Real Options and Asset Valuation in
Competitive Energy Markets

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
Abstract

The deregulation of energy markets around the world, including power markets has changed the way operating assets in these markets are managed. Independent power asset owners and even utilities operating in these markets no longer operate their assets based on the cost of service approach that prevailed under regulation. Just as in other competitive markets, the objectives of asset owners in power markets revolve around maximizing profit for their shareholders. To this end, financial valuation of physical assets in power markets should incorporate different strategies that are used by asset operators to maximize profit. A lot of observed strategies in power markets are driven by a number of factors, the key among which are:

- asset operators are no longer obligated to supply service or manage their assets in certain prescribed ways, rather they have rights to operate, within applicable market rules, using techniques that maximize their profits,
- revenues are driven by uncertain market factors, including power price, cost and/or availability of fuel stock and technical uncertainties, and
- power assets have physical operating and equipment constraints and limits.

Having flexibilities (“options”) to optimize their assets (inline with shareholders’ objectives), rational asset managers react strategically to gradual arrival of information\(^1\), given applicable equipment constraints, by revising previous decisions in such a way that only optimal (or near optimal) decisions are implemented. As a result, the appropriate approach to valuing power assets in competitive markets must account for managerial flexibilities or “real options” in the presence of uncertainties and technical constraints.

The focus of this work is to develop a robust valuation framework for physical power assets operating in competitive markets such as peaking or mid-merit thermal power plants and baseload power plants. The goal is to develop a modeling framework that can be adapted to different energy assets with different types of operating flexibilities and technical constraints and which can be employed for various purposes such as capital budgeting, business planning, risk management and strategic bidding

\(^1\) i.e. information on uncertain market and technical value drivers
planning among others. The valuation framework must also be able to capture the reality of power market rules and opportunities, as well as technical constraints of different assets.

The modeling framework developed conceptualizes operating flexibilities of power assets as “switching options” whereby the asset operator decides at every decision point whether to switch from one operating mode to another mutually exclusive mode, within the limits of the equipment constraints of the asset. As a current decision to switch operating modes (in the face of current realization of relevant uncertainty factors) may affect future operating flexibilities of the asset and hence cash flows\(^2\), a dynamic optimization framework is employed. The developed framework accounts for the uncertain nature of key value drivers by representing them with appropriate stochastic processes. Specifically, the framework developed conceptualizes the operation of a power asset as a multi-stage decision making problem where the operator has to make a decision at every stage to alter operating mode given currently available information about key value drivers. The problem is then solved dynamically by decomposing it into a series of two-stage sub-problems according to Bellman’s optimality principle. The solution algorithm employed is the Least Squares Monte Carlo (LSM) method.

The developed valuation framework was adapted for a gas-fired thermal power plant, a peaking hydroelectric power plant and a baseload power plant. This work built on previously published real options valuation methodologies for gas-fired thermal power plants by factoring in uncertainty from gas supply/consumption imbalance which is usually faced by gas-fired power generators. This source of uncertainty which has yet to be addressed in the literature, in the context of real options valuation, arises because of mismatch between natural gas and electricity wholesale markets. Natural gas markets in North America operate on a day-ahead basis while power plants are dispatched in real time. Inability of a power generator to match its gas supply and consumption in real time, leading to unauthorized gas over-run or under-run, attracts penalty charges from the gas supplier to the extent that the generator can not manage the imbalance through other means. A savvy gas-fired power plant operator will factor in the potential costs of gas imbalance into its operating strategies resulting in optimal operating decisions that may be different from when gas-imbalance is not considered. By considering an illustrative power plant operating in Ontario, we show effects of gas-imbalance on

\(^2\) i.e. cash flows are path dependent
dispatch strategies on a daily cycling operation basis and the resulting impact on net revenue. Results show that a gas-fired power plant is over-valued by ignoring the impacts of gas imbalance on valuation.

Similarly, we employ the developed valuation framework to value a peaking hydroelectric power plant. This application also builds on previous real options valuation work for peaking hydroelectric power plants by considering their operations in a joint energy and ancillary services market. Specifically, the valuation model is developed to capture the value of a peaking power plant whose owner has the flexibility to participate in a joint operating reserve market and an energy market, which is currently the case in the Ontario wholesale power market. The model factors in water inflow uncertainty into the reservoir forebay of a hydroelectric facility and also considers uncertain energy and operating reserve prices. The switching options considered include (i) a joint energy and operating reserve bid (ii) an energy only bid and (iii) a do nothing (idle) strategy. Being an energy limited power plant, by doing nothing at a decision interval, the power asset operator is able to time-shift scarce water for use at a future period when market situations are expected to be better. An illustrative example considered shows the impact of the different value drivers on the plant’s value and dispatch strategies. Results show that by ignoring the flexibility of the asset owner to participate in an operating reserve market, a peaking hydroelectric power plant is undervalued.

Finally, the developed valuation framework was employed to optimize life-cycle management decisions of a baseload power plant, such as a nuclear power plant. The applicability of real-options framework to the operations of baseload power plants has not attracted much attention in the literature given their inflexibility with respect to short-term operation. However, owners of baseload power plants, such as nuclear plants, have the right to optimize scheduling and spending of life cycle management projects such as preventative maintenance and equipment inspection. Given uncertainty of long-term value drivers, including power prices, equipment performance and the relationship between current life cycle spending and future equipment degradation, optimization is carried out with the objective of minimizing overall life-cycle related costs. These life-cycle costs include (i) lost revenue during planned and unplanned outages (ii) potential costs of future equipment degradation due to inadequate preventative maintenance and (iii) the direct costs of implementing the life-cycle projects. The switching options in this context include the option to shutdown the power plant in order to execute a given preventative maintenance and inspection project and the option to keep the
option “alive” by choosing to delay a planned life-cycle activity. Results of an illustrative example analyzed show that the flexibility of the asset owner to delay spending or to suspend it entirely affects the asset’s value accordingly and should be factored into valuation.

Applications can be found for the developed framework and models in different areas important to firms operating in competitive energy markets. These areas include capital budgeting, trading, risk management, business planning and strategic/tactical bidding among others.
Acknowledgements

It is only natural that I start by acknowledging the good Lord, the creator of all good “real” options who also made them available to mankind freely (i.e. at no premiums) even though we have developed sophisticated methods of charging one another hefty premiums for these rights e.g. the right to freedom of expression, equal opportunities, peaceful existence etc.

The completion of this thesis has greatly benefited from the support of several people, the most important being my supervisor, Prof. David Fuller who believed in me from the beginning and spent quality time providing many invaluable insights. Others include my examination committee members, practitioners and researchers in the Real Options field through their writings, Risk Services and Energy Markets colleagues at Ontario Power Generation (OPG) Inc., and my family and friends to mention a few. This is to thank everyone for their support and understanding through the years.
Dedication

Dedicated to my best friend and wife, Yemi.
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Chapter 1

Introduction

1.1 Background

The business of generating, transmitting and distributing power has undergone significant transformation in recent years due to electricity sector liberalization. In its simplest form, power sector liberalization has resulted in unbundling of government owned utilities that have traditionally monopolized electricity generation, transmission and distribution. In advanced liberalization schemes, electricity products such as energy, generation capacity and transmission rights are traded in competitive wholesale and retail markets as commodities. Given that most power markets are still considered as emerging, new challenges and opportunities are constantly evolving.

Several authors have attempted to catalogue the historical development of power sector liberalization schemes and some of the emerging challenges. Good examples of books on this subject include Weron [1], Bhattacharya et al. [2] and Shahidehpour et al. [3]. Power sector liberalization is believed to have been pioneered by Chile in 1982 but electricity trading through a wholesale market scheme was first introduced in England and Wales. The creation of the Nordic power pool, which originally included Norway but later extended to include Finland, Sweden and Denmark followed after the creation of the England and Wales electricity pool. The earliest markets in North America were created in the north east United States (i.e. New England Power Pool, Pennsylvania-New Jersey-Maryland (PJM) and New York Power Pool) in the late 1990s. Alberta, California, Texas and Ontario later followed.

Figure 1 shows the different bulk power system jurisdictions across North America. Four major interconnections are evident; the Eastern Interconnection, Electricity Reliability Council of Texas (ERCOT) Interconnection, Western Interconnection and Quebec Interconnection. The North America Electric Reliability Corporation (NERC) has the responsibility to ensure and improve the reliability and security of North America’s bulk power system including the development of reliability standards. Members of NERC include eight regional reliability councils as shown in Figure 1.

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Across the bulk power system and interconnections, there are several independent power markets (i.e. power pools), power exchanges and trading hubs. In most market jurisdictions, electricity products are traded internally and across inter-tie transmission lines connecting multiple jurisdictions.

Different types of market structure and design exist (Bhattacharya et al. [2]) but most of them have key commonalities. Typically, a jurisdiction consists of a power pool operated by an independent government-formed agency known as the Independent System Operator (ISO). Participants in a power pool include Independent Power Producers (IPPs), government-owned utilities, wholesale power consumers, power traders among others. The role of the ISO is to coordinate the market while ensuring power system reliability and integrity. In some liberalization schemes where power trading is done only through bilateral contracts (i.e. no power pools), the ISO’s role reduces to ensuring the reliability of the bulk power system. Where the ISO serves as the market coordinator, its objective is to maximize the social welfare of the entire market within the constraints of the system’s reliability.

In the United States, there are estimated 2800 IPPs and over 3100 public utilities⁴. Unlike the independent system/market operator, the goal of the individual power market participant is to maximize wealth for its own shareholders by engaging in market-based transactions as well as bilateral trades as opportunities arise. This objective is what drives investment and strategic decisions by individual power market participant. Problems related to strategic decision making have therefore become much more important to firms operating in power sectors. This is necessitated by the need to create, preserve and grow value.

⁴ Source: www.eia.doe.gov
Figure 1: North America Bulk Power Systems Interconnections

Given the uncertainties and opportunities that characterize competitive power market environments, several areas of strategic decision making have become key focal points for industry and academic studies. Some of these key areas include:

- Short-term generation planning and scheduling e.g. bidding strategies, peaking decisions, startup and shutdown strategies etc.

- Asset valuation i.e. pricing of derivative contracts, Power Purchase Agreements (PPA), Transmission Rights (TRs) etc.

- Portfolio planning and management i.e. optimization of multiple assets (physical and financial) to achieve desired risk-return objective.

- Risk management e.g. physical and financial hedging strategies

- Trading e.g. short and long-term trading strategies for energy, capacity, emissions, derivative instruments etc

Source: www.nerc.com
• Outage planning i.e. short and long term maintenance and fuelling outage planning

Underlying these (and other) strategic decision making problems in competitive power markets is the need for market participants to develop various analytical capabilities requiring knowledge and understanding of theoretical fundamentals in different subject areas such as power engineering, decision sciences, financial mathematics, statistics and investment management among others.

1.1.1 Economic Value Drivers in Power Markets

Electricity, being un-storable in large economic quantities, exhibits highly volatile market prices in line with short-term demand and supply imbalances. This behavior is evident in Figure 2 which shows the time series of Hourly Ontario Electricity Prices (HOEP) for years 2003 to 2006\(^6\). In effect, prices of electricity in competitive markets behave much like securities prices in financial markets and prices of other commodities competitively traded. In this context, future spot prices are uncertain and they tend to reflect historical volatilities. In addition, as shown in Figure 2, electricity prices also tend to exhibit the phenomenon of “mean reversion” and “jumps”. The frequent jumps (or spikes) occurring over short intervals (typically over few hours) are sudden price excursions usually caused by the occurrence of supply shortfalls and the need to balance supply and demand in real time which results in dispatch of expensive power plants. Price mean reversion on the other hand, is common to most commodities (Schwartz [4]). When prices are high, supply tends to increase while demand decreases with the overall downward pressure effect on prices. When prices are low, the opposite takes place leading to upward price pressures. In the aggregate, prices tend to settle around long-term averages which can be seasonal in nature.

Seasonality in long-term average prices of power tends to follow patterns of power consumption for space heating and for cooling. Typically, demand is higher during winter and summer compared to the shoulder months. This implies that very cold winter years may witness generally higher power prices compared to the years when the winter is mild. There may also be seasonality to the locally available energy capacity in a market jurisdiction depending on the type of generation that dominates the local capacity. For instance, in a market that has mostly hydroelectric power plants, local energy capacity will tend to vary with changes in local precipitation patterns.

There is also a daily pattern to electricity prices. Power consumption is lower during the night

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\(^6\) Data source: www.theimo.com
leading to correspondingly lower prices; this period is referred to as “off-peak”. As the day breaks, power consumption increases and typically peaks after noon corresponding to higher power prices. The periods of relatively higher prices during the day are referred to as the “peak” and the “super-peak” periods.

Apart from electricity, there are other uncertain value drivers important to power market participants. Fossil fuel prices, for instance also exhibit significant volatility, especially prices of natural gas and residual oil used for firing thermal power plants. Coal and uranium that were historically known to have stable prices have also been showing significant short-term volatilities in recent months. Other than fuel prices, examples of other value drivers exhibiting significant uncertainties include precipitation (for hydroelectric plants), frequency of forced equipment outages (power plants, distribution and transmission lines) as well as foreign exchange rates.

As in any market economy, the presence of uncertain economic value drivers constitutes both upside opportunities and downside risks to market participants. Savvy investment and operating strategies therefore take into consideration all available opportunities relevant to a power market participant (e.g. startups and shutdowns, planned outages, peaking etc) while considering also the impacts of downside exposures.

When value drivers are uncertain and investors/asset operators have future flexibilities to revise their decisions, downside risk exposures can be hedged and additional values derived. This assertion is intuitive when one thinks of these future operating or investment flexibilities as “options” where the downside is only the cost of exercising the option i.e. the sunk cost of the initial decision (Trigeorgis and Schwartz [5], Dixit and Pindyck [6], Copeland and Antikarov [7], Amram and Kulatilaka [8]). However, unlike financial options, exercise and feasibilities of these “real options” are subject to physical asset constraints such as equipment limits. The careful combination of the physical characteristics of power assets with concepts borrowed from financial markets to analyze and value investments in competitive energy markets is the focus of this work.
Figure 2: Historical Hourly Ontario Electricity Prices (HOEP): 2003-2006
1.2 Overall Thesis Objective

In a broad sense, the main goal of the work described in this thesis is the development of a robust modeling framework for investment valuation of physical assets in competitive power markets given uncertainty in key value drivers and availability of future flexibilities to operators and investors. Specific examples are developed covering the major types of power generating plants with different operating flexibilities. Illustrative studies are then carried out and presented to demonstrate the applications of the developed mathematical models.

1.3 Organization of Document

The remainder of this document is organized in six chapters as follows:

- Chapter 2 provides a general background on power markets and the different types of power generating plants. The Ontario wholesale electricity market is discussed as an illustrative example to shed light on power markets design and operations.
- In Chapter 3 a broad literature survey is presented covering various topics relevant to power asset investment analysis.
- In Chapter 4 we develop a real options analysis framework for valuing different types of power generating assets with flexibilities to switch between operating modes in response to resolution of underlying uncertainties.
- In Chapter 5 we develop a specific example for gas-fired power plants. Unlike existing applications of real options analysis to gas-fired power plants, we consider the impact of gas supply imbalance uncertainty on operating decisions and hence the market valuation of the plant’s output energy.
- Chapter 6 focuses on the application of the developed real options framework to the valuation of a hydroelectric power generating plant. We apply the framework to value a peaking hydroelectric facility where the operator has the flexibility to sell ancillary services competitively in a market in addition to energy.
- Chapter 7 focuses on the application of the developed framework to a nuclear baseload power plant given that the owners have flexibilities regarding Life Cycle Management (LCM)
spending and timing of planned shutdowns.

- In Chapter 8, we present some general conclusions and applications of the work. Opportunities for future work are also discussed.
Chapter 2
Introduction to Power Markets

2.1 Operations of Power Markets

There are different power market schemes currently in operation today. Most power markets operate as Power Pools. In addition to power pools, power products are also traded through bilateral arrangements (i.e. Over-the-Counter, OTC) between market participants or through organized exchanges. A power pool is operated by an Independent System Operator (ISO) to ensure that economic value is maximized fairly for all market participants. The ISO also ensures open and equal access to the transmission network by all market participants and overall system reliability. In liberalization schemes where only bilateral trading is done, the ISO serves a technical role i.e. that of the power grid coordinator. Unlike Power Pools, Power Exchanges are usually formed via private sector initiative for trading power related contracts, most especially forwards. To illustrate the operations of a power market, we discuss the Ontario wholesale electricity market in the next section.

2.1.1 Ontario Wholesale Electricity Market

In Ontario, the wholesale electricity market was created in May 2002 with the responsibility for market coordination held by the Independent Market Operator (IMO). The functions of the IMO relating to electricity market operation and short-term planning of Ontario’s electricity demand and resources were later transferred to a new organization called the Independent Electricity System Operator (IESO). Long term resource adequacy planning then became the responsibility of the Ontario Power Authority (OPA). The Ontario Energy Board (OEB) regulates the entire energy industry in Ontario and works with the Ministry of Energy to establish policies for both the power sector and the natural gas industry.

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7 Examples of ISOs in the North American eastern interconnection are:
Independent Electricity System Operator (IESO)
Alberta Electricity System Operator (AESO)
New York Electricity System Operator (NYISO)
Midwest Independent Electricity System Operator (MISO)

8 Examples of Power Exchanges include the Alberta Watt Exchange, the European Energy Exchange (EEX) and the UK Power Exchange (UKPX).
Currently, the Ontario wholesale electricity market administered by the IESO consists of real-time energy and operating reserve markets and a financial market for Transmission Rights (TRs). Other ancillary services besides operating reserve (i.e. voltage control, frequency regulation and black start services) are procured by the IESO from qualified market participants via bilateral contracts. Market participants are also able to engage in bilateral, over-the-counter trading activities with one another, within Ontario and with counterparties outside of the province across the interconnected network. The categories of market participants in Ontario as defined by the IESO\(^9\) are as follows:

- Transmitters
- Distributors
- Dispatchable and non-dispatchable generators
- Dispatchable and non-dispatchable loads
- Wholesalers
- Financial market participants
- Retailers

Wholesalers, retailers and financial market participants are those market participants that do not have physical facilities that are either connected to the IESO controlled grid or embedded in a distributor controlled network.

In terms of physical infrastructure, the Ontario power system consists of one 500 kV and one 230 kV transmission networks in addition to a number of smaller 115 kV lines. For reliability and stability planning purposes, the entire system is divided into 10 internal transmission zones (based on congestion areas) as shown in Error! Reference source not found.\(^{10}\). Also depicted in the figure are the seven inter-tie connections Ontario has with external jurisdictions such as New York, Michigan, Quebec etc.

Given the emerging nature of wholesale power trading, market rules and available market programs

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\(^9\) Source: www.theimo.com

\(^{10}\) The arrows in the figure are flow gates; the meanings of the acronyms are provided in Appendix A under list of acronyms.
are constantly evolving. For instance, a day-ahead market does not currently exist in Ontario but
discussions are ongoing for its creation. In place of a day-ahead market, certain programs exist to help
market participants plan the use of their resources day-ahead. Two examples of such programs are the
Day Ahead Commitment Process (DACP) and the Spare Generator Online (SGOL) program
(Zareipour [9]).

Figure 3: A Schematic Idealization of Ontario’s Internal Zones, Internal Interfaces and
External Connections11 (source: www.ieso.com)

11 The arrows show directions of power flows between the idealized zones; see Appendix A for the full
meaning of the acronyms BLIP, NBLIP, FN, FS etc.
To participate in the Ontario wholesale electricity market, dispatchable generators submit offers to the IESO for the hours they wish to operate, while dispatchable load entities (wholesale consumers) submit bids for the hours they wish to consume energy. A dispatchable generator can submit offers for both energy and operating reserve. Both offers and bids are submitted in price-quantity pairs using the formats specified by the IESO. All offers and bids are combined with non-dispatchable supply and the primary demand (i.e. non-dispatchable demand) to arrive at fair clearing prices for both energy and operating reserve. Market Clearing Prices (MCPs) and Operating Reserve (OR) prices are determined at every five minute intervals by the IESO by running an optimization software; the Dispatch System Optimizer (DSO). The DSO implements a marginal cost-based linear programming scheme that seeks to maximize economic benefits for all market participants. Economic value for all trades carried out at every interval across the market measures the difference between the value to consumers and costs to suppliers of the volume of energy and OR traded. Essentially, consumers bid the maximum price they are willing to pay for certain quantity of energy while suppliers offer the lowest price they are willing to receive for given energy quantities. Operating reserve requirements are specified by the IESO based on NERC requirements. To derive the demand and supply curve for an interval, the supply offers are stacked from lowest to highest price offers against the demand bids which are stacked from highest to lowest price bids. Non-dispatchable generators are allocated the minimum possible offer price while non-dispatchable loads are assigned the maximum possible bid price. The Market Clearing Price is the price that corresponds to the intersection of the supply and demand stacks. Due to their low marginal operating costs, baseload assets such nuclear and coal power plants are usually at the bottom of the supply stack while peaking assets (gas turbine and energy-limited hydro electric plants) are at the top. A single uniform price is calculated for the entire Ontario market but zonal prices are also calculated for the inter-tie zones to determine import and export prices. The extents to which zonal prices differ from the Ontario MCP is determined by congestion at the inter-tie zones. With regards to the pricing scheme used within Ontario, discussions are currently ongoing to change from the Uniform Marginal Pricing (UMP) scheme to a Location Marginal Pricing (LMP) scheme.

The IESO runs the DSO in two modes and two time-frames:

- Constrained and unconstrained modes
- Predispatch and real-time.
The unconstrained mode of the DSO does not include the physical transmission system hence it ignores transmission line limits and losses within Ontario but it does consider the limits of the inter-tie zones. Dispatchable generators are compensated at MCP which is the resulting clearing price from the unconstrained DSO run in real-time. Non-dispatchable generators however, receive HOEP which is the hourly average of 5 minute MCPs. Large commercial consumers pay HOEP for energy consumed while residential and small power consumers pay government regulated tariffs. Real time market dispatch is derived from the constrained version of the DSO run in real-time which factors in the physical characteristics of the transmission networks. The intent of running the DSO in both the constrained and the unconstrained mode is to ensure that market participants are not penalized for non-optimal dispatch created by the physical need to respect the limitations of the transmission network as opposed to the theoretically optimal dispatch (under a uniform pricing scheme) created when offers and bids are considered without those limitations. To the extent that the dispatch schedule from the unconstrained optimization run is different from that of the constrained run, market participants receive credit payments called Congestion Management Settlement Credits (CMSC) as compensation for being constrained on or off as the case may be. Details of the Ontario wholesale electricity market rules are readily available in several market procedures published by the IESO on its web site\(^\text{12}\).

### 2.1.2 Physical Assets in Power Markets

The term “physical asset” in the context of power markets refer to power generating plants of different types, power transmission and distribution lines, gas transportation pipelines and gas storage facilities. Power generating plants can be classified broadly as follows:

1. Hydroelectric
2. Fossil
3. Nuclear
4. Renewables

#### 2.1.2.1 Hydroelectric Plants

Hydroelectric plants generate electric power by employing the kinetic energy of running water to

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\(^\text{12}\) www.ieso.ca
drive turbines. There are three main types of hydroelectric schemes:

(i) run of the river

(ii) diversion

(iii) pumped storage

Run of the river hydroelectric schemes work by locating turbines and generators either in a dam or alongside it such that the dam uses the flow of the river to create a hydrostatic head. A diversion scheme works by diverting water from a river or lake via canals or tunnels to a reservoir forebay. Water from the reservoir is passed through a penstock which slopes down into a power house, to generate electricity via one or more turbines and generators. The difference between a diversion scheme and a pumped storage scheme is that a pumped storage plant incorporates one or more reversible pump-turbine units (or separate pumps and turbines) in such a way that water can be pumped into a storage reservoir when prices of power are relatively low (usually off-peak at night) to be used later for electricity generation at periods when power prices are relatively higher. Since pump motors require electricity to operate, during the pumping mode, an operator of a pump storage power generating plant essentially acts as both a load and a supplier during pumping and generation respectively.

Hydroelectric plants generally have low operating costs. Typically, there is no direct fuel cost for hydroelectric power plants but government authorities usually charge hydro operators water rental fees. In Ontario, water rental charge is known as the Guaranteed Rate Charge (GRC) which is a variable fee that corresponds to the volume of energy produced.

In terms of technical capability, hydroelectric plants typically have fast ramping rates making them suitable for provision of ancillary services and for operations as peaking generators. Plants with large water storage capability have more operational flexibilities and are able to cycle water over long intervals compared to those with very minimal storage. Run of the river plants are operated as baseload given that water can not be stored for later use.

Production volumes from hydroelectric plants are subject to seasonality, based on seasonal rainfall and snow melting patterns. The flexibility provided by reservoir storage is usually of more value during the dry summer months as opposed to the rainy spring and fall seasons (termed ‘freshet’
periods). A large reservoir storage can be filled during the freshet seasons when precipitation is high and prices typically low. The same reservoir can then be strategically drawn down during summer when precipitation is low and prices are typically relatively high and more volatile.

2.1.2.2 Fossil Power Plants

Fossil power plants make use of chemical energy in fossil fuels to produce steam that is used to drive steam turbines. The common fossil fuels are coal, natural gas and heating oil. While most plants can burn only one type of fuel, others are designed with the flexibility to be able to burn more than one fuel e.g. natural gas and heating oil. The efficiency of a fossil plant is determined by its heat rate which measures its capability to convert fuel to power. The heat rate is one of the main factors that determine the operating costs of fossil plants. The total operating costs are also affected by the equipment constraints such as boiler heating and cooling constraints. The flexibility to operate a fossil plant in peaking and baseload mode or only in a peaking mode directly depends on operating costs. For instance, a gas-fired plant with high heat rate and hence high operating costs will typically be “out-of-the-money” during most hours of the day except during the periods of peak power prices. Since a rational operator will not intentionally operate the plant out-of-the-money (except for the purpose of capturing value at a later time), such a plant is only operated when power prices are relatively high.

Another factor that plays an important role in the economics of fossil-fueled power plants is reliability of fuel supply. For instance, natural gas is usually supplied through transportation pipelines. As a result, easy access to flexible use of transportation services and possible gas storage facilities improves the economics of a gas-fired power plant. Similarly, transportation of coal to coal-burning power plants is often achieved via rail and sea; hence regional climate factors (such as winter effects on water transportation) make year round supply of coal to certain power plants, impossible.

Other factors of importance to fossil-fueled power generation (especially coal-fired power plants) relates to pollution control. Costs of emission control depend on a number of technical and regulatory

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13 During seasons of heavy rain, reservoirs are full and there maybe more water than the power plant can process resulting in spill of excess water flow.

14 The heat rate of a gas-fired power plant determines the required natural gas consumption (measured in MMBtu of Giga Joule) per MWh of energy produced, hence high heat rate indicates low efficiency (e.g. observed in single cycle gas-fired power plants).
factors such as the availability of pollution control technology (e.g. Sulphur dioxide scrubbers) including plant equipment technology, coal quality (high or low sulphur coal) and local/regional environmental policies.

2.1.2.3 Nuclear Power Plants

Nuclear power plants (NPP) make use of thermal energy produced by fission of nuclear materials to generate electricity. Due to the complexity of fission processes, nuclear power plants usually have long startup times making them suitable for baseload power generation only. Comparatively, operating costs of nuclear power generation are typically less compared to fossil-fueled power plants but their capital costs requirement can be prohibitively high. In addition to high capital costs, other costs related to long-term waste storage and plant decommission can significantly affect the economics of nuclear power plants. Costs of Life Cycle Management (LCM) for nuclear power plants can also be very significant. These costs include costs of equipment maintenance and inspection among others. Flexibilities to strategically allocate LCM spending and select shut-down periods in the presence of power price and materials costs uncertainties produce additional values for owners of NPP.

2.1.2.4 Renewable Energy

Recent environmental concerns are generating significant interests in the development of renewable energy technologies for power generation. However, the economics of most renewable energy sources make them unattractive for large scale power generation at least with the currently available technologies. To encourage investments in renewable energy power plants, special government programs are usually made available to investors, even within the context of competitive power markets. A good example of such special program is the Standard Offer Program (SOP) available in Ontario through the Ontario Power Authority (OPA) for investors interested in small-scale renewable power plants.

2.1.2.5 Other Physical Assets

Access to power transmission and distribution lines is vital to power market participants. From the perspective of an independent generator, financial value can be attached to firm and interruptible access on transmission lines (e.g. see www.miso.org), especially when such lines connect two trading hubs or zones. Short and long-term transmission rights are typically available for purchase in most markets, mostly through some sort of auction mechanism. The value of such rights depends on
whether they are firm or interruptible and on the locational price spreads between the two hubs or zones that they connect.

Similarly, access to natural gas is a key economic factor for gas-fired plants. As a result, energy marketers trade around the volatility of natural gas prices as well as locational and calendar price spreads, whether for hedging purposes or for pure profit speculation. The values of gas storage facilities/services and gas transportation facilities/services are therefore important in power markets and can be determined by considering the dynamics of gas prices along with the physical characteristics of storage reservoirs and transportation pipelines.
Chapter 3
Literature Review

3.1 Uncertainties, Risk and Opportunities in Competitive Energy Markets

Under regulated power systems, vertically integrated utilities or local distribution companies charged regulated tariffs which were in line with the costs of providing services. Under such compensation schemes, rate payers directly bore the risks of capital investments in the electricity sector, hence returns from power investments were generally considered guaranteed. With the introduction of market-based pricing, cash flows are subject to increased uncertainties for which government guarantees are not provided\textsuperscript{15}. This development has led to the growth of research in the areas of risk management and strategic asset optimization in power sectors.

There are several sources of uncertainties investors are faced with in a competitive power market environment. These can be classified broadly as follows (Eyedeland and Wolyniec [10], Kaminski [11], Ronn [12]):

(i) technical or operational uncertainties

(ii) market-based uncertainties

(iii) regulatory uncertainties, and

(iv) credit risk

As pointed out by several authors, the impacts of uncertainties on investment performance include both upside opportunities and downside risks (see Schwartz and Trigeorgis [5], Dixit and Pindyck [6], Ronn [12]). This is true for power markets where capital investments and asset operations essentially involve optimizing risks and strategic opportunities to maximize value for shareholders.

\textsuperscript{15} Even in partially deregulated markets (such as Ontario) where Power Purchase Agreements (PPAs) and other forms of long-term regulated energy contracts are employed to guarantee earnings, return on investments are still usually tied to optimized operating strategies.
3.1.1 Sources of Uncertainties

3.1.1.1 Technical and Operational Uncertainties

Technical and operational uncertainties in power markets can vary from broad operational factors affecting entire portfolio of assets owned by firms to asset specific factors. As in the financial sector, operational risks are defined\(^\text{16}\) for the energy industry as those risks arising from failure of systems, processes and people. For instance, failure of equipment could lead to forced outages of power plants and power transmission lines. Such outages results in unplanned financial losses and cash flow uncertainty. Similarly, faulty processes in different operational areas (e.g. trading, forecasting, operational safety etc) and failure of people can lead to unintended outcomes resulting in investment losses. There are also other more readily quantifiable sources of technical uncertainties such as variability in precipitation (affecting production of hydroelectric facilities) and weather (affecting load demand).

3.1.1.2 Market Based Uncertainties

Market-based uncertainties arise from macroeconomic factors that are usually outside of the control of each individual market participant. Examples include (i) electricity and fuel prices (ii) interest rate and (iii) foreign exchange rates.

3.1.1.3 Regulatory Uncertainties

Given that power markets are still generally considered as “emerging”, significant uncertainties exist with regards to future policy directions and market interventions by government regulatory bodies. This category of uncertainties is especially important within the context of long-term investments in power markets. Specific areas include market design, environmental regulations, re-regulation of power sectors and market rule changes among others.

3.1.1.4 Others

Credit risk refers to the risk that counterparty in a transaction will default on its obligation. Typically, the probability and magnitude of credit events depend on a number of factors including the counterparty’s credit rating and volume of transaction. In a power pool, there are usually many

\(^{16}\) See www.ccro.org; Committee of Chief Risk Officers - an industry alliance of leading risk officers for energy companies
participants, each contributing its own quota to potential credit events for the entire market. In Ontario, a credit risk prudential support system exists which requires each market participant on the demand side to contribute on an ongoing basis to a fund (based on market share and credit rating) that can be relied upon for partial hedging of credit default events.

In order to quantify the impact of uncertainties on investments, the key starting point is to be able to quantify those uncertainties. To this effect, a number of well established financial models exist for modeling time series evolutions of market-based factors such as interest rates and foreign exchange (e.g. see Hull [13]) and probability of credit events. Similarly, many established methods exist in the engineering literature for modeling technical uncertainties such as equipment failure rates. Regulatory uncertainties are not as easily quantifiable but their impacts can sometimes be forecasted and factored into analysis (Larsen and Bunn [14]).

3.2 Uncertainties and Value of Assets

Valuation is the process of quantifying the price of an asset or an investment project. The purpose and reason for asset valuation are diverse. Typical examples include financial reporting, capital budgeting, strategic planning and risk management (Brealey and Myers [15]).

There are a number of traditional methods for analyzing investment decisions. Among these, the Net Present Value (NPV) method is the most popular\(^\text{17}\). The basic premise of the conventional NPV method is to estimate future cash flows from an investment outlay (revenues and costs) and discount them to a common present time using a hurdle rate or a risk-adjusted rate of return. The convention is to determine the net difference between estimated discounted revenue and discounted cost such that if the net is greater than zero, the investment is considered viable (see Fraser et al. [16]). In the case of multiple investment opportunities, the alternative with the highest NPV is considered the most attractive.

In industrial applications of NPV, questions do arise regarding how future cash flows and an appropriate discount rate should be determined and how risks should be accounted for. Early critics of NPV (e.g. Hayes and Garvin [17], Kester [18] and Hertz [19]) provided different points of view on why traditional NPV is deficient and proposed methods of improvements. Some suggested the use of simulation to enhance the static NPV while some suggested incorporating decision trees. As pointed

\(^{17}\) Other examples include the Pay Back Period (PBP) method and Internal Rate of Return (IRR) among others
out by Trigeorgis [20] and Dixit [21] and many others who have written on this subject, there are three characteristics of investments that if present together make the use of traditional NPV inadequate for capturing true investment value. These include:

(i) Sunk costs - most investments have sunk costs, making decisions irreversible.

(ii) Future uncertainty of value drivers - most investments are subject to economic and technical uncertainties and information arrives gradually to resolve the uncertainties.

(iii) Non-disappearing investment opportunities - opportunities to invest do not always disappear if a decision is not taken immediately (i.e. decisions can be delayed), hence the investment question is not only if to invest but when to invest.

When there are uncertainties and irreversibility, and the opportunities to invest do not disappear immediately, two situations arise:

(i) it may be optimal to wait for new information arrival before investment decisions are made;

(ii) flexibility may be available to change a previously selected strategy as new information arrives.

In other words, in the presence of sunk costs, uncertain economic and technical factors and opportunities to wait, rational investors react strategically to gradual information arrival either by waiting to invest\(^{18}\) or by revising, as appropriate, previously chosen strategies. The premise of the traditional NPV however, is that investment decisions are made at the time of analysis (here and now approach), thereby leaving no room for future flexibility on the part of the decision maker as uncertainties become resolved in the future.

Managerial flexibilities of these forms in the context of investments in physical assets are referred to as “real options”. “Options” in this sense refer to choices on the part of the investor or managers to either defer decision making or change strategies along the way. The term “real options” was first introduced by Myers [22] to describe investment flexibilities of firms. In his paper, he proposed that real assets of firms can be viewed as “call options”. The term “real” was introduced to contrast these types of options with financial options which are simply referred to as options. Real options are non-

\(^{18}\) This approach is the so called “wait and see”
traded (unlike financial options) but their values, like financial options can depend on traded underlying assets. Also, unlike financial options, their values can also depend on non-traded factors such as technical uncertainties. Like the owner of a financial options contract, the owner or manager of a physical asset with future managerial flexibilities (or optionalities) has certain rights that he/she is not obligated to exercise. The value associated with this flexible decision making forms the basis of real options valuation as opposed to traditional investment analysis methods.

Early documentation of the applications of this type of thinking to investment valuation and strategy were provided by Myers [22], Mason and Merton [23], McDonald and Siegel [24], Brennan and Schwarz [25], Kulatilaka [26] among others. The idea is to value physical assets with embedded managerial flexibilities using techniques originally developed for financial options valuation. This method of thinking has now been extensively embraced by the academic corporate finance community, and it is also getting extensive attention among industry practitioners as a preferable alternative to traditional NPV (Schwartz and Trigeorgis [5]).

Different classes of real options have been defined in the literature as (Trigeorgis [27]) (i) abandonment option, (ii) switching option, (iii) deferral option, (iv) expansion option, etc. Several authors have attempted to define valuation frameworks and value these categories of real options and others. For example, Trigeorgis and Mason [28] examined the option to alter operating scale or capacity of an investment project. Kulatilaka and Trigeorgis [29] developed a framework to value the option to abandon a project for salvage. Ingersoll and Ross [30] examined waiting option in the presence of interest rate uncertainty. McDonald and Siegel [24] valued deferral option to temporarily shutdown and restart operations. Majd and Pindyck [32] developed a valuation framework for the deferral option in the context of construction projects.

3.2.1 Real Options in Energy Markets

The theory of real options provides an excellent framework for analyzing investments in physical energy assets. This realization has led to the development of different real options applications for power plants (Denton et al. [34], Thompson et al. [35], Tseng and Barz [36], Li et al., [37], and Leppard [38]), Transmission assets (Deng et al. [39] and Rosenberg et al. [40]) and gas storage facilities (Weston [41], Thompson et al. [42] and Maragos et al. [43]). Most of the value drivers in competitive power markets are uncertain. From the perspective of a power generator, values are derived through energy production and sales. In the presence of price uncertainties, the operator of a flexible power plant will switch operating strategies in response to power market fluctuations. If
technical operating constraints (such as switching costs) are ignored, such flexibilities can be conceptualized as simple call and put options on the spread between electricity and fuel. For instance, owning a gas-fired power plant can be considered “simplistically” as holding a long position in a series of financial ‘put options’ that are exercised only when the spot price of electricity is greater than the cost of natural gas. Under such a simplistic view, a rational asset manager, when faced with an option to generate below cost (i.e. negative spread\(^\text{19}\)), or idle the plant, will choose to idle his plant and allow the option to expire unexercised.

In reality, physical energy assets are characterized by multiple and interdependent real options. Among other things, the operator of a power plant has flexibilities with regards to (i) how to offer the plant’s output into the market, (ii) when to schedule inspection and maintenance outages, (iii) how much to spend on Life Cycle Management (LCM) and (iv) how to use emission credits. For example, the operator of an energy-limited hydroelectric plant has the option to strategically operate the plant by cycling its reservoir inventory. The reservoir can be drawn down to take advantage of immediate benefit from energy production (based on current market prices) or production can be deferred to later periods with the expectation that prices will be higher\(^\text{20}\). Also the owner may have the option to reserve some of its capacity for the provision of operating reserve to the market or offer all available capacity into the real–time energy market. This capacity reserve strategy can be thought of as selling (or “shorting”) call options to the system operator such that whether the system operator exercises its option or not (i.e. by activating the reserve or not), the generator gets to keep the premium it received. Also, the right amount of money to spend on plant maintenance and the time to shut down for maintenance are recurrent options that can affect future production costs and overall performance of a plant. Based on similar thinking, the decision to retire an aging plant or to make major capital additions to extend its useful life is a strategic option that depends on projected plant performance and market outlook.

\(^\text{19}\) The option with a payoff structure that depends on the difference between the values of two assets is known as ‘spread option’. The spread between the price of electricity and the price of the required fuel to generate it is referred to as ‘spark’ spread while the spread between crude oil price and refined petroleum products prices is referred to as ‘crack’ spread.

\(^\text{20}\) This strategic flexibility is also available to the owner of a gas storage asset
3.3 Valuing Financial and Energy Derivatives

3.3.1 Background on Derivative Instruments

A derivative instrument is a financial contract whose value or payoff depends on the value of an underlying asset e.g. stocks, commodities etc (Hull [13]). Typically, financial derivative instruments are used for one or more of the following purposes (Hull [13], Clewlow and Strickland [44], Eyedeland and Wolyniec [10], Skantze and Illic [45]):

• to reallocate risk exposures (i.e. hedging)

• to take advantage of arbitrage opportunities

• to speculate on future movements of market parameters for pure profit purposes.

Forwards/futures, swaps and options are the major classes of derivative instruments. A forward contract is an agreement between two counterparties to buy or sell an asset at a future date for an agreed price. Futures are like forwards, except that they are standardized contracts traded on exchanges (such as the New York Mercantile Exchange, NYMEX). Swaps are contractual agreements that allow two parties to exchange streams of cash flows e.g. a fixed interest rate for a floating interest rate (i.e. interest rate swap). Options involve the payment of a premium by one party to another to acquire a right (but without obligation) to buy or sell an asset to or from each other.

Historically, derivative instruments have been written on several types of underlying assets or market indices including stocks, bonds, weather, credit, foreign exchange and commodities among others. They are either traded over-the-counter (OTC) or on standardized exchanges. They may or may not include physical delivery of the underlying asset.

Energy market participants require derivative instruments on different types of underlying assets especially for hedging purposes. Examples include credit, foreign exchange, interest rate and weather derivatives. The term “energy derivatives”, however is commonly used in the literature to refer to those derivative instruments whose payoff depends on energy prices (or price indices) only (Clewlow and Strickland [44]).

The theoretical framework for valuing or pricing financial derivatives is based on the famous works of Black, Scholes and Merton (Black and Scholes [46], Merton [4]) on contingent claims analysis. The valuation framework is based on the premise that a claim on an asset (i.e. a derivative contract) is implicitly priced if the asset (i.e. the underlying) is traded. In their original work, Black and Scholes
[46] presented the value of a European call option and a European put option on the stocks of corporations by assuming that the price of stocks (i.e. the underlying asset) evolves randomly according to a stochastic diffusion process called the Wiener process or the Geometric Brownian Motion (GBM). They argued that in the absence of arbitrage, today’s fair price of a derivative contract is equal to the expected value of the future payoffs of the derivative, discounted at the risk-free interest rate. This conclusion was based on the assumption that the risk of a derivative contract could be hedged dynamically by continuously buying and selling the underlying asset. Based on this dynamic hedging strategy and the assumed stochastic process for the price of the underlying asset, the price of the contingent claim was shown via Ito’s Lemma\textsuperscript{21} (see Hull [13] for the derivation) to satisfy the following Partial Differential Equation (PDE) now called the Black-Scholes PDE:

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0
\]  

(3-1)

where \( V \) is the value of the claim on an underlying asset whose price is \( S \). The parameters \( t \), \( \sigma \) and \( r \) represent time, volatility of price return and the risk-free interest rate respectively. The evolution of \( S \) according to the GBM process is determined based on the following stochastic differential equation:

\[
dS = \mu Sdt + \sigma Sdz
\]  

(3-2)

Where \( \mu \) is the drift which is equal to the risk free interest rate and \( dz \) is the increment of the Wiener process in the interval \( dt \). By specifying appropriate boundary conditions, they were able to derive closed-form solutions to this PDE for European call and put options.

In general, all options whose payoff depends on the price of a single underlying asset and for which one can derive a closed form solution are generally referred to as “plain vanilla” options. All other options whose payoffs diverge from some of the basic assumptions of the Black-Scholes model are commonly referred to in the literature as exotic options. Examples include options (see Kaminski et al.[48]) with payoffs that depend on any of the following:

(i) the trajectory (i.e. path) of the price of the underlying asset as well as the price itself (e.g. Asian Options, Lookback Options, and Barrier Options),

\textsuperscript{21} Ito’s Lemma allows the finding of the differential of a function of a particular type of stochastic processes.
(ii) multiple underlying assets (e.g. Spread Options and Basket Options),

(iii) an underlying asset that is also an option (i.e. compound options).

Deng et al. [39] were among the first to present a methodology for valuing electricity derivatives by replicating their payoffs with futures contracts. They presented closed form expressions for the value of a range of derivatives, including spark spreads and locational spreads. Spread options are a general case of the exchange option\(^\text{22}\) first presented by Margrabe [49]. They are written to take advantage of price differential, either between two commodities (as in the case of spark spread and crack spread\(^\text{23}\)) or the same commodity sold at different locations (i.e. locational spreads) or at different calendar times (i.e. calendar spreads).

The swing option is another derivative instrument used for hedging commodity risks (Kaminski et al. [48]). By design, a swing option contract allows for multiple exercise rights during the life of the contracts. For instance, a swing contract on natural gas may allow the holder (e.g. a power generator) to deviate from taking a pre-agreed base volume of gas on more than one occasion (i.e. exercise periods) as requirement changes. Jaillet et al [50], Dahlgren [51] and Dörr [52] are among several authors that have presented valuation techniques for swing contracts. For valuing different types of spread options in general, Carmona and Durrleman [53] compiled an excellent review of available techniques for valuing different types of spread options. The books of Clewlow and Strickland [44], Eyedeland and Wolyniec [10] are excellent resources on energy derivatives and their valuation techniques.

### 3.4 Valuing Physical Assets

Physical assets in energy markets refer to power generating plants of different types, transmission and distribution lines, gas transportation pipelines, gas storage reservoirs and all other physical assets that are operated in energy markets. The market value today of an energy asset like a power plant is a function of the future cash flows derived by operating the asset. Future cash flows in turn depend on a number of factors including (but not limited to) electricity prices, variable costs of operation and

\(^{22}\) The exchange or swap option is the option to exchange one asset for another or the option to select the better of two assets.

\(^{23}\) Spark spread refers to the spread between electricity and natural gas prices while crack spread refers to the spread between crude oil and refined petroleum products.
maintenance, technical uncertainties and constraints as well as operating strategies. To determine future cash flows for an individual asset, the convention is to assume a perfectly competitive market where individual plant dispatch decisions do not have any noticeable impact on market prices (Ronn [12], Kaminski [11], Denton et al. [34], Thompson et al. [35], Tseng and Barz [36] etc ).

3.4.1 Valuation of Gas-Fired Power Plants

For an operator of a gas-fired power plant in a competitive power market, the desired objective is to maximize revenue by strategically operating the plant to capture value when the spread between power prices and gas prices (spark spread) are highest. Such an operator has the right to shut down the plant (by not offering to sell power) when the spread is unfavorable and start it up when favorable market conditions exist. Hence, a plant that is able to ramp up quickly in response to price movements has more value compared to a similar plant with low ramp rates. In reality, equipment constraints usually make it impractical to ramp up and down as often as price spread indication exists. One can interpret the impact of these constraints as adding to exercise costs for the flexibility options. This implies that switching decisions are inter-temporal in nature (i.e. current operating decisions affect the availability of future options). If one ignores these exercise costs, the spread option decision making at every period is independent, making the option values additive. Based on this approach, Deng et al. [39] proposed the following valuation formula for determining the value \( V \) of a gas-fired peaking plant:

\[
V = \sum_{t=1}^{T} E \left[ \max \left( \frac{S_e(t) - HR}{S_f(t)} \cdot S_f(t), 0 \right) \right]
\]

where \( S_e \) and \( S_f \) are the prices of electricity ($/MWh) and fuel ($/MMBtu) respectively, \( HR \) is the plant’s heat rate (MMBtu/MWh), \( t \) represents dispatch times and \( T \) is the valuation duration. The ratio \( \frac{S_e}{S_f} \) is referred to as the ‘market heat rate’ as compared to the ‘plant heat rate’ \( HR \) such that if the production process is thought of as a put option on the market heat rate with the strike price \( HR \), the plant is only turned on when the market heat rate is greater than the plant’s heat rate and turned off otherwise. The shortcoming of this simple spark-spread model proposed by Deng et al. [39] is that it leads to higher value than expected in reality as power plants cannot be turned on and off as often as desired without due consideration for physical operating constraints. For instance, thermal power
plants have physical limitations that constrain them to remain in an operating state for a minimum time period before the mode of operation can be changed.

To address this shortcoming, a number of authors have proposed more elaborate frameworks that incorporate the important operating constraints to different extents in the valuation problem. The inter-temporal nature of the decision making problem necessitates the use of a dynamic optimization technique in order to consider the impacts of current decisions on future decisions. For instance, Tseng and Barz [36] formulated the valuation problem as a multi-stage stochastic dynamic programming problem and solved it using a simulation based optimization technique. Similarly, Denton et al. [34] employed a trinomial lattice-tree backward induction technique. Thompson et al. [35] employed a continuous time framework and derived the value function as a partial integral differential equation (PIDE). The extent of operating constraints and power price characteristics incorporated by these different authors depend on the limitations of the numerical techniques employed. One key uncertainty factor that has not yet been considered in the literature however is the impact of operational differences between the gas market and electricity markets leading to gas supply/consumption imbalances for power generating plants. The occurrences of such imbalance on a frequent basis may result in additional costs and constraints that must be factored into valuation. By ignoring these costs and constraints, published models may result in over valuation. This issue will be analyzed in this thesis.

3.4.2 Valuation of Hydroelectric Plant

There are different types of hydroelectric schemes with the main uncertainties under regulation being precipitation (rainfall) and load demand. A quick survey of existing literature on hydroelectric plants optimization show an array of methodologies including linear programming, integer programming and dynamic optimization techniques with the objective typically being to maximize energy production or minimize costs (e.g. Chao-An et al. [54] and Sherkat et al. [55]).

Under market competition, the objective of the independent power asset owner is to maximize revenue. By applying options thinking, one can value a single hydroelectric scheme by considering the option of the asset operator to store water in a reservoir for future use with the expectation that market prices will be higher. Intuitively, the more flexibility there is to cycle water to capture market value, the more valuable a hydroelectric plant is. Based on this thinking, Thompson et al [42] and Doege et al [62] among others have developed real-options valuation models for valuing single (peaking type) hydroelectric plants operating in spot energy markets. While their frameworks capture
the option values of storage and cycling, there are additional flexibilities that were ignored. For instance, hydroelectric plants are highly suitable for the provision of ancillary services due to their fast ramping capability. In a market where a competitive ancillary services market exists (e.g. operating reserve market), a power generator will have the additional option of reserving some of its capacity for ancillary services. Such flexibility can be regarded as selling of call options to the Independent System Operator. Whether the system operator exercises the option to activate the reserve or not, the generator keeps the premium received and can make use of the reserved water at a later time. A strategic exercise of this type of option will increase the value of the plant and should be factored into economic evaluation of hydroelectric facilities. This issue is also addressed in this thesis.

3.4.3 Nuclear Power Plants Flexibilities

Nuclear Power Plants (NPPs) are typically baseload generating plants. Given their long startup and shutdown processes they are rarely shutdown except for period Life Cycle Management purposes which may include preventative maintenance and equipment inspection among others. The owner of a NPP therefore has the flexibility to strategically budget for LCM related activities. The amount of money spent on such activities is indirectly related to future plant performance and therefore may be a strong determinant of future cash flows. After operating for its initial design life, a NPP like other power plants may also be refurbished and the operator may seek an extension of its operating license. The alternative is for the plant to be retired and put in a “safe storage” mode leading to eventual decommissioning. The flexibility to choose plant life extension instead of safe storage and decommissioning is essentially a “deferral option” (Trigeorgis [27]. By choosing to refurbish a plant and re-license it, the owner is essentially deferring safe storage and decommissioning to future dates.

If one considers these flexibilities in the context of real options analysis, strategic decisions (including timing and spending decisions) around life cycle management and plant life extension can be assigned some values. For instance, if one assumes that the expectation of long-term market performance and future plant performance are the main drivers for these decision making problems, it is possible to optimize a plant’s operating strategy such that the present value of future cash flows can be maximized. Three recent publications from the International Atomic Energy Agency (IAEA) provide excellent discussions on several of the important issues regarding life cycle management and risk management for nuclear power plants (IAEA [56], IAEA [57], IAEA [58]). Pindyck [59] presented a real options analysis framework for investment decision making with regards to the
construction of Nuclear Power Plants (NPPs) given high uncertainty in construction costs.

3.5 Optimizing Asset Portfolios and Quantifying Risks

Unlike typical trading portfolios in equity markets, portfolios of power market participants typically include the following types of positions:

(i) financial-only positions (typically for hedging purposes)

(ii) asset-backed financial contracts i.e. contracts requiring physical asset delivery such as Power Purchase Agreements (PPAs), bilateral supply contracts etc

(iii) physical assets such as power generation plants and gas storage facilities.

The portfolio optimization problem in competitive electricity markets is therefore one of strategically dispatching power generation assets while taking appropriate positions in the financial markets such that total revenue is maximized subject to acceptable financial risk level. Typically, the suitable appropriate approach to risk hedging depends on the particular risk and availability of hedging instruments. Some risks exposure can be hedged using financial derivative contracts \(^{24}\) while others may only be hedged through the use of traditional insurance instruments. Modeling this problem in a consistent and robust fashion can be very challenging as recognized by various authors (Kleindorfer and Lide [60], Leppard [38], Clewlow and Strickland [44], Denton et al. [34], Doege [62]).

The common and the simplest approach to this portfolio optimization problem involves performing a single period (or a strip of single periods) optimization with the objective of selecting optimal weights of the different assets such that the value of the portfolio is maximized subject to a specified risk preference constraint. Doege et al. [62] used this type of approach to value a portfolio of a nuclear power plant and a pump storage hydroelectric facility. Theoretically, single period optimization results can be stacked to capture multi-period effects but the dynamic nature of the decision making will not be captured. A more advanced approach than single period linear optimization was presented by Sen et al. [61] to optimize dispatch and hedging decisions of a large power asset portfolio in a competitive market. The approach is based on a multi-stage stochastic integer programming framework that combines recursive optimization of power plant’s dispatch in a spot market with a forward-moving scenario analysis of hedging decisions via decision trees. It does not, however,

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\(^{24}\) A derivative instrument by definition is a financial contract whose value depends on the value(s) or prices of other, more basic underlying market variables (Hull [13]).
consider the interdependence between various hedging strategies. Another important factor to consider with portfolio optimization problems is the validity of the assumption of perfect competition where the activities of independent portfolio owners do not affect the behavior of the rest of the market and hence market prices. It is not uncommon to have markets with a dominant player (usually a large utility) along with a number of much smaller participants. When relative portfolio sizes are such that perfect competition can not be assumed, the concept of strategic games (simultaneous and sequential) may be required to solve portfolio optimization problems of this nature. Recently, a new concept called “option games” is being explored in the literature for combining real options analysis with game theory (Dias and Teixeira [63]).

### 3.5.1 Incorporating Risk Measures

A number of methods for measuring and reporting energy portfolio risks have been suggested by different authors. The common methods include Earnings at Risk (EaR) and Value at Risk (VaR). VaR is suitable for portfolios of purely financial assets (i.e. trading portfolios) and is defined as the maximum loss that a portfolio can sustain over a specified period of time at a given confidence level. The applications of VaR in energy markets have attracted a lot of discussions e.g. see Senor [64], Kleindorfer and Lide [60], Dahlgren et al. [68] and Ojanen et al. [67], Clewlow and Strickland [44]). The common approaches for calculating portfolio VaR include variance-covariance method, historical simulation, and Monte Carlo technique (Dahlgren et al [51]). Being a single period risk measure, one of the main challenges of using VaR has to do with incorporating VaR constraints into multi-period optimization modeling framework.

Cashflow or earnings based risk measures which have also been used traditionally in financial markets (see Artzner et al. [70]) have been proposed as being more appropriate for energy firms with physical power generating plants as opposed to pure financial instruments. These include ‘Earnings at Risk’ (EaR), ‘Profit-at-Risk (PaR)’ and ‘Cash flow-at-Risk (CFaR)’. Clewlow and Strickland [44] proposed a heuristic approach that uses EaR to assess energy portfolio risks. The approach involves simulating the total cashflows of the portfolio in a period of interest using asset specific techniques for individual assets. The cashflows are then aggregated into desired time intervals such that the net earnings distribution of the portfolio can be assessed at different intervals of time to understand the relative merits of the various assets in the portfolio. If the distribution of earnings to risk ratio is not satisfactory, the operating strategies can be adjusted and the hedging decisions revised. Leppard [38] had also suggested a similar approach based on VaR measure where a so called Technical VaR can be
defined for each key driving factor and used for stress testing on the portfolio’s value.

3.6 Modeling Market Prices and Other Uncertainties

Investment analyses in energy markets require modeling of market prices of power and fuel as well as other uncertainty factors such as load and precipitation. In general, the same models that are used for market prices can be adapted for other type of uncertainties. In the following subsection, we describe the common modeling approach in the literature from the perspective of market prices.

One can classify the common modeling methods as follows:

(i) quantitative models
(ii) statistical models
(iii) fundamental models and
(iv) hybrid models and others

3.6.1 Quantitative Models

Quantitative models for electricity and volatile fuel prices were inspired by finance methods for modeling the dynamics of stock prices. These models rely on stochastic processes to describe the evolution of prices (and other uncertain market drivers) by assuming that they evolve randomly. A stochastic process can be defined in continuous time or in discrete time frames depending on which is suitable for the particular problem being addressed.

The basic stochastic process for asset pricing in finance is the pure diffusion process or the Geometric Brownian Motion (GBM) process. The Geometric Brownian Motion process is a solution to the following stochastic differential equation:

\[ dS_t = \alpha S_t dt + \sigma S_t dz_t \]  \hspace{1cm} (3-4)

where the drift \( \alpha \) and volatility \( \sigma \) are constants. This equation models the change in the random variable \( x \) at time \( t \) over an infinitesimally short time increment \( dt \). The variable \( z \) is the standard Brownian Motion (or the Wiener process), hence \( dz \) is the increment of the Wiener process\(^{25}\) at

\(^{25}\) The Wiener process of a random variable \( z \) is defined mathematically as the set \( \{z_t, t \geq 0\} \) where the increments in \( z \) are stationary and independent (i.e. Markov) and \( z_t \sim N(0,t) \).
time $t$. More general forms of this process include time varying drift and volatility (3-5) and stochastic drift and volatility (3-6). When either one of the parameters $\alpha$ and $\sigma$ are themselves stochastic, the model is said to be a multi-factor model.

$$dx_i = \alpha(t)S_i\,dt + \sigma(t)S_i\,dz_i$$  \hspace{1cm} (3-5)

$$dx_i = \alpha(t,x_i)S_i\,dt + \sigma(t,x_i)S_i\,dz_i$$  \hspace{1cm} (3-6)

If $S$ in equation 3-4 is the spot price of a stock, $\alpha$ is the constant growth parameter (i.e. the drift) and $\sigma$ is the constant volatility of the stock’s return\(^{26}\), then the price increment $dS$ of the stock at an infinitesimally small time interval $dt$ according to 3-4 is given by the following stochastic differential equation:

$$\frac{dS_t}{S_t} = \alpha \, dt + \sigma \, dz$$  \hspace{1cm} (3-7)

The first term on the right hand side (RHS) of 3-7 captures the deterministic trend in the evolution of the stock price while the second term captures the uncertainty in the evolution. The price $S$ is log-normally distributed. In a risk-neutral world, the drift $\alpha$ becomes the risk-free interest rate $r$. If the stock is a dividend paying stock at rate $\delta$, then 3-7 becomes,

$$\frac{dS_t}{S_t} = (r - \delta) \, dt + \sigma \, dz$$  \hspace{1cm} (3-8)

Given the current price of the asset at $t$, the price $F_t$ of a forward contract on the stock asset can be derived based on a no-arbitrage argument since in the absence of arbitrage, the stock will earn the risk free rate between $t$ and $T$. Hence,

$$F_t = S_t e^{(T-t)}$$  \hspace{1cm} (3-9)

where $T$ is the contract maturity and $S_t$ is the integral of equation 3-8. For a dividend paying stock where $\delta$ is the dividend rate, the forward price is modified by deducting the dividend rate from the risk free rate.

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\(^{26}\) Volatility of a stock is a measure of the uncertainty of the stock’s return, typically on an annual basis. It is measured as the standard deviation of the log of the returns provided by the stock.
For commodities (such as gas and oil), it has been shown that the price dynamics do not obey the pure diffusion process (Schwartz [4]). This is because prices in commodities markets tend to revert to long-term averages, a phenomenon known as “mean reversion”. Essentially when prices are high, supply pressures and low demand tend to push them down with the opposite effects occurring when prices are low.

Instead of the pure diffusion process, the Ornstein-Uhlenbeck (OU) process is used to model the dynamics of commodity prices (Dixit and Pindyck [6], Schwartz [4]). The arithmetic OU process is the solution to the following stochastic differential equation where \( x_t \) follows a normal probability distribution:

\[
dx_t = \eta(\bar{x} - x_t)dt + \sigma dz_t,
\]

where the drift \( \alpha \) is now \( \eta(\bar{x} - x_t) \). The drift term is positive when \( \bar{x} > x_t \) and negative vice-versa. As a result, the process reverts back to a long-term mean of \( \bar{x} \) at the rate \( \eta \) called the mean reversion rate or the speed of adjustment.

In the absence of arbitrage, the forward price on a non-perishable commodity is not only a function of the risk free rate (or cost of financing the purchase) as in equation 3-9, but also the cost of storing it otherwise known as the cost-of-carry and the convenience yield (Eyedeland and Wolyniec [10]). The term “convenience yield” is used to represent the direction of the supply and demand fundamentals of a commodities market and it represents the term \( \delta \) in equation 3-9. If consumers are worried about future supply of a commodity, the convenience yield is low and the forward curve is upward sloping (since \( r > \delta \)); the curve is then said to be in “contango”. However, with an immediate supply crunch, the convenience yield is high and the forward curve becomes downward sloping, the phenomenon called “backwardation”.

Unlike other commodities, electricity can not be purchased and stored in large quantities to the extent that inventories can be used to absorb supply/demand shocks. As a result, the concept of convenience yield does not directly apply to electricity. Also, a realistic model of electricity price must be able to capture the frequent spikes observed in electricity prices which lead to fat tails in price distribution. To this effect, Clewlow and Strickland [44] proposed that a jump component be incorporated into the Schwartz [5] or the Dixit and Pindyck [6] model of commodity prices by incorporating another stochastic term (assumed to follow a Poison process) in the mean-reverting model. The ensuing
model, referred to as the Mean Reversion-Jump Diffusion (MRJD) model can be written as follows:

\[ dx_t = \eta(\bar{x} - x_t) dt + \sigma dz + \kappa dq \]  

(3-12)

where \( dq \) is the increment of a Poisson process. The Poisson process here models discrete jumps of random log size \( \kappa \). If \( dq \) equals 1, there is an occurrence of a jump event and vice-versa. If we define \( \phi \) as the intensity of the Poisson counter, then \( dq \) equals 1 with probability \( \phi dt \) or 0 with probability \( 1 - \phi dt \). The log jump size \( \kappa \) follows a normal distribution i.e.:

\[ \kappa \sim N(\bar{\kappa}, \gamma^2) \]  

(3-13)

where \( \gamma \) is the standard deviation (i.e. jump volatility) of the log jump size and \( \bar{\kappa} \) is the mean log jump size. The forward price of electricity can be derived by finding the expected value of the spot price at maturity (e.g. see Cartea and Figueroa [72]).

### 3.6.2 Fundamental Models

An alternative approach to modeling power prices is to consider the fundamental market drivers that determine price movements in the short and long time frames. This approach models price evolution by modeling the fundamental relationships between the key underlying factors that determine market equilibrium prices in the short and long term. These key market fundamental factors include (i) load demand (ii) weather (iii) fuel prices (iv) planned new build and planned plant retirement (v) forced and planned outages (vi) imports and exports of energy, (vi) transmission network parameters (vii) operating costs (viii) market rules etc. By considering the fundamental relationships between these key factors, market supply and demand curves can be derived as functions of time and solved to determine prices at equilibrium point.

The time evolution of the different fundamental factors is typically modeled using parametric methods. For instance, one can model load demand by capturing the relationship between load and weather. Similarly, one can combine available data (or models) of generators’ marginal costs, capacity availability (which depends on such factors as precipitation, planned and forced outages etc) and offer strategies to model the system supply curve. Similar approach can be used to model imports and exports into and out of the target market. An example of this modeling approach is discussed in Li and Chiu [73].

The major shortcoming of fundamental models is their reliance on data that are often not available.
publicly e.g. marginal costs data and planned outages of generators which makes them proprietary in nature. A firm developing such a price model will typically rely on their best judgment of other market participants’ offer strategy and proprietary information. Also, fundamental models do not typically capture price spikes. This is because price spikes are usually caused by sudden and temporary supply and demand imbalances; hence a model that relies on short to long-term behaviors of market fundamental factors will not be able to capture them.

3.6.3 Statistical Models

Like quantitative models, statistical models are also data driven and represent prices as functions of different parametric factors. However, the main premise of this class of models is to identify patterns and relationships from historical market data using time-series autoregression regression techniques (Weron [1], Zareipour [9]) such as the Autoregressive Conditional Heteroskedasticity (ARCH). The procedure usually involves identifying the model structure and variables and fitting the data to the desired model. The book of Weron [1] provides an excellent introduction to common statistical approaches for forecasting power prices and other market parameters such as load demand. The shortcoming of this type of models in the reliance on historical market data which does not necessarily represent future changes in market fundamentals.

3.6.4 Hybrid Models

Hybrid models, by their name seek to combine the different aspects of quantitative, statistical and fundamental models. For instance, Li and Chiu [37] proposed a so-called ‘time decomposition’ approach that relates electricity price dynamics based on stochastic processes to information arrival on market fundamentals. Davison et al. [74] also developed such a hybrid approach to modeling electricity prices. The premise of hybrid models is to combine the advantages of quantitative models which are based on stochastic processes with fundamental models. Derivative pricing techniques are based on quantitative models which assume that prices evolve randomly according to one or more stochastic processes. However, where reliable forward or futures prices are not readily available, quantitative models rely only on historical data for calibration and so may not capture future changes in market fundamentals.

3.7 Numerical Methods

For valuation problems for which analytical solutions can not be derived easily, a number of numerical techniques exist for solving them. One can broadly classify these techniques as follows:
• Finite difference method
• Lattice or tree methods i.e. binomial trees and trinomial trees
• Monte Carlo simulation (i.e. simulation-based optimization)

The finite difference method was first applied to solve derivative pricing problems by Brennan and Schwartz ([75],[76],[77]). The idea behind the finite difference approach is to approximate the value of a contingent claim as defined by the parabolic Black-Scholes PDE or its extensions (see equation 3-1) using the following steps:

• the state variables are discretized as desired
• The PDEs (first and second derivatives) are approximated using finite difference, transforming them into difference equations.
• The difference equations are solved iteratively, starting from the specified final value conditions.

While this approach can be computationally efficient (in terms of speed), it is highly limited in the number of underlying stochastic factors that can be considered or the number of uncertainties modeled without introducing great complexity (Barraquand and Martineau [80]). This limitation makes finite difference methods impractical for valuing assets with complex optionalities having multiple stochastic underlying variables. In addition, the complexity of the stochastic differential equations (e.g. when multi-factor variables are considered or a discontinuous jump process is involved) affects the practicality of implementation of finite difference approaches (Tsekrekos et al. [81], Sabour and Poulin [82]). For these reasons, it is difficult to implement a finite difference technique without having to make broad simplifying assumptions to the valuation model in order to manage complexity.

The lattice or tree approach was first introduced to option pricing by Cox et al [84]). It has been argued in the literature (Kamrad and Ritchken [85]) that the methodology is a special case of the finite difference method; hence it suffers the same limitations as finite difference. Particularly, the number of lattice nodes required for valuation grows geometrically with the number of specified time steps and underlying stochastic factors. However, it has the advantage of ease of implementation which makes it the most popular option pricing approach.
Monte Carlo simulation was first introduced by Boyle [86] for valuing European options. It works by directly simulating the time evolution of the underlying stochastic drivers of the asset using random number generation techniques. Several paths are simulated for each stochastic factor and the option payoff is computed for each simulated path. The value of the option is computed as the average of the values obtained from the different price paths.

Monte Carlo simulation was originally applied for valuing European options only. However, a number of simulation-based algorithms have recently appeared in the literature for valuing options with early exercise potentials and for solving dynamic programming problems (Glasserman [87]). The flexibility provided by simulation based approaches for options or real options valuation which makes it possible to avoid excessive modeling simplification is its main attraction. Since Monte Carlo methods directly simulate the stochastic processes of the underlying variables, it is not necessary to first transform those stochastic processes into a PDE.

The main shortcoming of Monte Carlo simulation is its relatively slow computational speed compared to finite difference and a nodal tree approaches. However, there are a number of techniques that have been developed over the years to improve computational speed and accuracy of Monte Carlo methods, including variance reduction techniques and control-variate approach among others (Hull [13], Glasserman [87]). In addition, given the significant reduction in costs of computing that has been achieved over the last decade, many practitioners are harnessing the power of parallel computing to further enhance computational speeds.

### 3.8 Contributions of Research Work

To analyze investments in competitive power markets, the value of flexible managerial decision making with regards to plant operations and capital budgeting must be considered and quantified. Real options analysis provides the appropriate framework for capturing and quantifying the values of such flexibilities. The particular approach depends on the context or purpose of analysis. For instance, if one is interested in the economic viability of new power plant construction project, the values of the following key managerial options may be relevant:

(i) option to defer construction (wait and see option)
(ii) option to abandon the project all together
(iii) option to scale up or scale down capacity as information regarding market and technical uncertainties arrives.
(iv) option to build the power plant capacity in modules (i.e. building smaller units in stages)

However, if one is interested in determining the value of energy output of a power plant for the purpose of negotiating a Power Purchase Agreement (PPA) or one is interested in business planning for an operating asset, the relevant managerial options to value are related to operating flexibilities. For example the operator of a hydroelectric facility with a storage reservoir has the option to strategically cycle the reservoir to peak and/or super-peak the plant and maximize revenue. Depending on storage size, he may also be able to time-shift water across months and seasons. These flexibilities increases the value of a plant.

With regards to operating flexibilities, current applications of real options theory to power assets tend to be limited to peaking power plants in terms of their flexibility to ramp up and down in response to market price movements. The tendency to focus on peaking plants seem to arise from the belief that real options valuation method applies only where the underlying uncertainties are driven by market traded factors only such as electricity and gas prices. However, as pointed out by a number of authors, technical uncertainties also drive optionalities (Schwartz and Trigeorgis [5]). For instance, the value of investments in baseload and mid-merit power plants are driven by life cycle management activities such as preventative maintenance, equipment inspection and refueling. Flexibilities around these activities are managerial optionalities that must be considered in valuing outputs of baseload and mid-merit power plants.

Similarly, published applications of real options to power assets tend to be limited with respect to the types of optionalities and uncertainties considered in valuation. The implication is that some of the developed applications do not fully reflect the reality of power markets environments. For instance, most of the existing real options applications to power plants only consider the asset operator’s option to optimize the plant’s energy output in an energy-only market. In reality, however, owners of qualified power plants may have the option to participate in an ancillary services market. This type of opportunity is available in Ontario where a joint energy and operating reserve market exists. Also, the owner of a power plant may have the opportunity to optimize the plant’s output across two market jurisdictions with different market rules. Neglecting the values of such options may lead to undervaluation of a power plant’s output.

Lastly, there is also a tendency in the literature to always attempt to adapt power plants valuation problems to the popular option valuation technique of contingent claims analysis which uses finite difference techniques to solve Black-Scholes type Partial Different Equation (PDE) that describes the
value of the contingent claim (in this case the value of the power asset). One reason for this approach is the desire to derive either analytical solutions or to use techniques with great computational speed (alternative methods usually involve simulation which until recently were only applicable for valuing European type options). The disadvantage with this approach is that the process of fitting valuation problems to a modeling technique typically involves over simplification that are not necessarily consistent with the true reality of power market environments. As pointed out by Longstaff and Schwartz [78] and several others, finite difference and nodal tree techniques are only convenient for solving option pricing problems with one to two state variables in one or two dimensions.

The focus of this work is to develop applications of real options valuation techniques for different power assets based on a general modeling framework. We extend previous work in this area with the intent of capturing the values of commonly available managerial flexibilities for both baseload and peaking power assets while taking into consideration the realities of competitive power markets and equipment/operating constraints. Illustrative studies are developed and solved using the recently developed Least Squares Monte Carlo technique for solving American-type options (Longstaff and Schwartz [78]).

Specifically, a general modeling framework is developed in Chapter 4. The framework is then applied to value three different power plant types operating in a competitive market environment using the Ontario wholesale electricity market as a backdrop. In chapter 5 we present a model to value gas-fired power plants in the presence of gas supply/consumption imbalance uncertainties. The costs and operating constraints posed by the imbalance between real time electricity markets and day-ahead gas markets have so far been ignored in gas plants valuation problems. In chapter 6, we present a valuation model that considers the operation of a peaking asset in a joint energy and operating reserve market. Similarly, in chapter 7 we develop a model for analyzing nuclear power plants given that the owners of such plants have the right to optimize life cycle management spending in response to cost or power price uncertainties. This includes the flexibilities of the owners with regards to scheduling maintenance and inspection outages and specific maintenance projects to undertake.

The general framework developed can be applied to value other assets such as gas storage facilities, and physical transmission rights among others.
Chapter 4
Proposed Valuation Framework

4.1 Real Options Valuation Framework

A general framework for valuing switching flexibilities in manufacturing systems was proposed by Kulatilaka and Trigeorgis [29] (based on early published works of Kulatilaka [26], Triantis and Hodder [79] among others) whereby future cash flows from such plant systems depend on the ability of the operators to switch between operating modes as uncertainties are resolved. The premise of the framework is that plant owners or operators observe the evolution of the underlying drivers of cash flow and then make the choice to switch from one operating mode to another whenever it is economic to do so. An example is a manufacturing system with two alternative input feed stocks whose prices are uncorrelated.

The same reasoning applies to power assets in general. For example, the owner of a peaking power plant can switch between “on” and “off” modes (i.e. startup and shutdown) in response to market price movements within the technical limitations of the plants. Similarly, the owner of a baseload power plant such as a Nuclear Power Plant (NPP) has the flexibility to temporarily shutdown operations for equipment inspection and/or maintenance. In the same manner, flexibility exists with regards to how to use the inventory of a gas storage facility or a hydroelectric reservoir or when to flow power over a transmission line for which a firm transmission right exists. By building on this concept of “switching options” values, one can analyze investments in different power assets by treating different operating modes as alternatives.

Following Kulatilaka and Trigeorgis [29], consider a power asset whose revenue fluctuates only with power prices. Assume that at every time period, price $S$ can either increase or decrease according to a binomial tree (Cox, Ross and Rubinstein [84]) as shown in Figure 4 for three decision periods. The binomial tree assumes that the price $S$ can only move up or down at every time step by a constant factor. If the operator of the power asset does not have any flexibility to switch operating

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27 The representation in Figure 4 is such that an upward movement is represented by a + sign while a downward movement is represented by a – sign. Hence two consecutive upward price movements are represented by ++ such that the price at that period will be $S^{++}$. The subscript of the price variable shown in Figure 4 represents
mode, future cash flows only depend on the evolution of the market price (i.e. linear derivative). If we
define net cash flows (or payoffs) at each period as \( B(k)^{\zeta_i} \), where \( i \) is the decision period, \( k \) is the
operating mode and \( \zeta \) is the price state i.e. +, -, ++, -- and +/-/+-, then the possible net cash flows at
each of the three decision periods are as shown in Figure 5. By discounting the expected cash flows
at each period back to the present, the value \( V \) of this asset is obtained as the present value \( PV(k) \) of
the discounted cash flows. In a complete market\(^{28}\), these cash flows will earn the same rate of return
as other cash flows (or securities) in the market which is the so-called “risk-free rate”. Hence the
present value depends only on the possible states of nature of the single underlying price (given that
the operating mode \( k \) is fixed) and the risk-free rate.

If two alternative operating modes are available to the operator, say \( k1 \) and \( k2 \) then that operator
has the flexibility (or real option) to switch from one mode to the other when it makes economic
sense to do so. At each decision period (assuming that the option is available at every decision node),
the operator will consider whether to switch operating modes with the objective of maximizing value
such that if there is no benefit to switching modes, the current operating mode will be continued.
Hence at every node of the binomial tree, the objective is to maximize the following nodal option
value function (see Figure 6).

\[
\text{Nodal Option Value} = \max\left(B^{\zeta_i}(k2) - B^{\zeta_i}(k1), 0\right) \quad (4-1)
\]

or

\[
\text{Nodal Option Value} = \max\left(B^{\zeta_i}(k1) - B^{\zeta_i}(k2), 0\right) \quad (4-2)
\]

That is, if there is no incentive to switch, the operator continues at the current mode.

In the absence of irreversible switching costs, Kulatilaka and Trigeorgis\[29\] argued that the
switching flexibility at each time period is a European type option expiring at that time. Hence, the
Real Option Value (\( ROV_i \)) for the entire time period is the sum of the discounted\(^{29}\) expected values

\( S^{++} \) will refer to the price in period 2 if a state ++ occurs

\(^{28}\) A market is said to be complete if it is possible to replicate the cash flows of all contingent claims by being
able to instantaneously bet on any future state of the market.

\(^{29}\) Again, under a complete market assumption, arbitrage is precluded which implies that the payoffs should be
discounted at the risk-free rate of return. Under this assumption, the probabilities of the up and down
movements under the binomial tree (as shown by Cox, Ross and Rubinstein\[84\]) are the risk-neutral probabilities.
of all in-the-money European options available in the time period.

This implies that the total real option value is the sum of the individual real option value at each decision node i.e.

\[ ROV = \sum_i ROV_i \] (4-3)

Therefore, the value of the asset in the presence of the flexibility to switch from operating mode \( k1 \) to \( k2 \) then becomes:

\[ PV = PV(k1) + ROV \] (4-4)

Where \( ROV \) is the option value to switch from \( k1 \) to \( k2 \). If we could switch back from \( k2 \) to \( k1 \), equation 4-4 will also be true in that case. This implies that:

\[ PV \geq PV(k1) \text{ or } PV(k2) \] (4-5)

i.e. the present value of the system output when switching is considered is at least equal to the present value when operation is carried out only in one mode.

In the presence of non-zero irreversible switching costs, the option additivity rule in equation 4-3 breaks down due to the so called “hysteris effect” (Dixit [21], Kulatilaka and Trigeorgis [29]). This is because the option values at the different decision nodes are no longer independent. With switching costs, exercise of the switching option at one decision period may affect available set of switching options at subsequent periods as well as cost of switching leading to compound style options. This implies that optimal operating policy may support waiting to exercise switching as opposed to performing an immediate switch.
Figure 4: Binomial Tree Evolution of Power Price

Figure 5: Evolution of Cash Flows for each of the three Decision Periods.
Figure 6: Option Decision Making at Every State of Nature

To apply this framework to a realistic physical asset such as a power plant, besides the price states (and other states of the world that occur independent of the asset operator’s decision making strategies), the physical state of the asset and operating constraints must also be considered. To illustrate this point, assume operating modes $k_1$ and $k_2$ represent ramp up and ramp down respectively. Also consider a third operating mode $k_3$ which represents an idle decision (i.e. do nothing). If we assume that the physical state of the asset at any point in time can be defined by three discrete states $U$, $M$ and $D$ representing “On” at maximum capacity (i.e. Up state), “On” at mid-capacity (i.e. Mid state) and “Off” (i.e. Down state) respectively, the decision making problem can be analyzed by examining the available flexibilities in conjunction with possible state transitions between three physical states and the asset’s operating constraints. Three key characteristics of the problem can be identified as follows (based on Figure 7):

(i) The same set of switching options is not necessarily available at all decision periods. For example, if the asset is at the physical state $U$ it is already at maximum capacity; hence the option to ramp-up to a higher capacity is not available. The same is true for the state $D$ where the option to ramp-down to a lower capacity is not available.

(ii) A choice of mode $k_2$ at the physical asset state $U$ will transition the asset to the state $M$ by the next decision period. Choosing $k_2$ at state $M$ will transition the asset to state $D$ while
choosing \( k_1 \) will transition to state U.

(iii) There may be operating constraints that will prevent an option from being feasible at a decision period. For instance, thermal power plants usually have requirements for minimum shutdown period and up-time period. This implies that the plant may be required to remain at state D for more than a single decision period and at state U for more than a single decision period.

Conceptually, one can imagine the presence of operating constraints and different physical asset states as contributing to switching costs resulting in the decision making at each period being dependent on the discounted expected value of the benefits from subsequent periods (this is the phenomenon referred to as “hysterisis” by Dixit [21]). The implication is that future cash flows are non-linear\(^{30}\) functions of the underlying state variables. Therefore, one cannot value this type of asset using a forward-looking approach unless the optimal operating strategy has already been pre-determined. This is why a dynamic optimization method is necessary as already pointed out by many authors e.g. Dixit [21], and Kulatilaka and Trigeorgis [29]. Also, since the future evolution of the key value drivers are stochastic in nature; a stochastic optimization method is required for valuing power assets with switching options.

\(^{30}\) i.e. path dependent
The other important distinction is with regards to the discount rate. Under a complete market assumption, in order to preclude arbitrage, future payoffs from any derivative (or claim) on a traded underlying asset must be discounted at the risk-free rate and the asset price future evolution modeled under the risk-neutral probability measure. This assumption is premised on the possibility of being able to replicate the cash flows of any derivative instrument, making it possible to continuously hedge any risk exposure of the derivative asset e.g. by continuously trading in the underlying asset as shown by Black and Scholes [46]. However, given that markets such as that of electricity cannot often be
argued to conform to the complete market assumption\(^{31}\) and that uncertainties driving real options on physical assets are not necessary derived from tradable cash flows, the assumption of risk-neutral valuation is often difficult to justify for real options problems.

A number of approaches have been suggested in the literature for dealing with the issue of discount rate in real options valuation (e.g. see Schwartz and Trigeorgis [5], Li and Chiu [37]). These include:

(i) Assume that the payoffs can be perfectly replicated with traded instruments e.g. by using futures contracts for electricity and gas.

(ii) The concept of investor utility function can be applied to incorporate the risk aversion of the asset owner.

(iii) The concept of hurdle rate or risk-adjusted discount rate (such as the Weighted Average Cost Capital (WACC) or a discount rate derived from the Capital Asset Pricing Model, CAPM) can also be used.

The question of which of these alternatives is the most appropriate for real-options valuation is a major subject of debate among researchers in the real options field. However from the perspective of industry practitioners, a risk-adjusted discount rate such as the WACC or a project specific hurdle rate is easily obtainable. While any one of the foregoing approaches can potentially be used within the context of the framework proposed here, for the purpose of this thesis we assume that an exogenously specified risk-adjusted discount rate is available. Even though it is still possible to employ risk-neutral valuation with a risk-adjusted discount rate if one can calculate the so called “market price of risk” (i.e. risk premium) for the derivative under valuation, for the purpose of this thesis, we use “real simulation” (as opposed to risk-neutralized simulation) of underlying stochastic processes and discount using an assumed risk-adjusted discount rate.

In summary, consider the operation of a physical power asset as a multi-stage decision making problem where at every decision stage, the operator has to make a decision to alter operating mode or remain at the current mode. If we assume that there is a finite set \(K\) of such mutually exclusive alternatives, one can make use of Bellman’s optimality condition to determine the value of the asset.

\(^{31}\) For example, most electricity markets are not completely deregulated and there may be significant illiquidity of financial instruments. Even if it is possible to hedge financial risks, physical risks may remain which can not be completely hedged away.
If an option \(k \in K\) is selected at every stage \((t)\), the operating profit (or the benefit) \(B_t(k)\) at stage \(t\) is a function of the states of the world at \(t\) and the operating option selected at \(t\). The current period benefit \(B_t(k)\) is known at time \(\tau\) but future benefits are unknown. The objective of the asset operator at time period \(\tau\) is to maximize the following value function:

\[
V_{\tau}(\xi_{\tau}) = \max_{k_{\tau}} \mathbb{E} \left[ \sum_{t=\tau}^{T} d^{t-\tau} \times (B_t(k_t))|\xi_{\tau} \right] \tag{4-6}
\]

where \(\xi_{\tau}\) represents the known state of the world at time \(t\), \(T-\tau\) is the valuation period, \(\tau\) is the current time and \(d^{t-\tau}\), the discount factor is:

\[
d^{t-\tau} = \frac{1}{(1+r)^{t-\tau}} \quad \text{where} \quad t = \tau, \ldots, T \tag{4-7}
\]

and \(r\) is the risk-free\(^{32}\) interest rate or the risk-adjusted rate of return as discussed earlier.

According to the Bellman’s equation, the expected value function for each alternative can be expressed as follows:

\[
V_t(\xi_t, k_t) = B_t(\xi_t, k_t) + d \times \mathbb{E}[V_{t+1}(\xi_{t+1})|\xi_t] \quad \text{for} \quad t \leq T-1 \tag{4-8}
\]

and

\[
V_T(\xi_T, k_T) = B_T(\xi_T, k_T) \tag{4-9}
\]

where \(k_t = 1, \ldots, K\)

i.e. at each decision point, other than the final period, the operator maximizes the sum of the current benefit and the discounted expected future benefits, conditional on the current states of nature. For instance, if 2 mutually exclusive operating modes are available, the objective at each decision point is to find the alternative that maximizes the following and select it as the operating mode:

\[
V_{1,t}(\xi_t) = B_{1,t}(\xi_t) + d \times \mathbb{E}[V_{t+1}(\xi_{t+1})|\xi_t] \tag{4-8a}
\]

\[
V_{2,t}(\xi_t) = B_{2,t}(\xi_t) + d \times \mathbb{E}[V_{t+1}(\xi_{t+1})|\xi_t] \tag{4-8b}
\]

\(^{32}\) In practice, the yield on a short-dated government bond e.g. US Treasury bill is used as a proxy for the risk-free interest rate.
At the final period $T$, as stated in equation 4-9, $V$ equals the intrinsic benefit at $T$ which can easily be calculated from the knowledge of $\mathcal{B}(\varsigma_T, k_T^*)$, hence the option that maximizes $V$ at that time is known. Therefore, one can start solving the alternative specific value functions and find the optimal decision at each decision time, starting at $T$ and stepping back (i.e. recursively) until the current time is reached. Once the set of optimal decisions are determined for every decision node, equation 4-6 can then be applied to value the asset.

In order to determine the value of the current benefit term at each decision node and to know which mode $k$ (or operator control action) is feasible at $t$ in state $\varsigma$, one has to model the dynamics of the physical asset and the underlying state variables (aggregated as $\varsigma$). Hence one has to know the possible state transitions given the constraints of the physical asset. The physical states of the assets for instance can mean possible output levels, storage levels or thermal states of a boiler (depending on the asset being modeled) which are typically subject to equipment or operating constraints. The states of nature of the underlying factors can refer to possible values of electricity prices or gas prices.

4.2 Numerical Solution – The Least Squares Monte Carlo Method

The set of equation depicted in 4-8 (for all available alternatives) has to be solved at every decision node to determine the optimal option at every node subject to all applicable constraints and conditional on currently available information regarding the states of nature. The physical constraints determine what option is available at each node.

The difficulty in solving equation 4-8 for each alternative arises from the conditional expectation term (i.e. continuation value) which is unknown at time $t$. One approach (among others) to this problem is to determine the state transition probabilities from the simulated paths of the underlying variables and to use the simulated probabilities to approximate the continuation value as proposed by Barraquand and Martineau [80].

Another simple approach proposed by Longstaff and Schwarz [78] involves directly approximating the conditional expectation function by regressing it onto a set of simple basis functions. That is, if $E[|\varsigma_t]$ represents the conditional expectation value at time $t$, Longstaff and Schwartz [78] proposed approximating it with a finite linear combination of a set of simple basis functions as follows:

$$ E[V_{t+1}|\varsigma_t] \approx \sum_{b=1}^{N} a_b \Psi_b(\varsigma_t) $$  \hspace{1cm} (4-10)
where $\psi_b \ (b = 1, \ldots, N)$ is a set of simple basis functions and $N$ is finite. The parameter $a_b \ (b = 1, \ldots, N)$ represents constant coefficients. This approach has been referred to as the Least Squares Monte Carlo (LSM) in the literature.

The idea of the LSM is that the coefficients $a_b \ (b = 1, \ldots, N)$ can be estimated using least-squares regression based on observation points from the simulated paths of the underlying variables. The concept of approximating the conditional expectation function by a linear combination of simple basis functions is based on the theory of Hilbert spaces in the sense that any function in a vector space can be approximated by a linear combination of bases for the vector space whereby the approximated value approaches the true value as $N \to \infty$.

Longstaff and Schwartz [78] originally applied this approach to price American options. However, a number of authors have extended the methodology to solve real options problems and multi-stage stochastic optimization problems in general (e.g. Tsekrekos et al. [81], Sabour and Poulin [82]).

The procedure starts at the final stage $T$ where the optimal decision is easily calculated. In our context, one would calculate equation 4-9 for each of the feasible alternatives at every discrete state of the asset. This calculation is repeated for each simulated price path (or simulated paths of the aggregate state of the market). Hence for a simulated path, one can determine the optimal alternative at every state of the asset as the alternative with the maximum value function. One can then step back in time (one stage) to $T - 1$, again to determine the optimal decision for each simulated path at every state of the asset. For example, if we consider two mutually exclusive alternatives as in equations 4-8a and 4-8b where $t$ is now $T - 1$, the optimal alternative at $T - 1$ is the one that yields the higher value among the two equations for each price path and each state of the asset. To achieve this, we use equation 4-10 to calculate the second term on the right-hand side of equations 4-8a and 4-8b.

Specifically, we choose the polynomial $\sum_{b=1}^{N} a_b \psi_b$ and determine the coefficients $a_b \ (b = 1, \ldots, N)$ by least-squares regression using the optimal values at $T$ as the sample outcomes of the dependent variable and the simulated prices at $T - 1$ as the sample outcomes of the independent variable. Once the regression coefficients are calculated, equation 4-10 can now be calculated for each alternative (i.e. equations 4-8a and 4-8b in this case), for each simulated price path and each state of the asset. The process is repeated until stage 1 is reached by which time all the optimal decisions would have been obtained. To determine the value of the plant, a forward-moving procedure can then be used to
calculate equation 4-6 since the values of all optimal benefits can now be calculated across the valuation period.

A number of authors have examined the robustness of the LSM methodology and compared it to other available numerical methods (e.g. Moreno and Navas [88], Stentoft [89] and Clement et al. [90]). Focusing on the two key approximations of the algorithm (i.e. replacing the conditional expectation by projections on a finite set of basis functions and using simulation and least squares regression to compute the value function), Clement et al. [90] have been able to prove that the LSM algorithm always converges under general conditions.

Moreno and Navas [88] as well as Stentoft [89] focused more on applications of the algorithm. Stentoft’s work showed that LSM is more computationally efficient and more robust than existing pricing techniques when there are multiple underlying stochastic factors. On their part, Moreno and Navas [88] showed that the technique is quite robust with respect to the choice of basis functions and works effectively with simple polynomials. By applying the technique to price different financial options, they found out that different polynomials produce similar results and similar option prices were obtained for polynomial degrees between 3 and 20. Above degree 20, they reported the occurrences of numerical instability in the least-squares regressions calculations.

4.3 Proposed Valuation Framework

We propose a broad framework that makes use of simulation-based optimization for valuing physical assets in competitive energy markets with complex embedded optionalities. The key steps of the framework are as follows:

(i) Derive a value function for the asset to be valued, as a function of available & relevant choice variables, underlying state variables, valuation time-frame, appropriate discount rate and relevant costs. The control variables reflect the choices (i.e. options) embedded in the operation of the asset from the perspective of the independent operator. The applicable state variables reflect all relevant uncertain underlying factors that affect the value of the asset. Broadly speaking, one can classify applicable state variables as (i) the states of the market and (ii) the states of the physical asset under valuation. The states of the market refer to possible occurrences of market-based factors such as power prices and fuel prices. Similarly, the states of the physical asset refer to possible occurrences of operating states such as output level or storage level.
(ii) Specify all constraints, the operating dynamics of the asset and the dynamics of the state variables. The operating dynamics of an asset depend on the particular type of asset e.g. the engineering parameters and relationships that govern the operation of a gas-fired plant are different from that of a hydroelectric plant. The dynamics of state variables refer to possible state transitions of the variables. For variables that model the states of the market, one can specify stochastic differential equations that can be calibrated and used to directly simulate the time evolution and probability distributions of the market parameters. For the variables used to represent the states of the asset, transition equations or rules which are functions of control variables and operating constraints can be formulated.

(iii) Once the value functions and all applicable variables are formulated, different paths of the market variables are simulated using forward-looking simulation techniques. This step involves discretizing the time state according to the solution granularity desired. The simulated scenarios of the key variables can also be taken as input, for instance in cases where a fundamental model has been used to derive forecasts of future spot prices.

(iv) To determine optimal control (i.e. operating) strategies, a simulation-based backward induction technique such as the Least Squares Monte Carlo (LSM) can then be applied.

(v) With the optimal operating strategy determined for each simulation path, the probability distribution of the asset value can then be determined easily as functions of the operating strategies and the simulated market parameters.

4.4 Simulating Stochastic Underlying Value Drivers

The process of using any simulation based methodology for options valuation involves simulating the underlying stochastic drivers of the asset. For power assets, there can be a number of uncertain value drivers depending on the asset type and the markets involved. For example, for a hydroelectric facility, one would have to model the future inflow of water into the plant’s forebay and also model market prices e.g. electricity price. Similarly, for a gas-fired thermal plant, the future evolution of gas price has to be modeled.

For derivatives pricing, the quantitative analysis models described in Section 3.6.1 are used. The idea is that an uncertain value driver can be represented as a random variable which evolves according to a
Markov process. To value an asset for which the random variable is an underlying parameter using a simulation-based approach, the future evolution of the variable has to be simulated using a forward-looking approach. The first step is to discretize the stochastic differential equations (as specified in Section 3.6.1) and then simulate several paths to obtain a probability distribution of the variable at each decision time. The appropriate stochastic process or combination of processes to employ for an exogenous state variable like water flow, electricity price or gas price depends on the observed nature of the variable as discussed earlier. For instance, to model the spot price of electricity, a mean-reverting jump-diffusion model is required. However, a mean reversion model is usually sufficient to capture the dynamics of water flow into a reservoir forebay and also the spot price of gas as neither typically exhibits (frequent) jumps. Depending on the level of complexity desired, one can also use single factor or multi-factor models as appropriate.

4.4.1 Simulating Mean Reversion Model

Consider an arithmetic Ornstein-Uhlenbeck (OU) process,

$$dx_t = \eta(\bar{x} - x_t)dt + \sigma dz_t, \quad (4-11)$$

where $x_t$ follows a normal distribution, $\bar{x}$ is the long-term mean of $x$ and $dz$ is the increment of the Wiener process. As described by Dixit and Pindyck [6], an exact discrete-time version of this process is as follows:

$$x(t + \Delta t) = x(t)e^{-\alpha \Delta t} + \bar{x}(1 - e^{-\alpha \Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \varepsilon \quad (4-12)$$

where $\varepsilon$ is an independent standard normal random variable and the other parameters are as defined in Section 3.6.1. This discrete-time format as shown by Dixit and Pindyck [6] is the “stationary autoregressive process” of order 1. Other methods of discretization have been used in the literature such as Euler’s method (Clewlow and Strickland [44]) but they have been shown to introduce discretization errors (see Dias [91]).

The parameter $x$ in equation 4-12 can assume a negative value but for our application, this is undesirable\(^{33}\). To avoid this undesirable behaviour of the OU process, we define $\bar{x}$ as the long-term

\(^{33}\) Electricity spot prices are sometimes negative during Excess Baseload Generation (EBG) situations but this is not a frequent occurrence.
mean value of the exogenous state variable required for our application and derive it as follows following Dias [91].

As shown by Dias [91], if we define,

\[
\bar{S} = \exp(x)
\]  \hspace{1cm} (4-13)

and

\[
E[S(t)] = \exp\{x(0)e^{-\eta t} + \bar{x}(1 - e^{-\eta t})\}
\]  \hspace{1cm} (4-14)

such that \( E[S(t)] = \exp\{E[x(t)]\} \) i.e. the drift terms are equal. Then given that

\[
S(t) = \exp\{x(t) - \frac{\sigma^2}{2}\}
\]  \hspace{1cm} (4-15)

for a variable \( S(t) \) that follows a log-normal distribution and \( x(t) \) that follows a normal distribution where \( \sigma^2 \) is the variance of \( x(t) \), then \( S(t) \) can be simulated directly using the following expression (Dias, M [91]):

\[
S(t + \Delta t) = \exp\left\{ \ln S(t)e^{-\eta \Delta t} + \ln(\bar{S})(1 - e^{-\eta \Delta t}) - \left(1 - e^{-2\eta \Delta t}\right)\frac{\omega^2}{4\eta} + \sigma^2 \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \right\}
\]  \hspace{1cm} (4-16)

Several paths of the “real” process of the random variable can be simulated using equation 4-16 starting from an initial value at time \( t = 0 \).

### 4.4.2 Simulating Mean Reverting-Jump Diffusion Model

The mean-reverting jump-diffusion model is a combination of the mean reversion model with a discrete jump Poisson process. As shown by Clewlow and Strickland [44], if \( U \) is a uniform random variable \([0,1]\), then the discrete jump process as specified in Section 3.6.1 can be simulated as follows:

\[
\left(\bar{k} + \gamma \epsilon'\right) \cdot \left(U > \phi \Delta t\right)
\]  \hspace{1cm} (4-17)

where the parameters are as defined in Section 3.6.1 and \( \epsilon' \) is an independent standard normal random variable, independent from \( \epsilon \) since the Poisson process is independent from the increment of the Wiener process. If \( U > \phi \Delta t \) is true, a jump occurs with a mean jump size of \( \bar{k} \).
Hence, to simulate independent paths of the “real process” of the random variable using JDMR model, we combine equations 4-16 and 4-17 as follows:

\[
S(t + \Delta t) = \exp\left\{ \ln S(t)e^{-\eta \Delta t} + \ln(S(0))\left(1 - e^{-\eta \Delta t}\right) - \left(1 - e^{-2\eta \Delta t}\right) \frac{\sigma^2}{4\eta} \right. \\
\left. + \frac{1 - e^{-2\eta \Delta t}}{2\eta} \varepsilon + \left(\bar{k} + \gamma', \varepsilon'\right) \cdot (U > \phi, \Delta t) \right\} 
\]

(4-18)

To simulate a risk-neutral process, one has to factor in excess return of the asset above the risk free rate (i.e. risk premium) by deducting it from the drift term.

### 4.4.3 Model Calibration

In order to determine the parameters of either the mean reversion model or the mean-reverting jump-diffusion model, the models have to be calibrated to available data. Typically, available historical data and current marker data are used to estimate the relevant parameters for each model. There are different schools of thought regarding the extent of historical data to use for model calibration. One approach is to go as far back in history as the future period for which the simulation applies, weighting each historical occurrence equally. One can also weigh more recent data higher than older information to appropriately reflect recent changes in market dynamics.

For a mean reversion model, the parameters to be estimated are the rate of mean reversion, historical volatility and the long-term average. One simple approach is the one described by Clewlow and Strickland [44]. If we imagine the time series data as consisting of two components, a deterministic trend and a random component, one can simultaneously determine the volatility, the long-term mean and the speed of mean reversion. Consider equation 4-11, if we ignore the random component, it becomes:

\[
dx_t = \eta(\bar{x} - x_t)dt
\]

(4-19)

For small $dt$, we have

\[
\frac{\Delta x_t}{\Delta t} = \eta \bar{x} - \eta x_t
\]

(4-20)
which implies that one can regress $\frac{\Delta x_t}{\Delta t}$ over $x_t$ to obtain $\eta$ and $\bar{x}$ as regression coefficients. The standard deviation of the residual should be due to the random component and hence it is the volatility.

More specifically, the first step in the calibration process is to determine the logarithm of the variables and then calculate $\ln S_t - \ln S_{t-1}$ (i.e. the returns, $\Delta x_t$). If we compare equation 4-20 to a standard polynomial of order 1 (i.e. $y = b + ax$) where $y = \frac{\Delta x_t}{\Delta t}$, then $\eta = -a$ and $\bar{x} = -\frac{b}{a}$.

The speed of mean reversion can be described conceptually as the half-life of a volatility shock to the long-term mean. Volatility introduces randomness around the long-term mean and the half-life is the time it takes for half the random shock to dissipate away. Mathematically, it can be shown that (see Clewlow and Strickland [44]) if $t_{1/2}$ represents the half-life of a mean reversion process with a speed of mean reversion $\eta$ (expressed in terms of % per time period) then:

$$t_{1/2} = \frac{\ln(2)}{\eta} \quad (4-19)$$

For instance, if we are simulating for time intervals of 1 year and $\eta$ is 1, then $t_{1/2} = 0.69$ which implies that it will only take in 0.69 years for half of a volatility shock to dissipate away.

For a random variable with jump components e.g. electricity, the entire dataset can not be used directly as stated above. One approach to estimating both the mean reversion parameters and the jump parameters is to use the recursive filtering approach proposed by Clewlow and Strickland [44]. The filtering approach recursively separates the jump components of the data from the diffusion components such that the jump parameters and the parameters of the mean reverting diffusion process can be estimated separately. This is achieved by arbitrarily determining a threshold level of the log returns beyond which the returns are considered as jumps. For instance, one can set this threshold as three times the standard deviation of the entire data such that returns that are larger than this at any time step are considered as jumps. The idea is to separate those portions of the entire data from the rest and repeat the procedure with the remaining data using three times the standard deviation of the remaining data as a new threshold. The procedure is repeated in this way until a convergence is achieved. Given the total number of jump components that are removed from the entire dataset, one
can adjust it based on the granularity of the data to obtain the jump frequency $\phi_s$. The standard deviation and the mean of the log of the separated jump data yields $\gamma_s$ and $\bar{k}_s$ respectively. The diffusion portion of the data (i.e. after excluding the jumps) is then used to obtain the long term mean reversion level ($\mu_s$), volatility ($\sigma_s$) and the speed of mean reversion ($\eta$) as described earlier.

Also, the framework of simulation allows the parameters to be estimated as functions of time. For instance, electricity prices not only exhibit seasonality but vary between off-peak and on-peak on a daily basis. In order to capture the different variations, one can separate the data set based on “off” and “on” peak hours. The parameter determination can then be carried out on a monthly basis for both “off” and “on” peak hours.
Chapter 5
Valuation of Gas Fired Power Plants with Gas Imbalance Uncertainty

A gas-fired thermal power plant converts the chemical energy stored in natural gas to electrical energy via thermal and mechanical processes (Kehlhofer, R. et al [65]). There are two broad categories, simple cycle plants and combined cycle plants. Simple cycle gas turbine generators have low thermal efficiency and are typically used for peaking. They require lower capital costs relative to combined cycle plants but higher operating costs. Combined cycle plants, however, are able to operate at higher efficiency by combining gas and steam turbines into the same thermal cycle. This is possible with the use of heat recovery steam generators (HRSG) which use exhaust gas heat from gas turbines to generate steam. Given their higher efficiencies, combined cycle plants are suitable for both baseload and peaking operations.

The economics of thermal power plants are influenced by a number of uncertain factors and engineering constraints. The key ones are:

- Commodity prices (electricity, gas or heating oil)
- Operating constraints and equipment limitations
- Equipment outages
- Gas supply

Future cash flows from operating a thermal plant depend on these key factors and available operating flexibilities. In the context of real options, these flexibilities include start-up and shut-down options; how often to start-up and shutdown the plant, the specific timing of these actions and the optimal output level per time among others. The availability of these options forms the basis of the challenges faced by the operator with regards to optimizing the market value of the plant. The objective is to operate the power plant in such a way that it is at a maximum output capacity when the spread between power prices and fuel costs (i.e. spark spread) is high and positive. If the operator has a fore-knowledge of these periods, he will simply offer to startup the plants correspondingly and shutdown at other times. However, it is often the case that both power and fuel prices are uncertain making
optimal operating decisions unknown ahead of time.

Other than electricity and gas prices, the ability to capitalize on the flexibility of the plant is directly impacted by the plant’s operating constraints and equipment limitations. For example, a unit with high ramp rate can be started up quickly when the spark spread is expected to be high and shutdown before the spread becomes negative. However, with a unit that ramps up relatively slowly, the plant may have to be started and operated at a loss to ensure that it is available later to capture expected high spread. A high ramp rate also makes a power plant suitable for the provision of ancillary services such as operating reserve and frequency regulation via automatic generation control (AGC).

Since net revenues depend on production volumes, frequency of forced outages is also important to the economics of power plants. Naturally, frequency of unplanned plant outages tends to correlate with equipment conditions and maintenance history of the plant. Equipment failures set in through wear and tear which makes it an important issue for operators of ageing power plants. A proper long-term analysis of a thermal plant should factor in the correlation between potential for forced outages and aggressive ramping decisions leading to excessive equipment wear and tear.

The location of a gas-fired thermal plant within the gas supply system (i.e. storage, transmission, and distribution) can have a significant impact on the overall costs of fuel, including costs associated with supply and reliability management (Devin, K [92], Bopp et al. [93], Chen and Baldick [94]). For instance, a power plant located far away from storage facilities and away from upstream gas transmission pipelines may have to procure more upstream transportation and load balancing services compared to a plant that is strategically located. Furthermore, ready access to a variety of electricity and gas markets trading opportunities improves the overall economics of gas-fired power plants.

Existing real options analysis applications to gas-fired power generation assets ignore potential costs and constraints associated with gas supply/consumption (e.g. Thompson et al [35], Tseng and Barz [36], Denton et al. [34]). Gas supply/consumption imbalances occur because of the timing differences between gas nominations (based on the gas industry operations and rules) and gas consumption for power generation (based on electricity market rules). Without carefully incorporating variable costs and operating constraints associated with this uncertainty factor, a gas fired power plant may be significantly over-valued.

5.1 Interplay between Electricity and Gas Markets

Both natural gas and electricity markets play important roles in the operation of gas-fired power
plants. To operate as planned, gas-fired power plants require reliable supply of natural gas, typically through a Local Distribution Company (LDC). Reliable supply of natural gas in this context implies that a power plant is able to receive gas at desired flow rates from its gas supplier when it requires it.

In North America, the natural gas industry operates on a day-ahead basis in compliance with North American Energy Standards Board (NAESB) rules. Day-ahead operation is considered necessary since most industrial natural gas consumers start up and shutdown their processes at similar times each day making overall gas consumption patterns readily predictable. Daily gas consumptions being predictable and to a good extent controllable, makes it easy for industrial gas consumers to order appropriate volume of natural gas day ahead and for transmission pipelines operators, storage operators and local distribution utilities (LDC) to plan for the delivery and receipt of required gas volumes on their respective systems. Due to this predictability, variations (hourly, daily or seasonally) between real-time and day-ahead scheduled volumes tend to be minor for industrial gas consumers. On the other hand, gas consumption patterns of electricity generators in competitive markets are not readily predictable.

Typically, the desired flow rates vary during the course of the day and from one day to another, depending on the states of nature in the electricity market. This is because power generators are dispatched in real time by a system or market operator, in increments of short intervals subject to their offers into the market. Typically, significant uncertainty exists with regards to the exact timing and volume of dispatch (i.e. real time) irrespective of a generator’s specific energy offer into the market.

In power markets, there are a number of reasons for dispatch uncertainties, the principal one being unplanned changes in real-time supply and demand curves mostly due to sudden forced outages of participating power plants, load entities and/or transmission lines. These unplanned changes create uncertainty in dispatch timing and output level for marginal generators. For instance, a generator expecting to be dispatched ‘on’ based on previous forecast of market conditions (supply and demand) may end up being instructed to remain shut down. Similarly, a plant expecting to run for only a short

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34 NAESB has two firm day-ahead gas nomination windows during which natural gas can be nominated for next day consumption by industrial gas consumers and power plants operators based on Eastern standard time:

- Timely window: 13:00 deadline on gas day (i.e. a day-ahead of consumption day)
- Evening window: 19:00 deadline on gas day (i.e. a day-ahead of consumption day)
period may be constrained to remain on for system reliability purposes. In order to be able to predict its real-time gas consumption, a power generator must therefore be able to accurately forecast real-time power prices and market operations, a day-ahead when gas nominations are processed. Since this is not always achievable, imbalances are bound to occur between real-time gas supply and gas consumption. In addition to these system induced uncertainties, the capability of a generator to accurately forecast short-term price movements irrespective of system related outages also has a significant impact on its ability to plan for future operation.

The plant operator can only wait until the last firm window day-ahead to make decisions about volume of gas to purchase for the next day of power plant operation. That decision has to be made based on his expectation of real-time production during the next operating day. As an illustration for the Ontario power market, if the IESO pre-dispatch price forecast is relied upon by a power generator to order gas day-ahead, that operator will have to base gas purchases on the 1st predispatch prices published by the IESO. Figure 8 shows a comparison of the first predispatch price set for August 09, 2006 with the actual HOEP for the same day. As indicated in Figure 9, HOEP was generally below the predispatch prices for that day by as much as $32 CAD in one of the hours. Figure 10 shows the differences between predispatch and HOEP since wholesale power market opening in Ontario. As indicated by the figure, estimates of prices in predispatch hours tend to be higher than actual prices.

From perspective of the gas supplier or distributor, the importance of gas supply/consumption imbalance is the physical impact it has on its pipelines and the need to either get rid of excess gas or make up for unplanned gas supply. In technical terms, volume of unused gas that is left in a gas distributor’s pipeline “packs” the pipeline by increasing system pressure. Similarly, gas over-run “drafts” the distributor’s pipeline by reducing system pressure. If required to get rid of unused gas (in case of an under-run by generator) or to make up for over-run gas volume, the Local Distribution Company (LDC) will penalize the power generator thereby adding to the overall costs of gas supply. For instance, the gas distributor may compel a power generator that has an over run to pay for the unscheduled gas at a predetermined premium based on the current spot price of gas. Naturally, depending on how significant these penalty charges are a power generator will consider them when making daily decisions to offer electricity products into the power market.
Figure 8: Comparison of 1st Pre-dispatch Prices in Ontario and HOEP for a Representative Market Day

August 09, 2006

Figure 9: Differences between HOEP and 1st Pre-dispatch Prices for a Representative Market Day in Ontario.
In some cases, gas suppliers offer services that power generators can purchase to manage their gas volume risks. Examples of such services include “load balancing” services and “high deliverability storage”. A load balancing service works like a virtual storage that allows a power generator to withdraw and inject gas as required (within the constraint of the service) to manage real-time over run or under run and avoid heavy penalty charges. The gas supplier is able to offer such service by making use of additional capacity in its pipeline network to temporarily store gas. On the other hand, high deliverability storage services require allocation of storage space in a physical storage facility typically located close to a gas market hub such as “Dawn” hub in Sarnia, Ontario and “Henry” hub in Louisiana. For a power generator, the decision to select one service over another depends on a number of factors, including costs (fixed and variable) and limitations (e.g. capacity, withdrawal limit, injection limit etc).

For illustrative purposes, consider a power generator with a day-ahead nomination of 48,000 MMBtu to be used for generation the next day. With a basic transportation contract (i.e. with no-intra day...
nomination window) the gas supplier may, for instance, supply equal volumes of gas per hour throughout the course of the day (i.e. 2000 MMBtu/hr), irrespective of the daily load shape desired by the generator. As a result, if the generator’s consumption in any hour is less or greater than 2000 mmBTU, an hourly imbalance results. Sometimes, imbalances are tracked on a daily basis, meaning that penalty only results when there is a net cumulative imbalance volume at the end of the day. With a load balancing service or high deliverability storage used in conjunction with transportation services that allow frequent and multiple intra-day gas nominations, the generator is able to inject or withdraw gas to adjust its day-ahead order. With one or more of these services, only imbalance volumes that can not be accommodated within their limits attract penalty charges.

In some power market jurisdictions, day-ahead electricity markets (DAM) exist in addition to real-time markets. In other markets such as Ontario where a day-ahead market does not currently exists, there are day-ahead unit commitment programs in place (e.g. the day ahead commitment process (DACP) and the Spare Generator Online program, SGOL) to help generators properly plan for real-time operation by providing guarantees for startup and minimum runtime costs. However, a “gas day” (as defined by the 24 hour period between the starts of daily gas delivery) is not consistent with “electricity day” (i.e. hour 0 to 24). As a result, even though the existence of a DAM improves the planning process for power generators with regards to day-ahead gas purchases, risk of imbalances still exists in real time. Similarly, the design of Ontario DACP only protects generators and power importers as it relates to costs of startup and operation at the minimum generation level (by providing them startup related cost recovery guarantee), it does not eliminate gas volume imbalance risks for gas-fired generators.

Hence, the specific impacts of the costs associated with gas consumption/imbalance risks on the economics of a gas-fired generator depend on a number of factors:

- The existence or lack thereof of a day-ahead market for electricity or a day-ahead unit commitment program.

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35 Day-ahead electricity markets are used to commit generators, importers and exporters a day-ahead of the dispatch day with the actual dispatch taking place in real time. Some market jurisdictions (e.g. MISO) operate both a day-ahead market and a real time market. However, others like Ontario (currently) operates only a real-time market.

36 See www.theimo.com
• The frequency at which volume imbalances are determined (hourly or daily)

• The availability, costs and constraints of gas imbalance risk hedging vehicles, such as load balancing, high deliverability storage and transportation services with multiple and frequent intra-day gas nomination windows.

From the perspective of real options analysis, the operator of a gas-fired thermal plant not only reacts to changes in price spread between electricity and gas to maximize revenue but also has to continually adjust operations in reaction to occurrences of imbalances. Once the uncertainty about real-time electricity prices and plant dispatch are revealed, the plant operator may have to adjust future energy offers with the intent of maximizing revenue not only by targeting periods of high spread between electricity and gas prices but also by reducing costs associated with gas imbalance penalties.

5.2 Operating Constraints

Mechanical characteristics of power plants contribute to operating costs by constraining possible operating actions at certain states of the plants. Hence, prior to making any operating decision, the operator of a plant must consider both immediate and future costs (including lost opportunities) associated with changing the current operating state.

For financial analytical purposes, the important operating costs and constraints are highlighted as follows (Thompson et al [35], Denton et al [34], Tseng and Barz [36], Kehlhofer et al [65]):

• Minimum generation level – the minimum output level at which a plant can begin to inject power into the electricity grid. This output level is dictated by the thermodynamics of the plant’s operation and can be sensitive to prevailing ambient conditions.

• Maximum operating capacity – the maximum output level at which a stable operation can be achieved. It depends on the capacity and efficiency of the turbines.

• Heat Rate – the efficiency at which heat energy of fuel is converted to electrical energy by a gas-fired plant. Expressed in MMBtu/MWh, it determines the unit volume of fuel required to produce a unit MWh of energy. The higher the heat rate at a particular power level, the less efficient the plant is. Heat rate also exhibits seasonal variation due to sensitivity of thermal plants performance to ambient air temperature conditions.
• Start-up time – the time required to bring a plant from a shutdown state to the minimum generation level. It depends on the initial temperature state of the plant’s boiler. A unit that is being started after the boiler has become completely cold will require more time before power output can be obtained.

• Shutdown time – the time required to bring a plant from an operating state at or above the minimum generation level to a shutdown state.

• Minimum run time – this is the minimum time required for a plant to operate at or above its minimum generation level before it can be shut down. This requirement is dictated by the need to reduce thermal and mechanical stresses on plant equipments due to cycling operation.

• Minimum down time – similar to minimum run time, this is the time that has to be satisfied before a plant that is originally in a shut down state can be started up. This requirement is necessary to minimize equipment wear and tear and probability of component failure due to thermal stresses.

• Ramp rate- the output level of a plant can only be changed at certain rates depending on the current output level.

• Start-up cost – the cost associated with bringing a plant from a shutdown state to the point at which energy can be injected into the power grid (i.e. the minimum generation level). Startup cost has a variable component (associated with fuel requirement and price of gas) and a fixed component (e.g. labour costs, estimated costs of equipment wear and tear etc). It is a function of the temperature state of the boiler at the time of startup as it takes longer and costs more to startup from a cold boiler state compared to when starting up when the boiler is still considered warm or hot.

• Shut-down cost- the cost associated with shutting down a plant from an operating state at or above the minimum generation level, to zero power.
5.3 Modeling Framework

For an independent power plant owner, the decision to operate\(^{37}\) or shutdown a flexible gas-fired power plant in real time depends on the immediate benefit of the decision as well as expected future benefit consequent to the decision. Given that future benefits are unknown ahead of time, operating decisions at each instant of time has to be taken based only on the currently available information about the state of the plant and the states of the relevant commodities markets (i.e., electricity and gas markets). Immediate operating decisions will affect the operating states that are feasible in the near future, as some of the key operating constraints have to be satisfied across operating time intervals. As a result, one can model the operating decisions of a flexible power plant as a continuous or discrete time dynamic decision making problem where the objective is to maximize expected value of future cash flows subject to all relevant constraints.

The valuation framework presented and discussed in section 4.1 is applied here to value a gas-fired thermal power plant with gas imbalance uncertainty. Consider the operation of a thermal power plant in which a control action \(k_t^\xi\) is taken at discrete times \(t\), given the information \((\xi)\) available at \(t\) and the set of mutually exclusive control actions that are feasible at \(t\) as determined by applicable operating constraints. If we assume that a control action is taken at each time \(t\) from a feasible set after market prices (i.e., price of electricity and price of gas) are revealed, the objective of the plant operator can then be stated as follows:

\[
V_t = \max_{k_t^\xi} \mathbb{E}\left[ \sum_{t=\tau}^{T} d^{t-\tau} \times B_t(k_t^\xi, S_{\epsilon,t}, S_{g,t}, q_t, \xi_t) \right]
\]

(5-1)

i.e. the objective of the plant operator is to maximize the current expected value \((V_t)\) of future cash flows \((B_t)\) given the set of all currently available information\(^{38}\) \(\xi_t\) at time \(t = \tau\).

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\(^{37}\) In competitive markets power plants are dispatched by the Independent System Operator. To ensure that its plant is dispatched at a particular hour, a market participant will offer to generate below the expected market price for that hour since all generators with accepted offer receive the market price. Hence for the purpose of valuation, independent power plants are modelled as price takers. For simplicity, we also assume that the capacity of the plant being valued is small enough such that its operating decisions do not have any effect on market price.

\(^{38}\) Relevant information in this case includes market information (prices) and the physical state of the power plant.
$B\left(k_t, S_{e,t}, S_{g,t}, q_t\right)$ is the benefit derived at time $t$ by taking a control action $k$ at that time. The parameters $q_t, S_{e,t}, S_{g,t}$ are the power output (MW), unit price of power ($$/MW-time) and unit cost of gas ($$/MMBtu) at time $t$ respectively. The factor $d^{t-r}$ is the discount factor given by:

$$d^{t-r} = \frac{1}{(1+r)^{t-r}}$$ (5-2)

where $r$ is the discount rate.

The set of benefits $B\left(k_t, S_{e,t}, S_{g,t}, q_t\right)$ captures the spread between revenues obtained at dispatch times $t$ when a power level $q$ is injected into the electricity grid, and the corresponding variable cost of producing the power.

Before defining the benefit parameter $B$, we examine the state variables that define the physical plant asset. For now we ignore any impact of gas supply/consumption imbalance uncertainty on the valuation.

To represent the physical state of the plant, we consider two state variables as follows:

(i) When a control action is taken, the output generation level of the plant is either changed or held constant. In reality, energy output is offered into power markets at discrete output levels; hence a state variable is needed to represent all possible discrete output outputs.

(ii) The flexibility to change operating states directly depends on equipment constraints. For instance, after a plant has been shutdown, it has to remain in that state for a while before it can be restarted in order to minimize thermal stresses on the plant’s equipment. Similarly, there is a minimum up-time requirement. In addition, the time it takes to startup a plant from a shutdown state depends on the initial thermal state of the plant’s boiler. If the boiler has become completely cold, it will take longer to warm it up (before power can be raised) compare to a situation where the plant is to be started back up when the boiler is still warm. Hence a second state variable is needed to represent the thermal state of the power plant’s boiler, whether the plant is shutdown or running.

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39 By definition, a control action leads to an operating mode, so one can either model the control actions or the operating modes directly.
Output Levels (MW)

To model the plant’s feasible output levels at operating intervals $t$, we define a set $Z$ that comprises all feasible output levels $z_t$ of the plant. Since a negative power output level is never feasible, then

$$z_t \in Z \equiv \{0, \ldots, q_{\text{max}}\} \quad \text{(MW)} \quad (5-3)$$

where $q_{\text{max}}$ represents the maximum power output level of the plant and $q_{\text{min}}$. It should be noted that below $q_{\text{min}}$, output power is not injected into the electricity grid but rather used to achieve what is called a no-load spin of the turbines and synchronize the power plant to the electricity grid. At $t = 0$, $z_t = z_0$ (initial condition).

Variable startup time, Minimum run time, Minimum down time

For a thermal power plant, variable start-up time, minimum run time and minimum down time requirements are related to the thermal state of the plant’s boiler. To avoid unnecessary complexity, following Tseng and Barz [36], we model these requirements with a single state variable. Instead of attempting to model the boiler temperature directly, one can model these constraints by tracking the elements of a set $Y$ of all time states $y_t$ between the time intervals $-t_c$ and $t_R$, where $t_c$ is the time it takes for a plant boiler to become completely cold after it has been shutdown and $t_R$ is the minimum run time requirement of the plant. Mathematically,

$$y_t \in Y \equiv \{-t_c, \ldots, t_R\} \quad y \neq 0 \quad (5-4)$$

Furthermore, if we define a set $Y^{\text{on}}$ to track the state of the boiler when the plant is running and $Y^{\text{off}}$ to track its state when the plant is shutdown, then the following is true:

$$Y^{\text{on}}, Y^{\text{off}} \subseteq Y \quad (5-5)$$

For modeling purposes, when the unit is running, we are only concerned with the states of the boiler between the time the plant reaches its minimum operating level and the time it satisfies its minimum run time requirement. Hence, we can define $Y^{\text{on}}$ as the following set:

$$Y^{\text{on}} = \{1, \ldots, t_R\} \quad (5-6)$$
Similarly, when the unit is shutdown, we are only concerned with the states of the boiler between the time the plant reaches zero power and the time at which the boiler becomes completely cold. Since the ‘on’ state starts at interval 1, we can represent the “off” state with an interval that starts at 1\(^{-1}\) such that the set \(Y^{\text{off}}\) can be defined as:

\[
Y^{\text{off}} = \{-t_c, \ldots, -1\}
\]  

(5-7)

In addition, if we define \(t_w\) and \(t_H\) as the times from shutdown that it takes for the boiler to cool down to a ‘warm’ state and a ‘hot’ state respectively, then both \(t_w\) and \(t_H\) lie within the set \(Y^{\text{off}}\). Hence:

\[
Y^{\text{off}} = \{-t_c, \ldots, t_w, \ldots, t_H, \ldots, -1\}
\]  

(5-8)

For simplicity of modeling, we can combine the sets \(Y^{\text{off}}\) and \(Y^{\text{on}}\) such that,

\[
y_t \in Y^{\text{on}} \cup Y^{\text{off}} \subseteq Y
\]  

(5-9)

where the sign of \(y_t\) indicates whether the plant is running (i.e. set \(Y^{\text{on}}\) is applicable) or shutdown (i.e. set \(Y^{\text{off}}\) is applicable) and its magnitude indicates its position in the \(Y^{\text{on}}\) and \(Y^{\text{off}}\) sets. At \(t = 0\), \(y_t = y_0\) (initial condition) represents the initial state of the plant boiler at start of valuation.

### Control Actions and Operating Benefits

Having specified the relevant state variables, the feasible set \(K_t\) of control actions can then be defined. If we define the entire control set \(K\) (including all control sets \(K_t\)) as containing three actions\(^{42}\), then one can represent these actions by three arbitrary integers as follows:

-1 \(\Rightarrow\) ramp down

---

\(^{40}\) Hence \(y = 0 \not\in Y\) as in equation 5-4.

\(^{41}\) For all practical purposes, operators of thermal power plants are concerned with startups from three shutdown states defined as ‘cold’, ‘warm’ and ‘hot’ states of the boiler.

\(^{42}\) Depending on the specific problem situation, more control actions can always be considered.
\( 0 \Rightarrow \) continue to operate at the same power level (idle at the same level)

\( 1 \Rightarrow \) ramp up to a higher power level (ramp up)

Then,

\[
K = \{-1, 0, 1\}
\]

(5-10)

Also, we can assume that the energy produced in an interval \( \Delta t \) is the arithmetic average of the power levels at the start and end of the interval. Then for each control action, the benefit \( B_t \) can be defined as follows:

\[
B_t = \begin{cases} 
\frac{1}{2} (z_{t+1} + z_t) \times (S_{c,t} - AHR(z_t, z_{t+1}) \times S_{g,t} - OM) \Delta t & q_{\text{min}} \leq z_t \leq q_{\text{max}} \\
\frac{1}{2} (z_{t+1} + z_t) \times (-SAHR(z_t, z_{t+1}) \times S_{g,t} - OM) \Delta t & 0 < z_t < q_{\text{min}} \\
0 & z_t = 0
\end{cases}
\]

(5-11)

where \( AHR \), \( SAHR \) and \( OM \) are “average heat rate”, “startup average heat rate” and “variable operations and maintenance costs” respectively. The first expression in equation (5-11) implies that when the plant output is at least at the minimum generation level, the net cash flow is the difference between revenue derived by injecting electricity into the power grid during the interval \( \Delta t \) (at a MW level given by the average output during the interval) and the cost of the energy injected (i.e. cost of natural gas and operations & maintenance cost). However, as indicated by the second expression, if the output level is less than the minimum generation level but greater than zero power, electricity is not injected into the grid and so there is no derived revenue from the power market even though fuel is consumed. The cost of this fuel, which is the fuel required to startup the power plant from a shutdown state to its minimum generation level and synchronize it to the grid, constitutes the variable startup cost. The variable operations and maintenance costs can include items such as labour costs, maintenance costs (to capture plant degradation) and other costs as deemed fit by a generator. It is possible to define a functional form for this cost but there is no commonly acceptable function, hence we focus only on the average heat rate and startup costs which are necessary to capture fuel related costs. The third expression indicates that at zero power, when the plant is completely shut down, there is neither revenue nor cost.

The average heat rate \( AHR \), measures the average fuel consumption per unit of energy required to
maintain a set output level of the power plant. To model $AHR$, the starting point is the power plant’s input-output curve which is derived by engineers from performance tests. The curve is derived by measuring required fuel consumption for various levels of plant outputs and fitting the results to a function. Since the input-output curve relates fuel consumption to power production, it implicitly captures the efficiency of the power plant in converting fuel to power. It is customary to approximate this input-output relationship with a polynomial function (typically a convex quadratic function) as follows (see Klein [66] for a detailed discussion on heat rates):

$$H = a + bz_z + cz_z^2 \quad (z_z \geq q_{\text{min}}) \quad (5-12)$$

where $H$ is the thermal energy consumed in MMBtu/h to produce power at level $z_z$ (MW) and the terms $a, b$ and $c$ are correlation coefficients. The coefficient $a$ (the intercept) represents a base fuel requirement irrespective of level of power output.

The average heat rate at a given output level $z_z$ is derived by dividing the input-output curve (equation 5-12) by the power output, i.e.

$$AHR = \frac{a}{z_z} + b + cz_z \quad (z_z \geq q_{\text{min}}) \quad (5-13)$$

For the purpose of our modeling, the output level during an interval $\Delta t$ is taken as the mid-point of the output at the start of the interval and the output at the end, hence we calculate the average heat rate at $0.5(z_{z_i} + z_{z_i+1})$ as indicated in equation 5-11.

To model the startup cost, which is a function the power plant’s boiler thermal state at startup, we make use of a startup average heat rate curve as follows:

$$SAHR = b_o + c_o z_z \quad (0 < z_z \leq q_{\text{min}}) \quad (5-14)$$

which is calibrated to ensure continuity of fuel requirement at the minimum generation level. Obviously when the power plant is shutdown, there is no fuel requirement. To capture different startup costs for different shutdown states (i.e. cold, warm or hot), we assume different intermediate ramp levels in-between zero power and the minimum generation level such that when the power plant is starting up from a hot shutdown state for instance, it will bypass some intermediate ramp levels thereby reaching the minimum generation level faster than when starting up from a cold boiler state.
As discussed earlier, startup and dispatch decisions for a thermal power plant do not only depend on market variables but also on the operating constraints of the plant. As a result, at every decision point, the available actions \( K_j \) depend on the current thermal state of the plant’s boiler (indicated by \( y_i \)) and the current output level \( z_i \). This set of feasible control actions is a sub-set of the entire control set available to the plant operator. These relationships are defined by equation (5-15) as follows, where the integers 1, 0 and -1 are as defined in equation 5-10.

\[
K_j = \begin{cases} 
\{1\} & \text{if } (y_i \geq 1 \text{ and } 0 \leq z_i < q_{\min}) \\
\{1, 0\} & \text{if } (1 \leq y_i < t_R \text{ and } q_{\min} \leq z_i \leq q_{\max}) \text{ or } (y_i = -t_c \text{ and } z_i = 0) \\
\{0\} & \text{if } (-t_c < y_i \leq -1 \text{ and } z_i = 0) \\
\{1, -1, 0\} & \text{if } (y_i = t_R \text{ and } q_{\min} \leq z_i < q_{\max}) \\
\{-1, 0\} & \text{if } (y_i = t_R \text{ and } z_i = q_{\max}) 
\end{cases}
\]

(5-15)

The five expressions in equation (5-15) imply the following respectively:

(i) The only option available is to ramp-up when the plant is already started up but the power level is still below the minimum generation level.

(ii) The operator can ramp-up or leave the plant at the current output state when (a) the output level is above the minimum generation level but the minimum run-time constraint has not yet been satisfied or (b) the plant is shutdown and the minimum shutdown constraint is already satisfied.

(iii) When the power plant is shut down and the minimum downtime constraint is not yet satisfied, the only alternative available to the operator is to leave the plant in its current state i.e. remain shutdown.

(iv) All three options are available when the minimum run time requirement has been satisfied and the plant is operating at or greater than its minimum generation level.

\(^43\) It should be noted that an expanded or reduced set of operating constraints can be specified depending on the particular application.

\(^44\) This is an imposed constraint that can be relaxed depending on the particular application. However, it makes intuitive sense that once a decision to start up has been made, it should not be reversed until the minimum output level is reached since the start-up costs cannot be recovered until the minimum output level is attained.
(v) The operator can only ramp down or leave the plant at its current state when the plant is operating at its maximum operating capacity. In the case of ramp down, the minimum run time requirement must also have been satisfied.

The dynamics of the state variables $y_t$ and $z_t$ can be modeled respectively as follows:

$$y_{t+1} = \begin{cases} \min(t_{\min}, \max(y_t,0) + 1) & \text{if } z_t \geq q_{\min} \\ y_t & \text{if } 0 < z_t < q_{\min} \\ \max(-t_c, \min(y_t,0) - 1) & \text{if } z_t = 0 \end{cases} \quad (5-16)$$

$$z_{t+1} = \begin{cases} z_t + \beta_u \Delta t & \text{if } k_t = 1 \\ z_t & \text{if } k_t = 0 \\ z_t + \beta_d \Delta t & \text{if } k_t = -1 \end{cases} \quad (5-17)$$

where $\beta_u$ and $\beta_d$ are the ramp-up rate and ramp-down rate respectively. The ramp rate (whether up or down) measures the change in output level over a decision time interval, i.e. $\frac{\Delta z}{\Delta t}$. If ramp-up rate and ramp-down rate are equal, $\beta_u = \beta_d$. It is also possible to model $\beta_u$ and $\beta_d$ as functions of $z_t$ implying that ramp rate varies with the output level of the power plant. The implication of equation 5-17 is that the elements of set $Z$ are separated by $\min(\beta_u, \beta_d)$ and either $\beta_u$ is an integer multiple of $\beta_d$ or $\beta_d$ is an integer multiple of $\beta_u$.

### 5.3.1 Incorporating Gas Supply/Imbalance Uncertainty

Gas supply/consumption imbalance arises because of the time delay between gas purchases and real time gas consumption. Since power plants are dispatched in real time, actual gas consumption takes place in real time depending on when a plant is dispatched and the output level of dispatch. Depending on the specific plant and the markets (power and gas) in which it operates, a number of gas management related scenarios are possible as follows:

**Gas Nomination**

- Gas nomination takes place at the natural gas day-ahead nomination window only (e.g. the
NAESB timely nomination window). The total volume ordered for the next “gas day” is divided into 24 equal amounts and flowed to the consumer hourly during consumption day.

- Gas nomination takes place at the day-ahead nomination window and at additional intra day windows (gas suppliers may offer intra-day gas nomination windows at additional cost to help the consumer reduce its imbalance uncertainty by having the ability to adjust gas nominations intra-day as more dispatch information becomes available).

**Volume Imbalance**

- Gas supply/consumption imbalance is tracked hourly
- Cumulative gas supply/consumption imbalance is tracked daily

**Load Balancing and Storage**

- There is no load balancing service or storage, all supply over run or under run attracts market-indexed\(^{45}\) penalty charges from gas supplier
- Hourly or daily supply over run or under run is first applied to a load balancing service or a storage service. Imbalance volumes generated beyond the capacity of load balancing or storage attracts market-indexed penalty charges from supplier.

In addition to the foregoing scenarios, specifics of gas transportation contracts, load balancing services and storage services are usually different from one supplier to another. For example, the types of constraints on the use of a load balancing account may vary from one supplier to another. These can be in terms of the number of allowable nominations into or out of the load balancing account and the maximum allowable imbalance volume. The specific requirements serve as constraints on the use of gas balancing service or storage. Also, cost structures for these services can differ. In some cases, most of the cost is fixed whether the service is used or not while in other cases, the services are priced at market value determined through open season bidding.

Let \( g_n \) be the gas flow rate (MMBtu/hr) supplied to the plant in real time such that if the rate is constant at \( c \),

\(^{45}\) i.e. penalty charges are calculated based on gas market prices at the time of imbalance
The rate of gas consumption by the plant at $t$ ($g_n(t)$) is a function of the output plant level at $t$ ($z_t$) i.e.

$$g_n(t) = c \text{ for } t = 1 \text{ to } T \quad (5-18)$$

The rate of supply-consumption imbalance $I$ at time $t$ can therefore be stated as:

$$I(t) = g_n(t) - g_b(z_t) \quad (5-20)$$

It then follows that the imbalance volume ($I_v$) over an interval $\Delta t$ is:

$$I_v = I(t)\Delta t \quad (5-21)$$

The cumulative imbalance volume ($IC_v$) over a period $T$ then becomes:

$$IC_v = \sum_{t=1}^{T} (g_n(t) - g_b(z_t))\Delta t \quad (5-22)$$

To model a gas load balancing service or a gas storage service, the parameters (constraints) of interest include the following:

- The capacity allocated to the service e.g. (i) a maximum positive imbalance volume and a minimum negative imbalance volume for a load balancing service (ii) a maximum storage capacity for a storage account. Typically, for the purpose of gas imbalance management, one will attempt to use a storage account by swinging the volume around a mid-point. In which case, if we define the mid-point capacity as zero, the range of storage allowed will be bounded by a maximum positive capacity and a minimum negative capacity just like a load balancing account. If we define the maximum positive capacity as $R_{max}$ and the minimum negative capacity as $-R_{max}$, the total volume capacity of a storage account or a load balancing account is then $2R_{max}$.

- The maximum volume that can be nominated into or out of a load balancing account or a storage account at a single time (also referred to as injectability and withdrawability respectively).

If we define injectability and withdrawability limits as $g_{in}$ and $g_{out}$, the dynamics of a storage account or a load balancing account can be modeled as follows:
\[ R_{t+1} = \begin{cases} \min\left( R_{\text{max}}, R_t + \min\left( g_{\text{in}} \Delta t, I_y \right) \right) & \text{if } I_y \geq 0 \\ \min\left( -R_{\text{max}}, R_t + \max\left( -g_{\text{out}} \Delta t, I_y \right) \right) & \text{if } I_y < 0 \end{cases} \] (5-23)

where \( R_t \) is the inventory of the load balancing or storage account at time \( t \) and \( I_y > 0 \) indicates that gas is being injected into the account (hence, \( I_y < 0 \) indicates that gas is being withdrawn).

The method of calculating the cost of volume imbalance varies, depending on a particular gas supply contract and if load balancing or storage\(^{46}\) is available or not. For instance, if a power generator does not have access to a load balancing service or gas storage; then all imbalance volumes generated attract penalty charges which are usually indexed to gas price (i.e. linked to published gas prices) at a predetermined hub. Under such a scenario, penalty can be charged on imbalance volumes generated at each supply interval or on a cumulative volume at the end of the supply period (e.g. end of gas day). If a load balancing or storage is available, only volumes that can not be accommodated within service attract penalty charges. There may also be variable costs to using such a load balancing or storage service, e.g. a variable carrying cost charge for gas volume in the account and a variable cost for using the service.

Assuming that the plant operator is penalized for a fraction \( \alpha \) of the unit price of gas whenever an imbalance is created (either by having to sell excess gas to the supplier at\(^{47}\) \((1 - \alpha)S_g \) or by paying for short fall at \((1 + \alpha)S_g \), the cost of imbalance \( I_C \) then becomes\(^{48}\):

\[ I_C = \alpha \times S_g \times I_y \] (5-24)

with a load balancing service or storage account in place, the penalty charge for imbalance volume created is:

\(^{46}\) For economic dispatch purpose, only the variable costs of storage or load balancing are relevant.

\(^{47}\) It is also possible that the penalty charge is indexed to gas price at a different hub than the hub of purchase in order to enable the gas supplier to maximize the amount of unit penalty charged to the generator.

\(^{48}\) For instance, if \( \alpha = 30\% \), excess gas used will be purchased from the LDC at 130\% of market price and unused gas sold to the LDC at 70\% of market price.
\[
I_C = \begin{cases} 
(1 - \alpha)S_g \times \max(R_t + I_Y - R_{\text{max}}, 0) & \text{if } I_Y \geq 0 \\
(1 + \alpha)S_g \times \max(R_t + I_Y + R_{\text{max}}, 0) & \text{if } I_Y < 0
\end{cases}
\quad (5-25)
\]

If the account is tracked on a cumulative basis, \( I_Y \) in equation (5-25) will be replaced by \( IC_Y \) as in equation 5-22. Equation 5-25 assumes that usage of gas load balancing service or gas storage does not require any variable operating costs. If this is the case, these variable operating costs must be added to this equation which will have the effect of reducing the potential benefit of load balancing.

For dispatch decision making, the cost of imbalance \( I_C \) has to be incorporated into equation 5-11.

### 5.3.2 Model Summary

The developed model is summarized in this section.

The value function of the gas-fired thermal power plant at the current time \( \tau \) is:

\[
V_\tau = \max_{k^*} E \left[ \sum_{t=\tau}^T d^{t-\tau} \times B_t \left( k^*_t, S_{c,t}, S_{g,t}, q_t \right) \zeta_t \right] 
\quad (5-1)
\]

where the feasible set of decision actions \( K_t \) at time \( t \) is determined by the plant’s operating constraints i.e.

\[
K_t = \begin{cases} 
\{1\} & \text{if } (y_t \geq 1 \text{ and } 0 \leq z_t < q_{\text{min}}) \\
\{1, 0\} & \text{if } (1 \leq y_t < t_R \text{ and } q_{\text{min}} \leq z_t \leq q_{\text{max}}) \text{or } (y_t = -t_c \text{ and } z_t = 0) \\
\{0\} & \text{if } (-t_c < y_t \leq -1 \text{ and } z_t = 0) \\
\{1, -1, 0\} & \text{if } (y_t = t_R \text{ and } q_{\text{min}} \leq z_t < q_{\text{max}}) \\
\{-1, 0\} & \text{if } (y_t = t_R \text{ and } z_t = q_{\text{max}})
\end{cases}
\quad (5-15)
\]

The benefit term \( B_t \), which incorporates the variable cost of gas supply/consumption imbalance, is then defined as follows:
As defined in equation (5-11), $B_t$ is a function of revenue derived from energy sale and variable operating costs where variable operating costs include (i) costs of fuel consumed which is a function of the average heat rate (ii) variable operations and maintenance cost ($OM$), and (iii) gas supply/consumption imbalance cost ($I_C$).

The average heat rate is modeled as follows:

$$AHR = \frac{a}{z_t} + b + cz_t \quad (z_t \geq q_{\min}) \quad (5-13)$$

where $a$, $b$, and $c$ are coefficients of the power plant’s input-output polynomial. The startup average heat rate is used to model variable startup costs based on a startup input-output curve defined as follows:

$$SAHR = b_o + c_o z_t \quad (0 < z_t \leq q_{\min}) \quad (5-14)$$

where the coefficients $b_o$ and $c_o$ are calibrated to ensure continuity of fuel requirement at the minimum generation level, $q_{\min}$.

The cost structure of gas supply/consumption imbalance varies between gas supply contracts and it depends on whether a load service or storage service is available for managing generated imbalance. Without load balancing service or storage, $I_C$ is modeled as follows (assuming imbalance is tracked at every period $t$):

$$I_C = \alpha \times S_g \times I_f \quad (5-24)$$

where $\alpha$ is the penalty factor and $S_g$ is the price of gas. The imbalance volume $I_f$ is defined as the
difference between real-time gas supply ($g_a$) and consumption ($g_b$) volumes:\footnote{The rate of gas supply (i.e. gas nomination) is taken as input into the model while gas consumption is determined intrinsically as a function of the output state. For instance, gas nomination can be determined from the knowledge of published pre-dispatch prices which generally represents the market’s forecast of near term spot prices.}

\[ I(t) = g_a(t) - g_b(z_t) \]  \hspace{1cm} (5-19)

With load balancing service or a storage service, the cost of imbalance is modeled as follows:

\[
I_c = \begin{cases} 
(1 - \alpha)S_g \times \max(R_t + I_V - R_{\text{max}}, 0) & \text{if } I_V \geq 0 \\
(1 + \alpha)S_g \times \max(R_t + I_V + R_{\text{max}}, 0) & \text{if } I_V < 0
\end{cases}
\]  \hspace{1cm} (5-24)

assuming that the load balancing service or storage service does not require variable operating costs of its own. $R_t$ is the storage or load balancing inventory at time $t$ and $R_{\text{max}}$ is the capacity.

The dynamics of a load balancing service or a storage service is modeled as follows:

\[
R_{t+1} = \begin{cases} 
\min(R_{\text{max}}, R_t + \min(g_{in} \Delta t, I_V)) & \text{if } I_V \geq 0 \\
\min(-R_{\text{max}}, R_t + \max(-g_{out} \Delta t, I_V)) & \text{if } I_V < 0
\end{cases}
\]  \hspace{1cm} (5-23)

To model market prices of gas, we use the discrete-time form of the mean reverting model as presented in Section 4.4.1, i.e.

\[
S(t + \Delta t) = \exp\left\{ \ln S(t)e^{-\eta \Delta t} + \ln(\bar{S})\left(1 - e^{-\eta \Delta t}\right) - \left(1 - e^{-2\eta \Delta t}\right)\frac{\sigma^2}{4\eta} + \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \varepsilon \right\}
\]  \hspace{1cm} (4-16)

It should be noted that all the parameters of equation 4-16 are defined for gas prices. Since gas is assumed to be purchased on a day-ahead basis, gas prices are modeled on a daily granularity. Hence for intra-day decisions, gas price is taken to be the same throughout the 24-hour operating period.

Similarly, we use the discrete-form of the mean-reverting jump-diffusion model as presented in Section 4.4.2 to simulate several paths of electricity prices, i.e.
\[ S(t + \Delta t) = \exp \left\{ \ln S(t) e^{-\eta \Delta t} + \ln(S) (1 - e^{-\eta \Delta t}) - \frac{(1 - e^{-2\eta \Delta t})}{2\eta} e + \sigma \sqrt{\frac{1}{2\eta} e + \sqrt{1 - \rho^2} \sigma \sqrt{\frac{1}{2\eta} e}} \right\} \]

(4-18)

The parameters of this equation (4-18) are defined for electricity prices and will be different from those of equation 4-16 when calibrated.

Since a joint probability distribution of gas and electricity prices has to be simulated (at every time step), correlations between the prices must be factored into the simulation. If we simulate different paths for electricity prices based on a standard normal random variable \( \varepsilon \) as indicated in equation 4-18, gas prices must be simulated with an adjusted version of equation 4-16 as follows:

\[ S(t + \Delta t) = \exp \left\{ \ln S(t) e^{-\eta \Delta t} + \ln(S) (1 - e^{-\eta \Delta t}) - \frac{(1 - e^{-2\eta \Delta t})}{2\eta} e + \sigma \sqrt{\frac{1}{2\eta} e + \sqrt{1 - \rho^2} \sigma \sqrt{\frac{1}{2\eta} e}} \right\} \]

(5-25)

where \( \varepsilon'' \) is a standard normal random variable independent from \( \varepsilon \) and \( \rho \) is the correlation coefficient between gas and electricity prices. If the prices of the two commodities are assumed to be perfectly correlated in the same direction (i.e. \( \rho = +1 \)), the last term of equation 5-25 disappears and the equation becomes 4-16.

The dynamics of the state variables \( y \) and \( z \) of the power plant are as follows:

\[
\begin{align*}
y_{t+1} &= \begin{cases} 
\min(t, \text{max}(y, 0) + 1) & \text{if } z_t \geq q_{\text{min}} \\
y_t & \text{if } 0 < z_t < q_{\text{min}} \\
\max(-t, \text{min}(y, 0) - 1) & \text{if } z_t = 0
\end{cases} \\
z_{t+1} &= \begin{cases} 
z_t + \beta_u \Delta t & \text{if } k_t = 1 \\
z_t & \text{if } k_t = 0 \\
z_t + \beta_d \Delta t & \text{if } k_t = -1
\end{cases}
\end{align*}
\]

(5-16)   (5-17)

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5.4 Illustrative Example

Consider a hypothetical, but realistic gas thermal power plant\(^{50}\). Let the input-output characteristics of the plant be given as shown in Figure 11 and the startup input-output curve be as shown in Figure 12. The key operating characteristics/constraints and valuation parameters are indicated in Table 1\(^{51}\). The plant’s startup profiles from three shutdown states (cold, warm and hot) and the assumed rampdown profile are as shown in Figure 13. We analyze daily dispatch strategies of the plant under different scenarios and compute market-based net revenues of the output energy.

**Scenario 1 (No gas imbalance):** The plant’s output is valued without any consideration for gas imbalance uncertainties. The effect of seasonality in power prices and gas prices is examined under this scenario.

**Scenario 2 (gas imbalance; no storage or load balancing):** Daily dispatch strategy of the plant is modeled with consideration given to hourly gas volume imbalance. A penalty charge equal to 50\(^{\circ}\)\(^{52}\) of the spot gas price is assumed to apply whenever an imbalance is created between supplied gas and consumed gas volumes. We examine different hourly gas nomination volumes which are assumed constant throughout the day.

**Case 3 (gas imbalance with load balance service):** Daily dispatch strategy of the plant is modeled considering potential penalty charges from hourly imbalance given that a load balancing service is available as a hedge for imbalance risk. We consider a load balancing account with two different capacities (±10,000 & ±20,000 MMBtu) and injection/withdrawal limits of 2000 MMBtu/hr.

The scenarios and the sub-cases are described in Table 2.

---

\(^{50}\) The characteristic and valuation parameters chosen are representative of a realistic gas thermal power plant even though the problem is only illustrative.

\(^{51}\) For the purpose of this illustration, we ignore variable operating and maintenance cost and also since computation is done for a 24-hour time frame, we ignore discount rate.

\(^{52}\) The example of 50\(^{\circ}\) penalty charge for gas consumption imbalance is taken from an actual gas supply contract in Ontario.
Figure 11: Input-Output Curve

Figure 12: Startup Input-Output Curve
The curves indicate that the plant can start-up from a hot shutdown state to the maximum capacity in 3 hours; from a warm state in 4 hours and from a cold state in 6 hours. Shutdown from maximum capacity is possible in 3 hours using 3 discrete steps.

---

53 Figure 13: Start-up Profiles from three Shutdown States

---

53 The curves indicate that the plant can start-up from a hot shutdown state to the maximum capacity in 3 hours; from a warm state in 4 hours and from a cold state in 6 hours. Shutdown from maximum capacity is possible in 3 hours using 3 discrete steps.
### Table 1: Operating Constraints and Costs Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Generation Level (base point)</td>
<td>120</td>
<td>MW</td>
</tr>
<tr>
<td>Maximum Generation Level (capacity)</td>
<td>460</td>
<td>MW</td>
</tr>
<tr>
<td>Minimum Run Time</td>
<td>3</td>
<td>hr</td>
</tr>
<tr>
<td>Minimum Down Time</td>
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<td>hr</td>
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</table>

**Valuation Parameters**

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<tr>
<td>Initial Plant Output</td>
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<td>MW</td>
</tr>
<tr>
<td>Initial Down Time</td>
<td>4</td>
<td>hr</td>
</tr>
<tr>
<td>Initial Load Balancing Account Level</td>
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<td>MMBtu</td>
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**Start-up Cost Coefficients**

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<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>1.5731</td>
<td>MMBtu/MWh</td>
</tr>
<tr>
<td>$c_0$</td>
<td>0.0355</td>
<td>MMBtu/MWh^2</td>
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</tbody>
</table>

**Cost Coefficients - Above Minimum Generation Level**

<table>
<thead>
<tr>
<th>Coefficient</th>
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<tr>
<td>$a$</td>
<td>314.86</td>
<td>MMBtu</td>
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<tr>
<td>$b$</td>
<td>1.9603</td>
<td>MMBtu/MWh</td>
</tr>
<tr>
<td>$c$</td>
<td>0.0104</td>
<td>MMBtu/MWh^2</td>
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**Load Balancing Service**

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<tr>
<td>Maximum Withdrawal Rate</td>
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<td>MMBtu/hr</td>
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<tr>
<td>Maximum Injection Rate</td>
<td>2000</td>
<td>MMBtu/hr</td>
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### Table 2: Scenario and Case Description

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<tr>
<th>SCENARIO</th>
<th>CASE #</th>
<th>DESCRIPTION</th>
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</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>Case 1a</td>
<td>No gas imbalance consideration, Season=Summer</td>
</tr>
<tr>
<td></td>
<td>Case 1b</td>
<td>No gas imbalance consideration, Season=Winter</td>
</tr>
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<td>Scenario 2</td>
<td>Case 2a</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=0 MMBtu</td>
</tr>
<tr>
<td></td>
<td>Case 2b</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=1000 MMBtu</td>
</tr>
<tr>
<td></td>
<td>Case 2c</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=2000 MMBtu</td>
</tr>
<tr>
<td></td>
<td>Case 2d</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=2500 MMBtu</td>
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<tr>
<td></td>
<td>Case 2e</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=3000 MMBtu</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>Case 3a</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=0 MMBtu, Load Balance Account capacity=+- 5,000 MMBtu</td>
</tr>
<tr>
<td></td>
<td>Case 3b</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=0 MMBtu, Load Balance Account capacity=+- 10,000 MMBtu</td>
</tr>
<tr>
<td></td>
<td>Case 3c</td>
<td>Hourly Gas imbalance consideration, Season=Summer, Hourly Day-ahead gas purchase=0 MMBtu, Load Balance Account capacity=+- 15,000 MMBtu</td>
</tr>
</tbody>
</table>
5.4.1 Market Data Calibration

The parameters of the mean-reverting jump diffusion model for modeling time-varying probability distribution of power prices are calibrated as discussed in chapter 4 using the 2006 HOEP data from the IESO. Similarly, the parameters of the mean-reverting model for modeling gas prices are calibrated using 2006 day-ahead gas price data at Dawn Hub. The 2006 HOEP data is presented in Figure 14 showing the range of prices observed including a few very significant jumps. A single day price data showing intra-day price variation is presented in Figure 15 showing off-peak and on-peak hours with moderate price spikes. To calibrate the power price model, the data is divided into 4 seasons as shown in Table 3 and into peak-off peak hours where peak hours cover 8 AM to 11 PM and off-peak hours cover 12 AM to 7 AM. The diffusion components of the data are separated from the jump components based on a criterion of 3 times standard deviation (as explained in section 4.4.3). The parameters of the model are then derived for on-peak and off-peak hours, for the different seasons (see Table 4). 1000 samples of simulated paths of HOEP are shown in Figure 16 and Figure 17 based on summer and winter 2006 data respectively. Figure 18 and Figure 19 show single simulation paths for 24 hour periods. The simulated price paths show on/off peak variations and price spikes as expected.

Table 5 shows the calibrated mean-reverting model parameters based on 2006 day-ahead gas price data (daily) at dawn Hub (see Figure 20). Figure 21 and Figure 22 show samples of simulated gas prices over 90 day periods based on winter and summer 2006 data respectively.

5.4.2 Sanity Checks

Coding implementations of the mathematical models developed for the purpose of this thesis (including the models discussed in chapters 6 and 7) were done in MATLAB. To ensure that the models were accurately implemented as intended, a number of sanity checks were carried out as appropriate. A big part of software coding quality assurance process involves knowing intuitively the characteristics of the results that one expects under certain assumptions. For instance, one can turn-off volatility and or jumps and examine the ensuing results of the simulation. Similarly, one can turn-off mean-reversion and observe the effect on the probability distributions of the simulated variables. These types of sanity checks were performed for all the coding implementation that was done for the purpose of this thesis.
Figure 14: 2006 Hourly Ontario Electricity Price Data

Figure 15: Single Day HOEP Data Showing Intra-Day Price Variation
Table 3: Description of Seasons\textsuperscript{54}

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov.</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Season #</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: Calibrated Electricity Price Model Parameters (Log Parameters)

<table>
<thead>
<tr>
<th>Calibrated Model Parameters – Hourly Data</th>
<th>On-Peak</th>
<th>Off-Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump</td>
<td>Diffusion</td>
</tr>
<tr>
<td></td>
<td>( \kappa )</td>
<td>( \bar{x} )</td>
</tr>
<tr>
<td>Winter</td>
<td>( \gamma )</td>
<td>0.296</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>( \bar{k} )</td>
<td>0.616</td>
</tr>
<tr>
<td></td>
<td>( \gamma )</td>
<td>0.504</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>( \bar{k} )</td>
<td>-0.218</td>
</tr>
<tr>
<td></td>
<td>( \gamma )</td>
<td>0.346</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>( \bar{k} )</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td>( \gamma )</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>0.033</td>
</tr>
</tbody>
</table>

\textsuperscript{54} We take season 1 to represent Winter, Season 2 - Spring, Season 3 – Summer and Season 4 - Fall
Figure 16: Sample Simulated Paths of Electricity Prices over a 5 Day Period Based on Parameters Calibrated from HOEP Data for Summer 2006
Figure 17: Sample Simulated Paths of Electricity Prices over 5 Day Period Based on Parameters Calibrated from HOEP Data for Winter 2006
Figure 18: A Sample of Simulated Electricity Price Paths over a 24-hour Period (sample 1)
Figure 19: A Sample of Simulated Electricity Price Paths over a 24-hour Period (sample 2)
Table 5: Calibrated Gas Price Model Parameters (Log Parameters – Daily Data)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>1.75</td>
<td>1.86</td>
<td>1.51</td>
<td>2.09</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.03</td>
<td>0.09</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.042</td>
<td>0.037</td>
<td>0.051</td>
<td>0.054</td>
</tr>
</tbody>
</table>

Figure 20: 2006 Day-Ahead Gas Prices at Dawn Hub
Figure 21: Samples of Simulated Gas Prices over a 30-day Period Based on Winter 2006 Data
Figure 22: Samples of Simulated Gas Prices over a 30-day Period Based on Summer 2006 Data
Figure 23: A Single Sample of Simulated Gas Prices over a 90-day Period Based on Summer 2006 Data (Sample 1)
5.4.3 Valuation Results

When the developed valuation modeled is solved numerically as described earlier, the outcome includes probability distributions of both the set of optimal operating decisions and the corresponding present value over the time horizon. In the short-term, the resulting optimal operating decisions can be used to offer the plant’s output energy into the wholesale market. In order to achieve that, one would re-run the model as new information about market uncertainties arrive. For medium to long-term planning purposes or for contractual asset valuation purposes (e.g. PPA, forward sales etc), the model can be run over the corresponding time frame considering long-term fundamental effects on prices (such as seasonality).

For the particular illustrative example presented here, we analyze the results over a 24-hour operating
period (i.e. one day cycling operation). For the base case (case 1a) which does not take into account the possibility of occurrence of gas imbalances and their effects on dispatch strategies, Figure 25 and Figure 26 show sample paths of resulting dispatch strategies. As expected, the power plant is started up during the off-peak hours in order to ensure that it is up and running at the minimum generation level, ready to be ramped up at the start of on-peak period. Since gas imbalance costs are not considered, dispatch decisions are only influenced by the spread between electricity and gas prices given the modeled operating constraints. Since there is no intra-day gas price variation, if we plot output production against power price, the dispatch should follow power price variation. This expected behaviour is shown in the samples of calculated dispatch strategies shown in Figure 27 to Figure 30.

With potential gas imbalance costs taken into consideration, hourly operating decisions will not only depend on the spread between power and gas prices (given operating constraints) but also on potential penalty costs from pre-purchased gas. For instance, as shown in Figure 31, for the case where gas was not ordered at all, day-ahead (case 2a), the operator will rely on the available gas in its load balancing service account (or storage) as the plant is being dispatched until the available capacity is used up. Figure 33 and Figure 34 compare operating decisions for the same simulation paths under scenario 1 (no imbalance, case 1a), scenario 2 (hourly imbalance, case 2a) and scenario 3 (hourly imbalance with load balancing account, case 3a) for the case where no gas was ordered day ahead. As indicated, optimal operating decision changes from one scenario to another. With potential gas imbalance penalties, for the situation when the operator did not order any gas ahead of time, the cost of imbalance will be factored into the spark spread before the plant is offered into the market. As indicated in the figures, for the case where gas was not ordered day ahead, it is likely that the plant will be dispatched at less output since the potential imbalance penalty will make the spark spread negative in some hours where it will otherwise be positive. When a hedging instrument like a load balancing account is available, the impact of imbalance is less as the operator is able to rely on the load balancing account depending on how much capacity is available.

In the context of a daily plant cycling operation, the following are considered true:

- The plant operator will start up the power plant and will incur necessary startup costs as long as the total net market revenue for the entire 24 hour period is positive.
- When gas imbalance cost is not considered, the plant operator will choose to idle the power plant in the shutdown state when the cumulative net market revenue for the 24 hour period is
negative or zero.

- If the plant operator did not order any gas day-ahead, it will also choose to idle the power plant in the shutdown state when the cumulative net market revenue for the 24 hour period is negative or zero. This is because, if gas was not ordered day-ahead, imbalance penalties cannot result unless the operator chooses to operate thereby drafting the pipeline of the gas supplier.

- If a non-zero volume of gas was ordered day-ahead, the maximum possible loss for the plant operator for the 24-hour period is the penalty factor multiplied by the volume of gas ordered. As a result, instead of zero net market revenue being the floor value for the day, the operator will operate the power plant if the cumulative 24-hour net revenue is greater than the maximum possible loss.

- If a non-zero volume of gas was ordered day-ahead, the plant operator may be forced to operate at certain hours when its spark spread is negative just to mitigate penalty charges that will result if the pre-ordered gas was not used.

- With a load balancing service or gas storage available, the maximum daily loss will be determined by the available capacity of load balancing or storage.

In Table 6 we compare the calculated values of daily plant output across the three different scenarios. When gas imbalance penalties are not factored into the valuation, under the real options valuation assumption, it is not possible for the plant operator to lose money on any given day since it will simply choose not to offer energy into the market when market conditions are not favourable. However with gas imbalance penalties being relevant, daily net market revenues will depend on how accurately the plant operator is able to predict dispatch day-ahead. For days when gas is ordered day-ahead, it is possible for the operator to record a net loss, with its loss being capped by its maximum possible gas imbalance penalty for that day (based on volume of gas ordered). However for days when gas was not ordered day ahead, the operator will only offer the plant into the market when the cost of producing energy, including the cost of drafting its gas LDC pipeline, is less than potential market revenue which implies that under the real options valuation approach, the plant operator can not lose money on such days. For the particular power plant considered in this illustration, the financial impacts of imbalance penalties are shown in Table 6 on an expected value basis. As indicated in the table, net revenue depends on volume of gas-ordered, availability of load balancing
and capacity of the load balancing account.

In conclusion, the uncertainty surrounding gas imbalance affects the plant’s operator’s flexibility to switch between operating modes. By not factoring these effects into valuation, the value of flexibility of the power plant is over-stated and the resulting optimal operating strategies from the valuation model may not actually maximize total net earnings.

Figure 25: Samples of 24-Hour Operating Strategies (Scenario 1- Case 1a)
Figure 26: Samples of Daily Operating Strategies when Imbalance Penalties are considered (case 3)
Figure 27: Sample Path of Dispatch Decisions against HOEP (sample 1)
Figure 28: Sample Path of Dispatch Decisions against HOEP (sample 2)
Figure 29: Sample Path of Dispatch Decisions against HOEP (sample 3)
Figure 30: Sample Path of Dispatch Decisions against HOEP (sample 4)
Figure 31: Samples of Load Balancing Account Usage when Gas was not ordered Day Ahead (sample 1)
Figure 32: Samples of Load Balancing Account Usage when Gas was not ordered Day Ahead (sample 2)
Figure 33: Comparison of Dispatch Strategies with No Imbalance, Hourly Imbalance and Hourly Imbalance plus Load Balancing Service (Sample 1)
Figure 34: Comparison of Dispatch Strategies with No Imbalance, Hourly Imbalance and Hourly Imbalance plus Load Balancing Service (Sample 2)
Table 6: Statistical Comparison of the three cases

<table>
<thead>
<tr>
<th>Nominated Gas (MMBtu)</th>
<th>Mean</th>
<th>5th Percentile</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1a N/A</td>
<td>$130,626</td>
<td>$23,920</td>
<td>$-</td>
<td>$478,519</td>
</tr>
<tr>
<td>Case 1b N/A</td>
<td>87,499</td>
<td>11,046</td>
<td>$-</td>
<td>373,938</td>
</tr>
<tr>
<td><strong>Scenario 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2a 0</td>
<td>$53,971</td>
<td>$-</td>
<td>$-</td>
<td>323,316</td>
</tr>
<tr>
<td>Case 2b 1000</td>
<td>82,883</td>
<td>2,786</td>
<td>(26,309)</td>
<td>382,999</td>
</tr>
<tr>
<td>Case 2c 2000</td>
<td>87,658</td>
<td>(13,998)</td>
<td>(54,673)</td>
<td>414,464</td>
</tr>
<tr>
<td>Case 2d 2500</td>
<td>90,961</td>
<td>$-</td>
<td>$-</td>
<td>431,439</td>
</tr>
<tr>
<td>Case 2e 3000</td>
<td>74,809</td>
<td>(48,997)</td>
<td>(101,362)</td>
<td>439,508</td>
</tr>
<tr>
<td>Case 2f 4000</td>
<td>44,899</td>
<td>(91,840)</td>
<td>(150,187)</td>
<td>422,734</td>
</tr>
<tr>
<td><strong>Scenario 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 3a ±10000</td>
<td>$76,620</td>
<td>$8,637</td>
<td>$-</td>
<td>341,775</td>
</tr>
<tr>
<td>Case 3b ±20000</td>
<td>$103,239</td>
<td>$17,983</td>
<td>320.5</td>
<td>388,703</td>
</tr>
</tbody>
</table>
5.5 Chapter Conclusions and Applications

The model developed in this section and the results of the illustrative case study can be valuable to a merchant energy producer in several ways. Here we summarize potential applications and discuss the contributions of this work with regards to the valuation of gas-fired power generators.

Given that power markets and power trading are still generally evolving, it is important that a valuation model be robust and adaptable. The modeling framework developed and presented in this work meets these criteria as it considers all the key value drivers of a gas-fired power generator and can be adapted to fit various power market rules. Even though fairly standard stochastic processes were employed in modeling the stochastic underlying value drivers, the framework can easily be used with other more complex stochastic processes and even forecast price scenarios from fundamental price forecasting software. To the best knowledge of the author, this is the first time in an academic literature that the issue of gas imbalance risk has been considered in the context of gas-fired power plant valuation.

From an applications perspective, here are a few examples of potential application areas for a merchant gas-fired power generator where this modeling framework and the type of analysis presented can find usefulness:

1. Quantification of the impact of a power market design on the economics of a gas-fired power plant. The type of analysis presented in this thesis can be carried out to quantify how gas imbalance risk impacts on the valuation of gas-fired power plants in different markets, e.g. a real time power market versus a day-ahead power market. It is expected that the availability of a day ahead power market can reduce the impact of the disconnect between the natural gas market and real time power market. This result can be demonstrated by analyzing a gas-fired power plant under the two different markets timelines (i.e. day ahead versus real time).

2. Earnings at risk analysis for a gas-fired power plant. Merchant energy firms use the concept of earnings at risk to measure the variability of expected earnings from market-based operations. Given that the framework developed in this work is based on a Monte-Carlo algorithm, one can easily calculate the “earnings at risk” metric over a desired time frame from the probability distribution of values resulting from the model. A simple way to do this is by performing a percentile ranking of the resulting values and subtracting the 5th
percentile value (as an example of worst case earning) from the expected value, to show potential deviation from expected earning.

3. Structuring of a long-term Power Purchase Agreement (PPA) deal. As a simple illustration, let us consider a PPA structuring arrangement for the hypothetical power plant example considered here based only on the results presented earlier in Table 6. For the purpose of this simple illustration, assume that the parameters of the model as calibrated are valid for the entire PPA period. If the merchant producer does not factor in potential gas imbalance risk into its valuation calculation, based on the results of scenario 1, it will expect a profit margin\(^{55}\) of \(~$12/MWh\) on average for every single hour of the day in the summer\(^{56}\). However in reality, considering the potential for gas imbalance penalties, the expected profit margin will range between $4/MWh and $8/MWh for every hour of the day in the summer. Having a short notice gas balancing account improves the situation and brings the expected daily average profit margin closer to the no-imbalance scenario. Given this kind of information that gas imbalance risk can reduce expected profit margin from $12/MWh to as low as $4/MWh, the merchant energy producer should be looking to demand for risk premiums from the PPA counterparty to cover its potential risk exposure.

4. Value of short-notice gas balancing account or storage. A gas-fired power generator may want to examine the advantage of having a hedging vehicle like a gas storage account for reducing the risks of imbalance penalties and also determine the appropriate capacity and how much it should pay for such services. For instance, for the illustrative case considered in this thesis, doubling the size of the short notice gas balancing account increases expected profit margin by 35% (based on case 3b versus 3a). Also, from the point of view of value of hedging, the $4/MWh profit margin scenario (see item 2) increases to $7/MWh with a \(\pm10,000\) MMBtu capacity and $9.4/MWh with a \(\pm20,000\) MMBtu capacity.

5. Optimal market operations planning. An asset optimization tool is usually required to determine the optimal market operation given available information on expected market

\(^{55}\) That is revenue net natural gas and gas related start-up costs

\(^{56}\) Note that profit margin per hour is different during the actual hours of operation. This illustration is based on daily margin divided by total possible energy production in a day assuming the plant is operated at full capacity (i.e. 460 MW) for every single hour of the day (i.e. for 24 hours).
prices and other value drivers. Dispatch results produced from a valuation model such as this for different price paths can be useful for such planning purposes.
Chapter 6

Valuation of Hydroelectric Power Plants with Ancillary Services Options

6.1 Introduction

6.1.1 Hydroelectric Schemes
There are three main types of hydroelectric schemes. These include: (i) run of the river, (ii) diversion and (iii) pumped storage. The first scheme involves locating one or more turbine(s) and generator(s) either in a dam or alongside it such that the dam uses the flow of the river to create a hydrostatic head. The diversion scheme works by diverting water from a dammed river or lake via canals or tunnels to a reservoir. Water from the reservoir forebay is then passed through a penstock (which slopes down into a power house) to generate electricity via one or more turbines and generators. The difference between this scheme and pumped storage scheme is that a pumped storage plant incorporates two reservoirs and one or more reversible pump-turbine units (or separate pumps and turbines as the case may be) in such a way that power can be purchased from the spot market during low price periods to pump water from the ground level reservoir into the raised storage reservoir for subsequent use (for power generation) during peak price period. Run of the river hydroelectric plants operate as baseload assets since water can not be stored for later use. With a reservoir, the other two schemes allow water to be stored and cycled over time, the possible length of cycling being a function of storage size.

6.1.2 Operations Strategies
Optimization of energy output from hydroelectric schemes has been extensively studied under regulated power systems. A number of such studies appeared in water resources, power systems and decision sciences literature (e.g. Pereira et al. [95], Georgakakos et al. [96]). Such studies mostly focus on scheduling hydroelectric reservoirs in hydrothermal systems with the objective of minimizing costs or maximize energy to satisfy load demand. Tariffs were fixed and known and operating flexibilities were limited as the goal was usually to meet demand.

In competitive markets however, the individual hydroelectric asset could be optimized to maximize
Two key uncertainties affect cash flows in such situations; power price and water inflow. With a reservoir available for temporary water storage, the operator is able to change operating modes in response to market price movement by making use of his ability to store water. Variability of water inflow from upstream river into the plant affects how much energy can be produced. For example see Figure 35 which shows water inflow variability into a hydroelectric power plant in Ontario. Prior to start of rainfall season (otherwise known as the freshet seasons) it makes intuitive sense to have the reservoir empty in anticipation of significant increase in water inflow. During the freshet season itself, the production flexibility that comes with storage is greatly reduced as the plant is typically running at full capacity and even possibly spilling water. However, during the dry season, the operator relies mostly on being able to strategically drawdown the reservoir to capture high market prices.

Figure 35: Time-Series of Water Inflow into a Hydroelectric Reservoir in Ontario

source: www.opg.com
The actions of the operator are governed by his current view of the underlying uncertainty factors (water inflow and power prices) and his available operating flexibilities. As uncertainties are resolved, the operator has the flexibility to change operating mode by making use of available storage space to temporarily store water (i.e. time-shifting of water or reservoir cycling). For instance, during off-peak hours when prices are low, the power plant units can be shutdown to build up the reservoir inventory which is then used during peak price hours to maximize production. Depending on the reservoir size, multi-day or seasonal time-shifting of water may also be possible. Intuitively, the more flexibility a hydroelectric asset has with regards to storage capability, the more should be its value.

A number of recent articles have focused on valuing hydroelectric assets by considering the value of the flexibility of the operator to time-shift water by switching operating modes and using the plant’s reservoir storage capability (e.g. Thompson et al. [35] and Doege et al. [62]). The objective of the asset owner in this case (i.e. under competition) is to maximize revenue using strategies that do not necessarily maximize production or minimize power system costs. This is the main difference between hydroelectric asset operations under regulated power systems and deregulated power markets.

A common flexibility that has been ignored in the existing applications of real options methodology for valuing hydroelectric assets is the option of the asset operator to provide ancillary services to the wholesale power market. For instance, hydroelectric power plants are suitable for providing Operating Reserve (OR) capacities given their capability to ramp up and down very quickly. The ISO will pay a premium to market participants that offer a portion of their production capacities as reserve to cover their opportunity costs of reserving capacity (Deb [97] and Hirst [98], Jacobs et al [99]). Depending on the actual events during the course of a market day, reserve may or may not be required but the generator gets to keep the premium received. One can think of this process as selling of call options to the ISO whereby the ISO has the right (but is not obligated) to call the reserve for which it has already paid a premium.

Clearly, operators of hydroelectric assets derive additional value by making use of the flexibility to provide ancillary services. Therefore by ignoring such flexibility, a hydroelectric asset with storage capability may be under valued. In a power market scheme where operating reserves are procured by the market operator through a competitive auction much like energy, future operating reserve premiums are uncertain, much like energy prices. Hence the number of uncertainty factors to be considered in the valuation of hydroelectric assets will increase considering that energy prices and
water inflow into reservoirs are also uncertain. In the following section, we discuss the operating reserve market in the Ontario wholesale power market which is operated jointly with the energy market, as an example.

6.2 Ontario Joint Real-Time Energy and Operating Reserves Market

In the IESO administered wholesale electricity market, supply offers and demand bids for energy and operating reserve are submitted by dispatchable market participants at the same time. These bids can be revised up to two hours prior to the dispatch hour without restrictions by the IESO. In addition, with special permission, the quantity of bids can be further revised 10 minutes before the actual dispatch hour.

The IESO offers three classes of operating reserve products to provide a market-based approach for expeditiously replacing any lost supply of planned energy (caused by unplanned generation or transmission outages) for a short period of time until supply from regular energy dispatch can be restored. The three operating reserve classes administered are as follows:

(i) 10 minute spinning reserve (synchronized reserve)
(ii) 10 minute non-spinning reserve and
(iii) 30 minute non-spinning reserve

The IESO determines appropriate premiums (called OR prices) for each class of operating reserve using the same approach that is used to determine Market Clearing Prices (MCPs) for energy. MCPs and dispatch instructions for energy are determined simultaneously along with the clearing prices and dispatch instructions for the three classes of operating reserves. All accepted offers for operating reserve are settled at the determined premium rate for the class of operating reserve competed for irrespective of whether or not the reserve capacity is later called. All activated reserves are then further settled at the market clearing price. Since unplanned outages are random in nature, the probability that a reserve will be called is uncertain and not controlled by any market player.

Figure 36 compares the premiums paid for the three classes of operating reserve in 2004. As expected, OR prices are typically much less than energy prices as the plant operator can always use the reserve water for later production (compare to Figure 2). Also, the category of operating reserve that attracts the highest premiums is the 10-minute spinning reserve.

All dispatchable generators participating in the energy market also have the option to offer any of the
three operating reserve classes. In order to be eligible to offer 10-minute spinning reserve, the generator must be synchronized to the grid, ready to ramp up power to the required capacity in no more than 10 minutes. As the name implies, for the non-spinning reserve, the reserve capacity does not have to be already synchronized to the grid but the plant must be able to synchronize and ramp up to the desired capacity within the required time frame.

![Graph showing daily average operating reserve premiums in Ontario 2004](image)

**Figure 36**: Daily Average Operating Reserve Premiums in Ontario 2004

### 6.3 Model Development

If we consider the operation of a hydroelectric power plant with a water storage capability as consisting of three modes as follows:

- (i) all output offered into the spot energy market,
- (ii) potential output energy is split between energy and operating reserve,
- (iii) the plant is shutdown, then the

58 The valuation framework developed in this work is amenable to different definitions of operating modes. For example, one can use ramp-up, ramp-down and idle or any combination and number of relevant operating modes.

59 Ideally this mode can refer to operating reserve output only. However in this case the Ontario wholesale market is used as a basis – a market rule requires generators to offer at least as much energy as operating reserve in any particular period. Hence going by this rule, it is not possible for a dispatchable generator to bid
operation of the plant can be thought of as a multi-stage dynamic decision making problem where the asset operator decides at every stage (e.g. hourly or daily) whether to exercise the switch from the current mode to another available mode. If the payoff from exercising this switching option is “out of the money” the exchange is not made and the option is left to expire.

Based on the framework developed earlier in Chapter 4, we define a set \( K \) whose elements are the three available operating/output modes. The revenue \( B(k_1) \) from output mode \( k_1 \) at every stage can be defined as follows:

\[
B(k_1) = H(t) \times S_e(t)
\]

(6-1)

where \( H \) is the total amount of potential energy available for release from the plant at the decision time \( t \), and \( S_e \) is the spot price of energy at time \( t \).

Similarly, the revenue \( B(k_2) \) from mode 2 can be defined as follows:

\[
B(k_2) = \xi H(t) \times S_e(t) + (1 - \xi)H(t) \times S_{or}(t) + (1 - \xi)H(t) \times S_e(t) \times \gamma(t)
\]

(6-2)

where \( S_{or} \) is the premium for the specific class of operating reserve, \( \xi \) is the fraction of the total available energy that is offered into the energy market at time \( t \) (hence the fraction that is reserved is \( 1 - \xi \)) and \( \gamma \) is the probability of reserve activation. For the shutdown mode, no output is produced, hence revenue is zero i.e.

\[
B(k_3) = 0
\]

(6-3)

One can also define net revenues for the three modes considering water rental charges and other variable operating costs if they are considered material enough.

Since the action of the ISO with regards to activation of operating reserve is uncertain, it constitutes an uncertainty factor for the asset operator. A simple approach to account for this uncertainty is to all of its capacity into a reserve market.

60 In order to ensure that bids are always accepted, price takers tend to bid much less than projected market price at the hour of interest. For instance, operators of baseload power plants often offer output close to zero price in order to avoid the risk of being asked to shutdown against their will. A price setter on the other hand uses bidding strategies that will make its plant the marginal plant but it also runs the risk of not being called up by the market operator. It is also possible for the market operator to reject offers due to transmission line outages or congestion.
define a random variable $\gamma(t)$ taking binary integer values. This variable is used to represent the action of the market operator such that $\gamma(t) = 1$ indicates that the reserve capacity is called up by the market operator\(^{61}\) at time $t$ and $\gamma(t) = 0$ indicates that the reserve is not called. We define this variable as follows:

$$
\gamma(t) = \begin{cases} 
1 & \text{if } u(t) < \varphi dt \\
0 & \text{if } u(t) > \varphi dt 
\end{cases}
$$

(6-4)

where $u$ is a uniform random variable used as a proxy to capture the random nature of reserve capacity activation by the market operator. The parameter $\varphi$ is the frequency of reserve activation (an estimate can potentially be determined from historical market data).

The other exogenous uncertainty variables driving the decision making process are (i) market price of energy (ii) premium rate for the specific class of operating reserve and (iii) rate of water inflow into the reservoir. Since the asset operator does not have a perfect foresight of these factors, the problem can only be solved recursively.

Specifically, the three operating modes defined for this problem are as follows\(^{62}\):

- **Mode 1** - offer all available output into the energy market i.e. receive immediate revenue.

- **Mode 2** – offer $\xi H(t)$ amount of output for energy and the remainder, $(1-\xi)H(t)$ for operating reserve. This implies that immediate revenue at stage $t$ corresponding to $\xi H(t)$ amount of energy at the current energy price and $(1-\xi)H(t)$ amount of reserve at the premium rate. If the reserve is activated by the market operator (depending on the random outcome of the parameter $\gamma(t)$), an additional revenue

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\(^{61}\) We assume for simplicity that total accepted reserve capacity bid is called whenever the reserve is activated by the market operator. Another parameter can potentially be specified to capture the additional uncertainty regarding how much reserved capacity is activated each time the market operator decides to activate the reserve.

\(^{62}\) For this problem, ramp rates or ramp constraints do not have to be explicitly modelled as hydroelectric plants are able to ramp up and down fairly quickly.
corresponding to $(1-\xi)H(t)$ at the current energy price is also received.

- Idle the plant (at a shutdown state) – if higher market prices are expected in the near future, depending on the size of available storage, it may be optimal to idle the plant at the current time and build up the reservoir to maximize production later during the periods of higher prices.

Hence, as detailed in Chapter 4, if $k$ represents the operating decision at time $t$ from the available set $K$, then neglecting variable operations and maintenance costs, the value of the plant’s output is given by the present value (PV) of all the future cash flows as follows:

$$V_t = \max_k E \left[ \sum_{\tau=t}^{\infty} d^{\tau-t} \times \left( B(k^{\tau_t}, S_{e,\tau}, S_{or,\tau}, f_t) \right) \right] \quad (6-5)$$

where $B$ is the revenue received at stage $t$ conditional on the decision taken at that stage i.e. $B$ is defined as in (6-1), (6-2) or (6-3) depending on the option selected at each decision stage $t$. Each time the operator observes the relevant states of nature and makes an appropriate choice among the three modes. The factor $d^{\tau-t}$ is the discount factor given by:

$$d^{\tau-t} = \frac{1}{(1+r)^{\tau-t}} \quad (6-6)$$

where $r$ is the appropriate discount rate and $\tau$ is the current time. $\Omega_t$ is the set of all currently (i.e. $t=\tau$) available information. From this point, to avoid cumbersome notation, we write $k^{\Omega_t}$ as $k_t$, $S_{e,\tau}$ as $S_e$ and $S_{or,\tau}$ as $S_{or}$. The constraints for the multi-stage decision problem are given by the following equations of motion for hydroelectric plant operation:

$$\Delta R(t) = \left( F(t) - Q(t) \right) \quad (6-7)$$

$$R_{\text{min}} \leq R(t) \leq R_{\text{max}} \quad (6-8)$$

$$0 \leq Q(t) \leq Q^{\max} \quad (6-9)$$

where $R(t)$ is the amount of water in the reservoir at stage $t$, $\Delta R(t)$ is the change in the reservoir water inventory at the end of stage $t$, $R_{\text{min}}$ is the minimum lower limit of the reservoir and $R_{\text{max}}$ is the reservoir water capacity or maximum allowable inventory. $Q(t)$ is the amount of water withdrawn.
from the reservoir during stage $t$ and $F(t)$ is the amount of water added to the reservoir via inflow (determined by precipitation) and flow from upstream rivers and lakes.

The state of the reservoir is defined as the volume of water available in it at stage $t$. The state transition depends on the control action taken at $t$ (which determines the water withdrawal amount - $Q(t)$) and the random water addition into the reservoir ($F(t)$) as represented by the mass balance relationship in (6-7).

The second constraint (6-8) represents allowable limits on the amount of water in the reservoir at every decision stage. Technically, the upper limit represents the capacity of the reservoir and the lower limit represents the minimum inventory below which the turbines may not be operated. It is also possible, that both the upper and lower reservoir limits are determined by environmental and water use regulations for the specific body of water on which the hydroelectric scheme is located.

The maximum amount of water withdrawn per stage given by the third constraint (6-9) represents the capacity of the turbine/penstock.

For small to medium scale hydroelectric plants, operation and maintenance costs have negligible impacts on dispatch decision making on the part of asset owners because avoidable costs in the short-term are usually insignificant.

The solution of this model, conditional on a set of initial conditions will yield a set of optimal operating decisions (dependent on the state of the world at each time step) which can then be used to estimate the value of the plant.

In the next subsections, we model the dynamics of the available energy for release from the reservoir and also the spot energy price and operating reserve premium.

### 6.4 Dynamics of Hydroelectric Plant Optimization

The instantaneous power ($P_{w}$) produced by a hydroelectric plant at time $t$ can be represented as

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63 For the purpose of this thesis, we have assumed a rectangular reservoir so that one can determine the amount of water in the reservoir either by specifying it as volume or by specifying the dimensions of the reservoir. In real application, hydroelectric reservoirs are not of standard geometry so operators usually create a chart or table that can be used to determine the volume of water in a reservoir by measuring depth.
follows:\(^6^4\):

\[ P_n(t) = \rho g h(t) \times q(h) \times \eta(q(h)) \]  \hspace{1cm} (6-10)

where
\[ \rho \] is the density of water (kg/m\(^3\))
\[ g \] is the acceleration due to gravity (m/s\(^2\))
\[ h \] is the variable vertical height of the reservoir water surface (m) - it is the addition of the variable water head in the reservoir and the vertical elevation of the penstock inlet.
\[ q \] is the instantaneous rate of water withdrawal (m\(^3\)/s) - it depends non-linearly on the total available water head according to 6-12.
\[ \eta \] is the efficiency of the turbine – it depends non-linearly on the volume flow rate \( q \). It is the ratio of the actual energy produced per time to the theoretical potential energy present in the reservoir.

Below, we shall also need the symbol \( f \) which is the instantaneous inflow of water into the reservoir (m\(^3\)/s) – it depends on precipitation.

To determine the relationship between the instantaneous water withdrawal rate \( q \) and the total available head \( h \), assume that the surface of the reservoir and the outlet of the turbine are both opened to the atmosphere, using Bernoulli’s equation:

\[ \frac{1}{2} \rho \nu_2^2(t) - \frac{1}{2} \rho \nu_1^2(t) = \rho g h(t) \]  \hspace{1cm} (6-11)

where \( \nu_1 \) is the velocity of water at the reservoir surface and \( \nu_2 \) is the flow velocity at the turbine outlet.

Since the surface area of reservoir is much larger than the penstock cross-sectional area,

\[ \nu_2 \gg \nu_1 \] such that if the cross-sectional area of the penstock is \( a \), then \[ q(t) = a \nu_2 \] can be calculated as:

\(^{64}\) The problem of draining a volume of water from a reservoir with variable inflow is a classical problem in fluid mechanics; several standard fluid mechanics textbooks can be consulted for details.
\[ q(t) = a \sqrt{2gh(t)} \] (6-12)

The instantaneous amount of water \( w(t) \) available in the reservoir at time \( t \) can be determined from (assuming vertical sides for the reservoir):

\[ w(t) = (h(t) - h_o)A \] (6-13)

where \( A \) is the surface area of the reservoir and \( h_o \) is the vertical elevation of the reservoir bottom from the tail water. The relationship between the instantaneous change in the amount of water in the reservoir \( dw(t) \), the instantaneous water withdrawal rate \( q(t) \) and the instantaneous inflow of water \( f(t) \) is given by:

\[ dw = (f(t) - q(t))dt \] (6-14)

Hence, the amount of water accumulated in the reservoir between two instantaneous times \( t_1 \) and \( t_2 \) is given by the integral of 6-14 as follows:

\[ w = \int_{t=t_1}^{t_2} (f(t) - q(t))dt \] (6-15)

If we assume that \( \Delta t \) represents a discrete stage \( t \) and that \( f \) and \( q \) are constant over that stage, then the amount of water withdrawn and the amount of water added (via inflow) per stage is given by:

\[ Q(t) = q\Delta t \] (6-16)

and \[ F(t) = f\Delta t \] (6-17)

Hence, the change in the reservoir water inventory during a stage \( t \) is given by 6-7. This implies that if \( W_i \) is the reservoir water inventory at the end of a stage 1, then at the end of the subsequent stage (stage 2), the reservoir water inventory \( (W_2) \) is determined by:

\[ W_2 = W_i + F(t) - Q(t) \] (6-18)

Similarly, the amount of potential energy \( H(t) \) available for release from the hydroelectric reservoir at stage \( t \) is given by:

---

65 We assume that the tail water elevation is constant
The model as described in the foregoing sections is summarized in this section.

The value function of the hydroelectric plant is given by:

\[ V_t = \max_{k^t} \mathbb{E} \left[ \sum_{\tau=t}^{\infty} \left( B(k^\tau, S_{e,t}, S_{or,t}, f_t) \right) (1 + r)^{-\tau} \right] \]  \hspace{1cm} (6-5)

where

\[ d^{t-\tau} = \frac{1}{(1 + r)^{t-\tau}} \]  \hspace{1cm} (6-6)

Subject to the following constraints:

\[ \Delta W(t) = (F(t) - Q(t)) \]  \hspace{1cm} (6-7)

\[ W_{\min} \leq W(t) \leq W_{\max} \]  \hspace{1cm} (6-8)

\[ 0 \leq Q(t) \leq Q_{\max} \]  \hspace{1cm} (6-9)

where \( B(k^\tau, S_{e,t}, S_{or,t}, f_t) \) is either \( B_1 \), \( B_2 \) or \( B_3 \) depending on the optimal mode at stage \( t \). The expressions for \( B_1 \), \( B_2 \) and \( B_3 \) are given as follows:

\[ B_1 = H(t) \times S_e(t) \]  \hspace{1cm} (6-1)

\[ B_2 = \xi H(t) \times S_e(t) + (1 - \xi) H(t) \times S_{or}(t) + (1 - \xi) H(t) \times S_e(t) \times \gamma(t) \]  \hspace{1cm} (6-2)

\[ B_3 = 0 \]  \hspace{1cm} (6-3)

\( B_3 \) is the immediate value derived by idling the plant at stage \( t \) (at a shutdown state), hence it is zero.

\( H(t) \) is the amount of energy (MWh) available for release at stage \( t \). It is given by

\[ H(t) = P_w(t) \Delta t \]  \hspace{1cm} (6-19)
where \( P_w \) is the power generated by the plant; it is assumed constant during a stage but it changes from stage to stage depending on the amount of water in the reservoir at each stage and the amount of water withdrawn. That is,

\[
P_w(t) = \rho \times g \left( \frac{W(t)}{A} + h_o \right) \times q(t) \times \eta \left( q(t) \right)
\]

(6-20)

where

\[
q(t) = a \sqrt{2g \left( \frac{W(t)}{A} + h_o \right)}
\]

(6-21)

The parameter \( \gamma(t) \) in equation 6-2 is a binary integer used to represent the uncertain action of the market operator with regards to its call option on the reserve capacity of the power plant. We model this action with a random variable as follows:

\[
\gamma(t) = \begin{cases} 
1 & \text{if } u(t) < \varphi dt \\
0 & \text{if } u(t) > \varphi dt
\end{cases}
\]

(6-4)

where \( u \) is a uniform random variable (i.e. assuming equal probability of reserve activation at all intervals).

(66) In reality there may be higher probability of reserve activation at some periods (e.g. seasons) than at others. For instance, reserve activation is likely to occur during peak summer demands than on a spring day.

There are three exogenous state variables that have to be modeled; water inflow into the reservoir forebay, energy prices and operating reserve premiums. Both energy and operating reserve prices exhibit mean reversion and jump diffusion, hence they are modeled with equation 4-18 presented in Section 4.4.1. Water inflow is modeled with the discrete form of the mean reversion model, i.e. equation 4-16.

Hence, replacing \( S \) in equation 4-16 with \( F \) where the rest of the parameters are as defined in Section 4.4.1, the discrete-form stochastic equation for modeling the evolution of water inflow into reservoir forebay is:

\[
F(t + \Delta t) = \exp \left\{ \ln F(t) e^{-\eta \Delta t} + \ln(\bar{F}) (1 - e^{-\eta \Delta t}) - \left( 1 - e^{-2\eta \Delta t} \right) \frac{\sigma^2}{4\eta} + \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \varepsilon \right\}
\]

(6-22)
A joint probability distribution of energy and operating reserve prices has to be simulated (at every time step), correlations between the prices must be factored into the simulation. If we simulate different paths for energy prices based on a standard normal random variable \( \varepsilon \) as indicated in equation 4-18, operating reserve prices must be simulated with an adjusted version of equation 4-16. Hence, the evolution of energy price is modeled using the following expression:

\[
S_s(t + \Delta t) = \exp\left\{ \ln S_s(t) e^{-\eta \Delta t} + \ln(\bar{S}_s) \left[ 1 - e^{-\eta \Delta t} \right] - \left( 1 - e^{-2\eta \Delta t} \right) \frac{\sigma^2}{4\eta} + \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \varepsilon \right\} + (\bar{\kappa}_s + \gamma_s \varepsilon^*) \cdot \left( U > \phi_s \Delta t \right)
\]

(4-18)

Whereby, the evolution of operating reserve prices is modeled as follows:

\[
S_{or}(t + \Delta t) = \exp\left\{ \ln S_{or}(t) e^{-\eta \Delta t} + \ln(\bar{S}_{or}) \left[ 1 - e^{-\eta \Delta t} \right] - \left( 1 - e^{-2\eta \Delta t} \right) \frac{\sigma^2}{4\eta} + \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \varepsilon^* \right\}
\]

(6-25)

where \( \varepsilon^* \) is a standard normal random variable independent from \( \varepsilon \). If the prices of the two commodities are assumed to be perfectly correlated in the same direction (i.e. \( \rho = +1 \)), the last term of equation 6-25 disappears.

**6.6 Hypothetical Hydroelectric Plant Example**

The following sections apply the developed model to a hypothetical but realistic example of a hydroelectric plant operating in the IESO administered market in Ontario.

**6.6.1 Problem Data and Definition**

Consider a hydroelectric plant with the characteristics listed in Table 7. Based on the stated flow rate at maximum capacity and the stated reservoir size, a full reservoir can be drained in about 12 hours if operated at maximum capacity with no water inflow i.e.

Reservoir inventory at full capacity = \( 10.9 \times 10^6 \text{ m}^2 \times (4 - 0.001) \text{ m} \)

Turbine flow rate at maximum capacity = \( 1054 \text{ m}^3 / s \)
Minimum number of days to drain reservoir at maximum capacity = \[ \frac{43.54 \times 10^6}{1054 \times 3600 \times 24} \]

Hence, another way to describe the storage capacity of the plant is in terms flow i.e. \( \sim 0.5 \text{ cms days} \) where “cms” refers to \( \text{m}^3/s \).

We value the plant’s output and determine optimal operating strategies using the scenarios presented in Table 8. Scenario 1 focuses on 24 hour cycling operation based on an energy only offer into the power market. Results of this scenario will enable us to illustrate the operator’s flexibility to time-shift water from off-peak price hours to on-peak hours (i.e. peaking operation) within the same day. Cases 1a and 1b cover two different seasons, summer and spring. The purpose of running the scenario for summer and spring seasons is to capture the effect of significant water inflow differences between the two seasons as shown in Figure 35. In scenario 2, we examine 5-day\(^{67}\) weekly cycling operation, also based on energy offers only (cases 2a and 2b cover two seasons of interest). This scenario illustrates the flexibility to time-shift water inter-day. Scenario 3 is designed to show the impact of a combined operating reserve and energy offer strategy.

\(^{67}\) Monday to Friday; weekends are considered off-peak in Ontario. Each week day has 16 hours of on-peak and 8 hours of off-peak (i.e. 8 AM-11 PM – on-peak, 11-7 AM- off-peak).
## Table 7: Hypothetical Hydroelectric Power Plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Forebay / Reservoir</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface Area</td>
<td>$10.9 \times 10^6$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Elevation of Reservoir from Tail Water (assumed constant) - $h_o$</td>
<td>20</td>
<td>m</td>
</tr>
<tr>
<td>Maximum reservoir height</td>
<td>4</td>
<td>m</td>
</tr>
<tr>
<td>Minimum reservoir level</td>
<td>0.001</td>
<td>m</td>
</tr>
<tr>
<td><strong>Penstock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross-sectional Area (a)</td>
<td>44</td>
<td>m$^2$</td>
</tr>
<tr>
<td><strong>Turbine</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turbine flow rate at maximum capacity, $q$</td>
<td>1054</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>Turbine peak efficiency, $\eta$</td>
<td>95</td>
<td>%</td>
</tr>
<tr>
<td>Metric Specific speed</td>
<td>88</td>
<td>-</td>
</tr>
<tr>
<td><strong>Initial and boundary conditions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial reservoir inventory</td>
<td>50% of reservoir capacity</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>Initial Plant output</td>
<td>0</td>
<td>MW</td>
</tr>
<tr>
<td>Terminal value of water</td>
<td>13</td>
<td>$/MWh</td>
</tr>
</tbody>
</table>

68 The efficiency of hydroelectric turbines can be determined from manufacturer supplied efficiency curves or from empirical correlations. Gordon [100] derived and presented such correlations for different types of hydroelectric turbines based on data collected from different turbines in use around the world.
Table 8: Description of Illustrative Cases

<table>
<thead>
<tr>
<th>SCENARIO</th>
<th>CASE #</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>Case 1a</td>
<td>24 hour cycling operation, energy bid only, Season=Summer</td>
</tr>
<tr>
<td></td>
<td>Case 1b</td>
<td>24 hour cycling operation, energy bid only Season=Spring</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>Case 2a</td>
<td>5 day cycling operation, energy bid only Season=Summer</td>
</tr>
<tr>
<td></td>
<td>Case 2b</td>
<td>5 day cycling operation, energy bid only Season=Spring</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>Case 3a</td>
<td>5 day cycling operation, energy &amp; OR bid, Season=Summer, ( \varepsilon = 0.5, \gamma = 1.0 )</td>
</tr>
<tr>
<td></td>
<td>Case 3b</td>
<td>5 day cycling operation, energy &amp; OR bid, Season=Summer, ( \varepsilon = 0.5, \gamma = 0.5 )</td>
</tr>
<tr>
<td></td>
<td>Case 3c</td>
<td>5 day cycling operation, energy &amp; OR bid, Season=Summer, ( \varepsilon = 0.5, \gamma = 1.0 ) &amp; average OR price is twice that of case 3a &amp; 3b</td>
</tr>
</tbody>
</table>
6.6.2 Solution to the Hypothetical Example

The solution to the problem is obtained by first performing a forward-looking Monte Carlo simulation of the uncertain underlying value drivers i.e. energy price, operating reserve premiums and water inflow into reservoir. The price models are calibrated using 4 years of historical market data available on IESO’s website. The flow model is also calibrated to 4 years of historical water inflow data for a river system in Ontario.

After simulating the stochastic factors as described, the dynamic optimization is then carried out recursively using a backward simulation starting from the final stage. For each possible reservoir state (the reservoir height is divided into small discrete states), the following were calculated:

\[
V_{1,t}(\zeta_t) = B_{1,t}(\zeta_t) + d \times E[V_{t+1}(\zeta_{t+1})|\zeta_t, k_1]
\]

\[
V_{2,t}(\zeta_t) = B_{2,t}(\zeta_t) + d \times E[V_{t+1}(\zeta_{t+1})|\zeta_t, k_2]
\]

\[
V_{3,t}(\zeta_t) = d \times E[V_{t+1}(\zeta_{t+1})|\zeta_t, k_3]
\]

for \((t \leq T - 1)\), where \(V\) represents the value. Subscripts 1, 2 and 3 represent the three available modes. For \(t = T\), \(V_{k,T}(\zeta_T) = B_{k,T}(\zeta_T)\) where \(k = 1,2,3\). We assume water left in the reservoir beyond \(T\) is worth a constant value called the terminal value of water as indicated in Table 7.

As stated earlier, the factor \(d\) is the discount factor but in this case, \(d\) is determined over \(t\) and \(t+1\) i.e.

\[
d = \frac{1}{(1+r)}
\]

where the interest rate \(r\) is adjusted to be consistent with the chosen interval \(\Delta t\) e.g. 1 hour. In reality, for an hourly granularity, the factor \(d\) can be neglected in equations 6-26 to 6-28.

To satisfy the capacity and operating constraints, we impose the following conditions on the available

\[V_t(S_t) = \max\{V_{1,t}(S_t), V_{2,t}(S_t), V_{3,t}(S_t)\}\]

\[69\] Note that \(V_t(S_t) = \max\{V_{1,t}(S_t), V_{2,t}(S_t), V_{3,t}(S_t)\}\)
operating mode at each decision step:

\[
K_t = \begin{cases} 
1 & \text{if } (W(t) = W_{\text{max}} \text{ or } W(t) + \Delta W > W_{\text{max}}) \\
0,1,2 & \text{if } (W_{\text{min}} < W(t) < W_{\text{max}} \text{ & } W_{\text{min}} < W(t) + \Delta W \leq W_{\text{max}}) \\
0 & \text{if } (W(t) = W_{\text{min}} \text{ or } W(t) + \Delta W < W_{\text{min}}) 
\end{cases}
\]

(6-30)

The implications of these conditions are as follows:

First condition – if the reservoir is currently filled to capacity or any change in water level within the current decision interval (due to production and water inflow) will result in spill, the only operating mode available is to “offer all available capacity into energy market”.

Second condition – if the reservoir constraints are satisfied over the decision interval, all three operating modes are available.

Third condition – if the reservoir is currently at minimum allowable capacity or any change in water level within the current decision interval (due to production and water inflow) will result violating the minimum reservoir level requirement, the only operating option available is to idle the plant.

6.6.2.1 Solution Steps

At each decision time step, the expressions 6-26 to 6-28 are calculated for each reservoir state space and all the feasible reservoir state transitions from that state space. The possible state transitions depend on the amount of water withdrawn at the current stage and the amounted of water inflow between the current stage and the next stage.

To solve the problem recursively, at the last time step, the conditional expectation value can be set to either zero (i.e. no continuation value) or to some positive value to capture the value of water e.g. water rental charge. For this particular illustrative case study, we select a fixed and known water rental charge to represent the continuation value of residual water inventory. Either way, \(V\) simply equals the cash flow at that stage. The decision at this stage is easy since the immediate cash flows can be compared. Once the optimal decision is taken for each possible reservoir state, we step back one stage and calculate 6-26 to 6-28 again. This time the conditional expectation must be calculated. This is done by using the LSM algorithm. In this example, following suggestions by Longstaff and Schwarz [78] and Tsekrekos [81], we use the following simple basis functions \(S_v, S_{or}, S^2_v, S^2_{or}, S, S_{or}\) such that
\[ \sum_{i=0}^{N} a_i \Psi_i(\xi_t) = a_1 + a_2 (S_s + S_{ao}) + a_3 (S_s + S_{ao})^2 \]  

(6-31)

The coefficients \( a_1, a_2 \) and \( a_3 \) are obtained by linear regression where the payoff from stage \( t+1 \) (discounted to \( t \)) is used as the dependent variable and the values of the basis functions at \( t \) are as the independent variables. After obtaining the coefficients, the continuation values (second terms of 6-26 to 6-28) are then determined. These calculations are done for each possible combination of reservoir state and state transition.

Following the same procedure, we step back all the way to stage 1. Now that the decision at every stage is known for each possible reservoir state, starting from a given initial reservoir state (level), the plant output can be determined by discounting all the future cash flows (based on the optimal decisions determined) to the present using the discount factor \( d \).

To model changes in turbine efficiency as reservoir head and flow rate changes, we use an empirical model presented by Gordon [100]. Gordon’s model, as presented in 6-32 to 6-34, for a “Francis” turbine was derived in his paper based on empirically observed relationships between turbine flows at peak efficiency and under “spin no load” conditions as well as turbine peak efficiency. With those information available, changes in efficiency (relative to the peak efficiency) can be determined for different turbine flow rates.

\[ \eta(q) = \eta_{peak} - \Delta \eta_{peak} \]  

(6-32)

\[ \Delta \eta_{peak} = \eta_{peak} \left( 1 - \frac{q_{SNL}}{q_{peak}} \right)^{-k} \left( 1 - \frac{q}{q_{peak}} \right)^k \]  

(6-33)

\[ k = 3.94 - 0.0195n \]  

(6-34)

where

\( \eta(q) \) is the turbine efficiency at flow rate \( q \)

\( \eta_{peak} \) is the turbine peak efficiency

\( \Delta \eta_{peak} \) is the change in turbine efficiency from peak efficiency (i.e. as the flow rate deviates from \( q_{peak} \))
$q_{SNL}$ is the “spin no load” flow (m$^3$/s)

$q_{peak}$ is the turbine flow at peak efficiency (m$^3$/s)

$n$ is the metric specific speed

For this hypothetical example, we assume a specific speed of 88 (Gordon [100]) and also a $q_{SNL}$ of 2% of $q_{peak}$.

### 6.6.3 Calibration of Underlying Variables

For this illustrative problem, we make use of the calibrated parameters of the mean-reverting jump diffusion model used in Chapter 5 to model the required probability distributions of power prices. The other uncertain factors of relevance to the problem are (i) water inflow into reservoir and (ii) operating reserve prices. Figure 37 and Figure 38 show actual water inflow into an Ontario based hydroelectric facility for 2006 by seasons. As discussed earlier, the high flow seasons are fall and spring (with the highest being spring) while the low flow seasons are winter and summer. Similarly, Figure 39 and Figure 40 show the 30 minute operating reserve prices for Ontario in 2006. For this particular year, the highest prices were observed in the spring were summer and fall witnessed very low prices. The parameters of the mean-reverting jump diffusion models are calibrated for water inflow and operating reserve prices using the data provided. The calibrated parameters are provided in Appendix A. Figure 41 to Figure 44 show samples of simulated water inflow for spring and winter seasons using the parameters calibrated from the data presented. Similarly, Figure 45 to Figure 48 show samples of simulated OR prices using the parameters calibrated from the 2006 Ontario 30 minute OR data set.
Table 9: Calibrated Parameters of Mean Reverting Model from Daily Flow Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{x})</td>
<td>5.215</td>
<td>6.767</td>
<td>4.654</td>
<td>5.379</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.32</td>
<td>0.06</td>
<td>0.35</td>
<td>0.09</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.3</td>
<td>0.2</td>
<td>0.39</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 10: Calibrated Parameters of Mean Reverting Model from IESO 10 Minute Spinning Operating Reserve Price Data (2006)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{x})</td>
<td>0.952</td>
<td>1.075</td>
<td>1.078</td>
<td>1.078</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.68</td>
<td>0.19</td>
<td>0.44</td>
<td>0.42</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.44</td>
<td>0.53</td>
<td>0.38</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Figure 37: 2006 Winter and Summer Reservoir Water Inflow Data

---

70 www.opg.com
Figure 38: 2006 Spring and Fall Reservoir Water Inflow Data

71 www.opg.com
Figure 39: 2006 Ontario Market 10 Minute Spinning Operating Reserve Prices
Figure 40: Seasonal Comparison of 2006 Ontario Market 30 Minute Spinning Operating Reserve Prices
Figure 41: Sample Simulated Paths of Reservoir Water Inflow over a 5 Day Period Based on Parameters Calibrated from 2006 Spring Flow Data.
Figure 42: Sample Simulated Paths of Reservoir Water Inflow over a 5 Day Period Based on Parameters Calibrated from Winter 2006 Flow Data.
Figure 43: Example of a Single Simulated Flow Path over a 5 day Period (sample 1)
Figure 44: Example of a Single Simulated Flow Path over a 5 day Period (sample 2)
Figure 45: Sample Simulated Paths of 10 Minute Spinning OR Price over a 5 Day Period Based on Parameters Calibrated from Spring 2006 IESO Data.
Figure 46: Sample Simulated Paths of 30 Minute OR Price over a 5 Day Period Based on Parameters Calibrated from Winter 2006 IESO Data.
Figure 47: An Example Simulated Path of OR Price over a 5 day Period (sample 1)
Figure 48: An Example Simulated Path of OR Price over a 5 day Period (sample 2)
6.6.4 Results

Being an energy limited resource, the value of a peaking hydroelectric power plant lies in its ability to transfer (or time-shift) water from low price hours to high price hours. This fact is illustrated by Figure 49 which compares power price, production (in MW) and reservoir height from a sample path of case 1a. For this 24-hour reservoir cycling case, the objective of the power plant operator is to make use of its available water (initial reservoir content and inflow across 24 hours) in such a way that profit is maximized. During this period, the operator draws down the reservoir as indicated by Figure 50 which shows samples of reservoir level trends across the 24 hour operating horizon. It is clear from the figures that the plant operator will “pond” the reservoir (i.e. allow it to build up) during the periods when prices are low (usually off-peak hours) so that production can be maximized during the hours when prices are higher (typically on-peak hours). This time-shifting of water is carried out taking into consideration the current reservoir inventory and expected inflow and also the impact of reservoir elevation on production efficiency. The best value for water is achieved when production is as the maximum turbine efficiency point (in this case 95%), inefficient production below or above the maximum efficiency points translate indirectly into water loss through reduction in total produced energy.

The flexibility to time-shift water is taken away when future water inflow exceeds the capacity of the power plant. This is the situation observed under most scenarios in case 1b as shown in Figure 51 and Figure 52. As shown in Figure 52, because of the high inflow experienced (since the season chosen for this case is Spring), the reservoir level is relatively unchanged or increasing not because the operator wants to “pond” intentionally but because of high water inflow. As shown in Figure 52, the power plant is producing power at its maximum capacity while the reservoir level is increasing. Under this scenario, operating decisions are insensitive to power prices which imply that the so called “value of flexibility” (i.e. real options) does not exist due to limited power plant capacity.

Cases 2a and 2b are similar to cases 1a and 1b except that the simulation is carried out over a 5 week day period\(^{72}\). For cases 2a and 2b, the objective of the power plant operator is to optimize the reservoir (initial inventory plus expected inflow) over the 5 day period with each day having 8 off-

\(^{72}\) In Ontario, weekends are considered as off-peak
peak and 16 on-peak hours. As shown in Figure 53 and Figure 54, the operator is able to take advantage of inter-day price differentials by drawing down the reservoir over the much longer period.

The cases under scenario 3 incorporate the power plant operator’s option to offer both energy and operating reserve into the market. Intuitively, the expected increase in value with this option should be proportional to the ratio of operating reserve price to that of energy price. For instance for this illustration, the long-term average operating reserve price in the summer is $3 while that of energy price is $52. So if one is certain about the operating reserve being called by the ISO and given the fact that only half of the available energy can be offered as operating reserve, one can expect an increase of $0.5 \times 6\%$. This intuition is validated by the results shown in Table 11 which compares the results on an expected value basis. This percentage increase would be less if the probability of the ISO calling the reserve is less than 1. Under this scenario, there may be less revenue from energy for example from having to produce reserved energy at a different hour which might be different from the optimal hour under an energy-only production strategy (see Figure 55). For a 50% probability of reserve activation, the result of case3b shows that the 3% incremental value reduces to 2%. If the long-term average operating reserve price in Ontario were double what it is in 2006 (the year on which model calibration was based), for instance as in 2004 spring (see Figure 36), one would expect a proportionately higher increase. Case 3c which was designed to capture this scenario shows an increase of 6%.

In conclusion, the value of a peaking hydroelectric power plant is influenced by operating flexibilities available to the plant operator and uncertain market factors. The valuation framework developed in this work captures these key parameters without introducing unneeded complexity.
Figure 49: Comparison of Hourly Operation with Power Price and Reservoir Level based on a Single Simulated Path from Case 1a
Figure 50: Samples of Simulated Hourly Reservoir Height Trend based on Production and Water Inflow across a 24 Hour Period (from Case 1a)
Figure 51: Comparison of Hourly Operation with Power Price and Reservoir Level based on a Single Simulated Path from Case 1b
Figure 52: Samples of Simulated Hourly Reservoir Height Trend based on Production and Water Inflow across a 24 Hour Period (from Case 1b)
Figure 53: Comparison of Hourly Operation with Power Price and Reservoir Level based on a Single Simulated Path from Case 2a (5 day cycling operation)
Figure 54: Samples of Simulated Hourly Reservoir Height Trend based on Production and Water Inflow across a 5-day Cycling Horizon (from Case 2a)
### Table 11: Calculated Revenues for Cases Considered

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Case</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Incremental Value of OR Option</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario 1</td>
<td>Case1a</td>
<td>$102,328</td>
<td>$27,271</td>
<td>N/A</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>Case2a</td>
<td>$308,261</td>
<td>$74,332</td>
<td>N/A</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>Case3a</td>
<td>$316,442</td>
<td>$76,770</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>Case3b</td>
<td>$315,690</td>
<td>$76,847</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>Case3c</td>
<td>$325,159</td>
<td>$84,006</td>
<td>5%</td>
</tr>
</tbody>
</table>
Figure 55: Comparison of Hourly Operation with Power Price and Reservoir Level based on a Single Simulated Path (Energy Only strategy vs. Energy & OR strategy, 5 day cycling operation)

6.7 Chapter Conclusions and Applications

The focus of this section has been on the valuation of a hydroelectric power plant. The types of applications of the modeling framework discussed in Chapter 5 are also largely valid here except for the gas-fired power plant specific issue of gas imbalance. The model developed and presented here for peaking hydroelectric facilities considers all the key value drivers for these facilities, including the value of storage reservoir and the opportunity to derive value from an ancillary services market. To the best of the author’s knowledge, this work is the first to address in a robust fashion, the opportunity for a peaking hydroelectric facility to derive additional value from its storage capability by playing the operating reserve market in addition to a real-time energy market. From an applications
perspective, the following are realistic scenarios where the model developed in this work for a peaking hydroelectric facility can find value:

1. Earnings at risk study for a hydroelectric power plant. Given that the model uses a Monte Carlo framework and it considers inflow variability as well as price uncertainty, low percentile earnings (from the resulting probability distribution of discounted earnings) represent a combination of low water inflow scenarios (i.e. low rainfall season or a generally poor precipitation year) and low power price scenarios. Knowledge of the impact on earnings of adverse water inflow and poor power price scenarios is valuable from risk hedging perspective, allowing the merchant energy producer to understand its hedging needs.

2. Value of storage reservoir and peaking capability. A flexible hydroelectric power plant is considered more valuable than a similar plant without storage since scarce water can be shifted from low price periods to high price period by making use of reservoir storage. The model developed in this work and the type of analysis presented can be used to determine the value of having storage and hence peaking capability. For instance, the valuation analysis can be repeated with the same set of inputs but different reservoir sizes. Any change in valuation observed should be as a result of reservoir size differences and the corresponding change in peaking capability.

3. Value of ancillary services. The developed model combines the opportunity to sell energy into a real time power market with the opportunity to provide ancillary services. Similar to the illustration presented in this thesis, one can combine valuation results based on an energy-only operating strategy with results from a combined operating reserve and energy strategy. The difference in valuation results should reflect the value of a combined strategy over an energy only strategy.

4. Optimal market operation planning. Given that the valuation model developed optimizes the operation of a hydroelectric power plant subject to available information on all relevant value drivers, the resulting dispatch results can be used for planning optimal market operations strategies.
Chapter 7
Flexibilities and Baseload Power Assets

7.1 Introduction
Existing applications of real options analysis to power plants assets have largely focused on peaking flexibilities. The premise of those applications is to account for the value of flexibility that a peaking power plant operator has with regards to his ability to revise operating decisions and switch operating modes as market-based uncertainties are resolved with time. Unlike peaking and mid-merit power plants, baseload power plants are always operated around the clock, including off-peak hours when power prices are low. Baseload power plants typically have one or more of the following characteristics:

- Low marginal operating costs
- Long start-up and or shutdown times
- Inability to time shift energy

For instance, all hydroelectric power plants generally have low marginal operating costs but for a hydroelectric power plant to be used as a peaking facility, it must have the capability to time-shift energy from one period to another by having storage reservoir to store water. A run-of the river hydroelectric scheme has no capability to reserve unneeded water and so is operated at all times with output only varying in response to upstream flow and direct precipitation. Unused water in a run-of the river hydroelectric facility has to be spilled, i.e. it has to bypass the turbines. Similarly, Nuclear Power Plants (NPPs) have long start-up procedures once they are shutdown and they also have relatively low marginal operating costs as most of their operating costs are fixed. Therefore, even in competitive wholesale power markets with volatile power prices, these power plants are operated around the clock.

However, when the operations of a baseload power plant are considered over a much longer time frame, operating flexibilities exist and are typically considered by owners or operators of such assets for decision-making purposes. These flexibilities have to do with long-term Life Cycle Management (LCM) of power plants. As it typical with real options, LCM investment decisions can be optimized
with regards to timing of key activities, budget and order of activities. The underlying uncertainties driving such strategic optimization will include lost revenue during outages (due to uncertain future price of power), costs of components and technical uncertainties such as equipment degradation. Typically, regulatory uncertainties are also considered such as future environmental laws and safety requirements. As these and other uncertainties are resolved, decisions can be revised or delayed resulting in value creation for the asset owner from costs savings. The flexibility to react to relevant conditions as they unravel constitutes optionalties on the part of power asset owners and the values of those options must be factored into valuation.

Over the life of a power plant, the owner will typically be faced with several choices regarding equipment maintenance and upgrades. Naturally, the higher the budget spent for maintaining and upgrading equipments, the better the plant should perform in the future. Inadequate equipment maintenance will generally result in degradation and failure leading to increase in forced or unplanned outages. Similarly, lack of timely components and equipments inspections will prevent early discovery of degradation thereby contributing to future equipment failure and poor performance. However, the question of marginal benefits always arises and rational investors will make decisions accordingly.

In reality, the relationship between LCM spending and future plant performance is complex and has been the subject of intensive engineering research. Extensive literature exists in several engineering subject areas dealing with optimal component and equipment replacement in ageing manufacturing systems, including power plants. However, the fact remains that owners of such engineering assets take into consideration several uncertain factors including economic and technical issues before key decisions are made. Hence, while baseload power plants are not optimized in response to short-term movements of key value drivers, over a long-enough time frame, there are flexibilities and those flexibilities do have values that can be attached to them to value such power plants. The static NPV approach to valuing baseload power plants is to assume a capacity factor\(^{73}\) and known variable costs of future maintenance activities such that net revenue can be estimated and discounted. A more realistic valuation framework will capture ongoing management reactions to key uncertainties as they evolve resulting in “option values” from the ensuing cost minimization.

---

\(^{73}\) At best, sensitivity analyses are carried out around such an assumption to reflect impact of different scenarios.
7.2 Conceptual Framework

Conceptually, the option to repeatedly shutdown a manufacturing system over the course of a planning horizon, either for maintenance or for other purposes can be likened to commodity “swing option”. Commodity delivery contracts with swing options are used in commodity markets, especially energy markets, to deal with uncertainty in the volume of a commodity required. For instance, the owner of a power plant may contract for fuel delivery (e.g. coal) with flexibility to “swing up” or “swing down” as its need for the fuel changes, given that it has a limited space for inventory. However to protect the seller, a swing contract will typically contains a penalty clause tied to the total volume of the commodity that the buyer takes during the life of the contract. Such a penalty clause will be activated if the cumulative volume taken by the buyer during the life of the contract is less or more than the agreed cumulative volume, hence they are called “take or pay” contracts.

The objective of the holder of a swing contract is to determine the optimal set of stopping times and swing volumes such that the value of its rights is maximized. Hence, swing options are a type of “Bermudan” option, allowing multiple early exercise rights. A number of authors have recently focused on the valuation of swing contracts, both in a continuous time and a discrete time framework. Examples include Jaillet et al. [50], Dahlgren [68], Ibanez [101] and Dorr [52].

The real option decisions of a baseload power plant owner or any power plant over a long time frame are conceptually similar to those of a swing contract holder. The swing contract holder has to optimally time the exercise of its swing rights. In the case of the baseload power plant operator, one can consider the life-cycle outage rights as some type of swing rights that have to be optimally exercised. Also, the volume of swings can be conceptually likened to size of budget during each outage exercise. Again, if we consider plant degradation as constituting a penalty for not executing enough LCM outages or spending enough money on maintenance and inspection during the life of the power plant, one can imaging costs of plant degradation as penalty payments to be made at the end of the plant life or at the end of some finite investment time horizon.

More specifically, consider the operation of a power plant over a time horizon [0, T] (where T is some finite time in the future) during which the owner has a finite number of key equipment maintenance and upgrade projects choices to be made. Each of those key project options can be characterized by (i) total costs/budget of implementation and (ii) length of plant outage required for
implementation. Given a set of discrete time windows in the valuation time interval at which the power plants can be shutdown for LCM purposes (i.e. exercise opportunities), a dynamic optimization model can be formulated to seek the set of optimal stopping times in such a way that the number of stopping times and the order of stopping times (i.e. which order of project execution) are optimized. In addition, the total amount of spending on maintenance and upgrades is also optimized taking into consideration penalty for inadequate spending which is plant degradation.

In general, the valuation problem of a baseload power plant falls within the valuation framework presented earlier in Chapter 4. However in this case, the intervals between exercise opportunities are much longer and instead of directly maximizing value, the objective of the optimization in the context of LCM is to minimize total costs thereby indirectly maximizing value. In the following section, the key costs associated with life cycle management are discussed.

7.3 Key Costs Parameters for Life Cycle Management

7.3.1 Lost Revenue during Outage
The revenue lost during a LCM outage depends on the contract structure under which the output energy of the specific baseload plant is sold. If the output energy of the plant has been pre-sold either through uninterruptible forward contract or a power purchase agreement (PPA), the asset owner may have to purchase replacement energy to meet its obligation during outages. Under this scenario, the lost revenue during a LCM outage is the cost of purchasing replacement energy.

If no such contract exists and the output of the plant is simply sold to the spot market, there may be no need on the part of the generator to purchase replacement power. The lost revenue (during the outage) in this case is the revenue that could have been received if the plant were operating at its normal capacity. Since revenues are contingent on market prices, they are uncertain.

7.3.2 Cost of Preventative Maintenance and Upgrades
The activities performed during a LCM outage could include (i) scheduled replacement of ageing components and equipment (ii) maintenance of existing equipment (iii) plant addition or equipment upgrades and (iv) component inspections for reliability purposes among other things. For all these and other types of projects, one can assume that the costs have two components (i) costs of hardware (components, equipment, machinery etc) and (ii) costs of labour. Costs of hardware will vary from
one piece of equipment to another, so also are the costs related to labour. Both of these costs are largely uncertain. The costs of labour can be related to the length of outage, as labour rates are typically determined per unit of time.

7.3.3 Penalty for Plant Degradation

As mentioned earlier, too little maintenance spending can result in increased frequency of forced outages in the future due to equipment ageing and component failures. Other than the revenue lost during forced outages, the cost of corrective maintenance also increases and plant capacity may degrade forcing operators to derate output. In the long term, the strategic flexibility to extend plant life and defer safe storage and decommissioning will also be impacted.

Degradation of plant performance due to inadequate maintenance is probabilistic in nature. One cannot be certain how much the plant will degrade or how many forced outages will occur due to inadequate maintenance. One can estimate costs associated with forced outages and corrective maintenance from historical plant performance data and relate them to cumulative spending on maintenance and upgrades. Such costs (given average and volatility) can be estimated and used as proxies to model penalty costs due to inadequate spending.

7.4 Valuation Model

Consider a multi-year operation of a baseload power plant within the interval \([\tau, T]\). Let the set \(\Theta\) be the set of exercise opportunities in the interval \([\tau, T]\) at which the asset owner can exercise its right to shutdown the power plant for LCM purposes\(^{74}\) where

\[
\Theta = \{t_j : j = 1, \ldots, N\}
\]  

(7-1)

\(^{74}\) Depending on the particular power market jurisdiction, this opportunity set can represent time periods that have been pre-approved by the Independent System Operator (ISO). For instance, in the Ontario wholesale power market, the IESO requires generators to request approval for any planned outages that are greater than 5 days in duration. For the purpose of system reliability, the IESO can reject requests if necessary. Such requests are required to be submitted at least 3 months in advance and confirmation is given between 33 and 21 calendar days before the outage is to commence. It is also possible for the ISO to request a deferral of an already pre-approved planned outage because of system reliability issues. In the context of real options valuation, the set of exercise opportunity intervals considered in modelling will depend on the information available or projected at decision time.
If the asset owner has the option to execute one LCM project within the interval, its objective is to find the optimal stopping time within the exercise opportunity set $\Theta$. At each decision time (or exercise opportunity), one can define two possible operating modes (in the context of the framework presented in chapter 4) $k_1$ and $k_2$ such that the alternatives of the asset owner is to either shutdown to execute the LCM project (mode $k_1$) or keep the option alive (mode $k_2$). That is, if $B_{t_j}$ is the immediate cost of executing the project at $t_j$, then we have the following alternative specific cost functions:

$$C_{t_j}(\xi_{t_j}, k_1) = B_{t_j}(\xi_{t_j}, k_1) + d \times E[C_{t_{j+1}}(\xi_{t_{j+1}}, k_1)] j < N \quad (7-2)$$

$$C_{t_j}(\xi_{t_j}, k_2) = d \times E[C_{t_{j+1}}(\xi_{t_{j+1}}, k_2)] j < N \quad (7-3)$$

where $C_{t_j}$ is the total cost of executing the LCM project at the exercise opportunity $t_j$ and the conditional expectation terms (in both 7-2 and 7-3) are the costs continuing beyond $t_j$. In equation 7-2, the conditional expectation term represents the LCM cost of the plant at $t_{j+1}$ when the project has already been executed at $t_j$ while the same term in equation 7-3 is the LCM costs at $t_{j+1}$ if the option to execute the project is kept alive until $t_{j+1}$. $\xi_{t_j}$ encapsulates all the information available to the decision maker at $t_j$ and $d$ is the discount factor given by:

$$d = \frac{1}{(1+r)} \quad (7-4)$$

where $r$ is the risk-free rate or risk-adjusted discount rate.

With multiple project alternatives available to the asset owner, the conditional expectation term in equation 7-2 is the total LCM cost for continuing at $t_{j+1}$ with one less project alternative while the same term in 7-3 is the total LCM cost of continuing with all the alternatives. If the project alternatives are unique (i.e. each project having different costs characteristics), the number of decision modes $k$ will increase accordingly with the decision maker having to solve equations 7-2 and 7-3 for each of the mutually exclusive project alternatives.
If there are \( n \) project alternatives and \( z \) is the counter for executed projects (i.e. \( z = 1, \ldots, n \)), then at the final exercise opportunity \( t_N \) (where \( N \) is the number of available exercise opportunities), the asset owner’s alternatives are as follows:

\[
C_{t_N}(\xi_{t_N}) = B_{t_N}(\xi_{t_N}, z + 1) + D_{t_N}(\xi_{t_N}, n - (z + 1)) \quad \text{(7-5)}
\]

\[
C_{t_N}(\xi_{t_N}) = D_{t_N}(\xi_{t_N}, n - z) \quad \text{(7-6)}
\]

where \( D_{t_j} \) represents the degradation penalty costs. The alternative specified by equation 7-5 implies that the asset owner can decide to execute one additional project and pay the corresponding degradation penalty for one less project. Alternatively, as specified by equation 7-6, the owner may decide not execute any further project and simply choose to pay the corresponding degradation penalty.

In general, for each available exercise opportunity, the cost minimization is carried out as stated above to determine the optimal decision. After determining the optimal decisions for the entire valuation interval, the number of optimal projects selected for execution will vary from 0 (if there is no minimum requirement) to the maximum under consideration and their timing will be a subset of \( \Theta \). Given the optimal projects and their corresponding outage timing, the total costs of LCM outages can be determined after which the plant can then be valued as the sum of the discounted future cash flows.

If we define \( H \) as the plant capacity, the cash flow at time \( t \) if the power plant is producing power is:

\[
B = \sum_{\Delta t} H \times S_t \quad \text{(7-7)}
\]

where \( S_t \) is power price at \( t \) and \( \Delta t \) is the duration of the continuous operation. Similarly, if the asset is shutdown for a LCM project, the cash flow is as follows:

\[
B = -(C^M + C^L + L^R(S_t)) \quad \text{(7-8)}
\]

where \( C^M \) is the costs of materials or components, \( C^L \) is the cost of labour and \( L^R \) is the lost
revenue or cost of replacement power during the LCM outage\(^{75}\) and it is a function of power price. The variable labour costs is a function of the number of outage man hours \((\text{hr})\) and total costs of labour per hour. Hence the cost of labour can be represented as follows:

\[
C^L = t_d \times r \tag{7-9}
\]

where \(t_d\) is outage duration and \(r\) is the aggregate labour rate per unit time\(^{76}\). Naturally, \(t_d \leq \Delta t_j\), implying that the maximum project duration must not be greater than the interval between two allowable exercise opportunities.

### 7.4.1 Solution Method

Following the valuation framework presented in Chapter 4, the first step in the solution process involves a forward-looking modeling of the underlying uncertainty factor(s). It is possible to assume that both costs of materials and power prices are random. Both of these can be modeled using a mean reverting stochastic process as detailed in section 4.4. After simulating different paths for the underlying value drivers, the alternative specific cost functions at each decision node can be evaluated using the Least Squares Monte Carlo (LSM) algorithm as discussed earlier. With the optimal decision selected for each decision node, the power plant value over the valuation time frame can be determined using a forward looking Monte Carlo approach, taking into consideration the appropriate cash flows for each period, including market revenue (during operation) and costs of LCM (during planned outages).

Since each project is unique, we define the problem state variable as the subset of projects that has been carried out as at the start of opportunity \(t_j\) such that if there are \(n\) projects under consideration the number of such subsets is \(2^n\). For instance, with 3 mutually exclusive projects under consideration by the asset owner, the relevant subsets are as follows: \(\{0\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\). To describe this in words, at the start of every exercise opportunity the following states are possible: (i) no project has been executed (ii) only project 1 has been executed (iii) only project 2 has been executed (iv) only project 3 has been executed.

\(^{75}\) Assuming that the plant is shutdown only for life-cycle management outage.

\(^{76}\) If the LCM outage costs X million dollars in labour charges and the outage duration is Y hours, then \(r = X / Y\).
executed etc. If the projects are not unique, then the elements of the subsets would be the number of projects. Figure 56 illustrates possible transitions when $n = 3$.

---

**Figure 56: Illustration of State Transitions between Exercise Opportunities**

The optimization procedure involves enumerating the alternative specific Bellman equations 7-2 and 7-3 (for $t_j < t_N$) and 7-5 and 7-6 (at $t_j = t_N$) for each project state considering the possible state transition. One can easily introduce technical constraints such as a requirement to (i) execute certain projects back to back (ii) ensure two specific projects are not executed after one another or (iii) execute a certain project first by constraining the appropriate transitions.

### 7.5 Model Summary

A summary of the model is summarized in this section. Given the capacity $H$ of a baseload power plant, the plant operates at all times within the interval $[\tau, T]$ except when it is shutdown for life cycle management (LCM) activities such as equipment maintenance and upgrades. During operation, future cash flows are given as $\left(\tau \leq t \leq T\right)$:
\[ B = \sum_{\Delta t} H \times S_t \]  

(7-7)

where \( S_t \) is power price at \( t \) and \( \Delta t \) is the length of continuous operation. When the power plant is shutdown for a desired LCM project, the immediate future cash flow during the shutdown period can be expressed as follows:

\[ B_t = - \left( C^M + C^L + L^R(S) \right) \]  

(7-9)

where \( C^M \) is the costs of materials or components, \( C^L \) is the cost of labour and \( L^R \) is the lost revenue/cost of replacement power during the decision stage. If \( t_d \) is the LCM outage duration the lost revenue \( L^R \) is given as follows:

\[ L^R = \sum_{t_d} H \times S_t \]  

(7-10)

i.e. as a function of future variable price of power. The variable labour cost can also be expressed as a function of the outage duration and labour rate per unit time such that:

\[ C^L = t_d \times r \]  

(7-9)

where \( r \) is the aggregate labour rate per unit time. If desired, \( C^M \) can be modeled as a random variable following a specified stochastic process such as the mean-reverting process presented in Chapter 4.

If the asset owner has \( n \) mutually exclusive LCM projects under consideration within the interval \([\tau, T]\) whereby one project at a time can be executed starting at the beginning of specified exercise opportunities \( t_j \) (\( j = 1, \ldots, N \) and \( t_d \leq \Delta t_j \)) and also knows the associated penalty for inadequate maintenance of the power plant (in terms of total costs of degradation), its objective is to minimize the total costs associated with LCM during the interval \([\tau, T]\).

In order to do this, the owner has to solve the following minimization problem at each decision stage:

\[ \min \left\{ B_t \left( \xi_{t,j}, k_1 \right) + d \times E\left[ C_{t,j,t,1} \left( \xi_{t,j,1} \right) S_{t,1}, k_1 \right], \ d \times E\left[ C_{t,j,t,2} \left( \xi_{t,j,2} \right) S_{t,2}, k_2 \right] \right\} \quad j < N \]  

(7-11)

\[ 77 \] If one decides to consider non LCM variable operating cost in the modeling, these will be factored into \( S_t \).
\[
\min \left(B_{t_N}(\xi_{t_N}, z+1) + D(\xi_{t_N}, n-(z+1)), D(\xi_{t_N}, n-z)\right) \quad j = N \quad (7-12)
\]

where \( T - t_N \geq t_{d, \text{max}} \) with \( t_{d, \text{max}} \) being the outage time required to execute the largest project in terms of outage time required. The discount factor \( d \) is given as follows:

\[
d = \frac{1}{(1 + r)} \quad (7-4)
\]

The immediate LCM cost \( B_j \) is as given in equation 7-8 and \( D \) is a specified degradation penalty cost function. The variable \( z \) is the project counter such that \( z = 1, \ldots, n \). The power price \( S \) in equation 7-10 can be modeled using the discrete form of the mean reversion model as presented in Chapter 4, i.e. \( \text{78} \).

\[
S(t + \Delta t) = \exp \left\{ \ln S(t) + \frac{\sigma^2}{4\eta} (1 - e^{-\eta \Delta t}) - (1 - e^{-2\eta \Delta}) + \sqrt{\frac{1 - e^{-2\eta \Delta}}{2\eta}} \epsilon \right\} \quad (4-16)
\]

### 7.5.1 Hypothetical Example

Consider the operation of a 550 MW capacity nuclear power plant over a five year horizon during which the owners have the flexibility to shutdown for three life-cycle management (LCM) projects. Suppose that costs of materials and labour are known and deterministic and that degradation penalty cost is an exponential function of aggregate LCM spending as follows \( \text{79} \):

\[
D = Ae^{-Bx} \quad (7-13)
\]

where \( A \) and \( B \) are constant coefficients, \( D \) is the penalty degradation cost and \( x \) is total amount spent on LCM activities by the end of the 5 year horizon (unit of million $). Assume monthly exercise opportunities.

#### 7.5.1.1 Price Model Calibration

Since exercise opportunities are of monthly granularity, we model power price with the same granularity. This level of granularity is reasonable for a baseload power plant as operation is not

---

78 It should be noted that all the key model parameters are time dependent and are calibrated as such.

79 This functional form is a reasonable assumption as it captures the concept of marginal LCM spending.
optimized to capture short-term price movements. Figure 34 compares monthly averages of power prices in Ontario between 2002 and 2006. It is obvious that the trends observed for all years show a level of mean reversion, however being monthly average prices, there are no jumps. As a result, a mean reversion price model should be sufficient to capture the observed trends.

If a liquid power futures market were available, the proper way to calibrate the price model would be to use futures prices in which case price simulation will be carried out using the risk-neutral version of the discrete mean reversion model. Due to lack of futures data, we calibrate the model to historical data as discussed in Chapter 4. The long-term average monthly price turns out to be $53.2/MWh, the speed of mean reversion is 0.4 and volatility is 17.6 % (see Appendix C). A mean reversion rate of 0.4 indicates that 50% of a volatility shock from the long-term mean will decay away in less than 2 months. Figure 58 shows the time series of the monthly Ontario prices and the estimated long-term average price. Based on the estimated model parameters, Figure 59 shows two independent simulated sample paths of monthly average prices for 5 years while Figure 60 shows a hundred paths. In order to highlight the effect of mean reversion speed, we simulate another set of prices using a mean reversion speed of 0.1 which corresponds to a half-life of 7 months in this case. One can see by comparing Figure 61 and Figure 62 (which are based on a lower mean reversion speed of 0.1) with Figure 59 and Figure 60 (which are based on a mean reversion speed of 0.4) that a stronger mean reversion force (i.e. $\eta = 0.4$) results in greater attenuation effect on volatility which leads to a tighter distribution around the mean.

7.5.1.2 Valuation Cases

The valuation parameters used are as shown in Table 12 and Table 13. For the degradation penalty cost function, if we set $A$ to $1$ billion and $B$ to $10$ million, the relationship between degradation penalty cost and aggregate LCM spending is as shown in Figure 63 which is an approximately linear relationship. However, setting $B$ to $90$ million instead results in a true exponential relationship (Figure 64) which indicates much lower penalty when more than 1 outage right is exercised.

We examine 6 cases with the parameters as specified in Table 8 with the base case being case 1. The difference between cases 1 and 2 is the shape of the degradation penalty cost function. Cases 3 and 4 compares the effects of mean reversion speed while cases 5 and 6 are designed to examine the effect discount rates on the results.
Figure 57: Average Monthly Ontario Power Prices from 2002 – 2006.
Figure 58: Time Series of Average Monthly Ontario Power Prices.
Figure 59: Two Samples of Simulated Monthly Average Price Paths ($\eta = 0.4$)
Figure 60: One hundred Samples of Simulated Monthly Average Price Paths ($\eta = 0.4$)
Figure 61: Two Samples of Simulated Monthly Average Price Paths ($\eta = 0.1$)
Figure 62: One hundred Samples of Simulated Monthly Average Price Paths ($\eta = 0.1$)
Figure 63: Degradation Penalty Cost Function (~ Linear)
Figure 64: Degradation Penalty Cost Function (Exponential)

\[ y = 1 \times 10^9 e^{-1 \times 10^{-7}x} \]
Table 12: Components and Labour Costs Data

<table>
<thead>
<tr>
<th>Item</th>
<th>Costs of Components &amp; Equipments</th>
<th>Required Down time (hours)</th>
<th>Labour Rate ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1,000,000</td>
<td>500</td>
<td>1500</td>
</tr>
<tr>
<td>2</td>
<td>$2,000,000</td>
<td>480</td>
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</tr>
<tr>
<td>3</td>
<td>$4,000,000</td>
<td>340</td>
<td>1200</td>
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</table>

Table 13: Valuation Parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>Base</td>
<td>A</td>
<td>$1 billion</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>90 (million $)(^1)</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>(\eta)</td>
<td>0.4</td>
<td>mean reversion speed</td>
</tr>
<tr>
<td></td>
<td>(\sigma)</td>
<td>17.6%</td>
<td>volatility</td>
</tr>
<tr>
<td></td>
<td>(\bar{S})</td>
<td>$53.2/MWh</td>
<td>long-term average price</td>
</tr>
<tr>
<td></td>
<td>(r)</td>
<td>5%</td>
<td>discount rate</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>$1 billion</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$10 million</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>(\eta)</td>
<td>0.4</td>
<td>mean reversion speed</td>
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<tr>
<td></td>
<td>(\sigma)</td>
<td>17.6%</td>
<td>volatility</td>
</tr>
<tr>
<td></td>
<td>(\bar{S})</td>
<td>$53.2/MWh</td>
<td>long-term average price</td>
</tr>
<tr>
<td></td>
<td>(r)</td>
<td>5%</td>
<td>discount rate</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>$1 billion</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$90 million</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>(\eta)</td>
<td>0.01</td>
<td>mean reversion speed</td>
</tr>
<tr>
<td></td>
<td>(\sigma)</td>
<td>17.6%</td>
<td>volatility</td>
</tr>
<tr>
<td>Case</td>
<td>Parameter</td>
<td>Value</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
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<td>------------------------</td>
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<tr>
<td></td>
<td>$\bar{S}$</td>
<td>$53.2$/MWh</td>
<td>long-term average price</td>
</tr>
<tr>
<td>4</td>
<td>$r$</td>
<td>5%</td>
<td>discount rate</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>$1$ billion</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$10$ million</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>$\eta$</td>
<td>0.01</td>
<td>mean reversion speed</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>17.6%</td>
<td>volatility</td>
</tr>
<tr>
<td></td>
<td>$\bar{S}$</td>
<td>$53.2$/MWh</td>
<td>long-term average price</td>
</tr>
<tr>
<td>5</td>
<td>$r$</td>
<td>5%</td>
<td>discount rate</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>$1$ billion</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$90$ million</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>$\eta$</td>
<td>0.01</td>
<td>mean reversion speed</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>17.6%</td>
<td>volatility</td>
</tr>
<tr>
<td></td>
<td>$\bar{S}$</td>
<td>$53.2$/MWh</td>
<td>long-term average price</td>
</tr>
<tr>
<td>6</td>
<td>$r$</td>
<td>1%</td>
<td>discount rate</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>$1$ billion</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$90$ million</td>
<td>constant coefficient</td>
</tr>
<tr>
<td></td>
<td>$\eta$</td>
<td>0.01</td>
<td>mean reversion speed</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>17.6%</td>
<td>volatility</td>
</tr>
<tr>
<td></td>
<td>$\bar{S}$</td>
<td>$53.2$/MWh</td>
<td>long-term average price</td>
</tr>
<tr>
<td></td>
<td>$r$</td>
<td>1%</td>
<td>discount rate</td>
</tr>
</tbody>
</table>
7.5.1.3 Results of Valuation

For the base case, the use of an exponential degradation penalty cost function implies that the severity of the penalty disappears as more than one LCM outage project is executed. As indicated in Figure 65 and Figure 66, given the currently available information, it is likely that the owner will wait before exercising any of its rights to shutdown temporarily and spend money for the maintenance of the power plant. When one compares the total LCM costs including lost revenue during an outage with the potential penalty due to degradation, the plant owner is better off allowing at least one of the three rights to expire unexercised. In this case, projects 2 and 3 are optimal and the right to execute project 1 is left to expire unexercised. This result can be explained by observing that the resulting penalty when projects 2 and 3 are executed is the least of the possible penalties for executing any two projects combination. Also, the immediate cost of exercising project 1 is the highest among the three project choices even though it has the lowest cost of materials. This observation is due to the length of outage time required to execute project 1 and the corresponding high labour rate compared to the other two available projects. Therefore the real option value in this case results from the money saved by not executing a third project within the valuation horizon.

On the other hand, when we adjust the coefficients of the exponential degradation penalty costs function such that the relationship between total LCM spending and future degradation costs become almost linear, the reduction in penalty cost observed with the truly exponential relationship when more than two maintenance projects are executed disappears. The result is that the likelihood of leaving a project right to expire unexercised becomes much smaller. As indicated in Figure 67 and Figure 68, all three projects have high likelihood of being exercised within the 5 year time frame. In terms of order, the results indicate that the power plant owner will likely execute project 1 last. Up until about the 40th month, project 2 appears to be the optimal choice to be executed first but that may change toward the later months with project 3 having higher likelihood of exercise before project 2.

The intent of cases 3 and 4 is to compare cases 1 and 2 but with a lesser effect of mean reversion on power prices. A low mean reversion speed allows the effect of price volatility to be stronger leading to higher uncertainty in future lost revenue during LCM outages. As indicated by the exercise probabilities in Figure 69 and Figure 70, the likelihood of waiting appears to increase with the increase in price uncertainty introduced by a lower force of mean reversion. With the truly exponential degradation penalty cost function, project 1 will still be let unexercised, however the
probability of waiting till the later months to exercise the other two projects increases, especially for project 3.

With cases 5 and 6, one can see the effect of discount rate on the low mean reversion cases (i.e. cases 3 and 4). If the discount rate is very low as in cases 5 and 6 (1% per annum), it becomes cheaper to borrow money in the short-term to execute projects. Also, future penalty costs are no longer heavily discounted from the point of view of the early months, so the likelihood of early exercise increases in both scenarios. Table 14 give the mean and standard deviation for all the 6 cases studied. It can be seen the cases with the low mean reversion speed has higher uncertainty around the mean values, in terms of the standard deviation as expected.

7.5.2 Chapter Conclusions and Model Application

Since baseload power plants are not flexible with regards to short-term operation, the general belief has been that their valuation is not amenable to the real options way of thinking. However, the modeling framework developed in this thesis has demonstrated that even baseload power plants can be valued from a real-options perspective when one considers their medium to long-term life cycle management and spending flexibilities. To the best of the author’s knowledge, this is the first real-options based modeling framework to consider the operations of a baseload power plant in a competitive power market as a parallel to the exercise of a commodity swing option. This type of thinking and the modeling framework developed in this thesis can be very useful not only from the perspective of outage planning but also when a baseload power plant refurbishment project (i.e. life extension) or a new build project is being considered. The ability to assign value to a decision to defer spending or to a decision to postpone a key maintenance activity can make a project that is initially considered uneconomical, an economically feasible one.

Framing the valuation problem of a baseload power plant as a real options problem can show the value of different flexibilities with regards to (i) budgeting for life cycle management projects (ii) timing of planned outages and (iii) order of project execution. Since these flexibilities exist in reality, one would expect rational investors to optimally exercise them hence valuation exercises of such power plants should capture these real option values. This indicates that a baseload power plant is actually worth more in reality than it has traditionally been reflected in static NPV valuation. As a result, it will not be inappropriate to reflect the values of these additional flexibilities for instance when developing a business case for a power plant refurbishment, new build or when negotiating a power purchase agreement.
Another benefit of this modeling framework is that it can be used to plan life cycle management such as outage cycles for a nuclear power plant. Given that the framework is based on dynamic optimization, it simultaneously finds the optimal life cycle management strategies, based on specified inputs, and values the plant outputs. For instance, the model can be used to develop a multi-year outage plan for a nuclear power plant. However, the penalty cost function will have to be derived from a rigorous analysis of the relationship between equipment failure and plant degradation. One way to achieve this is to combine a machine replacement model (an area that has been the focus of extensive engineering research) with a valuation framework of the kind developed here to realistically predict outages cycles and optimal maintenance spending.
Figure 65: Exercise Probabilities – Base Case
Figure 66: Two Sample Paths showing Simulated Price Evolution and the Corresponding Optimal Exercises – Case 1
Figure 67: Exercise Probabilities – Case 2
Figure 68: Two Sample Paths showing Simulated Price Evolution and the Corresponding Optimal Exercises - Case 2
Figure 69: Exercise Probabilities – Case 3
Figure 70: Exercise Probabilities – Case 4
Figure 71: Exercise Probabilities – Case 5
Figure 72: Exercise Probabilities – Case 5

Table 14: Mean and Standard Deviation of Net Present Values for all 6 Cases

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
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<tbody>
<tr>
<td>Mean</td>
<td>$1,070,800,000</td>
<td>$1,064,500,000</td>
<td>$1,076,600,000</td>
<td>$1,061,600,000</td>
<td>$1,186,100,000</td>
<td>$1,172,000,000</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$59,249,000</td>
<td>$57,632,000</td>
<td>$214,140,000</td>
<td>$201,860,000</td>
<td>$235,640,000</td>
<td>$223,350,000</td>
</tr>
</tbody>
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Chapter 8
Conclusions

The contributions of this thesis lie in the modeling and analysis of realistic physical energy assets given the complex nature of power markets and interactions with other commodity markets such as the natural gas market. The objective was to create a modeling framework, based on real options theory that can be easily adapted to various realistic valuation situations, given the constantly evolving nature of power markets and power trading. The focus therefore, was not on creating new stochastic processes for modeling uncertainty variables (such as power prices) or the creation of new optimization technique. Rather we have focused on the big picture of physical asset operations and valuation such as the impact of operating constraints, availability of market opportunities and inherent risk exposures.

Through the models developed for specific power plants and the analyses carried out, we have extended the existing understanding of the applications of stochastic optimal control for the valuation of power generating assets. The developed models are flexible enough such that one can easily incorporate different market rules to tailor an analysis to a specific market jurisdiction. Also, in place of the standard stochastic processes employed for modeling relevant uncertainty variables, user-generated scenarios (e.g. from a fundamental price model) can easily be incorporated into each of the developed model applications to tailor analysis as desired.

The overall modeling framework has been adapted for the valuation of a gas-fired thermal power plant, a peaking hydroelectric power plant and a baseload power plant. The main concept involves representing operating decisions as inter-related “switching options” that are exercised or not, depending on the state of the market and equipment/operating constraints of the asset. Given that the developed approach employs a simulation-based optimization, it is easily adaptable to different situations, including different energy markets (e.g. generation, gas and transmission), power asset types (e.g. nuclear, hydroelectric and thermal power plants), multiple underlying uncertainty factors, different asset operating constraints and different methods of modeling relevant uncertainty factors (e.g. can be used with hybrid, fundamental or quantitative approach of modeling market prices).
Specifically, three illustrative but realistic case studies were carried out to analyze the following:

(i) operating flexibilities of the operator of a gas-fired power plant in the face of price uncertainties (i.e. gas and power prices), equipment and operating constraints and uncertainty resulting from the mismatch of the operations of natural gas and electricity markets;

(ii) operating flexibilities of a peaking hydroelectric power generation plant, including the option of the owner to participate in an operating reserve market in addition to an energy market;

(iii) the flexibility of the owner of a baseload power generation plant to optimize life-cycle management spending and scheduling in the face of technical and market-based uncertainties.

For the operator of a gas-fired power generation plant, the mismatch between day-ahead operation of natural gas markets and real-time dispatch of power generators in electricity markets leaves a potential for gas supply/imbalance. This so-called “gas imbalance” constitutes additional variable cost and source of uncertainty for the operator of a gas-fired power plant. By incorporating this source of uncertainty and its potential costs in our valuation model, we have shown that different optimal operating strategies may result for the gas-fired power plant, compared to when the plant’s valuation does not take gas-imbalance into consideration. For instance, when one considers daily cycling operation of a gas-fired plant, resulting optimal operating decisions for a day may be such that will result in a loss in the power market for that day, if it achieves an overall reduction of the loss that would have occurred otherwise from costs related to gas purchase and penalty charges. With the availability of a gas storage facility or a gas load balancing service for intra-day management of gas imbalance, penalties charges can be mitigated or eliminated entirely depending on the capacity of the gas storage or load balancing facilities. Overall, the daily net value of the power plant is bounded on the lower end by the maximum possible loss that can arise from penalty charges due to gas imbalance for the day. The issue of gas imbalance, which is yet to be addressed in detail in the context of real options valuation before now, is one that has a potential to impact significantly the potential value of a gas-fired power generation plant.

For the operator of a peaking hydroelectric power plant, being able to participate in an ancillary services market, such as an operating reserve market has the potential to increase the value of the asset. As shown by the illustrative case study presented in chapter 6, the key source of flexibility for a
hydroelectric facility is its reservoir storage capacity, which makes it possible for the operator to store water and make use of it when price is highest. This ability to store water also makes it possible for the plant operator to sell the right to its reserve capacity (i.e. operating reserve) to the power market operator in exchange for a premium. Given that the operator will receive spot price for the energy reserved whether the reserve is called immediately or later, operating reserve premiums constitute additional source of revenue for a power generator. The more the available storage space, the more the operator of a hydroelectric asset can derive value from time-shifting water, including making use of operating reserve market. For a given reservoir storage size, the operator has more water time-shifting flexibility when inflow is low (winter and summer) compared to high rainfall seasons (spring and fall). By considering operation in a joint operating reserve and energy market, this application of the developed valuation framework extends existing applications of real-options valuation of hydroelectric power plants currently available in the literature.

We have also adapted the developed valuation framework to value the energy output of a baseload power generation asset such as a nuclear power plant. For the operator of a baseload asset, the key operating flexibility is not in short-term operation but rather in the long-term life-cycle management planning and spending. Most published applications of real-options valuation to power generation assets have focused on short-term operating flexibilities of peaking power plants. The illustrative case study developed and presented in this work has demonstrated the application of real-options thinking to the operation of a baseload power plant. The owner of a baseload power plant will strategically schedule life cycle shutdowns and optimize spending on preventative maintenance projects and inspections in such a way that overall costs are minimized. By strategically reacting to arrival of new information regarding market prices and equipment conditions, the owner may choose to delay planned life-cycle projects or reduce spending thereby maximizing net value of the asset in the planning horizon. What decisions are optimal in this case depend on the relationship between current life-cycle spending and future equipment degradation, and also on future evolution of key market factors.

8.1 Applications

The modeling framework developed, being a robust valuation framework can find application in capital budgeting for energy assets, business planning for asset owners, financial risk management, energy trading and valuation of new builds (e.g. design of power purchase agreements). For example,
the type of application developed here for a baseload power plant can be employed to assist in decision making regarding life-cycle management budgets and schedules. Similarly, one can use the gas-fired power plant valuation modeling approach developed in this work to assist in hedging gas imbalance risk e.g. by studying the impacts of different capacities or types of gas load balancing services or storage. Given that the models produce the probability distribution of optimal decisions and net values, they can also be employed in making short-term trading decisions and market bidding.

8.2 Further Development Opportunities

Further development opportunities exist for adapting the developed framework to value other energy assets such as a cascading hydroelectric facility, a refinery and an oil well among others. For instance, one can model the operation of a refinery as switching between different operating modes where each possible operating mode represents a given combination of refining output constrained by feasible crack-spread ratios. In addition, one can also study a situation where there is a portfolio of power generating assets with correlated value drivers, for example cascading hydroelectric power plants.
### Appendix A: LIST OF ACRONYMS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHR</td>
<td>Average Heat Rate</td>
</tr>
<tr>
<td>AGC</td>
<td>Automatic Generation Control</td>
</tr>
<tr>
<td>ARCH</td>
<td>Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>MW</td>
<td>Mega Watt</td>
</tr>
<tr>
<td>MISO</td>
<td>Midwest Independent Electricity System Operator</td>
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<tr>
<td>DAM</td>
<td>Day Ahead Market</td>
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<td>LDC</td>
<td>Load Distribution Company</td>
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<td>NAESB</td>
<td>North American Energy Standards Board</td>
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<td>HRSG</td>
<td>Heat Recovery Steam Generator</td>
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<tr>
<td>ROV</td>
<td>Real Options Value</td>
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<td>PV</td>
<td>Present Value</td>
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<td>MRJD</td>
<td>Mean Reverting Jump Diffusion</td>
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<td>OU</td>
<td>Ornstein Ulenbeck</td>
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<tr>
<td>CfaR</td>
<td>Cash Flow at Risk</td>
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<td>Value at Risk</td>
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<td>International Atomic Energy Agency</td>
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<td>MWh</td>
<td>Mega Watt Hour</td>
</tr>
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<td>MMBtu</td>
<td>Million British Thermal Unit</td>
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<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
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<tr>
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# Appendix B: LIST OF SYMBOLS

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<td>$t$</td>
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<td>$r$</td>
<td>Interest rate, discount rate, risk-free rate</td>
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<td>$S$</td>
<td>Price, representative random variable</td>
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<tr>
<td>$\sigma$</td>
<td>Standard deviation, volatility</td>
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<td>$\mu$</td>
<td>Drift, average return, growth rate</td>
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<td>Expectation</td>
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<td>$\alpha$</td>
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<td>$\delta$</td>
<td>Dividend rate, convenience yield</td>
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<td>$\theta$</td>
<td>Long-term average of a random variable</td>
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<td>$\eta$</td>
<td>Rate or speed of mean reversion, turbine efficiency</td>
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<td>$\bar{S}$</td>
<td>Long-term average of $S$</td>
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<td>$\kappa$</td>
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<td>$\bar{\kappa}$</td>
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<td>Poisson counter, power output</td>
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<td>$dq$</td>
<td>Change in Poisson process</td>
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\( \gamma \) Jump volatility, probability of Operating Reserve activation

\( N(\cdot, \cdot) \) Normal distribution

\( B \) Benefit, cash flow

\( k \) Operating mode, choice or control variable

\( \xi \) Fraction of energy offered into OR market

\( \tau \) Current time

\( \varsigma \) States of nature

\( d \) Discount factor

\( \rho \) Density

\( K \) Set of control elements

\( E[\cdot] \) Conditional expectation

\( \psi \) Basis function – for regression

\( N \) Number, counter

\( \Delta t \) Discrete time increment

\( z \) State variable, discrete output level

\( y \) State variable - time counter

\( OM \) Operations and Maintenance

\( a \) Correlation coefficient – for regression

\( b \) Correlation coefficient – for regression

\( c \) Correlation coefficient – for regression

\( \beta \) Ramp-down/ramp-up rate

\( g \) Gas flow rate, acceleration due to gravity

\( I \) Gas volume imbalance
$R$ Inventory of storage or load balance account, reservoir inventory (hydro)

$H$ Energy

$f$ Water flow rate

$F$ Volume of water inflow

$Q$ Volume of water withdrawal

$h$ Height – reservoir water head

$\omega$ Volume of water in hydro reservoir

$A$ Reservoir surface cross-sectional area

$v$ Velocity

$a$ Penstock cross-sectional area

$X$ Independent variable for regression

$\Theta$ Set of exercise opportunities

$snl$ Speed no load

$m$ Number of exercise rights

$Cont$ Continuation function

$hr$ Hour

$P$ Penalty
## Appendix C: SAMPLE CALIBRATION – MONTHLY POWER PRICES

**LINEST OUTPUTS**

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Appendix D: Implementation Codes

The models developed in this work are implemented in MATLAB. Input data and output results passing to and from MATLAB is done in VBA and Microsoft Excel. To request the codes for any of the models you can contact the author at oduntan2@gmail.com.
Bibliography


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