# Investigation of Piano Soundboard Voicing Techniques and Their Impact on Tone

by

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#### Abstract

This thesis acts as a first step in connecting the subjective description of piano tone with quantitative measurements of changes in the vibrational and acoustical properties of a piano. Voicing techniques used by piano technicians are applied to a fully strung test piano and a series of tests are undertaken to measure the effect they have on piano tone. The addition of weights and riblets to a piano soundboard. two commonly practiced methods used to change the tone of a piano and even out the transition between the low tenor and high bass bridges, are examined in detail in this thesis. Modal analysis is the first test method applied and is used to determine the mode shapes and the modal properties of the test soundboard. The addition of small masses and riblets is shown to have the ability to change modal properties to varying degrees and connections are made between the output sound, attachment locations, modification locations, and mode shapes of the soundboard. A new technique, piano tone mapping, is introduced as a visualization method that displays the harmonic structure of the entire piano in one image. Using the technique creates a unique fingerprint of a piano that can be compared in various ways to other pianos. The piano tone maps reveal that initial magnitude and decay rate of the output sound are both decreased for notes with attachment points close to modification locations where weights and riblets are added. Impedance testing is also conducted for a number of different modifications. These tests reveal that riblets and weights each have their own frequency bands of influence, with the weights creating impedance changes primarily below 800 Hz and riblets creating impedance changes above 500 Hz, with a region of overlap between 500 Hz and 800 Hz. Tests also revealed that changes in impedance created by these voicing techniques were local in effect, with a region of influence less than 10 cm in radius extending from the location where the weight or riblet is attached.

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### Chapter 1

### Introduction and Literature Review

### 1.1 Introduction

The modern piano is an incredible piece of technology that connects complex mechanical systems to human emotion through music. The soundboard of the piano is the component responsible for the conversion of the string's vibrational energy into sound and it is at this interface between the piano and the world that it gets its voice. The ability to change this voice is something desired by builders, players, and technicians alike to meet their individual needs. This thesis attempts to gain a deeper understanding of the piano soundboard and the relationship it has to the voice of the piano.

#### 1.1.1 Soundboard Construction

The soundboard can be broken down into several components that each play a specific role in transforming the energy from string vibration into sound. These components are the bass and tenor bridges, the panel, and the ribs seen in figure 1.1.

The bass and tenor bridges are the mechanical connection between the string and the panel. The tenor bridge is the termination point for the majority of strings that correspond to the higher pitches. The bass bridge has fewer strings that sound lower pitches, usually about 20-25 of the 88 notes. All modern pianos are overstrung, meaning that the strings attached to the bass bridge are physically seen to cross over top of some of the strings attached to the tenor bridge. As a result the bass bridge is not a direct continuation of the tenor bridge, but is instead offset behind the tenor on the soundboard. Typically the bridges are made of either a solid piece of hard maple or several laminated pieces. The bridge is meant to be a



Figure 1.1: Typical grand piano soundboard

solid, stiff connection that efficiently transmits vibrational energy to the panel with as little loss as possible.

Soundboard panels are constructed of thin pieces of quarter-sawn softwoods. One of the most common woods used is sitka spruce, but due to deforestation and poor logging practices there are limited supplies of sitka spruce currently available. Appropriate local softwood alternatives such as eastern white spruce are now used [1] extensively in modern piano construction. Four to six inch wide strips of spruce are planed to approximately 8-10 mm thickness and then edge-glued together to form a sheet large enough for the piano. In modern pianos the perimeter is usually thinned further to allow more flexibility [2]. Analogous to a speaker cone, the panel's large surface area and light weight allow it to produce compressions and rarefactions in the air that we hear as sound waves. The mass, stiffness, and efffective vibrational area are the most important vibrational parameters of the panel.

In the cross grain direction (i.e. perpendicular to the direction of the grain of the wood), the panel is quite flexible. Ribs are added to the panel in the cross grain direction to add stiffness and help support the static force downwards on the panel caused by string tension down bearing, a distributed force that ranges from 0-2500 N/m [3] along the length of the bridge that is the component of string tension normal to the soundboard due to the difference in height between the bridge and the mechanical end terminations of the string (see figure 1.2). The ribs can also play a role in forming a curvature on the panel called crown. There are two methods to attain this curvature in the panel: rib-crowning and compression-crowning. Rib-crowning requires the ribs to be shaped in an arch with the panel being forced to

the shape of the ribs to achieve crown. The other approach, compression-crowning, uses flat ribs and a dried panel that is brought back to standard moisture content which in turn causes the panel to expand and cause the ribs to curve and create the crown. Downbearing and crown are two very important parameters of soundboard design that can have a significant impact on the tone of the piano [2].



Figure 1.2: Downbearing due to string tension.

#### 1.1.2 Piano Tone

In terms of musical acoustics a tone can be thought to have "three variables: pitch, loudness and timbre" [4]. Pitch refers to the fundamental frequency of a tone and in the western tradition is defined most commonly in terms of an equal temperament scale. It does not in any way relate to the complexities of the higher harmonics or transient properties of a tone. An example of standardized pitch is A4, A above middle C, with a fundamental frequency of 440 Hz (see figure 1.3). The second variable, loudness, is a description of the amplitude of vibration of an object. The larger the amplitude of vibration, the louder the tone is that the instrument creates. Loudness is measured on a logarithmic scale (dB) due to the logarithmic nature of human hearing. The final variable, timbre, is not as easily defined as pitch and loudness. Timbre is often referred to as the the quality of a sound and is the property of tone that differentiates the sound of a guitar say from that of a piano or any other instrument. There are many complex factors that affect timbre such as spectral centroid, spectral flux, attack time, and harmonic envelope [5], factors that are predominantly related to the various frequency components of the tone.

Although tone is a mathematically complex characteristic to quantify, the human brain is exceptionally well trained to differentiate between tones in a subjective manner. The ability to distinguish between instruments that sound 'warm', 'brassy', or 'harsh' is an example of the subjective description of tonal properties. Musicians are quite willing to offer opinions on the quality of the sound an instrument makes, but almost exclusively with this kind of subjective terminology. This is where the greatest challenge is introduced for anyone studying musical acoustics. Making the connection between these subjective descriptions of sound and



#### NAME - NUMBER - FREQUENCY (Hz)

Figure 1.3: Standard note names, numbers, and fundamental frequencies.

the quantifiable physical characteristics of the instrument is truly what is needed to engineer the sound of a piano or any other instrument. To make this connection requires a complete understanding of the psychoacoustics, acoustics, and vibratory and mechanical properties of complex systems.

### 1.2 Literature Review

#### From the Forest to the Concert Hall - Fandrich, 2007 [1]

Del Fandrich, a well known and respected piano technician, delivered this presentation to a meeting of the Piano Technicians Guild in 2007. It details all aspects of the soundboard from ideal growth conditions for the trees to more modern attempts to improve soundboard output by moving away from the traditional designs common in the industry. The importance of old growth forests in the production of top quality tonewoods, the basic engineering concepts required to design soundboards, and a brief explanation of a new design that creates a floating bass region are the major points examined. Downbearing and its impact on tone is also discussed in some detail.

#### Piano Servicing, Tuning, and Rebuilding - Reblitz, 1993<sup>[2]</sup>

Reblitz's text on the maintenance and repair of pianos has become a standard reference work for anyone interested in understanding how a piano is designed and constructed. In this volume the basic construction of the piano is laid out in detail and the functions of each component are explained. From this work a basic understanding of the relationships between the many components in both a grand and an upright piano can be attained.

#### Acoustic correlates of timbre space dimensions: A confirmatory study using synthetic tones - Caclin, McAdams, et. al., 2005[5]

In this paper the authors use synthetic tones to determine which characteristics of sound are most directly linked to a listener's perception of timbre. The experiments showed that attack time, spectral centroid, spectral flux, and spectrum fine structure are all important parameters of timbre. All of these attributes can be applied to the study of the piano and represent quantifiable descriptions of piano tone that can be used to distinguish between different (or modified) pianos. Attack time and spectral center of gravity were found to have the most significant effects on perceived timbre. Attack time is defined as the amount of time taken to reach the initial maximum peak. Spectral center of gravity is given by:

$$SCG = \frac{\sum_{n} n \times A_{n}}{\sum_{n} A_{n}} \tag{1.1}$$

where n is the frequency and  $A_n$  is its amplitude.

#### Spectrum analysis and tone quality evaluation of piano sounds with hard and soft touches - Suzuki, 2007[6]

The method by which a pianist strikes the key is referred to as touch and in this paper Suzuki investigates whether any meaningful spectral difference are be seen between soft and hard touches. To ensure consistency and that spectral differences are not due to fluctuations in spectrum that occur due to the amount of input energy put into the strings, the overall sound levels of the soft and hard touches were kept as close as possible. From the results only a minor, almost insignificant variation in spectrum was found in the frequency analysis. Expert listeners are also asked if they were able to hear any difference between the hard and soft touches and it is shown that some are able to differentiate between the two touches. From this work the basic problem of the discussion of piano tone is evident: the minor fluctuations in tone are practically unnoticeable using standard frequency analysis, but expert listeners are able to tell them apart. The finesse required to quantify piano tone and the power of the human auditory system is evidenced by this.

# Vibration and sound radiation of a piano soundboard - Suzuki, 1986[7]

An early investigation into the vibrational and radiative properties of the piano soundboard, this paper identifies mode shapes and the sound intensity of a piano soundboard. Three main regions of radiative efficiency are identified: low efficiency below 80 Hz, fair efficiency from 100 Hz - 1 kHz, and high efficiency above 1.4 kHz. These regions correspond to frequency bands below the first mode of the soundboard panel, in the modal region, and above the modal region, respectively. The piano Suzuki experimented on was unstrung, with the plate removed. As a result the modal results he obtained cannot be directly compared to a fully strung piano. The work done in this paper serves as a basic foundation upon which more evolved analysis can be based using more modern equipment.

#### Acoustics of Pianos - Suzuki, Nakamura, 1990[8]

An overview of a variety of topics, this paper serves as a review of a variety of fields of piano research. The role of the action, hammers, the hammer-string interaction, string vibration, and the soundboard's vibration and radiation are all discussed. Along with Conklin's series in the Journal of the Acoustical Society of America (JASA), this paper serves as an excellent introduction to a large number of detailed piano design concepts. Early Finite Element Modeling (FEM) is compared to soundboard research with regards to the modal vibrations of the soundboard. The complexities of the strings are not included in the models and this affects the results. Reasonable agreement between the FEM and modal analysis is found, but it is concluded that much more physical testing is required to fully understand the radiative properties of the piano soundboard.

#### Design and tone in the mechanoacoustic piano. Part I-III. -Conklin, 1996[9, 10, 11]

Conklin's work in this series of papers in the Journal of the Acoustical Society of America (JASA) provides a detailed introduction to the concept of tone and its relation to the various parameters available to a piano designer. The series begins with a discussion of the role of the hammer, its mass, and its striking point and the relationship these have with the output sound of the piano. As the hammer is the input source of energy into the string it is logical that it plays a large role in a piano's tone. The other valuable result from this section is the confirmation of the importance of striking point on harmonic structure. In an ideal string an infinite attenuation of the *n*th harmonic is expected when the striking point is set at 1/nthe speaking length. In reality the attenuation is not infinite, but is significant, a result that will later be discussed in this thesis.

In the second paper Conklin discusses the soundboard with regards to its construction, material properties, modes of vibration, and mobility. Typically soundboards are constructed from sitka spruce or other softwood species with similar mechanical properties. The difficulties of using wood are observed in the large variation of mechanical properties within a single species. For 100 samples of quarter sawn sitka spruce the density was found to range from 0.343 to 0.501 g/cm<sup>3</sup> and the elastic modulus in the grain direction varied from 7.9 to 17 GPa. This variability is a major challenge for piano builders as these properties are not obvious from sight and touch alone. Extensive experience or a scientific method is required to ensure the most appropriate wood is chosen, but with the movement towards mass manufacturing techniques it becomes obvious that the ideal materials are not being consistently used.

Conklin then moves on to discuss the vibrational properties of the soundboard. Modal tests are conducted on an unstrung grand piano soundboard which is mounted in its rim and has had the bridge removed. Chladni patterns are created to determine modal frequencies and results are found to be similar to Suzuki's [8]. It is interesting to note that the first mode was found to be at 49 Hz, a value much lower than the 73 Hz obtained by Keane [12] for a fully strung grand piano. It is clear that the change in vibrational properties created by the addition of the bridge and the down bearing force of the strings is significant, and with this in mind that Conklin's results should be viewed. Mobility, the inverse of impedance, is then measured at several locations for a grand piano that is unstrung, but did have the bridge attached. It is shown that vibrations normal to the plane of the soundboard are not the only important factor in the output sound of the piano, the longitudinal vibrations along the soundboard are also found to be important contributors. Soundboard tapering is also examined with relationship to the modal frequencies and the response curve is found to shift significantly as tapering increases. Finally the duration of piano tones is discussed with regards to the 'Q' factor which is related to damping. Pianos tend to have two distinct regions of decay, the first region being rapid, with the second at a smaller decay rate. The transient development of piano tone is an important issue and will be examined further in this thesis.

Conklin's third paper discusses piano scale design; the design of the plate, string length, string dimensions, soundboard size, and bridges. Scale design is a compromise between the ideal condition and those prescribed by the space and materials available. In an ideal condition the length of strings would increase approximately logarithmically, allowing one diameter and one string tension to be used for all strings. In the ideal case typical string lengths for a 2.74 m piano would vary from  $5 \,\mathrm{cm}$  in the high treble to  $5.3 \,\mathrm{m}$  in the low bass. This creates a problem because the strings would need to be twice as long as the piano. This means that piano designers need to use special methods to allow everything to fit inside the piano. Decreasing string tension and increasing the linear density of the strings are two ways of decreasing the required string length, but both have their own problems. Decreasing tension reduces the power transfer and results in a loss of output sound pressure, while increasing linear density can have a negative impact on inharmonicity. The contribution of longitudinal string vibrations is also emphasized as this vibrational energy can have a significant impact on the harmonic structure of the output sound of a piano. Often these longitudinal vibrations do not coincide with the harmonic structure of the transverse vibrations of a string. As a result the longitudinal peaks will appear between the harmonic peaks of a particular note and will contribute unexpected frequencies to its output sound. It is possible, as Conklin suggests, to tune these longitudinal harmonics if the string properties are known, to allow them to better integrate with the harmonic structure of each note. Scale design is a complex and challenging task as many factors need to be taken into account and numerous compromises from the ideal need to be made. It becomes obvious that an intimate understanding of all aspects of the piano is required by the piano designer.

#### Tone Compensator for Piano Soundboards - Conklin, 1986[13]

Conklin patented (now expired) the idea of using small weights in the range of 50-200 g to compensate for tone discontinuities between the tenor and bass bridges in a piano. By directly attaching the mass to the bridge near to notes that have amplitudes greater than nearby notes he proposes a reduction in sound pressure level and an increase in sustain can be achieved. The contents of this patent will be tested directly in this thesis.

#### The strings and the soundboard - Wogram, 1990[14]

Wogram presented his experimental impedance results as a part of a lecture series on the acoustics of pianos sponsored by the Royal Swedish Academy of Music. His experiments consist of a systematic examination of the mechanical impedance along the bridge in 14 locations. The results show an initial region characterized by peaks up until 100 Hz, after which the impedance curve is seen to steadily decrease. These results were later questioned by Giordano [15], and it is convincingly shown that Wogram's measurement technique is flawed due to a physical decoupling of the impedance head from the bridge at high frequencies. Wogram also performed his tests on an unstrung upright piano, so their applicability directly to the output sound of a fully strung grand piano is questionable as they do not consider the physics of the additional mass and stiffness created by the strings and the downbearing force.

Modal tests are also conducted and the connection between modal nodes and impedance peaks is established. Wogram proposes the use of computer simulation to aid in scale design so that notes of certain frequencies do not end up being attached through the bridge to the soundboard at the anti-node of corresponding modes where impedance is very high and energy transfer is low. By doing this Wogram hopes to achieve a piano with a more even output sound loudness for all notes.

#### Voicing the Soundboard with Weights and Riblets - Richmond, Fandrich, et. al., 2007[16]

This article from the Piano Technicians Journal is a discussion with three well recognized technicians: Del Fandrich, Ron Nossaman, and John Rhodes. They discuss the problem of voicing a piano and the use of weights and 'riblets'. Similar to the method patented by Conklin [13], the addition of weights to the end of the tenor bridge is claimed to help even out the transition between notes on the low tenor and high bass by locally increasing mechanical impedance. One technician suggests adding weights from 50-200 g in mass until voicing problems 'go away'. Fandrich's approach is to use what he calls 'riblets' to locally add both mass and stiffness when voicing a piano. A riblet is a short piece of wood shaped in a similar manner to the ribs of the piano. Fandrich experimentally concluded that a 5 inch length, 7/8 inch with, and 7/8 inch height was most appropriate. Both techniques aim to reduce the rate at which string energy is transferred to the soundboard with the hopes of increasing sustain. The authors of this article consider the improvement in tone after the use of these techniques to be 'considerable and obvious'. The challenge of describing changes in piano tone is obvious here, as descriptions such as removing 'explosive sound' or 'boom' are all that are used. A quantitative analysis of these voicing techniques will be the focus of this thesis.

# Mechanical impedance of a piano soundboard - Giordano, 1998[15]

Measurements of mechanical impedance are reported by Giordano to determine whether or not the results reported by Wogram [14] are accurate and what role the ribs have on impedance in the kHz frequency range. Giordano takes issue with Wogram's measurement technique and convincingly argues that Wogram's results above 1 kHz are incorrect due to a decoupling between the impedance head and the soundboard that was caused by the use of wax to attach the impedance head to the soundboard. Giordano's measurement technique was emulated in this thesis and results similar to his were obtained. Impedance was found to increase in the modal region ( 100 Hz to 1 kHz), above which it held constant until approximately 3 kHz.

#### Simple model of a piano soundboard - Giordano, 1997[17]

In this paper Giordano sets out to explore the parameters required to create an accurate model of the piano soundboard. His earlier impedance results [15] serve as the baseline for the desired general behaviour of the system represented. Models are developed from the simplest case of an isotropic panel without ribs to an anisotropic panel with ribs attached. The best agreement with his experimental results was found in the latter case, but Giordano questions whether this is simply fortuitous given that the model still needs a great deal of improvement as it does not consider the bridges, three dimensional motion of the panel, or soundboard crown. From his model Giordano stresses the importance of the ribs to the low frequency impedance results.

#### Sound production by a vibrating piano soundboard: Experiment - Giordano, 1998[18]

There are many derived units that can be applied to understand vibrational motion and in this paper Giordano uses the ratio of the sound pressure level (p) over the velocity of the bridge  $(v_b)$ . Swept sinusoidal signals are input into the bridge of the piano using a shaker, with an accelerometer mounted to the bridge, and a microphone placed near the soundboard. The value of  $p/v_b$  is found to be frequency dependent and related to both the impedance and vibrational modes of the soundboard. A test with the shaker in the longitudinal direction of the string is also performed, once again confirming that longitudinal vibrations do play a significant role in the vibration and output sound of a piano as observed by Conklin [10].

# Motion of a piano string: Longitudinal vibrations and the role of the bridge - Giordano, Korty, 1996[19]

In this paper the authors examine which parameters need to be added to the soundboard model to create more realistic sounding strings. The influence of longitudinal vibrations and the three dimensional interface at the bridge are the major considerations examined. From experimental measurement Giordano shows that there are significant vibrations in all three orthogonal directions at the bridge string interface and that characteristics can be found that are common in the output sound signal, the transverse string vibration, and vibration in the three orthogonal directions. This work indicates that accurate string models need to consider the coupling of three dimensional modes of vibration through the bridge.

# On Hearing the 'Shape' of a Vibrating String - Giordano, 2002[20]

As an introduction to the string equation this paper provides a brief explanation of the importance of the striking point of a string on its harmonic structure. The fundamental string equation is examined and Giordano explains that if a string is excited at a point L/n distant from a termination point, where n is the mode number and L the string length, you are exciting the string at a node of the nth mode. In the ideal case the frequency that this mode corresponds to will receive no energy and its corresponding harmonic peak will be completely attenuated in the strings frequency response. In the real case attenuation is not complete, but is still significant [9]. The guitar is used as a primary example in this paper as it most closely resembles an ideal monochord. The frequency response of a piano is also examined and found to contain characteristic evenly spaced dips in the harmonic response due to the hammer strike point location. These results were confirmed experimentally in this thesis and are also visible as significant structures in the piano tone mapping technique described in the following chapters.

#### Numerical simulation of a piano soundboard under downbearing - Mamou-Mani, Frelat, et. al., 2008[3]

The authors present a model of the piano soundboard that includes ribs, bridges, the effect of crown, and downbearing in the modal region. The results are compared with simpler models and it is concluded that downbearing and crown both individually contribute to the modal properties of the piano soundboard. The frequency response curve tends to be shifted upwards with increasing downbearing force for soundboards modeled without crown. The opposite is true for soundboards modeled with crown, where the frequency response curve is shifted downwards with increased downbearing. Mamou-Mani acknowledges the value of these models, but also notes his main objective is to make a link between these changes and perceivable changes in timbre, something he believes is possible by using his model to create synthesized piano sounds.

# Piano soundboard: structural behavior, numerical and experimental study in the modal range - Berthaut, Ichchou, et. al., 2003[21]

The authors preparea an extensive FEM model of the piano soundboard which is compared to their own experimental study in the modal region. Analytical models of the soundboard are shown to be inaccurate due to the amount of complexity required to accurately predict the vibrational properties of a piano soundboard. Unlike analytical models, the FEM model presented in this paper was able to accurately include the complex geometry and material properties of a real soundboard. THe authors do state that the model is only valid for the low frequency, modal region, up to approximately 500 Hz. The mid frequency and high frequency performance of the soundboard is concluded to be beyond the capabilities of FEM or modal analysis, but is emphasized as being of great importance to a complete understanding of the sound of a piano. This paper serves as another indication of the limitations of FEM modeling and modal analysis in the understanding the mechanics of the piano soundboard in frequency bands above the modal region.

## Effects of relative phases on pitch and timbre in the piano bass range - Galembo, Askenfelt, et. al., 2001[22]

In this paper the authors experimentally test a listener's ability to distinguish timbral differences in synthesized bass notes by varying the phase of the various frequency components in the test signal. They use 100 harmonics in the creation of their test signals, as examination of frequency data shows significant contributions even at this high order harmonic, to convincingly reproduce the harmonic spectrum of real bass notes. Phase relationships are shown to have a clear effect on pitch and timbre, and when inharmonicity is factored into the harmonic content it was found that only random starting phases of the harmonics lead to a steady timbre for the duration of a note. Synthesized sounds played over headphones are found to have the most audible differences, as sounds played over loudspeakers in a room are subject to room acoustics that effectively randomize the signal. When applied to the piano the main conclusion is that phase randomization is required for constant pitch and tone perception by a listener.

#### Tonal properties of the pianoforte in relation to bass bridge mechanical impedance - Exley, 1969[23]

Exley provides a detailed analytical and experimental study of the relationship between mass, compliance, and piano tone. Mass loading similar to that presented by Conklin [13] and several piano technicians [16] is presented and its relationship to mechanical impedance in the bass region of the piano soundboard is discussed for small pianos that have inadequate low frequency response. Exley is subjectively able to improve the tone of the bass notes and is able to correct inharmonicity within the first few harmonics of the piano by adding two vibrating mass systems. The results are largely simplified analytical relationships that are then applied to the more detailed problem of the piano soundboard, but Exley does make some connections between his two-mass systems and the performance of large soundboards and the commonly used cantilevered bass bridge. This style of bridge is used to move the attachment point of the bass bridge away from the rim of the piano where it would encounter increased stiffness.

#### Spectral envelope sensitivity of musical instrument sounds -Gunawan, Sen, 2008[24]

In this paper the authors examine the perceptual sensitivity of a listener to changes in the harmonic envelope of a synthesized sound. Trumpet, clarinet, and viola sounds are sampled and then zero phase filters are applied to attenuate certain frequency bands of the sound. This creates a comparison between sounds such as a case where low frequencies are attenuated while high frequencies are left unaltered. A progression of filters is used on the signals to determine which region of the harmonic spectrum is the most important to the perception of timbre. It is found that lower order harmonics are more important to the perception of timbre and correspondingly require less attenuation for a listener to identify a change.

#### Interferometric studies of a piano soundboard - Moore, Zietlow, 2006[25]

One of the main challenges in studying the piano is its structural complexity which makes it difficult to gain access to its components while it is in an operational state. Gaining access to the soundboard for examination is complicated by the strings and the frame which cover the majority of the soundboard and make it difficult to attach instrumentation. Removing the strings and the frame allows access to the soundboard, but also removes the downbearing force, a force which has been shown to contribute significantly to the soundboards vibrational properties [3]. The use of interferometric techniques to visualize vibration modes is advantageous because it can be applied without requiring physical access to all regions of the soundboard panel. The authors examine an upright piano using the technique, which in a similar manner to Chladni patterns, highlights the shape and orientation of the physical vibration modes of a plate. Modes are determined for both a strung and unstrung piano and the unstrung results are compared to an FEM model. Reasonable agreement between the model and experimental results is achieved, but the authors emphasize that the point of this paper is to highlight the usefulness of the interferometric technique to the analysis of large vibrational bodies.

#### An evaluation of piano sound and vibration leading to improvements through modification of the material properties of the structure - Keane, 2006[12]

Keane undergoes a detailed study of the piano in this extensive PhD thesis. Topics include a statistical analysis of piano music, modal analysis of both upright and grand pianos, FEM modeling of an upright piano, and the measurement of the sound radiation properties of the piano case. Keane begins by examining the distribution of pitches in piano music and finds that the distribution is approximately normal. The notes at the extremities of the piano are the ones most likely to have been compromised in terms of scale design and with this understanding of the typical distribution of pitches it can be seen that the least compromised notes tend to appear most in composed music. In terms of modal analysis Keane uses two different techniques to measure upright and grand pianos. A laser vibrometer is used to measure the upright piano in combination with a shaker and a force transducer. This setup is effective because the soundboard is visible from the bottom of the piano and allows for easy use of the vibrometer. To measure the modal vibrations of the grand piano a roving accelerometer is used in conjunction with a fixed shaker and force transducer, with the accelerometer attached at various points under the piano. Keane's results indicate that there are significant vibrational differences between the grand and upright piano, and that some vibrational modes of the soundboard are aided by the deformation of the rim of the piano. The modal tests conducted for the present thesis followed a similar procedure to that of Keane and the results obtained are in agreement with those presented in Keane's work.

### **1.3** Contributions of This Thesis

The modern piano is the product of three centuries of experimentation and in most cases modifications in design were not the product of an analytical approach, but rather speculation and experience. As a result, the design of the modern piano is heavily influenced by traditional methods and reverence for certain builders like Steinway and Bösendorfer. With the development of modern tools for sound and vibration analysis we can begin to remove some of the mystique of these instruments and move forward with a more quantitative approach.

As discussed above, the description of piano tone is a major challenge for anyone trying to make changes to the instrument. The distance between the psychoacoustical understanding of piano tone and the physical parameters of piano design that affect it is still quite large. This thesis has two purposes: first, to provide a tool to assist in the visualization of piano tone for an entire piano; and second, to provide quantitative measurements of the piano after typical soundboard voicing techniques are used to influence piano tone. The objective of voicing is typically to even out the response of the piano from note to note, eliminating any unusual tone characteristics that would make an individual note stand out from the rest. In effect this thesis creates a dataset that links voicing techniques to the physical changes in the way the piano vibrates and produces sound. In future work this dataset could be connected to a psychoacoustical interpretation of the change in tone, thus creating a bridge between the physical changes in the piano and our subjective interpretation of the sound it creates.

The first component of this thesis will be the introduction of a mapping technique that allows the tone of the piano to be visualized in an intuitive and meaningful way. The technique, referred to hereafter as piano tone mapping, extracts information from recordings of piano notes and then plots this information as a surface for all 88 notes of the piano in one image. Piano tone mapping allows for a graphical representation, or fingerprint, of every piano's unique tone structure. It can also be used to visualize changes in a piano due to modifications or repairs. Transient analysis is also possible as maps can be created for different portions of the decay of the notes. Animations can be created from these maps that visualize the tonal decay of the piano. With this tool physical changes to the piano can be quantified in terms of salient acoustical and vibrational parameters.

The second portion of the thesis consists of experimental results related to

soundboards obtained through several different methods. The purpose of the experiments is to make a connection between standard piano soundboard voicing techniques used by piano technicians and the quantitative effect they have on the vibration and sound output of the piano. Modal analysis, impedance measurements, sound frequency analysis, and piano tone mapping will all be used to quantify the changes that result from the voicing techniques and conclusions will be drawn about the ability of the voicing techniques to affect piano tone.

Chapter 2 outlines the procedures and methods used. It details the development of the tone mapping technique, impedance measurements, and modal analysis. The test conditions and a description of the main test piano is also provided. In chapter 3 the modal analysis results are presented and discussed, followed by the tone mapping results in chapter 4, and the impedance results in chapter 5. Conclusions and future work are presented in chapter 6.

### Chapter 2

### Experimental Procedure and Analysis Methods

### 2.1 Equipment and Procedure

Modal testing and mechanical impedance measurements have been used extensively in previous research [7, 8, 15, 12, 3, 17, 21, 25] with a focus on the low modal region and impedance values along the bridge. These methods, along with piano tone mapping, are applied in this thesis to determine the impact of modifications to the soundboard on piano tone. The major difference between this work and most of that done previously is that testing in this thesis is conducted on a fully strung piano. It also differs in that this work is focused on quantifying the effect of standard voicing techniques. Pianos were typically tested unstrung in previous work to simplify the measurement process, but by removing the strings the downbearing force is eliminated and significant changes occur to the vibrational characteristics of the soundboard [3, 12]. Studying the strung piano is important to avoid these changes and examining voicing techniques used by piano technicians will also produce results with practical implications. The modifications and test methods are explained below.

#### 2.1.1 The Piano

The piano that was the main test subject in this thesis is a 2 m Hardman grand built in the 1890's. Unless otherwise stated, all testing in this thesis was conducted on this Hardman piano. The hammers had recently been replaced and the piano was in good condition. Some minor cracks were found in the soundboard, but these are quite typical of a piano of this age and are known to have no significant effect on soundboard behaviour [26]. The piano is shown in figure 2.1.

During testing the lid of the piano was removed to allow easier access to the soundboard and to minimize the impact of reflections off the lid on sound recording. The piano is overstrung, with standard bass (i.e. non-cantilevered) and tenor



Figure 2.1: 2 m Hardman grand piano.

bridges with notes 1-23 (A0-G2) attached to the bass bridge and notes 24-88 (Ab2-C8) attached to the tenor bridge. Of the notes on the bass bridge, notes 1-7 are single strung, notes 8-15 double strung, and notes 16-23 triple strung. All notes on the tenor bridge are triple strung with notes 24-64 having dampers and notes 65-88 undamped and therefore sympathetically vibrating. The locations of these transitions in stringing can be seen labeled in figure 2.2. The image presented is a tone map, a new technique which will be explained later.

### 2.1.2 Modifications Made to the Piano

One of the roles of the piano technician is to ensure the sound quality of a piano is even from note to note, a task that becomes a challenge due to the wide structural variation and complexity of the piano. The transition between notes attached to the tenor bridge and the bass bridge, where significant changes occur in physical location, mechanical impedance, and string type, is one of the most common problem areas of a piano. In terms of physical construction the low end of the tenor bridge and the high end of the bass bridge of the Hardman piano are located almost 40 cm apart, and attached to very different regions of the soundboard in terms of boundary induced stiffness. In an attempt to even out the transition piano technicians employ a number of different methods. Two of these techniques are examined in this thesis: the use of small brass weights and riblets. These techniques were taken from two articles discussed previously in the literature review, a patent by Conklin [13] and a discussion between several piano technicians in the Piano Technicians Journal [16].

Weights were made of 1.5 inch diameter brass rod in 25 g, 50 g, 100 g, 200 g, and 400 g sizes. Each weight was carefully drilled out until the mass was within  $\pm$  0.1 g to ensure accurate results. To make the riblets the method discussed by Fandrich [16] was followed, using the suggested dimensions rounded to equivalent



Figure 2.2: Bridge divisions.



Figure 2.3: Weights and riblets.

metric values. Riblets of the following dimensions were constructed of quarter sawn sitka spruce given as "height x width x length":

- 12.5 mm x 25 mm x 100 mm (mass of 20 g)
- 25 mm x 25 mm x 100 mm (mass of 40 g)
- 50 mm x 25 mm x 100 mm (mass of 80 g)
- 100 mm x 25 mm x 100 mm (mass of 158 g)
- 25 mm x 25 mm x 150 mm (mass of 60 g)
- 25 mm x 25 mm x 50 mm. (mass of 20 g)

The density of the sitka spruce was measured to be  $520 \text{ kg/m}^3$ , with a modulus of elasticity of approximately 10 GPa [27]. The riblets were not 'shouldered' (chamfered at the ends) as per the instructions given by Fandrich to ensure similarity between riblets of different height. The radial grain direction was kept approximately in line with the height axis of the riblets. The point of the experiments was to compare the effect of different blocks and shouldering the blocks in the style typical of piano ribs was considered to add a source of unnecessary variability. Mounting holes were drilled into the riblets and weights to allow a 2 inch wood screw to be used to attach them to the soundboard. The bridge of the piano is typically screwed and glued to the soundboard from underneath the panel. Soundboard buttons act as washers at these points and serve as logical attachment points for the riblets and weights. Four soundboard buttons were removed along with their screws in the bass end of the tenor bridge to serve as attachment points for the voicing modifications. These locations can be seen in figure 2.4. A 20 cm grid was marked on the rim of the piano similar to that shown in the figure.



Figure 2.4: Modification locations L1-L4 on soundboard bridge.

Initially it was hoped that the original screws and screw-holes could be reused, but it was found that the wood had degraded significantly and could no longer hold a screw with any force. The screw-holes were drilled out and dowelling was glued in the holes to provide new material for the screws to bite into. With the panel now prepared the modifications were screwed into place with the surface between the soundboard and the weight or riblet coated in wax to ensure a secure connection. In practice weights and riblets would be glued into place, but this would further complicate the process of examining different combinations of modifications and would cause progressively more damage to the soundboard as each modification was removed.

With six different riblets, five different weights, and four different attachment locations a large number of combinations was possible. When discussing which modifications were used in a particular test a shorthand labeling system was created as follows:

P-L1-L2-L3-L4

with P denoting the measurement position (as shown later in figure 2.17) and L denoting the type of modification at the appropriate modification location.

Weights were listed according to their mass, 25g, 50g, etc., and riblets were labeled according to their height, 12m, 25m, 50m, etc.. As an example the label 'P1-0-25m-0-50g' case would be a measurement conducted at location 1, with a 25 mm tall riblet attached at location 2 and a 50 g weight attached at location 4.

One final consideration made during the testing process was the consistency of atmospheric conditions. Changes in relative humidity are known to have an impact on the crown and down bearing of the piano [26], so it is essential that conditions remain constant throughout testing. All tests were performed during a one week period during the summer months in an air conditioned laboratory to ensure large variations in humidity were not encountered.

### 2.2 Modal Testing

Modal testing is a technique used to determine the modes of vibration of an object by applying an input force and measuring the output acceleration at several selected points on the object. Thanks to advancements in technology this technique can readily be performed on a personal computer using the proper software. Bruel and Kjaer's (B&K) Pulse system [28] was used in this thesis along with MEScope Modal Test Consultant [29] to perform the modal analysis. The geometry of the soundboard was entered into the software and 53 measurement points were identified where acceleration measurements would be conducted. This technique is referred to as a roving accelerometer, fixed shaker test. The location of the measurement points and the shaker can be seen in figure 2.5.

At each of the 53 points acceleration measurements were made with a bandwidth of 12 kHz with 2 Hz resolution. Linear averaging was used with 20 samples and a 66.67% overlap. At position 54 the shaker excited the soundboard with white noise and the force transducer measured values with the same bandwidth and averaging as the accelerometer. The shaker used in the modal tests was a B&K 4809 Vibration Exciter [30] and was powered by a B&K 2718 Power Amplifier [31]. An Endevco 256 HX-10 Accelerometer [32] and an Endevco 2311-100 Force Transducer [33] were used to take the necessary measurements. The force transducer was connected



Figure 2.5: Geometry of testing locations used in modal tests.

with wax to the soundboard and via a stringer to the shaker which was mounted hanging over the soundboard on an adjustable platform. This platform allowed for fine adjustment of the attachment of the force transducer to the soundboard and is similar to that described by Keane [12]. The modal tests were made for the following modifications and in all cases the setup of the shaker and force transducer were identical.

- 0-0-0-0
- 0-200g-0-0
- 0-400g-0-0
- 200g-200g-200g-0
- 0-25m-0-0
- 0-50m-0-0
- 25m-25m-25m-0

With the large number of possible combinations of modifications it was necessary to focus the more time intensive tests on a few select modification situations. This set of tests was chosen because it has the ability to illustrate trends in both the addition of mass and stiffness while allowing comparisons between the effect of weights and riblets. After the data was collected in the Pulse software it was then exported to MEScope Modal consultant where modal peaks and other data were extracted. This software calculates mode shapes, frequencies, and damping factors, and also creates animations that visualize the vibration of the soundboard at specific frequencies. To do this the frequency response functions (FRF) are calculated in Pulse which, along with the geometry of the soundboard and measurement locations, are exported to MEScope. This software can identify peaks that correspond to resonant modes of the soundboard using the imaginary part of the FRF. The residue, or mode shape, is then calculated using the data from all 53 of the measurement points [34]. Examples of these animations are included in an electronic appendix to this thesis.



Figure 2.6: Shaker setup for modal tests.

### 2.3 Piano Tone Mapping

#### 2.3.1 Concept

The basic concept behind the Piano Tone Map is to retrieve the harmonic envelope of each note and then plot these envelopes as a three dimensional surface for the entire piano. In essence this is a similar process to producing a spectrogram, but instead of plotting time, frequency, and magnitude the tone map plots note, harmonic, and magnitude. The term harmonic is used interchangeably with the term partial in the piano field, and will be used throughout this thesis to mean the same thing. To clarify, this means that the fundamental of a note is labelled as n = 1, with successive peaks labelled accordingly. This creates a slight distinction, as in this labeling system the nth harmonic would be equivalent to the (n-1)thpartial. As cited earlier, the harmonic envelope has been shown to be an important contributor to the perception of timbre [6, 5, 20]. To create the maps there are three main steps: recording a keystroke for every note, calculating its frequency spectrum and retrieving the harmonic peaks, and mapping the peaks for each note as part of a three dimensional surface. An example is seen in figure 2.7 as a three dimensional surface and in figure 2.8 in the style of a spectrogram. The three dimensional image tends to mask a large amount of data and was generally found to be inferior to the spectrogram style plot that is used throughout the rest of this thesis. The positive x-axis represents increasing note number, starting with 1 (A0) to 88 (C8). The positive y-axis represents increasing harmonic number from the fundamental labelled as '1', to the one-hundredth harmonic, '100'. Colours map harmonic magnitude with blue low, through cyan, yellow, and red high.



Figure 2.7: Three dimensional tone map of a Bösendorfer Model 214.

#### 2.3.2 Recording

Consistency in sound recording is an important consideration when trying to compare results between different pianos. Room acoustics and the force applied to the keys will change the harmonic structure of the sound being recorded by a microphone, so attempts must be made to ensure these effects are as minimized as much as possible. Ideally pianos would be tested in an anechoic chamber with a number of microphones, but due to the size of the piano and limited availability of anechoic chambers this is not a possibility for most researchers. In reality the testing needs to occur where the piano is located, be it in a concert hall, a workshop, or a research laboratory. This presents a problem as each of these locations would have their own room acoustics that would shape the way the piano sounds in the reverberant sound field. To minimize these deviations the microphone should be



Figure 2.8: Spectrogram style tone map of a Bösendorfer Model 214.

placed in the direct field so that the sound level created by direct sound energy is much greater than that created by reverberant energy. Care should also be taken to place the microphone at the outer limits of the direct field so that local changes in radiation near the soundboard do not affect the results. The following equations [35] can be used to determine an appropriate location for the microphone:

$$R_{rc} = \frac{\bar{\alpha}S}{1-\bar{\alpha}} \tag{2.1}$$

$$r_0 = \sqrt{\frac{R_{rc}Q_\theta}{16\pi}} \tag{2.2}$$

where:

 $r_0$  is the radius of reverberation the distance from the source at which the sound levels due to reverberant and direct sound are equal;

 $R_{rc}$  is the room constant derived empirically;

 $Q_{\theta}$  is the directivity factor and equals 1 for a spherical radiator;

 $\bar{\alpha}$  is the average absorptivity coefficient; and

S is the total surface area.

The  $r_0$  value for the lab in which the experiments in this thesis took place was calculated to be approximately 0.8 m using data from Pierce [35], and as a result the microphone was located 0.4 m from the center of the piano soundboard. Using this method allows for the impact of the room's acoustical signature to be minimized in the recorded sound, thus making the comparison of pianos recorded in different locations more valid.
The other important consideration to ensure consistency is to control the force applied to the keys in terms of repeatability. Suzuki [6] demonstrates that a hard versus a soft touch can create different spectrum results even if the overall sound level of the note is the same. Deviations in harmonic structure are thus to be expected if the force applied to the piano key is not kept relatively consistent. Once again the practicality of the situation is important to consider as well; for this reason a simple method was created that, although not ideal, produced consistent results and allowed for easy portability. The device consisted of a steel bar that was extended across the keybed of the piano. A bracket that would drop a short steel rod onto the keys was attached to this bar to allow easy adjustment from key to key while maintaining the same drop height of the rod. A small piece of felt was attached to the end of the rod to eliminate bouncing and double strikes of the keys. This experimental setup is shown in figure 2.9.



Figure 2.9: Key striking apparatus.

With this equipment in place every note on the piano was played and recorded using a Behringer ECM8000 microphone connected to a laptop through an M-Audio external sound card [36]. Python [37] and Octave [38] scripts that can be found in the appendix were created to help automate the process of recording and editing the recordings into individual files. Each note was saved in an uncompressed 16-bit, 44100 Hz, Sun Audio File, '.au', file format. To ensure that the starting point for all the notes was the same distance from the start of the file a simple method was applied similar to that used by Suzuki [6]. The Python script creates a directory containing 88 recordings of each individual note of the piano. The background noise level is determined and then the first instance where this level is exceeded is used to set the initial position of the key strike. To ensure the start of the note was not cut off in the recording the initial position is then moved backwards by 50 ms. An example waveform of a recording prepared in this manner is shown below in figure 2.10.

Tone maps for the following cases were made:

- 0-0-0-0
- 0-200g-0-0

- 0-400g-0-0
- 200g-200g-200g-0
- 0-25m-0-0
- 0-50m-0-0
- 25m-25m-25m-0

These modification combinations were chosen for the same reason that they were for the modal testing discussed earlier. A 12 s recording length was used for all notes because after this duration the recording had reached the noise floor of the equipment for the majority of notes. Each recording captured about 10 s of the note sounding due to the removal of silence from the sample due to the the initial delay in releasing the key striker. Some notes in the bass can sound for more than 30 s so this meant that certain notes were cut off before they had completely dissipated. The 10 s window was considered satisfactory given the minor role of notes greater than 10 s in duration in most forms of music [12]. With all the notes of the piano recorded the next step is to begin the spectral analysis of each note.



Time (s)

Figure 2.10: Edited recording of a single note.

#### 2.3.3 Frequency Analysis and Harmonic Peak Retrieval

One of the most commonly applied analysis techniques in the field of acoustics is the Fourier transform. This procedure transforms the time signal of the recorded sound into the frequency domain, revealing the magnitude and phase of the various different frequency components of the signal. The frequency domain representation of the signal is referred to as a frequency spectrum, or often just as the spectrum of the sound. To implement the Fourier transform computationally the Fast Fourier Transform (FFT) is applied to a digital signal.

Calculating the power spectrum from the FFT data creates a frequency response curve for each note. Piano notes have a fairly regular harmonic structure that



Figure 2.11: Frequency and time domain representation of note D3 (30), 146.8 Hz fundamental.

is typically characterized by a strong fundamental peak (absent in the low bass) followed by a fluctuating series of diminishing harmonic peaks with a structure related to the striking point of the hammer. These peaks do not occur at exact multiples of the fundamental because of inharmonicity due to the nature of real strings that have a diameter and bending stiffness. Instead they roughly obey the equation  $f_n = f_0 n \sqrt{1 + Bn^2}$  where  $f_0$  is the fundamental frequency, n is the number of the desired harmonic, and B is the inharmonicity coefficient [39] (typically B is in the range 0.0001-0.0005). The presence of peaks due to longitudinal string vibrations can further complicate things by creating peaks of similar magnitude to harmonic peaks, but that are located in between the locations of actual harmonic peaks of the fundamental frequency of the string. These features are shown in figure 2.12 that highlights the complexity of automatically retrieving harmonic peaks from this data.



Figure 2.12: Typical frequency response structure of piano tones.

The fundamental harmonic is easily identified in reasonably well tuned pianos because the frequency values for musical pitches are standardized. The algorithm developed to identify the magnitude and frequency of harmonic peaks begins by searching in a window around the fundamental frequency to determine the maximum magnitude value in the window and the frequency it corresponds to. Using this method allows for variations from the standardized pitches where the fundamental may not occur exactly where it is supposed to. The first twenty harmonic peaks are then found by predicting their frequency location,  $n \times f_0$ , and searching in a window around these frequencies for a maximum magnitude value. At this point inharmonicity has not been considered because estimating the inharmonicity coefficient, B, requires knowledge of the string properties. For the first twenty harmonics the method is robust enough to handle inharmonicity because it searches a window around the location where the harmonic should be. Above 30-40 harmonics the method needs to create better target values because inharmonicity has significantly shifted the location of the harmonic peaks upwards from their idealized values. Now that the location of the harmonic peaks is known for the first 20 harmonics the inharmonicity coefficient can be back calculated. It should be noted that the inharmonicity coefficient is not a constant value and will vary from harmonic to harmonic. As a result an average value of the back calculated B value is used to predict the location of the harmonic peaks 21-30. After these values are found a new B value is calculated from the most recently found peaks and new values are predicted for peaks 31-40. The algorithm continues in this manner until 100 peaks have been found or the value of the predicted peaks is above 20 kHz, the upper limit of human hearing. At higher order harmonics, typically above n = 50, the algorithm can fail due to the low magnitude of the harmonic peaks, the error in the inharmonicity prediction at high harmonics, and the relative magnitude of the noise to the signal. At this point peaks are still identified, but may not be correctly numbered (i.e. the 76th peak identified may actually be peak 75 that has been shifted due to inharmonicity). More than 100 harmonics are theoretically possible in bass notes, but for high treble notes there may only be 3 or 4 harmonics before the upper limit of human hearing is reached as the fundamental frequencies are in the kHz range above B5. This once again underlines the complexity of properly identifying higher order harmonic peaks. The magnitudes and frequency values are then stored in matrices that can be mapped as surfaces. Octave code for the peak finding algorithm can be found in the appendix.

### 2.3.4 Surface Mapping

The question of how to visualize this data emerges with the harmonic envelope for each note now retrieved and the appropriate FFT data saved in matrices. Most obviously, the magnitudes of the peaks can be mapped as colors or heights and a surface can be created by placing the data for each note side by side in the order of the 88 notes of the piano. A fully labeled tone map is shown in figure 2.13. The x-axis corresponds to increasing note number starting with note 1 (A0) increasing to note 88 (C8). The y-axis corresponds to increasing harmonic number, starting with the fundamental as 1 and increasing to the 100th harmonic labeled as 100. It should be noted that the *n*th harmonic for two different notes will not occur at the same frequency, as lines of equal frequency can be seen to increase logarithmically as shown in figure 2.13. Magnitudes are then mapped to colors using a standard colormap that varies from blue (low), to cyan, to yellow, and to red (high).



Figure 2.13: Fully labeled tone map.

The map of harmonic magnitude is the most commonly examined map in this thesis, but the mapping of different data from the FFT calculations is also an option. For each harmonic peak the percent inharmonicity, phase, and damping can be calculated and each one mapped individually using the same method. Looking at the difference between two maps can also be a useful tool as it can show the difference in harmonic structure between two different pianos or the change in harmonic structure within one piano after a modification has been made. To create these maps the magnitude of the harmonic peaks from the reference test are subtracted from the subsequent test. These are logarithmic differences, so in terms of energy this actually corresponds to a ratio of the two test situations. These difference maps will be used extensively to show the change in harmonic structure of a piano that occurs as a result of the modifications discussed by Fandrich [16], Conklin [13], and others.

#### 2.3.5 Transient Response

Another way that the tone maps can be useful is in examining the transient response of the piano. Typically the recorded notes are analyzed in 1 s windows to obtain an FFT result with a 1 Hz resolution. A time response can be created that shows the development of the piano's tone over time by shuffling the sample window along the full length of the recorded signal for each of the notes. An example of the windowing is seen in figure 2.14. The amount of overlap of the windows determines how many unique maps per second would be created. There is a significant computational cost associated with running high resolution FFT calculations for large frequency bandwidths, so a time resolution of 15 maps per second was used in the experimental work in this thesis.



Time (s)

Figure 2.14: Windowing method for transient response.

The time based maps are then quite readily turned into animations that can display the real time response of the tone of the piano. Selected frames from a time series of the response of the unmodified test piano used for research in this thesis are presented in figure 2.15. This data is best viewed as an animated video file that is available in the electronic appendix to this thesis [40]. These time response animations are an excellent visualization of the 'bloom' (i.e. development), of the piano's tone over time. Initially a large amount of energy is present in the upper harmonics, but can be seen to quickly decay. The first 10-20 harmonics have a much longer decay period with individual harmonics tending to increase and decrease over time. Connections between the transient response and mechanical impedance will be examined in this thesis, but the technique has many other applications.



Figure 2.15: Transient tone maps of a Hardman grand piano: 2s time step

### 2.4 Impedance Measurements

Mechanical impedance, Z, is defined as the ratio of the input force, F, to output velocity, v, at a single point:

$$Z = F/v \tag{2.3}$$

The impedance value is a function of the frequency, f, of the applied force, and can be thought to be similar to the resistance of an object to being moved. Along with its inverse value, mobility, impedance has been discussed at great length by Conklin [10] and others because it has a great deal of relevance to the transfer of energy between the strings, the bridge, and the soundboard. The impedance at the bridge is a function of bridge stiffness, the effective mass of the soundboard and bridge, and the modal properties of the soundboard. At a given position and frequency a high impedance value would indicate that it is difficult to transfer energy to the soundboard, whereas low impedance values indicate that it is easy to transfer energy. Neither extremely high nor low impedance values are desirable because both correspond to situations where energy transfer is innapropriate. To measure impedance requires a force transducer to measure the applied force and an accelerometer to determine the velocity of the soundboard. Both can be included in one device called an impedance head, or two individual units can be used. In this case a force transducer and accelerometer were used in a similar fashion to that of Giordano [15]. A small piece of aluminum was prepared with two holes tapped to allow the mounting of the force transducer and the accelerometer to ensure a high quality connection. The aluminum plate was then affixed to the soundboard using wax at five different locations that can be seen in figure 2.17. Three locations were located adjacent to the tenor bridge and two were located on the bass bridge.

The plate and strings made it difficult to take measurements from the top side of the piano for the tenor bridge locations P1, P2, and P3, so they were completed from underneath the soundboard. A small scissor lift was built to allow for fine adjustment of the shaker's position below the piano to accommodate this. P4 and P5 were accessible from top side of the soundboard and correspondingly the shaker and impedance head were hung from a support over the piano. The aluminum plate



Figure 2.16: Mounting plate used for impedance tests.



Figure 2.17: Measurement locations of impedance tests.

needs to be firmly attached to the soundboard, but if excessive force is used there will be a significant impact on the measurement results.



Figure 2.18: Impedance test.

An extensive set of modifications was carried out for each test location shown in figure 2.17. Velocity and force signals were recorded and analyzed using the B&K Pulse software package [28] for each location and each modification. To measure velocity the signal from the accelerometer was integrated, a process that can lead to errors in the measured time signal due to integration problems caused by noise. In terms of linear frequency response, integration does not present a significant problem as a noise signal will simply add a uniform increase throughout the spectrum. If the signal to noise ratio is high then the effect of noise will be relatively insignificant as it was in the results presented in this thesis.

Each set of modifications was completed during the same session to achieve consistency between results by ensuring that the shaker/plate/soundboard conditions were identical for a given measurement location. Table 2.1 lists all the combinations of modifications tested in this thesis. A 12.8 kHz bandwidth of measurement was chosen with a 2 Hz resolution was chosen to allow for comparison with other research [15]. Every measurement was a linear average of 50 samples with an overlap of 66.67% between samples. Velocity and force data were then saved and analyzed in Octave [38] to produce impedance results. Change in impedance can also be calculated from this data to illustrate the effect of modifications on the vibrational properties of the piano. The change in impedance was also compared to changes in the frequency spectrum of sound recordings.

0-0-0-0	25m-25m-0-0	25m-25m-25m-0	0-25m-25m-0
0-0-0-12m	0-0-12m-0	0-12m-0-0	12m-0-0-0
0-0-0-25m	0-0-25m-0	0-25m-0-0	25m-0-0-0
0-0-0-25 ms	0-0-25ms- $0$	0-25ms-0-0	25 ms-0-0-0
0-0-0-25ml	0-0-25ml- $0$	0-25ml- $0-0$	25ml- $0-0-0$
0-0-0-50m	0-0-50m-0	0-50m-0-0	50m-0-0-0
0-0-0-100m	0-0-100m-0	0-100m-0-0	100m-0-0-0
0-0-0-25g	0-0-25g-0	0-25g-0-0	25g-0-0-0
0-0-0-50g	0-0-50g-0	0-50g-0-0	50g-0-0-0
0-0-0-100g	0-0-100g-0	0-100g-0-0	100g-0-0-0
0-0-0-200g	0-0-200g-0	0-200g-0-0	200g-0-0-0
0-0-0-400g	0-0-400g-0	0-400g-0-0	400g-0-0-0
200g-200g-0-0	200g-200g-200g-0	0-200g-200g-0	

Table 2.1: Modifications measured with impedance tests.

## Chapter 3

# Modal Analysis Results and Discussion

Six unique modifications along with the unmodified Hardman piano were tested and analyzed using the B&K Pulse and MEScope software packages. MEScope outputs animations of the various modes of vibration and also identifies damping values for each mode. Each test does return slightly different results, but the general shapes of the modes were found to be common to all test situations. The mode shapes for the first six modes of the unmodified piano (0-0-0-0) are shown in figure 3.1. The complete set of animated mode shapes for all test situations can be found in the electronic appendix [40]. Modal amplitudes and damping values are calculated from the modal response curve, where peaks correspond to location of modes. An example curve is shown for the 0-200g-0-0 test in figure 3.2. It can be seen that some peaks are more isolated from neighbouring peaks than others and that the magnitude and variation between peaks changes considerably.

From these mode shapes it becomes very clear that in a strung piano the modes of vibration are far from the ideal case predicted by FEM modeling [3]. The mode shapes determined experimentally are much less symmetric compared to that of even a complex model. Boundaries are somewhat poorly defined and irregular when compared to Mamou-Mani's predictions and some modes tend to exhibit a wave-like flow of energy throughout the board. Mamou-Mani's [3] model considers anisotropic material properties, ribs, bridges, and the downbearing force, but even this detail of modeling has failed to capture the complex reality of the piano soundboard. Some possible reasons why the results vary include the modeling of the boundary conditions, the variability of the mechanical properties of wood, and the transfer of energy and interaction between the soundboard and the strings.

Keane also experimentally measured the mode shapes of a strung piano and his results [12] were similar to those found here. He observed that the case of the piano does play a role in the deformation of the soundboard, a result that is confirmed by this work by examining the deformation of the case at modal frequencies. From the modal animations it is seen that the rim does deform along with the rest of



Figure 3.1: Mode shapes of the unmodified piano.



Figure 3.2: Modal response curve of the 0-200g-0-0 test.

the soundboard and cannot be considered to be a fixed boundary condition. The case and attachment at the rim are two aspects that should be considered when developing improved models. The changes in local properties created by boundary conditions have been observed to be significant and would likely make mode shapes predicted by models more realistic.

A second point to consider is the natural variability of wood properties. Instrument builders and wood experts have long understood that the vibrational properties of wood can vary significantly from one piece to the next [41]. With an instrument as large as the piano it is extremely likely that there will be wide variations in radiative efficiency, damping, speed of sound, density, and modulus of elasticity in the three primary grain directions throughout the soundboard. This would act to randomize the mode shapes and, if considered in models, would be another way to achieve more realistic results. Samples of the soundboard in a number of regions would need to be taken, however, the most common methods of determining mechanical properties are destructive and require dimensionally large samples. New techniques need to be developed as current methods would effectively demand destruction of a soundboard to determine its properties.

A final aspect of the piano soundboard that could be included to improve models is the soundboard string interface. Even though strings are damped during the modal anlaysis tests it is likely that they are still an active part of the soundboard system. More experimental analysis is required to determine if soundboard energy is being transferred to the strings and the nature of this relationship. The one thing that is clear is that vibrational properties and mode shapes of strung pianos are much different than those of unstrung pianos [8]. Mamou-Mani's introduction of downbearing force into soundboard modeling is a good start, but it is clear that the interaction between the strings and the soundboard is more complex than just the addition of a static downward force.

The actual shape of the modes is just one characteristic to consider when examining the modal data. With the addition of weights and riblets to the soundboard it was observed that the frequency and damping of the mode shapes changed. A plot of the frequencies and damping values for the seven different test situations can be found on figures 3.3 and 3.4. Numerical values for this dataset are also provided in tables 3.1 and 3.2.

From the graphs and tables it can be seen that the first mode has the largest change in terms of frequency (except for one data point of the fourth mode which is considered to be an outlier). The addition of weights decreased the frequency value of the first mode, with the 200g-200g-200g-0 modification creating the largest decrease (8 Hz). This result is anticipated as adding mass to a vibrational system decreases its natural frequency. The riblets were seen to have very little impact on the first mode, with only the 25m-25m-0 modification creating a decrease of more than 1 Hz. Each 25m riblet has a mass of 40 g, so the total mass added in this situation is 120 g. The addition of a 200 g weight decreased the frequency value of the first mode by 2.4 Hz and the 120 g mass of the riblets created a decrease



Figure 3.3: Change in frequency referenced to the unmodified piano for M1-M6.



Figure 3.4: Change in percentage damping referenced to the unmodified piano for M1-M6.

Modification	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
0-0-0-0	79.95	138.33	193.37	262.36	285.23	312.98
Riblets						
0-25m-0-0	79.12	138.82	195.69	259.07	285.8	313.24
0-50m-0-0	79.21	138.51	195.52	251.13	282.64	309.26
25m-25m-25m-0	78.55	138.39	195.37	260.1	285.86	311.39
Weights						
0-200g-0-0	77.58	138.85	195.04	259.85	284.73	307.13
0-400g-0-0	74.86	138.65	194.43	263.78	284.93	309.26
200g-200g-200g-0	71.81	138.06	194.72	263.11	284.38	306.83

Table 3.1: Modal frequency values (Hz).

Table 3.2: Modal damping values (Percentage).

Modification	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
0-0-0-0	7.5	3.2	1.4	2.7	2.0	2.1
Riblets						
0-25m-0-0	8.8	3.2	3.5	2.8	1.5	2.2
0-50m-0-0	8.3	3.1	3.4	0.9	1.3	2.3
25m-25m-25m-0	9.2	2.9	1.8	2.8	1.5	2.2
Weights						
0-200g-0-0	9.5	3.0	3.8	2.4	1.3	2.4
0-400g-0-0	10.3	3.1	2.9	2.8	1.2	2.3
200g-200g-200g-0	9.7	2.8	3.3	2.7	1.1	1.6

of 1.4 Hz so it is arguable that the change in the first mode caused by riblets is due simply to their mass. Damping values for the first mode showed a similar pattern, with the weights creating the largest increase and the riblets inducing an intermediate change. The 0-400g-0-0 test increased damping by almost 3%, while the 200g-200g-200g-0 test increased it by about 2%. The riblets increased damping by 1-1.5%. The location of the modifications and mode shapes is relevant to the discussion and an overlay of soundboard features on the modes shapes is shown in figure 3.5. The nearer a modification is to an anti-node of a mode the more effect it will have on that mode of vibration.



Figure 3.5: Overlay of Hardman bridge and modification locations maps on mode shapes.

Aside from M1, M6 is the other mode that showed significant changes. Figure 3.5 shows that an anti-node of M6 is located near the modification locations and makes this mode susceptible to modifications at this point. The 0-400-0-0 test showed a 3.5 Hz reduction in frequency and a small 0.3% increase in damping and the 200g-200g-200g-0 modification created a 6 Hz decrease in frequency and a 0.4% decrease in damping. Riblets had a smaller effect on M6, with the 25m-25m-25m-0 test showing a 1.5 Hz decrease in frequency and 0.2% increase in damping.

These results confirm Fandrich's assertion [16] that riblets only slightly decrease the natural frequency of the soundboard, but it is not obvious that their effect on modes is significant enough to dramatically change the tone of the piano. Impedance results discussed later will show that the most significant changes in soundboard properties are at frequencies above the modal region, where the modal region is defined as the frequency region where modal density is low and modes are distinct and easily identifiable. It is generally assumed that vibration modes and modal properties are highly important to the acoustic output of a piano in the modal region (20 Hz to 500-1000 Hz). Above 500-1000 Hz the modal density increases to a degree that modes begin to stack up on top of each other and create a flatter response absent of the peaks and valleys found in the modal region. The technology exists to easily test and model vibration modes and in the literature much effort has been expended on these topics. For M2-M5 damping values were seen to be relatively consistent regardless of which modification was applied. It is clear that small weights (200 g), can have a significant impact on the modal response of the first mode and can be used by piano technicians to extend the bass range of the piano.

Modes theoretically should provide a boost in response at their respective frequency values, a boost which should be visible on the piano tone maps as regions of increase along lines of equal frequency. Examining figure 3.6 reveals a clear harmonic structure related to the first mode, but no other corresponding patterns for the other modes. As will be discussed later, the frequency line of the first mode separates the region where the magnitude of the fundamental is lower than the magnitude of the first few harmonics.



Figure 3.6: Modal frequencies overlaid on the unmodified Hardman tone map.

One explanation of the apparent lack of harmonic structures related to modes is that the impact of vibration modes on output sound may be minimized due to acoustical short circuiting, reflections, and phase cancellation occuring as sound radiates from the soundboard. Further tests would be required to fully confirm this hypothesis, but the results presented in this thesis suggest that the importance of vibration modes on the overall output sound of a piano is overemphasized in the research literature. That would imply that other phenomena such as impedance changes at the bridge are more important to change in piano tone than are changes to modal properties of the soundboard.

## Chapter 4

## Tone Mapping Results and Discussion

The tone mapping technique was introduced earlier in this thesis and has proven to be a valuable research tool. By presenting the magnitude of the harmonics of the piano in a single image a unique fingerprint of a piano's tone can be studied. The fact that perceptible changes in tone do not always correspond to large visible changes in the tone maps can make comparisons difficult. The most obvious examples of differences in harmonic structure are found when comparing the tone maps of two different models of piano, but one of the challenges of piano research is gaining access to a large number of different pianos on which to conduct a study. Fortunately a large sample of high quality grand pianos was found at Robert Lowery's Piano Experts in Toronto, Ontario, Canada, where field tests were conducted. These tests were of a preliminary nature and an experimental microphone was not used. The maps created from these recordings cannot be considered to be completely accurate, especially at low frequencies, but do provide a good picture of the harmonic fingerprint of the pianos tested. Included in the recordings were a Schimmel 1.89 m grand, a Bösendorfer 2.14 m Salon grand, and an Estonia 1.9 m grand. Three other pianos tested at this time in Waterloo, Ontario are also included, a Heintzman 1.9 m upright, the Hardman 2 m grand used throughout this thesis, and a Yamaha 48 inch upright. Tone maps of each of these pianos are shown in figure 4.1. In visual comparison these maps can be challenging; the best method is to download the tone map files from the electronic appendix [40] and rapidly flip through them on a computer. Persistence of vision allows the small changes to be highlighted when this approach is used.

By visualizing the harmonic content of an entire piano at one time a fingerprint of that piano's tone is created. Each piano will have its own characteristic map containing internal structures and patterns unique to itself. One clear example of a unique structure is the large amount of high order harmonic energy present in the Yamaha upright. Figure 4.2 illustrates this by subtracting the harmonic magnitudes of the Hardman piano from the Yamaha piano. Discussions with piano technician Will Horne [26] revealed that Yamaha pianos have a reputation as being 'noisy'.



Figure 4.1: Tone maps of six different pianos.

Excess energy in the higher harmonics is a likely explanation for this qualitative perception of the instrument.

Examining the maps of these pianos also reveals several common structures which are highlighted in figure 4.3. The first important thing to note is the region where the magnitude of the fundamental is lower than the magnitude of the first few harmonics. This occurs in the low bass of the piano and is due to the increased impedance of frequencies below the frequency of the first mode of the soundboard. With the first mode typically around 75 Hz for a grand piano it can be seen that notes below 19 (Eb2) are unable to effectively produce a fundamental harmonic. Lowering the frequency of the first mode of the piano is one way to improve the bass response of a piano, something that was shown to be achievable with the addition of weights.

The hammer strike location is also a phenomenon that has a visible structure in the tone maps. As mentioned previously, the strike point adds periodic increases and decreases in the harmonic structure of a note. As an example an attenuation will be seen in n multiples of the eighth harmonic for a string with a strike point at 1/8th of the speaking length. If a series of strings shares the same strike point a series of valleys become visible along lines of equal harmonic number. The Bösendorfer shows the most prominent example of this ridge structure, providing evidence to the precision of the construction of their instruments. A line denoting the ratio of the hammer strike point to string length is presented in figure 4.3 that clearly demonstrates this phenomenon in the Hardman piano.

Another ridge that is visible occurs along a line of equal frequency corresponding approximately to  $3 \,\mathrm{kHz}$  for notes on the tenor bridge from about 44 (E4) - 69 (F6). This small band of energy is present in all the tone maps to varying degrees. There is no evidence that this energy band has been previously studied and there is no current explanation for why it occurs. One possible explanation that was explored is that the undamped strings in the high tenor may be creating sympathetic vibrations. Figure 4.4 illustrates that this hypothesis was shown to be incorrect due to the presence of this 3 kHz energy band in a test where the normally undamped high tenor strings were damped using felt. Another phenomenon that may be related to this ridge structure is the "killer octave". Fandrich [42] describes the killer octave as a region in the fifth to sixth octave of the keyboard that is characterized by low sustain and is present in most pianos regardless of construction method or soundboard design. Examining figure 4.5 shows how the location of these notes corresponds to the termination point of this 3 kHz ridge. The lack of higher order harmonic energy for notes in this octave could lead to the perception that they lack sustain, but further experimentation is required to confirm or deny this hypothesis.

The final structures that are notable in the tone maps are the stringing transition points discussed earlier and shown in figure 2.2. At these locations abrupt changes are made in string type (i.e. monochord, bichord, and trichord) and the type of bridge terminations.

Maps of the phase of each harmonic can also be created and these results can be



Figure 4.2: Hardman tone map magnitudes subtracted from the Yamaha tone map magnitudes.



Figure 4.3: Typical harmonic structures visible in the Hardman grand piano.



Figure 4.4: Hardman tone map with high tenor strings damped.



Figure 4.5: Location of notes that are part of the killer octave.



Figure 4.6: Harmonic phase distribution of the Hardman grand piano.

connected to the work of Galembo [22]. By creating synthesized piano bass tones with various types of phase differences between the harmonics Galembo found that a random phase distribution was perceived to be the most similar to an actual piano's tone. The phase map shown in figure 4.6 demonstrates that the phase distribution of the harmonic structure of the Hardman piano is essentially random. This result is somewhat obvious, but does confirm the experimental result of Galembo.

Inharmonicity is also an important characteristic of piano tone. To minimize inharmonicity strings would be as long and thin as possible, but some compromises need to be made due to restrictions on the scale size of a piano. The low bass and high treble strings are especially susceptible to large amounts of inharmonicity caused by decreased ratios of string length to string diameter (i.e. shorter strings with larger diameters) due to scale restrictions. Strings with larger string diameters tend to behave more like a beam than an ideal string and this, along with other factors, causes the frequency values of each successive harmonic to increase. Piano builders have always had to make this compromise in scale length and as a result the standard perceived sound of a piano has been defined to include inharmonicity and accordingly zero inharmonicity is not considered qualitatively desirable. Similar to other aspects of piano tone, the goal of technicians and designers is to smooth the transition of changes in inharmonicity, particularly at the transition points mentioned earlier, rather than tying to eliminate it.

Exley suggests that the appropriate addition of masses can lead to changes



Figure 4.7: Percent inharmonicity - note 15 (B1).



Figure 4.8: Percent inharmonicity - note 22 ( $F\sharp 2$ ).

in inharmonicity [23] for the first few harmonics (up to harmonic n = 4). An undulatory vibrational motion is created when the soundboard and the bridge are considered to be a two mass system. Exley's analytical calculations indicate that the amount of string inharmonicity can be decreased if appropriate masses are added to the system. Choosing appropriate masses becomes somewhat difficult because each string requires different values and in the end he chooses an intermediate string number to base his mass choices on. The model considers the soundboard to have its own mass before any additional masses are loaded. The mass loading tests conducted in this thesis with the addition of weights can be considered to be similar to those conducted by Exley, but they are not directly examples of his technique.

Values for the percent inharmonicity (PI) defined in equation 4.1 for the first 10 harmonics of the 3 mass loading tests along with those of the reference piano are seen in figures 4.7 - 4.9, for three different notes.

$$PI = \frac{\text{mod}(f_n/f_0)}{f_0}$$
(4.1)

Small changes in the inharmonicity of these notes can be seen for the mass loadings tested lending some support to Exley's predictions. It should be noted that Exley was considering these modifications for small grand pianos where the amount of inharmonicity is greater than that of normal grands due to shorter scale lengths. Subjective testing was carried out by Exley and mass additions of 700 g and 900 g were agreed to improve the tone of the piano, however, the validity of this testing is questionable. Applying the techniques used in this thesis would allow for a numerical description of the inharmonicity of a piano and would provide a starting point for work focused on precisely testing Exley's theory.

Caclin has noted [5] that the spectral centre of gravity (SCG) is a perceptible aspect of timbre, a value defined earlier in the literature review 1.1 as:

$$SCG = \frac{\sum_{n} n \times A_{n}}{\sum_{n} A_{n}}$$
(4.2)

A calculation was made of the SCG for each note for the seven modifications used in the tone map tests. The percent change in SCG for each note is then presented for these modifications in figure 4.10. From this plot two things become clear. First, that a change in SCG is more prominent for notes in the middle of the keyboard; second, that modifications will both increase and decrease the SCG depending on which note is examined. If weights and riblets are intended to only modify the sound of notes that have string attachment points near the modification locations then notes 25-34 should have the largest change in SCG as these are the notes closest to where weights and riblets were added. Specifically, note 25 attaches at location 1, note 28/29 attach at location 2, note 31/32 attach at location 3, and note 34 attaches at location 4. This was not observed for this piano, which implies



Figure 4.9: Percent inharmonicity - note 32 (E3).



Figure 4.10: Percent change in spectral centre of gravity.

that changes in output harmonic spectrum of sound are not restricted to areas where modifications are added. It should be noted that these values are defined relative to their fundamental frequencies. Due to the large difference in frequency values of the various notes (27.5 Hz - 4186 Hz), the percent change in SCG may be perceptibly exaggerated in the bass region.



Figure 4.11: Transient response of a Hardman grand piano.

The development of the harmonic structure of a piano's tone, referred to as 'bloom', is yet another complex aspect contributing to its characteristic sound. The example time series of tone maps of the Hardman grand, shown in figure 4.11, illustrates some of the important characteristics of the transient response of a piano. Animated files are available in the electronic appendix [40] and serve as realtime representations of the bloom of the piano. The first important feature to notice is the rate of decay of different harmonics, where higher order harmonics decay much more rapidly than lower order harmonics across all notes. The second important feature is the increase and decrease of specific harmonics over time, with certain harmonics that have magnitudes that pulse up and down. Like any real object, the rate of absorption, energy storage, and energy transfer are all frequency dependent and lead to this complex behaviour. Mapping the transient response is a good first step, but it becomes difficult to compare the results of two different tests, especially when the differences are very subtle. The difference map technique presented in figure 4.2 can also be applied to a transient tone map series and will illustrate the change in harmonic structure over time of one piano referenced to another (or itself). Difference maps were created for the six tone map modifications referenced to the unmodified piano such that positive differences correspond to increases in harmonic magnitude due to the modification and negative differences correspond to decreases. Figures 4.12 and 4.13 show the changes created by the 200g-200g-200g-0 modification at time t = 0 s and t = 8 s respectively.

The modifications where weights were added to the soundboard are very similar to the situation described in the Conklin patent [13] and provide quantitative data to confirm his results. The 200g-200g-200g-0 modification added the most mass to the soundboard and also produced the most prominent change in harmonic structure of the piano, but all the modifications were found to have an impact on the initial magnitude and the decay rate of certain notes to varying degrees. Conklin



Figure 4.12: Harmonic difference due to 200g-200g-200g-0 at t = 0s.



Figure 4.13: Harmonic difference due to 200g-200g-200g-0 at t = 8 s

claims that by adding weights to the low tenor bridge the initial magnitude will be locally decreased for notes attached near to the point where the mass is added. He also claims that these notes will have increased sustain, two results that are confirmed by the experiments conducted in this thesis. Figure 4.12 clearly shows that in a local region near that attachment point of the weights there is an initial decrease in harmonic magnitude. At t = 8 s figure 4.13 shows a local region of increase that corresponds to a decreased rate of decay of the harmonic magnitude in a region near the modification locations. Adding mass to a vibrating system theoretically should reduce the amplitude of its response and decrease its decay rate, so these results are expected and confirm Conklin's claims [13]. Transient difference maps are presented for all the modifications tested in figures 4.14 to 4.19.



Figure 4.14: Harmonic change for the 0-25m-0-0 modification.



Figure 4.15: Harmonic change for the 0-50m-0-0 modification.



Figure 4.16: Harmonic change for the 25m-25m-0 modification.

The addition of weights creates an additive effect, with more weight causing more decrease in the initial magnitude and decay rates. Riblets also presented similar results to those of weights, but to a smaller degree that is shown in the transient



Figure 4.17: Harmonic change for the 0-200g-0-0 modification.



Figure 4.18: Harmonic change for the 0-400g-0-0 modification.



Figure 4.19: Harmonic change for the 200g-200g-200g-0 modification.

difference maps. This does confirm the assertion presented by John Rhodes [16] that riblets should reduce initial amplitude, as well as decreasing the decay rate in a local region near their attachment point. The effectiveness of riblets, however, can be called into question by our results, as it is unclear if the changes they induce are due to an increase in local stiffness and addition of mass, or if the change are dominated simply by the addition of mass. Further tests are required to fully understand the mechanism by which riblets create change in the tone of the piano. It can be concluded that for given typical modifications as discussed in the literature (a 25 mm riblet and a 200 g mass), the 200 g mass was found to create larger changes in the piano's harmonic structure compared to a 25 mm riblet. That does not necessarily mean that weights are better than riblets at changing the tone of a piano, but only that their effect is greater for typical applications.

## Chapter 5

## Impedance Results and Discussion

Impedance is a very important quantity to measure in a piano because the sole mechanical purpose of a piano soundboard is to act as a transducer that converts string motion into pressure waves in the air that we hear as sound. The rate and magnitude of string energy that is converted by the soundboard into sound is dependent on the impedance of the interface between the strings and the bridge, and the bridge and the soundboard. From equation 2.3, mechanical impedance is Z = F/v, where F is the applied force and v is the velocity. Measurements were conducted to determine the mechanical impedance at 5 different locations normal to the soundboard near the bridge. A series of modifications were made and new measurements were taken to determine the effect of these modifications on the mechanical impedance of the soundboard at these five points. Velocity and force were measured using the B&K Pulse system [28] and then processed in Octave [38]. Figure 5.1 shows the force, velocity, and impedance data for an example set of measurements at measurement position 1 (P1). Results for other measurement positions are similar to those of P1 and can be found in the appendix.

Lower impedance values correspond to frequencies at which the soundboard is more easily excited. These impedance valleys indicate frequencies at which energy transfer is rapid and vibration levels are high. Peaks or higher values in the impedance curve correspond to frequencies at which the soundboard is difficult to excite and energy transfer from the strings to the soundboard is slower. A short rod referred to as a 'stringer' was used to connect the force transducer to the shaker. The stringer has a natural frequency of approximately 3 kHz [43] and as a result data above 3 kHz was considered to be corrupted due to the effect of vibration modes of the stringer on the experimental data. A more detailed view of the impedance value measured at P1 for a series of different weight modifications can be found in figures 5.2 and 5.3.

Two important observations about the performance of weights in terms of their ability to modify the tone of a piano can be made. First, the effects of adding more weight are cumulative, with more weight either systematically increasing or decreasing the impedance values; second, that weights had very little effect on



Figure 5.1: Force, velocity, and impedance measurements at P1.



Figure 5.2: Impedance measurements at P1 20-2000 Hz - weights at L1.


Figure 5.3: Change in impedance measurements at P1 20-2000 Hz - weights at L1.

impedance above 800 Hz. Below approximately 100 Hz the weights can be seen to decrease the impedance values of the soundboard at P1, while above 100 Hz the impedance values are for the most part increased. From previous results it is clear that weights should do three things: decrease the initial amplitude of vibration, increase the sustain, and lower the frequency value of the first mode. These three things are all evidenced here as the increased impedance values above 100 Hz would create a lower amplitude response and would also decrease the rate of energy transfer from the string to the soundboard, thus creating more sustain. By lowering the frequency value of the first mode of vibration the impedance valley corresponding to this mode is shifted to the right, thus lowering the impedance curve below 100 Hz. The results also show that above 800 Hz the weights have very little effect on the impedance response curve. When compared to the results of riblet modifications shown in figures 5.4 and 5.5 at the same point, P1, it is clear that both the riblets and the weights have their most significant effect on impedance in different frequency bands. When testing the riblets there is very little deviation from the reference unmodified piano for any of the modifications below 300 Hz. The most significant changes can all be seen in the 500-2000 Hz range.

Fandrich commented that a 7/8 inch x 7/8 inch x 5 inch riblet "seems to work best" [16], something that is evidenced by the results in figure 5.4 and 5.5. The 25 mm thick riblet can be seen to create the largest change in the impedance for the frequencies from 500-2000 Hz, while having little to no effect on the lower frequencies related to the low order modes of vibration of the soundboard. The additional rib height and added stiffness of the 50 mm and 100 mm tall riblets does not appear



Figure 5.4: Impedance measurements at P1 20-2000 Hz - riblets at L1.



Figure 5.5: Change in impedance measurements at P1 20-2000 Hz - riblets at L1.

to add additional stiffness to the soundboard, but the additional height does add more mass to the riblet itself. It is shown in figure 5.6 to 5.8 that the addition of mass on the order of 200-400 g decreased impedance for frequencies between 800 Hz and 2000 Hz, a result that would explain why the 50 mm and 100 mm riblets respond the way that they do. Fandrich also suggests that riblets have little to no net effect on the first mode of the piano, a claim that is supported by these impedance results. If the riblets did have an effect on the frequency of the first mode of the piano the impedance curve below 100 Hz should change similar to the way it did when weights were added, something that does not occur. It is also claimed that riblets can locally increase sustain by stiffening the soundboard and reducing vibration amplitude [16]. Riblets are shown to increase impedance and correspondingly sustain, so this hypothesis is supported by the data.



Figure 5.6: Impedance measurements at P1 20-500 Hz - comparison of weights and riblets at L1.

The net effect of weights is seen to be an increase in impedance for frequencies between 100-600 Hz and a decrease in impedance in the region between 20-100 Hz and 600-2000 Hz. The opposite is true of riblets, which increase the impedance of higher frequencies and slightly decrease the impedance of lower frequencies. In terms of the ability to change the harmonic structure of output sound by changing the way string energy is transferred to the soundboard it is clear that weights and riblets each have their own applications. Another interesting result is the series of large spikes in the impedance curves in figure 5.8 in the 600-800 Hz region. It is unclear what the cause of these spikes are, but P1 is located at the tail end of the tenor bridge and one hypothesis is that the natural frequency of the bridge is in the



Figure 5.7: Impedance measurements at P1 20-2000 Hz - comparison of weights and riblets at L1.



Figure 5.8: Change in impedance at P1 20-2000 Hz - comparison of weights and riblets at L1.

600-800 Hz. If this is true weights and riblets would be directly affecting the first mode of the bridge at these frequencies. Further testing is required to investigate this hypothesis.

One final suggestion presented both in the Piano Technicians Journal article [16] and in Conklin's patent [13] is that any effect these modifications have should be local. Figures 5.9 and 5.10 present the results of measuring impedance at P1 while modifying the piano at each of the four modification locations with weights and riblets.



Figure 5.9: Impedance measurements at P1 20-500 Hz - weights at L1-L4.

The distance between neighbouring modification locations is typically 11-12 cm and Rhodes [16] claims that riblets only increase the stiffness of the soundboard in a 15-20 cm radius. It is seen that modifications at locations L2, L3, and L4 have very little effect on measured impedance and create response curves similar to the unmodified P1-0-0-0 reference condition. This implies that weights and riblets do have a small local region of effectiveness less than 10-15 cm in radius and confirms the statements of Rhodes and Conklin in terms of the localized effects of weights and riblets. With that said, it was shown earlier, using the piano tone mapping technique, that changes in harmonic spectrum and mode shapes are not restricted solely to notes with string attachment points in a small region near to the attachment point of a modification. The soundboard is a complex vibratory system and careful language must be used when discussing its response. It is clear that the addition of weights and riblets does have a local effect on mechanical impedance, but the impact on other vibrational and acoustic properties have been shown to be more global.



Figure 5.10: Impedance measurements at P1 20-2000 Hz - riblets at L1-L4.



Figure 5.11: Normalized change in impedance and sound level - P2 - 0-200g-0-0 modification - note 29.

The purpose of adding weights and riblets is to change the tone of the piano and it has been shown that impedance does relate to changes in initial amplitude and decay rate, two salient properties of timbre. The harmonic structure of certain notes should also be affected by changes in impedance because the impedance at the bridge acts as a filter for energy transfer between the strings and the soundboard. Changes in mechanical impedance should map to changes in the output sound of notes attached near to the modification locations. Figure 5.11 presents normalized curves of the change in frequency response for note 29 and the change in impedance measured at P2 when a 200 g weight is added at L2. The change in impedance and sound levels are both referenced to the unmodified piano and note 29 is chosen because its strings attach to the tenor bridge at L2. Curves have been fit to the data to better represent the general trends using a fourth degree polynomial interpolation. Changes in sound level map well to changes in impedance in the 20-400 Hz and 1500-2000 Hz bands with some deviation between 400 Hz and 1500 Hz. The maximum deviation occurs in the 800-1000 Hz range and could possibly be related to the bridge resonance discussed earlier. These results indicate that changes in impedance do map to changes in output sound for notes near to the modification point. Figure 5.12 shows the same change in impedance curve and 200 g modification at L2, but shows the change in sound level curve for note 40, a note outside the effective radius of the modification. The change in sound level curve for note 40 is close to zero for the majority of the spectrum and no connection is seen to the change in impedance curve.



Figure 5.12: Normalized change in impedance and sound level - P2 - 0-200g-0-0 Modification - note 40.

# Chapter 6

# Discussion, Conclusions, and Future Work

## 6.1 Discussion

#### 6.1.1 Modal Analysis

Modal analysis is one of the most commonly applied techniques in piano research and along with FEM modeling it allows researchers to gain an understanding of the low frequency vibrational properties of the piano. In this thesis it was observed that the mode shapes in a fully strung piano differ greatly from those reported in the literature for both unstrung pianos and the predictions of FEM modeling. The complexity of the piano is the obvious cause for these discrepancies, with the coupling between the soundboard, the case, the bridge, and the strings all playing significant roles in the low frequency vibrational characteristics of the soundboard. Another factor that complicates things is the natural variability of wood, a material that can greatly vary in material properties even for wood harvested from one tree. Including all of these factors in piano models will be a positive step towards achieving more accurate predictions of the piano's vibrational properties and output sound.

Examining commonly used voicing techniques was a major objective of this thesis and applying modal analysis after modifying the soundboard with these techniques allowed for their effect on modal properties to be determined. Riblets and weights were attached to the soundboard in the manner described in the Piano Technicians Journal [16] and tested in the laboratory. Riblets were observed to have only a minor effect on the modal properties of the soundboard, while weights were found to be effective in lowering the frequency of the first mode of vibration of the soundboard. The use of weights to extend the bass range of the piano is thus supported, along with the claim by Fandrich that riblets have little to no effect on the first vibrational mode of the soundboard.

In terms of frequency response it was assumed that vibration modes should play a significant role in the sound output of a piano, but this assumption is called into question when the modal analysis conducted in this thesis is combined with the tone mapping results. Theoretically large peaks along lines of equal frequency that correspond to the frequencies of the soundboard modes should be present in the tone maps. These peaks are not visible in the maps and their absence indicates that the overall harmonic structure of a piano is not highly dependent on its modal properties. The most prominent effects should be created by the first few modes, after which the modal density becomes so great that it becomes difficult to distinguish individual modes and the soundboard response becomes increasingly less frequency dependent. The sixth mode was found to have a frequency of approximately 300 Hz, but from figure 2.13 it can be seen that the frequency region below 300 Hz represents only a small portion of the harmonic structure of the piano. The impact of acoustical short circuiting, phase cancellation, reflections, and directivity will also act to alter the output sound spectrum of the piano and possibly minimize the importance of modes to its output sound. More detailed tests are needed to fully confirm these hypotheses, but it is suggested that the importance of modal properties to the overall sound of the piano is overemphasized in the literature.

#### 6.1.2 Tone Mapping

The power of the tone mapping technique is that it allows the patterns occurring in the harmonic structures of all the notes of the piano to be visualized in one image. A number of characteristic features were discovered using this new method for visualizing piano tone that were connected to the design of the piano and the vibrational properties of the soundboard. The relationship between hammer strike point and string length is traditionally set near 1/8th of the speaking length for most strings, and as a result n multiples of the eighth harmonic will be attenuated. These attenuations are clearly visible in the tone maps for all the pianos tested as valleys along lines of equal harmonic number. Examining the Bösendorfer map in figure 2.8 reveals the most prominent example of these structures.

Another unique feature that was discovered in all of the pianos occurs as a ridge along a line of equal frequency approximately equal to 3 kHz (visible in figure 4.3). Without the tone mapping visualization it would be very difficult to uncover this feature and it is believed that it has never been examined before. No explanation is available for why this occurs, but a hypothesis that it was related to sympathetic vibrations of the undamped high treble strings was tested and dismissed after a new tone map was created with these high treble strings damped (see figure 4.4). The "killer octave" was also hypothesized to be related to the termination point of this 3 kHz ridge, with the suggestion that the absence of higher order harmonic energy in this octave decreases sustain. Further experimentation is required to fully understand this ridge structure and how it relates to the perceived sound of a piano.

The locality of the effect of riblets and weights as voicing tehniques is claimed to be one of their major advantages, but it can clearly be seen from the difference maps (figure 4.12 - 4.19) that they do have an effect on the global harmonic structure of the piano. The most significant global effects were found immediately after hammer strike, with their impact becoming more local to the notes attached near to the modifications as time increases. Weights and riblets are both claimed [16, 13] to decrease initial amplitude and increase sustain, claims that were supported by the results obtained in this thesis.

#### 6.1.3 Impedance

In terms of mechanical impedance at the bridge, it was observed that weights and riblets both had a localized impact in a region less than 10 cm in radius from where they are attached. Weights were found to increase impedance between 100-600 Hz and decrease impedance between 20-100 Hz and 600-2000 Hz. The opposite was found to be true for riblets which increase impedance at higher frequencies and slightly decrease impedance at lower frequencies. These results confirm the statements of Rhodes and Conklin [16, 13] in terms of the localized effects of weights and riblets. Fandrich's [16] claim that riblets have little to no effect on the first mode of vibration of the soundboard was also confirmed by these impedance results. It must be kept in mind that changes in harmonic spectrum and mode shapes are not restricted solely to notes with string attachment points in a small region near to the attachment point of a modification. The complexity of the piano soundboard makes it important to use careful language when discussing its properties.

Another interesting result observed was a series of large spikes in the impedance curves in the 600-800 Hz frequency band for both riblets and weights. The cause of these spikes is unclear, but one hypothesis is that the natural frequency of the bridge occurs in this frequency band. Further testing is required to investigate this hypothesis.

## 6.2 Conclusions

The purpose of this thesis was to begin to understand the vibrational and acoustical properties of the piano soundboard. To accomplish this objective three analysis techniques were applied to a fully strung test piano. Modifications in the form of weights and riblets were made on the piano and measurements were taken in an attempt to quantify the change in vibrational and acoustical properties due to these modifications. Ideally the goal would be to connect these quantitative measurements with the subjective language used to describe musical instruments, but making a psychoacoustical connection was outside the scope of this thesis.

Modal analysis was the first technique applied and revealed that weights and riblets can have a significant effect on modal properties. The addition of mass was found to lower the frequency value of the first mode while increasing damping. Riblets also lowered the frequency value of the first mode, but not to the same degree that weights did. From the results it became clear that real soundboards in fully strung pianos behave quite differently from the predicted response of even very advanced computer models. It was recommended that the case and rim, randomized wood properties, and coupling between the bridge, strings, and soundboard be considered in models to make their predictions more comparable to real soundboards. An emphasis is also placed on the importance of mode shape in the identification of modification locations intended to change modal properties. Modifications near the anti-node of a mode were found to be the most effective.

The tone mapping technique provides an interesting new way to visualize the sound of the piano and also to document the global effect of small changes or modifications on piano tone. Phase maps, inharmonicity data, and other values can also be calculated and because the maps are created from recordings of output sound they have direct practical implications to piano tone, unlike other test methods like modal analysis or impedance measurements. Applying these maps to determine the effect of weights and riblets on the output sound of the piano revealed that while significant local effects were visible, a large region of the piano's harmonic structure was modified. A decrease in initial magnitude and an increase in sustain was also visible in the transient response of the piano.

Impedance measurements of the piano soundboard confirmed that the effect of weights and riblets is local in terms of impedance. It was also observed that they act to increase sustain in certain frequency bands by increasing impedance and slowing the transfer of energy from the strings to the soundboard. This increase in sustain comes at the cost of decreasing the initial amplitude of the output sound, but in most cases this is desirable. It was discovered that in terms of ability to change local impedance the most effective height of a riblet is approximately 25 mm and that riblets have their most significant effect in the 500-2000 Hz frequency band. Weights were found to be most effective below 800 Hz and to be cumulative in nature, with more weight creating more change in the impedance curve. Normalized changes in impedance were mapped to normalized changes in sound level of notes located near to a modification, with some deviation in the 600 Hz to 800 Hz frequency band that was hypothesized to be related to the natural resonance of the bridge.

## 6.3 Future Work

This preliminary understanding of how common voicing techniques affect the vibrational properties of a piano soundboard provides a clear path forward for future research. The next step is to link these changes to the subjective language used to describe musical instruments. To do this the recordings made during the tone mapping analysis could be employed in psychoacoustical testing. Every note of the piano was recorded for every modification so if a perceptible difference is created by adding weights and riblets it should be present in this data. Making this connection using psychoacoustical methods would serve as a starting point for further tests which could examine other piano design parameters including things such as

hammer voicing and material choices. Once the connection has been made between the subjective descriptions used by musicians and quantifiable piano properties the piano designer will be able to directly and accurately build pianos with a certain desired sound without the need for guesswork or experimentation. Making this link will also serve to demystify the piano, something that may open the doors to innovation an industry that is arguably stagnant and rooted too firmly in tradition.

Extending the application of the tone mapping techniques will also provide another positive avenue for future work. Three main problems exist with the method in its current form that need to be rectified before it can become a practical tool in the piano industry. The first problem is the accuracy of the peak finding algorithm which was found to break down after approximately 50 harmonics. As previously discussed, the problem of identifying harmonic peaks is complicated due to inharmonicity and the presence of non-harmonic peaks due to longitudinal string vibrations. Improving this algorithm would allow more accurate measurements to be made and would allow for the mapping of parameters like inharmonicity in more meaningful ways. The second problem is that the technique requires a user to run a complex series of Octave and Python scripts in a manner that is far from user friendly. Improving the usability would require a graphical front end being created that would allow a piano technician with minimal computer knowledge to use the software product to create tone maps. The final problem is the extensive computer resources currently required by the technique. High resolution FFT calculations are required for every note of the piano and when transient animations are created the system needs to run 150 or more sets of calculations to produce the final series of output images. Creating one 10s animation requires approximately 20 gigabytes of storage and over 4 hours of processing time on a well equipped computer (dual 3 GHz processors, 3 GB of RAM). With the seven different modifications tested in this thesis there was over 140 gigabytes of data produced and over a day of computer time spent processing the results. Reducing the size of the data sets and implementing more efficient FFT algorithms are two key tasks that would allow for the tone mapping technique to become more usable in an industrial setting.

Computer modeling of the soundboard and the entire piano can also benefit from the results of this thesis. The combination of impedance tests, modal analysis, and tone maps provide solid data upon which the accuracy of models can be compared and suggests ways to improve existing models such as including more realistic material properties and emphasizing the coupling that occurs between all parts of the piano. It is clear that the piano is an extremely complex system that not only connects mechanical and acoustical domains together, but also the perceptual and emotional domains of human experience. Producing models capable of engaging the emotions is a somewhat lofty goal, but a seemingly necessary one if models are to be considered satisfactory imitations of an emotive musical instrument like the piano. One of the greatest challenges in piano research, it would seem, is to never forgot the human connection to the instrument.

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# Appendices

## Piano Tone Maps

Tone maps are presented here for the initial t = 0 s frame for all of the pianos tested. These images are best viewed by rapidly flipping between different maps to make the subtle differences more obvious. Animations are also available in the electronic appendix along with all the raw image files. Tone maps of the Bösendorfer, Estonia, Schimmel, Heintzman, and Yamaha piano were created in preliminary tests and an experimental microphone was not used. The maps created from these recordings cannot be considered to be completely accurate, especially at low frequencies, but do provide a good picture of the harmonic fingerprint of the pianos tested. These pianos were tested on location in Toronto, Ontario, Canada, or in Waterloo, Ontario, Canada. Tone maps presented for the Hardman grand piano were created in the Piano Lab at the University of Waterloo and are of a higher standard in terms of equipment and recording methods.



Figure 1: Bosendorfer tone map.



Figure 2: Estonia tone map.



Figure 3: Hardman preliminary tone map.



Figure 4: Heintzman tone map.



Figure 5: Schimmel tone map.



Figure 6: Yamaha tone map.



Figure 7: Hardman 0-0-0-0 tone map - t0000.



Figure 8: Hardman 0-25m-0-0 tone map - t0000.



Figure 9: Hardman 0-50m-0-0 tone map - t0000.



Figure 10: Hardman 25m-25m-25m-0 tone map - t0000.



Figure 11: Hardman 0-200<br/>g-0-0 tone map - t<br/>0000.  $\,$ 



Figure 12: Hardman 0-400g-0-0 tone map - t0000.



Figure 13: Hardman 200g-200g-200g-0 tone map - t0000.

## Impedance Results

#### Position P1



Figure 14: Impedance at P1 - riblets.



Figure 15: Impedance at P1 - weights.



Figure 16: Impedance at P1 - weights and riblets.



Figure 17: Impedance at P1 - riblets at four locations.



Figure 18: Impedance at P1 - weights at four locations.



Figure 19: Impedance differences at P1 - riblets.



Figure 20: Impedance differences at P1 - weights.



Figure 21: Impedance differences at P1 - weights and riblets.




Figure 22: Impedance at P2 - riblets.



Figure 23: Impedance at P2 - weights.



Figure 24: Impedance at P2 - weights and riblets.



Figure 25: Impedance at P2 - riblets at four locations.



Figure 26: Impedance at P2 - weights at four locations.



Figure 27: Impedance differences at P2 - riblets.



Figure 28: Impedance differences at P2 - weights.



Figure 29: Impedance differences at P2 - weights and riblets.





Figure 30: Impedance at P3 - riblets.



Figure 31: Impedance at P3 - weights.



Figure 32: Impedance at P3 - weights and riblets.



Figure 33: Impedance at P3 - riblets at four locations.



Figure 34: Impedance at P3 - weights at four locations.



Figure 35: Impedance differences at P3 - riblets.



Figure 36: Impedance differences at P3 - weights.



Figure 37: Impedance differences at P3 - weights and riblets.

Position P4



Figure 38: Impedance at P4 - riblets at four locations.



Figure 39: Impedance at P4 - weights at four locations.



Figure 40: Impedance differences at P4 - riblets.



Figure 41: Impedance differences at P4 - weights.

Position P5



Figure 42: Impedance at P5 - riblets at four locations.



Figure 43: Impedance at P5 - weights at four locations.



Figure 44: Impedance differences at P5 - riblets.



Figure 45: Impedance differences at P5 - weights.

## Impedance and Sound Level Differences



Figure 46: Impedance and sound level differences at P2 - riblets - note 28.



Figure 47: Impedance and sound level differences at P2 - riblets - note 29.



Figure 48: Impedance and sound level differences at P2 - riblets - note 30  $\,$ 



Figure 49: Impedance and sound level differences at P2 - riblets - note 31



Figure 50: Impedance and sound level differences at P2 - riblets - note 32



Figure 51: Impedance and sound level differences at P2 - riblets - note 40  $\,$ 



Figure 52: Impedance and sound level differences at P2 - weights - note 28  $\,$ 



Figure 53: Impedance and sound level differences at P2 - weights - note 29  $\,$ 



Figure 54: Impedance and sound level differences at P2 - weights - note 30  $\,$ 



Figure 55: Impedance and sound level differences at P2 - weights - note 31  $\,$ 



Figure 56: Impedance and sound level differences at P2 - weights - note 32



Figure 57: Impedance and sound level differences at P2 - weights - note 40  $\,$ 

# Code

The three main software components are included here, with a complete listing of all Octave and Python scripts found in the electronic appendix.

### calcFFT.m

This script calculates the FFT for all the piano recordings in one set. It outputs a data file containing the results of the FFT for further analysis.

```
clear;
fpath = "damped/";
### Set the desired frequency resolution (Hz) in the frequency domain
ddf = 1;
### Set the upper and lower limit of frequency to be plotted
flow = 20;
fhigh = 20000;
### Individual files can be run by using the same value for fn and ln
fn = 1;
ln = 88;
#fftw('planner', 'patient');
### Start Loop for test files
for counter = fn:ln:
### load file
[x, Fs] = auload([fpath,"au/",num2str(counter),".au"]);
puts([fpath,"au/",num2str(counter),".au","\n"]);
fflush(stdout);
### Grab the portion of the signal to be analysed
x = x(1:floor(Fs/ddf), 1);
tsig = x(:,1);
t = 0:(1/Fs):(length(tsig)-1)/Fs;
### Do FFT
Yl = fft(tsig);
```

```
### Convert to Power
Pyyl = Yl.* conj(Yl) / length(Yl);
for n = 1:length(Pyyl);
Pyyl(n,1) = real(Pyyl(n,1));
endfor;
### Establish a vector corresponding to frequency values
ind = floor(length(Pyyl)/2);
df = (Fs/length(tsig));
freq = 0:df:(ind-1)*df;
indlow = freq > flow;
indhigh = freq > fhigh;
indlow = length(indlow)-sum(indlow);
indhigh = length(indhigh)-sum(indhigh);
freq = vec(freq(1,indlow:indhigh));
### Modify -inf values and take the log of Pyyl
logPyyl = 20*log(Pyyl);
for n = 1:length(logPyyl);
if logPyyl(n,1) == -inf;
logPyyl(n,1) = -100;
endif;
endfor;
logPyyl = logPyyl(indlow:indhigh,1);
Y1 = Y1(indlow:indhigh,1);
### Save data to an output file
save([fpath,"/fft/",num2str(counter),"fft.mat"], "logPyyl", "Yl", "freq");
endfor;
```

#### calcPEAKSTIME.m

This script searches for harmonic peaks by predicting the locaiton of the peaks using an adjustment for inharmonicity. It outputs a data file containing the frequency value and magnitude of the peaks along with other variables for further analysis.

```
clear;
### Choose folder
fpath = "hardman_Ot/";
### Number of harmonics to find (including fundamental)
np = 100;
### Fundamental Frequencies of the notes on a Piano
notef = [27.5; 29.135; 30.868; 32.703; 34.648; 36.708; 38.891; 41.203;
43.654; 46.249; 48.999; 51.913; 55; 58.270; 61.735; 65.406;
69.296; 73.416; 77.782; 82.407; 87.307; 92.499; 97.999; 103.83;
110; 116.54; 123.47; 130.81; 138.59; 146.83; 155.56; 164.81;
174.61; 185; 196; 207.65; 220; 233.08; 246.9; 261.63; 277.18;
293.66; 311.13; 329.63; 349.23; 369.99; 392.00; 415.30; 440;
466.16; 493.88; 523.25; 554.37; 587.33; 622.25; 659.26; 698.46;
739.99; 783.99; 830.61; 880; 932.33; 987.77; 1046.5; 1108.73;
1174.66; 1244.51; 1318.51; 1396.9; 1480; 1568; 1661.2; 1760.0;
1864.7; 1975.5; 2093; 2217.5; 2349.3; 2489; 2637; 2793.8; 2960.0;
3136; 3322.4; 3520; 3729.3; 3951.1; 4186;];
### Individual files can be run by using the same value for fn and ln
fn = 1;
ln = 88;
### Number of windows to create
nwin = 150;
for nw = 0:(nwin-1);
tfpath = ["time/",fpath, num2str(nw),"/"];
for counter = fn:ln;
load([tfpath,"/fft/",num2str(counter),"fft.mat"]);
```

```
puts([tfpath,"fft/",num2str(counter),"fft.mat","\n"]);
fflush(stdout);
### Find First nip=20 Peaks and calculate Inharmonicity B values
nip = 100;
F = rot90(freq);
P = logPyyl;
### Set the lower limit of frequency to be searched
flow = notef(counter,1)-500;
if flow < 20;
flow = 20;
endif;
### Set the upper limit of frequency to be searched
fhigh = (np+1)*notef(counter,1);
if fhigh > 20000;
fhigh = 20000;
endif;
FundFreq = notef(counter,1);
dallow = 0.35*FundFreq;
for i = 1:nip;
if ((i*FundFreq) < flow) || ((i*FundFreq) > fhigh);
Pk(1,i) = NaN;
PkV(1,i) = NaN;
elseif ((i*FundFreq - dallow) < flow);</pre>
[m,n] = find(F<=(flow), 1, 'last');</pre>
[p,q] = find(F<=(i*FundFreq+dallow), 1,'last');</pre>
Partialmax = max(P(n:q,1));
[Partial(1,i),z] = find(P(n:q,1)==Partialmax);
Pk(1,i) = F(Partial(1,i)+n-1);
PkV(1,i) = P(Partial(1,i)+n-1);
elseif ((i*FundFreq + dallow) > fhigh);
[m,n] = find(F<=(i*FundFreq-dallow), 1,'last');</pre>
[p,q] = find(F<=(fhigh), 1,'last');</pre>
```

```
Partialmax = max(P(n:q,1));
[Partial(1,i),z] = find(P(n:q,1)==Partialmax);
Pk(1,i) = F(Partial(1,i)+n-1);
PkV(1,i) = P(Partial(1,i)+n-1);
else
[m,n] = find(F<=(i*FundFreq-dallow), 1,'last');</pre>
[p,q] = find(F<=(i*FundFreq+dallow), 1,'last');</pre>
Partialmax = max(P(n:q,1));
[Partial(1,i),z] = find(P(n:q,1)==Partialmax);
Pk(1,i) = F(Partial(1,i)+n-1);
PkV(1,i) = P(Partial(1,i)+n-1);
endif;
endfor;
### Calculate Inharmonicity B values
for n = 1:nip-1;
B(n,1) = (n^2 * Pk(1,n+1)^2 - (n+1)^2 * Pk(1,n)^2) /
      ((n+1)^{4}*Pk(1,n)^{2}-n^{4}*Pk(1,n+1)^{2});
endfor;
B = sum(B(10:19,1))/10;
### Check for low magnitude fundamental (important in Bass notes)
#still needs to be implemented!
Pfund = Pk(1,20)/(20*(1+20^{2}*B)^{0.5});
### Predict peak frequencies using Inharmonicity Formula
for n = 1:np;
PPk(1,n) = n*Pfund*(1+n^2*B)^{0.5};
endfor;
### Save data to an output file
save([tfpath,"/peaks/",num2str(counter),"peaks.mat"], "Pk", "PkV");
endfor;
endfor;
```

### imMAGTIME.m

This script takes the data from calcPEAKSTIME.m and maps the magnitudes to a colormap. Tone map images are then saved for each time step.

```
clear;
fpath = "hardman_0t/";
cmap = jet(256);
divs = [7; 15; 23; 64;];
colormap(cmap);
### Individual files can be run by using the same value for fn and ln
fn = 1;
ln = 88;
### Number of windows to create
nwin = 75;
mkdir(["time/",fpath,"images"]);
for nw = 0:(nwin-1);
tfpath = ["time/",fpath, num2str(nw),"/"];
puts(["Window ",num2str(nw+1)," of ",num2str(nwin),"\n"]);
fflush(stdout);
for counter = fn:ln;
load([tfpath,"peaks/",num2str(counter),"peaks.mat"]);
Pk = Pk;
PkV = PkV;
MAG(counter,:) = PkV;
endfor;
[xs, ys] = size(MAG);
x = 1:xs;
y = 1:ys;
```

```
zlimits = [0 30];
MAG = MAG+20;
MAG(xs, ys) = zlimits(1, 2);
MAG(xs,ys-1) = zlimits(1,1);
# Pixels per block
bs = 20;
imax = max(max(MAG));
imin = min(min(MAG));
MAG(xs, ys) = NaN;
MAG(xs, ys-1) = NaN;
for x = 0: (xs-1);
for y = 0:(ys-1);
if isnan(MAG(x+1,y+1));
im((bs*x+1):(bs*x+bs),(bs*y+1):(bs*y+bs),1) = 1;
im((bs*x+1):(bs*x+bs),(bs*y+1):(bs*y+bs),2) = 1;
im((bs*x+1):(bs*x+bs),(bs*y+1):(bs*y+bs),3) = 1;
else
level = round(((MAG(x+1, y+1)-imin)/imax)*255)+1;
im((bs*x+1):(bs*x+bs),(bs*y+1):(bs*y+bs),1) = cmap(level,1);
im((bs*x+1):(bs*x+bs),(bs*y+1):(bs*y+bs),2) = cmap(level,2);
im((bs*x+1):(bs*x+bs),(bs*y+1):(bs*y+bs),3) = cmap(level,3);
endif;
endfor;
endfor;
im(1, 1:size(im, 2), 1) = 0;
im(1, 1:size(im, 2), 2) = 0;
im(1, 1:size(im, 2), 3) = 0;
im(size(im,1), 1:size(im,2), 1) = 0;
im(size(im,1), 1:size(im,2), 2) = 0;
im(size(im,1), 1:size(im,2), 3) = 0;
for n = 1:size(divs,1);
im((bs*divs(n,1)+2), 1:size(im,2),1) = 0;
```

```
im((bs*divs(n,1)+2), 1:size(im,2),2) = 0;
im((bs*divs(n,1)+2), 1:size(im,2),3) = 0;
endfor;
am = 255*ones(size(im,1),size(im,2));
im = 255*im;
pngwrite(["time/",fpath,"images/",num2str(nw),"hmagmap.png"],im(:,:,1),
im(:,:,2),im(:,:,3), am);
```

endfor;