A Stochastic Programming Model for a Day-Ahead Electricity Market: a Heuristic Methodology and Pricing

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

This thesis presents a multi-stage linear stochastic mixed integer programming (SMIP) model for planning power generation in a pool-type day-ahead electricity market. The model integrates a reserve demand curve and shares most of the features of a stochastic unit commitment (UC) problem, which is known to be NP-hard. We capture the stochastic nature of the problem through scenarios, resulting in a large-scale mixed integer programming (MIP) problem that is computationally challenging to solve. Given that an independent system operator (ISO) has to solve such a problem within a time requirement of an hour or so, in order to release operating schedules for the next day real-time market, the problem has to be solved efficiently. For that purpose, we use some approximations to maintain the linearity of the model, parsimoniously select a subset of scenarios, and invoke realistic assumptions to keep the size of the problem reasonable. Even with these measures, realistic-size SMIP models with binary variables in each stage are still hard to solve with exact methods. We, therefore, propose a scenario-rolling heuristic to solve the SMIP problem. In each iteration, the heuristic solves a subset of the scenarios, and uses part of the obtained solution to solve another group in the subsequent iterations until all scenarios are solved. Two numerical examples are provided to test the performance of the scenario-rolling heuristic, and to highlight the difference between the operative schedules of a deterministic model and the SMIP model.

Motivated by previous studies on pricing MIP problems and their applications to pricing electric power, we investigate pricing issues and compensation schemes using MIP formulations in the second part of the thesis. We show that some ideas from the literature can be applied to pricing energy/reserves for a relatively realistic model with binary variables, but some are found to be impractical in the real world. We propose two compensation schemes based on the SMIP that can be easily implemented in practice. We show that the compensation schemes with make-whole payments ensure that generators can have non-negative profits. We also prove that under some assumptions, one of the compensation schemes has the interesting theoretical property of minimizing the variance of the profit of generators to zero. Theoretical and numerical results of these compensation schemes are presented and discussed.

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Dedication

To my grandmother and parents.

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Nomenclature

A. *Indices*

i or j	Network buses.
g	Generating units.
s or a	Scenarios. s, a =0,1,…, A.
t or τ	Time period t, τ=1, 2, …, T.
n	Reserve shortage blocks. n=1, 2, …, N.
b	Blocks in generation offers. b=1, 2, … B.
m	Reserves types. Reserves types; $m=1,2,\ldots,M$: e.g. 1: 10-minute spinning reserve. 2: 10-minute non-spinning reserve.

B. *Sets*

I	All nodes in the power system.
$G(i)$	All units/load facilities associated with node i.
$G^f(i)$,$G^s(i)$,$G^r(i)$	Subsets of $G(i)$ for fast start-up, slow start-up and must-run units.
$M(g)$	All types of reserves that unit g can supply; $M(g) \subseteq \{1, 2\}$.

C. *Parameters*

C_{gb}^{E}	Energy offer price from unit g in block b (\$/MWh).
C_{mn}^{S}	Reserve shortage price of type m in block n (\$/MW).
C^{LS}	Penalty cost of load shedding (\$/MWh).

$C_{g\tau}$	Start-up cost for unit *g* with a down time τ ($).
C_g^{NL}	No-load cost for generating unit *g* ($/hour).
$Q_g^{\max}$	Maximum capacity offered by unit *g* (MW).
Q_{gb}^{E}	Generation offer in block *b* of unit *g* (MW).
Q_{mn}^{S}	Quantity of reserve shortage type *m* in block *n* (MW).
Q_g^{MSL}	Minimum generation level of unit *g* (MW).
Q_m^R, $Q_{m\_\min}^R$	Target and minimum reserve requirement of type *m* (MW).
Q_{it}^{D}	Power demand at node *i* during period *t* (MW).
π_s	Probability of scenario *s*; $\sum_{s\in S}\pi_s = 1$.
lk_{sat}	=1 if scenario *s* matches scenario *a* in period *t*; =0 otherwise.
μ_{gst}	Unit readiness level ($0 \le \mu_{gst} \le 1$; 0 down, 1 up, $0 < \mu_{gst} < 1$ derated).
$\lceil \mu_{gst} \rceil$	Returns the nearest integer greater than or equal to μ_{gst} : $\lceil \mu_{gst} \rceil$=1, if μ_{gst}>0; $\lceil \mu_{gst} \rceil$=0,if μ_{gst}=0.
U_{ij}	Line power capacity between nodes *i* and *j* (MW).
B_{ijt}	Line susceptance between nodes *i* and *j* in period *t* (Ω^{-1}).
υ_{ijst}	Transmission line contingency parameters between nodes *i* and *j* in scenario *s* in period *t* ($0 \le \upsilon_{ijst} \le 1$).
R_g^{up}, R_g^{dn}	Ramp up and ramp down limit of unit *g* (MW).

R_{gm}^{RP}	Upper limit on reserve type *m* for generator *g* (MW).
R_{g}^{su}, R_{g}^{sd}	Start-up and shut-down limit of unit *g* (MW).
T^{int}	Time interval (1 hour in this thesis).
T_{g}^{up}, T_{g}^{dn}	Minimum up and down time of unit *g* (Hour).
τ_{g}^{C}	Cold start time for generating unit *g* (Hour).
α	Limit on phase angles, $\alpha > 0$.

D. *Variables*

q_{gbst}^{E}	Power output in block *b* of unit *g* in scenario *s* in period *t* (MW).
q_{ist}^{LS}	Load shedding in scenario *s* in period *t* at node *i* (MW).
q_{gmst}^{R}	Reserve capacity for class *m* of unit *g* in scenario *s* in period *t* (MW).
q_{mnst}^{S}	System reserve shortage in type *m* in block *n* in scenario *s* in period *t* (MW).
ω_{gst}	1 if unit *g* in period *t* is online; 0 otherwise.
θ_{ist}	Voltage angle at node *i* in scenario *s* in period *t* (rad).
z_{gst}	Start-up costs of generating unit *g* in scenario *s* until period *t* ($).

Abbreviations and Terminology

CPU	Computer processing unit
DA	Day-ahead
DAM	Day-ahead market
GenCos	Generating companies
IP	Integer programming
ISO	Independent system operator
Load	Electricity demand
LP	Linear programming
LR	Lagrangian relaxation
MIN	Minimize
MIP	Mixed integer programming
MW	Megawatt
MWh	Megawatt-hour
NP	Nondeterministic polynomial
PJM	Pennsylvania-New Jersey-Maryland
RT	Real-time
RTM	Real-time market
SMIP	Stochastic mixed integer programming
UC	Unit commitment
VOLL	Value of lost load

Chapter 1
Introduction

In this chapter, we introduce some of the relevant concepts, terminologies, and polices in electricity markets. We then detail the motivations, objectives, and contributions of the thesis.

1.1 Relevant Background

1.1.1 Independent System Operator

Electric power industries around the world are affected by restructuring. The three components of the industry, i.e., generation, transmission, and distribution, that were vertically integrated, are operated separately under restructuring. With this, a neutral entity is required to guarantee the independent operation of the transmission grid, settle market price and maintain system reliability and security. An Independent System Operator (ISO) fills this requirement and serves as the market coordinator. It is important that an ISO be independent of all the market participants, such as generating companies (GenCos), transmission companies (Transcos), distribution companies (DisCos), and retailers. It is the role of the ISO to maintain the system security and dispatch power economically. To this end, it has the authority to call on GenCos to plan their power generation according to its instruction and to shed the load of customers in order to maintain supply-demand balance. The ISO also forecasts electricity demand and runs relevant models, e.g., unit commitment (UC) models, to ensure that systems are operating efficiently. In addition, the ISO has the authority to establish rules, set transmission tariffs and manage line congestion (Shahidehpour et al., 2002).

Both MinISOs and MaxISOs are responsible for transmission security, but MaxISOs have a broader range of responsibility. A MaxISO can coordinate market participants to ensure system reliability and security and can settle market prices. For example, a MaxISO can use data received from market participants, such as costs, prices and other variables, to run a UC model and obtain commitment states of generators. From this, the MaxISO can devise a power generation plan and set relevant prices. PJM (Pennsylvania-New Jersey-Maryland), for example, falls into this category of ISO. The ISO in this thesis is assumed to be a MaxISO, and it is in charge of system reliability and market settlement. On the other hand, MinISOs are mainly in charge of transmission security without any market roles. For example, California ISO is a MinISO (Shahidehpour et al., 2002).

1.1.2 GenCos

GenCOs own the actual power generating plants, and as such are very important market entities. The power generating plants they own may include different types of generating units. A GenCO may trade electricity with other market entities directly or sell electricity to an ISO. Buyers can then purchase electricity from an ISO to meet their demand, depending on the model of electricity market. If there is a scheduled outage, a GenCO needs to report it to the ISO in advance for approval. In a restructured electricity market, the objective of a GenCO is to maximize its own profit. It does not have to consider system-wide profits or costs since it is not integrated with transmission and distribution.

1.1.3 Market Models

There are three basic models within electrical market structures: the PoolCo model, the bilateral contract model and the hybrid model (Shahidehpour et al. 2002). In a PoolCo model, an ISO receives generation offers and demand bids from electric power generators and buyers. In general, the generation offer submitted by a generator is incremental, depending on a ratio of price and power quantity. As its power output increases, the price increases accordingly. Buyers submit their bids in a similar way, but in an opposite direction, leading to the supply-demand curve shown in Figure 1-1. The point where these two curves intersect represents the competitive price and competitive power quantity. Based on this information, the ISO will implement an economic dispatch to plan power generation efficiently and generate the price signals to both sellers and buyers. Ideally, the competitive market price is equal to the highest price submitted by a generator in a PoolCo model, assuming that its offer is accepted by the ISO.

In a bilateral contract model, two parties in the market can trade electricity independent of an ISO. Bilateral contracts give traders more flexibility to design their own contract terms but trading parties may face high negotiating costs and potential risks, such as default of counterparties.

A hybrid model, as its name suggests, lets market participants choose either a PoolCo model or a bilateral contract model depending on which market model they feel best meets their individual needs.

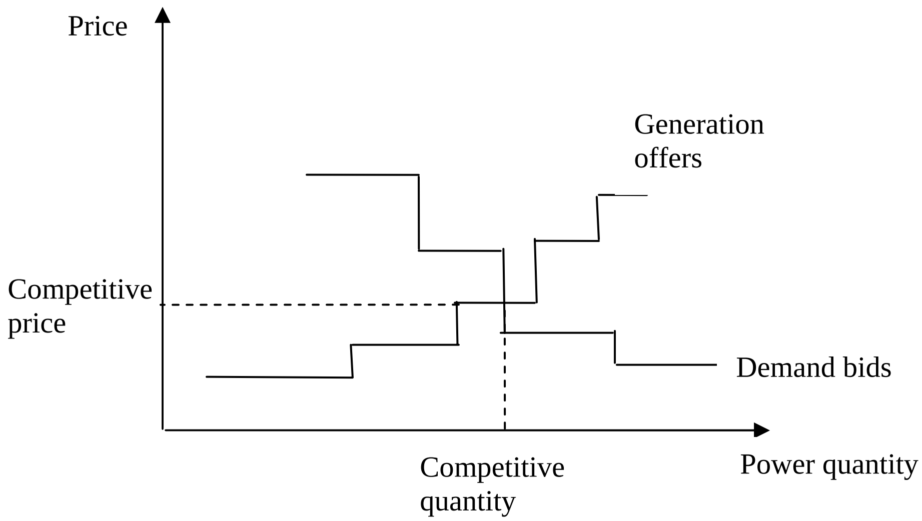


Figure 1-1 An illustrative example of generation offers and demand bids

1.1.4 Day-Ahead Electricity Market and Real-Time Market

Electricity markets can be classified as day-ahead market (DAM) and real-time market (RTM). DAM, as its name implies, refers to the market that exists during the 24 hours prior to commencement of the RTM; it is used for scheduling the resource for the next day. Both energy and ancillary service, e.g., reserves, can be traded in a DAM. In a DAM, an ISO receives day-ahead (DA) generation offers and demand bids from sellers and buyers, and then evaluates the information and plans the operating schedule of accepted generating units and produces a set of DA prices that can be a good predictor of spot price. PJM, NYISO (New York ISO), ISONE (ISO New England) all operate DAMs (Hunt, 2002).

In a DAM, generating units not only make power dispatch commitments but also financial commitments. Therefore, it can reduce the potential possibility of gaming, e.g., a GenCo could intentionally withdraw its capacity with short notice in RTM, and then the ISO would have to call upon expensive alternatives to supply energy. With such gaming in a RTM, the energy price would increase, depending on the marginal cost of the expensive generators. Therefore, the GenCo that breaks its promise will benefit from the high energy price. However, in a DAM the energy price is locked, and the ISO has more alternatives in DAM than in RTM; consequently, GenCos lose the incentive of gaming. In addition, a DAM can allow GenCos to update their predicted output level and commitment states in advance. In addition, generating units with high start-up costs that have to be turned on and turned off every day can benefit from a DAM; it can integrate their start-up or shut-down decision into their generation offers (Hunt, 2002).

An RTM is also called a balancing market; it adjusts the deviation between DAM and RTM if load, generation, and transmission in real-time (RT) are different from those in DAM. Any RT energy imbalance can be adjusted by automatic generation control, spinning, nonspinning, and supplemental reserves that have different response times and are subject to ramping limits. The so-called "two-settlement system" operated by PJM and NYISO consists of a DAM and a RTM. Figure 1-2 shows an overview of the timeline of the DAM and the RTM run by PJM (PJM Manual 11, 2009).

In this thesis, we only focus on a DAM where buyers and sellers bid for energy only, and assume that there are no separate reserve markets where market participants bid for reserves. With one model, we calculate both energy prices and reserve prices.

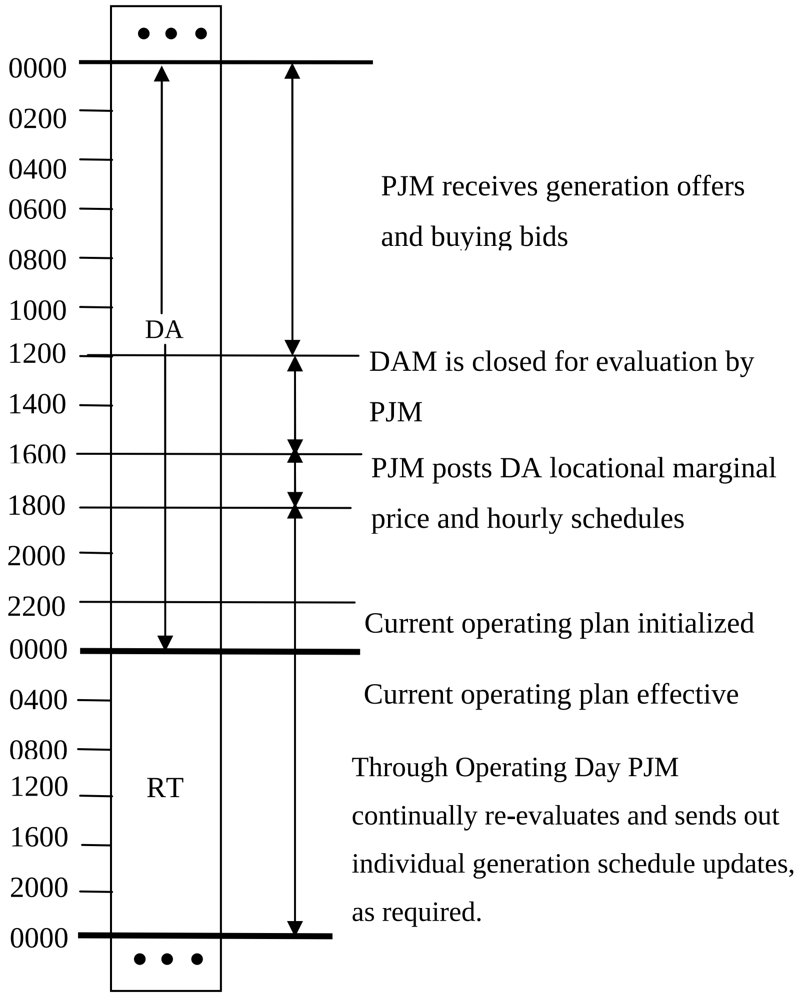


Figure 1-2 An overview of PJM scheduling timeline (PJM, 2009)

1.1.5 Relevant Characteristics in an Electric Power System

GenCos own various generating units; these generating units use different fuel and have different capacities and cost functions. A power generation system may include hydro plants, nuclear plants, thermal plants, and wind plants. Depending on the fuel they use, these plants have different cost functions. The power generation cost of thermal plants has two parts: fixed costs and fuel costs. Fixed costs include the costs that are used to start up or shut down a plant, no-load costs and relevant maintenance costs, labor costs, and so on. No-load costs refer to costs associated with maintaining a generating unit online while the generating unit does not have any power output. Variable costs are dependent on fuel consumption, which is a nonlinear function with respect to the output level of generators. In addition, nuclear plants have high fixed costs, since they are highly expensive to start up; however, their generation cost is very low compared with thermal units. Therefore, units with high start-up costs and low fuel costs are expected to run all the time except for necessary maintenance; they are called base plants. On the contrary, some thermal units have high fuel costs, but they are quick-start units and their start-up cost is low. These units are identified as peaking units or peakers, that is, they are turned on during peak demand hours when other cheap units cannot meet the electricity demand due to unexpected demand increase, generator failure or other contingencies.

1.1.6 Unit Commitment Problems

Power planning problems can be classified in three categories according to their planning time horizon (Ozturk, 2003). A long-term planning problem decides the number, type, and capacity of the generating units that GenCos should own in coming years; it is identified as the power capacity expansion problem. The second set consists of medium-term problems, running from a day to a few weeks. The goal of these problems is to schedule the existing units over the time horizon, i.e., when to turn on some of the generating units and when to turn them off. These problems are classified as UC problems. In the short term, the decision maker needs to decide how much power a generating unit should produce to meet the electricity demand in the RTM; the time horizon may extend from seconds to hours. These problems are identified as economic dispatch problems.

UC problems are used to decide the commitment states of a mix of various generating units and estimate their output level over a given time horizon, while the total generation costs of the generating units are minimized. In addition, the problem should satisfy relevant constraints, particularly some operational constraints. Ramp up/down limit constraints ensure that a generator can only increase its output level by its appropriate ramping rates during a certain time period. Minimum up/down constraints reflect the physical characteristic of generating units, i.e., once a generating unit is turned on or off, it has to stay in that state for a length of time; it cannot be started up and shut down again frequently in a short period of time. In addition, power balance constraints are very important; they ensure that the power system has adequate energy to satisfy the electricity demand (generally referred to as “load” in power engineering) in each time period. Reserve requirement constraints require that the system has surplus capacity to respond to contingencies, such as load spikes and equipment failures. Reserves considered in UC problems usually include spinning reserve and nonspinning reserve. Spinning reserve is defined as the extra capacity of generating units that is synchronized to the power system so that it can serve load immediately if required. On the other hand, nonspinning reserve refers to the capacity that is not connected to the system, but it can be ready to serve load within a certain amount of time, e.g., 30 minutes. Since spinning reserve is crucial to accommodate the imbalance of supply and demand in RT, it is required that a power system should have surplus supply as spinning reserve. The amount of spinning reserve requirement is usually deterministic in all applications of UC problems; the requirement is generally greater than or equal to the capacity of the largest generating unit in the system or a percentage of peak load, whichever has the greater value. In some applications of UC problems, stochastic elements or uncertainties are considered. The uncertainties mainly include load spikes and equipment failures. The resulting problems are identified as stochastic UC problems. In these problems commitment states and power output level are decided for numerous scenarios in contrast to one single scenario in the deterministic version of UC problems. In addition, to prepare for possible contingencies, the schedule of the scenario without any contingencies in the stochastic model is expected to differ from the schedule in the deterministic model, e.g., some generating units in the stochastic model are turned on earlier than in the deterministic model so that they are able to ramp up to higher generation levels in time during later time periods when a contingency occurs; or a generating unit at a location is turned on(off) in a stochastic model, while the same unit is turned off(on) and a different unit is turned on(off) in the corresponding deterministic model. In principle, expected cost of energy from reserves should be included.

As previously mentioned, UC problems are used to determine the commitment states and output levels in each time period of various generating units with different capacity and cost functions. As a result, a UC problem is a mixed integer programming (MIP) problem with binary variables and continuous variables, and it falls into the class of NP-hard problems (Garey and Johnson, 1979; Tseng et al., 1999). If we consider contingencies in a power system, the problem can easily become a stochastic MIP problem, which is equivalent to a large-scale deterministic MIP problem; the solution time may increase exponentially with the number of contingencies. Therefore, the computational challenges of UC problems require efficient solution methodologies.

Usually, in the real world, an ISO runs the UC problem first to determine the commitment states of acceptable generating units after receiving generation offers and demand bids from

GenCos and consumers. Then, it runs the economic dispatch problem in the RTM on a rolling time basis with a time window from 5 minutes to 15 minutes, changing the output of generators to reflect the deviation of loads in RT. For example, PJM, New England, New York, and Ontario Independent Electricity System Operator use a five-minute time window, California employs 10 minutes, and Electric Reliability Council of Texas uses 15 minutes (Hirst, 2001).

1.2 Research Motivation, Objectives and Scope

Reserves, particularly spinning reserves, play important roles in an electricity market because they are the resource used to guarantee the secure operation of the power system. The reserve requirement is set for ensuring that there is adequate capacity available in the system when there are contingencies. There is always a possibility that an ISO has to shed part of the load from customers when there is no extra capacity in the system to satisfy the electricity demand. During these shortage hours, reserve capacity is too low, and the reserve requirement cannot be satisfied. Consequently, the energy price and reserve price should be much higher than during a normal day when there is no contingency at all. High energy prices can also warn customers to be aware of the potential for an energy shortage, reserve scarcity, and blackouts. Therefore, the impact of reserve shortages on energy/reserve prices needs to be investigated. Based on some concepts proposed in Midwest ISO (2005), Cramton et al. (2005), and Hogan (2005), as well as a few related measures that are employed in some electricity markets, we propose a multi-stage stochastic mixed integer programming (SMIP) model incorporating these features into UC problems. However, the model in this thesis is for short-term planning. Thus, some issues discussed in the literature above are beyond the scope of this thesis, such as energy-only markets, installed capacity markets, capacity expansion, missing money problems, etc.

As mentioned in section 1.1, a UC problem is known to be NP-hard, and a UC problem with uncertainties can be easily extended to a large-scale MIP problem as numerous scenarios are considered. The resulting MIP problem is computationally challenging due to its size and the existence of discrete variables. However, an ISO has to solve such a complicated problem in a DAM, and the problem has to be solved within a time requirement, i.e., in an hour or so, so that the operating schedule can be finalized on time prior to the commencement of the RTM. Motivated by the computational difficulty, we use linear direct current (DC) power flow to approximate the actual alternative current power flow, only choose some major contingencies, and employ other assumptions to make the problem as small as possible. Even with these measures, realistic-size SMIP models with binary variables in each stage are still hard to solve with exact methods; therefore, we propose an efficient solution methodology to solve the underlying SMIP problem within a reasonable time limit.

In addition, how to price an MIP has been an intriguing problem due to its nonconvexity (Gomory and Baumol, 1960). The method of obtaining dual prices of a linear program (LP) cannot be applied to procuring commodity prices of an MIP. In a DAM, after an ISO solves the power planning problem with binary variables, it faces the same challenge of how to price electric power from an MIP. Motivated by some previous studies (O'Neill et al., 2005; Wong and Fuller, 2007; Sioshansi et al., 2008), we investigate the pricing issues of an MIP problem and propose appropriate compensation schemes.

The major objectives of this research are:

- Develop an SMIP model for a DAM setting that incorporates some features newly implemented in industry with a UC problem considering uncertainties.
- Propose an efficient heuristic methodology to solve the underlying multi-stage SMIP.
- Investigate the pricing issues of an MIP problem, and propose compensation schemes offering a way to reduce the financial risk for generators.

The focus of this research is mainly on how to solve an SMIP model in a DAM, although it is interrelated to an RTM, and its consequent pricing issues of the MIP. Since there are so many entities, issues, terms, components, and policies in an electricity market, we have to limit our scope to power generation and dispatch in a PoolCo model which involves GenCos and an ISO; issues related to transmission and distribution are beyond the scope of this research.

1.3 Contributions of this Thesis

The contributions of this thesis can be summarized as follows:

- We present a linear SMIP model that integrates a reserve demand curve and shares most of the features of a stochastic UC problem. The reserve demand curve can impact energy prices during shortage hours; from a modeling perspective, with this reserve demand curve an ISO does not have to increase energy prices to its cap just because a small amount of reserve requirement is not satisfied.
- We propose a heuristic methodology to solve the underlying multi-stage SMIP problem. The proposed heuristic is inexpensive and practical, and the algorithm has some flexibility so that an ISO can tailor it according to the size of the problem.

- We present a decision tree that only considers the most crucial scenarios over a 24-hour time horizon. We parsimoniously select a certain number of scenarios to limit the size of the model, since it is impossible to select all scenarios.
- We extend previous work on pricing MIP problems and its application to pricing electric power to show that these ideas can be applied to pricing energy/reserves for a relatively realistic model with binary variables. Nevertheless, some of the ideas in the previous work are proved not valid when they are applied to pricing electricity in practice.
- We propose two compensation schemes based on the SMIP that can be easily implemented in practice. We show that these compensation schemes can ensure that generators can have non-negative profit. We also prove that one of the compensation schemes can return interesting theoretical results; the variance of the profit of generators can be minimized to 0.

1.4 Organization of this Thesis

The remaining chapters of this thesis are organized in the following manner:

Chapter 2 provides a brief literature review on relevant models in electricity markets, stochastic programming and its solution methods, and electricity pricing. It first reviews mathematical models, particularly, stochastic programming models, that have been proposed in previous studies. Then it describes the importance of stochastic programming problems and introduces the methodologies used to solve stochastic programming problems according to their algorithms and features. Finally, it presents the relevant studies that have been completed on pricing of MIP problems and their applications to pricing of energy and reserves.

Chapter 3 presents a multi-stage linear SMIP model for an ISO to plan power generation efficiently in a DAM for the next day, i.e., the time horizon is over 24 hours. It introduces the concept of reserve demand curve, shows the relevant assumptions, and then proposes the formulation. Constraints and modeling details are discussed.

Chapter 4 proposes a heuristic method for solving the underlying SMIP. The detailed procedure of the algorithm is provided. Then it gives numerical examples to show the performance of the heuristic method. Relevant data are provided in this chapter. The overview performance of the heuristic method is evaluated. Next, two numerical examples are provided, and numerical results from some representatives of scenarios are analyzed and discussed.

Chapter 5 describes the details of obtaining commodity prices of the SMIP model based on the sub-optimal solution generated by the heuristic method in Chapter 4. Numerical results are given and discussed. Particularly, results without the reserve demand curve and results with the reserve demand curve are compared to show how the reserve demand impacts energy prices during shortage hours. In the second part of this section, we propose two compensation schemes on the assumption that all the scenarios are known to the ISO. One compensation scheme is an RT compensation scheme in which electricity prices are calculated based on the scenario that actually happens. The other compensation scheme is a hybrid scheme that uses the price and power quantities in the most probable scenario, and its imbalances between other contingency scenarios. Properties of both compensation schemes are described and proved; relevant numerical examples are provided.

Chapter 6 summarizes the thesis, highlights its contributions, and recommends some possible future research directions that could be explored.

Chapter 2
Literature Review

2.1 Introduction

This chapter presents a literature review of relevant research. It first reviews UC problems, optimization models of power generation planning that consider uncertainties, and some modeling techniques. Then it gives a brief introduction of stochastic programs and summarizes the solution methodologies used to solve stochastic programming problems. Finally it presents the previous work on pricing issues of MIP, integer programming (IP), LP problems, and their applications to pricing of energy and reserves.

2.2 UC Problems and Relevant Modeling Techniques

As previously described in Chapter 1, conventional UC problems are used to determine the schedule of generating units and estimate the generating level of each unit over a time horizon. Therefore, UC problems are multi-stage MIP problems; binary variables are employed to represent on/off of generating units. The objective is to minimize the total operation and generation costs that include fuel cost and fixed cost, e.g., start-up cost; constraints may include ramp limit constraints, minimum up/down constraints, power balance, and reserve requirement constraints. In addition, uncertainties have been considering in UC problems because unexpected load spikes and equipment failures can cause blackouts. These problems are identified as stochastic UC problems.

In these deterministic UC problems, the reserve requirement is usually a hard constraint, i.e., the reserve available in the system must be greater than or equal to a pre-set reserve requirement. However, some research has been conducted to apply different techniques to modeling the amount of reserve available in the system, instead of using a deterministic reserve requirement. Bouffard et al. (2005A) provide a multi-period SMIP model to optimize the total expected social welfare of a power generating system. The relevant numerical case studies are given in Bouffard et al. (2005B). Unlike the majority of approaches taken in research on unit commitment problems, which often adopts a fixed operating reserve requirement that is set up arbitrarily, e.g., as the largest unit in the system or 10% of historical peak demand, Bouffard et al. (2005A) penalize unserved load in the objective function and obtain reserve services as a result of optimization. They recognize that a deterministic reserve requirement does not explicitly consider RT uncertainties related to unit outages and line failures.

Wong and Fuller (2007) propose a single period stochastic linear programming model, also without an explicit operating reserve requirement. They determine the total capacity made ready for each generator as a DA decision, and define the reserve as the extra capacity when uncertainties in RT are resolved. This way, generators have to prepare enough capacity in the DA stage so that all RT constraints are satisfied in each scenario, allowing for the possibility of some load shedding, at a penalty. They additionally propose different compensation schemes for electricity markets, including energy-only RT pricing, as well as various DA schemes, to price energy and reserves. By avoiding deterministic reserve requirements and determining reserves as excess capacity in RT, the results in Bouffard et al. (2005B) and Wong and Fuller (2007) explicitly show where and how much reserves are needed in the system when contingencies occur. However, one drawback of the models in Bouffard et al. (2005A) and Wong and Fuller (2007) is that it is impossible for a model to cover all possible future scenarios due to computational limitations; therefore, a reserve requirement is still needed to prepare for the "missing scenarios" – the future states that are left out of the model.

In this thesis, we propose a model that extends the model in Wong and Fuller (2007) in several significant ways. Instead of a single-period model, we propose a multi-period model, in order to represent important features such as ramping limits and start-up costs that depend on down time. Unlike Wong and Fuller (2007), the present model includes binary variables for the on/off states of generating units to be able to represent unit commitment, start-up costs, and no-load costs. Because of the "missing scenarios," we include target and minimum reserve requirement constraints in each scenario, modified by the reserve demand curve (Midwest ISO 2005). These constraints allow that the total reserves available in the system are less than the target reserve requirement, but more than the minimum reserve requirement. See Chapter 3 for details.

Although the formulations of UC problems do not have much variety, different modeling techniques are used for some of the constraints. Suppose q_{gt} represents generation level of generator g in period t and ω_{gt} is the commitment state of generator g in period t, and it is a binary variable. $R_g^{up}, R_g^{dn}, R_g^{su}$, R_g^{sd} represent up-ramping limit and down-ramping limit, start-up limit (the ramping up limit by which a generating unit is started up), and shut down limit (the ramping down limit by which a generating unit is shut down), respectively. M_g is the maximum generation level of g. Due to the ramping limit, the ramp up/down amount between two consecutive periods must be constrained. In Bouffard (2005A), the time-coupled limitations between two periods are modeled as:

$$q_{gt} \le q_{g(t-1)} + R_g^{up}\omega_{g(t-1)} + R_g^{su}(\omega_{gt} - \omega_{g(t-1)}) + M_g(1-\omega_{gt}) \quad (2.1)$$

$$q_{gt} \ge q_{g(t-1)} - R_g^{dn}\omega_{gt} - R_g^{sd}(\omega_{g(t-1)} - \omega_{gt}) - M_g(1-\omega_{g(t-1)}) \quad (2.2)$$

Frangioni and Gentile (2006) provide a different formulation of the same constraint:

$$q_{g(t+1)} \le q_{gt} + R_g^{up}\omega_{gt} + R_g^{su}(1-\omega_{gt}) \quad (2.3)$$

$$q_{gt} \le q_{g(t+1)} + R_g^{dn}\omega_{g(t+1)} + R_g^{sd}(1-\omega_{g(t+1)}) \quad (2.4)$$

(2.3) and (2.4) are more compact in contrast with (2.1) and (2.2). In addition, Arroyo and Conejo (2004) provide a precise formulation of start-up and shut-down trajectories of thermal plants by introducing extra binary variables and constraints. To simplify the model and avoid extra binary variables, in this thesis we employ (2.3) and (2.4) when we model ramp up/down limitations between two consecutive periods. In contrast, most of the literature assumes, less realistically, that these are the same values for the start-up ramping limit and the ramping limit between periods, which means that no binary variables are required in the ramping up constraints; for example, see Shahidehpour et al. (2002).

Minimum up and minimum down constraints are also required in UC problems. As described in Chapter 1, a thermal plant has to stay in “on (off)” states for a certain period of time once it has been turned on (off). Arroyo and Conejo (2000) give a rigorous formulation of minimum up and minimum down constraints by adding extra binary variables as logic controls. In order to reduce the number of binary variables, in this thesis we use another formulation of the minimum up and minimum down constraints proposed in Takriti et al. (2000), Nowak and Romisch (2000), and Nowak and Schultz (2005):

$$\omega_{gt} - \omega_{g(t-1)} \leq 1 - \omega_{gk} \qquad \forall k = t+1, ..., \min\{T, t + T_g^{up} - 1\} \tag{2.5}$$

$$\omega_{g(t-1)} - \omega_{gt} \leq \omega_{gk} \qquad \forall k = t+1, ..., \min\{T, t + T_g^{dn} - 1\} \tag{2.6}$$

Where T_g^{up} and T_g^{dn} represent the minimum up time and minimum down time of generator *g*, and *T* is the last time period in the model.

There is another interesting modeling technique to formulate the start-up cost. The start-up cost function has been identified as a function of the time that a unit has been turned off. Mathematically (Bhattacharya et al., 2001; Shahidehpour et al., 2002),

$$C(t_g^{off}) = \alpha_g + \beta_g (1 - e^{-t_g^{off}/T_g}) \tag{2.7}$$

Where

t_g^{off} : time that generator *g* has been turned off.

α_g : fixed cost of start-up of generator *g*.

β_g : cold-start cost of generator *g*.

T_g : cooling speed of generator *g*.

$C(t_g^{off})$: start-up cost function of generator *g*.

As shown in (2.7), the start-up cost function of a thermal unit is an exponential function of the time that the unit has been shut down. The longer a thermal unit has been turned off, the more expensive it is to start up. The cold start time is the time interval, measured in hours, after which a unit has completely cooled off. A start-up has the same start-up cost as cold start time if the unit has been turned off longer than its cold start-up time. To simplify the formulation and avoid a nonlinear model, Nowak and Romisch (2000) use a step function to approximate the exponential cost function in (2.7):

$$\max(0, \max_{\tau=1,\ldots,\tau_g^C} C_{g\tau}(\omega_{gt} - \sum_{k=1}^{\tau} \omega_{g(t-k)})) \tag{2.8}$$

where $C_{g\tau}$ is the corresponding start-up cost from (2.7) if generator g has been actually turned off for τ time periods; τ_g^C is the cold start time. The value of (2.8) satisfies key properties to an approximation to (2.7):

1) if the unit is on at time t, $\omega_{gt} = 1$, and if it was also on in the previous period, $\omega_{gt} = 1$, then there is no start-up in period t and therefore start-up cost should be 0 in period t, i.e., the value of the inner maximand in (2.8) is 0 for $\tau = 1$, and it is less than or equal to 0 for all other values of τ ;

2) if the unit is off at time t, $\omega_{gt} = 0$, then there is no start-up at time t, and the start-up cost should be 0, i.e., the value of the inner maximand in (2.8) is less than or equal to 0, which makes the overall value of (2.8) equal to 0;

3) if the unit is on and has started up in period t, $\omega_{gt} = 1$, $\omega_{g,t-1} = 1$, and if it has been off for exactly τ' periods and τ' is less than τ_g^C, then the start-up cost at time t should be $C_{g\tau'}$, i.e., the inner maximand of (2.8) evaluates to 0 for $\tau > \tau'$, and to $C_{g\tau} (< C_{g\tau'})$ for $\tau < \tau'$; and

4) if the unit is on and has started up in period t, and it has been off for $\tau' \geq \tau_g^C$ periods, then the start-up cost at time t should be $C_{g\tau_g^C}$, i.e., the greatest value of the inner maximand in (2.8) is $C_{g\tau_g^C}$.

Figure 2-1 illustrates how (2.8) approximates the nonlinear start-up cost.

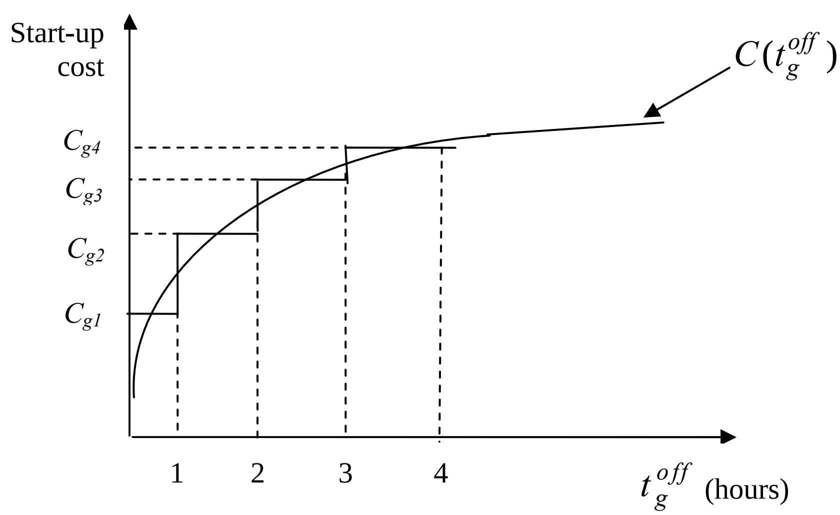


Figure 2-1 Stairwise approximation of nonlinear start-up cost function

2.3 Stochastic Programming and its Solution Methodologies

The model we propose in this thesis is a multiple period SMIP model. Therefore, in this section we will first briefly introduce stochastic programming, and then focus on the previous research on solution methods to multi-stage stochastic programming with binary variables, particularly on relevant stochastic UC problems.

2.3.1 Stochastic Programming

In general IP or MIP problems, stochastic issues are ignored, and it is assumed that parameters in the models are all known or can be precisely forecasted (Birge and Louveaux, 1997). This simplification can avoid the computational difficulties caused by the size of the problems and reduce the solution time. However, including randomness in a mathematical program with integer variables can generate more realistic results than otherwise, although it may significantly complicate the problems; decision makers can benefit from these realistic results obtained by considering uncertainties.

There are different ways to incorporate randomness into the models. One of them, which is widely used, is a recourse-based model. In this type of model, a decision can be made once the random value in the next stage is observed. Depending on the number of stages, they can be classified as two-stage models and multi-stage models. In both types of the models, the objective function is to minimize or maximize a nested sequence of conditional expectations, including first stage decisions and future decisions. In this thesis, we use an alternate formulation of the recourse-based model, namely, expectation-based model. In an expectation-based model, each outcome is associated with a weight or a probability in the objective function; this is the discrete probability of its occurrence, and the sum of them is equal to 1. Usually, it is impossible to include all the scenarios; therefore, only some of the possible scenarios are selected to be incorporated into the model. There is one feature in the multi-stage stochastic programming model that differs from its deterministic counterpart. A decision maker has to make decisions before moving to the next stage where there may be numerous different scenarios. However, the decision maker has to make decisions for the next stage immediately while in the current stage. Thus, the decision maker produces multiple branches of decision values for the next stage, one set for each possible scenario, from the current decision node. Once the random value of the next stage is known, the decision maker can follow one branch of the decision values. The multiple branches of decision values for the next stage derive from the same decision value in the incumbent stage; in other words, they share the same history up to the incumbent stage. In modeling of multi-stage stochastic programming problems, there is a set of constraints that are particularly used to ensure that some scenarios share decision values in a certain number of stages; the constraints are named nonanticipativity constraints (Birge and Louveaux, 1997). Figure 2-2 shows a four-stage decision tree to show how nonanticipativity constraints work. There are three scenarios in Figure 2-2, but they are not independent of each other. In the first three stages, the scenarios share the same decision values, i.e., the decision values of the three scenarios in these stages are equal to each other. The nonanticipativity constraints are needed to ensure this requirement. This thesis proposes a multi-stage SMIP; therefore, appropriate nonanticipativity constraints are required to satisfy the interrelation of decision values in some time periods.

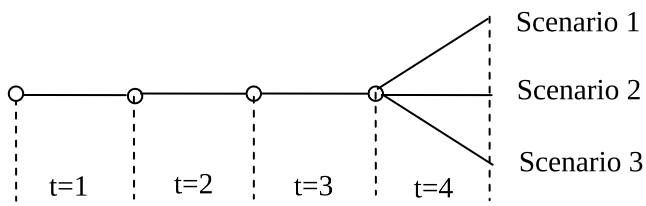


Figure 2-2 Scenario tree for a four-stage stochastic program

Nested Benders decomposition method and L-shaped method are usually applied to solve multistage recourse-based stochastic LP programs or MIPs with integer variables in the first stage. However, there are binary variables in each stage in the SMIP presented in this thesis, and in nested Benders method it is controversial to add optimality cuts or feasibility cuts to a master problem using the dual variables of MIP subproblems. Therefore, we did not use the recourse-based formulation. Instead, we employed the expectation-based formulation. This formulation has an advantage. If the nonanticipativity constraints are relaxed, the problem can be solved by each individual scenario. This feature may facilitate other decomposition methods or provide a good lower bound to the optimal solution that can be used to evaluate the quality of any feasible solution we can obtain.

Besides recourse-based models and expectation-based models, there is another way to incorporate randomness into a stochastic program: chance-constrained programming. In chance-constrained programming, some of the constraints are expressed in terms of confidence levels about first-stage decisions. This formulation is particularly for problems where costs or profits of future decisions are difficult to evaluate (Birge and Louveaux, 1997) and a relevant confidence level can be clearly defined. In the model proposed in this thesis, the costs of future decisions can be clearly assessed. Thus, we will not use chance-constrained programming to formulate the problem.

2.3.2 Solution Methods to Solve Stochastic Programming with Integer Variables

A lot of research has been produced to solve stochastic programming problems with integer variables. According to their features and algorithms, these methods can be classified as Benders decomposition methods (Benders, 1962), Lagrangian relaxation (LR) based decomposition methods, and other methods.

Benders decomposition is an efficient algorithm for solving two-stage stochastic programming problems with integer variables. Nested Benders decomposition is the classic approach to solve multi-stage versions. Nested Benders decomposition algorithm and its implementation of multi-stage linear programs without integer variables are shown and discussed by Birge (1985) and Birge et al. (1996). However, Benders decomposition and Nested Benders decomposition are only efficient for stochastic problems where integer variables are associated with the first stage. Their drawback is that if there are integer variables in each stage, the algorithms have to deal with nonconvex subproblems. On one hand, it is hard to solve a large-scale MIP master problem if all the integer variables are given to the master problem. On the other hand, it is intractable to generate optimality cuts or feasibility cuts when the subproblem is not convex. Therefore, nonconvexity makes the classic Benders decomposition methods inapplicable (Ruszczynski, 1997). Some extensions that are based on decomposition methods have been produced. Cerisola et al. (2009) propose a sophisticated sequential cut method based on generalized Benders decomposition to solve multi-stage stochastic programming problems with integer variables in each stage. They propose two solution methods and compare these methods with commercial software and a standard LR method using an application of stochastic UC problem. The conclusion is that the methods they propose are hard to implement, but they can ensure a feasible solution within a reasonable CPU time. Particularly, one of the methods requires a new Lagrangian function for each subproblem, which is a challenge for implementation in practice.

LR is also a well-known method, and with some enhancement it can be used for solving stochastic IPs, particularly working with decomposition methods. Depending on the constraints that are relaxed, there are three decomposition methods: scenario decomposition, nodal decomposition, and geographical decomposition (Dentcheva and Romisch, 2004). In scenario decomposition, the Lagrangian multipliers are associated with nonanticipativity constraints. The problem can be decomposed into small subproblems; each contains one single scenario. In nodal decomposition, the Lagrangian multipliers are associated with dynamic constraints, i.e., time-coupling constraints, at each node of the scenario tree. With this relaxation, the problem is decomposed by each node with one single time period. Geographical decomposition decomposes the whole problem by system components. For example, in a UC problem Lagrangian multipliers are associated with the supply-demand balance constraint; the sum of electricity supplied by generating units in the system should be greater than or equal to the demand. Through geographical decomposition, the problem can be decomposed into a much smaller decision space because there is only one system component in each subproblem. The first paper that applies LR to solving a scheduling problem in power generation systems is Muckstadt and Koenig (1976), although the model is deterministic. In this paper, the problem is solved using a geographical decomposition; it is decomposed into single generating unit problems. Then a subgradient method (Berksekas, 1999) is used to solve the problem. Numerical results show that the relaxation method can produce a tight lower bound, the technique can solve a large-scale problem, and the error tolerance is acceptable.

Takriti et al. (1996) use geographical decomposition to solve a multi-stage stochastic UC problem that considers generator failure and inaccurate load forecast as uncertainties. They decompose the whole problem into many single-generator problems, and then they employ dynamic programming to solve these subproblems using subgradient method to update the Lagrangian multipliers. They provide 3 numerical examples to validate their solution method; 10 generator outage scenarios, 22 generator outage scenarios, and 16 scenarios with different loads. The results indicate that the cost of the electric power system can be reduced significantly by a stochastic model. The size of the numerical examples used is relatively small; therefore, the method may not be applied to solving a large-scale problem (Ozturk, 2003).

Scenario decomposition is also an important decomposition method used to solve multi-stage stochastic programming models. Rockafellar and Wets (1991) propose a progressive hedging algorithm. It is based on ordinary Lagrangian relaxation and overcomes the nonseparability of augmented Lagrangian due to cross-multiplication of decision variables in two different scenarios in the quadratic penalty term in the objective function. It then solves the relatively much smaller subproblems that are single-scenario problems while updating the Lagrangian multipliers; the algorithm terminates when the preset stop criteria is satisfied. Nevertheless, this algorithm has some limitations. First, there are some implementation issues: it is difficult to select proper multipliers, and there is no conclusive theory that can be followed except for some empirical results (Mulvey and Valdimirou, 1991). Second, although convergence for convex problems can be achieved (Rockafellar and Wets, 1991; Ruszczynski, 1995), when it comes to solving stochastic programs with integer variables, convergence is not guaranteed (Rockafellar and Wets, 1991). There are some applications of this algorithm. Mulvey and Vladimirou (1991) investigate the performance of the algorithm by applying it to stochastic generalized networks. They also show their numerical results on how to set the penalty parameters. Comparisons to other existing solution methods are also provided. Other extensions based on a progressive hedging algorithm can be found in Helgason and Wallace (1991), Mulvey and Ruszczynski (1995), Lokketangen and Woodruff (1996), Caroe and Schultz (1999), and Liu et al. (2003).

Schultz (2003) summarizes the methods used to solve multi-stage stochastic IPs. He mentions that nonanticipativity constraints in a multi-stage model are more complicated than those in a two-stage model; therefore the existing subgradient methods are not applicable due to the high dimension of the resulting Lagrangian dual. Alonso-Auso et al. (2003) propose a branch-and-fix coordination approach. In this approach variables are split across scenarios. The branching nodes and branches are coordinated with the nonanticipativity constraints through a branch-and-fix scheme so that nonanticipativity constraints are satisfied. The methods presented are suitable for multi-stage stochastic pure 0-1 programs and two-stage mixed 0-1 programs.

Compared with other decomposition methods, nodal decomposition is still an open area (Schultz 2003). Some preliminary results can be found in Dentcheva and Romisch (2004), Romisch and Schultz (2001).

In addition to the approaches mentioned above, other methods that have been applied to solve stochastic IPs and their applications to power planning problems include: Dantzig-Wolfe decomposition method (Singh et al., 2009), column generation (Shiina and Birge, 2004), bundle methods (Borghetti et al., 2003; Bacaud et al., 2001), branch-and-price (Lulli and Sen, 2004), LR and dynamic programming (Bard, 1988), and heuristic-based methods (Ahmed and Sahinidis, 2003; Fan et al., 2002; Zhuang and Galiana, 1988).

The methods reviewed in this subsection offer some advantages when solving stochastic linear programs or stochastic IPs. However, the numerical examples used in some of the papers represent small- or medium-scale problems. For example, Takriti et al. (1996) use 10, 16, and 22 scenarios to test their methods. In addition, most efficient methods proposed only accommodate multi-stage stochastic IPs with integer variables in the first stage. Nevertheless, in some practical problems, e.g., stochastic UC problems or other power generation planning problems, integer (or binary) decision variables are associated with each stage of the model. This feature complicates the problem and can significantly increase the solution time. In this thesis, we propose a heuristic method to solve the underlying SMIP problem based on a scenario tree. The heuristic will be presented later in this thesis.

2.4 Reserve Demand Curve

The ISO can have a clear idea about the energy demand curve from customers' bids. However, an ISO has no way of measuring reserve demand as perceived by its customers because the reserve requirement depends on RT scheduling and RT contingencies. To make sure that the system reliability can be maintained all the time, the ISO usually sets a fixed reserve requirement according to the benchmark from the North America Electric Reliability Corporation (NERC). Once the reserve requirement cannot be met, the ISO considers load shedding to maintain the supply-demand balance and the reserve requirement. The drawback is that the ISO may have to shed load just because a small amount of reserve requirement is not satisfied, and energy prices soar to the capped price during the shortage hours. Thus, some ISOs in the United States implement an operating reserve demand curve based on reserve shortage. The basic idea is that when the reserve capacity is below the system target requirement but above the minimum requirement, the energy price and reserve price will increase accordingly. If the reserve capacity reduces to or below the minimum reserve requirement, the ISO has to adopt load shedding to recover the system reserve capacity level. As a result, the ISO sets up a high penalty cost or a capped price for reserve usage. When the reserve demand reaches to the minimum reserve requirement, the energy price will increase to its capped price. Then customers have to decide if they want to reduce their energy demand or face rotating blackouts. This price is usually set by the supply side and is an estimation of how much customers will pay for protecting themselves against blackouts.

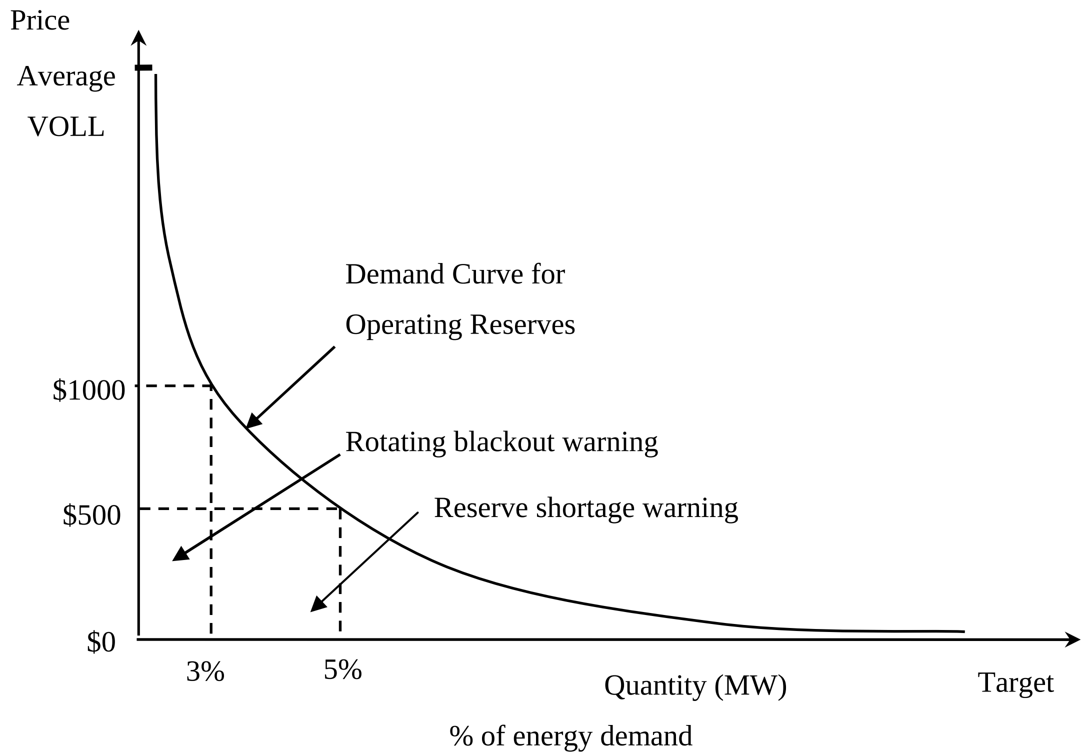


Figure 2-3 Illustrative reserve demand curve (Midwest ISO 2005)

The idea of using a reserve demand curve to implement shortage pricing is illustrated in Figure 2-3. In Figure 2-3, suppose that the ISO sets up a target reserve level which is an ideal level including all the reserves available in the system. This level accounts for a percentage of the energy demand, e.g., 17% of the energy demand. There is a minimum reserve requirement equal to 3% of the energy demand, for instance. If, in RT, the reserve level remains above this target level, the price for reserves is 0. Accordingly, the energy price is low because there is no energy scarcity at all in the system. Nevertheless, the reserve capacity cannot stay at or above this ideal level in RT due to unexpected high demand, unit outages and transmission line outages, etc. In case of these contingencies, the reserve capacity level will drop below the target level, say, to 80% of the target level (the target level is 17% of energy demand). Then a reserve price is applied to the 20% of target level traded for energy; but this price should not be too high because there are still plenty of reserves leftover in the system. 80% of the target level is still acceptable for the ISO. If there is a major outage occuring in the system, the reserve level in the system drops to 5% of the energy demand, which is close to the minimum reserve requirement. A much higher reserve price, $500/MWh, is charged to customers for the reserve capacity used to supply energy. The energy price in this scenario should also include this shortage price to come up with a high energy price. This is because in such a scenario, where reserve is critically short, every unit supplying energy or reserve is contributing to system reliability. These generating units are equally important. Without any one of them, the system could collapse instantly. If the reserve level reaches to the minimum requirement level of 3%, $1000/MWh is charged to warn customers that the reserves are in shortage and that the possibility of an intended blackout is very high. The high price of $1000/MWh is also used to suggest that customers may consider reducing their load to mitigate the reserve shortage. In the worst scenario where the reserve level shrinks to the minimum reserve requirement, the reserve price hits the capped price, which is supposed to be the maximum price that customers would like to pay to avoid load shedding. If this happens, although rare in practice, the ISO has to apply rotating blackouts to reduce the load so that the supply-demand balance can be maintained. Load shedding is always the last resort for the ISO. But in some extreme situations, without shedding load, the frequency of the electric power system cannot be maintained, which can directly result in a system-wide failure. The capped price poses one of the challenges in the operating reserve curve. It is hard to estimate how much customers are willing to pay to avoid taking the risk of blackouts. Above this price, customers would rather take the consequences of the blackout than pay an even higher price, because for them, the cost incurred by a blackout is lower than that price they pay to get more reserves, e.g., some extremely expensive imported reserves ISO buys from some external systems. Therefore, as a relatively safe and fair benchmark, Midwest ISO (2005) employs the value of lost load (VOLL) as the capped price. VOLL estimates the value of lost load from the customers based on historical data; it ranges from $2400/MWh to $20000/MWh (Cramton and Lien, 2000). Over VOLL, customers will not pay anything for extra reserves; instead, they would like the ISO to do the load shedding, usually by means of a rotating blackout. The reserve prices are not paid to the reserve capacity leftover in each time period in the RTM. Instead, they are paid to the reserve capacity traded for energy in RT. The marginal costs of reserve capacity not used to supply energy in RT is nearly 0 because their major costs are all fixed costs including start-up costs and no-load costs. Therefore, associated reserve prices are charged to reserves transformed into energy in RT.

With a different definition of the target reserve level, we can draw a similar reserve demand curve to Figure 2-3. This curve can be continuously convex, piece-wise linear or like a step-function. For example, NYISO uses a very simple reserve demand curve with a different setting of the target level. The total available 10-minute spinning and nonspinning reserves should be able to cover the first contingency in the system which is the largest capacity among all generators or the largest contingency. The 30-minute reserves can make at least 50% of the second contingency. Their demand curve appears to be a step-function. (Lynch, 2005). Hartshorn

(2001) describes the development of the demand curve for reserves and explains why a demand curve is needed.

In this thesis, we integrate a reserve demand curve into the SMIP model in order to investigate the impact of the reserve shortage level in the system on energy/reserve prices. We will focus on the impact of the reserve demand curve on energy and reserve prices and on how it can prevent an ISO from cutting electricity demand from a modeling perspective. These features and related numerical results will be shown in Chapter 4.

2.5 Electricity Pricing with Nonconvex Models

Pricing electric power has been a very challenging area. The proposed compensation schemes should not only guarantee that there is adequate revenue to cover GenCos' generation and operation costs, but should also minimize the financial risk faced by GenCos. In addition, in UC or similar power planning models, there are binary variables to represent the commitment states of generating units. Therefore, the resulting UC models are MIPs. While duality theory can be applied to price linear programs, it cannot be employed as the method to obtain dual prices for a MIP or IP. Gomory and Baumol (1960) discuss the details of pricing IPs and explore the dual prices of IPs. Particularly, they explain why the shadow prices (or the dual prices) of an IP can be different from the shadow prices of a conventional LP. An illustrative example is given in Figure 2-4. The feasible region of an LP maximization problem, i.e., *OABCD*, is given in Figure 2-4(a). Suppose that corner *C* is the optimal solution to the LP; *M* is the optimal solution of its corresponding IP; segment LL' is the level curve of the optimal LP objective function value. Figure 2-4(b) shows that constraint *CD* moves rightwards if we increase the right hand side of constraint *CD* by one unit. Then the optimal solution to the LP changes from *C* to *C'*; the shadow price (or the dual price) is positive. Nevertheless, the optimal solution to the IP is still at *M* because there are no new integers covered by the feasible region in Figure 2-4(b). Therefore, the shadow price (or the dual price) of the IP is 0, and we can safely conclude that the dual price of an LP can be different from the dual price of its corresponding IP. Thus, unlike with LPs, it is very difficult to procure the relevant prices from an MIP. The obstacle exists in the UC problems and similar problems in electricity market models containing binary variables. Other theoretical discussions can be found in Wolsey (1981, 1998).

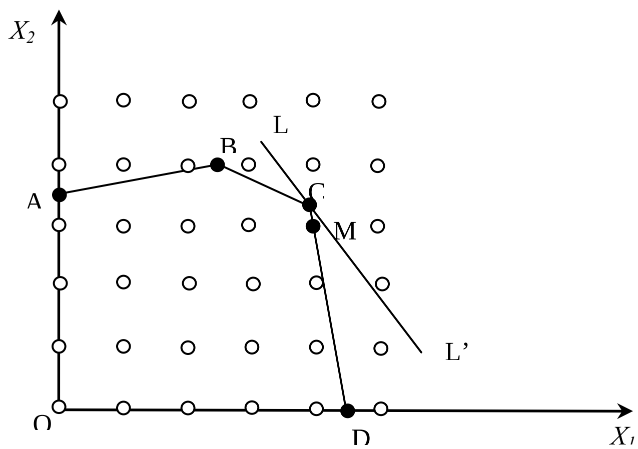


Figure 2-4 (a)

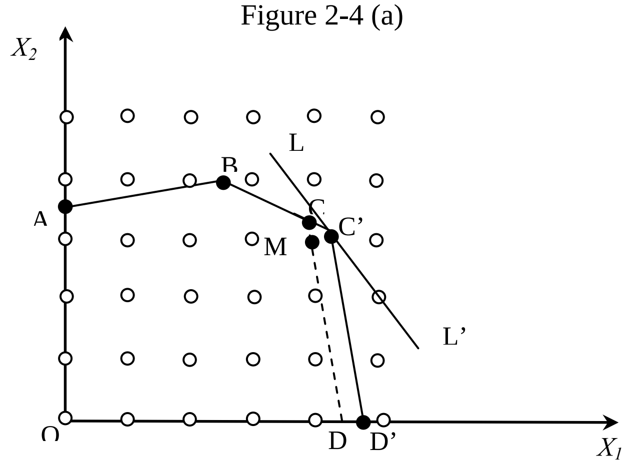


Figure 2-4 (b)

Figure 2-4 Illustrative examples of shadow price of LP and IP

Scarf (1990, 1994) studies the analogy of simplex algorithm for solving LPs and economic theory of finding equilibrium prices in competitive markets. The author notes that it is difficult to draw similar analogies to IP algorithms and entities with non-convex decisions, e.g., whether to start-up a new production process. Scarf (1994) gives a numerical example to show that presence of non-convexity causes failures to find competitive prices.

Some research on pricing MIPs has been conducted. O'Neill et al. (2005) provide a method based on an MIP where all the integer decision variables are binary. They first obtain the optimal solution to the MIP and then solve the whole problem as an LP after fixing the binary variables to their optimal values. To achieve this goal, they relax the integer constraints on the binary variables and add a group of new equality constraints, setting the value of binary variables to their optimal IP solution. The dual prices corresponding to the equality constraints are viewed as additional prices for the commitment of firms. They are used to cover the fixed costs associated with the binary variables in the MIP. The relevant dual prices of LP can be procured directly because the duality theory can be applied to an LP. By solving each producer's problem, the authors show that the price equilibrium is achieved and each individual producer is satisfied with the compensation scheme. The same numerical example used in Scarf (1994) is employed to support their theory on the price equilibrium and to show how the method interprets the solution.

Some studies have investigated and proposed compensation schemes and compensation plans for electricity markets based on either LPs or MIPs. Wong and Fuller (2007) propose different compensation schemes and reliability-relevant compensation for a single-period stochastic linear programming model. The compensation schemes include DA schemes, RT schemes, and hybrid schemes that use prices from both DAM and RTM. They show that the variance of profit across all the scenarios is 0 for each generating unit in the hybrid compensation schemes, while the variance of profits is high in other compensation schemes. It is beneficial to GenCos when their profit variance is 0; it can relieve their concerns about the uncertainties of their returns in the RTM due to contingencies. In addition, as an LP, the model does not contain any binary variables or integer variables. The property allows the authors to procure energy prices directly from the dual variables of power balance constraints; it is valid to apply the duality theory to price energy in the model, and the obtained prices are used to calculate the total revenues and the profits for the generating units.

Sioshansi et al. (2007) demonstrate that using branch and bound or LR to solve UC problems leads to payoff equity problems, if the problems cannot be solved to complete optimality within the required time frame. Furthermore, they show that the magnitude of payoff deviation does not positively correlate to the optimality gap of the sub-optimal solution, i.e., the magnitude of payoff deviation might not necessarily decrease even if the suboptimal solution is actually very close to the optimal solution. In other words, the payoff deviation could still be significant unless the problem is solved to its complete optimality. Therefore, they describe a lump sum payment called a make-whole payment to smooth out the payment difference among generators and compensate any start-up costs and no-load costs not recovered by inframarginal energy rents, if the underlying problem cannot be solved to its complete optimality. If a generating unit has a non-negative profit, its make-whole payment is 0; if it has a negative profit, the make-whole payment can lift its profit to 0, making this generating unit break even. The make-whole payment guarantees that each generating unit can have non-negative profit. To see the theoretical discussions about the optimality gap between a feasible solution and the optimal solution of an MIP, see Larsson and Patriksson (2006).

In general, the two-settlement system previously mentioned is applied to pricing electricity. In a DAM, an ISO will release the predicted energy price based on the bidding and generation offers it receives. The prices will be associated with the power quantities committed by generating units in the DAM. However, an RTM is always different from a DAM due to unexpected demand, forced outages and other contingencies for which the ISO has to adjust the generation levels of generating units in the RTM. The deviation of the DA generation level and the RT generation level of generators in the same period is paid at RT price; the ISO runs the economic dispatch model within a time window, e.g., every five minutes, to renew the electricity prices and operating schedules contingent on RT conditions.

In this thesis, we extend the ideas of O'Neill et al. (2005) and the compensation schemes based on an LP in Wong and Fuller (2007) to price electricity based on an MIP. Due to the nonconvexity and the size of the problem in this thesis, we cannot solve it to its complete optimality, although a sub-optimal solution can be obtained using a heuristic. As a result, we apply the idea of make-whole payment (Sioshansi et al., 2007) in order to mitigate the payment difference due to nonoptimal solution and to guarantee that generators can have non-negative profits in each scenario. In this thesis, we propose two compensation schemes, including an RT-based compensation scheme and a hybrid compensation scheme. In the hybrid compensation scheme we use a two-settlement payment mechanism different from the one used in industry; with deviations of DA and RT generation paid at the offer price instead of RT prices, we can achieve the interesting theoretical result of reducing the profit variance of generators.

2.6 Summary

This chapter first gave a brief review of relevant power planning models and their modeling techniques. It also provided a survey of previous solution methodologies used to solve stochastic programming problems, particularly stochastic IPs. These methodologies were classified according to their algorithms and features. While they have some limitations, they offer advantages when solving small- or medium-sized problems. We discussed Benders decomposition, Lagrangian Relaxation-based decomposition methods (particularly the scenario decomposition and the progressive hedging algorithm), and other methods. The formulation of the SMIP model and the solution method used to solve the model are shown in Chapter 3 and Chapter 4, respectively. We also discussed the concept of the reserve demand curve and its benefits.

Finally, we investigated the current research on pricing integer programs and its possible extensions to pricing electric power based on an MIP. Although the literature in this area is very limited, we focus on some recent work on MIP and electricity pricing and will combine and extend these ideas to price the MIP model with nonconvexity costs in this thesis. These issues will be addressed with numerical examples in Chapter 5.

Chapter 3
A Stochastic Formulation for a Day-Ahead Electricity Market

3.1 Introduction

In this chapter, we will propose a multi-stage SMIP model for planning power generation in a DAM. The model considers generator outages and transmission line failures as uncertainties. To avoid long computational time due to a large number of scenarios, we only focus on important scenarios where contingencies occur from different starting times.

3.2 Problem Statement and Assumptions

The model we propose is an SMIP that is run by an ISO to determine the schedule (0/1 binary variables) and the generation level of generators based on their bidding information in a DAM. Here, we consider a PoolCo electricity market, in which there is no bilateral contract-based electricity trading. After receiving generation offers from GenCos that own thermal generating units and other types of units, the ISO decides when each accepted generating unit is to be turned on or off and estimates its output level for the next day, i.e., the RTM. Therefore, we adopt a 24-hour time horizon. The generation offers for energy will follow a form of quantity-price pairs on an incremental basis. For example, a generation offer of a generating unit with a capacity of 100MW can be formed by three pairs: ($5.25/MWh, 50MWh), ($10.5/MWh, 40MWh), and ($20.5/MWh, 10MWh). This suggests that the price offered by this generator is $5.25/MWh when the generation level of the generating unit is up to 50MW for a full hour, $10.5 for the next 40MW, and $20.5 for the last 10MW of its capacity. In addition, as required, the GenCos will submit relevant information, such as minimum generation level, start-up costs, no-load costs, and ramp rates. Uncertainties considered in the model include generator outages and transmission line failures. Generation costs, start-up cost, and no-load costs are considered, while shutdown costs are not included, since they are less important than others (Bhattacharya, 2001). Constraints relevant to operations of thermal units are presented in the model, such as minimum up/down constraints, ramping limit constraints, and minimum generation level constraints. Meanwhile, fuel constraints and emission constraints are not included to reduce the size of the problem. Due to the complexity of a power generation model and an electricity market, one single model cannot accommodate all the features and characteristics. Furthermore, some parameter values cannot be estimated accurately in an academic environment. Therefore, we make the following assumptions to simplify the modeling and to narrow down the scope of the research:

1) Over the time horizon of the model, no customers can respond to energy prices by varying their demand. For this reason, ramping up or down limit for demand is not formulated.

2) Market participants submit the same generation offers for each generation facility for each dispatch period.

3) The only contingencies that the ISO is concerned about are generating unit and transmission line outages; other equipment failures are not considered.

4) Although demand varies over the time horizon, demand is deterministic within a time period. Therefore, the demand is identical in different scenarios at the same time period.

5) Quick-start units can be synchronized to the system so quickly that they are able to supply spinning reserve when offline.

6) Linear DC approximation is used to replace AC power flows.

7) Power loss along the transmission line is not considered. However, the loss can be roughly made up by increasing demand in each time period.

8) More than one element in a system might break down at the same time, e.g., "N-2" contingencies; this model can accommodate these contingencies by changing parameters μ_{gst} and υ_{ijst}. However, to avoid combinatorial explosions due to the existence of binary variables, only "N-1" contingencies are considered, i.e., only one element, either a generating unit or a transmission line, breaks down during a single time period.

9) Generator outages and transmission line failures may occur anytime during the time horizon, and they could be completely repaired and resume to function anytime after their failures. Although we can generate these scenarios with different repairing time by manipulating parameters μ_{gst} and υ_{ijst}, in this thesis, we only select the worst scenarios where a failure lasts for long hours in order to reduce the size of the problem. That is, once a unit or transmission line is down, it is down until the end of the planning horizon. This is also because repairing and restarting a unit or repairing a line can take more time than is covered by the model. In addition, for the same purpose of reducing the computational burden, we parsimoniously select some of the worst situations in which the outage unit or line is lost for a long time before it can work properly again. To make this assumption more clear, we give an illustrative example in Figure 3-1. This decision tree has 13 scenarios and 6 time periods (hours). In each hour, there are two contingencies. Scenario 1 is the most probable scenario with no contingencies. The remaining scenarios have two contingencies that start from

different time periods. For example, outages happen only during the last hour in scenarios 2 and 3, whereas contingencies take place starting from hour 2 in scenarios 12 and 13.

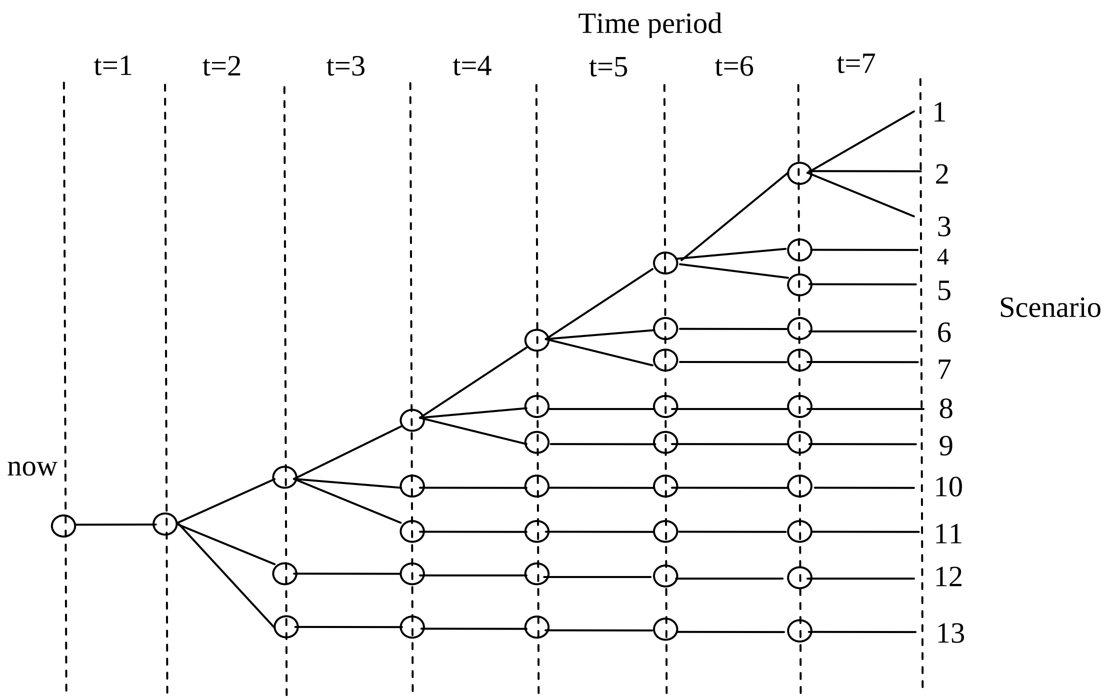


Figure 3-1 Illustration of scenario tree in the model

3.3 Integration of the Reserve Demand Curve

As discussed in the previous chapter, the basic idea of the reserve demand curve is that the ISO sets up a curve with respect to reserve shortage levels and their associated penalty costs. The curves can be continuous convex functions, piece-wise linear functions, or step-functions. For example, a step function is used in NYISO (NYISO 2001). In our model, the reserve demand curve is a step-function as illustrated in Figure 3-2. If the amount of reserve availability remains above the target level in RT, the penalty for reserve shortage is 0, as no further reserve is required. Accordingly, the price of energy is low, since there is no energy scarcity in the system.

In case of contingencies, the reserve availability may fall below the target level because part of the capacity is used to supply energy. If the available reserve level is between 50% and 100% of the target level, the amount of reserve shortage is penalized at $100/MW; $300/MW if it is between 20% and 50%; $600/MW if the available reserve drops to some point between 10% and 20%. The same penalties can be interpreted in terms of the reserve shortage, shown on the right side of Figure 3-2. If the reserve level falls below the minimum level, 10%, the penalty of reserve shortage rises to the capped energy price (converted from units of $/MWh to reserve prices in $/MW by assuming a duration of one hour). In general, the average VOLL is used as the capped price for load shedding (Brampton and Lien, 2000). The minimum reserve requirement is illustrated as 3% of the energy load. However, depending on some conditions of the system, the minimum reserve requirement can be allowed to drop to 0 (Midwest ISO, 2005).

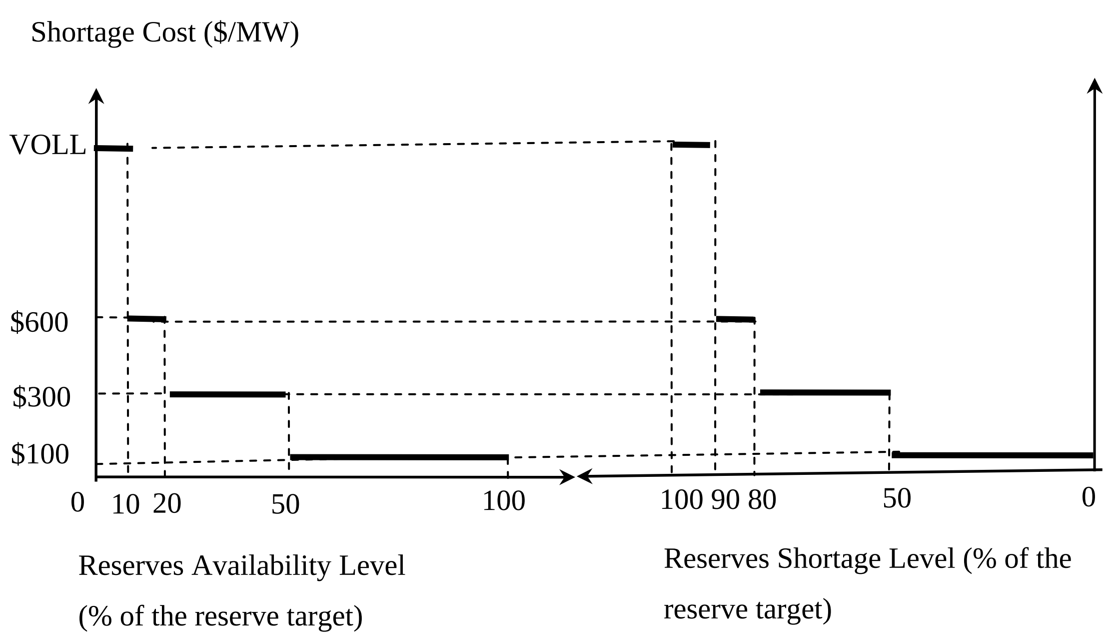


Figure 3-2 Four-step function of reserve demand curve

3.4 Stochastic Mixed Integer Programming Model Formulation

This section provides the formulation and the description of the model, i.e., objective function and constraints. In order to improve readability of the complex notation, we have adopted the following convention: all subscripts are indices drawn from sets, and all superscripts are abbreviations for verbal descriptions. Please refer to the nomenclature list on pages xiii to xv for definitions of all symbols.

The model is shown below.

$$\text{P:}\min\sum_{s=1}^{S}\sum_{t=1}^{T}\pi_s[\sum_{m=1}^{M}\sum_{n=1}^{N}C_{mn}^{S}T^{\text{int}}q_{mnst}^{S}+\sum_{i\in I}C^{LS}q_{ist}^{LS}+\sum_{i\in I}\sum_{g\in G(i)}(\sum_{b=1}^{B}C_{gb}^{E}T^{\text{int}}q_{gbst}^{E}+z_{gst}+C_{g}^{NL}T^{\text{int}}\omega_{gst})] \quad (3.1)$$

$$\text{s.t. } q_{gbst}^{E}\le Q_{gb}^{E}\omega_{gst} \qquad \forall i,\forall g\in G(i),\forall b,\forall s,\forall t \quad (3.2)$$

$$q_{mnst}^{S}\le Q_{mn}^{S} \qquad \forall m,\forall n,\forall s,\forall t \quad (3.3)$$

$$\sum_{b=1}^{B}q_{gbst}^{E}-Q_{g}^{MSL}\omega_{gst}\ge 0 \qquad \forall i,\forall g\in G(i),\forall s,\forall t \quad (3.4)$$

$$\sum_{b=1}^{B}q_{gbst}^{E}+\sum_{m\in M(g)}q_{gmst}^{R}\le Q_{g}^{\max}\mu_{gst} \qquad \forall i,\forall g\in G^{f}(i),\forall s,\forall t \quad (3.5)$$

$$\sum_{b=1}^{B}q_{gbst}^{E}+\sum_{m\in M(g)}q_{gmst}^{R}\le Q_{g}^{\max}\omega_{gst} \qquad \forall i,\forall g\in G^{s}(i),\forall s,\forall t \quad (3.6)$$

$$q_{gmst}^{R}\le R_{gm}^{RP} \qquad \forall i,\forall g\in G(i),\forall m,\forall s,\forall t \quad (3.7)$$

$$\sum_{g\in G(i)}\sum_{b=1}^{B}q_{gbst}^{E}+q_{ist}^{LS}-\sum_{j}\upsilon_{ijst}B_{ijt}(\theta_{ist}-\theta_{jst})=Q_{it}^{D} \qquad \forall i,\forall s,\forall t \quad (3.8)$$

$$\sum_{a=1}^{A}(lk_{sat}\pi_a\sum_{b=1}^{B}q_{gbat}^{E})-(\sum_{a=1}^{A}lk_{sat}\pi_a)\sum_{b=1}^{B}q_{gbst}^{E}=0 \qquad \forall i,\forall g\in G(i),\forall s,\forall t \quad (3.9)$$

$$\sum_{a=1}^{A}(lk_{sat}\pi_a q_{gmat}^{R})-(\sum_{a=1}^{A}lk_{sat}\pi_a)q_{gmst}^{R}=0 \qquad \forall i,\forall m,\forall g\in G(i),\forall s,\forall t \quad (3.10)$$

$$\upsilon_{ijst}B_{ijt}(\theta_{ist}-\theta_{jst})\le \upsilon_{ijst}U_{ij} \qquad \forall i,\forall j,\forall s,\forall t \quad (3.11)$$

$$\sum_{g\in G(i),i\in I} q_{gmst}^{R} + \sum_{n=1}^{N} q_{mnst}^{S} \geq Q_{m}^{R} \qquad \forall m, \forall s, \forall t \qquad (3.12)$$

$$\sum_{g\in G(i),i\in I} q_{gmst}^{R} \geq Q_{m\_\min}^{R} \qquad \forall m, \forall s, \forall t \qquad (3.13)$$

$$q_{ist}^{LS} \leq Q_{it}^{D} \qquad \forall i, \forall s, \forall t \qquad (3.14)$$

$$\omega_{gst} = \mu_{gst} \qquad \forall i, \forall g \in G^{r}(i), \forall s, \forall t \qquad (3.15)$$

$$\omega_{gst} \leq \lceil \mu_{gst} \rceil \qquad \forall i, \forall g \in G(i), \forall s, \forall t \qquad (3.16)$$

$$z_{gst} \geq C_{g\tau}(\omega_{gst} - \sum_{k=1}^{\tau} \omega_{gs(t-k)}) \qquad \forall i, \forall g \in G(i), \forall s, \forall t, \tau = 1,...,\tau_{g}^{C} \qquad (3.17)$$

$$\omega_{gs(t-1)} - \omega_{gst} \leq 1 - \omega_{gsk} \qquad \forall i, \forall g \in G(i), \forall s, \forall t, \forall k = t+1,...,\min\{T, t+T_{g}^{dn} - 1\} \qquad (3.18)$$

$$\omega_{gst} - \omega_{gs(t-1)} \leq \omega_{gsk} \qquad \forall i, \forall g \in G(i), \forall s, \forall t, \forall k = t+1,...,\min\{T, t+T_{g}^{up} - 1\}, \; \mu_{gsk} = 1 \qquad (3.19)$$

$$\sum_{b=1}^{B} q_{gbst}^{E} - \sum_{b=1}^{B} q_{gbs(t-1)}^{E} \leq R_{g}^{up} \omega_{gs(t-1)} + R_{g}^{su}(1 - \omega_{gs(t-1)}) \qquad \forall i, \forall g \in G(i), \forall s, \forall t \qquad (3.20)$$

$$\sum_{b=1}^{B} q_{gbs(t-1)}^{E} - \sum_{b=1}^{B} q_{gbst}^{E} \leq R_{g}^{dn} \omega_{gst} + R_{g}^{sd}(1 - \omega_{gst}) \qquad \forall i, \forall g \in G(i), \forall s, \forall t, \mu_{gst} = 1 \qquad (3.21)$$

$$q_{gbst}^{E}, q_{gmst}^{R}, q_{mnst}^{S}, z_{gst}, q_{ist}^{LS} \geq 0, \theta_{ist} \in [-\alpha, \alpha], \omega_{gst} \in \{0,1\} \qquad \forall i, \forall g \in G(i), \forall m, \forall n, \forall s, \forall t \qquad (3.22)$$

The objective of problem P is to minimize the total expected generation and operation costs including reserve shortage costs, load shedding costs, generation costs, start-up costs and no-load costs. Any unserved load is penalized at a high capped price. The reserve shortage is penalized with respect to the shortage level, following the four-step function shown in Figure 3-2. No-load costs refer to the costs to maintain generating units synchronized to the system, but without energy output.

Constraints (3.2)-(3.3) give the step widths for the step-functions of energy offer and reserve demand, respectively. The quantities appearing in the generation offers are actually upper bounds for the energy supplied within that offer block. For example, as previously illustrated, in a generation offer, ($5.25/MWh, 50MW), 50MW is the upper bound; any output level below or equal to 50MW for one hour is charged at $5.25/MWh. The two groups of constraints are identical, except that in (3.2) the right hand sides are associated with the binary variables, ensuring that the generation output of generating units may be positive at a time period within a scenario only when they are online, i.e., $\omega_{gst} = 1$.

Constraints (3.4) indicate that if a generator supplies power to the system, then it must be running at least at its minimum economical level. Although quick-start units can supply energy and reserve when they are available, slow-start units can supply energy and spinning reserve only when they are online. These restrictions are enforced by (3.5) and (3.6), respectively. In (3.5), if a fast-start unit does not break, $\mu_{gst} = 1$, it can supply both energy and reserve. (3.6) ensure that for a slow-start unit they can (cannot) delivery energy and prepare spinning reserve only when it is online (offline), i.e., $\omega_{gst} = 1$ ($\omega_{gst} = 0$). Constraints (3.7) are upper bounds on the reserve. For example, ten-minute spinning reserve supplied by a generator is limited by its ramp up rate and ramp up time, e.g., multiplying ramp up rates and ten minutes.

Constraints (3.8) are power balance constraints at nodes. As mentioned in the assumptions, in this model we do not consider power loss along transmission lines; but an estimate of losses can be included in the demand. We use a DC approximation to replace the nonlinear AC power flows. For the details of the approximation, see Fuller (2005).

Constraints (3.9) and (3.10) are nonanticipativity constraints. For each scenario s and period t constraints (3.9) ensure that the total power generation of a unit is equal to the expected value of the total output in all the scenarios to which s is matched in period t. Thus, the power outputs intended to be the same are all equal to the expected value. We use binary parameters, lk_{sat} to indicate if scenario s shares the same decision scenario a in period t. (3.10) are similar nonanticipativity constraints that enforce reserve decision variables equal to each other at these shared time periods. To make the size of the model small, we did not include all the variables in the nonanticipativity constraints except the power generation variables and reserve variables. If (3.9) and (3.10) hold, other decision variables naturally satisfy their own nonanticipativity constraints due to other constraints and cost minimization in the objective function. For example, the commitment state variables, ω_{gst} must be constrained by the nonanticipativity if (3.9) is satisfied; i.e., if $\sum_{b=1}^{B} q_{gbst}^{E} > 0$, $\omega_{gst} = 1$ because of (3.2). Since the objective function of the model is to minimize the total costs, z_{gst} must be equal to at least one of the right hand sides of (3.17) which are only associated with ω_{gst}. Thus, z_{gst} also satisfy their corresponding nonanticipativity constraints.

The power-carrying capacity limit constraints on any transmission lines are modeled by (3.11). System target operating reserve requirements are satisfied through (3.12) (Chao et al., 2005); for example, total ten-minute spinning reserve capacity plus its reserve shortage should be greater than or equal to ten-minute system operating reserve requirements at any time in any scenario. The ten-minute spinning reserve requirement is typically determined by a simple rule, e.g., the greater of the largest generator outage or a percentage more than the peak demand. California ISO (2003) presents a more complicated measure to calculate the spinning reserve requirement. (3.13) represent the minimum reserve requirement. Since the numerical examples presented in this thesis are assumed to be for an isolated system, a minimum reserve requirement other than 0 is still needed to avoid endangering operative reliability (Midwest ISO, 2005). In practice, reserve requirements are set by zones in the area, namely, zonal reserve requirements. To simplify the model we assume that there is one system-wide reserve requirement for the whole area. Zonal reserve requirements, however, can be incorporated into the model by modifying constraints (3.12) and (3.13), if we can have access to the relevant data.

Constraints (3.14) guarantee that the amount of load shedding cannot be more than the demand. Constraints (3.15) are only applied to must-run units. A must-run unit must supply energy unless it is in forced outage, i.e., $\mu_{gst} = 0$. Constraints (3.16) make sure that the UC state variables are consistent with the parameters representing the availability of each unit. The use of $\lceil \mu_{gst} \rceil$ models all output levels of generators, including derated output $0 < \mu_{gst} < 1$. $\lceil \mu_{gst} \rceil$ returns 1 unless $\mu_{gst} = 0$. That is to say, once μ_{gst} is other than 0, ω_{gst} is either 1 or 0; the underlying generating unit can still be selected to be online or offline. Constraints (3.17) model the downtime-dependent start-up costs (Nowak and Romisch, 2000) as described in Chapter 2. Here, we linearize the constraint (2.8) by adding an extra continuous variable, z_{gst}, representing the start-up cost. z_{gst} is greater than or equal to any of the values $C_{g\tau}(\omega_{gt} - \sum_{k=1}^{\tau} \omega_{g(t-k)}), \quad \tau = 1,\ldots,\tau_g^C$; it is equivalent that z_{gst} must be greater than or equal to the maximum value of $C_{g\tau}(\omega_{gt} - \sum_{k=1}^{\tau} \omega_{g(t-k)}), \quad \tau = 1,\ldots,\tau_g^C$, and z_{gst} is non-negative (see (3.22)). Because the problem is a minimization problem, at optimality z_{gst} must be equal to the maximum value of $C_{g\tau}(\omega_{gt}-\sum_{k=1}^{\tau}\omega_{g(t-k)})$, $\tau=1,...,\tau_g^C$, i.e., $\max_{\tau=1,...,\tau_g^C} C_{g\tau}(\omega_{gt}-\sum_{k=1}^{\tau}\omega_{g(t-k)})$. (3.18) and (3.19) are minimum down and minimum up time constraints, respectively (Nowak and Romisch, 2000). Constraints (3.20)-(3.21) hold for ramping up and ramping down limits between two consecutive time periods (Frangioni and Gentile, 2006). Phase angles are free variables, but in order to ensure that $|\theta_{ist}-\theta_{jst}|<<1$ for system stability, we assume that $\theta_{ist}\in[-\alpha,\alpha]$ where α is set to 0.05. This also ensures the validity of the nonlinear real power flow approximation.

3.5 Summary

In Chapter 3, we introduced a SMIP model to schedule electric power for the next day in a day-ahead electricity market, with allowance for outages. Unlike classic UC problems, we integrated a reserve demand curve into the model to show the reserve shortage level in the system; reserve shortage is penalized in the objective function according to its level. As a result, the energy/reserve prices are impacted by the reserve shortage in the system. In addition, we considered many scenarios where there are generator and/or transmission line outages. The resulting model is a large-scale stochastic programming model with binary variables in each stage, i.e., time period, as the number of scenarios increases. Solving such a large-scale problem is challenging particularly when in practice ISOs need to solve the problem in a day-ahead market within a time limit. Motivated by the computationally challenging large-scale problem, we propose a heuristic methodology to obtain a good sub-optimal solution in the next chapter.

Chapter 4
A Heuristic Methodology for Solving the SMIP Model

4.1 Introduction

The resulting model is a multi-stage SMIP problem that is challenging for commercial solvers. As described in Chapter 2, existing classical solution methods have some advantages solving stochastic IPs; but, they also have some limitations, such as that integer variables can only be associated with the first stage, or that numerical examples provided only have a small number of scenarios. The model proposed in this thesis can be easily extended to a large-scale optimization model if many scenarios are included; meanwhile, the ISO needs to solve the problem within some time requirement so that the DA schedule can be determined prior to the commencement of the RTM. Motivated by the need to solve the model within a reasonable time requirement, we propose a scenario-rolling heuristic to solve the problem based on the decision tree illustrated in Figure 3-1. This heuristic solves a small subproblem containing only some of the contingency scenarios at each iteration, and eventually obtains an overall near-optimal solution at the end.

4.2 Why a Heuristic Methodology

As previously mentioned in Chapter 2, major existing classical methods have difficulty solving a large-scale multi-stage SMIP with binary variables in each stage. Benders decomposition could be an option. However, according to the classical Benders algorithm, we have to put all the binary variables in the master problem in the underlying SMIP model, leaving the subproblem an LP. We have to solve the resulting large IP master problem at each iteration, and if there are many iterations, then the computational time could be very long. Solving the master problem is probably not too different from solving the original SMIP model in terms of computational difficulty, and it must be solved repeatedly. Meanwhile, the subproblem is still a multi-stage stochastic LP; it cannot be easily decomposed into small problems due to the existence of nonanticipativity constraints, dynamic constraints (constraints imposed on decision variables in two consecutive time periods), and geography-coupling constraints. On the other hand, since the model is expected to be solved within a time requirement prior to the commencement of the RTM, an algorithm that may take a long time to solve the problem is not desirable. Given all of these computational difficulties and the solution time requirement in practice, Benders is not selected to solve the problem. As a scenario decomposition method, a progressive hedging algorithm is also a possible candidate method. Nevertheless, as previously mentioned, there is no guarantee that a multi-stage stochastic program with discrete decision variables converges. Besides, it is difficult to decide the penalty coefficient in the objective function. Therefore, a progressive hedging algorithm is not adopted here.

We implemented both Benders decomposition and the progressive hedging algorithm in GAMS to evaluate the performance of the two methods given a large-scale numerical example. We found that both methods converge very slowly, i.e., after a long time, the stopping criteria are far from being satisfied. For the purpose of comparing the exact methods with the method proposed in this thesis, we will show the performance of Benders decomposition and progressive hedging algorithm at the end of this chapter.

Given the performance of classical methods, we provide a heuristic method that can return a feasible solution within the time requirement but without greatly affecting solution quality.

4.3 Solving the SMIP Problem

The heuristic is illustrated in Figure 4-1 (refer to Figure 3-1 on page 43). The iterations roll forward in time, one or more time periods per iteration, while fixing part of the solution found in previous iterations. However, to include some “look ahead” capability, the future of the most probable scenario is included in the subproblem at each iteration.

At each iteration, the heuristic solves a small subproblem including only a subset of the full scenario set. At iteration 1 (see Figure 4-1 (a)), only highlighted branches are solved in the first subproblem that contains the most probable scenario and the bottom four scenarios. Before the next iteration starts, these four outage scenarios are removed from the subset, since solutions to these scenarios have been found. The most probable scenario remains in the subset, and the solutions obtained in periods 1 and 2 are fixed at the values found in iteration 1. In iteration 2 (see Figure 4-1(b)), there are four new outage scenarios along with the most probable scenario. The scenario subset with these five scenarios is solved given that decisions in the first two periods are known. Again, at the end of iteration 2, all four outage scenarios are deleted from the subset, and solutions of time periods 3 and 4 in the most probable scenario are fixed. At this step, solutions of the most probable scenario in periods 1, 2, 3 and 4, which are known, are transferred to and used in iteration 3 (see Figure 4-1 (c)). The same procedure is repeated until all scenarios are removed from the decision tree.

As only a selected set of scenarios are use at each iteration, and the method rolls forward in time to a different bundle of scenarios, we call this heuristic a scenario-rolling heuristic.

To describe the heuristic in general, one can use a concise representation of the decision variables in the model. Suppose that $x_{st} = (\theta_{ist}, q_{ist}^{LS}, q_{mnst}^{S}, q_{gbst}^{E}, q_{gmst}^{R}, z_{gst})$ include all the continuous decision variables in scenario s during period t, and $y_{st} = (\omega_{gst})$ include all the binary variables in scenario s during period t. Denote an optimal solution of a subproblem by (x_{st}^*, y_{st}^*). Scenario “0” refers to the most probable scenario. The subset of outage scenarios solved in iteration k is defined as γ^k, and SP^k is used to denote its relevant subproblem that only contains scenarios in γ^k and scenario 0. t^k is defined as a subset of time periods; it contains time periods in the most probable scenario shared by the outage scenarios in γ^k, e.g. $\{x_{0t}^*, y_{0t}^*\} = \{x_{st}^*, y_{st}^*\}$, $s \in \gamma^k, t \in t^k$. SP^k, γ^k, and t^k are illustrated in Figure 4-1 (see the legends). In Figure 4-1, we solve an equal number of scenarios in each iteration, i.e., five scenarios. However, the number of scenarios solved in each iteration can be different, giving the user the flexibility to determine the size of subproblems. All γ^k in the numerical examples of this chapter contain the same number of contingency scenarios, except the last iteration that contains the remaining set of scenarios.

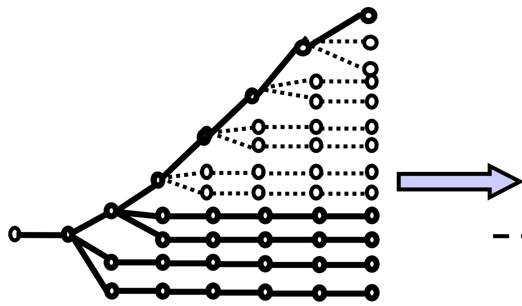


Figure 4-1 (a). Iteration 1 (k=1)
: γ^1= {9,10,11,12}, t^1={1,2},
scenarios in SP^1: 0, 9, 10, 11, 12

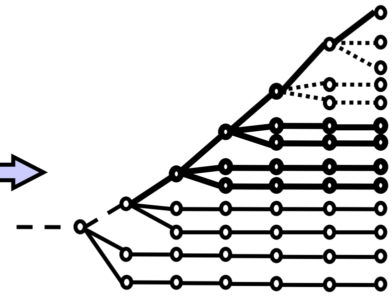


Figure 4-1 (b). Iteration 2 (k=2)
γ^2= {5,6,7,8}, t^2={3,4},
scenarios in SP^2: 0, 5, 6, 7, 8

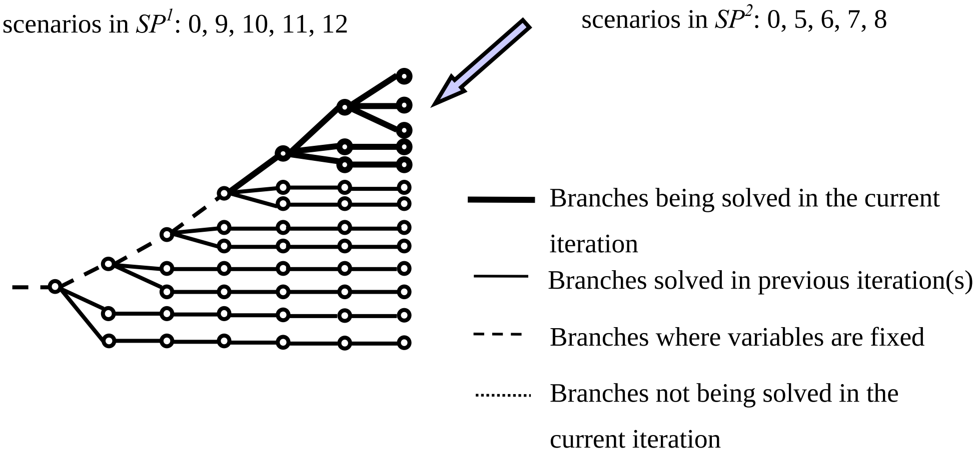


Figure 4-1 (c). Iteration 3 (k=3)
γ^3= {1,2,3,4}, t^3={5,6},
scenarios in SP^3: 0, 1, 2, 3, 4

Figure 4-1 Small illustrative example for scenario rolling heuristic

Suppose that the scenario-rolling heuristic starts from the bottom of the decision tree.

Step 1: k =1, initialize the outage scenario subset γ^k;

Step 2: Solve SP^k including $\{0 \cup \gamma^k\}$ and obtain x^*_{st} and y^*_{st};

Step 3: Fix variables in the most probable scenario: $x^*_{0t'}$, $y^*_{0t'}$ $s \in \gamma^k, t \in t^k$; record outage scenario variables x^*_{st}, y^*_{st}, for $\forall s \in \gamma^k, \forall t$;

Step 4: Empty the incumbent γ^k, then let $k=k+1$.

Update γ^k by introducing a new bundle of unsolved outage scenarios from the scenario tree into the subset.

If $\gamma^k = \varnothing$ after update, then go to step 5;

Otherwise go to Step 2.

Step 5: All the scenarios have been solved; a feasible solution to the overall problem is obtained.

In the next section, we give two numerical examples to show the performance of the heuristic.

4.4 Numerical Examples

In this section, we provide two numerical examples to test the performance of the heuristic method. The first example is a 6-bus small power system. We consider 70 scenarios including generating unit outages and a transmission line outage. The results are analyzed and discussed. The second example contains 32 generating units and 20 buses, and we consider 185 scenarios including generating outage scenarios only.

4.4.1 A 6-bus Power System

4.4.1.1 Data for 6-bus System

In the first numerical example, we use a small power network that includes 6 buses. The 6-bus power network topology is given in Figure 4-2. Tables 4-1 to 4-3 show the data used in this example. The system consists of one nuclear plant and five thermal generating units, nine lines, and three loads at nodes 2, 3, and 4, respectively. The nuclear plant is the base unit that is expected to run all the time unless it breaks down, thus it is the only must-run unit. The nuclear unit has the lowest fuel cost and highest start-up cost; three cyclers are generating units that will be turned on and off subject to minimum up/down constraints, depending on need; two peaking units or peakers are flexible units with expensive fuel costs and low start-up costs, and their start-up time is very short. Particularly, peaker 2 has the most expensive fuel cost and the lowest start-up cost. These units are needed when there is a contingency such as equipment failure or unexpected high demand. The generating units are allocated as follows: one base unit with the largest capacity at node 5, one cycler and one peaking unit at node 1, and one peaker and two cyclers at node 6. Among the two peakers, peak 2 associated at node 6 has the most expensive fuel cost and the smallest capacity. The system is scheduled over a 24-hour horizon. Time period 1 starts from 12:00 am on the delivery day. Each time period lasts one hour. We do not consider any GenCos that own groups of these generating units; each generating unit is independent and submits its own generation offer.

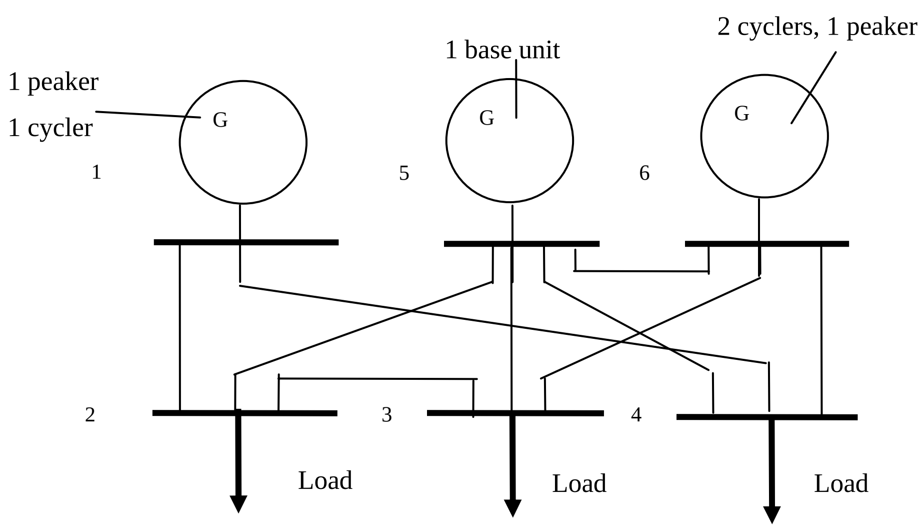


Figure 4-2 6-bus test system

Table 4-1 6-bus system generating unit data

Generating Units	Base Unit at Node 5	Cycler 1 at Node6	Cycler 2 at Node 6	Cycler 3 at Node 1	Peaker 1 at Node 1	Peaker 2 at Node 6
$Q_g^{\max}$ (MW)	400	350	155	100	100	70
Q_g^{MSL} (MW)	100	20	15	10	10	10
T_g^{dn} T_g^{up} (Hour)	24	5	3	1	1	1
R_g^{dn} R_g^{up} (MW)	200	150	75	40	80	60
R_g^{su} R_g^{sd} (MW)	100	20	15	10	20	10
τ_g^C (hour)	N/A	6	10	8	0	0
$C_{g\tau_g^C}$ ($)	(N/A)	225	225	225	175	90
C_g^{NL} ($/hr)	80	80	80	100	100	150
4-pair (C_{gb}^E, Q_{gb}^E) ($/MWh, MW)	(5.31,100)	(7.94,55)	(10.08,140)	(17.28,30)	(19.20,30)	(39.44, 30)
	(5.38,100)	(8.20, 40)	(10.68 ,90)	(18.29,30)	(20.32,30)	(40.56, 20)
	(5.53,120)	(8.54, 30)	(11.09, 70)	(19.10,30)	(21.22,30)	(57.09, 10)
	(5.66,80)	(9.01,30)	(11.72, 50)	(19.92,10)	(22.13,10)	(57.71,10)

Table 4-2 6-bus test system transmission line data

Node i	Node j	B_{ijst} (Ω^{-1})	U_{ij}(MW)
1	2	2.74	150
1	4	6.87	150
2	3	2.9	175
5	2	3.49	200
5	3	5.4	120
5	4	8.62	130
6	3	4.31	200
6	4	5.46	300
6	5	5.21	100

Table 4-3 6-bus test system load data

Period	Node 2	Node 3	Node 4	Total demand
1	71.136	144.723	146.604	362.463
2	78.660	149.559	160.740	388.959
3	78.660	152.428	160.740	391.828
4	114.000	152.000	161.500	427.500
5	114.000	154.404	166.250	434.654
6	114.000	160.028	171.000	445.028
7	114.000	168.245	195.073	477.318
8	95.057	190.380	254.505	539.942
9	134.708	237.782	320.325	692.815
10	168.456	237.910	322.483	728.849
11	170.157	288.492	319.140	777.789
12	168.456	285.492	317.978	771.926
13	157.320	277.396	298.680	733.396
14	157.320	277.396	311.836	746.552
15	165.053	291.337	309.110	765.500
16	163.351	288.338	324.522	776.211
17	163.351	288.338	310.935	762.624
18	158.246	279.327	307.592	745.166
19	144.734	265.700	295.762	706.196
20	144.734	265.700	298.976	709.411
21	146.308	268.584	233.073	647.965
22	136.868	251.256	192.888	581.012
23	83.904	173.280	175.750	432.934
24	94.392	154.024	171.456	419.872

For the sake of simplicity, only spinning reserves are considered in the 6-bus example. The target spinning reserve requirement is set to 400MW, equal to the largest unit capacity. The minimum reserve requirement is 25MW. We consider three contingencies in each time period;

two are generator outages, and one is a transmission line outage. The two generating units selected are the base unit at node 5 and the largest cycler at node 6, as these units have the more significant impact on the reliability of the system than other units. The line between nodes 5 and 2 is chosen as the line outage as it connects the base unit and one of the demand nodes. The most probable scenario is given the highest probability. We give a probability of 0.8 to the most probable scenario. The other 0.2 probability is evenly divided among the outage scenarios, assuming that they have identical probabilities. As to initial states of the generating units, the must-run unit and the cyclers at node 6 are in "on" state. In the real world, the states of generating units in period 1 depend on their states the day before, subject to minimum up/down constraints. However, in an academic environment, we have no knowledge of their states on the preceding day; therefore, we narrow down the time frame to a fixed 24 hours and assume that they are online.

Penalty costs for lost load are set to $10000/MWh. We use a four-step function to represent the reserve demand: $1000/MW for the first 100MW reserve loss, $3000/MW for the next 250MW, and $6000/MW for the reserve shortage between 250MW and 275MW. The last 25MW will be priced at $10000/MW. The function is similar to the one shown in Figure 3-2.

The decision tree with 70 scenarios for the 6-bus system is shown in Figure 4-3. As shown in Figure 4-2, we selected the largest base unit at bus 5 and the largest cycler at bus 6 as generator outages because these two largest units have more impact on the reliability of the system than other small generating units. We chose the line connecting the supply bus 5 and demand bus 2 as the transmission line outage since this line connects the base unit at bus 5, and this outage can restrict the output level of the largest unit. These three contingencies occur from time period 2 and last for 23 time periods. Therefore, we have 69 contingency scenarios with the assumption that a contingency, if it occurs, always remains until the end of the time period 24. Plus the most probable scenario, there are a total of 70 scenarios considered in the scenario tree. Scenario 0 represents the most probable scenario where nothing breaks down from time period 1 to period 24. In scenario 67, the unit with the second-largest capacity in the system is down from time period 2. The largest unit is offline from time period 2 in scenario 68. Scenario 69 represents a transmission line outage starting from the second time period. All outages hold until the end of time period 24, no matter when they occur. At the beginning of period 1, i.e., "now" in Figure 4-3, we assume that it is known to the ISO that all the equipment works perfectly without any failures in the next one hour. That is why the contingencies occur from time period 2.

We could incorporate demand scenarios into the scenario tree where actual demand levels deviate from the expected demand. However, an astronomical number of demand realizations can easily render the problem intractable. For example, assume we only have three demand realizations in each time period: a high demand, an expected demand, and a low demand. Then, we have a total of 94,143,178,827 scenarios at the end of time period 24. Thus, we did not consider demand variations, and assume that the ISO can have accurate forecast of the demand for the next 24 hours.

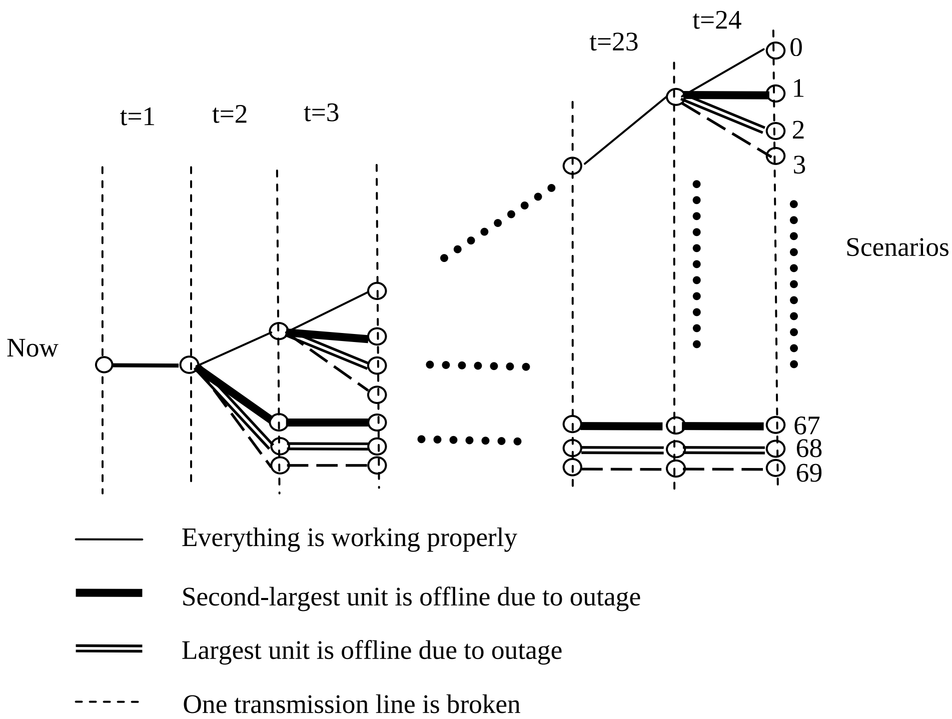


Figure 4-3 Scenario tree for 6-bus system

4.4.1.2 Performance of Heuristic on 6-bus System

The problem has 300,749 constraints and 92,401 variables including 10,080 binary variables. The scenario-rolling heuristic is implemented in GAMS and solved using Cplex 9.1 (see Appendix A for the GAMS coding of the numerical example). The program is run on a Sun Blade 2500 Workstation with 1.6GHz CPU and 5GB memory. We implemented the heuristic with different sizes of outage scenario subsets and compared the quality and efficiency of the resulting heuristics. The results are shown in Table 4-4.

Table 4-4 Performance of scenario rolling heuristic and Cplex for the 6-bus system

Maximum # of Scenarios Solved in Each Iteration	# of Iteration	Obj. Value	Solver Time (in sec.)	Gap with Lower Bound
4	23	1070059.337	34.276	0.080%
7	12	1069823.495	40.704	0.058%
10	8	1069823.735	51.625	0.058%
13	6	1069768.129	60.748	0.053%
19	4	1069797.568	108.448	0.056%
25	3	1069782.652	114.497	0.055%
37	2	1069330.810	215.950	0.012%
Cplex	1	1069343.264	18000.000	0.014%
Cplex lower bound	N/A	1069198.645		

The rows 2-8 show the results for the scenario-rolling heuristic with different numbers of outage scenarios in the subset. Row 9 gives the result when Cplex is used directly to solve the problem. In each iteration of the heuristics, we set the time limit to 1000 seconds and the optimality gap to 0.01%; as soon as one of the two stop criteria is satisfied, the incumbent iteration is terminated. When we use Cplex to solve the problem directly, the time limit is 18000 seconds and the optimality gap is 0. The columns display the maximum number of scenarios solved in each iteration in the heuristic, the number of iterations, the objective function value, solver time, and the gaps between heuristic solution, Cplex solution and the Cplex lower bound returned by Cplex.

We can see from Table 4-4 that the scenario-rolling heuristic gives good-quality solutions when compared with Cplex. Meanwhile, the heuristic offers a significant advantage in terms of computation time. The heuristic with 37 contingency scenarios in each iteration only takes

215.950 seconds, representing only 1.120% of the Cplex computation time, and its solution has a gap of 0.012% with respect to the Cplex lower bound, which is better than the best solution found by Cplex after 18000 seconds.

The heuristic grouping more scenarios in each iteration is expected to return better solutions than those solving fewer scenarios at each iteration. However, due the gap limit at each iteration, this is not necessarily true. All the heuristic solver times are very short; although this numerical example is not highly realistic, it is an example used to illustrate the performance of the scenario-rolling heuristic. In the real world, the time requirement for DA schedule is usually at most two hours as the RTM begins after DAM closes. A more realistic numerical example than the 6-bus system will be shown later in this chapter to further evaluate the performance of the heuristic method.

Next, we will take the 37-scenario heuristic (row 8 in Table 4-4) as an example to analyze the results of power output, load shedding and reserve shortage level in some selected scenarios, as it returns the minimum objective function value compared with others in Table 4-4.

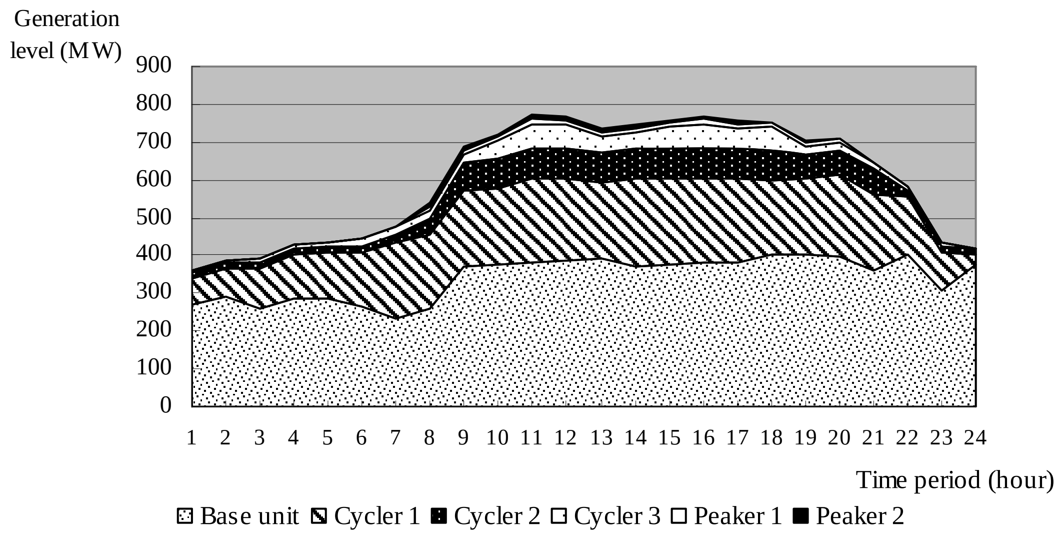


Figure 4-4 Power generation level of generating units in the most probable scenario

4.4.1.3 Discussion of Heuristic Solution to 6-bus System

In the most probable scenario (see Figure 4-4), all the generating units are turned on to supply power during peak hours during period 8 to period 17, i.e., 8:00AM to 5:00PM. Since the base unit is the cheapest, it runs up close to its capacity of 400MW all the time, while its idle capacity contributes to meeting reserve requirement. The power gap between its output and the demand is made up by the next cheapest available units, the three cyclers. As slow-start units, these generating units can provide spinning reserves only when they are online. Therefore, they have to be turned on and supply some amount of energy, making up the difference between the demand and the generation level of the base unit. During peak hours, both peakers are turned on to supply energy to avoid a blackout, although they have more expensive fuel costs than others. In the last few time periods the electricity demand is lower than the demand during the peak time of the day, and the expensive peakers are turned off; the base unit and the cyclers are in “on” states to maintain the supply-demand balance. These quick-start units are just offline to save their capacity for system reserves requirement. See Table 4-5 for the generation level of each generating unit in the most probable scenario.

Table 4-5 Generation plan in the most probable scenario

Time Period	Base Unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Load Shed	Demand
1	268.505	68.959	15.000	10.000	0.000	0.000	0.000	362.464
2	292.131	71.827	15.000	10.000	0.000	0.000	0.000	388.959
3	259.328	107.500	15.000	10.000	0.000	0.000	0.000	391.828
4	287.847	114.653	15.000	10.000	0.000	0.000	0.000	427.500
5	284.626	125.027	15.000	10.000	0.000	0.000	0.000	434.653
6	262.710	147.318	15.000	20.000	0.000	0.000	0.000	445.028
7	232.376	200.000	24.942	20.000	0.000	0.000	0.000	477.318
8	257.308	200.000	42.634	20.000	10.000	10.000	0.000	539.942
9	370.264	200.000	77.370	20.000	10.000	10.000	0.000	687.634
10	377.353	200.017	80.000	45.000	10.000	10.000	0.000	722.370
11	380.034	224.966	80.000	60.000	16.245	10.000	0.000	771.245
12	387.213	217.787	80.000	60.000	10.447	10.000	0.000	765.447
13	391.844	200.000	80.000	41.552	10.000	10.000	0.000	733.396
14	372.770	228.782	80.000	45.000	10.000	10.000	0.000	746.552
15	374.140	230.860	80.000	54.152	10.000	10.000	0.000	759.152
16	380.330	224.670	80.000	60.000	14.929	10.000	0.000	769.929
17	381.858	223.142	80.000	51.342	10.000	10.000	0.000	756.342
18	387.884	200.000	80.000	60.000	11.196	0.000	0.000	739.079
19	400.000	201.785	64.411	20.000	10.000	10.000	0.000	706.196
20	394.970	220.030	64.411	20.000	10.000	0.000	0.000	709.411
21	361.952	200.000	66.012	20.000	0.000	0.000	0.000	647.965
22	400.000	156.012	15.000	10.000	0.000	0.000	0.000	581.012
23	308.063	99.871	15.000	10.000	0.000	0.000	0.000	432.934
24	375.000	29.872	15.000	0.000	0.000	0.000	0.000	419.871

In Figure 4-5, we show the generation levels of the generating units in a contingency scenario where the base unit (the largest unit) is down from time period 9. We can see that all the units are online to supply energy from time period 9 to avoid blackouts as the penalty for load shedding is very high. The most expensive quick-start unit, peaker 2, is turned on due to the loss of the base unit and peak demand during period 8 to period 20 to avoid load shedding. It is called on only when other units cannot ramp up to a higher output level in time because of ramping limits during time period 9, when the largest generating unit breaks down. However, peaker 2 is turned on in period 8 (before it is known whether a contingency will occur in period 9) and runs at its minimum economical operating level, so that it can be ready to ramp up to 70MW in period 9 in case there is a contingency such as a breakdown of the base unit. In this scenario, the system loses 400MW of capacity in 16 hours that can be used to supply energy and prepare reserves when there is no contingency. As a result, there is not enough capacity available to meet the target reserve requirements during some of the time periods, while the minimum reserve requirement must be satisfied. Table 4-6 provides the generation plan of generating units in this scenario. We can see that in time period 9, the generation output of the base unit drops to 0 and every other generating unit ramps up subject to the ramping limit or maximum generation level limit to meet the demand that is supposed to be satisfied by the base unit. For example, the most expensive peaker (peaker 2) ramps up from 10MW to 70MW in time period 9; due to its capacity limit, that is the maximum generation level to which it can ramp up. In period 10, peaker 2 ramps down to 17.370MW because cheaper generating units can further ramp up to their maximum generation level from period 9; it just needs to delivery 17.370MW to satisfy the electricity demand. However, in next hour, all the generating units are running at their maximum operating level except for peaker 2; it saves a capacity of 25 MW to meet the minimum reserve requirement because it is the most expensive unit and the minimum reserve requirement must be satisfied all the time. As a result, the ISO has to shed load by 21.245MWh in period 11 to maintain the supply-demand balance when the reserve available in the system is only 25MW, barely satisfying the minimum reserve requirement. The numerical results demonstrate and interpret how the reserve demand curve works, as shown in Figure 3-2.

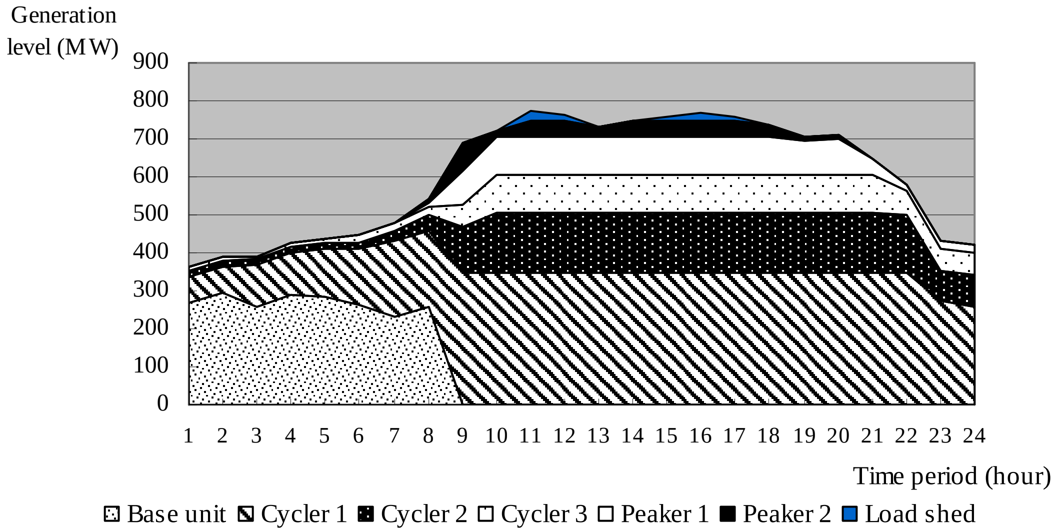


Figure 4-5 Power generation level of generating units in a contingency scenario (largest unit down from time period 9)

During some time periods, when the largest generating unit breaks down, there is no idle capacity; each of the functioning generating unit splits their capacity between supplying energy and providing reserve or commits all of its capacity to supplying energy. As the most expensive generating unit, peaker 2 supplies the least energy compared with others; most of its capacity is used to meet the reserve requirement except during the peak hours, i.e., period 8 to period 20. Particularly, it is always used to satisfy the minimum reserve requirement during the shortage hours while other committed units are running at their maximum operating level. Peakers 2 is turned off in the last few time periods when the demand is low and other generating units have adequate capacity to supply energy.

Table 4-6 Generation plan in a contingency scenario (largest unit down from time period 9)

Time Period	Base Unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Load Shed	Demand
1	268.505	68.959	15.000	10.000	0.000	0.000	0.000	362.464
2	292.131	71.828	15.000	10.000	0.000	0.000	0.000	388.959
3	259.328	107.500	15.000	10.000	0.000	0.000	0.000	391.828
4	287.847	114.653	15.000	10.000	0.000	0.000	0.000	427.500
5	284.626	125.027	15.000	10.000	0.000	0.000	0.000	434.653
6	262.710	147.318	15.000	20.000	0.000	0.000	0.000	445.028
7	232.376	200.000	24.942	20.000	0.000	0.000	0.000	477.318
8	257.308	200.000	42.634	20.000	10.000	10.000	0.000	539.942
9	0.000	350.000	117.634	60.000	90.000	70.000	0.000	687.634
10	0.000	350.000	155.000	100.000	100.000	17.370	0.000	722.370
11	0.000	350.000	155.000	100.000	100.000	45.000	21.245	771.245
12	0.000	350.000	155.000	100.000	100.000	45.000	15.447	765.447
13	0.000	350.000	155.000	100.000	100.000	28.396	0.000	733.396
14	0.000	350.000	155.000	100.000	100.000	41.552	0.000	746.552
15	0.000	350.000	155.000	100.000	100.000	45.000	9.152	759.152
16	0.000	350.000	155.000	100.000	100.000	45.000	19.929	769.929
17	0.000	350.000	155.000	100.000	100.000	45.000	6.342	756.342
18	0.000	350.000	155.000	100.000	100.000	34.079	0.000	739.079
19	0.000	350.000	155.000	100.000	91.196	10.000	0.000	706.196
20	0.000	350.000	155.000	100.000	94.411	10.000	0.000	709.411
21	0.000	350.000	155.000	100.000	42.965	0.000	0.000	647.965
22	0.000	350.000	151.012	60.000	20.000	0.000	0.000	581.012
23	0.000	272.934	80.000	60.000	20.000	0.000	0.000	432.934
24	0.000	259.871	80.000	60.000	20.000	0.000	0.000	419.871

In addition, node 2 is a demand node, and it is connected to supply nodes 1 and 5. In this scenario, the largest unit at node 5 breaks down. Therefore, the power injected in node 5 is equal to the power withdrawn from this node. There are two cyclers at node 6; part of the power output from the node goes through node 5 in order to supply demand at node 2. However, there are at most 200MW of power that can be delivered due to the power limit on the transmission line connecting node 5 and node 6. The 100MW going through node 5 will be used to inject power into node 2 and node 4. Nevertheless, the two cyclers cannot solely satisfy the total demand at the demand nodes. Therefore, in this scenario, peakers need to be turned on to supply energy. Since the peaker at node 1 is connected to both nodes 2 and 4 and is cheaper than the peaker at node 6, it runs at a high operating level in most of the time periods.

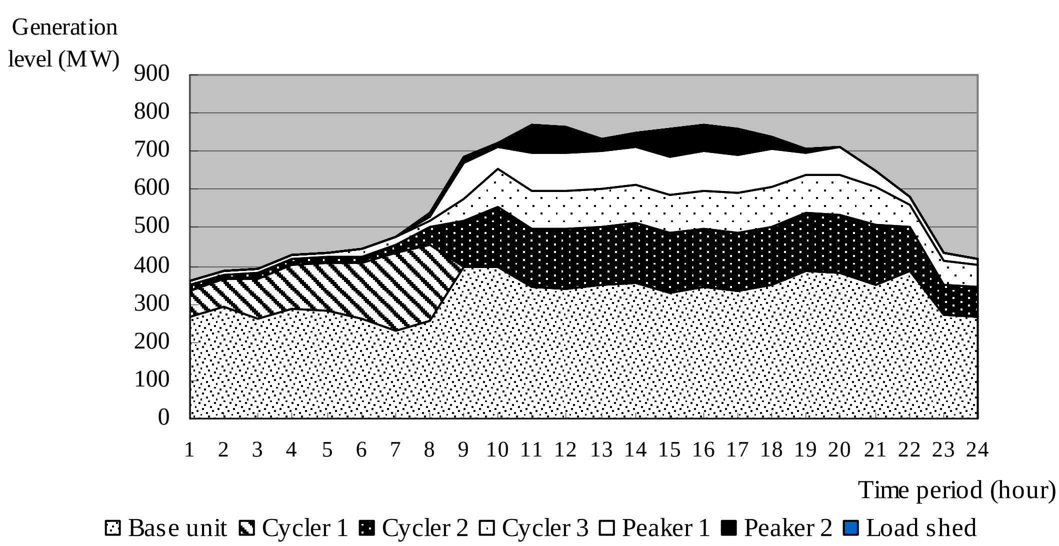


Figure 4-6 Power generation level of generating units in a contingency scenario (second largest unit down from time period 9)

Another generator outage scenario is shown in Figure 4-6; the second largest generating unit is down from time period 9. Table 4-7 shows the generation plan for this scenario. Similar to the largest unit outage, as the outage starts, every other generating unit ramps up to a higher generation level, making up the loss of the second-largest generating unit. In the first few time periods, due to lower demand, both peakers are turned off, since other generating units have adequate capacity to meet the electricity demand. There is a small amount of load shedding in a couple of time periods. For example, in period 11 there is 3.032MWh unserved load at node 3. We observe that the base unit at node 5 is not running at its maximum generation level, but it cannot ramp up its output level due to a power limit on the transmission line connecting node 3 and node 5, which is binding. Meanwhile, other generating units are operating at their maximum generation level constrained by their capacity limits. This causes load shedding in this time period. For the same reasons, there is unserved load during periods 15 and 16. Peakers are turned on in advance in time period 8 so that they can ramp up to a high generation level in case they are needed for contingencies during the following hours, e.g., to make up the loss of cycler 2. Peaker 2 is turned off in the last few hours because the demand at that point is lower than that in the previous time periods.

Table 4-7 Generation plan in a contingency scenario (second-largest unit down from time period 9)

Time Period	Base Unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Load Shed	Demand
1	268.505	68.959	15.000	10.000	0.000	0.000	0.000	362.464
2	292.131	71.828	15.000	10.000	0.000	0.000	0.000	388.959
3	259.328	107.500	15.000	10.000	0.000	0.000	0.000	391.828
4	287.847	114.653	15.000	10.000	0.000	0.000	0.000	427.500
5	284.626	125.027	15.000	10.000	0.000	0.000	0.000	434.653
6	262.710	147.318	15.000	20.000	0.000	0.000	0.000	445.028
7	232.376	200.000	24.942	20.000	0.000	0.000	0.000	477.318
8	257.308	200.000	42.634	20.000	10.000	10.000	0.000	539.942
9	400.000	0.000	117.634	60.000	90.000	20.000	0.000	687.634
10	400.000	0.000	155.000	100.000	57.370	10.000	0.000	722.370
11	343.212	0.000	155.000	100.000	100.000	70.000	3.032	771.244
12	342.722	0.000	155.000	100.000	100.000	67.725	0.000	765.447
13	348.280	0.000	155.000	100.000	100.000	30.116	0.000	733.396
14	355.175	0.000	155.000	100.000	100.000	36.377	0.000	746.552
15	331.831	0.000	155.000	100.000	100.000	70.000	2.320	759.151
16	343.686	0.000	155.000	100.000	100.000	70.000	1.242	769.928
17	334.240	0.000	155.000	100.000	100.000	67.102	0.000	756.342
18	349.899	0.000	155.000	100.000	100.000	34.180	0.000	739.079
19	385.117	0.000	155.000	100.000	56.079	10.000	0.000	706.196
20	380.771	0.000	155.000	100.000	73.640	0.000	0.000	709.411
21	351.818	0.000	155.000	100.000	41.147	0.000	0.000	647.965
22	385.241	0.000	115.771	60.000	20.000	0.000	0.000	581.012
23	272.934	0.000	80.000	60.000	20.000	0.000	0.000	432.934
24	265.000	0.000	80.000	60.000	14.871	0.000	0.000	419.871

Figure 4-7 shows the generation level of the generating units in a scenario where there is a transmission line failure. The line between node 2 and node 5 breaks down. As a result, 200MW capacity over this line cannot be utilized. Node 5 is a supply node associated with the base unit only, therefore the generation output of the base unit is significantly affected by this contingency; its output is expected to be lower than ususal. Thus, the ISO has to determine a different generation plan for all the generating units from the schedule in the most probable scenario.

Compared with Figure 4-4 showing the generation plan in the most probable scenario, the generation level of the base unit drops in the time period 9 and afterwards because it can only deliver energy through other transmission lines connected to node 5. Meanwhile, the three cyclers ramp up their output to replace the loss of capacity due to the ramping down of the base unit. They are associated with node 6 and node 1, and power capacity limits over the lines connecting these nodes are adequate for them to deliver more electricity than in the most probable scenario.

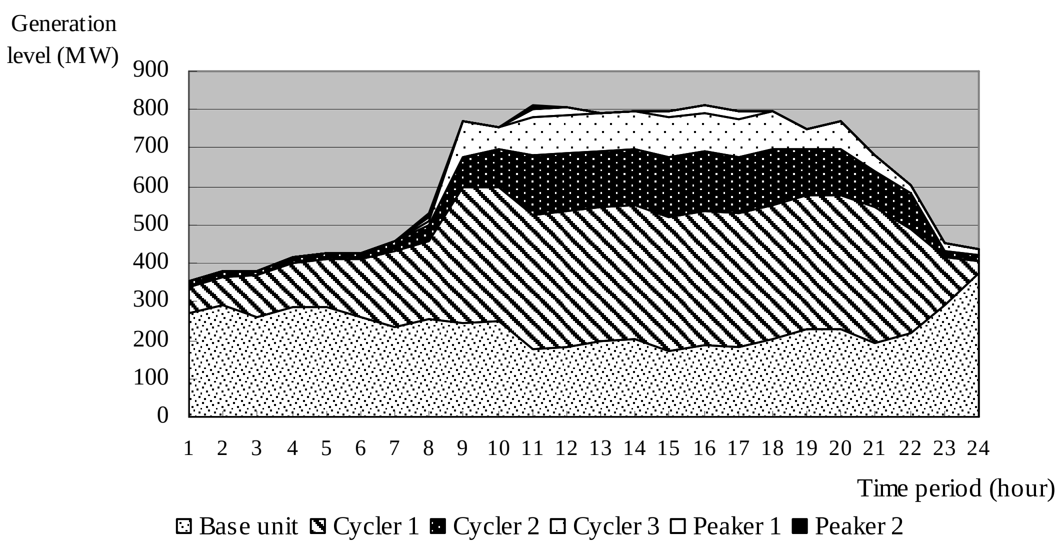


Figure 4-7 Power generation level of generating units in a contingency scenario (transmission line failure connecting node 2 and node 5 from time period 9)

Table 4-8 gives the generation plan in this scenario. We see that in time period 9, the base unit ramps down due to one transmission line loss. Cycler 1 operates at its maximum generation level in a few hours in the new schedule, generating more electricity to satisfy the demand, since the output of the base unit shrinks due to the line failure. In most time periods, the generation level of the three cyclers is higher than that in the most probable scenario. By rescheduling, the demand is still satisfied, and there is no load cut needed. Although there is no direct connection between node 6 and node 2, the cyclers can satisfy the demand at node 2 through node 3, as there is a connection between node 2 and node 3.

Table 4-8 Generation plan in a contingency scenario (transmission line failure from time period 9)

Time period	Base Unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Load Shed	Demand
1	268.504	68.959	15.000	10.000	0.000	0.000	0.000	362.463
2	292.131	71.827	15.000	10.000	0.000	0.000	0.000	388.958
3	259.328	107.500	15.000	10.000	0.000	0.000	0.000	391.828
4	287.847	114.653	15.000	10.000	0.000	0.000	0.000	427.500
5	284.626	125.027	15.000	10.000	0.000	0.000	0.000	434.653
6	262.709	147.318	15.000	20.000	0.000	0.000	0.000	445.027
7	232.376	200.000	24.942	20.000	0.000	0.000	0.000	477.318
8	257.308	200.000	42.634	20.000	10.000	10.000	0.000	539.942
9	245.781	350.000	81.853	10.000	0.000	0.000	0.000	687.634
10	250.762	350.000	94.238	27.370	0.000	0.000	0.000	722.370
11	177.877	350.000	153.368	60.000	20.000	10.000	0.000	771.245
12	184.068	350.000	151.379	60.000	20.000	0.000	0.000	765.447
13	196.258	350.000	145.000	42.138	0.000	0.000	0.000	733.396
14	201.370	350.000	143.630	51.552	0.000	0.000	0.000	746.552
15	172.770	350.000	155.000	61.381	20.000	0.000	0.000	759.151
16	186.183	350.000	153.746	60.000	20.000	0.000	0.000	769.929
17	180.330	350.000	146.012	60.000	20.000	0.000	0.000	756.342
18	201.102	350.000	143.898	44.079	0.000	0.000	0.000	739.079
19	228.338	350.000	116.662	11.196	0.000	0.000	0.000	706.196
20	229.534	350.000	115.466	14.411	0.000	0.000	0.000	709.411
21	194.970	350.000	92.995	10.000	0.000	0.000	0.000	647.965
22	216.896	274.116	90.000	0.000	0.000	0.000	0.000	581.012
23	293.818	124.116	15.000	0.000	0.000	0.000	0.000	432.934
24	375.000	29.871	15.000	0.000	0.000	0.000	0.000	419.871

Tables 4-9 to 4-12 present the power flows in the most probable scenario and the other three contingency scenarios, respectively. The values in brackets in the first row of each table represent the line capacity between nodes. If the value of the power flow is negative, it indicates that the power flows in the opposite direction. For example, in Table 4-9, power flow from node 5 to node 6 is -11.28MW in period 6, i.e., the power flows from node 6 to node 5 in the amount of 11.283MW.

Table 4-9 Line flows in the most probable scenario (all in MW)

Time Period	5 to 6 (200)	5 to 2 (200)	5 to 3 (150)	5 to 4 (180)	6 to 3 (300)	6 to 4 (300)	2 to 3 (175)	1 to 2 (150)	1 to 4 (150)
1	18.201	58.330	89.178	102.796	56.121	46.038	-0.577	12.230	-2.230
2	20.928	63.596	94.089	113.518	57.784	49.971	-2.315	12.749	-2.749
3	4.971	61.524	88.928	103.905	66.865	60.605	-3.366	13.771	-3.771
4	5.354	78.150	94.797	109.545	71.232	63.776	-14.029	21.821	-11.821
5	1.805	78.152	94.569	110.099	73.987	67.846	-14.153	21.696	-11.696
6	-11.283	77.077	91.959	104.956	82.730	78.305	-14.662	22.261	-12.261
7	-31.519	74.576	87.419	101.900	95.847	97.576	-15.021	24.403	-4.403
8	-33.656	69.418	94.305	127.240	103.112	115.866	-7.037	18.601	11.399
9	-26.371	95.550	126.484	174.601	122.769	138.231	-11.471	22.506	7.494
10	-27.625	106.351	129.938	168.707	126.563	135.812	-18.590	37.035	17.965
11	-33.577	107.783	150.000	155.828	147.499	133.891	-9.007	46.823	29.422
12	-30.297	108.128	150.000	159.382	144.785	132.705	-9.293	44.556	25.891
13	-24.458	108.137	148.604	159.561	138.842	126.700	-10.050	39.133	12.419
14	-36.039	106.560	143.941	158.308	144.700	138.043	-11.244	39.515	15.485
15	-36.168	106.939	150.000	153.369	149.643	135.049	-8.305	43.460	20.692
16	-33.029	105.318	149.251	158.791	146.447	135.194	-7.360	44.391	30.538
17	-32.582	106.979	150.000	157.461	146.676	133.883	-8.338	41.751	19.591
18	-21.467	103.452	148.824	157.075	136.542	121.990	-6.039	42.669	28.527
19	-18.747	105.423	144.653	168.671	130.963	126.485	-9.917	29.395	0.605
20	-22.010	105.120	143.390	168.470	132.654	129.777	-10.344	29.270	0.730
21	-22.496	104.069	144.078	136.301	133.606	109.910	-9.100	33.139	-13.139
22	11.953	102.360	148.254	137.434	108.440	74.525	-5.438	29.071	-19.071
23	15.369	68.643	104.074	119.976	70.353	59.888	-1.147	14.114	-4.114
24	46.065	77.566	109.877	141.492	49.591	41.346	-5.445	11.382	-11.382

Table 4-9 shows that in the most probable scenario, the constraints (3.11) are not binding except for the line connecting node 5 and node 3 in periods 11 and 12. In these two time periods, the power flow through this line is 150MW, equal to the line capacity between node 5 and node

3. From Table 4-9, we see that part of the power flowing out of node 5 comes from node 6, although the majority of it is injected by the base unit at node 5. For example, in period 11, 33.577MW from node 6 is delivered to node 5 (a supply node) and injected into the demand nodes through node 5.

Table 4-10 Line flows in the largest unit failure scenario (all in MW)

Time Period	5 to 6 (200)	5 to 2 (200)	5 to 3 (150)	5 to 4 (180)	6 to 3 (300)	6 to 4 (300)	2 to 3 (175)	1 to 2 (150)	1 to 4 (150)
1	18.201	58.330	89.178	102.796	56.121	46.038	-0.577	12.230	-2.230
2	20.928	63.596	94.089	113.518	57.784	49.971	-2.315	12.749	-2.749
3	4.971	61.524	88.928	103.905	66.865	60.605	-3.366	13.771	-3.771
4	5.354	78.150	94.797	109.545	71.232	63.776	-14.029	21.821	-11.821
5	1.805	78.152	94.569	110.099	73.987	67.846	-14.153	21.696	-11.696
6	-11.283	77.077	91.959	104.956	82.730	78.305	-14.662	22.261	-12.261
7	-31.519	74.576	87.419	101.900	95.847	97.576	-15.021	24.403	-4.403
8	-33.656	69.418	94.305	127.240	103.112	115.866	-7.037	18.601	11.399
9	-160.064	54.643	64.566	40.855	183.947	193.623	-10.731	64.153	85.847
10	-156.821	61.749	68.298	26.774	184.243	181.305	-14.631	85.597	114.403
11	-162.987	56.897	86.048	20.041	203.511	183.502	-1.067	84.404	115.596
12	-163.301	58.412	85.192	19.697	203.087	183.613	-2.786	85.332	114.668
13	-159.267	62.095	84.464	12.708	199.170	174.959	-6.237	88.987	111.013
14	-163.110	62.275	83.167	17.668	201.313	182.128	-7.084	87.961	112.039
15	-163.434	59.932	88.188	15.314	205.589	180.977	-2.440	87.181	112.819
16	-162.639	54.789	85.355	22.494	202.670	184.691	0.312	82.663	117.337
17	-163.548	60.246	86.975	16.326	204.715	181.737	-3.353	87.128	112.872
18	-160.560	60.201	84.159	16.199	199.995	178.524	-4.827	87.131	112.869
19	-153.357	57.185	79.802	16.370	190.559	171.084	-4.661	82.889	108.307
20	-153.295	56.817	79.693	16.786	190.421	171.284	-4.414	83.504	110.906
21	-151.427	63.354	83.916	4.157	192.246	161.327	-7.578	75.375	67.589
22	-151.035	65.813	77.509	7.714	186.808	163.169	-13.061	57.994	22.006
23	-105.151	38.433	50.630	16.088	127.396	120.387	-4.746	40.725	39.275
24	-102.263	41.460	44.483	16.320	120.102	117.507	-10.562	42.371	37.629

As suggested in Table 4-10, there is no line congestion in the scenario where the base unit breaks down in period 9. However, the base unit at node 5 does not inject any power into the system due to its breakdown from period 9. Thus, beginning in period 9, power delivered from node 6 to node 5 increases. It indicates that, through the line between node 6 and node 5, units at node 6 use the lines connecting node 5 to the demand nodes to deliver energy. For example, in period 9, the power flow from node 6 to node 5 is 160.064MW, and the power flows injected at node 5 to demand nodes 2, 3, and 4 are is 54.643MW, 64.566MW, and 40.855MW, respectively; the sum of these power flows is exactly 160.064MW. In other words, the 160.064MW injected from node 6 goes through node 5 and is split by the three demand nodes.

Table 4-11 Line flows in the second-largest unit failure scenario (all in MW)

Time Period	5 to 6 (200)	5 to 2 (200)	5 to 3 (150)	5 to 4 (180)	6 to 3 (300)	6 to 4 (300)	2 to 3 (175)	1 to 2 (150)	1 to 4 (150)
1	18.201	58.330	89.178	102.796	56.121	46.038	-0.577	12.230	-2.230
2	20.928	63.596	94.089	113.518	57.784	49.971	-2.315	12.749	-2.749
3	4.971	61.524	88.928	103.905	66.865	60.605	-3.366	13.771	-3.771
4	5.354	78.150	94.797	109.545	71.232	63.776	-14.029	21.821	-11.821
5	1.805	78.152	94.569	110.099	73.987	67.846	-14.153	21.696	-11.696
6	-11.283	77.077	91.959	104.956	82.730	78.305	-14.662	22.261	-12.261
7	-31.519	74.576	87.419	101.900	95.847	97.576	-15.021	24.403	-4.403
8	-33.656	69.418	94.305	127.240	103.112	115.866	-7.037	18.601	11.399
9	24.707	82.216	139.834	153.243	91.169	71.173	6.779	54.090	95.910
10	14.684	94.787	140.812	149.717	100.241	79.444	-3.142	64.048	93.323
11	-11.224	89.179	150.000	115.257	129.008	84.768	6.452	80.886	119.114
12	-10.533	88.452	150.000	114.803	128.436	83.756	7.056	80.581	119.419
13	1.613	86.103	150.000	110.564	118.388	68.342	9.009	80.225	119.775
14	0.955	86.758	150.000	117.461	118.932	73.400	8.464	79.026	120.974
15	-12.787	86.454	150.000	108.164	130.300	81.913	8.717	80.968	119.032
16	-10.687	86.676	150.000	117.698	128.563	85.750	8.533	78.926	121.074
17	-11.430	85.920	150.000	109.749	129.178	81.495	9.160	80.308	119.692
18	1.069	84.321	150.000	114.509	118.838	71.411	10.490	78.329	121.671
19	13.612	88.234	150.000	133.271	108.462	70.150	7.238	63.739	92.340
20	16.130	85.727	150.000	128.914	106.379	64.751	9.321	68.329	105.311
21	10.522	87.839	150.000	103.457	111.018	54.504	7.566	66.035	75.112
22	26.919	92.368	150.000	115.954	97.453	45.236	3.803	48.303	31.697
23	20.925	57.247	101.988	92.774	64.091	36.835	7.202	33.858	46.142
24	18.552	60.384	94.017	92.046	59.692	38.860	0.314	34.322	40.549

In Table 4-11, the line between node 5 and node 3 is congested from period 11 to period 22 because the base unit at node 5 has to increase its output level to make up the loss of the second-largest unit in this scenario. During periods 11 and 12, the base unit runs at its maximum generation level (see Table 4-7), and a small amount of the power injected at node 5 goes to node 6.

Table 4-12 Line flows in the transmission line failure scenario (all in MW)

Time Period	5 to 6 (200)	5 to 2 (200)	5 to 3 (150)	5 to 4 (180)	6 to 3 (300)	6 to 4 (300)	2 to 3 (175)	1 to 2 (150)	1 to 4 (150)
1	18.201	58.330	89.178	102.796	56.121	46.038	-0.577	12.230	-2.230
2	20.928	63.596	94.089	113.518	57.784	49.971	-2.315	12.749	-2.749
3	4.971	61.524	88.928	103.905	66.865	60.605	-3.366	13.771	-3.771
4	5.354	78.150	94.797	109.545	71.232	63.776	-14.029	21.821	-11.821
5	1.805	78.152	94.569	110.099	73.987	67.846	-14.153	21.696	-11.696
6	-11.283	77.077	91.959	104.956	82.730	78.305	-14.662	22.261	-12.261
7	-31.519	74.576	87.419	101.900	95.847	97.576	-15.021	24.403	-4.403
8	-33.656	69.418	94.305	127.240	103.112	115.866	-7.037	18.601	11.399
9	-72.232	0.000	138.013	180.000	169.909	189.712	-70.140	59.386	-49.386
10	-74.521	0.000	145.283	180.000	177.606	192.111	-84.978	76.998	-49.628
11	-107.160	0.000	150.000	135.037	208.371	197.836	-69.879	93.734	-13.734
12	-102.626	0.000	150.000	136.694	204.620	194.133	-69.127	92.849	-12.849
13	-98.607	0.000	150.000	144.865	201.295	195.098	-73.899	83.421	-41.282
14	-97.293	0.000	150.000	148.664	200.209	196.127	-72.813	84.507	-32.956
15	-105.697	0.000	150.000	128.467	207.161	192.142	-65.823	92.881	-11.500
16	-102.918	0.000	150.000	139.101	204.862	195.965	-66.524	90.544	-10.544
17	-101.771	0.000	150.000	132.101	203.913	190.328	-65.575	91.494	-11.494
18	-97.419	0.000	150.000	148.520	200.312	196.167	-70.985	81.175	-37.096
19	-84.739	0.000	150.000	163.077	189.823	192.100	-74.123	70.611	-59.415
20	-84.182	0.000	150.000	163.716	189.363	191.921	-73.663	71.072	-56.661
21	-84.018	0.000	150.000	128.987	189.226	169.751	-70.642	75.665	-65.665
22	-57.578	0.000	150.000	124.474	167.354	139.184	-66.098	70.770	-70.770
23	19.434	0.000	128.658	145.727	86.611	71.939	-41.989	41.915	-41.915
24	61.543	0.000	141.278	172.179	61.849	44.565	-49.104	45.288	-45.288

Table 4-12 presents the power flows on each branch in the transmission line scenario. The power flow on the line between node 5 and node 2 is 0 between period 9 and period 24 due to the line failure. Meanwhile, the constraints on line connecting node 5 and node 3 are binding between period 9 and period 22, and the line between node 5 and node 4 is congested in periods 9 and 10. This is because node 5 has to inject more power through other lines than in other scenarios since the line connecting it to node 2 does not work from period 9 in this scenario.

Figure 4-8 shows the relationship between the generation level of the outage unit, the system reserve shortage, and the reserve available in the scenario in which the largest generating unit breaks down from time period 9. Table 4-13 gives the spinning reserve provided by each generating unit and the reserve shortage over the time frame. Since the base unit breaks down in time period 9, and other generating units have to ramp up to supply more energy than they do in the most probable scenario, these generating units have to use some of their reserve capacity to supply energy. Thus, there is a reserve shortage of 312.634MW in this time period. For the same reason, there is a series of reserve shortage between time periods 9 and 24.

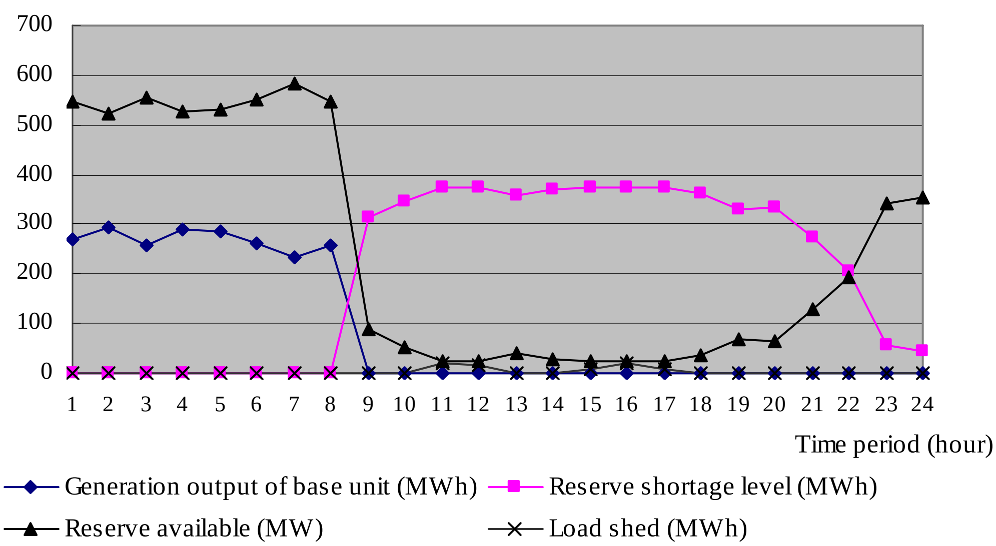


Figure 4-8 Reserve available, reserve shortage, load shed in a scenario (largest unit failure from time period 9)

Table 4-13 Reserve provided by each generating unit in a scenario (largest unit failure from time period 9)

Time period	Base unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Reserve available	Reserve shortage
1	131.495	150.000	75.000	40.000	80.000	70.000	546.495	0.000
2	107.869	150.000	75.000	40.000	80.000	70.000	522.869	0.000
3	140.672	150.000	75.000	40.000	80.000	70.000	555.672	0.000
4	112.153	150.000	75.000	40.000	80.000	70.000	527.153	0.000
5	115.374	150.000	75.000	40.000	80.000	70.000	530.374	0.000
6	137.290	150.000	75.000	40.000	80.000	70.000	552.290	0.000
7	167.624	150.000	75.000	40.000	80.000	70.000	582.624	0.000
8	142.692	150.000	75.000	40.000	80.000	60.000	547.692	0.000
9	0.000	0.000	37.366	40.000	10.000	0.000	87.366	312.634
10	0.000	0.000	0.000	0.000	0.000	52.630	52.630	347.370
11	0.000	0.000	0.000	0.000	0.000	25.000	25.000	375.000
12	0.000	0.000	0.000	0.000	0.000	25.000	25.000	375.000
13	0.000	0.000	0.000	0.000	0.000	41.604	41.604	358.396
14	0.000	0.000	0.000	0.000	0.000	28.448	28.448	371.552
15	0.000	0.000	0.000	0.000	0.000	25.000	25.000	375.000
16	0.000	0.000	0.000	0.000	0.000	25.000	25.000	375.000
17	0.000	0.000	0.000	0.000	0.000	25.000	25.000	375.000
18	0.000	0.000	0.000	0.000	0.000	35.921	35.921	364.079
19	0.000	0.000	0.000	0.000	8.804	60.000	68.804	331.196
20	0.000	0.000	0.000	0.000	5.589	60.000	65.589	334.411
21	0.000	0.000	0.000	0.000	57.035	70.000	127.035	272.965
22	0.000	0.000	3.988	40.000	80.000	70.000	193.988	206.012
23	0.000	77.066	75.000	40.000	80.000	70.000	342.066	57.934
24	0.000	90.129	75.000	40.000	80.000	70.000	355.129	44.871

Figure 4-9 shows a similar comparison in a different scenario, in which the second-largest generating unit breaks down from time period 9. Table 4-14 gives the reserve provided by each generating unit and the reserve shortage at each time period. The results suggest that there is no reserve shortage in time period 9, since the base unit acts as a replacement of cycler 2; it uses all of its capacity to supply energy (see Table 4-6), and there is no extra capacity for this generating unit to prepare any reserves. As demand increases from time period 9, other generating units have to ramp up to satisfy demand first. For example, cycler 2, cycler 3, and peaker 1 ramp up to their maximum operating level from period 11 to period 18; they cannot prepare any reserve during this time period.

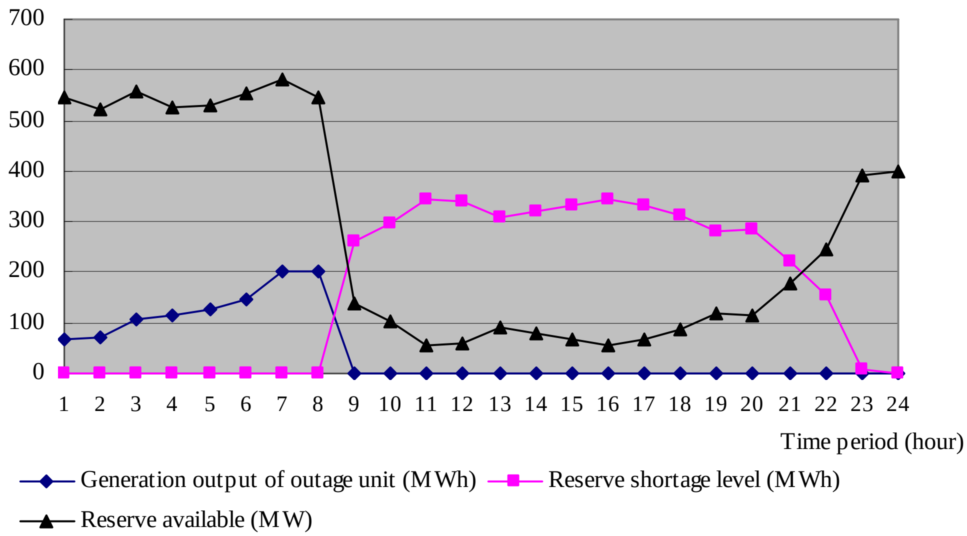


Figure 4-9 Reserve available vs. reserve shortage in a scenario (second-largest unit failure from time period 9)

Table 4-14 Reserve provided by each generating unit in a scenario (second-largest unit failure from time period 9)

Time period	Base unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Reserve available	Reserve shortage
1	131.496	150.000	75.000	40.000	80.000	70.000	546.496	0.000
2	107.869	150.000	75.000	40.000	80.000	70.000	522.869	0.000
3	140.672	150.000	75.000	40.000	80.000	70.000	555.672	0.000
4	112.153	150.000	75.000	40.000	80.000	70.000	527.153	0.000
5	115.374	150.000	75.000	40.000	80.000	70.000	530.374	0.000
6	137.291	150.000	75.000	40.000	80.000	70.000	552.291	0.000
7	167.624	150.000	75.000	40.000	80.000	70.000	582.624	0.000
8	142.692	150.000	75.000	40.000	80.000	60.000	547.692	0.000
9	0.000	0.000	37.366	40.000	10.000	50.000	137.366	262.634
10	0.000	0.000	0.000	0.000	42.630	60.000	102.630	297.370
11	56.788	0.000	0.000	0.000	0.000	0.000	56.788	343.212
12	57.278	0.000	0.000	0.000	0.000	2.275	59.553	340.447
13	51.720	0.000	0.000	0.000	0.000	39.884	91.604	308.396
14	44.825	0.000	0.000	0.000	0.000	33.623	78.448	321.552
15	68.169	0.000	0.000	0.000	0.000	0.000	68.169	331.831
16	56.314	0.000	0.000	0.000	0.000	0.000	56.314	343.686
17	65.760	0.000	0.000	0.000	0.000	2.898	68.658	331.342
18	50.101	0.000	0.000	0.000	0.000	35.820	85.921	314.079
19	14.883	0.000	0.000	0.000	43.921	60.000	118.804	281.196
20	19.229	0.000	0.000	0.000	26.360	70.000	115.589	284.411
21	48.182	0.000	0.000	0.000	58.853	70.000	177.035	222.965
22	14.759	0.000	39.229	40.000	80.000	70.000	243.988	156.012
23	127.066	0.000	75.000	40.000	80.000	70.000	392.066	7.934
24	135.000	0.000	75.000	40.000	80.000	70.000	400.000	0.000

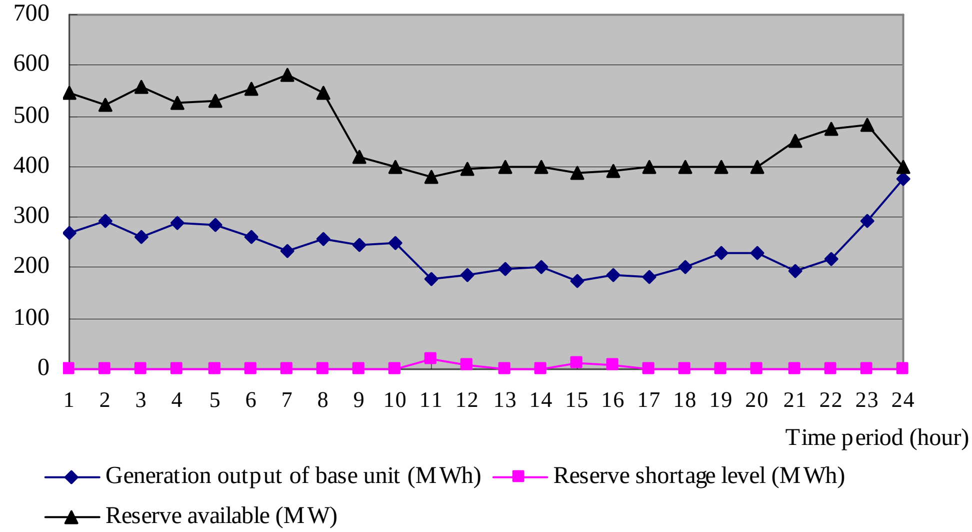


Figure 4-10 Reserve available vs. reserve shortage in a scenario (transmission line failure from time period 9)

Figure 4-10 and Table 4-15 show the same information about reserve prepared by the generating units and reserve shortage level in the transmission line failure scenario. The results demonstrate that there is no need for load shedding and no reserve shortage in each time period. In this model, we explicitly consider the line failure in the model; therefore, scheduling decisions must prepare for the possibility of the related generating unit being prevented from delivery due to a line failure.

Table 4-15 Reserve provided by each generating unit in a scenario (transmission line failure from time period 9)

Time	Base unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Reserve	Reserve
1	131.496	150.000	75.000	40.000	80.000	70.000	546.496	0.000
2	107.869	150.000	75.000	40.000	80.000	70.000	522.869	0.000
3	140.672	150.000	75.000	40.000	80.000	70.000	555.672	0.000
4	112.153	150.000	75.000	40.000	80.000	70.000	527.153	0.000
5	115.374	150.000	75.000	40.000	80.000	70.000	530.374	0.000
6	137.291	150.000	75.000	40.000	80.000	70.000	552.291	0.000
7	167.624	150.000	75.000	40.000	80.000	70.000	582.624	0.000
8	142.692	150.000	75.000	40.000	80.000	60.000	547.692	0.000
9	154.219	0.000	73.147	40.000	80.000	70.000	417.366	0.000
10	149.238	0.000	60.762	40.000	80.000	70.000	400.000	0.000
11	200.000	0.000	1.632	40.000	80.000	60.000	381.632	18.368
12	200.000	0.000	3.621	40.000	80.000	70.000	393.621	6.379
13	200.000	0.000	10.000	40.000	80.000	70.000	400.000	0.000
14	198.630	0.000	11.370	40.000	80.000	70.000	400.000	0.000
15	200.000	0.000	0.000	38.619	80.000	70.000	388.619	11.381
16	200.000	0.000	1.254	40.000	80.000	70.000	391.254	8.746
17	200.000	0.000	8.988	40.000	80.000	70.000	398.988	1.012
18	198.898	0.000	11.102	40.000	80.000	70.000	400.000	0.000
19	171.662	0.000	38.338	40.000	80.000	70.000	400.000	0.000
20	170.466	0.000	39.534	40.000	80.000	70.000	400.000	0.000
21	200.000	0.000	62.005	40.000	80.000	70.000	452.005	0.000
22	183.104	75.884	65.000	0.000	80.000	70.000	473.988	0.000
23	106.182	150.000	75.000	0.000	80.000	70.000	481.182	0.000
24	25.000	150.000	75.000	0.000	80.000	70.000	400.000	0.000

4.4.1.4 Comparison of Deterministic and Stochastic Models

Table 4-16 Generation plan in the most probable scenario: deterministic model

Time	Base unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2
1	317.463	20.000	15.000	10.000	0.000	0.000
2	343.958	20.000	15.000	10.000	0.000	0.000
3	346.828	20.000	15.000	10.000	0.000	0.000
4	382.500	20.000	15.000	10.000	0.000	0.000
5	389.654	20.000	15.000	10.000	0.000	0.000
6	400.000	20.028	15.000	10.000	0.000	0.000
7	400.000	52.318	15.000	10.000	0.000	0.000
8	400.000	114.942	15.000	10.000	0.000	0.000
9	376.904	215.000	80.000	15.730	0.000	0.000
10	396.393	215.000	80.000	30.977	0.000	0.000
11	378.745	232.500	80.000	60.000	20.000	0.000
12	383.932	221.515	80.000	60.000	20.000	0.000
13	400.000	215.000	80.000	38.396	0.000	0.000
14	400.000	215.000	80.000	51.552	0.000	0.000
15	368.697	230.455	80.000	60.000	20.000	0.000
16	382.571	227.358	80.000	60.000	20.000	0.000
17	375.450	220.891	80.000	60.000	20.000	0.000
18	400.000	215.000	80.000	44.079	0.000	0.000
19	400.000	215.000	80.000	11.196	0.000	0.000
20	400.000	215.000	80.000	14.411	0.000	0.000
21	396.857	215.000	26.108	10.000	0.000	0.000
22	400.000	156.012	15.000	10.000	0.000	0.000
23	387.934	20.000	15.000	10.000	0.000	0.000
24	374.871	20.000	15.000	10.000	0.000	0.000

Table 4-17 Generation plan in the most probable scenario: stochastic model

Time	Base unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2
1	317.463	20.000	15.000	10.000	0.000	0.000
2	343.959	20.000	15.000	10.000	0.000	0.000
3	346.827	20.000	15.000	10.000	0.000	0.000
4	382.500	20.000	15.000	10.000	0.000	0.000
5	389.653	20.000	15.000	10.000	0.000	0.000
6	400.000	20.028	15.000	10.000	0.000	0.000
7	400.000	52.318	15.000	10.000	0.000	0.000
8	257.308	200.000	42.634	20.000	10.000	10.000
9	370.264	200.000	77.370	20.000	10.000	10.000
10	396.393	215.000	80.000	30.977	0.000	0.000
11	378.745	232.500	80.000	60.000	20.000	0.000
12	383.932	221.515	80.000	60.000	20.000	0.000
13	400.000	215.000	80.000	38.396	0.000	0.000
14	400.000	215.000	80.000	51.552	0.000	0.000
15	368.697	230.455	80.000	60.000	20.000	0.000
16	382.571	227.358	80.000	60.000	20.000	0.000
17	375.450	220.891	80.000	60.000	20.000	0.000
18	400.000	215.000	80.000	44.079	0.000	0.000
19	400.000	215.000	80.000	11.196	0.000	0.000
20	400.000	215.000	80.000	14.411	0.000	0.000
21	396.857	215.000	26.108	10.000	0.000	0.000
22	400.000	156.012	15.000	10.000	0.000	0.000
23	387.934	20.000	15.000	10.000	0.000	0.000
24	374.871	20.000	15.000	10.000	0.000	0.000

To demonstrate the possible differences in the operating plans determined by a deterministic model and a stochastic model, we examine a deterministic model considering only one scenario and without any contingencies, and a stochastic model that considers seven scenarios. The contingencies built into the stochastic model include base unit and largest cycler outages, and the same transmission line outage we discussed; these equipment failures start from period 9 and period 10, and, together with the most probable scenario, make up a total of seven scenarios.

Table 4-16 and Table 4-17 show the optimal generation plan in the most probable scenario using the deterministic model and the stochastic model. The results suggest that the two plans are different from each other. In the stochastic model, because the most probable scenario is not independent of other contingency scenarios, the ISO has to coordinate the generation plan in the most probable scenario with other scenarios to prepare for the coming contingencies. For example, in the deterministic model, peaker 1 is turned on at period 11 and period 12, respectively, due to high demand, and peaker 2 is turned off all the time. On the other hand, in a stochastic setting, peaker 1 is turned on in period 8, earlier than the generation plan of the deterministic model, in order to be able to ramp up to its maximum generation level in hour 9 in case there is a contingency. For the same reason, the most expensive peaker 2 has to be turned on at the same time periods in the stochastic model to prepare for the future contingencies, while it is always off in the deterministic model. Table 4-18 and Table 4-19 show the reserve provided in the deterministic model and in the stochastic model, respectively. In Table 4-19, since peaker 2 is online at periods 8 and 9 and runs at its minimum generation level, 10MW (see Table 4-17)), it provides 60MW reserve in contrast with 70MW in Table 4-18.

Table 4-18 Reserve provided in the most probable scenario: deterministic model

Time	Base	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Total
1	100.000	150.000	0.000	0.000	80.000	70.000	400.000
2	46.000	150.000	75.000	0.000	80.000	70.000	421.000
3	43.200	150.000	75.000	0.000	80.000	70.000	418.200
4	25.000	150.000	75.000	0.000	80.000	70.000	400.000
5	25.000	150.000	75.000	0.000	80.000	70.000	400.000
6	25.000	150.000	75.000	0.000	80.000	70.000	400.000
7	25.000	150.000	75.000	0.000	80.000	70.000	400.000
8	25.000	150.000	75.000	0.000	80.000	70.000	400.000
9	28.100	106.900	75.000	40.000	80.000	70.000	400.000
10	6.800	128.200	75.000	40.000	80.000	70.000	400.000
11	20.000	115.000	75.000	40.000	80.000	70.000	400.000
12	12.800	122.200	75.000	40.000	80.000	70.000	400.000
13	0.000	135.000	75.000	40.000	80.000	70.000	400.000
14	0.000	135.000	75.000	40.000	80.000	70.000	400.000
15	25.900	109.100	75.000	40.000	80.000	70.000	400.000
16	15.700	119.300	75.000	40.000	80.000	70.000	400.000
17	18.100	116.900	75.000	40.000	80.000	70.000	400.000
18	0.000	135.000	75.000	40.000	80.000	70.000	400.000
19	0.000	135.000	75.000	40.000	80.000	70.000	400.000
20	0.000	135.000	75.000	40.000	80.000	70.000	400.000
21	3.100	131.900	75.000	40.000	80.000	70.000	400.000
22	25.000	150.000	75.000	0.000	80.000	70.000	400.000
23	25.000	150.000	75.000	0.000	80.000	70.000	400.000
24	25.000	150.000	75.000	0.000	80.000	70.000	400.000

Table 4-19 Reserve provided in the most probable scenario: stochastic model

Time	Base unit	Cycler 1	Cycler 2	Cycler 3	Peaker 1	Peaker 2	Total
1	100.000	150.000	0.000	0.000	80.000	70.000	400.000
2	46.000	150.000	75.000	0.000	80.000	70.000	421.000
3	43.200	150.000	75.000	0.000	80.000	70.000	418.200
4	25.000	150.000	75.000	0.000	80.000	70.000	400.000
5	25.000	150.000	75.000	0.000	80.000	70.000	400.000
6	25.000	150.000	75.000	0.000	80.000	70.000	400.000
7	0.000	150.000	75.000	40.000	80.000	70.000	415.000
8	142.700	150.000	75.000	40.000	80.000	60.000	547.700
9	29.700	150.000	75.000	40.000	80.000	60.000	434.700
10	6.800	128.200	75.000	40.000	80.000	70.000	400.000
11	20.000	115.000	75.000	40.000	80.000	70.000	400.000
12	12.800	122.200	75.000	40.000	80.000	70.000	400.000
13	0.000	135.000	75.000	40.000	80.000	70.000	400.000
14	0.000	135.000	75.000	40.000	80.000	70.000	400.000
15	25.900	109.100	75.000	40.000	80.000	70.000	400.000
16	15.700	119.300	75.000	40.000	80.000	70.000	400.000
17	18.100	116.900	75.000	40.000	80.000	70.000	400.000
18	0.000	135.000	75.000	40.000	80.000	70.000	400.000
19	0.000	135.000	75.000	40.000	80.000	70.000	400.000
20	0.000	135.000	75.000	40.000	80.000	70.000	400.000
21	3.100	131.900	75.000	40.000	80.000	70.000	400.000
22	25.000	150.000	75.000	0.000	80.000	70.000	400.000
23	25.000	150.000	75.000	0.000	80.000	70.000	400.000
24	25.000	150.000	75.000	0.000	80.000	70.000	400.000

4.4.2 A 20-bus Power System

In this example, we consider a system with 20 buses and 32 generating units. Part of the data used in this example is originally from Reliability Test System Task Force (1999); data obtained from the study include the basic topology of the power transmission system and demand in area B, heat rate for calculating marginal costs of generating units, and capacity of generating units. There are 24 buses in area B of this study; we combine some nodes, so that it has 20 nodes. We make up plausible values for the rest of the data needed for the model in this thesis. Nodes 221, 222, and 223 are supply nodes, i.e., they are associated with generating units, and there is no load at these nodes. Thus, they only inject power to the system and do not withdraw any. On the contrary, nodes 203, 204, 205, 206, 208, 209, 214, 219, and 220 are demand nodes; there is load at these nodes, and they only withdraw power from the system and do not supply any to the system. In addition, some nodes both supply and withdraw power from the system and have generating units and load. These nodes are 201, 202, 207, 213, 215, 216, and 218. There are 32 generating units in the system. The identification of a generating unit is represented by "(code)U(capacity)". For example, "1U20" represents a unit that has a capacity of 20MW. The "1" before "U20" is a code used to distinguish identical generating units connected to different nodes. The location of the generating unit is shown in Table 4-20. Among these generating units, 1U20, 2U20, 3U20, and 4U20 are peakers that can start up quickly; their fuel costs are the highest. 1U400, 2U400, 1U50, 2U50, 3U50, 4U50, 5U50, and 6U50 are taken as must-run units with cheap fuel cost. Other generating units are cyclers that are turned on or off based on need. In the original power system 1U50-6U50 are hydro units. To avoid complicating the formulation of the model because the hydro units need different constraints from thermal units, we just treat them as must-run and quick-start units.

Table 4-20 Generating units and their location in the 20-bus system

Nodes	Generating units
201	1U20, 2U20, 1U76, 2U76
202	3U20 ,4U20, 3U76, 4U76
207	1U100, 2U100, 3U100
213	1U197, 2U197, 3U197
215	1U12, 2U12, 3U12, 4U12, 5U12, 1U155
216	2U155
218	1U400
221	2U400
222	1U50, 2U50, 3U50, 4U50, 5U50, 6U50
223	3U155, 4U155, 1U350

185 scenarios over 24 time periods are considered and no transmission line outages are included. In this numerical example, for the purpose of illustration, we only consider generating unit outage scenarios. The 8 largest units, i.e., 1U400, 2U400, 1U350, 1U155, 2U155, 3U155, 1U197, and 1U100, are the possible unit contingencies in the system; their forced outages begin at different time periods in different scenarios, following the rule of "N-1" contingency. If there is more than 1 identical generating unit at the same location, only one of them is chosen as the outage generator. For example, there are two identical units at node 223, namely 3U155 and 4U155; only 3U155 is selected. Spinning reserve requirement is set to 500MW, slightly higher than the largest unit capacity. Load shedding is penalized at \$1000/MWh. A four-step penalty function is used to charge different reserve shortage levels: \$100/MW for the first 100MW, \$300/MW for every MW between 100MW and 325MW, and \$600/MW for the next 125MW. The remaining 50MW is the minimum reserve requirement priced at \$1000/MW (see Appendix B and C for the topology and data used in this example).

The problem has 3,408,901 constraints and 1,167,721 variables, including 142,080 binary variables. We run the scenario-rolling heuristic to solve the problem and compare the results with those obtained by Cplex. We set a uniform time limit on all iterations to compare the results: a maximum of 1 hour (3600 seconds) per iteration with an optimality gap of 0.1%, whichever condition is reached first. The numerical results are shown in Table 4-21.

According to Table 4-21, all the heuristic except the 93-scenario satisfy the 0.1% optimality gap within 1-hour time limit. The 93-scenario heuristic cannot return a good-quality feasible solution within the preset time limit. To have an idea of the performance of the heuristic, we relaxed the integer constraints and solved the resulting LP; it turned out that the LP lower bound is of very low quality, dropping to less than 500,000 after 10 hours (see row 10 of Table 4-21). As an alternative, we relaxed the nonanticipativity constraints, solved each individual scenario as a MIP to its optimality, and then summed up the weighted objective function values of all the scenarios. The summation is a lower bound of the optimal solution to the original SMIP problem designated as "wait-and-see" solution (Birge and Lauveaux, 1997). The lower bound turns out to be much better than the LP lower bound (see row 11). We show the gap between the heuristic solution and this lower bound in the last column of Table 4-21. In practice, DA schedules need to be prepared within a short period of time, for example, one hour. Taking this requirement into account, we can see that most heuristics are able to return feasible solutions within the time requirement. We can perform a similar analysis and discussion of the results of the large-scale numerical example. However, due to the dimension of the large-scale numerical example, we will not do so in this section. Some results are presented in Chapter 5 to help to understand pricing results. Also see Appendix D for the generation plans of generating units in a representative scenario.

Table 4-21 Results for a 20-bus system (scenario-rolling)

Max. # of contingency scenarios solved in each iteration	# of iterations	Objective. Value	Solver Time(sec.)	Gap with LB
9	23	654209.348	1200.583	0.165%
17	12	654335.721	1648.571	0.184%
25	8	654316.316	2093.074	0.181%
33	6	654111.422	2392.622	0.150%
49	4	654366.025	2130.827	0.189%
65	3	654223.441	3090.374	0.167%
93	2	1071438.765	5073.389	64.046%
Cplex	1	N/A solution	36,000	-
LP lower bound(LB)		<500,000	36,000	-
LB (Relaxing Nonanticipativity)		653133.693		

Table 4-22 Results for a 20-bus system (Benders decomposition)

Iteration	Objective Function Value of Master Problem	Solver Time (Sec.)	Objective Function Value Subproblem	Solver Time (Sec.)
1	60715.465	358.234	25154267.946	486.203
2	N/A	18000	N/A	N/A

Table 4-23 Results for a 20-bus system (progressive hedging)

Iteration	Objective Value	Solver Time(sec.)	Infeasible NA Constraints (%)
1	653133.693	1030.734	42.059%
2	653167.178	3386.202	42.059%
3	653148.126	2011.453	42.059%
4	653199.209	2204.432	42.059%
5	653235.539	2246.930	42.059%
6	653264.670	1967.818	42.054%
7	653296.503	2032.718	42.054%
8	653323.240	2273.929	42.054%
9	653355.905	2393.562	42.057%
10	653386.622	2196.899	42.057%
Total Time	N/A	21744.617	N/A

Table 4-22 shows the performance of Benders decomposition. We incorporated all the binary variables into the master problem, leaving the subproblem a LP. Due to the size of the master problem, Cplex cannot return an optimal solution after five hours in the second iteration. Table

4-23 provides the results of progressive hedging algorithm. The results demonstrate that there exist a number of infeasible nonanticipativity constraints after we ran the algorithm for more than six hours. We cannot easily obtain a feasible solution with this method.

4.5 Summary

In this chapter, we proposed a heuristic methodology for solving the SMIP model. According to the structure of the decision tree, we solved only a subset of the scenarios in each iteration of the heuristic. Therefore, the size of the problem we solve in each iteration is much smaller than the whole problem. We can solve such a small group of scenarios quickly and use part of the solution to solve another group of scenarios in the next iteration until all the scenarios are solved. The scenario-rolling algorithm can guarantee a feasible solution to the problem. We then provided two numerical examples to show the results of the model and to evaluate the performance of the scenario-rolling algorithm that can be used to solve the SMIP problem. The results indicated that the scenario-rolling algorithm can obtain a sub-optimal solution within the time limit; the gap between the heuristic results and the lower bound is acceptable, given the time limit. By performing result analyses in some representative scenarios, we found out that an equipment failure, such as a generating unit outage or a transmission line outage, can have a significant impact on the generation output level of generating units and the reserve available in the system. The results also showed an important difference between the stochastic and the deterministic model with regard to the operating schedules of their generating units as contingencies are introduced. In the stochastic model, some generating units have to start up ahead of time in order to prepare for the future contingencies due to ramp up limit.

In the next chapter, we discuss related pricing and compensation plans based on the SMIP model. We begin with how to obtain the dual variables of the SMIP, and then propose compensation plans based on these variables. Theoretical results and numerical results of these compensation schemes will be presented and discussed.

Chapter 5
Pricing Issues and Compensation Plans

5.1 Introduction

As mentioned in Chapter 2, previous research has been conducted in Wong and Fuller (2007), O'Neill et al. (2005), and Sioshansi et al. (2007). Wong and Fuller (2007) propose different compensation schemes based on a single-stage stochastic linear programming model, including DA pricing, RT pricing, and hybrid pricing that has some interesting properties. O'Neill et al. (2005) give a methodology to obtain the dual prices from an MIP that fixes binary variables to their optimal values, and then solves the resulting LP to procure the dual prices. Sioshansi et al (2007) show that generator payoff inequity problems exist because, usually, the UC problems cannot be solved to their optimality. They suggest that make-whole payment be imposed to eliminate this problem to ensure that generators make non-negative profits, i.e., the make-whole payment will bring the profit of a generator to 0 if the profit of a generator is otherwise negative; if the profit of a generator is non-negative, the make-whole payment is 0. Some recent work on this issue includes Sen and Genc (2008); the authors propose a highly challenging method to tackle the problem.

In this chapter, we extend these ideas to the SMIP in this thesis. We first explore the possibilities of applying them to a realistic SMIP that is closer to reality than the models proposed in Wong and Fuller (2007). With the method suggested in O'Neill et al. (2005), we first obtain the dual prices by solving an LP with fixed binary variables. Then we ignore the additional prices associated with fixed costs, which is suggested in O'Neill et al. (2005). Instead, we only use energy revenue and reserve revenue to generate the pricing scheme. However, because we use a heuristic method to solve the original MIP, we cannot solve the problem to its complete optimality. As a result, we then use make-whole payment to alleviate the payoff inequity due to the gap between the feasible solution and the optimal solution. We then propose two compensation schemes: one RT compensation scheme and one hybrid scheme. We will show that some properties proposed in Wong and Fuller (2007) are still valid. Numerical results are based on the 20-bus system (Reliability Task Force, 1999) discussed in Chapter 4.

5.2 Procurement of Dual Prices of the SMIP Model

In order to define the prices based on the SMIP model, we must first procure proper dual variables for marginal costs of energy and reserves, and possibly for the on/off status of generating units (following O'Neill et al., 2005). With the method proposed in O'Neill et al. (2005), we first apply the scenario-rolling heuristic to solve the SMIP model, and then we solve the corresponding LP model, which includes constraints that fix the continuous variables (which replace the binary variables in the SMIP) to their sub-optimal values from the heuristic. Since it is an LP model, we can take the dual variables directly from solving it. The dual variables associated with the power balance constraints are the energy marginal costs, and the dual variables corresponding to the reserve requirement constraints are the marginal costs for reserves. For the pricing of binary variables as proposed by O'Neill et al. (2005), we use the dual variables of the constraints which fix the continuous on/off variables to the value of the heuristic. The procedure of taking dual prices from the SMIP is shown below.

Step 1: Use the scenario-rolling algorithm to solve the SMIP to obtain a sub-optimal solution;

Step 2: Record the sub-optimal solution of the binary variables;

Step 3: Modify the original model to an LP by adding a set of equality constraints that fix the binary variables to their sub-optimal solution recorded in step 2, namely $\omega_{gst} = \omega_{gst}^{MIP*}$, where ω_{gst}^{MIP*} is the sub-optimal solution to the original SMIP.

Step 4: Solve the resulting LP to its complete optimality and record the proper dual variables.

We first revisit the SMIP model, and then show the corresponding LP problem, designated PLP, below. We define $\alpha^{(\bullet)}$ as the dual variables of constraints $(\bullet)$, where $(\bullet)$ corresponds to the constraint number in the PLP model. For example, the dual variable of constraint (5.8) is α_{ist}^{8}. α_{ist}^{8} is actually the marginal expected cost of energy, including the probability factor, π_s, as discussed in Wong and Fuller (2007), $p_{ist}^{E} = (1/\pi_s)\alpha_{ist}^{8}$ is the actual energy marginal cost at node *i* in scenario *s* during time period *t*, and it is interpreted as the prediction of the RT price if scenario *s* actually happens. Similarly, p_{mst}^{R} is the actual system-wide reserve marginal cost during period *t* in scenario *s* for reserve type *m*. We put these dual variables in the brackets to the right of the relevant constraints in the PLP model shown below. We convert the SMIP model to an LP by making the following modifications:

1. Since we will fix the binary variables to their optimal solution in the LP in this thesis, we delete redundant constraints (3.15), (3.16), (3.18), and (3.19) in Chapter 3 because these constraints only contain binary variables.

2. We add new constraints to fix the binary variables to their sub-optimal values, i.e., (5.17).
3. We relax the binary constraints on the commitment states in constraint (3.22).
4. To convert the model to its standard format, we change all the less than or equal to constraints in the SMIP model to greater than or equal to constraints so that all the dual variables of the constraints are non-negative in the LP model.

The LP used to obtain the dual variables is shown below.

PLP:min.

$$\sum_{s=1}^{S}\sum_{t=1}^{T}\pi_s[\sum_{m=1}^{M}\sum_{n=1}^{N}C_{mn}^{S}q_{mnst}^{S}+\sum_{i\in I}C^{LS}T^{\text{int}}q_{ist}^{LS}+\sum_{i\in I}\sum_{g\in G(i)}(\sum_{b=1}^{B}C_{gb}^{E}q_{gbst}^{E}+z_{gst}+C_{g}^{NL}T^{\text{int}}\omega_{gst})] \quad (5.1)$$

s.t.

$$-q_{gbst}^{E}\geq -Q_{gb}^{E}\omega_{gst} \qquad \forall i,\forall g\in G(i),\forall b,\forall s,\forall t \qquad (\alpha_{igbst}^{2}) \quad (5.2)$$

$$-q_{mnst}^{S}\geq -Q_{mn}^{S} \qquad \forall m,\forall n,\forall s,\forall t \qquad (\alpha_{mnst}^{3}) \quad (5.3)$$

$$\sum_{b=1}^{B}q_{gbst}^{E}-Q_{g}^{MSL}\omega_{gst}\geq 0 \qquad \forall i,\forall g\in G(i),\forall s,\forall t \qquad (\alpha_{igst}^{4}) \quad (5.4)$$

$$-\sum_{b=1}^{B}q_{gbst}^{E}-\sum_{m\in M(g)}q_{gmst}^{R}\geq -Q_{g}^{\max}\mu_{gst} \qquad \forall i,\forall g\in G^{f}(i),\forall s,\forall t \qquad (\alpha_{igst}^{5}) \quad (5.5)$$

$$-\sum_{b=1}^{B}q_{gbst}^{E}-\sum_{m\in M(g)}q_{gmst}^{R}\geq -Q_{g}^{\max}\omega_{gst} \qquad \forall i,\forall g\in G^{s}(i),\forall s,\forall t \qquad (\alpha_{igst}^{6}) \quad (5.6)$$

$$-q_{gmst}^{R}\geq -Q_{gm}^{RP} \qquad \forall i,\forall g\in G(i),\forall m,\forall s,\forall t \qquad (\alpha_{igmst}^{7}) \quad (5.7)$$

$$\sum_{g\in G(i)}\sum_{b=1}^{B}q_{gbst}^{E}+q_{ist}^{LS}-\sum_{j}\upsilon_{ijst}B_{ijst}(\theta_{ist}-\theta_{jst})=Q_{it}^{D} \qquad \forall i,\forall s,\forall t \qquad (\alpha_{ist}^{8}) \quad (5.8)$$

$$\sum_{a=1}^{A}(lk_{sat}\pi_a\sum_{b=1}^{B}q_{gbat}^{E})-(\sum_{a=1}^{A}lk_{sat}\pi_a)\sum_{b=1}^{B}q_{gbst}^{E}=0 \qquad \forall i,\forall g\in G(i),\forall s,\forall t \qquad (\alpha_{igst}^{9}) \quad (5.9)$$

$$\sum_{a=1}^{A}(lk_{sat}\pi_a q_{gmat}^{R})-(\sum_{a=1}^{A}lk_{sat}\pi_a)q_{gmst}^{R}=0 \qquad \forall i,\forall m,\forall g\in G(i),\forall s,\forall t \qquad (\alpha_{igmst}^{10}) \quad (5.10)$$

$$\upsilon_{ijst} Y_{ijst} (\theta_{ist} - \theta_{jst}) \le U_{ij} \qquad \forall i, \forall j, \forall s, \forall t \qquad (\alpha_{ist}^{11}) \qquad (5.11)$$

$$\sum_{g \in G(i)} q_{gmst}^{R} + \sum_{n=1}^{N} q_{mnst}^{S} \ge Q_m^R \qquad \forall m, \forall s, \forall t \qquad (\alpha_{mst}^{12}) \qquad (5.12)$$

$$\sum_{g \in G(i)} q_{gmst}^{R} \ge Q_{m\_\min}^{R} \qquad \forall m, \forall s, \forall t \qquad (\alpha_{mst}^{13}) \qquad (5.13)$$

$$-q_{ist}^{LS} \ge -Q_{it}^{D} \qquad \forall i, \forall s, \forall t \qquad (\alpha_{ist}^{14}) \qquad (5.14)$$

$$z_{gst} \ge C_{g\tau} (\omega_{gst} - \sum_{k=1}^{\tau} \omega_{gs(t-k)}) \qquad \forall i, \forall g \in G(i), \forall s, \forall t, \tau = 1, ..., \tau_g^C \qquad (\alpha_{igst\tau}^{15}) \qquad (5.15)$$

$$-\sum_{b=1}^{B} q_{gbst}^{E} + \sum_{b=1}^{B} q_{gbs(t-1)}^{E} \ge -R_g^{up} \omega_{gs(t-1)} - R_g^{su} (1 - \omega_{gs(t-1)}) \qquad \forall i, \forall g \in G(i), \forall s, \forall t \qquad (\alpha_{igst}^{16}) \qquad (5.16)$$

$$-\sum_{b=1}^{B} q_{gbs(t-1)}^{E} + \sum_{b=1}^{B} q_{gbst}^{E} \ge -R_g^{dn} \omega_{gst} - R_g^{sd} (1 - \omega_{gst}) \qquad \forall i, \forall g \in G(i), \forall s, \forall t, \; \mu_{gst} = 1 \qquad (\alpha_{igst}^{17}) \qquad (5.17)$$

$$\omega_{gst} = \omega_{gst}^{MIP*} \qquad \forall i, \forall g \in G(i), \forall s, \forall t \qquad (\alpha_{igst}^{18}) \qquad (5.18)$$

$$q_{gbst}^{E}, q_{gmst}^{R}, q_{mnst}^{S}, q_{ist}^{LS}, \omega_{gst} \ge 0, \theta_{ist} \in [-\alpha, \alpha] \qquad \forall i, \forall g \in G(i), \forall m, \forall n, \forall s, \forall t \qquad (5.19)$$

5.3 Numerical Example

Since the large-scale numerical example in Chapter 4 is more realistic, we use it to illustrate the dual prices. Tables 5-1, 5-2, 5-3, and 5-4 show the generation plan in a scenario where the largest unit at node 221, 2U400, breaks down starting at hour 9. The tables show us that every other generating unit ramps up to alleviate the energy loss caused by this equipment failure. The peakers 1U20, 2U20, 3U20, and 4U20 are all turned on, subject to their start-up limit.

Table 5-1 Generation plan of 20-bus system in a scenario (2U400 down from period 9)

Hour	Generators node 201				Generators at node 202				Generator at node 221
	1U20	2U20	1U76	2U76	3U20	4U20	3U76	4U76	2U400
1	0.000	0.000	60.800	60.800	0.000	0.000	60.800	60.800	400.000
2	0.000	0.000	38.000	38.000	0.000	0.000	56.112	60.800	400.000
3	0.000	0.000	38.000	38.000	0.000	0.000	38.000	38.000	400.000
4	0.000	0.000	38.000	29.403	0.000	0.000	38.000	38.000	400.000
5	0.000	0.000	36.349	38.000	0.000	0.000	38.000	38.000	400.000
6	0.000	0.000	38.000	38.000	0.000	0.000	38.000	38.000	400.000
7	0.000	0.000	60.800	60.800	0.000	0.000	60.800	60.800	400.000
8	0.000	0.000	58.414	60.800	0.000	0.000	60.800	60.800	400.000
9	10.000	10.000	76.000	76.000	10.000	10.000	76.000	76.000	0.000
10	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
11	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
12	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
13	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
14	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
15	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
16	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
17	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
18	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
19	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
20	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
21	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
22	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	0.000
23	0.000	0.000	68.480	76.000	0.000	0.000	76.000	76.000	0.000
24	0.000	0.000	60.800	60.800	0.000	0.000	60.800	60.800	0.000

Table 5-2 Generation plan of 20-bus system in a scenario (2U400 down from period 9) Con't

Hour	Generators node 207			Generators at node 213			Generators at node 223		
	1U100	2U100	3U100	1U197	2U197	3U197	3U155	4U155	1U350
1	15.000	15.000	15.000	40.000	40.000	40.000	54.250	72.248	140.000
2	15.000	15.000	15.000	40.000	40.000	40.000	54.250	54.250	100.000
3	15.000	15.000	15.000	40.000	40.000	40.000	54.250	43.372	100.000
4	15.000	15.000	15.000	40.000	40.000	40.000	25.000	25.000	100.000
5	15.000	15.000	15.000	40.000	40.000	40.000	25.000	25.000	100.000
6	15.000	15.000	15.000	40.000	40.000	40.000	54.250	51.180	100.000
7	15.000	15.000	15.000	40.000	40.000	40.000	54.250	80.056	140.000
8	15.000	19.960	15.000	91.350	88.650	88.650	93.000	98.500	187.500
9	30.000	30.000	30.000	160.300	157.600	157.600	147.250	152.750	310.000
10	65.000	65.000	65.000	173.500	167.000	167.000	155.000	155.000	350.000
11	80.000	86.900	100.000	167.000	197.000	167.000	155.000	155.000	350.000
12	80.000	100.000	80.000	194.400	197.000	167.000	155.000	155.000	350.000
13	100.000	80.000	100.000	167.000	167.000	183.100	155.000	155.000	350.000
14	100.000	80.000	80.000	167.000	197.000	192.000	155.000	155.000	350.000
15	80.000	100.000	100.000	197.000	167.000	181.600	155.000	155.000	350.000
16	80.000	80.000	80.000	167.000	167.000	167.000	155.000	155.000	350.000
17	80.000	80.000	60.200	167.000	167.000	167.000	155.000	155.000	350.000
18	80.000	53.000	80.000	167.000	167.000	167.000	155.000	155.000	350.000
19	50.000	46.100	50.000	167.000	167.000	167.000	155.000	155.000	350.000
20	40.600	50.000	30.000	167.000	167.000	167.000	155.000	155.000	350.000
21	30.000	50.000	50.000	167.000	167.000	167.000	155.000	155.000	350.000
22	50.000	50.000	49.100	167.000	167.000	167.000	155.000	155.000	350.000
23	37.520	15.000	30.000	167.000	157.600	165.900	125.000	155.000	310.000
24	15.000	0.000	23.120	98.050	88.650	96.950	125.000	125.000	280.000

Table 5-3 Generation plan of 20-bus system in a scenario (2U400 down from period 9) Con't

Hour	Generators at node 215						Generators at node 216
	1U12	2U12	3U12	4U12	5U12	1U155	2U155
1	2.000	2.000	2.000	2.000	2.000	25.000	25.000
2	2.000	2.000	2.000	2.000	2.000	25.000	25.000
3	2.000	2.000	2.000	2.000	2.000	25.000	25.000
4	2.000	2.000	2.000	2.000	2.000	25.000	25.000
5	0.000	0.000	0.000	0.000	0.000	25.000	25.000
6	0.000	0.000	0.000	0.000	0.000	25.000	25.000
7	0.000	0.000	0.000	0.000	0.000	25.000	25.000
8	0.000	0.000	0.000	0.000	0.000	70.750	70.750
9	2.000	2.000	2.000	2.000	2.000	125.000	125.000
10	6.200	6.200	6.200	6.200	6.200	155.000	155.000
11	10.400	9.600	10.400	9.600	9.600	155.000	155.000
12	9.600	12.000	12.000	12.000	12.000	155.000	155.000
13	12.000	9.600	9.600	9.600	9.600	155.000	155.000
14	12.000	12.000	12.000	12.000	12.000	155.000	155.000
15	9.600	12.000	9.600	9.600	9.600	155.000	155.000
16	9.600	9.600	10.200	9.600	10.500	155.000	155.000
17	9.600	6.000	6.000	9.600	9.600	155.000	155.000
18	9.600	9.600	9.600	9.600	9.600	155.000	155.000
19	5.400	6.000	6.000	6.000	6.000	155.000	155.000
20	2.400	6.000	6.000	6.000	6.000	155.000	155.000
21	3.800	2.400	2.400	6.000	2.400	155.000	155.000
22	6.000	2.400	6.000	6.000	6.000	155.000	155.000
23	2.000	2.000	2.000	2.000	2.000	155.000	155.000
24	0.000	0.000	0.000	0.000	0.000	132.030	125.000

Table 5-4 Generation plan of 20-bus system in a scenario (2U400 down from period 9) Con't

Hour	Generators at node 222						Generators at node 218
	1U50	2U50	3U50	4U50	5U50	6U50	1U400
1	45.000	50.000	49.302	50.000	50.000	45.000	400.000
2	45.000	45.000	48.588	50.000	50.000	45.000	400.000
3	45.000	45.000	45.000	48.378	50.000	45.000	400.000
4	45.000	45.000	45.000	47.597	50.000	45.000	400.000
5	45.000	45.000	45.000	50.000	50.000	45.651	400.000
6	45.000	45.000	45.570	50.000	50.000	45.000	400.000
7	45.000	46.494	50.000	50.000	50.000	50.000	400.000
8	45.000	45.000	50.000	50.000	50.000	46.076	400.000
9	50.000	50.000	50.000	50.000	50.000	50.000	400.000
10	50.000	50.000	50.000	50.000	50.000	50.000	400.000
11	50.000	50.000	50.000	50.000	50.000	50.000	400.000
12	50.000	50.000	50.000	50.000	50.000	50.000	400.000
13	50.000	50.000	50.000	50.000	50.000	50.000	400.000
14	50.000	50.000	50.000	50.000	50.000	50.000	400.000
15	50.000	50.000	50.000	50.000	50.000	50.000	400.000
16	50.000	50.000	50.000	50.000	50.000	50.000	400.000
17	50.000	50.000	50.000	50.000	50.000	50.000	400.000
18	50.000	50.000	50.000	50.000	50.000	50.000	400.000
19	50.000	50.000	50.000	50.000	50.000	50.000	400.000
20	50.000	50.000	50.000	50.000	50.000	50.000	400.000
21	50.000	50.000	50.000	50.000	50.000	50.000	400.000
22	50.000	50.000	50.000	50.000	50.000	50.000	400.000
23	50.000	50.000	50.000	50.000	50.000	50.000	400.000
24	50.000	50.000	50.000	50.000	50.000	50.000	400.000

The contingency causes most of the generating units to ramp up to their maximum generation level or to a highest generation level they can ramp. As a result, there is reserve shortage during some hours when 2U400 breaks down. Figure 5-1 and Table 5-5 show the system-wide reserve shortage, system-wide reserve available, and energy price at supply node 223 in each hour in this scenario. In Table 5-2, generating units at node 223 ramp up to a higher output in period 9 from period 8, subject to their ramp up limits. In hour 10, they all ramp up again and reach their maximum generation level; they cannot provide any reserves at this point. They keep their generation level at its maximum level until hour 23, because the demand is low enough for them to ramp down and have some reserve capacity available. Figure 5-1 shows that the energy prices at node 223 correlate with reserve shortage levels and reserve available in the system. During the energy and reserve shortage hours, the energy prices increase accordingly based on the pre-set reserve shortage penalty. For example, when the reserve shortage is above 325MWh at hours 12, 14, and 15, the energy prices in the same time periods soar above $600. In addition, Table 5-5 indicates that the reserve provided by generating units during period 9 and period 23 is exactly 500MW, the target reserve requirement, i.e., (5.12) is binding. Table 5-5 provides the reserve prices p_{mst}^{R} corresponding to (5.12) in each time period; it shows that when there are exactly 500MW of reserve available in the system, e.g., at hour 9 and hour 23, the reserve prices in these two hours are positive, $20.049/MW and $19.927/MW, respectively, which suggests that the system is about to be short of reserve and the reserve requirement is barely met. Meanwhile, if the reserve available is well above the reserve requirement, the reserve price is 0. Moreover, the energy price is always higher than the reserve price, even if the reserve price is high during the reserve shortage hours. For example, the reserve price is $600/MW during hour 15, and the energy price is $622.126/MWh. This indicates that during the times when the system loses the largest unit, not only reserve, but also energy is in shortage because a significant proportion of the reserve capacity in the system is used to supply energy. It therefore shows that the reserve demand curve has significant impact on the energy price, in addition to the reserve price; if the target reserve requirement cannot be satisfied, both reserve price and energy price are higher than otherwise. Since there is no load shedding in the scenario, the energy price is always below the capped price, $1000/MWh.

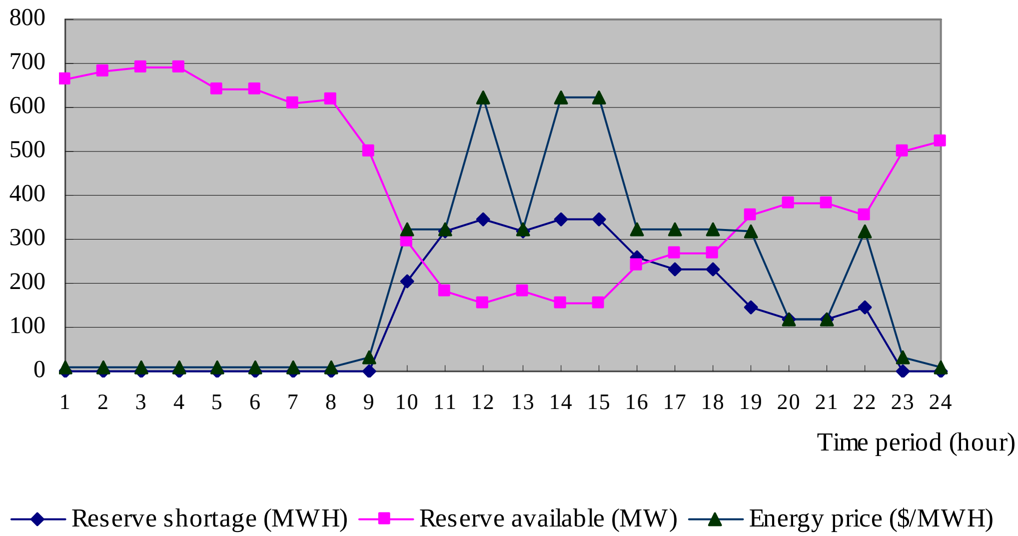


Figure 5-1 Reserve shortage, reserve available, and energy price at supply node 223 in a scenario (2U400 down from period 9)

At the same time, in the most probable scenario without any contingencies, there is no reserve shortage, and the energy price is just based on the generation offer submitted by generating units at node 223. In any hour in which the target reserve requirement is barely satisfied, reserve price is positive. In this scenario, the minimum reserve requirement is always met (see Figure 5-2 and Table 5-6).

Table 5-5 Reserve and prices at node 223 in a scenario (2U400 down from period 9)

Period	1	2	3	4	5	6	7	8
Reserve Shortage	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Reserve available	661.498	681.500	691.622	692.403	639.349	639.430	609.306	617.110
Load shed	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Energy Price at 223	10.249	10.044	9.918	9.704	9.704	9.918	10.249	10.087
Reserve Price	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Period	9	10	11	12	13	14	15	16
Reserve Shortage	0.000	202.500	316.500	345.000	316.500	345.000	345.000	259.500
Reserve available	500.000	297.500	183.500	155.000	183.500	155.000	155.000	240.500
Load shed	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Energy Price at 223	31.899	322.126	322.126	622.126	322.126	622.126	622.126	322.126
Reserve Price	20.049	300.000	300.000	600.000	300.000	600.000	600.000	300.000
Period	17	18	19	20	21	22	23	24
Reserve Shortage	231.000	231.000	145.500	117.000	117.000	145.500	0.000	0.000
Reserve available	269.000	269.000	354.500	383.000	383.000	354.500	500.000	523.770
Load shed	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Energy Price at 223	321.218	321.218	320.316	120.316	120.316	320.316	31.185	11.257
Reserve Price	300.000	300.000	300.000	100.000	100.000	300.000	19.927	0.000

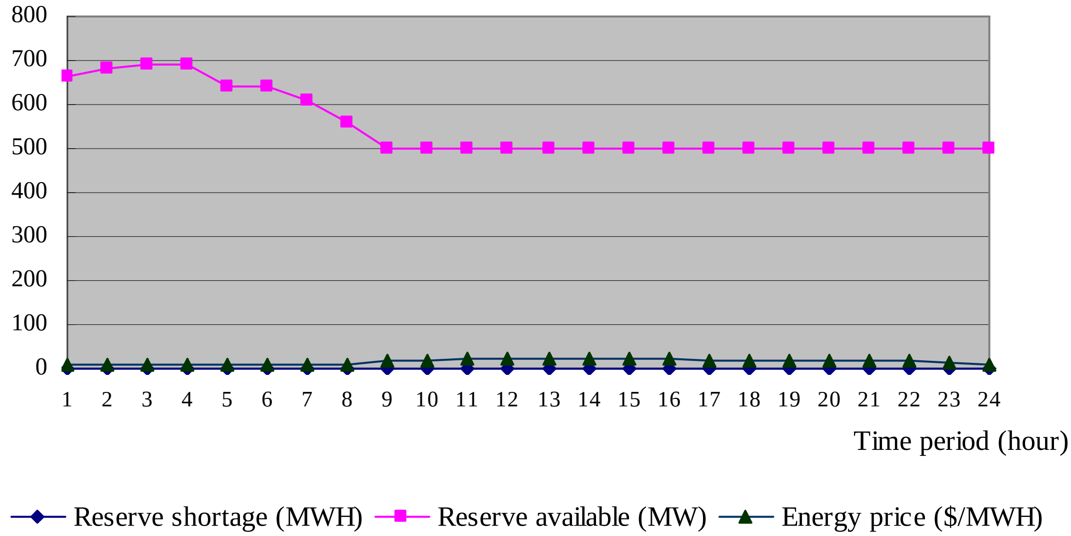


Figure 5-2 Reserve shortage, reserve available, and energy price at supply node 223 in the most probable scenario

Table 5-6 Reserve and prices at node 223 in the most probable scenario

Hours	1	2	3	4	5	6	7	8
Reserve Shortage	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Reserve available	661.498	681.500	691.622	692.403	639.349	639.430	609.494	559.954
Energy price	10.249	10.044	9.918	9.704	9.704	9.918	10.087	10.842
Reserve price	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hours	9	10	11	12	13	14	15	16
Reserve Shortage	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Reserve available	500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
Energy price	18.589	19.718	21.218	21.218	21.218	21.218	21.218	20.620
Reserve price	6.867	7.997	9.960	9.960	9.960	9.960	9.960	8.898
Hours	17	18	19	20	21	22	23	24
Reserve Shortage	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Reserve available	500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
Energy price	19.718	19.718	19.718	19.718	19.718	19.718	13.804	11.093
Reserve price	7.997	7.997	7.997	7.997	7.997	7.997	2.083	0.959

5.4 Results With and Without the Reserve Demand Curve

In most of the literature we reviewed in Chapter 2, a deterministic reserve requirement is imposed; this is a hard constraint that must be satisfied. In this setting, if there is a violation of the reserve requirement constraint, the ISO has to shed load to maintain the power supply-demand balance to ensure that there is adequate capacity in the system to meet the reserve requirement. The energy price may soar to the capped price, VOLL.

If we replace the reserve demand curve in our model with the fixed reserve requirement, the ISO will have to shed load even if there is only a small reserve shortage. On the other hand, the ISO does not have to shed load if there is a relatively small shortage with a reserve demand curve. We investigate this issue by using a fixed target reserve requirement without allowing any reserve shortage to replace the reserve demand curve in the model while maintaining load shed variables. We use the same scenario as a representative to illustrate the comparison between the two settings.

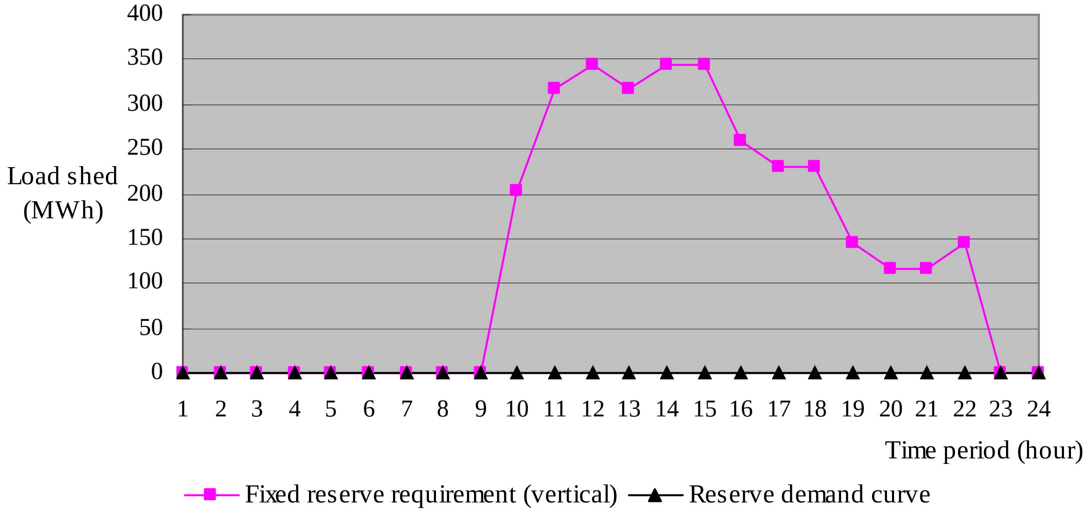


Figure 5-3 Load shed in a scenario (largest unit failure from time period 9)

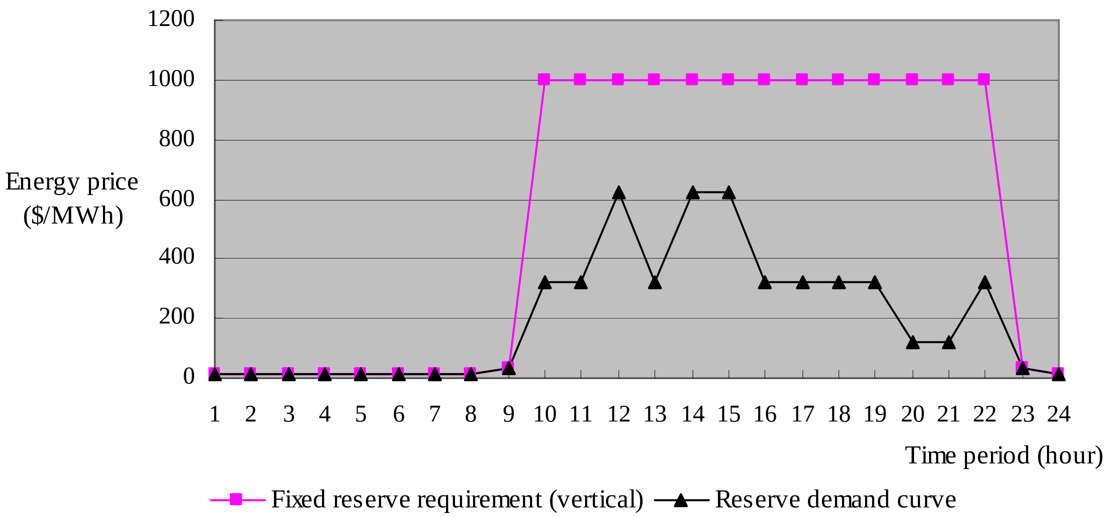


Figure 5-4 Energy price in a scenario (largest unit failure from time period 9)

Figure 5-3 compares the load shedding between two settings: with a reserve demand curve and without a reserve demand curve. Without a reserve demand curve, ISO has to shed load when there is a contingency and the target reserve requirement cannot be met. With the reserve demand, ISO can avoid load shedding by allowing the reserve available to be less than the target reserve requirement, and needs to perform load shedding only when there is no adequate capacity available in the system to supply energy or when the reserve available barely meets the minimum reserve requirement.

Figure 5-4 demonstrates that with a fixed reserve requirement, the energy prices will move up to the capped price during shortage hours. However, the reserve demand curve can mitigate the price spikes, and energy prices increase gradually based on the reserve shortage level. In Figure 5-4, with the fixed reserve requirement, the energy prices rise to the capped price, $1000/Mwh, whereas with the reserve demand curve, the energy prices rise to a level a little above the reserve shortage prices, i.e., $100, $300, and $600, depending on the reserve shortage levels. Thus, the reserve demand curve is important and necessary in stochastic settings; the fixed reserve requirement may not always be satisfied when we consider various contingencies.

5.5 Real Time Compensation Scheme (O'Neill et al. method)

In this section we propose a proper compensation plan for generating units. O'Neill et al. (2005) propose an RT compensation scheme for an MIP model on resource allocation. It includes two parts: a payment for resources provided and a commitment ticket. The former is the payment covering the generation cost, i.e., the product of the dual prices of the supply-demand balance constraints and the amount of reserve provided. The latter part of the payment aims at the fixed costs in the model; it is the product of the optimal solution value of the binary variable in the original MIP and the dual prices of the equality constraints that fix the binary variables to their optimal solution in the corresponding LP. In their paper, O'Neill et al. (2005) mention that the idea can be applied to UC-related problems because these problems have such fixed costs as start-up costs and no-load costs.

If we apply the method developed by O’Neill et al. (2005) to the SMIP in this thesis, we use three payments in the compensation plan: revenue for supplying energy, revenue for providing reserves, and revenue for committing online to provide energy.

In this scheme, payment to generators is based on RT information, i.e., the actual scenario that happens. ‘*’ represents the optimal solution value in the LP problems, either the primal problem or the dual problem.

Payment to generator g at node i in scenario s during time period t for energy delivered: $p_{ist}^{E*} q_{igst}^{E*}$, where $p_{ist}^{E*} = \frac{\alpha_{ist}^{8*}}{\pi_s}$, and $q_{igst}^{E*} = \sum_{b=1}^{B} q_{gbst}^{E*}$, i.e., the total power supplied is equal to the sum of power output from each energy offer block.

Payment to generator g at node i in scenario s during time period t for reserves (type m) prepared: $p_{mst}^{R*} q_{gmst}^{R*}$, where $p_{mst}^{R*} = \frac{\alpha_{mst}^{12*}}{\pi_s}$.

As O’Neill et al. (2005) point out in their general model, if the only prices paid are p_{ist}^{E*} and p_{mst}^{R*} for the commodities of energy and reserves, then some generators may be very disappointed with the optimal generation plan: some generators that are asked to supply energy or reserves would have to operate at a loss, because the revenue covers only their operating costs and not their fixed start-up costs; other generators that are asked to be in the “off” state may be resentful because, at the announced prices, they could have operated at a profit. The solution offered by O’Neill et al. (2005), applied to our model, is a third type of payment to generator g at node i in scenario s during time period t for commitment: $p_{igst}^{\omega *}\omega_{gst}^{MIP*}$, where $p_{igst}^{\omega *} = \frac{\alpha_{igst}^{18*}}{\pi_s}$. With this additional payment, all producers are satisfied.

By complementary slackness theory $p_{igst}^{\omega *}$ could be negative, 0 or positive since (5.18) is equality. Thus, it is possible that the additional price is negative even when $\omega_{gst} = 1$, i.e., the generating unit is online to supply energy, and there is no limit on the magnitude of the negative prices, should they occur. This is a potential problem that generators may disapprove of; their revenue can be deducted under this scheme, or part of their revenue from supplying energy and providing reserve can be taken back by this additional price that could be negative. It undermines the incentive of committing to supply energy during RT. Another possible objection to this scheme is that the commitment prices are discriminatory - $p_{igst}^{\omega *}$ is indexed by generator g, and so different generators can receive quite different payments for commitment, which could be seen as unfair treatment.

5.6 Real-Time Compensation Scheme with Make-Whole Payment

Given the potential for flaws in the RT compensation scheme provided by O'Neill et al. (2005), we propose a different RT compensation plan in which we replace the controversial additional prices $p_{igst}^{\omega *}$ by make-whole payments (Sioshansi et al., 2007), and maintain the energy revenue and reserve revenue in the scheme. A make-whole payment is a lump sum paid to a generator that would operate at a loss without the payment; the make-whole amount brings the generator's profit up to 0. Thus, the make-whole payment cannot be negative, which avoids one of the problems with the scheme of O'Neill et al. (2005). Some ISOs already use make-whole payments in their generator compensation scheme (Sioshansi et al., 2007).

Sioshansi et al. (2007) discuss another benefit of make-whole payments. Usually, the UC and relevant problems of a realistic size cannot be solved to complete optimality, and the gap between sub-optimal solutions and actual optimal solutions can cause significant differences in terms of operating schedules of generating units. When the sub-optimal operating schedule is implemented, profits of generators can, in a rather unfair way, be very different from profits under the optimal schedule. With make-whole payment, this inequity is greatly reduced.

In this thesis, the solution to the SMIP is a sub-optimal solution and brings with it the same payoff equity problem. In addition, we consider different contingencies in the model, and there is no guarantee that generating units can have non-negative profit in each scenario. Nevertheless, non-negative profit is crucial to generating units; it is an incentive to keep them online and commit in RT. As a result, we incorporate make-whole payment into the payment plan in addition to energy revenue and reserve revenue.

The make-whole payment, MH_{gs}, for generator g in scenario s (we adopt the acronym MH instead of MW to avoid confusion with Megawatts) is expressed by $MH_{gs} = \max\{0,\ CT_{gs} - RV_{gs}\}$, where CT_{gs} and RV_{gs} represent the total cost and total energy and reserve revenue of a generator, g, in scenario s, respectively. Mathematically, assuming T^{int}=1,

$$CT_{gs} = \sum_{t=1}^{T} [(\sum_{b=1}^{B} C_{gb}^{E} q_{gbst}^{E*}) + z_{gst}^{*} + C_{g}^{NL} \omega_{gst}^{*}] \tag{5.20}$$

$$RV_{gs} = \sum_{t=1}^{T} (p_{ist}^{E*} q_{gst}^{E*} + \sum_{m=1}^{M} p_{mst}^{R*} q_{gmst}^{R*}) \tag{5.21}$$

This scheme guarantees that for each generator i the profit in each period is non-negative, leading to Theorem 1.

Theorem 1: Under RT pricing with make-whole payment for each generator g the total value of revenue is greater than or equal to the total value of generation and operation costs in each scenario, i.e.,

$$RV_{gs} + \max\{0,\ CT_{gs} - RV_{gs}\} \geq CT_{gs} \tag{5.22}$$

Proof: If $CT_{gs} \leq RV_{gs}$, then $\max\{0,\ CT_{gs} - RV_{gs}\} = 0$, and thus $RV_{gs} + \max\{0,\ CT_{gs} - RV_{gs}\}$ $\geq CT_{gs}$

Else, if $CT_{gs} > RV_{gs}$, then $\max\{0,\ CT_{gs} - RV_{gs}\} = CT_{gs} - RV_{gs}$, and so $RV_{gs} + \max\{0,\ CT_{gs} - RV_{gs}\} = CT_{gs}$, satisfying (5.22).

5.7 Hybrid Compensation Scheme

As briefly mentioned previously in this thesis, a two-settlement system is implemented in practice. In a DAM, not only generation commitment but also financial commitment is made. The energy quantity committed in a DA market is locked at the DA energy price. The subsequent RTM is an adjustment market or a balance market; the deviation of energy quantity delivered in RT from the quantity contracted in the DA is paid at the RT price, depending on the RT demand and other contingencies. Mathematically,

$$Payoff = p^{DA} q^{DA} + p^{RT} (q^{RT} - q^{DA}) \tag{5.23}$$

Where p^{DA}, p^{RT} are the prices in DAM and RTM, and q^{DA}, q^{RT} are energy supplied in DAM, RTM, respectively.

The two-settlement system proposed in this thesis differs from (5.23). By modifying the mechanism in the two-settlement system (5.23), we can show that the financial risk of generating units can be reduced significantly. We also incorporate payments for reserves and make-whole payments, in a DA framework, with RT adjustments.

The compensation scheme in this thesis uses energy and reserve information from both the DAM and the RTM. Assuming for the moment that all the possible scenarios in RT are represented in the model, we use the price information calculated in the DA as predictors of the price in the RTM. There are three components in the compensation plan: most probable scenario payment, contingency adjustment, and an adder. In the most probable scenario payment, the ISO uses prices and power output from the most probable scenario to generate the payment, plus a make-whole payment that ensures that the profit of generating units is non-negative. If a contingency occurs and causes the power output of generating units to deviate from their most probable operating plan, then the ISO adjusts the compensation by considering the power output difference between the contingency scenario and the most probable scenario and paying the difference by pay-as-bid obtained from the generation offers of generating units. Furthermore, there is a DA adder on top of the most probable payment and contingency adjustment. If a generator chooses to be paid according to the hybrid scheme, it will contribute all its capacity to the DAM, and its payment will rely entirely on the following DA calculation:

Each generator *g* at node *i* is paid:

DA payment: $RV_{gs^0} + MH_{gs^0}$

Contingency adjustment: $CT_{gs} - CT_{gs^0}$

DA Adder: $E[RV_{gs} + MH_{gs} - CT_{gs}] - (RV_{gs^0} + MH_{gs^0} - CT_{gs^0})$

where s^0 represents the most probable scenario with no contingency, and MH_{gs} represents make-whole payment, i.e., $\max\{0, CT_{gs} - RV_{gs}\}$, and $E[\bullet]$ represents the expected value.

Under this compensation scheme, each generator is guaranteed to be paid in the DAM, based on its performance in the most probable scenario. The adder evaluates the overall performance of each generator in all possible scenarios compared with the most probable scenario. It ensures that the expected value of profit of each generator, if it chooses this compensation scheme, is the same as the expected value of profit in the RT compensation scheme (see Theorem 2).

Theorem 2: We assume that model P includes all possible scenarios. Under the most probable scenario-based hybrid compensation scheme, for each generator *g*, the expected value of profit is equal to the expected profit in the RT compensation scheme.

Proof:

$$
\begin{aligned}
& E[\text{DA Payment+Contingency Adjustment+ DA Adder} - CT_{gs}] \\
&= E[RV_{gs^0} + MH_{gs^0} + (CT_{gs} - CT_{gs^0}) + E[RV_{gs} + MH_{gs} - CT_{gs}] - (RV_{gs^0} + MH_{gs^0} - CT_{gs^0}) - CT_{gs}] \\
&= E[E[RV_{gs} + MH_{gs} - CT_{gs}]] \\
&= E[RV_{gs} + MH_{gs} - CT_{gs}]
\end{aligned}
$$

Theorem 1 and Theorem 2 lead to Theorem 3.

Theorem 3: We assume that model P includes all possible scenarios. Under the most probable scenario-based hybrid compensation scheme, for each generator *g*, and every scenario *s*, the total profit is greater than or equal to 0.

Proof: From the first 3 lines of the proof of Theorem 2, we know that the profit in each scenario s is $E[RV_{gs} + MH_{gs} - CT_{gs}]$, which is the same as the expected value of profit with RT compensation. According to Theorem 1, $E[RV_{gs} + MH_{gs} - CT_{gs}] \geq 0$

An additional property of this compensation scheme is that the variance of profit for each generating unit is 0, as shown in Theorem 4.

Theorem 4: We assume that model P includes all possible scenarios. Under the most probable scenario-based hybrid pricing, the variance of profit for each generating unit is 0.

Proof:

$$\begin{aligned}
&Variance(Profit) \\
&= E[RV_{gs^0} + MH_{gs^0} + (CT_{gs} - CT_{gs^0}) + E[RV_{gs} + MH_{gs} - CT_{gs}] - (RV_{gs^0} + MH_{gs^0} - CT_{gs^0}) \\
&\quad - CT_{gs} - E[RV_{gs} + MH_{gs} - CT_{gs}]]^2 \\
&= E[RV_{gs^0} + MH_{gs^0} + (CT_{gs} - CT_{gs^0}) - (RV_{gs^0} + MH_{gs^0} - CT_{gs^0}) - CT_{gs}]^2 \\
&= 0
\end{aligned}$$

Theorem 4 shows that the profit risk can be reduced to 0 under the hybrid compensation scheme while maintaining the expected value of profit of the RT compensation scheme (Theorem 2); the adder acts as an insurance used to cancel out the uncertainty of the profit of each generating unit. Note that make-whole payment MH_{gs} in the compensation plan can be replaced by any other terms, e.g., the commitment revenue used in O'Neill et al. (2005).

These properties depend on the assumption that all the possible scenarios can be accurately predicted in DA. With this assumption we can safely say that the profit variance can be reduced to 0 while maintaining the expected profit. Nevertheless, it is impossible for an ISO to predict all the possible scenarios that may happen in RT; even if an ISO can, the resulting problem will be computationally intractable due to its size. We suggest that, in a real-world implementation, these results may be approached but not precisely realized and leave the investigation of this for future research. In this thesis, we just select some important representatives of the scenarios to illustrate the theoretical results, e.g., unit and transmission line failures lasting for many hours. All expected values and variances are only computed over the scenarios included in the model. Moreover, the expected profits of the hybrid scheme are obtained without considering the payments charged by the entity that assumes the risk. Therefore, the expected profit values under the hybrid scheme are less than those under the RT scheme after these payments are deducted.

5.8 Numerical Examples for the Proposed Compensation Schemes

In this section, we will use the large-scale numerical example to illustrate the properties of the compensation plans.

We first apply the scenario-rolling heuristic algorithm to obtain the sub-optimal solution to the SMIP model. The sub-optimal solution to the binary variable is fixed in PLP, i.e., the commitment states of the generating units are known. Then we solve the PLP to its complete optimality. Following the steps given in the previous section, we can apply the duality theory to PLP and procure the energy prices, reserve prices and make-whole payments. Representative results about the compensation schemes are shown in Tables 5-7, 5-8, and 5-9.

Table 5-7 shows the expected values and standard deviations of profits of RT pricing and the hybrid compensation scheme. We can see that the standard deviation of profit of the RT compensation scheme is higher than that of the hybrid compensation scheme, i.e., 0. To compare the different compensation schemes, we also show the expected profit of each generator under the compensation scheme proposed in O'Neill et al. (2005) in Table 5-7. O'Neill et al. (2005), instead of using a make-whole payment in the RT payment, use commitment revenue that is represented by $p_{igst}^{\omega*}\omega_{gst}^{MIP*}$. The results in Table 5-7 demonstrate that most generators have higher expected profits under RT pricing with make-whole payment than under RT pricing with commitment revenue. Therefore, we can conclude that for most generators the additional commitment revenue is negative; this negative price reduces the profit of generators significantly by cutting part of their revenue. The total expected profit of the RT pricing with make-whole payment is $1237440.768 greater than the total expected profit of the RT pricing with commitment revenue, which is $290805.074. On the other hand, the total standard deviation of RT with make-whole payment, $2558571.535, is higher than that of RT pricing with commitment revenue, $381703.179.

Table 5-8 provides the expected values of revenue components under the RT compensation schemes and the hybrid compensation scheme for each generator. It shows that under RT pricing, the revenue of peaking units 1U20-4U20 mainly comes from preparing reserves, while they receive revenue from supplying energy during shortage hours. Base load units 1U400 and 2U400, if they choose RT compensation scheme, receive their revenue mainly from supplying energy. As expected, their revenue from preparing reserves is the lowest among all the generators. The make-whole payment in RT pricing is enforced to guarantee that the generators can at least break even in all the scenarios. In addition, we also compare the make-whole payment with the revenue from the additional prices proposed in O'Neill et al. (2005) (see columns 4-5). We can see that most of the generators have high negative commitment revenues. These negative revenues significantly reduce the total revenue received by the generators; this additional price is difficult to implement in practice, since generators will not accept a contract that would significantly cut their revenue when they commit. Comparatively, make-whole payment is more acceptable. It guarantees that each generator can have non-negative revenue by paying the generator to uplift its own profit to 0 if it has a negative profit in a scenario.

From Table 5-8 we observe that the expected values of make-whole payment for some of the cyclers are positive; these positive values indicate that in some scenarios, the make-whole payment of these generators must be positive and their profit is 0. The results from the hybrid compensation scheme indicate that in the most probable scenario, all the generators make positive revenue because there is no contingency in that scenario. Some generators, however, receive negative contingency adjustment payments due to outages or rescheduling in some scenarios. The expected loss of these generators can be fully covered by the adders.

Table 5-7 Expected profit and standard deviation of profit of each generator under different schemes

Generating Unit	Expectation(all in $)		Std. Deviation(all in $)		
	RT & Hybrid	RT with Commitment Revenue	RT with Make-whole	Hybrid	RT with Commitment
1U20	8052.248	8056.013	15441.390	0.000	15456.766
2U20	8050.757	8056.041	15436.233	0.000	15456.864
3U20	8052.338	8056.013	15440.380	0.000	15456.766
4U20	8051.260	8056.027	15438.750	0.000	15456.828
1U76	30983.024	2612.819	60411.801	0.000	1681.633
2U76	30983.054	2565.803	60411.953	0.000	1740.253
3U76	31511.306	3222.729	60360.050	0.000	1658.813
4U76	31511.308	3094.863	60360.064	0.000	1716.373
1U100	35558.429	999.218	77703.907	0.000	3670.351
2U100	37813.742	1072.722	77405.279	0.000	480.304
3U100	37815.593	1087.950	77396.607	0.000	513.223
1U197	44710.012	5582.567	149223.229	0.000	9498.320
2U197	56452.803	6349.257	155257.783	0.000	2822.893
3U197	56453.176	6333.285	155292.307	0.000	2792.806
1U12	3954.272	982.401	9307.260	0.000	238.301
2U12	3963.111	1036.815	9304.855	0.000	273.230
3U12	3925.174	958.496	9245.350	0.000	258.027
4U12	3954.215	963.416	9307.232	0.000	230.085
5U12	3962.459	980.872	9304.576	0.000	271.694
1U155	55067.866	2967.547	123855.086	0.000	5420.841
2U155	56158.034	2952.650	123799.324	0.000	4505.497
1U400	161918.995	34866.438	261549.462	0.000	10029.701
2U400	159836.836	33418.788	261650.710	0.000	9994.353
1U50	22037.011	22189.715	40327.745	0.000	40286.852
2U50	22037.058	22189.762	40327.826	0.000	40286.933
3U50	22037.089	22189.793	40327.840	0.000	40286.947
4U50	22037.012	22189.717	40327.757	0.000	40286.864
5U50	22037.071	22189.775	40327.859	0.000	40286.966
6U50	22037.055	22189.759	40327.861	0.000	40286.968
3U155	58239.532	3490.770	123020.248	0.000	3938.241
4U155	60179.557	3481.219	123434.343	0.000	2875.826
1U350	108059.371	6421.834	257246.468	0.000	13543.660
Total	1237440.768	290805.074	2558571.535	0.000	381703.17

Table 5-8 Revenue components of RT and hybrid compensation schemes (expected values)

Units	RT Compensation scheme (all in $)				Hybrid Compensation scheme(all in $)		
	Energy	Reserve	Make -Whole	Commitment Revenue	Most Probable Scenario	Contin-gency Adjustment	Adder
1U20	29.945	8028.321	0.000	3.765	2491.706	6.018	5560.542
2U20	61.766	7998.018	0.000	5.284	2491.706	9.027	5559.051
3U20	33.065	8025.291	0.000	3.675	2491.706	6.018	5560.632
4U20	45.913	8013.17	0.000	4.767	2491.706	7.823	5559.554
1U76	49710.45	291.957	0.000	-28370.205	28547.952	-103.923	21558.377
2U76	49483.693	321.244	0.000	-28417.251	28326.284	-79.756	21558.408
3U76	50774.38	106.707	0.000	-28288.578	29393.348	-35.608	21523.347
4U76	50764.691	75.973	0.000	-28416.446	29343.284	-25.970	21523.350
1U100	32142.241	17964.735	44.874	-34535.246	24719.299	-206.793	25618.435
2U100	35504.535	18234.978	0.000	-36741.020	25286.650	583.366	27869.497
3U100	34173.742	18962.873	0.000	-36727.642	24626.537	642.909	27867.169
1U197	75175.844	8891.786	139.012	-39046.316	40084.215	810.769	43253.776
2U197	86361.12	10588.178	0.000	-50103.546	38773.301	3179.430	54996.567
3U197	86576.463	10676.976	0.000	-50119.891	39118.659	3137.840	54996.940
1U12	2173.135	2971.84	0.000	-2971.544	1771.903	59.816	3313.583
2U12	2149.663	2962.11	0.000	-2925.969	1726.280	76.175	3309.645
3U12	2118.281	2952.636	0.000	-2966.188	1726.280	73.419	3271.707
4U12	2155.757	2986.79	0.000	-2990.472	1771.903	57.445	3313.526
5U12	2151.775	2958.983	0.000	-2981.260	1726.280	75.812	3308.992
1U155	78815.953	4528.808	32.139	-52074.618	42403.525	-374.546	41341.483
2U155	80011.216	4564.835	20.218	-53189.341	42166.115	-48.126	42474.105
1U400	223235.94	0.416	0.000	-127052.556	135808.200	-720.806	88148.971
2U400	221098.59	8.915	0.000	-126418.048	133561.503	-715.045	88261.053
1U50	27805.09	352.639	0.000	152.704	13635.623	11.239	14510.867
2U50	27938.535	210.238	0.000	152.704	13615.755	22.104	14510.915
3U50	28022.689	185.495	0.000	152.704	13682.923	14.316	14510.944
4U50	27881.518	277.309	0.000	152.704	13623.800	24.158	14510.869
5U50	27772.135	298.201	0.000	152.704	13531.244	28.163	14510.929
6U50	27921.798	224.599	0.000	152.704	13618.676	16.810	14510.911
3U155	92689.73	629.14	13.722	-54737.895	51783.864	-632.610	42178.483
4U155	95386.293	675.376	0.000	-56698.338	52122.401	-178.480	44117.749
1U350	190677.64	5060.052	117.894	-101584.054	113116.805	-1275.240	83949.614

We now choose a contingency scenario to present the numerical results of RT pricing and the hybrid compensation scheme. In this particular scenario, one of the base load units, 1U350, is down from hour 2 to hour 24, i.e., 23 hours out of 24 hours.

Table 5-9 Revenue components in a contingency scenario: RT and hybrid

Units	RT Pricing (all in $)				Hybrid Pricing (all in $)			
	Energy Revenue	Reserve Revenue	Make-whole	Total Profit	Most Probable Scenario	Contingency Adjustment	Adder	Total Profit
1U20	0.000	62730.062	0.000	62730.062	2491.71	0.000	5560.540	8052.246
2U20	0.000	62730.062	0.000	62730.062	2491.71	0.000	5559.049	8050.755
3U20	0.000	62730.062	0.000	62730.062	2491.71	0.000	5560.629	8052.335
4U20	0.000	62730.062	0.000	62730.062	2491.71	0.000	5559.551	8051.257
1U76	266228.631	0.000	0.000	246422.896	28513.61	682.428	21558.377	30983.024
2U76	266212.767	0.000	0.000	246422.896	28360.62	888.230	21558.408	30983.054
3U76	267328.143	0.000	0.000	246880.157	29332.03	1042.597	21523.347	31511.306
4U76	267141.248	0.000	0.000	246880.157	29393.35	905.766	21523.350	31511.309
1U100	246729.374	88091.217	0.000	311552.580	24725.59	8488.706	25618.425	35558.419
2U100	224468.094	108124.172	0.000	311552.570	25893.48	5697.291	27869.488	37813.733
3U100	216346.719	115711.646	0.000	311552.568	24013.42	5827.684	27867.159	37815.583
1U197	584217.193	85095.093	0.000	609900.810	38873.36	20783.498	43253.772	44710.008
2U197	574553.410	94095.093	0.000	609900.807	38962.63	21430.631	54996.563	56452.799
3U197	598867.025	70995.093	0.000	609900.814	40140.18	22298.881	54996.935	56453.171
1U12	24264.866	15139.557	0.000	37105.536	1726.28	1167.672	3313.582	3954.270
2U12	24586.083	14839.557	0.000	37105.536	1771.90	1247.291	3309.643	3963.110
3U12	24264.866	15139.557	0.000	37105.536	1726.28	1226.073	3271.706	3925.172
4U12	24294.283	15112.083	0.000	37105.536	1771.90	1169.615	3313.525	3954.214
5U12	24329.109	15079.557	0.000	37105.536	1726.28	1230.317	3308.991	3962.457
1U155	530380.105	1095.093	0.000	497763.647	42425.59	5034.410	41341.518	55067.901
2U155	527959.502	3057.491	0.000	497707.595	42144.05	4827.212	42474.066	56157.994
1U400	1388188.840	0.000	0.000	1326150.664	135808.20	0.000	88148.971	161918.995
2U400	1386379.520	0.000	0.000	1324341.344	133561.50	52.456	88261.055	159836.838
1U50	170652.801	176.429	0.000	164617.683	13631.92	102.068	14510.867	22037.011
2U50	170832.087	12.737	0.000	164617.683	13586.50	137.530	14510.915	22037.058
3U50	170812.537	12.737	0.000	164617.683	13621.44	50.812	14510.945	22037.089
4U50	170907.887	12.737	0.000	164617.685	13604.72	205.284	14510.869	22037.013
5U50	170888.604	12.737	0.000	164617.684	13650.31	278.555	14510.929	22037.071
6U50	170851.482	12.737	0.000	164617.684	13613.13	154.003	14510.912	22037.055
3U155	536706.515	396.236	0.000	499808.889	51783.86	1571.047	42178.484	58239.532
4U155	537193.937	19.740	0.000	499808.892	52133.66	1344.193	44117.749	60179.557
1U350	1434.888	0.000	976.648	0.000	113116.81	-86595.513	83949.608	108059.365

Table 5-9 gives the revenue components of RT pricing and the hybrid pricing in this scenario. Since 1U350 is the only unreliable generator in this scenario, in RT pricing it requires a make-whole payment of $976.648 to break even. Other generating units receive 0 make-whole payment because they are paid adequate energy and reserve revenue to cover their total costs. Peaking units 1U20-2U20 receive revenue only from preparing reserve; the magnitude of the reserve payment is large due to high reserve prices during the shortage hours. Base load units 1U400 and 2U400 contribute all their capacity to supplying energy; therefore, their revenue completely comes from supplying energy. Other cyclers receive either energy revenue or both energy and reserve revenue, depending on the rescheduled generation plan. In hybrid pricing, the contingency adjustment payment of 1U350 is negative, -$86595.513. However, as previously discussed, most of the loss is covered by the adder in the amount of $83949.608, which is based on the overall performance of this generator across all possible scenarios. The profit of 1U350 in this scenario is $108059.365, which is a significant improvement over barely breaking even in the RT compensation scheme. Most of the other generating units receive a positive contingency adjustment, since they are called on by ISO to supply more energy than that they do in the most probable scenario to make up the capacity loss due to the outage of 1U350. The calculation of the adder is tied to the overall performance of the generators in other scenarios to cover their total cost in the scenario. In this particular scenario, the profit of generators with the hybrid plan is lower than it would be for the RT compensation plan, except for the outage generator. The adder in the hybrid compensation plan acts as a form of insurance. If a generator is not confident of its equipment, it may prefer the hybrid plan, which can compensate part of the loss due to a contingency. However, the generator has to pay this insurance. Thus, the actual adder paid to the generators should be less than those values shown in Table 5-9 after being deducted by the insurance.

Table 5-10 Sensitivity analysis of expected profit and standard deviation of profit (low contingency probability)

Generating Unit	Expectation(all in $)		Std. Deviation(all in $)		
	RT & Hybrid	RT with Commitment Revenue	RT with Make-whole	Hybrid	RT with Commitment Revenue
1U20	4788.304	4788.741	5230.935	0.000	5236.658
2U20	4788.304	4788.741	5231.149	0.000	5236.652
3U20	4788.189	4788.743	5228.905	0.000	5236.670
4U20	4788.209	4788.741	5230.542	0.000	5236.652
1U76	18217.487	0.155	20440.698	0.000	4.023
2U76	18217.485	0.154	20440.698	0.000	4.023
3U76	18777.314	0.170	20412.450	0.000	4.003
4U76	18777.313	0.170	20412.450	0.000	4.003
1U100	21391.833	489.194	26210.206	0.000	729.637
2U100	21473.015	496.481	26200.242	0.000	215.297
3U100	21473.273	496.165	26195.281	0.000	259.268
1U197	12289.490	10069.613	52678.559	0.000	1939.660
2U197	12718.468	10090.771	53639.951	0.000	873.940
3U197	12714.826	10095.766	53647.689	0.000	886.013
1U12	870.655	1048.442	2905.859	0.000	141.062
2U12	2039.363	1041.990	2812.168	0.000	142.277
3U12	902.927	1049.202	3193.911	0.000	113.865
4U12	903.647	969.315	3194.429	0.000	113.357
5U12	907.804	963.633	2984.330	0.000	139.137
1U155	31459.666	364.685	41549.491	0.000	1182.090
2U155	31403.536	378.689	41555.444	0.000	1184.584
1U400	117195.117	0.127	86975.797	0.000	0.012
2U400	114963.768	0.066	86045.265	0.000	0.006
1U50	13329.121	13516.562	13709.343	0.000	13695.099
2U50	13329.124	13516.564	13709.343	0.000	13695.099
3U50	13329.121	13516.562	13709.343	0.000	13695.099
4U50	13329.121	13516.561	13709.343	0.000	13695.099
5U50	13329.121	13516.561	13709.343	0.000	13695.099
6U50	13329.123	13516.563	13709.343	0.000	13695.099
3U155	33785.688	33.168	41544.061	0.000	1011.755
4U155	33997.732	33.204	41798.891	0.000	13.451
1U350	62862.398	132.515	85843.746	0.000	4803.413
Total	706470.542	138008.014	863859.205	0.000	116882.102

Table 5-11 Sensitivity analysis of revenue components of RT and hybrid compensation schemes (low contingency probability)

Units	RT Compensation scheme (all in $)				Hybrid Compensation scheme(all in $)		
	Energy	Reserve	Make -Whole	Commitment Revenue	Most Probable Scenario	Contin-gency Adjustment	Adder
1U20	3.909	4785.107	0.000	0.437	4329.384	0.712	458.920
2U20	3.909	4785.107	0.000	0.437	4329.384	0.712	458.920
3U20	4.967	4784.129	0.000	0.554	4329.384	0.907	458.805
4U20	4.290	4784.760	0.000	0.532	4329.384	0.842	458.825
1U76	37252.377	15.153	0.000	-18217.331	35486.099	-6.801	1788.231
2U76	37163.998	32.809	0.000	-18217.331	35413.806	-5.232	1788.231
3U76	38167.686	11.085	0.000	-18777.144	36398.154	-3.933	1784.549
4U76	38118.043	11.222	0.000	-18777.144	36348.090	-3.375	1784.549
1U100	22041.995	14808.734	0.916	-20901.723	34626.249	7.856	2217.539
2U100	20828.300	15341.563	0.000	-20976.534	33819.452	51.684	2298.726
3U100	21280.598	15096.780	0.000	-20977.108	34028.958	49.437	2298.982
1U197	43435.847	6967.132	3.246	-2216.631	45859.661	156.176	4390.385
2U197	42947.727	7019.019	0.000	-2627.697	44896.477	250.904	4819.363
3U197	44127.226	7016.030	0.000	-2619.060	46107.419	220.114	4815.721
1U12	750.258	1256.255	0.334	178.122	1756.913	4.979	244.957
2U12	943.265	2231.392	0.299	-997.074	2946.912	4.378	223.666
3U12	773.606	1266.857	0.194	146.469	1756.913	6.516	277.228
4U12	773.569	1267.825	0.176	65.844	1756.913	6.708	277.949
5U12	735.103	1251.299	0.264	56.094	1726.280	6.049	254.337
1U155	53344.717	6433.850	1.471	-31093.509	56322.354	6.659	3451.024
2U155	53053.141	6437.960	1.475	-31023.372	56033.285	5.966	3453.323
1U400	179155.47	0.074	0.000	-117194.990	172617.520	-77.752	6615.773
2U400	176861.06	9.291	0.000	-114963.703	170363.779	-77.056	6583.625
1U50	18752.916	658.651	0.000	187.440	18200.463	2.321	1208.783
2U50	19495.038	53.797	0.000	187.440	18341.393	-1.342	1208.783
3U50	18775.595	638.329	0.000	187.440	18202.980	2.160	1208.783
4U50	18745.827	631.887	0.000	187.440	18166.202	2.728	1208.783
5U50	18803.959	583.520	0.000	187.440	18175.827	2.868	1208.783
6U50	18929.253	561.539	0.000	187.440	18281.104	0.903	1208.783
3U155	69841.313	61.983	1.233	-33751.287	66519.313	-64.252	3449.466
4U155	69957.412	83.768	0.000	-33964.528	66389.772	-10.105	3661.510
1U350	146813.97	5204.540	6.272	-62723.611	145612.131	-123.551	6536.195

Table 5-12 Sensitivity analysis of revenue components in a contingency scenario: RT and hybrid (low contingency probability)

Units	RT Pricing (all in $)				Hybrid Pricing (all in $)			
	Energy Revenue	Reserve Revenue	Make-whole	Total Profit	Most Probable Scenario	Contingency Adjustment	Adder	Total Profit
1U20	0.000	62730.064	0.000	62730.064	4329.384	0.000	458.92	4788.304
2U20	0.000	62730.064	0.000	62730.064	4329.384	0.000	458.92	4788.304
3U20	0.000	62730.064	0.000	62730.064	4329.384	0.000	458.805	4788.189
4U20	0.000	62730.064	0.000	62730.064	4329.384	0.000	458.825	4788.209
1U76	265931.551	184.514	0.000	246422.903	35486.099	636.318	1788.231	18217.487
2U76	266212.773	0.000	0.000	246422.904	35413.806	805.316	1788.231	18217.485
3U76	267232.202	0.000	0.000	246880.172	36398.154	946.641	1784.549	18777.314
4U76	267237.219	0.000	0.000	246880.171	36348.09	1001.723	1784.549	18777.313
1U100	235246.425	98253.224	0.000	311552.585	34626.249	6495.109	2217.539	21391.833
2U100	238991.59	94814.179	0.000	311552.587	33819.452	7608.019	2298.726	21473.015
3U100	227994.52	105182.179	0.000	311552.584	34028.958	6769.447	2298.982	21473.273
1U197	574063.327	94095.096	0.000	609900.82	45859.661	20297.046	4390.385	12289.49
2U197	576937.154	91875.096	0.000	609900.821	44896.477	21914.057	4819.363	12718.468
3U197	586955.276	82545.096	0.000	609900.824	46107.419	21391.234	4815.721	12714.826
1U12	25231.244	14239.558	0.000	37105.538	1756.913	1234.05	244.957	870.655
2U12	25102.212	14359.558	0.000	37105.537	2946.912	1225.018	223.666	2039.363
3U12	25102.212	14359.558	0.000	37105.537	1756.913	1225.018	277.228	902.927
4U12	25680.404	13819.558	0.000	37105.538	1756.913	1263.21	277.949	903.647
5U12	25616.705	13879.558	0.000	37105.538	1726.280	1317.912	254.337	907.804
1U155	530380.112	1095.096	0.000	497763.657	56322.354	5397.839	3451.024	31459.666
2U155	528289.938	2866.059	0.000	497707.606	56033.285	5365.319	3453.323	31403.536
1U400	1388188.88	0.000	0.000	1326150.704	172617.520	0	6615.773	117195.117
2U400	1386379.56	0.000	0.000	1324341.384	170363.779	54.54	6583.625	114963.768
1U50	170641.156	176.429	0.000	164617.683	18200.463	119.777	1208.783	13329.121
2U50	170931.117	12.737	0.000	164617.686	18341.393	105.115	1208.783	13329.124
3U50	170736.609	12.737	0.000	164617.682	18202.980	49.022	1208.783	13329.121
4U50	170908.798	12.737	0.000	164617.685	18166.202	257.985	1208.783	13329.121
5U50	170778.758	12.737	0.000	164617.683	18175.827	118.323	1208.783	13329.121
6U50	170948.965	12.737	0.000	164617.686	18281.104	183.251	1208.783	13329.123
3U155	537384.316	19.742	0.000	499808.904	66519.313	1412.064	3449.466	33785.688
4U155	536482.815	403.158	0.000	499808.901	66389.772	1023.522	3661.510	33997.732
1U350	1434.888	0.000	976.648	0.000	145612.131	-86874.392	6536.195	62862.398

Tables 5-10 to 5-12 present the sensitivity analysis of the results in Tables 5-7 to 5-9 when the probability of the most probable scenario is set to 0.98, and the contingency scenarios split the remaining probability of 0.2 evenly. We notice that the adders based on the new probabilities are much smaller than these in Tables 5-8 and 5-9. These indicate that if the probability that some equipment failure may occur in the next day is small, the adder as a form of insurance reduce accordingly.

5.9 Summary

In this chapter, we first applied the method proposed by O'Neill et al. (2005) to obtain the dual prices of the model: we used the scenario-rolling algorithm to obtain a feasible solution, then fixed the binary variables to their sub-optimal values and solved the resulting linear program to procure the dual prices. Then we gave a numerical example to show energy and reserve prices with this methodology. We also compared the load shed and the energy prices between two settings, with the reserve demand curve and without reserve demand curve, highlighting the difference between the results of the two models. The reserve demand curve can alleviate price spikes by allowing reserve shortages, while without the reserve demand curve the energy jumps to the pre-set cap price, even if there is only a small amount of reserve shortage.

We then proposed two compensation schemes: a RT compensation scheme and a hybrid compensation scheme. The RT compensation scheme uses the estimated RT prices in each scenario calculated in DA with the assumption that all the possible scenarios can be accurately predicted. We investigated the commitment revenue suggested by O'Neill et al. (2005) and found out that generating units have to pay back a proportion of their revenue because the dual prices associated with fixed commitment state constraints can be negative, and the magnitude is large. This drawback of the scheme cannot be accepted by generators in practice. We also proposed a RT compensation scheme with a lump sum make-whole payment that guarantees non-negative profit for each generator in each scenario. Its variance is high, since the output level of generating units varies significantly in different scenarios. Therefore, generators have to bear a high financial risk due to various contingencies. The hybrid compensation scheme relies on the price and generation information in the most probable scenario and financially compensates the power output deviation between the most probable scenario and other contingency scenarios. It has the same expected profit as the RT compensation scheme does. With the assumption that all the possible scenarios can be predicted and included in the model, the hybrid compensation scheme reduces the variance of profit of each generator to 0, reducing the financial risk of generators. Although not all of the scenarios can be predicted in the real world, the variance of profit is 0 in the scenarios considered in the model, no matter how many scenarios are considered in the model. The expected value of profit for each generating unit will, however, change accordingly.

The adder in the hybrid compensation plan works as a form of insurance. If a generator agrees to choose the hybrid compensation plan, it has to pay its insurance because it cannot be secured by the adder for free. The involved contract and other potential financial issues will make for interesting future research.

Chapter 6

Conclusions and Future Research

Reliable power generation is crucial to an electricity market. Incorporating possible uncertainties in RT in a DAM can help an ISO to better prepare for unexpected contingencies during the next day. This thesis presents a multi-stage SMIP model for determining unit commitment and allocation of the units to energy and reserves for a pool type of DA electricity market. It is challenging to solve a multi-stage SMIP model within a time requirement. Therefore, we propose a scenario-rolling heuristic to obtain a good sub-optimal solution to the large-scale SMIP problem. Given the sub-optimal solution, we extend the ideas in previous research to price the SMIP so that we can propose compensation plans for generating units with energy and reserve prices.

This thesis makes the following contributions:

- It incorporates a reserve demand curve into an SMIP model to associate the energy price with the reserve shortage level in the system. The model allows a reserve shortage while using a fixed reserve requirement in each scenario. From the perspective of modeling, an ISO does not have to shed load when a small amount of reserve requirement cannot be met.

- To avoid combinatorial explosion, we parsimoniously select a representative number of scenarios instead of incorporating all of them.

- We propose a scenario-rolling heuristic to solve the SMIP model within a short time period based on a scenario tree. The heuristic can generate good quality feasible solutions within reasonable time requirements.
- The numerical results show a difference between the operating schedules of the deterministic and stochastic models. Compared with the operating schedule determined by the deterministic model, some of the generating units need to be started up earlier in the operating schedule in anticipation of future forced equipment failures.
- We extend previous research on pricing commodity models with or without nonconvexities, by applying the ideas (O'Neill et al., 2005; Sioshansi et al., 2008; Wong and Fuller, 2007) to pricing energy and reserve based on the SMIP. Particularly, we conduct numerical tests to investigate the validity of additional prices associated with binary variables proposed in O'Neill et al. (2005). This proposal is shown to be impractical, and we advocate make-whole payments instead.
- We propose two compensation plans. Based on the assumption that all the possible scenarios can be accurately predicted by an ISO, we prove that one of the proposed plans has the desirable property of reducing the profit variance to 0.

6.1 Conclusions

The following conclusions are drawn from the results of this thesis:

- The proposed scenario-rolling heuristic method can solve large-scale numerical examples within the time requirement. Depending on the structure of the scenario trees, the heuristic can be used to solve large-scale multi-stage stochastic programs if a quick solution with good quality is required. The scenarios solved in each iteration can be selected intentionally according to the need of users.

- If contingencies are considered during the process of power generation planning, some generating units have to follow a different schedule from that determined by a deterministic model. For example, some generating units have to be started up early to prepare for future outages in the system. As a result, the allocation of energy and reserve changes accordingly.

- If we consider contingencies in the real world, we believe that a fixed reserve requirement is still needed. This conclusion differs from Galiana et al. (2005) and Wong and Fuller (2007).

- By providing numerical examples, we show that the reserve demand curve can mitigate the energy price spike. From the modeling perspective, allowance for reserve shortage can prevent an ISO from shedding load just because a small part of the reserve requirement is not met.

- We demonstrate that the additional price associated with binary variables in O'Neill et al. (2005) is impractical when pricing electricity in the real world. It can easily cause inequities and disagreement among generators.

- Assuming that an ISO can accurately predict RT scenarios, the profit variance of generating units can, in theory, be reduced to 0 under the compensation plan proposed in this thesis. In practice, because the ISO can only include a few of the major contingencies in scenarios of our model, we interpret this result to mean that profit variance can be reduced considerably, but not necessarily to 0. In future research, we will investigate the profit variance of generating units when more scenarios than a few of the important scenarios are included.

6.2 Future Research

The model and the solution methods presented in this thesis contain some limitations. To go beyond these limitations, we suggest the following future research:

- When the size of the problem increases as more scenarios are incorporated, or the scenario tree branches out further than the one presented in this thesis, the scenario-rolling heuristic method may need to work with other existing solution methods to improve the quality of the solution. We also need an algorithm to provide a good lower bound to evaluate the quality of the solution.
- We may consider sources of contingencies other than equipment failure. For example, load fluctuation is another important cause for contingencies. To limit the size of a multi-stage stochastic programming problem considering load uncertainties and equipment failure simultaneously, sampling and simulation techniques may be required. For example, the Monte-Carlo method may be needed to evaluate the quality of the solution (Mak et al., 1999).
- Since we already include the minimum reserve requirement and the target reserve requirement, we may move forward in the next step to consider responsive demand. When the energy prices and the reserve prices in the system tend to increase due to

contingencies, customers may consider ramping down their demand to avoid possible blackouts, especially for commercial customers with RT meters.

- To make the model more realistic, the next step in research may include other features of power system operation, such as emission constraints, fuel constraints, power loss along the lines, etc.
- As we introduce more scenarios in the model, we can further test the property of the hybrid compensation scheme to investigate the issues of expected profit and profit variance.
- We may investigate the contract issues relevant to the adder in the hybrid compensation plan or other forms of insurance that can cover the loss of generators due to contingencies.

Appendices

Appendix A

GAMS Codes of the Scenario-Rolling Method

```
* no overlap between time period when the algorithm moves forward
$TITLE  no overlap roll time period forward algorithm, version 1.00
$ontext
version: xx

date: Oct. 16 2008

author of this version: Jzhang

characteristics: ?

new in this version:

problem1: no improvement from Cplex from a feasible starting solution

note1: 73 scenarios, 6 units and 9 lines, 3 contingencies in each time period including two unit
outages and one line outage

note2: linkscenario, 1 stands for link on, 0 stands for no link

note 3:
        12 block: iter: 2, inner: 12, time block: 12
        8 block: iter: 5, inner: 4, time block: 8
        6 block: iter: 7, inner: 3, time block: 6
        4 block: iter: 11, inner: 2, time block: 4
```

```
    2 block: iter: 23, inner: 1, time block: 2

$offtext
$eolcom #
$inlinecom { }
$offsymxref
set tao time generator has been turned off
/0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24/;

set iter iterations /iter1*iter1/;
set inneriter inner iteration /23/;
scalar timeblock/23/;
Set i   buses  /201,223,218,202,203,204/;
set alls n-1 scenarios /1*73/;
set s(alls)   dynamic set;
set sdummy(alls) dynamic set;
set bs shortage blocks /1,2,3,4/;
set allt time periods /1*24/;


set b blocks of offers and bids /1*4/;
alias (alls,a);
alias (i,j);
alias (allt,t);
alias (allt,k);
scalar tcplexsolve/0/;
scalar tsolve/0/;
scalar big/1.0E9/;
scalar small/0.0000001/;

scalar sheddingcost/10000/;
```

```
scalar demandper/0.95/;

scalar Qrequire/400/;
Scalar MinRreq/25/;
sets
    isupply(i) /218,201,223/
    idemand(i) /202,203,204/
    g generating units /1U400,U350,U155,1U20,2U20,U197/


   gs(g) slow units /U350,2U20,1U400,U155/
   gf(g) fast units /U197,1U20/
    gr(g) units must run /1U400/

    igall(isupply,g) /
       201.(1U20,2U20)
       223.(U155,U197,U350)
       218.(1U400)
       /;

;

Parameter CS(bs)     reserve shortage capped price  /

1 1000
2 3000
3 6000
4 10000
 /;
```

```
Table BB(i,j)    negative of susceptance
            201     223     218     202     203     204
      201                           2.74            6.87
      223                   5.21            4.31    5.46
      218                          3.49     5.40    8.62
      202                             2.90
      203
      204                                        ;

Table U(i,j)    line capacity
            201     223     218     202     203     204
      201                          150              150
      223                     200            300     300
      218                           200     150     180
      202                                   175
      203
      204                                        ;


Scalar Tmax the max time period considered /24/;
Parameter mu(isupply,g,alls,allt)  generator scenarios /
$ondelim
$include MuLarge.csv
$offdelim
```

```
/;

Parameter link(alls,a,t)  nonanticitivity  /
$ondelim
$include LinkScenarioNew.csv
$offdelim
/;

Parameter vi(i,j,alls,allt) line outages   /
$ondelim
$include ViLarge.csv
$offdelim
/;
Parameters Cgen(isupply,g,b,allt)        generators' offer price /
$ondelim
$include GenOfferPriceLarge197.csv
$offdelim
/;
Cgen('201','2U20',b,allt)=0.9*Cgen('201','2U20',b,allt);
Cgen('223','U350',b,allt)=0.8*Cgen('223','U350',b,allt);
Parameters Demand(idemand,allt)      demand at demand bus in s during t/
$ondelim
$include DemandLarge.csv
$offdelim
/;
Demand(idemand,allt)$(ord(allt)>=9 and ord(allt)<=12)=1.04* Demand(idemand,allt);

Demand(idemand,allt)$(ord(allt)>=15 and ord(allt)<=18)=1.04* Demand(idemand,allt);

Parameter GenQuant(isupply,g,b)       quantities offered by generators /
```

```
$ondelim
$include GenQuantLarge197.csv
$offdelim
/;


Parameter Pi(alls)   probability of scenario s /
$ondelim
$include ScnProbSPi80.csv
$offdelim
/;

Parameter ShortQuant(allt,bs)    uppbound of shortage in blocks /
$ondelim
$include ShortQuantLarge.csv
$offdelim
/;

Parameter GenMax(isupply,g)    generator max energy and reserve capacity bid /

201.1U20       100
201.2U20       100
218.1U400       400
223.U155       155
223.U197       70
223.U350       350
/;

Parameter MinDnT(isupply,g)    generator min down time  /
201.1U20   1
```

201.2U20   2

218.1U400 0

223.U155   3

223.U197   0

223.U350   5

* change from 3 to 1 for 1u20 and 2u20, change from 3 to current value for u155 and from 5 to current value for U350

/;

Parameter MinUpT(isupply,g)   generator min up time /

201.1U20   1

201.2U20   2

218.1U400 0

223.U155   3

223.U197   0

223.U350   5

/;

Parameter ColdStartT(isupply,g)   cold start time   /

201.1U20   7

201.2U20   7

218.1U400 23

223.U155   11

223.U197   0

223.U350   10

/;

Parameter RampUpLimit(isupply,g)   ramp up rate*10 minutes   /

201.1U20   80

201.2U20   40

218.1U400 200

```
223.U155   75
223.U197   70
223.U350   150
/;

Parameter RampDnLimit(isupply,g)    rampdown rate*10 minutes    /
201.1U20   80
201.2U20   40
218.1U400  200
223.U155   75
223.U197   70
223.U350   150
/;

Parameter MSL(isupply,g)    Min stable level   /
201.1U20   10
201.2U20   10
218.1U400  100
223.U155   15
223.U197   10
223.U350   20
/;
Parameter SDR(isupply,g)    shut down ramp limit   /
201.1U20   20
201.2U20   10
218.1U400  100
223.U155   15
223.U197   10
223.U350   20
/;
```

```
Parameter SUR(isupply,g)    start up ramp limit   /
201.1U20   20
201.2U20   10
218.1U400  100
223.U155   15
223.U197   10
223.U350   20
/;

Parameter CNoLoad(isupply,g)    No-load costs   /
201.1U20   100
201.2U20   100
218.1U400  80
223.U155   80
223.U197   80
223.U350   80
/;

Parameter StartUpC(isupply,g,tao)       start-up costs /
$ondelim
$include StartUpCost197.csv
$offdelim
/;
Parameter isoqe(isupply,g,b,alls,t);
isoqe(isupply,g,b,alls,t)=0;
Parameter isow(isupply,g,alls,t);
isow(isupply,g,alls,t)=0;
Parameter isoz(isupply,g,alls,t);
isoz(isupply,g,alls,t)=0;
Parameter shedding(idemand,alls,t);
```

```
shedding(idemand,alls,t)=0;
Parameter resshortage(alls,t,bs);
resshortage(alls,t,bs)=0;
Parameter isototalqe(isupply,g,alls,t);
isototalqe(isupply,g,alls,t)=0;
parameter isoqr(isupply,g,alls,t);
isoqr(isupply,g,alls,t)=0;

parameter isoqsb(alls,t,bs);
isoqsb(alls,t,bs)=0;
Variables
        GenCost              expected gen cost($)
        lsGenCost            with load shedding($)
        GenCosttotal         total gen costs
        GenCosttotalcplex    total gen costs starting from a good starting point
        scencost(a)          cost of each scenario
        theta(i,alls,allt)   theta at bus i (voltage angle in radians)
        shortcost            cost of shortage
        energycost           costs related to energy
        loadshedding         load loss cost
        startupcost          startup cost in total
        noloadcost           no-load cost in total

        qeij(i,j,alls,allt)  power on line ij;

Positive variables
        qe(isupply,g,b,alls,allt)        output supplied by generators from block b in s during t(MW)
        totalqe(isupply,g,alls,allt)     total output supplied by generator in s during t(MW)
        qr(isupply,g,alls,allt)          reserve capacity available at igall in s during t for type m(MW)
         qs(alls,allt)                   reserve shortage in s during t for type m(MW)
```

z(isupply,g,alls,allt)	linearized variables
qsb(alls,allt,bs)	reserve shortage in different blocks(MW)
demandvar(idemand,alls,allt)	demand changes
supplygap(idemand,alls,t)	lost load;

Binary variables

w(isupply,g,alls,allt)	binary variables to represent the committment state of generators;

Equations

costls	cost with load shedding
offerquantub(isupply,g,b,alls,allt)	upper bound of energy quantity in each block in each offer
shortagequantub(alls,allt,bs)	upper bound of shortage in each block
MSLconstraint(isupply,g,alls,allt)	min stable level constraint
sreserve(isupply,gf,alls,allt)	spinning reserve supplied by online generators
sreserve1(isupply,gs,alls,allt)	spinning reserve from slow units
powerflows01(isupply,alls,allt)	supply node balance 01 without line outage
lspowerflowd(idemand,alls,allt)	with load shedding
scenariolink(isupply,g,alls,allt)	link between each scenario over time period t
powerlimit(i,j,alls,allt)	power limit over transmission lines without outage
reserverequirement(alls,allt)	system reserve requirement
minreserve(alls,allt)	minimum reserve requirement
rampupbetween(isupply,g,alls,allt)	ramp up limit between time periods
rampdnbetween(isupply,g,alls,allt)	ramp down limit between time periods
mindntime(isupply,g,alls,allt,k)	min down time
minuptime(isupply,g,alls,allt,k)	min up time
logic(isupply,g,alls,allt)	committment state vs generator outages
linearized(isupply,g,alls,allt,tao)	linearized constraints
u400mustrun(isupply,gr,alls,allt)	u400must run
reservelimit01(isupply,gs,alls,allt)	reserve limit for slow units

```
reservelimit02(isupply,gf,alls,allt)          reserve limit for fast units
initial(i,g,alls,allt)                        initial states of slow units
gapub1(isupply,g,idemand,alls,t)              upper bound of supply gap;




costls..    GenCost
            =e= sheddingcost*sum(idemand, sum(s, sum(t, Pi(s)*(supplygap(idemand,s,t)))))
                +sum(t,sum(s,Pi(s)*sum(bs,CS(bs)*qsb(s,t,bs))))
                +sum(t,sum(s,Pi(s)*sum(isupply, sum(g$igall(isupply,g),sum(b,
                        Cgen(isupply,g,b,t)*qe(isupply,g,b,s,t))))))
                +sum(t,sum(s,Pi(s)*sum((isupply,g)$igall(isupply,g),z(isupply,g,s,t))))
   +sum(t,sum(s,Pi(s)*sum((isupply,g)$igall(isupply,g),CNoLoad(isupply,g)*w(isupply,g,s,t))))
                -sum(t, sum(s,Pi(s)*sum((isupply,g)$igall(isupply,g),0.0001*qr(isupply,g,s,t))));


offerquantub(isupply,g,b,s,t)$igall(isupply,g)..      qe(isupply,g,b,s,t)=l=
                                                      GenQuant(isupply,g,b)*w(isupply,g,s,t);
shortagequantub(s,t,bs)..                        qsb(s,t,bs) =l= ShortQuant(t,bs);
MSLconstraint(isupply,g,s,t)$igall(isupply,g)..
                                           MSL(isupply,g)*w(isupply,g,s,t)=l=sum(b,qe(isupply,g,b,s,t));
sreserve(isupply,gf,s,t)$(GenMax(isupply,gf)>0)..
          sum(b,qe(isupply,gf,b,s,t))+qr(isupply,gf,s,t)=l=GenMax(isupply,gf)*mu(isupply,gf,s,t);
sreserve1(isupply,gs,s,t)$(GenMax(isupply,gs)>0)..
          sum(b,qe(isupply,gs,b,s,t))+qr(isupply,gs,s,t)=l=GenMax(isupply,gs)*w(isupply,gs,s,t);
powerflows01(isupply,s,t)..          sum(g$(GenMax(isupply,g)>0),
          sum(b,qe(isupply,g,b,s,t)))=e= sum(j$((U(isupply,j)>0 or
          U(j,isupply)>0)),(vi(isupply,j,s,t)+vi(j,isupply,s,t))*(BB(isupply,j)+BB(j,isupply))*(theta(isup
          ply,s,t)-theta(j,s,t)))*1000;
lspowerflowd(idemand,s,t)..
```

```
-Demand(idemand,t)*demandper +supplygap(idemand,s,t)=e= sum(j$((U(idemand,j)>0
or U(j,idemand)>0)),(vi(idemand,j,s,t)+vi(j,idemand,s,t))*(BB(idemand,j)
+BB(j,idemand))*(theta(idemand,s,t)-theta(j,s,t)))*1000;

gapub1(isupply,g,idemand,s,t)$(mu(isupply,g,s,t)=1)..

        supplygap(idemand,s,t)=l= demandper*Demand(idemand,t);

scenariolink(isupply,g,s,t)$(igall(isupply,g) and mu(isupply,g,s,t)=1)..
        sum(sdummy,link(s,sdummy,t)*Pi(sdummy)*sum(b,qe(isupply,g,b,sdummy,t)))-
        sum(sdummy,link(s,sdummy,t)*Pi(sdummy))*sum(b,qe(isupply,g,b,s,t))=e=0;

powerlimit(i,j,s,t)$(U(i,j)>0 or U(j,i)>0)..

        (vi(i,j,s,t)+vi(j,i,s,t))*(BB(i,j)+BB(j,i))*(theta(i,s,t)-theta(j,s,t))*1000=l=(U(i,j)+U(j,i));

reserverequirement(s,t)..

        sum((isupply,gs)$(GenMax(isupply,gs)>0), qr(isupply,gs,s,t))+
        sum((isupply,gf)$(GenMax(isupply,gf)>0), qr(isupply,gf,s,t))+sum(bs,qsb(s,t,bs))
        =g=Qrequire;

minreserve(s,t)..

        sum((isupply,gs)$(GenMax(isupply,gs)>0), qr(isupply,gs,s,t))+
        sum((isupply,gf)$(GenMax(isupply,gf)>0), qr(isupply,gf,s,t))=g= MinRreq;

rampdnbetween(isupply,g,s,t)$(igall(isupply,g) and ord(t) ge 2 and mu(isupply,g,s,t)=1)..
        sum(b,qe(isupply,g,b,s,t-1))-sum(b,qe(isupply,g,b,s,t)) =l=
        RampDnLimit(isupply,g)*w(isupply,g,s,t)+SDR(isupply,g)*(1-w(isupply,g,s,t));

rampupbetween(isupply,g,s,t)$(igall(isupply,g) and ord(t) ge 2 and mu(isupply,g,s,t)=1)..
        sum(b,qe(isupply,g,b,s,t))-sum(b,qe(isupply,g,b,s,t-1)) =l=
        RampUpLimit(isupply,g)*w(isupply,g,s,t-1)+SUR(isupply,g)*(1-w(isupply,g,s,t-1));

mindntime(isupply,g,s,t,k)$(not gr(g) and (MinDnT(isupply,g) ge 2) and  (mu(isupply,g,s,k)>0)
and (ord(t) ge 1) and igall(isupply,g) and (ord(k) ge ord(t)+1) and (ord(k) le Tmax) and (ord(k) le
ord(t)+MinDnT(isupply,g)-1))..

        w(isupply,g,s,t-1)-w(isupply,g,s,t)=l=1-w(isupply,g,s,k);

minuptime(isupply,g,s,t,k)$(not gr(g) and (MinUpT(isupply,g) ge 2) and (mu(isupply,g,s,k)>0)
and igall(isupply,g) and (ord(t) ge 1) and (ord(k) ge ord(t)+1) and (ord(k) le Tmax) and (ord(k) le
ord(t)+MinUpT(isupply,g)-1))..

        w(isupply,g,s,t)-w(isupply,g,s,t-1)=l=w(isupply,g,s,k);

logic(isupply,g,s,t)$igall(isupply,g)..                w(isupply,g,s,t)=l=mu(isupply,g,s,t);

u400mustrun(isupply,gr,s,t)$igall(isupply,gr)..        w(isupply,gr,s,t)=e=mu(isupply,gr,s,t);
```

```
linearized(isupply,g,s,t,tao)$(igall(isupply,g) and ord(tao) le ColdStartT(isupply,g))..
                  z(isupply,g,s,t)=g= StartUpC(isupply,g,tao)*(w(isupply,g,s,t)-sum(k$(ord(k) le
                  ColdStartT(isupply,g) and ord(k) lt ord(t)), w(isupply,g,s,t-ord(k))));
reservelimit01(isupply,gs,s,t)$igall(isupply,gs)..
                  qr(isupply,gs,s,t)=l= RampUpLimit(isupply,gs);
reservelimit02(isupply,gf,s,t)$igall(isupply,gf)..
                 qr(isupply,gf,s,t)=l= RampUpLimit(isupply,gf);
initial(isupply,gs,s,t)$(igall(isupply,gs) and not igall('201','2U20') and ord(t)=1 and
mu(isupply,gs,s,t)=1)..
                  w(isupply,gs,s,t)=e=1;


Model multimodel/all/;


option iterlim = 50000000;
OPTION RESLIM = 18000;
option limrow = 0;
option limcol = 0;
option solprint = off;
option sysout = off;
option mip=cplex;


if(timeblock=2,
  multimodel.optcr=0.0001;
elseif(timeblock=1),
  multimodel.optcr=0.0001;
elseif(timeblock=3),
  multimodel.optcr=0.0001;
elseif(timeblock=4),
  multimodel.optcr=0.0001;
elseif(timeblock=6),
  multimodel.optcr=0.0001;
```

```
elseif(timeblock=8),
  multimodel.optcr=0.0001;
elseif(timeblock=12),
  multimodel.optcr=0.0001;
elseif(timeblock=23),
  multimodel.optcr=0.000;
);
scalar  contingencies/3/;
scalar upperscen/70/;
scalar lowerscen/0/;
lowerscen=upperscen-timeblock*contingencies+1;
if(lowerscen<0,
  lowerscen=1;
 );
scalar lowertime/0/;
lowertime=timeblock;
scalar scenblock/0/;
scenblock=contingencies*timeblock;
scalar  backtime/0/;
backtime=timeblock-1;
theta.lo(i,alls,allt) = -0.05;
theta.up(i,alls,allt) =  0.05;
scalar  eachscen/1/;
scalar  basetime/1/;
scalar uppertime/24/;
scalar  backupperscen/0/;
scalar  backlowerscen/2/;
file fcpx Cplex Option file / cplex1.opt /;
multimodel.optfile = 1;
```

```
*****solve the scenario problems including base scenario and first N-worst bundles of scenarios
scalar counter/0/;
*****solve the MIP problems using scenario rolling heuristic algorithm
loop(iter,
        counter=counter+1;


        s(alls)$((ord(alls)>=lowerscen) and (ord(alls)<=upperscen))=yes;
         s('1')=yes;
        sdummy(alls)$((ord(alls)>=lowerscen) and (ord(alls)<=upperscen))=yes;
        if((counter gt 1),
        qe.fx(isupply,g,b,s,t)$(link('1',s,t)=1 and basetime<=ord(t) and ord(t)<=(lowertime-
timeblock) and mu(isupply,g,s,t)=0)= isoqe(isupply,g,b,'1',t);
        w.fx(isupply,g,s,t)$( link('1',s,t)=1 and basetime<=ord(t) and ord(t)<=(lowertime-timeblock)
and mu(isupply,g,s,t)=0)=isow(isupply,g,'1',t);
        qr.fx(isupply,g,s,t)$( link('1',s,t)=1 and basetime<=ord(t) and ord(t)<=(lowertime-timeblock)
and mu(isupply,g,s,t)=0)=isoqr(isupply,g,'1',t);
        z.fx(isupply,g,s,t)$( link('1',s,t)=1 and basetime<=ord(t) and ord(t)<=(lowertime-timeblock)
and mu(isupply,g,s,t)=0)=isoz(isupply,g,'1',t);
        qsb.fx(s,t,bs)$( link('1',s,t)=1 and basetime<=ord(t) and ord(t)<=(lowertime-
timeblock))=isoqsb('1',t,bs);
        );
         Solve multimodel using mip minimizing GenCost;
         display multimodel.modelstat;
         tsolve=tsolve+multimodel.resusd;
         display tsolve;
         if((multimodel.modelstat ne 8 and multimodel.modelstat ne 1),


            s(alls)$((ord(alls)>=lowerscen) and (ord(alls)<=upperscen) or (ord(alls)=1) )=no;
            sdummy(alls)$((ord(alls)>=lowerscen) and (ord(alls)<=upperscen) or (ord(alls)=1))=no;
          abort "we come across an infeasible solution";
```

```
        elseif((multimodel.modelstat = 8 or multimodel.modelstat = 1) and (upperscen-lowerscen)
>=(scenblock-1) and (scenblock <>2)),

            qe.fx(isupply,g,b,'1',t)$(basetime<=ord(t) and ord(t)<=lowertime)=qe.l(isupply,g,b,'1',t);

            totalqe.fx(isupply,g,'1',t)$(basetime<=ord(t) and
ord(t)<=lowertime)=sum(b,qe.l(isupply,g,b,'1',t));

            isoqe(isupply,g,b,'1',t)$(basetime<=ord(t) and
ord(t)<=lowertime)=qe.l(isupply,g,b,'1',t);

            w.fx(isupply,g,'1',t)$( basetime<=ord(t) and ord(t)<=lowertime)=w.l(isupply,g,'1',t);

            isow(isupply,g,'1',t)=w.l(isupply,g,'1',t);

            qr.fx(isupply,g,'1',t)$(basetime<=ord(t) and ord(t)<=lowertime)=qr.l(isupply,g,'1',t);

            z.fx(isupply,g,'1',t)$(basetime<=ord(t) and ord(t)<=lowertime)=z.l(isupply,g,'1',t);

            qsb.fx('1',t,bs)$(basetime<=ord(t) and ord(t)<=lowertime)=qsb.l('1',t,bs);

            isoqr(isupply,g,'1',t)$(basetime<=ord(t) and ord(t)<=lowertime)=qr.l(isupply,g,'1',t);

            isoz(isupply,g,'1',t)$(basetime<=ord(t) and ord(t)<=lowertime)=z.l(isupply,g,'1',t);

            isoqsb('1',t,bs)$(basetime<=ord(t) and ord(t)<=lowertime)=qsb.l('1',t,bs);


            s(alls)$((ord(alls)>=lowerscen) and (ord(alls)<=upperscen))=no;

            sdummy(alls)$((ord(alls)>=lowerscen) and (ord(alls)<=upperscen))=no;

            );

             lowertime=lowertime+timeblock;

             scenblock=contingencies*timeblock;


              upperscen=max(1,lowerscen-1);

              lowerscen=max(1,lowerscen-scenblock);


 );

*
_______________________________________________________________________________
__________________________________________

*output files
```

```
file lsnolap73_e generator energy quantities /lsnolap73_e.txt/;
put lsnolap73_e;
lsnolap73_e.pc=5;

loop(isupply,
put 'Report for supply buses ' , isupply.tl
put @25 '----------------------------------- Time Period ---------------------------------------' /;
put @10;
loop(t, put t.tl);
    put /;
        loop(g$igall(isupply,g),
              put g.tl;
              put /;
            loop(alls,
                put alls.tl;
                 loop(allt,
                      put sum(b,qe.l(isupply,g,b,alls,allt)):10:3);
put /;
)));

file lsnolap73_r generator re quantities /lsnolap73_r.txt/;
put lsnolap73_r;
lsnolap73_r.pc=5;
loop(isupply,
put 'Report for supply buses ' , isupply.tl
put @25 '----------------------------------- Time Period ---------------------------------------' /;
put @10;
loop(t, put t.tl);
    put /;
loop(g$igall(isupply,g),
```

```
      put g.tl;
      put /;
  loop(alls,
    put alls.tl;
        loop(allt,
             put qr.l(isupply,g,alls,allt):10:3);
put /;
)));

file lsnolap73_w generator comm states /lsnolap73_w.txt/;
put lsnolap73_w;
lsnolap73_w.nd=0;
lsnolap73_w.pc=5;
loop(isupply,
     loop(g$(igall(isupply,g)),
          loop((alls,allt),
               put isupply.tl g.tl alls.tl allt.tl w.l(isupply,g,alls,allt);
               put /;

               );
          );
     );

file lsnolap73short generator re shortage /lsnolap73short.txt/;
put lsnolap73short;
lsnolap73short.pc=5;
lsnolap73short.nd=3;
lsnolap73short.nr=1;
loop(t, put t.tl);
    put /;
```

```
            loop(alls,
                put alls.tl ;
                 loop(allt,
                      put sum(bs,qsb.l(alls,allt,bs)) :10:3
                      );
                      put /;
              );

file lsnolap73powergap generator load shedding /lsnolap73powergap.txt/;
put lsnolap73powergap;
lsnolap73powergap.pc=5;
lsnolap73powergap.nd=3;
lsnolap73powergap.nr=1;
loop(t, put t.tl:5);
    put /;
        loop(alls,
             put alls.tl;
                loop(allt,
                      put sum(idemand,supplygap.l(idemand,alls,allt)):10:3
                      );
                      put /;

              );



file lsnolap73scencost cost in each scenario /lsnolap73scencost.txt/;
put lsnolap73scencost;
lsnolap73scencost.pc=5;
loop(alls,
      put alls.tl;
```

```
    scencost.l(alls)= sheddingcost*sum(idemand, sum(a$(ord(a)=ord(alls)), sum(t,
Pi(a)*(supplygap.l(idemand,a,t)))))+sum(allt,sum(a$(ord(a)=ord(alls)),Pi(a)*sum(bs,CS(bs)*qsb.l
(a,allt,bs))))+sum(allt,sum(a$(ord(a)=ord(alls)),Pi(a)*sum((isupply,g)$igall(isupply,g),z.l(isupply,g
,a,allt)+sum(b,
Cgen(isupply,g,b,allt)*qe.l(isupply,g,b,a,allt))+CNoLoad(isupply,g)*w.l(isupply,g,a,allt))));

    put scencost.l(alls);

    put /;

   );


display tsolve;

s(alls)=yes;

sdummy(alls)=yes;

parameter GenCost01;


GenCosttotal.l= sheddingcost*sum(idemand, sum(alls, sum(t,
Pi(alls)*(supplygap.l(idemand,alls,t)))))

            +sum(allt,sum(alls,Pi(alls)*sum(bs,CS(bs)*qsb.l(alls,allt,bs))))

            +sum(allt,sum(alls,Pi(alls)*sum((isupply,g)$igall(isupply,g),z.l(isupply,g,alls,allt)

            +sum(b, Cgen(isupply,g,b,allt)*qe.l(isupply,g,b,alls,allt))

            +CNoLoad(isupply,g)*w.l(isupply,g,alls,allt))));

Pi(alls)$(ord(alls)=1)=1;

GenCost01= sheddingcost*sum(idemand, sum(alls$(ord(alls)=1), sum(t,
Pi(alls)*(supplygap.l(idemand,alls,t)))))

            +sum(allt,sum(alls$(ord(alls)=1),Pi(alls)*sum(bs,CS(bs)*qsb.l(alls,allt,bs))))

+sum(allt,sum(alls$(ord(alls)=1),Pi(alls)*sum((isupply,g)$igall(isupply,g),z.l(isupply,g,alls,allt)

            +sum(b, Cgen(isupply,g,b,allt)*qe.l(isupply,g,b,alls,allt))

            +CNoLoad(isupply,g)*w.l(isupply,g,alls,allt))));


display GenCosttotal.l, GenCost01;


loadshedding.l=sheddingcost*sum(idemand, sum(s, sum(t, Pi(s)*(supplygap.l(idemand,s,t)))));
```

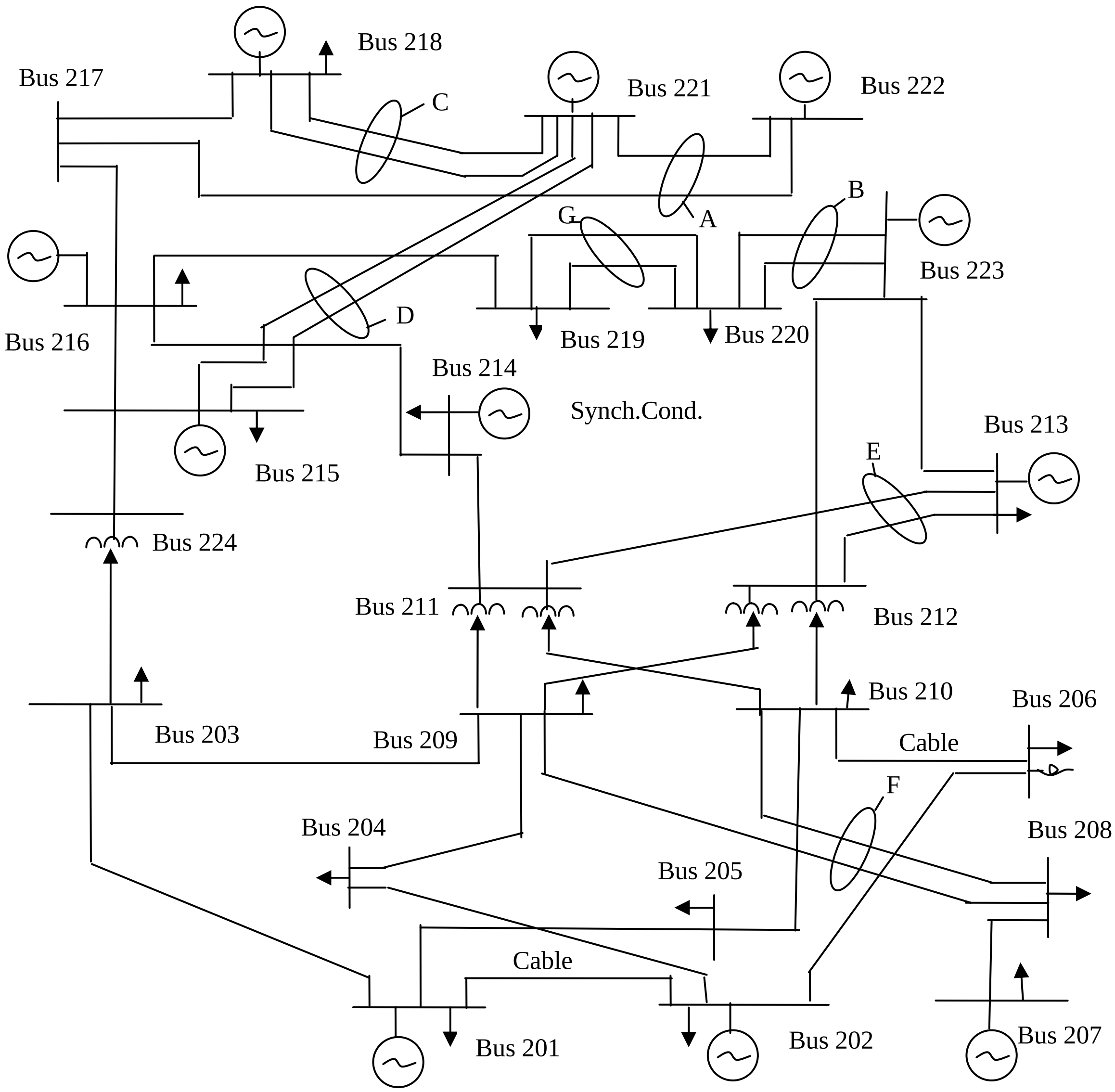
```
 isplay loadshedding.l;
shortcost.l=sum(allt,sum(alls,Pi(alls)*sum(bs,CS(bs)*qsb.l(alls,allt,bs))));
 display shortcost.l;
energycost.l=sum(allt,sum(alls,Pi(alls)*sum((isupply,g)$igall(isupply,g),sum(b,
Cgen(isupply,g,b,allt)*qe.l(isupply,g,b,alls,allt)))));

display energycost.l;
 startupcost.l=sum(allt,sum(alls,Pi(alls)*sum((isupply,g)$igall(isupply,g),z.l(isupply,g,alls,allt))));
display startupcost.l;

noloadcost.l=sum(allt,sum(alls,Pi(alls)*sum((isupply,g)$igall(isupply,g),CNoLoad(isupply,g)*w.l(i
supply,g,alls,allt))));
display noloadcost.l;
sdummy(alls)=no;
```

Appendix B The Topology of the Power System in the Large-Scale Example

(Area B, Reliability Test System Task Force, 1999)



Appendix C Data in the Large-Scale Example

Table C-1 Generator data

Node	Unit	Max. Output (MW)	Min. Output (MW)	Min Down Time (Hour)	Min. Up Time (Hour)	Cold Start-up (Hour)	Ramp Up/Down Limit (MW)	Start Up/Down Limit (MW)	Spinning reserve limit (MW)
201	1U20	20	10	1	1	0	7	10	30
201	2U20	20	10	1	1	0	7	10	30
201	1U76	76	10	4	8	8	26.6	10	20
201	2U76	76	10	4	8	8	26.6	10	20
202	3U20	20	10	1	1	0	7	10	30
202	4U20	20	10	1	1	0	7	10	30
202	3U76	76	10	4	8	8	26.6	10	20
202	4U76	76	10	4	8	8	26.6	10	20
207	1U100	100	15	8	8	8	35	15	70
207	2U100	100	15	8	8	8	35	15	70
207	3U100	100	15	8	8	8	35	15	70
213	1U197	197	40	10	12	7	68.95	40	30
213	2U197	197	40	10	12	7	68.95	40	30
213	3U197	197	40	10	12	7	68.95	40	30
215	1U12	12	2	2	4	4	4.2	2	10
215	2U12	12	2	2	4	4	4.2	2	10
215	3U12	12	2	2	4	4	4.2	2	10
215	4U12	12	2	2	4	4	4.2	2	10
215	5U12	12	2	2	4	4	4.2	2	10
215	1U155	155	25	8	8	8	54.25	25	30
216	2U155	155	25	8	8	8	54.25	25	30
218	1U400	400	80	48	48	48	140	200	200
221	2U400	400	80	48	48	48	140	200	200
222	1U50	50	45	0	0	0	17.5	0	10
222	2U50	50	45	0	0	0	17.5	0	10
222	3U50	50	45	0	0	0	17.5	0	10
222	4U50	50	45	0	0	0	17.5	0	10
222	5U50	50	45	0	0	0	17.5	0	10
222	6U50	50	45	0	0	0	17.5	0	10
223	3U155	155	25	8	8	8	54.25	25	30
223	4U155	155	25	8	8	8	54.25	25	30
223	1U350	350	100	12	12	12	122.5	100	40

Table C-2 Generation offers of generating units

Unit	Generation offer	
	Price($/MWh)	Quantity (MW)
1U20 – 4U20	29.577	15.800
	30.417	0.200
	42.816	3.800
	43.281	0.200
1U76-4U76	9.9180	15.200
	10.249	22.800
	10.680	22.800
	11.257	15.200
1U100-3U100	19.200	25.000
	20.316	25.000
	21.218	30.000
	22.126	20.000
1U197-3U197	19.200	68.950
	20.316	49.250
	21.218	39.400
	22.126	39.400
1U12-5U12	19.200	2.400
	20.316	3.600
	21.218	3.600
	22.126	2.400
1U155-3U155	9.918	54.250
	10.249	38.750
	10.68	31.000
	11.257	31.000
1U400-2U400	5.309	100.000
	5.379	100.000
	5.526	120.000
	5.663	80.000
1U50-6U50	5.309	45.000
	5.379	2.000
	5.526	2.000
	5.663	1.000
1U350	10.082	140.000
	10.675	87.500
	11.093	52.500
	11.722	70.000

Table C-3 Other relevant costs

Unit	cold start time (Hour)	cold start cost ($)	No Load Cost ($/Hour)
1U20-4U20	0	15	300
1U76-4U76	12	720	100
1U100-3U100	8	1380	80
1U197-3U197	13	1400	40
1U12-5U12	7	130	20
1U155-3U155	12	680	250
1U400-2U400	0	N/A	400
1U350	10	5300	1000

Table C-4 Line data

Node	Node	Susceptance (S)	Line Capacity (MW)
201	202	21.692	175
201	203	17.544	175
201	205	43.478	175
202	204	29.412	175
202	206	19.231	175
203	209	31.250	175
204	209	35.714	175
205	209	41.667	175
206	209	0.407	175
207	208	58.824	175
208	209	22.222	175
209	213	10.000	400
209	214	11.364	400
209	223	4.926	500
213	223	5.495	500
214	216	12.195	500
215	216	27.778	500
215	221	9.709	500
215	203	9.174	500
216	217	18.182	500
216	219	20.408	500
217	218	33.333	500
217	222	4.525	500
218	221	18.182	500
219	220	12.048	500
220	223	21.739	500
221	222	7.042	500

Table C-5 Demand data

Node / Period	201	202	203	204	205	206	207	208
1	69.12	62.08	115.20	47.36	45.44	87.04	80.00	109.44
2	64.80	58.20	108.00	44.40	42.60	81.60	75.00	102.60
3	62.64	56.26	104.40	42.92	41.18	78.88	72.50	99.18
4	60.48	54.32	100.80	41.44	39.76	76.16	70.00	95.76
5	60.48	54.32	100.80	41.44	39.76	76.16	70.00	95.76
6	62.64	56.26	104.40	42.92	41.18	78.88	72.50	99.18
7	69.12	62.08	115.20	47.36	45.44	87.04	80.00	109.44
8	82.08	73.72	136.80	56.24	53.96	103.36	95.00	129.96
9	93.96	84.39	156.60	64.38	61.77	118.32	108.75	148.77
10	102.60	92.15	171.00	70.30	67.45	129.20	118.75	162.45
11	106.92	96.03	178.20	73.26	70.29	134.64	123.75	169.29
12	108.00	97.00	180.00	74.00	71.00	136.00	125.00	171.00
13	106.92	96.03	178.20	73.26	70.29	134.64	123.75	169.29
14	108.00	97.00	180.00	74.00	71.00	136.00	125.00	171.00
15	108.00	97.00	180.00	74.00	71.00	136.00	125.00	171.00
16	104.76	94.09	174.60	71.78	68.87	131.92	121.25	165.87
17	103.68	93.12	172.80	71.04	68.16	130.56	120.00	164.16
18	103.68	93.12	172.80	71.04	68.16	130.56	120.00	164.16
19	100.44	90.21	167.40	68.82	66.03	126.48	116.25	159.03
20	99.36	89.24	165.60	68.08	65.32	125.12	115.00	157.32
21	99.36	89.24	165.60	68.08	65.32	125.12	115.00	157.32
22	100.44	90.21	167.40	68.82	66.03	126.48	116.25	159.03
23	93.96	84.39	156.60	64.38	61.77	118.32	108.75	148.77
24	77.76	69.84	129.60	53.28	51.12	97.92	90.00	123.12

Table C-5 Demand data (Con't)

Node / Period	209	213	214	215	216	218	219	220
1	236.80	169.60	124.16	202.88	64.00	213.12	115.84	81.92
2	222.00	159.00	116.40	190.20	60.00	199.80	108.60	76.80
3	214.60	153.70	112.52	183.86	58.00	193.14	104.98	74.24
4	207.20	148.40	108.64	177.52	56.00	186.48	101.36	71.68
5	207.20	148.40	108.64	177.52	56.00	186.48	101.36	71.68
6	214.60	153.70	112.52	183.86	58.00	193.14	104.98	74.24
7	236.80	169.60	124.16	202.88	64.00	213.12	115.84	81.92
8	281.20	201.40	147.44	240.92	76.00	253.08	137.56	97.28
9	321.90	230.55	168.78	275.79	87.00	289.71	157.47	111.36
10	351.50	251.75	184.30	301.15	95.00	316.35	171.95	121.60
11	366.30	262.35	192.06	313.83	99.00	329.67	179.19	126.72
12	370.00	265.00	194.00	317.00	100.00	333.00	181.00	128.00
13	366.30	262.35	192.06	313.83	99.00	329.67	179.19	126.72
14	370.00	265.00	194.00	317.00	100.00	333.00	181.00	128.00
15	370.00	265.00	194.00	317.00	100.00	333.00	181.00	128.00
16	358.90	257.05	188.18	307.49	97.00	323.01	175.57	124.16
17	355.20	254.40	186.24	304.32	96.00	319.68	173.76	122.88
18	355.20	254.40	186.24	304.32	96.00	319.68	173.76	122.88
19	344.10	246.45	180.42	294.81	93.00	309.69	168.33	119.04
20	340.40	243.80	178.48	291.64	92.00	306.36	166.52	117.76
21	340.40	243.80	178.48	291.64	92.00	306.36	166.52	117.76
22	344.10	246.45	180.42	294.81	93.00	309.69	168.33	119.04
23	321.90	230.55	168.78	275.79	87.00	289.71	157.47	111.36
24	266.40	190.80	139.68	228.24	72.00	239.76	130.32	92.16

Appendix D Data in the Large-Scale Example

Table D-1 Generation plan of generating units in a scenario (1U350 is down from period 9)

Hour	Generators at node 201				Generators at node 202				Generator at node 221
	1U20	2U20	1U76	2U76	3U20	4U20	3U76	4U76	2U400
1	0.000	0.000	60.800	60.800	0.000	0.000	60.800	60.800	400.000
2	0.000	0.000	38.000	38.000	0.000	0.000	60.800	56.112	400.000
3	0.000	0.000	38.000	38.000	0.000	0.000	38.000	38.000	400.000
4	0.000	0.000	38.000	29.403	0.000	0.000	38.000	38.000	400.000
5	0.000	0.000	38.000	36.349	0.000	0.000	38.000	38.000	400.000
6	0.000	0.000	38.000	38.000	0.000	0.000	38.000	38.000	400.000
7	0.000	0.000	56.000	55.508	0.000	0.000	60.800	60.800	400.000
8	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
9	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
10	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
11	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
12	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
13	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
14	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
15	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
16	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
17	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
18	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
19	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
20	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
21	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
22	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
23	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000
24	0.000	0.000	76.000	76.000	0.000	0.000	76.000	76.000	400.000

Table D-1 Generation plan of generating units in a scenario (1U350 is down from period 9) (Con't)

Hour	Generators node 207			Generators at node 213			Generators at node 223		
	1U100	2U100	3U100	1U197	2U197	3U197	3U155	4U155	1U350
1	15.000	15.000	15.000	40.000	40.000	40.000	54.250	72.248	140.000
2	15.000	15.000	15.000	40.000	40.000	40.000	54.250	54.250	100.000
3	15.000	15.000	15.000	40.000	40.000	40.000	54.250	43.372	100.000
4	15.000	15.000	15.000	40.000	40.000	40.000	25.000	25.000	100.000
5	15.000	15.000	15.000	40.000	40.000	40.000	25.000	25.000	100.000
6	15.000	15.000	15.000	40.000	40.000	40.000	51.180	54.250	100.000
7	15.000	15.000	15.000	40.000	40.000	40.000	69.750	75.900	140.000
8	15.000	15.000	19.960	49.250	45.150	49.250	100.750	108.504	227.500
9	44.960	50.000	50.000	118.200	114.100	118.200	155.000	155.000	0.000
10	52.500	50.000	50.000	167.000	167.000	167.000	155.000	155.000	0.000
11	80.000	80.000	80.000	167.000	167.000	175.500	155.000	155.000	0.000
12	80.000	87.000	80.000	167.000	197.000	167.000	155.000	155.000	0.000
13	80.000	80.000	80.000	167.000	167.000	175.500	155.000	155.000	0.000
14	80.000	87.000	80.000	167.000	197.000	167.000	155.000	155.000	0.000
15	85.000	80.000	80.000	169.000	197.000	167.000	155.000	155.000	0.000
16	50.000	79.500	80.000	167.000	167.000	167.000	155.000	155.000	0.000
17	80.000	50.000	50.000	167.000	167.000	167.000	155.000	155.000	0.000
18	65.000	50.000	66.000	167.000	167.000	167.000	155.000	155.000	0.000
19	30.000	50.000	35.810	167.000	167.000	167.000	155.000	155.000	0.000
20	37.320	30.000	30.000	167.000	167.000	167.000	155.000	155.000	0.000
21	37.320	30.000	30.000	167.000	167.000	167.000	155.000	155.000	0.000
22	30.000	35.810	50.000	167.000	167.000	167.000	155.000	155.000	0.000
23	30.000	30.000	30.000	137.900	137.900	149.700	145.000	155.000	0.000
24	15.000	15.000	15.000	68.950	68.950	80.750	125.000	130.461	0.000

Table D-1 Generation plan of generating units in a scenario (1U350 is down from period 9) (Con't)

Hour	Generators at node 215						Generators at node 216
	1U12	2U12	3U12	4U12	5U12	1U155	2U155
1	2.000	2.000	2.000	2.000	2.000	25.000	25.000
2	2.000	2.000	2.000	2.000	2.000	25.000	25.000
3	2.000	2.000	2.000	2.000	2.000	25.000	25.000
4	2.000	2.000	2.000	2.000	2.000	25.000	25.000
5	0.000	0.000	0.000	0.000	0.000	25.000	25.000
6	0.000	0.000	0.000	0.000	0.000	25.000	25.000
7	0.000	0.000	0.000	0.000	0.000	25.000	25.000
8	0.000	0.000	0.000	0.000	0.000	79.250	72.290
9	2.000	2.000	2.000	2.000	2.000	133.500	126.540
10	6.000	6.000	6.000	6.000	6.000	155.000	155.000
11	9.600	9.600	9.600	9.600	9.600	155.000	155.000
12	9.600	9.600	9.600	9.600	9.600	155.000	155.000
13	9.600	9.600	9.600	9.600	9.600	155.000	155.000
14	9.600	9.600	9.600	9.600	9.600	155.000	155.000
15	9.600	9.600	9.600	9.600	9.600	155.000	155.000
16	6.000	6.000	6.000	6.000	6.000	155.000	155.000
17	6.000	6.000	7.000	6.000	6.000	155.000	155.000
18	6.000	6.000	6.000	6.000	6.000	155.000	155.000
19	2.000	2.000	2.000	2.000	2.000	155.000	155.000
20	2.000	2.000	2.000	2.000	2.000	155.000	145.188
21	2.000	2.000	2.000	2.000	2.000	155.000	145.188
22	2.000	2.000	2.000	2.000	2.000	155.000	155.000
23	2.000	2.000	2.000	2.000	2.000	125.000	125.000
24	0.000	0.000	0.000	0.000	0.000	70.750	70.750

Table D-1 Generation plan of generating units in a scenario (1U350 is down from period 9) (Con’t)

Hour	Generators at node 222						Generators at node 218
	1U50	2U50	3U50	4U50	5U50	6U50	1U400
1	50.000	50.000	45.000	49.302	50.000	45.000	400.000
2	48.588	45.000	45.000	50.000	45.000	50.000	400.000
3	45.000	45.000	48.378	45.000	45.000	50.000	400.000
4	45.000	45.000	45.000	50.000	47.597	45.000	400.000
5	50.000	50.000	45.000	45.000	45.651	45.000	400.000
6	45.000	50.000	45.000	45.000	45.570	50.000	400.000
7	50.000	50.000	45.000	50.000	45.242	50.000	400.000
8	50.000	45.000	45.000	45.000	50.000	45.096	400.000
9	50.000	50.000	50.000	50.000	50.000	50.000	400.000
10	50.000	50.000	50.000	50.000	50.000	50.000	400.000
11	50.000	50.000	50.000	50.000	50.000	50.000	400.000
12	50.000	50.000	50.000	50.000	50.000	50.000	400.000
13	50.000	50.000	50.000	50.000	50.000	50.000	400.000
14	50.000	50.000	50.000	50.000	50.000	50.000	400.000
15	50.000	50.000	50.000	50.000	50.000	50.000	400.000
16	50.000	50.000	50.000	50.000	50.000	50.000	400.000
17	50.000	50.000	50.000	50.000	50.000	50.000	400.000
18	50.000	50.000	50.000	50.000	50.000	50.000	400.000
19	49.690	50.000	50.000	50.000	50.000	50.000	400.000
20	49.492	50.000	50.000	50.000	50.000	50.000	400.000
21	49.492	50.000	50.000	50.000	50.000	50.000	400.000
22	49.690	50.000	50.000	50.000	50.000	50.000	400.000
23	50.000	50.000	50.000	50.000	50.000	50.000	400.000
24	45.000	50.000	47.389	50.000	50.000	45.000	400.000

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