Economic Analysis of Hydrocarbon Exploration by Simulation with Geological Uncertainties

by

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ABSTRACT

This research continues the work of Chungcharoen (1994) to extend the benefit of Manly's Approximation Method in the area of hydrocarbon discovery process modeling, to incorporate the uncertainties in geologic parameters in order to provide an assessment of the distributions of total hydrocarbon discoveries that are expected to be recovered as a result of exploration activity, and to combine economic parameters into the evaluation of the economic worth of the results of multiple-wells exploration activity. This research is separated into two parts. In the first part, the uncertainties involved in the geological parameters are included in the initial field size distribution and the number of fields distribution. The Monte Carlo approach is used to sample data from the field size distribution with each number of fields selected from the number of fields distribution. A frequency-size distribution is constructed based on sampled data. Dry hole data are also added into the initial frequency-size distribution in order to reflect the exploration risk. After obtaining frequency-size distributions that define the uncertainties in geological parameters, the distributions of total hydrocarbon discoveries for a selected number of exploratory wells are constructed. The second part involves incorporating economic parameters, such as the price of oil/gas, and the costs of exploration, development, and production, into the distribution of the number of discoveries and the distribution of total hydrocarbon discoveries in order to produce a probability distribution of the net present value (NPV) of a proposed exploration program. The distributions of NPV are used as input to the expected utility analyses for determining multiple-wells exploration strategies.

The offshore Nova Scotia Shelf basin is selected for implementing the methodology. Several scenarios regarding changes in economic parameters are illustrated. The effect of increasing variability in field size in a basin on the forecast is also discussed.

This methodology could be used by a company as a part of a planning system for projecting exploration programs. It would provide insight into how a company makes a forecast of future discovery volumes that includes uncertainties in geological parameters and how the results are used in long-term planning to determine future development programs for these hydrocarbon reserves. In addition, results from this methodology could assist government departments by supporting their efforts to establish the potential of hydrocarbons discoveries and to aid in their analyses of policies concerning exploration programs regarding taxes and royalty regimes in any basin with various stages of exploration activity.

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DEDICATION

To my beloved mother, Mrs. Chavewon Chungcharoen. She is the great mother and teacher in my life. Her love, strength, constant warm support, and encouragement have inspired me to accomplish this goal. To my beloved father, Mr. Chitdee Chungcharoen. He is the great father and engineer in my life. His love and always warm support have propelled me to follow in his footsteps. To my beloved aunt, Mrs. Sawet Chungcharoen, and to my beloved sisters, Chulalak, Karmonwan, Piyachidd, and Sopita, for their love and inspiration throughout my studies and always.

"Forget about the days when it's been cloudy,

but don't forget your hours in the sun...

Forget about the times you've been defeated,

but don't forget the victories you've won...

Forget about mistakes that you can't change now,

but don't forget the lessons that you've learned...

Forget about misfortunes you've encountered,

but don't forget the times your luck has turned...

Forget about the days when you've been lonely,

but don't forget the friendly smiles you've seen...

Forget about the plans that didn't seem to work out right,

but don't forget to always have a dream."

- Amanda Bradley -

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CHAPTER 1

INTRODUCTION

1.1 Research Motivation

The motivation for this research originated from two principal interests. First, as we all know, hydrocarbons are the most important source of energy for industrialized development in our present day societies, and they will remain dominant in the framework of modern civilization for some time in the future. During the past several years the global search for hydrocarbon accumulations has been advanced and widened, especially in frontier regions, due to the major concern over the depleting inventory of hydrocarbons and the sharp decline in discovery rates in several well-established hydrocarbon basins around the world. Subsequently, several governments have made policy announcements encouraging oil and gas companies to search for and develop new hydrocarbon discoveries.

Second, there have always been tremendous financial risks attached to the exploration for hydrocarbons. These risks have, in turn, led to the remarkable rise, or occasional fall, of petroleum companies and, consequently, have led to economic impacts on nations. Because of the high degree of risk in the hydrocarbon exploration process and uncertainty in economic factors, it is essential for a company to try to appraise the potential results of future exploration and to develop an optimized strategy before making a decision to explore any region.

This research proposes a methodology which incorporates the uncertainties in geologic information into calculations to provide estimates of the distributions of total hydrocarbon discoveries that may be recovered as the exploration progresses, and combines these estimates with economic information to provide a probabilistic evaluation of the economic worth of exploration activities. This methodology would give a decision maker a better picture of the range of both reserve additions and economic potential of exploration efforts in the underlying region. Thus, it provides a useful exploration and policy-planning tool for used by government agencies as well as to oil and gas companies.

1.2 Background of Research

The decision on whether or not to explore a region is made under circumstances of great uncertainty in both geological and economic parameters. Today, even the most sophisticated estimation techniques used to assist decision makers in evaluating hydrocarbon reserves are limited by the accuracy of these parameters. Uncertainty in geological parameters involves the interpretation of geophysical, geological, and petrophysical data used in estimating hydrocarbon volume in potential or existing reservoirs (Robinson, 1990). Examples of these geological parameters with a high degree of uncertainty are field area, net pay thickness, discovery factor, and water saturation. Uncertainty in economic parameters encompasses, for example, hydrocarbon prices, costs, government royalties, and taxes. These uncertainties have added significant risks in evaluating hydrocarbon reserves and so can be of critical importance to investment and planning decisions for both government and companies that are involved in hydrocarbon exploration. An inadequate appreciation of the uncertainty surrounded in both geological and economic parameters can lead to misjudged policies, costly exploration failures, and other severe consequences for related organizations.

In order to cope with these uncertainties, various techniques based on several approaches have been developed and are being used to appraise the discovery rate, and to forecast future hydrocarbon discoveries, in an effort to reduce the risk factors. "Their approaches vary from the basin and play level of analysis to continental aggregations and from detailed structural and process models to simple extrapolations and curve-fitting. They also range from geologic-based attempts to estimate the in-situ resource base to the economic based estimates of supply" (Power and Fuller, 1992). Some of the common techniques are the judgmental prediction technique, the extrapolative method, discovery process models, econometric models, and probabilistic models. These techniques provide systems for evaluating the future discoveries of hydrocarbons and economic outcomes. However, some of these techniques do not yield objective quantitative appraisals because they involve an intuitive blending of geological qualities and subjective weighting qualities. Some of these techniques consider only "best estimate" or "most likely" values that give acceptable results in the cases where uncertainty is not large and the profit margin is not critical. Some of these techniques do not allow for quantitative assessment of risk and uncertainty in hydrocarbon exploration and they cannot account for other possible reserve levels and economic uncertainties.

Since the 1970's, a probabilistic model of the hydrocarbon discovery process has been widely accepted for evaluating the future discoveries of hydrocarbons due to its richness in using specific geological, technological, and economic attributes of the process of exploration. This probabilistic model benefited from the work of Arps and Roberts (1958), who first introduced the notions that the probability of discovering a field of a given size is proportional to the number of undiscovered fields of that size and to the areal extent of each field. Kaufman et al. (1975) and Barouch and Kaufman (1976) refined and extended the ideas of Arps and Roberts to use maximum likelihood techniques for the estimation of the field size distribution from the information in the historical discovery record. Other researchers have demonstrated that probabilistic formulations of the hydrocarbon exploration and discovery process are very useful in estimating the hydrocarbon reserves and in exploration policy analysis (Smith, 1980, O'Carroll and Smith, 1980, Lee and Wang, 1983a, 1983b, 1985, Schuenemeyer and Drew, 1983, Drew et al., 1988, Rabinowitz, 1991, Power and Fuller, 1991, Power and Jewkes, 1991, and Bickel et al.,1992). However, prolonged simulations have been required when using the probabilistic model to forecast the expected value and standard deviation of the volume of future discoveries due to a given exploratory effort.

In 1991 a new approach in discovery process modeling for estimating the future hydrocarbon discoveries was introduced. Fuller (1991) applied the Manly's Approximation Method, which was initially suggested by Manly (1974) in the context of biometrics experiments, to forecast the means and standard deviations of future discovery volumes as functions of the number of discoveries, or in some cases, of the number of wells drilled (including dry holes). Manly's Approximation Method assumes the same postulates as the probabilistic model of hydrocarbon discovery process. By using this method, running the model required only a few seconds on a computer, in comparison with several hours for traditional extensive simulation. On specific data sets, Fuller (1991) found that Manly's Approximation Method is 600 times faster than a simulation. The approximation method also gave satisfactorily accurate results for specific data with the parent population of fields having a lognormal distribution, exponential distribution, Weibull distribution, and gamma distribution (Ninpong et al., 1992). Later, the approximation method was further simplified by Fuller and Wang (1991) and it was used by Macdonald et al. (1994) in a regression approach to forecasting future discoveries.

For some purposes, the approximation of only the first two moments of the distribution of total discovery volume is insufficient. For example, Fuller (1991) showed that the construction of a 95% confidence interval as the approximate mean plus or minus two standard deviations produces the absurd result of negative total discovery volume within part of the confidence interval for small well numbers. The problem is that the probability density function is highly asymmetric for small well numbers (and also for large well numbers, as we discuss later), but the first two moments of a distribution say nothing about skewness. This lack of skewness information could also cause trouble in attempts to use an expected utility approach to decisions about proposed exploration programs.

In order to solve this problem, Chungcharoen (1994) and Chungcharoen and Fuller (1996) used the Manly approximations of mean and standard deviation, together with the calculated smallest and largest possible total discovery volumes at any well number, to set the parameters of a Beta distribution that then approximates the true distribution of total discovery volume for a given well number. The confidence intervals of the forecast, based on the Beta distributions, were constructed and compared to the confidence intervals of the forecast from the simulation distributions in order to support the investigation. Three real data sets; the Nova Scotia Shelf, the Bistcho Play, and the Zama Play, were chosen to verify the methodology developed. The sensitivity analyses were performed to show that the idea of using a family of Beta distributions is a robust approximation.

1.3 Research Objectives

The main objectives of this research are to continue the work of Chungcharoen (1994) to extend the benefit of Manly's Approximation Method in the area of hydrocarbon discovery process modeling, to incorporate the uncertainties in geologic parameters in order to provide an assessment of the distributions of total hydrocarbon discoveries that are expected to be recovered as a result of exploration activity, and to combine economic parameters into the evaluation of the economic worth of the results of multiple-wells exploration activity. This research can be separated into two parts.

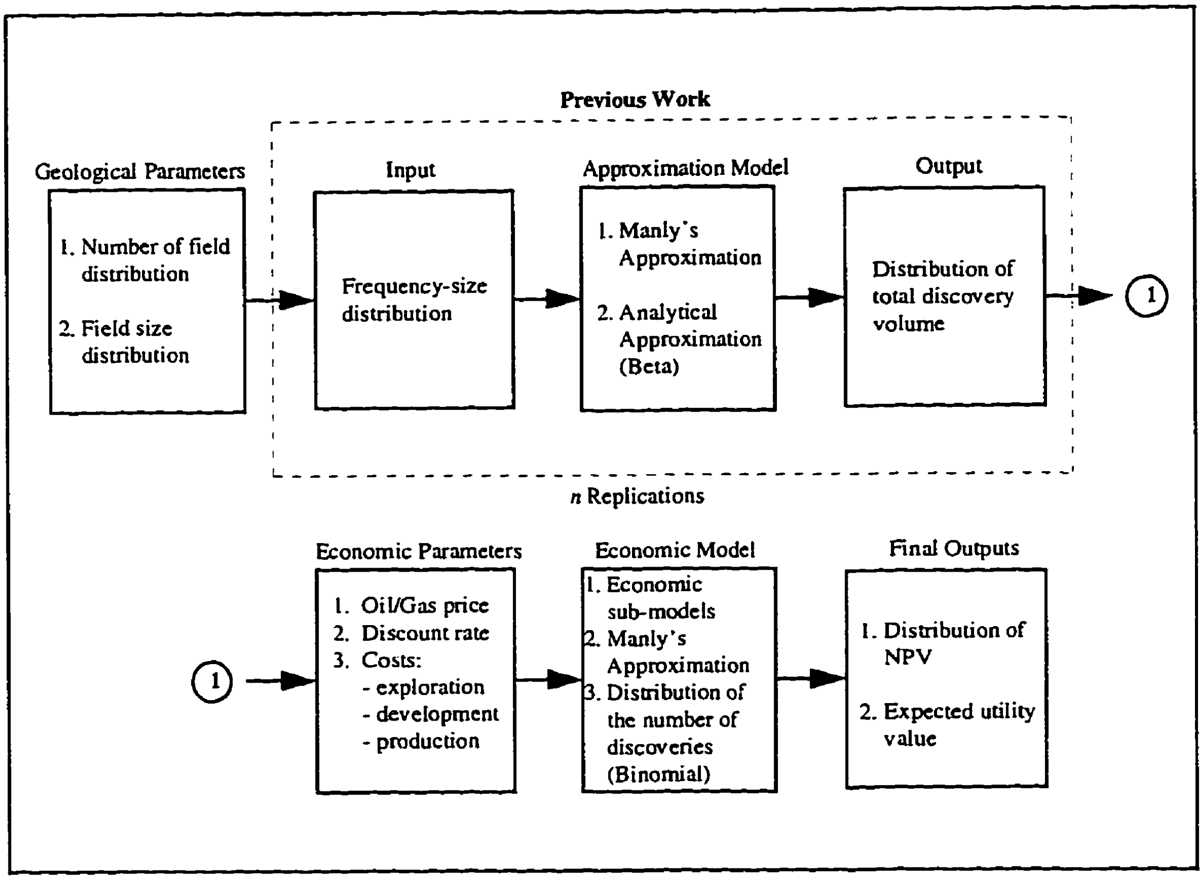
In the first part, the uncertainties involved in the geological parameters are included in the initial field size distribution. In addition to the uncertainty in geological parameters, there is also uncertainty in the number of fields. Therefore, the number of possible fields is also represented by a probabilistic distribution. After determining the field size distribution and estimating the number of fields distribution, the Monte Carlo approach will be used to sample data from the field size distribution with each number of fields selected from the number of fields distribution. Each frequency-size distribution is constructed based on each sampled data set and then by categorizing the sampled field data into size classes. Consequently, the average volume measured and the average areal extent for each size class are calculated. Finally, dry hole data will also be added into the initial frequency-size distribution in order to reflect the exploration risk. After obtaining n replications of the frequency-size distribution that define the uncertainties in geological parameters, the distributions of total hydrocarbon discoveries for selected number of exploratory wells will be constructed using the approximation approach as described in Chungcharoen (1994).

The second part of the research extends the benefits of using Manly's Approximation Method to approximate the distribution of number of discoveries. Then, the distribution of number of discoveries and the distribution of total discoveries from the first part together with economic parameters such as the price of oil/gas, the costs of exploration drilling, development, and production are incorporated into the economic model in order to produce a probability distribution of the net present value (NPV) of a proposed exploration program. The distribution on NPV can be used to produce confidence intervals, return measures, and risk measures, or, as input to expected utility analyses for determining multiple-wells exploration strategies.

Note that the methodology proposed in this research can be linked with the methods adopted by Geological Survey of Canada to estimate hydrocarbon resources. For a partly explored basin where discovery data are recorded, the frequency-size distribution used as an input in this methodology can be derived by the superpopulation concept and the pool-size-by-rank method (Lee and Wang, 1985, 1990). For a frontier basin, field size distribution as mentioned in part one of this research can be derived by field size equations and Monte Carlo approach (Roy, 1975, and Proctor and Taylor, 1984). This field size distribution together with the number of field distribution is used to obtain the frequency-size distributions in the model.

To represent the methodology developed, the Offshore Nova Scotia Shelf basin is selected for the study. This basin is suitable to the study because it is a partly explored, frontier basin lying off Canada's eastern seaboard which is an attractive area for exploration and development activities. According to the Nova Scotia Department of Natural Resources (1993), this basin contains some of the best frontier oil and natural gas plays yet to be explored in North America. A diagram of the research studies is shown in Figure 1.1.

Figure 1.1 Diagram of the research.



1.4 Contributions

The major contributions of this research are the extension of the benefits of Manly's Approximation Method to hydrocarbon discovery process modeling and the development of a complete package that goes from a frequency-size distribution through economic analysis. This work is especially valuable because of the paucity of work related to frontier exploration.

The methodology developed in this research uses the main benefit of Manly's Approximation Method which gives accurate results at a much faster speed than using the simulation approach. Hence, it could be possible to include uncertainties involved in the geological parameters that define the initial frequency-size distribution, yet complete the calculations in a reasonable amount of time. Similarly, economic parameters could be included in the model, to produce a distribution of the net present value of a proposed exploration program, within a short period of time. This allows a company or a government to thoroughly investigate the sensitivity of the results by varying economic parameters according to changes in economic conditions.

This methodology could be used by a company as a part of a planning system for projecting exploration programs. It would provide insight into how a company makes a forecast of future discovery volumes that includes uncertainties in geological parameters and how the results are used in long-term planning to determine future development programs for these hydrocarbon reserves. The resulting expenditures and forecast revenues from the model could be used to predict future income, cash flows, and profitability. On the basis of these results, the companies could set priorities and plan the allocation of exploration resources and manpower. The companies could plan the research needed to solve the technical problems associated with these basins. In addition, results from this methodology could assist government departments by supporting their efforts to establish the potential of hydrocarbons discoveries, to predict future exploration activity, and to aid in their analyses of policies concerning exploration programs regarding taxes and royalty regimes in any basin with various stages of exploration activity. Lastly, this model is open-ended for use in basins with sufficient historical drilling data as well as frontier basins.

1.5 Overview

Chapter 2 presents a review of the literature relevant to the issues in hydrocarbon exploration including field size distribution, probabilistic models of the hydrocarbon discovery process, and Manly's Approximation Method. Consequently, a review of the literature which investigates the methods of quantifying the probabilistic estimates of geological and/or economic parameters and the methods in economic analysis in hydrocarbon exploration is provided. Chapter 3 presents the mathematical descriptions of the probabilistic model of hydrocarbon discovery process, Manly's Approximation Method, and the approximation of the total hydrocarbon discoveries used in previous work. Chapter 4 is separated into two parts. In Section 4.2, a development of the methodology of the approximation of the total hydrocarbon discoveries that incorporates uncertainty in geological information is explained. Section 4.3 describes the methodology for incorporating economic parameters into the distributions of total hydrocarbon discoveries to obtain the distributions of net present value and the expected utility values of the exploration project. In Chapter 5, Nova Scotia Shelf basin is selected for implementing the methodology. The results in various cases are given and discussed. Chapter 6 provides the conclusion, summary, contributions, limitations, and suggestions for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The first three sections give specific reviews of the research literature on the principal methods used for estimating hydrocarbon resources. Section 2.2 discusses briefly the literature on field size distributions. Sections 2.3 and 2.4 review the literature on the probabilistic models of hydrocarbon discovery process and Manly's Approximation Method. These three approaches are directly related to the analytical model of estimating the total amount of hydrocarbon discoveries. Note that a general review of various approaches used in evaluating hydrocarbon's potential can be seen in Chungcharoen (1994). Subsequently, Section 2.5 examines the research literature on the methods of quantifying uncertainties in geological and economic parameters, and the methods of performing economic analysis for hydrocarbon exploration.

2.2 Field Size Distributions

The field size distributions play an important role in estimating hydrocarbon resources because they are parts of the important information used by stochastic drilling models to calculate the probability of hydrocarbon discovery. They deal directly with the natural units of hydrocarbon exploration, prospects and fields, in such a way that is useful for both geologic and economic analyses (Baker et al., 1984).

Several researchers have tried to use theoretical distributions to describe the size distributions of other mineral deposits such as gold, copper, and iron. The observed discovery size distribution of mineral deposits are typically described as lognormal (see Krige, 1951, Mathron, 1955, and Allais, 1957).

In hydrocarbon exploration, the lognormal distribution has also been used widely to describe the field size distribution of hydrocarbon discovery. For example, Arps and Roberts (1958) discussed the economic viewpoint of the Denver-Julesburg basin based on the past drilling activity and the production history, and showed that a histogram plot of the frequency size distribution of discoveries confirmed a lognormal distribution of observed field sizes. Kaufman et al. (1963) had demonstrated that empirical distribution of hydrocarbon fields in a nearly exhausted region was quite closely approximated by the lognormal distribution. McCrossan (1969) classified hydrocarbon fields discovered in Alberta, Western Canada, according to geological type and showed that the distribution of discovered hydrocarbon reserves were consistent with the lognormal distribution.

Other researchers, such as Barouch and Kaufman (1974, 1976), Kaufman et al. (1975), Lee and Wang (1983a, b), and Forman and Hinde (1985), had adopted the lognormal distribution as an integral part of hydrocarbon modeling frameworks.

Due to the petroleum crisis in the 70's and 80's, and concern for increasing usage of the world's depletable hydrocarbon resources, many researchers focused their attention on the number of smaller discovered hydrocarbon fields. They argued that there is a difference between the observed discovery field size distribution and parent field size distribution. Arps and Roberts (1958) first noted that the lognormal appearance of the observed field size distribution was the result of economic filtration, namely that the tapering-off on the left-hand side of the distribution must be largely caused by economic factors. Many small fields which contributed less than the economic feasible point were probably never completed and therefore escape the statistics.

Uhler and Bradley (1970) hypothesized that the number of hydrocarbon fields could be described by a negative binomial law after analyzing the spatial occurrence of hydrocarbon fields in Alberta. Drew (1972) studied the spatial distribution of petroleum within land tracts in Kansas which led to a probability law that was substantially different from the negative binomial. Cozzolino (1972) used the Gamma distribution to represent the field size deposits and explained that the discovered deposits have a flatter right tail than that of the underlying distribution using the lognormal distribution assumption.

O'Carroll and Smith (1980), and Smith and Ward (1981) examined the North Sea oil fields and used maximum likelihood estimates of the size distribution of them. The data showed that a 'J-shaped' field size distribution would better represent the geological process of hydrocarbon deposition in the North Sea.

Schuenemeyer and Drew (1983) stated that the problem of estimating the number of remaining fields subject to economic filtration was of practical importance. Many small hydrocarbon fields went unreported because they were not economical, which led to an underestimation of the number of undiscovered small fields. Their analysis of several well-explored hydrocarbon regions revealed that the large fields, when grouped into log base 2 size classes, were geometrically distributed.

Baker et al. (1984) suggested a class of "*J*-shaped" distributions as ideal candidates for the underlying parent size distributions. They also demonstrated that the observed lognormality in the size distribution of discovered fields can result from sampling bias other than economic filtration.

Attanasi and Drew (1985) considered field size distributions in several areas and time periods. They suggested that frequency distributions estimated with observed data and used to justify the hypothesis of lognormal parent field size distributions are conditional. The requirement of profitability of a commercial discovery, which depends on hydrocarbon prices and field development costs, tends to eliminate some new fields found at certain locations, depths, or water depths from being commercially developed. They called this filtering of new discoveries "economic truncation" and concluded that "the observed distribution should be regarded as conditional distributions since they represented the end result of an economic filtering process".

Drew et al. (1988) examined observed field size distributions in state and federal waters off Texas and demonstrated that the economic factors, such as significant hydrocarbon price changes and production cost differences, play important roles in the shape of the observed field size distributions. They pointed out that even though the hypothesis of lognormality cannot be rejected, changes of the discovery size distributions in response to increased prices showing that economic factors continued to truncate the actual discovery size distribution. The lognormal characterization of the observed field size distribution appears to be an unintentional consequence of finding large fields early and economic truncation of smaller fields (Attanasi and Drew, 1985). As a result, they recommended that one should not be confident when inferring the form and the specific parameters of the parent field size distribution from the observed distribution.

Power (1992) used the approach of Baker et al. (1984) and conducted the analysis of the effect of sampling bias on the observed discoveries size distribution by using the discovery process model and a series of Weibull parent field size distributions. He concluded that the lognormality arises as a result of the bias inherent in the discovery process over a wide range of resource exhaustion measures.

2.3 Probabilistic Methods

Probabilistic methods were developed because traditional forecasting techniques have a high degree of variability and do not perform well in increasing the confidence of the long-run forecasting of hydrocarbon reserves. Probabilistic models have gained popularity because they were developed from assumptions which include specific geological, technological, and economic attributes of the process of exploration in a petroleum basin. Based on the knowledge of the underlying population of deposits, the models provide probabilistic descriptions of future discovery trends which incorporate explicitly the physical law of resource depletion. By observing the past course of exploration in an area, the models permit statistical estimation of the underlying resource base, the extent of resource exhaustion, and the parameters of the hypothesized discovery process. These estimates then facilitate projections of any additional discoveries that are expected to occur as the process continues to evolve through time (O'Carroll and Smith, 1980). The probabilistic methods can be mainly categorized into two groups: the probabilistic models of the hydrocarbon discovery process and simulation models. Brief reviews follow Subsections 2.3.1 and 2.3.2.

2.3.1 The Probabilistic Models of the Hydrocarbon Discovery Process

The development of the probabilistic hydrocarbon discovery process model benefited from the work of Arps and Roberts (1958), who first introduced the notions that the probability of discovering a field of a given size is proportional to the number of undiscovered fields of that size and to the areal extent of each field. By adopting this concept, Barouch and Kaufman (1974, 1976) and Kaufman et al. (1975) had developed a probabilistic model of hydrocarbon discovery process by using statistical regularities in the number and size distribution of discovered fields as a model for predicting the distribution of undiscovered fields. This model involved the probabilistic characterization of the historical way in which a field develops in such a way that the largest field is usually discovered early in the play. The probability that the next discovery will be of size "A" is the ratio of the size of all "A" fields to the sum of the sizes of all undiscovered fields. From the above concept, the probabilistic model relies on two crucial postulates in order to describe the discovery process. First, the discovery of fields portrays the discovery phenomenon as a sampling process without replacement (Barouch and Kaufman, 1974, 1976). Second, the probability of discovery of an individual field is proportional to field size (Arps and Roberts, 1958). Analyses of the North Sea and the major Alberta plays were used as an empirical validation of the predictive character of the model by the above researchers.

Several authors have demonstrated that probabilistic formulations of the hydrocarbon exploration and discovery process are very useful in forecasting the hydrocarbon reserves, in decision making, and in policy analysis. Eckbo et al. (1978) further explored the probabilistic nature of the discovery process model and developed a point estimate of hydrocarbon discoveries and production.

Smith (1980) developed the probabilistic model as an extension of work initiated by Kaufman. O'Carroll and Smith (1980) explored the structural differences and comparative performance of the two models developed by Smith (1980) and by O'Carroll at British Petroleum. They used historical discovery data from the northern North Sea petroleum province. Smith and Ward (1981) continued the Smith, and O'Carroll and Smith research by exploring the influence of the probabilistic model specification on resulting estimates of the North Sea petroleum province.

Lee and Wang (1983a) refined the Barouch and Kaufman (1976)'s work and formulated the probabilistic model for Hydrocarbon Assessment System Processor (HASP), which was developed by the Geological Survey of Canada. They established a framework from which the distribution of the number of fields, the expected n^{th} largest field size and its distribution, and the generation of reservoir parameters for a given field size may be calculated. Combinations of data and expert judgment were expressed as probability distributions for population attributes. Their approach was based on fitting parametric probability distributions for various geological random variables and examining the conditional field size distribution implied by these fits.

Lee and Wang (1983b) used the same framework as in (1983a) and developed a feedback mechanism in an attempt to resolve discrepancies between different estimates, and to validate basic input ingredients such as the play risk, the number of prospects, and the conditional field size distribution. Play 10 from the East Coast of Canada was selected as an example to illustrate applications of the analytical method.

Lee and Wang (1985) presented a method using the probabilistic model of hydrocarbon discovery for checking the individual field sizes by utilizing a discovery record which consisted of observed field sizes and their perceived ranking in the play.

Power (1990) and Power and Jewkes (1992) examined the probabilistic model of hydrocarbon exploration and development in partially explored basins. They chose the Nova Scotian Shelf data and aimed to incorporate the geologic and econometric considerations in the study of the hydrocarbon exploration, development, and supply response of frontier basins. Their modeling framework consists of three sub-models: the drilling model uses geologic data to calculate the probabilities with discovering natural gas pools; the pool-cost model describes the costs and financial flows associated with exploring for, delineating, and developing natural gas pool on the Scotian shelf; the filtration model determines whether a given discovery is commercially viable and calculates the economic consequences of developing the pools judged to be commercially viable.

2.3.2 Simulation Methods

Simulation methods are widely accepted in the modeling of various kinds of real world processes and have gained popularity rapidly with the advance of computer technology. One good reason for this acceptance is that most real world systems are too complex to allow realistic models to be evaluated analytically, and therefore, these models must be studied by means of simulation which evaluate them numerically (Law and Kelton, 1991).

The simulation methods used in hydrocarbon exploration use a combination of approaches, as mentioned previously. Because exact estimates of the undiscovered hydrocarbon potential are not attainable, simulation methods involve considerable subjective input based on geological and geophysical data in order to define a series of estimated probability distributions of field characteristics and the likely distribution of possible field discovery sizes. The input and results are expressed as ranges of values in the form of probability distributions, rather than only as single values.

The most widely accepted simulation method is "the simulation of the probabilistic models of the hydrocarbon discovery process". Contributing works on simulation methods in hydrocarbon exploration are reviewed as follows:

The Geological Survey of Canada, GSC, (Roy, 1975; Energy, Mines, and Resources Canada, 1977; Procter et al., 1984) made significant advances in this approach by applying the "exploration play" method which; (1) incorporates uncertainty into the estimates of hydrocarbon potential; (2) indicates the possible range of estimates; (3) indicates the expected field size distribution; and (4) breaks the assessment procedures into parts, used a subjective probability and a Monte Carlo simulation to produce the assessment of hydrocarbon potential (Roy, 1975).

Roadifer (1975) evaluated the separate parameters of hydrocarbon accumulations from a hydrocarbon generation-trap system, and used the probability approach and Monte Carlo simulation to estimate future potential or volumes of undiscovered hydrocarbons. He credited these methods by stating that "Because the estimates are presented as probability distributions, they provide an evaluation of the chances that there may be large volumes of hydrocarbons as well as a near certainty of estimated small volumes".

White (1981) used the play approach and presented an integral part of a microeconomics simulation of hydrocarbon endowment, exploration, development, production, transportation, and distribution. He used the endowment model to simulate an inventory of prospects and generated an associated resource assessment. By using the exploration model to simulate the economic evaluation of these prospects and drilling decisions for each, he generated a sequence of discoveries over time that formed an inventory of deposits to be evaluated for development. White used this approach to assess the hydrocarbon potential and to forecast the associated exploration activity in a frontier or partly explored petroleum basin, such as in Alaska.

Baker et al (1984) constructed field size distributions from simulations of distributions of the play's prospect areas, reservoir parameters, and potential hydrocarbon relationships. They also used the Monte Carlo simulation for their final assessment curves which gave the exceedance probability versus the range of possible recoverable hydrocarbon potential.

West (1994) presented various aspects of Bayesian inference in selection and size biased sampling problems from both infinite and finite populations. Estimation of the size of finite populations and inference about superpopulation distributions when sampling is apparently informative was developed into two specific problems: truncated data analysis and discovery sampling in which units of a finite population were selected with probabilities proportional to some measure of size. Some details of simulation base Bayesian analysis were presented. Extensions of these approaches into multiparameter super populations, semi-parametric models, and problems of dealing with missing data in discovery sampling were suggested.

Simulation has been proven to be a valuable method, especially for large corporations which have the resources to meet the data, skills and computing requirements of implementing such resource appraisal systems (Power, 1990). These required resources may be a major obstacle in implementing this method in a small corporation.

2.4 Manly's Approximation Method

Manly (1974) introduced his approximation method in a biometrics context. In his article, Manly developed a model for the analysis of the Type Two Selection Experiment called "the sampling without replacement process". This process was the same as the second postulate used in the probabilistic models to describe the hydrocarbon discovery process. He defined a population containing N individuals in K distinct classes with A_i individuals in the i^{th} class ($\sum A_i = N$) and assumed that a selection process was observed in which n individuals were chosen, without being replaced, one-by-one from the population in such a way that individuals in the same class have the same probability of being chosen, with the possibility that this probability might vary from class to class. Manly derived equations for the estimation of the mean and other parameters and gave examples regarding predator-prey experiments.

Manly's Approximation Method was first applied to the hydrocarbon exploration model by Fuller et al. (1991). They focused on the use of this method which employed difference equations to approximate the expected values, variances, and covariances of future discoveries in various size classes in the hydrocarbon exploration model. They also found that calculations using this method were faster than the method used by Smith (1980) and it avoided the use of simulations. As stated in Fuller et al. (1991): "the approximation can be calculated much faster than a simulation, which has until now been the only method available for making forecasts with the model. Solution of the recursion relations involves simple arithmetic repeated for each well in the forecast. Manly's approximation is much faster than simulation which must generate a large number of discovery sequences". By using specific data sets, they found that Manly's Approximation Method is 600 times faster than simulation for an example of running Nova Scotian Shelf data.

Fuller (1991) and Fuller and Wang (1991) further simplified the model by converting the difference equation for the mean to a partial differential equation. The resulting formula for the solutions were compact and provided a convenient means to forecast future discoveries.

Ninpong (1992) and Ninpong et al. (1992) had examined the approximation method in comparison with the simulation method in the nature of hydrocarbon exploration with the parent population of fields being lognormal, exponential, Weibull, and gamma distributions and had concluded that the approximation method was very accurate. They developed regression models from the differences of results between these two methods in order that the accuracy of the Manly's Approximation Method could be improved.

Macdonald (1992) and Macdonald et al. (1994) further investigated the work of Fuller (1991) and Fuller and Wang (1991). They presented a new derivation of the differential equation model and a Taylor series approximation for its general case solution plus the second and third order Taylor series for the special case when a negative exponential distribution was used to describe the parent field size distribution of a hydrocarbon play. They used linear regression to estimate the coefficients and produce forecasting models which perform well compared with the simulation and the approximation method.

Finally, Chungcharoen (1994) and Chungcharoen and Fuller (1996) extended the usefulness of Manly's Approximation Method by developing an approximation model of the whole probability distribution of the total volume of hydrocarbons discovered. The mean and the standard deviation from Manly's approximation were used to help set the parameters of a family of Beta distributions, to represent the distributions of the total amount of hydrocarbons discovered from the beginning to the end of the exploration process in an area.

2.5 Methods of Incorporating Geological and Economic Uncertainties and the Economic Analysis

This section provides an overview of the research literature involving methods that incorporate uncertainties in geological and economical parameters into calculations and associate specific probabilities to possible outcomes (e.g. dry hole, or various levels of reserves). These methods are often called "Risk Analysis" in hydrocarbon exploration (Newendrop, 1975). This section also includes a literature review of economic analysis which plays a key role in the oil and gas industry for decisions regarding exploration, development, and production depending on the estimate of potential of undiscovered resources.

Since the 1960's, risk analysis has been applied to hydrocarbon exploration decisions and it has become a vital component in the decision making process. Today it is the view of many decision makers that risk analysis offers better ways to evaluate and compare drilling investment strategies and offers other advantages over the traditional "one-point" investment analysis for selecting drilling prospects. This is because risk analysis forces a more explicit look at the possible outcomes that could occur if the decision maker accepts a given prospect. It also helps explorers to answer to the "What if ?" questions. It explores the relative effects on both geologic and economic viability of drilling projects contributing to the outcomes such as the total reserve potential in the region, or cash flows. As a result, the decision makers in oil and gas firms are given a clearer insight of potential profitability and the likelihoods of achieving various levels of profitability.

Extensive research has been done in risk analysis of hydrocarbon exploration. Some articles investigate geological risk analysis. Others examine economic risk analysis. Many combine both approaches in their articles. Therefore, this review is presented in a chronological form so that a broader view of the application of risk analysis in hydrocarbon exploration will be clearly seen.

According to Smith (1970), the first article related to risk analysis in hydrocarbon exploration was published sixty years ago by Hayward (1934) with the title "Probabilities and Wildcats" for which he described probability methods for petroleum investment decisions. The evolution of techniques, however, was slow until Grayson (1960) stimulated renewed interest in this area with his classic "Decision Under Uncertainty: Drilling Decisions by Oil and Gas Operators". In his article, he described the nature of decision problems in drilling for hydrocarbons, where the uncertainties are exceptionally great. He explained that the factors that influence drilling decisions can be grouped under three broad categories: geological, economic, and personal, regardless of the size of a corporation. He also applied the expected utility criterion to help the decision maker in making decisions under these uncertainties.

From 1960 to 1995, several researchers have tried to incorporate uncertainties in geological and geophysical parameters into estimating hydrocarbon reserves, as well as to incorporate uncertainties in economic parameters into economic analysis for making drilling decisions. Their procedures could be justified as geological and economic risk analysis which involve quantifying the subjective judgments of experts on geological-geophysical parameters and economic parameters using probability distributions. Researchers have also applied statistical procedures and techniques in decision analysis to assist exploration firms to make better decisions for their hydrocarbon exploration projects. Of the techniques proposed, the decision trees approach, the utility theory, and the Monte Carlo method seem to represent the most promising for problems involving risk and uncertainty. These techniques have gained in popularity following the development of digital computer technology.

Walstrom et al. (1967) discussed how probability distributions could be used to describe uncertainty in geological and economic quantities. Uniform and triangular distributions were used to reasonably approximate the data. The Monte Carlo simulation technique was explained as the solution in determining the probability distribution result for a complicated expression involving one or more parameters, each of which has its associated uncertainty.

Smith (1968) suggested that risk and uncertainty associated with the petroleum prospect reserve calculation could be incorporated into a model of estimated reserves. The ranges of geological variables could be obtained from the judgment of an experienced explorer. He used a triangular distribution to represent the stochastic variables of area, thickness, and barrels per acre foot. He also used a discrete Bernoulli distribution to represent trap, the condition of pay presence, and oil saturation variables. The result of volumetric reserves for the productive zones was obtained by multiplying together all reserve variables using Monte Carlo simulations. He suggested that economic models designed along the same lines as his prospect reserve model would offer decision tools for the most complex questions facing exploration management.

Newendorp and Root (1968) presented a method for making drilling investment decisions which assesses the degree of uncertainty involved in the calculation of capital for drilling an oil or gas well. In their article, Newendorp and Root estimated the range and distribution of possible values for each independent variable that affects the ultimate profitability. They stated that these distributions may require the judgment of the geologist, engineer, geophysicist or economist based on their experiences. These distributions were combined into a final distribution of ultimate profitability, which could be arranged to determine the probability of obtaining various ranges of net profit.

Expected utility could also be computed from the distribution of profitability and the utility curve calculated to provide the final decision criterion for management.

McCray (1969) described areal extent, formation thickness, porosity of the rock, and fluid properties by using probability distributions. He used a triangular distribution to estimate the probable size of a discovery and converted it to the probabilistic distribution of the undeferred operating income by multiplying a net operating income with this triangular distribution. This distribution was then shifted downward by the amount of the initial investment to obtain the probabilistic distribution of the undeferred value which was further modified to give the distribution of present value. Monte Carlo simulation was used during each modification step.

Smith (1970) defined four distinct environments for exploration and production decisions: trend, prospect, development, and production phases. He explained that the most important decision in petroleum exploration and production must be made during the early phases when dry-hole risk and uncertainty about economic factors are greatest. The risk and uncertainty decreases as the environment changes from the trend to the production period. He established four probabilistic economic models, one for each of the four periods, and defined the risk and the uncertainty in each of them. The probability distributions of geological and economical parameters in each of the models are obtained by subjective judgments of the individual who is most familiar with the venture which represents the best information in the organization and by assuming uniform or triangular distributions. The Monte Carlo Method was chosen to obtain the results from each model.

Van Meurs (1971) explained the decision making in offshore exploration and production. He described the methods for prospect evaluation, for example, pay out time, internal rate of return, net present value, and the relationship between them. Geological, engineering, and economic risks in hydrocarbon exploration were discussed. The utility concept and attitudes towards risk were explained.

Newendorp (1975) represented the methods and logic of decision analysis, such as the expected value concept, decision tree analysis, preference theory, and risk analysis methods using simulation techniques. McCray (1975) explained applications of economics, probability, and statistics in the petroleum-producing industry. Return on investment, decision trees, economic models, the Monte Carlo method, evaluation of expected discoveries in mature regions, and Bayes strategies in estimating the distribution of sizes of hydrocarbon discoveries and in choosing decision rules in exploration were illustrated.

Roy (1975) presented the "exploration play" to assess the hydrocarbon potential in the frontier areas where little drilling has been done and data are sparse. This method incorporated uncertainty of the geological variables in the form of subjective frequency distributions using subjective opinions of experts. The estimate of hydrocarbon potential was considered as a solution to the equation relating a series of variables to hydrocarbon potential and the Monte Carlo method was used to combine these frequency distributions to arrive at the solution.

Newendrop (1976) stated that many publications regarding the use of Monte Carlo simulation methods for analyzing risk and uncertainty fail to realize that this procedure implies that each random variable is independent from all others. These publications explained only how to describe a distribution for each random variable and then sample a value from each distribution for each trial using random number as the entry point in a frequency distribution of the variables. He pointed out that, in fact, certain important random variables in drilling-prospect analysis are dependent, and a realistic estimate of risk and uncertainty must recognize such dependency relationships. He discussed two main issues relating to random-variable dependencies: (1) how to determine if two or more random variables are dependent; and (2) how to modify the normal sampling procedures on each simulation pass to account for observed partial dependencies between random variables. The results from a modified simulation program that takes into account the dependency among variables will be much more realistic model for appraising the risk associated with decision under uncertainty.

Harbaugh (1977) suggested the development of a fully integrated analytical system for petroleum exploration which uses conditional probability estimates of wildcat drilling success, based on geologic variables. These probabilities could be combined with financial analyses to produce assessments of optimum exploration strategies. He recommended the use of contour maps to display a variety of information such as geological features, to represent statistical measures applied to geological data, and to represent relationships that incorporate information from both geological and business sides. Four kinds of maps had been developed. They were Probability maps which could be used to express outcomes of specific acts over the area, Monetary maps which could express the financial consequences of particular outcomes stemming from a specific act, and Expected Monetary Value and Expected Utility maps which were the results from combining the first two maps.

Attanasi et al. (1981) introduced a methodology for incorporating economic considerations into resource appraisals for petroleum basins. A cost algorithm was used to calculate estimates of the costs of finding and developing undiscovered oil and gas fields in the Permian basin. It uses predictions of the size and depth distributions of fields that will be discovered based on the discovery-process model. The economic feasibility of developing a typical or representative field of a specific size and depth class for any given wellhead price and assumed rate of return is determined by carrying out a discounted cash-flow (DCF) analysis.

Jones et al. (1982) presented a model to pre-test exploration strategies. The model simulates the actual exploration process and evaluates the outcome. The scope of the model encompasses data analysis, including forecasts of future discovery sizes in the study area, and analysis of the relation between geologic and geophysical effort and test-well success. The evaluation capacities of the model include expected values and distributions for reserves, production, and economic indicators such as return on investment and net present value. A decision tree approach was used to model the actual exploration process for simulation purposes and a Monte Carlo simulation was used to build and implement the model.

Charreton and Bourdaire (1983) explained that risk parameters which define future hydrocarbon discovery can be classified into three families: geological risk (probability of success, type of hydrocarbon, reserves); technical risk (investment cost, planning delays, operating costs); and political risk (future hydrocarbon value, fiscal evaluation, political factors). Subjective probabilities were used to recognize these risk quantities. In their economic analysis, they used Savage's decision theory which associates measurement of risks by subjective probabilities, measurement of consequence by psychological utilities, and measurement of strategies by weighted average of utilities.

Megill (1984) reviewed the mathematical concepts and illustrated the applications of risk analysis in hydrocarbon exploration. The mechanics of prospect risking such as sizing the prospect, distribution of sizes, model for prospect, Monte Carlo technique, varying the dry risk, and the economic output were explained. In addition, problems in multiple-reservoir prospects were clarified.

Newendrop (1984) proposed five risk analysis models for analyzing drilling prospects. His models range from a simple two-outcome (dry or a discovery) analysis to a full Monte Carlo simulation risk model that takes into account all geologic and economic uncertainties (e.g. net pay thickness, areal closure, production rates, drilling costs, and crude prices, etc.). These five models were structured to yield the net present value (NPV) calculations and the expected monetary value (EMV) profits, and could be used for evaluating any management strategy for a given prospect such as drilling, farm out, electing to take a back-in option or dry hole contribution, etc. Even though these five models could be used for any type of drilling prospect whether gas, oil, offshore, onshore exploratory well, or development well, Newendrop suggested that level 4 or 5 model which takes the continuum of possible outcomes (such as recoverable reserve or profit) will be adequate for offshore and frontier exploration prospects in areas like Alaska and Canada.

Proctor and Taylor (1984) gave an example of a Geological Survey of Canada methodology in the evaluation of hydrocarbons potential of an offshore west coast Canada play. This methodology was based on the methodology explained by Roy (1975). It deals with high uncertainty in geological parameters in a frontier basin due to very limited information. An example from the Yakoun Play was given.

Holmes et al. (1985) presented a method of appraising prospect risk and reserves likely to be discovered based on the presumption that prospect reserves should be logarithmically distributed. The method involved estimating the largest reserve that might be found, determining the minimum economic reserve, estimating the geologic/engineering probability that the initial well will be successfully completed, and calculating the expected volume. Consequently, projected risk-adjusted reserves for the initial prospect wells, subsequent development wells, or for the total prospect program could be combined with costs and prices to predict economic outcome before any drilling begins. This approach had an advantage in use when abundant data from nearby or analogue properties were presented and it could be applied where only rudimentary data are available.

Ikoku (1985) illustrated economic analysis and investment decisions. He explained the estimation methods of oil and gas reserves, production decline curves, cash flow, time value of money, profitability of a venture, and valuation of oil and gas properties. Also, analysis of risk and uncertainty was given including mathematical expectation, utility theory, decision trees, contour maps (probability, monetary, expected monetary value, and expected utility value), and Monte Carlo simulation.

Kirkwood (1985) stated that the risk associated with an exploration adventure is a composite risk which can be analyzed in terms of the risk associated with each of the elements which produce a discovery, and these risks could be best estimated by experts. All independent and dependent variables were combined using the Monte Carlo method to represent a distribution of reserves. This distribution was combined with corresponding production costs and revenues, in order to obtain a value, normally the net present value (NPV), which represented statistically the value of the prospect. This approach had been widely used by large companies as a decision tool in petroleum exploration.

Sears and Phillips (1987) presented an approach to modeling the behavior of fractured hydrocarbon reservoirs. They developed a variation of the Arps rate-cumulative equation as a basic model for the determination of the distribution of original reserves and the decline rates. Monte Carlo simulation was used in this approach to determine the distribution of reserves. The results were used as input data into a cash-flow model to compute the net present value of the reserves and internal rate of return (IRR) by also using Monte Carlo simulation

Rose (1987) explained that successful exploration programs require a consistent consideration of risk aversion and an accurate perception of uncertainty, with continuing geotechnical effort to reduce uncertainty as much as possible. He pointed out that psychological biases are major reasons which contribute to the inconsistencies in decision making. Several quantitative methods such as presented by Arps and Arps (1974) and Cozzolino (1977) could be tested for consistency and help achieve a more systematic approach. Also, geotechnical estimates of prospect size, discovery probability, and finding costs, are classic examples of assessments made under uncertainty and followed recently identified patterns of heuristic bias. Awareness of such biases, different management programs, and structured programs to evaluate individual and group performance, could result in substantially improved accuracy in geological, geophysical, and engineering predictions.

Megill (1988) reviewed the procedures used in preparing an economic analysis for hydrocarbon exploration. Methods and terms in investment evaluation were clarified. Cash flow analysis, present value concepts, the minimum rate of return, and fundamentals of risk in exploration were explained in detail.

Wehrung (1989) examined the risk propensities of experienced executives in the oil and gas industry faced with a hypothetical risky business decision that involves significant profits and losses. The executives were asked to provide the minimum price their firm should accept before selling their share of a joint exploration venture whose future prospects were systematically varied to include profits only, losses only, and mixed profits and losses. He found out that the executives were more risk taking than risk averse over pure losses, consistent with the prediction of prospect theory. There was as much risk taking as risk aversion in pure profits and there was a tendency for certainty equivalents to show greater risk taking than probability equivalencies in mixed profit/loss situations. Wehrung summarized that more than half of the executives gave responses that were fully consistent with expected utility.

Lee and Wang (1990), in evaluating conceptual plays, explained the geological risk factors in both play and prospect level, which determine the accumulation of hydrocarbons, and demonstrated the construction of the probability distribution of each factor. Pool size equations were used with Monte Carlo simulation to obtain the pool size distribution. To reduce the effects of dependencies between geological parameters, they assumed lognormal distributions for all parameters. The pool size distribution was adjusted by exploration risk, then combined with the number-of-pool distribution to get the total resource estimation for economic analysis.

Clapp and Stibolt (1991) incorporated uncertainty surrounding the components that go into prospect reserve estimates by using the multiplicative aspect of a prospect reserve estimate, and defined probability distributions for each prospect consisting of a change factor and log-normal reserve distribution. The estimate of an entire exploration program was generated by the Monte Carlo simulation procedure. Consequently, the actual total reserves discovered and the number of discoveries could be compared with program probability distributions to determine whether they are within a reasonable range in terms of probability.

Murtha (1994) focused on using historical field data to help guide the probability distributions of the geological parameters and to incorporate them into the underlying the volumetric model by using the Monte Carlo simulation method. He considered the correlation among parameters such as area and net pay, porosity and permeability, or decline rate and pay thickness. According to his study, several bivariate dependencies among parameters were found. The comparison of the model outputs between using dependent and independent inputs showed dramatically different results. Consequently, he suggested that when two or more parameters in the underlying model appeared to depend on one another, the degree of dependence could be measured by regression and correlation tools. Also, any dependency of this sort could be included in the Monte Carlo simulation.

Dallaire (1994) included the exploration risk in the preparation of supply curves for the estimation of the economic potential for hydrocarbon plays when hydrocarbon resources are vertically superimposed on one another. In his article, the petroleum geology of hydrocarbon exploration plays and estimates of their discovered and undiscovered resources followed the "discovery-process model" described by Lee and Wang (1986), and the decision tree was used to help explain the "full-cycle" and "half-cycle" analyses. The full-cycle analysis assumed that a decision to drill an exploratory well is being considered. It included all exploration, development, and production costs. The half-cycle analysis assumed that a discovery has been made, and a decision to develop and produce it is being considered. It excluded all pre-development costs.

Harbaugh et al. (1994) pointed out that the Monte Carlo procedures for assessing risk and the financial attractiveness of hydrocarbon prospects are based on concepts of the genesis of petroleum deposits and attempt to incorporate these concepts into a probabilistic framework. However, this requires estimating probabilities relating to the occurrence of geologic events or assessing the state of geologic properties that cannot be observed. Such probabilities can only be based on subjective judgment, which makes it eminently difficult to appraise the validity of the resulting analyses. These difficulties are also compounded by the interdependencies between many geological properties. In addition, there is little information available on which to evaluate the possible significance of such independencies. As a result, most risk-assessment schemes simply ignore this possible problem. They suggested that improving assessment procedures requires either developing the necessary data to support relative-frequency estimates of geological properties and their interdependencies, or adopting alternative procedures that are less sensitive to these problems. Examples of alternative procedures are contingency-table analyses and discriminant function analysis of mappable geological variables.

Peterson et al. (1995) presented a method for developing an authorization-for-expenditure (AFE) time estimate, by generating a model and illustrating the technique with a specific offshore field development case study. Their model combines Monte Carlo

Simulation and statistical analysis of historical drilling data to predict the time of dry-hole drilling operations necessary to achieve the work planned in the AFE and thereby to meet dry-hole depth or geologic objectives. Examples were selected from analysis of drilling time and performance data on 27 wells drilled in the U.K. North Sea since 1990. They used history-matching software to match data to the best distribution by the chi-square "goodness-of-fit" criterion for each input distribution. Correlation among input parameters were taken into consideration. Simulations were run for 1,000 iterations and output distributions were plotted. The AFE results generated with risk analysis and Monte Carlo simulation were more accurate than conventionally (single-point) generated AFE estimates. The output distributions helped to clarify the uncertainty associated with drilling operations based on historical data and to quantify the contribution of problem days to total days. Finally, they reaffirmed that the range of possible drilling times from using this approach provides useful information for both budgeting and scheduling purposes.

Walls et al. (1995) developed a decision analysis software package called DISCOVERY that provides an exploration division of Phillips Petroleum Company an alternative means of evaluating a mix risky investments and selecting participation levels consistent with the firm's risk propensity. The software allows the user at Phillips to input ownership interests, product pricing, capital and operating expenditures, information on reservoir decline, and other suitable information concerning drilling project. It provides cash flow and net present value (NPV) modeling capacity for both onshore and offshore exploration projects. The user was able to model the uncertainties associated with individual investments in a number of formats designed to capture the user's subjective judgments about the prominent uncertainties, which are probably of success and oil and gas reserve outcomes. The software assumes probabilistic independence among each of the individually modeled projects but the user has the option to model projects on a prospect or play basis to account for certain dependencies. The software also uses the expected utility theory (EUT) to provide the framework for incorporating the firm's risk attitude into the exploration project analyses. It provides project valuations consistent with the division exploration manager's risk preferences by ranking projects in terms of overall preference, identifying the firm's appropriate level of participation, and staying within their division budgets. The linear and exponential preference functions were used in the software to represent the firm's risk-aversion. Walls and Dyer (1996) explored the differences in observed risk propensity among petroleum firms and their impact on performance. They developed a decision theoretic model which measures a firm's risk propensity in the form of an implied utility function, investigated changes in corporate risk propensity with respect to changes in firm size, and examined the relationships between firms' risk propensities and alternative dimensions of economic performance. They also developed a new risk propensity called Risk Tolerance Ratio (RTR) which controls for firm size and allows firms to be differentiated in terms of relative risk propensity. Their study substantiated the long held assumption that the degree of risk aversion decrease as wealth increases and suggested that corporate risk taking behavior affects firm performance in the petroleum industry.

From the above review, it has shown that the use of risk analysis in hydrocarbon exploration is an active area of research for which new papers on the development of methods of evaluating undiscovered hydrocarbon reserves continue to appear in the literature. The risk analysis involved in quantifying the uncertainty in geological and economic parameters is one major theme of the decision analysis studies, and economic analysis using decision trees, expected monetary value, and utility theory is another area of this research. Most of the researchers have concentrated on using these approaches to help a decision maker in evaluating hydrocarbon reserves for a single exploration field; a few have considered a multiple-wells drilling program. Therefore, a new approach in our research is proposed. This approach considers a multiple-wells drilling program using the above key components which incorporate the uncertainty in geological parameters into the initial frequency-size distribution, uses the benefit of Manly's Approximation Method to speed up the calculations in order to define a family of distributions of future hydrocarbon discovery volumes (as exploration progresses), and incorporates economic parameters into the model to produce a probability distribution of the net present value. The distribution of net present value can be used to produce confidence intervals, return measures, risk measures or as input to expected utility analyses for determining multiple-wells exploration strategies. As a result, this research will be another incentive in the study of applying these approaches to hydrocarbon exploration.

CHAPTER 3

PREVIOUS WORK

3.1 Introduction

This chapter explains the key methodology used by Chungcharoen (1994) in the previous research. Section 3.2 explains briefly the mathematical description of the probabilistic model of hydrocarbon discovery process. In Section 3.3, Manly's Approximation Method, which is used to obtain the expected values and the standard deviations of the total amount of hydrocarbons discovered as exploration progresses, is also briefly discussed. Then, Section 3.4 illustrates the main concept of using a family of Beta distributions as the approximate analytical model of the distribution of the total amount of hydrocarbon discoveries.

3.2 The Probabilistic Model of Hydrocarbon Discovery Process

The probabilistic model relies on two postulates. First, the model portrays the discovery phenomenon as a sampling process without replacement (Barouch and Kaufman, 1976). Second, the probability of discovery of an individual field is proportional to the field size (Arps and Roberts, 1958). Consider an unexplored area containing K possible field sizes S_1, S_2, ... , S_K (measured as areal extents) and suppose that each undiscovered field size occurs with frequency A_1, A_2, ... , A_K. By the second postulate, if the drilling locations are chosen randomly, the probability of discovering a field of size S_j with the first exploratory field W_1 can be expressed as the ratio of the product of its occurrence and size to the area available for exploration in the underlying basin. In some applications of the model, one of the field size classes represents dry holes, the area available for exploration includes all potential areas (including those that turn out to be dry), and the model tracks all exploratory wells (including dry holes). In other applications, all size classes represent actual hydrocarbon deposits, the total "exploration" area is just the sum of the areal extents of all deposits, and the model tracks only the discovery wells.

Based on evidence from the drilling histories of wells, researchers have found that there is more success in discovering fields than the random drilling rule would propose (e.g. Arps and Roberts, 1958, Attanasi et al., 1981). Hence the "discovery efficiency" parameter was introduced into the probabilistic model. This parameter measures the magnification of the influence of areal extent on the probability with which a field is discovered. The model is modified by raising the hydrocarbon field's size value S_j in the probability expression to the power of the discovery efficiency β. Thus, the probability of discovering a field of size S_j with the first exploratory field W_1 becomes

$$P(W_1 = S_j) = \frac{A_j S_j^{\beta}}{\sum_{k=1}^{K} A_k S_k^{\beta}} \tag{3.2.1}$$

As the exploration progresses, the number of fields of size S_j remaining prior to the i^{th} discovery, W_i, depends on the number of discoveries of size S_j, and on the numbers of discoveries of other sizes, for discoveries W_1 to W_{i-1}. Defining these number of discoveries as $D_{i-1,j}$ and $D_{i-1,k}$, respectively, the probability that the i^{th} well will result in the discovery of field size S_j can be written as:

$$P(W_i = S_j) = \frac{(A_j - D_{i-1,j})S_j^{\beta}}{\sum_{k=1}^{K}(A_k - D_{i-1,k})S_k^{\beta}}, \tag{3.2.2}$$

From the above equations, a simulation of the sampling without replacement process can be implemented. Power (1990) and Ninpong (1992) suggested that at least 10,000 replications of discovery sequences are needed in order to obtain reliable estimates of means and standard deviations.

3.3 Manly's Approximation Method

Manly (1974) derived an approximation of the first two moments of the process described by Equations (3.2.1) and (3.2.2), in a biometrics context. Interpreted in the hydrocarbon discovery setting, Manly's approximation gives the approximate means and variance-covariance matrices of the number of a field size class remaining undiscovered at a particular discovery number. In brief, the approximate mean number of fields remaining undiscovered in size class k after i discoveries is calculated by using recursion relations as follows:

$$\mu_{ki} = \mu_{ki-1} - \theta_{ki-1} \tag{3.3.1}$$

where μ_{ki} is the approximate mean value when i wells have been drilled for field size class k,

$$\theta_{ki-1} = \frac{S_k^{\beta}\mu_{ki-1}}{\mu_{i-1}^{*}}, \tag{3.3.2}$$

and

$$\mu_{i-1}^{*} = \sum_{j=1}^{K} S_j^{\beta}\mu_{ji-1} . \tag{3.3.3}$$

The recursion relations for the approximate variances and covariances are given as:

$$C_{kki} = (1-\frac{2S_k^{\beta}}{\mu_{i-1}^{*}})C_{kki-1} + \theta_{ki-1}(\frac{2\sum_{j=1}^{K} S_j^{\beta} C_{kji-1}}{\mu_{i-1}^{*}} + 1 - \theta_{ki-1}) \tag{3.3.4}$$

and

$$C_{kli} = (1-\frac{S_k^{\beta}+S_l^{\beta}}{\mu_{i-1}^{*}})C_{kli-1} - \theta_{ki-1}\theta_{li-1} + \frac{(\theta_{li-1}\sum_{j=1}^{K} S_j^{\beta} C_{kji-1} + \theta_{ki-1}\sum_{j=1}^{K} S_j^{\beta} C_{lji-1})}{\mu_{i-1}^{*}} \tag{3.3.5}$$

where C_{kki} is the approximate variance of the number of remaining fields of size k after i wells have been drilled, and C_{kli} is the approximate covariance between the numbers remaining of sizes k and l after i wells have been drilled.

It can be inferred that the total number of discovered fields in size class k at the i^{th} well is equal to the number of fields initially estimated in size class k less the number of fields in size class k remaining after the i^{th} well has been drilled. Hence, the mean value of the total volume discovered after i wells is estimated by

$$\sum_{j=1}^{K} V_j(A_j - \mu_{ji}), \tag{3.3.6}$$

where V_j is the volume per field in class j. The estimated standard deviation is the square root of the approximate variance of the total volume of hydrocarbons discovered and is estimated by

$$[\sum_{k=1}^{K}\sum_{l=1}^{K} V_k V_l C_{kli}]^{\frac{1}{2}} \tag{3.3.7}$$

See, for more details, Manly (1974), Fuller (1991), and Ninpong (1992).

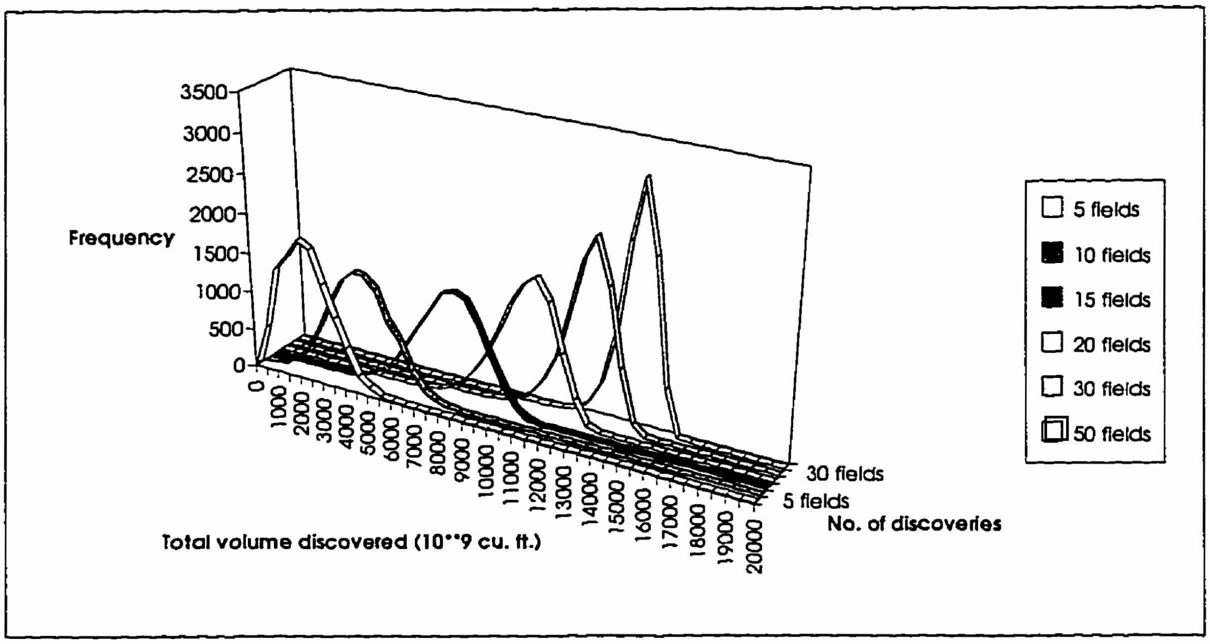
3.4 The Approximation of the Distribution of Total Hydrocarbon Discoveries

3.4.1 Hypothesizing the Family of Distributions

Based on careful but lengthy simulations, it is known that the probability density function of the total volume of hydrocarbons discovered is highly asymmetric (right-skewed) for small numbers of discovered fields. As the number of discovered fields increases, it becomes roughly normal in appearance. Later, when the number of fields gets close to being exhausted, the density function of the total amount of hydrocarbons discovered becomes asymmetric again but left-skewed. For a particular number of discovered fields, i, the maximum total amount of hydrocarbons discovered cannot exceed the sum of the volumes of the i largest fields. Also, the minimum total amount of hydrocarbons discovered cannot be less than the sum of the volumes of the i smallest fields. Figure 3.1 shows an example of six distributions of the total amount of hydrocarbons discovered for 5, 10, 15, 20, 30, and 50 discovered fields for fields of natural gas that are thought to exist in the Nova Scotia Shelf region off Canada's east coast. Each distribution is obtained from a simulation run with 10,000 replications and plotted by connecting the midpoints of intervals of a histogram.

Consider from Figure 3.1 that the shape of the density function of the total amount of hydrocarbons discovered changes with the increasing number of discovered fields. This shape looks somewhat similar to several continuous distributions such as lognormal, Beta, gamma, and Weibull. Note, however, that the range of each distribution is limited by the minimum and the maximum amount of hydrocarbons found for each discovered field. Since the Beta distribution fulfills this requirement, Chungcharoen (1994) hypothesized that the distributions of the total amount of hydrocarbons discovered might fit a family of Beta distributions. Actually, there are other distributions that satisfy the above requirement, such as Johnson's S_B distribution (the 4-parameter lognormal distribution), but his study was restricted to the family of Beta distributions for two reasons. First, the Beta distribution can accommodate a variety of skews, including positive, zero and negative. Second, the Beta distribution is well known and its properties are already well understood.

Figure 3.1 Example of six distributions of the total amount of hydrocarbons discovered as exploration progresses (Chungcharoen, 1994).



3.4.2 Background of the Beta Distribution

The family of Beta distributions is composed of all distributions with probability density functions of the form (Johnson and Kotz, 1970):

$$p_Y(y) = \frac{1}{B(p,q)} \frac{(y-a)^{p-1}(b-y)^{q-1}}{(b-a)^{p+q-1}} \qquad (a \le y \le b) \qquad (3.4.1)$$

where the two shape parameters, p and q, are greater than zero, and $B(p,q)$ is the Beta function, defined by

$$B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}\,dt \qquad (3.4.2)$$

If we make the transformation X=(Y-a)/(b-a), we obtain the probability density function

$$p_X(x) = \frac{1}{B(p,q)} x^{p-1}(1-x)^{q-1} \qquad (0 \le x \le 1) \tag{3.4.3}$$

which is the standard form of the Beta distribution with shape parameters p and q. The first and second moments of equation (3.4.1) are known to be

$$\mu_Y = a + (b-a)\frac{p}{(p+q)} \tag{3.4.4}$$

$$\sigma_Y^2 = (b-a)^2 \frac{pq}{(p+q)^2(p+q+1)} \tag{3.4.5}$$

If the values of a and b are known, then p and q can be estimated from knowledge of μ_Y and σ_Y^2, using equations (3.4.4) and (3.4.5):

$$\frac{\mu_Y - a}{b-a} = \frac{p}{p+q} \tag{3.4.6}$$

and

$$\frac{\sigma_Y^2}{(b-a)^2} = \frac{p}{p+q}\left(1 - \frac{p}{p+q}\right)\frac{1}{p+q+1} \quad . \tag{3.4.7}$$

Thus

$$p+q = \frac{\frac{\mu_Y - a}{b-a}\left(1 - \frac{\mu_Y - a}{b-a}\right)}{\left(\frac{\sigma_Y^2}{(b-a)^2}\right)} - 1 \tag{3.4.8}$$

and

$$p = \frac{\left(\frac{\mu_Y - a}{b-a}\right)^2\left(1 - \frac{\mu_Y - a}{b-a}\right)}{\frac{\sigma_Y^2}{(b-a)^2}} - \frac{\mu_Y - a}{b-a} \quad . \tag{3.4.9}$$

3.4.3 Procedures for Assessing the Accuracy of the Beta Approximation

Since the minimum and maximum (a and b) total amounts of hydrocarbons discovered for particular discovery well numbers are known, we can further hypothesize that the mean μ_Y and the standard deviation σ_Y from equations (3.4.4) and (3.4.5) can be obtained from Manly's approximation for each discovered field number. As a result, we are able to obtain values of the two shape parameters p and q with equations (3.4.8) and (3.4.9) in order to generate the Beta distributions.

The Beta distributions have been compared to the results of the simulations since the simulation is considered to be a good representation of the exact distribution, due to the large number (10,000) of replications. As we do not expect that the Beta distributions will exactly fit the underlying distributions, instead, we try to determine a family of distributions that is accurate enough for the purpose of generating forecasts, with ranges, and making decisions about exploration investment. A family of Beta distributions was investigated and found that it provided a good fit to the distributions of the total amount of hydrocarbons discovered from the beginning to the end of the exploration stages.

Procedures used for determining how representative the fitted Beta distributions are to the distributions of the total amount of hydrocarbons discovered are frequency comparisons and Kolmogorov-Smirnov (K-S) goodness-of-fit hypothesis tests. The hypotheses tested for goodness-of-fit are as follows.

The null hypothesis H_0: The sample (10,000 discovery sequences generated in the simulation) of the total amount of hydrocarbons discovered is from the Beta distribution in which its mean and standard deviation equal the mean and standard deviation obtained from Manly's approximation.

The alternative hypothesis H_1: The sample of the total amount of hydrocarbons discovered is not from the Beta distribution in which its mean and standard deviation equal the mean and standard deviation obtained from Manly's Approximation Method.

The methodology developed here was successfully applied to three real data sets; the Nova Scotia Shelf, the Bistcho Play, and the Zama Play. The Beta distributions were used to construct reasonably accurate confidence intervals of the forecast, which shows further support for this approach. More detail of the investigation can be found in Chungcharoen (1994).

CHAPTER 4

METHODOLOGY

4.1 Introduction

In this chapter, we extend the concept used in Chapter 3 to include geological uncertainties for evaluating exploration projects. This chapter is categorized into two parts. Section 4.2 explains the approximation of the distribution of total hydrocarbon discoveries including uncertainty in geological parameters. Firstly, uncertainties in field sizes and the number of fields are discussed. Secondly, frequency-size distributions that incorporate uncertainties in field sizes (including dryholes) and the number of fields are obtained, together with the determination of the data sets to be used for the evaluation. Section 4.3 describes the methodology to apply the results from Sections 3.4 and 4.2 to the calculations of the distributions of the net present value (NPV) and the expected utility value (EUV) of the exploration project.

4.2 The Approximation of the Total Hydrocarbon Discoveries Including Uncertainty in Geological Parameters

In an area where historical data are limited or not available such as offshore on the continental shelf and in the northern arctic regions, explorationists must deal with great uncertainties in the available geological information. They must rely on both objective methods and on the subjective judgment of experts based on their experience and intuition in order to obtain geological data that affect their hydrocarbon resource evaluations. The following procedures show the methodology of incorporating the uncertainties involved in the geological parameters into the field size distribution as well as into the number of fields distribution. These two distributions will be used to create several initial frequency-size distributions through a Monte Carlo approach. Each initial frequency-size distribution will be used as input data to obtain a family of Beta distributions that represent a robust approximation of the distributions of the total amount of hydrocarbon discoveries as demonstrated in Subsection 3.4. Dry hole data will be added into each initial frequency-size distribution in order to reflect the exploration risk.

4.2.1 Field Size Distribution

In order to describe the distribution of possible sizes of a field that may exist in an exploration play or basin and incorporate the uncertainty of the geological parameters, the field size equation based on Roy (1975), Proctor and Taylor (1984), and Lee and Wang (1990), is considered as follows:

Field size = (Area of closure) x (Reservoir thickness) x

(Reservoir fraction) x (Porosity) x (Hydrocarbon saturation) x

x (Recovery factor) x Constant (4.2.1)

where the constant can be the conversion factor, e.g. from hectare-meter to million cubic meters. The number of geological variables involved in Equation (4.2.1) can also be varied from one particular area to another. For example, Lee and Wang (1990) considered field area, average net pay, and average porosity as the three major parameters that affect the field size distribution for the Beaverhill Lake Play. Proctor and Taylor (1984) used all seven factors as in Equation (4.2.1) for Yakoun Play. Since there are high uncertainties in geological parameters and lack of data involved in some exploration regions, especially in a frontier area, geological data (such as net pay thickness, porosity, ..., etc.) are not precisely known. The degree of uncertainty may also vary from one variable to another. The uncertainty of a geological parameter may result from difficulty in directly and accurately measuring its quantity. As a result, there may be some range associated with each of the variables in the equation. This is equivalent to saying that each of these variables is probabilistic in nature and it would be appropriate to use a distribution for each of them. Distributions of variables such as field area, net pay, and so on, are based on interpretations by geologists and/or comparative studies; therefore, the subjective opinion of experts is the basis of estimates.

To use Equation (4.2.1), the various distributions of geological parameters are multiplied together using the Monte Carlo method to produce the field size distribution in the play or basin. The field size distribution is conditional in the sense that we are interested in identifying the distribution of field sizes given that hydrocarbons have in fact been generated, migrated, trapped and preserved in fields (Proctor and Taylor, 1984). Note that the Monte Carlo approach is, however, valid if the variables are independent or have no cross correlation. In reality, according to Proctor and Taylor (1984), as exploration proceeded and data was accumulated, correlation between variables was identified in several plays. Consequently, unacceptable errors and limitations associated with the Monte Carlo approach might be encountered if this correlation is not considered. Most risk-assessment schemes, however, simply ignore this problem (Harbaugh et al., 1994). Interdependencies between variables can be analyzed by the multiple-regression method (Sears and Phillips, 1987) and may be treated in several ways. One way to solve this problem between two dependent variables is by treating the first variable as an independent variable and the second variable as dependent on the first one (Walstrom et al., 1967, and Newendrop, 1975). The other way is by using lognormal approximations of each of the variables as described by Lee and Wang, (1990); this approach is capable of compensating for the effects of correlation among variables and also permits a more powerful analysis of the conditional field size distribution. The multiplication of variables using this approach has produced a lognormal field size distribution. Choosing the method to obtain field size distribution is crucial because it is the first step in the hydrocarbon resource analysis. As Proctor and Taylor (1984) stated, "Whatever method is used to prepare the distribution, it is important to recognize exactly what has been created, as the conditional pool size distribution is probably the most critical component of the analytical procedure and most useful in several forms of subsequent analysis".

Note that since the objective of this research is to demonstrate how to incorporate uncertainties into frequency-size distributions, we assume that all geological parameters (e.g., area closure, reservoir thickness, porosity, etc.) have been thoroughly examined by the expertise of geologists and their interdependencies have been taken into account. The field size distribution has already been obtained. How these are done is another area of work which we do not intend to investigate in detail.

4.2.2 Number of Fields Distribution

In addition to the uncertainty in geological parameters above, there is also uncertainty in the number of fields. Generally, the number of fields is usually determined by counting the number of closed anomalies on a structure contour map. However, the derivation of the number of prospects (or possible fields) will depend very much on the type and amount of geological or geophysical controls available (Proctor and Taylor, 1984). Therefore, the number of possible fields is also represented by a probabilistic distribution. In practice, it tends to be a relatively difficult distribution to prepare, as geologists consistently underestimate this number. There is a particular tendency to not recognize the very large number of relatively small fields that are associated with most areas (Proctor and Taylor, 1984).

4.2.3 Frequency-size Distribution

After determining the field size distribution and estimating the number of possible fields distribution, the Monte Carlo approach can be used to sample data from the field size distribution with each number of fields selected from the number of possible fields distribution. Each frequency-size distribution is constructed based on each sampled data set and then categorizing the sampled field data into size classes. Subsequently, the average volume in each size class is determined when all size classes are defined and the average areal extent for each size class is calculated using the relationship shown below (Harbaugh et al., 1977, and Power, 1990).

$$(\text{Average areal extent}) = \delta^*(\text{Average volume})^{\theta}$$

where δ and θ are constants that are observed in geologically similar, developed areas. Subsequently, the average areal extent for each size class is raised to the power of discovery efficiency β to reflect the magnification of the influence of areal extent on the probability with which a field is discovered.

4.2.4 Dry Holes Size Class

It is necessary to add a dry holes size class into the frequency-size distribution in order to reflect the exploration risk. The number of dry holes is calculated by subtracting the total area of all fields available (from sampled data) from the total area of the play or basin, and dividing by the drilling area. Note that the approximate effective drilling radius (the closest that one would drill to a known dry hole) used for calculation is around one mile which gives the drilling area of 3.14159 square miles (Ninpong, 1992).

4.2.5 Determining the Number of Replications

To determine the number of frequency-size distributions data used, the following approach is applied to the case of fifteen exploratory wells of each frequency-size distribution data.

a) For each frequency-size distribution which gives several Beta distributions representing the approximate total volume of discoveries as exploration progresses, we separate the Beta distribution of selected exploratory wells into several total discovery intervals with the interval width of 500 b.c.f., for example 0-500, 500-1000, ..., etc. We then integrate the Beta density function, as expressed in Equation (3.4.1), between these intervals to find the areas under the density function which represent the probabilities that the total discovery volumes would fall into these ranges. Since the minimum possible volume after a selected exploratory wells is zero (in this case, all exploratory wells are dry) and the maximum volume expected is *b*, the integration of Equation (3.4.1) between interval y_1 and y_2 can be written as

$$P(y_1 \le Y \le y_2) = \int_{y_1}^{y_2} \frac{1}{B(p,q)} \frac{(y-a)^{p-1}(b-y)^{q-1}}{(b-a)^{p+q-1}} dy$$

$$= \frac{1}{B(p,q)} \frac{1}{b^{p+q-1}} \int_{y_1}^{y_2} y^{p-1}(b-y)^{q-1} dy$$

where $0 \le y_1 \le y_2 \le b$ and $B(p,q)$ is obtained from Equation (3.4.2).

b) For a certain number of replications (500, 1000, ..., etc. replications) of a selected number of exploratory wells, calculate the average areas of intervals described in a).

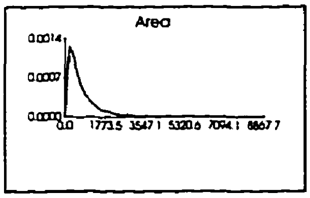
c) Determine the absolute percent differences of the average probabilities of intervals between adjacent numbers of replications such as between 500 and 1000, 1000 and 1500, ..., etc. The absolute percent differences of the average probabilities in all intervals were calculated by

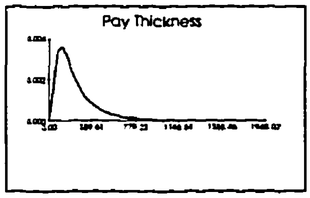
$$\textit{Absolute Percent Difference} = ABS\,[\frac{\overline{X}_m - \overline{X}_n}{\overline{X}_m}] * 100$$

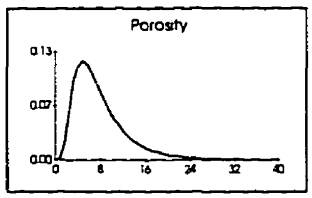
where $\overline{X}_m$ and $\overline{X}_n$ are the average probability in each interval for two adjacent replications. If they are less than the desired level, such as 5%, the minimum number of replications is obtained.

In brief, the procedure of incorporating uncertainty in geological parameters and dry holes into frequency-size distributions is shown as the following steps (Steps 1 and 2 are beyond the scope of this study).

<u>Step 1</u> Determine geological parameters using probability distributions. For example,



, 

, 

, etc.

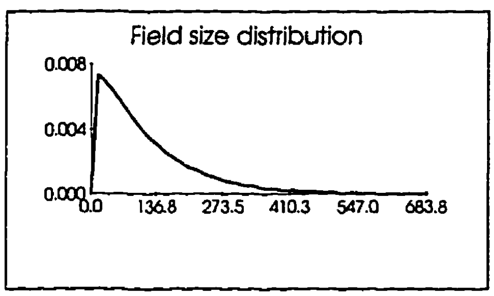
<u>Step 2</u> Obtain the field size distribution from Equation (4.2.1) by using the Monte Carlo method.

Field size = (Area of closure) x (Reservoir thickness) x (Porosity) x ...

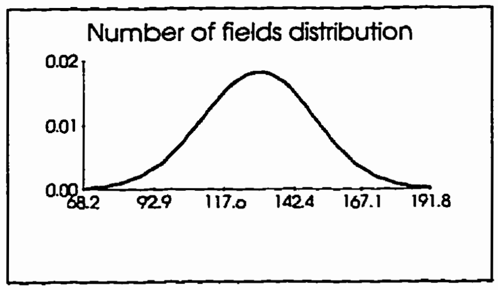
⇓

Monte Carlo Simulation

⇓



Step 3 For each replication, sample from the number of fields distribution.



Step 4 With the number of fields obtained from Step 3, sample from the field size distribution which is the result of Step 2.

Step 5 Categorize field data obtained from Step 4 into size classes.

Step 6 Calculate average volume, average areal extent, and average areal extent raised to the power of discovery efficiency β for each size class as described in Subsection 4.2.3.

Step 7 Calculate number of dry holes in dry holes size class as described in Subsection 4.2.4.

As a result of Steps 3 to 7, we obtain one frequency-size distribution. From this distribution, by using the methodology described in Subsection 3.4,

Step 8 Select Number of exploratory wells (e.g., 5, 10, 15,...), calculate the expected values and standard deviations for all number of exploratory wells using Manly's Approximation Method.

Step 9 Calculate the maximum total discovery volumes for all exploratory wells. Note that the minimum total discovery volume for these exploratory wells is zero, which means that all exploratory wells are dry.

Step 10 Use Equations 3.4.8 and 3.4.9 to compute the two shape parameters of the Beta distributions. Generate the Beta distribution for selected exploratory wells. For each selected exploratory wells, we separate the Beta distribution into several total discovery intervals with the interval width of 500 b.c.f., for example 0-500, 500-1000, ..., etc. We then integrate the Beta density function between these intervals to find the areas under the density function which represent the probabilities that the total discovery volumes would fall into these ranges.

Steps 3 to 10 are repeated n times to create n frequency-size distributions. Finally, the probabilities in all intervals from n Beta distributions for each selected exploratory wells are averaged to obtain the distribution of total hydrocarbon discoveries as a result of incorporating geological uncertainties.

4.3 Economics of Exploration

4.3.1 Incorporating Economic Parameters

After we incorporate uncertainties into a frequency-size distribution and determine a distribution of total volume of discoveries, we combine economic parameters into the calculations to examine the financial consequences of exploration, development, and production investments. The results of the economic evaluation will have a significant impact on the company's decision whether or not to invest in that particular basin.

In this part, the major economic parameters that play a key role in exploration economics will be discussed. These economic parameters are oil/gas prices and costs. Note that we will exclude taxes and royalties from the calculations as they are basically fixed rates that are applied against the company's gross revenue and net profit. Prices and costs for offshore Nova Scotia will be examined in the next chapter for the implementation of our methodology.

4.3.1.1 Oil/Gas Price

The price of oil or gas is the foremost factor that persuades companies to put their efforts into exploration programs. It determines the potential revenues that companies expect to receive after their successful exploration, development, and production. This parameter, however, is subject to instability due to the predominance of all the factors that influence the demand or supply of oil and gas and thus causes revenues to become uncertain. Uncertain revenues will affect the net present value of a company's cash flow and so affect the number of prospects drilled and, therefore, the total amount invested in exploration drilling.

To determine the price of oil or gas requires a discussion of supply and demand analysis in the petroleum or gas market. In a competitive market, like North America after deregulation in the mid 1980's, the price is usually the price-quantity combination that satisfies the desires of both consumers and sellers. This equilibrium point reflects prevailing supply and demand conditions, including wellhead deliverability (deliverable to pipelines), weather, storage levels, etc. Each market participant takes the price determined by the market equilibrium as a given. The price of oil or gas will be affected by all the factors that influence either demand or supply and determine equilibrium. With deregulation, regardless of the source of supply, buyers and sellers negotiate pricing mechanisms to address their perception of risk.

In general, the price received per unit of output at the point of retrieval, called "wellhead price", is used as a net-back price in determining producer returns and evaluating investment decisions with respect to exploration and development opportunities. The net-back price is the effective wellhead price to the producer of oil or gas based on the downstream market value less the charges for delivery to market. This price presumes the existence of markets for the oil or gas resource and easy access to distribution systems. However, in situations where there are no available facilities nor easy access to existing distribution systems, such as in frontier areas, an expensive transportation system must be constructed to deliver oil and gas from the fields to the point where it can connect to the distribution system. An example of where such a facility and transportation system is being considered is the Sable Offshore Energy Project (SOEP) for delivering natural gas from offshore Nova Scotia to the distribution system onshore. In this situation, the wellhead price is not suitable for use as a net-back price for investment decisions. The price referred to as the "landed price" will become more appropriate for consideration (Power, 1990). A landed price must be sufficient to cover the costs of exploring, developing, producing, and transporting the oil or gas to the existing pipelines.

In this research, we will not attempt to pursue a discussion in detail of all the factors that influence oil and gas prices as it is beyond the scope of our studies. Instead, we will observe the gas price in a specific market, especially in northeastern North America, and summarize expert opinion on future gas prices, in order to determine the net-back price to be used in our model. A discussion of the Northeastern market gas price is presented in Subsection 5.3.5 in the next chapter.

4.3.1.2 Discount Rate

The discount rate for use in oil and gas exploration projects is considered, in general, a compound discount rate comprised of three main components. The first component involves a cost of capital, or an opportunity cost of foregone rates of return from other potential low-risk investments, giving rise to the time preference of money: the desire to receive money earlier rather than later. The cost of capital is also influenced by the cost of the funds that a company borrows from a bank, or any lending institution, in order to invest in the exploration project. The overall cost of capital is an average weighted by the proportions of investment funds coming from the company's owners and from borrowing.

The second component of a compound discount rate is a time-related risk factor relating to exploration and production activities. This risk factor reflects the chance of discovering dryholes as well as many other sources of risk, such as the chance of considerable damage being done during the production stage such as damage caused by a storm in an offshore area. The time-related risk factor is generally expressed as an annual percentage rate and is included in the total discount factor.

The third component is the amount of inflation which oil and gas companies have to take into account. When inflation is low, prices are more stable, reducing the need of investors to be compensated for a declining value of their invested money, which reduces the interest rate acceptable to investors. On the other hand, when inflation is high, the value of money is depreciated quickly leading to higher interest rates.

From the three main components: the low-risk cost of capital, a time-related risk term, and a rate of inflation, a compound discount rate is constructed. As an example, suppose a company considers investing in its offshore exploration project with a 12% low-risk cost of capital, an 8% time-related risk, and a 5% inflation rate; the company would use a total discount rate of 25% to evaluate its venture. Since it is typical that oil and gas exploration projects are associated with very high risk and returns are received far in the future, several researchers have used high compound discount rates for their discounted cash flow analyses in their examples of hydrocarbon exploration projects (see for example, Newendrop, 1975, Van Meurs, 1981, Mcgill, 1988, and Lerch, 1992).

From the above, one can observe that the compound discount rate is high as it combines several aspects regarding the time preference of money, the risk involved, and inflation. Consequently, one could argue against using this type of discount rate by questioning whether or not it would give reliable results. One example of the arguments regarding the use of high compound discount rates was given by Van Meurs (1971) in which he argued that a high discount rate does not give proper weight to the element of risk, especially in petroleum exploration, because the uncertainty involved is mainly whether or not economically-recoverable oil or gas fields exist, and how large the reserves are; these issues are resolved before production begins. If an oil or gas field is economically recoverable and if markets are available, the uncertainty for selling the oil for a reasonable price is small. Once production begins, the fields' operating life may be more than twenty years. This is particularly true in the case of large oil or gas fields. By using a high compound discount rate to account for pre-production risk, the product produced twenty years after the start of exploration is grossly under-valued because of the high discount rate. Therefore, it is certainly an inferior evaluation procedure not to account for production twenty years or more after the beginning of the project. Finally, Van Meurs (1971) suggested that it is very important in evaluation work to detect, and properly take into account, possible large discoveries.

In this research we intend to estimate the real value of future cash flows; therefore, we will not take into account inflation in the discount rate. There will be no risk term in the discount rate; uncertainties about dryholes and discovery sizes will be reflected in the calculated distribution of NPV. The risk involved will be incorporated in the next step when we discuss the expected utility approach. Specific discussion of the discount rate for use with the offshore Nova Scotia exploration project will be given in the next chapter.

4.3.1.3 Costs

Costs related to exploration economics are generally classified as exploration, development, or production costs. Exploration costs are normally incurred on a yearly basis as basins are explored, whereas development and production costs are incurred only when there is a significant discovery and companies believe the potential revenue from developing a field will yield an acceptable profit. These three major costs are also variable; largely depending on many factors in different geological locations, such as onshore or offshore.

Based on several studies regarding the relationship between costs and influence factors, researchers have found that costs are prominently related to field size. Even though other important factors, such as well depth, and time spent on activities, also contributed to the costs of exploration, development, and production, they can be related to the size of the oil or gas field. Examples of these studies are publicized by Nystad (1981), Attanasi et al. (1981), Attanasi and Haynes (1984), and Power (1990), and are discussed briefly as follows.

Nystad (1981) examined the correlation between costs and several influence factors (e.g., recoverable reserve, production capacities, water depth, etc.) using different functional forms (linear, lognormal, polynomial) with North Sea data. He found the best fit to be a linear relationship between the component costs of field development and field size. Tests on the available North Sea data carried out by Power (1990) also agree with Nystad that depth is not a major factor of costs and field size was found to dominate the cost relationship. Comparing the Nova Scotia offshore with the North Sea experience, Power found that the influence of field size on costs in both areas is extremely high.

Since the geological information in the model is established in the form of a frequency-size distribution and given the above evidence on the relationship between costs and field size, it is therefore suitable for the cost models used in our methodology to be derived from the relationship between field size class and costs. Note that all costs regarding exploration and development wells can vary from one region to another depending on several influence factors. Exploration and development costs in an offshore region are generally higher than the same costs onshore. Among the same offshore categories, costs incurred in Alaska or in the North Sea are much higher than the costs incurred in the gulf of Mexico due to the more severe weather and the deeper water. The cost models developed in this subsection are intended to be used as basic models for the purpose of demonstrating our methodology. Also, these cost models represent relationships between costs and field size due to fairly aggregated information. To perform a more detailed economic study, disaggregated field data would have to be used.

In addition, the details of cost models used in other specific regions might be different from the ones used here.

4.3.1.3.1 Exploration Costs

Exploration costs basically can be categorized as: cost of exploration licenses, geological and geophysical costs, costs of drilling and equipping exploratory wells, and others. After acquiring an exploration license for any frontier region, a company has to perform extensive geological and geophysical surveys, such as aero-magnetic and seismic, to evaluate the sedimentary strata and identify potential traps in that region before exploration drilling begins. It could take a period of many years before the first wildcat drilling starts. As an example, in 1959 Mobile Oil acquired exploration licenses from the Nova Scotia government to conduct a survey of Sable Island and the surrounding offshore area. The company spent over seven years, from 1960 to 1966, acquiring aero-magnetic data and carrying out seismic surveys until it was ready to drill the first wildcat well in 1967.

The cost of acquiring exploration licenses and other miscellaneous costs are combined with geological and geophysical costs which are estimated as costs per well using the observed relationship between the average number of kilometers surveyed per well drilled. Consensus opinion using expert judgment together with historical records to estimate future geological and geophysical costs of exploration in new regions is also used.

According to Adelman and Ward (1980), exploration drilling and equipment costs are separated into depth dependent, time dependent, and non-classified groups. Depth dependent costs include drilling mud and additives, cementing, logs and wireline evaluations, casing and tubing, formation treating, drill bits, fuel, casing hardware, directional drilling services, and plugging. Time dependent costs are calculated on a day-rate basis and include payments to drilling contractors, transportation, special tool rentals, supervision and overhead, well site logging and monitoring, and other physical tests. Non-classified costs are normally figured as a per well average. They include site preparation, other equipment and supplies, wellhead equipment, perforating, and all other expenditures. These costs are calculated using the average day-rate, per-meter, or per-well average cost. The per-unit costs are then multiplied by the calculated average depth and time factors for dryholes and discovery wells.

To develop the model for exploration costs per well for dryholes and every discovery field size class, we consider the minimum exploration costs that are required to drill an exploratory well which comprises all the average costs mentioned above. If the well is dry, it is abandoned. Thus, to account for the fact that dryholes are, at best, only partially tested, it is assumed that a specific number of days of testing were completed before the decision to abandon them was taken (Power, 1990). In case of a discovery, extra work must be performed to further test a well and to construct the basic infrastructure for further development. This extra work translates into extra time spent at the site resulting in an increase of overhead costs. Assuming that each discovery field requires the same basic extra work, the exploration costs for a field size class k can be written as:

$$[\text{Exploration costs}]_k = \text{License Cost} + \text{G\&G Costs} + \text{Other Costs}$$
$$+ \text{Basic Drilling \& Equipment Costs}$$
$$+ \text{Extra Overhead Costs in Case of Discovery}$$

$$[\text{Exploration costs}]_k = a_1 + a_2 + a_3\,(DO) \qquad (4.3.1)$$

where a_1 represents the combined exploration licenses cost, geological and geophysical costs, and other costs per well; a_2 represents basic drilling and equipment costs; a_3 represents extra costs in case of discovery; and, DO is a variable equaling 0 if a well is dry and 1 if a well is a discovery.

4.3.1.3.2 Development Costs

If an exploratory well shows significant oil or gas discovery, the development costs must then be taken into account. These development costs include delineation, well completion (development drilling), surface facilities, and inter-field pipeline costs. Note that since this study intends to develop the model as an exploration tool for government and companies, we will not go into the details of development and production stages for each specifically developed field. Instead, the development and operation costs and the time period spent on these activities for each discovered field will be estimated using available information on fields in the same size class that have been developed and put into production in that basin, or in any basin with a similar environment.

(a) Delineation Costs

Every discovery well requires additional delineation wells before a decision to fully develop the discovered resources is taken. The number of delineation wells per discovered field depends on the size of the field. To determine the delineation costs associated with each field size class in any region, historical delineation data showing the relationship between a discovered field size class and the number of delineation wells is plotted. Consequently, the delineation costs associated with each field size class are determined and a regression model is used to represent the relationship between each field size class and delineation costs. In a frontier basin, where there is little information on delineation wells, the number of delineation wells from analogous well-established basins which the new basin most resembles is used in the delineation cost estimation. The linear regression model representing the relationship between a field size class k (with volume v_k) and delineation costs could be written as:

$$[\text{Delineation Costs}]_k \quad = \quad (b_1+b_2v_k)(DO) \tag{4.3.2}$$

where b_1 is the fixed component of the costs and b_2 is the variable component of the costs relative to the field size class k. DO is 0 for dryholes, and is 1 otherwise. Note that if an exploration well is dry, it is abandoned. Therefore, there is no delineation cost associated with dryholes. In this case both component costs are zero.

(b) Development Drilling Costs

Development drilling costs are a big part of the development investment. Several components contribute to make up the total development drilling costs. The comprehensive survey conducted by the Independent Petroleum Association of America (IPAA) every five years tracks 21 components and groups them into eight areas. These areas are site preparation, fuel, mud, logging, wireline, casing, overhead, and rig costs.

To determine the development drilling costs associated with each field size class historical data are used to show the relationships between discovered field size and each of the development drilling costs in the underlying region. A linear regression model representing the relationship between a field size class k and development drilling costs can be used, and is composed of fixed component costs and variable component costs relative to field size (in volume). This model could be written as:

$$[\text{Development Drilling Costs}]_k = (c_1 + c_2 v_k)(DO) \quad (4.3.3)$$

(c) Facilities Costs

Facilities include surface equipment that is necessary to operate an oil or gas field, such as treating facilities, pumping equipment, and permanent platforms to house drilling and production facilities. Costs related to this equipment comprise fixed and variable costs depending on the amount of oil or gas to be produced. The linear regression model for facilities costs for a field size class k could be written as:

$$[\text{Facilities Costs}]_k = (d_1 + d_2 v_k)(DO) \quad (4.3.4)$$

(d) Inter-field Pipeline Costs

Inter-field pipeline costs are costs associated with the construction of a pipeline from each discovery field to the common distribution center. According to Adelman and Ward (1980), pipeline costs frequently are an important component of the capital costs for expanding offshore, and, sometimes onshore production. The critical dimensions of a pipeline to be constructed are its diameter and its length. Presumably, material and assembly costs, such as fitting and welding, are proportional to the product of the two. These costs may differ between oil and gas pipelines due to their possibly different specific requirements. In addition, there may be a fixed component of costs and a cost associated with the laying down of the pipeline that is proportional to its length but does not depend on the pipeline's diameter. A capital cost function from the above, following from Adelman and Ward (1980), for a field size class k would look like:

$$[\text{Pipeline Costs}]_k = \{e_1 + (e_2 + e_3\ GAS)\ (\text{Diameter})_k\ (\text{Length})_k\} + e_4(\text{Length})_k$$

where $e_1,\ldots, e_4$ are coefficients of the cost function and $GAS = 1$ if a gas field, 0 otherwise.

From the above equation, the length and diameter factors directly influence the pipeline costs. Since the diameter of the line is selected based on the flow rate of each field and this flow rate will depend on field size, the field size measure can be used to replace the flow rate in an estimation of the cost relationships. The regression model will require assumptions about the diameter of the pipe used in pipeline construction which will vary depending on the size of the field feeding the line and the length of the line. By using the distance known discoveries are from the common distribution center of oil or gas, the average distance of each field size class can be obtained. The selected diameters for each field size class k used in the model are based on the inter-field pipeline summary data obtained from existing pipelines in analogous well-established basins. After obtaining data on the diameter and length of the pipeline, costs are calculated using the above equation. The relationship between costs and a field size class k are plotted. The final regression equation relating pipeline capital costs to a field size class k is as follows:

$$[\text{Inter-field Pipeline Costs}]_k = (f_1 + f_2 v_k)(DO) \tag{4.3.5}$$

In addition to the inter-field pipeline costs, the annual pipeline operating costs will be calculated based on a proportion of pipeline costs for a field in each size class. From Equation (4.3.5), the annual pipeline operation costs for field size class k can be written as:

$$\begin{aligned}[\text{Annual Inter-field Pipeline Operation Costs}]_k &= (z)(f_1 + f_2 v_k)(DO) \\ &= (f_3 + f_4 v_k)(DO) \end{aligned} \tag{4.3.6}$$

where z is a constant representing the proportion of the pipeline capital costs spent on yearly maintenance of the pipeline. Assuming a specific T years of operating life for all field size classes, the net present value of the annual pipeline operating costs for field size class k would be as following:

$$[\text{NPV of Annual Pipeline Operation Costs}]_k = (f_3 + f_4 v_k) \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t}(DO) \tag{4.3.7}$$

where r is the discount rate and t_o is the starting year of production after the exploration has been completed.

(e) Transportation Toll

In a frontier basin, such as offshore Nova Scotia, Newfoundland, or the Northwest Territories, where exploration activities have just taken place, there is no main transportation system to transfer oil or gas from the offshore developed fields to a common oil refinery or gas separation facility onshore. As mentioned earlier, the Sable Offshore Energy Project (SOEP) is an example of such a transportation system being created for offshore Nova Scotia gas fields. It is a joint development project of six major exploration companies for transporting gas to shore. These companies propose to develop six natural gas fields containing about 85 billion cubic meters of recoverable gas reserves around Sable Island in the Atlantic Ocean off Nova Scotia. The gas will be transported through a pipeline on the ocean bottom to onshore facilities for production and processing of natural gas and associated natural gas liquids for Canadian and U. S. markets.

As mentioned above, there is a need to incorporate the costs of construction of this transportation system when performing an economic evaluation. Accordingly, the general practice is assumed that this major transportation line would be constructed as a common facility and operators of developed fields would bear no direct costs for the line's construction or operation (Power, 1990). Instead, the line would be treated as a common carrier facility entitled to charge tariff rates based on the cost of service. An average capacity assumption is made based on the combined development reserves estimated. The line that is constructed must be large enough to transport the peak production of the fields as they are brought on stream. The toll rate is then fixed according to the volume of oil or gas transported to insure the revenue accruing to the operator of the line is sufficient to recover the capital invested and pay for the cost of service throughout the line's operation life. From the above, the annual toll rate equation of field size class k for any year t during its operating life could be written in the following form:

$$[\text{Annual Toll Rate}]_k \quad = h_1 v(t)_k\,(DO) \qquad (4.3.8)$$

where $v(t)_k$ is the volume of field size class k produced in year t.

4.3.1.3.3 Production Costs

Costs of production comprise mainly the annual costs of operation, maintenance, and overhead for a developed field. The same approach used in the development drilling and facilities costs was used to determine operation costs. Based on the U. S. Department of Energy - Energy Information Administration (DOE - EIA), analysis of data on several onshore regions and four major offshore regions (Louisiana, Texas, California, and Alaska) found that the annual operation costs depend little on well depth (Adelman and Ward, 1980). Power (1990) first examined the relationship between operating costs of any field in offshore Nova Scotia and water depth, facilities costs, and field size. From the regression of estimated operation costs, he found that the relationship between operation costs and well depth was negative, and well depth had little effect on operation costs as compared to field size and facilities costs. Among these three factors, the field's size was the most prominent influence on operation costs. As a result, when well depth is set to an average value for all discovery size classes and the facilities costs are determined, the regression model for annual operation costs for field size class k can be written in the following form:

$$[\text{Annual Operation Costs}]_k = (g_1 + g_2 v_k)(DO) \tag{4.3.9}$$

To determine the present value of the annual operating costs of a field, assuming that every field size class has constant annual operating costs and will have T years of operation life, the present value of operation costs in T years with a discount rate, r, for a field size class k is

$$\begin{aligned}[\text{NPV Annual Operation Costs}]_k &= (g_1 + g_2 v_k) \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t} (DO) \\ &= (g_3 + g_4 v_k)(DO) \end{aligned} \tag{4.3.10}$$

Note that the reason for assuming constant annual operation costs for each field is based on the fact that there are some costs that rise and others that fall corresponding to the decline in production.

4.3.2 Economic Evaluations

From the results in Subsection 4.2.1, we intend to extend the distributions of total hydrocarbon discoveries to include economic parameters, as mentioned in Subsection 4.3.1, in the model, and to perform key economic evaluations as follows.

4.3.2.1 The Distribution of Net Present Value (NPV)

We extend the results in Section 4.2 to perform the NPV calculations to demonstrate the usefulness of our approach as the NPV is one of the basic tools used to evaluate and compare investment opportunities by depicting expenditures for investments and subsequent revenues generated. The differences between cash inflows and outflows over a period of time are discounted to the present value and reveal the amount of the profit or loss. To determine the distributions of NPV for our proposed research, we will first look at the cash flows for the combined exploration, development, and production investment stream; however, income tax and royalty calculations will not be taken into consideration. The methodology to determine the distributions of NPV requires the same approach as used in determining the single NPV calculations relating to the exploration, development, and production stages. The methodology for this part is explained briefly below.

As exploration activity starts, companies spend a certain amount of money for geological and geophysical surveys and for drilling exploratory wells to obtain valuable information on a basin. Many exploratory wells will turn out to be dry as a result of the tremendous risk involved in this stage as described in Section 4.2. Few of the exploratory wells will result in discovery wells. NPV evaluations must be performed to justify whether each discovered field will be commercially viable to develop. There are cash inflows and outflows involved throughout these stages. For each discovered field, the major cash inflows are gross revenue from sales of oil and gas. The tax credits resulting from expensed items, depletion, and depreciation, which contribute minor inflows, will not be included for simplicity. The outflows resulting from a successful discovery are costs incurred in developing, producing, and operating. A field having an excess of discounted net revenue over discounted costs is declared economic (or commercial). A field having an excess of discounted costs over discounted net revenue is declared sub-economic and removed from the discoveries. "This situation is similar to what happens with prospects whose exploratory wells show only small hydrocarbon potential. They are not fully tested or delineated and do not enter the inventory of declared discoveries" (Power, 1990).

In addition to the above, when considering the overall exploration program, the discounted net revenue accruing to the company from the operation of these economic fields must exceed the discounted costs of developing and operating all economic fields plus the discounted costs of exploration in the basin in order to sustain the exploration program. By applying the aggregate costs and revenues information to the distributions of total hydrocarbon discoveries and by using the Monte Carlo approach, we are able to arrive at the distributions of net present value. The results of our model will be presented based on the proposed exploration program to see the overall picture of the exploration activity and its effort.

In dealing with this approach, there are two major difficulties presented in the calculations of the distributions of NPV. First, there is no specific rule that directs the time rate of exploration activity for each company. The exploration agreement between the government and a company, in general, governs only three basic tasks: defining the area a company can explore for oil and gas; specifying the amount of work the company must perform during the agreement; and, setting out the rate at which land must be returned, or relinquished, to the government (Nova Scotia Oil and Gas Report, 1984). For example, "A typical exploration agreement encompasses a block area averaging 600,000 acres, calls for work to be done over a three-year term, and returns 50 percent of the land to the crown mid-way through the term. The work commitment normally involves the shooting of a seismic program, the extent of which depends on the existing seismic data on the exploration agreement block, and the drilling of one or more wells. Shell and partners, for instance, have negotiated 13 exploration agreements covering 14 million acres and requiring 15 wells to be drilled" (Nova Scotia Oil and Gas Report, 1984). As a result, the company can plan an arbitrary number of exploratory wells each year depending on their interests and the amount of capital available. This presents some difficulty in determining net present value calculations.

To cope with this problem, one possible solution being used here is for the company who uses the model to specify their drilling schedule. In our studies, we assume a specific exploration program with a specific number of exploratory wells which must be drilled as corresponding to a real agreement for an exploration project mentioned earlier, and we assume that the company has a schedule to drill a certain number of exploratory wells equally in each of the total years of an exploration program. For example, for three years of an exploration program with the commitment of 15 exploratory wells, a company will drill 5 exploratory wells each year. This assumption is intended to simplify the calculations involved and to be used as a guideline for any exploration company. Later modifications can be done to suit each of the companies' exploration programs.

Second, for each discovered field which holds consequences for delineation and development decisions, there is also no obvious means of predicting the rate at which discovered fields will be developed. This also presents a difficulty in NPV calculations. Once a field has been discovered, there must be a development schedule which allows the incurred costs to be discounted back to the year in which the initial discovery well is drilled in order that all project costs and revenues have been discounted to a common point in the exploration and development cycle.

Again, one possibility that we selected to be used in our evaluations is for the company to begin developing the discovered field on a fixed schedule, such as within 3 months after the discovery. This assumption was chosen to simplify the calculations. Different development schedules can be implemented by any company later to suit their own available resources in order to make the exploration program more realistic to them.

From the above procedure, the distribution of NPV for a particular exploration agreement can be written as:

$$\text{NPV of the project } = \sum_{i=1}^{I} \frac{(\text{NPV of Total Annual Revenues - NPV of Total Costs})_i}{(1+r)^i} \tag{4.3.11}$$

where I is the total years of the exploration program, and r is a real discount rate. In this case, the company will drill J exploratory wells each year. Thus, the total number of exploratory wells drilled in the 1st, 2nd, 3rd,..., I^{th} years will be, J, $2J$, $3J$, ..., and IJ exploratory wells, respectively.

4.3.2.1.1 NPV of Total Annual Revenues

To compute the NPV of total annual revenue resulting from J exploratory wells each year, we consider the revenues from selling the amount of oil/gas that has been discovered as a result of these activities after its production has begun. In general, the production of an oil/gas field continues on a yearly basis and the production rate declines with time. Therefore, the production decline characteristics should be incorporated into the revenue calculations. Usually, there are many types of decline curves that have been used to represent these decline characteristics of oil/gas fields, such as exponential decline, harmonic decline, and hyperbolic decline. However, the exponential decline curve has been the most widely used to represent the production decline characteristics of oil/gas fields (see, for example, Newendrop, 1975, McCray, 1975, and Ikoku, 1985). According to Ikoku (1985), three major advantages of using the exponential decline curve are: first, the mathematics of the exponential decline are much simpler and easier to use than other types of decline curves; second, results from many observations show that fields actually follow this decline over a great portion of their productive life, and then only deviate significantly toward the end of this period; third, the divergence between the exponential decline and other types of decline is not usually significant when discounted to the present time as it occurs a few years in the future. Consequently, we adopt the exponential decline curve in order to emulate the production decline characteristics in the revenue calculations for the purpose of demonstration of our methodology. The exponential decline equation for a field size class k is defined as:

$$v(t)_k = vo_k e^{-\delta_k t}$$

where vo_k is the initial volume produced in the first year of production, $v(t)_k$ is the volume produced in year t, and δ_k is a decline rate for field size class k. Note that since we are interested in the total revenues as a result of the total volume produced to perform the economic evaluations of the exploration project, we do not intend to go into details of each specific field. Therefore, we will use the mean exponential decline rate, δ, for all fields. In reality, each field will have different decline characteristics depending on its geological and geophysical properties. However, the mean decline rate is the weighted average value of the decline rates of different field size classes. By replacing a decline rate δ_k with δ for each field size class k, the exponential decline equation can be written as:

$$v(t)_k = vo_k e^{-\delta t} \tag{4.3.12}$$

Next, the summation of oil/gas produced from each field size class k over the years is the volume size of the field, v_k, itself. Therefore, we can write

$$v_k = \sum_{t=0}^{\infty} v(t)_k = \sum_{t=0}^{\infty} vo_k e^{-\delta t} = vo_k \sum_{t=0}^{\infty} e^{-\delta t}$$

$$v_k = vo_k \sum_{t=0}^{\infty} \left(\frac{1}{e^{\delta}} \right)^t = vo_k \left(\frac{1}{1 - \frac{1}{e^{\delta}}} \right)$$

$$v_k = vo_k\left(\frac{e^{\delta}}{e^{\delta}-1}\right) \tag{4.3.13a}$$

Note that if we know the volume size of a field, we can determine the initial production as $vo_k = v_k\left(\frac{e^{\delta}-1}{e^{\delta}}\right)$. Also, in the real situations, the economic operation life of the field, T, will depend on the evaluations of annual revenues and annual operation costs. The decision to shut down the field operation normally comes when the annual operation costs are higher than the annual revenues. From the volume of field in size class k in Equation (4.3.13.a), the total discovery volume, V, from J exploratory wells can be written as:

$$V = \sum_{k=1}^{K} n_k v_k \tag{4.3.13.b}$$

where n_k is the number of discoveries in size class k (k = 1 is the dryholes size class and k = 2,...,K is the discovery size class k), and K is the total number of size classes in a frequency-size distribution. Substitute Equation (4.3.13.a) into Equation (4.3.13.b) and consider $v_1 = 0$ for $k = 1$; the above equation is reduced to the following:

$$\begin{aligned} V &= \sum_{k=2}^{K} n_k v_k \\ &= \sum_{k=2}^{K} n_k vo_k\left(\frac{e^{\delta}}{e^{\delta}-1}\right) \\ &= \left(\frac{e^{\delta}}{e^{\delta}-1}\right)\sum_{k=2}^{K} n_k vo_k \\ &= \left(\frac{e^{\delta}}{e^{\delta}-1}\right)V_0 \end{aligned} \tag{4.3.13.c}$$

where V_0 is the initial total volume produced in the first year of production. If the total discovery volume, V, and the exponential decline rate, δ, are known, we can compute V_0 by

$$V_0 = V\left(\frac{e^{\delta} - 1}{e^{\delta}}\right) \tag{4.3.13.d}$$

From the above, the annual revenue from a field size class k which comes from the amount of oil/gas produced can be written as:

$$[\text{Annual Revenue}]_k \quad = \quad \{(\text{Price})\, v(t)_k\}(\text{DO}) \tag{4.3.14}$$

where DO equals 0 for dryholes size class and 1 otherwise. For J exploratory wells, the total annual revenue is the result of selling the amount of oil/gas produced from all discovered fields and it is given by

$$\begin{aligned}\text{Total Annual Revenues} \quad &= \quad \sum_{k=1}^{K} n_k \{(\text{Price}) v(t)_k\}(DO) \\ &= \quad (\text{Price})\{\sum_{k=1}^{K} n_k v(t)_k\}(DO) \\ &= \quad (\text{Price})\{\sum_{k=1}^{K} n_k vo_k e^{-\delta t}\}(DO) \qquad (4.3.15)\end{aligned}$$

For dryholes size class, $k = 1$ and $v_1 = 0$. Also, $DO = 0$. Therefore, the above equation is reduced to the form:

$$\text{Total Annual Revenues} \quad = \quad (\text{Price}) \sum_{k=2}^{K} n_k vo_k e^{-\delta t} \tag{4.3.16}$$

Since $\sum_{k=2}^{K} n_k vo_k$ is the initial total volume produced, V_0, based on J exploratory wells, therefore, Equation (4.3.16) can be written as:

$$\begin{aligned}\text{Total Annual Revenues} &= (\text{Price})\, V_0 e^{-\delta t} \\ &= (\text{Price})\, V(t) \qquad (4.3.17)\end{aligned}$$

where $V(t)$ is the total volume produced from all discovered fields in year t. The net present value of total annual revenues from T years of the operating life, starting from year t_0 after exploration, becomes the total revenues resulting from J exploratory wells and is determined by:

$$\begin{aligned}\text{NPV of Total Annual Revenues} &= \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t} (\text{Price})\, V(t) \\ &= (\text{Price}) \sum_{t=t_0+1}^{T+t_0} \frac{V(t)}{(1+r)^t} \qquad (4.3.18)\end{aligned}$$

where t_0 is the lag time between exploration and production. Note that the economic operation life of all fields, T, will be determined by comparing total annual revenues in Equation (4.3.17) and total annual costs discussed in the next subsection: the production continues as long as the annual revenues are greater than total annual costs. When the total annual revenues are less than the total annual costs, the operation will be shut down. Detailed calculations of the operation life, T, are demonstrated in the next chapter.

4.3.2.1.2 NPV of Total Costs

Since the costs of exploration, development, and production can be estimated relative to the number of exploratory wells, the number of discoveries, and the total volume of discoveries using the regression relationships explained earlier, we are able to combine these costs. In addition, these costs can be categorized into the initial costs and the annual costs for a field size class k when the production of that field begins. By summarizing Equations (4.3.1) to (4.3.5), the initial costs comprise the exploration, delineation, development drilling, facilities, inter-field pipeline costs resulting from J exploratory wells for each field size class k ($k = 1, ..., K$). Note that these initial costs are obtained from the regression models that relate field sizes to the already discounted costs at the time of exploration. The initial costs can be written as:

$$\begin{aligned}
[\text{Initial Costs}]_k &= [\text{Exploration Costs}]_k + [\text{Delineation Costs}]_k \\
&\quad + [\text{Development Drilling Costs}]_k \\
&\quad + [\text{Facilities Costs}]_k + [\text{Inter-field Pipeline costs}]_k \\
&= \{a_1+a_2+a_3(DO)\} \\
&\quad + \{(b_1+c_1+d_1+f_1) + (b_2+c_2+d_2+f_2)v_k\}(DO) \qquad (4.3.19)
\end{aligned}$$

Since we can write the number of J exploratory wells as

$$J = \sum_{k=1}^{K} n_k ,$$

therefore, the total initial costs from all field size classes based on J exploratory wells can be written as follows:

$$\text{Total Initial Costs} = \sum_{k=1}^{K} n_k \{(a_1+a_2+a_3(DO)\}$$

$$+ \sum_{k=1}^{K} n_k \ (b_1+c_1+d_1+f_1)(DO)$$

$$+ \sum_{k=1}^{K} n_k \ (b_2+c_2+d_2+f_2)(\ v_k\ )(DO) \tag{4.3.20}$$

Subsequently, there is no development and production costs for the dryholes size class, therefore, $DO = 0$ for $k=1$. The above equation is reduced to the following:

$$\text{Total Initial Costs} = (a_1+a_2) \sum_{k=1}^{K} n_k + (a_3+b_1+c_1+d_1+f_1) \sum_{k=2}^{K} n_k$$

$$+ (b_2+c_2+d_2+f_2) \sum_{k=2}^{K} n_k \ v_k \tag{4.3.21}$$

In short, Equation (4.3.21) can be written as:

$$\text{Total Initial Costs} = A\,J + B \sum_{k=2}^{K} n_k + C \sum_{k=2}^{K} n_k \ v_k \tag{4.3.22}$$

where A, B, and C are coefficients shown in Equation (4.3.21). Because the number of exploratory wells minus the number of dryholes is the number of discovery wells (wetholes), therefore,

$$J - n_1 = \sum_{k=2}^{K} n_k$$

The total discovery volume, V, is the sum of the number of discoveries in each size class times the volume of that size. Thus,

$$V = \sum_{k=2}^{K} n_k v_k \, .$$

Substitute $(J - n_1)$ and V from the above into Equation (4.3.22), the total initial costs for J exploratory wells can be written as

$$\text{Total Initial Costs} = A\,J + B\,(J - n_1) + C\,V \tag{4.3.23}$$

To obtain the total annual costs of a field size class k, we combine Equations (4.3.6), (4.3.8), and (4.3.9) together as follows.

$$[\text{Total Annual costs}]_k = \{(f_3+g_1) + (f_4+g_2)v_k + (h_1 v(t)_k)\}(DO) \tag{4.3.24}$$

For J exploratory wells, $DO = 0$ for $k = 1$, so we obtain

$$\begin{aligned}\text{Total Annual Costs} &= \sum_{k=1}^{K} n_k \{(f_3+g_1) + (f_4+g_2)v_k + (h_1 v(t)_k)\}(\mathrm{DO}) \\ &= (f_3+g_1) \sum_{k=2}^{K} n_k + (f_4+g_2) \sum_{k=2}^{K} n_k\, v_k \\ &\quad + h_1 \sum_{k=2}^{K} n_k\, v(t)_k \end{aligned} \tag{4.3.25}$$

Substitute $(J - n_1)$, V, and $V(t)$ into Equation (4.3.25), we obtain

$$\begin{aligned}\text{Total Annual Costs} &= (f_3+g_1)(J - n_1) + (f_4+g_2)\,V + h_1\,V(t) \\ &= \mathrm{D}\,(J - n_1) + \mathrm{E}\,V + h_1\,V(t) \end{aligned} \tag{4.3.26}$$

From Equation (4.3.26), the net present value of the total annual costs assuming the average operation life time of T for all field sizes is calculated as

$$\text{NPV of Total Annual Costs} = \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t} \{\mathrm{D}(J - n_1) + \mathrm{E}\,V + h_1\,V(t)\} \tag{4.3.27}$$

Finally, the total costs are determined by adding the total initial costs in Equation (4.3.23) and the NPV of total annual costs in Equation (4.3.27) as follows:

$$\begin{aligned}\text{NPV of Total Costs} &= \text{Total Initial Costs} + \text{NPV Total Annual Costs} \\ &= \{A\,J + B\,(J - n_1) + C\,V\} \\ &\quad + \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t} \{\mathrm{D}\,(J - n_1) + \mathrm{E}\,V + h_1\,V(t)\} \qquad (4.3.28)\end{aligned}$$

4.3.2.1.3 Determining Economic Field Size Class

In the last two subsections, we consider that every field size class k ($k = 2, ..., K$) will be developed in the calculations of the distribution of net present value. In fact, the decision to develop a field in any field size class will depend on economic conditions involved at the time of exploration. In a situation where the perceived oil/gas price is low and the costs of developing and producing oil/gas are high, only large field size classes are likely to be developed as the revenues after their production can cover all expenses during stages of development and production, and yield profits to the a company. For this reason, the net present value calculations for each field size class must be carried out to justify whether each discovered field size will be economically viable to develop. A field size class having an excess of discounted costs over discounted net revenues will be declared sub-economic and will be treated as the dryholes size class in the calculations of the distribution of NPV for the overall exploration project. This is because there is no development and production for this field. We perform the net present value evaluations for each discovered field size class k to determine whether it is economic or sub-economic as follows. The total revenue from selling oil/gas from a field is the summation of discounted revenues over its operation life:

$$[\text{Total revenues}]_k = \sum_{t=t_0+1}^{T_k+t_0} [\text{Annual Revenue}]_k = \sum_{t=t_0+1}^{T_k+t_0} \{(\text{Price})v(t)_k\} \tag{4.3.29}$$

where $v(t)_k$ is the amount of oil or gas produced in year t as mentioned earlier, t_o is the time lag between exploration and production, and T_k is the economic life of a field size class k. The initial development costs involved are the extra exploration, development, and production costs obtained from Equation (4.3.19) without considering geological and geophysical surveys and basic drilling costs:

$$\begin{aligned}[\text{Initial Development Costs}]_k = \; & [\text{Extra Exploration Costs}]_k + [\text{Delineation Costs}]_k \\ & + [\text{Development Drilling Costs}]_k + [\text{Facilities Costs}]_k \\ & + [\text{Inter-field Pipeline Costs}]_k \\ = \; & (a_3 + b_1 + c_1 + d_1 + f_1) + (b_2 + c_2 + d_2 + f_2)\, v_k \end{aligned} \tag{4.3.30}$$

The total annual costs are the summation of the annual pipeline operation costs, operation costs, and transportation toll which can be obtained from Equation (4.3.24) as:

$$[\text{Total Annual costs}]_k = (f_3 + g_1) + (f_4 + g_2)\, v_k + h_1\, v(t)_k \tag{4.3.31}$$

By adding Equation (4.3.30) to the sum of the discounted total annual costs in Equation (4.3.31), the combined total development and total annual costs for a field size class k are obtained by:

$$[\text{Total Development \& Annual Costs}]_k = [\text{Initial Development Costs}]_k + \sum_{t=t_0+1}^{T_k+t_0} [\text{Total Annual Costs}]_k \qquad (4.3.32)$$

By comparing Equations (4.3.29) and (4.3.32), we are able to determine the economic and sub-economic field size classes as mentioned above. After knowing the these results, if d represents the number of sub-economic field size classes, therefore, DO parameter in all revenue and cost equations in the last two subsections equals 0 for dryholes and sub-economic field size classes ($k = 1$, ..., d), and equals 1 for economic field size classes ($k = d+1$, ..., K). Consequently, the number of economic discoveries from J exploratory wells becomes

$$(J - \sum_{k=1}^{d} n_k) = \sum_{k=d+1}^{K} n_k \qquad (4.3.34)$$

In this case, the total discovery volume as a result of J exploratory wells can be written as

$$V = \sum_{k=d+1}^{K} n_k v_k \qquad (4.3.35)$$

By substituting $(J - \sum_{k=1}^{d} n_k)$ in Equation (4.3.34) to replace (J - n_1) in all revenue and cost equations in the last two subsections and considering the number of economic discoveries size class $k = d+1,\ldots, K$, we are able to obtain the calculations of the distribution of NPV with the consideration of economic field size class.

4.3.2.1.4 Determining the Distribution of Number of Discoveries

In determining the distribution of total costs (Equation (4.3.28)), we need a distribution of the number of discoveries, $(J - \sum_{k=1}^{d} n_k)$. The dependence between the distributions of $(J - \sum_{k=1}^{d} n_k)$ and the volume V and $V(t)$ is handled by the conditional sampling procedure, discussed in the next subsection.

From Equation (4.3.28), if we know the number of dryholes and the number of fields in the sub-economic field size classes $(n_1, n_2, ..., n_d)$, we would be able to obtain the number of economic discoveries, $(J - \sum_{k=1}^{d} n_k)$, which contribute to the total volume. In order to determine the distribution of the number of economic discoveries, we ran the "exact" simulation program of the probabilistic model of hydrocarbon discovery process for several numbers of exploratory wells (e.g. 5, 10, and 15) with a sample size of 10,000 each. These simulation results are considered to be good representative of the real distributions of the number of discoveries as their sample sizes are very large (see Power, 1990, Ninpong, 1992, and Chungcharoen, 1994). We also observed the results from Manly's Approximation Method, because we need to quickly evaluate the parameters of a distribution of $(J - \sum_{k=1}^{d} n_k)$, i.e. without running the lengthy exact simulation. After comparing several discrete distributions to the distribution of the number of discoveries, we hypothesize that the distribution of the number of discoveries might fit a binomial distribution. A theoretical motivation for choosing the binomial distribution is the relationship between this distribution and the classical hypergeometric distribution as mentioned by Johnson et al. (1992). According to Johnson et al. (1992), for a finite population N, it is adequate to use the simple binomial approximation for hypergeometric distribution when a sample size is small as compared to the finite population. We next describe the use of Manly's approximation to estimate the parameter of the binomial distribution. Then we discuss the accuracy of the procedure.

Since Manly's Approximation Method gives the results of the approximate mean number of fields remaining undiscovered in size class k after J exploratory wells as explained in Equation (3.3.1), we are able to compute the approximate mean number of fields discovered in size class k by subtracting the mean number of fields remaining in size class k from the total number of fields of that size class. As a result, we can compute the approximate mean number of discovered dryholes and sub-economic field size classes (k = 1, 2,..., d) as mentioned above. Consequently, we can determine the approximate mean number of economic discoveries by subtracting the approximate mean of the number of dryholes and the approximate means of the numbers of sub-economic size classes from the total number of J exploratory wells. By using the approximate mean number of successful discoveries as an approximate mean number of success of the binomial distribution, we are able to calculate the probability of success for this binomial distribution by dividing this mean value by the number of J exploratory wells.

To verify the use of the binomial distribution to represent the distribution of the number of discoveries, we compare the binomial distribution to the distribution of the number of discoveries resulting from the exact simulation of the probabilistic model of hydrocarbon exploration process by using several frequency-size distributions as input data. In each frequency-size distribution, the comparisons between the two distribution functions are done for three numbers of exploratory wells ($J_x = J, 2J, 3J = 5, 10, 15$) with a sample size of 10,000 each. The hypotheses used for a comparison are as follows.

The null hypothesis H_o: The sample of the distribution of number of discoveries is from a binomial distribution in which its mean obtained from (J_x - the sum of means of dryholes and sub-economic size classes obtained from Manly's Approximation method).

The alternative hypothesis H_1: The sample of the distribution of the number of discoveries is <u>not</u> from a binomial distribution in which its mean obtained from (J_x - the sum of means of dryholes and sub-economic size classes obtained from Manly's Approximation method).

The comparisons between these two distributions are done by histograms and the chi-square (χ^2) tests. To test the null hypothesis, the χ^2 is calculated as follows.

$$\chi^2 = \frac{\sum_{j=1}^{L}(N_j - np_j)^2}{np_j}$$

where L is the total number of categories from $j = 1,\ldots, L$ and $L \leq J_x$. N_j is the frequency in category j from simulation distribution, and np_j is the expected frequency in category j from binomial distribution. The critical value is $\chi^2_{L-1,1-\alpha}$ with L-1 degree of freedom and significance level of α. We reject the null hypothesis if $\chi^2 > \chi^2_{L-1,1-\alpha}$. Results from these tests for a selected frequency-size distribution are shown in Chapter 5. They demonstrate that the binomial distribution is good fitted to represent the distribution of the number of discoveries.

4.3.2.1.5 Procedure for Generating the Distribution of NPV

Upon knowing the number of discoveries and the corresponding total volume from Section 4.2 for J, $2J$, $3J$,..., IJ exploratory wells, a Monte Carlo approach is used to generate distributions of total costs and revenues for each number of exploratory wells. To account for dependencies between distributions of the number of discoveries and total volume, and between distributions for different years, the conditional sampling process must be used as follows:

Step 1: From the input of n frequency-size distributions in Section 4.2, obtain the average probabilities of the total number of discoveries (approximate binomial distributions) for a specific number of J, $2J$, $3J$,...,IJ exploratory wells from first running Manly's approximation to attain the average means of the number of dryholes and sub-economic size class. Then, subtract these values from the corresponding number of exploratory wells to get the average mean numbers of successful discoveries. The success probabilities of the binomial distributions are calculated by dividing the average mean values by the corresponding number of the exploratory wells. The results from this step are given in terms of the accumulated total number of discoveries for *J*, *2J*, *3J*, ..., *IJ* exploratory wells in the 1st, 2nd,..., I^{th} years, respectively.

Step 2: Obtain the average values of the mean, standard deviation, minimum value, maximum value, and the two shape parameters of distributions of total volume (approximate Beta distributions) for 1, 2, ..., *IJ* discovery wells, as explained in the previous work and Subsection 4.2, based on *n* frequency-size distributions as input data to the model.

1st Year Exploration

Step 3: Sample from the binomial distribution using the probability parameter (*p*) from Step 1 to obtain the total number of discoveries from *J* exploratory wells.

Step 4: With this sampled value representing the number of discoveries, sample from the corresponding Beta distribution from *J* Beta distributions representing the distributions of total volume from 1 to *J* discoveries having parameters given in Step 2. After completing this step, we obtain a sample of the number of discoveries and the corresponding total volume from *J* exploratory wells in the 1st year.

2^{nd} Year Exploration

Step 5: For the 2^{nd} year of exploration with another J exploratory wells, sample from the binomial distribution representing the distribution of total number of discoveries based on a total of $2J$ exploratory wells using the probability of success (p) from Step 1 in order to obtain the number of discoveries. This value, however, gives the number of discoveries from a total of $2J$ exploratory wells independently from the 1^{st} year exploration. In order to represent the reality of a projected dependent exploration schedule by a company (e.g. drilling J exploratory wells in each year for an I-year exploration agreement), two constraints must be put into the sampling process to obtain the conditional sample of total number of discoveries from the total of $2J$ exploration wells.

First, the sample of the total number of discoveries cannot be less than the total number of discoveries in the 1^{st} year. For example, if the 1^{st} year total number of discoveries is 3 wells, the total number of discoveries after two years (from $2J$ exploration wells) cannot be less than 3 wells. Note that the total number of discoveries after two years can be 3 wells again. In this case, it means that there is no discovery in the 2^{nd} year.

Second, the sample cannot be greater than the total number of discoveries in the 1^{st} year plus the maximum number of J discoveries in the 2^{nd} year. This is because the company drills another J exploratory wells in the 2^{nd} year. The maximum number of discoveries in the 2^{nd} year is J discoveries which means that all exploratory wells in the 2^{nd} year are successful (though this is a very unlikely incident). For instance, if the sampled total number of discoveries in the 1^{st} year is 3, the sampled total number of discoveries after two years cannot exceed $3+J$. Note that if any of these two constraints is violated, this step is repeated.

Step 6: With each sample of total number of discoveries, sample from a corresponding Beta distribution from $2J$ Beta distributions representing 1 to $2J$ discoveries with parameters given in Step 2 to obtain the corresponding total volume. In this step, one more constraint must be added to the sampling process: the total volume obtained after two years must not be less than the total volume obtained from the first year. This follows from the fact that the total volume after two years of exploration must be at least equal to the total volume obtained from exploration activity in the first year. This includes the case of no discovery in the 2^{nd} year activity which makes the total volume after two years of exploration equal to the total volume obtained from the first year activity whether or not there is a discovery in the first year. If this constraint is violated, Step 5 and this step must be repeated. After completing this step, we obtain the total number of discoveries and the corresponding total volume after two years of $2J$ exploration wells.

Step 7: In order to obtain the number of discoveries and the corresponding total volume obtained, within the 2^{nd} year, subtract both the total number of discoveries and the corresponding total volume of the first year from the total number of discoveries and the total volume after 2 years (based on $2J$ exploratory wells).

3rd Year Exploration

For the 3rd year of exploration with another J wells drilled, the procedure is similar to Steps 5 to 7.

Step 8: Sample from the binomial distribution using the probability of successful discoveries obtained from Step 1 for a total of $3J$ exploratory wells. Again, this is the total number of discoveries based on $3J$ exploratory wells independent from the first and second years of exploration. In order to represent the dependent drilling of another J exploratory wells which continues from an earlier year, two constraints must be imposed into the sampling process to obtain the conditional sampling of the total number of discoveries.

First, the sample of the total number of discoveries cannot be less than the total number of discoveries in the second year. Second, the sample cannot be greater than the total number of discoveries in the second year plus the maximum number of J discoveries in the 3rd year. This is because the company drills another J exploratory wells in the third year. Therefore, the maximum number of discoveries in the 3rd year is J. If any of these two constraints is violated, this step must be repeated.

Step 9: With the sample of total number of discoveries from Step 8, sample from a corresponding Beta distribution from $3J$ Beta distributions that represent the distributions of total volume from 1 to $3J$ discoveries with parameters given in Step 2 to obtain the corresponding total volume. One more constraint must be added to the sampling process: the total volume obtained after three years must not be less than the total volume obtained from the two years of exploration. Again, this follows from the fact that the total volume after three years of exploration must be at least equal to the total volume obtained from exploration activity in the second year. This also includes the case of no discovery in the 3rd year's activity which makes the total volume discoveries after three years equal to the total volume obtained from the second year exploration whether or not there is any discovery in the second year. If this constraint is violated, Step 8 and this step must be repeated. After completing this step, we obtain the total number of discoveries and the corresponding total volume after three years of $3J$ exploratory wells.

Step 10: In order to obtain the number of discoveries and the total volume obtained, within the 3rd year, subtract both the total number of discoveries and the corresponding total volume after two years (based on $2J$ exploratory wells) from the total number of discoveries and the corresponding total volume obtained in three years (based on $3J$ exploratory wells).

Note that the conditional sampling procedure continues as explained above for the later years until we arrive at the I^{th} year which is the last year of the exploration program.

Step 11: After obtaining the number of discoveries and the total volume of discoveries in the 1st year, the conditional total number of discoveries and the conditional total volume discoveries in the 2^{nd} year, the conditional total number of discoveries and the conditional total volume discoveries in the 3^{rd} year, and so on, perform economic evaluations of each year's discoveries using total revenues and total costs as given in Equations (4.3.18) and (4.3.28). These two values are used to calculate the net profit for each year as a result of exploration activity. Finally, the NPV is calculated as the summation of the discounted net profit of every year during the exploration agreement by using Equation (4.3.11).

Step 12: Repeat Steps 3 to 11 for 10,000 replications to obtain the distribution of number of discoveries, the corresponding distribution of total volume, the distribution of NPV of total annual revenues, the distribution of NPV of total costs, the distribution of net profit for every year, and, finally, the distribution of NPV for I years exploration project.

Note that there are two alternative approaches to determine the average parameters of the distribution of number of discoveries and the distribution of total volume in Steps 1 and 2 from n distributions of number of discoveries and n distributions of total volume for each J, $2J$, $3J$, ..., IJ exploratory wells to be used in the conditional sampling process.

The first alternative approach is to categorize each distribution into several intervals. We then summarize each binomial mass function as wells as integrate each Beta density function between these intervals. Upon finishing n distributions, we average the probabilities within these intervals to determine the probabilities of the average binomial distributions and average Beta distributions. Subsequently, the parameters of this average distribution are determined.

The second alternative approach is to determine the average probabilities of success of n binomial distributions and the average minimum, maximum, and two shape parameters of n Beta distributions for each J, $2J$, ..., IJ exploratory wells, and use these average parameters directly in the conditional sampling purposes. This approach is much shorter than the first approach. The results of average parameters from both alternative procedures give almost the same values in case of binomial distribution and in case of standard Beta distribution (range from 0 to 1). However, when this range is transferred to a range between a and b, the results from both approaches are slightly different. How close the results from two approaches are to each other depends on how close the shapes of n Beta distributions to each other. The comparisons of these two approaches are shown in Appendix A.

4.3.2.2 The Expected Utility Value of the Exploration Project

In this section, we extend the results from the last section to the utility theory as this theory relates to an extension of the expected net present value concept in which the management's risk attitudes are incorporated into a quantitative decision parameter called expected utility value. This parameter would have all the features of the expected net present value plus having the additional benefit of accounting for the management's specific attitudes toward exploration activities. Thus, the management can maximize the expected net present value and, at the same time, minimize exposure to loss. The expected utility value should give a more realistic measure of the value of the exploration project than the expected net present value, and would lead to better investment decisions.

To implement this theory, the management's (acting as a representative for a company) attitude toward risk involved in exploration must be assessed. If he or she makes decisions rationally and consistently in such a way that his or her preferences meet criteria (or axioms) of decision making described by von Neumann-Morgenstern, the management's utility function which completely describes his or her attitudes and feeling regarding money can be determined. Subsequently, the expected utility value is calculated by multiplying the probabilities that the net present value falls into each interval by the utilities that are associated with that net present value and summing these values. The decision of selecting between two exploration projects will be judged by choosing the one with the highest expected utility value. Note that since we use a risk-free discount rate in the calculations of the net present value, there is no "double counting" of risk in the expected utility calculations.

In practice, however, direct assessment of the utility function is rather difficult as it is a time-consuming procedure requiring a highly experienced analyst and a very patient and educated executive (Cozzolino, 1979). For the purpose of demonstration, we will not intend to directly assess the utility function of an executive or management team of any oil/gas exploration company in particular. Instead, we will adopt a functional form of an exponential utility function as suggested by several researchers according to their studies of the management's risk attitudes of oil/gas companies toward their exploration investments (see for example, Cozzolino, 1977, Howard, 1988, Wilkerson, 1988, Walls et al., 1995, and Walls and Dyer, 1996). The reasons for using this utility function are that it has been widely used in both theoretical and applied works in the areas of decision theory and financial analysis for many years (see for example, Pratt, 1964, Raifa, 1968, Hax and Wiig, 1976, Denardo and Rothblum, 1979, Gheorghe and Babes, 1984, Pliska, 1986, and Taub, 1990). Evidence from studies of the vast majority of oil/gas exploration companies shows that the exponential utility function fitted empirical data of management's past allocation decisions under condition of risk and uncertainty very well. In addition, it provides a convenient measure of risk-taking behavior that can be studied at any point in time and can be used as a relative measure for comparing firms within an industry group. The single-parameter form of the exponential utility function, coupled with the empirical data available, enables researchers to capture risk-averse behavior at the level of the firm. From this reasoning, we define the exponential utility function as:

$$u(x) = 1 - e^{-cx}$$

where c is defined as a risk-aversion coefficient ($c > 0$) and x is the net present value of the proposed exploration investment. It is important to note that a company might make decisions when it is small that implies one utility function with specific c value and then when that company is larger, investment decisions may reflect a different c value. In addition, the risk attitudes of companies may vary over time, and may relate to firm size in a manner more consistent with decreasing rather than constant risk-aversion. However, the robust approximation of the exponential utility function at any point in time provides a good measure of risk attitudes that can be studied and used as a relative measure comparing investments within the exploration industry. Note that in this study, we will assume the risk-averse behavior of the management of a company investing in Nova Scotia exploration project as several empirical studies support that the risk taking behaviors of executives in organizations in many industries, including oil/gas exploration, are strongly risk-averse (e.g. the study of a group of 100 executives in large industrial organizations by Swalm, 1966, the study of 36 corporate executives in large industrial organizations by Spetzler, 1968, the study of 117 oil executives in risk taking over gains and losses by Wehrung, 1989, and the study of 55 independent and integrated oil companies over the period 1981-1990 in risk propensity and firm performance by Walls and Dyer, 1996).

In obtaining the risk-aversion coefficient of the exponential utility function, Cozzolino (1977) and Howard (1988) suggest that a relationship exists between certain financial measures, such as share-holder equity, net income, and capital budget size, and the firm's risk aversion coefficient. In an analysis of 60 investment opportunities with various degree of risk of offshore bidding projects for BP petroleum, Wilkerson (1988) suggested that the firm's implied risk-aversion coefficient, c, in exponential utility function was approximately 0.033×10^{-6} with the certainty equivalent in million dollars. He cautioned that the firm probably had some prior drilling commitments, biases about certain exploratory blocks, or other confounding issues that may have affected the estimation of the risk-aversion coefficient for this set of decision. According to Walls et al. (1995), a general finding from a group of 18 independent and integrated oil companies suggest a rule of thumb relating the firm's risk-aversion level of exponential utility function to the firm's budget level for the current period. His findings suggest that as a starting point for assessing an individual firm's risk-aversion coefficient, the firm's risk-aversion coefficient value is equal to the inverse of one-fourth of the firm's annual exploration budget.

Assuming that there exists an exponential utility function of the management's risk-taking behaviors in oil/gas exploration, the expected utility value can be determined as mentioned above. The risk-aversion coefficient chosen for companies participating in the offshore Nova Scotia Shelf will be described in the next chapter.

CHAPTER 5

IMPLEMENTATION: NOVA SCOTIA SHELF

5.1 Introduction

In this chapter, the methodology discussed in Chapters 3 and 4 is applied to the offshore Nova Scotia Shelf which is partly explored and yet still considered to be a frontier basin. The Nova Scotia Shelf data has been used by others for examining the simulation method and Manly's approximation previously and, therefore, it is suitable for use in this research for comparison purposes (see Power, 1990, Ninpong, 1992, and Chungcharoen, 1994). Sections 5.2, 5.3, and 5.4 explain the background information and give the results of geological and economic calculations for the Nova Scotia Shelf.

5.2 Background Information

According to Nova Scotia Offshore Petroleum Board (1991), the Nova Scotia offshore region covers an area approximately 400,000 square kilometers of offshore northeastern Canada, extending from the low water mark on the coast of Nova Scotia to the outer limit of the continental margin. This region is one of Canada's largest sedimentary basins. Potential for oil and gas was first recognized in the 1950's, and actual exploration began in 1959 with an aeromagnetic survey by Mobil in the vicinity of Sable Island. Since then, the geophysical surveys and drilling have been conducted throughout this area. The first deep exploratory well was drilled on Sable Island in 1967. It encountered traces of hydrocarbons at a depth of 4,604 meters. By 1969, the geological structures beneath the shelf were confirmed to be similar to the traps encountered in the northern Gulf of Mexico which is rich in hydrocarbons. Prior to June 1983, 79 wells had been drilled on the shelf testing 53 hydrocarbon structures (Procter et al., 1984). Up to 1991, the oil and gas industry has acquired over 300,000 kilometers of seismic and has drilled 125 wells. Eighty-eight separate structures have been tested by exploratory drilling resulting in 22 significant discoveries, most within a 40-km radius of Sable Island. This resulted in the discovery of some 162 billion cubic meters (5.7 trillion cubic feet) of gas and 22.9 million cubic meters (144 million barrels) of oil and condensate. It is estimated by the Geological Survey of Canada (GSC) and the Canada Oil and Gas Lands Administration (COGLA) that these discoveries represent only 32 percent of the total potential gas resources and 13 percent of the total potential oil and condensate resources predicted to exist in this area. The Scotia basin is still at a relatively immature stage of exploration. Current exploration activity is still being conducted essentially within the Sable Subbasin and the vicinity. There are still 200 identified structures that remain to be drilled, and their number will likely increase as the result of enhanced seismic acquisition, processing and interpretation. In addition, the Sable Offshore Energy Project (SOEP), which proposes to develop six natural gas fields in the Sable Island area containing 85 billion cubic meters (2.99 trillion cubic feet) of recoverable gas reserves and is planned to be put on production by the end of 1999, is currently in its regulatory approval phase (as of April 1997) following the filing of a comprehensive application with regulatory authorities.

Chungcharoen (1994) used Power's (1990) data on the Nova Scotia Shelf which were obtained from two principal sources. Data on the areal extent of declared significant discovery areas and the predicted sizes of the ten largest fields in each of the Shelf's seven plays were obtained through COGLA. Data such as field sizes and the number of fields were defined using the information on prospects and fields given by GSC in Wade et al. (1989). The geological information used in his study includes data up to December 31, 1987. It consists of a series of geological estimates, actual data, and statistically based estimates, which gives a description of the field frequency-size distribution for the whole Nova Scotia Shelf. The final geologic description used in comparing the simulation distributions and Beta distributions considers 113 fields, dividing into 12 field size classes with the number of fields, the average areal extent, and average volume of each size class. The average areal extent measure was calculated by using the data from 22 fields discovered before the end of 1987 and a non-linear estimation which produces the values δ equal to 3.780 and θ equal to 0.372 with the standard errors of 1.46 and 0.064, respectively (Power ,1990). Therefore, the relationship becomes

$$\text{Average areal extent in square miles} = 3.78*(\text{Volume in b.c.f.})^{0.371}.$$

5.3 The Approximation of the Total Discovery Volume Including Uncertainty in Geological Parameters

5.3.1 Field Size Distribution

In this research, we would like to see how the methodology of incorporating geological parameters as described in Chapter 4 works. Therefore, the estimates of frequency-size distribution and the number of fields as in Chungcharoen (1994) will not be used. Instead, we will follow the methodology in Section 4.2. Since geological information (e.g., porosity, water saturation, etc.) for the Nova Scotia Shelf is not available, we assumed that the field size distribution obtained by applying Monte Carlo methods to Equation (4.2.1) can be represented by the Weibull distribution, which is proven to be fitted to the Nova Scotia Shelf data by Power (1990). The use of the Weibull distribution is also justified, as described by Baker et al. (1984), because it is "*J*-shaped" which is more representative of the natural field size distribution. According to Power (1990), the shape (α) and scale (β) parameters of the fitted Weibull distribution to the Nova Scotia Shelf data were estimated to be 0.869 and 117.68, respectively. Also, the total productive area of the Nova Scotia shelf was averaged using information from a COGLA map and a Jansa and Wade map to be 30,132 square miles which cover an area around Sable Island that has been the center of exploration drilling.

5.3.2 Number of Fields Distribution

There are conflicting estimates of the number of fields thought to exist in Nova Scotia Shelf. For example, Wade et al. (1989) considered 105 fields to exist on the shelf, whereas COGLA information gave an estimate of 113 fields. These numbers result from the estimation using the data available of drilling 79 wells on the shelf testing 53 hydrocarbon structures in 1983. Until 1991, there were over 200 independent structures identified waiting to be explored. In order to represent the uncertainty of the number of fields for the preliminary studies, we assume a triangular distribution with the minimum, most likely, and the maximum values of 100, 113, and 130, respectively.

5.3.3 Generating Frequency-size Distributions and Obtaining the Distributions of Total Discovery Volume

The procedure described in Section 4.2.3 will start from Step 3 as follows.

Step 3 Sample the number of fields from the triangular distribution with the minimum, most likely, and maximum number of fields are 100, 113, and 130, respectively.

Step 4 With the number of fields from Step 3, sample from the Weibull distribution in which its shape and scale parameters are 0.869 and 117.68, respectively.

Step 5 Categorize these data into 12 size classes, starting from size class #2 with volume size between 0 and 100 billion cubic feet (b.c.f.). Each size class has an interval range of 100 b.c.f.. The last size class, size class #13, has a field size volume greater than 1200 b.c.f.. The number of fields falling into each size class is counted.

Step 6 Calculate the average volume for each size class. Consequently, the average areal extent is obtained by using the non-linear estimation as explained in Subsection 4.2.3.

$$\text{Average areal extent in square miles} = 3.78*(\text{Volume in b.c.f.})^{0.371}.$$

Step 7 Calculate the number of dryholes in the total productive area of the Nova Scotia Shelf by subtracting the sum of the total average area of all fields in each size class from the total productive area of 30,132 square miles and dividing the result by the effective drilling area of 3.1416 square miles. Then, assign size class #1 to it.

After finishing step 7, we obtain one frequency-size distribution for Nova Scotia Shelf used as one set of input data to calculate the distributions of total discovery volume. Note that we raise the areal extents of all field size classes to the power of the discovery efficiency of 2.2. This parameter was estimated by Power (1990) for Nova Scotia Shelf to reflect the magnification of the influence of areal extent on the probability with which a field is discovered. The following is applied for each frequency-size distribution.

Step 8 For each frequency-size distribution, calculate the estimated means and standard deviations of the total discovery volume for selected numbers of exploratory wells by using Manly's Approximation Method.

Note that these selected numbers of exploratory wells used here are 5, 10, and 15, respectively. These numbers are selected to correspond to a real agreement for an exploration project, issued by the Nova Scotia government. This agreement requires a company to drill 15 exploratory wells within 3 years period.

Step 9 Calculate the maximum total discovery volumes for 5, 10, and 15 exploratory wells. The minimum total discovery volume for these exploratory wells is zero, which means that all exploratory wells are dry.

Step 10 Calculate the two shape parameters for each Beta distribution and generate the Beta distributions for 5, 10, and 15 exploratory wells. This is done by using the procedure in Subsection 4.2.5 to integrate each Beta density function of the each selected number of exploratory wells between 0-500 b.c.f., 500-1000 b.c.f., ..., etc., intervals to find the probabilities that the total discovery volume falls into these ranges.

Steps 8-10 are repeated for *n* frequency-size distributions. Finally, the probabilities in all intervals for *n* replications are averaged in order to obtain the distribution of total hydrocarbon discoveries as a result of incorporating geological uncertainties. Two FORTRAN programs were written to implement the above steps, for simplicity in validation. These two programs will be combined with other programs in economic evaluations at later stages to become the main program that implements the overall methodology. The first program is for sampling data from the triangular distribution and the Weibull distribution (Steps 3 and 4). Then it categorizes these data into field size classes including dry holes, and calculates average volumes and average areal extents for all size classes in order to create the frequency-size distribution (Steps 5, 6, and 7). After completing *n* replications, we will have *n* frequency-size distribution data ready for use in the second program. The second program uses each frequency-size distribution to calculate the means and standard deviations of the total volume of discoveries using Manly's approach for 5, 10, and 15 exploratory wells (Step 8). For each selected number of exploratory wells, it determines maximum volume, calculates the two shape parameters for generating the Beta distribution, and determines the probabilities within 0-500 b.c.f., 500-1000 b.c.f., ..., etc. intervals of each Beta distribution (Steps 9 and 10). Notice that there are 3 selected numbers of exploratory wells (5, 10, and 15) and 3 Beta distributions generated for one frequency-size distribution. Upon *n* replications, the total of $3n$ Beta distributions will be generated. The program then averages the probabilities of all intervals for each number of exploratory wells. Hence, we obtain the distributions of total discovery volume that incorporate geological uncertainties for 5, 10, and 15 exploratory wells as final results.

5.3.4 Results and Discussions

The examples of the results, including frequency-size distributions, means and standard deviations obtained from Manly's Approximation Method, maximum values, and two shape parameters of Beta distributions, are shown in Appendices B and C, respectively.

5.3.4.1 Number of Replications

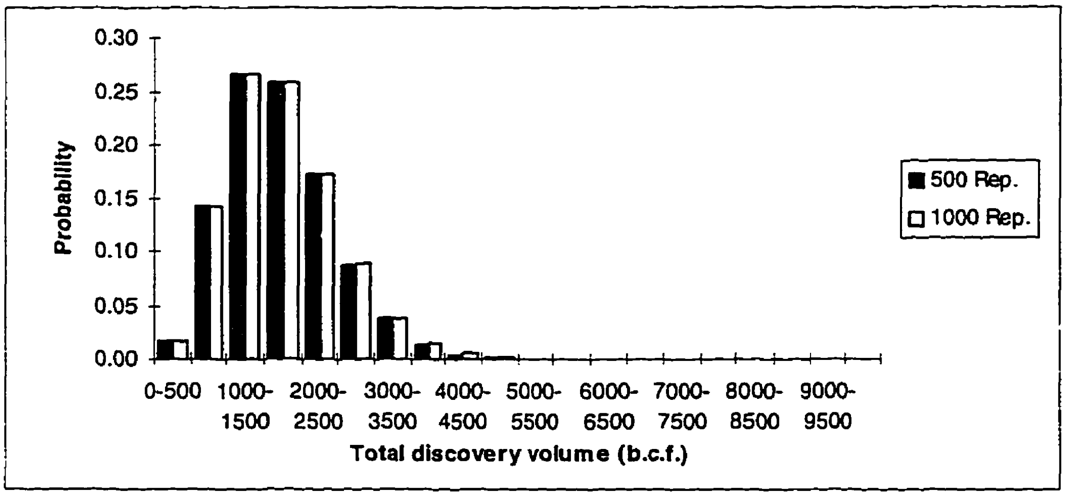
To determine the number of replications, the two programs were run several times using different numbers of replications (500, 1000, ..., and 5000) for the case of 15 exploratory wells. This number was selected because the distribution of total discovery volume has the highest degree of variation among the three distributions of total discovery volume (from 5, 10, and 15 exploratory wells). The results between 500 and 1000 replications are shown in Table 5.1.

Table 5.1 Absolute percent differences of the average probabilities between 500 and 1000 replications for 15 exploratory wells.

Interval (b.c.f.)	Probability. 500 Rep.	Cumulative Probability	Probability. 1000 Rep.	Cumulative Probability	Absolute % Difference
0-500	0.0168	0.0168	0.0166	0.0166	0.91
500-1000	0.1418	0.1586	0.1411	0.1578	0.49
1000-1500	0.2668	0.4254	0.2658	0.4235	0.38
1500-2000	0.2586	0.6840	0.2576	0.6811	0.38
2000-2500	0.1710	0.8550	0.1708	0.8519	0.15
2500-3000	0.0875	0.9425	0.0881	0.9400	0.69
3000-3500	0.0371	0.9796	0.0381	0.9781	2.60
3500-4000	0.0137	0.9934	0.0145	0.9926	5.77
4000-4500	0.0046	0.9980	0.0051	0.9977	9.75
4500-5000	0.0014	0.9994	0.0016	0.9993	13.36
5000-5500	4.24E-04	0.9998	4.88E-04	0.9998	15.20
5500-6000	1.17E-04	1.0000	1.35E-04	1.0000	15.07
6000-6500	2.92E-05	1.0000	3.37E-05	1.0000	15.43
6500-7000	6.02E-06	1.0000	7.22E-06	1.0000	20.04
7000-7500	8.99E-07	1.0000	1.18E-06	1.0000	31.56
7500-8000	7.48E-08	1.0000	1.10E-07	1.0000	47.63
8000-8500	1.94E-09	1.0000	2.81E-09	1.0000	45.01
8500-9000	6.70E-12	1.0000	3.59E-12	1.0000	46.47
9000-9500	2.62E-15	1.0000	1.31E-15	1.0000	50.00
9500-10000	4.62E-30	1.0000	2.31E-30	1.0000	50.00

Note that the cumulative probability of both replications become 1.0 from 5500-6000 interval when rounded to 4 digits. The results of Table 5.1 are also plotted in histograms for comparison in Figure 5.1.

Figure 5.1 Comparison between 500 and 1000 replications for 15 exploratory wells.



From Table 5.1 and Figure 5.1, one can see that the probabilities that total discovery volume falls into intervals greater than or equal to 2500-3000 b.c.f. are very small (less than 0.09). Table 5.1 shows that the absolute percent differences of the average probabilities between 500 and 1000 replications are less than 5% for 0-500, 500-1000, ..., 3000-3500 intervals and they are higher than 5% for other intervals beyond 3000-3500 b.c.f.. By looking at the cumulative probability, we can see that the probability that the total discovery volume would be less than or equal to 3500 b.c.f. is approximately 98% and the probability that the total discovery volume would be greater than 3500 b.c.f. is only 2 %. Therefore, we propose that only 7 intervals (0-500, 500-1000, ..., 3000-3500 b.c.f.) should be taken into consideration for determining the minimum number of replications. The reason that the absolute percent differences for intervals larger than 3500 b.c.f. are very large is that the probability of discovering large fields for every replication are usually so small. Therefore, their average probabilities within these interval are also so small, as seen in Figure 5.1, and any change in average probabilities between the two adjacent replications will be amplified which results in the large values of the absolute percent differences. As the result of this analysis, we selected 500 replications to be used in our studies.

It should be noted that this minimum number may vary depending on the uncertainties of the number of fields and the field size distributions. If the uncertainties are very large, more replications may be required before the absolute percent differences of the average probabilities of all intervals will become less than 5%. Therefore, it might be necessary to rerun the program for several numbers of replications to check the minimum sample size required for the model.

In this section, we examine 3 cases to illustrate the benefits of using the methodology in Section 4.2. Case 1 is for implementing our methodology. We use Weibull and triangular distributions with parameters described in Section 5.3.3 to generate 500 frequency-size distributions, which define geological uncertainties, to be used in the model. In Case 2, we generate 3 Beta distributions for 5, 10, and 15 exploratory wells, respectively using only a single frequency-size distribution. This frequency-size distribution is arbitrarily selected for the purpose of comparison to Case 1. It represents one possibility of frequency-size distributions that might occur in Nova Scotia Shelf basin as a result of the estimation of number of fields and field sizes by using statistical analysis based on limited data or by expert opinion. This frequency-size distribution does not include uncertainty in geological parameters. Case 3 is for comparing the results when uncertainties become large in order to see the effect of increasing uncertainties of geological parameters on the distributions of total discovery volume. To introduce more uncertainties of the geological parameters that can vary depending on knowledge and information of a company in the basin for our studies, we double the standard deviations of both the triangular distribution which represents the number of fields distribution and the Weibull distribution which represents the field size distribution in the Nova Scotia Shelf, and keep their means unchanged. The new calculated minimum, most likely, and maximum values of the triangular distribution become 85, 113, and 145, respectively. These values are obtained by using the formulas for calculating the mean and standard deviation of the triangular distribution and by keeping the mean and most likely value fixed, doubling the standard deviation, and solving for the new minimum and maximum values. Similarly, the new shape and scale parameters of Weibull distribution are obtained by using the formulas of the mean and standard deviation. The mean value is held constant but the standard deviation is double. The new estimates of shape and scale parameter values are 0.489 and 60.5670, respectively (see detailed calculations for both triangular and Weibull distributions in Appendix D). The discussions of these three cases are as follows.

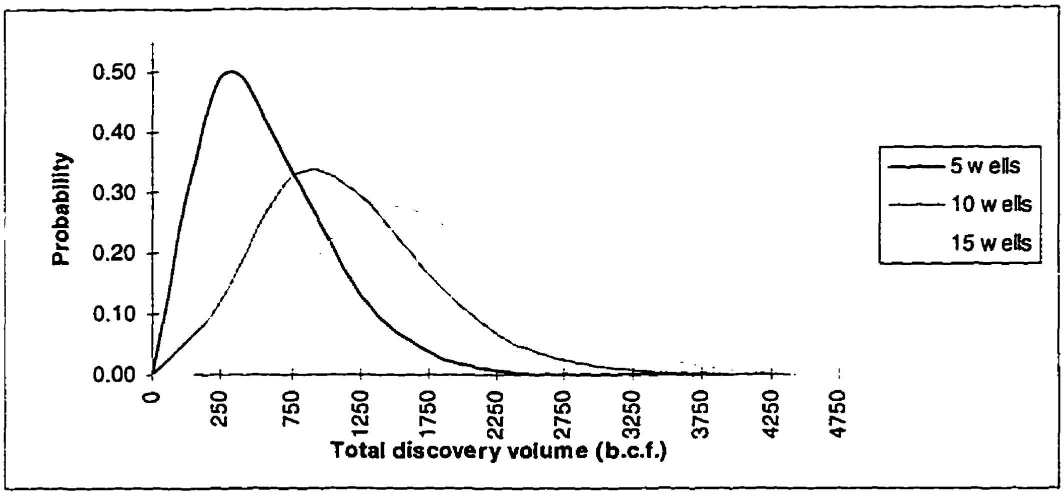
5.3.4.2 Case 1: Results of the Proposed Methodology for 5, 10, and 15 Exploratory Wells

Table 5.2 shows the probabilities that the total discovery volume falls into intervals ranging from 0 to 10,000 b.c.f. for 5, 10, and 15 exploratory wells, using the proposed methodology which includes uncertainty in geological parameters. Figure 5.2 shows the distributions of the total discovery volume based on the results from Table 5.2. Note that the line graph is the result of connecting the midpoint of average probabilities of each interval.

Table 5.2 Probabilities that the total discovery volume fall into each interval for 5, 10, and 15 exploratory wells

Interval	Probabilities		
(b.c.f.)	5 wells	10 wells	15 wells
0-500	0.4914	0.1194	0.0168
500-1000	0.3325	0.3282	0.1418
1000-1500	0.1312	0.2918	0.2668
1500-2000	0.0359	0.1618	0.2586
2000-2500	7.44E-03	0.0672	0.1710
2500-3000	1.33E-03	0.0227	0.0875
3000-3500	2.14E-04	6.66E-03	0.0371
3500-4000	2.87E-05	1.78E-03	0.0137
4000-4500	1.44E-06	4.40E-04	4.61E-03
4500-5000	2.37E-09	9.74E-05	1.44E-03
5000-5500	0	1.73E-05	4.24E-04
5500-6000	0	2.12E-06	1.17E-04
6000-6500	0	1.36E-07	2.92E-05
6500-7000	0	1.93E-09	6.02E-06
7000-7500	0	2.22E-12	8.99E-07
7500-8000	0	0	7.48E-08
8000-8500	0	0	1.94E-09
8500-9000	0	0	6.70E-12
9000-9500	0	0	2.62E-15
9500-10000	0	0	4.62E-30

Figure 5.2 Distributions of the total discovery volume in Nova Scotia Shelf as exploration progresses for 5, 10, and 15 exploratory wells.



5.3.4.3 Case 2: Results of 5, 10, and 15 Exploratory Wells from a Single Frequency- size Distribution

Table 5.3 shows a chosen frequency-size distribution generated from Weibull and triangular distributions for the purpose of comparison. Table 5.4 shows the probabilities that the total discovery volume fall into each interval using single frequency-size distribution for 5, 10, and 15 exploratory wells.

Table 5.3 Frequency-size distribution selected for comparison.

Size class	Number of fields	Average Volume (b.c.f.)	Average Areal Extent (mile^2)
1	8811	0	3.1416
2	63	42.1291	15.1998
3	25	138.3247	23.6541
4	14	258.2775	29.8393
5	8	336.3034	32.9182
6	5	457.3918	36.9079
7	1	507.3233	38.3582
8	0	0	0
9	0	0	0
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	0

Table 5.4 Probabilities that the total discovery volume fall into each interval using single frequency-size distribution for 5, 10, and 15 exploratory wells.

Interval (b.c.f.)	Probabilities		
	5 wells	10 wells	15 wells
0-500	0.4613	0.0789	5.77E-03
500-1000	0.4136	0.3699	0.1143
1000-1500	0.1148	0.3580	0.3083
1500-2000	0.0102	0.1552	0.3211
2000-2500	5.51E-05	0.0344	0.1790
2500-3000	0	3.46E-03	0.0590
3000-3500	0	1.05E-04	0.0114
3500-4000	0	2.76E-07	1.16E-03
4000-4500	0	2.11E-14	4.94E-05
4500-5000	0	0	5.35E-07
5000-5500	0	0	3.87E-10
5500-6000	0	0	3.16E-17

Figure 5.3 Comparison of the distributions of the total discovery volume between Case 1 and Case 2 for 5 exploratory wells.

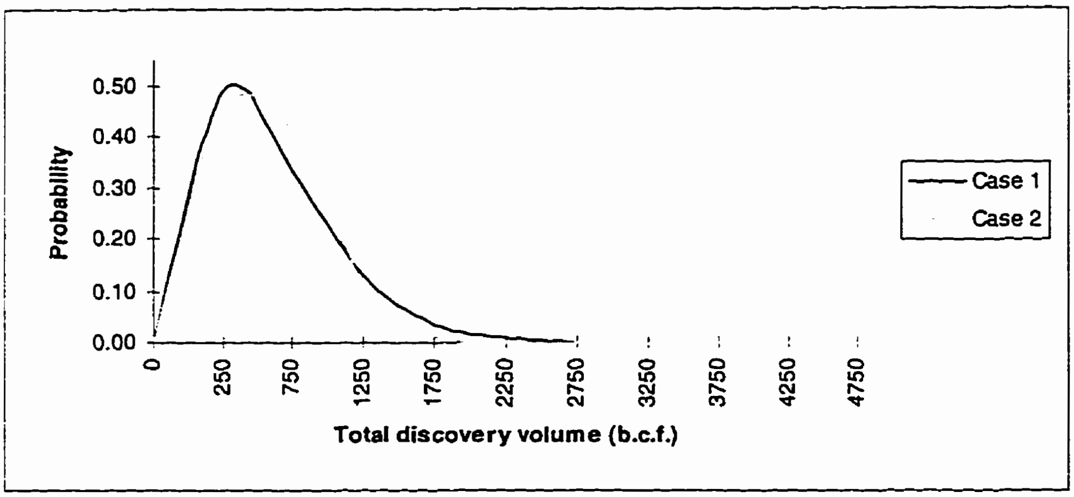


Figure 5.4 Comparison of the distributions of the total discovery volume between Case 1 and Case 2 for 10 exploratory wells.

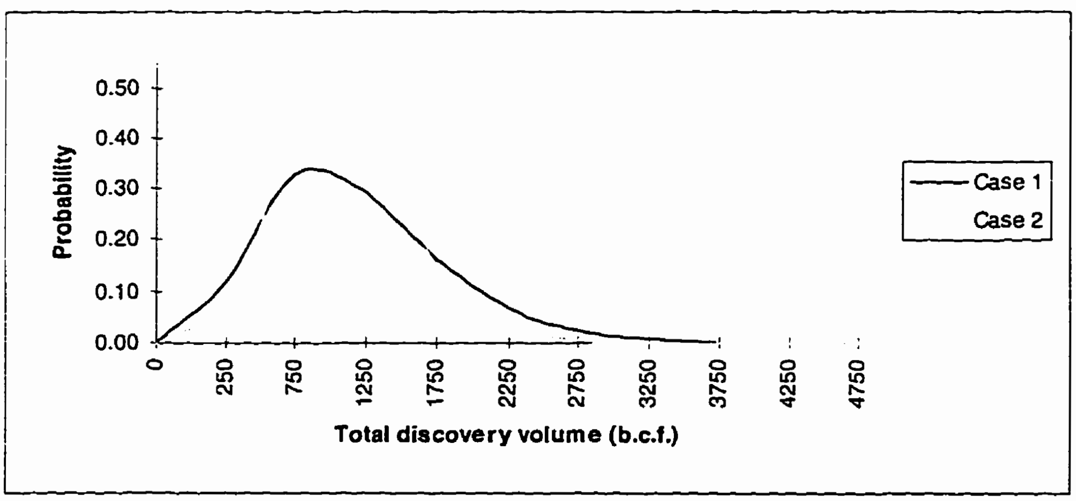
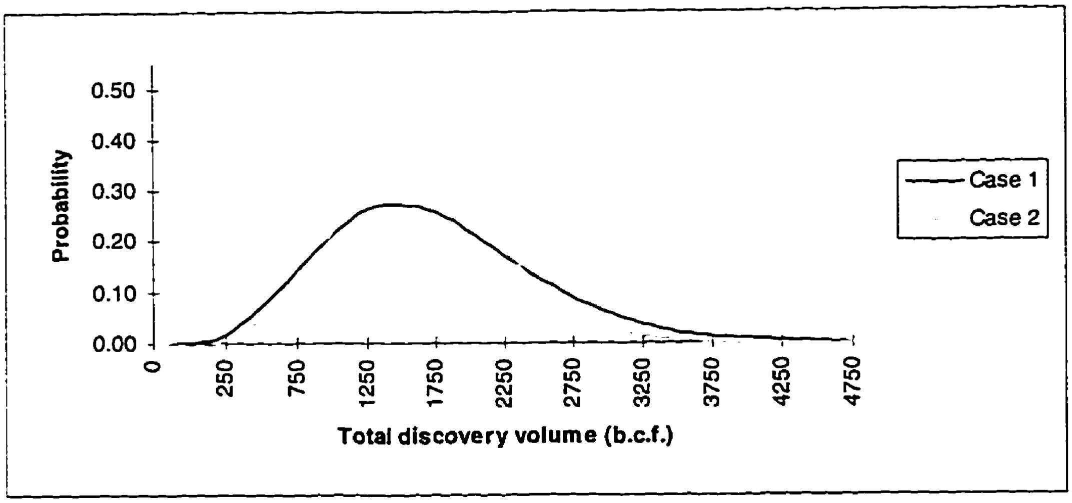


Figure 5.5 Comparison of the distributions of the total discovery volume between Case 1 and Case 2 for 15 exploratory wells.



Figures 5.3-5 show that there are differences between these two cases. As a single frequency-size distribution selected here presents only one possibility of a frequency-size distribution might occur in the Nova Scotia Shelf due to uncertainties in geological parameters, the distribution of total discovery volume resulting from incorporating the uncertainties by averaging all Beta distributions in Case 1 will be a better representation since it reflects the reality about uncertainties involved in exploration in the basin. Note that the differences between these two cases as shown in Figures 5.3 to 5.5 can become larger or smaller depending on which single frequency-size distribution is used. In case the uncertainties involved are not large, the differences between these two distributions are small. This is the case where a company has more information and gains experience in the basin. Therefore, in a well-established basin, a single frequency-size distribution might be sufficient to be used in the model . However, when uncertainties become very large in a

frontier basin where geological information is very limited, such as in the Nova Scotia Shelf, Newfoundland, or arctic regions, the distributions used to represent the number of fields and field size distribution of that basin could have wider ranges. As a result, the differences between the distributions of total discovery volume of Case 1 and Case 2 will become prominent, and there will be some benefit of including uncertainties in geological parameters. By using only a single frequency-size distribution without incorporating uncertainties, a company may misjudge by overestimating results of its exploration program and, therefore, make a wrong decision to bid for an exploration project and put a large amount of money to invest in the exploration activity. Consequently, it could face severe damage from the unsuccessful project. On the contrary, the company could underestimate results and turn down a profitable project.

5.3.4.4 Case 3: Results of 5, 10, and 15 Exploratory Wells When Uncertainties are Increased

The results of the probabilities in all intervals for 5, 10, and 15 exploratory wells are shown in Table 5.5. The comparisons between Case 1 and Case 3 are shown in Figure 5.6 to Figure 5.8.

Table 5.5 Probabilities that the total discovery volume fall into each interval for 5, 10, and 15 exploratory wells when uncertainty of field size is doubled.

Interval	Probabilities		
(b.c.f.)	5 wells	10 wells	15 wells
0-500	0.2766	0.0438	6.02E-03
500-1000	0.2116	0.1038	0.0316
1000-1500	0.1563	0.1308	0.0628
1500-2000	0.1123	0.1339	0.0874
2000-2500	0.0790	0.1233	0.1016
2500-3000	0.0546	0.1064	0.1059
3000-3500	0.0371	0.0875	0.1027
3500-4000	0.0248	0.0695	0.0942
4000-4500	0.0164	0.0536	0.0828
4500-5000	0.0108	0.0404	0.0701
5000-5500	7.18E-03	0.0300	0.0576
5500-6000	4.75E-03	0.0219	0.0462
6000-6500	3.14E-03	0.0159	0.0363
6500-7000	2.09E-03	0.0114	0.0281
7000-7500	1.37E-03	8.14E-03	0.0214
7500-8000	8.39E-04	5.77E-03	0.0162
8000-8500	4.75E-04	4.09E-03	0.0122
8500-9000	3.08E-04	2.89E-03	9.12E-03
9000-9500	1.97E-04	2.05E-03	6.83E-03
9500-10000	1.24E-04	1.46E-03	5.13E-03
10000-10500	7.35E-05	1.04E-03	3.87E-03
10500-11000	3.98E-05	7.41E-04	2.94E-03
11000-11500	2.23E-05	5.26E-04	2.25E-03
11500-20000	1.22E-05	3.68E-04	1.72E-03
20000-20500	6.34E-06	2.53E-04	1.32E-03

Table 5.5 Continued

20500-21000	2.91E-06	1.68E-04	1.00E-03
21000-21500	1.12E-06	1.08E-04	7.57E-04
21500-22000	3.14E-07	6.64E-05	5.63E-04
22000-22500	3.63E-08	3.94E-05	4.12E-04
22500-23000	9.66E-11	2.23E-05	2.95E-04
23000-23500	0	1.19E-05	2.08E-04
23500-24000	0	5.97E-06	1.43E-04
24000-24500	0	2.77E-06	9.71E-05
24500-25000	0	1.19E-06	6.42E-05
25000-25500	0	4.30E-07	4.09E-05
25500-26000	0	1.11E-07	2.49E-05
26000-26500	0	1.57E-08	1.44E-05
26500-27000	0	6.97E-10	7.60E-06
27000-27500	0	4.81E-12	3.54E-06
27500-28000	0	0	1.36E-06

Figure 5.6 Comparison of the distributions of the total discovery volume between Case 1 and Case 3 for 5 exploratory wells.

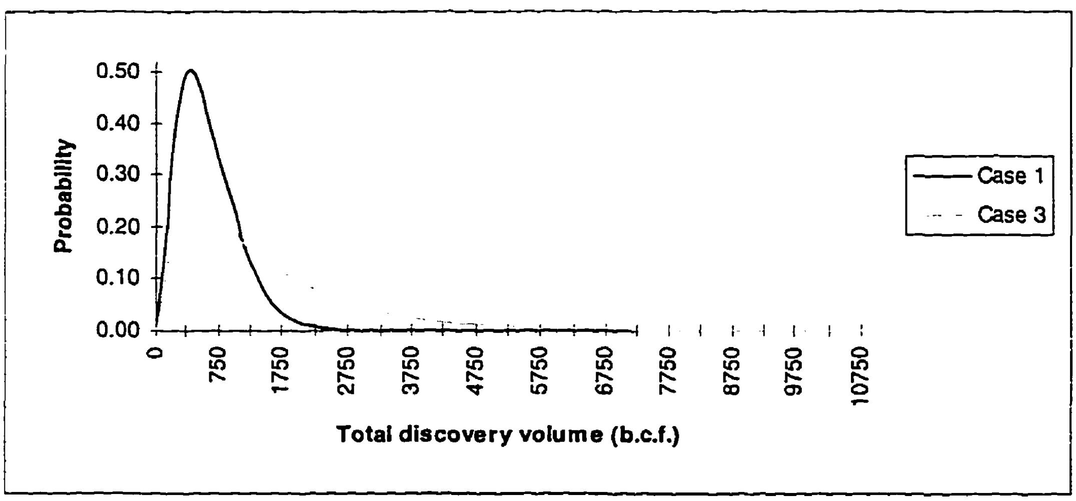


Figure 5.7 Comparison of the distributions of the total discovery volume between Case 1 and Case 3 for 10 exploratory wells.

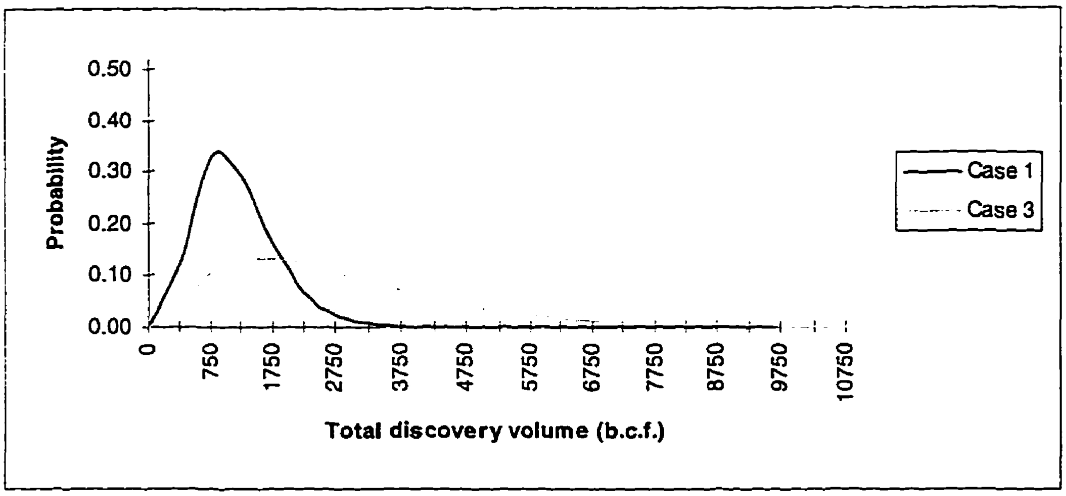
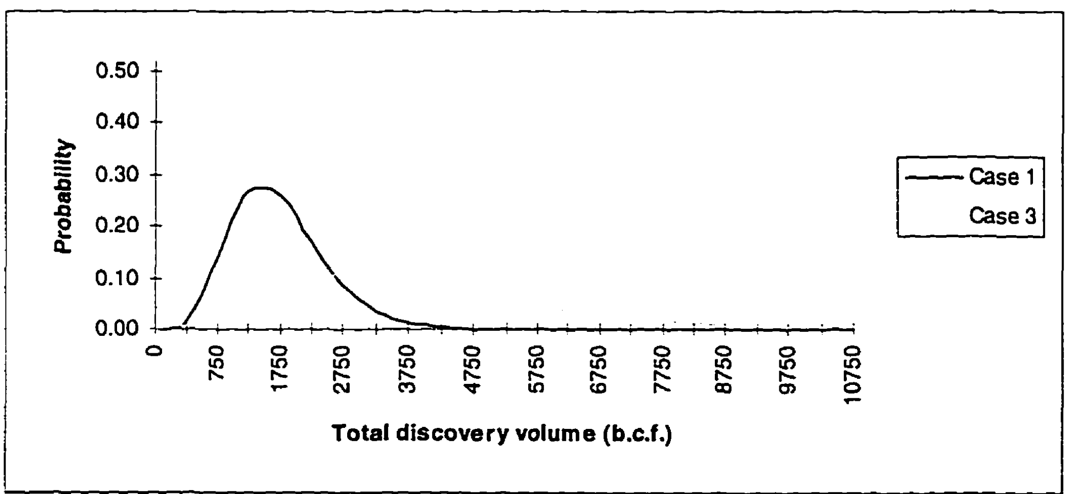


Figure 5.8 Comparison of the distributions of the total discovery volume between Case 1 and Case 3 for 15 exploratory wells.



From Figures 5.6-5.8, we can see that when the uncertainties are doubled, the distributions of the total discovery volume for 5, 10 and 15 exploratory wells are more spread out over wider ranges as a result of more uncertain information. Consequently, the company will have higher risk associated with its exploration project. In addition, the expected total discovery volumes for 5, 10, and 15 exploratory wells in Case 1 obtained from Table 5.2 are 620.1975 b.c.f., 1168.1680 b.c.f., and 1723.5190 b.c.f., respectively, whereas the expected total discovery volumes in Case 3 calculated from Table 5.5, for 5, 10, and 15 exploratory wells are 1408.717 b.c.f., 2715.766 b.c.f., and 3891.226 b.c.f., respectively. Notice that there is a substantial increase in the expected values when uncertainties increase, even though the means of the two underlying distributions are held constant. The changes in the expected values when uncertainties change point out a major advantage of including uncertainties of geological parameters.

To further investigate that whether the uncertainty of the number of fields or of the field size distribution has more effect on the distribution of total discovery volume, we double the uncertainty of each underlying distribution one at a time. First, we double the standard deviation of the triangular distribution and keep its mean and the Weibull distribution unchanged. The results show no change in the shape of the distributions of total discovery volume and their expected values for all numbers of exploratory wells. The results are similar to the results in Case 1, Table 5.2 and Figure 5.2.

Second, we increase the uncertainty in the field size by doubling the standard deviation of the Weibull distribution and keep its mean and the triangular distribution unchanged. The results are similar to the results shown in Table 5.5 and Figures 5.6 to 5.8. These results point out that an increase in the uncertainty in field size alone causes a substantial increase in expected total discovery volume. To see the difference between the expected values of Case 1 and Case 3 clearly, we run the model with a wide range of numbers of exploratory wells. Figures 5.9 and 5.10 show the trends of the expected value and the standard deviation of the distribution of the total discovery volume for Case 1 and Case 3 from 10 to 100 exploratory wells, respectively.

Figure 5.9 Trends of the expected discovery volume of Case 1 and Case 3 for 10 to 100 exploratory wells.

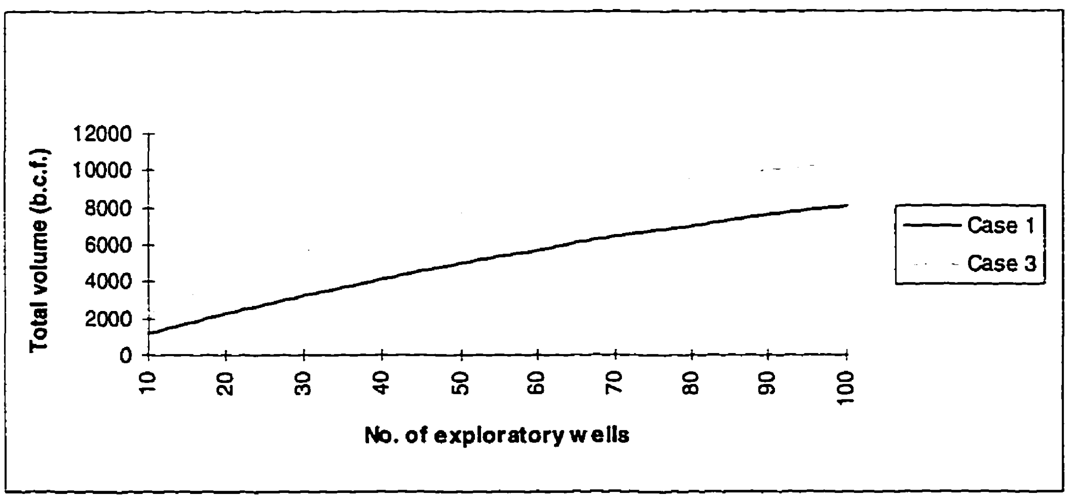
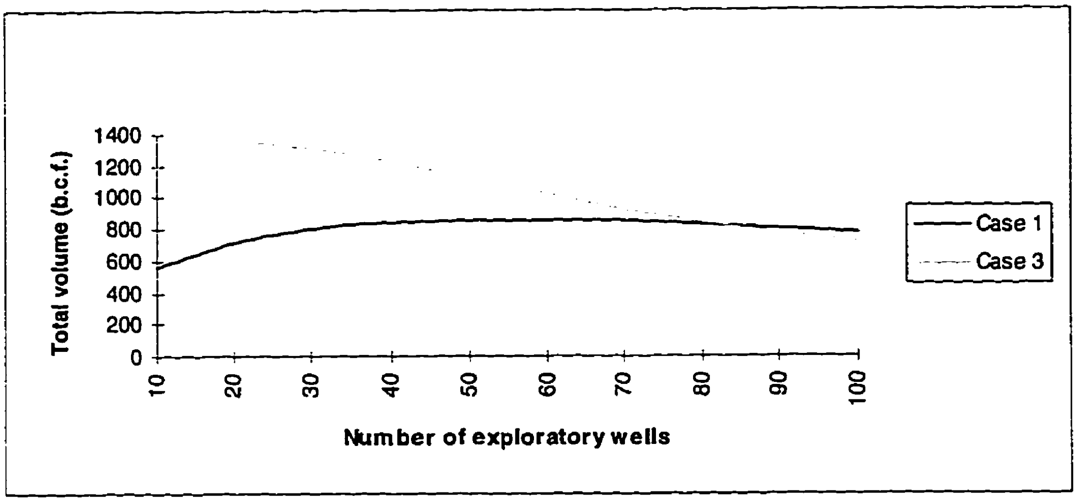


Figure 5.10 Trends of the standard deviation of the distributions of total discovery volume for 10 to 100 exploratory wells.



An increase in the uncertainty in field size which causes a substantial increase in the expected total discovery volume can be explained by the nature of the field size distribution and the probabilistic model of hydrocarbon discovery process. Since the field size distribution is highly asymmetric (highly right-skewed), as we double its standard deviation to increase the uncertainty and keep its mean unchanged, its density will spread out toward the right tail. Hence, the probability associated with very large field sizes on the right tail is increased. By using the probabilistic model of discovery process (as described in Section 3.2), the probability of discovering a field of one particular size class is proportional to the number of fields of that size class and its area raised to the power of discovery efficiency. As a result, there will be a higher probability that large fields will be discovered at early exploration stages than at later stages. This statement was supported by Barouch and Kaufman (1978) and Eremenko et al. (1979) who reported that the largest field in a region is discovered between the fifth and twentieth discovery. Therefore, the expected total discovery volume in Case 3 with double uncertainty is higher than the expected total discovery volume in Case 1 at the early stage of exploration. Figure 5.9 shows that the expected total discovery volume in Case 3 is approximately double the value of the expected total discovery volume of Case 1 at the beginning. Similarly, Figure 5.10 shows that the standard deviation of the distribution of total discovery volume in Case 3 is approximately twice the standard deviation in Case 1, at the beginning, due to higher uncertainty. As exploration progresses, large fields are becoming depleted and small fields remain to be found. The percent differences of the expected total discovery volumes and of the standard deviations between these two cases are reduced as shown from the trends in Figures 5.9 and 5.10. (See also, Gerchak et al., 1997, who provides the mathematical verification that the expected size of the first field discovered is increasing in the variability of field sizes.)

The above results draw attention for further investigation. If we use the expected utility of monetary value as a criterion for exploration investment decisions, the distributions of net present value, generated from the distributions of total discovery volume through economic analysis, could favor high uncertainty, because the expected net present values in case of high uncertainty could be higher than the expected net present values in case of low uncertainty. Consequently, the company might decide to invest in the exploration project with higher uncertainty. Further investigation on this issue will be discussed in the next section.

5.4 Economics of Exploration

Major parameters for determining the economics of exploration for offshore Nova Scotia are discussed as follows.

5.4.1 Nova Scotia Gas Price

Nova Scotia is strategically located near the major gas markets of northeastern North America which includes the Maritime Provinces and the northeastern United States. According to the National Energy Board (1992), the US Northeast market for Canadian gas has seen dramatic changes since deregulation in 1986 both in terms of its growth and in the emergence of new market sectors for natural gas, such as electricity co-generation. In 1985, the Northeast US was a market that held considerable potential for western Canadian gas but there were relatively few contracts to serve this market. By 1995, total Canadian natural gas exports to the Northeast US increased to 647 b.c.f., up 13.8% from 1994's level of 569 b.c.f. Currently, the Canadian share of the Northeast US natural gas market remains stable at around 20% due to export growth that kept pace with incremental increases in demand in the region; 80% of the market share belongs to natural gas from the Gulf of Mexico. Also, Canadian natural gas represents only 11 percent of total US supply, thus Canada is considered a price taker in a competitive North American natural gas market (Natural Resources Canada, 1997). The price for offshore Nova Scotia natural gas, once produced, will be determined largely by the Northeast US market since the market in the Maritime Provinces is unlikely to absorb all the offshore gas produced.

To determine the natural gas price for offshore Nova Scotia producers, we first consider the Sable Offshore Energy Project (SOEP) that is currently in its regulatory approval phase. According to the SOEP Overview (1997), the Sable Offshore Energy Project is currently planned for development in conjunction with a sales gas pipeline, the Maritimes and Northeast Pipeline Project, to be built from a gas plant at Country Harbor through Nova Scotia and New Brunswick and into the New England states by a consortium of Canadian and American companies. This pipeline will tie into the North America gas grid. With this information, the price for Nova Scotia offshore natural gas producers, the net-back price, would, therefore, be determined by using the Northeast US natural gas price less the transportation toll that covers the distance from the distribution system in the vicinity of Country Harbor, onshore Nova Scotia, to the Northeast US cities in the New England region. From a discussion with the Natural Gas Division, Natural Resources Canada, the transportation toll is approximately C$1.35 per m.c.f. (1995 dollars). With this information, we select the average price delivered to local distribution companies (LDCs), called the "citygate" price, of natural gas in the New England region for use as the Northeast US price in order to calculate the expected net-back price for offshore Nova Scotia gas producers. The New England region covers 6 States: Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, and Vermont. This average citygate price represents the total cost paid by local distribution companies for gas received at a point where the gas is transferred from a pipeline company or transmission system. This price is intended to reflect the gas commodity costs and the expense of transporting, storing, and managing gas supplies for delivery to the citygate. It is generally used to represent the wholesale cost of gas in scattered end-use customer markets.

Following from above, we consider the historical trend of the New England citygate price and the outlook for this price in the long-term when selecting the appropriate figures for use in the long-term price pattern for offshore gas producers. This price pattern will influence their decisions to invest in exploration activities. To do that we need to observe the relationship between the wellhead and citygate prices of natural gas in the United States during the past decade, and, to look at the forecast for these prices in the future since the New England citygate price follows these trends. Note that we recognize that there are several institutes that publish price information, including forecasts, such as Energy Information Administration (EIA), Gas Research Institute (GRI), Data Resources, Inc./McGraw Hill, and Wharton Econometric Forecasting Associates (WEFA). Diversity among published forecasts of natural gas prices, production, consumption, and imports indicates the uncertainty of future market trends and the conditions imposed in the forecasts. Most forecasters, however, predict results within close ranges. We decide to use information from the Energy Information Administration (EIA) in our studies as it can be easily accessed with great detailed discussion including the effects of technology progress and economic growth. Brief observations of average wellhead and citygate prices are outlined as follows.

According to the EIA Issues and Trends (1995), the gas industry has moved from a highly regulated environment, dominated by long-term contracts, to one where markets respond quickly to short-term shifts in supply and demand since its restructuring began in the mid-1980's. The national inflation-adjusted average natural gas prices of all segments of the industry have been falling, although volatile in the short-term due to supply and demand conditions, while volumes of gas delivered have increased over this period. Between 1985 and 1994, the real average wellhead price dropped by US$1.52 per m.c.f. (45%) and the average citygate price dropped by US$1.93 per m.c.f. (39%). This has resulted in falling average end-user prices, to varying degrees, in the different market sectors comprising residential, commercial, industrial, transportation, and electric utility. Regional average prices, however, show significant variation in the United States due to the differences in transportation costs and the differences in local distribution company procurement and management policies. Among all regions, the New England citygate price is the highest (about one-third higher than the national average), reflecting the farther distance of this market from natural gas sources (currently about 80% of the natural gas delivered to the Northeast market comes from the Gulf of Mexico and another 20% comes from western Canada). In this region, the average citygate price has increased from US$3.42 per m.c.f. in 1991 to US$3.82 per m.c.f. in 1995, corresponding to the upward movement of national average citygate prices.

According to the Annual Energy Outlook, AEO, (1997), EIA recognizes that there is considerable uncertainty about the future price and supply of natural gas which is heavily influenced by the rate of technological advances in the industry and by the uncertainty of the gas resources as they state that "Within natural gas supply markets, technological progress has historically expanded the economically recoverable resource base and reduced effective exploration and development costs". Technological progress was assumed to slow the decline in finding rates-reserves discovered per new well and even reverses declining finding rates in some regions. Consequently, natural gas production is increased, with less drilling activity and at lower cost, particularly in offshore regions, where technological progress has a greater impact on the development of relatively immature fields. Therefore, in their forecasts, EIA reflects key assumptions about the progress of oil and gas supply technologies, such as annual improvement rates for onshore and offshore drilling; lease equipment, and operating costs; inferred reserves and undiscovered resources; and, finding and success rates. The AEO 1997 uses a single set of assumptions about these rates, derived from an analysis of historical trends which are generally assumed to continue throughout the period and projects the reference average wellhead price of natural gas to be increased from US$1.61 per m.c.f. in 1995 to US$1.82 per m.c.f., US$2.01 per m.c.f., and US$2.13 per m.c.f. (1995 dollars) with forecasted average citygate prices of US$2.78 per m.c.f., US$2.78 per m.c.f., US$2.99 per m.c.f., and US$3.11 per m.c.f. corresponding to the wellhead price in the years 1995, 2000, 2010, and 2015, respectively.

In order to test and quantify the sensitivity of the AEO 1997 projections to changes in assumptions about future technological progress, assumptions of more rapid and slower technological progress were used. Whereas the reference case assumes that the most likely estimate of this historical influence will continue throughout the forecast period, the rapid technology cases assume a higher estimate, and the slow technology case assumes a lower estimate. In addition, market-driven demand-side adjustments in the gas supply sector that cause consumption and production to deviate from reference case levels, the demand feedback, are applied to the forecast models. According to AEO 1997, the impact of changes in technological progress and production assumptions on projected natural gas prices will not be seen clearly in the next few years. However, shortly after the year 2000, natural gas prices begin to diverge noticeably from their reference case values. In rapid technological progress, the lower prices made possible by more rapid technological progress tend to stimulate additional natural gas consumption. As a result, higher levels of domestic production are projected, imposing upward pressure on natural gas prices, which partially offset the price reductions that result from improved technology. Consequently, the natural gas wellhead price is expected to be 27 percent lower than the reference price in 2015 (at US$1.55 per m.c.f.). In the slow technology case, the opposite effect occurs and causes the expected wellhead price in 2015 to be 30 percent higher than the reference price (at US$2.77 per m.c.f.). When looking at the average citygate price, we expect the same effects will happen, as it follows the trend of the price at the wellhead (the difference between wellhead price and the citygate price is the transportation cost). For the New England region, since there is only slight pipeline capacity expansion between 1995 and 2015, we do not anticipate that this expansion and the changing policy regarding the pipeline system to affect the pipeline costs. As a result, we assume the same influence on the average wellhead and citygate prices to apply to the New England citygate price. In order to determine the forecast of the New England citygate price, we average the differences between the average citygate price and the New England citygate price for the years between 1991 and 1995. The result shows the average difference to be US$0.76 per m.c.f. We then add the forecasts of the average citygate prices in the years 2000, 2010, and 2015 to obtain the forecast for the New England citygate prices. Table 5.6 shows the average annual wellhead, citygate, and New England citygate prices from 1991 to 1995, and the forecasts of these prices in the years 2000, 2010, and 2015, respectively.

Table 5.6 Average wellhead, average citygate, and New England citygate prices from 1991 to 1995, and the forecast prices from 1995 to 2015.

Year	Average Wellhead Price(US$)/m.c.f.	Average Citygate Price(US$)/m.c.f.	New England Price(US$)/m.c.f.
1991	1.64	2.90	3.42
1992	1.74	3.01	3.59
1993	2.04	3.21	3.95
1994	1.85	3.07	4.00
1995	1.61	2.78	3.82
2000	1.82	2.78	3.54
2010	2.01	2.99	3.75
2015	2.13	3.11	3.87

From the table, we calculate the average New England citygate price from 1995 to 2015 to be US$3.75 per m.c.f. in order to use it as the perceived citygate price for offshore producers. For the sensitivity analysis, we assume the same effect of technological progress as occurred on wellhead prices and average citygate prices to occur on the New England citygate prices in 2010 and 2015. In the rapid technological progress case, we assume the New England citygate prices in 2010 and 2015 to be 27 percent lower than the reference price of US$2.74 per m.c.f. in 2010 and US$2.83 per m.c.f. in 2015. Whereas in slow technological progress, we assume the price in 2010 and 2015 to be 30% higher than the reference prices in the same two years (US$4.88 per m.c.f. in 2010 and US$5.03 per m.c.f. in 2015). As a result, we obtain the average forecasts of the New England citygate prices from 1995 to 2015 in the case of high technological progress to be US$3.23 per m.c.f. and in the case of slow technological progress to be US$4.32 per m.c.f..

From the above discussion, we attain the low, reference, and high New England citygate prices to be US$3.23, US$3.75, and US$4.32, respectively. By using the 1995 exchange rate from Statistics Canada (US$.7286 per C$1), the average low, reference, and high citygate prices are converted to C$4.43, C$5.15, and C$5.93, respectively. Subtracting these prices by the transportation toll of C$1.35 per m.c.f. as mentioned earlier, we obtain the perceived low, reference, and high net-back prices for offshore Nova Scotia producers to be C$3.08, C$3.80, and C$4.58 per m.c.f., respectively. These prices should cover the worst, the reference, and the best expectation of prices perceived in long-run by offshore Nova Scotia producers.

5.4.2 Discount Rate

As recommended by the Nova Scotia Offshore Petroleum Board, the rate of return for the costs of money invested in the exploration project should be calculated at the same rate as long-term Government of Canada bonds plus 10 percentage points for unrecovered costs. In our evaluation, however, we intend to determine the real value of future cash flows. Therefore, we will not use the rate of long-term (30 years) Government of Canada Bonds as it is a nominal rate which incorporates the influence of inflation expectation. Also, we will not take into account the 10 percentage points for unrecovered costs as this risk will be dealt with when we consider the preference function of a company for its exploration project. Instead, we will consider the real rate of return from the Real Return Bonds (RRBs) as this rate is the real interest rate that is determined by financial market participants and it excludes the influence of inflation according to the Bank of Canada Review (1996). Note that if the quoted real yield on RRBs can be assumed to be equal to the expected real return incorporated in a conventional bond's yield, inflation expectations can be measured as the difference between the (nominal) yield on a conventional bond and the (real) yield on an RRB with a similar term to maturity. Currently, there are two Real Return Bond maturities: one due on December 1st, 2021 and the other due on December 1st, 2026. Both have a real rate of return of 4.25%. As a result, a real rate of return of

4.25% is selected as a reference discount rate for our studies. Sensitivity analysis will be performed by varying this reference discount rate by ±20%.

5.4.3 Cost Models

The cost models along the line of Power's (1990) models are examined for the purpose of implementing our methodology for offshore Nova Scotia. All costs are converted to 1995 dollars. Brief descriptions are given below.

5.4.3.1 Exploration Costs

Power's (1990) data for seismic surveys came from the Canadian Oil and Gas Lands Administration (COGLA). The average number of kilometers surveyed per well drilled was calculated from historical data. These data were obtained by using the consensus opinion of expert judgments from both COGLA and the Nova Scotia Department of Mines and Energy. It was estimated that the geological and geophysical costs of exploration were C$ 2 million per well (1984 dollars). Exploration drilling costs were grouped into three dependent categories as mentioned in Subsection 4.3.1.2.1. Those items which were contracted for on a day-rate basis were classified as time dependent cost factors; those items which were purchased on a unit cost basis were treated as either depth dependent cost factors or average per well cost factors. Knowledge of the drilling process was used to sort the items into the appropriate category. In each case, the unit cost of the item was calculated using either the average day-rate or the per meter or per well average cost implied by the cost which the

Husky/Bow Valley (HBV) company reported to COGLA. The per unit costs were then multiplied by the calculated average depth and time factors for dryholes, discovery wells, and delineation wells. Upon completion of an exploration drilling, it was estimated that another 15 days of testing were performed before a decision to abandon the well was taken. Every discovery well requires additional testing and, thus, extra costs were added. The total dryholes costs per well were estimated to be C$31.767 million and the discovery drilling costs per well were estimated to be C$45.821 million (1984 dollars). From the relationship given in Equation (4.3.1), the exploration costs in millions of dollars for field size class k is as follows:

$$
\begin{aligned}
[\text{Exploration costs}]_k &= 2.0 + 31.767 + (45.821 - 31.767)(DO) \\
&= 33.767 + (14.054)(DO)
\end{aligned}
$$

where DO equals 0 for dryholes and sub-economic size classes and DO equals 1 for an economic discovery.

5.4.3.2 Development Costs

(a) Delineation Costs

Data on the delineation wells per discovery on the Scotian Shelf are incomplete because some discoveries have not been fully delineated and a few have not yet begun to be delineated at all. With this short, incomplete history, the delineation pattern of the North Sea experience was used to guide the delineation process. In addition, the development cost studies from the Government of Nova Scotia, Department of Mines and

Energy, indicate that fields under 250 b.c.f. are unlikely to be developed. Accordingly, the assumptions adopted by Power (1990) are used. These assumptions are: no delineation drilling for field sizes less than 200 b.c.f., two delineation wells are required for fields in the range of 200 to 400 b.c.f., three delineation wells are required for fields in the range 400 to 1000 b.c.f., and four delineation wells are needed for fields in excess of 1000 b.c.f.. Note that these assumptions are consistent with the delineation drilling completed to date on the Scotian Shelf. To determine the relationship between delineation costs and field size, the frequency-size distribution estimated by Power (1990) was used. The number of delineation wells required by each field size class is multiplied by the delineation costs per well which was estimated to be C$36.214 million (1984 dollars). Then the volume in billion cubic feet (b.c.f.) of each field size class is plotted against the corresponding delineation costs. A linear regression model representing the relationship between field size class k and delineation costs in million dollars is determined as follows:

$$[\text{Delineation costs}]_k = (59.287668 + 0.058854 v_k)(DO)$$

$$(\text{adjusted } R^2 = 0.62, \text{ standard error} = 24.87)$$

where v_k is the field size in b.c.f. and DO equals 0 for dryholes and sub-economic discoveries and equals 1 otherwise.

(b) Development Drilling Costs

From the data set made available by the Nova Scotia Government, development drilling costs were initially hypothesized as being a function of field size, water depth, and pressure factor. Power (1990) had examined the relationships between drilling costs and these variables. The regression equations showed that there was no significant difference between the results obtained using aggregated data and the results obtained using separate normal and geo-pressured data sets. As a result, he decided to drop the pressure factor for his simulation model as it had no means to take into account the effect of this variable. He also found that water depth, though significant, was less important as an explanatory variable than field size. Therefore, it was suitable to calculate an average water depth of all discovery and delineation wells and assign this average value to all regression equations requiring a water depth value so as to calculate the average influence of water depth on costs. This intention was to minimize the impact of unavailable information on the final estimation of costs. From the above discussion, Power's regression equation of development drilling costs in million dollars is written as:

$$\text{Drilling Costs} = -\,4.612 + 0.051\ (\text{Depth in feet}) + 0.214547\ (\text{Field Size in b.c.f})$$

$$(\text{adjusted } R^2 = 0.995,\ \text{standard error} = 5.578,\ 0.013,\ 2.747)$$

By multiplying by the average depth of 181 feet, the final regression equation of drilling costs for field size class k using Equation (4.3.3) becomes

$$[\text{Development Drilling Costs}]_k \;=\; (4.619 + 0.214547 v_k)(DO)$$

where v_k is the field size in b.c.f. and DO equals 0 for dryholes and sub-economic discoveries and equals 1 otherwise.

(c) Facilities Costs

Similar to the development drilling costs, facilities costs were initially hypothesized as being a function of both field size and water depth. Field size will affect the number of slots in the production template and the volumetric handling capacity of the production system. As both variables increase, costs are expected to increase. In the same way, as water depth increases, more quantities of steel and concrete used in platform construction are needed and, therefore, costs will rise. By setting the water depth variable at the average level for all discovery and delineation wells, the regression equation for facilities costs in million dollars is reduced from

$$\text{Facilities Costs} = 14.766 + 0.261\ (\text{Depth in feet}) + 0.17584\ (\text{Field Size in b.c.f.})$$

$$(\text{adjusted } R^2 = 0.93, \text{ standard error} = 18.828, 0.045, 9.273)$$

to

$$\text{Facilities Costs} = 62.007 + 0.175841(\text{Field Size in b.c.f.})$$

Writing the above equation in the form of Equation (4.3.4) for a field size class k gives

$$[\text{Facilities Costs}]_k = (62.007 + 0.175841 v_k)(DO)$$

where v_k is the field size in b.c.f. and $DO = 0$ for dryholes and sub-economic discoveries and equals 1 otherwise.

(d) Inter-field Pipeline Costs

Power's (1990) inter-field pipeline model also assumed a common water depth and assumed that only gas would be carried by the line. The costs of inter-field pipeline construction were estimated from the data provided by the Nova Scotia Department of Mines and Energy. As mentioned in Subsection 4.3.1.2.2 (d), the critical dimensions of pipeline construction costs are the diameter of the pipe and the length of the line. Power's (1990) regression model is used as a basis for constructing pipeline costs as follows:

$$\text{Pipeline Cost} = 13071 + \{65.081\ (\text{Length in Miles})(\text{Diameter in inches})\}$$

$$(\text{Adjusted } R^2 = 0.996,\ \text{std. error} = 2918,\ 1.140)$$

The lengths of the inter-field lines were estimated using the average distance of known discoveries from the proposed Venture pipeline corridor around Sable Island. The diameter of the pipe used in pipeline construction will vary depending on the flow rate feeding the line and the length of the line. The flow rate is directly related to field size. The selected diameters used in the model were based on the inter-field pipeline summary data obtained through the Nova Scotia Department of Mines and Energy. The selected diameters ranged in size from 8 inches, for the smallest field size class, to 24 inches for the largest field size class. Since there is still no real data on diameters of the pipelines used in offshore Nova Scotia, it is arbitrarily assumed that the minimum diameter of 8 inches is required for field size class two, 12 inches for field size classes three and four, 16 inches for size classes five and six, 20 inches for size classes seven and eight, 22 inches for size classes nine and ten, and 24 inches for sizes classes eleven and larger. The selected discovery sizes assume an average pipeline length of 14.4 miles which is a distance equal to the average distance between all known discoveries and the proposed shoreline route.

By using the above data, the inter-field pipeline costs in million dollars were plotted against field sizes. The result is written in the form of Equation (4.3.5) for field size class k as

$$[\text{Pipeline Costs}]_k = (24.16318 + 0.008273 v_k)(DO)$$

$$(\text{Adjusted } R^2 = 0.78901, \text{standard error } = 2.34589)$$

where v_k is the field size in b.c.f. and DO equals 0 for dryholes and sub-economic discoveries and equals 1 otherwise. The costs of operating pipeline facilities were estimated by the Nova Scotia Department of Mines and Energy at 2.2 percent of incurred capital costs (Power, 1990). Therefore, the annual pipeline operation costs for field size class k are obtained using Equation (4.3.6) as

$$\begin{aligned}[\text{Annual Inter-field Pipeline Operation Costs}]_k &= (0.022)(24.16318 + 0.008273 v_k)(DO) \\ &= (0.53159 + 0.000182 v_k)(DO)\end{aligned}$$

As a result, the net present value of the annual pipeline operation costs is determined using Equation (4.3.7) for approximately T years of operation life assuming the pipeline operation starts 3 years after the exploration (based on the production schedule of the SOEP project) and using a real discount rate of 4.25% as follows.

$$[\text{NPV of Annual Pipeline Operation Costs}]_k = (0.53139 + 0.000182 v_k)\sum_{t=4}^{T+3} \frac{1}{(1+.0425)^t}(DO)$$

where v_k is the field size in b.c.f. and DO equals 0 for dryholes and sub-economic discoveries and equals 1 otherwise.

(e) Transportation Toll

The approach adopted by Power (1990) was based on the regulation of pipeline tolls and tariffs for determining the cost of service issued by the National Energy Board. The procedure comprises two steps. The first step is to determine what constitutes reasonable costs incurred in providing the transportation service to the customer. The second step involves the design of a toll that will create enough revenue to meet the pipeline company's cost service. It is assumed that there is only one class of customer; the offshore producer. The costs incurred by the pipeline are termed the cost of service which includes operating expenses, income taxes, depreciation, amortization, and the rate of return allowed on the pipeline's rate base (the sum of the net value, after depreciation, of the capital assets used in providing the service to customers). The value of the capital assets are the construction costs of the pipeline, the construction of the gas plant and terminals, and the costs of the overland lines necessary to connect the shoreline to the export lines. The capital assets also include the working capital necessary to run the day-to-day operations of the line for each single year. The fixed tolls are set so that sufficient revenue is generated from the line's likely throughput to cover the total cost of service and return on capital invested in the project. In constructing the scenario for the shoreline, an average capacity assumption was made based on the combined development reserves estimated from Mobil's discovered fields at Venture, South Venture, and Thebaud, and from Shell's discovered fields at Alma, Glenelg, and North Triumph. The line was intended to be used as a common facility pipeline to transport the gas to shore and it must be large enough to transport the peak production of the fields as they are brought on stream. Power (1990) assumed that in the thirty-third year of operation additional maintenance and repair work would be undertaken to upgrade pipeline facilities. The subjected costs of the work were assumed to equal 10 percent of the original installation cost and are intended only to extend pipeline life. It was assumed that the average lifetime capacity utilization of the line would not exceed 85 percent. He further simulated the toll rate model over several operation life spans from thirty-two to seventy-five years. After considering the major aspects of shoreline construction and a number of operational life spans of the pipeline, Power had decided to use seventy-five years of operation life as a basis for toll rate calculation as this number yields the lowest toll rate and succeeds in transporting the amount of gas viewed as the most likely to be economic. The result gave the estimated toll rate as C$0.365 (1984 dollars) per m.c.f. of gas transported. At this moment, no actual development has occurred, therefore, this value is used as an estimate toll rate for the purpose of our study as it is impossible to measure the accuracy of this selection. The modified annual transportation toll in million dollars for field size class k when considering the production decline follows Equation (4.3.8) as shown below.

$$[\text{Annual Transportation Toll}]_k = (0.365\ v(t)_k)(DO)$$

where $v(t)_k$ is the volume of field size class k produced in year t, and DO equals 0 for dryholes and sub-economic discoveries, and is equal to 1 otherwise.

5.4.3.3 Production Costs

The same approach as used in the development drilling and facilities costs was used to determined operation costs. The operating costs of any field depend largely on water depth, facilities costs, and field size. Based on costs against water depth, facilities costs, and field size data from the Nova Scotia Department of Mines and Energy development scenarios, the Power's (1990) regression of estimated operation costs produces the following result:

$$\text{Annual Operation Costs} = 6.912 - 0.013\,(\text{Depth in feet}) - 0.005545\,(\text{Field Size in b.c.f.}) + 0.118\,(\text{Facilities Costs})$$

(adjusted R^2 = 0.988, standard error = 0.700, 0.002, 1.290, and 0.007)

As with development drilling and facilities costs, water depth was factored into the calculation of operations costs by setting water depth at the average level of all discovery and delineation wells drilled on the shelf. When the average water depth of 181 feet and the facilities costs estimated are entered into the operation costs regression, the regression equation reduces to the form given below:

$$\text{Annual Operation Costs} = 11.876 + 0.015204\,(\text{Field Size in b.c.f.})$$

The above equation gives the relationship between annual operation costs in million dollars and field size in b.c.f. Writing this equation in term of annual operation costs for a field size class k as in Equation (4.3.9), we obtain the following equation.

$$[\text{Annual Operation Costs}]_k = (11.876 + 0.015204 v_k)(\text{DO})$$

When considering T years of operating life for each economic field size class k, the net present value of annual operation costs following Equation (4.3.10), with the real discount rate of 4.25 percent and with the time lag of 3 years between exploration and production, becomes

$$[\text{NPV Annual Operation Costs}]_k = (11.876 + 0.015204\ v_k)\sum_{t=4}^{T+3}\frac{1}{(1+0.0425)^t}(DO)$$

where costs is in millions of dollars, v_k is the field size in b.c.f., and $DO = 0$ for dryholes and sub-economic discoveries, and equals 1 otherwise.

Note that the statistical robustness of the regression cost models was investigated by Power (1990) by examining probability and auto-correlation plots of the model residuals. He found the residuals to be normally distributed and lacking in any significant auto-correlation. Consequently, he suggested that the North Sea costs data are highly aggregated, and no detailed data are available for the Nova Scotia Shelf. Therefore, the Scotian Shelf results can only be accepted on the basis of their statistical properties such as high adjusted R-squared values, significant parameter estimates, and random residuals". Finally, all costs mentioned above were determined in 1984 dollars and they cover the period between 1981 and 1983. This period is the time of high world-wide demand for drilling rigs and supply vessels following the peak of the world's petroleum prices. Power (1990) suggested that the costs quoted for exploration and development activities during this time should be viewed as unusually high and reflective more of the shortage of equipment than the real costs of obtaining and using such equipment. After the world's oil and gas prices collapsed in 1986, exploration and development activities dropped rapidly resulting in the supply over demand situation of this equipment which brought the dropped in equipment costs. In addition, the changes in drilling technology that have occurred in the late 1980's and early 1990's, such as an introduction of a new class of jack-up rig capable of working in the hostile environment, resulted in the more efficient drilling activities. This leads to the reduction in spending costs on exploration activities. Since we would like to perform the evaluation that represents current economic conditions, we converted these costs in 1984 dollars to 1995 dollars by considering the cost indices from Drilling Cost Analysis issued by Gas Research Institute (1989). The results from GRI report in 1989 also agree with Power's (1990) statement. The overall exploration, development, and production expenditures in the United States were at the highest values during 1981 and 1984 period, and declined rapidly to the lowest values in 1987 after the collapsed of the petroleum prices in 1986. From 1987 to 1995, these expenditures climbed up slowly with a steady rate. By considering the GRI 1989 baseline offshore drilling cost indices which consider 1987 as a reference year (1987 = 1), the overall average exploration, development, and production costs in 1995 were appraised to be approximately 80 percent of the spending costs with the same activities as in 1984.

5.4.4 Determining the Distribution of NPV

5.4.4.1 NPV of Total Annual Revenues

The NPV of total annual revenues in 1995 dollars for each J exploratory wells, assuming that the production starts 3 years after exploration, are determined by using Equation (4.3.18) as

$$\text{NPV of Total Annual Revenues} = (\text{Price})\sum_{t=4}^{T+3}\frac{V(t)}{(1+0.0425)^t}$$

where T is the average operation life of all fields and $V(t)$ is the total volume produced from all discovery fields at year t.

5.4.4.2 NPV of Total Costs

The initial costs for each field size class k resulting from J exploratory wells each year are calculated by summarizing all the exploration, delineation, development drilling, facilities, and inter-field pipeline costs from the last subsection as follows.

$$\begin{aligned}[\text{Initial Costs}]_k = {} & \{33.767 + (14.054)(DO)\} + (59.287668 + 0.058854v_k)(DO) \\ & + (4.619 + 0.214547\,v_k)(DO) + (62.007 + 0.175841v_k)(DO) \\ & + (24.16318 + 0.008273\,v_k)(DO) \\ = {} & 33.767 + (164.130848 + 0.457515\,v_k)(DO)\end{aligned}$$

The total initial costs for J exploratory wells are obtained by multiplying n_k by the above equation summing over k, as shown in Equations (4.3.21) to (4.3.23), and considering $DO = 0$ for dryholes and sub-economic field size classes (k = 1, 2, ..., d) as follows.

$$\text{Total Initial Costs} = 33.767 \sum_{k=1}^{K} n_k + 164.130848 \sum_{k=d+1}^{K} n_k + 0.457515 \sum_{k=d+1}^{K} n_k \, v_k$$

$$= 33.767\, J + 164.130848 \, (J - \sum_{k=1}^{d} n_k) + 0.457515 \, V$$

The total annual costs for each field size class k are obtained by summarizing the annual pipeline operation costs, annual operation costs, and annual transportation toll as explained in Equation (4.3.24) as follows.

$$[\text{Total Annual Costs}]_k = (0.53159 + 0.000182 v_k)(DO) + (11.876 + 0.015204 v_k)(\mathrm{DO}) + (0.365\, v(t)_k)(DO)$$

$$= \{12.40759 + 0.015386\, v_k + 0.365\, v(t)_k\}(DO)$$

For J exploratory wells, $DO = 0$ for k = 1, 2, ..., d, the total annual costs become

$$\text{Total Annual Costs} = \sum_{k=1}^{K} n_k \{12.40759 + 0.015386\, v_k + 0.365\, v(t)_k\}(DO)$$

$$= 12.40759 \sum_{k=d+1}^{K} n_k + 0.015386 \sum_{k=d+1}^{K} n_k v_k + 0.365 \sum_{k=d+1}^{K} n_k v(t)_k$$

$$= 12.40759\, (J - \sum_{k=1}^{d} n_k) + 0.015386\, V + 0.365 V(t)$$

The NPV of total annual costs for average economic operation life of all fields, T, are obtained from Equation (4.3.27) as:

$$\text{NPV of Total Annual Costs} = \sum_{t=4}^{T+3} \frac{1}{(1+0.0425)^t} \{12.40759\,(J - \sum_{k=1}^{d} n_k) + 0.015386\,V + 0.365V(t)\}$$

From the above equations, the NPV of total costs are determined using Equation (4.3.28) as:

$$\begin{aligned}\text{NPV of Total Costs} &= \text{Total Initial costs} + \text{NPV of Total Annual Costs}\\ &= 33.767\,J + 164.130848\,(J - \sum_{k=1}^{d} n_k) + 0.457515\,V\\ &\quad + \sum_{t=4}^{T+3} \frac{1}{(1+0.0425)^t} \{12.40759\,(J - \sum_{k=1}^{d} n_k)\\ &\quad + 0.015386\,V + 0.365\,V(t)\}\end{aligned}$$

5.4.4.3 Generating the Distribution of NPV

To determine the economic field size classes and the number of sub-economic field size classes, d, we compare the total revenues of each field size class with its total development and total annual costs as mentioned in Equations (4.3.29) and (4.3.32). the total revenues for field size class k is given by

$$[\text{Total revenues}]_k = \sum_{t=t_0+1}^{T_k+t_0} [\text{Annual Revenue}]_k = \sum_{t=t_0+1}^{T_k+t_0} \{(3.80)v(t)_k\}$$

$$[\text{Total Development \& Annual Costs}]_k = 164.130848 + 0.457515v_k$$

$$+\sum_{t=4}^{T_k+3} \frac{1}{(1+0.0425)^t} \{12.40759$$

$$+ 0.015386\, v_k + 0.365\, v(t)_k\}$$

To determine the average economic operation life of all fields, we compare the total annual revenue with the total annual costs, Equations (4.3.17) and (4.3.26), as follows:

$$\text{Total Annual Profits} = \{(\text{Price})V(t)\} - \{12.40759\,(J - \sum_{k=1}^{d} n_k) + 0.015386\, V + 0.365V(t)\}$$

where $V(t) = V_o e^{-\delta\, t}$ and $V_0 = V\left(\frac{e^{\delta} - 1}{e^{\delta}}\right)$ as explained in Equation (4.3.13). If the total annual costs are greater than annual revenues, then the operations are shut down. Notice that the above equation implies that the production from all discovery fields will be shut down simultaneously if the total annual costs are greater than the total annual revenues from all fields. This contradicts to the real situations where each field is operated separately. Each field has its own different volume size, geological and geophysical properties, and, thus, has different production decline rate. However, the purpose of our methodology is to perform economic evaluations of the overall exploration project, therefore, we assume the average operation life of all fields to replace each individual operation life. This average value compensates various operation lives of all fields. More details of the evaluations can be done for each particular field at later stages. Finally, we multiply the costs by 0.80 to convert the total cost equations in 1984 dollars into 1995 dollars.

By using Manly's Approximation Method to determine the mean number of discoveries for J_x exploratory wells (J_x = 5, 10, 15), we are able to calculate the probability of success of the corresponding binomial distribution which is the approximate distribution of the number of discoveries. After obtaining the distributions of the number of discoveries, $(J - \sum_{k=1}^{d} n_k)$, and the corresponding distributions of total discovery volume, V, for 5, 10, and 15 exploratory wells, the conditional sampling process is performed to determine the distribution of number of discoveries and the distribution of total discovery volume for 5 exploratory wells in the first year, the conditional distribution of number of discoveries and the conditional distribution of total discovery volume for another 5 exploratory wells in the second year, and the conditional distribution of number of discoveries and the conditional distribution of total discovery volume another 5 exploratory wells in the third year. This conditional sampling process is to replicate the sampling without replacement of the real situations in hydrocarbon discovery process. Consequently, the distributions of NPV of total annual revenues and NPV of total costs for each 5 exploratory wells are determined. Finally, the distribution of NPV based on an exploration agreement of 3 years with the total of 15 exploratory wells (5 wells drilled per year) is achieved by using Equation (4.3.11) as follows.

$$\text{NPV} = \sum_{i=1}^{3} \frac{(\text{NPV of Total Annual Revenues - NPV of Total Costs})_i}{(1+0.0425)^i}$$

The sampling procedure follows Steps 1 to 12 in Subsection 4.3.2.1.5 for J = 5 exploratory wells per year, I = 3 years, and n = 500 frequency-size distributions.

In order to implement the procedure, the main program has been written by using Microsoft FORTRAN (Microsoft, 1987). This program combines the two earlier subprograms, as explained in Section 4.2, which incorporate geological uncertainties into frequency-size distribution and determine the distributions of total volume. In addition, it contains subprograms (Manly1 and Manly2) calling Manly's approximation to determine the probabilities of successful discoveries of binomial distributions for 5, 10, and 15 exploratory wells and the parameters for corresponding Beta distributions for 5, 10, and 15 discovery wells as explained in Steps 1 and 2. Note that we considered the "discovery" wells for Beta distributions as these are the estimate distributions of total discovery volume corresponding to the number of discoveries. Later, this program performs the calculations of the distribution of NPV from Steps 3 to 12. The flowcharts of these programs can be seen in Appendix E.

Note that the binomial random numbers in the sampling process are generated by the RNBNML function in the Mathematical Function Library for Microsoft FORTRAN, MAF3.LIB, (United Laboratories, 1989). The Beta random numbers are generated by the BETARN subprogram which uses Cheng's (1978) BB algorithm. This subprogram also uses the G05GAF and G05CBF functions supported by the NAG FORTRAN Libraries, NAGG and NAGX, from the Numerical Algorithms Group (1983) to generate uniform random numbers. Since the random numbers are generated by functions in commercial FORTRAN libraries, the independence among random variables can be warranted. In addition, the distributions of total number of discoveries and the distributions of total discovery volume for 5, 10, and 15 exploratory wells have been compared to the theoretical binomial and Beta distributions by using BestFit software from Palisade (1995). The results of comparisons using histograms and formal statistical tests, such as Chi-square and K-S tests, show that the results fit the theoretical distributions. This supports the validation of our program. Notice that only the distribution of number of discoveries in the 1st year is binomial. The conditional distributions of number of discoveries in the 2nd and 3rd years are no longer binomial as they are the by-products of the conditional sampling process. The distributions of total discovery volume in each of three years are the by-products of the sampling process from the 15 Beta distributions which represent the approximate distributions of total discovery volume from 1 to 15 discovery wells.

5.4.5 Distribution of the Number of discoveries

Results from the comparisons between simulation distribution and binomial distribution for the frequency-size distribution described in Table 5.3, as an example, are given in Tables 5.7 to 5.9 and Figures 5.11 to 5.13. Note that n_1, n_2, n_3, ..., n_K represent the number of dryholes, the number of discoveries in field size class 2, the number of discoveries in size class 3, ..., the number of discoveries in size class K, respectively, based on J_x exploratory wells ($J_x = J$, $2J$, $3J = 5$, 10, and 15 exploratory wells). Thus, (J_x-n_1) represents the number of discoveries when all size classes are economic, (J_x-n_1-n_2) represents the number of discoveries when field size class 2 is sub-economic, and (J_x-n_1-n_2-n_3) represents the number of discoveries when field size class 2 and 3 are sub-economic.

Table 5.7 Probabilities of success of binomial distributions for 5, 10 and 15 exploratory wells.

Exploratory Wells (J_x)	Probability of Success		
	(J_x-n_1)	(J_x-n_1-n_2)	(J_x-n_1-n_2-n_3)
5	0.4992	0.3847	0.2653
10	0.4949	0.3800	0.2609
15	0.4863	0.3753	0.2566

Table 5.8 Comparisons of the means, standard deviations, mode, median, skewness, and kurtosis between simulation and binomial distributions for 5, 10, and 15 exploratory wells.

(a) Number of discoveries equals (J_x-n_1) when all size classes are economic.

Statistics	5 Exploratory Wells		10 Exploratory Wells		15 Exploratory Wells	
	Simulation	Binomial	Simulation	Binomial	Simulation	Binomial
Mean	2.4929	2.4961	4.9446	4.9492	7.3554	7.3594
Std.	1.1022	1.1180	1.5473	1.5811	1.8780	1.9362
Mode	2	2	5	5	7	7
Median	2	2	5	5	7	7
Skewness	-0.0018	0.0014	-0.0121	0.0064	0.0288	0.0097
Kurtosis	2.6097	2.6000	2.7858	2.8000	2.9054	2.8668

(b) Number of discoveries equals (J_x-n_1-n_2) when size class 2 is sub-economic.

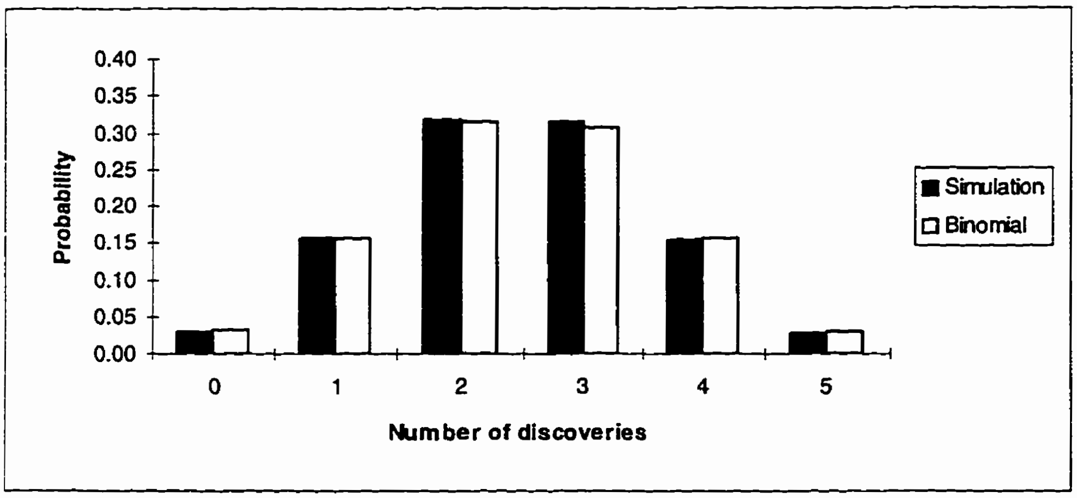
Statistics	5 Exploratory Wells		10 Exploratory Wells		15 Exploratory Wells	
	Simulation	Binomial	Simulation	Binomial	Simulation	Binomial
Mean	1.9252	1.9237	3.7991	3.7997	5.6213	5.6283
Std.	1.0754	1.0879	1.4967	1.5349	1.8118	1.8752
Mode	2	2	4	4	6	6
Median	2	2	4	4	6	6
Skewness	0.2917	0.2119	0.2150	0.1564	0.1853	0.1331
Kurtosis	2.6636	2.6449	2.8093	2.8245	2.9078	2.8844

(c) Number of discoveries equals (J_x-n_1-n_2-n_3) when size classes 2 and 3 are sub-economic.

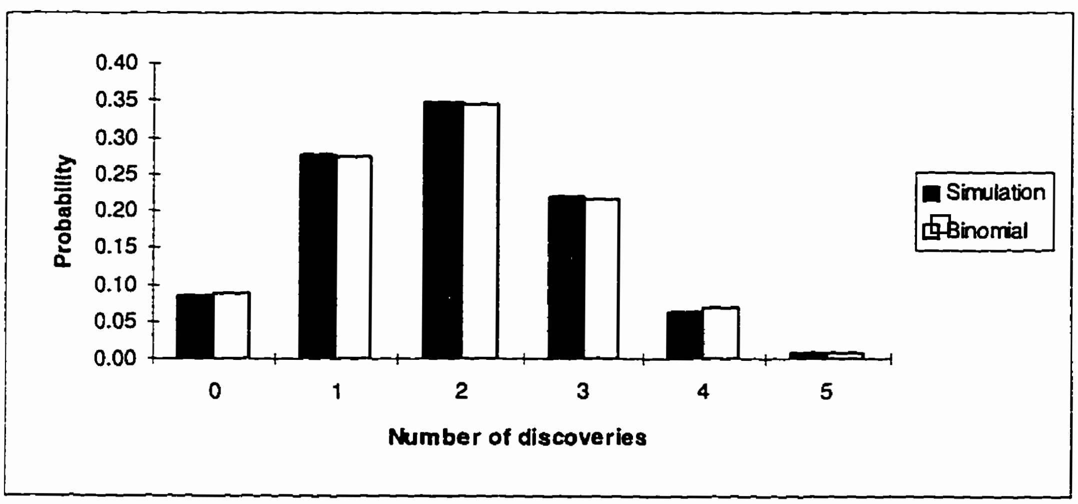
Statistics	5 Exploratory Wells		10 Exploratory Wells		15 Exploratory Wells	
	Simulation	Binomial	Simulation	Binomial	Simulation	Binomial
Mean	1.3250	1.3263	2.6054	2.6091	3.8417	3.8490
Std.	0.9749	0.9872	1.3427	1.3887	1.6063	1.6916
Mode	1	1	2	2	4	4
Median	1	1	3	3	4	4
Skewness	0.6421	0.4756	0.4205	0.3443	0.3413	0.2878
Kurtosis	2.8029	2.8262	2.9088	2.9186	2.9593	2.9495

Figure 5.11 Frequency comparison between simulation and binomial distributions for 5 exploratory wells.

(a) Number of discoveries equals $(5\text{-}n_1)$ when all size classes are economic.



(b) Number of discoveries equals $(5\text{-}n_1\text{-}n_2)$ when size class 2 is sub-economic.



(c) Number of discoveries equals ($5-n_1-n_2-n_3$) when size classes 2 and 3 are sub-economic.

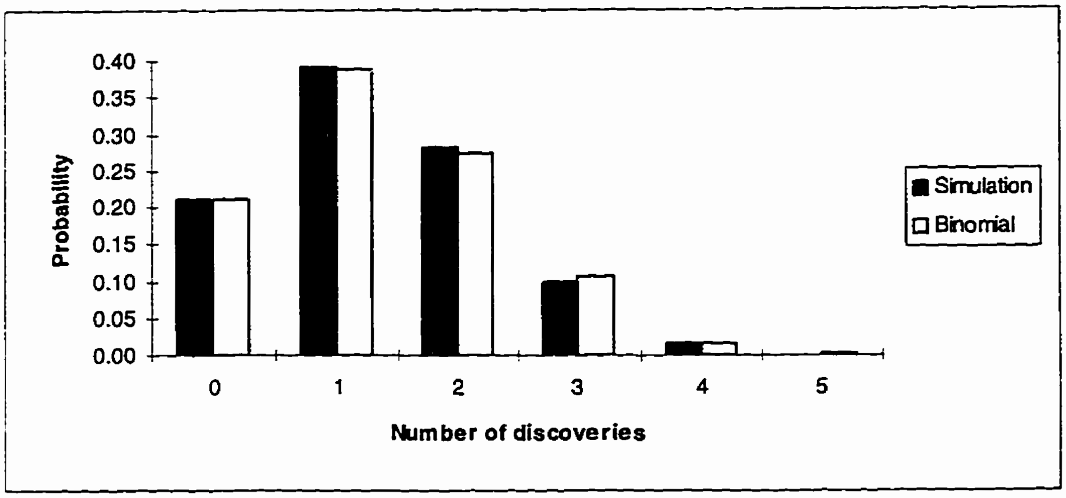
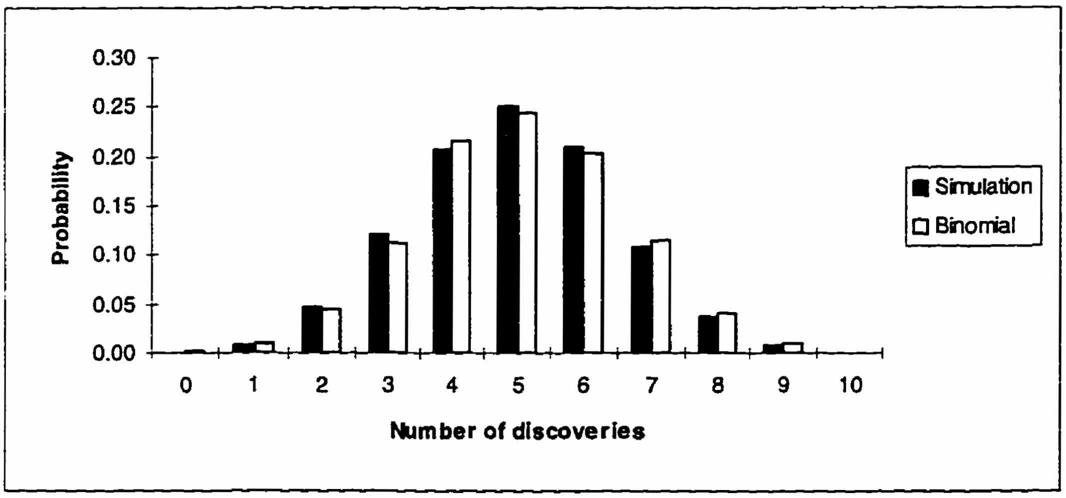
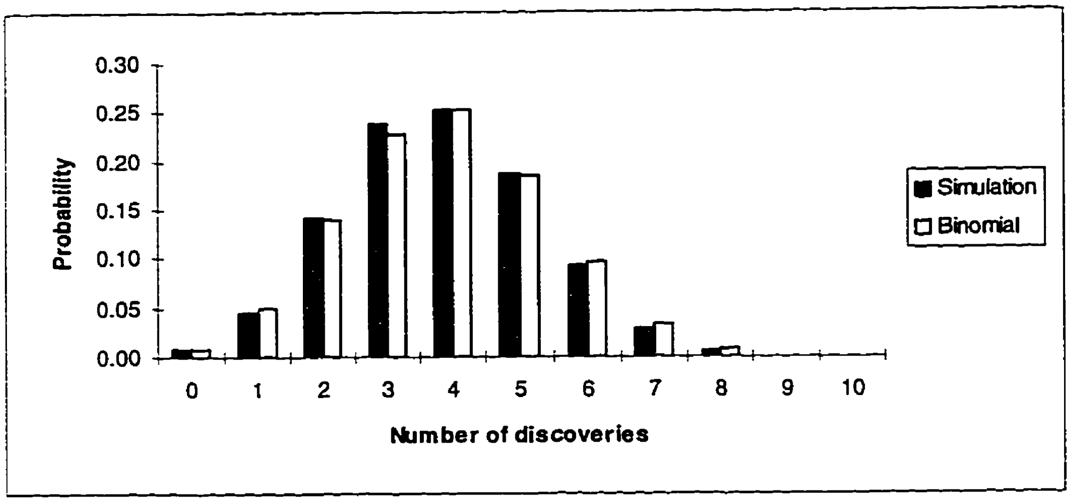


Figure 5.12 Frequency comparison between simulation and binomial distributions for 10 exploratory wells.

(a) Number of discoveries equals ($10-n_1$) when all size classes are economic.



(b) Number of discoveries equals $(10-n_1-n_2)$ when size class 2 is sub-economic.



(c) Number of discoveries equals $(10-n_1-n_2-n_3)$ when size classes 2 and 3 are sub-economic.

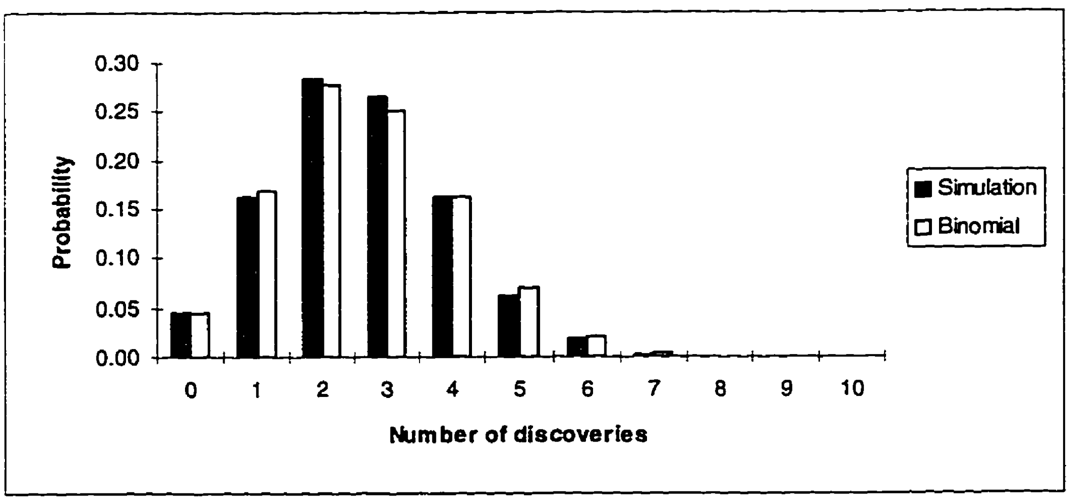
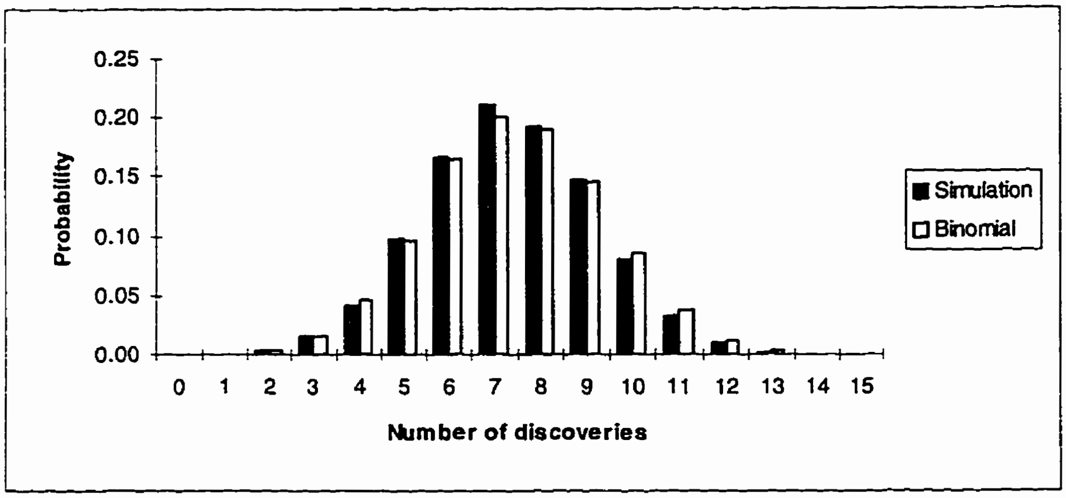
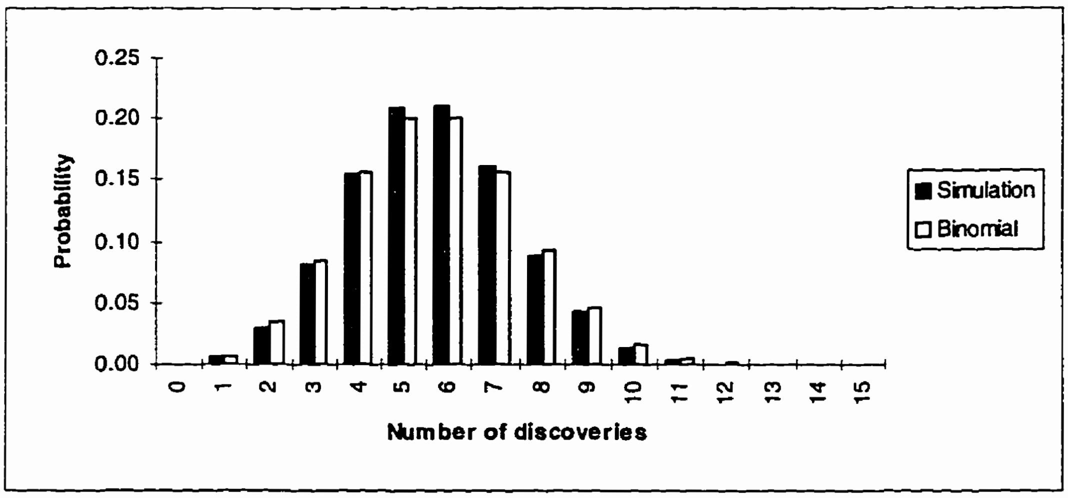


Figure 5.13 Frequency comparison between simulation and binomial distributions for 15 exploratory wells.

(a) Number of discoveries equals ($15-n_1$) when all size classes are economic.



(b) Number of discoveries equals ($15-n_1-n_2$) when size class 2 is sub-economic.



(c) Number of discoveries equals $(15-n_1-n_2-n_3)$ when size classes 2 and 3 are sub-economic.

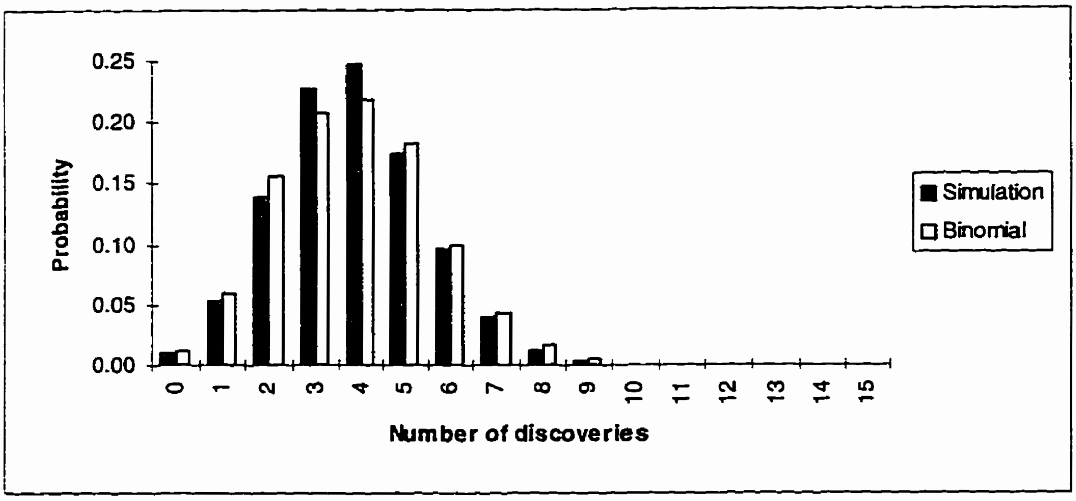


Table 5.9 Results of chi-squared tests for 5, 10, and 15 exploratory wells.

(a) chi-squared values

Exploratory Wells (J_x)	χ^2 values		
	(J_x-n_1)	$(J_x-n_1-n_2)$	$(J_x-n_1-n_2-n_3)$
5	5.3862	4.3867	3.8703
10	17.5134	18.9781	28.3513
15	31.9323	28.9056	57.3452

(b) critical values

Exploratory Wells (J_x)	Critical values		
	(J_x-n_1)	$(J_x-n_1-n_2)$	$(J_x-n_1-n_2-n_3)$
5	$\chi^2_{5,0.99}$ 15.0863	$\chi^2_{5,0.99}$ 15.0863	$\chi^2_{5,0.99}$ 15.0863
10	$\chi^2_{10,0.99}$ 23.2093	$\chi^2_{10,0.99}$ 23.2093	$\chi^2_{8,0.99}$ 20.0902
15	$\chi^2_{15,0.99}$ 30.5780	$\chi^2_{12,0.99}$ 26.2170	$\chi^2_{11,0.99}$ 24.7250

Since we use a very large sample size of 10,000 for both simulation and binomial distributions, we can justify that they are very good approximation of the exact population distribution as explained in Chungcharoen (1994). To select a significance level for the tests, we follow Labovitz's (1969) suggestion that as a sample size increases, there is a greater probability of correctly rejecting the null hypothesis. In addition, the standard error varies inversely with sample size. Therefore, with a large sample size a small difference is likely to be statistically significant. As a result, a small significance level should be used (0.01 or 0.001). On the other hand, a large significance level (0.1 or 0.05) should be used with a small sample size since large differences may not be detected by the predetermined level. Therefore, we select a significance level of 0.01 for the χ^2-tests in order to clearly distinguish between the two distribution functions. Notice that a significance level is a probability of Type I error (reject H_o, but H_o is true). In general, a Type II error (do not reject H_o, but H_o is false) should also be taken into consideration when a small significance level is chosen. However, since a sample size is very large, the Type II error will be reduced to a negligible value.

When considering 5 exploratory wells in Table 5.9, the χ^2 values of all three cases are less than the critical value $\chi^2_{5,0.99}$ of 15.0868. Therefore, we accept the null hypothesis that the sample of the distribution of number of discoveries is from a binomial distribution in which its mean obtained from $(J_x\text{-}n_1)$ when all field sizes are economic, $(J_x\text{-}n_1\text{-}n_2)$ when size class 2 is sub-economic, and $(J_x\text{-}n_1\text{-}n_2\text{-}n_3)$ when both size classes 2 and 3 are sub-economic, respectively. Similar results can be seen in 10 exploratory wells in $(J_x\text{-}n_1)$ and $(J_x\text{-}n_1\text{-}n_2)$ cases where their χ^2 values are less than the critical value $\chi^2_{10,0.99}$ of 23.2093. Therefore, we accept the null hypothesis. However, when size classes 2 and 3 become sub-economic, we reject the null hypothesis as its χ^2 value of 28.3513 is greater than the critical value $\chi^2_{8,0.99}$ of 26.2170. In 15 exploratory wells, the χ^2 values of 31.9323, 28.9056, and 57.3452 are greater than the critical value $\chi^2_{15,0.99}$ of 30.5780, $\chi^2_{15,0.99}$ of 26.2170, and $\chi^2_{11,0.99}$ of 24.7250, respectively. As a result, we reject the null hypothesis for all results even though the comparison from descriptive statistics and from histograms show that the binomial distribution is close fitted to the distribution of number of discoveries. In these rejecting cases, we observed that the difference between the two tails of simulation and binomial distributions has an important effect in enlarging the χ^2 values, even though the probabilities of these two tails are very small. Notice that the worst case in the comparison occur at 15 exploratory wells when size classes 2 and 3 are sub-economic as the χ^2 value is far away from the accepting region. Two main reasons for rejecting the null hypothesis are explained below.

First, it is known that Manly's Approximation Method produces small errors on the order of 1% for the mean and as much as 5% for the standard deviation (Ninpong, 1992). Chungcharoen (1994) also showed that the percent differences between the means of the simulation method and the means of Manly's approximation Method are within

0.5%, and the percent differences between the standard deviation of the two methods are within 5% for 5 to 80 discovered fields. These percent differences are the errors carried into the estimation of the parameters of the binomial distribution. Since we use such a large sample size, there is a high probability that the test will result in rejecting the null hypothesis. Notice that the calculated χ^2 value is still not far away from the critical value. One approach to avoid this problem is by selecting a smaller sample size to allow a larger difference to occur between the two distributions. Another approach is to reduce the significance level to be a smaller value, such as 0.001. This will allow a larger difference to occur between the two distribution functions and bring the χ^2 value closer to the accepting region. For example, when a significance level of 0.005 is selected, the critical value $\chi^2_{15,0.995}$ is 32.8010 resulting in accepting the null hypothesis for 15 exploratory wells when all size classes are economic.

Second, and the most important, reason for rejecting the null hypothesis in higher number of exploratory wells and in higher number of sub-economic field size classes originate from the difference between the hydrocarbon discovery process and the binomial distribution. Hydrocarbon discovery process is a sampling without replacement process from a finite population that follows a non-central multivariate hypergeometric distribution as mentioned in Fuller (1991) and Chungcharoen (1994). The probabilities of finding a dryhole and a discovery in different field size classes depends on the number of fields remaining in each size class and its size raised to the power of discovery efficiency as explained in Equations (3.2.1) and (3.2.2) and these probabilities change as exploration progresses. On the other hand, binomial distribution follows a sampling with replacement process which requires that the probability of success in each sample to be constant. There is also no weight (various sizes of the field) involved in the calculations of the probability of success. As mentioned in Chapter 4, the idea of using binomial distribution to approximate the distribution of the number of discoveries comes from difficulties in the determination of the probability of discoveries from 12 fields size classes by using a non-central multivariate hypergeometric distribution. We adopted Johnson et al.'s (1992) recommendation regarding the approximation of the classical hypergeometric distribution. According to Johnson et al. (1992), for a finite population N, it is adequate to use the simple binomial distribution with

$$P(X = x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

when $n < 0.1\,N$, where n is a sample size (or number of exploratory wells), x is the number of success, and p is the probability of success. This approach is suitable for approximating the classical hypergeometric distribution. However, it does not deal directly with an approximation of the underlying non-central multivariate hypergeometric distribution. As a sample size n increases when compared to $0.1N$, the difference between these two distributions will become more prominent as is the case of higher number of exploratory wells or higher number of sub-economic field size classes. This is because N is less whereas n *is* larger, causing $n > 0.1N$. Therefore, the accuracy of the approximation will deteriorate as the number of exploratory wells or the number of sub-economic field size class increase. The accuracy of the approximation could be improved by modifying p and n parameters in the above equation or by modifying the probability mass function of the binomial distribution as suggested by Sandiford (1960), Ord (1968a), and Johnson et al. (1992).

From the above reasoning, we acknowledge that there is some difference between the distribution of the number of discoveries from simulation and binomial distributions. We also recognize that the accuracy of the approximation approach diminishes as the number of fields remained to be discovered in the basin reduced. However, we still can see a good fit between the two distribution functions from descriptive statistics and histogram comparisons. Also, this effect should not happen in large basins which have large numbers of fields with different sizes. In addition, further modifications could be done to acquire greater accuracy of the approximation as mentioned above. Therefore, we propose that the binomial distribution should be justified for using as an approximate distribution of the number of discoveries.

5.4.6 Results and Discussions

Continuing from the three cases in Section 5.3, we examine six more cases to illustrate the overall methodology in this part. Case 4 presents the results from the calculations of the distribution of NPV that incorporate geological uncertainty into the frequency-size distribution for a reference price of \$3.80/m.c.f. and a reference real discount rate of 4.25%. Since the sensitivity of results by varying one input parameter may depend upon the level of another input parameter, in Case 5, we investigate the effect of changing prices on the distribution of NPV when a real discount rate is held constant at a low (3.40%), a reference (4.25%), and a high (5.10%) discount rate. Case 6 presents the effect of changing real discount rates when price is held at three different figures. Case 7 shows the results of the distribution of NPV when the total costs change by ± 20% while price and a real discount rate are held constant at reference values. As demonstrated in Case 3, there is an effect of increasing uncertainties of geological parameters on the distributions of total discovery volume. In Case 8, we continue our investigation of this effect on the distribution of NPV when uncertainty is increased with a reference price of $3.80/m.c.f. and a reference real discount rate of 4.25%. Finally, Case 9 presents the results and implications of applying expected utility analysis to the distribution of NPV in case of regular uncertainty and increasing uncertainty.

5.4.6.1 Case 4: Results of the Overall Methodology for the Exploration Project

Table 5.10 shows the probabilities of success for binomial distributions representing the distributions of the number of economic discoveries for 5, 10, and 15 exploratory wells after running Step 1. These probabilities are the average probabilities of success of 500 binomial distributions representing the distributions of number of discoveries from 500 frequency-size distribution data sets which represent the geological uncertainty in the Nova Scotia Shelf. Table 5.11 shows the minimum, maximum, and two shape parameters of the Beta distributions representing the distributions of total discovery volume for 1 to 15 discovery wells as a result of averaging 500 Beta distributions for each number of discovery wells.

Table 5.10 Probabilities of success for binomial distributions representing the distributions of the number of economic discoveries for 5, 10, and 15 exploratory wells.

Number of Exploratory Wells	Probabilities of success
5	.3644
10	.3592
15	.3540

Table 5.11 Minimum, maximum, and two shape parameters of the Beta distributions representing the distributions of total discovery volume for 1 to 15 economic discovery wells.

Discovery Number	Total Volume Range		Shape Parameters	
	Minimum (b.c.f.)	Maximum (b.c.f.)	p	q
1	143.1121	793.8480	0.3419	0.9016
2	286.2243	1422.7400	0.9236	2.0509
3	429.3357	1970.1600	1.5050	2.9420
4	572.4485	2463.7400	2.0932	3.6776
5	715.5601	2914.0000	2.6888	4.2921
6	858.6714	3331.8400	3.2930	4.8113
7	1001.7840	3721.1000	3.9079	5.2543
8	1144.8970	4084.8600	4.5308	5.6202
9	1288.0080	4431.5300	5.1725	5.9477
10	1431.1200	4762.4800	5.8340	6.2388
11	1574.2330	5076.7300	6.5132	6.4884
12	1717.3430	5374.5800	7.2085	6.6923
13	1860.6400	5659.4900	7.9250	6.8662
14	2004.1410	5933.7900	8.6678	7.0192
15	2147.6410	6196.9900	9.4367	7.1414

Table 5.12 shows probabilities that the number of discoveries fall into each interval for 5, 10, and 15 exploratory wells. Figure 5.14 shows the distributions of the number of discoveries for 5, 10, and 15 exploratory wells. These distributions are obtained from a conditional sampling process from binomial distributions which have probabilities of success from Table 5.10.

Table 5.12 Probabilities that the number of economic discoveries fall into each interval for 5, 10, and 15 exploratory wells.

Number of Discoveries	Probabilities		
	5 wells	10 wells	15 wells
0	0.1032	0.0009	0.0000
1	0.2992	0.0188	0.0000
2	0.3375	0.0895	0.0028
3	0.1988	0.2244	0.0250
4	0.0557	0.2988	0.0941
5	0.0056	0.2218	0.2001
6	0.0000	0.1126	0.2482
7	0.0000	0.0285	0.2328
8	0.0000	0.0046	0.1248
9	0.0000	0.0001	0.0581
10	0.0000	0.0000	0.0123
11	0.0000	0.0000	0.0017
12	0.0000	0.0000	0.0001
13	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000

Figure 5.14 Distributions of the number of economic discoveries for 5, 10, and 15 exploratory wells.

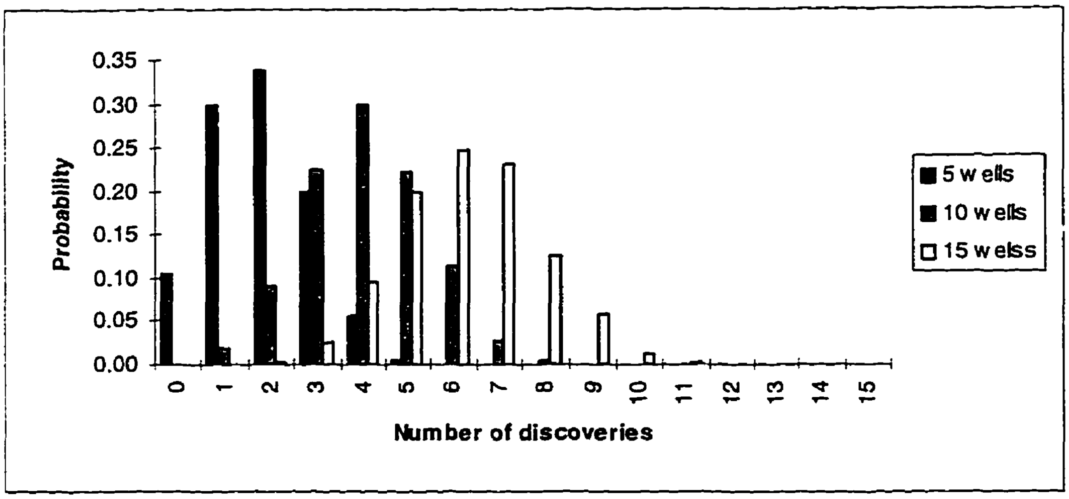


Table 5.13 and Figure 5.15 show probabilities that the total discovery volume falls into each interval and the distributions of total discovery volume for 5, 10, and 15 exploratory wells as a result of the conditional sampling process from Beta distributions using the parameters obtained in Tables 5.11.

Table 5.13 Probabilities that the total discovery volume falls into each interval for 5, 10, and 15 exploratory wells.

Interval	Probabilities		
(b.c.f.)	5 wells	10 wells	15 wells
000- 500	0.4684	0.0482	0.0004
500-1000	0.3939	0.2703	0.0325
1000-1500	0.1145	0.3613	0.1765
1500-2000	0.0205	0.2321	0.3102
2000-2500	0.0026	0.0748	0.2976
2500-3000	0.0001	0.0121	0.1376
3000-3500	0.0000	0.0012	0.0375
3500-4000	0.0000	0.0000	0.0072
4000-4500	0.0000	0.0000	0.0005
4500-5000	0.0000	0.0000	0.0000

Figure 5.15 Distributions of the total discovery volume for 5, 10, and 15 exploratory wells.

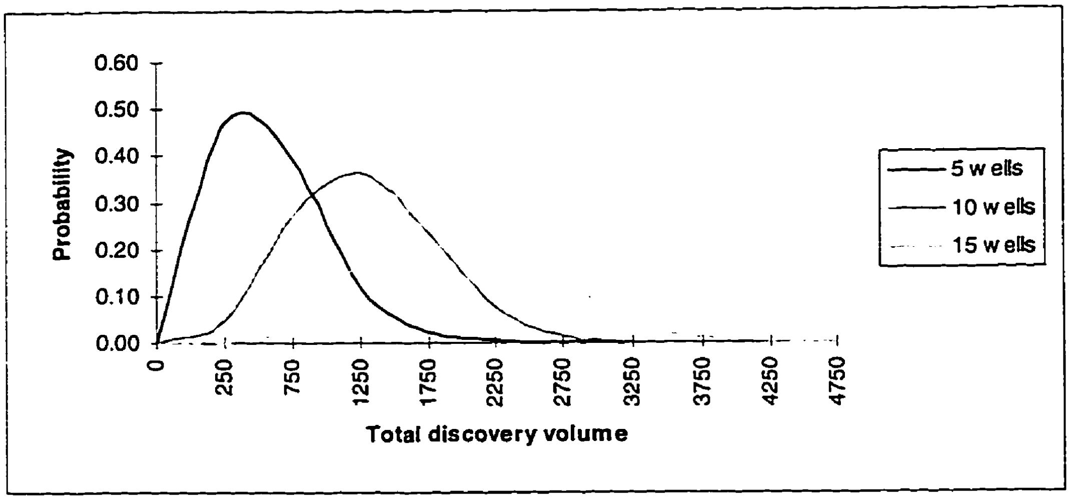


Table 5.14 and Figure 5.16 show the probabilities that the number of discoveries falls into each interval and the distributions of total volume in the 1st, 2nd, and 3rd year of the exploration project. These are the distributions of the number of economic discoveries from drilling 5 exploratory wells each year.

Table 5.14 Probabilities that the number of economic discoveries falls into each interval in the 1st, 2nd, and 3rd year of the exploration project.

Number of	Probabilities		
Discoveries	1st Year	2nd Year	3rd Year
0	0.1032	0.0992	0.1433
1	0.2992	0.2133	0.2112
2	0.3375	0.2694	0.2482
3	0.1988	0.2219	0.2072
4	0.0557	0.1322	0.1272
5	0.0056	0.0640	0.0629

Figure 5.16 Distributions of the number of economic discoveries in the 1st, 2nd, and 3rd year of the exploration project.

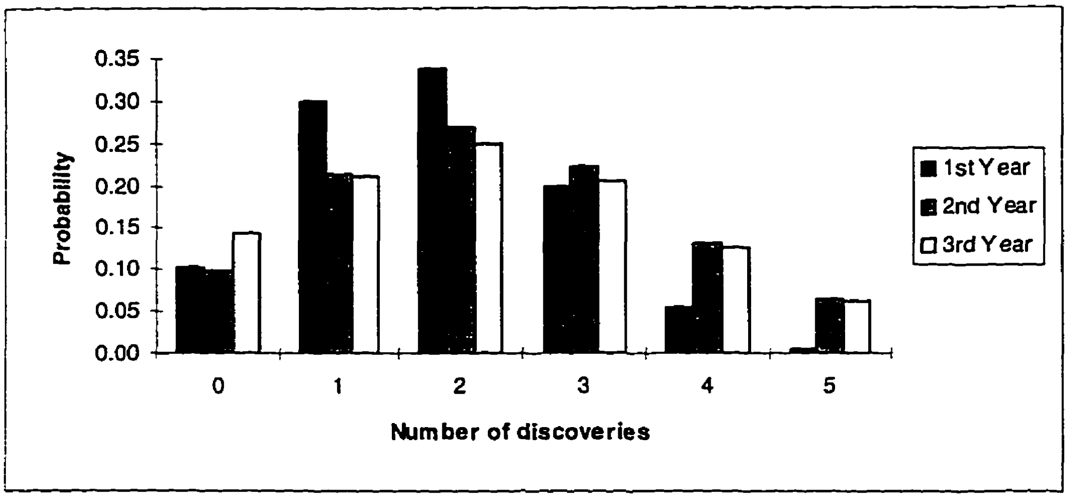


Table 5.15 and Figure 5.17 show probabilities that the total discovery volume falls into each interval and the distributions of total volumes in the 1st, 2nd, and 3rd year of the exploration project. Notice that due to the geological and geophysical nature of the Nova Scotia Shelf and the sampling without replacement which follows the probabilistic model of discovery process, the numbers of discovery wells and the total volume discovered in the 2nd and 3rd year are close to each other.

Table 5.15 Probabilities that the total discovery volume falls into each interval in the 1st, 2nd, and 3rd year of the exploration project.

Interval (b.c.f.)	Probabilities		
	1st Year	2nd Year	3rd Year
000- 500	0.4684	0.4108	0.4086
500-1000	0.3939	0.3320	0.3331
1000-1500	0.1145	0.1818	0.1755
1500-2000	0.0205	0.0648	0.0655
2000-2500	0.0026	0.0103	0.0148
2500-3000	0.0001	0.0003	0.0022
3000-3500	0.0000	0.0000	0.0003
3500-4000	0.0000	0.0000	0.0000
4000-4500	0.0000	0.0000	0.0000
4500-5000	0.0000	0.0000	0.0000

Figure 5.17 Distributions of the total discovery volume in the 1st, 2nd, and 3rd year of the exploration project.

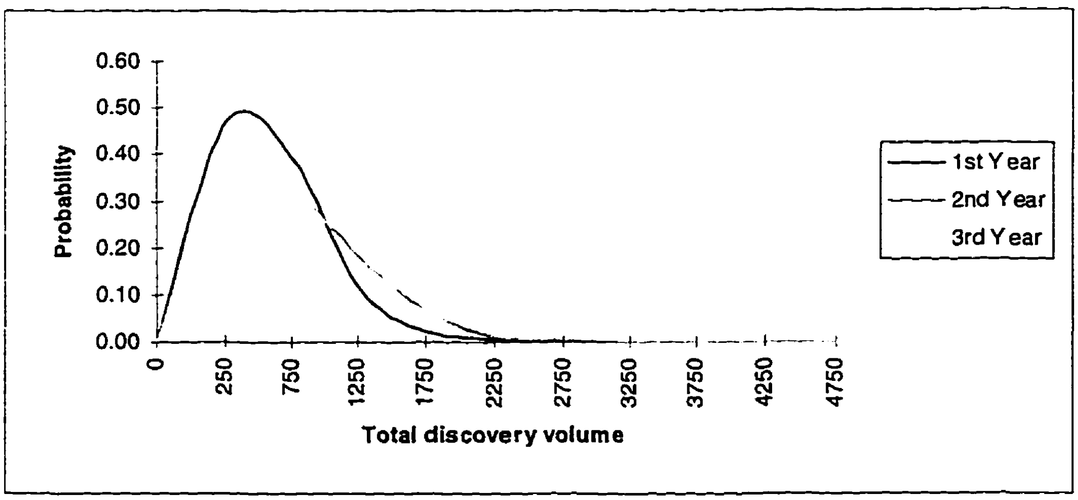


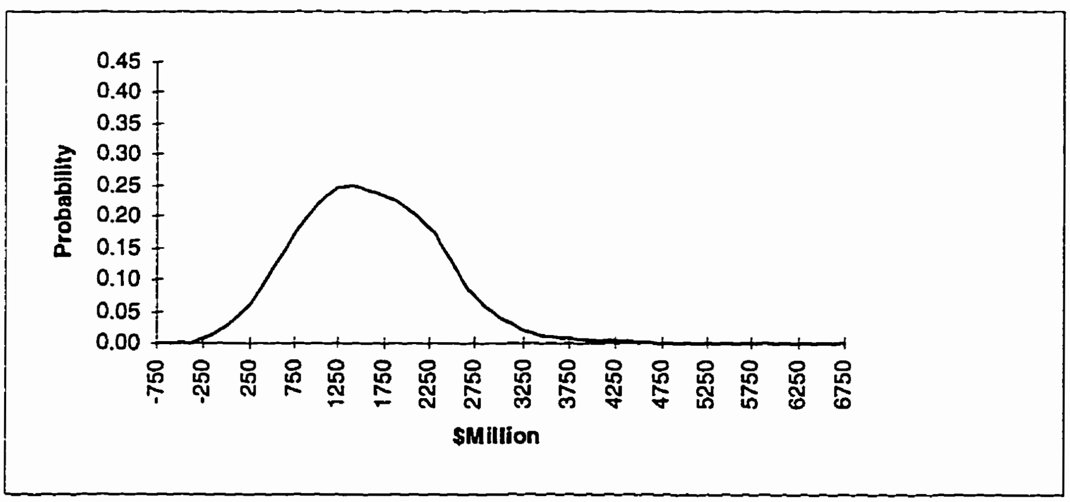
Table 5.16 and Figure 5.18 show the final result of the distribution of NPV of the exploration project after we apply the results of the distributions of number of discoveries and the distributions of total discovery volume for the 1st, 2nd, and 3rd year into the NPV of total annual revenues and NPV of total costs calculations in Equation (4.3.11). Note that

the expected net present value (or the expected monetary value, EMV) of the exploration project is calculated by multiplying the midpoint of each interval by the probability that the net present value falls in that interval, and by summing these results.

Table 5.16 Probabilities that the net present value falls into an interval and the expected monetary value of the exploration project.

Interval ($million)	Probability	MV ($million)
-500- 000	0.0053	-1.3250
000- 500	0.0618	15.4500
500-1000	0.1732	129.9000
1000-1500	0.2461	307.6250
1500-2000	0.2335	408.6250
2000-2500	0.1798	404.5500
2500-3000	0.0737	202.6750
3000-3500	0.0196	63.7000
3500-4000	0.0049	18.3750
4000-4500	0.0019	8.0750
4500-5000	0.0002	0.9500
	EMV	1558.6000

Figure 5.18 Distribution of NPV of the exploration project at the price of $3.80/m.c.f. and the real discount rate of 4.25%.

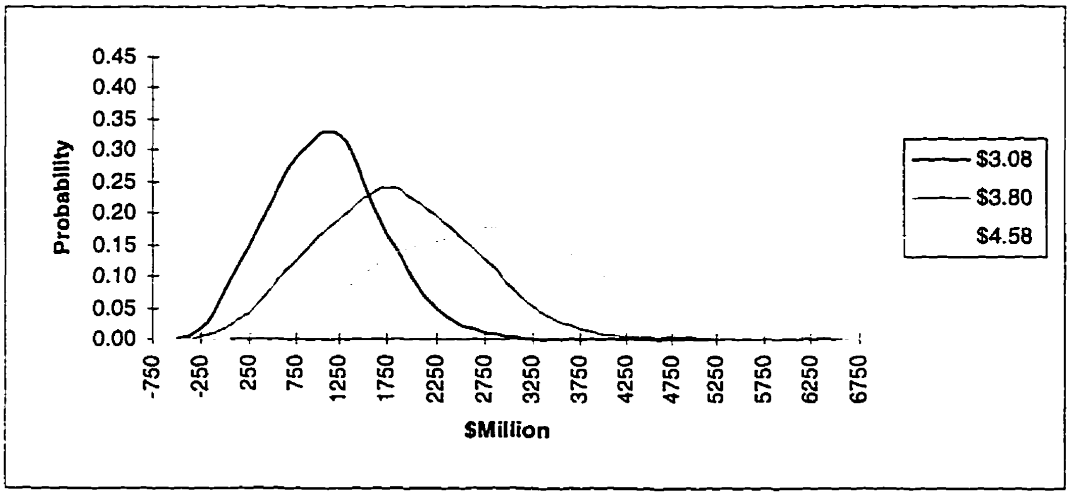


5.4.6.2 Case 5: Evaluation of the Exploration Project When Price Changes

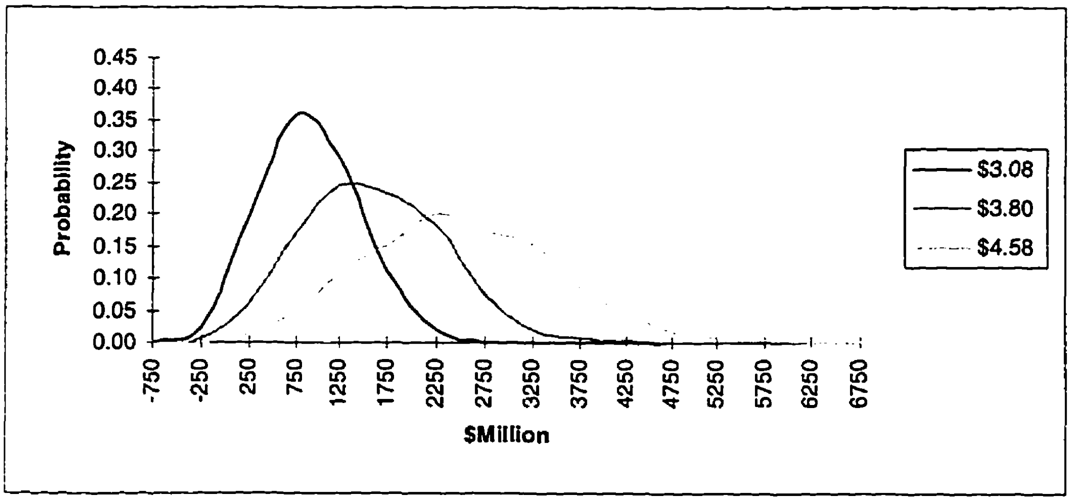
Figure 5.19 shows the distributions of NPV with the price of \$3.08/m.c.f., \$3.80/m.c.f., and \$4.58/m.c.f. at each of three discount rates.

Figure 5.19 Distributions of NPV with the price of \$3.08/m.c.f., \$3.80/m.c.f., and \$4.58/m.c.f. at three constant discount rate.

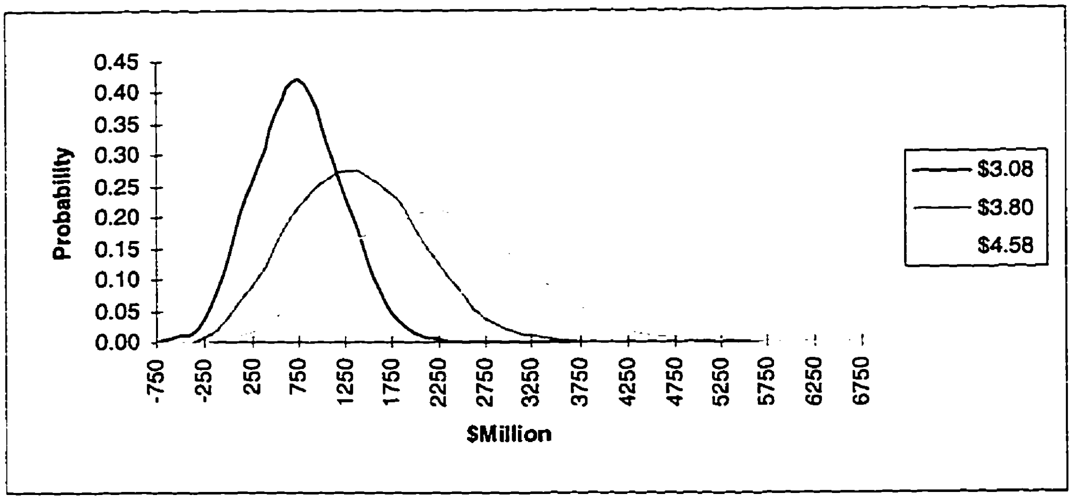
(a) low discount rate (3.4%)



(b) reference discount rate (4.25%)



(c) high discount rate (5.10%)



From Figures 5.19 (a), (b) and (c), we can see clearly that changing the price in any direction has a similar effect on the distribution of NPV in all three discount rates. Reducing the price from \$3.80/m.c.f. to \$3.08/m.c.f. causes the distribution of NPV to move toward the left-hand side of the graph and to have a narrower range. This results in the reduction of expected net present value of the exploration project. On the other hand, rising the price from \$3.80/m.c.f. to \$4.58/m.c.f. causes the distribution of NPV to spread more toward the right-hand side resulting in an increasing expected net present value of the project. Notice that there is only slight movement at the left tail of the distribution of NPV as price changes. These results can be explained by considering the effect of changes in price on the minimum economic field size class as shown in Table 5.17 and on the economic life of already commercially declared fields as explained below.

Table 5.17 Minimum economic field size classes when price changes at three different real discount rates.

Price/m.c.f.	Minimum Economic Field Size Class		
	DC=3.40%	DC=4.25%	DC=5.10%
\$3.08	≥ class 4	≥ class 4	≥ class 4
\$3.80	≥ class 3	≥ class 3	≥ class 3 (484) ≥ class 4 (16)
\$4.58	≥ class 3	≥ class 3	≥ class 3

From Table 5.17, dropping the price from \$3.80/m.c.f. to \$3.08/m.c.f. will cause fields in size class 3 to become sub-economic, and, hence, are not developed. This will affect the distribution of the number of economic discoveries and, subsequently, the distribution of total volume in each frequency-size distribution. As these distributions are used in the economic evaluations, the revenues generated from fields in size class 3 will be absent. In addition, the revenues from already commercially declared fields are also reduced due to the reduction in operation life of these fields. On the other hand, raising the price from \$3.80/m.c.f. to \$4.58/m.c.f. will have no effect on the minimum economic field size class as it will remain at size class 3. Note that even though there are 16 frequency-size data sets that have minimum economic field size classes changed from class 4 to class 3, they will have almost no effect on the final result as there are 484 frequency-size data sets for which the minimum economic field size class remains at class 3. However, raising the price will increase the revenues from additional production resulting from the extensions to the economic life of already commercially viable fields. Notice that the exploration company will have more benefit in larger fields than in smaller fields when price rises as the marginal cost of producing gas is less in large fields than in small fields, whereas the marginal revenues are higher in large fields than in small fields. Also, observe that the revenues generated from fields in size class 3 when it becomes economic at price \$3.80/m.c.f. and \$4.58/m.c.f. contribute only a minor part in the total revenues of the exploration project as compared to the revenues from the extension of economic life of large fields. The expected net present values when the price changes at three different discount rates are shown in Table 5.18 below.

Table 5.18 Expected net present values when the price changes at each of three real discount rates.

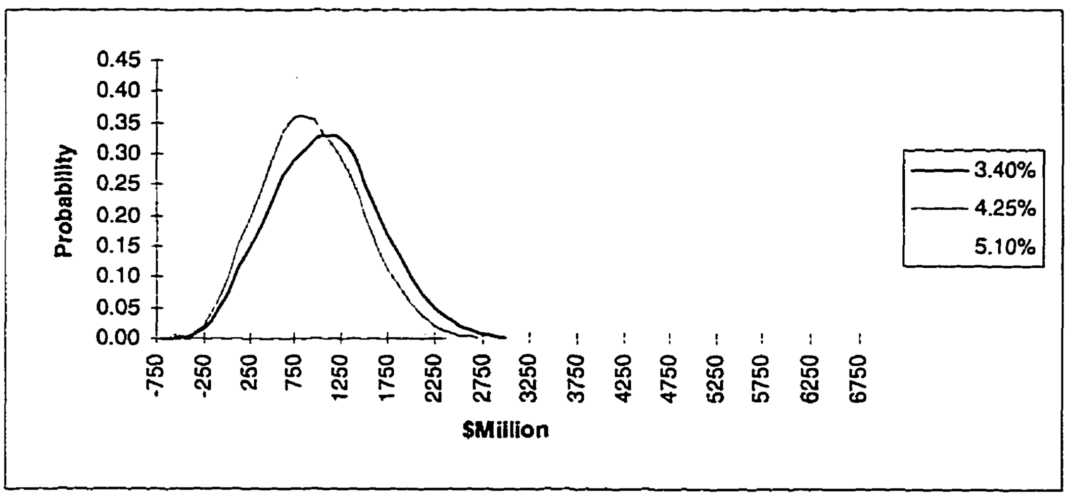
Price/m.c.f	Expected Net Present Value (\$million)		
	DC=3.40%	DC=4.25%	DC=5.10%
\$3.08	1076.85	913.55	752.75
\$3.80	1804.85	1558.60	1362.05
\$4.58	2811.10	2488.90	2198.70

5.4.6.3 Case 6: Evaluation of the Exploration Project when Real Discount Rate Changes

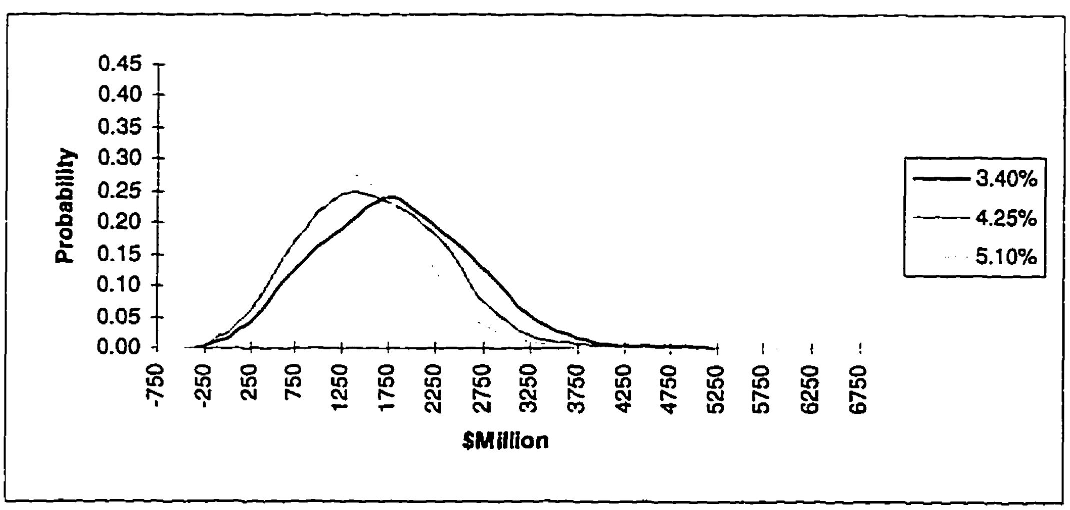
Figure 5.20 shows the distributions of NPV when the real discount rate changes at each of three different prices.

Figure 5.20 Distributions of the net present value with the real discount rates of 3.4%, 4.25%, and 5.10%, respectively.

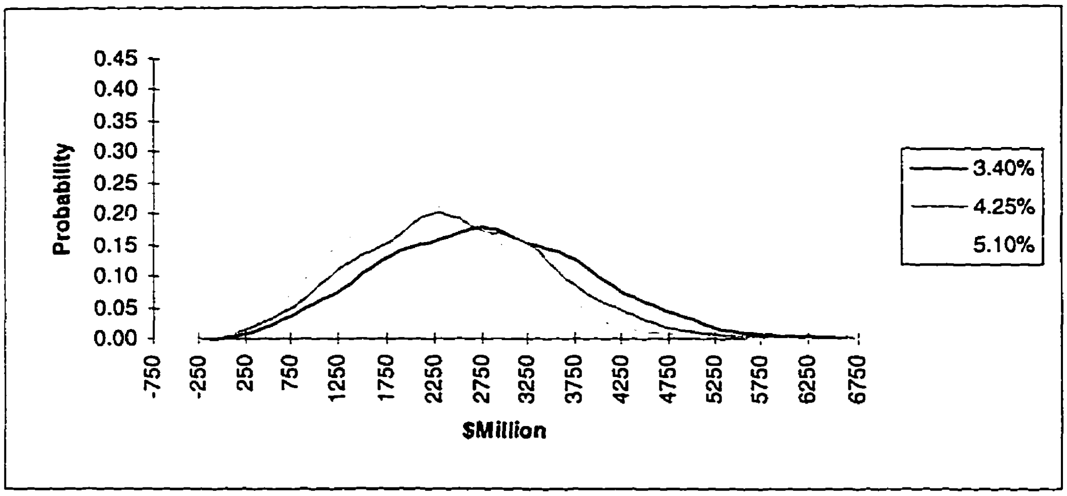
(a) low price ($3.08/m.c.f.)



(b) reference price ($3.80/m.c.f.)



(c) high price ($4.58/m.c.f.)



From Figures 5.20 (a), (b), and (c), when a real discount rate is reduced from 4.25% to 3.40%, the distribution of NPV spreads toward the right-hand side of the graph resulting in the increase in the expected net present value. When a real discount rate is increased from 4.25% to 5.1%, the distribution of NPV moves toward the left-hand side of the graph causing the expected net present value to reduced. However, we can see that the change in a real discount rate by ±20% from a reference value when price is held constant has less effect on the shape of the distribution of NPV than the change in price when a real discount rate is held constant. The reasons for these results can be explained as follows.

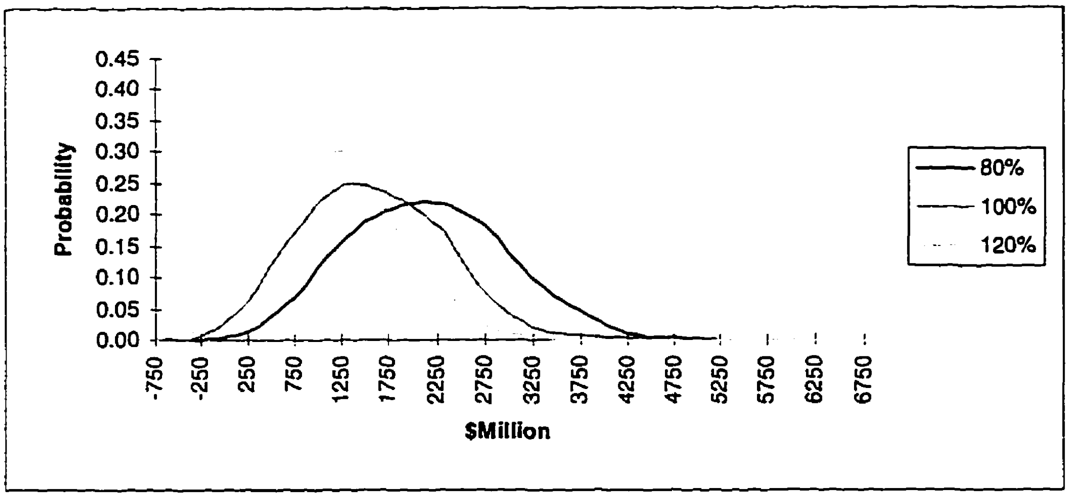
As can be seen in Table 5.17, there is almost no change in the minimum economic field size class when the discount rate is reduced or increased when price is held constant. The minimum economic field size class remains at class 4 for price $3.08/m.c.f., at class 3 for prices $3.80/m.c.f. and $4.58/m.c.f. Even though there are 16 frequency-size distributions that have the minimum economic field size class changed from class 3 to class 4 when a discount rate rises from 4.25% to 5.10% at price $3.80/m.c.f., there are 484 frequency-size distribution data sets that remain at the minimum economic field size class of 3. The effect of 16 data sets is minor comparing to 484 data sets. As a result, they do not affect the average distributions of the number of economic discoveries and the average distributions of total volume that are generated. We can say that the changes in discount rate have virtually no effect on the minimum economic field size class. Since there is no new minimum economic field size class entering or leaving the model as the discount rate decreases or increases, it will have no effect on the distribution of NPV.

The reduction of a real discount rate will result in an increase in values of the discounted annual revenues as well as the discounted total annual costs. However, this reduction has no effect on the initial costs of exploring, developing, and producing the fields in our methodology as these costs are obtained from regression equations of the already discounted costs and volume size of fields. Although different forms of initial cost calculations take a real discount rate into account, they would have minor effect on initial costs as the time lag between exploration and development is usually a few years. Increasing total revenues results in an extension of the economic life of already commercially declared fields. This has more effect on the net present value than the rising total annual operation costs. As a result, the distribution of NPV spreads toward the right-hand side of the graph and the expected net present value increases. On the other hand, when a real discount rate rises, it causes the annual revenues and annual costs to be more discounted and, subsequently, shortens the economic life of already commercially declared fields. As a result, the distribution of NPV moves toward the left-hand side reducing the expected net present value of the exploration project. Notice that the effect of revenues reduction caused by the shorten economic life of fields has more effect on the net present value than the effect of total annual costs reduction. Also, an increase in discount rate has no effect on the total initial costs as mentioned above. Therefore, there is only slight change on the left tail of the distribution. Since a discount rate used in our studies is a real discount rate that does not incorporate other risk factors, it is relatively low. The effect of changing a discount rate on the distribution of NPV by ±20 percent is not pronounced. In addition, we can see that a change in price when a discount rate is held constant has more prominent effect on the distribution of NPV than a change in discount rate when price is held constant.

5.4.6.4 Case 7: Evaluation of the Exploration Project When Total Costs Change

To see the effect of changes in total costs, we vary total costs, comprising the total initial costs and the total annual costs, by ± 20% from its original value by keeping the price and a real discount rate unchanged at \$3.80/m.c.f. and 4.25%, respectively. The results are shown in Figure 5.21.

Figure 5.21 Distributions of NPV of the exploration project when total costs change at the price \$3.80/m.c.f. and a real discount rate 4.25%



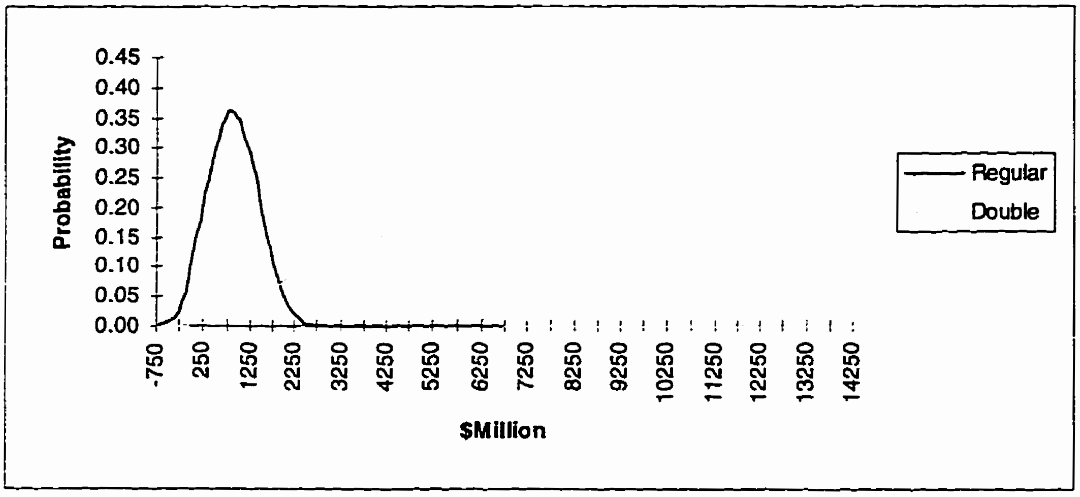
From Figure 5.21, the effect of total costs rising or dropping when price and a real discount rate are held constant has the same effect on the distribution of NPV as when price dropped or rose when total costs and discount rate are held constant. As total costs decrease, the distribution of NPV moves toward the right-hand side of the graph with its shape close to normal causing an increase in the expected net present value from its original value of \$1,558.60 million to \$2,153.40 million. When total costs increase, the distribution of NPV will become more right-skewed and move toward the left-hand side of the graph causing the expected net present value to decrease. Consequently, the expected net present value decreases from \$1,558.60 million to \$1,208.25 million. There is no effect of a new economic field size class entering the model as total costs drop by 20 percent because all minimum economic field size classes from 500 frequency-size distributions remain at class 3. However, there is an effect of changing the new minimum economic field size classes when total costs rise since size class 3 becomes sub-economic. As total costs decrease, there is an increase in total revenues from the extensions of the economic life of already commercially declared fields. As total costs increase, there is an effect of total revenues dropping as a result of changing minimum economic field size class as well as shortening economic life of the already commercial declared fields. Finally, we can see that the drop in total costs has more effect on the distribution of NPV than the rise of total costs

5.4.6.5 Case 8: Evaluation of an Exploration Project When Uncertainties are Increased

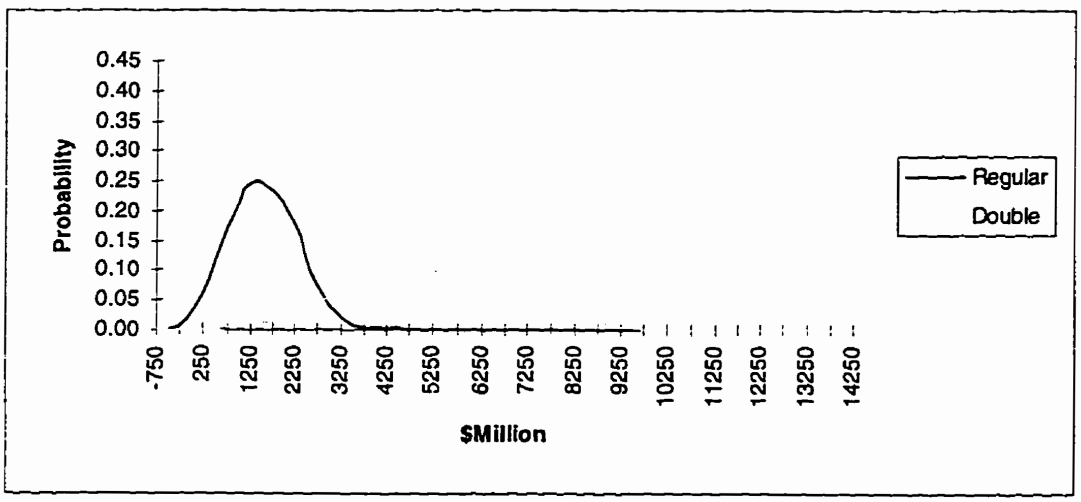
The comparisons of the distributions of NPV of the exploration project between the regular case and when the uncertainty of the field size distribution of the Nova Scotia Shelf is doubled are shown in Figure 5.22. As explained in Case 5 and Case 6, changes in price have a prominent effect on the distribution of NPV. Therefore, the comparisons are illustrated at three different prices with a real discount rate of 4.25%. Note that the results of comparison at other real discount rates are similar to Figure 5.22. Therefore, they will not be displayed here.

Figure 5.22 Comparison of the distributions of NPV between regular and double uncertainty for the exploration project when a real discount rate is 4.25%

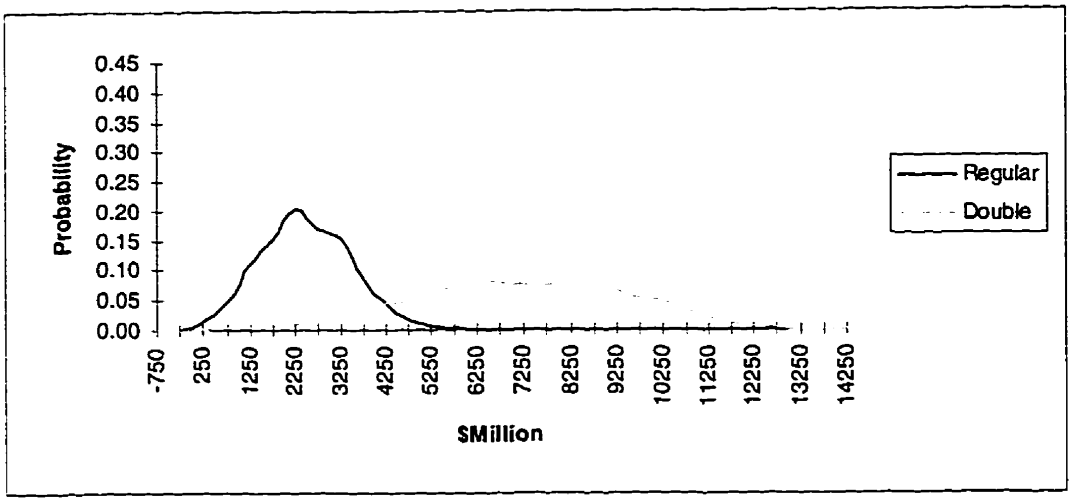
(a) at low price ($3.08/m.c.f.)



(b) at reference price ($3.80/m.c.f.)



(c) at high price ($4.58/m.c.f.)



From Figure 5.22 (a), (b), and (c), the distributions of NPV in case of double uncertainty spread toward the right-hand side of the graph more than the distribution of NPV in case of regular uncertainty. The expected net present values of the exploration project in both cases are $913.55 million and $3,561.80 million at price $3.08/m.c.f., $1,558.6 million and $5,251.65 million at price $3.80/m.c.f., and $2,488.90 million and $7,192.25 million at price $4.58/m.c.f., respectively. We can see that the expected net present values in case of double uncertainty are 4 times, 3.4 times, and 2.9 times the expected net present values in case of regular uncertainty at low price, reference, and high prices. Explanations of these results are given below.

As explained in Case 3, Subsection 5.3.4.4, an increase in the uncertainty in field size causes a substantial increase in expected total discovery volume. Since the field size distribution is highly right-skewed, as we double its standard deviation to increase the uncertainty and keep its mean unchanged, its density will spread out toward the right tail. Hence, the probability associated with very large field size on the right tail is increased. By using the probabilistic model of discovery process, the probability of discovering a field of one particular size class is proportional to the number of fields of that size class and its area raised to the power of discovery efficiency. As a result, there will be a higher possibility that large fields will be discovered at an early exploration stage. Therefore, the distribution of total volume in Case 3 with double uncertainty is much more spread out than the distribution of total volume in Case 1. Therefore, the expected total volume in Case 3 is higher than the expected total discovery volume in Case 1 at the early stage of exploration. As the exploration process progresses, large fields are becoming depleted and small fields remained to be found. This phenomenon was carried out in the calculations of the distribution of NPV. As we use the distributions of total volume in the economic evaluations, the results show the distribution of NPV in case of double uncertainty more spread out than the distribution of NPV in case of regular uncertainty. As a result, the expected net present value of the exploration project when uncertainty is doubled is much higher than the expected net present value of the same project in case of regular uncertainty.

To see the effect of increasing uncertainty of field size distribution on the minimum economic field size class, Table 5.20 shows the minimum economic field size classes when price changes at three different discount rates from 500 frequency-size distribution data sets in case of double uncertainty.

Table 5.20 Minimum economic field size classes when price changes at three real discount rates of 4.25% in case of double uncertainty.

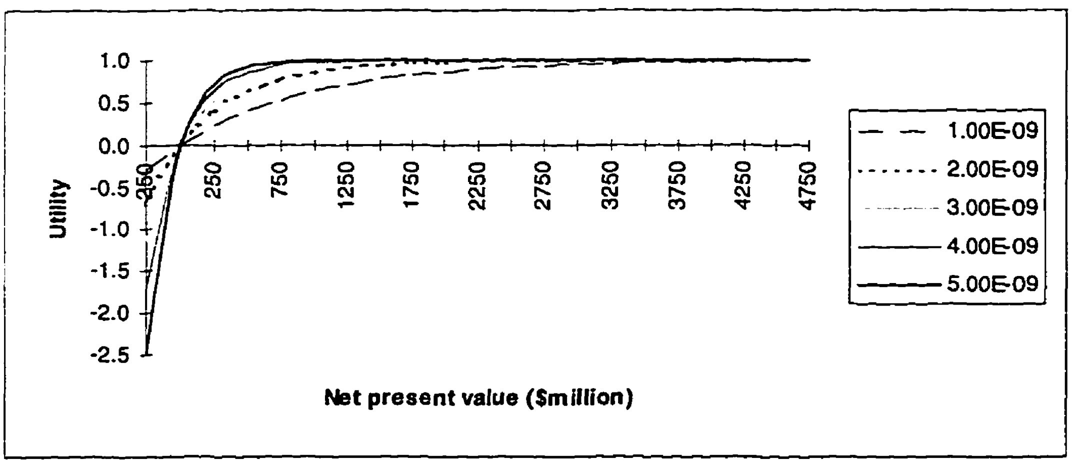
Price/m.c.f.	Minimum Economic Field Size Class		
	DC=3.40%	DC=4.25%	DC=5.10%
\$3.08	≥ class 3 (2) ≥ class 4 (498)	≥ class 3 (1) ≥ class 4 (499)	≥ class 4
\$3.80	≥ class 3 (498) ≥ class 4 (2)	≥ class 3 (498) ≥ class 4 (2)	≥ class 3 (445) ≥ class 4 (55)
\$4.58	≥ class 3	≥ class 3	≥ class 3

From Table 5.20, we see that changing the discount rate when price and total costs are held constant in the case of double uncertainty in field size distribution has little effect on the minimum economic field size class. This finding is similar to the finding in Case 5. At price \$3.08/m.c.f., almost all of 500 frequency-size distributions have the minimum economic size class at class 4. At price \$3.80/m.c.f., the majority of 500 data sets have minimum economic field size classes at class 3, except at high discount rate where there are 445 data sets having minimum economic field size classes at class 3 and another 55 data sets have minimum economic field size classes at class 4. Though, this will have minor effect on the calculations of the distribution of the number of economic discoveries and the distribution of total volume and, hence, the distribution of NPV. At high price, all data sets remain the minimum economic field size classes at class 3. Notice that even though the probability associated with small field size on the left tail of field size distribution is also increased when its standard deviation is doubled, it has almost no effect on the minimum economic field size class in a frequency-size distribution, which is obtained by sampling from the number of fields distribution and the field size distribution, as these small fields are categorized into size class 2 which is a sub-economic size class. If they are discovered, they are treated as dryholes and are not developed.

5.4.6.6 Case 9: Results of the Expected Utility Analysis

Since detailed financial information regarding the Nova Scotia project among companies is not available to the public, we decided to apply the exponential utility function with various chosen degrees of risk-aversion as displayed in Figure 5.23. Note that the coefficient of 1.0×10^{-9} depicts the lowest degree of risk-aversion, whereas the coefficient of 5.0×10^{-9} depicts the highest degree of risk-aversion in this investigation.

Figure 5.23 Exponential utility function with risk-aversion coefficient varies from 1.0×10^{-9} to 5.0×10^{-9}.



Continuing from Figure 5.22, Table 5.21 displays the probabilities that the net present values fall into intervals between the regular case and when the standard deviation of field size distribution of Nova Scotia Shelf is doubled (hereafter referred to as "double uncertainty"). Notice that the probabilities associated with losses from investing in this particular project are very small when compared to the gains from discovering economic fields, especially when the price is high and when uncertainty of field size is increased. There are much higher benefits when economic fields are discovered than the expenses from exploring and developing spending these fields.

Table 5.21 Probabilities that the net present value falls into intervals for three different prices at reference discount rate.

Interval	$3.08/m.c.f.		$3.80/m.c.f.		$4.58/m.c.f.	
($million)	Regular	Double	Regular	Double	Regular	Double
-500- 000	0.0229	0.0005	0.0053	0.0002	0.0002	0.0001
000- 500	0.1981	0.0045	0.0618	0.0011	0.0120	0.0005
500-1000	0.3594	0.0128	0.1732	0.0043	0.0481	0.0019
1000-1500	0.2896	0.0343	0.2461	0.0088	0.1108	0.0025
1500-2000	0.1098	0.0584	0.2335	0.0142	0.1510	0.0070
2000-2500	0.0190	0.0904	0.1798	0.0280	0.2019	0.0089
2500-3000	0.0012	0.1237	0.0737	0.0477	0.1702	0.0132
3000-3500		0.1421	0.0196	0.0612	0.1508	0.0194
3500-4000		0.1550	0.0049	0.0769	0.0852	0.0325
4000-4500		0.1479	0.0019	0.0950	0.0454	0.0433
4500-5000		0.1038	0.0002	0.1090	0.0154	0.0460
5000-5500		0.0763		0.0988	0.0062	0.0630
5500-6000		0.0360		0.1053	0.0021	0.0698
6000-6500		0.0126		0.0969	0.0005	0.0816
6500-7000		0.0016		0.0812	0.0002	0.0785
7000-7500		0.0001		0.0711		0.0742
7500-8000				0.0497		0.0757
8000-8500				0.0318		0.0797
8500-9000				0.0124		0.0674
9000-9500				0.0050		0.0656
9500-10000				0.0012		0.0516
10000-10500				0.0002		0.0479
10500-11000						0.0305
11000-11500						0.0190
11500-12000						0.0102
12000-12500						0.0057
12500-13000						0.0034
13000-13500						0.0005
13500-14000						0.0003
14000-14500						0.0001

Table 5.22 Comparisons of the expected utility values and certainty equivalents between regular and double uncertainty at different risk-aversion coefficients.

a) Expected utility values

Risk-Averse Coefficient ($x\ 10^{-9}$)	Expected Utility Value					
	$3.08/m.c.f.		$3.80/m.c.f.		$4.58/m.c.f.	
	Regular	Double	Regular	Double	Regular	Double
1.0	0.5244	0.9397	0.7276	0.9782	0.8689	0.9914
2.0	0.7346	0.9872	0.8855	0.9962	0.9648	0.9985
3.0	0.8127	0.9942	0.9341	0.9983	0.9851	0.9992
4.0	0.8449	0.9961	0.9523	0.9988	0.9917	0.9994
5.0	0.8543	0.9966	0.9592	0.9989	0.9945	0.9995

b) Certainty equivalents

Risk-Averse Coefficient ($x\ 10^{-9}$)	Certainty Equivalent ($million)					
	$3.08/m.c.f.		$3.80/m.c.f.		$4.58/m.c.f.	
	Regular	Double	Regular	Double	Regular	Double
1.0	743.18	2808.42	1300.48	3825.85	2031.79	4755.99
2.0	663.26	2179.16	1083.59	2786.38	1673.35	3251.15
3.0	558.35	1716.63	906.54	2125.71	1402.13	2376.97
4.0	465.92	1386.69	760.71	1681.36	1197.87	1854.65
5.0	385.24	1136.80	639.81	1362.49	1040.60	1520.18

From Table 5.22 (a), it is not surprising that the expected utility values of the exploration project when uncertainty in field size is doubled are higher than the expected utility values of the project at regular uncertainty at all levels of price and risk-averse coefficients. At a high price, we can see that the utility values of these two cases are close to each other. This is because the management's risk preferences become less different due to high profits (the utility function levels off at high profits). The expected utility values in Table 5.22 (a) were converted to the certainty equivalents in Table 5.22 (b) in order that the comparison can be done in monetary values. The certainty equivalent results agree with the expected net present values obtained from the last subsections which point out the apparent benefit of investing in the exploration project with higher uncertainty. Nonetheless, these results show some implications as discussed below.

At a low price, the certainty equivalents of doubled uncertainty are considerably higher than the certainty equivalents of the regular uncertainty. Consequently, these results might lead the company to invest in the project with higher uncertainty. As price rises, the certainty equivalents from both cases increase. The certainty equivalents of the regular uncertainty case are slightly closer to the certainty equivalents obtained from the doubled uncertainty case corresponding to the utility values that level off at high profits. As the degree of risk-aversion increases (higher risk-aversion coefficient), the certainty equivalents in the regular uncertainty case also rise and closer to the certainty equivalents of the double uncertainty case. At a high price ($4.58/m.c.f.) and high degree of risk-aversion ($c = 5.0 \times 10^{-9}$), the certainty equivalents of these two cases are $1,040.60 and $1,520.18 millions, respectively.

Although the results from our investigation are obtained by using the theoretical form of an exponential utility function and by assuming different degrees of risk-aversion to represent the management's risk behavior toward the exploration investment in offshore Nova Scotia, these results have some insightful benefits, because various risk-aversion coefficients cover different degrees of risk-averse behavior of the management toward the exploration investment when there is uncertainty involved. We anticipate that the results obtained from using other forms of risk-averse utility functions, either from a direct assessment of the management's risk preferences or from empirical available data, would behave in a similar way. Since we consider the sub-economic field size as dryholes in the calculations of net present values, the maximum loss from discovering these fields is equal to the expenses of dryholes. In some situations when the price drops lower and when there are higher expenses involved in drilling dryholes (or sub-economic fields) and in developing and producing economic fields, the strong risk-averse behavior should cause the expected utility value of the exploration project in the underlying basin with less uncertainty to have higher value than the expected utility value of the same or different project with higher uncertainty. More investigation is needed to confirm this anticipation.

In addition to the discussions in Cases 3, 8, and 9, there is an interesting implication regarding the variability of field size (e.g. the standard deviation of the field size distribution). According to Fuller[1], we could interpret the meaning of the variability of field sizes in two ways. First, the variability could be "real", i.e. that there are in fact fields of various sizes in unknown locations. Second, the standard deviation could represent, in part, the "real" variability, but also, to some extent, our ignorance about field sizes. If the first interpretation is the valid one, then it makes perfectly good sense to prefer to explore in one basin that has a larger variability but the same mean value of field sizes. This is because the size-biased search process will tend to find the more prevalent

[1] Based on the discussion and suggestion by JD Fuller

large fields that in fact exist in the higher-variability area. However, if the second meaning is more valid in some situation, then to conclude that one area is preferable to another for exploration, just on the basis of a greater variability measure for field sizes, could be a terribly costly mistake. The larger "expected value" associated with the higher variability basin could be misleading, if the reason for the larger standard deviation of field size is just ignorance. In this case, it would be more wise to conduct further geological tests to reduce the degree of ignorance about the "real" field size distribution.

CHAPTER 6

CONCLUSIONS

6.1 Conclusions

A major benefit of the methodology developed in this thesis is that it allows the model to run much faster than the regular simulation method of the probabilistic model of hydrocarbon exploration. The much faster speed of our approach allows the model to incorporate uncertainty in geological parameters into the frequency-size distribution data, and to obtain the distribution of results within a short period of time. For an IBM PC Pentium 100 MHz, with 16 MB RAM, generating 500 frequency-size distributions for the model requires only 8 seconds. Running one frequency-size distribution with 3 full simulations (for 5, 10, and 15 exploratory wells) requires approximately 2 minutes (time required for each number of exploratory wells is approximately 36 seconds, 40 seconds, and 44 seconds, respectively), whereas using our approach for the same three number of exploratory wells requires only approximately less than a second. It is clear that running 1,500 simulations to account for the uncertainty with 500 frequency-size distributions by using the full simulation method would require as much as 16 hours and 40 minutes while our approach using Manly's approximation and approximate Beta distribution would require only 11 minutes and 15 seconds. The differences in time will become more pronounced, when we perform the analysis over a longer range exploration program (such as 5 to 50 exploratory wells) for policy analysis, and when larger uncertainties in geological parameters, that require more than 500 frequency-size distributions, are involved. In addition, running each scenario in the economic model requires 2 minutes and 23 seconds. Within this short period of time, the model allows a company to thoroughly analyze a variety of exploration and development scenarios with changes in geological and economic conditions.

6.2 Summary

The main objective of this research is to extend my previous work (Chungcharoen, 1994) to incorporate the uncertainties in geological parameters in order to provide an assessment of the distributions of total hydrocarbon discoveries that are expected to be recovered as a result of exploration activity and to combine the economic parameters into the evaluation of the economic worth of the results of exploration activity. This research can be separated into two parts.

In the first part, the uncertainties involved in the geological parameters are included in the initial field size distribution. In addition to the uncertainty in geological parameters, there is also uncertainty in the number of fields. Therefore, the number of possible fields is also represented by a probability distribution. After determining the field size distribution and estimating the number of fields distribution, the Monte Carlo approach is used to sample data from the field size distribution with each number of fields selected from the number of fields distribution. Each frequency-size distribution is constructed based on each sampled data set and then by categorizing the sampled field data into size classes. Consequently, the average volume measured and the average areal extent for each size class are calculated. Finally, dry hole data is also added into the initial frequency-size distribution in order to reflect the exploration risk. After obtaining n replications of the frequency-size distribution that define the uncertainties in geological parameters, the distributions of total hydrocarbon discoveries for selected number of exploratory wells are constructed using the analytical approximation approach as described in my previous work.

The second part of the research extends the benefits of Manly's Approximation Method to estimate the distribution of number of discoveries. Subsequently, the distribution of number of discoveries and the distribution of total discovery volume from the first part together with economic parameters, such as the price of oil or gas, the costs of exploration drilling, development, and production are incorporated into the economic model in order to produce a probability distribution of the net present value of a proposed exploration program. The distribution on NPV can be used to produce confidence intervals, return measures, and risk measures for determining multiple-wells exploration strategies. In addition, the investigation of the influence on distributions of NPV of increasing uncertainty in field size with a constant mean is extended to the utility approach to see how these results affect the expected utility values of the exploration project. To illustrate the methodology developed, the Offshore Nova Scotia Shelf basin is selected for the study. This basin is suitable to the study because it is a partly explored, frontier basin lying off Canada's eastern seaboard which is currently an attractive area for exploration and development activities.

6.3 Contributions

The major contributions of this methodology for research are the extension of the benefits of Manly's Approximation Method to hydrocarbon discovery process modeling and the development of a complete package that goes from a frequency-size distribution through economic analysis. This work is especially valuable because of the paucity of work related to frontier exploration.

This methodology could be used by a company as a part of a planning system for projecting exploration programs. It would provide insight into how a company makes a forecast of future discovery volumes that includes uncertainties in geological parameters and how the results are used in long-term planning to determine future development programs for these hydrocarbon reserves. The resulting expenditures and forecast revenues from the model could be used to predict future income, cash flows, and profitability. On the basis of these results, the companies could set priorities and plan the allocation of exploration resources and manpower. The companies could plan the research needed to solve the technical problems associated with these basins. In addition, results from this methodology could assist government departments by supporting their efforts to establish the potential of hydrocarbons discoveries, to predict future exploration activity, and to aid in their analyses of policies concerning exploration programs regarding taxes and royalty regimes in any basin with various stages of exploration activity.

6.3 Limitations

The main limitation of this methodology is in the accuracy of the approximation model using a binomial distribution to represent the distribution of the number of discoveries. The accuracy of the model deteriorates as the number of exploratory wells increases (see the discussion in Subsection 5.4.5 and Appendix F).

In addition, the accuracy of the model predictions will depend on the accuracy of all geological and economic parameters involved. The economic sub-models used in the evaluations of the distribution of NPV are aggregated using the linear relationships between field size and costs. Assumptions regarding exploration and development as well as production decline characteristics are made to simplify the calculations and the situations when no detailed information is available.

This methodology can be applied to predict the distribution of total discovery volume as exploration progresses or the distribution of net present value of the exploration project in a frontier basin as well as the well-established basin where aggregated geological economic data are available. This methodology assumes the average depth of oil and gas fields. Therefore, it is suitable for a basin where field depth has a minor effect on the economic evaluations. However, this methodology is not suitable to be implemented in a basin where field depth become a major factor and where oil and gas fields are vertically superimposed on one another.

6.4 Suggestions for Future Research

1. As explained in Chungcharoen (1994), Subsection 5.4.5, and Appendix F, there are small errors of the means and the standard deviations between the simulation method and the approximation method. Since we use the means and the standard deviations of the Manly's Approximation Method to estimate the two shape parameters of the Beta distribution as well as the probability of success of the binomial distribution, these errors are also carried into the estimation. Therefore, a system approach to propagation of errors in the approximation method should be thoroughly investigated. To reduce these errors and increase the accuracy of the results of the approximation method, a regression model, as suggested by Ninpong (1992), might be used to adjust the approximated means and standard deviations in order that the Beta distributions and the binomial distributions can provide a better fit to the distributions of the total discovery volume and the distributions of the number of discoveries, respectively. In addition, the accuracy of the approximation could be improved by modifying parameters or the probability mass function of the binomial distribution as suggested by Sandiford (1960), Ord (1968a), and Johnson et al. (1992). An alternative approach to improve the accuracy of the approximation is to find a theoretical distribution that provides a better fit to the distribution of the number of discoveries than binomial distribution.

2. In this study, the exploration project is assumed to be conducted in three years starting at the beginning of exploration process in the basin. Several stages of exploration in the basin should be further investigated in order to completely demonstrate

the validity of this methodology throughout the exploration process. In addition, a more formal model evaluation including comparing model predictions with a few real data sets should be considered.(see the discussion on the model validity in Appendix G).

3. This methodology should be applied to other frontier basins in order to firmly support the usefulness of Manly's Approximation Method and the claim of using a family of Beta distributions to represent the approximate model of the total hydrocarbon discoveries as well as using a binomial distribution to represent the approximate model of the number of discoveries.

4. As we all know, developing a complete forecasting model for the hydrocarbon discovery process is a complicated task. There are several geological and economic parameters as well as their uncertainties involved in the model which make precise answers not possible. The accuracy of the model predictions will depend on the accuracy of all geological and economic parameters involved. In case of no directly available data, expert's opinions must be used. Assumptions have to be made in order to simplify the situations. Therefore, better access to the available geological and economic data would improve the predictive capacities of the model.

5. The economic sub-models presented in this research are intended to be basic models for the purpose of demonstrating the methodology. These models represent relationships between production decline characteristics and field size, price and revenues, and costs and field size due to fairly aggregated information. To perform a more detailed economic study, more elaborated detail of production decline characteristics, prices, costs,

and disaggregated field data would have to be fine-tuned. The details of revenue and cost models used in other regions might be different from the ones used here. Therefore, some modifications must be made. Moreover, the exploration and development schedules used in this thesis are fixed to simplify the calculations for the purpose of the demonstration. More complicated schedules such as taking the time rate of exploration as uncertain quantity and predicting this rate by using the forward-looking dynamic optimization approach as suggested by Kaufman et al. (1981) might be another subject for future study.

6. Applications of this methodology to the decision analysis should be further explored. It is often the case that there are several participants who co-invest development and production in frontier regions. Therefore, it is interesting to see how one could apply this approach to the investment decisions with different levels of participation.

7. The implication of two possible meanings for a measure of variability of field sizes, "real" versus "degree of ignorance", as described in Subsection 5.4.6.6 should be further investigated. Finally, this methodology could be applied to other size-biased search processes.

APPENDIX A

Approaches to Obtain the Average Distribution of the Number of Discoveries and the Average Distribution of Total Discovery Volume

As mentioned in Section 4.3.2.1.5, there are two alternative approaches to obtain the average parameters of the distribution of number of discoveries and the distribution of total discovery volume, as a result of incorporating geological uncertainties into frequency-size distribution, to be used in Steps 1 and 2 of the conditional sampling process. In the first approach, we determine the probabilities that the number of discoveries from J exploratory wells fall into 0, 1, 2,..., and J discoveries by summarizing the binomial mass function. After performing 500 frequency-size distributions, we average the probabilities at each number of discoveries and obtain the average distribution of number of discoveries. The average distribution of total discovery volume is determined in a similar way. For each frequency-size distribution, we generate the distribution of total discovery volume according to J exploratory wells and integrate the Beta density function between 0-500 b.c.f., 500 -1000 b.c.f.,..., etc., intervals to find the probabilities that the total discovery volume fall into these ranges. After performing 500 frequency-size distributions, we average the probabilities in each interval in order to obtain the average distribution of total volume discoveries. Finally, we determine the probabilities of success for the average binomial distribution, and determine the minimum, maximum, and 2 shape parameters of the average Beta distribution to be used in the conditional sampling process by fitting these average distributions to the theoretical binomial and Beta distributions using BestFit software.

The second alternative approach is performed by running the approximation method to obtain the means number of discoveries in each field size class and the means and standard deviations of the total discovery volume as exploration progresses for each frequency-size distribution for a particular exploratory wells. Instead of determining the probabilities in each interval like the first approach, we calculate the probability of success for the binomial distribution, and calculate the minimum, maximum, and the two shape parameters of the Beta distribution. After performing 500 frequency-size distribution data sets, we average these parameters for binomial distribution and Beta distribution directly. Consequently, we use these average parameters directly in Steps 1 and 2 for conditional sampling process.

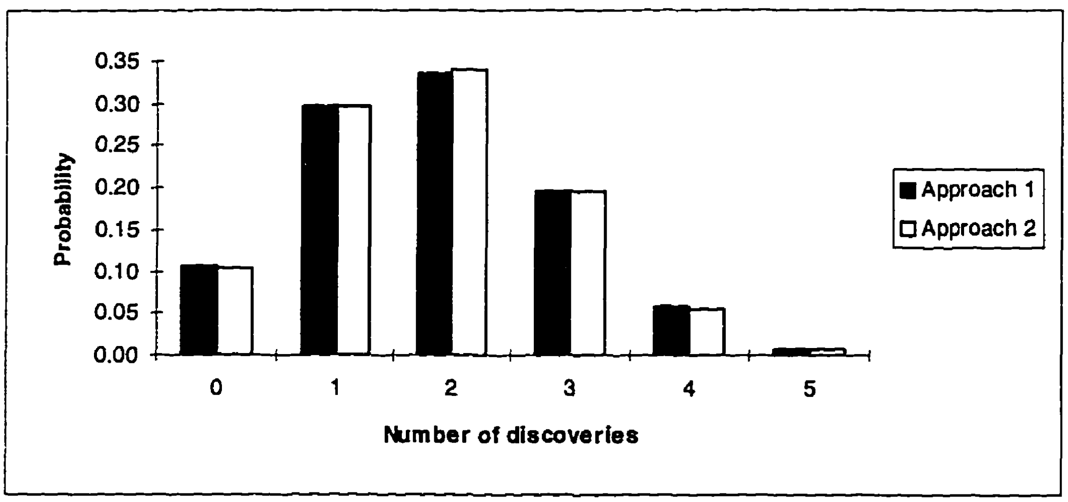
The results from comparisons of these two approaches show that the average binomial distribution and the average Beta distribution obtained from these two approaches are very close. In case of the binomial distribution and the standard Beta distribution (range from 0 to 1), the results of average parameters from both approaches give almost the same values in all cases. However, when the range of Beta distribution is transferred to a and b, the results from both approaches are slightly different. How close the results from both approaches are to each other depends on how close the shapes of 500 Beta distributions are to each other. Note that the second approach is a short-cut to obtain parameters for conditional sampling process in Steps 1 and 2. Therefore, running the main program with the second approach is faster than with the first approach as we do not need to perform the summation and the integration as well as to determine the parameters of these distributions in the last stage before using them in the conditional sampling process. The example of comparisons between these two approaches at price of \$3.80/m.c.f. and a real discount rate of 4.25%, where the minimum economic field size is size class 3 for 500 frequency-size distribution data sets, are shown below.

A.1 Binomial Distribution

Table A.1 Probability comparison of the distributions of the number of discoveries between Approach 1 and Approach 2 for 5 exploratory wells.

Number of	5 exploratory wells	
Discoveries	Approach. 1	Approach. 2
0	0.1069	0.1037
1	0.2963	0.2973
2	0.3364	0.3410
3	0.1953	0.1955
4	0.0580	0.0561
5	0.0070	0.0064

Figure A.1 Frequency comparison of the distributions of the number of discoveries between Approach 1 and Approach 2 for 5 exploratory wells.

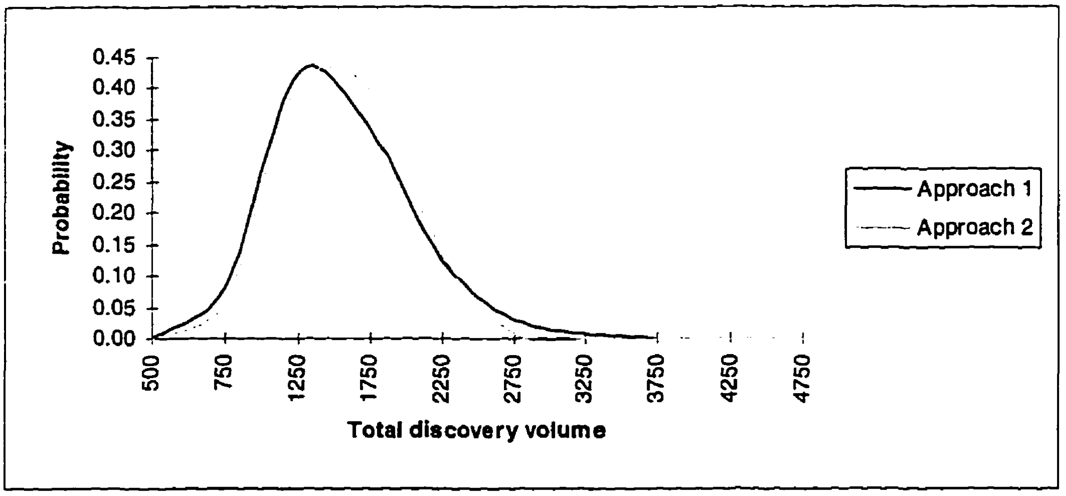


A.2 Beta Distribution

Table A.2 Probability comparison of the distributions of total discovery volume between Approach 1 and Approach 2 for 5 exploratory wells.

Total Volume (b.c.f.)	5 Discovery Wells	
	Approach 1	Approach 2
0000-0500	0.0000	0.000
0500-1000	0.0806	0.0571
1000-1500	0.4246	0.4073
1500-2000	0.3347	0.3955
2000-2500	0.1236	0.1330
2500-3000	0.0297	0.0071
3000-3500	0.0057	0.0000
3500-4000	0.0010	0.0000
4000-4500	0.0001	0.0000
4500-5000	1.36E-06	0.0000

Figure A.2 Frequency comparison of the distributions of total discovery volume between Approach 1 and Approach 2 for 5 discovery wells.



APPENDIX B

Examples of Frequency-size Distribution.

Iteration # 1

8811	.00000000	12.408780
63	42.129120	398.14410
25	138.32470	1053.4030
14	258.27750	1756.0370
8	336.30340	2179.5100
5	457.39180	2803.2510
1	507.32330	3051.3160
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000

Iteration # 2

8875	.00000000	12.408780
60	36.671290	355.40750
28	141.97730	1076.1140
7	246.78140	1691.8060
5	323.35440	2110.5870
5	449.38650	2763.0350
2	563.70760	3326.1710
2	666.89840	3816.7400
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000

Iteration # 3

8829	.00000000	12.408780
67	32.773780	324.18200
31	150.97820	1131.6330
13	246.04320	1687.6630
4	362.51280	2317.5670
1	411.45500	2570.6530
4	537.76140	3200.3440
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000

Iteration # 4

8840	.00000000	12.408780
62	38.807620	372.26460
28	143.74180	1087.0460
12	239.78780	1652.4660
3	336.88120	2182.5750
4	468.26440	2857.6700
2	571.04170	3361.5460
1	679.78910	3877.0120
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
0	.00000000	.00000000
1	1352.7000	6808.5950

APPENDIX C

Example of Manly's Approximation, Maximum Volume, and Two Shape Parameters.

Start running iteration # 1
Total number of fields = 116
Results from Manly-Approximation

5,	559.08870	, 332.29040	,
10,	1101.7450	, 457.46540	,
15,	1628.0670	, 545.11490	,
20,	2138.1690	, 612.07000	,
25,	2632.1890	, 665.05970	,
30,	3110.2850	, 707.64980	,
35,	3572.6330	, 742.03500	,
40,	4019.4290	, 769.70000	,
45,	4450.8890	, 791.71970	,
50,	4867.2470	, 808.91140	,

Maximum vol for 5-50 exploratory wells

5	2336.8910
10	4139.4960
15	5742.9860
20	7034.3730
25	8325.7600
30	9377.2400
35	10068.860
40	10760.480
45	11452.100
50	12143.720

2 Shape parameters

1.9143840	6.0873980
3.9903270	11.002200
6.1078510	15.437520
8.1901120	18.754580
10.395930	22.486970
12.578880	25.345350
14.600960	26.549370
16.710160	28.024890
18.932760	29.781080
21.292920	31.832670

APPENDIX D

Approach to Double the Standard Deviations of the Triangular and Weibull Distributions while Keeping their Means Unchanged

D.1 Triangular Distribution

The mean and the standard deviation of the triangular distribution are given by (Law and Kelton, 1991):

$$\mu = \frac{a+b+c}{3} \tag{D.1.1}$$

$$\sigma = \sqrt{\frac{a^2+b^2+c^2-ab-ac-bc}{18}} \tag{D.1.2}$$

where a, b, and c are the minimum, maximum, and mode, respectively. By substituting $a = 100$, $b = 130$, and $c = 113$, in Equations (D.1.1) and (D.1.2), we obtain

$$\mu_1 = \frac{100+130+113}{3} = 114.3333 \tag{D.1.3}$$

$$\begin{aligned}\sigma_1 &= \sqrt{\frac{100^2+130^2+113^2-(100)(130)-(100)(113)-(130)(113)}{18}} \\ &= \sqrt{\frac{679}{18}} = 6.1418\end{aligned} \tag{D.1.4}$$

By keeping the mean and the mode of the distribution unchanged and doubling the standard deviation, the new minimum and maximum values can be obtained as follows.

$$\begin{aligned}\mu_2 &= \mu_1 \\ &= 114.33 = \frac{a_2+b_2+113}{3}\end{aligned} \tag{D.1.5}$$

Therefore,

$$a_2 = 229.99 - b_2 \tag{D.1.6}$$

$$\sigma_2 = 2\sigma_1 = 12.2836 = \sqrt{\frac{a_2{}^2 + b_2{}^2 + 113^2 - ab - 113a - 113b}{18}}$$

$$150.89 \quad = \frac{(229.99 - b_2)^2 + b_2^2 + 113^2 - (229.99 - b_2)b_2 - 113(229.99 - b_2) - 113b}{18}$$

$$\begin{aligned} 2715.96 &= 52895.40 - 459.98b_2 + b_2^2 + b_2^2 + 12769 - 229.99b_2 + b_2^2 \\ &\quad - 25988.87 + 113b_2 - 113b_2 \\ 0 &= 3b_2^2 - 689.97b_2 + 36959.57 \\ 0 &= \ b_2^2 - 229.99b_2 + 12319.86 \end{aligned}$$

From $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, therefore

$$\begin{aligned} b_2 &= \frac{229.99 \pm \sqrt{52895.40 - 49279.42}}{2} \\ &= 145.06,\ 84.93 \ \approx \ 145,\ 85 \end{aligned}$$

By choosing $b_2 = 145.06$ into Equation (D.1.6), we obtain a_2 as follows.

$$a_2 = 229.99 - 145.06 = 84.93 \ \approx \ 85$$

Finally, the random variables are generated from Triang(85,113,145) to represent the the number of fields distribution in the case of double standard deviation.

D.2 Weibull Distribution

The mean and the standard deviation of Weibull distribution are given by (Law and Kelton, 1991):

$$\mu = \frac{\beta}{\alpha}\Gamma\left(\frac{1}{\alpha}\right) \tag{D.2.1}$$

$$\sigma^2 = \frac{\beta^2}{\alpha}\left\{2\Gamma\left(\frac{2}{\alpha}\right) - \frac{1}{\alpha}\left[\Gamma\left(\frac{1}{\alpha}\right)\right]^2\right\} \tag{D.2.2}$$

For $\alpha_1 = 0.869$ and $\beta_1 = 117.68$,

$$\begin{aligned}\mu_1 &= \frac{117.68}{0.869}\Gamma\left(\frac{1}{0.869}\right)\\ &= 135.4200(0.9328)\\ &= 126.3189\end{aligned} \tag{D.2.3}$$

$$\begin{aligned}\sigma_1^2 &= \frac{117.68^2}{0.869}\left\{2\Gamma\left(\frac{2}{0.869}\right) - \frac{1}{0.869}\left[\Gamma\left(\frac{1}{0.869}\right)\right]^2\right\}\\ &= \frac{117.68^2}{0.869}\left\{2\Gamma(2.3015) - \frac{1}{0.869}[\Gamma(1.1508)]^2\right\}\\ &= \frac{117.68^2}{0.869}\left\{2(1.1678) - \frac{1}{0.869}[0.9328]^2\right\}\\ &= 15936.2283\{2.3355 - 1.0013\}\\ &= 21262.1158\\ \sigma_1 &= \sqrt{21262.1158} = 145.8154\end{aligned} \tag{D.2.4}$$

By keeping the mean unchanged and doubling the standard deviation, we obtain

$$\begin{aligned}\mu_2 = \mu_1 = 126.3189 &= \frac{\beta_2}{\alpha_2}\Gamma\left(\frac{1}{\alpha_2}\right)\\ \beta_2 &= \frac{126.3189\alpha_2}{\Gamma\left(\frac{1}{\alpha_2}\right)}\end{aligned} \tag{D.2.5}$$

By substituting $\sigma_2 = 2\sigma_1 = 2(145.8154) = 291.6308$ into Equation (D.2.2), we obtain

$$(291.6308)^2 = (126.3189)^2 \alpha_2^2 \frac{1}{\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2} \frac{1}{\alpha_2} \left\{ 2\Gamma\left(\frac{2}{\alpha_2}\right) - \frac{1}{\alpha_2}\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2 \right\}$$

$$\frac{(291.6308)^2}{(126.3189)^2} = \frac{\alpha_2}{\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2} \left\{ 2\Gamma\left(\frac{2}{\alpha_2}\right) - \frac{1}{\alpha_2}\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2 \right\} \qquad \text{(D.2.6)}$$

$$5.3302 = 2\frac{\alpha_2}{\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2} \Gamma\left(\frac{2}{\alpha_2}\right) - 1$$

$$3.1651 = \frac{\alpha_2}{\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2} \Gamma\left(\frac{2}{\alpha_2}\right) \qquad \text{(D.2.7)}$$

Therefore,

$$\frac{\alpha_2}{\left[\Gamma\left(\frac{1}{\alpha_2}\right)\right]^2} \Gamma\left(\frac{2}{\alpha_2}\right) - 3.1651 = 0 \qquad \text{(D.2.8)}$$

By selecting α_2 that satisfies Equation (D.2.8), we are able to obtain the value of α_2. By substituting this α_2 value into Equation (D.2.5), we obtain β_2. The final solutions are $\alpha_2 \approx 0.489$ and $\beta_2 \approx 60.5670$. Note that the value of gamma function $\Gamma(\cdot)$ is determined by using MAPLE software. Finally, the random variables are generated from Weibull(0.489,60.5670) to represent the the field size distribution in the case of double standard deviation.

APPENDIX E

Flowcharts of the Main and Subprograms.

Figure E.1 Flowchart of the subprogram creating frequency-size distribution.

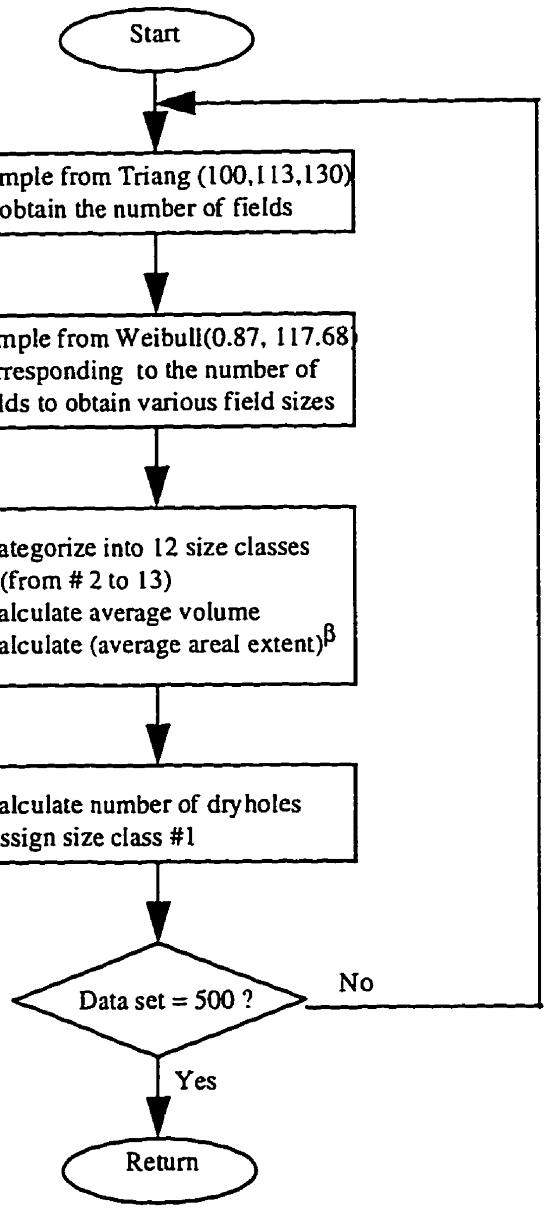


Figure E.2 Flowchart of Manly1 subprogram.

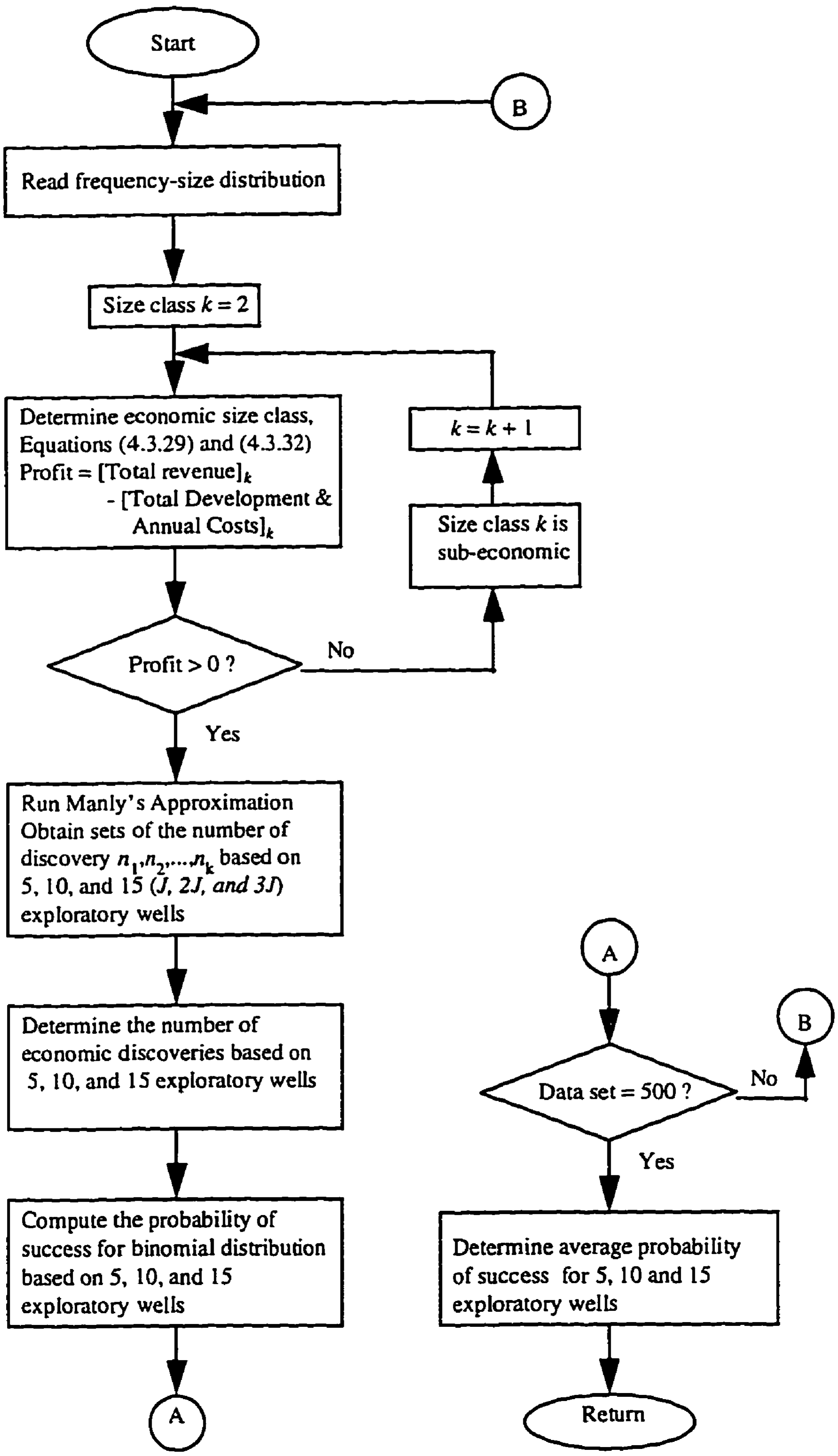


Figure E.3 Flowchart of Manly2 subprogram.

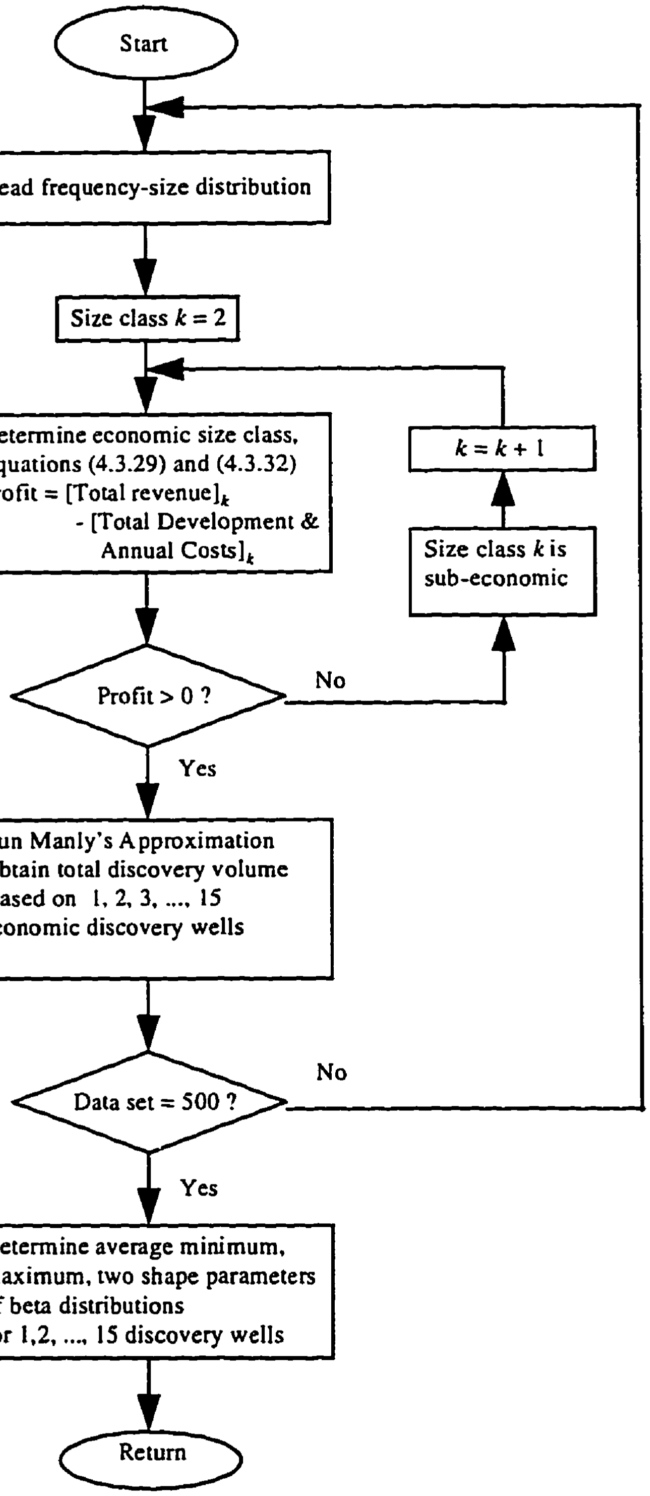
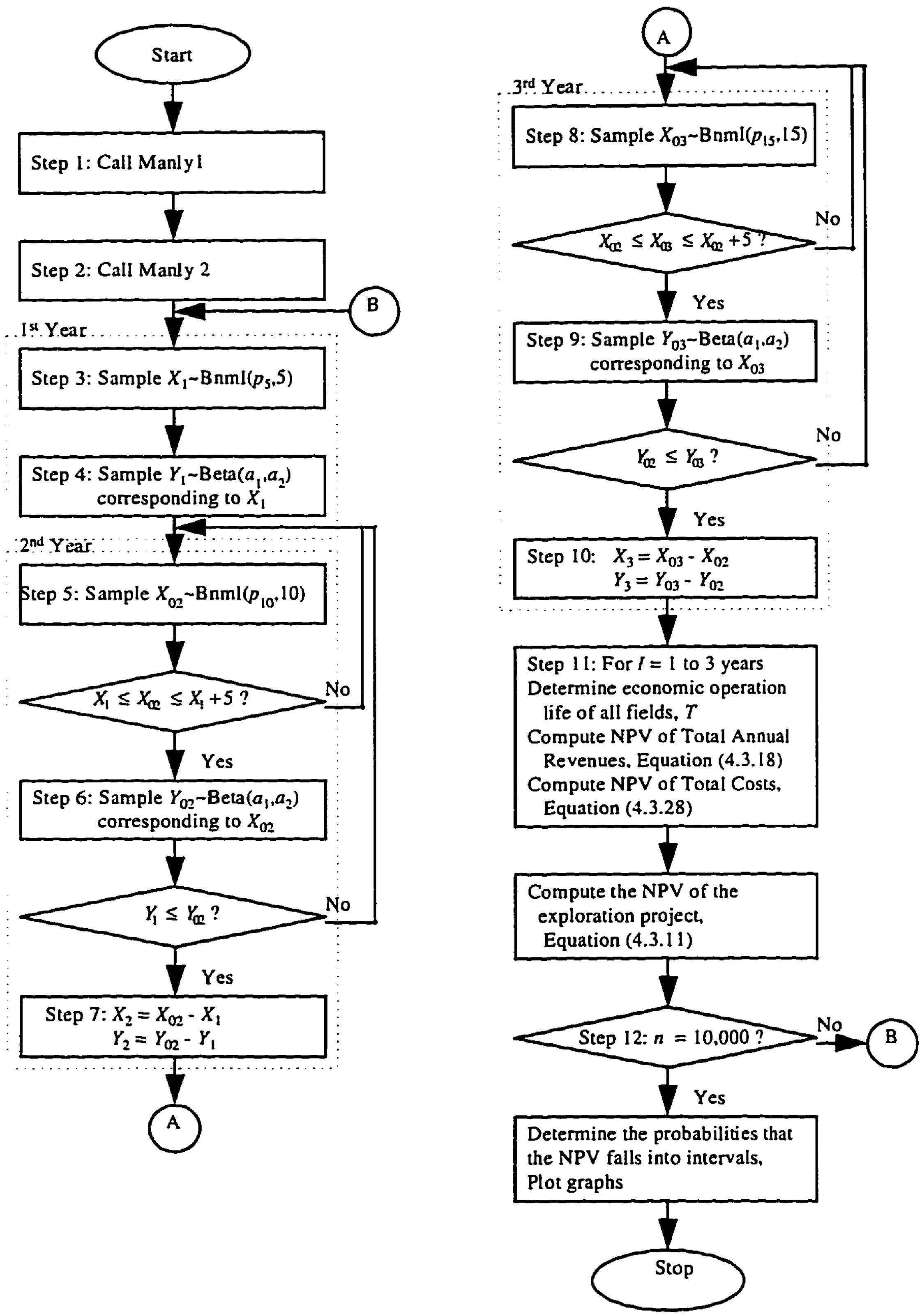


Figure E.4 Flowchart of the main program.



APPENDIX F

Propagation of System Errors

To consider the propagation of system errors, we consider 2 key components in our methodology: the distribution of total discovery volume and the distribution of number of discoveries.

As discussed in Chungcharoen (1994) and in Section 5.4.5, it is known that Manly's Approximation Method produces small errors on the order of 1% for the mean and as much as 5% for the standard deviation (Ninpong, 1992). Chungcharoen (1994) also showed that the percent differences between the means of the simulation method and the means of Manly's approximation Method are within 0.5%, and the percent differences between the standard deviation of the two methods are within 5% for 5 to 80 discovered fields. These percent differences are the errors carried into the estimation of the parameters of the distribution of total discovery volume. Since we use such a large sample size, there is a high probability that the goodness-of-fit test will result in rejecting the null hypothesis. However, the results from frequency comparisons, descriptive statistics, and goodness-of-fit tests still show that the Beta distribution is a good fit to the distribution of total discovery volume obtained from the full simulation results. Note that the same small errors are also carried into the approximation of the distribution of the number of discoveries. However, as evidence has shown, these errors contribute only a minor part in the error of the distribution of the number of discoveries.

To further investigate the propagation of system errors in our model, we compare the distribution of total discovery volume obtained without the conditional sampling procedure (the result from Case 1) to the distribution of total discovery volume obtained from the conditional sampling procedure (the result from Case 4). Note that in the case without the conditional sampling, we assume that all 15 exploratory wells are drilled in the same year. The parameters of Beta distributions for 5, 10 , and 15 exploratory wells in this case are given in Table F.1. In the case with the conditional sampling, we assume that 5 exploratory wells are drilled in each year for three years of exploration. We also increase the price of natural gas in this case to a high enough level (e.g. $100/m.c.f.) so that we can compare the two cases directly without including the effect of sub-economic field size classes. The parameters of binomial distributions for 5, 10, and 15 exploratory wells and of Beta distributions for 1 to 15 discovery wells are given in Tables F.2 and F.3, respectively. Recalling the conditional sampling procedure, we sample from the binomial distribution for J_x exploratory wells (J_x = 5, 10, and 15) to obtain the number of discoveries, and, then, sample from the corresponding Beta distribution to obtain the total discovery volume. The conditional sampling procedure also follows constraints imposed in the 2nd and 3rd year as described in Subsection 4.3.2.1.5.

Figure F.1 shows the distributions of total discovery volume for 5, 10, and 15 exploratory wells: (a) without the conditional sampling procedure and (b) with the conditional sampling procedure. Notice that Figure 1 (a) is similar to Figure 5.2 in Case 1 (The slightly difference between these two figures occurs because Figure 5.2 is the result obtained from using Approach 1 and Figure F.1 (a) is the result obtained from using Approach 2 as explained in Appendix A).

Table F.1 Minimum, maximum, and two shape parameters of the Beta distributions representing the distributions of total discovery volume for 5, 10, and 15 exploratory wells.

Number of	Total Volume Range		Shape Parameters	
Exploratory wells	Minimum	Maximum	p	q
5	0.0	2914.00	1.5169	5.8038
10	0.0	4762.48	3.1867	9.7454
15	0.0	6196.99	4.8976	12.7781

Table F.2 Probabilities of success for binomial distributions representing the distributions of the number of discoveries for 5, 10, and 15 exploratory wells.

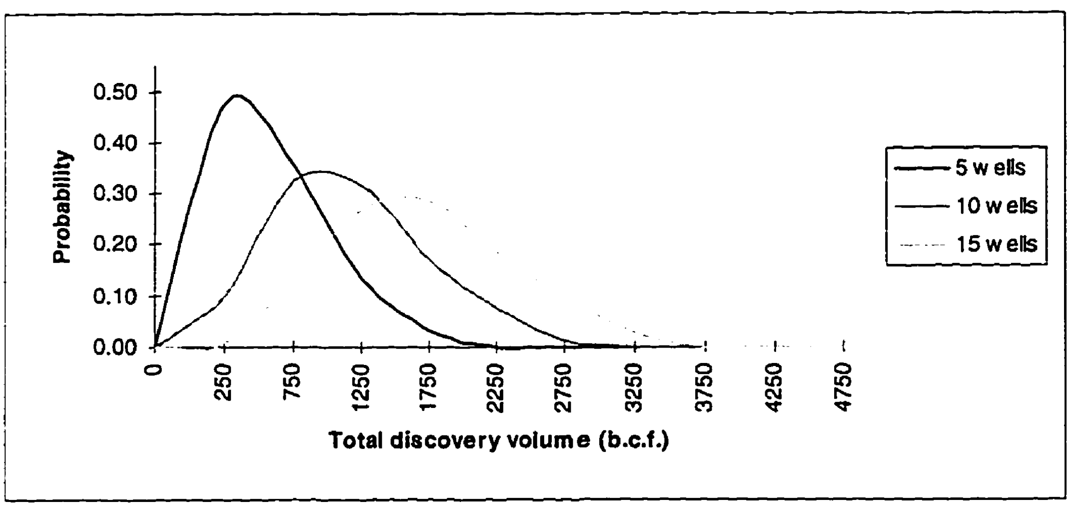
Number of Exploratory Wells	Probabilities of success
5	.481945
10	.477238
15	.472536

Table F.3 Minimum, maximum, and two shape parameters of the Beta distributions representing the distributions of total discovery volume for 1 to 15 economic discovery wells.

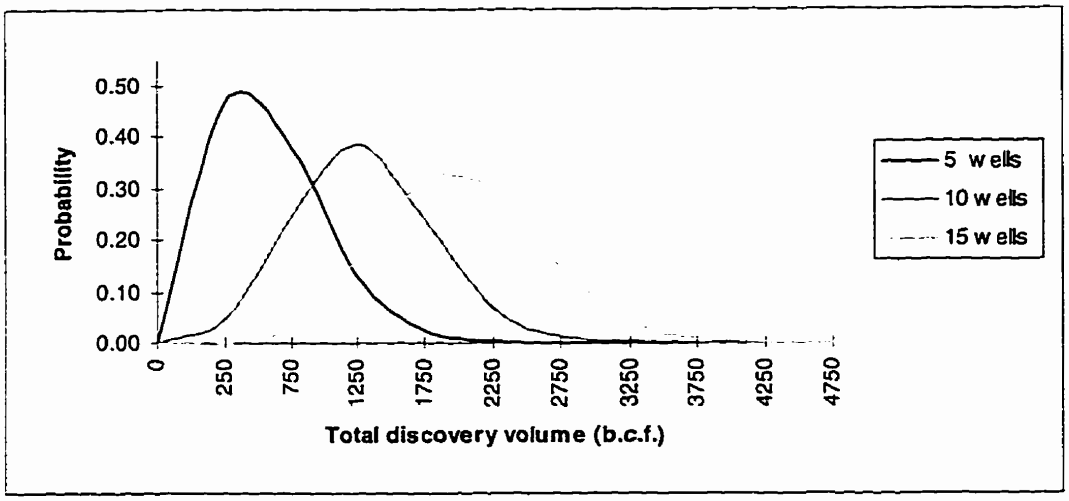
Discovery Number	Total Volume Range		Shape Parameters	
	Minimum (b.c.f.)	Maximum (b.c.f.)	p	q
1	39.4275	793.8480	0.4898	1.2163
2	78.8550	1422.7400	1.2195	2.5995
3	118.2825	1970.1600	1.9380	3.6971
4	157.7100	2463.7400	2.6527	4.6210
5	197.1376	2914.0000	3.3649	5.4120
6	236.5651	3331.8400	4.0765	6.1025
7	275.9926	3721.1000	4.7894	6.7112
8	315.4200	4084.8600	5.5025	7.2431
9	354.8478	4431.5300	6.2227	7.7339
10	394.2753	4762.4800	6.9507	8.1874
11	433.7027	5076.7300	7.6844	8.5986
12	473.1302	5374.5800	8.4235	8.9660
13	512.5578	5659.4900	9.1716	9.3055
14	551.9851	5933.7900	9.9318	9.6267
15	591.4129	6196.9900	10.7016	9.9217

Figure F.1 Distributions of total discovery volume from 5, 10, and 15 exploratory wells.

(a) without the conditional sampling procedure



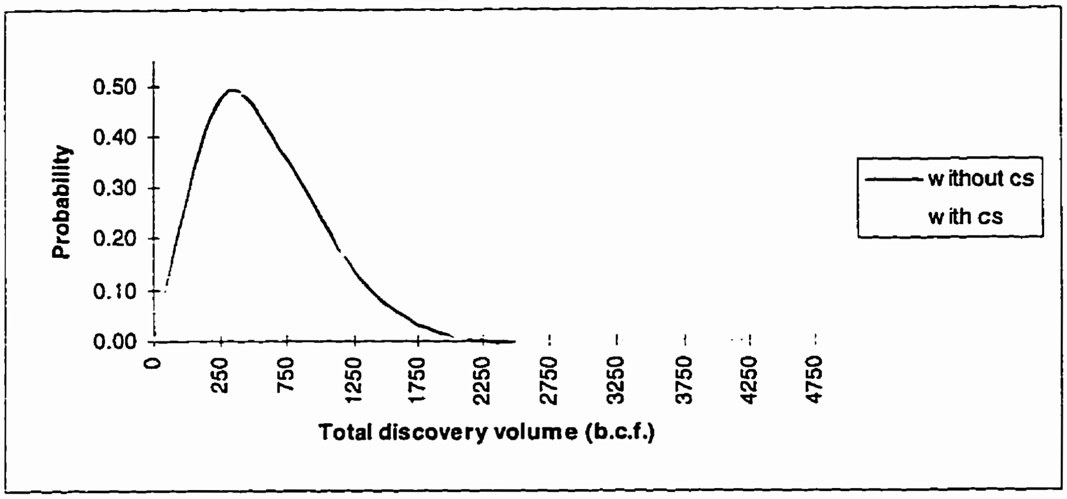
(b) with the conditional sampling procedure.



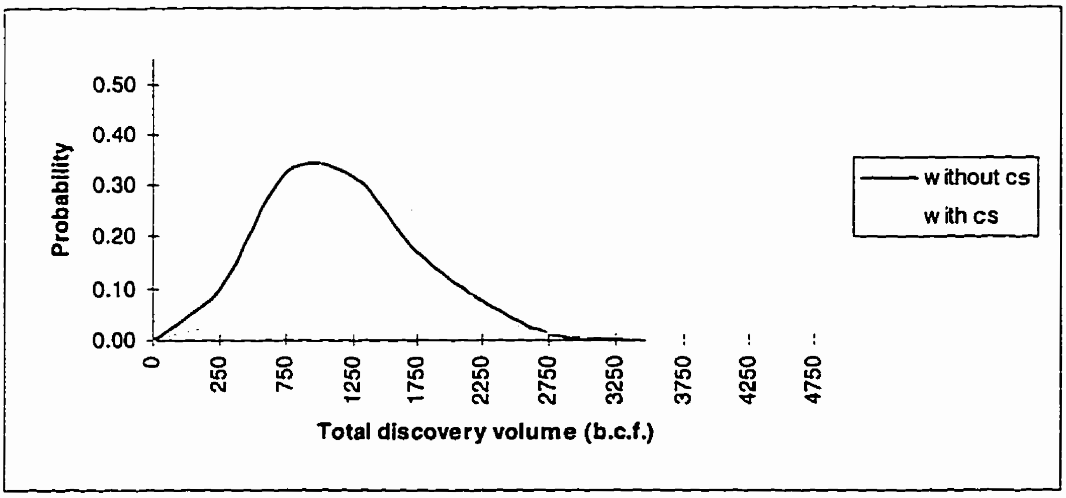
To see the difference between the distributions of total discovery volume in Figure F.1 (a) and in Figure F.1 (b) clearly, we compare these two distributions by the number of exploratory wells. Figures F.2 (a), (b), and (c) show the comparison between the distributions of total discovery volume without the conditional sampling procedure and with the conditional sampling procedure for 5, 10, and 15 exploratory wells, respectively.

Figure F.2 Comparison of the distributions of total discovery volume obtained without the conditional sampling (cs) procedure and with the conditional sampling procedure.

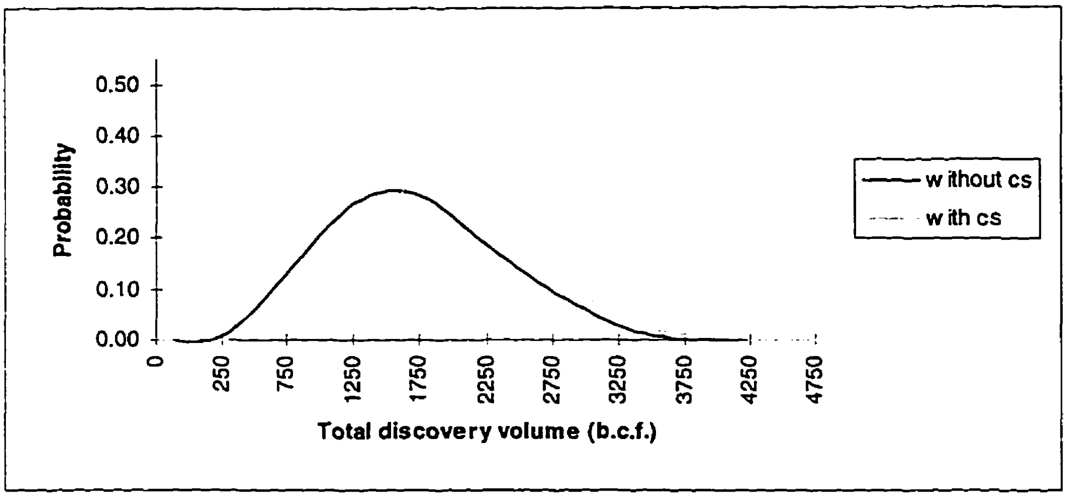
(a) 5 exploratory wells



(b) 10 exploratory wells



(c) 15 exploratory wells



Theoretically, the distributions of total discovery volume obtained without the conditional sampling procedure and with the conditional sampling procedure should be identical as both of them are the distributions of total discovery volume that incorporate the risk of dryholes resulting from J_x exploratory wells (J_x = 5, 10, and 15). In Figure F.2 (a), the distributions of total discovery volume with and without the conditional sampling procedure for 5 exploratory wells are almost identical. These results confirm the robustness of the conditional sampling procedure in the case of a small number of exploratory wells. However, the difference between these two distributions becomes greater when the number of exploratory wells is increased as in the cases of 10 and 15 exploratory wells in Figures F.2 (b) and (c), respectively.

The main source of inaccuracy of the approximation obtained from the conditional sampling procedure originates from the discrepancy in the assumption of using the binomial distribution to represent the distribution of number of discoveries in the hydrocarbon discovery process. As mentioned in Subsection 5.4.5, the hydrocarbon discovery process is a sampling without replacement process from a finite population that follows a non-central multivariate hypergeometric distribution. On the other hand, the binomial distribution follows a sampling with replacement process which requires that the probability of success in each sample be constant. There is also no weight (various sizes of the field) involved in the calculations of the probability of success. Therefore, the accuracy of the approximation will deteriorate as the number of exploratory wells or the number of sub-economic field size classes increases as compared to the total number of economic fields in a basin. The approximation gives satisfactorily accurate results when the number of exploratory wells is not larger than 10 percent of the number of economic fields in a basin (see Subsection 5.4.5).

From our investigation, we can see that the assumption of using a binomial distribution to represent the distribution of the number of discoveries produces accurate results for a small number of wells (e.g. 5 exploratory wells in our case). If the exploration project involves drilling a small number of exploratory wells, our methodology should provide satisfactorily accurate estimates of the distributions of total discovery volume to be used in the economic sub-model in order to determine the distribution of NPV and the expected utility value of the exploration project. However, if the exploration project involves a large number of exploratory wells over a long period of time, we must be careful in implementing our model. In this case, the accuracy of the approximation must be improved. Methods suggested by Sandiford (1960), Ord (1968a), and Johnson et al. (1992) should be implemented or another theoretical distribution that provides a better fit to the distribution of the number of discoveries than the binomial distribution should be used. These two improvement issues are subject to future research.

APPENDIX G

Model Validity

Manly's Approximation Method has been used in hydrocarbon discovery process modeling for approximating the mean and the standard deviation of the total discovery volume by several researchers before (see Fuller, 1991, Fuller and Wang, 1991, Macdonald, 1992, Ninpong, 1992, and Macdonald et al., 1994). The comparisons between the results from this method and the results from simulation showed that Manly's Approximation Method is very accurate. In addition, Chungcharoen (1994) has further investigated the approach of using the results from Manly's Approximation Method to determine the two shape parameters of a Beta distribution that represents the distribution of total discovery volume by using several real data sets. His results supported the conclusion that this approach is a robust approximation. Therefore, the validity and credibility of Manly's Approximation Method are clearly seen.

In this research, the benefit of Manly's Approximation Method is extended to the approximation of the distribution of total discovery volume that incorporates the uncertainty in geological parameters and the distribution of the number of discoveries in order to determine the distribution of NPV of the exploration project. In the simulation program implementing the methodology, the binomial random numbers in the sampling process are generated by the RNBNML function in the Mathematical Function Library for Microsoft FORTRAN, MAF3.LIB, (United Laboratories, 1989). The Beta random numbers are generated by the BETARN subprogram that uses Cheng's (1978) BB algorithm. This subprogram also uses the G05GAF and G05CBF functions supported by the NAG FORTRAN Libraries, NAGG and NAGX, from the Numerical Algorithms Group (1983) to generate uniform random numbers. Since the random numbers are generated by functions in commercial FORTRAN libraries, the independence among random variables can be warranted. In addition, the distributions of the number of discoveries and the distributions of total discovery volume for 5, 10, and 15 exploratory wells have been compared to the theoretical binomial and Beta distributions by using BestFit software from Palisade (1995). The results of comparisons using histograms and formal statistical tests, such as chi-square and K-S tests, show that the results fit the theoretical distributions. Notice that only the distribution of number of discoveries in the 1st year is binomial. The conditional distributions of number of discoveries in the 2nd and 3rd year are no longer binomial as they are the by-products of the conditional sampling process. The distributions of total discovery volume in each of three years are the by-products of the sampling process from the 15 Beta distributions which represent the approximate distributions of total discovery volume from 1 to 15 discovery wells. In addition, the programs have been run under a variety of input parameters to check that the results are reasonable. Interim results have been compared with similar results obtained from manual calculations and the results from commercial statistical software, such as MAPLE and Bestfit. The results support the validation of our program.

To validate whether the results obtained from our methodology accurately represent the results from the real exploration activity in offshore Nova Scotia, two major factors must be taken into consideration. First, there is currently no real development and production of Nova Scotia "significant" discovered gas fields, except for the small volume of oil produced at the Panuke and Cohasset fields. There is also no existing main pipeline transportation system to transport the natural gas from these discovered fields onshore (the small amount of oil produced from the two fields is transported to shore by ship). The possible Sable Island Offshore Energy Project is also under consideration. Therefore, there is no way to prove that the results forecasted by our model are absolutely correct unless there exists the real exploration, development, and production of economic offshore Nova Scotia fields. Once production is started, we must then wait for the end of the operation life of these discovered fields so that we can assess the accuracy of our prediction. Note that, depending on economic conditions, the operation life of each field could extend more than 20 years before production is cut off.

Second, the economic assumptions used in our methodology, such as exploration and economic schedules and production decline characteristics, have been made to simplify the situations. Different exploration companies may have their own working schedules due to their available capital resources and interests. These assumptions are also made based on selected public information regarding current and future economic conditions. This information is also varied from one source to the another. In addition, the real economic conditions will, of course, be changed over time depending on the world and regional supply and demand conditions of hydrocarbons.

From the above, we can see that validating the results of an exploration project from our model is a very difficult task (if possible at all). In terms of validating the results of the distribution of total discovery volume and the distribution of number of discoveries without considering the economic evaluations, there are also several obstructions as follows.

First, available information obtained from the evaluations from both public and private institutions for both frontier and well-established basins are probabilistic in nature. This is because there is no way to know exactly how much hydrocarbons exist within a basin and in each field. The recoverable hydrocarbon's potentials are estimated based on testing geological and geophysical properties of oil and gas fields. These estimates come with uncertainty involved in determining geological and geophysical parameters. In addition, there is evidence of a phenomenon called "field growth" in several basins in the United States as a result of the investigation by Drew and Schuenemeyer (1992) and Root (1982). Thus, there is no "exact" information to use for comparison purposes.

Second, the public information regarding discovered fields in offshore Nova Scotia is not completed. Many "gas show" discovered fields that are sub-economic due to the current economic condition are not further tested and evaluated. As a result, there is no information on these fields regarding total hydrocarbon potential. Some available public data are aggregated data as a result of combining several "significant" fields in that area. Data evaluated by private companies are usually kept confidential for their own uses.

Third, several organizations use different approaches and sources of information in estimating the hydrocarbon discovery volume. Therefore, their forecasts are varied with wide ranges. Considering information on seventeen point estimates of aggregated recoverable oil in US beyond know reserves made from 1965-1974 obtained by Cook (1975), the highest estimate was fifteen times the lowest. According to Smith (1980), the estimates of reserves in the North Sea from British Petroleum, Conoco, Mobil, and Shell are range from 38.0 - 67.0 billion barrels. The results of predictions by Power's (1990) model was differentiated from that of the Nova Scotia Department of Mines and Energy and the Canadian Oil and Gas Lands Administration due to unpredictable factors. However, he stated that these do not reduce the credibility of his model based on expert opinion. In the publication of the Technical Summaries of Scotian Shelf Significant Commercial Discoveries in 1997, the Canada Nova Scotia Offshore Petroleum Board (C-NSOPB) also pointed out in their assessment of the recoverable hydrocarbon resource that their information, while believed to be accurate, is not warranted to be so. In addition, it should be appreciated that many of the statements contained in the publication, particularly with respect to technical matters, are based on assumptions, opinions or interpretations. To summarize the variation of results from different approaches, Barouch and Kaufman (1976) stated that "difference in the amount and quality of geological information employed and in its interpretation, differences in the economic and technical scenarios implicitly or explicitly assumed, and differences in the methods employed for computing forecasts account in part for this enormous range".

From the above reasons, we recognize the difficulties in the validation of the results of our methodology due to the incomplete available information. Changes in the perceptions of oil and gas companies as well as government sections due to changes in economic conditions and technology will have effects on future exploration, development, and production of hydrocarbons. However, the model proposed here allows all kinds of experiments to be conducted. One may easily examine a wide variety of situations before appropriate policies will be deployed. Note that the distribution of total discovery volume or the distribution of NPV of the exploration project is, necessarily, broad, but the reality after the exploration will be narrow as it is just one realization of what could have existed in a basin.

Based on the statistical robustness of the model and reasonable results, there are rational grounds to believe that our model is valid. The alternative approach of validation could be done by comparing the results of our methodology to the results obtained by running a full simulation. However, this is a very time consuming process. Due to the limited time of this study, the alternative validation approaches are suggested for future research.

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