

**Optimal Power Allocation for a
Successive Refinable Source with
Multiple Descriptions over a Fading
Relay Channel Using
Broadcast/Multicast Strategies**

by

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Abstract

In a wireless fading relay system with multicast/broadcast transmission, one of the most crucial challenges is the optimization of a transmission rate under multi-user channel diversity. Previously reported solutions for mitigating the vicious effect due to multi-user channel diversity have been mainly based on superposition coded multicast [1, 2, 3], where an optimal power allocation to each layer of modulated signals is determined. The studies in [4, 5, 6, 7, 8, 9, 10] investigated a harmonic interplay between the successively refinable (SR) content source and a layered modulation via superposition coding (SPC) over the multicast/broadcast channels. By jointly considering the successive refinement characteristic at the source and the dependency of the layered modulation at the channel, a graceful flexibility can be achieved on a group of users with different channel realizations. Here most of the receivers are supposed to obtain the base quality layer information modulated in a lower rate, while the receivers with better channel realizations will obtain more information by refining the base quality layer information using the enhancement quality layer information. In particular, the optimal power allocation for a SR source over a fading relay channel using broadcast/multicast strategy can be determined such that the minimum distortion of total received information is produced.

However, a quality layer of data in a successively refined source may not be decodable if there is any loss of channel codewords, even if the corresponding long-term channel realization is sufficient for decoding. To overcome this problem, the studies in [11] introduced a framework of coded video multicast, where multiple description coding (MDC) is applied to an SR content source and is further mapped into a layered modulation via SPC at the channel. Up till now, there has not been a rigorous proof provided on the benefit of manipulating the two coding techniques, (i.e. MDC and SPC), nor has any systematic optimization approach been developed for quantifying the parameter selection.

Cooperative relaying in wireless networks has recently received much atten-

tion [12, 13]. Because the received signal can be severely degraded due to fading in wireless communications, time, frequency and spatial diversity techniques are introduced to overcome fading. Spatial diversity is typically envisioned as having multiple transmit and/or receive antennas. Cooperation can be used here to provide higher rates and results in a more robust system. Recently proposed cooperation schemes, which take into account the practical constraint that the relay cannot transmit and receive at the same time, include amplify-forward(AF), decode-forward(DF), and compress-forward(CF).

In this study, in a fading relay scenario, a proposed framework is investigated to tackle the task of layered power allocation, where an in-depth study is conducted on achieving an optimal power allocation in SPC, such that the information distortion perceived at the users can be minimized. This thesis provides a comprehensive formulation on the information distortion at the receivers and a suite of solution approaches for the developed optimization problem by jointly considering MDC and SPC parameter selection over the fading relay channel.

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List of Abbreviations

16QAM	16 Quadrature Amplitude Modulation
BPSK	Binary Phase-Shift Keying
BS	Base Station
CAPEX	Capital Expenditures
CSI	Channel State Information
GoFs	Group of Frames
IDMA	Interleave Division Multiple Access
IPTV	Internet Protocol Television
KKT	Karush-Kuhn-Tucker
MDC	Multiple Description Coding
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
OPEX	Operation Expense
PMF	Probability Mass Function
RD	Rate-Distortion
RS	Reed-Solomon (code)
SIC	Signal-Interference Cancellation
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SNR	Signal-to-Noise Ratio
SPC	Superposition Coding
SR	Successive Refinement (code)
SS	Subscriber Station

Chapter 1

Introduction

One of the most critical challenges in a wireless fading relay system with multi-cast/broadcast transmission is the determination of an optimal transmission rate under multi-user channel diversity. Previously reported solutions for mitigating the vicious effect due to multi-user channel diversity have been mainly based on superposition coded multicast [1, 2, 3], where an optimal power allocation to each layer of modulated signals is determined. The studies in [4, 5, 6, 7, 8, 9, 10] investigated a harmonic interplay between the successively refinable (SR) content source and a layered modulation via superposition coding (SPC) over the multicast/broadcast channels. By jointly considering the successive refinement characteristic at the source and the dependency of the layered modulation at the channel, a graceful flexibility can be achieved on a group of users with different channel realizations. Here most of the receivers are supposed to obtain the base quality layer information modulated in a lower rate, while the receivers with better channel realizations will obtain more information by refining the base quality layer information using the enhancement quality layer information. In particular, the optimal power allocation for a SR source over a fading relay channel using broadcast/multicast strategy can be determined such that the minimum distortion of total received information is produced.

According to the source-channel separation theorem [16], it is optimal to first

compress the source and incur the associated distortion at a rate equal to the channel capacity, then send the compressed information through the channel at the capacity with asymptotically small error. However, when delay constraints stipulate that the receiver decodes within a single realization of a slowly fading channel, without channel state information (CSI) at the transmitter, the channel becomes non-ergodic and source-channel separation is not necessarily optimal. In this case, it is possible to reduce the end-to-end distortion of the reconstructed source by jointly optimizing the source-coding rate and the transmit power allocation based on the characteristics of the source and the channel. In particular, it is a good approach using the layered broadcast coding approach with successive refinement in the transmission of a Gaussian source over a slowly fading channel, which has a finite number of discrete fading states, in the absence of CSI at the transmitter. The source is coded in layers, with each layer successively refining the description in the previous one. The transmitter simultaneously transmits the codewords of all layers to the receiver by superimposing them with an appropriate power allocation. The receiver successfully decodes the layers supported by the channel realization, and combines the descriptions in the decoded layers to reconstruct the source up to a distortion. In this thesis, minimizing the expected distortion of the reconstructed source by optimally allocating the transmit power among the layers of codewords is conducted. The system model is applicable to communication systems with real-time traffic where it is difficult for the transmitter to learn the channel condition. For example, in a satellite voice system, it is desirable to consider the efficient transmission of the voice streams over uncertain channels that minimize the end-to-end distortion.

The broadcast/multicast strategy is proposed in [1] to characterize the set of achievable rates when the channel state is unknown at the transmitter. In the case of a Gaussian channel under Rayleigh fading, [2] describes the layered broadcast coding approach, and derives the optimal power allocation that maximizes expected capacity when the channel has a single-antenna transmitter and receiver. The layered broadcast approach is extended to multiple-antenna channels and the corre-

sponding achievable rates are presented in [3]. In the transmission of a Gaussian source over a Gaussian channel, uncoded transmission is optimal [17] in the special case when the source bandwidth equals the channel bandwidth [18]. For other bandwidth ratios, hybrid digital-analog joint source-channel transmission schemes are studied in [19, 20, 21]; in these works, the codes are designed to be optimal at a target SNR but degrade gracefully should the realized SNR deviate from the target. In particular, [20] conjectures that no code is simultaneously optimal at different SNRs when the source and channel bandwidths are not equal. In this paper, the code considered is not targeted for a specific fading state; we minimize the expected distortion over the fading distribution of the channel.

In [22], the minimum distortion is investigated in the transmission of a source over two independently fading channels in terms of the distortion exponent, which is defined as the exponential decay rate of the expected distortion in the high SNR regime. Upper bounds on the distortion exponent and achievable joint source-channel schemes are presented in [23] for a single-antenna quasi-static Rayleigh fading channel, and later in [24], [25] for multiple-antenna channels. One of the proposed schemes in [24], layered source coding with progressive transmission, is analyzed in terms of expected distortion for a finite number of layers at a finite SNR in [26].

The results in [23], [24] show that the broadcast/multicast strategy with layered source coding under an appropriate power allocation scheme is optimal for multiple-input/single-output (MISO) and single-input/multiple-output (SIMO) systems in terms of the distortion exponent. Numerical optimization of the power allocation with constant rate among the layers is examined in [27], while [28] considers the optimization of power and rate allocation and presents approximate solutions in the high SNR regime.

1.1 Research Motivation, Objectives and Contributions

In general, the aforementioned SR-SPC based approaches in [4, 5, 6, 7, 8, 9, 10] focused on a countermeasure of slow fading diversity of multiple users, in which the average channel realization at each receiver is assumed to stay at a level for the long run. The evaluation of distortion in these works is based on the expected number of layers that can be decoded under the constant channel realizations, where no loss is assumed in the layers that are decoded. However, the truth is that a quality layer of data in a successively refined source may not be decodable if there is any loss of the channel codeword even if the corresponding long-term channel realization is sufficient. Such a loss of the codeword could be easily due to the short-term Rayleigh fading effect, which leads to failure of achieving the required instantaneous channel realization from time to time. This situation is particularly an issue when user mobility is considered. Note that retransmission of lost information/packets is only an effective countermeasure in the case of point-to-point transmission instead of multicast/broadcast.

To overcome the aforementioned problem, the studies in [11, 15] introduced a framework of coded video multicast/broadcast, where multiple description coding (MDC) is applied to a SR content source and is further mapped into layered modulation via SPC at the channel. Each source layer is encoded into multiple descriptions based on Reed-Solomon (RS) codes for protection such that a source layer can resist some extent of loss due to the fast fading channel. Then, each source layer belonging to a common description is modulated differently but superimposed as a broadcast signal via SPC at the channel. The receiver decodes the layers supported by their channel realizations, recovers the source layers by the corresponding number of multiple descriptions, and finally reconstructs the source. A layer can be fully recovered if the number of received descriptions of the layer is larger than a pre-defined threshold.

By additionally employing MDC on the data of each layer in the content source, the coded video multicast framework can significantly improve the robustness of transmitting each layer via a suite of parameter selection under MDC. However, the studies of [11], despite conducting extensive simulations to verify the proposed framework and approaches, did not provide a rigorous proof and formulation on the benefit of distortion by manipulating the joint source-channel coding techniques for different layers, nor have they developed any systematic closed-form optimization approach for quantifying the power allocation.

Motivated by the above observations, this thesis aims to provide a systematic approach for the optimization of overall information distortion over a fading relay channel using a broadcast/multicast strategy with SR information through the interplay of MDC at the content source and SPC at the channel. The contributions can be categorized as follows: 1) formulate the information distortion under the interplay of MDC at an SR content source and SPC at the channel; 2) provide an efficient solution approach for the formulated optimization problem for minimizing the overall distortion by determining the optimal redundancy at the source and power allocations at the channel.

1.2 Thesis Outline

The rest of the paper is organized as follows. The background concepts as well as a literature survey of scheduling schemes are presented in Chapter 2. In Chapter 3, the system model and problem definition are provided, where the coded source multicast/broadcast framework for the MDC and SPC interplay are reviewed. In Chapter 4, an optimization problem is formulated and its optimality is proved; the dual problem of minimizing the power consumption subject to an expected distortion constraint is also considered. Numerical results are presented to prove the benefits of the proposed cross-layer framework in Chapter 5. Finally, conclusions are drawn and future work is discussed in Chapter 6.

Chapter 2

Background and Related Work

Multiple description codes (MDC) address the problem of unreliable channels by means of independent descriptions, while layered codes address the problems of heterogeneous client bandwidths and dynamic network congestion by means of sequences of layers. It has been proposed for use in packet audio and video transmission systems as a means of combatting both packet loss and component failure, in a variety of application scenarios. Many methods of multiple description coding have been developed over the years [29, 30]. Specifically, several problems can be recognized retrospectively as special cases of MDC. One of them is known as successive refinement (SR) coding [29].

Superposition coding (SPC) is a physical layer technique that allows a transmitter to simultaneously send independent packets to multiple receivers. For a long time, it was introduced to increase the overall user capacity of a wireless communication system by exploiting the spatial or temporal power disparities perceived by multiple users for common broadcast signals [1, 14].

Since its proposition nearly 40 years ago in [31], the relay channel has been investigated by a number of scholars. Transmission over wireless channels suffers from random fluctuations in signal level known as fading and from co-channel interference. Diversity is introduced as a powerful technique to mitigate fading and improve robustness to interference [32, 33, 34].

2.1 Multiple Description Coding(MDC)

The MD coding problem was not created as a pure information-theoretical puzzle. As the author stated in [29], Multiple Description coding has come full circle from explicit practical motivation to theoretical novelty and back to engineering application. MD coding was invented at Bell Laboratories during the 1970s in connection with communicating speech over the telephone network. At that time, though the telephone network enjoyed good reliability, outages of transmission were inevitable, mainly due to device failures, routine maintenance or upgrades. Rather than diverting calls to standby transmission links in the case of transmission outage, it was preferable to split the information from a single call onto two separate links or paths. Some early attempts at channel splitting are summarized in [29].

The channel splitting idea inspires the following question: "If an information source is described with two separate descriptions, what are the concurrent limitations on qualities of these descriptions taken separately and jointly?" [29]. This question eventually came to be known as the MD coding problem.

Before presenting the problem formulation of MD coding, we shall first introduce the basic definitions of rate-distortion theory and state Shannons rate-distortion theorem.

2.1.1 Shannons Rate-Distortion Theory

When information is transferred over a channel at a rate above the channels capacity, distortion in the recovery of the information is inevitable. The branch of information theory devoted to characterizing the relationship between achievable distortion and required rate is called the rate-distortion theory. The most important result in the rate-distortion theory is perhaps Shannons rate-distortion theorem [35], which is restated in this subsection.

We assume that $X_i, i = 1, 2, \dots$ is a sequence of i.i.d. discrete random variables drawn according to a common probability mass function $p(x), x \in \mathcal{X}$. We are given

a reconstruction space \hat{X} , together with an associated distortion measure

$$d : \mathcal{X} \times \hat{\mathcal{X}} \rightarrow \mathbb{R} \quad (2.1)$$

A description of $\underline{x} \in \hat{\mathcal{X}}^n = \hat{\mathcal{X}} \times \cdots \times \hat{\mathcal{X}}$ is a map $i : \hat{\mathcal{X}}^n \rightarrow \{1, \dots, 2^{nR}\}$, where R is the rate of description in bits per source symbol of \underline{x} . A reconstruction of \underline{x} is a map $\hat{x} : \{1, \dots, 2^{nR}\} \rightarrow \hat{\mathcal{X}}^n$. The distortion incurred through this pair of description and reconstruction is defined by

$$d^n = E \left[\frac{1}{n} \sum_{k=1}^n d(X_k, \hat{x}_k(i(\underline{X}))) \right] \quad (2.2)$$

The distortion d is said to be achievable with rate R for the source sequence $\{X_i\}_{i=1}^n$ if for $n = 1, 2, \dots$, there exists a sequence of rate R descriptions $i : \hat{\mathcal{X}}^n \rightarrow \{1, \dots, 2^{nR}\}$ and reconstructions $\hat{x} : \{1, \dots, 2^{nR}\} \rightarrow \hat{\mathcal{X}}^n$ such that $d^n \leq d$, for all n sufficiently large.

Rate-Distortion Function The rate-distortion function $R(d)$ is the infimum of all rates R achieving distortion d on a given stochastic process $\{X_i\}_{i=1}^\infty$.

Theorem 1 (Shannons Rate-Distortion Theorem) [35] *If $\{X_i\}_{i=1}^\infty$ are i.i.d. discrete finite alphabet random variables with probability mass function $p(x)$, then*

$$R(d) = \inf_{\mathcal{P}(d)} I(X; \hat{X}) \quad (2.3)$$

where

$$\mathcal{P}(d) = \{p(\hat{x}|x) : \sum_{x, \hat{x}} p(x)p(\hat{x}|x)d(x, \hat{x}) \leq d\} \quad (2.4)$$

We can calculate the rate-distortion function for several special sources and distortion measures.

Corollary 1 (Bernoulli Source with Hamming Distortion) *The rate-distortion function for a Bernoulli(α) Source with Hamming Distortion is*

$$R(d) = \begin{cases} H(\alpha) - H(d), & 0 \leq d \leq \min\{\alpha, 1 - \alpha\}; \\ 0, & d > \min\{\alpha, 1 - \alpha\}. \end{cases} \quad (2.5)$$

where $H(\cdot)$ is the entropy function of a binary random variable.

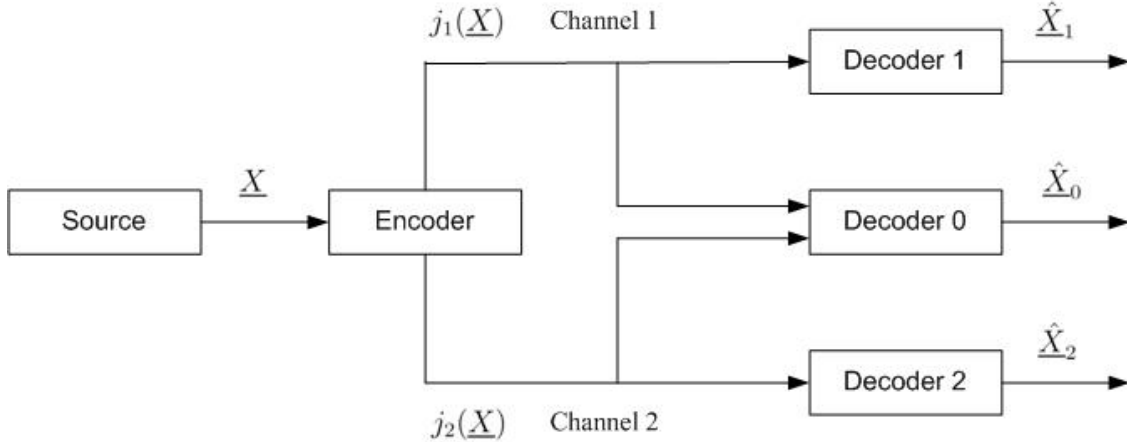


Figure 2.1: 2-Channel 3-Receiver MDC Problem

Corollary 2 (Gaussian Source with Squared Error Distortion) *The rate-distortion function for a $\mathcal{N}(0, \sigma^2)$ source with squared error distortion is*

$$R(d) = \begin{cases} \frac{1}{2} \log \frac{\sigma^2}{d}, & 0 \leq d \leq \sigma^2; \\ 0, & d > \sigma^2. \end{cases} \quad (2.6)$$

2.1.2 MD Coding: Problem Formulation

The 2-channel 3-receiver MD coding problem is represented in Fig. 2.1. The encoder is presented with a sequence of i.i.d. source symbols $\{X_i\}_{i=1}^{\infty}$. Each source symbol is distributed according to a probability mass function $p(x), x \in \mathcal{X}$. We are given three reconstruction spaces $\hat{\mathcal{X}}_1, \hat{\mathcal{X}}_2, \hat{\mathcal{X}}_0$, together with the associated distortion measures

$$d_t : \mathcal{X} \times \hat{\mathcal{X}}_t \rightarrow \mathbb{R}, t = 1, 2, 0 \quad (2.7)$$

The distortion measure on n -sequences is defined by the average per-symbol distortion

$$d_t^n(\underline{x}, \underline{\hat{x}}_t) = 1/n \sum_{k=1}^n d_t(x_k, \hat{x}_{tk}), t = 1, 2, 0 \quad (2.8)$$

where $\underline{x} = \{x_1, \dots, x_n\} \in \mathcal{X}^n$ and $\hat{\underline{x}}_t = \{\hat{x}_{t1}, \dots, \hat{x}_{tn}\} \in \mathcal{X}_t^n$. The encoding and decoding functions are defined by

$$f_t : \mathcal{X}_n \rightarrow \{1, \dots, M_t\}, t = 1, 2 \quad (2.9a)$$

$$g_t : \{1, \dots, M_t\} \rightarrow \hat{\mathcal{X}}_t^n, t = 1, 2 \quad (2.9b)$$

$$g_0 : \{1, \dots, M_1\} \times \{1, \dots, M_2\} \rightarrow \hat{\mathcal{X}}_{1,2} \quad (2.9c)$$

Denote $\underline{X} = (X_1, \dots, X_n) \in \mathcal{X}^n$. Define

$$\hat{\underline{X}}_t = g_t(f_t(\underline{X})), t = 1, 2 \quad (2.10a)$$

$$\hat{\underline{X}}_0 = g_0(f_1(\underline{X}), f_2(\underline{X})) \quad (2.10b)$$

and

$$D_t : E \left[d_t^n(\underline{X}, \hat{\underline{X}}_t) \right], t = 1, 2, 0 \quad (2.11)$$

The quintuple $(f_1, f_2, g_1, g_2, g_0)$ is called a code with parameters $(M_1, M_2, D_1, D_2, D_0)$.

Achievable Rate-Distortion Vector We shall say (R_1, R_2) is achievable for distortion $\underline{d} = (d_1, d_2, d_0)$ if, for all $\epsilon > 0$, there exists for n sufficiently large, a code with parameters $(M_1, M_2, D_1, D_2, D_0)$, where

$$M_t < 2^{(R_t + \epsilon)n}, t = 1, 2 \quad (2.12a)$$

$$D_t < d_t + \epsilon, t = 1, 2, 0 \quad (2.12b)$$

Rate-Distortion Region The rate-distortion region $\mathcal{R}(\underline{d})$ for distortion $\underline{d} = (d_1, d_2, d_0)$ is the closure of the set of achievable rate vectors (R_1, R_2) inducing distortions $\leq \underline{d}$.

Achievable Rate-Distortion Region Any subset of the rate-distortion region is called an achievable rate-distortion region. Another common name for the achievable rate-distortion region is the inner bound to the rate-distortion region.

2.1.3 Results on Achievable Rate-Distortion Region

The following two sets of sufficient conditions for $(R_1, R_2, d_1, d_2, d_0)$ to be achievable was deduced by El Gammal and Cover in [36] which is referred to as the EGC* resp. EGC region.

Theorem 2 (EGC* Achievable RD Region) *Let X_1, X_2, \dots be a sequence of i.i.d. finite alphabet random variables drawn according to a probability mass function $p(x)$. Let $d_m(\cdot, \cdot)$ be bounded. An achievable rate distortion region for distortion $\underline{d} = (d_1, d_2, d_0)$ is given by the convex hull of all (R_1, R_2) such that*

$$R_1 \geq I(X; U) \tag{2.13a}$$

$$R_2 \geq I(X; V) \tag{2.13b}$$

$$R_1 + R_2 \geq I(X; U, V) + I(U; V) \tag{2.13c}$$

for some random variables U and V jointly distributed with a generic source random variable X such that there exist random variables of the forms,

$$\hat{X}_1 = g_1(U) \tag{2.14a}$$

$$\hat{X}_2 = g_2(V) \tag{2.14b}$$

$$\hat{X}_0 = g_1(U, V) \tag{2.14c}$$

such that $E \left[d_t(\underline{X}, \underline{\hat{X}}_t) \right] \leq d_t, t = 1, 2, 0$.

Theorem 3 (EGC Achievable RD Region) *The quintuple $(R_1, R_2, d_1, d_2, d_0)$ is achievable if there exist random variables $\hat{X}_1, \hat{X}_2, \hat{X}_0$ jointly distributed with a generic source random variable X such that*

$$R_1 \geq I(X; \hat{X}_t), t = 1, 2 \tag{2.15a}$$

$$R_1 + R_2 \geq I(X; \hat{X}_1, \hat{X}_2, \hat{X}_0) + I(\hat{X}_1; \hat{X}_2) \tag{2.15b}$$

$$d_t \geq E \left[d_t(\underline{X}, \underline{\hat{X}}_t) \right], t = 1, 2, 0 \tag{2.15c}$$

for some random variables U and V jointly distributed with a generic source random variable X such that there exist random variables of the forms,

$$R_1 \geq I(X; U) \tag{2.16a}$$

$$R_2 \geq I(X; V) \tag{2.16b}$$

$$R_1 + R_2 \geq I(X; U, V) + I(U; V), t = 1, 2, 0 \tag{2.16c}$$

Let \mathcal{R}_{EGC^*} resp. \mathcal{R}_{EGC} denote the EGC^* resp. EGC achievable rate-distortion region. Actually, \mathcal{R}_{EGC^*} and \mathcal{R}_{EGC} are closely related, as stated in the following theorem.

Theorem 4 $\mathcal{R}_{EGC^*} \subset \mathcal{R}_{EGC}$

The EGC^* region is also included here since it (and also the EGC region) turns out to be optimal in the special case of Gaussian source and squared error distortion measure, which will be summarized next.

2.1.4 Special Cases: Gaussian Source with Squared Error Distortion

For the special case of Gaussian source with squared error distortion, the MD coding rate-distortion region was preliminarily deduced in [37] and [36]. The authors of [38] fixed some remaining inconsistencies and characterized the entire rate-distortion region.

Theorem 5 (MD Coding RD Region: Gaussian Source with a Squared Error Distortion [37] [36] [38]) *For i.i.d. Gaussian source sequence, $X_i \sim \mathcal{N}(1, \sigma^2)$, with a squared error distortion measure, the MD coding rate distortion region for Fig. 2.1 is the set of quintuples $(R_1, R_2, d_1, d_2, d_0)$ satisfying the following conditions:*

1. Given that $0 \leq d_0 \leq d_1 + d_2 - \sigma^2$, then the rate pair (R_1, R_2) is achievable if

$$R_1 \geq \frac{1}{2} \log \frac{1}{d_1} \quad (2.17a)$$

$$R_2 \geq \frac{1}{2} \log \frac{1}{d_2} \quad (2.17b)$$

$$R_1 + R_2 \geq \frac{1}{2} \log \frac{1}{d_0} \quad (2.17c)$$

2. Given that $d_1 + d_2 - \sigma^2 \leq d_0 \leq \left(\frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{\sigma^2}\right)^{-1}$, then the rate pair (R_1, R_2) is achievable if

$$R_1 \geq \frac{1}{2} \log \frac{1}{d_1} \quad (2.18a)$$

$$R_2 \geq \frac{1}{2} \log \frac{1}{d_2} \quad (2.18b)$$

$$R_1 + R_2 \geq \frac{1}{2} \log \frac{1}{d_0} + \frac{1}{2} \log \frac{(\sigma^2 - d_0)^2}{(\sigma^2 - d_0)^2 - (\sqrt{(\sigma^2 - d_1)(\sigma^2 - d_2)} - \sqrt{(d_1 - d_0)(d_2 - d_0)})^2} \quad (2.18c)$$

3. Given that $\left(\frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{\sigma^2}\right)^{-1} \leq d_0 \leq +\infty$, then the rate pair (R_1, R_2) is achievable if

$$R_1 \geq \frac{1}{2} \log \frac{1}{d_1} \quad (2.19a)$$

$$R_2 \geq \frac{1}{2} \log \frac{1}{d_2} \quad (2.19b)$$

Remark 1 For Gaussian sources and squared error distortion measure, Theorem. 5 not only characterizes an achievable rate-distortion region but also states that this region is the best achievable rate-distortion region. The rate-distortion region in Theorem. 5 is obtained by evaluating and optimizing the EGC* rate-distortion region in Theorem. 2. The converse part (showing the optimality of the region) was presented in [37]. The main technicality of the converse part shows a lower bound to the mutual information between the two side receivers reconstructions (Fig. 2.1), $I(\hat{X}_1^n; \hat{X}_2^n)$, for a given rate pair (R_1, R_2) and side receiver's distortions (d_1, d_2) .

Denote the reconstructions at the two side receivers as \hat{X}_1^n and \hat{X}_2^n . The lower bound is,

$$I(\hat{X}_1^n; \hat{X}_2^n) \geq \frac{n}{2} \log \frac{\sigma^2}{\sigma^2 - (\sqrt{(\sigma^2 - d_1)(\sigma^2 - d_2)} - \sqrt{(d_1 - d_0)(d_2 - d_0)})^2} \quad (2.20)$$

The above lower bound is the cause for the tradeoff between the central receiver and side receivers. On the one hand, if the central decoder needs to approach Shannons rate-distortion bound (Theorem. 1), the two descriptions need to be approximately independent, which renders the two side reconstructions also approximately independent. Therefore, the approximate independence condition $I(\hat{X}_1^n; \hat{X}_2^n) \approx 0$ necessitates that $\sigma^2 + d_0 \approx d_1 + d_2$, which means at least one of d_1 and d_2 is close to the sources variance (i.e. performs poorly). On the other hand, if both side decoders need to approach Shannons rate-distortion bound, then the above lower bound on $I(\hat{X}_1^n; \hat{X}_2^n)$ is far away from zero, which means that the two descriptions sent over two channels are highly correlated. Therefore, the central decoders distortion performance is bounded away from the rate-distortion bound since receiving two highly correlated descriptions is not much better than receiving only one description.

2.2 Successive Refinement(SR)

In Fig. 2.1, removing Decoder 2, as in Fig. 2.2, gives what is known as successive refinement (SR) coding. Unlike in MD coding, the channels in SR coding play asymmetric roles. One channel (Channel 1) is received by both decoders, while the other (Channel 2) is received by only one decoder. Thus the information sent on Channel 2 need not be useful in isolation (i.e., without Channel 1). The description on Channel 2 is said to refine the information on Channel 1. The theoretical bounds for SR coding are established in [39, 40, 41, 42, 43].

The SR coding abstraction applies to layered broadcasting when the decoders represent different users. The two classes of users are labeled 0 and 1. Both receive Channel 1, but only Class 0 receives Channel 2. This situation may occur in wireless

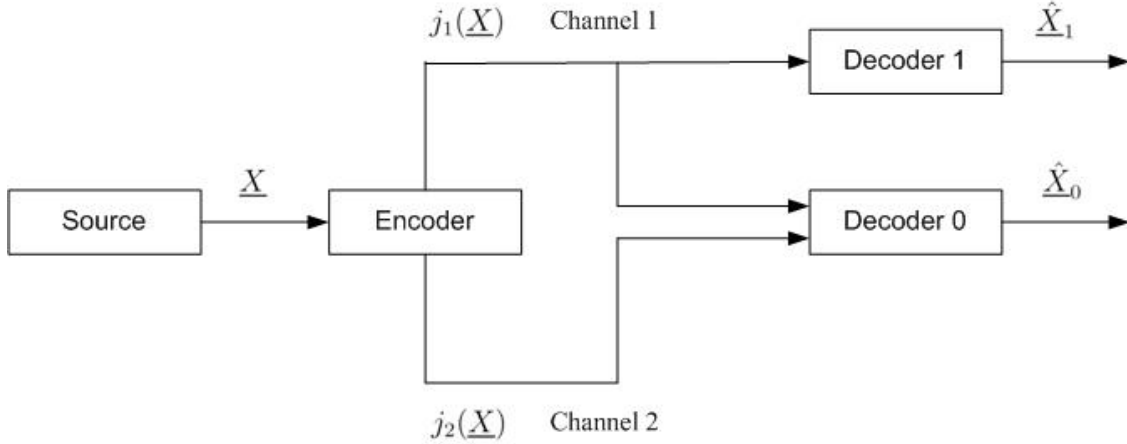


Figure 2.2: 2-Channel 2-Receiver SR Problem

transmission with multiresolution constellations [44] or in packet communication when multicasting to users with different available bandwidths [45].

When the decoders in Fig. 2.2 represent two different states of the same user, SR coding can be used for progressive transmission. The information on Channel 1 is sent first and the receiver uses Decoder 1. If communication is not terminated at this point, the information on Channel 2 is then sent, and the receiver uses Decoder 0.

Good SR source codes share the following characteristic: The most important data is sent on Channel 1 (thus to all users in a broadcast or first in sequence) and additional data to improve the reconstruction quality is sent on Channel 2. For example, most significant bits can be sent on Channel 1 and least significant on Channel 2. For images, a coarse or low pass version can be sent on Channel 1 with additional details sent on Channel 2; this is easy and common with wavelet representations [46, 47, 48].

Another antecedent network communication problem was introduced by Gray and Wyner in 1974 [49]. Instead of having a single source sequence to communicate over two channels to three receivers, they have a sequence of pairs of random variables $(\underline{X}, \underline{Y})$ to communicate to two receivers over three channels. Receiver 1 is interested only in \underline{X} and forms its estimate from Channel 1 and a common channel.

Receiver 2 has its own private channel and is interested in the other sequence \underline{Y} . This is a special case of MD coding with three channels and seven receivers [50].

2.3 Superposition Coding(SPC)

The superposition coding technique is a bandwidth efficient coding approach recently applied to high throughput transmission [51, 52]. In this approach, several independent coded sequences, called layers, are linearly superimposed before transmitting [53, 54, 55, 56]. As a result, the transmitted signal has an approximate Gaussian distribution, a perfect shape to achieve the theoretical capacity. This provides the shaping gain [57, 58, 59, 60], which is usually achieved by special complicated shaping codes or shaping algorithms [59, 60]. In addition, as shown in [55, 56], together with random layer-specific interleavers introduced in the encoding process, the superposition coding scheme is similar to one of the multiple-access systems, the Interleave Division Multiple Access (IDMA) [61], where one layer is viewed as one user. Hence, most of the advanced techniques and nice features provided by IDMA systems can be applied to superposition coding such as the low complexity iterative receiver, the fast SNR evolution to predict the system performance or even the linear programming technique for power allocation to enhance the system throughput [62].

Recently, superposition coding technique has been considered for a number of wireless communication systems such as broadcast channels [53, 54] or full-duplex relay channels [63, 64, 65, 66]. This coding technique is also flexible to combine with adaptive modulation, which is achieved by adjusting the number of layers according to channel conditions based on the soft information delivered by the iterative receiver [67]. In contrast to conventional adaptive modulation techniques, this technique can use a fixed modulation scheme with a binary channel code for all layers. This means the scheme is more flexible to change rates and a lower complexity receiver can be employed. In addition, superposition coding is applicable

to MIMO systems with a variable number of transmit/receive antennas [68]. No orthogonal or arithmetic space-time code design is necessary.

2.3.1 Two-Level Superposition Coded Multicasting

For a long time, superposition coding was introduced to increase the overall user capacity of a wireless communication system by exploiting the spatial or temporal power disparities perceived by multiple users for common broadcast signals [1, 14]. An example of superposition coding is illustrated in Fig. 2.3, cited from [14]. The nodes are indexed in an increasing order according to their distance from the BS. As shown in the figure, when the BS transmits signals to M_3 with the targeted signal noise ratio (SNR) level, the SNR experienced by both M_1 and M_2 is much greater than their targeted SNR levels (by the amount of $A + B$ and C , respectively). Similarly, when the BS transmits signal to M_2 , M_1 receives additional A dB of power above its targeted SNR level. This implies that M_1 has sufficient SNR to decode the messages intended for both M_2 and M_3 , and M_2 has sufficient SNR to decode the messages intended for M_3 . The power disparities at nodes M_1 , M_2 , and M_3 suggest that the information for M_1 can be included in the transmission to M_2 or M_3 through the adoption of superposition coding. Similarly, the information for M_2 can be included while transmitting information to M_3 . The dotted line in Fig. 2.3 indicates that the BS transmits information to M_2 while transmitting to M_3 at the targeted SNR level by employing superposition coding.

By leveraging the superposition coding, the proposed two-level SCM scheme is as follows. Instead of using one modulation scheme at a time, multicast signals in a time slot are generated by performing superposition on data modulated by BPSK and data modulated by 16QAM in the physical layer as shown in Fig. 2.4. In this work, we assume all the SS can support BPSK in their worst channel conditions. For ensuring the video quality, the base quality data of each video frame is modulated by BPSK, and the enhancement quality data is modulated by 16QAM. With such superposition coded multicast signals, each SS can at least decode and obtain the

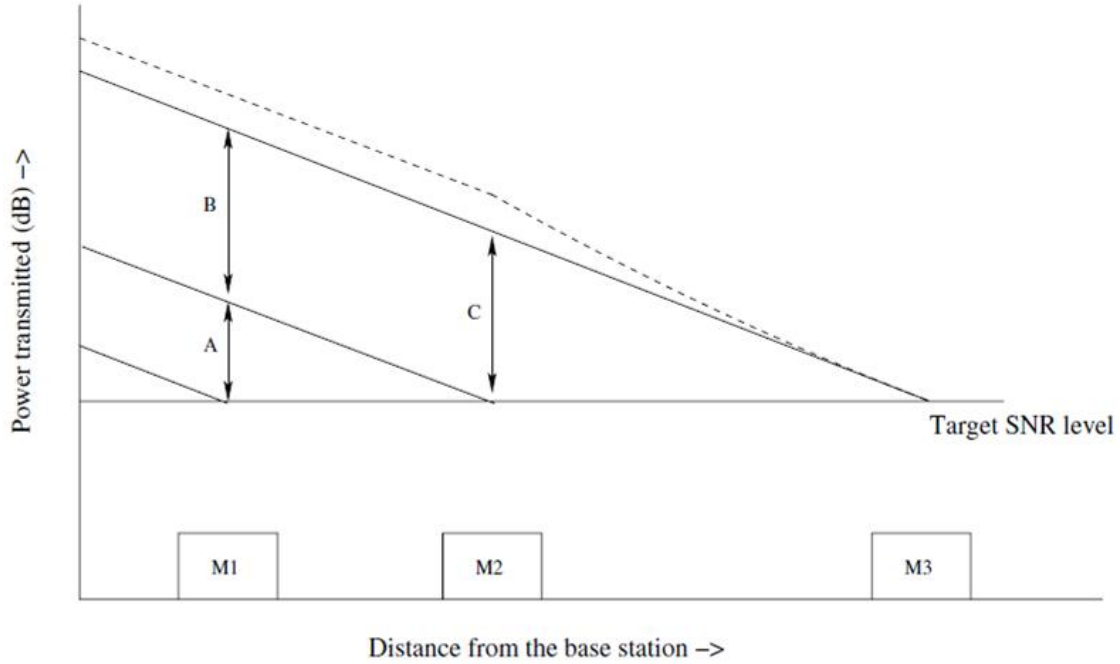


Figure 2.3: Power Disparities in a Wireless Network (Cited from [14])

base video quality data of an IPTV channel modulated with BPSK when it is in the bad channel state, or achieve the full video quality with all the data modulated by both BPSK and 16QAM when it is in the good channel state. Even in the presence of fluctuation of channel conditions in all the SS between good and bad from time to time, an acceptable video quality of an IPTV channel to every SS can be achieved. An optimal transmission strategy at the BS for an IPTV channel can be unhooked from such deadlock situations due to the multicasting over heterogeneous channel conditions.

2.4 Relay Channel

In a relay channel, between the sender X and the the receiver Y lies at least one relay. Generally, the relay can both transmit its own information and help forwarding other sources' information. This summary considers the latter case (i.e., the relay intends solely to help the receiver). The relay and the transmitter

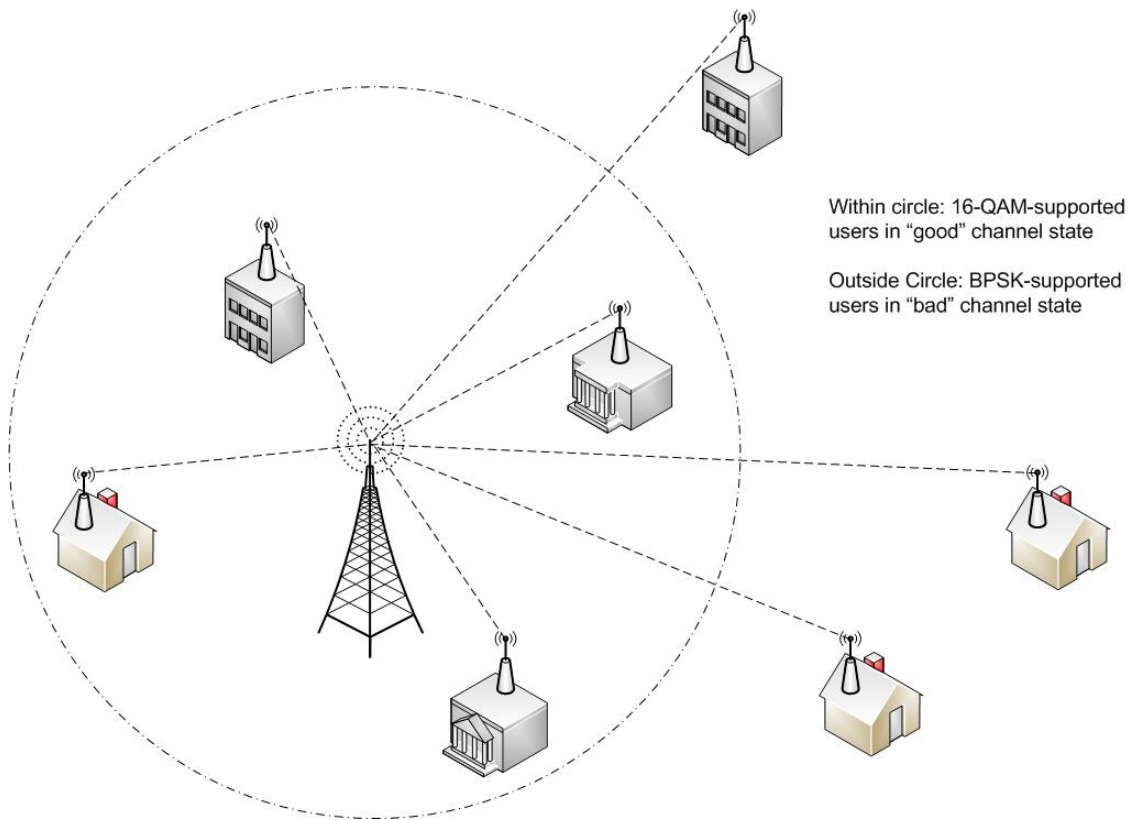


Figure 2.4: The Multicast Signal Based on Two-Level Superposition Coded Multicasting Scheme (16 QAM / BPSK)

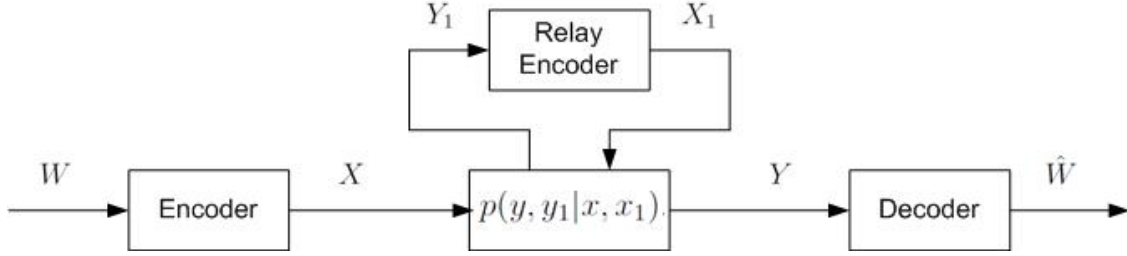


Figure 2.5: Relay Channel

cooperate to resolve the receiver's uncertainty. Due to the presence of relay, the relay channel capacity is difficult to determine. The capacity is known only for some special cases (e.g., physically degraded relay channel [53, 69]), Gaussian relay channel [70, 71] (asymptotic capacity).

2.4.1 General Relay Channel

Fig. 2.5 illustrates the simplest general relay channel which has only one relay. The channel consists of four finite sets \mathcal{X} , \mathcal{X}_1 , \mathcal{Y} and \mathcal{Y}_1 , and a collection of probability mass functions $p(y, y_1|x, x_1)$. x is the input to the channel, y is the output of the channel; y_1 is the relay's observation and x_1 is the input chosen by the relay and depends only on the past observation $(y_{11}, y_{12}, \dots, y_{1i-1})$. The capacity problem is to find the channel capacity between X and Y .

An (M, n) code for the relay channel consists of a set of integers $M = \{1, 2, \dots, M\}$, an encoding function $X : \mathcal{M} \rightarrow \mathcal{X}^n$, a set of relay functions $\{f_i\}_{i=1}^n$ such that

$$x_{1i} = f_i(Y_{11}, Y_{11}, \dots, Y_{1i-1})$$

and a decoding function

$$g : \mathcal{Y}^n \rightarrow \mathcal{M}$$

The channel is memoryless in the sense that (Y_i, Y_{1i}) depends on the past only through the current transmitted symbols (X_i, X_{1i}) . Thus, for any choice $p(w)$, $w \in \mathcal{M}$, code choice $x : \mathcal{M} \rightarrow \mathcal{X}^n$ and relay functions $\{f_i\}_{i=1}^n$, the joint probability

mass function on $\mathcal{M} \times \mathcal{X}^n \times \mathcal{X}_1^n \times \mathcal{Y}^n \times \mathcal{Y}_1^n$ is given by

$$p(w, x, x_1, y, y_1) = p(w) \prod_{i=1}^n p(x_i|w)p(x_{1i}|y_{11}, y_{12}, \dots, y_{1i-1}) \cdot p(y_i, y_{1i}|x_i, x_{1i}) \quad (2.21)$$

The average probability of error is defined as follows:

$$P_e^{(n)} = \frac{1}{2^{nR}} \sum_{w \in \mathcal{M}} \Pr\{g(Y) \neq w | w \text{ sent}\} \triangleq \frac{1}{2^{nR}} \sum_{w \in \mathcal{M}} \lambda(w) \quad (2.22)$$

The relay channel combines a broadcast channel (X to Y and Y_1) and a multiple access channel (X_1 and X to Y). Directly applying the max-flow-min-cut theorem for general multiterminal networks to the relay channel, an upper bound of the capacity is obtained.

Theorem 6 *For any relay channel, the capacity is bounded above by*

$$C \leq \sup_{p(x, x_1)} \min\{I(X, X_1; Y), I(X; Y, Y_1|X_1)\} \quad (2.23)$$

The first term in Eq. 2.23 upper bounds the maximum rate of information transfer from senders X and X_1 to receiver Y (Multiple Access Channel); the second term bounds the rate from X to Y and Y_1 (Broadcast Channel, but the ultimate receiver Y should first decode the relay signal X_1 before decoding X , which contributes to the conditioning term X_1 in $I(X; Y, Y_1|X_1)$). The proof is given in [69].

2.4.2 Degraded Relay Channel

The degraded relay channel, similar to the degraded broadcast channel, implies that one receiver is a degraded version of the other receiver. There are two degradednesses in the relay channel. One of interest is called degraded relay channel, in which the relay receiver y_1 is better than the ultimate receiver y and thus the relay can cooperate to send x . The other case, in which the relay y_1 is worse than y , is less interesting, because the relay can contribute no new information to the receiver.

Definition 1 *The relay channel $(\mathcal{X} \times \mathcal{X}_1, p(y, y_1|x, x_1), \mathcal{Y} \times \mathcal{Y}_1)$ is said to be degraded if*

$$p(y, y_1|x, x_1) = p(y_1|x, x_1)p(y|x_1) \quad (2.24)$$

Equivalently, a relay channel is degraded if $p(y|y_1, x, x_1) = p(y|y_1, x_1)$ (i.e. $X \rightarrow (X_1, Y_1) \rightarrow Y$ form a Markov chain). A degraded relay channel can be regarded as a family of physically degraded broadcast channels indexed by x_1 .

Theorem 7 *The capacity C of the degraded relay channel is given by*

$$C = \sup_{p(x, x_1)} \min\{I(X, X_1; Y), I(X; Y_1|X_1)\} \quad (2.25)$$

where the supremum is over all joint distributions $p(x, x_1)$ on $\mathcal{X} \times \mathcal{X}_1$.

Here, due to the degradedness,

$$I(X; Y, Y_1|X_1) = I(X; Y_1|X_1) \quad (2.26)$$

The proof of the converse directly follows from Theorem 6.

2.4.3 Gaussian Degraded Relay Channel

Consider a Gaussian degraded relay channel, shown in Fig. 2.6, where Z_1 and Z_2 are sequences of i.i.d. normal random variables with zero mean and variance N_1 and N_2 , respectively. The ultimate receiver Y is a corrupted version of the relay Y_1 , conditioning on X_1 .

$$Y_1 = X + Z_1 \quad (2.27a)$$

$$Y = Y_1 + X_1 + Z_2 \quad (2.27b)$$

In addition, the transmitted power is constrained by

$$\frac{1}{n} \sum_{i=1}^n x_i^2(w) \leq P, w \in \{1, 2, \dots, M\}$$

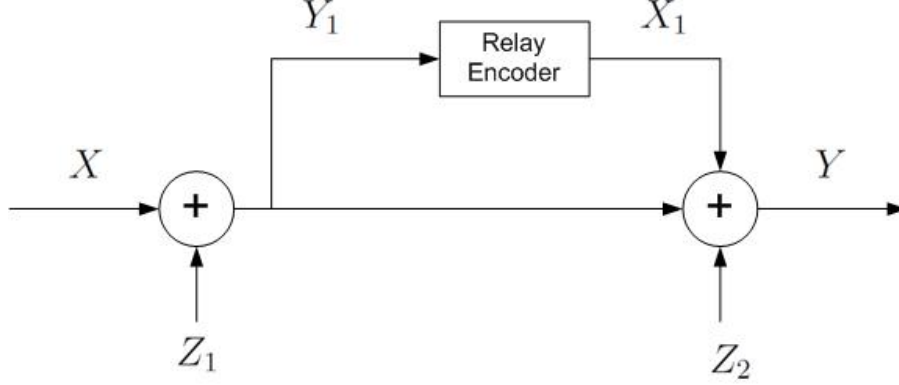


Figure 2.6: Gaussian Degraded Relay Channel

$$\frac{1}{n} \sum_{i=1}^n x_{1i}^2(y_{11}, y_{12}, \dots, y_{1i-1}) \leq P_1, (y_{11}, y_{12}, \dots, y_{1n}) \in \mathbb{R}^n$$

Theorem 8 *The capacity C^* of the Gaussian degraded relay channel is given by*

$$C^* = \max_{0 \leq \alpha \leq 1} \min \left\{ C\left(\frac{P + P_1 + 2\sqrt{\bar{\alpha} P P_1}}{N_1 + N_2}\right), C\left(\frac{\alpha P}{N_1}\right) \right\} \quad (2.28)$$

where $\bar{\alpha} = 1 - \alpha$ and $C(x) = \frac{1}{2} \log(1 + x)$.

We just sketch the random code that achieves C^* . For $0 \leq \alpha \leq 1$, let $X_1 \sim N(0, P_1)$, $\tilde{X} \sim N(0, \alpha P)$, with \tilde{X} , X_1 independent. The transmitter x generates the first codebook $\mathcal{M} = \{1, 2, \dots, 2^{nR}\}$ according to $N_n(0, \alpha P)$ (i.e., given a message index $w \in \mathcal{M}$, the transmitter sends the codeword $\tilde{x}(w)$ with power αP). Given $R < C(\frac{\alpha P}{N_1})$, the relay knows w correctly, but the receiver (without the relay) has ambiguity because its capacity $C(\frac{\alpha P}{N_1 + N_2}) < C(\frac{\alpha P}{N_1})$. It has a list of possible words of size $2^{n[R - C(\alpha P / (N_1 + N_2))]}$.

In the next block, the transmitter and the relay cooperate to resolve the receiver's uncertainty. As mentioned before, they partition the first codebook M into 2^{nR_0} mutually exclusive subsets, which is exactly the second codebook with 2^{nR_0} codewords. They coherently transmit the partition index s through the codeword $x_1(s)$. Specifically, the transmitter sends s with power $\bar{\alpha} P$, and the relay sends with power P_1 . The power seen by the receiver y is therefore $(\sqrt{\bar{\alpha} P} + \sqrt{P_1})^2$, which

leads to

$$R_0 < C \left(\frac{(\sqrt{\alpha P} + \sqrt{P_1})^2}{\alpha P + N_1 + N_2} \right)$$

Chapter 3

System Model

A group of frames (GoFs) from a video source is assumed to be encoded into a set of successively refinable (SR) information, which will be further encoded with a SR based Multiple Description Coding (MDC) before being transmitted to a group of receivers over fading relay channel using a broadcast/multicast strategy with superposition coding at the channel as illustrated in Fig. 3.1.

3.1 Broadcast Approach

3.1.1 SR, MDC Encoding and Channel Modulation

The set of SR information contains L quality layers, where the information of each layer can be refined by the data of higher layers. b_{l-1} and b_l are the bit boundaries of a quality layer l bitstream. For enhancing the robustness of the data in each SR layer, a RS code (N, K_l) is applied to the SR layer l , in which a block with K_l bytes from layer l is encoded into a N -byte long protected bitstream as shown in Fig. 3.1. With the given size of each layer, the value of K_l determines the number of rows in layer l (i.e., $(b_l - b_{l-1})/(8K_l)$). Each row of a layer is then packetized into a set of MDC descriptions (or packets), where the 1st byte in each row of all quality layers is assigned into the 1st description, and so on. N equally important

cation to layer l . Such transmit symbol x , contains symbols of L layers, which is broadcasted wirelessly to a group of receivers.

3.1.2 Channel Demodulation, MDC and SR Decoding

With a single antenna at the transmitter and each receiver, the wireless channel is described by:

$$y = Hx + z \quad (3.2)$$

where y is the received signal, H is the channel gain under fading, x is the transmit signal, and z is a unit variance noise. The probability mass function (PMF) of a fading distribution in a broadcast network with L discrete states [4] is assumed, such that a receiver has a channel power gain realization as $\gamma_L > \dots > \gamma_l > \dots > \gamma_1 \geq \gamma_0$, where $\gamma_l = |h_l|^2$. h_l is a realization of H with probability p_l for $l = 1, \dots, L$, and States L and 1 are referred as the highest state and the lowest state, respectively. In other words, a receiver in a wireless broadcast network has L possible channel conditions to decode a MDC description, V_l , completely or partially from the received SPC broadcast signal.

With successive SPC demodulation using a signal-interference cancellation (SIC) technique [5], the receiver first decodes the information of the lower layer(s) and cancels the lower layer(s) information from the original received signal before decoding its target layer, where the undecodable higher layers are treated as noise. Thus, the rate at the channel intended for a receiver to decode layer l is denoted as R_l , and can be expressed as:

$$R_l = \log\left(1 + \frac{\gamma_l \pi_l}{\gamma_l \sum_{j=l+1}^L \pi_j + 1}\right) \quad (3.3)$$

where the term $\gamma_l \sum_{j=l+1}^L \pi_j + 1$ represents the interference power from the higher layers under fading when the l -th layer data is being decoded.

In order to reconstruct a MDC description \hat{V}_l , where $l = 1 \dots N$, from the received SPC broadcast signal, the channel output for each layer is collected through the SPC demodulation process depending on the instantaneous channel realization at the receiver. Suppose γ_k is the realized channel power gain at a receiver in receiving a SPC broadcast signal, then the receiver can reconstruct a MDC description \hat{V}_l , with decoded data up to layer k from the received broadcast SPC signal. Hence, the realized channel rate at the receiver is $R_1 + \dots + R_l$ from this SPC broadcast transmission. After receiving and reconstructing N MDC descriptions, a better set of estimated SR layers can be obtained, where a certain amount of data originally lost in some of the SR layers due to the fading channel can be potentially recovered by MDC decoder due to the RS codes across all the MDC descriptions. Recall that the data of a layer from a SR information source is only useful if information of all the lower layers are decoded. Therefore, a better estimate of \hat{S} of the source can be generated due to the more data/number of complete SR layers successfully recovered from the MDC decoder, which are never achievable by only using SR and SPC. A schematic diagram is shown in Fig. 3.2 to illustrate the encoding and decoding processes in this cross-layer framework:

We consider a fading channel model, where each channel codeword consists of a length- l_c channel symbol block, and the realization of the channel gain is independent across symbol blocks. A source symbol of block length- l_s is encoded into multiple channel codewords, and there is a source channel bandwidth ratio defined as $b = l_c/l_s$; this ratio can also be interpreted as the bandwidth expansion/compression factor. Each channel block is assumed to be sufficiently long to approach the channel capacity as well as the rate-distortion limit, which is, however, much shorter than the dynamics of the fast fading process. For the feasibility and simplicity of analytical study for such emerging cross-layer designed coded wireless multicast/broadcast, a complex Gaussian source [53] is employed in this work. Since the source is successively refinable, a receiver with a channel realization γ_l to achieve the channel rate of R_l can therefore reconstruct the original set of informa-

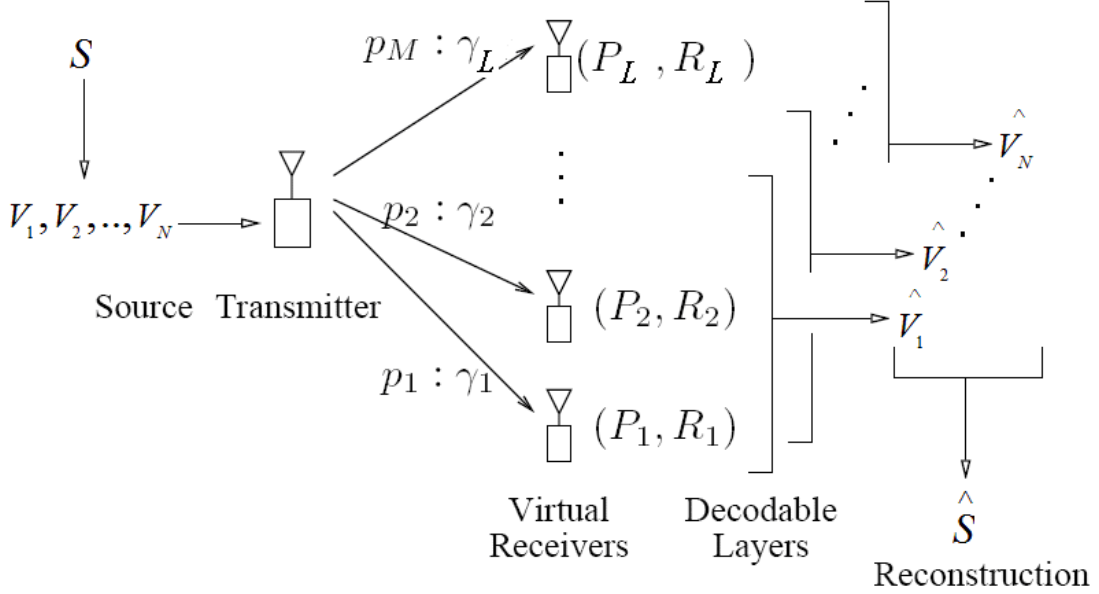


Figure 3.2: Successive Refined and Multiple Description Coded Source Passing through the Superimposed Broadcast Channel (Cited from [15])

tion with a distortion \hat{S} , up to layer l . The realized distortion D_l is:

$$D_l = 2^{-b \sum_{j=l}^L R_j} \quad (3.4)$$

Recall that a channel realization γ_l happens with a PMF p_l , for $l = 1, \dots, L$. If the channel realization of each receiver in a wireless broadcast/multicast network stay unchanged for a long run, which are generally assumed in the previous works [4] [5] [6] for the simplicity, the overall expected distortion $E[D]$ can be well described as:

$$E[D] = \sum_{l=1}^L p_l D_l \quad (3.5)$$

where p_l is probability in the fading distribution PMF to obtain the channel gain realization as γ_l . As one of the contributions of this thesis, by employing MDC as a protection means before sending SR information over layered broadcast codes, the overall expected distortion $E[D]$ can be further minimized which is investigated next.

A symbol block in layer l will be lost at the receiver whenever the channel gain in receiving any of the N descriptions is not realized up to γ_l , which fails to reconstruct the SR layer l for the final information. Due to the RS code (N, K_l) employed in that layer l , such loss may be potentially mitigated if any K_l of N symbol blocks of layer l are successfully decoded from the N received MDC descriptions. Thus, the expected distortion $E[D]$ in Eq. 3.5 will be no longer proportional to p_l only. The effective rate of the channel at the receiver in receiving symbols of layer l is facilitated by the effect of error recovery due to the MDC. By considering the effect of MDC, the expected distortion $E[D]$ in Eq. 3.5 becomes:

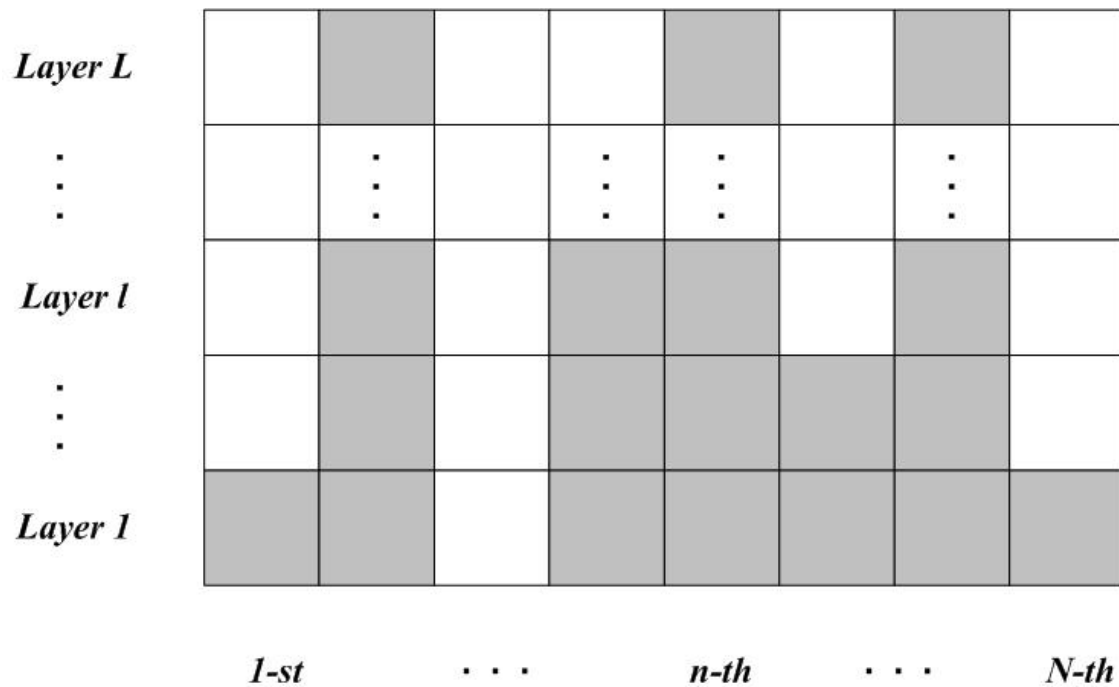
$$E[D] = \sum_{l=1}^L \tilde{p}_l D_l \quad (3.6)$$

where \tilde{p}_l is the effective probability of achieving a new effective rate of the channel in receiving symbols of layer l due to the effect of MDC. Instead of solely due to the distribution of channel gain realizations, \tilde{p}_l is also affected by the RS code (N, K_l) employed in each layer. This relation is formulated in the following section.

3.1.3 Effects of MDC Towards the Expected Distortion

Due to the RS codes in each MDC description, the probability of successfully decoding layer l in an individual SPC broadcast signal now strongly depends on the number of symbols that were successfully received within layer l among the N received descriptions, as well as the fact that all layers below layer l are also decoded. This idea is illustrated in Fig. 3.3 with a grid of N columns and L rows, which represent all the N MDC descriptions with each containing the symbols of L layers that are possibly decodable by a receiver for reconstructing the estimated SR layers. A shaded box at the $l - th$ row and $n - th$ column indicates that the layer l symbol in $n - th$ description is decoded.

Due to the intrinsic nature of SPC demodulation using SIC, a layer l symbol can be decoded only if all the lower layer symbols in the same column have been successfully decoded when the channel realization is at γ_l with a probability of p_l . In



MDC descriptions

Figure 3.3: All Symbols That Are Possibly Receiveable from N MDC Descriptions with l Layers by a Receiver (Cited from [15])

particular, assume that the probability that a layer l symbol in any column is independently decodable, is denoted as q_l . Layer l is claimed successfully reconstructed only if at least K_l symbols in layer l are decoded within the N columns. Let n_l denote a random variable for the total number of symbols of layer l successfully decoded in N columns. To get an overview of all symbols possibly received from all the N descriptions, the probability $\Psi(N_1 = n_1, \dots, N_l = n_l, \dots, N_L = n_L)$ that a receiver decoded $n_1, \dots, n_l, \dots, n_L$ symbols in layers $1, \dots, l, \dots, L$, respectively, can be expressed as:

$$\Psi(N_1 = n_1, \dots, N_l = n_l, \dots, N_L = n_L) \quad (3.7)$$

$$= \Psi(N_L = n_L) \times \dots \times \Psi(N_l = n_l | N_{l+1} = n_{l+1}, \dots, N_L = n_L) \quad (3.8)$$

$$\times \dots \times \Psi(N_1 = n_1 | N_2 = n_2, \dots, N_L = n_L)$$

$$= \Psi(N_L = n_L) \times \dots \times \Psi(N_l = n_l | N_{l+1} = n_{l+1}) \quad (3.9)$$

$$\times \dots \times \Psi(N_1 = n_1 | N_2 = n_2)$$

$$= \binom{N}{n_L} q_L^{n_L} (1 - q_L)^{N - n_L} \times \dots \times \binom{N - n_{l+1}}{n_l - n_{l+1}} q_l^{n_l - n_{l+1}} (1 - q_l)^{N - n_{l+1}} \quad (3.10)$$

$$\times \dots \times \binom{N - n_2}{n_1 - n_2} q_1^{n_1 - n_2} (1 - q_1)^{N - n_1}$$

Since the probability q_l that a layer l symbol is decoded within a column can also be affected by the channel gain realizations from γ_{L-l} to γ_L with probabilities of p_{L-l}, \dots, p_L . Hence, relations of p_l and associated q_l , $l = 0, 1, \dots, L$ are:

$$\left\{ \begin{array}{l} q_1 = p_1 + p_2 + \dots + p_l + \dots + p_L \\ q_2 = p_2 + \dots + p_l + \dots + p_L \\ \vdots \\ q_l = p_l + \dots + p_L \\ \vdots \\ q_L = p_L \\ \sum_{l=0}^L p_l = 1 \end{array} \right. \quad (3.11)$$

The term \tilde{p}_l , in Eq. 3.6 can be expanded by considering the parameters q_j , N , and K_j for $j = 1, \dots, l$, due to the MDC as follows:

$$\tilde{p}_l = \underbrace{\sum_{n_1=K_1}^N \cdots \sum_{n_l=K_l}^N}_{1, \dots, l} \underbrace{\sum_{n_{l+1}=0}^{K_{l+1}-1} \cdots \sum_{n_L=0}^{K_L-1}}_{l+1, \dots, L} \Psi(N_1 = n_1, \dots, N_L = n_L) \quad (3.12a)$$

$$\begin{aligned} &= \underbrace{\sum_{n_1=K_1}^N \cdots \sum_{n_l=K_l}^N}_{1, \dots, l} \underbrace{\sum_{n_{l+1}=0}^{K_{l+1}-1} \cdots \sum_{n_L=0}^{K_L-1}}_{l+1, \dots, L} \binom{N}{n_L} q_L^{n_L} (1 - q_L)^{N-n_L} \\ &\times \cdots \times \binom{N - n_{l+1}}{n_l - n_{l+1}} q_l^{n_l - n_{l+1}} (1 - q_l)^{N - n_{l+1}} \\ &\times \cdots \times \binom{N - n_2}{n_1 - n_2} q_1^{n_1 - n_2} (1 - q_1)^{N - n_1} \end{aligned} \quad (3.12b)$$

subject to $x_1 > \cdots > x_l > x_{l+1} > \cdots > x_L$.

A power constraint is imposed on each transmit symbol at the transmitter: $E[|x|^2] \leq P$, where the expectation is taken over the duration of each fading block. Power allocation in each fading block is considered here. By combining Eqs. 3.3 3.4 3.6 3.10 3.11 3.12, an optimization problem can be formulated to minimize the power allocation $(1, 2, \dots, L)$, given N, K_1, K_2, \dots, K_L , subject to:

$$\begin{aligned} \pi_l &\geq 0, l = 1, 2, \dots, L, \\ \pi &= \sum_{l=1}^L \pi_l. \end{aligned} \quad (3.13)$$

3.2 Fading Relay Channel with Broadcast/Multicast Strategy

This section considers a basic 'Two-Layer, Three-Node' fading relay channel scenario, which means there are two layers of SR layers, and three transmission nodes: the source, the relay, and the destination.

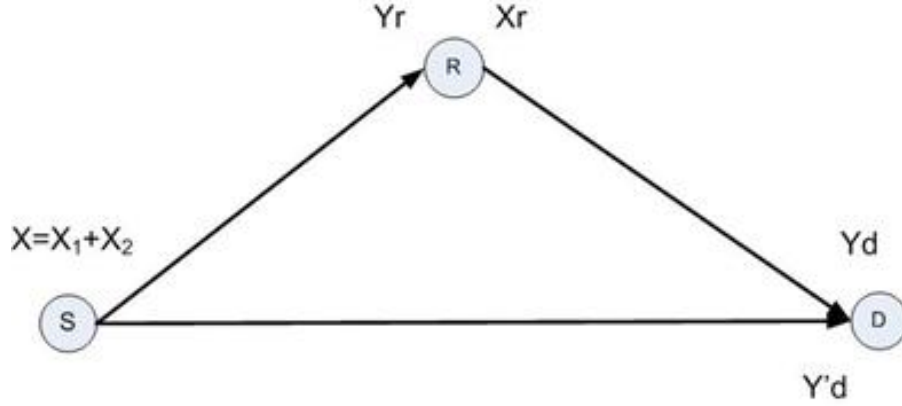


Figure 3.4: Basic 'Two-Layer, Three-Node' Fading Relay Channel

3.2.1 Calculation of Transmit Rates

For simplicity purpose, here assume $\pi_s = \pi_r$, which indicates that the power from the source is the same as that from the relay. The source encodes its information using 2-level superposition coding. Therefore, the source signal can be written as the sum of two signals X_1 and X_2 :

$$X = X_1 + X_2 \quad (3.14)$$

Here X_1 is the base and X_2 is the additional information. This superposition of information enables both the relay and the destination to decode the information successively at two different rates, R_1 and R_2 :

$$R_1 = \log \left(1 + \frac{\gamma_1(1 - \alpha)\pi_s}{1 + \gamma_1\alpha\pi_s} \right) \quad (3.15a)$$

$$R_2 = \log(1 + \gamma_2\alpha\pi_s) \quad (3.15b)$$

To decode X_1 , X_2 is treated as noise. After decoding X_1 , it is subtracted from the received signal and X_2 can be decoded afterwards. If only X_1 is decoded reliably, but X_2 is in error, then the received rate is R_1 . On the other hand, after decoding X_1 correctly, if in addition X_2 is decoded reliably, then the total received rate becomes $R_1 + R_2$.

3.2.2 Idea of Broadcast Strategies for the Fading Relay Channel

We assume the total block length is M . In the first time slot, or first half of the block consisting of $M/2$ transmissions, the source transmits the signal X in Eq. 3.14. The source allocates $\alpha\pi_s$ power to X_2 and $(1 - \alpha)\pi_s$ power to X_1 , where $\alpha \in [0, 1]$. The corresponding received signals by the relay and the destination are Y_r and Y_d respectively. We have

$$Y_r = h_{sr}X + Z_r \quad (3.16a)$$

$$Y_d = h_{sd}X + Z_d \quad (3.16b)$$

After receiving Y_r in the first time slot, the relay sends X_r , resulting in the received signal Y'_d at the destination. If the relay receives multiple description packets with the quantities less than K_1 in the first layer and the quantities less than K_2 in the second layer, then it cannot understand anything and hence cannot transmit any information during its time slot. If the numbers of received multiple description packets from the source to relay are greater than K_1 in the first layer while less than K_2 in the second layer, then the relay can reliably decode X_1 and can allocate all of its power to X_1 . If the relay understands everything, then it transmits both parts of the information X_1 and X_2 assigning power levels $(1 - \alpha)\pi_r$ and $\alpha\pi_r$ respectively.

$$X_r = \begin{cases} 0, & 0 \leq n_1 < K_1, 0 \leq n_2 < K_2 \\ \sqrt{\frac{\pi_r}{(1-\alpha)\pi_s}}X_0, & K_1 \leq n_1 \leq N, 0 \leq n_2 < K_2 \\ \sqrt{\frac{\pi_r}{\pi_s}}(X_0 + X_1), & K_1 \leq n_1 \leq N, K_2 \leq n_2 \leq N \end{cases} \quad (3.17)$$

$$Y'_d = h_{rd}X_r + Z'_d \quad (3.18)$$

Z_r , Z_d and Z'_d denote complex Gaussian noise at the relay and at the destination respectively with variance N_0 .

Destination attempts to decode X_1 first, combining X and X_r , which depends on the source to relay channel quality as in Eq. 3.17, and treating X_2 as noise. We

assume the destination knows which signal the relay has transmitted in Eq. 3.17. If the destination can successfully decode X_1 then it attempts to decode X_2 by subtracting off X_1 first. This ordering is due to the layering of information in broadcast codes. If the destination cannot decode X_1 , then it cannot decode X_2 either.

3.2.3 Calculation of Probabilities

As discussed before, we continue the assumption of a Gaussian source with zero mean and unit variance passing through a "Two-Layer, Three-Node" fading relay channel and the source and the relay share equal power levels. When the broadcast/multicast strategy is employed, the overall expected distortion expression can be written as:

$$\begin{aligned}
 E_{relay}[D] = & \Pr(\text{cannot decode either } X_1 \text{ or } X_2) + \\
 & \Pr(\text{can decode } X_1, \text{ cannot decode } X_2)2^{-bR_1} + \\
 & \Pr(\text{can decode both } X_1 \text{ and } X_2)2^{-b(R_1+R_2)}
 \end{aligned} \tag{3.19}$$

For a simpler notation, cannot decode either X_1 or X_2 , can decode X_1 cannot decode X_2 , and can decode both X_1 and X_2 are denoted by $\bar{X}_1\bar{X}_2$, $X_1\bar{X}_2$, and X_1X_2 respectively. For each single channel, we have three transmit information status, cannot support either R_1 or R_2 , can support R_1 cannot support R_2 , and can support both R_1 and R_2 , denoted by 0, 1, 2, respectively.

A table of received signals at destination is shown above.

Table 3.1: Received information at destination

SD	SR	RD	info@dest.	SD	SR	RD	info@dest.	SD	SR	RD	info@dest.
0	0	0	$\bar{X}_1\bar{X}_2$	1	0	0	$X_1\bar{X}_2$	2	0	0	X_1X_2
0	0	1	$\bar{X}_1\bar{X}_2$	1	0	1	$X_1\bar{X}_2$	2	0	1	X_1X_2
0	0	2	$\bar{X}_1\bar{X}_2$	1	0	2	$X_1\bar{X}_2$	2	0	2	X_1X_2
0	1	0	$\bar{X}_1\bar{X}_2$	1	1	0	$X_1\bar{X}_2$	2	1	0	X_1X_2
0	1	1	$X_1\bar{X}_2$	1	1	1	$X_1\bar{X}_2$	2	1	1	X_1X_2
0	1	2	$X_1\bar{X}_2$	1	1	2	$X_1\bar{X}_2$	2	1	2	X_1X_2
0	2	0	$\bar{X}_1\bar{X}_2$	1	2	0	$X_1\bar{X}_2$	2	2	0	X_1X_2
0	2	1	$X_1\bar{X}_2$	1	2	1	$X_1\bar{X}_2$	2	2	1	X_1X_2
0	2	2	X_1X_2	1	2	2	X_1X_2	2	2	2	X_1X_2

Thus, we can obtain three transmit probabilities:

$$\begin{aligned} \tilde{p}_0 &= \Pr(\text{cannot decode either } X_1 \text{ or } X_2) \\ &= \Phi_{sd}(\bar{X}_1\bar{X}_2)[\Phi_{sr}(\bar{X}_1\bar{X}_2) + (\Phi_{sr}(X_1\bar{X}_2) + \Phi_{sr}(X_1X_2))\Phi_{rd}(\bar{X}_1\bar{X}_2)] \end{aligned} \quad (3.20a)$$

$$\begin{aligned} \tilde{p}_1 &= \Pr(\text{can decode } X_1, \text{ cannot decode } X_2) \\ &= \Phi_{sd}(\bar{X}_1\bar{X}_2)[\Phi_{sr}(X_1\bar{X}_2)(\Phi_{rd}(X_1\bar{X}_2) + \Phi_{rd}(X_1X_2)) + \Phi_{sr}(X_1X_2) \\ &\quad \Phi_{rd}(X_1\bar{X}_2)] + \Phi_{sd}(X_1\bar{X}_2)[1 - \Phi_{sr}(X_1X_2)\Phi_{rd}(X_1X_2)] \end{aligned} \quad (3.20b)$$

$$\begin{aligned} \tilde{p}_2 &= \Pr(\text{can decode both } X_1 \text{ and } X_2) \\ &= [\Phi_{sd}(\bar{X}_1\bar{X}_2) + \Phi_{sd}(X_1\bar{X}_2)]\Phi_{sr}(X_1X_2)\Phi_{rd}(X_1X_2) + \Phi_{sd}(X_1X_2) \end{aligned} \quad (3.20c)$$

Here the Φ function presents the probability of received information status at each node through a specific channel. For example, $\Phi_{sd}(\bar{X}_1\bar{X}_2)$ indicates the probability that the destination node receives neither X_1 nor X_2 through the SD channel. Now

for the total 9 probabilities, we have

$$\left\{ \begin{array}{l} \Phi_{ij}(\bar{X}_1\bar{X}_2) = \sum_{n_1=0}^{K_1-1} \sum_{n_2=0}^{K_2-1} \Psi_{ij}(N_1 = n_1, N_2 = n_2) \\ \Phi_{ij}(X_1\bar{X}_2) = \sum_{n_1=K_1}^N \sum_{n_2=0}^{K_2-1} \Psi_{ij}(N_1 = n_1, N_2 = n_2) \\ \Phi_{ij}(X_1X_2) = \sum_{n_1=K_1}^N \sum_{n_2=K_2}^N \Psi_{ij}(N_1 = n_1, N_2 = n_2) \end{array} \right. \quad (3.21)$$

where $ij \in \{sd, sr, rd\}$.

Applying the analysis of Ψ function in the last section, we obtain

$$\begin{aligned} \Psi_{ij}(N_1 = n_1, N_2 = n_2) &= \binom{N}{n_2} (q_{ij}^{X_2})^{n_2} (1 - q_{ij}^{X_2})^{N-n_2} \times \\ &\quad \binom{N - n_2}{n_1 - n_2} (q_{ij}^{X_1})^{n_1 - n_2} (1 - q_{ij}^{X_1})^{N - n_1} \end{aligned} \quad (3.22)$$

Here $q_{ij}^{X_l}, l = 1, 2$ presents the independently decodable probability that the j node can receive X_l through the channel from node i to node j . Note that this probability covers both the probability of $X_1\bar{X}_2$ and that of X_1X_2 . Thus,

$$\left\{ \begin{array}{l} q_{ij}^{X_1} = p_{ij}^{X_1\bar{X}_2} + p_{ij}^{X_1X_2} \\ q_{ij}^{X_2} = p_{ij}^{X_1X_2} \end{array} \right. \quad (3.23)$$

Chapter 4

Optimal Power Allocation for Superimposed SR Source with MDC

4.1 Two-Layer Optimal Power Allocation

In this section, we first consider two layers of superimposed SR layers with MDC passing through the three nodes relay. The channel fading distribution also only have two states (i.e., $L = 2$). The channel power gain realization is assumed either to be α or β , where $\beta > \alpha \geq 0$. The transmitter sends two layers of symbols at once in a broadcast SPC signal with a total transmit power constraint through each relay channel, $\pi = \pi_1 + \pi_2$. According to the discussion in the previous section, the decodable rates of layer 1 and 2, denoted as R_1 and R_2 respectively, at the receiver's channel with SIC are given as follows:

$$R_1 = \log \left(1 + \frac{\gamma_1(1 - \alpha)\pi_s}{1 + \gamma_1\alpha\pi_s} \right) \quad (4.1a)$$

$$R_2 = \log(1 + \gamma_2\alpha\pi_s) \quad (4.1b)$$

Then, the expected distortion with R_1 and R_2 is as below:

$$E_{relay}[D] = \tilde{p}_0 + \tilde{p}_1 \cdot 2^{-bR_1} + \tilde{p}_2 \cdot 2^{-b(R_1+R_2)} \quad (4.2)$$

The minimum expected distortion with two layers can be derived by solving the optimization problem below as a standard convex programming problem:

$$E_{\text{relay}}[D]^* = \min_{0 \leq \alpha \leq 1} \left\{ \tilde{p}_0 + \left(1 + \frac{\gamma_1(1-\alpha)\pi_s}{1 + \gamma_1\alpha\pi_s} \right)^{-b} \cdot (\tilde{p}_1 + \tilde{p}_2 \cdot (1 + \gamma_2\alpha\pi_s))^{-b} \right\} \quad (4.3)$$

4.1.1 KKT Conditions

Given N, K_1, K_2, p_1 and p_2 , the term \tilde{p}_l becomes a specific statistical constant for $l = 1, 2$. The minimization can be solved by evaluating the Lagrangian form below:

$$L(\alpha, \lambda_1, \lambda_2) = E_{\text{relay}}[D]^* - \lambda_1\alpha - \lambda_2(1 - \alpha) \quad (4.4)$$

Applying the Karush-Kuhn-Tucker (KKT) conditions, the gradient of the Lagrangian vanishes at the optimal power allocation α^* . Specifically, the KKT conditions stipulate that at α^* , either one of the inequality constraints in Eq. 4.4 is active, or $dE_{\text{relay}}[D]^*/d\alpha = 0$, which subsequently yields to the solution:

$$\alpha^* = \frac{1}{\gamma_2\pi_s} \left(\left(\frac{\tilde{p}_2}{\tilde{p}_1} ((\gamma_1 - \gamma_2)\pi_s) \right)^{\frac{1}{b+1}} - 1 \right) \quad (4.5)$$

4.1.2 Achievement of Global Minimum

In this subsection, we show that for the optimal α^* , $E_{\text{relay}}[D]^*$ can achieve global minimum. To show the global minimum, we need to prove $E_{\text{relay}}[D]$ is a convex function. Consider the second-order derivative of $E_{\text{relay}}[D]$. As long as the second-order derivative of $E_{\text{relay}}[D]$ is smaller than 0, the conclusion can be drawn that $E_{\text{relay}}[D]$ is a convex function.

Proof 1 Denote $E_{\text{relay}}[D]$ as $f(\alpha)$:

$$\begin{aligned} f(\alpha) &= \tilde{p}_0 + \left(1 + \frac{\gamma_1(1-\alpha)\pi_s}{1 + \gamma_1\pi_s\alpha} \right)^{-b} \cdot (\tilde{p}_1 + \tilde{p}_2 \cdot (1 + \gamma_2\pi_s\alpha))^{-b} \\ &= \tilde{p}_0 + \tilde{p}_1 \cdot \left(\frac{1 + \gamma_1\pi_s\alpha}{\gamma_1\pi_s} \right)^b + \tilde{p}_2 \cdot \left(\frac{1 + \gamma_1\pi_s\alpha}{\gamma_1\pi_s(1 + \gamma_2\pi_s\alpha)} \right)^b \end{aligned} \quad (4.6)$$

The first-order derivative of $f(\alpha)$ is:

$$f(\alpha)' = \frac{\tilde{p}_1 b}{(\gamma_1 \pi_s)^{b-1}} (1 + \gamma_1 \pi_s \alpha)^{b-1} + \frac{\tilde{p}_2 (\gamma_1 - \gamma_2) b}{\gamma_1^b \pi_s^{b_1}} \cdot \frac{(1 + \gamma_1 \pi_s \alpha)^{b-1}}{(1 + \gamma_2 \pi_s \alpha)^{b+1}} \quad (4.7)$$

The second-order derivative of $f(\alpha)$ is:

$$f(\alpha)'' = \frac{\tilde{p}_1 b(b-1)}{(\gamma_1 \pi_s)^{b-2}} (1 + \gamma_1 \pi_s \alpha)^{b-2} + \frac{\tilde{p}_2 (\gamma_1 - \gamma_2) b}{\gamma_1^b \pi_s^{b-2}} \cdot \frac{(1 + \gamma_1 \pi_s \alpha)^{b-2}}{(1 + \gamma_2 \pi_s \alpha)^{b+2}} \cdot \{b(\gamma_1 - \gamma_2) - (\gamma_1 + \gamma_2) - 2\gamma_1 \gamma_2 \pi_s \alpha\} \quad (4.8)$$

Since the previous assumption $\gamma_2 > \gamma_1 \geq 0$, we can derive

$$\gamma_1 - \gamma_2 < 0 \quad (4.9a)$$

$$b(\gamma_1 - \gamma_2) - (\gamma_1 + \gamma_2) - 2\gamma_1 \gamma_2 \pi_s \alpha < 0 \quad (4.9b)$$

From Eq. 4.8 4.9a 4.9b, we can easily acquire $f(\alpha)'' \geq 0$. Since the function $f(\alpha)''$ is always greater than or equal to 0, we can draw the conclusion that $E_{\text{relay}}[D]$ is a convex function. Therefore, $E_{\text{relay}}[D]$ can achieve a global minimum for the optimal α^* .

4.2 Recursive Relations and Multiple-Layer Optimal Power Allocation

The optimization for the system with more than two quality layers is obviously subject to very high complexity and is computationally intractable when system dynamics are considered. The approach is to go through a recursive process, in which the 2-layer optimization is performed iteratively and recursively on the L -layers power allocation problem. All the layers between the 1st and the $(L-1)$ -th are treated as a single aggregated layer such that the optimal power allocation to the aggregated layer and the L -th layer becomes a 2-layer optimization problem in the first recursion. With the results of π_L and $\pi_{1..(L-1)}$, the next 2-layer optimization problem is formed by considering a new aggregated layer from the 1st to $(L-2)$ -th layer and the individual $(L-1)$ -th layer, where the new total power for allocation

becomes $\pi_s - \pi_L$. This recursive process continues until all the layers are specifically allocated.

Let T_l denote the cumulative transmit power for the layers from layer l to layer L (i.e. $T_l \triangleq \sum_{j=l}^L \pi_j$, for $l = 1, \dots, L$). According to this definition, we can directly obtain $T_1 = \pi_s$; hence, the optimization problem is over $L - 1$ variables of power allocation, T_2, \dots, T_L . The channel rate R_l , in the receiving layer l at a receiver with successive decoding using SIC is calculated as follows:

$$R_l = \log\left(1 + \frac{\gamma_l(T_l - T_{l+1})}{1 + \gamma_l T_{l+1}}\right) \quad (4.10)$$

Similarly, given $N, K_1, \dots, K_L, p_1, \dots, p_L$, the term \tilde{p}_l becomes a specific statistical constant for $l = 1, \dots, L$. The minimum expected distortion can be rewritten as:

$$E_{\text{relay}}[D]^* = \min_{T_1, T_2, \dots, T_L} \tilde{p}_0 + \sum_{l=1}^L \left(\prod_{j=1}^l \left(\frac{1 + \gamma_j T_j}{1 + \gamma_j T_{j+1}} \right) \right)^{-b} \quad (4.11)$$

subject to:

$$0 \leq T_{l+1} \leq T_l \text{ for } l = 1 \dots L,$$

$$T_1 = \pi_s,$$

$$T_{L+1} \triangleq 0.$$

Factor the sum of cumulative products in Eq. 4.11 and rewrite the expected distortion as a set of recursive relations:

$$D_L \triangleq \tilde{p}_L (1 + \gamma_L T_L)^{-b} \quad (4.12a)$$

$$D_l = \left(\frac{1 + \gamma_l T_l}{1 + \gamma_l T_{l+1}} \right)^{-b} (\tilde{p}_l + D_{l+1}) \quad (4.12b)$$

where l runs from $L - 1$ down to 1. The term D_l can be now interpreted as the cumulative distortion from layer $(L - 1)$ down to layer 1. Particularly, $D_1 = E_{\text{relay}}[D]$.

In general, in the $l - th$ recursive step, the power allocation between layer l and layer $l + 1$ can be found by the optimization:

$$D_l^* = \min_{T_l, T_{l+1}} \left(\frac{1 + \gamma_l T_l}{1 + \gamma_l T_{l+1}} \right)^{-b} \{ \tilde{p}_l + \tilde{p}_{l+1} (1 + \gamma_{l+1} T_{l+1})^{-b} \} \quad (4.13)$$

subject to the same constraints as that for Eq. 4.11.

Define the minimum distortion at the top layer as $D_l^* \triangleq D_L$. The solution of Eq. 4.13 is derived as:

$$D_l^* = G_l \cdot (1 + \gamma_l T_l)^{-b} \quad (4.14)$$

where

$$G_l = (1 + \gamma_l T_{l+1}^*)^b \{ \tilde{p}_l + \tilde{p}_{l+1} (1 + \gamma_{l+1} T_{l+1})^{-b} \} \quad (4.15)$$

in which l runs from $L - 1$ down to 1.

As previously described, at the top layer, the minimum distortion is defined by Eq. 4.14. Thus, for the next recurrence step $L - 1$, we can obtain:

$$D_{l-1}^* = \min_{T_{l-1}, T_l} \left(\frac{1 + \gamma_{l-1} T_{l-1}}{1 + \gamma_{l-1} T_l} \right)^{-b} \{ \tilde{p}_{l-1} + \tilde{p}_l (1 + \gamma_l T_l)^{-b} \} \quad (4.16)$$

subject to the same constraints as that for Eq. 4.11.

Note that Eq. 4.16. has the same form as the one of Eq. 4.13, with the following parameters substitution:

$$\begin{aligned} \tilde{p}_l &\rightarrow \tilde{p}_{l-1}, \\ \tilde{p}_{l+1} &\rightarrow \tilde{p}_l, \\ \gamma_l &\rightarrow \gamma_{l-1}, \\ \gamma_{l+1} &\rightarrow \gamma_l \end{aligned} \quad (4.17)$$

Through such recursive iteration, the two-layer optimization procedure described in Chapter 5 can be iteratively applied to find the minimum distortion and the optimal power allocation between the current layer and the aggregate higher layer.

Chapter 5

Numerical Results and Comparisons

This chapter investigate the optimal power allocations and the expected distortion under various factors of channels and the total power constraint. After that, a comparison is conducted to evaluate the expected distortions between the proposed scheme (denoted as SR-MDC-SPC) and a counterpart scheme (denoted as SR-SPC) only using SR at the source and SPC at the channel that is mostly studied in the previous literature with the assumption of no or very slow fading.

Starting with a two-layer case, assume that $\gamma_1 = 1, N = 32, K_1 = 28$ and $K_2 = 24$. The optimal power allocation and the minimum expected distortion for a two-layer case are verified under different probabilities for the receiver to successfully decode the second layer data (i.e., $p_1 = 0.12, p_2 = 0.76$ and $p_1 = 0.24, p_2 = 0.68$). The results are shown in Fig. 5.1 and Fig. 5.2.

As shown in Fig. 5.1, the optimal power allocation to the layer 2 has a sharp increase in the small range of channel power gain realization when $p_1 = 0.12$ and $p_2 = 0.76$. In the cases of $b = 0.5$ and $b = 1$, this increase can be up to the total power π_s . As γ_2 increases, the power allocation assigned to layer 2 begins to decrease, which is because the overall expected distortion would be dominated by layer 1 when layer 2 has a stronger channel. When there is a smaller chance to

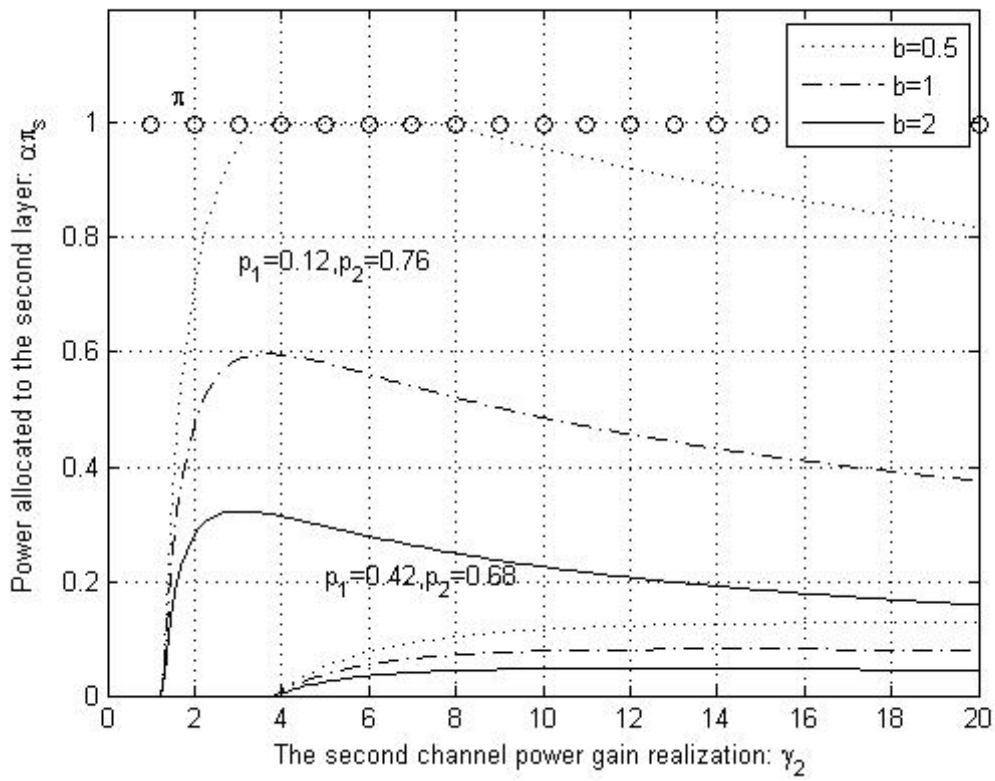


Figure 5.1: Optimal Power Allocation between Two Layers ($\pi_s = 0$ dB)

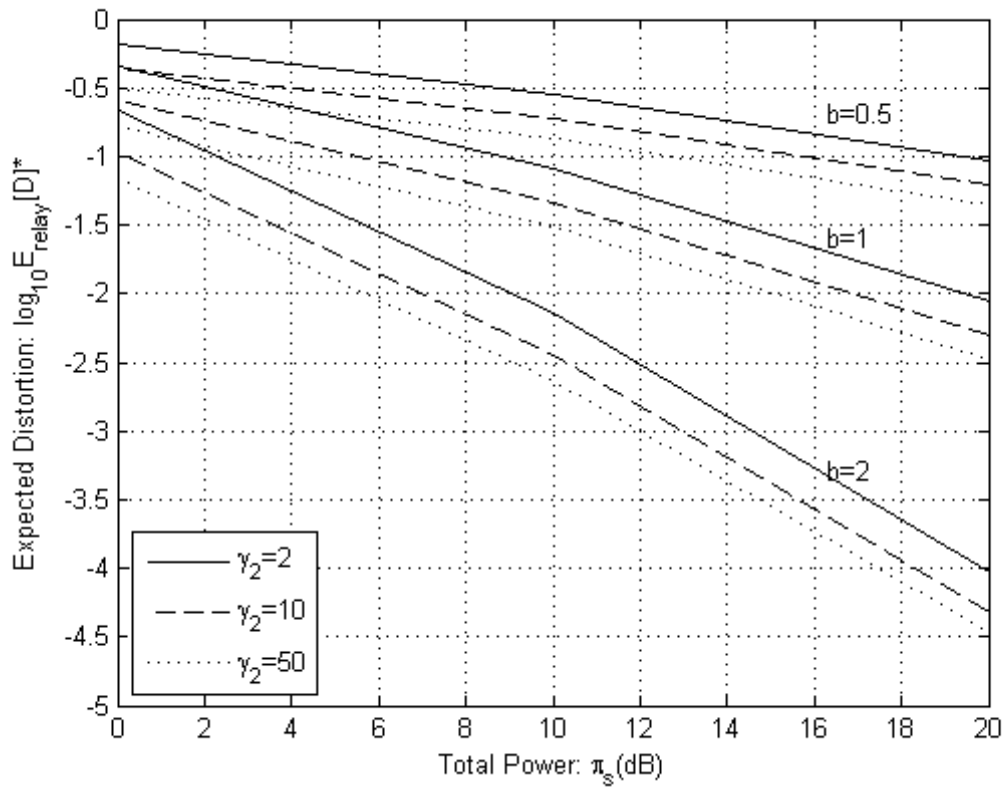


Figure 5.2: Two-Layer Minimum Expected Distortion ($p_1 = 0.12, p_2 = 0.76$)

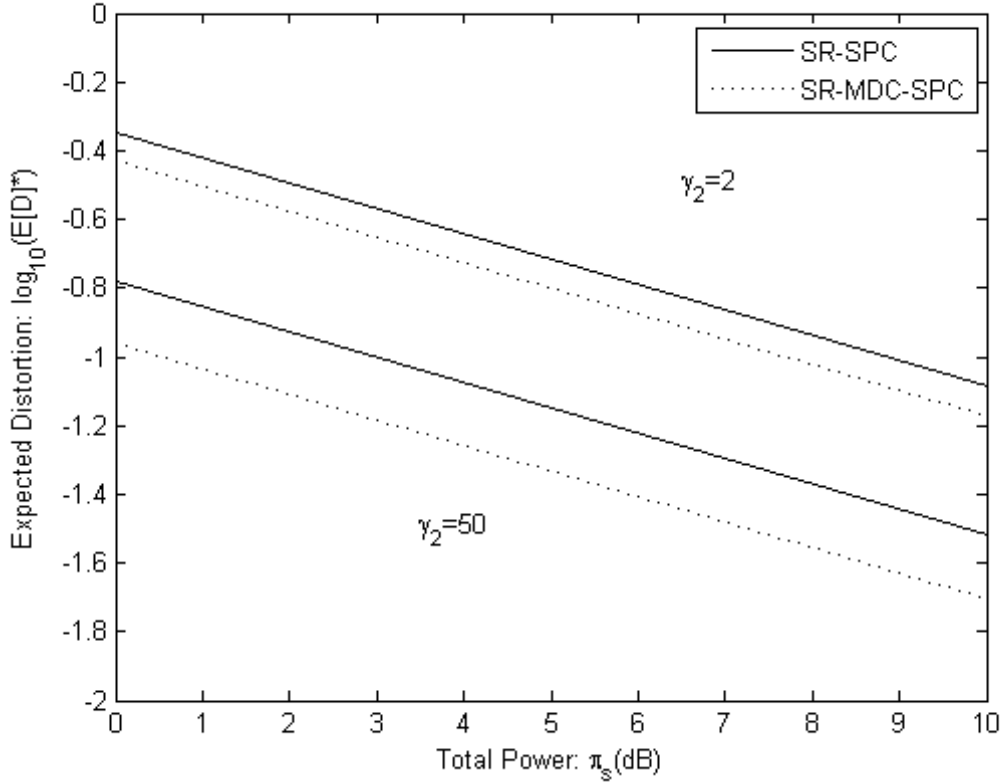


Figure 5.3: The Expected Distortions of Both Schemes for Two Layers

decode layer 2 (i.e. $p_1 = 0.24$, $p_2 = 0.68$), $\alpha\pi_2$ is expected to be smaller.

Regarding the expected distortion, Fig. 5.2 shows that an increase of total power leads to a decrease of the expected distortion, which is expected. However, it is found that a stronger channel in decoding layer 2 does not improve much on the expected distortion, since the overall distortion is mainly dominated by layer 1.

Fig. 5.3 shows the result of expected distortion in a two-layer case under fading with bandwidth expansion/compression factor $b = 1$ and the probability of channel gain realization γ_2 as $p_2 = 0.9$. The proposed scheme yields the smaller minimum distortion, when γ_2 becomes larger, the advantage of the proposed scheme is getting more significant.

We also evaluate and compare the expected distortions under the optimal power allocations for the case of multiple-layer (i.e., $L = 5$) under fading where assuming

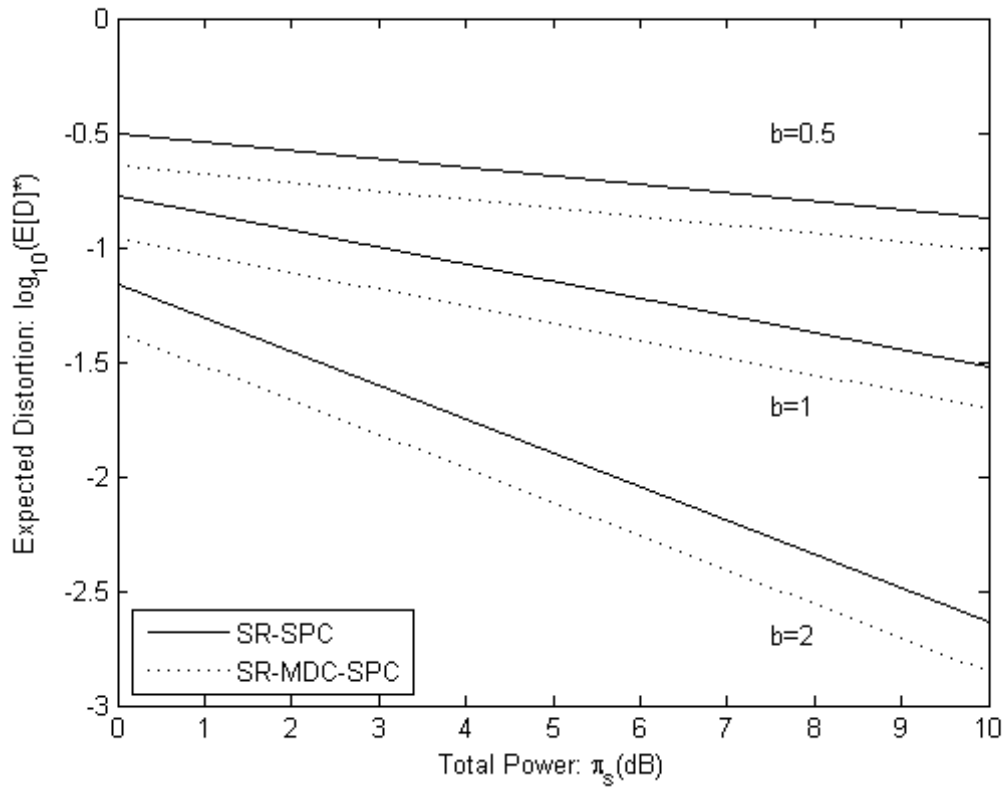


Figure 5.4: The Expected Distortions of Both Schemes for Multiple Layers (i.e.. $L = 5$)

$N = 32$, $K_1 = 28$, $K_2 = 26$, $K_3 = 24$, $K_4 = 22$, and $K_5 = 20$. For the multiple-layer power allocation, use the recursive algorithms described in Chapter 4 to compute the power allocation for each layer and derive the minimum expected distortion. Fig. 5.4 shows the result of expected distortions under both schemes for different bandwidth expansion/compression factors b , which again shows that a smaller distortion can be achieved under corresponding optimal power allocation in both schemes. The higher value of b results with a lower expected distortion, because more source symbol in a channel symbol can be recovered by the MDC despite the fading.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This thesis has investigated a cross-layer framework using SR information encoded with MDC at the source and SPC at the channel for wireless coded fading relay channel using broadcast/multicast strategy. The advantages of this framework can tackle the impairments to SR layers due to the fast fading effect by strategically exploiting protection properties of the RS-code based MDC in mitigating the channel outage for some higher layers. Lost data in some higher layers are therefore recovered to minimize the distortion.

A comprehensive formulation is derived for the framework. An optimization problem for minimizing the overall expected distortion is developed, which can be used to obtain the optimal power allocation for each quality layer. By taking the solution with two layers as the basis, a recursive algorithm is developed which can solve a general case for the number of layers greater than two.

Numerical evaluations were conducted to verify the proposed solutions to the formulated optimization problem. The cross-layer framework also proved to achieve a smaller minimum expected distortion when compared to the previous common approach using SR at the source and SPC at the channel only.

Finally, it is claimed that this work is one of the earliest investigations on a cross-layer framework jointly using SR-MDC-SPC to tackle the impairments under the coded fading relay channel in wireless systems, which is not addressed in previous literature so far; and the proposed formulation and solution can well enable the framework to be a realizable coded wireless broadcast/multicast strategy for emerging wireless broadband transmission applications.

6.2 Future Work

This research focused on the optimal power allocation for successive refinable source with multiple descriptions over fading relay channel using broadcast strategies. A 'Two-Layer, Three-Node' fading relay channel has been considered.

- To generalize our results, the channels which contain multi-level cooperative relays (CR) in between the source and the destination (multiple nodes) will be explored. The multi-level CR in this research refers to a multi-hop transmission that both the source and internal relays cooperatively transmit to a destination. Different from the traditional multi-hop non-cooperative transmission (NCT) in which the received data are simply forwarded at each internal node to the next-hop node, it exploits the concurrent transmission of signals to the destination, instead of taking the signals sent by different relay nodes as interference with each other as that in the traditional multi-hop NCT.
- Moreover, besides power allocation, other topics such as location planning and frequency planning for multiple relays also pose more challenging issues. The location planning problem (or placement problem) is the foremost critical issue for network planning and deployment, which has a direct impact on the subsequent QoS provisioning, especially the data rate and transmission delay. On the other hand, for a large-scale wireless access network which

covers metropolitan areas, an optimal frequency reuse scheme is important to increase the capacity of a wireless system. Although frequency planning can be conducted after location planning, a joint design of location planning and frequency planning can achieve better performance. However, it is much more challenging. It can be extended the current research by relaxing the assumption of frequency reuse factor of one. In other words, another design dimension can be considered together with location planning to further increase the system performance in terms of wireless bandwidth utilization.

6.2.1 Mathematic Techniques and Tools

One of the key mathematical methods that is used in our research is optimization. We will optimize the system parameter settings through a cross-layer design, and the corresponding optimization tools such as GAMS, CPLEX, and search heuristics [72] will be utilized to solve problems in parameter selection and tuning. The parameters we are interested include associated signal powers allocated to each of the quality layers and the distance for location planning within a certain determined fading area, as well as thresholds for the received symbol package data (N, K_1, \dots, K_L) . Extensive simulations will be conducted using C/C++ language combined with MATLAB to build the event-driven simulation environment.

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