Coupled Numerical Modelling of Vacuum Consolidation with Nonuniform Pore Pressure Distribution

by

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A thesis

presented to the University of Waterloo

in fulfilment of the

thesis requirement for the degree of

Master of Applied Science

in

Civil Engineering

Waterloo, Ontario, Canada, 2021

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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ABSTRACT

In this study, Biot's type hydro-mechanical coupled numerical models are used to examine ground improvement of fine-grained soft soil deposits using prefabricated vertical drains (PVD) and vacuum assisted consolidation methods in combination with embankment preloading. Fully coupled numerical simulations are developed in the context of the traditional unit cell radial consolidation theory commonly applied to PVD and vacuum assisted consolidation. The theoretical justification of nonuniform stress and porewater pressure distribution under an embankment of finite dimension is examined, with reference to field observations from full-scale case studies of PVD/vacuum consolidation in the literature. The impact of nonuniform porewater pressure distribution on the traditional unit cell radial consolidation theory are examined through numerical modelling, and the theoretical compatibility of nonuniform porewater pressure distributions and unit cell radial consolidation theory is discussed.

Through numerical modelling, it is observed that the traditional unit cell model of radial consolidation theory, which PVD and vacuum consolidation solutions were developed from, is functionally constrained to the assumption of uniform surcharge in the soil as the initial undrained condition. Deviations from the uniform surcharge assumption, such as nonuniform porewater pressure distribution in the soil that leads to variable porewater pressure gradients with respect to depth below the preloading embankment, or nonuniform applied vacuum pressure with depth, will effectively highlight the theoretical limitation of the traditional unit cell radial consolidation. To adequately address nonuniform stress and porewater pressure distribution in the soil, fundamental revisions to the traditional linear governing equations for PVD and vacuum consolidation are needed considering nonlinearity of the consolidation equation arising from evolving permeability and compressibility of the soil due to change in void ratio during consolidation; non-Darcian flow regime for low permeability soil; and large strain

elasto-plastic behavior of the soil. In this study, considering the nonlinear soil stress-strain relationship are approximated using the Modified Cam-Clay model.

ACKNOWLEDGEMENTS

I would like to acknowledge the guidance and support from my supervisor Professor Dipanjan Basu.

I would also like to acknowledge the guidance and supports from Professor Leo Rothenberg and Professor William Annable, whose suggestions and encouragements were invaluable in all aspect of this research project.

I would like to extend a heartfelt acknowledgement to my spouse. This thesis and my graduate study would not have been possible without her constant love and support over the years.

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1. Introduction

Since its introduction in the mid 1900's, the method of vertical drains for improving saturated soft soil deposits have undergone a number of iterations from sand drains; to band shaped Prefabricated Vertical Drains (PVD); and to the recent popularity of vacuum drains. Today, PVD combined with preloading remains a widely utilized method of ground improvement, particularly for soft clay deposits situated in coastal regions. The seminal work by Barron (Barron, 1948) provided the first closed form solution for "equal-strain hypothesis" theory of a single vertical drain unit cell radial consolidation. Hansbo later iterates on Barron's unit cell radial consolidation theory and proposed a closed form analytical solution of radial consolidation applied to band-shaped PVDs (Hansbo, 1982). Over the past three decades, the popularity of vacuum assisted consolidation has grown steadily to now it is seen as a cost-effective alternative to PVD consolidation. Currently available closed form analytical solutions of vacuum assisted consolidation proposed by Indraratna and Chai were also developed using the same framework of unit cell radial consolidation theory, and largely serves as an extension to Hansbo's solution for PVD consolidation.

Although Hansbo's solution remains widely used in ground improvement via PVD combined with preloading, particularly for estimating the rate of consolidation of the improved soil, recent publications in literature have been forgoing the traditional unit cell radial consolidation theory for an increased emphasis on Biot's type fully coupled numerical models for the study of PVD and vacuum consolidation. This is apparent in the publications on topics of vacuum assisted consolidation in the past decade (Rujikiatkamjorn et al, 2008; Saowapakpiboon et al, 2010; Chai et al, 2014; Liu et al, 2015; Vu et al, 2018), where there has been an effort to improve the numerical models of the conventional unit cell models of radial consolidation that were presented up to the mid 2000's (Bergado et al, 1992; Hird et al, 1992; Chai et al, 1995; Indraratna et al, 2005a; Baek et al, 2006), towards consolidation models that incorporates multiple (or a field of) PVD/vacuum drains inside plane strain; axisymmetric; or fully 3-D

semi-infinite domain. These multi-drains models of consolidation typically feature multiple (or a field of) PVD/vacuum drains installed in various patterns (usually triangular or square patter) and drain spacings; as well as the dimensions of the embankment providing preloading to the soil, resulting in a model domain that resembles the conceptual model for a footing problem commonly found in geotechnical engineering. These new consolidation models almost exclusively feature Biot's type hydro-mechanical (H-M) coupling scheme, with Darcian flow properties for the dissipation of porewater, and utilizing Modified Cam-Clay model for the soft soil. This new approach to modeling PVD and vacuum consolidation in fine grained soft soils offers several theoretical and practical advantages compared to the traditional unit cell model of radial consolidation. To start, the conceptual model of PVD and vacuum drains with preload from embankment can be better represented by a semi-infinite domain of either plane strain, axisymmetric or fully 3-D, which allows multiple drains or a field of drains of various spacing and spatial patterns to be included in the model on consolidation, and the analysis of consolidation induced strain and deformation of the soft soil at any given location below the embankment. Setting up the model domain in this way means the finite dimensions of the embankment above the soil can be included, multiple drains placed in the soil below the embankment means both vertical settlement and lateral deformation as a result of consolidation can be simulated, thus providing the analysis of strain and deformation at the toe of the embankment. Another improvement comes from utilization of an elasto-plastic soil model such as Modified Cam-Clay in the H-M coupled scheme. For a fine-grained soft soil such as normally consolidation soft clay, which PVD and vacuum consolidation methods are commonly applied to, the inclusion of non-linear stress-strain relationship in the soil is more realistic compared to the traditional unit cell theory derived from loosely coupled poroelastic model of the porous medium, which can be an oversimplification when applied to clay rich soils.

1.1 Research Needs

With the aforementioned improvements to the conceptual models of PVD/vacuum consolidation and its application to numerical modeling studies, recent studies of PVD/vacuum consolidation provided more rigorous numerical simulation of the problem, and researchers often presented better correlation of the numerical results to that of observed in full-scale case studies of PVD/vacuum plus preload consolidation in the field, whereas the traditional unit cell radial consolidation models typically are verified through laboratory studies. However, in the author's opinion, even though aspects of these new numerical models of PVD/vacuum assisted consolidation presented in the literature certainly provide a step forward, in several other aspects, the new numerical models are still utilizing many of the core assumptions of the traditional unit cell model of radial consolidation. This is evident in the traditional governing equations of radial consolidation, which Hansbo, Indraratna and Chai's solutions of PVD/vacuum consolidation were derived from, are still being utilized in most of the coupled consolidation models presented in the past decade. Recent studies from the likes of Walker and Indraratna (Walker et al. 2012; Indraratna et al. 2017) made strides to update the governing equation of radial consolidation to include considerations for non-Darcian flow, evolving permeability and void ratio of the soil, and large strain analysis of consolidation.

Another aspect that can be improved upon is the conceptual model of PVD/vacuum consolidation, and the central focus of this thesis, is the concept of preloading the soft soil through the construction of an embankment on top, and the common assumptions to the stress and porewater pressure in the soil layer below the embankment. In the traditional conceptual model of consolidation, it is traditionally assumed that preloading the soil resulted in a surcharge of excess pore pressure of equal magnitude in the soil at the undrained stage and leads to porewater dissipation and ultimately consolidation under the additional effective stress in the soil. The phenomenon of the surcharge is often observed in laboratory testing, however in many full-scale case studies of PVD and vacuum consolidations, it was

observed that if the soil is preloaded by an embankment of finite dimensions, the assumed surcharge is limited to the regions of the soil immediately below the embankment and the uniform distribution of excess pore pressure in the soil can not be easily distinguished (Long 1990, Chu et al. 2001, Indraratna et al. 2012, Deng et al. 2017), the distribution could vary with depth below the embankment. This has some researchers question the limitation of the surcharge assumption in consolidation problems, and how pore pressure distribution in the soil layer effects the dissipation of the porewater towards the PVD/vacuum drains during consolidation. To the author's knowledge, the question of embankment induced excess stress and pore pressure distribution has not been specifically addressed in any of the recent numerical models of PVD/vacuum consolidation. One must also bring to question if the currently accepted conceptual model of PVD/vacuum consolidation, which led to the existing numerical models of consolidation presented in the literature, are fully capable of analyzing, or compatible to, the concept of stress and pore pressure distribution in the soil, that is not assuming surcharge, or uniformly distributed with depth. Ultimately the impacts of the nonuniform distribution of stress and pore pressure have on PVD/vacuum consolidation needs to be examined.

1.2 Research Objectives

In this study, several Biot's type fully H-M coupled numerical models are developed to examine the theoretical distribution of excess stress and pore pressure under an embankment, which provides the preloading to a fine-grained soft soil deposit. The theoretical validity of the nonuniform stress and pore pressure distribution under an embankment will be examined through numerical simulations, as well as reference to observation made in prior full-scale case studies of PVD/vacuum consolidation. The nonuniform distribution of pore pressure is also examined in the context of the traditional unit cell radial consolidation model, which remains the popular framework for the analysis of PVD/vacuum consolidation. The impact of nonuniform pore pressure distribution have on traditional unit cell radial consolidation theory are illustrated by numerical simulations, and their theoretical compatibility are

discussed. The justification for utilizing unit cell model, instead of the more recent multi-drain model of PVD/vacuum consolidation for this study is that despite the improvements made by through the recent numerical models, ultimately the majority of which still utilizes the traditional governing equation of unit cell radial consolidation. In this aspect, the new models can generally be interpreted as several unit cell consolidation models placed side by side in a semi-infinite domain, because they still share many of the concepts and assumptions about permeability, compressibility, and the coefficient of radial consolidation. Throughout this study, it is found the theoretical limitations of the unit cell model of consolidation will likely also be present in the new methods of modelling PVD/vacuum consolidation, and fundamental updates to the governing equation for vacuum consolidation is needed to address nonuniform stress and pore pressure distribution. It is hypothesized that the governing equation of radial consolidation should include considerations for non-Darcian flow, evolving permeability and void ratio, and large strain analysis of consolidation.

2. Literature Review

2.1 Development of Prefabricated Vertical Drain (PVD) and Vacuum Consolidation for Improving Soft Soils

PVD combined with preloading is a commonly utilized method for ground improvement in saturated soft soils, particularly for soft clay deposits situated in coastal regions. Since it was first introduced in 1937 by Walter Kjellman, the method of vertical drains for improving strength of soft soils have gone through several evolutions from sand drains; to band shaped PVD; to the recent popularity of vacuum drains. The seminal work by Barron (Barron, 1948) provided the first closed form solution for “equal strain hypothesis” theory of radial consolidation which describes drainage towards a single central vertical drain inside a cylindrical unit cell, where the settlement of the soil is constraint to the 1-D uniaxial

direction. The single drain/PVD cylindrical unit cell theory assumes small strain conditions and ideally below the center of the embankment, where the lateral strain is assumed to be zero. Yoshikuni and Nakanodo developed the “free strain” model of radial consolidation in a unit cell model (Yoshikuni. H, Nakanodo. H, 1974) with consideration for Biot’s theory of consolidation. Hansbo proposed an equivalent drain radius for converting band shaped PVDs and presented an iteration to Barron’s free strain radial consolidation solution (Hansbo, 1981, 1997, 2001). Subsequently development introduced iterations on Hansbo’s original solution with additional considerations for smear zones, vertical drainage, preloading via ramp function, time-dependent embankment loading; non-Darcian porewater drainage and variable horizontal coefficient of consolidation. (Hansbo, 1997, 2001 b, Leo et al. 2004, Basu et al. 2000, Conte et al. 2009; Walker et al. 2012). Due to its simplicity of use, the original Hansbo’s solution remains the most utilized solution for PVD consolidation in ground improvement projects.

Since the early 1990’s, the popularity of vacuum consolidation has steadily grown over the last three decades. Vacuum consolidation has several advantages over the traditional PVD and preloading method of ground improvement. Most notably, vacuum consolidation does not require a backfill embankment to be built on top of the soft soil, thus providing a cost-efficient alternative. With its growing popularity, numerous case studies and recent research publications on vacuum consolidation are available (Bergado et al. 1998; Tang and Shang 2000; Indraratna et al. 2005a; Chai et al. 2007; Chu et al. 2000; Saowapakpiboon et al. 2010; Kumar et al. 2013; Vu et al. 2018; Cao et al. 2019) Although preloading via backfill embankment is not theoretically necessary for vacuum consolidation, in practice, vacuum consolidation is often combined with preloading to achieve faster consolidation time. Similar to the premise of consolidation by PVD, the radial consolidation theory serves as the basis for the currently applied conceptual model of vacuum assisted consolidation, which also includes the assumption for surcharge in the soil resulting from preloading. This is evident in several developed close form solutions of vacuum assisted consolidation available today.

Currently the two common methods of applying vacuum pressure into the ground are via either surface sealing sheet method, or capped PVD method (Chai et al. 2010). It is also common to apply additional preloading in the vacuum consolidation improved soil via backfill embankment above. Depending on whether PVDs and/or preloading embankments are present, their combinations with the two aforementioned methods for applying vacuum pressure to the ground can result in different conceptual models and interpretations of vacuum consolidation. For practical purposes, two conceptual designs for vacuum consolidation are theoretically the most efficient application for vacuum consolidation and of the most interests by researchers. They are:

1) Surface sealing sheet method combined with PVD and preloading embankment
2) Capped PVD method combined with preloading embankment

Depending on subsurface stratigraphy and the presence of drainage boundaries that allows for drainage of excess porewater pressure, the two methods of applying vacuum pressure to the soft soil will differs in how the vacuum pressure is thought to be distributed in the subsurface.

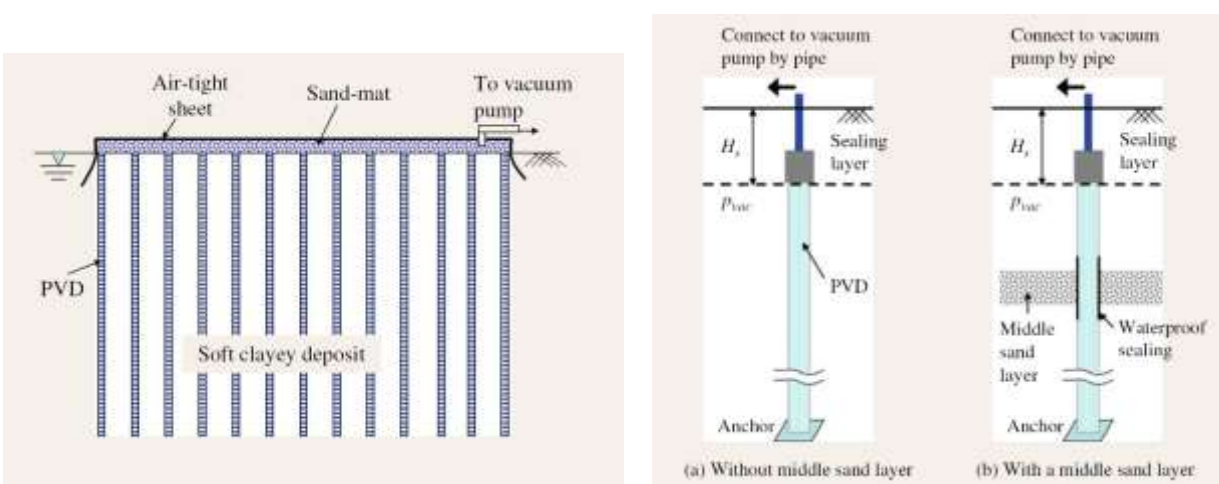

Figure 1 Vacuum Assisted Consolidation: 1) Surface Sealing Method, 2) Capped PVD Method (Chai et al, 2010)

Indraratna et al. proposed a closed form analytical solution to vacuum consolidation in a cylindrical unit cell (Indraratna et al. 2005a), where vacuum pressure is applied via uniform pressure in a surface sealing layer above the soil layer being improved. Indraratna's proposed vacuum consolidation solution is an extension of the traditional PVD radial consolidation solutions (from the likes of Barron and Hansbo) to account for vacuum pressure distribution propagating downward along the PVDs, in addition to a uniformly applied vacuum pressure at the top of the soil layer. Traditionally, the applied vacuum suction via surface sealing sheet method is typically converted to an equivalent uniform surcharge load onto the soil layer under vacuum consolidation. Radial consolidation is then assumed to also follow Hansbo's solution as mentioned earlier (Chai et al. 2006). Indraratna proposes that due to the extent that vacuum pressure can propagate downward along the PVDs, there is an additional negative pressure inside the PVD and thus generating additional radial flow gradient. Indraratna argues that the additional vacuum pressure propagating in the PVD should be considered due to the potentially of increasing rate of consolidation. Indraratna presented two scenarios for vacuum pressure distribution, "short drain" and "long drain" cases. For either case, Indraratna presented a constant "K' to describe the linear variation of vacuum pressure with depth.

Indraratna further proposed the application of a conversion factor that converts his vacuum consolidation solution from axisymmetric model to a plane model of consolidation where the PVD drain radius is converted to an equivalent plane strain thickness of PVD. With his proposed equivalent plane strain conversion of drain radius, Indraratna was able to develop a numerical model for a multiple vacuum drains domain in plane strain and examine the effect of consolidation inside a field of vacuum drains. Subsequently, Indraratna proposed an additional conversion factor for multiple equivalent PVD drain radius in an axisymmetric model, where the PVD/ vacuum drain radius is converted to an equivalent concentric ring thickness considering its radial distance from the center of the domain (Indraratna et al. 2016).

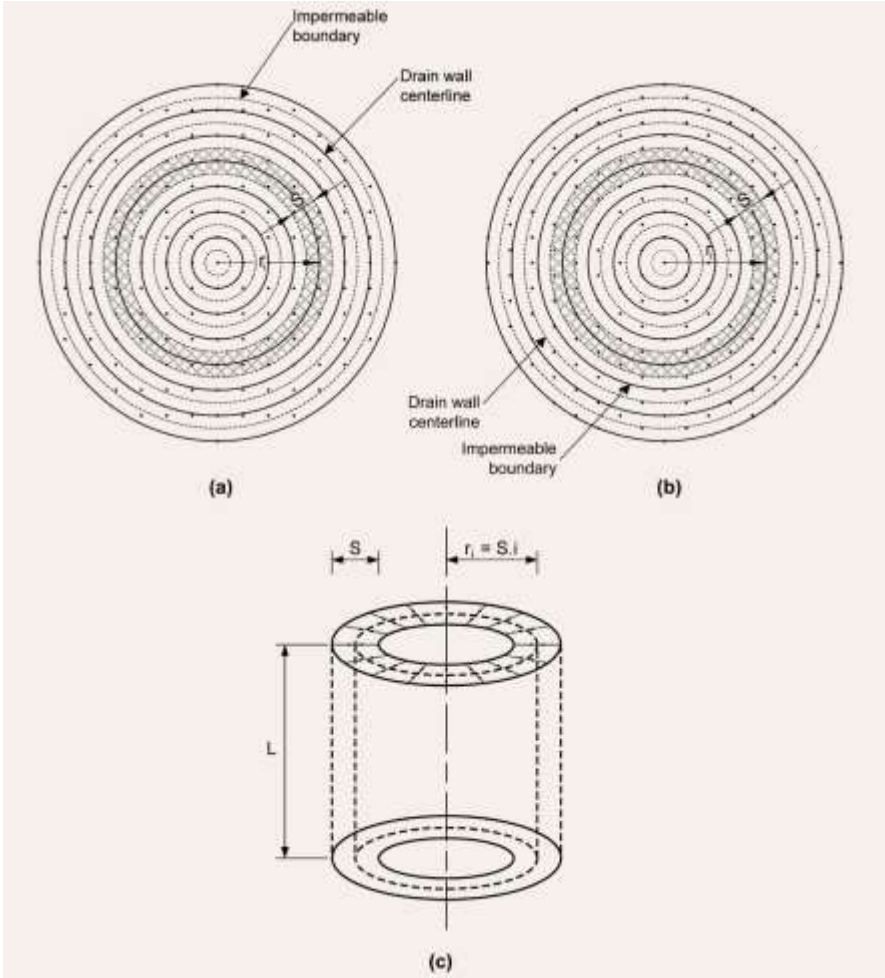

Figure 2. Converting Multiple PVD Unit Cell into Axisymmetric Model (Indraratna et al, 2005a)

In the literature, it is often shown that the observed consolidation settlement results from the bench tests as well as those observed in the field tests tend to present a different conceptual model of vacuum consolidation than the ones presented by the analytical solutions above. Most notably, Asaoka's graphical method (Asaoka, 1978) is a commonly used method in laboratory testing of consolidation, because Asaoska's graphical method requires observed settlement data during consolidation. Saowapakpiboon found that Asaoka's method can provide a reasonable settlement estimate for a PVD improved soft soil compared to field observed data, however it tends to overpredict settlement for the same soft soil under vacuum consolidation (Saowapakpiboon et al. 2009). Saowapakpiboon also observed that under vacuum assisted consolidation, the rate of settlement (hence rate of consolidation)

was faster by 1/3 compared to the consolidation rate under PVD only. Saowapakpiboon attributed the faster consolidation rate to an increase in the average horizontal coefficient of consolidation (C_h) value induced by the applied vacuum pressure. Using Asaoka's graphical method, Saowapakpiboon back calculated the average horizontal coefficient of consolidation (C_h) values for both PVD and vacuum assisted consolidation and found a noticeable difference. The considerations for the effect of applied vacuum pressure on the coefficient of consolidation in the unit cell was notably absent in the analytical solutions presented by Chai and Indraranta, and in their respective conceptual model of vacuum consolidation.

2.2 Development of Numerical Modelling for Vacuum Consolidation

The cylindrical unit cell theory of radial consolidation is the most common method for determining consolidation by vertical drains (Barron 1948; Yoshikuni and Nakanodo 1974; Hansbo 1981), of which, due to its simplicity, Hansbo's solution for radial consolidation in PVD has been widely used in practical design since the 1980s. Because of the popularity of Hansbo's solution, the early numerical models for PVD improved ground were largely focusing on simulating radial consolidation in a cylindrical unit cell and comparing results with developed closed form solutions from Barron and Hansbo (Hird et al. 1992; Chai et al. 1995).

Chai et al presented a fully coupled finite element model of unit cell radial consolidation (Chai et al. 1995) in both 2-D plane strain and 3-D axisymmetric, and with consideration for soil deformation in both elastic and the elasto-plastic range. Chai found the numerically simulated "free strain" radial degree of consolidation (via excess porewater pressure) coupled with linear elastic deformation in the soil resulted in a good match with Hansbo's solution. Chai presented a second fully coupled radial consolidation unit cell model using the modified cam-clay (Roscoe and Burland, 1970) model for soil deformation that extends into the elasto-plastic range. Chai found the numerical simulated results for settlement and

lateral displacement under elasto-plastic deformation provided a reasonably match with field measured results from the real-world project.

Chai subsequently presented his solutions for vacuum consolidation in a cylindrical unit cell, as well as a coupled finite element model of vacuum consolidation that is an extension of the PVD model (Chai et al. 2006). Chai's coupled FEM model again considers elasto-plastic deformation and lateral displacement that occurs in the soft soil, and he found this method of numerical modeling to reasonably good match to real world vacuum consolidation results for ground settlement. Rujikiatkamjorn developed a similar fully coupled elasto-plastic numerical model of vacuum consolidation combined with preloading embankment (Rujikiatkamjorn et al. 2008) that is also able to incorporate multiple vacuum drains under the embankment loading. Rujikiatkamjorn presented two multi-vacuum drain consolidation models, a 2-D plane strain model and a fully 3-D model. The vacuum drain model presented by Rujikiatkamjorn was found to be able to reasonably simulate the consolidation process in a soft clay deposit where dozens of capped vacuum drains (CPVD) are installed in a square pattern, in combination with preloading pressure from an embankment above the improved soft soil. When compared with field measured vertical settlement results below the embankment, Rujikiatkamjorn's 3-D finite element model with coupled Modified Cam Clay model was able to provide the best numerically simulated results for the vacuum consolidation improved soil when compared to in-situ field condition. To the author's knowledge, the vacuum consolidation model presented by Rujikiatkamjorn et al. in 2008 is the first time a 3-D coupled Modified Cam Clay model that incorporated multiple vacuum PVD (CPVD) in designated spacing, as well as time dependent embankment stress and vacuum pressure, and shown to produce good matching with the field measured settlement data.

Since the mid 2000s, numerical modelling studies of PVD and vacuum assisted consolidation have undergone a shift in focus from the traditional single vertical drain unit cell theory of consolidation in an axisymmetric setting, to increasingly more sophisticated 2-D plane strain or 3-D numerical models

incorporating multiple vertical drains and various drain spacings and embankment dimensions in addition. The recent multi-drain models are set up for a semi-infinite half space domain that incorporates the entire embankment dimensions and the field of vertical drains underneath, which most often includes the soil adjacent to the embankment and drains, so that the embankment and vertical drains can be examined in the context of a footing problem. Indraratna proposed a geometric conversion factor that could convert the geometries of a single drain cylindrical unit cell in the axisymmetric coordinate system into an equivalent single drain unit cell model in 2-D plane strain (Indraratna, 2005a), which allows for the development of 2-D plane strain models that can simulate the consolidation process when multiple drains are present under a large embankment. And it allows for the simulation of more realistic lateral deformations and consolidation settlements that develops in the soil at any given locations below (or adjacent to) the embankment, which was previously not possible to simulated in the single unit cell model of consolidation.

2.3 Hansbo's Radial Consolidation Solution for PVD Consolidation

Hansbo's solution (Hansbo 1981) is a commonly used method for estimating ground improvement time for a PVD improved soft soil. It is essentially an iteration of Barron's "equal strain" radial consolidation model and under the exact same premise. An embankment above the soft soil deposit is assumed to provide an applied stress/surcharge that is uniformly distributed to the soft soil deposit, and subsequently converted to an excess porewater pressure in the soft soil deposit. This surcharge induced excess pore pressure is assumed also be uniformly distributed, and equal to the magnitude of the surcharge, at everywhere in the soft soil. In addition, Hansbo solution takes into account a zone of lowered permeability adjacent to the PVD, called the smear zone. The smear zone is widely accepted as a by-product of PVD installation, where the soil column immediately to the PVD have been disturbed and compressed, resulting in reduced permeability.

Hansbo's solution for radial consolidation in a cylindrical unit cell with considerations for smear zone is given by:

$$U_h = 1 - exp\left(-\frac{8T_h}{\mu_s}\right)$$

Equation 1: Hansbo's Radial Consolidation Solution

U_h *is the degree of consolidation*

$T_h = \frac{C_h \cdot t}{4 \cdot R^2}$ is the time factor in radial consolidation

$\mu_s = \ln\left(\frac{n}{s}\right) + \left(\frac{k}{k_{smear}}\right)\ln(s) - \frac{3}{4} + \pi z(2l - z)k/q_w$

$n = \frac{R}{r_{drain}} = \frac{D}{d_{drain}}$

$s = \frac{r_{smear}}{r_{drain}}$

$q_w = k_w A_w k_w \pi d^2/4$ is the discharge capacity of the drain well/PVD

2.4 Equivalent Drain Diameter for Band Shaped PVD

With the use of PVD replacing traditional sand drains, there is a need to convert the dimensions of the band shaped PVD into an equivalent circular drain diameter. In Hansbo's solution outlined in the previous section, the parameter r_{drain} represents the equivalent radius of a band shaped PVD.

Currently there are several proposed methods for determining equivalent radius of band shape PVDs. These methods typically are based on finding equivalent cross-sectional area and consideration for flow pattern in the vicinity of the band shape PVDs. Abuel-Naga presented a numerical study (Abuel-Naga et al. 2012) to examine the performance of five proposed conversion factor for equivalent diameters of a band shaped PVD presented by: Hansbo; Atkinson and Eldred (Atkinson and Edldred, 1981); Fellenius and Castonguay (Fellenius and Castonguary, 1985); Long and Covo (Long and Covo, 1994); Abuel-Naga

and Bouazza (Abuel-Naga and Bouazza, 2009). Abuel-Naga presented a 2-D plane strain finite element model of a cylindrical unit cell (circular domain in plane strain) to examine radial consolidation using the difference equivalent radius method for a band shape PVD. The difference in the simulated excess porewater pressure degree of consolidation under various equivalent radius is then compare with the original band shaped PVD. Abuel-Naga found that Long and Covo's method for equivalent radius of PVD provided the best result for unit cell radial consolidation, a slight improvement over the commonly used Hansbo's method. Although Abuel-Naga found that the maximum difference in average excess porewater pressure between the two methods were both within 3% from the actual band shape PVD in this application. Hence, Abuel-Naga concluded that Hansbo's equivalent PVD radius is sufficient in actual practice.

Hansbo proposed a band shaped PVD that can be converted to an equivalent diameter when applied to unit cell radial consolidation theory via the conversion:

$$d_{drain} = 2(\omega + t)/\pi$$

Equation 2: Hansbo's Equivalent Diameter for Band-Shaped PVD

d_{drain} is the equivalent diameter of a band shaped PVD

ω, t are the width and thickness of the PVD, respectively

2.5 Analytical Solutions for Vacuum Assisted Consolidation

In ground improvement projects, vacuum consolidation is generally applied to a soft soil deposit via either the surface sealing sheet method, or the capped PVD method. For the two distinct methods of vacuum consolidation, closed form solutions in the literature differs in their conceptual model of vacuum consolidation. As a result, currently, there is a lack of universal adoption of a closed form solution for vacuum consolidation, in contrast to the Hansbo's solution for traditional PVD consolidation.

Chai proposes that vacuum consolidation solution for porewater pressure in the improved soft soil consists of two parts: a combination of an initial transient state, followed by a latter steady state, each with its own respective solution. The overall solution for porewater pressure is the sum of the two solutions.

$$U(z,t) = -p_{vac}[Y(z) - v(z,t)]$$

Equation 3: Average Porewater Pressure in Vacuum Consolidation (Chai, 2010)

$U(z,t)$ Average porewater pressure at depth (z) and at consolidation time (t)

p_{vac} Applied vacuum pressure

$p_{vac}Y(z)$ Final steady state porewater pressure distribution

$p_{vac}v(z,t)$ Transient component of the porewater pressure distribution

According to Chai, the transient component of vacuum consolidation behaves similarity to traditional Terzaghi and radial consolidation theory. If vacuum pressure is applied via the sealing sheet method and absent of vertical drains (PVD) to promote radial drainage pathways in the improved soil deposit, vacuum consolidation would then behave similarly to Terzaghi's 1-D consolidation with "one-way" and "two-way" drainage to the soil layer(s) above or below the improved soil deposit. In this case, applying a vacuum pressure to the soil is then assumed to convert to equal magnitude of excess porewater pressure, as if the soil is under an applied surcharge stress. The drainage of this equivalent "excess porewater pressure" is then assumed to follow Terzaghi's solution for one-way or two-way drainage boundaries (Chai et al. 2010).

If vacuum pressure is applied via the capped PVD method (CPVD), in which a uniform vacuum pressure is distributed along the length of the PVD, Chai proposes a conceptual model of vacuum consolidation is similar to radial consolidation theory. In this case, Chai proposes the applied vacuum pressure can be

converted to an equivalent applied surcharge stress above the soil layer. The induced excess porewater pressure in the soil layer as a result of the surcharge would be equal to magnitude of the applied vacuum pressure and distributed uniformly throughout the soil layer. According to Chai, vacuum consolidation via CPVD method could be converted to an equivalent PVD combined with surcharge preloading scenario, where the drainage of the "induced excess porewater pressure" would be governed by radial consolidation and Hansbo's solution can be applied to determine degree of consolidation.

To the author's knowledge, there is no widely agreed upon adoption of the method of converting CPVD vacuum pressure to equivalent surcharge pressure, and subsequently simplifying CPVD vacuum consolidation to conventional surcharge and PVD consolidation. Instead, the literature acknowledge the vacuum to equivalent surcharge conversion method has been applied to surface sealing sheet method (Indraratna et al. 2005a) of vacuum assisted PVD consolidation. However, under this premise, the vacuum pressure is intended to assist in conventional PVD consolidation by reducing the necessary height of surcharge embankment in order to achieve the same consolidation settlement (Shang et al. 1998) under the equivalent surcharge preload. The method's application in vacuum consolidation in ground improvement projects remains a topic of study at this time.

Indraratna presented a closed form solution for vacuum consolidation via the surface sealing sheet method, where PVDs are also present in the vacuum improved soil to encourage radial consolidation (Indraratna et al. 2005a).

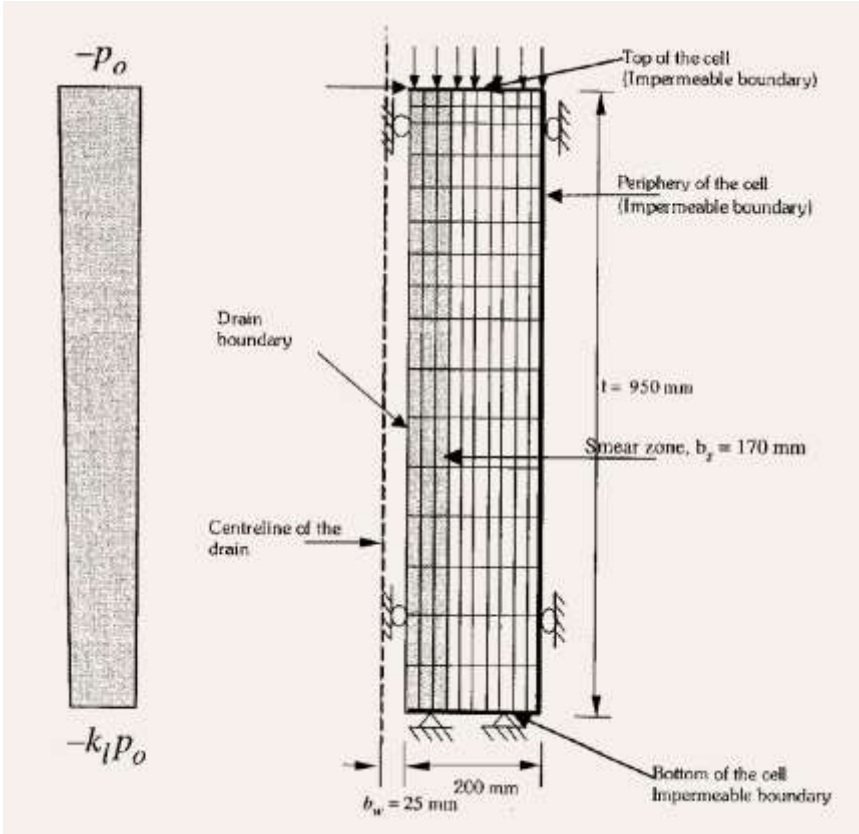

Figure 3. Indraratna's "Short-Drain" Model of Unit Cell Vacuum Consolidation (Indraratna et al. 2005a)

Indraratna argues that the vacuum pressure propagation along the PVD is linear with depth. With the vacuum pressure at the top of PVD is assumed to equal to the applied vacuum pressure under the sealing sheet, then propagates downward and decrease linearly along the length of the PVD. Indraratna proposed a linear k_1 function that describes the propagation of vacuum pressure which decreases linearly with depth. The propagation of vacuum pressure and $k_1$function can be divided into either "short drain" or "long drain" cases. Indraratna presented an example of "short drain" as PVD that is less than 1m in length, where the k_1 function is between 0 and 1. The "long drain" example is for a PVD that is 10m in length, where the k_1 function is equal to 0, which leads to no vacuum pressure at the bottom end of the PVD. Chu observed that for the magnitude of the applied vacuum pressure that is also able to be sustained long term, it is unlikely to purge the porewater inside the PVD for a depth greater than 10m (Chu et al. 2000), hence this could be verification for the "long drain" assumption that is applied to

10m long PVDs. However, Indraratna did not propose an exact range of the length of the PVD under 10m that could be classified as “long drain”, therefore in actual practice the categorization is often left to interpretation.

Indraratna’s closed form solution for vacuum assisted PVD consolidation via the surface sealing sheet method is give in axisymmetric condition, with consideration for smear zone and well resistance:

$$Short\ Drain: \qquad \frac{\tilde{u}}{\sigma_1} = \left[1 + \frac{(1+k_1)p_0}{2\sigma_1}\right] exp\left[\frac{-8T_h}{\mu}\right] - \frac{(1+k_1)p_0}{2\sigma_1}$$

$$Long\ Drain: \qquad \frac{\tilde{u}}{\sigma_1} = \left[1 + \frac{p_0}{2\sigma_1}\right] exp\left[\frac{-8T_h}{\mu}\right] - \frac{p_0}{2\sigma_1}$$

$$\mu = \frac{n^2}{(n^2-1)}\left[ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s} ln(s) - \frac{3}{4}\right] + \frac{s^2}{(n^2-1)}\left[1 - \frac{s^2}{4n^2}\right]$$

$$+ \frac{k_h}{k_s}\frac{1}{(n^2-1)}\left[\frac{s^4}{4n^2} - s^2 + 1\right] + \frac{2\pi k_h}{3q_w} l^2 \left[1 - \frac{1}{n^2}\right]$$

$$n = \frac{d_e}{d_w} or \frac{r_e}{r_w} \qquad s = \frac{d_s}{d_w} or \frac{r_s}{r_w}$$

Equation 4: Indraratna's Solution for Unit Cell Vacuum Consolidation

σ_1 initial excess porewater pressure in the soil at the start of consolidation. Indraratna’s analytical model assumes this value is equal to the combined preloading pressure and equivalent applied vacuum pressure surcharge

$\tilde{u}$ average porewater pressure inside the cylindrical unit cell, where the porewater pressure u(r,z,t) is a time dependent function given in axisymmetric domain

$T_h = \frac{C_h \cdot time}{4r^2}$ is the time factor for radial consolidation

μ drain geometry factor, it is of the same form as the geometry factor found in conventional unit cell PVD radial consolidation solutions of the likes of Barron, and Hansbo

r_e radius of influence, or cylindrical unit cell radius

r_w sand drain radius or equivalent drain radius of a band shaped PVD

r_s radius of the smear zone

k_h radial hydraulic conductivity in the undisturbed zone (m/s)

k_s radial hydraulic conductivity in the smear zone (m/s)

q_w flow term for well discharge capacity or well resistance. Well discharge is typically neglected for most PVD types (Holtz et al. 1991; Indraratna et al. 2000)

k_1 Indraratna's k function for distribution of vacuum pressure in the PVD. Value between 0 and 1

p_0 Applied vacuum pressure via surface sealing sheet

2.6 Approximation of Preload Effects and Initial Conditions Around Drains Below Circular Embankment

If a semi-infinite soil layer subjected to a circular load at the surface, with no lateral boundary constraints, and drainage towards the surface, the closed form solution by Gibson & McNamee provides the transient solution for the vertical displacement below the center of the circular footing (McNamee and Gibson, 1960). It is important to note that Gibson & McNamee solution is only valid for an elastic porous media with Poisson's Ratio $\nu = 0.0$ therefore material intrinsic lateral strain is neglected. Gibson & McNamee solution correlates well with linear poroelastic theory or Biot's theory of consolidation, and it is often applied to elastic consolidation.

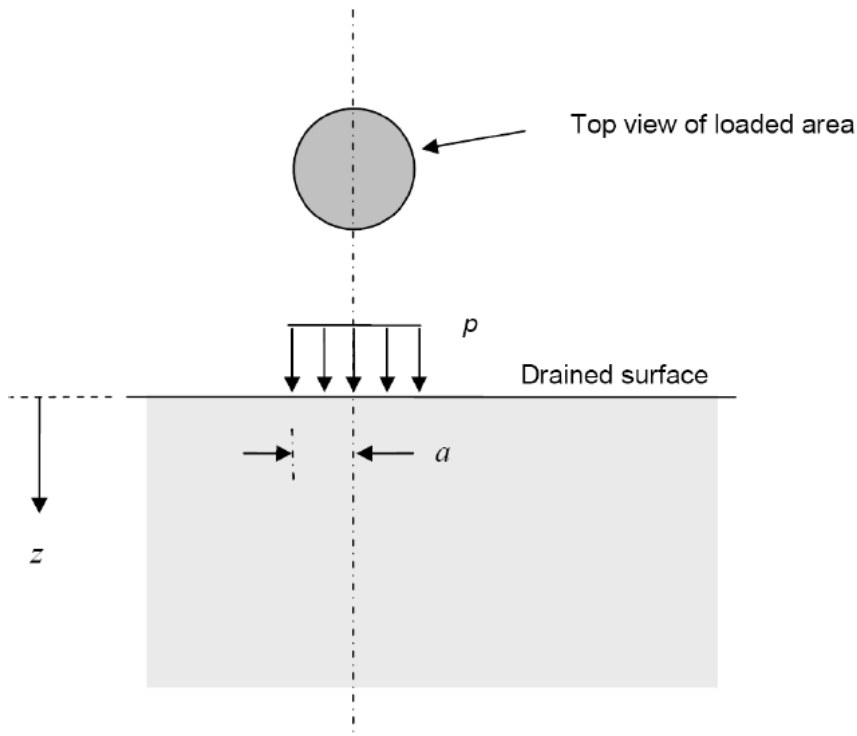

Figure 4. Finite Circular Embankment in a Semi-Infinite Domain (Itasca, FLAC Tutorial)

The Gibson & McNamee solution for an elastic material with $\nu = 0.0$ and subjected to a circular embankment load is given by:

$$\frac{2G}{pa}[\omega - \omega_{\tau=0}] = erfc\left(\frac{1}{2\sqrt{\tau}}\right) + 2\sqrt{\frac{\tau}{\pi}}(1 - e^{-\frac{1}{4\tau}})$$

$$\tau = \frac{c_v \cdot time}{a^2}$$

Equation 5: Gibson & McNamee Solution

G shear modulus

ρ Stress exerted by the circular embankment

a Radius of the circular embankment

ω Total vertical displacement, time dependent

$\omega_{\tau=0}$ Instantaneous displacement during undrained phase at consolidation time = 0

τ Normalized time factor

For a poroelastic material, which the Gibson & McNamee solution is applied to, the total vertical settlement (ω) consists of instantaneous settlement ($\omega_{\tau=0}$) and primary consolidation settlement. When compared with primary consolidation settlement, typically, instantaneous settlement ($\omega_{\tau=0}$) can be assumed to exhibit small strain properties (elastic) due to the fact that the porous media is in an undrained state, so the bulk property can be assumed to be elastic. Therefore, Poulos and Davis solution for instantaneous displacement in an elastic material can be used to determine the magnitude of instantaneous displacement under a circular embankment load (Poulos and Davis, 1974), by simply substituting for undrained poroelastic soil properties.

$$\omega_{\tau=0} = \frac{2pa(1 - {v_u}^2)}{E_u}$$

Equation 6: Poulos & Davis Solution

v_u undrained Poisson's Ratio = 0.5

E_u Undrained Young's Modulus (Elastic)

3. Methodology

3.1 Hydro-Mechanical Coupling Scheme of Consolidation

The physics of consolidation in a porous media is most often thought to be represented by a simple type of hydro-mechanical (H-M) coupling scheme, which solves for the quasi-static equilibrium in Biot's theory of static poroelasticity (Biot, 1941). According to Biot's theory of consolidation, the drainage of a single-phase fluid is governed by Darcy's law of fluid diffusion via hydraulic head gradients. The fluid diffusivity component in Biot's theory is time dependent, starting at a transient state and eventually reaching steady state when constrained by boundary conditions. In contrast, Biot's theory assumes any mechanical deformation in the porous media to takes place instantaneously and therefore at a state of static equilibrium at any time during consolidation (alternatively, dynamic analysis provides considerations for wave propagation and is a more rigorous H-M coupling scheme). This instantaneous mechanical response, combined with transient fluid diffusion, forms the quasi-static equilibrium framework of Biot's consolidation theory. In practice, the quasi-steady state equilibrium assumption is reasonably applied to consolidation problems largely because of the timescale in which porewater drainage induced primary consolidation settlement typically takes place, which gives plenty of time for the porous media to reach mechanical equilibrium.

For consolidation problems in soft clay type deposits, H-M coupling theory with consideration for elasto-plastic strain is widely accepted and utilized. Particularly in ongoing studies of PVD assisted consolidation and vacuum PVD assisted consolidation in soft clay soil, there are numerous publications on the application of H-M coupled numerical models with considerations for elasto-plastic deformations (Chai et al. 1995; Chai et al. 2006; Baek et al. 2006; Rujikiatkamjorn et al. 2008; Saowapakpiboon et al. 2009; Chai et al. 2010; Liu et al. 2015; Vu et al. 2018; Pham et al. 2019), all of which used the Modified

Cam-Clay model (Roscoe and Burland, 1968) to simulate consolidation settlements which provided the best calibrated results to real world case studies involving PVD and/or vacuum consolidation.

The current analytical solutions for PVD and vacuum consolidation (Barron, Hansbo, Chai, Indraratna) were mainly derived with the intentions of solving for the fluid diffusion component of a consolidation problem, similar to the Terzaghi's equation of consolidation. This is apparent in the analytical solutions above, all of which solves for porewater pressure and degree of consolidation in the unit cell, while vertical settlement and lateral deformation that develops during the process of consolidation were not specifically accounted for in the solutions. Instead, consolidation settlement is typically determined via Asaoka's graphical method (Asaoka, 1978), which requires observed settlement data during consolidation and has been shown to be reasonably effective for PVD improved consolidation (Saowapapkpiboon et al. 2009).

The currently available analytical solutions are all bounded by the cylindrical unit cell theory of radial consolidation, which inherently limits their application to settlement analysis. Thus, they were derived to only focused on estimating the timescale of consolidation, rather than the magnitude of consolidation settlement. As a result, analytical solutions for various methods of vacuum assisted consolidation remains an ongoing study. At this time, predicting the vertical settlements and lateral displacements that develops during a PVD and vacuum assisted consolidation project is typically done through numerical modeling analysis.

3.2 The Explicit Finite Difference Method

The hydro-mechanical coupled numerical models used in this study were developed using the explicit finite difference software FLAC (Itasca Consulting Canada, 2010). FLAC utilizes a 2-D mesh grid discretization composed of quadrilateral elements, and further divides each quadrilateral element into

two overlaid sets of two constant-strain triangles. The resultant four triangular sub-elements discretization is then used by FLAC in the finite difference equations for triangular grid mesh.

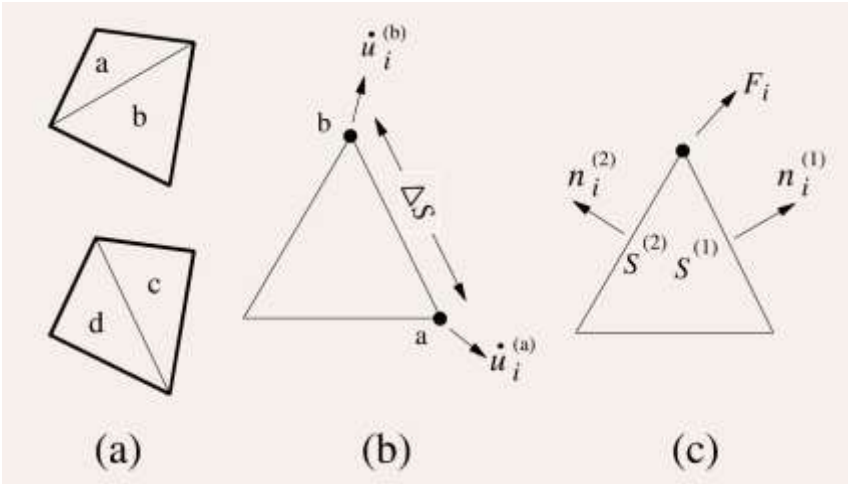

Figure 5. FLAC Finite Difference Model Discretization (Itasca Consulting Canada, 2010)

For the study of PVD and vacuum assisted consolidation, two types of H-M coupled finite difference model in FLAC were used. Firstly, the fluid coupled small strain elastic material scheme in FLAC, which utilizes linear poroelasticity theory, mostly resembles the frameworks of the conceptual analytical models used in deriving several existing analytical solutions of consolidation. Therefore, the linear poroelasticity coupling scheme in FLAC was used to develop verification models to match the small strain assumptions in the existing analytical solutions. Secondly, the fluid coupled Modified Cam-Clay model was used for its elasto-plastic strain constitutive relationship, which is generally accepted as more applicable when it comes to soft clay type soil.

Typically, FLAC discretize the domain into either 2-D plane strain or axisymmetric mesh. For this study, axisymmetric models were created to simulate cylindrical unit cells with a single PVD at its center. Setting up the axisymmetric domain this way provides the best match for the cylindrical unit cell model of radial consolidation which the analytical solutions were derived from.

3.3 Vertical Settlement Under a Circular Embankment Verification Model

A fully coupled poroelastic model developed using FLAC is used to demonstrate the feasibility of the fully coupled linear poroelastic scheme when compared to McNamee and Gibson's analytical solution for linear elastic deformation in a saturated semi-infinite porous media under a circular embankment.

Figure 6 shows the set up of the circular embankment as a footing problem on top of a semi-infinite soil layer that is homogeneous and isotropic. The radius of the circular embankment is set to 6m, and the applied stress is set to 50 kPa. The 3-D axisymmetric soil domain has a radius of 100m and depth of 60m. This dimension of the soil domain is assumed to be large enough compared to the circular embankment to therefore be considered semi-infinite domain.

The domain is characterized by a saturated homogeneous and isotropic soft clay type soil. The drained Young's modulus of the soil is 450 kPa, the dry density is 1182 kg/m^3, the porosity is 0.5, the drained Poisson's ratio is zero. Drainage of excess porewater pressure is towards the top of the saturated domain (ala water table). Roller boundary is set up at the central axis in order to simulate uniaxial compression below the center of the circular embankment. Therefore, the vertical coefficient of consolidation (C_v) is assumed to be 1.8 $m^2/year$. Considering uniaxial elastic deformation below the embankment and assuming bulk modulus of porewater is 2 GPa (2e9 Pa), the vertical coefficient of consolidation (C_v) corresponds to a vertical hydraulic conductivity of $1.24x10^{-9}$m/s. The coupled numerical model domain is illustrated in Figure 6.

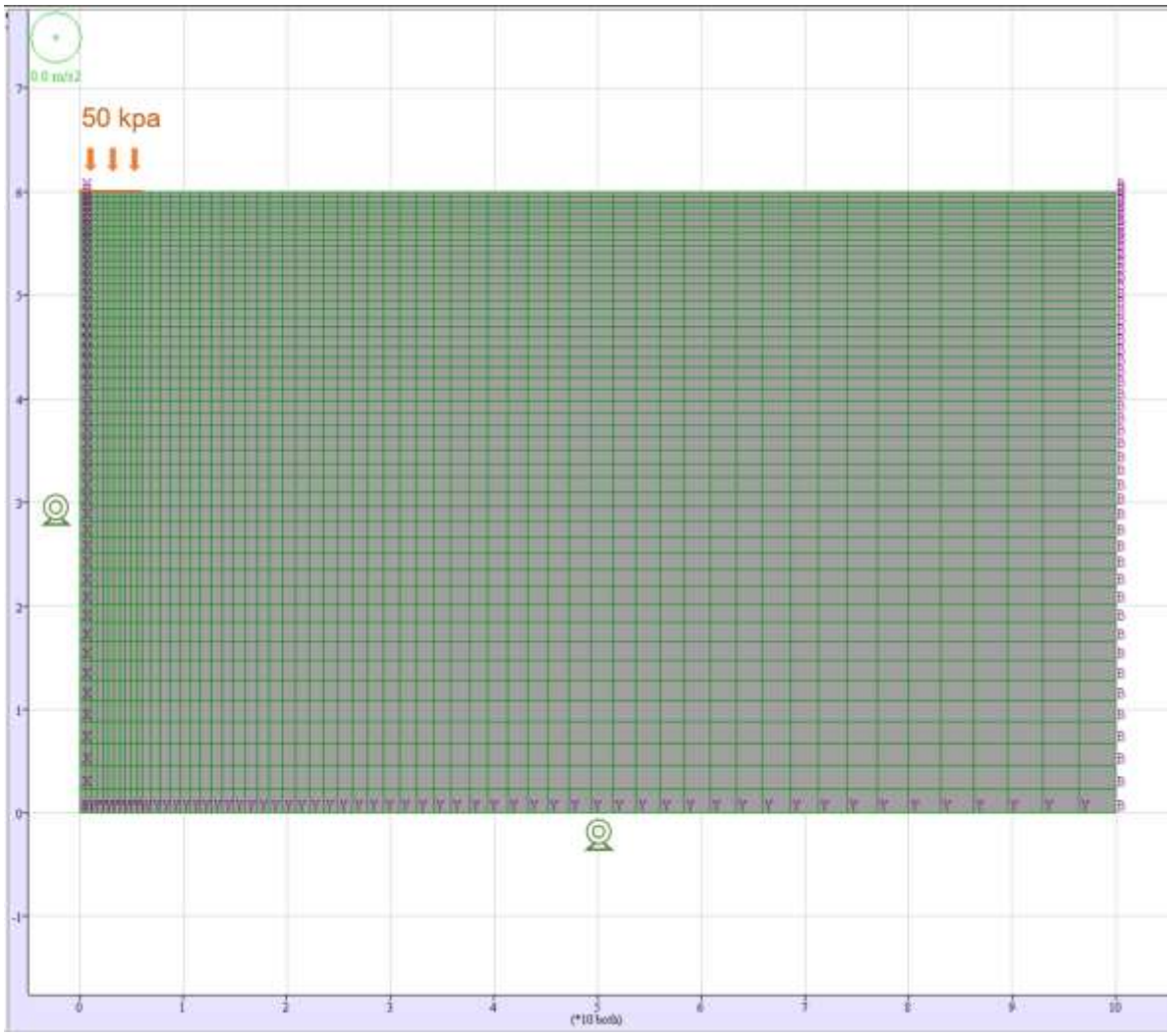

Figure 6. Verification Model: Circular Embankment Preload on a Semi-Infinite Soil Domain

The coupled numerical model is first ran to determine static equilibrium at the undrained state. At this initial stage of consolidation (consolidation time = 0), the applied stress from the embankment is assumed to be instantaneously distributed into the subsurface, leading to an increase in excess porewater pressure below the embankment. The settlement that develops during the undrained stage is a result of bulk deformation of the porous media that combines the soil matrix and the fluid bulk modulus (soil grain deformation is assumed to be much greater than matrix, and therefore typically neglected for consolidation problems), resulting in instantaneous settlements. Typical, given the magnitude of applied stress from the embankment, the strain and instantaneous settlements developed

during undrained stage is very small compared to those that develops during the drained stage (primary consolidation) in this example, one can reasonably assume the instantaneous settlement is small strain. With this assumption in mind, the magnitude of instantaneous settlement under the circular embankment can be estimated using Poulus and Davis' analytical solution for the deformation of an elastic material under the center of a circular load (Poulos and Davis, 1974). For Poulos and Davis's solution, undrained elastic parameters such as undrained Young's modulus (E_u) can be derived from well known empirical equation. The result is then plugged into Gibson and McNamee solution as instantaneous settlement.

Because Poulos and Davis's analytical solution, as well as McNamee and Gibson's solution only solves for elastic deformations directly under the embankment, the distribution of excess porewater pressure in the domain during undrained and drained consolidation stages are often unknowns. Here, taking advantage of the capabilities of the coupled numerical models, the excess porewater pressure distributions in the semi-infinite domain can be determined. Figure 7&8 illustrates the numerically simulated excess porewater pressure in the domain at the initial undrained stage, corresponding to instantaneous settlement value of 0.626m directly below the embankment. While Poulos and Davis's solution gives an instantaneous settlement value of 0.667m.

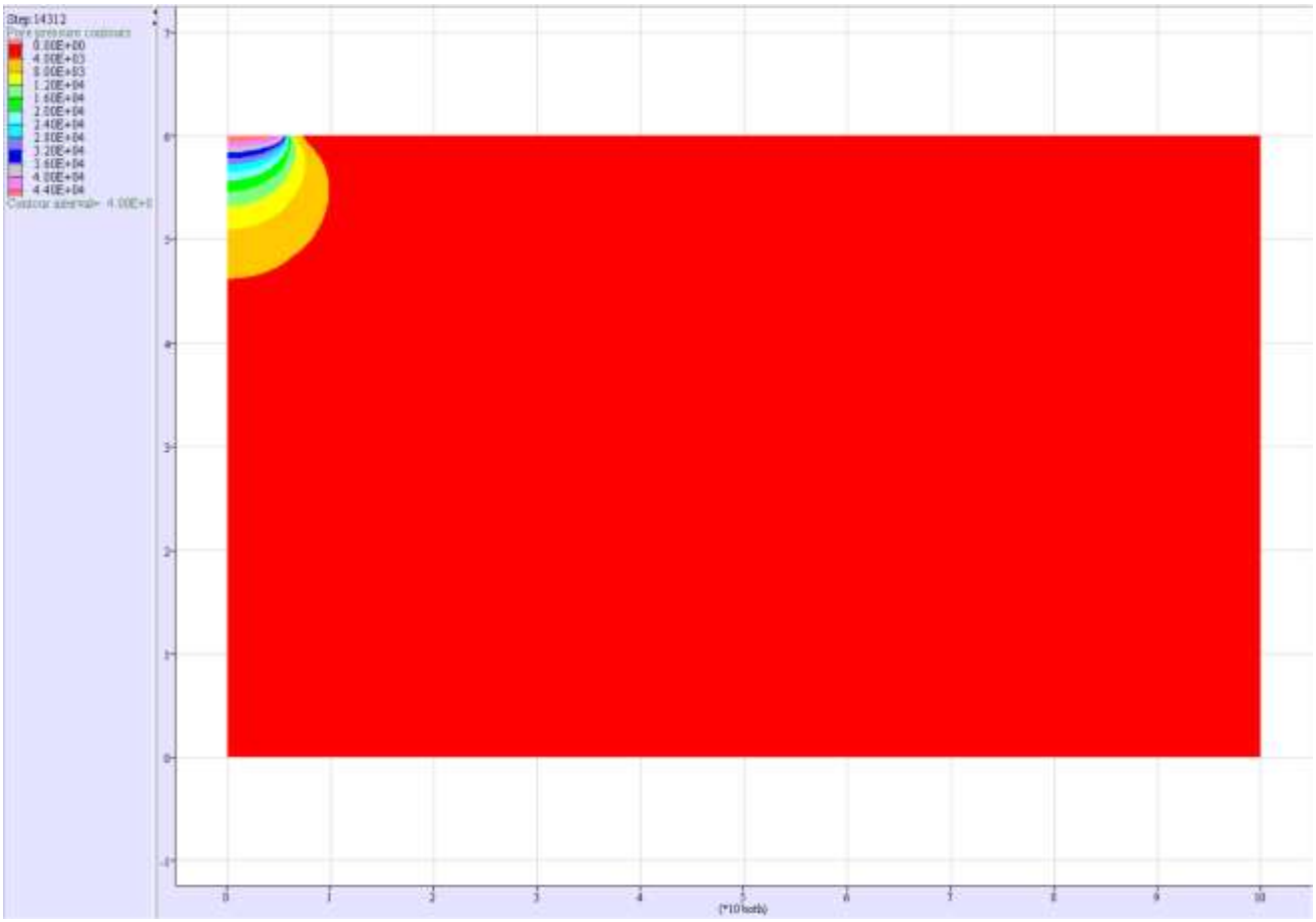

Figure 7. Embankment Induced Excess Pore pressure at the Undrained Stage

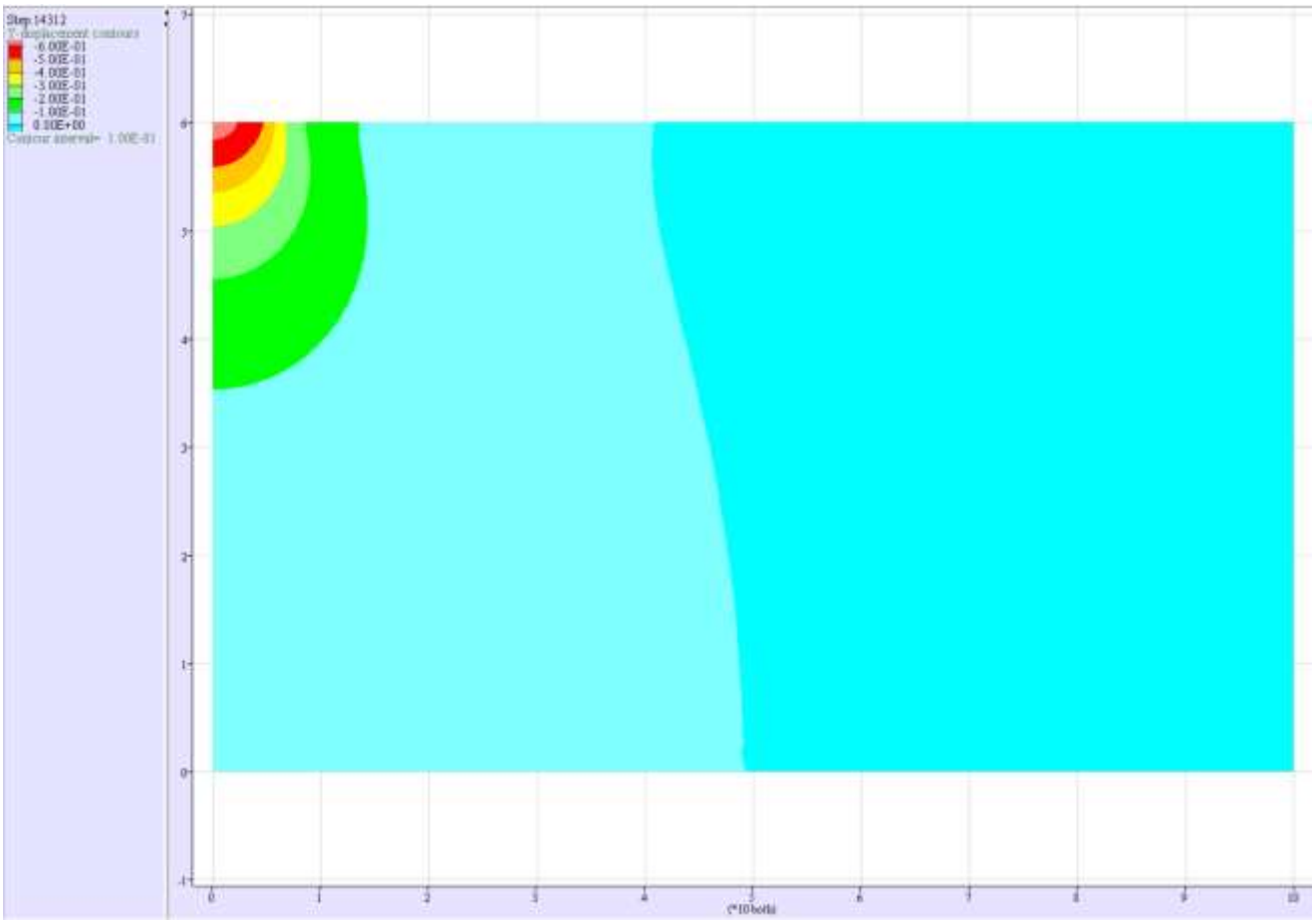

Figure 8. Embankment Induced Total Vertical Stress at the Undrained Stage

Modeled analysis is conducted over a 1,000-year period, the simulated primary consolidation results are then compared with the McNamee and Gibson's solution. The simulated deformation result is reasonably well matched to McNamee & Gibson's solution. The discrepancies in the results can be attributed to the instantaneous settlement result obtained from Poulos and Davis's solution, and the simulated undrained soil parameters.

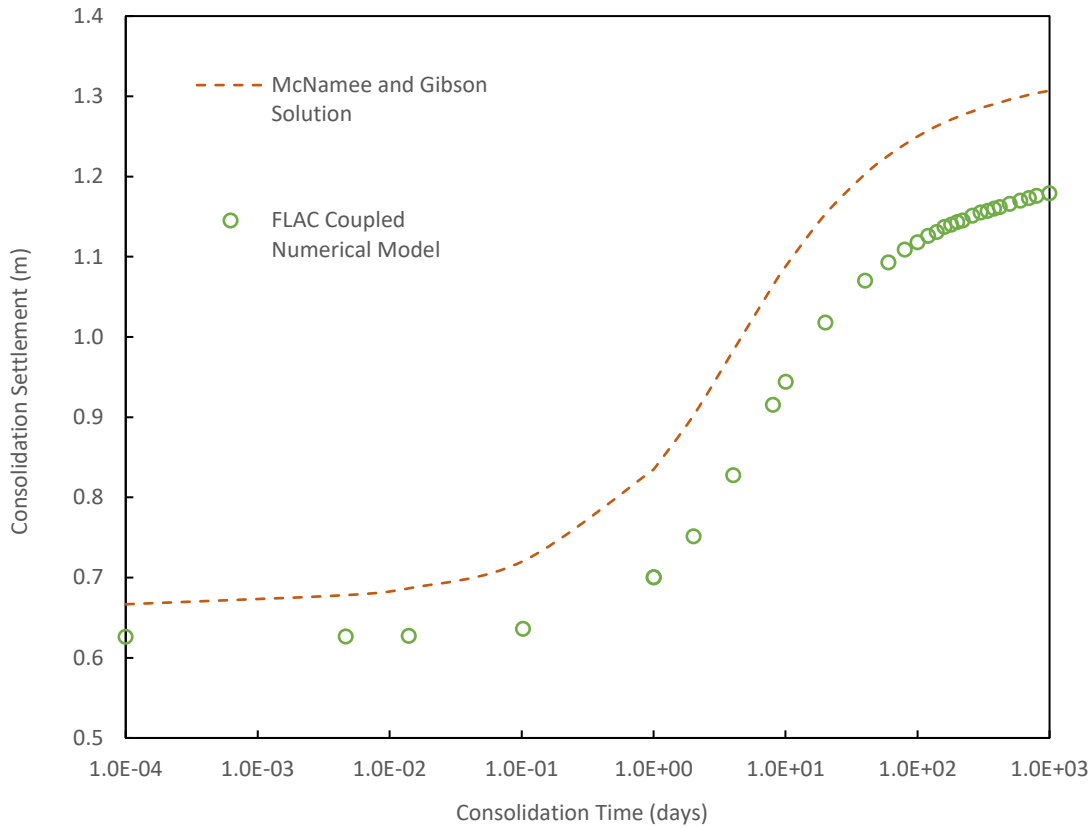

Figure 9. Verification Model: Vertical Settlement Below the Center of the Circular Embankment - McNamee&Gibson versus FLAC Numerical Simulated

3.4 Nonuniform Stress and Porewater Pressure Distribution Below a Circular Embankment

The conventional conceptual model for preloading the soil through the construction of a backfill embankment has been to equate the body force exerted by the embankment to a uniform surcharge that develops in the soil below the embankment. This preload to equivalent surcharge conversion is present in the classic Terzagh's 1-D consolidation solution, and many other well known consolidation solutions that followed, including radial consolidation solutions from the likes of Barron and Hansbo. While this assumption can be confirmed in the laboratory setting, many researchers have noted in actual field applications with an embankment of finite dimensions, the resultant preloading stress and excess porewater pressure distribution in the soil below does not form a uniform surcharge, but varies with depth (Long 1990, Holtz et al. 1991, Chu et al. 2001, Indraratna et al. 2012, Deng et al. 2017).

Consider the previous example of NcNamee and Gibson's solution. Figure 7&8 shows that the applied stress and excess pore pressure at the undrained phase is not a uniform surcharge. Rather, the nonuniform distribution of the pore pressure is highly dependent on both the depth and the location below the circular embankment.

The build up and distribution of excess pore pressure can be illustrated through a coupled numerical model. If a circular embankment with radius of 7m is placed on a semi-infinite domain consist of a homogeneous and isotropic soft soil. The preload from the circular embankment exerts a uniform 50 kPa of stress onto the soil. Figures 10&11s shows the induced total vertical stress and excess porewater pressure along vertical profiles below the center of the circular embankment; at 3.5m away from the center; and at the toe of the embankment. The coupled numerical model is developed using the same poroelastic framework verified in section 3.3.

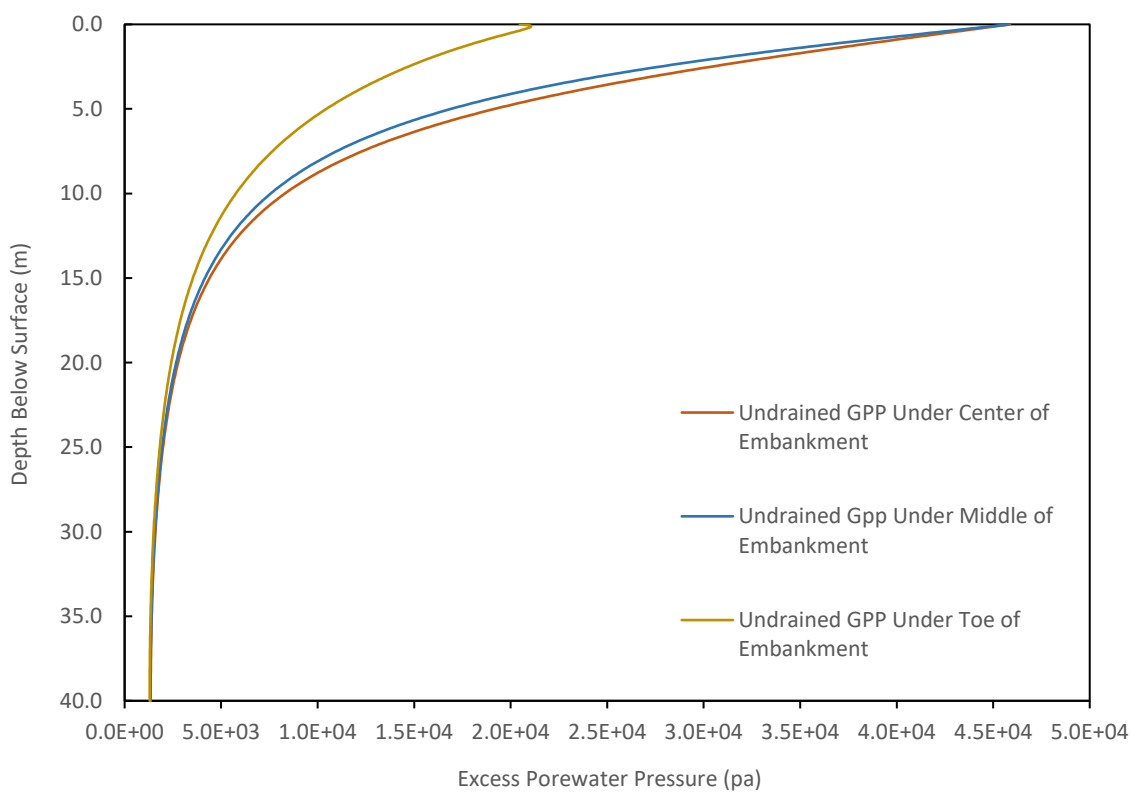

Figure 10. Embankment Induced Excess Pore Pressure Vertical Profiles Below Embankment Radius of 7m

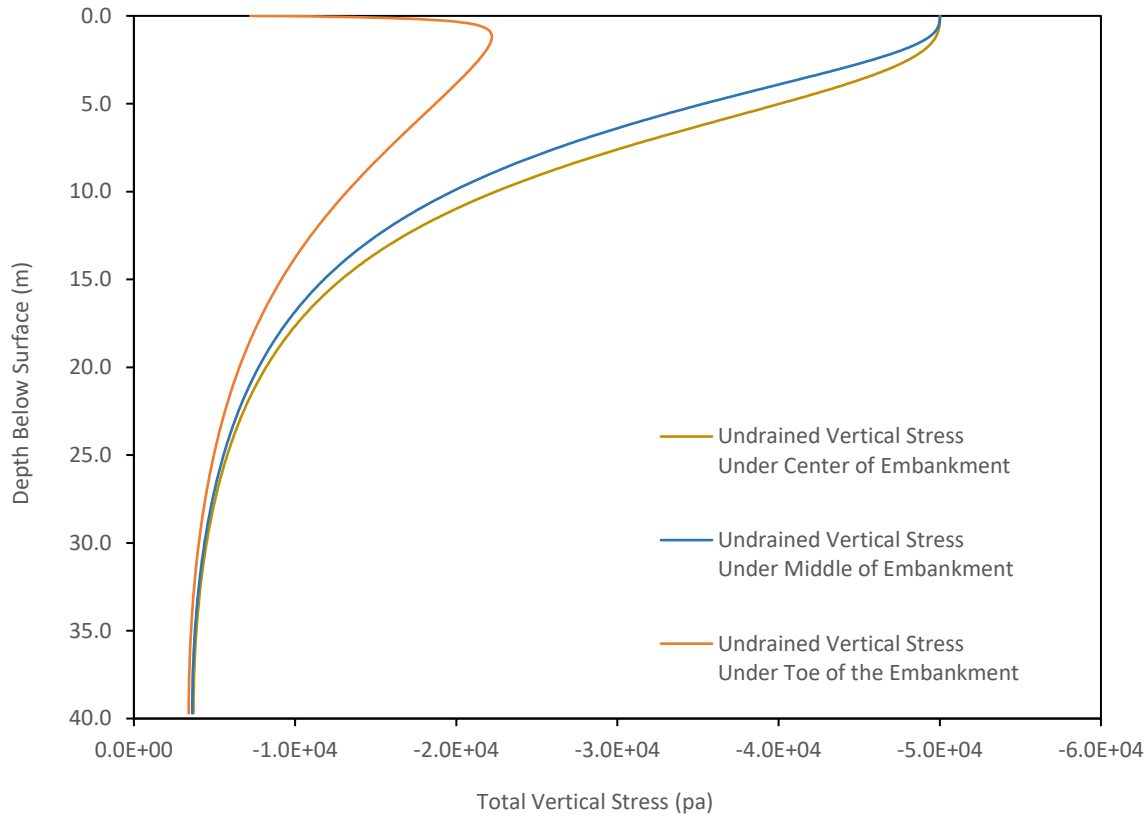

Figure 11. Embankment Induced Vertical Stress Vertical Profiles Below the Embankment Radius of 7m

For radial consolidation problems involving vertical drains (PVD and vacuum drains), the nonuniform distribution of stress and excess pore pressure at the undrained stage could lead to a noticeable deviation from the conventional unit cell radial consolidation theories presented by the likes of Barron and Hansbo, which assumes uniform surcharge under the embankment and uniform excess pore pressure distribution.

3.5 Insitu At Rest Stress State and Hydrostatic Pore Pressure Distribution

The in'situ stress state of a soil at rest, and the hydrostatic porewater pressure ($\rho_w gz$) in a saturated soil layer, are often omitted from analysis in the traditional conceptual model of consolidation. This is largely due to the prevalence of the methodology framework to which examines consolidation. For example, the classical 1-D consolidation theory developed by Terzaghi had introduced the concepts of describing consolidation through the process of applied stress induced surcharge and dissipation of

excess pore pressure. The popular radial consolidation solutions maintained the same concepts of applied stress and induced surcharge in the soil, and dissipation of excess pore pressure towards a central drain as the controlling process of consolidation, while omitting the in'situ stress state and hydrostatic pore pressure distribution that exists in the soil prior to the application of preloading the soil.

In examples of cases studies of PVD and vacuum assisted consolidation carried out in the field, several researchers have observed and incorporated in'situ hydrostatic pore pressure (hydrostatic pore pressure and in'situ pore pressure are used interchangeably in this section) into the results (Choa et al. 1989, Chu et al. 2000). Conventional consolidation theories typically examine consolidation in terms of excess pore pressure and effective stress of the soil, thus omitting the in'situ hydrostatic pore pressure and stress state in the soil absent of preload. While in'situ stress state and pore pressure in the soil can be reasonably omitted with the assumptions that: 1) the magnitude of the preload is significantly greater than the at-rest in'situ stress of the soil, over the entire thickness of the soil layer; 2) the preload induced excess pore pressure (whether in the form of uniform surcharge or nonuniform distribution with depth) is much greater than the in'situ hydrostatic pore pressure that already exists in the soil. Consider in actual practice the preloading embankment has finite dimensions, and the idea of surcharge is likely only applicable to the soil near the surface (immediately below the embankment). The assumptions above are also less likely to hold true if the thickness of the consolidating soil layer is on par with the dimensions of the embankment, or if the soil layer only starts at a noticeable depth below the embankment. Therefore, the magnitude of in'situ stress state and hydrostatic pore pressure in the soil can potentially becomes greater than the preload induced stress relatively quickly as depth increase below the embankment.

To demonstrate this process, a coupled Biot's type poroelastic model is developed using FLAC to simulate preloading from a circular embankment with applied load of 50 kPa and embankment radius of

7 m on top of a semi-infinite soil domain. Including a simple consideration for pre-embankment in'situ stress state and pore pressure via gravitational forces such that $\rho_{dry}gz$ represents in'situ effective vertical stress (σ_v) and $\rho_{water}gz$ represents in'situ hydrostatic pore pressure. The embankment is then placed on top the at-rest soil and allowed to reach stress equilibrium while the excess pore pressure is undrained. The resultant undrained pore pressure distribution below the center of the embankment is illustrate in Figure 12. For demonstrative purposes, the soil in the example is set up as poroelastic material with a Poisson's ratio equal to zero, which means at-rest lateral earth pressure is omitted from the model for simplicity. The soil has a dry density of 1182 kg/m^3, and the porewater has a density of 1000 kg/m^3.

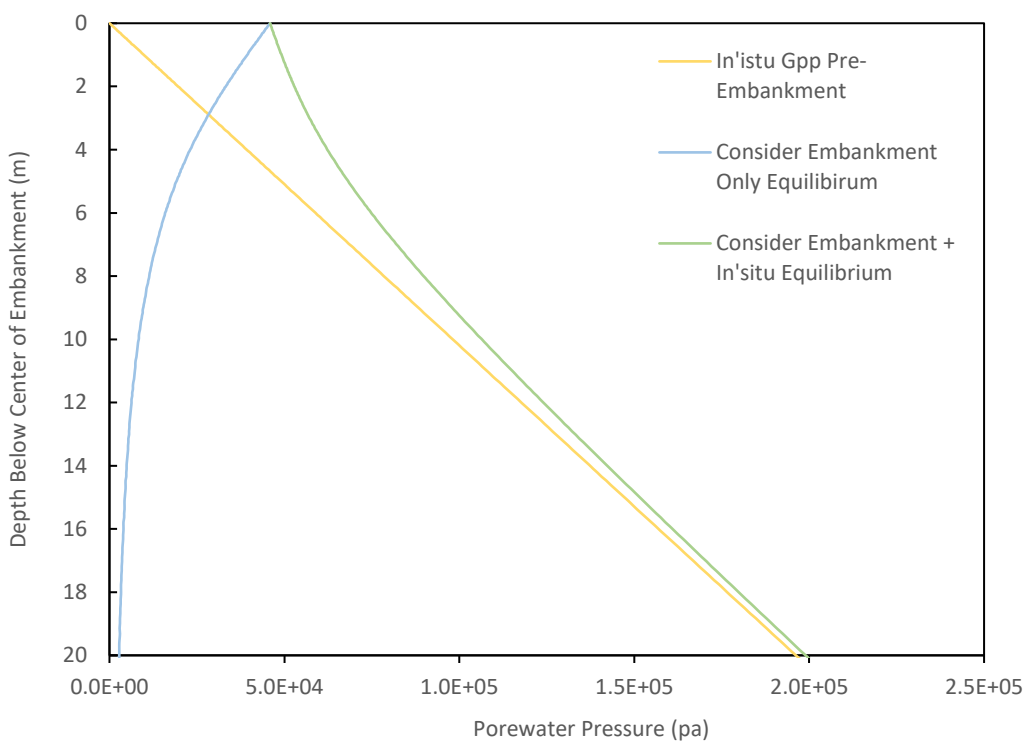

Figure 12. Undrained Pore pressure Below the Center of the Embankment of 7m Radius: Embankment Induced Excess Pore pressure, Hydrostatic Pore pressure

For a circular embankment with radius equal to 7m, with an equivalent applied preload of 50 kPa, the induced undrained excess pore pressure distribution exponentially decreases with depth below the embankment. At a depth of around 4m below the center of the embankment, the preload induced

excess pore pressure is just under 24 kPa, already a decrease of over 50% compared to the pore pressure at the surface. At the depth of only 4m below the center of the embankment, the in'situ hydrostatic pore pressure ($\rho_w gz$) is already much greater than the induced excess porepressure.

Consider the potential for the magnitude of in'situ hydrostatic pore pressure in the soil overtaking the induced excess pore pressure at relative shallow depth below the embankment, one must ask the question of whether the in'situ hydrostatic pore pressure portion of the total undrained pore pressure will drain towards the central vertical and in turn contributes to the radial degree of consolidation. Although there have been indications to support the idea that the hydrostatic pore pressure has a role in the overall degree of consolidation in several case studies of PVD and vacuum consolidation (Chu et al. 2000, Bergado et al. 2002). To the author's knowledge, there has not been a dedicated case study on the effect of hydrostatic pore pressure in PVD and vacuum consolidation, nor there has been studies examining PVD consolidation without the use of preloading, that could potentially isolate the effects of in'situ stress and hydrostatic pressure have on radial consolidation.

4. Results: Nonuniform Pore pressure Distributions and PVD Consolidation Theory

In this chapter, a numerical modelling study is performed to demonstrate the theoretical impact of incorporating nonuniform stress and pore pressure distribution in the analysis of conventional vertical drains (PVD) plus preloading method of ground improvement to promote consolidation. Several Biot's type fluid-mechanical coupled models of PVD consolidation are developed according to the traditional cylindrical unit cell theory of radial consolidation (from the likes of Barron, and Hansbo), and the impact of nonuniform stress and excess pore pressure distribution in the unit cell have on the radial consolidation process is examined.

4.1 Hansbo's Solution Verification Model

Hansbo's equation for a PVD and surcharge improved soil is the most commonly used solution for estimating the consolidation timeline of a PVD improved soil. Hansbo's equation is a closed form solution of radial consolidation in a cylindrical unit cell with the following assumptions:

- A surcharge is applied throughout the cylindrical unit cell that is converted to an equivalent excess porewater pressure
- The soil in the unit cell is isotropic and homogeneous
- Strain and deformations are uniaxial (vertical direction) only, no lateral strain occurs during consolidation
- Barron's "equal strain" assumption is applied
- Porewater drainage is only towards the PVD at the center of the cylindrical unit cell. Radial flow only
- The band shaped PVD can be converted to an equivalent radius in the cylindrical unit cell

- Accounts for presence of smear zone adjacent to the PVD, where the effective horizontal (radial) hydraulic conductivity is noticeably reduced compared to the rest of the soil in the unit cell. It is widely accepted that the smear zone surrounding a PVD develops during to the installation of the PVD and the disturbance to the adjacent soil led to a decrease in void ratio and permeability (Basu et al. 2000).

A numerical model for PVD and surcharge consolidation is developed in FLAC to compare the software's fully coupled numerical simulation to Hansbo's analytical solution. In order to best match the cylindrical unit cell radial consolidation premise of Hansbo's solution, the FLAC model utilizes a 3-D axisymmetric domain, and a fully coupled linear poroelastic constitutive relationship for its porous media. The axisymmetric domain is discretized into a 30x75 grid mesh, which consists of 31 horizontal (radial) grid points and 76 vertical (axial) grid points. The model domain has a dimension of 0.677m x 2.4m, which resembles the cylindrical unit cell in the analytical solution. However, the analytical solution model has to account for the equivalent radius of the PVD, which in this case is assumed to be 0.033m according to Hansbo's equivalent drain radius. Therefore, the dimension of the cylindrical unit cell in Hansbo's solution is taken to be 0.7m x 2.4m.

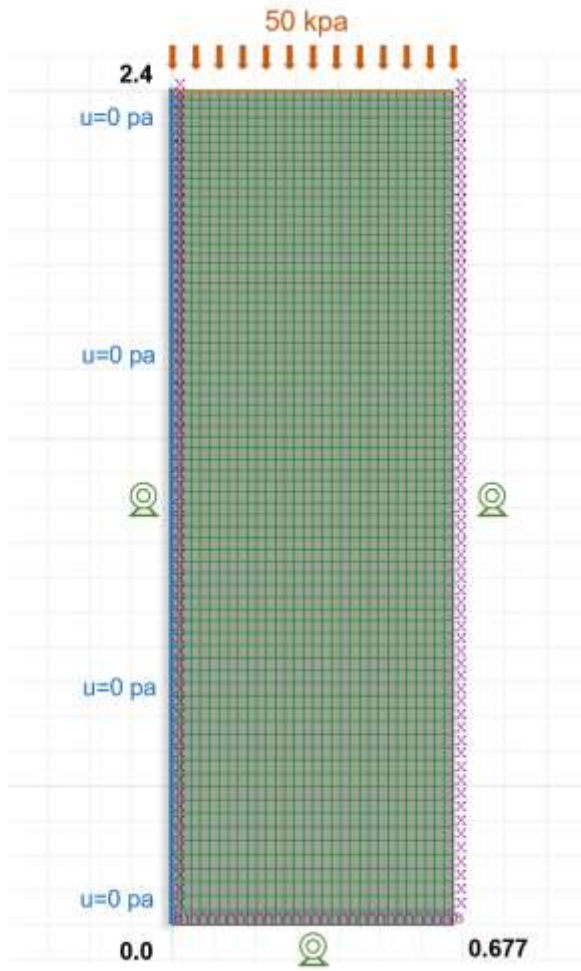

Figure 13. Verification Model: Hansbo's PVD Unit Cell Radial Consolidation

To simulate uniaxial compression during consolidation, roller boundaries (constraint laterally) were set up at the two sides of the domain. Another roller boundary (constraint vertical) at the bottom of the domain acts as the vertical constraint boundary.

For the poroelastic model to simulate the initial conditions assumed in Hansbo's solution, 50 kPa of stress is applied the top boundary of the model domain. The initial total vertical stress is then set at -50 kPa at every grid points. And the initial porewater pressure is then set to 50 kPa at every grid points. Defining the initial stress state in this way results in an undrained porewater pressure of 50 kPa at every grid point in the domain, which is equivalent to the 50 kPa excess porepwater pressure assumption

made by the analytical solution. The 50 kPa applied stress at the top boundary is also being equated as a surcharge of -50 kPa total vertical stress at every grid point, combined with 50 kPa of porewater pressure at the same time, which means the vertical effective stress in the domain equals to zero at the initial undrained stage of consolidation. Therefore, there is no strain that develops prior to drainage taking place (instantaneous settlement is neglected). Thus, the model domain matches with all the initial conditions assumed by the analytical solution.

During consolidation, the numerical unit cell domain follows the "free-strain" model of deformation, whereas the Hansbo's solution follows Kjellman's "equal-strain" hypothesis. However, it has been observed that for a cylindrical unit cell model of consolidation, the free-strain and equal-strain hypothesis does not affect the average degree of consolidation to any significant degree (Barron, 1948; Leo et al. 2004).

The properties of the porous media used in the analytical solution and the numerical model are the exact same. The soil in the unit cell is a homogeneous soft marine clay, with a porosity of 0.5, drained Poisson's Ratio (υ') is 0.4, and drained Young's modulus (E') equal to 9.36 Mpa.

A single PVD at the center of the cylindrical unit cell is simulated by a constant direct boundary condition, where the pore pressure is equal to zero at the boundary grid points. The zero pore pressure boundary at the center of axisymmetric domain is then fixed for the entire duration of the simulation of consolidation. A Radial flow only scheme is simulated by setting vertical permeability to zero and only allowing horizontal (radial) permeability to be non-zero values. The horizontal hydraulic conductivity (k_h) of the soil is set as 1x10^{-10}m/s in the undisturbed zone. The horizontal hydraulic conductivity in the smear zone is assumed to be reduced to 20% of k_h. The radius of the smear zone is assumed to be 2x of the equivalent drain radius. The hydraulic conductivity in the undisturbed zone and smear zone, as well as coefficient of radial consolidation (C_h) will remain constant during the consolidation process.

The coupled numerical model is allowed to consolidate to 1160 days, during which, average porewater pressure in the domain is obtained by numerical integration (2-D composite trapezoid method, using the same discretized mesh zones) and compared with the result of Hansbo's solution.

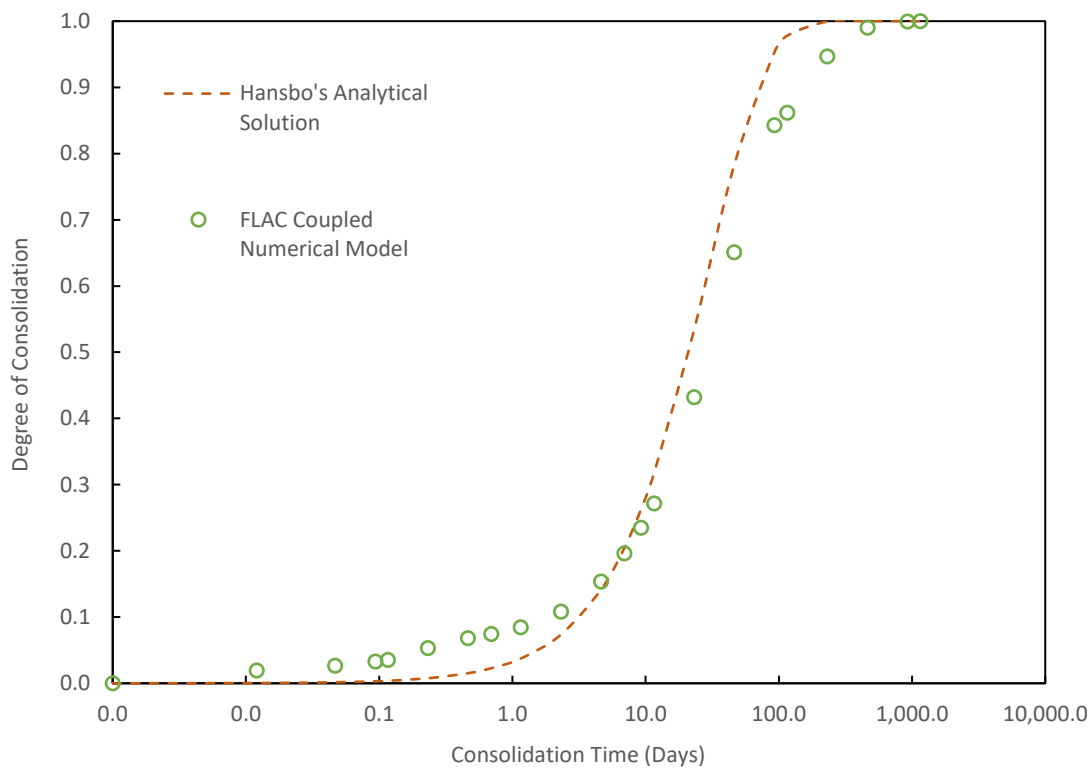

Figure 14. PVD Consolidation Verification: Degree of Consolidation Hansbo's Analytical Solution versus FLAC Numerical Simulated

The coupled numerical model simulated degree of consolidation compared well with the results obtained from Hansbo's analytical solution. The maximum discrepancy at any time during the consolidation process is less than 10%.

4.2 Undrained Pore Pressure Under a Circular Embankment

To determine the nonuniform stress and excess pore pressure distribution prior to the start of the primary consolidation process, a Biot's type coupled poroelastic model is developed with an axisymmetric semi-infinite domain, and a circular embankment is placed above the soil to provide the

preloading stress to the domain. This model (hence referred to as Circular Embankment Model) is set up in such a way that closely resembles what one would find on a typical ground improvement project for a soft marine clay deposits, that utilizes conventional PVDs and preloading methods. The porous media in the domain is a homogeneous soft marine clay type of soil. The property of the soil is summarized in Table 1 below.

Table 1. Elastic Soil Properties of a Fine-Grained Soft Soil

Poroelastic Soil Properties of Soft Marine Clay Sample			
Soil Properties		**Unit**	
Saturated Unit Weight	γ_{Sat}	16.50	KN/m^3
Dry Unit Weight	γ_{Dry}	11.60	KN/m^3
Porosity	n	0.50	-
Poisson Ratio - Undrained	υ_u	0.50	-
Poisson Ratio - Drained	υ'	0.40	-
Young's Modulus - Undrained	E_u	10.00	Mpa
Young's Modulus - Drained	E'	9.36	MPa
Bulk Modulus - Undrained	K_u	333.33	MPa
Bulk Modulus - Drained	K'	15.61	MPa
Shear Modulus	G	3.34	MPa

The semi-infinite axisymmetric model consists of the aforementioned homogeneous soft marine clay with isotropic permeability. The axisymmetric domain has a radius of 100m and a depth of 60m. The circular embankment is placed on top of the domain at the central axis and has a radius of 7m and exerts a body force equivalent to a uniform 50 kPa of vertical stress to the soil domain below.

Figure 15. Circular Embankment on Semi-Infinite Domain

Prior to preloading pressure from the embankment above, the soil domain is assigned hydrostatic ($\rho_{water} gz$) in'situ pore pressure distribution. The footing model is then ran with flow turned off until undrained equilibrium is reached, in order to simulate the stress and pore pressure in the soil domain under undrained conditions.

Without simulating the flow of porewater, the bulk material property (ie. bulk modulus) of the porous media is theoretically assumed to also include the fluid moduli of the porewater. Taking a realistic value for the moduli of water (k_w) at $2.0e^9$ Pa. The applied stress from the embankment is $5.0e^4$ Pa (50 kPa), which is considerably less compared to the moduli of the porewater, and consequently, the bulk moduli of the porous media in the undrained state. Hence it provides the justification for applying a poroelastic

model to a soft marine clay type soil, where the magnitude of strain in the porous media (or instantaneous settlement) is expected to be very small at the undrained stage.

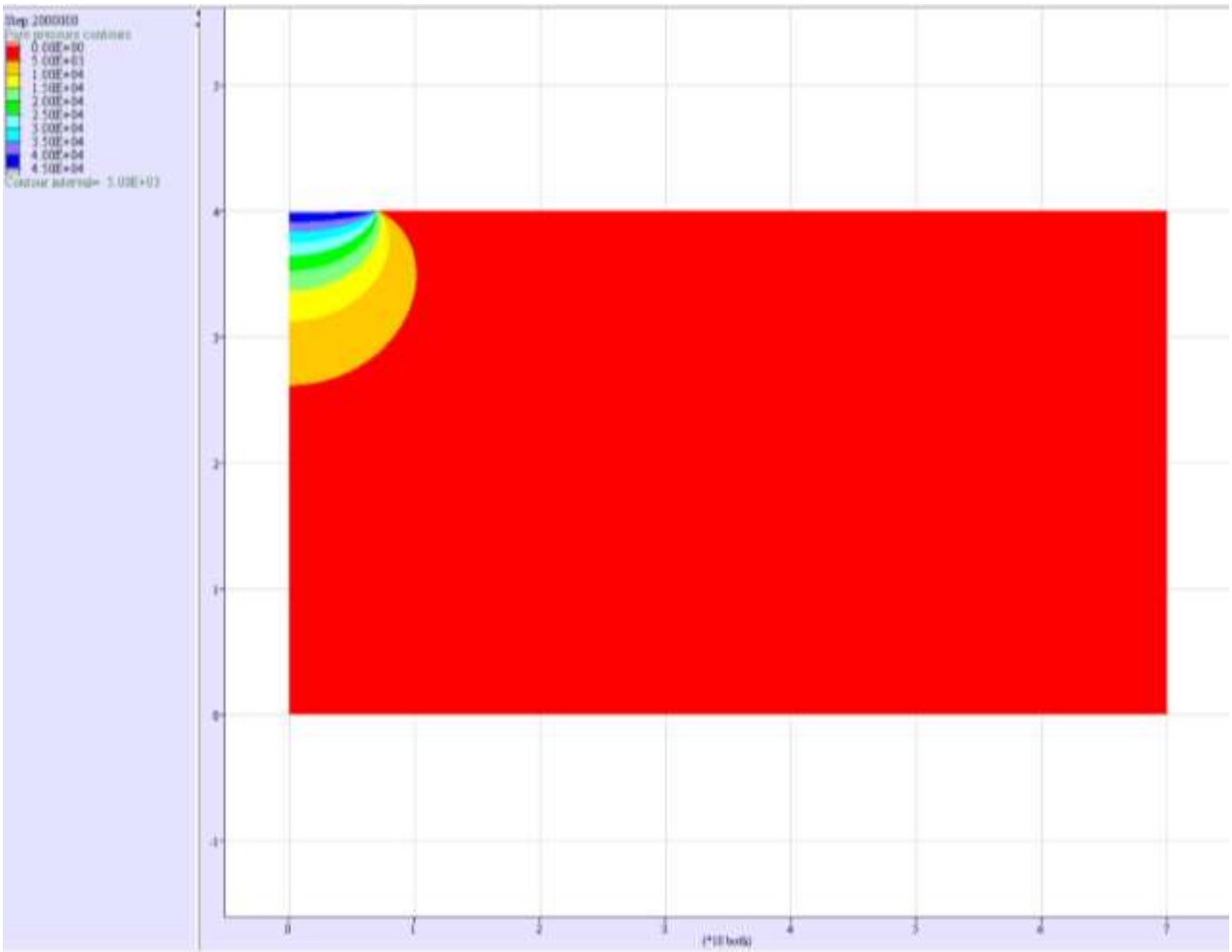

Figure 16. Embankment Induced Excess Pore Pressure at the Undrained Stage

Figure 17&18 shows the two vertical profiles of undrained pore pressure below the center of the circular embankment. The first vertical profile of pore pressure illustrates the component of excess pore pressure formed as a result of the induced stress by the circular embankment above. The second vertical profile of pore pressure shows the combined induced excess pore pressure and the hydrostatic pore pressure in the soil. It can be illustrated that generally with greater magnitude of applied stress, or larger embankment dimensions, the influence of the embankment applied stress and excess pore pressure can exerts greater influence with depth. However, this serves to demonstrate that the concepts of uniform

surcharge of stress and excess pore pressure below the embankment, which is ubiquitous in many consolidation solutions (Terzaghi, Barron, Hansbo, Indraratna etc), are limited by the dimensions of the embankment, and the depth and thickness of the soft soil deposit being improved.

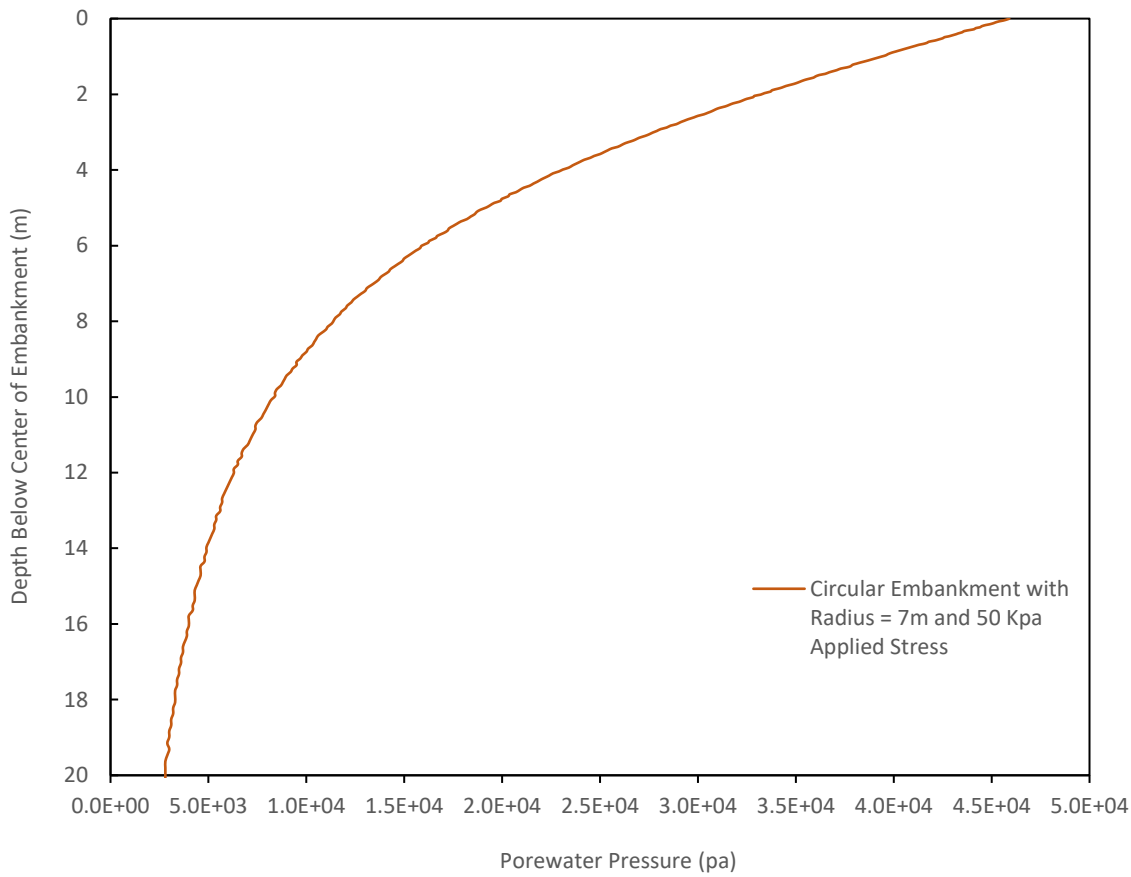

Figure 17. Embankment Induced Excess Pore Pressure Profile Below the Center of the Embankment Radius of 7m

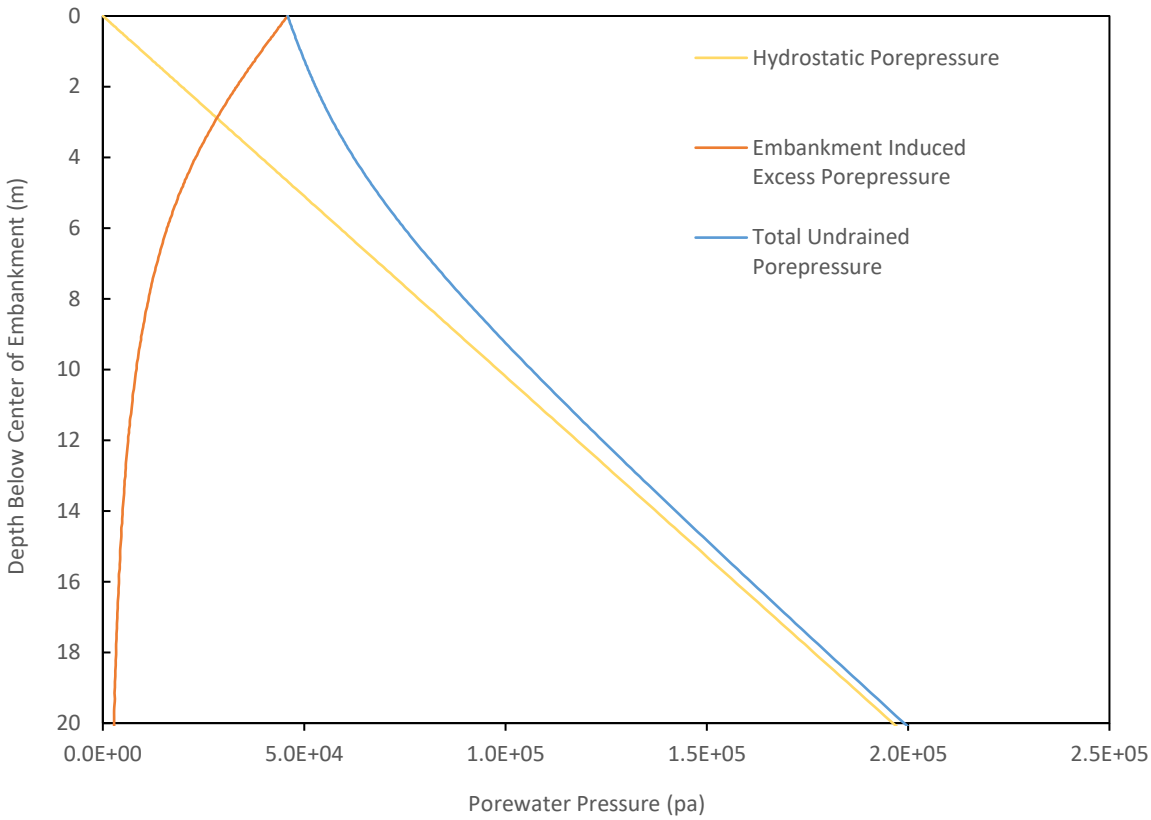

Figure 18. Total Pore Pressure Below the Center of the Embankment Radius of 7m: Embankment Induced Excess Pore Pressure + Hydrostatic Pore Pressure

4.3 Model 1: Unit Cell PVD Radial Consolidation Model - Poroelastic Soil

The purpose of model 1 is to examine nonuniform pore pressure distribution in the process of cylindrical unit cell radial consolidation theory and compatibility with the analytical solution presented by Hansbo. Model 1 is a cylindrical unit cell model with a radius of 0.677m and length (depth) extending to 6m below the center of the embankment. The unit cell is made up of the same homogeneous soft clay type soil as the footing model in the previous Section 4.2, the properties of which are given in Table 1. The soft clay type soil is assigned a horizontal hydraulic conductivity value of $1x10^{-10}$m/s. A single PVD is placed at the center (axial) of the cylindrical unit cell where it provides the only pathway for porewater to dissipates to, thus the pore pressure is limited to flow in the radial direction. The unit cell is located below the center of the circular embankment (radius=7m) serving as preload to the soil by applying a uniform 50 kPa of stress to the soil below. The initial undrained pore pressure distribution inside the unit cell at the start of primary consolidation (t=0) is determined by extracting the undrained pore pressure distribution from the footing model in Section 4.2. As previously stated, the initial undrained pore pressure distribution is the combination of the embankment induced nonuniform excess pore pressure below the center of the footing, and the hydrostatic pore pressure in the soil.

The coupled numerical model of the unit cell consists of an axisymmetric domain discretized into a 31x144 grid. Roller boundaries are placed at the two sides of the domain, and the bottom of the domain is fixed. A single PVD is placed at the center (axial) of the cylindrical domain. Dissipation of porewater is only toward the central PVD, where the pore pressure is fixed to 0 kPa. The radial flow only condition is simulated by assigning anisotropic permeability to the soil, where the only non-zero value is assigned to the horizontal (radial) permeability. Figure 19 shows the domain, boundary, and mesh of Model 1.

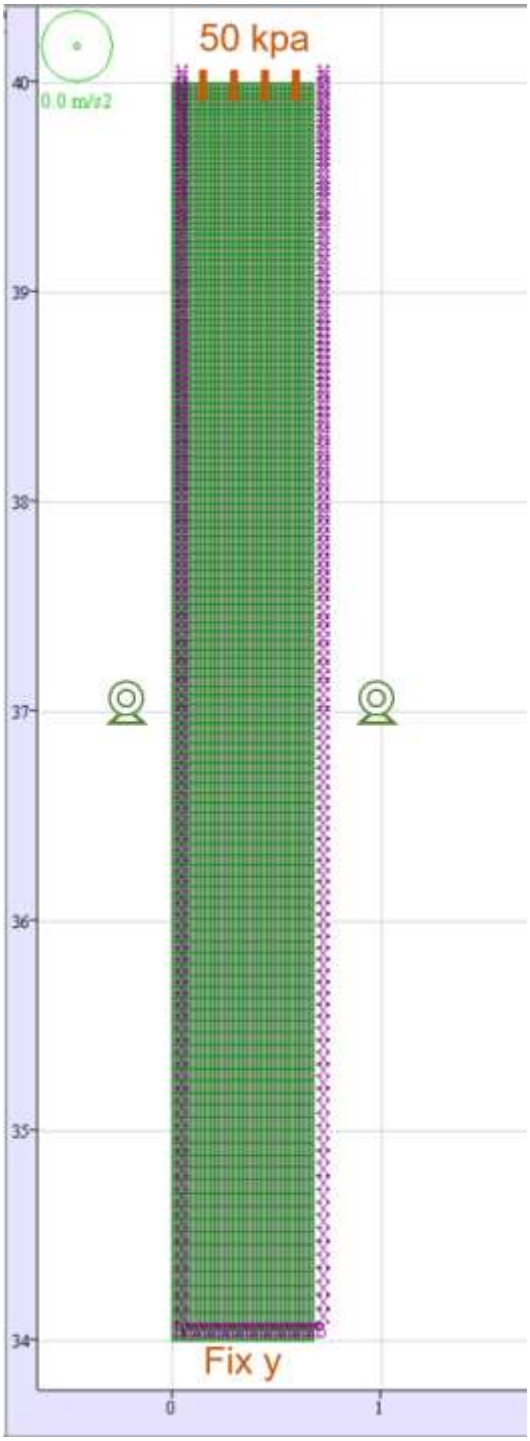

Figure 19. Model 1 Domain: Unit Cell PVD Consolidation

At the start of the drained stage, pore pressure in the unit cell starts to dissipate at the central PVD (pore pressure = 0 kPa) and the unit cell is assumed to be constraint to uniaxial compression under the

50 kPa applied stress from the embankment. The model simulates the consolidation process for a total of 4,600 days of consolidation time. The average pore pressure over the entire cylindrical domain, along with the total vertical settlement at the ground surface are recorded for the entire duration of consolidation. The transient pore pressure during consolidation, in the form of average pore pressure in the cylindrical domain, is shown in Figure 20.

In Hansbo's conceptual model for radial consolidation solution the preloading stress from the embankment is assumed to result in an equivalent surcharge and induced excess pore pressure in the soil below. Hence 50 kPa of preloading stress applied to the top of the unit cell subsequently becomes a uniform 50 kPa of excess pore pressure at every grid point in the unit cell prior to the start of the consolidation process. This is apparent through the average pore pressure recorded in the unit cell equal to 50 kPa at the start of the consolidation.

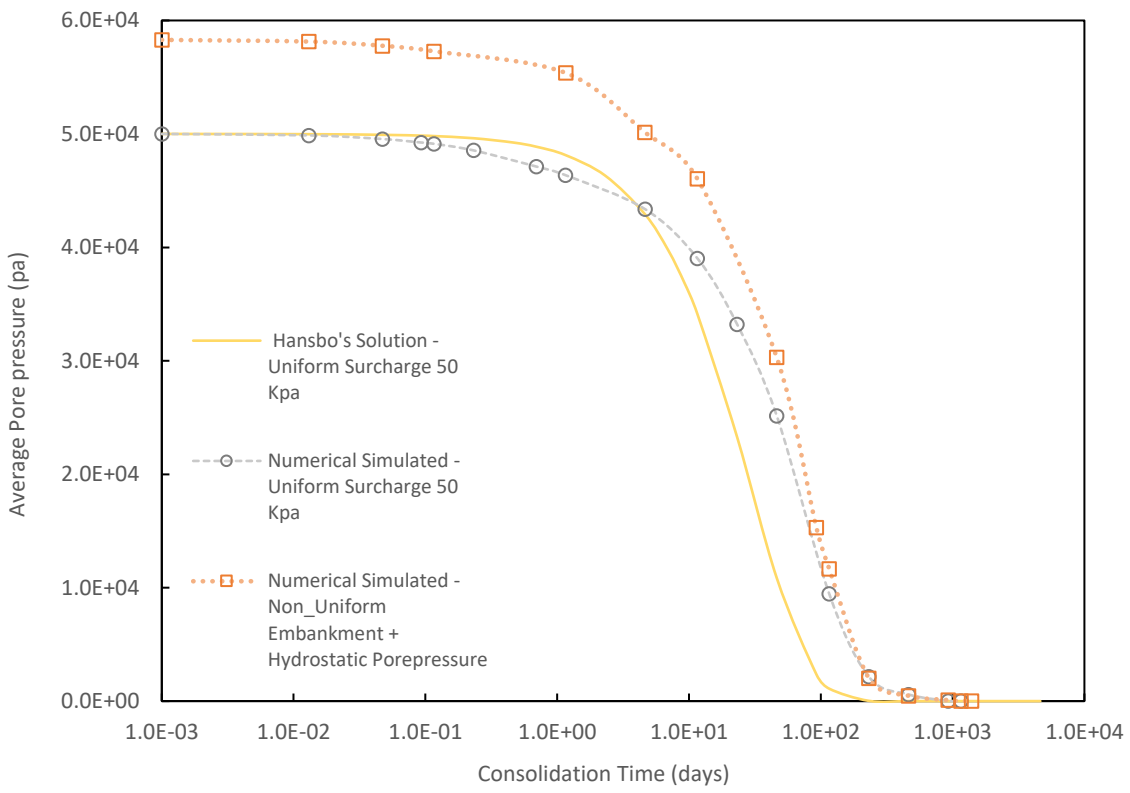

Figure 20. Model 1: Average Pore Pressure in the Unit Cell – Uniform Surcharge versus Nonuniform Pore Pressure Distribution

For this case, the nonuniform pore pressure distribution resulted in the average pore pressure in the unit cell at the start of the consolidation of around 58 kPa, which is noticeably greater than 50 kPa of uniform surcharge assumed by Hansbo. Depending on the dimensions of the embankment, the locations as well as length (depth below embankment and thickness) of the unit cell, the average pore pressure in the unit cell could potentially be significantly greater or less than the assumption for uniform surcharge/pore pressure in the unit cell. Consequently, the conventional method of uniform surcharge could significantly overestimate or underestimate the amount of pore pressure in the unit cell that ends up dissipating through the PVD. Due to the discrepancy in average pore pressure in the unit cell, the total vertical settlements that occurs in the unit cell during primary consolidation will be different as well, and it serves to highlight the limitation of Hansbo's solution being not a fully coupled solution. Figure 21 compares the ultimate vertical settlements at 100% consolidation of the unit cell corresponding to the uniform pore pressure assumption (starting at average 50 kPa of pore pressure) and the nonuniform pore pressure (starting at average 58 kPa of pore pressure) distribution in this example.

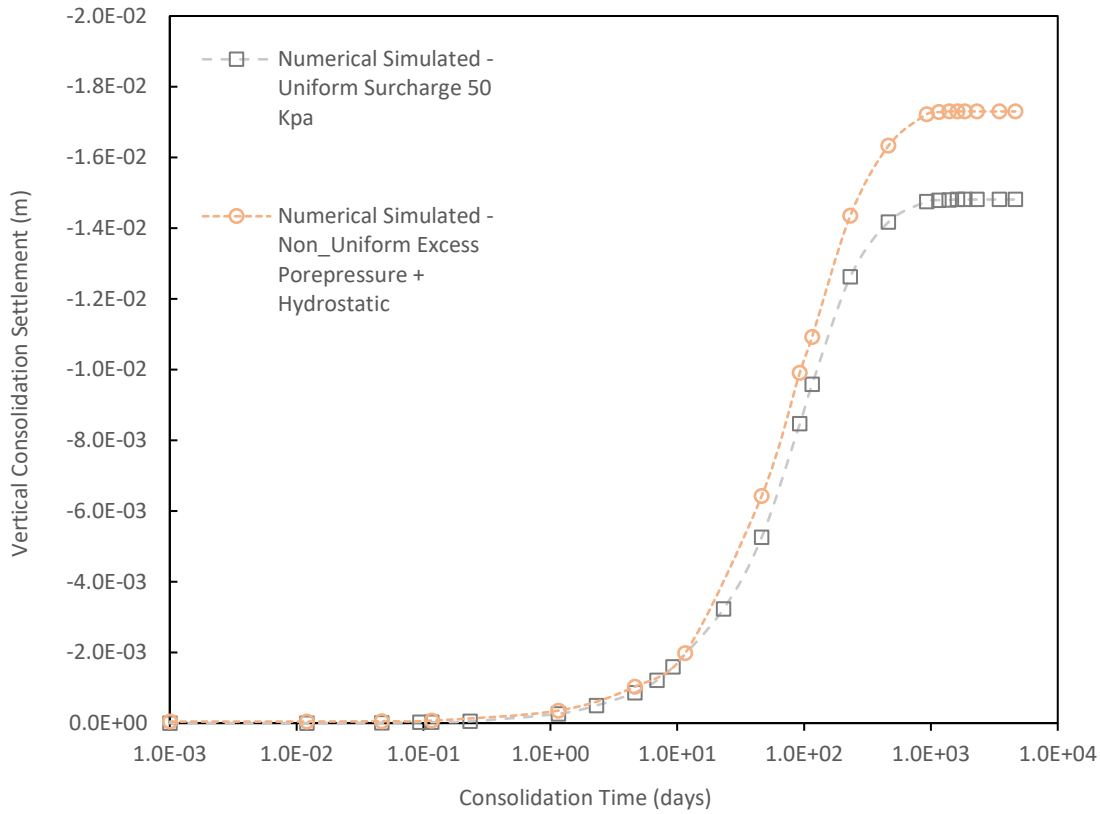

Figure 21. Model 1: Total Vertical Settlement - Uniform Surcharge versus Nonuniform Pore Pressure Distribution

Degree of consolidation (DOC) is typically determined by taking the average excess pore pressure in the unit cell at a given time during drained primary consolidation process and subtracting it from the maximum average excess pore pressure at the start of consolidation and dividing the result by the maximum average excess pore pressure. Essentially, the conventional method of quantifying DOC by looking at average excess pore pressure is simply performing normalization on the process of the transient diffusion of excess pore pressure in the unit cell.

Using the same definition of degree of consolidation applied to the nonuniform pore pressure distribution in this example. The DOC determined for the nonuniform pore pressure distribution in Model 1, compared to that of the traditional uniform surcharge assumption, is presented in Figure 22.

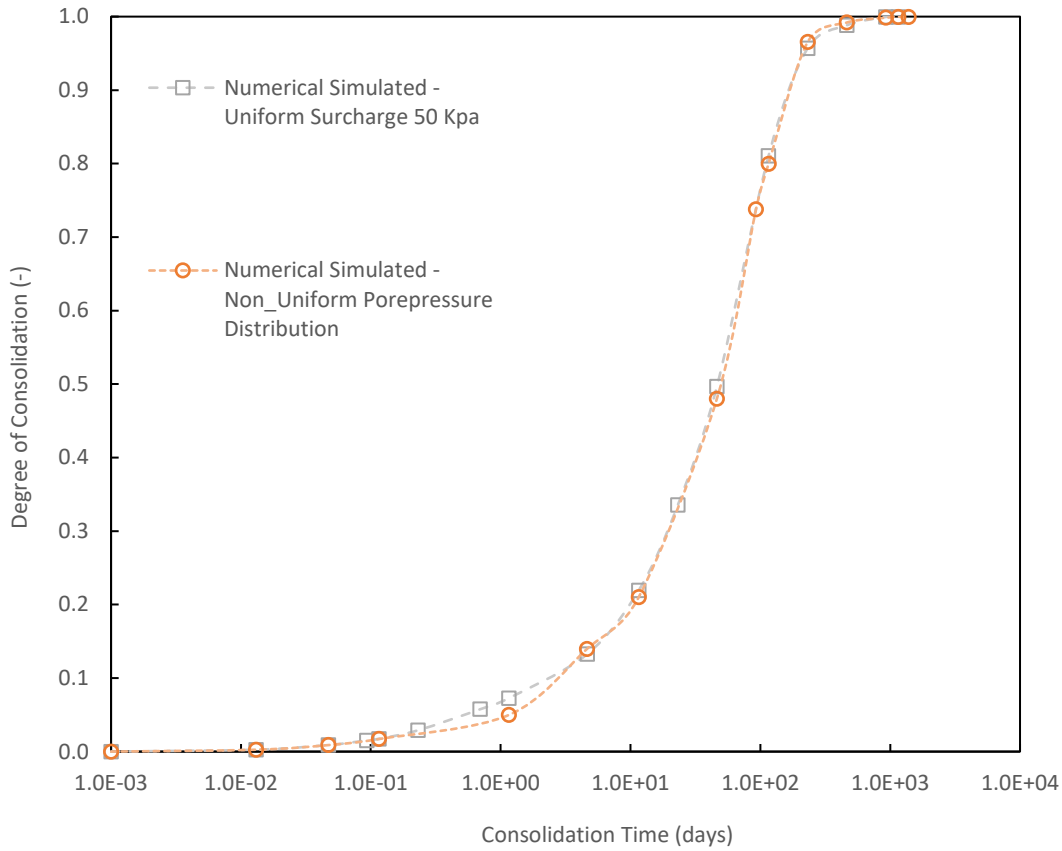

Figure 22. Model 1: Degree of Consolidation - Uniform Surcharge versus Nonuniform Pore Pressure Distribution

The example with uniform surcharge (initial average pore pressure 50 kPa) and the example with nonuniform pore pressure distribution (initial average pore pressure 58 kPa) have nearly identical degree of consolidation value at any time in the consolidation process. This identifies that for the traditional unit cell radial consolidation theory presented by Hansbo, the rate of consolidation in the unit cell is solely determined by the coefficient of horizontal (or radial) consolidation (C_h), which is assumed to be a constant value through the entire consolidation process according to Hansbo's theory. Regardless of the initial average pore pressure and distribution of the pore pressure in the unit cell, the framework provided by the traditional unit cell radial consolidation theories would result in identical DOC and rate of consolidation. This inherent limitations in the conventional unit cell radial consolidation theory makes the Hansbo's original analytical solution incompatible for analyzing nonuniform pore pressure distribution in the unit cell.

4.4 Model 2: Unit Cell Radial Consolidation - Poroelastic Model

A second fully coupled poroelastic unit cell model is developed for an example where the initial average pore pressure in a unit cell with nonuniform pore pressure distribution is noticeably less than that of the uniform surcharge/pore pressure assumption. For this case, the length of the unit cell/domain is reduced to 1.6m long, and the applied stress from the circular embankment above the unit cell is increase to 70 kPa. The dimension of the circular embankment remains the same at radius of 7m. Model 2 also uses the same homogeneous soft clay type soil as the previous models, the properties of which are summarized in Table 1. The soft clay type soil is assigned a horizontal hydraulic conductivity value of $1x10^{-10}$m/s. A single PVD is placed at the center (axial) of the cylindrical unit cell where it provides the only pathway for porewater to dissipates to, thus the pore pressure is limited to flow in the radial direction. The unit cell is located below the center of the circular embankment (radius=7m) serving as preload to the soil by applying a uniform 70 kPa of stress to the soil below. The initial undrained pore pressure distribution below the embankment is determined through the same process outlined in Section 4.2.

The cylindrical unit cell model (Model 2) has an axisymmetric domain with radius of 0.677m, but with height of only 1.6m. Model 2 domain is discretized into 41x55 grid points, and for the 1.6m length, this corresponds to the same aspect ratio of the mesh grid as that of Model 1, only now with a shorter domain length. Roller boundaries are placed at the two sides of the domain, and the bottom of the domain is fixed. A single PVD is placed at the center (axial) of the cylindrical domain. Dissipation of porewater is only toward the central PVD, where the pore pressure is fixed to 0 kPa. Figure 23 shows the domain, boundary, and mesh of Model 2.

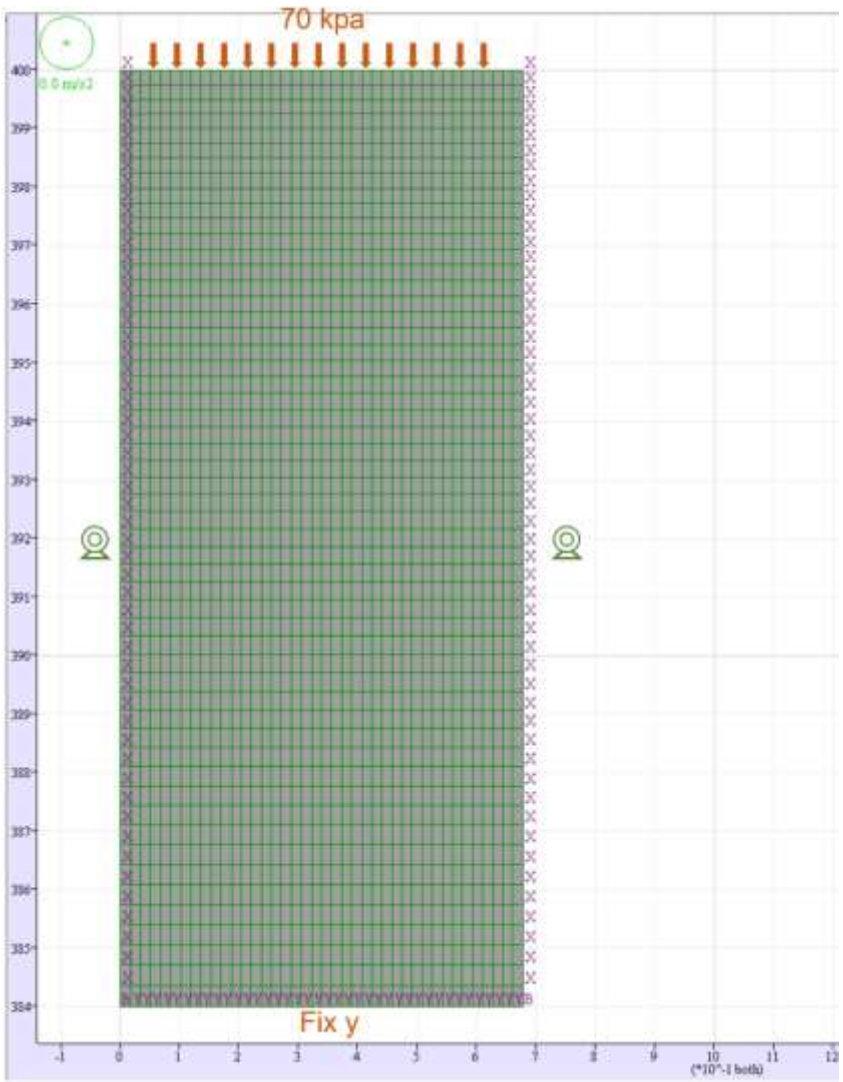

Figure 23. Model 2: PVD Consolidation

Figure 24 shows the initial pore pressure distribution for the 1.6m unit cell at the start of primary consolidation. The unit cell pore pressure distribution is extracted from the undrained footing model and consist of the combination of embankment induced nonuniform excess pore pressure and the hydrostatic pore pressure. The model simulates the consolidation process for a total of 2,300 days of consolidation time. The transient pore pressure during consolidation, in the form of average pore pressure in the cylindrical domain, is shown in Figure 25.

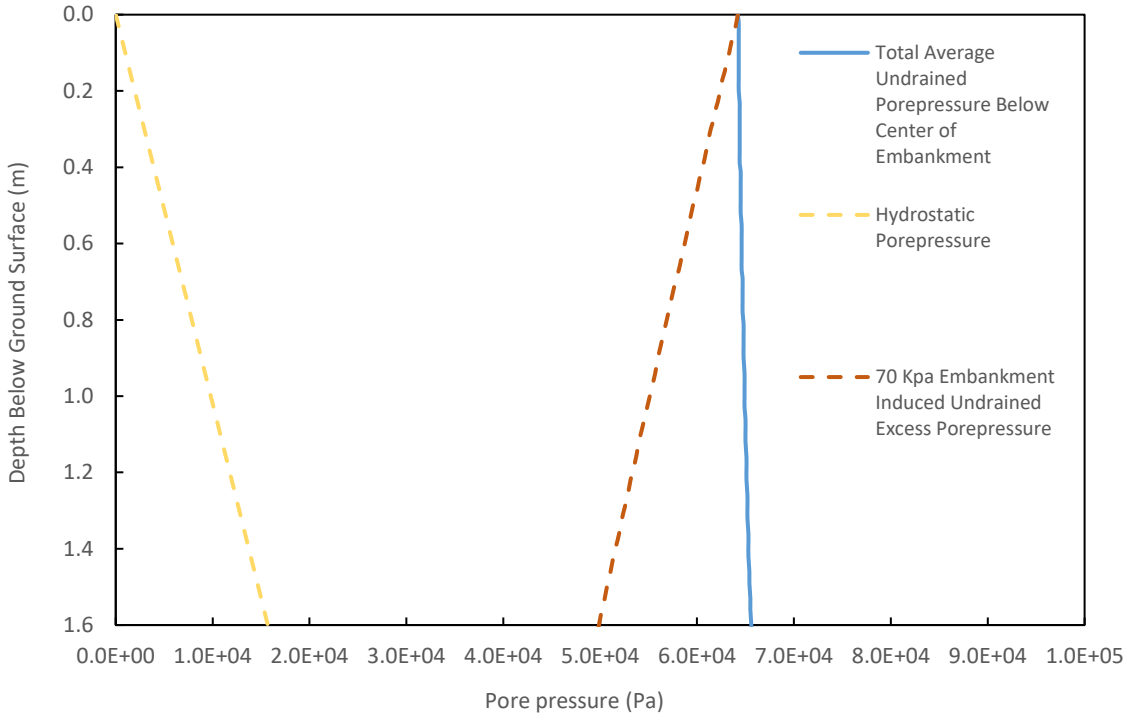

Figure 24. Model 2: Nonuniform Pore Pressure Distribution in the Unit Cell at the Undrained Stage

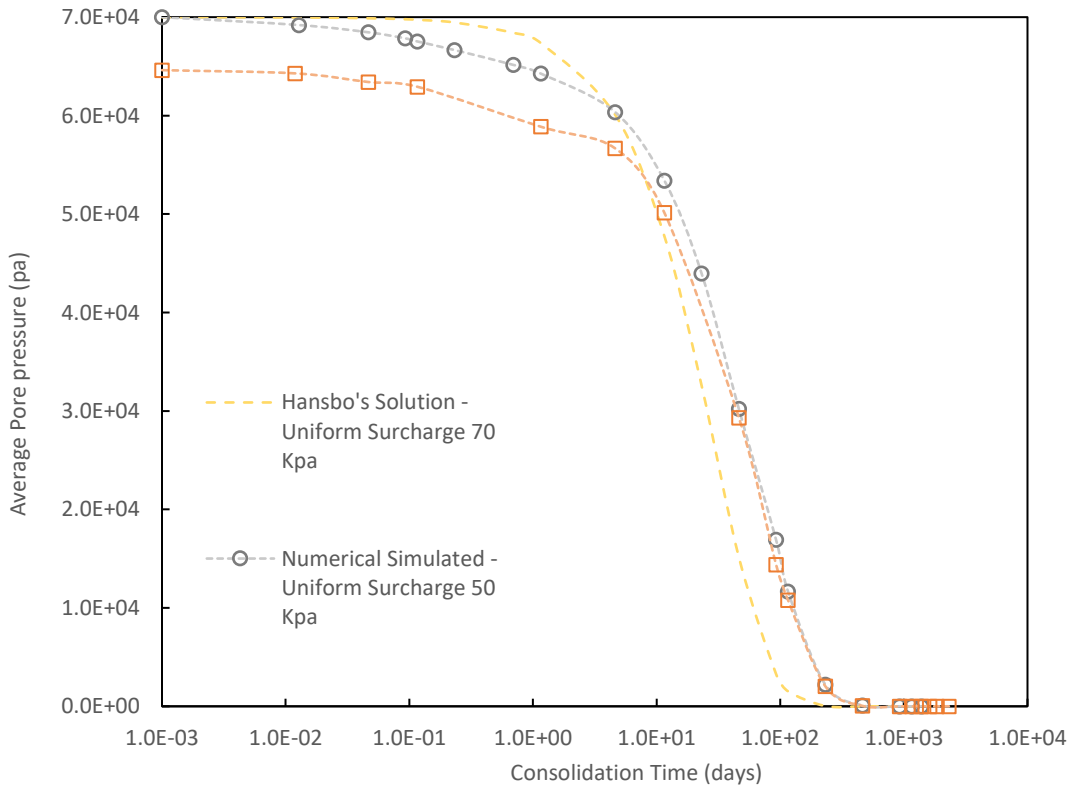

Figure 25. Model 2: Average Pore Pressure in the Unit Cell - Uniform Surcharge versus Nonuniform Pore Pressure Distribution

For this case, the initial average pore pressure in the unit cell with nonuniform pore pressure distribution started at 64.6 kPa prior to consolidation, which is noticeably less than 70 kPa, if one is to assume uniform surcharge. As a result, similar to the previous case (Model 1), the discrepancies in the total pore pressure being drained during the primary consolidation process leads to a difference in the magnitude of deformation and vertical settlement that could potentially form in the soil. Figure 26 shows the theoretical total vertical settlement potential of the unit cell if 100% consolidation is reached. As expected, because there is theoretically less pore pressure being drained to the PVD, the result is that there will be noticeably less consolidation settlement as a result.

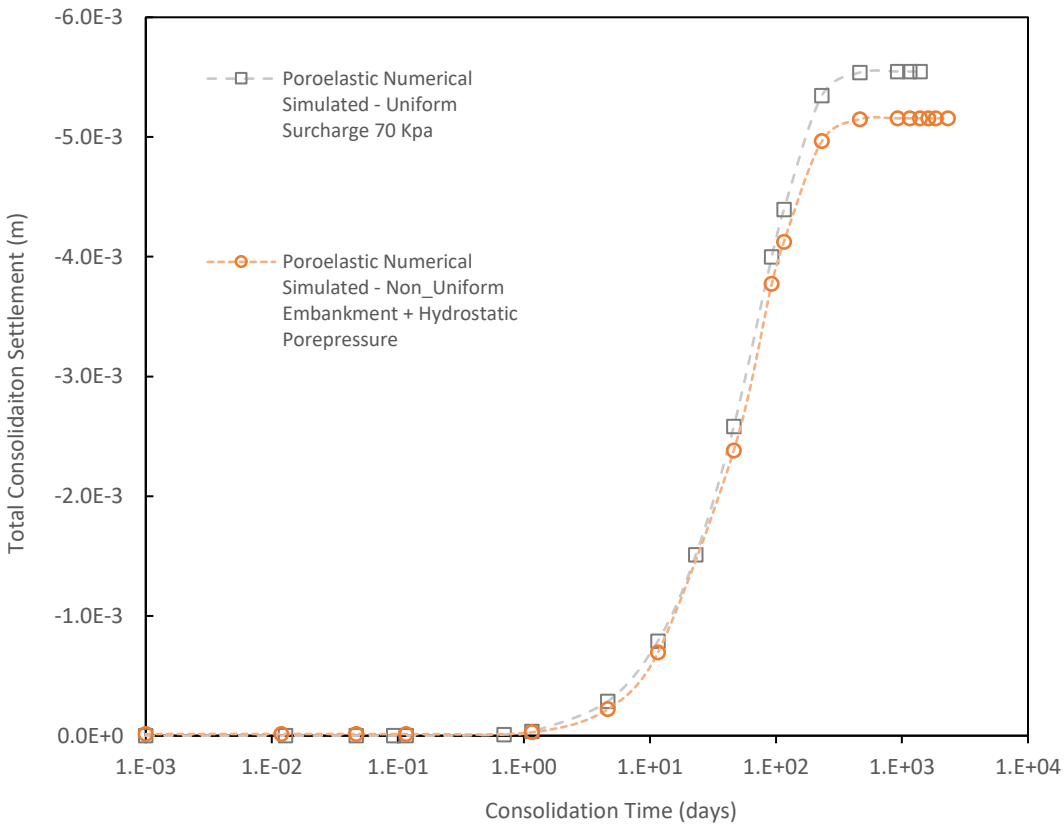

Figure 26. Model 2: Total Surface Settlement - Uniform Surcharge versus Nonuniform Pore Pressure Distribution

Once again, similar to the previous example (Model 1), when consolidation is presented in terms of degree of consolidation by examining the rate of pore pressure dissipation in the unit cell, the amount

of pore pressure in the unit cell at the start of consolidation appears to be irrelevant, and the degree of consolidation are nearly identical for uniform and nonuniform pore pressure.

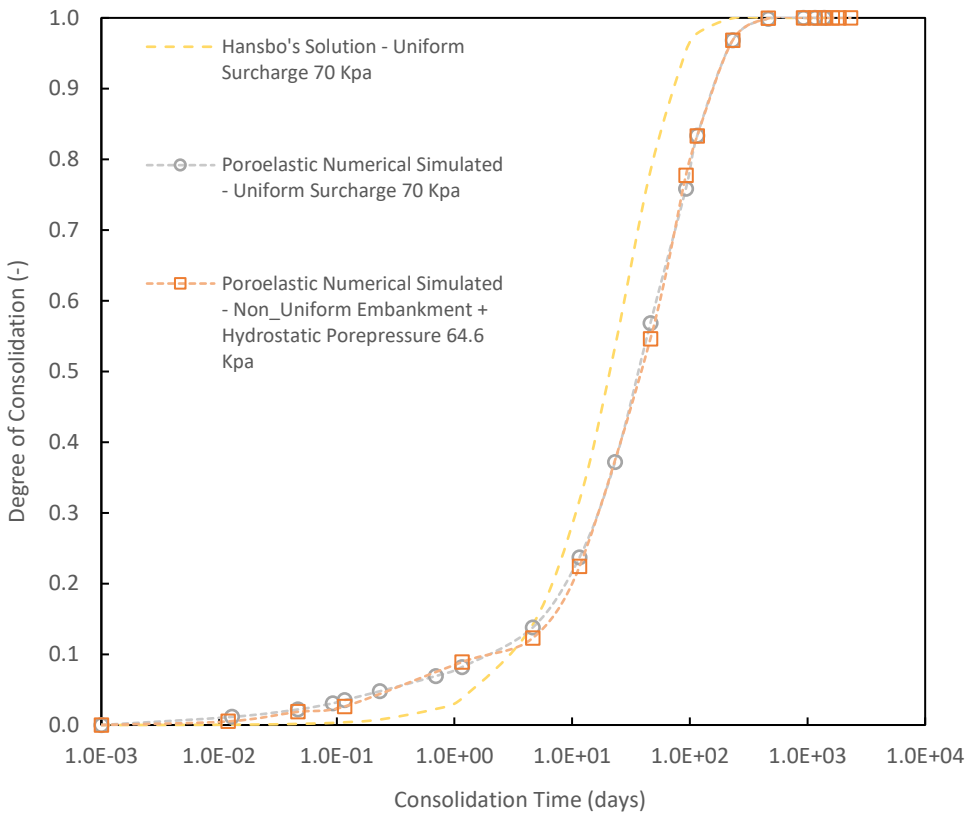

Figure 27. Model 2: Degree of Consolidation - Uniform Surcharge versus Nonuniform Pore Pressure Distribution

4.5 Model 3: Unit Cell PVD Radial Consolidation - Modified Cam-Clay Model

The previous poroelastic models (Model 1&2) highlighted the theoretical limitation of the conventional unit cell radial consolidation analytical solution presented by Hansbo, which remains widely used for PVD consolidation to this day. In Hansbo's original solution (Hansbo, 1982), the process of radial pore pressure diffusion is governed by Darcy's Law, and the permeability of the soil is assumed to be a constant material property independent of the consolidation process. As a result, the horizontal (radial) coefficient of consolidation (C_h) of the unit cell will also be a constant value that is independent of the consolidation process. The assumptions for constant permeability and compressibility in Hansbo's solution are rather antiquated concepts and overly simplified view on consolidation. These concepts

have long been revised and updated in more rigorous studies of soil mechanics, yet they remain in many of the currently used models of consolidation. For example, the permeability of a soil can evolves and vary during consolidation, and in soil mechanics it is common to consider evolving permeability that is coupled to changes to the void ratio of the soil. Similarly, strains and deformations that forms in a fine-grained soil during the primary consolidation process often can not be described by a linear poroelastic model. This is due to fine grained soil rich in silt and clay which typically exhibits non-linear stress-strain relationships, and the large-strain that forms during consolidation of fine-grained soil are typically outside of the ranges of elastic deformation. Studies on consolidation of soft soils, particularly in numerical modelling studies of PVD and vacuum assisted consolidation in the literature over the past 20 years, the most utilized method is the Modified Cam-Clay model (Roscoe and Burland, 1970) for a soft clay type soil. Utilizing a non-linear stress-strain relationship of the soil means the moduli/compressibility of the soil is no longer a constant during consolidation, hence introducing another evolving variable to the coefficient of consolidation, as well as nonlinearity to the governing equation of consolidation.

These two methods for the evolution of permeability and the compressibility of the soil during consolidation are expansion of the unit cell radial consolidation theories that theoretically allows it to incorporate nonuniform stress and pore pressure distribution in the unit cell. For the scope of this study, only the nonlinear stress-strain relationship of the soil is utilized to examine the effect of nonuniform pore pressure distribution applied to unit cell radial consolidation. The method for evolving permeability values during consolidation will be addressed in subsequent studies.

To demonstrate the non-linear stress-strain relationship of a soft clay type soil and its effect on the unit cell radial consolidation theory, particularly how nonlinear governing equation of radial consolidation is able to incorporate nonuniform pore pressure distribution in the unit cell, Model 3 is developed as a fully coupled unit cell radial consolidation model utilizing Modified Cam-Clay model of the soil. To

contrast and compare the theoretical difference between the conventional consolidation theories and the Modified Cam-Clay model of consolidation, Model 3 has the same domain, boundary and parameters as the previous example Model 2, with the only difference being that Model 3 utilizes the Modified Cam-Clay model for a slightly over-consolidated soil. While Model 3 shares the same elastic soil properties as the previous models, with the additional Modified Cam-Clay soil properties that are summarized in Table 2 below.

Table 2. Modified Cam-Clay Soil Properties

Modified Cam-Clay Soil Properties of Soft Marine Clay Sample			
Soil Properties		**Unit**	
Soil Constant	M	0.888	-
Slope of Normal Consolidation Line	λ	0.161	-
Slope of Elastic Swelling Line	κ	0.062	-
Reference Pressure	ρ'_1	100	Pa
Specific Volume at Reference Pressure	υ_λ	2.858	-
Initial Value of Cap Pressure	ρ_c	$1.6x10^5$	Pa

Once again Model 3 has an axisymmetric domain with radius of 0.677m, but with height of only 1.6m, and discretized into 41x55 grid points. Roller boundaries are placed at the two sides of the domain, and the bottom of the domain is fixed. A single PVD is placed at the center (axial) of the cylindrical domain. Dissipation of porewater is only toward the central PVD, where the pore pressure is fixed to 0 kPa. The radial flow only condition is simulated by assigning anisotropic permeability to the soil, where the only non-zero value is assigned to the horizontal (radial) permeability. The unit cell is located below the center of the circular embankment (radius=7m) serving as preload to the soil by applying a uniform 70 kPa of stress to the soil below. The initial undrained pore pressure distribution below the embankment is identical to that of Model 2, as well as the domain, mesh, and boundary conditions.

Model 3 simulates the consolidation process for a total of 2,300 days of consolidation time, reaching 99% consolidation in the unit cell. The average pore pressure over the cylindrical domain, along with the total vertical settlement at the ground surface are recorded for the entire duration of consolidation. The

transient pore pressure compared to the poroelastic model (Model 2) during consolidation, in the form of average pore pressure in the cylindrical domain, is shown in Figure 28.

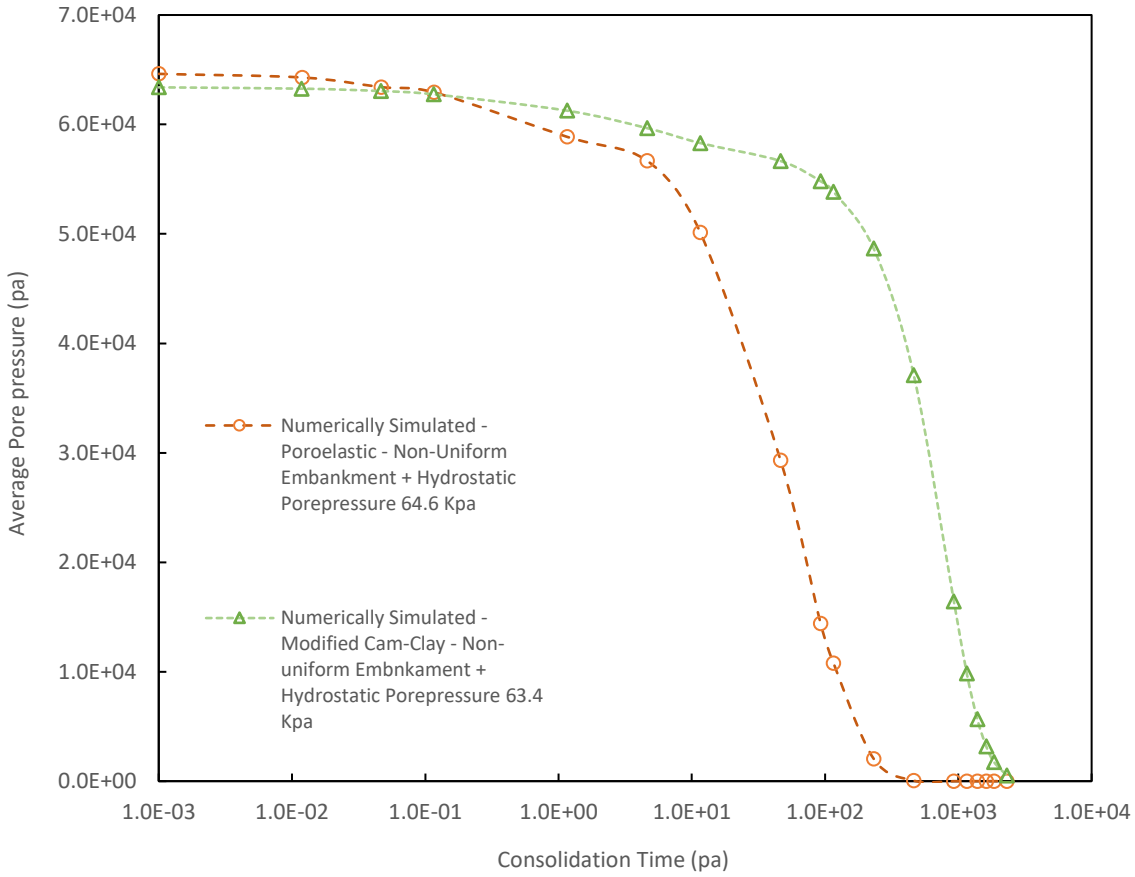

Figure 28. Model 3: Average Pore Pressure in the Unit Cell - Modified Cam Clay Model versus Poroelastic Model

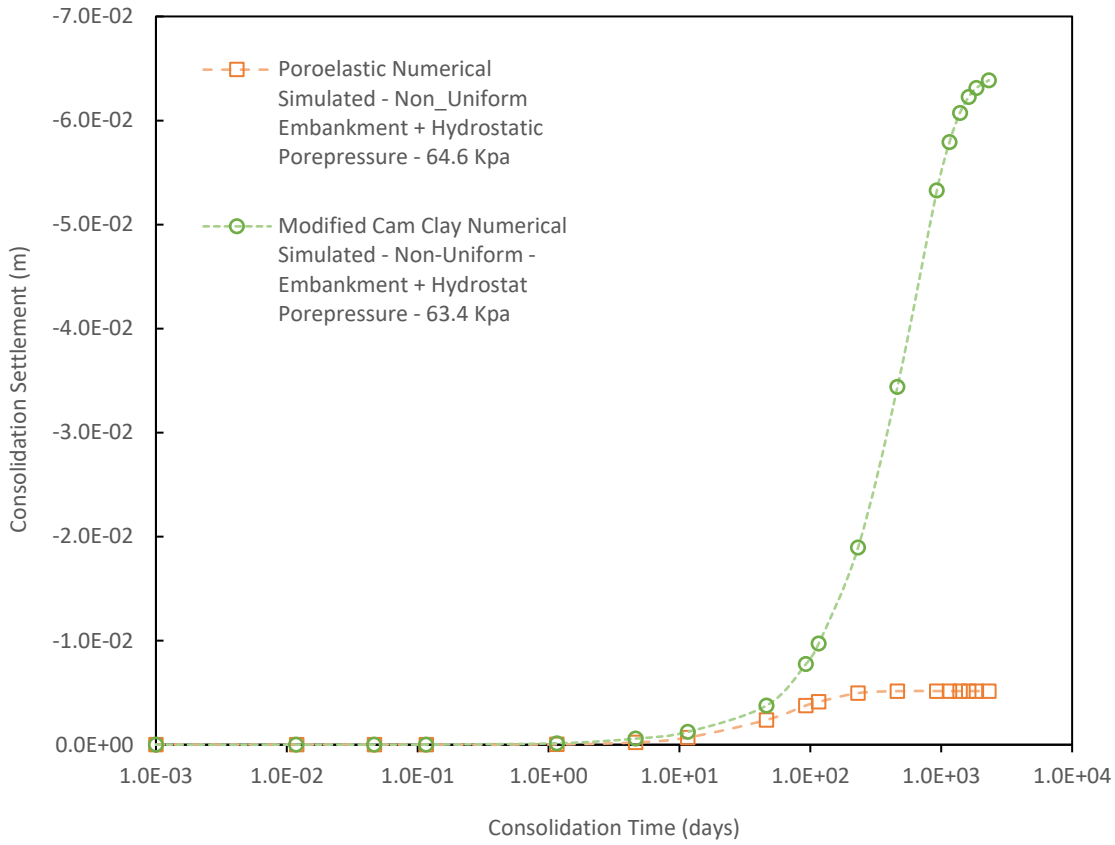

Figure 29. Model 3: Total Surface Settlement: Modified Cam Clay Model versus Poroelastic Model

Under nonuniform pore pressure distribution in the unit cell, the initial average pore pressure are nearly identical between the poroelastic (64.6 kPa) and the Modified Cam-Clay (63.4 kPa) model. However, incorporating elasto-plastic strain in the soil led to much greater (more than 10 times more) vertical settlement in the unit cell compared to the poroelastic model of consolidation.

With Modified Cam-Clay model, not only does the elasto-plastic strain lead to larger consolidation settlement, but also a noticeably slower rate of consolidation compared to conventional unit cell radial consolidation theory. Figure 29 shows the degree of consolidation of the Modified Cam-Clay model compared to poroelastic model used in conventional unit cell radial consolidation theories. Recall that previous it was shown that under the framework of the conventional unit cell consolidation theories, either applying uniform surcharge to the unit cell, or taking nonuniform pore pressure distribution in the unit cell, ultimately resulted in identical degree of consolidation for both scenarios. This is because the

coefficient of consolidation was assumed to be a constant. In this case, employing the Modified Cam-Clay model, the compressibility/moduli of the soil will evolve during consolidation, thus providing some variability to the coefficient of consolidation.

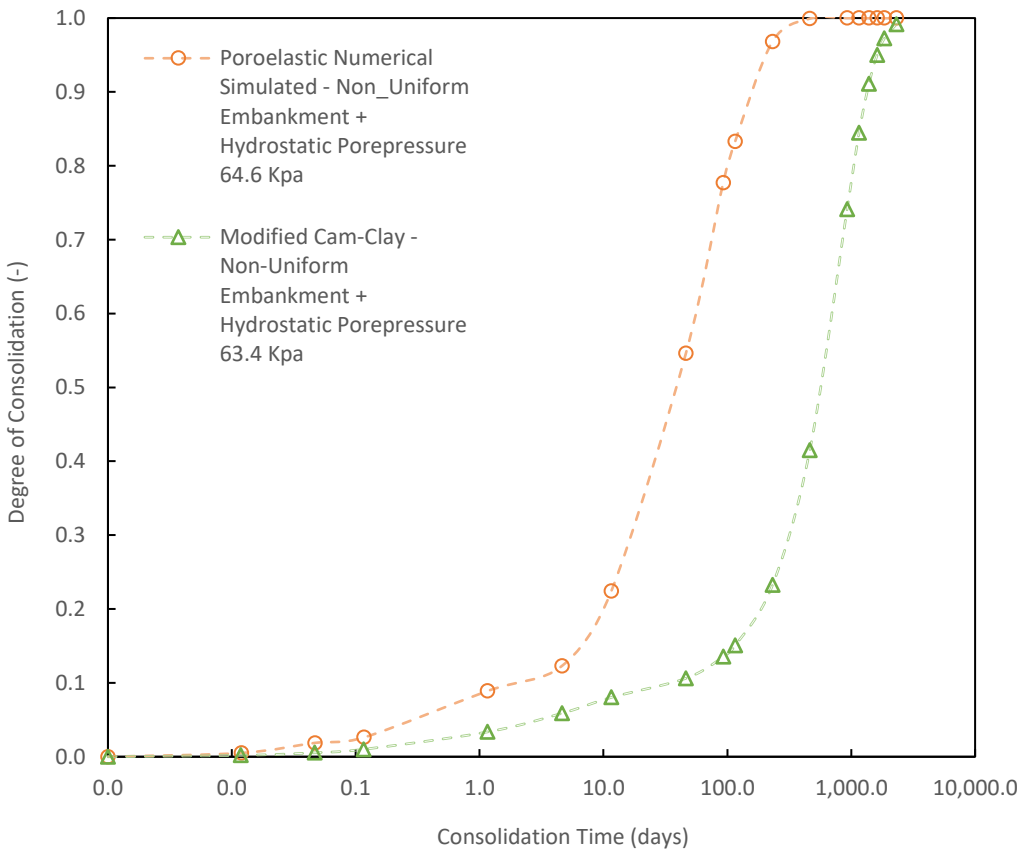

Figure 30. Model 3: Degree of Consolidation - Poroelastic Model (Model 2) versus Modified Cam Clay Model (Model 3)

The interactions between elasto-plastic stress-strain relationships, volumetric strain, void ratio and permeability of the soil during the consolidation process should be incorporated in the fully coupled governing equation of unit cell radial consolidation. The research on PVD and vacuum radial consolidation conducted by Walker and Indraratna (Walker et al. 2012; Indraratna et al. 2017) attempted to address this problem by proposing non-linear radial consolidation governing equations that incorporates Non-Darcian porewater diffusion, void ratio dependent permeability and compressibility. Walker observed that under a constant applied stress, the rate of consolidation in the

cylindrical unit cell could be faster or slower depending on compressibility to permeability ratio and the pre-consolidation pressure, which could lead to increasing or decreasing coefficient of consolidation as the degree of consolidation progresses. Walker's finding could potentially provide the basis for the examination of nonuniform pore pressure distribution during consolidation. The same sentiment is also echoed by Indraratna, whose recent publications also stressed the importance of nonlinear governing equation of consolidation (Indraratna et al, 2017) in numerical models of consolidation, particularly the inclusion of evolving permeability and compressibility of the soil, large strain, and non-Darcian flow scheme. Indraratna summarized the currently available analytical and numerical models of both PVD and vacuum assisted consolidation, which examined one or more components of nonlinear governing equation of radial consolidation. Table 3 is taken from a recent publication from Indraratna.

Table 3. Summary of Nonlinear Models of Radial Consolidation (Indraratna et al, 2017)

Summary of Nonlinear Models of Radial Consolidation				
	Factors Included			
Models	Varying Permeability and Compressibility	Non-Darcian Flow	Large-Strain Effect	Vacuum Preloading
Hansbo (1997)	No	Yes	No	No
Fox et al. (2003)	Yes	No	Yes	No
Indraratna et al. (2005)	Yes	No	No	No
Sathananthan and Indraratna (2006)	No	Yes	No	No
Walker et al. (2012)	Yes	Yes	No	Yes
Kianfar et al. (2013)	No	Yes	No	Yes
Hu et al. (2014)	Yes	No	Yes	No

As previously stated, currently there are several numerical studies that examines PVD/vacuum consolidation together with large strain via Modified Cam-Clay model. To the author's knowledge, there has not been a numerical simulation study that is able to validate Walker's proposed solutions for non-linear vacuum consolidation, or able to include the combination of varying permeability/compressibility

and non-Darcian flow. This is likely due to the novel nature and the complexities of simulating several coupling processes together with non-linear governing equation of consolidation.

5. Results: Nonuniform Pore Pressure and Vacuum Consolidation Theory

In this chapter, theoretical compatibilities of available vacuum consolidation theories to incorporating nonuniform stress and pore pressure distributions in the soil is examined using the same numerical modelling methods deployed in the previous chapter for PVD consolidation. The currently available methods for developing numerical models and procedures to simulate vacuum consolidation are not well defined and largely left to interpretation, which many researchers opted to resort to applying the same conventional unit cell vertical drain consolidation procedures used for simulating PVD consolidation problems. The recent numerical studies on vacuum consolidations also considered finite embankment, semi-infinite domain, and multiple drains to offer a more realistic picture of consolidation under preloading embankment due the added capabilities of determining lateral strain and shear strain in the soil at any location below the embankment, which makes the simulation of lateral deformation at the toe of embankment, or tension cracks around the vertical drains possible. In contrast, application of traditional unit cell consolidation models is limited to vertical strain below the center of the embankment. However, most of the vacuum consolidation models largely still follows the conventional radial consolidation theory as applied by the likes of Chai and Indraratna. Therefore, the limitations of conventional radial consolidation theory and its inherent incompatibility with nonuniform pore pressure distribution in a fine-grained soft soil, as shown in Section 4, also applies to the vacuum consolidation solutions presented by the likes of Chai and Indraratna. Many of the previous discussions on the coefficient of consolidation, evolving permeability and compressibility of the soil that were brought up for PVD consolidation in Section 4, are also valid concerns for vacuum consolidation solutions.

In this chapter, several numerical models of vacuum consolidation are developed in the framework described by the unit cell theory of radial consolidation, and the theoretical impact of nonuniform stress and excess pore pressure distribution in the unit cell on the vacuum consolidation process is examined.

Because there is no widely accepted industry standard method of solution dedicated to the two most common methods of vacuum assisted consolidation (surface sealing sheet method, capped PVD method) found in literature at this time, two currently well known proposed analytical solutions, from the likes of Indraratna and Chai, are used as reference in this study.

5.1 Vacuum Consolidation Solutions Verification Models

Close form analytical solution of vacuum assisted consolidation remains an ongoing topic of study. For the application of vacuum assisted consolidation, currently there exists several methods. Currently the two common methods:

1) Surface sealing sheet method combined with PVD and preloading embankment
2) Capped PVD method combined with preloading embankment

Previously it was shown that the current available close form solutions for vacuum assisted consolidation of a cylindrical unit cell, from the likes of Chai and Indraratna, are an implicit solution of the conventional radial consolidation solutions from Barron, Hansbo and others, which have been widely applied to the PVD and preloading method. The main differentiation for vacuum consolidation solutions from the conventional PVD solution is how the applied vacuum pressure is treated in the conceptual models of consolidation, and consequently, in the numerical models of vacuum consolidation as well.

Current conceptual model of vacuum consolidation shares a central theoretical assumption that a uniformly applied vacuum pressure can be converted to an equivalent magnitude surcharge in the soil layer where the vacuum suction propagates to. This vacuum to equivalent surcharge conceptual model was ubiquitous in the studies of vacuum consolidation to the present day, as early researchers examined vacuum consolidation as an extension of the cylindrical unit cell theory of radial consolidation of Barron and Hansbo (Indraratna et al. 2005a; Chai et al. 2006; Rujikiatkamjorn et al. 2008).

To validate the accuracy of the coupled poroelastic model developed in FLAC for the purpose of simulating vacuum assisted consolidation, three conceptual models of vacuum assisted consolidation are simulated, each utilizing a different method of applying vacuum pressure, thus consequently the analytical solutions are unique to their respective conceptual model of vacuum consolidation.

5.1.1 Indraratna Solution for Vacuum Assisted PVD Consolidation

In Indraratna's conceptual model (surface sealing sheet + PVDs) of vacuum assisted consolidation, applied vacuum pressure at the top of the soil column and is assumed to be uniform under the sealing sheet, which can be converted to an equivalent surcharge in the improved soil, a conventional assumption for vacuum consolidation in literature. Additionally, Indraratna proposes that the PVDs installed in the soil will act as conduits for further vacuum pressure propagation along the length of the PVDs. In another word, Indraratna's conceptual analytical model for vacuum consolidation interprets applied vacuum pressure (surface sealing sheet method) as an equivalent surcharge on the soil, while vacuum pressure propagation along the PVDs were interpreted as negative pressure.

The cylindrical unit cell theory is maintained in Indraratna's conceptual model, and thus the coupled numerical model is once again developed in 3-D axisymmetric domain for a single PVD located at its central axis. The axisymmetric domain is discretized into a 30x75 grid mesh, which consists of 31 horizontal (radial) grid points and 76 vertical (axial) grid points. The model domain has a dimension of 0.677m x 2.4m, which resembles the cylindrical unit cell in the analytical solution. However, the analytical solution model has to account for the equivalent radius of the PVD, which in this case is assumed to be 0.033m according to Hansbo's equivalent drain radius. Therefore, the dimension of the cylindrical unit cell in Indrartna's solution is taken to be 0.7m x 2.4m.

Roller boundaries (constraint laterally) were set up at the two sides of the domain. Fluid drainage is constrained to radial direction only, vertical flow in the soil is neglected. Discharge capacity/well

resistance is also neglected. The presence of low permeability smear zones in the soil adjacent to the PVD was previous examined in the validation model for PVD consolidation, therefore smear zone is omitted here. The porous media in the cylindrical domain is homogeneous and anisotropic (only non-zero lateral permeability to enforce radial flow), soft marine clay, with a porosity of 0.5, drained Poisson's Ratio (υ') is 0.4, and drained Young's modulus (E') equal to 9.36 Mpa. The horizontal hydraulic conductivity (k_h) of the soft clay soil is set to 1x10^{-10}m/s.

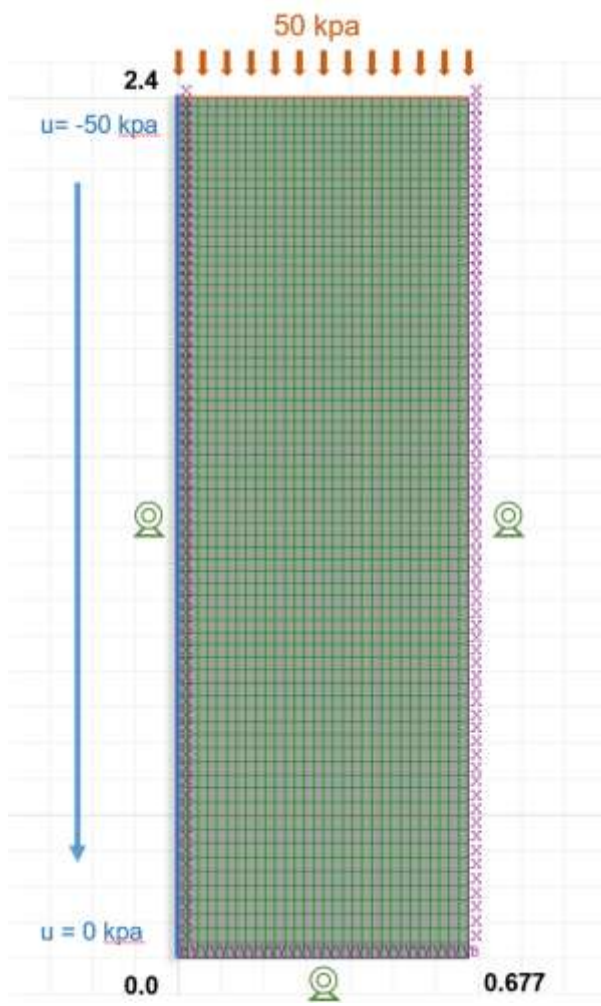

Figure 31. Verification Model: Indraratna's "Long-Drain" Vacuum Consolidation

The vacuum suction is set to 50 kPa. According to Indraratna's conceptual model, the applied vacuum pressure at the top boundary of the cylindrical unit cell is then converted to an equivalent surcharge of

50 kPa in the soil, and consequently, the initial excess porewater pressure in the soil prior to any drainage is 50 kPa throughout the unit cell. For a 2.4m long PVD in this example, vacuum pressure propagation is assumed to be linear and following Indraratna's "long-drain" hypothesis, where the vacuum pressure is 50 kPa at the top of the PVD and linearly decreasing until 0 kPa at the bottom of the PVD. For the FLAC numerical model, the assumption for vacuum pressure to equivalent surcharge preloading conversion is simulated by simply setting up an apply pressure of 50 kPa at the top of the unit cell domain. During the initial undrained phase, the porewater pressure is set to 50 kPa at every grid point in the domain, and the initial total vertical stress is set to -50 kPa at every grid points. Together with the apply boundary pressure and defined boundary conditions, the initial pore pressure and stress definitions at the undrained phase forms an equilibrium for the poroealstic soil in the domain, where the initial effective vertical stress is 0 kPa and thus bypassing immediate settlements. Vacuum pressure propagation is introduced by setting a constant boundary condition at the central axis grid points to representing the negative pressure created by vacuum suction. The porewater pressure along the central axis grid points are defined as 50 kPa at the top of the domain, decreasing linearly to 0 kPa at the bottom of the domain. These pore pressure values are then fixed for the entire duration of consolidation to simulate a constant applied vacuum presssure propagating along the length of the PVDs.

The coupled numerical model is allowed to consolidate to 1160 days, during which, average porewater pressure in the domain is obtained by numerical integration (2-D composite trapezoid method, using the same discretized mesh zones) and compared with the result of Indraratna's solution.

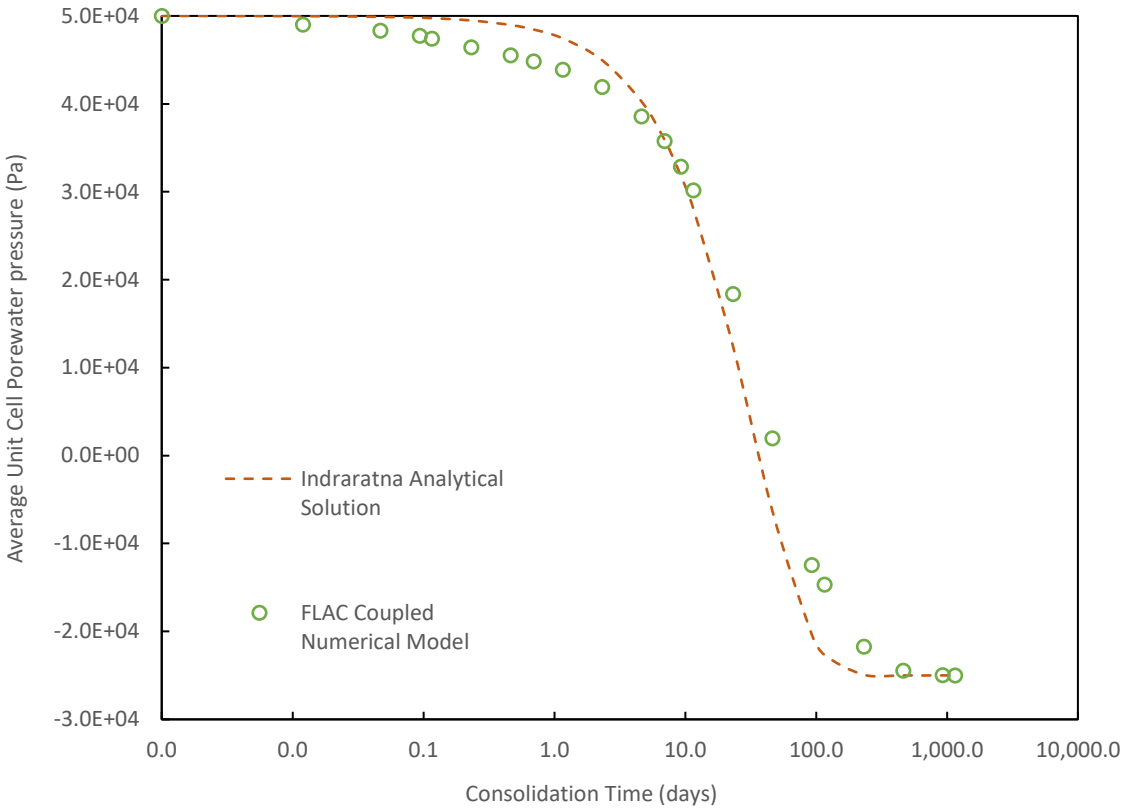

Figure 32. Vacuum Consolidation Verification: Average Unit Cell Pore Pressure Indraranta versus FLAC Numerical Simulated

The fully coupled poroelastic model is able to reasonably simulate Indraratna's model of vacuum assisted PVD consolidation. To the authors knowledge, there is no widely agreed upon way to represent degree of consolidation for a vacuum assisted consolidation at this time. This could largely be due to the fact that the current literature do not offer a sufficiently rigorous examination of vacuum induced negative porewater pressure gradients. Indraratna opted to present the average porewater pressure over the entire unit cell domain (Indraratna et al. 2005a).

Operating within the framework of cylindrical unit cell radial consolidation theory (which Indraratna's solution is derived from), the pore pressure distribution in the unit cell domain at very large consolidation time should ultimately be the unit cell in steady state with the boundary value. In the numerical model, it is shown that the ultimately steady state pore pressure distribution is reached when

the pore pressure in the unit cell is equal to the boundary condition (absent of any other drainage boundaries, according to unit cell theory of consolidation). Taking this steady state pore pressure distribution as the ultimate state of consolidation in the unit cell, it is possible to form a normalized degree of consolidation, where the negative porewater pressure zones are still considered in the consolidation process in reaching the final steady state pore pressure distribution. Interestingly, forming normalized degree of consolidation for Indraratna's conceptual of vacuum assisted PVD consolidation and analytical solution, showed a striking similarity to earlier solutions for unit cell radial consolidation via PVD only, most notably, Hansbo's solution.

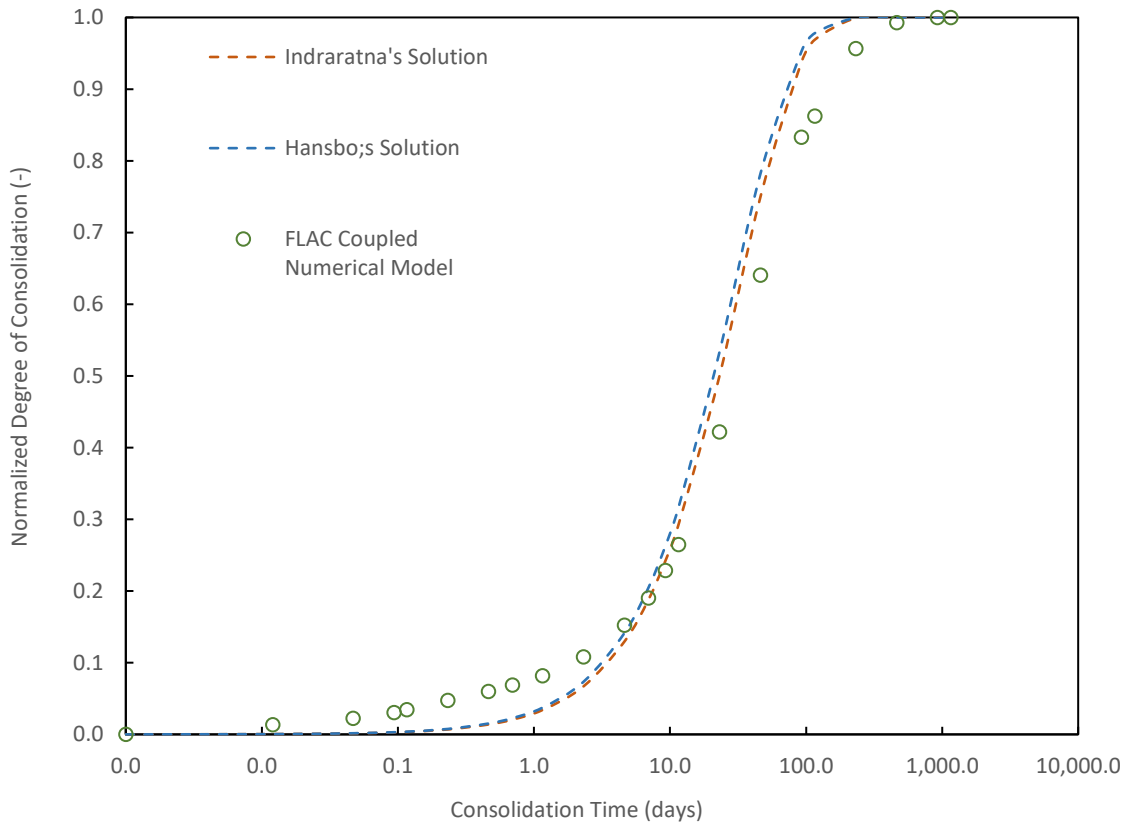

Figure 33. Model Verification: Normalized Degree of Consolidation Indraratna's Solution, Hansbo's Solution and FLAC Numerical Simulated

Under normalized degree of consolidation, the similarity between the numerical simulated results, Indraratna's solution for vacuum assisted PVD consolidation and Hansbo's solution for PVD

consolidation is quite apparent. This shows that Indraratna's conceptual model and analytical solution for vacuum assisted consolidation is very similar to Hansbo's solution for PVD consolidation, because both solutions are following unit cell radial consolidation theory developed by Barron. And the "semi-coupled" nature of these solutions is very similar to and can be reasonably approximated by a linear poroelastic model. The process of radial consolidation is ultimately governed by the radial coefficient of consolidation (C_h). The presence of vacuum pressure (as negative pore pressure boundary) only affects the pore pressure gradient and the final steady state pore pressure distribution in the unit cell domain and does not affect the rate of consolidation. This is the reason why Indraratna's vacuum consolidation solution and Hansbo's PVD consolidation solution have the very similar rate of consolidation, when applied to the same domain.

5.1.2 Chai Solution for Vacuum Consolidation via Capped PVD

The capped PVD (CPVD) method of vacuum consolidation (Chai et al. 2010) simply assumes that a uniform vacuum suction is applied directly through the length of the CPVDs, or through sections of the CPVDs, and into the soil layer(s).

Chai's proposed close form analytical solution for vacuum consolidation via CPVD describes the distribution of porewater pressure in the unit cell domain at any time duration consolidation as the sum of two components: an initial transient component; and an ultimate steady state component. The steady state component represents the ultimate vacuum pressure distribution in the subsurface, and it's determined with consideration for nearby porewater pressure boundary conditions (as evident by the conceptual one-way and two-way drainage hypothesis) at very large consolidation time. Therefore, even though Chai's solution is also derived from unit cell radial consolidation theory, same as Indraratna's solution, the addition of the steady state solution in Chai's conceptual model of vacuum consolidation takes into account the applied vacuum suction as a negative boundary condition and examine how the

vacuum pressure form steady state with the other boundary conditions in the unit cell domain. Meanwhile, the transient component of Chai's solution is very similar to Indraratna's solution. Chai makes the general assumptions that the rate of porepwater dissipation/drainage is not affected by the magnitude of applied vacuum pressure (negative pore pressure). Much like Indraratna, Chai's conceptual model for CPVD is simply extending the unit cell radial consolidation theory by incorporating the negative pressure induced by vacuum suction as a boundary condition. Consequently, the average coefficient of radial consolidation is a property of the porous medium and remains constant regardless of the magnitude of applied vacuum pressure. This assumption is in contrast with the findings presented by the likes of Bergado and Saowapakpiboon, whose conceptual models of vacuum consolidation were developed from observed settlements in laboratory bench tests, and found vacuum pressure increases the rate of consolidation compared to the conventional PVD and surcharge only setup.

The transient component in Chai's CPVD solution also assumes the conversion between the negative pressures from vacuum suction, to an equivalent surcharge in the soil, in the form of an uniform initial excess porewater pressure at the start of the consolidation.

Three scenarios of vacuum consolidation are simulated: vacuum pressure in CPVD only with no preload; preload surcharge with conventional PVD; and vacuum pressure in CPVD with preloading surcharge.

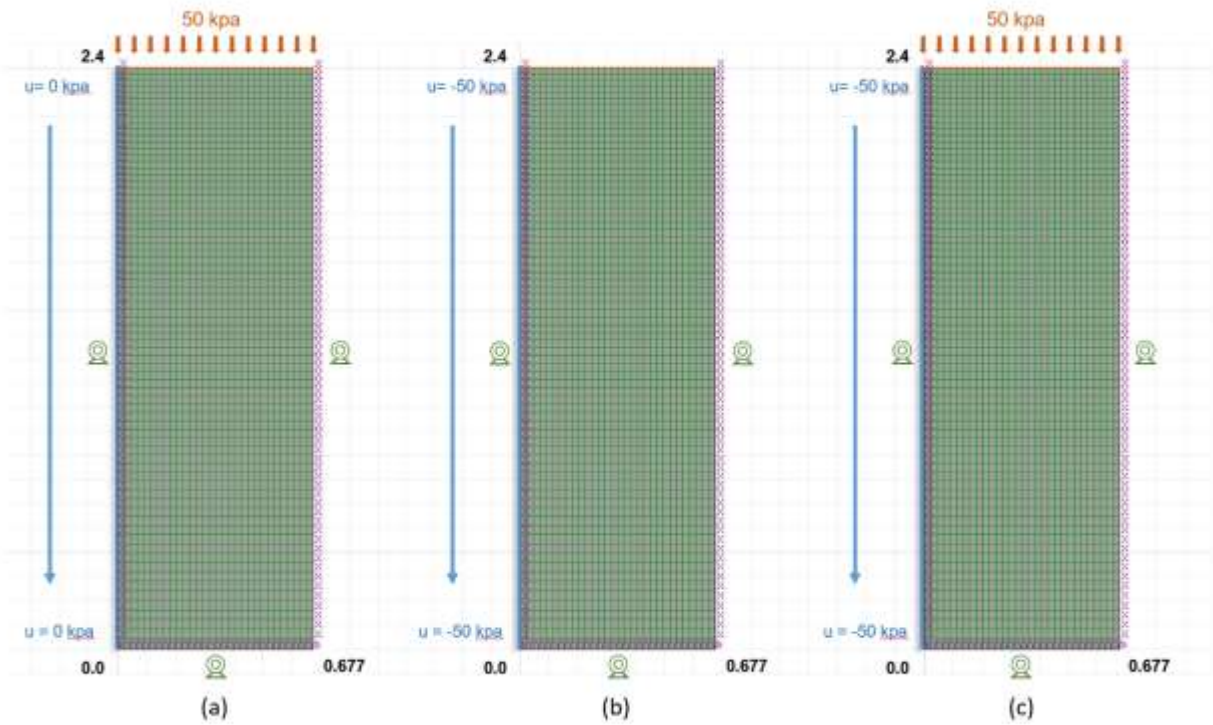

Figure 34. Model Verification: Conceptual Models Representing 3 Methods of Vacuum Consolidation

The vacuum pressure being applied in the CPVD is set to -50 kPa. In scenario (a) and (c) 50 kPa of preloading in the form of surcharge is applied to the soil in the unit cell domain. Scenario (a) is the conventional PVD and preload surcharge case examined earlier. According to Chai's conceptual model, no porewater recharge is entering the unit cell, and consolidation would develop in a dewatering scenario. The goal of the verification is to demonstrate the similarities in the rate of consolidation in the transient solution, regardless of applied vacuum pressure, which is an inherent limitation of the current unit cell theory of consolidation, particularly when applied to analyze vacuum consolidation.

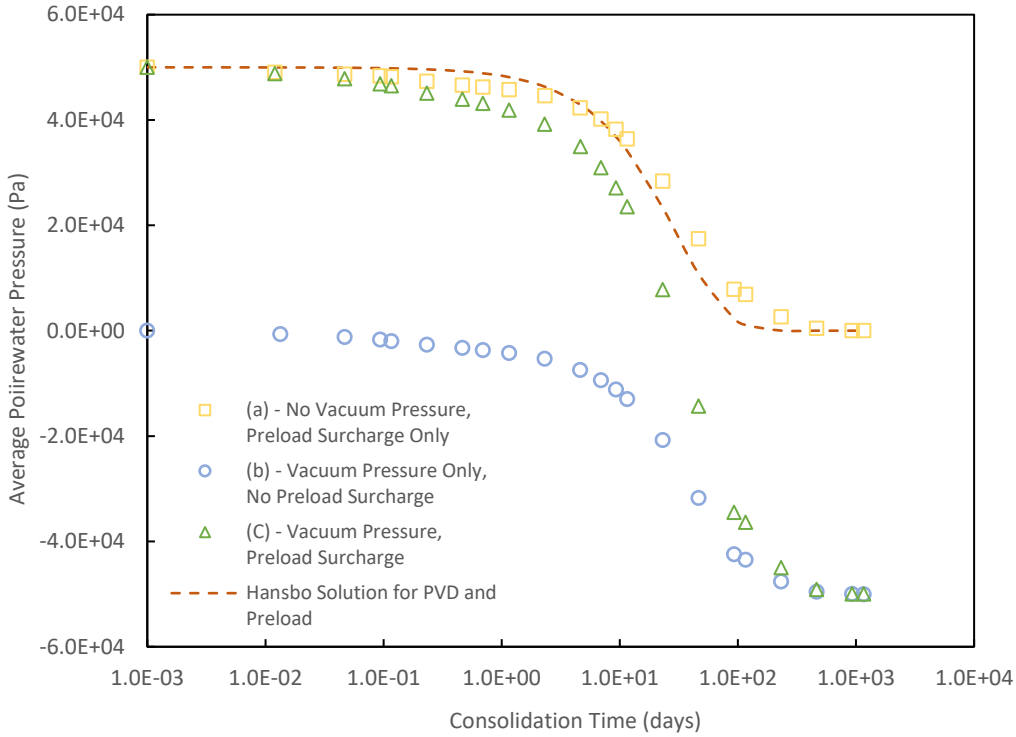

Figure 35. PVD and Vacuum Consolidation Verification: Average Unit Cell Pore Pressure FLAC Numerical Simulated

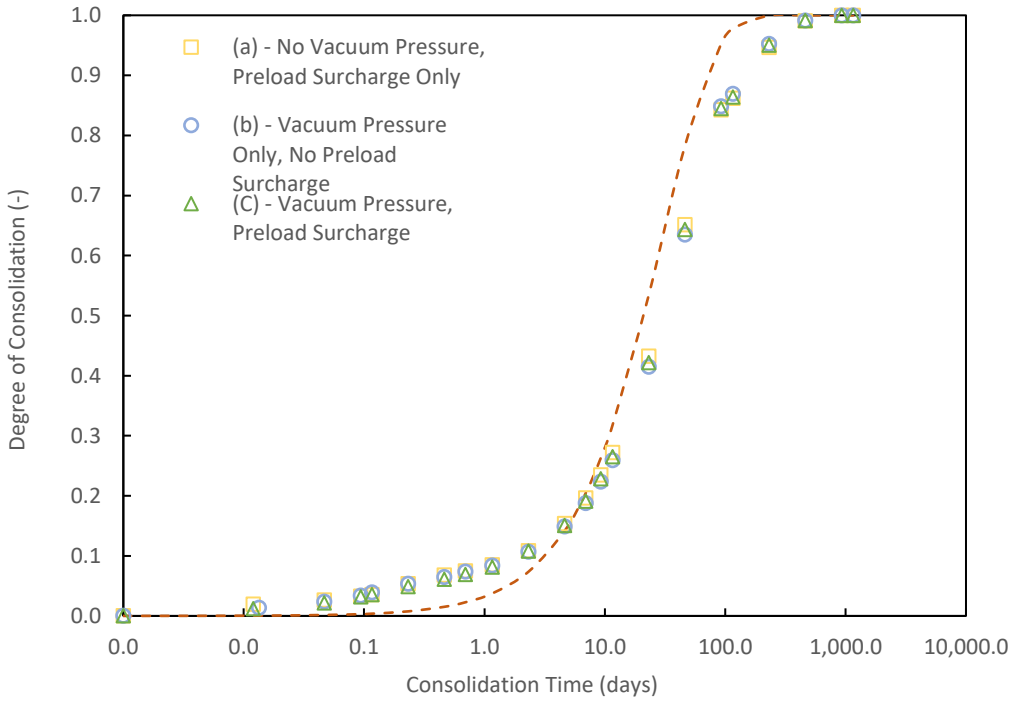

Figure 36. PVD and Vacuum Consolidation Verification: Degree of Consolidation FLAC Numerical Simulated

5.2 Model 4: Unit Cell Vacuum Consolidation With Nonuniform Pore Pressure Distribution

The purpose of model 4 is to examine nonuniform pore pressure distribution in the process of vacuum consolidation theory and compatibility with the analytical solution presented by Indraratna. Model 4 is a cylindrical unit cell model with a radius of 0.677m and length (depth) extending to 2.4m below the center of the embankment. The unit cell is made up of the same homogeneous soft clay type soil as the footing model in Section 4.2, the properties of which are given in Table 1 and the soil is assumed to exhibit poroelastic properties under consolidation. The soft clay type soil is assigned a horizontal hydraulic conductivity value of $1x10^{-10}$m/s. A single PVD is placed at the center (axial) of the cylindrical unit cell where it provides the only pathway for porewater to dissipates to, thus the pore pressure is limited to flow in the radial direction.

Model 4 differs from the previous unit cell models of consolidation by having a theoretical "embankment preloading stress" converted from the vacuum pressure applied to the top of the unit cell. For this example, the applied vacuum suction under the sealing sheet is a constant -50 kPa throughout the consolidation process. According to Indraratna's conceptual model, this vacuum pressure converts to an equivalent 50 kPa of surcharge stress and excess pore pressure uniformly distributed to every point in the unit cell. Therefore, if one is to follow Indraratna's conceptual model, and solution of vacuum consolidation, or the method of converting vacuum pressure to surcharge in general, it essentially means the previously established concepts for the embankment induced nonuniform excess pore pressure distribution in the unit cell is incompatible with the current model of vacuum consolidation. And to the author's knowledge, there is no available full-scale field study on the equivalency of stress and pore pressure distribution in the soil as a result of vacuum suction (as negative pressure applied to the top of the soil) and to that of results from preloading by an embankment of finite dimension at the top of the soil. Because of the adoption of the vacuum pressure to equivalent

surcharge assumption, Indraratna's proposed model of vacuum consolidation is limited to the examination of uniform surcharge in the soil. It is evident that there is a theoretical incompatibility between Indraratna's model of vacuum consolidation and that of the previously established nonuniform stress and pore pressure distribution below the embankment serving as preload to the soil. Recall the inclusion of negative pressure propagation with depth along the PVD. In Indraratna's model of vacuum consolidation, this is represented by a linear distribution of negative pore pressure boundary condition (according Indraratna's function K) along the PVD at the center of the unit cell. The linear distribution of negative pore pressure at the boundary condition essentially leads to nonuniform pore pressure distribution because it causes differential horizontal pore pressure gradients with depth during the consolidation process. Therefore, the variable gradient with depth will eventually lead to nonuniform pore pressure distribution in the unit cell as more porewater begins to dissipate toward the central PVD, even though a uniform surcharge of excess pore pressure is assumed at the start of consolidation. Indraratna believes that due to the presence of the negative pore pressure boundary and the differential gradients that it causes, consolidation in the unit cell should be theoretically faster than that of conventional PVD, without the vacuum pressure propagation. However, as established in the previous chapter, under the framework of unit cell radial consolidation theory, differential horizontal hydraulic gradients do not always lead to change in the rate of the transient consolidation.

Model 4 consists of an axisymmetric domain discretized into a 31x76 grid. Roller boundaries are placed at the two sides of the domain, and the bottom of the domain is fixed. A single PVD is placed at the center (axial) of the domain, where the pore pressure is fixed at -50 kPa at the top of the PVD and decreases linearly to 0 kPa at the bottom of the domain, according to Indraratna's "long-drain" hypothesis. Radial flow only condition is simulated by assigning anisotropic permeability to the soil, where the only non-zero value is assigned to the horizontal (radial) permeability. Figure 39 shows the domain, boundary, and mesh of Model 4.

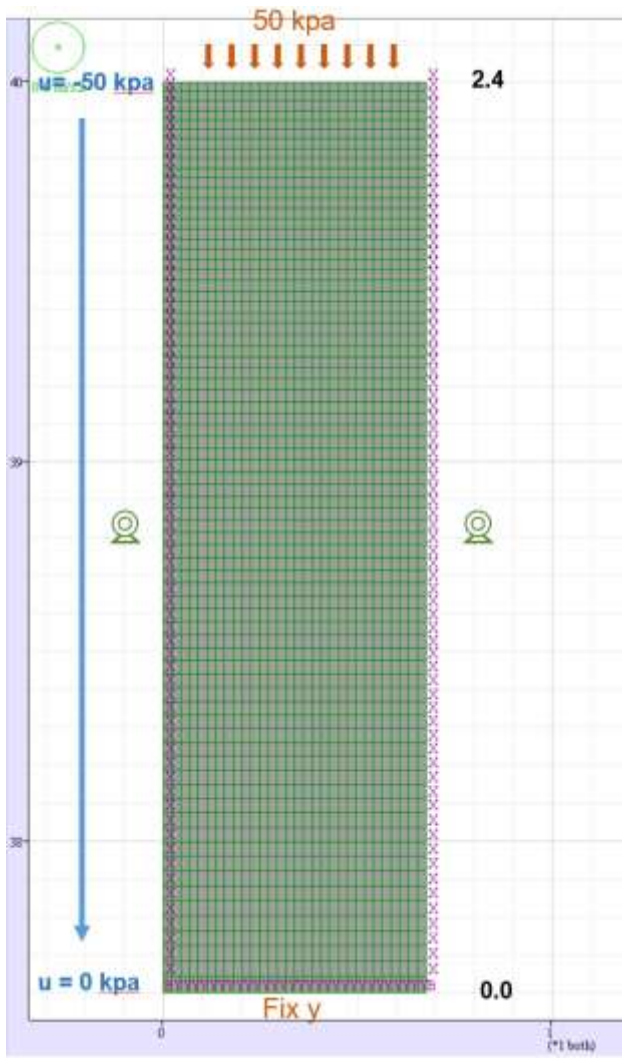

Figure 37. Model 4: Vacuum Consolidation Unit Cell Model Domain, Boundary and Mesh

At the start of the drained phase, pore pressure in the unit cell is assumed to be a uniform 50 kPa, as converted from vacuum pressure to an equivalent surcharge of excess pore pressure. The initial pore pressure surcharge in the domain is set up by applying an equivalent 50 kPa of compressive stress to the top of the domain, while the initial undrained pore pressure in the unit cell is set to 50 kPa, and the initial total vertical stress in the domain is set to -50 kPa. During consolidation, the unit cell is assumed to be constraint to uniaxial compression when the excess pore pressure dissipates towards the central PVD. The model simulates the coupled consolidation process for a total of 4,600 days of consolidation time. The average pore pressure over the entire cylindrical domain, along with the total vertical

settlement at the ground surface are recorded for the entire duration of consolidation. The transient pore pressure during vacuum consolidation, in the form of average pore pressure in the cylindrical domain, is shown in Figure 38.

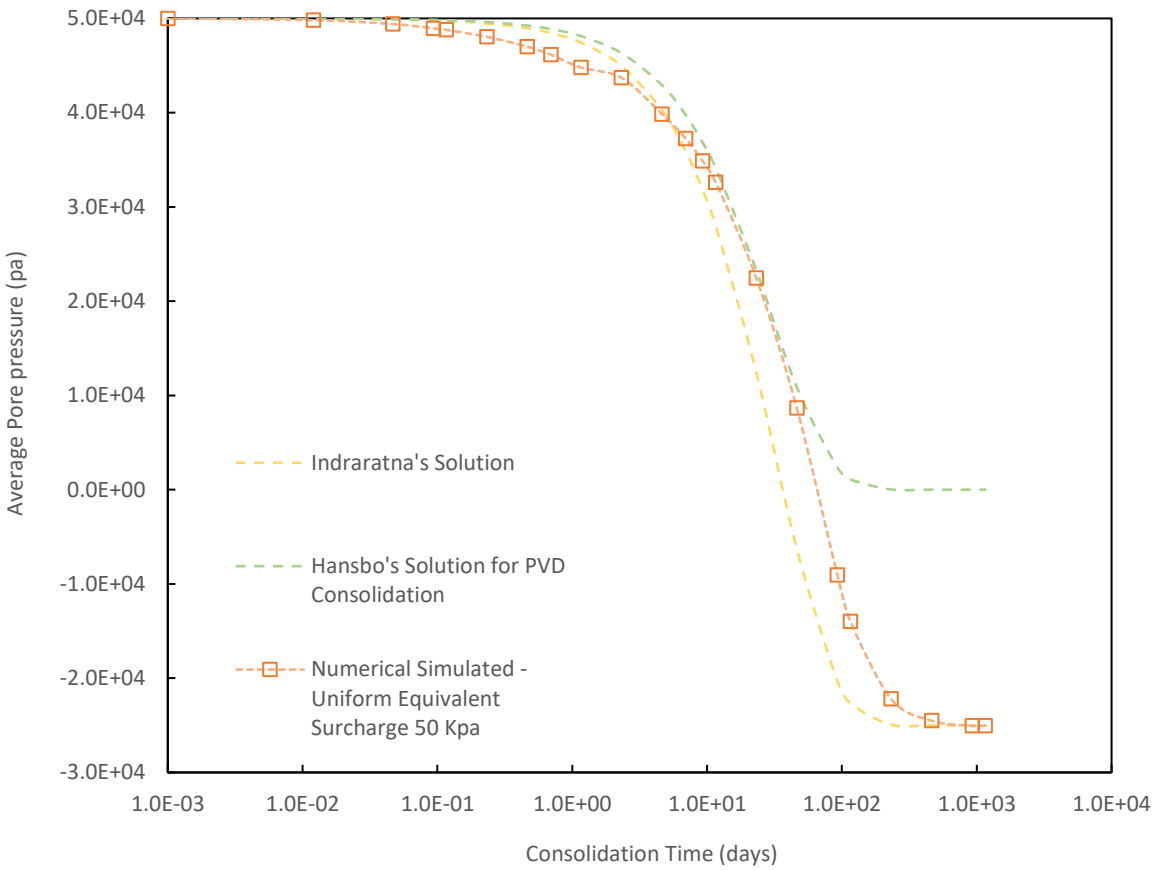

Figure 38. Model 4 Vacuum Consolidation: Average Pore Pressure in the Unit Cell - Indraratna's Solution versus FLAC Numerical Simulated

The fully coupled unit cell model of vacuum consolidation with poroelastic soil properties produces a reasonable match with Indraratna's solution in terms of average pore pressure in the unit cell during consolidation. When the result of vacuum consolidation is compared to Hansbo's solution for PVD consolidation, it is evident that when negative pore pressure boundary condition is present in the domain, the final steady state pore pressure distribution in the unit cell will be a negative value, compared to 0 kPa in the case of PVD consolidation. Despite this fact, when the transient process of consolidation, as determined through the average pore pressure in the unit cell, is translated into the

degree of consolidation, which essentially represents the rate of the consolidation. Once again, it is evident that Indraratna's model of vacuum consolidation produces identical degree of consolidation as Hansbo's model for a PVD consolidation. In another word, regardless of the vacuum pressure propagation along the PVD, the vacuum consolidation model takes the exact same amount of time to reach 100% consolidation as the PVD consolidation model. The inclusion of the negative pore pressure in the PVD has no impact on the rate of consolidation, instead, only the final steady state solution accounts for the negative pore pressure. This result is unsurprising, because as previously established, unit cell radial consolidation theory is inherently limited by the assumption of constant permeability and compressibility during consolidation. The concept of negative pore pressure to represent vacuum suction only serves to further highlight the limitation of unit cell consolidation theory, which the currently available vacuum consolidation solutions are derived from, therefore they are not able to fully incorporated negative pore pressure boundaries into the solution, for the same reasons that nonuniform pore pressure distribution can not be properly addressed.

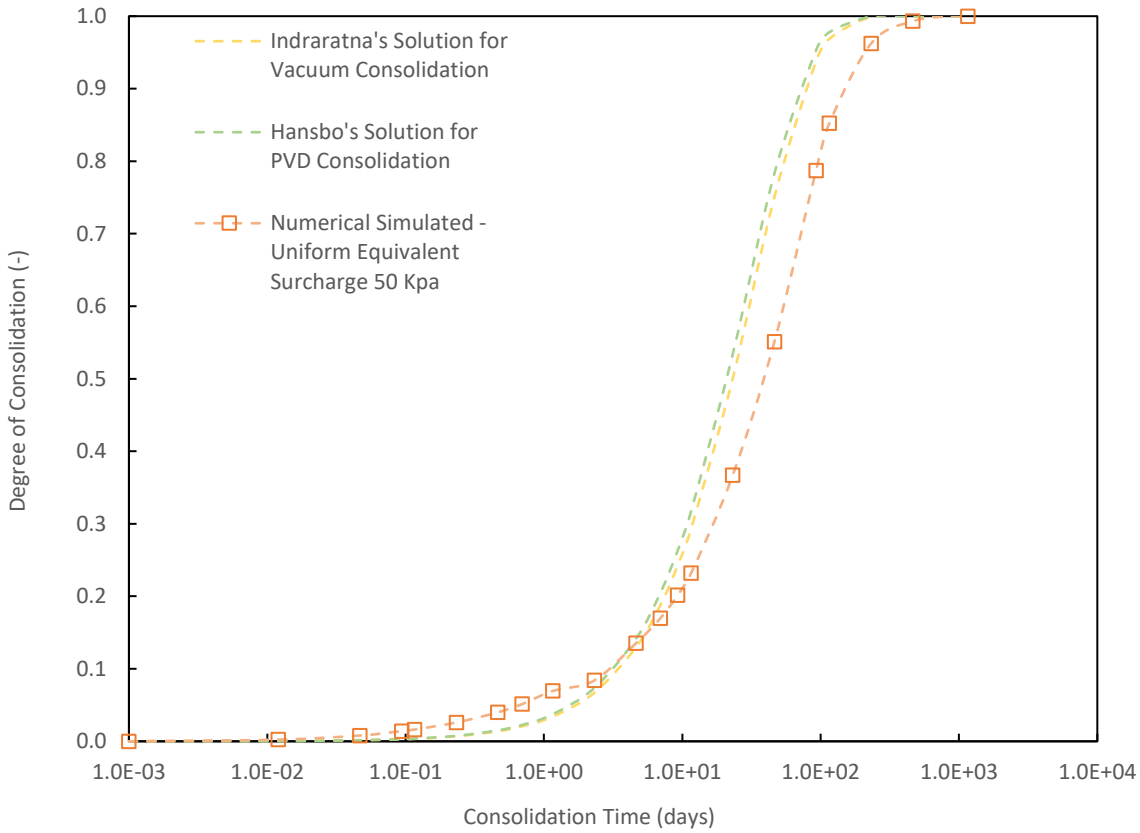

Figure 39. Model 4 Vacuum Consolidation: Degree of Consolidation - Indraratna's Solution versus FLAC Numerical Simulated

In Model 4, the result of Indraratna's model of vacuum consolidation at very large consolidation time reaches steady state average pore pressure of -25 kPa in the domain. It is evident that Indraratna's solution (as well as Chai's solution) accounts for the negative pore pressure as part of the transient solution to vacuum consolidation. The theoretical justification for negative pore pressure in consolidation is not often discussed in the literature, even though this concept has often been presented in vacuum consolidation studies. Consider that the steady state negative pore pressure distribution in the soil has never been observed in case studies of vacuum consolidation. There are also apparent theoretical incompatibilities between the concept of negative pore pressure zone in the soil and the unit cell radial consolidation theory that Indraratna's solution is based on. In theory, unit cell model of consolidation is only applicable to a single PVD unit cell below the center of the embankment, and the assumption is that the unit cell in question is located at the center of a larger field of PVDs (and

unit cells) surround it. Therefore, one can assume the unit cell has only one pore pressure boundary condition situated at the center to represent the PVD/vacuum drain, while every other boundary is no-flow, because the assumption is that at the outer boundary of the PVD, porewater diffusion will be toward the PVD of an adjacent unit cell. Hence, there is no porewater flowing into the unit cell as drainage occurs, and the consolidation takes place throughout the entire unit cell domain as it is dewatered by the central PVD. This is the theoretical assumption of unit cell consolidation. However when negative pore pressure boundary condition is introduced by the vacuum consolidation models, the presence of negative pore pressure in the unit cell is not theoretically justified, because once pore pressure in the soil has dissipated to 0 kPa and unsaturated condition has been reached, the effective stress of the soil will have reached its maximum value, therefore the inclusion of further negative pore pressure build up in the unit cell theoretically does not translate into further consolidation settlement, to the same degree as drainage pore pressure would lead to consolidation. To the author's knowledge, there has not been case study that examines negative pore pressure in vacuum consolidation, and it remains unclear whether the negative pore pressure component seen in Indraratna's solution contributes to further consolidation, if one is to follow the framework set out by the unit cell theory of radial consolidation.

5.3 Model 5: Unit Cell Vacuum Consolidation – Omitting Negative Pore Pressure Zones

Consider the same unit cell model of vacuum consolidation presented in Section 5.2. By omitting the negative pore pressure zones that forms in the unit cell during vacuum consolidation, the theoretical impact of negative pore pressure on the degree of consolidation in the unit cell can be seen. A simple theoretical justification for omitting the negative pore pressure zones in the unit cell is the fact that the unit cell is being dewatered during consolidation, the maximum effective stress in the soil is reached when the porewater has completely drained and degree of saturation in the soil is zero. Therefore,

when the pore pressure in an area in the unit cell has reached 0 kPa, the region of the unit cell would be considered 100% consolidated. In this example, by numerically simulating vacuum consolidation and omitting the negative pore pressure zones in the calculation of degree of consolidation, the numerical model essentially normalizes the transient portion of the vacuum consolidation solution between the initial pore pressure (which is assumed as uniform surcharge in the soil) in the soil, and the completely dewatered soil (where pore pressure is 0 kPa).

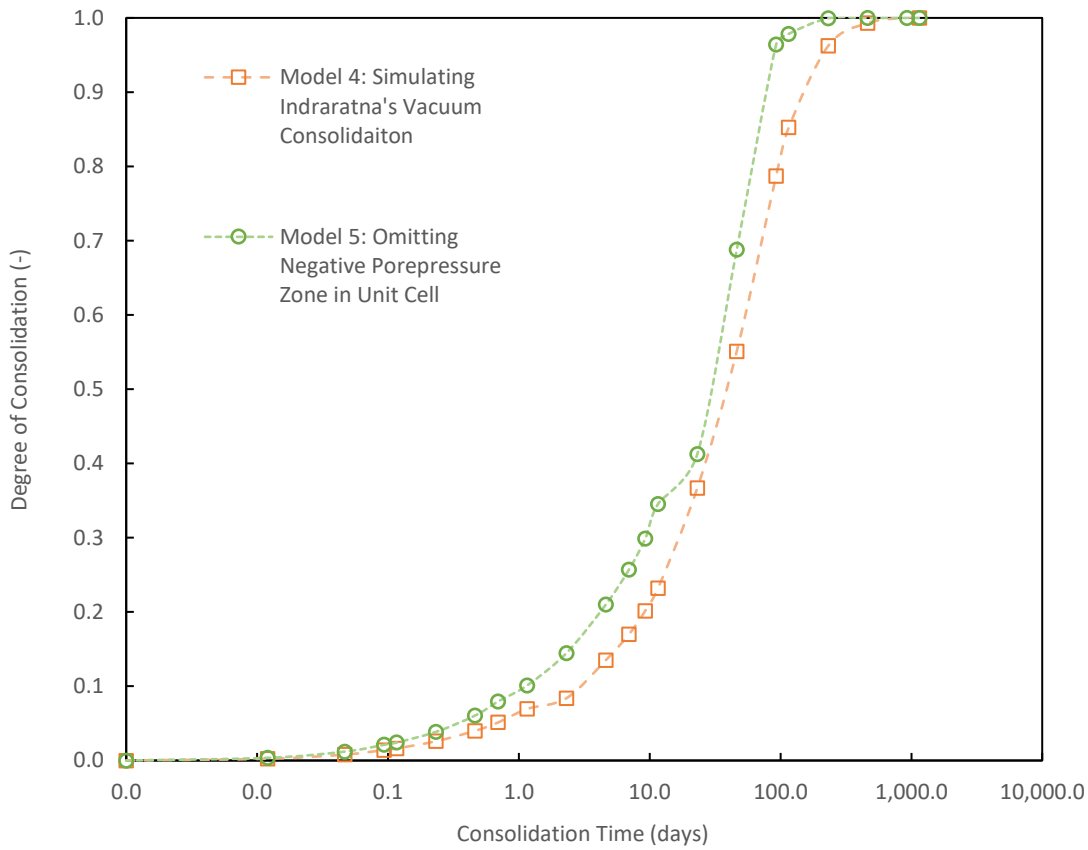

Figure 40. Model 5: Vacuum Consolidation - Degree of Consolidation Omitting Negative Pore Pressure Zones in the Unit Cell

Figure 40 compares the degree of consolidation of Model 4 and Model 5. Recall that Model 4 is developed to simulate Indraratna's solution for vacuum consolidation, and Model 5 have identical properties as Model 4, with the exception that Model 5 omits negative pore pressure in the average

pore pressure calculations. Model 5 produced noticeably faster degree of consolidation compared to Model 4, which corresponds to an average of 36% faster rate of consolidation.

6. Conclusion and Recommendations

6.1 Research Summary

Utilizing Biot's type hydro-mechanical coupled numerical models of consolidation, the theoretical justifications for nonuniform stress and pore pressure distributions in a fine-grained soft soil deposit under preloading by an embankment of finite dimension is presented. The Biot's type fully coupled model has an axisymmetric semi-infinite domain, and a circular embankment of finite dimension is placed above the soil thus provides the preloading stress to the soil domain. The conceptual model is set up to resemble a typical ground improvement project for a soft marine clay deposit applying PVDs and preloading embankment to speed up the process of consolidation in the soil. The embankment induced excess stress and porewater pressure in the soil is simulated for the undrained stage, prior to the start of consolidation, and found to be both exponentially decreasing with depth under the embankment. It is hypothesized that the undrained pore pressure and initial stress state in the soil is a combination of embankment induced excess pore pressure, and the hydrostatic pore pressure in the soil.

A vertical profile of the initial undrained pore pressure and stress distribution in the soil below the center of the embankment is extracted from the footing model and serves as the initial condition for fully coupled cylindrical unit cell models of radial consolidation. In Section 4 of the study, a unit cell radial consolidation model is developed for an axisymmetric domain and simulates a single PVD cylindrical unit cell conceptual model presented by Hansbo's closed form solution for PVD consolidation, which itself is derived from the popular "equal-strain" theory of unit cell radial consolidation presented by Barron. However, the inclusion of the nonuniform undrained pore pressure distribution in a unit cell below the center of the embankment provide a noticeable deviation from the assumption of uniform

surcharge of stress and excess pore pressure in the unit cell, which were present by the traditional consolidation theories. Otherwise, the unit cell numerical model behaves largely identical to that of the analytical model of radial consolidation and includes the following: a single PVD at the center of the axisymmetric domain; dissipation of pore pressure via radial flow towards the central PVD; deformation due to consolidation is constrained to uniaxial (vertical) settlement; the permeability and compressibility governs porewater dissipation and coefficient of consolidation are assumed to be material properties of the soil, and independent of consolidation. The numerical model simulates primary consolidation until the unit cell achieves 99% degree of consolidation, determined via pore pressure. The numerically simulated average pore pressure, and the vertical settlement in the unit cell is recorded at regular intervals during the consolidation process. The resultant average pore pressure and degree of consolidation is compared with that determined with Hansbo's analytical solution, and the impact of nonuniform pore pressure distributions, in the framework of unit cell radial consolidation theory is examined.

In Section 5 of the study, the impact of nonuniform pore pressure distribution is examined in the context of unit cell vacuum consolidation theories. Two common methods of applying vacuum assisted PVD consolidation are referenced: the surface sealing sheet method, with analytical solution proposed by Indraratna; and the Capped PVD (CPVD) method, with analytical solution proposed by Chai. The conceptual models of vacuum consolidation presented by both Indraratna, and Chai's analytical solutions are by and large extensions of unit cell radial consolidation theory. The coupled numerical model simulates unit cell vacuum assisted consolidation following the frameworks set out by Indraratna and Chai. Using the numerical modelling analysis, the conceptual models of vacuum consolidation is examined in the context of the unit cell radial consolidation framework. The impact of nonuniform initial undrained pore pressure distribution in the unit cell on the result of vacuum consolidation is examined.

6.2 Results

According to unit cell radial consolidation theories, proposed by the likes of Barron and Hansbo, primary consolidation is ultimately determined through the process of porewater pressure dissipation in the unit cell. During primary consolidation process, the pore pressure is limited to radial flow towards the central PVD, which effectively serves as a constant head boundary at 0 KPa. Therefore, according to unit cell theory, a fully consolidated soil deposit corresponds to the average pore pressure in the unit cell reaching steady state with the central PVD, which equates to an average pore pressure of 0 KPa in the unit cell.

Following the framework set out by unit cell radial consolidation theory, fitting nonuniform undrained pore pressure distributions into the unit cell resulted in an initial average pore pressure in the unit cell that could be either noticeably greater or less than the uniform surcharge assumption. This is demonstrated by Model 1 and 2, both are fully coupled poroelastic models of unit cell radial consolidation. Model 1 exhibits nonuniform pore pressure distribution below a 50 kPa preloading stress, at the undrained stage, prior to consolidation, it started with an average pore pressure of 58 kPa in the unit cell domain, a noticeably deviation from the surcharge assumption, which would have resulted in an average undrained pore pressure of 50 kPa in the domain. Model 2 also accounted for nonuniform pore pressure distribution in the unit cell domain. However, in this case, the unit cell domain is under 70 kPa of preloading stress, while the Model 2 simulates a shorter (shallower) 1.4m length of PVD unit cell, relative to the dimension of the preloading embankment. The initial undrained average pore pressure in Model 2 is 64.6 kPa prior to consolidation, noticeably less than the assumption for surcharge, which equate to an average pore pressure of 70 kPa.

If the transient solution of PVD consolidation is represented by the average pore pressure in unit cell still to be drained, then it appears that the unit cell with higher initial average pore pressure such as Model 1 will require more consolidation time. However, if one is to present the transient portion of pore

pressure dissipation (therefore primary consolidation according to Hansbo) in terms of degree of consolidation of the unit cell, it becomes apparent that despite the difference in the starting initial average pore pressure, both in terms of quantity and the spatial distribution of said pore pressure, ultimately, under unit cell radial consolidation theory, the rate of consolidation in the unit cell is independent of pore pressure. This is illustrated in Figure 30 and Figure 35, which shows the unit cell radial consolidation in terms of degree of consolidation compared to Hansbo's analytical solution, for Model 1 and Model 2, respectively.

The nonuniform pore pressure distribution in Model 1 and Model 2 serves to highlight the theoretical limitation of the conventional unit cell radial consolidation theory. In Hansbo's original solution, the process of radial pore pressure dissipation is governed through Darcy's Law, while the permeability of the soil is assumed to be a constant material property independent of the consolidation process. In addition, the "semi-coupled" nature of unit cell consolidation means the soil in the unit cell resembles a linear poroelastic porous media, whose material compressibility is also assumed to be constant throughout the consolidation process, and the stress-strain relationship of the soil is assumed to be small strain, or linear elastic. The inherit traits of unit cell theory means that the horizontal (radial) coefficient of consolidation (C_h) of the unit cell is a constant value that is independent of the consolidation process. This ultimately led to the two models resulting in identical degree of consolidation, regardless of the quantity and spatial distribution of the initial undrained pore pressure in each unit cell. Despite this theoretical limitation in the determination of degree of consolidation via average pore pressure, impact of nonuniform pore pressure, in terms of initial undrained pore pressure quantity and spatial distribution of said porepresuure in the unit cell, are instead reflected in the magnitude of total vertical settlement that forms in the unit cell during consolidation. Model 1 showed that a greater initial average pore pressure in the unit cell led to greater vertical settlement as a result of consolidation. For example, the nonuniform pore pressure distribution in Model 1 corresponds to an

initial average pore pressure of 58 kPa, compared to 50 kPa if one is to assume uniform surcharge, as per Hansbo's solution. The difference between the two starting average pore pressure is 16%, and the simulated consolidation model showed that the greater initial average pore pressure corresponded to 16% more vertical settlement that formed during consolidation. The same correlation between initial average pore pressure in the unit cell and ultimate vertical settlement due to consolidation can also be observed in Model 2, albeit this time the model parameter led to nonuniform pore pressure distribution in the unit having less initial average pore pressure in the unit cell compared to that of applying uniform surcharge assumption. Once again, the relative difference between the initial average pore pressure and the ultimate consolidation settlements is the same in Model 2. This is because Model 1 and Model 2 both are linear poroelastic models.

Model 1 and Model 2 showed that even though the framework of unit cell radial consolidation theory is inadequate for examining nonuniform pore pressure distributions in terms of average pore pressure and the degree of the consolidation, the discrepancy in the initial average pore pressure in the unit cell is ultimately still reflected by the differences in the magnitude of vertical settlement that formed as a result of consolidation.

The current governing equation of radial consolidation should be expanded to include the following considerations:

1) Non-Darcian flow in radial consolidation theory, a form of which was proposed by Hansbo in a follow up update to his original analytical solution.
2) Permeability and compressibility of the soil should not be constants throughout the process of consolidation, but instead evolves with respect to the void ratio of the soil during consolidation. The evolving permeability and compressibility at any time during consolidation can be represented by a function of the current void ratio, as proposed in Walker's study (Walker et al, 2012).

3) The consolidation model should account for large-strain properties of the soil and employ elasto-plastic models.

These considerations introduce nonlinearity to the current governing equation of consolidation. The author recognizes that in order for unit cell radial consolidation models (whether it's single PVD unit cell model, or multiple PVD unit cells model) to adequately incorporate the effect of nonuniform pore pressure distribution into the determination for rate of consolidation, the governing equation of radial consolidation has to be nonlinear in nature.

Model 3 demonstrates the importance of nonlinear governing equation of radial consolidation by including Modified Cam-Clay model of the soil into the consolidation model for the considerations large-strain deformations. The inclusion of Modified Cam-Clay model, which corresponds to one of the three nonlinear considerations shown above, has a noticeable impact on simulated consolidation results. Model 3 produced noticeably faster degree of consolidation compared to Model 2, even though the models have nearly identical initial average pore pressure. Figure 30 shows that because Model 3 utilizes a non-linear governing equation of consolidation, it is able to represent the bulk effects of nonuniform pore pressure distribution in conjunction with large strain deformations in the consolidation process. And through further additions of non-Darcian flow regime and the evolving permeability and compressibility during consolidation, additional nonlinearity is added to the governing equation of consolidation and allows for a more rigours examination of the impact nonuniform pore pressure distributions have on PVD consolidation theories.

An examination of current vacuum consolidation theories by numerical simulations showed the same theoretical incompatibility to nonuniform pore pressure distributions that exists in the PVD consolidation theories, are also present in the current vacuum consolidation theories. The current conceptual models of vacuum consolidation and the analytical solutions, presented by the likes of Indraratna and Chai, are found to be largely extensions of the unit cell radial consolidation theories of

Hansbo. By the addition of vacuum pressure propagation in the PVD(s) that varies with depth in the unit cell domain, Indraratna's vacuum consolidation solution further revealed the theoretical limitations of the unit cell radial consolidation theory. When applied to vacuum consolidation scenarios, this theoretical limitation remains largely the same as was shown for the PVD consolidation case, in that, the difference in the pore pressure gradient (whether it's due to spatial distribution of pore pressure in the unit cell, or the presence of a vacuum pressure boundary) in the unit cell is irrelevant to the degree of consolidation. Once again, this limitation can be attributed to the constant coefficient of consolidation, and the linear governing equation of consolidation that is at the basis of both Chai and Indraratna's solutions. This is demonstrated in Model 4 and in Figure 39, which shows that for the same unit cell model, numerical simulations for Indraratna's solution and Hansbo's solution ends up producing the same degree of consolidation, despite the presence of vacuum suction inside the central PVD.

6.3 Recommendations

The established unit cell consolidation theories are shown to be inadequate for the examination of nonuniform stress and pore pressure distribution in the unit cell. The theoretical incompatibility can be largely attributed to the linear governing equation of radial consolidation. In order for the governing equation of consolidation to include nonuniform stress and pore pressure in the consolidation process, the author recognizes the following factors that needs to be included in the conceptual model of unit cell consolidation:

1) Evolving permeability with respective to the void ratio of the consolidating soil
2) Evolving compressibility with respective to the void ratio of the consolidating soil
3) Non-Darcian flow governing porewater dissipation in soft clay soil with low permeability
4) Large strain elasto-plastic model in soft clay soil

In the past two decades, there has been on-going research to update the governing equation of radial consolidation, and recent publications from the likes of Walker, Indraratna, Hu, Lu (Lu et al, 2018) and

others have included several of the nonlinear consolidation concepts listed above in the study of PVD and vacuum assisted consolidation. A particular standout is Walker and Indraratna's proposed closed form analytical solution for unit cell vacuum consolidation which includes the evolving permeability and compressibility; as well as Non-Darcian flow scheme.

There is on-going research into numerical modeling of the nonlinear radial consolidation. To the author's knowledge, Indraratna is the first to present a numerical model of unit cell non-linear vacuum consolidation that is capable of incorporating the combination of varying permeability/compressibility and non-Darcian flow (Indraratna et al, 2017). Indraratna's proposed numerical model for unit cell non-linear vacuum consolidation appears to be a promising iteration for numerical solutions that is capable of examination nonuniform stress and pore pressure distribution in the unit cell. It remains to be seen if this type of unit cell vacuum consolidation model can be expanded to examine large scale multiple PVDs or vacuum drains consolidation scenario. And due to the novel nature of the nonlinear governing equation of consolidation and the complexities derived from the coupling process of several nonlinear material properties due to consolidation, PVD and vacuum assisted consolidation of soft soil remains a topic for on-going study at this time.

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