Generation Capacity Expansion Planning in Deregulated Electricity Markets

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
Abstract

With increasing demand of electric power in the context of deregulated electricity markets, a good strategic planning for the growth of the power system is critical for our tomorrow. There is a need to build new resources in the form of generation plants and transmission lines while considering the effects of these new resources on power system operations, market economics and the long-term dynamics of the economy. In deregulation, the exercise of generation planning has undergone a paradigm shift. The first stage of generation planning is now undertaken by the individual investors. These investors see investments in generation capacity as an increasing business opportunity because of the increasing market prices. Therefore, the main objective of such a planning exercise, carried out by individual investors, is typically that of long-term profit maximization.

This thesis presents some modeling frameworks for generation capacity expansion planning applicable to independent investor firms in the context of power industry deregulation. These modeling frameworks include various technical and financing issues within the process of power system planning. The proposed modeling frameworks consider the long-term decision making process of investor firms, the discrete nature of generation capacity addition and incorporates transmission network modeling. Studies have been carried out to examine the impact of the optimal investment plans on transmission network loadings in the long-run by integrating the generation capacity expansion planning framework within a modified IEEE 30-bus transmission system network.

The work assesses the importance of arriving at an optimal IRR at which the firm’s profit maximization objective attains an extremum value. The mathematical model is further improved to incorporate binary variables while considering discrete unit sizes, and subsequently to include the detailed transmission network representation. The proposed models are novel in the sense that the planning horizon is split into plan sub-periods so as to minimize the overall risks associated with long-term plan models, particularly in the context of deregulation.
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Dedication

This thesis is dedicated to my parents and teachers.
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1.1 Background

It is difficult to imagine our life in present times without the presence of electric power. Its forms and modes of usages are increasing day-by-day because it is the easiest and cheapest transportable form of energy. But as the demand of electric power keeps on increasing every day, a good strategic planning for the growth of the power system is critical for our tomorrow. There is a need to build new resources in the form of generation plants and transmission lines while considering the effects of these new resources on power system operations, market economics and the long-term dynamics of the economy.

With deregulation of the electricity market worldwide, the strategies for generation capacity expansion planning are not the same anymore. Earlier, generation planning was strongly correlated to future demand growth projections, reliability concerns of the system as a whole and minimization of the total investment costs. Such a planning exercise was typically undertaken by the central planner and the objective was minimization of the total system-wide plan costs while meeting the forecasted peak demand growth and energy demand growth. Associated constraint such as ensuring a specified system reliability level, would also be considered in the planning framework.

In the context of deregulated electricity markets, the exercise of generation planning has undergone a paradigm shift. The first stage of generation planning is now undertaken by the individual investors. These investors see investments in generation capacity as an increasing business opportunity because of the increasing market prices. Therefore, the main objective of such a planning exercise, carried out by individual investors, is typically that of long-term profit maximization. The main driving force of such a planning exercise is the forecast of electricity price trends and the return on investments [1], [2].

The second stage of the planning process is the responsibility of a central authority which receives the individual plans from all investors. Such an entity usually has access to information on available transmission resources and incorporates the Independent System Operator (ISO) specified security and reliability standards and guidelines. The outcome of the second stage is the approval or non-approval of the proposed investments and in some cases there may be iterative mechanisms between the two stages which provide the individual investors an opportunity to revise their investment offers
Fig. 1 shows the schematic overview of the process of generation planning, as discussed above, in the context of deregulation.

Some of the important issues that need to be taken into account by the central planning authority in its plan approval and coordination process are listed below:

- A long-term generation expansion plan should provide an acceptable level of reliability and meet the future demand growth of the power system taking into account all individual submission of investment proposals from independent parties,
- The plan should be able to convince governmental regulating agencies that it will improve the overall market efficiency,
- The final approved plan should not deteriorate the security of the transmission network,
- Generation expansion plan coordination carried out by the central planner should be in a way that it does not give rise to market power to any single entity,
- Uncertainties and risk factors arising from electricity market price volatilities, regulatory changes, demand growth, etc., need to be considered in the coordination process,
- It should be mentioned here that the main focus of the present thesis is to specifically examine and dwell upon the first stage of the planning process only. Therefore, in this thesis we will not discuss the issues of plan coordination or systems level generation expansion, rather the work will examine the generation expansion planning tasks from the perspective of an individual investor.
1.2 Literature Review

In this section a modest attempt is being made to examine a few pertinent research papers that have addressed the problem of generation expansion planning in the context of deregulation. It should be pointed out that this subject has not yet received much attention in the research literature, and hence the body of published works available in this subject is still not too large.

A study of market based planning is reported in [3] wherein coordination between generation and transmission expansion plans and ISO’s security assessment is carried out. The study provides locational signals to investors for the selection of new generation and transmission facilities. In this model, energy, transmission auction and capacity mechanism are correlated. It facilitates competition between generation and transmission resources.

An integrated analysis of generation capacity expansion and financial planning is presented in [4] where the financial constraints are modeled in the form of balance of in-flow and out-flow of funds for each year. An optimal investment planning study was reported in [1] incorporating uncertainties in demand and future electricity prices. Stochastic dynamic optimization was used to determine the optimal investments in new generation in both centralized and decentralized environment.

A non-cooperative game theoretic model for generation expansion planning is presented in [5] which uses a Cournot model of oligopoly behaviour. This work mainly focuses on generation expansion planning in a pool dominated electricity market. Research work carried out in [6] provides a study based on minimum cost assessment for system adequacy and operational economics of system expansion at different load growth levels.

Two well-known modeling approaches which study the competitive behavior of generation firms for investments in the medium-term in the context of deregulated electricity markets are the Cournot model and the Supply Function Equilibrium (SFE) models, as discussed in [7]. In the Cournot model, generation firms compete in terms of total capacity investment quantities which thereby affect the electricity market price to maximize their profits. In the SFE model, generation firms compete by offering supply curves that include the supply as well as the price. Both of these modeling approaches are based on the concept of Nash Equilibrium. Cournot models are simple in comparison to the SFE model because the Cournot model requires less computation, although such models are different from the exact functioning of the medium-term power market. Supply function models are complex in terms of computability, but these models closely represent the medium-term power market behavior.

Some newer optimization techniques such as expert systems, fuzzy logic, neural networks, analytic hierarchy process, network flow, decomposition method, simulated annealing, and genetic algorithms
are discussed in [8]. The merits and demerits of each of these techniques applied to solving generation expansion planning problems have been discussed.

The work in [9] examines and includes the uncertainties and risks associated in generation expansion planning, at two levels. The first uses a multiple criteria model for minimizing the environmental risk while the second level involves a risk analysis to cover a full range of uncertainties. The results show large reduction in risk with little increase in cost, and that incorporation of additional criteria produces much more flexible and efficient strategies.

The issues related to the responsibility of the generation capacity addition are discussed in [10] from the perspective of Swedish electric power system and its legislation. The study also discusses the issue of reserve capacity provision in power systems. It is stated in the paper that “since the future, in reality, is a sum of infinity short-term periods, the ISO can essentially be considered as responsible also for the future balance, i.e., capacity addition”. The work in [10] proposes that the load imbalance should be paid by the company that causes such imbalance.

A modeling framework for transmission expansion planning while considering the impact of generation expansion on congestion over a multi-year horizon is presented in [11]. A Benders decomposition approach is used to decompose transmission expansion planning into a master problem and two sub-problems representing security and optimal operation. It is concluded in [11] that potential generation investment decisions are important, because they help alleviate over-investment in transmission or non-optimal generation investments that may lead to increased total social cost in the planning horizon.

Murphy and Smeers [12] have examined three different models for generation capacity expansion-the first being the traditional case which considers a competitive equilibrium. The second model considers investments in an oligopolistic market where capacity is simultaneously build and sold in long-term contracts when there is no spot-market. The third model is a two-stage Cournot game where the first stage deals with investment decisions and the second stage with spot-market sales.

In a recent thesis [13] an optimization model based on real options theory is developed for devising investment strategies for a profit maximizing investor. The load growth is modeled as a stochastic variable and a stochastic dynamic programming algorithm is applied to solve the investment problem. An agent based model is proposed in [14] for generation expansion planning where a multi-agent simulation system is used to simulate the interactions between various agents in an electricity market.

An optimal generation resource planning model is presented in [15] which include expected level of revenue, transmission charges, load curtailment and expected level of system reliability. The model
considers volatility of electricity and fuel prices, various options for securing investment loans and transmission congestion costs as major incentives for adding generating capacity to power systems.

A security-based competitive generation resource planning model is proposed in [16] where the impact of transmission security is incorporated in a multi-genco framework. The solution approach is based on Lagrangian relaxation and Benders decomposition techniques.

Investment decisions under time-of-use rates and their relationship to power system planning has been examined in [17]. Optimal investment and prices under social welfare maximization are derived in the paper.

The development and experiences from the Nordic region with regard to investment incentives are discussed in [18]. The paper concludes that investment in generating capacity is a problem in competitive markets, where investors are exposed to high risks and uncertainty. The authors conclude that price elasticity of demand is important in an open market, and additional incentive mechanisms can be used to achieve capacity adequacy, when the market fails to provide sufficient signals for new investments.

1.3 Motivation and Objectives of the Research

This research seeks to develop a comprehensive long-term modeling framework for generation capacity expansion planning, specifically pertaining to independent investor firms in the context of deregulation. The model should be capable of incorporating the technical and economic parameters of investor firms and electricity market effects, in order to arrive at optimal investment decisions that seek to maximize the investor’s profit while determining the optimal internal rate of return (IRR). No earlier work has been reported in the literature that addresses the issue of how to find the best (or optimal) rate of return for investors.

In order to address the above, the overall modeling framework presented in this thesis is developed in three stages- the first stage model is relatively simple in nature wherein generation investments are considered to be continuous variables and the model is a linear programming model. A long-term plan horizon of 25-year is considered to examine how the model performs, when the investment cost recovery and the rate of return constraints are incorporated. A new approach is adopted in the modeling framework by splitting the plan horizon into plan sub-periods in order to minimize the overall risks associated with long-term planning models, particularly in the context of deregulation. A new approach to determining the optimal IRR is further presented, which can help the investor firms
to arrive at the optimal decisions on project approvals, when the optimal IRR meets a Minimum Acceptable Rate of Return (MARR) barrier.

It is important to consider the impact of environmental constraints on investment decisions of the investor. Similarly, it is important to examine the electricity market effects such as market price and fuel price scenarios on investment decisions, which have not been reported in the literature. In view of the above, annual emission constraints are included in the proposed planning models and their impact on the investment plans is examined. The work also considers the sensitivity of such emission constraints vis-à-vis the budgetary allocations of investor firms.

In order to render the proposed models more practical and realizable, there is a need to consider discrete unit sizes instead of treating the generation capacities as continuous variables, which requires introduction of binary decision variables. The second stage of model development addresses this aspect and a dynamic, mixed-integer linear programming model is developed wherein the selection of generation capacities are now considered in discrete sizes. While such modeling frameworks have been proposed by researchers, no investigation has been provided on how the IRR would be affected when discrete (binary) variables are present in the generation planning model. Furthermore, how the IRR so obtained, would compare with that obtained in the continuous case. This thesis presents a new perspective on this issue.

There is also a need to examine how generation investments impact the transmission system in the long run. Conversely, it is also necessary to examine how the inclusion of the transmission network impacts generation investments, and in particular, when location specific resource constraints are introduced. Keeping the above in mind, the proposed investment planning model is further improved in the third stage to include the transmission network, as a dc load flow model. This model presents a new perspective on the optimal investments and helps the investor to understand the impact of fuel prices and other parameters on line overloads.

The proposed plan models are envisaged to provide a promising direction towards power system planning in the context of competitive electricity markets with consideration of many of the technical and economic parameters. Multiple scenarios and case studies are developed and tested in the thesis, each with their own effects on the financial balance and economics of the firm.

1.4 Outline of the Thesis

Chapter-2 presents a linear programming model from the perspective of investor firms to determine the optimal generation investments over the long-term. The chapter presents an iterative method to
arrive at the optimal IRR from the investment plans. The proposed modeling framework also incorporates an emissions cap on the investor and the effects of variations of the emissions cap on its financial returns are also examined.

In **Chapter-3** the planning model proposed in Chapter-2 is upgraded to take into consideration discrete unit sizes. The consideration of discrete unit sizes makes the planning model a mixed integer linear programming framework. Various scenarios and cases are considered for the analysis.

In **Chapter-4** the generation expansion planning model incorporates the transmission network model in detail. A dc load flow representation is used to model the transmission system and the long-term planning model is modified to develop the investment plans that seek to match the location demand and the demand growth in the long run.

**Chapter-5** presents the main conclusions from the thesis and identifies the scope for future work in this area.
Chapter 2
A Generation Planning Model for Investor Firms and Financial Analysis

In this work a modeling approach is proposed that pertains to an individual investor participating in the first-stage exercise of generation planning. The mathematical framework developed herein is a dynamic, linear programming model which evaluates the effects of internal rate of return (IRR), electricity market prices and budget constraints on an investor’s financial parameters and its optimal investment decisions.

This work also touches upon the issue of energy dispatch and optimal capacity factor of different generation technologies at the planning stage itself so as to maximize the total profit of the investor. The proposed model is novel in the sense that the planning horizon is split into plan sub-periods so as to minimize the overall risks associated with long-term plan models, particularly in the context of deregulation.

2.1 Features of the Developed Model

2.1.1 General Characteristics

This study examines the investment planning problem from the prospective of an independent investor willing to invest in the power sector. It analyzes the related aspects of investment planning by considering rate of return variations as well as market price and budget scenarios. The investor’s main criterion in arriving at the optimal decisions is maximization of its profits. The decision variables are optimal sizing and timing of investments, while taking into consideration various constraints.

2.1.2 Technologies

It is assumed in this chapter that the investor is willing to invest in three different technologies namely gas-fired, coal-fired and combine cycle plants. It should be pointed out that the model developed is generic enough to accommodate other technologies with certain modifications in the operational constraints. For example, if hydro resources are considered than, there is a need to include
reservoir water balance equations, hydro energy availability and related constraints. Similarly, if wind technologies are to be included, then the model should take into account the intermittent variations in wind speed based on locations, relational constraints of wind speed to wind energy production and associated constraints. Apart from such specific constraints, the basic model structure would nevertheless, remain the same, as that described herein.

2.1.3 Electricity Market Price

Price duration curves are used for each year, classified into three categories, base price, intermediate price and peak price. These price blocks are in the same order as the demand blocks of similar categories, making up the system load duration curve for a given year.

Future electricity price estimates are generated using a price escalation factor on base-year prices. In this work a 3% per year of price escalation is used without any loss of generality, and this is a fairly conservative estimate. Such an estimate was made because of the absence of any clear information on historical price trends in Ontario.

2.1.4 Energy sale in the market

It is assumed that the investor can sell all its generated electricity to the electricity market, thereby implying that there is enough demand in the system to absorb all the energy generated by this investor.

2.1.5 Risk Averse Plan

New investors are usually exposed to various risk parameters arising from market competition [1]. In order to reduce the risk, the long-term plan period of the investor is segregated as 5-year sub-periods and the investment cost recovery for new capacity is considered to be completed within 5 years.

2.1.6 Internal Rate of Return (IRR)

The proposed model determines the optimal IRR at which the total profit of the firm is maximized over the plan period. In order to do so, different rate of returns are tested for the fixed electricity price to check the effect of different IRR requirements on the economics of operation in planning period.
2.1.7 Assumptions Made in Modeling

- Adequate transmission resources are in place to transfer the power generated by the new units to the demand locations. This assumption basically implies that the investor’s decisions are not affected by transmission constraints. In a later chapter, we will consider the case where this assumption is removed, and the presence of the transmission network will affect the investor’s decisions.

- The investment decisions of the investors do not influence the electricity market prices. This assumption ensures that no investor holds market power or is likely to hold market power through large scale investments. This implies that the central planning authority, in its coordination process, ensures a level playing field in the investment sector.

- The investor firm is the sole owner and operator of the plants - which means that the dispatch decisions are taken by the firm and that the firm’s sell bids are always accepted in the market.

2.2 Generation Investment Planning Model

2.2.1 Objective Function

The objective function is the maximization of the present worth of the total profit of the firm over the plan period and the present worth of the salvage value that the firm will receive at the end of plan period. This is given in (1) below:

\[ J = \sum_{k=1}^{P} \frac{A_p(k)}{(1 + A)^k} + \sum_{k=1}^{P} \sum_{m} \frac{Svl(m, k)}{(1 + A)^P} \]  

(1)

\( J \) Objective function of the firm, $

\( A_p(k) \) Annual profit of firm in year \( k \), $

\( A \) Discount rate, %

\( Svl(m, k) \) Salvage value of generating unit of technology \( m \) commissioned in year \( k \), $/MW

\( P \) Planning period, years
Salvage value $Svl(m,k)$ is the depreciated value of the commissioned plant of technology $m$ at year $k$, and is given by (2).

$$Svl(m,k) = Nc(m,k) \times 1000 \times Pc(m,k) \times \left(1 - D_{rate}(P - k + 1)\right)$$  \hspace{1cm} (2)

$Nc(m, k)$ \hspace{1cm} New capacity investment of technology $m$ in year $k$, MW

$Pc(m, k)$ \hspace{1cm} Capital cost of plant of technology $m$ in year $k$, $$/kW

$D_{rate}$ \hspace{1cm} Depreciation rate for plant value

The annual profit of the firm, $Ap(k)$, is the revenue earned by it from sell of energy net of its total annual cost (3).

$$Ap(k) = R(k) - Ac(k)$$ \hspace{1cm} (3)

$R(k)$ \hspace{1cm} Revenue earned by the firm in year $k$, $

$Ac(k)$ \hspace{1cm} Annual cost of the firm in year $k$, $

The revenue earned by the firm by selling energy generated by its units at the market price, is given by (4).

$$R(k) = \sum_m \sum_b E(m, b, k) \times Pr(b, k)$$ \hspace{1cm} (4)

$E(m, b, k)$ \hspace{1cm} Energy generated by the firm from plants of technology $m$ in demand block $b$ of year $k$, MWh

$Pr(b, k)$ \hspace{1cm} Electricity price in time block $b$ of year $k$, $$/MWh

The parameter $Pr(b,k)$ is typically a long-term estimate of the expected price trend in the electricity market. It is very complex exercise to arrive at such an estimate in the long-term. Typically the investor firm would have to rely on market trends, economic indications judiciously choose there values. It may also be possible to revise the estimates as the plan progress.

In (3), $Ac(k)$ denotes the total annual operations, maintenance and capital investment costs of all plants to produce the energy $E(m, b, k)$. The components of $Ac(k)$ are fuel cost, variable O&M cost, fixed O&M cost and investment cost, and is given by (5).
\[ Ac(k) = \sum_m \sum_n \left[ \left\{ E(m,b,k) \cdot hr(m) \cdot fp(m,k) \div 1000 \right\} + \left\{ E(m,b,k) \cdot vom(m,k) \right\} + \left\{ C(m,k) \cdot 1000 \cdot fom(m,k) \right\} + \left\{ Pbcap(m,k) \cdot Pc(m,k) \cdot lfcr(m) \cdot 1000 \right\} \right] \]  \hspace{1cm} (5)

- \textit{hr(m)} \quad \text{Heat rate of plant, of technology } m, \text{ BTU/kWh}
- \textit{fp(m, k)} \quad \text{Fuel price associated with technology } m \text{ in year } k, \text{$/MBTU} \n- \textit{vom(m, k)} \quad \text{Variable O&M cost for plants of technology } m \text{ in year } k, \text{$/MWh} \n- \textit{fom(m, k)} \quad \text{Fixed O&M cost for plants of technology } m \text{ in year } k, \text{$/kW} \n- \textit{lfcr(m)} \quad \text{Levelized fixed cost rate for plants of technology } m, \% \n
In (5), \( Pbcap(m, k) \) is the cumulative plant capacity of technology \( m \) that is installed by the firm over a plan sub-period \( Pb \). The notion of plan sub-period has been used in this work in order to ensure that the capital cost is received within the sub-period. In this work, \( Pb \) is considered to be 5 years.

\[ Pbcap(m, k) = \sum_{n=1}^{Pb} Nc(m, k - n + 1) \]  \hspace{1cm} (6)

\( Pb \quad \text{Plan sub-period for payback of plant costs, years} \n
2.2.2 Constraints

2.2.2.1 Dynamic capacity update

This constraint relates the total investments made by the firm in a given year for technology \( m \), to existing investments in the same technology, using an inter-temporal constraint (7).

\[ C(m,k) = Nc(m,k) + C(m,k - 1) \]  \hspace{1cm} (7)

In (7), \( C(m,k) \) is the total capacity in year \( k \) of technology \( m \), in the portfolio of the investor. The investment decision variable \( Nc(m,k) \) depends on available budget in year \( k \) and is only selected when the value of the project exceeds the value of deferring the decision to invest in the future [1].
2.2.2.2 Asset Recovery Constraint

New investments can only be made up to a certain year of the planning horizon, and no investments can be made beyond that because there would be insufficient time left to recover the costs. The asset recovery constraint introduces an upper limit on investment capacity, feasible for the firm, for each specific technology $m$.

$$Nc(m,k) \leq Nc^{\text{Max}}(m) \cdot \text{bin}(k) \quad (8)$$

$Nc^{\text{Max}}(m)$: Upper limit on capacity that can be installed in a year, of technology $m$, MW

$\text{bin}(k)$: Binary Parameter on new capacity investment in year $k$

$$\begin{align*}
\text{bin}(k) &= \begin{cases} 
1 & \text{if } P - k + 1 \geq pb \\
0 & \text{if } P - k + 1 < pb
\end{cases}
\end{align*} \quad (9)$$

2.2.2.3 Energy Generation Constraints

Energy generation depends on the capacity available in a particular year and the duration for which plants are dispatched in particular load block, i.e., base, intermediate and peak (10).

$$E(m,b,k) \leq C(m,k) \cdot D(b) \quad (10)$$

In (10), $D(b)$ denotes the duration of generation dispatch in a given load block $b$, in hours. $D(b)$ is based on average load profile of different days of the year.

2.2.2.4 Budget constraints

This constraint specifies the upper limit on total spending by the firm over the plan sub-period of five years.
\[ \sum_{k=5i-4}^{5i} Ac(k) \leq Bd(i) \quad \forall i = 1,2,...,5 \quad (11) \]

- \( i \)  
  Index for a plan sub-period

- \( Bd(i) \)  
  Total available budget over a plan sub-period

### 2.2.2.5 Emission constraint

The total annual emissions by the different plants owned by the firm within a pre-specified emission limit needs to be limited within a certain maximum value (12).

\[ Em(k) \leq E_{cap} \quad (12) \]

Where

\[ Em(k) = \sum_{m} \sum_{b} E(m,b,k) \times Ec(m) \]

\[ (13) \]

- \( Em(k) \)  
  Total emission by all generators owned by the firm, in year \( k \), tons

- \( Ec(m) \)  
  Emission coefficient of generation technology \( m \), tons/MWh

- \( E_{cap} \)  
  Emission cap imposed on the firm by regulator, tons/year

In this work, we have considered \( E_{cap} = 10^7 \) tons/year.

### 2.2.2.6 IRR Constraint

The IRR constraint is an equality constraint that is imposed on the firm to ensure that for a specified IRR, the present values of investment costs are equated to the returns (14). By varying the value of IRR over a range, the optimal combinations to the profit maximized investment decisions can be obtained.

\[ \sum_{k=1}^{p} \frac{R(k)}{(1 + IRR)^k} + \sum_{k=1}^{p} \sum_{m} \frac{svl(m,k)}{(1 + IRR)^p} = \sum_{k=1}^{p} \frac{Ac(k)}{(1 + IRR)^k} \quad (14) \]
2.3 Case Study

2.3.1 System Data

The mathematical model discussed in the previous section is a linear programming model which is programmed in the GAMS [19] environment. As discussed earlier, this model is designed from the perspective of an investor. In the base case the optimal value of IRR is determined while the objective function is maximization of the present value of total profits and salvage.

Data used in the base case scenario are given in Table I, which are adapted from [20]. It should be mentioned here that the data used here, although being of 1990 values, are generally within the close range of the currently reported figures in [21]. It should be noted that some of the data specific to the technologies considered in this work, such as cost data, have a certain range and would vary depending on the time, location and market.

<table>
<thead>
<tr>
<th></th>
<th>Gas-fired</th>
<th>Coal-Fired</th>
<th>Combine-cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel price, $/MBtu</td>
<td>6</td>
<td>2.8</td>
<td>6</td>
</tr>
<tr>
<td>Heat rate, Btu/kWh</td>
<td>11000</td>
<td>9900</td>
<td>7800</td>
</tr>
<tr>
<td>Capital cost, $/kW</td>
<td>350</td>
<td>1400</td>
<td>700</td>
</tr>
<tr>
<td>Fixed O&amp;M cost, $/kW</td>
<td>1</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td>Variable O&amp;M, $/MWh</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Price escalation of all costs related to plant, %</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Depreciation rate, %</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Levelized FCR, %</td>
<td>27</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>Present worth rate, %</td>
<td></td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>Fuel price escalation, %</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Planning period, years</td>
<td></td>
<td></td>
<td>25</td>
</tr>
</tbody>
</table>
Table II presents the electricity market price for the different demand blocks, i.e., base, intermediate and peak, during the base year. In this thesis we have used standard assumptions for the three price blocks, which are fairly similar to that observed in the Ontario electricity market prices, the Hourly Ontario Energy Price (HOEP) [23].

The duration of each demand block is also provided in the table. As mentioned earlier, this information on demand block duration can be extracted from historical load duration curve data of the system.

<table>
<thead>
<tr>
<th>Investment capacity size, MW</th>
<th>500</th>
<th>500</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emission coefficient, tons/MWh</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Capacity Factor, %</td>
<td>50</td>
<td>80</td>
<td>50</td>
</tr>
</tbody>
</table>

Table II Price and Demand Duration by Demand Blocks

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>Intermediate</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price, $/MWh</td>
<td>40</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td>Duration, hours/year</td>
<td>3285</td>
<td>3285</td>
<td>2190</td>
</tr>
</tbody>
</table>

Table III Budget Allocated to Different Plan Sub Periods (IN $)

<table>
<thead>
<tr>
<th>Sub-period (Year)</th>
<th>1 (1-5)</th>
<th>2 (6-10)</th>
<th>3 (11-15)</th>
<th>4 (16-20)</th>
<th>5 (21-25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget, Million $</td>
<td>100</td>
<td>200</td>
<td>400</td>
<td>800</td>
<td>1,600</td>
</tr>
</tbody>
</table>

As mentioned in Section-2.1.5, the firm’s budget constraint is applied in every sub-period in order to arrive at a risk averse plan. The budget allocations considered for the firm, over the plan sub-periods, given in Table III. These budgetary allocations are chosen for the model studies keeping in mind that the investor is not a dominant player in the investment business, and does not intent to hold
any market power, but at the same time has enough budgets to invest in some reasonable generating capacity within a 5-year time frame. In this work, the first sub-period (5-year) budgetary allocation has been assumed to be $100 million because at this value the active investments take place for the investor, while for any lower budgetary allocation, there are no investments. It is further assumed that the budgetary allocation for later sub-periods (each 5-year period) is twice the amount of the previous sub-period. This assumption is fairly generic in nature.

2.3.2 Base Case

The optimal investment decisions of the firm are obtained from the solution of the model discussed in Section-2.2. First, the model is solved by choosing a low value of IRR and optimal decisions are obtained while maximizing the firm’s profit, J. Let us denote the maximum value of J, obtained for a given IRR as $J^*$. The IRR is increased in small steps and the model is solved to maximize J and the corresponding $J^*$ is obtained. It is observed that the firm’s maximum profit, $J^*$, increases as the IRR is increased, attains a maximum, and with further increase in IRR, starts decreasing. This value of IRR where $J^*$ attains a maximum, denoted by $J_{\text{Max}}$, represents the optimal solution in the base case.

Fig. 1 Effect of IRR on firm’s profit

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Fig. 1 shows the variation of $J^*$ with IRR for a range of IRR values. The optimal investment plan corresponding to this IRR (denoted by IRR*) represents the Base Case solution, and is discussed in detail. It can be seen from Fig. 1 that the optimal IRR so obtained is IRR* = 33.12%.

Table IV provides the firm’s optimal investment plan over the 25-year planning horizon. It can be observed that the firm concentrates its investment decisions at the beginning of each plan sub-period so as to allow the maximum possible time for cost recovery from the investment.

**TABLE IV GROSS OPTIMAL INVESTMENT DECISIONS IN THE BASE CASE**

<table>
<thead>
<tr>
<th>Year of Installation</th>
<th>Gas-Fired (MW)</th>
<th>Coal-Fired (MW)</th>
<th>Combine-Cycle (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>48</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>75</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>60</td>
<td>47</td>
</tr>
<tr>
<td>21</td>
<td>456</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table V summarizes the present value of the financial balance of the firm in the base case solution, where the firm yields an IRR* of 33.12%. The total present value of its profit over the planning horizon is 190.631 M$.

**TABLE V PRESENT VALUE OF FINANCIAL BALANCE OF THE FIRM OVER PLAN HORIZON**

<table>
<thead>
<tr>
<th>Total revenue, M$</th>
<th>Total cost, M$</th>
<th>Total profit, M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>740.10</td>
<td>571.35</td>
<td>190.63</td>
</tr>
</tbody>
</table>

Fig. 2 shows a plot of the firm’s total cost, total revenue and salvage value, for a range of IRRs. It is observed that when the firm’s IRR is low, the salvage value is an important parameter in its financial balance since the revenue earnings are lower than total costs. For higher values of IRR, the salvage value remains more or less constant.
2.3.3 Scenario of Electricity Price Variations

In this scenario the price of electricity is varied over a range of 85% to 115% of the base price and its effect on optimal IRR, total profit, total revenue and total firm cost is studied. Fig. 3 shows the relationship between electricity price and the firm’s optimal IRR. It can be seen from Fig. 3 and Fig. 4 that if the price of electricity increases, both optimal IRR and the total profit starts increasing linearly. Around a price value of 95% of the base price, the optimal IRR increases with a steeper slope (Fig. 3). Because the firm does not have any further budget for investment, its investment costs attains a maximum limit while its revenue keeps on increasing.

Moreover, since the firm’s emission cap is also reached at 95% of base price, its energy generation is constrained and thus limiting the generating costs while revenue keeps increasing because of price increase, as shown in Fig. 5.
Fig. 3 Effect of electricity price variations on optimal IRR

Fig. 4 Effect of electricity price variations on present value of firm profit
2.3.4 Scenario of Variations in Budget Limits

In this scenario the available budget for a plan sub-period is varied over a range of 75% to 125% of the base budget and its effect on optimal IRR, total firm profit, total firm revenue, total cost and salvage value is studied. Fig. 6 shows that the optimal IRR is significantly affected by variations in available budget of the firm. As the budget limit is reduced from the base budget (denoted by 100%, in Fig. 6), the IRR increases slightly up to 95% of base budget because of some reduction in the firm’s investment, but electricity production remain almost same. At 90% and below of base budget, the firm’s IRR reduces, which can be attributed to reduction in investments and generating costs exceeding the lost revenue from reduced investments. When the budget is increased beyond 100% of the base budget, more coal based capacity is added. Because of the high investment cost involved, the IRR declines linearly (as shown in Fig. 6). With increased coal based capacity, the emission level will also increase and at about 120% of base budget, the emission constraint is binding, which consequently restricts any further coal based capacity addition.

When budget is increased beyond 120% of base, there is a shift and gas-fired capacity is added. This being a low capital intensive investment, the IRR will see a steep increase, as shown in Fig. 6.
In Fig. 7 it is shown that with increase in budget availability, the total revenue and total cost of the firm increases linearly but the rate of increase of total revenue is higher than that of total cost, which indicates a progressively increasing profit of the firm.
It is further shown in Fig. 8 that the firm’s profit increases linearly with budget, this is because more coal based capacity is added, as mentioned earlier, which consequently results in increased generation from coal unit, being comparatively of lower costs.

![Graph showing the relationship between budget and firm's total profit.](image)

**Fig. 8 Effect of change in budget on PV of total profit**

### 2.3.5 Emission Cap Scenario

It should be noted that the emissions cap imposed on the individual investor was considered to be $10^7$ tons/year and the optimal investment plan was obtained accordingly. It would be interesting to examine how the investor’s decisions would change if the emissions cap was varied. For example, if the investor was allowed a higher quota of emissions- would it invest in more of coal; based capacity? On the other hand, if the emission cap was tightened, how will it alter its investment decisions. It is also important to examine the consequent impacts on the firm’s IRR and on its total profit.

In order to carry out the studies, the emission cap is varied from the base case quota of $10^7$ ton/year to a lower limit of $10^5$ tons/year, which signifies the limit up to which the investor can make some investments while below which no investments are possible.

On the higher side the emissions cap is increased up to the point new capacity is selected (which is 108 tons/year), and beyond this limit no new capacity is added because budget constraints become the limiting parameter.
Fig. 9 shows the effect of imposing an emission cap (tons/year) on actual emission from the generators in the portfolio of the investment for firm. These actual emissions are calculated for the horizon year of the plan (i.e., year-25). It can be seen from the figure that as the emission cap is increased up to 1.0E+7 tons/year the actual emission of the firm is always at the level of emission cap imposed, but beyond that the firm’s emission levels are lower than the cap because the firm is bound by the budget constraint, that does not allow it to produce more energy and associated emission.
Fig. 10 shows that the optimal value of IRR is not affected by the emission cap beyond a value of 5.0E+6 tons/year, because the available budget is used for more expensive plants like coal to produce cheaper electricity, which leaves lesser budget for energy production, to increase revenue and hence IRR. On the other hand when the emission cap is reduced from 5.0E+6 tons/year to 1.0E+6 tons/year, maximum optimal value of IRR is obtained, it can be attributed to the investment in lesser expensive gas-fired and combine-cycle plants. Also budget does allow them to produce more electricity to increase earned revenue and hence optimal IRR is increased by a significant amount. Below 5.0E+6 tons/year value of emission cap, investment in all type of technology plants is reduced, hence electricity production and the Optimal IRR value.
Fig. 11 shows the total profit of the firm with change in the emission cap. It can be seen that up to a certain emission cap of $1.0E+7$ tons/year, the firm’s profit increases continuously. This increased profit is attributed to increased investment in coal-based capacity which consequently results in cheap generation, and hence more revenue. However beyond $1.0E+07$ tons/year of emission cap the allowable budget becomes a binding constraint for further coal-based capacity addition, and firm’s profit see a gradual steady level.

### 2.3.6 Trade-off between financial risk and financial return

As mentioned before, in order to alleviate the financial risk, the cost recovery period for new capacity investment has been considered to be 5 years in previous sections, since the recovery period directly influences the financial risks involved in the investment. In order to have a trade off between financial risk and return, the recovery period is varied from 5 to 8 years and its impact on IRR and firm’s profit is analyzed. Fig. 12 shows that the IRR increases if the investor is ready to take more risk by allowing a longer cost recovery period. This is because with a longer cost recovery period a significant amount of annual cost gets shifted from investment to operation to increasing energy production. So, rather
then going for new capacity investment, firm prefers to use budget in energy production, and hence obtain higher value of IRR.

Fig. 12 Effect of financial risk (cost recovery period) on optimal IRR

2.3.7 High Fuel Price Scenario

The fuel prices are generally quite volatile and are influenced by various external factors. In order to take into consideration the escalation in fuel prices, a uniform rate of increase of 3% and 2% per year is assumed for gas and coal prices, respectively.

In order to carry out a scenario study considering high fuel prices, a 10% and 15% increase in fuel price for gas and coal prices respectively are used, over and above their base prices, to assess the effect of high fuel prices on optimal investment plan (Fig. 13).
Table VI provides the firm’s corresponding optimal investment plan over the 25-year planning horizon. It can be observed from a comparison of the firm’s optimal investment decisions from Table IV and Table VI that there is some shift in the optimal investment decisions when high fuel prices are considered. It can be observed from Table VII that the IRR and firm’s revenue is significantly lower in the high fuel price scenario.
### TABLE VI GROSS OPTIMAL INVESTMENT DECISIONS WITH HIGH FUEL PRICE

<table>
<thead>
<tr>
<th>Year of Installation</th>
<th>Gas-Fired (MW)</th>
<th>Coal-Fired (MW)</th>
<th>Combine-Cycle (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>29</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>66</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>115</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE VII COMPARISON OF IRR, REVENUE AND TOTAL COST OVER THE PLANNING PERIOD WITH BASE CASE AND HIGHER FUEL PRICE

<table>
<thead>
<tr>
<th></th>
<th>IRR, %</th>
<th>Revenue, M$</th>
<th>Total Cost, M$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base Price</td>
<td>High Price</td>
<td>Base Price</td>
</tr>
<tr>
<td></td>
<td>33.12</td>
<td>18.81</td>
<td>740.10</td>
</tr>
<tr>
<td></td>
<td>571.35</td>
<td>486.72</td>
<td></td>
</tr>
</tbody>
</table>

#### 2.4 Conclusions

This work presents a generation investment planning model from the perspective of an investing firm seeking to participate in electricity markets. The work assesses the importance of arriving at an optimal internal rate of return (IRR) at which the firm’s profit maximization objective attains an extremum value. Scenarios for electricity market price variations and plan sub-period budget limit variations are carried out. It is observed that as electricity price increases the optimal IRR starts increasing linearly up to a certain point and thereafter it increases non-linearly. The effect of price increase or budget limit increase has very similar impact on the firm’s profit which increases linearly.
Chapter 3

A Generation Planning Model for Investor Firms: Mixed Integer Programming Framework

In Chapter-2 a generalized long-term planning model pertaining to an investor firm was presented. The plan period was considered to be twenty-five years and the capacity addition variables were considered to be continuous variables. A problem with such a model was that the generating unit sizes that were determined were not of the standard sizes available in the market. Therefore, the investor would not be able to use such a model in practical application. Therefore, there is a need to revise the model and introduce proper unit sizes that are available from the manufacturers.

In view of the above, in the present chapter, a more realistic model is developed that considers discrete unit sizes for capacity addition. In order to introduce discrete unit sizes, there is a need to include binary decision variables in the mathematical model formulation.

Furthermore, in order to make the investment planning more practical, the planning horizon is now reduced to 15 years, in all the analysis reported in the subsequent part of this thesis.

3.1 Features of the Developed Model

In Chapter-2, the basic features of the planning model were discussed in detail. The additional features that emerge in this chapter are discussed below.

3.1.1 Discrete Unit Sizes

In this work it is assumed that the investor firm invests with discrete size of units, i.e., only certain specified units capacity can be commissioned. This requires the introduction of binary variables and an associated set of new constraints pertaining to the binary selection variables.

3.1.2 Assumptions Made in Modeling

- The basic assumptions made in the investment modeling framework have been discussed in the previous chapter, and are valid in the present work as well. The additional assumption in this work is:
• The discrete sizes of the generating units have been assumed arbitrarily for the purpose of these studies without any loss of generality.

3.2 Generation Investment Planning Model with Discrete Unit Sizes

The basic modeling aspects, including the investor’s objective function and associated constraints, remain same as presented in the previous chapter. The additional constraints that emerge from the introduction of discrete unit sizes are given below:

3.2.1 Asset Recovery Constraint

New investments can only be made up to a certain year of the plan horizon, and no investments can be made beyond that because there would be insufficient time left to recover the costs. The asset recovery constraint introduces an upper limit on investments by the firm, \(DNc(m)\), for each specific technology \(m\).

\[
Nc(m,k) = DNc(m) \cdot ID(m,k) \cdot bin(k)
\]

\(DNc(m)\)  
Capacity size that can be commissioned in a year of technology \(m\), MW

\(ID(m,k)\)  
Binary investment decision variable for commissioning generating units of technology \(m\) in year \(k\), 0 or 1

\(bin(k)\)  
Binary parameter on new capacity investment in year \(k\)

\[
bin(k) = \begin{cases} 1 & \text{if } P - k + 1 \geq pb \\ 0 & \text{if } P - k + 1 < pb \end{cases}
\]

\(ID(m,k) = \begin{cases} 1 & \text{Binary Variable denoting selection of capacity} \\ 0 & \text{Otherwise} \end{cases}
\)

3.2.2 Budget constraints

This constraint specifies the upper limit on total spending by the firm over the plan sub-period of five years.
\[
\sum_{k=5i-4}^{2i} Ac(k) \leq Bd(i) \quad \forall \; i = 1, 2 & 3
\]  

\(i\) \hspace{1cm} \text{Index for a plan sub-period}

\(Bd(i)\) \hspace{1cm} \text{Total available budget over a plan sub-period}

### 3.3 Case Study

#### 3.3.1 System Data

The mathematical model discussed in Section-3.2 is a Mixed-Integer linear programming (MILP) model which is programmed in the GAMS [19] environment. As discussed earlier, this model is designed for financial analysis for a firm willing to invest in discrete sized generation units with three options of technologies. In the base case the optimal value of IRR is determined while the objective function is maximization of the present value of total profits. Data used in the base case scenario has been given in Chapter-2, Table I. The discrete unit sizes considered in this chapter is chosen arbitrarily, without any loss of generality, to demonstrate the functioning of the proposed model.

- Gas-based = 100 MW
- Coal-based = 300 MW
- Combine-cycle plants = 200 MW

The assumptions pertaining to electricity market prices remain the same, as discussed in Chapter-2, Section-2.3.1, Table II. The firm’s budget constraint is applied every sub-period in order to arrive at a risk averse plan. The budgetary allocations of the firm over different plan sub-periods are given in Table VIII. The justifications with regard to the budgetary allocations were discussed in Chapter-2.

It should be noted that in this chapter, the firm’s budget available over a given sub-period has been significantly increased, as compared that used in Chapter-2. This increase of budget was necessary because of the discrete nature of unit sizes which are added, and the minimum unit size considered being 100 MW.
### TABLE VIII BUDGET ALLOCATED TO DIFFERENT PLAN SUB PERIODS (IN $)

<table>
<thead>
<tr>
<th>Sub-period (Year)</th>
<th>1 (1-5)</th>
<th>2 (6-10)</th>
<th>3 (11-15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget, Million $</td>
<td>600</td>
<td>1200</td>
<td>2400</td>
</tr>
</tbody>
</table>

#### 3.3.2 Base Case

The optimal investment decisions of the firm are obtained from the solution of the model discussed in Section 3.2. As explained in Chapter-2, the similar procedure is adapted here to arrive at the optimal value of IRR. The model is solved by choosing a low value of IRR and optimal decisions are obtained while maximizing the firm’s profit.

The IRR is increased in small steps to maximize J and the corresponding $J^*$ is obtained. The value of IRR where $J^*$ attains a maximum, represents the optimal IRR (Fig. 14).

![Effect of IRR on firm’s profit with MILP framework](image-url)
Fig. 14 shows the variation of $J^*$ with IRR for a range of IRR values. The optimal investment plan corresponding to this IRR (denoted by $\text{IRR}^*$) represents the Base Case solution, and is discussed in detail in the coming Sections. It can be seen from Figure that the optimal $\text{IRR}^*$ so obtained is 45.1%.

Table IX provides the firm’s optimal investment plan over the 15-year plan horizon. It can be observed from this table that investment decisions are made in such a way that the firm can efficiently utilize its available budget to maximize the total profit. It is seen that most of the investment decisions appear either in the beginning or at the end of a plan sub-period so as to utilize the available budget effectively.

One noticeable difference of the optimal plan, of the base case, arrived in this Chapter with that in Chapter-2 is the complete absence of coal-based generation addition when discrete unit sizes are considered.

### Table IX Base Case Gross Optimal Investment Decisions for MILP Framework

<table>
<thead>
<tr>
<th>Year of Installation</th>
<th>Gas-Fired (MW)</th>
<th>Coal-Fired (MW)</th>
<th>Combine-Cycle (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>0</td>
<td>200</td>
</tr>
</tbody>
</table>

Table X summarizes the present value of the financial balance of the firm in the base case solution with discrete size of units coming during plan period, where the firm yields an $\text{IRR}^*$ of 45.1%. The total present value of its profit over the planning horizon is 287.69 M$.

There is a noticeable increase in IRR when discrete unit sizes are considered in this Chapter, as compared to the results in Chapter-2. The justification for this increase is the higher allowable budget in the present Chapter, resulting in more investments, and large capacity unit additions because of specified discrete unit sizes of 100 MW, 200 MW and 300 MW only.

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Table X Present Value Financial Balance Summary for MILP Framework

<table>
<thead>
<tr>
<th>Total revenue, M$</th>
<th>Total cost, M$</th>
<th>Total profit, M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1793.23</td>
<td>1622.24</td>
<td>287.69</td>
</tr>
</tbody>
</table>

Fig. 15 shows a plot of the firm’s total revenue, total cost and salvage value, for a range of IRRs. It is observed that when the firm’s IRR is low, the salvage value is an important parameter in its financial balance since the revenue earnings are lower than total costs and for higher values of IRR, the salvage value remains more or less constant. The same observation was made in Chapter-2 in the context of the planning framework for 25-years. The plot in Fig. 15 are however not smooth curves, unlike those obtained in Chapter-2 because of the presence of binary variables associated with discrete unit sizes.
3.3.3 Scenario of Electricity Price Variations

In this scenario the price of electricity is varied over a range of 85% to 115% of the base price and its effect on optimal IRR, total profit, total revenue and total firm cost is studied. Fig. 16 shows the relationship between electricity price and the firm’s optimal IRR. It can be seen from Fig. 16, when electricity price is reduce from 100% of the base price, up to 90% of price, the optimal value of IRR reduces linearly. The reduction in IRR is attributed to reduced capacity factor operation of generation thus resulting in received revenue. When price is reduced further to 85%, the drop in prices induces a reduction in investments and hence fall in IRR. As electricity price is increased above the base price, the IRR increases because of high capacity factor operation without any change in investment. Beyond 105% of the base prices, the IRR drops in value because of new unit addition and generation from expensive units.

![Graph showing the relationship between electricity price and optimal IRR](image)

Fig. 16 Effect of electricity price variations on optimal IRR with MILP framework

Fig. 17 shows the variation of total revenue and total cost as prices are increased. It is observed that the total cost increases with price, up to 95 % of base price, because of new investments and increased generation; and beyond that attains a steady-state level when investments are constrained by the emission limits. On the other hand, the revenue continue to increase with price, there by increased the profit.
3.3.4 Scenario of Variations in Budget Limits

In this scenario the available budget for a plan sub-period is varied over a range of 85% to 115% of the base budget and its effect on optimal IRR, total firm profit, total firm revenue, total cost and salvage value is studied. Fig. 18 shows that the optimal IRR is affected by variations in available budget in a sub-period.
As the budget limit is reduced from the base budget (denoted by 100% in Fig. 18) up to 95% of base budget, the IRR reduces from a value of 45.1% to 43.5% because of some reduction in the firm’s investment and in production of electricity. At below 95% of base budget, the firm’s IRR increases because of further reduction in investment, while production and hence revenue earning remains same. While the budget is increased to 105% of base budget, the IRR is increased, because more investments are made in cheaper generation plants like combine-cycle, which can produce more electricity with higher available budget and hence earn higher revenue.

As observed from Fig. 18, the IRR of the firm is quite sensitive and dependent on the budgetary allocations, and the investment decisions are impacted. However, the firms profit has a linear relationship with the budget. As budget is increased, the profit increases monotonically.

![Fig. 19 Effect of budget on present value of total profit with MILP framework](image)

3.3.5 Emission cap scenario

To analyze the effect of the emission constraint in investment planning with discrete unit sizes, in terms of optimal IRR and total firm’s profit, the emission cap is varied from 1.0E+5 tons/year to 1.0E+8 tons/year.

Fig. 20 compares the effect of emission cap (tons/year) on firm’s annual emission with actual emission in the last year of the plan horizon (maximum emission is expected). It can be seen from the figure that as the emission cap is increased up to 1.0E+7 tons/year the firm’s actual emission is always equal to the emission cap, beyond that, the firm’s emission level is lower than the cap. This can be attributed to the fact that beyond the emission cap of 1.0E+7 tons/year, the firm is bound by the budget constraint, that does not allow it to produce more energy and hence restricts the emissions.
Analysis has been carried out to examine the variation of optimal IRR with emission cap. It is seen (Fig. 21) that as the emission cap is increased, the IRR increases, attains a maximum of 45.1% at 1.0E+7 tons/year of cap, and then decreases the increase in IRR with emission cap, increase can be attributed to new investments and additional revenue generation. The decrease in IRR beyond the emission cap of 1.0E+7 tons/year is because of shift in investment towards coal-based plants and consequent reduction in energy generation because of budget constraints.
Fig. 21 Effect of emission cap on the optimal value of IRR with MILP framework

Fig. 22 shows the net profit of the firm with change in the emission cap. It can be seen that up to an emission cap of 5.0E+7 tons/year, the firm’s profit increases but after that it attains a steady state level is unaffected by change in emission cap.

Fig. 22 Effect of emission cap on firm’s total profit with MILP framework
3.3.6 Trade-off between financial risk and financial return

In order to consider financial risk, the cost recovery time for new capacity investment has considered being 5 years in all previous Sections of this chapter. Investment cost recovery period directly influences the financial risk involved in the investments. In order to have a trade off between financial risk and return the cost recovery period varied from 5 to 8 years and its impact on the return is analyzed. Fig. 23 shows that return on the investment is expected to increase almost linearly if cost recovery time is increased up to 7-years, which can be attributed to increase in the investment towards cheaper generation plants, which results in more revenue margin with lower total cost. If cost recovery period is increased beyond 7-years, the IRR is expected to reduce because with longer recovery time the firm is expected to invest in cheaper generation plants like coal to get higher profit, but since such plants have higher emission coefficients, the emission cap and budget constraints come into play and constraints the energy generation from these units.

Fig. 23 Effect of financial risk (Cost recovery time) on IRR with MILP framework

In Fig. 24 it is shown that with increase in cost recovery period the firm’s total profit is expected to decrease, as lesser number of units are expected to be installed, resulting in lower energy production, and hence lower revenue. Also because lesser number of units are expected to be installed, the
salvage value will also reduced. But if cost recovery period is increased from 7-years to 8-years, the investment shifts towards cheaper generation plants like coal to earn more revenue and profit. But such plants have high capital cost, so during the cost recovery period of these units, only few new units will be installed with discrete size, so all budget will not get used, and hence IRR get reduced (Fig. 24).

![Graph showing the effect of financial risk (Cost recovery time) on firm’s profit with MILP framework](image)

**Fig. 24 Effect of financial risk (Cost recovery time) on firm’s profit with MILP framework**

### 3.4 Conclusions

This chapter presents a new planning framework for investor firms in the generation sector taking into consideration investment decisions specified to discrete unit sizes only. A mixed-integer linear programming model is developed that analysis various scenarios of budget and price increase to examine their impact on the optimal plan.

It is observed from the studies that the optimal plan of the firm with discrete unit sizes yields a higher IRR as compared to that obtained in Chapter-2. It can also be concluded that if electricity prices are expected to increase, then firms with discrete unit size options are expected to invest more even if their budget are limited.
Chapter 4
Generation Capacity Expansion Planning in Coordination with Transmission Resources

The main backbone of generation expansion planning exercise in power systems is the availability of adequate transmission resources. As electricity demand growth takes place over the long term driven by the growth of industries and the economy as a whole, there is a need to take into account the individual demand growths specific to load centers and load regions. Moreover it is also important to consider the feasibility of the selection of a particular generation technology at a certain location.

In the previous chapter, the generation investments selected, did not take into consideration the availability of the resources. For example, if a coal-based generation is selected at a location, that location should be expected to have adequate coal supply linkages within its vicinity for all practical purposes. Similarly, an investment decision in hydro cannot be made without adequate water resources at the location. In view of this, it is extremely important to introduce the concept of ‘location’ in the planning framework and arrive at long-terms plans that provide information on where, when, how much, and what type of investment, in a comprehensive manner.

The long-term generation expansion plans must also provide some inputs to the transmission system as to its adequacy and reliability and how it will be loaded when the new plans are introduced in a dynamic manner, year by year. It is important to know whether the transmission system will be able to provide secure and reliable transmission services or, it will require upgrades, and if transmission system requires upgrades, which are the transmission circuits that require augmentation.

In this chapter, a 15-year generation expansion planning model is presented that takes into consideration the transmission system model, and seeks to meet the location specific demand growth over the long-term. The investor’s objective remains the same- that of maximizing its total profit over the plan horizon.

The well-known IEEE 30-bus system has been used in the studies presented in this chapter; the bus loads have been considerably increased to suit the present planning problem. The model incorporates uncertain fuel prices for different technologies. Moreover, the individual buses have been identified as candidate locations for specific generation technologies based on an assumption of availability of appropriate resources at these buses. An expected bus-wise demand growth rate of 3% per year is considered. This outcome of this proposed model are the determination of optimal timing, capacity and location of specific technology generating units for investment as well as the long-term energy dispatch from the investor firm’s perspective. The electricity prices are modeled as price duration
curves, as in previous chapters, on the basis of three demand blocks namely base, intermediate and peak. The proposed planning model and the outcome from the model are examined to study their effects on transmission line loading for existing transmission lines.

4.1 Features of the Proposed Model

The salient features of the planning model are the same as that presented in Chapters-2 and 3. The additional issues that are introduced in this chapter, as discussed here.

4.1.1 Location Specific Options

The candidate locations for specific technologies are now introduced in this chapter that provides information for each bus, whether a specific technology option is feasible for investment or not. As shown in Table XI, a “YES” means that the technology option for investment is open to investor firms at a specific bus. Such a Table is constructed and used in this chapter, but it can be safely assumed that the investor will already have such information, in practical system planning. As in Chapter-3, it is assumed that the firm is willing to invest in three technologies- gas, combined-cycle and coal and the investments will be in discrete unit sizes.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Gas Plant</th>
<th>Coal Plant</th>
<th>Combine-cycle Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>YES</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
<tr>
<td>17</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
<tr>
<td>19</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
<tr>
<td>20</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
<tr>
<td>23</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
<tr>
<td>29</td>
<td>YES</td>
<td>-</td>
<td>YES</td>
</tr>
</tbody>
</table>
4.1.2 Modified IEEE 30-Bus System

As mentioned earlier, a modified IEEE 30-Bus system data is used for the studies presented in this chapter. The system is modified by increasing the bus active power loads by 5 times, for the base year. There are three existing generators at bus-1, bus-2 and bus-8 as shown in Fig. 25. The expected demand blocks (Base, Intermediate and Peak) and technical parameters for this modified system are given in Appendix A.

4.1.3 Risk Averse Plan

New investors are usually exposed to various risk parameters arising from market competition [1]. In order to reduce the risk, the 15-year plan budget of the investor is segregated as 5-year budgets and the investment cost recovery period for new capacity investment is considered to be 5 years. That means a new capacity plan has to recover its total investment costs in 5 years.

4.1.4 Assumptions Made for Modeling

- Assumptions are same as previous chapter work

4.2 Mathematical Model

4.2.1 Objective Function

The objective function is the maximization of the present worth of the total profit of the firm over the plan horizon and the present worth of the salvage value that the firm will receive at the end of plan period. This is given in (19) below:

\[
J = \sum_i \sum_b \sum_k \frac{Ap(i,b,k)}{(1 + a)^k} + \sum_i \sum_k \sum_m \frac{Svl(i,k,m)}{(1 + a)^p}
\]  

(19)
Ap(i,b,k) \hspace{1cm} Annual Profit from bus I during demand block b in year k, $

A \hspace{1cm} Discount rate, \% 

Svl(i,k,m) \hspace{1cm} Salvage value of m technology unit commissioned in year k at bus i, $

P \hspace{1cm} Planning period, years 

The annual profit of the firm, Ap(i,b,k), is the revenue earned by it from sell of energy net of its total annual costs (20).

\[ Ap(i,b,k) = R(i,b,k) - Ac(i,b,k) \] (20)

R(i,b,k) \hspace{1cm} Revenue earned from bus i during demand block b in year k, $

Ac(i,b,k) \hspace{1cm} Annual cost at bus i during demand block b in year k, $

Salvage value Svl(i,k,m) is the depreciated value at the end of the planning period for the plant based on technology m that commissioned in year k at bus i, and is given by:

\[ Sval(i,k,m) = Nc(i,k,m) \times pc(m,k) \times 1000 \times \left[ 1 - D_{rate} \times (P - k + 1) \right] \] (21)

Nc(i,k,m) \hspace{1cm} New plant capacity commissioned at bus i in year k of technology m, MW

pc(m,k) \hspace{1cm} Plant cost for plants of technology m in year k, $/kW

D_{rate} \hspace{1cm} Depreciation rate, \%

The revenue, R(i,b,k), earned by the firm by selling generated energy at the market price is given by (22).

\[ R(i,b,k) = \left[ \sum_{m} Pg(i,b,k,m) \times Dr(b) \right] + EP(i,b,k) \times Dr(b) \times Pr(b,k) \] (22)

Pg(i,b,k,m) \hspace{1cm} Power dispatch at bus i during demand block b in year k from m technology plant, MW

Dr(b) \hspace{1cm} Duration of demand block b every year, Hours
\(EP(i,b,k)\) \hspace{1cm} \text{Power dispatch at bus } i \text{ during demand block } b \text{ in year } k \text{ from pre-installed plants, MW}

\(Pr(b,k)\) \hspace{1cm} \text{Price of electricity during demand block } b \text{ in year } k, \$/MWh

In (20), \(Ac(i,b,k)\) denotes the total operations and maintenance costs of all plants to produce the energy. The components of \(Ac(i,b,k)\) are generation cost for pre-installed plants, fuel cost, variable O&M cost, fixed O&M cost and investment cost of new installed plants during plan horizon, and is given by (23).

\[
Ac(i,b,k) = [(B(i) \times EP(i,b,k) + C(i)) \times Dr(b)] \div (1 + a)^k \\
+ \sum_m \{Pg(i,b,k,m) \times Dr(b) \times hr(m) \times fp(m,k) / 1000\} \div (1 + a)^k \\
+ \sum_m \{Pg(i,b,k,m) \times Dr(b) \times vom(m,k)\} \div (1 + a)^k \\
+ \sum_m \{Tc(i,k,m) \times 1000 \times fom(m,k)\} \div (1 + a)^k \\
+ \sum_m \{Pbcap(i,k,m) \times 1000 \times pc(m,k) \times lfcr(m)\} \div (1 + a)^k
\] (23)

\(B(i), C(i)\) \hspace{1cm} \text{Cost coefficient for pre-installed plants, \$/MWh, \$/h respectively}

\(hr(m)\) \hspace{1cm} \text{Heat rate of plant based on } m \text{ technology, Btu/KWh}

\(fp(m,k)\) \hspace{1cm} \text{Fuel price for } m \text{ technology plants in year } k, \$/MBtu

\(vom(m,k)\) \hspace{1cm} \text{Variable Operation and maintenance cost for } m \text{ technology plants in year } k, \$/MWh

\(Tc(i,k,m)\) \hspace{1cm} \text{Total available capacity from } m \text{ technology plants at bus } i \text{ in year } k, \text{ MW}

\(fom(m,k)\) \hspace{1cm} \text{Fixed Operation and maintenance cost for } m \text{ technology plants in year } k, \$/MWh

\(Pbcap(i,k,m)\) \hspace{1cm} \text{Total } m \text{ technology plants capacity at bus } i \text{ in year } k \text{ for which cost recovery is remaining, MW}

\(lfcr(m)\) \hspace{1cm} \text{Levelized fixed cost rate for plants of technology } m, \%

In (23), \(Pbcap(i,k,m)\) is the cumulative plant capacity of technology \(m\) that is installed by the firm over a plan sub-period \(Pb\) at bus \(i\). In this work we have considered \(Pb = 5\) years.
\[ P_{\text{bcap}}(i, k, m) = \sum_{n=1}^{P_b} Nc(i, k - n + 1, m) \]  

\[ Pb \quad \text{Plan sub-period for cost recovery of plant costs, years} \]

### 4.2.2 Load Flow Equations

In this planning model the DC load flow equations are used for the purpose of keeping the model as a mixed-integer linear programming model. Detailed load flow equations including the reactive power and voltage variables are not considered critical here because of the study time-frame of 15 years, and this helps avoid the introduction of non-linear constraints in the model.

\[
\left\{ \sum_m P_g(i, b, k, m) \right\} + EP(i, b, k) - BD(i, b, k) = -\sum_j b(i, j) \ast \{\delta(i, b, k) - \delta(j, b, k)\}
\]  

\[ BD(i, b, k) \quad \text{Active power demand at bus } i \text{ during demand block } e \text{ in year } k, \text{ MW} \]

\[ b(i, j) \quad Y_{\text{bus}} \text{ Susceptance between transmission line } i-j, \Omega^{-1} \]

\[ \delta(i, b, k) \quad \text{Voltage angle at bus } i \text{ during demand block } e \text{ in year } k, \text{ Radians} \]

### 4.2.3 Line Flow Equations

As discussed in the previous section, in the present model the DC load flow equations are used. Accordingly, the active power line flows in the network are determined by the DC line flow equations, as given below:

\[
P_{\text{flow}}(i, j, b, k) = b(i, j) \ast \{\delta(i, b, k) - \delta(j, b, k)\}
\]  

\[ P_{\text{flow}}(i, j, b, k) \quad \text{Active power flow from bus } i \text{ to bus } j \text{ during demand block } b \text{ in year } k, \text{ MW} \]

It is to be noted here that the present model includes the line flow equations only for the purpose of computing the line flows and overloads resulting from new generating units. Line flow limits are not considered in this work and therefore, they do not impact the investment decisions of the investor.
The issue of imposing line flow limits is essentially the responsibility of the central planning authority which has to take into account all investment proposals and incorporate them into its operations studies with security constraints, and hence examine whether such proposals are acceptable or not. Such studies are beyond the scope of this thesis, and need to be taken up in the future.

4.2.4 New Capacity Installation variable

In this modeling approach the new capacity \( Nc(i,k,m) \), based on \( m \) technology plant that is commissioning at bus \( i \) in year \( k \), is a discrete size as considered in Chapter-3, the selection of these units to be installed based on a binary variable that decides whether the unit should be installed in particular year or not to maximize the objective function. The modeling equation for this variable is given below:

\[
Nc(i,k,m) = ID(i,k,m) \times ADCS(m) \times ALO(i,k,m) \times \text{bin}(k) \tag{27}
\]

\( ID(i,k,m) \)  
Binary decision variable to decide whether \( m \) technology unit should be installed at bus \( i \) in year \( k \) or not

\( ADCS(m) \)  
Available discrete capacity size of \( m \) technology plant that can be commissioned in a year, MW

\( ALO(i,k,m) \)  
Available Location options to install \( m \) technology plant at bus \( i \) in year \( k \), 0 or 1

\( \text{bin}(k) \)  
Binary decision parameter for the capacity installation in year \( k \)

\[
\text{bin}(k) = \begin{cases} 
1 & \text{if } P - k + 1 \geq pb \\
0 & \text{if } P - k + 1 < pb 
\end{cases} \tag{28}
\]

\[
ID(i,k,m) = \begin{cases} 
1 & \text{Binary Variable denoting selection of capacity} \\
0 & \text{Otherwise} 
\end{cases} \tag{29}
\]
4.2.5 Constraints

4.2.5.1 Generation constraint on pre-installed plants in the system
Since there are three existing generating units in the system, these units will continue to operate within their operating constraints which are basically their respective maximum and minimum limits on generation, which are expressed as follows:

\[ EP(i)^{\text{Min}} \geq EP(i,b,k) \geq EP(i)^{\text{Max}} \]  
\[ EP(i)^{\text{Max}} \text{ Upper limit on pre-installed generation units at bus } i \]
\[ EP(i)^{\text{Min}} \text{ Lower limit on pre-installed generation units at bus } i. \]

4.2.5.2 Generation constraint on new plants coming into the system
The new generating units installed at specific buses in different years will be constrained in their dispatch by different factors such as maximum capacity factors of respective technologies and maximum available capacity at a bus in a year of such technology. All these constraints are modeled as follows:

\[ \sum_b Pg(i,b,k,m) \cdot Dr(b) \leq Tc(i,k,m) \cdot cf(m) \cdot 8760 \]  
\[ Pg(i,b,k,m) \leq Tc(i,k,m) \]

\[ cf(m) \text{ Capacity factor of the plants based on } m \text{ technology} \]
\[ Tc(I,k,m) \text{ Available capacity at bus } i \text{ in year } k, \text{ of technology } m \]

4.2.5.3 Emission constraint
To impose a cap on total annual emissions by different generation plants so as to ensure environmentally friendly investment plans, a pre-specified maximum annual emission limit is considered, that is modeled as (33) and (34).
\[ Em(k) \leq E_{cap} \]  

Where

\[ Em(k) = \sum_i \sum_b \left[ \sum_m \left( P_g(i, b, k, m) \times Dr(b) \times Ec(m) \right) + \left\{ E(i) \times EP(i, b, k) + F(i) \right\} \right] \times Dr(b) \]  

\[ E(i), F(i) \] Emission coefficients of pre-installed generation plants at bus \( i \), tons/MWh and tons/h respectively

\[ Em(k) \] Total emission by all generators in year \( k \), tons

\[ Ec(m) \] Emission coefficient of new installed generation plants based on technology \( m \), ton/MWh

\[ E_{cap} \] Emission limit imposed by regulator, tons

In this work, we have considered \( E_{cap} = 1 \times 10^{10} \) tons.

4.2.5.4 Dynamic capacity update

This constraint relates the total investments made at each bus by the firm in a given year for technology \( m \), to existing investments in the same technology and the same bus, using an inter-temporal constraint (35).

\[ Tc(i, k, m) = Nc(i, k, m) + Tc(i, k-1, m) \]  

In (35), \( Tc(i, k, m) \) is the total capacity in year \( k \), at bus \( i \), of technology \( m \), which depends on the available capacity of year \( k-1 \), \( Tc(i, k-1, m) \), and decision on new capacity investment in year \( k \) at bus \( i \), \( Nc(i, k, m) \). The investment decision variable \( Nc(i, k, m) \) depends on the objective function while satisfying all constraints and is only selected when the value of the project exceeds the value of keeping the option to invest in the future [1].
4.3 Case Study

4.3.1 System Data

The mathematical model discussed in the previous section is a Mix-Integer linear programming model which is programmed in the GAMS [19] environment, GAMS is used because of its flexibility and available tools in it [22]. This model is designed from the prospective of an individual investor willing to invest in generation sector with discrete unit sizes, while the location option and transmission resources are given to it and firm can give its conditions to centralize planning authority for invest in the generation sector. In which firm can put its conditions for specific transmission line capacity expansion during planning horizon.

In this work the objective of the firm remains maximizing profit. In the case study objective is maximized while recording change in power flows of transmission lines during the plan period, on the basis of those flows, firm can take its decision and can put its conditions while investing in the generation sector that transmission capacity should be increase during plan period according to the requirement, while all other constraints related to plants parameter, future demand in the system and environment constraint should be satisfied by this plan. For transmission system data, modified IEEE 30-Bus system is used in this work, whose pre-installed generators and line parameters are given in Appendix A. It is assumed that the pre-installed generators in this modified IEEE 30-Bus system are providing Active power to the system before planning and will continue serve the system as per their capabilities. To calculate their cost of generation and emission every hour, cost coefficients and emission coefficients are taken in to consideration as mentioned in Appendix A, Table XIX.

Table XVII in Appendix A, shows the technical specifications of transmission lines in the modified IEEE 30-Bus network, the pre-installed line flow capacities are assumed according to the KV of the individual lines, same line flow capacity is considered for the first year operation of the planning horizon, as shown in Table XII.
### Table XII Line Flow Capacities during First Year Operation on the Basis of Line KV

<table>
<thead>
<tr>
<th>Line KV</th>
<th>Line Flow Capacity, MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>132</td>
<td>200</td>
</tr>
<tr>
<td>33, 11</td>
<td>75</td>
</tr>
</tbody>
</table>

**Fig. 25 Modified IEEE 30-Bus Transmission system before investment**

Fig. 25 shows modified IEEE 30-Bus transmission system’s view; it shows 41 available transmission lines between the buses and three pre-installed generators, bold arrows on the buses show loads in the system those are increasing every year at growth rate of 3%. Square boxes shows...
the circuit-breakers between the lines, transformer between different voltage buses are also shown in the figure.

4.3.2 Investment vs. Transmission line loading: Base case study

The optimal investment decisions of the firm are obtained from the solution of the model discussed in Section 4.2. The model is solved for the objective function that is maximization of firm’s profit while using the available transmission resources without capacity cap on the transmission line of given transmission system, optimal decisions are obtained, as shown in Table XIII. Fuel prices are assumed to be increasing at 3% and 2% per year for gas and coal price respectively. Here IRR parameter is not considered to make framework less complex, IRR issues are considered for the future work.

<table>
<thead>
<tr>
<th>Year of Commissioning</th>
<th>At Bus-7 Coal Unit (MW)</th>
<th>At Bus-29 Gas-Fired Unit (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>

These capacity investment decisions assume that transmission capacity expansion would be provided by the centralized planning authorities to support the investment plan in a cost effective manner. After obtaining the optimal decision during the plan horizon their effect on transmission line loading is studied, which shows that some of the transmission line get overloaded during the 15-year planning period as demand growing 3% every year, the transmission lines capacities that are expected to be overloaded during the plan period are shown in Table XIV and Fig. 26.

It is to be noted that this analysis is being carried out from the perspective of one single investor, and how its investment decisions will cause transmission line overloads. In practical systems, there will be multiple investors, and hence their respective decisions will also impact the transmission line loadings. The central planning authority needs to take into account all such individual plans and
examine the overall impact on the transmission system and hence develop the appropriate transmission expansion plan for the system.

### Table XIV Transmission Line Overload Due to New Investments

<table>
<thead>
<tr>
<th>Line</th>
<th>Line KV</th>
<th>Capacity Available (MW)</th>
<th>Max Flow (MW)</th>
<th>Overload (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7—5</td>
<td>132</td>
<td>200.00</td>
<td>299.40</td>
<td>99.40</td>
</tr>
<tr>
<td>7—6</td>
<td>132</td>
<td>200.00</td>
<td>260.80</td>
<td>60.80</td>
</tr>
<tr>
<td>9—10</td>
<td>33</td>
<td>75.00</td>
<td>124.60</td>
<td>49.60</td>
</tr>
<tr>
<td>27—25</td>
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<td>33</td>
<td>75.00</td>
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<td>18.50</td>
</tr>
</tbody>
</table>

Fig. 26 Overloading on the IEEE 30-Bus system from investments
4.3.3 Effect of high fuel price on investment and line overloading

A study is carried out on the same modeling framework as discussed in Section-4.2, with high fuel prices, where on top of fuel price annual escalations there is a further increase by 10% and 15% for gas and coal respectively. Optimal investment plan obtain from this study is shown in Table XV. After obtaining the optimal decision during the plan horizon their effect on transmission line loading is studied, it is concluded that the transmission lines that get overloaded (Table XVI) during the 15-year of planning period are lesser than the base case, shown in Fig. 27, and it can be attributed to the change in the investment decisions for two cases, which significantly affect the line flows in the transmission system.

### Table XV Optimal Investment Decisions during the Planning Horizon

<table>
<thead>
<tr>
<th>Year of Commissioning</th>
<th>At Bus-7 Coal Unit (MW)</th>
<th>At Bus-14 Gas-Fired Unit (MW)</th>
<th>At Bus-20 Gas-Fired Unit (MW)</th>
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<tr>
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</table>

### Table XVI Transmission Line Overload in High Price Scenario

<table>
<thead>
<tr>
<th>Line</th>
<th>Line KV</th>
<th>Capacity Available (MW)</th>
<th>Max Flow (MW)</th>
<th>Overload (MW)</th>
</tr>
</thead>
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<td>7—5</td>
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<td>96.90</td>
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<td>60.80</td>
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<td>33</td>
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<td>49.10</td>
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</table>
4.4 Conclusion

The work presented in this chapter deals with the investment planning by an individual investor firm in the generation sector with certain candidate investment options specified in terms of capacity size, location specific technologies, and available transmission resources. The modeling framework includes dc power flow representation of the transmission network alongside the long-term generation expansion planning model. It is concluded from the studies that with demand growth, the investor’s investment decisions will introduce overloading on some of the transmission lines and the central planning authority will have to take a holistic view of the entire system taking into consideration the investment decisions of all parties and how they impact the system. A coordinated transmission
expansion plan will be eventually required to support the new investment decisions from all the investors, which will be forthcoming over the 15-year horizon.
Chapter 5
Conclusions and Future Work

5.1 Conclusions

This thesis presents a comprehensive modeling framework for generation capacity expansion planning utilizing dynamic linear programming models. The proposed models are capable to take into account the technical and economical parameters of the investor firms and the electricity market effects, and determine the optimal investment plans that seek to maximize the investor’s profit while determining the optimal internal rate of return (IRR). Multiple scenarios and case studies have been constructed and tested in the thesis, each with their own effects on the financial balance and economics of the firm.

In Chapter-2 a linear programming model is formulated from the perspective of investor firms to determine the optimal generation investments over the long-term. Since the investor is a competitive market player, and not a centralized planner, it has no obligation to seek a demand-supply balance. Its main driver for investment decisions are profit and the rate of return from the project. The chapter presents a method to arrive at the optimal IRR from the investment plans. The proposed modeling framework also incorporates an emissions cap on the investor and the effects of variations of the emissions cap on its financial returns are also examined.

In Chapter-3 the planning model proposed in the earlier chapter is upgraded to take into consideration discrete unit sizes. The consideration of discrete unit sizes makes the planning model a mixed integer linear programming framework and only three specific unit sizes are considered, 100 MW, 200 MW and 300 MW. This necessitates higher budgetary allocations. The model is solved for a 15-year plan horizon to examine the effects in a more practical manner. Various scenarios and cases are considered for analysis.

To further analyze the generation expansion planning framework for the investor, in coordination with the transmission resources of the whole system, a new model is developed and presented in Chapter-4 that incorporates the transmission network model in detail. A dc load flow representation is used to model the transmission system and the long-term planning model is now modified to develop the investment plans that seek to match the location demand and the demand growth in the long run. The investment decisions so obtained are now locational plan decisions and provide the planner with information on where, when, what capacity, and what technology of generation to be
invested in. These optimal plans also provide information to the transmission system operator on line overloads arising in the long-run, and how the transmission system has to be augmented to cater to the generation investments.

These different planning frameworks provide a promising direction towards integrated power system planning in the context of competitive electricity markets with consideration of many of the technical and non-technical parameters of electric power systems.

### 5.2 Main Contributions from the Thesis

The thesis presents new modelling approaches which are specifically geared towards the investors in the context of competitive electricity markets. Different technical constraints as well as finance parameters are simultaneously considered in the long-term decision making frameworks. Comprehensive sensitivity studies have been carried out to examine the impact of various parameters on investment decisions. The following are the main contributions of the thesis:

- A mathematical model for generation planning specific to investor firms has been proposed. The mathematical model is further improved to incorporate binary variables in order to consider discrete unit sizes, and subsequently to include the detailed transmission network representation. The proposed models are novel in the sense that the planning horizon is split into plan sub-periods so as to minimize the overall risks associated with long-term plan models in the context of deregulation.

- The work assesses the importance of arriving at an optimal IRR at which the firm’s profit maximization objective attains an extremum value. This is a significant contribution of the thesis since no work has been reported that addresses the optimal rate of return for investors. Using the proposed approach, decisions on project approvals can be made, when the optimal IRR meets a Minimum Acceptable Rate of Return (MARR) barrier.

- The analytical studies reported in the thesis provide a new insight into the electricity market effects on individual investors and their decision making process on generation investment, which have not been reported in the literature. It is observed from the studies that as electricity prices increase, the optimal IRR increases linearly up to a point and then it increases with a steeper slope. The effect of budget limit increase has a similar impact on the firm’s profit, which also increases linearly. However their impact on IRR is significantly different.
Although in the literature, generation planning using discrete unit sizes has been considered, but no analysis is reported on how the IRR would be affected when such discontinuous variables are present. Furthermore, how the IRR so obtained, would compare with that obtained in the continuous case. This thesis presented a detailed and new perspective on this subject. It is observed from the studies that the optimal plan of the firm with discrete unit sizes yields a higher IRR as compared to that with continuous unit sizes. It is observed that if electricity prices increase then the firm is expected to invest more even if budget is limited.

A contribution of the thesis is the novel development of the investor’s planning framework considering transmission network flows, and location specific resource constraints. Such a model is important from the view point of determining the optimal investments and also to understand the impact of fuel prices and other parameters on line overloads.

5.3 Scope for Future Work

This work can be further extended to examine various other pertaining issues such as-

- Consideration of the volatility of different market parameters such as fuel prices and the electricity market price. In the present work sensitive analysis has been carried out to examine their effects, but a more comprehensive analysis is needed in the future.

- The uncertainty associated with price and demand variations in the long-term may also be considered in detail, and appropriate stochastic modeling framework need to be developed to examine their effects.

- There is a need to coordinate the individual investor’s plans and arrive at a system’s level expansion plan. This work can be extended to formulate such a coordination scheme for the central planning authority so as to determine the overall optimal plan for the system after taking into account the plan submissions from multiple independent investors.

- There is a need to develop a coordinated plan for the generation and the transmission system in the long-term. Such a coordinated plan must take into account the individual investors’ plans as determined from their respective planning models, of the type presented in this thesis, and integrate them in a comprehensive transmission expansion planning model.
### Appendix A

#### Technical Parameters of IEEE 30-Bus System

**TABLE XVII Line Parameters of IEEE 30-Bus System Transmission Network**

<table>
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<tr>
<th>Line</th>
<th>Resistance, P.U.</th>
<th>Reactance, P.U.</th>
<th>Shunt, P.U.</th>
<th>Line KV</th>
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TABLE XVIII BASE YEAR DEMANDS ACCORDING TO THE DEMAND BLOCKS

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<th>Peak Demand (MW)</th>
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Appendix B
Publication From This Work

This work resulted in the following publication.

References


