Throughput Limits of Wireless Networks With Fading Channels

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Wireless Networks have been the topic of fundamental research in recent years with the aim of achieving reliable and efficient communications. However, due to their complexity, there are still many aspects of such configurations that remain as open problems. The focus of this thesis is to investigate some throughput limits of wireless networks. The network under consideration consists of n source-destination pairs (links) operating in a single-hop fashion. In Chapters 2 and 3, it is assumed that each link can be active and transmit with a constant power P or remain silent. Also, fading is assumed to be the dominant factor affecting the strength of the channels between transmitter and receiver terminals. The objective is to choose a set of active links such that the throughput is maximized, where the rate of active links are either unconstrained or constrained. For the unconstrained throughput maximization, by deriving an upper bound and a lower bound, it is shown that in the case of Rayleigh fading: (i) the maximum throughput scales like $\log n$, (ii) the maximum throughput is achievable in a distributed fashion. The upper bound is obtained using probabilistic methods, where the key point is to upper bound the throughput of any random set of active links by a chi-squared random variable. To obtain the lower bound, a threshold-based link activation strategy (TBLAS) is proposed and analyzed. The achieved throughput of TBLAS is by a factor of four larger than what was obtained in previous works with centralized methods and with multihop communications. When the active links are constrained to transmit with a constant rate λ, an upper bound is derived that shows the number of active links scales at most like $\frac{1}{\lambda}\log n$. It is proved that TBLAS *asymptotically almost surely*

(a.a.s.) yields a feasible solution for the constrained throughput maximization problem. This solution, which is suboptimal in general, performs close to the upper bound for small values of λ. To improve the suboptimal solution, a double-threshold-based link activation strategy (DTBLAS) is proposed and analyzed based on some results from random graph theory. It is demonstrated that DTBLAS performs very close to the optimum. Specifically, DTBLAS is a.a.s. optimum when λ approaches ∞ or 0. The optimality results are obtained in an interference-limited regime. However, it is shown that, by proper selection of the algorithm parameters, DTBLAS also allows the network to operate in a noise-limited regime in which the transmission rates can be adjusted by the transmission powers. The price for this flexibility is a decrease in the throughput scaling law by a factor of $\log \log n$. In Chapter 4, the problem of throughput maximization by means of power allocation is considered. It is demonstrated that under individual power constraints, in the optimum solution, the power of at least one link should take its maximum value. Then, for the special case of $n = 2$ links, it is shown that the optimum power allocation strategy for throughput maximization is such that either both links use their maximum power or one of them uses its maximum power and the other keeps silent.

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Abbreviations

a.a.s.	asymptotically almost surely
AWGN	Additive White Gaussian Noise
SNR	Signal-to-Noise Ratio
SINR	Signal to Interference-plus-Noise Ratio
CDMA	Code Division Multiple Access
LAS	Link Activation Strategy
TBLAS	Threshold-Based Link Activation Strategy
DTBLA	Double-Threshold-Based Link Activation Strategy
i.i.d.	independent identically distributed
pdf	probability density function
cdf	cumulative distribution function
ccdf	complementary cumulative distribution function
r.v.	random variable
RHS	Right-Hand-Side
LHS	Left-Hand-Side
CLT	Central Limit Theorem

Notations

Bold face lower case letters	Vectors
Bold face upper case letters	Matrices
$\mathcal{N}_n$	the set of natural numbers less than or equal to n
A_n holds a.a.s	$\mathrm{P}(A_n) \to 1$ as $n \to \infty$
$\log(\cdot)$	the natural logarithm function
$\mathrm{P}(A)$	probability of event A
$\mathrm{E}(x)$	expected value of the random variable x
$\mathrm{Var}(x)$	variance of the random variable x
$\approx$	approximate equality
$h(n) = O(f(n))$	$\lim_{n\to\infty} \|h(n)/f(n)\| < \infty$
$h(n) = \Omega(f(n))$	$\lim_{n\to\infty} \|h(n)/f(n)\| > 0$
$h(n) = o(f(n))$	$\lim_{n\to\infty} \|h(n)/f(n)\| = 0$
$h(n) = \omega(f(n))$	$\lim_{n\to\infty} \|h(n)/f(n)\| = \infty$
$h(n) = \Theta(f(n))$	$\lim_{n\to\infty} \|h(n)/f(n)\| = c$, where $0 < c < \infty$
$h(n) \sim f(n)$	$\lim_{n\to\infty} h(n)/f(n) = 1$
$\chi^2(m)$	chi-squared distribution with m degrees of freedom
$\Phi(\cdot)$	cdf of normal distribution
$G(V, \boldsymbol{E})$	a graph with vertex set V and the adjacency matrix $\boldsymbol{E}$

$\mathcal{G}(m,p)$	the family of m-vertex *random graphs* with edge probability p
$\mathrm{cl}(G)$	clique number of a graph G
$\lfloor \cdot \rfloor$	floor function
$\mathbf{0}_n$	all-zero column vector of length n
$\mathbf{x} \leq \mathbf{y}$ or $\mathbf{x} < \mathbf{y}$	element-wise inequality for the vectors $\mathbf{x}$ and $\mathbf{y}$
$\boldsymbol{x}_{-i}$	a vector obtained by eliminating the ith element of $\boldsymbol{x}$

Chapter 1

Introduction

Wireless Networks have been the topic of fundamental research in recent years with the aim of achieving reliable and efficient communications. This has been done assuming different network topologies, traffic patterns, protocol schemes, and channel models [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

In a wireless network, a number of source nodes transmit data to their designated destination nodes through a shared wireless channel. From the information theoretic point of view, a wireless network is a generalized version of the *interference channel* [13], whose capacity region has not been fully characterized yet. Consequently, only a small fraction of the works in this field take a pure information theoretic approach to the throughput of wireless networks [2, 3, 4]; instead, most researchers base their throughput analyses on certain simplifying assumptions including Gaussian signal transmission, linear receiver structures (which excludes interference cancelation), and point-to-point coding (which excludes, for example, multi-access and broadcast schemes). In this case, interference at each receiver is treated as *additive white Gaussian noise* (AWGN) and the *signal to interference-plus-noise ratio* (*SINR*), along with the Shannon capacity formula, determine the achievable rate of each link. We will follow this paradigm throughout this dissertation.

1.1 Large Single-Hop Wireless Networks

In a general network model, the information flow is routed through some intermediate nodes, named relay or router, to reach the final destination [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Such a strategy is called multihop communications. Despite its potential in enabling more efficient communications, routing adds complexity to the communication systems. Furthermore, it induces delay in the network that cannot be tolerated in some applications. Also, implicit in multihop communication is the excess power consumption by the relay nodes. This latter issue can be a critical disadvantage in applications where the total power is constrained. In this work, we consider a network model in which data is transmitted directly from sources to their corresponding receivers without utilizing any other nodes as routers. This model includes single-hop ad hoc networks, cellular networks, and code division multiple access (CDMA) systems as its special cases and has been extensively studied in the literature. Due to the shared environment, simultaneous transmissions act as interference for each other. This issue is the main challenge in the design of single-hop wireless networks. For works in the context of large wireless networks that have adopted this model see [14, 15, 16].

The focus of this dissertation is on investigating throughput scaling laws of *large* single-hop wireless networks. The study of how the throughput of large wireless networks scales with n, the number of nodes, was initiated by Gupta and Kumar [1]. In that work, it is shown that a rate of $1/\sqrt{n}$ and $1/\sqrt{n \log(n)}$ per node is achievable in arbitrary and random networks[1], respectively. An extension of this result for the case that nodes are mobile or under delay constraints is provided in [9]. The result of [1] was later improved in [5] by showing that even random networks can achieve the rate of $1/\sqrt{n}$ per node.

1.2 Wireless Networks in Fading Environment

Most of the works analyzing the throughput of large wireless networks consider a channel model in which the signal power decays according to a distance-based attenuation law [1, 2, 3, 4, 5, 6, 7, 8]. However, in a wireless environment, the presence of obstacles and scatterers adds some randomness to the received signal. In addition, the power attenuation laws may not be valid when the receiver is not in the far field of the transmitter, as in the dense networks. This random behaviour of the channel, known as fading, can drastically change the scaling laws of a network in both multihop [9, 10, 11, 12] and single-hop scenarios [14, Chapter 8],[15, 16, 17].

In [9], it is shown that for the same setup as in [1], the presence of fading de-

[1]In an arbitrary network, the nodes locations can be chosen optimally, but, in a random network the nodes are located randomly.

creases the order of the lower bound on rate-per-node by a factor of $\log n$. In [10], transmissions occur with a same constant rate and the objective is to maximize the throughput. The proposed method is based on finding some noncolliding paths between the source-destination pairs such that only *good* enough links are traversed. The number of these paths, the transmission rate, and the channel *goodness* criterion are chosen such that the total throughput of the network is maximized. It is shown that the throughput of the network strongly depends on the fading channel distribution. A generalization of this work has appeared in [11], where the authors consider a general fading model on top of a power decay law. The transport capacity of wireless networks over fading channels has been investigated in [12].

In the context of single-hop wireless networks, [14, Chapter 8] considers a frequency selective Rayleigh fading channel and seeks the minimum bandwidth such that all communication links can support a certain rate. In [15], outage probability and transmission capacity have been considered as performance metrics and the effect of fading has been studied. Their study is based on some heuristic methods including random transmission and threshold based scheduling with or without power control. In [16], the achievable throughput of a cellular system over fading channel is studied when the number of cells (links) is limited, but each cell has an infinitely large number of users to choose from. In [17], in addition to the randomness coming from the fading channels, the nodes' locations follow a Poisson point process. The objective is to accommodate the maximum density of users such that a certain outage probability is not exceeded.

In this dissertation, we follow the model of [10, 14], where fading is assumed to be the dominant factor affecting the strength of the channels between nodes. Of course, a realistic model of wireless network channels should take into account both randomness and distance-based effects (see e.g. [11, 15]). However, this simplifying assumption makes it possible to precisely analyze the effect of fading distribution on the achievable throughput and obtain optimality results. Considering the more general channel model can be the topic of future research.

1.3 Small Wireless Networks

Although the focus of this dissertation is on the large wireless networks, it is worth mentioning that there are numerous works in the literature investigating the throughput optimization in networks with arbitrarily small sizes. Based on the network structure, throughput optimization can be executed in different ways, e.g. by power control [18], bandwidth allocation [19, 20], transmission scheduling [21], routing [22, 23], base station selection [24], etc. Among these various challenging problems, power control has a prominent role in the past and ongoing research in this area. In the last chapter of this dissertation, we address the problem of power allocation for wireless networks.

1.4 Overview of the Dissertation

In Chapters 2 and 3, we consider a single-hop wireless network with fading channels. Despite the randomness of the channel, we are only interested in events that occur with high probability, i.e., with probability tending to one as $n \to \infty$. The objective is to maximize the throughput over all possible sets of active links, i.e., links which are allowed to transmit simultaneously with a constant power P.

In Chapters 2, by deriving an upper bound and a lower bound, it is shown that in the case of Rayleigh fading (i) the maximum throughput scales like $\log n$ (ii) the maximum throughput is achievable in a distributed fashion. The upper bound is obtained using probabilistic methods, where the key point is to upper bound the throughput of any random set of active links by a chi-squared random variable. To obtain the lower bound, a decentralized method for choosing the active links is proposed and analyzed.

The unconstrained throughput maximization using the threshold based distributed method yields an average rate per active link that approaches zero as $n \to \infty$. In Chapter 3, it is assumed that each active link transmits with a constant rate λ. An upper bound is derived that shows the number of active links scales at most like $\frac{1}{\lambda} \log n$. To obtain a lower bound, the decentralized method of Chapter 2 is adopted and analyzed. It is shown that for small values of λ, the number of supported links by this strategy meets the upper bound; however, as λ grows, this number becomes far below the upper bound. To shrink the gap between the upper bound and the achievability result, a modified method for choosing the active links is proposed and analyzed based on some results from random graph theory. It is shown that this modified strategy performs very close to the optimum. Specifically, this strategy is *asymptotically almost surely* optimum when λ approaches ∞ or 0. It turns out the optimality results are obtained in an interference-limited regime.

It is demonstrated that, by proper selection of the algorithm parameters, the proposed scheme also allows the network to operate in a noise-limited regime in which the transmission rates can be adjusted by the transmission powers. The price for this flexibility is a decrease in the throughput scaling law by a factor of $\log\log n$.

Chapter 4 addresses the problem of throughput maximization by means of power allocation. In specific, it is shown that for $n = 2$ interfering links, the maximum throughput is achieved when one of the links transmit with the maximum power and the other one remains silent or both links transmit with the maximum power.

Chapter 5 presents a summary of the thesis contributions and provides some possible directions for future research.

Chapter 2

Unconstrained Throughput Maximization

2.1 Introduction

In this chapter and the next one, we follow the model of [10, 14], where fading is assumed to be the dominant factor affecting the strength of the channels between nodes. Despite the randomness of the channel, we are only interested in events that occur asymptotically almost surely, i.e., with probability tending to one as $n \to \infty$. Such a deterministic approach to random wireless networks has been also adopted in [5, 8], where the nodes' locations are random.

We consider a single-hop scenario, i.e., a network structure in which the transmitters send data to their corresponding receivers directly and without utilizing other nodes as routers. It is assumed that each link can be active and transmit with a constant power P or remain silent. The objective is to maximize the throughput over all sets of active links. We propose a threshold-based LAS in which each link is active if and only if its channel gain is above some predetermined threshold. The decision on being active can be made at the receivers, where their own channel gains are estimated and a single-bit command data is fed back to the transmitters. Hence, there is no need for the exchange of information between links. Consequently, this method can be implemented in a decentralized fashion. We analyze this method for a general fading model and show how to obtain the value of the activation threshold to maximize the throughput. As examples, we derive closed form expressions for the achievable throughput in Rayleigh, log-normal, and shadow fading environments.

Using probabilistic methods, we derive an upper bound on the achievable throughput when the channel is Rayleigh fading. Interestingly, this upper bound scales the same as the lower bound achieved by the proposed strategy. This proves the asymptotic optimality of the proposed technique among all link activation strategies.

In addition to the channel modeling, [10] is a relevant work in the sense that transmissions occur with the same power and the objective is to maximize the throughput. However, they allow multihop communication in their scheme. Their proposed scheme requires a central unit which is aware of all channel conditions and decides on active source-destination pairs and the paths between them. Despite this complexity, the achievable throughput of their method in the popular model of Rayleigh fading is by a factor of 4 less than the value obtained in this work for a more restricted configuration, i.e., single-hop networks with decentralized management.

The rest of the chapter is organized as follows: In Section 2.2, the network model and problem formulation are presented. By proposing a decentralized LAS, a lower bound on the network throughput is derived in Section 2.3. In Section 2.4, we prove the optimality of the proposed decentralized method in a Rayleigh fading environment. Finally, we conclude the chapter in Section 2.5.

2.2 Network Model and Problem Formulation

We consider a wireless communication network with n pairs of transmitters and receivers. These n communication links are indexed by the elements of $\mathcal{N}_n$. Each transmitter aims to send data to its corresponding receiver in a single-hop fashion. The transmit power of link i is denoted by p_i. It is assumed that the links follow an on-off paradigm, i.e., $p_i \in \{0, P\}$, where P is a constant. Hence, any power allocation scheme translates to choosing the set of active links, which is denoted by $\mathcal{A}$. In other words,

$$p_i = \begin{cases} P & \text{if} \quad i \in \mathcal{A} \\ 0 & \text{if} \quad i \notin \mathcal{A} \end{cases}. \tag{2.1}$$

The process in which the set $\mathcal{A}$ is chosen is called a *link activation strategy (LAS)*.

The channel between transmitter j and receiver i is characterized by the coefficient g_{ji}. This means the received power from transmitter j at the receiver i equals $g_{ji}p_j$. We refer to the coefficients g_{ii} and g_{ji} ($j \neq i$) as *direct channel coefficients* and *cross channel coefficients*, respectively. In this chapter and the next one, we assume that the channel coefficients are *independent identically distributed* (i.i.d.)

random variables drawn from a pdf $f(x)$ with mean μ and variance σ^2.

We consider an additive white Gaussian noise (AWGN) with limited variance η at the receivers. The transmit *signal-to-noise ratio* (*SNR*) of the network is defined as

$$\rho = \frac{P}{\eta}. \tag{2.2}$$

The receivers are conventional linear receivers, i.e., without multiuser detection. Since the transmissions occur simultaneously within the same environment, the signal from each transmitter acts as interference for other links. Assuming Gaussian signal transmission from all links, the distribution of the interference will be Gaussian as well. Thus, according to the Shannon capacity formula [13], the maximum supportable rate of link $i \in \mathcal{A}$ is obtained as

$$r_i(\mathcal{A}) = \log\left(1 + \gamma_i(\mathcal{A})\right) \text{ nats/channel use}, \tag{2.3}$$

where

$$\gamma_i(\mathcal{A}) = \frac{g_{ii}}{1/\rho + \sum_{\substack{j \in \mathcal{A} \\ j \neq i}} g_{ji}} \tag{2.4}$$

is the *signal-to-interference-plus-noise ratio* (*SINR*) of link i.

As a measure of performance, in this chapter we consider the throughput of the network, which is defined as

$$T(\mathcal{A}) = \sum_{i \in \mathcal{A}} r_i(\mathcal{A}). \tag{2.5}$$

Also, the average rate per active link, or simply rate-per-link, is defined as

$$\bar{r}(\mathcal{A}) = \frac{T(\mathcal{A})}{|\mathcal{A}|}. \tag{2.6}$$

In this chapter, wherever there is no ambiguity, we drop the functionality of $\mathcal{A}$ from the network parameters and simply refer to them as r_i, γ_i, T, or $\bar{r}$.

The problem of throughput maximization is described as

$$\max_{\mathcal{A}\subseteq\mathcal{N}_n} \quad T(\mathcal{A}). \tag{2.7}$$

This problem is referred to as the *unconstrained throughput maximization*. We denote the maximum value of this problem by T_u^*. Due to the nonconvex and integral nature of the throughput maximization problem, its solution is computationally intensive. However, in this chapter we propose and analyze a decentralized LAS which leads to efficient solutions for the above problem. Indeed, we show that the proposed strategy a.a.s. achieves the optimum solution of the throughput maximization problem in Rayleigh fading environment.

2.3 Achievability Results

In this section, to derive a lower bound on the network throughput, we propose a simple heuristic LAS, which we call a *threshold-based LAS (TBLAS)*. Due to the randomness of the channel, the achievable throughput of the proposed strategy is a random variable; however, our analyses yields a lower bound which is a.a.s. achievable.

TBLAS: For a threshold Δ, choose the set of active links according to the following rule

$$i \in \mathcal{A} \quad \text{iff} \quad g_{ii} > \Delta. \tag{2.8}$$

In this strategy, the quality of the direct channel of each link determines whether that link is active or not. If each transmitter is aware of the threshold Δ and its direct channel coefficient, it can individually determine its transmit power. Hence, TBLAS can be implemented in a *decentralized* fashion.

The performance of TBLAS depends on the value of the threshold Δ; if Δ is very large, the quality of the selected links will be very good, but the number of such links is small and as a result, the achieved throughput will be small. On the other hand, if Δ is very small, many links are chosen, but it causes a large interference and again the throughput will be small. Thus, it is crucial to choose a proper value for Δ. To obtain the optimum value of Δ, we should first know the achievable throughput of TBLAS in terms of Δ.

Let $k = |\mathcal{A}|$ denote the number of active links chosen by TBLAS. Without loss of generality, we assume that $\mathcal{A} = \{1, 2, \cdots, k\}$. By defining $I_i = \sum_{\substack{j=1 \\ j \neq i}}^{k} g_{ji}$ and using (2.3), (2.4), and (2.5), the throughput can be lower bounded as

$$
\begin{aligned}
T &> \sum_{i=1}^{k} \log\left(1 + \frac{\Delta}{1/\rho + I_i}\right) && (2.9)\\
&\geq k \log\left(1 + \frac{\Delta}{1/\rho + \frac{1}{k}\sum_{i=1}^{k} I_i}\right), && (2.10)
\end{aligned}
$$

where the first equality is based on the fact that $g_{ii} > \Delta$ for the active links and the second one is the result of applying the Jensen's inequality.

Let us define $I = \frac{1}{k}\sum_{i=1}^{k} I_i$, which is the empirical average of the interference terms. Unfortunately, since $\mathrm{Var}(I_i) = (k-1)\sigma^2$ grows with k, we cannot apply the law of large numbers to I. However, by applying the Chebyshev inequality, we obtain the upper bound

$$I < (k-1)\mu + \psi, \tag{2.11}$$

which holds a.a.s. for any $\psi = \omega(1)$. By using this upper bound in (2.10) we obtain

$$T > k \log\left(1 + \frac{\Delta}{\mu k + \psi}\right), \quad a.a.s. \tag{2.12}$$

Note that the constant $1/\rho - \mu$ is absorbed in the function ψ. To make the lower bound (2.12) as tight as possible, we should choose ψ as small as possible, e.g. $\psi = o(k)$. For such values of ψ, the lower bound (2.12) becomes an increasing function of k. More precisely, we have the following lemma.

Lemma 1. *Assume ψ is such that the function $\frac{\psi}{k}$ is nonincreasing in k. For any Δ and μ the function $k \log\left(1 + \frac{\Delta}{\mu k + \psi}\right)$ is increasing in k.*

Proof. See Appendix A. □

The lower bound in (2.12) is a function of Δ, which is a deterministic parameter to be chosen optimally later, and k, which is a random variable. To remove the randomness from this lower bound, we can replace k by a deterministic lower bound and use Lemma 1 to obtain a deterministic lower bound on the throughput. We do this in the following.

Assume the probability of a link being active is denoted by q. Due to our LAS, which selects links with direct channel coefficients larger than Δ, we have

$$q = 1 - F(\Delta), \tag{2.13}$$

where $F(x)$ is the cumulative distribution function (cdf) of the channel coefficients. Since the links are selected independently and with probability q, the number of active links, k, is a binomial random variable with parameters n and q. Using the Chebyshev inequality, it can be shown that k a.a.s. satisfies the lower bound

$$k > nq - \xi\sqrt{nq}, \tag{2.14}$$

for any $\xi = \omega(1)$. By using (2.12), (2.14), and Lemma 1, we obtain the main result of this section, which is an achievability result on the throughput.

Theorem 2. *Consider a wireless network with n links and i.i.d. random channel coefficients with pdf $f(x)$, cdf $F(x)$, and mean μ. Choose any $\Delta > 0$ and define $q = 1 - F(\Delta)$. Then, a throughput of*

$$T_{TBLAS}(\Delta) = (nq - \xi\sqrt{nq}) \log\left(1 + \frac{\Delta}{\mu(nq - \xi\sqrt{nq}) + \psi}\right) \tag{2.15}$$

is a.a.s. achievable by TBLAS for any $\xi = \omega(1)$ that satisfies $\xi = o(\sqrt{nq})$ and any $\psi = \omega(1)$ that makes $\frac{\psi}{k}$ a nonincreasing function in k.

Note that the achievable throughput $T_{TBLAS}(\Delta)$ is a deterministic value. It easily follows that under the conditions described in Theorem 2, the number of active links and the achievable average rate-per-link in TBLAS scale as

$$k \sim nq \quad a.a.s. \tag{2.16}$$

$$\bar{r} \sim \log\left(1 + \frac{\Delta}{\mu(nq - \xi\sqrt{nq}) + \psi}\right) \quad a.a.s. \tag{2.17}$$

Note that the functions ξ and ψ can be chosen arbitrarily small. Hence, they do not affect the order of the achievable throughput. As a result, (2.15) can be simplified to

$$T_{TBLAS}(\Delta) \sim nq \log\left(1 + \frac{\Delta}{\mu nq}\right). \tag{2.18}$$

We will use (2.18) in the calculations in the rest of the chapter, except for the special case of the Rayleigh fading model, where we use the original formula (2.15) to show how ξ and ψ affect the order of the achievable throughput.

As specified in Theorem 2, the achievable throughput of TBLAS is a function of the parameter Δ. Thus, Δ can be chosen such that the achievable throughput is maximized. Let us define

$$\Delta^* = \arg\max_{\Delta} T_{TBLAS}(\Delta), \tag{2.19}$$

and

$$T^*_{TBLAS} = \max_{\Delta} T_{TBLAS}(\Delta) \tag{2.20}$$

to be the optimum threshold and the maximum achievable throughput, respectively. We also define k^*_{DTBLAS} and $\bar{r}^*_{DTBLAS}$ to be the number of users and the rate-per-link corresponding to Δ^*.

It is worth mentioning that the above result on a.a.s. achievability of T^*_{TBLAS} was derived for static channels. However, in time-varying channels, where the expected value of T determines the actual throughput, we can easily conclude that

$$\mathrm{E}[T] > T^*_{TBLAS}, \tag{2.21}$$

which holds asymptotically.

In general, the values of Δ^* and T^*_{TBLAS} are functions of n, but how they scale with n strongly depends on the channel distribution function $f(x)$. Specifically, one needs to know the relation between q and Δ as well as the value of μ to obtain Δ^* and T^*_{TBLAS}. In the following, we provide some examples and show how the achievable throughputs depend on the fading model.

2.3.1 Case Study

Rayleigh Fading

In a Rayleigh fading channel, the coefficients g_{ji} are exponentially distributed with $f(x) = e^{-x}$ and $\mu = 1$. Thus, using the corresponding cumulative distributed function, the relation between q and Δ is described as $q = e^{-\Delta}$. By substituting this value in (2.15), we obtain the function to be maximized as

$$T_{TBLAS}(\Delta)=\left(ne^{-\Delta} - \xi\sqrt{ne^{-\Delta}}\right)\log\left(1 + \frac{\Delta}{ne^{-\Delta} - \xi\sqrt{ne^{-\Delta}} + \psi}\right). \tag{2.22}$$

The result of maximizing this function over Δ is given in the following corollary.

Corollary 3. *In a wireless network with Rayleigh fading channels and n links, under optimum operation conditions of TBLAS, we have*

$$\Delta^* = \log n - 2\log\log n + \log 2 + O\left(\frac{\log\log n}{\log n}\right), \tag{2.23}$$

$$T^*_{TBLAS} = \log n - 2\log\log n + \log(2/e) + O\left(\frac{\log\log n}{\log n}\right), \tag{2.24}$$

$$k^*_{TBLAS} = \frac{1}{2}\log^2 n(1 + o(1)), \quad a.a.s., \tag{2.25}$$

$$\bar{r}^*_{TBLAS} = \frac{2}{\log n}(1 + o(1)), \quad a.a.s. \tag{2.26}$$

Proof. See Appendix B.1. □

As it is seen, ξ and ψ do not affect the dominant terms of the network parameters. Hence, instead of (2.22), we could start the optimization process with

$$T_{_{TBLAS}}(\Delta) \sim ne^{-\Delta} \log\left(1 + \frac{\Delta}{ne^{-\Delta}}\right). \tag{2.27}$$

We will use this equation in the next chapter.

The throughput scaling order of $\log n$ is, by a factor of 4, larger than the value obtained in [10] in a centralized and multihop scenario. It should also be noted that the average rate-per-link approaches zero az $n \to \infty$. We will address the issue of nonzero-approaching rates in the next chapter.

Log-Normal Fading

Consider a network with channel coefficients drawn i.i.d. from a log-normal distribution with pdf

$$f(x) = \frac{1}{\sqrt{2\pi}Sx} e^{-\dfrac{(\log x - M)^2}{2S^2}}, \; x \geq 0 \tag{2.28}$$

with S and M being the parameters of the distribution [10]. The following proposition establishes the relation between q and Δ in the log-normal fading model for large values of Δ.

Proposition 4. *Assume X is a log-normal random variable with the pdf given in (2.28) and let $q = P(X > \Delta)$. Then, for large values of Δ, we have*

$$q \approx \frac{S}{\sqrt{2\pi}(\log \Delta - M)} e^{-\dfrac{(\log \Delta - M)^2}{2S^2}}. \tag{2.29}$$

Proof. By the definition of q, we have

$$q = \int_{\Delta}^{\infty} \frac{1}{\sqrt{2\pi}Sx} e^{-\frac{(\log x - M)^2}{2S^2}} dx \tag{2.30}$$

$$= \int_{\log\Delta}^{\infty} \frac{1}{\sqrt{2\pi}S} e^{-\frac{(x-M)^2}{2S^2}} dx \tag{2.31}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{\log\Delta - M}{\sqrt{2}S}\right), \tag{2.32}$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-z^2} dz$ is the complementary error function. By using the approximation $\mathrm{erfc}(x) \approx \frac{1}{\sqrt{\pi}x} e^{-x^2}$ for large values of x, the result is obtained. ☐

By substituting the value of q from Proposition 4 in (2.18), the throughput, as a function of Δ, is obtained as

$$T_{TBLAS}(\Delta) = \frac{S}{\sqrt{2\pi}} \frac{n}{u} e^{-\frac{u^2}{2S^2}} \log\left(1 + \frac{Bue^u e^{\frac{u^2}{2S^2}}}{n}\right), \tag{2.33}$$

where, for the brevity of notation, we have defined $u = \log\Delta - M$. Also, B is a constant depending on the distribution parameters.

Lemma 5. *In a wireless network with log-normal fading channels and n links, under optimum operation conditions of TBLAS, we have*

$$\Delta^* \sim e^{M-S^2} e^{\sqrt{2}S\sqrt{\log n}}, \tag{2.34}$$

$$T^*_{TBLAS} \sim e^{-\frac{3S^2}{2}} e^{\sqrt{2}S\sqrt{\log n}} \tag{2.35}$$

$$k^*_{TBLAS} \sim \frac{e^{-\frac{3S^2}{2}}}{\sqrt{8}S} \sqrt{\log n}\, e^{\sqrt{2}S\sqrt{\log n}}, \quad a.a.s. \tag{2.36}$$

$$\bar{r}^*_{TBLAS} \sim \frac{\sqrt{8}S}{\sqrt{\log n}}, \quad a.a.s. \tag{2.37}$$

Proof. See Appendix B.2. ☐

The throughput scaling order of $e^{\sqrt{2S}\sqrt{\log n}}$ is, by a multiplicative factor of $\frac{\log n}{\log \log n}$, larger than what is obtained in [10] in a centralized and multihop scenario. Also, the average rate-per-link approaches zero as $n \to \infty$.

Shadow Fading

Consider a network whose channel coefficients are drawn i.i.d. from the pdf [10]

$$f(x) = (1 - \varrho) \cdot \delta(x) + \varrho \cdot \delta(x - 1), \tag{2.38}$$

where $\delta(\cdot)$ is the Dirac's delta function. For this distribution $\mu = \varrho$. This pdf simply models a shadow fading environment in which, for any links, with probability $1 - \varrho$ there exists an obstacle that completely suppresses the signal; with probability ϱ such an obstacle does not exist and the transmitted signal is received without any fading effect.

For a constant ϱ, since there are only two possibilities for the channel coefficients, the threshold optimization is trivial; one should choose $\Delta^* = 1$ to maximize the throughput. This gives $q = \varrho$ and consequently,

$$T_{_{TBLAS}} \sim n\varrho \log \left(1 + \frac{1}{n\varrho^2}\right). \tag{2.39}$$

To complete the comparison with [10], we consider the scenario in which it is possible to choose ϱ as a function of n such that the throughput is maximized. Intuitively, when ϱ is very small, the effect of interference is low, but the number of unblocked links is low as well, resulting in a small throughput. On the other hand, if ϱ is very large, there are many unblocked links, but the number of links interfering with each link is also high and the achieved throughput will be small again. Thus, there should be some optimum value for ϱ in between.

Lemma 6. *In a wireless network with shadow fading channels and n links, the optimum value of ϱ for TBLAS and the corresponding throughput, number of active links, and rate-per-link are obtained as*

$$\varrho^* = \frac{c}{\sqrt{n}}, \tag{2.40}$$

$$T^*_{_{TBLAS}} \sim c\log\left(1+\frac{1}{c^2}\right)\sqrt{n}, \quad a.a.s. \tag{2.41}$$

$$k^*_{_{TBLAS}} \sim c\sqrt{n}, \tag{2.42}$$

$$\bar{r}^*_{_{TBLAS}} \sim \log\left(1+\frac{1}{c^2}\right). \tag{2.43}$$

where $c \approx 0.5050$ is a constant.

Proof. See Appendix B.3. □

As observed, the throughput scales as $\sqrt{n}$, which is smaller than the linear scaling with n that was obtained in [10]. This shows the necessity of a multihop scheme to achieve higher orders of throughput for the case of shadow fading. Interestingly, with this network set-up, the average rate-per-link does not approach zero.

2.4 Optimality Result

In this section, we provide an upper bound on the throughput of the wireless network. Hereafter, we restrict the discussions to the Rayleigh fading model. First, we need the following lemma that provides a lower bound on the number of active links.

Lemma 7. *In the optimum set of active links for the unconstrained throughput maximization* (2.7)*, the number of active links,* k_u^**, a.a.s. satisfies*

$$k_u^* \geq \frac{\log n}{\log\log n}\left(1 + O\left(\frac{1}{\log\log n}\right)\right). \tag{2.44}$$

Proof. It can be shown that a.a.s.

$$g_{ii} \leq \log n + \varphi, \quad \forall i \in \mathcal{N}_n, \tag{2.45}$$

for any $\varphi = \omega(1)$. By ignoring the interference term and using the above upper bound, we obtain

$$T_u^* \leq k_u^* \log\left(1 + \frac{\log n + \varphi}{1/\rho}\right). \tag{2.46}$$

Combining this upper bound with the lower bound in (2.24), we obtain

$$k_u^* \geq \frac{\log n + O(\log\log n)}{\log\left(1 + \frac{\log n + \varphi}{1/\rho}\right)} \tag{2.47}$$

$$= \frac{\log n}{\log\log n}\left(1 + O\left(\frac{1}{\log\log n}\right)\right), \tag{2.48}$$

where it is assumed $\varphi = o(\log n)$. □

Theorem 8. *In a Rayleigh fading environment, the solution of the unconstrained throughput maximization problem* (2.7) *is a.a.s. upper bounded as*

$$T_u^* \leq \log n + \log\log n(1 + o(1)). \tag{2.49}$$

Proof. Similar to Lemma 7, assume the number of active links yielding T_u^* is denoted by k_u^*. For a randomly selected set of active links $\mathcal{A}$ with $|\mathcal{A}| = k_u^*$, the interference term $I_i = \sum_{\substack{j\in\mathcal{A}\\ j\neq i}} g_{ji}$ in the denominator of (2.4) has $\chi^2(2k_u^*-2)$ distribution. Hence, we have

$$\begin{aligned}
\mathrm{P}(\gamma_i > x) &= \int_0^\infty \mathrm{P}\left(\gamma_i > x | I_i = z\right) f_{I_i}(z)dz \\
&= \int_0^\infty e^{-x(1/\rho+z)} \frac{z^{k_u^*-2}e^{-z}}{(k_u^*-2)!}dz \\
&= \frac{e^{-x/\rho}}{(1+x)^{k_u^*-1}}. \qquad (2.50)
\end{aligned}$$

Consequently, by using (2.3), we obtain

$$\begin{aligned}
\mathrm{P}(r_i > x) &= \mathrm{P}(\gamma_i > e^x - 1) \\
&= \frac{e^{-(e^x-1)/\rho}}{e^{(k_u^*-1)x}}. \qquad (2.51)
\end{aligned}$$

By defining $X_i = r_i + \dfrac{e^{r_i}-1}{\rho(k_u^*-1)}$ and using (2.51), it can be shown that X_i is exponentially distributed with mean $\frac{1}{k_u^*-1}$. On the other hand, from the definition of X_i it is clear that $X_i \geq r_i$. Thus, the throughput $T(\mathcal{A}) = \sum_{i\in\mathcal{A}} r_i$ is upper bounded as

$$T(\mathcal{A}) \leq \sum_{i\in\mathcal{A}} X_i. \qquad (2.52)$$

Consequently, we have

$$\begin{aligned}
\mathrm{P}\left(T(\mathcal{A}) > x\right) &\leq \mathrm{P}\left(\sum_{i\in\mathcal{A}} X_i > x\right) \qquad (2.53) \\
&\overset{(a)}{=} e^{-(k_u^*-1)x} \sum_{m=0}^{k_u^*-1} \frac{((k_u^*-1)x)^m}{m!} \qquad (2.54) \\
&\overset{(b)}{<} k_u^* e^{-(k_u^*-1)x} \frac{((k_u^*-1)x)^{k_u^*-1}}{(k_u^*-1)!} \qquad (2.55) \\
&\overset{(c)}{\approx} \sqrt{k_u^*} e^{-(k_u^*-1)(x-1)} x^{k_u^*-1}, \qquad (2.56)
\end{aligned}$$

where (a) is because $\sum_{i\in\mathcal{A}} X_i$ has $\chi^2(2k_u^*)$ distribution, (b) is because the maximum of the summand terms occurs at $m = k_u^*-1$ for large enough x [1], and (c) is obtained by applying the Stirling's approximation for the factorial, i.e., $m! \approx \sqrt{2\pi m} m^m e^{-m}$.

Assume $\mathcal{L}_1$ is the event that there exists at least one set $\mathcal{A} \subseteq \mathcal{N}_n$ with $|\mathcal{A}| = k_u^*$ such that $T(\mathcal{A}) > x$. We have

$$\begin{aligned} \mathrm{p}(\mathcal{L}_1) &\leq \binom{n}{k_u^*} \mathrm{P}\left(T(\mathcal{A}) > x\right) && (2.57) \\ &< \left(\frac{ne}{k_u^*}\right)^{k_u^*} \sqrt{k_u^*} e^{-(k_u^*-1)(x-1)} x^{k_u^*-1} && (2.58) \\ &< \exp(\mathcal{E}(x,\, k_u^*)), && (2.59) \end{aligned}$$

where the first inequality is due to the union bound, the second inequality is due to (2.56) and the Stirling's approximation, and $\mathcal{E}(x,\, k_u^*)$ is defined as

$$\mathcal{E}(x,\, k_u^*) = k_u^*(\log n - x - \log k_u^* + \log x + 2) + \frac{1}{2}\log k_u^* + x. \qquad (2.60)$$

For $x = \log n + \log\log n + 2\log\log\log n$, we have

$$\begin{aligned} \mathcal{E}(x,\, k_u^*) \;\approx\; & -k_u^*(2\log\log\log n + \log k_u^* - 2) + \frac{1}{2}\log k_u^* \\ & + \log n + \log\log n(1 + o(1)). && (2.61) \end{aligned}$$

Noting that the RHS of (2.61) is a decreasing function in k_u^*, we can replace k_u^* by its lower bound from Lemma 7 to obtain the upper bound

$$\mathcal{E}(x,\, k_u^*) \leq -\frac{\log\log\log n}{\log\log n} \log n(1 + o(1)). \qquad (2.62)$$

[1]Since we are seeking an upper bound on the throughput, x is at least of order $\log n$. This value is large enough to satisfy the mentioned condition.

Since the RHS of (2.62) goes to $-\infty$ when $n \to \infty$, from (2.59) we conclude that $\mathrm{p}(\mathcal{L}_1) \to 0$. This means, with probability approaching 1, there does not exist any set $\mathcal{A}$ that achieves a throughput larger than $x = \log n + \log\log n(1 + o(1))$. This completes the proof. $\square$

Comparison between the achievability result in Corollary 3 and the upper bound in Theorem 8 reveals the following result.

Theorem 9. *Consider a wireless network with n links and i.i.d. random channel coefficients drawn from an exponential distribution with mean $\mu = 1$. Then, the maximum throughput a.a.s. satisfies*

$$T_u^* \sim \log n. \tag{2.63}$$

Moreover, this maximum throughput scaling law is a.a.s. achieved by the distributed TBLAS presented in Section 2.3.

2.5 Conclusion

In this chapter, the throughput of single-hop wireless networks with on-off strategy is investigated in a fading environment. To obtain a lower bound on the throughput, a decentralized LAS is proposed and analyzed for a general fading model. It is shown that in the popular model of Rayleigh fading a throughput of order $\log n$ is achievable, which is by a factor of four larger than what was obtained in previous works with centralized methods [10]. Moreover, for the Rayleigh fading model, an upper bound of order $\log n$ is obtained that shows the optimality of the proposed LAS.

Chapter 3

Constrained Throughput Maximization

3.1 Introduction

In Chapter 2, the unconstrained throughput maximization problem in a Rayleigh fading environment has been investigated. It is shown that the maximum throughput scales like $\log n$. Also, a decentralized LAS, called the threshold-based link activation strategy (TBLAS), is proposed that achieves this scaling law. The unconstrained throughput maximization using the threshold based distributed method yields an average rate per active link that approaches zero as $n \to \infty$. The same phenomenon has been observed in [1, 5, 9, 10]. Since most of the existing efficient channel codes are designed for moderate rates, it is a drawback for a system to have zero-approaching rates. Thus, from a practical point of view, it is appealing to assign constant rates to active communication links. In [6], it is shown that a nondecreasing rate per node is achievable when nodes are mobile.

In this chapter, we consider the problem of constrained throughput maximization in a Rayleigh fading environment. More specifically, the objective is to maximize the number of active links such that each active link can transmit with a constant rate λ. We derive an upper bound that shows the number of active links scales at most like $\frac{1}{\lambda}\log n$. To obtain a lower bound, first, we examine the simple TBLAS of Chapter 2 and show that it is capable of guaranteeing rate-per-links equal to λ. The number of active links provided by this method scales like $\Theta(\log n)$. The scaling factor is close to the optimum when λ is small. However, as λ grows large, the scaling factor decays exponentially with λ, making it far below the upper bound $\frac{1}{\lambda}$. This inspires developing an improved LAS that works well for large values of desired rates, as well. To this end, we propose a double-threshold-based link activation strategy (DTBLAS).

DTBLAS is attained by adding an interference management phase to TBLAS. This is done by choosing from good enough links only those with small enough mutual interference. The analysis of DTBLAS is more complicated than that of TBLAS. However, it can be carried out using some results from the random graph theory. It is shown that DTBLAS performs very close to the optimum. Indeed, its performance achieves the upper bound for large values of the demanded rate. This shows the asymptotic optimality of DTBLAS for the constrained throughput maximization problem.

In the scenarios described above, the interference power is much larger than the noise power and the rates become independent of SNR. In other words, the network performs in an interference-limited regime. A natural question is whether it is possible to have rate-per-links which depend on the SNR. The importance of this scenario, which is called the noise-limited regime, is that the transmission rate λ can be adjusted by adjusting the transmission power P. We show that the answer to the above question is affirmative and the noise-limited regime can be realized by using DTBLAS. However, the throughput achieved by this method scales like $\frac{\log n}{\log \log n}$, which is by a factor of $\log \log n$ less than what is achievable in an interference-limited regime.

The rest of the chapter is organized as follows: In Section 3.2, network model and problem formulation are presented. An upper bound on the throughput is derived in Section 3.3. In Sections 3.4 and 3.5, achievability results via decentralized and centralized schemes are presented. Some optimality results are provided in Section 3.6. The operation of the network in a noise-limited regime is investigated in Section 3.7. Finally, the chapter is concluded in Section 3.8.

3.2 Network Model and Problem Formulation

The network model is the same as in the previous chapter, except that here we only consider a Rayleigh fading model, i.e. $f(x) = e^{-x}$, with mean $\mu = 1$ and variance $\sigma^2 = 1$.

Throughout this chapter, we assume all active links transmit with a same rate λ. In this case, the throughput becomes proportional to the number of active links, i.e., $T(\mathcal{A}) = |\mathcal{A}|\lambda$. Hence, the problem of throughput maximization becomes equivalent to maximizing the number of active links subject to a constraint on the rate of active links, i.e.,

$$\begin{array}{lc} \max\limits_{\mathcal{A}\subseteq\mathcal{N}_n} & |\mathcal{A}| \\ \text{s.t.} & r_i(\mathcal{A}) \geq \lambda, \quad \forall i \in \mathcal{A} \end{array}. \tag{3.1}$$

This problem is referred to as the *constrained throughput maximization*. We denote the throughput corresponding to the maximum value of this problem by T_c^*.

Due to the nonconvex and integral nature of the throughput maximization problem, its solution is computationally intensive. However, in this chapter we propose and analyze LASs which lead to efficient solutions for the above problem. Indeed, we first show that the decentralized method of Chapter 2 is a.a.s. optimum when λ is vanishingly small. Then, we propose a new LAS which is asymptotically optimum for large values as well as small values of λ. Also, for moderate values of λ, there is a small gap between the performance of the proposed LAS and a derived upper bound. This shows the closeness of its performance to the optimum.

As in the previous chapter, we denote the number of active links by k instead of $|\mathcal{A}|$. Motivated by the result of Chapter 2 that shows the maximum throughput scales like $\log n$, we introduce the following definitions. The scaling factor of the throughput is defined as

$$\tau = \lim_{n\to\infty} \frac{T}{\log n}, \tag{3.2}$$

Similarly, the scaling factor of the number of active links is defined as

$$\kappa = \lim_{n\to\infty} \frac{k}{\log n}. \tag{3.3}$$

3.3 Upper Bound

In this section, we obtain an upper bound on the optimum solution of (3.1). This upper bound can be either presented as an upper bound on the throughput or as an upper bound on the number of active links.

Theorem 10. *Assume* $\mathcal{A}_c^*$ *is the solution to the constrained throughput maximization* (3.1) *and* $k_c^* = |\mathcal{A}_c^*|$. *Then, the associated throughput and the scaling factor of* k_c^* *a.a.s. satisfy*

$$T_c^* < \log n - \log\log n + c, \tag{3.4}$$

$$\kappa_c^* < \frac{1}{\lambda}, \tag{3.5}$$

for some constant c.

Proof. For a randomly selected set of active links $\mathcal{A}$ with $|\mathcal{A}| = k$, similar to (2.50), we have

$$\mathrm{P}(\gamma_i > x) = \frac{e^{-x/\rho}}{(1+x)^{k-1}}. \tag{3.6}$$

Assume $\mathcal{L}_2$ is the event that there exists at least one set $\mathcal{A} \subseteq \mathcal{N}_n$ with $|\mathcal{A}| = k$ such that the constraints in (3.1) are satisfied. Also, assume γ_0 is a quantity that satisfies $\lambda = \log(1+\gamma_0)$. We have

$$\begin{aligned}
\mathrm{P}(\mathcal{L}_2) &\overset{(a)}{\leq} \binom{n}{k}\left(\mathrm{P}(r_i \geq \lambda)\right)^k && (3.7)\\
&= \binom{n}{k}\left(\mathrm{P}(\gamma_i \geq \gamma_0)\right)^k && (3.8)\\
&\overset{(3.6)}{=} \binom{n}{k}\frac{e^{-\gamma_0 k/\rho}}{(1+\gamma_0)^{k(k-1)}} && (3.9)\\
&\overset{(b)}{\leq} \left(\frac{ne}{k}\right)^k \frac{e^{-\gamma_0 k/\rho}}{(1+\gamma_0)^{k(k-1)}} && (3.10)\\
&= e^{k(\log n - \log k - \lambda k + \lambda + 1 - \gamma_0/\rho)}, && (3.11)
\end{aligned}$$

where (a) is due to the union bound and (b) is the result of applying the Stirling's approximation for the factorial. It can be verified that there exists a constant c such that if $k\lambda = \log n - \log\log n + c$, then, the above upper bound approaches zero for $n \to \infty$. Hence, for the event $\mathcal{L}_2$ to have non-zero probability, we should a.a.s. have

$$k\lambda < \log n - \log\log n + c. \tag{3.12}$$

This inequality holds for any feasible number of active links. By choosing $k = k_c^*$, the upper bounds in the lemma are proved. □

3.4 Lower Bound: A Decentralized Approach

To derive a lower bound, in this section, we consider the decentralized method of Chapter 2, i.e. TBLAS, which was introduced in (2.8).

As before, $T_{_{TBLAS}}$ denotes the achieved throughput of TBLAS. The following results, which were obtained in Chapter 2, are repeated here for simplicity.

$$
\begin{aligned}
T_{TBLAS} &\sim ne^{-\Delta}\log\left(1+\frac{\Delta}{ne^{-\Delta}}\right), && (3.13)\\
k_{TBLAS} &\sim ne^{-\Delta}, && (3.14)\\
|k_{TBLAS}-ne^{-\Delta}| &< \xi\sqrt{ne^{-\Delta}}, \quad a.a.s. && (3.15)
\end{aligned}
$$

If the rate of active links equal λ, the average rate per active link equals λ, as well. To have $\bar{r}_{TBLAS}=\lambda$, we should choose Δ such that the throughput and the number of active links both become proportional to $\log n$. The following lemma shows how to realize such a scenario.

Lemma 11. *Assume the activation threshold for TBLAS is chosen to be* $\Delta = \log n - \log\log n - \log\alpha$ *for some* $\alpha > 0$. *Then, a.a.s. we have*

$$
\begin{aligned}
\tau_{TBLAS} &= \alpha\log\left(1+\frac{1}{\alpha}\right) && (3.16)\\
\kappa_{TBLAS} &= \alpha && (3.17)\\
\bar{r}_{TBLAS} &= \log\left(1+\frac{1}{\alpha}\right)+o(1). && (3.18)
\end{aligned}
$$

Proof. With the specified value of Δ, we have $ne^{-\Delta}=\alpha\log n$. The values of τ_{TBLAS} and κ_{TBLAS} are readily obtained by substituting this value in (3.13) and (3.14) and using the definitions (3.2) and (3.3), respectively. The value of $\bar{r}_{TBLAS}$ is obtained by using the definition (2.6). □

Lemma 11 indicates that by a proper choose of α, an average rate-per-link equal to λ is achievable; however, it does not guarantee that all active links are supported by this rate. In other words, one may ask whether TBLAS is capable of satisfying the constraints in problem (3.1). The following lemma addresses this issue and shows that a.a.s. the rate of all active links are highly concentrated around the average value $\bar{r}$.

Lemma 12. *Assume the activation threshold for TBLAS is chosen to be* $\Delta = \log n - \log\log n - \log\alpha$ *for some* $\alpha > 0$*. Then, a.a.s. we have*

$$|r_i - \bar{r}| < 2\sqrt{\frac{\log\log n}{\alpha^3 \log n}}(1 + o(1)), \quad \forall i \in \mathcal{A}, \tag{3.19}$$

where $\bar{r} = \log\left(1 + \dfrac{1}{\alpha}\right)$.

To prove the lemma, we need the following result about the *central limit theorem* (*CLT*) for large deviations.

Theorem 13 ([25]). *Let* $\{Y_m\}$ *be a sequence of i.i.d. random variables. Suppose that* Y_1 *has zero mean and finite positive variance* ν *and satisfies the Cramér's condition*[1]*. For* $Z_m = \frac{1}{\sqrt{m\nu}} \sum_{j=1}^{m} Y_j$*, define* $F_m(y) = P(Z_m < y)$*. If* $y \geq 0$*,* $y = O(m^{1/6})$*, then*

$$1 - F_m(y) = [1 - \Phi(y)] \exp\left(\frac{\theta_3 y^3}{6\sqrt{m\nu^3}}\right) + O\left(\frac{e^{-y^2/2}}{\sqrt{m}}\right), \tag{3.20}$$

where $\Phi(y)$ *is the cdf of the normal distribution and* $\theta_3 = E(Y_1^3)$*.*

Proof of Lemma 12. From the definition of r_i and the concavity of the $\log(\cdot)$ func-

[1] A random variable Y satisfies the Cramér's condition if its moment-generating function exists in some interval with the center at the origin.

tion, we have

$$|r_i - \bar{r}| = \left|\log(1+\gamma_i) - \log\left(1+\frac{1}{\alpha}\right)\right| \quad (3.21)$$

$$\leq \left|\gamma_i - \frac{1}{\alpha}\right|. \quad (3.22)$$

Thus, to prove the lemma, it is sufficient to prove that a.a.s.

$$\left|\gamma_i - \frac{1}{\alpha}\right| < 2\sqrt{\frac{\log\log n}{\alpha^3 \log n}}(1+o(1)), \quad \forall i \in \mathcal{A}, \quad (3.23)$$

or equivalently

$$x_- < \gamma_i < x_+, \quad (3.24)$$

where

$$x_\pm = \frac{1}{\alpha}\left(1 \pm 2\sqrt{\frac{\log\log n}{\alpha \log n}}(1+o(1))\right). \quad (3.25)$$

Here, we just prove the left-side inequality in (3.24). The other side can be proved in a similar manner.

Let $\mathcal{L}_3$ denote the event that

$$\gamma_i > x_-, \quad \forall i \in \mathcal{A}. \quad (3.26)$$

In the following, we show that $\mathrm{P}(\mathcal{L}_3) \to 1$ as $n \to \infty$.

Denoting the cdf of γ_i conditioned on $|\mathcal{A}| = k$ by $F_\gamma(x,k)$, the probability of the event $\mathcal{L}_3$ is obtained as

$$\begin{aligned}
\mathrm{P}(\mathcal{L}_3) &= \sum_{k=0}^{n} \mathrm{P}(|\mathcal{A}|=k)\mathrm{P}(\mathcal{L}_3||\mathcal{A}|=k) && (3.27)\\
&\overset{(a)}{=} \sum_{k=0}^{n} \mathrm{P}(|\mathcal{A}|=k)\left(1-F_\gamma(x_-,k)\right)^k && (3.28)\\
&\overset{(b)}{\geq} \sum_{k=k_-}^{k_+} \mathrm{P}(|\mathcal{A}|=k)\left(1-F_\gamma(x_-,k)\right)^k && (3.29)\\
&\overset{(c)}{>} \left(1-F_\gamma(x_-,k_+)\right)^{k_+} \sum_{k=k_-}^{k_+} \mathrm{P}(|\mathcal{A}|=k) && (3.30)\\
&= \left(1-F_\gamma(x_-,k_+)\right)^{k_+} \mathrm{P}(k_- \leq |\mathcal{A}| \leq k_+), && (3.31)
\end{aligned}$$

where (a) is because the channel gains are independent, (b) is valid for any $0 \leq k_- \leq k_+ \leq n$ and (c) is due to the fact that $\left(1-F_\gamma(x,k)\right)^k$ is a decreasing function of k. According to (3.15), by choosing

$$\begin{aligned}
k_\pm &= ne^{-\Delta} \pm \xi\sqrt{ne^{-\Delta}} && (3.32)\\
&= \alpha\log n \pm \xi\sqrt{\alpha\log n}, && (3.33)
\end{aligned}$$

for some $\xi \to \infty$, we have $\mathrm{P}(k_- \leq |\mathcal{A}| \leq k_+) \to 1$. Hence, to prove $\mathrm{P}(\mathcal{L}_3) \to 1$, it is enough to show that $\left(1-F_\gamma(x_-,k_+)\right)^{k_+} \to 1$. However, due to the inequality

$$\left(1-F_\gamma(x_-,k_+)\right)^{k_+} \geq 1 - k_+F_\gamma(x_-,k_+), \tag{3.34}$$

it is enough to show that

$$k_+F_\gamma(x_-,k_+) \to 0. \tag{3.35}$$

To prove (3.35), we provide an upper bound on $k_+F_\gamma(x_-,k_+)$ and show that it approaches zero as $n \to \infty$. We have

$$\begin{aligned} F_\gamma(x_-, k_+) &= \mathrm{P}(\gamma_i \le x_- \| \mathcal{A}| = k_+) \\ &\overset{(a)}{=} \mathrm{P}\left(\frac{g_{ii}}{1/\rho + \sum_{\substack{j=1 \\ j \ne i}}^{k_+} g_{ji}} \le x_- \right) \\ &= \mathrm{P}\left(\sum_{\substack{j=1 \\ j \ne i}}^{k_+} g_{ji} \ge \frac{g_{ii}}{x_-} - \frac{1}{\rho} \right) \\ &\overset{(b)}{<} \mathrm{P}\left(\sum_{\substack{j=1 \\ j \ne i}}^{k_+} g_{ji} \ge \frac{\Delta}{x_-} - \frac{1}{\rho} \right), \end{aligned} \tag{3.36}$$

where (a) is based on $\mathcal{A} = \{1, \cdots, k_+\}$, which has been assumed for simplicity of notation, and (b) is due to the fact that, in TBLAS, $g_{ii} > \Delta$ for any $i \in \mathcal{A}$. Let us define $Y_j = g_{ji} - 1$, which has the variance $\nu = 1$. Thus, the right-hand-side (RHS) of (3.36) translates to the complementary cdf (ccdf) of $Z = \frac{1}{\sqrt{k_+ - 1}} \sum_{\substack{j=1 \\ j \ne i}}^{k_+} Y_j$, i.e. (3.36) can be rewritten as

$$F_\gamma(x_-, k_+) < 1 - \mathrm{P}(Z \le y), \tag{3.37}$$

where

$$y = \frac{\frac{\Delta}{x_-} - \frac{1}{\rho} - (k_+ - 1)}{\sqrt{k_+ - 1}}. \tag{3.38}$$

By substituting $\Delta = \log n - \log\log n - \log \alpha$ and the value of x_- from (3.25) into (3.38), we obtain

$$y = 2\sqrt{\log\log n}(1 + o(1)). \tag{3.39}$$

Since Y_j is a shifted exponential random variable, its moment-generating function exists around zero and the Cramér's condition is satisfied. Also, by choosing

$m = k_+ - 1$ we have $y = O(m^{1/6})$. Hence, the result of Theorem 13 can be applied to calculate the ccdf of Z. Consequently, by using (3.20) with $\theta_3 = \mathrm{E}(Y_j^3) = 2$, (3.37) can be rewritten as

$$F_\gamma(x_-, k_+) < [1 - \Phi(y)] \exp\left(\frac{y^3}{3\sqrt{k_+ - 1}}\right) + O\left(\frac{e^{-y^2/2}}{\sqrt{k_+ - 1}}\right). \tag{3.40}$$

Noting that $y^3 = o(\sqrt{k_+})$ and using the inequality $1 - \Phi(y) < \frac{e^{-y^2/2}}{y}$, from (3.40) and (3.39), we conclude that

$$k_+ F_\gamma(x_-, k_+) < k_+ \frac{e^{-y^2/2}}{y} \tag{3.41}$$

$$= \exp\left(-\log\log n(1 + o(1))\right). \tag{3.42}$$

It is clear that the above upper bound approaches zero as $n \to \infty$. Hence, $\mathrm{P}(\mathcal{L}_3) \to 1$ and the proof is complete. □

Lemma 12 shows that with the specified threshold for TBLAS, all active links can transmit with rate $\lambda = \log(1 + \frac{1}{\alpha})$. Hence, TBLAS provides a solution, albeit suboptimum, for the problem (3.1). Lemmas 11 and 12 reveal the following relation between the demanded rate λ and κ_{TBLAS} as well as τ_{TBLAS}

$$\kappa_{TBLAS} = \frac{1}{e^\lambda - 1}, \tag{3.43}$$

$$\tau_{TBLAS} = \frac{\lambda}{e^\lambda - 1}. \tag{3.44}$$

Noting that for small values of λ, the RHS of (3.43) can be approximated as $\frac{1}{\lambda}$ and using the upper bound in Theorem 10, it turns out that TBLAS is close to the optimum for small values of λ.

3.5 Lower Bound: A Centralized Approach

Although TBLAS enjoys the simplicity of decentralized implementation, its performance is far from the optimum. This can be seen by comparing the upper bound in Theorem 10 and the achievability result in (3.43). A reason for this suboptimality is that the mutual interference of the active links is not considered in choosing $\mathcal{A}$. In this section, we provide an LAS that performs close to the upper bound in Theorem 10 and turns out to be asymptotically optimum when λ is very large or very small. We name this method double-threshold-based LAS (DTBLAS).

DTBLAS: For the thresholds Δ and δ

1. Choose the largest set $\mathcal{A}_1 \subseteq \mathcal{N}_n$ such that $g_{ii} > \Delta$ for all $i \in \mathcal{A}_1$.

2. Choose the largest set $\mathcal{A}_2 \subseteq \mathcal{A}_1$ such that $g_{ij} \leq \delta$ and $g_{ji} \leq \delta$ for all $i,\ j \in \mathcal{A}_2$.

The set of active links is $\mathcal{A} = \mathcal{A}_2$.

This strategy chooses the links to be active in a two-phase selection process; in the first phase, which is basically similar to TBLAS, a subset $\mathcal{A}_1$ of the links with good enough direct channel coefficients is chosen. In the second phase, which is the interference management phase, a subset of links in $\mathcal{A}_1$ is chosen such that their mutual interferences are small enough. Note that the second phase of the strategy requires full knowledge of the channel coefficients. Hence, this scheme should be implemented in a centralized fashion.

We aim to find Δ and δ such that the throughput is maximized subject to the rate constraints of the active links.

For simplicity, we use the notation $k_i = |\mathcal{A}_i|$ for $i = 1, 2$. Without loss of generality, assume $\mathcal{A}_i = \{1, \cdots, k_i\}$. By using (2.3), (2.4), and (2.5), and applying the Jensen's inequality, the throughput is lower bounded as

$$T \geq k_2 \log \left(1 + \frac{\Delta}{1/\rho + \frac{1}{k_2} \sum_{i=1}^{k_2} I_i} \right), \tag{3.45}$$

where $I_i = \sum_{\substack{j=1 \\ j \neq i}}^{k_2} g_{ji}$. Since $g_{ji} \leq \delta$, the mean and variance of I_i depend on δ. More precisely, we have

$$\begin{aligned} \mathrm{E}(I_i) &= (k_2 - 1)\hat{\mu}, & (3.46) \\ \mathrm{Var}(I_i) &= (k_2 - 1)\hat{\sigma}^2, & (3.47) \end{aligned}$$

where

$$\begin{aligned} \hat{\mu} &= \mathrm{E}\{g_{ji} | g_{ji} \leq \delta\} = 1 - \frac{\delta e^{-\delta}}{1 - e^{-\delta}}, & (3.48) \\ \hat{\sigma}^2 &= \mathrm{Var}\{g_{ji} | g_{ji} \leq \delta\} = 1 - \frac{\delta^2 e^{-\delta}}{(1 - e^{-\delta})^2}. & (3.49) \end{aligned}$$

Assume δ is a constant and $k_2 \to \infty$ as $n \to \infty$. To simplify the RHS of (3.45), we apply the Chebyshev inequality to obtain the upper bound

$$\frac{1}{k_2} \sum_{i=1}^{k_2} I_i < (k_2 - 1)\hat{\mu} + \psi, \tag{3.50}$$

which holds a.a.s. for any $\psi = \omega(1)$. Consequently, the lower bound (3.45) becomes

$$T \geq k_2 \log \left(1 + \frac{\Delta}{\hat{\mu} k_2 + \psi} \right) \qquad a.a.s. \tag{3.51}$$

Note that the constant $1/\rho - \hat{\mu}$ is absorbed in the function ψ. Since ψ can be chosen arbitrarily small, say with an order smaller than $\hat{\mu} k_2$, we can rewrite (3.51) as

$$T \geq T_{DTBLAS}, \tag{3.52}$$

where

$$T_{_{DTBLAS}} = k_2 \left(\log \left(1 + \frac{\Delta}{\hat{\mu} k_2} \right) + o(1) \right) \qquad a.a.s. \tag{3.53}$$

denotes the throughput achievable by DTBLAS.

Since k_2 is a random variable, the right hand side of (3.53) is a random variable as well. However, the following discussion shows that k_2 is highly concentrated around a certain value. Hence, it can be treated as a deterministic value.

Construct an undirected graph $G(\mathcal{A}_1, \boldsymbol{E})$ with vertex set $\mathcal{A}_1$ and the adjacency matrix $\boldsymbol{E} = [e_{ij}]$ defined as

$$e_{ij} = \begin{cases} 1 & ; \quad g_{ij} \leq \delta \text{ and } g_{ji} \leq \delta \\ 0 & ; \quad \text{otherwise} \end{cases} .$$

The probability of having an edge between vertices i and j, when g_{ji} and g_{ij} have exponential distribution, equals

$$p = \left(1 - e^{-\delta}\right)^2 . \tag{3.54}$$

The definition of G implies that $G \in \mathcal{G}(k_1, p)$, where $\mathcal{G}(k_1, p)$, which is a well-studied object in the literature [26], is the family of k_1-vertex *random graphs* with edge probability p.

In the second phase of DTBLAS, we are interested to choose the maximum number of links whose cross channel coefficients are smaller than δ. This is equivalent to choosing the largest complete subgraph[2] of G. The size of the largest complete subgraph of G is called its clique number and is denoted by $\text{cl}(G)$. The

[2] A complete graph is a graph in which every pair of vertices are connected by an edge.

above discussion yields

$$k_2 = \mathrm{cl}(G), \quad \text{for some} \quad G \in \mathcal{G}(k_1, p). \tag{3.55}$$

Although the clique number of a random graph G is a random variable, the following result from random graph theory states that it is concentrated in a certain interval.

Theorem 14. *Let $0 < p < 1$ and $\epsilon > 0$ be fixed. The clique number $cl(G)$ of $G \in \mathcal{G}(m,p)$, for large values of m, a.a.s. satisfies $s_1 \leq cl(G) \leq s_2$ where*

$$s_i = \lfloor 2\log_b m - 2\log_b \log_b m(1-p) + 2\log_b(e/2) + 1 + (-1)^i \epsilon/p \rfloor, \quad i = 1,\ 2, \tag{3.56}$$

$b = 1/p$.

Proof. The theorem is a direct result of Theorem 7.1 in [27], which states a similar result for the stability number of random graphs. Using the fact that the stability number of a random graph $\mathcal{G}(m,\ p)$ is the same as the clique number of a random graph $\mathcal{G}(m,\ 1-p)$, the theorem is proved. □

Corollary 15. *Consider DTBLAS with parameters Δ and δ. The number of active links, $k_{DTBLAS} = k_2$, a.a.s. satisfies $k'_- \leq k_{DTBLAS} \leq k'_+$, where*

$$k'_\pm = 2\log_b ne^{-\Delta} - 2\log_b \log_b ne^{-\Delta}(1 - \frac{1}{b}) + 2\log_b(e/2) + 1 \pm \epsilon/p + o(1) \tag{3.57}$$

and $b = (1 - e^{-\delta})^{-2}$.

Proof. According to (3.15), a.a.s. we have $k_1 = ne^{-\Delta} + O(\xi\sqrt{ne^{-\Delta}})$. Assuming $\xi = o(\sqrt{ne^{-\Delta}})$, and by substituting this value of k_1 into (3.55) and using Theorem 14, the corollary is proved. □

The next lemma indicates how to choose the thresholds Δ and δ such that the throughput and the number of active links both become proportional to $\log n$. As a result, a constant average rate-per-link is achieved.

Lemma 16. *Assume the threshold Δ for DTBLAS is chosen to be*

$$\Delta = (1-\alpha')\log n(1+o(1)), \tag{3.58}$$

for some $\alpha' > 0$ and δ is a constant. Then, a.a.s. we have

$$\kappa_{DTBLAS} = \frac{-\alpha'}{\log\left(1-e^{-\delta}\right)}, \tag{3.59}$$

$$\tau_{DTBLAS} = \frac{-\alpha'}{\log\left(1-e^{-\delta}\right)}\log\left(1-\frac{(1-\alpha')\log\left(1-e^{-\delta}\right)}{\alpha'\left(1-\dfrac{\delta e^{-\delta}}{1-e^{-\delta}}\right)}\right), \tag{3.60}$$

$$\bar{r}_{DTBLAS} = \log\left(1-\frac{(1-\alpha')\log(1-e^{-\delta})}{\alpha'\hat{\mu}}\right)+o(1). \tag{3.61}$$

Proof. For the number of active links, we have

$$k_{DTBLAS} \overset{(a)}{\sim} 2\log_b ne^{-\Delta} \tag{3.62}$$

$$\overset{(b)}{=} \frac{-\alpha'(1+o(1))}{\log\left(1-e^{-\delta}\right)}\log n, \tag{3.63}$$

where (a) is based on Corollary 15 and (b) is obtained by using (3.58). From (3.63), and by using the definition (3.3), κ_{DTBLAS} is obtained as given in (3.59).

The number of active links in (3.63) can be used along with the value of Δ in (3.58) to rewrite (3.53) as

$$T_{DTBLAS} = \left[\frac{-\alpha'}{\log\left(1-e^{-\delta}\right)}\log\left(1-\frac{(1-\alpha')\log\left(1-e^{-\delta}\right)}{\alpha'\hat{\mu}}\right)+o(1)\right]\log n. \tag{3.64}$$

The scaling factor τ_{DTBLAS}, as given in the Lemma, is obtained by using the value of $\hat{\mu}$ from (3.48) and applying the definition (3.2). The value of $\bar{r}_{DTBLAS}$ is obtained by using the definition (2.6). This completes the proof. $\square$

According to this lemma, by proper choose of the constants α' and δ, the average rate-per-link $\bar{r}_{DTBLAS}$ can be adjusted to be equal to the required rate λ. A natural question is whether, under the specified conditions in DTBLAS, all active links can support the rate λ. The following lemma addresses this issue and shows that a.a.s. the rate of all active links are highly concentrated around the average value $\bar{r}_{DTBLAS}$.

Lemma 17. *Consider DTBLAS with thresholds δ and $\Delta = (1-\alpha')\log n$ for some $\alpha' > 0$. Then, a.a.s. we have*

$$|r_i - \bar{r}| < c\sqrt{\frac{\log\log n}{\log n}}(1+o(1)), \quad \forall i \in \mathcal{A}, \tag{3.65}$$

for some constant $c > 0$, where

$$\bar{r} = \log\left(1 - \frac{(1-\alpha')\log(1-e^{-\delta})}{\alpha'\hat{\mu}}\right).$$

Proof. The proof is based on the same arguments as in the proof of Lemma 12. Thus, here we just highlight the differences.

Let us define $\bar{\gamma}$ as

$$\bar{\gamma} = -\frac{(1-\alpha')\log(1-e^{-\delta})}{\alpha'\hat{\mu}}. \tag{3.66}$$

Similar to the proof of Lemma 12, it is enough to show that a.a.s.

$$x'_- < \gamma_i < x'_+, \tag{3.67}$$

where

$$x'_{\pm} = \bar{\gamma}\left(1 \pm c'\sqrt{\frac{\log\log n}{\log n}}(1+o(1))\right), \tag{3.68}$$

with $c' = c/\bar{\gamma}$. We only prove the left side inequality in (3.67); the other inequality can be proved in a similar manner.

Let $\mathcal{L}_4$ denote the event that

$$\gamma_i > x'_{-}, \quad \forall i \in \mathcal{A}, \tag{3.69}$$

In the following, we show that $\mathrm{P}(\mathcal{L}_4) \to 1$ for some $c' > 0$.

Note that with $\Delta = (1-\alpha')\log n$, the parameter k'_{+} in Corollary 15 is obtained as

$$\begin{aligned} k'_{+} &= \kappa_{DTBLAS}\log n - a\log\log n && (3.70)\\ &< \kappa_{DTBLAS}\log n, && (3.71)\end{aligned}$$

where κ_{DTBLAS} is given in (3.59) and $a > 0$ is a constant. Denoting the cdf of γ_i conditioned on $|\mathcal{A}| = k$ by $F_{\gamma}(x, k)$, we have

$$\begin{aligned} \mathrm{P}(\mathcal{L}_4) &\overset{(a)}{>} \left(1 - F_{\gamma}(x'_{-}, k'_{+})\right)^{k'_{+}} \mathrm{P}\left(k'_{-} \leq |\mathcal{A}| \leq k'_{+}\right) && (3.72)\\ &\overset{(b)}{\approx} \left(1 - F_{\gamma}(x'_{-}, k'_{+})\right)^{k'_{+}} && (3.73)\\ &\overset{(c)}{>} \left(1 - F_{\gamma}(x'_{-}, \kappa_{DTBLAS}\log n)\right)^{\kappa_{DTBLAS}\log n}, && (3.74)\end{aligned}$$

where (a) is obtained in the same manner as (3.31), (b) results from Corollary 15, and (c) is due to (3.71) and the fact that $(1 - F_{\gamma}(x, k))^{k}$ is a decreasing function of k. To show that the RHS of (3.74) tends to one, we upper bound $\kappa_{DTBLAS}\log n F_{\gamma}(x'_{-}, \kappa_{DTBLAS}\log n)$ and show that it approaches zero.

Similar to the derivation of (3.36), it can be shown that

$$F_\gamma(x'_-, \kappa_{DTBLAS} \log n) < \mathrm{P}\left(\sum_{\substack{j=1 \\ j\neq i}}^{\kappa_{DTBLAS} \log n} g_{ji} \geq \frac{\Delta}{x'_-} - \frac{1}{\rho} \right). \tag{3.75}$$

Let us define $Y_j = g_{ji} - \hat{\mu}$, where $\hat{\mu}$ is obtained from (3.48). Random variable Y_j has the variance $\nu = \hat{\sigma}^2$, where $\hat{\sigma}^2$ is given in (3.49). By defining $Z = \frac{1}{\sqrt{\nu(\kappa_{DTBLAS} \log n - 1)}} \sum_{\substack{j=1 \\ j\neq i}}^{\kappa_{DTBLAS} \log n} Y_j$, (3.75) can be reformulated as

$$F_\gamma(x'_-, \kappa_{DTBLAS} \log n) < 1 - \mathrm{P}(Z \leq y), \tag{3.76}$$

where

$$y = \frac{\frac{\Delta}{x'_-} - \frac{1}{\rho} - (\kappa_{DTBLAS} \log n - 1)\hat{\mu}}{\sqrt{(\kappa_{DTBLAS} \log n - 1)\hat{\sigma}^2}}. \tag{3.77}$$

By substituting $\Delta = (1 - \alpha') \log n$ and the value of x'_- from (3.68) into (3.77), we obtain

$$y \;=\; c' \sqrt{\frac{\kappa_{DTBLAS} \hat{\mu}^2}{\hat{\sigma}^2}} \sqrt{\log \log n} \left(1 + O\left(\frac{1}{\sqrt{\log n \log \log n}} \right) \right). \tag{3.78}$$

It is straightforward to show that the moment-generating function of Y_j exists around zero. Hence, the Cramér's condition is satisfied. Also, by choosing $m = \kappa_{DTBLAS} \log n - 1$, the condition $y = O(m^{1/6})$ is satisfied, as well. As a result, Theorem 13 can be utilized to calculate the RHS(3.76) as

$$1 - \mathrm{P}(Z \leq y) = [1 - \Phi(y)] \exp\left(\frac{\theta_3 y^3}{6\sqrt{\nu^3 \kappa_{DTBLAS} \log n}} \right) + O\left(\frac{e^{-\frac{y^2}{2}}}{\sqrt{\kappa_{DTBLAS} \log n}} \right) \tag{3.79}$$

By combining (3.76), (3.79), and (3.78), and noting that θ_3 is a constant, $y^3 = o(\sqrt{\kappa_{DTBLAS} \log n})$, and $1 - \Phi(y) < \frac{e^{-y^2/2}}{y}$, we conclude that

$$\kappa_{DTBLAS} \log n F_\gamma(x, \kappa_{DTBLAS} \log n) < \kappa_{DTBLAS} \log n \frac{e^{-\frac{y^2}{2}}}{y} \tag{3.80}$$

$$= \exp\left((1 - \frac{c'^2 \kappa_{DTBLAS} \hat{\mu}^2}{2\hat{\sigma}^2}) \log\log n + O(\log\log\log n) \right)$$

It is clear that if c' is chosen large enough, the above upper bound approaches zero as $n \to \infty$. This completes the proof. $\square$

According to Lemmas 16 and 17, when maximizing the throughput of DTBLAS, δ should be a constant and Δ is obtained from another constant α'. Hence, the constrained throughput maximization for DTBLAS simplifies to an optimization problem with constant parameters α' and δ. Assume γ_0 is a quantity that satisfies $\lambda = \log(1+\gamma_0)$, i.e., γ_0 is the required *SINR* by the active links. Instead of the number of active links, we can maximize the scaling factor of the number of active links given in Lemma 16. Hence, the constrained throughput maximization problem (3.1) is converted for DTBLAS to the following optimization problem

$$\max_{\alpha', \delta} \quad \frac{-\alpha'}{\log\left(1 - e^{-\delta}\right)} \tag{3.81}$$

$$\text{s.t.} \quad -\frac{(1-\alpha')\log\left(1 - e^{-\delta}\right)}{\alpha'\left(1 - \frac{\delta e^{-\delta}}{1 - e^{-\delta}}\right)} = \gamma_0. \tag{3.82}$$

Note that in contrast to problem (3.1), there is only one constraint in this problem. However, according to Lemma 17, this single constraint guarantees the required rate for all active links. From the equality constraint (3.82), parameter α' can be found in terms of δ as

$$\alpha' = \frac{-\log\left(1 - e^{-\delta}\right)}{\gamma_0\left(1 - \frac{\delta e^{-\delta}}{1 - e^{-\delta}}\right) - \log\left(1 - e^{-\delta}\right)}. \tag{3.83}$$

By substituting this value in the objective function (3.81), we obtain the following equivalent unconstrained optimization problem

$$\min_{\delta} \gamma_0 \left(1 - \frac{\delta e^{-\delta}}{1 - e^{-\delta}}\right) - \log\left(1 - e^{-\delta}\right). \tag{3.84}$$

Consequently, (α'^*, δ^*), the solution of (3.81), can be obtained by first finding δ^* from (3.84) and then substituting it into (3.83) to obtain α'^*. Due to the complicated form of (3.84), it is not possible to find δ^* analytically and it should be found numerically.

Fig. 3.1 shows δ^* and α'^* versus λ. The values of δ^* and α'^* can be replaced in (3.60) and (3.59) to obtain the maximum throughput scaling factor (τ^*_{DTBLAS}) as well as the maximum scaling factor for the number of active links (κ^*_{DTBLAS}). The value τ^*_{DTBLAS} is shown in Fig. 3.2. Depicted in the figure is also the throughput scaling factor of TBLAS obtained from (3.44). As it is observed, for small values of λ, the performance of TBLAS and DTBLAS are almost the same. However, as λ grows larger, the scaling factor of TBLAS approaches zero, but the scaling factor of DTBLAS approaches 1. This shows some kind of optimality for DTBLAS which will be later proven formally. Figure 3.3 demonstrates the tradeoff between the number of supported links and the demanded rate-per-link for TBLAS and DTBLAS. The tradeoff curve for TBLAS is obtained from (3.43). The upper bound from Theorem 10 is also plotted for comparison. As observed, for a ceratin value of λ, DTBLAS can support larger number of users, especially for larger values of λ. Indeed, the tradeoff curve of DTBLAS is very close to the upper bound. Specifically, for large values of λ, these two curves coincide. This will be later proven formally.

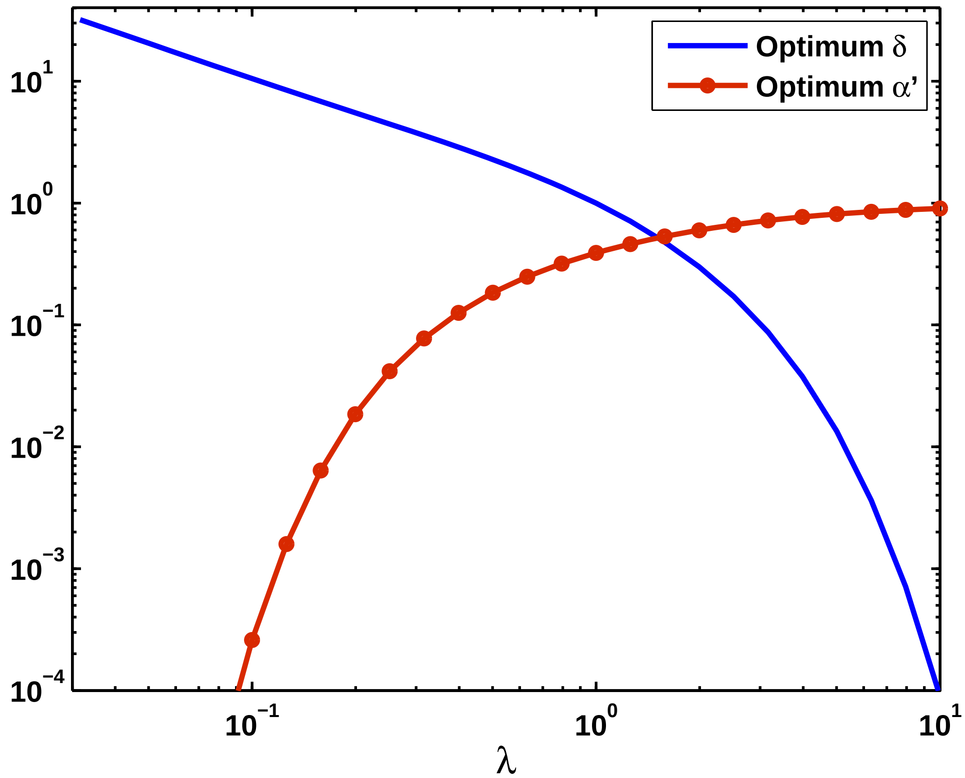

Figure 3.1: Optimum of the threshold δ and the parameter α' vs. the demanded rate λ.

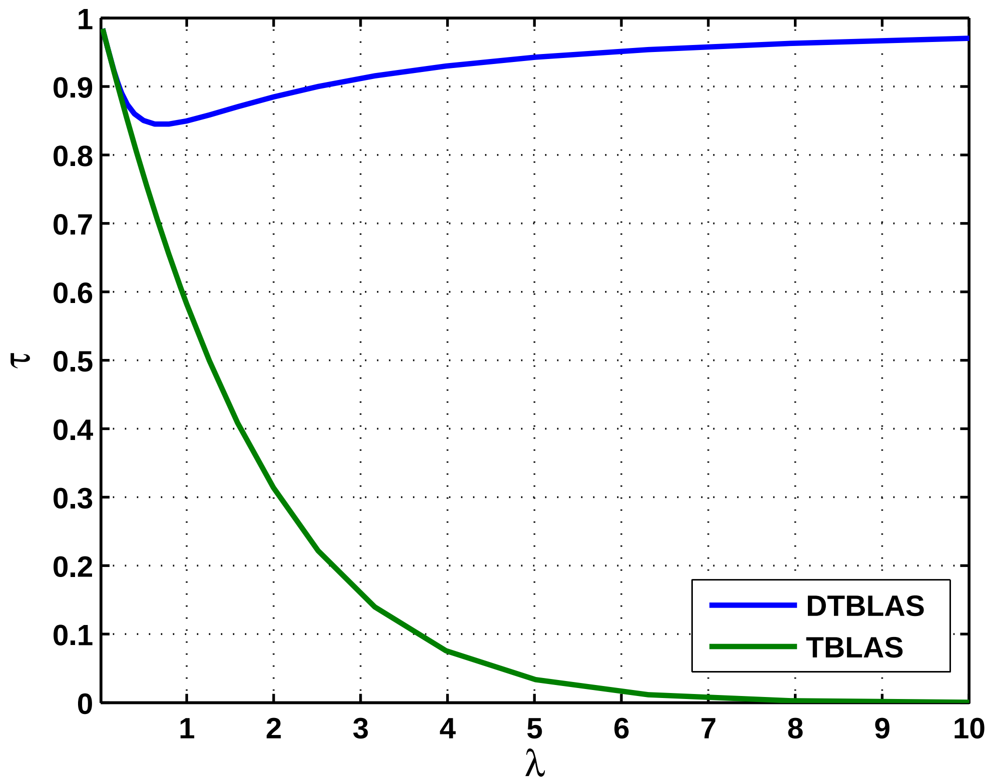

Figure 3.2: Maximum throughput scaling factor vs. the demanded rate λ.

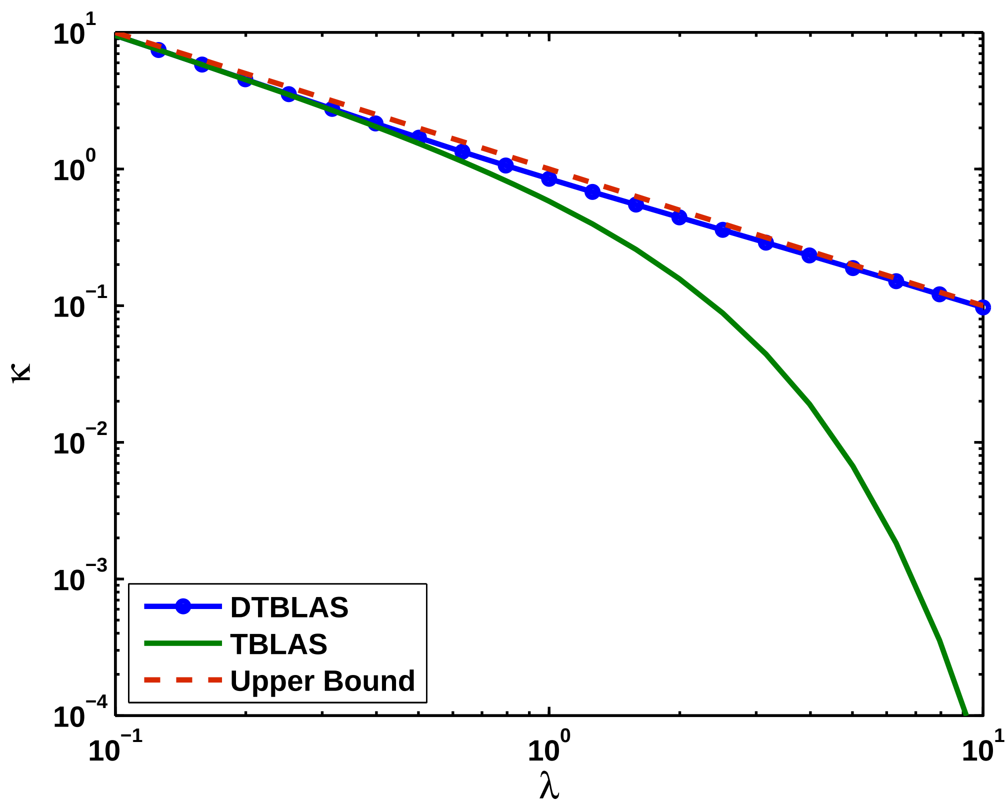

Figure 3.3: Tradeoff between rate-per-link and the number of active links.

3.6 Optimality Results

Although the behaviour of DTBLAS is numerically described in Figs. 3.1, 3.2, and 3.3, it is possible and also insightful to obtain closed form expressions for δ^* and α'^* as well as κ^*_{DTBLAS} and τ^*_{DTBLAS} when λ is very small or very large. An important result of these extreme-case analyses is the asymptotic optimality of DTBLAS.

Setting the derivative of the objective function (3.84) equal to zero reveals that, at the optimum point, δ, satisfies

$$e^{\lambda}(1 - e^{-\delta} - \delta) + \delta = 0. \tag{3.85}$$

Two extreme cases of large λ and small λ are discussed separately in the following.

Large λ: In this case, solving (3.85) yields

$$\delta^* = 2e^{-\lambda} + O\left(e^{-2\lambda}\right). \tag{3.86}$$

Consequently, α'^*, τ^*_{DTBLAS}, and κ^*_{DTBLAS} are obtained as

$$\alpha'^* = 1 - \frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right) \tag{3.87}$$

$$\tau^*_{DTBLAS} = 1 - \frac{\log(e/2)}{\lambda} + O\left(\frac{1}{\lambda^2}\right) \tag{3.88}$$

$$\kappa^*_{DTBLAS} = \frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right). \tag{3.89}$$

As it is seen from the above equations, for large values of λ, δ^* becomes very small and α'^* approaches one. This means, when large rate-per-links are demanded, it is more crucial to manage the interference than to choose links with high direct gain.

Small λ: In this case, solving (3.85) yields

$$\delta^* = \frac{1}{\lambda} + \frac{1}{2} + O(\lambda). \tag{3.90}$$

Consequently, α'^*, $\tau^*_{\scriptscriptstyle DTBLAS}$, and $\kappa^*_{\scriptscriptstyle DTBLAS}$ are obtained as

$$\begin{aligned}
\alpha'^* &= e^{-\frac{1}{\lambda}-\frac{1}{2}}\left(\frac{1}{\lambda} + \frac{1}{2} + O(\lambda)\right) && (3.91)\\
\tau^*_{\scriptscriptstyle DTBLAS} &= 1 - \frac{\lambda}{2} + O\left(\lambda^2\right) && (3.92)\\
\kappa^*_{\scriptscriptstyle DTBLAS} &= \frac{1}{\lambda} - \frac{1}{2} + O\left(\lambda\right). && (3.93)
\end{aligned}$$

The above equations show that for small values of λ, δ^* is very large and α'^* is very small. In other words, DTBLAS is converted to its special case, TBLAS.

The above discussion yields the following optimality result on DTBLAS.

Theorem 18. *Consider the constrained throughput maximization problem* (3.1)*. Assume T^*_c and κ^*_c are the maximum achievable throughput and the maximum scaling factor of the number of supported links, respectively. Also, assume $T^*_{\scriptscriptstyle DTBLAS}$ and $\kappa^*_{\scriptscriptstyle DTBLAS}$ are the maximum throughput and the maximum scaling factor of the number of active links when DTBLAS is deployed. Then, a.a.s. we have*

$$\begin{aligned}
\lim_{\lambda\to\infty} \quad \frac{T^*_{\scriptscriptstyle DTBLAS}}{T^*_c} &= 1, && (3.94)\\
\lim_{\lambda\to\infty} \quad \left(\kappa^*_{\scriptscriptstyle DTBLAS} - \kappa^*_c\right) &= 0, && (3.95)\\
\lim_{\lambda\to 0} \quad \frac{T^*_{\scriptscriptstyle DTBLAS}}{T^*_c} &= 1, && (3.96)\\
\lim_{\lambda\to 0} \quad \frac{\kappa^*_{\scriptscriptstyle DTBLAS}}{\kappa^*_c} &= 1. && (3.97)
\end{aligned}$$

Proof. The proof of the theorem is straightforward by using the upper bounds provided in Theorem 10 and the asymptotic achievability results provided in this section. □

3.7 Noise-Limited Regime

In the previous sections, we considered an interference-limited regime in which the noise power is negligible in comparison with the interference power. In this case, the achievable throughput is not a function of the network *SNR*. In other words, changing the transmission powers does not affect the supportable rate of each link. However, in a practical scenario, it is appealing to have rates which scale by increasing ρ. This way, the transmission rates can be easily adjusted by changing the transmission powers. Specifically, it is desirable that the rate of active links a.a.s. scale as

$$r_i = \log\left(1 + \frac{g_{ii}}{1/\rho + \beta_i}\right), \quad \forall i \in \mathcal{A}, \tag{3.98}$$

for some $\beta_i = O(1)$, which are the design parameters. At the same time, we require the conditions of problem (3.1), i.e. $r_i \geq \lambda$, be satisfied. In this section, we show how to realize such a situation by using DTBLAS.

According to (3.98), we should a.a.s. have $I_i = \beta_i$, where I_i is the interference observed by active link i and is defined in (3.45). However, this requires that $\mathrm{E}(I_i) = \beta_i$. Noting that $\mathrm{E}(I_i) = (k_2 - 1)\hat{\mu}$ (see (3.46)), we conclude that all β_is should take a same value, say β. Hence, a necessary condition for being in the noise-limited regime is

$$(k_2 - 1)\hat{\mu} = \beta, \tag{3.99}$$

where $\beta = O(1)$ is a design parameters. Later, we show that (3.99) is also a sufficient condition for operating in a noise-limited regime.

Based on the above discussion, we propose the following scheme for choosing the parameters of DTBLAS for a noise-limited regime: For a given required rate $\lambda = \log(1 + \gamma_0)$ and the interference β,

1. choose Δ as

$$\Delta = \Delta_0 = \gamma_0(1/\rho + \beta). \tag{3.100}$$

2. choose δ such that (3.99) is satisfied.

Note that the selection of Δ is such that the rate constraints $r_i \geq \lambda$ are satisfied. Also, as will be shown later, the selection of δ is such that operation in the noise-limited regime is guaranteed.

The next step is to solve (3.99) to obtain the value of δ and the corresponding number of active links k_2. By using (3.48), which gives the value of $\hat{\mu}$ in terms of δ, it is clear that (3.99) holds only if $\delta \to 0$ as $k_2 \to \infty$. In this case, (3.48) converts to $\hat{\mu} = \dfrac{\delta}{2} + O(\delta^2)$ and (3.99) simplifies to

$$k_2 \delta = 2\beta \quad a.a.s \tag{3.101}$$

To solve (3.101) and obtain δ, we should first obtain the value of k_2 in terms of n and δ. From (3.15) and condition (3.100), the number of links chosen by phase (i) of DTBLAS is obtained as

$$k_1 = ne^{-\Delta_0} + O\left(\xi\sqrt{ne^{-\Delta_0}}\right). \tag{3.102}$$

Also, recall from (3.55) that k_2 is the clique number of a random graph $\mathcal{G}(k_1, p)$, where p is obtained from (3.54). Since $\delta \to 0$, (3.54) can be rewritten as

$$p = \delta^2 + O(\delta^3), \tag{3.103}$$

which approaches zero as well. Note that Theorem 14, which was adopted from [27], and a similar result that appears in [28], are valid only for a fixed value of p. A natural question is whether a similar concentration result on the clique number of random graphs holds when p approaches zero. In the following lemma, we address this issue and obtain a concentration result on the clique number for zero-approaching values of p.

Lemma 19. *Let $p = p(m)$ be such that $p = o(1)$ and $p = \omega(m^{-a})$ for all $a > 0$. For fixed $\epsilon > 0$, the clique number $cl(G)$ of $G \in \mathcal{G}(m, p)$ a.a.s. satisfies $\lfloor s \rfloor \leq cl(G) \leq \lfloor s \rfloor + 1$, where*

$$s = 2\log_b m - 2\log_b \log_b m + 1 - 4\log_b 2 - \frac{\epsilon}{\log b},$$

$b = 1/p$.

Proof. See the Appendix. □

By using this lemma, (3.102), (3.103), and assuming $\xi = o(\sqrt{n})$, the number of active links a.a.s. becomes

$$k_2 = \left\lfloor \frac{\log n - \log\log n}{-\log \delta} \right\rfloor. \tag{3.104}$$

Thus, (3.101) can be rewritten as

$$\frac{\log n - \log\log n}{-\log \delta} \cdot \delta = 2\beta. \tag{3.105}$$

Assuming $|\log \beta| = o(\log\log n)$, it can be verified that the solution of (3.105) is

$$\delta = \frac{2\beta \log\log n}{\log n}(1 + o(1)). \tag{3.106}$$

With this value of δ, the number of active links is obtained from (3.104) as

$$k_2 = \left\lfloor \frac{\log n}{\log\log n}(1 + o(1)) \right\rfloor. \tag{3.107}$$

As mentioned before, we should show that the selected values of δ and Δ for DTBLAS, yields the network to operate in the noise-limited regime. The following theorem addresses this issue.

Theorem 20. *For the values of Δ and δ given in* (3.100) *and* (3.106)*, respectively, the interference of active links a.a.s. satisfy*

$$|I_i - \beta| \to 0, \quad \forall i \in \mathcal{A}. \tag{3.108}$$

Proof. By using the central limit theorem it can be shown that

$$|I_i - \beta| < \frac{\beta \log\log n}{\sqrt{\log n}}, \quad \forall i \in \mathcal{A}, \tag{3.109}$$

which readily yields the desired result. Since the calculations are similar to those in the proof of Lemmas 12 and 17, we omit them for brevity. □

Lemma 21. *Let $T_{_{NL}}$ denote the throughput achieved by DTBLAS in the noise-limited regime described above. Then, almost surely we have*

$$\log\left(1 + \frac{\Delta_0}{1/\rho + \beta}\right) \leq \lim_{n\to\infty} \frac{\log\log n}{\log n} T_{_{NL}} \leq \log\left(1 + \frac{\Delta_0 + 1}{1/\rho + \beta}\right). \tag{3.110}$$

Proof. According to Theorem 20, the throughput is obtained as

$$T_{_{NL}} = \sum_{i=1}^{k_2} \log\left(1 + \frac{g_{ii}}{1/\rho + \beta}\right). \tag{3.111}$$

Due to the fact that $g_{ii} > \Delta_0$, we have

$$T_{_{NL}} \geq k_2 \log\left(1 + \frac{\Delta_0}{1/\rho + \beta}\right). \tag{3.112}$$

The left-hand-side inequality in the lemma is readily obtained by using this inequality and the value of k_2 from (3.107). For the right-hand-side inequality, by utilizing the Jensen's inequality in (3.111), we obtain

$$T_{_{NL}} \leq k_2 \log\left(1 + \frac{\frac{1}{k_2}\sum_{i=1}^{k_2} g_{ii}}{1/\rho + \beta}\right). \tag{3.113}$$

According to the law of large numbers and due to the fact that $g_{ii} > \Delta_0$, we have

$$\frac{1}{k_2}\sum_{i=1}^{k_2} g_{ii} \to \mathrm{E}(g_{ii}) = 1 + \Delta_0. \tag{3.114}$$

The result is obtained by using (3.113), (3.114), and the value of k_2 from (3.107). $\square$

It is observed that the price for operating in the noise-limited regime is a decrease in the throughput by a factor of $\log\log n$.

3.8 Conclusion

In this chapter, wireless networks in Rayleigh fading environments are studied in terms of their achievable throughput. It is assumed that each link is either active and transmits with power P and rate λ, or remains silent. The objective is to maximize the network throughput or equivalently the number of active links. First, an upper bound is derived that shows the throughput and the number of active links scale at most like $\log n$ and $\frac{1}{\lambda} \log n$, respectively. To obtain lower bounds, we propose two LASs (TBLAS and DTBLAS) and prove that both of them a.a.s. yield feasible solutions for the throughput maximization problem. In TBLAS, which is adopted from a previous work, the activeness of each link is solely determined by the quality of its direct channel. TBLAS, which can be implemented in a decentralized fashion, performs very close to the upper bound for small values of λ. However, its performance falls below the upper bound when λ grows large. In DTBLAS, the mutual interference of the links are also taken into account when choosing the active links. It is demonstrated that DTBLAS not only performs close to the upper bound for $\lambda \to 0$, but its performance meets the upper bound when $\lambda \to \infty$. The above discussions take place in an interference-limited regime in which the transmission power P does not affect the transmission rate λ. However, we show that by a proper choose of the DTBLAS parameters, the rate-constrained network can also operate in a noise-limited regime; this feature of the DTBLAS comes at the price of decreasing the network throughput by a factor of $\log \log n$.

Chapter 4

Power Allocation

4.1 Introduction

The focus of the previous chapters is on the large wireless networks. However, it is useful and interesting to investigate the throughput optimization in networks with arbitrarily small sizes. In the previous chapters, we assume that the links follow an on-off paradigm. In this chapter, we consider power allocation for throughput maximization.

In the context of power allocation, there are roughly two groups of works studying the capacity of wireless networks. In one group, the objective is to minimize the transmit power while satisfying some quality of service (QoS) requirements of the users [29]. Such problems can be usually formulated as linear programs [21] and can be even solved in a decentralized fashion deterministically [18, 30], or stochastically [31]. In the other group of power allocation problems, which is of interest to us in this work, the objective is to use the limited power resources efficiently in order to improve some measures of QoS like throughput or minimum rate. Unfortunately, these problems usually entail some nonlinear optimization problems for which a systematic solution method may not exist.

An important measure of the network performance is the throughput. The problem of throughput maximization has been frequently appeared in the literature [29, 32, 33, 34, 35]. This problem translates to the problem of maximizing a product of linear fractional functions, which is a non-convex problem. Although there are algorithms to find the global optimum of such problems [36], their complexity precludes them from being implemented practically. Thus, one should think of finding suboptimum methods which are simple and yet their performance is not far from the optimum. One approach is to utilize numerical optimization methods [37] to solve the problem (see e.g. [34]). Since the problem is non-convex, these methods may converge to local optimum solutions. Another approach is to adopt an approximation of the objective function such that the problem can be converted to a convex program. Specifically, one common technique that has been utilized in [32, 33, 34] is the assumption of large *SINR*; with this assumption, the 1 in the Shannon capacity formula is neglected and the rate of each link becomes proportional to the logarithm of the corresponding *SINR* (i.e. $\log(1 + \mathit{SINR}) \approx \log(\mathit{SINR})$). As a result, the problem is easily converted to a convex program [38]. Unfortunately, in the interference channel, the assumption of large *SINR* is not valid. The reason is that due to the presence of interference the solution of the optimization problem is not guaranteed to satisfy this condition even when the noise power is quite low.

In this chapter, we consider the special case of $n = 2$ interfering links with arbitrary channel coefficients. It is shown that in this case, when the transmit power of the links are individually constrained, the optimum power allocation is such that either both transmitters transmit with maximum power or one of them remains off and the other transmits with maximum power.

Our discussions in this chapter and the references mentioned above are based on the assumption that a central unit, which is aware of all links resource limitations, QoS requirements, noise levels, and channels coefficients, performs the optimization procedure and assigns a transmit power to each link. In an effort to make the power allocation procedure decentralized, some authors have invoked game theoretic concepts [39] for power allocation [20, 40]. Although such techniques provide simplicity, their performance is usually far from the optimum. Another class of works include characterization of the the *feasible rates region*, when power allocation is allowed [33, 41, 42, 43]. None of these issues is of interest to us in this thesis.

The rest of the chapter is organized as follows. In Section 4.2, the system description is presented . We investigate the problem of throughput maximization via power allocation in Section 4.3. Finally, the chapter is concluded in Section 4.4.

4.2 System Description

The network structure we consider in this chapter is different from the previous chapters in the following matters.

- Instead of following an on-off scheme, the transmit powers are subject to individual power constraints. In other words, the vector of transmit powers $\boldsymbol{p} = (p_1, \cdots, p_n)$ satisfies

$$\mathbf{0}_n \leq \boldsymbol{p} \leq \mathbf{a}, \tag{4.1}$$

where $\mathbf{a} = (a_1, \cdots, a_n)$ represents the vector of maximum allowed transmit powers.

- The channel coefficients g_{ji} are known (not random) parameters. The channel gains, in general, depend on small scale and large scale fadings, path attenuation, processing gain of the CDMA system, etc.

- We consider an additive white Gaussian noise (AWGN) with variance η_i at the receiver i.

In all other matters, the model is the same as in the previous chapters. The *SINR* of the receiver i is defined as

$$\gamma_i(\boldsymbol{p}) = \frac{g_{ii}p_i}{\eta_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{ji}p_j}. \tag{4.2}$$

Throughout the chapter, we occasionally use γ_i instead of $\gamma_i(\boldsymbol{p})$. According to the Shannon capacity formula [13], the rate of link i is equal to

$$r_i(\boldsymbol{p}) = \log\left(1 + \gamma_i(\boldsymbol{p})\right) \quad \textit{nats/channel use.} \tag{4.3}$$

The network rate vector is defined as $\mathbf{r} = (r_1, \cdots, r_n)$. In a network, we desire to have all rates as large as possible. However, due to the interplay between the rates of different links (see (4.2) and (4.3)), it is not possible to maximize all the rates simultaneously. Instead, one may consider maximizing a utility function of the network which is increasing in all rates. A common utility function is the throughput of the network.

4.3 Throughput Maximization

The problem of throughput maximization is formulated as follows:

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \sum_{i=1}^{n} r_i(\boldsymbol{p}), \\ \text{s.t.} \quad & \mathbf{0}_n \leq \boldsymbol{p} \leq \mathbf{a}, \end{aligned} \tag{4.4}$$

which is a non-convex optimization problem. Thus, the algorithms developed for convex problems may converge to local optimum points for this problem. The main result of this chapter is a statement about the above problem for $n = 2$. However, the discussions prior to the proof of the main result are applicable for any values of n.

Lemma 22. *For any $a < 1$ and any power vector $\boldsymbol{p}$, we have $\boldsymbol{r}(a\boldsymbol{p}) \leq \boldsymbol{r}(\boldsymbol{p})$ with equality only for the indices $i \in \mathcal{N}_n$ for which $p_i = 0$.*

Proof. If the power of some of the links is zero, scaling the power vector keeps the power and consequently the rate of such links unchanged and equal to zero. Thus, we can consider only the case when all powers are nonzero. First, note that the functions

$$\gamma_i(a\boldsymbol{p}) = \frac{a g_{ii} p_i}{\eta_i + a \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ji} p_j}, \quad \forall i \in \mathcal{N}_n, \tag{4.5}$$

are all increasing in a. Using this property and the fact that $a < 1$, we have

$$\begin{aligned} r_i(a\boldsymbol{p}) &= \log\left(1 + \gamma_i(a\boldsymbol{p})\right) \\ &< \log\left(1 + \gamma_i(\boldsymbol{p})\right) \\ &= r_i(\boldsymbol{p}). \end{aligned} \tag{4.6}$$

This completes the proof. □

The following lemma identifies one of the characteristics of problem (4.4).

Lemma 23. *In the optimum solution* $\boldsymbol{p}^*$ *of (4.4), the power of at least one link takes its maximum allowed value.*

Proof. Consider the index set $\mathcal{I} = \{i \in \mathcal{N}_n : p_i^* > 0\}$ and define

$$a^* = \min_{i \in \mathcal{I}} \left\{ \frac{a_i}{p_i^*} \right\}. \tag{4.7}$$

For the sake of contradiction, assume $\boldsymbol{p}^* < \mathbf{a}$. Hence, we have $a^* > 1$. Choose a new power vector $\hat{\boldsymbol{p}} = a^* \boldsymbol{p}^*$, which obviously satisfies the constraints $\mathbf{0}_n \leq \hat{\boldsymbol{p}} \leq \mathbf{a}$. According to Lemma 22, we have

$$r_i(\hat{\boldsymbol{p}}) = r_i(a^* \boldsymbol{p}^*) > r_i(\boldsymbol{p}^*), \qquad \forall i \in \mathcal{I}, \tag{4.8}$$

which implies

$$\sum_{i=1}^{n} r_i(\hat{\boldsymbol{p}}) > \sum_{i=1}^{n} r_i(\boldsymbol{p}^*). \tag{4.9}$$

This is in contradiction to the optimality of $\boldsymbol{p}^*$. Thus, we should have $p_i^* = a_i$ for at least one $i \in \mathcal{I}$. □

There are some special cases, where even stronger statements can be expressed about the solution of (4.4). One of them, which is the main result of this chapter, is about the case of $n = 2$.

Theorem 24. *The optimum solution of (4.4) for $n = 2$ is obtained when one of the transmitters transmits with maximum power and the other one is silent, or both transmitters transmit with maximum power*[1]*.*

Proof. Assume for simplicity that channel coefficients and noise powers are scaled such that the maximum allowed power of both links and also the direct channel coefficients g_{ii} are equal to one. According to Lemma 23, in the optimum solution of (4.4) the power of at least one link should be equal to one; without loss of generality assume $p_2 = 1$. It suffices to show that the maximum of the function

$$f(p_1) = \log\left(1 + \frac{p_1}{\eta_1 + g_{21}}\right) + \log\left(1 + \frac{1}{\eta_2 + g_{12}p_1}\right) \tag{4.10}$$

is obtained either at $p_1 = 0$ or $p_1 = 1$. By computing the derivative of $f(p_1)$ and simplifying it we obtain

$$f'(p_1) = \frac{Ap_1^2 + Bp_1 + C}{d(p_1)}, \tag{4.11}$$

where $A = g_{12}^2$, $B = 2\eta_2 g_{12}$, $C = \eta_2(\eta_2+1) - g_{12}(\eta_1+g_{21})$, and $d(p_1)$ is a polynomial in p_1 with all coefficients non-negative. Thus, the sign of $f'(p_1)$ is determined by the sign of its numerator. Note that A, $B \geq 0$. If $C \geq 0$, the numerator (and thus $f'(p_1)$) is always non-negative for $p_1 \geq 0$. Thus, $f(p_1)$ is increasing in p_1 and achieves its maximum at $p_1 = 1$. If $C < 0$, the numerator has exactly one positive

[1]After publishing this result in [44], it was independently reported in [45].

root (p_1') and one negative root (p_1''). Thus, $f(p_1)$ has a minimum at p_1' and attains its maximum at 0 or 1. □

Another noteworthy special case is the *low SINR regime*; if we know that the *SINR* of all links are small, we can use the approximation $\log(1+x) \approx x$ to write (4.4) as

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \sum_{i=1}^{n} \gamma_i(\boldsymbol{p}), \\ \text{s.t.} \quad & \mathbf{0}_n \leq \boldsymbol{p} \leq \mathbf{a}. \end{aligned} \tag{4.12}$$

This problem is still non-convex; however, the following result can be concluded that allows for obtaining the optimum solution only by examining the vertices of the hypercube $\mathbf{0}_n \leq \boldsymbol{p} \leq \mathbf{a}$. This result has been already touched on in [46] in the context of uplink CDMA.

Theorem 25. *In the optimum solution $\boldsymbol{p}^*$ of (4.12), all transmit powers are either zero or the maximum allowed value, i.e., $p_i^* \in \{0, a_i\}$ for all $i \in \mathcal{N}_n$.*

Proof. Define the objective function

$$B(\boldsymbol{p}) = \sum_{i=1}^{n} \gamma_i(\boldsymbol{p}). \tag{4.13}$$

Obviously, for some $i \in \mathcal{N}_n$, we have $p_i^* \neq 0$. If $\boldsymbol{p}_{-i}^* = \mathbf{0}_{n-1}$, clearly $p_i^* = a_i$ maximizes the throughput and the proof is complete. If $\boldsymbol{p}_{-i}^* \neq \mathbf{0}_{n-1}$, by substituting the values of $\gamma_i(\boldsymbol{p})$ from (4.2) in the objective function (4.13) and computing the second order partial derivative with respect to p_i we obtain

$$\frac{\partial^2 B(\boldsymbol{p})}{\partial p_i^2} = 2 \sum_{j \neq i} G_{ij}^2 \frac{\gamma_j(\boldsymbol{p})}{d_j^2(\boldsymbol{p})}, \tag{4.14}$$

which is positive for all $\boldsymbol{p}_{-i} \neq \mathbf{0}_{n-1}$. Thus, $B(\boldsymbol{p})$ is convex with respect to p_i. As a result, the maximizing value of p_i lies on one end of the interval $[0,\, a_i]$. $\square$

Another special case pertains to uplink CDMA, where $g_{ji} = g_j$ for all $j \in \mathcal{N}_n$. This scenarion has been investigated in [35] and it is proved that the power of all links take the value of zero or the maximum allowed value except for at most one link.

4.4 Conclusion

In this chapter, some properties of the throughput maximization problem via power allocation, which is a nonconvex optimization problem, is investigated. It is demonstrated that under individual power constraints, in the optimum solution, the power of at least one link should take its maximum value. Then, for the special case of $n = 2$ links, it is shown that the optimum power allocation strategy for throughput maximization is such that either both links use their maximum power or one of them uses its maximum power and the other keeps silent. Also, in the low *SINR* regime, in which the throughput maximization problem converts to maximizing the sum-*SINR*, the power of all links should take either the value of 0 or the maximum allowed value.

Chapter 5

Concluding Remarks

5.1 Summary of Contributions

The dissertation focuses on throughput maximization for wireless networks. This is done by active link selection for large wireless networks and power allocation for small wireless networks.

In Chapter 2, the throughput of single-hop wireless networks with on-off strategy is investigated in a fading environment and without any constraints on the transmission rates. Despite the random behaviour of the channel, we present our results as the events that happen asymptotically almost surely. To obtain a lower bound on the throughput, a decentralized link activation strategy is proposed and analyzed for a general fading model. It is shown that in the popular model of Rayleigh fading a throughput of order $\log n$ is achievable a.a.s., which is by a factor of four larger than what was obtained in previous works with centralized methods and with multihop communications [10]. Moreover, for the Rayleigh fading model, an upper bound of order $\log n$ is obtained by using probabilistic methods. This shows the optimality of the proposed link activation strategy. The throughput maximization leads to rate-per-links that approach zero as $n \to \infty$.

In Chapter 3, throughput maximization via active link selection is investigated for wireless networks with fading channels, when the active links are restricted to transmit with a rate λ. First, we prove that the proposed distributed method in Chapter 2 is a solution, though suboptimum. We modify this method by adding an interference management phase to it. This modified LAS not only improves the performance of the network, but achieves the optimum solution of the constrained throughput maximization for large and small values of λ. We also demonstrate that by utilizing this latter strategy, the network can operate in a noise-limited regime at the price of a decrease in its throughput by a factor of $\log \log n$.

In Chapter 4, we address the problem of throughput maximization via power allocation. It is mentioned that this problem is a nonconvex optimization problem that cannot be solved in a systematic manner. However, as a preliminary result, we show that in the solution of this problem, at least one link should utilize its maximum allowed power. More importantly, for the special case of $n = 2$ links, we prove that the optimum solution is such that each link is active and transmits with full power or remains silent.

5.2 Future Research Directions

The dissertation can be extended in several directions. In the following, we briefly explain some of these possible directions.

Our results in Chapters 2 and 3 are restricted to single-antenna transmitters and receivers. A promising work is to investigate the throughput scaling law when using multiple-antennas. The study of the scaling law of MIMO broadcast systems has been already appeared in the literature [47, 48]. However, to the author's best knowledge, there is no work on the scaling laws of MIMO interfering links. Since there are several dimensions and parameters involved in the design of MIMO wireless systems, obtaining optimality results for such systems is a challenging problem.

We have considered linear receivers. Recently, interference cancelation for ad hoc networks has been considered in [49]. It would be interesting to incorporate interference-cancelation in investigating the throughput of the network in the fading environment. It is also possible to consider joint coding/decoding schemes like multiple-access and broadcast channels in our analysis.

In this dissertation, we considered a single-hop wireless network. In a more general scenario, nodes may cooperate to route each others data to the final destinations. A preamble to study the effect of multihop on the scaling law, is to investigate the matching of transmitter-receiver pairs; in each hop, there is a flexibility of choosing a proper receiver for each transmitting node. Finding the optimum matching can help design efficient multihop schemes.

In the context of power allocation (Chapter 4), it is appealing to characterize the feasible rates region of the network. For the special case of $n = 2$ links, it is possible to obtain closed form expressions describing the feasible rates region [50]. Also, for other values of of n, the feasible rates region has been characterized parametrically [51]. A possible extension is to study the feasible rates region when multiple antennas are deployed at the terminals.

Appendix A

Proof of Lemma 1

Define

$$g(k) = k \log\left(1 + \frac{\Delta}{k\mu + \psi_k}\right). \tag{A.1}$$

It is enough to show that $g'(k) > 0$. We have

$$\begin{aligned}
g'(k) &= \log\left(1 + \frac{\Delta}{k\mu + \psi_k}\right) - \frac{k}{1 + \dfrac{\Delta}{k\mu + \psi_k}} \cdot \frac{\Delta(\mu + \psi_k')}{(k\mu + \psi_k)^2} && \text{(A.2)}\\
&\geq \log\left(1 + \frac{\Delta}{k\mu + \psi_k}\right) - \frac{k}{1 + \dfrac{\Delta}{k\mu + \psi_k}} \cdot \frac{\Delta(\mu + \psi_k/k)}{(k\mu + \psi_k)^2} && \text{(A.3)}\\
&= \log\left(1 + \frac{\Delta}{k\mu + \psi_k}\right) - \frac{1}{1 + \dfrac{k\mu + \psi_k}{\Delta}} && \text{(A.4)}\\
&> 0. && \text{(A.5)}
\end{aligned}$$

The first inequality is due the assumption that $h(k) = \dfrac{\psi_k}{k}$ is nonincreasing in k, which yields $h'(k) \geq 0$ or equivalently $\psi_k' \leq \dfrac{\psi_k}{k}$. The last inequality is based on the inequality $\log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x} > 0$ for $x > 0$.

Appendix B

Optimum Parameters for TBLAS

B.1 Optimum Threshold for Rayleigh Fading Model

The optimum value of the threshold, Δ^*, is the value that maximizes the achievable throughput in (2.22). As it is seen, $T_{_{TBLAS}}(\Delta)$ is a complicated function of Δ. However, since ξ can grow as slow as desired, we can set $\xi = 0$ to obtain a more tractable form for $T_{_{TBLAS}}(\Delta)$ from which a *zero order approximation* of the solution is obtained. In the next stage, we will improve the solution using this zero order approximation.

Zero order approximation By setting $\xi = 0$, the objective function in (2.22) is transformed to

$$\hat{T}_{_{TBLAS}}(\Delta) = ne^{-\Delta} \log\left(1 + \frac{\Delta}{ne^{-\Delta}}\right). \tag{B.1.1}$$

Using the approximation $\log(1+x) \approx x - \frac{x^2}{2}$, the above function can be approximated as

$$\hat{T}_{TBLAS}(\Delta) \approx \Delta - \frac{\Delta^2}{2ne^{-\Delta}}. \tag{B.1.2}$$

The maximum of this function can be found using the first derivative test as follows. By taking derivative of both sides of (B.1.2), we obtain

$$\hat{T}'_{TBLAS}(\Delta) \approx 1 - \frac{t}{ne^{-\Delta}} - \frac{\Delta^2}{2ne^{-\Delta}}, \tag{B.1.3}$$

which is an increasing function in Δ. Consequently, the root of the equation $\hat{T}'_{TBLAS}(\Delta) = 0$ gives the value $\Delta^*_{(0)}$. This equation is equivalent to

$$2ne^{-\Delta} = 2\Delta + \Delta^2. \tag{B.1.4}$$

Noting that the solution to this equation is increasing with n (i.e. $\Delta^*_{(0)}$ is large), by taking logarithm of both sides of (B.1.4), we arrive at the following equation

$$\Delta = \log n - 2\log t + \log 2 - \frac{2}{\Delta}, \tag{B.1.5}$$

whose solution can be verified to be

$$\Delta^*_{(0)} = \log n - 2\log\log n + \log 2 + O\left(\frac{\log\log n}{\log n}\right). \tag{B.1.6}$$

First order approximation Using $\Delta^*_{(0)}$ in (B.1.6), the term containing ξ in (2.22) is approximated as[1]

$$\xi\sqrt{ne^{-t}} = \xi \log n. \tag{B.1.7}$$

[1] With a little abuse of notation we have replaced $\frac{\xi}{\sqrt{2}}$ by ξ. This is acceptable, because we are only interested in the order of the term that ξ introduces to the solution.

Since φ_n can be chosen of order $o(\log n)$, it is negligible in comparison with $\xi \log n$. Thus, the function to be maximized takes the form

$$T_{TBLAS}(\Delta) = \left(ne^{-\Delta} - \xi \log n\right) \log \left(1 + \frac{\Delta}{ne^{-\Delta} - \xi \log n}\right). \tag{B.1.8}$$

Assuming $\xi = o(\log \log n)$, and taking the same approach as for obtaining $\Delta^*_{(0)}$ in the zero order approximation, we can obtain

$$\Delta^*_{(1)} = \log n - 2 \log \log n + \log 2 + \frac{4 \log \log n}{\log n} + O\left(\frac{\xi}{\log n}\right). \tag{B.1.9}$$

As it is observed, as long as $\xi = o(\log \log n)$, the parameter ξ does not contribute in the dominant terms of Δ^*. Thus, we have

$$\Delta^* = \log n - 2 \log \log n + \log 2 + O\left(\frac{\log \log n}{\log n}\right). \tag{B.1.10}$$

By substituting the value of Δ^* in (2.22), the achievable throughput is obtained as mentioned in the lemma. The number of active links and the rate-per-link are obtained by using (2.16) and (2.17).

B.2 Optimum Threshold for Log-Normal Fading Model

The function to be maximized is given in (2.33). We first consider it as a function of u. Once we have obtained u^*, the value that maximizes this function, we will be able to find Δ^* from

$$\Delta^* = e^{u^* + M}. \tag{B.2.11}$$

Using the approximation $\log(1+x) \approx x - \frac{1}{2}x^2$, the objective function is written as

$$g(u) = \frac{SB}{\sqrt{2\pi}} \left(e^u - \frac{Bue^{2u}e^{\frac{u^2}{2S^2}}}{n} \right). \tag{B.2.12}$$

Setting the derivative of $g(u)$ equal to zero, we obtain

$$Be^u e^{\frac{u^2}{2S^2}} \left(1 + 2u + \frac{u^2}{S^2} \right) = n. \tag{B.2.13}$$

Since the right hand side of this equatin is n, which is assumed to be large, the solution u^* should be large as well. Thus, we can simplify the equation as

$$\frac{Bu^2 e^u e^{\frac{u^2}{2S^2}}}{S^2} = n. \tag{B.2.14}$$

By taking the logarithm of both sides of this equation and rearranging the terms, we obtain

$$\frac{u^2}{2S^2} = \log n - u - 2\log u - \log \frac{B}{S^2}. \tag{B.2.15}$$

The solution of this equation can be verified to be

$$u^* = \sqrt{2}S\sqrt{\log n} - S^2 + O\left(\frac{\log\log n}{\sqrt{\log n}} \right). \tag{B.2.16}$$

Then, from (B.2.11), Δ^* is obtained as given in (2.34). By substituting the value of Δ^* in (2.33) the maximum throughput is obtained. The number of active links and the rate-per-link are obtained by using (2.16) and (2.17).

B.3 Optimum ϱ for Shadow Fading Model

By defining $x = n\varrho$, the function to be maximized is

$$g(x) = x \log\left(1 + \frac{n}{x^2} \right). \tag{B.3.17}$$

Setting the derivative of $g(x)$ equal to zero, we obtain $\frac{x^2}{n} = c^2$ or equivalently

$$\varrho = \frac{c}{\sqrt{n}}, \tag{B.3.18}$$

where $c \approx 0.5050$ is a constant. It is easy to verify that for the aforementioned value of x, $g''(x) > 0$. Thus, ϱ given in (B.3.18) is the maximizing argument of the function under consideration. The maximum value is obtained by substituting ϱ from (B.3.18) into (2.40). The number of active links and the rate-per-link are obtained by using (2.16) and (2.17).

Appendix C

Proof of Lemma 19

The proof is based on the standard second moment method.

Suppose that V is the vertex set of G. For $S \subseteq V$, assume $\mathcal{C}_S$ is the event that the induced subgraph of G on S is a clique. Also, assume X_S is the indicator random variable of $\mathcal{C}_S$. If $|S| = s$, we have

$$\mathrm{E}(X_S) = \mathrm{P}\{\mathcal{C}_S\} = p^{\binom{s}{2}}. \tag{C.0.1}$$

Denote by Y_s the number of cliques of size s in G. In mathematical notation, we can write

$$Y_s = \sum_{\substack{S:\, S \subseteq V \\ |S|=s}} X_S. \tag{C.0.2}$$

In the sequel, we first provide some calculations which are required for the proof of the lemma. The main body of the proof is presented in subsection C.

C.0.1 Mean and Variance of Y_s

For brevity, hereafter, we will omit the description $S \subseteq V$. From (C.0.1), (C.0.2), and the linearity of expectation, we can calculate μ_s, the mean of Y_s, as

$$\mu_s = \mathrm{E}(Y_s) = \sum_{S:\,|S|=s} \mathrm{E}(X_S) = \binom{m}{s} p^{\binom{s}{2}}. \tag{C.0.3}$$

Also, the second moment of Y_s can be calculated as

$$\begin{aligned}
\mathrm{E}(Y_s^2) &= \mathrm{E}\left(\left(\sum_{S:\,|S|=s} X_S\right)^2\right) \\
&= \sum_{S:\,|S|=s}\ \sum_{S':\,|S'|=s} \mathrm{E}(X_S X_{S'}) \\
&= \sum_{S:\,|S|=s}\ \sum_{S':\,|S'|=s} \mathrm{P}(\mathcal{C}_S \cap \mathcal{C}_{S'}).
\end{aligned} \tag{C.0.4}$$

By defining $\ell = |S \cap S'|$, the double summation in (C.0.4) can be simplified as

$$\mathrm{E}(Y_s^2) = \sum_{\ell=0}^{s} \binom{m}{s} \binom{s}{\ell} \binom{m-s}{s-\ell} p^{2\binom{s}{2}-\binom{\ell}{2}}. \tag{C.0.5}$$

By using (C.0.3) and (C.0.5), σ_s^2, the variance of Y_s, is obtained as

$$\sigma_s^2 = \sum_{\ell=2}^{s} \binom{m}{s} \binom{s}{\ell} \binom{m-s}{s-\ell} p^{2\binom{s}{2}} (b^{\binom{\ell}{2}} - 1), \tag{C.0.6}$$

where $b = 1/p$. For a later reference, we also need to calculate σ_s^2/μ_s^2; from (C.0.3) and (C.0.6), we have

$$\frac{\sigma_s^2}{\mu_s^2} = \sum_{\ell=2}^{s} \frac{\binom{s}{\ell}\binom{m-s}{s-\ell}}{\binom{m}{s}} (b^{\binom{\ell}{2}} - 1), \tag{C.0.7}$$

C.0.2 Upper bounds on μ_s and σ_s^2/μ_s^2

By applying the Stirling's approximation to (C.0.3), we obtain

$$\begin{aligned} \mu_s &= \frac{m^{m+\frac{1}{2}}}{\sqrt{2\pi} s^{s+\frac{1}{2}} (m-s)^{m-s+\frac{1}{2}}} p^{\frac{s(s-1)}{2}} && \text{(C.0.8)} \\ &\leq \frac{1}{\left(\frac{s}{m}\right)^s \left(1-\frac{s}{m}\right)^m} p^{\frac{s(s-1)}{2}} && \text{(C.0.9)} \end{aligned}$$

For any $\epsilon > 0$, the inequality $1 - x \geq e^{-(1+\epsilon)x}$ holds for sufficiently small values of x. Since we are interested in small values of s/m, from this inequality and (C.0.8), we obtain

$$\mu_s \leq e^{s(\log m - \log s + (1+\epsilon) - \frac{s-1}{2} \log b)} \tag{C.0.10}$$

Equation (C.0.7) is readily converted to the following inequality

$$\frac{\sigma_s^2}{\mu_s^2} \leq \sum_{\ell=2}^{s} F_\ell, \tag{C.0.11}$$

where

$$F_\ell = \frac{\binom{s}{\ell}\binom{m-s}{s-\ell}}{\binom{m}{s}} b^{\binom{\ell}{2}}. \tag{C.0.12}$$

By using the definition of the binomial coefficients, we obtain

$$\begin{aligned} F_\ell &\leq 2^s \cdot \frac{(m-s)!}{m!} \cdot \frac{(m-s)!}{(m-2s+\ell)!} \cdot \frac{s!}{(s-\ell)!} \cdot b^{\frac{\ell(\ell-1)}{2}} && \text{(C.0.13)} \\ &\leq \frac{2^s \cdot (m-s)^{s-\ell} \cdot s^\ell}{(m-s)^s} \cdot b^{\frac{\ell(\ell-1)}{2}} && \text{(C.0.14)} \\ &= 2^s \cdot \left(\frac{m}{s} - 1\right)^{-\ell} \cdot b^{\frac{\ell(\ell-1)}{2}} && \text{(C.0.15)} \end{aligned}$$

Noting that $\frac{m}{s} \gg 1$, the above inequality can be approximately written as

$$F_\ell \leq 2^s \cdot \left(\frac{s}{m}\right)^\ell \cdot b^{\frac{\ell(\ell-1)}{2}}. \tag{C.0.16}$$

Using (C.0.11) and (C.0.16), we obtain

$$\frac{\sigma_s^2}{\mu_s^2} \leq \sum_{\ell=2}^{s} e^{g(\ell)}, \tag{C.0.17}$$

where

$$g(\ell) = s \log 2 + \ell(\log s - \log m + \frac{\ell}{2} \log b - \frac{1}{2} \log b) \tag{C.0.18}$$

is a quadratic convex function with a minimum at $\ell_0 = \frac{\log m}{\log b} - \frac{\log s}{\log b} + \frac{1}{2}$. Define

$$s_0 = 2 \log_b m - 2 \log_b \log_b m - 2 \log_b 2. \tag{C.0.19}$$

It is easy to show that if $s > s_0$, then $g(s) > g(2)$. Hence, (C.0.17) can be simplified as

$$\frac{\sigma_s^2}{\mu_s^2} \leq e^{\log s + g(s)}. \tag{C.0.20}$$

C.0.3 Proof

According to the Markov's inequality, we have

$$\mathrm{P}\{Y_s \geq 1\} \leq \mu_s. \tag{C.0.21}$$

For a fixed $\epsilon > 0$, define

$$s_1 = 2 \log_b m - 2 \log_b \log_b m + 1 + 2 \log_b(e/2) + \frac{\epsilon}{\log b}. \tag{C.0.22}$$

Using (C.0.10), it is easy to verify that for $s \geq s_1$, we have $\mu_s \to 0$ as $m \to \infty$. Hence, from (C.0.21), we conclude that

$$\mathrm{P}\{Y_s \geq 1\} \to 0, \quad \text{for} \quad s \geq s_1 \tag{C.0.23}$$

as $m \to \infty$. This means a.a.s. the clique number of G is less than s_1, i.e., we have the following upper bound on $\mathrm{cl}(G)$

$$\mathrm{cl}(G) < s_1 \quad a.a.s. \tag{C.0.24}$$

According to the Chebyshev's inequality, we have

$$\mathrm{P}\{Y_s = 0\} \le \frac{\sigma_s^2}{\mu_s^2}. \tag{C.0.25}$$

For a fixed $\epsilon > 0$, define

$$s_2 = 2\log_b m - 2\log_b \log_b m + 1 - 4\log_b 2 - \frac{\epsilon}{\log b}. \tag{C.0.26}$$

Using (C.0.20), it is easy to verify that for $s \le s_2$, we have $\sigma_s^2/\mu_s^2 \to 0$ as $m \to \infty$. Hence, from (C.0.25), we conclude that

$$\mathrm{P}\{Y_s = 0\} \to 0, \quad \text{for} \quad s \le s_2 \tag{C.0.27}$$

as $m \to \infty$. This means a.a.s. the clique number of G is not less than $\lfloor s_2 \rfloor$, i.e., we have the following lower bound on $\mathrm{cl}(G)$

$$\mathrm{cl}(G) \ge \lfloor s_2 \rfloor \quad a.a.s. \tag{C.0.28}$$

For sufficiently small ϵ, the difference between the upper bound s_1 and the lower bound s_2 is less than one. Hence, from (C.0.24) and (C.0.28) we can conclude that

$$\lfloor s_2 \rfloor \le \mathrm{cl}(G) \le \lfloor s_2 \rfloor + 1 \quad a.a.s. \tag{C.0.29}$$

List of References

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.

[2] M. Gastpar and M. Vetterli, "On the capacity of large gaussian relay networks," *IEEE Trans. Information Theory*, vol. 51, no. 3, pp. 765–779, March 2005.

[3] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: scaling laws and optimal operation," *IEEE Trans. Information Theory*, vol. 50, no. 5, pp. 748–767, May 2004.

[4] O. Lévêque and E. Telatar, "Information theoretic upper bounds on the capacity of large extended ad hoc wireless networks," *IEEE Trans. Information Theory*, vol. 51, no. 3, pp. 858–865, March 2005.

[5] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Trans. Information Theory*, vol. 53, no. 3, pp. 1009–1018, March 2007.

[6] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Networking*, vol. 10, no. 4, pp. 477–486, August 2002.

[7] O. Dousse, M. Franceschetti, and P. Thiran, "On the throughput scaling of wireless relay networks," *IEEE Trans. Information Theory*, vol. 52, no. 6, pp. 2756–2761, June 2006.

[8] S. R. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling in wireless networks," *IEEE Trans. Information Theory*, vol. 50, no. 6, pp. 1041–1049, June 2004.

[9] S. Toumpis and A. J. Goldsmith, "Large wireless networks under fading, mobility, and delay constraints," in *Proc. IEEE Infocom*, vol. 1, Hong Kong, 2004, pp. 609–619.

[10] R. Gowaikar, B. Hochwald, and B. Hassibi, "Communication over a wireless network with random connections," *IEEE Trans. Information Theory*, vol. 52, no. 7, pp. 2857–2871, July 2006.

[11] R. Gowaikar and B. Hassibi, "On the achievable throughput in two-scale wireless networks," in *Proc. IEEE ISIT*, 2006.

[12] F. Xue, L.-L. Xie, and P. R. Kumar, "The transport capacity of wireless networks over fading channels," *IEEE Trans. Information Theory*, vol. 51, no. 3, pp. 834–847, March 2005.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* New York: John Wiley & Sons, Inc., 1991.

[14] R. Etkin, "Spectrum sharing: Fundamental limits, scaling laws, and self-enforcing protocols," Ph.D. dissertation, EECS Department, University of California, Berkeley, 2006.

[15] S. Webere, J. G. Andrews, and N. Jindal, "The effect of fading, channel inversion, and threshold scheduling on ad hoc networks," *to appear in IEEE Trans. Information Theory*, November 2007.

[16] D. Gesbert and M. Kountouris, "Resource allocation in multicell wireless networks: Some capacity scaling laws," in *Proc. Workshop on Resource Allocation in Wireless NETworks (RAWNET '07)*, 2007.

[17] N. Jindal, J. G. Andrews, and S. Weber, "Bandwidth-SINR tradeoffs in spatial networks," in *Proc. IEEE International Symposium on Information Theory*, Nice, France, June 2007.

[18] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Selec. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.

[19] K. Kumaran and H. Viswanathan, "Joint power and bandwidth allocation in downlink transmission," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1008 – 1016, May 2005.

[20] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," in *Proc. IEEE DySPAN*, Baltimore, MD, Nov. 8-11 2005, pp. 338–345.

[21] T. ElBatt and A. Ephremides, “Joint scheduling and power control for wireless ad hoc networks,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 74–85, Jan. 2004.

[22] S. Toumpis and A. J. Goldsmith, “Capacity regions for wireless ad hoc networks,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 736–748, July 2003.

[23] L. Xiao, M. Johansson, and S. P. Boyd, “Simultaneous routing and resource allocation via dual decomposition,” *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1136–1144, July 2004.

[24] S. V. Hanly, “An algorithm for combined cell-cite selection and power control to maximize cellular spread spectrum capacity,” *IEEE J. Selec. Areas Commun.*, vol. 13, no. 7, pp. 1332–1340, Sep. 1995.

[25] V. V. Petrov, *Limit Theorems of Probability Theory: Sequences of Independent Random Variables*, ser. Oxford Studies in Probability. Oxford University Press, 1995.

[26] N. Wormald, “Random graphs and asymptotics,” in *Handbook of Graph Theory*, J. Gross and J. Yellen, Eds. Boca Raton: CRC, 2004, ch. 8.2, pp. 817–836.

[27] S. Janson, T. Luczak, and A. Rucinski, *Random Graphs.* John Wiley & Sons, Inc., 2000.

[28] B. Bollobás, *Random Graphs.* Cambridge University Press, 2001.

[29] A. Sampath, P. S. Kumar, and J. M. Holtzman, "Power control and resource management for a multimedia CDMA wireless system," in *Proc. IEEE Sixth International Symposium on Personal, Indoor and Mobile Radio Communications*, 1995, pp. 21–25.

[30] S. V. Hanly and D. N. Tse, "Power control and capacity of spread spectrum wireless networks," *Automatica*, vol. 35, no. 12, pp. 1987–2012, 1999.

[31] J. Luo, S. Ulukus, and A. Ephremides, "Standard and quasi-standard stochastic power control algorithms," *IEEE Trans. Information Theory*, vol. 51, no. 7, pp. 2612–2624, July 2005.

[32] D. Julian, M. Chiang, D. O'Neill, and S. Boyd, "QoS and fairness constrained convex optimization of resource allocation for wireless cellular and ad hoc networks," in *Proc. IEEE Infocom*, vol. 2, New York, NY, Jun. 23-27 2002, pp. 477–486.

[33] H. Boche and S. Stanczak, "Optimal QoS tradeoff and power control in CDMA systems," in *Proc. IEEE Infocom*, vol. 2, March 2004, pp. 1078–1088.

[34] X. Qiu and K. Chawla, "On the performance of adaptive modulation in cellular systems," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 884–895, June 1999.

[35] S.-J. Oh and A. C. K. Soong, "QoS-constrained information-theoretic sum capacity of reverse link CDMA systems," *IEEE Trans. Wirless Commun.*, vol. 5, no. 1, pp. 3–7, Jan. 2006.

[36] H. Konno and H. Yamashita, "Minimizing sums and products of linear fractional functions over a polytope," *Naval Research Logistics*, vol. 46, no. 5, pp. 583–596, Aug. 1999.

[37] J. Nocedal and S. J. Wright, *Numerical Optimization*. Springer-Verlag, 1999.

[38] M. Chiang, *Geometric Programming for Communication Systems*, ser. Foundations and Trends in Communications and Information Theory. now Publishers Inc., July 2005, vol. 2, no. 1-2.

[39] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: The MIT Press, 1991.

[40] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[41] L. A. Imhof and R. Mathar, "Capacity regions and optimal power allocation for CDMA cellular radio," *IEEE Trans. Information Theory*, vol. 51, no. 6, pp. 2011–2019, June 2005.

[42] C. W. Sung, "Log-convexity property of the feasible SIR region in power-controlled cellular systems," *IEEE Commun. Letters*, vol. 6, no. 6, pp. 248–249, Jun. 2002.

[43] H. Boche and S. Stanczak, "Log-convexity of the minimum total power in cdma systems with certain quality-of-service guaranteed," *IEEE Trans. Information Theory*, vol. 51, no. 1, pp. 374–381, Jan. 2005.

[44] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, "Power allocation and asymptotic achievable sum-rates in single-hop wireless networks," in *Proc. Conference on Information Sciences and Systems*, Princeton, NJ, March 2006.

[45] Gjendemsjoe, D. Gesbert, G. Oien, and S. Kiani, "Optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. Workshop on Resource Allocation in Wireless NETworks (RAWNET '06)*, Boston, MA, April 2006.

[46] S. Ulukus and L. J. Greenstein, "Throughput maximization in CDMA uplinks using adaptive spreading and power control," in *Proc. IEEE Sixth International Symposium on Spread Spectrum Techniques and Applications*, 2000, pp. 565 – 569.

[47] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Information Theory*, vol. 51, no. 2, pp. 506–522, February 2005.

[48] A. Bayesteh and A. K. Khandani, "On the user selection for mimo broadcast channels," *Submitted to IEEE Trans. Information Theory*, 2007.

[49] S. P. Weber, J. G. Andrews, X. Yang, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancellation," *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2799–2814, Aug. 2007.

[50] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, "On the achievable rates of wireless networks," in *Proc. Conference on Information Sciences and Systems*, Baltimore, MD, March 16-18 2005.

[51] H. Mahdavi-Doost, M. Ebrahimi, and A. K. Khandani, "Characterization of rate region in interference channels with constrained power," in *Proc. IEEE International Symposium on Information Theory*, Nice, France, June 2007, pp. 2441–2445.