because when MPP is applied in his ‘tonk-context,’ a whole bunch of statements that were previously expressible (via tonk) in the system become invalid. And since the tonk-reasoner previously endorsed the system that contains tonk, he would find any acceptance of MPP to unfavourably alter the expressive power of his logical system. As we have seen, tonk enables us to infer any statement from any other, while MPP is much more restrictive. So why do we accept MPP and reject tonk? Analytic validity is an inadequate criterion for distinguishing bad from good rules of

inference. Furthermore, we are no better ahead in vindicating one rule of inference over another in the framework of deduction solely on the basis of analytic validity. Other criteria shall therefore be required.

2.3 Carroll, Regress, and Scepticism

It will be useful at this point to elaborate on the lessons drawn by Pascal Engel from his examination of the Lewis Carroll problem discussed in Chapter 1. He identifies at least four morals to be drawn from the Carroll paper:

1. The need to preserve the distinction between rules and premises;
2. The need to distinguish between propositional and practical knowledge;
3. The need to recognise and deal with an unavoidable circularity in any attempt to justify a basic rule of inference.
4. Scepticism about the force of logical reasons.

Per 1. Engel expresses the view exhorting many in response to the Carroll paper that regress does not start if we prohibit rules from being countenanced as premises. Referring back to the structure of the Euclidean argument, it is apparent that A , B , and C are treated equally as premises leading to the conclusion Z . ‘Equally’ here means that they all have the same force in their status as premises playing a specific (assertoric) role in a deductive argument. But observe that A and B , whether countenanced as statements or sequents, represent assertions (in this case pertaining to geometry), while C articulates a rule expressing the validity of the sequence. Engel suggests that in a conditional argument, the major and minor premises, as well as the rule, are each distinct in status. If we countenance the rule as something distinct, and subsequently disallow rules from figuring in arguments in that way, then the regress cannot begin.

Let us flesh out this possibility proposed in moral 1. It suggests the prohibition of logical rules from figuring as premises, thereby sidestepping the regress problem. In a typical valid deductive inference, if the premises are true, the conclusion necessarily follows. Were the rule of inference to figure in statement form (*cf.* statement C) and held to be true, then this would be an inferential justification of our right to move from premises A and B to conclusion Z , thereby expressing an argument that uses (and explicitly expresses) MPP as its rule of inference. C would *propositionally* express how the deduction is valid. In other words, it would solidify the relationship existing between A and B , forcing the conclusion Z . So what can we do when the rule C remains unexpressed? What then could justify the procedure of this deduction? Dummett explains his stance on the matter of the justification of deduction in the following quote.

This is the philosopher's question. In everyday life, we do not wait upon a justification, or ask for one. It is not rational to entertain any serious doubt about the matter; both thought and discourse would break down if we attempted to eschew all deductive inference until a justification of the practice was forthcoming, which it would never be, because any such justification must involve some deductive argument.¹¹

We can distill at least two messages from this quote. First, Dummett points out that in common discourse, the justification of simple inferences just does not occur. If it were a requirement in such a circumstance, communication would be difficult and awkward. The second message is that by acceding that a justification for a basic rule of inference not be required, one must be fairly certain and convinced of the correctness of the rule; for if not, why would its application be permitted unchecked? This ties fittingly into another of his theses, to be discussed at length in the next section, that in satisfying *the philosopher's curiosity* about the correctness of the rule, a curiosity that demands the 'checking' just mentioned, we must demonstrate that correctness by argument, and that argument will plainly be circular. In the case presently considered, we are examining the possibility of justifying a rule of inference. We may do so by applying the very rule we seek to justify, but we may also apply some other rule to arrive at our proof. While the justification of a rule of inference by means of the application of *some other rule* may not initially seem to be circular, we are reminded that our concern is for *basic* rules of inference such as MPP, and so any non-basic rule should conceivably be justifiable by a basic rule after the requisite number of line-items in a proof. Once again, the next section will examine Dummett's theory explaining exactly when we might be permitted to champion a circular argument as a justification for a rule.

Leaving Dummett's work aside now, what other possibilities exist, aside from circular or *petitio principii* arguments, that might serve to justify the correctness of rules of inference? We recall that we are presently attempting to do this in accordance with moral 1, that is, without allowing the rule to figure as a premise. The thesis that rules of inference are 'self-justified' is therefore also worthy of consideration, and countenancing a rule as such has a few advantages. It avoids circularity because no argument is required to vindicate correctness. It avoids regress because it is held to be a bottom-certitude of sorts. Engel recounts Dummett's suggestion that a rule is self-justified when it satisfies the requirement of harmony¹² and conservative extension, and that such a proposal is among the most forceful responses to the Tortoisean challenge.¹³ Dummett's solution is challenged however by the responsibility to correlate a rule that is self-justified with a rule that is conservative and in harmony. Our entitlement to countenance a rule as self-justified will depend on a reliable method to verify correctness without using an argument containing an expression of the rule's correctness among its premises.

¹¹Dummett (1994), pg. 194.

¹²A metalinguistic cognate of conservatism explained in the next section.

¹³Engel, pg. 730 - 731.

Per 2. The Tortoise’s main argument is that no thinker of any persuasion is under obligation to accept the necessity of the claim labeled C of the Carroll argument: if the premises of an instance of MPP are true, so is the conclusion. The problem resulting from her puzzle shows that the sort of knowledge required to carry us from premises to conclusion is not the sort of knowledge that can be expressed in a proposition—it is not a form of knowledge *that* or propositional knowledge, but rather a form of knowledge *how*, or practical knowledge. So far, it does not seem that practical knowledge is the sort that can be expressed in a proposition. Some philosophers attribute the sort of knowledge required to move from premises to conclusion to a faculty of the *understanding*. Comprehension of how MPP works and that it facilitates valid inferences is a function of that faculty—it is possession of the practical knowledge required. To know ‘if P then Q ’ is to know that when we know P we are able to ‘see’ the consequence Q . If someone were to know that P , yet be unable to see Q , then that would be what it is not to know that ‘if P then Q ’. This moral therefore sees the Tortoise arguing that a particular type of understanding is required to apply MPP in inference, and that application of it is not the sort of move that can be articulated by a proposition. Practical knowledge of MPP’s necessity is therefore something very different from an item of propositional knowledge which attempts, yet fails, to express MPP’s necessity.

Per 3. We have already seen indications of the likelihood that circular arguments are going to be involved in the justification of BLK. Engel presents an argument that this is unavoidable. Given the infinitude of instances of MPP and our ability to acknowledge only a finite number of them, our ground for logical truths cannot simply lie in a finite number of conventions because then we would be using a rule of inference to derive truths from only finitely many conventions; For grasp of the rule involves grasp of its applicability to the infinitely many instances. Recognising that a particular argument is an instance will involve reasoning akin to:

- P1. This argument has the form $A \rightarrow B, A$, therefore B ;
- P2. Any argument of the form ‘ $A \rightarrow B, A$, therefore B ’ is an instance of MPP;
therefore,
- C. this is an instance of MPP.

Of course, if we recast this argument in another form, say as a disjunctive syllogism, the problem simply becomes one of the circularity of the justification of that rule, which would then be basic in place of MPP. This would mean that we would be using logic to derive logic from convention. In Engel’s words, ‘...if we want to explain the nature of logical truths or rules from the existence of conventions, we must presuppose these logical truths and rules to derive them from conventions, and our derivation is thus circular.’¹⁴

¹⁴Engel (2004), 727.

Per 4. Unlike the other cited morals where the Tortoise has a grievance with premise C , viz., MPP in statement form, Engel considers the possibility that the Tortoise *endorses* C , but does not accept the conclusion Z , or more specifically, does not accept the logical force or necessity that takes us to the conclusion Z . Some cited reasons for this refusal are ignorance, stubbornness, recalcitrance, akrasia, and outright scepticism. If the reason for the Tortoise's refusal to accept the logical truth C is actually that she does not understand C , then this brings us back to moral 2 which discusses the epistemology of understanding. If she actually does understand C but still refuses to accept its logical force, then we can label her as a sceptic about the force of logical reasons.

Scepticism can give rise to some philosophically damaging results. Moral 4. seems to show that the position of the stalwart sceptic leaves us at an impasse in the task of justifying our warrant to apply MPP in either inferential or non-inferential fashion. The sceptic argues that there does not seem to be any attitude, disposition, principle, or rule that allows a reasoner to bridge the gap existing between premises and conclusion inferred by means of conditional elimination. In other words, practical knowledge for our warrant to reason according to MPP is unavailable or non-existent, and warrants expressed propositionally engender regressive results. Our discussion of Dummett's treatment of this problem beginning in the next section will figure integrally in opposing this sceptical claim.

The final comment I would like to make about MPP in this section pertains to the integral way in which it figures into any logical system. Above I referred to paraconsistent logics that exclude MPP as one of its admissible rules of inference, and observed that for any valid rule actually admitted by a logical framework, we can offer up a deductive argument that employs MPP in order to assert that rule's validity. It's not much of a feat:

(P1.) If rule x is one of the rules of inference of language y , then it is valid,

(P2.) Rule x is part of y ,

therefore,

(C.) Rule x is valid.

The paraconsistent logician, who does not admit MPP as a rule in her system, cannot deny this piece of reasoning which employs MPP to arrive at its conclusion. It has figured into her system whether she likes it or not. In this way MPP plays a role similar to transitivity in Gentzen's characterisation of deduction; Prior overlooked transitivity in his characterisation of tonk. However it was implicit or unavoidable as a constituent rule of deduction. MPP seems to possess that same status: we know it belongs in any deductive system, and it seems as though it belongs to *any* consistent logic as some sort of an antecedent or implicit context, *even in cases where MPP is programatically excluded as a permissible rule* as in the case of paraconsistency. So whence comes this universality or unavoidable applicability of

MPP? In the section focusing on Hale’s work below we will see that he argues that use of MPP in a consistent system is unavoidable. He takes this finding and uses it as a foundation for a non-inferential account of our knowledge of the correctness of basic rules of inference.

Since any argument about the truth, meaning, or validity of a particular rule can be framed in terms of MPP, and any argument that attempts to explain the validity of MPP has the problems named by the morals in this subsection, then perhaps a special status needs to be assigned to MPP that considers our unreflective intuition that it is valid, despite our inability to offer up proof of its necessity. One argument for the special status of MPP might be that we are predisposed to reason in that way. This could mean one of at least two things. First, that our reasoning faculties are ‘hard-coded’ to behave in such a way that propels us to arrive at conclusions using a strategy such as MPP. This argument is weak though, because we are at a loss to conclusively explain what this ‘hard-codedness’ would consist in. Even more problematic is that were we actually to characterise this hard-codedness, there is no guarantee that the hard-coded rule is actually correct—we should only be able to point to its correctness in an inductive way, that is, by pointing to as many instances of our application of this rule that have helped us deduce sound and consistent results. Presently, as we come closer to understanding the nature of quantum science, the hypothesis that the principle of bivalence, for example, is not a universally applicable principle becomes plausible. If ever the mind was hard coded with a rule, the principle of bivalence should qualify as one, yet quantum science shows that this principle should be called into question. The second argument might be that given the way our languages and histories of thought have unfolded, the status of MPP as *the* underived rule of inference might have evolved as their product. To this we might respond with an analogy to Neurath’s ship, such that the way we reason could have turned out to be otherwise had our ascent into language and epistemology a few millenia ago unfolded differently. This might shed some light on the necessity of MPP, such that if a language system could be developed by a sufficiently large group of people who know nothing about language or theory of knowledge, people who have essentially been subjected to the conditions described in Plato’s cave in book IX of *The Republic*,¹⁵ then we might gain some insight into the necessity of MPP insofar as that rule of inference would either emerge as a natural output of their linguistic and reasoning practice or it would not—and this would be useful knowledge gained about the necessity of MPP. However, to begin with a veritable *tabula rasa* in some linguistic community in an attempt to see if MPP would figure integrally in some newly-contrived system in the same way that it has in our current system(s) would be impossible. I doubt such an attempt could even be conducted via thought-experiment because implicit and unavoidable usage of concepts that we already possess will be involved in, and thus inform the outcome of our enquiry. The subjects of such an experiment would truly have to

¹⁵And the conditions for us as observers, and for the subjects, as the objects of our enquiry would not be the same still because our language and theory of knowledge emerged as a result of groups of people developing them *in this world*, rather than off in some Platonic cave.

be non-lingual for such an experiment to be possible.

MPP figures integrally in our reasoning processes and seems to enjoy a special status as a rule of inference. We have run into difficulty trying to come up with a justification for our entitlement to reason according to MPP, and appeals to the make-up of our minds or of our linguistic practice fall short of providing the justification we seek. Now does moral 1 propose a potential solution to the Carroll problem? Did he intend that the readers of his paper distill the important distinction between rules and premises from Achilles' and the Tortoise's banter? The rules-are-not-premises will have an important place in my concluding findings presented in Chapter 3.

2.4 Circularity, Consistency, Harmony, Scepticism

The way in which the Carroll story is laid out shows how premise C is required to justify $\ulcorner(A \wedge B) \rightarrow Z\urcorner$, and then how premise D is required to justify $\ulcorner(A \wedge B \wedge C) \rightarrow Z\urcorner$, and ultimately how this leads to an infinite regress of justification that seeks to, but never succeeds in vindicating the conclusion, Z . A regress of this sort is fatal to the logician's task of offering up a justification for our entitlement to reason according to this rule where he seeks to demonstrate the rule's correctness. Perhaps then, a circular approach to justifying MPP should be reexamined to see if a plausible justification for this rule of inference can be exacted from a strategy, viz., circularity, which we were quick to dismiss earlier on.

Dummett begins his enquiry into the possibility of circular justifications from the important observation that there really is no reasoning with or proof available as response to the stalwart sceptic. Appealing to the Carroll morals (3.) and (4.) of the previous section, the sceptic argues that there is no satisfactory justification to be had for MPP as a rule that preserves truth. But this denial of the possibility of justifying MPP is a categorical statement that can be turned into an argument framed in terms of MPP. The sceptic would have to acknowledge our rule in order to proffer the conclusion he bolsters, which renders the sceptic's view self-defeating.

The crux of scepticism is relevant to Dummett's work in metaphysics because there¹⁶ he is concerned ultimately to defend intuitionistic instead of classical logic as *the logic*. He champions such a defense by arguing that classical logic is informed by metaphysical presuppositions. For instance, when we consider an object, and suppose that there is a fact-of-the-matter about any statement that mentions it, we are taking the metaphysical stance of realism. In this case classical logic is the accepted framework we use to reason about the members of this domain. But an anti-realist stance about a particular domain emerges from an adherence to intuitionistic logic for that domain; if *it* is regarded as the logic of choice to discuss the objects in some domain, then we do not, for example, admit the principle of bivalence to figure in our reasoning —*for any proposition P , either P is true or P is*

¹⁶in his book *The Logical Basis of Metaphysics*, Harvard University Press, (1991).

false, for the nature of this principle implies a metaphysical stance. Realism is supposed to mirror a correspondence to reality exclusive or regardless of our access to a domain. Realism also entails belief in a fact-of-the-matter about any proposition of the domain in question. It is a presupposition of which intuitionism in mathematics arguably dispenses, and Dummett extends the lesson to non-mathematical domains. A sceptic (categorically) denies the possibility of justification thereby suggesting a fact-of-the-matter in itself and thus proffers a realist thesis. Intuitionism repudiates the fact-of-the-matter way of reasoning and is therefore not amenable to the realist metaphysical stance (of bivalence for one) presupposed by classical logic. The categorical statement made by the sceptic agrees with a realist attitude about justification, and so in the broader context of Dummett's entire book (and life's work for that matter), which seeks to defend intuitionistic logic as the logic of choice, one that attempts to circumvent metaphysical presuppositions, the reader evolves to understand how the sceptic's thesis is dissatisfying and untenable according to Dummett.

Continuing with his enquiry into the possibility of the existence of a plausible circular justification for our entitlement to reason according to MPP, Dummett distinguishes circularities of two sorts:

Gross Circularity. A grossly circular argument contains among its premises the very claim sought to be proven in the conclusion. In the case of justifying a basic rule of inference such as MPP, one of the premises in a deductive inference will be the expression that MPP is valid. Such an argument might resemble the following:

(P1.) MPP is a correct rule of inference. *(Premise)*

(P2.) MPP is a correct rule of inference or the sun sets at 18h03.¹⁷ *(Or-introduction)*

Therefore,

(C.) MPP is a correct rule of inference. *(Or-elimination)*

The principal issues facing gross circularities however touch on the fact that a grossly circular argument assumes what it seeks to prove, and subsequently, by assuming what it seeks to prove, such an argument can prove anything, even falsehoods or inconsistencies. The gross circularity manifests in the fact that the conclusion is identical to one of the stated premises, *and*, in this particular case, the argument arrives at a conclusion about the rule MPP itself. A gross or viciously circular argument is immune to failure, and by extension, is empty: in reasoning about a rule, if we state its correctness in a premise and then conclude that that rule is correct, then the argument cannot fail. But the conclusion doesn't tell us anything more than what we already

knew in our premises/assumptions, and so nothing has been gained—this is what I mean by calling a grossly circular argument an empty one.

These two aspects of grossly circular arguments described in the first sentence of the paragraph above are strong enough reasons to impel the inferentialist to seek out a more satisfactory account of justification than gross circularity is capable of providing.

Pragmatic Circularity. A pragmatically circular argument that justifies a basic rule of inference is one that reasons *according to* that rule but does not contain an expression of its validity among the premises of the argument. In this way, the argument is not guaranteed to succeed, and because it is not guaranteed to succeed we thereby actually prove something.

I believe that this distinction between vicious and pragmatic circularities can be read as part of Dummett's response to the Tortoise and sceptic. Consider the remark Dummett makes about the stalwart sceptic:

Since a justification of a logical law will take the form of a deductive argument, there can be no justification that appeals to no other laws whatever; but that does not matter, since there is no sceptic who denies the validity of *all* principles of deductive reasoning, and, if there was, there would obviously be no reasoning with him.¹⁸

This quote is almost a *reductio*, highlighting the largest problem confronting the sceptic because it leads to the conclusion that the sceptic's attitude is unreasonable. The Tortoise systematically refuses to accept any conclusion arrived at by inference, and Dummett's quote reiterates that there simply isn't any reasoning with an attitude of that ilk. The Tortoise is trying to point out the futility of grounding any warrant for reasoning according to MPP because regress ensues, while Dummett hones in on the fact that if we are to accept that sceptical thesis, then our entire science of deductive reasoning cannot rest on firm foundations. But this is unreasonable. Dummett's suggestion is that our inferential practice can be justified via pragmatically circular reasoning that has the potential to avoid regress, and at least be enlightening in some way. Dummett's task then, should be to build on his defense of pragmatic circularities and develop a theory that does not admit defeat in the face of susceptibility to inconsistency and unsoundness.

Let us begin by looking more closely at Dummett's claim that, unlike viciously circular arguments, pragmatically circular ones are not guaranteed to succeed. Our earlier discussion of tonk highlights this case. Consider the following pragmatically circular argument.

¹⁸Dummett (1994), 203.

(P1.) Stephen Harper is current Prime Minister of Canada; (Premise)

(P2.) Stephen Harper is current Prime Minister of Canada, *tonk*, Céline Dion was his predecessor; (*tonk-introduction*)

Therefore,

(C.) Céline Dion is former Prime Minister of Canada. (*tonk-elimination*)

In this case, The introduction and elimination rules for *tonk* allow us to infer, in an analytically valid way, a conclusion that is false. The inference shows us that it is possible to reason according to *tonk* (viz., it is possible for such a sequence of reasoning to be *valid*) and still generate an argument that fails. A pragmatically circular argument is therefore enlightening in the sense that some such valid arguments will yield true conclusions, and others will be false. If some arguments fail because they are unsound, yet some others obtain, then we can legitimately countenance pragmatically circular reasoning as fruitful and contingent on the meanings of operators and rules involved. *Tonk* was shown to be meaningless above, and therefore the meaning of any sentence employing it should be difficult to ascertain.

Now exactly what aspect of *tonk* renders it incorrect and causes some arguments that apply it to fail? The examples under consideration are arguments such that a true premise is followed by two correct applications of the *tonk* rules (viz., its introduction and elimination rules) which lead to a false conclusion. Because of the false conclusion, we know that one of the inferential steps taken is incorrect, despite being validly applied. This tells us that there is a problem with *tonk*—we’ve used it properly, only to infer a false conclusion. The problem is with *tonk*’s rules. Taken separately, both rules are correct, for, as we have seen, the *tonk*-introduction rule is identical to the rule for *and*-introduction and *tonk*-elimination is identical to *or*-elimination. So both rules are correct when applied in other contexts, yet when used together to characterise the meaning of *tonk*, they define a bogus operator.

As expressed in the metalanguage, it is not immediately apparent that *tonk* is contingent.¹⁹ *Tonk*’s failure emerges through exemplification in the object language. First consider its rules in the metalanguage as expressed in §2.1; We then observe the expressions in the object language below. They demonstrate that arguments applying *tonk* result in sentences that are inconsistent with the system of natural deduction, namely,

$\mathbf{P} \vdash \mathbf{Q}$ and $\mathbf{Q} \vdash \mathbf{P}$.

¹⁹The contingent nature of *tonk* might be apparent to a seasoned logician or somebody with sharp analytic skills. Yet it is possible to characterise some other operator in the metalanguage possessing introduction and elimination rules that are too complex for a reasoner to immediately intuit its status as valid.

Whereas a sound and consistent characterisation of natural deduction allows $\mathbf{P} \vdash \mathbf{P}$, it does not permit $\mathbf{P} \vdash \mathbf{Q}$, for the latter does not express a universal truth. It is thus that we reason according to tonk in a pragmatically circular way, and find that the rules governing this operator fail a conservativeness test, which must be passed if we are to adopt the rule in natural deduction. Our justification of tonk by pragmatic circularity has therefore failed. A different operator that is characterised in such a way that it *is* conservative will be a better candidate for admission into a system of natural deduction.

Now in their status as circular patterns of reasoning, are pragmatically circular arguments as useless as the grossly circular ones in their attempts to justify logical rules? It depends, Dummett holds, on the purpose of the argument put forth. If its purpose is *explanatory*, whereby the argument serves to clarify the reasons an explainer has for holding that rule to be valid, and whereby the explainees endorses the correctness of that rule as well, but merely seeks a *demonstration* of that correctness, then pragmatically circular arguments will have achieved their intended purpose. However if the purpose of a pragmatically circular argument is *suasive*, whereby the explainer intends to use a pragmatically circular argument to convince or persuade a doubtful explainees (such as the Tortoise) of the correctness of that rule, then the argument will fail because the explainees shall not allow the explainer to employ that rule of inference to generate a justification for that very rule—it would be question-begging. The explanatory campaign takes for granted that the rule in question is correct, and subsequently delivers a circular vindication of this belief. But the intuition just mentioned is the very state that the sceptic, and the Tortoise for that matter, take issue with. For as we have seen in the Carroll story, the only sort of inferential justification available is an infinitely regressive one.

An inferentialist endeavour with explanatory rather than suasive objectives takes a leap of faith in assuming the validity of MPP prior to delivering his explanation. It's a small leap, because even the (natural) language from which the rule derives its characterisation and use²⁰ countenances that rule as correct.²¹ The difference between a thinker who accepts pragmatic circularities and the sceptic who denies them lies in the former's willingness to allow certain types of faithful leaps by accepting that a rule be applicable in inference *ab initio*. A proponent of pragmatically circular arguments will then use that supposition to formulate an argument that justifies our entitlement to use that particular inferential rule. By contrast, the tortoisean (or sceptic) prohibits those same faithful leaps. This is an example of a pragmatically circular argument arguing for the correctness of MPP:

²⁰I am referring here to the way introduction and elimination rules are delivered in English, as presented for the case of tonk in §2.1.

²¹In his paper *A Counterexample to Modus Ponens*. The Journal of Philosophy, Vol. 82, No. 9. 1985. pp. 462 - 471. Hartry Field argues that counterexamples to MPP exist. I do not discuss his observations here because I do not think it will help this project to study a counterexample to a rule, viz., MPP, that is uncontestedly valid to the extent that it is. Additionally, the counterexample Fields uses contains another conditional embedded into its antecedent, and I think that I would be straying too far afield to enter into an enquiry that enumerates the differences existing between the example Fields proposes compared to a more conventional exemplar of MPP.

(P1.) A rule of inference that is conservative and truth-preserving is a correct rule of inference.

(P2.) MPP is conservative and truth-preserving

Therefore,

(C.) MPP is a correct rule of inference

So we have come some way in establishing what sort of circularities might be permissible in arriving at legitimate justifications for logical laws and operators according to Dummett, namely pragmatic ones. But we also saw that some validly characterised operators such as tonk were problematic insofar as they generated inconsistencies. Since illegitimate operators can still be characterised in a pragmatically circular way, how might we then circumscribe those operators that preserve the soundness and consistency of any argument containing true premises? We have yet to bridge the leap of faith I mentioned above, and must consider whether a conventionalist attitude about the meanings of operators might secure a justification where circularities fall short. Under conventionalism, we might hold that the logical constants or rules are valid simply in virtue of being the ones we treat as valid, thereby replacing the leap of faith at the beginning of the inference pedigree (viz., where the justification for the correctness of a rule of inference is delivered prior to the application of that rule of inference in argument) by accepting those rules as basic, regarding them as default-reasonable, or countenancing them as unjustifiable. But any rule system can be conjured up, such as the system that admits of tonk—if we are to speak of systems that we simply *adopt* as valid. Dummett likens conventionalism and its shortcomings to following the rules of a game. If all the rules of a particular game are understood by some player, and the overall objective of the game was also understood by that player, then he would have a complete understanding of the game. But the game could also be played by a player who knew the rules perfectly well, but didn't understand its purpose, objective or strategy. It is questionable then whether the person has a full grasp of the game despite her strict adherence to the rules. This is analogous to a context where, hypothetically, a reasoner understands the rules governing tonk, and adopts them conventionally. The reasoner could apply the tonk rules but he would generate inconsistencies. And because the reasoner endorses a system (in the case of logical expression) that generates inconsistencies, we can say that perhaps this reasoner does not really know what he is doing—that is, does not seem to understand the purpose of reasoning. When we use a logical operator, we do not always use it in the context of deductive inference—sometimes our usage is non-assertoric, and our conception of the operator is therefore richer than a purely formal conception is capable of supporting. The *understanding* of the game's purpose and strategy figures integrally in helping us know which are the appropriate rules to the game, and this understanding seems to be the very thing we have yet to describe. This understanding is intuitive or inchoate. We barely know we have it, but intuition

does do its work when we come to understand that we have been playing a game lacking of objective or strategy in the case of a game, *or reasoning according to an inconsistent rule of inference* in the case of an argument.

The important question in the preceding paragraph therefore asks after a method of circumscribing those pragmatically circular justifications of logical laws that are sound and valid from those that are not, such as tonk. We saw that metalinguistic characterisation was insufficient, for it was possible to characterise tonk in analytically valid fashion, despite our awareness of its shortcomings. Dummett's response to this problem begins by acknowledging that the ends of natural language are internal. We presumably need natural language in order to express our thoughts. The meanings of our words and linguistic devices are determined by the rules governing their use. Failures such as inconsistency arise in our linguistic tools when the multiplicity of principles governing our linguistic practice are discordant. 'For the language to function as intended, these principles must be in harmony with each other.'²² We shall first identify Dummett's distinction of the two categories of meaning-constituting rules of use, and then we shall investigate exactly what he means for rules to be in harmony:

Verificationist Principles. Are those that have to do with the circumstances that warrant assertion, indicating when we are entitled or required to assert a particular statement. This is established both by inference (in the case say, of mathematics) or by observation, and often our entitlement is based on a hybrid of the two. The verificationist component of the meaning of the logical operators are particularly easy to specify: they are the introduction rules. And if we know the introduction rules, it can be said that we possess the complete *meaning* of the operator in question because we may verify the meaning against the rules—something akin to analytic validity.

However if we were to rely solely on what is required to *verify* a particular statement in order to be entitled to assert it, then '... we should be skilled at making assertions but incapable of responding to the assertions of others.'²³ This is because we do not all know the same things—we are not all in possession of the same fact and meaning base, and what a thinker knows about any one particular thing ultimately depends on everything else he knows. And therefore given knowledge of the rules that characterise a logical operator (admittedly it gets trickier with the non-logical terms of natural language), we can form statements and verify their conformity to the rules, however this principle is incapable of considering the *consequences* of our choice and application of rules.

We therefore counterbalance verificationist principles with;

Pragmatic Principles. These principles are the ones that determine what we can warrantably infer from a claim. They prescribe that the content of a statement

²²Dummett (1991), 210.

²³Dummett (1991), 211.

should be regarded as determined by its *consequences* for one who accepts it as true. This principle asks what the *difference made* to a thinker consists of should she choose to accept the statement. In the case of a logical operator, the pragmatist component of the meaning is once again easily specified: it is given by the elimination rules. The pragmatic approach to governing our linguistic practice therefore assesses the logical consequence of an application of an elimination rule.

Suppose we interpret formalism generally as the principle that allows us to assemble the rules for *any* language, and that a language so created is nothing but a series of symbols and rules telling us what may be expressed in the system. It is nothing but such a series because it does not express statements about anything in particular. In order for such a system to have meaning, we must assign it a semantics, and this is where pragmatist principles figure: once the semantics is assigned, we can decide whether the statements inferred in that system agree with what we hold to be a correct view of the domain of discourse. It decides about the consequences of being allowed to assert a particular statement.

The principles of verificationism and pragmatism account for essential aspects of meaning, and neither of them is singly sufficient for a complete meaning theory. Dummett held that a complete meaning theory must be informed by both principles, and that if a defined operator is going to be correct in a logical system, its introduction and elimination rules must be in harmony. The introduction rules prescribe which statements involving the logical vocabulary are expressible in the language, and the elimination rules prescribe what inferences we are permitted to draw from those statements. The introduction and elimination rules for a particular logical operator are in harmony when the elimination rules allow us to derive from a sentence with that operator as its main connective all and only those statements that the introduction rules require that we know in order to warrantably infer that statement in the first place. Harmony guarantees the absence of inconsistency when we consider the addition of any operator to a system that is consistent to start with, because it guarantees conservativeness.

The logical conception of a conservative extension best reflects the semantic, syntactic, and meta-theoretical requirements prescribed by the attribute of harmony. Conservative extensions were discussed in detail in §2.2 above, and what we must carry forward from that discussion involves the requirement that any rules we wish to introduce into a system that characterise an operator be a conservative extension of that system in order to preserve validity and consistency, and that harmony preserves this conservativeness. When we add a new constant to a natural deduction system, we threaten the consistency of the newly anointed system. If we can prove that the addition of the new constant does not generate inconsistencies in the augmented system (i.e. the natural deduction system augmented by the new constant), then the newly anointed constant will be correct in the new system. The anointed system will not yet be a conservative extension of the original one however, for conservativeness carries with it the additional requirement that we should not

be able to express conclusions in the new system that are unexpressible in the old system, save for conclusions that contain the new operator themselves. Conservativeness is therefore a stricter criterion than harmony—its stringency surpasses that of consistency’s with its additional requirements pertaining to new expressions expressible in the anointed system: that they do not facilitate new expressions unless the new operator is included in that expression.

Dummett believes that harmony implies conservativeness because of the sharp correlate existing between the metalanguage and the object language in logic. Suppose some consistent system of natural deduction. We then augment that system by an operator whose rules are in harmony. We can then see, by observing stepwise proofs in a corresponding object language, that the use of that operator is superfluous. We never required the operator to be added to the system in the first place. A step of a proof that utilises such an operator introduces it by its rule. An inference immediately ensues by applying that operator’s elimination rule. Dummett calls such an occurrence in a proof a ‘peak’. The superfluosity just mentioned is apparent when we consider that the propositions (represented by variables) that we admit into a deductive proof are already ones that we accept. What remains unknown as yet are the properties of the resultant inference - the conclusion. But we were already able to articulate the premises and effect all the possible valid operations on them using the original system; we were able to do away with the new operator; to *level those peaks*, much in the same way as we eliminate steps of a proof using the cut-rule in the predicate calculus. Harmony therefore implies conservativeness because no new statements are inferable via the conservatively augmented system that we couldn’t infer in the original one.

What lesson might we draw about BLK from Dummett’s enquiry into harmony? Acknowledging that the basic rules must be introduced first into any system, prior to the more complex rules that will likely employ BLK in their own characterisations, it is obvious that basic logical rules will comprise a consistent natural deduction system. This is because, given nothing but basic operators, there are no other components of the system with which they may be inconsistent.

Which rules are basic? In other words, when do we know that we have enumerated them all? The concept of harmony answers this question with its requirements of consistency and parsimony. If we can raze a logical system of an operator without sacrificing explanatory power then the item razed is not basic. The criterion of conservativeness is therefore useful and effective to the end of circumscribing which elements of a system are to be basic.

2.5 Boghossian and Conceptual Role Semantics

Boghossian’s treatment of the problem of characterising the logical operators takes an epistemological approach, which is to say that he examines our practice of belief in order to ascertain what epistemological stances we should hold, or are permitted

to hold, in the case of the basic rule of inference MPP. He promotes a theory of justification for BLK that argues in favour of our entitlement to do so, provided that the meanings of the logical operators may be fixed, and once again, provided we are not attempting to demonstrate this entitlement to a stalwart sceptic.

As we have seen, gross circularities give rise to two familiar fallacies in argumentative reasoning, namely, the fallacy of *begging the question*, which takes the strategy of assuming in the premises of an argument that which we seek to prove in its conclusion. The other fallacy that arises from a grossly circular argument is the problem of *bad company*, whereby in assuming what we seek to prove, we become capable of proving anything.

Boghossian agrees with Dummett's view on the issue of circularity by distinguishing the pragmatic from the gross, and sets up his endorsement of the former as a justificatory strategy with promise:

... it is not immediately clear that we should say that an argument relies on its implicated rule of inference in the same way as we say that it relies on its premises.²⁴

The problem of begging the question gives rise to two principal (epistemological) challenges according to Boghossian.

1. '... is to say in what the entitlement to use a rule of inference consists, if not in one's justified belief that that rule is truth-preserving;'
2. '... is to say how a rule-circular argument can confer warrant on its conclusion even if it is powerless to move the relevant sceptic.'²⁵

If we are obliged to justify our belief that a rule is correct before we can legitimately employ it in argument, then the sceptic's challenge will indeed block our entitlement to reason according to that rule. For according to him, our belief in its validity is not justified or worse, unjustifiable. However, rather than focus on our *beliefs* about validity, an alternative response to the sceptic's prohibitions (who, as we know already, repudiates pragmatically circular justifications) consists in the delivery of a correct account of what determines the *meanings* of our logical constants. With the meanings of those constants fixed, Boghossian argues that we are entitled to employ the rule without first being justified. It is proposed that meanings be delivered by way of a conceptual role semantics (CRS), which seeks to demonstrate the meaning of an operator or rule by enumerating the role it plays in inference. That role is thereby *meaning-constitutive* of the logical operator in question. To wit, of the entire set of statements employing the 'if-then' rule in a thinker's repertoire, a specific subset of those statements will constitute the meaning of that operator.

²⁴Boghossian (2001) 245.

²⁵Boghossian, (2001), 246.

We have referred to these throughout this text as introduction and elimination rules.

At least two challenges can be mounted against a CRS. A semantic indeterminist might oppose this conceptual-role model on the basis that only an infinite (and therefore unquantifiable) number of demonstrations can fix the meaning of an operator. But a CRS implies that a *specific subset* of inferences involving some operator fix its meaning. If that specific subset is quantifiable, and we have reason to believe that it is in consideration of characterisations of logical constants through specifically designated introduction and elimination rules, then the implications of the challenge of semantic indeterminacy change because the indeterminist is no longer capable of challenging the practice of prosecuting an infinite set as determinative of meaning. Another angle to these opposing views might be understood along the following lines; a semantic indeterminist would argue that an infinite number of logical truths represent the meaning of an operator, while the proponent of a CRS believes a finite subset is sufficient for the job. The upshot of these opposing views is that the latter shall be able to ostensibly generalise the meanings of logical laws or operators by pointing to a subset of rules that make up a meaning, while the indeterminist is unable to express the same generalisations if he maintains that the *total set* of logical truths make up that meaning. As we shall see in this section, Boghossian will have to make a strong case for a CRS in order to persuade an enquirer that meaning indeterminacy can be overcome.

The second challenge is that Boghossian's proposal of employing a CRS is threatened by the potentially lax results permitted by the acceptance of introduction and elimination rules as constitutive of meaning. As we have already seen in the case of tonk, nothing blocks us from nominating any collection of introduction and elimination rules to characterise meaning. When used in a rule circular justification however, we saw how tonk enabled us to legitimately derive any statement—even inconsistencies and falsehoods. But Boghossian wants to say that not every collection of introduction and elimination rules determines a meaning, and in his story tonk would be an operator that fails to do so. Certain criteria must be satisfied in a CRS in order for an operator to have meaning by characterisation through introduction and elimination rules—truth-preservation is one such criterion. Some sets of rules are truth-preserving, such as Belnap's presentation of the rules for the operator 'and', while other sets of rules are not, again tonk. To avoid the lax results facilitated by tonk and similarly characterised operators, Boghossian explains that for an operator to have meaning, the given subset of (introduction and elimination) rules '... means that unique logical concept, *if any* whose semantic value makes the inferences in that subset truth-preserving.'²⁶ Tonk and other operators devoid of meaning do not satisfy this last requirement.

Supposing that a CRS succeeds in delivering the meanings we seek, Boghossian suggests the following epistemological principle:

(L) If M is a genuinely meaning-constituting rule for S, then S is entitled to infer

²⁶Boghossian (2001), 249.

according to M, independently of having supplied an explicit justification for M.²⁷

He then remarks that this principle,

... does not require that S *know* that M is meaning-constituting for S if S is to be entitled to infer according to M but only that M *be* meaning-constituting for S.²⁸

(L) he argues, is intuitively plausible. That we have the disposition to reason according to some rule should stimulate an entitlement to reason according to it that is prior to, and independent of, an explicit justification for that rule. Without the disposition or intuition to reason as such in the first place, there would be no rule of inference for which we would need to seek out a justification. Boghossian seems to be saying something along the following lines; ‘we’ve used some rule in our inferential practice and it is indeed as yet unjustified, but something compelled us to conceive of and use *that* rule, rather than some other. We should recognise this fact as significant, and give this rule that we’ve been disposed to use some credit as being a rule that is correct.’ If we didn’t allow these dispositions to stimulate inferences, then the only thing we would be capable of doing in a sceptic’s world is assert ‘contentful’ items (and not transitions between contentful items), which in my view amounts to a license to only make statements about what is directly observable in the case of physicalia, or knowable in the case of memory, belief, or intuition.

Entitlement to use a rule of inference ensues from a CRS in Boghossian’s story. A CRS is a theory whereby introduction and elimination rules determine the meanings of operators. But he also holds that not every rule set determines a meaning. His claim is that only operators with genuine meanings may be used to justify rules of inference in an argument. The fundamental issue then is the problem of determining what makes a class of rules genuinely meaning-constituting. The sceptic may continue to deny beliefs (namely those beliefs hanging on to the correct nature of some rule) from figuring into any justification of a rule of inference, but Boghossian holds that there are forms of reasoning that emerge *a fortiori* from our propensity to infer, that is, we are disposed to reason in these ways. Provided that characterisation in terms of introduction and elimination rules in the case of the logical constants can be delivered and can establish an operator as genuinely meaning-constituting, then the CRS delivers the entitlement to reason according to a rule. Our entitlement therefore reduces to the problem of determining what makes a rule meaning-constituting. Boghossian’s first epistemological challenge listed above, (1.), is thus resolved. In order for Boghossian’s story to work, he would need to give us a worked-out account explaining exactly how he links the conceptual role an operator plays in inference with its meaning. That is the very

²⁷ *ibid.*

²⁸ *ibid.*

large proviso mentioned five sentences above, and this Boghossian attempts in his paper *Analyticity Reconsidered*.²⁹

His second challenge, (2.) at the beginning of this section, treats of the problem of persuading the sceptic. Supposing the sceptic were stalwart, is there anything we could do to persuade him? He blocks all explicit categorical statements to the effect that a particular rule of inference is correct. By extension, our entitlement to reason according to a rule of inference is nullified as well, on pain of the lack of justification for the belief that the rule is correct. We would also have difficulty persuading him that a rule is meaning-constituting because that would involve an argument, and that argument would have to employ MPP. So Boghossian asks the important question, *what is the epistemological significance of the fact that we are unable to persuade the sceptic about MPP?*

Boghossian evaluates Dummett's response to the problem of replying to the sceptic where Dummett distinguishes between the distinct projects of quelling the sceptic's doubts versus explaining to a non-sceptic why MPP is valid. The key to Dummett's response is, as we have seen, the distinction he makes between two sorts of arguments, which he baptised with the terms *suasive* and *explanatory*. But Boghossian argues that he does not understand what it would be to explain *why* a given law is true, short of adopting some (unfavourable) conventionalism about it. Instead;

The question that we need to be asking, I think, is rather this: Can we say that something is a real warrant for believing that *p* if it cannot be used to answer a sceptic about *p*? Is it criterial for my having a genuine warrant for believing that *p* that I be able to use it to persuade someone who doubts whether *p*?³⁰

This question travels with the thought that if we are genuinely justified in believing *p*, then we should be able to convince others as well—to 'see as we do', so to speak. Boghossian believes that it is this propensity, one that supposes the possibility of intersubjective and unanimous assent, that informs our theory of knowledge, and that lies behind the persistence of the feeling that we must satisfy the sceptic—that having a genuine warrant includes the sceptic's blessing. But Boghossian holds that this thought is false, and the following vein of reasoning which I have distilled from page 253 of his paper elucidates exactly why:

- P1. We deny that warrants do not exist for BLK;
- P2. We don't know what that warrant might consist in if not rule-circular inference.
- P3. The sceptic denies rule-circular accounts of BLK.

²⁹Paul Boghossian, *Analyticity Reconsidered*, *Noûs*, Vol. 30, No.3, 373 - 387.

³⁰Boghossian (2001), 252.

Therefore,

- C. There must be genuine warrants that do not carry any sway with the sceptic.

Although this argument expresses Boghossian’s view on the status of warrants for BLK, it is an argument that would not satisfy sceptical scrutiny. P1. is a premise that opposes the sceptical thesis, and essentially names that thesis as incorrect. It is only on the basis of acceptance of P1. that we may arrive at the stated conclusion. Boghossian presumes—he begs the question—concerning the existence of such a warrant via P1., and this presumption conspires to deliver an argument in favour of the possibility of justifying a warrant for BLK that is as effective as an *explanatory* argument, but not a *suasive* one. On this issue then, Dummett and Boghossian seem to arrive at the same result.

Once we accept the concessions an explanatory argument forces us to make, we may enjoy an entitlement to reason according to MPP because we are able to deliver the meanings of the logical constants via introduction and elimination rules that preserve truth. Truth preservation is the criterion, according to Boghossian, that demarcates operators with genuine meaning from those devoid of it. Finally, we do not need to satisfy the sceptic if we agree to allow just one premise citing the validity of MPP into an argument that justifies our warrant for applying it in inference; with that one assumption we may present a deductively valid justification that even the most stalwart sceptic would have difficulty denying.

2.6 Hale’s Non-Inferential Route

In his paper *Basic Logical Knowledge*,³¹ Hale argues that there must be *some* non-inferential knowledge underlying knowledge derived inferentially. This constitutes his greatest departure from the views of Dummett, Boghossian, and other inferentialists. It is possible to define conditions delivering a characterisation of what an inferential justification for a basic principle of logic consists in, but Hale does not see any way for these conditions to be met. Recalling from §1.2, those conditions are:

- That an argument to the effect that some basic rule of inference is sound does not use that rule, nor any other rule to corroborate that argument.
- That an argument not assume that some inference rule is sound in order to corroborate an argument that some other rule is sound.

Obstructing the non-inferential route from arriving at a plausible account of BLK is its lack of a satisfactory answer to the question: what is to constitute the fabric of non-inferential knowledge? Again, §1.2 discusses the problems associated with

³¹Hale (2002).

grounding this sort of knowledge in sense-perception, introspection, self-evidence, and other similar ideas. In response to this challenging question, Hale argues that we can answer it by directing enquiry towards two distinctly different projects:

Project A Explaining *how we can come to know* that basic rules such as *modus ponens* are sound.

Project B Explaining *why it is not possible intelligently* (i.e. clear-headedly and coherently) *to doubt* the soundness of basic rules...³²

Dismissing rule-circular attempts to arrive at BLK, Hale sees promise in a strategy that explains BLK as grounded in the conditions for understanding the logical constants. In order to understand the functioning of a particular operator, we must at the very least *accept* certain basic patterns of inference as criterial and constitutive of the meaning of that operator. Of course, this by itself does not seem sufficient to ground a claim that we have logical knowledge; Hale cites Paul Horwich with noting that acceptance of a rule is distinct from its truth or correctness. But Horwich's point only threatens project A.

Project B is the more modest of the two, and Hale believes that success in that project will permit advancement to the more ambitious project of explaining BLK identified in Project A. By accepting certain simple patterns of inference such as MPP as constitutive of understanding, Hale suggests we commit ourselves to the claim that it is impossible to countenance an (accepted) inference as unsound, and also commit ourselves to the claim that no counter-examples to that rule exist.

But acceptance as arbiter of our understanding of meaning fails to be a *sufficient* condition for that understanding, for it is possible to accept rules that are, unbeknownst to us, defective in some way. Acceptance can be at most constitutively necessary for understanding. A sceptic can oppose our present train of thought about the practice of acceptance; she might charge that if acceptance of certain basic patterns of inference is constitutive of (a.) understanding the meaning of a constant, and (b.) being sound and immune to counterexample (viz., impossible to doubt), then in the same way we accept MPP, so too must an operator such as tonk enjoy these qualities in a case where somebody erroneously *accepts* that operator as well. But we know that there is something wrong with the operator tonk, and therefore something amiss with the sceptic's reductio argument. There must be a codifiable difference between accepting MPP and accepting tonk.

The sceptic's charge is correct in that we should be able to entertain doubts about an operator such as tonk. It is also correct that if acceptance entails immunity to counterexample and doubt, then that immunity must extend to all rules of inference we accept: both to sound and unsound ones. Yet the sceptic is wrong to suppose that a sound rule of inference (such as MPP) should be doubtable through counterexample. He would only be able to make this claim if it were possible to find

³²Hale (2002), 289.

a counterexample, say, in the case of MPP, where the premises $\lceil A, A \rightarrow B \rceil$ are true but the conclusion B is not. But such an instance of counterexample does not exist. In the case of tonk, the abundance of counterexamples we are able to infer from it, as well as the proof above that shows we can infer $A \vdash B$ by tonk, amounts to the possibility of inferring any statement whatever, or of inferring contradictory statements. Tonk therefore has no fixed or coherent meaning because its rules are non-conservative and inconsistent. Hale draws two morals here, the first is that:

‘... whilst there is undoubtedly something wrong with the tonk rules, their fault is not most happily or illuminatingly characterized in terms of failure to preserve truth—neither can properly be convicted separately of this failing...’³³

What goes wrong, he suggests, might be that tonk fails a conservativeness constraint on rules of deduction. The second moral is that examples such as tonk do not corrupt the idea that it is impossible to infer counterexamples or inconsistencies using logical operators whose characterisations we accept as constitutive of their meanings. Hale shares this belief with Boghossian, and both would agree that conservativeness (according to the former), and truth-preservation (*per* the latter) distinguish the meaningful operators from the meaningless.

Hale maintains that the sceptic’s challenge, which ultimately says that we *can* doubt the correctness of basic logical rules, does not disturb the prospect of succeeding at project (B). Suppose we wish to verify that an operator satisfies a conservativeness requirement. Tonk fails, and we must demonstrate this in an argument. If tonk is not conservative, then we certainly cannot use tonk as a rule of inference in argument, for tonk is unsound. But we can reason *about* tonk, which is different from reasoning *using* it. And reasoning *about* does not require that we reason *according to* in an inference. Consider the following argument charging tonk of non-conservativeness:³⁴

The introduction and elimination rules for tonk are not conservative: take any pair of tonkless propositions, say, ‘ $7 + 5 = 12$ ’ and ‘*Hilary Putnam is a brain in a vat.*’

(P1.) $7 + 5 = 12$

(P2.) ‘ $7 + 5 = 12$ ’ tonk ‘*Hilary Putnam is a brain in a vat.*’ (Tonk-Introduction).

(C1.) ‘*Hilary Putnam is a brain in a vat.*’ (Tonk-Elimination).

Since the undischarged premise and final conclusion are tonkless, and the latter cannot be inferred from the former without using

³³Hale (2002), 293. By ‘neither’, Hale must undoubtedly be referring to the introduction and elimination rules that characterise tonk.

³⁴My adaption of Hale (2002), 295.

the tonk rules, those rules are non-conservative. ³⁵

The above argument does not *use* tonk to reach its conclusion that that operator is bunk, it merely reasons *about* the operator, and concludes something important: that it is non-conservative. This is a metatheoretical statement about our knowledge of tonk - specifically about its non-conservativeness, and by extension, its meaningfulness in the context of deduction. This argument about Putnam cannot be considered a counterexample to tonk because we do not give an argument applying tonk any clout: if a reasoner is to accept a given argument, she must accept the applied rule of inference, but she doesn't because tonk is inconsistent and therefore meaningless. Hale has shown that any vindication of a doubt about conservativeness must use rules other than those whose conservativeness is in question—rules whose reliability is assumed during the reasoning.

It does not, of course, follow from this that there must be *some* rules whose reliability must, and may properly, be assumed in *any* demonstration we can give of the conservativeness or non-conservativeness (more generally, soundness or unsoundness) of *any* (other) rules. It does not *follow*, but it is—or so I believe—*true*. (Hale's *emphases*.)³⁶

Hale proceeds by discussing this explicit proof of the non-conservativeness of tonk at length, arguing that in giving any such proof, he cannot help but use MPP, universal quantifier introduction and elimination. Part of the proof involves asserting that some particular inference is an instantiation of its general form (viz., '*p, so p tonk q*' is of the general form '*A, so A tonk B*'). In a page-long footnote³⁷ regarding this last inference he observes that despite the obviousness of the content it asserts, we should still ask after some sort of verification of its correctness. But we would fall to an infinite regress of the Tortoisean sort to ask for an inferential justification, and so the correctness we seek is, he suggests, the result of a *seeing*—a species of non-inferential intellectual recognition. Hale reminds us that this insight cannot provide us with a view into something more complex such as validity, but it does allow us to mediate recognition of some particular inference as exemplifying some general rule. If we are to evaluate a rule of inference for soundness, that rule must be both general and conditional: general in the sense that some conclusion of some specified general form may be drawn from premises of some specified general form; conditional in the sense that given premises of the specified form, a conclusion of the specified form may be drawn. There must exist a *minimal kit*

³⁵This argument can be interpreted as making the following point(s): '(P1.) For a rule to be conservative, the following qualities must obtain. . . , (P2.) Here is an argument that uses the operator in question. (P3.) The operator's rule(s) do not satisfy the conditions for conservativeness in this argument. (C) Therefore, This operator is bogus.' Framed correctly, the argument contained in this footnote could be turned into a deductive one that reasons according to (i.e. *uses*) MPP.

³⁶Hale (2002), 297.

³⁷Hale(2002), 298.

of inference rules that consists, according to Hale, of the above-mentioned MPP, universal quantifier introduction, and elimination.

Hale has therefore pointed out two ways in which it might be thought possible to doubt the soundness of an inference rule:

1. We might envisage a counterexample to it.
2. We might doubt its possession of other properties required for soundness such as conservativeness.

He has shown *per* 1., that because we initially accept the rule of inference we have no right to doubt it by counterexample. But acceptance is insufficient to guarantee correctness because if we were to hold the same attitude about an operator such as *tonk*, we would have reason not to doubt this rule either. But this clearly isn't the right way to view the situation. We do indeed need to be able to doubt the soundness of *tonk*, but not through supposing that counterexamples to it exist. Rather it should be *per* 2., through a demonstration showing how it lacks other properties such as conservativeness. The reasoning we employ to prove non-conservativeness avoids circularity by not *using* the rule of inference in question, and furthermore employs a minimal kit of inference rules, consisting of MPP for one, that is required to reason about the soundness of any inference rule. 2. is inapplicable to sound rules of inference, but perfectly applicable to unsound ones. If a minimal kit exists, then it shall enable us to articulate doubts about unsound rules while preserving the charge that the sound ones are immune to doubt.

To complete this survey of Hale's views, we now address the objective mentioned in project A above, the explanation of *how we can come to know* that basic rules are sound. We recall Hale's suggestion that we might build upon the success realised in project B in order to realise success in A. A legitimate concern exists in this regard about the prospect of mitigating the connection between these two projects without the use of those items contained in the *minimal kit*, for in utilising the kit we shall have employed circular reasoning to achieve the goal, and Hale has avoided circularity at all costs thus far. The answer to this dilemma Hale suggests, is that rather than assuming that some theory renders project B successful, and then, armed with the resultant knowledge that the operators of the minimal kit are impossible to doubt, we proceed to leverage this knowledge by using those operators to argue in defense of a positive conclusion in support of project A. Instead we here obviate the need to express their indubitability in a premise.

Hale is, unsurprisingly, less comfortable with this suggestion but speculates,

Perhaps it can be maintained that there is, simply, no inferential gap to be traversed—that is, that knowledge can, much as Descartes thought, consist in believing truly something which it is impossible rationally to doubt.³⁸

³⁸Hale (2002), 302.

It is interesting to note that treating basic rules as a foundationalist agrees with the first moral mentioned by Engel in §2.3, where he draws the distinction between rules and premises, and suggests that we might consider prohibiting the former from being expressed assertorically in a deductive inference. Without them explicitly expressed, there doesn't seem to be much else to do other than accept them implicitly, which would suggest that we have license to reason according to them without justification—that they are *default reasonable*.

If this is the case, we should be able to countenance basic logical knowledge as non-inferential, different from the problematic sorts of non-inferential knowledge discussed in §1.2 insofar as it is, in this way, general and necessary, rather than subjective and contingent in the way memories, intuitions, and beliefs can be.

2.7 Summary of Findings in this Chapter

In §1.6 I enumerated a series of questions and problems that an account of our entitlement to reason according to BLK must address. In this section I shall assess whether or not those questions have been answered, or rather, to what degree they have been answered by our survey of philosophical views presented in this chapter. Recalling from Chapter 1, the principal problems are described thus:

- (i.) Carroll shows us that paradox ensues from an inferential argument offered to justify the correctness of a rule. In the case of MPP, such an argument makes use of that rule to arrive at its conclusion. A subsequent argument is then required to defend our warrant to reason according to the rule, and this repeats itself *ad infinitum*.
- (ii.) There are problems associated with all manner of non-inferential support for the basic logical constants: such support is subjective and arbitrary rather than universal and necessary as we need it to be.

Also in §1.6 it was stated that any treatment of these problems would have to address the following questions and issues:

- (a.) That inferentialism and non-inferentialism constitute the two strategies we have at our disposal in order to arrive at knowledge. In the scope of this thesis, that knowledge is about the correctness of basic logical rules.
- (b.) Inferentialism *seems* like a more rigorous strategy for characterising the logical constants because it gives us reasons in the form of valid arguments that conclude that a logical constant is correct. Non-inferentialism is only as rigorous as the intuitions it codifies, yet intuition about logical constants seems like the very sort of knowledge we seek to explain, and therefore non-inferentialism has an air of circularity to it.

- (c.) That a strategy for demonstrating the correctness of a rule exists and does not succumb to regress or vicious circularity. We extrapolate from this that the Sceptic's position on this matter is to be rejected.

Upon completion of our survey, it is obvious that (b.) is wrong-headed. It seems as though (a.) is strengthened, and we have also improved the likelihood that (c.) is correct, with qualification. As for (b.), both reason-based and non-inferential approaches to justification can be interpreted as yielding circular results: Carroll showed, as we saw, that given an inferential argument about the correctness of MPP, an infinite regress of justifications followed. If instead we hold that justification terminates (by accepting rule-circular, or pragmatically circular sequences of reasoning), then the terminating inference will be the circular supposition that the rule in question is correct. A non-inferential justification of rules can be equally interpreted as circular, for such an explanation is grounded in, as we saw, intuition, belief, definition, etc. The correctness of a logical constant under a non-inferential characterisation is itself an intuition - a hunch we seek to vindicate, and so in the case of the basic logical constants, a non-inferential justification also appeals to the very principles sought to be justified. Therefore the illusion that presented itself to me in Chapter 1 suggesting that an inferential justification is more rigorous than a non-inferential one because it offers *reasons* is just not right because the reasons are couched in circular justifications, in the same way as an intuition.

I do not think that (a.) can be proven as a cold hard fact just yet, if ever at all, but intuitively it does seem as though these are the only two strategies for arriving at knowledge. I exclude the thesis of scepticism from these strategies because it rejects the possibility of justifying knowledge. I think many of us would like to admit or begin with the initial assumption in epistemological enquiry that knowledge exists *and* that much of it—if not all of it—is capturable via one of our justificatory strategies. Regardless of the domain and whether we call the strategy inferentialist or non-inferentialist, proof-theoretic or model-theoretic, or some other account that amounts to the same fundamental principles, I can think of no other positive accounts of justification than the ones described by inferentialism and non-inferentialism.

The qualification that needs to be made regarding (c.) is exactly addressed by Dummett's suasive/explanatory distinction. He is correct about suasive justifications; there is no convincing a stalwart sceptic by argument, because that argument will ultimately use the rule of inference under question in a pragmatically circular way. Nor will the sceptic accept stipulations, decrees, definitions, or others discussed throughout because no manner of verification for their correctness will satisfy her. Therefore, so long as our purpose for demonstrating correctness is explanatory in nature, then we should be allowed to admit pragmatically circular justifications in a defense of the correctness of a rule of inference. But Hale's point needs to be considered here. For if we are going to admit a pragmatically circular justification, then we must, for at least a fundamental collection of rules, admit of their correctness. If we do, and assume their correctness *ab initio*—which is es-

essentially what the admission of pragmatically circular arguments consists in, then perhaps we can countenance them as default-justified. A viable non-inferential defense of the correctness of basic rules may then be launched from these assumptions. I will have more to say about this in Chapter 3.

How have the proposed solutions to problems (i.) and (ii.) unfolded? In the case of (i.) we are confronted with the difficulties suggested by both regress and circularity. Someone who addresses these would have to present an account that circumvents them. Focus on *meaning* has been thought to be an effective strategy: if we can fix the meaning of an operator, then perhaps we shall be able to stave off regress and circularity. Dummett's and Boghossian's accounts may both be interpreted as being grounded in meaning by virtue of developing a theory couched in analytic validity, although their strategies explaining how to arrive at meaning differ significantly.

We have yet to discuss problem (ii.) pertaining to non-inferentialism, mentioned at the beginning of this section. Hale's piece is the one that makes a significant contribution to the progress of enquiry into that problem. To avoid repetition, I shall save discussion of Hale for the next Chapter, for it is his contribution that figures chiefly in my concluding findings.

Chapter 3

The Primacy of Non-Inferential Justification

Obvious questions to ask at this point, subsequent to my enquiry into the problem of justifying basic logical rules and thereby arriving at BLK, is the question that asks, *which justificatory strategy?* Should inferentialism or non-inferentialism be our epistemic attitude when justifying warrant? Which is better capable of delivering *the correct* characterisation of BLK, if such a thing exists? In this concluding chapter, I answer these questions with a defense of non-inferentialist approaches to justifying rules of inference. I shall argue that non-inferential knowledge of basic logic should be distinguished from other forms of non-inferential knowledge, and that while non-inferential knowledge in general is typically dubious, non-inferential knowledge of basic logic should be accepted. The thesis ends with my most important argument in favour of non-inferentialism: that this sort of knowledge is more deeply grounded in a chain of reasoning than knowledge arrived at inferentially. In light of this, non-inferential knowledge of basic logic remains possible, while inferential knowledge of basic logic is not.

3.1 Distinguishing Types of Non-Inferential Knowledge

Let us take stock of the kinds of non-inferential knowledge previously discussed: beliefs, memories, intuitions, stipulations and a few others fall into one category. They are basic in the sense of being the starting point for an item of knowledge—and in the cases under consideration, the items didn't exist until thought-up or recalled by someone. We generally regard this sort of knowledge as dubious because of its subjective and oftentimes non-transmittable¹ nature. If an item of knowledge depends on a single individual in order to be completely understood, or if it is

¹Examples of non-transmittable knowledge might be knowledge of one's own pain or pleasure. A somewhat more tangible example, which I credit to Tim Kenyon, is a person's knowledge

simply non-transmittable, then the item of knowledge does not rest on the firm foundations we aspire to lay down for logic, nor can they serve as foundations themselves.

But non-inferential knowledge *of logic*—the second category—fares differently for two reasons. First, I have demonstrated in this thesis how we may countenance our knowledge of basic logical rules as universal, absolute, general and necessary. Furthermore, knowledge that they are conservative and in harmony lends strength to our knowledge of their correctness. We cannot, and do not, unanimously assign these logical properties to the first sort of non-inferential knowledge. The second reason that non-inferential knowledge of logic is exceptional is because of its indispensability to deduction and reasoning in general. This is one of the principal messages in Hale’s paper. In this chapter I take the indispensability thesis further. I demonstrate, by extrapolating from Hale’s paper, exactly how and where in the reasoning process we use basic knowledge, why that knowledge should be countenanced as logical knowledge, and why this species of non-inferential knowledge of logic is distinct from other non-inferential knowledge. We should conclude with Hale, that non-inferential knowledge of logic is essential to the reasoning process. I shall then expand on this in the final section by arguing that not only is non-inferential knowledge essential, but non-inferentialism is also the *only* method available to us suitable for knowing the correctness of basic rules of logic.

3.2 Indispensability²

We cannot, as we have seen, simply suppose that an operator is impossible to doubt—it should be demonstrated in an argument. In §2.6, I presented my adapted argument from Hale which showed, via a chain of reasoning, that tonk is non-conservative. That argument is not circular if we acknowledge the difference between an argument that *uses* or *reasons according to* an operator versus one that *reasons about* that operator. When we reason about an operator, we do not use it in the same way Achilles proposes we do with his premise (*C*): we may therefore conclude something about the operator without reasoning with it viciously. The following is the chain of reasoning that Hale uses to demonstrate the non-conservativeness of tonk. It is a more explicit version of the argument I presented in §2.6, and it is important and useful to present here because it highlights the application of basic rules in their most primitive forms:

of their ‘lost sock circle’, viz., the collection of socks a person has lost in their lifetime. The circle has a distinct and detailed status to its owner, but only represents a vaguely defined and undifferentiated collection to anyone else.

²In this section I introduce the abbreviations UQI, UQE, and UQI&E, UQIorE which shall stand *mutatis mutandis* for universal quantification introduction and/or elimination.

- (1) Tonk-introduction allows you to make inferences of the form ‘ A , so A tonk B ’.
- so:
 - (2) If the inference: ‘ p so p tonk q ’ is of the form ‘ A , so A tonk B ’, then tonk-introduction allows you to make it.
 - (3) The inference: ‘ p , so p tonk q ’ is of the form ‘ A , so A tonk B ’.
- Hence:
 - (4) Tonk-introduction allows you to infer ‘ p tonk q ’ from ‘ p ’.
- Further:
 - (5) Tonk-elimination allows you to make any inference of the form ‘ A tonk B , so B ’.
- So:
 - (6) If the inference: ‘ p tonk q , so q ’ is of the form ‘ A tonk B , so B ’, then tonk-elimination allows you to make it.
 - (7) The inference: ‘ p tonk q , so q ’ is of the form ‘ A tonk B , so B ’.
- Hence:
 - (8) Tonk-elimination allows you to infer ‘ q ’ from ‘ p tonk q ’.
- Hence:
 - (9) Tonk-introduction and tonk-elimination allow you to infer ‘ q ’ from ‘ p ’.
- Hence:
 - (10) The tonk rules together allow you to derive any conclusion from any premise.³

Among the observations we might make about this chain of reasoning, of note for our purposes is the fact that (2) and (6) are the result of an application of universal quantifier elimination such that we recognise a particular statement to be an instance of its universal form. Furthermore, MPP is used to derive the conclusions found in (4) and (8) from the conditionals expressed in (2) and (6), *via the minor premises expressed in (3) and (7)*.

We have already discussed how Hale seeks to avoid inferential reasoning that uses basic logical operators because of an inability to offer-up a satisfactory inferential explanation of their correctness. Another aspect of Hale’s non-inferentialism can be clearly understood by examining his view about what is actually going on in premises (3) and (7); despite the obviousness of these two premises, we should still enquire into their justification, that is, a justification of our entitlement to mediate recognition of a particular statement as being an instance of some general rule. That would essentially constitute a justification of UQI&E. But what sort of premises should we put forth in such a defense? There do not seem to be any. Additionally problematic is the fact that an inferential justification would lead to the usual regress or circularity. In a footnote Hale therefore argues:

Our recognition of their correctness must, it seems, be a non-inferential matter. That is, the right answer is just the one we should naturally give, viz. that we can just *see* that the particular inference is of the displayed general form... what is involved here is a species of non-inferential intellectual recognition—which we may call rational insight, and which has an indispensable role to play whenever we operate with rules of inference.⁴

⁴*ibid.* Hale’s *emphases*.

The demonstration contained in (1) - (10), as well as this last quote reveal the significant aspects of Hale's epistemology that I wish to highlight: correctness of a small number of items of logical knowledge is a non-inferential matter. It is *natural* to make a few of the basic inferences that we do, and we just 'see' that these inferences are correct. This amounts to a species of 'non-inferential intellectual recognition'. As I mentioned in §2.6, Hale does not think that the recognition of something more complex such as validity may be realised in this way, yet simple inferences such as those inferred by UQIorE may be countenanced as the products of *rational insight*.

If we subscribe to this point of view thus far, then it is not a large leap to conclude that the sort of inference pattern just discussed plays an indispensable role in an argument for the non-correctness of tonk. It does not prove that these basic inferences *must* be used in reasoning, but I cannot think of a non-circular way of expressing tonk's non-correctness without using them, and that takes us a good way towards the qualification of being indispensable.

Now I must answer my own question: how does the above figure into a justification for MPP? It may be justified in the same manner as UQIorE. §2.6 discussed the stipulation that a sound rule of inference will be general in the sense that a conclusion of some specified general form may be drawn from premises of some specified form. A sound rule shall also be conditional such that given premises of a particular form, we may draw conclusions of a corresponding form. If we are going to analyse the pedigree of an argument to its most primitive level as demonstrated in (1) - (10), then Hale points out that we are going to have to reason from explicit formulations of those rules (exemplified in (1) - (10) by lines (1) and (5)). Mediating particular instances of UQI&E or MPP from their explicit formulations will, once again, be an indispensable step in the reasoning process. MPP and UQI&E therefore constitute a *minimal kit* of inference rules required to reason about the soundness of rules of inference.

And how does (1) - (10) and the preceding vein of reasoning show that the items in the minimal kit are impossible to doubt? Well, we showed how they are impossible to doubt by counterexample because they were the rules that we accepted. Furthermore, we cannot accept a bogus rule of inference if we can demonstrate its inconsistency and/or non-conservativeness. With this achieved, we have a viable way to distinguish the rules of inference that yield sound conclusions from those that don't. The former shall be the correct rules. In consideration of an argument presented to refute the correctness of the rules contained in the minimal kit, we shall have had to *use* those rules to argue for their non-correctness, and this is something that we are not permitted to do. It therefore seems as though a viable defense of the indispensability of MPP and UQI&E has been presented.

The final task in this project involves linking indispensability with correctness. If Hale is right, it would be the most significant achievement of his paper. How do we move from the former to the latter? This question motivates Hale's thesis about the possibility of leveraging the findings of Project B to successfully achieve

the goal set out in Project A. But it seems as though we would have to present an argument in order to support this claim, an argument something similar to the following: ‘because certain basic rules are immune to doubt or are indispensable, claims about their correctness will be true’. This argument cannot be supported, as we have seen, without applying some sort of pragmatically-circular reasoning. To avoid this problem, Hale suggests a solution that has already been aptly described in this thesis by the first moral presented in §1.5, the one that says that rules are not premises. Something that is impossible to doubt need not be taken as a premise. Knowledge of the correctness of simple logical constants is impossible to doubt, and instead of proffering this fact as a premise in an argument, we take a non-inferential stance on this sort of knowledge by claiming that there is no inferential gap to be traversed. I reiterate one of Hale’s final comments in his paper to conclude my summary of his arguments:

Perhaps it can be maintained that there is, simply, no inferential gap to be traversed—that is, that knowledge can, much as Descartes thought, consist in believing truly something which it is impossible rationally to doubt.⁵

I have shown that non-inferential knowledge can be bifurcated into two distinct types: the logical, and the non-logical. I also explained why knowledge of non-inferential, non-logical statements is dubious: because none of it is universal, necessary, and completely general. Basic logical rules do have these qualities however. And when we tie this into the thesis that they are indispensable in reasoning, we come away with a viable non-inferential grounding of basic logical rules.

3.3 The Primacy of Non-Inferential Knowledge of Logical Laws

In a chain of reasoning, if something comes first then anything that follows from it cannot be first as well. In this sense, whatever comes first is basic. The strength of my thesis—that non-inferential knowledge of logic is the only sort we can have about the basic rules of logic—comes from (a.) the weakness I find in the argument for inferentialism with respect to its inability to deliver anything but circular or regressive results, and (b.) the strength of Hale’s account.

Boghossian and Dummett share an endorsement for the suasive versus explanatory distinction and both believe that the latter notion advances their cause. We recall from Chapter 2 that suasive justifications *convince* a doubtful explainee, in this case of correctness. Explanatory justifications demonstrate correctness to an explainee who already endorses that correctness, but simply wants to see the justification. The doubtful explainee, like the Tortoise, refuses to accept circular or

⁵Hale (2002), 302.

regressive demonstrations. Both Dummett and Boghossian agree that there is no persuading this sort of sceptical attitude. But explanatory justifications facilitate inferential justifications. Dummett says of the non-suasive explainee:

He does not need to be persuaded of the truth of the conclusion; what he is seeking is an *explanation* of its being true. An explanation frequently takes the form of a deductive argument, in which the conclusion is the fact to be explained. There is therefore no uncertainty about the conclusion, which we already know; and often the best reason for believing the premisses is that they offer an explanation for the conclusion's being true.⁶

Dummett goes on to argue that an explanatory justification, which is a pragmatically circular justification, has some value. In fact, he says that it's the best we can do, or, to use other words, it's the most basic sort of justification for basic logic that is possible. Whether he believes an explanatory and pragmatic justification is the most basic of all justifications or merely the most basic sort of inferential justification Dummett does not say. But there is little if anything in his paper *Circularity, Consistency, and Harmony* that endorses non-inferential justification. So I take it that Dummett's position on justification for basic knowledge is anchored by his explanatory stance about justification described in this most recent quote. Boghossian's story leads him to a near-identical conclusion:

To put this point another way: we must recognize a distinction between two different sorts of reason—suasive and *non-suasive reasons*. And we have to reconcile ourselves to the fact that in certain areas of knowledge, logic featuring prominently among them, our warrant can be at most non-suasive, powerless to quell sceptical doubts.⁷

Boghossian's and Dummett's stories are right-on to somebody who accepts (pragmatically) circular reasoning. One reason for this might be because pragmatically circular reasoning that uses the basic rules endorsed by classical logic yields valid results. But I do not believe that circular reasoning has any place in a justification of basic logic because the justification falls short of rigour. Rigour requires that proof-checking can be effected through application of rules of inference or axioms. But the justification sought in this case is one for the very rule of inference accepted. There are numerous paradoxes associated with the quality of being self-referential, and a pragmatically circular justification of a basic rule of logic is just that. My conviction is that this denies the reasoner the right to accept pragmatic circularities as justification for basic laws of logic. The moment we prohibit circularity, Boghossian's and Dummett's arguments fizzle.

⁶Dummett (1994), 202, his *emphasis*.

⁷Boghossian, 253, his *emphasis*.

I shall now describe the flaw in their reasoning, and I believe what follows here to be the most important contribution this thesis makes to enquiry into basic logical knowledge. The simplest way to explain the flaw is that the supposition that MPP is correct, held by an explainee under explanatory—but not suasive—circumstances is tantamount to non-inferential knowledge of correctness. This is why: if a reasoner is to ask for an inferential justification of the correctness of an item of knowledge, an item about which she is already convinced of correctness, then her conviction is prior to any argument that demonstrates that correctness. Believing or knowing that something is correct prior to a demonstration of the correctness is non-inferential knowledge. Knowledge arrived at by reason is, as we know, inferential knowledge, but knowledge we possess prior to reasoning must be non-inferential knowledge. Non-inferential knowledge about basic logical rules is therefore basic because it lies prior to the inferential knowledge that depends on it.

It is possible to establish basic knowledge as correct in a non-inferential way without depending on any inferential steps or attitude in the process. Yet it is impossible to achieve the opposite, that is, to justify basic knowledge in inferential fashion without the application of some non-inferential knowledge. For consider: Hale showed us how we may countenance the basic rules as correct by showing that they are impossible to doubt and subsequently that their correctness need not be enumerated as premises in reasoning. No inferential step figures here. If we examine Dummett's inferential strategy on the other hand, explanatory rather than suasively circular explanations are the best outcomes we can hope for. Furthermore, the explanatory account, being pragmatically circular, presupposes the correctness of the rule.

For these reasons, an inferential determination of the correctness of basic rules of inference cannot be basic because that determination depends on non-inferential knowledge. Hale has described non-inferential knowledge as a 'minimal kit' of default-justified rules. This kit, which we have shown to be indispensable to deduction is therefore the only sort of logical knowledge we may countenance as basic with any confidence.

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