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Vortex break-down during the impact of a starting subsonic compressible gas jet on a multi-plume spray

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Abstract The impact process and the consequent two-phase interaction for a compressible subsonic starting gas jet colliding on a multi-plume spray are investigated using large eddy simulation with Eulerian/Lagrangian multiphase approach, and the λ_2 criterion is used to visualize the temporal and spatial evolution of the vortical structures in the gas field. It is shown that before the impact a leading tip vortex ring is followed by smaller vortex rings in the quasi-steady region of the starting jet while the vortical structures inside the spray plumes known as spray-induced air jets are formed. After the impact the leading tip vortex ring and the following rings as well as spray-induced air jet vortices start to deform and eventually break down into smaller elongated vortex filaments. Unlike the injection of multi-plume sprays into the core of a steady cross flow gas jet, spray droplets are dispersed in a larger volume in all directions when impacted by the starting gas jet, beneficial for two-phase mixing enhancement. A pair of vortex rings is also observed merging into a new ring before reaching the impact zone.

Keywords: *S*pray, large eddy simulation (LES), starting jet, vortex ring

1 Introduction

One of the main objectives of producing liquid sprays in gaseous media is to provide fine droplets and enhance the mixing efficiently. While ambient and injection conditions such as pressure (Roisman et al, 2007) and temperature (Park et al, 2010) control the physical processes in a single plume spray in stagnant ambient, additional factors become important in the case of multi-plume sprays or when a cross flow is present. When multi-plume spray arrangements are implemented in stagnant ambient air, the main difference is associated if the individual plumes are interacting (Ghasemi et al, 2014) or evolve independently (Cárdenas et al, 2009; Eagle et al, 2014). However, in non-quiescent environments a cross flow imposed on single-plume sprays is found to deflect the spray axis, deform the cross section and eventually promote the atomization by introducing additional instabilities to the liquid column (Leong et al, 2000; Mashayek and Ashgriz, 2011; Desantes et al, 2006; Amighi et al, 2009; Kim et al, 2010; Costa et al 2006). The presence of multi-plume sprays creates more complicated flow features such as sheltering effect of sprays on each other (Yu et al 2006).

The present study is part of our work on multi-plume sprays in quiescent and steady cross airflow jets. In the quiescent ambient the multi-plume sprays evolve independently and behave similarly to single-plume sprays because of the large spacing between the plumes. When injected into the core of a steady turbulent compressible subsonic air jet, the multi-plume sprays merge into a single plume and are deflected downstream. However, in practice sprays may be impacted by the tip of a transitioning starting jet rather than being injected into the core of a fully developed steady jet. The presence of a leading tip vortex in starting jets (Kruegera and Gharib, 2003) results in a significantly different flow-field compared to steady jets (Ghasemi et al, 2013). In the present study, Eulerian/Lagrangian large eddy simulation (LES) is used to study the impact of a compressible turbulent subsonic starting jet on multi-plume sprays. Gas/liquid interaction in terms of formation, evolution and break-down of gas phase vortical structures and their effect on liquid droplet dispersion is investigated. Turbulent compressible gas jet is started from a circular orifice with a Mach number of Ma = 0.58 and a Reynolds number of $Re = 2.7 \times 10^5$ at the orifice exit. The multi-plume sprays are issued with the injection pressure of $P_{inj} = 15\,MPa$ from six holes elliptically distributed on the injector.

2 **Model Formulation**

Lagrangian-Eulerian (LE) multiphase approach is implemented to account for the interaction of liquid spray with the ambient gas (Subramaniam, 2013). Liquid droplets are issued into the domain as Lagrangian discrete particles while the Eulerian definition of the continuous phase accounts for the transport of spatial x_i $(x_1 = X, x_2 = Y, x_3 = Z)$ and temporal (t) transport of mass, momentum and energy, which are solved using a control volume method. Large scale variables of continuous phase transport equations are directly resolved using large eddy simulation (LES) (Pope, 2000). Flow scales smaller than the filter width are evaluated using a dynamic Smagorinsk-Lilly sub-grid scale model (Pope, 2000). The information obtained based on the calculations for the resolved field is implemented in dynamic updating of Smagorinsky model constant (Smagorinsky, 1963) according to Germano et al. (1991) and Lilly (1992). For the LES of compressible flows, any flow variable can be Favre-averaged (density weighted) as $\tilde{\varphi} = \frac{\overline{\rho\varphi}}{\bar{\rho}}$ where ρ is the density. The resolved field unsteady compressible viscous Navier-Stokes equations for mass, momentum and energy are:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho}\widetilde{u_i})}{\partial x_i} = S_m \qquad (1)$$

$$\frac{\partial \bar{\rho}\widetilde{u_i}}{\partial t} + \frac{\partial \bar{\rho}\widetilde{u_i}\widetilde{u_j}}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} - \frac{\partial \bar{\rho}\tau_{ij}^{r}}{\partial x_j} + F \qquad (2)$$

$$\frac{\partial \bar{\rho}\tilde{e}}{\partial t} + \frac{\partial \widetilde{u_j}\bar{\rho}\tilde{e}}{\partial x_j} = -\bar{p}\frac{\partial \widetilde{u_j}}{\partial x_j} + \bar{\sigma}_{ij}\frac{\partial \widetilde{u_i}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(K\frac{\partial \tilde{T}}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}D_a \sum_m h_m \frac{\partial \widetilde{y_m}}{\partial x_j}\right) + S \tag{3}$$

where u_i and $\bar{p}$ present the continuous phase velocity vector and the resolved pressure, respectively. Gas and liquid phases exchange mass, momentum and energy through the source terms S_m, F and S, respectively. In equation (3), e, D_a, K, h_m and y_m describe the specific internal energy, diffusion coefficient, gas thermal conductivity, specific enthalpy and species mass fraction, respectively. Second order accurate central scheme is implemented to discretize the convective and diffusion terms of the transport equations in space. Pressure is coupled with density by the equation of state and corrected by the pressure implicit with splitting of operators (PISO) scheme (Issa, 1986).

Time derivatives are discretized using Crank-Nicolson scheme which is second order accurate. Time step resolution can be limited by maintaining the Courant–Friedrichs–Lewy (CFL) criterion. Liquid injection velocity can be estimated using $(U_{inj} = \sqrt{2(P_{inj} - P_o)/\rho_l})$. Selecting a time step $\delta\mathrm{t} = 1 \times 10^{-7}\, s$, CFL number corresponding to the convection of liquid spray can be obtained as $CFL = \frac{U_{inj}\,\delta\mathrm{t}}{\Delta} = 0.17$ where Δ is the LES filter width. For the convection of gas phase, the above time step gives $CFL = \frac{U_j\,\delta\mathrm{t}}{\Delta} = 0.18$ based on the jet velocity at the nozzle exit U_j.

The liquid spray droplets are discretely tracked using the following Lagrangian equation of motion:

$$\frac{du_p}{dt} = \frac{3}{4}\frac{\rho_a}{\rho_l}C_D\frac{|u_{rel}|}{d_p}\,u_{rel} + g_i + F_x \tag{4}$$

where u_{rel} is the relative velocity between the gas and liquid phase, and C_D is the drag coefficient. Gravity is accounted for by g_i and F_x is the source term, including any other forces that might be present in the flow system. Various sub-models are used to account for the droplet evaporation, collision, break-up and turbulent dispersion (Baumgarten, 2006).

3 Numerical

Eulerian/Lagrangian LES of the present multiphase problem is carried out using CONVERGE CFD solver. Compressible subsonic turbulent air jet is issued from a circular orifice into the computational domain shown in Figure 1. While expanding with downstream distance the computational domain extends 16 times jet diameter (D_j), or 1951 spray nozzle diameter (D), downstream. It should be noted that the spray injector includes six nozzles each with a diameter (D) distributed in an elliptic pattern to produce the six spray plumes in different orientations. Axis of the injector (NOT sprays) is oriented normal to the jet shear layer at the edge of the air jet and a downstream distance of $X/D_j = 1.36$.

Initially generated Cartesian base grid is refined during the simulation using adaptive mesh refinement (AMR) which is triggered by the threshold set for the sub-grid scale of flow quantities (Bedford and Yeo, 1993; Pomraning, 2000). Proper resolving of the small scales using LES can be conducted by reducing the filter width Δ as low as 12 times Kolmogorov micro scale (Pope, 2006). For the present study this would be a very cumbersome task since the liquid sprays penetrate several hundreds of spray nozzle diameters. Alternatively a grid resolution of $\frac{D}{\Delta} = 1.32$ is adopted, following the suggestions of Senecal et al. (2013) for grid requirements in the Lagrangian/Eulerian LES of sprays who suggested a spray nozzle to filter width ratio of $\frac{D}{\Delta} = 0.72 - 1.44$ to adequately resolve the spray flow field.

Fig. 1 Computational domain: dimensions in terms of gas jet orifice diameter (D_j) and spray nozzle diameter (D)

4 Experimental and numerical and flow visualization

The flow structures obtained from the present study includes experimental visualization of the multi-plume sprays and numerical visualization of the spray and the cross flow gas jet.

In the experiments, multi-plume sprays are generated by the fuel delivery system and using a commercial six-hole injector with elliptic nozzle distribution. A volumetric illumination approach is conducted which uses triggered-stroboscopic lighting to illuminate the entire viewable spray surface.

In the simulations, the liquid phase associated with the spray formation is simply presented by a white color cloud of droplets. The formation and evolution of the spray clouds are properly marked with arrows in the Figures of the following sections. On the other hand, visualization of the gas field vortical structures demands further considerations. Perhaps the most intuitive definition of a vortex is a region of the flow in which fluid particles rotate around a common center causing a low pressure region (Robinson, 1991). However, it would be very difficult to attribute an appropriate value of pressure to identify the vortices. Another commonly used technique is to implement instantaneous vorticity fields which suffer from not being able to distinguish between irrotational shear and pure rotation of the flow elements (Robinson, 1991). For instance, in order to distinguish between shear layer and the vortex cores formed in a turbulent jet, Ghasemi et al (2013) compared vorticity field and swirling strength criterion. For the present study, the λ_2 criterion proposed by Jeong and Hussain (1995) is implemented to identify the vortical structures in the gas field. To this end, a gradient is operated on the Navier-Stokes equations to obtain the acceleration gradient tensor. The antisymmetric component of the acceleration gradient tensor provides information on the vorticity transport while the symmetric part represents pressure field. The λ_2 criterion associates the vortex cores with local pressure minimum, while the contributions of unsteady irrotational straining and viscous terms are neglected in the symmetric component of the acceleration gradient tensor. This results in real eigenvalues for the symmetric component as $\lambda_1, \lambda_2, \lambda_3$. Assuming $\lambda_1 \geq \lambda_2 \geq \lambda_3$, the flow region with $\lambda_2 < 0$ is defined to characterize the vortex core.

5 Results and discussion

Before starting the discussion of the results it would be helpful to briefly explain different cases of spray and gas jet injection in experiments and the simulations. For the first case, the multi-plume spray is injected into a quiescent ambient reaching its maximum flow rate in a short time. For the instant shown in the present study, experiments and simulations maintain the maximum flow rate. In the second case, both of the experiments and simulations allow for the gas jet to reach a steady state and then the spray injection into the core of the jet is started. For the third case which is purely simulations, both the starting gaseous jet and the multi-plume spray are issued into the domain at the same instant and maintain their maximum flow rate during the simulation time.

For comparison, a sample shape of the multi-plume spray is visualized first in the quiescent ambient air as well as in the steady cross airflow jet. Then, the temporal evolution of the multi-plume spray impacted by the starting jet is presented for three time intervals: before the impact, during the impact and after the start of the impact. The time after the start of air/liquid spray injection from the spray nozzle is rendered non-dimensional by using the spray injection velocity (U_{inj}) and spray nozzle diameter D as $t^* = \frac{U_{inj}\,\mathrm{t}}{\mathrm{D}}$.

As shown in Figure 2, multi-plume spray images obtained by the present experimental (EXP) and large eddy simulation (LES) show independent evolution of spray plumes in quiescent ambient air. In such a scenario individual spray plumes evolve similarly to a single plume spray expanding as traveling downstream due to the ambient air entrainment. As observed in previous studies of the multi-plume sprays (Cárdenas et al, 2009; Eagle et al, 2014), the interaction between the plumes separated by orientation angles larger than 15^o (which is the case in present study) can be considered negligible.

Fig. 2 Images of the multi-plume spray (at the injection pressure of P_{inj} = 15 MPa) in quiescent air (U_j = 0) at the non-dimensional time of $t^* = 303$ after the start of injection) (or ASOI). The image marked "EXP" represents the experimental result while "LES" stands for the result of large eddy simulation (LES).

Figure 3 illustrates the multi-plume spray injected into the core of a steady air jet. Both the experimental (marked with EXP) and LES images show the individual spray plumes merging into one tail-shaped plume which is deflected downstream while penetrating into the cross flow air jet. The deflection of the spray plumes towards the downstream if the cross flow gas jet is due to the momentum transfer from the gas to the liquid. The additional momentum exerted by the cross flow gas jet forms finer droplets and accelerates the liquid break-up process compared to the spray in quiescent ambient.

Fig. 3 Images of the multi-plume spray (at the injection pressure of P_{inj} = 15 MPa) injected into a steady cross flow air jet (at the Mach number of Ma = 0.58) at the non-dimensional time of $t^* = 825$ after the start of injection) (or ASOI). The image marked "EXP" represents the experimental result while "LES" stands for the result of large eddy simulation (LES).

For easy reference to the images of spray-jet interaction, the image number $N = t^*/T^*$ and the non-dimensional time interval between two consecutive images ($T^* = \frac{U_{inj}\,\Delta \mathrm{t}}{\mathrm{D}}$) are defined where $\Delta\mathrm{t}$ and T^* are the physical and the non-dimensional time intervals between two consecutive images, respectively; and the non-dimensional time $t^* = \frac{U_{inj}\,\mathrm{t}}{\mathrm{D}}$. For the results shown in this study, $T^* = 10.24$ is chosen for the best illustration of the interaction between the starting air jet and the spray. Figures 4 presents the evolution of the starting gas jet and multi-plume spray for N = 2-30 when both phases are injected simultaneously. Liquid phase is presented as white color spray droplets. To visualize the gas field, λ_2 criterion (Jeong and Hussain, 1995) is used to identify the vortical structures. It is seen that before the impact the starting gas jet and the multi-plume spray are evolving independently. The six spray plumes evolve independently as well similarly to the case in a quiescent ambient air as shown in Figure 2 earlier. Vortical structures in the gas jets are important since they are the location where gas/liquid mixing occurs. Inside the individual spray plumes, vortical structures associated with the spray-induced air jets are formed. On the other hand, the starting gas jet starts to form a leading tip vortex. Starting jets differ from steady jets due to the presence of a leading tip vortex ring followed by a quasi-steady region. Until N = 10, the first vortex ring (leading tip vortex) is shed from the gas jet orifice. During N = 12-30, the leading tip vortex ring grows in size and travels downstream towards the injected spray plumes. During this time only one vortex ring (the leading vortex ring) is formed in the gas jet. As suggested by the experimental work of Didden (1979), vortex ring formation is due to the separation of a vortex sheet at the nozzle edge where the internal boundary layer flow transits into a free shear flow which rolls up. The generated vortex ring travels downstream with a velocity including convective and self-induced components.

Fig. 4 Evolution of the multi-plume spray (at the injection pressure of P_{inj} = 15 MPa) and the starting gas jet (at the Mach number of Ma = 0.58) before the impact; the gas field vortical structures for both the starting air jet and the spray-induced air jets are shown by λ_2 criterion in red and the spray droplets in white color. The red color in the spray plumes represents the spray-induced air jets.

In Figure 5, velocity streamlines are superimposed on Y-vorticity (ω_y) contour in the central plane of the starting gas jet (Y = 0) at N = 020. It should be noted that, the (Y = 0) plane goes only through spray plumes A and D. Therefore, only the spray-induced gas jets corresponding to the plumes A and D are observed in this Figure. At N = 020 which is an instant before the impact, the leading vortex ring is formed due to the roll-up of the shear layer of the gas jet. The leading tip vortex is followed by a shear layer which is still under development. The leading vortex ring is formed due to the deceleration of the tip of the gas jet by the ambient air. The velocity streamlines show the expansion zone (EZ) of the ambient air in the downstream of the leading vortex ring as well as recirculation zones (RZ) caused by the ambient air being entrained into the ring.

Fig. 5 Velocity streamlines superimposed on Y-vorticity (ω_y) contour in the central plane of the starting gas jet (Y = 0) for multi-plume spray (at the injection pressure of P_{inj} = 15 MPa) and the starting air jet (at the Mach number of Ma = 0.58) at N = 020; (EZ: Expansion zone; RZ: Recirculation zone).

Presented in Figure 6 is the interaction of the starting gas jet with the multi-plume spray for N = 32-88. At the beginning of this time interval, the leading tip vortex starts to impact on the multi-plume spray and two sets of interactions start to take place. The first is due to the gaseous leading tip vortex ring impacting on the spray creating the dispersion of the spray droplets. The second is the single-phase interaction of the leading tip vortex ring with the vortical structures associated with the spray-induced gas jets. It can be seen that unlike the spray evolution into the steady gas jet (Figure 3) where spray plumes merged into one and deflected downstream as a tail-shape, liquid droplets are blasted in all directions due to the impact of the starting gas jet. As the vortex ring grows, it pushes the droplets outward and downstream. Also some of the droplets are entrained into the vortex ring due to the low pressure in the core of the ring. In addition, some droplets surrounding the periphery of the vortex ring are recirculated towards the upstream. Compared to the case of the steady gas jet, starting jet scatters the liquid droplets in a larger volume of space contributing to a spatially enhanced liquid/air mixture distribution. From N = 32-48, the leading tip vortex starts to deform and break-down the spray-induced air jet vortices. However, due to its strength the leading tip vortex ring is not significantly deformed. But from N = 52 onwards the leading tip starts to deform and eventually break down into multiple elongated vortex filaments. Formation of the many fine scale vortex filaments with complex topologies distributed in a large volume enhances the mixing between the two phases, illustrating the advantage of using a starting gas jet over a steady gas jet in practical applications.

Fig. 6 Interaction of the multi-plume spray (at the injection pressure of P_{inj} = 15 MPa) with the starting air jet (at the Mach number Ma = 0.58); the gas field vortical structures for both the starting air jet and the spray-induced air jets are shown by λ_2 criterion in red and the spray droplets in white color.

As shown in Figure 7 for N = 90-104, after the leading tip vortex ring has broken down the newly generated vortex rings which are not as strong as the first ring travel towards the impact zone. These new vortex rings are formed as a result of the shear layer becoming unstable due to the strong velocity gradients at the interface and decelerating the edges of the jet. This phenomenon is known as the Kelvin-Helmholtz instability which rolls-up the shear layer and grows into the large structure ring vortices. It can be seen that the second ring formed follows the leading vortex ring to the impact zone and eventually breaks down into smaller scales. Another interesting observation is the interaction of the two ring vortices marked as V_1 and V_2 in Figure 6. It is known that when the vortex rings expand they are decelerated. This is because at a larger ring radius (smaller curvature) self-induced velocity of the ring V_2 becomes smaller according to the Biot-Savart law (Margerit and Barncher, 2001). Therefore the following ring V_1 can catch up and join the ring V_2. This can result in leap-frogging of the rings or as the case of Figure 6 vortex ring pairing. From N = 90-96 the two rings V_1 and V_2 are still separate while their distance becomes shorter with time. At N = 98 and 100 the two rings start to interact but they are not completely merged. At N = 102 the pairing process is complete and the combined new ring V_3 is formed. Beyond this merging point the passage frequency of the vortices becomes smaller than the vortex generation frequency in the upstream where the shear layer is rolled-up due to the Kelvin-Helmholtz instability.

Fig. 7 Vortex pairing in the starting gas jet (Ma = 0.58) impacting on the multi-plume spray (P_{inj} = 15 MPa); the gas field vortical structures for both the starting air jet and the spray-induced air jets are shown by λ_2 criterion in red and the spray droplets in white color.

5 Conclusions

Large eddy simulation (LES) of a compressible subsonic starting jet colliding on a multi-plume spray is conducted by Eulerian/Lagrangian multiphase methodology. Identification of gas phase vortex cores before the impact using the λ_2 criterion reveals the independent formation and evolution of vortex rings in the starting jet as well as spray-induced air jet vortices. During the impact, the leading tip vortex ring followed by the smaller ring vortices in the starting jet arrive in the impact zone, deform and break-down due to interaction with the spray and the spray-induced air jet vortices. This impact and interaction between the starting jet and the spray disperse the liquid droplets in a much larger volume in all direction, significantly different from that of multi-plume sprays injected into a steady cross flow gas jet. This large spatial distribution of the liquid droplets as well as small scale elongated vortex filaments after vortex ring break-down creates enhanced mixing region around the impact zone. A pair of vortex ring is also observed merging before reaching the impact zone.

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