Unifying Thermal Big Bang and Black Holes

by

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Statement of Contributions

This MSc thesis has been entirely written by me, and has not been published elsewhere. Chapters 2 and 3 are mainly overviews of previous literature, while Chapter 4 is original research that first appear in this thesis. The research has been supervised by N. Afshordi, who has also edited the current text.

Abstract

Theories that stand the test of constant bombardment of new (and old) ideas, observations and unexplained phenomena are hard to come by. Despite countless attempts over many years of trying, cosmology and gravitational theories are of those lacking in a full unified description. A tachyacoustic model of the thermal big bang has been proposed which has a remarkable prediction for the scalar index parameter of primordial fluctuations. In this work we provide a brief review of the motivations leading up to this tachyacoustice big bang model as well as review problems with dark energy and quantum black holes and proposed solutions. In this thesis, we tie these ideas together by finding black hole solutions of the underlying tachyacoustic theory. This also leads to an explanation for current cosmic acceleration, resulting in a three-in-one unified potential model of big bang, black holes, and dark energy.

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Chapter 1

Introduction

As will become abundantly clear in the upcoming chapters, cosmology, and physics in general, face numerous problems, puzzles and paradoxes with every new (and old) advancement. Every time we think we've gotten close to *something good*, an influx of "what ifs" comes pouring in. In many cases, a theory's ability to answer all of the "what ifs" posed will make or break it. In some cases, the theory may just be "good enough except for...", and can eventually gain traction as well as attracting new resolutions to the new proposed problems.

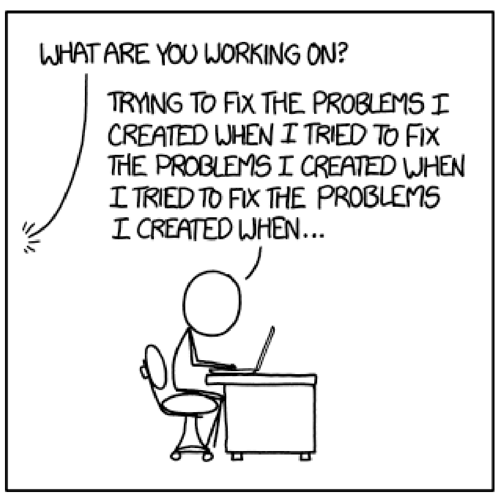

Figure 1.1: Solving problems in cosmology
https://xkcd.com/1739/

Chapter 2 introduces the standard model of Big Bang (SBB) cosmology (also referred to as the ΛCDM model "Lambda cold dark matter"), which among its many successes, still has some unanswered questions and issues. Of these issues, one that tends to rub people the wrong way is that of *fine tuning*. The SBB has six parameters: physical baryon density ρ_b, physical dark matter density ρ_{DM}, the age of the universe t_0, scalar spectral index n_S, curvature fluctuation amplitude Δ_R^2 and reionization optical depth τ. These parameters are not predicted by the SBB and are given a value strictly based on observations, our hope is that there can be a theory to predict these initial conditions. Chapter 2 includes a discussion of Cosmic inflation which can make a close prediction for the value of n_S, as well as introduces a competing theory Varying Speed of Light (VSL) and it's different guises. The final section introduces a proposed model of a thermal big bang which makes use of the VSL theories.

Chapter 3 introduces the dark energy "identity crisis" and looks at some proposed solutions which include the cosmological constant, quintessence and gravitational aether. Observations of the universe today tell us that the total matter density should be close to that of the critical density ρ_{crit}, however this is not even close to true as ρ_m only makes up close to 31% of that total. This is where dark energy comes into play, it should be responsible for the remaining energy density. What this dark energy could really BE is still up in the air. The second half of chapter 3 takes an exciting peek into the realm of quantum black holes and the possible exotic compact objects (ECOs) they could be. Many of these objects are posed as a solution to a well known problem with black hole physics — the information paradox. By suggesting modifications to the classical ideas of black hole horizons, this paradox can be amended or avoided all together. Lastly, with the observation of gravitational waves back in 2016, new efforts are being put forward to observe and understand gravitational wave "echoes" which may shed light on any modifications of black holes.

Chapter 4 builds upon the Thermal big bang model discussed in chapter 2 and follows suit of the models of modified black holes in chapter 3 to look for black hole solutions within the proposed thermal big bang model. It is hoped that a "three-in-one" model can be made from this already promising big bang model to include static black holes as well as offer a prediction for another of the unknown parameters in the SBB, the dark energy parameter. An unexpected result is that there is a real singularity for which pressure blows up and our g^{00} goes to zero, but the acoustic metric hasn't the slightest care. It is shown that the acoustic metric is indeed non-singular while the gravity metric seems to have a distinct singularity.

Lastly chapter 5 provides a conclusion and recap of everything that has been covered, as well as what hasn't. It includes a brief future to-do list offering some suggestions for future work.

Chapter 2

Can light chase gravity at the Big Bang?

The standard Big Bang (SBB) model, though well popularized, comes with it's fair share of missing links. The main issue is that of the initial conditions; cosmologists have the responsibility of putting in by hand what these should be in order to match current observations. Though there's nothing wrong with this, it would be nice to have a model of the universe that does not need to manually adjust the parameters in order to to match current observations. Fingers can be pointed at the Horizon problem for this gap as it leaves out any causal explanation for how two seemingly causally disconnected regions of spacetime can exhibit the same properties without set initial conditions. The SBB's inability to explain the horizon problem leaves cosmologists with fine tuning issues and in need for perhaps a more predictive model of the early universe. Another concern is known as the Flatness Problem. Current observations show that the universe is nearly flat and has an energy density ρ with a value almost that of the critical density ρ_{crit}, in fact, $|\Omega - 1| < 0.1$ where $\Omega = \frac{\rho}{\rho_{crit}}$ [30]. In the case that $\rho = \rho_{crit}$ the universe would be completely flat. What makes this strange, is that any deviation from the critical value in the early universe would have increased drastically over the past ~ 14 billion years from expansion. The universe seems to have figured out a way for the density of the early universe to be around one part in 10^{62} of the critical density [20]. The SBB cannot explain why the curvature at the beginning of the universe would have been so small, and so one is left to their own devices to put in by hand the necessary initial conditions to match what is seen today. Another key issue to point out is how the large-scale structures in the universe could have been formed. Although the universe is homogeneous and isotropic in the large scale picture, the universe is filled with structures whose origins can not been explained by the SBB. These inhomogeneities in an otherwise perfectly homogeneous universe can be looked at from the point of view of density perturbations. The fluctuations arising from the perturbations will have all scales of wavelengths. The wavelengths will be stretched by the expansion of the universe, and so they must have corresponded to fluctuations with much smaller wavelengths in the early universe. The issue is at one point or another, some of these wavelengths would have still been larger than the causal horizon of the universe during the radiation and matter dominated periods of the universe. Such fluctuations are said to be scale invariant, with spectral scale index $n_s \approx 1$. This is similar to the horizon problem in that there is no mechanism to generate such fluctuations outside of the horizon.

2.1 Inflation

Many options have been introduced in hope of avoiding the above problems. The most notable of which is cosmic inflation (”old inflation” [20] and ”new inflation” [10][22]) which suggests a brief period of accelerated expansion of the universe shortly after it's birth. Inflation allows the universe to have started out in a small region of space before expanding exponentially accelerating outwards. Thus, regions that would appear to be causally disconnected today, would have actually been in contact and had time to be in equilibrium prior to the expansion driving them apart.

Inflation is driven by the inflaton field, which is a hypothesized scalar field with a large vacuum energy, referred to as a cosmological constant or a dense dark-energy. With this, inflation also offers an explanation for the fluctuations which give rise to the large scale structures. The inflaton field being a quantum field, will exhibit quantum fluctuations. These quantum fluctuations can exhibit a near scale invariant spectrum and appropriate amplitude to match observations, allowing them to be seeds for the large scale structure. This is however, assuming there is an unspecified process for transforming quantum fluctuations into classical fluctuations [29]. What's more is that the density of the inflaton field does not decrease and dissipate as space expands, instead it remains constant. This in turn forces the value $|\Omega - 1|$ to decrease towards the required value of 10^{-62} during inflation, in order to then increase to the current observed value of $\simeq 0.01$ with today's rate of expansion [42].

An alternative model to inflation, which of interest for the upcoming model development, is Varying Speed of Light (VSL) theories[32][9][25]. Here, rather than the universe undergoing a rapid expansion, VSL proposes that the speed of light in vacuum was actually faster in the early universe compared to it's current value.

Although both Inflation and VSL models (along with other theories not mentioned here), seem to check most boxes when it comes to filling in the blanks of the SBB, they still require some fine-tuning of initial conditions in order for them to take off. In particular, observations show that the spectral index n_s, and amplitude A_s of the density fluctuations mentioned above are 0.9649 and 2.101×10^{-9} respectively [18]. In hopes of getting a predictive model for these initial conditions, it was proposed in [6] that one can avoid fine-tuning at least the spectral index of the primordial density fluctuations n_s by utilizing a class of VSL theories. A discontinuity leading to a critical solution is found and further explored, leading to a fully predicted spectral index value $n_s = 0.96478(64)$, which is in very close range to the observational value without any fine-tuning [6].

2.2 Varying Speed of Light

The mechanism for Inflation is the scalar *Inflaton* field which has a negative pressure such that the matter content of the universe is modified in order for Einstein gravity to be repulsive and drive the expansion. In this way, the inflaton field violates the strong energy condition. What VSL models suggest is "simply" change the speed of light in the early universe, leaving matter content to be that of the SBB and Einstein gravity to be as is [9]. VSL theories have gained traction and have shown promising progress in solving the problems faced in cosmology [9],[31] and references there in. This is not intended to be an exhaustive review of VSL theories, instead just an introduction of the main ideas, and for completeness, briefly discuss the ways in which VSL theories can be used as solution to the above mentioned problems. It is then look at how one particular mechanism can be used to induce a dynamical speed of light. For consistency with the referenced work the rest of this chapter assumes a metric with $[+---]$ signature, however this will change in Chapter 4 when introducing new research.

2.2.1 VSL and the Three Puzzles

A simple solution to the Horizon problem is to imagine speed of light in the early universe which is faster than the present value. In this case, there would be no concern of information not reaching from one point to another in some allotted time if the speed of signal travel could just be increased. For this to work, it has been proposed that there was a phase transition at some time $t = t_c$ in the past, such that the speed of light changes from c^- to c^+(with $c^- > c^+$). Today's past light cone would intersect the horizon at $t = t_c$ with a much smaller comoving radius than that of the horizon [25],[9] (see Fig.2.1-2.3)

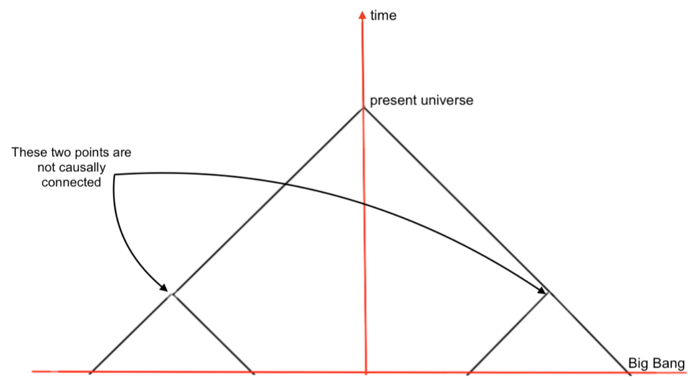

Figure 2.1: As it stands the SBB cannot account for these two points being in causal contact.

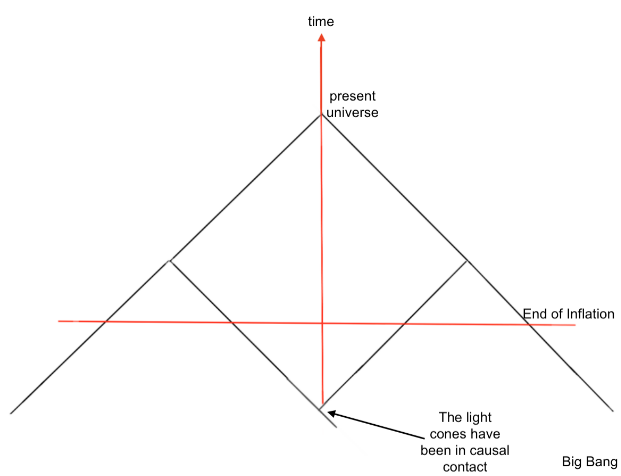

Figure 2.2: Inflation extends to negative conformal time, so the two points past light cones would have intersected

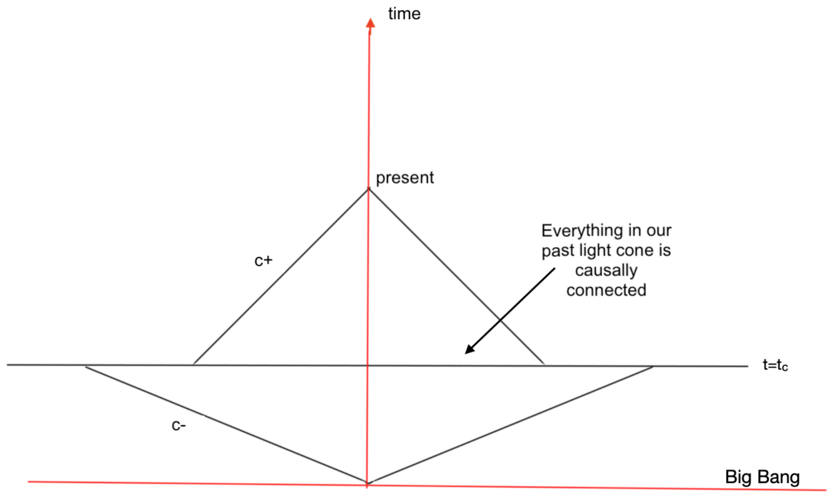

Figure 2.3: If the speed of light were to be faster before $t = t_c$ the horizon of our past light cone would fit within the horizon of the light cone of c^-

Though VSL theories do not require violating the strong energy condition as in inflation, they do violate the weak equivalence principle while c is changing. This however is not observable in present experiments if c was only dynamical in the early universe and occurred for only a short period of time such as a very fast phase transition which avoids issues in causality [9].

In regards to the flatness problem, it is shown in [9] and also [25], that a decreasing speed of light ($\dot{c}/c < 0$) after $t = t_c$ can drive the energy density $\rho \to \rho_{crit}$. If the speed of light were to change in a sufficiently fast phase transition such that $|\dot{c}/c| \gg \dot{a}/a$, it can be calculated that $(\Omega - 1) \propto c^2$. As mentioned above, $(\Omega - 1)$ would have had to be $\simeq 10^{-62}$ at early times in order to match observations today. This would indeed be the case if the speed of light were to decrease by more than 32 orders of magnitude during the phase transition [25]. What's more is that if $\dot{c}/c < 0$ for a period of time, then a flat universe (i.e $k = 0$) is the only stable option and thus the universe will eventually tend to $k = 0$ and energy density to ρ_{crit} [9], solving the Flatness problem.

VSL theories also try their hand at solving the large scale structure formation puzzle. In contrast with the inflationary model which makes use of quantum vacuum fluctuations, it has been suggested that the seeds for large scale structure developed from thermal fluctuations [34] and is further explored in [24]. Highlighting the main results here, [24] points out that thermal fluctuations are not necessarily Gaussian but can in fact be approximately Gaussian under the same conditions which ensure thermalization. The thermal fluctuations however have a white noise spectrum of $n_s = 0$, as opposed to the required scale invariant spectrum $n_s = 1$. This issue can be resolved if the thermalization only applies to modes within the horizon, and as the modes leave the horizon they freeze and become non-thermal, it is the super horizon spectrum which shows to be scale invariant [24],[25]. In general, the amplitude of these fluctuations will be of order 1, contrary to the observed order of 10^{-5} but by introducing a dynamical speed of light, it is believed that $c(t)$ can be designed such that the fluctuations have the appropriate amplitude to explain structure formation [9],[25],[24]. The structure formation puzzle can also be tackled via VSL theories but it is constructive to first introduce the mechanism for what actually formulates a varying speed of light.

2.2.2 VSL and the Two Metrics

A number of mechanisms for inducing a varying speed of light have been proposed since the theory has gained popularity. A full discussion and exploration of all such mechanisms can be found in [25] and the references there in.

Of concern will be the *Bimetric theories* suggested by [16] in which there are two non-conformal spacetime metrics, one for gravity and one which will describe the geometry of ordinary matter, following the scalar-tensor model proposed in [16] and further studied in [17] and [25]. This model introduces a scalar field ϕ as opposed to the vector model in which a vector field is introduced (the main focus of [16]).

The matter metric $\hat{g}_{\mu\nu}$ is then given by

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B[\phi]\partial_\mu\phi\partial_\nu\phi \tag{2.1}$$

Where $B(\phi)$ is referred to as the *warp factor* and has units of M_P^{-4} while ϕ has units of M_P ($M_P = \sqrt{\frac{\hbar c}{8\pi G}} = \frac{m_p}{8\pi}$ is the reduced Planck mass [1]). If $B[\phi] = B$ a constant, the filed equations for ϕ avoid complicated terms and is referred to as the *minimal bimetric theory* which will come up again later. In order for the speed of light and other massless particles to be greater than that of gravity it is required that $B > 0$ so as to have two seperate light cones that do not overlap (one for gravity and one for light).

[1] m_p is the regular Planck mass $\sqrt{\frac{\hbar c}{G}}$

The action can then be written in the general form

$$S = S_H + S_\phi + \hat{S}_M \tag{2.2}$$

Where S_H is the usual Einstein-Hilbert action, S_ϕ is the scalar field action, both with dependence on the gravity metric $g_{\mu\nu}$. $\hat{S}_M$ is written as it normally would be for all matter fields in spacetime, but instead depends on the matter metric $\hat{g}_{\mu\nu}$.

Above it was mentioned that VSL theories may violate the Weak Equivalence Principle, but from the view point of the bimetric models the matter fields couple to $\hat{g}_{\mu\nu}$ in the same way, and thus saving the theory from such a violation [17]. It is however noted that the Strong Equivalence Principle may be violated if one considers the expansion of the matter and gravitational fields in a region where $\hat{g}_{\mu\nu} \approx \eta_{\mu\nu}$ as the perturbation equations for $\hat{g}_{\mu\nu}$ and ϕ will not be of proper special relativistic form [17]. It is further explored in [17], as well as in[25] that the bimetric mechanism of VSL theories maintains success in solving the horizon and flatness problem as discussed above.

It is speculated that the structure formation problem is still not sufficiently dealt with by VSL theories. This is due to the fact that the fluctuation modes *start off* outside the horizon, so one cannot follow the fluctuations from inside the horizon to outside the horizon allowing for the set up of the initial conditions[27]. Instead, it was suggested in [26] that if the speed of sound was taken to be larger in the earlier universe than in the present, this could generate the near scale invariant density fluctuations needed. The idea uses the bimetric VSL theories introduced above and proves to be a useful union for solving the problems faced by the SBB as we will see next.

2.3 Varying Speed of Sound

During the early universe, it was filled with a hot plasma which was a cosmic soup of photons, unbound electrons and atomic nuclei. Also present in the universe during this time was dark matter. The multitude of collisions between the photons and unbound electrons made it difficult for the light to travel anywhere during this time, making the plasma opaque. The photons exert a pressure on the plasma during these collisions, but the dark matter does not interact with the photons and is left unbotherd.

The density fluctuations from the early universe lead to the inhomegeneous distribution of mass, causing a gravitational pull on the dark matter and the plasma itself. This gravitational pull is then counteracted by the pressure by the photons on the plasma driving the region apart little by little. This tug-of-war between gravity and pressure results in oscillations referred to as acoustic oscillations or *sound waves* of these disturbances. Once the region has spread out enough for collisions to settle down, the temperature begins to cool and the electrons and atomic nuclei can combine into atoms. This is referred to as recombination. The stable atoms no longer bother the photons, allowing them to freely propagate throughout the universe as CMB radiation, carrying with them energy from the regions they resided in. Photons that were stuck in denser regions had more energy than those left outside the denser regions and the variation due to the sound waves, is imprinted in the CMB observed today.

Within SBB, before recombination the speed of sound was 60% the speed of light, but once the electrons were able to form atoms and the pressure dropped and the speed of sound decreased. The sound waves froze and the radius of the sound horizon became fixed with the rate of expansion of the universe. The initial quantum fluctuations predicted by inflation can explain the source of the sound waves, as well as explain the scale invariance and consistent amplitude of the oscillations.

In order for VSL theories to compete with inflationary theories they need a sufficient explanation for structure formation and scale invariant fluctuations. The proposal of a larger speed of sound before recombination investigated in [26] finds that scale invariant fluctuations could have indeed been produced. The prospect of a varying speed of sound proves to be a simpler framework for structure formation within VSL models and is further explored in a follow-up paper [27]. This shows a promising solution to the structure formation problem while still holding onto bimetric VSL solutions to horizon and and flatness problems.

Scale invariance follows from quantum or thermal fluctuations which start off inside the sound horizon and will freeze as they cross the sound horizon (as opposed to the Hubble horizon as in inflation). In the case of quantum initial conditions, the speed of sound should be proportional to the density ($c_s \propto \rho$) for any constant equation of state $\omega = p/\rho$ [27]. In the case of thermal initial conditions it is required that c_s decrease rapidly in a very short instance i.e a rapid phase transition. As the sound horizon shrinks and the oscillations freeze-out, they are left imprinted outside the horizon. It should be noted that a rapid phase transition in the case of quantum fluctuations will cause $n_s > 1$ [26]. The case of thermal initial conditions is further studied in [7] and suggests that the speed of sound would have decreased by 25 orders of magnitude during the phase transition which translates to the ~ 60 orders of magnitude change required during inflation as mentioned above. What follows is a basic overview and summary of how one can realize the speed of sound mechanism with bimetric VSL theories as discussed in [27].

2.3.1 Speed of Sound Meets K-Essence

So far we have discussed the general VSL theory and its competition with inflation, the more specific bimetric VSL model and have now introduced the idea that a faster speed of sound in the early universe may be the solution we need for scale invariant fluctuations and structure formation. In order to tie this all together we need one more piece of the puzzle, K-essence.

K-essence models are scalar field theories which have a non-standard (non-linear) kinetic term in their Lagrangian, i.e some function $K(X)$ where $X = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi$. The scalar field action can then be written

$$\begin{aligned} S_\phi &= \int d^4x\sqrt{-g}\mathscr{L}(X,\phi) \\ &= \int d^4x\sqrt{-g}\big(K(X) - V\big) \end{aligned} \tag{2.3}$$

Following the usual methodology, variation with respect to ϕ ,the Euler-Lagrange equations in curved spacetime

$$\frac{\partial\mathscr{L}}{\partial\phi} - \nabla_\mu\frac{\partial\mathscr{L}}{\partial(\nabla_\mu\phi)} = 0$$

gives the equation of motion for the scalar field

$$0 = (\mathscr{L}_{,X}g^{\mu\nu} + \mathscr{L}_{,XX}\phi^{,\mu}\phi^{,\nu})\phi_{;\mu\nu} + 2X\mathscr{L}_{,X\phi} - \mathscr{L}_{,\phi} \tag{2.4}$$

where $_{,X}$ represents the partial derivative with respect to X. Variation with respect to the metric $g_{\mu\nu}$ gives the Stress-Energy tensor

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \mathscr{L}_{,X}\phi_{,\mu}\phi_{,\nu} + g_{\mu\nu}\mathscr{L} \tag{2.5}$$

which can be written in terms of a perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$ if $u_\mu \equiv \sigma\frac{\nabla_\mu\phi}{\sqrt{2X}}$ is the fluid four-velocity and $\sigma = sgn(\partial_0\phi)$ [40]. From this the pressure and density are given by

$$\begin{aligned} p &= \mathscr{L} \\ &= K - V \end{aligned} \tag{2.6}$$

$$\begin{aligned} \rho &= 2X\mathscr{L}_{,X} - \mathscr{L} \\ &= 2XK_{,X} - K + V \end{aligned} \tag{2.7}$$

The speed of sound as defined in [19] by $c_s^2 \equiv \frac{p,_X}{\rho,_X}$ is then given to be

$$c_s^2 = \frac{K,_X}{K,_X + 2XK,_{XX}} \tag{2.8}$$

The importance of the K-essence model here is the realization that they can be modified in order to exhibit a varying speed of sound, in the first paper [26] this avenue is explored, however it was noted in [27] that a simpler approach had been found. The K-essence theory originally proposed was later realized to be more like a "anti-" Dirac-Born-Infeld (DBI) model in a limiting case. The feat of this realization being that the resulting DBI model is one known to be associated with scale invariance, and it just so happens to be the minimal bimetric theory mentioned before! [2] Here however, the important information to know about DBI models is that they have the following Lagrangian

$$\mathscr{L} = -\frac{1}{f(\phi)}\sqrt{1 - 2f(\phi)X} + \frac{1}{f(\phi)} - V(\phi), \tag{2.9}$$

and in order to have an increasing speed of sound at high energies as is desired here, one must have $f = -C$ where C is a positive constant (as opposed to the "regular" DBI in which$f > 0$). In the limit of $X \gg 1/C$(in regular DBI models X cannot be greater than $1/2f$, but since there are no worries this limit is worthy of exploration[27]), the Lagrangian becomes $\mathscr{L} = \sqrt{\frac{2}{C}}\sqrt{X} + \frac{1}{\sqrt{2C^3}}\frac{1}{\sqrt{X}} - V(\phi)$which, with constraints on the coefficients turns out to be the exact K-essence model that was "tediously" constructed in the original paper [26] which gives rise to the necessary condition $c_s^2 \propto \rho$ for scale invariance in the case of quantum fluctuations. This can be recognized as the Cuscuton model proposed by Afshordi in [5] . The results also lead to a relation between the parameters and observables $\frac{(5+3\omega)^2}{\sqrt{2}(1+\omega)}\frac{2}{CM_{Pl}^4} \sim 10^{-10}$(where ω is the equation of state parameter relating pressure and density $P = \omega\rho$) [27].

Reiterating the point here for clarity — the "anti"-DBI model was used as a tool to implement the varying speed of sound idea for which it is known near scale invariant fluctuations with appropriate amplitudes can be produced. In the limit of $X \gg 1/C$ it is found that the resulting Lagrangian is actually the same as the Lagrangian found when the author constructed it by hand. The next thing to do is relate all of this to the bimetric VSL models previously introduced.

[2]For more about DBI and DBI-inflation and relations to this model, the reader is encouraged to take a look at [11], [36] and references therein.

2.3.2 Bimetric VSL Theories Meets K-Essence

What makes the bimetric VSL theory a competitor to inflation is that VSL already solves the previously introduced problems faced in cosmology, it is however, still missing a piece of the puzzle before it can stand it's ground and that is — structure formation. What has been done above suggests that the structure formation puzzle could be solved if the speed of sound were to be much larger in the early universe. In order to implement a varying speed of sound, the K-essence model was implemented and results in being a special variation of the known DBI-models. DBI models are generally mixed in with inflation and work to *reduce* the speed of sound. However, by switching the sign of the "warp factor" $f(\phi)$, the model works to have an increasing speed of sound at high energies and this is what is referred to as the "anti-DBI" model in the referenced work.

As it stands this anti-DBI model really only gives solutions to the scale invariance issue, and so it is wed to the bimetric VSL model in hopes of having solutions to all cosmological problems.

Recall the action for bimetric VSL theories 2.2, as well as the new matter metric 2.1, $\hat{g}_{\mu\nu} = g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$ with $B = $ const in the minimal theory. The scalar field can be made to have a Klein-Gordon Lagrangian in either the Einstein frame $g_{\mu\nu}$, or matter frame $\hat{g}_{\mu\nu}$, however the field equations for ϕ will not be a Klein-Gordon equation of motion ($\nabla^2\phi + V'(\phi) = 0$ in the Einstein frame or $\hat{\nabla}^2\phi + V'(\phi) = 0$ in the matter frame), due to the addition of an extra term from the variation of S with respect to either metric, i.e $\delta\hat{g}_{\mu\nu} \neq \delta g_{\mu\nu}$[17],[27]. This extra term is related to the stress energy tensor in the matter frame $\hat{T}^{\mu\nu}$ and the covariant derivatives $\hat{\nabla}_\mu$ defined with respect to the matter metric. It is suggested that if one were to define a third metric $\bar{g}^{\mu\nu}$ in terms of $\hat{g}^{\mu\nu}$ and $B\hat{T}^{\mu\nu}$, the field equation can be made to be a Klein-Gordon equation of motion, $\bar{g}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu\phi + V'(\phi) = 0$. It is stressed in [27] however, that $\bar{g}^{\mu\nu}$ is not a proper spacetime structure but rather a nice way to describe the propagation of ϕ in terms of a Klein-Gordon field. An important realization is that if no matter is present, the scalar field action S_ϕ that will produce a Klein-Gordon equation in the matter frame is actually a cosmological constant $\hat{\Lambda}$. In this case the scalar field action in the matter metric will be of the form

$$S_\phi = \int d^4x\sqrt{-\hat{g}}(-2\hat{\Lambda}) \tag{2.10}$$

and the stress-energy tensor will be $\hat{T}^{\mu\nu} = \hat{\Lambda}\hat{g}^{\mu\nu}$. The field equation in the matter frame is then

$$\hat{T}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu\phi = \hat{\Lambda}\hat{g}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu\phi = 0.$$

Which is Klein-Gordon in the matter frame. It is the simplest non-trivial S_ϕ to result in a Klein-Gordon equation of motion[27].

To check if this action is appropriate for the scale invariance, it must be looked at in the Einstein frame. To do this, we have from 2.1 the determinant

$$\hat{g} = g(1 + 2B(\phi)X)$$

so the action 2.10 becomes

$$S_\phi = \int d^4x\sqrt{-g}\sqrt{1 + 2B(\phi)X}(-2\hat{\Lambda})$$

In the high energy limit, this already has the desired scale invariance and what's left is to determine the low energy limit for which the bimetric theory is the minimal bimetric VSL theory.

K-essence models can be reinterpreted as a bimetric theory, and for a full derivation of the next result, the reader is directed to [27] Sec.∼ V. The exciting result is that the bimetric VSL theory with $B(\phi) = B$, has the k-essence kinetic term in the Einstein frame

$$K = \frac{1}{B}\sqrt{1 + 2BX} - \frac{1}{B}. \tag{2.11}$$

Comparing with 2.9 and defining $B = C = -f(\phi)$, so that B must be a positive constant in order to give the anti-DBI model. This result leads to the choice of $\hat{\Lambda} = -\frac{1}{2B}$ in order to match the low energy limit. It also requires that there be an opposite balancing term Λ in the Einstein frame. Giving us a bimetric theory which in turn is the (anti) DBI action in the Einstein frame

$$\begin{aligned} S_\phi &= \int d^4x\sqrt{-\hat{g}}(-2\hat{\Lambda}) + \int d^4x\sqrt{-g}(-2\Lambda) \\ &= \int d^4x\sqrt{-g}(\frac{1}{B}\sqrt{1 + 2BX} - \frac{1}{B}) \end{aligned} \tag{2.12}$$

It is the anti-DBI behaviour in the Einstein frame which induces a varying speed of sound which has been shown to have the proper characteristics for structure formation[27].

2.4 A Thermal Origin

The model reviewed above was primarily discussed for the case of quantum initial conditions, however, it was also mentioned that the universe could have had a simpler history if the initial conditions were thermal. In this case, scale invariance follows from a very fast phase transition and eliminates the need for a reheating period, as required by inflation. Thermal initial conditions with the "speedy sound" a.k.a tachyacoustic cosmology is studied in [7] and [6]. The model presented in [6] determines that the reason that perfectly scale invariant fluctuations are unreachable is due to a discontinuity in the theories presented above. This discontinuity leads to a critical solution which is to be regarded as the preferential model for the required phase transition[6].

The model follows the bimetric theory presented in 2.3.2 with the metric for the matter frame 2.1 and B as a general function of ϕ. A more general approach to 2.12 allows the scalar field action to consist of *non-constant* cosmological terms in both the matter and Einstein frame, so the action has the form

$$S_\phi = \int d^4x\sqrt{-\hat{g}}(-2\hat{\Lambda}(\phi)) + \int d^4x\sqrt{-g}(-2\Lambda(\phi)). \tag{2.13}$$

Following the same recipe as done for 2.12, but allowing for a general potential, $2\Lambda(\phi) = V(\phi)$ (as opposed to the constant $\frac{1}{B}$ in 2.12) the still anti-DBI action as given in the Einstein frame is

$$S_\phi = \int d^4x\sqrt{-g}\Big(\frac{1}{B(\phi)}\sqrt{1+2B(\phi)X} - V(\phi)\Big) \tag{2.14}$$

The speed of sound as given by eqn. 2.8 in the Einstein frame is then

$$c_s^2 = 1 + 2BX \tag{2.15}$$

In the high energy limit $X \gg 1$, the Lagrangian of 2.14 becomes $\mathscr{L}_\phi \approx \sqrt{\frac{2X}{B}} - V$ which can be recognized as the Cuscuton Lagrangian as defined in [5] which has an infinite speed of sound, $c_s \to \infty$ for $X \gg 1$. From this model spatial flatness is determined to be compulsory such that V is no longer a free parameter. Then $V \approx \rho$ and $P + \rho \approx K$ where $K = \dot{\phi}/\sqrt{B}$ is the kinetic term, and ρ and P are the density and pressure [6]. It is shown that V is fully fixed to be a function of B through the Friedmann equations with $k = 0$ (this also leads to a natural solution of the flatness problem in SBB). Leaving $B(\phi)$ as the only free function in the high energy limit and the potential given as

$$V(\phi) = \frac{3}{4M_P^2}\Big(\int \frac{d\phi}{\sqrt{B(\phi)}}\Big)^2 \tag{2.16}$$

[6]. Near scale invariance follows from $B(\phi) \propto \phi^n$ when $n \sim 2$ as pointed out in [6],[7]. However, the absolute scale invariant limit $n_s = 1$ is unattainable due to 2.16 becoming a

non-power law potential at $B \propto \phi^2$. This becomes the critical theory as found in [6] since every potential on either side of $n = 2$ is still a power law with an abrupt variation in c_s and scaling cosmological solutions. The scaling cosmological solutions are what lead to thermal fluctuations with a constant n_s. However, the result of the critical theory is that natural deviations from scale invariance are induced has a non-scaling speed of sound. As calculated in [6] the critical theory is found to have

$$B_{crit}(\phi) = B_0(\frac{\phi}{M_P})^2 \tag{2.17}$$

$$V_{crit}(\phi) = \frac{3}{4B_0}\ln^2(\frac{\phi}{M_P}) \tag{2.18}$$

where $B_0 = \frac{1}{6M_P^4} 9.0 \times 10^{14}$ which can be determined via the observed amplitude A_s. The significance of this model is that the observed value of n_s is fully predictable from the amplitude A_S for a given scale, cutting down on one fine tuned parameter in SBB[6]. The model also disposes of the reheating phase at the end of the varying-c phase as required by inflation. By incorporating the VSL model prior to the phase transition, the usual cosmological problems are skirted. This model also does not require introducing a new field such as the inflaton field, as the scalar field in this model can be thought of as the already existing plasma field.

The prediction for the spectral index of the scalar fluctuations as given by the critical model for the observed amplitude is

$$n_s = 0.96478(64)$$

with current observations by Planck [18] of the spectral index to be

$$n_s = 0.965 \pm 0.004.$$

A more detailed comparison of critical thermal big bang and inflationary predictions with Planck 2018 observational constraints is shown in Figure 2.4.

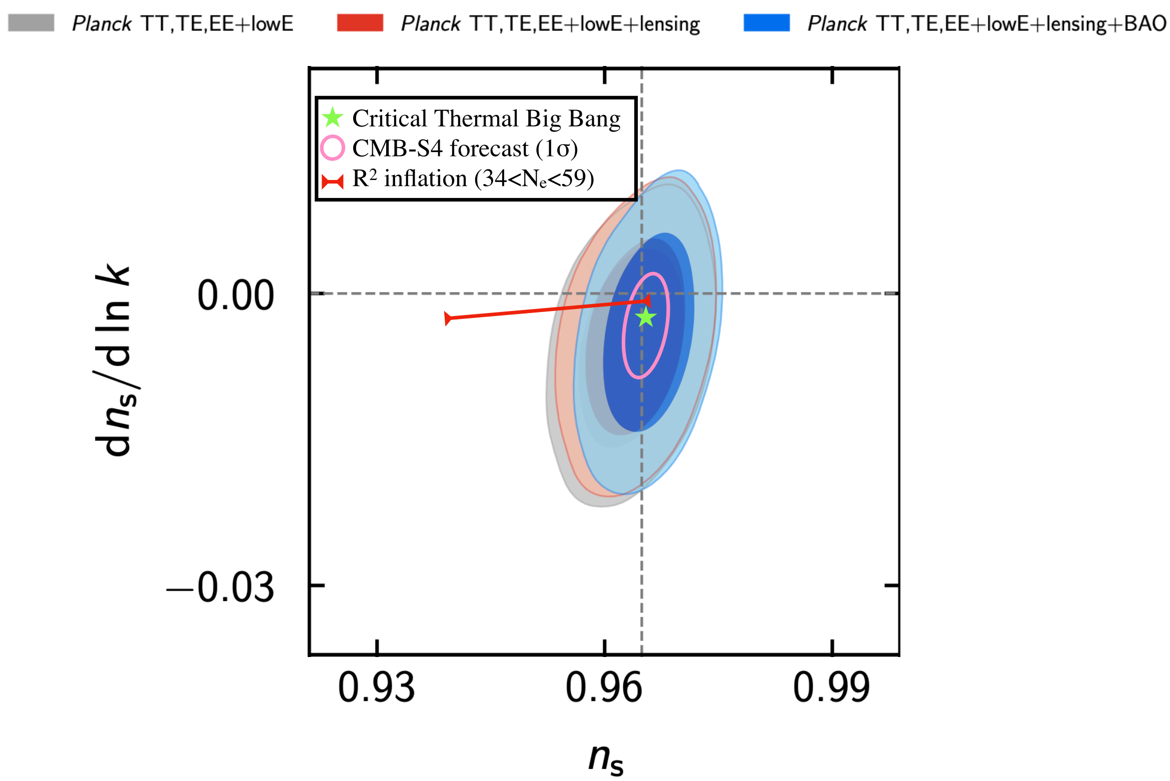

Figure 2.4: Observational constraints on spectral index, n_s and its running $dn_s/d\ln k$ for scalar perturbations from Planck 2018 data release [18]. We compare this against predictions for critical thermal big bang (see text) and the R^2 inflation model [8]. The pink ellipse shows the forecasted constraints for CMB-S4 experiment [1] (figure courtesy of N. Afshordi).

Chapter 3

Quantum Gravity at the Door: From Dark Energy to Firewalls

3.1 Dark Energy and its Possible Forms

The existence of an extra unknown energy density within the universe is needed when one looks at the current state of the universe. According to observations the universe is expanding, and not at a constant rate. In order for the universe to have such an accelerated expansion, presence of a homogeneous and evenly distributed energy density is required.

What's more, observations of cosmic microwave background (CMB) temperature anisotropies have found that primordial density perturbations should be nearly scale invariant Gaussian perturbations in a flat universe. As noted above, a flat universe requires that the energy density of the universe ρ be that of the critical density ρ_{crit} making the density parameter $\Omega = \frac{\rho}{\rho_{crit}} \approx 1$. However, the matter in the universe which can be accounted for, only makes up about 30% ($\Omega_m = 0.315$ [18]) and the rest is left to be filled by this unknown energy density, given the fitting name, dark energy.

Contestants for dark energy include a cosmological constant, quintessence- a scalar field with a varying density in space and time, or even abandoning Einstein's General Relativity and replacing it with a model of modified gravity.

3.1.1 The Cosmological Constant

The most popular and natural option for dark energy is the cosmological constant. The idea of a cosmological constant takes into account that there should be a vacuum energy density as a result of vacuum fluctuations in empty space. The cosmological constant can then be seen as an intrinsic property of spacetime rather than an additional piece to an already complicated puzzle. However, things are never that simple, the problem now is comparing the expectation versus reality of the value of the vacuum energy density. The energy density that the universe requires is only around $\rho_{vac} \approx (10^{-3}eV)^4$ but upon "guesstimating" ρ_{vac} via field theory, the result is an astonishing $\rho_{vac} \approx (10^{27}eV)^4$ leading to the 120 orders of magnitude difference, earning the title of the cosmological constant problem[14]. This extra energy density would need to be balanced by another equally as large term.

Another issue is the coincidence problem. It appears that the universe has chosen the perfect time to begin an accelerated expansion. Had this acceleration started any earlier or later large scale structure would not have been capable of existing. This is related to the fact that the vacuum energy density and matter energy density are almost equal. The early universe was not phased by the vacuum energy when matter and radiation were so dominant, but at later times when the matter and radiation has become so dispersed, the vacuum energy becomes dominant. In between these two stages is only a very brief period in which the trade off between dominance is very much in favour of the current state of large scale structure of the universe[14]. As noted int [37], the cosmological constant is static, and so it is as it always will be. Quintessence however is dynamical, allowing things to evolve over time, giving some leeway to its values.

3.1.2 Quintessence

Quintessence allows a way around the cosmological constant problem and the coincidence problem, but of course still comes with its own fair share of complexities. The quintessence model approaches cosmic acceleration by suggesting it is a result of the potential energy of a dynamic scalar field termed the quintessence field [39]. In comparison to the cosmological constant which has an equation of state parameter $\omega \equiv \frac{P}{\rho} = -1$, quintessence has an equation of state parameter which may evolve over time along with the pressure and density. The different value of ω means a different prediction of cosmic acceleration, and in this case, quintessence models tend to predict a slower acceleration rate than the cosmological constant does. The varying energy density could very well evolve to zero, being rid of the vacuum energy density and solving the cosmological constant problem [14]. A possible solution to the coincidence problem is that the evolving density of the field closely "tracks" the density of radiation but never catches it until radiation-matter equality. At the point of radiation-matter equality the quintessence field will begin to behave like dark energy[37]. This allows the current energy density to be independent of initial conditions of the field, but still depends on the potential [14]. Unfortunately, fine tuning of the scalar field mass is needed to bring it down to an acceptable range for matching the observations of energy density. This is the fault of renormalization of the fields, which predict that scalar fields tend to acquire high masses, pushing them out of the acceptable range[14].

3.1.3 Gravitational Aether

It's easy to point fingers at the source of the issues mentioned above; the equations of general relativity. The doubt in general relativity leads to another option for dark energy- a modified theory of gravity. This carries with it that general relativity is good enough to explain gravity in the current universe, but perhaps general relativity breaks down in the early universe, leaving some underlying theory of gravity that is yet to be determined.

Of interest here is the *Gravitational Aether* proposed by N. Afshordi in his paper [4]. Afshordi argues that introducing an incompressible fluid dubbed gravitational ether, would cause the vacuum energy to decouple from gravity. This is done by modifying the energy momentum tensor (in contrast to the Einstein tensor as in most alternative theories) so that the Einstein field equation follows

$$(8\pi G'')^{-1}G_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}T^{\alpha}_{\alpha}g_{\mu\nu} + p'(u'_{\mu}u'_{\nu} + g_{\mu\nu}) \tag{3.1}$$

with $G'' = 4/3G$ and p',u'_{μ} are aether pressure and four velocity fixed by conservation of $T_{\mu\nu}$ [4][35]. What's interesting about this model is that if interpreted as a thermodynamic description of gravity, formation of stellar black holes in this theory could be an explanation of the cosmic acceleration [4]. If this were the case, no fine tuning is required to match observations, and so avoids the cosmological constant problem.

Intrigued by the possibilities of gravitational aether, a natural question to ask is whether or not this model permits black hole solutions. This question was tackled in [35] and has been motivation for the research to be presented in this work. The authors of [35] find that the gravitational aether couples the spacetime metric close to black hole horizon to the spacetime metric far away from the black hole horizon. Using the assumptions of a spacetime with no matter and spherical symmetry along side with the aether taking the form of a fluid the model takes on the same form of a static and spherically symmetric

metric inside a star, with zero density. The stress-energy tensor is then

$$T_{\mu\nu} = p(u_\mu u_\nu + g_{\mu\nu}) \tag{3.2}$$

and the metric is determined to be

$$ds^2 = -e^{2\phi}dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{3.3}$$

With

$$e^{\phi(r)} = \left(1 - \frac{2m}{r}\right)^{-1/2} (4\pi p_0 f(r) + 1) \tag{3.4}$$

p_0 is an integration constant introduced via the pressure solution to the Tolman–Oppenheimer –Volkoff (TOV) equation, $p = p_0 e^{-\phi}$. The function $f(r)$ is found to be

$$f(r) = \frac{r^2}{2} + 3mr + \mathscr{O}(m^2) \tag{3.5}$$

in a far away regime where $r >> m$ and

$$f(r) = -8\frac{\sqrt{2}m^{5/2}}{\sqrt{r-2m}} + \mathscr{O}(m^{3/2}(r-2m)^{1/2}) \tag{3.6}$$

close to the Schwarzschild horizon $r_S = 2m$.

The metric may look like a Schwarzschild metric with perturbations, but as [35] points out, the above functions f dominate both close to and far away from the black hole horizon, respectively. In this way the gravitational aether model is capable of explaining both the formation of black horizon and cosmology far from the black hole. The metric becomes complex inside the Schwarzschild radius and is thus only defined for outside the Schwarzschild radius. Interestingly, the pressure is inversely proportional to e^ϕ thus when $e^\phi \to 0$ (i.e when approaches the horizon) the pressure blows up as it would for a singularity. Looking at the Ricci scalar, this also $\to \infty$, suggesting that the horizon is in fact a real singularity and not just a coordinate singularity as may be expected. As it turns out, static event horizons cannot exist in the gravitational aether model with a UV completion strictly due to the solution $p = p_0 e^{-\phi}$. The value of the integration constant is further explored and for $p_0 < 0$, approaching the Schwarzschild radius is attainable within finite coordinate time due to a finite redshift at this point. Within r_S a change of coordinates can be made so that the metric is real and an event horizon does actually exist for small values of p_0 when $e^\phi = 0$ in the new coordinates.

p_0 is speculated to be fixed by quantum gravity effects due largely in part to the horizon being a real curvature singularity. Making use of the maximum redshift at r_S given by the ratio of the Planck temperature to the Hawking temperature (this is the Trans-Planckian ansatz), a value of p_0 is determined. The Planck temperature is used as the maximum rest frame temperature of a source falling into r_S, with $T_{max} = \Theta_P$ which is of order one in Planck units and Θ_P is the Trans-Planckian parameter. The Hawking temperature is the temperature of source falling in as observed from far away. The integration constant is then found to be $p_0 = -\frac{1}{256\pi^2\Theta_P m^3}$ and the event horizon is found to be about a Planck length away from r_S. In other words, the corrections to the Schwarzschild metric due to the gravitational aether in the close regime only become important a Planck distance from the horizon [35]. With this value of p_0 the pressure of the aether in the far away regime is comparable to the density of the dark energy as given by the cosmological constant (recall $\omega_\Lambda = -1$). Coming full circle to the suggestion that the formation of stellar mass black holes could be responsible for the cosmic acceleration[35] [4].

3.2 A Taste of Quantum Black Holes

General relativity makes the idea of black holes seem relatively simple- the only characteristics (i.e hair) black holes are expected to have are their mass, angular momentum and charge. In some cases, such as in astrophysical black holes, this number can even go down two by ignoring the very negligible charge. At this point, it is not surprising to mention that things are never this simple. Despite the three parameters one has to describe a black hole, they remain a large and complex puzzle. It is thought that black holes carry with them the missing key ingredient needed for a quantum theory of gravity.

Observations of black holes with two simple parameters may be the "smoking gun" for quantum or classical modifications to GR since any observed deviations from the standard predictions may help lead the way to such modifications[3]. A nice list of such models of "near-horizon" modified gravity was given by Abedi et al. in their detailed review paper on Quantum Black holes [3]. Of the models mentioned included here are gravitational aether black holes introduced above [35], firewalls [12],[13] and Gravistars [28], [15]. Echoes of gravitational waves are dubbed a "smoking gun" for testing these models[2]. Following in this order for consistency with the detailed review by Abedi et al. a brief overview of these models will be given.

3.2.1 Information Paradox

Increased attention on Hawking radiation of black holes has lead to some interesting debates about what happens to information that has fallen victim to the black hole. Hawking recognized that radiating black holes lose mass over time and eventually succumb to this constant loss.

The black holes lifetime is calculated to be $t_{lifetime} \sim 10^5 m^3 \sim 10^{75}(\frac{m}{m_\odot})^3$seconds, which is far older than the age for astrophysical black holes with mass $\geq m_\odot$. The evaporation could happen as early as Page time where the black hole is much bigger than m_P and will happen until the black hole reaches Planck mass and the curvature is of Planck scale where a classical treatment can no longer be trusted [3]. The information paradox posits that if at the end of the black holes life there is nothing but Hawking radiation, and the matter that had entered the black hole was a pure state, the end state is inevitably a mixed state radiation, destroying the unitarity required by quantum mechanics [23]. Thus any information about the original pure state is no where to be found.

3.2.2 Black Hole Complementarity

Black hole complementarity is a proposed solution to the information paradox, postulating that for an observer far from the black hole, the formation and evaporation can be viewed entirely within the realm of standard quantum theory [38]. What makes this proposal quite remarkable is that it is the best of both worlds. An observer far away will watch as the information falls toward the black hole, but is stopped at the horizon and takes an infinite time to ever cross. As the infalling information gets close to the horizon, it enters a membrane just outside the event horizon which, as seen by the observer far away, heats up the information and radiates it back out as Hawking radiation. Thus no violations of information conservation have been committed. This membrane is referred to as the *stretched horizon*. For an observer falling towards the black hole along with the information, they do not record anything out of the ordinary occurring when passing through the stretched and event horizon. The infalling observer and information continue their journey undisturbed all the way to the singularity. Of course, the infalling observer can never tell the outside observer of their adventure and vice versa. Only if one attempts to have a combined description valid for both observers is there an issue[38].

3.2.3 Firewalls

An argument was put forward against complementarity stating that not all of the following statements from complementarity could be simultaneously true 1) Hawking radiation is in a pure state, 2) the information carried by the radiation is emitted from the region near the horizon, with low energy effective field theory valid beyond some microscopic distance from the horizon, and 3) the infalling observer encounters nothing unusual at the horizon. A rather crude solution is then proposed to just have the infalling observer "burn up at the horizon" [13].

In this case, the black hole horizon is replaced by a firewall, whose job is to break entanglement of the Hawking particle pairs. Hawking radiation is considered by quantum field theory to consist of entangled particle-antiparticle pairs being created and annihilated in spacetime outside of the black hole. When this happens sufficiently close to the horizon, one of these particles may fall to its demise into the black hole while the other escapes off to infinity- earning the title of Hawking radiation. What's more is that the emitted particle must also be entangled with all the Hawking radiation that has been emitted before it. However, the outgoing particle is already "bound" to it's partner, which is now a victim of the black hole singularity. Being entangled with two independent states is not only frowned upon, it also contradicts the *principle of monogamy of entanglement* [3]. To avoid the controversy of a two-timing entanglement, the authors of [13] (referred to as AMPS) suggest that the entanglement should be broken between the pair of Hawking particles by replacing the black hole horizon by high-energy boundaries dubbed *firewalls*. Thus, any object falling into the black hole burns up at the new boundary, contradicting the equivalence principle and replaces black hole complementarity [3]. Also noted by Abedi et al. is that in general if quantum effects do lead to a high-energy boundary at the stretched horizon, it could contribute to the reflectivity of the black hole which may be observable by merger events leading to the formation of black holes.

3.2.4 Gravastars

Another alternative model to the astrophysical black hole suggested by Mazur and Mottola [28], the interior is chosen to be de Sitter space with a cosmological constant equation of state ($P = -\rho$) and an outer region which consists of a thin shell of matter in the form of a perfect fluid ($P = \rho$) and finally is surrounded by Schwarzschild vacuum ($P = \rho = 0$). The Dark energy interior prevents collapse to a singularity and the thin shell replaces the horizon. Thus there is no horizon and no singularity, it is also thermodynamically stable and as such has no information paradox [28]. It has also been shown by Cattoen et al. in

[15], that gravastars cannot be perfect fluids due to the fact that anisotropic pressures are unavoidable.

3.3 Gravitational Wave Echoes

It is suggested that the models described above (and more which were not mentioned, but see [3] for a full review) may produce gravitational waves similar to those produced in binary black hole mergers as detected by the LIGO-Virgo collaboration. These gravitational waves should be followed by delayed repeating “echoes”. It is suggested that such gravitational wave observations could reveal any near-horizon modifications of black holes, as any modifications would show themselves through these delayed echoes [41]. As stated by Abedi et al., in order to model these echoes a full knowledge of quantum black hole nonlinear dynamics is needed, this however, has yet to happen. The search is on for a sufficient model in which the majority of people can agree on[1].

[1]For more information on current standings of gravitational wave echoes the reader is again refererred to [2] and [3] Chapter 5

Chapter 4

The Big Bang and the Black Hole

Chapter 2 introduces the critical model of a thermal big bang in which, using thermal fluctuations in a bimetric scenario, with the propagation speed of light and other massless particles being greater than that of gravity, an abrupt phase transition in the speed of sound in the early universe leads to nearly scale-invariant fluctuations. The successful prediction of n_s through this critical model takes the list of fine-tuned cosmological parameters in the SBB (or ΛCDM) model down from 6 to 5. Is there a possibility of checking more off this list with this model, namely the Dark Energy density $\frac{\rho_{DE}}{\rho_{crit}} = \Omega_\Lambda$? What is presented next is the core of this project, determining whether or not the model introduced above can provide non-standard black hole solutions that could potentially shed light on the cosmological constant problem, discussed in Chapter 3. In this case, the model introduced by Afshordi and Magueijo in [6] could be a three in one theory, giving a predictive model of the thermal big bang for the early universe, as well as a model for black holes in the current universe, and even possibly cosmic Dark Energy. It is not a coincidence that these are also the astrophysical processes where significant gaps in understanding are expected to be filled by a quantum theory of gravity.

In order to look for black hole solutions, I shall make similar assumptions to those that lead to the Schwarzschild solution.

4.1 The Characters

Recall that chapter 2.2 followed the referenced texts by using a metric with signature $[+---]$, here as is done in most cases when looking at black hole solutions, we adopt the $[-+++]$ signature and have applied a negative sign where necessary.

To recap, the final critical bimetric scalar field theory that was presented in Sec.2.4 has the matter metric (note the change in sign)

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - B(\phi)\partial_\mu\phi\partial_\nu\phi \tag{4.1}$$

such that $B > 0$ implies that the propagation speed of light (and other massless matter particles) is faster than gravity which is described by the gravity metric $g_{\mu\nu}$ (this has been referred to as the Einstein frame throughout this paper). The metrics $\hat{g}_{\mu\nu}$ and $g_{\mu\nu}$ are non-conformally related in order for the light cones to not coincide. The action is given as

$$\begin{aligned} S_\phi &= \int d^4x\sqrt{-\hat{g}}\frac{1}{B(\phi)} - \int d^4x\sqrt{-g}V(\phi) \\ &= \int d^4x\sqrt{-g}\left(\frac{M_P^2}{B_0\phi^2}\sqrt{1+2\frac{B_0\phi^2}{M_P^2}X} - V(\phi)\right), \end{aligned} \tag{4.2}$$

with $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ (note the change in sign) and $V(\phi) \simeq \frac{3}{4B_0}\ln^2(\frac{\phi}{M_P})$ for $\phi \gg M_P$ in the critical model. However. the potential $V(\phi)$ will remain unconstrained from early universe for smaller field values. It will also keep things aesthetically simple by letting $M_P = 1/\sqrt{8\pi}$ for the rest of the paper ($G = \hbar = c = 1$). To avoid the carrying the factor of $1/8\pi$ throughout, we redefine the constant $\tilde{B}_0 = \frac{B_0}{8\pi}$ and drop the tilde moving forward.

4.1.1 The Perfect Fluid

It is easy to see that, as long as the scalar field gradient is time-like, $X > 0$, its energy-momentum tensor has the form of a perfect fluid with isotropic pressure and density. If we define the k-essence kinetic term to be $K = \frac{1}{B_0\phi^2}\sqrt{1+2B_0\phi^2X}$, the stress-energy tensor can be calculated via Eqn. 2.5

$$T_{\mu\nu} = \frac{1}{\sqrt{1-B_0\phi^2\partial_\alpha\phi\partial^\alpha\phi}}\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}[\frac{1}{B_0\phi^2}\sqrt{1-B_0\phi^2\partial_\alpha\phi\partial^\alpha\phi} - V(\phi)], \tag{4.3}$$

and the pressure and density can then be calculated from Eqns. 2.6 and 2.7 to be respectively

$$P = \frac{1}{B_0\phi^2}\sqrt{1-B_0\phi^2\partial_\alpha\phi\partial^\alpha\phi} - V(\phi) \tag{4.4}$$

$$\rho = -\frac{1}{B_0\phi^2}\frac{1}{\sqrt{1-B_0\phi^2\partial_\alpha\phi\partial^\alpha\phi}} + V(\phi), \tag{4.5}$$

with a fluid four-velocity $u_\mu = -\frac{\nabla_\mu\phi}{\sqrt{2X}}$, such that in the rest frame (i.e with $u^\mu u_\mu = -1$ and $u_i = 0$) of a (locally) isotropic scalar field the above correspond to $T^0_0 = -\rho$ and $T^1_1 = P$.

4.1.2 The Metric

In order to look for black hole solutions, similar assumptions are made to those that lead to the Schwarzschild solution. A spherically symmetric and (quasi-)static spacetime in general relativity is assumed to have the gravitational metric $g_{\mu\nu}$ of the following form

$$ds^2 = -\exp\big[-2\int_r^\infty g(r)dr\big]dt^2 + \frac{dr^2}{1-\frac{2m(r)}{r}} + r^2 d\Omega^2, \tag{4.6}$$

where $g(r)$ represents the locally measured gravitational acceleration, pointing inwards for positive $g(r)$ and $m(r)$ is the total enclosed mass-energy within in a sphere of radius r [15]. The relevant Einstein tensor G^μ_ν components which will be of use are

$$G^0_0 = \frac{-2}{r^2}\frac{dm(r)}{dr}$$

$$G^1_1 = \frac{1}{r^3}\big\{\big[-4rm(r) + 2r^2\big]g(r) - 2m(r)\big\}$$

4.1.3 Field Equations

The first two Einstein-field equations are given to be

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho, \tag{4.7}$$

and

$$g(r) = \frac{4\pi r^3 P + m(r)}{r^2\big[1-\frac{2m(r)}{r}\big]}, \tag{4.8}$$

and a third as given by conservation of the energy-momentum tensor and Bianchi identities

$$\frac{dP}{dr} = -(\rho + P)g(r), \tag{4.9}$$

which is nothing other than the isotropic Tolman-Oppenheimer-Volkoff (TOV) equations [33] . The last equation, Eqn. 4.9, is the relativistic version of the hydrostatic equilibrium equation, indicating that pressure increases as we get deeper into the gravitational potential.

The goal is to now solve TOV equations, Eqns. 4.7, 4.8 and 4.9 analytically, and verify the findings with numerical solutions. In order to find the analytical solutions, the region of spacetime of interest can be broken into 3 separate regimes which will match at each boundary. The different regimes to look at are; the "faraway regime", the "luminal regime" and the "superluminal regime". An unknown regime which may account for a singularity is also included [see figure 4.1].

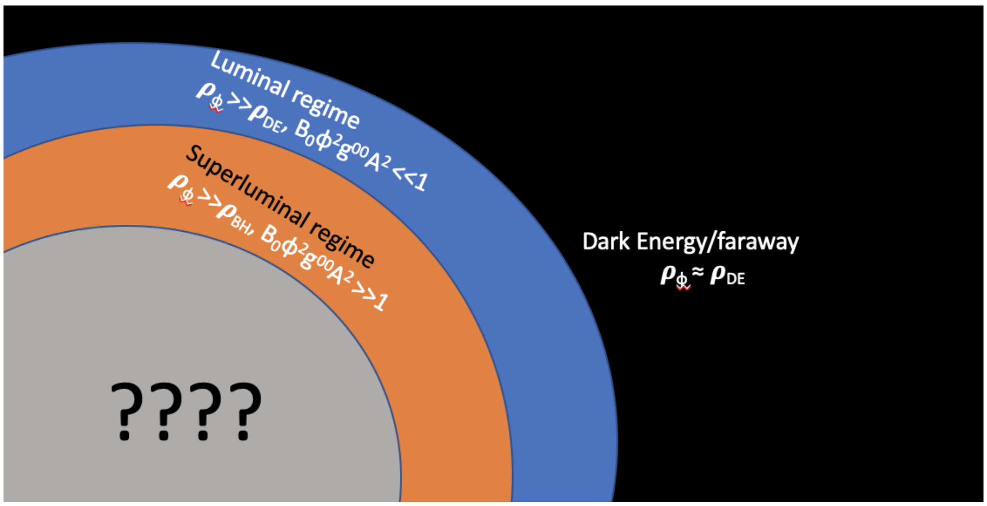

Figure 4.1: The three different regimes of the model, with a potential singularity.

For simplicity, the scalar field is chosen to only slowly evolve in time, with no spatial dependence so that $X = -\frac{1}{2}g^{00}\dot{\phi}$ and let $\dot{\phi} = A$ where A is a constant to be determined. This is equivalent to saying that the fluid is quasi-static in the rest frame of the black hole. This assumption, i.e. that the fluid does not accrete into the black hole differentiates our set-up from the standard one, and requires modifications of spacetime at/near the black hole horizon.

4.2 Far Far Away, $r \gtrsim 2m$

In the region far away from the black hole, $r \gtrsim 2m$ (but much smaller than the cosmological horizon), the metric should have a temporal component $g^{00} \to -1$ with a density and pressure expected to be comparable to those of the dark energy, denoted as ρ_{DE} and P_{DE} respectively. In this regime, indicated by $_\infty$ subscript, from Eqns.4.5 and 4.4, we shall find that the density and pressure can be determined as

$$\rho_\infty = -\frac{1}{B_0\phi^2}\frac{1}{\sqrt{1 + B_0\phi^2\dot{\phi}^2}} + V(\phi), \tag{4.10}$$

$$P_\infty = \frac{1}{B_0\phi^2}\sqrt{1 + B_0\phi^2\dot{\phi}^2} - V(\phi). \tag{4.11}$$

To be precise, the current observational 68%-level constraints on the density and pressure of dark energy is given by [18]

$$\rho_{DE} = \rho_\infty = (1.13 \pm 0.03) \times 10^{-123}, \tag{4.12}$$

$$|\rho_{DE} + P_{DE}| = |\rho_\infty + P_\infty| < 1.1 \times 10^{-124}. \tag{4.13}$$

Comparing this result with the expansion of 4.10 about small $\dot{\phi} = A$ then yields

$$\rho_\infty \simeq -\frac{1}{B_0\phi^2} + V(\phi) \quad \simeq \quad (1.13 \pm 0.03) \times 10^{-123}, \tag{4.14}$$

$$\rho_\infty + P_\infty \simeq A^2 \quad \lesssim \quad 1.1 \times 10^{-124}. \tag{4.15}$$

With the above equations for ρ_∞ and P_∞ and small A, Equations 4.5 and 4.4 can be expanded

$$\begin{aligned} \rho(r) &\simeq \frac{-1}{B_0\phi^2} + \frac{1}{2}g^{00}(r)A^2 + V(\phi) \\ P(r) &\simeq \frac{1}{B_0\phi^2} + \frac{1}{2}g^{00}(r)A^2 - V(\phi) \end{aligned} \tag{4.16}$$

and written in terms of the far away density and pressure

$$\begin{aligned} \rho(r) &= \frac{1}{2}(\rho_\infty - P_\infty) - \frac{1}{2}(\rho_\infty + P_\infty)g^{00}(r) \\ P(r) &= -\frac{1}{2}(\rho_\infty - P_\infty) - \frac{1}{2}(\rho_\infty + P_\infty)g^{00}(r) \end{aligned} \tag{4.17}$$

which will be of use later on. Some equation gymnastics between 4.10 and 4.11 allows one to write

$$V(\phi) = \rho_\infty + \frac{1}{B_0^2\phi^4}\frac{1}{[P_\infty + V(\phi)]}. \tag{4.18}$$

Substituting this into $\rho = -\frac{1}{B_0^2\phi^4}\frac{1}{[P+V(\phi)]} + V(\phi)$ (achieved through similar equation gymnastics between 4.4 and 4.5) and 4.7 results in

$$\frac{dm}{dr} = \frac{4\pi r^2}{B_0^2\phi^4}\Big(\rho_\infty B_0^2\phi^4 + \frac{(P - P_\infty)}{(P + V(\phi))(P_\infty + V(\phi))}\Big) \tag{4.19}$$

Eqn.4.4 with these substitutions becomes

$$\frac{dP}{dr} = \frac{-1}{B_0^2\phi^4}\Big(\frac{P - P_\infty}{(P + V)(P_\infty + V)} + B_0^2\phi^4 P + B_0^2\phi^4\rho_\infty\Big)g(r) \tag{4.20}$$

By moving in closer to the horizon of the black hole, the above equations can be further simplified and solved.

4.3 Luminal Regime, $\rho \simeq P$

As we saw in Eqn. 4.16 above, while the density and pressure of the scalar field approaches constants (i.e. those of dark energy) in the far away regime, they tend to diverge as we approach the black hole horizon, where $g^{00} \to \infty$. What happens first is that when

$$g^{00}(r) \gtrsim \frac{\rho_\infty - P_\infty}{\rho_\infty + P_\infty} \gtrsim 20, \tag{4.21}$$

we approach a regime where $\rho(r) \simeq P(r) \simeq A^2 g^{00}(r)/2$. We call this the "luminal regime". The equations 4.19 and 4.20 can be immediately simplified

$$\begin{aligned} \frac{dm}{dr} &\approx \frac{4\pi r^2}{B_0^2\phi^4}\left[\frac{P}{V^2(1+P/V(\phi))}\right] \\ &\approx \frac{4\pi r^2}{B_0^2\phi^4}\frac{P}{V(\phi)^2} \\ &= 4\pi r^2 P, \end{aligned} \tag{4.22}$$

and

$$\begin{aligned} \frac{dP}{dr} &\approx -\frac{1}{B_0^2\phi^2}\frac{2P}{V^2(\phi)}g(r) \\ &= -2P\left[\frac{4\pi r^3 P + m(r)}{r^2(1-\frac{2m(r)}{r})}\right]. \end{aligned} \tag{4.23}$$

With the above equations, 4.22 and 4.23 one can arrive at a single second order differential equation

$$m''(r) = -2m'\left[\frac{3m - r - rm'}{r^2(1-\frac{2m}{r})}\right] \tag{4.24}$$

We can further simplify Eqn. 4.24 by introducing κ that quantifies (negative coordinate) distance from the horizon

$$\kappa(r) \equiv m(r) - \frac{r}{2}. \tag{4.25}$$

Now, Eqn.4.24 can be written as

$$\begin{aligned} 2\kappa\kappa'' &= (1+2\kappa')\left[\frac{3\kappa}{r} + \kappa' + 1\right] \\ &\Downarrow \\ \frac{d\kappa'^2}{d\ln\kappa} &= (1+2\kappa')(\kappa'+1) \simeq (1+2\kappa')(\kappa'+1), \end{aligned} \tag{4.26}$$

where in the last step, we used $\frac{3\kappa}{r} \ll 1$, which is ensured in the luminal regime from Eqn. 4.21 (using the Schwarzschild metric with $g_{00} = 1 - \frac{2m}{r} = -\frac{2\kappa}{r}$). Integrating gives

$$\kappa = a\frac{(\kappa' + 1)^2}{(2\kappa' + 1)}, \tag{4.27}$$

where a is an integration constant to be determined.

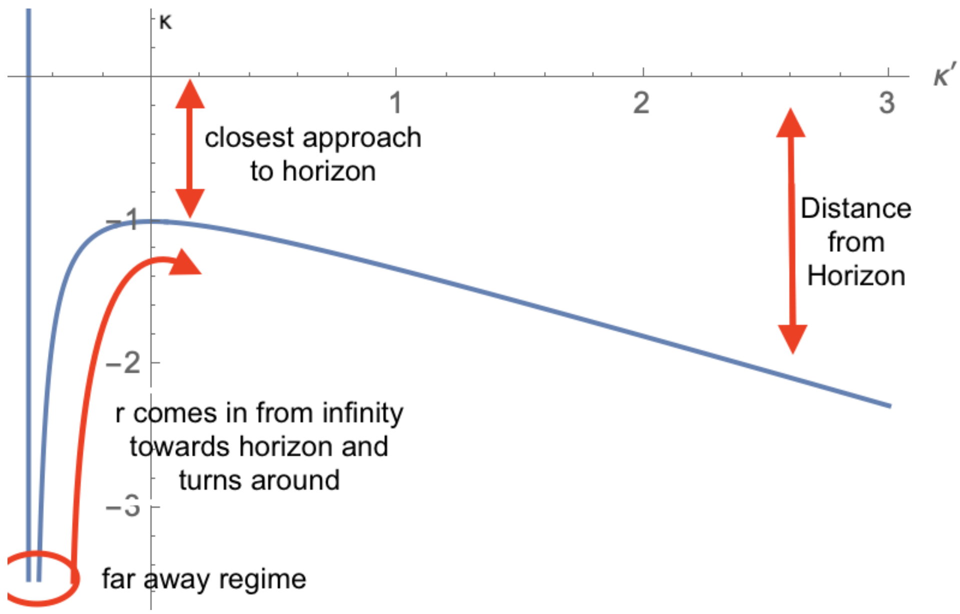

Figure 4.2: By introducing the parameter $\kappa(r) = m(r) - \frac{r}{2}$, representing the distance to the horizon, solutions can be determined within the luminal regime. In the far away regime, $(\kappa, \kappa') \to (-\infty, -\frac{1}{2})$. Moving into the luminal regime, κ approaches a maximum of a (set to -1 here, in Planck units) and begins to decrease as mass becomes negative.

In order to have regions of $r > 2m$, from $r = 2m - 2\kappa$ it is clear that κ needs to be negative, and thus $a < 0$.

In the case of the far away regime, we expect the Schwarzschild metric with $\kappa' = -\frac{1}{2}$ and the graph in Fig. 4.2 would simply be a vertical line up to $\kappa = 0$. We see that indeed the solution asymptotes to $\kappa' = -\frac{1}{2}$ as $r \simeq -2\kappa \to \infty$, i.e. the "far away" regime.

However, in the exact solution, as one approaches the horizon the backreaction of the field becomes important and so the deviations from Schwarzschild metric become significant. Therefore, coming in from the far away regime, κ increases toward zero but only ever makes to a maximum of a (see Fig.4.2). At this point, it then turns around and starts its descent to more negative values, eventually approaching the superluminal regime, which we shall discuss in the next section.

Through matching the solutions between the far away and the luminal regimes, the value of a can be related to the dark energy density and pressure:

$$a = -64\pi m_\infty^3 (\rho_\infty + P_\infty), \tag{4.28}$$

Here, m_∞ is the mass of the enclosed field/fluid and black hole in the far away regime.

It is interesting to compare this with a pressure integration constant p_0 found in [35] whose value is determined to be $p_0 = -\frac{1}{256\pi^2 \theta_P m^3}$ where θ_P, the so-called *Trans-Planckian parameter*, is a dimensionless constant that measures the maximum rest frame temperature of a source in units of Planck temperature and was conjectured to be $\mathscr{O}(1)$. With this assumption, the constant p_0, the gravitational aether pressure far from a black hole has a similar value to the pressure of dark energy for stellar mass black holes.

Similar to the above proposal, we may expect that the value of $|a| = \mathscr{O}(1)$, i.e. we need to approach to within a Planck length of the horizon for the scalar field backreaction to alter metric significantly. Plugging in for the dark energy density and pressure (Eqn. 4.12), we get:

$$a = -(0.179 \pm 0.005)\left(\frac{m_\infty}{100\ M_\odot}\right)^3 (1 + w_{DE}), \tag{4.29}$$

where $w_{DE} \equiv P_{DE}/\rho_{DE}$ is the equation of state of dark energy. We note that, while the coincidence of the stellar mass black hole scale, the dark energy scale, and $a = \mathscr{O}(1)$ is suggestive, realizing this scenario for a population of black holes with different masses and spins remains an open problem [35].

Let us get back to Eqn. 4.27, which can be solved in closed form to find the mass in terms of κ

$$m(r) = m_\infty + \frac{\kappa}{2} \pm \frac{1}{2}\sqrt{\kappa(\kappa - a)} \pm \frac{a}{4}\ln\left(\frac{2\sqrt{\kappa(\kappa-a)} - a + 2\kappa}{a}\right) + \frac{a}{4}\ln\left(\frac{4\kappa_*}{a}\right) - \frac{a}{4}, \tag{4.30}$$

where

$$\kappa_* \simeq m_\infty - r_*/2 \simeq -\frac{1}{2}(1 + w_{DE})m_\infty \tag{4.31}$$

denotes where we match the luminal to the far away (or Schwarzschild) regime (Eqn. 4.21). We switch from the plus to the minus sign solution, as κ' goes from negative to positive

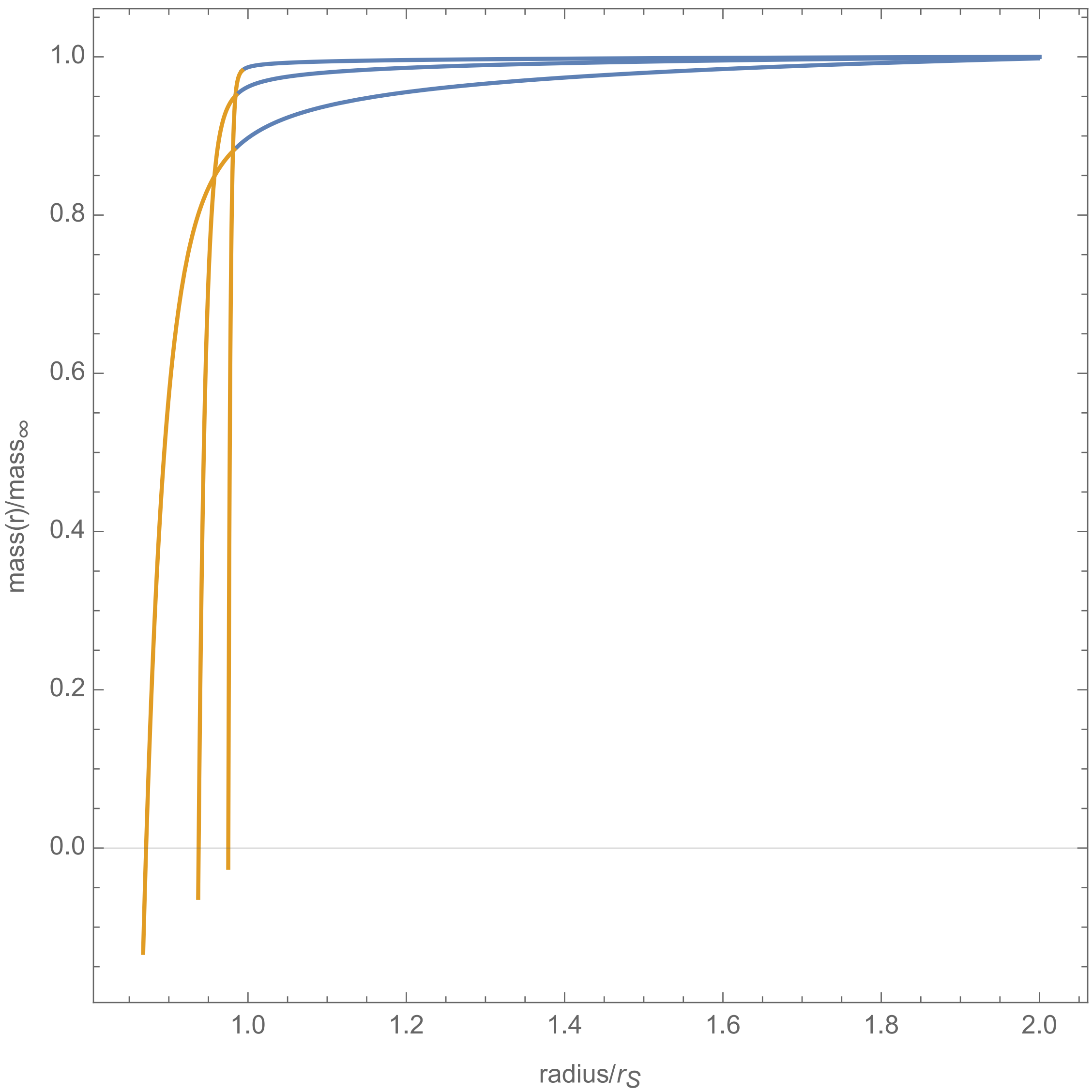

Figure 4.3: The analytic solutions for $m(r)/m_\infty$ (Eqn. A.11) as a function of $r/(2m_\infty)$ for three values of $m_\infty = 10, 30$ and 100, and $a = -1$ (in Planck units). Different colors indicates different branches of the solutions in Eqn. A.11. The near horizon-strucure appear on the scale of $r - 2m \sim |a|$, and that become sharper for larger black hole mass (or Schwarzschild radius, r_S).

values. Now, combining Eqn. A.11 with $r(\kappa) = 2m(\kappa) - 2\kappa$, we can show some examples of the resulting mass profiles in Fig. 4.3.

Furthermore, the density/pressure/metric can be found by computing $m' = \kappa' + 1/2$ and plugging into Eqs. 4.7 and 4.17

$$\rho \simeq P \simeq \frac{1}{2}(\rho_\infty + P_\infty) g^{00}(r) \simeq \frac{1}{4\pi r(\kappa)^2}\left(-\frac{1}{2} + \frac{\kappa \pm \sqrt{\kappa(\kappa - a)}}{a} \right) \tag{4.32}$$

It is interesting to note that there is a point on the $\kappa' > 0$ branch where mass becomes negative. From $m = \kappa + \frac{r}{2}$, so long as $\kappa > -\frac{r}{2}$, m remains positive (κ is a negative number). However, if $\kappa < -\frac{r}{2} \simeq -m_\infty$ the mass becomes negative, and indeed asymptotically diverges as:

$$m(r) \simeq -\frac{a^2}{8(1 + w_{DE})m_\infty} \exp\left[2 - \frac{2(r - 2m_\infty)}{|a|}\right]. \tag{4.33}$$

It is instructive to determine whether or not the solution in the luminal regime should permit a negative mass. To do so, we should check if one has already crossed into the superluminal regime at the crossing point $m(r) = 0$, $r = r_{\text{zero-mass}} \approx 2m_\infty$ and $\kappa = -\frac{1}{2} r_{\text{zero-mass}}$. Eqn. 4.32 then implies:

$$\rho(r_{\text{zero-mass}}) \simeq \frac{1}{2}(\rho_\infty + P_\infty) g^{00}(r_{\text{zero-mass}}) \simeq \frac{1}{8\pi |a| m_\infty} \simeq 4 \times 10^{-42} |a|^{-1} \left(\frac{m_\infty}{100\ M_\odot} \right)^{-1}. \tag{4.34}$$

However, as the speed of sound is given by $c_s^2 = 1 + 2BX = 1 + \frac{1}{2} B_0 \phi^2 (\rho_\infty + P_\infty) g^{00}(r)$, the propagation only becomes significantly superluminal when

$$\rho(r_{\text{super}}) \simeq \frac{1}{2}(\rho_\infty + P_\infty) g^{00}(r_{\text{super}}) \gg \frac{1}{B_0 \phi^2} \simeq 10^{-17} \phi^{-2} \tag{4.35}$$

Therefore, we see that for typical astrophysical black hole masses, and assuming ϕ and a of order unity, the mass becomes negative well within the luminal regime.

At what r then is one in the superluminal regime? To find this, we can take the radial derivative of Eqn. 4.33 and use Eqn. 4.7 to find density. The density crosses the superluminal threshold (4.35) at

$$r_{\text{super}} \simeq 2m_\infty - a + \frac{a}{2} \ln\left[\frac{64\pi(1 + w_{DE}) m_\infty^3}{|a| B_0 \phi^2} \right]. \tag{4.36}$$

Upon crossing into $r \lesssim r_{\text{super}}$, one has left the luminal, and entered the highly superluminal regime, which we shall discuss next.

4.4 Superluminal Regime

The superluminal regime is referred to as such because this is where $c_s^2 = 1 + B_0\phi^2 g^{00}\dot{\phi}^2 \gg 1$. It is easy to see that, deep into the superluminal regime, the density approaches a constant, while pressure is blowing up:

$$\begin{aligned} \rho \to \rho_{BB} \equiv V(\phi) \simeq \frac{1}{B_0\phi^2} - \rho_\infty \simeq \frac{1}{B_0\phi^2} \\ P \simeq \sqrt{\rho_{BB}(\rho_\infty + P_\infty)g^{00}(r)} \to \infty \end{aligned} \tag{4.37}$$

The subscript $_{BB}$ represents the notion that this density should be similar to the cosmic density at the end of the critical big bang phase (Chapter 2). This makes determining the mass equation 4.7 relatively simple

$$m(r) \simeq m_0 + \frac{4\pi}{3} r^3 \rho_{BB} \tag{4.38}$$

where the integration constant m_0 can be found by matching $m(r)$ from Eqn. 4.33 at the boundary of the superluminal regime 4.36, which yields:

$$m(r) \simeq 4\pi\rho_{BB} r_{\text{super}}^2 \left(r - r_{\text{super}} + \frac{a}{2}\right). \tag{4.39}$$

In the superluminal regime, we can now use the following approximations:

$$r_{\text{super}} \simeq 2m_\infty, \; |r - r_{\text{super}}| \ll r_{\text{super}}, \; |m(r)| \sim \rho_{BB}|a| r_{\text{super}}^2 \gg m_\infty, \tag{4.40}$$

to simplify the hydrostatic equilibrium Eqn. 4.9, which combined with Eqn. 4.39 yield:

$$\frac{dP}{dr} \simeq \frac{P(\rho_{BB} + P)}{\rho_{BB}\left[2(r - r_{BB}) + a\right]}. \tag{4.41}$$

If we set the boundary condition $P \simeq \rho_{BB}$, at the boundary of luminal and superluminal regimes, $r = r_{\text{super}}$, we find a closed-form solution:

$$P(r) \simeq \rho_{BB}\left\{2\left[\frac{2}{a}(r - r_{\text{super}}) + 1\right]^{-1/2} - 1\right\}^{-1}. \tag{4.42}$$

Plugging into Eqn. 4.4 using $X = -\frac{1}{2}\partial^\alpha\phi\partial_\alpha\phi = g^{00}(r)(\rho_{DE} + P_{DE})/\rho_{BB}$, we can also find $g^{00}(r)$:

$$g^{00}(r) \simeq \frac{\rho_{BB}}{\rho_{DE} + P_{DE}}\left\{\frac{4}{\left[-2 + \sqrt{\frac{2}{a}(r - r_{\text{super}}) + 1}\right]^2} - 1\right\}. \tag{4.43}$$

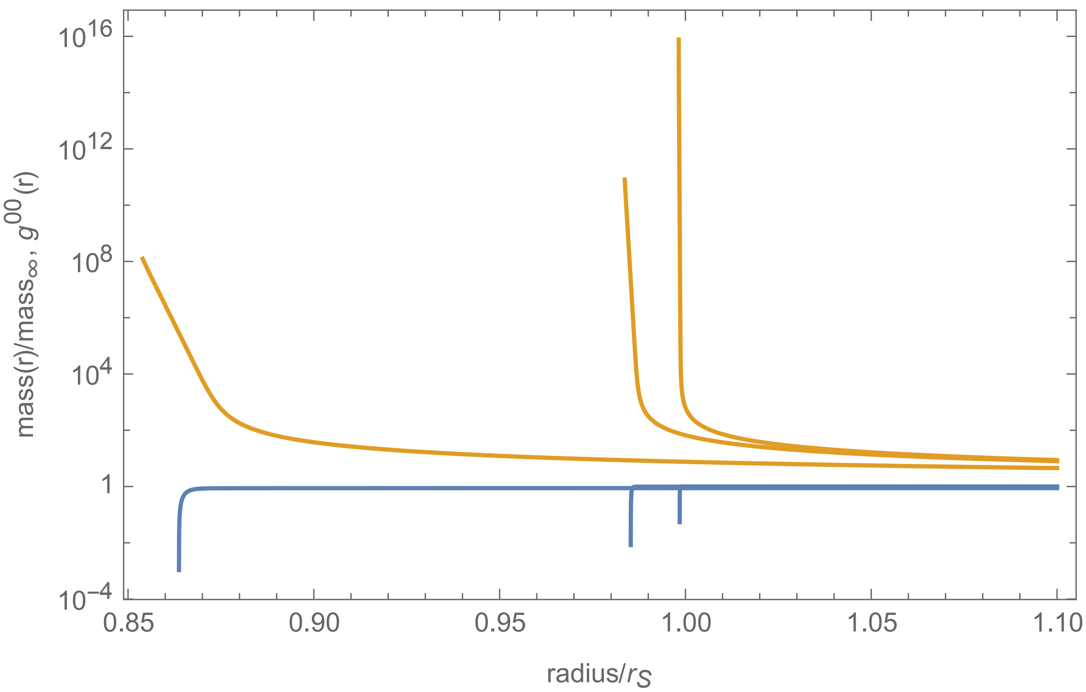

Figure 4.4: Numerical solutions to the TOV equations 4.7-4.9 near horizon for black hole masses of $10^2, 10^3$ and 10^4 Planck masses, and $-a = \rho_{BB} = 1$. The metric component $g^{00}(r)$ is plotted in orange, while the enclosed mass (normalized by its value at infinity) is in blue. As discussed in the text (also Fig. 4.3), the mass goes negative in the luminal regime, while the metric becomes singular deeper, within the superluminal regime. The divergence happens more abruptly (in scaled radius), and closer to the Schwarzschild radius, r_S for more massive black holes.

We notice that both pressure and the metric component $g^{00}(r)$ diverge at $r = r_{\rm super} - \frac{3}{2}|a|$. Since the density is constant, this implies that the Ricci scalar $\propto \rho - 3P$ also blows up at this radius, i.e. this is not just a coordinate singularity. Indeed, this pressure singularity is very much similar to what found around Gravitational Aether black holes of Section 3.1.3. Fig. 4.4 shows the emergence of this singularity in the superluminal regime. In the final section of this chapter, we shall speculate on the physical meaning and characteristic of this emergent singularity.

4.5 The Singularity/Firewall

As we saw in the previous section, solutions to the Einstein+field equations for the critical thermal Big Bang action lead to a singularity, just inside the Schwarzschild radius:

$$r_{\text{singularity}} \simeq 2m_\infty + \frac{a}{2} + \frac{a}{2}\ln\left[\frac{64\pi(1+w_{DE})m_\infty^3}{|a|B_0\phi^2}\right]. \tag{4.44}$$

While singularities are often thought of as pathologies, we remind the reader that Chapter 3 provides a comprehensive list of proposals, with motivations ranging from the Dark Energy to the Information Paradox, that suggest non-classical structure at/or near black hole horizons. This provides some fodder to think that maybe we should just as well embrace the singularity that we find here, a *firewall* for lack of better word.

However, perhaps our most surprising result is that while the Einstein metric diverges at $r_{\text{singularity}}$, the acoustic metric is actually non-singular everywhere. The results above have shown that there is a singularity when $P \to \infty$, however the acoustic metric will not see this. Recall the speed of sound is given as

$$\begin{aligned} c_s^2 &= 1 + B_0\phi^2\partial^\mu\phi\partial_\mu\phi \\ &\simeq B_0\phi^2 g^{00}A^2, \end{aligned}$$

in the superluminal regime. The acoustic metric is then given by

$$\begin{aligned} ds^2 &= -c_s^2 g_{00}dt^2 + g_{rr}dr^2 + r^2 d\Omega^2 \\ &= -\left(\frac{\rho_{DE}+P_{DE}}{\rho_{BB}}\right)dt^2 + \frac{dr^2}{1-\frac{2m(r)}{r}} + r^2 d\Omega^2, \end{aligned}$$

where $m(r)$ is given by Eqn. 4.38, and is also perfectly regular. This is reminiscent of how VSL solves the horizon problem in cosmology (Chapter 2). The same superluminal action that allowed communication across the cosmos at Big Bang, enables signal from inside the black hole horizons/firewalls. The fact this works for both Big Bang and Black Holes is a special characteristic of the anti-DBI, or cuscuton-like square-root action (Eqn. 4.2), which exhibits $c_s^2 \propto g^{00}$ leading to the regularity of the acoustic metric.

One may wonder the fate of an infalling observer in this peculiar scene. Although the singularity is avoided for the acoustic metric, the density of the superluminal regime is still extremely high. It is thought that an infalling observer would feel nothing out of the ordinary until they hit the Schwarzschild horizon with high density. Presumably, due to this intense density the outcome would not be a nice one, despite the fact the "singularity" is non-existent.

Chapter 5

Conclusion and Future Prospects

In this work, we introduced a general framework to unite approaches to a thermal tachyacoustic Big Bang, dark energy, and firewalls.

Starting with the motivation of finding an alternative theory to compete with cosmic inflation, varying speed of light theories have been giving inflation a run for its money. Thermal fluctuations within VSL theories eliminate the extra step of somehow thermalizing the quantum vacuum fluctuations for which inflation depends on for its predictability of n_S. By introducing a second metric for the matter which is non-conformally related to the normal gravity metric, bimetric VSL models provide an insightful mechanism for varying the speed of light in the early universe. As discussed in chapter 2.2, this alone is not enough to explain large scale structure. To mend the issue the bimetric VSL model face, it is then suggested one look at the speedy sound as the new VSL. Within this model a bimetric VSL theory with a sufficiently fast phase transition for the speed of sound and thermal fluctuations can predict a near scale invariant n_s. The exact scale invariance of the spectral index can never be reached due to a seemingly overlooked discontinuity. This discontinuity was addressed by N. Afshordi and J. Magueijo and leads to a critical model of a thermal big bang also called tachyacoustic Big Bang). The critical model makes a prediction for n_s which is in strong agreement with the observed spectral index without the need of any adjustments or tweaking.

In hopes of having a consistent and predictable theory with perhaps only four unknown parameters (in contrast to the SBBs 6 mentioned in the introduction) the tachyacoustic big bang model should give insight into the dark energy problem and perhaps offer a prediction for Ω_{DE}. Introduced in 3 were some current models which attempt to do just that, these included the cosmological constant, quintessence and gravitational aether. The cosmological constant is probably the most well known option for dark energy, however it faces the critique of still needing to add in by hand some adjustments in order to match observations (*the cosmological constant problem*). Quintessence offers up a solution by suggesting that dark energy can be explained by a dynamic scalar field with a energy density which may vary over time eventually becoming 0. Unfortunately the scalar field is at the mercy of renormalization which predicts a high mass for the quintessence field which does not agree with observations so far. The last model that is introduced is the gravitational aether model which suggests a modification to the right side of Einsteins field equations. If interpreted as a thermodynamic description of gravity, formation of stellar black holes in this theory could be an explanation of the cosmic acceleration. Further interest in the gravitational aether model sparks the question of whether or not it can permit static black hole solutions. It turns out that the gravitational aether couples metric solutions found close to the horizon with metric solutions far from the horizon, opting for placing responsibility of cosmic acceleration on formation stellar mass black holes.

Lastly, in building up to the feature research presented in this work, we took a brief look at possible options for quantum black holes and how they tackle the information paradox. It has been questioned for a while about what happens to information that falls into a black hole after it has fully evaporated. Is the information that found its way past the black hole horizon banished from the universe forever? or will it make it back out from the depths of the black hole? Black hole complemenarity, firewalls and gravastars are just a couple options for finding our way around this paradox.

Gravitational wave echoes have been suggested to be a "smoking gun" [3] for any such near-horizon modifications to black holes. If there are any modifications to what is predicted from a standard black hole, the gravitational waves should be followed by delayed repeating echoes, thus providing insight into what more could be happening at the black hole horizon.

Finally, the feature work is presented. Black hole solutions to the Einstein field equations with an energy-momentum tensor in the form of a perfect fluid as given by the action of the critical model for the thermal big bang 4.2 are sought out. Using similar assumptions to those which lead to the Schwarzschild solution, the metric in the Einstein frame is assumed to be spherically symmetric and quasi-static given by 4.6. Solving the resulting field equations is made simpler if the the near-horizon structure of black hole horizons can be split into three regimes:

1. Far away: Here we have regular GR solutions, with the pressure of Dark Energy approximately negative its density $P \simeq -\rho$

2. Luminal Regime: Density and Pressure of dark energy are approximately the same: $P \simeq \rho$. Deep into the luminal regime, the gravity of Dark Energy dominates over that of the black hole

3. Superluminal regime: Here the speed of sound becomes much larger than 1. Solutions here show that deep in this regime, there is a singularity in the classical metric. Peculiarly however, we discovered that the acoustic metric remains well-behaved and sees no such singularity.

The singularity as seen by the classical metric could be viewed as a firewall, which would also eliminate the information paradox all together. Furthermore, if this is the case, observations of gravitational wave echoes [3] could probe this near horizon structure and further credit or discredit the proposal. Simple preliminary estimates suggest that the model has an echo time-delay of:

$$\Delta t_{\text{echo}} \simeq 4m_\infty \ln(m_\infty/|a|) \simeq 4m_\infty \ln\left(\frac{m_\infty^2}{\rho_{DE} + P_{DE}}\right), \tag{5.1}$$

connecting the equation of state of dark energy with gravitational wave observations of gravitational wave echoes.

We also note that this model has only considered a single black hole. Future interest may lie in determining the state of the model with multiple black holes.

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APPENDIX

Appendix A

Detailed Calculations For Chapter 4

In order to go from eqn.4.22 and 4.23 to 4.24, isolate for P in 4.22

$$P = \frac{1}{4\pi r^2}\frac{dm}{dr} \tag{A.1}$$

Play with 4.23 to get

$$\begin{aligned}
\frac{1}{P}\frac{dP}{dr} &= -2\left(\frac{4\pi r^3 P + m(r)}{r^2(1-\frac{2m(r)}{r})}\right) \\
\frac{d\ln P}{dr} &= -2\left(\frac{4\pi r^3 P + m(r)}{r^2(1-\frac{2m(r)}{r})}\right)
\end{aligned} \tag{A.2}$$

Combining the two

$$\begin{aligned}
\frac{d}{dr}(\ln\frac{dm}{dr} - \ln(4\pi r^2)) &= -2\left(\frac{4\pi r^3 P + m(r)}{r^2(1-\frac{2m(r)}{r})}\right) \\
m''(r) &= -2m'(r)\frac{-rm'(r) + 3m(r) - r}{r^2(1-2m(r)/r)}
\end{aligned} \tag{A.3}$$

The equation 4.25 gives

$$\begin{aligned}
m(r) &= \kappa(r) + \frac{r}{2} \\
m'(r) &= \kappa'(r) + \frac{1}{2} \\
m''(r) &= \kappa''(r)
\end{aligned} \tag{A.4}$$

Plugging this into eqn.4.24 gives

$$\begin{aligned}
\kappa''(r) &= -(1+2\kappa')\left(\frac{3(\frac{r}{2}+\kappa)+r(\frac{1}{2}+\kappa')-r}{-2r\kappa}\right) \\
&= (1+2\kappa')\left(\frac{\frac{r}{2}+3\kappa+r(\frac{1}{2}+\kappa')}{2r\kappa}\right) \\
2\kappa\kappa'' &= (1+2\kappa')\left[\frac{3\kappa}{r}+\kappa'+1\right]
\end{aligned} \tag{A.5}$$

Then by writing

$$\kappa'' = \frac{d\kappa'}{dr} = \frac{d\kappa}{dr}\frac{d\kappa'}{d\kappa} = \kappa'\frac{d\kappa'}{d\kappa} = \frac{1}{2}\frac{d\kappa'^2}{d\kappa}$$

the left hand side of A.5 can be written

$$2\kappa\kappa'' = 2\kappa(\frac{1}{2}\frac{d\kappa'^2}{d\kappa}) = \kappa\frac{d\kappa'^2}{d\kappa} = \frac{d\kappa'^2}{d\ln\kappa}$$

using $\frac{d\ln\kappa}{d\kappa} = \frac{1}{\kappa}$, so

$$\frac{d\kappa'^2}{d\ln\kappa} = (1+2\kappa')\left[\frac{3\kappa}{r}+\kappa'+1\right]$$

Integrating this results in

$$\begin{aligned}
\ln\kappa &= 2\ln(\kappa'+1) - \ln(2\kappa'+1) + const \\
\kappa &= a\frac{(\kappa'+1)^2}{(2\kappa'+1)}
\end{aligned} \tag{A.6}$$

and a is the integration constant introduced in the process. The value of a can be determined by going into the Schwarzschild radius with $m'(r) = \kappa'(r) + \frac{1}{2}$ on the left hand side of 4.22, $P = \rho$. Using eqn 4.17 with the term involving g^{00} dominating over the other, and $g^{00} = -(1-2m_\infty/r)^{-1}$

$$\begin{aligned}
\frac{dm}{dr} &= 4\pi r^2\rho \\
\frac{2\kappa'(r)+1}{2} &= 4\pi r^2\left(\frac{-\frac{1}{2}(\rho_\infty+P_\infty)}{-(1-\frac{2m_\infty}{r})}\right)
\end{aligned}$$

From A.6 notice $2\kappa'(r) + 1 = a\frac{(\kappa'+1)^2}{\kappa}$, and $\kappa = m_\infty - \frac{r}{2}$, $\kappa' = -1/2$. All together this gives

$$\begin{aligned} \frac{a(\frac{1}{4})}{m_\infty - r/2} &= 4\pi r^2 \frac{(\rho_\infty + P_\infty)}{1 - \frac{2m_\infty}{r}} \\ a &= -8\pi r^3 \frac{(\rho_\infty + P_\infty)}{1 - \frac{2m_\infty}{r}} (1 - \frac{2m_\infty}{r}) \\ a &= -8\pi r^3 (\rho_\infty + P_\infty) \end{aligned} \tag{A.7}$$

Then with $r = 2m_\infty$ the value of a is

$$a = -64\pi m_\infty^3 (\rho_\infty + P_\infty) \tag{A.8}$$

which is negative as expected.

The solution to A.6 is found via Mathematica [21] and has two results

$$\begin{aligned} r_+ &= -\kappa(r) + a\frac{\sqrt{\kappa(r)(\kappa(r) - a)}}{a} + \frac{a}{2}\ln\left(2\sqrt{\kappa(r)(\kappa(r) - a)} - a + 2\kappa(r)\right) + ab \\ r_- &= -\kappa(r) - a\frac{\sqrt{\kappa(r)(\kappa(r) - a)}}{a} - \frac{a}{2}\ln\left(2\sqrt{\kappa(r)(\kappa(r) - a)} - a + 2\kappa(r)\right) + ab \end{aligned} \tag{A.9}$$

and with $r = 2m - 2\kappa$

$$\begin{aligned} m(r) &= \frac{\kappa}{2} + \frac{1}{2}\sqrt{\kappa(\kappa - a)} + \frac{a}{4}\ln\left(2\sqrt{\kappa(\kappa - a)} - a + 2\kappa\right) + \frac{ab_1}{2} \\ m(r) &= \frac{\kappa}{2} - \frac{1}{2}\sqrt{\kappa(\kappa - a)} - \frac{a}{4}\ln\left(2\sqrt{\kappa(\kappa - a)} - a + 2\kappa\right) + \frac{ab_2}{2} \end{aligned} \tag{A.10}$$

Where b are new integration constants which can be determined by matching these two solutions when $\kappa = a$ and $\kappa = \infty$ at resulting in

$$m(r) = m_\infty + \frac{\kappa}{2} \pm \frac{1}{2}\sqrt{\kappa(\kappa - a)} \pm \frac{a}{4}\ln\left(\frac{2\sqrt{\kappa(\kappa - a)} - a + 2\kappa}{a}\right) + \frac{a}{4}\ln\left(\frac{4\kappa_*}{a}\right) - \frac{a}{4}, \tag{A.11}$$

where

$$\kappa_* \simeq m_\infty - r_*/2 \simeq -\frac{1}{2}(1 + w_{DE})m_\infty \tag{A.12}$$

r_* is where the matching happens and the k_* is from the condition for luminal condition 4.21.

From the above two solutions $m(\kappa)$ and $r(\kappa)$ one can determine the pressure from the simple fact that $\frac{dm}{dr} = 4\pi r^2 P$, and $\frac{dm}{dr} = \frac{dm}{d\kappa}\frac{d\kappa}{dr}$ with

$$\frac{dm}{d\kappa} = \frac{1}{2}(1 + \frac{\kappa}{\sqrt{\kappa(\kappa - a)}})$$

$$\frac{dr}{d\kappa} = -1 + \frac{\kappa}{\sqrt{\kappa(\kappa - a)}}$$

This yields

$$\begin{aligned}\frac{dm}{dr} &= -\frac{1}{2} + \frac{k + \sqrt{k(k-a)}}{a} \\ &= -\frac{1}{2} + \frac{m - r/2 + \sqrt{(m - r/2)(m - r/2 - a)}}{a}\end{aligned}$$

equating this with $4\pi r^2 P$ results in the equation for pressure (and density since $\rho \approx P$

$$P = \frac{1}{4\pi r^2}\left(-\frac{1}{2} + \frac{m - r/2 + \sqrt{(m - r/2)(m - r/2 - a)}}{a}\right) \tag{A.13}$$

We can also determine the metric component from $\rho \simeq \frac{1}{2}(\rho_\infty + P_\infty)g^{00}(r)$

Expansion of $m_-(\kappa)$ around $a = 0$ above with $\kappa = m - r/2$, gives

$$m(r) = \frac{1}{4}a(-2\log(-a) + \log(r - 2m(r)) + \log(-k_*) - 2 + 3\log(2)) + \left(m + m_\infty - \frac{r}{2}\right) \tag{A.14}$$

Solving for mass gives

$$m(r) \simeq \frac{-a^2}{8(1 + w_{DE})m_\infty} e^{2 - \frac{2(r - 2m_\infty)}{|a|}} \tag{A.15}$$

To determine whether or not a negative mass should be permitted by the luminal solutions, check if at the point of $m = 0$, $r = r_{zero-mass} \approx 2m_\infty$, $\kappa = -\frac{r_{zero-mass}}{2}$ one has already crossed into the super luminal regime. We can determine what the density at this point should be

$$\begin{aligned}\rho(r_{zero-mass}) &\simeq \frac{1}{4\pi r_{zero-mass}^2}(-\frac{1}{2} + \frac{(-m_\infty) - \sqrt{(-m_\infty)((-m_\infty) - a)}}{a}) \\ &\simeq \frac{1}{8\pi |a| m_\infty}\end{aligned} \tag{A.16}$$

Speed of sound is $c_s^2 = 1 + B_0\phi^2 A^2 g^{00} = 1 + B_0\phi^2(\rho_\infty + P_\infty)g^{00}$ which needs to be larger than one for the superluminal regime and so we see that the density in the superluminal regime should be

$$\rho(r_{super}) \simeq 1/2(\rho_\infty + P_\infty)g^{00} >> \frac{1}{B_0\phi^2} \tag{A.17}$$

This density at zero-mass is smaller than what one would expect for ρ_{BB}, thus nothing is stopping r from becoming less than $r_{zer-mass}$ and thus mass becomes negative. Now taking radial derivative of A.15 gives

$$\frac{dm}{dr} \simeq \frac{-a}{4(1+w_{DE})m_\infty} e^{2-\frac{2(r-2m_\infty)}{|a|}} \tag{A.18}$$

then from $\frac{dm}{dr} = 4\pi r^2\rho$ when $\rho = \rho(r_{super})$ we can find that the radius at which we cross into the superluminal regime is

$$\begin{aligned} \frac{-a}{4(1+w_{DE})m_\infty} e^{2-\frac{2(r-2m_\infty)}{|a|}} &= 4\pi\rho_{BB}r_{sup}^2 \simeq 4\pi\rho_{sup}(2m_{\text{inf})^2} \\ r_{sup} &\simeq 2m_\infty - a + \frac{a}{2}\ln\left[\frac{64\pi(1+w_{DE})m_\infty^3}{|a|B_0\phi^2}\right] \end{aligned} \tag{A.19}$$

In the Superluminal Regime we can easily solve $\frac{dm}{dr} = 4\pi r^2\rho_{BB}$ to give

$$m(r) \simeq m_0 + \frac{4\pi}{3}r^3\rho_{BB} \tag{A.20}$$

Matching with A.15 at $r = r_{sup}$

$$\frac{-a^2}{8(1+w_{DE})m_\infty} e^{2-\frac{2(r_{sup}-2m_\infty)}{|a|}} - \frac{4\pi}{3}r_{sup}^3\rho_{BB} \simeq m_0 \tag{A.21}$$

g

$$m(r) \simeq 4\pi\rho_{BB}r_{sup}^2(r - r_{sup} + \frac{a}{2}) \tag{A.22}$$

Then $\frac{dP}{dr} = -(\rho + P)g(r)$ can be solved with the following assumptions $r_{sup} \approx 2m_\infty, |r - r_{sup}| << r_{sup}, m \sim \rho_{BB}|a|r_{sup}^2 >> m_\infty$ with this we see that the $4\pi r^3 P$ term in numerator of g(r) is dominant, and we have $\rho = \rho_{BB}$ we can write

$$
\begin{aligned}
\frac{dP}{dr} &= -(\rho_{BB}+P)\frac{4\pi(2m_\infty)^3 P}{-2m(r)(2m_\infty)} \\
&= (\rho_{BB}+P)\frac{4\pi(2m_\infty)^2 P}{2(4\pi\rho_{BB}r_{sup}^2(r-r_{sup}+a/2))} \\
&= (\rho_{BB}+P)\frac{P}{2\rho_{BB}(r-r_{sup}+a/2)}
\end{aligned}
\tag{A.23}
$$

Solving this

$$
\begin{aligned}
\frac{1}{P(\rho_{BB}+P)}dP &= \frac{dr}{2\rho_{BB}(r-r_{sup}+a/2)} \\
P(r) &\simeq \rho_{BB}\left\{2[\frac{2}{a}(r-r_{sup}+1]^{-1/2}-1\right\}^{-1}
\end{aligned}
\tag{A.24}
$$

Where we've matched this at the boundary of the luminal and superluminal regime where $P=\rho_{BB}$ and $r=r_{sup}$.

With our pressure this pressure and from $P=\frac{1}{B_0\phi^2}\sqrt{1-B_0\phi^2\partial_\alpha\phi\partial^\alpha\phi}-V(\phi)$ where here we can write $\rho_{BB}=V(\phi)\approx\frac{1}{B_0\phi^2}$ and $A^2=(P_{DE}+\rho_{DE})$ we find that

$$
P \simeq \rho_{BB}\sqrt{1-\frac{(P_{DE}+\rho_{DE})g^{00}(r)}{\rho_{BB}}}
\tag{A.25}
$$

Matching this with A.13 and solving for $g^{00}(r)$ gives

$$
\begin{aligned}
\rho_{BB}\left\{2[\frac{2}{a}(r-r_{sup}+1]^{-1/2}-1\right\}^{-1} &= \rho_{BB}\sqrt{1-\frac{(P_{DE}+\rho_{DE})g^{00}(r)}{\rho_{BB}}} \\
\left\{2[\frac{2}{a}(r-r_{sup}+1]^{-1/2}-1\right\}^{-2} &= 1-\frac{(P_{DE}+\rho_{DE})g^{00}(r)}{\rho_{BB}} \\
\frac{(P_{DE}+\rho_{DE})g^{00}(r)}{\rho_{BB}} &= 1-\left\{2[\frac{2}{a}(r-r_{sup}+1]^{-1/2}-1\right\}^{-2} \\
g^{00}(r) &= \frac{\rho_{BB}}{(P_{DE}+\rho_{DE})}\left\{1-\left\{2[\frac{2}{a}(r-r_{sup}+1]^{-1/2}-1\right\}^{-2}\right\}
\end{aligned}
\tag{A.26}
$$

So

$$g^{00}(r) \simeq \frac{\rho_{BB}}{\rho_{DE}+P_{DE}}\left\{\frac{4}{\left[-2+\sqrt{\frac{2}{a}(r-r_{\text{super}})+1}\right]^2}-1\right\} \tag{A.27}$$

A singular point is obvious at $r = r_{sup} - 3/2|a|$, the Ricci scalar $R \propto (\rho - 3P)$ will blow up when P blows up.

$$\begin{aligned} r_{sing} &\simeq 2m_\infty - a + \frac{a}{2}\ln\left[\frac{64\pi(1+w_{DE})m_\infty^3}{|a|B_0\phi^2}\right] - 3/2|a| \\ &\simeq 2m_\infty + \frac{a}{2} + \frac{a}{2}\ln\left[\frac{64\pi(1+w_{DE})m_\infty^3}{|a|B_0\phi^2}\right] \end{aligned} \tag{A.28}$$

Echo time Delay can be roughly estimated. The closest we can get to the horizon is a, making this the cut off

$$\begin{aligned} \Delta t_{echo} &= 2\times\int g^{rr}dr \\ &= 2\times\int_{3m_\infty}^{|a|}\frac{1}{1-2m_\infty/r}dr \\ &\simeq 4m_\infty\ln[\frac{2m_\infty}{|a|}] \end{aligned} \tag{A.29}$$

and $a \propto m_\infty^3(\rho_{DE}+P_{DE})$so $\Delta t_{echo} \simeq 4m_\infty \ln[\frac{m\infty^2}{(\rho_{DE}+P_{DE})}]$ and so if we know the dark energy density and pressure we could estimate the echo time we would be searching for.

Numerical Solutions: Using mathematica NDSolve command, we solve

$$\frac{dm}{dr} = 4\pi r^2(-\frac{1}{\sqrt{1+g^{00}(r)A^2}}+1)$$

and

$$\begin{aligned} g^{00} &= e^{\int g(r)dr} \\ \ln g^{00} &= \int g(r)dr \\ \frac{d\ln g^{00}}{dr} &= g(r) \\ \frac{1}{g^{00}}\frac{dg^{00}}{dr} &= g(r) \end{aligned}$$

$$
\begin{aligned}
\frac{dg^{00}}{dr} &= -g^{00}g(r) \\
&= -g^{00}\frac{4\pi r^3\left[\frac{1}{B_0\phi^2}\sqrt{1+B_0\phi^2 g^{00}\dot{\phi}^2}-V(\phi)\right]+m(r)}{r^2\left[1-\frac{2m(r)}{r}\right]} \\
&= -2g^{00}\frac{(4\pi r^3\sqrt{1+g^{00}A^2}-1)+m(r)}{r^2(1-2m(r)/r)}
\end{aligned}
\tag{A.30}
$$

With $B_0 \approx -a = 1$, $A = \sqrt{\frac{1}{64\pi {m_\infty}^3}} = 2 \times 10^{-4}$.