THREE ESSAYS ON VENDOR MANAGED INVENTORY IN SUPPLY CHAINS

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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ABSTRACT

Vendor Managed Inventory (VMI), Consignment Inventory (CI) and a combination of both (C&VMI) are supply-chain sourcing agreements between a vendor and customer. VMI allows the vendor to initiate orders on behalf of the customer. In CI, the customer pays for the goods supplied by the vendor only upon use. The vendor under C&VMI decides customer-replenishments, and owns the goods replenished until they are deployed by the customer. Our thesis studies these agreements in three essays.

The first essay considers a vendor *V* that manufactures a particular product at a unique location. That item is sold to a single retailer, the customer *C*. Three cases are treated in detail: Independent decision making (no agreement between the parties); VMI, whereby the supplier *V* initiates orders on behalf of *C*; and Central decision making (both Vendor and Customer are controlled by the same corporate entity).

Values of some cost parameters may vary between the three cases, and each case may cause a different actor to be responsible for particular expenses. Under a constant demand rate, optimal solutions are obtained analytically for the customer's order quantity, the vendor's production quantity, hence the parties' individual and total costs in the three cases. Inequalities are obtained to delineate those situations in which VMI is beneficial.

The problem setting in the second essay is the same with that of Essay 1, but the sourcing agreements investigated are now CI and C&VMI. In CI, as in the usual independent-sourcing approach, the customer has authority over the timing and quantity of replenishments. CI seems to favour the customer because, in addition, he pays for the goods only upon use. Under a C&VMI agreement, the vendor still owns the goods at the customer's premises, but at least can determine how much to store there.

The second essay thus contrasts the cases CI and C&VMI, and compares each of them to a no-agreement case. General conditions under which those cases create benefits for the vendor, the customer and the whole chain are determined.

Essay 3 investigates VMI and C&VMI separately for a vendor and multiple customers who face time-varying, but deterministic demand for a single product. In any of those agreements, the vendor seeks the best set of customers to achieve economies of scale. MIP models are developed to find that set of customers, and to determine the vendor’s optimal production, transportation, and customer-replenishment quantities. The model for VMI is solved using a heuristic that produces two sub-models, and uses hierarchical solution approach for production, customer-replenishment and transportation decisions. C&VMI model is solved using Lagrangian relaxation. Various numerical examples are used to test the solution approaches used.

In the mean time, the customers can guarantee to be no worse off under VMI or C&VMI than the no-agreement case by setting the right levels of maximum inventory. A model to determine those levels and a solution algorithm are also proposed in Essay 3.

The first two essays can help a vendor or customer in a supply chain to determine the least costly sourcing option, which depends on the relative values of various cost parameters. A vendor with multiple customers can make use of the results in the third essay, which reveal the best possible economies of scale under VMI or C&VMI. Those customers can guarantee to be no worse of than traditional sourcing when they set the proposed levels of maximum inventory.

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1. Introduction

1.1 Coordinating the Operations in a Supply Chain

The essential operational processes of a firm in industry are sourcing (procurement of required materials), making (production of goods using the sourced materials), and delivering (transportation of those goods to customers). One of the main concerns of every company involved in a *supply chain* is to plan these processes, and to minimize its operational costs while maintaining the best possible efficiency.

A supply chain describes the flow of materials and information from suppliers through manufacturing plants and depots to customers. In general, a supply chain is thus a *network* whose nodes represent locations of suppliers or members of the chain that carry out processing or manufacturing operations. Each arc denotes a flow of materials and information between nodes.

Consider a simple example of a supply chain (Figure 1.1), in which node *V* is the vendor or supplier of materials or products to a customer, *C*. The customer's processes are composed of planning his requirements, sourcing goods from the vendor, and releasing those goods to end-consumers. The vendor, who similarly plans her requirements, sources materials/parts for production, manufactures goods, and releases those goods to the customer. The operational processes of these two independent firms are linked as in Figure 1.2.

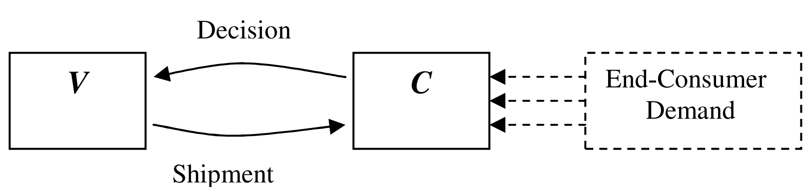

Figure 1.1: Independent decision making in a supply chain: *V* vendor, *C* customer

Although part of a supply chain, a firm may still focus on optimizing its own costs. In that case, the decisions concerning production, purchase and shipment are made separately and independently by members of that chain. When put into a sequence of events, the customer first develops his requirements plan and sourcing method based on his costs. The vendor then reacts to fulfill the customer's requirements.

As a result, replenishment decisions made by the customer do not necessarily consider its upstream business-partner's preferences. His choices of the quantity and timing of replenishments may create inflexibility in the vendor's operations, resulting in higher costs for her and the entire supply chain.

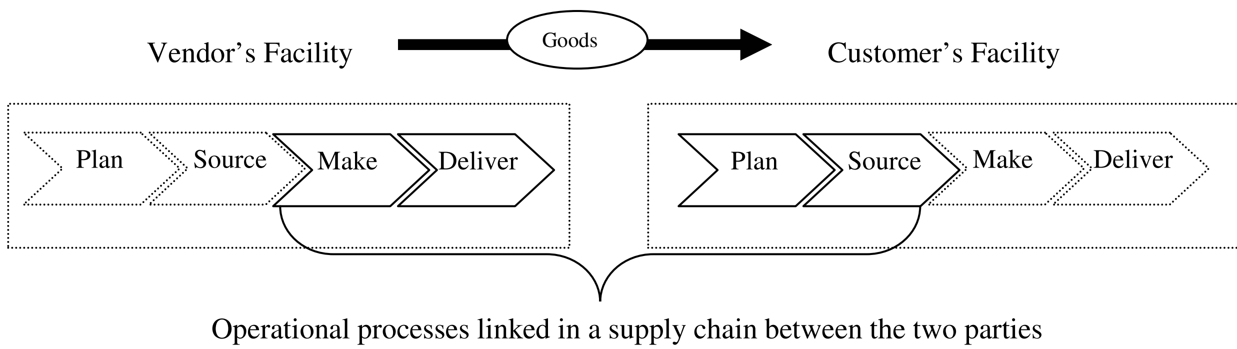

Figure 1.2: The supply chain between the vendor and the customer: The primary operations that are interrelated are the customer's *plan* and *source* choices, and the corresponding *make* and *deliver* decisions of the vendor.

It is therefore important to align the decisions in a supply chain, even when its members have different operational goals. In fact, performance of that chain depends not only on how well each member manages its operational processes, but also on how well the members coordinate their decisions (Achabal et al. 2000). Coordinated decision making (Figure 1.3) may reduce the need for inventories and lower the shipping costs, or enable improved utilization of resources at the manufacturer.

Various degrees of coordination are possible, depending on the business arrangement between the vendor and customer. Papers included in Tayur et al. (1999) discuss a number of such arrangements. One example, Vendor Managed Inventory (VMI), will be the main subject of this thesis.

Authors such as Karonis (1997) or Szymankiewicz (1997) have emphasized strategic partnerships between a manufacturer and retailer. The parties would work together, as a team, to maximize supply-chain efficiency. The common goal is to deliver better value to the customer. This would be achieved by coordinated decision making to enable smooth movement of product from manufacturer to customer, as in Continuous Replenishment Programs, or CRP (e.g. Kurt Salmon Associates, 1993).

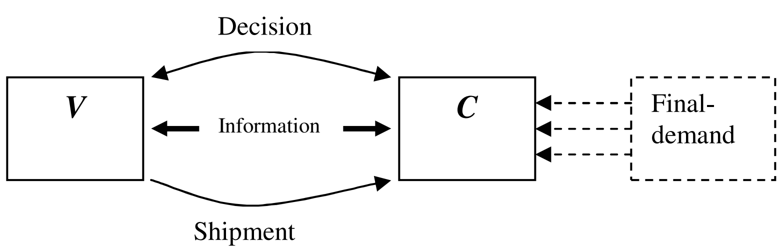

Figure 1.3: Coordinated decision making in a supply chain: *V* vendor, *C* customer

Coordinating the decisions by means of vertical and horizontal integration, where one supply chain member acquires the others or various members merge, is regarded as a traditional approach that often fails (Aviv and Federgruen 1998). The reasons stem from the organizational difficulties in integrating independent players and from different organizational cultures and incentives.

Vendor Managed Inventory (VMI) emerged in the late 1980's as a partnership to coordinate replenishment decisions in a supply chain while maintaining the independence of chain members. An important part of continuous replenishment program (CRP), VMI is also referred to as a program of supplier-managed inventory, or direct replenishment. In this partnership between a vendor and customer, it is the *vendor* that decides when and in what quantity the customer's stock is replenished. With such an agreement, the vendor may be able to share the customer's point-of-sale and inventory-level data.

From the vendor's perspective, VMI entails (e.g. Intentia, 2001):

- Receiving (through EDI, fax or the internet) information on customer stock levels, sales, and any sales forecasts that have been made
- Generating replenishment orders as needed
- Sending dispatch advice (electronically) to the customer, and then the invoice.

The VMI agreement may also specify a consignment inventory (CI), whereby the customer will not be invoiced right at shipment, but only after selling the goods to its end consumer. Whether part of VMI or not, CI thus leaves ownership of the products shipped with the vendor, until the customer sells those items. Hence, the consignment inventory is not shown as an asset on the customer's books, and the inventory turnover ratio will be higher.

Intentia (2001) has summarized the benefits possible under VMI. Not only does a customer obtain relief from placing a purchase order, but he/she may increase the inventory turns while being guaranteed a service level. The vendor can potentially enhance the gross margin by reducing the costs of manufacturing (a stabilized schedule via lessened demand-uncertainty) and transportation (economies of scale in shipment quantities).

Successful VMI arrangements include Wal-Mart/Procter & Gamble (Waller et al., 1999), Campbell Soup Company (Clark, 1994), Barilla SpA (Hammond, 1994), Intel (Kanellos, 1998) and Shell Chemical (Hibbard 1998).

However, there are many who question whether VMI is beneficial. For example, Burke (1996) claims that vendors are unwillingly forced into a VMI agreement by powerful customers. Saccomano (1997) argues that VMI is just a way to transfer the risks involved in inventory management from customers to vendors. Betts (1994) mentions that the vendor may be overwhelmed since, to make VMI work, more technological investment is required there than at the customer. According to Copacino (1993), a poorly designed VMI agreement can harm the supplier who ships more often to satisfy the inventory turns required at the retailer.

Disputes over the benefits of VMI stem from the fact that few quantitative analyses are available. General attributes of those agreements are fully understood in only some settings, making it difficult to assess and justify even conceptual models of VMI contracts.

In Section 1.2, we thus summarize information gathered through our industry contacts. An overall qualitative evaluation of VMI agreements is presented in Section1.3. In light of these two sections, we then define the problems undertaken and our research scope in Section 1.4. Thesis outline is described in Section 1.5.

1.2 VMI/CI Examples from Industry

Siemens Automation and Drives, Controls and Distribution (A&D CD) in Germany supplies products, systems and solutions, starting from switching devices for load feeders or for power distribution, via control circuit devices, through to complete cabinet systems. The Purchasing department of Siemens A&D CD considers a full range of sourcing methods including VMI and CI.

Their “Standard Parts Management” requires the supplier to manage the planning and control of inventory. They usually consider low cost items for this type of sourcing. The company emphasizes the importance of collaboration when the sourcing type is CI, which is perceived as a very effective way to reduce inventory costs. CI is used more for items with high purchasing volume.

Parmalat Canada, which offers milk and dairy products, fruit juices, table spreads and cookies, is one of the country's largest food companies. Parmalat manages the inventory of its products sold to customers who have agreed to a VMI relationship. Those customers take possession of the goods, as received on their premises.

The company has control over how much to ship to the customer and when. Through the VMI agreement, both parties set targets for service-levels as well as inventory turns. These measures are reviewed periodically by the customer to ensure effective management of stock. The firm has reduced costs through more effective truck utilization and stable production.

Arcelik-Eskisehir is Turkey's largest cooling-compressor plant, established in 1975. Besides meeting domestic demand, the company exports an important part of its products. Although there is not a formal VMI agreement, one or more representatives of the supplier of semi-finished products visit Arcelik periodically to view stock levels and report unexpected fluctuations in manufacturing. The aim is to synchronise the vendor's manufacturing with Arcelik's, who is responsible for periodic orders.

The company is not in favour of VMI because of the complexity of products and automated production (that the vendor may not handle) and unwillingness to share related information. The firm has a "well running" MRP and "already very low" ordering costs through improved technology. While Arcelik seeks increased inventory turns, it believes that VMI will not change system requirements but only ownership of inventory.

Eti-group is the second-largest food company in Turkey, producing and distributing biscuits, chocolate, and chewing-gum products to its retailers. The company has initiated VMI agreements with its vendor, who agreed to build its own depot at Eti-group's manufacturing site. In this case, the vendor will handle ordering, and the customer will not pay for goods until they are taken from the depot and used in a manufacturing line.

Tepe Home is a large home-improvement retailer in Turkey. It sells decoration products and also manufactures its own brand of home/office furniture. For the suppliers of known brands, the firm uses a VMI agreement where a representative of the vendor is responsible for ordering. In this case, the customer Tepe Home allocates space and sometimes owns the goods, sometimes not. For small-unknown brands, Tepe Home also uses CI, where now the customer is responsible for ordering but makes no payment to the vendor until goods are sold. The firm comments that both types of agreements are easy to implement for independent products, but CI is more favourable for the customer when that company is relatively powerful.

1.3 Characteristics of VMI and CI agreements

Our research will focus on quantitative analyses of VMI and CI agreements in supply chain settings. To understand the nature of those agreements, we provide in this section a further discussion based on our industry observations.

When there is a CI agreement, the customer sends purchase orders to the vendor for a specific time period. After the goods are delivered to a depot at his premises, he then takes the required amount at any time from the *vendor's* stock (i.e., the customer does not pay for goods until they are used). Therefore, CI is regarded as more beneficial for the customer: a) CI requires no information sharing; b) inventory management takes a minimum effort for the customer, who pays no opportunity cost of capital in inventory; c) the customer can order as much as storage space permits, yet pays just upon use. The only benefit foreseen for the vendor is continued business with the customer.

If there is a *VMI* agreement, however, the vendor is not controlled by the customer. The vendor can simultaneously manage the total inventory (its own and the customer's), and produce more effectively because of increased flexibility in timing and quantity. She thus can use economies of scale in her operations.

Under a VMI agreement, inventory and warehousing costs for the items supplied are generally charged to the customer. However, in some cases, a strong customer may force the vendor to assume those costs as well. The latter agreement could be termed "consignment and vendor managed inventory (C&VMI)", where the vendor *owns* as well as controls the inventory of the customer.

The general opinion in industry seems to be that VMI (but without CI) is more favourable for the vendor, who would consolidate orders and ship larger amounts without worrying about the average inventory level of customers. Table 1.1 summarizes qualitatively some characteristics found in industrial VMI and CI agreements.

Naturally, this thesis is concerned with *academic* issues in VMI. We will adopt the point of view throughout that consignment stock should be considered distinctly from VMI (although the contract may include both). Dong and Xu (2002), who study the economics of consignment inventory in the long-term and short-term, and others, however, have taken VMI to be synonymous with the consignment arrangement. There is thus not unanimous agreement about our interpretation. The following section will amplify the types of models we will analyze.

Aspect	VMI	CI
Inventory	Inventory is owned by the vendor **or** by the customer	Inventory is owned by the vendor
Warehouse	Warehouse is owned by the vendor **or** by the customer	Warehouse is owned by the customer
Ordering	Performed by the vendor	Performed by the customer
Power Relations	Vendor and customer have almost equal power	Customer is more powerful
Industry	More common in retail sector where goods are "end products"	Common both in retail and manufacturing sectors
Role in Supply Chain	Vendor may be a raw material supplier, a semi-finished goods manufacturer, end-products manufacturer or distributor	Same as VMI, but no distributor

Table 1.1: Attributes of VMI and CI agreements observed in industry

1.4 Problem Definition and Research Scope

In this thesis, we study VMI, CI, and C&VMI agreements in three independent essays, each written in a "paper" form. First two essays assume stationary demand, and consider various agreements in a supply chain composed of a single vendor and customer. The last essay is concerned with VMI and C&VMI agreements in a supply chain of a single vendor and multiple customers facing time-varying, but deterministic demand. We shall now describe the problems and research goals in those essays separately.

1.4.1 Calculating the Benefits of Vendor Managed Inventory in a Manufacturer-Retailer System

The models in this essay will concern one vendor (manufacturer or supplier) who produces a single product at a sole manufacturing plant and furnishes it to a particular customer (buyer or retailer). A conceptual framework of the problem is depicted in Fig. 1.4.

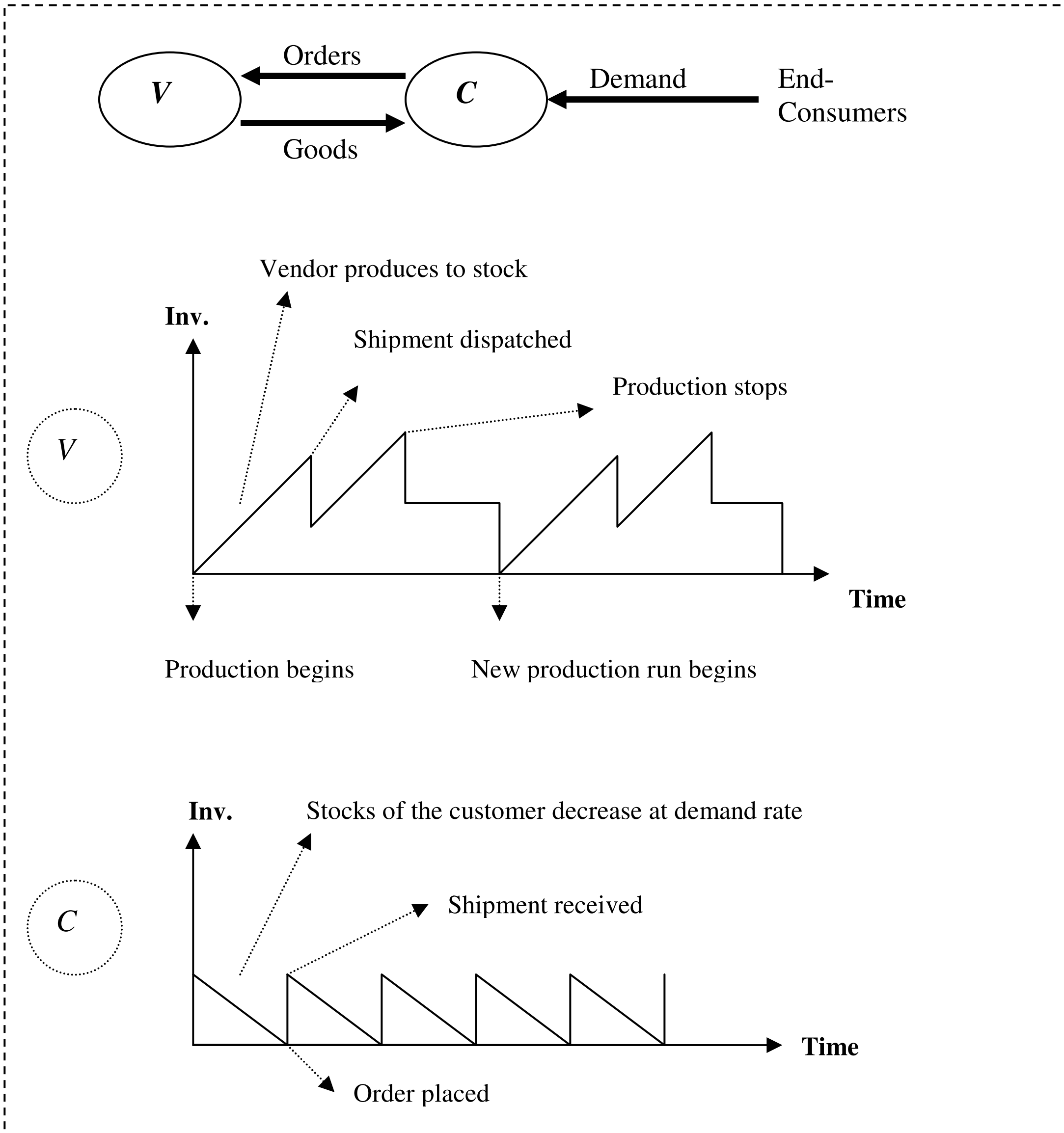

Figure 1.4: Vendor-customer inventory system

The customer faces a constant demand which is known. When the parties act independently, the customer decides its replenishment orders. Suppose there is no lead time and all customer orders are known instantaneously by the vendor. At any moment of time, the vendor's plant is either idle (actually, producing other SKUs not part of this analysis), or manufacturing the given item at a constant production rate which is larger than the customer's demand rate. The vendor thus produces in batches at a finite rate. When the customer is replenished, those units are shipped from the vendor's inventory to the customer's.

Because of constant prices, the vendor's total production cost and the customer's overall revenue are both linear, and will be omitted since all demands have to be satisfied. We assume that the vendor's fixed costs of setup and of shipment dispatch, and the customer's fixed cost per order are independent of the quantities involved. Both parties' inventory costs are directly proportional to the average stock levels.

The performance criterion we use in our models is the same as in EOQ models, namely the total cost of inventory holding plus ordering (including shipment dispatch). We begin with the simplest situation where the vendor and customer make independent decisions. This forms the base case. We then develop and analyze a VMI agreement and compare it to the preceding, so as to obtain insights into the potential benefits of vendor managed inventory. We also develop a model for central decision making where all parties belong to the same firm, and compare it to the base case and VMI.

Determination of the optimal order quantity and number of orders, where each party minimizes his own cost, is the well known EOQ approach for the customer and modified Economic Production Quantity (EPQ) for the manufacturer. Taking this as the base case and carefully identifying the cost parameters of each party, our aim is to develop and analyze quantitative cost models through which the economic value of VMI can be estimated. In light of those analyses, we will provide insights from the point of view of the vendor, the customer and the whole system. Inventory control policies of the following cases will be investigated in Essay 3:

1. *No agreement between the parties*. Vendor and customer act separately as in traditional systems. Hence, each party is responsible for its own inventory control. The customer decides the quantity and timing of replenishments. The vendor produces any quantity demanded in an optimal way.
2. *Vendor Managed Inventory*. The vendor and customer act based on a VMI agreement where each party is responsible for its own inventory holding costs, but the vendor establishes and manages the inventory control policy of the customer. The vendor therefore pays an ordering cost on behalf of the customer.
3. *Central Decision Making*. The analysis is similar to Joint Economic Lot Sizing. System-wide cost, which is merely the sum of all costs associated with each party, is minimized. As a result, the customer's optimal order quantity is found, and the vendor's optimal batch production quantity is an integer multiple of it.

1.4.2 Impact of Consignment Inventory and Vendor Managed Inventory for a Two-Echelon Supply Chain

The problem setting and description in this essay are the same as in Essay 1, but now we analyze CI and C&VMI agreements. Those agreements will be compared within themselves, and also to the traditional way of doing business (base case). In Essays 2 and 3, we shall refer to the base case as "Inventory Sourcing" (IS).

Under CI, since goods belong to the vendor until used by the customer, the vendor pays the inventory-holding expense of goods stored at the customer's site. However, it is still the customer who makes replenishment orders. Expenses for physical storage of stock are still borne by the customer; the vendor is responsible only for the opportunity-cost of capital, which may not be the same with that of customer.

C&VMI refers to VMI coupled with CI. Therefore, it is now the vendor who makes orders on behalf of the customer, but still owns the goods until at the customer's location until they are used. Our aim is to identify the conditions under which IS, CI, or C&VMI are preferred sourcing options for the vendor and customer. The three cases we look at are summarized below.

1. *Inventory Sourcing (IS)*. This is exactly the same with the base case in Essay 1. There is no agreement between parties.
2. *Consignment Inventory (CI)*. The vendor and customer act based on a consignment agreement, where the quantity and timing of customer's replenishment are decided by the customer itself. (The customer thus pays its own ordering cost.) Any inventory supplied by the vendor is owned by her until used. The customer pays the physical storage cost of those goods, whereas the vendor incurs opportunity cost of capital.
3. *VMI and CI together (C&VMI)*. The vendor and customer act based on a consignment agreement (the vendor still pays the inventory holding costs of the customer), where now the vendor decides the timing and quantity of customer replenishments.

1.4.3 Analysis of VMI for a Single Vendor and Multiple Customers under Deterministic, Time-Varying Demands

The previous papers evaluate various agreements between a vendor and customer to understand under what conditions an agreement can create benefits. Those benefits to the vendor, customer and the entire system depend on the cost parameters of the parties involved. However, VMI can be more advantageous to achieve economies of scale in production and transportation when multiple customers are involved. In Essay 3, we therefore study VMI agreements for a vendor and multiple customers in a supply chain where the vendor can make better use of her replenishment authority.

It is customary in industry to exercise *time periods* for the realization of operational decisions. In many real life situations, demands are satisfied at the beginning or end of certain time periods such as days, weeks or months. Accordingly, purchasing materials and finished goods, releasing and receiving shipments, scheduling production and storage are based on those time periods. We employ this idea in Essay 3, and use a time horizon composed of 12 periods. Demand in each period varies, but it is deterministic.

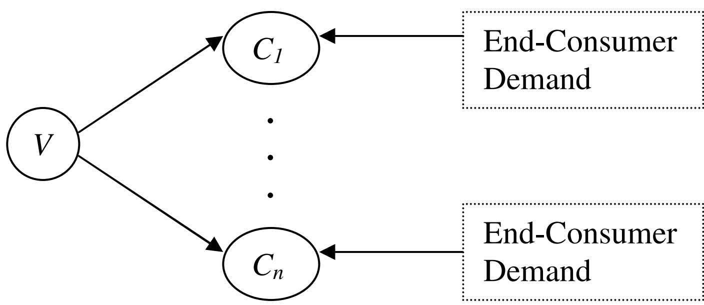

Figure 1.5: Single vendor, multi-customer supply chain (Distribution System)

The supply chain we consider in Essay 3 includes a single vendor who produces a unique item for her multiple customers (Figure 1.5). End-consumer demand, which is different for each customer, is realized only at the customers. Customers are independent; each must meet the demand of his end-consumers, and the vendor must meet the replenishment orders issued by customers. No backlogging is allowed.

In the traditional way of doing business (IS), each customer orders from the vendor based on his costs of inventory holding and ordering. That ordering cost includes the costs of replenishment-decision making and of shipment receipt. Orders are placed at the beginning of a period and received immediately (i.e., lead time is assumed zero). Inventory holding costs are incurred for stocks on hand at the end of any period.

Under IS, the vendor has to fulfill the orders specified by her customers. We assume that the vendor's production capacity exceeds total demand in any period. Customer orders are shipped at the beginning of periods, and inventory holding cost is charged at the end of each period for the items on hand. Production-lead time is assumed to be zero. Production and sale costs are linear with constants prices, and can be neglected since all demands must be met. In addition to the inventory holding cost, the vendor pays a production setup cost each time she initiates manufacturing; incurs a cost per shipment released to one or more customers; and transportation costs.

To evaluate the impacts of VMI on shipment consolidation, we assume that the transportation cost is paid by the vendor. (That is, a vendor in control of the timing and quantity of shipments can combine the small orders of different customers to achieve economies of scale in transportation, if the transportation cost is paid by her). We further assume that the vendor engages the transportation services of a common carrier, i.e. a public, for-hire trucking company.

That carrier, taking into account an *all-units discount* scheme, offers a piece-wise linear freight rate as a function of the total weight in a given shipment. At the end of a period, a vehicle dispatched from the vendor may carry an amount b of goods to be dropped to a total of i customers. The carrier then charges the vendor a fixed cost for every stop the truck makes. We do not consider shipment routing in this paper.

Under IS, each customer has his own ordering plan, and replenishes separately from the vendor. That plan depends on inventory holding charges and ordering costs. Once all customers determine their replenishment policies over the planning horizon, the vendor collects them to plan her production. The vendor is aware of the customers' ordering plans, but not the actual end-consumer demand.

When the customers make decisions regarding the timing and quantity of replenishments, it is difficult for the vendor (who must meet the customers' requirements) to seek economies of scale in her operations. The vendor may prefer alternative replenishment quantities and/or different ordering times than the customers specify. That is, their decisions may act as constraints against the vendor's flexibility which she requires to decrease her production, inventory and transportation costs.

A VMI agreement between a vendor and customer gives her that flexibility, but at a cost. Under VMI, the vendor makes replenishment decisions on behalf of the customer, and pays the cost of replenishment decision-making. The agreement also provides the vendor with data on the end-consumer demand.

When there are $n \geq 2$ customers, the vendor may choose to implement VMI with fewer than n. By a "VMI-customer", we shall mean one who implements VMI with the vendor. VMI-customers are relieved of paying expenses associated with making replenishments. Although this may not guarantee them lower costs compared to IS, no customer would implement VMI unless he is no worse off. Under VMI, the vendor may wish to send large quantities to achieve economies of scale in transportation. Then, should the customer accept any quantity determined by the vendor, or should he set some limits to it? A customer that is offered VMI must answer this question, before accepting the agreement, so his costs do not go up.

The vendor, on the other hand, is concerned first with the *right* set of customers to offer VMI. Before implementing the agreement, the vendor must foresee the savings it can create in production, replenishment and transportation. The vendor, too, would not wish to undertake any VMI agreements that create no cost savings.

When Consignment Inventory is coupled with Vendor Managed Inventory (C&VMI), the vendor now owns the stock at a C&VMI-customer's premises until the items are used. Hence, the vendor pays the opportunity cost of those goods. In this case, even when there is no restriction specified by the customer, it may be in the vendor's best interest not to send large quantities. As in VMI, both the vendor and customer would want to get the best from this agreement. Therefore, either party will cast aside implementing C&VMI if the agreement increases their total costs compared to IS.

Similar to the first two essays, IS will be the base case to which we compare VMI and C&VMI. Although the outcomes of these two agreements will be contrasted, choosing to implement any agreement is based on comparison of total costs under that agreement vs IS. Values of decision variables and total costs of the vendor and customers in each case will be determined using Mixed Integer Programming. Each of the following sourcing option will be modeled and solved in separate sections.

1. *Inventory Sourcing (IS):* Each customer decides on replenishments first based on his minimum total costs. A customer's cost stems from replenishment decision-making, shipment-received, physical storage and opportunity cost of inventory. The vendor receives the order quantities from each customer, and optimizes her operations based on her costs of production setup, inventory holding and transportation. Under IS, the customers' optimal replenishment decisions are input to the vendor's model.

2. *Vendor Managed Inventory (VMI):* The vendor now makes replenishment orders on behalf of customers. Hence, there is only one model that determines the production, customer-replenishment, and transportation quantities and timing. The vendor pays a cost for each decision made to replenish, which is assumed to be the same as what the customer pays in IS. The vendor's problem is solved by a heuristic which 1. separates VMI decisions (concern the right set of customers with whom to implement VMI) from the model, and 2. solves hierarchically the remaining problem of integrated production, replenishment and transportation by three different decomposition methods. We also determine whether a customer should set a maximum-inventory level that the vendor can keep at his premises.
3. *Consignment and Vendor Managed Inventory (C&VMI):* The vendor determines each customer's replenishment quantity as in VMI, but now pays, in addition to the cost of replenishment decision making, the opportunity cost of goods stored at the customers' locations. Hence, there is still one model to find optimal production, replenishment and transportation quantities, and timing. Of course, the vendor should also identify the best set of C&VMI-customers. The vendor's model is solved using Lagrangian relaxation. The Lagrange multipliers are determined using the cutting-plane approach of Kelly (1960).

In all the models of Essay 3, we assume that at time zero and at the end of the planning horizon, there is no inventory anywhere in the supply chain. Moreover, end-consumer demand of a customer is revealed to the vendor when VMI becomes an option for those two parties. Our aim is to evaluate the cost impacts of the agreement for the vendor before implementing it. We will also show the maximum level of inventory that should be allowed by the customer under VMI.

The three essays described in this section form the main chapters of our thesis. Each of them is written in a paper form and incorporated into to thesis body as it is. Therefore, format of this thesis differs from the usual. We explain that format more in detail in the next section.

1.5 Thesis Outline

The previous sections of Chapter 1 provided an overview of the topics and models we will cover. Since we included the major chapters of this thesis in the essay format, we shall now provide a brief outline of them, as well the rest of the chapters involved.

Each of chapters three to five are written as independent essays. Therefore, each has its own abstract, introduction, literature review, analysis, conclusions and reference list. As such, equations and analysis employed in one essay do not necessarily build up on a previous one. Corresponding numbers of those equations, as well as the proposition and lemma numbers, start from one in every essay. Table and figure numbers, on the other hand, follow a chapter-specific sequence, which is also reflected in the Table of Contents.

Appendices that provide various proofs are also included within essays. Appendix A at the end of Essay 1 (Chapter 3) details the proofs of four propositions stated in that essay. Similarly, Appendix B at the end of Chapter 4 explains the proof of a proposition specific to Essay 2. There is no appendix to Essay 3, which is Chapter 5.

Chapter 6 is the last chapter of the thesis, and summarizes our conclusions derived from the analysis provided in three essays. Possible future-research streams are also included there.

Chapter 2, which will be discussed next, provides a *general* survey of the literature related to VMI agreements we consider. The reference list at the back of this thesis document corresponds to that general survey. Naturally, the literature review provided in each essay may include some of the papers introduced in Chapter 2. However, each of those essays will also study additional papers specific to the topic of that essay and to the modeling approach considered in it (e.g., Dynamic Lot Sizing in Essay 3). Comparison of our work with the previous studies is also described more in detail in those essays.

2. Literature Review

The literature survey provided in this chapter relates to VMI agreements in general, and serves as a seed for our essays. We do not necessarily detail our contributions here.

We have identified three categories of literature that have ties to VMI research. The first concerns the joint economic lot sizing problem. Though not apparently related to VMI, it forms the starting point of our analysis, and can be considered a form of coordinated decision making. The second category depicts a VMI agreement as a mechanism to coordinate the supply chain, while the third identifies literature that is more focused on operational benefits of VMI contracts.

2.1 Joint Economic Lot-Sizing (JELS) Models

Also called integrated vendor-buyer models, research in this category minimizes the overall cost of a two echelon inventory system composed of a single supplier and one or multiple customers. Based on deterministic EOQ models, the cost function of the parties at each echelon is the sum of inventory holding and ordering costs. Instead of separately optimizing each actor's cost, studies in this area minimize a *total*-cost function, adding up the cost of each party.

Banarjee (1986) was the first to analyze an integrated vendor-buyer model where the vendor produced items at a finite rate. He examined a lot-for-lot model in which the vendor manufactures each shipment as a separate batch. As an extension, Goyal (1988) formulated a joint total-relevant-cost model for a single vendor and buyer production-inventory system, where the vendor's lot size is an integer multiple of the buyer's order size. He still assumed that the vendor can ship to the purchaser only after the entire lot is completed. A batch that constitutes an integral number of equal shipments furnished a lower-cost solution. A summary of research to that date on integrated vendor-buyer models can be found in Goyal and Gupta (1989).

Lu (1995) extends Goyal's (1988) work by allowing the vendor to supply some quantity to the purchaser before completing the entire lot. Lu gives an optimal solution for the case of a single vendor and buyer, again based on the assumption that a batch provides an integral number of equal-sized shipments. She also investigates heuristics for the single-vendor, multiple-buyer problem.

Goyal (1995) employed the example provided by Lu for the single vendor and buyer, but showed that a different shipment policy, other than equal-size, could give a better solution. The policy assumed that each successive shipment within a production batch increases by a factor (ratio of production rate to demand rate). This was also based on Goyal (1977) who solved a very similar problem in a slightly different setting.

Hill (1997, 1998) considered a single vendor who manufactures a product at a finite rate and in batches, and supplies a sole buyer whose external demand is level and fixed. Each batch is sent to the buyer in a number of shipments. The vendor incurs a batch setup cost and a fixed order or delivery cost associated with each shipment. The four parameters in his model are thus inventory holding for the vendor and for the buyer, a fixed production set up cost, and fixed cost per delivery. The study's goal is to show, in general, that neither Lu's (1995) nor Goyal's (1995) shipment policies are optimal. Hill's policy assumes that successive shipment sizes increase by a factor whose value lies between one and the ratio of manufacturing rate to the product's demand rate. Considering the system as an integrated whole, the objective is minimization of the total costs of all parties. Hill (1997) concludes that Goyal's (1995) policy may perform much better than Lu's equal-size-shipment policy, but his policy outperforms all. Later, Goyal (2000) proposed a procedure to modify the shipment size in Hill (1997) to obtain a still-lower cost.

Viswanathan (1998) considers a model to minimize the sum of overall costs of production setup, customer ordering, and vendor's and buyer's inventory carrying. He shows that the performance of Lu's (1995) and Goyal's (1995) policies depend on the problem parameters.

Similar joint economic-lot-sizing problems, with small variations in structure and parameters used, are also investigated in two working papers. For a single vendor and customer, Ongsakul (1998) studies a joint lot-sizing model that also includes pipeline-inventory cost. Kosadat (2000), in a similar vein, considers the impacts of backordering.

In general, the more-recent research on JELS showed numerically that a lower total cost could be achieved compared to earlier work. The joint cost function used in each study is simply the sum of the vendor's production-setup and shipment-dispatch or ordering costs, and the inventory holding cost of each actor. However, real case examples show that vendors and customers are rarely willing to divulge their cost-related information in full. Except in the case of merger or acquisition, the vendor would generally be unable to learn the customer's inventory holding and ordering costs. Hence, it is very unusual that one party alone could find system-wide optimality.

Moreover, the focus of JELS studies is more on numerical solution and overall cost comparison of integrated and separate systems. Little has been done on questions concerning the best type of *relationship* between two parties. Note also that any type of agreement would require the shift of some cost parameters from one actor to another.

None of the papers reviewed in this section notes that there are actually three types of setup costs, one for customer's ordering and two for the vendor's production-setup and shipment dispatch. All three are included explicitly in our models, whether it is joint decision-making or not (see Essay 1).

2.2 Supply Chain Coordination through VMI

Traditionally, the independent companies in a supply chain have not chosen policies that optimize overall supply-chain performance. Each firm has instead attempted to optimize its own objective. Coordination within a supply chain then mainly refers to finding the optimal actions for chain members who need to align their decisions to achieve optimal chain costs. The incentive to apply those optimal actions can be set by transfer payments. More on coordination can be found in Thomas and Griffin (1996), Corbett and Tang (1999), Boyaci and Callego (2000), Aviv (2001), Agrawal and Seshadri (2001), and Chen et al. (2001). We shall then *summarize* here a variety of research related to supply chain coordination.

Coordination is facilitated when entities in the supply chain will abide by a set of incentives that specify their activities; system-wide optimality may then be achieved (Cachon, 1998). There are several examples of policies used to avoid deviations from system-wide optimal conditions. Buy-back contracts specify a price at which the supplier will purchase unsold items from the retailer (Tsay and Lovejoy, 1999). Quantity discount is the price discount from the supplier whenever the retailer orders a greater amount of product (Weng 1995; Klastorin et al., 2002). Revenue sharing first maximizes the total profit of the system and then finds the best allocation (Gerchak and Wang, 2002). A review of incentive methods for coordination is provided by Cho (2002).

One example of coordination through price discounts is presented by Viswanathan and Piplani (2001). In a single vendor and multi-buyer setting where the demand is deterministic, they consider the case where the vendor specifies common replenishment periods. Each buyer must replenish at (only) those times. The authors use a joint lot-sizing model, employing also a Stackelberg game: The vendor makes a first decision, and the customer then acts on this to make its own decision.

In other research that will be mentioned shortly, VMI agreements are conceived to be means of obtaining supply-chain coordination. Those authors usually consider a single vendor and one or multiple customers who experience stochastic demand. The objective function may be cost minimization or profit maximization, based on parameters of inventory holding, setup, shortage-penalty costs and selling price.

Note that although the academic interest in VMI agreements has developed only recently, the roots of such research go back to multi-echelon inventory papers starting with Clark and Scarf (1960). Assuming the entire system (consisting of a single vendor and customer) is controlled by a single person, they developed an optimal policy for a finite planning horizon. Federgruen and Zipkin (1984) then extended and simplified that policy for the case of infinite horizon. More information on development of this type of multi-echelon inventory control can be found in Lee and Whang (1999) and in Dong and Lee (2001).

Cachon (2001) studies VMI in a single supplier and multi-retailer setting. Several different strategies are analyzed with the aim of coordinating the channel. In each case, he employs game theory to find the equilibrium for each party of the supply chain. With a VMI agreement, the supplier can set all reorder points. Cachon remarks that VMI alone does not guarantee an optimal supply-chain solution; both the vendor and retailers must also agree to make fixed transfer payments to participate in the VMI contract, and then be willing to share the benefits. He employs a numerical study to show that no improvement under VMI can be achieved if fixed payments are not allowed.

Aviv and Federgruen (1998), with the aim of investigating impacts of information sharing, consider a single vendor plus multiple retailers. They assume a VMI agreement that leads to a fully centralized planning model where the vendor minimizes the system-wide total cost of inventory holding and distribution. Using a combination of Markov decision process and non-linear programming, they construct approximate policies for the vendor and the retailers under both information-sharing alone and information sharing in conjunction with VMI. They find that VMI (with that sharing) is always more beneficial than information sharing alone.

Bernstein and Federgruen (2003) study a constant-demand-rate VMI setting characterized as a partially centralized model (the retailer retains decision rights on pricing and sales target). The supplier determines a replenishment strategy for the entire supply chain. They show that channel coordination can be achieved under VMI. In their model, the vendor incurs all inventory holding costs including those at the retailer. Hence, the agreement they consider should be regarded as VMI and CI together, rather than a pure VMI.

Narayanan and Raman (1997) analyze VMI agreements between a single vendor and retailer. They compare traditional retailer-managed-inventory to VMI, identifying situations where stocking-decision rights should be transferred from retailer to vendor.

Narayanan and Raman investigate how an inability to observe and include certain variables (such as sales effort) in an agreement can affect supply-chain performance.

Corbett (2001) studies the impact of (cost) information asymmetries between supplier and customer, where there are also incentive conflicts between them. In a principal-agent framework, he shows how consignment inventory can help reduce cycle stock, but may simultaneously increase the safety stock. (We note in passing that the usual discussions of "information sharing" do not extend to knowledge of the cost parameters.)

Aside from papers reviewed previously in this section, Gavirneni et al. (1996), Cachon and Fisher (2000), and Lee et al. (2000) look at how a supplier can use customer-demand information for better sales forecasting and inventory control. These models show significant direct and indirect benefits to the supplier. (Indirect benefit refers to the possibility that the supplier will pass some of its own benefits to the retailers.) However, retailers receive no direct benefit.

2.3 VMI for Operational Benefits

Research in this category focuses on benefits offered by flexibility in delivery and other operational decisions under VMI agreements. That flexibility may enable a supplier to combine routes from multiple origins and delay stock assignments, consolidate shipments to two or more customers, or postpone a decision on the quantity destined for each of them.

Campbell et al. (1998) and Kleywegt et al. (2000) analyze a stochastic inventory-routing problem by a Markov decision process. Both investigate the benefits of allowing the supplier to construct better delivery routes for multiple retailers.

Cheung and Lee (2002) consider a single supplier serving multiple retailers who face random demand. The supplier (replenished by an outside source with ample stock) follows a continuous review (Q,r) policy; lead time is constant and unfilled demand is backordered. The authors analyze two information-based supply-chain efforts:

1. Knowledge of retailers' inventory position to coordinate and achieve truck load shipments
2. Use of that same information to balance retailers' stocking positions.

The research of Cheung and Lee focuses on benefits in terms of shipment coordination and stock rebalancing. This is done through upper and lower bounds, and by simulating the costs of the joint replenishment model.

In a similar study, Cetinkaya and Lee (2000) synchronize inventory and transportation decisions. For the case of Poisson demand observed at retailers, an analytical model based on renewal theory enables determination of the optimal replenishment quantity and dispatch frequency. Their contribution is based on an idealized application of VMI, whereby the vendor has the autonomy of holding orders until a suitable dispatch time at which orders can be economically consolidated.

Aviv and Federgruen (1998) quantify the benefits of inventory sharing and VMI programs in a periodic review setting. VMI allows the supplier to determine the optimal timing and quantity of replenishments. As opposed to Cheung and Lee (2002), their formulation does not include shipment constraints.

Chaouch (2001) analyzes a single powerful retailer and a supplier who wants quicker replenishment at lower costs (see also Fisher 1997 for the shift in power towards the retailer). His study can be regarded as a transportation-inventory problem whose tradeoffs are investment in inventory, delivery rates and shortages. The supplier's performance measures are the frequency of shipment dispatch and the frequency of retailer stockouts. Time between deliveries is a stochastic variable; the retailer's demand is fairly stable but with Poissondistributed jumps. Under a VMI agreement, the supplier is fully responsible for shortage and delivery costs and can choose the delivery interval. The supplier is allowed to stock at most M units at the retailer, who must bear the inventory carrying cost. The author's cost minimization models find the shipment rate that balances delivery and shortage costs.

Fry et al. (2000) also examine VMI as a means of offering a single supplier and retailer some operational flexibility. The supplier follows a fixed production schedule, but can ship to the retailer in each or any period. They assume that VMI is initiated by a contract which transfers decision rights to the supplier, but that supplier must maintain certain stock levels at the retailer. Performance of traditional retailer-managed-inventory with information sharing and VMI are compared. Through a periodic review inventory model, they show that VMI is beneficial in most scenarios but not all, and that its effectiveness depends strongly on the initiating contract.

Waller et al. (1999) follow a simulation-study approach to analyze the impacts of VMI under various levels of demand variability, limited manufacturing capacity, and partial channel coordination. They demonstrate that inventory-reduction achieved in VMI is due to more frequent reviews of stock and shorter intervals between deliveries. Associated costs are not discussed.

Chapter 2 has thus summarized the several important streams of literature with ties to VMI research. We will now provide the three essays in Chapters 3-5.

3. Calculating the Benefits of Vendor Managed Inventory in a Manufacturer-Retailer System (Essay 1)

Abstract

Firms such as Wal-Mart and Campbell's Soup have successfully implemented Vendor Managed Inventory (VMI). Articles in the trade press and in academic literature often begin with the premise that VMI is "beneficial." But beneficial to which party? Under what conditions?

We consider in this paper a vendor *V* that manufactures a particular product at a unique location. That item is sold to a single retailer, the customer *C*. Three cases are treated in detail: Independent decision making (no agreement between the parties); VMI, whereby the supplier *V* initiates orders on behalf of *C*; and Central decision making (both Vendor and Customer are controlled by the same corporate entity).

Values of some cost parameters may vary between the three cases, and each case may cause a different actor to be responsible for particular expenses. Under a constant demand rate, optimal solutions are obtained analytically for the customer's order quantity, the vendor's production quantity, hence the parties' individual and total costs in the three cases. Inequalities are obtained to delineate those situations in which VMI is beneficial.

3.1 Introduction

A supply *chain* implies interactions of different firms that seek decreased costs and greater market share. However, when the companies are managed independently, decisions made by individual firms downstream in the chain can impose constraints on those upstream, resulting in additional costs. Consider the simplest example of a supply chain where there is a manufacturer (called the vendor, *V*) who supplies materials or products, and a customer *C* that orders from *V* (Figure 3.1). When each party makes decisions independently, the customer determines a replenishment based on minimizing his own operational costs. However, since the customer's decisions on timing and quantity neglect the vendor's costs, the resulting quantities might not be preferred by the vendor.

On the other hand, coordinated decision making (Figure 3.2) fosters potential benefits for the individual organizations. It may reduce the need for inventories and lower the shipping costs, or enable improved utilization of resources at the manufacturer.

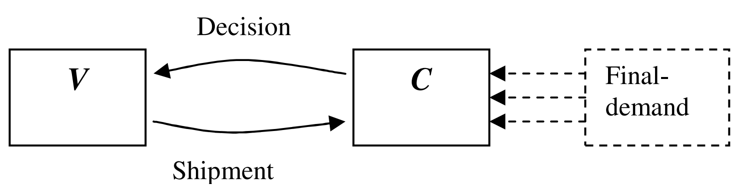

Figure 3.1: Independent decision making

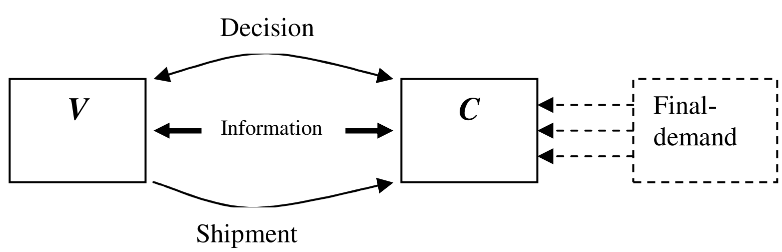

Figure 3.2: Coordinated decision making

Two forms of coordination identified in the literature are vertical and virtual integration. In the former, one supply chain member acquires the others or various members merge. However, that ends the independence of the firms, and can fail (Aviv and Federgruen 1998) because of behavioral difficulties in integrating the distinct organizational cultures.

The second form of co-ordination, virtual integration, maintains the independence of those firms, yet harmonizes their decisions by means of a business arrangement between them. Chapters included in Tayur et al. (1999) discuss a number of such approaches. Vendor Managed Inventory (VMI), the subject of the present paper, is one example.

VMI, also referred to as a program of supplier-managed inventory or direct replenishment, emerged in the late 1980's as a partnership to coordinate replenishment decisions in a supply chain while maintaining the independence of chain members. In this relationship between a vendor and customer, it is the *vendor* that decides when and in what quantity the customer's stock is replenished. VMI was successfully implemented by numerous firms including Wal-Mart and Procter & Gamble (Waller et al. 1999), Campbell Soup Company (Clark 1994), Barilla SpA (Hammond 1994), Intel (Kanellos 1998) and Shell Chemical (Hibbard 1998).

Despite the range of such examples of VMI relationships, there are researchers who question whether VMI is beneficial to all parties. For example, Burke (1996) claims that vendors are unwillingly forced into a VMI agreement by powerful customers. Saccomano (1997) argues that VMI is just a way to transfer the risks involved in inventory management from customers to vendors. Betts (1994) mentions that the vendor may be overwhelmed since, to make VMI work, more technological investment is required there than at the customer. According to Copacino (1993), a poorly designed VMI agreement can harm the supplier who ships more often to satisfy the inventory turns required at the retailer.

Disputes over the benefits of VMI arise because few quantitative analyses are available, and in those, general attributes of the agreements are fully understood in only some instances. That makes it difficult to assess and justify even conceptual models of VMI contracts.

Our aim in this paper is thus to develop and compare replenishment models by considering carefully the costs incurred by the vendor and the customer in various settings. We start with the traditional uncoordinated scenario where the customer makes the ordering decisions and the vendor reacts (Case 1).

Without a VMI agreement, the customer is responsible for inventory holding cost, transportation expense, and ordering charges: the cost of issuing the order and the cost of receiving those goods. "Issuing the order" relates to writing up the purchase request and determining the size of order, and thus, it is the cost of having the authority over replenishment planning. The vendor's expenses are those of production setup, inventory holding and shipment release.

We next assume that the vendor is not content in simply reacting, and wants to get involved in replenishment decision-making. With VMI (Case 2), the vendor takes over the ordering decision and hence the issuing-cost related to it, which might not be the same as what the customer used to pay. We analyze under what circumstances VMI is beneficial for one of the parties, or for both of them.

Ignoring any organizational difficulty or investment implication, we finally consider central decision making (Case 3). We will also term this, "vertical integration," where both parties are assumed to belong to the same company. Cost differences between vertical integration and VMI, and between vertical integration and independent decision making, are then explored.

There are publications that investigate how a certain type of VMI agreement impacts supply chain coordination. Examples include Aviv and Federgruen (1998), who analyze VMI in terms of information sharing, and find that VMI with information sharing is always more beneficial than information sharing alone. Cachon (2001) suggests fixed transfer payments in addition to VMI in a single supplier and multi-retailer setting. Bernstein and Federgruen (2003) study a partially centralized VMI model and conclude that channel coordination can be achieved under VMI.

Our research, on the other hand, analyzes tradeoffs between independent versus coordinated decision making. In a broader context, we try to understand what VMI is, and under what circumstances it works or fails.

VMI has also been conceived as a means of enabling operational benefits. Through the "flexibility" that VMI offers, the supplier may combine routes from multiple origins (Campbell et al. 1998, Kleywegt et al. 2002) and delay stock assignments, consolidate shipments to two or more customers (Cheung and Lee 2002), or postpone a decision on the quantity destined for each of them (Cetinkaya and Lee 2000). VMI may also come up in a transportation-inventory problem whose tradeoffs are investment in inventory, delivery rates and shortages (Chaouch 2001), or in a simulation-study that analyzes the impacts of demand variability, limited manufacturing capacity, and partial channel coordination (Waller et al. 1999).

The preceding stream of literature, "VMI for operational benefits," investigates the gains when decisions are supported by a presumed contract. However, we are aware of no publication on VMI which considers the cost implications of changing the decision-making authority from one party to another.

The Joint Economic Lot-Sizing (JELS) problem, although not apparently related to VMI, forms the starting point of our analysis, and can be considered a form of coordinated decision making. Also called "integrated vendor-buyer models," research in this category minimizes the overall cost of a two-echelon inventory system composed of a single supplier and one or multiple customers. The cost function of the parties at each echelon is the sum of inventory holding and ordering costs. Instead of separately optimizing each party's cost, studies in this area minimize a *total*-cost function, adding up the cost to each of them.

Banarjee (1986) was first to analyze the integrated vendor-buyer case, examining a lot-for-lot model in which *V* manufactures each shipment as a separate batch. As an extension, Goyal (1988) formulated a joint total-relevant-cost model for a single vendor and customer production-inventory system, where *V*'s lot size is an integer multiple of *C*'s order size.

Lu (1995) extended Goyal's (1988) work by allowing the vendor to supply some quantity to the purchaser before completing the entire lot. Lu gives an optimal solution for the case of a single vendor and buyer, and investigates heuristics for the single-vendor, multiple-buyer problem. Goyal (1995) employed the example provided by Lu for the single vendor and buyer, but showed that a different shipment policy could give a better solution.

Hill (1997, 1998) considers a single vendor who manufactures a product at a finite rate and in batches, and supplies a sole buyer whose external demand is level and fixed. Each batch is sent to the buyer in a number of shipments. The vendor incurs a batch setup cost and a fixed order or delivery cost associated with each shipment. Hill's policy assumes that successive shipment sizes increase by a factor whose value lies between one and the ratio of manufacturing rate to the product's demand rate. Hill (1997) concludes that, although Goyal's (1995) policy may perform much better than Lu's equal-size-shipment policy, his own policy outperforms all. Goyal (2000) proposed a procedure to modify the shipment size in Hill (1997) to obtain a still-lower cost.

Studies on general coordination are not always conclusive. Suitable incentives for coordination may not have been discussed, and numerical examples in those papers show that cost reduction might not be that significant. Total cost of the coordinated system might have been underestimated, e.g. by ignoring the customer's expense for ordering. That is often seen (e.g. Hill 1997), resulting in unrealistically lower costs. We also remark that changes to any system require adjustments in the relevant parameters.

Moreover, the sharing of cost-related information by two independent parties hardly occurs unless *C* and *V* belong to the same firm, making general-coordination difficult to achieve. A VMI contract, on the other hand, enables coordination based on cost *reallocation*, and leaving each party still independent.

Having thus summarized the several important streams of literature with ties to our research, the following sections will amplify the types of models we analyze. As opposed to Dong and Xu (2002), who study the economics of consignment inventory in the long-term and short-term, we will adopt the point of view throughout that consignment inventory (CI) should be treated distinctly from VMI. Under CI, the vendor still owns the products shipped, until the customer sells those items. Consignment inventory will be considered no further in what follows.

3.2 Problem Definition and Research Scope

Our models will concern one vendor *V* who produces a single product at one manufacturing plant, and furnishes it to a particular customer (retailer). The customer *C* faces a constant, deterministic demand which is known. Suppose there is no lead time and all customer orders are transmitted instantaneously to the vendor. At any moment of time, the vendor's plant is either idle (actually, producing other SKUs not part of this analysis), or manufacturing the given item at a constant production rate which is larger than the customer's demand rate. The vendor thus produces in batches at a finite rate. When the customer is replenished, those units are shipped from the vendor's inventory to the customer's.

Because of constant prices, the vendor's total production cost and the customer's overall revenue are both linear, and will be omitted since all demands have to be satisfied. We assume that the vendor's fixed costs of setup and of shipment dispatch, and the customer's fixed cost per order, are independent of the quantities involved. Both parties' inventory costs are directly proportional to the average stock levels. The performance criterion we use in our models is the total cost of inventory holding plus ordering.

Independent decision making (Case 1) is the traditional way of doing business between the vendor and the customer. Taking this as the base case and carefully identifying the cost parameters of each party, our aim is to develop and analyze quantitative cost models through which the economic value of VMI agreements can be estimated. In light of those calculations, we will provide insights on desirable agreements.

Inventory control policies for the following cases will be investigated in this paper:

1. *No agreement between the parties*. Vendor and customer act separately. Hence, each independent party is responsible for its own inventory control. The customer determines a replenishment quantity and passes it to the vendor. The vendor then optimizes her production quantity in satisfying the customer's order. But the actors are otherwise engaged in independent decision making.

2. *VMI*. The vendor and customer are governed by a VMI agreement: Each party is responsible for its own inventory holding costs, but the vendor establishes and manages the inventory control policy of the customer. VMI thus requires shifting some costs from the customer to the vendor. We will compare to the case with no agreement, to see if VMI is efficient (both parties realize costs savings), potentially efficient (system-wide cost savings are achieved although one party is worse off), or inefficient (no system-wide cost savings).
3. *Central decision making*. The vendor and the customer belong to the same corporate entity who manages the inventory of both parties. The model considered is similar to JELS models, and our aim is to identify any potential benefits in this vertical integration compared to no-agreement (independent decision making) and VMI.

These cases will be analyzed in Sections 4-6, and then numerical examples and further interpretation will follow in Sections 7 and 8. In the final section, we provide a summary and conclusions.

3.3 Notation

Let us begin with the basic notation that will be employed throughout our models.

A_c: Customer's fixed cost of ordering ($ per order). $A_c = a_o + a_t + a_r$, where

a_o : cost of issuing the order

a_t : transportation cost

a_r : cost of receiving the goods ordered

h_c : Annual cost to carry one unit in stock at customer's retail store ($/unit/year). This has two parts in it, *viz* h: cost of capital per item; h_s: physical storage cost of an item

The customer's inventory holding cost is thus $h_c = h + h_s$

S : Vendor's fixed production setup cost incurred at the start of each cycle ($ per setup)

a_v : Vendor's cost per shipment release ($ per shipment to the customer)

h_v : Annual cost of holding a unit in inventory at the vendor's production site ($/unit/year)

p : Vendor's annual production rate (units/year)

d : Annual demand rate at the customer (units/year).

k_i : Number of shipments to customer between successive production runs

(i.e. during the vendor's cycle time) in Case i, i = 1, 2, 3

For feasibility, it is assumed throughout that $p \geq d$. But, as opposed to JELS models in general, we do not require $h_c \geq h_v$. Note that any type of agreement between the parties may require a shift in expenses from one actor to the other. But unless explicitly stated, it should not be assumed that a cost parameter of the vendor includes another one of the customer.

The cost of receiving the goods shipped is incurred by the customer, independent of which party initiates the replenishment order. That expense includes the costs related to the arrival of product at the store, receipt of the vendor's invoice and further processing (by the customer) of that invoice. Likewise, the vendor pays the costs related to receipt of the order information and the processing of it, and is charged for release of goods to the customer.

Let us begin in the next section by looking at the traditional way of doing business between the vendor and the customer. We call it "independent decision making," with no agreement between the parties. That case is the building block for VMI analyses.

3.4 Independent Decision Making (Case 1)

This first case thus assumes that C and V, separately, each plan their own replenishments or production, respectively. End-user demand d is realized at the customer, who must decide, based on that demand, how often and in what quantity he should order from the vendor. While doing so, the customer considers the ordering cost A_c and inventory holding cost h_c.

In light of the preceding costs, the customer in Case 1 orders from the vendor a quantity q_1 = EOQ = $\sqrt{\frac{2A_c d}{h_c}}$. It follows that the customer's total cost in Case 1 is then TC_{c1} = $\sqrt{2A_c dh_c}$. Now, the vendor is informed by the customer of the ordering quantity q_1. The vendor has production rate $p \geq d$, and should satisfy the customer's order fully since no backorders are allowed. In choosing her batch size Q_1, the vendor considers the production setup cost (S), inventory holding cost (h_v), and the cost per shipment release (a_v).

We assume without loss of generality that the vendor begins producing when the customer's inventory level is q_1. (This facilitates comparison of the several cases we consider.) During each cycle of length $T^{'}$ in Case 1, the vendor produces initially at a rate p, and total system inventory increases at rate $p - d$ during the uptime T. After production stops, the vendor supplies goods to the customer from her stock (until there are none left); system-wide inventory decreases at a rate d until the end of the vendor's cycle (see Figure 3.3). All items that are carried over, i.e. that stay in the vendor's cycle (during the uptime plus the downtime) are charged h_v for holding inventory. Note that $p = \frac{k_1 q_1}{T}$ where the production time $T < T^{'}$. Total average-inventory in the system is thus $q_1 + (p-d)\frac{Q_1}{2p}$, and the vendor's mean stock level is $\frac{q_1}{2} + (p-d)\frac{Q_1}{2p}$.

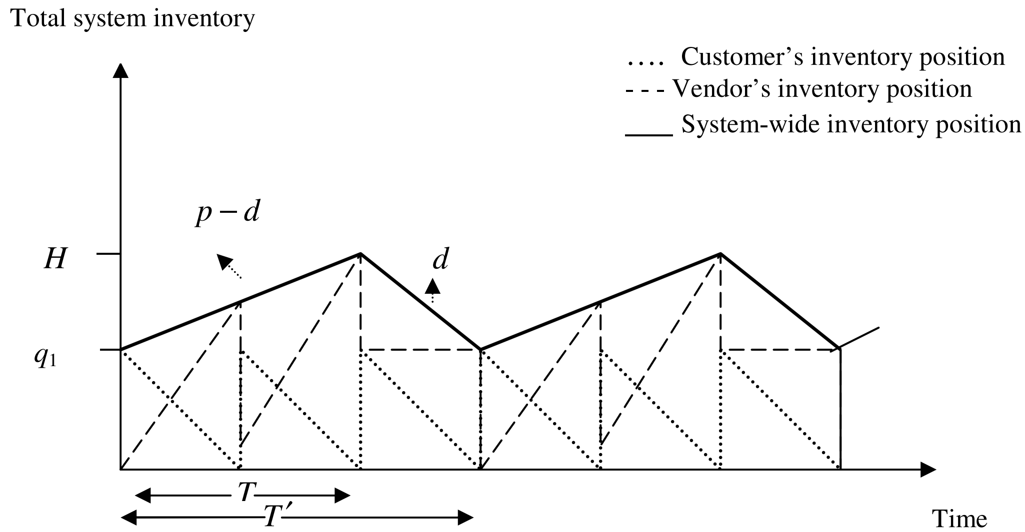

Figure 3.3: Inventory positions over time. H denotes the maximal system inventory, and T' the time between successive production start-ups.

Since we require $Q_1 \geq q_1$, we state that there is a number of shipments k_1 from the vendor to customer during the vendor's cycle: $Q_1 = k_1 q_1$. Note also that the transportation cost is paid by the customer, but the vendor pays a_v for every shipment released. Then, the vendor's total cost is

$$TC_{v1} = d\,[S/Q_1 + a_v/q_1] + h_v\,[q_1 + (1-d/p)\,Q_1] \,\big/\, 2 \qquad (1)$$

The first two terms in (1) correspond to production setup and shipment release costs, and the third and fourth to inventory carrying cost. The only variable in that equation is Q_1. Treating the number of shipments as continuous, rather than discrete (which will be the assumption throughout; see also Proposition 1), the optimal production quantity in one cycle is then $Q_1 = \sqrt{\dfrac{2Sd}{h_v(1-d/p)}}$ = EPQ. We remark that since TC_{v1} is a strictly convex function in the interval $(0, \infty)$, the optimal integer value for k is

$$k^{\text{int}} = \underset{TC_{v1}(k^*)}{Min}\left\{ \lfloor EPQ/q_1 \rfloor, \lceil EPQ/q_1 \rceil \right\}.$$

Based on the values of Q_1 and q_1, we then have

$$TC_{v1} = \sqrt{2Sdh_v(1-d/p)} + \frac{da_v}{\sqrt{2A_c d/h_c}} + h_v\sqrt{2A_c d/h_c}\Big/ 2$$

Letting $\gamma = \dfrac{a_v}{A_c}$ and $\phi = \dfrac{h_v}{h_c}$, and denoting $C' = \sqrt{2Sdh_v(1-d/p)}$, the result is

$$TC_{v1} = C' + (\gamma+\phi)\sqrt{\frac{dA_c h_c}{2}}.$$

Let us consider once more $TC_{v1} = dS/Q_1 + \dfrac{h_v}{2}(1-d/p)Q_1 + \left(d\ a_v\ /q_1 + \dfrac{h_v q_1}{2} \right)$. We remark that Q_1 is constant no matter what quantity q_1 the customer orders, hence C' is constant independent of q_1. The second (circled) part of this total cost is a *forced* cost: The vendor has no influence on it. The customer's decision q_1 determines how much the vendor must pay. This explains a major motivation behind a VMI agreement, whereby V seeks a way to get involved in ordering decisions to see if that second part of her total cost can be decreased.

Although the customer's decision could be near-optimal, we will suppose that the vendor is not happy with the customer's order quantity, and she wants to make replenishment decisions herself. V then offers C a VMI partnership that states: The vendor will make replenishment decisions on behalf of the customer, and will be responsible to pay any cost associated with it. Here, an "associated cost" does not include the expense for transportation, which is still assumed to be paid by the customer. (That will be relaxed later in our analysis.)

3.5 Vendor-Managed Inventory (Case 2)

With VMI, the vendor takes over from the customer the responsibility for replenishment. The customer does not place any order, hence pays no ordering charge, although the customer does pay its cost of holding stock.

The expense associated with the replenishment decision, i.e. the cost of issuing an order, was a_o, as paid by the customer when he makes that decision. This parameter might be a different value for the vendor. Let us write that under the proposed VMI partnership, *V* will need to pay $\beta_1 a_o$ for issuing an order, where $\beta_1 \geq 0$ can be interpreted as the vendor's efficiency factor. (*C* will then be exempt from paying a_o.)

Under VMI, the vendor pays $\beta_1 a_o$ plus her costs that were discussed in Case 1. As such, the customer pays all his costs from Case 1 except a_o. The proposed VMI partnership does not include sharing the transportation cost; it is still paid by the customer. Let Q_2 be the production quantity in Case 2 and q_2 be the replenishment quantity, which is now determined by the vendor on behalf of the customer. The vendor can then find optimal values of Q_2 and q_2 that minimize her total cost TC_{v2}, where

$$TC_{v2} = \frac{d}{Q_2}\left[S + k_2\ (a_v\ + \beta_1 a_0)\right] + \frac{1}{2}\ h_v\ \left[q_2 + (1-d/p)\ Q_2\right].$$

Note that q_2 is now also a decision variable for the vendor.

Proposition 1: For a continuous number of shipments k_2, the optimal $Q_2 = Q_1 =$ EPQ independent of q_2, the minimum system-wide inventory.

Proof: Replace k_2 by Q_2 / q_2. Then, $TC_{v2} = f\,(Q_2) + \bar{f}(q_2)$ where

$f(Q_2) = dS/Q_2 + \frac{h_v(1-d/p)Q_2}{2}$, and $\bar{f}(q_2) = d(a_v + \beta_1 a_o)/q_2 + \frac{h_v q_2}{2}$. These functions, each convex over $(0, \infty)$, can be optimized separately.

The optimal value for f thus occurs when

$$Q_2 = \sqrt{\frac{2Sd}{h_v(1-d/p)}}. \qquad \blacksquare$$

Minimizing $\bar{f}(q_2)$, the vendor finds the replenishment quantity under VMI as

$$q_2 = \sqrt{\frac{2(a_v + \beta_1 a_o)d}{h_v}}.$$

Let $\delta_1 = \frac{a_o}{A_c}$. We previously defined $\gamma = \frac{a_v}{A_c}$ and $\phi = \frac{h_v}{h_c}$. Then

$$q_2 = \sqrt{\frac{\gamma + \beta_1 \delta_1}{\phi}}\; q_1 = m\, q_1,$$

where we define $m = \sqrt{\frac{\psi}{\phi}}$ and $\psi = \gamma + \beta_1 \delta_1$.

Now we want to find TC_{v2} and TC_{c2}, and to see how they compare with results from Case 1. Basically, we want to know if this VMI partnership can help us achieve some of the following:

i. $TC_{v2} < TC_{v1}$: cost saving for the vendor

ii. $TC_{c2} < TC_{c1}$: cost saving for the customer

iii. $TC_{v2} + TC_{c2} < TC_{v1} + TC_{c1}$: system-wide cost savings

To categorize the results of the two systems, we will state that

- VMI is an *efficient* system if both the vendor and the customer are better off compared to Case 1: Both (i) and (ii) hold. The VMI partnership is clearly acceptable to both parties.
- In a *potentially-efficient* system, although one party is better off, the other is worse off while VMI achieves system-wide cost savings: Both (iii), and either (i) or (ii), hold. If so, we can look for a way to adjust the partnership so that no party is worse off.
- An *inefficient* system means system-wide cost under VMI exceeds that of Case 1, hence (iii) does not hold.

Note that each statement (i) – (iii) is a strict inequality. We shall often emphasize this by saying "*positive* cost savings." Based on cost comparisons, we can infer Propositions 2 through 5, proofs of which are contained in the Appendix A.

Proposition 2: Under VMI, the vendor achieves positive cost savings if and only if $\gamma + \phi > 2\sqrt{\phi}\ \sqrt{\gamma + \beta_1\ \delta_1}$, i.e. if and only if $\gamma\ >\ (2m-1)\,\phi$.

Proposition 3: Under VMI, the customer achieves positive cost savings if and only if $2\sqrt{\phi}\ \sqrt{\gamma + \beta_1\,\delta_1}\ > \phi\ (\delta_2 + \delta_3) + \gamma + \beta_1\ \delta_1$, that is, if and only if $\delta_1\ >\ (m-1)^2$, where $\delta_2 = a_t\ /\ A_c$, $\delta_3 = a_r\ /\ A_c$, and $\delta_1 + \delta_2 + \delta_3 = 1$. Equivalently, for a fixed value of δ_1, the customer will achieve positive cost savings under VMI if and only if $1-\sqrt{\delta_1} < m\ < 1+\sqrt{\delta_1}$.

Proposition 4: If $\gamma + \beta_1\,\delta_1 < \phi$ when $\phi > \gamma$, or if $\phi < \gamma$ when $\gamma + \beta_1\,\delta_1 > \phi$, then VMI will yield positive system-wide cost savings. That is, VMI will enable positive system-wide cost savings if and only if $(2\phi + 1)m^2\ - (\gamma + \phi + 2\,)m + 1 - \delta_1 < 0$.

Lemma 1: If $\phi < \beta_1$**,** both parties cannot simultaneously be better off.

Proof: A necessary (but not sufficient) condition that both parties be better off together is

$$\gamma + \phi > \phi\,(\delta_2 + \delta_3) + \gamma + \beta_1\ \delta_1$$

$\Rightarrow \phi > \phi\,(1-\delta_1) + \beta_1\ \delta_1$, which implies $\phi > \beta_1$ ■

We remark that if $\beta_1 = 1$, VMI *cannot* be an efficient system if the vendor's inventory holding cost is smaller than the customer's. Note also that, to tell when (i) and (ii) hold together, requires knowledge of at least the ranges of parameters. We will provide examples later.

Consider the case when (iii) is true, but only (i) or (ii) holds. This means either

- <u>The customer is better off</u>: Here a VMI partnership is not applicable since *V* (who offered the partnership) is worse off, and there is no incentive for *C* to share his cost savings with the vendor (the customer already pays the transportation cost).
- <u>The vendor is better off</u>: *C*, now worse off, will not consent to VMI unless *V* offers an additional incentive, so that the customer's cost is no greater than in Case 1. One such incentive is "transportation cost sharing": The vendor shares *C*'s transportation cost (a_t) so that, overall, the customer does not suffer under VMI. That is, the vendor pays $(1-\alpha_2)a_t$, and the customer pays $\alpha_2 a_t$ per shipment, where $0 < \alpha_2 < 1$.

Proposition 5: A potentially-efficient VMI arrangement, where the vendor is better off, can be turned into an efficient system by setting

$$1-\alpha_2 = \left[\gamma + \beta_1\delta_1 + \phi(1-\delta_1) - 2\sqrt{\phi}\sqrt{\gamma+\beta_1\delta_1}\,\right]\frac{1}{\phi\,\delta_2} = \frac{(m-1)^2 - \delta_1}{\delta_2}$$ ■

Note that, when the customer pays just the fraction α_2 of transportation cost, he is now no worse off than in Case 1; the vendor is still better off, and all the savings are captured by the vendor.

As another way of sharing the cost savings, consider a payment from the vendor to the customer in the form of a price discount. Suppose that the customer (originally) pays \$ c per item purchased. A discount of y % offered by the vendor will make VMI an efficient system when $y = \frac{100}{cd}(TC_{c2} - TC_{c1})$. Such a price discount is an alternative to the sharing of transportation cost. Either incentive can turn a potentially-efficient VMI system, where the vendor is better off, into an efficient one, benefitting both actors. (We remark that, in light of our cost assumptions, those two incentives are the only means available to share the savings in total cost.)

Our analyses up to now were for a vendor and customer that were independent decision makers in a supply chain. We played the role of an outside observer to see the impacts of VMI. That is, we investigated if it was possible to keep the independence of the actors and achieve efficiency at the same time.

Let us next assume that there is a third party who has control over both vendor and customer, and also has enough information on each of their particular cost parameters. This will be true if the vendor and customer belong to a single corporate entity, hence are vertically integrated.

3.6 Central Decision Making (Case 3)

We now analyze the system from the point of view of this third party, and call it "central decision making." As in JELS models, there is a single total cost function denoted by TC_{sys} that includes all expenses of both the vendor and customer. Assume also that $\beta_1 = 1$ when comparing Cases 2 and 3, since a vertical integration implies the capture of all possible efficiencies created by any of the supply chain members. Total cost in Case 3 is then

$$TC_{sys} = \frac{dS}{Q_3} + \frac{1}{2} h_v (1 - d/p) Q_3 + \frac{d(A_c + a_v)}{q_3} + \frac{1}{2}(h_c + h_v) q_3 .$$

Observe that Proposition 1 still holds. When TC_{sys} is minimized, optimal production and replenishment quantities are then

$$Q_3 = \sqrt{\frac{2Sd}{h_v(1-d/p)}} = \text{EPQ}, \quad \text{and} \quad q_3 = \sqrt{\frac{2(A_c + a_v)d}{h_c + h_v}} = \sqrt{\frac{1+\gamma}{1+\phi}}\; q_1$$

Note that if $\gamma = \phi$, then $q_3 = q_1$: The customer is replenishing at the system-wide optimal quantity anyway. There is then no need for a contract to decrease overall total costs; they are already at their minimum. Comparing Case 3 and Case 1, we see that

$$\left(\sqrt{1+\phi} - \sqrt{1+\gamma}\right)^2 \sqrt{\frac{A_c d h_c}{2}} = TC_{v1} + TC_{c1} - TC_{sys} \geq 0 .$$

Computing the total costs in Case 3 and Case 2, it is found that

$$\left(\sqrt{1+\phi}\sqrt{\gamma+\delta_1} - \sqrt{\phi}\sqrt{1+\gamma}\right)^2 \sqrt{\frac{A_c d h_c}{2\phi(\gamma+\delta_1)}} = TC_{v2} + TC_{c2} - TC_{sys} \geq 0 .$$

Both of the equations above show that the lowest system-wide cost can be achieved through central decision making. In the next section, computational examples will highlight this point as well as the previous analytical results.

3.7 Numerical Examples

We now consider a series of examples to contrast the Cases 1 - 3. The following values are taken throughout: $\delta_1 = 0.2$, A_c = \$100 per order, h_c = \$1.5 per item stored, p=1600 items/year and d = 1300 items/year in each example. Dollar values of total costs require only the preceding parameters, plus of course the “ratios” defined in our analysis: ϕ, γ, β_1. In Figs. 3.4 through 3.11, generally two of those ratios are fixed, while the third is varied. In each case, that range of variation has encompassed a factor of 40: $[0.1\ (0.1)\ 4]$, i.e. between values of 0.1 and 4.0, in steps of 0.1.

When a ratio, say ϕ, is fixed at level ϕ_1 in one set of graphs, it may be fixed at level ϕ_2 in the next set. The levels ϕ_1, ϕ_2 (and similarly for the γ_i in their respective graphs) are chosen strategically, such that qualitatively different behaviour is observed for ϕ_1 vs ϕ_2. (We remark that $\beta_1 = 1$ in every figure except Figs. 3.8 and 3.9.) In discussing Figs. 3.4 – 3.11, we usually first compare Case 2 to Case 1 and then comment on the differences between each of those and Case 3. TC$_i$ in those figures denotes the total system cost for Case *i*.

VMI vs Independent Decision Making

Example 1: $\phi = 1.5$, $\gamma = [0.1\ (0.1)\ 4]$

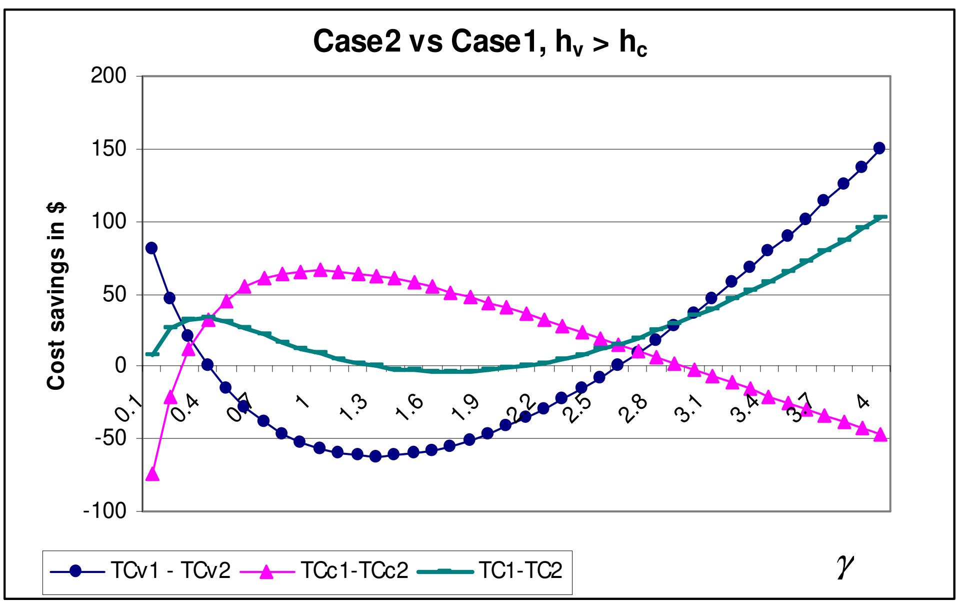

Figure 3.4: Comparison of Cases 1 and 2; $\phi = 1.5$

Since $\phi > 1$, $h_v > h_c$. It is then possible to observe some intervals where both parties are better off. Figure 3.4 shows that VMI is an efficient system if γ is within [0.3, 0.4] or within [2.6, 2.9]. It is inefficient when γ lies within [1.4, 2], but potentially efficient for the remaining γ.

Example 2: $\phi = 0.8$, $\gamma = [0.1\ (0.1)\ 4]$

Here, $h_v < h_c$, so VMI cannot be efficient (Fig. 3.5). System-wide cost savings occur when γ is within [0.1, 0.6] or [2 , 4]. In these ranges, there are two possibilities:

- *C* is better off while *V* is worse off: $\gamma \in$ [0.1, 0.6]. Not much can be done, since (as discussed before) there is no incentive for the customer to share his cost savings.
- The vendor is better off while the customer is worse off: $\gamma \in$ [2, 4]. Here, *V* can share her cost savings with *C* to achieve an efficient system. Suppose γ = 2.4, δ_2 = 0.7, and the mechanism chosen is transportation-cost sharing. By Proposition 5, 1- α_2 = 0.635 : VMI can be an efficient system if *V* pays 63.5 % of total transportation cost.

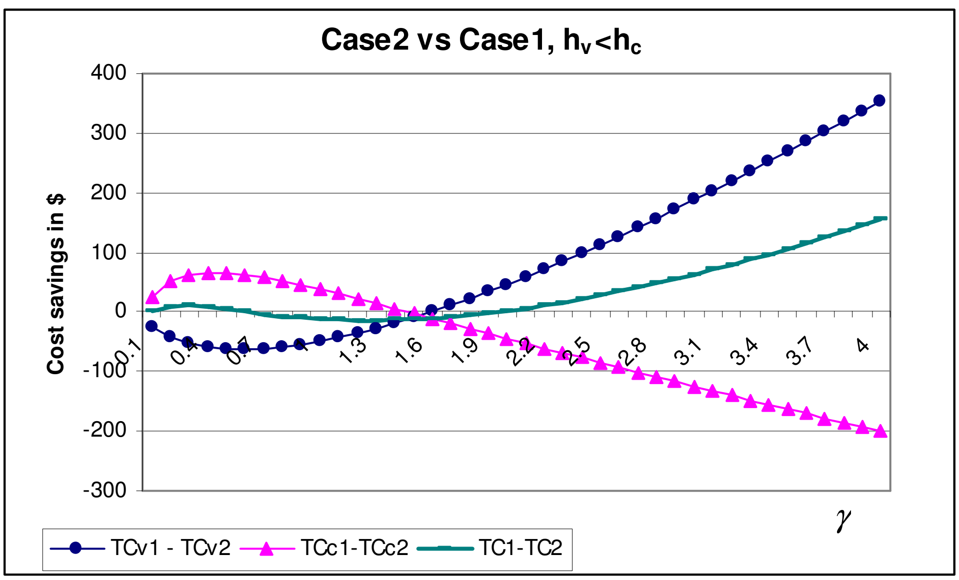

Figure 3.5: Comparison of Cases 1 and 2; $\phi = 0.8$

Vertical Integration vs VMI or Base Case

We previously concluded that Case 3 provides the minimum system-wide cost, hence a lower bound for the cost of any virtual integration between the vendor and customer. Figs. 3.6 and 3.7, on Examples 1 and 2, show that (TC_1 - TC_3) and (TC_2 - TC_3) are non-negative. Differences between graphs in each figure basically give the cost improvement from Base Case to VMI. Hence, interpretation of ranges that create potentially-efficient and inefficient systems remains the same.

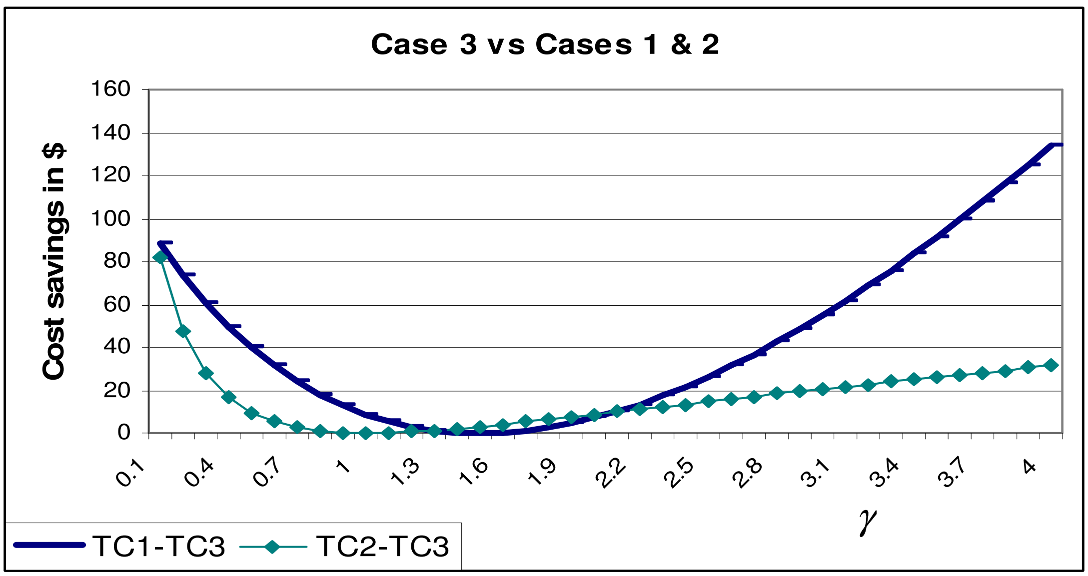

Figure 3.6: Comparison of Cases 1-3; ϕ = 1.5

The numerical examples we provided are thus in line with our analytical results. In the next two sections, these will be summarized and conclusions will be presented, following additional discussion of our findings.

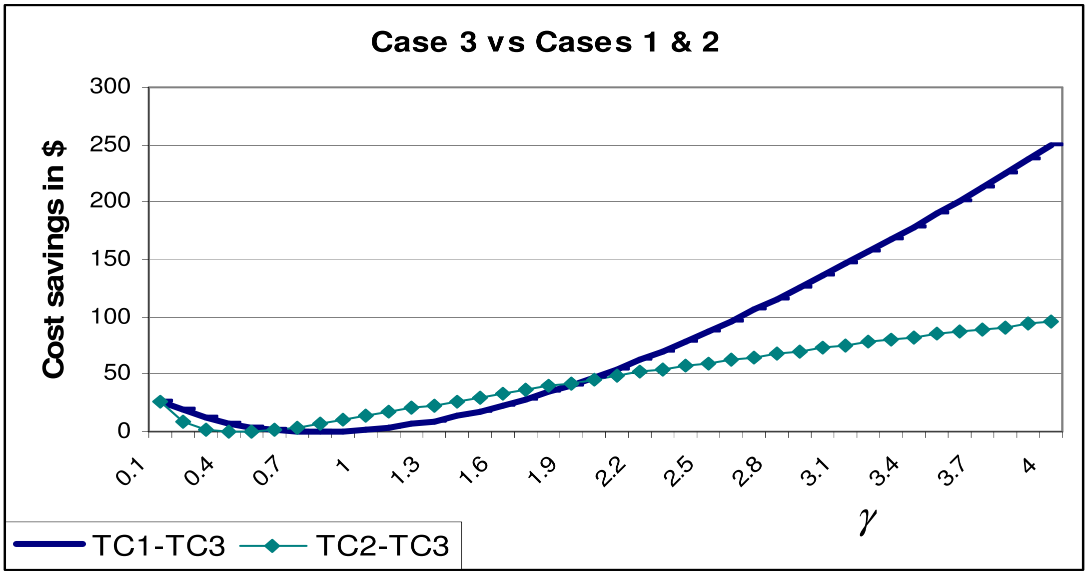

Figure 3.7: Comparison of Cases 1-3; $\phi = 0.8$

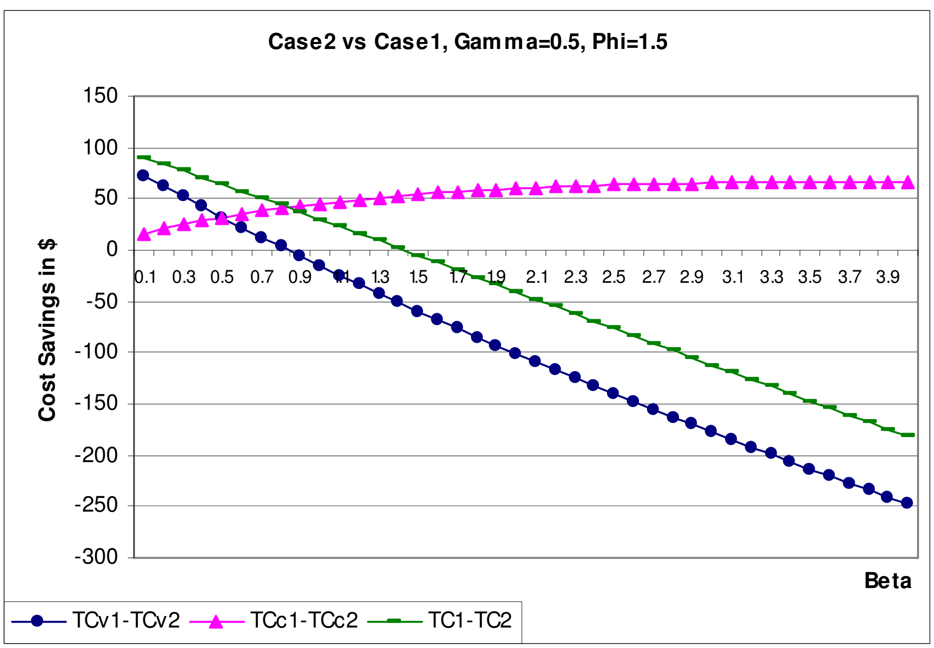

Figure 3.8: Impacts of β_1 when $\phi = 1.5$

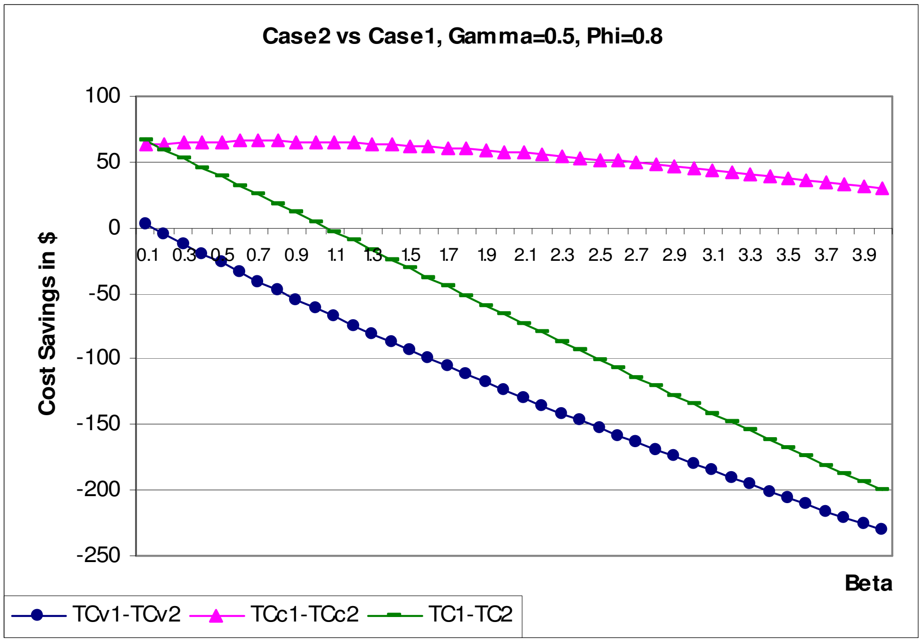

Figure 3.9: Impacts of β_1 when $\phi = 0.8$

3.8 Further Discussion

Most observers, whether academics or practitioners, would feel that VMI is, *in general*, either better or worse than the traditional business approach. Rather, the results of this paper show that the success of VMI depends greatly on the cost parameters of the parties involved. Hence, there are cases where both actors are better off (requires $\phi = h_v / h_c > \beta_1$, the vendor's efficiency factor). There are also cases where

only the customer is better off: $[\gamma \leq (2m-1)\,\phi$ and $1-\sqrt{\delta_1} < m < 1+\sqrt{\delta_1}\ ]$;

only the vendor is better off : $[\gamma > (2m-1)\,\phi$ and one of : $m \leq 1-\sqrt{\delta_1}$ or $m \geq 1+\sqrt{\delta_1}\ ]$; or neither party is better off: $[\gamma \leq (2m-1)\,\phi$ and one of $m \leq 1-\sqrt{\delta_1}$ or $m \geq 1+\sqrt{\delta_1}\ ]$. Recall that $m = \sqrt{\dfrac{\psi}{\phi}} = \sqrt{\dfrac{\gamma + \beta_1\,\delta_1}{\phi}}$. We emphasize that each of the conditions in square brackets is *both necessary and sufficient* for that particular case. Those general inequalities can be verified on the given regions in any of Figs. 3.4 – 3.11.

More can be said, in terms of individual cost parameters, if we go back to our closed-form results of Propositions 2-5. We are particularly interested in Outcome (iii), system-wide cost savings under VMI, where the vendor's costs have decreased more than the customer's costs have increased. This situation is more likely if ϕ is <u>much larger</u> than $\gamma \left(= a_v / A_c \right)$, which can be seen after some algebra. That is, the customer's efforts to decrease his own inventory-carrying costs, combined with VMI, result in greater savings.

VMI may be efficient, potentially efficient, or inefficient, when ϕ and γ have other relative values. In those cases, the difference in system costs due to VMI depends strongly on β_1. This is observed in Figs. 3.8 and 3.9, where ϕ / γ is respectively 3.0 and 1.6.

Recall that, when the vendor orders on behalf of the customer, it costs her $\beta_1 a_o$, compared to simply a_o when the customer orders on his own. It is thus reasonable to view $(1-\beta_1)$ as the degree to which the vendor is "more efficient."

We see that in Fig. 3.8, even when $\beta_1 = 1.4$, there are system-wide cost savings. That situation, namely a potentially efficient system under VMI, requires in Fig. 3.9 that the vendor *V* be at least as efficient as the customer *C*.

As *V* becomes more efficient, i.e. as β_1 decreases, her costs clearly decrease. System-wide costs decrease as well. In fact, the latter is true even when the customer's savings are *increasing* in β_1 (Fig. 3.8). As far as concerns the vendor's savings, we observe in Fig. 3.8

that V need only be 20% more efficient than C, to achieve savings for herself. Contrast this with Fig. 3.9, where she must be 90% more efficient.

Let us now turn to the impact of ϕ. This parameter is allowed to vary in Figs. 3.10 and 3.11, where γ is respectively fixed at 1.5 and 0.8, all other data remaining unchanged from previous examples. In both figures, the VMI system is inefficient for $\phi < 1.0$ or so. Figure 3.10 exhibits a wider range of ϕ for which the system is potentially efficient. The wider range in Fig. 3.11 corresponds to the system being *efficient*: The smaller value of γ permits the savings of each party under VMI to respond more quickly to an increment in ϕ.

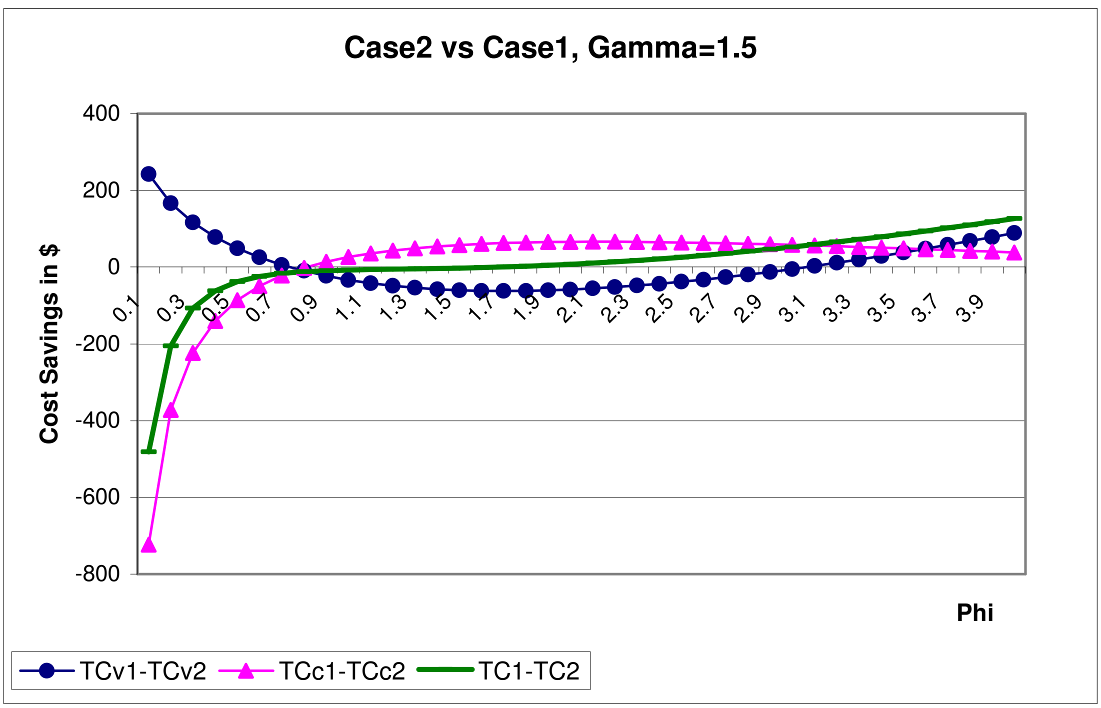

Figure 3.10: Impacts of ϕ when $\gamma = 1.5$

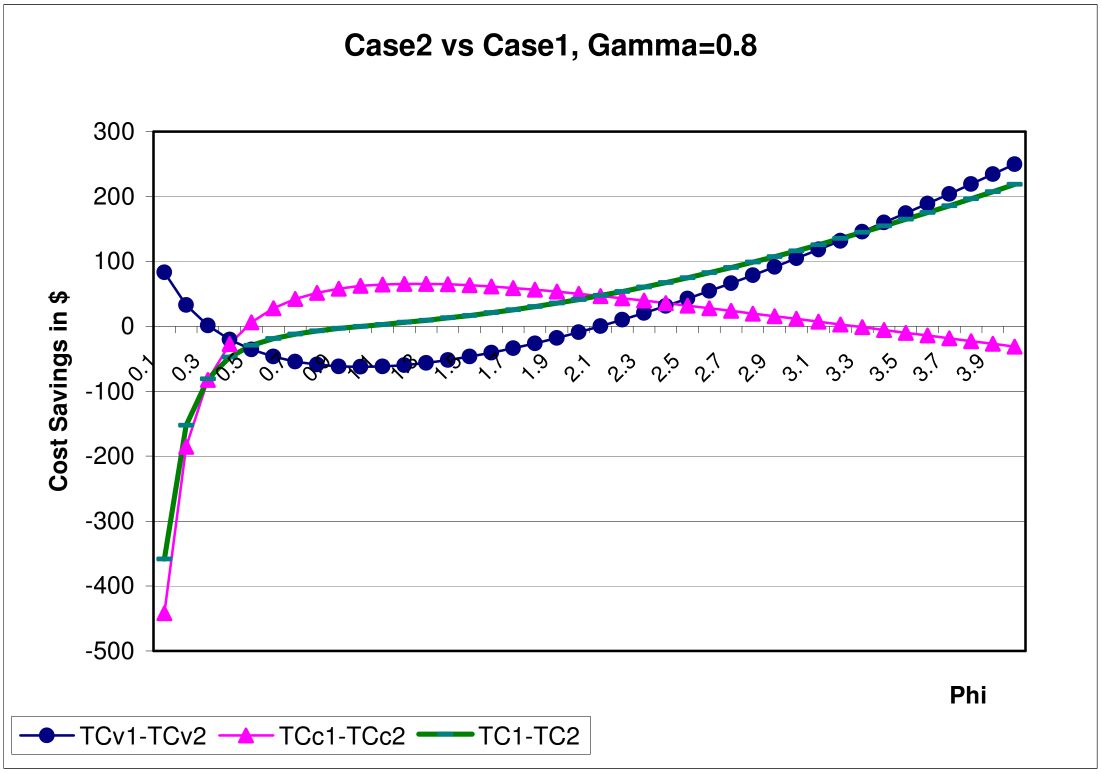

Figure 3.11: Impacts of ϕ when $\gamma = 0.8$

3.9 Summary and Conclusions

We have considered a VMI agreement between a vendor *V* and customer *C* who initially acted independently. With that agreement, *V* could make replenishment decisions on behalf of *C,* but would incur the cost to issue an order. We identified three possible outcomes of VMI:

- An *efficient system* where both the vendor and the customer are better off. This is possible only when $\phi > \beta_1$.
- A *potentially-efficient* system if there are system-wide cost savings, and either the customer or the vendor is better off. If we get a potentially-efficient system, we can turn it into an efficient one (when the vendor is the better-off party) through transportation-cost sharing or a price discount.

- An *inefficient* system if $TC_{c1} + TC_{v1} < TC_{c2} + TC_{v2}$. VMI causes an increase in the system's total cost.

System-wide total cost function changes when β_1 is included in Case 2. Even when $\beta_1 =$ 1, that function takes different values depending on which party is responsible for ordering. In each of the three cases we considered, the decision maker is another party of the same supply chain. Although the cost parameters may remain the same, total costs of the supply chain and its members differ based on who makes replenishment decisions.

Assuming that it is financially and culturally feasible, Case 3 (Central decision making) would provide the best possible system-wide cost. Table 3.1 summarizes the analytical results we obtained in the three cases. Proposition 1 and its proof explain why the vendor's production batch size remains the same in Cases 1-3.

We remark that our findings are "general," in the following sense. The formulations account for relevant cost parameters of each party; no inequalities between parameter values have been assumed in advance. In a particular application, there will be specific numerical figures. The preceding results permit determination of whether VMI is efficient or potentially efficient or not.

Our analyses indicate, in many instances, that either the customer alone or the vendor alone captures the savings generated by VMI. Even so, a change from independent decision making is often worthwhile. VMI is beneficial overall (Proposition 4) if and only if

$$(2\phi + 1)m^2 - (\gamma + \phi + 2)m + 1 - \delta_1 < 0 .$$

The better-off vendor can compensate the customer to the point that his losses are neutralized (Proposition 5).

Future research might include several products or customers. If there were two products, both managed under VMI by *V* for *C*, consolidated shipments of a *mixed load* could be dispatched to *C*. But the non-VMI case now is also more interesting, namely "coordinated inventory control," i.e. joint replenishment by *C* of SKUs ordered from the same supplier, *V*.

In the case of two customers C_1, C_2, even a single product could be shipped from *V* to a cross-dock (CD; e.g. Gümüş and Bookbinder 2004), followed by transport over shorter distances to each C_i individually. And whether or not a CD is employed, a *route* that combines deliveries to the two C_i is a separate option.

The point is that two products and/or two customers would allow additional economies in inventory or transportation decisions, both for VMI and non-VMI. To capitalize on the richness of the new examples, however, will again require precise treatment of the cost parameters, and care in allocating particular expenses to each actor.

	Case 1	Case 2	Case 3
q_i	$\sqrt{\dfrac{2\,A_c\,d}{h_c}} = EOQ$	$m\,q_1$	$\sqrt{\dfrac{1+\gamma}{1+\phi}}\;q_1$
Q_i	$\sqrt{\dfrac{2Sd}{h_v\,(1-d/p)}} = EPQ$	Q_1	Q_1
Customer's cost TC_{ci}	$\sqrt{2\,A_c\,d\,h_c}$	$\left(\dfrac{m^2\;+\;1-\delta_1}{2m}\right)TC_{c1}$	
Vendor's cost*	$(\dfrac{\gamma+\phi}{2})\;TC_{c1}$	$m\phi TC_{c1}$	
System-wide cost*	$TC_{sys,1} = \left(1+\dfrac{\gamma+\phi}{2}\right)TC_{c1}$	$\left[\dfrac{m^2(2\phi+1)+(1-\delta_1)}{m(\gamma+\phi+2)}\right]TC_{sys,1}$	$\sqrt{(1+\gamma)(1+\phi)}\,TC_{c1}$

Table 3.1: Summary of analytical results. * Excludes the fixed cost C'

Appendix A

Proof of Proposition 2: We showed (Sec. 4) that $TC_{v1} = C' + (\gamma + \phi)\sqrt{\frac{A_c\, d\, h_c}{2}}$. Also,

$$TC_{v2} = C' + \frac{d\,(a_v + \beta_1 a_o)}{q_2} + \frac{1}{2} h_v\, q_2$$

Since $q_2 = \sqrt{\frac{\gamma + \beta_1 \delta_1}{\phi}}\sqrt{\frac{2 A_c\, d}{h_c}}$, $a_0 = \delta_1 A_c$, $a_v = \gamma A_c$ and $h_v = \phi h_c$ **,** we then have

$$TC_{v2} = C' + \sqrt{\phi(\gamma + \beta_1 \delta_1)}\sqrt{2 A_c\, d\, h_c}\,.$$

Let $TC_{v1} - TC_{v2} = \Gamma_v$ denote the vendor's savings, the decrease in costs due to VMI.

Because $\sqrt{A_c d h_c} > 0$, $\Gamma_v > 0$ if and only if $\sqrt{\frac{1}{2}}\,(\gamma + \phi) - \sqrt{2}\sqrt{\phi\,(\gamma + \beta_1 \delta_1)} > 0$

That is, the vendor's savings are positive if and only if $\gamma + \phi > 2\sqrt{\phi}\sqrt{\gamma + \beta_1 \delta_1}$, i.e. if and only if $\gamma > (2m - 1)\,\phi$. ■

Proof of Proposition 3: $TC_{c1} = \sqrt{2 A_c\, d h_c}$

$$TC_{c2} = \frac{d\,(a_r + a_t)}{q_2} + \frac{1}{2} h_c\, q_2 = \frac{d\,(A_c - a_o)}{q_2} + \frac{1}{2} h_c\, q_2$$

Based on q_2 as well as $\delta_1 + \delta_2 + \delta_3 = 1$,

$$TC_{c2} = \left[\gamma + \beta_1 \delta_1 + \phi(1 - \delta_1)\right]\sqrt{\frac{A_c\, d\, h_c}{2\phi(\gamma + \beta_1 \delta_1)}}$$

Let $TC_{c1} - TC_{c2} = \Gamma_c$ denote the customer's savings under VMI. We find $\Gamma_c > 0$ if and only if $2\sqrt{\phi(\gamma + \beta_1 \delta_1)} - \left[\gamma + \beta_1 \delta_1 + \phi(1 - \delta_1)\right] > 0$, i.e. if and only if $\delta_1 > (m - 1)^2$.

The latter is easily seen to be equivalent to $1 - \sqrt{\delta_1} < m < 1 + \sqrt{\delta_1}$. ■

Proof of Proposition 4:

The system's overall savings are $\Gamma_{sys} = (TC_{v1} + TC_{c1}) - (TC_{v2} + TC_{c2})$, which is

$$\Gamma_{sys} = \sqrt{\frac{A_c\, dh_c}{2}}\left[(\gamma + \phi) + 2 - \left(\frac{2\phi(\gamma + \beta_1\,\delta_1) + \gamma + \beta_1\,\delta_1 + \phi(1 - \delta_1)}{\sqrt{\phi(\gamma + \beta_1\,\delta_1)}}\right)\right]$$

We will have $\Gamma_{sys} > 0$ (system-wide cost savings) if and only if

$(\gamma + \phi + 2)\sqrt{\phi\,\psi} - \left[(2\phi + 1)\,\psi + \phi(1 - \delta_1)\right] > 0$. That condition is equivalent to

$$\left(\sqrt{\phi} - \sqrt{\gamma + \beta_1\,\delta_1}\right)\left(\sqrt{\gamma + \beta_1\,\delta_1}\,(\phi + 1) - \sqrt{\phi}\,(\gamma + 1)\right) > \phi\,\delta_1(\beta_1 - 1) \qquad (1)$$

One sees from the right-hand side of (1) that, if $\beta_1 = 1$, a necessary and sufficient condition for system-wide cost savings is that both factors on the left have the same sign. When both factors are positive, that necessary and sufficient condition reduces to

$$\frac{\phi + 1}{\gamma + 1} > \frac{\sqrt{\phi}}{\sqrt{\gamma + \beta_1\,\delta_1}} > 1\,, \qquad (2)$$

while if both factors are negative, the corresponding condition is

$$\frac{\phi + 1}{\gamma + 1} < \frac{\sqrt{\phi}}{\sqrt{\gamma + \beta_1\,\delta_1}} < 1. \qquad (3)$$

When $\beta_1 < 1$, the right-hand side of inequality (1) is negative, hence each of conditions (2) and (3) is *sufficient* (but not necessary) for system-wide cost savings. In the case that $\beta_1 > 1$, inequalities (2) and (3) are now alternative statements of *necessary* (but not sufficient) conditions for overall cost savings under VMI.

To combine the three cases, recall that $m = \sqrt{\frac{\psi}{\phi}}$. We have after some algebra that VMI enables positive system-wide cost savings if and only if

$$\left(2\phi + 1\right) m^2 - \left(\gamma + \phi + 2\right) m + 1 - \delta_1 < 0. \qquad \blacksquare$$

Proof of Proposition 5:

Initially, $T_{c2} - T_{c1} > 0$. Now the vendor pays $(1-\alpha_2)\, a_t$ so that the customer is not worse off than in Case 1. Then,

$$(1-\alpha_2)\, a_t \frac{d}{q_2} = TC_{c2} - TC_{c1} = \frac{[\gamma + \beta_1\delta_1 + \phi(1-\delta_1)]}{\sqrt{2\phi(\gamma + \beta_1\delta_1)}}\sqrt{A_c\, dh_c} - \sqrt{2A_c\, dh_c}$$

We have $(1-\alpha_2)\delta_2\sqrt{\dfrac{\phi}{\gamma + \beta_1\delta_1}}\sqrt{\dfrac{A_c\, dh_c}{2}} = \sqrt{\dfrac{A_c\, dh_c}{2}}\left(\dfrac{[\gamma + \beta_1\delta_1 + \phi(1-\delta_1)]}{\sqrt{\phi(\gamma + \beta_1\delta_1)}} - 2\right)$, i.e.

$$(1-\alpha_2) = \left[\gamma + \beta_1\delta_1 + \phi(1-\delta_1) - 2\sqrt{\phi(\gamma + \beta_1\delta_1)}\,\right]\frac{1}{\phi\delta_2}$$

After some algebra, this is shown to be

$$1 - \alpha_2 = \frac{(m-1)^2 - \delta_1}{\delta_2} \qquad \blacksquare$$

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4. Impact of Consignment Inventory and Vendor Managed Inventory for a Two-Echelon Supply Chain (Essay 2)

Abstract

Vendor Managed Inventory (VMI) and Consignment Inventory (CI) are supply-chain sourcing practices between a vendor and customer. VMI allows the vendor to initiate orders on behalf of the customer. This presumably benefits the vendor who can then make replenishment decisions according to her own preferences. In CI, as in the usual independent-sourcing approach to doing business, the customer has authority over the timing and quantity of replenishments. CI seems to favour the customer because, in addition, he pays for the goods only upon use. Our main aim in this paper is to analyze CI in supply chains under deterministic demand, and provide some general conditions under which CI creates benefits for the vendor, for the customer, and the whole chain. We also consider similar issues for the combined use of CI and VMI.

4.1 Introduction

Planning, sourcing raw materials, making the product and delivering to customers are typical operational processes for a company within a supply chain. Here we consider a customer who purchases goods from a vendor. The customer's processes comprise the planning of his requirements; sourcing goods from the vendor; and releasing those goods to end-consumers. The vendor, similarly, plans her requirements and sources materials/parts for production, manufactures goods, and releases those goods to the customer.

When these two firms are independent and linked in a supply chain as in Figure 4.1, decisions concerning operational processes are, in general, made individually. In the usual sequence of events, the customer first develops his requirements plan and sourcing method based on his own costs. The vendor then reacts to fulfill the customer's requirements. Hence, replenishment decisions made by the customer do not necessarily consider his upstream business-partner's choices.

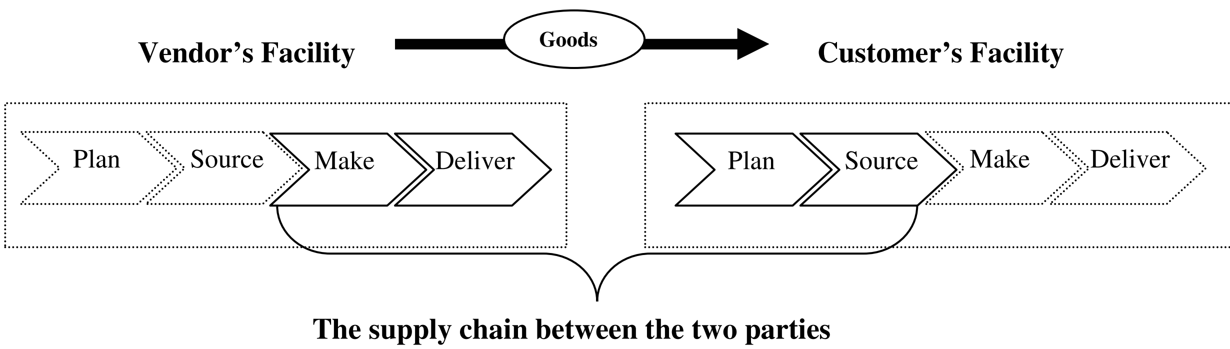

Figure 4.1: The supply chain between the vendor and customer: The primary interrelated operations are the customer's *plan* and *source* choices, and the corresponding *make* and *deliver* decisions of the vendor.

A common focus of research and supply chain practice is to seek mechanisms to align the decisions of chain members by means of contracts or agreements. Those arrangements aim to increase the overall supply chain performance. Vendor Managed Inventory (VMI), one such agreement, was analyzed by Gümüş et al. (2006) to obtain conditions under which it may lower the costs of each party and of the chain.

There are, however, other practices that *seem* to unbalance the total costs of supply-chain members. In this paper, we will analyze in detail one of those practices, Consignment Inventory (CI). Our aim is, similarly, to determine conditions whereby consignment stocks create benefits for the customer, the vendor, or for both parties.

In CI, goods are owned by the vendor until they are used by the customer. Those goods are stored at the customer's premises. Although the customer may have authority over the timing and quantity of orders, he pays for the goods only upon use. Hence, the customer does not tie up his capital in inventory.

In the traditional way of doing business, which we will call "Inventory Sourcing (IS)" throughout, the customer orders from the vendor based on his total inventory holding costs (both costs of opportunity and physical storage, where opportunity cost refers to the cost of capital), and costs of ordering. Inventory sourcing is generally characterized by a purchasing contract including shipment terms, annual demand specified by the customer, and the price per unit purchased by him. Under this practice, which will be our base case for analysis, the customer makes a payment to the vendor once the goods arrive at his premises (see Figure 4.2).

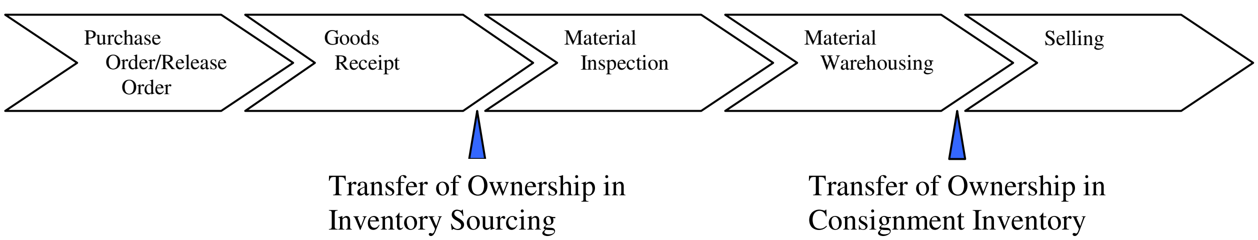

Figure 4.2: The customer's sourcing activities: Transfer of ownership in Inventory Sourcing and Consignment Inventory

In CI, ownership of goods is transferred to the customer only after they leave his in-house warehouse for production. If other terms of the purchasing contract stay the same as in IS, one major benefit to the customer is deferral of payment until production. When end-consumer demand is unknown, CI also allows the customer to hedge against uncertainties in production and sales. This will influence his total inventory carrying cost. Because the customer's inventory assets are now off his balance sheet, conventional wisdom holds that the *customer* benefits most from CI.

The benefits of CI are less clear for the vendor. One situation which favours CI is where the vendor offers new products that the customer hesitates to buy, or expensive items difficult for the customer to own. In that case, the vendor can use CI as a strategic means to create new sales channels (Piasecki, 2004). This motivation, however, does not explain why a vendor would accept a CI contract when demand is stable and the material purchased is not new.

An example of such is seen in the Automation and Drives division of Siemens, where standard parts such as metal springs and nuts can be consigned from suppliers even though the demand during a year can be quite stable. Other scenarios when a vendor might accept a CI contract include a power differential between a strong customer and a 'weaker' vendor who needs to accommodate the customer's wishes, or when the vendor at least has sufficient power to negotiate more favourable terms in the CI agreement.

There appears to be very little previous work that examines analytically the impact of CI. The focus of the present paper is to establish analytical results that specify general conditions under which CI is beneficial to one or both parties. To the best of our knowledge, there is no academic work that treats CI in this context.

In the literature, CI is mostly taken to be synonymous with VMI or with CI plus VMI ("C&VMI"). In VMI, replenishment decisions are made by the vendor on behalf of the customer. In CI, even though the vendor is informed about the consumption of goods at the customer's premises, it is still the customer who finalizes the timing and quantity of orders. We will consider both types of agreements in this paper.

The framework we use is similar to those in Joint Economic Lot Sizing (JELS) decisions. The JELS literature generally assumes a central decision maker that can optimize the sum of total costs of the vendor plus the customer. The context is very similar in each paper, and the contributions are incremental.

Banarjee (1986), the first to analyze the integrated vendor-buyer case, examines a lot-for-lot model in which the vendor *V* manufactures each shipment as a separate batch. Goyal (1988) extends this work in that he formulates a joint total-relevant-cost model for a single vendor and customer production-inventory system, where the vendor's lot size is an integer multiple of the customer's order size. Lu (1995) extended Goyal's (1988) work by allowing *V* to supply some quantity to the purchaser before completing the entire lot. Goyal (1995) employed the example provided by Lu for the single vendor and buyer but showed that a different shipment policy could result in a better solution.

Hill (1997) considers a single vendor who manufactures a product at a finite rate and in batches, and supplies a sole buyer whose external demand is level and fixed. Each batch is sent to the buyer in a number of shipments. The vendor incurs a batch setup cost and a fixed order or delivery cost associated with each shipment. Hill's policy assumes that successive shipment sizes increase by a factor whose value lies between one and the ratio of manufacturing rate to the product's demand rate. He concludes that, although Goyal's (1995) policy may perform much better than Lu's equal-size-shipment policy, his own policy outperforms all.

Similar to the JELS literature, we use a base case (IS) for comparison purposes, contrasting that to other models which assume that the parties in the supply chain still make decisions independently (whether coordinated or not).

JELS studies do not discuss how the savings created by central decision making should be divided between the parties involved. Benefits achieved are difficult to generalize and the cost models are not analyzed in detail. For example, the customer's ordering cost is not explicit in those models. The CI or C&VMI sourcing models that we consider require a shift of certain costs from one actor to another to reflect changes in decision-making responsibility or ownership of inventory. We provide a breakdown of cost parameters so as to identify the impact of such changes on each member.

Sucky (2005) extends EOQ and JELS to a *bargaining* model, where the vendor offers a side payment to the customer whose costs under JELS go up compared to individual decision making. It is assumed that the vendor, who achieves cost savings under JELS, makes a "take-it-or-leave-it" offer of joint policy with a side-payment. The customer may accept the vendor's offer, or if he is not satisfied with it, can enforce his EOQ. The bargaining then ends. Sucky assumes that the vendor has full information regarding the customer's costs.

A number of papers have also been written on combined use of CI and VMI. This literature discusses various C&VMI systems that differ in the costs considered, the demand structure, and the nature and number of supply-chain members involved.

Boyaci and Gallego (2002) study a system of a single wholesaler and retailer under deterministic but price-sensitive demand. They analyze the impacts of coordinating pricing and replenishment when decisions are made jointly. They use wholesaler-owned inventory with delayed payment vs consignment inventory to extend the models of Crowther (1964) and Monahan (1984). They conclude that pricing and inventory decisions are best made with a coordinated-channel's profit function.

In our paper, we analyze the impacts of CI from the operational point of view. That is, under CI, there is no change in pricing terms from those in the purchasing contract under inventory sourcing. This enables us to focus on operational benefits to both parties. If one party is not satisfied with the outcome, a price change may then become an option, as it would be in industry.

Valentini and Zavanella (2003) describe the technique of consignment stock by a case study of a manufacturer providing parts to the automotive industry. In that example, the vendor manages the inventory of her customer using an (s, S) policy. While the authors' main aim was to qualitatively analyze the advantages and disadvantages of this sourcing practice, they compare it numerically with Hill's (1997) solution, using the same deterministic model. Although they include the customer's opportunity and storage cost of inventory separately, they omit his ordering cost totally. Based on numerical examples only, they come to the conclusion that consignment stock outperforms the usual inventory models.

Persona et al. (2005) build on the analysis provided by Valentini and Zavanella (2003) using the same assumptions concerning characteristics of the agreement. In their paper, they analyze the consequences of product obsolescence, concluding that obsolescence decreases the optimal level of consignment stock.

There are also publications that examine C&VMI in various contexts. For example, Dong and Xu (2002) explore the economics of C&VMI in the short and long terms. Moreover, Gerchak and Khmelnitsky (2003) provide an interesting example of C&VMI when demand is uncertain. They consider a retailer selling newspapers and his vendor (a publisher) under VMI and revenue sharing. They analyze the impacts of retailer's sales report (to the publisher) on coordination.

Although we take both CI and VMI into account in this paper, the problem setting, the approach we use, and our goal are quite distinct from those of Dong and Xu(2002) or of Gerchak and Khmelnitsky (2003). We consider a well-known problem but analyze it under different partnerships, accounting for changes in certain cost parameters. We provide closed-form solutions to see under what conditions a partnership is more favourable than others.

4.2 Problem Definition

Suppose a customer purchases a standard good from a vendor. Yearly demand is constant and is realized at the customer; there is no backordering. The vendor and customer are independent firms, each with the goal of minimizing their own total cost.

Under IS, the customer orders from the vendor based on his total cost of planning (fixed cost per order), sourcing (fixed cost per shipment received) and inventory holding (physical storage and opportunity cost of inventory). The vendor bears productioon setup costs, costs per shipment released to the customer, and inventory-holding costs for both work-in-process and finished goods not yet shipped to the customer.

The customer buys goods from the vendor based on a purchasing contract that specifies the (minimum) annual quantity, the price per item, and shipment terms. We assume that the price per item as well as shipment terms were negotiated between the two parties based on yearly requirements, and a shipment destination was set by the customer. Our aim is not to optimize these parameters by arranging a new purchasing contract between the two parties. Rather, we will compare different business-partnerships to see if any of them creates more benefits when the contract parameters are the same.

The customer in IS plans the optimal quantity and timing of his orders, and performs the sourcing from the vendor based on this plan. The vendor releases shipments based on the customer’s ordering decisions. When the customer receives the goods, he makes a payment to the vendor and thus owns the product from that point on. Until such items are sold to end-consumers, inventory holding costs are accumulated at the customer.

Under CI, goods are owned by the vendor until they are *used* by the customer, i.e. until sold or employed as inputs in the customer’s manufacturing process. Although the customer pays physical storage costs (such as rent and electricity), he does not own the inventory and hence does not incur capital costs for holding that stock. Those carrying costs accrue to vendor. It is still the customer who sets the timing and quantity of orders. We will determine under what conditions consigning stocks creates benefits for the customer, the vendor and for both.

We will also look at the use of CI and VMI combined. When CI is coupled with VMI, even though it is the vendor who pays the opportunity cost of goods stored at the customer, the vendor now also takes over responsibility for setting the quantity and timing of shipments released to the customer. This transfer of authority also shifts the decision-making costs to the vendor, but the vendor may benefit from this agreement by decreasing her total inventory holding cost. Table 4.1 identifies the three cases we consider and their major differences.

	IS	**CI**	**C&VMI**
Ordering decision made by	C	C	V
Bearer of ordering cost	C	C	V
Ownership of stock at customer	C	V	V
Bearer of opportunity cost	C	V	V

Table 4.1: Comparison of the basic characteristics of IS, CI, and C&VMI. C: the customer, V: the vendor

The remainder of this paper is as follows. Section 3 introduces our notation. In Section 4, we develop a model for IS and find the analytical solution for our base case. We then extend the base-case model to incorporate CI (Section 5) and C&VMI (Section 6), and compare those solutions to that of the base case. We provide numerical examples in Section 7, while Section 8 includes a summary and our conclusions.

4.3 Notation

In developing our models for IS, CI, and C&VMI, the following basic notations are used.

A_c: Customer's fixed cost of ordering ($ per order).

A_c consists of the cost of issuing an order, a_o, and the cost per shipment received. The latter does not need to be defined separately.

h_c : Annual cost to carry one unit in stock at customer's retail store ($/unit/year).

This per-item inventory holding cost is composed of h_o, the opportunity cost, and the h_s, the physical storage cost: $h_c = h_o + h_s$.

S : Vendor's fixed production setup cost incurred at the start of each cycle ($ per setup)

a_v : Vendor's cost per shipment release ($ per shipment to the customer)

h_v : Annual cost of holding a unit in inventory at the vendor's production site ($/unit/year)

p : Vendor's annual production rate (units/year)

d : Annual demand rate at the customer (units/year).

The vendor is assumed to have sufficient capacity to meet the customer's demand (i.e., $p \geq d$). In IS, each party pays its own costs as defined above. In CI and C&VMI, portions of A_c and/or h_c are paid by the vendor on behalf of the customer.

In all our formulation, the subscripts v and c refer to the vendor and customer respectively. Moreover, the subscripts 1, 2, and 3 are used both for variables and total costs in IS, CI, and C&VMI respectively.

In the next section, we analyze inventory sourcing, where there is no agreement between vendor and customer. Since it is the traditional way of doing business, we will take IS as the base case to contrast with CI and C&VMI. We note that in our analysis and comparisons of different agreements, the terms "better off" and "worse off" will respectively mean strictly lower and strictly higher costs the party in question.

4.4 Inventory Sourcing (IS)

In IS, the customer first makes replenishment plans based on his costs A_c and h_c, and the end-user demand d. The customer's decisions concern the frequency and in what quantity to order from the vendor. His optimal economic order quantity is q_1 = EOQ = $\sqrt{\frac{2A_c d}{h_c}}$ and his optimal total cost is $TC_{c1} = \sqrt{2A_c d h_c}$. The customer passes the replenishment decision to the vendor, who produces at a rate $p \geq d$.

The vendor, who must satisfy the customer's orders fully, finds her economic production quantity (Q_1) based on her costs of production setup (S), inventory holding (h_v), and shipment release (a_v).

To describe system inventory levels, we assume that the vendor switches from other SKUs and begins producing this item when the customer's inventory level is q_1. Starting at that moment, the vendor produces at a rate p during an interval $T = \frac{kq}{p}$, where k is the number of shipments from the vendor to the customer during the vendor's production cycle.

When the vendor is producing, the total system inventory increases at rate $p - d$. After production stops, the vendor supplies goods to the customer from her stock until that is depleted. When the vendor is not producing, the system-wide inventory decreases at a rate d. We denote the time between successive production runs at the vendor by T' (see Figure 4.3). The vendor's total production quantity in her cycle is $Q_1 = kq_1$. All items carried by the vendor are charged holding costs at a rate h_v.

We can see from Fig. 4.3 that the total average inventory in the system is $q_1 + (p-d)\frac{Q_1}{2p}$, and the vendor's mean stock level is $\frac{q_1}{2} + (p-d)\frac{Q_1}{2p}$. The vendor's total cost per period is then

$$TC_{v1} = d\,[S/Q_1 + a_v/q_1] + h_v\,[q_1 + (1-d/p)\,Q_1]\,/2 \qquad (1)$$

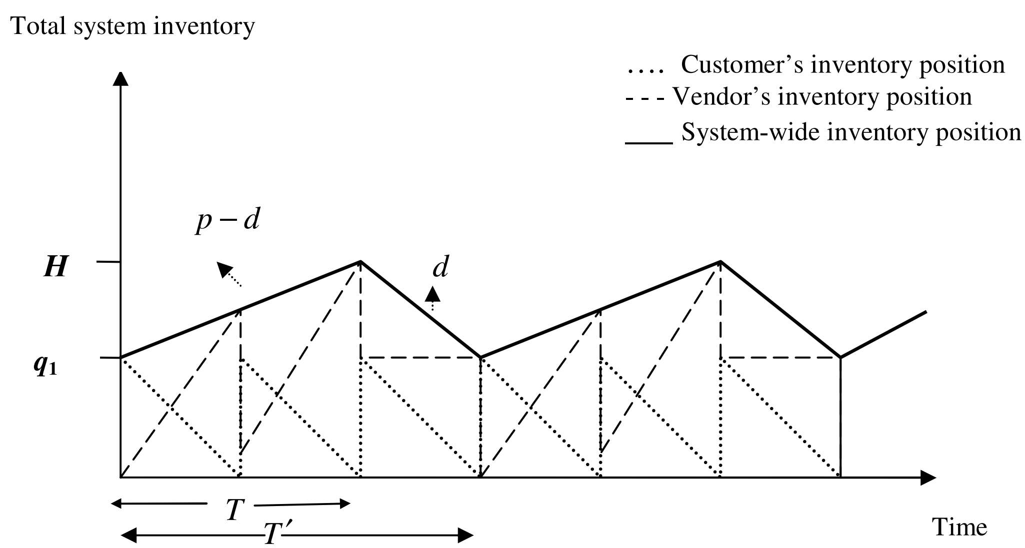

Figure 4.3: Inventory positions over time. H denotes the maximal system inventory, and T' the time between successive production start-ups.

The first two terms in (1) are the total production setup and shipment release costs, and the third and fourth terms are inventory carrying costs. To be able to compare different partnerships analytically, will assume throughout that the number of shipments per cycle is a continuous variable. The optimal value of Q_1, the production quantity in one cycle, is then $\sqrt{\dfrac{2Sd}{h_v(1-d/p)}}$ = EPQ. We remark that since TC_{v1} is a strictly convex function in the interval $(0, \infty)$, the optimal integer value for k is

$$k^{\text{int}} = \underset{TC_{v1}(k^*)}{Min}\left\{ \lfloor EPQ/q_1 \rfloor, \lceil EPQ/q_1 \rceil \right\}.$$

Based on the optimal values of Q_1 and q_1, the vendor's optimal cost is

$$TC_{v1} = \sqrt{2Sdh_v(1-d/p)} + \frac{da_v}{\sqrt{2A_c d/h_c}} + h_v\sqrt{2A_c d/h_c} \Big/ 2$$

Setting $\gamma = \dfrac{a_v}{A_c}$ and $\phi = \dfrac{h_v}{h_c}$, and denoting $C' = \sqrt{2Sdh_v(1-d/p)}$, the total costs for the vendor can be rewritten as $TC_{v1} = C' + (\gamma+\phi)\sqrt{\dfrac{dA_c h_c}{2}}$. The system-wide cost under inventory sourcing ($TC_{c1} + TC_{v1}$) is therefore $TC_1 = C' + [1 + (\gamma+\phi)/2]\sqrt{2A_c dh_c}$.

The preceding model developed for the base case assumed no agreement between customer and vendor. When there is a CI or C&VMI agreement, its benefits will be reckoned with respect to total costs found under inventory sourcing. The first type of agreement, CI, is the topic of next section.

4.5 Consignment Inventory (CI)

The customer maintains control over the timing and quantity of orders under a CI agreement, and pays A_c every time he places an order. However, he does not incur the opportunity-cost portion of carrying inventory, since the vendor owns the goods at the customer's premises until they are used.

For i = 1 and 2, let ε_i ($0 < \varepsilon_i < 1$) denote the ratios $\varepsilon_1 = \frac{h_s}{h_c}$ and $\varepsilon_2 = \frac{h_o}{h_c}$ (where $\varepsilon_1 + \varepsilon_2 = 1$) of portions of the customer's inventory holding cost per item (h_c) under IS. The customer's total cost in CI is the sum of ordering and pyhsical storage costs: $TC_{c2} = \frac{A_c d}{q_2} + \frac{1}{2} h_s q_2$. Based upon those costs, his optimal order size is $q_2 = \sqrt{\frac{2A_c d}{h_s}} = \frac{1}{\sqrt{\varepsilon_1}} q_1$ which is strictly greater than q_1 since $\varepsilon_1 < 1$. His optimal total cost under CI is then $\sqrt{2A_c dh_s} = \sqrt{\varepsilon_1}\, TC_{c1}$, which is strictly less than TC_{c1}. Therefore, the customer is always better off under CI when compared to IS.

The vendor, who bears the opportunity cost of goods stored at the customer, faces less-frequent shipments under CI than IS. (We assume for now that when the vendor orders on behalf of the customer, there is no "efficiency factor," that is, she pays the same opportunity cost h_o as the customer.) If we denote the vendor's production batch size by Q_2, then,

$$TC_{v2} = \frac{Sd}{Q_2} + \frac{a_v d}{q_2} + \frac{1}{2} h_v [q_2 + (1 - d/p) Q_2] + \frac{1}{2} h_o q_2$$

Since backordering is not allowed, the vendor's optimal production batch size is $max\left\{q_2, \sqrt{\frac{2Sd}{h_v(1-d/p)}}\right\}$. We assume that $\sqrt{\frac{S}{h_v(1-d/p)}} > \sqrt{\frac{A_c}{h_s}}$, and hence her optimal production quantity is $Q_2 = \sqrt{\frac{2Sd}{h_v(1-d/p)}} = Q_1$. Then,

$TC_{v2} = C' + \frac{a_v d}{q_2} + \frac{1}{2}h_v q_2 + \frac{1}{2}h_o q_2$ (where C' is as defined in IS). We can also write this cost as $TC_{v2} = C' + \frac{1}{\sqrt{\varepsilon_1}}\sqrt{\frac{A_c dh_c}{2}}(\varepsilon_1\gamma + \phi + \varepsilon_2)$. Recall from IS that $TC_{v1} = C' + (\gamma + \phi)\sqrt{A_c dh_c / 2}$. The vendor is better off under CI if and only if $TC_{v1} > TC_{v2}$. This requires (since $\sqrt{A_c dh_c / 2}$ and $\frac{1}{\sqrt{\varepsilon_1}}$ >0) :

$\sqrt{\varepsilon_1}(\gamma + \phi) > (\varepsilon_1\gamma + \phi + 1 - \varepsilon_1)$. Then, $\gamma(\sqrt{\varepsilon_1} - \varepsilon_1) > \phi(1 - \sqrt{\varepsilon_1}) + (1 - \sqrt{\varepsilon_1})(1 + \sqrt{\varepsilon_1})$. Since $1 - \sqrt{\varepsilon_1} > 0$,

$$\sqrt{\varepsilon_1}\gamma > \phi + 1 + \sqrt{\varepsilon_1} \qquad (2)$$

Proposition 1: A necessary condition for the vendor to be better off in a CI agreement is $\gamma > \phi + 2$

Proof: We see in (2) that $\sqrt{\varepsilon_1}(\gamma - 1) > \phi + 1$. Since ϕ and ε_1 are greater than zero, $(\gamma - 1)$ must be greater than zero for the inequality to hold. Therefore, $\gamma > 1$ and we rewrite (2) as $\sqrt{\varepsilon_1} > \frac{\phi + 1}{\gamma - 1}$. Since ε_1 (and thus $\sqrt{\varepsilon_1}$) is less than one, $1 > \frac{\phi + 1}{\gamma - 1}$. Hence,

$\gamma > \phi + 2$. ■

Proposition 1 states that the vendor will be better off under a CI agreement if her cost per shipment released is greater than $2A_c + \frac{h_v}{h_c}A_c$, where A_c is the customer's ordering cost. We also see that (2) is more likely to hold as the ratio h_v / h_c decreases. Consider, for example, an inventory sourcing agreement where the vendor delivers the goods to the customer's premises and pays transportation costs. Let a_v' be the vendor's total fixed cost of transporation and shipment released per vehicle dispathced from her premises. (Note that a_v'

can replace a_v in the models without changing the nature of analysis.) That a_v' is expected to be much greater than the customer's cost per shipment-received.

Moreover, it is likely that some of the vendor's shipment costs are passed on to the customer through an increased price per item. Hence, his inventory holding cost can be much higher than the vendor's. A consignment agreement in such a setting is then more likely to create benefits for both parties.

What happens if condition (2) does not hold? There are two possible cases:

(i) CI achieves system-wide cost savings where the customer is no worse off but the vendor is worse off. In practice, there is recourse for the vendor if this happens: If the vendor has sufficient bargaining power, she may be able to negotiate a better price. Alternatively, if the vendor does not have this power, she may simply accept the terms to maintain her business with the customer.

(ii) The System-wide cost is greater in CI than in IS. Then, it is in neither party's interest to change the traditional way of doing business.

To explore these two possible situations, we formulate the total cost under a CI agreement and compare it with inventory sourcing. The system-wide cost under CI is

$$TC_2 = C' + \frac{1}{2\sqrt{\varepsilon_1}}\sqrt{2A_c dh_c}\,(\varepsilon_1\gamma + \phi + \varepsilon_2) + \sqrt{\varepsilon_1}\sqrt{2A_c dh_c}$$

Recall from IS that $TC_1 = C' + \frac{1}{2}(\gamma + \phi)\sqrt{2A_c dh_c} + \sqrt{2A_c dh_c}$

Therefore, CI leads to system-wide cost savings if $TC_1 > TC_2$, which requires

$$\left(\sqrt{2A_c dh_c}\right)\left(\frac{1}{2}(\gamma + \phi) + 1\right) > \left(\sqrt{2A_c dh_c}\right)\left(\frac{1}{2\sqrt{\varepsilon_1}}(\varepsilon_1\gamma + \phi + 1 - \varepsilon_1) + \sqrt{\varepsilon_1}\right) \qquad (3)$$

Proposition 2: A necessary condition to achieve system-wide cost savings under CI is $\phi < \gamma$.

Proof: Since $\sqrt{2A_c dh_c}$ and $\sqrt{\varepsilon_1}$ are greater than zero, the following sequence of inequalities must be satisfied for (3) to hold:

$$\sqrt{\varepsilon_1}(\gamma+\phi+2) > (\varepsilon_1\gamma+\phi+1-\varepsilon_1)+2\varepsilon_1$$

$$\Rightarrow \sqrt{\varepsilon_1}(\gamma+1)+\sqrt{\varepsilon_1}(\phi+1) > \varepsilon_1(\gamma+1)+(\phi+1)$$

$$\Rightarrow \sqrt{\varepsilon_1}(\gamma+1)(1-\sqrt{\varepsilon_1}) > (\phi+1)(1-\sqrt{\varepsilon_1}) .$$

Since $(1-\sqrt{\varepsilon_1}) > 0$, $\sqrt{\varepsilon_1} > \frac{\phi+1}{\gamma+1}$. Since $1 > \varepsilon_1$, $1 > \frac{\phi+1}{\gamma+1}$. Therefore, it is necessary that $\phi < \gamma$. ■

Proposition 2 implies that if the vendor is relatively more efficient in inventory holding costs than for shipment release costs, it is more likely she achieves costs savings under CI. Intuitively, the customer's replenishment quantities increase under CI compared to IS. That increase can be beneficial for the vendor, who prefers fewer shipments if her cost per shipment relase is high.

This concludes the analytical results for a *basic* CI agreement, where the vendor pays exactly the same opportunity cost per item, h_o, that the customer pays in IS. Also, we assume that the wholesale price of an item does not change when the type of sourcing is CI. A summary of our findings is presented in Table 4.2 (note that $m = (\phi+1)/\sqrt{\varepsilon_1}$).

In the next two subsections, we will analyze the impacts of the vendor's efficiency on the opportunity cost of an item, and of cost sharing through a wholesale price adjustment.

Necessary and Sufficient Condition	Benefits under CI compared to IS		
	Customer	**Vendor**	**Supply Chain**
$m-1<\gamma<m+1$	Better off	Worse off	Better off
$\gamma=m+1$	Better off	No worse off	Better off
$\gamma>m+1$	Better off	Better off	Better off
$\gamma=m-1$	Better off	Worse off	No worse off
$\gamma<m-1$	Better off	Worse off	Worse off

Table 4.2: Summary of conditions when CI is beneficial for the customer, the vendor, and the whole system; $m=(\phi+1)/\sqrt{\varepsilon_1}$

4.5.1 Impacts of the vendor's efficiency factor

We previously assumed that the vendor pays h_o on behalf of the customer in a CI agreement. However, various considerations might create a situation where the capital costs of holding inventory for the vendor and the customer may not be the same. For example, an organization's capabilities in financing, and the firm's relative power in industry, can make tremendous changes in capital costs.

In a CI agreement, let $\beta_2 h_o$ be what the vendor pays per unit held at the customer's premises. β_2 > 0 represents the vendor's capital cost efficicency compared to the customer. We will now determine how this parameter changes our cost analysis.

Whatever the value of β_2, the customer's order quantity and total cost are $\frac{1}{\sqrt{\varepsilon_1}}q_1$ and $\sqrt{\varepsilon_1}\,TC_{c1}$ respectively. However, the vendor's total cost is now $TC_{v2}=C'+\frac{a_v d}{q_2}+\frac{1}{2}h_v q_2+\frac{1}{2}\beta_2 h_o q_2$, which can be written as

$$TC_{v2}=C'+\frac{1}{\sqrt{\varepsilon_1}}\sqrt{\frac{A_c d h_c}{2}}\left(\varepsilon_1\gamma+\phi+\beta_2\varepsilon_2\right).$$

Comparison of TC_{v2} and TC_{v1} shows that the vendor is better off if the following sequence of inequalities holds.

$\sqrt{\varepsilon_1}(\gamma+\phi) > \varepsilon_1\gamma+\phi+\beta_2(1-\varepsilon_1)$

$\Rightarrow \sqrt{\varepsilon_1}\gamma\left(1-\sqrt{\varepsilon_1}\right) > \phi\left(1-\sqrt{\varepsilon_1}\right)+\beta_2\left(1-\sqrt{\varepsilon_1}\right)\left(1+\sqrt{\varepsilon_1}\right)$. Since $1-\sqrt{\varepsilon_1}>0$,

$\sqrt{\varepsilon_1}\gamma > \phi+\beta_2(1+\sqrt{\varepsilon_1})$. A necessary condition for this inequality to be satisfied is $\gamma > \beta_2$.

We then have $\sqrt{\varepsilon_1} > \dfrac{\phi+\beta_2}{\gamma-\beta_2}$.

Similar to the proof of Proposition 1, we now see that a neccesary condition for the vendor to be better off is $\gamma > \phi + 2\beta_2$.

The system-wide costs now become

$$TC_2 = C' + \sqrt{\frac{A_c d h_c}{2}}\left(\frac{1}{\sqrt{\varepsilon_1}}(\varepsilon_1\gamma+\phi+\beta_2\varepsilon_2)+2\sqrt{\varepsilon_1}\right).$$

As compared to TC_1, we see that system-wide cost savings are achieved when the following holds: $\sqrt{\varepsilon_1} > \dfrac{\phi+\beta_2}{\gamma+2-\beta_2}$. Again as in the proof of Prospostion 2, it is necessary that $\gamma+2 > \phi+2\beta_2$.

The above analysis shows that the system-wide costs, as well as the vendor's costs improve as β_2, the vendor's cost factor, gets smaller. A CI agreement is more promising for both parties when the vendor can develop efficiencies in the opportunity cost of capital.

Even if the vendor is unable to develop these efficiencies, CI can create a situation where there is *potential* to lower system-wide costs, but not at the expense of one party. We call this a "potentially efficient system". We will examine this in detail in the next subsection.

4.5.2 Cost Sharing

We showed that, compared to IS, the customer is always better off in a CI agreement. Consequently, a potentially efficient system in CI is a situation where the vendor is worse off but there are system-wide costs savings.

A potentially efficient system can be turned into an efficient one by some sort of an incentive offered by the customer to transfer a portion of his benefits to the vendor. When CI is applied, it is customary in industry that the vendor be allowed to increase the unit price to share total savings. Without getting into details on cost-sharing research, we briefly explain how this could work.

Let c be the original price per item paid by the customer. The vendor suggests a price increment over c in order to make CI beneficial to herself. Let y^{high} be the maximum percentage increase in price acceptable to the customer.

$$\frac{y^{high}cd}{100} = \left(1-\sqrt{\varepsilon_1}\right)\sqrt{2A_c dh_c}\text{ , which means } y^{high} = \frac{100}{c}\left(1-\sqrt{\varepsilon_1}\right)\sqrt{\frac{2A_c h_c}{d}}$$

Without information sharing between parties, the customer may be unsure that a price increase is in his best interest (e.g. when he receives an equal, or even smaller, share of system-wide savings due to CI).

We now determine the smallest price increment acceptable to the vendor, the value that makes her no worse off than under IS. We assume that inequality (2) does not hold; this is why the vendor is motivated to ask for a price change. Taking $\beta_2 = 1$ results in

$$\frac{y^{low}cd}{100} = \sqrt{\frac{A_c d h_c}{2}}\left(\frac{\varepsilon_1\gamma + \phi + \varepsilon_2}{\sqrt{\varepsilon_1}} - (\gamma + \phi)\right)\text{, which means}$$

$$y^{low} = \frac{100}{c}\sqrt{\frac{A_c h_c}{2d}}\left(\frac{\varepsilon_1\gamma + \phi + \varepsilon_2}{\sqrt{\varepsilon_1}} - (\gamma + \phi)\right)$$

Note that the maximum price increase the customer will accept is y^{high}, and it is the upper bound on the price increase that would erase his benefits under CI. On the other hand, y^{low} is the lower bound that would compensate the vendor for her increase in costs, but still leave the customer with some benefit. Therefore, when CI creates a potentially efficient system, a wholesale price increment $\in (y^{low}, y^{high}]$ will make the vendor willing to accept the CI agreement rather than inventory sourcing. The customer will be in favour of CI as long as price increments are in the range $[y^{low}, y^{high})$.

Another means of creating possible cost savings for both vendor and customer may be the use of CI and VMI combined. While CI always benefits the customer, VMI has the potential of creating benefits for the vendor. CI plus VMI will be the subject of next section.

4.6 Consignment and Vendor Managed Inventory (C&VMI)

In a C&VMI agreement, the vendor owns the goods at the customer's location until they are sold, but also manages the ordering on behalf of the customer. As for her associated costs, the vendor pays h_o per item stored at the customer, and a_o for every order she places on his behalf. The customer is then exempt from those expenses. Taking these changes into consideration, we now formulate the total costs under a C&VMI agreement for the vendor and the customer. Those totals will subsequently be compared to the costs under IS.

The vendor's total cost in C&VMI is

$$TC_{v3} = \frac{Sd}{Q_3} + \frac{a_v d}{q_3} + \frac{1}{2} h_v [q_3 + (1 - d/p) Q_3] + \frac{a_o d}{q_3} + \frac{1}{2} h_o q_3 \qquad (4)$$

This total cost is also equal to $C' + (a_v + a_o)\frac{d}{q_3} + \frac{1}{2}(h_v + h_o) q_3$ where C' is as explained in IS. Using the ratios defined previously, we can re-write the vendor's total cost as

$$TC_{v3} = C' + (\gamma + \delta_1)\frac{A_c d}{q_3} + \frac{1}{2}(\phi + \varepsilon_2) h_c q_3$$

Therefore, the optimal order quantity determined by the vendor on behalf of the customer is

$$q_3 = \sqrt{\frac{2(\gamma + \delta_1) A_c d}{(\phi + \varepsilon_2) h_c}} = \sqrt{\frac{\gamma + \delta_1}{\phi + \varepsilon_2}}\, q_1$$

Incorporaing the optimal order quantity in (4), we get $TC_{v3} = C' + \sqrt{\gamma + \delta_1}\sqrt{\phi + \varepsilon_2}\sqrt{2 A_c d h_c}$.

Recall for IS that $TC_{v1} = C' + (\gamma + \phi)\sqrt{\frac{d A_c h_c}{2}}$. Therefore, the vendor's cost under C&VMI is less than her cost of IS if $TC_{v3} < TC_{v1}$, which reduces to

$$\sqrt{\gamma + \delta_1}\sqrt{\phi + \varepsilon_2} < \frac{\gamma + \phi}{2} \qquad (5)$$

After some algebra, (5) can be written in the form

$$4[(\gamma + \delta_1)\varepsilon_2 + \phi\delta_1] < (\gamma - \phi)^2 \qquad (6)$$

The right-hand side of (6) is zero when $\gamma = \phi$. Therfore, C&VMI would create benefits for the vendor if she has, compared to the customer, efficiency or inefficiency either in her ordering or inventory holding, but not in both costs. That is, the vendor can make better use of the ordering authority created by C&VMI when she has an advantage or disadvantage in either her ordering or inventory holding costs.

For example, if the vendor's ordering cost is too high compared to the customer but her inventory holding cost per item is around what the customer pays, then the vendor can ship larger quantities to decrease her total ordering cost. This holds true if her inventory holding cost is lower but there is not a clear efficiency in her ordering costs, relative to the customer. On the other hand, if the vendor's inventory holding cost is too high, she can replenish the customer frequently in small quantities to achieve cost savings.

In the meantime, the vendor's costs associated with C&VMI influence the benefits that the agreement can create for her. The vendor's costs under C&VMI increase linearly in the ratios δ_1 and ε_2. Hence, as those parameters they get lower, it is more likely that the vendor achieves costs savings, since there is a decrease in the left-hand side of (6).

Now, the customer would accept CI plus VMI if his costs under this agreement were not higher than his costs in IS. The customer's total cost under C&VMI is

$$TC_{c3} = \frac{(A_c - a_o)d}{q_3} + \frac{1}{2}h_s q_3 \;=\; \frac{(1-\delta_1)A_c d}{q_3} + \frac{1}{2}\varepsilon_1 h_c q_3 .$$

The optimal ordering quantity q_3 was determined by the vendor on behalf of the customer. Incorporating that optimal quantity in the customer's cost function yields

$$TC_{c3} = \sqrt{\frac{A_c d h_c}{2}}\left(\frac{(1-\delta_1)(\phi+\varepsilon_2)+\varepsilon_1(\gamma+\delta_1)}{\sqrt{(\gamma+\delta_1)(\phi+\varepsilon_2)}}\right).$$

The customer's total cost in IS is $TC_{c1} = \sqrt{2A_c d h_c}$. C&VMI is thus beneficial for the customer ($TC_{c3}<TC_{c1}$) if

$$(1-\delta_1)(\phi+\varepsilon_2)+\varepsilon_1(\gamma+\delta_1) < 2\sqrt{(\gamma+\delta_1)(\phi+\varepsilon_2)} \tag{7}$$

Let $\sqrt{\gamma+\delta_1} = m_1$ and $\sqrt{\phi+\varepsilon_2} = m_2$. Then (7) reduces to $\dfrac{(1-\delta_1)}{m_1/m_2} + \varepsilon_1\, m_1/m_2 < 2$.

Note that $(1-\delta_1)$ and ε_1 are both less than one. If the vendor's replenishment quantity q_3 is higher than the customer's replenishment quantity q_1 under IS (m_1/m_2 > 1), it is more likely for the customer to achieve cost savings under C&VMI when ε_1 is low. That is to say, the customer would not mind large order quantities as long as his physical storage cost per item is low.

Similarly, the customer can still achieve cost savings when the vendor replenishes him very frequently (m_1/m_2 < 1), if his cost per shipment-received is not high. In general, the customer is more likely to achieve costs savings under C&VMI because he does not pay the opportunity cost of items in stock nor the cost of placing orders. We can now check whether *both* parties can be better off under C&VMI.

Proposition 3: If $2\delta_1\varepsilon_2 > \varepsilon_2(1-\gamma)+\delta_1(1-\phi)$, then C&VMI can create an efficient system.

Proof: Inequalities (5) and (7) together imply that $(1-\delta_1)(\phi+\varepsilon_2)+\varepsilon_1(\gamma+\delta_1) < \gamma+\phi$, which is a necessary (but not sufficient) condition for both parties to be better off compared to IS. With some algebra, this condition reduces to $\delta_1\phi+2\delta_1\varepsilon_2+\varepsilon_2\gamma > \varepsilon_2+\delta_1$, and then to $2\delta_1\varepsilon_2 > \varepsilon_2(1-\gamma)+\delta_1(1-\phi)$. ■

We see in the proof of Proposition 3 that this necesarry condition (required to achieve an efficient system) holds when $\gamma > 1$ and $\phi > 1$, and also when $\gamma >> 1$ or $\phi >> 1$. The latter is more likely the case where both parties are better off. This can be explained by our analytical results on C&VMI for the vendor and customer.

We observed previously that the vendor can make use of the C&VMI agreement to offset inefficiency in one of her costs. Depending on which cost parameter is high, the vendor can decrease or increase the order quantity to achieve cost savings. That order quantity is also acceptable to the customer, as long as the costs from which he is exempt (cost of placing orders and opportunity cost of inventory) compensate his increased costs resulting from ordering decisions made by the vendor for him.

It may be less likely to achieve an efficient system than a potentially efficient system which can be worked out to satisfy both parties. Recall that a system is potentially efficient if there are system-wide cost savings.

Proposition 4: C&VMI creates system-wide costs savings relative to IS if

$$\frac{1+\phi}{1+\gamma} < \frac{m_2}{m_1} < 1, \text{ or } \frac{1+\gamma}{1+\phi} < \frac{m_1}{m_2} < 1.$$

The proof of Proposition 4 is provided in Appendix B. We will use numerical examples in the next section to see when C&VMI creates a potentially efficient system. Those examples will also highlight the analytical results found in the inventory sourcing and CI models considered. We note in passing that the cost sharing argument discussed for CI in Sec. 5.2 can also be applied to C&VMI.

4.7 Numerical Examples

In this section, we provide figures to contrast IS, CI, and C&VMI numerically when certain parameters are varied. In all those examples, A_c = \$100 per order, h_c=\$1.5 per item stored, d = 1300 items/year, and p = 1600 items/year. We do not assume any efficiency of the vendor over h_o or a_o in case of a CI or C&VMI agreement.

Figures 4.4 – 4.6 test the impact of γ on different sourcing options. In those examples, we use the fixed values ε_1 = 0.4, δ_1 = 0.1, and ϕ = 0.8, while γ is between (0, 5]. In the next three figures, we vary ϕ over the interval (0, 5], but set γ = 1.5, ε_1 = 0.4, δ_1 = 0.1. In

Figures 4.10 – 4.12, we change the value of ε_1 over (0, 1) while γ = 1.5, ϕ = 0.8, and δ_1 = 0.1. In the last two figures, δ_1 varies between (0, 1), γ = 1.5, ϕ = 0.8, and ε_1 = 0.4.

In line with our analytical results, we observe in Figure 4.4 that the customer's cost savings under CI is fixed, yet the system-wide and the vendor's savings increase linearly as γ increases. CI is benefical for the vendor when her cost per shipment is at least 3.8 times the customer's cost per order. System-wides savings can be achieved for lower values of γ.

We see in Figure 4.5 that cost savings are possible for the vendor when γ is very low or very high. When $\gamma \leq 0.02$, the vendor replenishes the customer frequently to save on inventory holding costs. However, the large number of shipments (when γ is small) increases the customer's and the system-wide total cost compared to IS. On the other hand, higher values of γ enable system-wide cost savings; both parties are better off under C&VMI when $\gamma \geq 4.0$.

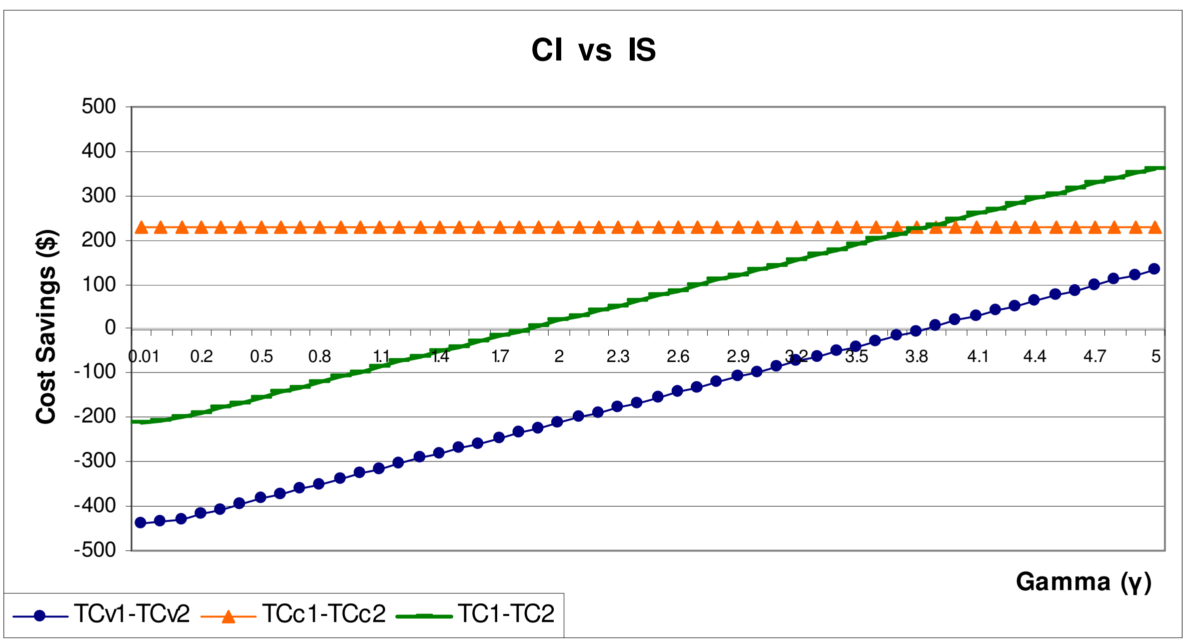

Figure 4.4: CI versus IS; ε_1 = 0.4, δ_1 = 0.1, and ϕ = 0.8; γ is between (0, 5]

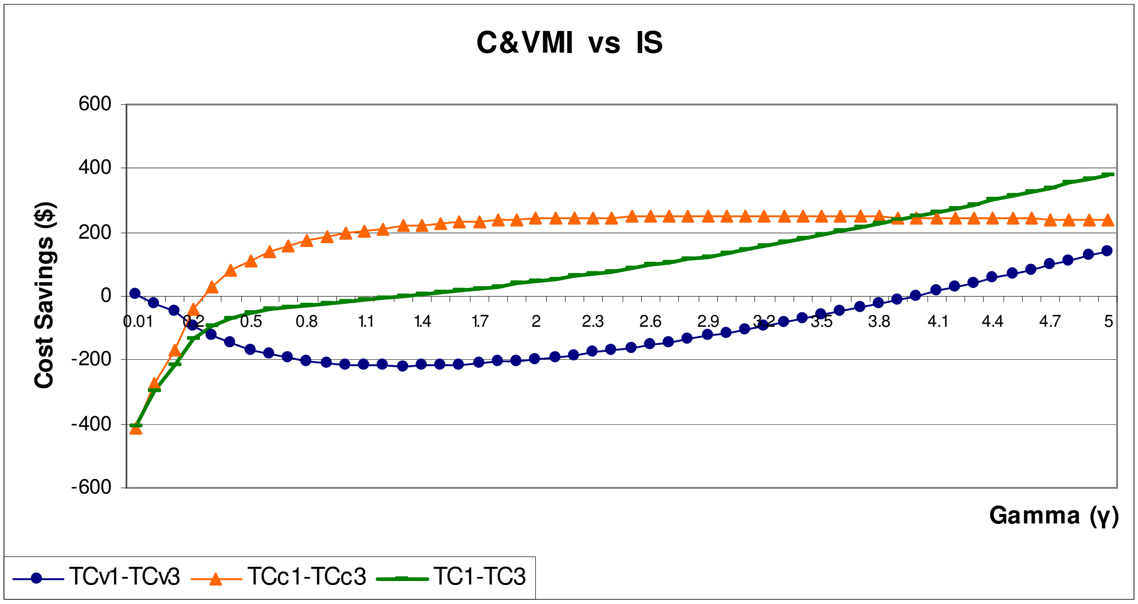

Figure 4.5: C&VMI versus IS; ε_1 = 0.4, δ_1 = 0.1, and ϕ = 0.8; γ is between (0, 5]

When we compare C&VMI to CI for varying values of γ (Fig. 4.6), we observe that C&VMI almost always generates more system-wide savings, although one party may sometimes be worse off. (Compared to CI, the vendor is worse off when $2.3 < \gamma < 4.7$, the customer is worse off when $\gamma < 1.6$). Then, if a cost-sharing that splits the benefits equally can be negotiated, C&VMI is most of the time a better option for both actors. We also see in Figure 4.6 that as γ increases, both parties become indifferent between C&VMI and CI (relative cost savings are around zero). This is logical: Compared to IS, the customer under CI orders larger quantities, and this is what the vendor would do under C&VMI if her shipment costs were high.

In Proposition 1, we stated that $\gamma > \phi + 2$ is a necessary condition for the vendor to be better off under CI. Therefore, the vendor never achieves cost savings in Figure 4.7, where γ = 1.5 and ϕ varies between (0, 5]. As ϕ increases, the vendor's total cost increases.

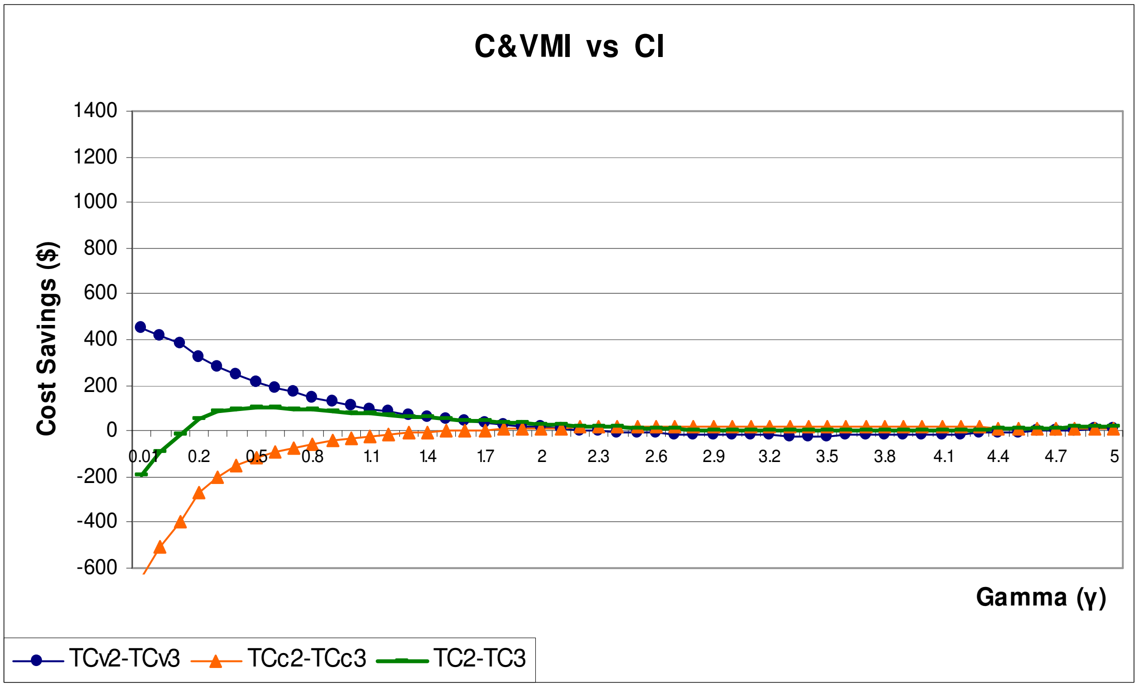

Figure 4.6: C&VMI versus CI; $\varepsilon_1 = 0.4$, $\delta_1 = 0.1$, and $\phi = 0.8$; γ is between (0, 5]

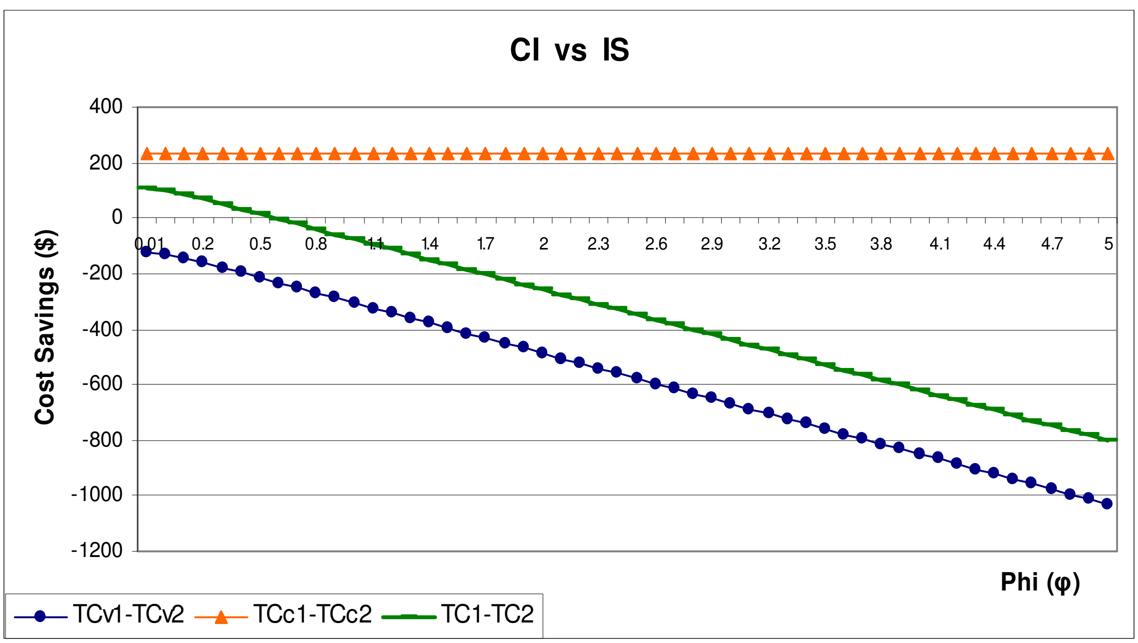

Figure 4.7: CI versus IS; $\varepsilon_1 = 0.4$, $\delta_1 = 0.1$, $\gamma = 1.5$; ϕ is between (0, 5]

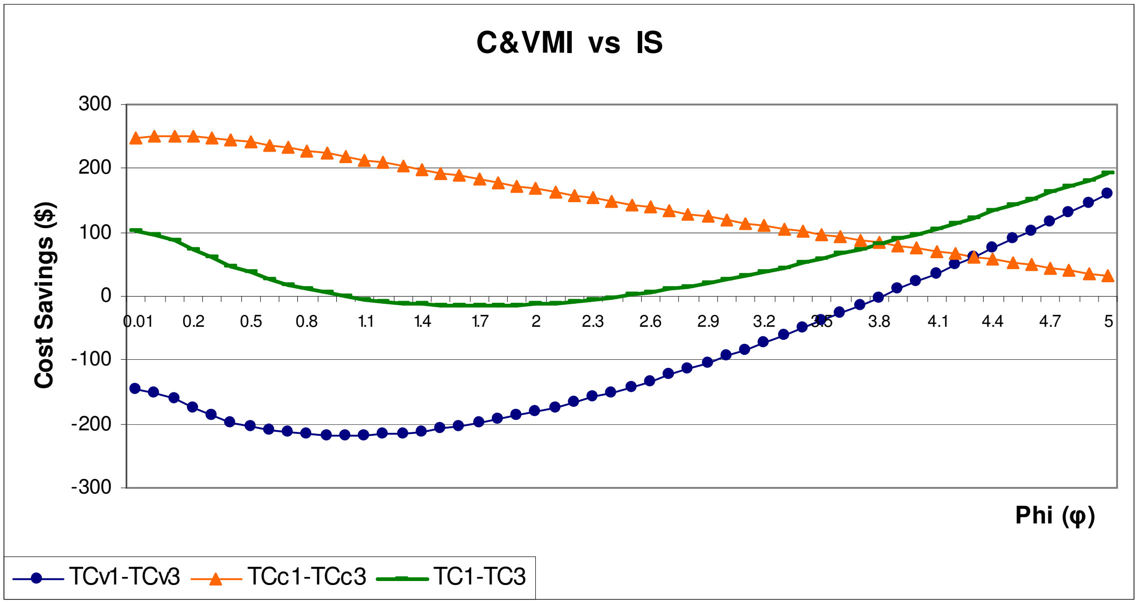

Figure 4.8: C&VMI versus IS; $\varepsilon_1 = 0.4$, $\delta_1 = 0.1$, $\gamma = 1.5$; ϕ is between (0, 5]

Under a C&VMI agreement, however, it is possible for all parties to achieve cost savings when ϕ is high enough, namely $\phi \geq 3.7$ (Figure 4.8). As discussed in the analytical formulations, C&VMI becomes an opportunity for the vendor, who is relatively inefficient in inventory holding cost ($\phi >> \gamma$), to decrease her carrying costs by sending frequent shipments.

When we compare C&VMI to CI for varying ϕ values (Figure 4.9), we see that C&VMI becomes a much better option for the vendor and the whole system as ϕ increases. This makes sense since the customer under CI increases the order quantity, which in turn increases the average system inventory. The vendor, on the other hand, prefers more frequent shipments and less inventory when ϕ is high, and she can decide so under C&VMI.

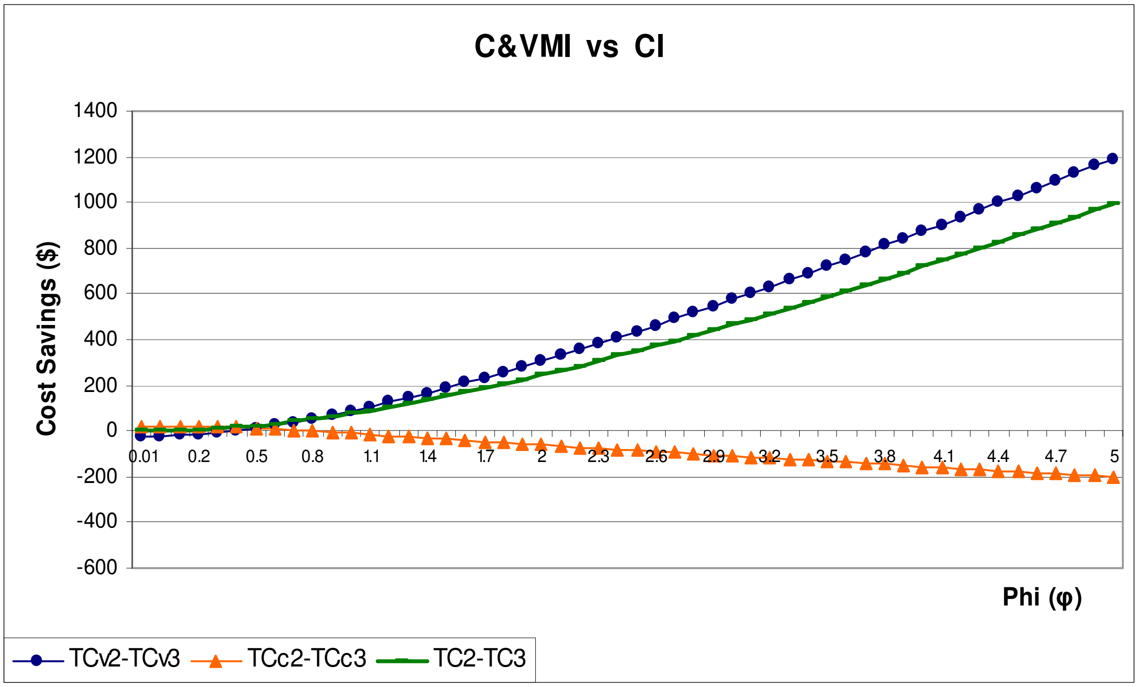

Figure 4.9: C&VMI versus CI; $\varepsilon_1 = 0.4$, $\delta_1 = 0.1$, $\gamma = 1.5$; ϕ is between (0, 5]

In line with our analytical results, we see in Figure 4.10 that a change in the value ε_1 changes all cost savings nonlinearly. As ε_1 aproaches one, the system returns to the costs under IS. While the customer's savings decrease, the vendor's as well as system-wide savings increase as $\varepsilon_1 \rightarrow 1$. We also observe in this example that no ε_1 value creates an efficient system; the customer is always better off. The system is potentially efficient when $\varepsilon_1 \geq 0.52$, but as ε_1 approaches zero, the system-wide and vendor's costs increase enormously.

Figure 4.11 compares C&VMI to IS. As ε_1 increases, the vendor's costs and the customer's savings decrease. An efficient system is never achieved. The vendor does achieve cost savings, but only when $\varepsilon_1 \geq 0.98$. (Compare this to CI in Fig. 4.10 where the vendor *never* achieves cost savings.) System-wide costs in Fig. 4.11 do not change much as ε_1 varies.

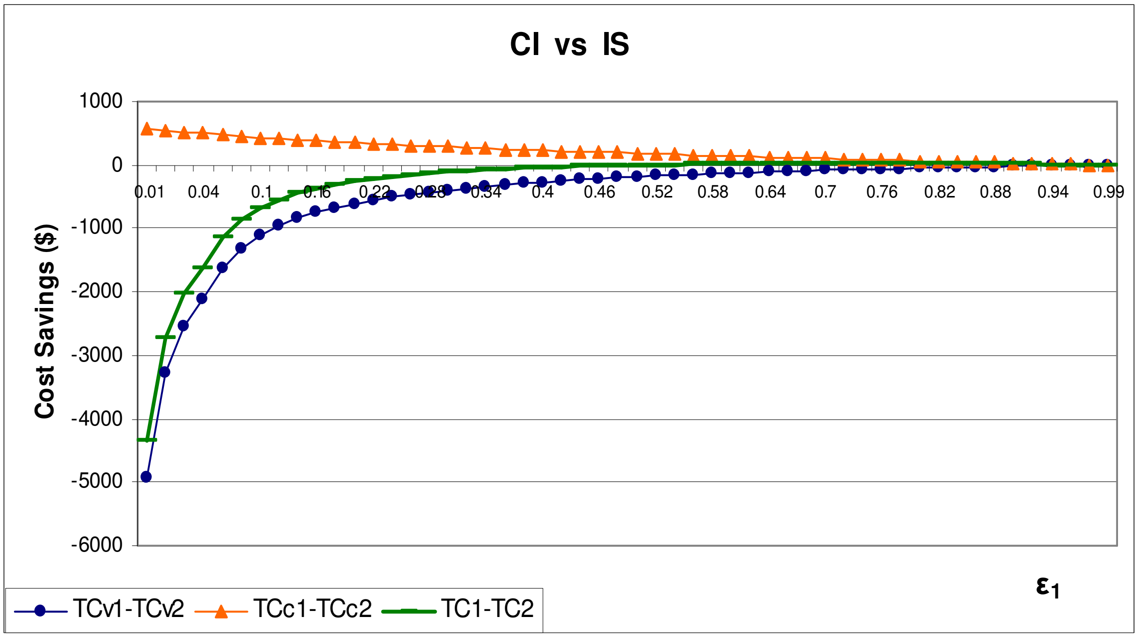

Figure 4.10: CI versus IS; $\delta_1 = 0.1$, $\phi = 0.8$, $\gamma = 1.5$; ε_1 is between (0, 1)

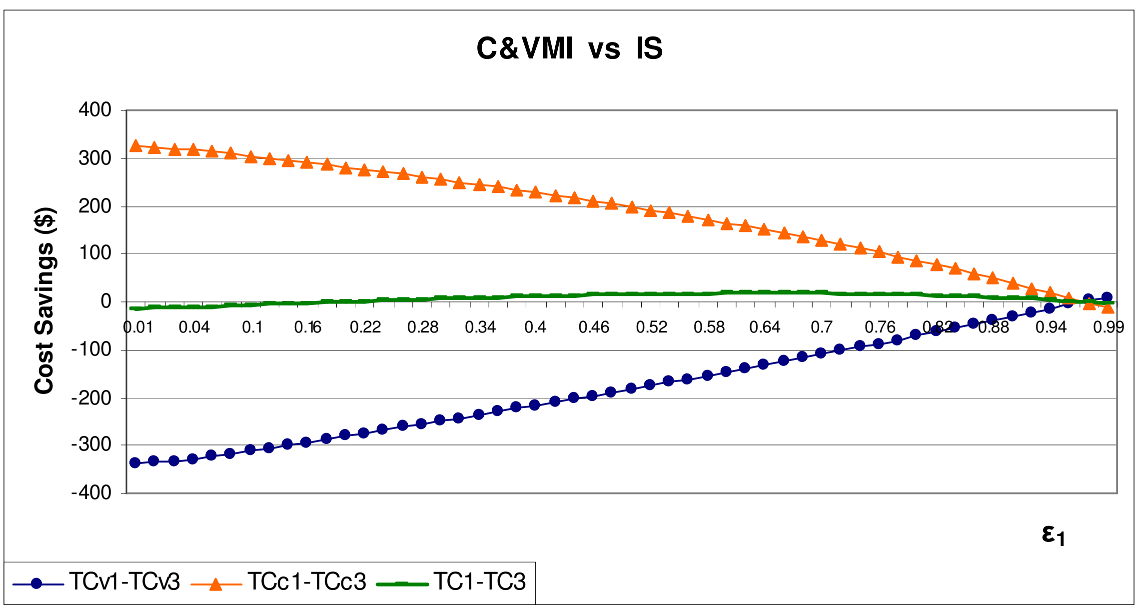

Figure 4.11: C&VMI versus IS; $\delta_1 = 0.1$, $\phi = 0.8$, $\gamma = 1.5$; ε_1 is between (0, 1)

It can be seen in Figure 4.12 that low values of ε_1 make a big difference for the vendor's and stytem-wide costs under C&VMI when compared to CI. However, CI becomes a preferred option for the whole system when $\varepsilon_1 \geq 0.69$, and for the vendor when $0.95 \geq \varepsilon_1 \geq 0.51$. Note that the customer favours CI over C&VMI when $0.41 \geq \varepsilon_1$.

We present only in a single graph (Fig. 4.13) the implication of varying δ_1 values, since they do not influence the costs for IS or CI. When we compare C&VMI to IS in Figure 4.13, we see that the customer's savings and vendor's costs under C&VMI are increasing in δ_1. System-wide savings, on the other hand, do not change much, remaining near zero.

The numerical examples we have provided in this section tested the parameters (δ_1 being the last) that influence the vendor's and the customer's costs in different sourcing options. In the next section, we provide a summary in addition to some conclusions.

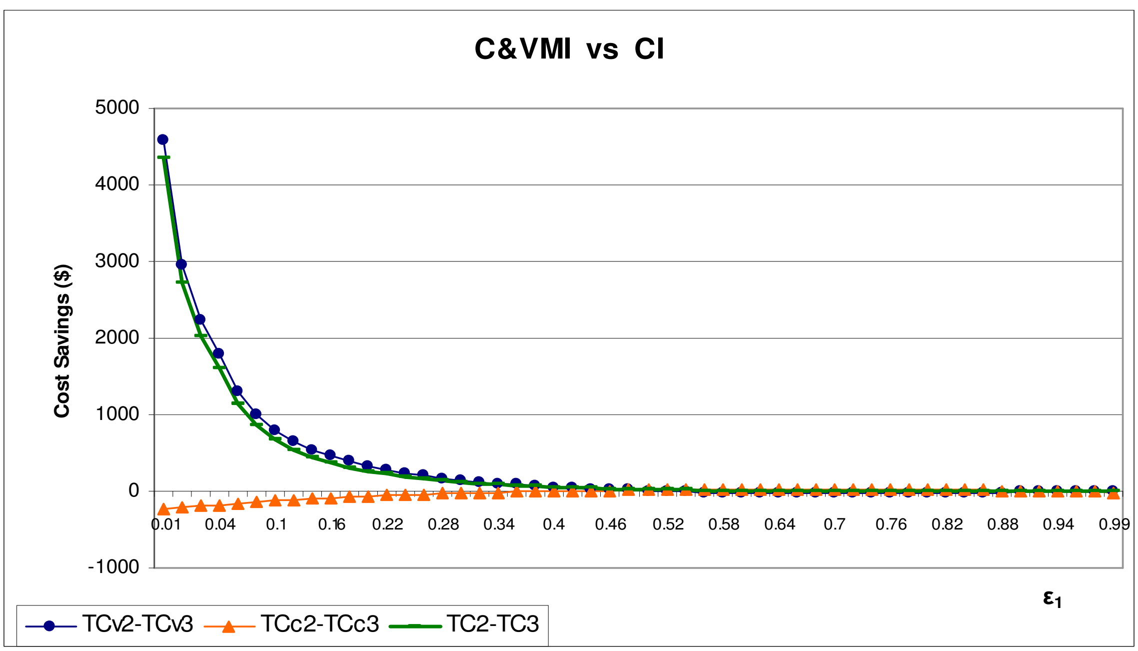

Figure 4.12: C&VMI versus CI; $\delta_1 = 0.1$, $\phi = 0.8$, $\gamma = 1.5$; ε_1 is between (0, 1)

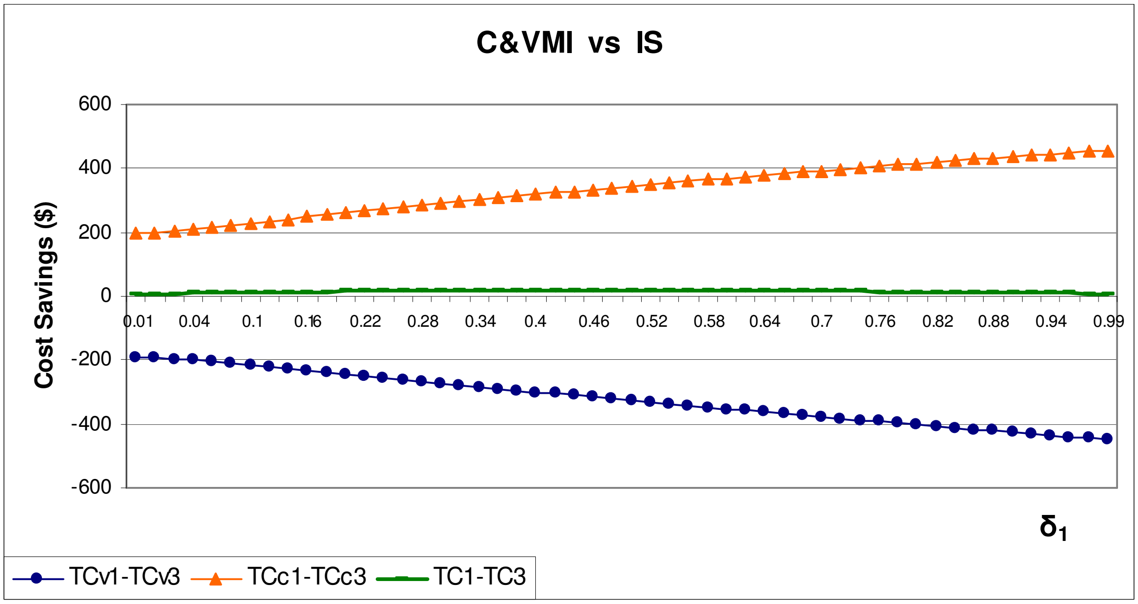

Figure 4.13: C&VMI versus IS; $\varepsilon_1 = 0.4$, $\phi = 0.8$, $\gamma = 1.5$; δ_1 is between (0, 1)

4.8 Summary and Conclusions

In this paper, we studied a case where a customer and vendor initially consider consignment inventory for a single item. Comparing it to our base case, which is inventory sourcing, we obtained analytical conditions under which CI creates benefits for one or more parties. In contrast to the general belief that CI is beneficial only for the customer, we showed that it may be favourable for the vendor as well. Depending on the costs of shipment, and who pays for transportation, CI can be beneficial for both parties.

We showed that if the CI agreement results in a potentially efficient system, it can be turned into an efficient one. To achieve that, we found the minimum and the maximum amounts to which the wholesale price may increase, such that the customer may accept to share his benefits with the vendor. When the system is inefficient under CI, the vendor can offer a C&VMI agreement to realize savings for her and for the system.

We considered the option of CI plus VMI and extended our analysis to find the optimal costs and saving-potentials under that agreement. We showed the vendor can make use of C&VMI to improve her costs in areas in which she is inefficient. Although the customer prefers in general CI rather than C&VMI, and the vendor vice versa, we observed that C&VMI is more likely to generate system-wide cost savings.

This paper provided closed-form analytical results for different sourcing options. We showed that the outcome of any of those options depends on the cost parameters of the parties involved. We identified conditions under which option one is preferred to another. Our findings can help a vendor or a customer decide a priori if CI or C&VMI create benefits for them.

Future research may evolve in two streams. First, it is possible to study the economies of scale created by a C&VMI agreement when there are multiple customers: The vendor, whose goal is to achieve flexibility in production and to reduce her operational costs (such as shipment expenses), offers C&VMI to some of her customers. In the mean time, the customers under that agreement should not be worse off compared to IS.

Secondly, a model can be developed for a customer to choose vendors for CI when there are multiple suppliers of certain items. CI is always beneficial for the customer without any change in the wholesale price. But various suppliers could enforce a price adjustment when CI is offered. In that case, the customer should carefully select the CI-vendors to maximize her savings.

Both of those possible extensions consider multiple customers or vendors. One can simulate the resulting models when end-consumer demand is uncertain. In settings where demand is more stable, it may be possible to find closed-form solutions.

Appendix B

Proof of Proposition 4: Our aim is to see when $TC_3 < TC_1$

$$TC_1 = TC_{c1} + TC_{v1} = C' + \frac{1}{\sqrt{2}}\sqrt{A_c d h_c}\left(2 + \gamma + \phi\right)$$

$$TC_3 = TC_{c3} + TC_{v3} = C' + \frac{1}{\sqrt{2}}\sqrt{A_c d h_c}\left((1-\delta_1)\frac{m_2}{m_1} + (1-\varepsilon_2)\frac{m_1}{m_2} + 2m_1 m_2 \right)$$

Then, $TC_3 < TC_1$ if $(1-\delta_1)m_2^2 + (1-\varepsilon_2)m_1^2 + 2m_1^2 m_2^2 < m_1 m_2 (2+\gamma+\phi)$. After some algebra, this reduces to $(1+\gamma)m_2^2 + (1+\phi)m_1^2 < m_1 m_2 \left[(1+\phi) + (1+\gamma)\right]$, and then to

$(1+\phi)m_1(m_1 - m_2) < (1+\gamma)m_2(m_1 - m_2)$

If $(m_1 - m_2) > 0$, then $\dfrac{1+\phi}{1+\gamma} < \dfrac{m_2}{m_1} < 1$.

If $(m_1 - m_2) < 0$, then $\dfrac{1+\gamma}{1+\phi} < \dfrac{m_1}{m_2} < 1$. ■

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5. Analysis of VMI for a Single Vendor and Multiple Customers under Deterministic, Time-Varying Demands (Essay 3)

Abstract

Vendor Managed Inventory (VMI) is a partnership that enables the vendor to order on behalf of customers. When coupled with consignment inventory (C&VMI), the vendor also owns the goods at the customer's premises until they are used. In this paper, we study these supply chain practices for a vendor and multiple customers who face time-varying, but deterministic, external demand for a single product.

We develop MIP models for different sourcing options, propose a heuristic for the vendor's model under VMI, and a Lagrangian-relaxation method for her model under C&VMI. The two-part heuristic first determines the best set of customers to offer VMI, and then solves the remaining problem of integrated production, customer-replenishment and transportation through decomposition. For customers, we show that it is in their best interest to establish the maximum inventory for vendor-replenishments when VMI or C&VMI are options. A model to determine those levels and a solution algorithm are proposed.

Our heuristics and the Lagrangian-relaxation approach are tested via numerical examples, which reveal that the heuristic for the VMI model performs well. The optimality gap resulting from Lagrangian relaxation for the C&VMI model is reasonable in 32 test problems. We also find that C&VMI is a better option than VMI for any customer. The vendor, on the other hand, prefers C&VMI over VMI only when it results in a greater number of agreements.

5.1 Introduction

A company involved in supply chain operations must accomplish the processes of sourcing (procurement of required materials), making (production of goods using the sourced materials), and delivering (transportation of those goods to customers). In industry, one of the main concerns of every company is to minimize its operational costs while maintaining the best possible efficiency.

Although part of a supply chain, a firm may still focus on optimizing its own costs. In that case, the decisions concerning production, replenishment and transportation are made separately and independently by members of that chain. However, a supply *chain* implies the interaction of its members, even when they have different operational goals. In fact, the performance of that chain depends not only on how well each member manages its operational processes, but also on how well the members coordinate their decisions (Achabal et al. 2000).

Consider a supply chain composed of a vendor who manufactures product, and customers who purchase those goods and deliver them to end consumers. The sourcing plans of customers affect the vendor's manufacturing and delivery decisions. Customers' decisions on the quantity and timing of replenishments may create inflexibility in the vendor's operation, resulting in higher costs for her and the entire supply chain.

Industry regards "coordinated decision making" as working together to maximize the efficiency of the whole supply chain (Karonis, 1997). With the help of information technology, various chain members can collaborate to decrease costs and deliver better value to end consumers. With that common goal in mind, a supply chain becomes a single entity that competes with other supply chains, rather than chain members competing with each other (Marshall et al. 1999). The benefit of coordinated replenishments was estimated to be $30 billion in the grocery industry (Kurt Salmon Associates, 1993) and $14 billion in the foodservice industry (Troyer, 1996).

Such figures have motivated researchers to devise contracts that enable coordination within a supply chain. The main idea is to find the optimal actions for chain members who need to align their decisions to achieve optimal chain costs. The incentive to apply those optimal actions is set by transfer payments. A number of examples in that research area are presented by Cachon (2003).

Although research in supply chain contracts has been extended to various settings, the industry has preferred supply-chain agreements that are more general (better known) and practical (easier to understand and implement). Vendor Managed Inventory (VMI), one such agreement, will be analyzed in this paper for a supply chain containing a single manufacturer and its multiple customers.

VMI, a partnership agreement between vendor and customer, allows the vendor decide when that customer will be replenished, and in what quantity. VMI enables upstream and downstream supply-chain members to coordinate their decisions, while staying independent as separate companies. In an ideal VMI partnership, the customer is relieved of the effort and cost of placing purchase orders. The vendor, who now can be informed about the point-of-sale data and the customer's inventory levels, achieves savings through economies of scale in production and transportation.

VMI has been in use since the late 1980's, and has become an alternative sourcing option that is easy to understand and relatively easy to implement. Successful applications of VMI, including Wal-Mart and Procter & Gamble (Waller et al., 1999), Campbell Soup

Company (Clark, 1994), Barilla SpA (Hammond, 1994), Intel (Kanellos, 1998) and Shell Chemical (Hibbard, 1998) have motivated some other companies to seek cost savings.

Through contacting business firms, we observed that VMI does not belong to a single type of industry. For example, Parmalat Canada, one of the largest dairy producers in Canada, uses VMI with its retailers. Making use of the point of sales data, the company can react quicker to changes in the end-consumer demand (which can be quite *uncertain*), and can make better utilization of its fleet and manufacturing facilities.

On the other hand, the Automation and Drives section of Siemens AG, which produces low-voltage power distributors and industrial controls, uses VMI with some of its suppliers under the name Standard Parts Management and for products with fairly *certain demand*. The company saves some costs of replenishment and inventory control, and strengthens the ties with its VMI suppliers. The literature also identifies implementation of VMI in various sectors. De Toni and Zamalo (2005) and Holmstrom (1998) provide examples for household electrical appliances and food in the grocery sector, respectively.

However, VMI is not always a success. In addition to some failure-stories (Schreibfeder, 1997), there are also claims that vendors are unwillingly forced to a VMI agreement by powerful customers (Burke, 1996), and that VMI is only a way to transfer inventory risks from customers to vendors (Saccomano, 1997). The dispute over whether VMI is beneficial or not, combined with a shortage of quantitative models, causes hesitation for many firms that consider implementing VMI.

Why does VMI work for some companies but fail for some others? In an attempt to answer this question, Gumus et al. (2006) studied the VMI agreement of a vendor and a customer. Under stationary demand, they identified the general conditions under which VMI does or does not succeed.

Although VMI is an agreement between one vendor and a single customer, it is natural that the supplier can offer VMI to a number of its customers. While the outcome of such a decision may not make a big difference for the customers, the vendor may save more through economies of scale. In this paper, we will study a system consisting of a single vendor and multiple customers, under time-varying but deterministic demand. The vendor, who has the choice to offer VMI to one or more of her customers, should determine the *right* set of customers to minimize her total operational costs resulting from production, replenishment of customers, and transportation.

When VMI is coupled with Consignment Inventory (C&VMI), the consequent agreement forces the vendor, in addition to arranging replenishments on behalf of the customer, to own the goods replenished until they are used by that customer. In Section 6, we will look at the C&VMI agreement as an alternative to VMI.

In the traditional way of doing business (which we will call Inventory Sourcing (IS) throughout), the customers order from the vendor, and all parties make decisions separately. We will assume that IS is our base case, and will use the vendor’s and the customers’ costs under it as a benchmark to compare them with the costs under VMI and C&VMI. It is natural that, before any such agreement can be accepted by a customer or the vendor, it must create a lower total cost compared to the base case.

There are two distinct, yet interrelated sets of decisions the vendor has to make when VMI or C&VMI is an option. First, to which of its customer should the vendor offer an agreement? Since she incurs a cost associated with each and every agreement, the vendor should select the right customers and the proper number of agreements. Those decisions should ensure that the vendor, as well as each customer, should have total costs no worse than respective costs under IS. (Throughout this paper, the term "worse off" refers to a "strictly less than" condition, whereas the term "better off" means "strictly greater than").

But this then depends on the second set of decisions: the choices that the vendor makes for her operational processes, namely production, customer replenishment, and transportation. When the vendor offers to make replenishments on behalf of a customer, that customer's main concern should be to evaluate the implications of accepting the agreement. Rationale is that the customer wants to maintain his total cost not higher than what it is under IS.

To classify the topic of this paper, it lies within that area of VMI research containing studies that consider the coordination aspect of VMI or the operational benefits it creates. We use Mixed Integer Programming (MIP) models for the cases we consider, and devise algorithms and heuristics for the vendor and the customer for their operational decisions in a multi-period setting. Hence, in terms of formulation, our study can be considered to be in the multi-echelon dynamic lot-sizing research area.

The literature includes a number of publications on VMI coordination.Cachon (2001) investigates whether VMI can coordinate the supply chain of a single supplier and multiple retailers. He concludes that both the vendor and retailers must also agree to make fixed transfer payments in addition to VMI, and must be willing to share the benefits. Bernstein and Federgruen (2003) show that channel coordination can be achieved under VMI, when there is a constant-demand-rate and a single retailer retains the decision rights on pricing and sales target.

Aviv and Federgruen (1998) assume a VMI agreement that leads to a fully centralized planning model where the vendor minimizes the system-wide costs. They find that information sharing in conjunction with VMI is always more beneficial than information sharing alone. Aside from that, Cachon and Fisher (2000) and Lee et al. (2000) look at how a supplier can use customer-demand information for better sales forecasting and inventory control. These models show significant direct and indirect benefits to the supplier, yet no direct benefits to the retailers.

In our models, the vendor has the choice to implement VMI with a *subset* of its customers, which is new in the VMI literature. We don't consider pricing, nor central decision making in our paper. While the vendor optimizes her operational costs based on the right set of VMI decisions, the customers ensure (by means of adding a constraint to the vendor's model, which will be discussed in the next sections) that their costs do not go up under VMI. As in industry, we assume that VMI is a better choice than IS if none of the parties involved is worse off.

In addition to the impacts of VMI on channel coordination, VMI has also previously been examined in the context of operational benefits that it may create for the vendor. Research in this category focuses on benefits offered by the flexibility that VMI enables, mainly in delivery. Examples include combining routes from multiple origins and consolidating shipments to two or more customers.

Campbell et al. (1998) and Kleywegt et al. (2000) analyze a stochastic inventory-routing problem by a Markov decision process. Both investigate the benefits of allowing the supplier to construct better delivery routes for multiple retailers. Cheung and Lee (2002) compare two information-based supply-chain efforts: Knowledge of retailers' inventory position to coordinate and achieve truck load-shipments, and use of that same information to balance retailers' stocking positions.

In a similar vein, Cetinkaya and Lee (2000) analyze how a vendor under VMI can synchronize inventory and transportation decisions. They assume that the vendor under VMI can hold orders until a suitable dispatch time, at which orders can be economically consolidated. Chaouch (2001) aims at finding the vendor's shipment rate under VMI, provided that the vendor can change shipment frequencies to balance shortage cost.

Previous studies related to the operational benefits of VMI assume that VMI is already implemented, and focus on the vendor's cost savings when she has control over the customers. The impacts of VMI on the customers are not considered. Moreover, those studies do not address the question of whether VMI should be in use at all. In our paper, we determine if VMI is a better option than IS for supply chain members, and optimize the operational costs of the vendor when there are VMI and non-VMI customers. We evaluate the impacts of an agreement and the operational decisions on all parties involved.

As opposed to what may be assumed in some publications (e.g. Bernstein and Federgruen 2003, or Shah and Goh, 2006), we remark that VMI and C&VMI are different agreements with distinct impacts on the vendor and customers. Thus, we will treat these two agreements in separate models. Some parts of those models and formulations therein have roots in Dynamic Lot Sizing (DLS), the second part of our literature survey.

DLS is one of the fields in Production and Operations Management in which there is considerable volume of publications. An overview of a number of those papers is contained in Drexl and Kimms (1997) and in Erenguc et al. (1999). Here, we will give examples of papers related to uncapacitated and capacitated models that motivate, in one way or another, some of the formulations in our models.

The Uncapacitated Lot Sizing Problem (ULSP) deals with a sole decision maker who optimizes the replenishment of a single product under unlimited supply. Wagner and Within (1958) present a dynamic programming algorithm for the solution of ULPS. Examples of extensions to this algorithm (for efficient implementation) include Federgruen and Tzur (1991) and Aggarwal and Park (1993). The stochastic version of the problem was also considered by several authors (e.g. Bookbinder and Tan, 1988).

The Capacitated Lot Sizing Problem (CLSP) considers factors, such as labor and equipment, that can limit production. Florian et al. (1980) show that CLSP is NP-hard, which indicates the computational complexity of the problem. Therefore, special cases of CLSP were addressed and solved by researchers. Examples of such are Baker et al. (1978), Erenguc and Aksoy (1990), Shaw and Wagelmans (1998), and Sox and Gao (1999). Gopalakrishnan et al. (2001) provide a more general form of the problem by considering carryover of setups into adjacent periods.

We do not consider any production capacity, but there may be a *replenishment* capacity specified by a customer, when the vendor manages his inventory. We will evaluate the implications of customer-specified restrictions in our models.

The lot-sizing literature deals mainly with production and inventory management, without considering the transportation aspect explicitly (Diaby and Martel, 1993). Coordinated production and distribution within DLS, on the other hand, extends the traditional lot sizing models to see the tradeoffs between production and transportation decisions. Examples include Federgruen and Zipkin (1984) who study inventory routing, and

Cohen and Lee (1988) who analyze cost implications of inventory distribution in a supply chain (but do not consider the impacts on physical distribution).

In addition to those, Chandra and Fisher (1994), in research which may be the most related to some parts of our paper, investigate the value of coordinated production and distribution. They model the production and delivery options of a manufacturer who must satisfy external demand over a finite horizon, and who has her own fleet. In their computational study, they solve the production and transportation problems separately. They conclude that the value of coordinating production and distribution can be high under the "right" conditions.

In our models, we consider not only the production and distribution decisions, but also the question of which customers to offer VMI/C&VMI, and replenishment to them under those agreements. Moreover, instead of private carrier, the transportation portion of our models assume "common carrier," a for-hire outside trucking company. (That case is more typical in industry than is transport in a manufacturer's own truck.) Finally, we evaluate additional heuristics to solve our VMI model.

Most of the literature mentioned on ULSP and CLSP looks at a single firm that optimizes its operations over a planning horizon. When multiple echelons are considered, the research focuses on joint optimal lot sizing, but for an infinite horizon. According to the flow of materials from the origins to destinations in a production facility or a supply chain, the problems in this area are categorized as serial, assembly, distribution or general system.

Common to all those problems is a central decision maker who optimizes the system-wide costs, mostly using the concept of echelon inventory. A good number of examples and references in this are provided by Axsater (2003). Since a central decision maker is assumed, full knowledge of cost parameters of all parties is also assumed to be available to the decision maker. In VMI or C&VMI, however, only the demand information, not the cost parameters, is shared. Moreover, the companies involved remain independent.

It can be difficult to find a solution to the multi-echelon lot sizing problem even when the demand is stationary. The dynamic version of the problem is even more difficult, making heuristic approaches the only way to deal with them. The reader is referred to Silver, Pyke and Peterson (1998), Zipkin (2000) and the references therein for heuristic approaches to multi-echelon dynamic and joint lot sizing problems (MDLSP).

Our formulations for VMI and for C&VMI differ from MDLSP because of the decisions we need to make. Hence, to find solutions, our models require an alternative treatment that considers special structures in the formulations. Although the underlying idea of our VMI or C&VMI models is the same as in other references mentioned in the literature survey, there is no single paper to which we can truly compare our work. The next section will highlight the problem characteristics that we take into account, and describe the steps we take for our formulations and solution approaches.

5.2 Problem Definition and Research Scope

We consider a supply chain in which a single vendor produces a unique item and satisfies multiple customers. End-consumer demand is different for each customer; it is time-varying but known, and realized only at the customers. We assume a planning horizon with 12 periods. Customers are independent; each must meet the demand of its end-consumers, and the vendor must meet the replenishment orders coming from her customers.

In the traditional way of doing business (IS), each customer orders from the vendor based on his costs of inventory holding and ordering. That ordering cost includes the costs of replenishment-decision making and of shipment receipt. Orders are placed at the beginning of a period and received immediately (i.e., lead time is assumed zero). No backlogging is allowed. In every period, each customer has the choice of ordering from the vendor, or satisfying his demand from stock. Inventory holding costs are incurred for stocks on hand at the end of any period.

Under IS, the vendor has to fulfill the orders specified by her customers. We assume that the vendor's production capacity exceeds total demand in any period. Customer orders are shipped at the beginning of periods, and inventory holding cost is charged at the end of each period for the items on hand. We assume that production-lead time is zero, and in any period the vendor can decide to produce, or to satisfy an order from her stock. The vendor's production cost and sales price per item are fixed, and hence will be omitted in our models (recall that all demand must be met). In addition to the inventory holding cost, the vendor pays a production setup cost each time she initiates manufacturing; incurs a cost per shipment released to one or more customers; and transportation costs.

To evaluate the impacts of VMI on shipment consolidation, we assume that the transportation cost is paid by the vendor. (That is, a vendor in control of the timing and quantity of shipments can combine the small orders of different customers to achieve economies of scale in transportation, if the transportation cost is paid by her). We further assume that the vendor engages the transportation services of a common carrier, i.e. a public, for-hire trucking company.

That carrier, taking into account an *all-units discount* scheme, offers a piece-wise linear freight rate as a function of the total weight in a given shipment. At the end of a period, a vehicle dispatched from the vendor may carry an amount b of goods to be dropped to a total of i customers. The carrier then charges the vendor a fixed cost for every stop the truck makes.

In the piece-wise linear freight rate structure, there is a variable cost per unit transported, and a fixed cost per segment, for every linear segment separated by breakpoints. Total quantity transported can only be in one segment. The reader is referred to Balakrishnan and Graves (1989) for a more detailed discussion of this cost structure, and to Higginson (1993) and Croxton et al. (2003) for formulations and discussions of various transportation cost structures. The fixed cost per stop at a customer is charged independent of the sequence of stops (e.g., if a shipment released from the vendor includes consolidated orders of two customers, then there are two stops and two fixed charges). We do not consider shipment routing in this paper.

Under IS, each customer has his own ordering plan, and replenishes separately from the vendor. That plan depends on inventory holding charges and ordering costs. Once all customers determine their replenishment policies over the planning horizon, the vendor collects them to plan her production. Note that the vendor is aware of the customers' ordering plans, but not the actual end-consumer demand.

When the customers make decisions regarding the timing and quantity of replenishments, it is difficult for the vendor (who must meet the customers' requirements) to seek economies of scale in her operations. The vendor may prefer alternative replenishment quantities and/or different ordering times than the customers specify. That is, their decisions may act as constraints against the vendor's flexibility which she requires to decrease her production, inventory and transportation costs.

A VMI agreement between a vendor and customer gives her that flexibility, but at a cost. Under VMI, the vendor makes replenishment decisions on behalf of the customer, and pays the cost of replenishment decision-making. The agreement also provides the vendor with data on the end-consumer demand.

VMI is a tactical decision, yet its success depends on potential savings in operational activities. Whether to choose VMI or to stay in IS requires a thorough evaluation of operational costs. The total costs in each case must be compared. Those costs for the vendor, either fixed or variable, relate to production, replenishment and transportation. The customer pays a fixed amount for every shipment received, and a variable cost per item he stores in stock.

Note that when there are $n \geq 2$ customers, the vendor may choose to implement VMI with fewer than n. By a "VMI-customer", we shall mean one who implements VMI with the vendor. VMI-customers are relieved of paying expenses associated with making replenishments. Although this may not guarantee them lower costs compared to IS, no customer would implement VMI unless he is no worse off. Under VMI, the vendor may wish to send large quantities to achieve economies of scale in transportation. Then, should the customer accept any quantity determined by the vendor, or should he set some limits to it? A customer that is offered VMI must answer this question, before accepting the agreement, so his costs do not go up.

The vendor, on the other hand, is concerned first with the *right* set of customers to offer VMI. Before implementing the agreement, the vendor must foresee the savings it can create in production, replenishment and transportation. The vendor, too, would not wish to undertake any VMI agreements that create no cost savings.

When Consignment Inventory is coupled with Vendor Managed Inventory (C&VMI), the vendor now owns the stock at a C&VMI-customer's premises until the items are used. Hence, the vendor pays the opportunity cost of those goods. In this case, even when there is no restriction specified by the customer, it may be in the vendor's best interest not to send large quantities. As in VMI, both the vendor and customer would want to get the best from this agreement. Therefore, either party will cast aside implementing C&VMI if the agreement increases their total costs compared to IS.

As we emphasized in the previous discussions, IS is our base case to which we compare VMI and C&VMI. Although we will contrast the outcomes of these two agreements, choosing to implement any agreement is based on comparison of total costs under that agreement vs IS. Values of decision variables and total costs of the vendor and customers in each case will be determined using Mixed Integer Programming. We will describe the notation used in those models in Section 3. Numerical examples will be provided in Section 7, and a summary and conclusions in Section 8. Sections 4-6 will cover various models and solution algorithms, which we briefly describe here.

In Section 4 (IS), each customer decides on replenishments first based on his minimum total costs. A customer's cost stems from replenishment decision-making, shipment-received, physical storage and opportunity cost of inventory. The vendor receives the order quantities from each customer, and optimizes her operations based on her costs of production setup, inventory holding and transportation. Under IS, the customers' optimal replenishment decisions are input to the vendor's model.

In Section 5 (VMI), it is the vendor who makes replenishment orders on behalf of customers. Hence, there is only one model that determines the production, customer-replenishment, and transportation quantities and timing. The vendor pays a cost for each decision made to replenish, which is assumed to be the same as what the customer pays in IS. We propose a heuristic to solve the vendor's model. In the first part of that heuristic, VMI decisions (which concern the right set of customers with whom to implement VMI) are separated from the model. In the second part, we propose three decomposition methods to solve the remaining problem of integrated production, replenishment and transportation. Those methods generate two or more sub-problems which are solved hierarchically.

We also determine whether a customer should set a maximum-inventory level that the vendor can keep at his premises. In case he should, we propose a model that decides the optimal level of stock that can be allowed by each and every customer, and provide a solution algorithm.

In Section 6(C&VMI), the vendor determines each customer's replenishment quantity as in Section 5, but now pays (in addition to the cost of replenishment decision making), the opportunity cost of goods stored at the customers' locations. Hence, there is still one model to find optimal production, replenishment and transportation quantities, and timing. Of course, the vendor should also identify the best set of C&VMI-customers. The vendor's model is solved using Lagrangian relaxation: By relaxing one set of constraints in the model, we decompose it into two sub-problems, each are easier to solve. To determine the Lagrange multipliers, we use the cutting-plane approach of Kelly (1960).

In all our models, we assume that at time zero, there is no inventory anywhere in the supply chain. Also, closing inventory at the end of the planning horizon is zero. We further assume that when VMI is an option for a vendor and customer, the end-consumer demand of that customer is passed to the vendor. The idea is to give the vendor a fair chance to evaluate the cost impacts of the agreement before implementing it.

We will next define the model indices, parameters, and variables used throughout this paper. Note that for all of our parameters and variables, any subscript will be an *index* relating to a customer, time period, or segment of the freight-discount scheme (See Sec. 3). Superscripts are used to differentiate which party owns that parameter or variable: If a parameter/variable's superscript starts with a "v", then it belongs to the vendor, whereas a "c" means it is the customer's. Similarly, the notation V and C_i used in the text refer to the vendor and customer i respectively.

5.3 Notation

Let us now "name" the models we will develop and use. (In what follows, it should be recalled that each customer's replenishment under VMI or C&VMI is determined by the vendor. Hence, there is no replenishment model for *customers* under those cases.)

IS- C_i and *IS-V*: C_i's and the V's models for IS respectively.

VMI-V and *CVMI-V*: V's model for VMI and C&VMI respectively.

Imax C_i *-VMI* and *Imax* C_i *-CVMI*: C_i's model for his maximum inventory level under VMI and C&VMI respectively.

To keep the notation compact and easy to follow, we provide a single notation for each of the decision variables of the customers and the vendor, although it may be used in multiple models. To compare and contrast decision variables in two or more models, we will use the models' names as identifiers (e.g. the vendor's production quantities in *VMI-V* and *CVMI-V*).

Indices

$i = 1, ..., I$ (customers)

$t = 1, ..., T$ (time period)

$s = 1, ..., S$ (particular quantity segment for transportation)

Parameters

d_{it} : end-consumer demand at C_i in period t

a_i^{cs} : cost per shipment received, paid by customer C_i

a_i^{co} : cost per ordering decision made, paid by C_i

$a_i^{vo} = a_i^{co}$: cost per ordering decision made on behalf of C_i, paid by the vendor, V

a^v : V's cost per shipment released

S^v : V's cost per production setup

h_i^c : C_i's cost per inventory, charged at the end of periods. $h_i^c = h_i^{cs} + h_i^{co}$ where

h_i^{cs} : physical storage cost of an item in stock

h_i^{co} : opportunity cost of an item in stock

h^v : V's cost per unit held in inventory, charged at the end of periods

$I_{it}^{c\,MaxVMI}$: maximum level of stock allowed by C_i in period t under VMI

$I_{it}^{c\,MaxCVMI}$: maximum level of stock allowed by C_i in period t under C&VMI

c_s : cost per item transported

f_s : fixed cost of transportation (fixed cost per dispatch from the vendor's facility)

$\bar{f}_i$: fixed cost per stop at C_i during transportation

M_1, M_2, M_3: "big numbers" used in MIP formulation

$$M_1 = \sum_{t \in T} d_{it}\ ,\ M_2 = \sum_{i \in I} \sum_{t \in T} d_{it}\ ,\ M_3 = 12$$

Continuous Decision Variables

Q_t^v : the vendor's production quantity in period t

q_{it}^c : quantity ordered by C_i in period t

q_{it}^v : quantity ordered by V on behalf of C_i in period t

I_{it}^{c} : Inventory level of C_i at the end of period *t*, decided by C_i

I_{it}^{vc} : Inventory level of C_i at the end of period *t*, determined by *V* on behalf of C_i

I_{t}^{v} : Inventory level of *V* at the end of period *t*

0 – 1 Variables

α_i^v : equals 1 if there is a VMI agreement between *V* and C_i, 0 otherwise

y_{it}^{c} : equals 1 if C_i orders in period *t*, 0 otherwise

y_{it}^{vc} : equals 1 if *V* orders on behalf of C_i in period *t* (requires α_i^v =1), 0 otherwise

y_{t}^{v} : equals 1 if V produces in period *t*, 0 otherwise

y_{st} : equals 1 if quantity transported in period *t* is in segment *s* of freight tariff, 0 otherwise

Total Costs

$TC_{IS\_i}^{c}$ and TC_{IS}^{v} : C_i's and *V*'s total cost under IS

$TC_{VMI\_i}^{c}$ and TC_{VMI}^{v} : C_i's and *V*'s total cost under VMI

$TC_{CVMI\_i}^{c}$ and TC_{CVMI}^{v} : C_i's and *V*'s total cost under C&VMI

Note that we will adopt a superscript "*" at several occasions. When used, it refers to the optimal value of a decision variable, which will then serve as a parameter. Having described the notation, we can now proceed to formulate the several MIP models. Let us begin with the first case, that of Inventory Sourcing.

5.4 Inventory Sourcing (IS)

Under IS, each and every customer separately minimizes his total cost of ordering and inventory holding over the planning horizon, and orders individually from the vendor. The resulting formulation is as follows.

Model *IS-C*$_i$: For every *i*, *i* = 1, ..., *I*,

$$\text{Min } TC^{c}_{IS\_i} = \sum_{t\in T}[(a^{cs}_i + a^{co}_i)y^{c}_{it} + h^{c}_i I^{c}_{it}] \qquad (1.1)$$

Subject to:

$$q^{c}_{it} \le M_1 y^{c}_{it} \qquad \forall t \qquad (1.2)$$

$$I^{c}_{it} = I^{c}_{it-1} + q^{c}_{it} - d_{it} \qquad \forall t \qquad (1.3)$$

$$I^{c}_{i'0'} = 0 \qquad (1.4)$$

$$y^{c}_{it} \in \{0,1\}; \quad q^{c}_{it},\ I^{c}_{it} \ge 0 \qquad \forall t \qquad (1.5)$$

Our aim is to minimize the objective function (1.1), which is the total cost of ordering and shipment-received, $\sum_{t\in T}(a^{cs}_i + a^{co}_i)y^{c}_{it}$, plus inventory holding ($\sum_{t\in T} h^{c}_i I^{c}_{it}$). Constraint (1.2) is a forcing constraint, and ensures that the quantity ordered is zero if there is no order placed. The equality (1.3) is the demand balance, determining the inventory on hand at the end of each period after satisfying that period's demand. At the beginning of the planning horizon, each customer has zero inventory (1.4). Constraint (1.5) shows the types of variables. Variable y^{c}_{it} equals one if customer *i* orders from the vendor in period *t.*

IS-C$_i$ is a dynamic economic order quantity model, and was solved by Wagner and Within (1958). They developed a deterministic dynamic programming algorithm (now known as the Wagner-Within algorithm), which solves the problem for a finite planning horizon. It guarantees an optimal selection of replenishment quantities.

After each customer minimizes his cost function, the optimal decision variables q^{*c}_{it} and y^{*c}_{it} become inputs for the vendor, who incorporates them in his model. Let us now formulate the vendor's cost-minimization model under IS.

Model *IS-V*

$$\text{Min} \quad TC_{IS}^{v} = \sum_{t\in T}\sum_{i\in I} a^{v} y_{it}^{*c} + \sum_{t\in T}\left(h^{v} I_{t}^{v} + S^{v} y_{t}^{v}\right)$$

$$+ \sum_{t\in T}\sum_{i\in I} \bar{f}_{i} y_{it}^{*c} + \sum_{t\in T}\sum_{s\in S}\left(f_{s} y_{st} + c_{s} x_{st}\right) \tag{1.6}$$

Subject to:

$$Q_{t}^{v} \le M_{2} y_{t}^{v} \qquad \forall t \tag{1.7}$$

$$I_{t}^{v} = I_{t-1}^{v} + Q_{t}^{v} - \sum_{i\in I} q_{it}^{*c} \qquad \forall t \tag{1.8}$$

$$\sum_{i\in I} q_{it}^{*c} = \sum_{s\in S} x_{st} \qquad \forall t \tag{1.9}$$

$$b_{s-1} y_{st} \le x_{st} \qquad \forall s,t \tag{1.10}$$

$$x_{st} \le b_{s} y_{st} \qquad \forall s,t \tag{1.11}$$

$$\sum_{s\in S} y_{st} \le 1 \qquad \forall t \tag{1.12}$$

$$b_{0} = I_{0}^{v} = 0 \tag{1.13}$$

$$y_{st},\ y_{t}^{v} \in \{0,1\}; \quad Q_{t}^{v}, I_{t}^{v}, x_{st} \ge 0 \quad \forall s,t \tag{1.14}$$

The vendor's objective function (1.6) is composed of the total costs of shipment-release ($\sum_{t\in T}\sum_{i\in I} a^{v} y_{it}^{*c}$), inventory holding and production-setup ($\sum_{t\in T}\left(h^{v} I_{t}^{v} + S^{v} y_{t}^{v}\right)$), and transportation ($\sum_{t\in T}\sum_{i\in I} \bar{f}_{i} y_{it}^{*c} + \sum_{t\in T}\sum_{s\in S}\left(f_{s} y_{st} + c_{s} x_{st}\right)$, where the first part denotes the fixed cost per stop during transportation). Constraints (1.7) and (1.8) are similar to (1.2) and (1.3) respectively: (1.7) is a forcing constraint for production, whereas (1.8) is the demand balance constraint. Constraint set [1.9, 1.12] is for transportation: (1.9) equates, for every period, the total quantity ordered to total quantity transported. (1.10) and (1.11) identify the quantity segments for LTL pricing. Constraint (1.12) states that the quantity transported can only be in one segment. Equalities in (1.13) set the starting conditions for the first quantity segment of transportation, and for the vendor's opening inventory at time zero. Finally, (1.14) specifies the types of variables used in the model.

Note that the customers' decisions q_{it}^{*c} and y_{it}^{*c} are parameters in *IS-V*. Therefore, constraint (1.9) fixes the total quantity to be transported in period t to $\sum_{i\in I} q_{it}^{*c}$. As a result, constraints (1.9) to (1.12) do not influence the vendor decisions, hence shipment-release costs and transportation costs serve as constants in the objective function.

The vendor's problem then reduces to finding when and how much to produce, so that her total cost of inventory holding and production setup is minimized. That is to say, the vendor's problem under IS is also a dynamic economic order quantity model, and can be solved by the Wagner-Whitin algorithm.

We see in *IS-V* that the customers' total order quantity $\sum_{i\in I} q_{it}^{*c}$ in any period t is the demand that the vendor faces. Yet, the vendor is not aware of the actual demand realized at customers. It is the customers who decide the timing and quantity of orders (and hence shipments, since no backlogging is allowed). Therefore, inventory sourcing reduces the vendor's flexibility in production, and can eliminate potential savings in transportation.

In an attempt to decrease her costs, the vendor can offer VMI to one or more customers. In the next section, we will develop models that can help her decide to whom VMI should be offered, and how to make use of the resulting authority over customer-replenishment.

5.5 Vendor Managed Inventory (VMI)

A VMI agreement allows the vendor decide replenishment quantities on behalf of a customer. Through VMI, the vendor acquires information regarding the customer's actual demand, and satisfies that demand while considering possible savings for herself.

This agreement between the vendor and customer entails the payment by *V* of a new cost associated with the ordering decision. But for *C*, VMI eliminates the effort (and hence costs) of making replenishments.

VMI is not a requirement, but rather an *option* that can replace IS. When there are multiple customers, the vendor has a choice, to implement VMI for a subset of her customers. The key concern for *V* is the right set of agreements that minimize her total costs. From a VMI-customer's point of view, however, VMI is not favorable if his total cost is lower under IS.

There are various questions for the vendor when VMI is an option. The major one concerns the right choice of customers with whom to implement VMI. Decisions on production setup, replenishments on behalf of VMI-customers and transportation are the others. Note that decisions for VMI-customers are made by the vendor, hence those customers do not require an optimization model. Their total costs will be calculated based on the replenishment quantities that the vendor determines. Let us now formulate the vendor's model.

Model *VMI-V*

$$\text{Min} \quad TC^{v}_{VMI} = \sum_{t\in T}\sum_{i\in I} a^{co}_{i}\, y^{vc}_{it} + \sum_{t\in T}\left(h^{v} I^{v}_{t} + S^{v} y^{v}_{t}\right) + \sum_{t\in T}\sum_{i\in I} a^{v}\left(y^{vc}_{it} + (1-\alpha^{v}_{i})\, y^{*c}_{it}\right)$$

$$+ \sum_{t\in T}\sum_{i\in I} \bar{f}_{i}\left(y^{vc}_{it} + (1-\alpha^{v}_{i})\, y^{*c}_{it}\right) + \sum_{t\in T}\sum_{s\in S}\left(f_{s} y_{st} + c_{s} x_{st}\right) \tag{2.1}$$

Subject to

$$Q_t^v \le M_2 y_t^v \qquad \forall t \qquad (2.2)$$

$$q_{it}^v \le M_1 y_{it}^{vc} \qquad \forall i,t \qquad (2.3)$$

$$\sum_{t\in T} y_{it}^{vc} \le M_3 \alpha_i^v \qquad \forall i \qquad (2.4)$$

$$I_t^v = I_{t-1}^v + Q_t^v - \sum_{i\in I} q_{it}^v - \sum_{i\in I} q_{it}^{*c}(1-\alpha_i^v) \qquad \forall t \qquad (2.5)$$

$$I_{it}^{vc} = I_{it-1}^{vc} + q_{it}^v - d_{it}\alpha_i^v \qquad \forall i,t \qquad (2.6)$$

$$I_{it}^{vc} \le I_{it}^{c\,MaxVMI} \qquad \forall i,t \qquad (2.7)$$

$$\sum_{i\in I}(q_{it}^v + (1-\alpha_i^v)q_{it}^{*c}) = \sum_{s\in S} x_{st} \qquad \forall t \qquad (2.8)$$

$$b_{s-1} y_{st} \le x_{st} \qquad \forall s,t \qquad (2.9)$$

$$x_{st} \le b_s y_{st} \qquad \forall s,t \qquad (2.10)$$

$$\sum_{s\in S} y_{st} \le 1 \qquad \forall t \qquad (2.11)$$

$$b_0 = I_0^v = I_{i,'12'}^{cv} = 0 \qquad (2.12)$$

$$y_{st},\ y_t^v,\ y_{it}^{vc},\ \alpha_i^v \in \{0,1\}; \quad q_{it}^v,\ Q_t^v, I_t^v, I_{it}^{vc},\ x_{st} \ge 0 \qquad (2.13)$$

There are many similarities in this model with *IS-V*, which includes fewer cost parameters, decision variables, and constraints. The vendor's objective is to minimize the total of the following costs in (2.1): cost of making replenishment decisions (first term), inventory holding and production setup (second), shipment release (third term), and transportation costs (last two terms).

Constraints (2.2) and (2.3) are forcing constraints for production-setup and customer-replenishment respectively. Inequality (2.4) ensures that the vendor can replenish only the

VMI-customers. Equations (2.5) and (2.6) are the demand-balance constraints for the vendor and customers respectively. We remark that (2.6) covers the VMI-customers only: If α_i^v is zero for C_i, then $q_{it}^v = 0$; if not, *V* determines the replenishment quantity for customer *i*. Any customer's inventory level under VMI cannot exceed the maximum stock level specified by that customer (2.7). Constraints (2.8) to (2.11) concern transportation, as explained in *IS-V*. Inventories are set to zero at the beginning and end of the planning horizon, in (2.12). Types of the variables in the model are defined in (2.13).

In Model *VMI-V*, the vendor determines the VMI-customers' replenishment quantities and their timing (y_{it}^{*vc}), as well as the amount of inventory these customers carry in each and every period (I_{it}^{*vc}). Therefore, C_i has a total cost

$TC_{VMI\_i}^{c} = \sum_{t\in T} a_i^{cs} y_{it}^{*vc} + \sum_{t\in T} h_i^c I_{it}^{*vc}$ if $\alpha_i^* = 1$, i.e. under VMI. Otherwise, that cost is $TC_{IS\_i}^{*c}$.

Before we move on to solve the model *VMI-V*, let us investigate whether customer *i* should set any $I_{it}^{c\,MaxVMI}$ level and enforce constraint (2.7) on the vendor when VMI is an option. Let us denote by *Imax* the level of maximum inventory for the vendor's replenishments. We state the following lemma:

Proposition 1: If C_i does not specify an *Imax* level, and if his $q_{i..'1'}^{*c} \neq \sum_{t\in T} d_{it}$, the vendor implements VMI with him as long as $a_i^{co} < 2(\bar{f}_i + a^v)$.

We provide two lemmas to prove Proposition 1.

Lemma 1: If a VMI-customer does not specify an *Imax* level, $q_{i'1'}^{v}$ for that customer is $\sum_{t\in T} d_{it}$.

Proof: $y_{i'1'}^{vc} = 1$ and $\sum_{t\in T} y_{it}^{vc} = 1$ are optimal for the vendor who does not pay inventory holding cost of goods that she replenishes on behalf of that customer. Intuitively, because the vendor is not concerned about the customer's inventory carrying cost, she would want to satisfy that customer's total demand all at once in the first period to minimize her costs related to ordering, shipment release, and transportation. Note that the vendor would ship the total quantity produced to the customer, and would keep no inventory herself.

Lemma 2: If a VMI-customer does not specify an *Imax* level, and if his $q^{*c}_{i..'1'} \neq \sum_{t \in T} d_{it}$, it is definite that the vendor is better off, and the customer is worse off, if $a_i^{co} < 2(\bar{f}_i + a^v)$.

Proof: If $q^{*c}_{i..'1'} \neq \sum_{t \in T} d_{it}$, then y_{it}^{*c}>1, which means that the vendor pays at least $2(\bar{f}_i + a^v)$ under IS, whereas she pays only $(\bar{f}_i + a^v)$ under VMI (see Lemma 1). Intuitively, the customer in IS would have multiple orders since he tries to balance inventory holding costs with his ordering costs. In VMI, the vendor replenishes the customer all at once in period one. Note that compared to IS, the vendor is better off, not only because of the fixed costs of transportation and shipment release, but also due to economies of scale in production and transportation. (That is, the vendor would have fewer production setups and larger quantities shipped to the customer). Therefore, the vendor may achieve cost savings even if a_i^{co} is not less than $2(\bar{f}_i + a^v)$. Then, $a_i^{co} < 2(\bar{f}_i + a^v)$ is a sufficient condition.

The two preceding proofs, together, prove Proposition 1, and lay out the importance of $I_{it}^{c\,MaxVMI}$ from a customer's point of view. In case of VMI, every customer's major concern must be to establish maximum levels of inventory for each period in the planning horizon.

The easiest way to set *Imax* levels is to designate a fraction of the customer's demand in any period *t* (say 20% of d_{it}). However, this does not guarantee benefits for the customer: If $I_{it}^{c\,MaxVMI}$ is too low, the vendor may not implement VMI with him in the first place. If it is too high, the customer's inventory holding costs under VMI may be excessive. Therefore, each and every customer should determine the appropriate *Imax* level that guarantees at least indifference, if not reduction, in his total cost under VMI compared to IS. We propose the following model to find the values of $I_{it}^{c\,MaxVMI}$.

Model *ImaxC$_i$ -VMI*: For every i, $i = 1,..,I$:

$$\text{Max } \sum_t h_i^c I_{it}^c \tag{2.14}$$

Subject to:

$$q_{it}^c \leq M_1 y_{it}^c \qquad \forall t \tag{2.15}$$

$$I_{it}^c = I_{it-1}^c + q_{it}^c - d_{it} \qquad \forall t \tag{2.16}$$

$$\sum_t (a_i^{cs} y_{it}^c + h_i^c I_{it}^c) \leq TC_{IS\_i}^{*c} \tag{2.17}$$

$$I_{i'0'}^c = I_{i'12'}^c = 0 \qquad \forall i \tag{2.18}$$

$$I_{it}^{c\,MaxVMI} = I_{it}^c \tag{2.19}$$

$$y_{it}^c \in \{0,1\}; \quad q_{it}^c,\ I_{it}^c \geq 0 \qquad \forall i,t \tag{2.20}$$

In this model, each customer maximizes his inventory holding cost (equation 2.14), provided that his total cost of shipments-received and inventory holding must be less than or equal to his total cost in *IS-C$_i$* (constraint 2.17), and that his starting and ending inventories are zero (constraints 2.18). Note that the left hand side of (2.17) excludes a_i^{co}, cost of making a replenishment decision, which is now paid by the vendor under VMI. Constraints (2.15)-(2.16), and (2.20) are as explained for constraints (1.2)-(1.3), and (1.5) in *IS-C$_i$*.

Proposition 2: The customer is never worse off under VMI, if he allows a maximum inventory level of $I_{it}^{*c\,MaxVMI}$

Proof: Let n and n' be the total number of orders in the optimal solutions of $ImaxC_i$ *-VMI* and *IS-*C_i respectively. Then, n is always less than or equal to n'. Under VMI, the vendor cannot send fewer than n orders because of feasibility required by $I_{it}^{*c\,MaxVMI}$. She would not send n' or more orders, but would rather prefer remaining under IS.

To maximize the total inventory over the planning horizon, a customer must minimize the number of orders while satisfying all demand, without backlogging. Meanwhile, his total cost in $ImaxC_i$ *-VMI* cannot exceed $TC_{IS\_i}^{*c}$. Note that "lot-for-lot" ordering is the worst possible solution to $ImaxC_i$ *-VMI*. We then propose (for every i) the following algorithm which optimally finds $\sum_t I_{it}^{c}$.

Let n be the number of demand periods. In the least-favorable case, lot-for-lot ordering, we would need n orders to meet total demand. We then start with the first period of demand to see if the order size at that time can be increased to cover requirements upcoming periods, provided that total cost does not exceed $TC_{IS\_i}^{*c}$.

Once we find the order quantity in that period, we then proceed to the successive period where we will need to order again. Every time we must order, we determine the maximum upcoming demand that can be covered with a single replenishment, satisfying the constraint (2.17) that we have enough budget to achieve lot-for-lot ordering for the demand remaining.

Note that if $n\,a_i^{cs}$ is less than $TC_{IS\_i}^{*c}$, the algorithm will increase the total cost of ordering and inventory holding to the point where it equals $TC_{IS\_i}^{*c}$. Otherwise, the algorithm will decrease that total cost further below $n\,a_i^{cs}$, which will then converge to $TC_{IS\_i}^{*c}$. We now provide details of the algorithm.

Algorithm ImaxC$_i$ -VMI

Initiate the algorithm: Start with the first period t' in which demand is non-zero. Define $t'' = t'+1$, and $k = 1$. Define and initiate $Surplus_i = n\, a_i^{cs}$, $Slack_i = 0$.

Step 1: Check the demand of next period:

If $d_{it''} = 0$, $t'' = t'' + 1$, go to Step 1

If not, go to next step

Step 2: Define $Cost_i = (n- k)\ a_i^{cs} + h_i^c \sum_{t=t'+1}^{t''} (t - t')d_{it}$

If $n\, a_i^{cs} \leq TC_{IS\_i}^{*c}$

If $Cost_i < TC_{IS\_i}^{*c}$, go to Step 3

Else go to Step 4

If $n\, a_i^{cs} > TC_{IS\_i}^{*c}$

If $TC_{IS\_i}^{*c} < Cost_i < Surplus_i$

Set $Surplus_i = Cost_i$, go to Step 3

If $Slack_i < Cost_i < TC_{IS\_i}^{*c}$

Set $Slack_i = Cost_i$, go to Step 3

Else, go to Step 4

Step 3: $I_{it'}^{c\,MaxVMI} = \sum_{t=t'+1}^{t''} d_{it}$; $t'' = t'' + 1$, $k = k + 1$

If $t'' = 12$, set number of orders = $n- k$, go to Step 5

Else go to Step 1

Step 4: $t' = t''$, $t'' = t' + 1$

If $t' = 12$, stop, set number of orders = $n- k$

Else go to Step 1

Step 5: Define $residual\_cost = TC_{IS\_i}^{*c}$ - $(n - k)\, a_i^{cs}$ - $h_i^c \sum_{t=1}^{12} I_{it'}^{c\,MaxVMI}$

Define $residual\_inventory = (residual\_cost) /\ h_i^c$

For every successive order, let t' be the period of placing an order that covers demand until period t''. Select the minimum of $(t'- t'')$'s. Add

($residual\_inventory$) / (t' - t'') to the inventory levels of periods [t', t''-1]. **STOP**.

This algorithm gives the minimum number of orders, and the optimal amount of total inventory, for customer i over the planning horizon. We can now return to solution of the Model *VMI-V*.

Solution to *VMI-V*

The problem *VMI-V* is complicated by the interrelated decisions of production, customer-replenishment and transportation, and of course with whom to implement VMI. First of all, the vendor has 2^I-1 choices of VMI agreements with I customers. (Note that the case of "*No* VMI agreements" is excluded). Assume for a moment that those choices are given in advance to the vendor. Her remaining problem is still difficult: the vendor's production and replenishment choices not only depend on the relevant costs, but also on how much the vendor can ship to customers.

Although it seems that V would take advantage of the maximum inventory levels of customers, shipping as much as possible in any period, that may not be optimal because of possible economies of scale in transportation in future periods. This means, for that specific period, the vendor can utilize only that fraction of $I_{it}^{*c\,MaxVMI}$ which optimizes the cost of shipment dispatch and transportation in that period plus the periods thereafter.

We therefore provide a heuristic to solve the *VMI-V* model. Note that the vendor must make four sets of decisions. The first is the right set of VMI-customers, identified with optimal values of α_i^v. If we knew the correct set of VMI-customers in advance, then we could run the model *VMI-V* once instead of evaluating all 2^I-1 VMI options. Hence, the first part of our heuristic starts by identifying those customers best for VMI implementation.

The remaining decisions are related to production (P), replenishment (R), and transportation (T). Although these are interrelated, we can decompose them into sub-problems that are easier to solve. The tradeoff of doing so is the optimality gap. The more we decompose, the less time it takes to solve, yet the solution quality worsens. In the second part of our heuristic, we solve P, R, and T problems in four different ways. In order of increasing complexity, they are:

Method 1 (P-R-T): Solve P, R, and T separately [decompose by constraints (2.5) and (2.8)]
Method 2 (P-RT): Solve P separately; R and T together [decompose by constraint (2.5)]
Method 3 (PR-T): Solve P and R together; T separately [decompose by constraint (2.8)]
Method 4 (PRT): Solve P, R, and T together (no decomposition)

Solutions to sub-problems are obtained in a hierarchical order: P decisions comes first, R decisions next, and T decisions last. (This hierarchy applies, whatever the decomposition scheme.) We can now provide the details of our heuristic.

Heuristic for VMI – Part 1

Assume for the moment that there is only *one customer*, customer *i*, to serve. There are two options for the vendor: a VMI agreement or not. Calculate the vendor's total cost for each of those options. The total cost of no VMI is TC_{IS}^{v}, which is obtained from *IS-V*. Total cost of VMI (TC_{VMI}^{v}) is determined by solving P, R, and T together in *VMI-V* where now α_i^v=1 for that specific customer *i*. If $TC_{VMI}^{v} < TC_{IS}^{v}$, set C_i as a VMI-customer. If not, set α_i^v = 0. Repeat the process for every *i*, $i \in I$. Part 1 of the heuristic determines α_i^{*v} (heuristic solutions), which are then used as parameters in Part 2.

Heuristic for VMI – Part 2

Method 1: P-R-T

Solve P, R, and T separately [decompose by constraint sets (2.5) and (2.8)].

Problem P: Determine the production periods and quantities based on actual demands of the VMI-customers and the order quantities of non-VMI customers:

$$\text{Min} \quad \sum_{t \in T} \left(h^v I_t^v + S^v y_t^v \right)$$

Subject to

$$Q_t^v \le M_2 y_t^v \qquad \forall t \qquad (2.2)$$

$$I_t^v = I_{t-1}^v + Q_t^v - \sum_{i \in I} \alpha_i^{*v} d_{it} - \sum_{i \in I} q_{it}^{*c} (1 - \alpha_i^{*v}) \qquad \forall t \qquad (\text{P2.5})$$

$$I_0^v = 0 \ ; \ \ y_t^v \in \{0,1\}; \quad Q_t^v, I_t^v \ge 0 \qquad (\text{P2.13})$$

This is the multi period lot sizing problem for the vendor as discussed in the *IS-V* model, but now q_{it}^v in equality (2.5) is replaced by d_{it}, resulting in P2.5. We can then find the optimal production periods and quantities using the Wagner-Within algorithm. The optimal values Q_t^{*v} and y_t^{*v} then become parameters for the replenishment problem, which is next discussed.

Problem R: Using the optimal production periods and quantities found in Problem P, find the best replenishment policy by solving

$$\text{Min} \quad \sum_{t \in T} \sum_{i \in I} a_i^{co} y_{it}^{vc} + \sum_{t \in T} \sum_{i \in I} (a^v + \bar{f}_i) \left(y_{it}^{vc} + (1 - \alpha_i^{*v}) y_{it}^{*c} \right)$$

Subject to

$$q_{it}^v \le M_1 y_{it}^{vc} \qquad \forall i, t \qquad (2.3)$$

$$\sum_{t \in T} y_{it}^{vc} \le M_3 \alpha_i^{*v} \qquad \forall i \qquad (2.4)$$

$$RI_t^v = RI_{t-1}^v + Q_t^{*v} - \sum_{i \in I} q_{it}^v - \sum_{i \in I} q_{it}^{*c}(1-\alpha_i^{*v}) \qquad \forall t \qquad \text{(R2.5)}$$

$$I_{it}^{vc} = I_{it-1}^{vc} + q_{it}^v - d_{it}\alpha_i^{*v} \qquad \forall i,t \qquad (2.6)$$

$$I_{it}^{vc} \le I_{it}^{c\,MaxVMI} \qquad \forall i,t \qquad (2.7)$$

$$I_{i,'12'}^{cv} = 0\,;\;\; y_{it}^{vc} \in \{0,1\}; \quad q_{it}^v,\, I_{it}^{vc}, RI_t^v \ge 0 \qquad \text{(R2.13)}$$

Note that we introduced a new variable RI_t^v in (R2.5) to determine the true value of the vendor's inventory level. I_t^v in (P2.5) does not take into account the replenishment quantities that depends on customers' requirements and their maximum inventory levels.

Problem R is not a difficult problem. We first identify the consecutive production periods and quantities from Problem P. For every two consecutive production periods, we can then use a simple algorithm to find the replenishment policy for each customer *i*:

Initialize: Let *t'* and *t''* be the two consecutive production periods considered, with *t'* the first demand period. Let *n* be the total number of replenishments until t'. From *t*=*t'* to *t*=*t''*-1, compute the following steps. **Stop when updated *t*=*t''* or when *t*=12.**

Step 1: Determine the replenishment quantity in period *t*

$q_{it}^v = d_{it} + Extra_t$, where $Extra_t$ = Min ($I_{it}^{c\,MaxVMI}$, $d_{it+1} + I_{it+1}^{c\,MaxVMI}$)

Step 2: Update the on-hand inventory in the next period:

$Extra_t = Extra_t - d_{it+1}$

Step 3: Evaluate the value of $Extra_t$

if $Extra_t < 0$, $n = n+1$, $d_{it+1} = d_{it+1} - Extra_t$, $t = t+1$, go to Step 1

if $Extra_t \ge 0$, $t = t+1$, go to Step 2

Once we solve Problem R, the optimal replenishment quantities q_{it}^{*v} become parameters for the transportation problem, which we will detail now.

Problem T: Using the optimal replenishment quantities, determine the transportation decisions by solving

$$\text{Min} \quad \sum_{t\in T}\sum_{s\in S}\left(f_s y_{st} + c_s x_{st}\right)$$

Subject to

$$\sum_{i\in I}(q_{it}^{*v} + (1-\alpha_i^{*v})q_{it}^{*c}) = \sum_{s\in S} x_{st} \qquad \forall t \qquad \text{(T2.8)}$$

$$b_{s-1} y_{st} \le x_{st} \qquad \forall s,t \qquad \text{(2.9)}$$

$$x_{st} \le b_s y_{st} \qquad \forall s,t \qquad \text{(2.10)}$$

$$\sum_{s\in S} y_{st} \le 1 \qquad \forall t \qquad \text{(2.11)}$$

$$b_0 = 0; \quad y_{st} \in \{0,1\}; \quad x_{st} \ge 0 \qquad \text{(T2.13)}$$

When we know the optimal replenishment quantities q_{it}^{*v}, we can simply determine the transportation quantity ($\sum_{s} x_{st'}$) for every period using equation (T2.8). That quantity also identifies the associated transportation segment.

Finally, we calculate the total cost of production, replenishment and transportation in Method 1, using the optimal solutions found in problems P, R, and T:

$$\text{TC of Method 1} = \sum_{t\in T}\left(h^v RI_t^{*v} + S^v y_t^{*v}\right) + \sum_{t\in T}\sum_{i\in I} a_i^{co} y_{it}^{*vc} +$$

$$\sum_{t\in T}\sum_{i\in I}(a^v + \bar{f}_i)\left(y_{it}^{*vc} + (1-\alpha_i^{*v})y_{it}^{*c}\right) + \sum_{t\in T}\sum_{s\in S}\left(f_s y_{st}^* + c_s x_{st}^*\right) \qquad \text{(TCM1)}$$

Note that the true value of vendor's inventory level RI_t^v is used to calculate the total cost of production. Having analyzed the P-R-T decomposition method, we can now easily detail the remaining three methods.

Method 2: P-RT

Solve P separately, and R and T together [decompose by constraint set (2.5)].

Problem P:

Min $\sum_{t \in T} \left(h^v I_t^v + S^v y_t^v \right)$

s.t. (2.2), (P2.5), and (P2.13)

Problem RT:

Min $\sum_{t \in T} \sum_{i \in I} a_i^{co} y_{it}^{vc} + \sum_{t \in T} \sum_{i \in I} (a^v + \bar{f}_i) \left(y_{it}^{vc} + (1 - \alpha_i^{*v}) y_{it}^{*c} \right) + \sum_{t \in T} \sum_{s \in S} (f_s y_{st} + c_s x_{st})$

s.t. (2.3), (2.4), (R2.5), (2.6), (2.7), (R2.13), (2.8) - (2.11), (T2.13)

Total cost of Method 2 can be calculated in the same way used for Method 1 [refer to equation (TCM1)].

Method 3: PR-T

Solve P and R together, and T separately [decompose by constraint set (2.8)].

Problem PR:

Min $\sum_{t \in T} \left(h^v I_t^v + S^v y_t^v \right) + \sum_{t \in T} \sum_{i \in I} a_i^{co} y_{it}^{vc} + \sum_{t \in T} \sum_{i \in I} (a^v + \bar{f}_i) \left(y_{it}^{vc} + (1 - \alpha_i^{*v}) y_{it}^{*c} \right)$

s.t. (2.2) - (2.7), (P2.13), and (R2.13)

Problem T:

Min $\sum_{t \in T} \sum_{s \in S} (f_s y_{st} + c_s x_{st})$

s.t. (T2.8), (2.9) - (2.11), (T2.13)

Using the optimal values of variables found in problems PR and T, the total cost of Method 3 can be calculated in the same way as for TC_{VMI}^v [refer to equation (2.1)].

Method 4: PRT

The model under this method is the same as for the model *VMI-V* (α_i^{*v} values from Heuristic Part1 are of course set as parameter values).

Computational requirements of these methods reveal the tradeoff between finding a good solution the cost minimization problem and the time required to find that solution. For example, Method 1 is easiest to solve, but is anticipated to generate the worst solution. Numerical examples in Section 7 will contrast the four methods in terms of their solution quality and the time required to find a solution. But let us first look at the C&VMI case.

5.6 Consignment and Vendor Managed Inventory (C&VMI)

C&VMI is an alternative sourcing option to VMI where now the vendor, in addition to managing her customer's replenishments, assumes ownership of goods at the customer's premises until they are used. Although the customer pays physical storage cost (h_i^{cs}) of those goods, it is the vendor who pays for opportunity cost of inventory stored at the customer.

Therefore, as opposed to VMI, now it may not be optimal for the vendor to replenish the customer to the greatest extend possible in a given period. The vendor's model under C&VMI is very similar to *VMI-V*. As we will see, the two differences are an extra term in the objective function and a modification to one set of constraints:

Model *CVMI-V*

$$\text{Min } TC_{CVMI}^{v} = \sum_{t\in T}\sum_{i\in I} a_i^{co} y_{it}^{vc} + \sum_{t\in T}\left(h^v I_t^v + S^v y_t^v\right) + \sum_{t\in T}\sum_{i\in I} a^v\left(y_{it}^{vc} + (1-\alpha_i^v) y_{it}^{*c}\right)$$

$$+ \sum_{t\in T}\sum_{i\in I} \bar{f}_i\left(y_{it}^{vc} + (1-\alpha_i^v) y_{it}^{*c}\right) + \sum_{t\in T}\sum_{s\in S}\left(f_s y_{st} + c_s x_{st}\right) + \sum_{t\in T}\sum_{i\in I} h_i^{co} I_{it}^{vc} \qquad \text{(CV2.1)}$$

Subject to

(2.2) - (2.6) production and replenishment constraints

$I_{it}^{vc} \le I_{it}^{c\,MaxCVMI}$ $\forall\, i,t$ (CV2.7)

(2.8) - (2.11) transportation constraints

(2.12) - (2.13) starting/ending conditions and variable types

Note that in (CV2.7), we use the maximum level of inventory set by C_i for C&VMI ($I_{it}^{c\,MaxCVMI}$), instead of $I_{it}^{c\,MaxVMI}$ employed in the inequality (2.7) in the VMI-V model. The total cost of C_i under C&VMI is $TC_{IS\_i}^{*c}$ if α_i^v=0, and it is $TC_{CVMI\_i}^{c} = \sum_{t\in T} a_i^{cs} y_{it}^{*vc} + \sum_{t\in T} h_i^{cs} I_{it}^{*vc}$ otherwise.

As in Section 5, the vendor has to make four sets of decisions when C&VMI is an option: C&VMI (the best set of customers with which to implement C&VMI), production, replenishment, and transportation. Assume for a moment that there is a single customer, and there is no transportation question. The remaining problem of production and replenishment is a dynamic, joint economic lot-sizing model for the vendor. Total cost of the first echelon is $\sum_{t\in T}\left(h^v I_t^v + S^v y_t^v\right)$, and the total cost of second echelon is $\sum_{t\in T}\left(h_i^{co} I_{it}^{vc} + (a_i^{co} + a_v + \bar{f}_i)\, y_{it}^{vc}\right)$. The resulting replenishment quantities are optimal for the vendor, yet they may not be so for the customer who still has to pay h_i^{cs} per unit of inventory and a_i^{cs} for every order. Then, it is in the best interest of customer i to set a maximum inventory level when the vendor replenishes on behalf of him under C&VMI. The *Imax* C_i*-VMI* model that we proposed for VMI can be modified to find those *Imax* levels:

Model *Imax* C_i*-CVMI*: For every *i, i* = 1, ..., *I*:

$$\text{Max } \sum_t h_i^{cs} I_{it}^c \qquad \text{(CV2.14)}$$

Subject to:

(2.15) - (2.16)

$$\sum_t (a_i^{cs} y_{it}^c + h_i^{cs} I_{it}^c) \le TC_{IS\_i}^{*c} \qquad \text{(CV2.17)}$$

$$I_{it}^{c\,MaxCVMI} = I_{it}^c \qquad \text{(CV2.19)}$$

(2.18), and (2.20)

The algorithm that was developed for *ImaxC$_i$ –VMI* can also be used to determine the *Imax* levels under C&VMI.

Lagrangian Relaxation

Lagrangian relaxation is a widely used method to solve large-scale MIP problems. The underlying idea in this method is to obtain smaller problems (that are easier to solve) by means of relaxing some sets of constraints. The relaxation generates a lower bound (in a minimization problem) which in general outperforms the LP lower bound. The reader is referred to Fisher (1981) and Pirkul and Jayaraman (1998) for detailed discussions of the method.

Lagrangian relaxation has been applied to dynamic lot sizing problems by several researchers. Examples include Thizy and Van Wassenhove (1985), Trigeiro (1987), Diaby and Martel (1993), Millar and Yang (1994), and Jans and Degraeve (2004). In those studies, it is generally the capacity constraints that are relaxed. Our model structure, however, is quite different from their DLSP formulations. First of all, we have four sets of decisions (C&VMI or not, production, replenishment, and transportation) in one model. Moreover, we have inventory restrictions specified by customers. Finally, we use an LTL formulation for transportation, which is more realistic, and which includes not only a fixed cost per shipment but also a variable cost per unit.

We will employ Lagrangian relaxation to find a good lower bound to the model *CVMI-V*. Constraint set (2.8) will be relaxed to obtain two sub-problems, one for transportation, and the other for production and replenishment together. Note that, as discussed in Section 5, we could further decompose the latter into two, separating production and replenishment problems. However, this would worsen the lower bound obtained.

Let λ_t be the Lagrange multipliers for the constraint set (2.8). In light of the preceding discussion, the *CVMI-V* problem is decomposed into the following two sub-problems.

Sub Problem 1 (SP1): Production and Replenishment

$$\text{Min } TC_{SP1} = \sum_{t\in T}\sum_{i\in I}(h_i^{co} I_{it}^{vc} + a_i^{co} y_{it}^{vc}) + \sum_{t\in T}\left(h^v I_t^v + S^v y_t^v\right) + \sum_{t\in T}\sum_{i\in I}(a^v + \bar{f}_i)\left(y_{it}^{vc} + (1-\alpha_i^v) y_{it}^{*c}\right)$$

$$+ \sum_{t\in T}\lambda_t\left(\sum_{i\in I}(q_{it}^v + (1-\alpha_i^v) q_{it}^{*c})\right) \qquad \text{(SP1.1)}$$

Subject to:

$$Q_t^v \le M_2 y_t^v \qquad \forall t \qquad (2.2)$$

$$q_{it}^v \le M_1 y_{it}^{vc} \qquad \forall i,t \qquad (2.3)$$

$$\sum_{t\in T} y_{it}^{vc} \le M_3 \alpha_i^v \qquad \forall i \qquad (2.4)$$

$$I_t^v = I_{t-1}^v + Q_t^v - \sum_{i\in I} q_{it}^v - \sum_{i\in I} q_{it}^{*c}(1-\alpha_i^v) \qquad \forall t \qquad (2.5)$$

$$I_{it}^{vc} = I_{it-1}^{vc} + q_{it}^v - d_{it}\alpha_i \qquad \forall i,t \qquad (2.6)$$

$$I_{it}^{vc} \le I_{it}^{c\,MaxCVMI} \qquad \forall i,t \qquad \text{(CV2.7)}$$

$$I_0^v = I_{i,'12'}^{cv} = 0; \quad y_t^v,\ y_{it}^{vc},\ \alpha_i^v \in \{0,1\}; \quad q_{it}^v, Q_t^v, I_t^v, I_{it}^{vc} \ge 0$$

Sub Problem 2 (SP2): Transportation

$$\text{Min } TC_{SP2} = \sum_{t\in T}\sum_{s\in S}\left(f_s y_{st} + c_s x_{st}\right) - \sum_{t\in T}\lambda_t\left(\sum_{s\in S} x_{st}\right) \qquad \text{(SP2.1)}$$

Subject to

$$b_{s-1} y_{st} \le x_{st} \qquad \forall s,t \qquad (2.9)$$

$$x_{st} \le b_s y_{st} \qquad \forall s,t \qquad (2.10)$$

$$\sum_{s\in S} y_{st} \le 1 \qquad \forall t \qquad (2.11)$$

$$b_0 = 0\,; \quad y_{st} \in \{0,1\}; \;\; x_{st} \ge 0$$

When we fix λ_t, we can find solutions to SP1 and SP2, and determine TC_{SP1} + TC_{SP2}, which gives us a Lagrangian lower bound. To update the values λ_t and to find a Lagrangian upper bound, we use cutting plane approach of Kelly (1960). We define a Master Problem (MP) that takes the optimal solution of production, replenishment, and transportation variables from SP1 and SP2, and generates new Lagrange multipliers, to be used in SP1 and SP2. Employing this iterative approach, the algorithm terminates when the Lagrangian lower and upper bounds converge. The solution is then a lower bound to *CVMI-V*.

Master Problem (MP)

The sum of the objective functions for the sub-problems generates a Lagrangian lower bound. In each iteration *k*, the master problem selects the maximum of those lower bounds. Two new constraints, one for each sub problem, are then added to the master problem (see Kelly, 1960). MP, which is a linear model, is then:

$$\underset{\lambda_t}{\text{Max}} \quad \theta_1 + \theta_2 \tag{MP1.1}$$

Subject to:

$$\theta_1 \le \sum_{t\in T}\sum_{i\in I}(h_i^{co}\ {}_kI_{it}^{vc} + a_i^{co}\ {}_ky_{it}^{vc}) + \sum_{t\in T}\left(h^v\ {}_kI_t^v + S^v\ {}_ky_t^v\right) +$$

$$\sum_{t\in T}\sum_{i\in I}(a^v + \bar{f}_i)\left({}_ky_{it}^{vc} + (1-{}_k\alpha_i^v)y_{it}^{*c}\right) + \sum_{t\in T}\lambda_t\left(\sum_{i\in I}({}_kq_{it}^v + (1-{}_k\alpha_i^v)q_{it}^{*c})\right) \tag{MP1.2}$$

$$\theta_2 \le \sum_{t\in T}\sum_{s\in S}\left(f_s\ {}_ky_{st} + c_s\ {}_kx_{st}\right) - \sum_{t\in T}\lambda_t\left(\sum_{s\in S}{}_kx_{st}\right) \tag{MP1.3}$$

The Lagrangian relaxation explained above gives us a lower bound for the *CVMI-V* model. In what follows, we devise a heuristic to find an upper bound to that model.

Upper-Bound Heuristic

SP1 solves the production and replenishment problems together. The quantity shipped in any period must be the same as the total amount replenished to all customers in that period. Lagrangian relaxation furnishes the values of replenishment quantities that are also used to find the best lower bound to the model. To construct a feasible solution to *CVMI-V*, and to determine the upper bound, we do the following: Given the solution of SP1, solve SP2 such that $\sum_{i\in I}(q_{it}^{*v} + (1-\alpha_i^{*v})q_{it}^{*c}) = \sum_{s\in S}x_{st}$.

By the "optimality gap" of C&VMI, we shall mean the percentage difference between the best lower bound found by the Lagrangian-relaxation method and the upper bound determined by the heuristic. We test the performance of the relaxation and the heuristic in 32 large scale problems. Results of those, as well as numerical examples in connection with the models we developed in this paper, are presented in the next section.

5.7 Numerical Analyses

Various computational results are provided here to gain further insights. The first set of examples concerns the impacts of the maximum-inventory levels set by customers. For a small number of customers (up to six), we then compare and contrast exact solutions to cases IS, VMI, and C&VMI. Using that same set, we test our heuristics developed for the *VMI-V* model. Finally, we obtain lower and upper bounds for the *CVMI-V* model in 32 large-size problems.

We coded all of our models in GAMS 20.5, and employed CPLEX 7.5 as the MIP solver. The machine used for computations had an Intel Pentium M 1.6 GHz processor and 512 SD RAM.

While setting values to production and replenishment parameters, we referred to various example-problems of Silver, Pyke and Peterson (1998). We generated parameters of each customer randomly: d_{it} = uniform (0, 200) units, a_i^{cs} = uniform (5, 30) \$/shipment received, a_i^{co} = uniform (10, 45) \$/order, h_i^c = uniform (0.6, 2) \$/unit. We set a^v = \$15/shipment released, S^v = \$ 300 /production run, h^v = \$ 0.6/unit. We assumed that h_i^{co} is fixed to 85% of h^v. Furthermore, the quantity segmentation and cost parameters in our transportation models are based on the example problem in Swenseth and Godfrey (2002). Each example is 12 periods in length. There are five transportation-quantity segments in all examples except the Lagrangian-relaxation test problems.

In six examples we tested the impacts of the customers' maximum inventory levels allowed in VMI and C&VMI agreements. Each of those examples included six customers. Results are presented in Tables 5.1 and 5.2.

Six other examples were solved to compare and contrast the optimal solutions under IS, VMI, and C&VMI. We had only a single customer in the first, and increased the number of customers by one in each successive example. Tables 5.3 – 5.6 summarize the findings.

Those same six examples served to compare the Heuristic Methods 1-4 devised for Model *VMI-V*. Efficiency and effectiveness of those methods are highlighted in Tables 5.7 – 5.9. Finally, the Lagrangian-relaxation approach that we introduced for Model *CVMI-V* was studied for 32 test problems, solutions of which are depicted in Table 5.10.

	% Savings under VMI								
Customer's *Imax* level	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**V**	**Total**	**Total # of contracts**
Infinity	-612	-444	-676	-323	-903	-729	73	-79	6
d_{it}	13	8	-4	25	-3	-15	1	2	6
50% d_{it}	0	0	0	0	0	0	0	0	0
20% d_{it}	0	0	0	0	0	0	0	0	0
$I_{it}^{*c\,MaxCVMI}$	-62	-36	-44	-48	-21	-51	31	15	6
$I_{it}^{*c\,MaxVMI}$	0	18	0	9	2	0	14	12	6

Table 5.1: Impacts of customers' maximum inventory (*Imax*) level in VMI.

We see in Table 5.1 that if customers do not set an *Imax* level under VMI, their losses can be extremely high; the vendor's savings are negligible compared to the total loss of all customers. Therefore, it is very important that each customer determine a maximum level for his stock under VMI. For a low value of *Imax* (e.g. 20% of d_{it}), it is not beneficial for the vendor to offer VMI. Too high a level (e.g. $I_{it}^{*c\,MaxCVMI}$), on the other hand, may harm *all* customers. A choice such as d_{it} will harm at least some of them. The customers can get the best out of a VMI opportunity when they set the "right" level of maximum inventory, which is $I_{it}^{*c\,MaxVMI}$.

In C&VMI, however, a high level of *Imax* may not be disadvantageous for a given customer, since the vendor owns the goods at his facility. Yet, even though the vendor pays the opportunity cost of those goods, customers that have high physical storage costs may be harmed by the vendor's replenishment decisions (e.g. C_5 in Table 5.2). Therefore, it is important, for those customers in particular and for all customers in general, to set an *Imax* level for C&VMI.

	% Savings under C&VMI								
Customer's *Imax* Level	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**V**	**Total**	**Total # of contracts**
Infinity	46	32	17	72	-11	39	21	26	6
d_{it}	0	0	38	0	0	0	1	2	1
50% d_{it}	0	0	0	0	0	0	0	0	0
20% d_{it}	0	0	0	0	0	0	0	0	0
$I_{it}^{*c\,MaxCVMI}$	21	23	6	47	13	39	15	18	6
$I_{it}^{*c\,MaxVMI}$	41	0	27	52	29	41	4	11	5

Table 5.2: Impacts of customers' maximum inventory (*Imax*) value in C&VMI

Similar to the case of VMI, as the value of *Imax* decreases (Table 5.2), it becomes more difficult for the vendor to realize operational benefits, which implies fewer C&VMI agreements. When we compare the last two rows of Table 5.2, we see that $I_{it}^{*c\,MaxVMI}$ results in a smaller number of agreements, lower savings for the vendor, but higher savings for the C&VMI customers. Although $I_{it}^{*c\,MaxVMI}$ under C&VMI seems to be a good option, it fails for C_2. On the other hand, an *Imax* level of $I_{it}^{*c\,MaxCVMI}$ increases the vendor's savings, and also increases the number of C&VMI-customers. This means a more even distribution of savings among the customers.

Table 5.3 summarizes the cost comparisons for both the vendor and customers in three cases. We see that C&VMI is definitely a better option for customers than *VMI*. Yet, it may not be so for the vendor: she owns inventory at customers' premises and pays the associated inventory costs under C&VMI. We observe that in a given example, the total number of agreements influence which type of agreement is more beneficial for the vendor. In Examples 4 and 6, the number of VMI agreements and the savings under VMI is greater. On the other hand, there are more agreements and higher savings under C&VMI in Examples 3 and 5. When the number of VMI and C&VMI agreements is the same (Examples 1 and 2), VMI tends to be more beneficial than C&VMI for the vendor.

			Costs ($) under IS		**Average % savings per customer**		**% savings for *V***	
# of customers	# of VMI contracts	# of C&VMI contracts	Average per customer	*V*	VMI	C&VMI	VMI	C&VMI
1	1	1	493	4,469	0	3	2	1
2	2	2	405	6,470	0	5	3	2
3	2	3	435	8,062	5	7	8	13
4	3	2	477	10,353	4	8	13	12
5	3	4	572	12,002	1	3	8	12
6	6	5	408	12,635	1	3	12	11

Table 5.3: Percentage of total savings in six examples that are solved optimally

Observing the cost breakdowns of customers under different cases (Table 5.4), we see that total ordering costs are far greater than total inventory costs under IS (note that we aggregate the costs of all customers to perform the analysis here). Of course, the assumed values of parameters determine that breakdown; however, we are only concerned with the impacts of VMI and C&VMI, given the costs under IS. Then, we note that those two agreements increase the percentage of inventory costs for customers. This indicates that most of their savings come from efficiencies in ordering. We also see that customers prefer C&VMI to VMI, since that the former provides additional savings in inventory (i.e., percentage of inventory costs is in general lower under C&VMI than VMI).

# of customers	**IS**		**VMI**		**C&VMI**	
	Ordering	Inventory	Ordering	Inventory	Ordering	Inventory
1	91	9	26	74	27	73
2	95	5	24	76	24	76
3	87	13	56	44	57	43
4	87	13	44	56	65	35
5	90	10	56	44	52	48
6	93	7	41	59	56	44

Table 5.4: Cost breakdown (in %) of customers in all cases (aggregated for customers)

Table 5.5 depicts the production (P), replenishment (R), and transportation (T) cost breakdown of the vendor in the three cases considered. As the number of customers increases under IS, one sees an obvious decrease in the percentage of P costs, and an increase in the percentage R.

# of customers	**IS**			**VMI**			**C&VMI**		
	P	R	T	P	R	T	P	R	T
1	41	11	48	39	14	47	38	17	45
2	35	11	54	33	15	52	31	17	52
3	31	11	58	30	15	55	27	26	47
4	27	16	57	29	22	49	27	25	48
5	27	15	58	27	22	51	26	28	46
6	27	19	54	29	25	46	25	28	47

Table 5.5: Cost breakdown (in %) of vendor in all cases

It is observed in Table 5.5 that compared to IS, VMI helps the vendor achieve economies of scale in production and distribution, although the percentage of the costs R increases. The same argument is true for C&VMI, but now the impacts are greater (that is, the percentage of costs P and T is lower, and the R cost is higher compared to VMI).

	Computation Times (in seconds)			
# of customers	*IS-C*	*IS-V*	*VMI-V*	*CVMI-V*
1	0.9	1.5	3.1	5.4
2	0.9	1.5	4.5	4.8
3	2.25	1.5	6.9	18.3
4	2.55	1.5	8.4	16.8
5	1.5	1.8	16.5	29.7
6	3	4.5	41.7	217.5

Table 5.6: Computation times for six example problems, solved optimally.

The computation times included in Table 5.6 serve as a benchmark for the relative difficulty of finding a solution to our models. We see that the CPU time to solve a *CVMI-V* model always exceeds the solution time required for *VMI-V*. As the problem size increases, the absolute CPU gap between those two tends to increase, becoming very high when there are six customers.

The remaining tables concern the heuristic methods we proposed for solution of VMI and C&VMI models. In Table 5.7, we present the performance of Method 4, employed in our heuristic developed for VMI. Recall that the first part of that heuristic initially selects VMI-customers. Method 4 in the second part then solves the remaining problem of coordinated production, replenishment and transportation together.

We notice that the number of agreements found by part one of the heuristic is greater than or equal to the optimal number of agreements in all examples (see the first two columns of Table 5.7). This makes sense, since the optimal solution considers all VMI alternatives, but eliminates some: Economies of scale can be achieved by selecting fewer customers for VMI, and adjusting their replenishments and the transportation to non-VMI customers. As a result, part one of the heuristic finds an upper-bound for the number of agreements. Solving the remaining problem of P, R, and T together by Method 4, one sees that the gap between that upper-bound and the exact solution is very low (Table 5.7). Commparing CPU times, we see that the heuristic saves more time as the number of customers increases (in Example 6, the computation time required to solve the heuristic's part one and two is around 14 times less than finding an optimal solution to the *VMI-V* model).

total # of agreements		**% Gap between costs: 100(opt H – opt E) / opt E**				**CPU time ratio**
E	H	Prod	Rep	Trans	Total Cost	cpu(E)/cpu(H)
1	1	0	0	0	0	1.3
2	2	0	0	0	0	1.7
2	3	0.02	0.13	-0.01	1.98	1.5
3	4	-0.02	0.08	-0.01	0.73	2.5
3	5	-0.02	0.16	-0.01	2.68	7.9
6	6	0	0	0	0	13.9

Table 5.7: Exact versus Method 4 (heuristic) solution of VMI. E: Exact, H: Heuristic, opt: optimal objection function value.

Having shown that part one of the heuristic performs reasonably well, let us now compare the decomposition methods proposed for solving the remaining problem of coordinated production, replenishment and transportation. Since Method 4 solves P, R and T together without any decomposition, it is the base method to which other approaches involving decomposition can be compared to evaluate their performances. Method 3 solves P and R together and T separately, whereas Method 2 solves P separately and R and T together. Method 1 decomposes the problem into three as P, R, and T and solves each separately. Also remember that in any case, we employ the hierarchical solution approach where P comes first, R second, and T the last.

	TC^{v}_{VMI} ($)	**% Gap compared to Method 4**		
# of customers	Method 4	Method 3	Method 2	Method 1
1	4,389	0	0.1	0.1
2	6,262	0	0.4	0.4
3	7,942	0.3	2.4	2.8
4	9,075	5.1	10.3	11.3
5	11,293	5.4	**12.4**	**12.4**
6	11,148	9.8	10.5	10.6

Table 5.8: Comparison of heuristic methods 1-4 in terms of the vendor's total cost

We see (Table 5.8) that Method 3 performs very well when compared to the methods two and one, and yields a total cost of VMI which deviates from optimality by at most 9.8 percent (but still less than the total cost under IS). Method 2 performs only slightly better than Method 1 in some examples, leading to the conclusion that the performance of one over the other is negligible. It is important to note that, as opposed to Method 3, these two methods may not always provide a total cost still lower than that of IS. In example 5, the 12.4 % gap for Methods 2 and 1 leads to a total cost exceeding the total cost under IS. Finally, we remark that Method 3 performs well in determining replenishments costs, but poorly for the costs of transportations. Methods 1 and 2 do well in production, yet replenishment and transportation costs are very high.

Comparing Methods 1-3 in terms of computation times (Table 5.9), Method 1 clearly fastest. Method 3 takes more solution time than Methods 1-2 (but performs better, as revealed in Table 5.8). However, the former is still less time-consuming than Method 4. These results also justify the reasoning in Lagrangian relaxation, which we used to separate the problem into two as in Method 3.

# of customers	CPU time (in seconds)	CPU-time ratio of Method(n)/Method4		
	Method 4	Method 3	Method 2	Method 1
1	2.4	0.87	0.73	0.66
2	2.6	0.88	0.74	0.68
3	4.6	0.91	0.8	0.77
4	3.4	0.9	0.77	0.73
5	2.1	0.86	0.71	0.63
6	3	0.89	0.76	0.71

Table 5.9: CPU time comparison of heuristics methods 1-4

Table 5.10 presents the results of 32 test problems solved by the Lagrangian relaxation method we used for Model *CVMI-V*. The table includes the problem sizes (indices i and s), percentages of cost breakdown in each problem, LP lower bound, heuristic upper bound, and optimality gap (all comparisons assume that the lower bound obtained from Lagrangian relaxation is fixed at 100%), and the computation times (in seconds) required to solve each problem.

We see in those 32 problems that the LP bound varies between 41% and 49%, which indicates that the Lagrangian relaxation performs well in finding a better lower bound. The percentage gap is around 6.4 at most, and lower than 3.0 in majority of the problems. This indicates the good quality of the heuristic solution, with very reasonable upper bounds. SP1 requires the greatest amount of CPU time, followed by SP2, MP, and the heuristic. Total time required to solve a problem varies between two and six minutes.

This concludes our numerical analysis. In the next section, we will provide a summary and conclusions obtained from our work.

	Index		Breakdown of Costs (%)			% Bounds (assume Lag. Bound =100)			CPU TIMES (in 10*seconds)				
Pro. #	*i*	*S*	Prod.	Repl.	Trans.	LP	Heur.	% GAP	SP1	SP2	MP	Heur.	TOTAL
1	15	5	28	14.3	57.7	44.0	102.4	2.4	23.46	0.6	0.29	0.02	24.35
2	16	5	26.8	14.8	58.4	44.5	102.3	2.3	22.69	0.55	0.29	0.02	23.53
3	17	5	25.6	15.6	58.8	43.9	101.9	1.9	29.21	0.79	0.21	0.02	30.21
4	18	5	25	15.3	59.7	45.6	101.8	1.8	20.63	0.67	0.41	0	21.71
5	19	5	24.3	15.6	60.1	45.5	101.8	1.8	19.71	0.64	0.26	0.02	20.6
6	20	5	23.8	15.5	60.7	47.2	101.8	1.8	19.08	0.63	0.24	0.02	19.95
7	21	5	22.6	16.2	61.2	46.4	101.5	1.5	15.74	0.47	0.21	0.02	16.42
8	22	5	22.1	16	62	47.6	101.4	1.4	17.47	0.49	0.26	0.11	18.22
9	23	5	21.8	16.1	62.1	48.8	101.4	1.4	15.29	0.46	0.2	0.02	15.94
10	24	5	20.4	16.3	63.3	47.8	101.3	1.3	17.6	0.46	0.18	0.09	18.24
11	25	5	19.7	16.1	64.2	47.9	101.3	1.3	16.5	0.41	0.18	0.03	17.1
12	26	5	19.2	16	64.7	49.1	101.2	1.2	16.12	0.38	0.11	0.11	16.6
13	27	5	18.3	16.7	65.1	48.5	101.2	1.2	18.17	0.47	0.18	0.11	18.83
14	28	5	17.6	17.2	65.2	48.1	101.1	1.1	22.28	0.36	0.17	0.09	22.81
15	29	5	17.4	17.3	65.2	49.1	101.1	1.1	20.11	0.47	0.24	0.09	20.83
16	30	5	16.8	17.2	66	49.1	101.1	1.1	17.46	0.43	0.21	0.09	18.1
17	15	6	28	14.3	57.7	41.9	106.4	6.4	21.44	0.65	0.26	0.02	22.34
18	16	6	26.8	14.8	58.4	42.3	106.3	6.3	21.54	0.64	0.24	0	22.42
19	17	6	25.6	15.6	58.8	41.8	106.0	6.0	30.18	0.52	0.24	0.09	30.93
20	18	6	25	15.3	59.7	43.4	106.0	6.0	19.89	0.61	0.26	0.02	20.76
21	19	6	24.3	15.6	60.1	43.3	106.0	6.0	24.91	0.47	0.2	0	25.58
22	20	6	23.9	15.5	60.5	45.0	105.6	5.6	19.88	0.69	0.15	0.09	20.72
23	21	6	22.9	16.4	60.8	44.2	104.5	4.5	18.05	0.38	0.15	0.03	18.58
24	22	6	22.5	16.3	61.2	45.3	103.7	3.7	22.63	0.38	0.21	0.11	23.22
25	23	6	22.2	16.4	61.4	46.6	103.8	3.8	18.38	0.39	0.15	0.03	18.92
26	24	6	20.8	16.6	62.6	45.6	103.6	3.6	19.24	0.32	0.18	0.06	19.74
27	25	6	20.3	16.6	63.1	45.7	102.5	2.5	19.4	0.35	0.21	0.03	19.95
28	26	6	19.9	16.6	63.5	46.8	102.2	2.2	17.36	0.43	0.17	0.09	17.96
29	27	6	19	17.4	63.6	46.2	101.5	1.5	18.47	0.24	0.24	0.08	18.95
30	28	6	18.3	17.9	63.8	45.8	101.4	1.4	22.71	0.77	0.2	0.09	23.67
31	29	6	18.1	18.1	63.8	46.7	101.4	1.4	19.54	0.35	0.14	0.09	20.02
32	30	6	17.5	17.9	64.6	46.7	101.4	1.4	19.22	0.38	0.14	0.03	19.74

Table 5.10: Lagrangian relaxation results for 32 test problems

5.8 Summary and Conclusions

In this paper, we investigated a supply chain composed of a single vendor and multiple customers who face time-varying external demand. In the traditional way of doing business, each and every customer replenished from the vendor, and the vendor optimized her costs independently. We modeled the vendor's and the customers' dynamic lot sizing models as an MIP, and found the optimal values of their decision variables.

We then proposed that the vendor may decrease her costs if she implements VMI with a subset of customers. We formulated an MIP model for VMI so that the vendor could select the right set of VMI-customers, and could optimize her production, customer-replenishment, and transportation processes. To solve the model developed, we introduced a heuristic with two parts. The first decided the best set of customers with which to implement VMI. The second part included four methods to solve the remaining problem of integrated production, replenishment, and transportation. These three operational activities could be decomposed into two or more separate problems, as suggested by several of the methods.

As for customers, we showed how important it is that they set maximum inventory levels when VMI is an option. We proposed a model to find the optimal value of those levels, and an algorithm to solve it.

Afterwards, we analyzed C&VMI in Section 6 as an alternative to VMI. Under this agreement, the vendor paid the opportunity cost of goods stored on the premises of any C&VMI-customers. The model we formulated for the vendor under C&VMI was very similar to the one under VMI, yet it proved more difficult to solve. We then used Lagrangian relaxation, with the cutting-plane approach of Kelly (1960), to find a good lower bound on the solution. We devised a heuristic to obtain an upper bound, which generated reasonable optimality gaps.

Under C&VMI, it was not as crucial as in VMI that a customer specify a maximum level of inventory that the vendor could employ. Nevertheless, we showed that it is in a customer's best interest to do so. We modified for C&VMI the maximum-inventory-level model developed for VMI. Both models could use the same solution algorithm.

We can derive several managerial insights based on our analytical and numerical work. First of all, success of VMI for a customer depends on the maximum inventory level he allows. Too low a value results in no VMI agreement, hence the customer loses the chance to reduce his costs. Too high a value, on the other hand, likely increases his costs compared to the traditional way of doing business, causing VMI to fail.

Under C&VMI, since the vendor owns the goods replenished to the customer, that level of inventory is not as important as it is under VMI. Customers with high physical-storage costs, however, should be careful about the large quantities the vendor may prefer to ship. In any case, a customer would eliminate the possibility of losing money under C&VMI if he sets maximum inventory levels beforehand.

Our results show that, for any customer, C&VMI is definitely a better option than VMI. This is true for the vendor only when the number of C&VMI agreements can exceed the number of VMI agreements in a given setting. In any other case, the vendor saves more under VMI. However, the difference in savings between the two agreements tends not to be large. Therefore, the vendor may offer C&VMI to those customers less willing to accept VMI.

We can also provide conclusions on the heuristics proposed and employed in this paper. The first part of the heuristic developed for the vendor's model under VMI separated from the rest of the model the decisions concerning whom to offer VMI. Numerical examples reveal that solutions found by that heuristic form an upper bound on the exact solution, and the optimality gap is not high.

The four methods we proposed to solve the second part of the heuristic for integrated production (P), customer-replenishment (R), and transportation(T) can be compared within themselves. In three of those methods, we decomposed that integrated problem into sub-problems as P-R-T, P-RT and PR-T, where a dash indicates the separation. Computational testing showed the obvious tradeoff between getting a solution and spending more time to get a good solution. The separation PR-T clearly outperforms the others in finding a good solution, but does not consume much more time in doing so.

For the vendor's model under C&VMI, Lagrangian relaxation decomposed the problem into two, again as PR-T. When compared to LP bounds, the relaxation performed well within reasonable times. The heuristic that we developed could also be executed quickly to find a good upper bound. The relaxation method and heuristic solution yielded acceptable results, as revealed by the optimality gap.

Future research could investigate additional heuristic methods for the C&VMI problem either in the same setting, or when there are multiple products. An extension of the latter might include a customer who wishes to select VMI, C&VMI or Consignment Inventory agreements for various types of items he purchases from multiple vendors.

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6. Conclusion

We have investigated various supply-chain sourcing practices in three essays. The first essay included VMI and central decision making for a single vendor and customer under stationary demand. The second evaluated CI and C&VMI in the same setting. The third considered VMI and C&VMI in a supply chain composed of a vendor and multiple customers who face time-varying but deterministic demand. In all the essays, the traditional way of doing business was used as a base case.

VMI was the main theme of Essay 1. With that agreement, the vendor could make replenishment decisions on behalf of the customer, but would incur the cost to issue an order. The three possible outcomes of VMI are an efficient, inefficient, or a potentially efficient system. There are no system-wide cost savings in an inefficient system. Both the vendor and customer are better of if the system is efficient. In a potentially efficient system, only one party is better off, yet there are system-wide cost savings. Central decision making, on the other hand, would provide the best possible system-wide cost.

Some general conditions as to what type a system VMI creates were discussed in Essay1. Our analyses indicate, in many instances, that either the customer alone or the vendor alone captures the savings generated by VMI. Even so, a change from independent decision making is often worthwhile.

In a potentially-efficient system, the better-off vendor can compensate the customer to the point that his losses are neutralized. We used transportation-cost sharing and price discounts to demonstrate how that compensation could be achieved. We showed that when the benefits are shared in the right way, a potentially efficient system can be turned into an efficient one.

In the second essay, we initially compared CI with the base case (inventory sourcing) and obtained analytical conditions under which CI creates benefits for the vendor, the customer and the whole system. In contrast to the general belief that CI is beneficial only for the customer, our results reveal that it may be favourable for the vendor, too, depending on the costs of shipment, and who pays for transportation.

When a CI agreement results in a potentially efficient system, it can be turned into an efficient one through a price discount. We found the minimum and the maximum amounts to which the wholesale price may increase, such that the customer may accept to share his benefits with the vendor. When the system is inefficient under CI, the vendor can offer a C&VMI agreement to realize savings for her and for the system.

We showed the vendor can make use of C&VMI to improve her costs in areas in which she is inefficient. Although in general the vendor prefers C&VMI rather than CI, and the customer vice versa, we observed that C&VMI is more likely to generate system-wide cost savings.

In Essay 3, we investigated a supply chain composed of a single vendor and multiple customers under time-varying and deterministic demand. In the traditional way of doing business, each and every customer replenished from the vendor, and the vendor optimized her costs independently. We modeled the vendor's and the customers' dynamic lot sizing models as an MIP, and found the optimal values of their decision variables.

Through VMI, the vendor could achieve economies of scale in her operations. We formulated an MIP model for VMI so that the vendor could select the right set of VMI-customers, and could optimize her production, customer-replenishment, and transportation processes. A heuristic with two parts was introduced to solve that model. The first part decided the best set of customers with which to implement VMI. The second included four methods to solve the remaining problem of integrated production, replenishment, and transportation. These three operational activities could be decomposed into two or more separate problems, and solved hierarchically.

Customers could guarantee at least not to be worse off under VMI when they set the right levels of maximum inventory. We proposed a model to find the optimal value of those levels, and an algorithm to solve it.

C&VMI was analyzed in a separate section of the third essay. The vendor's model proposed for this agreement was more difficult to solve compared to the VMI model. We then used Lagrangian relaxation, with the cutting-plane approach of Kelly (1960), to find a good lower bound on the solution. We devised a heuristic to obtain an upper bound, which generated reasonable optimality gaps. As for the customers, it was not very crucial, as in VMI, for them to set maximum levels of inventory. Yet, we showed that it is in their best interest to do so. The algorithm used to find those levels under VMI would work under C&VMI as well.

The results obtained in Essay 3 reveal that C&VMI is a better option than VMI for the customers. This is true for the vendor only when the number of C&VMI agreements can exceed the number of VMI agreements in a given setting. In any other case, the vendor saves more under VMI. However, the difference in savings between the two agreements tends not to be large. Therefore, the vendor may offer C&VMI to those customers less willing to accept VMI.

In terms of the solution approaches used, the heuristic developed for the vendor's VMI model performed well. The first part of the heuristic, which dealt with whom to offer VMI, found solutions which formed an upper bound on the exact solution; the optimality gap was not high. The second part handled the remaining problem of integrated production (P), customer-replenishment (R) and transportation (T). Among the three decomposition methods P-RT; PR-T; P-R-T (where a dash indicates the decomposition), which solved the subproblems hierarchically, PR-T performed the best in terms of finding a good solution.

The vendor's model under C&VMI was solved using Lagrangian relaxation which decomposed that model into two as PR-T. When compared to LP bounds, the relaxation performed well within reasonable times. A simple heuristic could also be executed quickly to find a good upper bound. The relaxation method and heuristic solution yielded acceptable results, as revealed by the optimality gap.

Future research might consider a customer who needs to determine the proper sourcing method for various items purchased from different vendors. For example, CI is always beneficial for the customer without any change in the wholesale price. However, various suppliers could enforce a price increase when CI is offered. In that case, the customer could carefully select the CI-vendors to maximize her savings, or could consider VMI or C&VMI under which the vendor would offer different prices.

One can simulate the resulting models when end-consumer demand is uncertain. In settings where demand is more stable, it may be likely to find closed-form solutions. If this is not possible, heuristic methods could be developed.

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