Production Scheduling Optimization
of a Plastics Compounding Plant with Quality Constraints

by

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Author's Declaration

I hereby declare I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Michelle Leung

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Abstract

Production scheduling is a common problem that occurs in multi-product manufacturing facilities where a wide range of products are produced in small quantities, resulting in frequent changeovers. A plastics compounding plant offering tailor-made resins is a representative case. This kind of scheduling problem has already been extensively researched and published in the past. However, the concept of incorporating quality of the finished product has never been visited previously.

There are many different factors that may affect the quality of polymer resins produced by extrusion. One such factor is temperature. A production schedule cannot be related to the temperature or quality in any direct manner, and any other indirect relationships are not very apparent. The key to a correlation between the temperature of the processed material and the production schedule is the extruder flow rate. The flow rate affects the temperature of the molten plastic inside the extruder barrel, which means it also directly affects the quality of the final resin. Furthermore, the extruder is the critical machine in the extrusion process. Therefore, it determines the processing time of an order, serving as the basis for the scheduling problem.

The extruded polymer resin must undergo quality control testing to ensure that quantitative quality measurements must meet specifications. This is formulated as a constraint, where the extruder flow rate is determined to generate an optimized production schedule while ensuring the quality is within range. The general scheduling problem at a plastics compounding plant is formulated as a mixed integer linear programming (MILP) model for a semi-continuous, multi-product plant with parallel production lines. The incorporation of quality considerations renders the problem a mixed integer nonlinear program (MINLP).

Another objective of the proposed research deals with providing insight into the economic aspects of the scheduling process under consideration. The scheduling problem is analyzed and relations for its various cost components are developed. A total opportunity cost function was suggested for use as the comprehensive criterion of optimality in scheduling problems. Sensitivity analysis showed that none of the individual criteria gives optimal or near optimal results when compared to the total opportunity cost.

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1.1 Background

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1.1.1 Plant Operation

The plant that is being studied specializes in compounding of two different types of plastic, polycarbonate (PC) and polybutylene terephthalate (PBT), which are offered in a variety of grades and colours. This is a made-to-order plant, meaning that the products are tailor-made for each individual customer. The customer order specifying the quantity, grade, and colour initiates production on the line. Inventory is not kept by the plant to fulfill orders.

The plant operates on a 3-shift schedule, 24 hours a day, and 7 days a week. The compounding process is performed on an extrusion line. A job starts and ends on the same line and cannot be interchanged for quality assurance purposes. The equipment set up on each line is generally the same across all lines except for maybe a few minor differences. A typical line consists of a mixer, feeder, single or twin screw extruder, water bath, and a pelletizer. This will be discussed in further detail in the following section.

After each batch is completed, the remaining material inside the extruder is purged with material from the succeeding batch until all remnants have been cleared. It is usually not necessary to change the screws in the extruder in between runs on the same line. A colour matrix governs the length of purging (also referred to as changeover) time required between each pair of product colours. The darker the colour of the succeeding batch, the greater the increase in changeover time. For example, the time required to purge black with white, is much greater than if white were to be purged by black.

Once the batch has passed quality control, it is shipped off to the customer as soon as possible. There is limited warehouse space available for storage. Keeping finished product inventory will only increase costs to the company. Secondly, it is very important to the company to meet customer's deadline and there is no reason why delivery to customer should not be immediate, providing that it has met all necessary quality control requirements.

1.1.2 Process Description

A product is defined by its type, grade and colour. Each product has its unique recipe which specifies the required raw materials, the corresponding quantity, and the range of processing parameters for the equipment. Generally, processing parameters are not a function of product colour and depend only on the type and grade. The compounding process for different products shares the same basic structure (Figure 1-1). It is the operating parameters and certain steps that may differ.

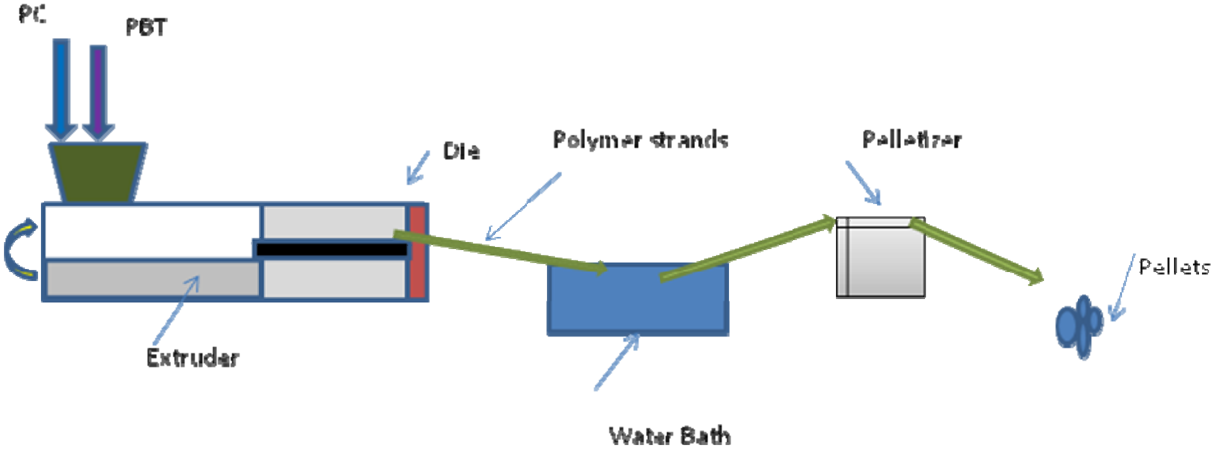


Figure 1-1 Polymer Compounding Line[1]

In the preparation step, the raw materials (polymers, colour pigments, fillers, additives) are weighed according to the formulation in the recipe by an operator at the weighing station. Once all the ingredients are assembled, it is brought to the assigned compounding line and loaded into the mixer. If the size of the job is greater than the capacity of the mixer, the job is divided into batches for mixing. The following formula is used to determine the batch size:

$$\text{Batch Size (kg)} = \frac{\text{Size of job (kg)} + \text{Safety factor (kg)}}{\text{Size of mixer (kg)}} \qquad (1.1)$$

The purpose of the safety factor is to ensure sufficient space in the mixer for achieving an even distribution of materials and to prevent overloading of the mixer.

The resulting mixture is fed via a gravimetric feeder to a single or twin screw extruder, depending on the degree of mixing and shearing the particular product requires. At the same time, a subsequent batch of raw materials can be loaded into the mixer while the previous batch is being processed in the extruder. As the molten plastic is forced through the die at high pressure, it is extruded as continuous strands of spaghetti-like plastic and cooled in a temperature-controlled water bath.

The type of extruder (single or twin screw) on each line is set and cannot be interchanged. However, the type and design of screw can be changed depending on the mixing, shearing, and kneading required. For the most part, there is a dedicated screw for each extruder, and are rarely changed in order to minimize downtime.

The last step in the compounding process is pelletization. It consists of a large blade that cuts the plastic strands into pellets. The speed of the blade is synchronized with the rate at which the strands are fed in order to achieve a relatively uniform pellet size.

The feed is a batch process while extrusion, cooling and pelletization are performed continuously. Therefore, this process can be classified as a semi-continuous process.

1.2 Thesis Overview

The structure of this thesis is as follows: in Chapter 1, a brief overview is given of the background, research initiatives, objective and scope of this study. Chapter 2 is a review of existing literature on the various aspects of scheduling that are relevant to this study to provide some background information. In Chapter 3, the different cost functions that compose the opportunity cost as defined are visited, as well as methods of comparison to determine the effectiveness of a schedule. Chapter 4 explains the quality characteristics that are taken into consideration in the development of the mathematical model as well as the correlations that were developed to relate these quality parameters with the production schedule. Chapter 5 is dedicated to presenting the mathematical formulation of the MINLP scheduling model. In Chapter 6, a case study is presented to illustrate the precisional efficiency of various cost components relative to using the total opportunity cost as the optimization criterion, and the shortfalls of the MINLP formulation. Lastly, conclusions of this thesis and recommendations for further research are presented in Chapter 7.

Chapter 2 Literature Review

The topic of production scheduling has been extensively researched over the years. When using a mathematical approach to solving industrial scheduling problems, different techniques such as constraint programming (CP), mixed integer linear programming (MILP), mixed integer non-linear programming (MINLP), or even a hybrid of CP and MILP formulations can be used, depending on the type of problem. MILP models are typically solved by brand and bound algorithms, and tend to solve general scheduling problems more efficiently than CP formulations. CP models are "solved through implicit enumeration techniques for domain reduction of variables, which in turn are based on constraint propagation techniques", and solve discrete manufacturing scheduling problems more effectively.[2] Generally, MILP models are used when optimization is the main goal, and CP models are used when feasibility is the main concern.[2]

2.1 Classification of Scheduling Problem

The first step to approaching any problem is to understand and define the problem. One general framework used for classification of production scheduling is defined by the following three criteria: [3,4]

1) Production requirements
2) Process structure
3) Scheduling Objectives

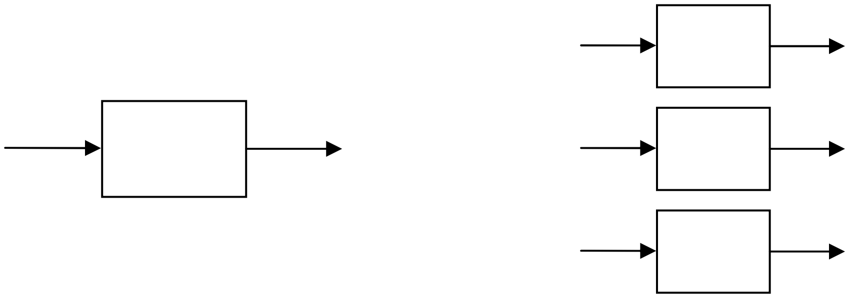
2.1.1 Production Requirements

A plant can be classified as a closed shop or an open shop based on its production requirements. In a closed shop, all customer orders are satisfied by products in inventory and a cyclic operation mode is employed.[5] In the cyclic operation mode, the production lines produce batches in the same sequence over a time period of fixed length. Production is initiated by the need to replenish inventory and not directly from customer's orders. In such a problem, the scheduling decision primarily involves determining the size and quantity of batches (lot sizing problem) to be produced on each processing unit, as well as the sequence in which they are to be processed (sequencing problem) in order to optimize a chosen objective function.[4] An example of an algorithm for optimization of cyclic scheduling was proposed by Pinto and Grossman.[6] The presented problem was formulated as a MINLP, consisting of a MINLP sub-problem that optimized the cycle time and inventory levels for a fixed sequence of products, and an overall MINLP problem that determined the optimal production order of those fixed sequences.

In an open shop, all production orders are initiated by customer orders, and finished product inventory is not stocked. In the simplest case, the open shop scheduling problem would involve only sequencing decisions.[3,4] This type of operation mode often use short term scheduling, and is typically found in plants that manufacture a large number of low volume, high-value-added or tailor-made products, such as polymer and specialty chemical plants because customers' orders cannot be effectively satisfied from product inventories.[5] Different formulations have been proposed to deal with the open shop problem. Kayis and Cheng Beng[7] developed a PC-based production scheduling system for the injection moulding process at a plastic manufacturing plant. The plant operates in an open shop manner, where the orders, the size of the order and the expected due dates are specified by the customer. More general models based on a single-stage, multi-product, batch plant with parallel lines were developed by Cerdá and Henning[5], Hui and Gupta[8], and Chen et al.[9]

2.1.2 Process Structure

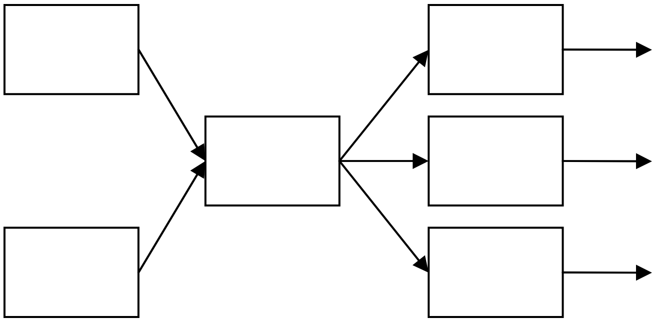
The second criterion addresses the complexity of the process, specifically the number of processing steps, and the configuration of the process units for each task.[2] Processes can generally be categorized into four configurations: single-unit process, parallel unit process, serial unit process, and generalized serial / parallel unit[4] as shown in Figure 2-1.



a) Single-unit process

b) Parallel unit process

c) Serial unit process



d) Generalized serial / parallel process

Figure 2-1 Different process configurations

Although the classical scheduling problem and many of its variants have been extensively studied by researchers, specific applications to the polymer industry with its numerous and complicated constraints are seldom reported in literature. Schulz and Engell[10] presented two different mathematical formulations, a continuous time representation model and a fixed-grid model, for scheduling of a multi-product polymer batch plant. It takes into account that the production process consists of both batch and continuous stages, resulting in a large, non-convex MINLP problem. Their main focus was to compare the effectiveness (the computation effort required) to compute the optimal schedule between the two models. The polymerization process that they had studied was combination of serial and parallel processes, similar to the process structure shown in Figure 2-1d. Wang et al.[11] studied a problem similar to the one presented by Schulz and Engell[10], but using an augmented genetic algorithm (GA) to solve. Castro et al.[12] addressed the short-term scheduling problem of a three parallel production line polymer compounding plant. The MILP model they proposed was based on a resource task network (RTN) discrete time formulation. By using the GAMS software in conjunction with the capabilities of Microsoft Excel for data handling and analysis, the user is able to obtain an optimal schedule based for a number of different objectives.

2.1.3 Scheduling Objectives

In Heuristics Scheduling Systems, Morton and Pentico defined a scheduling system as:

> “*A scheduling system dynamically makes decisions about matching activities and resources in order to finish jobs and projects needing these activities in a timely and high-quality fashion while simultaneously maximizing throughput and minimizing direct operating costs.*” [13]

To state this simply, scheduling is the process of allocating available production resources to complete a certain set of tasks in a given time period while satisfying one or more objectives. These objectives can typically be categorized as either performance-based or economic-based. Performance-based objectives include, but not limited to, minimization of:

- Maximum or average tardiness, defined as the positive difference between the actual completion time of the order and the due date.
- Number of tardy orders, to prevent customer dissatisfaction with orders that are not filled by the due date.

- Maximum or average flow time, where flow time is the duration between when the first job is released to the production floor until the last job in the given array is completed.
- Makespan, is a similar measure to flow time, but begins at the beginning of the scheduling horizon instead of when the first job is released.
- Mean earliness, in order to complete the order as close to the due date as possible.

Economic-based objectives aim to minimize costs which include: [3,4]

- Sequence dependent costs due to equipment set-up and product changeovers
- Inventory holding costs
- Shortage costs due to missing specified deadlines or for stocking out (in the case of a closed shop plant)
- Overhead and labour costs

In actual practice, schedules are evaluated on both performance and economic criteria. However, in academic and theoretical literature, models for open shop plants focus primarily on optimizing performance while models for closed shop plants are usually more concerned with minimizing costs.[3]

Cerdá and Henning[5] presented a multi-objective problem, with the aim to minimize makespan, total tardiness and the total number of tardy orders in three separate objective functions. Similarly, the formulation that Hui and Gupta[8] developed consisted of two separate objective functions to minimize tardiness and makespan, whereas the number of objectives for the same type of problem increases to five functions in the works of Chen et al.[9]

Kayis and Cheng Beng[7] proposed a single objective function that minimizes all of changeover, processing (good and defects), and inventory costs. A different approach to an economic-based measure was used by Kondili et al.[14] Their MILP formulation for a short-term scheduling batch operation problem maximizes the total profit, which is the difference between the value of the products and costs.

While many theoretical mathematical formulations have been proposed for the production scheduling problem, few are capable of handling an actual industrial-sized problem. The number of pieces of equipment, the array of products offered, and the many other production constraints result in a large-scale problem. In addition, most of the models that were proposed take into consideration only a few separate objectives, such as minimization of makespan or tardiness. In reality, it is highly unlikely that management would be satisfied in a production schedule that would only satisfy individual criteria. To tackle this, Janak et al.[15] presented a mathematical formulation for the production scheduling of a multi-purpose, multi-product industrial batch plant. The model is capable of handling over 80 pieces of equipment and can take into account the processing recipes of hundreds of different products. Furthermore, the multi-objective function of the model combines both performance-based and economic-based objectives. The overall objective function is composed of weighted individual functions with the main goal to maximize sales while minimizing starting times of tasks, number of binary variables, overall demand satisfaction, orders satisfaction, individual order amount, individual order due date, raw material demand, and minimum tank inventory.

Another attempt to breakthrough the use of conventional objective functions commonly aiming to minimize makespan, tardiness, or total cost of production was presented by Gupta in his PhD thesis entitled Economic Aspects of Scheduling Theory.[16] His research showed that the measure of performance that should be optimized is the total opportunity cost function. In addition, the use of minimization of maximum flow time as the optimality criterion may actually increase production cost for the company rather than optimizing it. Using sensitivity analysis, he concluded that none of the individual criteria (minimum operational costs, raw material costs, inventory costs, penalty costs, or utilities costs) gives optimal results when compared to the total opportunity cost. Thus, a modified version of the total opportunity cost function presented by Gupta will be used as the criterion of optimality for the scheduling model developed in later sections.

2.2 Time Domain Representation

Another important consideration that has to be taken when developing a scheduling model is the concept of time domain representation. There are generally two types of approaches: discrete and continuous-time. A graphical illustration of the two approaches is shown in Figure 2-2.

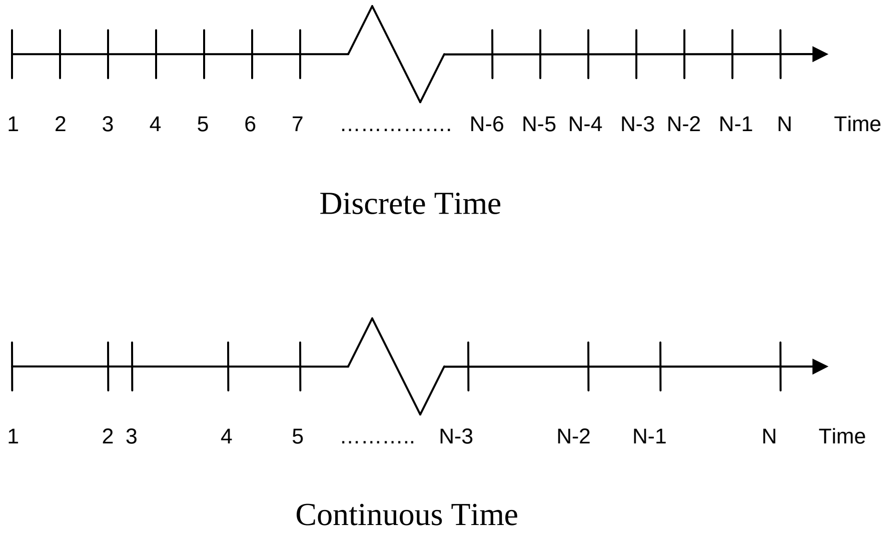


Figure 2-2 Discrete and continuous time representation

Early attempts at solving scheduling problems employed discrete time representation. In discrete time representation, the time domain is divided into time intervals of uniform duration. The start and end of events in the system are forced to coincide with the boundaries of the intervals. In order to achieve a certain degree of accuracy, the time intervals have to be sufficiently small such that accuracy is not greatly compromised. The greatest common factor (GCF) of the processing times is often used for determining the duration of the time intervals.[17] For an industrial size problem, this formulation may result in several thousands of binary variables, making the scheduling problem inefficient to solve using general-solution methods.[14,17] Therefore, discrete time representation is only an approximation and there exists a trade-off between accuracy and computability.

Discrete time representation has many advantages. Its main advantage is to provide a reference time grid for all operations competing for shared resources, such as equipment. This enables the scheduling problem to be formulated in simple and straight forward manner.[17] Kondili et al.[14] proposed a general formulation for the short term scheduling of batch operations based on the concept of State Task Network (STN). A STN is a graph that is composed of three elements: (1) state nodes, which represent the feed, intermediate and final product materials; (2) task nodes, which represent the process operations that transform materials from its input state to its output state; and (3) arcs, which indicate the flow of material in between the state and task nodes.[14] Uniform time discretization, where the time domain was divided into intervals of uniform duration, was used for time representation in this MILP model. As mentioned earlier, the major drawback with this type of mathematical formulation is the size of the resulting problem. Even with state of the art MILP solver, the solution of a small size problem requires significant amount of computation effort.[14]

Several techniques to decrease the size of the problem presented by Kondili et al.[14] have since been presented. These include reformulation of the allocation constraints from operations to machines by Shah et al.,[18] which greatly reduced the gap between MILP and the optimal solution of the corresponding relaxed linear program (LP), resulting in reduction of computational time; introducing cut constraints that will minimize the relaxation gap by Shah et al.[18] which is suitable for problems with long changeover times in comparison to processing times; and decomposition of the scheduling problem into smaller sub-problems by spatial and temporal decomposition, then solved sequentially by Elkamel et al.[19]

Due to the limitations presented by discrete time representation, the majority of scheduling algorithms found in literature are based on a continuous time domain. Pinto and Grossman[20] presented a continuous-time MILP model for short term scheduling of multistage batch plants using the concept of parallel time coordinates for units and tasks. The MILP model proposed by Cerdá et al.[5] is for the scheduling of single-stage multi-product batch plants with parallel lines in continuous-time. Tri-index variables and the concept of job predecessors / successors are used to handle the sequencing aspect of the schedule. Hui and Gupta[8] improved upon the Cerdá et al.[5] model by replacing tri-index variables with bi-index variables. This resulted in a decrease of the overall number of binary decision variables, and consequently, the computational effort required finding the optimal solution. There are many other scheduling models based on a continuous time domain including Ierapetritou and Floudas[21], Pinto and Grossman[20], Chen et al.[9], Castro and Grossmann[2], and Janak et al.[15]

For a more comprehensive investigation into time representation in scheduling problems, Munawar et al.[22] had completed a comparative study for continuous-time models for short-term scheduling of multi-purpose batch plants. Floudas and Lin[17] conducted a review of both continuous and discrete time approaches for scheduling of chemical processes.

2.3 Deterministic vs. Stochastic

Two other criteria that can be also used to classify a problem are the nature of the requirement specification and the scheduling environment.[3] Requirement specifications can be deterministic or stochastic. Deterministic processes can be defined as situations where the next state depends solely on the current state. An example would be an extruder operating at a known speed, and the throughput can be accurately calculated based on the operating speed.

A stochastic process is defined as a process in which “behavior is non-deterministic in that the next state of the environment is not fully determined by the previous state of the environment.”[23] For instance, the amount of defects in a given batch cannot be accurately predicted without some variability and uncertainty. The scheduling environment can be static or dynamic. In a static environment, requirements and specifications will no longer be added or changed at the time the scheduling problem is defined. In a dynamic environment, the scheduling problem is defined with respect to known requirements and specifications, as well as the need to anticipate and accommodate changes in future time periods within the scheduling horizon.[3] Sudden equipment failure or electricity outage or the arrival of “rush” orders that must be incorporated into the current schedule are events that can occur in a dynamic environment.

In a practical industrial setting, the scheduling problem is both stochastic and dynamic. However, most literature on the topic of production scheduling are deterministic and static in nature, because introducing uncertainties into the problem would make it much more complex.

2.4 Heuristics Approach

There are different software programs available (such as GAMS or SAS) that are capable of solving MILP and MINLP problems. However, even with the powerful capabilities and high throughput of today’s computers, it still takes a substantial amount of time to find the optimal solution for a problem with a size of industrial relevance. The use of heuristics will help decrease the number of variables and unknowns, resulting in a more efficient scheduling process. As a result, the use of heuristics may decrease the time it requires to solve a problem, but the tradeoff is optimality in the solution.

Musier and Evans[4] used an evolutionary strategy that requires an initial solution, then employs a method referred to as a heuristic improvement method (HIM) to systematically improve the solution until the objective value cannot be further improved upon. Cerdá and Henning[5] successfully employed heuristics to reduce the size of their proposed model. The heuristic rule reduces the number of feasible predecessors by discarding those that are not likely to result in an optimal schedule. For a problem involving 4 units and 20 orders, the number of binary variables and linear constraints in the model decrease from 194 and 446 respectively without the use of heuristics, to 125 and 270 when the proposed heuristics were used.

Hui and Gupta[8] later on improved on the model proposed by Cerdá and Henning[5] by using three sets of bi-index decision variables instead of the traditional tri-index decision variables. The model requires significantly less binary variables than earlier tri-index models, resulting in less computation time. However, the proposed model requires a greater number of constraints and continuous variables that may be compensated with the performance of the formulation. Similar heuristics as the ones proposed by Cerdá and Henning[5] were also used to reduce the model size. The authors pointed out that optimality of the solution cannot be guaranteed because the heuristic may have discarded an order sequence from the original problem that may otherwise have resulted in the optimal solution.

Chen et al.[9] introduced two heuristic rules by exploiting the characteristics of their model and multi-product single-stage batch plants. They have shown that the optimality of the resulting solution was not compromised through the use of heuristics.

2.5 Literature Review Conclusion and Research Direction

The literature reviewed in this section is by no means an exhaustive evaluation of all the research that has been conducted in the topic of production scheduling. Most academic literature on the mathematical formulation of the production scheduling problem is presented as a generic model. Only a certain few are targeted towards solving scheduling problems for specific operating processes. In many instances, researchers are primarily concerned with the scheduling problem itself, specifically sequencing, lot-sizing, assignment and allocation of resources, and/or improving computability. There are other factors within the operating processes that can be incorporated into the scheduling problem. For instance, Adonyi et al.[24] incorporated heat integration into batch production scheduling. The proposed method uses a branch and bound algorithm, and solve both the scheduling and heat integration optimization problem simultaneously.

Another possible process element that can be incorporated into the scheduling problem is the concept of controlling quality of the finished product. In the compounding of plastics, the quality of the finished product is a function of many parameters, and one such parameter is the extruder processing rate. There is a given speed range that the extruder must operate at in order to comply with the standards as stated by the recipe to ensure the quality of the finished products. This can serve as one of the constraints in the scheduling model. Most scheduling models in literature assume a specific processing time, but in the proposed model, the processing time is a function of the processing rates, which is within an allowable range. Therefore, the processing rate can be optimized to produce an optimal schedule while optimizing quality.

Chapter 3 Scheduling Objective

The total opportunity cost function proposed by Gupta[16] to be used as the optimization criterion for a scheduling problem is defined as the sum of operation cost, job waiting cost, machine idle cost, and penalty cost of jobs. This will serve as the primary basis of the objective function in the proposed model. The individual cost components presented below are slightly modified versions of the ones found in Gupta's model.[16] The objective functions in both models aim to achieve the same goal – to minimize total opportunity cost. The only notable difference between the two is the use of variable processing rate in the proposed model, whereas in Gupta's model[16], the output rate of the process is assumed to be fixed.

3.1 Operation Cost

The operation cost incurred in the span of a scheduling horizon is composed of two parts: setup / changeover cost and processing cost. The cost for changeover is assigned to the preceding job in a pair of consecutive jobs. For example, the changeover cost incurred when switching from job *i* to job *j* will be assigned to job *i*.

$$SC_i = \sum_{j \in PS_{ij}} \left(CC * C_{ij} X_{ij} \right) \qquad \forall i \in I \qquad (3.1)$$

where SC_i = setup or changeover cost of job *i* in dollars

CC = changeover cost per unit time in dollars

C_{ij} = changeover time from job i to job j

X_{ij} = binary variable indicating assignment of job j after job i

The processing cost of job i can be defined as the cost incurred directly by the actual compounding process, such as utilities and manpower but not including cost of raw materials. This is dependent on the output rate of the extruder and can be calculated as:

$$PC_i = PRC \cdot \frac{Q_i}{PR_{iu}} \cdot W_{iu} \qquad \forall i \in I, u \in U_i \qquad (3.2)$$

where PC_i = processing cost of job i in dollars

PRC = processing cost per unit time in dollars

Q_i = size of job i in kg

PR_{iu} = processing (or output) rate of job i on unit u in kg / hr

W_{iu} = binary variable indicating assignment of job i to unit u

Taking the total sum of the set up cost and the processing cost of each individual job result in the total operation cost, OC in dollars, of all jobs in the scheduling horizon.

$$OC = \sum_i (SC_i + PC_i) \qquad (3.3)$$

3.2 Job Waiting Cost

There is a loss of opportunity cost for the plant when raw materials and unfinished goods sit on the plant floor, waiting to be processed due to machine unavailability. Capital that is otherwise available to generate revenue is tied up in the in-process goods. Gupta (1969) called this cost job waiting cost, or otherwise known as in-process inventory cost. He defined the job waiting cost as the sum of the value of the raw materials required for the job and the subsequent value that has been added to it by the production steps prior to the step that require waiting due to machine unavailability.

In this particular plant, the entire process can be treated as one long process, since there is no disruption between each of the stages. The batch mixing process proceeds directly to extrusion, to cooling in the water bath, and to pelletization. Therefore, when calculating the job waiting cost, only the cost of the raw materials has to be taken into account and the additional value added on by previous processing steps can be omitted:

$$WC = \sum_i \left(WT_i \cdot R_i \cdot MC_i \right) \tag{3.4}$$

where WC = total waiting cost of all jobs in the scheduling horizon in dollars

WT_i = wait time of job i in unit time

R_i = expected return of raw materials used in job i

MC_i = raw material cost of job i

3.3 Machine Idle Cost

Similar to the concept of opportunity lost due to tied up occupied capital for in-process inventory, when a machine sits idling and not producing, there is an opportunity cost. This machine idling cost is directly proportional to the length of time that the machine is not utilized and represents the average revenue that the machine could have otherwise been generating by producing goods during that period of idling time.

$$IC = \sum_u R_u \cdot IT_u \tag{3.5}$$

where IC = total idling cost of all units u during the scheduling horizon in dollars

R_u = expected rate of return on unit u per unit time

IT_u = idling time of unit u

3.4 Job Penalty Cost

The last cost component that the total opportunity cost is composed of is the penalty cost of jobs that are late. This is difficult to quantify as not all the consequences of completion of job past the due date can be directly measured. Gere[25] suggests that the penalty costs incurred due to lateness of a job should include contractual penalty clauses, additional expenses incurred for dealing with customer, cost of expediting tardy jobs, and loss of good will cost.

The most difficult factor in determining job penalty cost is the loss of good will [26]. When a customer is dissatisfied with the services provided by the company, it results in customer dissatisfaction, and consequently, may result in lost sales in the future and/or loss of other potential customers due to damaged reputation. This cannot be accurately predicted, as it is solely based on the behavioural tendencies of the customer in question and can only be estimated with a degree of error.

It will be assumed that the penalty cost is directly proportional and increasing with the tardiness of the job. Expressed mathematically,

$$PnC = \sum_i D_i \cdot Pn_i \tag{3.6}$$

where PnC = total penalty cost of all late jobs in scheduling horizon in dollars

D_i = tardiness of job i in unit time

Pn_i = penalty cost for late job *i* per unit time

3.5 Total Opportunity Cost

By taking the sum of Equations 3.3 – 3.6 inclusively, the total opportunity cost, *TOC*, of a schedule can be expressed as:

$$Z = OC + WC + IC + PnC \qquad (3.7)$$

Theoretically, another term representing the loss from defective products, defined as products that do not meet quality specifications, can be included in the total opportunity cost (Equation 3.7). However, it would be assumed that the plant would aim to achieve producing polymers that are within quality limits 100% of the time. If a term denoting loss due to quality issues were to be included in the objective function, it can potentially allow the mathematical model to generate a solution that result in a lower total opportunity cost, but compromising on quality while increasing production costs due to discarding and/or recycling of defective products. This approach may be feasible, but would not make sense from a management perspective. By excluding this term in the calculation of the total opportunity cost, the model is obligated to enforce the quality constraints without any exceptions.

3.6 Method of Comparison

The method of measurement that Gupta[16] used for comparison of the total cost of a schedule obtained by minimizing one of the optimization criteria and one obtained by minimizing the total opportunity cost is called the precisional efficiency. Let C_{no} be the total cost of the schedule as defined by Equation 3.7, obtained by optimizing the n^{th} criterion, and let C_{1o} represent the total cost of the schedule obtained by minimizing the total opportunity cost, then the precisional efficiency of the n^{th} criterion of optimality is defined by Equation 3.8.

$$\Delta_n = \frac{C_{no} - C_{1o}}{C_{no}} \times 100 \tag{3.8}$$

The precisional efficiency evaluates the percentage deviation of the total cost of a schedule from the total cost of the optimal schedule obtained using minimization of the total opportunity cost as the optimality criterion.

A related measurement is the effectiveness, defined by Equation 3.9. If the total cost of a schedule obtained by optimizing the n^{th} criterion is the same as the one obtained by optimizing the total opportunity cost, then $C_{no} = C_{1o}$. The precisional efficiency would be 0 and the effectiveness of such a schedule would be 100%, yielding the same results cost-wise as a schedule generated by using the total opportunity cost as the criterion of optimality. The lower the precisional efficiency, the more effective the n^{th} criterion is.

$$\xi_n = 100 - \Delta_n \tag{3.9}$$

Chapter 4 Quality

A major difference between the proposed model and various models presented by other researchers on the subject of production scheduling is the incorporation of quality control into the model. With relevance to polymers, there are many chemical and physical properties that have to adhere to quality specifications. For the purpose of this thesis, only characteristics that are dependent on the extruder processing rate will be investigated, as the throughput rate of the process have a direct impact on the production schedule.

The correlations (Equation 4.1 – 4.3) for melt volumetric flow rate, impact, and specific energy consumption with respect to the independent variables: processing rate and screw speed, were derived by Vahid Noeei for his Master's thesis entitled *A Study of Polycarbonate / Poly(butylene terephthalate) Compounding in a Twin Screw Extruder*[1], and are specific to the equipment and polymer being compounded. Experiments were performed at the SABIC Innovative Plastics, Cobourg plant, using a Wernier Pfleiderer 58mm co-rating twin-screw extruder to compound Xenoy®, a blend consisting of polycarbonate (PC) and polybutylene terephthalate (PBT). For compounding of different materials and/or utilization of different extruders, additional experiments have to be conducted to determine the appropriate correlations.

4.1 Melt Volumetric Flow Rate

The Melt Flow Index (MFI) is a standard rheological quality control test specified in the ISO, BS, and ASTM. It was primarily used in the past for specification and characterization of polyethylenes, but has since been extended for use for other thermoplastics. It can be used to predict physical, mechanical, thermal, and certain optical properties.

MFI is basically defined as the weight of the polymer (g) extruded in 10 minutes through a capillary of specific diameter and length by pressure applied through dead weight under prescribed temperature conditions.[27] The standards that SABIC Innovative Plastics adhere to were used in the development of the correlation described later on in the section.

Instead of reporting the MFI, SABIC Innovative Plastics uses a similar parameter for quality control, the melt volumetric flow rate (MVR). The MVR is defined as the MFI divided by the polymer melt density:

$$MVR = \frac{MFI}{\rho} [=] \frac{\frac{g}{10\min}}{\frac{g}{cm^3}} [=] \frac{cm^3}{10\min} \tag{4.1}$$

The product melt volumetric flow rate (MVR) is a quality characteristic that is usually measured offline in a laboratory. As with many other product parameters, a target value is usually set, with an upper and lower limit. Samples are drawn from the production line, and taken to the laboratory for analysis using an extrusion plastometer. Due to the time lag between sampling time and the relay of analytical results back to the production line, any deviation from the specification limits will not be detected and reported until a certain time period later, resulting in continual production of defective material that must be either reworked or discarded. As an attempt to resolve this problem, Ancheh [1] proposed a mathematical model for inferential estimation of MVR from die pressure model parameters. If N_u is the screw speed in rpm and PR_{iu} is the processing rate of job i on unit u, then the MVR may be estimated by the following equation:

$$MVR_{iu} = 28.8 + 0.0856 N_u - 0.0812 PR_{iu} \qquad \forall i \in I, u \in U_i \qquad (4.2)$$

This multiple regression model has a coefficient of determination of 0.899 and an adjusted coefficient of determination of 0.887.

4.2 Impact Strength

The impact strength is a measure of the toughness of a material. There are several different tests that are used to determine the impact strength. The standard test that is generally used in North America is ASTM D256[28], the Notch Izod Impact Test (Figure 4-1). A notched sample of the material is clamped in place in a vertical position with the notch facing the direction of impact. A swinging pendulum held at a specific height is released and breaks the specimen. The impact strength is the amount of energy required to break the sample, determined by calculating the energy lost by the pendulum per unit of sample thickness or cross sectional area at the notch, based on the mass, release height, and return height of the pendulum after impact. The dimensions of the specimen as stipulated in ASTM D256[29] are 12.7mm x 64mm x 3.2 mm (½” x 2-½” x 1/8”), with a notch in the middle of the specimen (Figure 4-2).

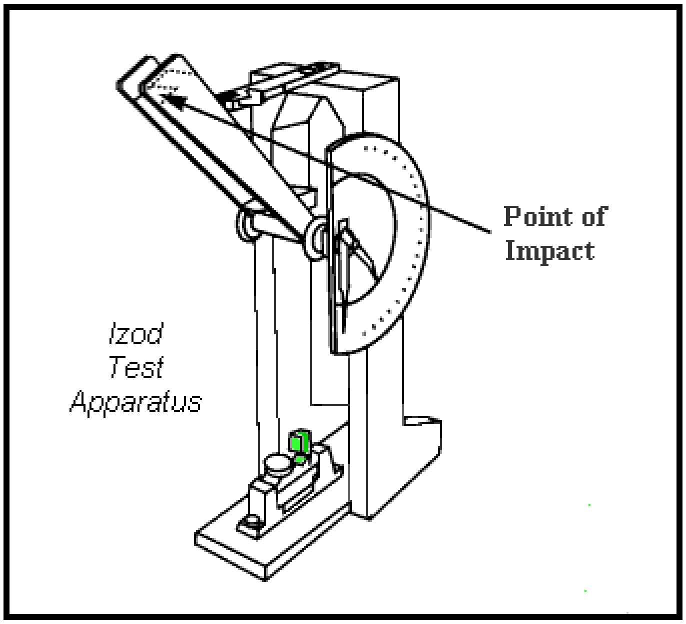


Figure 4-1 Notch Izod Impact Test Set-up[30]

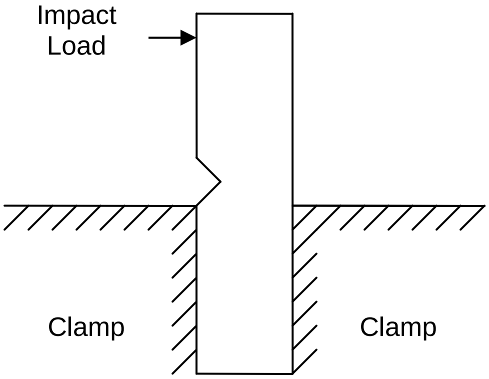


Figure 4-2 Notch Izod Impact Test Sample

Ancheh[1] showed that impact strength is dependent on screw speed and processing rate of the extruder. Let impact strength be *IMP* in J/m^2, then it can be estimated by:

$$IMP_{iu} = 14.5 - 0.0152N_u + 0.0150PR_{iu} \qquad \forall i \in I, u \in U_i \qquad (4.3)$$

This multiple regression model has a coefficient of determination of 0.528 and an adjusted coefficient of determination of 0.472.

4.3 Specific Energy Consumption

One additional parameter that is incorporated into the scheduling model is the specific energy consumption (SEC). The SEC which represents the amount of energy required per unit mass of material can be derived by dividing load by the throughput. Unit of SEC is kW·h/kg. SEC is a measure of the total deformation that the material is subjected to during the extrusion process and the stress that is required to bring about this deformation.[31]

The regression model for SEC in kW·h/kg developed by Ancheh [1] was given as:

$$SEC_u = 0.179 + 0.00191 N_u - 0.000120 PR_{iu} \qquad \forall i \in I, u \in U_i \qquad (4.4)$$

This multiple regression model has a coefficient of determination of 0.887 and an adjusted coefficient of determination of 0.874.

Chapter 5 The Scheduling Model

5.1 Problem Definition

The objective of this model is to generate a production schedule for a multi-product plastics compounding plant. The plant operates 24 hours a day, 7 days a week and run on a 3-shift schedule. For scheduling purposes, it is assumed that there is a smooth transition in between shifts, with no break in production. There are a fixed number of extrusion lines and a set of jobs that have to be processed within the scheduling horizon. No additional jobs may be added during the current scheduling period.

The processing units on each line are not identical. They vary by manufacturer, model, screw design, and age. Pre-emption, or splitting of jobs among different extrusion lines are not allowed. This is to ensure quality consistency within the batch and to eliminate any errors that may occur when packaging. It is assumed that there are no resource constraints, manpower or material-wise, unless otherwise stated in the form of delayed job or unit release time.

In between consecutive production orders, a changeover is required. During this time, any purging and cleaning of the equipment is executed. The processing line is product-type dedicated; therefore, the screws inside the extruder do not normally have to be changed in between jobs. The time allotted for changeovers in the schedule is governed by a matrix. The matrix is compiled based on an average of past changeover times. For accounting purposes, the cost incurred for changeovers will be allocated to the preceding job in the sequence. It is assumed that any preventative maintenance will be performed outside of the scheduling period.

5.2 Mathematical Formulation

The scheduling problem described above can be modeled as a general mixed integer non-linear programming (MINLP) model for a single-stage, non-identical parallel line, multi-product, semi-continuous plant with sequence-dependent and quality constraints. This bi-index formulation in continuous time domain representation was based primarily on the mixed integer linear programming (MILP) model proposed by Hui and Gupta (2001). The researchers have demonstrated that the use of the bi-index model for job sequencing and job allocation to various processing lines, requires fewer numbers of binary variables as compared to the traditional tri-index model, resulting in shorter solution times.

There are three major differences between the proposed formulation and that of Hui and Gupta's which are the objective function of the model, the use of variable processing (or output) rates, and the incorporation of product quality constraints. The objective that Hui and Gupta (2001) had chosen was based on conventional performance measures, specifically the minimization of total tardiness and the minimization of makespan. They had assumed a fixed processing time for each order. Although, depending on the unit that the job is assigned to, the processing time may vary. The integration of quality control aspects into production scheduling is a new concept. There are also a few other minor modifications and additions to the constraints to account for the specific requirements of the particular problem.

The notation used in the mathematical formulation for the problem sets, variables and parameters are explained in the following sections.

5.2.1 Definition of Sets, Variables, and Parameters

Indices

The two main entities that have to be kept track of are the array of orders that have to be processed in the given time period, and the number of extrusion lines that the compounding plant is equipped with for this particular process.

i, j	jobs / orders
u, v	units / lines

Note: The terms *jobs / orders* and *units / lines* are used interchangeably throughout this thesis.

Problem Sets

Within the set of jobs and units, there are subsets that are used for indicating the domain of which certain constraints apply.

I	set of orders to be processed in the scheduling horizon
I_u	set of orders that can be processed on unit u in the scheduling horizon
PP_{ij}	set of orders i that are feasible predecessors to order j
PS_{ij}	set of orders j that are feasible successors of order i
U	set of units available for processing
U_i	set of units capable of processing order i

Binary Problem Variables

Three sets of binary problem variables are used to make the discrete operational decisions of the production schedule, specifically to obtain the optimal sequencing of jobs, allocation of jobs to the appropriate processing lines, and to dictate which job will be first processed on the line.

X_{ij}	assignment of job j to be processed directly after job i on the same unit
W_{iu}	assignment of job i to be processed on unit u
S_{iu}	assignment of job i as first job to be processed on unit u

Non-Negative Problem Variables

A number of continuous non-negative variables are used to define the specific details of the production schedule.

D_i	tardiness (or delay in the completion after due date) of job i in days
H	makespan (or timespan between start of first order to completion of last order) in days
IT_u	idle time of unit u in days
LTF	latest completion time of the set of jobs in days
PR_i	output rate of job i on unit u in kg/hr
TF_i	completion time of job i in days
TS_i	starting time of job i in days
TT	total tardiness in days
WT_i	waiting time of job i in days

The several additional continuous non-negative variables which are used to define the various cost components of the objective function were introduced in Chapter 4. To briefly reiterate:

IC	total idle cost of all units during the scheduling horizon in dollars
PC_i	processing cost of job *i* in dollars
PnC_i	total penalty cost for all late jobs in scheduling horizon in dollars
SC_i	setup or changeover cost of job *i* in dollars
OC	total operation cost in dollars
WC	total waiting cost of all jobs in the scheduling horizon in dollars
Z	total opportunity cost in dollars

The following non-negative continuous variables are used to capture the quality characteristics of the products that depend on the output rate of the process.

IMP_{iu}	Izod impact energy of material produced from job *i* on unit *u* in kJ/m^2
MVR_{iu}	melt volumetric flow rate of material produced from job *i* on unit *u* in cm^3/10 min
N_u	extruder screw speed of unit *u* in rotations per minute
SEC_u	specific energy consumption of unit *u* in kW/(kg/hr)

Order / Line Parameters

The remaining information required for determination of the production schedule are categorized as parameters. These are fixed values that are dependent on the job and/or extrusion line.

C_{ij}	changeover time from job *i* to job *j* in days
CC	changeover cost in dollars per day
LB_{iu}	lower bound on processing rate of job *i* on line *u* in kg/hr
MC_i	raw material cost for order *i* in dollars per kg
PC_i	penalty cost of late order *i* per day
Q_i	size of job *i*
R_i	expected rate of return on raw materials
R_u	expected rate of return of machine
RTO_i	release time of job *i* in days
RTU_u	release time of line *u* in days
τ_i	due date of job *i* in days
UB_{iu}	upper bound on processing rate of job *i* on line *u* in kg/hr

5.2.2 Problem Constraints

a) Assignment of consecutive orders

This constraint ensures that consecutive orders are processed on the same unit. If job *i* is assigned to unit *u*, and job *j* succeeds job *i* in the job sequence, then job *j* must also be assigned to be processed on unit *u*. This can be enforced by the following constraint:

$$W_{iu} + \sum_{\substack{v \in U_j \\ v \neq u}} W_{jv} + X_{ij} - 2 \leq 0 \qquad \forall i \in I, j \in PP_{ij}, u \in U_i \qquad (5.1)$$

b) Each order can have a maximum of one direct succeeding order

Every order can at most, have only one unique successor in the job sequence that is within the subset of possible successors unless it is the last order assigned to the unit. If job *i* is indeed assigned to be the last processed on the unit, then any value of X_{ij} or the left hand side (LHS) of equation 5.2, would equal 0, and the inequality holds.

$$\sum_{j \in PS_{ij}} X_{ij} \leq 1 \qquad \forall i \in I \qquad (5.2)$$

c) Each order can have a maximum of one direct preceding order

Every order must either be assigned as the first job to be processed on the line, or have a unique direct predecessor in the assigned job sequence which is in the subset of possible predecessors.

$$\sum_{j \in PP_{ij}} X_{ji} + \sum_{u \in U_i} S_{iu} = 1 \qquad \forall i \in I \qquad (5.3)$$

d) Each unit must have exactly one order assigned to it as the starting order

This constraint is based on the assumption that production capacity will be fully utilized, meaning that every production line will have at least one job assigned to it. There can only be one unique order that is assigned as the first order to be processed on each unit.

$$\sum_{i \in I_u} S_{iu} = 1 \qquad \forall u \in U \qquad (5.4)$$

e) Each order must be processed

For each order that exists, it must be processed. In other words, every job must be assigned to exactly one unit. This can be achieved by stating that the summation over the elements in U_i equates to 1 for each order *i*.

$$\sum_{u \in U_i} W_{iu} = 1 \qquad \forall i \in I \qquad (5.5)$$

f) Definition of starting times of consecutive orders

If order *j* is assigned to be processed directly after order *i* on the same unit, then the starting time of order *j* must be greater than the sum of the starting time and processing time of order *i*, in addition to the time required for the changeover from job *i* to job *j*, C_{ij}. The changeover time is independent of the unit, and is only a function of the jobs *i* and *j*.

The processing time of job *i* is defined by the division of the quantity size by the processing rate. The processing rate of job *i* is variable across units, but remains constant while processing any given job. In the case where job *j* is not assigned to be consecutive to job *i*, the very large number *M*, will allow the starting time of job *j* to have any value but continue ensuring the inequality to hold true.

$$(1 - X_{ij})M + TS_j \geq TS_i + \sum_{u \in U_i} \left(W_{iu} \cdot \frac{Q_i}{PR_{iu}} \right) + C_{ij} \qquad \forall i \in I, j \in PS_i \qquad (5.6)$$

g) Definition of starting time of first order on the unit

The starting time of a job must be greater than either the release time of the assigned unit or the release time of the job, whichever has a larger value. In the event that a job is assigned to be first processed on the unit, then the starting time of the job must be equal to the maximum of the unit release time and job release time, hence the use of inequality to take both scenarios into consideration.

$$TS_i \geq \sum_{u \in U_i} \left[W_{iu} \cdot Max\left(RTU_u , RTO_i \right) \right] \qquad \forall i \in I \qquad (5.7)$$

h) Relation between the variables W_{iu} and S_{iu}

By adding the constraint that W_{iu} must be greater than or equal to S_{iu} for all elements in I and U_i, it ensures that any order that is designated as the first order to be processed on unit *u* indeed assigned to that particular unit.

$$W_{iu} \geq S_{iu} \qquad \forall i \in I , u \in U_i \qquad (5.8)$$

i) Definition of tardiness

Tardiness is the positive difference between actual completion time of an order and its target due date. If for example, an order is completed prior to the due date, then it would have a tardiness of 0. Whereas if the target due date of an order was *n*, and the actual completion time of the order was *n*+3, the tardiness of the job would be 3.

$$D_i \geq TS_i + \sum_{u \in U_i} \left(W_{iu} \cdot \frac{Q_i}{PR_{iu}} \right) - \tau_i \qquad \forall i \in I \qquad (5.9)$$

j) Determination of order completion time

The completion time of an order is simply defined as the sum of the starting time and its processing time.

$$TF_i = TS_i + W_{iu} \cdot \frac{Q_i}{PR_{iu}} \qquad \forall i \in I, u \in U_i \qquad (5.10)$$

k) Definition of job wait time

The calculation of job wait time is necessary for determination of the opportunity cost of jobs waiting for processing. From the moment the job is released for processing until the actual starting time of the job, the materials are tied up as in-process inventory costs instead of being transformed into the much more valuable finished product.

$$WT_i \geq TS_i - RTO_i \qquad \forall i \in I \qquad (5.11)$$

l) Determination of completion time of last job processed

The latest completion time of all jobs *i* can be derived from the following equation:

$$LTF \geq TF_i \qquad \forall i \in I \qquad (5.12)$$

m) Definition of unit idle time

Simply stated, the idle time of a unit is the actual time that the unit is not processing any jobs. Therefore, the idle time of unit *u*, is the latest completion time of the last job in the entire production schedule minus the processing times of all jobs allotted to unit *u* while taking into account the release time of the unit. The justification in using the latest completion time of the last job in the entire schedule instead of the last job assigned to that particular unit is that, even though one particular unit may have complete processing all jobs assigned to it, it is still sitting idle while other units are still running.

$$IT_u \geq LTF - \sum_{i \in I_u} \left(W_{iu} \cdot \frac{Q_i}{PR_{iu}} \right) - RTU_u \qquad \forall u \in U \qquad (5.13)$$

n) Definition of total tardiness

Total tardiness is a performance-based measure commonly used as optimization criterion for a production schedule. It is the sum of the tardiness of all jobs. This calculation is included in the scheduling model for comparison to using the total opportunity cost as the optimization criterion.

$$TT \geq \sum_i D_i \qquad (5.14)$$

o) Determination of schedule makespan

Similar to total tardiness, the minimization of makespan is a performance-based measure often used as the scheduling objective in academic literature. Makespan is defined as the total time required for completion of an array of jobs, from the beginning of the scheduling horizon to the completion time of the last job.

$$H \geq TF_i \qquad \forall i \in I \qquad (5.15)$$

In order to calculate the individual cost functions, equations 3.1 to 3.6 are used. Note that the equal sign in the equations shown in Chapter 3 has been changed to a sign of greater than or equal to. This is due to the fact that the program used for solving the MINLP model, GAMS, has difficulty solving the scheduling model if equalities were used. Due to the nature of the model, it will not affect the resulting schedule because by minimizing the total opportunity cost objective function, it forces each of the individual cost function to take on the smallest possible value. To briefly reiterate:

p) Definition of set up cost

$$SC_i \geq \sum_{j \in PS_{ij}} \left(CC * C_{ij} X_{ij} \right) \qquad \forall i \in I \qquad (5.16)$$

q) Definition of processing cost

$$PC_i \geq PRC \cdot \frac{Q_i}{PR_{iu}} \cdot W_{iu} \qquad \forall i \in I, u \in U_i \qquad (5.17)$$

r) Definition of total operation cost

$$OC \geq \sum_i (SC_i + PC_i) \qquad (5.18)$$

s) Definition of job waiting cost

$$WC \geq \sum_i \left(WT_i \cdot R_i \cdot MC_i \right) \qquad (5.19)$$

t) Definition of unit idle cost

$$IC \geq \sum_u R_u \cdot IT_u \qquad (5.20)$$

u) Definition of penalty cost

$$PnC \geq \sum_i D_i \cdot Pn_i \tag{5.21}$$

As discussed in the previous chapter, the concept of quality is introduced into the scheduling model. The three main quality measures that can be related to the production schedule are melt volumetric flow rate, impact strength and specific energy consumption. These three measures are all dependent on the output rate and the screw speed of the extruder, which affects the processing time of an order and ultimately, the production schedule.

v) Definition of melt volumetric flow rate

$$MVR_{iu} = 28.8 + 0.0857 N_u - 0.0812 PR_{iu} \qquad \forall i \in I, u \in U_i \tag{4.2}$$

w) Definition of impact strength

$$IMP_{iu} = 14.5 - 0.012 N_u + 0.0150 PR_{iu} \qquad \forall i \in I, u \in U_i \tag{4.3}$$

x) Definition of specific energy consumption

$$SEC_u = 0.179 + 0.00191 N_u - 0.000120 PR_{iu} \qquad \forall i \in I, u \in U_i \tag{4.4}$$

5.2.3 Objective Function

The optimization criterion for this scheduling problem is the total opportunity cost, *Z*. An optimal schedule will be a feasible schedule that minimizes the total opportunity cost, while taking into consideration all the relative constraints from a) to x).

$$Z = OC + WC + IC + PnC \tag{3.7}$$

Chapter 6 Case Study

6.1 Problem Statement

The MINLP model consisting of the objective function and all constraints proposed in Chapter 5 is formulated in GAMS and solved using the BARON solver for global optimality. The model was optimized for 16 different objective functions: the 15 exhaustive combinations of the cost functions that compose the total opportunity cost and the makespan (Table 6.1).

Table 6.1 List of optimization criteria for minimizing

n	Optimization Criteria	Abbreviation
1	Total Opportunity Cost	Z
2	Operation Cost + Idle Cost + Penalty Cost	OIP
3	Operation Cost + Waiting Cost + Penalty Cost	OWP
4	Operation Cost + Waiting Cost + Idle Cost	OWI
5	Waiting Cost + Idle Cost + Penalty Cost	WIP
6	Operation Cost + Idle Cost	OI
7	Operation Cost + Penalty Cost	OP
8	Operation Cost + Waiting Cost	OW
9	Waiting Cost + Idle Cost	WI
10	Waiting Cost + Penalty Cost	WP
11	Idle Cost + Penalty Cost	IP
12	Operation Cost	O
13	Wait Cost	W
14	Idle Cost	I
15	Penalty Cost	P
16	Makespan	H

A set of hypothetical data were used in the case study. The data were taken from Cerda et al.[5] with minor modifications and additions to account for the additional requirements of the problem. The example consists of a number of jobs that have to be assigned and sequenced on 4 processing units. Information about each order such as order size, due dates, release times of orders and/or unit, as well as the unit costs for the different cost functions are specified. See Appendix A for raw data.

The sequence-dependent setup times are shown in Table A2 in Appendix A. This value is independent of the unit in which it is processed. A blank indicates that job *i* cannot be processed before job *j*. In Table A3, a value of 1 indicates that a job *i* can be processed on unit *u*, and a blank indicates that it cannot.

Due to limitations on correlations availability, it will be assumed that the 4 processing units are identical but have different operating ranges (Table A4) and will compound only Xenoy resins. As more correlations for other materials and extruders are developed via laboratory studies, the MINLP model can easily be modified by replacing and/or adding the necessary constraints. The specifications for melt volumetric flow rate, impact, specific energy consumption and the limitations on screw speed can be found in Table A5. The various unit costs used to calculate the different cost functions are shown in Table A6.

6.2 Results and Discussions

The precisional efficiency was calculated for all the schedules that were generated. Graphical representations of the results are shown in Figure 6.1 and all the data for the figures can be located in Appendix B. The optimization criteria are shown in order of decreasing effectiveness.

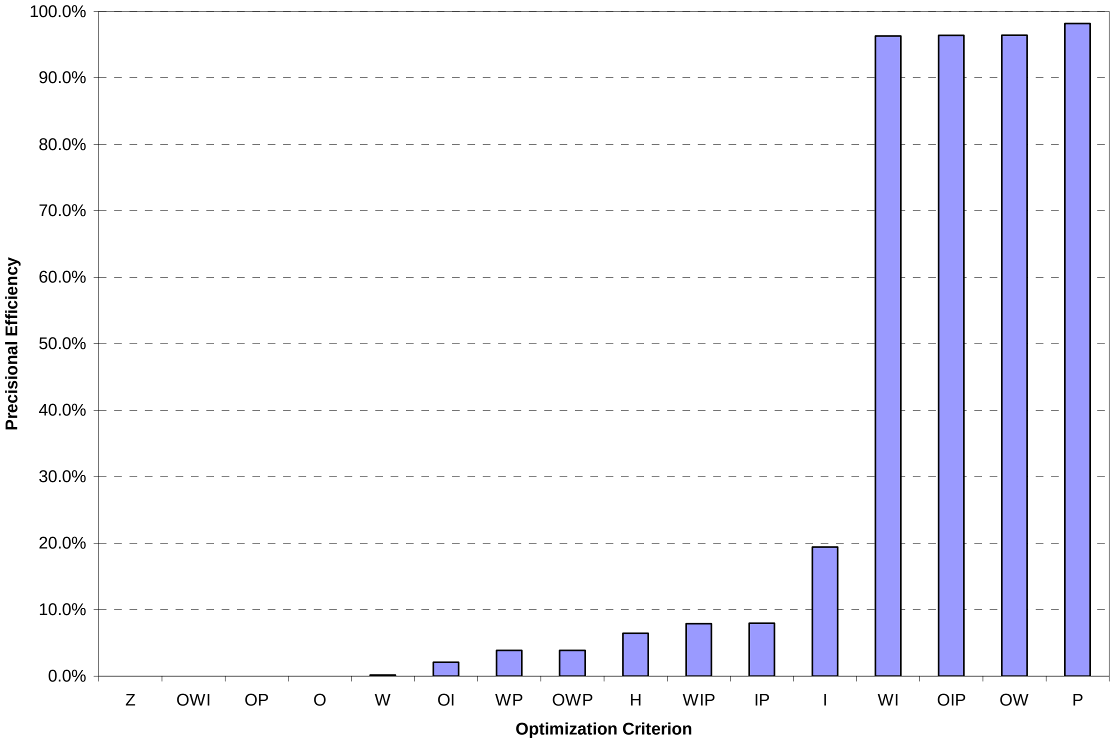


Figure 6-1 Precisional efficiency for each optimization criterion

It is evident that aside from optimizing the total opportunity cost, minimization of subsets of other cost functions may also generate an optimal schedule. In this particular data set, the minimization of the (1) sum of operating cost, waiting cost, idling cost; (2) sum of operating cost, penalty cost; (3) operating cost; and 4) waiting cost also generated the same value for the total opportunity cost, resulting in an optimal schedule. A look at the individual schedules reveals that the sequence of jobs is not the same for all 5 schedules, indicating that more than one optimal schedule can exist.

The optimal schedule obtained in the case of total opportunity cost (Z) and its details (start time and end time) is given in Table B1. As can be seen, the schedule is feasible and all tasks starting on a given unit do not coincide with any other task during the same processing times. For illustration purposes, the schedule obtained in the case of makespan (H) is given in Table B16. As can be seen, when the focus is only on makespan, the schedule tries to have the orders performed as soon as possible without paying attention to other objectives. This results in an overall opportunity cost greater than the case when the optimization considers all costs into consideration.

MINLP problems are often very difficult to solve because of the complexities of their sub-classes: the combinatorial nature of mixed integer programs (MIP) and the difficulty in solving non-convex non-linear programs (NLP).[32] It is not always possible to obtain a solution. There are situations where a solution may be infeasible, or when GAMS is unable to converge to a solution and continue to perform iterations without convergence within a reasonable timeframe, if at all.

A time limit of 3600s was set in the model in case GAMS was not able to find the optimal solution within the timeframe. The reason why a time limit was imposed was because in an industrial setting, it would not be possible to wait for an extended period of time for the computer to generate a production schedule. Although in 100% of the scenarios investigated, GAMS was able to generate a solution in under 1 hour with the maximum amount of time taken being slightly over 15 minutes, this is still inefficient and unacceptable for a real life application. This example consists of only 10 jobs and 4 processing units. In an industrial setting, the number of jobs is significantly larger, requiring sequencing and allocation to an increased number of processing lines. The computational effort required to solve such problems would increase dramatically.

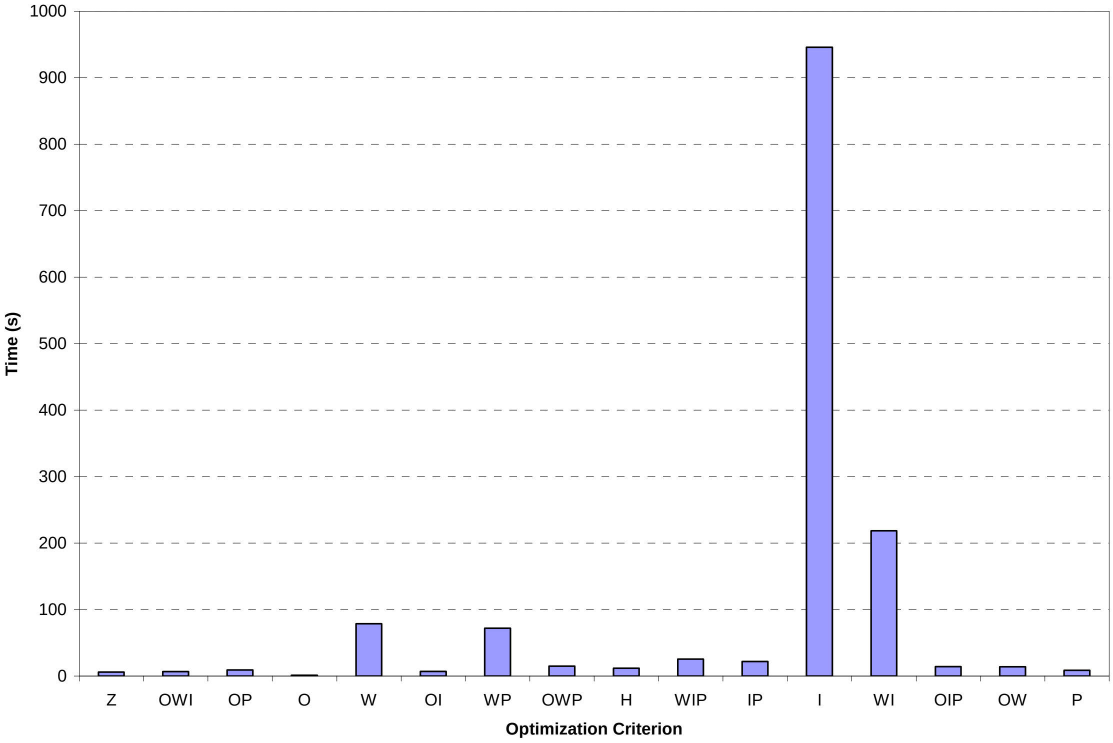


Figure 6-2 Time required to solve MINLP problem

As dictated by the correlations, the resulting MVR, SEC and impact strength of the samples based on the processing rate fall within the specifications shown in Table A5. Although these quality parameters are within the upper and lower bound, other factors outside of the screw speed and throughput rate may have affected the integrity of the resin. Therefore, just because the calculated MVR, impact strength and SEC is in spec, does not guarantee that an actual physical test performed on samples would pass quality control.

Chapter 7 Conclusions and Recommendations

In this thesis, the scheduling of parallel production units was considered. A mixed integer program was developed and implemented in GAMS. The model allows that jobs performed on different units may be shifted or resequenced according to the quantity of demand and the product the job performs. The focus was on the criterion of optimization. Various criteria were discussed and a mathematical relationship for them was given. Sensitivity analysis showed that the use of a single criterion alone (e.g. minimize inventory) is not enough for optimizing operation. In fact, such use might even increase the overall production cost rather than optimizing it. A new criterion of optimality was therefore used. This criterion was termed the total opportunity cost and takes into account the different single criterion in a weighted sum.

One major novelty of the present work is the consideration of quality constraints. Correlations between the temperature of the processed material and the production schedule are the extruder flow rate were appropriately incorporated in the scheduling model. The flow rate affects the temperature of the molten plastic inside the extruder barrel, which means it also directly affects the quality of the final resin. Furthermore, the extruder is the critical machine in the extrusion process. Therefore, it determines the processing time of an order, serving as the basis for the scheduling problem. The extruded polymer resin must undergo quality control testing to ensure that quantitative quality measurements must meet specifications. Through the use of the model proposed in this thesis, these considerations are automatically taken care of during the scheduling stage.

One drawback of the present model is the relatively large computational effort associated with it. Future work should be directed for example towards the generation of valid inequalities and heuristics to tight the bounds on the problem solution and to provide initial feasible solutions.

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Appendix A: Raw Data

Table A1: Batch Size, Due Date, Release Time of Order, Processing Rate and Raw Material Unit Cost

Order	Q (kg)	Tau (days)	RTO (days)	P ($/day)	RM ($/kg)
I1	550	10	0	100	5.5
I2	850	22	5	100	8.5
I3	700	25	0	100	7
I4	900	20	6	100	9
I5	500	28	0	100	5
I6	1050	30	2	100	10.5
I7	950	17	3	100	9.5
I8	850	23	0	100	8.5
I9	450	30	2	100	4.5
I10	650	30	6	100	6.5

Table A2: Set-up Times (in days) between Jobs Used

i/j	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
I1						0.65			0.85	0.4
I2			1.1							
I3	1	0.15					0.3		1.6	0.2
I4					0.05					0.5
I5				0.3		0.7	0.9	0.6		
I6	1.4		0.3	0.7					1.2	
I7		1.8			0.85			0.45		
I8							1.65			
I9	2.1		1.25			0.8				0.65
I10	1.5		0.6	0.75	0.5				0.7	

Table A3: Unit Capability of Processing Order

i/u	U1	U2	U3	U4
I1	1			
I2			1	
I3	1		1	
I4		1		
I5		1		1
I6	1	1		
I7			1	1
I8				1
I9	1			
I10	1	1		

Table A4: Operating Ranges of Processing Units

Unit No.	Maximum Processing Rate (kg/days)
U1	50
U2	80
U3	100
U4	100

Table A5: Specifications for Melt Volumetric Flow Rate, Impact Strength, Specific Energy Consumption and Limitations on Screw Speed

Quality Characteristic	Lower Limit	Upper Limit
MVR (cm^3/10 min)	25.5	37.5
Impact Strength (J/m^2)	12	40
SEC (kW·h/kg)	0.15	0.45
Screw Speed (rpm)	30	60

Table A6: Unit costs used to calculate various cost functions

Costs / Rates	$ or Rate
Processing cost per unit time	30
Setup cost per unit time	50
Expected rate of return on raw materials	1
Expected rate of return on machine	5

Appendix B: Optimal schedules and details generated from GAMS using different objective functions

Table B1: Optimal schedule obtained when total opportunity cost is used as objective (n = 1)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	11.85	20.85	30	50	9	21	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.82	28.07	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U2	I10	650	28.57	36.7	30	80	8.13	21.88	6.7	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	60	15.28	25.82	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	60	15.28	25.82	0.18
U4	I5	500	3	8	28	100	5	23	0	56.24	15.33	25.5	0.18
U4	I7	950	8.9	18.4	17	100	9.5	7.5	1.4	56.24	15.33	25.5	0.18
U4	I8	850	18.85	27.35	23	100	8.5	14.5	4.35	56.24	15.33	25.5	0.18

Table B2: Optimal schedule obtained when the sum of operation, idle, and penalty cost is used as objective (n = 2)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	60	14.98	27.45	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	60	14.98	27.45	0.18
U2	I6	1050	24.25	37.37	30	80	13.13	16.88	7.37	60	14.98	27.45	0.18
U3	I3	700	6.35	13.35	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	13.5	22	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	56.24	15.33	25.5	0.18
U4	I8	850	12.95	21.45	23	100	8.5	14.5	0	56.24	15.33	25.5	0.18

Table B3: Optimal schedule obtained when the sum of operation, waiting, and penalty cost is used as objective (n = 3)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	37.29	15.25	25.5	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	37.29	15.25	25.5	0.18
U2	I6	1050	24.25	37.38	30	80	13.13	16.87	7.37	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	56.24	15.33	25.5	0.18
U4	I8	850	12.95	21.45	23	100	8.5	14.5	0	56.24	15.33	25.5	0.18

Table B4: Optimal schedule obtained when the sum of operation, waiting, and idle cost is used as objective (n = 4)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	11.85	20.85	30	50	9	21	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.82	28.07	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U2	I10	650	28.57	36.7	30	80	8.13	21.88	6.7	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	60	15.28	25.82	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	60	15.28	25.82	0.18
U4	I5	500	3	8	28	100	5	23	0	56.24	15.33	25.5	0.18
U4	I7	950	8.9	18.4	17	100	9.5	7.5	1.4	56.24	15.33	25.5	0.18
U4	I8	850	18.85	27.35	23	100	8.5	14.5	4.35	56.24	15.33	25.5	0.18

Table B5: Optimal schedule obtained when the sum of waiting, idle, and penalty cost is used as objective (n = 5)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	37.29	15.25	25.5	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	37.29	15.25	25.5	0.18
U2	I6	1050	24.25	37.37	30	80	13.13	16.88	7.37	37.29	15.25	25.5	0.18
U3	I3	700	2	10.93	25	78.37	8.93	16.07	0	35.75	15.25	25.5	0.18
U3	I2	850	11.08	21.93	22	78.37	10.85	11.15	0	35.75	15.25	25.5	0.18
U4	I7	950	3	13.33	17	92	10.33	6.67	0	48.67	15.3	25.5	0.18
U4	I8	850	13.78	23.01	23	92	9.24	13.76	0.01	48.67	15.3	25.5	0.18

Table B6: Optimal schedule obtained when the sum of operating and idle cost is used as objective (n = 6)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	11.85	20.85	30	50	9	21	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.82	28.07	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U2	I10	650	28.57	36.7	30	80	8.13	21.88	6.7	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I5	500	3	8	28	100	5	23	0	56.24	15.33	25.5	0.18
U4	I7	950	8.9	18.4	17	100	9.5	7.5	1.4	56.24	15.33	25.5	0.18
U4	I8	850	28.2	36.7	23	100	8.5	14.5	982.83	56.24	15.33	25.5	0.18

Table B7: Optimal schedule obtained when the sum of operating and penalty cost is used as objective (n = 7)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	37.29	15.25	25.5	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	37.29	15.25	25.5	0.18
U2	I6	1050	24.25	37.38	30	80	13.13	16.87	7.37	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	56.24	15.33	25.5	0.18
U4	I8	850	12.95	21.45	23	100	8.5	14.5	0	56.24	15.33	25.5	0.18

Table B8: Optimal schedule obtained when the sum of operating and waiting cost is used as objective (n = 8)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	11.85	20.85	30	50	9	21	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	60	14.98	27.45	0.18
U2	I4	900	16.82	28.07	20	80	11.25	8.75	8.07	60	14.98	27.45	0.18
U2	I10	650	28.57	36.7	30	80	8.13	21.87	6.7	60	14.98	27.45	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	60	15.28	25.82	0.18
U4	I5	500	13.35	18.35	28	100	5	23	0	60	15.28	25.82	0.18
U4	I8	850	18.95	27.45	23	100	8.5	14.5	4.45	60	15.28	25.82	0.18

Table B9: Optimal schedule obtained when the sum of waiting and idle cost is used as objective (n = 9)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.82	28.07	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U2	I5	500	28.12	34.37	28	80	6.25	21.75	6.37	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	20.6	29.1	22	100	8.5	13.5	7.1	56.24	15.33	25.5	0.18
U3	I7	950	9.3	18.8	17	100	9.5	7.5	1.8	56.24	15.33	25.5	0.18
U4	I8	850	3	34.39	23	27.08	31.39	-8.39	971.55	30	14.55	29.17	0.18

Table B10: Optimal schedule obtained when the sum of waiting and penalty cost is used as objective (n = 10)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	60	14.53	29.88	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	60	14.53	29.88	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	60	14.53	29.88	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	60	14.98	27.45	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	60	14.98	27.45	0.18
U2	I6	1050	24.25	37.38	30	80	13.13	16.87	7.37	60	14.98	27.45	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	56.24	15.33	25.5	0.18
U4	I8	850	12.95	21.45	23	100	8.5	14.5	0	56.24	15.33	25.5	0.18

Table B11: Optimal schedule obtained when the sum of idle and penalty cost is used as objective (n = 11)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	37.29	15.25	25.5	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	37.29	15.25	25.5	0.18
U2	I6	1050	24.25	37.37	30	80	13.13	16.88	7.37	37.29	15.25	25.5	0.18
U3	I3	700	2	10.96	25	78.09	8.96	16.04	0	37.09	15.23	25.64	0.18
U3	I2	850	11.11	22	22	78.09	10.89	11.11	0	37.09	15.23	25.64	0.18
U4	I7	950	3	13.32	17	92.07	10.32	6.68	0	48.73	15.3	25.5	0.18
U4	I8	850	13.77	23	23	92.07	9.23	13.77	0	48.73	15.3	25.5	0.18

Table B12: Optimal schedule obtained when the operating cost is used as objective (n = 12)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	11.85	20.85	30	50	9	21	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.83	28.08	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U2	I10	650	28.58	36.7	30	80	8.13	21.87	6.7	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I5	500	3	8	28	100	5	23	0	56.24	15.33	25.5	0.18
U4	I7	950	8.9	18.4	17	100	9.5	7.5	1.4	56.24	15.33	25.5	0.18
U4	I8	850	18.85	27.35	23	100	8.5	14.5	4.35	56.24	15.33	25.5	0.18

Table B13: Optimal schedule obtained when the waiting cost is used as objective (n = 13)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.82	28.07	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U3	I3	700	2	9	25	100	7	18	0	56.24	15.33	25.5	0.18
U3	I2	850	9.15	17.65	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	56.24	15.33	25.5	0.18
U4	I5	500	13.35	18.35	28	100	5	23	0	56.24	15.33	25.5	0.18
U4	I8	850	18.95	27.45	23	100	8.5	14.5	4.45	56.24	15.33	25.5	0.18

Table B14: Optimal schedule obtained when the idle cost is used as objective (n = 14)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11.1	10	49.54	11.1	-1.1	1.1	30	14.88	27.35	0.18
U1	I9	450	25.32	34.41	30	49.54	9.08	20.92	4.42	30	14.88	27.35	0.18
U1	I10	650	11.5	24.62	30	49.54	13.12	16.88	0	30	14.88	27.35	0.18
U2	I6	1050	3	16.14	30	79.92	13.14	16.86	0	37.21	15.25	25.5	0.18
U2	I4	900	16.84	28.1	20	79.92	11.26	8.74	8.1	37.21	15.25	25.5	0.18
U2	I5	500	28.15	34.41	28	79.92	6.26	21.74	6.42	37.21	15.25	25.5	0.18
U3	I3	700	2	16.57	25	48.05	14.57	10.43	0	30.01	14.86	27.47	0.18
U3	I2	850	16.72	34.41	22	48.05	17.69	4.31	12.43	30.01	14.86	27.47	0.18
U4	I7	950	3	19.34	17	58.15	16.34	0.66	2.34	30.05	15.01	26.65	0.18
U4	I8	850	19.79	34.41	23	58.15	14.62	8.38	11.42	30.05	15.01	26.65	0.18

Table B15: Optimal schedule obtained when the penalty cost is used as objective (n = 15)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I4	900	6	17.25	20	80	11.25	8.75	0	37.29	15.25	25.5	0.18
U2	I5	500	17.3	23.55	28	80	6.25	21.75	0	37.29	15.25	25.5	0.18
U2	I6	1050	24.25	37.38	30	80	13.13	16.87	7.37	37.29	15.25	25.5	0.18
U3	I2	850	5	13.5	22	100	8.5	13.5	0	56.24	15.33	25.5	0.18
U3	I3	700	14.6	21.6	25	100	7	18	0	56.24	15.33	25.5	0.18
U4	I7	950	3	12.5	17	100	9.5	7.5	0	60	15.28	25.82	0.18
U4	I8	850	12.95	21.45	23	100	8.5	14.5	0	60	15.28	25.82	0.18

Table B16: Optimal schedule obtained when the makespan is used as objective (n = 16)

Unit	Order	Size	Start	End	Due Date	Proc. Rate	Proc. Time	Slack Time	Tardi -ness	Screw Speed	Impact	MVR	SEC
U1	I1	550	0	11	10	50	11	-1	1	30	14.89	27.31	0.18
U1	I9	450	25.1	34.1	30	50	9	21	4.1	30	14.89	27.31	0.18
U1	I10	650	11.4	24.4	30	50	13	17	0	30	14.89	27.31	0.18
U2	I6	1050	3	16.13	30	80	13.13	16.88	0	37.29	15.25	25.5	0.18
U2	I4	900	16.83	28.08	20	80	11.25	8.75	8.07	37.29	15.25	25.5	0.18
U3	I2	850	5	15.12	22	84.03	10.12	11.88	0	41.11	15.27	25.5	0.18
U3	I3	700	16.22	24.55	25	84.03	8.33	16.67	0	41.11	15.27	25.5	0.18
U4	I8	850	3	11.5	23	100	8.5	14.5	0	56.24	15.33	25.5	0.18
U4	I5	500	23.5	28.5	28	100	5	23	0.5	56.24	15.33	25.5	0.18
U4	I7	950	13.15	22.65	17	100	9.5	7.5	5.65	56.24	15.33	25.5	0.18

Table B17: Cost breakdown and details of each schedule

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Objective Function	**Z**	**OIP**	**OWP**	**OWI**	**WIP**	**OI**	**OP**	**OW**
Total Opportunity Cost	3726	103258	3876.7	3726	4046.2	3805.5	3876.7	103489
Operating Cost	2907.5	3036.3	3036.3	2907.5	3211.5	2907.5	3036.3	2912.5
Job Waiting Cost	579.54	100000	618.52	579.54	641.97	659.01	618.53	576.09
Machine Idling Cost	239	221.87	221.88	239	192.66	239	221.88	100000
Penalty Cost	2152.5	1247.5	1247.5	2152.5	1248.9	100000	1247.5	100000
Makespan	36.7	37.37	37.38	36.7	37.37	36.7	37.38	36.7

n	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Objective Function	**WI**	**WP**	**IP**	**O**	**W**	**I**	**P**	**H**
Total Opportunity Cost	100667	3876.7	4047.8	3726	3733	4623.7	203084	3982.4
Operating Cost	100000	3036.3	3213.3	2907.5	3046.3	3939.8	3083.8	3234.6
Job Waiting Cost	619.55	618.52	642.18	579.54	524.07	671.6	100000	599.91
Machine Idling Cost	47.75	221.88	192.37	239	162.63	12.25	100000	147.89
Penalty Cost	100000	1247.5	1247.5	2152.5	1762.5	4624.4	1247.5	1932.5
Makespan	34.39	37.38	37.37	36.7	34.1	34.41	37.38	34.1

Table B18: Precisional efficiency and time usage for solving MINLP

Optimization Criterion	Precisional Efficiency	Time Used (s)
Z	0.0%	6.22
OWI	0.0%	6.78
OP	0.0%	9.11
O	0.0%	1.24
W	0.2%	78.66
OI	2.1%	6.89
WP	3.9%	72.16
OWP	3.9%	14.99
H	6.4%	11.81
WIP	7.9%	25.49
IP	7.9%	21.85
I	19.4%	945.92
WI	96.3%	218.4
OIP	96.4%	14.35
OW	96.4%	13.88
P	98.2%	8.68