Characterization of Rate Region and User Removal in Interference Channels with Constrained Power

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

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Abstract

Channel sharing is known as a unique solution to satisfy the increasing demand for the spectral-efficient communication. In the channel sharing technique, several users concurrently communicate through a shared wireless medium. In such a scheme, the interference of users over each other is the main source of impairment. The task of performance evaluation and signaling design in the presence of such interference is known as a challenging problem. In this thesis, a system including n parallel interfering AWGN transmission paths is considered, where the power of the transmitters are subject to some upper-bounds. For such a system, we obtain a closed form for the boundaries of the rate region based on the Perron-Frobenius eigenvalue of some non-negative matrices. While the boundary of the rate region for the case of unconstrained power is a well-established result, this is the first result for the case of constrained power. This result is utilized to develop an efficient user removal algorithm for congested networks. In these networks, it may not be possible for all users to attain a required Quality of Service (QoS). In this case, the solution is to remove some of the users from the set of active ones. The problem of finding the set of removed users with the minimum cardinality is claimed to be an NP-complete problem [3]. In this thesis, a novel sub-optimal removal algorithm is proposed, which relies on the derived boundary of the rate region in the first part of the thesis. Simulation results show that the proposed algorithm outperforms other known schemes.

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Chapter 1

Introduction

1.1 Background

In a wireless network, a number of source nodes transmit data to their designated destination nodes through a shared wireless channel. Such a wireless network is known as an interference channel [13]. Many systems like cellular networks, sensor networks, and ad-hoc networks fall in this category. The capacity of such a channel has not been characterized yet, but it has been investigated under some simplifying conditions. A typical assumption is to treat the interference as Gaussian noise. In this case, the Shannon capacity would be proportional to the logarithm of signal-to-interference-plus-noise ratio (SINR) plus one. We will follow this assumption throughout this thesis.

Different aspects of interference channels have been investigated in the literature. Power control over such channels is one of the prominent challenges in this area. In the context of power allocation, there are roughly two groups of work studying the capacity of wireless networks. In one group, the objective is to minimize the transmit power while satisfying some quality of service (QoS) requirements, e.g., [8]. Such problems can be usually formulated as linear programs [16] and can be even solved in a decentralized fashion [51], [47]. In the other group of power allocation problems, which is of interest to us in this thesis, the objective is to use the limited power resources efficiently in order to improve some measures of QoS like sum-rate or minimum rate. Unfortunately, these usually entail some nonlinear optimization problems for which a systematic solution method may not exist. As a result, most authors have obtained an approximate or a suboptimum solution.

The first power control schemes were proposed in $60'$s for the purpose of the voice service in the broadcast context. In [1], Aein investigated the problem of interference management and proposed the scheme of signal to interference (SIR) balancing in which the power control is done in order to balance the received SIR ratios in the receivers. This work which was based on Perron-Frobenius theorem, presented in the context of satellite systems, and later this result was extended to the spread spectrum cellular networks in [2].

The algorithms related to power control problems can be performed in a distributed or centralized fashion. In the distributed power control algorithms, the local information of the channel is utilized to update the power. Zander [52] suggested the distributed balancing algorithm (DBA) which maximizes and balances the SIR through allocating appropriate powers to the transmitters. Moreover, in [22] Grandhi et al. presented a distributed power control (DPC) algorithm which was shown to converge faster than DBA. Foschini and Miljanic in [19] proposed a novel algorithm which updates the power in a distributed fashion while considering a non-zero noise to attain a predetermined SINR target. This algorithm was shown to converge synchronously [19] or asynchronously [38]. Grandhi et al. extended the Foschni's algorithm to the constrained power control in a distributed constrained power control algorithm (DCPC) [24]. In [51], Yates established a unified framework to analyze the convergence of the distributed power control algorithms. This framework has recently been generalized in [36] and [44]. In [44] a new framework for distributed power control is established, which is applicable to systems supporting opportunistic communications and with heterogeneous service requirements.

The centralized algorithms are more favorable when the channel information is available in a central controller. Many power control algorithms utilize this scenario to update the powers fast and accurately. In [53] and [23], a centralized power control (CPC) with zero noise is presented. In this scheme, which is based on the Perron-Frobenius theorem, the power vector is expressed as the Perron-Frobenius eigenvalue (PF-eigenvalue) of a non-negative matrix.

When the system is congested, all the users can not attain the desired QoS, and the system becomes infeasible. Therefore, some users should be removed in order to make the remaining users feasible (all the remaining users satisfy the required QoS). A common objective is to find the subset of users with maximum cardinality while they achieve the required QoS. Such a problem is claimed to be NP-complete in [3]. There are a few heuristics for this problem which yield suboptimal results. In [53], a stepwise removal algorithm (SRA) has been proposed for the unbounded power system. In SRA, in every iteration, for each user, the maximum of total normalized channel gain from that user to the others and the total normalized channel gain from the other users to the same user is computed. Then, the user with the largest computed parameter is removed. The removal algorithm continues until the maximum achievable SIR (signal-to-interference-ratio), obtained from the Perron-Frobenius theorem, overtake the required QoS. Later, in [52] a distributed power control algorithm for a noiseless system was proposed (DBA). Then, based on this power allocation scheme, a Limited Information SRA (LI-SRA) was proposed in which in each iteration the power is updated with DBA. If the users are not satisfied with the QoS, the user with the smallest SIR for a fixed power is removed. In [37], another algorithm named as stepwise-maximum-interference-removal-algorithm (SMIRA) is proposed, in which the maximum of the aggregate interference power from each user to the other ones and from the other users to that user is computed and the user with maximum computed value is removed. This procedure continues iteratively until the maximum achievable SIR meets the target SIR. The simulation results show that this algorithm outperforms SRA.

For congested systems with constraint on the power of the individual transmitters, an algorithm known as gradually-removal-distributed-constrained-power-control (GRX-DCPC) is presented in [3]. In this algorithm, the power of the transmitters are updated based on the DCPC algorithm presented in [24, 19] and the removal is performed based on a predetermined criterion. The presented removal algorithm can be performed in a restricted or a non-restricted fashion. In the restricted algorithm, known as GRR-DCPC, the user to be removed is selected from the users attaining the maximum power in the power updating procedure. Whereas, in the non-restricted algorithm (GRN-DCPC), the user to be removed is selected from all the active users. The removal criterion can be based on SMIRA or some other alternatives presented in that work. GRX-DCPC can be performed in a distributed fashion in which a user is removed with a certain probability in each iteration. In addition, this algorithm is capable of removing multiple users at each iteration. The simulation results show that GRN-DCPC (centralized non-restricted) outperforms other mentioned schemes in [3].

The notion of the feasible rate region is in close relation with power allocation problems. Due to the interplay between rate of different links, the rates that can be simultaneously achieved by all links are bounded within a subset of the n-dimensional space. Hence, when the power allocation is aimed to improve the minimum rate of the network, it is helpful to investigate the characteristics of the feasible rate region.

There have been some efforts to evaluate the maximum achievable SINR in the interference channels. In [1], the maximum achievable SINR of a system with no constraint on the power is expressed in terms of the Perron-Frobenius eigenvalue of a non-negative matrix and this result is utilized to develop an SINR-balancing scheme for satellite networks. This formulation for the maximum achievable SINR is deployed in many other wireless communication applications such as [2, 53, 52, 47] afterwards.

Recently, the rate region of interference channels and its properties has been investigated in the literature. In [9], it is shown that the capacity region when the power is unbounded is convex. The capacity region in [9] is defined as the set of feasible processing gains while for a constant bandwidth, the processing gain is inversely proportional to the rate. In [28], some topological properties of the capacity region (with the aforementioned definition) of CDMA systems are investigated for the cases when there are constraints on the power of individual users and when there is no constraint on the power. The authors in [28] show that the boundary of the capacity region with one user's power fixed and the rest unbounded is a shift of the boundary of some capacity region with modified parameters, but unlimited power. However, this result is not in a closed form and can not be extended to the other forms of power constraints.

It is shown that the feasible SINR region is not convex, in general [7, 8, 14]. In [43], it is shown that in the case of unlimited power, the feasible SINR region is log-convex. The authors in [9] also consider a CDMA system without power constraints, and show that the feasible inverse-SINR region is a convex set. In [7], it is proved that the feasible QoS region is a convex set, if the SINR is a log-convex function of the corresponding QoS parameter. Reference [42] shows that under a total power constraint, the infeasible SINR region is not convex.

Based on the requirement of the users and structure of the network many other problems related to power control are considered in the literature which are beyond the scope of this thesis, including bandwidth allocation [35], [17], [48] , [26], transmission scheduling [16], [6],

[4], routing [46], [21], [50], base station selection [25], dynamic resource allocation [39], user capacity [49], combined rate and power control [33], [41], [30], convergence improvement [31], [32], energy saving [5], soft computing [10], [11], [20], and game theoretical approach to power control [45], [18].

1.2 Contributions and Outline of the Thesis

In this thesis, a system including n parallel interfering links is considered. It is assumed that each user considers the signal of the rest as interference. In addition, the power of users is subject to some constraints. For such a system, we obtain a closed form for the rate region of the system, based on the PF-eigenvalue of some non-negative matrices. We address the problem where there are upper-bounds on the power of the individual users and/or over the total power of a subset of users. This result is extended to a time-varying system, where the channel gain is selected from a limited-cardinality set, and the average of the total power of a subset of users is upper-bounded.

After characterizing the rate region of the interference channel with certain constraints on power in Chapter 2, this result is utilized in Chapter 3 to develop an efficient user removal algorithm for congested networks. In these networks, it may not be possible for all users to attain a required QoS. In this case, the solution is to remove some of the users from the set of active ones. The problem of finding the set of removed users with the minimum cardinality is an NP-complete problem. In this thesis, a novel sub-optimal removal algorithm is proposed, which relies on the derived boundary of the rate region in the first part of the thesis. Simulation results show that the proposed algorithm outperforms any other scheme in terms of the number of active users. Chapter 4 concludes the thesis and finally the plan for the future research is presented in Chapter 5.

1.3 Notations

- All boldface letters indicate column vectors (lower case) or matrices (upper case). The transpose of $\mathbf{X}$ is denoted by $\mathbf{X}'$. x_{ij} and $\mathbf{x}_i$ represent the entry (i, j) and column i of the matrix $\mathbf{X}$, respectively.

- A matrix $\mathbf{X}_{n\times m}$ is called *non-negative* if

$$x_{ij} \geq 0 \quad \forall i \in \{1, 2, \ldots, n\}, j \in \{1, 2, \ldots, m\}, \tag{1.1}$$

  which is denoted by $\mathbf{X} \geq \mathbf{0}$ or *positive*, denoted by $\mathbf{X} > \mathbf{0}$ if

$$x_{ij} > 0 \quad \forall i \in \{1, 2, \ldots, n\}, j \in \{1, 2, \ldots, m\}. \tag{1.2}$$

  This definition is extended to row vectors and column vectors, and also to expressions such as, e.g.

$$\mathbf{X} \geq \mathbf{Y} \Longleftrightarrow \mathbf{X} - \mathbf{Y} \geq \mathbf{0}, \tag{1.3}$$

  where $\mathbf{X}, \mathbf{Y}$ and $\mathbf{0}$ are non-negative matrices of compatible dimensions [40].

- $\det(\mathbf{X})$, where $\mathbf{X}$ is a square matrix, denotes the determinant of $\mathbf{X}$.

- $\mathrm{Tr}(\mathbf{X})$, where $\mathbf{X}$ is a square matrix, is the trace of $\mathbf{X}$.

$$\mathrm{Tr}(\mathbf{X}) = \sum_i x_{ii}. \tag{1.4}$$

- $|\mathbf{X}|$ denotes the norm of $\mathbf{X}$.

- $\otimes$ represents the Kronecker product operator.

$$\mathbf{X}_{k\times l} \otimes \mathbf{Y}_{m\times n} = \begin{bmatrix} x_{11}\mathbf{Y} & x_{12}\mathbf{Y} & \ldots & x_{1l}\mathbf{Y} \\ x_{21}\mathbf{Y} & x_{22}\mathbf{Y} & \ldots & x_{2l}\mathbf{Y} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k1}\mathbf{Y} & x_{k2}\mathbf{Y} & \ldots & x_{kl}\mathbf{Y} \end{bmatrix}. \tag{1.5}$$

- diag($\mathbf{x}$) is a diagonal matrix with the following definition,

$$\text{diag}(\mathbf{x}) = \begin{bmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n \end{bmatrix}., \tag{1.6}$$

  where $\mathbf{x} = [x_i]_{n\times 1}$.

- $\mathbf{I}$ is an identity matrix with compatible size.

- The reciprocal of polynomial $\mathrm{q}(x)$ of degree m is defined as $x^m \mathrm{q}(\frac{1}{x})$.

- $\psi(\mathbf{X}, \mathbf{y}, \mathcal{S})$ is a matrix defined as a function of three parameters, which are respectively a matrix, a vector and a set of indices,

$$\psi(\mathbf{X}, \mathbf{y}, \mathcal{S}) = \mathbf{Z} = [\mathbf{z}_j], \quad \mathbf{z}_j = \begin{cases} \mathbf{x}_j + \mathbf{y} & j \in \mathcal{S} \\ \mathbf{x}_j & \text{otherwise} \end{cases}$$

  In other words, this function adds $\mathbf{y}$ to $\mathbf{X}$ in the columns with indices in $\mathcal{S}$.

- $\mathbf{X}^{i^-}$ is the matrix $\mathbf{X}$ whose i^{th} column and row is removed. We use a similar notation for a vector whose i^{th} element is removed.

Chapter 2

Characterization of Rate Region for an Interference Channel

2.1 System Model

In an interference channel, a number of non-cooperating transmitters try to communicate their separate information to designated receivers via a common channel. Transmission of information from each transmitter to its corresponding receiver interferes with the communication between the other transmitters and receivers. We call a pair of transmitter and corresponding receiver a *link* or a *user*. We consider an interference channel including n links. The vector of transmit powers is represented by

$$\mathbf{p} = [p_i]_{n\times 1}, \tag{2.1}$$

where p_i is the power of transmitter i.

This system is represented by the gain matrix $\mathbf{G} = [g_{ij}]_{n\times n}$ where g_{ij} is the attenuation gain of the power from transmitter j to receiver i. This attenuation can be the result of fading, shadowing, or the processing gain of the CDMA system. A white Gaussian noise with zero mean and variance σ_i^2 is added to each signal at the receiver i terminal. The signal-to-interference-plus-noise ratio (SINR) of the receiver i denoted by γ_i is equal to

$$\gamma_i = \frac{g_{ii}p_i}{\sigma_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ij}p_j}, \quad \forall i \in \{1, \ldots, n\}. \tag{2.2}$$

In practice, the power vector is subject to a set of constraints. A natural requirement is to expect non-negative power for the transmitters, i.e.,

$$\mathbf{p} \geq \mathbf{0}. \tag{2.3}$$

In addition, sometimes it is desirable to limit the total power of the users with indices in a subset $\Omega \subseteq \{1, 2, \ldots n\}$, i.e.,

$$\sum_{i \in \Omega} p_i \leq \overline{p}_\Omega. \tag{2.4}$$

A special case of constraint (2.4) is when the power of user i, p_i, is constrained by $\overline{p}_i$ (individual power constraint) as

$$p_i \leq \overline{p}_i \tag{2.5}$$

where $\overline{p}_\Omega$ and $\overline{p}_i$ are the upper-bounds for the total power on Ω and maximum value for the power of user i, respectively. If any of these constraints does not exist, the corresponding upper-bound can be considered as infinity.

2.2 Problem Formulation

In many applications, the QoS of the system is measured by an increasing function of SINR, e.g., rate which has the following relationship with SINR,

$$r_i = \log_2(1 + \gamma_i), \tag{2.6}$$

where r_i is the rate of user i. In this chapter, we focus on the problem of finding the region of achievable SINR in the aforementioned channel. To this end, we solve the following optimization problem

$$\max \gamma \tag{2.7}$$

$$\text{s.t.} \quad \gamma_i \geq \mu_i \gamma, \tag{2.8}$$

while considering the power constraints in (2.3), (2.4), (2.5) . The γ_i is given in (2.2), and $\boldsymbol{\mu}$ is a vector with $\mu_i \geq 0$ and $|\boldsymbol{\mu}| = 1$. The vector $\boldsymbol{\mu}$ provides the flexibility of satisfying different rate services for different users. According to Fig. 2.1, the solution of (2.7) yields the maximum achievable SINR in the direction of vector $\boldsymbol{\mu}$. Therefore, to obtain the whole boundary, $\boldsymbol{\mu}$ is changed in order to sweep all the rate ratios and for each, the aforementioned problem is solved. Although the numerical solution of this problem is already obtained through geometric programming [12], [34], we propose a novel approach which leads to a closed-form solution for the optimization problem (2.7).

2.3 Maximum Achievable SINR

By defining the normalized gain matrix, $\mathbf{A}$, as

$$\mathbf{A} = [a_{ij}]_{n \times n}, \quad a_{ij} = \begin{cases} \frac{g_{ij}}{g_{ii}} & i \neq j \\ 0 & i = j \end{cases} \tag{2.9}$$

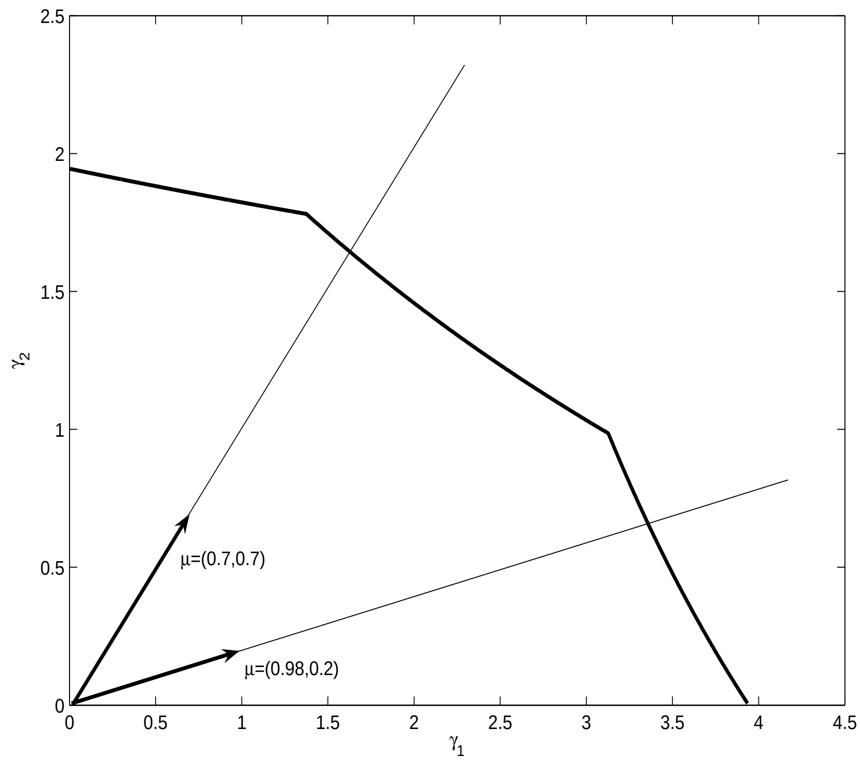

Figure 2.1: The SINR Region for an interference channel with 2 users

The constraint (2.8) is rewritten as

$$\frac{p_i}{\eta_i + \sum_{j=1}^{n} \mu_i a_{ij} p_j} \geq \gamma, \quad \forall i \in \{1, \ldots, n\}, \tag{2.10}$$

where

$$\eta_i = \frac{\mu_i \sigma_i^2}{g_{ii}}. \tag{2.11}$$

Since we are interested in the maximum possible γ, if SINR of one user is more than that of the others, it can reduce its power in order to increase the SINR of the others, while its own SINR is decreased; finally all the users achieve a balanced SINR which is the maximum achievable γ. Therefore, equality holds in (2.10) as

$$\frac{p_i}{\eta_i + \sum_{j=1}^{n} \mu_i a_{ij} p_j} = \gamma, \quad \forall i \in \{1, \ldots, n\}. \tag{2.12}$$

After reformulating the problem in a matrix form we will have

$$\big(\frac{1}{\gamma}\mathbf{I}_{n\times n} - \mathrm{diag}(\boldsymbol{\mu})\mathbf{A}\big)\mathbf{p} = \boldsymbol{\eta}, \tag{2.13}$$

where

$$\boldsymbol{\eta} = [\eta_i]_{n\times 1}. \tag{2.14}$$

So, our goal is to find the maximum γ while the system of linear equations in (2.13) yields a power vector in the desired range. In what follows, we derive a closed form formula for the the maximum γ, while the power is subject to one or more of the aforementioned constraints.

When there is no constraint on the power vector rather than the trivial constraint of $\mathbf{p} \geq \mathbf{0}$, the maximum achievable γ is characterized based on the Perron-Frobenius theorem (Theorem A.5).

This result which takes advantage of the non-negativity and irreducibility (Definition A.2) of the channel gain matrix $\mathbf{A}$, shows that the maximum achievable SINR in such a system is equal to

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{A})}, \tag{2.15}$$

where $\lambda^*(\mathbf{A})$ is the PF-eigenvalue (see Theorem A.3 for definition) of $\mathbf{A}$. The PF-eigenvalue of a non-negative irreducible matrix is a positive real value whose magnitude (norm) is greater than or equal to the norm of other eigenvalues of the matrix (see Theorem A.4). This result can be extended to the case when the users have different rate requirements, i.e., $\gamma_i = \mu_i \gamma$. In this case, γ^* would be

$$\gamma^* = \frac{1}{\lambda^*(\text{diag}(\boldsymbol{\mu})\mathbf{A})}, \tag{2.16}$$

where diag is defined in Section 1.3.

This paradigm was exploited in [1] for the first time for SINR balancing in a satellite network. Afterwards, it was followed and extended to many other systems and applications [47], [2], [53], [52]. When the system is noiseless, the eigenvector corresponding to the PF-eigenvalue of $\mathbf{A}$ would be the power vector of the system which achieves γ^*. This property is utilized in [52] to develop a distributed SIR-balancing algorithm.

This thesis is devoted to the characterization of the maximum achievable SINR under some constraints on the power vector. The proposed scheme is simple and can be easily extended for different constraints on power. In addition, it leads to a closed form solution. The discussions start with the system where there is a constraint on the power of one user and then it is extended to the total power constraint.

2.3.1 Constraint on the Power of Individual Transmitters

Assume that the power vector is subject to the following constraints

$$\mathbf{p} \geq \mathbf{0}, \tag{2.17}$$

$$p_i \leq \overline{p}_i, \tag{2.18}$$

for an arbitrary $i \in \{1, 2, \ldots n\}$. In other words, we desire to limit the power of user i to $\overline{p}_i$. The objective is to compute the maximum achievable SINR in such a system.

Let us define $\mathbf{F}$ as

$$\mathbf{F} = \mathbf{I}_{n\times n} - \gamma \mathrm{diag}(\boldsymbol{\mu})\mathbf{A}. \tag{2.19}$$

Then, the system of linear equations in (2.13) is reformulated as

$$\mathbf{F}\mathbf{p} = \gamma \boldsymbol{\eta}, \tag{2.20}$$

where $\boldsymbol{\eta}$ is defined in (2.14). According to the Cramer's rule, the solution to (2.20) is obtained by

$$p_i = \frac{\det(\mathbf{H}^{(i)})}{\det(\mathbf{F})}, \tag{2.21}$$

where

$$\mathbf{H}^{(i)} = [\mathbf{h}_j^{(i)}]_{n\times n}, \quad \mathbf{h}_j^{(i)} = \begin{cases} \gamma\boldsymbol{\eta} & j = i \\ \mathbf{f}_j & j \neq i \end{cases} \tag{2.22}$$

Defining

$$\mathrm{h}^{(i)}(\gamma) = \det(\mathbf{H}^{(\mathbf{i})}) \tag{2.23}$$

and

$$\mathrm{f}(\gamma) = \det(\mathbf{F}), \tag{2.24}$$

we have

$$p_i = \frac{\mathrm{h}^{(i)}(\gamma)}{\mathrm{f}(\gamma)}. \tag{2.25}$$

Considering constraint (2.18), we have

$$\frac{\mathrm{h}^{(i)}(\gamma)}{\mathrm{f}(\gamma)} \leq \overline{p}_i. \tag{2.26}$$

This constraint can be reformulated as

$$\frac{\mathrm{u}^{(i)}(\gamma)}{\mathrm{f}(\gamma)} \geq 0, \tag{2.27}$$

where

$$\mathrm{u}^{(i)}(\gamma) = \overline{p}_i \mathrm{f}(\gamma) - \mathrm{h}^{(i)}(\gamma). \tag{2.28}$$

The objective is to find the largest possible interval where both the numerator and the denominator have the same sign. In [15, Lemma 2], it is shown that if a rate vector is achievable, any rate vector smaller than that is achievable, as well. As a result, the aforementioned interval on γ is connected and adjacent to zero. Apparently, $\mathrm{u}^{(i)}(0) = 0$ and $\mathrm{f}(0) > 0$. It is easy to show that $\frac{\partial \mathrm{u}^{(i)}(\gamma)}{\partial \gamma}|_{\gamma=0} > 0$, as well. Consequently,

$$\exists\ \epsilon > 0 \quad : \quad \mathrm{f}(\epsilon) > 0 \quad \text{and} \quad \mathrm{u}_{\Omega}(\epsilon) > 0.$$

Therefore, both the numerator and the denominator are positive in the positive neighborhood of zero. To satisfy (2.27), we have to find the smallest positive real simple root of the numerator and the denominator, $r(\mathrm{u}^{(i)})$ and $r(\mathrm{f})$, and take the minimum of the two as

$$\hat{\gamma} = \min \{r(\mathrm{f}), r(\mathrm{u}^{(i)})\}. \tag{2.29}$$

where $r(\mathrm{f})$ and $r(\mathrm{u}^{(i)})$ denote the smallest positive real simple root of f and $\mathrm{u}^{(i)}$ respectively.

For deriving $r(\mathrm{u}^{(i)})$, using (2.28), we have

$$\begin{aligned} \mathrm{u}^{(i)}(\gamma) &= \overline{p}_i \det(\mathbf{F}) - \det(\mathbf{H}^{(i)}) \\ &= \overline{p}_i(\det(\mathbf{F}) - \det(\hat{\mathbf{H}}^{(i)})), \end{aligned} \tag{2.30}$$

where $\hat{\mathbf{H}}^{(i)}$ is obtained by dividing the column i of $\mathbf{H}^{(\mathbf{i})}$ by $\overline{p}_i$, i.e.,

$$\hat{\mathbf{H}}^{(i)} = [\hat{\mathbf{h}}_j^{(i)}]_{n\times n}, \quad \hat{\mathbf{h}}_j^{(i)} = \begin{cases} \dfrac{\gamma\boldsymbol{\eta}}{\overline{p}_i} & j = i \\ \mathbf{f}_j & j \neq i \end{cases} . \tag{2.31}$$

Lemma 2.1 *If square matrices* $\mathbf{X}$ *and* $\mathbf{Y}$ *differ only in column* i*, i.e.,*

$$\begin{cases} \mathbf{x}_j \neq \mathbf{y}_j & j = i \\ \mathbf{x}_j = \mathbf{y}_j & j \neq i \end{cases} ,$$

then

$$\begin{aligned} \det\big(\mathbf{X}) + \det(\mathbf{Y}\big) &= \det\big(\psi(\mathbf{X}, \mathbf{y}_i, \{i\})\big) \\ &= \det\big(\psi(\mathbf{Y}, \mathbf{x}_i, \{i\})\big). \end{aligned}$$

According to (2.31), $\mathbf{F}$ and $\hat{\mathbf{H}}^{(i)}$ are the same except in column i. Applying Lemma 2.1 to (2.30), we have

$$\begin{aligned} \mathrm{u}^{(i)}(\gamma) &= \overline{p}_i \det(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\overline{p}_i}, \{i\})) && (2.32) \\ &= \overline{p}_i \det(\psi(\mathbf{I} - \gamma\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, -\frac{\gamma\boldsymbol{\eta}}{\overline{p}_i}, \{i\})) && (2.33) \\ &= \overline{p}_i\gamma^n \det(\psi(\frac{1}{\gamma}\mathbf{I} - \mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, -\frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})) && (2.34) \\ &= \overline{p}_i\gamma^n \det(\frac{1}{\gamma}\mathbf{I}_{n\times n} - \psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})), && (2.35) \end{aligned}$$

Therefore, $\frac{1}{\overline{p}_i}\mathrm{u}^{(i)}(\gamma)$ is the reciprocal of the characteristic polynomial of $\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})$. Also, since $\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})$ is a primitive matrix (see Definition A.1), according to Theorem A.3, this matrix has one real positive eigenvalue with the largest norm among all eigenvalues which is the inverse of the simple root of the above characteristic polynomial; therefore the inverse of that gives the smallest positive simple root of $\mathrm{u}^{(i)}$, and consequently, the following lemma is concluded.

Lemma 2.2 *The smallest positive simple root of* $\mathrm{u}^{(i)}(\gamma)$, $r(\mathrm{u}^{(i)})$, *is*

$$r(\mathrm{u}^{(i)}) = \frac{1}{\lambda^*(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \{i\}))}. \tag{2.36}$$

For the denominator, we have

$$\begin{aligned} \mathrm{f}(\gamma) &= \det(\mathbf{F}) = \det(\mathbf{I} - \gamma \mathrm{diag}(\boldsymbol{\mu})\mathbf{A}) \\ &= \gamma^n \det(\frac{1}{\gamma}\mathbf{I} - \mathrm{diag}(\boldsymbol{\mu})\mathbf{A}). \end{aligned} \tag{2.37}$$

Therefore, $\mathrm{f}(\gamma)$ is the reciprocal of the characteristic polynomial of $\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}$. On the other hand, according to Theorem A.3, $\lambda^*(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A})$, the PF-eigenvalue of $\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}$, is real and positive and takes the largest magnitude (norm) among the eigenvalues of the matrix. Moreover, it is the simple root of the characteristic polynomial of the associated matrix. Therefore, $\lambda^*(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A})$ is the inverse of the smallest positive simple root of $\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}$. Thus,

$$r(\mathrm{f}) = \frac{1}{\lambda^*(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A})}. \tag{2.38}$$

On the other hand, according to (2.16), $r(\mathrm{f})$ is also the maximum achievable SINR to guarantee positive values for the power (maximum achievable SINR in the unbounded power case). Consequently, using (2.29), (2.38) and Lemma(2.2) the maximum achievable SINR to satisfy all constraints is

$$\gamma^* = \min\{r(\mathrm{f}), r(\mathrm{u}^{(i)})\} \tag{2.39}$$

$$= \min\{\frac{1}{\lambda^*(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A})}, \frac{1}{\lambda^*(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \{i\}))}\}. \tag{2.40}$$

Since $\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \{i\}) \geq \mathrm{diag}(\boldsymbol{\mu})\mathbf{A} \geq \mathbf{0}$, using Theorem A.3, we have

$$\lambda^*(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \{i\})) \geq \lambda^*(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}), \tag{2.41}$$

and consequently the maximum achievable γ for a system with upper-bound on the power of one user is achieved.

Theorem 2.3 *The maximum achievable γ in (2.7), where power vector is subject to the following constraints,*

$$\mathbf{p} \geq \mathbf{0} \tag{2.42}$$

$$p_i \leq \overline{p}_i, \text{ for given } i \in \{1, 2, \ldots, n\} \tag{2.43}$$

is equal to

$$\gamma^* = \frac{1}{\lambda^*(\psi(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\}))}. \tag{2.44}$$

When multiple constraints on power exist, it is obvious that the maximum achievable SINR is the minimum of the maximum achievable SINR when each of the constraints is applied separately, i.e.,

$$\gamma^* = \min_i \gamma_i^*, \tag{2.45}$$

where γ_i^* is the maximum achievable SINR for the constraint i on power. The following corollary yields the maximum achievable SINR when the power of individual users and the total power are constrained.

Therefore, if the power of all users is bounded individually as

$$p_i \leq \overline{p}_i, \ \forall i \in \{1, 2, \ldots, n\}, \tag{2.46}$$

then we have $\gamma^* = \min_i \{\frac{1}{\lambda^*(\psi(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\}))}\}$.

2.3.2 General Form

We can generalize the previous discussion on finding the maximum achievable SINR with constraints on the power of individual transmitters to the case when there is a constraint on the total power of a subset of the links as

$$\mathbf{p} \geq \mathbf{0} \tag{2.47}$$

$$\sum_{i\in\Omega} p_i \leq \overline{p}_\Omega, \tag{2.48}$$

where $\Omega \subseteq \{1, 2, \ldots, n\}$ is an arbitrary subset of the users with k elements. When Ω has just one element, it corresponds to the individual constrained power problem which was discussed in the previous section.

According to (2.23), (2.24), and (2.25), the constraint (2.48) can be written as

$$\frac{\sum_{i\in\Omega} \mathrm{h}^{(i)}(\gamma)}{\mathrm{f}(\gamma)} \leq \overline{p}_\Omega. \tag{2.49}$$

Defining $\mathrm{u}_\Omega(\gamma)$ as

$$\mathrm{u}_\Omega(\gamma) = \overline{p}_\Omega \mathrm{f}(\gamma) - \sum_{i\in\Omega} \mathrm{h}^{(i)}(\gamma), \tag{2.50}$$

the equation (2.49) is equivalent to

$$\frac{\mathrm{u}_\Omega(\gamma)}{\mathrm{f}(\gamma)} \geq 0. \tag{2.51}$$

Similar to the previous section, the objective is to find the largest possible interval where both the numerator and the denominator have the same sign. As mentioned before, in [15, Lemma 2], it is shown that if a rate vector is achievable, any rate vector smaller than that is achievable, as well. As a result, the aforementioned interval on γ is connected and adjacent to zero. Apparently, $\mathrm{u}_\Omega(0) > 0, \quad \text{and} \quad \mathrm{f}(0) > 0$. Consequently,

$$\exists\, \epsilon > 0 \quad : \quad \mathrm{f}(\epsilon) > 0 \quad \text{and} \quad \mathrm{u}_\Omega(\epsilon) > 0.$$

Therefore, both the numerator and the denominator are positive in the positive neighborhood of zero. To satisfy (2.51), we have to find the smallest positive real simple root of the numerator and the denominator, $r(\mathrm{u}_\Omega)$ and $r(\mathrm{f})$, and take the minimum of the two as

$$\hat{\gamma} = \min\{r(\mathrm{f}), r(\mathrm{u}_\Omega)\}. \tag{2.52}$$

For the sake of simplicity, without loss of generality, we assume that $\Omega = \{1, \ldots, k\}$, $k \leq n$, i.e., the first k users are subject to the sum power constraint. For the numerator, we have

$$\begin{aligned} \mathrm{u}_\Omega(\gamma) &= \overline{p}_\Omega \det(\mathbf{F}) - \sum_{i=1}^{k} \det(\mathbf{H}^{(i)}) \\ &= \overline{p}_\Omega\big(\det(\mathbf{F}) - \sum_{i=1}^{k} \det(\hat{\mathbf{H}}^{(i)})\big), \end{aligned} \tag{2.53}$$

where $\hat{\mathbf{H}}^{(i)}$ is defined as

$$\hat{\mathbf{H}}^{(i)} = [\hat{\mathbf{h}}_j^{(i)}]_{n\times n}, \quad \hat{\mathbf{h}}_j^{(i)} = \begin{cases} \dfrac{\gamma\boldsymbol{\eta}}{\overline{p}_\Omega} & j = i \\ \mathbf{f}_j & j \neq i \end{cases}.$$

Equation (2.53) is rewritten as

$$\mathrm{u}_\Omega(\gamma) = \overline{p}_\Omega\big(\det(\mathbf{F}) - \det(\hat{\mathbf{H}}^{(1)}) - \sum_{i=2}^{k} \det(\hat{\mathbf{H}}^{(i)})\big). \tag{2.54}$$

Since $\mathbf{F}$ and $\hat{\mathbf{H}}^{(1)}$ are the same except for the first column, using Lemma 2.1 , we will have

$$\det(\mathbf{F}) - \det(\hat{\mathbf{H}}^{(1)}) = \det\big(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\overline{p}_\Omega}, \{1\})\big). \tag{2.55}$$

On the the other hand, using the fact that addition or substraction of columns does not change the value of the determinant, we have

$$\det(\hat{\mathbf{H}}^{(i)}) = \det\big(\psi(\hat{\mathbf{H}}^{(i)}, -\hat{\mathbf{h}}_i^{(i)}, \{1, \ldots, i-1\})\big). \tag{2.56}$$

Then, using (2.55) and (2.56) and regarding $\hat{\mathbf{h}}_i^{(i)} = \dfrac{\gamma\boldsymbol{\eta}}{\overline{p}_\Omega}$, we can rewrite (2.54) as

$$\begin{aligned} \mathrm{u}_\Omega(\gamma) = \overline{p}_\Omega\Big(&\det\big(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\overline{p}_\Omega}, \{1\})\big) \\ &- \sum_{i=2}^{k} \det(\psi(\hat{\mathbf{H}}^{(i)}, -\frac{\gamma\boldsymbol{\eta}}{\overline{p}_\Omega}, \{1, \ldots, i-1\}))\Big). \end{aligned} \tag{2.57}$$

Since $\mathbf{F}$ and $\hat{\mathbf{H}}^{(i)}$ are the same except for the column i, we have $\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \{1,\ldots,i-1\})$ and $\psi(\hat{\mathbf{H}}^{(i)}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \{1,\ldots,i-1\})$ are the same except for the i^{th} column. Therefore,

$$\begin{aligned}\det\big(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \{1,\ldots,i-1\})\big) - \det\big(\psi(\hat{\mathbf{H}}^{(i)}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \{1,\ldots,i-1\})\big)\\ = \det\big(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \{1,\ldots,i\})\big).\end{aligned}$$

Applying this result to (2.57) successively yields the following lemma.

Lemma 2.4

$$\mathrm{u}_\Omega(\gamma) = \bar{p}_\Omega \det\big(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega)\big).$$

We utilize the result in Lemma 2.4 to find the smallest positive simple root of u_Ω using Perron-Frobenius theorem (Theorem A.3).

Lemma 2.5 *The smallest positive root of* $\mathrm{u}_\Omega(\gamma)$ *is*

$$r(\mathrm{u}_\Omega) = \frac{1}{\lambda^*\big(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega)\big)}.$$

Proof

$$\begin{aligned}\mathrm{u}_\Omega(\gamma) &= \bar{p}_\Omega \det\big(\psi(\mathbf{F}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega)\big)\\ &= \bar{p}_\Omega \det\big(\psi(\mathbf{I} - \gamma\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, -\frac{\gamma\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega)\big)\\ &= \bar{p}_\Omega\gamma^n \det\big(\psi(\frac{1}{\gamma}\mathbf{I} - \mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, -\frac{\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega)\big)\\ &= \bar{p}_\Omega\gamma^n \det\big(\frac{1}{\gamma}\mathbf{I} - \psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega)\big).\end{aligned}$$

Consequently, $\frac{\mathrm{u}_\Omega(\gamma)}{\bar{p}_\Omega}$ is the reciprocal of the characteristic polynomial of the matrix $\psi\big(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega\big)$. Therefore, the roots of this polynomial are equal to the inverse of the eigenvalues of

$\psi\big(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega\big)$. On the other hand, according to Theorem A.3, since $\psi\big(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_\Omega}, \Omega\big)$ is a primitive matrix, the PF-eigenvalue of this matrix is real and positive and has the largest norm among all eigenvalues. Also, this is a simple root of the characteristic polynomial of the aforementioned matrix. Therefore, the inverse of this eigenvalue gives the smallest positive simple root of $\text{u}_\Omega(\gamma)$ and the claim is proved. ∎

According to (2.38), the smallest positive simple root of f, is equal to the inverse of the PF-eigenvalue of a non-negative matrix. On the other hand, according to (2.16), $r(\text{f})$ is also the maximum achievable SINR for the system with unbounded power satisfying constraint (2.47). Consequently, using (2.52), (2.38) and Lemma(2.5), the maximum achievable SINR to satisfy all the constraints on power (constraints (2.47) and (2.48)) is

$$\begin{aligned}\gamma^* &= \min\{r(\text{f}), r(\text{u}^{(i)})\} \\ &= \min\{\frac{1}{\lambda^*\big(\text{diag}(\boldsymbol{\mu})\mathbf{A}\big)}, \frac{1}{\lambda^*\big(\psi(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \{i\})\big)}\}.\end{aligned}$$

Since $\psi\big(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \Omega\big) \geq \text{diag}(\boldsymbol{\mu})\mathbf{A} \geq \mathbf{0}$, using Theorem A.3, we have

$$\lambda^*\Big(\psi\big(\text{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\bar{p}_i}, \{i\}\big)\Big) \geq \lambda^*\big(\text{diag}(\boldsymbol{\mu})\mathbf{A}\big),$$

and consequently the maximum achievable γ for a system with constraint on the sum power of any subset of the users is achieved. This discussion leads to the following theorem.

Theorem 2.6 *The maximum achievable γ in an interference channel with n links and gain matrix $\mathbf{A}$, where power vector is subject to the following constraints,*

$$\begin{aligned}&\mathbf{p} \geq \mathbf{0}, \\ &\sum_{i\in\Omega} p_i \leq \bar{p}_\Omega\end{aligned}$$

is equal to

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_\Omega}, \Omega)\big)},$$

where $\Omega \subseteq \{1, \ldots, n\}$ *is an arbitrary subset of the users.*

Based on (2.45), Theorem 2.3, and Theorem 2.6, the following corollary is concluded.

Corollary 2.7 *The maximum achievable* γ *in* (2.7)*, where power vector is subject to the following constraints,*

$$\mathbf{p} \geq \mathbf{0},$$
$$\mathbf{p} \leq \overline{\mathbf{p}},$$
$$\sum_{i=1}^{n} p_i \leq \overline{p}_t$$

is equal to $\gamma^* =$

$$\min\{\frac{1}{\lambda^*\big(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, \ldots, n\})\big)}, \frac{1}{\lambda^*\big(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_1}, \{1\})\big)}, \frac{1}{\lambda^*\big(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_2}, \{2\})\big)}, \ldots, \frac{1}{\lambda^*\big(\psi(\mathrm{diag}(\boldsymbol{\mu})\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_n}, \{n\})\big)}\}. \tag{2.58}$$

The boundary of the SINR region in any direction can be obtained by choosing $\boldsymbol{\mu}$, accordingly. Due to the explicit relationship between the SINR and the rate in Gaussian channels, obtaining the SINR region in these channels amounts to the rate region characterization.

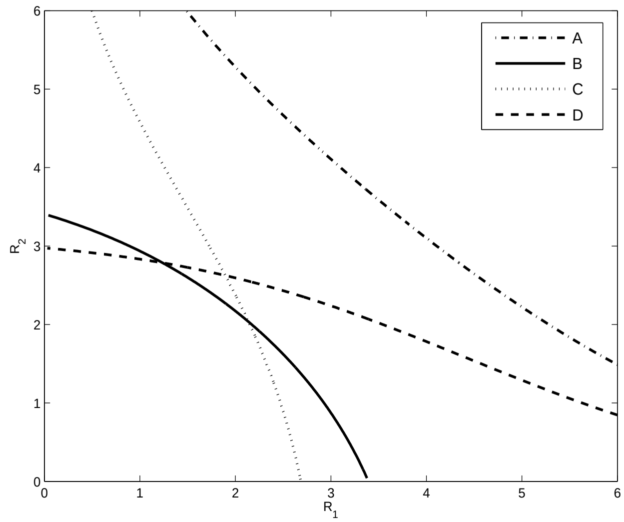

Figure 2.2: The rate region for a 2-user interference channel with the following constraints on the power, A: $p_1 \geq 0$, $p_2 \geq 0$, B: $p_1 + p_2 \leq \bar{p}_t$, $p_1 \geq 0$, $p_2 \geq 0$ C: $0 \leq p_1 \leq \bar{p}_1$, $p_2 \geq 0$, D: $0 \leq p_2 \leq \bar{p}_2$, $p_1 \geq 0$

As an example, Fig. 2.2 and 2.3, respectively, depict the rate region and SINR region of a system with the gain matrix $\mathbf{G}$ as

$$\mathbf{G} = \begin{bmatrix} 0.6791 & 0.0999 \\ 0.0411 & 0.6864 \end{bmatrix},$$

while the power of individual users and the total power are upper-bounded as $\overline{p}_1 = 0.8, \quad \overline{p}_2 = 1, \quad \overline{p}_t = 1.4$, and $\sigma_1^2 = \sigma_2^2 = 10^{-1}$.

The rate region is simply the intersection of all the rate regions resulted from applying each constraint separately. As shown in Fig. 2.2 and Fig. 2.3, the boundary of SINR and rate region, when there is no upper-bound on powers is always above the other boundaries. This is due to the fact that the maximum achievable SINR for the unbounded-power system is the inverse of PF-eigenvalue of diag($\boldsymbol{\mu}$)$\mathbf{A}$; while the maximum achievable SINR when the power is bounded, is the inverse of PF-eigenvalue of a matrix which is definitely greater

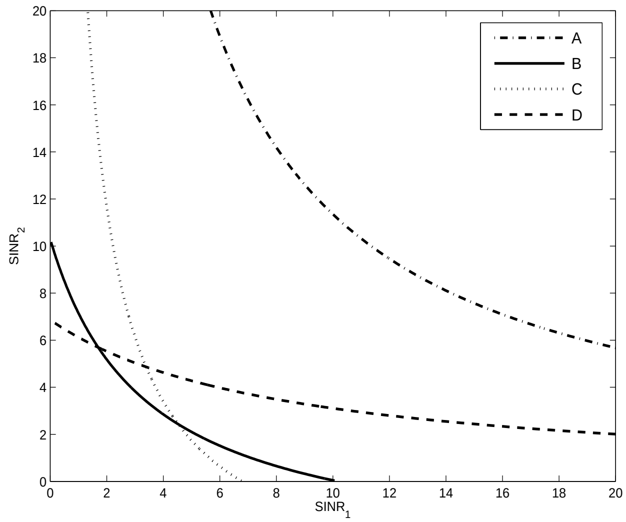

Figure 2.3: The rate region for a 2-user interference channel with the following constraints on the power, A: $p_1 \geq 0$, $p_2 \geq 0$, B: $p_1 + p_2 \leq \bar{p}_t$, $p_1 \geq 0$, $p_2 \geq 0$ C: $0 \leq p_1 \leq \bar{p}_1$, $p_2 \geq 0$, D: $0 \leq p_2 \leq \bar{p}_2$, $p_1 \geq 0$

than $\text{diag}(\boldsymbol{\mu})\mathbf{A}$. Therefore, based on Theorem A.3, the unbounded SINR boundary would be above the bounded-power systems. Thus, this boundary doesn't have any direct role in forming the main boundary. An interesting observation is that if the $\overline{p}_i$'s or $\overline{p}_t$ are increased the boundaries of bounded-power systems tend to the unbounded-power system boundary; the extreme case is when the maximum power goes to infinity which means the power is unbounded, then the matrices whose inverse of PF-eigenvalue form the boundaries become equal and these boundaries touch each other.

As another observation, the rate and SINR regions for a 2-user channel with weaker cross links are shown in Fig. 2.4 and 2.5. The gain matrix in this system is assumed to be

$$\mathbf{G} = \begin{bmatrix} 2.0430 & 0.0359 \\ 0.0134 & 1.3313 \end{bmatrix}, \tag{2.59}$$

while the power of individual users and the total power are upper-bounded as $\overline{p}_1 = 1, \quad \overline{p}_2 =$

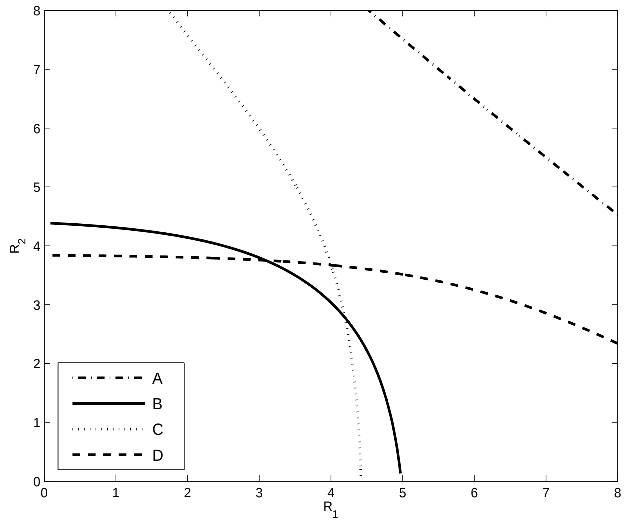

Figure 2.4: The rate region for a 2-user interference channel with the following constraints on the power, A: $p_1 \geq 0$, $p_2 \geq 0$, B: $p_1 + p_2 \leq \bar{p}_t$, $p_1 \geq 0$, $p_2 \geq 0$ C: $0 \leq p_1 \leq \bar{p}_1$, $p_2 \geq 0$, D: $0 \leq p_2 \leq \bar{p}_2$, $p_1 \geq 0$

1, $\overline{p}_t = 1.5$, and $\sigma_1^2 = \sigma_2^2 = 10^{-1}$. The extreme point of this situation is when the links have no interference on each other, and consequently, the maximum SINR for each user considering the individual constraints would be $SINR_i = \dfrac{\overline{p}_i g(i,i)}{\sigma^2}$. We can see in Fig. 2.4 and Fig. 2.5 that these boundaries are more straight than the ones in Fig. 2.2 and Fig. 2.3 which confirms our conjecture.

2.3.3 Time-Varying Channel

So far, we have assumed that the channel gains are fixed with time. However, in practice, channel gains vary with time due to the users' movement or changes in the environment.

In this section, we consider an interference channel with n co-channel links whose channel gain matrix is randomly selected from a finite set $\{\mathbf{G}_1, \ldots, \mathbf{G}_l\}$ with probability

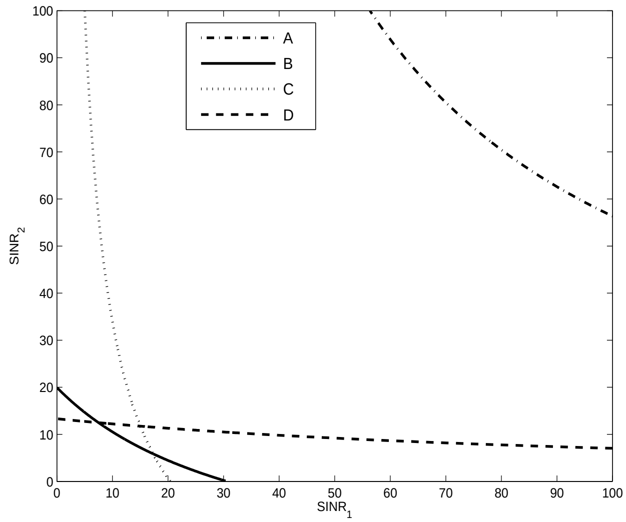

Figure 2.5: The rate region for a 2-user interference channel with the following constraints on the power, A: $p_1 \geq 0$, $p_2 \geq 0$, B: $p_1 + p_2 \leq \bar{p}_t$, $p_1 \geq 0$, $p_2 \geq 0$ C: $0 \leq p_1 \leq \bar{p}_1$, $p_2 \geq 0$, D: $0 \leq p_2 \leq \bar{p}_2$, $p_1 \geq 0$

$\rho_1, \ldots, \rho_l$, respectively. The matrix $\mathbf{A}_i$ denotes the normalized gain matrix in the state i, $i \in \{1, \ldots, l\}$. The objective is to find the maximum γ which is achievable by all users in all channel states, while the average power of the users are constrained, i.e.,

$$\begin{aligned} & \max \gamma \\ \text{s.t.} \quad & \gamma_{j,i} \geq \mu_j \gamma, \quad \forall j \in \Omega, i \in \{1, \ldots, l\} \\ & p_{j,i} \geq 0, \quad \forall j \in \Omega, i \in \{1, \ldots, l\} \qquad (2.60) \\ & E_i[\sum_{j \in \Omega} p_{j,i}] \leq \overline{p}_\Omega, \qquad (2.61) \end{aligned}$$

where $\gamma_{j,i}$ and $p_{j,i}$ are the SINR and the power of transmitter j respectively, when the channel gain matrix is $\mathbf{G}_i$. We define an expanded system including ln users with block diagonal matrices $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{A}}$ as the channel gain matrix and the normalized gain matrix, respectively.Matrices $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{A}}$ are block diagonal matrices, where the i^{th} submatrix on the diagonal is $\mathbf{G}_i$ and $\mathbf{A}_i$, respectively. It is clear that block diagonal format of these matrices reflects the fact that there is no interference between the virtual links associated with different states. In the new system, $p_{j+(i-1)n}$ denotes the power of transmitter j, when the channel gain matrix is $\mathbf{G}_i$. Similar to the previous discussions, the requirements on these links form a system of linear equations with the following formulation in a matrix form,

$$(\frac{1}{\gamma}\mathbf{I}_{nl\times nl} - \text{diag}(\mathbf{1}_{l\times 1} \otimes \boldsymbol{\mu})\tilde{\mathbf{A}})\mathbf{p} = \boldsymbol{\eta}, \tag{2.62}$$

where

$$\eta_{j+(i-1)n} = \frac{\mu_j \sigma_j^2}{g_{j+(i-1)n,j+(i-1)n}}, j \in \Omega, i \in \{1, \ldots, l\}.$$

According to (2.19), we define $\mathbf{F}$ as

$$\mathbf{F} = \frac{1}{\gamma}\mathbf{I}_{nl\times nl} - \text{diag}(\mathbf{1}_{l\times 1} \otimes \boldsymbol{\mu})\tilde{\mathbf{A}}.$$

Then, we have

$$\mathbf{F}\mathbf{p} = \gamma\boldsymbol{\eta}.$$

Using Cramer's rule, we will have

$$p_{j+(i-1)n} = \frac{\det(\mathbf{H}^{(j+(i-1)n)})}{\det(\mathbf{F})},$$

where $\mathbf{H}^{(j+(i-1)n)}$ according to (2.22) is the matrix $\mathbf{F}$ whose column $j+(i-1)n$ is substituted by $\gamma\boldsymbol{\eta}$. The average of the total power of the users in Ω is equivalent to

$$\begin{aligned} E_i[\sum_{j\in\Omega} p_{j+(i-1)n}] &= \sum_{i=1}^{l} \rho_i \sum_{j\in\Omega} p_{j+(i-1)n} \\ &= \sum_{i=1}^{l} \rho_i \sum_{j\in\Omega} \frac{\det(\mathbf{H}^{(j+(i-1)n)})}{\det(\mathbf{F})} \\ &= \frac{1}{\det(\mathbf{F})} \sum_{i=1}^{l} \rho_i \sum_{j\in\Omega} \det(\mathbf{H}^{(j+(i-1)n)}). \end{aligned} \tag{2.63}$$

Based on (2.63), we define

$$\mathrm{u}_\Omega(\gamma) = \overline{p}_\Omega \det(\mathbf{F}) - \sum_{i=1}^{l} \rho_i \sum_{j\in\Omega} \det(\mathbf{H}^{(j+(i-1)n)}),$$

and

$$\mathrm{f}(\gamma) = \det(\mathbf{F}).$$

Therefore, the constraint in (2.61) is equivalent to

$$\frac{\mathrm{u}_\Omega(\gamma)}{\mathrm{f}(\gamma)} \geq 0.$$

Like before, it is easy to show that the maximum achievable SINR satisfying constraints (2.60) and (2.61) is

$$\gamma^* = \min\{r(\mathrm{f}), r(\mathrm{u}_\Omega)\}. \tag{2.64}$$

To simplify $\mathrm{u}_\Omega(\gamma)$, we have

$$\begin{aligned}\mathrm{u}_\Omega(\gamma) &= \overline{p}_\Omega \det(\mathbf{F}) - \sum_{i=1}^{l} \rho_i \sum_{j\in\Omega} \det(\mathbf{H}^{(j+(i-1)n)}) \\ &= \overline{p}_\Omega(\det(\mathbf{F}) - \sum_{i=1}^{l} \sum_{j\in\Omega} \det(\hat{\mathbf{H}}^{(j+(i-1)n)})),\end{aligned}$$

where $\hat{\mathbf{H}}^{(j+(i-1)n)}$ is $\mathbf{H}^{(j+(i-1)n)}$ whose column $j+(i-1)n$ is multiplied by $\frac{\rho_i}{\overline{p}_\Omega}$. Using the same procedure as before, we obtain

$$\mathrm{u}_\Omega(\gamma) = \overline{p}_\Omega \det(\mathbf{F} - \mathbf{D}),$$

where

$$\mathbf{D} = \sum_{i=1}^{l} \psi(\mathbf{0}_{nl\times nl}, \frac{\rho_i \gamma \boldsymbol{\eta}}{\overline{p}_\Omega}, \{j+(i-1)n : j\in\Omega\})$$

According to Theorem A.3, it is easy to see that

$$r(\mathrm{u}_\Omega) = \frac{1}{\lambda^*\big(\mathrm{diag}(\mathbf{1}_{l\times 1}\otimes\boldsymbol{\mu})\tilde{\mathbf{A}} + \sum_{i=1}^{l}\psi(\mathbf{0}_{nl\times nl}, \frac{\rho_i\boldsymbol{\eta}}{\overline{p}_\Omega}, \{j+(i-1)n : j\in\Omega\})\big)}.$$

and

$$r(\mathrm{f}) = \frac{1}{\lambda^*(\mathrm{diag}(\mathbf{1}_{l\times 1}\otimes\boldsymbol{\mu})\tilde{\mathbf{A}})}.$$

Therefore, using Theorem A.3 and equation (2.64), we have the following theorem.

Theorem 2.8 *The maximum achievable γ in a time-varying interference channel with n links and probability vector $\boldsymbol{\rho}_{l\times 1}$, with the following constraints on power,*

$$p_{j,i} \geq 0, \forall j\in\Omega, i\in\{1,\dots,l\},$$
$$E_i[\sum_{j\in\Omega} p_{j,i}] \leq \overline{p}_\Omega$$

is equal to

$$\gamma^* = \frac{1}{\lambda^*\big(\mathrm{diag}(\mathbf{1}_{l\times 1}\otimes\boldsymbol{\mu})\tilde{\mathbf{A}} + \sum_{i=1}^{l}\psi(\mathbf{0}_{nl\times nl}, \frac{\rho_i\boldsymbol{\eta}}{\overline{p}_\Omega}, \{j+(i-1)n : j\in\Omega\})\big)},$$

where $\tilde{\mathbf{A}}$ is an $nl\times nl$ block diagonal matrix whose i^{th} diagonal submatrix is the normalized gain matrix at the state i. Also, $\otimes$ represents the Kronecker product operator.

Apparently, if there are multiple constraints on the power, the maximum achievable SINR γ^* is computed by

$$\gamma^* = \min_i \gamma_i^*,$$

where γ_i^* is the maximum achievable SINR obtained by Theorem 2.8 while only the constraint i is considered for the system.

Chapter 3

Removal Algorithm

In a congested system, all the users can not satisfy the QoS requirement. Therefore, some of the users should be dropped in order to reduce effective interference on the active users and consequently ameliorate the achievable SINR. As a result, we are interested to find the maximum subset of the users which can meet the minimum required QoS. Unfortunately, this problem is claimed to be NP-complete [3]. In what follows, we propose a suboptimal algorithm for obtaining a subset of the users with maximum cardinality satisfying the rate requirement, based on the equation (2.16) and Theorems 2.3 and 2.6.

3.1 Removal Algorithm

We keep the users “on” or “active” if the SINR of that user exceeds a required threshold. Otherwise, it is “off” or “inactive” and its power is zero. To find the optimal set of active users, satisfying the QoS requirement, we have to examine all the combinations of the users and select the feasible one with the maximum cardinality. Clearly, this scheme is computationally exponential. As a suboptimal alternative scheme, we show that removing the users in a greedy manner yields a result which is very close to the optimum solution. The main idea behind the presented algorithm is as follows. At each step, if the active users do not satisfy the required SINR, one user is removed. This user is the one which provides the highest increase in the maximum achievable SINR if it is removed. We call this user the *worst user*. The proposed algorithm is presented for different types of constraints on the transmit powers.

According to (2.16) and Theorem 2.6, in general, the maximum γ is equal to the inverse of the PF-eigenvalue of a non-negative (or irreducible) matrix $\mathbf{X}$, i.e.,

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{X})}.$$

In a system with a large number of users, computing the PF-eigenvalue is computationally extensive. In this case, we use an approximation of the PF-eigenvalue. When a matrix is raised to a power, its eigenvalues are raised to the same power as well [27], i.e.,

$$\lambda(\mathbf{X}^q) = \lambda^q(\mathbf{X}).$$

On the other hand, the trace of a matrix is equal to the summation of the eigenvalues of that matrix [27]; therefore,

$$Tr(\mathbf{X}^q) = \sum_i \lambda_i^q.$$

Since the PF-eigenvalue of a non-negative primitive (or irreducible) matrix has the largest norm among all the eigenvalues of that matrix (Theorem A.3), we can approximate $\lambda^{*q}(\mathbf{X})$ with the $Tr(\mathbf{X}^q)$, i.e.,

$$\lambda^{*q}(\mathbf{X}) \approx Tr(\mathbf{X}^q).$$

This approximation is stronger if the power q is larger. However, the simulation results show that $q = 2$ yields a very good approximation for the purpose of the proposed removal algorithm. Therefore, we use

$$\gamma^* \approx \frac{1}{\sqrt{Tr(\mathbf{X}^2)}} \tag{3.1}$$

as an approximate value for γ^*. In what follows, we investigate the problem of user removal for different power constraints and give an efficient algorithm for each case.

Case One: No Power Constraint

Based on the previous discussions on the worst link determination and using (2.16), when there is no power constraint, the index of the user to be removed, $\hat{i}$, is obtained as

$$\hat{i} = \arg\max_i \{\frac{1}{\lambda^*(\mathbf{A}^{i-})}\}.$$

If this link is removed and still the maximum achievable SINR computed through (2.16) does not meet the required SINR, additional links are removed in a recursive manner till the remaining users become feasible. This algorithm is called the *Removal Algorithm I-A* throughout this thesis.

Algorithm I-A

1. Set $\mathbf{A}$ as in (2.9), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2. Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*(\mathbf{A})}$.
3. If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.
4. Find the worst link as $\hat{i} = \arg\max_i \dfrac{1}{\lambda^*(\mathbf{A}^{i-})}$.
5. Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $m \leftarrow m - 1$, and go to step 2.

where $\leftarrow$ is a substitution notation.

To avoid the complexity of computing PF-eigenvalues in each iteration, we present the following algorithm which is an approximate version of algorithm I-A. According to (2.16) and (3.1) for the unconstrained power scenario, we have

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{A})} \approx \frac{1}{\sqrt{Tr(\mathbf{A}^2)}} = \frac{1}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji}}} \ . \tag{3.2}$$

We define vector $\mathbf{w}$ as

$$\mathbf{w} = [w_i]_{n\times 1}, \ \ w_i = \sum_{j=1}^{n} a_{ij}a_{ji} \ .$$

Then we have

$$\gamma^* \approx \frac{1}{\sqrt{\sum_{i=1}^{n} w_i}} \ .$$

It is easy to show that by removing user i, $2w_i$ is subtracted from the trace of $\mathbf{A}^2$. An immediate conclusion is that if we want to remove one link to obtain the largest increase in the maximum achievable SINR, the best choice (worst link) is to remove the one with the largest w_i. Therefore, $\hat{i} = \arg\max_i w_i$. Based on this result, an efficient algorithm for gradually removing the users is presented as follows. In each iteration, we find the maximum achievable γ using (2.16) and if this amount is greater than γ_{th}, all the links can be active. Otherwise, the worst link is determined and removed. This algorithm repeats iteratively until the remaining users satisfy the required threshold.

Algorithm I-B

1. Set $\mathbf{A}$ as in (2.9), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.

2. Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*(\mathbf{A})}$.

3. If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.

4. Update the vector $\mathbf{w}_{m\times 1}$ as $w_i = \sum_{j=1}^{m} a_{ij}a_{ji}$.

5. Determine the worst link as $\hat{i} = \arg\max_i w_i$.

6. Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $m \leftarrow m-1$, and go to step 2.

Case Two: Constraints on the Power of Individual Transmitters

When the power of each transmitter is subject to an upper-bound constraint, based on Theorem 2.3, we design an efficient suboptimal algorithm to find the maximum cardinality subset of the users satisfying a minimum SINR requirement. We define the matrix $\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})$ as the matrix $\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})$ whose i^{th} column and row are removed. Therefore, the worst link is

$$\hat{i} = \arg\max_i \min_{\substack{j \\ j\neq i}} \frac{1}{\lambda^*\big(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}. \tag{3.3}$$

The users are removed one by one based on (3.3) until all of the active users satisfy the rate requirement. We call this algorithm the *Removal Algorithm II-A*.

Algorithm II-A

1. Set $\mathbf{A}$ as in (2.9), $\overline{\mathbf{p}}$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.

2. Find the maximum achievable as SINR $\gamma^* = \min_j \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}$.

3. If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.

4. Find the worst link as $\hat{i} = \arg\max_i \min_j \frac{1}{\lambda^*\big(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}$.

5. Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $\overline{\mathbf{p}} \leftarrow \overline{\mathbf{p}}^{\hat{i}-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}-}$, and $m \leftarrow m-1$, and go to step 2.

To reduce the complexity of this algorithm, we use the following approximation scheme. According to Theorem 2.3 and (3.1), we have

$$\gamma^* = \min_j \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)} \approx \min_j \frac{1}{\sqrt{Tr\big(\psi^2(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}},$$

which can be rewritten as

$$\gamma^* \approx \min_j \frac{1}{\sqrt{(\frac{\eta_j}{\overline{p}_j})^2 + \sum_{k=1}^{n}\sum_{l=1}^{n} a_{kl}a_{lk} + 2\sum_{k=1}^{n}\frac{\eta_k}{\overline{p}_k}a_{jk}}}.$$

We define the matrix $\mathbf{W}$ as $\mathbf{W} = [w_{ij}]_{n\times n}$,

$$w_{ij} = \begin{cases} (\frac{\eta_j}{\overline{p}_j})^2 + \sum\limits_{\substack{k=1\\k\neq i}}^{m}\sum\limits_{\substack{l=1\\l\neq i}}^{m} a_{kl}a_{lk} + 2\sum\limits_{\substack{k=1\\k\neq i}}^{m}\frac{\eta_k}{\overline{p}_k}a_{jk} & j \neq i \\ 0 & j = i \end{cases}.$$

We can show that (3.3) can be simplified to $\hat{i} = \arg_j \min\limits_j \max\limits_i w_{ij}$. Based on this result, the following algorithm is developed.

Algorithm II-B

1. Set $\mathbf{A}$ as in (2.9), $\overline{\mathbf{p}} = [\overline{p}_i]$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.

2. Find the maximum achievable SINR as

$$\gamma^* = \min_j \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}.$$

3. If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.

4. Update $\mathbf{W}_{m\times m}$ as

$$w_{ij} = \begin{cases} (\frac{\eta_j}{\overline{p}_j})^2 + \sum_{\substack{k=1\\k\neq i}}^{m} \sum_{\substack{l=1\\l\neq i}}^{m} a_{kl}a_{lk} + 2\sum_{\substack{k=1\\k\neq i}}^{m} \frac{\eta_k}{\overline{p}_k} a_{jk} & j \neq i \\ 0 & j = i \end{cases}.$$

5. Determine the worst link as $\hat{i} = \arg\min_i \max_j w_{ij}$.

6. Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $\overline{\mathbf{p}} \leftarrow \overline{\mathbf{p}}^{\hat{i}-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}-}$, and $m \leftarrow m-1$, and go to step 2.

Case Three: Total Transmit Power Constraint

When the total power is constrained by $\overline{p}_t$, the maximum achievable SINR is computed through Theorem 2.6. In this case, the worst user is determined as

$$\hat{i} = \arg\max_i \{\frac{1}{\lambda^*\big(\psi^{i-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1,2,\ldots,n\})\big)}\}. \tag{3.4}$$

We call this algorithm the *Removal Algorithm III-A*.

Algorithm III-A

1. Set $\mathbf{A}$ as in (2.9), $\overline{\mathbf{p}}$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1,2,\ldots,n]'$.

2. Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1,2,\ldots,m\})\big)}$.

3. If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.

4. Find the worst link as $\hat{i} = \arg\max_i \{\dfrac{1}{\lambda^*\big(\psi^{i-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1,2,\ldots,m\})\big)}\}$.

5. Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}-}$, $m \leftarrow m-1$, and go to step 2.

To reduce the complexity of the algorithm III-A, we use the following method. According to Theorem 2.6 and (3.1), we have

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1,2,\dots,n\})\big)} \approx \frac{1}{\sqrt{Tr\big(\psi^2\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1,2,\dots,n\})\big)}}.$$

Therefore, we have

$$\gamma^* \approx \big(\sum_{i=1}^{n} (\frac{\eta_i}{\overline{p}_t})^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji} + 2\sum_{i=1}^{n}\frac{\eta_i}{\overline{p}_t}\sum_{j=1}^{n} a_{ji} + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\eta_i\eta_j}{\overline{p}_t^{\,2}}\big)^{-\frac{1}{2}}.$$

We define $\mathbf{w}$ as

$$\begin{aligned} w_i =& (\frac{\eta_i}{\overline{p}_t})^2 + 2\sum_{j=1}^{n} a_{ij}a_{ji} + 2\frac{\eta_i}{\overline{p}_t}\sum_{j=1}^{n} a_{ji} \\ &+ 2\sum_{j=1}^{n}\frac{\eta_j}{\overline{p}_t}a_{ij} + 2\frac{\eta_i}{\overline{p}_t}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\eta_j}{\overline{p}_t}. \end{aligned}$$

We can show that the worst user can be found by,

$$\hat{i} = \arg\max_i w_i.$$

According to this result, we have the following algorithm.

Algorithm III-B

1. Set $\mathbf{A}$ as in (2.9), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \dots, n]'$.

2. Find the maximum achievable SINR as

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1,\dots,m\})\big)}.$$

3. If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.

4. Update the vector $\mathbf{w}_{m\times 1}$ as $w_i = (\frac{\eta_i}{\overline{p}_t})^2 + 2\sum_{j=1}^{m} a_{ij}a_{ji} + 2\frac{\eta_i}{\overline{p}_t}\sum_{j=1}^{m} a_{ji} + 2\sum_{j=1}^{m}\frac{\eta_j}{\overline{p}_t}a_{ij} + 2\frac{\eta_i}{\overline{p}_t}\sum_{\substack{j=1\\ j\neq i}}^{m}\frac{\eta_j}{\overline{p}_t}$.

5. Determine the worst link as $\hat{\imath} = \arg\max_i w_i$.

6. Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{\imath}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{\imath}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{\imath}-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{\imath}-}$, $m \leftarrow m-1$, and go to step 2.

In the following section, we will demonstrate the performance of the proposed algorithms via simulation and compare the results with the performance of the other schemes.

3.2 Numerical Results

The simulation results are presented for two environments of cellular networks and Rayleigh fading channels. In each environment, three cases are considered; (i) No constraint on the power, (ii) Constraint on the power of individual users, and (iii) Constraint on the total transmit power. The proposed algorithms are compared with other schemes for the aformentioned environments and constraints on power.

We focus on the uplink ISI-free transmission in a diamond structure cellular network. We consider one channel which is a certain time slot or a frequency interval and discuss the inter-cell interference on the co-channel users in that specific channel. We assume that in each cell there is one user that desires to send data to that cell's base station. The location of each user is uniformly distributed over the assigned cell. We define a cluster as a group of cells with different frequencies in which all the available frequencies are used and no two cells have the same frequency, i.e., the cluster size in Fig.3.2 and Fig.3.1 is 4. We used the

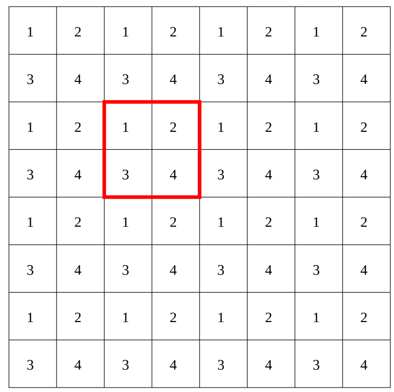

Figure 3.1: An 8X8 cellular network with cluster size 4

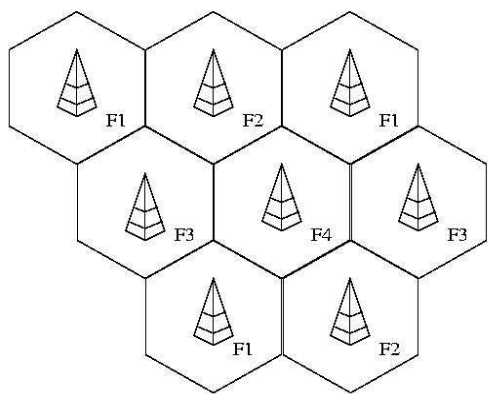

Figure 3.2: A hexagonal cellular network with cluster size 4 [1]

diamond-shaped (square) clusters in the simulations. In this case, all the co-channel cells are placed symmetrically in a sparse square pattern (Fig. 3.2). To generate the link gains, we use a simple model which is well accepted in the analysis of cellular networks [52], [3], and [29]. g_{ij}, which is the gain of power from transmitter j to the receiver i, is modelled as

$$g_{ij} = \frac{\beta_{ij}}{\nu_{ij}^{\alpha}},$$

where β_{ij} is the shadow fading term which models the irregularities in the terrain, such as mountains, hills, buildings, etc. $1/\nu_{ij}^{\alpha}$ models the large scale propagation loss in which ν_{ij} is the distance between transmitter j and receiver i and α is the propagation constant. For the simulations, we consider the shadow fading term as a log-normal random variable, where

$$E[10 \log \beta_{ij}] = 0 \tag{3.5}$$

$$Var[10 \log \beta_{ij}] = \varsigma^2. \tag{3.6}$$

The parameters ς and α depend on the environment and change in the range of $4 - 10$ dB and $3 - 5$, respectively. We assume $\varsigma = 6$ dB and $\alpha = 3$ in our simulations. Moreover, the radius of each micro-cell is assumed to be 1 km [3].

In the Rayleigh fading channel, we assume that the parameters g_{ij} follow an exponential distribution with average and variance one for the forward gains, and average 10^{-2} and variance 10^{-4} for the cross gains.

We define *Outage Probability* as the ratio between the number of the inactive users to the total number of the users. This probability shows the percentage of the users that fail to attain the required QoS. We use this function as a metric to compare different algorithms, as it is used in [53], [52].

For the case that there is no constraint on the users' power, the curves of the outage probability for different user removal algorithms are depicted in Fig. 3.3, Fig. 3.4, and Fig. 3.5. The results for the algorithm I-A which uses the exact values for the PF-eigenvalues and also algorithm I-B which uses the approximation for the PF-eigenvalues are compared with the performance of SMIRA (stepwise-maximum-interference-removal-algorithm) in [37] and SRA (stepwise removal algorithm) in [53]. Since in SMIRA and SRA algorithms, the noise power is considered zero, we assigned a very small value to the noise power to be able to compare the different algorithms. As shown in Fig. 3.5, in Rayleigh fading channel which has strong cross gains and consequently high interference, algorithms

I-A and I-B outperform SMIRA and SRA algorithm. In addition, in Fig. 3.3 and Fig. 3.4, it is easy to see that algorithms I-A, I-B and SMIRA have a very close-to-optimal outage probability while SRA is very far from the optimal value, compared to the others. Another observation is that the performance of algorithm I-B is very close to that of algorithm I-A, while it enjoys much less operational complexity.

In [3], a number of removal algorithms when the power of transmitters are individually constrained are proposed. We selected centralized GRN-DCPC to compare it with our results since according to [3], it outperforms the other presented algorithms in that work. The simulation results in Fig. 3.6, Fig. 3.7, and Fig. 3.8 show a significant improvement in the outage probability of the algorithms II-A and II-B compared to GRN-DCPC.

As depicted in Fig. 3.9, Fig. 3.10, and Fig. 3.11, when the total power is bounded, the performance of algorithms III-A and IIII-B is very close to the optimal result. Up to our knowledge, there is no alternative algorithms for the case that the total power is upper-bounded.

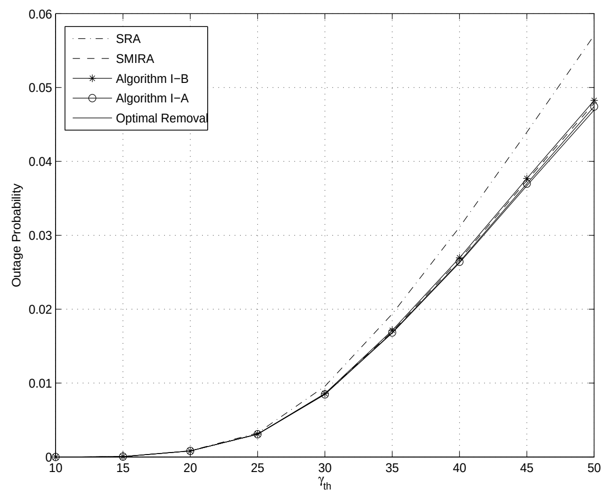

Figure 3.3: No Constraint on Power in an 8×8 Cellular Network with Cluster Size= 4, $n = 16, {\sigma_i}^2 = 10^{-16}\ \forall i$

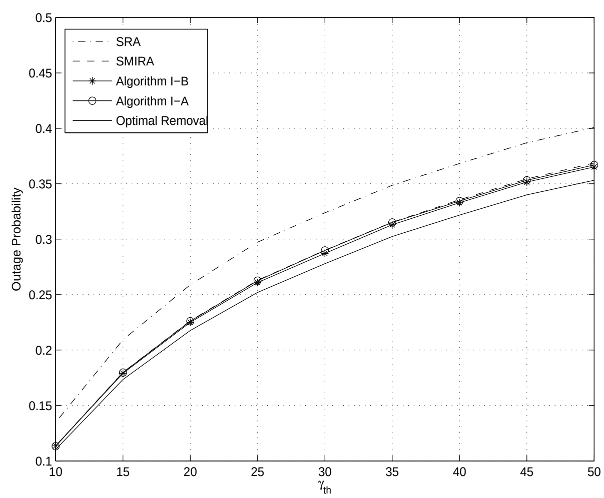

Figure 3.4: No Constraint on the Power in a 4×4 Cellular Network with Cluster Size= 1, $n = 16, {\sigma_i}^2 = 10^{-16}\ \forall i$

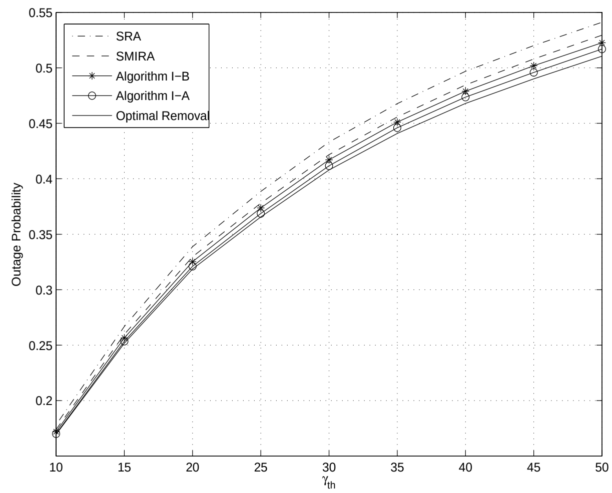

Figure 3.5: No Constraint on the Power in a Rayleigh Fading Channel, $n = 8, {\sigma_i}^2 = 10^{-16}\ \forall i$

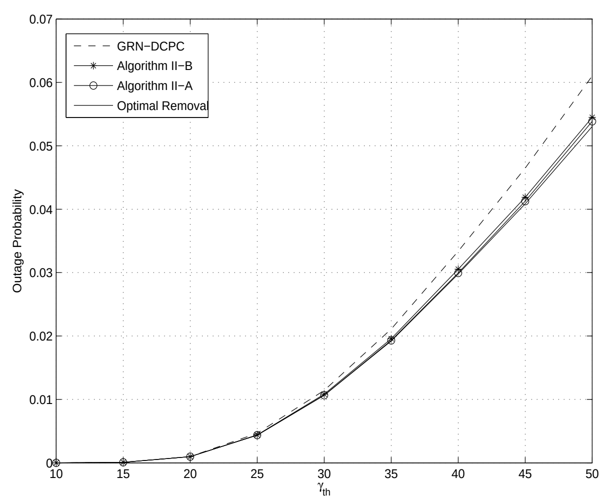

Figure 3.6: Constraints on the Power of Individual Transmitters in an 8×8 Cellular Network with Cluster Size= 4, $n = 16, {\sigma_i}^2 = 10^{-12}$, $\overline{p}_i = 1\text{w}\ \forall i$

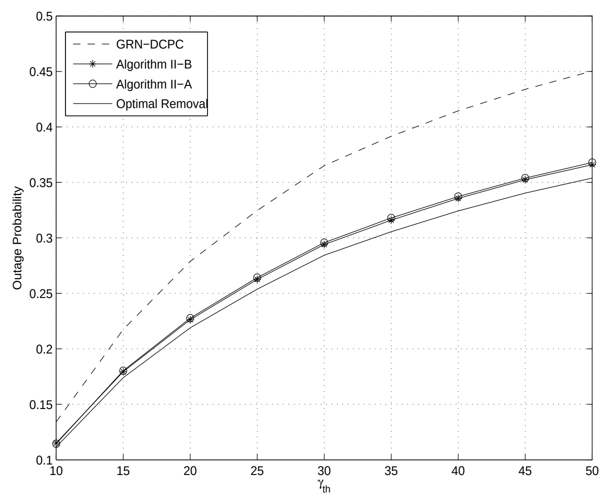

Figure 3.7: Constraints on the Power of Individual Transmitters in a 4×4 Cellular Network with Cluster Size= 1, $n = 16, {\sigma_i}^2 = 10^{-12}$, $\overline{p}_i = 1\text{w} \ \forall i$

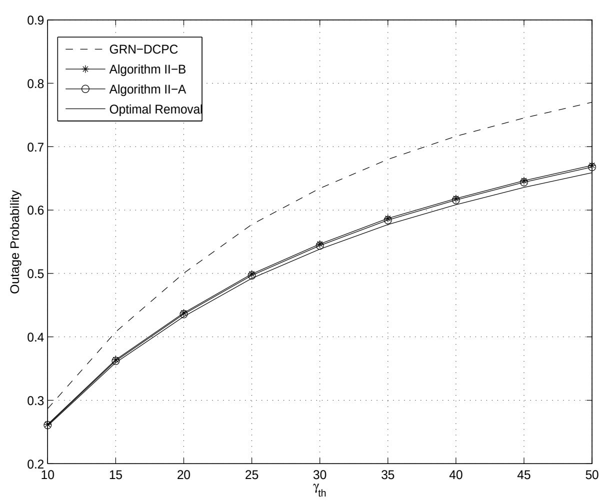

Figure 3.8: Constraints on the Power of Individual Transmitters in a Rayleigh Fading Channel, $n = 8, {\sigma_i}^2 = 10^{-2}$, $\overline{p}_i = 1\text{w} \ \forall i$

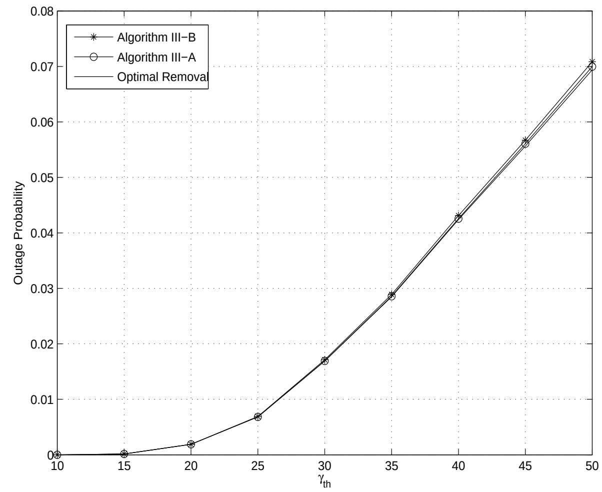

Figure 3.9: Constraint on the Total Power in an 8×8 Cellular Network with Cluster Size= 4, $n = 16, {\sigma_i}^2 = 10^{-12}\ \forall i,\ \overline{p}_t = 1$w

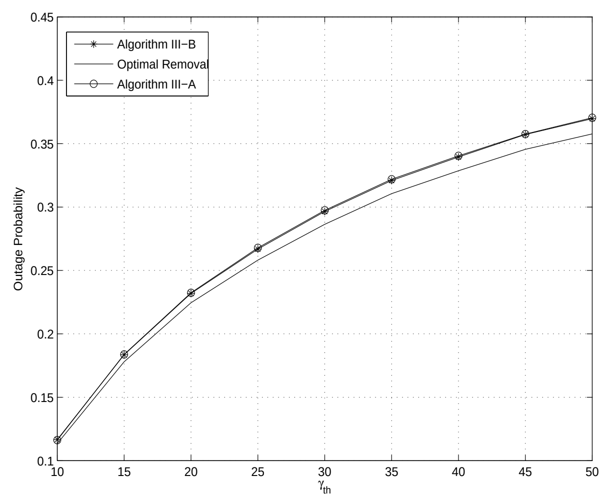

Figure 3.10: Constraint on the Total Power in a 4×4 Cellular Network with Cluster Size= 1, $n = 16, {\sigma_i}^2 = 10^{-12}\ \forall i,\ \overline{p}_t = 1$w

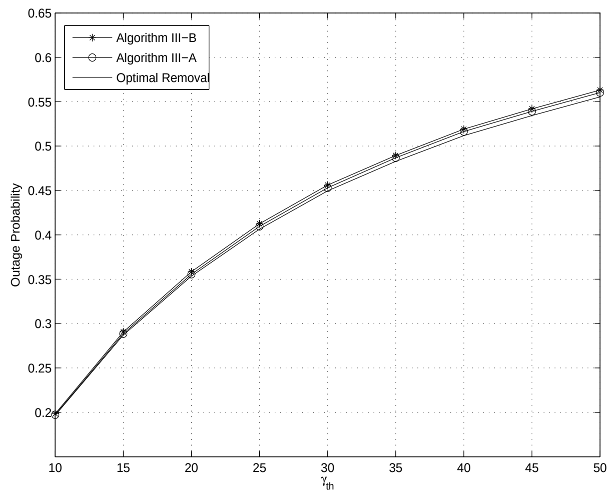

Figure 3.11: Total Transmit Power Constraint in a Rayleigh Fading Channel, $n = 8, {\sigma_i}^2 = 10^{-3}\ \forall i$, $\overline{p}_t = 1$w

Chapter 4

Conclusion

Interference channels and their application have been emerging in the new wireless communication networks technology ubiquitously. In spite of the benefits of high capacity and coverage, the interference on the co-channel signals cause a deterioration in the performance of the system. Although many interference reducing techniques such as sectorization, smart antennas, interference averaging, multiuser detection, and interference precancellation try to mitigate this problem, the large complexity of such systems make them impractical. Moreover, many of these methods can not remove the interference completely and still the system's performance is compromised. Consequently, the existence of such interference limits the QoS to a maximum value.

In this thesis, we have obtained a closed form for the maximum achievable SINR in an interference channel utilizing the Perron-Frobenious theorem when there is a total power constraints on any subsets of the system. This result leads to achieving the boundary of rate region for the aforementioned constraints on the power. While the boundary of the rate region for the case of unconstrained power is a well-established result, this is the first result for the case of constrained power. Using this relationship between the maximum achievable SINR and the parameters of the network, many challenging problems for the interference channels with constraints on the power can be addressed, e.g., we considered a time-varying interference channel where the total average power of an arbitrary subset of the system is subject to an upper-bound. We utilized the aforementioned scenario to obtain an explicit solution for the maximum achievable SINR of such a system based on the Perron-Frobenius theorem.

Congestion problem is one of the challenging problems in the interference channels in which all the users can not achieve the required SINR target simultaneously. For solving this problem one or more users should be removed in order to the remaining users attain the required threshold. Usually, it is desired to minimize the number of removed users to achieve maximum possible sum-rate when the SINR balancing is considered. This problem is NP-complete. Based on the results for the maximum achievable SINR, we have proposed a novel sub-optimal algorithm to obtain the largest possible subset of the users which satisfy the QoS requirement while the power may be constrained to individual or total upper-bounds. Moreover, we presented an approximate algorithm which achieves the same performance as the proposed sub-optimal algorithm, while enjoying a much less complexity. We have shown that our algorithm outperforms the available algorithms which consider constraints on the power.

Chapter 5

Future Research

- This thesis investigates the rate region of some parallel channels which are working in the same frequency band. As an extension, one can consider the system where more than one frequency band is available to each transmitter and transition concurrently takes place in all frequencies (OFDM). The rate region for such a scenario is desirable.

- The current work can be extended to the case where the users have the possibility of successive or joint detection. In this scenario, users have the possibility of decoding the data of other transmitters to reduce the effective interference and increase the resulting SINR. The rate region of such channels for different scenarios is desirable.

- Sometimes the required QoS necessitates a rate vector which is outside the rate region. In this case, it is desired to find a rate vector on the boundary of the rate region which has the closest distance from the desired rate vector.

- The obtained rate region is not convex. By incorporating time sharing strategies to the system, one can obtain the convex hull of the rate region. Finding the maxmin point in a specific direction in a closed form is desirable.

- The proposed algorithms in this thesis are performed in a centralized fashion. One direction to extend this work is to decentralize the proposed algorithms.

Appendix A

Some basic definitions and theorems which are related to the thesis are quoted from [40].

Definition A.1 *A square non-negative matrix* $\mathbf{X}$ *is said to be primitive if there exists a positive integer* k *such that* $\mathbf{X}^k > \mathbf{0}$.

Definition A.2 *An* $n \times n$ *non-negative matrix* X *is irreducible if for every pair* i, j *of its index set, there exists a positive integer* $m \equiv m(i, j)$ *such that* $x_{ij}^{(m)} > 0$ *which* $x_{ij}^{(m)}$ *is the* ij^{th} *element of* X^m.

It is clear that any primitive matrix is an irreducible matrix.

Theorem A.3 *(The Perron-Frobenius Theorem for primitive matrices) Suppose* $\mathbf{X}$ *is an* $n \times n$ *non-negative primitive matrix. Then there exists an eigenvalue* $\lambda^*(\mathbf{X})$ *(Perron-Frobenius eigenvalue or PF-eigenvalue) such that*

(i) $\lambda^*(\mathbf{X}) > 0$ *and it is real.*

(ii) there is a positive vector $\mathbf{v}$ *such that* $\mathbf{Xv} = \lambda^*(\mathbf{X})\mathbf{v}$.

(iii) $\lambda^*(\mathbf{X}) > |\lambda(\mathbf{X})|$ *for any eigenvalue* $\lambda(\mathbf{X}) \neq \lambda^*(\mathbf{X})$.

(iv) If $\mathbf{X} \geq \mathbf{Y} \geq \mathbf{0}$, *then* $\lambda^*(\mathbf{X}) \geq |\lambda(\mathbf{Y})|$ *for any eigenvalue of* $\mathbf{Y}$.

(v) $\lambda^*(\mathbf{X})$ *is a simple root of the characteristic polynomial of* $\mathbf{X}$*.*

Theorem A.4 *(The Perron-Frobenius Theorem for irreducible matrices) Suppose* $\mathbf{X}$ *is an* $m \times m$ *non-negative irreducible matrix. Then, all of the assertions (i)-(v) of Theorem A.3 holds except that (iii) is replaced by the weaker statement:* $\lambda^*(\mathbf{X}) \geq |\lambda(\mathbf{X})|$ *for any eigenvalue* $\lambda(\mathbf{X})$*.*

Theorem A.5 *If* $\mathbf{Y}$ *is a non-negative irreducible matrix, a necessary and sufficient condition for a solution* $\mathbf{x}(\mathbf{x} \geq \mathbf{0}, \mathbf{x} \neq \mathbf{0})$ *to the equations*

$$(\beta \mathbf{I} - \mathbf{Y})\mathbf{x} = \mathbf{z} \tag{A.1}$$

to exist for any $\mathbf{z} \geq \mathbf{0}, \mathbf{z} \neq \mathbf{0}$ *is that* $\lambda^*(\mathbf{Y}) < \beta$*. In this case there is only one solution* $\mathbf{x}$*, which is strictly positive and given by*

$$\mathbf{x} = (\beta \mathbf{I} - \mathbf{Y})^{-1}\mathbf{z}. \tag{A.2}$$

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