The Economics of Water Conservation Regulations in Mining: An Application to Alberta's Lower Athabasca River Region

by

Yichun Huang

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Applied Economics

Waterloo, Ontario, Canada, 2020

Examining Committee Membership

The following served on the Examining Committee for this thesis. The decision of the Examining Committee is by majority vote.

External Examiner: Jinhua Zhao
Professor,
Department of Economics,
Department of Agricultural, Food, and Resource Economics,
Michigan State University

Supervisor: Margaret Insley
Associate Professor,
Department of Economics, University of Waterloo

Internal Members: Alain-Désiré Nimubona
Associate Professor,
Department of Economics, University of Waterloo

Dinghai Xu
Associate Professor,
Department of Economics, University of Waterloo

Internal-External Member: Chengguo Weng
Associate Professor,
Department of Statistics and Actuarial Science,
University of Waterloo

Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contributions

The second and third chapters are co-authored with my supervisor, Dr. Margaret Insley. I have contributed to all aspects of the research, including the development of the research topic and the economic model, customizing the numerical solution approach, and analyzing the results.

I am the sole author of the fourth chapter.

Abstract

Large demands for water by the mining industry are of increasing concern around the world and access to water is seen as a significant constraint on future mine development. Citizens, environmental groups and other non-governmental organizations have called for better regulation of water consumption by the mining industry in many regions across the globe. This thesis analyzes the efficiency of a specific command and control water management policy in the Lower Athabasca River Region in Alberta, Canada applied to oil sands mining operations. This policy imposes different restrictions on water withdrawals from the river according to the severity of threat to the aquatic ecosystem due to low water levels. In developing the policy, the Alberta government focused on the potential environmental impacts of projected water use by the oil sands industry. Economic cost was considered only in terms of the cost to the oil sands industry of constructing water storage facilities. This thesis undertakes a more robust examination of economic cost by developing a stochastic optimal control model for an oil sands firm choosing production and water use rates, as well as the optimal timing to build a water storage facility. A Hamilton Jacobi Bellman equation is specified which incorporates uncertain oil prices as well as uncertain water flow volumes in the Athabasca River and a numerical solution is implemented using a finite difference approach. The price of oil is modelled as a log-mean reverting stochastic process. Uncertainty in river flows is captured by modelling the restrictions on water withdrawals as a regime switching stochastic process. The thesis estimates the economic cost of the restrictions in terms of the difference in value of the oil-producing asset with and without water restrictions.

In Chapter 2, the model is used to analyze the Phase 1 water regulations, which were first applied in 2007. The Phase 1 regulations classified river water flows into green, yellow, or red zones with green implying abundant water and red implying reduced water flows. The water restrictions varied depending on river flow zone. In the thesis, the impact of these restrictions is captured by modelling the zones as different regimes with the probability of switching between regimes based on historical river flow data. The analysis also considers a number of cases in which the future river flow conditions are lower than those experienced historically. In Chapter 2, the total cost of the regulations is estimated as well as the marginal cost of increasing the water restrictions. For the Phase 1 restrictions, no information was available regarding the potential environmental benefits of the restrictions. Our conclusions show that the cost of the Phase 1 restrictions was quite small given the current reserve base and capacity of the industry. The chapter demonstrates how the marginal cost of tightening restrictions depends on the state variables, including resource stock and price. It is also shown that marginal cost is nonmonotonic with respect

to price volatility. The marginal cost is shown to vary across individual oil sands projects depending on reserve levels and lease length.

Chapter 3 undertakes an analysis of the Phase 2 regulations, implemented in 2015 as an update of the Phase 1 regulations. The development of the Phase 2 regulations was supported by a detailed scientific report (Phase 2 Framework Committee (P2FC) Report) outlining the likely environmental benefits of a suite of different water restrictions (rule sets) in terms of wetted area around the river. Wetted area was used as the indicator of ecosystem disturbance. The suite of water restrictions considered encompassed a much finer delineation of different water zones than in the Phase 1 regulations. The P2FC report presented an efficient frontier contrasting the effect on wetted area with the cost of water storage implied by the different restrictions. Based on their analysis the Committee chose one of the rule sets as the preferred option. This chapter uses the model developed in Chapter 2 to create a similar efficient frontier, comparing the change in wetted area with the economic cost to the oil sands. Assumptions regarding future river flows, operating costs, oil prices, future production and storage capacity and remaining established oil reserves are examined to determine their impact on the efficient frontier and the relative cost-effectiveness of the various options. The most important factors in determining the cost of the water restrictions are found to be the assumed storage capacity, cost of storage, projected river flow conditions, productive capacity and reserves. It is also found that given the significant growth of oil sands productive capacity assumed in the P2FC report, the recommended water restriction rule set is robust. However, for a smaller assumed growth in oil sands capacity, the proposed water restrictions impose very little cost on the oil sands industry. In this case, a different rule set would be recommended based on its better expected outcome in terms of maintaining the chosen ecosystem indicator.

A key input to the analysis is the assumed model for oil prices. Chapter 4 applies different versions of the Kalman filter to estimate three one-factor stochastic models. The regime switching model turns out to outperform the other two single-regime models. However, the single-regime log mean-reverting model is judged to be adequate for the analysis in Chapters 2 and 3 and is preferred because it greatly reduces the complexity of the numerical computation and the interpretation of results.

Acknowledgements

Firstly, I would like to express my sincere gratitude to my supervisor Professor Margaret Insley for her patient instruction, complete trust, continuous support, and enthusiastic encouragement. Her willingness to give her time and to share her wisdom so generously has been very much appreciated. Without her persistent help, the goal of this project would not have been realized.

Besides my supervisor, I would like to thank the rest of my thesis committee: Professor Jinhua Zhao, Professor Alain-Désiré Nimubona, Professor Dinghai Xu, and Professor Chengguo Weng, for taking the time to read this thisis and providing their insightful comments and inspirations, which motivated me to widen my research from various perspectives.

I am also grateful to Professor Charles Séguin and Professor Mireille Chiroleu Assouline, who contributed helpful comments for the preliminary version of Chapter 2 in Canadian Economics Association 2016 conference and in Canadian Resource and Environmental Economics Association 2016 conference.

My sincere thanks also goes to Professor Pierre Chaussé, who offered me valuable advice on stochastic process study as well as computing skills and resources.

I would like to acknowledge the support of Dr. Irene Huang, who inspired me and unreservedly provided me computer hardware and software resources and relevant knowledge.

My special thanks are extended to Patricia Shaw, Graduate Administrative Co-ordinator in our department, who has offered me endless help from various aspects.

I also thank all my fellow students for the enlightening discussions, the stimulating conversations, and all the great memories they have left in my mind.

Last but not the least, I must express my very profound gratitude to my family for providing me unfailing support and continuous encouragement throughout writing this thesis and my life in general. This accomplishment would not have been possible without them.

Dedication

I dedicate this thesis to my family for offering me unconditional love, unwavering support and utter devotion.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introductory Chapter

1.1 Overview of the Thesis

Water is an invaluable and indispensable gift of nature. However, with the growth of human population and rapid pace of economic development, human activities impose huge negative impacts on the quantity and quality of water resources. Although there is not a global water shortage, water scarcity has been a serious concern in specific countries and regions. We can glimpse the importance of the water resource problem from the fact that the United Nations set sustainable management of water as one of 17 Sustainable Development Goals in the 2030 Agenda for Sustainable Development (United Nations 2015). In this context, regulators in nations and regions across the globe are responsible for creating water management policies to relieve water scarcity while balancing rights of important stakeholders. This thesis examines one of these practices: a water management regulation in the Lower Athabasca River Region developed by the Alberta government in Canada.

The Athabasca River runs through the province of Alberta, Canada flowing from southwest to northeast. The Athabasca oil sands, the largest deposit in the world (Masliyah et al. 2004), is located in northeastern Alberta, which is around the lower course of the Athabasca River. The large amount of water required by oil sands production is provided by the Athabasca River. Compared to other water usage such as agricultural and domestic use, oil sands mining operations are the dominant water user from the Athabasca River.[1] Although the water allocation to the oil sands sector in 2018 was under 2.5% of the river's average annual flow and the actual withdrawal is even less[2], seasonal water fluctuations mean that potential impacts on the ecosystem are a concern. This will be exacerbated if oil sands production increases in future, combined with the potential impacts of climate change. In 2007, the Alberta government began to implement restrictions on water use, referred to as the Water Management Framework. This framework had two phases. Each phase prescribed a "rule set", which is composed of a rule to classify river flow conditions into different zones and a rule to impose associated water withdrawal limits for different zones. The intent of the framework is to protect the aquatic ecosystem by imposing relatively strict withdrawal constraints in dry seasons and loosening these restrictions when the river flow is abundant. The Phase 1 rule set defined green, yellow, and red zones for river flows associated with abundant, mildly dry, and very dry conditions. For each zone, there is a criterion for setting the water withdrawal limit by the oil sands mining projects. The Phase 2 Framework, which was implemented in 2015, further refines the Phase 1 Framework in that it defines multiple different river flow zones for different periods during a year and sets water withdrawal limits for those zones.

The development of environmental regulations such as the Water Management Framework, is a complex task involving the balancing of the interests of different stakeholders while maintaining the integrity of the ecosystem. Based on the standard economics paradigm, a regulation would be judged to be welfare improving, and hence desirable, if the benefits of the regulation outweigh its costs. The focus of the development of the Water Management Framework was maintaining an undisturbed ecosystem, which implies keeping river flows to within historical norms. No attempt was made to put an economic value on an undisturbed ecosystem. The Phase 1 Framework was developed fairly quickly as a stopgap before more serious investigations could be done regarding the benefits of different alternative rule sets. No study was made of the economic costs of the rule set. The Phase 2 Rule set considered a number of different possible rule sets and calculated economic cost in terms of the cost for oil sands firms to install storage. The benefit of the various rule sets was summarized by a particular ecosystem indicator, and a cost-effectiveness analysis was undertaken whereby the costs of storage for each rule set were contrasted with their effectiveness at maintaining the ecosystem indicator.

[1] Information is provided on the websites of Alberta Environment and Parks (https://www.environment.alberta.ca/apps/OSEM/) and the Athabasca River Basin Research Institute (http://arbri.athabascau.ca/About-the-Athabasca-River-basin/Index.php) (accessed on January 11, 2020).

[2] Information is provided on the website of Alberta Environment and Parks (https://www.environment.alberta.ca/apps/OSEM/) (accessed on January 11, 2020).

The objective of this thesis is to reevaluate the economic costs of the Phase 1 and Phase 2 Frameworks using a more robust methodology for estimating economic cost. The cost of storage is only one component of the economic cost of the regulations and may not capture the full opportunity cost of the regulations. The thesis does not attempt to put a monetary valuation on the ecosystem benefits of the regulations. For the Phase 1 Framework, we infer the marginal costs of the water withdrawal limits in the applied rule set. The efficient water withdrawal limits would be those that equate the marginal economic costs and the marginal economic environmental benefits. For the Phase 2 Framework, we compare the total economic costs of multiple candidate rule sets. The optimal rule set is the most cost effective one, i.e. the least costly one when the environmental benefits are at the same level.

The study of the Phase 1 Framework is carried out in Chapter 2. We apply a stochastic optimal control approach to investigate the economic costs to the oil sands industry due to the water constraints imposed by the Phase 1 Framework. In particular, we develop a dynamic economic model of optimal exploitation of the oil resource given uncertain oil prices and stochastic river flows and we implement a numerical solution. We calculate the total and marginal costs of tightened restrictions corresponding to different water withdrawal limit levels given the specific river flow classification. As noted, we have no information on the marginal benefit of tightened restrictions, However, the marginal cost curve can give a lower bound on what marginal benefits should be to justify tightening restrictions. Apart from this, our study also suggests the best timing to invest in water storage facilities.

The examination of the Phase 2 Framework (Alberta 2015) is conducted in Chapter 3. In 2015, the Phase 2 Framework replaced the Phase 1 Framework. In fact, the Phase 2 Framework Committee (P2FC) was established in 2008 to develop and assess multiple candidate water constraint rule sets and recommend one of them for the Phase 2 Framework. The final report by the P2FC was released in 2010. The Alberta government fine tuned the rule set recommended in the report to correct some mistakes and adopted it for the Phase 2 Framework ("the Phase 2 Choice"). The economic costs due to the water constraint imposed by the Phase 2 Choice can be inferred as was done for the Phase 1 Framework. However, we do not repeat the process in Chapter 3. Instead, we focus on the justification of the assessment of the candidate rule sets by the P2FC. In the P2FC's research, a detailed analysis is given to assess candidate rule sets, especially from the environmental effect perspective. For each rule set, the P2FC used a software tool (the Flow Calculator) to determine its environmental benefit in terms of maintaining wetted area along the river. The P2FC also estimated the water storage levels for alternative rule sets that are able to provide sufficient water to maintain the oil sands production at full production capacity.

The economic costs are inferred simply according to the estimated water storage capacities. The P2FC report comparing the different rule sets in terms of their impact on an ecosystem indicator provides a starting point for a more detailed assessment of the cost effectiveness of the regulation. This chapter explores the economic costs of the different rule sets, considering the potential effects of stochastic oil prices, stochastic river flows, and associated optimal production levels, in a dynamic setting. Using the stochastic optimal model developed for Chapter 2, we estimate economic costs for the suite of different rule sets proposed by the P2FC based on their assumptions for productive capacity, required storage levels, etc. Furthermore, we also determine optimal water storage levels for each candidate rule set in different river flow scenarios. By checking multiple factors affecting the economic costs, we conclude that the rule set recommended by the P2FC is not necessarily always the optimal choice. In the chapter we discuss what factors have the biggest impact on economic cost, and in what cases our conclusions agree with those of the P2FC.

A key component of the research in Chapters 2 and 3 is the modelling of the stochastic process of the price of the underlying asset (i.e. the oil price). Chapter 4 focuses on the estimation of the dynamics of oil prices, a topic which has been the focus of much research in the literature. In that our optimal control problem involves two stochastic factors: the oil prices and the river flows, with the intention of avoiding unnecessary computational complexity, we adopt a model of oil prices with only one stochastic factor. In this chapter, we consider three one-factor stochastic models, respectively in level mean-reverting, log mean-reverting, and regime switching log mean-reverting forms. We apply different versions of the Kalman filter to estimate these models. The results show that generally the regime switching model outperforms the other two. The log mean-reverting model is better than the level mean-reverting models for forecasting long term oil futures contracts. Considering that the project lifespans in Chapters 2 and 3 are more than 1 year, and the regime switching model significantly increases the complexity of calculation and interpretation of our optimal control problem without significantly changing the results, we adopt the log mean-reverting model for the analysis in Chapters 2 and 3.

1.2 Main Contributions

- We develop an economic model incorporating uncertainties in oil prices and water withdrawal limits to investigate the oil sands sector's stochastic optimal control problem under water management regulations and numerically solve the derived two-factor Hamiltonian-Jacobi-Bellman equation.

- We use the aforesaid model to examine the efficiency of a particular command and control regulation. We delineate a marginal cost curve for water restrictions and examine key factors that affect the marginal cost. An interesting result is that the marginal cost curve is non-monotonic with respect to price volatility. Our investigation also shows that the marginal costs of the same regulation vary across projects depending on the lease length and remaining reserves. This finding demonstrates the well known result that a uniform command and control regulation is unlikely to achieve efficiency.

- Our estimates of the full economic cost of a suite of candidate water restrictions allows us to examine their cost-effectiveness in terms of maintaining wetted area along the river. In our example, changing price uncertainty does not affect the relative cost-effectiveness of the different candidate restrictions. Different assumptions about future river conditions affects the magnitude of economic cost, but again does not change the relative cost-effectiveness. Changing assumptions about reserves and productive capacity does change the ranking of alternative restrictions.

- We apply two extensions of the Kalman filter (the extended Kalman filter and the Kim filter) to estimate a level mean-reverting model and a regime switching log mean-reverting model respectively.

Chapter 2

An Economic Analysis of Alberta's Water Management Regulations for Oil Sands Mining

2.1 Introduction

The management of scarce water supplies is an issue of increasing concern in many areas of the world and is exacerbated by uncertainty surrounding the impacts of a warming planet on water availability. Surface and ground water sources are typically exploited as common pool resources meeting diverse needs. As noted by Libecap & Barbara (2012), the fluid nature of water and the fact that it is used sequentially or simultaneously by many parties hinder our ability to define an efficient property rights system. Externalities and third party effects of water diversion are pervasive. The resource extraction industry is responsible for large withdrawals of water around the world, and competition for water supplies may put industry operations into conflict with local communities. These conflicts arise when the water demands for resource extraction encroach on the water supplies used for other human activities or compromise aquatic ecosystems. Protection of the public interest requires that governments around the world specify limits on water withdrawals and enforce legal and regulatory requirements regarding water access rights.

Media and industry reports make it clear that competition for water supplies is of increasing concern for firms involved in resource extraction. Water availability is reported as being one of the biggest problems facing mining firms today.[1] Similar concerns have been raised regarding shale gas development.[2] Regulatory responses vary across jurisdictions, depending on the state of water supplies, the nature of other competing uses, as well as the existing political, legal and regulatory frameworks. Thomashausen et al. (2018) review the legal framework regulating water use for gold and copper mining in eight different countries. All countries surveyed required mining firms to obtain water licenses or permits as well as undertake some sort of environmental assessment. The basis for allocating water shares varies, and is typically some combination of riparian or prior appropriation rights, as well as rules about the transfer or trading of water rights.

The focus of this chapter is on the assessment of the economics of water regulations imposed on resource extraction activities. To this end, we examine the regulation of mining operations in the Alberta oil sands limiting fresh water withdrawals from the Lower Athabasca River. Our modelling approach and conclusions provide insights for public policy in Alberta's oil sands, as well as in other similar industries throughout the world.

The large ramping up in scale of oil sands activity in the past decade has brought public attention to the quantity of both surface and groundwater withdrawals, as well as the many other environmental impacts that have been well documented in the literature.[3] Fresh water withdrawals from the Athabasca River by open pit mining operations have the potential to impact the health of the river ecosystem, particularly during low-flow periods. In addition, the river sustains the livelihood and culture of First Nations and Metis communities in the area, and low flow hinders navigation on the river. Predictions of continued industrial expansion and growing population as well as expectations that water flows will be reduced in the future due to the effects of a warming climate have exacerbated concerns. In response to these concerns, the Alberta government drafted a river management plan for the Lower Athabasca River to limit withdrawals according to river conditions. The management plan was first imposed in 2007 and is described in the Phase 1 Framework (Alberta and Canada 2007). This Phase 1 Framework was intended to address immediate needs for water protection based on currently available evidence, with the intention that the regulations would be revised in future based on the results of further research. Additional research and consultation with stakeholders were done over the subsequent seven years, resulting in a revision to the water regulations released in 2015 as the Phase 2 Framework (Alberta 2015). The focus of this chapter is on the regulations

[1] See for example a July 27 2014 Financial Times article "Water scarcity and rising energy costs threaten mining industry"; a Moody's Investor Service report "Global Mining Industry: Water scarcity could increase rating pressure on global mining companies", February 14, 2013; and Toledano & Roorda (2014).

[2] See discussions in Vengosh et al. (2014) and Holding et al. (2017).

[3] See Griffiths et al. (2006), Gosselin et al. (2010), Squires et al. (2010), and Bruce (2006) for details.

specified by the Phase 1 Framework. In the next chapter the impact of the changes made with the imposition of the Phase 2 Framework are examined.

The Alberta Framework (both Phases 1 and 2) specifies aggregate permitted water withdrawals by oil sands mining firms depending on river conditions. When river flows are below certain specified thresholds cutbacks in water diversions are required. In the Phase 1 Framework, river conditions are categorized as being in any of the red, yellow or green zone which signifies a low, medium and abundant water flows, respectively. The Phase 2 Framework has a finer classification of water flow conditions. Alberta's water management Framework is layered upon an existing prior allocation (or appropriation) regime whereby senior licence holders are given priority over more junior water license holders. Oil sands firms are asked to submit annually a water sharing agreement to the government which specifies how cumulative withdrawals will be shared within the terms of existing water licenses in event of a shortfall. River flows are highly seasonal and the Phase 1 Framework encourages firms to store water during times of high water availability for use during times of shortfall.

The stated objective of the Alberta Framework is to "manage cumulative water withdrawals to support both human and ecosystem needs, while balancing social, environmental, and economics interests" (Alberta 2015, p. 3). An assessment of the efficiency of Alberta's water regulations requires analysis of the marginal benefits and costs of the water restrictions. The costs will be felt mainly by oil sands firms in terms of lost profits. The benefits of water quantity restrictions are more diverse, reflecting the benefit of leaving additional units of water in the river. These may be benefits to the ecosystem or benefits to other users of the river. There is considerable scientific uncertainty in how much water can be safely diverted from the river without harming the aquatic ecosystem[4], making it very difficult to pin down a reasonable estimate of the value of leaving more water in the river. While the marginal cost of water restrictions to firms is also unknown, it is easier to assess than the marginal benefit of an additional unit of water to the ecosystem. A careful assessment of the marginal cost is useful in that it provides a lower bound for the marginal benefit in order for the regulation to be judged to be welfare improving.

For efficiency, water regulations should ensure equal marginal costs of compliance across

[4]See for example a CTV news report from March 19 2014, "Alberta's plan for Athabasca River 'pathetic,' not science-based: critics." by Bob Weber, The Canadian Press. This article quotes David Schindler, a University of Alberta ecologist who claims a lack of scientific evidence for the chosen water restrictions and argues that even a couple of inches less in the river can have a critical impact on fish habitat, bug populations, water quality, ground water etc. http://www.ctvnews.ca/sci-tech/alberta-s-plan-for-athabasca-river-pathetic-not-science-based-critics-1.1735778 (accessed on January 11, 2020)

individual firms in the industry. The Alberta water regulations are 'command and control' with no mechanism to promote an efficient allocation of water across regulated firms. According to Alberta's current regulations, when water withdrawals are restricted, the limited available water is allocated equally to all existing oil sands plants, without considering the needs of new entrants nor the differing efficiency of water use of existing firms. Also noteworthy is that there are no stated financial penalties for oil sands firms violating water sharing agreements to distribute the allowed amount of water. According to Braathen & Johnstone (2003), the efficiency of voluntary approaches is generally low.[5] However even with no penalties, the literature suggests that some compliance with voluntary regulations is expected as a result of firms' caring about their public image and loss of good will if they are observed as being poor environmental stewards, among other reasons (Arora & Cason (1995),Arora & Cason (1996), Khanna & Damon (1999)).

The broad purpose of this chapter is to contribute to our understanding of the efficiency of 'command and control' regulations for the industrial use of water in cases where the specific restrictions are tied with a stochastic environmental indicator. The specific contribution is to model the profit maximizing decisions of a typical oil sands project subject to of Alberta's Phase 1 Framework and from this to construct the firm's marginal cost curve for a range of water withdrawal restrictions. A further objective is to consider the effect of the option to invest in water storage facilities on the firm's marginal cost curve. From the individual project marginal cost curve, a characterization of the industry marginal cost curve can be obtained and contrasted with possible marginal benefit curves.

The modelling of economic cost is challenging because of the particular characteristics of the firm's decision problem. Water demands by oil sands firms are determined by their decisions about oil production. A firm chooses optimal production levels in the context of the optimal timing of depletion of the stock of oil reserves of a particular project. In the case of Alberta, oil reserves are publicly owned, and firms pay for the right to extract resources over specified time according to the terms of a license agreement. Firms also face restrictions on water extraction levels which are stochastic in nature, depending on river flows. If a firm chooses to install water storage, then the management of water storage levels is another component of the firm's decisions problem. Finally oil production, and hence water use, is affected by volatile oil prices determined in world markets. In summary, the firm's problem involves the optimal choice of oil production and the timing to install water storage facilities, given stochastic oil prices and water withdrawal restrictions, and given path dependent state variables - oil reserves and water inventory levels.

[5] Information is provided on the website of the Organisation for Economic Co-operation and Development (OECD) (http://www.oecd.org/env/tools-evaluation/voluntaryapproachesforenvironmentalpolicy.htm)(accessed on January 11, 2020).

In this chapter, the firm's decision is modelled as an optimal control problem, with oil prices described by a stochastic differential equation and water restrictions modelled as a Poisson process. The firm chooses at each time period over the life of a project whether to comply with water restrictions or invest in a water storage unit, which relaxes the constraint imposed by water restrictions. A system of Hamiltonian-Jacobi-Bellman (HJB) equations is specified which describes the firm's optimal decision problem. The system of HJB equations is solved using a numerical method, as there is no closed form solution. The solution provides estimates of the value of the a hypothetical oil sands project for a range of water restrictions, as well as the critical level for oil prices at which it would be optimal for firms to invest in water storage facilities. By varying water restriction levels we are able to estimate the marginal cost in terms of the value to the firm of relaxing the restrictions at the margin. While this model is applied to a specific example in Alberta, the approach and conclusions are of relevance for other mining projects world-wide, where water availability is becoming a significant constraint on development.

This paper contributes to the literature on optimal natural resource use under uncertainty as exemplified by papers such as Brennan & Schwartz (1985), Mason (2001), Slade (2001), and Chen & Insley (2012). It extends the analysis in these papers by including uncertain regulatory constraints resulting from natural variability in the environment. It also contributes to the environmental economics literature addressing water issues specifically. A paper with a similar motivation is Mannix et al. (2014) which examines the efficiency of Alberta's water regulations for the oil sands using a deterministic model. We will contrast our conclusions to their results. There are also some papers addressing the optimal use of publicly owned water storage facilities in agricultural operations (Alaouze (1991), Brennan (2010), Dudley & Hearn (1993)). A more detailed review of the literature is presented in Section 2.3.

As a preview, some key findings of the chapter are summarized below.

- Phase 1 water regulations impose only a very small cost on our hypothetical oil sands firm. Costs only become significant under drier river conditions and more strict regulations than specified in the Alberta Framework.

- Oil price volatility affects the decision to invest in water storage facilities in an interesting way, depending on the tightness of water withdrawal restrictions. When water withdrawals are tightly restricted, an increase in price volatility makes the investment in storage more likely (i.e. the critical oil price for investment is reduced). In contrast, when water restrictions are not binding an increase in oil price volatility makes it optimal to delay investment in water storage.

- Stricter regulations on water withdrawals may cause a firm to delay the permanent abandonment of a project.
- The marginal cost of water restrictions is lower when storage is in place and, as expected, rises as water restrictions are tightened. The shape of the marginal cost curve is affected by the option to install storage, and is non-monotonic when it becomes optimal to install storage.

The rest of this paper is structured as follows. Section 2.2 provides background information related to the oil sands industry and Alberta's oil water use regulations. Section 2.3 conducts a review of the relevant literature. Sections 2.4 and 2.5 develop a model for the stochastic optimal control problem. Section 2.6 describes the determination of parameters values in the model. Section 2.7 elaborates on the results. Section 2.8 carries out the sensitivity analysis, while Section 2.9 concludes. Appendices A.1, A.2, and A.3 describe the details of numerical solution approach. Appendix A.4 provides more details on sensitivity analyses.

2.2 Regulation of Water Use In the Alberta Oil Sands

Alberta's oil sands comprise the third-largest proven crude oil reserve in the world, next to Saudi Arabia and Venezuela[6]. With current extraction techniques, production from the oil sands depends heavily on water as an input. There are two main methods of producing oil from oil sands. One is open-pit mining, which is used to develop shallow oil sands reserves. In an open-pit mining project, oil sands are mined by huge shovels and transported by trucks to a plant, where oil is extracted from the oil sands by using warm water. The other one is in-situ (Latin, meaning "in place") drilling, which is used to develop deep reserves. In an in-situ project, steam-assisted gravity drainage (SAGD) technology is employed, i.e. steam is injected into the reservoir to reduce the viscosity of oil sands so that bitumen can be pumped out. Due to technical limitations, bitumen mining uses only fresh water, while SAGD can use saline water as well as fresh water. According to Kuwayama et al. (2013), the minimum water intensity in open-pit mining for oil sands is 14 gal/MMBtu, i.e. around 1.94 barrels of water needed for producing one barrel of oil[7] (hereinafter referred

[6]Source: the Government of Alberta, https://www.alberta.ca/oil-sands-facts-and-statistics.aspx (accessed on January 11, 2020)

[7]MMBtu stands for million British thermal units. To provide 1 MMBtu of energy, taking distillate no. 2 fuel oil for example, 7.2 gallons of oil are required. 14 gal/MMBtu represents that 14 gallons of water are needed for producing 7.2 gallons of oil. That is to say, 14/7.2=1.94 gallons of water/gallon of oil, or, in terms of barrel, 1.94 barrels of water/barrel of oil.

to using the form of 1.94:1), while the maximum is 47 gal/MMBtu (around 6.53:1), with an average of 29 gal/MMBtu (around 4.03:1). For in-situ drilling, the range of water intensity is from 9 gal/MMBtu (around 1.25:1) to 23 gal/MMBtu (around 3.19:1), with an average of 16 gal/MMBtu (around 2.22:1). It is interesting to note that the maximum water intensity for oil sands mining operations, classed as an unconventional oil resource, is less than for conventional oil (primary and secondary) or enhanced oil recovery as is reported in Kuwayama et al. (2013). This paper focuses on the relevant problems raised by water management regulations in the context of open-pit mining projects, since it is a significant concern for environmental protection in the Lower Athabasca River, which is the source of the fresh water used in these projects, as well as the Peace-Athabasca Delta, where flow from the Athabasca River converges with flow from the Peace river.

In Alberta, water rights are granted according to the "First in time, first in right" principle. Before 1999, licenses to withdraw water were issued without expiry dates according to the Water Resources Act. Since the Water Act took effect in 1999, new water licenses have a fixed time of validity (usually ten years). In practice, the amount of water permitted to be withdrawn by the licenses is more than enough to meet current production levels. Since the water allocated to the oil sands industry greatly exceeds the amount actually withdrawn, oil sands water users are not bound by licenses. Consequently, there arise some concerns about the impact of surface fresh water withdrawals on the aquatic ecosystem, should firms ever decide to make full use of the water they are licensed to use. According to Lunn et al. (2013), in the Lower Athabasca River, the collective withdrawals constitute only a tiny percentage of the river flow (less than 0.6% of average total river flows and about 3% of the lowest weekly winter flows). However, since the river flows vary significantly between seasons while the oil sands production has less seasonal variation, in water short seasons, there are risks that the withdrawals will exceed the sustainable level and irreversibly damage aquatic habitat. Moreover, in view of forecasts for ongoing increases in oil sands production, there have been significant concerns about the impacts of water withdrawals on the aquatic ecosystem (National Energy Board 2006, Griffiths & Woynillowicz 2003, Jensen 2010, Toman et al. 2008, Woynillowicz et al. 2005, Peters et al. 2013, Mannix et al. 2010, Ivanhoe Energy Inc. 2012). Combined with the conclusions drawn by some scholars (Wolfe et al. 2012, Schindler & Donahue 2006, Squires et al. 2010, Wolfe et al. 2008, Bawden et al. 2014, Rasouli et al. 2013, Peters et al. 2013) that there is a declining trend of the river flow in the Athabasca catchment, the recent focus on impacts on the aquatic ecosystem is unsurprising. The Peace-Athabasca Delta is a landscape of great ecological significance and is located within one of Canada's 15 UNESCO World Heritage Sites. Its ecosystem is heavily dependent on the river flow level of the Athabasca River (Wolfe et al. 2012).

As noted in Section 2.1, in order to assure the sustainability of the Lower Athabasca River as well as Peace-Athabasca Delta, the Alberta government (Alberta Environment) and the federal government (Fisheries and Oceans Canada) developed Alberta's 2007 Water Management Framework[8]. The Phase 1 Framework was in effect from 2007 to 2015, after which the Phase 2 Framework was implemented. According to the Phase 1 Framework, the river flows are classified into three conditions (green, yellow or red) based on the weekly river flow measurements. In the green zone, the water flow is regarded as abundant, and there is negligible impact of withdrawals on the aquatic ecosystem. In the yellow zone, the river flow is considered as low, and it is assumed that the aquatic ecosystem may experience stress from a 15% withdrawal. In the red zone, the river flow is regarded as too low for habitat health. There are different withdrawal limits for three different river flow conditions. In the green zone, up to 15% of instantaneous flow is allowed to be cumulatively withdrawn by the five oil sands firms. In the yellow zone, the maximum amount of water allowed to be withdrawn is 10% of the average of HDA80 [9] and Q95[10]. In the red zone, only a maximum 5.2% of the historical median flow in each week can be withdrawn.

Figure 2.1 depicts average, minimum and maximum river flows in the Athabasca River since 1957 compared to the three regimes set by the Phase 1 Framework. It also shows the frequency with which river flows would be classified in the green, yellow or red zones over that 60 year period. It will be observed that the river did fall into the yellow or red zones with a significant frequency over this period.

After the Framework came into effect, oil sands firms could be asked to curb water usage in winter, affecting industry profitability. In the short term, this could be accomplished by scaling back production. Over the longer term, if a firm expects more frequent or more severe restrictions, consideration may be given to investing in water storage or searching for new technology to conserve water. Imperial Oil's Kearn Lake project was the first to invest in water storage in order to eliminate the need to withdraw water from the river during low flow seasons.[11] Constructing an on-site pond is one feasible choice.[12] According to Alberta Energy Regulator (hereinafter referred to as "AER")'s Oil Sands and Coal Exploration

[8]Water Management Framework: Instream Flow Needs and Water Management System for the Lower Athabasca River. https://open.alberta.ca/publications/3990727(accessed on January 11, 2020)

[9]HDA80 is the river flow level corresponding to a habitat area level that is equalled or exceeded 80% of the time.

[10]Q95 is the flow level that is equalled or exceeded 95% of the time.

[11]page 19 of Imperial Oils 2012 summary annual report

[12]According to an on-line article from Suncor Energy Inc. entitled "Athabasca River water use: 5 things you need to know." http://osqar.suncor.com/2014/07/athabasca-river-water-use-5-things-you-need-to-know.html(accessed on January 11, 2020)

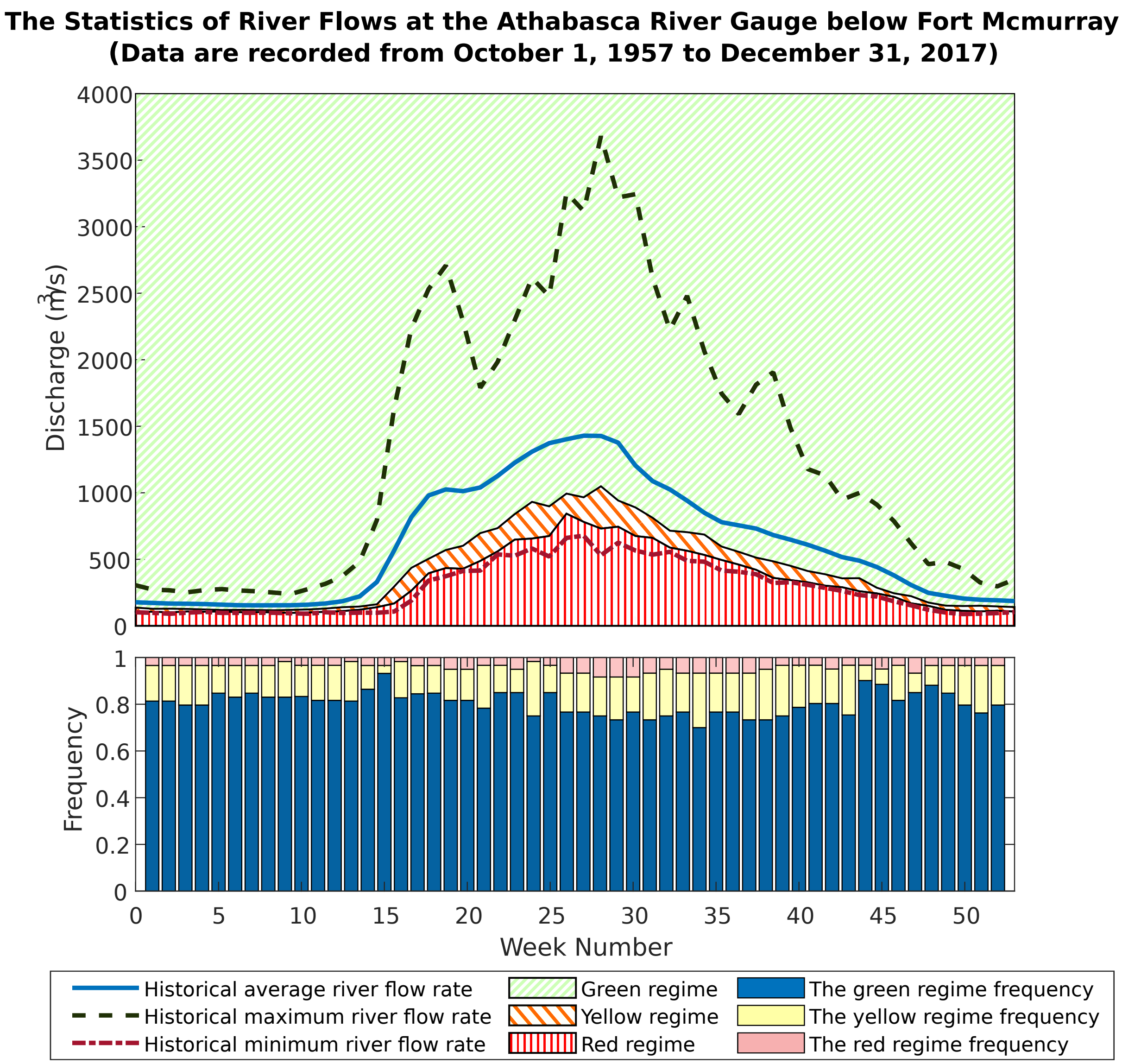

Figure 2.1: River Flows at the Athabasca River Gauge below Fort McMurray Station 07DA001 Compared to the Three Regimes Set by Alberta's 2007 Water Management Framework (The data are recorded from October 1, 1957 to December 31, 2017)

Application Guide[13], operators require permission from the AER if there are changes to exploration or operation locations. The construction of on-site water storage facilities will affect the status of the original location, so without the regulator permission, these cannot be built. However, from the above mentioned article, we can see that there have been operators who have taken water facilities construction into consideration when applying for new oil sands operation licenses.

Currently, oil sands companies are complying with the water withdrawal limits via the implementation of an annually renewed Oil Sands Water Management Agreement for the Winter Period[14] (herein after referred to as "the Agreement"), which is a voluntary agreement among five main oil firms in the Athabasca oil area. This agreement allocates the restricted water quantity almost equally among five oil sands extraction operators during the yellow and red zones. It stipulates that when the amount withdrawn by any individual operator exceeds the assigned allotment, the operator should report this to the relevant Alberta government department. However, there is no punishment specified for exceeding the agreed to allotment.

To improve the efficiency of water usage, the Government of Canada has also introduced certain rules about the minimum recycle rate. In 2009, the Energy Resources Conservation Board introduced the mandatory produced-water recycle rate of 90 percent for new SAGD projects. However, there is no recycle standard for open-pit mining so far.

2.3 Literature Review

There are several strands of literature which are relevant for this paper. One strand examines natural resource extraction under uncertainty; the second deals with policy and property rights issues for water; the third is about inter-temporal water storage issues.

2.3.1 Natural Resource Extraction Under Uncertainty

The decision framework used in this paper is based on the seminal papers of Pindyck (1980) and Brennan & Schwartz (1985), which spawned a long line of literature examining optimal behaviour of natural resource firms. Some literature focuses on stylized models

[13] Oil Sands and Coal Exploration Application Guide. https://www.aer.ca/documents/manuals/Manual008.pdf(accessed on January 11, 2020)

[14] For example: Oil Sands Water Management Agreement for the 2014-2015 Winter Period. http://osip.alberta.ca/library/Dataset/Details/562(accessed on January 11, 2020)

with closed form solutions (e.g. Miller & Voltaire (1983)), while others analyse more complex models requiring numerical solutions (e.g. Paddock et al. (1988)). Some of the literature characterizes these decision problems as real options, drawing on the finance literature. As noted by Vollert (2012), for real options, whose characteristics are not necessarily represented by mere isolated options of either European or perpetual American type, closed form solution is rarely applicable. The model developed in this thesis depicts both the timing of an investment as well as the optimal production of a non-renewable resource. A numerical solution is required.

Much of this literature focuses on how various relevant stochastic factors affect investment decisions. For example, Schwartz (1997) utilised a mean-reverting process to model output prices, and also added two other uncertain factors into his price model to examine implications for evaluating investment opportunities. Slade (2001) analysed the respective impacts on optimal decisions regarding a mining project of choosing either General Brownian Motion (hereinafter referred to as "GBM") process or a mean-reverting process to approximate the dynamics of output prices. Inspired by Hamilton (1989) who used a regime switching stochastic process for simulating economic variables, researchers started to capture the cyclic property of output prices by using regime switching models and address subsequent impacts on optimal investment timing. Hardy (2001) used a regime-switching lognormal model to simulate stock prices dynamics and derives the value of financial instruments. Parallel applications of regime-switching models can also be found in the analysis of investment in real assets. Chen & Insley (2012) investigated the advantage of a regime switching model of stochastic lumber prices over single regime models. Insley (2017) examined the best management and operation decision in the context of multi-stage investment using a regime-switching stochastic process to simulate the oil prices. Our work is similar to existing literature in modelling resource output prices as an exogenous stochastic process. In Chapter 4 of the thesis we examine several alternative processes to describe oil prices.

Prior to Brennan & Schwartz (1985) and McDonald & Siegel (1985), there was little discussion about the impact of managerial options (i.e. to suspend or reopen a project) on investment decisions. However for a long term project, such as an oil sands extraction project, it is necessary to take managerial options into consideration when determining optimal investment behaviour. In our work several managerial options are considered, including the option to suspend construction of the water storage facility and the option to abandon the oil sands project.

The non-renewable nature of the oil sands resource requires considering the limitation of available reserves. Mason (2001) improved predecessors' research regarding stochastic optimal control of resource extraction projects by introducing a finite resource stock instead of an infinite stock. Slade (2001) modelled reserves as a stochastic process to reflect uncertainty about recoverable reserve stock. In our work, oil sands reserves are assumed to be finite, and represent a path dependent variable, since the rate of extraction depends on uncertain oil prices.

Another strand of literature examines the impact of uncertain input costs. Pindyck (1993) dealt with an investment decision problem with two types of cost uncertainty. Slade (2001) investigated the impact on the valuation of a copper mine when taking cost uncertainty into account. Almansour & Insley (2016) also took the stochastic property of natural gas prices into account when examining investments in the oil sands industry. Our work is unique in introducing a restricted input (i.e. water) to the production process, with the input restrictions modelled as an uncertain stochastic process.

There are numerous papers in the stochastic optimal control literature addressing the timing of firms' investments in environmental protection in response to regulations. In some of these papers, other than building particular facilities, there are no alternative strategies to employ to achieve regulatory compliance while maintaining production capacity. Cortazar et al. (1998) explored the optimal investment decision when an environmental regulation restricts a smelter's production capacity from a higher level to a lower level. In contrast to their work, in this chapter, the restricted production level is not fixed. Instead, it depends on the stochastic variation of natural river flow. In other papers, firms can also resort to market based strategies to achieve the compliance. Insley (2003) examined the timing of building a scrubber in an electric power company in the presence of tradable emissions permits in order to comply with the U.S. Acid Rain Program.

A strand of the literature examines the impact of regulatory uncertainties upon investment decisions including the papers of Teisberg (1993) and Brennan & Schwartz (1982), among others. Teisberg (1993) used an Ito process to reflect the changes in the value of a project rate-of-return regulation arising from uncertain outcome of the regulation. In his model, the variation in regulations was captured by a stochastic regulatory term, whose value depends on the value of completed project, which is modelled as an exogenous stochastic process. In contrast, in our work, the uncertainty of the regulation is caused by natural flow conditions, and the model captures the uncertainty by introducing a stochastic input constraint factor. As an early paper examining the valuation of a regulated firm in a dynamic regulatory decision context, Brennan & Schwartz uses two variables (the probability of holding a hearing for deciding the proper rate of return and the outcome of the hearing, i.e. the allowed rate of return) to capture the uncertainties of the rate-of-return regulation, where these two variables are dependent on the value of current rate of return, which is a stochastic variable. Both above papers ascribe regulation uncertainties to regulators, whereas in this paper, the uncertainty arise from natural forces.

The methodology for the numerical solution of the optimal decision problem examined in this thesis are available in the current literature. We model the decision problem as a stochastic optimal control problem, which is solved numerically using a semi-Lagrangian approach as described in the works of d'Halluin et al. (2005), Chen & Forsyth (2007), and Chen & Forsyth (2010).

2.3.2 Efficient Regulatory Design for Water Policy

As noted in Section 2.1, there are many challenges to defining an efficient property rights scheme for water. Libecap & Barbara (2012) discuss the pervasive externalities and third party effects of water use. They note that because water supplies are stochastic, the extent of third party effects will change over time and location along a water course. Libecap & Barbara (2012) refer to the costs of bounding or partitioning water, the costs of measurement, and the interconnected private and public good qualities of water as factors which make efficient usage problematic. Government involvement or some type of collective agreement, rather than private markets, is the rule rather than the exception in water allocation.

Efficient water regulations would, in theory, maximize the total value of water use, which requires equalizing the marginal value of water in its different uses. Johnson et al. (1981) show that an efficient water allocation can be achieved when water rights are defined in terms of consumptive use (i.e. water diversion less water returned to a water course) and constraints are imposed to prevent third party effects.

Existing property rights regimes for water based on prior appropriation and/or riparian rights are known to generate significant inefficiencies in many cases, in particular when the transfer of water rights from lower to higher valued uses is inhibited by existing regulation. Prior appropriation allows users to withdraw a specified quantity from a body of water for use at a location which may be remote from water source. Older claims have priority over newer claims. In times of insufficient water the older claims have first right to the water, while new claims may have their allocations cut back or reduced to zero. In contrast riparian water rights give the water access right to land owners adjacent to the body of water and in times of drought all parties share equally in any reduced water allocation. The extent of inefficiencies of these systems, in practice, has been the subject of debate (see, for example, Bennett et al. (2000), Libecap (2011), Libecap & Barbara (2012)). Barring a mechanism for water transfers, the efficiency of prior allocation or riparian schemes will depend on the extent to which the rule for sharing in any cutback requires the largest curtailment from those with the lowest marginal value of water use (Weber & Cutlac 2014). Efficiency will also depend critically on the protection given to in-stream river flows.

Mannix et al. (2014) addresses the efficiency of water regulations in the Alberta oil industry. Mannix et al. (2014) examined the economic costs of the current regulations and the reduction in costs by applying more efficient allocation policies and technological improvements. In a deterministic model solved with a linear programming approach, they find that the economic costs of water restrictions can be reduced by water storage technology, and can be further reduced by policies that allocate water more efficiently. The aspect of the current regulation Mannix et al. focus on is water allocation across firms within the oil sands industry based on the priority for senior licenses. In particular, the article estimates the gain that would result if seniority of water licenses was based on efficiency of water use.

Most of the literature on efficient water regulations addresses municipal household water demand (Olmstead & Stavins 2009, Mansur & Olmstead 2012, Boldt-Van Rooy 2003). There is great political sensitivity about raising water rates for households to an efficient level as this is seen as regressive. Imposition of levies for industrial water use has mainly been suggested for the bottled water industry[15]. The use of water pricing mechanisms for industries besides agriculture is rare. Most industries do not pay any fee to extract water from the environment.

2.3.3 Inter-temporal Water Storage

Water storage has been examined in the literature in the context of water irrigation projects in agriculture. Agricultural irrigation has an apparent seasonal feature, as a result, consistent with production, there is little demand for water during the water short seasons. However, irrigation also faces water shortage in some drought years. Inter-yearly water storage for agriculture has been studied in an Australian context. Since the weather conditions in Australia vary significantly and droughts occur from time to time, inter-yearly water storage facilities are already in use. In contrast to our case, those facilities are publicly owned infrastructure, instead of industry owned facilities. Some of the literature demonstrates the feasibility of the inter-yearly water storage system for irrigation purpose (Alaouze 1991). Some concludes that to leave the water storage decisions to market is more efficient than using engineering rules of thumb (Brennan 2010). There are also studies about how to maximize profits through the annual choice of irrigation area in the presence of storage facilities (Dudley & Hearn 1993, Dudley 1972, 1988). However, the

[15] Water-bottling fees to be re-examined, says B.C. Premier Christy Clark. http://www.cbc.ca/news/canada/british-columbia/water-bottling-fees-to-be-re-examined-says-b-c-premier-christy-clark-1.3150529(accessed on January 11, 2020)

existing relevant literature does not address the investment in construction of such storage facilities, taking the existence of storage facilities for granted.

2.4 Model description

We analyze the case of a typical oil sands firm in the Lower Athabasca River region. We assume the operation is large enough that a single water storage pond will serve only one operation. Our goals are to determine the best timing for this firm to construct a water storage facility to maximize profits under the water restrictions set by the Framework as well as to explore the marginal cost of the restrictions for a typical firm.

2.4.1 Oil Production and Water Usage

We assume that the firm is already producing bitumen from its oil sands development and that there is a fixed oil to water ratio. In practice this is a reasonable assumption as, given a certain technology choice, the ratio is highly stable. For example, net fresh water use in oil sands production in 2013 averaged about 3.2 barrels of water per barrel of oil produced by mining operations.[16] Accordingly, we assume a linear production function:

$$Q(W_p(t), t) = \eta W_p(t) \qquad \eta > 0,\, W_p(t) \geqslant 0,\, 0 \leq Q(W_p(t), t) \leq \bar{q} \tag{2.1}$$

where Q is output, η is the number of barrels of bitumen that can be produced using one barrel of fresh water, $W_p(t)$ is the water used in production at time t, and $\bar{q}$ is a fixed upper limit on the rate of production. We assume that the production technology is stable so that η is a constant independent of t.

With no water management regulations, the firm can produce up to its full capacity by using water without any restriction. In the presence of the Framework, in the absence of water storage capacity, the firm has to cut back production during the yellow and red zones, in which case profits will be impaired. To maintain its profitability, the firm will consider installing a water storage facility. The inventory of water in storage, I, will be augmented by water withdrawals from the river, W_w and reduced as water is drawn out of storage for use in oil production, W_p. The change in water inventory is given by the following differential equation:

$$\mathrm{d}I = (W_w(t) - W_p(t))\mathrm{d}t \tag{2.2}$$

[16] Source: Responsible Canadian Energy 2014 Progress Report, http://www.capp.ca/publications-and-statistics/publications/(accessed on January 11, 2020)

The level of the water inventory in storage is constrained to be a positive number which is less than the storage capacity $\bar{I}$:

$$I(t) = I(t_0) + \int_{t_0}^{t} (W_w(t') - W_p(t'))\,\mathrm{d}t' \geqslant 0, \qquad I(t_0) = I_0, \quad 0 \leq I(t) \leq \bar{I} \tag{2.3}$$

2.4.2 Water Withdrawals From the River

According to the Framework, a weekly constraint on fresh water withdrawals is set for the oil sands industry. Through the examination of the Agreement[17], we also find that the restricted cumulative withdrawal in the yellow and red conditions is allocated among five oil sands firms roughly evenly.

The rate of water withdrawal, W_w, is restricted to be no greater than $\bar{W}$ where $\bar{W} \in \{\bar{W}_1, \bar{W}_2, \bar{W}_3\}$. The subscripts $k = 1, 2, 3,$ represent the river flow condition or water zone where $k = 1$ is the green zone, $k = 2$ is the yellow zone, and $k = 3$ is the red zone. It is assumed that the change of water constraint from the current zone k to another u can be described by a stochastic differential equation.

$$\mathrm{d}\bar{W} = \sum_{u=1}^{3} \big(\bar{W}_u(t) - \bar{W}_k(t)\big) \times \mathrm{d}X_{k\to u} \qquad k = 1, 2, 3 \tag{2.4}$$

where $\mathrm{d}X_{k\to u}$ is a Poisson Process:

$$\mathrm{d}X_{k\to u} = \begin{cases} 1 & \text{with probability } (\lambda^{k\to u}\mathrm{d}t), \\ 0 & \text{with probability } (1 - \lambda^{k\to u}\mathrm{d}t). \end{cases} \qquad k = 1, 2, 3 \,;\, u = 1, 2, 3 \tag{2.5}$$

The Poisson process is intended to reflect the natural variability in river flows.

2.4.3 Oil Resource Stock

Production depletes the resource stock S:

$$\mathrm{d}S = -Q(W_p(t), t)\mathrm{d}t, \; S(t_0) = s_0 \tag{2.6}$$

given

$$\int_{t_0}^{T} Q(W_p(t), t)\mathrm{d}t \leq S(t_0) \tag{2.7}$$

[17]Each year the Agreement updates the assignment of water. Only the current year's Agreement is publicly available. This information was taken from the Agreement for the 2014-2015 winter period.

where $S(t_0)$ is the available oil reserves at t_0, t_0 is starting time, and T is the lease end date.

2.4.4 Project Stages

To investigate the investment behaviour of this firm, we consider following 5 project stages. In the first stage, there is no water storage facility, and the firm holds the option to suspend production (stage 2) or to move on to stage 3, in which the water storage facility is installed and put into use. With the presence of the water storage facility, the firm can choose to stay in stage 3, or suspend the production temporarily (stage 4). The final stage, stage 5, is the permanent abandonment of the project. When in stages 1 to 4, the firm can decide to abandon (switching to stage 5) by paying an abandonment cost. Let δ_m be the notation for each stage, where m stands for the sequence number of stages and $m = 1, ..., M$. In our example $M = 5$. Stages are summarized in the following table:

Stage, δ	Description
1	Producing oil, no storage
2	Suspended, no storage
3	Producing oil, storage installed
4	Suspended, storage installed
5	Permanently abandoned

2.4.5 Oil Prices

As noted in Section 2.3.1, there is a substantial existing literature examining alternative models for stochastic resource prices. The best model choice depends on the context in which it will be used. For this thesis the goal is to find a parsimonious model that provides a reasonable depiction of the behaviour of oil prices, but does not involve additional stochastic factors which unnecessarily complicate the solution of the HJB equation. Chapter 4 examines two single-regime mean-reverting process models and a two-regime mean-reverting process model to explore their performance in capturing the characteristics of oil price dynamics. It turns out that the the two-regime model performs the best overall, but the single regime logarithmic mean-reverting model also performs well. The analysis for this chapter was done using both the single and two regime processes and the results were found to be qualitatively similar. We therefore present the results for the log mean-reverting model to avoid unnecessary complexity of two price regimes as well as three water zones. The log mean-reverting process stochastic differential equation is given as follows:

$$\mathrm{d}P = \epsilon(\mu - \ln P(t))P(t)\mathrm{d}t + \sigma P(t)\mathrm{d}z \tag{2.8}$$

where $P(t)$ is the crude oil spot price at time t, μ is the long run mean log price that $\ln P(t)$ tends to, ϵ is the speed of the mean reversion, σ is the volatility, and $\mathrm{d}z$ is the increment of a Wiener process. $\epsilon(\mu - \ln P(t))P(t)$ and $\sigma P(t)$ are called the drift term and the volatility term respectively. $\mathrm{d}z$ and $\mathrm{d}X_{k\to u}$ (defined in Equation (2.5)) are assumed to be independent of each other.[18]

2.4.6 Cash Flows

Annual cash flows are derived from revenue from the production and sale of oil reserves less fixed, variable costs and taxes. Both revenues and costs depend on the stage of operation, whether the project is operating, temporarily suspended or permanently abandoned. At time t, the realized profits will be:

$$\begin{aligned}
&\pi\left(P(t), S(t), \bar{W}(t), I(t), \delta(t)\right) \\
=&\left[P(t)\cdot\rho - (c^o_{v_e} + c^o_{v_{ne}})\cdot \mathbb{1}_{\{\delta=1,3\}}\right]\cdot\eta\cdot W_p\left(P(t), S(t), \bar{W}(t), I(t), \delta(t)\right) \\
&-c^o_f\cdot\mathbb{1}_{\{\delta=1,3\}} - c_s\cdot\mathbb{1}_{\{\delta=1,2,3,4\}} - \left[c^s_f + c^s_v(I)\right]\cdot\mathbb{1}_{\{\delta=3,4\}} - \Lambda(t)\cdot\mathbb{1}_{\{\delta=1,2,3,4\}}
\end{aligned} \tag{2.9}$$

where ρ is the discount of bitumen prices against WTI prices. As will be discussed in Section 2.5, in our optimal control problem, the value of W_p depends on five state variables P, S, $\bar{W}$, I, and δ. $\mathbb{1}_{\delta=\delta_m}$ is the indicator function which equals one if $\delta = \delta_m$ and zero otherwise. $c^o_{v_e}$ is the energy variable operating cost of oil production, $c^o_{v_{ne}}$ is the non-energy variable operating cost, c^o_f is the fixed operating cost, c_s is the sustaining capital cost no matter whether the operating is carried on or suspended, c^s_f is the fixed cost of water storage, $c^s_v(I)$ is the variable cost of water storage, which depends on the water inventory I, and Λ is the sum of all applicable taxes:

- Carbon tax (\$/barrel)[19] = Carbon tax rate (\$/tonne) $\times$ Carbon emissions (Tonnes/barrel);

[18] This assumption is justified because it is reasonable to treat oil price shocks and river flows as independent factors.

[19] Unless otherwise specified in this thesis, all references to "\$" or "dollars" herein refer to United States (U.S.) dollars.

- Royalty (\$/barrel) = Royalty rate[20] $\times$ $P(t)$(\$/barrel) $\times$ ρ;
- Income tax (\$/barrel) =max{0, Income tax rate $\times$ [$P(t)$(\$/barrel) $\times$ ρ - Royalty (\$/barrel) - Carbon tax (\$/barrel) - Operating cost (\$/barrel)]}.

That is to say, $\Lambda(t)$ = carbon tax + royalty + income tax.

In addition to annual cash flows, there are one time costs incurred to move from one stage to another. To go from an operating stage without storage to one with storage, the cost of constructing storage facilities must be incurred, which we denote as C. To switch from an operating stage to a suspending stage, the mothball cost, C_m is incurred. To move back from a suspending stage to an operating stage, the reactivating cost, C_{re} is incurred. Similarly, to move from any stage to permanent abandonment, an abandonment cost, C_r is incurred. We also assume that it is not possible to move from a stage with water storage back to a stage without water storage or move from permanent abandonment back to any other stage. This is implemented by setting the costs to these relevant stage switches as a very large number C_{large}.

Table 2.1 summarizes the costs incurred in or between stages.

Table 2.1: Project Costs

	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
Annual costs					
Fixed operating cost c_f^o	✓		✓		
Sustaining capital cost c_s	✓	✓	✓	✓	
Energy variable operating cost $c_{v_e}^o$	✓		✓		
Non-energy variable operating cost $c_{v_{ne}}^o$	✓		✓		
The fixed cost of water storage c_f^s			✓	✓	
The variable cost of water storage $c_v^s(I)$			✓	✓	
One time costs					
Construction cost of water storage C			✓		
Mothball cost C_m		✓		✓	
Reactivating cost C_{re}	✓		✓		
Abandonment costs C_r					✓

[20]The royalty rate differs between the pre-payout and the post-payout phases of a project. Before the point that a project's cumulative revenues start to cover its cumulative costs, it is in the pre-payout phase. After this point, it is in the post-payout phase. Without altering the qualitative results of our research, we assume that the studied project is in the pre-payout phase.

2.5 Specification of the Decision Problem

The firm's objective is to maximize the expected present value of cash flows from its oil sands operation over T years. There are three control variables: water withdrawals (W_w) from the river, oil production Q (which determines the water used in production, W_p), and the decision to switch project stages which we denote (δ^+). Control variables depend on five state variables: the oil price (P), the resource stock (S), the water withdrawal limit ($\bar{W}$), the water inventory in storage (I), and the current project stage (δ).

2.5.1 Admissible Sets for Control Variables

Admissible sets are now specified for the control variables. Let Z_{δ^+} denote the admissible set for δ^+ where

$$Z_{\delta^+} = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}. \tag{2.10}$$

The admissible set for oil production, Q, depends on the resource stock, water storage inventory, project stage, and water withdrawals from the river. Denote this admissible set as $Z_Q(S, I, \delta, W_w)$, which is given as follows:

$$Q \in Z_Q(S, I, \delta, W_w) \tag{2.11a}$$

$$Z_Q = \left[0, \min\left[S, \bar{q}, \eta W_w\right]\right], \quad \text{if} \quad S > 0,\ \delta = \delta_1. \tag{2.11b}$$

$$Z_Q = \left[0, \min\left[S, \bar{q}, \eta(W_w + I)\right]\right], \quad \text{if} \quad S > 0,\ \delta = \delta_3. \tag{2.11c}$$

$$Z_Q = 0, \quad \text{if} \quad S = 0,\ \delta = \delta_m,\ m = 1, 3. \tag{2.11d}$$

$$Z_Q = 0, \quad \text{if} \quad \delta = \delta_m,\ m = 2, 4, 5,\ \forall S. \tag{2.11e}$$

Equation (2.11b) states that in stage δ_1, oil production is constrained by the stock of oil reserves, the maximum oil production limit, and the amount of water withdrawn from the river multiplied by the water productivity coefficient. In stage 3, described in Equation (2.11c), water from the existing storage inventory is added to the current water withdrawal from the river as a constraint on water available for oil production.

Define an admissible set for water withdrawals, W_w, denoted $Z_W(\bar{W}, \delta)$, as follows:

$$\begin{aligned}
&W_w \in Z_W(\bar{W}, \delta) \\
&Z_W = [0, \bar{W}_1], \quad \text{if} \quad \bar{W} = \bar{W}_1, \ \delta = \delta_1, \delta_3 \\
&Z_W = [0, \bar{W}_2], \quad \text{if} \quad \bar{W} = \bar{W}_2, \ \delta = \delta_1, \delta_3 \\
&Z_W = [0, \bar{W}_3], \quad \text{if} \quad \bar{W} = \bar{W}_3, \ \delta = \delta_1, \delta_3 \\
&Z_W = 0, \quad \text{if} \quad \delta = \delta_2, \delta_4, \delta_5
\end{aligned} \tag{2.12}$$

2.5.2 Optimal Controls and Value Function

It is assumed that at predetermined, fixed times, the firm makes a decision about whether to change to a different project stage. These fixed times are denoted by $\mathcal{T}_d$:

$$\mathcal{T}_d \equiv \{t_0 = 0 < t_1 < ... < t_m <, ..., t_M = T - 1\} \tag{2.13}$$

The firm can switch stages instantaneously at $t \in \mathcal{T}_d$. At time T, the project must be terminated and clean up costs are incurred. In the numerical example in this paper, the time between fixed decisions dates is set as a week.

Choices regarding the rate of water withdrawal, W_w, and oil production, Q, are made in continuous time at time intervals given as follows:

$$\mathcal{T}_c \equiv \{(t_0, t_1), ..., (t_{m-1}, t_m), ..., (t_{M-1}, t_M)\}. \tag{2.14}$$

Controls are specified as functions of state variables as follows:

$$\begin{aligned}
&Q^+(P, S, \bar{W}, I, \delta, t), \quad W_w^+(P, S, \bar{W}, I, \delta, t), \ t \in \mathcal{T}_c \\
&\delta^+(P, S, \bar{W}, I, \delta, t), \ t \in \mathcal{T}_d.
\end{aligned}$$

Let K denote the set of particular choices for the controls for all t_m.

$$K = \{(\delta^+)_{t \in \mathcal{T}_d} \ ; \ (Q^+, W_w^+)_{t \in \mathcal{T}_c}\} \tag{2.15}$$

For any particular K, the value function $V(p, s, \bar{w}, \bar{\delta}, t)$, can be written as the expected discounted value of the integral of future cash flows with the expectation taken over the controls, given the state variables, where $p, \ s, \ \bar{w}, \ \iota, \ \bar{\delta}$ denote particular realizations of the state variables P, S, $\bar{W}$, I, and δ.

$$
\begin{aligned}
V(p,s,\bar{w},\iota,\bar{\delta},t) \;=\; \mathbb{E}_K \Bigg[& \int_{t'=t}^{t'=T} e^{-rt'}\; \pi(P(t'),S(t'),\bar{W}(t'),I(t'),(\delta(t'))\; dt' \\
& + e^{-r(T-t)} V(P(T),S(T),\bar{W}(T),I(T),\delta(T),T) \\
& \Bigg| P(t)=p, S(t)=s, \bar{W}(t)=\bar{w}, I(t)=\iota, \delta(t)=\bar{\delta} \Bigg].
\end{aligned} \tag{2.16}
$$

r is the risk free interest rate, which is assumed to be 0.02, and $\mathbb{E}[\cdot]$ is the expectation operator. Note that the expectation is taken under the risk neutral or $\mathcal{Q}$ measure.[21] In our numerical example the value in the final time period, $V(P(T),S(T),\bar{W}(T),I(T),\delta(T),T)$, is assumed to be the cost of clean up if the project had not been abandoned before T ($\delta=\delta_m,\;\; m=1,2,3,4$), or is equal to zero if the firm has already abandoned the project ($\delta=\delta_5$).

Equation (2.16) is solved for the optimal controls contained in the admissible sets (Equations (2.10), (2.11), and (2.12) and subject to Equations for dS, d$\bar{W}$, dI, and dP ((2.6), (2.4), (2.2), and (2.8)). A dynamic programming algorithm is implemented solving backwards in time and proceeding in two phases: (1) the decision to switch stages made at fixed time points, t_m, and (2) the choice of water withdrawals and oil production made in continuous time in the interval $t \in (t_m^+, t_{m+1}^-)$, where t_m^+ denotes the instant after t_m and t_{m+1}^- denotes the instant before time t_{m+1}.

2.5.3 Solution at Fixed Decision Dates

At any $t_m \in \mathcal{T}_d$, the decision on the optimal stage from t_m^+ should be the one in which the project value minus switching cost is the maximum, other things equal.

$$
\delta^+(p,s,\bar{w},\iota,\bar{\delta},t_m) \;=\; \arg\max_{\delta} \left(V(p,s,\bar{w},\iota,\delta,t_m) - C_{\bar{\delta}\to\delta}\right) \tag{2.17}
$$

where $C_{\bar{\delta}\to\delta}$ denotes the cost for switching from stage $\bar{\delta}$ at time t_m to stage δ at time t_m^+. Table 2.2 specifies $C_{\bar{\delta}\to\delta}$ at the intersection of $\bar{\delta}^{\text{th}}$ row and the δ^{th} column.

[21] We assume that the risk of uncertain water flows is not correlated with the economy and the stock market. Therefore, it is a diversifiable risk and the $\mathcal{P}$ measure can be used.

Table 2.2: Switching Costs

Stage	1	2	3	4	5
1	0	C_m	C	C_{large}	C_r
2	C_{re}	0	C_{large}	C_{large}	C_r
3	C_{large}	C_{large}	0	C_m	C_r
4	C_{large}	C_{large}	C_{re}	0	C_r
5	C_{large}	C_{large}	C_{large}	C_{large}	0

2.5.4 Solution between Fixed Decision Dates, Going Backward In Time From t_{m+1}^- to t_m^+.

In this section we describe the solution going backwards in time between decision dates, i.e. $t_{m+1}^- \rightarrow t_m^+$. Define the differential operator $\mathcal{L}$ as follows:

$$\mathcal{L}V =$$

$$\frac{1}{2}b^2\frac{\partial^2 V}{\partial P^2} + a\frac{\partial V}{\partial P} - Q\frac{\partial V}{\partial S} + (W_w - W_p)\frac{\partial V}{\partial I} + \sum_{u=1,u\neq k}^{3} \lambda^{k\rightarrow u}(V(\bar{w} = \bar{W}_u) - V(\bar{w} = \bar{W}_k)) - rV \tag{2.18}$$

where $a \equiv \epsilon(\mu - \ln P)P$; and $b \equiv \sigma P$.

Recall that there is a fixed relationship between water used in production, W_p, and the rate of oil production $W_p = Q/\eta$.

Define a small time interval h where $h < (t_{m+1} - t_m)$. For $t \in (t_m^+, t_{m+1}^- - h)$, according to the dynamic programming principle, for small h we know that

$$V(p, s, \bar{w}, \iota, \bar{\delta}, t) = e^{-rh}\mathbb{E}\Big[V(P(t+h), S(t+h), \bar{W}(t+h), I(t+h), \delta(t), (t+h))\Big| \tag{2.19}$$

$$P(t) = p, S(t) = s, \bar{W}(t) = \bar{w}, I(t) = \iota, \delta(t) = \bar{\delta}\Big] \tag{2.20}$$

Letting $h \rightarrow 0$ and applying Ito's Lemma[22], the value function can be shown to satisfy the

[22]See Björk (2009) for a rigorous overview of optimal decisions under uncertainty characterized by an Ito process in a finance context. Dixit & Pindyck (1994) provides an introductory overview

following Hamilton-Jacobi-Bellman equation:

$$\frac{\partial V}{\partial t} + \pi(p, s, \bar{w}, \iota, \bar{\delta}, t) + \max_{Q, W_w} \left(\mathcal{L}V \right) = 0 \tag{2.21}$$

Equation (2.21) is defined on the domain $(p, s, \bar{w}, \iota, \bar{\delta}, t) \in \Omega^\infty$, where

$$\begin{aligned}
&\Omega^\infty \equiv [0, \infty] \ \times \ [0, S_0] \ \times \ Z_{\bar{W}} \ \times \ [0, I^{\max}] \ \times \ Z_\delta \ \times \ [0, T]. \\
&Z_{\bar{W}} = \{\bar{W}_1, \bar{W}_2, \bar{W}_3\} \\
&Z_\delta = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}
\end{aligned}$$

T reflects the length of the lease to operate the project. For computational purposes the domain Ω^∞ is truncated to Ω where

$$\Omega \equiv [0, p_{\max}] \ \times \ [0, S_0] \ \times \ Z_{\bar{W}} \ \times \ [0, I^{\max}] \ \times \ Z_\delta \ \times \ [0, T]. \tag{2.22}$$

$p_{\max}$ is chosen to be large enough to represent a very high oil price in relation to historical prices.

2.5.5 Boundary Conditions

Boundary conditions must be established for the state variables t, P, S, and I.

- At $t = T$ if the project has not previously been abandoned, reclamation costs will be paid of amount $-C_r$. Therefore $V = -C_r$ for $\delta \in [\delta_1, \delta_2, \delta_3, \delta_4]$. For $\delta = \delta_5$, $V = 0$ at $t = T$ as reclamation will already have been carried out so that the value will not change.

- As $P \to 0$, the volatility term of the stochastic differential equation describing P (Equation (2.8)), goes to zero. Hence we can just solve the HJB equation along the boundary at $P = 0$. The differential operator becomes:

$$\mathcal{L}V = -Q\frac{\partial V}{\partial S} + (W_w - W_p)\frac{\partial V}{\partial I} + \sum_{u=1, u \neq k}^{3} \lambda^{k \to u}(V(\bar{w} = \bar{W}_u) - V(\bar{w} = \bar{W}_k)) - rV \tag{2.23}$$

- At $P = p_{\max}$ it is assumed that the value of the project will be linear in the oil price, implying $\frac{\partial^2 V}{\partial p^2} = 0$. The implicit assumption is that volatility is unimportant at very high prices and is commonly assumed in the finance literature (Wilmott 1998). In this case the differential operator becomes:

$$\mathcal{L}V = a\frac{\partial V}{\partial P} - Q\frac{\partial V}{\partial S} + (W_w - W_p)\frac{\partial V}{\partial I} + \sum_{u=1,u\neq k}^{3} \lambda^{k\rightarrow u}(V(\bar{w} = \bar{W}_u) - V(\bar{w} = \bar{W}_k)) - rV \tag{2.24}$$

where $a \equiv \epsilon(\mu - \ln P)P$; and $b \equiv \sigma P$.

Since $a = \epsilon(\mu - \ln P)P \leq 0$, according to the discussion of boundary conditions by Chen & Forsyth (2007), we know that characteristics are outgoing in the P direction at $P \rightarrow p_{\max}$. Hence no additional information is needed from outside of the domain of P and we can solve the PDE at the boundary.[23]

- As $S \rightarrow 0$, the oil production converges to zero: $Q \rightarrow 0$. At this point, the project ends, and the land must be reclaimed according to regulations.
- At $S = S_0$, we solve the HJB equation at this boundary, and no special boundary condition is needed.
- As $I = 0$, we can not withdraw water from the storage facility, but can only add water into the facility through water withdrawals from the river. Hence $(W_w - W_p) \geqslant 0$. Accordingly there are outgoing characteristics in the I direction. We do not need additional information from outside of the domain of I and can just solve the HJB equation along the boundary.
- When $I = I_{max}$, we cannot add any additional water to storage which means $(W_w - W_p) \leq 0$. Hence there are outgoing characteristics in the I direction. No additional information is needed from outside of the domain of I.

2.5.6 Numerical Solution Details

As mentioned in the literature review (Section 2.3.1, Page 18), a number of papers (d'Halluin et al. 2005, Chen & Forsyth 2007, 2010) introduce the standard method for solving stochastic optimal control problems. More details can be found in theses d'Halluin (2004) and

[23] A detailed discussion about the information propagation direction along characteristics can be found in Strikwerda (2004).

Chen (2008). We tailor the standard method to accommodate our particular problem in MATLAB. The detailed semi-Lagrangian time stepping and fully implicit discretization scheme are elaborated in Appendices A.1 and A.2. Tests for the accuracy of the numerical solution are provided in Appendix A.3.

2.6 Specification of the Parameters

2.6.1 Price Dynamic Process Related Parameters

The estimation of the parameters of Equation (2.8) is described in Chapter 4. Specifically, $\epsilon = 0.14$, $\mu = 4.59$, $\sigma = 0.31$. Accordingly, the oil SDE followed by oil prices is given as:

$$\mathrm{d}P \;=\; 0.14(4.59 - \ln P) \times P\mathrm{d}t \;+\; 0.31P\mathrm{d}z.$$

According to these estimates, if volatility were zero the time for the log of price to revert to its long run mean is approximately $\frac{1}{\epsilon} = \frac{1}{0.14} = 7.14$ years. The long run mean for the benchmark oil price (West Texas Intermediate) is $e^{4.59} = \$98/\text{barrel}$. Recall that these estimates are risk adjusted under the $\mathcal{Q}$ measure.

With regard to the discount of bitumen prices against WTI prices, ρ (see Equation (2.9)), as in Insley (2017), we fix it at the level of 83%. In other words, we fix the oil sands price in Canadian dollars at 83% of the WTI price in US dollars. In reality, the bitumen price discount is highly variable and could itself be modelled as a second stochastic factor.

2.6.2 Water Withdrawal Limits

The Framework sets the rules for determining these water withdrawal limits in different zones, and also explicitly lists for each week how many cubic meters of water per second the oil sands industry is permitted to remove from the Athabasca River in the yellow and red zones based on the historical flow record up to 2007. The 'Alberta Oil Sands Industry Quarterly Update' (spring 2015) Economic Development and Trade (2015) (hereinafter referred to as "AOSIQU") shows that as of spring 2015 there were 12 surface mining projects in operation. Assuming the available water during the yellow and red zones is allocated evenly among those projects, according to the Framework, the resulting specific weekly water restrictions in the yellow and red zones for the whole industry and the amount assigned to each project are listed in Table 2.3. The weekly water limits in the yellow and red zones for the entire oil sands industry are also depicted in Figure 2.2. According to the AOSIQU, the total production capacity of the the oil sands mining sector was 9.975 million barrels/week. If production is at full capacity, the weekly water required is about 33.3 million barrels.[24] From Table 2.3, it is shown that even if the river flow condition is in the yellow or the red zone, the production of the industry would not be bound by the constraints. That said, we know that projects do differ in terms of their water licenses so that when the river is in the red or yellow zone, some projects with less generous license provisions may experience water shortage. From the AOSIQU, the average production capacity of each of the 12 surface mining projects is about 0.83 million barrels/week implying the hypothetical project's water intake is not restricted even in the red zone. However, from AOSIQU we know that in fact there are 4 of those 12 projects whose production may be restricted in yellow or red zones (2 of which cannot achieve the full capacity of production in the yellow zone.) In this case the water sharing agreement would apply so that those firms with more generous licenses would need to give up some of their water to share with firms in short supply. Although the restrictions do not currently appear to be binding, they might bind in the future if the industry grows or if river flows become less abundant over time. For the purposes of this thesis, we assume parameters for a hypothetical oil sands plant which is constrained by the water restrictions. Specifically the hypothetical project is assumed to have a production capacity of 1.38 million barrels/week. If there were 12 equally sized projects this implies oil production capacity of 16.62 million barrels/week. Then the weekly water needed would be about 55.5 million barrels. In this hypothetical case, the water constraint due to the framework would have an effect in some dry periods.

The parameter $\lambda^{k\to u}\mathrm{d}t$ in Equation (2.5) refers to the hazard rate of switching from river flow zone k to u in the period of $\mathrm{d}t$. We examined the historical data of Athabasca river flows and found that in the recent years the river flows are lower compared to the average historical level. To show a more obvious effect on the oil sands industry of the Framework, we adopt the relatively low river flows condition of 2015 for estimating the hazard rates. According to the 2015 data of Athabasca river flows provided by Alberta Environment, we estimate average values for $\lambda^{i\to j}$ (for all $i = 1, 2, 3$ and $j = 1, 2, 3$, where 1 corresponds to the green zone, 2 the yellow zone, and 3 the red zone.) as follows:

$$\lambda^{i\to j} \;=\; \frac{N_{i\to j}}{N_i}\cdot\frac{1}{\mathrm{d}t}$$

[24] This amount of water required is derived by using the water productivity specified in Section 2.6.3, page 35: 9.975 million barrels of bitumen/week × 3.34 barrels of water/barrel of bitumen = 33.3 million barrels of water/week.

Table 2.3: Water Withdrawal Limit (million Barrels/week)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Total (yellow zone)	58.1	58.1	53.2	58.1	58.1	53.2	53.2	53.2	53.2	53.2	53.2	58.1	62.9
Total (red zone)	48.4	43.5	43.5	43.5	43.5	38.7	38.7	38.7	38.7	38.7	38.7	43.5	43.5
Individual project (yellow zone)	4.8	4.8	4.4	4.8	4.8	4.4	4.4	4.4	4.4	4.4	4.4	4.8	5.2
Individual project (red zone)	4	3.6	3.6	3.6	3.6	3.2	3.2	3.2	3.2	3.2	3.2	3.6	3.6
Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Total (yellow zone)	62.9	72.6	72.6	106.4	121	135.5	145.2	164.5	164.5	164.5	164.5	164.5	164.5
Total (red zone)	48.4	62.9	72.6	106.4	121	135.5	145.2	164.5	164.5	164.5	164.5	164.5	164.5
Individual project (yellow zone)	5.2	6	6	8.9	10.1	11.3	12.1	13.7	13.7	13.7	13.7	13.7	13.7
Individual project (red zone)	4	5.2	6	8.9	10.1	11.3	12.1	13.7	13.7	13.7	13.7	13.7	13.7
Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Total (yellow zone)	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	159.7
Total (red zone)	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	164.5	159.7
Individual project (yellow zone)	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.3
Individual project (red zone)	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.3
Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Total (yellow zone)	154.8	150	135.5	130.6	72.6	72.6	72.6	72.6	67.7	67.7	62.9	62.9	62.9
Total (red zone)	154.8	150	135.5	130.6	72.6	72.6	72.6	58.1	53.2	48.4	48.4	48.4	48.4
Individual project (yellow zone)	12.9	12.5	11.3	10.9	6	6	6	6	5.6	5.6	5.6	5.6	5.6
Individual project (red zone)	12.9	12.5	11.3	10.9	6	6	6	4.8	4.4	4	4	4	4

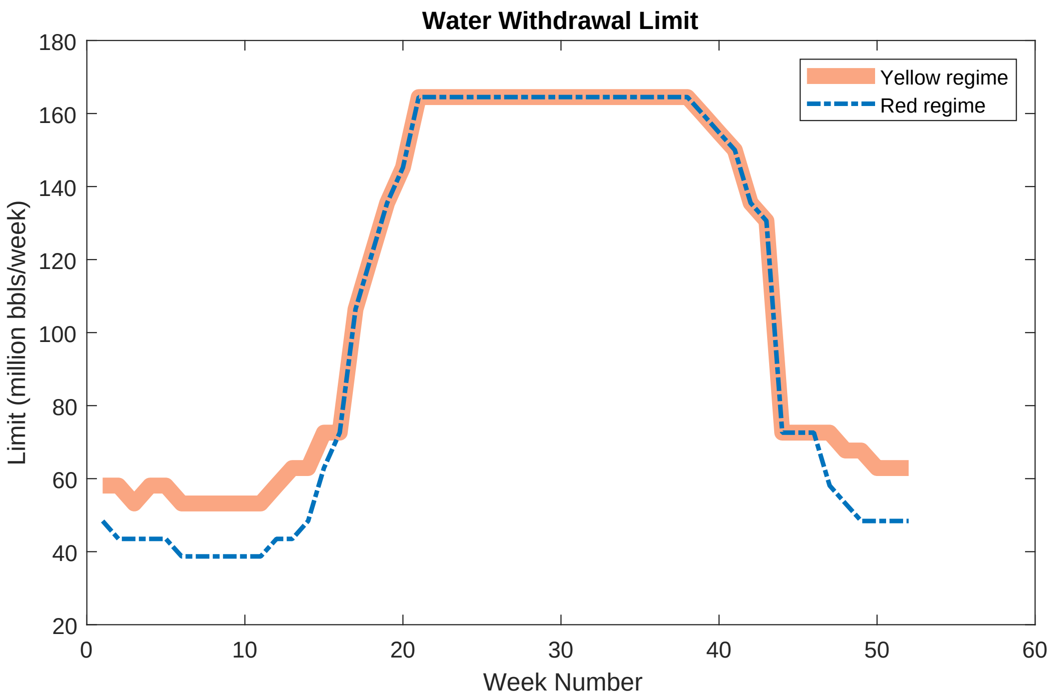

Figure 2.2: Weekly water withdrawal limits in the yellow and red zones

where N_i is the number of weeks in 2015 that are in the zone specified by i, $N_{i\to j}$ is the number of times that the zone switches from i to j in 2015, and $\mathrm{d}t$ is $\frac{1}{52}$ of a year or 1 week.

The hazard rate matrix is as follows.

$$\begin{bmatrix} 40.7 & 11.3 & 0 \\ 12.2 & 36.7 & 3.1 \\ 0 & 4.3 & 47.7 \end{bmatrix}$$

where the entry at the i$^{\text{th}}$ row and the j$^{\text{th}}$ column stands for $\lambda^{i\to j}$.

2.6.3 Production Related Parameters

We assume that the project is already in operation and the remaining lifespan is 10 years. Water requirements per barrel for each year are provided in the Responsible Canadian Energy 2014 Progress Report[25] by the Canadian Association of Petroleum Producers. Herein we adopt the average level of fresh water withdrawal per barrel of production during the period of 2003∼2013, i.e. 3.34 barrels of water/barrel of oil. Therefore, $\eta = 1/3.34 \approx 0.3$.

Section 2.6.2 (Table 2.3) shows that in the driest week (the 10$^{\text{th}}$ week) of each year, the limitation in water availability confines a project's production to a maximum of 1.32 million barrels of oil per week[26] in the yellow zone and 0.96 million barrels/week[27] in the red zone. As noted in the previous section we choose a hypothetical oil sands project with a production capacity of 1.38 million barrels/week so that the production is affected by both the yellow and red zones.[28]

Information on water storage capacity was obtained from Imperial Oil's description of their Kearl oil sands project which commenced production on April 27, 2013[29]. Like the

[25] Source: information provided on the website of Canadian Association of Petroleum Producers (CAPP) (http://www.capp.ca/publications-and-statistics/publications) (accessed on January 11, 2020).

[26] Each week the maximum amount of available water is 4.4 million barrels. Considering that 3.34 barrels of water can produce one barrel of oil, the weekly oil production is under 4.4/3.34 million barrels.

[27] Similarly derived by 3.2 million barrels/3.34

[28] In this hypothetical case, we presume that the oil sands project's production is constrained by the availability of fresh water. The rationality of this assumption can be supported by Mannix et al. (2010), where the authors showed that the water constraint is possible to be binding on oil sands production in the future. As a forward-looking regulation, it is helpful to anticipate the implications in the setting of future problems occurring.

[29] Source: information provided on the website of Imperial Oil (http://www.imperialoil.ca/Canada-English/operations_sands_kearl_environment.aspx) (accessed on January 11, 2020).

Kearl project it is assumed that storage can sustain 30 day's production during the dry season, which implies a capacity of about 20 million barrels. A report of Golder Associates Ltd. (2015) showed that the capital cost for fresh water storage is \$16/m^3 and the annual operating costs for the storage is 5% of capital cost plus relevant power costs. Accordingly we assume that the storage facility in our case costs 39 million dollars and the fixed cost of running the facility is 2.1 million dollars/year. In addition, due to the lack of publicly available information, the construction duration and variable operating costs for the facility are arbitrary assumptions. We assume that the construction of the storage pond can be built immediately (which has little impact on results) and the variable cost of the storage capacity is \$0.0028/barrel.

We require an estimate of the initial reserves of the hypothetical project. From Alberta's Energy Reserves 2014 and Supply/Demand Outlook 2015-2024 (Alberta Energy Regulator 2015) we know that the remaining established reserves of crude bitumen are 166 billion barrels in Alberta, and 20% of this is recoverable by surface mining approach. According to the AOSIQU, there are 33 projects so far (including approved, operating, on hold, cancelled, and in-application). Based on the above information, we assume that the hypothetical project has less than 900 million barrels recoverable resource stock. Given the above assumed production capacity of 1.38 million barrels/week (i.e. 72 million barrels/year), the upper limit of resource stock that a 10-year life project would extract is 720 million barrels if operating at full capacity. Therefore we assume the remaining resource stock for the hypothetical project is 720 million barrels.[30]

With regard to the various cost values for operation, we used estimates provided by the Canadian Energy Research Institute (hereinafter referred to as "CERI") (Millington & Murillo (2015)). Table 2.1 details when the various costs are incurred.

Table 2.4 lists all necessary parameter values for the hypothetical project in the base case.

2.7 Results

We examined four different scenarios which reflect current regulations and river conditions, as well as stricter regulations and drier river conditions. We denote these scenarios as follows.

[30] In fact, the hypothetical project's scale, in term of its production capacity and the remaining established reserves, is quite close to the scale of North Steepbank Extension project of Suncor Energy Inc.

Table 2.4: Base Case Parameter Values

Parameter	Description	Reference	Assigned Value	Source
	Extraction method		Surface mining	$* * *$
$T - t_0$	Remaining lifespan of the project (years)	Equation (2.7)	10	$*$
$\bar{q}$	Production capacity (million barrels/year)	Equation (2.1)	72	$*$
s_0	Remaining established reserves (million barrels)	Equation (2.7)	720	$*$
η	Productivity of water (barrels of bitumen/barrel of water)	Equation (2.1)	0.3	$**$
$\bar{W}_1$	Water withdrawal constraint in the green zone (million barrels/week)	Equation (2.4)	$+\infty$	$* * *$
$\bar{W}_2$, $\bar{W}_3$	Water withdrawal constraint in the yellow zone and the red zone (million barrels/week)	Equation (2.4)	refer to Table 2.3	$*$
ρ	Discount of bitumen prices against WTI prices	Equation (2.9)	83%	$*$
C	The construction cost of the water storage (million dollars)	Table 2.1	39	$*$
$\bar{I}$	Water storage capacity (million barrels)	Equation (2.3)	20	$*$
c_f^s	The fixed cost of water storage (million $/year)	Equation (2.9)	2.1	$*$
c_v^s	The variable cost of water storage ($/barrel)	Equation (2.9)	0.0028	$*$
	Carbon emissions (tonnes/barrel)	Equation (2.9)	0.091	$**$
$c_{v_e}^o$	Energy variable operating cost (% of the WTI price)	Equation (2.9)	1.62	$**$
$c_{v_{ne}}^o$	Non-energy variable operating cost ($/barrel)	Equation (2.9)	7.98	$**$
c_f^o	Fixed operating cost (million $/year)	Equation (2.9)	470	$**$
c_s	Sustaining capital cost (million $/year)	Equation (2.9)	468	$* * *$
	Income tax rate (%)	Equation (2.9)	25	$* * *$
	Carbon tax ($/tonne)	Equation (2.9)	40	$* * *$
	Royalty rate (%)	Equation (2.9)	1 when $P <$\$55/barrel 9 when $P >$\$120/barrel (0.12P-5.77) otherwise	$* * *$
C_m	Mothball cost (million $)	Table 2.1	0	$*$
C_{re}	Reactivating cost (million $)	Table 2.1	0	$*$
C_{large}	A large number to prevent stage switching (million $)	Page 24	10^9	$*$
C_r	Abandonment cost (million $)	Table 2.1	278	$*$
ϵ	Speed of reverting to the mean log oil price	Equation (2.8)	0.14	$* * *$
μ	Long run mean log oil price	Equation (2.8)	4.59	$* * *$
σ	Volatility of oil prices	Equation (2.8)	0.31	$* * *$
 $\lambda^{1\to2}$ $\lambda^{1\to3}$ $\lambda^{2\to1}$ $\lambda^{2\to3}$ $\lambda^{3\to1}$ $\lambda^{3\to2}$	Hazard rate of switching from the green zone to the yellow zone, from the green zone to the red zone, from the yellow zone to the green zone, from the yellow zone to the red zone, from the red zone to the green zone, and from the red zone to the yellow zone	Equation (2.5)	 11.3 0 12.2 3.1 0 4.3	$* * *$
r	Risk free interest rate	Equation (2.16)	0.02	$*$

The column "Reliablility" indicates the reliablity of the given parameter values. $* * *$ means these values are publicly available or are estimated from empirical evidence. $**$ means these values are derived according to AOSIQU, Alberta Energy Regulator (2015), or CERI's report ((Millington & Murillo 2015) . $*$ means these values are assumed by referring to miscellaneous sources, which are specified in the text.

- W_L (wet lenient scenario): The river flow is in the current condition (i.e. a fairly wet condition), as shown by the solid curve in Figure 2.3.[31] The water withdrawal limit is regulated by the Phase 1 Framework, which is relatively lenient compared to the other case we examine.

- W_S (wet strict scenario): The river flow is in the current condition. The water withdrawal limit is tightened by 1.35 million barrels/week (i.e. up to 30% and 42% of the weekly withdrawal limit set by the Phase 1 Framework for the yellow zone and the red zone respectively)[32] based on the Phase 1 Framework.

- D_L (dry lenient scenario): The river flow is in such a dry condition that it falls in the red zone all the time, as shown by the dash-dot curve in Figure 2.3. The water withdrawal limit is regulated by the Phase 1 Framework.

- D_S (dry strict scenario): The river flow is in the red zone all the time. The water withdrawal limit is tightened by 1.35 million barrels/week based on the Phase 1 Framework which amounts to a 30% to 40% reduction in allowed water withdrawals over the year compared to the more lenient regulations of the Phase 1 Framework.

Figure 2.3 shows the levels of the weekly river flow rates for the wet (represented by "W" in the scenario nomenclature) and dry (represented by "D" in the scenario nomenclature) river flow conditions compared to the historical river flow rates. The boxplots in Figure 2.3 indicate the first quartile (represented by the lower edge of each box), the third quartile (the upper edge of each box), the median (the short red horizontal bar cutting through each box), the maximum level (the highest tip of the dashed whisker), the minimum level (the lowest tip of the dashed whisker), and outliers (the plus signs) of the historical weekly river flow rate.

2.7.1 Economic Impact of Water Restrictions for the Firm

In this section we examine the impact of the regulation on the oil sands project if no action can be taken to alleviate the water shortage pressure, i.e. there is no storage option available. This case indicates the maximum effect of the water restrictions. Note also that

[31] We call this scenario "wet" because it is relatively wet compared to other scenarios listed herein. It does not mean that it is in the wet spectrum of the historical river flow record. In fact, from Figure 2.3, we can see that this scenario is fairly dry if we use the historical record as a reference.

[32] The reason we choose 1.35 million barrels/week is that this number is suitable for our numerical analysis while is appropriate for the policy analysis.

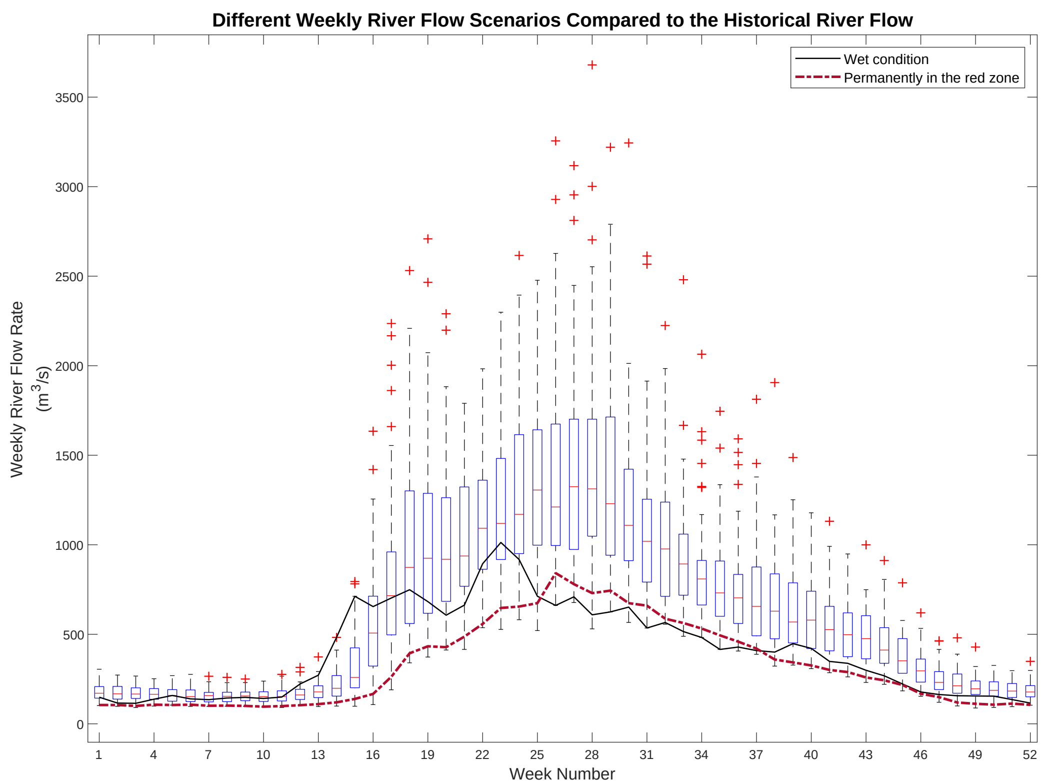

Figure 2.3: Curves Showing the Assumed Wet and Dry Weekly River Flow Rates versus the Box Plots of Historical Weekly River Flow Rates.

a reliance on water storage to alleviate water shortfall has been the subject of controversy in certain cases due to potential negative consequences as discussed in Di Baldassarre et al. (2018).

2.7.1.1 Solution Surface for the Value of the Project

Figure 2.4 depicts the solution surface for W_L, which shows the project's values corresponding to different combinations of the oil sands resource stock and crude oil price when the present river flow condition is in the green zone. Note that these graphs depict the value of the project at time zero for different values of the state variables, assuming the project owner acts optimally in the choice of controls until the lease end date at time T. As expected, other things equal, the project's value rises with an increase in oil price as well as with an increase in resource stock. When the present river flow condition is in either of the other zones, the shape of the solution surface is very similar to that in Figure 2.4, and hence is not shown. For any specific combination of oil price and stock, the project value if currently in the red zone is slightly less than that in the yellow zone, which is slightly less than that in the green zone. The current river zone is not a large determinant of project value since the applied water constraint is not very tight. Therefore, even if the river flow condition is in the yellow or the red zone, there will not be a major impact on production.

2.7.1.2 Project Value Comparison between the Four Scenarios

To compare the project values across the four scenarios, we adopt one particular value for the resource stock dimension and examine how the project present value varies with the current oil price. We select 720 million barrels, where the resource stock is at its highest level but it can be shown that if any other spot is selected, the comparison result also holds. Since for D_L and D_S, the river flow is always in the red zone, our comparison is done for the current river flow in the red zone. Figure 2.5 exhibits the comparison result. As expected, the stricter the water withdrawal restriction or the drier the river flow condition, the lower the project's value. In the scenarios with dry river conditions, a significant difference is now observed between the scenarios. At an oil price of \$100/barrel, the project's value is lower by 11.1% in the D_S scenario compared to D_L.

2.7.1.3 Critical Prices To Abandon the Project

In general project abandonment will occur when reserves run out, when the lease ends, or when the oil price is so low that the firm is better off abandoning rather than maintaining

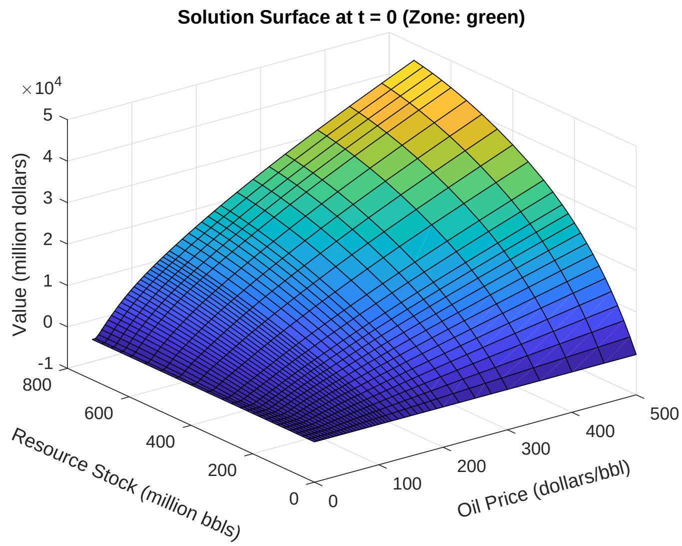

Figure 2.4: W_L: Project present value versus present price and resource stock if the current river flow condition is in the green zone and there is no option to install a water storage facility

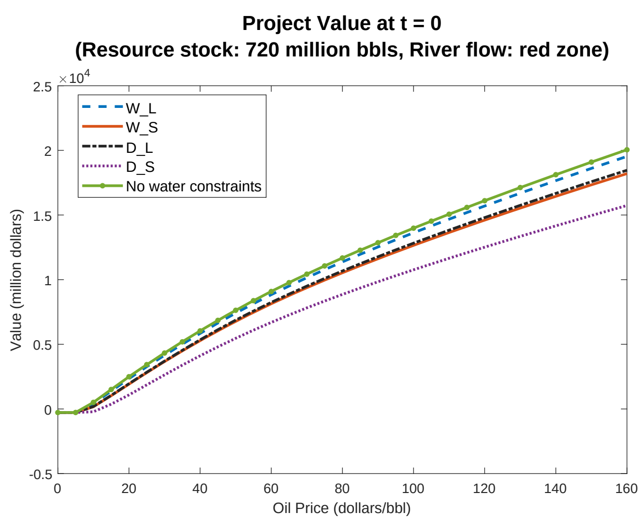

Figure 2.5: Comparison between scenarios: Project present value versus present price if the present resource stock level is 720 million barrels, the current river flow condition is in the red zone, and there is no option to install a water storage facility

an active project. Abandonment requires the firm to pay rehabilitation costs, but the firm thereby avoids the fixed costs of the oil sands operation. Fixed costs at \$470 million per year exceed rehabilitation costs of \$278 million. Rather than abandoning the firm has also has the option to suspend production but still incurs fixed costs of \$468 million per year.

We might expect that stricter water withdrawal restrictions will affect a firm's decision about when to permanently abandon a project. Intuitively, stricter limitations on water withdrawals will require reduced production in dry periods which the firm will try to make it up in wetter periods. As a result the expected abandonment time would be delayed if restrictions become tighter. We investigate this effect for our hypothetical project by examining critical prices to abandon the project. If the oil price is greater than the critical price, the firm's optimal choice is to continue the project. Otherwise, it should shut down the project permanently. Table 2.5 lists the critical prices to abandon the project for the four scenarios for different levels of oil reserves. It shows that the crude oil prices for abandonment are fairly low (\$45 per barrel and less) and are lower for higher reserve levels. Across scenarios, the change in critical prices is quite slight implying that stricter regulations and drier river conditions are having insignificant impact on the decision to abandon the hypothetical project examined here. In our example, the river flow is quite abundant even in the red zone. The water withdrawal limit is high enough for the oil sands project to produce at close to its full capacity. To be specific, without using a water storage facility to shift available water between weeks, in scenario W_L, there are 51 out 52 weeks that the project can produce at its full capacity, in D_L, 33 out of 52, in W_S, 47 out of 52, and in D_S, 31 out of 52 weeks. We do observe some higher critical prices in the W_S, D_L, or D_S cases compared to W_L when the resource stock is below 500 million barrels. When the resource stock is higher than 500 million barrels, it is not optimal to abandon the project except for at oil prices less than \$5/barrel.

The critical prices to suspend the project are shown in Table 2.6. It will be observed that, except for low reserve levels (below 140 million barrels), critical prices to suspend are always higher than those to abandon the project. This implies that the project will always be suspended before it is abandoned. For low reserves, the project will not move from the operating stage to the suspended stage. Instead, it is just permanently abandoned.

2.7.1.4 Main Findings

Water is a crucial input for an oil sands operation. Concerns have been expressed in the public discourse about the effect of oil sands water withdrawals on the Athabasca River ecosystem as well as the effect of water restrictions on the profitability of oil sands operations. The results in this section show that the Phase 1 Water Restrictions would

Table 2.5: Critical Prices To Abandon the Project While There Is No Option To Install Water Storage To Mitigate the Impacts of Water Restriction Regulation

From operating stages to abandonment								
	W_L			W_S			D_L	D_S
Resource stock	green	yellow	red	green	yellow	red	red	red
(million barrels)	stage 1→5	stage 1→5	stage 1→5	stage 1→5	stage 1→5	stage 1→5	stage 1→5	stage 1→5
0	H	H	H	H	H	H	H	H
20	35	35	35	35	35	40	35	45
40	30	30	35	35	35	35	35	40
60	30	30	30	30	30	35	35	35
80	30	30	30	30	30	30	30	35
120	30	30	30	30	30	30	30	30
140	25	25	25	25	25	25	25	30
180	25	25	25	25	25	25	25	25
200	20	20	20	20	20	20	20	25
240	15	15	15	15	15	15	15	20
300	10	10	10	10	10	10	10	15
350	5	5	5	10	10	10	10	10
450	5	5	5	5	5	5	5	10
500	5	5	5	5	5	5	5	10
600	5	5	5	5	5	5	5	5
660	5	5	5	5	5	5	5	5
720	5	5	5	5	5	5	5	5
From suspending stages to abandonment								
	W_L			W_S			D_L	D_S
Resource stock	green	yellow	red	green	yellow	red	red	red
(million barrels)	stage 2→5	stage 2→5	stage 2→5	stage 2→5	stage 2→5	stage 2→5	stage 2→5	stage 2→5
0	H	H	H	H	H	H	H	H
20	35	35	35	35	40	40	40	45
40	35	35	35	35	35	35	35	40
60	30	30	30	30	35	35	35	35
80	30	30	30	30	30	30	30	35
120	30	30	30	30	30	30	30	30
140	25	25	25	25	25	25	25	30
180	25	25	25	25	25	25	25	25
200	20	20	20	20	20	20	20	25
240	15	15	15	15	15	15	15	20
300	10	10	10	10	10	10	10	15
350	5	5	5	10	10	10	10	10
450	5	5	5	5	5	5	5	10
500	5	5	5	5	5	5	5	5
600	5	5	5	5	5	5	5	5
660	5	5	5	5	5	5	5	5
720	5	5	5	5	5	5	5	5

"H" means that the critical prices is higher than \$500/barrel. Since in the history the oil price never exceeded \$500/barrel, it implies that it is never optimal to abandon the project.

Table 2.6: Critical Prices To Suspend the Project While There Is No Option To Install Water Storage To Mitigate the Impacts of Water Restriction Regulation

From operating stages to suspending stages								
	W_L			W_S			D_L	D_S
Resource stock	green	yellow	red	green	yellow	red	red	red
(million barrels)	stage 1→2	stage 1→2	stage 1→2	stage 1→2	stage 1→2	stage 1→2	stage 1→2	stage 1→2
0	H	H	H	H	H	H	H	H
20	25	25	30	25	30	35	30	35
40	25	25	30	25	30	35	30	35
60	25	25	30	25	30	40	30	35
80	25	25	30	25	30	45	30	40
100	25	25	30	25	35	45	30	45
120	25	25	35	25	35	50	35	45
140	35	35	40	30	40	50	35	45
160	40	40	40	35	45	50	40	50
180	45	45	45	40	45	55	45	50
200	45	45	50	45	50	55	50	50
220	50	50	55	50	50	60	50	55
240	55	55	55	50	55	60	55	55
270	55	55	55	55	55	60	55	55
300	55	55	60	55	60	60	55	60
350	55	55	60	55	55	60	55	55
400	55	55	55	50	55	60	55	55
450	50	50	55	50	55	60	50	50
500	50	50	50	45	50	55	50	50
550	45	45	45	40	45	50	45	45
600	40	40	45	40	40	45	40	40
630	40	40	40	35	40	45	40	40
660	35	35	40	35	40	45	35	40
690	35	35	35	35	35	40	35	40
720	35	35	35	30	35	40	35	35

"H" means that the critical prices is higher than \$500/barrel. Since in the history the oil price never exceeded \$500/barrel, it implies that it is never optimal to abandon the project.

not have a significant detrimental effect on our hypothetical oil sands operation. This is reinforced by comparing the project values when no water constraints at all are imposed to those for scenario W_L in Figure 2.5. It will be observed that the two curves are very close together. The impact of the regulations only becomes significant under drier river conditions or stricter regulations.

2.7.2 Option To Install a Water Storage Facility

When confronted with restrictions on water usage, firms may seek some technological option to reduce the impact of the restrictions. Among the possible options, the installation of a water storage facility is currently feasible and has been recommended by Alberta regulatory authorities. This section introduces the option to install a water storage facility to the model and we examine its effects on the behaviour of the firm and project value.

2.7.2.1 Project Values With and Without the Storage Option

Figure 2.6 shows the project‘s values for the four scenarios with and without the option to install a water storage facility. As expected when the option to install storage is present, the project becomes more valuable. However we see that the difference in values with and without the storage option becomes significant only for the D_S case where values differ by 15.1% at an oil price of $90/barrel. (The differences are 2.1%, 7.5%, 8.2% respectively for W_L, W_S, and D_L.)

To ground-truth our result about the project value, we compare the sector’s total value hereby derived to that derived from 2014 $\sim$ 2016 financial statements of the five existing oil sands mining operators.[33] This will provide a rough comparison of the orders of magnitude. The annual total profit from the oil sands mining business reflected by the financial statements is about 7.4 billion dollars. Treating it as an annuity lasting for 10 years, the present value is about 66 billion dollars. The total value derived from our model is about 147 billion dollars, which is calculated by 12.239 billion dollars/project $\times$ 12 projects. Recall that the hypothetical project's production capacity is 1.38 million barrels/week rather than the actual 0.83 million barrels/week. This increase in production capacity can explain part of the difference between the above values derived from the financial statements and from our model. In addition the project value calculated is this thesis is an expected value based on an assumption about the future price path of oil, which we are comparing to an annuity base on historical profit levels.

2.7.2.2 Critical Prices To Install Water Storage

Considering the stochastic features of the market and the river flow conditions, the firm chooses the timing of installing the water storage facility to optimize the present value of the project. The critical prices to switch from operating stage 1 to stage 3 indicate the optimal strategy for the decision to invest in water storage. If the crude oil price on the decision day is greater than the critical price, it is optimal to invest, otherwise the investment should be delayed. The critical prices depend on the state variables including present river flow condition as well as the resource stock level. Assuming that the resource stock is at its full level (i.e. 720 million barrels), figure 2.7 shows for the four scenarios, the value of switching from operating stage 1 to stage 3 (i.e. stage 3 value less stage 1 value, hereinafter referred to as "switching values") for different oil prices at time zero. When the

[33] Because for each operator, the ownership of mining blocks can vary over time, we choose 2014 to 2016 to calculate the average annual total profit in order to keep a relative stable profile of blocks, which could be comparable to the hypothetical case that we examine. The five companies are: Canadian Natural Resources Limited, Imperial Oil Limited, Shell Albian Sands, Suncor Energy Inc., and Syncrude Canada Ltd.. Their financial statements can be found in the following websites (accessed on January 11, 2020).
https://www.cnrl.com/investor-information/annual-documents
https://www.imperialoil.ca/en-CA/Investors/Investor-relations/Annual-and-quarterly-reports-and-filings
https://www.shell.com/investors/financial-reporting/annual-publications/annual-reports-download-centre.html
https://www.suncor.com/en-ca/investor-centre/financial-reports/archived-annual-reports
https://www.syncrude.ca/our-news/sustainability-report/

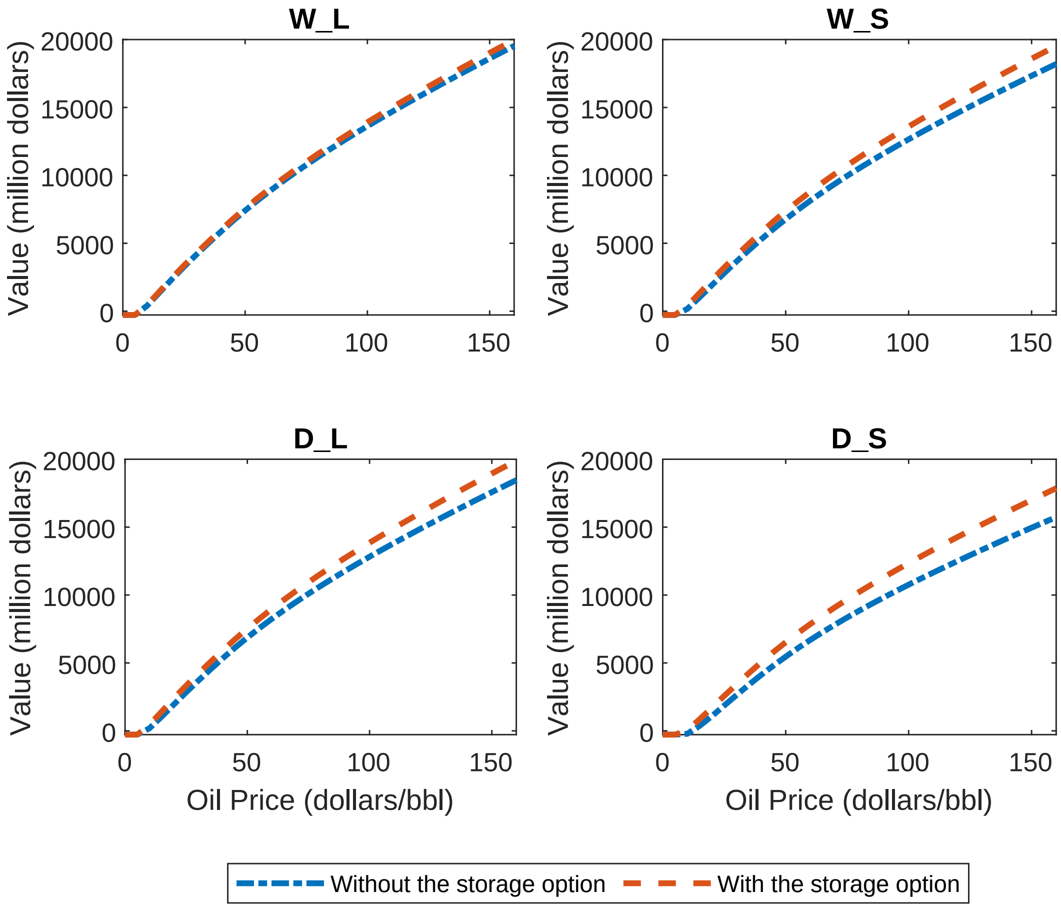

Figure 2.6: Comparing the project values in different scenarios between two cases: there is an option to install a water storage facility & there is no such an option (the present resource stock level is 720 million barrels, the current river flow condition is in the red zone)

switching values are positive, it is optimal to switch to stage 3 by installing storage. The oil price at which the switching value changes from negative to positive is the critical price. For instance, in the top left panel of Figure 2.7, when the current river flow condition is in the green zone, as long as the oil price is higher than \$50/barrel, it is optimal to begin the construction of the water storage facility. So \$50/barrel is the critical price. Similarly, in this scenario, when the present river flow condition is in the yellow or the red zone, the critical prices are \$40/barrel and \$35/barrel respectively.

Table 2.7 (depicted in Figure 2.8) provides a complete reference of the critical prices to proceed to operating stage 3 at different resource stock levels for the four scenarios. It is notable that the critical prices rise quite quickly as the resource stock is depleted, increasing from around \$40 per barrel at full reserves in the W_L case in the green zone to \$140 per barrel when reserves are at 350 million barrels. We also observe critical prices falling significantly when river conditions are drier and water restrictions are more severe.

2.7.2.3 Critical Prices To Abandon the Project

In Section 2.7.1.3 we claim that even without the option to install storage, the critical prices for abandoning the project are fairly low and are not very sensitive to different scenarios. When the option to install storage is available it will be even less likely that the project will be abandoned before the end of the lease at time T. Table 2.8 and Table 2.9 confirms this showing critical prices for abandonment that are the same or lower than when there is no storage option. As in the no storage option case, except for reserve levels below 140 million barrels, the project will always be suspended before it is abandoned.

2.7.2.4 Main Findings

The above results indicate that the application of a water storage facility can generally increase the project's value. The drier the river flow condition or the tighter the water restriction, the more valuable the investment in a water storage facility. However, in the four scenarios, the abandonment decision is not affected much by the installation of a water storage facility. We also investigate some scenarios where the project's production is further limited by the water constraints. It turns out that only when annual production is cut back to much less than the maximum production capacity does the installation of a water storage facility have a significant impact on the critical prices of abandonment. Given the assumed oil price process, the project is generally profitable when producing oil. Stricter water restrictions imply the project may need to stay in operation longer in

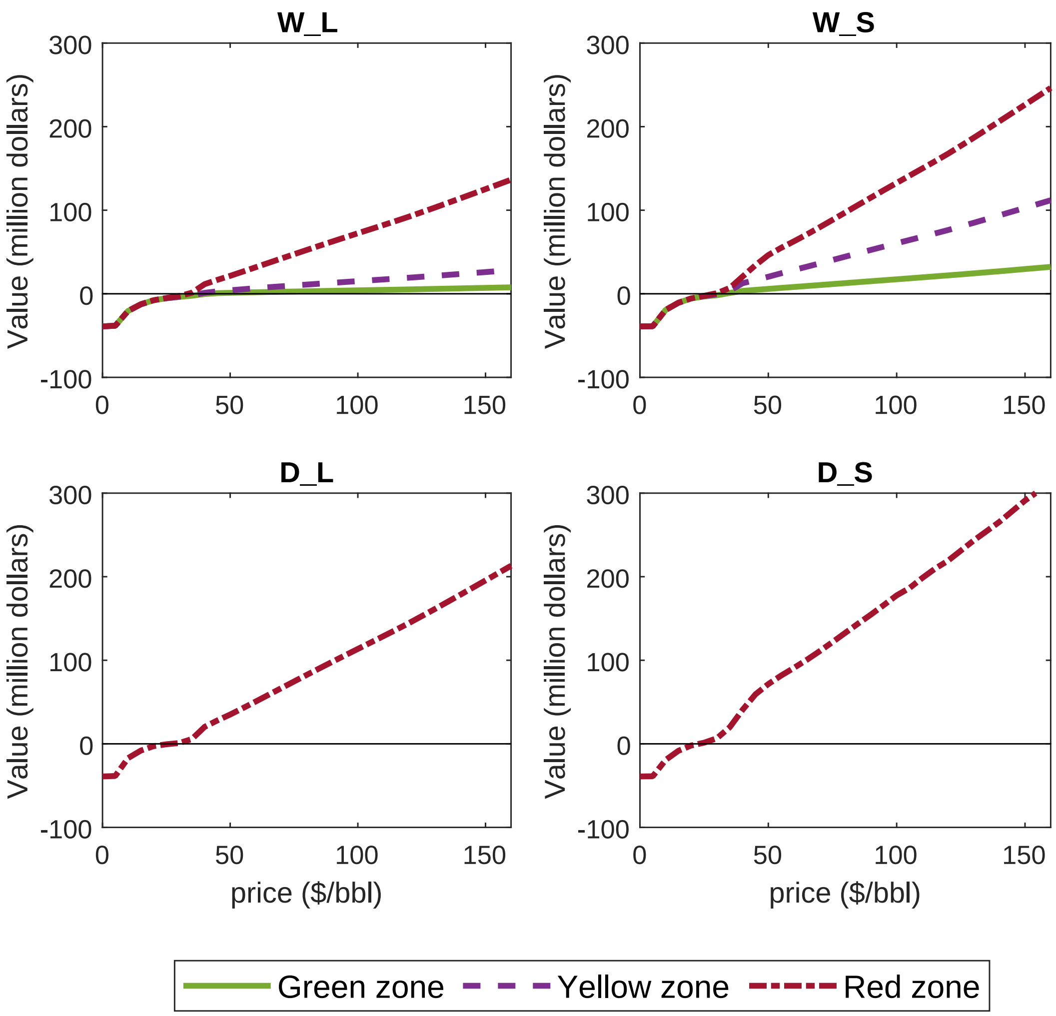

Figure 2.7: Values of switching from stage 1 to stage 3 by installing storage when the resource stock is at the full level (i.e. 720 million barrels)

Table 2.7: Critical Prices To Proceed To Operating Stage 3 While There Is an Option To Install a Water Storage Facility

Resource stock	W_L			W_S			D_L	D_S
(million barrels)	green	yellow	red	green	yellow	red	red	red
0	H	H	H	H	H	H	H	0
20	H	H	300	400	200	40	160	40
40	H	H	160	180	85	35	40	35
60	H	400	130	100	45	35	35	35
80	400	275	105	50	35	35	35	30
100	300	225	90	40	35	35	35	35
120	225	190	80	40	35	35	35	35
140	190	160	70	40	35	35	35	35
160	160	130	65	40	40	35	40	35
180	140	120	65	45	45	40	40	35
200	120	105	60	45	45	45	45	40
220	110	95	60	50	50	45	45	40
240	100	90	60	55	50	50	50	45
270	90	80	60	55	50	50	50	45
300	85	75	60	55	55	50	50	45
350	75	70	60	55	55	50	50	45
400	70	65	55	55	50	50	50	40
450	65	60	55	50	50	45	45	35
500	60	55	50	50	45	40	45	30
550	55	50	45	45	40	35	40	25
600	50	45	40	40	35	35	35	25
630	50	45	40	35	35	30	30	25
660	45	40	35	35	30	30	30	20
690	40	40	35	35	30	30	30	20
720	40	35	30	30	30	25	25	20

"H" means that the critical prices is higher than \$500/barrel. Since in the history the oil price never exceeded \$500/barrel, it implies that it is never optimal to proceed from operating stage 1 to stage 2.

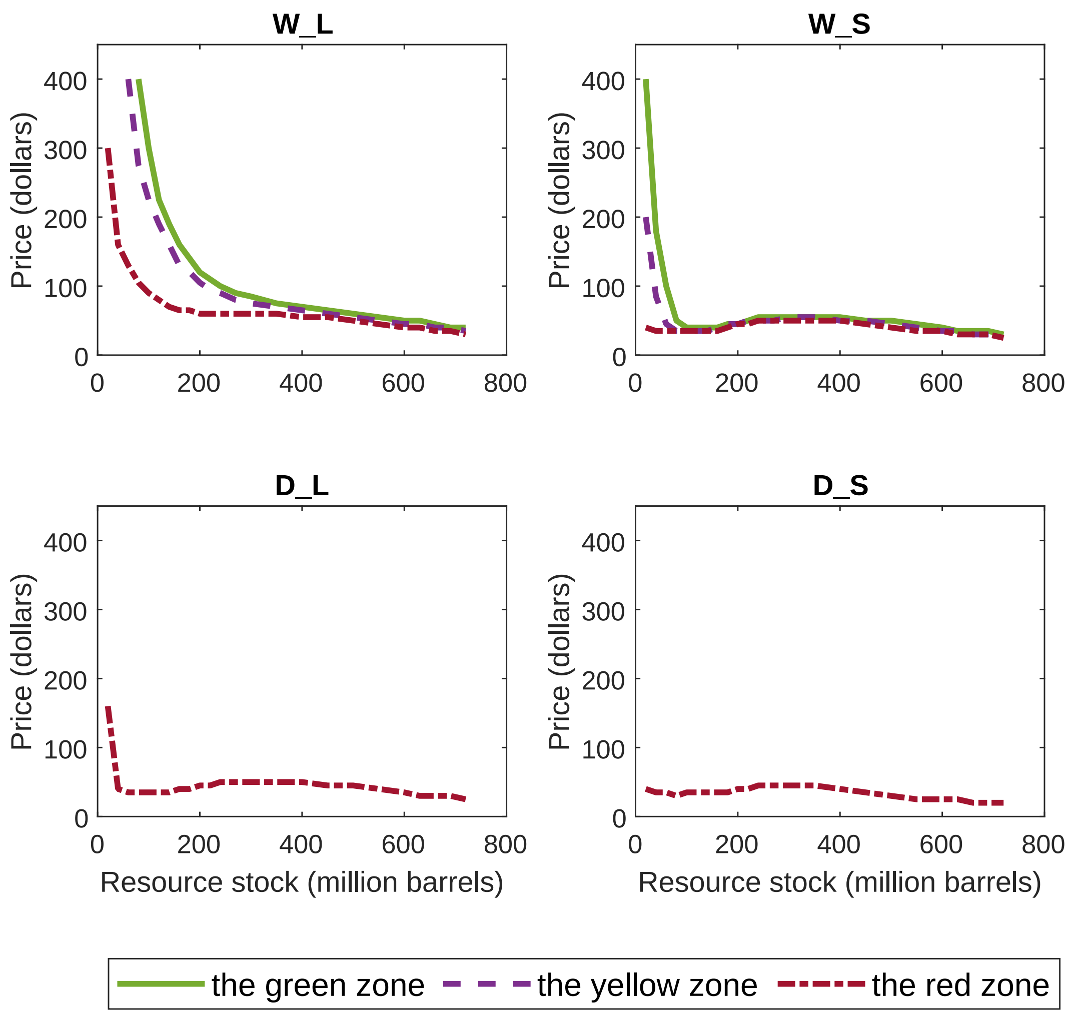

Figure 2.8: Critical prices to proceed from operating stage 1 to stage 3 for different present resource stock levels in the four scenarios

Table 2.8: Critical Prices To Abandon the Project While There Is an Option To Install Water Storage To Mitigate the Impacts of the Water Restriction Regulation

From operating stages to abandonment

Resource stock (million barrels)	W_L						W_S						D_L		D_S	
	green		yellow		red		green		yellow		red		red		red	
	stage		stage		stage		stage		stage		stage		stage		stage	
	1→5	3→5	1→5	3→5	1→5	3→5	1→5	3→5	1→5	3→5	1→5	3→5	1→5	3→5	1→5	3→5
0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
20	35	35	35	35	35	35	35	35	35	35	40	35	35	35	45	35
40	30	30	30	30	35	30	35	30	35	30	35	35	35	30	40	35
60	30	30	30	30	30	30	30	30	30	30	35	30	35	30	35	30
80	30	30	30	30	30	30	30	30	30	30	30	30	30	30	35	30
120	30	25	30	25	30	25	30	30	30	30	30	30	30	30	30	30
140	25	25	25	25	25	25	25	25	25	25	25	25	25	25	30	25
180	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
200	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
240	15	15	15	15	15	15	15	15	15	15	15	15	15	15	20	20
300	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
350	5	5	5	5	5	5	10	10	10	10	10	10	5	5	10	10
450	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
500	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
600	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
660	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
720	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

From suspending stages to abandonment

Resource stock (million barrels)	W_L						W_S						D_L		D_S	
	green		yellow		red		green		yellow		red		red		red	
	stage		stage		stage		stage		stage		stage		stage		stage	
	2→5	4→5	2→5	4→5	2→5	4→5	2→5	4→5	2→5	4→5	2→5	4→5	2→5	4→5	2→5	4→5
0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
20	35	35	35	35	35	35	35	35	40	35	40	35	40	35	45	35
40	35	35	35	35	35	35	35	35	35	35	35	35	35	35	40	35
60	30	30	30	30	30	30	30	30	35	30	35	30	35	30	35	30
80	30	30	30	30	30	30	30	30	30	30	30	30	30	30	35	30
120	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
140	25	25	25	25	25	25	25	25	25	25	25	25	25	25	30	25
180	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
200	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
240	15	15	15	15	15	15	15	15	15	15	15	15	15	15	20	20
300	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
350	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10	10
450	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
500	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
600	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
660	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
720	0	0	0	0	0	0	5	5	5	5	5	5	0	0	5	5

"H" means that the critical prices is higher than \$500/barrel. Since in the history the oil price never exceeded \$500/barrel, it implies that it is never optimal to abandon the project.

Table 2.9: Critical Prices To Suspend the Project While There Is an Option To Install Water Storage To Mitigate the Impacts of Water Restriction Regulation

From operating stages to suspending stages																
	W_L						W_S						D_L		D_S	
Resource stock	green		yellow		red		green		yellow		red		red		red	
(million barrels)	stage		stage		stage		stage		stage		stage		stage		stage	
	1→2	3→4	1→2	3→4	1→2	3→4	1→2	3→4	1→2	3→4	1→2	3→4	1→2	3→4	1→2	3→4
0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
20	25	25	25	25	30	25	25	25	30	25	35	25	30	25	35	25
40	25	25	25	25	30	25	25	25	30	25	35	25	30	25	35	25
60	25	25	25	25	30	25	25	25	30	25	40	25	30	25	40	25
80	25	25	25	25	30	25	25	25	30	25	45	25	30	25	40	25
100	25	25	25	25	30	25	25	25	35	25	45	25	30	25	45	25
120	25	25	25	25	35	25	25	25	40	25	50	25	35	25	50	25
140	35	35	35	35	40	35	35	35	40	35	50	35	40	35	50	35
160	40	40	40	40	40	40	40	35	45	35	55	40	40	40	50	40
180	45	45	45	45	45	45	45	40	45	45	55	45	45	45	55	45
200	45	45	45	45	50	45	45	45	50	45	60	50	50	45	55	50
220	50	50	50	50	55	50	50	50	55	50	60	50	55	50	60	50
240	55	55	55	55	55	55	50	50	55	55	60	55	55	55	60	55
270	55	55	55	55	55	55	55	55	60	55	65	55	55	55	60	55
300	55	55	55	55	60	55	55	55	60	55	65	60	60	55	60	55
350	55	55	55	55	60	55	55	55	60	55	65	60	60	55	60	55
400	55	55	55	55	55	55	55	55	60	55	65	55	55	55	60	55
450	55	55	55	55	55	55	50	50	55	55	60	55	55	55	55	50
500	50	50	50	50	50	50	50	50	50	50	55	50	50	50	50	50
550	45	45	45	45	50	45	45	45	50	45	55	45	50	45	50	45
600	45	45	45	45	45	45	40	40	45	40	50	45	45	45	45	40
630	40	40	40	40	40	40	40	40	45	40	50	40	40	40	45	40
660	40	40	40	40	40	40	35	35	40	35	45	40	40	40	45	35
690	35	35	35	35	40	35	35	35	40	35	45	35	40	35	40	35
720	35	35	35	35	35	35	35	35	35	35	45	35	35	35	40	35

"H" means that the critical prices is higher than $500/barrel. Since in the history the oil price never exceeded $500/barrel, it implies that it is never optimal to abandon the project.

order to extract the resource. Hence even when the oil price drops to very low levels, the project will be temporarily suspended but not abandoned. For instance, if the resource stock is at the current level, when the project is forced to produce at the level of 20% of the maximum capacity, the critical price to abandon falls from \$15/barrel to \$5/barrel. When the project is forced to produce at a lower level of 10% of the maximum capacity, the critical price to abandon is reduced from around \$25/barrel to \$10/barrel. And when the project's production is further cut back to 5% of the maximum capacity, the critical price to abandon is reduced from around \$30/barrel to \$15/barrel.

2.7.3 The Marginal Effect of the Phase 1 Water Management Framework and Efficiency of Water Withdrawal Constraints

In this section we calculate the marginal costs of water withdrawal restrictions in order to consider the efficiency of the regulation. We define marginal cost to be the change in the expected value of the project to the firm, at time zero, caused by a marginal reduction in allowed water withdrawals in all future time periods (i.e. a marginal increase in water restrictions). This is a dynamic definition of marginal cost, in that it is assumed the firm will respond optimally to the change in water restrictions, and may adopt new technology through the installation of storage.

In theory, the goal of government policy is to set an efficient level of water restrictions that maximizes the total benefits of the regulation, which will be at a point that equates marginal costs and marginal benefits. The marginal benefits reflect the value to society of increased water flows in the Athabasca River. Although we have no monetary estimate of the benefit of increase river flows, the marginal cost estimate provides a lower limit for the marginal benefits in order for the regulation to be welfare enhancing. The marginal cost also indicates a firm's willingness to pay for water, and hence would be the price expected if water trading were permitted.

2.7.3.1 The Marginal Costs Under the Phase 1 Water Management Framework

The water withdrawal limits are defined over 52 weeks of the year and in three different water zones. For the purposes of this chapter, we define an increase in water withdrawal restrictions to be a reduction of permitted water withdrawal rates of 70 million barrels per year in all weeks of the year when the river is in the yellow and red water zones over the lifetime of the project. We denote the reduction of withdrawal rates as $\Delta \bar{w}$.

The marginal cost of increased restrictions depends on the current value of the state variables. We estimate the marginal cost of the Phase 1 Water Management Framework to the hypothetical firm, MC, by taking the present value of the hypothetical firm $V(p, s, \bar{w}, \iota, \bar{\delta}, t)$, in a given river zone where $\bar{W} = \bar{w}$, at a specific oil price level, $P = p$, at a certain oil stock level, $S = s$, and finding the change in $V(p, s, \bar{w}, \iota, \bar{\delta}, t)$, when the annual constraints on water withdrawal rates in the yellow and red zones are further restricted by $\Delta\bar{w}$[34] over the lifespan of the project, i.e. $T - t_0$. That is to say, $MC = \frac{\Delta V(p,s,\bar{w},\iota,\bar{\delta},t)}{\Delta\bar{w}\cdot(T-t_0)}$.

A list of the marginal costs corresponding to different current oil prices in the four scenarios when there is an option to build a water storage facility are reported in Table 2.10 and depicted in the panels on the left hand side of Figure 2.9. The marginal costs for the cases where there is no storage option to mitigate the impact of the water constraints are reported in Table 2.11 and depicted in the panels on the right hand side of Figure 2.9.

We observe that the marginal cost of increasing the water restrictions is very low in the W_L scenario when there is an option to install storage - no more than 21 cents for oil prices lower than \$150 per barrel. Without a storage option the marginal cost in this case is higher, ranging from 27 cents for the lowest oil prices to \$1.35 in the red zone at \$150 per barrel for the price of oil. The marginal costs of the regulation are substantially higher when the river is assumed to be in drier conditions and when the base case regulations are stricter - that is in W_S, D_L and D_S.

2.7.3.2 The Marginal Costs Under the Different Water Withdrawal Constraints and the Efficient Level of Constraints

In this section we map out the marginal cost of increased restrictions for a range of initial water restrictions. If all firms in the industry were like this typical firm depicted in our example, we could map out the marginal cost of water restrictions for the oil sands industry. In reality oil sands firms have different efficiencies and cost curves. The calculation of an aggregate marginal cost curve must be viewed as illustrative.

Figure 2.10 below shows the marginal cost curve for the industry composed of 12 same scale projects (as is the number of operating projects in 2015). The figure is shown for an initial oil price of \$50 per barrel and assuming the oil stock is at its maximum level for each firm. The horizontal axis shows the adjustment of the level of year-round water withdrawal restrictions (i.e. the annually available water for oil industry), with water

[34] Due to the accuracy of the numerical method the smallest marginal change that can be examined is 1 million barrels of water per week over the lifespan of the project. The change in the firm's present value is in millions of dollars.

Table 2.10: Marginal Cost To the Project While There Is an Option To Build Storage ($/barrel)

Oil price	W_L			W_S			D_L	D_S
($/barrel)	Green	Yellow	Red	Green	Yellow	Red	Red	Red
10	0.05	0.05	0.05	0.23	0.23	0.23	0.36	0.67
15	0.07	0.07	0.07	0.32	0.32	0.32	0.50	1.06
20	0.08	0.08	0.08	0.38	0.38	0.38	0.59	1.33
25	0.08	0.08	0.08	0.42	0.42	0.42	0.64	1.50
30	0.08	0.08	0.08	0.44	0.44	0.44	0.68	1.63
35	0.09	0.09	0.08	0.46	0.46	0.46	0.70	1.74
40	0.09	0.08	0.09	0.47	0.47	0.49	0.72	1.88
45	0.09	0.09	0.09	0.50	0.50	0.53	0.75	2.02
50	0.09	0.09	0.10	0.52	0.53	0.57	0.79	2.16
55	0.10	0.10	0.11	0.55	0.56	0.61	0.83	2.30
60	0.10	0.10	0.11	0.58	0.59	0.65	0.87	2.41
65	0.10	0.11	0.12	0.61	0.62	0.69	0.91	2.52
70	0.11	0.11	0.12	0.63	0.65	0.72	0.95	2.63
75	0.11	0.11	0.13	0.66	0.67	0.75	0.99	2.74
80	0.12	0.12	0.14	0.68	0.70	0.78	1.03	2.83
85	0.12	0.12	0.14	0.71	0.73	0.81	1.07	2.94
90	0.12	0.13	0.15	0.73	0.75	0.85	1.11	3.02
95	0.13	0.13	0.15	0.75	0.78	0.88	1.14	3.13
100	0.13	0.13	0.16	0.78	0.80	0.91	1.18	3.23
105	0.14	0.14	0.16	0.80	0.82	0.94	1.22	3.31
110	0.14	0.14	0.17	0.82	0.85	0.97	1.25	3.41
115	0.14	0.15	0.18	0.84	0.87	1.00	1.29	3.52
120	0.15	0.15	0.18	0.86	0.90	1.03	1.32	3.58
130	0.15	0.16	0.19	0.91	0.94	1.09	1.39	3.78
140	0.16	0.16	0.20	0.95	0.99	1.15	1.46	3.93
150	0.17	0.17	0.21	0.99	1.03	1.21	1.52	4.13
160	0.17	0.18	0.22	1.03	1.08	1.27	1.59	4.26
170	0.18	0.19	0.23	1.07	1.12	1.33	1.65	4.46
180	0.19	0.19	0.24	1.11	1.16	1.39	1.71	4.62
190	0.19	0.20	0.25	1.15	1.20	1.45	1.77	4.77
200	0.20	0.21	0.26	1.19	1.24	1.50	1.83	4.94
225	0.22	0.22	0.29	1.28	1.35	1.64	1.98	5.33
250	0.23	0.24	0.31	1.37	1.44	1.78	2.13	5.71
275	0.24	0.25	0.34	1.45	1.54	1.91	2.26	6.09
300	0.26	0.27	0.36	1.54	1.63	2.04	2.40	6.44
350	0.28	0.29	0.41	1.70	1.81	2.29	2.66	7.14
400	0.31	0.32	0.45	1.85	1.98	2.54	2.91	7.80
450	0.33	0.35	0.49	2.00	2.15	2.78	3.15	8.45
500	0.36	0.37	0.54	2.15	2.31	3.02	3.39	9.09

The remaining resource stock is 720 million barrels.

Table 2.11: Marginal Cost To the Project While There Is No Option To Build Storage ($/barrel)

Oil price	W_L			W_S			D_L	D_S
($/barrel)	Green	Yellow	Red	Green	Yellow	Red	Red	Red
10	0.27	0.27	0.27	0.48	0.48	0.48	0.64	0.36
15	0.38	0.38	0.38	0.72	0.72	0.72	0.95	1.04
20	0.45	0.45	0.45	0.88	0.88	0.88	1.16	1.44
25	0.49	0.49	0.49	0.98	0.98	0.98	1.28	1.67
30	0.51	0.51	0.51	1.05	1.05	1.05	1.37	1.87
35	0.53	0.53	0.54	1.10	1.11	1.11	1.45	2.06
40	0.55	0.56	0.57	1.17	1.18	1.19	1.55	2.24
45	0.58	0.58	0.62	1.24	1.26	1.27	1.65	2.44
50	0.61	0.61	0.65	1.31	1.33	1.37	1.73	2.62
55	0.64	0.65	0.68	1.38	1.41	1.46	1.82	2.78
60	0.67	0.68	0.72	1.45	1.48	1.54	1.91	2.92
65	0.70	0.71	0.76	1.51	1.54	1.60	2.00	3.04
70	0.73	0.74	0.80	1.58	1.61	1.67	2.09	3.16
75	0.76	0.78	0.83	1.64	1.67	1.73	2.17	3.27
80	0.79	0.81	0.87	1.70	1.74	1.80	2.26	3.38
85	0.82	0.84	0.91	1.75	1.80	1.86	2.34	3.49
90	0.85	0.87	0.94	1.81	1.86	1.92	2.42	3.59
95	0.88	0.90	0.98	1.87	1.92	1.99	2.50	3.70
100	0.91	0.93	1.01	1.92	1.97	2.05	2.58	3.80
105	0.93	0.96	1.05	1.98	2.03	2.11	2.65	3.90
110	0.96	0.99	1.08	2.03	2.09	2.17	2.73	4.00
115	0.99	1.02	1.12	2.09	2.14	2.23	2.80	4.10
120	1.02	1.04	1.15	2.14	2.20	2.29	2.88	4.19
130	1.07	1.10	1.22	2.24	2.31	2.41	3.03	4.39
140	1.12	1.16	1.29	2.34	2.42	2.52	3.17	4.58
150	1.17	1.21	1.35	2.44	2.52	2.64	3.31	4.76
160	1.22	1.27	1.42	2.54	2.63	2.75	3.45	4.94
170	1.27	1.32	1.48	2.64	2.73	2.86	3.59	5.12
180	1.32	1.37	1.55	2.73	2.83	2.97	3.73	5.30
190	1.37	1.42	1.61	2.83	2.93	3.08	3.86	5.47
200	1.42	1.48	1.68	2.92	3.03	3.19	4.00	5.64
225	1.54	1.60	1.83	3.15	3.27	3.45	4.32	6.06
250	1.65	1.72	1.98	3.37	3.51	3.71	4.63	6.46
275	1.76	1.84	2.13	3.58	3.74	3.96	4.94	6.85
300	1.87	1.96	2.28	3.78	3.96	4.20	5.24	7.23
350	2.08	2.18	2.56	4.18	4.39	4.67	5.82	7.97
400	2.28	2.40	2.84	4.56	4.80	5.12	6.37	8.67
450	2.48	2.61	3.11	4.94	5.20	5.57	6.91	9.35
500	2.67	2.82	3.38	5.30	5.60	6.00	7.45	10.02

The remaining resource stock is 720 million barrels.

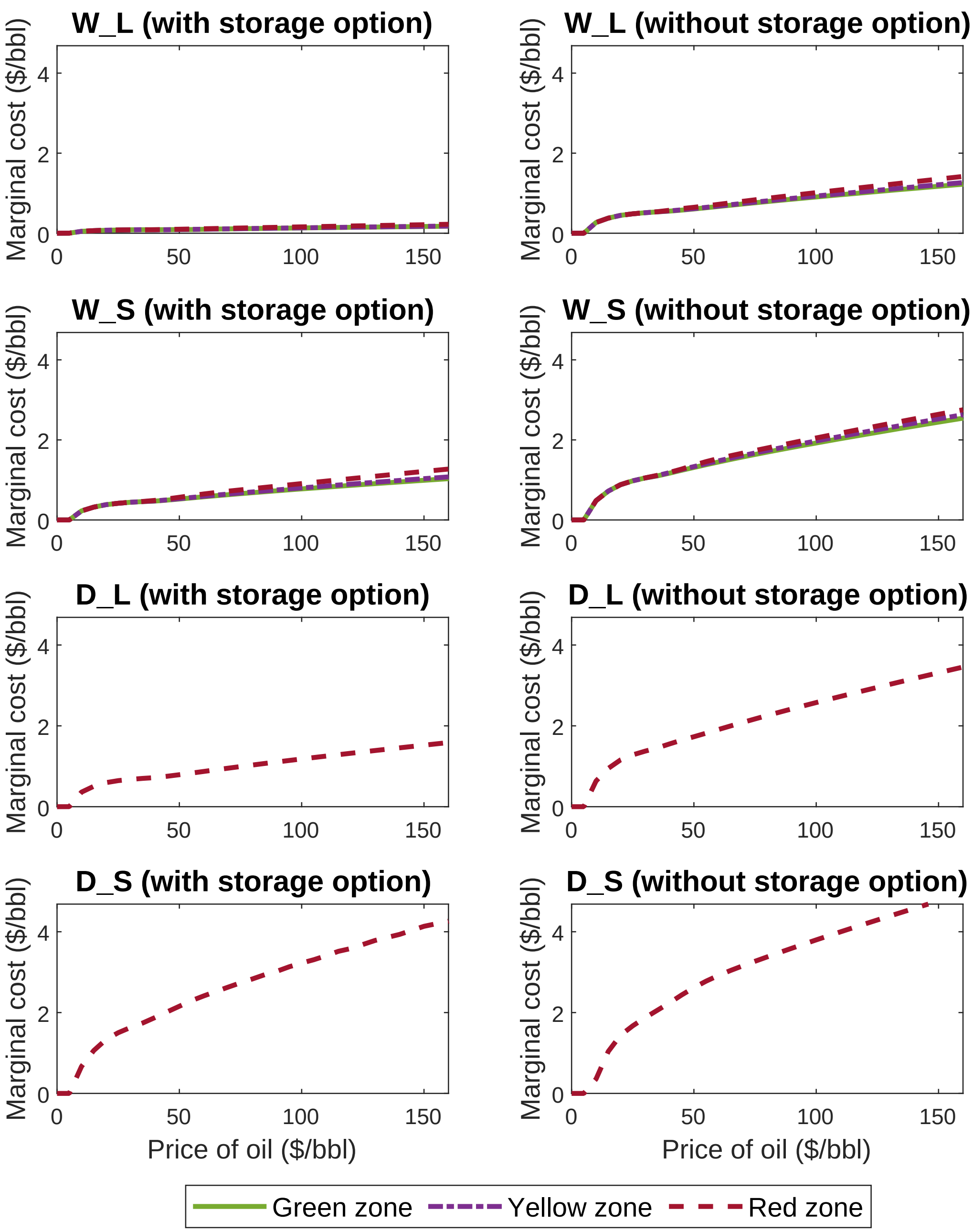

Figure 2.9: Comparison of marginal cost between the cases of being with and without a storage option

constraint regulations becoming less strict moving from left to right, in all future time periods. For example, -168 means that the water withdrawal limits in the red and yellow zones have been reduced by 168 million barrels per year for all future time periods. We do not follow the regular way of depicting marginal cost curves using the quantity of available water as the horizontal axis. The reason is that the available water varies across weeks while depending on the river flow zones. For each point on the horizontal axis, there is not a single number that can represent the available water quantity. Instead, there is a specific combination of available water quantities for different weeks and different river flow zones mapping each point on the horizontal axis. One of those points represents the combination set by the Phase 1 Framework. We specify this point as a reference, labeled as 0. The labels of other points reflect the changes of available water quantity of the corresponding regulated combination compared to that of the Phase 1 Framework, which are aforementioned $\Delta\bar{w}$. For a given stage of operation we would expect the marginal cost of water restrictions to decline as restrictions become less onerous, moving from left to right on the graph. However this marginal cost curve depicts the situation of a firm initially in stage 1 and assuming optimal decisions are made regarding the installation of storage technology. We see that starting from a point of lenient restrictions on the right the marginal cost curve initially rises, then between +672 and +168 the cost curve falls. For restrictions of +672 and more lenient (going right) it is not optimal to install storage at any critical price. For restrictions of +672 and more stringent (going to the left), it is optimal to install storage in the future for some critical prices. When storage is installed the marginal cost of restrictions drops, hence we see a portion of the marginal cost curve which falls and then starts rising again to the left of +168. This curve traces out a long run dynamic marginal cost curve[35] which captures a change in technology happening between +168 and +672. For further intuition we plot on the same graph the marginal cost curves for when there is no storage available (blue dashed curve) and when storage is freely available (red dashed curve) (and hence is a free option which will always be exercised.) It can be seen that the marginal cost curve in stage 1 falls between these two other cases.

We are unable to determine the efficient level of water restrictions as we do not have an estimate of the benefits to the ecosystem of an additional unit of water flowing in the river. Three hypothetical marginal benefit curves are shown in Figure 2.10 - one (# 1) in which $\bar{w}$ (reflecting the Framework's restrictions) is the efficient level where marginal benefit equals marginal cost and the other two where $\bar{w}$ is above or below the socially optimal point (# 2 and # 3 respectively). The efficiency loss when the restrictions are not at the optimal levels depends on the slopes and locations of the marginal benefit curve and the marginal

[35]This is a long run marginal cost curve in the sense that technology is allowed to change through the addition of storage.

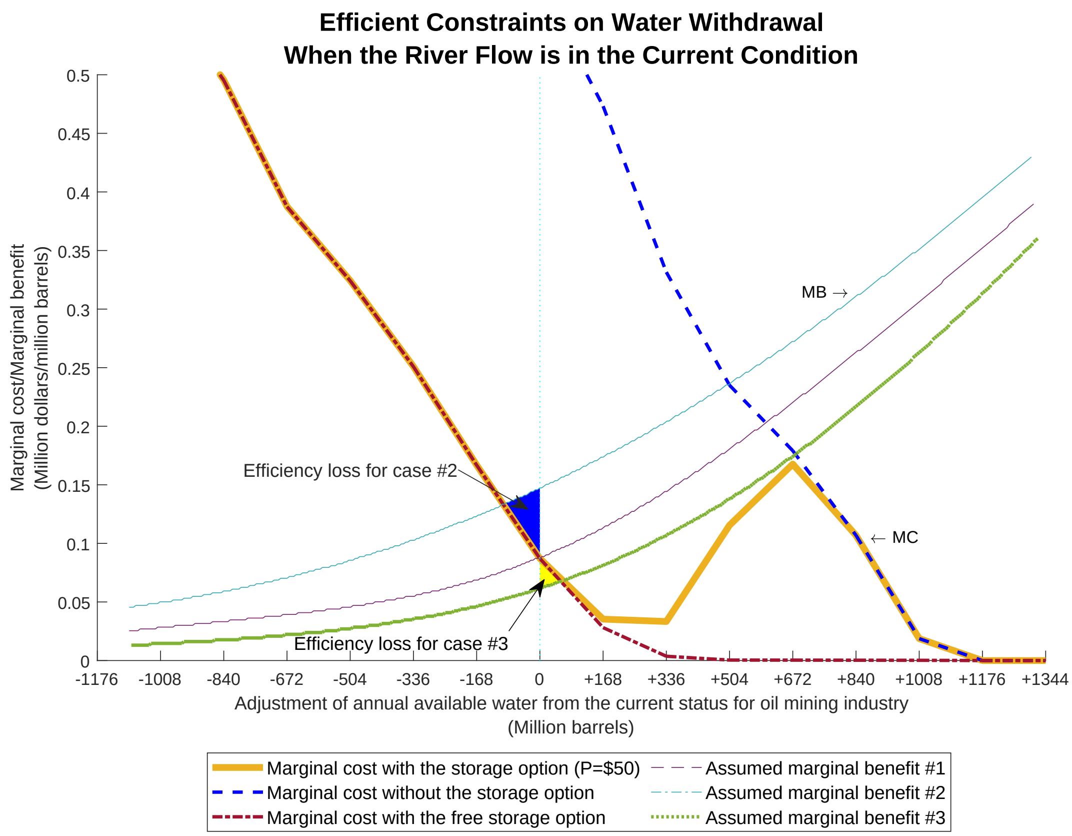

Figure 2.10: Marginal cost vs. water constraint levels when the present oil price is \$50/barrel, the resource stock is at the full level, and the present river flow condition is in the green zone

cost curve.

Note that if the marginal benefit curve crossed the rising portion of the marginal cost curve, then there would be no unique point where MB=MC. In this case, the total benefits and total costs would need to be examined for a range of restrictions to find the optimum.

The marginal benefit curve will not change with the variation of oil price or resource stock. However the marginal cost of restrictions will depend on the state variables, i.e. the oil price P and the oil reserve S, in particular. In Figure 2.11, we assume that the current constraint level is efficient, that is to say, when the oil price is \$50/barrel, the resource stock is at the full level, and the river condition is in the green zone, the marginal benefit equals the marginal cost. From this figure, we can see that different levels of the current oil price imply a different efficient water constraint. It is clearly impractical to change the level of water restrictions based on changing economic conditions in the oil industry which shift the marginal cost curve. However this highlights the fact that quantitative restrictions such as these have a highly variable cost for firms, depending on the value of key state variables such as the price of oil.

2.7.3.3 Comparison With Previous Estimates

The marginal costs due to the water withdrawal constraints imply the marginal values of water to the firm (i.e. the marginal willingness to pay for water, or the implied shadow price of water). Mannix et al. (2014) measured the willingness to pay for water by oil sands firms including both in-situ and surface mining projects. In addition to some specific assumptions for projects regarding productivity, costs, and project life-cycle, their assumptions about the oil price and the river flow condition are somewhat different from ours. Specifically, they assume that the oil price is at a constant level: \$70/barrel. And their assumed river flow condition is 10% drier than the historic condition. Under their assumptions, when water is assigned according to license priorities, they found the highest marginal willingness to pay is \$180/m^3 (\$22.5/barrel). Then they derive an efficient water distribution mechanism among oil sands firms by solving a linear programming problem and find in this case the highest marginal willingness to pay among firms with a shortfall of water decreases to a maximum of \$78/m^3 (\$9.75/barrel) and on average \$6.7/m^3 (\$0.84/barrel). Furthermore, when a consolidated tailings technology is in use together with the efficient allocation policy, the average willingness to pay decreases to \$4.15/m^3 (\$0.52/barrel). Instead, if a storage technology is in use under an efficient allocation policy, the average willingness to pay will further decrease, although the paper does not provide an estimate. In our analysis, since we focus on an industry where every firm can share the available water evenly, there is no less senior firm that is allocated less water coming in. It implies that

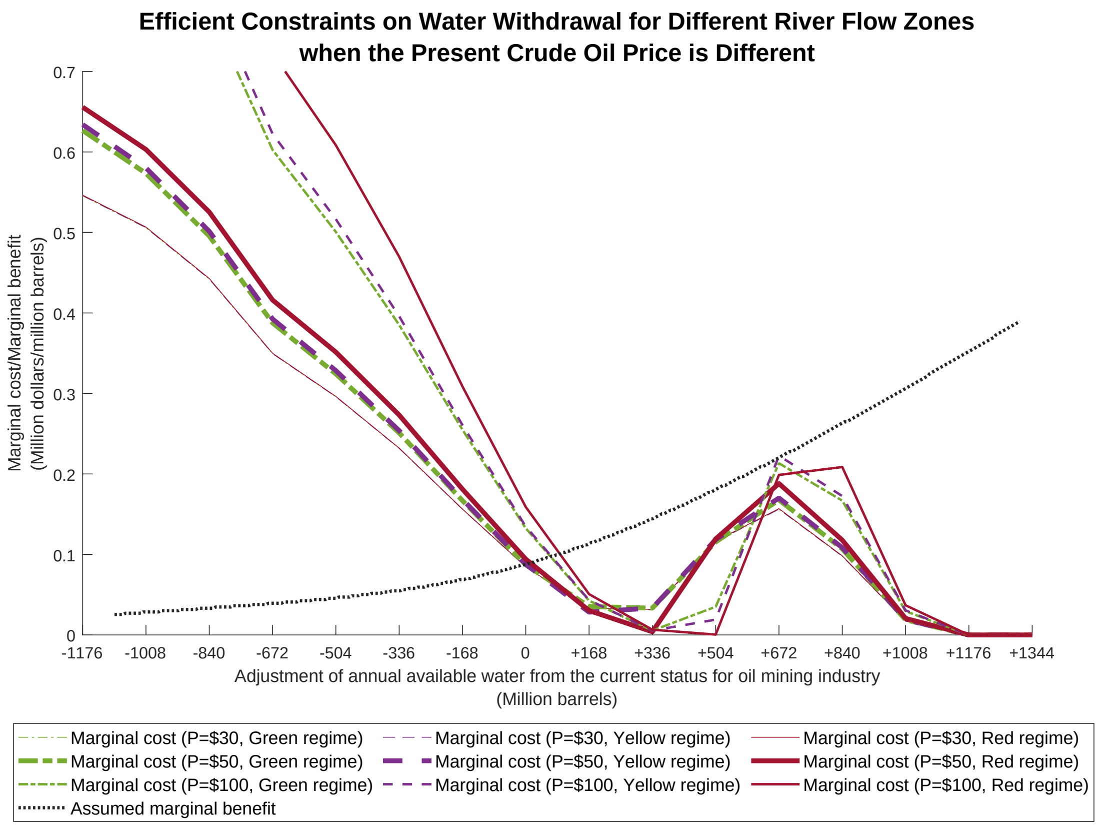

Figure 2.11: Marginal cost vs. water constraint levels for different present oil prices when the resource stock is at the full level and the present river flow condition is in the green zone

we are examining the cost due to the water constraint regulations without the added complication of inefficient allocation across firms. When there is no water compensating technology applied, given the present oil price is \$70/barrel, the implied shadow price of water is from \$0.73/barrel to \$0.80/barrel depending on the specific river flow zone it is in. To compare more closely to Mannix et al.'s result, we undertake a sensitivity analysis in which adopt their assumption regarding river flow condition. The marginal cost when the present oil price is \$70/barrel (which is the assumed constant oil price in Mannix et al. (2014)) is from \$0.28/barrel to \$0.29/barrel, depending on which river flow zone it is in. The result is still lower than that of Mannix et al. (2014), which is \$0.84/barrel, when they eliminate the impact of inefficient allocation due to the prior allocation (by applying an efficient allocation policy). Furthermore, in our study, when there is an option to build a water storage facility, the implied shadow price of water decreases to a very insignificant level that less than \$0.12/barrel. This is much lower than the results given by Mannix et al. (2014).

The approaches and main purposes of Mannix et al. (2014) and our study are quite different. Mannix et al. (2014) investigates several firms with different assumed water productivities and water seniority, giving them a constant oil price and fixed water availabilities throughout their life-cycle. Its main purpose is to examine the allocation efficiency and recommend auxiliary policies (e.g. an efficient allocation policy) and technologies. Our study examines one typical firm, assuming that it has the seniority like those existing companies who can obtain an equal share from the available water supply. We also allow the oil price to be a stochastic process and river flow condition vary according to its historic pattern and inspect the costs to the firm due to the water constraint policy under a dynamic background. Our purpose is to examine the average impact of phase 1 water management framework on the current oil sands industry and try to improve the water management regulation *per se*.

2.8 Sensitivity Analyses

The stochastic process assumed for crude oil prices plays an critical role in the optimal decision about investment in a water storage facility. In this section, we check how the outcomes are affected by different assumptions for the mean log crude oil price, water productivity, and volatility. More detailed tables and graphs for the following subsections can be found in Appendix A.4.

2.8.1 The Effects of Price Volatility

Figures 2.12 - 2.14 plot critical prices to install storage versus volatility for several cases. Looking first at the D_S scenario (Figure 2.12), the critical prices are observed to fall as volatility increases. This is interesting as for simple investment options, an increase in volatility results in the delay of an investment (Majd & Pindyck (1987)). However in this case, when water flows are reduced and water withdrawals are heavily constrained an increase in price volatility makes storage more valuable to the firm. Without storage, and under strict water constraints the firm may not be able to take advantage of a sudden upswing in prices. Hence the more volatile prices increase the desirability of storage. We see a similar effect under base case water restrictions - W_L (Figures 2.13 and 2.14). However in both the diagrams showing the W_L case, we observe an increase in critical prices for some reserve levels as volatility reaches levels higher than 0.47. In this case where water withdrawals are only mildly constrained, once volatility reaches a certain level, further increases tend to delay investment, as per the normal effect of uncertainty. Table A.9 and Figure A.5 in the appendix provide more details.

In the wet scenarios, the marginal and total costs of the regulations do not change substantially under different volatility assumptions. This can be seen in Tables A.10, A.11, Figures A.6, A.7, and A.8 in Appendix A.4. The reason is that the option to install a water storage facility reduces the impact of price volatility. Storage allows the firm to respond as desired to changing prices. However, for D_S, the economic costs (total costs and marginal costs) do increase significantly when oil price volatility increases. This is because even in the presence of water storage, with the more limited water availability in the D_S scenario the firm is unable to take advantage of high oil prices that might occur with a more volatile oil price.

2.8.2 The Mean Log Crude Oil Price Effects, μ

In Figure 2.15 below and Table A.3 in Appendix D, we compare the optimal timing to switch between operating stages for different mean oil prices. Figure 2.15 exhibits the critical prices under different mean log oil prices for all possible resource stock levels when the current river flow is in the green zone. Here we see the familiar pattern of critical prices to install storage increasing for smaller oil reserves. For the cases where the current state is in the yellow or red zone, the patterns are similar and are not included here. The relationship between the mean log oil price and critical prices to install storage is easier to see in Figure 2.16 which is drawn for a particular reserve level. In this graph we see

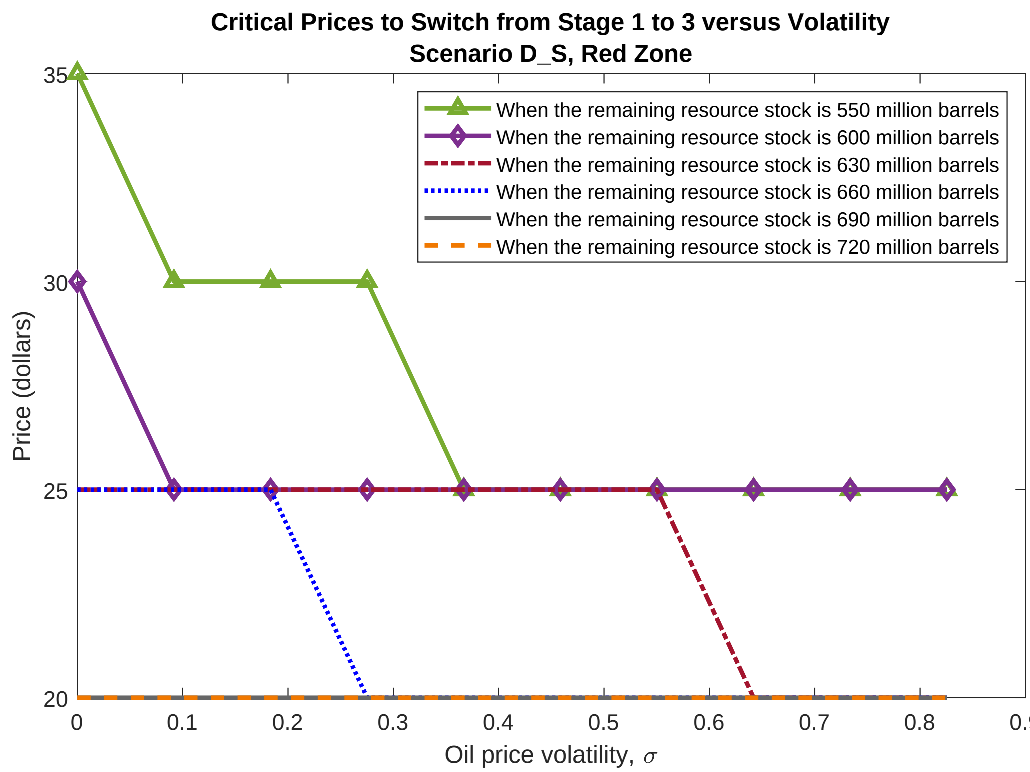

Figure 2.12: Critical prices to install storage versus volatility for scenario D_S in the red zone

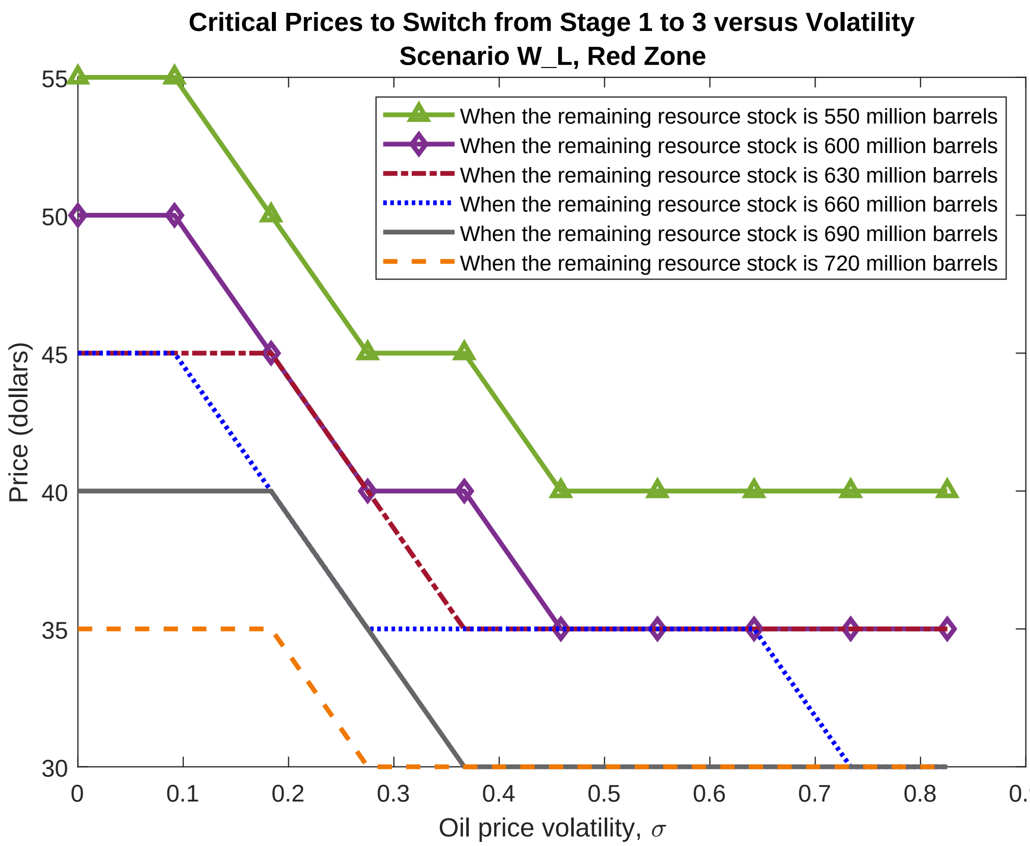

Figure 2.13: Critical prices to install storage versus volatility for scenario W_L in the red zone

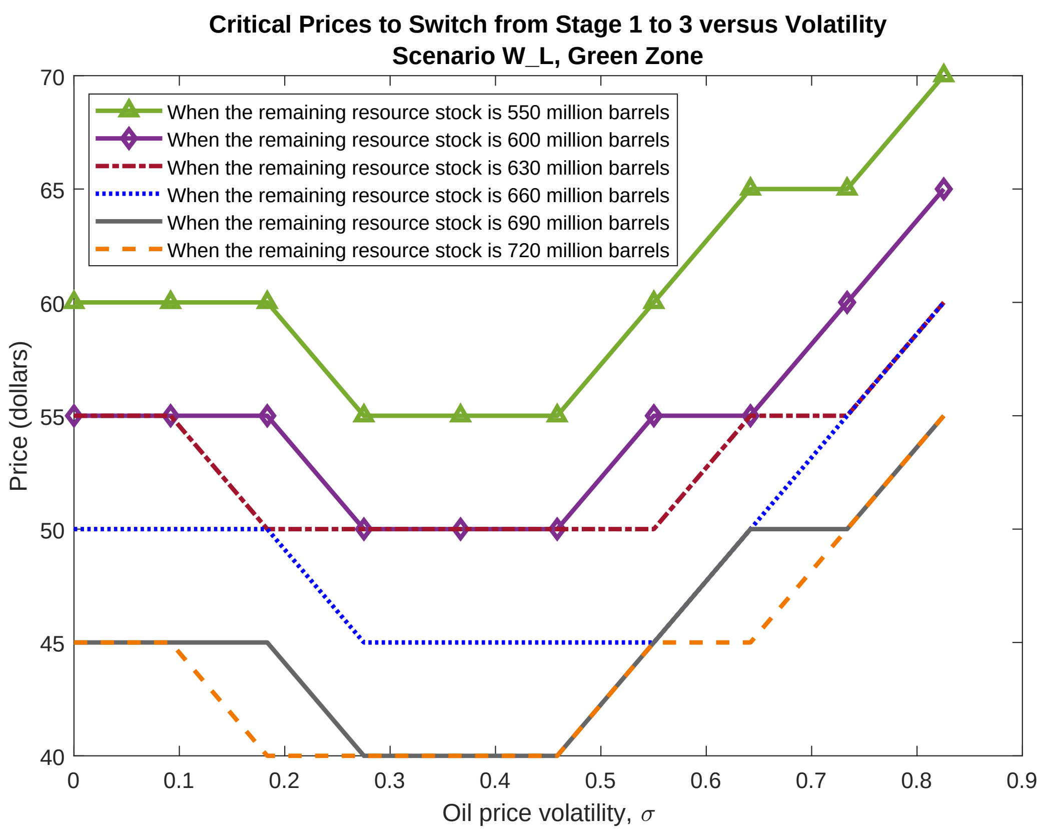

Figure 2.14: Critical prices to install storage versus volatility for scenario W_L in the green zone

that critical prices have a U-shaped relationship with the mean oil price. Recall that the speed of mean reversion is $\epsilon = 0.14$ which implies the expected time to revert to the long run mean is 7.14 years. At values of μ below about 3.5 ($33 per barrel for WTI), the critical price to install storage is inversely related to μ. Since price will be pulled down to a low level in future, it is worthwhile waiting for a relatively higher price before making the investment to install storage. At levels of μ above 4 (or $55 per barrel for WTI) critical prices are positively related to μ. In this case prices are expected to be pulled up to a relatively high level in future, and there is an option value to waiting for higher prices before beginning installation.

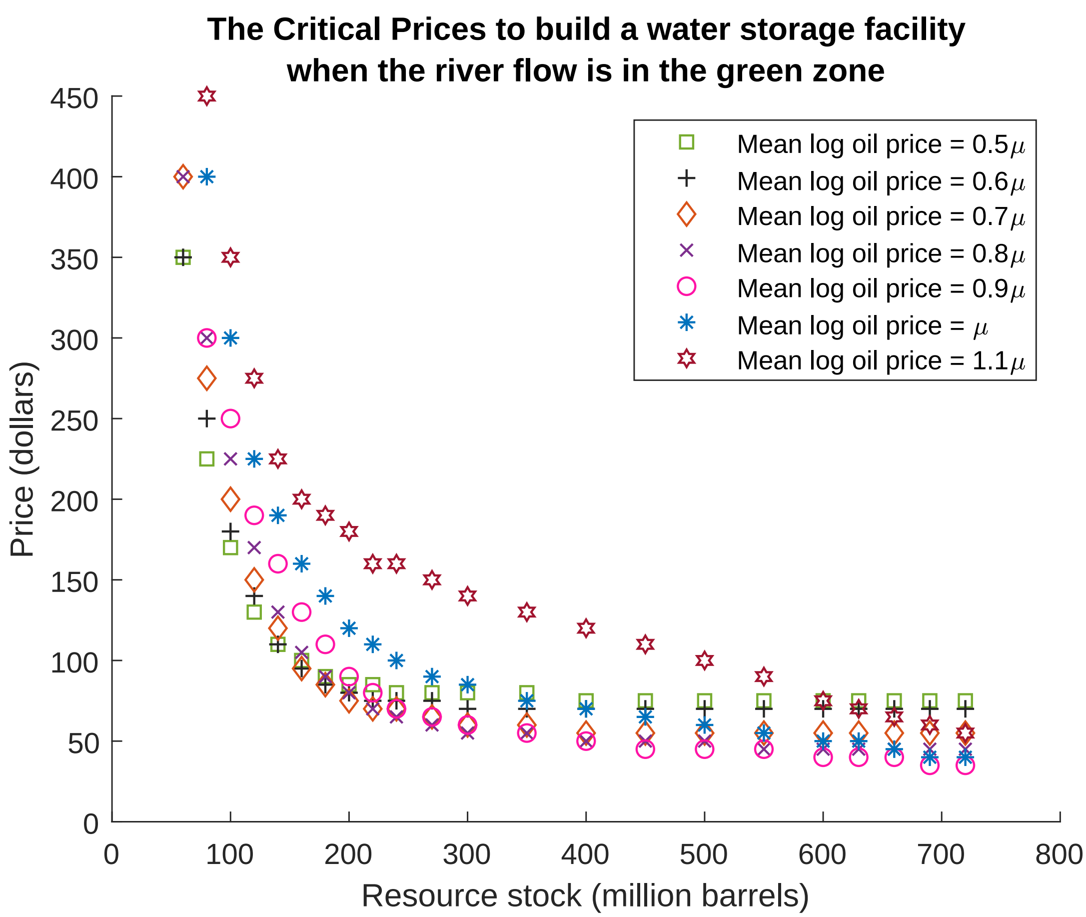

Figure 2.15: Critical prices to build a water storage facility when the current river flow is in the green zone under different mean log oil prices. Base case mean log oil price is 4.59.

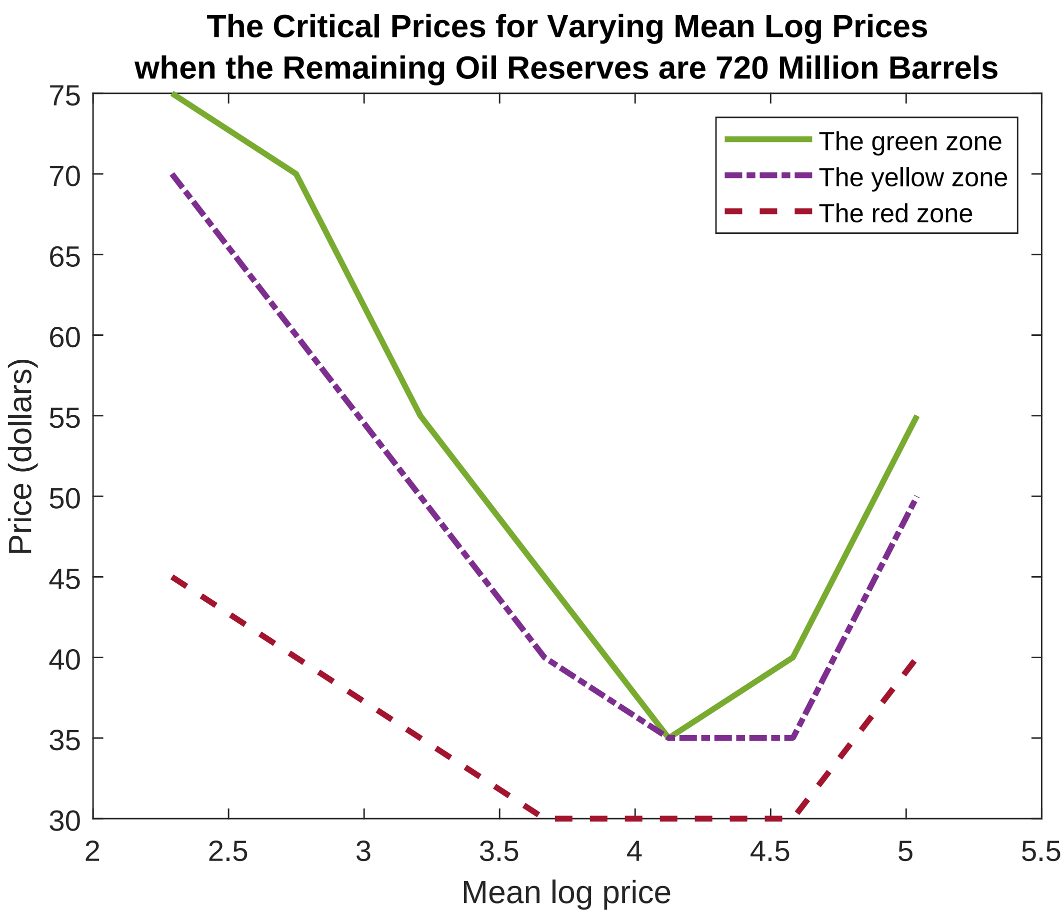

Figure 2.16: The critical prices for varying mean log oil prices when the remaining established oil reserves are 720 million barrels

By examining values of the project in different cases where the mean oil prices vary, we can compare the total economic cost, the relative economic loss, and the marginal cost, which is the shadow price of water to the firm. The results are listed in Table A.4, Table A.5, Figure A.1 and A.2. The assumed long run mean oil price level affects the marginal cost and the total cost significantly. The marginal cost to the firm of increasing water restrictions is higher for a higher long run mean price. On a $ per barrel basis the marginal cost is less than $0.2 for all cases except D_S, which is dry river conditions under very strict water withdrawal limits. The total cost of the regulations, defined as the difference in the project value with and without any water restrictions is also higher for a higher long run mean price. When the current oil price is $100 per barrel, it ranges from $47 to $72 million in all cases except for D_S, when the cost is up to $1.9 billion. These amount to 2.7%, 4%, and 10.3% of total project value, respectively.

2.8.3 The Effects of Water Productivity

Table A.6 in Appendix A.4 details the effects of water productivity on the critical prices for switching from one stage to another. The table shows that with the enhancement of water productivity, the desirability of installing a water storage facility declines. Figure A.3 in Appendix A.4 also shows the critical prices under different water productivity levels. Due to the limited space, we skip critical prices for water productivity levels between 0.3 and 0.4. These critical prices rise steadily, rather than jump, with increasing water productivity. Our main purpose is to show the water productivity threshold over which the water constraints are not binding. When the water productivity is greater than 0.5 barrels of bitumen/barrel of water, (i.e. one barrel of oil needs less than two barrels of water), for W_L scenario, there is no need to invest in a water storage facility no matter which river flow zone it is in. The reason is straight forward: when the water productivity is high, even in dry seasons, the allowed withdrawal amount can satisfy the production demands. Therefore, the water constraint regulation does not make the firm short of water. The water productivity we used in previous sections is the average level for oil sands industry. For individual projects, there are a range of water productivity levels.

Figures 2.17, and 2.18 show how economic costs of water restrictions are affected by the different water productivity levels. Further details are given in Appendix A.4, Tables A.7, A.8, and Figure A.4. When the water productivity is greater than 0.5 barrels of bitumen/barrel of water, there will be no economic cost caused by the water constraint regulation.

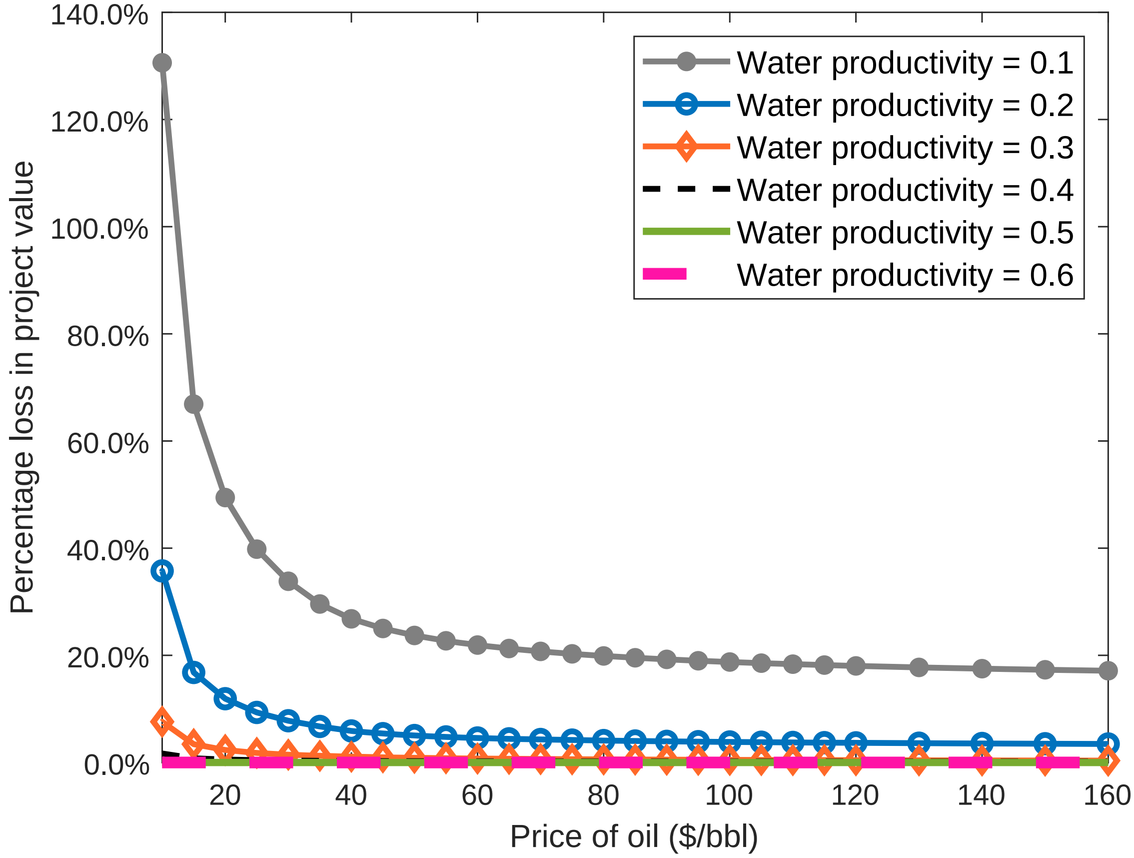

Figure 2.17: The percentage loss to the oil sands project due to the water constraints when the river flow is in the green zone under various water productivity levels for scenario W_L (water productivity is in barrels of bitumen/barrel of water. The percentage loss refers to the reduction in total project value when restrictions are imposed.)

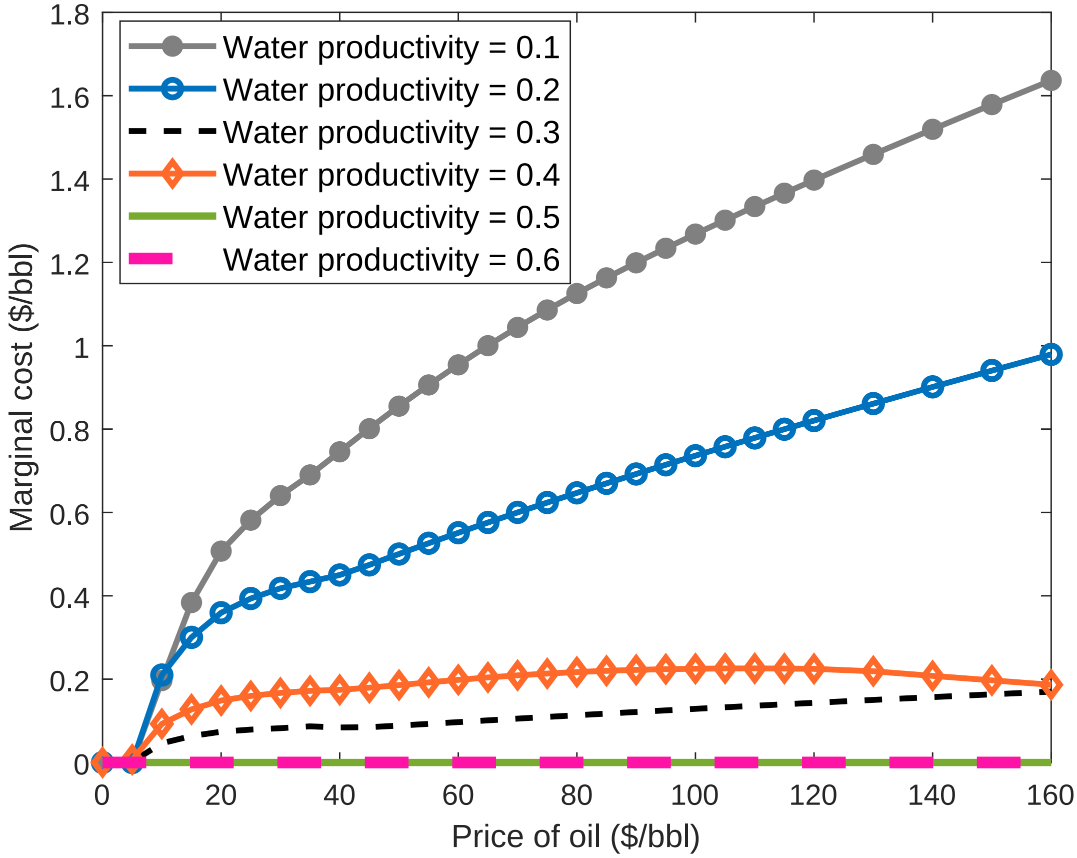

Figure 2.18: The marginal costs when the river flow is in the green zone under different water productivity levels for scenario W_L (Water productivity is in barrels of bitumen/barrel of water. The marginal cost refers to the loss in value to the project on a $/barrel basis of an increase in water withdrawal restrictions as outlined in Section 2.6.2, page 33.)

2.9 Conclusions

An efficient policy is one which maximizes the total net benefits from the resource in question, which in this case is water in the Athabasca River. While the estimation of the environmental benefits of restrictions on water withdrawals is beyond the scope of this thesis, it is possible to get a handle on the costs of these restrictions to firms. Currently there is not much relevant research about the estimation of the costs. We fill this gap by estimating costs of water limits on the oil sands industry in Alberta.

The estimated marginal costs can be also seen as implied shadow prices of water for the firm and may be considered a minimum values required for the environmental benefits to justify the regulation. These estimates also provide a reference for further research about market based water conservation instruments, e.g. water pricing. The methodology and conclusions from this analysis can inform the regulation of other resource extraction projects.

Some key findings of this chapter are summarized below.

- Low cost of the regulations. The Phase 1 Water Management Framework does not impose a large cost on firms. But if changing climate resulted in drier conditions and/or if the Phase 1 regulations were made stricter, there would be a larger cost for firms. Nevertheless there appears to be scope for adopting stricter regulations if future research determines there is significant ecological benefit or benefit to other stakeholders in the area from doing so.

- Non-monotonic impact of increasing price volatility. It is well known in the literature that for a simple investment option, increased price volatility is likely to delay an investment. However with strict water regulations increased volatility initially reduces the critical price required to install storage, implying that the expected time for the investment is sooner. With high volatility the restrictions are more costly to the firm, making storage more valuable. In contrast, when water restrictions are lenient and are not binding on the firm, an increase in oil price volatility delays investment in water storage as per the normal effect.

- Delay in project abandonment. Stricter regulations on water withdrawals may cause a firm to delay the permanent abandonment of a project. This follows because water restrictions means it may take longer to extract the resource. For the hypothetical project examined in this chapter, there is no strong effect on the abandonment decision.

- Long run dynamic marginal cost curve. A long run dynamic marginal cost curve shows the impact of making water restrictions more restrictive when the firm has the option to install technology that limits the impact of the restrictions. Marginal cost is generally increasing with tighter restrictions, but the marginal cost falls as restrictions are relaxed over a particular range. This indicates the importance of examining a range of restrictions to allow for changing technology.

- Marginal cost depends on state variables. The marginal cost of stricter regulations depends on the values of key state such as the price of oil and the resource stock. The higher is the price of oil the higher is the marginal cost of restrictions. The marginal cost is higher for larger resource levels.

Due to the heterogeneity of individual oil sands project, which in this thesis is reflected by different production capacities and remaining resource stocks, the marginal costs (shadow prices) for different projects are not the same. In addition according to Mannix et al. (2014), there is heterogeneity in the efficiency of water usage. Using a deterministic model Mannix et al. found that by allocating water preferentially to the firms with the highest productivity of water use, a higher level of efficiency is obtained. Hence an even allocation of water resources is not cost-efficient. It may be of interest to investigate optimal allocation mechanisms. Command and control regulations are not efficient in terms of allocating limited resources. A tradable permits scheme is a more promising approach for addressing this problem.

Chapter 3

Assessing the Trade off between Environmental Objectives and Economic Cost: a Study of the Phase 2 Water Management Rules for Oil Sands Mining

3.1 Introduction

In Chapter 2, we examined the economic implications of the water constraint rules imposed by the Phase 1 Framework (Alberta and Canada 2007). In 2015, this framework was updated to the Phase 2 Framework as is described in Alberta (2015). The Phase 2 Framework was intended to improve upon the Phase 1 Framework through the use of more sound scientific evidence than had been possible for Phase 1. In developing the new regulations, the Phase 2 Framework Committee (P2FC) compared a number of different regulations in terms of a number of indicators of environmental and economic impact. In the end, the environmental indicator chosen for their analysis was the percentage reduction in wetted area around the Athabasca River. The chosen economic indicator was the cost of water storage required to maintain full production throughout the year, despite water withdrawal restrictions. The P2FC evaluated a number of alternative regulations based on the trade off between these two indicators. The P2FC presented its report to the Alberta government in 2010. After a few corrections were made to some of the technical analysis, a preferred regulatory option was chosen. The new regulations took effect in 2015.

Significant time and effort were put into developing the Phase 2 regulations, including detailed scientific research to determine the change in wetted area that would result from different regulations (Alberta 2015). However, the use of storage costs as the measure of economic cost is quite narrow, although easy to calculate. The full cost of the regulations to the industry is determined by the cost to oil sands firms of having to change their behaviour in response to the regulations. The need to install storage (or any other type of capital) is just one of those costs. Additional costs are incurred if firms have to cut back production in response to water restrictions, even in the presence of storage. As already discussed in Chapter 2, an estimate of the true cost of restrictions requires a detailed economic model of firm behavior. Whether the cost of storage is an adequate proxy for the true economic cost will depend on economic factors affecting the industry such as oil prices and costs, as well as environmental factors such as future river flow conditions. The P2FC report is based on deterministic projections of future river flows which ignores optimal decisions of oil sands firms in the face of uncertain river flows.

This chapter applies a fully dynamic stochastic optimal control model to investigate the economic costs due to the water management rules taking into consideration the optimal reaction by oil sands firms to changes in oil prices and expected river flows. When oil sands firms have the flexibility to determine production levels and the timing of investment in water storage, the economic cost will not be purely determined by the storage volume, but rather by the full cost of changes made by firms in response to water restrictions. In this chapter, we contribute the current literature by investigating this economic cost in a more rigorous manner and consider whether this will result in different recommendations than those of the P2FC report. In a broader context we seek to draw some conclusions about those factors which have an important effect on economic cost of regulations. The remainder of the chapter is organized as follows. Section 3.2 provides a detailed overview of the Phase 2 Framework and compares it to Phase 1. Section 3.3 describes the key assumptions for the analysis as well as parameter estimates. Section 3.4 characterizes alternative river flow states. Section 3.5 presents the environmental benefits of different water restriction rules. Section 3.6 presents the results and Section 3.7 provides concluding comments.

3.2 Overview of the Phase 2 Framework management rule

3.2.1 Phase 2 Framework Description and Comparison With Phase 1

We can recall from Chapter 2 that in the Phase 1 Framework, the river flow condition was classified into three zones based on water flow levels. The criterion for being in each zone varied depending on the week. Similarly, in the Phase 2 Framework, in different seasons, the river flow condition is classified into different numbers of zones according to different thresholds or triggers in river flows. A specific water withdrawal limit is stipulated for each zone as defined by the river flow triggers which vary by season. The division of zones according to river flow triggers and the corresponding water withdrawal limits for Phase 2 are exhibited in Table 3.1.

Table 3.1: Weekly Flow Triggers and Cumulative Water Use Limits On the Lower Athabasca River for Oil Sands Operations

Mid Winter (Weeks 1-15)		**Early Spring (Weeks 16-18)**		**Late Spring (Weeks 19-23)**		**Summer/Fall (Weeks 24-43)**		**Early Winter (Weeks 44-52)**	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>270	16							>200	16
150∼270	6% of RF	>98.6	16	>102.6	20	>111.6	29	150∼200	8% of RF
91.6∼150	9							94.6∼150	12
87∼91.6	RF-82.6	87∼98.6	RF-82.6	87∼102.6	RF-82.6	87∼111.6	RF-82.6	87∼94.6	RF-82.6
<87	4.4	<87	4.4	<87	4.4	<87	4.4	<87	4.4

1. “RF” stands for “the river’s flow”;
2. “Limits” means “cumulative water withdrawal limits”;
3. “RF Triggers” and “Limits” are both measured in cubic metres per second.

The river flow triggers specified in Table 3.1 for 5 seasons can be summarized by defining 10 different zones as specified in Table 3.2. Zone 1 represents the driest condition while zone 10 corresponds to the wettest. The weekly water withdrawal limits for different zones are listed in Table B.3 in Appendix. Table B.3 fully characterizes the constraints of Table 3.1. Different from the 3 zones in Chapter 2, the definition of each river flow zone is constant over time. For example, regardless of the season, according to the Phase

2 Framework, as long as the river flow rate is in the interval of [200,270), the zone is defined as 9. While in the Phase 1 Framework, when the river flow falls in the interval of [200,270), its zone classification depends on the particular week of the year. Figure 3.1 exhibits the mapping relationship between the 10 zones of the Phase 2 Framework and the 3 zones of the Phase 1 Framework by representing each of the 10 zones in terms of the 3 zones. We also graph the weekly water withdrawal limits triggered by different river flow thresholds stipulated in the two frameworks in a single picture in Figure 3.2. Figure 3.1 and Figure 3.2 jointly reflect the following changes made in the updated framework.

- Generally speaking, the constraints on water withdrawals in all seasons become tighter.
- For some extremely dry situations in winter, which is rare, the constraints turn out to be eased.
- There is a maximum withdrawal limit ($29m^3/s$)[1] as assumed by the P2FC in its report[2] even in the water abundant seasons.

Table 3.2: The Ranges of Weekly Flow for 10 River Flow Zones of the Phase 2 Framework

Zone	1	2	3	4	5	6	7	8	9	10
Weekly flow (m^3/s)	$(-\infty,87)$	[87,91.6)	[91.6,94.6)	[94.6,98.6)	[98.6,102.6)	[102.6,111.6)	[111.6,150)	[150,200)	[200,270)	$[270,+\infty)$

3.2.2 The Development of the Phase 2 Framework

The rule set specified in the Phase 2 Framework was an update of a prototype rule set recommended by the Phase 2 Framework Committee ("P2FC") as described in Ohlson et al. (2010). The update corrects some mistakes in the original recommendations. Ohlson et al. (2010) provides the detailed process for deciding on the recommended rule set. The P2FC began with a collection of hundreds of candidate rule sets. Those rule sets laid down different combinations of constraint trigger thresholds and the corresponding triggered water

[1] It is reflected in Figure 3.2 by imposing a withdrawal limit of 147 million barrels per week from the 24^{th} week to the 43^{rd} week. It follows from the fact that weekly withdrawal limit = 7 days/week × 24 hours/day × 3600 seconds/hour × 29 m^3/second × 8.3864 barrels/m^3 = 147 million barrels/week.

[2] The P2FC assumed that the withdrawal amount has an upper limit determined by the physical, or technical, limit of intake capacity, such as the diameters of the water pump pipes and delivery pipes.

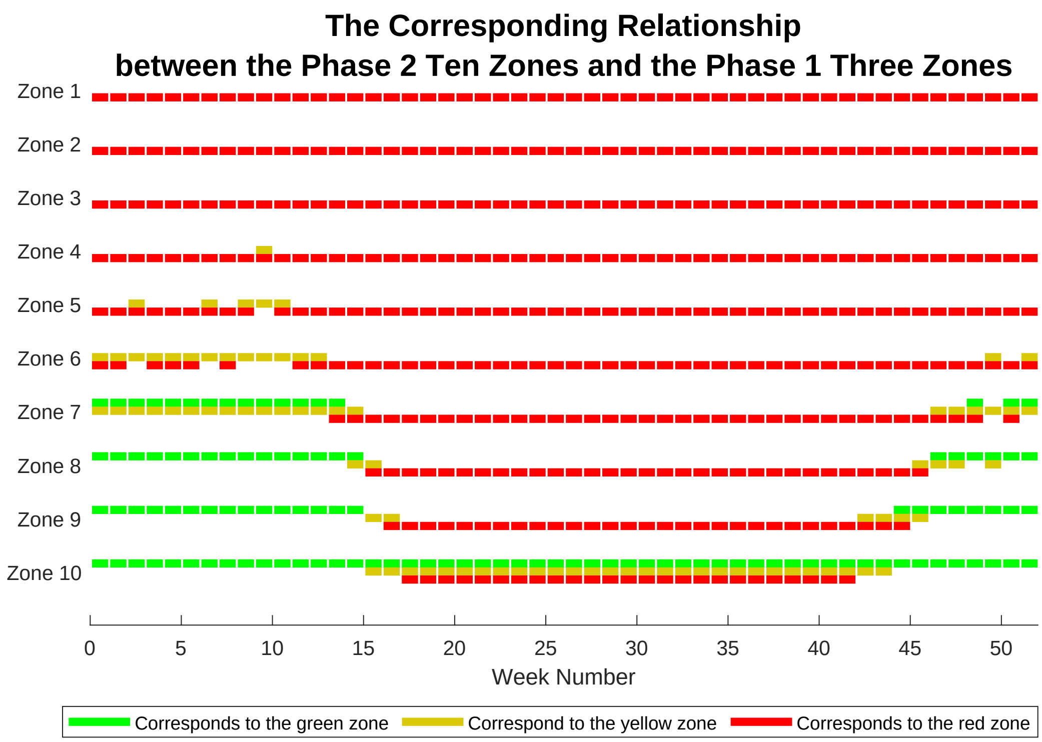

Figure 3.1: The corresponding relationship between the Phase 2 ten zones and the Phase 1 three zones

Figure 3.2: Comparison of the Phase 1 and 2 Frameworks in terms of the cumulative weekly water withdrawal limits on the Lower Athabasca River for oil sands operations

withdrawal limits. To select among all competing rule sets, the P2FC developed a spreadsheet tool named “Flow Calculator” to examine the environmental and economic impacts of each rule set. Using each of the rule sets, the predicted water required by the oil sands mining projects and the projected river flow condition as the inputs to the Flow Calculator, the P2FC obtained the impacts on a series of environmental criteria as well as the water storage required for the oil sands industry to maintain water availability at $16\text{m}^3/\text{s}$. This represents water demands by the oil sands mining industry at a projected production level 240% higher than the current level. In the end the chosen environmental criterion was the percentage reduction in wetted area under the different possible regulations. Then the P2FC created what they referred to as an efficient frontier to compare the impacts of all the alternative rule sets. The environmental and economic criteria were examined for three different (deterministic) river flow scenarios (hereinafter referred to as “P2FC scenarios” to differentiate them from the river flow scenarios that we will analyze in this chapter) shown in Table 3.3.

Table 3.3: The P2FC Scenarios of the Athabasca River Flows

Scenarios	Description
P2FC scenario 1A	**50 year historical case**: The historical river flows of the 1958 to 2007 period were projected to recur over the next 50 years.
P2FC scenario 1B	**1 in 100 year low flow case**: The historical river flows from 1958 to 2007 are used, except that the winter river flows in the driest year are replaced by river flow data for a 1 in 100 year low flow rate.
P2FC scenario 1C	**1 in 200 year low flow case**: The historical river flows from 1958 to 2007 are used, except that the winter river flows in the driest year are replaced by river flow data for a 1 in 200 year low flow rate.

There were several outstanding alternatives judged to be superior to the others examined, denoted as “Alt 19”, ‘’Alt 20”, “Alt 21”, “Alt 22”, “Option A”, and “Option H”[3], for which the efficiencies are plotted in Figure 3.3[4]. The different alternatives vary in terms of the maximum withdrawal rates and the profile of restrictions over a year, as presented in Table B.3 through B.9 in Appendix B.

Each of the diagrams in Figure 3.3 shows an efficient frontier reflecting the trade off relationship between two impacts: percentage reduction in wetted area as the indicator of environmental cost and water storage cost as an indicator of economic cost. The horizontal axis measures the water storage capacity required to be built and the corresponding cost. The first row of the horizontal axis label shows the water storage capacity. The second row shows the economic cost. The vertical axis measures the reduction in winter wetted area after water withdrawals compared to before withdrawals. Ohlson et al. (2010) argues that the reduction in winter wetted area is a good proxy for other environmental criteria so that it is a proper measure of ecosystem response. From either direction, the closer the point is to the origin, the better the rules set is, because it reflects less impact on the environment and less economic cost to the oil sands sector. However because the horizontal and vertical axes are not measured in comparable units, there is no obvious way to choose the best rule set amongst the six alternatives. In the end, the P2FC committee recommended Option H because it represented a smoother path for restrictions from week to week. However, the final Phase 2 regulation that was adopted was a variant of Option H, as some adjustments and corrections were made to the final report.

3.2.3 Challenges To the Phase 2 Framework

Unsurprisingly, not all interested parties were happy with the recommendations in the P2FC report. Due to the time constraint for the report’s completion, when it was submitted to Alberta Government, there were still some controversies about the extent of environmental protection and the rights of First Nations, to name just two examples. Furthermore, one of the dry climate conditions used by P2FC for climate change sensitivity analysis had some errors and some corrections had to be made. The Phase 2 Framework, including corrections made, is described in Alberta (2015). One year after the Phase 2

[3] The weekly flow triggers and cumulative water use limits, the ranges of weekly flow for river flow zones, the water withdrawal limits for each week corresponding to different zones, the corresponding relationship between the Phase 2 zones and the three zones of the Phase 1 Framework, and a comparison to the Phase 1 Framework in terms of the cumulative weekly water withdrawal limits for “Alt 19”, ‘’Alt 20”, “Alt 21”, “Alt 22”, “Option A”, and “Option H” (their counterparts in the Phase 2 Framework are Table 3.1, Table 3.2, Table B.3, Figure 3.1 and Figure 3.2.) can be found in Appendix B.

[4] The Phase 1 rule set is also depicted in the graphs for reference.

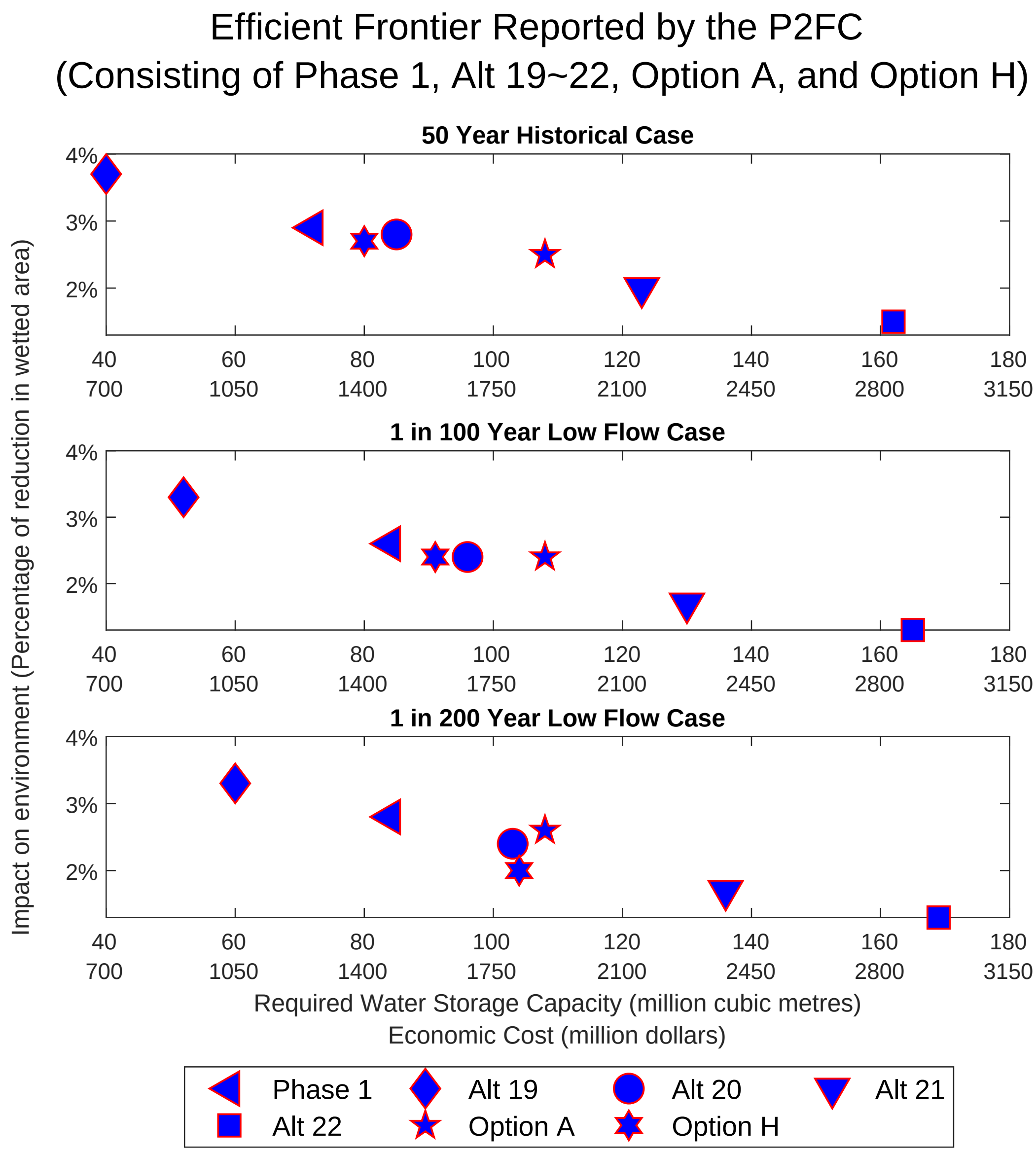

Figure 3.3: The efficient frontier according to the P2FC's study about Alternative Water Management rule sets

Framework took effect, Leong & Donner (2016) used different climate forecasting models for simulating the river flows and obtained different river flow simulation paths which changed the trade off relationship between environmental impacts and storage costs.

Most of the controversies surrounding the Phase 2 Framework came from an environmental perspective. The P2FC also mainly focused on the environment impacts of the alternative rule sets. When the P2FC evaluated the alternatives, it assumed that the oil sands industry had a total water demand of 16 m^3/s, which implies a high rate of oil sands production based on a forecast of peak oil sands production between 2015 and 2025. In addition, P2FC's study was based on the belief that the oil sands industry would build the water storage needed to maintain the 16 m^3/s water supply. Leong & Donner (2016) raised the concern about the P2FC's assumption of a constant high rate of the oil sands production over time. In response they reexamined the environmental and industry cost trade off adopting several exogenously picked levels of the production.

Taking the P2FC's assumed production capacity as given, actual production will be affected by available water as well as the oil price. The P2FC assumed that the cost of reducing production due to water restrictions is always greater than the cost of building necessary water storage facilities. In fact, the loss due to the water constraint rules has two components: the cost to install and maintain the water storage facility and the loss because of any possible production cutbacks. Both these components should be taken into account when analyzing the trade off between environmental constraints and economic cost.

Leong & Donner (2016) used the 2012 WTI oil price to estimate the costs of reducing production and compare to the costs of establishing specific capacities for water storage. They conclude that when the water management rule prioritizes the environment by applying stricter water withdrawal rules during times of water short age, the cost of adding water storage capacity will be greater than the cost of reducing production; while when the rule prioritizes industry, the cost of water storage will be less than the cost of reducing production.

3.3 Key Assumptions and Parameter Estimates

In Chapter 2 Section 2.4, we fully described a stochastic optimal control model to evaluate the expected value of an oil sands operation under various water restrictions. In this chapter we use the same model to determine optimal decisions and valuation of oil sands operations for the suite of regulations proposed in the P2FC report. We adopt the assumptions of the P2FC report and then vary key assumptions one by one to isolate which factors have the largest impact on the optimal choice of water regulations. Below we describe the assumptions which are the focus of our analysis.

- **Assumption 1: Stochastic or deterministic oil prices.** In using storage cost as the economic indicator, the P2FC ignored the impact of different water restrictions on firm revenues. The possible impact of oil prices on the cost of water restrictions was not considered. As is well known in the literature, volatile output prices provide an incentive to delay investments, reflecting a positive option value of waiting to make a decision. This option value will affect decisions about storage investment. In addition, volatile output prices may affect the economic cost of water restrictions which reduce a firm's flexibility to respond quickly to changing oil prices. Introducing the stochastic oil price model might change the relative ranking of different regulatory rules.

- **Assumption 2: Oil sands production.** Regardless of which alternative rule set is applied, the P2FC analysis assumes that the total annual water demand by the oil sands industry is 16 m^3/s. This represented a plateau in mean water demand projected in the P2FC's Growth Case Scenario. The reasoning by the P2FC committee was that the alternative rules should be tested against the largest projected demand from the industry (Alberta 2015). Given the P2FC's forecast about average water productivity, the production of one barrel of bitumen requires 2.4 barrels of water. This water demand is consistent with industry production capacity of 1764 million barrels of oil per year.[5] The assumed production level is at a very high rate compared to current levels as is indicated in Table 3.4. To understand the economic cost of the regulations, optimal firm production responses to changing water restrictions needs to be considered. In addition, it is of interest to consider the cost of restrictions given current productive capacity of the industry.

  The constant high rate of annual production requires current remaining established reserves of 88 billion barrels (see Table 3.4). This is much higher than current estimates of remaining reserves. These reserves are depleted at the end of the 50 year period of analysis. To be consistent with the P2FC report we also assume a 50 lifetime of production. However, in the optimal control model, operations may be shut in temporarily if oil prices are unfavourable. Hence in our modelling exercise there is a possibility that not all reserves will be used up, if future oil prices turn out to be

[5] The annual water demand can be derived as 16 $m^3/s \times 3600$ s/hour $\times$ 24 hours/day $\times$ 365 days/year $= 504\,576\,000$ m^3/year. Given 1 $m^3 = 8.3864$ barrels for water, the demand of water is converted to 4 231 576 166 barrels per year. This amount of water can be used for producing $4\,231\,576\,166/2.4 = 1764$ million barrels of bitumen.

Table 3.4: The Actual and Projected Levels of Cumulative Remaining Reserves and Production Capacity of the Mining Firms In the Lower Athabasca River Region

	Remaining established reserves (million barrels)	Production capacity (million barrels/year)
Actual level in 2015	19 197	521
Projected level by P2FC	88 200	1 764

Note:

– The projected production capacity is derived from the water demand of 16 m^3/s. The projected remaining established reserves are derived by multiplying the projected production capacity with 50, which is the time span that is examined by the P2FC.

– Source of the actual level of the remaining established reserves: Alberta's Energy Reserves 2014 and Supply/Demand Outlook 2015-2024 (Alberta Energy Regulator 2015)

– Source of the actual level of the production capacity: Alberta Oil Sands Industry Quarterly Update (spring 2015) (Economic Development and Trade 2015)

depressed for long periods of time. This is one source of any differences in the P2FC report and our own analysis.

- **Assumption 3: Operating costs for the water storage facilities.** The P2FC report considered only the capital costs of storage. Annual operating costs for storage maintenance can be significant and are affected by decisions about storage usage. Hence we consider storage operating costs in our analysis.

- **Assumption 4: Forecast river flow conditions.** The P2FC report examined three deterministic river flow scenarios for the next 50 years, as is described in Table 3.3. When examining optimal decisions it may be more realistic to consider uncertain future river conditions by modelling river flow as a stochastic process, which oil sands operators take into account in making investment and operating decisions. In our analysis, we examine several possibilities for river flow conditions which are summarized below.

    - Scenario 1A: Deterministic river flows according to the P2FC report, using historical data for the 1958 to 2007 period.
    - River flows modelled as a Poisson process, as described in Equations 2.5 in Chapter 2. Several different hazard rate matrices are examined based on:
        - Scenario 2A: river flow data from 1958 to 2007
        - Scenario 2B: river flow data of 2015

- Scenario 2C: river flow rate stays at the lowest weekly level during the period of 1957 to 2017
- Scenario 2D: river flow rate is always in the driest zones.

Scenarios 2A through 2D allow us to portray the impact on economic cost of extreme river flow conditions. This is of interest since with changing climate, we do not expect a repeat of the past five decades of river flow conditions. Recall Figure 2.3, in which the solid dark curve reflects the 2015 river flow condition. Other than some late winter and early spring weeks (from week 5 to week 19), the flow rate is generally lower than the historical average condition. The specific hazard rate matrices can be found in Table B.10 of Appendix B.

A problem with this step of the analysis is that the environmental benefit from applying the alternative rules sets was calculated by the P2FC based on historical river flow data as described in Table 3.3. By using new river flow assumptions (scenarios 2A through 2D), the environmental benefits will not be the same as the P2FC estimates. Although we have no environmental benefit estimates that are precisely consistent with our scenarios 2A through 2D, we presume that the environmental benefit estimates for the P2FC scenario 1A provide a lower bound. We can compare this with the environmental benefits from P2FC scenarios 1B and 1C, which represent drier conditions. However, the P2FC did not provide data to define these scenarios so we have no way of knowing how these compare with our scenarios 2A through 2D. In summary, we will use the P2FC environmental benefits estimates in our analysis but we acknowledge these are not entirely consistent with river flow conditions assumed in scenarios 2A through 2D.

- **Assumption 5: Water storage capacity.** The P2FC report assumed a very high rate of storage capacity would be installed by firms, so that production could always be maintained at full capacity even in the face of water restrictions. Table 3.5 lists the water storage capacities required under different competing rule sets for the cases used by the P2FC, which were described in Table 3.3. In our analysis we examine the following alternatives:
  - the P2FC storage assumption
  - storage capacity that can maintain four weeks of production
  - an optimal level of storage capacity.

Table 3.5: Water Storage Capacity Assumed by P2FC (in Million Barrels)

Alternative rule sets	Phase 1	Alt 19	Alt 20	Alt 21	Alt 22	Option A	Option H
P2FC scenario 1A: 50 year historical flow	604	335	713	1032	1359	906	671
P2FC scenario 1B: 1 in 100 year low flow case	704	436	805	1090	1384	906	763
P2FC scenario 1C: 1 in 200 year low flow case	704	503	864	1141	1417	906	872

- **Assumption 6: Remaining reserves level.** The P2FC report analysis assumes the whole oil sands mining industry will last 50 years with an annual production capacity of 1 764 million barrels. This implies 88.2 billion barrels of recoverable reserves. However, the actual recoverable reserves are less than half of this level.[6] This overestimation of reserves will affect the optimal operations including the decision to install water storage, and will thus affect the economic cost of the regulation. In our analysis we apply the actual remaining established reserves to examine the problem.

Our goal is to determine whether the conclusions of the stochastic optimal control model will differ from those of the P2FC report and to determine the cause of any difference. To accomplish this we examine the above assumptions one-by-one in a systematic manner and compare with the P2FC results. This results in 8 different cases. A summary of the assumptions used in each of the eight cases is provided in Table 3.6. For each case, the table highlights the particular assumptions used. Other than these six assumptions listed above, all other assumptions are based on the P2FC report.

Tables 3.7 and 3.8 summarize relevant parameter values for those cases which use the P2FC assumption regarding storage capacity (cases 1 through 4 in Table 3.6), reserves and productive capacity. Parameter values relevant for cases with differing project reserves and storage capacity (cases 5 through 8) are provided in Table B.11 and B.12 in Appendix B. The shaded rows in these tables are the parameters that remain unchanged across all cases.

[6] According to Alberta Energy Regulator (2015), the remaining established reserves for the Alberta oil sands is 166 billion barrels, 20% of which is recoverable by mining. (In Alberta Energy Regulator (2015), the definition of established reserves is: "those reserves recoverable under current technology and present and anticipated economic conditions, specifically proved by drilling, testing, or production, plus that judgement portion of contiguous recoverable reserves that are interpreted to exist, from geological, geophysical or similar information, with reasonable certainty". Initial established reserves are defined as "established reserves prior to the deduction of any production". Remaining established reserves are defined as "initial established reserves less cumulative production". The currently estimated remaining established reserves could well be revised upward and move closer to what the P2FC committee assumed.)

Table 3.6: The Comparison of the Assumptions Adopted by the P2FC and This Chapter

	Assumption 1	Assumption 2	Assumption 3	Assumption 4	Assumption 5	Assumption 6
P2FC	**Price is not relevant** Stochastic price	**High productivity** Current productivity	**No maintenance costs** With maintenance costs	**Historical river data** **1 in 100 case occurs** **1 in 200 case occurs** Historical hazard rates 2015 hazard rates Historical lowest flows State 1 flows	**Full water storage** 4 weeks storage Optimal storage	**High reserves level** Moderate reserves level
Case 1	Price is not relevant **Stochastic price**	**High productivity** Current productivity	**No maintenance costs** With maintenance costs	**Historical river data** 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates 2015 hazard rates Historical lowest flows State 1 flows	**Full water storage** 4 weeks storage Optimal storage	**High reserves level** Moderate reserves level
Case 2	Price is not relevant **Stochastic price**	**High productivity** Current productivity	No maintenance costs **With maintenance costs**	**Historical river data** 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates 2015 hazard rates Historical lowest flows State 1 flows	**Full water storage** 4 weeks storage Optimal storage	**High reserves level** Moderate reserves level
Case 3	Price is not relevant **Stochastic price**	**High productivity** Current productivity	No maintenance costs **With maintenance costs**	Historical river data 1 in 100 case occurs 1 in 200 case occurs **Historical hazard rates** 2015 hazard rates Historical lowest flows State 1 flows	**Full water storage** 4 weeks storage Optimal storage	**High reserves level** Moderate reserves level
Case 4	Price is not relevant **Stochastic price**	**High productivity** Current productivity	No maintenance costs **With maintenance costs**	Historical river data 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates **2015 hazard rates** **Historical lowest flows** **State 1 flows**	**Full water storage** 4 weeks storage Optimal storage	**High reserves level** Moderate reserves level
Case 5	Price is not relevant **Stochastic price**	**High productivity** Current productivity	No maintenance costs **With maintenance costs**	Historical river data 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates **2015 hazard rates** **Historical lowest flows** **State 1 flows**	**Full water storage** 4 weeks storage Optimal storage	High reserves level **Moderate reserves level**
Case 6	Price is not relevant **Stochastic price**	**High productivity** Current productivity	No maintenance costs **With maintenance costs**	Historical river data 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates **2015 hazard rates** **Historical lowest flows** **State 1 flows**	Full water storage **4 weeks storage** Optimal storage	High reserves level **Moderate reserves level**
Case 7	Price is not relevant **Stochastic price**	**High productivity** Current productivity	No maintenance costs **With maintenance costs**	Historical river data 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates **2015 hazard rates** **Historical lowest flows** **State 1 flows**	Full water storage 4 weeks storage **Optimal storage**	High reserves level **Moderate reserves level**
Case 8	Price is not relevant **Stochastic price**	High productivity **Current productivity**	No maintenance costs **With maintenance costs**	Historical river data 1 in 100 case occurs 1 in 200 case occurs Historical hazard rates **2015 hazard rates** **Historical lowest flows** **State 1 flows**	Full water storage **4 weeks storage** Optimal storage	High reserves level **Moderate reserves level**

Table 3.7: Parameter Values for Cases 1 To 4

Parameter	Description	Reference	Assigned Value	Source	Cases
	Extraction method		Surface mining	$***$	
$T - t_0$	Remaining lifespan of the project (years)	Equation (2.7)	50	$*$	1,2,3,4
$\bar{q}$	Production capacity (million barrels/year)	Equation (2.1)	1764	$*$	1,2,3,4
s_0	Remaining established reserves (million barrels)	Equation (2.7)	88 200	$*$	1,2,3,4
η	Productivity of water (barrels of bitumen/barrel of water)	Equation (2.1)	0.4	$**$	1,2,3,4
$\bar{W}$	Water withdrawal constraint (million barrels/week)	Equation (2.4)	It is equal to $\bar{W}_c$ that is specified in Table 3.8		1,2,3,4
C	The construction cost of the water storage (million dollar)	Equation (2.9)	Specified in Table 3.8	$*$	1,2,3,4
$\bar{I}$	Water storage capacity (million barrels)	Equation (2.3)	Specified in Table 3.8	$*$	1,2,3,4
c_f^s	The fixed cost of water storage (million \$/year)	Equation (2.9)	Specified in Table 3.8	$*$	2,3,4
c_v^s	The variable cost of water storage (\$/barrel)	Equation (2.9)	0.0028	$*$	2,3,4
	Carbon emissions (tonnes/barrel)	Equation (2.9)	0.091	$**$	
$c_{v_e}^o$	Energy variable operating cost (% of the WTI price)	Equation (2.9)	1.62	$**$	
$c_{v_{ne}}^o$	Non-energy variable operating cost (\$/barrel)	Equation (2.9)	7.98	$**$	
c_f^o	Fixed operating cost (million \$/year)	Equation (2.9)	11515	$**$	1,2,3,4
c_s	Sustaining capital cost (million \$/year)	Equation (2.9)	11466	$***$	1,2,3,4
	Income tax rate (%)	Equation (2.9)	25	$***$	
	Carbon tax (\$/tonne)	Equation (2.9)	40	$***$	
C_m	Mothball cost (million \$)	Table 2.1	0	$*$	
C_{re}	Reactivating cost (million \$)	Table 2.1	0	$*$	
C_{large}	A large number to prevent stage switching (million \$)	Page 24	10^9	$*$	
C_r	Abandonment cost (million \$)	Equation (2.9)	6811	$*$	1,2,3,4
ϵ	Speed of reverting to the mean log oil price	Equation (2.8)	0.14	$***$	
μ	Long run mean log oil price	Equation (2.8)	4.59	$***$	
σ	Volatility of oil prices	Equation (2.8)	0.31	$***$	
	River flows		Historical data from 1957 to 2017	$***$	1,2
			Hazard risk matrices according to 2015 data	$***$	3,4
r	Risk free interest rate	Equation (2.16)	0.02	$*$	

The column "Reliablility" indicates the reliablity of the given parameter values. $***$ means these values are publicly available or are estimated from empirical evidence. $**$ means these values are derived according to AOSIQU, Alberta Energy Regulator (2015), or CERI's report ((Millington & Murillo 2015) . $*$ means these values are assumed by referring to miscellaneous sources which are documented in the text.

Table 3.8: Parameter Values for Alternative Rule Sets

Parameter	k	$\bar{W}_c$	P2FC scenario 1A: 50 year historical flow			P2FC scenario 1B: 1 in 100 year low flow case			P2FC scenario 1C: 1 in 200 year low flow case		
			C	$\bar{I}$	c_f^s	C	$\bar{I}$	c_f^s	C	$\bar{I}$	c_f^s
Description	River flow zone	Cumulative water withdrawal limit									
Equation Reference	(2.4)		(2.9)	(2.3)	(2.9)	(2.9)	(2.3)	(2.9)	(2.9)	(2.3)	(2.9)
Phase 1	1,2,3	Refer to Table 2.3	1177	604	63	1374	704	74	1374	704	74
Alt 19	1,2,3	Refer to Table B.4	654	335	35	850	436	46	981	503	53
Alt 20	1,2	Refer to Table B.5	1390	713	75	1570	805	85	1684	864	91
Alt 21	1,2	Refer to Table B.6	2012	1032	108	2126	1090	114	2224	1141	120
Alt 22	1,2	Refer to Table B.7	2649	1359	143	2698	1384	145	2764	1417	149
Option A	1,2,3	Refer to Table B.8	1766	906	95	1766	906	95	1766	906	95
Option H	1,2,. . .,5	Refer to Table B.9	1308	671	70	1488	763	80	1701	872.19	92

3.4 Specification of River Flow States

Since we need to compare the costs of different rule sets, we define a collection of river flow states ranging from 1 to 15, with state 15 having the most abundant water flows and state 1 the least abundant water flows. The term "state" is used in order to differentiate it from "zones" defined for each rule set. Recall that for a given rule set, zones define the weekly water withdrawal limitation for each river flow state. For a specific rule set, each river flow state maps to one zone. The mapping between states and zones is shown in Table 3.9. Note that this table shows the Phase 1 rule set, the main alternatives considered in the P2FC report as well as the final regulations adopted for Phase 2, which was a corrected version of Alternative H. This definition of states serves to unify the classification of river flow conditions across alternative rule sets, except for the Phase 1 rule set. This definition applies to the Phase 1 rule set only when the decision time is in the first week of a year. To devise a classification system so that the Phase 1 and Phase 2 rules sets are fully comparable requires 106 states. We have not done so as the computational complexity is increased considerably, for not a large reward in terms of our analysis. For a comparison of Phase 1 and Phase 2 regulations we therefore set the time to evaluate the economic costs at the first week of a year so that the three zones defined in the Phase 1 Framework approximately fit into the 15-state system. It turns out that if other weeks of a year is considered to be the decision making time, the result won't change significantly. Note however that the zones for each rule set alternative apply different water withdrawal limits. So although zone 1 always refers to the strictest limitation, the definition of each zone varies across the alternative rule sets and across the weeks of the year. The specific weekly water withdrawal limits are laid out in Table B.4 through Table B.9.

3.5 Environmental Benefits

According to the P2FC report, when no water management rules are applied, the percentage reduction in wetted area is 4.5% compared to the case when there are zero water withdrawals by the oil sand industry. The benefit to the environment for each alternative rule set can therefore be reflected by the difference between the percentage reduction in wetted area due to the rule set and 4.5%. The benefit to the environment in terms of saved wetted area for three river flow scenarios as defined by the P2FC and for each alternative rule set is shown in Table 3.10.

We note that except for cases 1 through 3, these P2FC scenarios are not the same as those used in our analysis of economic cost, but these environmental benefits estimates are

Table 3.9: The Mapping between River Flow States and River Flow Zones

<table>
<tr><th></th><th>RF Triggers</th><th>Phase 1</th><th>Alt 19</th><th>Alt 20</th><th>Alt 21</th><th>Alt 22</th><th>Option A</th><th>Option H</th><th>Phase 2</th></tr>
<tr><td>State 15</td><td>>355</td><td rowspan="7">Green zone</td><td rowspan="6">Zone 3</td><td rowspan="4">Zone 2</td><td rowspan="2">Zone 2</td><td>Zone 2</td><td rowspan="2">Zone 3</td><td rowspan="2">Zone 5</td><td rowspan="2">Zone 10</td></tr>
<tr><td>State 14</td><td>270∼355</td><td rowspan="14">Zone 1</td></tr>
<tr><td>State 13</td><td>200∼270</td><td rowspan="13">Zone 1</td><td rowspan="5">Zone 2</td><td>Zone 4</td><td>Zone 9</td></tr>
<tr><td>State 12</td><td>185∼200</td><td rowspan="2">Zone 3</td><td rowspan="2">Zone 8</td></tr>
<tr><td>State 11</td><td>150∼185</td><td rowspan="11">Zone 1</td></tr>
<tr><td>State 10</td><td>140∼150</td><td rowspan="9">Zone 2</td><td rowspan="3">Zone 7</td></tr>
<tr><td>State 9</td><td>133∼140</td><td rowspan="3">Zone 2</td></tr>
<tr><td>State 8</td><td>111.6∼133</td><td rowspan="3">Yellow zone</td><td rowspan="8">Zone 1</td></tr>
<tr><td>State 7</td><td>110∼111.6</td><td rowspan="2">Zone 6</td></tr>
<tr><td>State 6</td><td>102.6∼110</td><td rowspan="6">Zone 1</td></tr>
<tr><td>State 5</td><td>98.6∼102.6</td><td rowspan="5">Red zone</td><td>Zone 5</td></tr>
<tr><td>State 4</td><td>94.6∼98.6</td><td>Zone 4</td></tr>
<tr><td>State 3</td><td>91.6∼94.6</td><td>Zone 3</td></tr>
<tr><td>State 2</td><td>87∼91.6</td><td>Zone 2</td></tr>
<tr><td>State 1</td><td><87</td><td>Zone 1</td><td>Zone 1</td></tr>
</table>

1. "RF" stands for "the river's flow";
2. "RF Triggers" are measured in cubic metres per second.

the only ones available. We use the environmental benefits for the P2FC scenarios with the full economic cost estimated in the subsequent sections for our cost-effectiveness analysis. However we must acknowledge that the environmental benefits of the scenarios we have defined may differ from the P2FC estimates. For the purposes of the cost-effectiveness analysis, we compare the P2FC scenario 1A with our scenario 2B, the P2FC scenario 1B with our scenario 2C and the P2FC scenario 1C with our scenario 2D. In general, starting from case 4, the scenarios we have defined are drier than the ones used by the P2FC, so we would expect these benefit estimates to be a lower bound.

3.6 Results

The economic cost of a particular rule set is defined as the difference in expected value of the oil sands operations under that rule set compared to the case of no water restrictions. Given a specific rule set and a river flow scenario, the economic costs will vary with the model state variables which are the current oil price, the current reserves, and the current river flow condition. To keep the discussion concise, we present results for current reserves at the full level. It turns out that at other levels of the reserves, the results are not

Table 3.10: Environmental Benefits of Alternative Rule Sets (Percentage of Increase In Wetted Area)

Alternative rule sets	Phase 1	Alt 19	Alt 20	Alt 21	Alt 22	Option A	Option H
Percentage of increase in wetted area (P2FC scenario 1A: 50 year historical river flow)	1.6	0.8	1.7	2.5	3	2	1.8
Percentage of increase in wetted area (P2FC scenario 1B: 1 in 100 year low flow case)	1.9	1.2	2.1	2.8	3.2	2.1	2.1
Percentage of increase in wetted area (P2FC scenario 1C: 1 in 200 year low flow case)	1.7	1.2	2.1	2.8	3.2	1.9	2.5

Data from the P2FC report.

significantly different. We present results for a selection of different oil prices and current river flow conditions.

We can compare the cost-effectiveness of each alternative using the full costs to the oil sand industry and the data in Table 3.10. We will plot the environmental benefit of each alternative (vertical axis) versus the economic cost (horizontal axis). (Note that this differs from the presentation in Figure 3.3 in which the vertical axis showed the environmental cost of each alternative.)

3.6.1 Cases 1 To 3: Varying Price, Storage Cost, and River Flow Assumptions

To examine the impacts of introducing an optimal control approach on the selection of the optimal water management rule set, we consider the case where the river flow condition is the same as that assumed by the P2FC, i.e. the river flow condition in the future 50 years will reproduce the historical levels from 1958 to 2007. In cases 1 to 3, as in the P2FC report storage levels accommodate full production levels so that there is no cost from production cut backs. The P2FC considered only the capital costs for building the water storage facilities and did not consider the water maintenance cost for water storage. In Case 1 we consider optimal decisions of the firm given stochastic oil prices, but other assumptions are as in the P2FC report. In case 2 we add in the maintenance costs to see the effect on economic cost of restrictions. Then in case 3, we use the hazard rate matrices to replace the deterministic river flow data to see the effect of capturing uncertainty of river flow by a Poisson process.

The economic costs comparison for these three cases for different current reserves and current oil prices are depicted in Figure 3.4. All three graphs show a steep rise in economic cost as the oil prices rises from 0 to about \$35 per barrel; above \$35 costs are much less sensitive to oil prices. In all three diagrams Alt 19 appears to be the least costly while Alt 22 is the most costly. In Figure 3.5 we pick a combination of the current river flow state (i.e. state 10) and oil price (i.e. \$90/barrel) to create the cost-effectiveness graph. Other combinations of the current river flow state and oil price turn out to create very similar shapes of the cost-effectiveness graph. It is clear that any point in the environmental benefit - economic cost plane (or cost-effectiveness plane) is dominated by points which fall north-west of that point. All north-west points are more cost effective in that they achieve greater environmental benefit for the same or lower cost.

The results of case 1 can be seen by comparing the upper graphs in both Figure 3.3 and Figure 3.5. We see that Alt 19 is judged to be the least cost in both the P2FC report and in case 1 with an economic cost of close to \$700 million. Note that Alt 19 has the highest environmental cost (Figure 3.3), which implies the lowest environmental benefit (Figure 3.5). The ranking and magnitude of economic costs for the other rule sets is the same in the two figures. The results of case 1 show that introducing the stochastic oil price model does not change relative cost-effectiveness performance of the alternative rule sets. Option H, the preferred option in the P2FC report, is not rejected as a desirable alternative in the case 1 analysis. The main reason for this is that the assumed storage level is large enough that the water constraints are never binding, and with zero maintenance costs for storage, it is costless to use the storage. In this case, the only costs imposed by the water restrictions are the costs to build storage. Note that in the optimal control model, storage is installed only once prices is above a particular critical value. Our analysis shows that the critical price to install storage is below \$90 when reserve levels are still abundant. Hence at \$90 per barrel as the time zero price of oil in Figure 3.5, storage would be installed immediately. The magnitude of the costs vary to some extent when the current oil price changes. When the current oil price is less than \$35/barrel, the costs of the alternative rule sets range from between 500 and 2200 million dollars at low oil prices to between 650 and 2650 million dollars at \$35 per barrel. When the current oil price is over \$35/barrel, the costs of the alternatives do not change much as price increases further, and remain in the range roughly between 650 and 2650 million dollars. The reason why costs are lower at lower oil prices is that for some very low oil prices it is not economic to install storage and oil production may be shut in for significant periods of time.

When water storage maintenance costs are included in case 2, the economic costs of restrictions are increased. When the current oil price is less than \$35/barrel, the costs of all the alternatives range from between 1600 and 6650 million dollars to between 2000 and 8500 million dollars. When the current oil price is greater than \$35/barrel, the costs are quite flat in relation to oil prices, ranging between around 2000 and 8500 million

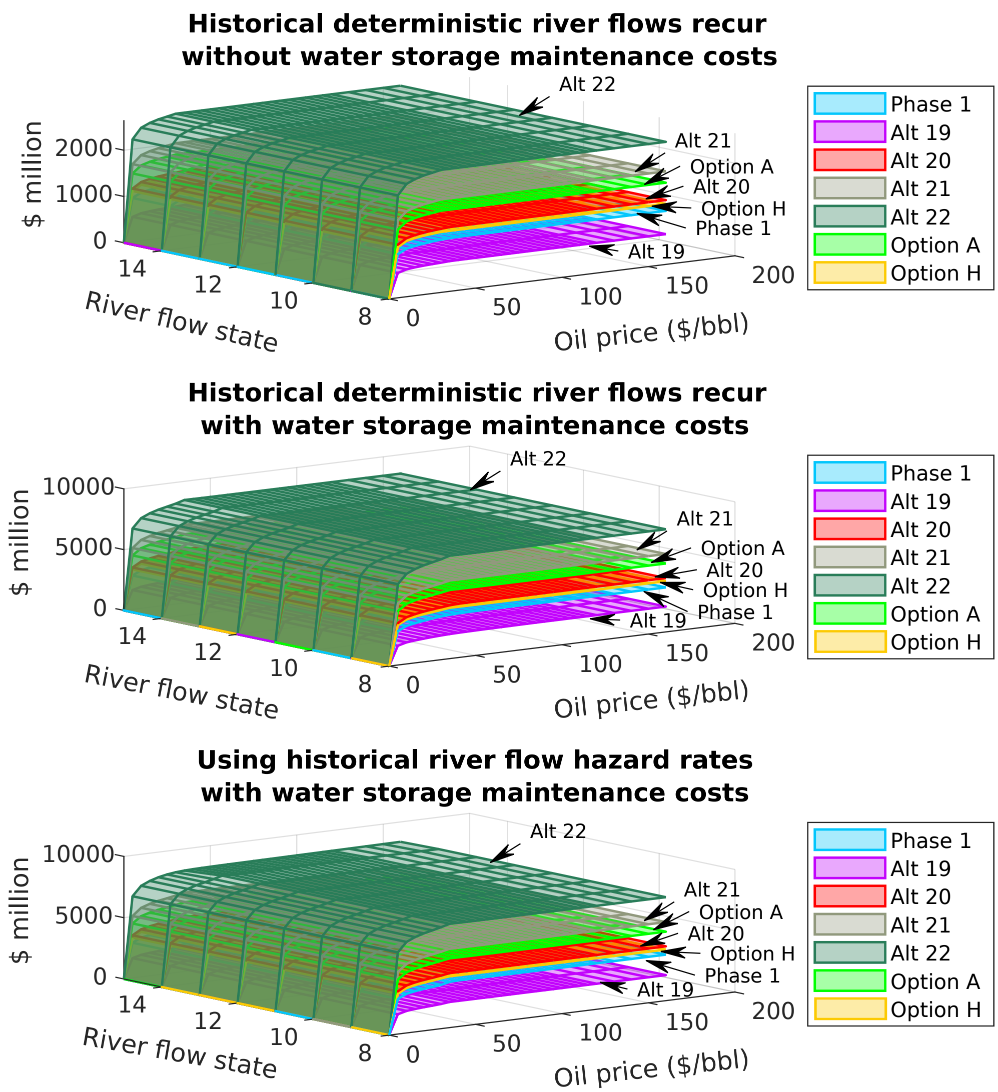

Figure 3.4: The economic cost of the oil sands industry due to the alternative rule sets for Cases 1 to 3. (Note that in this river flow condition, the river is always in states 8 to 15.)

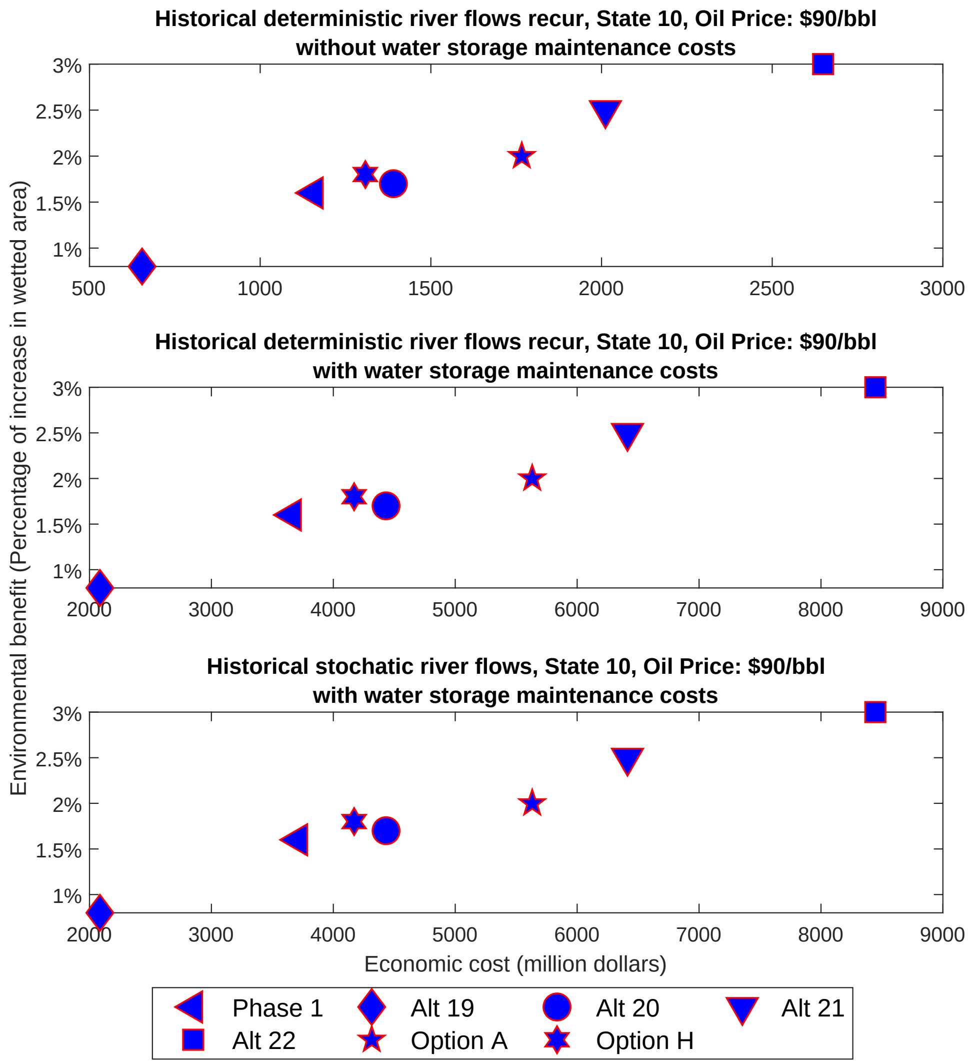

Figure 3.5: Representative cost-effectiveness graphs when the current river flow is in state 10 and the current oil price is $90/barrel for Cases 1 to 3

dollars. Nevertheless this amplification in the magnitude does not alter the relative cost-effectiveness performances of the alternatives. The selection of Option H cannot be rejected.

In case 3, when we use a Poisson process to describe the river flow condition, the result is very close to the result derived from using the deterministic river flow data set. The degree of uncertainty reflected in historical river flows, as described by the Poisson process, is not sufficient to significantly alter the economics of installing storage.

3.6.2 Case 4: Using the Hazard Rate Matrices Derived From Drier River Flow Data

In this case, we start to examine the problem under the 2015 river flow condition (referred to as "Scenario 2B"). We also investigate two drier scenarios: the weekly river flows are always at the historical lowest levels during the period 1957 to 2017 (referred to as "Scenario 2C"); the weekly river flow rate is always at the level of 86 m^3/s (which means always in state 1) (referred to as "Scenario 2D").

As mentioned before, the environmental benefits of the alternative rules sets given by the P2FC are based on the river flow inputs of the P2FC scenarios: the historical 50 years river flow condition, the 1 in 100 year dry case, and the 1 in 200 year dry case. For the three scenarios we examine, there is no available information about the environmental benefits. To proceed with our research, we just use the environmental benefit corresponding to the P2FC scenario 1A to approximate those for our scenario 2B. For the other two scenarios of our research, we use the environmental benefits corresponding to the P2FC scenarios 1B and 1C respectively. However, we acknowledge that since the three scenarios in our research are all drier than the three in the P2FC's research, the environmental benefits we apply this way are most probably the lower bound of the true benefits, especially for the scenarios 2C and 2D since these two scenarios in our research are much drier compared to corresponding P2FC scenarios.

We depict the costs for various combinations of the current oil price and the river flow state for the alternative rule sets in Figure 3.6. The figure shows that in general the economic cost of any given rule set increases steeply with price of oil when the current oil price is less than \$5/barrel. In scenario 2B, at a price of \$5/barrel, the costs range between 1600 (Alt 19) and 6500 (Alt 22) million dollars. The costs increase gradually when the oil price rises from \$5 to \$35 per barrel. Over \$35/barrel, the costs are quite flat in relation to the price of oil, ranging between 2000 (Alt 19) and 8500 (Alt 22) million dollars. The economic costs decrease only slightly as the current river flow state becomes more abundant although this is difficult to see give the scale of the diagrams. In scenario

2C, the costs of the alternative rules sets range from 2100 to 6800 million dollars at a price of \$5/barrel. Over \$35 per barrel the range is from 2700 to 8600 million dollars. As with scenario 2B, the current river flow state almost does not change the costs significantly. In scenario 2D, the costs of the alternative rules sets increase, ranging between 2500 and 7000 million dollars for the current oil price of \$5/barrel. Costs range between 3100 and 8800 million dollars when the current oil price is over \$35/barrel.

To provide a better perspective of the magnitudes involved, Table 3.11 shows expected costs at a current oil price of \$90 per barrel as well as the ratio of each alternative's costs to Alt 19 costs across the three scenarios. Recall that scenario 2B reflects river conditions in 2015, scenario 2C reflects the lowest weekly flow rates over the past 60 years and scenario 2D reflects extremely dry conditions not seen historically.)

Table 3.11: Base Case Economic Cost Comparison for P=\$90/barrel

	Phase 1	**Alt 19**	**Alt 20**	**Alt 21**	**Alt 22**	**Option A**	**Option H**
Costs in million dollars							
Scenario 2B	3 755	2 085	4 433	6 413	8 446	5 631	4 171
Scenario 2C	4 379	2 711	5 005	6 778	8 603	5 631	4 745
Scenario 2D	4 379	3 129	5 371	7 091	8 812	5 631	5 423
Ratio of costs to Alt 19 cost							
Scenario 2B	1.80	1	2.13	3.08	4.05	2.70	2.00
Scenario 2C	1.62	1	1.85	2.50	3.17	2.08	1.75
Scenario 2D	1.40	1	1.72	2.27	2.82	1.80	1.73

Note: Scenario 2B costs are for current river flow state 10, scenario 2C costs are for current river flow state 5, while scenario 2D costs are for current river flow state 1.

By comparing the lowest panel of Figure 3.5 and the upmost panel of Figure 3.7, we can see that by using 2015 river flow condition, the economic costs of the alternative rule sets do not change a lot compared to case 3 when we use historical hazard rate matrices to reflect the river flow condition. When the river flow condition is in scenario 2C and 2D, the economic costs are greater than or equal to the wetter scenario. The relative cost-effectiveness performances of the alternatives when the current oil price is \$90/barrel are shown in Figure 3.7. From the figure, it shows that Option H is not dominated by other alternatives. We cannot reject the P2FC's conclusion of choosing Option H. It turns out that for other combinations of current river flow states and oil prices, the shapes of the graphs do not change significantly.

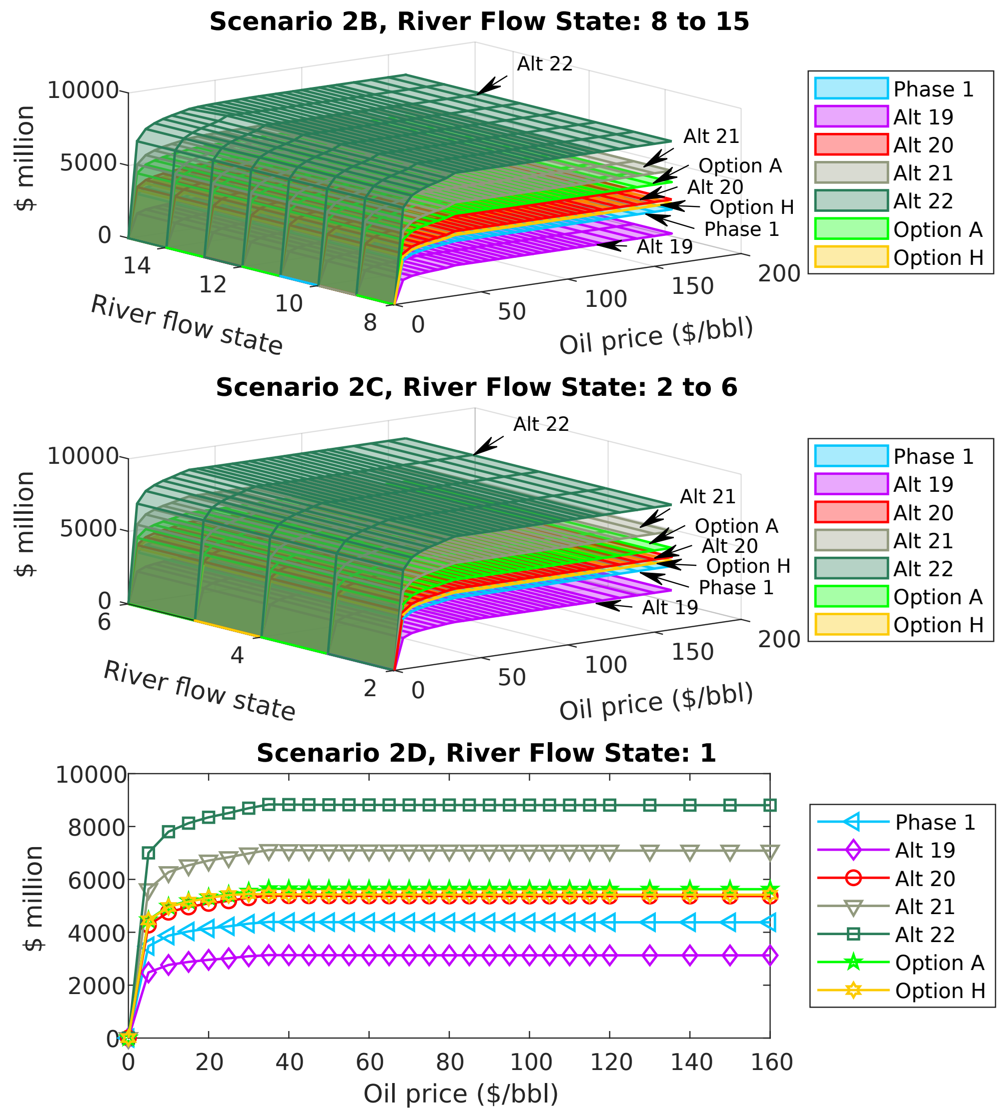

Figure 3.6: The economic cost of the oil sands industry due to the alternative rule sets for different river flow scenarios for Case 4. Note that in scenario 2B the river is always in states 8 to 15, while in scenario 2C the river may be in states 2 to 6 while in scenario 2D the river is always in state 1.

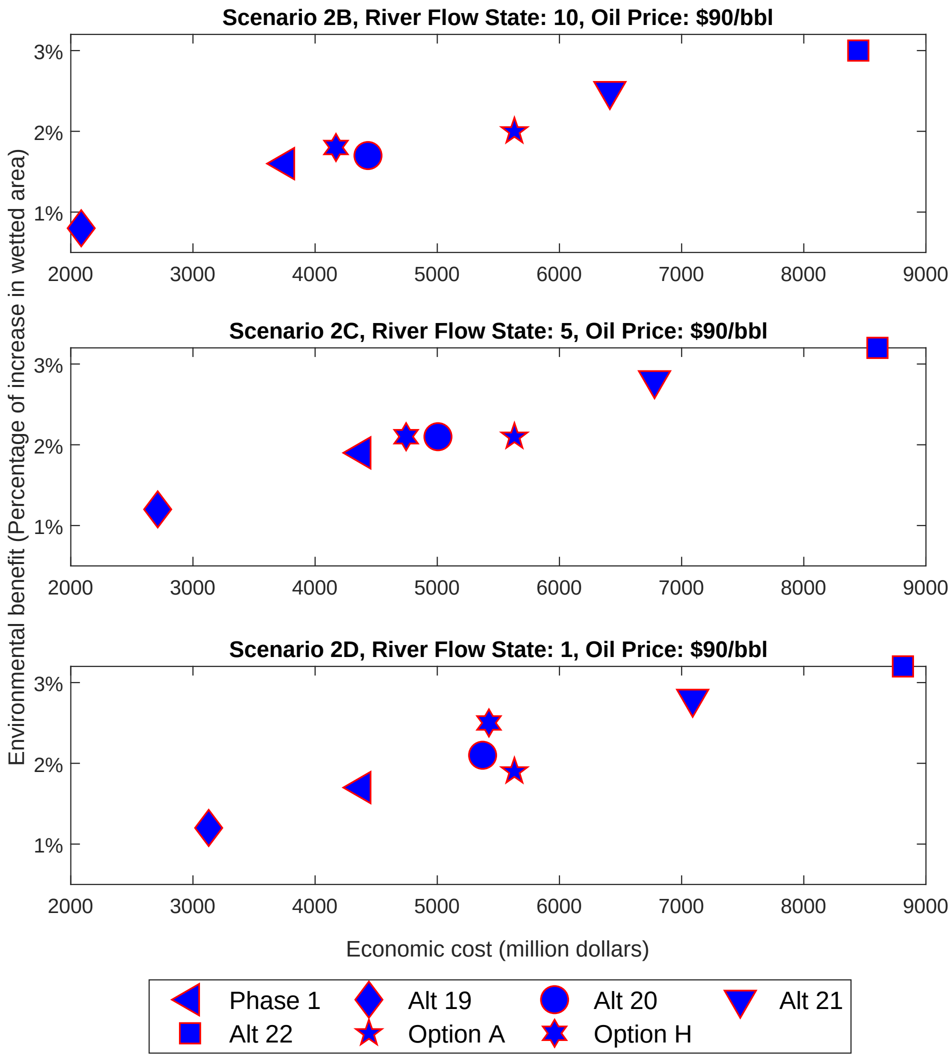

Figure 3.7: The cost-effectiveness for three scenarios given the specific current oil prices and river flow states for Case 4

3.6.3 Case 5: Lower Reserve Levels

The previous cases show that given the assumed capacity of storage and production capacity of the oil sands operations, introducing a stochastic oil price and the river flow conditions does not change the relative performances of the water management rule sets. Nevertheless, the object studied in the previous cases is the entire oil sands industry with a constant production capacity as high as 1 764 million barrels per year lasting for 50 years. This requires a reserve amount to 88 200 million barrels, which is much higher than currently estimated remaining established reserves. As noted earlier, remaining established reserves for the Alberta oil sands is 166 billion barrels but only 20% of that (33.2 billion barrels) is considered recoverable by mining (Alberta Energy Regulator 2015). In this section we conduct our analysis assuming remaining reserves are equal to those reported for current individual projects in the Athabasca region.

Our analysis assumes that the lifetime of an oil sands operation is just sufficient so that remaining reserves will be depleted if annual production is always at full capacity. Given this assumption, the larger the level of reserves the higher is the cost of water constraints. This is because in a stochastic setting (for price and river flow conditions), firms may not always produce at full capacity, implying that some reserves would be left unexploited at the end of the project. The larger the reserves, the more may be left unexploited. The length of a mining project also has an effect on the costs of water constraints. The length of the mining project may be determined by the terms of a firm's lease agreement with the resource owner. Given a specific level of remaining reserves and a specific production capacity, the longer a project lifespan, the higher the costs of water constraints. The intuition behind this phenomenon can be explained as follows. The present value of a project increases with its lifespan but the rate of increase for the project with and without water constraints differs. The increase of the present value for the project without water constraints is faster than that for the project with water constraints. Hence, their difference, which is exactly the costs due to the water constraints, increases with the lifespan. Figure 3.8 depicts the project values for two reserves levels with and without water constraints when the project's life varies. One reserves level is 88 200 million barrels, which is applied in cases 1 to 4. The other reserves level is the actual reserves level for active mining projects in 2015, i.e. 19 197 million barrels. Here the entire industry is treated as a single project. The current oil price adopted to create this figure is $90/barrel, the set of water constraint rules is Option H, the river flow scenario is 2B, and the current river flow state is assumed to be 10. For other levels of the oil price, other rule sets, other river flow scenarios, and other river flow states, the same phenomenon also exists. Figure 3.8 shows the increase in the cost of the constraints as project lifespan is increased for the given level of reserves and productive capacity. We also found that if productive capacity scales up on its own, the cost of restrictions decreases. The reason of this is that the higher the productive capacity, the less reserves may be left unexploited at the end of the project. In addition, the higher the productive capacity, the less water is needed for production, so that the less binding is the water restriction. While when the productivity capacity increases and the reserves scales up at the same time, other things being equal, the cost scales up. This combined effect implies that the effect of more reserves dominate the effect of higher productive capacity.

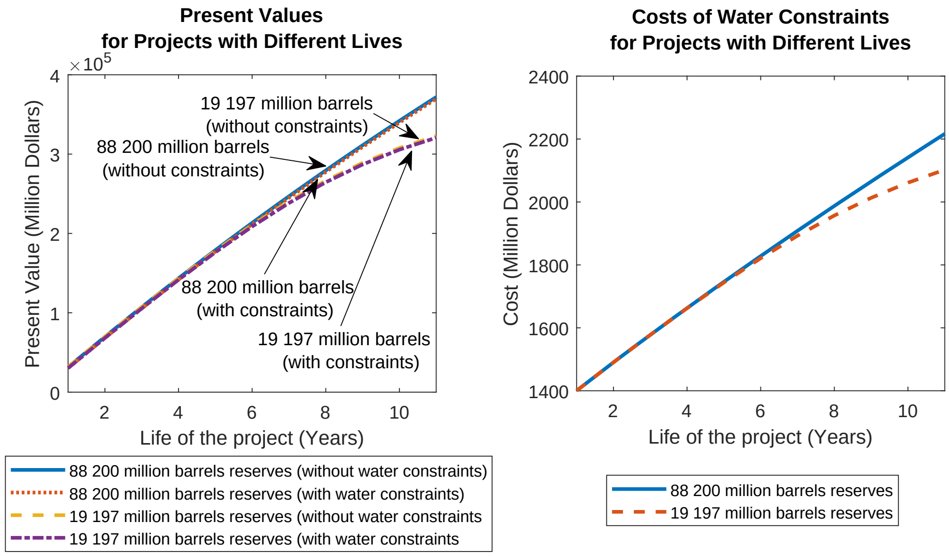

Figure 3.8: The present values and costs due to the water constraints vs. lifespans of the project for the entire oil sands mining industry with 88200 and 19197 million barrels of reserves without and with the water constraints imposed by Option H when the current oil Price is \$90/barrel, the current river flow state is 10 for scenario 2B

Up to this point the analysis has ignored the differing projects in the oil sands, and treated the entire reserve base as one large project. The oil sands industry in the Lower Athabasca River Region is composed of multiple projects run by several companies. According to Alberta's Energy Reserves 2014 and Supply/Demand Outlook 2015-2024 (Alberta Energy Regulator 2015) and 'Alberta Oil Sands Industry Quarterly Update' (spring 2015) (Economic Development and Trade 2015), in 2015, there were 5 oil sands firms running 6 projects. Table 3.12 outlines different projects in the industry. These projects have different production capacities and remaining established reserves. The upper portion of the table shows the actual reserves and production capacities in 2015 for those projects and the derived water demands and lifespans accordingly. (These values will be adopted to study case 8 in Section 3.6.6.) The lower portion of the table lists the values that are applied to this case (Case 5). The remaining established reserves are at 2015 actual levels of 19 197 million barrels, in contrast with Cases 1 through 4 in which remaining reserves are assumed to be at the much higher level of 88 200 million barrels. We maintain the water demand assumed in the P2FC report associated with the high productive capacity, but distribute that demand across different projects in the proportions indicated in Table 3.12. The production capacity for each project is derived from its water demand. Considering the possibility of renewing oil sands projects licenses, we assume that the lifespan of an oil sands project is sufficient so that the remaining established reserves will be depleted if the production is at the full capacity. Consistent with the assumption of lower total reserves, we observe in the lower portion of Table 3.12 that all projects have shorter lifespans than the 50 years considered in previous cases. We expect that the costs of the restrictions in this case would fall accordingly, which will be shortly shown to be the case. As mentioned in the above paragraph, the production capacity also affects the cost. In this case, we also explore whether different production capacity and lifespans of projects alter the optimal selection of the alternative rule sets.

The parameters specifications for this case are shown in Table B.11 in Appendix B. The water storage capacities for different projects in different river flow scenarios are shown in Table B.12 in Appendix B. One thing of note is that since the water sharing agreement allows water to be shared almost evenly across firms while allowing water exchanges as needed, we distribute the available water when the withdrawal is limited *pro rata* to all projects in accordance with their respective production capacities. The water storage capacity is also allocated to projects according to their production capacities.

The loss surfaces for the alternative rule sets are shown in Figure 3.9. The cost-effectiveness planes are depicted in Figure 3.10. Compared to Figure 3.7 (i.e. the cost-effectiveness planes for case 4), we can see that the magnitude of the total costs of all alternative rule sets is almost halved. Nevertheless, the relative performances of the cost-effectiveness of the alternatives are almost unchanged. We observe that the cost goes down significantly while the total productive capacity for the industry is still the same. The reason is that the projects are no longer as profitable with lower reserves. So restrictions now cause a smaller loss in value.

Figure 3.11 depicts a break down of the total costs of water constraints into different projects for Case 4, Case 5, and another case which modifies Case 4 by shrinking the lifespan to 11 years so that the remaining reserves conform to the 2015 true level, i.e. 19197 million barrels. From the figure, we can see that the reduced lifespan of the projects implies significantly lower costs of restrictions. In addition, we see that reflecting actual

Table 3.12: The Operating Projects' Remained Established Reserves and Production Capacities In the Lower Athabasca River Region In 2015

With the actual water demand				
Oil Sands Project	Remaining established reserves* (million barrels)	Production capacity** (million barrels/year)	Water required (million barrels/year)	Time to expiry (years)
CNRL Horizon	3164	55	132	57
Imperial Kearl	5447	40	96	136
Shell Muskeg River	2044	57	137	36
Shell Jackpine	1245	37	89	34
Suncor	2139	183	439	12
Syncrude	5158	149	358	35
With the higher water demand adopted in the P2FC report (16 m^3/s for the entire oil sands industry)				
Oil Sands Project	Remaining established reserves (million barrels)	Production capacity (million barrels/year)	Water required (million barrels/year)	Time to expiry (years)
CNRL Horizon	3164	188	451	17
Imperial Kearl	5447	136	327	40
Shell Muskeg River	2044	192	460	11
Shell Jackpine	1245	124	297	10
Suncor	2139	620	1488	3
Syncrude	5158	504	1209	10

Note:

- For the actual water demand, the water required is calculated by multiplying the corresponding production capacity with 2.4, which is the number of barrels of water required to produce one barrel of bitumen.
- For the higher water demand, the water required for each projects is a *pro rata* distribution from the total available 16 m^3/s according to their 2015 production capacities. The production capacities are derived by dividing the available water by 2.4.
- The time to expiry is derived by dividing the remaining established reserves by the corresponding production capacity.

* Source: Alberta's Energy Reserves 2014 and Supply/Demand Outlook 2015-2024 (Alberta Energy Regulator 2015)

** Source: Alberta Oil Sands Industry Quarterly Update (spring 2015) (Economic Development and Trade 2015)

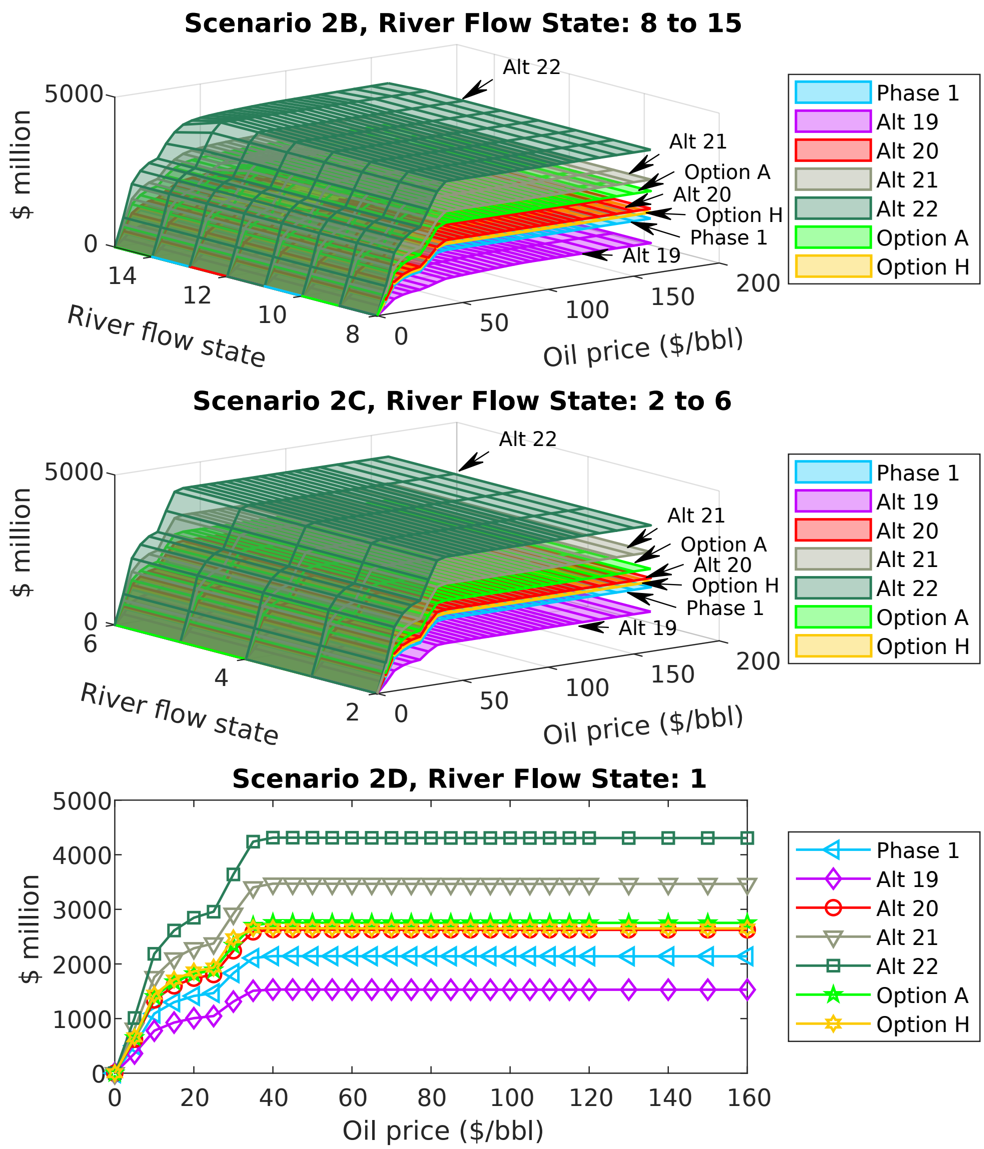

Figure 3.9: The economic cost of the oil sands industry due to the alternative rule sets for different river flow scenarios for Case 5, when the differences in the production capacity and the remaining reserves across oil sands mining projects are considered

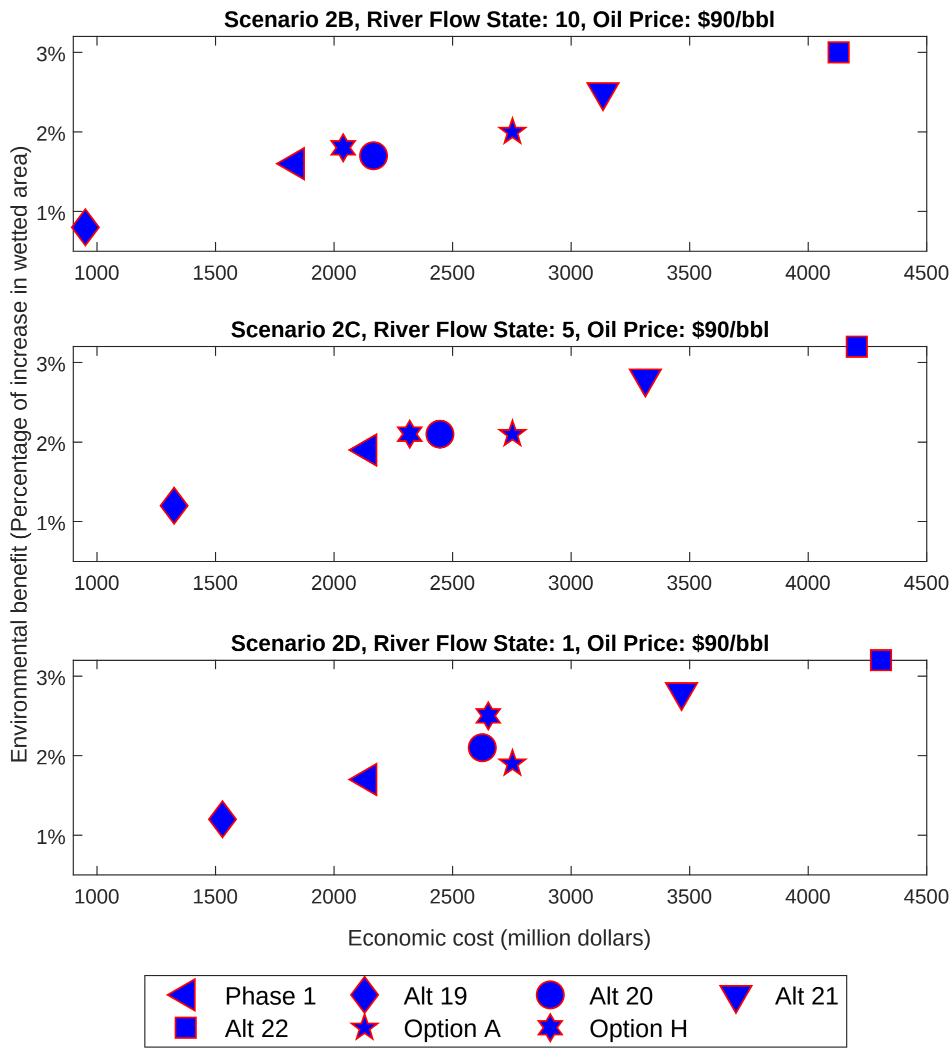

Figure 3.10: Three representative cost-effectiveness graphs in three river flow scenarios for Case 5, when the differences in the production capacity and the remaining reserves across oil sands mining projects are considered

reserves of individual projects as is done in case 5 affects the distribution of costs across projects and has a small effect on the total cost.

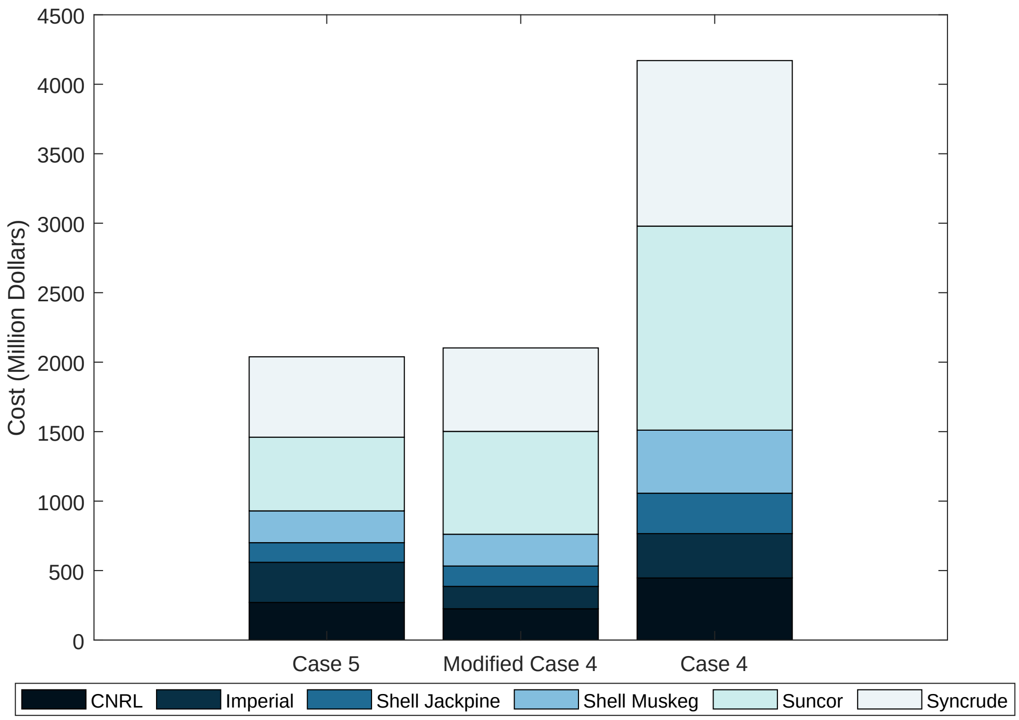

Figure 3.11: The breakdown costs imposed by Option H for each project in Case 5, modified Case 4, and Case 4 when the current oil price is \$90/barrel, the current river flow state is 10 for scenario 2B (Case 5 is to examine the heterogeneous projects with different production capacities and lifespans, Case 4 is to examine the entire oil sands mining industry with 88,200 million barrels of reserves, and modified Case 4 is to examine the entire oil sands mining industry with 19197 million barrels of reserves)

3.6.4 Case 6: Using a Fixed Water Storage Capacity Across the Alternative Rule Sets

As noted in the previous cases, it is assumed that enough storage capacity will be built so that water restrictions will never require firms to reduce production levels due to water restrictions. The only cause of reduced production would be low oil prices. We note that this assumed storage capacity is derived based on the assumption of a full production rate. Construction of such large storage capacity may not be feasible or optimal for the firms. In this case and the next we examine if the selection of the water management rule set will change under different choices for water storage. In Case 6 we assume water storage can support four weeks of production. This is the amount of storage chosen by the Kearl project. In the next case, the optimal water storage and its effects will be investigated.

Figure 3.12 shows the economic costs for the alternative rules sets in three river flow scenarios. Note that for the first time we include that rule set that was finally implemented by the Alberta government (hereinafter referred to as the "Phase 2 Choice") into the figures because from this case to Case 8, the information about the water storage volume for the Phase 2 Choice is available. We do not have an estimate of environmental benefits for the Phase 2 Choice. It can be seen from the figure that when the current oil price is over a specific level, other than the Alt 19 in scenario 2B, the costs for all of alternative rule sets' costs are almost identical. Figure 3.13, Figure 3.14, and Figure 3.15 show when the current oil price is at different levels, in different scenarios, how the cost-effectiveness performances of the alternative rule sets vary. We see in Figure 3.13 that when the river flow condition is scenario 2B, and the current oil price is below \$35/barrel, Option H appears to be on the efficient frontier. When the current oil price is over \$45/barrel, all alternatives' costs, excluding Alt 19, are identical. Now the Option H is dominated by all other alternatives that have higher environmental benefits than it (i.e. Alt 22, Alt 21 and Option A). When the current oil price is greater than \$45/barrel the only cost of the restrictions is the cost of building storage. There are no additional costs due to production cutbacks. This is why the economic costs are identical for six of the seven rule sets. Rule set 19 has lower cost because it imposes generally weaker restrictions[7], and hence the optimal timing for building storage is delayed. For scenario 2C, when the current oil price is over \$45/barrel, all the alternative rule sets, including Alt 19, have identical economics costs. So Option H is not the optimal one because it is dominated by Alt 22 and Alt 21. For scenario 3, it shows that Option H and Phase 2 rule set are always dominated by Alt 21 and Alt 22.

It turns out that other than for extremely dry conditions as in scenario 2D, when the current oil price is less than \$35/barrel, Option H can be the optimal choice. However, when the current oil price is higher than \$35/barrel, the advantage of Option H in terms of

[7]From Figure B.2 in Appendix B, it looks to be the case that the constraints put by Alt 19 are weaker than those of other alternatives proposed by the P2FC. It can be observed that from week 17 to week 40, (except for weeks 19 and 20), the water withdrawal constraints imposed by Alt 19 are stricter than those imposed by the Phase 1 Framework. Nevertheless, during this period, the river flow level is fairly high so that the constraints are not binding. Therefore, we can still say that overall the Alt 19 restrictions are weaker than those of the Phase 1 Framework.

economic costs will disappear and it will be dominated by other alternative rule sets. When the river flow condition is extremely dry (scenario 2D), Option H will have no advantage over others regardless of the current oil price.

3.6.5 Case 7: Adopting the Optimal Water Storage Capacities

We derive the economic costs due to each alternative rule set given a sequence of capacities for the water storage facility. The capacity that make the costs the least is the optimal one. Limited by the computational capability, we use a fairly sparse sequence: the storage capacity that can supply water required by 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 2, 3, 4, and 5 weeks' oil sands production. Nevertheless, we get an indication of the approximate range of the optimal water storage capacity for each alternative rule set.

Figures B.3, B.4, and B.5 in Appendix B depict the economic total costs corresponding to various water storage capacities for the alternative water management rule sets in three river flow scenarios. The optimal water storage capacities for all alternative rules sets in different river flow scenarios are also shown in Table 3.13. This table shows the storage capacity in terms of how many weeks' production it can maintain. The specific volumes for water storage by project are derived from this and shown in Table B.12 in Appendix B.

Table 3.13: Without Any Withdrawal From the Athabasca River, the Number of Weeks That the Production Can Continue With the Water Supply From the Water Storage Facility

		Phase 1	Alt 19	Alt 20	Alt 21	Alt 22	Option A	Option H	Phase 2
Scenario 2B	case 5	7	4	8	12	15	10	8	n/a
	case 6, case 8	4	4	4	4	4	4	4	4
	case 7	0.5	0.2	0.4	0.5	0.7	0.5	0.5	0.5
Scenario 2C	case 5	8	5	9	12	16	10	9	n/a
	case 6, case 8	4	4	4	4	4	4	4	4
	case 7	0.5	0.4	0.5	0.7	1	0.5	0.5	0.5
Scenario 2D	case 5	8	6	10	13	16	10	10	n/a
	case 6, case 8	4	4	4	4	4	4	4	4
	case 7	0.5	0.5	0.7	0.7	1	0.5	1	1

It turns out that in all river flow scenarios, the optimal water storage capacities for the alternative rules sets are much less than the levels that the P2FC proposed. This is due, in part, to the fact that the P2FC report assumes a much larger reserve base.

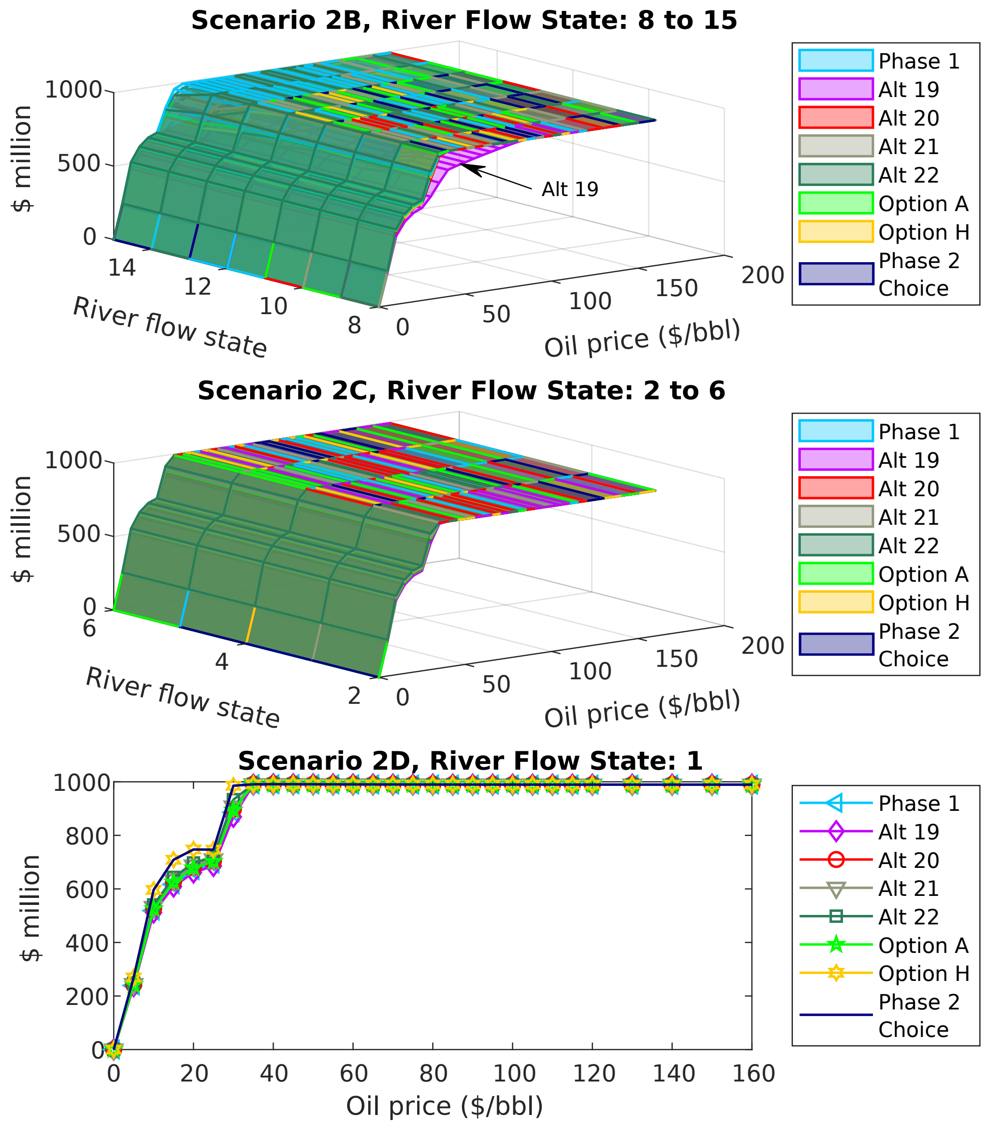

Figure 3.12: The economic cost of the oil sands industry due to the alternative rule sets for different river flow scenarios for Case 6

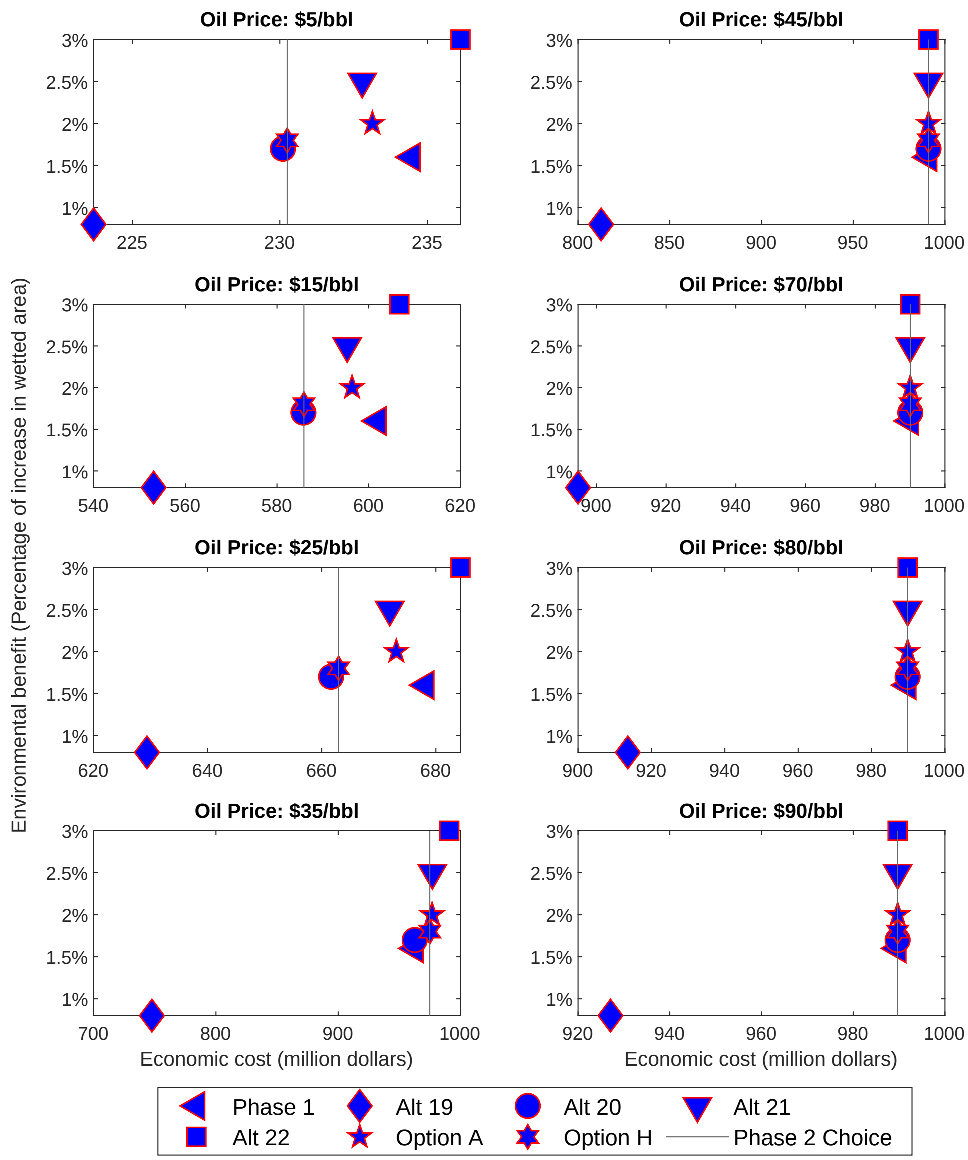

Figure 3.13: Cost-effectiveness graphs under different present oil prices for Case 6, where the water storage is fixed at the level capable of sustaining 4 weeks production, when the river flow condition is in scenario 2B

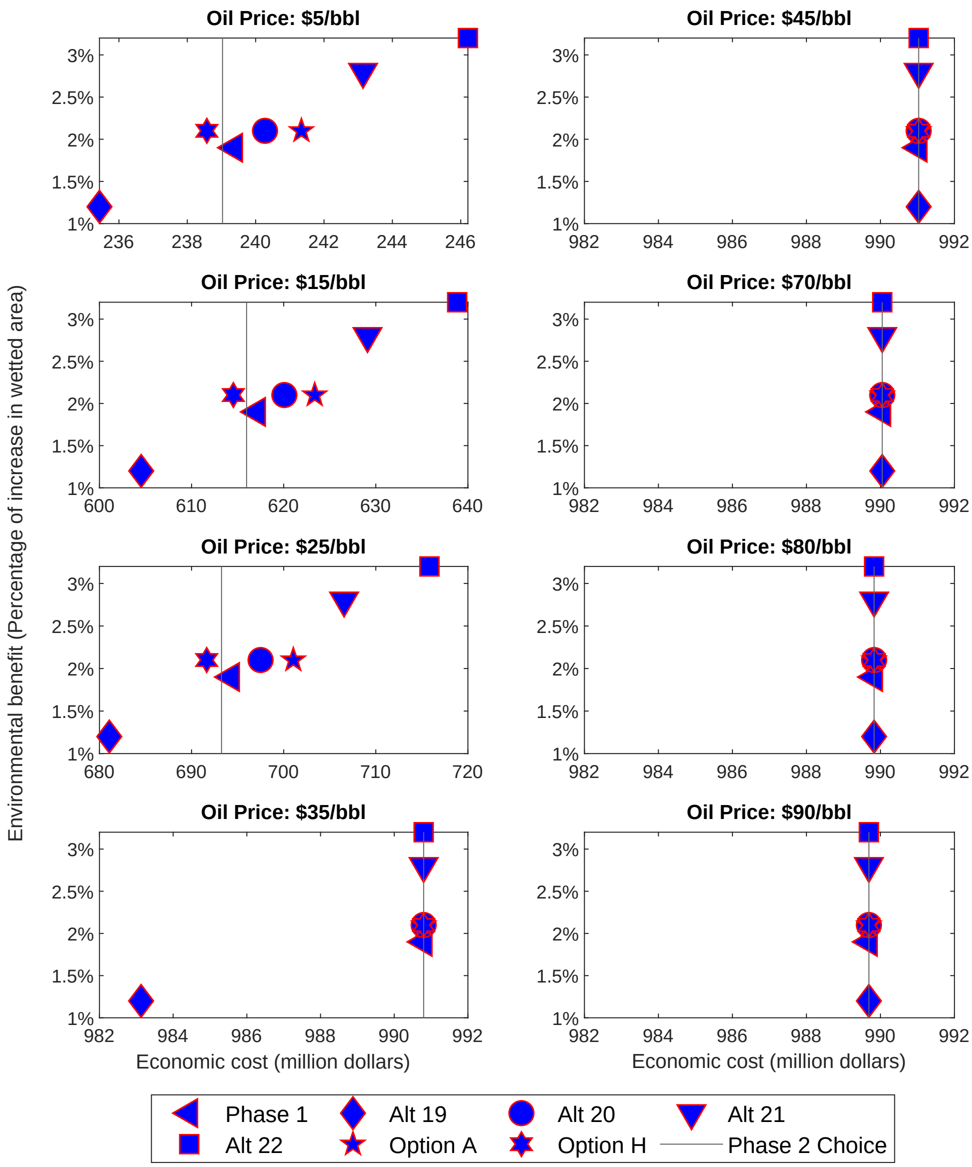

Figure 3.14: Cost-effectiveness graphs under different present oil prices for Case 6, where the water storage is fixed at the level capable of sustaining 4 weeks production, when the river flow condition is in scenario 2C

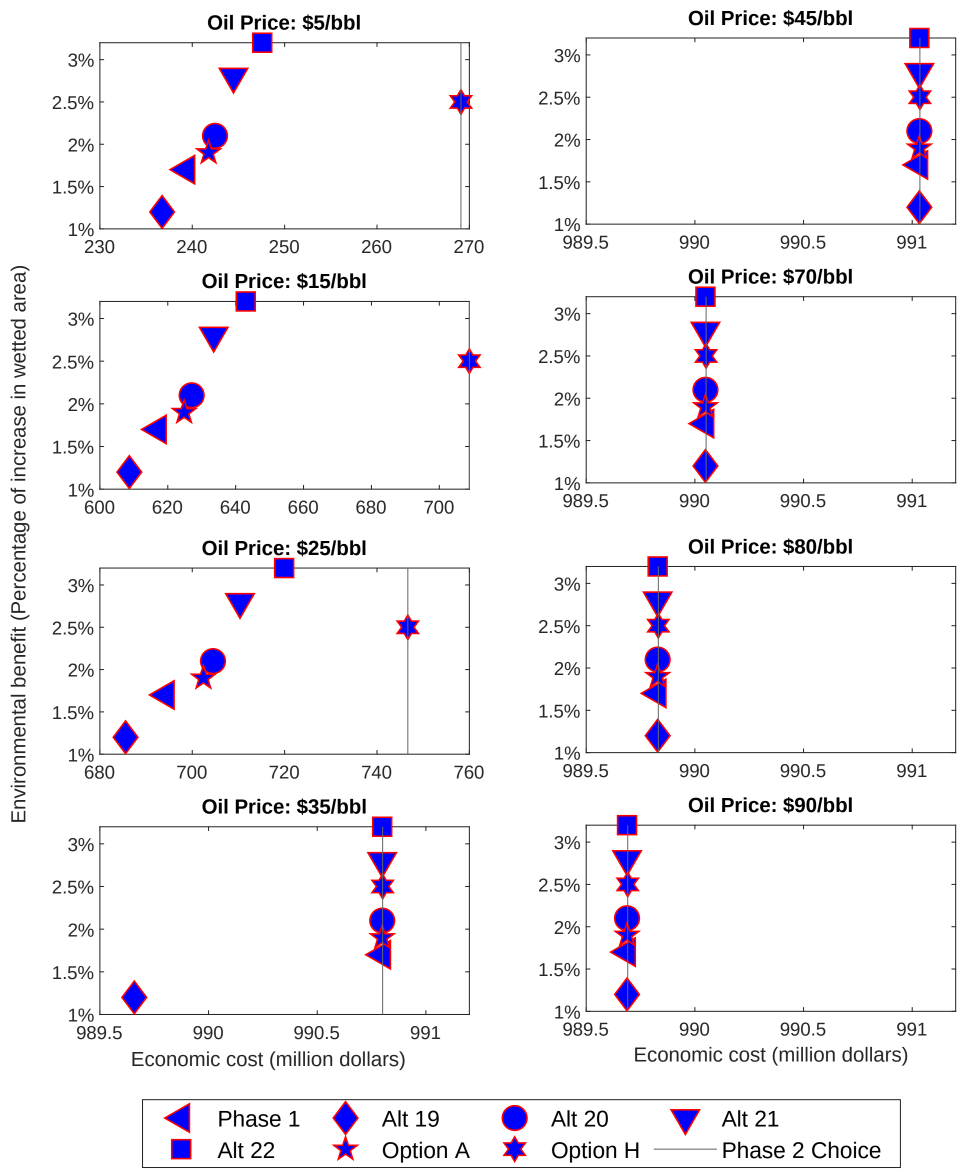

Figure 3.15: Cost-effectiveness graphs under different present oil prices for case 6, where the water storage is fixed at the level capable of sustaining 4 weeks production, when the river flow condition is in scenario 2D

Assuming the optimal water storage capacities are adopted, we depict the economic costs for the alternative rule sets shown in Figure 3.16 and investigate their cost-effectiveness when the current oil price is \$90/barrel in Figure 3.17. With different current river flows and oil prices, the layout of the alternative rule sets in the cost-effectiveness plane is similar to that in Figure 3.17.

The figure shows that in scenario 2B, Option H is dominated by Alt 21. In scenario 2C, Option H is indifferent with Alt 20 and Option A. In scenario 2D, Option H is dominated by Alt 22 and Alt 21. This indicates that the relative cost-effectiveness performances of the alternative rule sets can be affected by the river flow scenarios. Even considering only the current river flow condition, Option H is not the optimal choice.

3.6.6 Case 8: the Performance of the Rule Sets Under the Current Productive Capacity and Reserves

Another question that might be of interest is how the performances of the cost-effectiveness of the alternative rule sets will be based on the current oil sands industry production capacity. We assume that the water storage is able to supply four weeks water required by the industry. The investigation shows that in the scenario 2B river flow condition, the economic costs due to all alternatives are zero. It means that there are no expected cutbacks in production because the water constraints are not binding.

A comparison of economic costs for the alternative rule sets is shown in Figure 3.18. It is shown that when the river flow condition is in scenario 2C, Alt 21, Alt 22, and the Phase 2 Choice incur some costs, while other alternatives have no costs because the water constraints are not binding and it is not necessary to build water storage. When the river flow condition is in scenario 2D, Option H, Phase 2 Choice, Alt 21, and Alt 22 incur identical costs while other alternatives have no costs. The cost-effectiveness plane is exhibited in Figure 3.19. Note that when the costs of the relevant rule sets are zero, it implies that it is not necessary to build water storage. Hence the relevant rule sets will not cause any effects on the oil sands production compared to the case where there are no water management rules. Therefore, the environmental benefits are also zero. For those rule sets that have some economic costs, since the production levels are less than the levels forecasted in the P2FC report, the loss in winter wetted area is less than that under the forecasted high production levels. However, we are unable to determine the environmental benefit because the loss in winter wetted area when there are no water constraints decreases as well. For the purpose of illustration, we use the environmental benefits given by the P2FC as a reference. We acknowledge that the true environmental

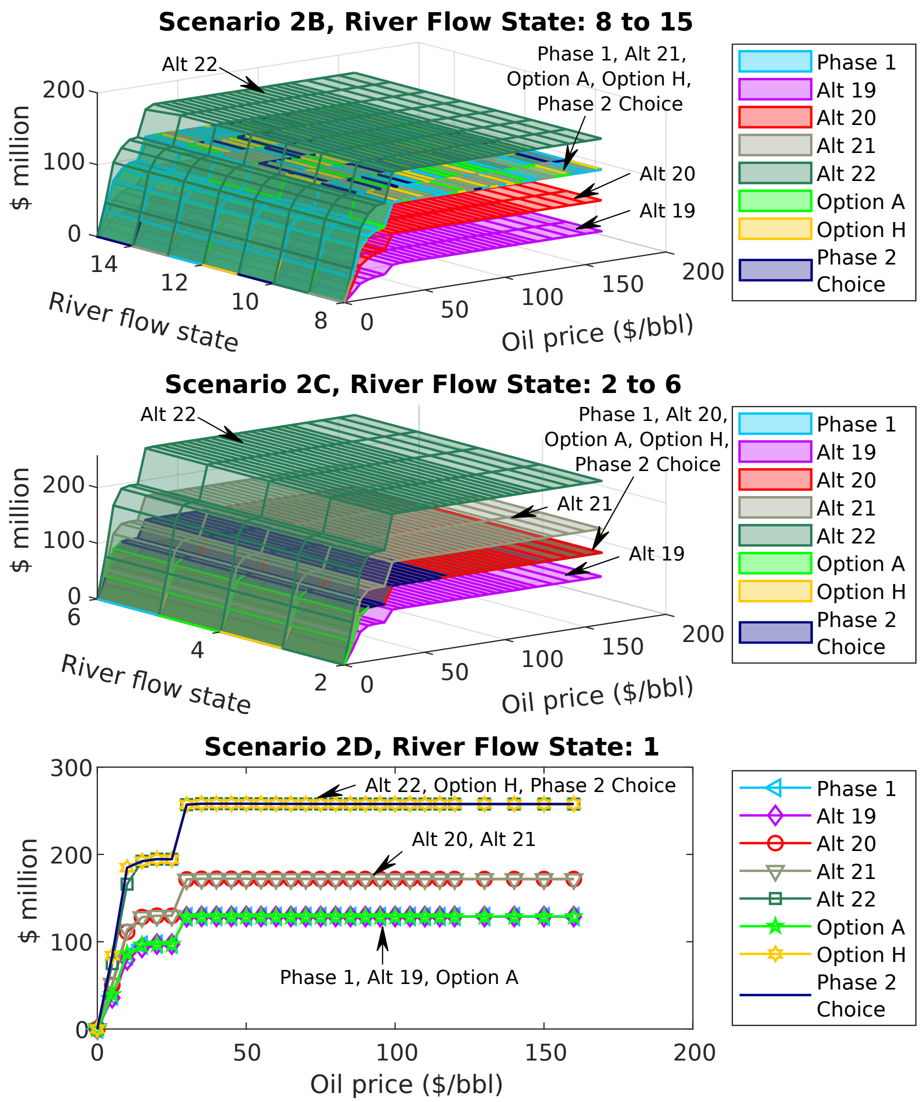

Figure 3.16: The economic cost of the oil sands industry due to the alternative rule sets for different river flow scenarios for Case 7, where the optimal water storage capacities are adopted

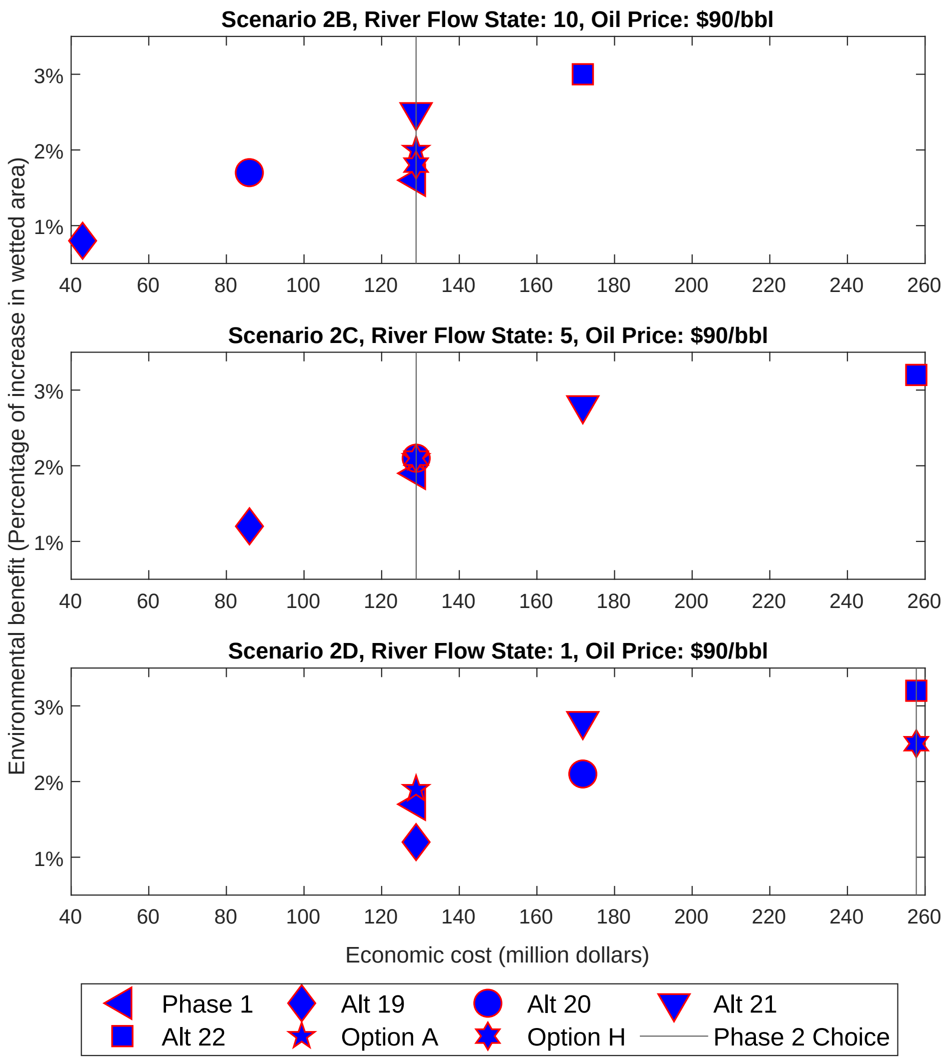

Figure 3.17: Representative cost-effectiveness graphs for different river flow scenarios for Case 7, where the optimal water storage capacities are adopted

benefits might cause the relative performance of the alternative rule sets different from what is shown in Figure 3.19. Nevertheless, regardless of the environmental benefits of the alternative rule sets, it is obvious that with the current oil sand production level, it is not optimal to apply Option H to manage water withdrawal by the oil sands industry.

3.6.7 Summary of the Results and Findings

Table 3.14 summarizes the results for all cases investigated in previous sections.

Table 3.14: Summary of Results of Cases

	Wet (50 year historical river flow or 2015 condition)	Drier (river flow is at the historical weekly lowest levels)	Driest (river flow is at the driest state)
Case 1	The magnitude of costs unchanged the relative C-E* unchanged P2FC results hold	-	-
Case 2	The magnitude of costs increased The relative C-E unchanged P2FC results hold	-	-
Case 3	Almost unchanged compared to Case 2 P2FC results hold	-	-
Case 4	Almost unchanged compared to Case 3 P2FC results hold	Compared to the wet scenario, the magnitude unchanged the relative C-E do not change significantly P2FC results hold	Compared to the wet scenario, the magnitude unchanged the relative C-E do not change significantly P2FC results hold
Case 5	Compared to Case 4, the magnitude is halved the relative C-E does not change significantly P2FC results hold	Compared to Case 4, the magnitude is halved the relative C-E does not change significantly P2FC results hold	Compared to Case 4, the magnitude is halved the relative C-E does not change significantly P2FC results hold
Case 6	Compared to Case 5, the magnitude shrinks further when $P<35$, the P2FC results can still hold when $P>35$, Option H is dominated P2FC results do not always hold	Compared to Case 5, the magnitude shrinks further when $P<35$, the P2FC results can still hold when $P>35$, Option H is dominated P2FC results do not always hold	Compared to Case 5, the magnitude shrinks further Option H is always dominated P2FC results do not hold
Case 7	The magnitude of costs is around 100 million Option H is dominated by Alt 21 P2FC results do not hold	The magnitude of costs is around 100 million P2FC results hold	The magnitude of costs is around 100 million Option H is dominated P2FC results do not hold
Case 8	All rule sets costs zero P2FC results do not hold	Option H can be the optimal since it costs zero Phase 2 cannot be the optimal because it costs more than 500 million P2FC results do not always hold	Option H and Phase 2 cannot be optimal because they cost more than 700 million while Phase 1, Alt 19,20, Option A cost zero P2FC results do not always hold

* C-E refers to cost effectiveness.

From the results, we have some main findings about the alternative rule sets studied by the P2FC to develop the Phase 2 Framework.

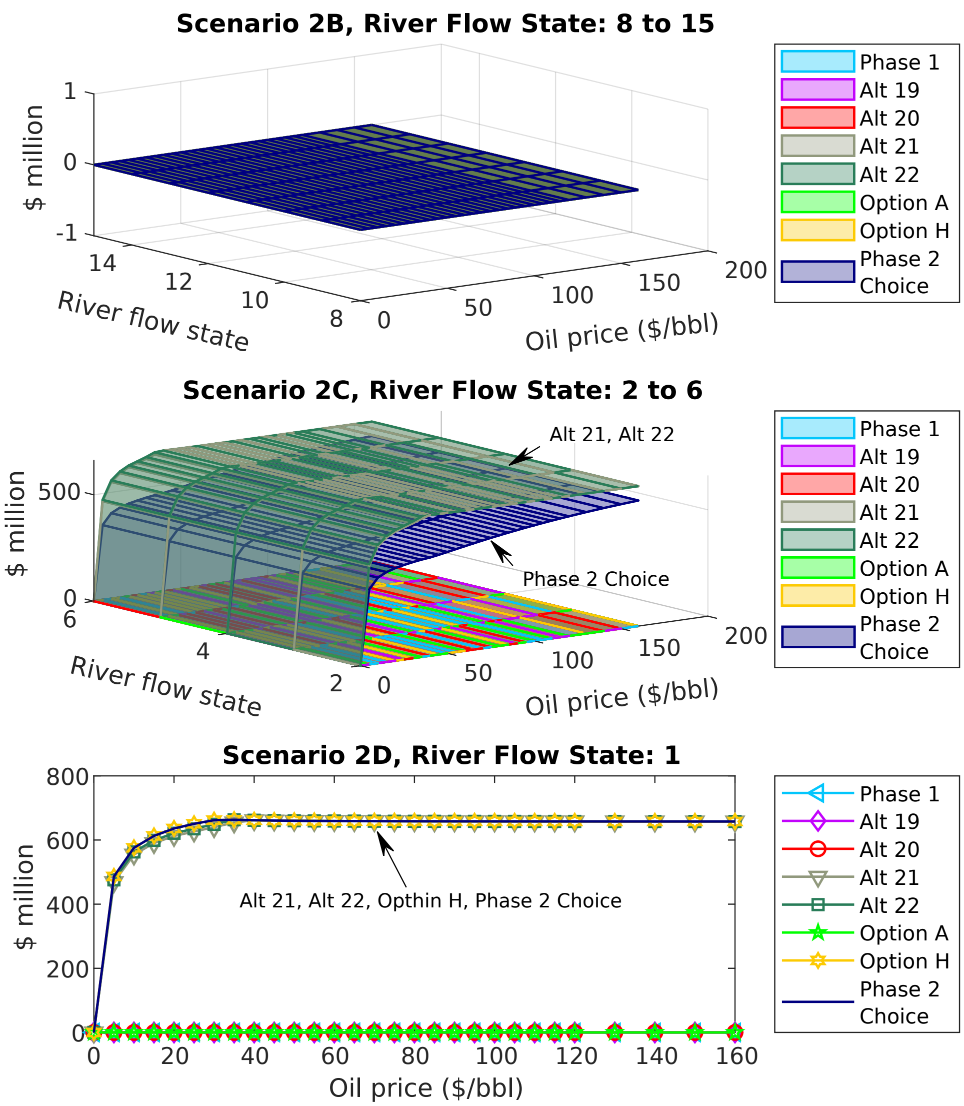

Figure 3.18: The economic cost of the oil sands industry due to the alternative rule sets for Case 8, i.e. when the oil sands production is at 2015 status, for three river flow scenarios

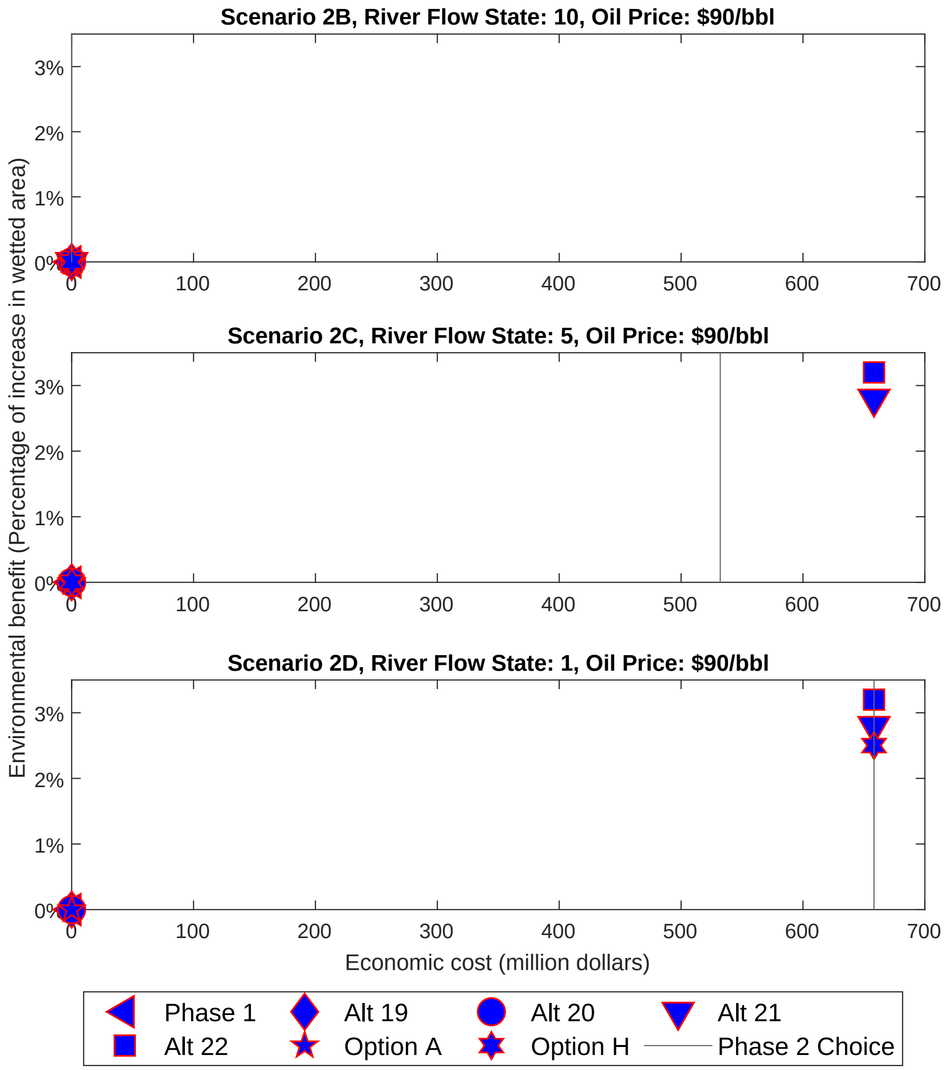

Figure 3.19: The representative cost-effectiveness graphs for Case 8, i.e. when the oil sands production is at 2015 status, for three river flow scenarios

When the water storage is at the full volume proposed by the P2FC, when introducing stochasticity to the oil price or river flows, the relative cost-effectiveness for the alternative rule sets does not change much. Furthermore, the addition of the water storage maintenance costs or the consideration of the different reserve levels across different oil sands projects does not change the relative cost-effectiveness of the different rule sets. The results are robust under extremely dry scenarios. However the magnitude of economic costs do change significantly when remaining established reserves are reduced to reflect actual reserve estimates of the individual oil sands projects.

Once the water storage capacities are changed to other values, the relative performances of cost-effectiveness for the alternative rule sets are different from those given in the P2FC's report. Two cases are examined. One is with a storage level that can supply four weeks of water demanded by the oil sands mining projects. This is much smaller than that assumed by the P2FC report. For this case, for scenario 2B and 2C, when the current oil price is less than the critical price to install water storage, Option H and the Phase 2 Choice are on the efficient frontier (referred to by the P2FC). When the current oil price is over the critical price to install the water storage, all the alternative rule sets, except for Alt 19, have identical economic costs which reflect the installation and maintenance of the water storage.[8] Therefore, Option H and the Phase 2 Choice are not optimal choices. When the river flow is in scenario 2D, even when the current oil price is below the critical price for building the water storage immediately, Option H and Phase 2 Choice are dominated by other alternative rule sets.

Our study shows that the water storage capacity applied plays an important role in the relative performances of the cost-effectiveness of the alternative rule sets. The assumption that firms will always produce at full capacity implies a storage capacity that is inefficiently large. Firms acting optimally in response to oil prices and river flows will choose to invest in less storage. The optimal water storage that accommodates the optimal production turns out to be much lower than the level proposed by the P2FC. The ordering of the economic costs of the alternative rule sets are consistent with the ordering of the costs to build and maintain the optimal water storage. When the projected river scenario is the same as the condition in 2015 (scenario 2B), or is always as low as the rate of 86 m^3/s (scenario 2D) (an extremely dry condition that has never happened historically), Option H and the Phase 2 Choice are not the optimal choices. When the river flow is always at the lowest level recorded over the last 50 years (scenario 2C), Option H is on the efficient frontier (referred to by the P2FC) and cannot be rejected. Alt 21 and Alt 22 are always on the efficient frontier regardless of the river flow scenarios.

[8]For Alt 19 it is not optimal to build storage at any oil price when in scenario 2D.

Alt 22 is found to never be dominated by any other rule sets throughout our research, although its economic cost is never less than the others either. Nevertheless, when the optimal water storage is adopted, the amount by which Alt 22's costs is more than others is less than \$100 million. When the optimal water storage is applied, another competing rule set that is not dominated is Alt 21, the cost of which is at around the middle of all alternative rule sets.

If the oil sands industry's production level is at the 2015 level, none of the alternative rule sets will be binding under the wet river flow scenario so that no rule set is necessary. Under scenario 2C, other than Alt 21, no rule sets are dominated by others so that all can be selected depending on the preference with regard to the environment-economic trade off. Under scenario 3, Alt 21, Option H, Phase 2 Choice are dominated by Alt 22 so that should be ruled out. However, since in the near future from 2015, it is not likely to be in scenario 3, this case would not be used as the basis for the decision.

3.7 Conclusions

Water regulations that restrict water usage are costly to an oil sands firm to the extent that they change a firm's production or investment decisions compared to when there are no water regulations.

The full economic cost can be estimated through an optimal control model that describes the firm's decision problem in a dynamic setting, taking into account the main factors that will affect the cost of the regulations. These factors include anything that affects the extent or cost of the firm's response to the regulation.

In the case of oil sands operations, these factors include:

- **Expected future river flow levels:** With the lower expected river flow levels, the water regulations will more frequently be binding on the firm and will reduce production levels.
- **Uncertainty in future river flow patterns:** Volatile, uncertain future river flow patterns can affect a firm's optimal decisions about the timing of investment in water storage. Even if average expected river flow flows are plentiful, if there is a significant probability of very dry conditions, it may be optimal for firms to invest in storage. Hence modelling the stochasticity of river conditions is potentially an important component of the analysis. However in our analysis we did not find a

significant difference between the results using deterministic water availability based on the past 50 years versus modelling stochastic river flows using a hazard matrix based on the data from the past 50 years. This is understandable since the last 50 years was a time of plentiful water supply.

- **The expected price of oil:** The higher the expected price of oil, the more costly the regulations as the opportunity cost of lost production is higher. Note that the same holds for any major operating cost, with higher costs reducing the cost of the regulation.

- **Uncertainty in oil prices:** Water restrictions reduce a firm's ability to respond to volatile prices. Volatile prices imply that a firm may shut in production in periods when prices are low and produce as much as possible when price are higher. Water restrictions reduce the firm's flexibility to respond optimally to changing prices or costs. In this analysis we focused on stochastic prices and found that price uncertainty did not have a significant effect on the results. This follows because the assumed price process implies that firms should produce at full capacity most of the time. However in general it would be important to consider the impact of stochastic prices (or costs) in an analysis of the economic cost of regulations.

- **The cost of investment in storage:** The cost of capital investments such as for storage helps alleviate the impact of restrictions. We have seen that this factor is a key determinant of the costs of the P2FC study.

- **Productive capacity of the oil sands:** The larger is productive capacity the more often the water regulations will be binding and the greater the cost of the regulations. This was also a key determinant of the magnitude and ranking of costs in the P2FC study.

- **The size of reserves and the remaining life of the project:** Binding water restrictions imply that the firm cannot exploit the oil field as quickly as desired. Production is therefore pushed out to the future, which imposes a cost in that revenue is received later. If there is a fixed end date for a particular project, perhaps because of an oil lease, it is possible that binding water regulations would result in a particular firm having to leave some reserves in the ground. What happens in reality will depend on polices regarding lease renewals. In our study it was assumed that all reserves could be produced within the assumed lifetime of the project, if production is at full capacity. Water restrictions may imply that not all reserves can be produced during the life of the project.

All of the above points may be influential in any particular analysis of the costs of water restrictions on the industry. In the particular case examined in this chapter the most important factors were the storage capacity, cost of storage, river flow conditions, productive capacity and reserves. In general, this study points to the importance of considering multiple factors that might affect the cost to industry of regulations, and not just the directly related capital expenditures.

Chapter 4

Estimation of the Stochastic Process of Crude Oil Prices

4.1 Introduction

As has been shown in Chapter 2, the specification of the dynamic process of crude oil prices has an important effect on the decision making problem. In this chapter we use a Kalman filter methodology to estimate the parameters of three alternative models of crude oil prices. This analysis provides the background for the choice of model used in Chapter 2. The chapter makes a contribution to the literature on modelling oil price dynamics in two ways. Firstly the chapter provides updated estimates for a model previously studied in the literature. Secondly, the chapter demonstrates the use of two extensions of the Kalman filter to estimate alternative oil price models.

In the literature, there is much research investigating the dynamic behaviour of crude oil prices. There is no general agreement about what constitutes the best model, in part because the choice of model depends on the context in which it will be used. Further, what is deemed to be the appropriate model may change as key features of oil markets shift over time. In what follows we mention some past research which is relevant for our analysis. A focus of several key papers from the 1990s was the modelling of crude oil prices for the purpose of valuing oil-related contingent claims. Gibson & Schwartz (1990) propose a model for crude oil prices with two stochastic factors. The spot price of oil follows a GBM process correlated with the convenience yield which follows an Ornstein-Uhlenbeck ("OU") process. They find it seemingly reliable for valuing short term financial instruments. Schwartz (1997) extends Gibson & Schwartz (1990)'s model by examining three alternative models of crude oil prices. His first model is an Ito process whereby the log oil price is mean-reverting. His second model includes a stochastic convenience yield in the drift term of the oil price model. Finally, he examines a three-factor model by including a stochastic interest rate which is correlated with the price of oil and the convenience yield. Schwartz's conclusion is that the two and three-factor models outperform the one-factor model. Schwartz & Smith (2000) suggest a two-factor model combining a long-run equilibrium and a short-run deviation from equilibrium, which follow a GBM and an Ornstein-Uhlenbeck process respectively. Chen (2010) proposes a regime-switching model based on Schwartz (1997)'s log mean-reverting model. Al Mansour (2012) applies an extended model of Brennan & Schwartz (1985). In this extension, the logarithm of the spot price follows a Brownian motion process with drift. He concludes that this model performs better than Gibson & Schwartz (1990)'s two factor model in the long-run. Insley (2017) applies a regime switching model based on the level mean-reverting process to capture the dynamics of crude oil prices which is used as an input to a stochastic optimal control problem.

There is a large number of papers in the financial econometrics literature which study the estimation of oil price models. Campbell et al. (1997), Schorfheide (2003), Panik (2017) introduce econometrics approaches such as the maximum likelihood method, method of moments, and the Shoji-Ozaki routine to estimate the parameters of various oil price models. The data used in this strand of literature are spot prices, which are neither directly observable (as will be explained shortly) nor able to reflect the arbitrage free feature in contingent claims markets. Those approaches result in crude oil price model estimates in the physical measure ("$\mathcal{P}$ measure"). However, for pricing contingent claims, since the pricing process is based on the assumption of arbitrage free markets, it is a norm to apply the prices process in the martingale equivalent measure ("$\mathcal{Q}$ measure") in order to get a correct estimate of asset value. Prices of futures contracts allow for the estimation of price models for the underlying asset in the Q-measure.

As noted, the observability of crude oil spot prices data is also a concern raised in the literature such as in Gibson (1991), Gibson & Schwartz (1990), Schwartz (1997). The spot price is defined as the price of a good for immediate delivery. For crude oil the demand for immediate delivery is small relative to the demand for future delivery, due in part to the logistics of transporting oil. Hence futures contracts are more commonly used to purchase oil.[1] For the purposes of estimating price model parameters, spot prices of crude oil have typically been represented by the corresponding one month futures contract (Gibson & Schwartz (1990)) or estimated from futures prices data by applying the Kalman

[1]See information at Investopedia.com. https://www.investopedia.com/terms/c/crude-oil.asp (accessed on January 11, 2020)

filter (Schwartz (1997)), or through calibration (Chen (2010)).

Fattouh (2011) provides helpful background information about the development of crude oil markets and pricing practices. Along with the collapse between 1986 and 1988 of the pricing system dominated by OPEC, spot markets for crude oil have emerged and matured. Since spot contracts are made between parties and cannot be observed by outsiders, to identify the value of each barrel of crude oil in spot markets, oil pricing reporting agencies ("PRAs"), such as S&P Global Platts and Argus Media, assess the benchmark spot prices on a daily basis based on specific methodologies described on their websites. Although there has been some concern about the accuracy and integrity of the assessed spot prices for crude oil (see Fattouh (2011)), the assessed spot price is applied worldwide as the benchmark price for futures and spot oil transactions. These spot prices of crude oil are quoted on a daily basis by EIA (U.S. Energy Information Administration). From Figure 4.1, we can see that the quoted spot prices by PRAs and the proxy of the corresponding first month futures contract prices are very close to each other. Nevertheless, in papers most closely related to our research, quoted oil spot prices are not used directly for model estimation. Gibson & Schwartz (1990) used the price of the closest maturity crude oil futures contract as the proxy of the spot price. Schwartz (1997), Schwartz & Smith (2000), Chen (2010), Insley (2017), and Al Mansour (2012) all used futures prices for estimating the dynamic of oil prices. No spot price data was needed.

With regard to the estimating methodologies, Gibson & Schwartz (1990) applied a seemingly unrelated regression and a "guess and check" approach to estimate a GBM process of crude oil prices and an OU process of convenience yields. Schwartz (1997) applied a Kalman Filter to estimate a GBM process for crude oil prices and an OU process for convenience yields. Schwartz & Smith (2000) applied a Kalman filter to estimate a process for crude oil combining a long-term Brownian Motion trend and a short-term OU oscillation. Chen (2010) and Insley (2017) conducted calibration for estimating the crude oil prices following regime switching mean-reverting processes. Al Mansour (2012) estimated the parameters of a regime switching Brownian Motion process for crude oil prices by applying a Kim filter. In all of these papers, spot prices are assumed to be unobservable and are estimated from futures prices.

To support the analysis in Chapter 2, in this chapter, we examine three alternative models of oil prices and compare their performance at fitting historical data. The parameters of the three models are estimated via either a Kalman or Kim filter. The first process will be referred to as a level mean-reverting process. Insley (2017) adopted this process in their analysis, but use a calibration procedure rather than the Kalman filter. The second model is a log mean-reverting process, which has been used by others such Schwartz (1997) and Chen (2010). As in Schwartz (1997) we adopt a Kalman filter for estimation. The

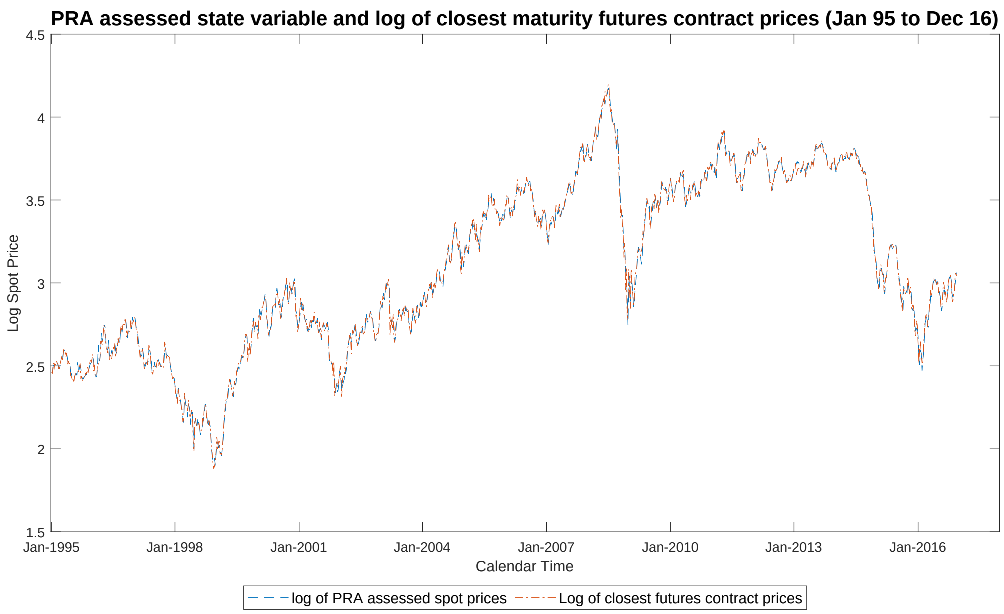

Figure 4.1: Comparison of PRA assessed crude oil spot prices and the front month futures contract prices (The data source: U.S. EIA https://www.eia.gov/dnav/pet/pet_pri_spt_s1_w.htm(accessed on January 11, 2020))

third model features regime switching and is the same model proposed by Chen (2010). Different from Chen (2010), we use the Kim filter, which is an extension of Kalman filter.

4.2 Models Specification

4.2.1 Model 1: Level Mean-reverting Process

This process is introduced in Brennan & Schwartz (1980) to model the dynamics of the interest rate and the value of the studied firm. Some applications of this model can be found in Insley & Rollins (2005) for modelling timber prices, in Chen & Forsyth (2007) for modelling natural gas prices, and in Insley (2017) for modelling the specific regime of crude oil price. According to this process, the spot prices of crude oil follow a mean-reverting process given by

$$\mathrm{d}P = \epsilon(\bar{P} - P_t)\mathrm{d}t + \sigma P_t \mathrm{d}z \tag{4.1}$$

where P_t is the spot price at time t, $\epsilon>0$ is the speed of reversion, $\bar{P}$ is the spot price level to which P tends to revert, σ is the volatility, and $\mathrm{d}z$ is the increment of a Wiener process.

4.2.2 Model 2: Log Mean-reverting Process

As noted, the log mean-reverting process is estimated by Schwartz (1997) using data from 1985 to 1995. We update his analysis using more recent data (i.e. from 1995 to 2016).

The model is given as follows:

$$\mathrm{d}P = \epsilon(\mu - \ln P_t)P_t \mathrm{d}t + \sigma P_t \mathrm{d}z \tag{4.2}$$

where P_t is the spot price at time t, $\epsilon>0$ is the speed of reversion, μ is a parameter that reflects the long run mean log spot price, σ is the volatility, and $\mathrm{d}z$ is the increment of a Wiener process.

4.2.3 Model 3: Regime Switching Log Mean-reverting Process

In the third model, we allow the μ and σ in the second model to switch between two regimes 1 and 2. That is to say, the spot price of crude oil follows a regime switching log mean-reverting process given by

$$\mathrm{d}P = \epsilon(\mu_{s_t} - \ln P_t)P_t\mathrm{d}t + \sigma_{s_t}P_t\mathrm{d}z. \tag{4.3}$$

where P_t is the crude oil spot price at time t, ϵ is the mean-reverting speed, μ is the long run mean log spot price, σ is the volatility, and $s_t = 1, 2$ is the regime status at time t. s_t switches between 1 and 2 according to a transition matrix as follows:

$$\begin{bmatrix} p^{\mathcal{P}} & 1-p^{\mathcal{P}} \\ 1-q^{\mathcal{P}} & q^{\mathcal{P}} \end{bmatrix}$$

where $p^{\mathcal{P}}$ denotes the probability of staying in regime 1 under the $\mathcal{P}$ measure and $q^{\mathcal{P}}$ denotes the probability of staying regime 2 under the $\mathcal{P}$ measure. ϵ is assumed constant across different regimes to ensure that the estimation is tractable, as is explained in Section 4.4.3, page 143.

4.3 Methodology

The development and application of the Kalman filter is well documented in the literature. In this section, we just provide a brief introduction of it. The Kalman filter (Kalman 1960, Kalman & Bucy 1961, Kalman 1963) was developed for estimation and control of aerospace systems during the Cold War period from around 1950s to 1960s. Its first application is to track the spacecraft trajectory in Apollo Moon Project. It was also soon be used in radar tracking of other aircraft such as missiles and satellites. (Mohinder & Angus 2001, Grewal & Andrews 2010) After several decades' development, the Kalman filter was applied in more areas, including physics, engineering, finance, health, and economics. The main idea of the Kalman filter is to estimate an unobservable state variable by using another observable measurement which can be expressed as a function of the unobservable one. A "state-space" is created composed of the modelled stochastic process of the unobservable state variable and the relationship between the unobservable state variable and the observable measurement. For example, in the aerospace area, the unobservable variable could the real trace of the spacecraft, and the observable measurement is the echo signal received by receiving facilities. In our research, the unobserved variable is the spot price of crude oil while the observed measurement is the futures price. Since the measurement is updated over time, which brings new information (innovation) over time, this provides information to determine the dynamic of the unobserved variable of interest. By applying the Kalman filter, the dynamic model specifying the unobserved variable can be estimated and the time series of the unobserved variable can be derived. The three models examined have different features. This fact requires the use of different filters in the "family" of Kalman Filter.

The regular Kalman filter is suitable for the cases where there is only one regime and the relationship between the state variable and measurable variable is linear. The second model has this feature. However, as will be shown later, the first model cannot been written as a linear state-space - the relationship between the state variable and the measurement variable is nonlinear - and the third model has two regimes. To cope with these two special features, we apply two extensions of the regular Kalman filter. The extended Kalman filter (described in Jazwinski (2007) and Sorenson (1985)) can be applied to models with a nonlinear state-space. The Kim filter (proposed by Kim (1994)), another extension of regular Kalman filter, is appropriate for the regime switching model.

4.4 Estimation of the Models

Kalman filters are used to estimate unobserved state variables' values whenever new information about the observable variable becomes available. In other words, a Kalman filter estimates the values of the state variables at time t when observable information is updated from time $t-1$ to t. To carry out this process, a so-called state-space form is created. The state-space includes a transition equation and a measurement equation. The transition equation predicts the state variables' values at time t given their values at time $t-1$. The measurement equation updates the state variables' values given the latest measurable variables' values. With the state-space specified, the filtering process can be carried out by conducting prediction and updating repeatedly as time goes by. In this section we briefly describe the process of obtaining the measurement and transition equations for each model. For more details on the Kalman filter, extended Kalman filter and Kim filter the reader is referred to Harvey (1990), Haykin (2004), and Kim (1994).

4.4.1 Model 1

4.4.1.1 State-space Construction

The level mean-reverting process cannot be expressed in a linear discrete form. Hence we transform it into a nonlinear local volatility form which is used as the transition equation for composing the state-space system.
Define $X_t = \ln P_t$, then applying Ito's Lemma,

$$\mathrm{d}X = \left(\epsilon \bar{P} e^{-X_t} - \epsilon - \frac{\sigma^2}{2}\right)\mathrm{d}t + \sigma \mathrm{d}Z_t$$

Applying the Euler Maruyama approximation, we can write the above process as the following discrete form:

$$X_t = X_{t-1} + \left(\epsilon \bar{P} e^{-X_{t-1}} - \epsilon - \frac{\sigma^2}{2}\right)\Delta t + v_t$$

where $v_t \sim \mathcal{N}(0, \sigma^2 \Delta t)$. Therefore, the transition equation is of the following form:

$$\underset{(1\times 1)}{X_t} = \underset{(1\times 1)}{g(X_{t-1})} + \underset{(1\times 1)}{v_t}$$

where

$$g(X_t) = X_t + \left(\epsilon \bar{P} e^{-X_t} - \epsilon - \frac{\sigma^2}{2}\right)\Delta t, \quad v_t \sim \mathcal{N}(0, \sigma^2 \Delta t) \tag{4.4}$$

To derive the relationship between spot prices and futures prices, we transform the stochastic process for the spot price, expressed as Equation (4.1), to the risk-neutral form:

$$\mathrm{d}P = \epsilon(\bar{P} - \lambda - P_t)\mathrm{d}t + \sigma P_t \mathrm{d}Z_t^*$$

where λ is the market price of risk, and $\mathrm{d}Z_t^*$ is the increment of a Wiener process under the $\mathcal{Q}$ measure.
Let $F_{T,t}$ denote the futures price at time t with time to maturity of T. It can be shown that the futures price $F_{T,t}$ is the expected spot price at time T, in the risk neutral world. The following relationship can be derived. (The details of the derivation are shown in Appedix C.)

$$F_{T,t} = \mathbb{E}_t(P_{t+T}) = (\bar{P} - \lambda)(1 - e^{-\epsilon T}) + P_t e^{-\epsilon T}$$

Hence the measurement equation, for a vector of N futures prices, is

$$\underset{(N\times 1)}{Y_t} = \underset{(N\times 1)}{h(X_t)} + \underset{(N\times 1)}{\boldsymbol{w}_t}$$

where

$$Y_t = \begin{pmatrix} F_{T_1,t} \\ F_{T_2,t} \\ \vdots \\ F_{T_N,t} \end{pmatrix}, \quad h(X_t) = \begin{pmatrix} (\bar{P}-\lambda)(1-e^{-\epsilon T_1}) + e^{X_t-\epsilon T_1} \\ (\bar{P}-\lambda)(1-e^{-\epsilon T_2}) + e^{X_t-\epsilon T_2} \\ \vdots \\ (\bar{P}-\lambda)(1-e^{-\epsilon T_N}) + e^{X_t-\epsilon T_N} \end{pmatrix}, \quad \boldsymbol{w}_t = \begin{pmatrix} w_{t1} \\ w_{t2} \\ \vdots \\ w_{tN} \end{pmatrix}, \tag{4.5}$$

where N is the number of futures contracts. $\boldsymbol{w}_t$ is the vector of measurement errors with the mean of $\mathbb{E}[\boldsymbol{w}_t] = \mathbf{0}$ and the covariance matrix of $\Sigma(\boldsymbol{w}_t) = R$. In addition, we assume that $\boldsymbol{w}_t$ follows a multivariate normal distribution.

Therefore, the state-space system of model 1 can be written as:

$$\text{Transition equation:} \qquad \underset{(1\times 1)}{X_{t+1}} = \underset{(1\times 1)}{g(X_t)} + \underset{(1\times 1)}{v_{t+1}}$$

$$\text{Measurement equation:} \qquad \underset{(N\times 1)}{Y_t} = \underset{(N\times 1)}{h(X_t)} + \underset{(N\times 1)}{\boldsymbol{w}_t}$$

where $g(X_t)$ is defined in Equation (4.4) and $h(X_t)$ is defined in Equation (4.5).

$$\mathbb{E}(v_t v_\tau') = \begin{cases} \underset{(1\times 1)}{Q} & \text{for } t=\tau \\ \underset{(1\times 1)}{0} & \text{otherwise} \end{cases}$$

$$\mathbb{E}(\boldsymbol{w}_t \boldsymbol{w}_\tau') = \begin{cases} \underset{(N\times N)}{R} & \text{for } t=\tau \\ \underset{(N\times N)}{\mathbf{0}} & \text{otherwise} \end{cases}$$

where $Q = \sigma^2 \Delta t$ is the variance of v_t and R is an $(N \times N)$ covariance matrix of the $(N \times 1)$ random vector $\boldsymbol{w}_t$.

4.4.1.2 Filtering Process

The filtering process applying the extended Kalman filter is conducted in following two steps (refer to Haykin (2004) for details).

- Updating step

Start with a initial value for the unobservable state variable $\hat{X}_{1|0}$, where $1|0$ in the subscript means that this initial value is the predicted value at time 1 given the available information up to time 0 (i.e. there are not any observed values for the measurement variables. (We use the logarithm price of the futures contract closest to maturity at the beginning of the examined period as the initial value.). The state variable X is updated by the following formula given the new information provided by measurement Y_t:

$$\underset{(1\times 1)}{\hat{X}_{t|t}} = \underset{(1\times 1)}{\hat{X}_{t|t-1}} + \underset{(1\times 1)}{\Gamma_{t|t-1}} \cdot \underset{(1\times N)}{J_t^{h\prime}} \cdot \underset{(N\times N)}{MSE(\hat{Y}_{t|t-1})^{-1}} \cdot (\underset{(N\times 1)}{Y_t} - \underset{(N\times 1)}{\hat{Y}_{t|t-1}})$$

where the estimate of the measurement $Y_{1|0}$ can be obtained by using the measurement equation in the state-space system:

$$\underset{(N\times 1)}{\hat{Y}_{t|t-1}} = \underset{(N\times 1)}{h(\hat{X}_{t|t-1})} \tag{4.6}$$

$\Gamma_{t|t-1}$ is the mean squared error (MSE) of the state variable's forecast and $MSE(\hat{Y}_{t|t-1})$ is the MSE of the measurement variable's forecast:

$$\underset{(1\times 1)}{\Gamma_{t|t-1}} \equiv \mathbb{E}[(\underset{(1\times 1)}{X_t} - \underset{(1\times 1)}{\hat{X}_{t|t-1}})^2]$$

$$\underset{(N\times N)}{MSE(\hat{Y}_{t|t-1})} \equiv \mathbb{E}\Big[(\underset{(N\times 1)}{Y_t} - \underset{(N\times 1)}{\hat{Y}_{t|t-1}})(\underset{(N\times 1)}{Y_t} - \underset{(N\times 1)}{\hat{Y}_{t|t-1}})'\Big] = \underset{(N\times 1)}{J_t^h} \cdot \underset{(1\times 1)}{\Gamma_{t|t-1}} \cdot \underset{(1\times N)}{J_t^{h\prime}} + \underset{(N\times N)}{R}$$

where J_t^h is the Jacobian matrix of $h(X_t)$:

$$J_t^h = \frac{\partial h}{\partial x}\bigg|_{\hat{X}_{t|t-1}}$$

For the initial value of $\Gamma_{1|0}$, we tried various values and choose the one that leads to the highest likelihood.

Then the MSE of $\hat{X}_{t|t}$ can be obtained as follows:

$$\underset{(1\times 1)}{\Gamma_{t|t}} = \underset{(1\times 1)}{\Gamma_{t|t-1}} - \underset{(1\times 1)}{\Gamma_{t|t-1}} \cdot \underset{(1\times N)}{J_t^{h\prime}} \cdot \underset{(N\times N)}{MSE(\hat{Y}_{t|t-1})}^{-1} \cdot \underset{(N\times 1)}{J_t^h} \cdot \underset{(1\times 1)}{\Gamma_{t|t-1}}$$

- Prediction step

Having obtained the updated state variable and its MSE embodying the information up to the current time t, the predicted state variable X for time $t+1$ can be obtained by applying the transition equation in the state-space system:

$$\underset{(1\times 1)}{\hat{X}_{t+1|t}} = \underset{(1\times 1)}{g(\hat{X}_{t|t})}$$

The MSE of predicted X for time $t+1$ can be derived applying the following formula:

$$\underset{(1\times 1)}{\Gamma_{t+1|t}} = \underset{(1\times 1)}{J_{t+1}^g} \cdot \underset{(1\times 1)}{\Gamma_{t|t}} \cdot \underset{(1\times 1)}{J_{t+1}^{g}{}'} + \underset{(1\times 1)}{Q}$$

where J_{t+1}^g is the Jacobian matrix of $g(X_t)$:

$$J_{t+1}^g = \left.\frac{\partial g}{\partial x}\right|_{\hat{X}_{t|t}}$$

By applying above filters, the predicted Y, which is produced from updated estimates for the state variable, and the corresponding MSE can be produced iteratively for each time step t, where $t = 1, 2, \cdots, T$. Since $\boldsymbol{w}_t$ follows a multivariate normal distribution, according to the measurement equation and Equation (4.6), $Y_t - \hat{Y}_{t|t-1}$ also follows a multivariate normal distribution $\mathcal{N}(0, MSE)$. Then the density function of $Y_t - \hat{Y}_{t|t-1}$ for each time step is as follows:

$$f(Y_t - \hat{Y}_{t|t-1}) = \frac{1}{\sqrt{(2\pi)^N \cdot \left|MSE(\hat{Y}_{t|t-1})\right|}} \exp\left(-\frac{(Y_t - \hat{Y}_{t|t-1})^2}{2 \cdot MSE(\hat{Y}_{t|t-1})}\right)$$

For the whole period that is examined, the likelihood is:

$$\prod_{t=1}^{T} f(Y_t - \hat{Y}_{t|t-1}) = \prod_{t=1}^{T} \frac{1}{\sqrt{(2\pi)^N \cdot \left| MSE(\hat{Y}_{t|t-1}) \right|}} \exp\left(-\frac{(Y_t - \hat{Y}_{t|t-1})^2}{2 \cdot MSE(\hat{Y}_{t|t-1})} \right)$$

Taking logarithm of the likelihood function, we obtain the following log-likelihood function:

$$LL = \sum_{t=1}^{T} \left[-\frac{N}{2} \cdot \ln(2\pi) - \frac{1}{2} \cdot \ln \left| MSE(\hat{Y}_{t|t-1}) \right| - \frac{1}{2} \cdot (Y_t - \hat{Y}_{t|t-1})' \cdot MSE(\hat{Y}_{t|t-1})^{-1} \cdot (Y_t - \hat{Y}_{t|t-1}) \right]$$

The estimates of the parameters, i.e. $\bar{P}$, ϵ, σ, and λ, are the parameter values that maximize the log-likelihood function. This log-likelihood function cannot be maximized analytically. Instead, the unknown parameters are estimated numerically.

4.4.2 Model 2

4.4.2.1 State-space Construction

First, in order to derive the transition equation, we define $X_t = \ln P_t$, and apply Ito's lemma,

$$\mathrm{d}X = \epsilon \left[\left(\mu - \frac{\sigma^2}{2\epsilon} \right) - X_t \right] \mathrm{d}t + \sigma \mathrm{d}Z_t$$

Then the above stochastic differential equation can be written in an exact discrete form:

$$X_t = \left(\mu - \frac{\sigma^2}{2\epsilon} \right) \left(1 - e^{-\epsilon \Delta t} \right) + e^{-\epsilon \Delta t} X_{t-1} + v_t$$

where $v_t \sim \mathcal{N}\left(0, \frac{(1-e^{-2\epsilon\Delta t})\sigma^2}{2\epsilon}\right)$,

or an approximate discrete form:

$$X_t = \epsilon \left(\mu - \frac{\sigma^2}{2\epsilon} \right) \Delta t + (1 - \epsilon \Delta t) X_{t-1} + v_t$$

where $\mathbb{E}[v_t] = 0$ and $Var[v_t] = \sigma^2 \Delta t$.

Following Schwartz (1997), we choose the approximate version for the subsequent discussion.

Therefore, the transition equation can be written as

$$\underset{(1\times 1)}{X_t} = \underset{(1\times 1)}{C} + \underset{(1\times 1)}{G} \cdot \underset{(1\times 1)}{X_{t-1}} + \underset{(1\times 1)}{v_t}$$

where

$$C = \epsilon\left(\mu - \frac{\sigma^2}{2\epsilon}\right)\Delta t, \qquad G = (1-\epsilon\Delta t)\,, \qquad v_t \sim \mathcal{N}\left(0, \sigma^2\Delta t\right) \tag{4.7}$$

To acquire the measurement equation, first, convert the stochastic process to a risk-neutral form by subtracting the risk premium λ from the original mode:

$$\mathrm{d}X_t = \epsilon\left[\left(\mu - \frac{\sigma^2}{2\epsilon} - \lambda\right) - X_t\right]\mathrm{d}t + \sigma \mathrm{d}Z_t{}^*$$

where $\mathrm{d}Z_t{}^*$ is the increment of a Wiener process under the $\mathcal{Q}$ measure. According to Dixit & Pindyck (1994), the mean and variance of X_T at time t can be derived as:

$$\mathbb{E}_t\left[X_T\right] = e^{-\epsilon T}X_t + \left(1 - e^{-\epsilon T}\right)\left(\mu - \frac{\sigma^2}{2\epsilon} - \lambda\right)$$

$$Var\left[X_T\right] = \frac{\sigma^2}{2\epsilon}\left(1 - e^{-2\epsilon T}\right)$$

As in Schwartz (1997), by taking expectation of P_T from the perspective of time t in the risk-neutral environment, the price of a futures contract with a time to maturity of T can be expressed as follows.

$$F_{T,t} = \mathbb{E}_t^{\mathcal{Q}}\left[P_T\right] = \exp\left(\mathbb{E}_t^{\mathcal{Q}}\left[\ln P_T\right] + \frac{1}{2}Var^{\mathcal{Q}}\left[\ln P_T\right]\right)$$

where the superscript $\mathcal{Q}$ means that the expectation and variance are taken with regard to the risk neutral process.

The second equal sign comes from the fact that P_T follows a log normal distribution. Taking logarithm on both sides of the above equation, we can get:

$$\begin{aligned}
\ln F_{T,t} &= \mathbb{E}_t^{\mathcal{Q}}\left[\ln P_T\right] + \frac{1}{2}\cdot Var^{\mathcal{Q}}\left[\ln P_T\right] \\
&= \mathbb{E}_t\left[X_T\right] + \frac{1}{2}\cdot Var\left[X_T\right] \\
&= e^{-\epsilon T}X_t + \left(1 - e^{-\epsilon T}\right)\left(\mu - \frac{\sigma^2}{2\epsilon} - \lambda\right) + \frac{\sigma^2}{4\epsilon}\left(1 - e^{-2\epsilon T}\right)
\end{aligned}$$

The measurement equation can be constructed accordingly:

$$\underset{(N\times 1)}{Y_t} = \underset{(N\times 1)}{d} + \underset{(N\times 1)}{H'} \cdot \underset{(1\times 1)}{X_t} + \underset{(N\times 1)}{\boldsymbol{w}_t}$$

where

$$Y_t = \begin{pmatrix} \ln F_{T_1,t} \\ \ln F_{T_2,t} \\ \vdots \\ \ln F_{T_N,t} \end{pmatrix}, \; d = \begin{pmatrix} (1-e^{-\epsilon T_1})(\mu - \frac{\sigma^2}{2\epsilon} - \lambda) + \frac{\sigma^2}{4\epsilon}(1-e^{-2\epsilon T_1}) \\ (1-e^{-\epsilon T_2})(\mu - \frac{\sigma^2}{2\epsilon} - \lambda) + \frac{\sigma^2}{4\epsilon}(1-e^{-2\epsilon T_2}) \\ \vdots \\ (1-e^{-\epsilon T_N})(\mu - \frac{\sigma^2}{2\epsilon} - \lambda) + \frac{\sigma^2}{4\epsilon}(1-e^{-2\epsilon T_N}) \end{pmatrix}, \; H' = \begin{pmatrix} e^{-\epsilon T_1} \\ e^{-\epsilon T_2} \\ \vdots \\ e^{-\epsilon T_N} \end{pmatrix}, \; \boldsymbol{w}_t = \begin{pmatrix} w_{t1} \\ w_{t2} \\ \vdots \\ w_{tN} \end{pmatrix}, \tag{4.8}$$

and N is the number of futures contracts.

Therefore, the state-space system representing Model 2 is as follows:

$$\text{Transition equation:} \qquad \underset{(1\times 1)}{X_{t+1}} = \underset{(1\times 1)}{C} + \underset{(1\times 1)}{G} \cdot \underset{(1\times 1)}{X_t} + \underset{(1\times 1)}{v_{t+1}}$$

$$\text{Measurement equation:} \qquad \underset{(N\times 1)}{Y_t} = \underset{(N\times 1)}{d} + \underset{(N\times 1)}{H'} \cdot \underset{(1\times 1)}{X_t} + \underset{(N\times 1)}{\boldsymbol{w}_t}$$

where C and G are defined in Equations 4.7 and d and H' are defined in Equations 4.8.

$$\mathbb{E}(v_t v_\tau') = \begin{cases} \underset{(1\times 1)}{Q} & \text{for } t = \tau \\ \underset{(1\times 1)}{0} & \text{otherwise} \end{cases}$$

$$\mathbb{E}(\boldsymbol{w}_t \boldsymbol{w}_\tau') = \begin{cases} \underset{(N\times N)}{R} & \text{for } t = \tau \\ \underset{(N\times N)}{\mathbf{0}} & \text{otherwise} \end{cases}$$

Since the state-space system is composed of linear equations, the regular Kalman filter is capable of estimating this model.

4.4.2.2 Filtering Process

The filtering process is conducted according to the following steps as detailed in Harvey (1990):

- Updating step:

We set the logarithm price of the futures contract closest to maturity as the initial value for $\hat{X}_{t|t-1}$ for $t = 1$: $\hat{X}_{1|0}$. In addition we choose the initial value of the MSE of the initial state variable prediction $\Gamma_{t|t-1}$ when $t = 1$ that makes the log-likelihood highest: $\Gamma_{1|0}$. The following formula is adopted to update the value of the state variable X and the MSE Γ of updated X when the measurement at time t (i.e. the new information) is available.

$$\underset{(1\times 1)}{X_{t|t}} = \underset{(1\times 1)}{\hat{X}_{t|t-1}} + \underset{(1\times 1)}{\Gamma_{t|t-1}} \cdot \underset{(1\times N)}{H} \cdot \underset{(N\times N)}{MSE(\hat{Y}_{t|t-1})^{-1}} \cdot (\underset{(N\times 1)}{Y_t} - \underset{(N\times 1)}{\hat{Y}_{t|t-1}})$$

$$\underset{(1\times 1)}{\Gamma_{t|t}} = \underset{(1\times 1)}{\Gamma_{t|t-1}} - \underset{(1\times 1)}{\Gamma_{t|t-1}} \cdot \underset{(N\times 1)}{H} \cdot \underset{(N\times N)}{MSE(\hat{Y}_{t|t-1})^{-1}} \cdot \underset{(N\times 1)}{H'} \cdot \underset{(1\times 1)}{\Gamma_{t|t-1}}$$

where Y_t can be obtained from the actual futures prices. $\hat{Y}_{t|t-1}$ can be obtained by using the measurement equation:

$$\underset{(N\times 1)}{\hat{Y}_{t|t-1}} = \underset{(N\times 1)}{d} + \underset{(N\times 1)}{H'} \cdot \underset{(1\times 1)}{\hat{X}_{t|t-1}}$$

$MSE(\hat{Y}_{t|t-1})$ can be obtained by applying the following formula:

$$\underset{(N\times N)}{MSE(\hat{Y}_{t|t-1})} = \underset{(N\times 1)}{H'} \cdot \underset{(1\times 1)}{\Gamma_{t|t-1}} \cdot \underset{(1\times N)}{H} + \underset{(N\times N)}{R}$$

- Prediction step:

Given $X_{t|t}$ and $\Gamma_{t|t}$ by the updating step, the prediction of the value of X at time $t + 1$ based on all information up to time t can be obtained by applying the transition equation:

$$\underset{(1\times 1)}{\hat{X}_{t+1|t}} = \underset{(1\times 1)}{C} + \underset{(1\times 1)}{G} \cdot \underset{(1\times 1)}{X_{t|t}}$$

The MSE of $\hat{X}_{t+1|t}$ can be calculated using the following formula:

$$\underset{(1\times 1)}{\Gamma_{t+1|t}} = \underset{(1\times 1)}{G} \cdot \underset{(1\times 1)}{\Gamma_{t|t}} \cdot \underset{(1\times 1)}{G'} + \underset{(1\times 1)}{Q}$$

By the above filtering process, in each time step, $\hat{Y}_{t|t-1}$ and $MSE(\hat{Y}_{t|t-1})$ can be obtained. Since $\boldsymbol{w}_t$ is normally distributed, similarly as the inferring in Model 1, the log-likelihood function is the same as the one for Model 1.

$$LL = \sum_{t=1}^{T} \left[-\frac{N}{2} \cdot \ln(2\pi) - \frac{1}{2} \cdot \ln \left| MSE(\hat{Y}_{t|t-1}) \right| - \frac{1}{2} \cdot (Y_t - \hat{Y}_{t|t-1})' \cdot MSE(\hat{Y}_{t|t-1})^{-1} \cdot (Y_t - \hat{Y}_{t|t-1}) \right]$$

The estimates of the parameters, i.e. μ, ϵ, σ, and λ, are the parameter values that maximize the log-likelihood function.

4.4.3 Model 3

4.4.3.1 State-space Construction

In this model, to derive the transition equation, we first define $X_t = \ln P_t$ and apply Ito's lemma,

$$\mathrm{d}X = \epsilon \left[\left(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} \right) - X_t \right] \mathrm{d}t + \sigma_{s_t} \mathrm{d}Z_t$$

(Note: $\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} = \ln \bar{P}_{s_t}$, where $\bar{P}$ is the long run mean crude oil price.)

Then the above stochastic differential equation can be written in an approximate discrete form[2]:

$$X_t^{(s_t)} = \epsilon \left(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} \right) \Delta t + (1 - \epsilon \Delta t)\, X_{t-1} + v_t$$

where $\mathbb{E}[v_t] = 0$ and $Var[v_t] = \sigma_{s_t}^2 \Delta t$.

$X_t^{(s_t)}$ represents the logarithm of price at time t if the regime is s_t.

Therefore, the transition equation can be written as follows.

$$\underset{(1\times 1)}{X_t^{(s_t)}} = \underset{(1\times 1)}{\alpha^{(s_t)}} + \underset{(1\times 1)}{\beta^{(s_t)}}\, \underset{(1\times 1)}{X_{t-1}} + \underset{(1\times 1)}{v_t}$$

[2] We apply the approximate form following Schwartz (1997) as we did for Model 2 in Section 4.4.2.1.

where

$$\alpha^{(s_t)} = \epsilon\left(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon}\right)\Delta t, \qquad \beta^{(s_t)} = (1 - \epsilon\Delta t), \qquad v_t \sim \mathcal{N}\left(0, \sigma_{s_t}^2 \Delta t\right)$$

To infer the measurement equation, we relate the logarithm of futures prices $\ln F_{T,t}^{(s_t)}$ (where T is the time to expiry of the futures contract at time t, when the futures price is calculated, and the superscript (s_t) denotes the regime at time t) to the logarithm of spot prices $X_t^{(s_t)}$. To this end, we need to use the expression of $X_t^{(s_t)}$ in the risk neutral world.

$$\underset{(1\times1)}{X_t^{(s_t)}} = \underset{(1\times1)}{\alpha^{(s_t)*}} + \underset{(1\times1)}{\beta^{(s_t)}}\,\underset{(1\times1)}{X_{t-1}} + \underset{(1\times1)}{v_t}$$

where

$$\alpha^{(s_t)*} = \epsilon\left(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\right)\Delta t, \qquad \beta^{(s_t)} = (1 - \epsilon\Delta t), \qquad v_t \sim \mathcal{N}\left(0, \sigma_{s_t}^2 \Delta t\right) \tag{4.9}$$

where λ_{s_t} is the risk premium for regime s_t under the $\mathcal{Q}$ measure. The transition probability matrix under $\mathcal{Q}$ measure is:

$$\begin{bmatrix} \pi^{11} & \pi^{12} \\ \pi^{21} & \pi^{22} \end{bmatrix} = \begin{bmatrix} p^{\mathcal{Q}} & 1 - p^{\mathcal{Q}} \\ 1 - q^{\mathcal{Q}} & q^{\mathcal{Q}} \end{bmatrix} \tag{4.10}$$

Where π^{ij} refers to the probability of switching from regime i to regime j under the $\mathcal{Q}$ measure. Then, assume there is an affine relationship between $\ln F_{T,t}^{(s_t)}$ and $X_t^{(s_t)}$

$$\ln F_{T,t}^{(s_t)} = A_T^{(s_t)} + B_T^{(s_t)} \cdot X_t^{(s_t)} \tag{4.11}$$

i.e.

$$F_{T,t}^{(s_t)} = \exp(A_T^{(s_t)} + B_T^{(s_t)} \cdot X_t^{(s_t)}) \tag{4.12}$$

The superscript (s_t) of A_T and B_T reflects the fact that both coefficients vary over time, depending on the realization of s_t. Therefore we need to express A_T and B_T in terms of the set of parameters (i.e. ϵ, μ, λ, and σ) corresponding to the regime at time t.

To find out the explicit form of $A_T^{(s_t)}$ and $B_T^{(s_t)}$, we use the following comparing coefficients method as is used in Al Mansour (2012).

As explained in Al Mansour (2012), in an arbitrage free market, we can get the relationship between two consecutive futures prices shown in Equation (4.13). This follows from the following derivation. A futures contract entered at time t has a price of $F_{T,t}^{(s_t)}$ at time t and has a price of $F_{T-1,t+1}^{(s_{t+1})}$ at time $t+1$. The payoff of the futures contract is $F_{T-1,t+1}^{(s_{t+1})} - F_{T,t}^{(s_t)}$ at time $t+1$. Under the martingale measurement, the discounted value of the expected payoff must equal the cost of entering the futures contract, which is zero. Therefore, $0 = e^{-r\Delta t} \cdot \mathbb{E}^{\mathcal{Q}}\left[F_{T-1,t+1}^{(s_{t+1})} - F_{T,t}^{(s_t)}|X_t^{(s_t)}\right]$, where r is the risk free interest rate. Rearranging the equation, we can obtain the aforesaid relationship between two consecutive futures prices.

$$F_{T,t}^{(s_t)} = \mathbb{E}^{\mathcal{Q}}\left[F_{T-1,t+1}^{(s_{t+1})}|X_t^{(s_t)}\right] \tag{4.13}$$

The left hand side of Equation (4.13) can be expressed as Equation (4.12). The right hand side of Equation (4.13) can be expressed as follows.

$$\begin{aligned}
&\mathbb{E}^{\mathcal{Q}}\left[F_{T-1,t+1}^{(s_{t+1})}\middle|X_t^{(s_t)}\right] \\
&= \mathbb{E}^{\mathcal{Q}}\left[\exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}^{(s_{t+1})}X_{t+1}\right)\middle|X_t^{(s_t)}\right] \\
&= \sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}}\mathbb{E}^{\mathcal{Q}}\left[\exp(A_{T-1}^{(s_{t+1})} + B_{T-1}^{(s_{t+1})}X_{t+1}^{(s_{t+1})})\middle|X_t^{(s_t)}\right] \\
&= \sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}}\mathbb{E}^{\mathcal{Q}}\left[\exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}^{(s_{t+1})}\Big(\epsilon\big(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\big)\Delta t + \big(1-\epsilon\Delta t\big)X_t^{(s_t)} + v_{t+1}\Big)\right)\right] \\
&= \sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}}\exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}^{(s_{t+1})}\Big(\epsilon\big(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\big)\Delta t + \big(1-\epsilon\Delta t\big)X_t^{(s_t)}\Big)\right)\mathbb{E}^{\mathcal{Q}}\left[e^{v_{t+1}}\right] \\
&= \sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}}\exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}^{(s_{t+1})}\Big(\epsilon\big(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\big)\Delta t + \big(1-\epsilon\Delta t\big)X_t^{(s_t)}\Big)\right)\exp\left(\frac{\sigma_{s_{t+1}}^2\Delta t}{2}\right)
\end{aligned} \tag{4.14}$$

where $\pi^{s_t s_{t+1}}$ denotes the element on row s_t and column s_{t+1} of the $\mathcal{Q}$ measure transition matrix (4.10).

Since the left hand sides of Equations (4.12) and (4.14) are equal according to Equation (4.13), the right hand sides, and in turn the log of the right hand sides must be equal, too.

$$A_T^{(s_t)} + B_T^{(s_t)} X_t^{(s_t)}$$

$$= \ln\left(\sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}} \exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}^{(s_{t+1})}\Big(\epsilon\big(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\big)\Delta t + \big(1-\epsilon\Delta t\big)X_t^{(s_t)}\Big)\right)\exp\left(\frac{\sigma_{s_{t+1}}^2 \Delta t}{2}\right)\right) \tag{4.15}$$

Our task is to derive expressions for A_T and B_T in terms of A_{T-1}, B_{T-1}, and parameters for the specific regime at time t. We hope we can reformulate the right hand side of Equation (4.15) so that it can be expressed as a summation of a constant term and a term with $X_t^{(s_t)}$. Then by directly comparing the constant term and the coefficient of $X_t^{(s_t)}$ of both sides, we can derive the expression of A_T and B_T.

To this end, we assume that B_{T-1} is regime independent, i.e. $B_{T-1}^{(1)} = B_{T-1}^{(2)}$. Recall thet in Section 4.2.2 (page 130) we assume the speed of mean-reverting ϵ is regime independent. The intention of this assumption is also for the mathematical derivation of A_T and B_T. With these assumptions, we can transform Equation (4.15) to the following form.

$$A_T^{(s_t)} + B_T^{(s_t)} X_t^{(s_t)}$$
$$= \ln\left(\sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}} \exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}\epsilon\Big(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\Big)\Delta t + \frac{\sigma_{s_{t+1}}^2 \Delta t}{2}\right)\right) + B_{n-1}(1-\epsilon\Delta t)X_t^{(s_t)}$$

Comparing the constant terms on both sides, we obtain:

$$A_T^{(s_t)} = \ln\left(\sum_{s_{t+1}=1}^{2} \pi^{s_t s_{t+1}} \exp\left(A_{T-1}^{(s_{t+1})} + B_{T-1}\epsilon\Big(\mu_{s_t} - \frac{\sigma_{s_t}^2}{2\epsilon} - \lambda_{s_t}\Big)\Delta t + \frac{\sigma_{s_{t+1}}^2 \Delta t}{2}\right)\right)$$

Comparing the coefficients of X_t, we obtain:

$$B_T = B_{T-1}(1-\epsilon\Delta t)$$

Since at the expiry date $t+T$, the futures price equals the spot price, we have

$$\ln F_{0,t+T}^{(s_{t+T})} = X_{t+T}^{(s_{t+T})}$$

according to Equation (4.11), we can obtain

$$A_0^{s_{t+T}} = 0, \quad B_0 = 1$$

Then given specific sets of parameters (i.e. ϵ, μ_{s_t}, λ_{s_t}, σ_{s_t}, $p^{\mathcal{Q}}$, and $q^{\mathcal{Q}}$), $A_i^{(s_{t+T-i})}$ and B_i $(i = 1, 2, \ldots, T)$ can be obtained.

We acknowledge that the assumptions of regime independent B_i and ϵ are at the probable cost of the accuracy of the estimation if they actually vary across regimes. It is a problem that has a potential to be improved in the future research. Nevertheless, it turns out, which will be shown in the subsequent sections, that the performance of Model 3 in forecasting futures prices is an improvement compared to Models 1 and Model 2.

The measurement equation can be written as follows.

$$\underset{(N\times 1)}{Y_t^{(s_t)}} = \underset{(N\times 1)}{A^{(s_t)}} + \underset{(N\times 1)}{B} \cdot \underset{(1\times 1)}{X_t^{(s_t)}} + \underset{(N\times 1)}{\boldsymbol{w}_t}$$

where

$$Y_t = \begin{pmatrix} \ln F_{T_1,t} \\ \ln F_{T_2,t} \\ \vdots \\ \ln F_{T_N,t} \end{pmatrix}, \quad A^{(s_t)} = \begin{pmatrix} A_{T_1}^{(s_t)} \\ A_{T_2}^{(s_t)} \\ \vdots \\ A_{T_N}^{(s_t)} \end{pmatrix}, \quad B = \begin{pmatrix} B_{T_1} \\ B_{T_2} \\ \vdots \\ B_{T_N} \end{pmatrix}, \quad \boldsymbol{w}_t = \begin{pmatrix} w_{t1} \\ w_{t2} \\ \vdots \\ w_{tN} \end{pmatrix},$$

and $\boldsymbol{w}_t \sim \mathcal{N}(\mathbf{0}, R)$, N is the number of futures contracts, and $T_1, T_2, \ldots, T_n$ are the times to maturity of N futures contracts respectively.

Therefore, the state-space system of Model 3 is as follows.

$$\text{Transition equation:} \qquad \underset{(1\times 1)}{X_t^{(s_t)}} = \underset{(1\times 1)}{\alpha^{(s_t)}} + \underset{(1\times 1)}{\beta^{(s_t)}} \underset{(1\times 1)}{X_{t-1}} + \underset{(1\times 1)}{v_t}$$

$$\text{Measurement equation:} \qquad \underset{(N\times 1)}{Y_t^{(s_t)}} = \underset{(N\times 1)}{A^{(s_t)}} + \underset{(N\times 1)}{B} \cdot \underset{(1\times 1)}{X_t^{(s_t)}} + \underset{(N\times 1)}{\boldsymbol{w}_t}$$

where

$$\mathbb{E}(v_t v_\tau') = \begin{cases} \underset{(1\times 1)}{Q} & \text{for } t = \tau \\ \underset{(1\times 1)}{0} & \text{otherwise} \end{cases}$$

$$\mathbb{E}(\boldsymbol{w}_t \boldsymbol{w}_\tau') = \begin{cases} \underset{(N \times N)}{R} & \text{for } t = \tau \\ \underset{(N \times N)}{\mathbf{0}} & \text{otherwise} \end{cases}$$

where $Q = \sigma^2 \Delta t$ is the variance of v_t and R is an $(N \times N)$ covariance matrix of the $(N \times 1)$ random vector $\boldsymbol{w}_t$.

4.4.3.2 Filtering Process

According to Kim (1994), the filtering algorithm is as follows.

For both regimes $s_0 = 1, 2$, the initial values of the state variable $X_{0|0}^{(s_0)}$ and the MSE of it $\Gamma_{0|0}^{(s_0)}$ are set to be the logarithm of the price of futures contract closest to maturity at the beginning of the examined period and the value that makes the log-likelihood the highest respectively.

- Prediction step

By applying the transition equation, we obtain the predicted value of the state variable at time $t+1$ based on the information available at time t:

$$\underset{(1 \times 1)}{\hat{X}_{t+1|t}^{(s_t, s_{t+1})}} = \underset{(1 \times 1)}{\alpha^{(s_{t+1})}} + \underset{(1 \times 1)}{\beta^{(s_{t+1})}} \cdot \underset{(1 \times 1)}{X_{t|t}^{(s_t)}} \tag{4.16}$$

The reason we use (s_t, s_{t+1}) rather than (s_{t+1}) as the superscript of the term on the left hand side and add a superscript (s_t) to $X_{t|t}$ on the right hand side is that other than when $t = 0$, $X_{t|t}$ will be different for two regimes based on the prediction that the filter made in the past time period so that for a specific s_{t+1}, there are two different $\hat{X}_{t+1|t}$ determined by $X_{t|t}^{(1)}$ and $X_{t|t}^{(2)}$ respectively.

The MSE of the predicted state variable $\hat{X}_{t+1|t}^{(s_t, s_{t+1})}$ is inferred to be:

$$\underset{(1 \times 1)}{\Gamma_{t+1|t}^{(s_t, s_{t+1})}} = \underset{(1 \times 1)}{\beta^{(s_{t+1})}} \cdot \underset{(1 \times 1)}{\Gamma_{t|t}^{(s_{t+1})}} \cdot \underset{(1 \times 1)}{\beta^{(s_{t+1})\prime}} + \underset{(1 \times 1)}{Q} \tag{4.17}$$

The second step is to update $\hat{X}^{(s_t,s_{t+1})}_{t+1|t}$ and $\Gamma^{(s_t,s_{t+1})}_{t+1|t}$ to $X^{(s_t,s_{t+1})}_{t+1|t+1}$ and $\Gamma^{(s_t,s_{t+1})}_{t+1|t+1}$ by embodying the information up to time $t+1$ (i.e. by using the measurement at time $t+1$):

To compute the updated values, we need to compute the forecast error for the measurement variable and Kalman gain:

The predicted value $\hat{Y}^{(s_t,s_{t+1})}_{t|t-1}$ can be obtained by using the measurement equation:

$$\underset{(N\times 1)}{\hat{Y}^{(s_t,s_{t+1})}_{t+1|t}} = \underset{(N\times 1)}{A^{(s_{t+1})}} + \underset{(N\times 1)}{B} \cdot \underset{(1\times 1)}{\hat{X}^{(s_t,s_{t+1})}_{t+1|t}}$$

- Updating step

After computing Y_{t+1} by using actual data for futures contracts, we can obtain the forecast error:

$$\underset{(N\times 1)}{\xi^{(s_t,s_{t+1})}_{t+1|t}} = \underset{(N\times 1)}{Y_{t+1}} - \underset{(N\times 1)}{\hat{Y}^{(s_t,s_{t+1})}_{t+1|t}}$$

The variance of $\xi^{(s_t,s_{t+1})}_{t+1|t}$ can be inferred to be:

$$\underset{(N\times N)}{\Xi^{(s_t,s_{t+1})}_{t+1}} = \underset{(N\times 1)}{B} \cdot \underset{(1\times N)}{\Gamma^{(s_t,s_{t+1})}_{t+1|t}} \cdot \underset{(1\times N)}{B'} + \underset{(N\times N)}{R}$$

The Kalman gain is defined as:

$$\underset{(1\times N)}{K^{(s_t,s_{t+1})}_{t+1}} \equiv \underset{(1\times 1)}{\Gamma^{(s_t,s_{t+1})}_{t+1|t}} \cdot \underset{(1\times N)}{B'} \cdot \underset{(N\times N)}{[\Xi^{(s_t,s_{t+1})}_{t+1}]^{-1}}$$

Then the updating can be carried out using the following formulas:

$$\underset{(1\times 1)}{X^{(s_t,s_{t+1})}_{t+1|t+1}} = \underset{(1\times 1)}{X^{(s_t,s_{t+1})}_{t+1|t}} + \underset{(1\times N)}{K^{(s_t,s_{t+1})}_{t+1}} \underset{(N\times 1)}{\xi^{(s_t,s_{t+1})}_{t+1|t}}$$

$$\underset{(1\times 1)}{\Gamma^{(s_t,s_{t+1})}_{t+1|t+1}} = (1 - \underset{(1\times N)}{K^{(s_t,s_{t+1})}_{t+1}} \cdot \underset{(N\times 1)}{B}) \cdot \underset{(1\times 1)}{\Gamma^{(s_t,s_{t+1})}_{t+1|t}}$$

Then the updated values can be used to carry out the prediction step for time $t+1$. By carrying out the two steps recursively, the state variable can be derived for two regimes for each time step.

However, the updated $X_{t+1|t+1}^{(s_t,s_{t+1})}$ and $\Gamma_{t+1|t+1}^{(s_t,s_{t+1})}$ are conditional on the regime at time t while in Equations 4.16 and 4.17, the corresponding values applied (i.e. $X_{t|t}^{(s_t)}$ and $\Gamma_{t|t}^{(s_t)}$) are not so. In fact, if in each recursive step, we apply the updated valued conditional on the lagged regime, the number of possible outcomes will reach 2^T and the filtering will become intractable. To avoid this circumstance, we follow Kim (1994) to get approximations for the two updated values not conditional on the lagged regime in each recursive step:

$$X_{t+1|t+1}^{(j)} = \frac{\sum_{i=1}^{2} Pr\left[s_t = i, s_{t+1} = j|\psi_{t+1}\right] X_{t+1|t+1}^{(i,j)}}{Pr\left[s_{t+1} = j|\psi_{t+1}\right]} \tag{4.18}$$

$$\Gamma_{t+1|t+1}^{(j)} = \frac{\sum_{i=1}^{2} Pr\left[s_t = i, s_{t+1} = j|\psi_{t+1}\right] \cdot \left[\Gamma_{t+1|t+1}^{(i,j)} + \left(X_{t+1|t+1}^{(j)} - X_{t+1|t+1}^{(i,j)}\right)\left(X_{t+1|t+1}^{(j)} - X_{t+1|t+1}^{(i,j)}\right)'\right]}{Pr\left[s_{t+1} = j|\psi_{t+1}\right]} \tag{4.19}$$

where $j = 1, 2$ is the regime at time $t+1$, and ψ_{t+1} is the information up to time $t+1$: $\psi_{t+1} = \{Y_{t+1}, Y_t, \ldots, Y_0\}$.

In order to get the value of relevant probabilities in above equations, we follow Kim (1994)'s four steps:

- Step 1
  Calculate the joint conditional probability mass function of consecutive regimes:

  $$Pr\left[s_t = i, s_{t+1} = j|\psi_t\right] = Pr\left[s_{t+1} = j|s_t = i\right] \cdot Pr\left[s_t = i|\psi_t\right]$$

  where $Pr\left[s_{t+1} = j|s_t = i\right]$ is the element at the i^{th} row and j^{th} column in the $\mathcal{P}$ measure transition probability matrix. The initial value of $Pr\left[s_t = i|\psi_t\right]$ is to be estimated.

- Step 2
  Calculate the joint conditional probability density function:

  $$f\left(Y_{t+1}, s_t = i, s_{t+1} = j|\psi_t\right) = f\left(Y_{t+1}|s_t = i, s_{t+1} = j, \psi_t\right) \cdot Pr\left[s_t = i, s_{t+1} = j|\psi_t\right]$$

  where $f\left(Y_{t+1}|s_t = i, s_{t+1} = j, \psi_t\right) = (2\pi)^{-\frac{N}{2}} \left|\Xi_{t+1}^{(i,j)}\right|^{-\frac{1}{2}} \cdot \exp\left(-\frac{1}{2} \cdot \xi_{t+1|t}^{(i,j)\,\prime} \cdot \left(\Xi_{t+1}^{(i,j)}\right)^{-1} \cdot \xi_{t+1|t}^{(i,j)}\right)$

- Step 3
  Update the joint conditional probability mass function of consecutive regimes:

$$Pr\left[s_t = i, s_{t+1} = j|\psi_{t+1}\right] = \frac{f\left(Y_{t+1}, s_t = i, s_{t+1} = j|\psi_t\right)}{f\left(Y_{t+1}|\psi_t\right)}$$

  where $f\left(Y_{t+1}|\psi_t\right) = \sum_{j=1}^{2}\sum_{i=1}^{2} f\left(Y_{t+1}, s_t = i, s_{t+1} = j|\psi_t\right)$

- Step 4
  Update the conditional probability mass function of the current regime

$$Pr\left[s_{t+1} = j|\psi_{t+1}\right] = \sum_{i=1}^{2} Pr\left[s_t = i, s_{t+1} = j|\psi_{t+1}\right]$$

By conducting the above four steps at each time step, we can collapse the 2 outcomes for $X_{t+1|t+1}$ and the 2 for $\Gamma_{t+1|t+1}$ both to one in each iteration by using the formulas 4.18 and 4.19.

We can also derive the sample conditional log-likelihood from step 3:

$$LL = \ln\left(f\left(Y_T, Y_{t-1}, ..., Y_1; \theta\middle|\psi_0\right)\right) = \sum_{t=1}^{T} \ln\left(f\left(Y_t; \theta|\psi_{t-1}\right)\right)$$

where θ is a vector composed of the unknown parameters (i.e. ϵ, μ_{s_t}, λ_{s_t}, σ_{s_t}, $p^{\mathcal{P}}$, $p^{\mathcal{Q}}$, $q^{\mathcal{P}}$, and $q^{\mathcal{Q}}$). Therefore the estimates for those parameters can be obtained by using a nonlinear optimization procedure. We use @fmincon in MATLAB to obtain the estimate.

4.5 Data

Futures contract prices are for West Texas Intermediate (WTI) as reported by DataStream. We use data for 17 contracts with different times to maturity from less than 1 month to 17 months. We apply the notation "F1" to represent the futures contract that is closest to the maturity, "F2" to represent the futures contract that is the second closest to the maturity, and so on. For a specific contract, the time to maturity changes over time. For example, one contract that currently has two months life span will have the life span of only one month a month from now on. This contrasts with the estimation procedure which requires sets of futures contracts with constant time to maturity. The time to maturity for all contracts is depicted in Figure 4.2. From the graph, we can see that each contract's time to maturity is within a small range. For example if we want to look at contracts with a time to maturity of 1 year, we can consider those contracts whose time to maturity varies between 0.9 and 1.0 year(s). We may expect the stochastic process that best describes historical data will change depending on the particular time period considered. The estimates of risk premium reflected by different futures contracts with different times to maturity are usually not the same. To observe the impacts of different studied periods and times to maturity on our estimation results, we use three data sets to proceed with our examination.

1. Contracts F1, F3, F5, F7, F9 from January 6, 1995 to December 16, 2016;
2. Contracts F1, F3, F5, F7, F9 from January 6, 2006 to December 16, 2016;
3. Contracts F1, F5, F9, F13, F17 from January 6, 2006 to December 16, 2016.

The data sets 1 and 2 above use identical groups of futures contracts in different periods. While data sets 2 and 3 examine the same time period but using different groups of contracts. Table 4.1 describes the means and standard errors of the futures contracts' prices and times to maturity. Since the purpose of our research is to find out the stochastic behaviour of real crude oil spot prices, we use Consumer Price Index ("CPI") for All Urban Consumers reported by the U.S. Bureau of Labor Statistics to convert nominal futures prices into real prices. We use data from January 6^{th}, 1995 to December 16^{th}, 2016 to create time series for the 17 contracts.

It was observed in the introduction that the WTI spot price as reported by the U.S. EIA is very similar to the one month futures contract price. Hence when examining the models' performance, we use the one month futures contract prices to approximate the true oil spot prices.

4.6 Empirical results

Table 4.2 shows the estimates for Model 1. From the results, we can see that Model 1 does not appear to provide a good fit for the data. For the whole sample period, the estimated mean reverting speed is a negative number, which makes no economic sense. As indicated by Balvers et al. (2000), without a significant finding of the reverting speed, we cannot confirm mean reversion. This negative speed implies that there is no significant

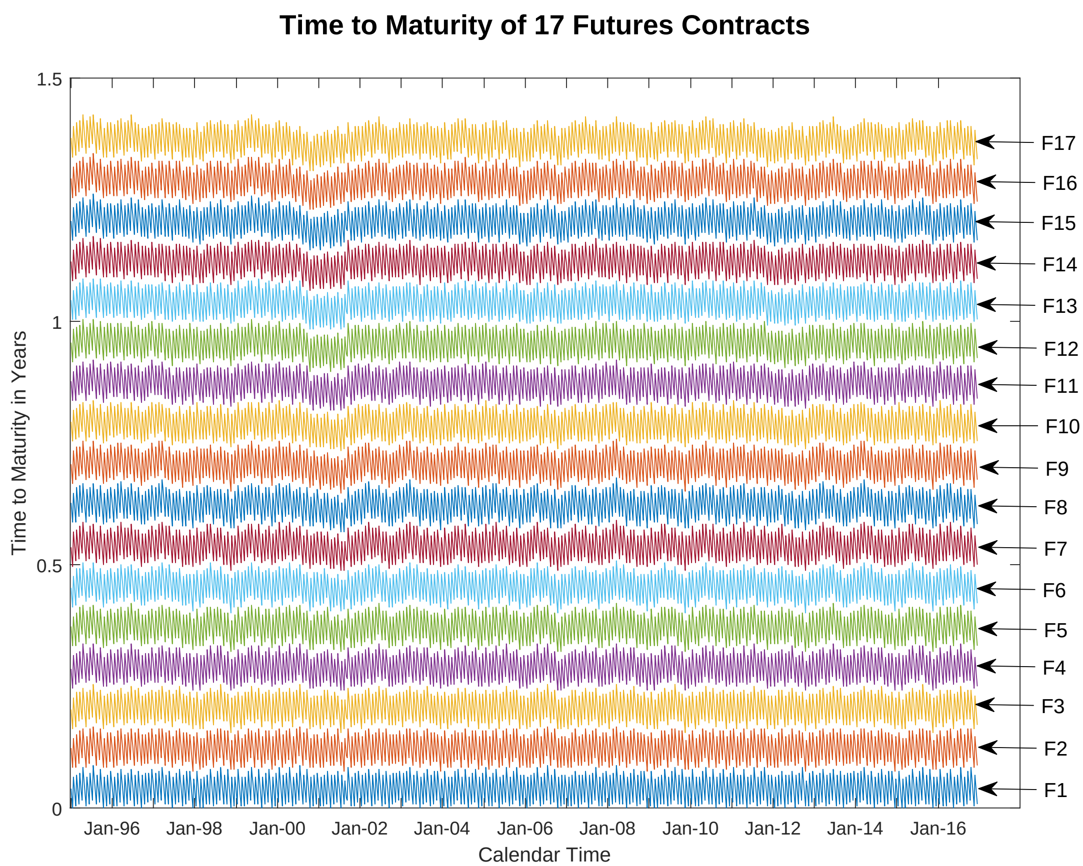

Figure 4.2: Time to maturity each week for the 17 futures contracts

Table 4.1: Summary of Futures Contracts (price in $/barrel)

Futures Contract	Mean Price ($) (Standard Error)	Mean Maturity (year) (Standard Error)
Data set 1: From January 6th, 1995 to December 16th, 2016: 1146 weekly observations		
F1	61.46(0.90)	0.04(0.00069)
F3	61.73(0.91)	0.20(0.00070)
F5	61.72(0.92)	0.36(0.00071)
F7	61.71(0.92)	0.52(0.00071)
F9	61.57(0.93)	0.68(0.00071)
Data set 2: From January 6th, 2006 to December 16th, 2016: 572 weekly observations		
F1	84.57(1.04)	0.04(0.00069)
F3	85.65(1.00)	0.20(0.00070)
F5	86.24(0.97)	0.36(0.00071)
F7	86.78(0.95)	0.52(0.00071)
F9	87.01(0.93)	0.68(0.00071)
Data set 2: From January 6th, 2006 to December 16th, 2016: 572 weekly observations		
F1	84.57(1.04)	0.04(0.00069)
F5	86.24(0.97)	0.36(0.00070)
F9	87.01(0.93)	0.68(0.00071)
F13	87.17(0.89)	1.01(0.00071)
F17	87.12(0.86)	1.33(0.00071)

evidence of mean reversion in the whole sample period. We note that the estimated speed of mean reversion does have the expected sign in the two sub-sample periods. The negative signs of the log-likelihoods for all three data sets imply that the MSEs of the measurement variable's forecast are not low. These non-trivial MSEs might be caused by either the linear approximation of the predicted state variable's MSEs during filtering or the unfitness of a regular mean-reverting model.

Table 4.3 shows the estimates for Model 2. For the full sample period (i.e. from January 1995 to December 2016), the estimated speed of reverting is not significantly different from zero. Therefore the dynamics of oil prices in this whole sample period show little evidence of log mean reversion. For sub-sample period from January 2006 to December 2016, the estimates show that the prices evolve following a log mean-reverting process. The log-likelihoods for all three data sets are positive. Nevertheless, we cannot conclude that Model 2 outperforms Models 1 simply based on the log-likelihoods comparison. First, we can not determine the exact cause of the negative log-likelihoods in Model 1. Secondly, since Model 1 and 2 are not nested, the log-likelihoods derived from them are not comparable.

Schwartz (1997) also applied a Kalman filter to estimate Model 2, using data sets from an earlier period. One of the data sets he used is from January 1990 to February 1995, composed of contracts F1, F5, F9, F13, and F17. It is comparable to one data set we used, which is from January 2006 to December 2017, composed of the same set of contracts. The mean reversion speed estimated by Schwartz (1997) is 0.428, which is higher than our result, which is 0.14. It shows a weaker mean reversion from 2006 to 2016 compared to the period between 1990 and 1995. In addition, the long run mean log spot price estimated by Schwartz (1997) is 2.991, lower than our estimate, 4.46. His estimates for market price of risk and volatility are 0.002 and 0.257 respectively, compared to our results, -0.13 and 0.31.

For both Model 1 and Model 2, in the sub-sample period, the estimates of mean reversion speed are less when the contracts with shorter times to maturity are used. This phenomenon is the same as what Schwartz (1997) found in his one factor log mean-reverting model.

The estimates for Model 3 are exhibited in Table 4.4. The results show that the regime switching log mean-reverting model provides a reasonable description of the data in all three periods examined. In addition, when the contracts with shorter times to maturity are used for the sub-sample period estimation, the estimated reverting speed is higher. This is different from Models 1 and 2. However, since we assume the two regimes have the same reverting speeds, we cannot determine how the estimates of speeds for the two regimes (if they are different) change while longer contracts are used.

About Model 3, in the existing literature, Chen (2010) applied a non-linear least square approach to calibrate its parameter values using data with a different time interval and different futures contracts. The data are between January 1997 and May 2009. The futures contracts are F2, F5, F8, F12, F18, F24, F36, F48, F60, and F72. The mean reversion speed estimate using this data set is 0.06, which is not greater than all three estimates listed in Table 4.4. It shows that for the period and contracts examined in this chapter, the mean reversion is not weaker than those examined in Chen (2010). In Chen (2010), the long run mean log spot price for regime 1 is estimated to be 4.87, fairly close to three estimates reported in Table 4.4. However, for the regime 2, the estimate in Chen (2010) is 0.92, which is much lower than those reported in Table 4.4. With regard to the volatility, the estimate by Chen (2010) for regime 1 is 0.98, much higher than those listed in Table 4.4. The volatility estimate for regime 2 in Chen (2010) is 0.20, which is relatively close to those listed in Table 4.4. The probabilities of switching from regime 1 to regime 2 and from regime 2 to regime 1 estimated by Chen (2010) are 0.36 and 0.08 respectively, which are both much higher than the probabilities we estimate.

Furthermore, it can be observed that for all three models, the estimated volatilities do not vary much across different data sets.

Table 4.2: Results for Model 1

Futures contracts	F1, F3, F5, F7, F9	F1, F3, F5, F7, F9	F1, F5, F9, F13, F17
Sample period	Jan 1995 – Dec 2016	Jan 2006 – Dec 2016	Jan 2006 – Dec 2016
Number of observations	1146	572	572
ϵ	-0.04 (0.0000)	0.14 (0.0000)	0.12 (0.0000)
$\bar{P}$	63.00 (0.0006)	108.00 (0.0013)	97.02 (0.0006)
σ	0.25 (0.0002)	0.32 (0.0000)	0.31 (0.0001)
λ	0.50 (0.0006)	0.50 (0.0013)	1.48 (0.0011)
log-likelihood	-11877	-6366	-6725

The numbers in parentheses are standard errors of the associated estimates.

Figure 4.3 shows the estimated state variable (i.e. log of spot prices of crude oil) according to the three models and log of the actual spot prices approximated by log of the prices of the closest maturity futures contract F1. It can be observed that the estimated values are quite close to, although not exactly the same as, the actual values.

Table 4.3: Results for Model 2

Futures contracts	F1, F3, F5, F7, F9	F1, F3, F5, F7, F9	F1, F5, F9, F13, F17
Sample period	Jan 1995 – Dec 2016	Jan 2006 – Dec 2016	Jan 2006 – Dec 2016
Number of observations	1146	572	572
ϵ	1.65e-08 (4.14e-04)	0.20 (0.0000)	0.14 (0.0000)
μ	4.90 (0.0000)	4.47 (0.0001)	4.46 (0.0000)
σ	0.29 (0.0000)	0.33 (0.0000)	0.31 (0.0000)
λ	1.06 (0.0000)	-0.20 (0.0001)	-0.13 (0.0000)
log-likelihood	10704	6361	6027

The numbers in parentheses are standard errors of the associated estimates.

Table 4.4: Results for Model 3

Futures contracts	F1, F3, F5, F7, F9	F1, F3, F5, F7, F9	F1, F5, F9, F13, F17
Sample period	Jan 1995 – Dec 2016	Jan 2006 – Dec 2016	Jan 2006 – Dec 2016
Number of observations	1146	572	572
ϵ	0.06 (0.0000)	0.16 (0.0000)	0.20 (0.0000)
μ_{s1}	4.23 (0.0000)	4.21 (0.0000)	4.77 (0.0000)
μ_{s2}	4.98 (0.0000)	2.87 (0.0000)	3.79 (0.0000)
σ_{s1}	0.32 (0.0000)	0.33 (0.0000)	0.33 (0.0000)
σ_{s2}	0.23 (0.0000)	0.27 (0.0000)	0.28 (0.0000)
λ_{s1}	-3.26 (0.0000)	-1.09 (0.0000)	-0.40 (0.0000)
λ_{s2}	3.74 (0.0000)	-1.46 (0.0000)	-0.51 (0.0000)
$p^{\mathcal{P}}$	0.9919 (0.0000)	0.9885 (0.0000)	0.9765 (0.0000)
$q^{\mathcal{P}}$	0.9923 (0.0000)	0.9521 (0.0000)	0.9749 (0.0000)
$p^{\mathcal{Q}}$	0.9821 (0.0000)	0.9575 (0.0000)	0.9775 (0.0000)
$q^{\mathcal{Q}}$	0.9954 (0.0000)	0.9863 (0.0000)	0.9999 (0.0000)
log-likelihood	13190	7426	6884

The numbers in parentheses are standard errors of the associated estimates.

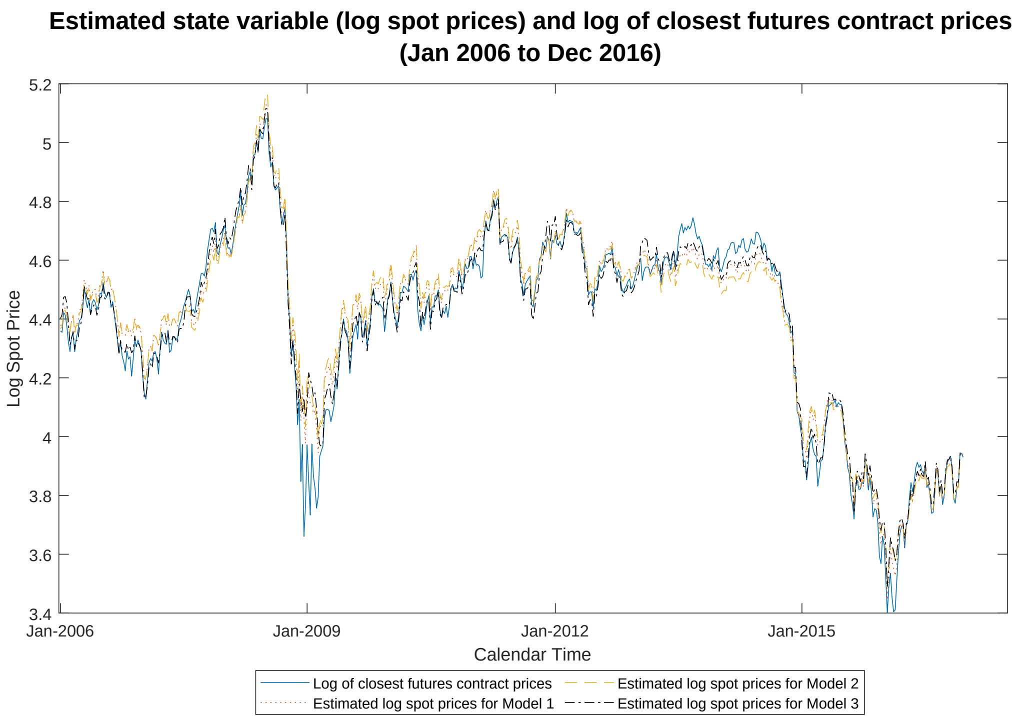

Figure 4.3: Estimated state variable vs. log of prices of the futures contract closest to maturity

4.7 Model Comparison

As mentioned in the previous section, the three models are not nested, so the log-likelihoods are not comparable. In this section, we compare the three models' performance from two perspectives: the forecast errors for the futures prices, and the term structure of the futures prices.

One motivation of our study of oil prices is to determine the best model to us in our stochastic optimal control problems examined in Chapters 2 and 3. The lifespans of the projects to be evaluated are all more than 1 year. Specifically, in Chapter 2 the life of projects are 10 years, in Chapter 3, the lives of projects vary from 5 years to 136 years. Therefore, in this section, we focus on the results based on the data set with longer futures contracts.

4.7.1 Forecast Errors for the Futures Prices

The forecast error refers to the difference between the actual value of a variable (in this section, the measurement variable Y) observed at a point in time and the forecasted value of this variable based on the information obtained up to the prior point in time. We compare the forecast errors of the three models by using both in-sample criteria and out-of-sample criteria.

4.7.1.1 Forecast Errors Comparison Using In-sample Criteria

To do the in-sample forecast comparison, for each time point t in the sample period except for time 0, we forecast the measurement variables' values in market based on the information up to time $t-1$ (i.e. the prior information) and obtain the difference between the observed values of the measurement variables and the forecasted values. Then we compute the mean absolute error (MAE) of the forecast.

Figure 4.4 shows the forecast errors of the three models using the data of five futures contracts F1, F5, F9, F13, and F17 from January 2006 to December 2016. The figure shows that the forecast errors are quite close for the three models. The MAE of the forecast for three models are 0.0369, 0.0390, and 0.0339 respectively. They amount to 0.85%, 0.90%, and 0.78% of the average[3] of the log of futures prices.

[3]We calculated the percentage for each time step and calculated the MAE for those percentages.

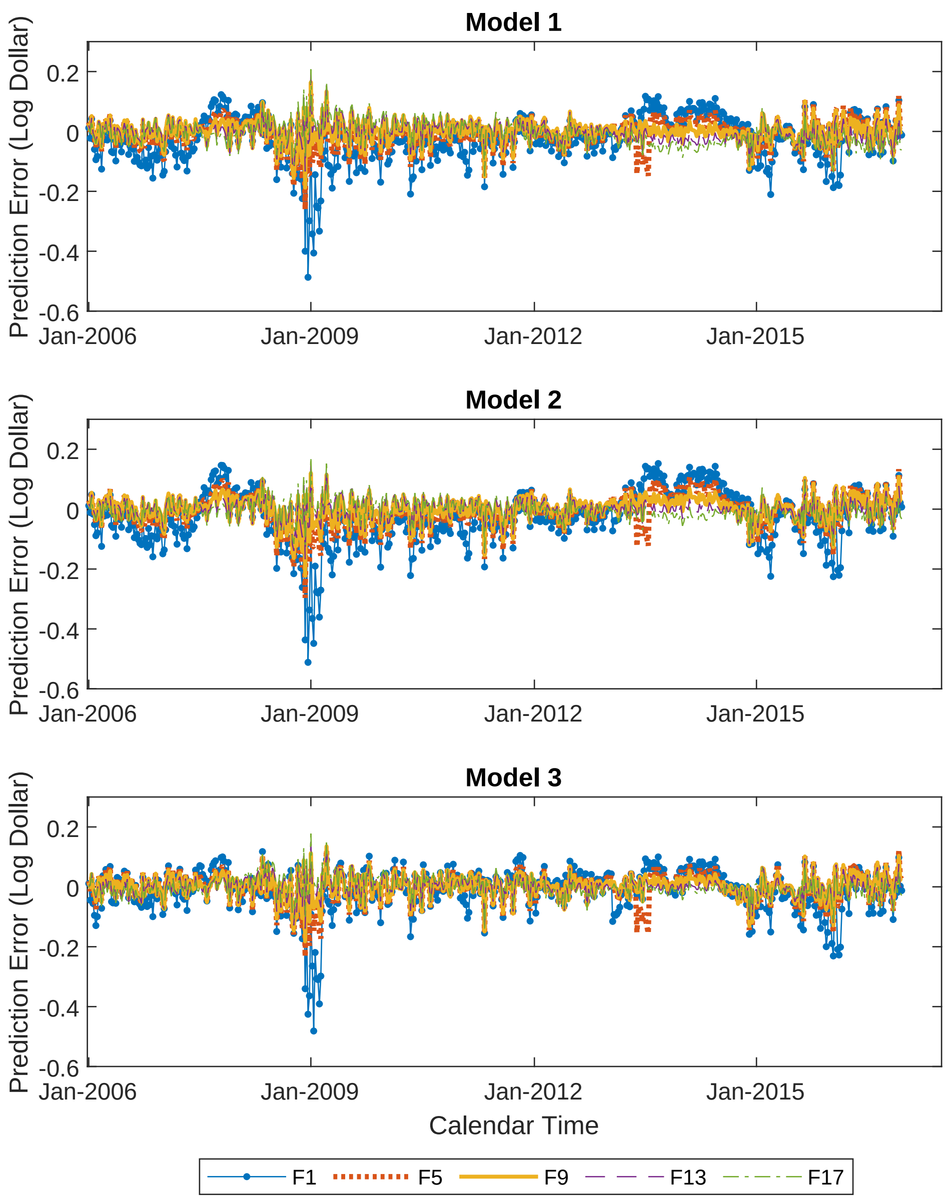

Figure 4.4: Log of futures prices forecast errors comparison for three models

Table 4.5 lists the MAE values for the three models, for single futures contracts and all of them. It is observed that, consistent with what is shown in Figure 4.4, when taking into consideration all the futures contracts, the MAEs for the three models are close. Model 3 has the lowest MAE and Model 2 has the highest MAE. Nevertheless when considering the individual futures contracts, it is observed that for the longer contracts such as F13 and F17, the MAEs for Model 2 are lower than the MAEs for Model 1.

Table 4.5: In-sample Futures Prices Prediction MAE

	Model 1	Model 2	Model 3
In Log Dollars			
F1	0.0579	0.0676	0.0506
F5	0.0365	0.0432	0.0369
F9	0.0279	0.0318	0.0294
F13	0.0289	0.0260	0.0261
F17	0.0334	0.0265	0.0264
All	0.0369	0.0390	0.0339
In Percentage			
F1	1.3573	1.5793	1.1991
F5	0.8408	0.9915	0.8534
F9	0.6416	0.7285	0.6770
F13	0.6599	0.5936	0.5964
F17	0.7604	0.6026	0.5983
All	0.8520	0.8991	0.7848

Chen (2010) also did an in-sample test for the regime switching log mean-reverting model, i.e. Model 3, after calibrating it. Her results of MAE are generally larger than those listed in Table 4.5. In addition, in Chen (2010), the forecast errors for longer term contracts are larger. In Table 4.5, it is observed that the forecast errors decrease with the contract term when the term is no more than 13 months and begin to increase when the term is longer.

The MAEs of the in-sample forecast obtained by Schwartz (1997) for Model 2 is 0.33 (around 1% of the log of futures price of the contract closest to maturity). This result is very close to our result, which is 0.0339 (0.78% of the average of the log of futures prices).

4.7.1.2 Forecast Errors Comparison Using Out-of-sample Criteria

To examine the performances of the three models for out-of-sample forecast of futures prices, we do two types of tests as Schwartz (1997) did in his work. One is the cross-section test to test the accuracy of forecast on the futures contracts that are not used in estimation. Since the contracts we use to estimate the three models are F1, F5, F9, F13, and F17, the cross-section test examines the forecast errors for F2, F3, F4, F6, F7, F8, F10, F11, F12, F14, F15, and F16. The other is the one-step-ahead out-of-sample test. In this test, we forecast the most recent 50 log futures prices in the sample period by using the parameters estimated based on the information up to the prior point in time. Specifically, assuming that in the sample period, the most recent 50 futures prices are observed at time $t-49, t-48, \ldots, t$, we use estimated parameters based on the information up to time $t-50$ to forecast the log futures price at time $t-49$, and use estimated parameters based on the information up to $t-49$ to forecast the log futures price at time $t-48$, and so on. Then we obtain the forecast errors between these forecasted values and log of the observed futures prices. Based on these forecast errors, we compute the root mean squared errors (RMSEs) and MAEs for the three models.

The cross-section forecast test results are reported in Table 4.6. Table 4.7 reports the results for one-step-ahead forecast test. The results shows that the RMSE and MAE comparison is similar to the comparison of the in-sample forecast errors described in the previous section. When considering all the futures contracts, Model 2 leads to the highest RMSE and MAE and Model 3 produces the least RMSE and MAE. When examine individual futures contracts, it is observed that for the futures contracts with time maturities more than 12 months, Models 2 and 3 give lower RMSEs and MAEs than Model 1. Furthermore, when the futures contract is longer than 15 months, Model 2's RMSEs and MAEs are the lowest among the three models.

Chen (2010) did the cross-section out-of-sample test for Model 3. The RMSEs and MAEs derived by her are generally larger than those in our results. In addition, similar to the phenomenon found in the in-sample test, in Chen (2010), with the increase in contracts' times to maturity, the RMSE and MAE become larger. In contrast, our observation is that the RMSE and MAE decrease with the contract's length when the contract is no more than 15 months; and when the contract is longer than 15 months, the RMSE and MAE begin to increase. Schwartz (1997) did the cross-section out-of-sample test for Model 2. He observed that the RMSE and MAE decrease with the contract's length when the contract's time to maturity is no more than 12 months; and when the contract is longer than 12 months, the RMSE and MAE begin to increase. In our result, the similar pattern occurs but the turning point is around 15 months. Moreover, for contracts with shorter terms, the RMSE and

MAE are higher in the results in Schwartz (1997). For longer term contracts, our results about RMSE and MAE are higher. This implies that compared to the model estimated by Schwartz (1997), our estimated model performs better for shorter term futures contracts price forecast and is weaker for predicting longer term contracts prices.

4.7.2 Term Structure of the Futures Prices

This section compares the three models by examining the in-sample fit in terms of the term structure of the futures prices. By using the three models, we can forecast the prices for different futures contracts with various times to maturity at each time point t except for the time point 0 based on the information obtained up to time $t-1$. It turns out that generally speaking, Model 3 can capture the actual term structure better than the other two models. We choose four representative observation dates from the beginning part, the intermediate part, and the latter part of the sample period respectively, including both contango and backwardation scenarios, to illustrate the comparison of the term structures of the futures prices implied by the three models to the observed actual term structure in Figure 4.5.

It is shown that Model 1 and Model 2 work fairly well when the term structure is in contango (the diagrams for 2006-01-06 and 2016-12-16). But they are not as good as Model 3 for capturing the price dynamics when it is in backwardation (the diagram for 2007-07-27) or in the transitional period from contango to backwardation (the diagram for 2012-03-16). This finding about Model 2 is similar to that observed by Schwartz (1997). Model 3's capability of capturing both contango and backwardation outperforms the other two.

4.8 Conclusion

In this chapter, we investigate three stochastic models for the crude oil prices. We use different members in the Kalman filter family to estimate the parameters for the three stochastic process models. It is shown that Model 3 (the regime switching log mean-reverting process) can capture the dynamics of oil prices in either a 20-year period or a 10-year period. Model 1 (level mean-reverting process) and Model 2 (log mean-reverting process) are capable of capturing the 10-year period price dynamics but are not able to capture the 20-year period dynamics. While the forecast errors of three models are very close, Model 3 has the least forecast errors and Model 1 has the highest forecast errors

Table 4.6: Cross-section Forecast RMSE and MAE

	RMSE			MAE		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
In Log Dollars						
F2	0.0662	0.0774	0.0621	0.0492	0.0583	0.0446
F3	0.0582	0.0680	0.0562	0.0437	0.0516	0.0413
F4	0.0533	0.0615	0.0530	0.0399	0.0469	0.0391
F6	0.0449	0.0521	0.0462	0.0333	0.0398	0.0347
F7	0.0420	0.0483	0.0434	0.0306	0.0368	0.0325
F8	0.0398	0.0450	0.0412	0.0289	0.0341	0.0308
F10	0.0379	0.0402	0.0384	0.0276	0.0298	0.0283
F11	0.0377	0.0384	0.0373	0.0277	0.0281	0.0273
F12	0.0379	0.0370	0.0364	0.0282	0.0268	0.0266
F14	0.0391	0.0354	0.0355	0.0298	0.0256	0.0259
F15	0.0401	0.0351	0.0353	0.0308	0.0256	0.0258
F16	0.0414	0.0352	0.0354	0.0321	0.0259	0.0260
All	0.0458	0.0497	0.0442	0.0335	0.0358	0.0319
In Percentage						
F2	1.5774	1.8345	1.5112	1.1433	1.3490	1.0445
F3	1.3730	1.5956	1.3439	1.0112	1.1898	0.9602
F4	1.2494	1.4358	1.2544	0.9216	1.0794	0.9069
F6	1.0482	1.2095	1.0838	0.7656	0.9122	0.8000
F7	0.9781	1.1207	1.0165	0.7051	0.8424	0.7486
F8	0.9276	1.0451	0.9644	0.6632	0.7812	0.7083
F10	0.8794	0.9313	0.8935	0.6318	0.6818	0.6499
F11	0.8713	0.8884	0.8645	0.6334	0.6425	0.6260
F12	0.8733	0.8551	0.8419	0.6449	0.6132	0.6087
F14	0.9007	0.8159	0.8152	0.6785	0.5845	0.5902
F15	0.9232	0.8078	0.8088	0.7023	0.5825	0.5870
F16	0.9514	0.8076	0.8088	0.7306	0.5898	0.5904
All	1.0690	1.1598	1.0420	0.7693	0.8207	0.7351

Table 4.7: One-step-ahead Forecast RMSE and MAE

	RMSE			MAE		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
In Log Dollars						
F1	0.0663	0.0746	0.0779	0.0527	0.0574	0.0555
F5	0.0544	0.0620	0.0574	0.0450	0.0535	0.0491
F9	0.0438	0.0509	0.0491	0.0351	0.0428	0.0411
F13	0.0454	0.0411	0.0414	0.0359	0.0328	0.0330
F17	0.0611	0.0416	0.0397	0.0522	0.0333	0.0311
All	0.0549	0.0555	0.0549	0.0442	0.0440	0.0420
In Percentage						
F1	1.8289	2.0711	2.1846	1.4242	1.5527	1.5120
F5	1.4347	1.6257	1.5189	1.1804	1.3989	1.2900
F9	1.1481	1.3279	1.2850	0.9146	1.1114	1.0696
F13	1.1878	1.0727	1.0808	0.9282	0.8506	0.8560
F17	1.5852	1.0826	1.0338	1.3456	0.8579	0.8015
All	1.4591	1.4845	1.4811	1.1586	1.1534	1.1058

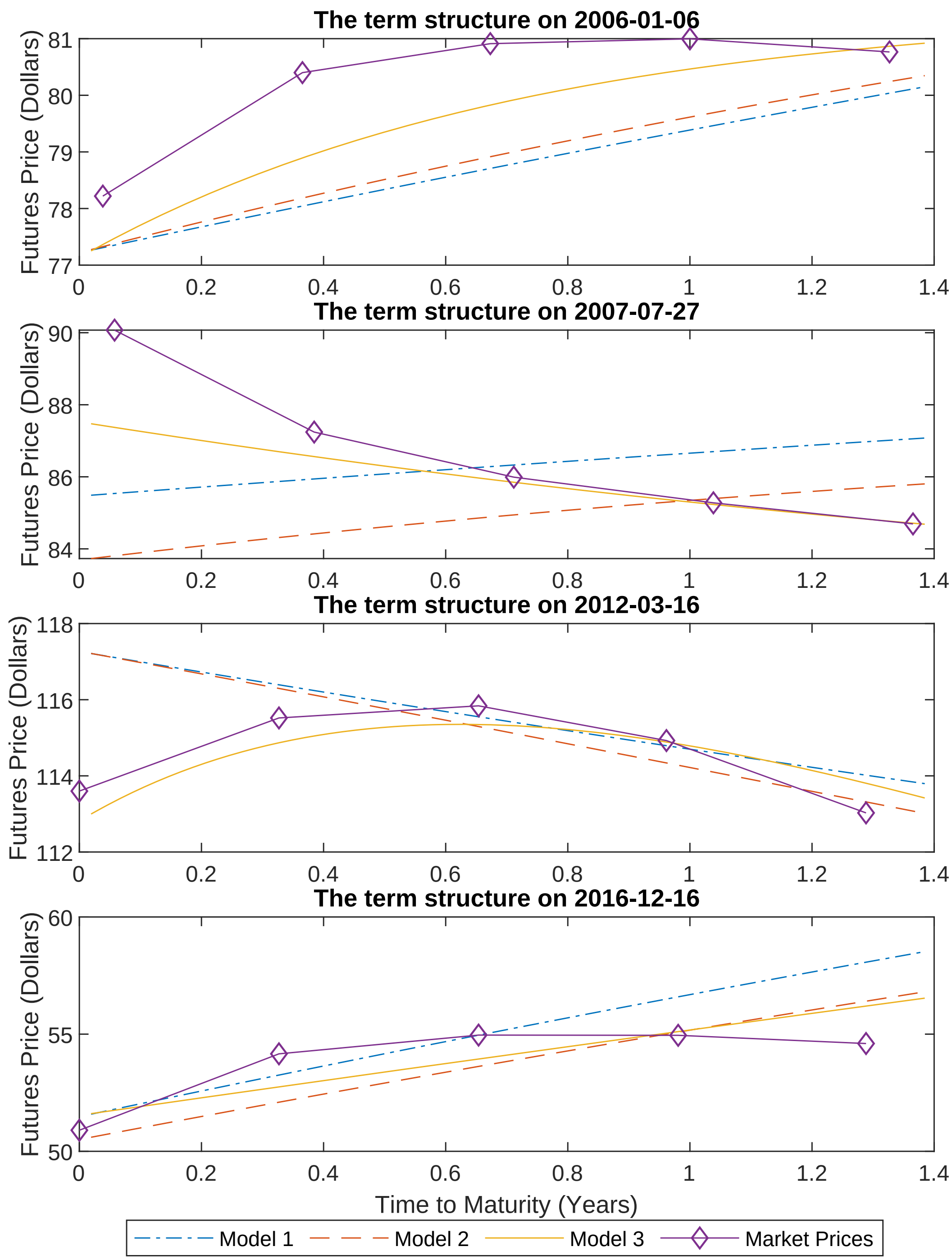

Figure 4.5: The futures term structure implied by three models on four different dates (Using the data for F1, F5, F9, F13, and F17 from January 2006 to December 2016)

taking all futures contracts into consideration. However, when individual contracts are considered separately, Model 1's performance in terms of forecast errors is dominated by the other two. It is also observed that Model 3's performance of capturing the term structure of the futures prices is better than the other two models.

When the stochastic process is for solving a problem with short-term period of interest, Model 1 and Model 3 are preferred than Model 2. However, when a long-term investment is concerned, Model 1 is no match for the other two.

Since Chapters 2 and 3 address investments for more than 1 year, we prefer Model 3 for the study. Nonetheless, as mentioned in Chapter 2, for our study, Model 3 can lead to more detailed results but no more insights to our problem than does Model 2. Hence what we finally adopt for our stochastic optimal control problems is Model 2.

In summary, none of the models stands out as being clearly superior to the others. For studying long term investments, models 2 and 3 do appear to have some advantage over model 1 based on the fact that Models 2 and 3 create less forecast errors for long term futures contracts than does Model 1. Model 3 can provide a better description of prices in backwardation. As noted in Chapter 2, the inclusion in our stochastic optimal control problem of two price regimes as required for price model 3, would considerably complicate our interpretation of the results. Hence we have adopted Model 2 in our analysis in Chapter 2.

Chapter 5

Conclusion

This thesis systematically studies a command and control policy addressing the management of a key natural resource, i.e. surface water, in a stochastic optimal control setting. The policy of interest, i.e. the Water Management Framework issued by the Alberta government, includes two phases which are examined in Chapters 2 and 3, respectively.

The Phase 1 Framework came into force in 2007. It was in some sense a interim phase with an intent to initiate water quantity protection for the Athabasca River. There is limited information about the development process of the rule set proposed by the Phase 1 Framework. We know little about the environmental effects of it. To investigate the framework's impact on economy, in Chapter 2, we examine the potential costs of this framework to the oil sands sector through modelling the profit maximizing behavior of a hypothetical oil sands firm using a stochastic optimal control approach. We contribute to the literature by introducing an uncertain regulation influenced by a varying natural environmental factor into the optimal natural resource extraction problem. Another contribution is to apply a stochastic optimal control model to study the water policy's cost in the oil sands mining sector. Chapter 2 shows that the optimal production behavior of the oil sands firm is closely associated with oil price, river flow condition, and oil sands reserves. Under the current circumstance (i.e. 2015 status), generally speaking, the Phase 1 Water Management Framework does not impose a large cost on oil sands firms. We derive the marginal cost curve of fresh water constraint, which is a non-monotonic curve due to the possibility of applying a new technology (i.e. water storage). This marginal cost curve provides a reference from economic perspective for measuring the social welfare condition and determining the efficient level of water constraints. However, a marginal benefit curve from the perspective of environment is not available for now, which is a problem that is beyond our research scope.

While the Phase 1 Framework was in effect, the Alberta government began to improve the framework by delegating the development of the Phase 2 Framework to the Phase 2 Framework Committee (P2FC). In 2015, the Phase 2 Framework was implemented. Its development was based on more rigorous scientific research about the environmental benefits and a simple estimation of the economic costs. One of multiple candidate rule sets was finally selected as the official regulation. The P2FC's working report provides solid evidence of environmental benefits of the candidate rule sets while leaving some gaps regarding the economic cost assessment, which need to be filled by further investigation. The assessment of costs of candidate rule sets determines the cost-effectiveness comparison among them and in turn affects the final choice of the best rule set. To address this problem, in Chapter 3, we apply the stochastic optimal control model developed in Chapter 2 to examine the economic costs of multiple candidate rule sets for the Phase 2 Framework in a stochastic setting rather than using the deterministic approach applied by the P2FC. Our results show that the economic cost of the policy is affected mostly by river flow conditions, water storage capabilities, and oil sands production capacities among other relevant factors. When the relevant factors alter, the rule set selected by the P2FC dose not always dominate others in terms of the cost-effectiveness.

The main task of Chapter 4 is to examine three stochastic process models of oil prices to select one to solve the stochastic optimal control problems raised in Chapters 2 and 3. The first model is a level mean-reverting process. Since it has one regime but cannot be transformed into a linear state-space, an extended Kalman filter is used to estimate it. As far as we know, no estimation of this model has been done by using a Kalman filter in the literature. The second model is a log mean-reverting process. It has one regime and can be transformed into a linear state-space, so a regular Kalman filter is used. The second model was estimated by Schwartz (1997), using data from 1985 to 1995. we update his analysis using data from 1995 to 2016. The third model is a regime switching log mean-reverting process. This model was estimated by Chen (2010) by conducting calibration. Our work is different in that we estimate it adopting a Kim filter, which is suitable for stochastic processes with two regimes and being able to transformed into a linear state-space. Our study shows that for the long-term forecasting, the second and third models perform better. Considering the third model would complicate the interpretation of results for stochastic optimal control problems without bringing too many insights, we choose the second model for solving the stochastic optimal control problems in Chapters 2 and 3.

Although this thesis examines a specific regulation implemented by a certain administrative division (in this thesis, the province of Alberta, Canada), the methodology developed herein can be adopted in many other contexts of water management such as shale gas hydraulic fracturing in the United States[1], copper and molybdenum mining in Rio Chili watershed in Peru[2], copper and gold mining in the Great Artesian Basin in Australia[3], irrigation in the Guadalquivir river Basin in Spain[4], etc.

This thesis focuses on a command and control instrument's economic effects. As expected, it turns out that the command and control instrument is unlikely to realize an efficient water allocation. Market-based instruments are known to be efficient. Which type of market-based instrument is better for efficient water allocation? Possibilities include water withdrawal taxes, water saving subsidies, and tradable water permit. These are all interesting topics that we do not address in this thesis but need to be further examined.

Admittedly, due to limited space and time, we simplify some conditions and assumptions. For example, when we discuss the economic costs of the Water Management Framework, we confine the scope to only the costs to oil sands firms. However, in fact, there are other costs such as the administrative costs associated with the regulation, incurred by the regulatory agency, and the costs associated with uncertainty in terms of efficiency loss. These costs could also have significant effects on social welfare. These costs are worthwhile to be considered in the future research. Another assumption that can be further investigated is the justification of using $\mathcal{P}$ measure river flow process in the stochastic optimal control problem.

[1] The background can be found in Rahm (2011).
[2] The background can be found in Garner et al. (2012).
[3] The background can be found in Garner et al. (2012).
[4] The background can be found in Berbel et al. (2011).

References

Al Mansour, A. (2012), Essays in risk management for crude oil markets, PhD thesis, University of Waterloo, Waterloo, Ontario, Canada.

Alaouze, C. M. (1991), 'Intertemporal water transfers and drought', *Australian Economic Papers* **30**(56), 114–127.

Alberta (2015), Lower Athabasca region: surface water quantity management framework for the Lower Athabasca River, Policy, Environmental and Sustainable Resource Development, Edmonton, Canada. ISBN: 9781460121733.

Alberta and Canada (2007), Water management framework: Instream flow needs and water management system for the Lower Athabasca River, Policy, Alberta Environment and Fisheries and Oceans Canada.

Alberta Energy Regulator (2015), ST98-2015, Alberta's energy reserves 2014 and supply/demand outlook 2015–2024, Technical report, Alberta Energy Regulator, Calgary, Canada.

Almansour, A. & Insley, M. C. (2016), 'The impact of stochastic extraction cost on the value of an exhaustible resource: An application to the Alberta oil sands', *Energy Journal* **37**(2).

Arora, S. & Cason, T. N. (1995), 'An experiment in voluntary environmental regulation: Participation in EPA's 33/50 program', *Journal of Environmental Economics and Management* **28**(3), 271–286.

Arora, S. & Cason, T. N. (1996), 'Why do firms volunteer to exceed environmental regulations? Understanding participation in EPA's 33/50 program', *Land Economics* **72**(4), 413–432.

Balvers, R., Wu, Y. & Gilliland, E. (2000), 'Mean reversion across national stock markets and parametric contrarian investment strategies', *The Journal of Finance* **55**(2), 745–772.

Bawden, A. J., Linton, H. C., Burn, D. H. & Prowse, T. D. (2014), 'A spatiotemporal analysis of hydrological trends and variability in the Athabasca River region, Canada', *Journal of Hydrology* **509**, 333–342.

Bennett, L., Howe, C. & Shope, J. (2000), 'The interstate river compact as a water allocation mechanism: Efficiency aspects', *American Journal of Agricultural Economics* **82**(4), 1006–1015.

Berbel, J., Martin-Ortega, J. & Mesa, P. (2011), 'A cost-effectiveness analysis of water-saving measures for the water framework directive: the case of the Guadalquivir River Basin in Southern Spain', *Water Resources Management* **25**(2), 623–640.

Björk, T. (2009), *Arbitrage Theory in Continuous Time*, Oxford University Press.

Boldt-Van Rooy, T. (2003), 'Bottling up our natural resources: the fight over bottled water extraction in the United States', *Journal of Land Use & Environmental Law* pp. 267–298.

Braathen, N. A. & Johnstone, N. (2003), *Voluntary approaches for environmental policy: effectiveness, efficiency and usage in policy mixes*, Organisation for Economic Co-operation and Development (OECD).

Brennan, D. (2010), 'Economic potential of market-oriented water storage decisions: Evidence from Australia', *Water Resources Research* **46**.

Brennan, M. J. (1991), The price of convenience and the valuation of commodity contingent claims, *in* D. Lund & B. Øksendal, eds, 'Stochastic Models and Option Values - Application to Resources, Environment and Investment Problems', Elsevier Science Publishers B.V., Amsterdam, The Netherlands, pp. 33–71.

Brennan, M. J. & Schwartz, E. S. (1980), 'Analyzing convertible bonds', *Journal of Financial and Quantitative Analysis* **15**(4), 907–929.

Brennan, M. J. & Schwartz, E. S. (1982), 'Consistent regulatory policy under uncertainty', *The Bell Journal of Economics* pp. 506–521.

Brennan, M. J. & Schwartz, E. S. (1985), 'Evaluating natural resource investments', *Journal of Business* pp. 135–157.

Bruce, J. P. (2006), Oil and water - Will they mix in a changing climate? The Athabasca River story, *in* 'Implications of a 2°C Global Temperature Rise on Canada's Water Resources, Athabasca River and Oil Sands Development, Great Lakes and Hydropower Production', pp. 12–34.

Campbell, J. Y., Lo, A. W.-C., MacKinlay, A. C. et al. (1997), *The econometrics of financial markets*, Princeton University Press.

Chan, K. C., Karolyi, G. A., Longstaff, F. A. & Sanders, A. B. (1992), 'An empirical comparison of alternative models of the short-term interest rate', *The Journal of Finance* **47**(3), 1209–1227.

Chen, S. (2010), Modelling the dynamics of commodity prices for investment decisions under uncertainty, PhD thesis, University of Waterloo, Waterloo, Ontario, Canada.

Chen, S. & Insley, M. (2012), 'Regime switching in stochastic models of commodity prices: An application to an optimal tree harvesting problem', *Journal of Economic Dynamics and Control* **36**(2), 201–219.

Chen, Z. (2008), Numerical methods for optimal stochastic control in finance, PhD thesis, University of Waterloo, Waterloo, Ontario, Canada.

Chen, Z. & Forsyth, P. A. (2007), 'A semi-Lagrangian approach for natural gas storage valuation and optimal operation', *SIAM Journal on Scientific Computing* **30**(1), 339–368.

Chen, Z. & Forsyth, P. A. (2010), 'Implications of a regime-switching model on natural gas storage valuation and optimal operation', *Quantitative Finance* **10**(2), 159–176.

Cortazar, G., Schwartz, E. S. & Salinas, M. (1998), 'Evaluating environmental investments: A real options approach', *Management Science* **44**(8), 1059–1070.

d'Halluin, Y. (2004), Numerical methods for real options in telecommunications, PhD thesis, University of Waterloo, Waterloo, Ontario, Canada.

d'Halluin, Y., Forsyth, P. A. & Labahn, G. (2005), 'A semi-Lagrangian approach for American Asian options under jump diffusion', *SIAM Journal on Scientific Computing* **27**(1), 315–345.

Di Baldassarre, G., Wanders, N., AghaKouchak, A., Kuil, L., Rangecroft, S., Veldkamp, T. I. E., Garcia, M., van Oel, P. R., Breinl, K. & Van Loon, A. F. (2018), 'Water shortages worsened by reservoir effects', *Nature Sustainability* **1**(11), 617–622.

Dinar, A. & Subramanian, A. (1998), 'Policy implications from water pricing experiences in various countries', *Water Policy* **1**(2), 239–250.

Dixit, A. K. & Pindyck, R. S. (1994), *Investment under Uncertainty*, Princeton University Press.

Dudley, N. J. (1972), 'Irrigation planning: 4. optimal interseasonal water allocation', *Water Resources Research* **8**(3), 586–594.

Dudley, N. J. (1988), 'A single decision-maker approach to irrigation reservoir and farm management decision making', *Water Resources Research* **24**(5), 633–640.

Dudley, N. J. & Hearn, A. B. (1993), 'Systems modeling to integrate river valley water supply and irrigation decision making under uncertainty', *Agricultural Systems* **42**(1), 3–23.

Economic Development and Trade (2015), Alberta oil sands industry quarterly update (spring 2015) - Reporting on the period: December 5, 2014 to March 20, 2015, Policy, Economic Development and Trade, Edmonton, Canada. retrieved from URL:https://open.alberta.ca/dataset/b70a79b3-387f-475a-be38-6fe4cd5bb007/resource/5fd600ff-ce99-41e3-8235-8eb6bcbe414d/download/2614198-2015-sring-aosid-quarterlyupdate.pdf.

Fattouh, B. (2011), *An anatomy of the crude oil pricing system*, University of Oxford.

Garner, R., Naidu, T., Saavedra, C., Matamoros, P. & Lacroix, E. (2012), Water management in mining: A selection of case studies, Report, International Council on Mining & Metals, London, London, United Kingdom. retrieved from URL:https://www.icmm.com/website/publications/pdfs/water/water-management-in-mining_case-studies.

Gibson, R. (1991), Valuation of long term oil-linked assets, *in* D. Lund & B. Øksendal, eds, 'Stochastic models and option values - Application to resources, environment and investment problems', Elsevier Science Publishers B.V., Amsterdam, The Netherlands, pp. 73–101.

Gibson, R. & Schwartz, E. S. (1990), 'Stochastic convenience yield and the pricing of oil contingent claims', *The Journal of Finance* **45**(3), 959–976.

Golder Associates Ltd. (2015), Engineering mitigation options for meeting the Athabasca River Water Management Framework, Private report for the benefit of the client 07-1345-0027.5000, Golder Associates Ltd.

Gosselin, P., Hrudey, S. E., Naeth, M., Plourde, A., Therrien, R., Van Der Kraak, G. & Xu, Z. (2010), Environmental and health impacts of Canada's oil sands industry, Royal Society of Canada Ottawa, Ontario, Canada.

Grewal, M. S. & Andrews, A. P. (2010), 'Applications of Kalman filtering in aerospace 1960 to the present', *IEEE Control Systems Magazine* **30**(3), 69–78.

Griffiths, M., Taylor, A. & Woynillowicz, D. (2006), Troubled waters, troubling trends - Technology and policy options to reduce water use in oil and oil sands development in Alberta, Technical report, Pembina Institute, Drayton Valley, Alberta, Canada. ISBN 0-921719-91-4, retrieved from https://www.pembina.org/pub/troubled-waters-troubling-trends.

Griffiths, M. & Woynillowicz, D. (2003), Oil and troubled waters - reducing the impact of the oil and gas industry on Alberta's water resources, Technical report, Pembina Institute, Drayton Valley, Alberta, Canada.

Hamilton, J. D. (1989), 'A new approach to the economic analysis of nonstationary time series and the business cycle', *Econometrica: Journal of the Econometric Society* pp. 357–384.

Hardy, M. R. (2001), 'A regime-switching model of long-term stock returns', *North American Actuarial Journal* **5**(2), 41–53.

Harrington, W. (1988), 'Enforcement leverage when penalties are restricted', *Journal of Public Economics* **37**(1), 29–53.

Harvey, A. C. (1990), *Forecasting, structural time series models and the Kalman filter*, Cambridge university press.

Haykin, S. (2004), *Kalman filtering and neural networks*, Vol. 47, John Wiley & Sons.

Holding, S., Allen, D. M., Notte, C. & Olewiler, N. (2017), 'Enhancing water security in a rapidly developing shale gas region', *Journal of Hydrology: Regional Studies* **11**(SI), 266–277.

Insley, M. (2017), 'Resource extraction with a carbon tax and regime switching prices: Exercising your options', *Energy Economics* **67**, 1–16.

Insley, M. C. (2003), 'On the option to invest in pollution control under a regime of tradable emissions allowances', *Canadian Journal of Economics/Revue canadienne d'économique* **36**(4), 860–883.

Insley, M. & Rollins, K. (2005), 'On solving the multirotational timber harvesting problem with stochastic prices: a linear complementarity formulation', *American Journal of Agricultural Economics* **87**(3), 735–755.

Ivanhoe Energy Inc. (2012), 'Application for approval of the Tamarack integrated oil sands project (Volume 6: Supplemental information request #3)'. retrieved from https://open.alberta.ca/publications/4926022.

Jazwinski, A. H. (2007), *Stochastic processes and filtering theory*, Dover Publication Inc.

Jensen, K. (2010), 'Environmental impact of the oil and gas industry's consumption of water from the Athabasca River during the predicted water shortage for Canada's Western Prairie Provinces'. retrieved from https://www.queensu.ca/ensc/sites/webpublish.queensu.ca.enscwww/files/files/501/Jensen.pdf.

Johnson, R., Gisser, M. & Werner, M. (1981), 'The definition of a surface-water right and transferability', *Journal of Law & Economics* **24**(2), 273–288.

Kalman, R. E. (1960), 'A new approach to linear filtering and prediction problems', *Journal of basic Engineering* **82**(1), 35–45.

Kalman, R. E. (1963), New methods in Wiener filtering theory, *in* 'Proceedings of the First Symposium on Engineering Applications of Random Function Theory and Probability, edited by JL Bogdanoff and F. Kozin, John Wiley & Sons, New York'.

Kalman, R. E. & Bucy, R. S. (1961), 'New results in linear filtering and prediction theory', *Journal of basic engineering* **83**(1), 95–108.

Khanna, M. & Damon, L. A. (1999), 'EPA's voluntary 33/50 program: Impact on toxic releases and economic performance of firms', *Journal of Environmental Economics and Management* **37**(1), 1–25.

Kim, C.-J. (1994), 'Dynamic linear models with Markov-switching', *Journal of Econometrics* **60**(1-2), 1–22.

Kuwayama, Y., Olmstead, S. M. & Krupnick, A. (2013), Water resources and unconventional fossil fuel development: Linking physical impacts to social costs. Discussion paper, Resources for the Future, retrieved from https://media.rff.org/archive/files/sharepoint/WorkImages/Download/RFF-DP-13-34.pdf.

Leong, D. & Donner, S. (2016), 'Future water supply and demand management options in the Athabasca oil sands', *River Research and Applications* **32**(9), 1853–1861.

Libecap, G. D. (2011), 'Institutional path dependence in climate adaptation: Coman's "some unsettled problems of irrigation"', *American Economic Review* **101**(1), 64–80.

Libecap, G. D. & Barbara, S. (2012), Water rights and markets in the U.S. semiarid west - Efficiency and equity issues. ICER working paper, retrieved from http://ssrn.com/abstract=1738371.

Liu, X., Chen, X. & Wang, S. (2009), 'Evaluating and predicting shadow prices of water resources in China and its nine major river basins', *Water Resources Management* **23**(8), 1467–1478.

Longstaff, F. & Schwartz, E. (2001), 'Valuing American options by simulation: A simple least-squares approach', *Review of Financial Studies* **14**(1), 113–147.

Lunn, S. et al. (2013), 'Water use in Canada's oil-sands industry: The facts', *SPE Economics & Management* **5**(01), 17–27.

Majd, S. & Pindyck, R. S. (1987), 'Time to build, option value, and investment decisions', *Journal of Financial Economics* **18**(1), 7–27.

Mannix, A. E., Adamowicz, W. L. & Dridi, C. (2014), 'Solutions to the high costs of future water restrictions for new oil sands industry along the Athabasca River', *Canadian Water Resources Journal* **39**(4), 395–408.

Mannix, A. E., Dridi, C. & Adamowicz, W. L. (2010), 'Water availability in the oil sands under projections of increasing demands and a changing climate: An assessment of the Lower Athabasca Water Management Framework (Phase 1)', *Canadian Water Resources Journal* **35**(1), 29–52.

Mansur, E. T. & Olmstead, S. M. (2012), 'The value of scarce water: Measuring the inefficiency of municipal regulations', *Journal of Urban Economics* **71**(3), 332–346.

Masliyah, J., Zhou, Z. J., Xu, Z., Czarnecki, J. & Hamza, H. (2004), 'Understanding water-based bitumen extraction from Athabasca oil sands', *The Canadian Journal of Chemical Engineering* **82**(4), 628–654.

Mason, C. F. (2001), 'Nonrenewable resources with switching costs', *Journal of Environmental Economics and Management* **42**(1), 65–81.

McDonald, R. L. & Siegel, D. R. (1985), 'Investment and the valuation of firms when there is an option to shut down', *International Economic Review* pp. 331–349.

Miller, R. A. & Voltaire, K. (1983), 'A stochastic analysis of the tree paradigm', *Journal of Economic Dynamics and Control* **6**, 371–386.

Millington, D. & Murillo, C. A. (2015), Canadian oil sands supply costs and development projects (2015-2035), , Canadian Energy Research Institute.

Mohinder, S. G. & Angus, P. A. (2001), *Kalman filtering: theory and practice using MATLAB*, John Wiley & Sons. ISBN 0-471-26638-8.

National Energy Board (2006), 'Canada's oil sands - opportunities and challenges to 2015: An update'.

Ohlson, D., Long, G. & Hatfield, T. (2010), Phase 2 Framework Committee report, Report submitted to alberta environment/fisheries and oceans canada, and the cumulative environmental management association, Cumulative Environmental Management Association, fort McMurray, Alberta, Canada. Copyright: #1075698.

Olmstead, S. M. & Stavins, R. N. (2009), 'Comparing price and nonprice approaches to urban water conservation', *Water Resources Research* **45**(4).

Paddock, J. L., Siegel, D. R. & Smith, J. L. (1988), 'Option valuation of claims on real assets: The case of offshore petroleum leases', *The Quarterly Journal of Economics* pp. 479–508.

Panik, M. J. (2017), *Stochastic Differential Equations: An Introduction with Applications in Population Dynamics Modeling*, John Wiley & Sons.

Peters, D. L., Atkinson, D., Monk, W. A., Tenenbaum, D. E. & Baird, D. J. (2013), 'A multi-scale hydroclimatic analysis of runoff generation in the Athabasca River, western Canada', *Hydrological Processes* **27**(13), 1915–1934.

Pindyck, R. S. (1980), 'Uncertainty and exhaustible resource markets', *The Journal of Political Economy* pp. 1203–1225.

Pindyck, R. S. (1993), 'Investments of uncertain cost', *Journal of Financial Economics* **34**(1), 53–76.

Press, W. H., Flannery, B. P., Teukolsky, S. A. & Vetterling, W. T. (1987), *Numerical Recipes: The art of scientific computing*, Vol. 2, Cambridge University Press London.

Rahm, D. (2011), 'Regulating hydraulic fracturing in shale gas plays: The case of Texas', *Energy Policy* **39**(5), 2974–2981.

Rasouli, K., Hernández-Henriquez, M. & Déry, S. (2013), 'Streamflow input to Lake Athabasca, Canada', *Hydrology and Earth System Sciences* **17**(5), 1681–1691.

Schindler, D. W. & Donahue, W. F. (2006), 'An impending water crisis in Canada's western prairie provinces', *Proceedings of the National Academy of Sciences* **103**(19), 7210–7216.

Schorfheide, F. (2003), 'Book review - Financial econometrics, by Christian Gourieroux and Joann Jasiak, Princeton University Press, 2001', *Econometric Theory* **19**(2), 401–409.

Schwartz, E. S. (1997), 'The stochastic behavior of commodity prices: Implications for valuation and hedging', *The Journal of Finance* **52**(3), 923–973.

Schwartz, E. & Smith, J. E. (2000), 'Short-term variations and long-term dynamics in commodity prices', *Management Science* **46**(7), 893–911.

Slade, M. E. (2001), 'Valuing managerial flexibility: An application of real-option theory to mining investments', *Journal of Environmental Economics and Management* **41**(2), 193–233.

Sorenson, H. W. (1985), *Kalman filtering: theory and application*, The Institute of Electrical and Electronics Engineers Press.

Squires, A. J., Westbrook, C. J. & Dubé, M. G. (2010), 'An approach for assessing cumulative effects in a model river, the Athabasca River basin', *Integrated Environmental Assessment and Management* **6**(1), 119–134.

Strikwerda, J. C. (2004), *Finite difference schemes and partial differential equations*, Vol. 88, Siam.

Susquehanna River Basin Commission (2003), Guidelines for using and determining passby flows and conservation releases for surface-water and ground-water withdrawal approvals, Policy, Harrisburg, Pennsylvania. No. 2003-01.

Tavella, D. & Randall, C. (2000), *Pricing financial instruments: The finite difference method*, Vol. 13, John Wiley & Sons.

Teisberg, E. O. (1993), 'Capital investment strategies under uncertain regulation', *The RAND Journal of Economics* pp. 591–604.

Thomashausen, S., Maennling, N. & Mebratu-Tsegaye, T. (2018), 'A comparative overview of legal frameworks governing water use and waste water discharge in the mining sector', *Resources Policy* **55**, 143–151.

Toledano, P. & Roorda, C. (2014), Leveraging mining investments in water infrastructure for broad economic development: Models, opportunities and challenges, Technical report, Columbia Center on Sustainable Investment, Columbia University. retrieved from http://ccsi.columbia.edu/work/projects/leveraging-infrastructure-investments-for-development/.

Toman, M., Curtright, A. E., Ortiz, D. S., Darmstadter, J. & Shannon, B. (2008), *Unconventional fossil-based fuels: Economic and environmental trade-offs*, Rand Corporation.

United Nations (2015), 'Transforming our world: The 2030 agenda for sustainable development'.

Vengosh, A., Jackson, R. B., Warner, N., Darrah, T. H. & Kondash, A. (2014), 'A critical review of the risks to water resources from unconventional shale gas development and hydraulic fracturing in the United States', *Environmental Science & Technology* **48**(15, SI), 8334–8348.

Vollert, A. (2012), *A stochastic control framework for real options in strategic evaluation*, Springer Science & Business Media.

Weber, M. & Cutlac, M. (2014), 'Economic and environmental tradeoffs from alternative water allocation policies in the South Saskatchewan River Basin', *Canadian Water Resources Journal / Revue canadienne des ressources hydriques* **39**(4), 409–420.

Wilmott, P. (1998), *Derivatives - The theory and practice of financial engineering*, Wiley Chichester.

Wolfe, B. B., Hall, R. I., Edwards, T. W. & Johnston, J. W. (2012), 'Developing temporal hydroecological perspectives to inform stewardship of a northern floodplain landscape subject to multiple stressors: paleolimnological investigations of the Peace–Athabasca Delta', *Environmental Reviews* **20**(3), 191–210.

Wolfe, B. B., Hall, R. I., Edwards, T. W., Vardy, S. R., Falcone, M. D., Sjunneskog, C., Sylvestre, F., McGowan, S., Leavitt, P. R. & van Driel, P. (2008), 'Hydroecological responses of the Athabasca Delta, Canada, to changes in river flow and climate during the 20th century', *Ecohydrology* **1**(2), 131–148.

Woynillowicz, D., Severson-Baker, C. & Raynolds, M. (2005), *Oil sands fever: The environmental implications of Canada's oil sands rush*, Pembina Institute Edmonton.

APPENDICES

Appendix A

Numerical Solution Details and Sensitivity Analysis Results for Chapter 2

A.1 Semi-Lagrangian Discretization

For simplicity, we only elaborate the problem in a certain stage m. Hence the expressions at this stage can be simplified by eliminating the subscript m temporarily.

There are two types of approaches to solve this kind of problem. One is to do simulations, and the other one is to solve PDEs. Since the latter one is more accurate, we adopt it in this thesis. To solve the PDEs (2.16), we apply the extended semi-Lagrangian discretiztion approach introduced in Chen & Forsyth (2007) (with no regime switching) and Chen & Forsyth (2010) (with regime switching).

$$\frac{\partial V^k}{\partial \tau} = \max_{W_w, W_p} \left\{ \frac{1}{2} b^2 \frac{\partial^2 V^k}{\partial P^2} + a \frac{\partial V^k}{\partial P} - Q^k \frac{\partial V^k}{\partial S} + Z^k \frac{\partial V^k}{\partial I} + \pi^k + \sum_{u=1, u\neq k}^{3} \lambda^{k \to u} (V^u - V^k) - rV^k \right\} \tag{A.1}$$

Since control variables W_w and W_p only have effects on state variables Z and Q, we can rewrite the above expression as:

$$\frac{\partial V^k}{\partial \tau}=\frac{1}{2}b^2\frac{\partial^2 V^k}{\partial P^2}+a\frac{\partial V^k}{\partial P}+\max_{W_w,W_p}\left\{\pi^k+Z^k\frac{\partial V^k}{\partial I}-Q^k\frac{\partial V^k}{\partial S}\right\}+\sum_{u=1,u\neq k}^{3}\lambda^{k\to u}(V^u-V^k)-rV^k \tag{A.2}$$

Denote $\mathcal{L}V^k=\frac{1}{2}b^2\frac{\partial^2 V^k}{\partial P^2}+a\frac{\partial V^k}{\partial P}+\sum_{u=1,u\neq k}^{3}\lambda^{k\to u}(V^u-V^k)-rV^k$,

hence (A.2) can be simplified as:

$$\frac{\partial V^k}{\partial \tau}-\max_{W_w,W_p}\left\{\pi^k+Z^k\frac{\partial V^k}{\partial I}-Q^k\frac{\partial V^k}{\partial S}\right\}-\mathcal{L}V^k=0 \tag{A.3}$$

That is:

$$\max_{W_w,W_p}\left\{\left[\frac{\partial V^k}{\partial \tau}+Q^k\frac{\partial V^k}{\partial S}-Z^k\frac{\partial V^k}{\partial I}\right]-\pi^k\right\}-\mathcal{L}V^k=0 \tag{A.4}$$

The terms in the square brackets in the above equation can be written as a lagrange directional derivative by identify semi-Lagrangian trajectories:

$$\frac{\mathrm{d}\mathcal{S}}{\mathrm{d}\tau}=-Q^k \tag{A.5}$$

$$\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}\tau}=-Z^k \tag{A.6}$$

The expression in the aforesaid square bracket can be expressed as the following lagrange directional derivative:

$$\frac{\mathrm{D}V^k}{\mathrm{D}\tau}=\frac{\partial V^k}{\partial \tau}-\frac{\partial V^k}{\partial \mathcal{S}}\cdot\frac{\mathrm{d}\mathcal{S}}{\mathrm{d}\tau}+\frac{\partial V^k}{\partial \mathcal{I}}\cdot\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}\tau} \tag{A.7}$$

Then the PDEs (A.4) can be transformed to:

$$\max_{W_w,W_p}\left\{\frac{\mathrm{D}V^k}{\mathrm{D}\tau}-\pi^k\right\}-\mathcal{L}V^k=0 \tag{A.8}$$

We apply unequally spaced grids in the directions of P, I, S, and τ: $[P_1,P_2,\ldots,P_{i_{max}}]$, $[I_0,I_1,\ldots,I_{j_{max}}]$, $[S_0,S_0,\ldots,S_{l_{max}}]$, and $0=\tau^0\le\tau^1\le\cdots\le\tau^N=T$ $(\Delta\tau^n=\tau^{n+1}-\tau^n)$, where T is the lifespan of a development license. Normally a lifespan is several decades. Since a certain set of control variables varies both Z and Q, we must treat these two state variables as a pair. With a certain path of control variables $(\mathcal{W}_w, \mathcal{W}_p)$, the set of I and S follows a certain path.

Suppose P is fixed at the grid P_i, if at τ^{n+1} the set of I and S reaches a discrete grid point (P_i, I_j, S_l), the departure point at $\tau = \tau^n$, $\mathcal{I}(\tau^n; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau^n), \mathcal{W}_p(\tau^n))$ and $\mathcal{S}(\tau^n; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau^n), \mathcal{W}_p(\tau^n))$ can be solved by:

$$\begin{cases} \frac{\mathrm{d}\mathcal{S}}{\mathrm{d}\tau}(\tau; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau), \mathcal{W}_p(\tau)) = -Q(\mathcal{W}_p^k(\tau)) & \text{for } \tau < \tau^{n+1}, \\ \frac{\mathrm{d}\mathcal{I}}{\mathrm{d}\tau}(\tau; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau), \mathcal{W}_p(\tau)) = \mathcal{W}_p^k(\tau) - \mathcal{W}_w^k(\tau) & \text{for } \tau < \tau^{n+1}, \\ \mathcal{S}(\tau; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau), \mathcal{W}_p(\tau)) = S_l & \text{for } \tau = \tau^{n+1}, \\ \mathcal{I}(\tau; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau), \mathcal{W}_p(\tau)) = I_j & \text{for } \tau = \tau^{n+1}. \end{cases} \tag{A.9}$$

Solving $\mathcal{S}(\tau = \tau^n; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau = \tau^n), \mathcal{W}_p(\tau = \tau^n))$ (simply written as $\mathcal{S}(\tau^n)$) and $\mathcal{I}(\tau = \tau^n; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau = \tau^n), \mathcal{W}_p(\tau = \tau^n))$ (simply written as $\mathcal{I}(\tau^n)$), we get:

$$\mathcal{S}(\tau^n) = S_l - \int_{\tau^n}^{\tau^{n+1}} -Q(\mathcal{W}_p^k(\tau))\,\mathrm{d}\tau \tag{A.10}$$

$$\mathcal{I}(\tau^n) = I_j + \int_{\tau^n}^{\tau^{n+1}} (\mathcal{W}_p^k(\tau) - \mathcal{W}_w^k(\tau))\,\mathrm{d}\tau \tag{A.11}$$

Note that $\mathcal{S}(\tau^n)$, $\mathcal{I}(\tau^n)$ do not necessarily reside exactly at a grid point. Later we will use a two dimensional bilinear interpolation introduced by Press, William, et al. (1987) to address this problem.

Integrating both sides of (A.8) from $\tau = \tau^n$ to $\tau = \tau^{n+1}$ with P fixed at P_i and W_w, W_p following the path of $(\mathcal{W}_w(\tau), \mathcal{W}_p(\tau))$, we have:

$$\int_{\tau^n}^{\tau^{n+1}} \max_{W_w, W_p} \left\{ \frac{\mathrm{D}V^k}{\mathrm{D}\tau} - \pi^k \left(P_i, \mathcal{W}_p(\tau)\right) \right\} - \mathcal{L}V^k \left(P_i, \mathcal{S}(\tau), \mathcal{I}(\tau), \tau\right)\, \mathrm{d}\tau \tag{A.12}$$

Since

$$\int_{\tau^n}^{\tau^{n+1}} \frac{\mathrm{D}V^k}{\mathrm{D}\tau}\,\mathrm{d}\tau = V^k(P_i, S_l, I_j, \tau^{n+1}) - V^k(P_i, \mathcal{S}(\tau^n), \mathcal{I}(\tau), \tau^n) \tag{A.13}$$

it gives:

$$V^k(P_i, S_l, I_j, \tau^{n+1}) = \max_{W_w, W_p} \{V^k(P_i, \mathcal{S}(\tau^n), \mathcal{I}(\tau^n), \tau^n)) + \int_{\tau^n}^{\tau^{n+1}} \pi^k(P_i, \mathcal{W}_2(\tau))\,\mathrm{d}\tau\} \\ + \int_{\tau^n}^{\tau^{n+1}} \mathcal{L}V^k(P_i, \mathcal{S}(\tau), \mathcal{I}(\tau), \tau)\,\mathrm{d}\tau \tag{A.14}$$

where $\mathcal{S}(\tau) = \mathcal{S}(\tau; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau), \mathcal{W}_p(\tau))$ and $\mathcal{I}(\tau) = \mathcal{I}(\tau; P_i, I_j, S_l, \tau^{n+1}, \mathcal{W}_w(\tau), \mathcal{W}_p(\tau))$.

A.2 Fully Implicit Timestepping

A fully implicit timestepping is used to solve the PDEs (2.16).

From Chen & Forsyth (2010) we know that fully implicit timestepping approach is sufficient to get a satisfactory result. So in this thesis, we only use fully implicit timestepping to address our problem.

We assume:

$$(\mathcal{W}_w)_{i,j,l,(k)}(\tau) = (\mathcal{W}_w)_{i,j,l,(k)}(\tau^{n+1})$$
$$(\mathcal{W}_p)_{i,j,l,(k)}(\tau) = (\mathcal{W}_p)_{i,j,l,(k)}(\tau^{n+1})$$

where $\tau \in (\tau^n, \tau^{n+1})$.

Then we can obtain:

$$\mathcal{I}^n_{j(i,l,n+1)} = I_{j(l)} - \Delta\tau \left[(\mathcal{W}_p)_{i,j,l,(k)}(\tau^{n+1}) - (\mathcal{W}_w)_{i,j,l,(k)}(\tau^{n+1})\right]$$

while

$$\mathcal{S}^n_{l(i,j,n+1)} = S_{l(j)} + \Delta\tau \cdot Q \cdot (\mathcal{W}_p)_{i,j,l,(k)}(\tau^{n+1})$$

.

Then (A.14) can be discretized as:

$$V_{i,j,l}^{n+1} = \min_{\mathcal{W}_w, \mathcal{W}_p} V_{i,j,l(i,j,l,n+1)}^{n} + \Delta\tau \cdot \pi(P_i, \mathcal{W}_p(\tau)) + \Delta\tau^n (\mathcal{L}_h V)_{i,j,l}^{n+1} \tag{A.15}$$

As aforesaid, $\mathcal{S}(\tau^n)$, $\mathcal{I}(\tau^n)$ do not necessarily reside exactly at a grid point. Hence the corresponding $V_{i,j,l(i,j,l,n+1)}^{n}$ should be calculated by interpolating a set of values $V_{i,j,l}^{n}$. We use bilinear interpolation method herein.

Now we introduce a column vector

$V^n = [V_{1,1,1(1)}, V_{2,1,1(1)}, \ldots, V_{i_{max},1,1(1)}, V_{1,2,1(1)}, V_{2,2,1(1)}, \ldots, V_{i_{max},2,1(1)}, \ldots, V_{i_{max},j_{max},1(1)}, V_{1,1,2(1)}, \ldots, V_{i_{max},j_{max},2(1)}, \ldots, V_{i_{max},j_{max},l_{max}(1)}, V_{1,1,1(2)}, \ldots, V_{i_{max},j_{max},l_{max}(2)}, \ldots, V_{1,1,1(3)}, \ldots, V_{i_{max},j_{max},l_{max}(3)}]^T$

with $i_{max} \times j_{max} \times l_{max} \times 3$ entries.

Let L be a matrix such that

$$\begin{aligned}
[LV^n]_{i,j,l(k)} &= (\mathcal{L}_h V)_{i,j,l(k)}^{n} \\
&= \alpha_i^n V_{i-1,j,l(k)}^{n} + \beta_i^n V_{i+1,j,l(k)}^{n} - (\alpha_i^n + \beta_i^n + r) V_{i,j,l(k)}^{n} + \textstyle\sum_{u=1,u\neq k}^{3} \lambda^{k\to u} (V_{i,j,l(u)}^{n} - V_{i,j,l(k)}^{n})
\end{aligned}$$

Let Φ^{n+1} be the interpolation matrix such that

$$[\Phi^{n+1} V^n]_{i,j,l(k)} = V_{i,j,l(i,j,l,n+1)}^{n} + \text{interpolation error.}$$

Hence the matrix form of the PDEs can be written as:

$$V_{j,l(k)}^{n+1} - \Delta\tau^n [LV^{n+1}]_{j,l(k)} = [\Phi^{n+1} V^n]_{j,l(k)} + \Delta\tau^n \pi(P, W_p(\tau))$$

where $V_{i,j,l(k)}^{n+1} = \underset{W_w, W_p}{\operatorname{argmin}} (V_{i,j,l(i,j,l,n+1)(k)}^{n} + \Delta\tau^n \pi(P_i, W_p(\tau)) + \Delta\tau^n (\mathcal{L}_h V)_{i,j,l(k)}^{n+1})$

for $j = 0, ..., j_{max}$, $l = 0, ..., l_{max}$ and $k = 1, 2, 3$.

The above discrete optimization problem is solved numerically.

A.3 Accuracy Test

Since it is not feasible to solve the partial differential equation continuously, the extent to which we refine the discretized nodes is important to the solution accuracy. However, the smaller the mesh sizes are, the higher the algorithm complexity is. To balance the computation cost and accuracy, we test various mesh sizes and choose the discretization method shown in Table A.1.

Table A.1: The Discretized Nodes Without Refining

Coarse mesh size		
Oil price nodes	Resource nodes	Water inventory nodes
0	0	0
10	40	4
20	80	8
30	120	12
40	160	16
53	200	20
60	240	
70	300	
80	400	
90	500	
100	600	
120	660	
140	720	
160		
180		
200		
250		
300		
400		
500		

By refining the spacing between nodes along all these three dimensions once and twice, we get the convergence performance as Table A.2 shows.

Table A.2: Project Value (in Million Dollars) Convergence Performance

Current oil price ($/barrel)	Coarse	Refine1	Refine2	Refine2 − Coarse	Refine2 − Refine1	$\frac{\text{Refine1}-\text{Coarse}}{\text{Refine2}-\text{Refine1}}$	$\frac{\text{Refine2}-\text{Refine1}}{\text{Refine1}}$
				Green zone			
30	4739	4816	4875	77	59	1.31	1.2%
40	5700	5793	5861	93	68	1.37	1.2%
60	8299	8426	8514	127	88	1.44	1.0%
120	15526	15757	15888	231	131	1.76	0.8%
				Yellow zone			
30	4732	4809	4868	77	59	1.31	1.2%
40	5691	5785	5853	94	68	1.38	1.2%
60	8286	8412	8500	126	88	1.43	1.0%
120	15496	15727	15858	231	131	1.76	0.8%
				Red zone			
30	4698	4773	4831	75	58	1.29	1.2%
40	5647	5739	5806	92	67	1.37	1.2%
60	8216	8340	8427	124	87	1.43	1.0%
120	15337	15564	15693	227	129	1.76	0.8%

The values in column 2, 3, and 4 are the project's value when decision is making. By refining nodes spacings twice, the relative changes in values are less than 3%. This is an acceptable convergence performance. Considering the computation cost, we choose the refine once results as the base of our discussion.

A.4 Tables and Figures for Sensitivity Analysis

This appendix shows the details of the sensitivity analyses carried out for the case when there is an option to install the water storage facility. Sensitivities are analysed for the long-run mean log price, water productivity, and volatility.

Table A.3: The Hypothetical Project's Critical Prices ($/barrel)-changing Mean Log Oil Price

Resource stock (million barrels)	Mean log oil price	W_L									D_S		
		Green			Yellow			Red			Red		
		Stage			Stage			Stage			Stage		
		1→3	1→5	3→5	1→3	1→5	3→5	1→3	1→5	3→5	1→3	1→5	3→5
550		70	35	35	60	35	35	40	35	35	35	40	35
600		75	35	35	70	35	35	45	35	35	40	45	35
630	0.5μ	75	35	35	70	35	35	45	35	35	40	45	35
660		75	35	35	70	35	35	45	35	35	40	45	35
690		75	35	35	70	35	35	45	35	35	40	45	35
720		75	35	35	70	35	35	45	35	35	40	45	35
550		70	35	35	60	35	35	40	35	35	35	40	35
600		70	35	35	60	35	35	40	35	35	35	40	35
630	0.6μ	70	35	35	60	35	35	40	35	35	35	40	35
660		70	35	35	60	35	35	40	35	35	35	40	35
690		70	35	35	60	35	35	40	35	35	35	40	35
720		70	35	35	60	35	35	40	35	35	35	40	35
550		55	30	30	50	30	30	35	30	30	35	35	30
600		55	30	30	50	30	30	35	30	30	35	35	30
630	0.7μ	55	30	30	50	30	30	35	30	30	35	35	30
660		55	30	30	50	30	30	35	30	30	35	35	30
690		55	30	30	50	30	30	35	30	30	35	35	30
720		55	30	30	50	30	30	35	30	30	35	35	30
550		45	25	25	40	25	25	30	25	25	30	30	30
600		45	25	25	40	25	25	30	25	25	30	30	30
630	0.8μ	45	25	25	40	25	25	30	25	25	30	30	30
660		45	25	25	40	25	25	30	25	25	30	30	30
690		45	25	25	40	25	25	30	25	25	30	30	30
720		45	25	25	40	25	25	30	25	25	30	30	30
550		45	15	15	40	15	15	35	15	15	25	20	20
600		40	15	15	35	15	15	30	15	15	25	20	20
630	0.9μ	40	15	15	35	15	15	30	15	15	25	20	20
660		40	15	15	35	15	15	30	15	15	25	20	20
690		35	15	15	35	15	15	30	15	15	25	20	20
720		35	15	15	35	15	15	30	15	15	25	20	20
550		55	5	5	50	5	5	45	5	5	25	5	5
600		50	5	5	45	5	5	40	5	5	25	5	5
630	μ	50	5	5	45	5	5	40	5	5	25	5	5
660		45	5	5	40	5	5	35	5	5	20	5	5
690		40	5	5	40	5	5	35	5	5	20	5	5
720		40	5	5	35	5	5	30	5	5	20	5	5
550		90	0	0	80	0	0	70	0	0	35	0	0
600		75	0	0	70	0	0	60	0	0	25	0	0
630	1.1μ	70	0	0	65	0	0	55	0	0	25	0	0
660		65	0	0	60	0	0	50	0	0	20	0	0
690		60	0	0	55	0	0	45	0	0	20	0	0
720		55	0	0	50	0	0	40	0	0	15	0	0

Table A.4: Marginal Costs of Increased Water Restrictions Under Various Mean Log Oil Prices ($/barrel)

Mean log price	Oil price ($/barrel)	W_L Green	W_L Yellow	W_L Red	D_S Red
0.5μ	20	0	0	0	0
	30	0	0	0	0
	50	0.06	0.07	0.02	0.18
	70	0.06	0.02	0.03	0.42
	100	0.03	0.03	0.06	0.74
0.6μ	20	0	0	0	0
	30	0	0	0	0
	50	0.06	0.08	0.02	0.26
	70	0.02	0.02	0.04	0.57
	100	0.04	0.04	0.07	0.95
0.7μ	20	0	0	0	0
	30	0	0	0	0
	50	0.06	0.03	0.04	0.44
	70	0.04	0.04	0.06	0.85
	100	0.06	0.06	0.09	1.33
0.8μ	20	0	0	0	0
	30	0.03	0.03	0.03	0
	50	0.04	0.04	0.06	0.86
	70	0.06	0.06	0.08	1.37
	100	0.08	0.09	0.12	1.91
0.9μ	20	0.03	0.03	0.03	0
	30	0.06	0.06	0.06	0.81
	50	0.07	0.07	0.08	1.56
	70	0.09	0.09	0.11	2.04
	100	0.11	0.11	0.14	2.59
μ	20	0.08	0.08	0.08	1.33
	30	0.08	0.08	0.08	1.63
	50	0.09	0.09	0.10	2.16
	70	0.11	0.11	0.12	2.63
	100	0.13	0.13	0.16	3.23
1.1μ	20	0.11	0.11	0.11	2.07
	30	0.12	0.12	0.12	2.36
	50	0.13	0.12	0.12	2.77
	70	0.13	0.13	0.14	3.28
	100	0.16	0.16	0.18	3.98

The remaining resource stock is 720 million barrels. $\mu = 4.59$. Stage 1, prior to building water storage.

Table A.5: Total Costs To the Oil Sands Project Due To the Water Constraints Under Various Mean Log Oil Prices (in Million Dollars)

Mean log price	Oil price ($/barrel)	W_L			D_S
		Green	Yellow	Red	Red
0.5μ	20	0	0	0	0
	30	0	0	0	0
	50	17.61	21.85	42.63	146.80
	70	37.69	44.32	45.56	267.36
	100	47.04	47.16	48.62	437.92
0.6μ	20	0	0	0	0
	30	0	0	0	0
	50	23.46	27.77	43.65	192.18
	70	45.70	46.28	47.27	351.08
	100	49.28	49.40	50.89	559.42
0.7μ	20	0	0	0	0
	30	0	0	0	0
	50	41.72	46.11	48.18	325.09
	70	51.45	51.53	52.54	530.54
	100	54.93	55.05	56.57	780.53
0.8μ	20	0	0	0	0
	30	21.13	22.13	27.64	116.40
	50	54.31	54.37	55.02	540.41
	70	57.84	57.92	58.92	781.25
	100	60.77	60.89	62.39	1062.65
0.9μ	20	29.80	29.81	29.84	185.54
	30	50.81	51.21	53.64	523.97
	50	62.63	62.68	63.25	859.98
	70	64.47	64.54	65.44	1087.48
	100	66.24	66.36	67.74	1370.38
μ	20	58.39	58.39	58.40	781.91
	30	63.11	63.17	63.56	906.39
	50	66.99	67.02	67.42	1110.61
	70	67.77	67.83	68.54	1332.22
	100	68.97	69.07	70.26	1634.74
1.1μ	20	62.53	62.53	62.54	1134.39
	30	65.74	65.74	65.76	1255.55
	50	68.12	68.90	69.40	1375.49
	70	69.61	69.65	70.11	1591.20
	100	70.75	70.82	71.72	1928.82

The remaining resource stock is 720 million barrels.
Total cost is the project value without water withdrawal restrictions less project value with water restrictions in the W_L scenario.

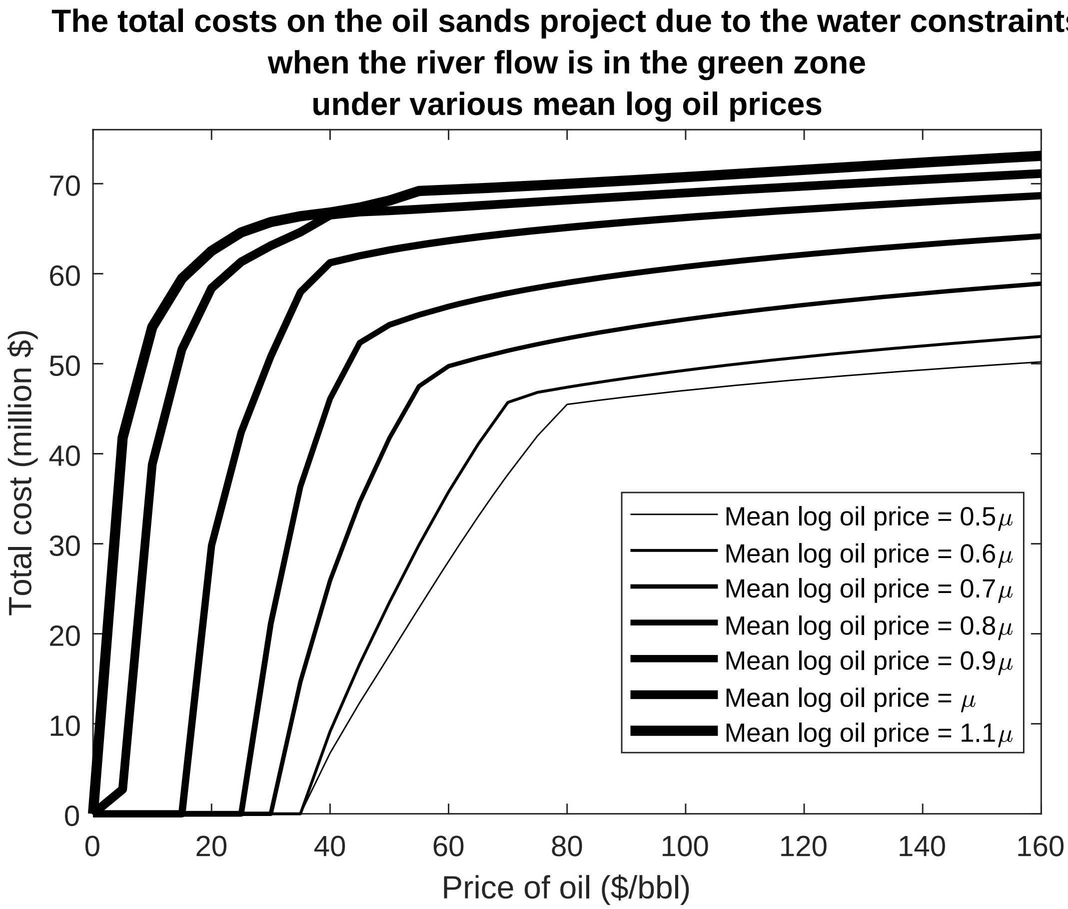

Figure A.1: Total cost under different mean log oil prices (Total cost is the project value without water withdrawal restrictions less project value with water restrictions in the W_L scenario.)

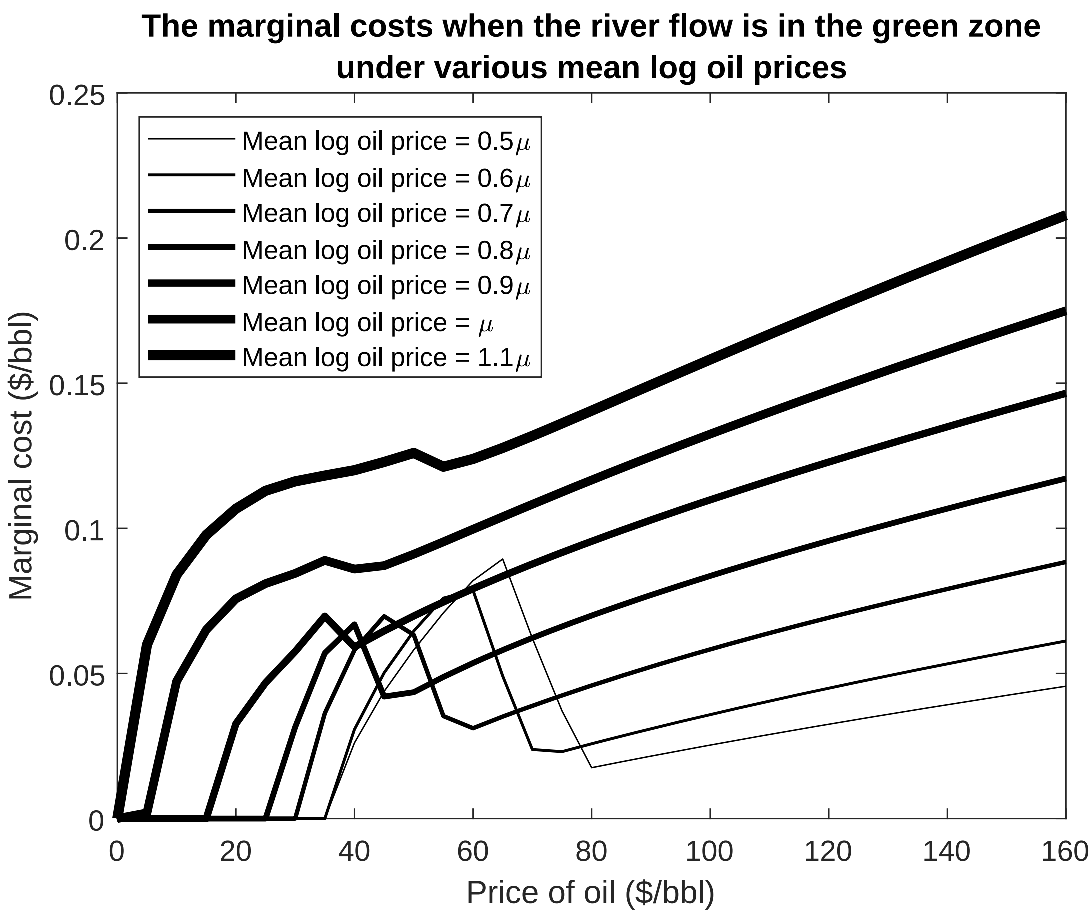

Figure A.2: Marginal cost under different mean oil prices (The marginal cost is the loss in value to the project on a $/barrel basis of an increase in water withdrawal restrictions as outlined in Section 2.6.2, page 33 in the W_L scenario.)

Table A.6: The Hypothetical Project's Critical Prices ($/barrel)-changing Water Productivity

Resource stock (million barrels)	Water productivity (barrels of bitumen/ barrel of water	W_L									D_S		
		Green			Yellow			Red			Red		
		Stage			Stage			Stage			Stage		
		1→3	1→5	3→5	1→3	1→5	3→5	1→3	1→5	3→5	1→3	1→5	3→5
550		30	5	5	25	5	5	25	5	5	25	20	20
600		30	5	5	25	5	5	25	5	5	25	20	20
630	0.1	25	5	5	25	5	5	25	5	5	25	20	20
660		25	5	5	25	5	5	25	5	5	25	20	20
690		25	5	5	20	5	5	20	5	5	25	20	20
720		25	5	5	20	5	5	20	5	5	25	20	20
550		40	5	5	40	5	5	35	5	5	25	10	10
600		40	5	5	35	5	5	35	5	5	20	10	10
630	0.2	35	5	5	30	5	5	30	5	5	20	10	10
660		35	5	5	30	5	5	30	5	5	20	10	10
690		30	5	5	30	5	5	25	5	5	20	10	10
720		30	5	5	25	5	5	25	5	5	20	10	10
550		55	5	5	50	5	5	45	5	5	25	5	5
600		50	5	5	45	5	5	40	5	5	25	5	5
630	0.3	50	5	5	45	5	5	40	5	5	25	5	5
660	(base case)	45	5	5	40	5	5	35	5	5	20	5	5
690		40	5	5	40	5	5	35	5	5	20	5	5
720		40	5	5	35	5	5	30	5	5	20	5	5
550		H	5	5	450	5	5	275	5	5	35	5	5
600		H	5	5	400	5	5	250	5	5	30	5	5
630	0.4	450	5	5	400	5	5	250	5	5	25	5	5
660		450	5	5	350	5	5	225	5	5	25	5	5
690		400	5	5	350	5	5	225	5	5	25	5	5
720		400	5	5	350	5	5	200	5	5	20	5	5
550		H	5	5	H	5	5	H	5	5	40	5	5
600		H	5	5	H	5	5	H	5	5	40	5	5
630	0.5	H	5	5	H	5	5	H	5	5	35	5	5
660		H	5	5	H	5	5	H	5	5	35	5	5
690		H	5	5	H	5	5	H	5	5	30	5	5
720		H	5	5	H	5	5	H	5	5	30	5	5
550		H	5	5	H	5	5	H	5	5	45	5	5
600		H	5	5	H	5	5	H	5	5	45	5	5
630	0.6	H	5	5	H	5	5	H	5	5	40	5	5
660		H	5	5	H	5	5	H	5	5	40	5	5
690		H	5	5	H	5	5	H	5	5	35	5	5
720		H	5	5	H	5	5	H	5	5	35	5	5

"H" means that the critical prices is higher than $500/barrel. Since in the history the oil price never exceeded $500/barrel, it implies that it is never optimal to proceed from operating stage 1 to stage 2.

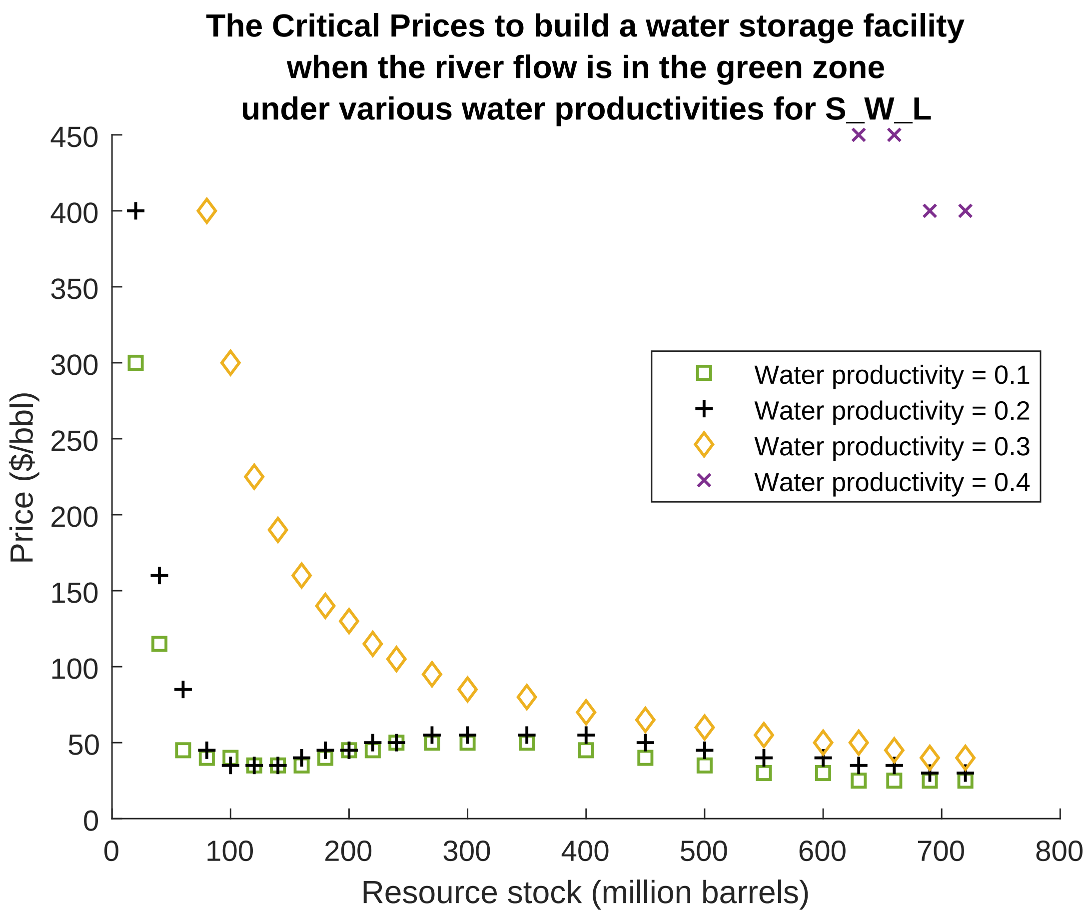

Figure A.3: Critical prices to switch from stage 1 to 3 (i.e. to construct water storage) under different water productivity levels

Table A.7: Marginal Costs To the Oil Sands Project Due To the Water Constraints Under Various Levels of Water Productivity ($/barrel)

Water	Oil price	W_L			D_S
productivity	($/barrel)	Green	Yellow	Red	Red
0.1	20	0.51	0.51	0.51	0.00
	30	0.64	0.64	0.64	0.78
	50	0.86	0.86	0.87	1.55
	70	1.04	1.06	1.09	2.04
	100	1.27	1.29	1.34	2.71
0.2	20	0.36	0.36	0.36	0.91
	30	0.42	0.42	0.42	1.22
	50	0.50	0.51	0.54	1.78
	70	0.60	0.61	0.66	2.28
	100	0.74	0.76	0.83	2.78
0.3	20	0.07	0.07	0.07	1.32
	30	0.08	0.08	0.08	1.62
	50	0.09	0.09	0.10	2.15
	70	0.11	0.11	0.12	2.62
	100	0.13	0.13	0.15	3.22
0.4	20	0.15	0.15	0.15	1.13
	30	0.17	0.17	0.17	1.30
	50	0.19	0.19	0.21	1.57
	70	0.21	0.22	0.26	1.90
	100	0.23	0.24	0.21	2.35
0.5	20	0	0	0	0.08
	30	0	0	0	0.08
	50	0	0	0	0.08
	70	0	0	0	0.10
	100	0	0	0	0.12
0.6	20	0	0	0	0.02
	30	0	0	0	0.01
	50	0	0	0	0
	70	0	0	0	0
	100	0	0	0	0

The remaining resource stock is 720 million barrels.
The marginal cost is the loss in value to the project on a $/barrel basis of an increase in water withdrawal restrictions as outlined in Section 2.6.2, page 33.

Table A.8: Total Costs To the Oil Sands Project Due To the Water Constraints Under Various Levels of Water Productivity (in Million Dollars)

Water productivity	Oil price ($/barrel)	W_L Green	W_L Yellow	W_L Red	D_S Red
0.1	20	1232.22	1232.26	1232.47	2770.14
	30	1462.68	1463.57	1466.90	4206.06
	50	1809.07	1834.28	1891.49	5893.26
	70	2161.75	2202.04	2307.71	7249.87
	100	2621.66	2681.44	2836.09	8902.98
0.2	20	296.73	296.73	296.78	1686.30
	30	337.04	337.23	337.96	2050.92
	50	384.13	389.24	413.62	2660.98
	70	447.51	456.16	497.49	3215.89
	100	535.95	550.01	616.97	3925.90
0.3	20	58.17	58.17	58.18	777.81
	30	62.87	62.93	63.32	901.40
	50	66.74	66.77	67.14	1103.97
	70	67.47	67.53	68.21	1324.01
	100	68.61	68.70	69.83	1624.37
0.4	20	13.78	13.78	13.78	172.96
	30	15.65	15.66	15.73	190.81
	50	17.91	18.20	20.06	211.71
	70	21.53	22.07	25.52	241.95
	100	26.73	27.64	33.42	284.53
0.5	20	0	0	0	58.61
	30	0	0	0	62.24
	50	0	0	0	62.65
	70	0	0	0	62.61
	100	0	0	0	62.67
0.6	20	0	0	0	55.00
	30	0	0	0	59.32
	50	0	0	0	62.29
	70	0	0	0	62.20
	100	0	0	0	62.18

The remaining resource stock is 720 million barrels.
Total cost is the project value without water withdrawal restrictions less project value with water restrictions in the W_L scenario.

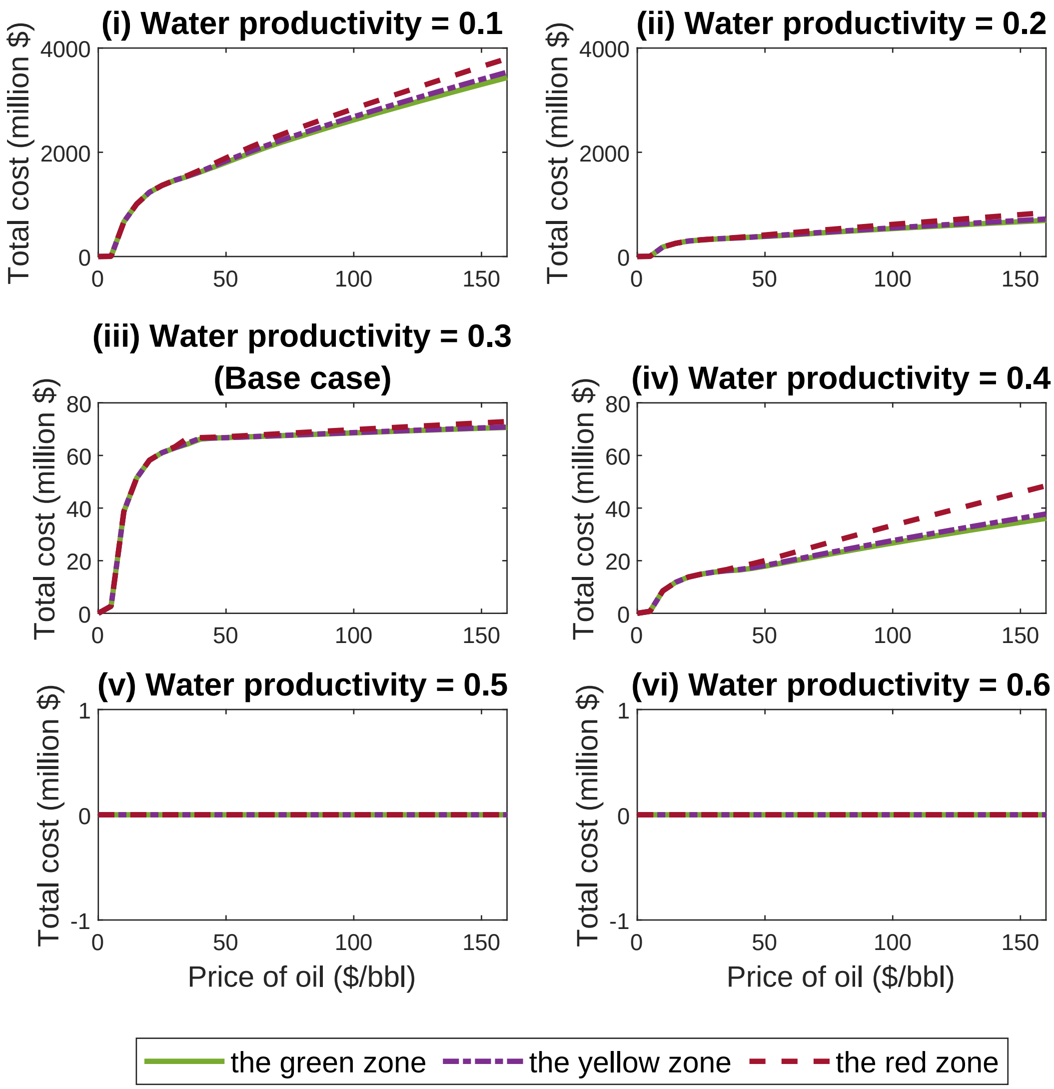

Figure A.4: Total cost to the oil sands project due to the water constraints under various water productivity levels for the scenario W_L (Water productivity is in barrels of bitumen/barrel of water. Total cost is defined as the project value without water withdrawal restrictions less project value with water restrictions.)

Table A.9: The Hypothetical Project's Critical Prices ($/barrel)-changing Oil Price Volatility

Resource stock (million barrels)	Oil price volatility	W_L									D_S		
		Green			Yellow			Red			Red		
		Stage			Stage			Stage			Stage		
		1→3	1→5	3→5	1→3	1→5	3→5	1→3	1→5	3→5	1→3	1→5	3→5
550	0	60	5	5	60	5	5	55	5	5	35	5	5
600		55	5	5	55	5	5	50	5	5	30	5	5
630		55	5	5	50	5	5	45	5	5	25	5	5
660		50	5	5	45	5	5	45	5	5	25	5	5
690		45	5	5	45	5	5	40	5	5	20	5	5
720		45	5	5	40	5	5	35	5	5	20	5	5
550	0.3σ	60	5	5	60	5	5	55	5	5	30	5	5
600		55	5	5	55	5	5	50	5	5	25	5	5
630		55	5	5	50	5	5	45	5	5	25	5	5
660		50	5	5	45	5	5	45	5	5	25	5	5
690		45	5	5	45	5	5	40	5	5	20	5	5
720		45	5	5	40	5	5	35	5	5	20	5	5
550	0.6σ	60	5	5	55	5	5	50	5	5	30	5	5
600		55	5	5	50	5	5	45	5	5	25	5	5
630		50	5	5	50	5	5	45	5	5	25	5	5
660		50	5	5	45	5	5	40	5	5	25	5	5
690		45	5	5	40	5	5	40	5	5	20	5	5
720		40	5	5	40	5	5	35	5	5	20	5	5
550	0.9σ	55	5	5	55	5	5	45	5	5	30	5	5
600		50	5	5	50	5	5	40	5	5	25	5	5
630		50	5	5	45	5	5	40	5	5	25	5	5
660		45	5	5	40	5	5	35	5	5	20	5	5
690		40	5	5	40	5	5	35	5	5	20	5	5
720		40	5	5	35	5	5	30	5	5	20	5	5
550	1.2σ	55	5	5	50	5	5	45	5	5	25	5	5
600		50	5	5	45	5	5	40	5	5	25	5	5
630		50	5	5	45	5	5	35	5	5	25	5	5
660		45	5	5	40	5	5	35	5	5	20	5	5
690		40	5	5	35	5	5	30	5	5	20	5	5
720		40	5	5	35	5	5	30	5	5	20	5	5
550	1.5σ	55	5	5	50	5	5	40	5	5	25	5	5
600		50	5	5	45	5	5	35	5	5	25	5	5
630		50	5	5	45	5	5	35	5	5	25	5	5
660		45	5	5	40	5	5	35	5	5	20	5	5
690		40	5	5	35	5	5	30	5	5	20	5	5
720		40	0	0	35	0	0	30	0	0	20	5	5
550	1.8σ	60	5	5	50	5	5	40	5	5	25	5	5
600		55	5	5	45	5	5	35	5	5	25	5	5
630		50	5	5	45	5	5	35	5	5	25	5	5
660		45	5	5	40	5	5	35	5	5	20	5	5
690		45	5	5	40	5	5	30	5	5	20	5	5
720		45	5	5	35	5	5	30	5	5	20	5	5
550	2.1σ	65	5	5	55	5	5	40	5	5	25	5	5
600		55	0	0	50	0	0	35	0	0	25	5	5
630		55	0	0	45	0	0	35	0	0	20	5	5
660		50	0	0	45	0	0	35	0	0	20	5	5
690		50	0	0	40	0	0	30	0	0	20	5	5
720		45	0	0	40	0	0	30	0	0	20	5	5
550	2.4σ	65	0	0	55	0	0	40	0	0	25	5	5
600		60	0	0	50	0	0	35	0	0	25	5	5
630		55	0	0	50	0	0	35	0	0	20	5	5
660		55	0	0	45	0	0	30	0	0	20	5	5
690		50	0	0	45	0	0	30	0	0	20	5	5
720		50	0	0	40	0	0	30	0	0	20	5	5
550	2.7σ	70	5	5	60	5	5	40	5	5	25	5	5
600		65	5	5	55	5	5	35	5	5	25	5	5
630		60	5	5	50	5	5	35	5	5	20	5	5
660		60	5	5	50	5	5	30	5	5	20	5	5
690		55	5	5	45	5	5	30	5	5	20	5	5
720		55	5	5	45	5	5	30	5	5	20	5	5

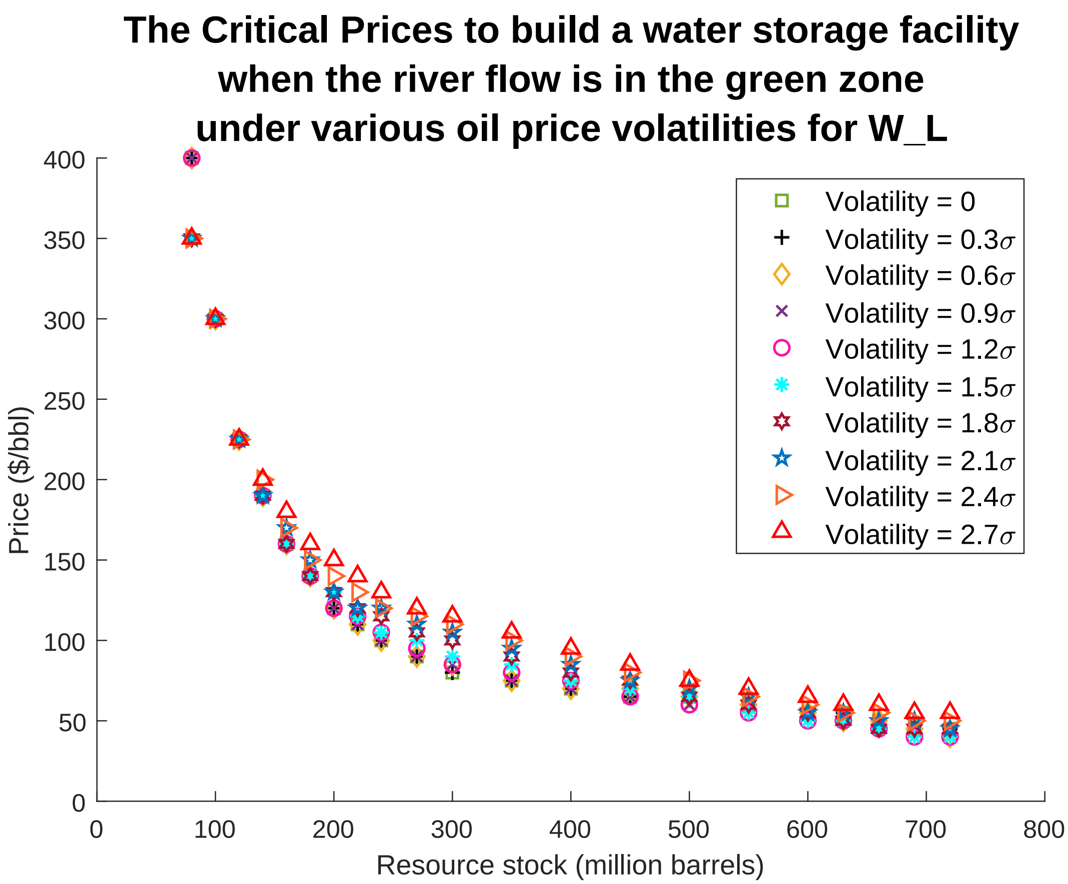

Figure A.5: Critical prices under different volatility levels

Table A.10: Marginal Costs To the Oil Sands Project Due To the Water Constraints Under Various Levels of Oil Price Volatility ($/barrel)

Volatility	Oil price	W_L			D_S
	($/barrel)	Green	Yellow	Red	Red
	20	0.08	0.08	0.08	1.52
	30	0.09	0.09	0.09	1.78
0	50	0.09	0.09	0.10	2.27
	70	0.11	0.11	0.12	2.73
	100	0.13	0.14	0.16	3.35
	20	0.08	0.08	0.08	1.49
	30	0.09	0.09	0.09	1.77
0.3σ	50	0.09	0.09	0.10	2.26
	70	0.11	0.11	0.12	2.72
	100	0.13	0.14	0.16	3.33
	20	0.08	0.08	0.08	1.45
	30	0.09	0.09	0.09	1.74
0.6σ	50	0.09	0.09	0.10	2.26
	70	0.11	0.11	0.12	2.72
	100	0.13	0.14	0.16	3.31
	20	0.08	0.08	0.08	1.35
	30	0.09	0.09	0.09	1.67
0.9σ	50	0.09	0.09	0.10	2.19
	70	0.11	0.11	0.12	2.65
	100	0.13	0.13	0.16	3.25
	20	0.07	0.07	0.07	1.24
	30	0.08	0.08	0.08	1.55
1.2σ	50	0.09	0.09	0.10	2.10
	70	0.11	0.11	0.12	2.58
	100	0.13	0.13	0.16	3.16
	20	0.07	0.07	0.07	1.13
	30	0.08	0.08	0.08	1.44
1.5σ	50	0.09	0.09	0.10	1.99
	70	0.11	0.11	0.12	2.46
	100	0.13	0.13	0.16	3.06
	20	0.07	0.07	0.07	1.02
	30	0.08	0.08	0.08	1.33
1.8σ	50	0.08	0.09	0.10	1.88
	70	0.10	0.10	0.12	2.35
	100	0.13	0.13	0.16	2.93
	20	0.06	0.06	0.06	0.95
	30	0.08	0.08	0.08	1.25
2.1σ	50	0.08	0.08	0.09	1.78
	70	0.10	0.10	0.12	2.23
	100	0.12	0.13	0.15	2.81
	20	0.06	0.06	0.06	0.88
	30	0.08	0.08	0.08	1.17
2.4σ	50	0.08	0.08	0.09	1.69
	70	0.10	0.10	0.12	2.14
	100	0.12	0.12	0.15	2.71
	20	0.06	0.06	0.06	0.81
	30	0.08	0.08	0.08	1.09
2.7σ	50	0.09	0.08	0.09	1.61
	70	0.09	0.09	0.11	2.04
	100	0.11	0.12	0.15	2.60

The remaining resource stock is 720 million barrels.

Marginal cost refers to the loss in value to the project on a $/barrel basis of an increase in water withdrawal restrictions as outlined in Section 2.6.2, page 33.

Table A.11: Total Costs To the Oil Sands Project Due To the Water Constraints Under Various Levels of Oil Price Volatility (in Million Dollars)

Volatility	Oil price	W_L			D_S	Volatility	Oil price	W_L			D_S
	($/barrel)	Green	Yellow	Red	Red		($/barrel)	Green	Yellow	Red	Red
	20	60.85	60.85	60.85	865.49		20	51.16	51.17	51.22	671.27
	30	64.20	64.21	64.23	969.40		30	58.40	58.67	60.14	813.85
0	50	66.97	66.99	67.30	1128.66	1.5σ	50	65.30	65.34	65.82	1056.34
	70	67.90	67.96	68.60	1374.55		70	66.40	66.47	67.26	1277.02
	100	69.16	69.25	70.38	1689.82		100	67.81	67.91	69.18	1577.54
	20	60.48	60.48	60.48	854.77		20	46.39	46.41	46.54	612.44
	30	64.28	64.29	64.33	965.63		30	54.68	55.15	57.90	761.07
0.3σ	50	67.02	67.04	67.35	1128.33	1.8σ	50	63.83	63.88	64.39	1015.16
	70	67.90	67.95	68.60	1369.69		70	65.12	65.19	66.03	1236.09
	100	69.14	69.23	70.37	1681.52		100	66.68	66.79	68.11	1534.70
	20	59.18	59.18	59.19	832.09		20	42.28	42.32	42.58	568.58
	30	64.11	64.12	64.19	951.59		30	51.17	51.83	55.83	717.33
0.6σ	50	67.14	67.16	67.50	1125.05	2.1σ	50	62.38	62.43	62.97	973.56
	70	67.92	67.98	68.65	1355.15		70	63.77	63.84	64.71	1192.91
	100	69.13	69.22	70.37	1663.22		100	65.41	65.52	66.89	1488.36
	20	57.92	57.92	57.93	787.02		20	38.27	38.34	38.77	525.34
	30	63.57	63.62	63.88	923.67		30	47.45	48.26	53.37	675.23
0.9σ	50	67.13	67.16	67.54	1117.46	2.4σ	50	60.13	60.93	61.50	932.49
	70	67.88	67.94	68.64	1340.43		70	62.37	62.44	63.35	1149.37
	100	69.06	69.15	70.33	1644.56		100	64.09	64.20	65.60	1442.15
	20	55.77	55.77	55.79	735.28		20	33.40	33.51	34.15	477.37
	30	61.36	61.47	62.16	868.07		30	43.14	44.08	50.00	630.36
1.2σ	50	66.46	66.50	66.92	1091.92	2.7σ	50	56.70	59.21	59.79	889.56
	70	67.36	67.42	68.17	1311.88		70	60.81	60.89	61.82	1105.40
	100	68.64	68.74	69.96	1614.81		100	62.65	62.77	64.20	1396.18

The remaining resource stock is 720 million barrels.
Total cost is defined as the project value without water withdrawal restrictions less project value with water restrictions.

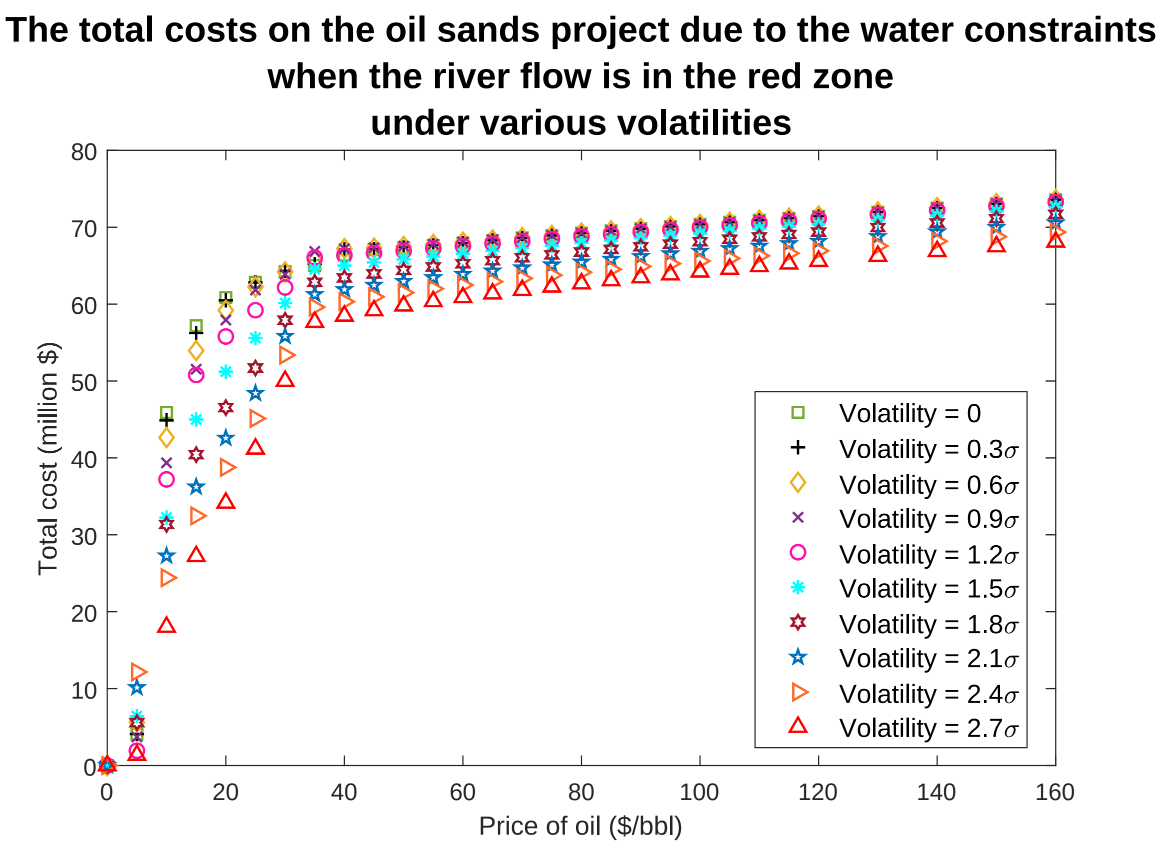

Figure A.6: Total cost to the oil sands project due to the water constraints under various oil price volatility levels for the scenario W_L (Total cost is defined as the project value without water withdrawal restrictions less project value with water restrictions.)

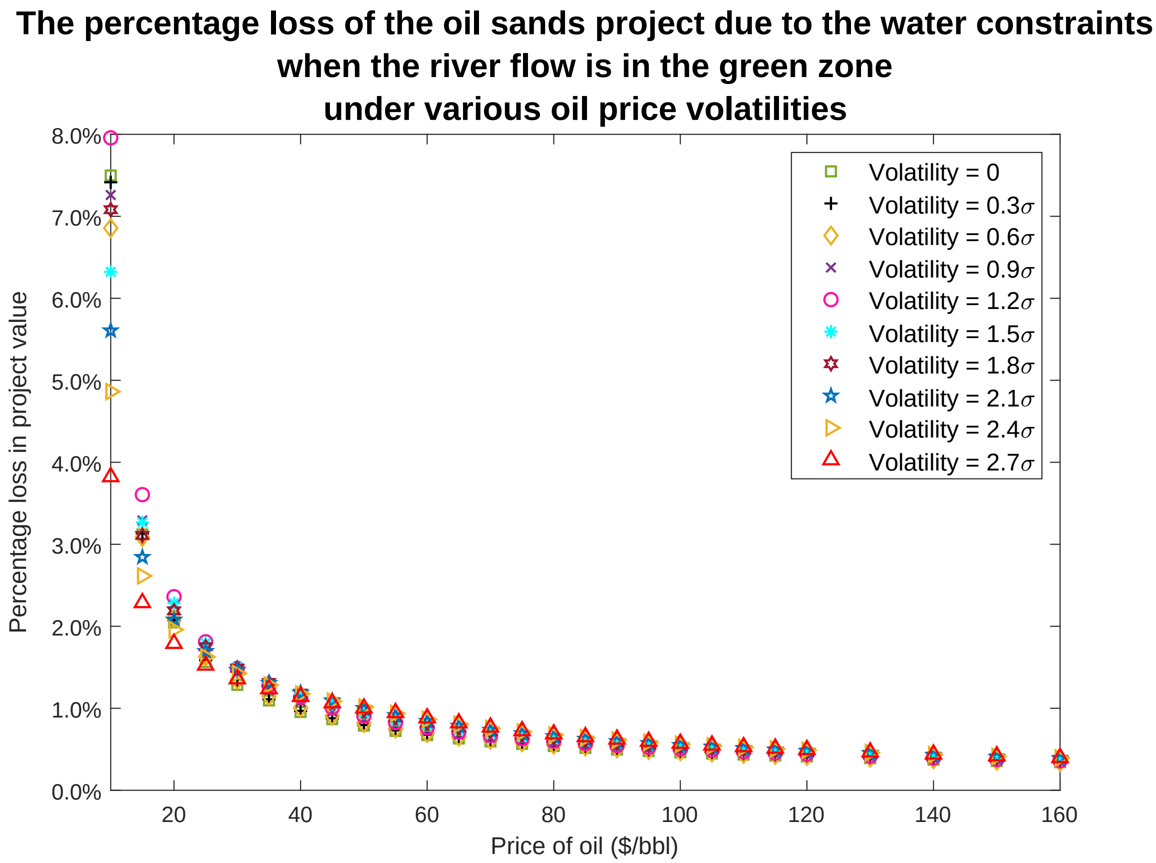

Figure A.7: The percentage loss to the oil sands project due to the water constraints when the river flow is in the green zone under various oil price volatility levels for the scenario W_L (The percentage loss refers to the reduction in total project value when restrictions are imposed.)

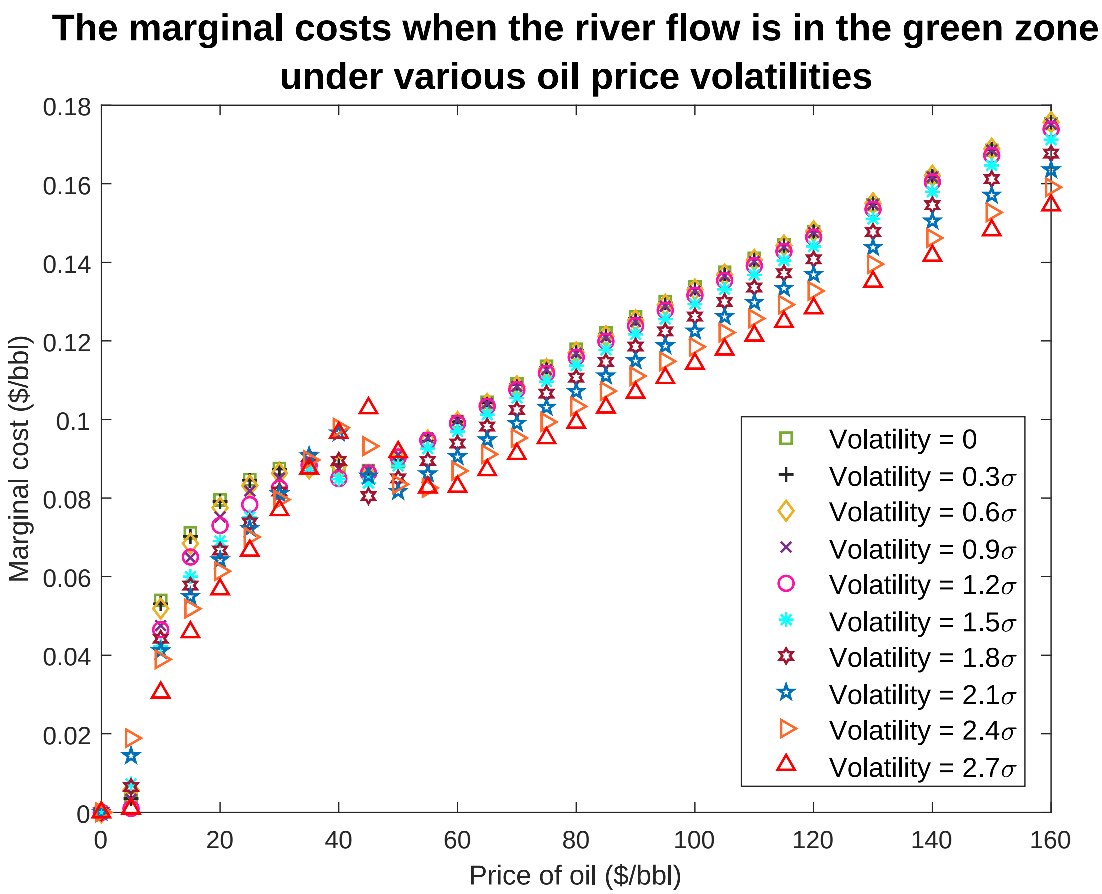

Figure A.8: The marginal costs when the river flow is in the green zone under various oil price volatility levels for the scenario W_L (Marginal cost refers to the loss in value to the project on a $/barrel basis of an increase in water withdrawal restrictions as outlined in Section 2.6.2, page 33.)

Appendix B

Tables and Figures for Chapter 3

Table B.1: Weekly Flow Triggers and Cumulative Water Use Limits On the Lower Athabasca River for Oil Sands Operations for Alternative Rule Sets

Alt 19

Period 1 (Weeks 1-15)		Period 2 (Weeks 16-18)		Period 3 (Weeks 19-45)		Period 4 (Weeks 46-49)		Period 5 (Weeks 50-52)	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>140	16	None	16	None	29	None	16	>110	16
<140	11.5% of RF							<110	11.5% of RF

Alt 20

Period 1 (Weeks 1-15)		Period 2 (Weeks 16-23)		Period 3 (Weeks 24-43)		Period 4 (Weeks 44-52)	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>185	16	None	16	None	29	>185	16
<185	8.5% of RF					<185	8.5% of RF

Alt 21

Period 1 (Weeks 1-15)		Period 2 (Weeks 16-23)		Period 3 (Weeks 24-43)		Period 4 (Weeks 44-52)	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>270	16	None	16	None	29	>270	16
<270	6% of RF					<270	6% of RF

Alt 22

Period 1 (Weeks 1-18)		Period 2 (Weeks 19-23)		Period 3 (Weeks 24-43)		Period 4 (Weeks 44-52)	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>355	16	None	16	None	29	>355	16
<355	4.5% of RF					<355	4.5% of RF

Option A

Period 1 (Weeks 1-15)		Period 2 (Weeks 16-23)		Period 3 (Weeks 24-43)		Period 4 (Weeks 44-52)	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>270	16					>270	16
133∼270	6% of RF	None	16	None	29	133∼270	6% of RF
<133	8					<133	8

Option H

Period 1 (Weeks 1-15)		Period 2 (Weeks 16-18)		Period 3 (Weeks 19-23)		Period 4 (Weeks 24-43)		Period 5 (Weeks 44-52)	
RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits	RF Triggers	Limits
>270	16							>200	16
150∼270	6% of RF	>87	16	>87	20	>87	29	150∼200	8% of RF
87∼150	9							87∼150	12
<87	4.4	<87	4.4	<87	4.4	<87	4.4	<87	4.4

1. “RF” stands for “the river’s flow”;
2. “Limits” means “cumulative water withdrawal limits”;
3. “RF Triggers” and “Limits” are both measured in cubic metres per second.

Table B.2: The Ranges of Weekly Flow for River Flow Regimes Defined In Different Water Management Rule Sets

Alt 19

Zone	1	2	3
Weekly flow (m^3/s)	$(-\infty,110)$	$[110,140)$	$[140,+\infty)$

Alt 20

Zone	1	2
Weekly flow (m^3/s)	$(-\infty,185)$	$[185,+\infty)$

Alt 21

Zone	1	2
Weekly flow (m^3/s)	$(-\infty,270)$	$[270,+\infty)$

Alt 22

Zone	1	2
Weekly flow (m^3/s)	$(-\infty,355)$	$[355,+\infty)$

Option A

Zone	1	2	3
Weekly flow (m^3/s)	$(-\infty,133)$	$[133,270)$	$[270,+\infty)$

Option H

Zone	1	2	3	4	5
Weekly flow (m^3/s)	$(-\infty,87)$	$[87,150)$	$[150,200)$	$[200,270)$	$[270,+\infty)$

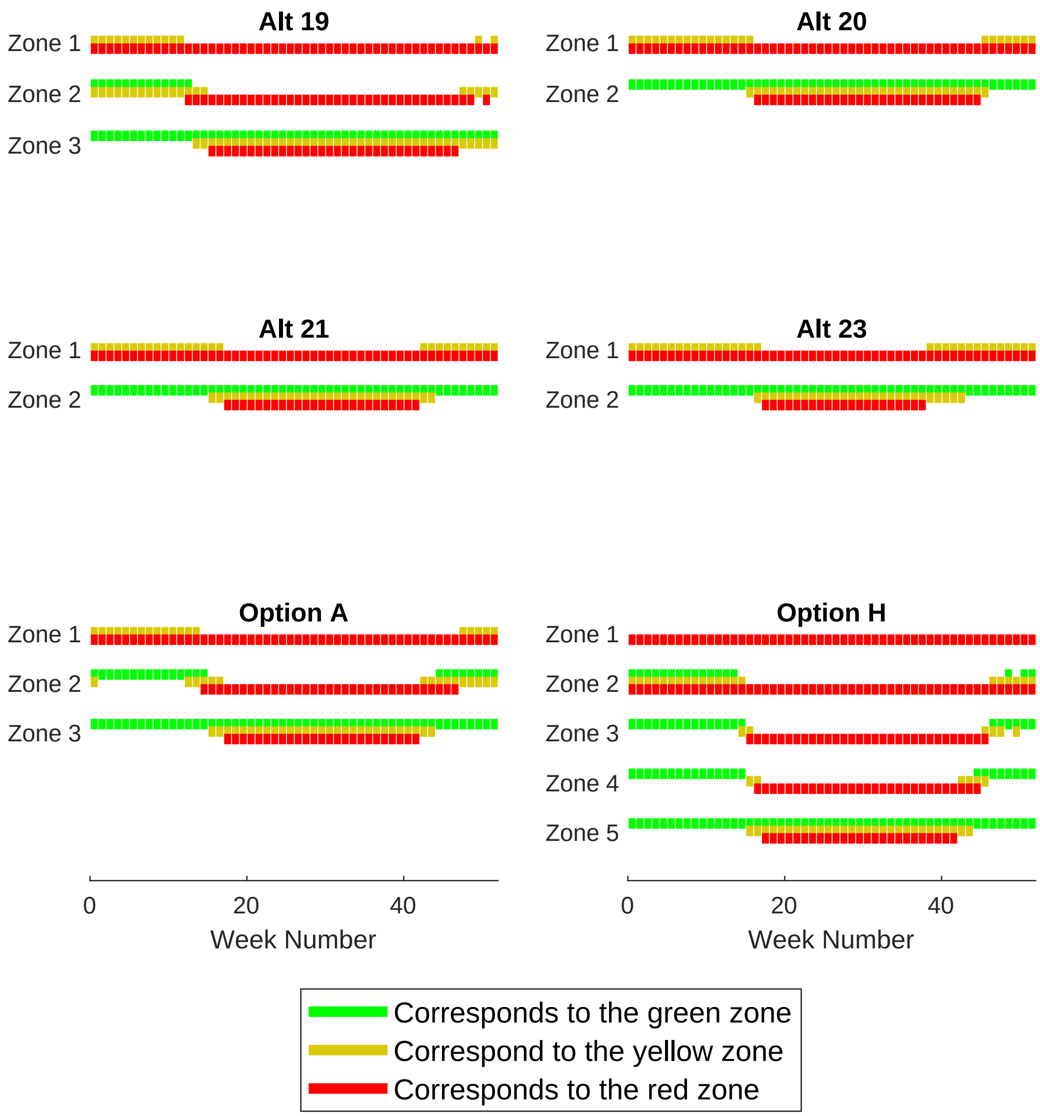

Figure B.1: The corresponding relationship between the regimes defined by different alternative rule sets and the three zones in the Phase 1 Framework

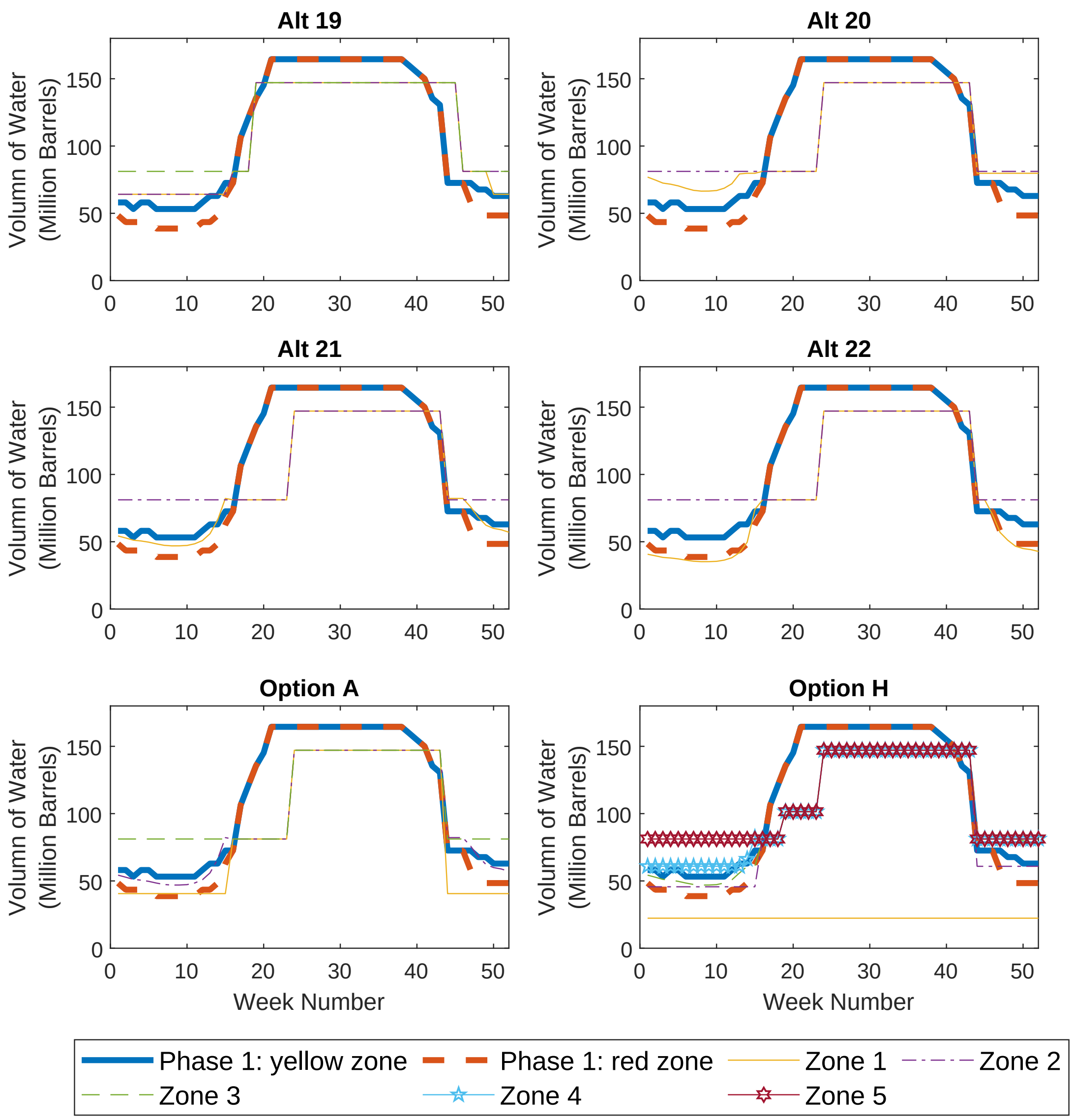

Figure B.2: Comparison of the Phase 1 rules and alternative rule sets for the Phase 2 Framework in terms of the cumulative weekly water withdrawal limits on the Lower Athabasca River for oil sands operations

Table B.3: Cumulative Weekly Withdrawal Limits for Each Regime On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0
Zone 3	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6
Zone 4	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6
Zone 5	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6
Zone 6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6
Zone 7	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6
Zone 8	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3
Zone 9	71.5	71.5	71.5	71.5	71.5	71.5	71.5	71.5	71.5	71.5	71.5	71.5	71.5
Zone 10	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0
Zone 3	45.6	45.6	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3
Zone 4	45.6	45.6	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0
Zone 5	45.6	45.6	81.2	81.2	81.2	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3
Zone 6	45.6	45.6	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	124.3	124.3	124.3
Zone 7	45.6	45.6	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1
Zone 8	53.3	53.3	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1
Zone 9	71.5	71.5	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1
Zone 10	81.2	81.2	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1

Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0
Zone 3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3
Zone 4	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0
Zone 5	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3	91.3
Zone 6	124.3	124.3	124.3	124.3	124.3	124.3	124.3	124.3	124.3	124.3	124.3	124.3	124.3
Zone 7	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 8	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 9	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 10	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1

Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0	34.0
Zone 3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3
Zone 4	71.0	71.0	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9
Zone 5	91.3	91.3	91.3	91.3	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9
Zone 6	124.3	124.3	124.3	124.3	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9
Zone 7	147.1	147.1	147.1	147.1	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9
Zone 8	147.1	147.1	147.1	147.1	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0	71.0
Zone 9	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Zone 10	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Table B.4: Cumulative Weekly Withdrawal Limits for Each Regime Defined In Alt 19 On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2
Zone 2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2	64.2
Zone 3	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	64.2	64.2	81.2	81.2	81.2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 2	64.2	64.2	81.2	81.2	81.2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 3	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 3	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	147.1	147.1	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	64.2	64.2	64.2
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Zone 3	147.1	147.1	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Table B.5: Cumulative Weekly Withdrawal Limits for Each Regime Defined In Alt 20 On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	76.9	74.8	72.5	71.7	70.4	68.6	67.1	66.5	66.5	66.9	68.7	72.0	79.3
Zone 2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	79.8	79.8	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Zone 2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	147.1	147.1	147.1	147.1	79.8	79.8	79.8	79.8	79.8	79.8	79.8	79.8	79.8
Zone 2	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Table B.6: Cumulative Weekly Withdrawal Limits for Each Regime Defined In Alt 21 On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	54.3	52.8	51.2	50.6	49.7	48.4	47.4	46.9	47.0	47.3	48.5	50.8	56.0
Zone 2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	66.2	82.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Zone 2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	147.1	147.1	147.1	147.1	82.2	82.2	82.2	75.9	68.2	62.3	59.9	58.9	57.1
Zone 2	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Table B.7: Cumulative Weekly Withdrawal Limits for Each Regime Defined In Alt 22 On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	40.7	39.6	38.4	37.9	37.3	36.3	35.5	35.2	35.2	35.4	36.4	38.1	42.0
Zone 2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	49.7	73.9	81.0	81.0	81.0	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Zone 2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	147.1	147.1	147.1	147.1	81.0	81.0	70.4	56.9	51.1	46.7	44.9	44.2	42.8
Zone 2	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Table B.8: Cumulative Weekly Withdrawal Limits for Each Regime Defined In Option A On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6
Zone 2	54.3	52.8	51.2	50.6	49.7	48.4	47.4	46.9	47.0	47.3	48.5	50.8	56.0
Zone 3	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	40.6	40.6	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Zone 2	66.2	82.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Zone 3	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	147.1	147.1	147.1
Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 3	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	147.1	147.1	147.1	147.1	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6	40.6
Zone 2	147.1	147.1	147.1	147.1	82.2	82.2	82.2	75.9	68.2	62.3	59.9	58.9	57.1
Zone 3	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Table B.9: Cumulative Weekly Withdrawal Limits for Each Regime Defined In Option H On the Lower Athabasca River for Oil Sands Operations (in Million Barrels)

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6	45.6
Zone 3	54.3	52.8	51.2	50.6	49.7	48.4	47.4	46.9	47.0	47.3	48.5	50.8	56.0
Zone 4	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9
Zone 5	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

Week	14	15	16	17	18	19	20	21	22	23	24	25	26
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	45.6	45.6	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1
Zone 3	60.9	60.9	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1
Zone 4	66.2	82.2	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1
Zone 5	81.2	81.2	81.2	81.2	81.2	101.4	101.4	101.4	101.4	101.4	147.1	147.1	147.1

Week	27	28	29	30	31	32	33	34	35	36	37	38	39
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 3	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 4	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1
Zone 5	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1	147.1

Week	40	41	42	43	44	45	46	47	48	49	50	51	52
Zone 1	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
Zone 2	147.1	147.1	147.1	147.1	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9	60.9
Zone 3	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	79.9	78.5	76.1
Zone 4	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2
Zone 5	147.1	147.1	147.1	147.1	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2	81.2

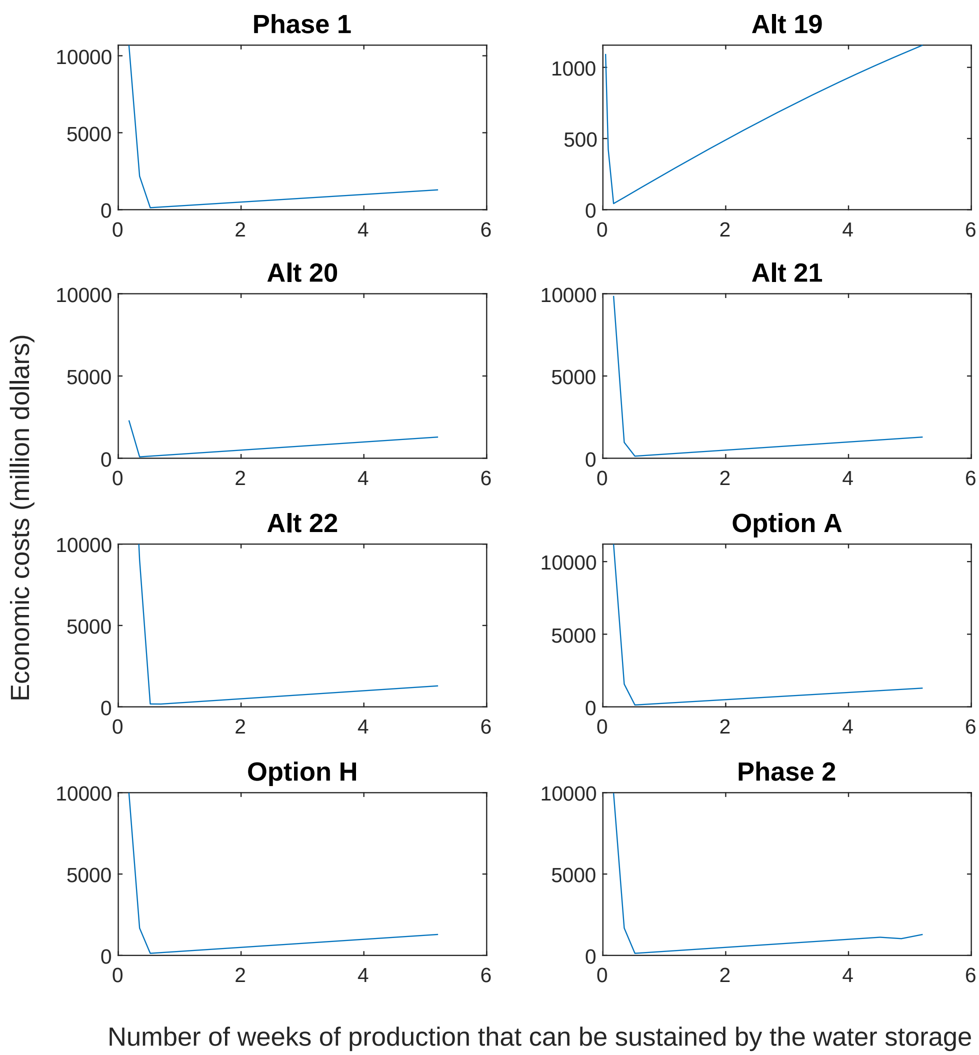

Figure B.3: The relationship between the total economic costs due to water management rule sets and the water storage capacities when the river flow condition is in scenario 2B

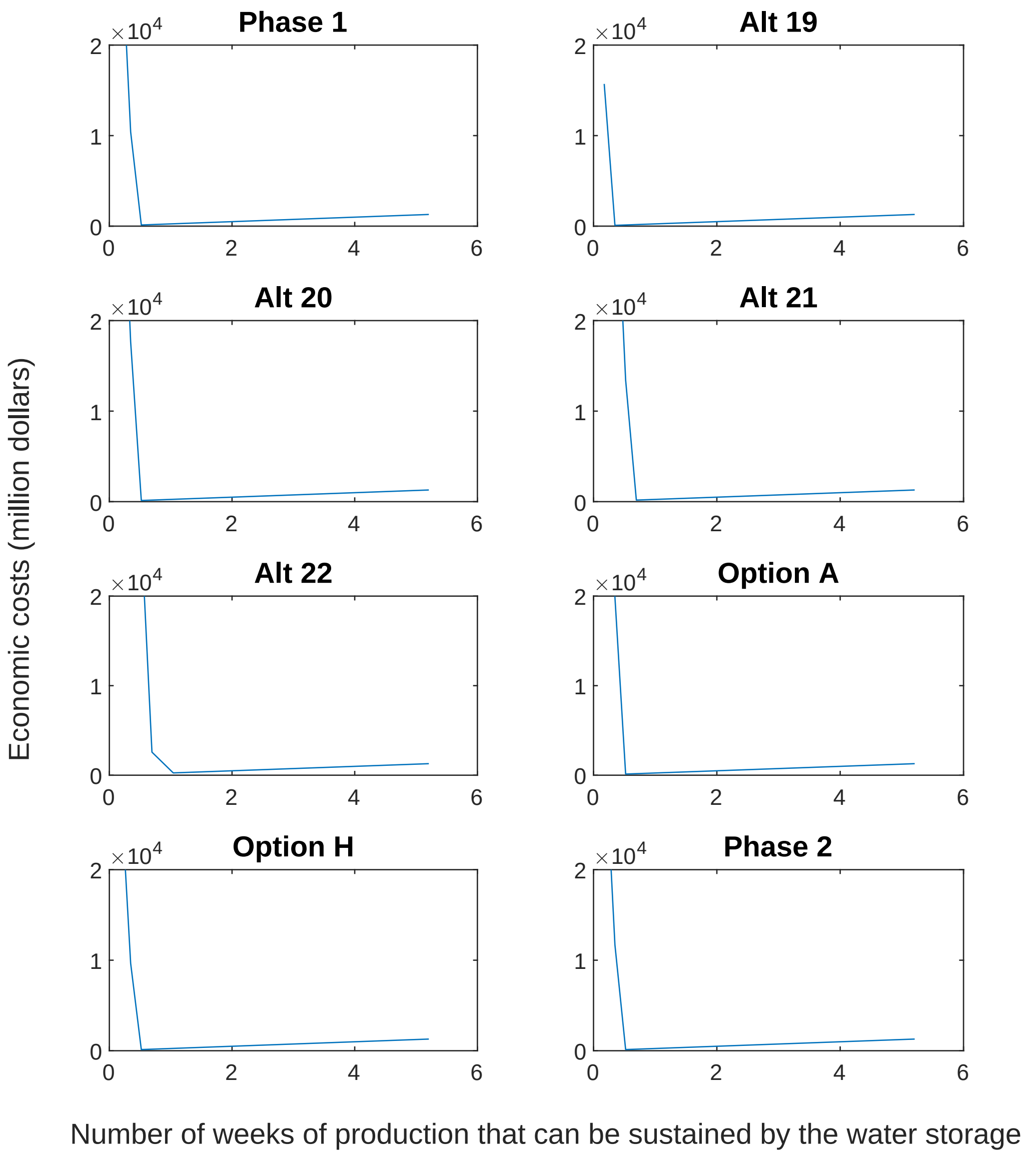

Figure B.4: The relationship between the total economic costs due to water management rule sets and the water storage capacities when the river flow condition is in scenario 2C

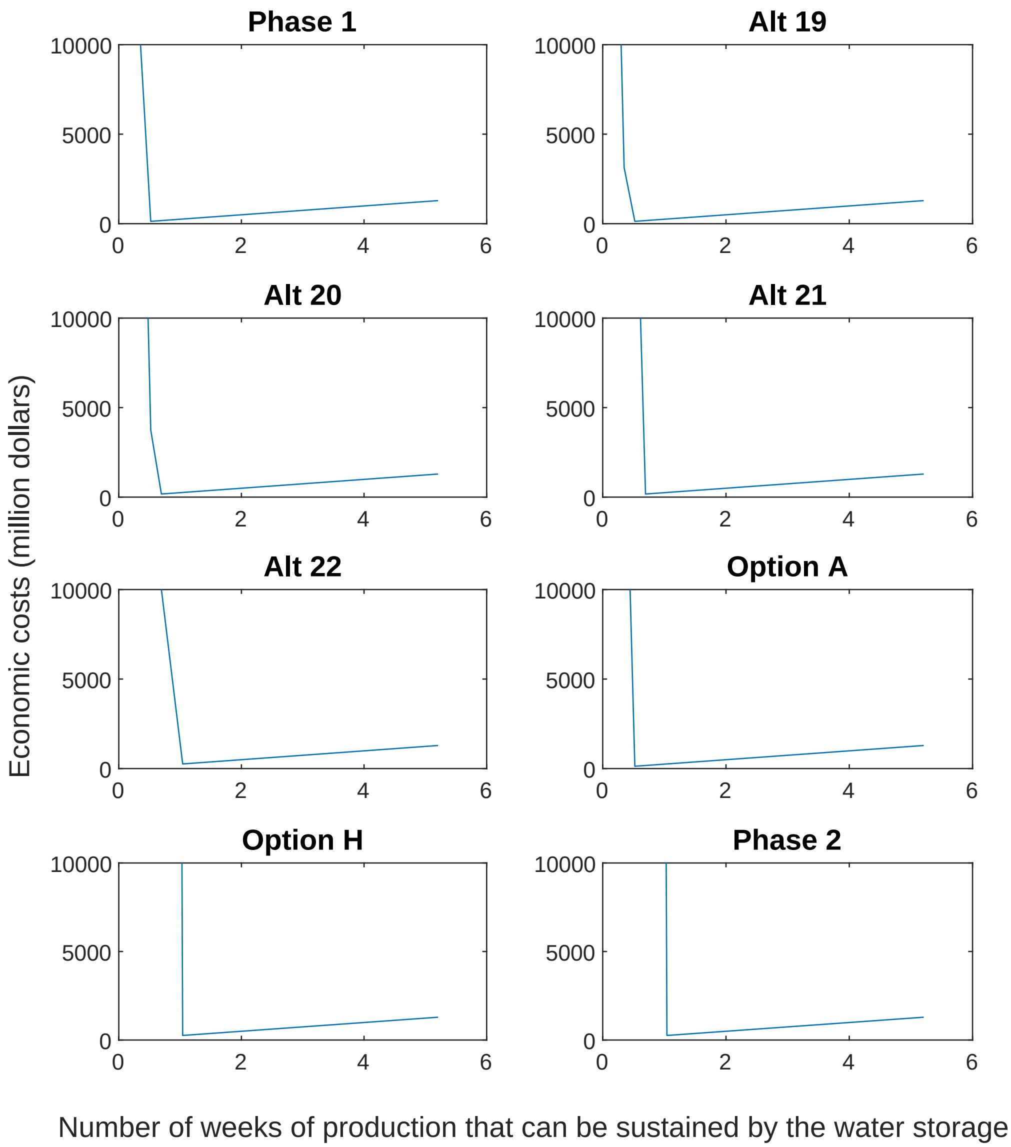

Figure B.5: The relationship between the total economic costs due to water management rule sets and the water storage capacities when the river flow condition is in scenario 2D

Table B.10: Hazard Rate Matrices In the Three River Flow Scenarios for Alternative Rule Sets

	Scenario 2B	Scenario 2C	Scenario 2D
Alt 19	$\begin{bmatrix} \text{n/a} & \text{n/a} & \text{n/a} \\ 0 & 37.14 & 14.86 \\ 0 & 3.47 & 48.53 \end{bmatrix}$	$\begin{bmatrix} 49.52 & 0 & 2.48 \\ 52 & 0 & 0 \\ 0 & 1.73 & 50.27 \end{bmatrix}$	$\begin{bmatrix} 52 & 0 & 0 \\ \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} \end{bmatrix}$
Alt 20	$\begin{bmatrix} 49.11 & 2.89 \\ 1.53 & 50.47 \end{bmatrix}$	$\begin{bmatrix} 49.83 & 2.17 \\ 1.86 & 50.14 \end{bmatrix}$	$\begin{bmatrix} 49.11 & 2.89 \\ 1.53 & 50.47 \end{bmatrix}$
Alt 21	$\begin{bmatrix} 49.52 & 2.48 \\ 1.68 & 50.32 \end{bmatrix}$	$\begin{bmatrix} 49.52 & 2.48 \\ 1.68 & 50.32 \end{bmatrix}$	$\begin{bmatrix} 52 & 0 \\ \text{n/a} & \text{n/a} \end{bmatrix}$
Alt 22	$\begin{bmatrix} 49.92 & 2.08 \\ 1.93 & 50.07 \end{bmatrix}$	$\begin{bmatrix} 50.42 & 1.58 \\ 2.74 & 49.26 \end{bmatrix}$	$\begin{bmatrix} 52 & 0 \\ \text{n/a} & \text{n/a} \end{bmatrix}$
Option A	$\begin{bmatrix} 34.67 & 17.33 & 0 \\ 5.78 & 43.33 & 2.89 \\ 0 & 1.68 & 50.32 \end{bmatrix}$	$\begin{bmatrix} 49.64 & 2.36 & 0 \\ 8.67 & 34.67 & 8.67 \\ 0 & 2.17 & 49.83 \end{bmatrix}$	$\begin{bmatrix} 52 & 0 & 0 \\ \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} \end{bmatrix}$
Option H	$\begin{bmatrix} \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ 43.33 & 0 & 4.33 & 4.33 & 0 \\ 0 & 17.33 & 34.67 & 0 & 0 \\ 0 & 0 & 17.33 & 17.33 & 17.33 \\ 0 & 0 & 0 & 1.68 & 50.32 \end{bmatrix}$	$\begin{bmatrix} \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ 0 & 49.64 & 2.36 & 0 & 0 \\ 0 & 17.33 & 17.33 & 0 & 17.33 \\ 0 & 0 & 17.33 & 34.67 & 0 \\ 0 & 0 & 0 & 2.17 & 49.83 \end{bmatrix}$	$\begin{bmatrix} 52 & 0 & 0 & 0 & 0 \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \end{bmatrix}$
Phase 2 choice	$\begin{bmatrix} \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ 0 & 0 & 0 & 0 & 0 & 0 & 43.33 & 4.33 & 4.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17.33 & 34.67 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17.33 & 17.33 & 17.33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.68 & 50.32 \end{bmatrix}$	$\begin{bmatrix} \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ 0 & 13 & 0 & 13 & 26 & 0 & 0 & 0 & 0 & 0 \\ 0 & 52 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6.5 & 6.5 & 19.5 & 6.5 & 13 & 0 & 0 & 0 & 0 \\ 0 & 10.4 & 0 & 31.2 & 0 & 10.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 17.33 & 0 & 17.33 & 0 & 17.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 52 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17.33 & 17.33 & 0 & 17.33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17.33 & 34.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.17 & 49.83 \end{bmatrix}$	$\begin{bmatrix} 52 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \\ \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} & \text{n/a} \end{bmatrix}$

Table B.11: Parameter Values for Study Cases 5 To 8

Parameter	Description	Reference	Production Level	Assigned Value						Case
				Horizon	Kearl	Muskeg	Jackpine	Base	Mildred	
	Extraction method			Surface mining						
$T - t_0$	Remaining lifespan of the project (years)	Equation (2.7)	H L	23 57	56 136	15 36	14 34	5 12	14 35	5,6,7 8
$\bar{q}$	Production capacity (million barrels/year)	Equation (2.1)	H L	135 55	98 40	138 57	89 37	446 183	362 149	5,6,7 8
s_0	Remaining established reserves (million barrels)	Equation (2.7)		3164	5447	2044	1245	2139	5158	5,6,7,8
η	Productivity of water (barrels of bitumen/barrel of water)	Equation (2.1)	H L	0.4 0.3	0.4 0.3	0.4 0.3	0.4 0.3	0.4 0.3	0.4 0.3	5,6,7 8
$\bar{W}$	Water withdrawal constraints (million barrels/week)	Equation (2.4)		$11\%\bar{W}_c$	$8\%\bar{W}_c$	$11\%\bar{W}_c$	$7\%\bar{W}_c$	$35\%\bar{W}_c$	$29\%\bar{W}_c$	5,6,7,8
ρ	Discount of bitumen prices against WTI prices	Equation (2.9)		83%	83%	83%	83%	83%	83%	
C	The construction cost of the water storage(million dollar)	Table 2.1		Specified in Table B.12						5,6,7,8
$\bar{I}$	Water storage capacity (million barrels)	Equation (2.3)		Specified in Table B.12						5,6,7,8
c_f^s	The fixed cost of water storage (million $/year)	Equation (2.9)		Specified in Table B.12						5,6,7,8
c_v^s	The variable cost of water storage ($/barrel)	Equation (2.9)		0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	
	Carbon emissions (tonnes/barrel)	Equation (2.9)		0.091	0.091	0.091	0.091	0.091	0.091	
$c_{v_e}^o$	Energy variable operating cost (% of the WTI price)	Equation (2.9)		1.62	1.62	1.62	1.62	1.62	1.62	
$c_{v_{ne}}^o$	Non-energy variable operating lost ($/barrel)	Equation (2.9)		7.98	7.98	7.98	7.98	7.98	7.98	
c_f^o	Fixed operating cost (million $/year)	Equation (2.9)	H L	311 127	226 92	318 131	205 85	1028 822	835 344	5,6,7 8
c_s	Sustaining capital cost (million $/year)	Equation (2.9)	H L	177.2 72.2	128.6 52.5	181.1 74.8	116.8 48.6	585.4 240.2	475.1 195.6	5,6,7 8
	Income tax rate (%)	Equation (2.9)		25	25	25	25	25	25	
	Carbon tax ($/tonne)	Equation (2.9)		40	40	40	40	40	40	
C_m	Mothball cost (million $)	Table 2.1		0	0	0	0	0	0	
C_{re}	Reactivating cost (million $)	Table 2.1		0	0	0	0	0	0	
C_{large}	A large number to prevent stage switching (million $)	Page 24		10^9	10^9	10^9	10^9	10^9	10^9	
C_r	Abandonment cost (million $)	Table 2.1	H L	38 15	27 11	38 16	25 10	124 51	101 41	5,6,7 8
ϵ	Speed of reverting to the mean log oil price	Equation (2.8)		0.14	0.14	0.14	0.14	0.14	0.14	
μ	Long run mean log oil price	Equation (2.8)		4.59	4.59	4.59	4.59	4.59	4.59	
σ	Volatility of oil prices	Equation (2.8)		0.31	0.31	0.31	0.31	0.31	0.31	
	River flows			Refer to the hazard matrices given in Table B.10						5,6,7,8
r	Risk free interest rate	Equation (2.16)		0.02	0.02	0.02	0.02	0.02	0.02	

$\bar{W}_c$ denotes the weekly cumulative water withdrawal limit, i.e. the total amount of water that is allowed to be withdrawn by all oil sands projects each week. It is specified in Table 3.8.

The column of "Production Level" shows the categarization into "H" or "L" of the production capacity. "H" means that the production capcity is at the high level assumed by the P2FC. "L" represents the current relative low production level.

Table B.12: The Volume of the Water Storage Facilities Built by Oil Sands Projects In Alternative Rule Sets In Different Cases and Different River Flow Scenarios

			Phase 1			Alt 19			Alt 20			Alt 21			Alt 22			Option A			Option H			Phase 2		
			C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s	C	$\bar{I}$	C_f^s
Case 5	Scenario 2B	CNRL Horizon	126	64	6.8	70	36	3.8	148	76	8.0	215	110	11.6	283	145	15.2	189	97	10.2	140	72	7.5	n/a	n/a	n/a
		Imperial Kearl	91	47	4.9	51	26	2.7	107	55	5.8	155	80	8.4	205	105	11.0	136	70	7.3	101	52	5.4	n/a	n/a	n/a
		Shell Muskeg River	128	66	6.9	71	37	3.8	151	78	8.1	219	112	11.8	288	148	15.5	192	99	10.4	142	73	7.7	n/a	n/a	n/a
		Shell Jackpine	83	42	4.5	46	24	2.5	98	50	5.3	141	72	7.6	186	95	10.0	124	64	6.7	92	47	4.9	n/a	n/a	n/a
		Suncor	414	212	22.3	230	118	12.4	489	251	26.3	708	363	38.1	932	478	50.2	622	319	33.5	460	236	24.8	n/a	n/a	n/a
		Syncrude	337	173	18.1	187	96	10.1	397	204	21.4	575	295	31.0	757	388	40.8	505	259	27.2	374	192	20.1	n/a	n/a	n/a
	Scenario 2C	CNRL Horizon	147	75	7.9	91	47	4.9	168	86	9.0	227	116	12.2	288	148	15.5	189	97	10.2	159	81	8.6	n/a	n/a	n/a
		Imperial Kearl	106	54	5.7	66	34	3.5	121	62	6.5	164	84	8.8	208	107	11.2	136	70	7.3	115	59	6.2	n/a	n/a	n/a
		Shell Muskeg River	150	77	8.1	93	47	5.0	171	88	9.2	231	119	12.5	294	151	15.8	192	99	10.4	162	83	8.7	n/a	n/a	n/a
		Shell Jackpine	96	49	5.2	60	31	3.2	110	57	5.9	149	77	8.0	190	97	10.2	124	64	6.7	105	54	5.6	n/a	n/a	n/a
		Suncor	483	248	26.0	299	153	16.1	552	283	29.7	748	384	40.3	950	487	51.1	622	319	33.5	524	269	28.2	n/a	n/a	n/a
		Syncrude	393	201	21.1	243	125	13.1	449	230	24.2	608	312	32.7	771	396	41.5	505	259	27.2	425	218	22.9	n/a	n/a	n/a
	Scenario 2D	CNRL Horizon	147	75	7.9	105	54	5.6	180	92	9.7	237	122	12.8	295	151	15.9	189	97	10.2	182	93	9.8	n/a	n/a	n/a
		Imperial Kearl	106	54	5.7	76	39	4.1	130	67	7.0	172	88	9.3	214	110	11.5	136	70	7.3	131	67	7.1	n/a	n/a	n/a
		Shell Muskeg River	150	77	8.1	107	55	5.8	183	94	9.9	242	124	13.0	301	154	16.2	192	99	10.4	185	95	10.0	n/a	n/a	n/a
		Shell Jackpine	96	49	5.2	69	35	3.7	118	61	6.4	156	80	8.4	194	100	10.5	124	64	6.7	119	61	6.4	n/a	n/a	n/a
		Suncor	483	248	26.0	345	177	18.6	593	304	31.9	783	401	42.1	973	499	52.4	622	319	33.5	599	307	32.2	n/a	n/a	n/a
		Syncrude	393	201	21.1	281	144	15.1	482	247	25.9	636	326	34.2	790	405	42.5	505	259	27.2	486	249	26.2	n/a	n/a	n/a
Case 6	Scenarios 2B, 2C, 2D	CNRL Horizon	68	35	3.6	68	35	3.6	68	35	3.6	68	35	3.6	68	35	3.6	68	35	3.6	68	35	3.6	68	35	3.6
		Imperial Kearl	49	25	2.6	49	25	2.6	49	25	2.6	49	25	2.6	49	25	2.6	49	25	2.6	49	25	2.6	49	25	2.6
		Shell Muskeg River	69	35	3.7	69	35	3.7	69	35	3.7	69	35	3.7	69	35	3.7	69	35	3.7	69	35	3.7	69	35	3.7
		Shell Jackpine	45	23	2.4	45	23	2.4	45	23	2.4	45	23	2.4	45	23	2.4	45	23	2.4	45	23	2.4	45	23	2.4
		Suncor	223	114	12.0	223	114	12.0	223	114	12.0	223	114	12.0	223	114	12.0	223	114	12.0	223	114	12.0	223	114	12.0
		Syncrude	181	93	9.8	181	93	9.8	181	93	9.8	181	93	9.8	181	93	9.8	181	93	9.8	181	93	9.8	181	93	9.8
Case 7	Scenario 2B	CNRL Horizon	8	4	0.5	3	2	0.2	7	3	0.4	8	4	0.5	12	6	0.6	8	4	0.5	8	4	0.5	8	4	0.5
		Imperial Kearl	6	3	0.3	2	1	0.1	5	3	0.3	6	3	0.3	9	4	0.5	6	3	0.3	6	3	0.3	6	3	0.3
		Shell Muskeg River	9	4	0.5	3	2	0.2	7	4	0.4	9	4	0.5	12	6	0.7	9	4	0.5	9	4	0.5	9	4	0.5
		Shell Jackpine	6	3	0.3	2	1	0.1	4	2	0.2	6	3	0.3	8	4	0.4	6	3	0.3	6	3	0.3	6	3	0.3
		Suncor	28	14	1.5	11	6	0.6	22	11	1.2	28	14	1.5	39	20	2.1	28	14	1.5	28	14	1.5	28	14	1.5
		Syncrude	23	12	1.2	9	5	0.5	18	9	1.0	23	12	1.2	32	16	1.7	23	12	1.2	23	12	1.2	23	12	1.2
	Scenario 2C	CNRL Horizon	8	4	0.5	7	3	0.4	8	4	0.5	12	6	0.6	17	9	0.9	8	4	0.5	8	4	0.5	8	4	0.5
		Imperial Kearl	6	3	0.3	5	3	0.3	6	3	0.3	9	4	0.5	12	6	0.7	6	3	0.3	6	3	0.3	6	3	0.3
		Shell Muskeg River	9	4	0.5	7	4	0.4	9	4	0.5	12	6	0.7	17	9	0.9	9	4	0.5	9	4	0.5	9	4	0.5
		Shell Jackpine	6	3	0.3	4	2	0.2	6	3	0.3	8	4	0.4	11	6	0.6	6	3	0.3	6	3	0.3	6	3	0.3
		Suncor	28	14	1.5	22	11	1.2	28	14	1.5	39	20	2.1	56	29	3.0	28	14	1.5	28	14	1.5	28	14	1.5
		Syncrude	23	12	1.2	18	9	1.0	23	12	1.2	32	16	1.7	45	23	2.4	23	12	1.2	23	12	1.2	23	12	1.2
	Scenario 2D	CNRL Horizon	8	4	0.5	8	4	0.5	12	6	0.6	12	6	0.6	17	9	0.9	8	4	0.5	17	9	0.9	17	9	0.9
		Imperial Kearl	6	3	0.3	6	3	0.3	9	4	0.5	9	4	0.5	12	6	0.7	6	3	0.3	12	6	0.7	12	6	0.7
		Shell Muskeg River	9	4	0.5	9	4	0.5	12	6	0.7	12	6	0.7	17	9	0.9	9	4	0.5	17	9	0.9	17	9	0.9
		Shell Jackpine	6	3	0.3	6	3	0.3	8	4	0.4	8	4	0.4	11	6	0.6	6	3	0.3	11	6	0.6	11	6	0.6
		Suncor	28	14	1.5	28	14	1.5	39	20	2.1	39	20	2.1	56	29	3.0	28	14	1.5	56	29	3.0	56	29	3.0
		Syncrude	23	12	1.2	23	12	1.2	32	16	1.7	32	16	1.7	45	23	2.4	23	12	1.2	45	23	2.4	45	23	2.4
Case 8	Scenarios 2B, 2C, 2D	CNRL Horizon	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1
		Imperial Kearl	14	7	0.8	14	7	0.8	14	7	0.8	14	7	0.8	14	7	0.8	14	7	0.8	14	7	0.8	14	7	0.8
		Shell Muskeg River	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1	20	10	1.1
		Shell Jackpine	13	7	1.1	13	7	1.1	13	7	1.1	13	7	1.1	13	7	1.1	13	7	1.1	13	7	1.1	13	7	1.1
		Suncor	66	34	3.5	66	34	3.5	66	34	3.5	66	34	3.5	66	34	3.5	66	34	3.5	66	34	3.5	66	34	3.5
		Syncrude	53	27	2.9	53	27	2.9	53	27	2.9	53	27	2.9	53	27	2.9	53	27	2.9	53	27	2.9	53	27	2.9

Appendix C

The Derivation of the Expectation of Oil Price for Model 1 In Chapter 4

In this appendix we show the derivation of the expected value of price, P, when P is described by the following stochastic differential equation.

$$\mathrm{d}P = \epsilon\left(\bar{P} - \lambda - P\right)\mathrm{d}t + \sigma P \mathrm{d}Z^*$$

$$\begin{aligned}
\mathbb{E}_t\left[P_{t+T}\right] &= \mathbb{E}_t\left[P_t + \int_t^{t+T}\mathrm{d}P\right] \\
&= \mathbb{E}_t\left[P_t\right] + \mathbb{E}_t\left[\int_t^{t+T}\epsilon(\bar{P} - \lambda - P)\mathrm{d}\zeta + \int_t^{t+T}\sigma P\mathrm{d}Z^*\right] \\
&= P_t + \mathbb{E}_t\left[\int_t^{t+T}\epsilon(\bar{P} - \lambda)\mathrm{d}\zeta - \int_t^{t+T}\epsilon P\mathrm{d}\zeta + \int_t^{t+T}\sigma P\mathrm{d}Z^*\right] \\
&= P_t + \epsilon(\bar{P} - \lambda)\cdot T - \epsilon\cdot\int_t^{t+T}\mathbb{E}_t\left[P\right]\mathrm{d}\zeta + \mathbb{E}_t\left[\int_t^{t+T}\sigma P\mathrm{d}Z^*\right] \\
&= P_t + \epsilon(\bar{P} - \lambda)\cdot T - \epsilon\cdot\int_t^{t+T}\mathbb{E}_t\left[P\right]\mathrm{d}\zeta
\end{aligned}$$

Taking derivative with regard to variable T on both sides, we obtain

$$\frac{\mathrm{d}\left(\mathbb{E}_t\left[P_{t+T}\right]\right)}{\mathrm{d}T} = \epsilon\left(\bar{P} - \lambda - \mathbb{E}_t\left[P_{t+T}\right]\right)$$

Arranging the above equation, we obtain the following linear differential equation of first order:

$$\frac{\mathrm{d}\left(\mathbb{E}_t\left[P_{t+T}\right]\right)}{\mathrm{d}T} + \epsilon \cdot \mathbb{E}_t\left[P_{t+T}\right] = \epsilon\left(\bar{P} - \lambda\right) \tag{C.1}$$

We apply the method of variation of a constant to solve this linear differential equation of first order.

First we consider the homogeneous equation:

$$\frac{\mathrm{d}\left(\mathbb{E}_t\left[P_{t+T}\right]\right)}{\mathrm{d}T} + \epsilon \cdot \mathbb{E}_t\left[P_{t+T}\right] = 0 \tag{C.2}$$

We transform the above equation to:

$$\frac{\mathrm{d}\left(\mathbb{E}_t\left[P_{t+T}\right]\right)}{\mathbb{E}_t\left[P_{t+T}\right]} + \epsilon \cdot \mathrm{d}T = 0$$

$$\mathrm{d}\left(\ln \mathbb{E}_t\left[P_{t+T}\right]\right) = -\epsilon \cdot \mathrm{d}T$$

$$\int \mathrm{d}\left(\ln \mathbb{E}_t\left[P_{t+T}\right]\right) = -\int \epsilon \,\mathrm{d}T$$

$$\ln \mathbb{E}_t\left[P_{t+T}\right] = -\epsilon \cdot T + C$$

where C is an arbitrary constant (the constant of integration).

Then we get the general solution of the Equation (C.2):

$$\mathbb{E}_t\left[P_{t+T}\right] = C' \cdot e^{-\epsilon \cdot T}$$

where $C' = e^C$

Replacing constant C' by $v(T)$, which is an unknown function of T, we obtain:

$$\mathbb{E}_t\left[P_{t+T}\right] = v(T) \cdot e^{-\epsilon \cdot T} \tag{C.3}$$

Substituting Equation (C.3) into the non-homogeneous differential equation (C.1), we obtain:

$$\frac{\mathrm{d}\left(v(T) \cdot e^{-\epsilon \cdot T}\right)}{\mathrm{d}T} + \epsilon \cdot v(T) \cdot e^{-\epsilon \cdot T} = \epsilon \left(\bar{P} - \lambda\right)$$

Solving for $v(T)$, we obtain:

$$v(T) = \left(\bar{P} - \lambda\right) \cdot e^{\epsilon \cdot T} + C''$$

where C'' is the constant of integration.

Substituting the expression of $v(T)$ into Equation (C.3), we obtain the general solution of Equation (C.1):

$$\mathbb{E}_t\left[P_{t+T}\right] = \left(\bar{P} - \lambda\right) + C'' \cdot e^{-\epsilon \cdot T}$$

Since $\mathbb{E}_t\left[P_t\right] = P_t$, substitute T in the above equation with 0, we obtain:

$$\mathbb{E}_t\left[P_t\right] = \left(\bar{P} - \lambda\right) + C'' = P_t$$

Therefore,

$$C'' = P_t - (\bar{P} - \lambda)$$

Then the solution of Equation (C.1) is

$$\mathbb{E}_t\left[P_{t+T}\right] = \left(\bar{P} - \lambda\right) + \left(P_t - (\bar{P} - \lambda)\right) \cdot e^{-\epsilon \cdot T}$$

Re-arranging the above equation, we get the following equation:

$$\mathbb{E}_t\left[P_{t+T}\right] = \left(\bar{P} - \lambda\right)\left(1 - e^{-\epsilon \cdot T}\right) + P_t \cdot e^{-\epsilon \cdot T}$$