Studies on Trade-off Between Throughput and Reliability in Wireless Systems

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In the first part of the thesis, we study the trade-off between the transmission reliability and data rate in high signal-to-noise ratio regime in ad-hoc wireless networks. Bandwidth allocation plays a significant role in this trade-off, since dividing bandwidth reduces the number of users on each band and consequently decreases the interference level, however it also decreases the data rate. Noting that the interference power is substantially influenced by the network density, this trade-off introduces a measure for appropriate bandwidth allocation among users considering the network density. The diversity-multiplexing trade-off is derived for a one-dimensional regular ad-hoc network. In the second part of the thesis, we study the performance of point-to-point and broadcast systems with partial channel state information at the transmitter in a time-varying environment. First, the capacity of time-varying channels with periodic feedback at the transmitter is evaluated. It is assumed that the channel state information is perfectly known at the receiver and is fed back to the transmitter at the regular time-intervals. The system capacity is investigated in two cases: i) finite state Markov channel, and ii) additive white Gaussian noise channel with time-correlated fading. In a multiuser scenario, we consider a downlink system in which a single-antenna base station communicates with single antenna users, over a time-correlated fading channel. It is assumed that channel state information is perfectly known at each receiver, while the rate of channel variations and the fading gain at the beginning of each frame are known to the transmitter. The asymptotic throughput of the scheduling that transmits to the user with the maximum signal to noise ratio is examined applying variable code rate and/or variable codeword length signaling. It is shown that by selecting a fixed codeword length for all users, the order of the maximum possible throughput (corresponding to quasi-static fading) is achieved.

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.

To My Parents

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Chapter 1

Introduction

The rapidly increasing number of wireless users and new bandwidth-consuming applications fuel the growing demand for more bandwidth and higher data rates. This, in turn, tends to exhaust the valuable resource of frequency spectrum, and necessitates the development of spectrally efficient signaling schemes.

One of the most effective techniques to increase the spectral efficiency in wireless systems is *channel reuse* in which several links communicate at the same time and at the same frequency through a shared channel. The main source of impairment in this scheme is the interference of the users to each other, which is called the *co-channel interference*. In channel reuse schemes, the links sharing the same channel are graphically separated such that co-channel interference is minimized. As a result, the overall data rate in the shared bandwidth increases. Applying channel reuse by sharing bandwidth between users decreases the number of users in each frequency band. Although this results in decreasing the co-channel interference and increasing the transmission reliability, the data rate decreases due to the smaller allocated bandwidth. Thus, improving the spectral efficiency in this scheme requires investigating the optimal trade-off between the transmission reliability and data rate. In the first part of this thesis, we study this trade-off in a one-dimensional regular network and use it as a measure to determine the optimal bandwidth allocation among users. *Diversity-multiplexing* trade-off which quantifies the trade-off between the transmission reliability and data rate in high signal-to-noise ratio (SNR) regime was first developed for multiple-input multiple-output (MIMO) systems by Zheng and Tse [29]. Let $C(\mathrm{SNR})$ be a family of codes indexed by SNR and P_e be the error probability. If

$$\lim_{SNR\to\infty} \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}} \leq -d$$

then this family of codes achieves a diversity gain of d. The higher the diversity gain, the faster P_e decays as SNR increases. Let $R(\mathrm{SNR})$ be the transmission rate. If

$$\lim_{SNR\to\infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}} \geq r$$

then this family of codes achieves multiplexing gains of r. The diversity-multiplexing trade-off has been established for some wireless networks including multiple-access channel [16] and relay network [25]. We study diversity-multiplexing trade-off in Z-channel and one-dimensional interference channel.

Adaptive transmission is another efficient technique to increase the spectral efficiency of time-varying wireless channels by adaptively modifying the transmission rate, power, etc., according to the state of the channel seen by the receiver. Adaptive transmission with perfect channel state information (CSI) at the transmitter and the receiver, was first proposed in the late 1960's [23]. In practice, the CSI is not perfectly available at the transmitter or at the receiver. Practical implementation of adaptive transmission schemes with partial CSI at the transmitter or at the receiver has been the subject of numerous research works (see [4, 10, 51, 58] and references therein). In these works, various feedback models have been adopted that reflect the limitations to perfectly delivering the CSI to the transmitter. In our work, we consider a different form of feedback, namely periodic feedback, that updates the CSI in regular intervals. As a special case, the CSI can be updated at the beginning of each transmission frame of data. We consider a point-to-point time-varying channel with perfectly known CSI at the receiver. The CSI is provided at the transmitter through a noiseless feedback link at regularly-spaced time intervals. Every T channel use, the CSI of the current channel use is fed back to the transmitter. We obtain the channel capacity of the system and show that it is achievable by multiplexing T codebooks across the channel.

In a multiuser scenario, we consider a downlink system in which a single-antenna base station (BS) communicates with single antenna users, over a time-correlated fading channel. It is assumed that the CSI of users is known at the beginning of each transmission frame. In this system, we analyze the system throughput by utilizing adaptive code rate and adaptive codeword length. The throughput is defined as the average rate of information that is reliably transmitted to the receiver. We use *error exponent* as a reliability measure. Error exponent expresses the trade-off between the probability of error and the codeword length. More precisely, the reliability function quantifies the exponential behavior of the probability of error for the best coding schemes, as the coding delay is increased with the rate of transmission held fixed. It is well known that increasing the codeword length results in improving the error probability in additive white Gaussian noise (AWGN) channel. However, in a time-varying channel with partial CSI at the transmitter, it is not always possible to obtain arbitrary small error probabilities by increasing the codeword length. In fact, increasing the codeword length also results in increasing the fading fluctuations over the frame, and consequently, the throughput will decrease. In other words, a finite codeword length maximizes the throughput. We evaluate the error exponent and derive the system throughput based on the rate of channel variation.

The rest of this chapter is devoted to a brief summary of the materials presented in the following chapters.

Thesis Outline

(Chapter 2) is devoted to the diversity-multiplexing trade-off analysis in interference channels. The diversity-multiplexing tradeoff in Z-channel is investigated assuming the interference channel is known/unknown at the receiver.

(Chapter 3) investigates diversity-multiplexing trade-off in a one-dimensional wireless network to optimally allocate bandwidth among users.

(Chapter 4) obtains the capacity of the Markovian time-varying channel and time-correlated flat fading channel assuming the availability of periodic feedback.

(Chapter 5) is devoted to the evaluation of the reliability measure for the time-varying channels assuming partial CSI at the transmitter. We evaluate the random coding error exponent of the time-correlated flat fading channel in a wireless downlink system. The data rate and codeword length is optimized based on the rate of channel variation of users to maximize the system throughput.

(Chapter 6) presents a summary of the thesis contributions and future works.

(Appendix A) proves Theorem 5.1.

Chapter 2

Diversity-Multiplexing Trade-off in Z-channel

The fundamental trade-off between the diversity and multiplexing gains has been characterized for MIMO systems in [29] and is extended for MIMO multiple-access channel in [16]. This approach has been applied for other wireless channels and networks [21,25,37]; In [25], the trade-off between the rate and the reliability is studied for different strategies in a wireless relay network. In [37], diversity-multiplexing trade-off upper bounds are obtained for cooperative diversity protocols in a wireless network.

We consider a simple Z-channel with an arbitrary interference power and derive the diversity-multiplexing trade-off of this channel. Throughout this chapter, all channels are assumed to be flat Rayleigh fading and quasi-static, i.e., the channel gains remain constant during a coherence interval and change independently from one coherence interval to another. We assume the use of random Gaussian codebooks, where a codeword spans the entire coherence interval of the channel. This assumption has been applied in similar works such as [25].

The rest of the chapter is organized as follows; Section 2.1 introduces the structure of the Z-channel. In Section 2.2, the diversity-multiplexing trade-off in Z-channel is studied. In Section 2.3, the diversity-multiplexing trade-off for the multiple-access channel and z-channel with unknown interference is derived. Moreover, the comparison between the diversity-multiplexing trade-off between Z-channel with known interference channel, unknown interference channel and multiple-access channel is studied. Section 2.4 concludes the chapter.

2.1 System Model

We consider a Z-Channel (Fig. 2.1) in which the transmitters and receivers are equipped with single antennas. The received signal at the first receiver can be written as

$$\mathbf{y}_1 = \mathbf{h}_1\mathbf{x}_1 + \mathbf{h}_0\mathbf{x}_2 + \mathbf{n}_1, \tag{2.1}$$

and the second link will be a standard point-to-point link as

$$\mathbf{y}_2 = \mathbf{h}_2\mathbf{x}_2 + \mathbf{n}_2, \tag{2.2}$$

where $\mathbf{x}_i \sim \mathcal{CN}(0, \rho_i)$ is the transmitted signal from the i^{th} transmitter with the power constraint $\mathbb{E}\{\|\mathbf{x}_i\|^2\} \leq \rho_i$, and $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}, 1)$ is the AWGN at i^{th} receiver for $i = 1, 2$ ($\mathcal{CN}(m, v)$ denotes complex Gaussian distribution with mean m and variance v). All the channels are assumed to be quasi-static Rayleigh fading, i.e. the channel coefficients $\mathbf{h}_0$, $\mathbf{h}_1$ and $\mathbf{h}_2$ have complex Gaussian distribution with zero mean and unit variance. It is assumed that $\mathbf{h}_0$ and $\mathbf{h}_1$ are perfectly known at the first receiver and unknown at both transmitters. The channel coefficient $\mathbf{h}_2$ is assumed to be perfectly known at the second receiver and unknown at both transmitters.

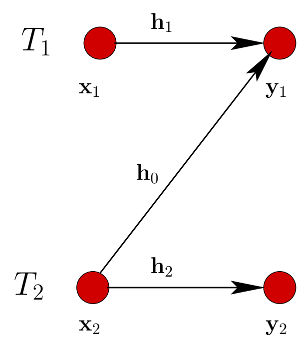

Figure 2.1: Z-channel

2.2 Analysis of Diversity-Multiplexing Trade-off

Investigating the diversity-multiplexing trade-off curves for this setup requires the assumption of $\rho_1, \rho_2 \to \infty$. Hence, using the definition of the diversity and multiplexing in [29], for each link, we have

$$\begin{aligned} r_i &= \lim_{\rho_i \to \infty} \frac{\mathcal{R}_i(\rho_i)}{\log \rho_i}, \\ d_i &= \lim_{\rho_i \to \infty} -\frac{\log \mathbb{P}_i(\rho_i)}{\log \rho_i}, \quad i = 1, 2, \end{aligned} \tag{2.3}$$

where $\mathcal{R}_i(\rho_i)$ denotes the transmission rate, and $\mathbb{P}_i(\rho_i)$ represents the average error probability of link i. Assuming large enough block lengths, from [29] it is realized that the average error probability is equal to the outage probability almost surely, i.e.

$$\mathbb{P}_i(\rho_i) = \Pr\{C(\rho_i) < \mathcal{R}_i(\rho_i)\}, \tag{2.4}$$

where $C(\rho_i)$ denotes the capacity of link i. We focus on characterizing the diversity-multiplexing trade-off curve for the first link, since the second link is an ordinary point-to-point link. For this purpose, we derive the outage probability for this link, for any given multiplexing gain vector (r_1, r_2).

From the first receiver's point of view, the channel is a multiple-access channel (MAC). However, since the first receiver is not interested in the data of the second transmitter, the outage event is different from that of MAC. In fact, the outage in this case can be written as the intersection of the following events:

$$\begin{aligned} \mathscr{B}_1 &\triangleq \{(\mathcal{R}_1, \mathcal{R}_2) \notin C_{MAC}\}, \\ \mathscr{B}_2 &\triangleq \{\mathcal{R}_1 > I(\mathbf{x}_1; \mathbf{y}_1)\}, \end{aligned} \tag{2.5}$$

where C_{MAC} denotes the capacity region of the MAC at the first receiver. The first event corresponds to the case that the first receiver can not decode both $\mathbf{x}_1$ and $\mathbf{x}_2$. The second event corresponds to the case that the first receiver can not decode $\mathbf{x}_1$, considering $\mathbf{x}_2$ as noise. The above events can be written in terms of the channel fading gains as follows:

$$\begin{aligned} \mathscr{B}_1 &= \left\{ \begin{array}{ll} \log(1+\rho_1 h_1) < \mathcal{R}_1 & \bigcup \\ \log(1+\rho_2 h_0) < \mathcal{R}_2 & \bigcup \\ \log(1+\rho_2 h_0 + \rho_1 h_1) < \mathcal{R}_1 + \mathcal{R}_2 & \end{array} \right\}, \\ \mathscr{B}_2 &= \left\{ \log(1 + \frac{\rho_1 h_1}{1+\rho_2 h_0}) < \mathcal{R}_1 \right\}, \end{aligned} \tag{2.6}$$

where $h_i \triangleq \|\mathbf{h}_i\|^2$, $i = 0, 1$. The intersection of these two events is depicted in Fig. 2.2. As can be observed, the outage event can be expressed as the union of the events $\mathcal{A}_1$ and $\mathcal{A}_2$. $\mathcal{A}_1$ is the outage event when there is no interference. The effect of interference from the second user is captured in $\mathcal{A}_2$.

Theorem 2.1. *Assuming $\rho_1 = \rho$ and $\rho_2 = \rho^{\beta}$, $0 \leq \beta$, the diversity-multiplexing trade-off for the first user of Z-channel is*

$$d_Z^*(r_1, r_2) = \left\{ \begin{array}{ll} \min(1 - r_1, (1+\beta) - 2(r_1 + \beta r_2)) & r_1 + \beta r_2 < \eta \\ \mu - r_1 - \beta r_2 & \eta < r_1 + \beta r_2 < \mu \end{array} \right. \tag{2.7}$$

where $\eta \triangleq \min(1, \beta)$ *and* $\mu \triangleq \max(1, \beta)$.

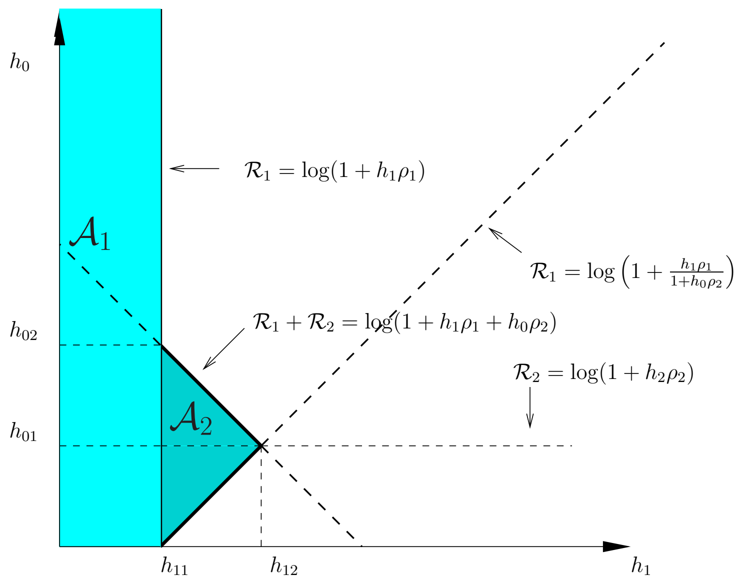

Figure 2.2: Outage region of Z-channel

Proof. Defining the total outage event as $\mathscr{B}$, we have

$$\Pr\{\mathscr{B}\} = \Pr\{\mathcal{A}_1\} + \Pr\{\mathcal{A}_2\}. \tag{2.8}$$

Considering Fig. 2.2, we formulate the probability of $\mathcal{A}_1$ and $\mathcal{A}_2$ as follows:

$$\Pr\{\mathcal{A}_1\} = \int_0^{h_{11}} f(h_1)dh_1, \tag{2.9}$$

$$\Pr\{\mathcal{A}_2\} = \int_{h_{11}}^{h_{12}} \int_{m_1(h_1-h_{11})}^{h_{02}+m_2(h_{11}-h_1)} f(h_1)f(h_0)\mathrm{d}h_0\mathrm{d}h_1, \tag{2.10}$$

where $h_{11} \triangleq \frac{e^{\mathcal{R}_1}-1}{\rho_1}$, $h_{01} \triangleq \frac{e^{\mathcal{R}_2}-1}{\rho_2}$, $h_{12} \triangleq \frac{e^{\mathcal{R}_2}(e^{\mathcal{R}_1}-1)}{\rho_1}$, $h_{02} \triangleq \frac{e^{\mathcal{R}_1}(e^{\mathcal{R}_2}-1)}{\rho_2}$, $m_1 \triangleq \frac{h_{01}}{h_{12}-h_{11}}$, $m_2 \triangleq \frac{h_{02}-h_{01}}{h_{12}-h_{11}}$, and $f(h_0)$ and $f(h_1)$ are the probability distribution function (pdf) of h_0 and h_1, respectively. Since $f(h_0) = e^{-h_0}$ and $f(h_1) = e^{-h_1}$, we derive (2.9) and (2.10) as follows:

$$\Pr\{\mathcal{A}_1\} = 1 - e^{-h_{11}}, \tag{2.11}$$

$$
\begin{aligned}
\Pr\{\mathcal{A}_2\} &= \int_{h_{11}}^{h_{12}} \int_{m_1(h_1-h_{11})}^{h_{02}+m_2(h_{11}-h_1)} e^{-h_0} e^{-h_1} \mathrm{d}h_0 \mathrm{d}h_1 \\
&= \int_{h_{11}}^{h_{12}} e^{-h_1} \left(e^{-m_1(h_1-h_{11})} - e^{-(h_{02}+m_2(h_{11}-h_1))}\right) \mathrm{d}h_1 \\
&= \begin{cases}
\frac{1}{m_1+1}\left[e^{-h_{11}} - e^{-(h_{12}+h_{01})}\right] & \\
-\frac{1}{1-m_2}\left[e^{-(h_{02}+h_{11})} - e^{-(h_{12}+h_{01})}\right] & m_2 \neq 1 \\
\frac{1}{m_1+1}\left[e^{-h_{11}} - e^{-(h_{12}+h_{01})}\right] & \\
-(h_{12}-h_{11})e^{-(h_{11}+h_{02})} & m_2 = 1.
\end{cases} \\
&= \begin{cases}
\frac{1}{m_1+1} e^{-h_{11}} \left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) & \\
\frac{1}{m_2-1} e^{-(h_{12}+h_{01})} \left(1 - e^{-(m_2-1)(h_{12}-h_{11})}\right) & m_2 \neq 1 \\
\frac{1}{m_1+1} e^{-h_{11}} \left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) & \\
-(h_{12}-h_{11})e^{-(h_{11}+h_{02})} & m_2 = 1.
\end{cases}
\end{aligned}
\tag{2.12}
$$

Setting $\rho_1 = \rho$ and $\rho_2 = \rho^{\beta}$, where β is a constant and using (2.3), we have

$$
\begin{aligned}
h_{11} &= \rho^{r_1-1}\left(1-\rho^{-r_1}\right), \\
h_{12} &= \rho^{r_1+\beta r_2-1}\left(1-\rho^{-r_1}\right), \\
h_{01} &= \rho^{\beta(r_2-1)}\left(1-\rho^{-\beta r_2}\right), \\
h_{02} &= \rho^{r_1+\beta r_2-\beta}\left(1-\rho^{-\beta r_2}\right), \\
m_1 &= \rho^{1-\beta-r_1}\left(1-\rho^{-r_1}\right)^{-1}, \\
m_2 &= \rho^{1-\beta}.
\end{aligned}
\tag{2.13}
$$

Noting (2.11) and (2.13), we can write

$$
\Pr\{\mathcal{A}_1\} \doteq \rho^{r_1-1}, \quad r_1 < 1 \tag{2.14}
$$

where $b \doteq \rho^a$ is equivalent to $\lim_{\rho\to\infty} \frac{\log b}{\log \rho} = a$. We consider 3 scenarios to derive $\Pr\{\mathcal{A}_2\}$:

2.2.1 Weak interference ($\beta < 1$)

From (2.13), it follows that $m_2 \to \infty$ as $\rho \to \infty$. Hence, (2.12) can be written as

$$\begin{aligned}
\Pr\{\mathcal{A}_2\} &= \frac{1}{m_1+1} e^{-h_{11}} \left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) - \\
&\quad \frac{1}{m_2-1} e^{-(h_{12}+h_{01})} \left(1 - e^{-(m_2-1)(h_{12}-h_{11})}\right) \\
&\overset{(a)}{\backsimeq} e^{-h_{11}}(h_{12}-h_{11}) - \\
&\quad \frac{1}{m_2-1} e^{-(h_{12}+h_{01})} \left(1 - e^{-(m_2-1)(h_{12}-h_{11})}\right), \qquad (2.15)
\end{aligned}$$

where (a) comes from applying the approximation $1 - e^{-(1+m_1)(h_{12}-h_{11})} \backsimeq (1+m_1)(h_{12} - h_{11})$, since $(1+m_1)(h_{12}-h_{11}) \doteq \rho^{1-\beta-r_1}\rho^{r_1+\beta r_2-1} = \rho^{\beta(r_2-1)}$, and as a result $(1 + m_1)(h_{12}-h_{11}) \to 0$ noting that $r_2 < 1$. We consider two scenarios:

i) $r_1 + \beta r_2 > \beta$: Using (2.15), we can write

$$\begin{aligned}
\Pr\{\mathcal{A}_2\} &\overset{(a)}{\backsimeq} e^{-h_{11}}(h_{12}-h_{11}) - \frac{1}{m_2-1} \qquad (2.16)\\
&\doteq \rho^{r_1+\beta r_2-1} - \rho^{\beta-1} \\
&\doteq \rho^{r_1+\beta r_2-1}, \qquad (2.17)
\end{aligned}$$

where (a) comes from the fact that $(m_2-1)(h_{12}-h_{11}) \doteq \rho^{1-\beta}\rho^{r_1+\beta r_2-1} = \rho^{r_1+\beta r_2-\beta} \to \infty$.

ii) $r_1 + \beta r_2 < \beta$: Noting (2.15), we have

$$\begin{aligned}
\Pr\{\mathcal{A}_2\} &\backsimeq e^{-h_{11}}(h_{12}-h_{11}) - \\
&\quad \frac{1}{m_2-1} e^{-(h_{11}+h_{02})} \left(e^{(m_2-1)(h_{12}-h_{11})} - 1\right), \\
&\overset{(a)}{\backsimeq} e^{-h_{11}} \left(1 - e^{-h_{02}}\right)(h_{12}-h_{11}) \\
&\overset{(b)}{\backsimeq} h_{02}(h_{12}-h_{11}) \qquad (2.18)\\
&\doteq \rho^{2(r_1+\beta r_2)-(1+\beta)}, \qquad (2.19)
\end{aligned}$$

where (a) results from the fact that $(m_2 - 1)(h_{12} - h_{11}) \to 0$, and subsequently $\frac{e^{(m_2-1)(h_{12}-h_{11})}-1}{m_2-1} \backsimeq (h_{12} - h_{11})$ and (b) comes from the fact that $e^{-h_{11}} \backsimeq 1$ and $e^{-h_{02}} \backsimeq 1 - h_{02}$, noting $h_{11}, h_{02} \to 0$. In summary, we have

$$\Pr\{\mathcal{A}_2\} \doteq \begin{cases} \rho^{2(r_1+\beta r_2)-(1+\beta)} & r_1 + \beta r_2 < \beta \\ \rho^{r_1+\beta r_2-1} & \beta < r_1 + \beta r_2 < 1 \end{cases}. \tag{2.20}$$

2.2.2 Moderate interference ($\beta = 1$)

Noting that $m_2 = \rho^{1-\beta} = 1$ in this scenario, from (2.12), we can write

$$\begin{aligned}\Pr\{\mathcal{A}_2\} = & \frac{1}{m_1+1} e^{-h_{11}} \left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) \\ & - (h_{12} - h_{11}) e^{-(h_{11}+h_{02})}.\end{aligned} \tag{2.21}$$

Having the fact that $m_1 \to 0$ and $0 < h_{11} < h_{12}$, the necessary condition to have $\Pr\{\mathcal{A}_2\} \to 0$ is $h_{12} \to 0$, which incurs $r_1 + r_2 < 1$ (otherwise the diversity gain is zero). Using (2.21) and the approximation $\frac{1}{m_1+1}\left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) \backsimeq (h_{12} - h_{11})$, we have

$$\begin{aligned}\Pr\{\mathcal{A}_2\} & \backsimeq e^{-h_{11}} \left(1 - e^{-h_{02}}\right) (h_{12} - h_{11}) \\ & \backsimeq h_{02}(h_{12} - h_{11}) \\ & \doteq \rho^{2(r_1+r_2-1)}.\end{aligned} \tag{2.22}$$

2.2.3 Strong interference ($\beta > 1$)

From (2.13), it follows that in this case $m_1 \to 0$ and as a result (2.12) can be written as

$$\begin{aligned}\Pr\{\mathcal{A}_2\} = & \frac{1}{m_1+1} e^{-h_{11}} \left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) - \\ & \frac{1}{1-m_2} e^{-(h_{11}+h_{02})} \left(1 - e^{-(1-m_2)(h_{12}-h_{11})}\right). \end{aligned} \tag{2.23}$$

Here, we consider two cases:

i) $r_1 + \beta r_2 < 1$: In this case, it is easy to show that the first term and the second term in (2.23) behaves as $(h_{12} - h_{11})$ and $e^{-h_{02}}(h_{12} - h_{11})$, respectively. Since $\beta > 1$, this condition also incurs that $h_{02} \sim \rho^{r_1+\beta r_2-\beta} \to 0$, and as a result,

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &\simeq \left(1 - e^{-h_{02}}\right)(h_{12} - h_{11}) \\ &\simeq h_{02}(h_{12} - h_{11}) \\ &\doteq \rho^{2(r_1+\beta r_2)-(1+\beta)}. \end{aligned} \tag{2.24}$$

ii) $1 < r_1 + \beta r_2 < \beta$: In this case, noting (2.13), it is easy to see that $h_{12} - h_{11} \to \infty$. Therefore, we can write

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &\simeq \frac{1}{1+m_1} - \frac{1}{1-m_2} e^{-h_{02}} \\ &\overset{(a)}{\simeq} (1 - m_1) - (1 + m_2) e^{-h_{02}} \\ &\overset{(b)}{\simeq} h_{02} - (m_1 + m_2 e^{-h_{02}}) \\ &\overset{(c)}{\doteq} \rho^{r_1+\beta r_2-\beta}, \end{aligned} \tag{2.25}$$

where (a) comes from using the approximation $\frac{1}{1-x} \simeq 1 + x$ for $x \ll 1$, (b) results from the assumption of $r_1 + \beta r_2 < \beta$ which incurs $h_{02} \to 0$, and (c) results from the fact

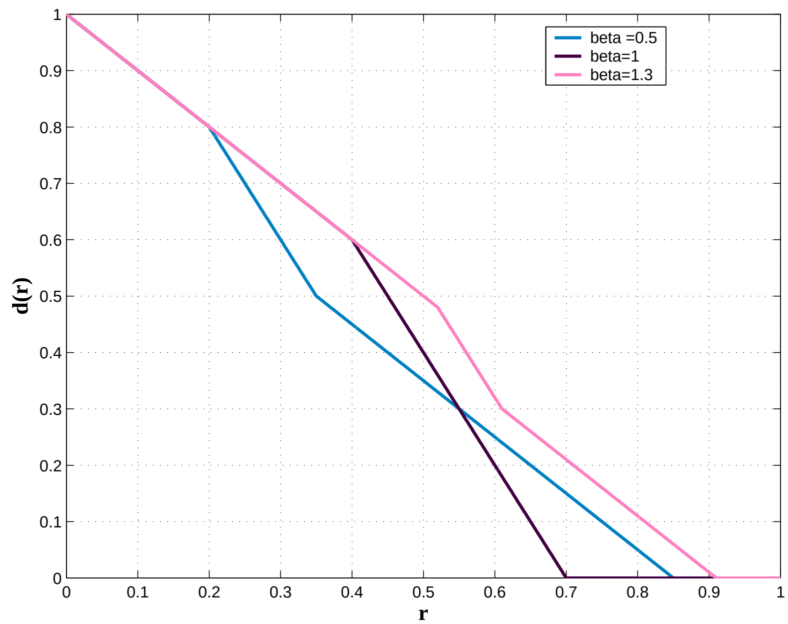

Figure 2.3: Diversity-Multiplexing trade-off for various values of β, $r_2 = 0.3$.

that since $r_1 + \beta r_2 > 1$, $h_{02} \sim \rho^{r_1+\beta r_2-\beta}$ dominates m_1 and m_2. Unifying (2.20), (2.22), (2.24), and (2.25), we obtain

$$\Pr\{\mathcal{A}_2\} \doteq \begin{cases} \rho^{2(r_1+\beta r_2)-(1+\beta)} & r_1 + \beta r_2 < \eta \\ \rho^{r_1+\beta r_2-\mu} & \eta < r_1 + \beta r_2 < \mu \end{cases} \tag{2.26}$$

where $\eta \triangleq \min(1, \beta)$, $\mu \triangleq \max(1, \beta)$. Using (2.8), (2.14) and (2.26), the result of the theorem is achieved. $\square$

Fig. 2.3 depicts the optimal diversity-multiplexing trade-off curve for $\beta = 0.5$, $\beta = 1$ and $\beta = 1.3$. As can be observed, the curve corresponding to $\beta = 1.3$ outperforms the other curves. Moreover, comparing the two curves corresponding to $\beta = 0.5$ and $\beta = 1$, we realize that for moderate values of multiplexing gain, the curve corresponding to $\beta = 1$ yields the higher diversity gain, while for high multiplexing gain values, $\beta = 0.5$ is preferable. Fig. 2.4 shows the maximum diversity gain versus β, for the fixed

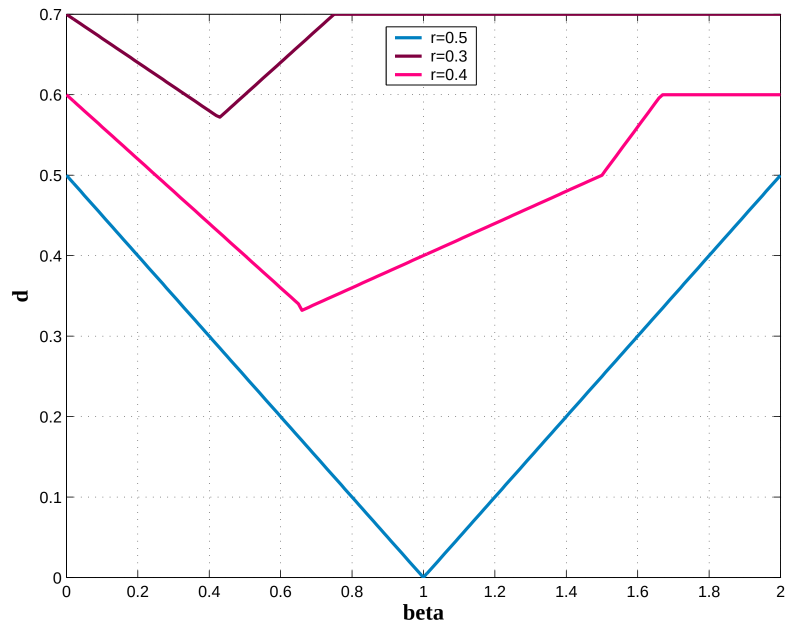

Figure 2.4: Diversity vs. β for various values of multiplexing gains, $r_1 = r_2 = r$.

multiplexing gain values of 0.3, 0.4, and 0.5, assuming $r_1 = r_2$. As can be observed, all the curves have a global minimum, depending on the value of the multiplexing gain.

2.3 Multiple-Access Channel and Unknown Interference Channel

In this section, we compare the diversity-multiplexing trade-off curve, derived in the previous section, with two other scenarios; i) Multiple-Access channel (MAC), where the first receiver decodes the transmitted data from both senders, and ii) Z-channel assuming that interference fading channel $\mathbf{h}_0$ is not known at the first receiver. The former is studied in [16], for the case that all links have the same power constraint. In the following, we derive diversity-multiplexing trade-off for the MAC assuming $\rho_1 = \rho$ and $\rho_2 = \rho^{\beta}$.

2.3.1 Multiple-Access channel

The outage event probability of the MAC denoted as $\mathscr{B}_{MAC}$ (Fig. 2.5) can be written as

$$\begin{aligned}
\Pr\{\mathscr{B}_{MAC}\} = & \int_0^{h_{11}} e^{-h_1} \mathrm{d}h_1 \\
& + \int_{h_{11}}^{h_{12}} \int_0^{h_{02}+m_2(h_{11}-h_1)} e^{-h_0} e^{-h_1} \mathrm{d}h_0 \mathrm{d}h_1 \\
& + \int_{h_{12}}^{\infty} \int_0^{h_{01}} e^{-h_0} e^{-h_1} \mathrm{d}h_0 \mathrm{d}h_1 \\
= & 1 - e^{-h_{11}} + \chi + (1 - e^{-h_{01}}) e^{-h_{12}},
\end{aligned} \tag{2.27}$$

where

$$\chi = \begin{cases} \left(e^{-h_{11}} - e^{-h_{12}}\right) & \\ -\frac{1}{m_2 - 1}\left[e^{-(h_{01}+h_{12})} - e^{-(h_{02}+h_{11})}\right] & m_2 \neq 1 \\ \left(e^{-h_{11}} - e^{-h_{12}}\right) & \\ -e^{-(h_{01}+h_{12})}\left(h_{12} - h_{11}\right) & m_2 = 1 \end{cases} \tag{2.28}$$

Following similar arguments in the proof of Theorem 2.1, we can easily show that

$$\chi \doteq \begin{cases} \rho^{2(r_1+\beta r_2)-(1+\beta)} & r_1 + \beta r_2 < \eta \\ \rho^{r_1+\beta r_2-\mu} & \eta < r_1 + \beta r_2 < \mu \end{cases}, \tag{2.29}$$

where $\eta = \min(1, \beta)$ and $\mu = \max(1, \beta)$. Using (2.27) and (2.29), we can write

$$d^*_{MAC}(r_1, r_2) = \begin{cases} \min(1 - r_1, 1 + \beta - 2(r_1 + \beta r_2), \beta(1 - r_2)) & r_1 + \beta r_2 < \eta \\ \min(1 - r_1, \mu - r_1 - \beta r_2, \beta(1 - r_2)) & \eta < r_1 + \beta r_2 < \mu \end{cases}$$

We interpret the diversity gain of MAC in terms of that of Z-channel as follows:

$$d^*_{MAC}(r_1, r_2) = \begin{cases} d^*_Z(r_1, r_2) & 1 < r_1 + \beta r_2 < \beta \\ \min(d^*_Z(r_1, r_2), \beta(1 - r_2)) & \text{Otherwise} \end{cases}$$

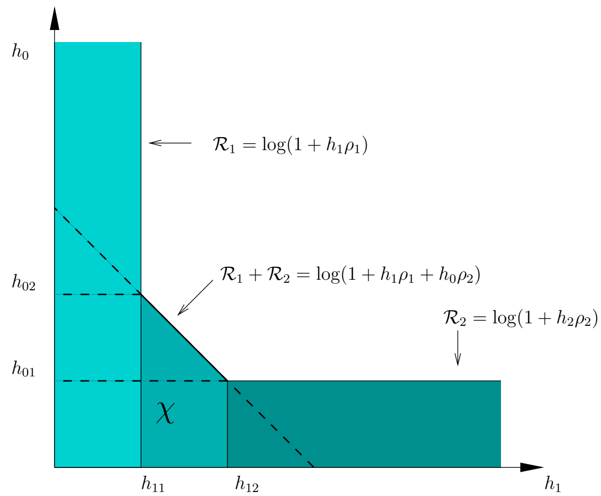

Figure 2.5: Outage region of multiple-access channel

2.3.2 Z-channel with unknown interference

In this scenario, we consider the system model of Z-channel described in Section 2.1. However, we assume that the first receiver only knows the direct channel ($\mathbf{h}_1$) and does not have any information about the interference channel ($\mathbf{h}_0$).

Lemma 2.2. *The outage event of a point-to-point Rayleigh fading channel with unknown interference in high-SNR regime is*

$$\mathscr{B}_{Z-NCSI} = \Pr\{\mathcal{R}_1 > \log(1 + \rho^{1-\beta} h_1)\}, \tag{2.30}$$

where $h_1 = \|\mathbf{h}_1\|^2$ and ρ and ρ^β are SNR of the direct link and the interference link, respectively.

Proof. Let us define

$$\mathscr{B}^U_{Z-NCSI} \triangleq \{\mathcal{R}_1 > I(\mathbf{x}_1; \mathbf{y}_1 | \mathbf{h}_1)\}, \tag{2.31}$$

and

$$\mathscr{B}^L_{Z-NCSI} \triangleq \{\mathcal{R}_1 > I(\mathbf{x}_1; \mathbf{y}_1 | \mathbf{x}_2, \mathbf{h}_1)\}. \tag{2.32}$$

$\mathscr{B}^L_{Z-NCSI}$ denotes the outage event when the first receiver considers $\mathbf{x}_2$ as noise and $\mathscr{B}^U_{Z-NCSI}$ denotes the outage event when the second users data is decoded correctly at the first receiver. Following the above definitions, we have

$$\Pr\{\mathscr{B}^L_{Z-NCSI}\} \leq \Pr\{\mathscr{B}_{Z-NCSI}\} \leq \Pr\{\mathscr{B}^U_{Z-NCSI}\}. \tag{2.33}$$

We derive the mutual information in (2.31) and (2.32) as follows:

$$\begin{aligned}
I(\mathbf{x}_1; \mathbf{y}_1|\mathbf{h}_1) &= H(\mathbf{y}_1|\mathbf{h}_1) - H(\mathbf{y}_1|\mathbf{h}_1, \mathbf{x}_1) \\
&= H(\mathbf{h}_1\mathbf{x}_1 + \mathbf{h}_0\mathbf{x}_2 + \mathbf{n}_1|\mathbf{h}_1) \\
&- H(\mathbf{h}_0\mathbf{x}_2 + \mathbf{n}_1) \\
&\overset{(a)}{\geq} H(\mathbf{h}_1\mathbf{x}_1 + \mathbf{n}_1|\mathbf{h}_1) \\
&- \log\left(2\pi e \mathrm{Var}(\mathbf{h}_0\mathbf{x}_2 + \mathbf{n}_1)\right) & (2.34) \\
&= \log(2\pi e(\rho h_1 + 1)) - \log(2\pi e(\rho^{\beta} + 1)) \\
&\backsimeq \log(1 + \rho^{1-\beta} h_1), & (2.35)
\end{aligned}$$

where (a) comes from the fact that $H(X+Y) > H(X)$, for independent X and Y, and $H(X) \leq \log(2\pi e\mathrm{Var}(\mathrm{X}))$. Substituting (2.34) in (2.31), we obtain

$$\Pr\{\mathscr{B}^U_{Z-NCSI}\} \leq \Pr\{\mathcal{R}_1 > \log(1 + \rho^{1-\beta} h_1)\}. \tag{2.36}$$

For calculating the lower-bound, we first compute $I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{x}_2,\mathbf{h}_1)$ as follows:

$$\begin{aligned}
I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{x}_2,\mathbf{h}_1)\} &= H(\mathbf{y}_1|\mathbf{x}_2,\mathbf{h}_1) - H(\mathbf{y}_1|\mathbf{x}_1,\mathbf{x}_2,\mathbf{h}_1) \\
&= H(\mathbf{h}_1\mathbf{x}_1 + \mathbf{h}_0\mathbf{x}_2 + \mathbf{n}_1|\mathbf{x}_2,\mathbf{h}_1) \\
&- H(\mathbf{h}_0\mathbf{x}_2 + \mathbf{n}_1|\mathbf{x}_2) \\
&\overset{(a)}{=} \mathbb{E}_{\mathbf{x}_2}\left\{\log(2\pi e(\rho h_1 + |\mathbf{x}_2|^2 + 1))\right\} \\
&- \mathbb{E}_{\mathbf{x}_2}\left\{\log(2\pi e(|\mathbf{x}_2|^2 + 1))\right\} \qquad (2.37) \\
&= e^{\frac{1+\rho h_1}{\rho^\beta}} E_1(\frac{1+\rho h_1}{\rho^\beta}) + \log(1+\rho h_1) - e^{\rho^{-\beta}} E_1(\rho^{-\beta}) \\
&\overset{(b)}{\simeq} \log(1+\rho^{1-\beta}h_1), \qquad (2.38)
\end{aligned}$$

where $E_1(x) = \int_x^\infty \frac{e^{-u}du}{u}$. In (2.37), (a) results from the fact that conditioned on $\mathbf{h}_1$ and $\mathbf{x}_2$, $\mathbf{h}_1\mathbf{x}_1$ and $\mathbf{h}_0\mathbf{x}_2$ are independent Gaussian variables with variances $h_1\rho_1 = |\mathbf{h}_1|^2\rho_1$ and $|\mathbf{x}_2|^2$, respectively and (b) results following the equality $E_1(x) = -\gamma - \log x - \sum_{n=1}^\infty \frac{(-1)^n x^n}{n!n}$ and the asymptotic expansion $E_1(x) = \frac{e^{-x}}{x}[1 - \frac{1}{x} + \frac{2}{x^2} + \cdots]$. As a result, the lower-bound on the outage probability can be expressed as

$$\Pr\{\mathscr{B}^L_{Z-NCSI}\} \geq \Pr\{\mathcal{R}_1 > \log(1+\rho^{1-\beta}h_1)\}. \qquad (2.39)$$

Considering (2.33), (2.36) and (2.39), the result of the lemma follows. □

From Lemma 2.2, it is concluded that

$$\Pr\{\mathscr{B}_{Z-NCSI}\} \doteq \rho^{1-\beta-r_1}, \quad r_1 < 1-\beta, \qquad (2.40)$$

or equivalently,

$$d^*_{Z-NCSI}(r_1) = \max(1-\beta-r_1, 0). \qquad (2.41)$$

Hence, the diversity gain in this scenario is equivalent to the diversity gain in (2.7) with $r_2 = 1$. Note that in this scenario, β is limited to be strictly less than one (otherwise the diversity gain is zero). It is realized that in this case, the diversity-multiplexing curve is strictly below the curve when the interference channel is known at the receiver.

2.4 Conclusion

The diversity-multiplexing trade-off curve is characterized for Z-channel assuming known and unknown interference channel at the receiver. The diversity-multiplexing gain of MAC is obtained assuming different scenarios for the SNR of the links and is interpreted in terms of the diversity-multiplexing gain of Z-channel.

Chapter 3

Applications of Diversity-Multiplexing Trade-off in Network Design

One of the most effective scheme to increase the spectral efficiency is the scheme known as channel reuse. In this scheme, time slots (frequency channels) can be shared by wireless users geographically separated so that small interference is obtained [19, 24, 40]. It is of interest to find the channel reuse factor which yields the highest spectral efficiency while maintaining a required quality of service. In this regard, there is an inherent tradeoff between the reliability of reception and the rate of communication. Dividing bandwidth reduces the number of users and decreases the interference level. While this boosts the reliability of the network, it decreases the achievable data rate. On the other hand, the rate can be increased by sharing the bandwidth among users at the cost of a considerable interference level.

In wireless ad-hoc networks, the performance of channel reuse has been studied and the impact of the this scheme on the performance of the network capacity is investigated [35, 36, 49, 52]. Implementing the channel reuse scheme requires studying bandwidth allocation among users. Algorithms for spectrum sharing in ad-hoc wireless network are proposed in [28, 57, 59] using a game theory approach. Optimal spectrum sharing for a single-hop wireless network is studied and power allocation among different users has been determined in [44].

We propose a different approach to optimally allocating bandwidth in a wireless network in high-SNR regime. Investigating the diversity-multiplexing trade-off of interference channels, we determine the optimal time-division multiple-access scheme which maximizes the network spectral efficiency considering the network density in a one-dimensional network. It is assumed that the interference from each link to the other links grows exponentially with the distance, such that the interference between two neighboring links declines as $\rho^{-\alpha_0}$ where ρ is the received SNR and α_0 is a constant. We study this model in two following scenarios; i) the interference channel is not known at the receiver. ii) the interference channel of the corresponding strongest interferer is known at each receiver. It is shown that for any given multiplexing gain r, the maximum diversity gain is achieved by utilizing a general time-sharing scheme where the active users form equal-size equally-spaced clusters (a group of adjacent nodes) with the size at most 3 and the distance spread of at most $\lceil \frac{1}{\alpha_0} \rceil$. The maximum diversity gain for each value of r is obtained by taking the maximum diversity gain among all the time-sharing strategies. We analyze the effect of network density on the performance by deriving the diversity gain in terms of the distances. Throughout this chapter, all channels are assumed to be flat Rayleigh fading and quasi-static, i.e., the channel gains remain constant during a coherence interval and change independently from one coherence interval to another.

The rest of the chapter is organized as follows; Section 3.1 introduces the structure of the one-dimensional network and the channel model. Section 3.2 is devoted to the analysis of the diversity-multiplexing trade-off for a one-dimensional ad-hoc network. In Section 3.3, the diversity gain is derived in terms of the network density for a one-dimensional network which utilizes a general TDMA scheme. Section 3.4 concludes the chapter.

3.1 System Model

We consider a homogeneous one-dimensional network, consisting of n pairs of transmitters and receivers. The nodes are equally spaced on two parallel lines such that the corresponding transmitter-receiver links are parallel (Fig. 3.1). The network utilizes a general TDMA scheme which assigns transmission rights to the links, i.e. transmitter/receiver pairs such that the i^{th} link is active in δ_i portion of the time. It is assumed that transmitters are centrally synchronized and use the same length data blocks. The objective is to find the optimum $\delta = (\delta_1, \cdots, \delta_n)$ and $\eta(t) = (\eta_1(t), \cdots, \eta_n(t))$, where $\eta_i(t)$ is one, if the i^{th} link is active at the t^{th} transmission block and otherwise is zero. We study this network in the following two scenarios; i) the interference channels are not available at the receiver nodes, ii) the strongest interference channels are available at the receiver nodes.

The received signal at the i^{th} link's receiver at the t^{th} transmission block can be written as

$$\mathbf{y}_i(t) = \sqrt{\alpha_{ii}}\mathbf{h}_i(t)\mathbf{x}_i(t)\eta_i(t) + \sum_{j \neq i} \sqrt{\alpha_{ji}}\mathbf{h}_{ji}(t)\mathbf{x}_j(t)\eta_j(t) + \mathbf{n}_i(t), \tag{3.1}$$

where $\mathbf{h}_i(t) \sim \mathcal{CN}(0,1)$, $\mathbf{x}_i(t) \sim \mathcal{CN}(0,\rho)$, $\mathbf{h}_{ji}(t) \sim \mathcal{CN}(0,1)$, $\mathbf{n}_i(t) \sim \mathcal{CN}(0,1)$, α_{ii}, and α_{ji} denote the direct channel, the transmitted signal, the interference channel from the j^{th} transmitter to the i^{th} receiver, additive Gaussian noise, the attenuation from the i^{th} transmitter to the i^{th} receiver, and the attenuation factor from the j^{th} transmitter to the

i^{th} receiver, respectively. The power constraint for the i^{th} transmitter is $\mathbb{E}\{\|\mathbf{x}_i\|^2\} \leq \rho$. The channel between each transmitter and each receiver node is assumed to be Rayleigh fading. The attenuation factor from each transmitter node to its receiver is assumed to be negligible, i.e. $\alpha_{ii} = 1$. The attenuation factor from each transmitter node to its receiver neighbors is assumed to be ρ^{α_0}, i.e. $\alpha_{ij} = \rho^{-\alpha_0}$ for $|i-j| = 1$. Since the attenuation model is assumed to be exponentially related to the distance, the closest active link will dominate the interference.

3.2 Diversity-Multiplexing Trade-off Analysis

The network consists of n pairs of transmitters and receivers. For each link, we define

$$r_i = \lim_{\rho \to \infty} \frac{\mathcal{R}_i(\rho)}{\log \rho}, \tag{3.2}$$

where $\mathcal{R}_i(\rho)$ denotes the transmission rate of link i. The optimal diversity-multiplexing tradeoff curve for this setup is defined as an $(n{+}1)$-dimensional vector $(r_1, \cdots, r_n, d^*(\mathbf{r}))$, $\mathbf{r} \triangleq (r_1, \cdots, r_n)$, such that

$$d^*(\mathbf{r}) = \max_{\eta,\delta} \lim_{\rho \to \infty} \frac{\log \Pr\{\mathscr{B}(\mathbf{r})\}}{\log \rho}, \tag{3.3}$$

where $\mathscr{B}(\mathbf{r}) \triangleq \bigcup_{i=1}^{n} \mathscr{B}_i(\mathbf{r})$ and $\mathscr{B}_i(\mathbf{r})$ denotes the outage event in link i, and the maximization is taken over all time-sharing strategies. For simplicity, we study a special case in which the multiplexing gains of all the links are the same, i.e., $r_1 = \cdots = r_n = r$. Hence, we can express $d^*(\mathbf{r})$ as $d^*(r)$. The outage probability of the network can be bounded as

$$\Pr\{\mathscr{B}^{\max}(\mathbf{r})\} \leq \Pr\{\mathscr{B}(\mathbf{r})\} \leq n\Pr\{\mathscr{B}^{\max}(\mathbf{r})\}, \tag{3.4}$$

where $\mathscr{B}^{\max}(\mathbf{r}) \triangleq \mathscr{B}_j(\mathbf{r})$ and $j = \arg\max_i \Pr\{\mathscr{B}_i(\mathbf{r})\}$. Assuming $\log n \sim o(\log \rho)$ and $\Pr\{\mathscr{B}^{\max}(\mathbf{r})\} = \rho^{\xi}$, we have

$$\begin{aligned} n\Pr\{\mathscr{B}(\mathbf{r})\} &= \rho^{\xi + \frac{\log n}{\log \rho}} \\ &\backsim \rho^{\xi}. \end{aligned} \tag{3.5}$$

Using (3.4) and (3.5), we can write

$$\Pr\{\mathscr{B}(\mathbf{r})\} \backsim \Pr\{\mathscr{B}^{\max}(\mathbf{r})\}. \tag{3.6}$$

In the following, we focus on deriving $\Pr\{\mathscr{B}^{\max}(\mathbf{r})\}$ assuming known/unknown interference channel at the receiver.

3.2.1 Unknown interference channel

In this part, it is assumed that each receiver perfectly knows the channel to its corresponding transmitter, however, it does not have any information about the interference channel. We assume that the i^{th} link receives the dominant interference from the m^{th} transmitter. Let us define $\alpha_\nu \triangleq \alpha_{mi} = \rho^{-|m-i|\alpha_0}$.

Theorem 3.1. *The diversity-multiplexing trade-off of the i^{th} link is given by*

$$d_i(r) = 1 - \beta_\nu - \frac{r}{\delta_i}, \tag{3.7}$$

where $\beta_\nu \triangleq (1 + \frac{\log \alpha_\nu}{\log \rho})^+$.

Proof. Similar to the approach of proving Lemma 2.2, we derive the upper-bound and lower-bound of the outage probability. We note that depending on TDMA strategy, each link can have one or two dominant interferers (Fig. 3.1). Using (2.34) and (3.1), we can write

$$
\begin{aligned}
I(\mathbf{x}_i;\mathbf{y}_i|\mathbf{h}_i) &\geq H(\mathbf{h}_i\mathbf{x}_i+\mathbf{n}_i|\mathbf{h}_i) \\
&\quad -\log\left(2\pi e\mathrm{Var}(\sum_{j\neq i}\alpha_{ji}\mathbf{h}_{ji}\mathbf{x}_j+\mathbf{n}_i)\right) \\
&= \log(2\pi e(\rho h_i+1)) - \log(2\pi e(\sum_{j\neq i}\eta_j\alpha_{ji}\rho_j+1)) \\
&\gtrsim \log(1+\frac{\rho^{1-\beta_\nu}h_i}{n}). \qquad (3.8)
\end{aligned}
$$

Using (2.37) and (3.1), we have

$$
\begin{aligned}
I(\mathbf{x}_i;\mathbf{y}_i|\mathbf{V}_i,\mathbf{h}_i)\} &= \mathbb{E}\left\{\log(2\pi e(h_i\rho+\sum_{j\neq i}\eta_j\alpha_{ji}|\mathbf{x}_j|^2+1))\right\} \\
&\quad - \mathbb{E}\left\{\log(2\pi e(\sum_{j\neq i}\eta_j\alpha_{ji}|\mathbf{x}_j|^2+1))\right\} \\
&\leq \mathbb{E}\left\{\log(2\pi e(h_i\rho+\sum_{j}\alpha_{mi}|\mathbf{x}_j|^2+1))\right\} \\
&\quad - \mathbb{E}\left\{\log(2\pi e(\alpha_{mi}|\mathbf{x}_m|^2+1))\right\} \\
&\overset{(a)}{\lesssim} \log(1+\rho h_i) - e^{\rho^{-\beta_\nu}}E_1(\rho^{-\beta_\nu}) \\
&\backsimeq \log(1+h_i\rho^{1-\beta_\nu}), \qquad (3.9)
\end{aligned}
$$

where $\mathbf{V}_i=\{\mathbf{x}_j|1\leq j\leq n, j\neq i\}$ and (a) results from the following

$$
\begin{aligned}
\mathbb{E}\left\{\log(h_i\rho+\sum_{j}\alpha_{mi}|\mathbf{x}_j|^2+1)\right\} &= \int_0^\infty \frac{e^{-u}u^{n-1}}{(n-1)!}\log(1+\rho h_i+\rho^{\beta_\nu}u)du \\
&= \log(1+\rho h_i)+\mathcal{I}, \qquad (3.10)
\end{aligned}
$$

since $\sum_j \alpha_{mi}|\mathbf{x}_j|^2$ has a χ_n^2 distribution and $\mathcal{I}$ can be bounded as follows:

$$\begin{aligned}
\mathcal{I} &= \int_0^\infty \frac{e^{-u}}{(n-1)!(u+\varphi)}((n-1)! + \sum_{k=0}^{n-2} \psi(n,k)u^{n-k-1})du \\
&= \sum \psi(n,k) \int_0^\infty \frac{e^{-u}u^{n-k-1}}{(n-1)!(u+\varphi)}du + \int_0^\infty \frac{e^{-u}}{u+\varphi}du \\
&= \sum \psi(n,k)(\int_0^{\log\varphi} \frac{e^{-u}u^{n-k-1}}{(n-1)!(u+\varphi)}du + \int_{\log\varphi}^\infty \frac{e^{-u}u^{n-k-1}}{(n-1)!(u+\varphi)}du) \\
&+ e^{\varphi}E_1(\varphi) \\
&\leq \sum \psi(n,k)(\frac{(\log\varphi)^{n-k-1}}{(n-1)!(\log\varphi+\varphi)} \int_0^{\log\varphi} e^{-u}du + \int_{\log\varphi}^\infty \frac{e^{-u}u^{n-k-1}}{(n-1)!(\log\varphi+\varphi)}du) \\
&= O(\frac{(\log\varphi)^{n-1}}{\varphi}),
\end{aligned} \tag{3.11}$$

where $\varphi = \frac{1+\rho h_i}{\rho^{\beta_\nu}}$. Noting (3.8) and (3.9), we have

$$\Pr\{\mathcal{R}_i > \delta_i \log(1+\rho^{1-\beta_\nu}h_i)\} \leq \Pr\{\mathscr{B}_i\} \leq \Pr\{\mathcal{R}_i > \delta_i \log(1+\frac{\rho^{1-\beta_\nu}h_i}{n})\}. \tag{3.12}$$

Using (3.12), the result of the theorem follows. □

3.2.2 Known interference channel

In this part, it is assumed that each receiver perfectly knows the channel to its corresponding transmitter, as well as the channel corresponding to the strongest interference.

In the following, $\mathbf{X}_0(t)$ and $\mathbf{H}_0$ represent the strongest interference signal and the corresponding interference channel for the i^{th} link, respectively. Let us define τ_i as the number of i^{th} link's strong interferes. We distinguish between two scenarios:

- One strong interferer ($\tau_i = 1$): The i^{th} link receives the dominant interference from the m^{th} transmitter. In this case, $\mathbf{X}_0(t) \triangleq \mathbf{x}_m(t)$, $\mathbf{H}_0 \triangleq \mathbf{h}_{mi}$, $\mathcal{R}_0 \triangleq \mathcal{R}_m$ and $\alpha_\nu \triangleq \alpha_{mi} = \rho^{-|m-i|\alpha_0}$.

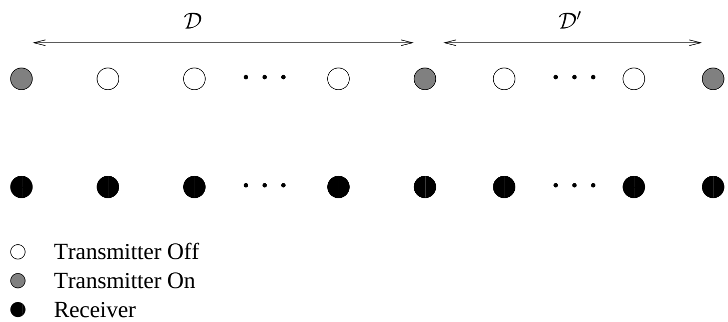

Figure 3.1: $\mathcal{D} > \mathcal{D}'$: One strong interferer; $\mathcal{D} = \mathcal{D}'$: Two strong interferers.

- Two strong interferers ($\tau_i = 2$): The i^{th} link receives two equal interference from m^{th} and l^{th} transmitters such that $m - i = i - l$. In this case, we have $\mathbf{X}_0(t) \triangleq [\mathbf{x}_l(t), \mathbf{x}_m(t)]^T$, $\mathbf{H}_0 \triangleq [\mathbf{h}_{li}, \mathbf{h}_{mi}]$, $\mathcal{R}_0 \triangleq \mathcal{R}_l + \mathcal{R}_m$, and $\alpha_\nu \triangleq \alpha_{li} = \alpha_{mi} = \rho^{-|m-i|\alpha_0}$.

Lemma 3.2. *The probability of the outage event of the* i^{th} *user is as follows:*

$$Pr\{\mathscr{B}_i\} = Pr\left\{\left[\mathcal{R}_i > \delta_i \log\left(\frac{h_i\rho}{1+\alpha_\phi\rho}\right)\right] \bigcup \left[\mathcal{R}_i + \mathcal{R}_0 > \delta_i \log\left(\frac{\alpha_\nu\rho\|\mathbf{H}_0\|^2 + h_i\rho}{1+\alpha_\phi\rho}\right)\right] \bigcap \left[\mathcal{R}_i > \delta_i \log\left(1 + \frac{\rho h_i}{1+\alpha_\nu\rho\|\mathbf{H}_0\|^2 + \alpha_\phi\rho}\right)\right]\right\}, \tag{3.13}$$

where $\alpha_\phi = \max_{j, j \notin [2i-m, m], \eta_j = 1} \alpha_{ji}$.

Proof. Noting (3.1), for all the links (except the first and last ones), we can write

$$\begin{aligned} \mathbf{y}_i(t) &= \mathbf{H}_i(t)\mathbf{x}_i(t)\eta_i(t) + \sqrt{\alpha_\nu}\mathbf{H}_0(t)\mathbf{X}_0(t) \\ &+ \sum_{j, j \notin [2i-m, m]} \sqrt{\alpha_{ji}}\mathbf{H}_{ji}(t)\mathbf{x}_j(t) + \mathbf{n}_i(t), \end{aligned} \tag{3.14}$$

Let us define $\mathscr{B}_i^L$ and $\mathscr{B}_i^U$ as follows:

$$\begin{aligned} \mathscr{B}_i^L \triangleq & \Big\{ (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0, \mathbf{V}_0)) \bigcup \\ & (\mathcal{R}_i + \mathcal{R}_0 > I(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0)) \\ & \bigcap (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0)) \Big\}, \end{aligned} \tag{3.15}$$

where $\mathbf{V}_0$ includes all the transmitted signals except $\mathbf{x}_i$ and $\mathbf{X}_0$.

$$\begin{aligned} \mathscr{B}_i^U \quad \triangleq \quad & \Big\{ (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0)) \bigcup \\ & (\mathcal{R}_i + \mathcal{R}_0 > I(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0)) \\ & \bigcap (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0)) \Big\}. \end{aligned} \tag{3.16}$$

In fact, $\mathscr{B}_i^L$ denotes the outage event for the i^{th} link, when the receiver has full access to the other users data, and $\mathscr{B}_i^U$ stands for the outage event in the i^{th} link, when the receiver treats all the users data, except the dominant interference, as noise. It is clear that

$$\Pr\{\mathscr{B}_i^L\} \leq \Pr\{\mathscr{B}_i\} \leq \Pr\{\mathscr{B}_i^U\}. \tag{3.17}$$

We compute the mutual information in (3.15) and (3.16) as follows:

$$\begin{aligned} & I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0) \\ & = \quad \Pr\{\eta_i = 1\} I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0, \eta_i = 1) \\ & \geq \quad \delta_i H(\mathbf{H}_i \mathbf{x}_i + \sqrt{\alpha_\nu} \mathbf{H}_0 \mathbf{X}_0 + \mathbf{n}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0) \\ & - \quad \delta_i \log \left(2\pi e \mathrm{Var} \left(\sum_{j, j \notin [2i-m, m]} \sqrt{\alpha_{ji}} \mathbf{H}_{ji} \mathbf{x}_j \eta_j + \mathbf{n}_i \right) \right) \\ & = \quad \delta_i \log(2\pi e(\rho h_i + 1)) \\ & - \quad \delta_i \log \left(2\pi e \left(\sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j + 1 \right) \right) \\ & = \quad \delta_i \log \left(\frac{h_i \rho + 1}{\sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j + 1} \right). \end{aligned} \tag{3.18}$$

$$\begin{aligned}
&I\left(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0\right) \\
&\geq \delta_i H(\mathbf{H}_i \mathbf{x}_i + \sqrt{\alpha_\nu} \mathbf{H}_0 \mathbf{X}_0 + \mathbf{n}_i | \mathbf{H}_i, \mathbf{H}_0) \\
&- \delta_i \log\left(2\pi e \mathrm{Var}\left(\sum_{j, j \notin [2i-m, m]} \sqrt{\alpha_{ji}} \mathbf{H}_{ji} \mathbf{x}_j \eta_j + \mathbf{n}_i\right)\right) \\
&= \delta_i \log\left(\frac{h_i \rho + \alpha_\nu \rho \|\mathbf{H}_0\|^2 + 1}{\sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j + 1}\right).
\end{aligned} \tag{3.19}$$

$$\begin{aligned}
&I\left(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0\right) \geq \\
&\delta_i \log\left(\frac{h_i \rho + \alpha_\nu \rho \|\mathbf{H}_0\|^2 + 1}{\alpha_\nu \rho \|\mathbf{H}_0\|^2 + \sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j + 1}\right).
\end{aligned} \tag{3.20}$$

$$\begin{aligned}
&I\left(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0, \mathbf{V}_0\right) \geq \\
&\delta_i \log\left(1 + \frac{\rho h_i}{1 + \sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j}\right).
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
&I\left(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0\right) \geq \\
&\delta_i \log\left(1 + \frac{\rho h_i + \alpha_\nu \rho \|\mathbf{H}_0\|^2}{1 + \sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j}\right).
\end{aligned} \tag{3.22}$$

$$\begin{aligned}
&I\left(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0\right) \geq \\
&\delta_i \log\left(1 + \frac{\rho h_i}{1 + \alpha_\nu \rho \|\mathbf{H}_0\|^2 + \sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j}\right).
\end{aligned} \tag{3.23}$$

We define $\alpha_\phi \triangleq \max_{j, j \notin [2i-m, m], \eta_j = 1} \alpha_{ji}$. As $\rho \to \infty$, we have $\sum_{j, j \notin [2i-m, m]} \alpha_{ji} \rho \eta_j \backsimeq \alpha_\phi \rho$. From the equations (3.18)-(3.23), we can see that $\Pr\{\mathscr{B}_i^L\} \backsimeq \Pr\{\mathscr{B}_i^U\}$ and the result of the lemma follows. $\square$

Lemma 3.2 determines the outage probability region. We derive the corresponding diversity gain using the same approach applied in the previous section by computing the probability of the equivalent outage events $\mathcal{A}_1$ and $\mathcal{A}_2$ in Fig. 2.2.

Theorem 3.3. *The diversity-multiplexing trade-off of the* i^{th} *link is given by*

$$d_i(r) = \begin{cases} \min(1-\beta_\phi - \frac{r}{\delta_i}, (1+\tau_i\beta_\nu - (\tau_i+1)\beta_\phi) & \\ -\frac{(\tau_i+1)r}{\delta_i}(1+\tau_i\beta_\nu)) & \mathcal{F}_1 \\ 1-\beta_\phi - \frac{r}{\delta_i}(1+\tau_i\beta_\nu) & \mathcal{F}_2 \\ 0 & \text{Otherwise} \end{cases}, \tag{3.24}$$

where $\beta_\nu \triangleq (1+\frac{\log\alpha_\nu}{\log\rho})^+$, $\beta_\phi \triangleq (1+\frac{\log\alpha_\phi}{\log\rho})^+$, $\mathcal{F}_1 \equiv r < \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+\tau_i\beta_\nu}$, *and* $\mathcal{F}_2 \equiv \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+\tau_i\beta_\nu} < r < \frac{\delta_i(1-\beta_\phi)}{1+\tau_i\beta_\nu}$.

Proof. Let us define

$$\begin{aligned} h_{11} &\triangleq \frac{1+\alpha_\phi\rho}{\rho}(e^{\frac{\mathcal{R}_i}{\delta_i}}-1), \\ h_{01} &\triangleq \frac{1+\alpha_\phi\rho}{\alpha_\nu\rho}(e^{\frac{\mathcal{R}_0}{\delta_i}}-1+e^{-\frac{\mathcal{R}_i}{\delta_i}}), \\ h_{12} &\triangleq \frac{1+\alpha_\phi\rho}{\rho}(e^{\frac{\mathcal{R}_i+\mathcal{R}_0}{\delta_i}}-e^{\frac{\mathcal{R}_0}{\delta_i}}+e^{-\frac{\mathcal{R}_i}{\delta_i}}-1), \\ h_{02} &\triangleq \frac{1+\alpha_\phi\rho}{\alpha_\nu\rho}(e^{\frac{\mathcal{R}_i+\mathcal{R}_0}{\delta_i}}-e^{\frac{\mathcal{R}_i}{\delta_i}}+1), \\ m_1 &\triangleq \frac{h_{01}}{h_{12}-h_{11}}, \\ m_2 &\triangleq \frac{h_{02}-h_{01}}{h_{12}-h_{11}}. \end{aligned} \tag{3.25}$$

Considering the result of Lemma 3.2 and noting Fig. 2.2, the outage probability in (3.13) can be written as

$$\begin{aligned} \Pr\{\mathscr{B}_i\} &= \Pr\{\mathcal{A}_1\} + \Pr\{\mathcal{A}_2\} \\ &= 1 - e^{-h_{11}} \\ &+ \int_{h_{11}}^{h_{12}} \int_{m_1(h_1-h_{11})}^{h_{02}+m_2(h_{11}-h_1)} f(h_1)f(h_0)\mathrm{d}h_0\mathrm{d}h_1, \end{aligned} \tag{3.26}$$

where $h_0 \triangleq \|\mathbf{H}_0\|^2$. We consider the following two scenarios:

- *Single strong interferer*: In this case, $f(h_0) = e^{-h_0}$ and $f(h_1) = e^{-h_1}$. The results in (3.24) is obtained following the approach of the proof of Theorem 2.1 in Section 2.2.1. Noting (3.25), for the case $r > \frac{\delta_i(\beta_\nu - \beta_\phi)}{1+\beta_\nu}$, the approximation in (2.16) is true for $\Pr\{\mathcal{A}_2\}$. Substituting (3.25) in (2.16), we have

$$\begin{aligned} \Pr\{\mathcal{A}_2\} &\backsimeq e^{-h_{11}}(h_{12} - h_{11}) - \frac{1}{m_2 - 1} \\ &\doteq \rho^{\frac{r}{\delta_i}(1+\beta_\nu)+\beta_\phi-1}. \end{aligned} \tag{3.27}$$

For the case that $r < \frac{\delta_i(\beta_\nu - \beta_\phi)}{1+\beta_\nu}$, substituting (3.25) in (2.18), we have

$$\begin{aligned} \Pr\{\mathcal{A}_2\} &\backsimeq h_{02}(h_{12} - h_{11}) \\ &\doteq \rho^{\frac{2r}{\delta_i}(1+\beta_\nu)-1-\beta_\nu+2\beta_\phi}. \end{aligned} \tag{3.28}$$

- *Two strong interferers*: In this case, $f(h_0) = h_0 e^{-h_0}$ and $f(h_1) = e^{-h_1}$. In (3.26), defining $\mathscr{I} \triangleq \Pr\{A_2\}$, we upper-bound and lower-bound $\mathscr{I}$ as follows:

$$\begin{aligned} \mathscr{I} \leq \mathscr{I}^U &\triangleq \int_{h_{11}}^{h_{12}} \int_0^{h_{02}} h_0 e^{-h_0} e^{-h_1} \mathrm{d}h_0 \mathrm{d}h_1 \\ &= \left(e^{-h_{11}} - e^{-h_{12}}\right)\left(1 - (1 + h_{02})e^{-h_{02}}\right), \end{aligned} \tag{3.29}$$

$$\begin{aligned} \mathscr{I} \geq \mathscr{I}^L &\triangleq \int_{h_{11}}^{h_{12}} [e^{-m_1(h_1 - h_{11})} \\ &\quad - (h_{02}+1)e^{-(h_{02}+m_2(h_{11}-h_1))}]e^{-h_1}\mathrm{d}h_1 \\ &= e^{-h_{11}}[\frac{1}{m_1+1}\left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) \\ &\quad - e^{-h_{02}}\frac{(h_{02}+1)}{m_2 - 1}\left(e^{(m_2-1)(h_{12}-h_{11})} - 1\right)]. \end{aligned} \tag{3.30}$$

For $r < \frac{\delta_i(\beta_\nu - \beta_\phi)}{1+2\beta_\nu}$, the first and the second terms in (3.29) can be approximated by $(h_{12} - h_{11})$ and h_{02}^2, respectively. For $r > \frac{\delta_i(\beta_\nu - \beta_\phi)}{1+2\beta_\nu}$, (3.29) can be approximated by

$h_{12} - h_{11}$. It can be easily shown that the same result is true for the lower-bound. Therefore, $\Pr\{\mathcal{A}_2\}$ can be summarized as follows:

$$\Pr\{\mathcal{A}_2\} \doteq \begin{cases} \rho^{\frac{3r}{\delta_i}(1+2\beta_\nu)+3\beta_\phi-1-2\beta_\nu} & r < \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+2\beta_\nu} \\ \rho^{\frac{r}{\delta_i}(1+2\beta_\nu)+\beta_\phi-1} & \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+2\beta_\nu} < r < \frac{\delta_i(1-\beta_\phi)}{1+\tau_i\beta_\nu} \end{cases} \tag{3.31}$$

Using (3.26), (3.27), (3.28) and (3.31), we obtain the diversity gain in (3.24).

□

3.3 Network Design

In the previous part, we have derived the maximum diversity gain for the i^{th} link, conditioned on having a fixed δ_i and η. In this part, we obtain the optimum values for $\delta = (\delta_1, \delta_2, \cdots, \delta_n)$ and η, based on α_0 and r. Let us consider the following special cases:

- $\alpha_0 > 1$: In this case, it is easy to see that the interference from all the links are negligible with respect to the noise. Therefore, we can consider this case as a parallel non-interfering ad-hoc Network, where the optimal values of η and δ are equal to $\mathbf{1}$ and $\mathbf{1}$, respectively. The maximum diversity gain of the network can be obtained as

$$d^*(r) = 1 - r, \quad 0 \leq r < 1. \tag{3.32}$$

- $\alpha_0 = 0$: In this case, the attenuation of all interference channels is 1. Assuming that the interference channel is not available at the receiver nodes, the diversity gain is zero (noting the result of Theorem 3.1). Assuming that all the receiver nodes know all their corresponding channels (direct channel and interference channels), similar to (3.33), the outage probability for the i^{th} link can be written as

$$\begin{aligned}\Pr\{\mathscr{B}_i\} = \Pr\Big\{ & [r\log\rho > \delta_i \log(1+h_i\rho)] \bigcup \\ & \left[n' r\log\rho > \delta_i \log(1+\rho\|\mathbf{H}_0\|^2)\right] \bigcap \\ & \left[r\log\rho > \delta_i \log\left(1+\frac{\rho h_i}{\rho\|\mathbf{H}_0\|^2+1}\right)\right]\Big\},\end{aligned} \tag{3.33}$$

where $\mathbf{H}_0 \triangleq [\mathbf{h}_{\pi_1 i}, \cdots, \mathbf{h}_{\pi_{n'} i}]$, π_j's are active links and n' denotes the number of active links in the network. Due to the symmetry between the links, we have $\delta_i = \delta$, $\forall i$. As a consequence, $n' = n\delta$. Following (3.26), (3.29), and (3.30), and noting that $f(h_0) = \frac{h_0^{n'-1}e^{-h_0}}{(n'-1)!}$, we have

$$\begin{aligned} d_i(r) &= \begin{cases} \min\left(1-\frac{r}{\delta}, n'(1-\frac{n'r}{\delta})\right) & r < \frac{\delta}{n'} \\ 0 & \text{Otherwise} \end{cases} \\ &= \begin{cases} \min\left(1-\frac{r}{\delta}, n\delta(1-nr)\right) & r < \frac{1}{n} \\ 0 & \text{Otherwise} \end{cases} \end{aligned} \tag{3.34}$$

The value of δ which maximizes the diversity gain in (3.34) is 1. Hence,

$$d^*(r) = \begin{cases} \min\left(1-r, n(1-nr)\right) & r < \frac{1}{n} \\ 0 & \text{Otherwise} \end{cases} \tag{3.35}$$

For $0 < \alpha_0 < 1$, we make the following observations:

- Since all the links are in the same situation (we ignore the edge effect), as a result of the symmetry, we have $\delta_i = \delta$, $\forall i$. This suggests that we only need to derive the diversity-multiplexing trade-off for one link.

- We can categorize the links into *clusters*, where each cluster consists of some neighboring links, which are active simultaneously. Because of the symmetry in the network, all the clusters have the same number of links.

- As observed in Theorems 3.1 and 3.3, the diversity gain of one link depends only on the corresponding dominant interferers received power. Considering (3.6), we note that the diversity gain of a cluster is the minimum of the diversity gains of the links in that cluster. In the unknown-interference scenario, following Theorem 3.1, the diversity gain corresponding to a cluster with two or more links is the same as the diversity gain if all links are active simultaneously. In the known-interference scenario, noting Theorem 3.3, the diversity gain corresponding to a cluster with more than 3 links is the same as the diversity if all the links are active. The number of the links in each cluster must be one in unknown-interference scenario and less than or equal to 3 in known-interference scenario, since otherwise diversity gain is upper-bounded by that of the all-active case and time-sharing is not necessary.

- Let us define the distance between two clusters as

$$\mathscr{D}(\mathcal{C}_1, \mathcal{C}_2) \triangleq \min_{l_1 \in \mathcal{C}_1, l_2 \in \mathcal{C}_2} \mathscr{D}(l_1, l_2), \tag{3.36}$$

  where l_1 and l_2 are two links in the clusters $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively, and $\mathscr{D}(l_1, l_2)$ denotes the distance between these links (normalized in terms of the distance between two neighboring links). For $\mathscr{D}(\mathcal{C}_i, \mathcal{C}_j) > \lceil \frac{1}{\alpha_0} \rceil$, the interference from neighboring clusters is negligible with respect to the noise and increasing the distance between the clusters while incurs that less portion of the time is allocated to the clusters, it does not effect the diversity gain. Therefore, we consider

$$\mathscr{D}(\mathcal{C}_i, \mathcal{C}_j) \leq \lceil \frac{1}{\alpha_0} \rceil, \tag{3.37}$$

  for any active neighboring clusters $\mathcal{C}_i$ and $\mathcal{C}_j$.

From all the above observations, it follows that the diversity gain is a function of k, the number of links in a cluster, and s, the distance between two neighboring clusters.

We denote the diversity gain of the specified cluster by $d^{(k,s)}(r)$.

3.3.1 Unknown interference channel

Following Theorem 3.1, we have

$$d^{(1,s)}(r) = 1 - (1 - s\alpha_0)^+ - rs, \tag{3.38}$$

where $1 \leq s \leq \lceil \frac{1}{\alpha_0} \rceil$. The network diversity is optimized on the distance between clusters as follows:

$$d^*(r) = \max_s d^{(1,s)}(r). \tag{3.39}$$

Noting (3.38) and (3.39), we can write

$$d^*(r) = \begin{cases} 1 - (s^* + 1)r & r \leq 1 - s^*\alpha_0 \\ s^*(\alpha_0 - r) & 1 - s^*\alpha_0 \leq r \leq \alpha_0 \\ 0 & \text{Otherwise} \end{cases} \tag{3.40}$$

where $s^* = \lfloor \frac{1}{\alpha_0} \rfloor$. The diversity-multiplexing trade-off of the network is depicted in Fig. 3.3.1 for $\alpha_0 = 0.32$. It is compared with the diversity-multiplexing trade-off of two scenarios; i) all the links are active simultaneously (all-active), ii) the active links do not see interference from each other (orthogonal-transmission). It can be observed that orthogonal-transmission scheme is optimum for low multiplexing gains while all-active scheme is closer to optimum for high multiplexing gains.

3.3.2 Known interference channel

Defining $\mathfrak{F}_\tau(\frac{r}{\delta}, \beta_\nu, \beta_\phi)$ as the diversity gain in (3.24), we interpret $d^{(k,s)}(r)$ in terms of $\mathfrak{F}$ as follows:

$$d^{(1,s)}(r) = \mathfrak{F}_2(rs, (1 - s\alpha_0)^+, (1 - 2s\alpha_0)^+). \tag{3.41}$$

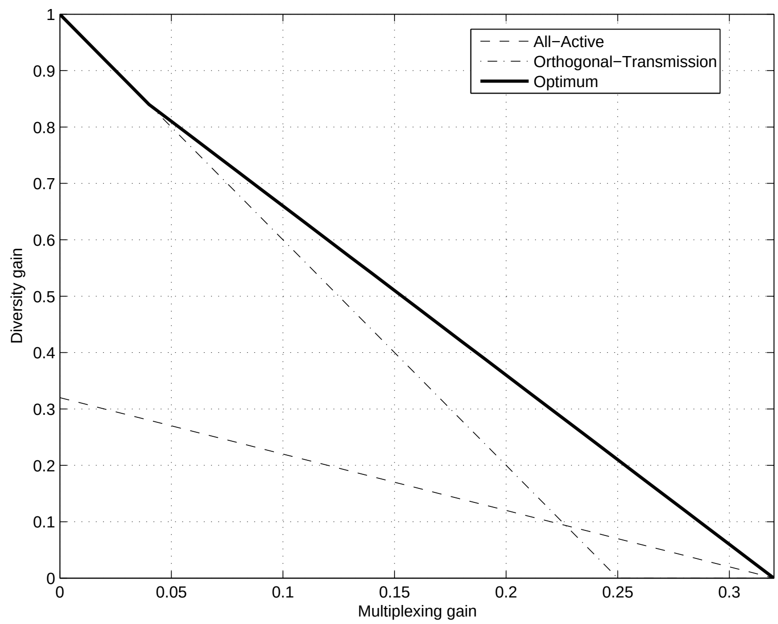

Figure 3.2: The diversity gain of a one-dimensional network vs. multiplexing gain for $\alpha_0 = 0.32$.

$$d^{(2,s)}(r) = \mathfrak{F}_1(\frac{r(s+1)}{2}, (1-\alpha_0)^+, (1-s\alpha_0)^+). \tag{3.42}$$

$$d^{(3,s)}(r) = \min(d'(r), d''(r)), \tag{3.43}$$

where

$$d'(r) = \mathfrak{F}_1(\frac{r(s+2)}{3}, (1-\alpha_0)^+, (1-s\alpha_0)^+), \tag{3.44}$$

and

$$d''(r) = \mathfrak{F}_2(\frac{r(s+2)}{3}, (1-\alpha_0)^+, (1-(s+1)\alpha_0)^+). \tag{3.45}$$

The maximum diversity gain can be obtained as

$$d^*(r) = \max_{k,s} d^{(k,s)}(r). \tag{3.46}$$

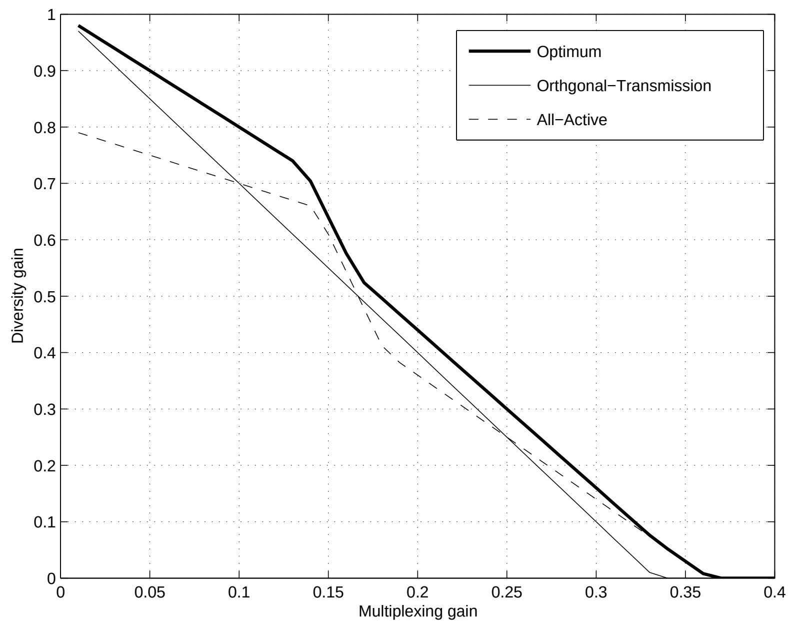

Figure 3.3: The diversity gain of a one-dimensional network vs. multiplexing gain for $\alpha_0 = .4$.

The diversity-multiplexing trade-off of the network is depicted in Fig. 3.3.2 for $\alpha_0 = 0.4$ and is compared with all-active and orthogonal-transmission scenarios. Fig. 3.3.2 clearly shows that the optimum scheme depends on the rate of transmission, e.g. for low multiplexing gains, the performance of orthogonal-transmission scheme is close to optimum while for high multiplexing gains, the performance of all-active scheme is optimum.

3.4 Conclusion

This chapter introduces a measure for optimally allocating bandwidth among users considering network's infrastructure density in wireless networks. The diversity-multiplexing trade-off curve is characterized for one-dimensional equally-spaced Rayleigh fading network. The results show that we can significantly improve the performance if we carefully select the TDMA scheme based on the desired data rate and network density.

Chapter 4

Time-varying Single-User Channel with Partial CSI

Communications theory over time-varying channels has been widely studied from different perspectives regarding the availability of the channel state information (CSI) at the transmitter and/or the receiver. Communication with perfect CSI at the transmitter is studied by Shannon in [13], where the capacity is expressed as that of an equivalent memoryless channel without side information at either the transmitter or the receiver. Communication with perfect CSI at the receiver is investigated, for example, in [26]. With the assumption of perfect CSI at both the transmitter and the receiver, the capacity of finite state Markov channels (FSMCs) and compound channels is studied in [9] and [55], respectively.

In practice, the assumption of perfect CSI is not practical due to estimation inaccuracy, limited feedback channel capacity, or feedback delay. Communication with imperfect side information is well investigated in the literature [6,18,32,33,47]. In [18], the capacity of FSMCs is evaluated based on the assumed statistical relationship of the channel state and side information at the transmitter. The channel capacity, when feedback delay is taken into account, is studied in [22, 53]. The optimal transmission and feedback strategies with finite feedback alphabet cardinality is investigated in [54].

In this chapter, we consider a point-to-point time-varying channel with perfectly known CSI at the receiver. It is assumed that the channel is constant during a channel use and varies from one channel use to the next, based on a Markov random process. The CSI is provided at the transmitter through a noiseless feedback link at regularly-spaced time intervals. Every T channel use, the CSI of the current channel use is fed back to the transmitter. We obtain the channel capacity of the system and show that it is achievable by multiplexing T codebooks across the channel. It is worth mentioning that for FSMCs, the results of [18] apply directly to compute the channel capacity, if the CSI at the transmitter and receiver are jointly stationary. However, in our model, the CSI at the transmitter is not stationary. Moreover, we utilize the introduced feedback model to obtain the capacity of additive white Gaussian noise (AWGN) channel with time-correlated fading. It is shown that the capacity is achievable using a single codebook with adaptively allocating power based on the side information at the transmitter. Also, the optimum power allocation is derived.

The rest of the chapter is organized as follows; In Section 4.1, the system model is described and the channel capacity is obtained. The capacity of time-correlated fading channel with periodic feedback is derived in Section 4.2. The impact of channel correlation and feedback error on the capacity is evaluated in Section 4.3. Finally, the chapter is concluded in Section 4.4.

4.1 Markov Channel With Feedback State

We consider a channel with discrete input $X_n \in \mathcal{X}$ and discrete output $Y_n \in \mathcal{Y}$ at time instant n. The channel state is characterized as a finite-state first order Markov process:

$$\Pr(u_n|u_1^{n-k}) = \Pr(u_n|u_{n-k}). \tag{4.1}$$

The channel output at time n is assumed to depend only on the channel input and state at time n, i.e. $\Pr(y_n|x_1^N, u_1^N) = \Pr(y_n|x_n, u_n)$. Hence, the block transition probability of the channel is

$$\Pr(y_1^N|x_1^N, u_1^N) = \prod_{n=1}^{N} \Pr(y_n|x_n, u_n), \tag{4.2}$$

which implies that the channel is memoryless given the state process $U_n \in \mathcal{U}$. It is assumed that CSI is perfectly known at the receiver. The CSI is provided at the transmitter through a noiseless feedback link periodically at every T symbols, i.e., $U_1, U_{T+1}, U_{2T+1}, \cdots$ are sent over the feedback link and instantly received at the transmitter. Assume that the codeword length, N, is an integer factor of T and $M \triangleq \frac{N}{T}$. Let us define $V_i \triangleq U_{T(i-1)+1}$ for $1 \leq i \leq M$ and $\tilde{n} \triangleq \lfloor \frac{n}{T} \rfloor + 1$.

Encoding and Decoding

Assume that $W \in \mathcal{W}$ is the message to be sent by the transmitter and $A_w = 2^{NR}$ is the cardinality of $\mathcal{W}$. A codeword of length N is a sequence of the encoding function φ_n which maps the set of messages to the channel input alphabets. The input codeword at time n depends on the message w and the CSI at the transmitter up to time n, i.e. $v_1^{\tilde{n}}$,

$$x_n = \varphi_n(w, v_1^{\tilde{n}}). \tag{4.3}$$

The decoding function, ϕ, maps a received sequence of N channel outputs using CSI at the receiver to the message set such that the decoded message is $\hat{w} = \phi(y_1^N, u_1^N)$.

Theorem 4.1. *The capacity of a finite state Markov channel with periodic feedback is* *given by*

$$\frac{1}{T}\sum_{t=1}^{T}\sum_{v}\Pr(v)\max_{q_t(x|v)}\sum_{u}P_t(u|v)I(X;Y|u,v), \tag{4.4}$$

where T is the feedback period, $P_t(u|v) = \Pr_{u_i|u_{i-t+1}}(u|v)$ and $q_t(x|v)$ is the random coding probability distribution function (PDF) parametrized with subscript t to reflect the dependency on time.

4.1.1 Achievability

We state a result on the capacity of FSMCs, which we then apply in the proof. It is shown that the capacity of FSMCs with perfectly known CSI, U, at the receiver and side information V at the transmitter is [18]

$$C = \sum_{v}\Pr(v)\max_{q(x|v)}\sum_{u}\Pr(u|v)I(X;Y|u,v), \tag{4.5}$$

where U and V are jointly stationary and ergodic with joint PDF $\Pr(U,V)$ and V is a deterministic function of U.

We consider the channel as T parallel subchannels where the t^{th} subchannel ($1 \leq t \leq T$) occurs in time instances $(i-1)T+t, 1 \leq i \leq M$. Noting that the channel state of the t^{th} subchannel $\{U_{(i-1)T+t}\}_{i=1}^{M}$ and the side information at the transmitter $\{V_i\}_{i=1}^{M} = \{U_{(i-1)T+1}\}_{i=1}^{M}$ are jointly stationary and ergodic, we define $P_t(u|v) = \Pr_{u_i|u_{i-t+1}}(u|v)$ for $1 \leq t \leq T$. Using (4.5), the achievable rate of the t^{th} subchannel is

$$R_t = \sum_{v}\Pr(v)\max_{q_t(x|v)}\sum_{u}P_t(u|v)I(X;Y|u,v). \tag{4.6}$$

T codebooks are designed corresponding to R_t for $1 \leq t \leq T$ and multiplexed across the T subchannels, i.e., at time instants $(i-1)T+t$ for $1 \leq i \leq M$, the channel inputs from the t^{th} codebook are sent over the channel. Therefore, the achievable rate is

$$R = \frac{1}{T}\sum_{t=1}^{T}\sum_{v}\Pr(v)\max_{q_t(x|v)}\sum_{u}P_t(u|v)I(X;Y|u,v). \tag{4.7}$$

4.1.2 Converse

In this part, we prove the converse to the capacity theorem. The proof is motivated by the proof in [18]. From the Fano's inequality [20], we have

$$H(W|Y_1^N, U_1^N) \leq P_e \log A_w + h(P_e) = N\epsilon_N, \tag{4.8}$$

where $P_e = \Pr(W \neq \hat{W})$ and $\epsilon_N \to 0$ as $N \to \infty$.

$$\begin{aligned} H(W|Y_1^N, U_1^N) &= H(W|U_1^N) - I(W; Y_1^N|U_1^N) \\ &= NR - I(W; Y_1^N|U_1^N). \end{aligned} \tag{4.9}$$

Using (4.8) and (4.9), we can write

$$R \leq \frac{1}{N} I(W; Y_1^N|U_1^N) + \epsilon_N. \tag{4.10}$$

Then we have,

$$\begin{aligned} &I(W; Y_1^N|U_1^N) \\ &= \sum_{n=1}^{N} I(W; Y_n|U_1^N, Y_1^{n-1}) \\ &= \sum_{n=1}^{N} H(Y_n|U_1^N, Y_1^{n-1}) - H(Y_n|U_1^N, Y_1^{n-1}, W) \\ &\leq \sum_{n=1}^{N} H(Y_n|U_n, V_1^{\tilde{n}}) - H(Y_n|U_1^N, Y_1^{n-1}, W) \\ &\stackrel{a}{\leq} \sum_{n=1}^{N} H(Y_n|U_n, V_1^{\tilde{n}}) - H(Y_n|U_n, X_n, V_1^{\tilde{n}}) \qquad (4.11) \\ &= \sum_{n=1}^{N} I(X_n; Y_n|U_n, V_1^{\tilde{n}}), \qquad (4.12) \end{aligned}$$

where (a) follows from the fact that the channel output is independent of the message and past channel outputs given the state of the channel and the channel input. On the other hand, for a given n, we have

$$\begin{aligned}
&I(X_n;Y_n|U_n,V_1^{\tilde{n}})\\
=&\sum_{u_n,v_1^{\tilde{n}}}\Pr(u_n|v_{\tilde{n}},v_1^{\tilde{n}-1})\Pr(v_1^{\tilde{n}-1}|v_{\tilde{n}})\Pr(v_{\tilde{n}})I(X_n;Y_n|u_n,v_1^{\tilde{n}-1},v_{\tilde{n}})\\
\overset{b}{=}&\sum_{u_n,v_{\tilde{n}}}\Pr(u_n|v_{\tilde{n}})\Pr(v_{\tilde{n}})\sum_{v_1^{\tilde{n}-1}}\Pr(v_1^{\tilde{n}-1}|v_{\tilde{n}})I(X_n;Y_n|u_n,v_1^{\tilde{n}-1},v_{\tilde{n}})\\
\overset{c}{\leq}&\sum_{u_n,v_{\tilde{n}}}\Pr(u_n|v_{\tilde{n}})\Pr(v_{\tilde{n}})\max_{q(x_n|v_{\tilde{n}})}I(X_n;Y_n|u_n,v_{\tilde{n}}),
\end{aligned}\tag{4.13}$$

where $q(x_n|v_{\tilde{n}}) \triangleq \sum_{v_1^{\tilde{n}-1}}\Pr(v_1^{\tilde{n}-1}|v_{\tilde{n}})\Pr(x_n|v_1^{\tilde{n}})$, (b) follows from the property in (4.1), and (c) results from the concavity of mutual information with respect to the input distribution. Replacing $n=(\tilde{n}-1)T+t$ in (4.13) and using (4.12), we have

$$\begin{aligned}
&I(W;Y_1^N|U_1^N)\\
\leq&\sum_{\tilde{n}=1}^{M}\sum_{t=1}^{T}\sum_{v_{\tilde{n}}}\sum_{u_{(\tilde{n}-1)T+t}}\Pr(u_{(\tilde{n}-1)T+t}|v_{\tilde{n}})\Pr(v_{\tilde{n}})\times\\
&\max_{q(x_{(\tilde{n}-1)T+t}|v_{\tilde{n}})}I(X_{(\tilde{n}-1)T+t};Y_{(\tilde{n}-1)T+t}|u_{(\tilde{n}-1)T+t},v_{\tilde{n}})
\end{aligned}\tag{4.14}$$

$$=M\sum_{t=1}^{T}\sum_{u,v}P_t(u|v)\Pr(v)\max_{q_t(x|v)}I(X;Y|u,v),\tag{4.15}$$

where (4.15) follows from the fact that $\{V_i\}_{i=1}^M$ and $\{U_{(i-1)T+t}\}_{i=1}^M$ are jointly stationary and ergodic and the right-hand side of (4.14) does not depend on $\tilde{n}$. Using (4.10) and (4.15), we have

$$R\leq\frac{1}{T}\sum_{t=1}^{T}\sum_{v}\Pr(v)\max_{q_t(x|v)}\sum_{u}P_t(u|v)I(X;Y|u,v)+\epsilon_N.\tag{4.16}$$

■

4.2 Gaussian Channel

In this section, we consider a point to point transmission over a time-correlated fading channel. It is assumed that the channel gain is constant over each channel use (symbol) and varies from symbol to symbol, following a first order Markovian random process. The signal at the receiver is

$$r_n = h_n x_n + z_n, \tag{4.17}$$

where $h_n \in \mathbb{C}$ is the fading gain and z_n is AWGN with zero mean and unit variance. It is assumed that the CSI is perfectly known to the receiver. Every T channel use, the instantaneous fading gain is sent to the transmitter through a noiseless feedback link, i.e., $|h_1|, |h_{T+1}|, \cdots, |h_{(M-1)T+1}|$ are fed back and instantly received at the transmitter.

Let us define $u_n \triangleq |h_n|^2$ for $1 \leq n \leq N$, $v_i \triangleq |h_{(i-1)T+1}|^2$ for $1 \leq i \leq M$ and $P_t(u|v) \triangleq \Pr_{u_i|u_{i-t+1}}(u|v)$. The average input power is subject to the constraint $\mathbb{E}[|x_n|^2] \leq \mathcal{P}$. In the following, $\mathbb{E}_t[g(U,V)]$ denotes the expectation value over $g(u,v)$ where U and V have joint PDF $P_t(u,v)$.

Theorem 4.2. *The capacity of time-correlated fading channel with periodic feedback is*

$$\max_{\overline{\rho}_1 \cdots \overline{\rho}_T} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_t[\log(1 + U\overline{\rho}_t(V))], \tag{4.18}$$

subject to $\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[\overline{\rho}_t(V)] \leq \mathcal{P}$, *where* T *is the feedback period.*

First, we recount some results on the capacity of single user channels, which is applied in the proof. A general formula for the capacity of single user channels which is not necessarily information stable or stationary is obtained in [50]. Consider input X and output Y as sequences of finite-dimensional distribution, where Y is induced by X via a channel which is an arbitrary sequence of finite-dimensional conditional output distribution from input alphabets to the output alphabets. The general formula for the channel capacity is as follows:

$$C = \sup_{X} \underline{I}(X;Y), \tag{4.19}$$

where $\underline{I}(X;Y)$ is defined as the liminf in probability of the normalized information density [50]

$$i_N(X_1^N;Y_1^N) = \frac{1}{N}\log\frac{\Pr(Y_1^N|X_1^N)}{\Pr(Y_1^N)}. \tag{4.20}$$

Assume that the channel state information, Q, is available at the receiver. Considering Q as an additional output, the channel capacity is $C = \sup_X \underline{I}(X;Y,Q)$. If Q is not available at the transmitter and is consequently independent of X, then the capacity is [8]

$$C = \sup_{X} \underline{I}(X;Y|Q), \tag{4.21}$$

where $\underline{I}(X;Y|Q)$ is the liminf in probability of the normalized conditional information density

$$i_N(X_1^N;Y_1^N|Q_1^N) = \frac{1}{N}\log\frac{\Pr(Y_1^N|X_1^N,Q_1^N)}{\Pr(Y_1^N|Q_1^N)}. \tag{4.22}$$

Now, we are ready to prove Theorem 2, where the proof is motivated by the proof in [18].

4.2.1 Achievability

Noting (4.17), the processed received signal at time n is

$$y_n = r_n\frac{h_n^*}{|h_n|} = |h_n|x_n + z_n', \tag{4.23}$$

where $z_n' = \frac{h_n^*}{|h_n|}z_n$, which has the same distribution as z_n. The transmitter sends

$$x_n = \sqrt{\rho_n(v_{\tilde{n}})}s_n, \tag{4.24}$$

over the channel where s_n is an i.i.d. Gaussian codebook with zero mean and unit variance, and $\rho_n : \mathbb{R}_+ \to \mathbb{R}_+$ is the power allocation function. Using (4.23) and (4.24), we can write

$$y_n = \sqrt{q_n} s_n + z'_n, \tag{4.25}$$

where $q_n = \rho_n(v_{\tilde{n}})|h_n|^2 = \rho_n(v_{\tilde{n}})u_n$. Noting (4.25), we have a channel with input S and output Y and channel state Q, which is known at the receiver. Since Q_1^N is independent of S_1^N, we can use (4.21) to obtain the achievable rate.

$$\begin{aligned} i_N(S_1^N; Y_1^N | Q_1^N) &= \frac{1}{N} \log \frac{\Pr(Y_1^N | S_1^N, Q_1^N)}{\Pr(Y_1^N | Q_1^N)} \\ &\overset{d}{=} \frac{1}{N} \sum_{n=1}^{N} \log \frac{\Pr(Y_n | S_n, Q_n)}{\Pr(Y_n | Q_n)} \\ &= \frac{1}{N} \sum_{n=1}^{N} \left(\log(1 + Q_n) + \frac{|Y_n|^2}{1 + Q_n} - |Z'_n|^2 \right), \end{aligned} \tag{4.26}$$

where (d) results from the fact that S_1^N and ${Z'}_1^N$ are i.i.d. sequences and the last line follows from the fact that Y_n conditioned on Q_n is Gaussian with zero mean and variance $1 + Q_n$. Note that as $N \to \infty$, $\frac{1}{N} \sum_{n=1}^{N} \frac{|Y_n|^2}{1+Q_n} = \frac{1}{N} \sum_{n=1}^{N} |Z'_n|^2 = 1$ with probability one. Therefore, with probability one, we have

$$\begin{aligned} & i_N(S_1^N; Y_1^N | Q_1^N) \\ &= \frac{1}{N} \sum_{n=1}^{N} \log(1 + Q_n) \\ &= \frac{1}{MT} \sum_{t=1}^{T} \sum_{i=1}^{M} \log(1 + Q_{(i-1)T+t}) \\ &= \frac{1}{T} \sum_{t=1}^{T} \frac{1}{M} \sum_{i=1}^{M} \log\left(1 + U_{(i-1)T+t} \rho_{(i-1)T+t}(V_i)\right). \end{aligned} \tag{4.27}$$

Noting that $\{U_{(i-1)T+t}\}_{i=1}^M$ and $\{V_i\}_{i=1}^M$ are jointly stationary and ergodic for $1 \le t \le T$, we define $P_t(u, v)$ to be their joint PDF. We set $\rho_{(i-1)T+t} = \overline{\rho}_t$ for $1 \le i \le M$ and $1 \le t \le T$. As $M \to \infty$ in (4.27), the sample mean converges in probability to the expectation. Therefore, the achievable rate is

$$R = \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}_t[\log(1+U\overline{\rho}_t(V))]. \tag{4.28}$$

4.2.2 Converse

Using (4.11), we have

$$\begin{aligned} I(W;Y_1^N|U_1^N) &\leq \sum_{n=1}^{N} H(Y_n|U_n,V_1^{\tilde{n}}) - H(Y_n|U_n,X_n,V_1^{\tilde{n}}) \\ &\leq \sum_{n=1}^{N}\mathbb{E}[\log(1+U_n\mathbb{E}[|X_n|^2|V_1^{\tilde{n}}])]. \end{aligned} \tag{4.29}$$

The above inequality relies on the facts that

$$H(Y_n|U_n,X_n,V_1^{\tilde{n}}) = H(Z_n) = \log 2\pi e \tag{4.30}$$

and

$$H(Y_n|U_n,V_1^{\tilde{n}}) \leq \mathbb{E}[\log(2\pi e(1+U_n\mathbb{E}[|X_n|^2|V_1^{\tilde{n}}]))]. \tag{4.31}$$

The upper-bound in (4.31) is achieved if X_n conditioned on $V_1^{\tilde{n}}$ has a Gaussian distribution. We set $x_n = \sqrt{f_n(v_1^{\tilde{n}})}s_n$ where $f_n : \mathbb{R}_+^{\tilde{n}} \to \mathbb{R}_+$ and S_1^N is an i.i.d. Gaussian sequence with zero mean and unit variance. On the other hand,

$$\begin{aligned} \mathbb{E}[\log(1+U_nf_n(V_1^{\tilde{n}}))] &= \mathbb{E}[\mathbb{E}[\log(1+U_nf_n(V_1^{\tilde{n}}))|U_n,V_{\tilde{n}}]] \\ &\overset{d}{\leq} \mathbb{E}[\log(1+\mathbb{E}[U_nf_n(V_1^{\tilde{n}})|U_n,V_{\tilde{n}}])] \\ &= \mathbb{E}[\log(1+U_n\mathbb{E}[f_n(V_1^{\tilde{n}})|V_{\tilde{n}}])], \end{aligned} \tag{4.32}$$

where (d) follows from the concavity of the logarithm. Let us define $\rho_n(V_{\tilde{n}}) \triangleq \mathbb{E}[f_n(V_1^{\tilde{n}}))|V_{\tilde{n}}]$. By using (4.29) and (4.32), we obtain

$$\begin{aligned}
&\frac{1}{N} I(W; Y_1^N | U_1^N) \\
&\leq \frac{1}{N} \sum_{n=1}^{N} \mathbb{E}[\log(1 + U_n \rho_n(V_{\tilde{n}}))] \\
&= \frac{1}{T} \sum_{t=1}^{T} \frac{1}{M} \sum_{i=1}^{M} \mathbb{E}[\log(1 + U_{(i-1)T+t} \rho_{(i-1)T+t}(V_i))] \\
&\leq \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\log(1 + \frac{1}{M} \sum_{i=1}^{M} U_{(i-1)T+t} \rho_{(i-1)T+t}(V_i))]. \qquad (4.33)
\end{aligned}$$

Using (4.33) and noting the fact that that $\{U_{(i-1)T+t}\}_{i=1}^M$ and $\{V_i\}_{i=1}^M$ are jointly stationary and ergodic for $1 \leq t \leq T$, we can write

$$\frac{1}{N} I(W; Y_1^N | U_1^N) \leq \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_t[\log(1 + U \overline{\rho}_t(V))], \qquad (4.34)$$

where $\overline{\rho}_t(.) \triangleq \frac{1}{M} \sum_{i=1}^{M} \rho_{(i-1)T+t}(.)$. Combining (4.10) and (4.34), we conclude that

$$R \leq \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_t[\log(1 + U \overline{\rho}_t(V))], \qquad (4.35)$$

subject to $\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\overline{\rho}_t(V)] \leq \mathcal{P}$.

$\blacksquare$

Remark: In Section 4.1, we proved that the capacity of Markov channels is generally achieved by using multiple code multiplexing technique. However, for AWGN channel with time-correlated fading, the proof relies on using one Gaussian codebook, where the symbols are adaptively scaled by the appropriate power allocation function based on the side information at the transmitter.

4.3 Performance Evaluation

We study the impact of the channel correlation and feedback period on the capacity of the time-correlated Rayleigh fading channel. Let us assume that time-correlated Rayleigh fading channel is a Markov random process with the following PDF [11]:

$$\Pr(u) = \begin{cases} e^{-u} & u \geq 0 \\ 0 & \text{Otherwise} \end{cases} . \tag{4.36}$$

$$\begin{aligned} P_1(u|v) &= \delta(v), \\ P_t(u|v) &= \Phi(u, v, \alpha^{t-1}), \end{aligned} \tag{4.37}$$

where

$$\Phi(u,v,\sigma) = \begin{cases} \frac{1}{1-\sigma^2} \exp\left(-\frac{u+\sigma^2 v}{1-\sigma^2}\right) \mathcal{I}_0\left(\frac{2\sigma\sqrt{uv}}{1-\sigma^2}\right) & u \geq 0, \\ 0 & \text{Otherwise.} \end{cases} \tag{4.38}$$

In (4.38), $0 < \sigma < 1$ describes the channel correlation coefficient and $\mathcal{I}_0(.)$ denotes the modified Bessel function of order zero. Noting that the capacity in (4.18) is a strictly concave region of $\overline{\rho}_t, 1 \leq t \leq T$, we numerically solve the convex optimization problem.

In Figure 4.1, the capacity is depicted versus the feedback period for various channel correlation coefficients and compared to the capacity when no CSI is available at the transmitter.

4.4 Conclusion

We have obtained the capacity of finite state Markov channel with periodic feedback at the transmitter. Also, the channel capacity and optimal adaptive coding is derived

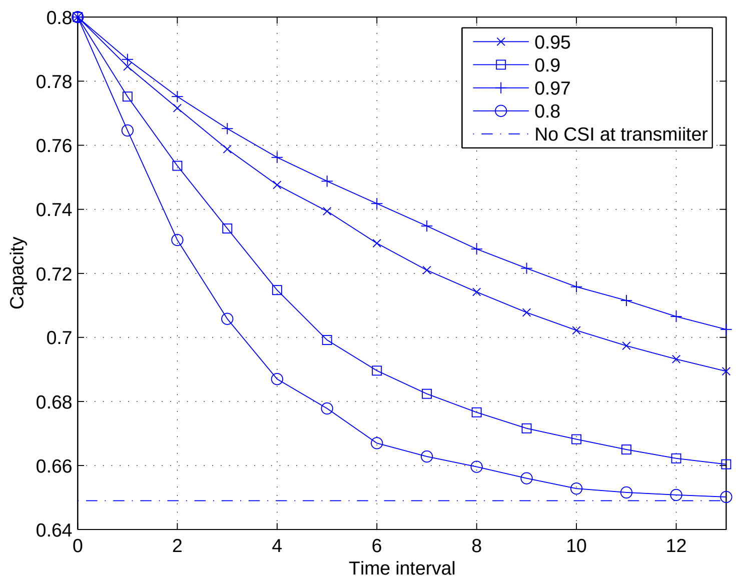

Figure 4.1: Capacity of time-correlated Rayleigh fading channel versus T for $\mathrm{SNR} = 1$ and channel correlation coefficients $\alpha = 0.97, 0.95, 0.9, 0.8$. The dash-dot line is the capacity with no side information at the transmitter.

for the time-correlated fading channel with periodic feedback. It is shown that the optimal adaptation can be achieved by a single Gaussian codebook, while scaling by the appropriate power.

Chapter 5

Time-varying Multi-User Channel with Partial CSI

The time-varying fading inherent to the wireless link is one of the main challenges in designing wireless communication systems. Adaptive signaling schemes which use knowledge of the current channel fading values to optimize the transmitted signal, can significantly improve the performance of communications systems operating over fading channels. Utilizing optimal dynamic power and rate allocation strategies, the ergodic capacities with CSI at both the transmitter and the receiver of a single-user fading channel, a fading multiple access channel, and a fading broadcast channel are obtained in [1], [2] and [27], respectively.

In practice, the assumption of perfect CSI is not feasible due to estimation inaccuracy, limited feedback channel capacity, or feedback delay. Communication with partial CSI in time-varying point-to-point channel is well investigated in the literature [6,18,32,33,47]. However, there are few results on the performance of time-varying broadcast channels with partial CSI at the transmitter. The capacity region for a certain class of fading Gaussian broadcast channels when the receivers have the perfect

CSI while the transmitter has no CSI is derived in [15]. In [42], the ergodic capacity region for a fading broadcast channel is obtained assuming the transmitter knows only the channel ordering information of the users. In this work, applying adaptive signaling, we study the asymptotic throughput of a time-varying broadcast channel with partial CSI at the transmitter assuming large number of users in the system.

We confine the signaling scheme to opportunistic scheduling which employs *multiuser diversity*. In a broadcast channel where users have independent fading and feed back their SNR to the base station (BS), system throughput is maximized by transmitting to the user with the strongest SNR [14, 41]. Multiuser diversity underlies much of the recent works for downlink scheduling [7, 39, 43, 46, 56]. Multiuser diversity has also been studied in the context of multiple antenna systems [38] and ad-hoc networks [31].

We consider a broadcast channel in which a BS transmits data to a large number of users in a time-correlated flat fading environment. It is assumed that CSI is perfectly known to the receivers, while the BS only knows the statistical characteristics of the fading process for all users (which is assumed to be constant during a long period). Moreover, each user feeds back its channel gain to the BS at the beginning of each frame (initial fading gain). Based on this information, the BS selects the user with maximum initial fading gain for transmission in each frame. The BS adapts the rate and/or the codeword length of the selected user based on the available information.

We analyze different adaptive signaling schemes; i) the BS adapts the rate of the selected user to its initial fading gain, ii) the BS adapts the rate based on both the initial fading gain and statistical characteristics of the fading process of the selected user, and iii) the BS adapts the rate and codeword length based on the initial fading gain and statistical characteristics of the fading process of the selected user. We characterize the asymptotic throughput of the system applying the underlying adaptive techniques assuming large number of users.

The rest of the chapter is organized as follows; In Section 5.1, the system model is described. Section 5.2 is devoted to study of the error exponent of time-correlated fading channel in the downlink system described in Section 5.1. In Section 5.3, asymptotic throughput of various adaptive signaling strategies are derived for large number of users. Finally, in Section 5.4, we conclude the chapter.

Throughout this chapter, $\mathbb{E}\{.\}$ and $\text{var}\{.\}$ represents the expectation and variance, respectively, "log" is used for the natural logarithm, and rate is expressed in *nats*. For given functions $f(N)$ and $g(N)$, $f(N) = O(g(N))$ is equivalent to $\lim_{N\to\infty} \left|\frac{f(N)}{g(N)}\right| < \infty$, $f(N) = o(g(N))$ is equivalent to $\lim_{N\to\infty} \left|\frac{f(N)}{g(N)}\right| = 0$, $f(N) = \omega(g(N))$ is equivalent to $\lim_{N\to\infty} \frac{f(N)}{g(N)} = \infty$, and $f(N) = \Omega(g(N))$ is equivalent to $\lim_{N\to\infty} \frac{f(N)}{g(N)} = c$, where $0 < c < \infty$.

5.1 System model

The channel of any given user is modeled as a time-correlated fading process. It is assumed that the channel gain is constant over each channel use (symbol) and varies from symbol to symbol, following a Markovian random process. Assume that the fading gain of k^{th} user is $\boldsymbol{h}_k = [h_{1,k}, \ldots, h_{N_k,k}]^T$ where $h_{i,k}, 1 \leq i \leq N_k$ are complex Gaussian random variables with zero mean and unit variance and N_k is the codeword length of the k^{th} user. The received signal for the k^{th} user is given by

$$\boldsymbol{r}_k = \boldsymbol{X}_k \boldsymbol{h}_k + \boldsymbol{n}_k, \tag{5.1}$$

where $\boldsymbol{X}_k = \text{diag}(x_{1,k}, x_{2,k}, \ldots, x_{N_k,k})$ is the transmitted codeword with the power constraint[1] $\mathbb{E}\{|x_{i,k}|^2\} \leq P$, and $\boldsymbol{n}_k$ is AWGN with zero mean and covariance matrix $\boldsymbol{I}$. Assume that $h_{0,k}$ is the fading gain at the time instant before $\boldsymbol{X}_k$ is transmitted. The

[1]Obviously, for maximizing the throughput, the power constraint translates to $\mathbb{E}\{|x_{i,k}|^2\} = P$.

sequence $u_{i,k} = |h_{i,k}|$, $0 \leq i \leq N_k$, is assumed to be a stationary ergodic chain with the following probability density function [11]:

$$f_{u_{0,k}}(u) = \begin{cases} 2ue^{-u^2}, & u \geq 0 \\ 0. & \text{otherwise} \end{cases} \tag{5.2}$$

$$f(u_{1,k}, u_{2,k}, \cdots, u_{N_k,k}|u_{0,k}) = \prod_{i=1}^{N_k} q_k(u_{i,k}|u_{i-1,k}), \tag{5.3}$$

where,

$$q_k(u|v) = \begin{cases} \frac{2u}{1-\alpha_k^2} \exp\left(-\frac{u^2+\alpha_k^2 v^2}{1-\alpha_k^2}\right) \mathcal{I}_0(\frac{2\alpha_k uv}{1-\alpha_k^2}) & u \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

in which $0 < \alpha_k < 1$ describes the channel correlation coefficient of the k^{th} user and $\mathcal{I}_0(.)$ denotes the modified Bessel function of order zero. It is assumed that CSI is perfectly known at each receiver, while $u_{0,k}$, $1 \leq k \leq K$ are known at the transmitter. It is assumed that α_k, $1 \leq k \leq K$, are i.i.d. random variables which remain fixed during the entire transmission. We assume that $\alpha_{\min} < \alpha_k < \alpha_{\max}$ where $0 < \alpha_{\min} < \alpha_{\max} < 1$, i.e. we exclude the i.i.d. fast fading and quasi-static fading model. To obtain the result in a closed form, we assume that α_k has a uniform distribution. Generalizing the results for other distributions is straightforward.

5.2 Time-Correlated Fading Channel's Throughput

We use average throughput per channel use as our measure of performance to compare the performance of various adaptation schemes. Average throughput per channel use is frequently applied to study the performance of communication systems, e.g. [12, 48]. First, we evaluate the error exponent of the channel model described in previous section and derive the corresponding achievable throughput. Using the concept of random coding error exponent [20], the frame error probability, p_e, can be upper-bounded as

$$p_e \leq \inf_{0 \leq \rho \leq 1} e^{-N(E(\rho)-\rho R)}, \tag{5.4}$$

where R is the transmitted rate per channel use and $E(\rho)$ is the corresponding error exponent. This bound is tight for rates close to the capacity as used in [3, 5, 17]. We define the k^{th} user's average throughput per channel use, denoted by T_k, as

$$T_k \triangleq R_k(1 - \inf_{0 \leq \rho \leq 1} e^{-N(E_k(\rho)-\rho R_k)}). \tag{5.5}$$

Assuming $s_{i,k}, 1 \leq i \leq N_k$, are Gaussian and i.i.d., it is shown that the random coding error exponent for the k^{th} user, $E_k(\rho)$, is given by [3],

$$E_k(\rho) = -\frac{1}{N_k} \log \mathbb{E}_{\boldsymbol{u}_k} \left\{ \prod_{i=1}^{N_k} \left(\frac{1}{1 + \frac{P}{1+\rho} u_{i,k}^2} \right)^{\rho} \right\}, \tag{5.6}$$

where $\boldsymbol{u}_k = [u_{1,k}, \ldots, u_{N_k,k}]$.

In the following, we assume that $u_{0,k} \gg 1$. Since in schemes introduced in this work, a user is selected if the corresponding initial fading gain is maximum, this assumption is valid when the number of users is large.

Theorem 5.1. *For the channel model described in the previous section, and assuming $u_{0,k}$ is known, we have*

$$E_k(\rho) = \frac{1}{N_k} \sum_{i=1}^{N_k} \rho \log \left(1 + \frac{P u_{0,k}^2 \alpha_k^{2i}}{(1+\rho)} \right) + O\left(\frac{1}{\sqrt{u_{0,k}}} \right) - O\left(e^{-u_{0,k}^2} \right). \tag{5.7}$$

Proof. See Appendix A. □

Noting (5.7), we have

$$\begin{aligned}
E_k(\rho) - \rho R_k &= \frac{1}{N_k}\sum_{i=1}^{N_k}\rho\log\left(1+\frac{Pu_{0,k}^2\alpha_k^{2i}}{(1+\rho)}\right) - \rho R_k + O\left(\frac{1}{\sqrt{u_{0,k}}}\right) - O\left(e^{-u_{0,k}^2}\right) \\
&= \frac{1}{N_k}\sum_{i=1}^{N_k}\rho\log\left(\frac{Pu_{0,k}^2\alpha_k^{2i}}{(1+\rho)}\right) - \rho R_k + \frac{1}{N_k}\sum_{i=1}^{N_k}\rho\log\left(1+\frac{(1+\rho)}{Pu_{0,k}^2\alpha_k^{2i}}\right) \\
&+ O\left(\frac{1}{\sqrt{u_{0,k}}}\right) - O\left(e^{-u_{0,k}^2}\right) \\
&= \frac{\rho}{N_k}\sum_{i=1}^{N_k}\left(\log\left(Pu_{0,k}^2\right) + 2i\log(\alpha_k) - \log(1+\rho)\right) - \rho R_k \\
&+ \frac{1}{N_k}\sum_{i=1}^{N_k} O\left(\frac{1}{u_{0,k}^2}\right) + O\left(\frac{1}{\sqrt{u_{0,k}}}\right) - O\left(e^{-u_{0,k}^2}\right) \\
&= \rho[\log(Pu_{0,k}^2) + (N_k+1)\log(\alpha_k) - \log(\rho+1) - R_k] \\
&+ O\left(\frac{1}{\sqrt{u_{0,k}}}\right) - O\left(e^{-u_{0,k}^2}\right). \qquad (5.8)
\end{aligned}$$

It is easy to show that ρ_k^{opt} which minimizes the bound in (5.4) or equivalently maximizes (5.8) for large values of $u_{0,k}$ is

$$\begin{aligned}
\log(1+\rho_k^{\text{opt}}) + \tfrac{\rho_k^{\text{opt}}}{1+\rho_k^{\text{opt}}} = \beta_k, \quad \beta_k < \log(2) + \tfrac{1}{2} \\
\rho_k^{\text{opt}} = 1, \quad \beta_k \geq \log(2) + \tfrac{1}{2}
\end{aligned} \qquad (5.9)$$

where

$$\beta_k = \log(Pu_{0,k}^2) + (N_k+1)\log(\alpha_k) - R_k. \qquad (5.10)$$

Using (5.4), (5.5), (5.7) and (5.8), we have

$$\begin{aligned}
T_k &= R_k\left[1 - e^{-N_k\left(E_k(\rho_k^{\text{opt}}) - \rho_k^{\text{opt}} R_k\right)}\right] \\
&= R_k\left[1 - e^{-\rho_k^{\text{opt}} N_k\left(\log(Pu_{0,k}^2) + (N_k+1)\log(\alpha_k) - \log(\rho_k^{\text{opt}}+1) - R_k\right)}\right]. \qquad (5.11)
\end{aligned}$$

5.3 Adaptive Signaling Schemes: Performance Analysis

In this part, we study the asymptotic throughput of various adaptive signaling schemes in a time-correlated fading broadcast channel assuming large number of users. In these schemes, the BS transmits to the user with the maximum initial fading gain applying code rate and/or codeword length adaptation. The adaptation is performed based on the initial fading gain and/or channel correlation coefficient of the selected user. For simplicity of notation, let us define υ as the initial fading gain of the selected user, i.e. $\upsilon \triangleq \max_{1\leq k\leq K} u_{0,k}^2$ and α as the fading correlation coefficient of the corresponding user.

5.3.1 Adaptive Code Rate

In this scheme, the rate of the selected user is adapted to its initial fading gain. The codeword length is assumed to be fixed for all users at all time, i.e., $N_1 = N_2 = \cdots = N_K = N$, where N is selected such that the system throughput is maximized.

Theorem 5.2. *The asymptotic throughput of the system scales as*

$$\overline{\mathcal{T}}_1 \sim \log\left(\frac{P\log K}{2}\right) - 2\sqrt{\log(\alpha_{\min}^{-1})\log\log\left(P\log K\right)}, \tag{5.12}$$

as $K \to \infty$.

Proof. We derive the lower-bound for the throughput of the selected user by setting $\rho^{\text{opt}} = 1$. Using (5.11), we can write[2]

$$\mathcal{T}_1(\upsilon, \alpha) \quad \geq \quad R\left[1 - e^{-N\left(\log\left(\frac{P\upsilon}{2}\right) + (N+1)\log(\alpha) - R\right)}\right]. \tag{5.13}$$

[2]We drop user index of parameters R and ρ^{opt} for the selected user.

By averaging over α, we have

$$\begin{aligned}
\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)\} &\geq R\left[1 - e^{-N\left(\log\left(\frac{P\upsilon}{2}\right)-R\right)}\frac{\alpha_{\min}^{-N(N+1)+1} - \alpha_{\max}^{-N(N+1)+1}}{(N(N+1)-1)(\alpha_{\max}-\alpha_{\min})}\right] \\
&\geq R\left[1 - \frac{e^{-N\left(\log\left(\frac{P\upsilon}{2}\right)+(N+1)\log(\alpha_{\min})-R\right)}}{(N(N+1)-1)\varphi}\right], \qquad (5.14)
\end{aligned}$$

where $\varphi = \frac{\alpha_{\max}-\alpha_{\min}}{\alpha_{\min}}$. The lower-bound can be tightened by maximizing the right-hand side in (5.14) over N. Consequently, we have

$$\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)\} \geq \log\left(\frac{P\upsilon}{2}\right) - 2\sqrt{\log(\alpha_{\min}^{-1})\log\log\left(P\upsilon\right)}. \qquad (5.15)$$

Noting that $\upsilon \sim \log K + O(\log\log K)$, with probability one [34], we obtain the following lower-bound on the achievable throughput of the system.

$$\overline{\mathcal{T}}_1 \gtrsim \log\left(\frac{P\log K}{2}\right) - 2\sqrt{\log(\alpha_{\min}^{-1})\log\log\left(P\log K\right)}. \qquad (5.16)$$

To derive the upper-bound, we re-write the averaged throughput as follows:

$$\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)\} = \mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)|\mathcal{B}\}P(\mathcal{B}) + \mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)|\mathcal{B}^c\}P(\mathcal{B}^c). \qquad (5.17)$$

where $\mathcal{B} \equiv \{\rho_k^{\text{opt}} = 1\}$. We characterize the region of $\mathcal{B}$ over $[\alpha_{\min}, \alpha_{\max}]$ by formulating ρ_k^{opt} as follows:

$$\begin{aligned}
\log(1+\rho^{\text{opt}}) + \tfrac{\rho^{\text{opt}}}{1+\rho^{\text{opt}}} = \beta, &\quad \alpha_{\min} < \alpha < \alpha_0 \\
\rho^{\text{opt}} = 1, &\quad \alpha_0 < \alpha < \alpha_{\max}
\end{aligned} \qquad (5.18)$$

where α_0 can be optimized to maximize the throughput of the selected user.

$$\begin{aligned}
\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)|\mathcal{B}^c\} &= \mathbb{E}_{\alpha}\{R(1 - e^{-\frac{N(\rho^{\text{opt}})^2}{1+\rho^{\text{opt}}}})\} \\
&\leq R(1 - e^{-\mathbb{E}_{\alpha}\{\frac{(\rho^{\text{opt}})^2}{1+\rho^{\text{opt}}}\}N}) \\
&\leq R(1 - e^{-\frac{N}{2}}) \\
&\overset{(a)}{\leq} R(1 - e^{-\frac{\log(P\upsilon)-R}{2\log(\alpha_0^{-1})}}), \qquad (5.19)
\end{aligned}$$

where (a) follows from the fact that $\beta_k \geq 0$ in (5.10) and therefore $N \leq \frac{\log(P\upsilon)-R}{\log(\alpha^{-1})}$. Maximizing the right-hand side of (5.19) over R, for large values of υ, we have

$$\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)|\mathcal{B}^c\} \leq \log(P\upsilon) - 2\log(\alpha_0^{-1})\log\log(P\upsilon). \tag{5.20}$$

On the other hand, we have

$$\begin{aligned}\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)|\mathcal{B}\} &= \mathbb{E}_{\alpha}\left\{R\left[1-e^{-N\left(\log\left(\frac{P\upsilon}{2}\right)+(N+1)\log(\alpha)-R\right)}\right]\right\}\\ &\leq \log\left(\frac{P\upsilon}{2}\right) - 2\sqrt{\log(\alpha_{\min}^{-1})\log\log\left(P\upsilon\right)}.\end{aligned} \tag{5.21}$$

Noting (5.17), (5.20) and (5.21)

$$\mathbb{E}_{\alpha}\{\mathcal{T}_1(\upsilon,\alpha)\} \lesssim \log\left(\frac{P\upsilon}{2}\right) - 2\sqrt{\log(\alpha_{\min}^{-1})\log\log\left(P\upsilon\right)}. \tag{5.22}$$

$$\overline{\mathcal{T}}_1 = \mathbb{E}\{\mathcal{T}_1(\upsilon,\alpha)\} \lesssim \log\left(\frac{P\log K}{2}\right) - 2\sqrt{\log(\alpha_{\min}^{-1})\log\log\left(P\log K\right)}. \tag{5.23}$$

Noting (5.16) and (5.23), the result of the theorem is obtained. □

5.3.2 Rate Adaptation based on Correlation Coefficient

The BS adapts the data rate of the selected user with respected to both the initial fading and fading correlation coefficient of its channel. The codeword length of all users is fixed and is selected such that the throughput of the system is maximized.

Theorem 5.3. *The asymptotic throughput of the system scales as*

$$\overline{\mathcal{T}}_2 \sim \log\left(\frac{P\log K}{2}\right) - 2\sqrt{\mathbb{E}\{\log(\alpha^{-1})\}\log\log(P\log K)}, \tag{5.24}$$

as $K \to \infty$.

Setting the derivative of (5.11) with respect to R to zero, we find the rate of the selected user and the corresponding throughput in terms of υ and α as follows:

$$R = \log\left(\frac{P\upsilon}{1+\rho^{\text{opt}}}\right) + (N+1)\log(\alpha) - \frac{\log(1+\rho^{\text{opt}}NR)}{\rho^{\text{opt}}N}, \tag{5.25}$$

$$\begin{aligned}\mathcal{T}_2(\upsilon,\alpha) &= \left[\log\left(\frac{P\upsilon}{1+\rho^{\text{opt}}}\right) + (N+1)\log(\alpha) - \frac{\log(1+\rho^{\text{opt}}NR)}{\rho^{\text{opt}}N}\right] \times \\ &\quad \left[1 - \frac{1}{1+\rho^{\text{opt}}NR}\right],\end{aligned} \tag{5.26}$$

where noting (5.9), ρ^{opt} is determined as follows:

$$\begin{aligned}\log(1+\rho^{\text{opt}}) + \tfrac{\rho^{\text{opt}}}{1+\rho^{\text{opt}}} = \beta, &\quad \beta < \log(2) + \tfrac{1}{2} \\ \rho^{\text{opt}} = 1, &\quad \beta \geq \log(2) + \tfrac{1}{2}\end{aligned} \tag{5.27}$$

Using (5.10) and (5.25), we have

$$\beta = \log(1+\rho^{\text{opt}}) + \frac{\log(1+\rho^{\text{opt}}NR)}{\rho^{\text{opt}}N}. \tag{5.28}$$

Let us define R^* and event $\mathcal{A}$ as follows:

$$\begin{aligned}R^* &\triangleq \log(P\upsilon) + (N+1)\log(\alpha) \\ &\overset{(5.25),(5.28)}{=} R + \beta,\end{aligned} \tag{5.29}$$

$$\mathcal{A} \equiv \{R^* > \frac{1}{2}\log(P\log K)\}. \tag{5.30}$$

In the following, we derive upper-bounds for the throughput of the system in terms of R^* and $\mathcal{A}$ which we use later in Lemma 5.4 and Lemma 5.5.

$$\begin{aligned}\overline{\mathcal{T}}_2 &= \mathbb{E}\{\mathcal{T}_2(\upsilon,\alpha)\} \\ &\overset{(5.25),(5.26)}{\leq} \mathbb{E}\{R\} \\ &= \mathbb{E}\{R|\mathcal{A}\}\Pr\{\mathcal{A}\} + \mathbb{E}\{R|\mathcal{A}^C\}\Pr\{\mathcal{A}^C\} \\ &\overset{(5.29)}{=} \left(\mathbb{E}\{R^*|\mathcal{A}\} - \mathbb{E}\{\beta|\mathcal{A}\}\right)\Pr\{\mathcal{A}\} + \mathbb{E}\{R|\mathcal{A}^C\}\Pr\{\mathcal{A}^C\} \\ &\leq \mathbb{E}\{R^*\} - \mathbb{E}\{\beta|\mathcal{A}\}\Pr\{\mathcal{A}\} \qquad (5.31) \\ &\leq \mathbb{E}\{R^*\}, \qquad (5.32)\end{aligned}$$

where (5.31) is derived by replacing R with R^*, noting $R \leq R^*$, and $\Pr\{\mathcal{A}\}$ can be computed as follows:

$$\begin{aligned}
\Pr\{\mathcal{A}\} &= \Pr\{\log(P\upsilon) + (N+1)\log\alpha > \frac{1}{2}\log(P\log K)\} \\
&= 1 - \Pr\{\log(P\upsilon) + (N+1)\log\alpha \leq \frac{1}{2}\log(P\log K)\} \\
&= 1 - \int_0^\infty \Pr\left\{\log\alpha < \frac{1}{N+1}\left(\frac{1}{2}\log(P\log K) - \log(Px)\right)\Big| x\right\} f_\upsilon(x)dx \\
&= 1 - \int_0^\infty F_\alpha\left(e^{\frac{1}{N+1}\left(\frac{1}{2}\log(P\log K) - \log(Px)\right)}\right) f_\upsilon(x)dx, \qquad (5.33)
\end{aligned}$$

where $f_y(.)$ and $F_y(.)$ are probability density function and cumulative density function of random variable y, respectively. Noting that α has a uniform distribution, we have

$$\begin{aligned}
\Pr\{\mathcal{A}\} &= 1 - e^{\frac{\log\log K}{2(N+1)}} \int_0^\infty e^{-\frac{\log(Px)}{N+1}} f_\upsilon(x)dx \\
&\simeq 1 - \max\{0, \frac{e^{-\frac{\log(P\log K)}{2(N+1)}} - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}}\} \\
&= \min\{1, 1 + \frac{\alpha_{\min} - e^{-\frac{\log(P\log K)}{2(N+1)}}}{\alpha_{\max} - \alpha_{\min}}\}, \qquad (5.34)
\end{aligned}$$

where the second line follows from the fact that $\upsilon \sim \log K + O(\log\log K)$, with probability one [34].

According to (5.27), there are two regions for ρ^{opt} of the selected user. To obtain the throughput of the system, we upper-bound the throughput in these two regions in Lemma 5.4 and Lemma 5.5, respectively. Then, we derive a lower-bound for the throughput of the system in Lemma 5.6.

Lemma 5.4. *Assuming $\beta < \log(2) + \frac{1}{2}$, the throughput is upper-bounded as follows:*

$$\overline{\mathcal{T}}_{21} \lesssim \log(P\log K) - (\log(\log(P\log K) - 2\log(2)))\mathbb{E}\{\log(\alpha^{-1})\}Pr\{\mathcal{A}\}. \qquad (5.35)$$

Proof. Using (5.27) and (5.28) and noting $\beta < \log(2) + \frac{1}{2}$, we obtain

$$(\rho^{\text{opt}})^{-1} + (\rho^{\text{opt}})^{-2} = \frac{N}{\log(1 + \rho^{\text{opt}}NR)}. \qquad (5.36)$$

Noting $\rho^{\mathrm{opt}} < 1$, it follows from (5.36) that

$$N > \log(1 + \rho^{\mathrm{opt}} N R). \tag{5.37}$$

Assuming R is large enough, from (5.36), we have

$$\frac{\rho^{\mathrm{opt}} N R}{\log(1 + \rho^{\mathrm{opt}} N R)} > 2R \Rightarrow \rho^{\mathrm{opt}} N > 2. \tag{5.38}$$

Using (5.37) and (5.38), we can write

$$\begin{aligned} N &\overset{(5.37)}{>} \mathbb{E}\left\{\log(1 + \rho^{\mathrm{opt}} N R) | \mathcal{A}\right\} \Pr\{\mathcal{A}\} \\ &\overset{(5.38)}{>} \mathbb{E}\{(\log(1 + 2R)) | \mathcal{A}\} \Pr(\mathcal{A}) \\ &\overset{a}{>} (\log(\log(P \log K) - 2\log(2))) \Pr\{\mathcal{A}\}, \end{aligned} \tag{5.39}$$

where (a) results from the fact that conditioned on $\mathcal{A}$, we have $R = R^* - \beta > \frac{1}{2}\log(P \log K) - \frac{1}{2} - \log(2)$. Noting that $\upsilon \sim \log K + O(\log\log K)$, we can write the throughput of the system as follows:

$$\begin{aligned} \overline{\mathcal{T}}_{21} &\overset{(5.29),(5.32)}{\leq} \mathbb{E}\{\log(P\upsilon) + (N+1)\log(\alpha)\} \\ &= \mathbb{E}\{\log(P\upsilon)\} - (N+1)\mathbb{E}\{\log(\alpha^{-1})\} \\ &\overset{(5.39)}{\lesssim} \log(P \log K) - (\log(\log(P \log K) - 2\log(2)))\mathbb{E}\{\log \alpha^{-1}\}\Pr\{\mathcal{A}\} \end{aligned} \tag{5.40}$$

□

Lemma 5.5. *Assuming $\beta \geq \log(2) + \frac{1}{2}$, the throughput is upper-bounded as follows:*

$$\overline{\mathcal{T}}_{22} \lesssim \log\left(\frac{P \log K}{2}\right) - 2\sqrt{\mathbb{E}\{\log \alpha^{-1}\}}\sqrt{\log\log(P \log K)}. \tag{5.41}$$

Proof. Noting $\beta \geq \log(2) + \frac{1}{2}$, from (5.27), we have $\rho^{\text{opt}} = 1$. Hence, using (5.29) and (5.31) and noting $\upsilon \sim \log K$, we can write

$$\begin{aligned}
\overline{\mathcal{T}}_{22} &\leq \log\left(P \log K\right) - (N+1)\mathbb{E}\{\log(\alpha^{-1})\} - \mathbb{E}\{\beta|\mathcal{A}\}\Pr\{\mathcal{A}\} \\
&\overset{(5.28)}{\lesssim} \log\left(P \log K\right) - (N+1)\mathbb{E}\{\log(\alpha^{-1})\} - \mathbb{E}\{\log(2) + \frac{\log(1+NR)}{N}|\mathcal{A}\}\Pr\{\mathcal{A}\} \\
&\overset{a}{\lesssim} \log\left(P \log K\right) - (N+1)\mathbb{E}\{\log(\alpha^{-1})\} \\
&\quad - \left[\frac{\log(\frac{1}{2}\log(P\log K) - \frac{\log(1+N\log(P\log K))}{N} - \log(2))}{N} + \frac{\log N}{N} + \log(2)\right]\Pr\{\mathcal{A}\},
\end{aligned} \tag{5.42}$$

where (a) follows from the fact that conditioned on $\mathcal{A}$, we have

$$\begin{aligned}
R &= R^* - \beta \\
&> \frac{1}{2}\log(P\log K) - \frac{\log(1+NR)}{N} - \log(2) \\
&> \frac{1}{2}\log(P\log K) - \frac{\log(1+N\log(P\log K))}{N} - \log(2).
\end{aligned} \tag{5.43}$$

The last line results from the fact that $R < \log(P\log K)$ which follows from (5.25). Substituting (5.34) in (5.42), and setting the derivative of $\overline{\mathcal{T}}_{22}$ to zero with respect to N, we obtain

$$N^{\text{opt}} \sim \sqrt{\frac{\log\log(P\log K)}{\mathbb{E}\{\log\alpha^{-1}\}}}[1+o(1)]. \tag{5.44}$$

Substituting (5.34) and (5.44) in (5.42), the result of the lemma follows. □

Lemma 5.6. *The throughput can be lower-bounded as follows:*

$$\overline{\mathcal{T}}_2 \gtrsim \log\left(\frac{P\log K}{2}\right) - 2\sqrt{\mathbb{E}\{\log\alpha^{-1}\}\log\log(P\log K)}. \tag{5.45}$$

Proof. The throughput can be lower-bounded by setting $\rho^{\text{opt}} = 1$ for all values of υ and α in (5.26). Using (5.25) and (5.26), we can write

$$\overline{\mathcal{T}}_2 \;\geq\; \mathbb{E}\{R - \frac{R}{1+NR}\}, \tag{5.46}$$

where

$$R \;=\; \log\left(\frac{P\upsilon}{2}\right) + (N+1)\log(\alpha) - \frac{\log(1+NR)}{N}. \tag{5.47}$$

Averaging the right-hand side of (5.46) and setting $N = \sqrt{\frac{\log\log(P\log K)}{\mathbb{E}\{\log\alpha^{-1}\}}}$, the result of the lemma is obtained. □

The Proof of Theorem 5.3: Lemma (5.4) and Lemma (5.5) provide upper-bounds on two complementary cases where ρ^{opt} of the selected user is either less than 1 or equal to 1 in (5.35) and (5.41), respectively. Lemma (5.6) lower-bounds the throughput of the system as in (5.45). Comparing (5.35), (5.41) and (5.45), we conclude the result of the theorem.

■

5.3.3 Adaptive Code Rate and Adaptive Codeword Length

In this scheme, both the code rate and codeword length of the selected user are adapted to maximize the corresponding throughput. First, we determine the corresponding code rate and codeword length. We re-write the throughput of k^{th} user from (5.11):

$$T_k \;=\; R_k\left[1 - e^{-\rho_k^{\text{opt}} N_k\left(\log(Pu_{0,k}^2)+(N_k+1)\log(\alpha_k)-\log(\rho_k^{\text{opt}}+1)-R_k\right)}\right]. \tag{5.48}$$

It is easy to show that T_k is a concave function of variables R_k and N_k, and the values of R_k and N_k which maximize the throughput (R_k^{opt} and N_k^{opt}) satisfy the following equations:

$$R_k^{\text{opt}} = \log(Pu_{0,k}^2) + (2N_k^{\text{opt}}+1)\log(\alpha_k) - \log(\rho_k^{\text{opt}}+1), \tag{5.49}$$

$$N_k^{\text{opt}} = \sqrt{\frac{\log\left(1+\rho_k^{\text{opt}} N_k^{\text{opt}} R_k^{\text{opt}}\right)}{\rho_k^{\text{opt}}\log(\alpha_k^{-1})}}. \tag{5.50}$$

It follows that $N_k^{\text{opt}} \to \infty$ and $R_k^{\text{opt}} \to \infty$ as $u_{0,k} \to \infty$. Using (5.49) and (5.50), (5.48) can be re-written as follows:

$$T_k = \left(\log\left(\frac{Pu_{0,k}^2}{\rho_k^{\text{opt}}+1}\right) + (2N_k^{\text{opt}}+1)\log(\alpha_k)\right)\left(1 - \frac{1}{1+\rho_k^{\text{opt}} N_k^{\text{opt}} R_k^{\text{opt}}}\right). \quad (5.51)$$

Substituting (5.49) in (5.10), we have

$$\beta_k = N_k^{\text{opt}} \log(\alpha_k^{-1}) + \log(\rho_k^{\text{opt}}+1). \quad (5.52)$$

From (5.9) and (5.52), it is concluded that

$$\rho_k^{\text{opt}} = \begin{cases} \frac{N_k^{\text{opt}}\log(\alpha_k^{-1})}{1-N_k^{\text{opt}}\log(\alpha_k^{-1})} & N_k^{\text{opt}}\log(\alpha_k^{-1}) < \frac{1}{2} \\ 1 & N_k^{\text{opt}}\log(\alpha_k^{-1}) \geq \frac{1}{2} \end{cases} \quad (5.53)$$

Note that since α_k is fixed and $\alpha_k < 1$, then $N_k^{\text{opt}}\log(\alpha_k^{-1}) \geq \frac{1}{2}$ for large values of $u_{0,k}$ with probability one. In the case $\rho_k^{\text{opt}} = 1$, the asymptotic throughput is obtained by substituting (5.49) and (5.50) in (5.51) as follows:

$$\begin{aligned} T_k &= \log\left(\frac{Pu_{0,k}^2}{2}\right) - 2\sqrt{\log(\alpha_k^{-1})\log\log\left(Pu_{0,k}^2\right)} \times \\ &\quad \left(1 + O\left(\frac{\log\log\log(u_{0,k})}{\log\log(u_{0,k})}\right)\right). \end{aligned} \quad (5.54)$$

From (5.50), the optimum codeword length scales as follows:

$$N_k^{\text{opt}} \sim \sqrt{\frac{\log\log(Pu_{0,k}^2)}{\log\alpha_k^{-1}}}. \quad (5.55)$$

Theorem 5.7. *Assuming $K \to \infty$, the asymptotic throughput of the system scales as follows:*

$$\overline{\mathcal{T}}_3 \sim \log\left(\frac{P\log K}{2}\right) - 2\mathbb{E}\{\sqrt{\log(\alpha^{-1})}\}\sqrt{\log\log\left(P\log K\right)}. \quad (5.56)$$

Proof. Noting that the case $\rho^{\text{opt}} = 1$ happens with probability one and using (5.54), we can write

$$\overline{\mathcal{T}}_2 = \mathbb{E}\left\{\log\left(\frac{P\upsilon}{2}\right) - 2\sqrt{\log(\alpha^{-1})\log\log\left(\frac{P\upsilon}{2}\right)}\left(1 + O\left(\frac{\log\log\log(\upsilon)}{\log\log(\upsilon)}\right)\right)\right\} \quad (5.57)$$

Noting that $\upsilon \sim \log K + O(\log \log K)$ with probability one, and υ and α are independent, we obtain the result of the theorem. $\square$

Remark 1- Although $\lim_{K\to\infty} \frac{\overline{\mathcal{T}}_1}{\overline{\mathcal{T}}_{\max}} = \lim_{K\to\infty} \frac{\overline{\mathcal{T}}_2}{\overline{\mathcal{T}}_{\max}} = \lim_{K\to\infty} \frac{\overline{\mathcal{T}}_3}{\overline{\mathcal{T}}_{\max}} = 1$, where $\overline{\mathcal{T}}_{\max} \sim \log\left(P \log K\right)$ is the maximum achievable throughput for a quasi-static fading channel [34], there exists a gap of $\Omega(\sqrt{\log \log (P \log K)})$ between the achievable throughput of these schemes and the maximum throughput.

Remark 2- Comparing the performance of the underlying schemes, in general we have $\overline{\mathcal{T}}_1 \leq \overline{\mathcal{T}}_2 \leq \overline{\mathcal{T}}_3$. As an example, for the case of $\alpha_{\min} = .2$ and $\alpha_{\max} = .95$, we have

$$\overline{\mathcal{T}}_1 \sim \log\left(\frac{P \log K}{2}\right) - 2.5\sqrt{\log \log (P \log K)}. \tag{5.58}$$

$$\overline{\mathcal{T}}_2 \sim \log\left(\frac{P \log K}{2}\right) - 1.6\sqrt{\log \log (P \log K)}. \tag{5.59}$$

$$\overline{\mathcal{T}}_3 \sim \log\left(\frac{P \log K}{2}\right) - 1.5\sqrt{\log \log (P \log K)}. \tag{5.60}$$

5.4 Conclusion

A multiuser downlink communication over a time-correlated fading channel has been considered. We have studied the asymptotic throughput of various adaptive signaling schemes. Assuming a large number of users in the system, we show that using the scheduling that transmits to the user with maximum SNR with fixed codeword length optimized over fading correlation coefficient of the users achieves the order of maximum throughput. Although the system performance improves by using both adaptive code rate and adaptive codeword length at the price of extra complexity, it is shown that the order of the gap between the maximum throughput and the achievable throughput remains the same.

Chapter 6

Summary and Future Work

In the first part of thesis, we introduced the diversity-multiplexing trade-off as a measure for optimally allocating bandwidth among users considering network density. The diversity-multiplexing trade-off is characterized for Z-channel and multiple-access channel assuming different scenarios for the SNR of the transmitters. Moreover, we assessed the performance of a one-dimensional regular wireless network under a general TDMA scheme. This work can be extended in both theoretical and practical aspects. The channel model and network model can be changed to fit the practical requirements. The assumption of regularity in the network (the transmitter and receivers are equally spaced) can be relaxed. A random distribution model can be considered for users locations.

We have obtained the diversity-multiplexing trade-off of Z-channel which is a special case of two-user interference channel. Although the capacity region of two-user interference channel is still unknown, there are some results that show the outer-bound for the interference channel is tight in high SNR regime [45]. In continuation of this work, the diversity-multiplexing trade-off analysis can be generalized for two-user interference channel by defining the cross-channel coefficients (interference channels) as the multiplication of Rayleigh fading and attenuation factor which is an exponential function of SNR.

Multiple-antenna systems with two well-known benefits, i.e. diversity gain and multiplexing gain, play a significant role in rate-reliability trade-off in a MIMO wireless network. The problem of bandwidth allocation in a wireless system in which transmitters and receivers are equipped with multiple antennas is substantially important in developing next generation communications systems. In this respect, the problem can be approached through diversity-multiplexing trade-off analysis of MIMO wireless networks.

In the second part of the thesis, the performance of time-varying channels with partial CSI at the transmitter is investigated. First, we obtained the capacity of finite state Markov channel with periodic feedback at the transmitter and we designed a codebook that achieves the capacity. The channel capacity and optimal adaptive coding are also derived for the time-correlated fading channel with periodic feedback. It is shown that the optimal adaptation can be achieved by a single Gaussian codebook, while scaling by the appropriate power. In this part, We have assumed that the CSI is known perfectly at the receiver. Moreover, the feedback link has unlimited capacity and consequently, the CSI is received perfectly at the transmitter in regular time instants. To complete this work, we can consider the channel estimation error at the receiver in our model. Also, we can assume a limited capacity for the feedback link. By assuming power constraint for sending the CSI through feedback link and limited feedback capacity, there exists a trade-off between the frequency and the accuracy of transferring the CSI. The more frequently the CSI is fed back, the less precisely the side information is received at the transmitter. On the other hand, long period feedback results in outdated CSI and degrades the performance. Therefore, there is an optimal feedback period that maximizes the capacity.

In Chapter 5, a multiuser downlink communication over a time-correlated fading channel has been considered. We have studied the asymptotic throughput of various adaptive signaling schemes. Assuming a large number of users in the system, we have shown that using the scheduling that transmits to the user with maximum SNR with fixed codeword length optimized over fading correlation coefficient of the users achieves the order of maximum throughput. Although the system performance improves by using both adaptive code rate and adaptive codeword length at the price of extra complexity, it is shown that the order of the gap between the maximum throughput and the achievable throughput remains the same. In this part, we analyze the performance of the broadcast channel when users have different channel statistics. A very basic problem can be defined as follows; Consider a Rayleigh fading broadcast channel with two users. Assume that the channels corresponding to the first user and the second user are slowly fading and fast fading, respectively. In this scenario, channel coding design for sum-rate maximization is an interesting problem.

Appendix A

For simplicity, we drop the user index. Noting (5.6), we have $E_0(\rho) = -\frac{1}{N}\log I_N$, where

$$I_N = \int_{u_N} \dots \int_{u_1} \prod_{i=1}^{N} \left(\frac{1}{1 + \frac{P}{1+\rho} u_i^2} \right)^{\rho} p(\mathbf{u}|u_0) du_i. \tag{A.1}$$

Using (5.3), we have

$$I_N = \int_{u_N} \dots \int_{u_1} \prod_{i=1}^{N} \frac{2u_i}{1-\alpha^2} \exp\left\{ -\frac{u_i^2 + \alpha^2 u_{i-1}^2}{1-\alpha^2} \right\} \mathcal{I}_0\left(\frac{2\alpha u_i u_{i-1}}{1-\alpha^2} \right) \left(\frac{1}{1 + \frac{P}{1+\rho} u_i^2} \right)^{\rho} du_i. \tag{A.2}$$

Substituting $v_i = \frac{u_i}{u_0\sqrt{(1-\alpha^2)/2}}$, $0 \leq i \leq N$, we have

$$\begin{aligned} I_N &= \int_{v_N} \dots \int_{v_1} \prod_{i=1}^{N} u_0^2 v_i e^{-\frac{v_i^2 + \alpha^2 v_{i-1}^2}{2/u_0^2}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i \\ &= \int_{v_N} \dots \int_{v_1} \prod_{i=1}^{N} u_0^2 v_i e^{-\frac{(v_i - \alpha v_{i-1})^2}{2/u_0^2}} e^{-\alpha u_0^2 v_i v_{i-1}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i, \end{aligned} \tag{A.3}$$

where,

$$f(v_i) = \left(\frac{1}{1 + \frac{P u_0^2 (1-\alpha^2)}{2(1+\rho)} v_i^2} \right)^{\rho}. \tag{A.4}$$

For large values of u_0, we evaluate the following integral.

$$\begin{aligned} I &= u_0^2 v e^{-\frac{(v-\mu)^2}{2/u_0^2}} e^{-u_0^2 v\mu} \mathcal{I}_0(u_0^2 v \mu) \varphi(v) \\ &= \int_0^{\infty} g(v) \frac{1}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv, \end{aligned} \tag{A.5}$$

where $g(v) \triangleq \sqrt{2\pi} v u_0 \mathcal{I}_0(u_0^2 v\mu) e^{-u_0^2 v\mu}\varphi(v)$ and $\varphi(v)$ is differentiable and satisfies $0 \leq \varphi(v) \leq 1$ and $\varphi(v) \sim O(\frac{1}{u_0^\rho})$. Noting that [30]

$$\mathcal{I}_0(z)e^{-z}\sqrt{2\pi z} = 1 + O\left(\frac{1}{z}\right), \qquad z \gg 1 \tag{A.6}$$

it is easy to show that $g^{(n)}(\mu)$ is bounded for $\mu \geq 0$ and $n \geq 1$. Using Taylor series of $g(v)$ about μ, we have

$$\begin{aligned}
I &= \int_0^\infty \left(g(\mu) + \sum_{n=1}^\infty \frac{g^{(n)}(\mu)}{n!}(v-\mu)^n \right) \frac{1}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\
&= g(\mu)(1 - Q(\mu u_0)) + \int_0^\infty \sum_{n=1}^\infty \frac{g^{(n)}(\mu)}{n!}(v-\mu)^n \frac{1}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\
&= g(\mu)(1 - Q(\mu u_0)) + \int_{[\mu - \frac{1}{\sqrt{u_0}}]^+}^{\mu + \frac{1}{\sqrt{u_0}}} \sum_{n=1}^\infty \frac{g^{(n)}(\mu)}{n!}(v-\mu)^n \frac{1}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv + \varepsilon \\
&= g(\mu)(1 - Q(\mu u_0)) + O\left(\frac{g'(\mu)}{\sqrt{u_0}}\right) + \varepsilon.
\end{aligned} \tag{A.7}$$

where ε can be bounded as follows:

$$\begin{aligned}
\varepsilon &\overset{a}{\leq} \int_0^{[\mu - \frac{1}{\sqrt{u_0}}]^+} \frac{g(v)}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv + \int_{\mu + \frac{1}{\sqrt{u_0}}}^\infty \frac{g(v)}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\
&\overset{b}{\leq} \sqrt{2\pi} u_0 \int_0^{[\mu - \frac{1}{\sqrt{u_0}}]^+} \frac{v}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv + \sqrt{2\pi} u_0 \int_{\mu + \frac{1}{\sqrt{u_0}}}^\infty \frac{v}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\
&\leq 2\sqrt{2\pi} u_0 \int_{\mu + \frac{1}{\sqrt{u_0}}}^\infty \frac{v}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\
&= 2\sqrt{2\pi} u_0 \left(Q(\sqrt{u_0}) \left(\mu + \frac{1}{\sqrt{u_0}}\right) + \int_{\sqrt{u_0}}^\infty Q(z) dz \right) \\
&\overset{c}{\leq} 2\sqrt{2\pi} u_0 \left(\left(\mu + \frac{1}{\sqrt{u_0}}\right) e^{-\frac{u_0}{2}} + \int_{\sqrt{u_0}}^\infty e^{-\frac{z^2}{2}} dz \right) \\
&\overset{d}{\leq} 2\sqrt{2\pi} e^{-\frac{u_0}{2}} u_0 \left(\left(\mu + \frac{1}{\sqrt{u_0}}\right) + \sqrt{2\pi} \right) \\
&\leq O\left(u_0 e^{-\frac{u_0}{2}}\right)
\end{aligned} \tag{A.8}$$

where (a) results from the fact that $g(\mu) \geq 0$, (b) is valid because $\mathcal{I}_0(\mu z)e^{-\mu z} \leq 1$ for $\mu \geq 0$ and $z \geq 0$, and (c) and (d) follow from the fact that $Q(z) \triangleq \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-t^2/2} dt \leq e^{-z^2/2}$. Moreover, using (A.6), we can write

$$\begin{aligned} g(\mu) &= \varphi(\mu)\sqrt{2\pi}\mu u_0 \mathcal{I}_0(u_0^2\mu^2)e^{-u_0^2\mu^2} \\ &= \varphi(\mu)\left(1 + O\left(\frac{1}{u_0^2}\right)\right). \end{aligned} \tag{A.9}$$

Also, using (A.6) and noting $\varphi(v) \sim O(\frac{1}{u_0^\rho})$, we have

$$\begin{aligned} g(v) = \varphi(v)\sqrt{\frac{v}{\mu}}\left(1 + O\left(\frac{1}{u_0^2}\right)\right) \\ \Rightarrow O(g'(v)) = O\left(\frac{\varphi(v)}{2\sqrt{v\mu}} + \varphi'(v)\sqrt{\frac{v}{\mu}}\right) \\ \Rightarrow O(g'(\mu)) = O(\varphi(\mu)). \end{aligned} \tag{A.10}$$

Using (A.7), (A.8), (A.9) and (A.10), we have

$$\begin{aligned} I &= \varphi(\mu)\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right) + O\left(u_0 e^{-\frac{u_0}{2}}\right) \\ &\overset{a}{=} \varphi(\mu)\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right), \end{aligned} \tag{A.11}$$

where (a) follows from the fact that $\varphi(\mu) = O\left(\frac{1}{u_0^\rho}\right)$. Applying (A.11) in (A.3), we have

$$\begin{aligned} I_N &= \int_{v_{N-1}} \cdots \int_{v_1} f(\alpha v_{N-1})\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right)(1 - Q(\alpha v_{N-1}u_0)) \times \\ &\quad \prod_{i=1}^{N-1} u_0^2 v_i e^{-\frac{(v_i - \alpha v_{i-1})^2}{2/u_0^2} - \alpha u_0^2 v_i v_{i-1}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i \\ &= \int_{v_{N-2}} \cdots \int_{v_1} f(\alpha^2 v_{N-2}) f(\alpha v_{N-2})\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right)^2 \left(1 - Q(\alpha^2 v_{N-2}u_0)\right) \times \\ &\quad (1 - Q(\alpha v_{N-2}u_0)) \prod_{i=1}^{N-2} u_0^2 v_i e^{-\frac{(v_i - \alpha v_{i-1})^2}{2/u_0^2} - \alpha u_0^2 v_i v_{i-1}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i \\ &= \cdots = \prod_{i=1}^{N} f(\alpha^i v_0)\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right)\left(1 - Q(\alpha^i v_0 u_0)\right) \end{aligned} \tag{A.12}$$

Substituting $v_0 = \frac{1}{\sqrt{(1-\alpha^2)/2}}$, we have

$$\begin{aligned} I_N &= \prod_{i=1}^{N} f\left(\frac{\sqrt{2}\alpha^i}{\sqrt{(1-\alpha^2)}}\right)\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right)\left(1 - Q\left(\frac{\sqrt{2}\alpha^i u_0}{\sqrt{(1-\alpha^2)}}\right)\right) \\ &= \prod_{i=1}^{N} f\left(\frac{\sqrt{2}\alpha^i}{\sqrt{(1-\alpha^2)}}\right)\left(1 + O\left(\frac{1}{\sqrt{u_0}}\right)\right)\left(1 - O\left(e^{-u_0^2}\right)\right) \end{aligned} \tag{A.13}$$

Using (A.4) and (A.13) and noting $E_0(\rho) = -\frac{1}{N}\log I_N$, we conclude Theorem 5.1.

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