An Architectural Implementation of Topology Optimization Guided Discrete Structures with Customized Geometric Constraints

by

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A thesis presented to the University of Waterloo in fulfillment of the thesis requirements for the degree of Master of Architecture

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
Abstract

This thesis explores the use of Topology Optimization (abbreviated to TO) in architectural design by implementing a Bidirectional Evolutionary Structural Optimization (abbreviated to BESO) type TO script as a guide to create a composition of discrete members with complex geometries. TO is an efficient tool for generating an optimal spatial arrangement of structural members along a load path. In the field of computational design, TO has been employed for form-generation of a range of assembled structures that employ discrete units, as well as continuum structures that employ unified and continuous materials. The most advanced current architectural implementations for continuum structures appear in the design of connections, and for discrete structures within space truss designs. Yet, the use of TO in atypical discrete frame structures with complex geometries remain relatively undeveloped in contemporary practice.

This thesis contributes a case study where TO is implemented at two key scales: at the component level, geometrically constrained discrete components are assembled using TO, at the macro level, these components are arranged over a TO-designed body. A review of literature from computational design and structural engineering fields, discussing current TO implementations, as well as presenting case studies, is included.

The demonstration within the thesis presents a contemporary architectural design process by using existing Karamba BESO code components within a Grasshopper parametric script. Fine-grained components employed within the facade system are combined using TO to produce a cellular lattice architectonic assembly that refers to traditional Korean ornamental pattern found near the site. This demonstration is evaluated structurally and aesthetically. Analyses of comparative structural models with varying configurations are used to demonstrate the structural efficiency of the proposed design. For the aesthetic evaluation, a series of drawings are included to
demonstrate what type of spatial qualities the customized lattice structure would look like.

The goal of this thesis is to demonstrate architectural and structural qualities resulting from a hybrid exercise where a TO process is applied to geometrically constrained discrete structures. The approach in this thesis provides compromises where structural efficiency and aesthetics are both reasonably achieved, and may lead to novel designs. Future work could be to create a new TO algorithm that can automate this process for increased structural efficiency.
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Also, thank you to Elizabeth English, without your introduction of me to the Structure Certificate program, this research would not have been made.
Dedication

For my husband Richard Mui, who has shown me the greatest support with continuing love then and now.

For my parents Youngsun Yeo and Sungeun Woo and my sisters Deumji Woo and Hojung Woo, without whom nothing would be possible.
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Part 1.

Introduction
Topology optimization (abbreviated to TO) is a computational tool used to derive an optimal layout of material based on a given set of forces and support conditions. Originally, it was employed in the mechanical engineering field for making medical devices or aircraft elements due to their demand on creating lighter structures. In collaboration with Altair, aircraft company Airbus proposed a TO generated structural cross section of an airplane wing that provided a significant weight reduction (fig. 1.1). In architectural design, TO has been used to generate organic structural elements. A prime example of this is the 2011 Qatar National Convention Center, a convention center located in Doha, Qatar designed by Priztker prize laureate Arata Isozaki. (fig.1.2) The most prominent feature of the design is a pair of massive tree-like organic structures that support the roof over the main facade. These structural elements were the product of a type of topology optimization algorithm known as Evolutionary Structural Optimization (abbreviated to ESO). This project also represents one of the first times any TO algorithm had been used to generate architectural form. Yet, since then, there are still only a handful of realized examples. Thus, the goal of my thesis is to push forward the development of TO’s applicability to architecture by contributing a case study where TO is involved in an architectural design process of structural systems.

This thesis presents relevant theoretical background information on Topology Optimization algorithms and their implementation in current practice. The TO process varies depending on the mathematical approach, but schematically they all consist of three parts: application of finite element analysis, sensitivity analysis, and optimization criteria that drive an iterative feedback loop. The core of any TO algorithm process is an objective function, which is a mathematical function that aims to represent the value and configuration of the elements to be optimized. Depending on the methods, the design variable in the objective function varies. Solid Isotropic Material with Penalization
method (abbreviated to SIMP), the most classical approach, considers a density associated stiffness as a design variable. The SIMP method was initially proposed by Bendsoe and Kikuchi (1988) in the book, Topology Optimization: Theory, Methods, and Applications. Through an iterative optimization process, material in the domain is removed to achieve the optimal form. There are intermediate density values penalized to steer the solution towards a pure bifurcation between solid and void. An alternative topology optimization method is Bi-directional Evolutionary Structural Optimization (abbreviated to BESO), which is widely used in most commercial software. As its name suggests, this algorithm either removes or adds material to evolve the structure to an optimum. Unlike SIMP which has an intermediate density, BESO treats the design variable as a discrete variable which takes a value of either ‘1’ when an element is present or ‘0’ when completely removed (fig.1.3).

Due to the inherent difference in consideration of the discrete function between SIMP and BESO methods, the types of structures in which these methods are applied are also different: one is continuum structures and the other is discrete structures. Continuum structures refer to unified structures that employ continuous materials with isotropic characteristics, while discrete structures refer to assembled structures that employ discrete units. For continuum structures, SIMP methods are mostly used, while BESO methods are preferred in most cases for discrete structures.

Recent examples of continuum structures include research projects from ETH Zurich where Topology Optimization was used to find smooth forms of continuum structural components. Their Digital Building Technology lab focuses on creating design with up-to-date digital fabrication tools. Smart Slab, a project from the lab, is a lightweight concrete slab, displaying three-dimensional geometric differentiation on multiple scales. This is the first concrete slab fabricated with a TO algorithm generated 3D-printed formwork. Another example from ETH Zurich is the series of 3D printed columns called Concrete Choreography (fig.1.4) that were used as a stage set for a dance festival in the middle of the Alps. Unlike Smart Slab, these columns are printed hollow with filling, so formwork was not required. When it comes to examples of discrete structure design using Topology Optimization, there are not many realized projects. Some of them are on-going research, while others exist only at the conceptual stage.

The majority of research in discrete structures using the TO algorithm focuses on truss design. This is inevitable because the triangular shape of truss bays provide the most efficient load path, and as a result, the outcome of TO follows funicular forms. Yet some framing systems are not primarily interested in structural performance. Especially, when the structural members are exposed to be appreciated by building users, other parameters such as: cultural, social and aesthetic aspects govern design decisions. Exoskeleton structures are load bearing systems, which are typically hybrid lateral and gravity force resisting structures, that are prominently featured on the building’s exterior. The Centre Pompidou in Paris, France by then architecture firm Piano and Rogers uses an exoskeleton to create its culture machine aesthetic, as well as freeing up space for flexible galleries on its interior. (fig. 1.6) More recently, designers practicing the Parametric Architecture have used exoskeleton structures to create a computationally derived aesthetic. One Thousand Museum, a residential tower by Zaha Hadid architects, prominently features a reinforced concrete exoskeleton that shows members thickening and thinning in reflection of structural demand. Sometimes these exoskeletons are geometrically constrained. Exposed Vierendeel frames are a prime example of these types of structures.
A Vierendeel frame consists of members joined in orthogonal bays because of an architectural desire to have an unobstructed view (fig. 1.7). For this reason, the joints in a Vierendeel frame are required to provide a significant amount of moment resistance, which is always less efficient from an engineering standpoint. In this thesis, these types of suboptimal systems are defined as atypical structures. The choice to use these atypical structure systems is typically driven by the architectural parameters that are determined by the designer. These parameters are not general and vary by projects; sometimes they are based on the site, and sometimes they are based on the cultural and social dimensions. Topology Optimization can be used by designers to augment their creative agency by mediating between structural and architectural criteria in an atypical structure. Therefore, my thesis aims to contribute a case study that demonstrates how TO can be implemented in atypical discrete structure design.

The architectural parameter for the choice of structural system is framed through a hypothetical architectural intervention. This project is conceived of as an addition to the Rafael Vinoly designed Jongno Tower in Seoul, Korea. The existing building is designed in the high-tech style and features an exoskeleton. Jongno Tower provides a prime opportunity for the combined technique application of TO because the building already features a prominent exoskeleton, but the design suffers from having a lack of symbolic link with the adjacent historic district (fig. 1.8). The tower sits across the street to a traditional Korean temple, and neighbours a heritage district; however, the building does not iconographically relate to the characteristic Korean vernacular buildings that are nearby. The vernacular quality from these buildings that I want to incorporate into my design process, as an architectural parameter, is that Korean architecture is constructed in a highly tectonic manner. A particularly interesting tectonic element of vernacular Korean architecture is the Gongpo system, which is a bearing beam system. The Gongpo assembly is composed of orthogonally interlocking bars formed in an inverted step pyramid shape. Another fascinating tectonic element in vernacular Korean architecture is the wooden frame on windows and screen doors known as Chang. Typically, this frame is infilled with a lattice of simple grid geometry, but sometimes this infill pattern is complex. From inside, the view of the landscape is framed by this decorative frame and creates the experience that the frame becomes part of the landscape. Therefore, this intervention intends to mend the visual disconnection between the two types of buildings on the site by applying the TO process to generate an addition to the building featuring an exoskeleton composed of components that have been subjected to these contextual parameters. The proposition is to modify the tower with the addition of a cultural hub within the existing void in the tower. Like the existing building, this addition will prominently feature an exoskeleton, but, in contrast to the existing design, this new exoskeleton will consist of multiple scales of elements that are iconographically similar to the aforementioned vernacular Korean architectural elements.

The goal of this design experiment is to speculate on the result of applying TO to the design of a building’s structure using the following approach in two key scales: optimizing the distribution of structural components at the macro scale, and optimizing the distribution of material within members at the component scale. Furthermore, the component scale consists of three parts: the distribution of the discretization of each member, reinforcements of the nodes within member discretization, and the design of the joints between these members. The initial step is to perform macro level TO on the domain of the intervention based on a gravity loading scenario. The generated options will be used to inform the placement of conventional structural beam and column members. This strategy is reasonable because TO identifies an optimal load path between the supports and load, so aligning members along this load path should lead to a good structural solution.
Afterwards, a second round of TO is applied, this time to the individual members themselves. The boundary conditions of each element are derived from the element forces from the macro level analysis, allowing each element to be set up as a discrete TO problem. In a similar vein to the macro level analysis, the generated region is used to inform the reinforcement within each beam or column member. Rather than conventional solid members, these members are re-imagined as architecturally driven ornamental pattern lattices. The TO domain is used as a stencil to select members along the critical load path for reinforcement.

Stencil method used in this thesis refers to a selection criteria of the individual lattice filaments in the assembly of structural components for reinforcement. In other words, the lattice filaments lying along the body of the TO outcome, that indicates the critical load path of force within the member, are chosen to be thickened. This group of critical filaments were made thicker than the filament of the surrounding lattice. The thickness of the critical filaments is increased in diameter based force requirement. As a result, the stencil method creates a lattice frame with a clear visual interpretation of the load path within the structural element.

While this thesis considers the form generation up to this point in detail, the last two steps within the component scale method are only nominally developed, so that the final design could be visualized. Due to the discrepancy between the dimensions of the region and the density of the lattice, certain parts of the reinforced region were only reinforced with one or two filaments. This places a concentrated force on the nodes, and thus has to be further reinforced. A rigorous method for determining this local reinforcement is outside the scope of this thesis because it would require creating new TO software. Thus, specific weak points in the lattice components were intuitively identified and thickened for the purpose of creating more accurate visualizations. The last step in generating the exoskeleton is to reassemble these lattice columns and beams into the Vierendeel. This requires consideration of how the joints between these members will be assembled and how they would transfer bending moment. Since the topic of joints was not a primary focus of this thesis, a conventional bolted splice connection was adapted to complete the system.

Although this lattice member Vierendeel exoskeleton was generated through an adaptation of topology optimization, it is not an objectively optimal structural solution because it lacks funicular members. Yet, there are plenty of architecturally appropriate uses for structurally non-optimal frames. My position is that, when using inherently non-optimal structures, it is still valuable to make them as optimal as possible within the constraints of the system you are using. Frequently, designers are asked to make designs equipped with economical, social and cultural values. In such a design space, it is impossible to relentlessly optimize a single aspect without simultaneously diminishing another. In this vein, the pure pursuit of structural optimization will inevitably lead to a project that is suboptimal in its other aspects. Rather, the practice of design tends to favour working towards a scheme that performs well in many aspects for a greater aggregate sum of value. The challenge is the relative importance of each of these values are
different for each person. Thus, this work has presented a method for trying to optimize an inherently non-optimal structure, and balance that with other non-quantifiable objectives.

The choices that guide the process in this thesis were either structural, architectural, or arbitrary. For instance, the choice of a Vierendeel frame was architecturally driven, and the TO itself is structurally driven, but the TO target void ratio was arbitrarily determined. The challenge with this method is that the architectural and arbitrary parameters have the greatest impact on the overall structural efficiency of the result. This method is not meant to make all Vierendeel exoskeletons structurally equivalent to diagrids; rather, it is meant to be a rational process for navigating the design space of such structures. The performance of the design, based on the specific set of architectural, structural and arbitrary choices, should be evaluated after the exoskeleton has been generated.

In this case, this intervention can be evaluated both structurally and aesthetically. Structural evaluation can be carried out by performing structural analysis on the design, and comparing it with reference designs. The structural evaluation was carried out in a relative fashion rather than absolute because of two reasons: the choice of a Vierendeel frame makes the entire structure less optimal than other structures, and the current method for generating lattice members does not yet produce structurally valid designs. The criteria of comparison is structural weight versus stiffness and strength and these comparisons are made at both building and structural component scales. The project can be evaluated aesthetically by creating a series of drawings and renders, and seeing what type of spatial quality the contextually optimized lattice structure creates.

At the macro level, it was found that the design structurally performed better than a reference design that was composed of a typical grid of Vierendeel members made without TO input. Yet, at the component level, the selectively reinforced lattice was shown to be less efficient than a uniformly thickened lattice. Furthermore, it was found that the lattice column members would not be able to resist the required loads. This failure was due to the known incongruence between the density of the lattice and the proportions of the TO generated guide. Aesthetically, the contextually optimized latticework structural components possess a visual quality similar to the ornamental screens found in traditional Korean architecture. The approach to apply TO in this thesis lays the foundation for rationalizing the compromise between: structural efficiency, architectural context, and creative agency.

Through the process of implementing the design method proposed in this thesis, several key areas for future work have been identified. In particular, the nodes within components and the joints between them require further investigation. It would be valuable to expand upon the proposed stencil method of component reinforcement by integrating it with size optimization. Furthermore, optimization of members without uniform cross sections would allow a finer level of control of reinforcement around areas of stress concentration. This research could help to fix the structural failures occurring in designs generated through the current method. The current method also relies on several manual processes making it unsuitable for exploring multiple iterations. Automating these processes will allow designers to explore many design options, which will ultimately lead to more optimal results (fig. 1.10). Further research into methods of joining components together would be valuable for the progression of this work. New construction methods, such as large scale additive manufacturing, might provide better methods for joining components on site.

![figure 1.10] Future recommendation for TO algorithm modification
Part 2.

Context

2.1. Topology Optimization
   2.1.1. Theoretical Background of Topology Optimization
   2.1.2. Current Applications of Topology Optimization

2.2. Exposed Structural Elements as Ornamentation
   2.2.1. Exoskeleton Structures
   2.2.2. The Vierendeel
Part 2.1.

Topology Optimization

2.1.1. Theoretical Background of Topology Optimization
2.1.2. Current Applications of Topology Optimization
Theoretical Background of Topology Optimization

From an engineering standpoint, Topology optimization is a technique for finding the form of a structure that maximizes material efficiency. It is a special process of generative design that leverages the computer’s ability to perform rapid calculations in order to derive a form.

Topology Optimization has a wide range of applications in aerospace, mechanical, bio-chemical and civil engineering. In collaboration with Altair, an aircraft company, Airbus, proposed a cross section that provides a significant weight reduction of overall wing design. Finite element based topology, sizing and shape optimization tools were involved in a two-phase design process: firstly, Topology Optimization is performed to obtain an optimal load path of an initial design. Then, this form is recreated by using a sizing and shape optimization after considering manufacturability. As seen in this example, a multi-phase design process is required due to the free forms that naturally occur, and the result of TO algorithms is often difficult to manufacture. For that reason, the result emerging from TO is often fine-tuned for manufacturability. Adding constraints to the formulation in order to increase the manufacturability is an active field of research. In some cases results from TO can be directly manufactured using additive manufacturing; TO is thus a key part of design for additive manufacturing.

2.1.1.1. Mathematical Background of Topology Optimization

The core of any TO algorithm process is an objective function, which is a mathematical function that aims to represent the value and configuration of the elements to be optimized. In structural analysis, this function is usually a function of compliance, which is the reciprocal of stiffness. The formulation of this objective function varies depending on the mathematical approach, but there are typically 3 parts that constitute the algorithm (fig. 2.1.1): Finite Element Analysis, Sensitivity analysis, and Optimization. TO algorithms often include solving a differential equation. And these equations do not have a known analytical solution, so...
they can only be solved numerically through Finite Element Analysis. Through FEA, the domain of a given design is subdivided into simpler parts for which the displacement field and the strain energy are computed. Next, it goes through sensitivity analysis. Sensitivity is defined as the rate of change of the objective function in response to changes in the design variables. By this standard, the direction in which design variables are changed is determined. The only design variable in solid topology optimization is geometric density. Geometric density is an abstract value used to express the layout of material in a space. Usually it is represented as 0, or a small value close enough to 0, or close enough to 1. Using this scale, if a geometric density of value between 0 and 1 is determined for all elements, then the overall material distribution is determined. The optimization problem is then to minimize compliance, in other words, to maximize Stiffness. With the assigned design variables, the algorithm plots a curve based on the relationship between objective function and the design variables, and seeks to find the minimum value of this curve. One of mathematical expressions for topology optimization is presented below:

$$\min \; c(x) = U^T K U - \sum_{e=1}^{N} E_e(x_e) u_e^T k_0 u_e$$

subject to : $$V(x)/V_0 = f$$

$$K U = F$$

$$0 \leq x \leq 1$$

where $c$ is the compliance
$U$ is the global displacement
$F$ is the force vectors
$K$ is the global stiffness matrix
$U_e$ is the element displacement vector
$k_e$ is the element stiffness matrix
$x$ is the element densities
$N$ is the number of elements
$V(x)$ and $V_0$ are the material volume and design domain volume, respectively
$f$ is the prescribed volume fraction.

2.1.1.2. Different Techniques in Topology Optimization

Martin P. Bendsøe pioneered the field of Topology Optimization, his book: Topology Optimization: Theory, Methods and Application introduced a definition of Topology optimization and its principles in detail. The book is mainly divided into two parts: Firstly, he explains the optimization of solid isotropic material, while the second part focuses on compliance design. The aspect of the book that is of particular interest is Bendsøe's explicit 99 line code Solid Isotropic Material with Penalization or SIMP method. SIMP method is a principle optimization algorithm applied to the result of a finite element model. The design variable of SIMP method varies as fractals between 0 and 1; then, it is penalized by associated stiffness to become discrete. As its name suggests, the method is applied to isotropic material such as steel or concrete. In the later chapter, homogenization method is adopted to operate SIMP method to anisotropic material such as wood. Following, the book, Efficient topology optimization in MATLAB using 88 lines of code was published by Bendsøe et al. The article contains an 88 line of MATLAB code that is substantially more efficient than the previous 99 lines version. The new version is more computationally efficient making it superior for practical application. This code is the basis for almost all commercial topology optimization software today including Solidworks and Ansys.

SIMP provides an efficient method for generating optimal topology for continuum structures, yet there has been a parallel research into whether TO could be applied to research in discrete structures. The basic topology optimization problem uses discrete variables. Hence, it is reasonable to deal with it by formulating it instantly in discrete variables. However, this mathematical solution can be very challenging. In addition, this approach has some
limitations with respect to size of problems and structures. Nevertheless, there are some notable discrete approaches, such as the evolutionary structural optimization (ESO), additive evolutionary structural optimization (AESO) and the bidirectional evolutionary structural optimization (BESO), which have considerable efficiency. In 1993, Y.M. Xie and G.P. Steven introduced an approach called evolutionary structural optimization (ESO)\(^5\). ESO is based on the simple idea that the optimal structure can be produced by gradually removing the ineffective material from the design domain. ESO has received extensive attention because it can be easily implemented and linked to existing finite element analysis packages.

Xie, Steven and Querin continued developing ESO and introduced the BESO method, which either removes or adds material to evolve the structure to an optimum. Unlike SIMP, which has an intermediate density, BESO treats the design variable as a discrete variable which takes a value of either ‘1’ when an element is present or ‘0’ when completely removed. Now, BESO is widely used in most commercial software including the Karamba 3D parametric engineering tool.

SIMP and BESO are the most widely used TO algorithms, because of their efficiency and simplicity. Both methods have the same goal of minimizing compliance while minimizing material by identifying whether each element should consist of solid material or void, but how it is executed is different. Through an iterative optimization process, SIMP slowly removes material. The intermediate values are penalized to steer the solution towards a pure solid and void. On the other hand, the BESO algorithm either removes or adds material to evolve the structure to an optimum and that is why it is called bidirectional. Unlike SIMP, which has an intermediate density, BESO treats the design variable as a discrete variable which takes a value of either ‘1’ when an element is present or ‘0’ when completely removed. (fig. 2.1.2) Because of this inherent difference in considering the discrete variable, the use of methods also takes a different path as well. For continuum structures, SIMP methods are mostly used, while BESO methods are preferred in most cases for discrete structural design. The figure 2.1.5. shows the continuum and discrete structures using Topology Optimization, which will be discussed in detail in the next sub-chapter. One important note is that BESO is also used to analyze continuum structures.

2.1.1.3. State-of-Art Topology Optimization Analysis

In the article, Topology Optimization of Hierarchical Lattice Structures with Substructuring, by L. Xia et al., a novel method that builds upon the homogenization method from Sigmund is proposed\(^7\). Recently, these methods were further expanded upon in the book Topology Optimization of Hierarchical Lattice Structures with Substructuring by Xia et al. Of particular interest is the chapter that introduces what Xia refers to as meso-scale optimization. The method referred as Approximation of Reduced Substructure with Penalization (fig. 2.1.3). The entire structure is divided into a sub-mesh of squares, which are analyzed with finite element method. This method conducts a concurrent analysis between local to global structure. Looking at a different scale of analysis is important to the discrete structures design, since this can potentially contribute to the manufacturability and fabrication of single structural elements.

The paper Structure and Appearance Optimization for Controllable Shape Design proposed a method for optimizing shapes for both their structural properties and their appearance by using existing topology optimization tools\(^8\). The paper refers to appearance as a means for the authors to adjust the surface pattern to conform to a user provided input. The paper presents several examples including a form based on a spiderweb. (fig. 2.1.4) Unlike the other engineering research of this field, these researches did not seek optimal material usage; the unique aspect of this
research is they attempted to produce sufficient rigidity while allowing for freedom to control the aesthetic of the frame. Their research in attempt to incorporate a desired pattern opens up a possibility to introduce a tactile approach into structural analysis.

[figure 2.1.4] Outcome of Appearance Optimization Using Texture Mapping Technique in the TO Algorithm
### Continuum Structures

Topology Optimization is applied to continuum structures that employ unified and continuous materials

![Pictogram of continuum structure]

#### Examples

- Qatar National Convention Centre by Arata Isozaki
- Alucotugawa Riverside Building in Osaka, Japan
- 3D Printed Concrete Slab by ETH Zurich
- 3D Printed Joint Analysis by ARUP

### Discrete Structures

Topology Optimization is applied to assembled structures that employ discrete members

![Pictogram of discrete structure]

#### Examples

- Design Space Exploration Using Interactive Evolutionary Algorithm
- Cailtin Mueller at MIT Digital Structures Lab
- Size, Shape, and Topology Optimization of Planar and Space Trusses
- Using Mutation-based Improved Metaheuristics
- MIT Digital Structures Lab
2.1.2. Current Applications of Topology Optimization

Digital tectonics is a methodology that integrates traditional construction methods with computer design software. It takes advantage of algorithmic form generation in a way that builds upon contemporary construction techniques. Since Topology Optimization is a computer based tool that generates iterative forms, there has been extensive research and implementation on using Topology Optimization as an expression of digital tectonics. Beghini et al. proposes that topology optimization can offer architects digital models that are valuable to both architecture and engineering disciplines. In their paper Connecting Architecture and Engineering through Structural Topology Optimization, it is stated that architects evaluate digital models visually, while engineers evaluate models numerically. Models built by architects strive for good visual representation, while engineers build models to accurately capture the behaviour of structures, this leads each discipline to create rather different computer models. Since buildings are designed as a collaboration between architects and engineers, interdisciplinary methodologies are paramount to achieving a synergistic result. While topology optimization can be used as a means to minimize material usage, it has the added benefit of creating design options beneficial to architectural exploration. Furthermore, these optimization software combine structural analysis with 3d model generations, and these models can be used directly for studying architectural qualities.

Another benefit of TO is that it can be used as a method to analyze existing buildings. Burry et al., at research conducted at RMIT, collaborated on the reverse-engineering of Gaudi’s Passion Facade of the Sagrada Familia using the ESO method. ESO is a process where the most efficient structural form is created through iterative addition and removal of material. These researchers sought to understand the structural performance factors that have a significant impact on determining architectural form. Gaudi’s work was the most appropriate to examine in this manner because he used analogue force driven from finding to determine the shape of the vaults. Yet, the Sagrada Familia goes beyond pure force driven form, the aesthetic agenda of the project is equally apparent in the design. The researcher’s saw this quality as highly desirable, and hoped that TO could be used to bring a balance of structural function and beauty to new architectural projects.

Not only can TO be used to analyze existing building forms, it can be used in the creation of novel architectural forms as well. In their article, Architectural Morphogenesis Through Topology Optimization, Naboni and Paoletti connect the idea of topology optimization with architectural design through form-finding techniques. The authors propose that performance criteria should shift from a method of evaluation to a generator of form. They note that Topology Optimization in particular has the potential to create the most interesting designs. They experimented on the application of Topology Optimization in creating a shear wall.

The novel form generation capabilities of TO is what is explored in this thesis. Work in applying TO to physical buildings can be sorted into two scales: the first is at the building scale, and the second is at the component scale. At the building scale, TO generated forms constitute most of a building’s exposed structure. At the component scale, TO is used to design discrete elements that can be used within a larger structural system in a building.

At the building scale, Januszkiewicz and Banachowicz, in their paper Nonlinear Shaping Architecture Designed with Using Evolutionary Structural Optimization Tools, present an overview of realized projects that used TO in their design process. Of these projects presented in their paper, the projects that employed TO in the most visually distinctive manner are the Akutagawa Riverside building, and the Qatar National Convention Center. The Akutagawa building was the first project implementing TO to be built. (Fig. 2.1.6) In this case, TO was used to design the lateral load resisting shear walls. Using TO allowed for significant porosity in the structure’s design, because of this, the structural walls could be moved...
to the building's facade and featured as an aesthetic feature of the building. The Qatar National Convention Center features the largest realized application of TO. In this project, TO was used to design a branching tree-like structure supporting the roof. Due to its size, this structure had to be fabricated in parts from steel, then clad in metal panels to achieve its smooth look. Since the size of building scale TO application is so immense, there is always a transformation between the ideal mono material blob from TO and the actual built structure.

Recently, a number of research at the component scale design has been produced due to the manufacturability of large scale application of TO generated forms. Arup in collaboration with Autodesk invented a 3D printed joint generated by a TO algorithm. An existing structural node is recreated with TO algorithms which is part of a tensegrity structure. The three structural nodes shown are all designed to carry the same structural loads, which is tensile forces in this experiment. The initial element is the original connection accepting jaw fittings and the regenerated form is designed to accept swage studs using the very latest optimization and design methods applied by Arup. This redesigned joint features an optimization algorithm and an integrated FEM solver, which is an extended analysis tool of SIMP method. A full set of material tests was executed for specifying products using additive manufacturing processes in the building industry.

Another major research group producing TO building components is ETH Zurich. Their digital building technology lab focuses on creating design with up to date digital fabrication tools. Jipa et al. propose one method for implementing TO to create concrete forms in their paper 3D-Printed Stay-in-Place Formwork for Topologically Optimized Concrete Slabs. The authors present an experiment showcasing a practical application of topology optimized structures in roof slabs. The author asserts that Topology optimization can be used as a design method to reduce material without affecting the functionality of an object; however, the constructability of such computational models make it hard to use in practice.

He stresses this is because computational optimization algorithms produce solutions which are difficult to fabricate, especially at a large scale. Therefore, his research investigates the feasibility of using additive manufacturing to produce large-scale building components with optimized material distribution. Two large horizontal load-bearing slab prototypes are shown that are generated with different topology optimization algorithms. As a continuing research, Smart Slab was implemented by the same lab. Smart Slab is a lightweight concrete slab, presenting three-dimensional geometric differentiation on multiple scales. The Smart Slab uses 3D-printed formwork for casting and spraying concrete in geometrically complex shapes, which added benefits that geometric complexity and differentiation come at no additional production cost. The computational design process uses the structural grid as a starting point to generate a basic mesh geometry with several dozen faces. Unlike the single element analysis of the previous experiment, these slabs are assembled with the rest of building components in order to envision the overall building structures. The most recent work of ETH Zurich is the 3D printed column called Concrete Choreography. 3D-printed concrete columns are used as a stage set for a dance festival in the middle of the Alps. Unlike the smart slab, these columns are printed hollow with fillings, and because of this reason, formwork was not required.
Topology optimization is well suited to additive manufacturing. Speculating on future 3D printing techniques, Jerome Frumer from RMIT proposed that TO could be applied to material scale and the element scale concurrently. The section Evolutionary Plasticity from the article, An Energy Centric Approach to Architecture, considers how evolutionary structural optimization can be used to generate context specific and materially efficient components for use in construction. (fig. 2.1.11) This project was developed with the Bidirectional Evolutionary Structural Optimization algorithm. The approach they used was to optimize both the macro level material distribution and the micro level. At the macro level, there is a significant removal of material, effectively dividing the solid into members. On the micro level, varying porosity is used to reduce member weight. Final form was driven by the desire to achieve the maximum strength to weight ratio. Although the researchers did not pursue fabrication, They believed a new computer aided technique known as lost foam casting would be the most efficient technique to realize this shape. The authors found that structural efficiency was linked to aesthetics. They proposed the idea of a performative ornament. Resultant forms tended to be periodic and filigree. Furthermore, manipulation of the micro-scale optimization yielded different macro-scale results.

Further into the topic of 3D printing and building TO generated forms, Dutch company MX3D has completed the largest 3D printed structure in the world. (fig. 2.1.12) The MX3D Bridge is an experimental project developed by structural engineers and designers in the Netherlands, using the up-to-date robotic 3D printing technology. Fully 3D printed in stainless steel, the bridge welds traditional steelwork and advanced digital modelling into a structurally sound piece of public-urban infrastructure. MX3D bridge is the first example of using Topology Optimization algorithm to the design of civil infrastructure. With Arup involved as lead structural engineer, MX3D created software that generates the analysis and iterates optimal shape options for the design.

On the other end of the spectrum of TO application, TO has also been considered in discrete structures design. MIT digital structures lab is the most active research group looking into a novel application of the evolutionary algorithm such as TO to discrete structures. (fig. 2.1.13) According to Mueller, the leader of Digital Structures group, by having structures slightly less optimal, the architectural variety of viable solutions greatly increases, and evolutionary design tools like TO can help this process. She stresses that the interactive evolutionary algorithm with parametric modelling opens a great potential to create various structural solutions in the early design process. Two computational design strategies, parametric modeling and interactive evolutionary optimization, offer compelling alternatives to the previously mentioned techniques. Indeed, these approaches are naturally oriented towards creative exploration, are theoretically applicable to any design problem and are well-suited to make ill-defined criteria meet quantifiable objectives in architectural design. The implemented design tool capitalizes on Grasshopper’s versatility for geometry generation but supplements the visual programming interface with a flexible GUI portal, increasing the designer’s creative freedom through enhanced interactivity. These allow the user to script complex generative algorithms without prior programming knowledge and can help steering design space exploration. Exploring different solutions can
be done in a timely manner as the parametric design process is by essence non-destructive, meaning that one model contains all the previously explored solutions as well as the ones yet to evaluate. Furthermore, these parametric modeling environments can be used in combination with analysis and optimization components to constitute integrated design environments. Thus, such environments are not solely dedicated to computer-aided drawing but benefit from the numerous available plug-ins to assess the performance of architectural designs according to a wide range of criteria, from building envelope performance to daylighting availability.

When it comes to the design of lattice structures, it is more preferred to use a sizing or shape optimization; there are a few realized projects using TO algorithms in discrete structures. Through the experimentation, a creation or alteration of the existing algorithm is not intended, yet a speculative implementation of Mueller’s idea on application of the evolutionary algorithm to an architectural design is present.

Endnotes


Part 2.2.

Exposed Structural Elements as Ornamentation

2.2.1. Exoskeleton Structures
2.2.2. The Vierendeel
The word exoskeleton was first used in the field of zoology to describe animals that had their rigid supporting structure on the outside of their bodies rather than on the inside. The word comes from the combination of the Greek words: exo- meaning outer, and skeletons meaning dried up. Exoskeletons in architecture are load bearing structures constructed outside of the building envelope. In this case the exoskeleton is exposed and becomes a primary component of the building’s facade design.

The architectural implication of exoskeleton construction is largely prevalent in the adaptive reuse discourse. In the Paper Adaptive Socio-Technical Devices, Francesca Guidolin underlines the need for contemporary cities to rehabilitate their aging multi story post war buildings. These buildings require upgrading from more than a pure maintenance point of view; they also need to be upgraded to suit contemporary social situations. Guidolin proposes exoskeleton structures as the design solution to this problem. Such structures allow for improvement in four areas: accessibility, customization, social innovation, and a participative construction process. Accessibility can be improved by adding wider hallways, elevators and ramps to existing buildings. Customization is facilitated by allowing residents to choose what types of additional rooms they need. The process can be socially innovative if residence are involved in the renovation design process. The participative construction process described by Guidolin is one where residents are not displaced by construction. By building the addition with exoskeleton framing, work can be done without resettling residents. The renovation of the Tour Bois le Pretre in Paris is an example of some of these benefits. (fig. 2.2.1 & fig. 2.2.2) The building involved its inhabitants in the design process and construction was facilitated in stages. An exemplary unit was completed first, so residents could see the types of upgrades that would be taking place. Furthermore, additional stairwells and elevators with fire security compartmentations were added to bring the building up to contemporary accessibility standards.
From a structural engineering design perspective, the most difficult aspect of integrating an exterior structure was fire protection. In the design of the John Hancock Centre in Chicago, engineered by SOM, the structure was coated in an unsightly fire protective coating, therefore the structure had to be clad in aluminum panels to have the look of an exposed structure. Over two decades later, the engineers at SOM utilized recent advancements in fire engineering to prove that placing the steel megaframe of the Hotel de las Atres less than 5 ft outboard of the build allow the structure to satisfy the code specified fire rating without any fire protective coating. This allowed the actual wide flange and cruciform structural members to stay exposed, leading to a true architectural expression of the structure.

The thesis largely focuses on exoskeleton as a major constituent of facade design. This type of exoskeleton structures does not primarily pursue structural performance, yet its function is greatly affected by other parameters such as cultural, social and aesthetic aspects. Exterior expression of structure is most prevalent in the high tech design movement. Early high tech architecture focused on maximum flexibility of its interior, leading designers to place the building services and superstructure outside. The Reliance Controls building by team 4 is one of the earliest examples of the style. The building used a flexible modular bay construction to create a factory for the Reliance Controls company. High tech architecture rhetoric grew largely out of modernism emphasizing communication of the function of a building throughout its interior and exterior. In this way, the structure becomes an aesthetic device, and its detailing and finish suddenly become important.

The following buildings are works of architecture by prominent designers that feature exoskeleton structures: the Pompidou Centre by Renzo Piano and Richard Rogers, and One Thousand Museum, by Zaha Hadid. Although both structures prominently feature exoskeletons, they use different formal languages. The Pompidou centre uses an industrial tectonic aesthetic, while One Thousand museum uses a fluid stereotomic aesthetic. The Pompidou Centre was the result of an open architectural competition for a new cultural center in Paris. The competition was won by the then fledgling architecture duo of Renzo Piano and Richard Rogers. Their proposal was considered radical, a building with a facade of steel bracing and mechanical pipes on full display within the historic city of Paris. The exterior structure and systems likened the building to a construction site. The architects considered this representative of the constant state of cultural evolution. The building’s exoskeleton played a major role in achieving this aesthetic.

One Thousand Museum is a residential skyscraper in Miami designed by Zaha Hadid architects. According to the designer’s website, the major feature of the design is its reinforced concrete exoskeleton structure. This exoskeleton's curvilinear form is both aesthetic and functional, because of its funicular shape for allowing it to transfer lateral loads. Lateral wind loads are significant in Miami, according to the ASCE 7 design loads for structures, the city has some of the highest wind loads in the continental US. Concentrating the lateral system to the perimeter frees up the layout of the tower, allowing for more architectural variation in floor plan. The structural exoskeleton is the major architectural feature of the tower. It serves as the defining aesthetic, formal, and plan generating technique. The fluidity of the exoskeleton serves to aestheticize the structural optimization of the frame. The thickening and thinning of the form matches the utilization of the members.
2.2.2. The Vierendeel

2.2.2.1. History of Vierendeel

The Vierendeel frame represents a prime example of popular structurally non-optimal systems. From a structural engineering standpoint, a Vierendeel frame is never more efficient than any type of braced frame system; however, its ability to integrate with architectural space and mechanical systems make it an appropriate system for certain conditions. The Vierendeel frame is named after Belgian engineer Arthur Vierendeel, who invented the system in 1896. The major advantage of this system is the removal of diagonal bracing members within structural bays that block views and openings.

David Wickersheimer, from the Journal of the Society of Architectural Historians, described five categories of vierendeel frame use. The first is to accommodate large and complicated mechanical systems. The second is when rectangular openings are required for circulation and fenestration. The third is when a rigid foundation system is required in exceptionally poor soil conditions. In these cases, the floors and columns can be used together to create a vierendeel frame. The fourth category is use of the frame as support for multiistory buildings. The final category is for creating cubic voids for aesthetic purposes. The following case study projects: the Salk Institute, the Beinecke Library, and Pancras Square tower, representing the first, second, and fourth of Wickersheimer’s categories respectively, are presented in the following paragraphs.

The Salk Institute in San Diego by architect Louis Khan is the most prominent example of how a vierendeel frame can be used to integrate a building’s service systems and its superstructure. The goal was to create column free labs for future flexibility, so huge girders were required to support the long spanning floor. These girders were built as 2.7 meter deep prestressed concrete vierendeel frames. The depth of these girders effectively created interstitial floors between labs to run mechanical systems. The use of vierendeel frames to remove the diagonal members allowed easy human access of this service space. This allowed these mechanical systems to be changed as research requirements evolved. The design of the vierendeel strut members incorporated manual optimization of their cross sections. For example: the top and bottom chords increase in depth towards the support because global bending stress in the chords are the greatest there. The thickness of the vertical struts vary in thickness following corresponding to the stress distribution in the girder. In addition to the functional reasons for adopting the vierendeel system, the structure also reinforced Kahn’s architectural concept of served and servant spaces. The space created by the depth of the girders, and its relationship to the lab spaces above and below, is a manifestation of Kahn’s spatial concept.

The Beinecke library of Yale in New Haven by architect Gordon Bunshaft features a large donut shaped hall around its central book tower. This massive hall spans between the central tower and only four concrete supports around its perimeter. To support this impressive span, four large vierendeel frames were used. These frames span 40 meters, and extend the entire height of the facade to support the roof as well. The advantage of the system was to allow for the frame to be integrated with the architectural cladding system. If a braced bay system was used, the
orthogonal marble grid on the facade would have been interrupted by steel diagonals on the inside, ruining the architectural expression. Furthermore, Bunshaft mirrored the bending distribution of the frame in the thickening and thinning of the structure’s cladding. The cladding becomes thinnest at the location of zero moment, the point of inflection, in each structural bay. (fig. 2.2.7)

The Four Pancras Square tower in London by Eric Perry uses a vierendeel frame as an exoskeleton. (fig. 2.2.8) The overall structural system consists of elements in both steel and concrete, with an exposed weathering steel frame that provides support to the slab edges. This facade works as the support structure, picking up the floor around the edge through the height of the whole tower. At the base, it is used to support a 27m span over the entryway10. Given that the frame sits in front of the facade, the vierendeel configuration is beneficial because it does not obstruct the views outward from the units. From the exterior, the weathered steel frame is visually evocative of the industrial revolution that enabled the expansion of London.

2.2.2.2. The Engineering Treatment of the Vierendeel Frame

A truss is an assemblage of structural elements that are organized in such a way that they are not required to take any bending to be stable under load. On the other hand a frame is an assemblage that is designed to use the bending resistance of its joints to achieve stability. A Vierendeel frame is a type of frame used to achieve the structural function of a beam. Most other structural assemblies used as beams are trusses, therefore the Vierendeel frames have often been referred to as trusses, but this is incorrect.

The benefit of a truss is that it allows the members that it consists of to carry load only in tension and compression. This is referred to as axial load. The reason this is beneficial is because materials are more efficient at carrying load axial than through bending, therefore making a truss configuration is often the most economical configuration. Typical truss types are the: Pratt, Howe, and Warren types. A Pratt truss is designed so its bracing elements resist load in tension. A Howe truss is the opposite where its bracing takes compression. A Warren truss is a combination where braces alternate between tension and compression. What all trusses have in common is that they have diagonal members bracing each bay of the truss.

The benefit of the Vierendeel frame, when compared to a truss, is the removal of the diagonal members. While this comes at the cost of material efficiency, the benefit of this system is the architectural capabilities it offers. The Vierendeel also poses a unique engineering challenge because it deforms greater under load when compared to a truss.

When observing a hypothetical section of a truss member, one would find that the axial stress would be uniformly distributed over its cross section. Since a Vierendeel frame carries load in bending, the stress across one of its member cross sections would be varying linearly thereby making it less efficient.

The bending distribution in a Vierendeel frame can be approximated by magnifying its deformed shape. Under load, each member of each bay is bent into an “S” like shape, showing that the direction of bending inverts along each member. The point of inversion is known as the point of inflection, and there is a point of zero stress. It is possible to place hinges at these points and still maintain structural stability.

When a Vierendeel frame is used as a spanning member, it acts globally similarly to a beam. Yet, the distribution of force in a Vierendeel frame is quite different, unlike a beam where the distribution is uniform, in a Vierendeel frame the compression and tension forces are concentrated in the top and bottom chords. A good approximation of the
A Vierendeel girder or truss is an open web girder consisting of top and bottom chords and diagonal members, thus their moment is equal to the difference of adjacent chord moments.

Typical results for an elastic structural analysis of joints shown in Figure 4. The majority of the girder are then checked for the presence of points of inflection in the middle region of the members.

The calculation of the shear force in many web panel zones must resist, to be determined by the design engineer. This AD provides advice on the design of the web panel zone. As a result of the end moments in the web panel zone, the local moment taken by a chord can be found with this formula:

\[ M = \frac{(V \times e)}{2} \]

Where:
- \( M \) is the bending moment in a chord
- \( V \) is the shear
- \( e \) is the length of the chord.

Therefore, the maximum chord moment is found near the support where the shear is equal to the reaction. (fig. 2.2.9)

Bending in the web members of the frame can be visualized as a series of “S” shapes. The bending moments vary from positive to negative in each strut. If a horizontal force was exerted on the frame, the chords carry about half the total shear each. Therefore, if we assume the point of contraflexure to be approximately in the center of each chord, the local moment taken by a chord can be found with this formula:

\[ C = \frac{T}{D} = \frac{M}{D} \]

Where:
- \( C \) is the compression force
- \( T \) is the tension force
- \( M \) is the maximum bending moment
- \( D \) is the overall depth.

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- \( M \) is the maximum bending moment
- \( D \) is the overall depth.
Endnotes


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Part 3.

Design Synthesis

3.1. Introduction

3.2. Site: Jongro Tower in Seoul, South Korea

3.3. Methodology
   3.3.1. Introduction
   3.3.2. Application at Building Scale
   3.3.3. Application at Component Scale
3.1. Introduction

The design intervention is about the use of contemporary generative design tools coupled with historic Korean architectural iconography, to renovate the Jongno Tower in Seoul. This chapter divides into two parts: site and methodology. The purpose of introducing a site in design exercise is to determine a structural system which is framed by architectural qualities based on cultural and aesthetic parameters. Then, a methodology to integrate Topology Optimization to this structural system is explained.

For this design exercise, the architectural qualities are derived from the greater context of the site: The Jongno district in Seoul consists of two polar opposite building typologies: contemporary skyscrapers, and traditional low rise Korean buildings. This urban dichotomy is most prevalent near the surface of Jonggak station, where the contemporary Jongno tower sits across the street from a traditional Korean pavilion, Boshingak. Korean iconography is integrated through pattern mapping the domains generated through topology optimization. These patterns are derived from compositions found on Korean vernacular architecture, especially inspired by the vernacular construction method of the Gong-po system and the geometric patterns of Chang.

This thesis presents a method that integrates these contextual considerations into the design of TO structural elements. This is achieved through pattern mapping the domains generated through topology optimization. The pattern mapped domain is then reanalyzed for its structural capacity. The product of this process is then used as input for another round of generative structural optimization. This process continues until a generation of designs with highly efficient structural performance, and whose forms abstractly allude to Korean vernacular buildings, are arrived at.

The background motivation of this process is to rectify the international style of contemporary skyscrapers with historic Korean architecture. The existing Jongno tower was designed by architect Raphael Vinoly in the high-tech
or ultramodern style. The current facade design of Jongno Tower is dominated by its diagrid exterior structure. For the new facade to serve as a viable replacement, it must be designed to be structural as well. While the diagrid layout is highly structurally efficient, it has a substantial impact on the visual quality of a building. To create a new structural facade, there should be a technique to guide the design toward a structurally viable solution. By using the generative structural design technique topology optimization, it is possible to identify the optimal material distribution for each load condition. These distributions are used as the underlying template to guide the design of the new facades of an additional space.
3.2.
Site

The Jongno Tower in Seoul, Korea was chosen for the site for design intervention (fig. 3.2.1). This High-tech style tower sits across the street to a traditional Korean temple, and neighbours a heritage district: however, the building does not iconographically relate to the characteristic Korean vernacular buildings that are nearby. The design intervention aims to mend the disconnection between these two buildings by considering contextual parameters from the site when applying up-to-date technology like Topology Optimization into architectural design.

Jongno District in Seoul has been the center of the city for 600 years, because it is where the Joseon dynasty established its capital city. Since this district played important roles in culture and history as the capital city, there has been a lot of development. As a result, Jongno district consists of two polar opposite building typologies: contemporary skyscrapers, and traditional low rise Korean buildings (fig. 3.2.3) This urban dichotomy is most prevalent near the surface of Jonggak station, where the contemporary Jongno tower sits across the street from a traditional Korean pavilion, Boshingak (fig. 3.2.2.) This temple was originally constructed in 1396. This phenomena is inevitable when comparing the height of the towers and the surrounding low rise buildings built before the development of the city. (fig. 3.2.4) Since Jongno tower was completed, there has been criticism questioning the cultural integration of the design. Thus, my question involves how a designer can integrate a historical and cultural aspect into a high-tech oriented design like Jongno Tower. The mimicry of the same style may not be sufficient. (fig. 3.2.5) Thus, my design seeks to propose a sustainable way to adopt an emerging technology such as topology optimization, when implemented, while keeping the identity of the site.

The current design of Jongno tower prominently features a structural exoskeleton, while the Korean pavilion is made from ornamental wood construction. The design problem posed by this thesis is to create an addition to Jongno
Height Differences between the Jongno Tower and Adjacent Buildings

134 m
Skyscrapers in Jongno District
During the urban development of the 1960s, high-rise building construction began taking place in the old part of the city. Some of these buildings' facades were designed to reference the vernacular aspect of the city, but there were many modern designs that have been criticized for lack of contextual referencing.

Gyeongbokgung Palace
Gyeongbokgung was the main royal palace of the Joseon dynasty. Built in 1395, it is located in northern Seoul, South Korea.

Bukchon Village
Bukchon Hanok Village is a traditional Korean village in Seoul located on the top of a hill between Gyeongbok Palace, Changdeok Palace and Jongmyo Royal Shrine. The traditional village features many narrow alleys, and hanok construction. It is preserved to show the urban environment of Korean from 650 years ago.

Gamgodang Road
Gamgodang road is the main street of the heritage district. It has become a popular tourist destination for observing the urban fabric of ancient Korea.

Bosingak Pavilion
Bosingak is a large bell pavilion in the Jongno district of Seoul. Jongno translates to bell street, and gets its name from the very bell in Bosingak. In the Joseon Dynasty, this bell was the center of the town, and rang every day to signify the opening and closing of the four city gates.

[Figure 3.2.4] Overview Mapping Diagram of Building Types in Jongno
Using topology optimization to generate a form, then subjecting it to further TO with geometric constraints based on architectural context is reasonable to apply to this design problem, because fashioning a new addition featuring an exoskeleton is in line with the existing building, and mediate the addition's appearance with the adjacent historic building adds visual cohesion to the district. Jongno tower consists of 3 circulation shafts straddling oval-shaped floorplates. These shafts have an exterior diagrid truss structure that serves as part of the building's lateral load resisting system. Atop the three shafts sits a floating 3-story ring that is framed like a bridge. This floating ring houses a restaurant, bar, and viewing deck for views over the district. The design by Rafael Vinoly is itself a renovation, built around a pre-existing design for a 12 story office.

Two vernacular qualities are referenced in this design exercise as architectural parameters: the Gong-po system (fig. 3.2.6) and Chang patterns (fig. 3.2.7). Vernacular Korean architecture features interlocking wooden bearing beams on top of its columns in what is known as Gongpo assemblage. The Gongpo system is a bearing beam system that you can easily see in Korean architecture. Known as a corbel bracket set, Gongpo system functions as a structural safety buffer by distributing or concentrating a weight of a building roof. The Gongpo assembly structure is composed of orthogonally

interlocking bars formed in an inverted step pyramid shape. Like an inverted pyramid, these members allow for increased roof overhang and spans by stepping outwards in layers, and relying on the weight of the roof to hold them in place without fasteners. This structure creates the iconic dense layer of structural members within the entablature region of traditional buildings. Another feature of traditional Korean architecture is seen on the Openings in traditional Korean buildings are furnished with wooden lattice screens known as Chang (fig. 3.2.8). The lattice within these Chang is organized into ornamental patterns known as Geumcho. These patterns are the most iconographically representative aspects of the Korean architectural style.

In conclusion, these two vernacular aspects are considered as contextual parameters which decides structural systems that will incorporate TO-systems. At building scale, the design proposal is to create an exoskeleton structure within the void of the tower that has the tectonic qualities of Gongpo construction. Then, This exoskeleton will be composed of structural components that have structural framing constrained into Chang patterns.
Part 3.3.

Methodology

3.3.1. Introduction

3.3.2. Application at Building Scale
   3.3.2.1. Domain Setup
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3.3.3. Application at Component Scale
   3.3.3.1. Domain Setup
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   3.3.3.3. Topology Optimization Outcome
   3.3.3.4. Geometric Constraints by Contextual References
   3.3.3.5. Structural Evaluation
3.3.1. Introduction

This chapter introduces a method of combining outputs of Topology Optimization and a desired structural system. TO is an efficient tool for generating structural forms with optimal material distribution; therefore, by combining TO and an existing structural system, a design that has advantages of both methods may be found.

Furthermore, this method of combination is applied at two different scales as shown in the flow chart (fig. 3.3.1): at the buildings scale, and at the component scale, for the design of the exoskeleton and lattice system respectively. This base diagram is used as a navigation guide through the process in this chapter and placed at the side margin. The structural members in each system are constrained with geometric forms that are based on an architectural parameter, which is iconographically relevant to the immediate context of the project.

The process for the building scale implementation begins with a building massing determined by the author. This form was chosen to match with constraints of tower geometry and TO output. Next, a grasshopper script, implementing Karamba 3D BESO TO components, was executed over the surface of the domain. The force input for TO was the live load within the building that contributes to the load on the perimeter exoskeleton. Finally, the resultant material domain from TO is used as a guide to organize the chosen exoskeleton system of structural members. In this case, the Vierendeel frame system was chosen because its orthogonal lines share a visual connection with the traditional Korean Gong-po tectonic system.

The component scale implementation begins with an analysis of the frame created from the previous process. To get these results, linear elastic finite element analysis (FEA) is performed on the frame using Karamba components in a Grasshopper script. This FEA model is loaded in the same manner as the previous TO generation setup. The result of the FEA is the internal force values for every member.
in the structure. Afterwards, these internal forces can be used as input for TO of each structural member. Next, the TO member results are used as guides for the thickening of structural lattice system chosen for the components. The lattice was chosen because of its similarity to a Geumcho pattern found within traditional Chang panels. Finally, these components are reassembled into their respective positions within the Vierendeel frame, completing the exoskeleton structure.

All Grasshopper scripts used for generating TO forms at each scale are included as appendices from the page 141.

In this process, the choices that guide this process were either structural, architectural, or arbitrary. These assumptions are remarked as a footnote in each page when the decision is made. For instance, the choice of a Vierendeel frame was architecturally driven, and the TO itself is structurally driven, but the TO target void ratio was arbitrarily determined. The challenge with this method is that the architectural and arbitrary parameters have the greatest impact on the overall structural efficiency of the result. This method is not meant to make all Vierendeel exoskeletons structurally equivalent to diagrids; rather, it is meant to be a rational process for navigating the design space of such structures. The performance of the design, based on the specific set of architectural, structural and arbitrary choices, should be evaluated after the exoskeleton has been generated.
[figure 3.3.1] Flow Chart Diagram of Design Process
This subchapter presents a method for using a TO generated form as a guide to arrange structural members in a Vierendeel frame exoskeleton. This process will be presented under the following subheadings: domain setup, Grasshopper script setup for topology optimization, contextual geometric constraints, and structural analysis of the frame.

### 3.3.2.1 Domain Setup

The domain input for the TO script is the surface of the new addition. From the site review chapter, the addition was chosen to fill the void in between the upper deck and the rest of Jongno tower. (fig. 3.3.2.1) There are several reasons for the tapered form of the addition, the first is to mediate the change in floor plate dimensions. The floor plan raised bridge deck level of Jongno tower is about twice as large as the portion of the tower below. Another reason for the taper was to create a more visually interesting load path. If the massing was a straight extrusion, the optimal gravity load path would simply be a series of straight lines. The tapering form creates interesting branching load lines, these are more appropriate for showing off this techniques ability to create a visually interesting facade1.

1 Architectural decision made by author

### 3.3.2.2 Topology Optimization Setup

This subheading overviews the input parameters of the parametric TO script, then presents some sample TO generated forms. These samples are created using the base domain.

In any mathematical model, there is a known number of input and output quantities. Furthermore, there is a known relationship between these inputs and outputs. In this particular TO set up, the input parameters are: the mesh density, target surface area reduction, thickness, number of iterations, external loads, and support conditions. The mesh density is the measure of the number of shell finite elements that can be packed into the domain. In any FEA based code, the accuracy of the analysis is related to the density of the mesh, the more dense the mesh, the better the analysis, but the more time it will take to run the analysis. The target surface area is desired surface area expressed as a ratio between the starting surface area and the resultant surface area from TO. It is important to note that, in this research, results from TO were bifurcated into solid and void. While Karamba executes BESO by manipulating the thickness of shell elements, this research treats very small thickness values as zero. The thickness is the depth of the structure, it can also be understood as the offset distance from the input surface. The number of iterations is the number of trials the Karamba BESO algorithm will run in attempts to hit the target volume. More iterations give a higher chance of hitting the target, but lead to longer computation time. The external load is a series of vectors that represent an

[figure 3.3.2.1] Current Photograph of the Jongno Tower (left) and 3D Modelling with the Domain Setup (right)

[figure 3.3.2.2] Navigation Diagram 1

Input: Building Domain

[figure 3.3.2.3] Navigation Diagram 2

Topology Optimization Setup 1

[figure 3.3.2.4] 3D View of Building Domain (right) and Load and Support Conditions (left)
approximation of how the structure will be loaded in real life. Finally, the support conditions are an approximation of how the structure of the addition is supported by the existing structure. (fig. 3.3.2.4)

In this thesis, not every combination of input variables are studied. The external load and support condition can be determined from consideration of the design problem. In terms of gravity loading, the most significant load on a structure is often the live load. This is the combined load of all the occupants and furniture in the building. The building code conservatively approximates this load at 4.8 kPa, applied to every floorplate. For the support condition, since this thesis studies only the effect of gravity loads, all resistance would come from the bottom of the addition. (fig. 3.3.2.5)

Furthermore, there are input parameters that have a general effect on the accuracy of the analysis, and not meant to influence the form. These are the mesh density and the number of iterations performed. Through trial and error, optimal values for these quantities were found and used in all subsequent analysis. These values can be considered optimal so long as the boundary shape, load, and support conditions are held constant.

Therefore, in this thesis, the parameters that can be varied to manipulate form are the thickness and target surface area. The product of thickness and surface area is volume, thus the volume is the control variable in creating forms through TO in this situation. Target volume reduction to 25% 75% is manipulated to achieve visually dynamic results3; however, there has to be limits set on the range of volumes that are reasonable to consider. This value was deduced from structural weight. Considering the material to be steel, the volume multiplied by the density of steel yields the

2  ■ Structural assumption made by author

3  ▲ The arbitrary assumption of 25% material reduction is determined based on author’s previous experiment of aesthetic variable forms using Topology Optimization.
approximate weight of the structure. Given that TO is meant to reduce material required to carry a load, it made sense to set the upper boundary of volume from the weight of a uniform Vierendeel frame system. Such a frame could be designed through conventional engineering methods rather simply, therefore the TO generated frame should weigh, at a maximum, less than a conventional frame.

As a result, the script displays a progressive material reduction through iterative form generations (fig. 3.3.2.6). The shaded area represents material concentration according to the load distribution. The final result of shaded area represents the optimal form layout after removing unnecessary material shown as the last image in the matrix. The iteration occurs until the final volume reaches the targeted volume reduction. This final result has been used as a guide to re-arrange structural members within the shaded area.

3.3.2.3. Building Scale Geometric Constraints by contextual references
The geometric constraint to the building scale TO form is an orthogonal grid system of beams and columns. The rationale behind this choice was its visual connection to traditional Gong-po construction studied in the site analysis chapter. (fig. 3.3.2.9) The bearing beams of the Gong-po system form an inverted step pyramid massing. The tapered domain, once rationalized into an orthogonal grid system, creates a similar inverted step pyramid form.

As a result, the frame follows the Vierendeel structural system on the surface of facade. This transformation from the direct TO outcome to the desired orthogonal system is shown in the figure. 3.3.2.11 below. Firstly, the regular grid system boolean-unions to the TO outcome. This grid system contains beams and columns arranged following the grid lines; then, these members are re-arranged over the shaded area of the TO-body. For this thesis, the output of TO is used as a guide to manually place the columns and beams in the exoskeleton frame. The rest of beams and columns are connected with pin joints.
3.3.2.4. Structural Evaluation

This exoskeleton system requires to be linked to the rest of building system (fig. 3.3.2.13). After the system is assembled, linear elastic FEA could be conducted on it. The purpose of this analysis was to find the interior forces each member experiences in the frame. Although FEA is part of the TO algorithm, internal forces are not an output from TO. Furthermore, the system has been transformed to frame, therefore the internal forces would be different. For these reasons, it is necessary to run FEA on the frame, using the same input loads and supports, to find the internal frame forces. All nodal conditions are considered fixed while conducting this analysis. (fig.3.3.2.16)

To more accurately capture the load distribution between the exterior and interior structure, it was necessary to model the interior columns and beams. The amount of load taken by the exoskeleton, known as the tributary area, is the portion of floor plate halfway between the exterior frame and the first row of internal columns. Offsetting a line internally from the exterior of the plan, the first row of columns would be the first set of columns that offset meets. The internal forces are shown on the following graphic plots where: N is the axial force, My and Mz are the moments about the y and z element axes, Vy and Vz are the shear about the element axes. (fig. 3.3.2.14) Karamba program displays the individual plots of these forces (fig.3.3.2.15).

6 Structural assumption made by author due to the availability of current software.
3.3.3. Component Scale Implementation

3.3.3.1. Component Scale Domain Setup

The member that carries the most load is selected to determine the base domain size that will be applied to the TO process. The critical beam and column, at the global level, are located on the first level. For the purpose of leveraging TO’s ability to create interesting forms, the column was chosen to determine the base size because the outcome of TO for beam analysis always results in a predictable truss-like form. (fig. 3.3.3.2)

From Karamba linear elastic finite element analysis, the column undergoes following forces:

- $N_z$ (Axial load) = 5000 kN
- $M_{y,weak}$ = 425 kNm
- $M_{z, strong}$ = 234 kNm
- $V_{y, strong}$ = 475 kN
- $V_{x, weak}$ = 805 kN

For this column, the axial load has the largest magnitude, therefore it is reasonable to approximately a column size considering only axial resistance. According to the square hollow section (HSS) chart from CSA S16-09, the compressive resistance for an unbraced member length of 4.6m will be 4000 kN for a 305x305x13 section, which is the largest section size in the chart. So long as a member is not slender, meaning it has a low height to width ratio, there is a linear relation between the size of the member and its compressive resistance. Therefore it was determined that a 375 x 375 x 13 section, at the specified unbraced length, would resist an axial load of 5000kN.

Given that the later component design will increase the porosity of the structural members and reduce the material by 50 %, a size double of the aforementioned cross section is applied as a TO domain. Therefore, the domain of the column applied to the TO script has dimensions of 750 mm x 750 mm (fig. 3.3.3.3) To the current industrial standard, this size is inevitably large: however, this experiment is based on the assumption that in the near future the development of additive manufacturing benefits some constraints regarding sizing and fabrication of structural members.

This implementation aims to have the same material weight as an equivalent regular hollow section. In order to achieve this, the targeted volume reduction of TO should be 50% due to the doubled size of the column. There are two parameters

7 The assumption of 50% material reduction is determined based on author’s previous experiment of aesthetic variable forms using Topology Optimization.
that affect the volume reduction: thickness of the member and surface area reduction. When the thickness remains the same as a structurally equivalent HSS member, the targeted surface area reduction should be 50% in order to achieve the same volume. When the thickness is doubled, the area should be reduced by 75% (fig. 3.3.3.3)

3.3.3.2. Component Scale Topology Optimization Parameter Setup

Although the internal force values are taken from the centre line frame analysis, these values cannot be directly applied to the component model. The reason for this is that the building scale analysis was conducted with beam elements; however, the BESO method used in this thesis requires shell elements. Furthermore, each structural member was modeled as a single beam element in the building scale model, but the component TO process requires finer discretization. While it is possible to apply a distributed moment to the edge of a shell element, this would not accurately reflect the effect of the moment on the component as a whole. Applying a linearly distributed moment to the edges of the component model would result in a force that curls the edges of the tube, rather than bending it.

To translate these forces onto the component model, a reasonable approximation should be made. The approximation in this case is to create a lateral force on the free end of the column. The value of this force is such that its product with the height of the column will yield the correct moment. This load will also induce a base shear, that may or may not be higher than the target shear. The target shear and the equivalent moment force are compared, then the greater of the two was chosen to be applied. In this way, one of the forces will be accurate, and the other will be conservative. Given that the column is subjected to biaxial bending and shear, the higher value of each axis is applied to both axes.

The critical bending moment of 1300 kNm occurs at the bottom cantilevered beam above the support (fig. 3.3.3.7). The following calculation demonstrates that an HSS beam, of the chosen domain size of 750 mm x 750 mm, is capable of carrying the required bending moment:

For Class 1 and Class 2 HSS section (CSA S16 clause 13.5),

\[
M_r = \Phi Z F_y \\
Z_{req} = M_r / (\Phi F_y) \\
= 1300 \text{ kNm} / (0.9 \times 345 \text{ Mpa}) \\
= 4186.7 \times 10^3 \text{ mm}^3
\]

8 Conservative assumption made by author
For the beam of 750 mm x 750 mm x 13mm section,
\[ Z_f = \frac{(d^3 - d_1^3)}{4} \]
where \( d_1 = d - 2t \)
\[ = \frac{(750^3 - (750 - 2 \times 13)^3)}{4} \]
\[ = 10 \,600 \times 10^3 \text{ mm}^3 \]
\[ Z_f < Z_{req} \]
Therefore, a 750x750mm HSS beam can carry over 1300 kNm bending moment. The beam splices should be placed at the points of inflection in the beam diagram so they only have to be designed to resist shear. This means that the beam should be continuous from the cantilever over the support, then be spliced midway on the first bay of its backspan. Each subsequent backspan bay can be spliced in the center, since the bending moment is zero at those locations.

The bending of the members in the plane of the facade is due to the fact that they are acting as a Vierendeel frame in that plane. The moments are significantly lower in this plane; the maximum bending in the plane has 570kNm, which is less than half of the bending occurring out of the plane. (fig. 3.3.3.10) Therefore, splices are inserted near the joints and these connections are excluded from the TO domains. If the splices occurred at the points on inflections in this plane, each piece would have to be fabricated as a three dimensional cruciform. This would be disadvantageous for shipping, because the long beams cannot lay flat during transport. The joints in this plane are required to transmit bending and shear. This connection detail references an existing HSS moment connection product called the Diablo™ Bolted Splice by CAST CONNEX® (fig. 3.3.3.9) The diagram of how these connections would integrate around the critical beam is shown in the figure 3.3.3.10.

Given that the current scheme is optimized for gravity load, the maximum facade plane moment occurs near the bottom of the structure. In this plane, the frame acts as a Vierendeel, so moment has to be transferred to the columns. A simplified model for designing the moment resistance of a Diablo is to imagine that the two rows of bolts act as a moment couple to resist the bending on the connection.
This assumption imagines that the bolts are engaged in compression, where in reality a portion of the plate bearing on the surface will provide the compressive portion of the couple. Nonetheless, this assumption is an accepted simplifying assumption in the design engineering industry. It follows that the assumed limit state would be tensile yield of the bolts. Assuming 1” bolts and 1.5 bolt diameter edge distance, the effective moment arm would be around 0.65m. Assuming 4 bolts per row:

\[ D = \text{moment arm} \]
\[ T_b = \text{tension per bolt} \]
\[ n = \text{number of bolts per row} \]

\[ T = \frac{M}{D} = \frac{570}{0.65} = 877 \text{kN} \]
\[ T_b = \frac{T}{n} = \frac{877}{4} = 220 \text{kN} \]

From Table 3-4 in the S16 handbook, \( T_r = 251 \text{kN/bolt} \)

From this simplified calculation, we can see that the proposed connection is within the realm of reason.
3.3.3.3. Component Scale Topology Optimization Outcome

A series of trials were performed to identify the range of forms that could be achieved in TO design of a member. For column design, various ratios of vertical and horizontal forces were applied at the top of the column. The vertical and horizontal forces were varied, and the results were plotted in a matrix (fig. 3.3.3.13). About the plot image, the column is unfolded to show all sides of the column elevation where there are 4 panels in each profile. The far right profile is the final result when the material reduction reaches the targeted volume of 50% in the first matrix, and of 75% in the second matrix through the iterative process.

From this study, it seemed that the ratio between horizontal and vertical force had the greatest effect on form. Interestingly, the magnitude of force did not seem to affect results. The TO column presented in this design was subjected to the loads presented earlier in this section. The target volume was based on the reference HSS and the parameters varied for maximum visual interest.
Then, these 5 different types of TO-Columns based on the matrix corresponding five different ratios listed are distributed through the TO-Vierendeel structure. There are total 127 columns in the entire structure, placed on the exterior. 68 of them has 1 to 1 ratio between vertical and horizontal forces, 28 in 2 to 1 ratio, 14 in 5 to 1 ratio, 7 in 10 to 1 ratio, and 6 in 20 to 1 ratio. The figure 3.3.3.14 shows where these columns with these TO profiles will be placed according to the ratio between vertical and horizontal forces.
Similar to the columns, a series of trials were performed to identify the range of forms that could be achieved in TO design of a beam. (fig. 3.3.3.15) Unlike the column analysis, where axial load to bending moment ratios governed the TO result, in the beams, the slenderness of a beam was the controlling parameter in the execution of the script. The reason for this is that the bending moment on a beam is far larger than the axial force, varying the scaling up the axial load hardly changes the formal results. Thus, for the beam form study, various span proportions were studied and plotted in figure 3.3.3.15. In the matrix shown here, the TO forms are displayed in iterations for four different proportions of slenderness between length and width dimension.

![figure 3.3.3.15] Plot Matrix of TO-Beam Design Progress

\[ \text{L : Member Length (m)} \]
\[ \text{W : Member width (m)} \]
\[ \text{t : Thickness (mm)} \]

Support: Fixed Condition

\[
\begin{array}{|c|}
\hline
\text{L : W} & \text{iteration process} \\
\hline
4 : 1 &  \\
\hline
3 : 1 &  \\
\hline
2 : 1 &  \\
\hline
1 : 1 &  \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
\text{t = 13 mm, Area Reduction = 50\%} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
\text{t = 26 mm, Area Reduction = 75\%} \\
\hline
\end{array}
\]
Similar to the column distribution, these outcomes of TO-Beam are distributed through the TO-Vierendeel structure. There are total 222 beams in the entire structure, placed on the exterior. 127 of them has slenderness ratio of 4 or greater than 4 to 1, 39 in 3 or greater than 3 to 1, 35 in 2 or greater than 2 to 1, 21 in less than 1 or equal to 1 to 1 ratio. The figure 3.3.3.16 shows where these beams with these TO profiles will be placed according to the ratio assigned.

[figure 3.3.3.16] Location of the 5 Types of TO-Beam in the TO-Vierendeel Structure
3.3.4. Component Scale Contextual Referencing

After executing the TO component implementation, the resultant TO forms are used, again, as guides for reinforcing a lattice structure. The lattice was created by combining Korean Geumcho patterns found on traditional Chang panels (fig. 3.3.3.18). The idea was to replace a solid walled HSS column or beam with a Geumcho patterned lattice. Since this pattern is not inherently structural, it needs reinforcement to resist the required loads. To accomplish this, the TO form is used as a guide to thicken material along load paths within the member. This operation was performed on the example column and presented in this research. (fig. 3.3.3.19) The members overlapping more than half of the recommended TO domain area of the shell are selected to be thickened. This process described above was performed manually for this thesis.

Currently, Karamba 3D does not support optimization for non-funicular forms. As a compromise, the results from
shell BESO were mapped on top of the desired pattern. This result serves as a stencil for picking out the members within the frame that lie along the critical load path. The lattice members overlapping more than half of the shaded TO-domain are selected as the reinforcements. (fig.3.3.3.20) This group of critical members were made thicker than the surrounding lattice. When comparing the result with a uniformly thickened frame, one can see that the proposed method produces a frame with significantly more porosity than a uniformly thickened one. Furthermore, the proposed frame possesses a dynamic visual quality that the base pattern does not. (fig.3.3.3.22)
3.3.3.5. Structural Evaluation

The issue with this method is that it does not consider the granularity of the input pattern, and this leads to areas of weak connection within the critical region. Certain parts within this region are only connected by a single lattice member. This is a shortcoming of the software, since it was not designed to be used in this type of application. Future software development should include a parameter considering the grain size which would affect the stiffness of the outcome. Furthermore, the software should iterate between the domain stiffness and frame stiffness to minimize the deviation between these two entities. The following is an illustration of what I believe the outcome of such an algorithm would look like.

It should be noted that BESO only considers Von Mesis stress as the only structural criteria for optimization. In reality, there are many situations where Von Mesis stress is not the limiting state. For this reason, I present the following diagrams showing increased material near the joints and non-porous joint nodes. These areas are subjected to high shear and bearing stress, furthermore they need to accommodate splices for erection. Thus, it was assumed that, if a more complete engineering study of all possible failure modes were conducted, the resultant material distribution would be as indicated in these diagrams.

Due to the amount of manual processing required to execute the current method, it was not feasible to perform component optimization on every member in the structure. Some assumption was required in order to extrapolate the results from several select members to the entire structure. It is known that gravity loads accumulate at the base of the structure, therefore it is reasonable to assume that the amount of material present on the lower levels will be more than on upper levels. If the optimization process were executed on the entire frame, it will most likely have a gradient of material distribution, where the material density increases towards the base. (fig 3.3.25)

Furthermore, the choice of this vernacular pattern will inevitably compromise the structural capability of the column. While the goal of this method is to optimize the lattice component within the bounds of the chosen pattern, it will, unfortunately, end up less optimal than a conventional structural member. The reason for this is the layout of the pattern deviates from the optimal load path for a column. In any structure, columns are primarily responsible for

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2 The selection of dense area is based on structural intuition. In order to achieve more accurate density distribution, advanced structural analysis on indeterminate structures is required. Further recommendations on this issue is described in the conclusion chapter.
taking axial loads to the ground, and the optimal load path to achieve this is a straight line. From figure 3.3.3.26, we can see that, if a straight bar is subjected to an axial force 'P', the ground will push back with the equal force of 'P'. Yet, within the Guemcho pattern lattice, there are many offset 'L' conditions. From figure 3.3.3.26, we can see that, when a "L" bar is subjected to the same axial force 'P', the ground has to react with 'P' and an additional 'M_e' due to the offset e. The additional reaction has to be translated through the 'L' bar as bending and shear stresses. These stresses are not present in the straight bar. Thus, when subjected to an equivalent load, a straight bar will always take less stress than a 'L' bar of the same height. Since the Guemcho pattern consists of many 'L' bars, it will always be less efficient than a conventional straight structural member.
1. Base thickness lattice members
2. Gradually thickening lattice members towards joint connection
3. Bolted stiffened splice
4. Elevation of cover panels with lattice members beyond.

[figure 3.3.3.27] Sectional Diagram of Member to Joint Transition

[figure 3.3.3.28] Perspective View of Member to Joint Transition
Part 4.

Evaluation

4.1. Introduction

4.2. Structural Validity Report
   4.2.1. Comparative Building System Analysis
   4.2.2. Comparative Component Analysis

4.3. Architectural Demonstration
4.1. Introduction

This chapter presents the final result of combined system of TO-Columns and TO-beams into the TO-Vierendeel. This system can be evaluated structurally and aesthetically; thus, analysis provided in this chapter is to validate the structural and architectural merit of the proposed design process.

Given that the TO for both scales of analysis considered the structure to be made of continuous shell, the transformation between the direct output of TO and the frame systems makes them less structurally optimal. This is not necessarily a problem, because the purpose of this thesis is to seek an aesthetic variety of TO generated forms rather than pursuing minimal structure weight. The question is whether these frames perform better than a regularly distributed frame or not. To explore this point, two sets of studies have been conducted: first, a comparison between the TO informed overall frame and a uniform Vierendeel frame, and second, a comparison between the selectively thickened lattice column versus a uniformly thickened one.

Architecturally, the scheme cannot be evaluated in the same numeric manner as structural performance. A series of drawings and perspective views are generated for interrogating the quality of space created.
4.2. Structural Validity Report

4.2.1. Building Frame Structural Validity Test

A comparative analysis between the proposed, coloured in blue, and typically arranged Vierendeel frame, coloured in red, was conducted using Karamba 3D linear elastic finite element analysis. Both frames had the same external input load, and boundary conditions. A live load of 4.8 kpa was applied at each floor and the bottom nodes were fixed (fig. 4.2.1).

It was found that the locations of critical members were the same in the two systems. In both scenarios, the bottom column and beam carried the critical axial load and bending moment respectively.

The numeric chart shows the axial load and bending moment distribution in order of magnitude. The number at the top or bottom represents the maximum load that occurs in the critical members. (fig. 4.2.3 & fig. 4.2.5)

As the results show, in this model comparison, the proposed design provides better structural performance in terms of reducing maximum axial load; however, in terms of maximum bending moment, it was inferior to the regular frame.

[figure 4.2.1] Uniform Vierendeel System (left) and Proposed TO Guided Vierendeel System (right)
[figure 4.2.2] Location of the Critical Members (left) and Load and Support Condition for Uniform Vierendeel System (right)

[figure 4.2.3] Numeric Chart for Axial load (left) and Bending Moment (right) Distribution for the Uniform Vierendeel at the Critical Column and Beam Respectively.

[figure 4.2.4] Location of the Critical Members (left) and Load and Support Condition for Uniform Vierendeel System (right)

[figure 4.2.5] Numeric Chart for Axial load (left) and Bending Moment (right) Distribution for the TO-Vierendeel at the Critical Column and Beam Respectively.
4.2.2. Component Structural Validity Test

For the structural analysis of the components, a comparison between the proposed lattice column and a continuously densified lattice column was made. (fig. 4.2.6) In both cases, the Von Mises utilization is greater than 100%, so neither structure is technically capable of resisting the required loads. Thus, this analysis only intends to provide a model scenario to facilitate the comparison between a regular lattice structure and a TO-guided lattice. For future development, investigation of an interior support system is required for meeting the required structural capacity.

The external load is limited to axial load for this analysis since a column is primarily loaded in compression. Other loads, such as shear and bending, are excluded to simplify the comparison. The total gravity load of about 5000 kN is distributed evenly to each node at the top of the column model. The bottom has a fixed support condition.

The specific modulus of each column is the evaluation criteria for comparison since these lattice structures contain varying thickness. The specific modulus is a material property defined as the elastic modulus per unit of mass density of a material. In other words, it is a measure of the stiffness to weight ratio or specific stiffness. Since Karamba analysis does not provide a direct value of stiffness, the model strain energy was used instead. Strain energy is defined as the energy stored in a body due to deformation, and denoted by the letter ‘U’. This value can be obtained directly from the Karamba 3D analysis. Since stiffness is proportional to reciprocal of strain energy and weight, we can write it as following:

\[
Eval = \frac{Stiffness}{Weight}
\]

Stiffness is in proportion to Strain Energy, and Eval is in proportion to Strain Energy. Thus,

\[
\therefore Eval = \frac{Strain\ Energy}{Weight}
\]
Weight can be converted to a form of Volume as follow:

\[ W = m \times g \]

where \( W \) is weight

\( m \) is mass

\( g \) is acceleration of gravity

Mass can be written as a product of volume and density,

\[ m = V \times \rho \]

where \( V \) is volume

\( \rho \) is density of steel

\( l \) is length of each tube

\( A \) is sectional area of each tube

\[ W = V \times \rho \times g \]

\( \rho \) and \( g \) are constant in this case, with values of 8.05 g/cm³ and 9.8 m/s². Thus, \( W \) is proportional to the volume of the entire assembly.

Also, total volume of the lattice frame is a product of sectional area and length of each tubes:

\[ \text{Total Volume} = \sum_{n} A_{n} \times l_{n} \]

Where \( n \) is the number of lattice members

\( A \) is sectional area of each tube

\( l \) is length of each tube

In conclusion, the evaluation function is written as follows:

\[ \text{Eval} = \frac{\text{Stiffness}}{\text{Total Volume}} = \frac{\sum_{n} A_{n} \times l_{n}}{\sum_{n} A_{n} \times l_{n}} \]

The comparison is made between a set of lattices with constant member diameters varying from 2cm to 8cm, and the set of proposed structures consisting of specific members with diameters from 2cm to 8cm and infill members of a constant 2cm diameter. This comparison shows that the proposed frame of a 2cm diameter selectively reinforced with 8cm diameter elements has roughly the same strain energy as a uniform 3cm diameter frame (fig 4.2.7 & fig. 4.2.8). The total volume of the proposed frame was greater than the equivalent uniform frame. Thus, the proposed design had a lower specific modulus then a uniform frame, in turn, less economical value; however, the proposed frame is more visually dynamic and the moduli are comparable.

The reason why the proposed frame did not perform better than the reference frame might be due to the fact that, although the member reinforcement followed the load path generated through TO, the discretization caused the members along that path to experience bending. The most efficient way to resist load is always axial, therefore, when members within the region of reinforcement deviate in angle to the axis of the load path, their efficiency drops dramatically. There are some points where the members are perpendicular to the load path, meaning they have to transmit force entirely through bending.
4.3. Architectural Demonstration

This chapter presents a series of architectural drawings as a demonstration of the aesthetic evaluation for the experiment: a design intervention using Topology Optimization as a guide to create a structural system based on contextual references. One goal of the project architecturally was to modify the tower so that it was more contextually relevant to the neighbourhood. From the image on the right (fig. 4.3.1 & fig. 4.3.2), we can see that the addition is successful in being iconographically Korean. The exoskeleton on the addition also serves to tie it together with the architecture of the existing building.

The interior layout of the addition takes advantage of its location high up on the tower (fig. 4.3.3). The top floor of the existing building is already a successful viewing deck in the city. The extension proposes to extend this space into a multi-tiered space, tied together with a vertical atrium. This space could serve as a co-working and canteen space, where occupants can enjoy the view and engage in a collaborative convivial environment.

The diagram shows the relationship between the exterior Vierendeel and the floor plates (fig.4.3.4). The following views explore the quality of the extension. The exterior view shows the texture of the decorative, yet structural frame (fig.4.3.5). The interior view shows these structural lattices in scale with building occupants (fig.4.3.6). Though these members are large, they appear less oppressive due to their fine grain texture. The final view shows how the frame reaches out into the space beyond, simultaneously encapsulating the space within and without (fig. 4.3.7).
[Figure 4.3.3.] Progress Diagram of Interior Space Arrangement
[figure 4.3.4] Exploded Axonometric Illustration of the Proposed Design
[figure 4.3.5] Exterior View of the Proposed Design
[figure 4.3.6] Interior View of the Proposed Design
[figure 4.3.7] Frame View of the Proposed Design
Part 5.

Conclusion
This thesis presented an architectural implementation of an engineering tool, Topology Optimization, in atypical discrete structures. Atypical structures refer to structural systems of which optimal performance is not their primary interest; rather, these structures are designed with cultural, social, and aesthetic considerations in compromise with structural efficiency. Since TO research has been primarily undertaken in pursuit of maximum structural efficiency, there has been limited research of its application to non-funicular structural design. Therefore, this implementation intends to contribute a case study of designing a structural system in negotiation between structural and architectural demands, through the use of emerging technology such as TO.

The site chosen for this implementation was Jongno tower in Seoul, South Korea, of which the existing facade features an exposed diagrid that has been criticized for lack of contextual integration. The implementation was conceived of as an addition to the existing building that incorporated TO informed Vierendeel frame and lattice systems at building and component scales respectively. The final design is the result of combining TO generated forms with a structural system chosen by the designer. This combination opens a potential for further investigation of how structural tools could be used as guides to create novel architectural designs.

This implementation was evaluated both structurally and architecturally to prove the merit of this design process. A series of drawings demonstrates that these structures create similar spatial qualities found in vernacular Korean architecture. Structurally, linear elastic finite element analysis was used to demonstrate the structural validity of the TO-Vierendeel and TO-lattice structures. It was noted that the proposed design did not satisfy the Von Mies utilization criteria; thus, it requires further development to reach adequate structural performance. The structural validity presented in this thesis only served as a comparison between the proposed structure and a typical structural
layout for the same scenario. Similarly, the joint design was meant to propose a plausible solution within the constraints of current construction logistics. It was not my intention to tout this design as the best possible configuration. I believe that future construction techniques, such as large scale additive manufacturing, are what will ultimately make designs, such as the proposed, viable. The goal for the joint design was to create more tactility in the design, so it made sense to ground it with techniques closely resembling ones from current engineering practice.

For future development, an extension of the proposed exoskeleton design to the rest of Jongno tower’s diagrid could be explored. The current design was conducted in the void of the tower to create a visual comparison with the existing diagrid structure. The second phase of this thought experiment might be to re-imagine the existing diagrid as a TO guided structure (fig. 5.1). Although this may not be a realistic proposal, this conceptual design will allow us to compare the technique in the greater urban context and provide comprehensive understanding of the project in urban analysis.

The main important area to develop is structural performance. From the structural analysis, the current TO-Lattice design is overstressed under gravity load. The proposed elements have axial stress utilizations of greater than 100%, meaning that these elements are not structurally sufficient. One strategy to rectify this is to create a more robust method of reinforcing the input lattice pattern. The current method designates regions within the lattice for uniform reinforcement. It does not take the relative grain of the input lattice into account. (fig.5.2) This leads to undesirable situations where reinforced regions are only connected by a single thickened member. One possible solution to this problem is to create a minimum region width based on the granularity of the input lattice. Another issue is the angles of the frame relative to the actual load path. Some of the frame members are perpendicular to the load path, causing them to be engaged in bending. A solution to this issue could be a non uniform size optimization of the reinforced region that takes into account the angle derivation of the frame member and the principal stress angle.

These suggestions made above require a modification of existing TO algorithms, which has not been explored in this thesis. By integrating the geometric constraint process within the algorithm, less time could be spent modeling, thus more architectural options could be explored. Furthermore, once the process is automated, it becomes easier to analyze more structural options. As with every iterative process, exploration of more options will inevitably lead to a better performance.
Given the benefits of performing multiple design iterations, it is recommended that a parametric process that controls the geometry of the structure be inserted in the TO algorithm (fig. 5.3). Potentially, there are two possible ways which parametric control could be incorporated. First, the desired geometric language of the structure can be used as a constraint in the algorithm. The process might be functionally similar to Truss Topology Optimization (TTO), where truss geometry is set before any mathematical analysis is executed, and iterations are performed by adjusting the angles and cross sections of truss members until an optimal layout is reached. Since the proposed methodology in this thesis is targeting Vierendeel frames, the member angles should not change. Therefore, it might be possible to modify an existing TTO algorithm and remove the angle variation parameter. Second, the geometric constraint could be a part of the optimization function itself. This requires a mathematical quantification of the geometry. Martínez et al., demonstrated a potential method for this in their research paper Structure and Appearance Optimization for Controllable Shape Design. In their paper, they propose a model which introduces a texture mapping technique into the optimization function. This modified optimization algorithm incorporates a desired pattern into the TO generated structures, and as a result, creates novel structures that are visually similar to the input texture. An adaptation of the algorithm from their paper might be the next step in developing the methodology proposed in this thesis.

Significant portions of the process proposed in this thesis were carried out manually. This is problematic because it is too cumbersome to cycle through iterations. There are several areas that could be scripted for reduced manual input and improved structural performance. Firstly, the current process involves manual placement of structural members at building and component scales. At the building scale, columns and beams are arranged at the centre of the mass over the TO outcome to create the Vierendeel frame. At the component scale, the lattice members overlapping with the TO generated surfaces are manually selected. Another manual adjustment was performed after the initial creation of the TO-Lattice structures. Some members were selected to be thickened to provide additional support toward the connection based on engineering judgement. In order to improve structural efficiency, this step should also be integrated within the form generation algorithm. There should be an iterative loop between determining the regions requiring reinforcement, translating these regions to discrete components, re-evaluating the discrete structure and noting the deviation in performance between this and the initial analysis, and finally iterating this process to minimize the performance difference between the continuous region and the discrete frame.

This thesis has been my attempt at creating a method for dealing with the often contradictory requirements placed on architectural design. Frequently, architects are asked to design a building to satisfy aspects other than structural integrity such as: aesthetic, social and cultural integrity. In turn, this creates a dilemma in today's design practice which necessitates the reconciliation of opposites. Thus, this work has presented a method for trying to optimize an inherently non-optimal structure, and balance that with other unqualifiable objectives. My hope is that the methods proposed in this theses serve as a tool for others trying to navigate contradictory design spaces.
Bibliography
Bibliography


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Appendix

This appendix chapter contains a series of figures of Grasshopper scripts conducted for the Topology Optimization form generation presented in the methodology chapter. The figures presented here also help you guide through the procedure of the script execution step by step, and build future scripts upon the current version. Each of the following sub-appendices include expanded scripts in detail:

A-1: Script for Building Scale Topology Optimization
A-2: Script for Component Scale Topology Optimization
A-1
Script for Building Scale Topology Optimization

A-1.1. Overview of building scale Topology Optimization script
Domain Setup

A-1.2. Domain setup of building scale Topology Optimization script

Force Parameter Setup

A-1.3. Force Parameter setup of building scale Topology Optimization script
A-1.4. Optimization solver of building scale Topology Optimization script

A-1.5. Graphic setting of building scale Topology Optimization script
A-2
Script for Component Scale Topology Optimization

A-2.1. Overview of component scale Topology Optimization script
A-2.2. Domain setup of component scale Topology Optimization script

A-2.3. Force Parameter setup of component scale Topology Optimization script
A-2.4. Optimization solver of component scale Topology Optimization script

A-2.5. Graphic setting of component scale Topology Optimization script