# Some Aspects of Static and Dynamic Distribution System State Estimation with Optimal Meter Placement Studies

by

Sreedevi Valsan Kandenkavil

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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## Abstract

In a power distribution system, due to the evolution of Active Distribution Networks (ADNs), there is a possibility of violation of the system operational constraints. A state estimator provides an approximate snapshot of the distribution system operation when the bus voltages and power measurements are available. Thus it plays a key role in monitoring the system, thereby ensuring a safe state of operation. According to the nature of the system, Distribution System State Estimation (DSSE) can be classified in to static DSSE and dynamic DSSE. Static DSSE is commonly designed as a Weighted Least Square (WLS) estimator using either bus voltages or branch currents as system states. For dynamic DSSE, the performance of static state estimators are limited. A Kalman filter based state estimator can be used in such time varying systems. A study of the algorithms used for these two DSSE methods is necessary in order to analyze the factors affecting the estimation accuracy. In a power distribution system, with limited availability of measurements, and additional measurements being expensive, careful selection of the location for the placement of meters becomes important. The measurement meters typically considered are Phasor Measurement Units (PMUs) and power (PQ) meters. The existing placement problems lay more emphasis on minimizing the cost of installing such meters, while the quality of estimation remains ignored. Thus there is a need to formulate methods for optimal allocation of meters in a cost effective way without altering the accuracy of DSSE.

In this work, a detailed study is conducted on the two static DSSE algorithms, Node Voltage based State Estimation (NVSE) and Branch Current based State Estimation (BCSE) and the DSSE performance is compared based on Average Root Mean Square (ARMSE) Value of state estimates. The thesis also analyzes the impact of the number of PMU measurements available on DSSE performance. Several optimization based approaches are proposed to address the optimal meter placement problem considering different objectives such as minimization of cost, WLS residual estimate, a multi-objective function comprising cost and WLS, and the ARMSE of the estimated bus voltage. An Iterative Extended Kalman Filter (IEKF) is used for performing dynamic DSSE. The dependency of various parameters such as selection of time frame, apriori estimate information length and PMU measurement errors on the accuracy acquired by DSSE is also presented.

The studies and proposed models are simulated in a 33-bus distribution feeder. The results illustrating the efficiency and speed of convergence of different static and dynamic DSSE methods are discussed. The various optimization models for meter allocation are formulated and compared based on meter placement cost and ARMSE of voltage estimates.

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## Dedication

This thesis is dedicated to my husband Nitin Padmanabhan, daughter Parvathy Nitin, my parents Sreevalsan K and Usha K, my sister Sreerekha Valsan K and my mother-in-law Kamala Padmanabhan.

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# Nomenclature

## ***Sets*** & ***Indices***

| | |
|---|---|
| $i, j$ | Index for the buses, $i \in I$ |
| $k$ | Index for the state vector entries $\{1, 2, ...2N-2\}$ |
| $R$ | Set of buses where PQ meters are located |
| $S$ | Set of buses where PMUs are located |
| $t$ | Index for time [min] |

## ***Parameters***

| | |
|---|---|
| $A_{i,j}$ | Connectivity matrix of dimension NxN indicating connected branches between the buses $i$ and $j$ |
| $b$ | Unity column matrix of dimension Nx1 |
| $C_{PMU}$ | Cost of PMU installation [$] |
| $C_{PQ}$ | Cost of PQ installation [$] |
| $F_t$ | The system model of dimension 2N-2 x 2N-2 at time $t$ |
| $Im\{I_{ij}\}^*$ | True value of imaginary part of current magnitude at $ij^{th}$ branch [pu] |
| $\|I_{ij}\|^*$ | True value of current magnitude at $ij^{th}$ branch [pu] |
| $Im\{I_{ij}\}^m$ | Imaginary part of current magnitude measurement at $ij^{th}$ branch [pu] |
| $\|I_{ij}\|^m$ | Current magnitude measurement at $ij^{th}$ branch [pu] |
| $P_i^*$ | True value of real power injection at bus $i$ [pu] |
| $P_i^m$ | Real power injection measurements at bus $i$ [pu] |
| $Q_i^m$ | Reactive power injection measurements at bus $i$ [pu] |
| $Q_i^*$ | True value of reactive power injection at bus $i$ [pu] |

| | |
|---|---|
| $Re\{I_{ij}\}^*$ | True value of real part of current magnitude at $ij^{th}$ branch [pu] |
| $Re\{I_{ij}\}^m$ | Real part of current magnitude measurement at $ij^{th}$ branch [pu] |
| $R_t$ | Error covariance matrix associated with observation noise at time $t$ |
| $S_t$ | Error covariance matrix associted with process noise at time $t$ |
| $u_t$ | Power injection vector consisting the active and reactive power injections at time $t$ [pu] |
| $V_i^*$ | True value of bus voltage at bus $i$ [pu] |
| $V_i^{l*}$ | True voltage at bus $i$ of the $l^{th}$ run [pu] |
| $V_i^m$ | Bus voltage measurement at bus $i$ [pu] |
| $X_{k,t}^*$ | $(k,t)^{th}$ element of true state matrix |
| $X_t$ | True system state vector at time $t$ |
| $Y_{ij}$ | The $ij^{th}$ element of Y matrix [pu] |
| $Z_t$ | The measurement vector at time $t$ |
| $\delta_i^*$ | True value of voltage angles at bus $i$ [rad] |
| $\delta_i^m$ | Voltage angle measurement at bus $i$ [rad] |
| $\epsilon_{Im\{I_{ij}\}}$ | Imaginary part of current magnitude measurement error at $ij^{th}$ branch [pu] |
| $\epsilon_{\|I_{ij}\|}$ | Current magnitude measurement error at $ij^{th}$ branch [pu] |
| $\epsilon_{P_i}$ | Real power injection measurement error at bus $i$ [pu] |
| $\epsilon_{Q_i}$ | Reactive power injection measurement error at bus $i$ [rad] |
| $\epsilon_{Re\{I_{ij}\}}$ | Real part of current magnitude measurement error at $ij^{th}$ branch [pu] |
| $\epsilon_t$ | Process noise or random noise at time $t$ |
| $\epsilon_v$ | Bus voltage preset threshold [pu] |
| $\epsilon_{V_i}$ | Bus voltage measurement error at bus $i$ [pu] |
| $\epsilon_\delta$ | Bus voltage angle preset threshold [rad] |
| $\epsilon_{\delta_i}$ | Bus voltage angle measurement error at bus $i$ [rad] |
| $\sigma_{Im\{I_{ij}\}}$ | Standard deviation of imaginary part of current measurement at $ij^{th}$ branch [pu] |

| | |
|---|---|
| $\sigma_{P_i}$ | Standard deviation of real power injection measurement error at bus $i$ [pu] |
| $\sigma_{Q_i}$ | Standard deviation of reactive power injection measurement error at bus $i$ [pu] |
| $\sigma_{Re\{I_{ij}\}}$ | Standard deviation of real part of current magnitude measurement at $ij^{th}$ branch[pu] |
| $\sigma_{V_i}$ | Standard deviation of voltage measurement error at bus $i$ [pu] |
| $\sigma_{\delta_i}$ | Standard deviation of voltage angle measurement error at bus $i$ [rad] |
| $\sigma_{\lvert I_{ij}\rvert}$ | Standard deviation of current magnitude measurement at $ij^{th}$ branch [pu] |
| $\theta_{ij}$ | The phase shift from bus $i$ to $i$ [rad] |
| $\nu_t$ | Measurement residual vector at time $t$ |

## *Variables*

| | |
|---|---|
| $\bar{C}_t$ | Predicted state error covariance matrix at time $t$ |
| $\hat{C}_t$ | Estimated state error covariance at time $t$ |
| $\mathfrak{f}(\hat{X}_{t-1})$ | Non-linear function to compute the predicted state at time $t$ from the previous estimates $\hat{X}_{t-1}$ |
| $h(x)$ | Non-linear function which expresses the measured quantity at bus $i$ in terms of the state variables x |
| $h(X_t)$ | Measurement function relating the predicted value of measurements to system state at time $t$ |
| $H_t$ | Jacobian of meaurement function $h(X_t)$ at time $t$ |
| $Im\{\widehat{I_{ij}}\}$ | Estimated imaginary part of current at $ij^{th}$ branch [pu] |
| $\lvert\widehat{I_{ij}}\rvert$ | Estimated magnitude of current at $ij^{th}$ branch [pu] |
| $J_{BCSE}$ | WLS estimator based objective function for BCSE [pu] |
| $J_{Cost}$ | Cost based objective function [$] |
| $\underline{J}_{Cost}$ | Lowest possible value of cost estimate [$] |
| $\overline{J}_{Cost}$ | Highest possible values of the cost estimate [$] |

| | |
|---|---|
| $J_{Goal}$ | Goal programming based objective function [pu] |
| $J_{NVSE}$ | WLS estimator based objective function for NVSE [pu] |
| $J_{Pareto}$ | Pareto optimization based objective function [pu] |
| $J_{WLS}$ | WLS residue based objective function [pu] |
| $\underline{J}_{WLS}$ | Lowest possible value of WLS estimate [pu] |
| $\overline{J}_{WLS}$ | Highest possible values of the WLS estimate [pu] |
| $J_t$ | Jacobian of load flow equations at time $t$ |
| $K_t$ | Optimal Kalman gain matrix at time $t$ |
| $Re\{\widehat{I_{ij}}\}$ | Estimated real part of current at $ij^{th}$ branch [pu] |
| $T$ | Length of apriori information [min] |
| $\hat{V}_i$ | Estimated bus voltage at bus $i$ [pu] |
| $\hat{V}_i^l$ | Estimated voltage at bus $i$ for the $l^{th}$ run [pu] |
| $W_{PMU}(i)$ | PMU decision variable at bus $i$ [1=PMU meter present] and [0=No PMU] |
| $W_{PQ}(i)$ | PQ meter decision variable at bus $i$ [1=PQ meter present] and [0=No PQ meter] |
| $\bar{X}_t$ | Apriori state estimate or predicted state vector at time $t$ |
| $\hat{X}_t$ | Aposteriori state estimate or updated state vector at time $t$ |
| $\bar{X}_{k,t}$ | $(k,t)^{th}$ element of apriori state estimate or predicted state matrix |
| $\hat{X}_{k,t}$ | $(k,t)^{th}$ element of aposteriori state estimate or updated state matrix |
| $\hat{\delta}_i$ | Estimated bus voltage angle at bus $i$ [rad] |

# List of Acronyms

| | |
|---|---|
| ADNs | Active Distribution Networks. |
| ARMSE | Average Root Mean Square Error. |
| BCSE | Branch Current based State Estimation. |
| DER | Distributed Energy Resources. |
| DMS | Distribution Management System. |
| DSE | Dynamic State Estimation. |
| DSSE | Distribution System State Estimation. |
| EKF | Extended Kalman Filter. |
| EMS | Energy Management System. |
| GA | Genetic Algorithm. |
| GPS | Gobal Positioning System. |
| GRV | Gaussian Random Variable. |
| IEKF | Iterative Extended Kalman Filter. |
| IKF | Iterated Kalman Filter. |
| LP | Linear Programming. |
| MINLP | Mixed Ineger Non-Linear Programming. |
| NLP | Non-Linear Programming. |
| NVSE | Node Voltage based State Estimation. |

| | |
|---|---|
| OPF | Optimal Power Flow. |
| PMU | Phasor Measurement Units. |
| PQ | Power meters. |
| RTU | Remote Terminal Units. |
| SCADA | Supervisory Control And Data Acquisition. |
| SSE | Static State Estimation. |
| UKF | Unscented Kalman Filter. |
| WLS | Weighted Least Squares. |

# Chapter 1

# Introduction

## 1.1 Motivation

The most important requirement of a power distribution system is to provide reliable and secure supply of electricity to customers. The penetration of distributed energy resources (DERs) and flexible loads have resulted in the evolution of active distribution networks. Thus there is a growing possibility of violation of system operational constraints, referred to as unsafe mode operation. The distribution system therefore, need be monitored in a timely manner and necessary control actions be initiated in order to take the system back to its safe operation mode. This is a critical responsibility of the distribution system control center.

A state estimator is an important component of the distribution control center operation as it provides an approximate snapshot of the system operation. It acts as an intermediate process, functioning between the real-time measurement acquisition process and the distribution management system (DMS). The real-time measurements are usually available from the measurement device interfaced to remote terminal units (RTUs) installed at various points in the distribution network. These acquired measurements, along with the measurement error of the device is passed on to the state estimator in the control center, where the system states are computed and made available for monitoring and verification by the DMS, and hence determine the control actions, if necessary. Thus the estimation of the state variables of the network is essential for monitoring, protecting and controlling the operations of a power distribution system. With the growing number of controllable devices, and emergence of the smart grid paradigm, Distribution System State Estimation (DSSE) is becoming increasingly important.

The conventional state estimation problem in power systems was formulated for static systems, *i.e.,* where the system was considered to be in a steady-state. The fact that the existing systems are time varying or dynamic in nature, necessitates the formulation of dynamic state estimation techniques. Although the conventional or static estimation techniques can be applied to dynamic systems by considering each state independent, it was observed that the performance criteria could not be met [1], hence the need for dynamic state estimation.

Since state estimators act as a filter that reduces the effect of measurement noise on the real-time measurements, the quality of estimation is greatly impacted by the availability of these input measurements. However it is not cost effective to take measurements from each and every node of the distribution network, and therefore a proper optimization of meter placement locations should be carried out, prior to the state estimation process. The optimally placed meters should ensure complete network observability.

As the state estimator is an important tool in power distribution systems, there is a need for careful study and analysis of the available techniques related to static and dynamic DSSE with efficient allocation of measurement devices.

## 1.2 Literature Review

### 1.2.1 Distribution System State Estimation

The importance of state estimation in power systems is discussed in [2] as it is the key function in determining real-time models for interconnected networks. The paper provides a brief insight to the conventional state estimation process and network modelling, and reviews the principal developments in state estimation, observability analysis, bad data processing, network topology processing, topology estimation, and parameter estimation.

The use of state estimators is proposed in [3] to improve the data needed for real-time monitoring and control of distribution feeders. A three-phase state estimation method is developed for this purpose, with bus voltage magnitudes and phase angles as the states. This method is known as Node Voltage based State Estimation (NVSE). The effectiveness of state estimation mainly depends on the accuracy of the forecasted loads when there are only limited real-time measurements. It was also noted that power flow measurements are more effective in bad data identification than measurements of the feeder currents.

The application of state estimation for real-time monitoring of distribution systems is presented in [4]. The paper summarizes the enhancements that need to be made to address the challenges of field implementation. Even when the availability of measurements on the feeder are few, such a method can provide acceptable estimates of the operating conditions in a distribution system.

A three-phase DSSE algorithm is proposed in [5], wherein the normal equation method is used to compute the real-time states. This paper uses a Weighted Least Square (WLS) state estimator using bus voltages in rectangular form as state variables. A current based formulation is introduced and is compared with the conventional estimation method. Observability analysis for the proposed DSSE techniques is carried out to investigate the effects of measurement types and redundancy on the performance of the state estimator.

For state estimation in transmission networks, an usual practice is to assume a set of redundant measurement data, which is not possible in DSSE problems as the real-time measurement redundancy is very low. To address this challenge, a state calculation algorithm for distribution systems was formulated in [6], where a load adjustment model is proposed that takes all the available measurements from the buses to determine the steady state operating condition of the distribution system. The algorithm is simple to implement and has proven to be very robust under field testing with real-time data.

A practical method for field testing of real-time state estimation in distribution networks is proposed in [7]. The estimation is based on available real-time data as well as historical data of loads. The measurements could be current magnitudes and power factors, active and reactive powers and voltage magnitudes at any network location.

A comprehensive survey on power system state estimation techniques is presented in [8]; the algorithms used for finding the system states under both static and dynamic state estimations are discussed in brief. The paper discusses the state estimation with Phasor Measurement Units (PMUs) and Supervisory Control and Data Acquisition (SCADA) measurements using a WLS algorithm which minimizes the sum of the squares of the weighted deviations of the estimated measurements from the actual measurements.

A WLS based state estimator considering different measurements such as voltage magnitude and phase-angle, branch current magnitude and phase angle, load current magnitude and phase angle and real and reactive power flow is proposed in [9]. This estimator revised the traditional WLS based estimator by adding new measurements. It is noted that the estimation error can be greatly reduced if the PMU measurements are selected appropriately.

In [10] a Branch Current based State Estimation (BCSE) is proposed for a radial three phase weakly meshed distribution system. The method has superior performance compared to the conventional voltage based method in terms of computational speed, memory requirement and bad data handling performance which is a requirement for a practical DSSE.

In [11] a revised BCSE for a three-phase distribution system is proposed. The main objective is to provide an accurate snapshot of the state of the distribution system, using all the available information on the system. This BCSE algorithm uses current magnitudes and phase angles as the state variables and decouples the three phases to improve the computational speed. The impact of meter placement locations on the estimators performance is also discussed in detail.

An efficient BCSE based DSSE technique is proposed in [12] using synchronized phasor measurements provided by PMUs. In this method, the state variables used for branch currents estimation are the currents amplitude and phase angles or their real and imaginary parts. In addition, the state model is extended so that the knowledge of the voltage profile is significantly improved.

While transmission system state estimation allows measurement redundancy, it is not practically possible in DSSE. So the measurements from each bus should be real-time measurements which can be obtained from PQ meters. The static WLS based algorithms such as NVSE and BCSE can be formulated as optimization problems to obtain the best possible results. Performance comparison of BCSE and NVSE pertaining to DSSE problems are not reported in the literature, to the best of the author's knowledge.

### 1.2.2 Optimal Meter Placement

At the earlier stages of development the measurements for state estimation were provided by the SCADA system which included active and reactive power flows, active and reactive power injections and bus voltage magnitudes and angles. Thereafter, as discussed in [14], the utilization of Global Positioning System (GPS) along with sampled data processing techniques led to the development of PMUs. PMU monitors provide real-time, synchronized, highly accurate phasor measurements from different locations in the distribution network such as voltage phasor at the bus and the current phasors of some or all the branches incident on a bus. As noted in [15], the application of PMUs has been attracting more attention in recent times, in power system monitoring, security and control. This has lead to the development of a meter placement scheme, providing full observability of the network. The high cost of PMUs and the higher cost of required communication facilities make the optimal PMU placement problem an important challenge. In [16] these challenges are explained, as it is not economical to place PMUs at every bus to detect the voltages. This is the reason behind the development of various PMU placement techniques.

There exists various algorithms to determine the number, location, and type of meters to be placed on distribution feeders, such that the state estimation with these measurements will achieve desired performance. The technique of optimizing the meter placements uses a minimum set of phasor measurements which minimizes the installation cost while providing full network observability. For power system topology observability, a fast analysis method by optimal PMU placement was presented in [17].

A Genetic Algorithm (GA) based approach to achieve a trade-off between investment cost and reliability of the state estimation process under different topology scenarios is proposed in [18]. This is done by formulating a fitness function where the cost of the metered system is minimized, while no critical measurements are allowed in the optimal solution. This objective function can be formulated as a cost minimization problem, which includes the cost of meters and remote terminal units RTUs to be installed, subject to performance requirements or state estimation process constraints.

To solve the optimal placement problem, Linear Programming (LP), Non Linear Programming (NLP), dynamic programming and combinatorial optimization based models have been proposed in [19]. In [20] an LP model with the objective to minimize the number of PMU locations provided the full system observability is proposed. The model is further extended to incorporate the injection measurements along with existing PMUs. The same formulation is used while considering measurement redundancies. It also discusses the measurement losses, bad data identification and elimination. In [21] an integer programming approach to PMU placement is presented, with a constraint on budget, and considering power injection measurements. In addition to minimizing the error related to state estimation, failure of single PMUs are accounted too. In [22] a generalized formulation of the optimal PMU placement problem for different cases such as redundant PMU placement, full network observability and incomplete network observability was proposed.

Past works on optimal meter placement focuses more on PMU placement cost without placing much emphasis on the accuracy of the obtained DSSE results. Rather than approaching the problem using cost based objective functions, other minimization problems can be formulated considering objective functions based on WLS estimate, a combination of cost and WLS residue or using performance metrics such as Average Root Mean Square Error (ARMSE) of system states so as to obtain an improved accuracy for DSSE results while optimally placing the meters in the distribution system.

### 1.2.3 Kalman Filtering for DSSE

Kalman filter is a recursive method that utilizes a set of mathematical equations to perform state estimation. The method was proposed by R. E. Kalman [23] in 1960. It can estimate the previous, present, and even the future states. Kalman filter uses the present measurement values to verify the previous prediction values, thereby improving the reliability of the obtained estimates while only storing the information of the previous step, so it has the advantages of small storage space and easy calculation.

The Kalman filter is a linear state estimation method and cannot be directly applied to distribution systems which are highly non-linear. In order to apply the Kalman filter to such non-linear systems, there is a need to linearize the system. In [24], an Extended Kalman Filter (EKF), is proposed that is based on the linearization of the non-linear power flow equations, by using Taylor Series, where quadratic and higher order terms are omitted.

A forecast and filtering method to track the dynamic states for measuring Gaussian noises with zero mean is proposed in [25]. An Unscented Kalman Filtering (UKF) is based on Unscented Transformation (UT) theory integrated with the kalman filter technique which provides estimates of the projections of mean and covariance of the state vector by applying unscented nonlinear transformation to the probability distribution.

The EKF, which simply linearizes all non-linear models, is one of the most widely used methods for tracking and estimating. In [26] a comparison is made between the WLS and the EKF methods and proposes a first analysis of the relative importance of the process and measurement covariance matrices.

An improved version of the Kalman filter is the Iterated Kalman Filter (IKF), presented in [27] which is characterized by an iterative application of the EKF for the case of non-linear measurement models along with non-linear system model. The IKF is noted to improve the performance of the EKF, where the update method of the IKF reduces to that of the EKF in the case of a single iteration. At the expense of more computation, the IKF has better performance than the EKF.

In [28] an improved IKF has been proposed to reduce the sensitivity of the filter to the initial estimate error. Here the Gauss-Newton method is used to approximate a maximum likelihood estimate, thus a new update method is obtained. Since the improved IKF does not utilize the initial estimates during every iteration, the influence of the initial estimates for the whole iterative process is decreased, which is a major drawback of actual IKF and thus better performance is achieved.

A procedure based on the use of the IKF in a straightforward manner is proposed in [29] to perform state estimation of ADNs integrating PMU measurements. The paper discusses the sensitivity analysis of the performances of WLS and IKF methods as a function of the measurements and process covariance matrices. In particular it is shown that the performances of WLS and IKF state estimation methods are largely dependent on the process and measurement covariance matrices and that their evaluation plays an important role in the selection of the proper estimation algorithm.

A proper investigation is required on the dependency of various parameters such as selection of time frame and the length of apriori information on the performance of Kalman filtering based DSSE. The importance of Kalman gain on updating the estimate values in each time step also requires detailing. The different requirements while using Iterative Extended Kalman Filter (IEKF) for various measurement error conditions also need be addressed properly.

## 1.3 Research Objectives

In view of the literature review and discussion presented in the previous section, the several objectives of this thesis are outlined as follows:

- Compare the performance of a conventional WLS driven NVSE which provides the best possible estimate of the operating bus voltages and voltage angles with a BCSE method that uses branch current components as system states.
- Develop optimization based approaches to address the problem of optimal placement of PMUs and power meters, considering different objectives such as minimization of cost, WLS residual estimate, a multi-objective function comprising the two, and the ARMSE of the estimated state vector.
- Carry out detailed analysis of the performance of an IEKF for estimating the time-varying states of a distribution system to study the impact of parameters such as total time frame, apriori information length, PMU measurement errors on DSSE.

## 1.4 Thesis Outline

A general overview of the DSSE problem and the optimal meter placement technique are presented in Chapter 2. In chapter 3, the conventional DSSE problem based on the WLS algorithm using node voltages and branch currents are discussed in detail and the performance of the two algorithms are compared based on their ARMSE values and the squared error values of system states. Chapter 4 presents different optimization approaches for placement of PMUs and PQ meters considering various objectives. Chapter 5 presents the IEKF approach for dynamic DSSE and discusses the effect of various parameters on the estimated states.

# Chapter 2

# BACKGROUND

This chapter presents an overview of the tools and models that form the basis of this thesis. In Section 2.1, a overview of static DSSE technique is presented. Further, a brief overview of existing optimal meter placement techniques are discussed in Section 2.2. Finally, in section 2.3 the basic concept and mathematical formulations of linear Kalman filtering and EKF for state estimation are presented.

## 2.1 Distribution System State Estimation

Power system operators have been using transmission system state estimation in their control centers since the seventies decade, after being first introduced by Prof. Fred Schweppe and team [13]. Over the years, it has became the backbone of Energy Management Systems (EMS), playing an important role in monitoring and controlling power systems for reliable operations. The EMS in power system seek to preserve the system in its normal and optimum economic operation state. For this a series of measurement data is transmitted by SCADA. If the data received from SCADA are adequate and correct then state estimation is not required. Thus state estimation became an interface of SCADA and EMS. In short, power system state estimation is an essential tool used by system operators for real-time analysis of the power systems which estimates the state variables such as optimal voltage magnitudes, voltage angles, line flows etc., based on the available redundant measurements.

In a modern distribution system, the presence of distributed energy resources and flexible loads resulted in an evolution from passive distribution networks toward active distribution network. Since these changes lead to frequent violations of operational constraints like voltage limits and line ampacities, there is a need for distribution management systems. Thus for an electric distribution system similar to power systems one of the major challenges faced at the control station is monitoring and controlling these real-time operations. The distribution system control station is responsible for collecting various real time measurements and analyzing whether the operational constraints are within the bound or not. If not necessary corrective actions should be carried out so as to maintain an efficient distribution of power over the distribution system. This process requires a series of operations to be carried out at the control station which includes a state estimator. Distribution system state estimation (DSSE) can be defined as the calculation of the system state estimates obtained by optimizing set of equations in terms of real time measurements and measurement noise information constrained by basic power flow equations.

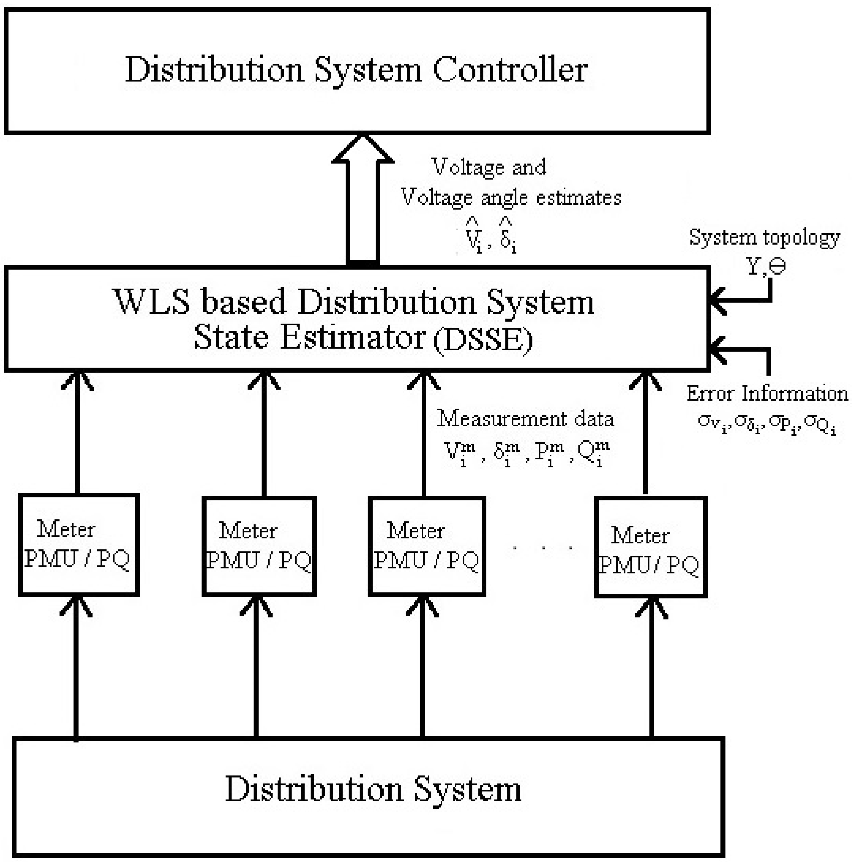


Figure 2.1: Information flow in DSSE

As shown in Figure 2.1, the DSSE process uses measurement data ($V_i^m$, $\delta_i^m$, $P_i^m$, $Q_i^m$, $I_{ij}^m$), which is corrupted by measurement device error, the system topology including bus admittance matrix (Y, $\theta$) and the standard deviation of measurement errors ($\sigma_{V_i}$, $\sigma_{\delta_i}$, $\sigma_{P_i}$, $\sigma_{Q_i}$, $\sigma_{I_{ij}}$) as inputs and provides the estimate of system states *i.e.*, bus voltages, voltage angles and branch current components ($\hat{V}_i$, $\hat{\delta}_i$, $\hat{I}_{ij}$). The WLS estimation seeks to minimize the measurement errors *i.e.*, the residue $\epsilon_i$ = ($V_i^*$ - $V_i^m$) which is the difference between the true and measured bus voltages, such that the estimated states are close enough to the true states of the system. Choose the buses where measurement devices such as PMUs or PQ meters are to be installed. The measurement device errors associated with PMUs (bus voltages and voltage angles) and PQ meters (real and reactive power injections) are $\epsilon_{V_i}$, $\epsilon_{\delta_i}$, $\epsilon_{P_i}$, $\epsilon_{Q_i}$ respectively.

The two possible state estimation techniques are Static State Estimation (SSE) and Dynamic State Estimation (DSE). In SSE the state model is build on the assumption that the state variable is in steady state or quasi steady state *i.e.,* it remains constant with respect to time. SSEs [12] removes the errors from the measurements and converts them into a form which a central control system can readily use to take decisions on system quality and security. In dynamic state estimation the model is build on the assumption of changing behavior of state variable with respect to time. As the system always cannot remain constant and its parameter may change with time, real time state estimation can be possible with dynamic state estimators.

WLS estimators are the most popular and considerable efforts have been devoted to reduce the computational requirements. Mostly the state estimation with measurement dependencies is solved with WLS technique. In the WLS method, the objective is to minimize the sum of the squares of the weighted deviations of the estimated measurements from the actual measurements. The system states are estimated from the available measurements. Weights associated with the actual measurements are proportional to the accuracy of the measurements. The objective function for the WLS estimator is

$$J(\vec{x}) = \sum_{i=1}^{M} \frac{(Z_i - h_i(\vec{x}))^2}{\sigma_i^2} \tag{2.1}$$

where,
M is the number of measurements
$\vec{x}$ denotes the state vector
$Z_i$ denotes the measurement data
$\sigma_i$ denotes the standard deviation of error
$h_i(\vec{x})$ is a measurement function that relates the measurement to state variables.

The two main categories for the choice of state variables in WLS are node voltage (NVSE) and branch current based state estimators (BCSE). Both can be formulated in polar and rectangular coordinates. Node voltage based state estimation uses the bus voltages and voltage angles as state variables $x$. *i.e.,* $x_i = [V_{est}(i), \delta_{est}(i)]$ where $V_{est}(i)$ is the voltage estimate and $\delta_{est}(i)$ is the angle estimate. The only difference between the node voltage based state esimation and BCSE is the measurement functions associated with the type of measurements to be processed.The branch current based state estimation method, like conventional node voltage based state estimation method, is based on the WLS approach. Rather than using the node voltages as the system state, the method uses the branch current components, *i.e.,* $x_i = [Re\{I_{ij}\}, Im\{I_{ij}\}]$ where $Re\{I_{ij}\}$ is branch current real part and $Im\{I_{ij}\}$ is branch current imaginary part. Incorporating voltage measurements increases the complexity in BCSE as the relation between the node voltages and branch currents are non-linear.

## 2.2 Optimal Meter Placement Problem

For a distribution system in order to rectify the measurement and topology erors, the measurements from substations are passed on to control centers to conduct state estimation thus providing an estimate for all metered and unmetered electrical quantities and network parameters. Usually measurements are provided by the SCADA system which includes active and reactive power flows, active and reactive power injections, bus voltage magnitudes, and angles. Furthermore the utilization of global positioning system (GPS) along with sampled data processing techniques have led to the development of PMUs.

PMUs monitor real-time, synchronized, highly accurate measurements of electrical quantities in phasor form, such as bus voltage phasor and branch current phasors. Recently, the application of PMUs has been attracting attention in power system monitoring, security and control. This requirement of a sufficient and accurate control, lead to the development of an adequate meter placement scheme, providing full observability of the network. The relatively high cost of PMUs and the higher cost of required communication facilities make the optimal PMU placement problem an important challenge.

It is not economical to place PMUs at every bus to detect the voltages in a distribution system; this paved the way for the development of PMU placement techniques. Researchers have proposed various algorithms to determine the number, location, and type of meters to be placed on distribution feeders, such that the state estimation with these measurements will achieve desired performance. These placement techniques depend on the PMUs installation, application, system characteristics and restrictions. Most of the proposed works aim to develop a technique or a series of methods that uses a minimum set of phasor measurements which minimizes the installation cost while providing full network observability. The measurement meters typically considered are phasor measurement units (PMUs) and power (PQ) meters which returns the voltage measurements such as bus voltages and angles and power measurements such as real and reactive power injections.

The meter placement as an optimization problem [30] is generally formulated in the form of the following minimization problem.

$$\begin{gathered} min\ J = \sum_{i=1}^{N} W_{PMU}(i) f(x) \\ Subject\ to: \text{Observability constraint} \end{gathered} \tag{2.2}$$

Where the observability constraint is used to meet an important requirement that every bus in the network must be observed at least once by PMUs. The constraint is defined as follows.

$$\sum_{j} A_{i,j} W_{PMU}(i) \geq\ b \qquad \forall\ i\ and\ j \in N \tag{2.3}$$

The $W_{PMU}(i)$ is a decision variable that set to 1 and 0 if PMU is present at a bus i or not respectively. b is a unity column matrix of dimension Nx1. The $f(x)$ represents the function to be reduced like PMU placement cost function for the system. $A_{i,j}$ represents a NxN connectivity matrix that indicates the connected branches between the buses.

## 2.3 Kalman Filtering for DSSE

Traditionally, power system state estimators implemented in EMS are static in nature and state estimations are performed by approaches which have a single set of measurements to estimate the system states. There are alternative estimation approaches capable of extracting valuable information from a set of independent states. This class of estimators is called dynamic state estimators (DSE) [31]. The DSE have the capability of tracking the current system states and also predicting the state vector at the next sampling time. The predicted values help to identify topology errors, gross bad data and sudden change of states. Although DSE were explored at the same time as static state estimation, the technology was too primitive to develop and improve it. However, with the new advancement in computer science and communication technologies, DSE is at its raise nowadays.

Despite the fact that the WLS method can be used to accurately solve the unknown state variables of a power system, it remains a static approach. This means that the WLS method uses measurements from the power network at a certain snapshot of time, and calculates or estimates values close to the true value for the unknown state variable. However, WLS cannot be used for DSE. The Kalman filter is an effective tool for DSE [32].

Kalman filter, as shown in Figure 2.2, is a recursive method that utilizes a set of mathematical equations to perform estimation. The method was proposed by R. E. Kalman in 1960. It can estimate the previous and present states, and even the future state. In the optimal state estimate, the observer gain is denoted as Kalman gain. The non-linear observer is applied for finding the states of single machine- infinite busbar power system.

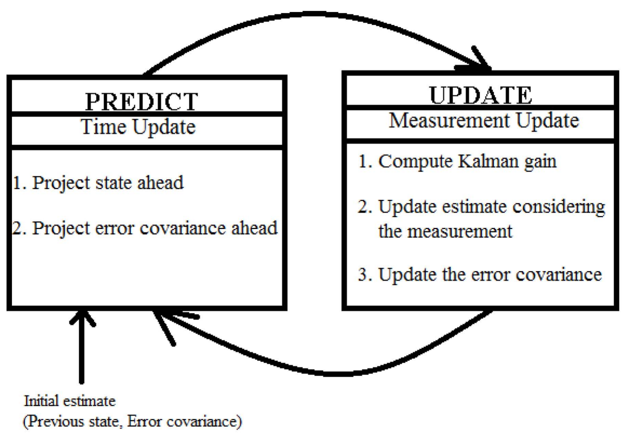


Figure 2.2: Kalman filtering a recursive approach

### 2.3.1 Linear Kalman Filter

Linear Kalman filter is a method used for estimating the instantaneous state of a linear dynamic system which accomplishes the prediction and correction of system states. The main assumption is that the system state at time $t$ can be represented in terms of the state of the system at time *t-1* as follows:

$$\begin{gathered} X_t = FX_{t-1} + \epsilon_t \\ where\ \epsilon_t \sim (0, S_t) \end{gathered} \tag{2.4}$$

Where, $X_t$ is the system state vector at time $t$, F is the system model, $X_{t-1}$ the system state vector at time *t-1* and $\epsilon_t$ is the process noise or random noise (can be considered as white noise with zero mean, and $S_t$ is the error covariance matrix).

The measurement vector $Z_t$ at time $t$ can be written as

$$\begin{aligned} Z_t &= HX_t + \nu_t \\ where\ \nu_t &\sim (0, R_t) \end{aligned} \tag{2.5}$$

Where, H is the observation model that maps the observed space from true space and $\nu_t$ is the measurement noise or observation noise (can be considered as white noise with zero mean and $R_t$ is the error covariance matrix).

The two step process for the linear Kalman filter [23] approach is given as follows:

Step-1: Prediction

This step is to predict the current state vector and the error covariance matrix of the predicted state based on the knowledge of a previously estimated state vector, the error covariance matrix of the previous state estimate and the system state model. The step includes:

1. The apriori state estimate or predicted state vector at time $t$, given as: $\bar{X}_t = F\hat{X}_{t-1}$
2. The predicted state error covariance matrix at time $t$, given as: $\bar{C}_t = F\widehat{C}_{t-1}F^T + S_t$

Where, $\hat{X}_{t-1}$ is the system state at time *t-1* and $\widehat{C}_{t-1}$ is the error covariance matrix at time *t-1*.

Step-2: Update

This step is to correct the predicted state vectors based on a Kalman gain weight matrix which is calculated using the error covaraince matrices and the measurement model. The associated steps are as follows:

1. The measurement residual vector at time $t$ is given as: $\nu_t = Z_t - H_t\bar{X}_t$
2. The optimal Kalman gain matrix at time $t$ is given as: $K_t = \bar{C}_t {H_t}^T (H_t\bar{C}_t{H_t}^T + R_t)^{-1}$
3. The aposteriori state estimate or updated state vector at time $t$ is given as: $\hat{X}_t = \bar{X}_t + K\nu_t$
4. The estimated state error covariance matrix at time $t$ is given as: $\widehat{C_t} = \bar{C}_t + KH_t\bar{C}_t$

### 2.3.2 Extended Kalman Filtering

The simplest form of Kalman filtering is the linear Kalman filter, where the state model and observation model are linear functions of the states. However, as most of the systems are nonlinear in nature, the two improvements of the classical Kalman Filter method are used, known as Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) [33]. The Extended Kalman Filter (EKF) [34] is used where the state transition model is a set of differentiable, non-linear functions of the states. The non-linear form of (2.4) can be thus expressed as,

$$\bar{X}_t = \mathfrak{f}(\hat{X}_{t-1}) + \epsilon_t \tag{2.6}$$

Where, $\mathfrak{f}(\hat{X}_{t-1})$ is a non-linear function to compute the predicted states from the previous estimates $\hat{X}_{t-1}$. However, $\mathfrak{f}$ cannot be directly applied in (2.6), instead Jacobian matrix of the partial derivatives of $\mathfrak{f}$ are used as given below.

$$F_{t-1} = \frac{\partial \mathfrak{f}}{\partial \hat{X}_{t-1}} \tag{2.7}$$

This process linearizes the non-linear function around the current estimate. Each time the Jacobian is evaluated with the current predicted states, which can be used in the Kalman filtering equations. Thus, the two step process associated with EKF is as follows

Step-1: Predict

1. The apriori state estimate or predicted state vector at time $t$, is given as: $\bar{X}_t = \mathfrak{f}(\hat{X}_{t-1})$
2. The predicted state error covariance matrix at time $t$, is given as: $\bar{C}_t = F_{t-1}\widehat{C}_{t-1}F_{t-1}^T + S_t$

Step-2: Update

1. The measurement residual vector at time $t$ is given as: $\nu_t = Z_t - H_t\bar{X}_t$
2. The optimal Kalman gain matrix at time $t$ is given as: $K_t = \bar{C}_t {H_t}^T (H_t\bar{C}_t {H_t}^T + R_t)^{-1}$
3. The aposteriori state estimate or updated state vector at time $t$ is given as: $\hat{X}_t = \bar{X}_t + K_t\nu_t$
4. The estimated state error covariance matrix at time $t$ is given as: $\widehat{C}_t = \bar{C}_t + K_tH_t\bar{C}_t$

The Jacobian matrix of the non-linear function $\mathfrak{f}(\hat{X}_{t-1})$ where $\hat{X}_{t-1}$ is the estimated state vector that includes voltage magnitudes and angle estimates $\hat{V}_{i,t},\ \hat{\delta}_{i,t}$ comprises the partial derivatives of the load flow equations, as given below.

$$\begin{aligned} P_{i,t} &= \sum_{j=1}^{N} \widehat{V}_{i,t}\widehat{V}_{j,t}Y_{ij}cos(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t}) \qquad \forall\ i,\ i \neq slack \\ Q_{i,t} &= -\sum_{j=1}^{N} \widehat{V}_{i,t}\widehat{V}_{j,t}Y_{ij}sin(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t}) \quad \forall\ i,\ i \neq slack, \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i \neq NPV \end{aligned} \tag{2.8}$$

Where, NPV is the number of PV buses in the system. Thus the Jacobian matrix $J$ based on (2.8) can be mathematically formulated as follows,

$$J_t = \begin{bmatrix} \frac{\partial P_{i,t}}{\partial \widehat{\delta}} & \frac{\partial P_{i,t}}{\partial |\widehat{V}|} \\ \frac{\partial Q_{i,t}}{\partial \widehat{\delta}} & \frac{\partial Q_{i,t}}{\partial |\widehat{V}|} \end{bmatrix} \tag{2.9}$$

$$J_t = \begin{cases} \frac{\partial P_{i,t}}{\partial \widehat{\delta}_{i,t}} = \sum_j |\widehat{V}_{i,t}||\widehat{V}_{j,t}||Y_{ij}|sin(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t}) \\ \frac{\partial P_{i,t}}{\partial \widehat{\delta}_{j,t}} = -|\widehat{V}_{i,t}||\widehat{V}_{j,t}||Y_{ij}|sin(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t})\ \forall\ j \neq i \\ \frac{\partial P_{i,t}}{\partial |\widehat{V}_{i,t}|} = 2|\widehat{V}_{i,t}|Y_{ii}|cos\theta_{ii} + \sum_j |\widehat{V}_{j,t}||Y_{ij}|cos(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t}) \\ \frac{\partial P_{i,t}}{\partial |\widehat{V}_{j,t}|} = |\widehat{V}_{i,t}||Y_{ij}|cos(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t})\ \forall\ j \neq i \\ \frac{\partial Q_{i,t}}{\partial \widehat{\delta}_{i,t}} = \sum_j |\widehat{V}_{i,t}||\widehat{V}_{j,t}||Y_{ij}|cos(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t}) \\ \frac{\partial Q_{i,t}}{\partial \widehat{\delta}_{j,t}} = -|\widehat{V}_{i,t}||\widehat{V}_{j,t}||Y_{ij}|cos(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t})\ \forall\ j \neq i \\ \frac{\partial Q_{i,t}}{\partial |\widehat{V}_{i,t}|} = -2|\widehat{V}_{i,t}|Y_{ii}|sin\theta_{ii} - \sum_j |\widehat{V}_{j,t}||Y_{ij}|sin(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t}) \\ \frac{\partial Q_{i,t}}{\partial |\widehat{V}_{j,t}|} = -|\widehat{V}_{i,t}||Y_{ij}|sin(\theta_{ij}+\widehat{\delta}_{j,t}-\widehat{\delta}_{i,t})\ \forall\ j \neq i \end{cases} \tag{2.10}$$

For an N-bus system where bus 1 is the slack bus, the linearization of load flow equations in the prediction step is as follows:

$$\begin{bmatrix} \Delta\ P \\ \Delta\ Q \end{bmatrix} = [J] \begin{bmatrix} \Delta\ V \\ \Delta\ \delta \end{bmatrix} - e \tag{2.11}$$

$$\Delta\ u = [J]\Delta\ X - e \tag{2.12}$$

$$[I]\Delta u - [J]\Delta X + e = 0 \tag{2.13}$$

Where J is the Jacobian, $u$ is the power injection vector consisting the active and reactive power injections given as: $u = [P_2....P_N\ Q_2....Q_N]^T$, $\Delta X$ is the change of state, $\Delta u$ is the vector of power injection mismatch, which effectively denotes the error associated with bus load measurements and I is an identity matrix. Rearranging this equation and solving for $X_t$ resulting in the following prediction step formulation,

$$X_t = X_{t-1} + J_{t-1}^{-1}[u_t - u_{t-1}] + J_{t-1}^{-1}e \tag{2.14}$$

The above equation is a pseudo-dynamic model of state $X_t$ due to an applied input $u_t$ thus relating the state vectors at time $t$ with $t-1$. The term $J_{t-1}^{-1}e$ is the process noise resulting from the linearization process. Once the pseudo-dynamic model is formulated the prediction-correction cycles can be applied as explained before. The steps of EKF and mathematical equations are as follows.
Step-1: Prediction

1. The apriori state estimate or predicted state at time $t$, is given as:

$$\bar{X}_{k,t} = \hat{X}_{k,t-1} + J_{t-1}^{-1}[u_{k,t} - u_{k,t-1}] \tag{2.15}$$

2.The predicted state error covariance matrix at time $t$, is given as:

$$\bar{C}_t = \widehat{C}_{t-1} + S_t \tag{2.16}$$

Step-2: Correction

1. The optimal Kalman gain matrix at time $t$ is given as:

$$K_t = \bar{C}_t H_t^T (H_t \bar{C}_t H_t^T + R_t)^{-1} \tag{2.17}$$

2. The aposteriori state estimate or updated state at time $t$ is given as:

$$\hat{X}_{k,t} = \bar{X}_{k,t} + K_t[Z_t - H_t\bar{X}_{k,t}] \tag{2.18}$$

3.The estimated state error covariance matrix at time $t$ is given as:

$$\widehat{C_t} = \bar{C}_t + K_t H_t \bar{C}_t \tag{2.19}$$

In the prediction step, the apriori states are predicted using previous information of states and the transition model. The state $\bar{X}_{i,t}$ and error covariance matrix $\bar{C}_t$ at time step $t$ can be predicted using the state variable information at time step *t-1* plus the dynamics between $t$ and *t-1* which uses the load data $u_t = [P_i \;\; Q_i]$, $\forall \;\; i \in \{1, ...N\}$, $i \neq$ slack bus. Subsequently, in the correction step, which is an iterative process, the prediction errors are corrected using a Kalman gain $K_t$ and the measurement innovation term $\nu_t$. Updated $\widehat{C}_t$ is also calculated at this step.

To improve the quality of DSE the filtering process can be iteratively performed inorder to facilitate the convergence to the most accurate value. To give a further extension for EKF this iterativeness can be coupled together with EKF to form an IEKF. The concept of IEKF as explained in [29], [36] will improve the efficiency of estimation of a normal EKF.

## 2.4 Conclusions

In this chapter the basic information flow in DSSE method based on a WLS estimator have been discussed. The concept of static DSSE algorithms such as NVSE and BCSE are also explained. The need of optimal planning of meter placement is further discussed. The section shows the general formulation of a optimal meter placement problem based on cost minimization. Further, the idea of Kalman filtering for DSE has also been presented. The section briefly discusses about the steps involved in linear Kalman filtering process. The detailed algorithm and mathematical model of a non-linear EKF for DSE is also presented.

# Chapter 3

# Studies on WLS Based NVSE and BCSE Algorithms

This chapter discusses the concept of static DSSE algorithms such as NVSE and BCSE. In Section 3.1 objective of DSSE is discussed. The basics of WLS estimation is explained in Section 3.2. The NVSE method and the steps involved in it are presented in Section 3.3. The mathematical formulation of BCSE and the detailed algorithm is discussed in Section 3.4. In Section 3.5 the performance comparison of two algorithms is presented in detail. Section 3.6 summarizes the work presented in this chapter.

## 3.1 Static DSSE Algorithms

State estimation can be defined as the method of obtaining the best estimate of the state of the system based on a set of measurements and system topology. Measurements can be a set of data available from PMUs or power meters. Based on a set of nonlinear equations relating the measurements and the system states, *i.e.,* bus voltages and phase angles of a power distribution system, a state estimator estimates the system states by minimizing the sum of residual squares. The estimated states can be used to analyze whether the system is in a secure state.

The objective of DSSE is to provide a robust estimate of the operating point (state) for a distribution system on a feeder basis. The two commonly used state estimation techniques are NVSE and BCSE. The NVSE method can handle power and voltage measurements, and uses bus voltages and angles as state variables. The BCSE method uses branch measurements as state variables.

## 3.2 Weighted Least Square (WLS) Estimation

Static state estimation in distribution systems is the process of estimating the state from measurements of voltage and branch current magnitudes, real and reactive branch power flows and real and reactive node power injections. In the WLS method, the objective is to minimize the sum of the squares of the weighted deviations of the estimated measurements from the actual measurements.

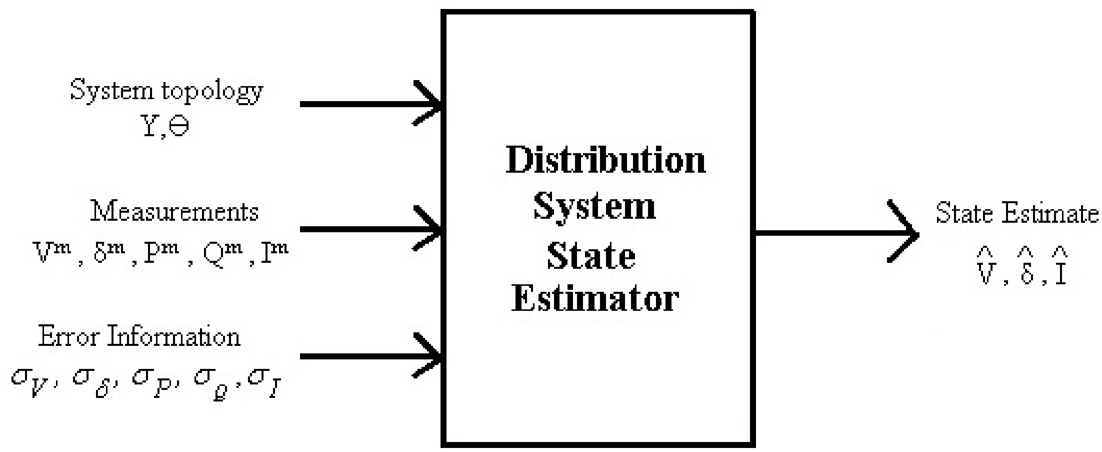


Figure 3.1: Genaral Block Diagram of WLS based DSSE

As shown in Figure 3.1, the WLS based DSSE takes measurement data ($V_m$, $\delta_m$, $P_m$, $Q_m$, $I_m$) as inputs, which is corrupted by measurement device error with standard deviation of measurement errors ($\sigma_V$, $\sigma_\delta$, $\sigma_P$, $\sigma_Q$, $\sigma_I$), and the system topology, which is the bus admittance matrix (Y, $\theta$) and provides the system states *i.e.,* bus voltages, voltage angles and branch current components ($\hat{V}_i$, $\hat{\delta}_i$, $\hat{I}_{ij}$). The purpose of WLS based state estimation is to minimize the measurement error $\epsilon_i$ = ($V_i^*$ - $V_i^m$ ). The two common DSSE algorithms NVSE and BCSE are discussed in the following sections.

## 3.3 Node Voltage Based State Estimation (NVSE)

The NVSE method is based on the WLS approach and can handle power and voltage measurements, and uses bus voltages and voltage angles as state variables. For an N-bus system, $V_i^*$, $\delta_i^*$, $P_i^*$, $Q_i^*$ be the true states of bus voltages, angles, active and reactive power injections at bus $i$ respectively.

Choose the buses where measurement devices like PMU or power meters are located. The PMUs will return the voltage and angle measurements, the active and reactive power injections at bus $i$ $(P_i, Q_i)$ will be provided by the PQ meters. Let $\sigma_{V_i}$, $\sigma_{\delta_i}$, $\sigma_{P_i}$, $\sigma_{Q_i}$ be the measurement noise standard deviation associated with voltages and angles and the measurement device errors associated with the PMUs and PQ meters are $\epsilon_{V_i}$, $\epsilon_{\delta_i}$, $\epsilon_{P_i}$, $\epsilon_{Q_i}$. Then we have,

$$V_i^m = V_i^* + \epsilon_{V_i} \qquad \forall\ PMUs \tag{3.1}$$

$$\delta_i^m = \delta_i^* + \epsilon_{\delta_i} \qquad \forall\ PMUs \tag{3.2}$$

$$P_i^m = P_i^* + \epsilon_{P_i} \qquad \forall\ PQs \tag{3.3}$$

$$Q_i^m = Q_i^* + \epsilon_{Q_i} \qquad \forall\ PQs \tag{3.4}$$

where the measurement errors for the state variables are given as follows,

$$\epsilon_{V_i} = normal(0, \sigma_{V_i}) \qquad \forall\ PMUs \tag{3.5}$$

$$\epsilon_{\delta_i} = normal(0, \sigma_{\delta_i}) \qquad \forall\ PMUs \tag{3.6}$$

$$\epsilon_{P_i} = normal(0, \sigma_{P_i}) \qquad \forall\ PQs \tag{3.7}$$

$$\epsilon_{Q_i} = normal(0, \sigma_{Q_i}) \qquad \forall\ PQs \tag{3.8}$$

For the NVSE method the objective function is given as follows,

$$J_{NVSE} = \sum_{i \in S} \Big(\frac{V_i^m - \hat{V}_i}{\sigma_{V_i}}\Big)^2 + \sum_{i \in S} \Big(\frac{\delta_i^m - \hat{\delta}_i}{\sigma_{\delta_i}}\Big)^2 + \sum_{i \in R} \Big(\frac{P_i^m - \hat{P}_i}{\sigma_{P_i}}\Big)^2 + \sum_{i \in R} \Big(\frac{Q_i^m - \hat{Q}_i}{\sigma_{Q_i}}\Big)^2 \tag{3.9}$$

Where,
The system state $x = (V_i,\ \delta_i) \qquad \forall\ i$.

The measurement set Z = ($V_i^m$, $\delta_i^m$, $P_i^m$, $Q_i^m$) $\quad \forall\, i$.
S: Set of buses where PMUs are located.
R: Set of buses where PQ meters are located.

The objective function (3.9) is minimized subject to the measurement function constraint h(x). This non-linear function relates the predicted value of measurements at bus $i$ to system state $x$ given as follows,

$$h(x) = \begin{cases} \hat{V}_i & \forall\, i \\ \hat{\delta}_i & \forall\, i \\ \sum\limits_j \hat{V}_i \hat{V}_j Y_{ij} cos(\theta_{ij} + \hat{\delta}_j - \hat{\delta}_i) & \forall\, i \\ -\sum\limits_j \hat{V}_i \hat{V}_j Y_{ij} sin(\theta_{ij} + \hat{\delta}_j - \hat{\delta}_i) & \forall\, i \end{cases} \tag{3.10}$$

Where $Y_{ij}\angle\theta_{ij}$ is the bus branch admittance matrix.

### 3.3.1 Performance Metric

The measure the effectiveness of DSSE, the performance metric used in this work is the ARMSE value of state estimates. ARMSE corresponds to an average prediction error. This performance metric is defined as the square root of the mean squared error of the true and estimated system states, averaged over the number of buses (N). ARMSE ($\mathcal{M}$) is a monotonically non increasing function of $\mathcal{M}$, the number of PMUs. The ARMSE calculation for bus voltages is a follows:

$$ARMSE(\mathcal{M}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N} E[V_i^* - \hat{V}_i]^2} \tag{3.11}$$

### 3.3.2 NVSE Algorithm Steps

1. For the given N bus radial distribution system, solve OPF to obtain the true state $V_i^*$, $\delta_i^*$, $P_i^*$, $Q_i^*$.
2. Select the buses where measurement taken using PMU units which returns the $V_i^m$, $\delta_i^m$

and buses where power meters are kept which returns the active and reactive power injections $P_i^m$, $Q_i^m$. For GAMS simulation, using the optimal results create the measurement data by adding error $\epsilon_{V_i}$, $\epsilon_{\delta_i}$, $\epsilon_{P_i}$, $\epsilon_{Q_i}$ to it. PMU and PQ meter error are chosen as white noise of zero mean and $\sigma_{V_i}$, $\sigma_{\delta_i}$, $\sigma_{P_i}$, $\sigma_{Q_i}$ standard deviation.
3. Execute WLS model for NVSE using GAMS optimization tool by minimizing $J_{NVSE}$ to obtain state variables $\hat{V}_i$, $\hat{\delta}_i$.
4. Compute ARMSE to compute average estimation error comparing optimal and estimated results.

## 3.4 Branch Current Based State Estimation (BCSE)

The branch current based DSSE method is used for radial and weakly meshed distribution feeders, using real and reactive components of branch current as state variables. In this state estimator based on WLS approach, rather than using node voltage as system state, this method uses branch currents. ie, $x = (Re\{I_{ij}\}, Im\{I_{ij}\}, |I_{ij}|)$.
where
$Re\{I_{ij}\}$ is the real part of branch current.
$Im\{I_{ij}\}$ is the imaginary part of branch current.
$|I_{ij}|$ is the branch current magnitude.

let $Re\{I_{ij}\}^m$, $Im\{I_{ij}\}^m$, $|I_{ij}|^m$ be the branch current real and imaginary part measurements obtained from PMUs and the branch current magnitude measurements obtained from SCADA respectively which are the branch current true states branch current real, imaginary part and the branch current magnitude measurements, corrupted by measurement error $\epsilon_{Re\{I_{ij}\}}$, $\epsilon_{Im\{I_{ij}\}}$, $\epsilon_{|I_{ij}|}$ of standard deviation $\sigma_{Re\{I_{ij}\}}$, $\sigma_{Im\{I_{ij}\}}$, $\sigma_{|I_{ij}|}$.

$$Re\{I_{ij}\}^m = Re\{I_{ij}\}^* + \epsilon_{Re\{I_{ij}\}} \quad \forall\ PMUs \tag{3.12}$$

$$Im\{I_{ij}\}^m = Im\{I_{ij}\}^* + \epsilon_{Im\{I_{ij}\}} \quad \forall\ PMUs \tag{3.13}$$

$$|I_{ij}|^m = |I_{ij}|^* + \epsilon_{|I_{ij}|} \quad \forall\ SCADA \tag{3.14}$$

where measurement error for each state variables are as follows,

$$\epsilon_{Re\{I_{ij}\}} = normal(0, \sigma_{Re\{I_{ij}\}}) \tag{3.15}$$

$$\epsilon_{Im\{I_{ij}\}} = normal(0, \sigma_{Im\{I_{ij}\}}) \tag{3.16}$$

$$\epsilon_{|I_{ij}|} = normal(0, \sigma_{|I_{ij}|}) \tag{3.17}$$

The Objective function for WLS based BCSE is as follows,

$$\begin{aligned} J_{BCSE} = \sum_{i,j=1}^{N} \Big(\frac{Re\{I_{ij}\}^m - Re\{\widehat{I_{ij}}\}}{\sigma_{Re\{I_{ij}\}}}\Big)^2 + \sum_{i,j=1}^{N} \Big(\frac{Im\{I_{ij}\}^m - Im\{\widehat{I_{ij}}\}}{\sigma_{Im\{I_{ij}\}}}\Big)^2 + \\ \sum_{i,j=1}^{N} \Big(\frac{|I_{ij}|^m - |\widehat{I_{ij}}|}{\sigma_{|I_{ij}|}}\Big)^2 \end{aligned} \tag{3.18}$$

The objective function (3.18) is minimized subject to the measurement function constraints as given below.

$$h(x) = \begin{cases} Re\{\widehat{I_{ij}}\} = \hat{V}_i Y_{ij} cos(\hat{\delta}_i + \theta_{ij}) - \hat{V}_j Y_{ij} cos(\hat{\delta}_j + \theta_{ij}) \\ Im\{\widehat{I_{ij}}\} = \hat{V}_i Y_{ij} sin(\hat{\delta}_i + \theta_{ij}) - \hat{V}_j Y_{ij} sin(\hat{\delta}_j + \theta_{ij}) \\ |\widehat{I_{ij}}| = \sqrt{(Re\{\widehat{I_{ij}}\})^2 + (Im\{\widehat{I_{ij}}\})^2} \end{cases} \tag{3.19}$$

### 3.4.1 BCSE Algorithm steps

1. For the given IEEE 33 bus radial distribution system, solve OPF to obtain the optimal result $Re\{I_{ij}\}^*$, $Im\{I_{ij}\}^*$, $|I_{ij}|^*$.
2. For the N bus system, create the measurement data $Re\{I_{ij}\}^m$, $Im\{I_{ij}\}^m$, $|I_{ij}|^m$ using the optimal results by adding error $\epsilon_{Re\{I_{ij}\}}$, $\epsilon_{Im\{I_{ij}\}}$, $\epsilon_{|I_{ij}|}$ to it.
3. Execute WLS model for BCSE to minimize $J_{BCSE}$ using GAMS optimization tool to obtain state variables $Re\{\widehat{I_{ij}}\}$, $Im\{\widehat{I_{ij}}\}$, $|\widehat{I_{ij}}|$.
4. Compute ARMSE comparing optimal and estimated results of state variables.

## 3.5 Results and Discussions

The IEEE 33-bus radial distribution system used for the studies is shown in Figure 3.2. An Optimal Power Flow (OPF) [37] is executed to determine the true states of the system. The state estimates are initialized as $V_{est} = 1\ pu$ and $\delta_{est} = 0\ rad$. For simulation purposes, a

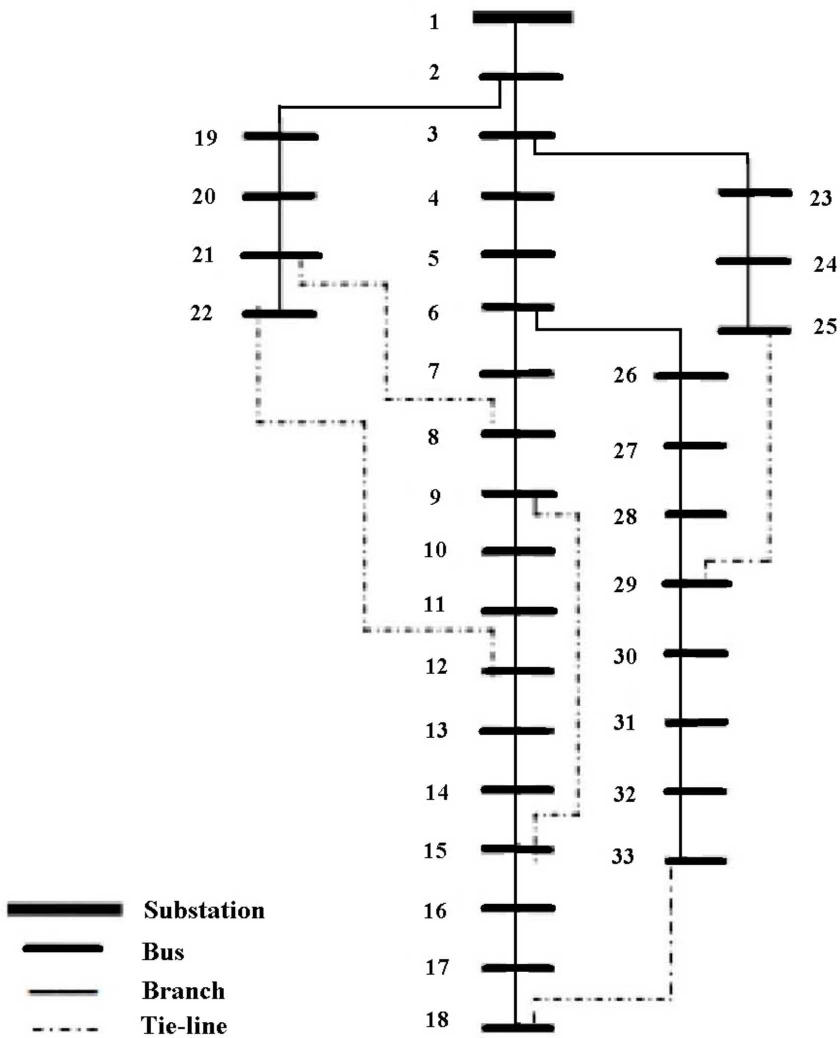


Figure 3.2: Layout of IEEE 33 bus radial distribution system

normally distributed white noise of zero mean and an error variance of 1% on the $V_i$, $\delta_i$ obtained from OPF solution are considered; and for active and reactive power, a normally distributed white noise of zero mean and an error variance of 3% on the power injections $P_i, Q_i$ obtained from OPF solution are considered. For current measurements, the zero mean white noise error standard deviation $\sigma_{Re\{I_{ij}\}}$, $\sigma_{Im\{I_{ij}\}}$, $\sigma_{|I_{ij}|}$ are chosen as 5% of the corresponding optimal values. The two algorithms are simulated as optimization problems, where the objective functions are minimized subjected to measurement function constraints.

A Monte Carlo simulation is carried out for 500 iterations so as to observe the state variable convergence to the best possible value. The optimization models are formulated as NLP problems and solved using the MINOS solver in GAMS [38]. The obtained results for NVSE and BCSE methods are discussed and compared in the following sections.

### 3.5.1 NVSE and BCSE Results

The various state parameters such as bus voltages, voltage angles and magnitude of branch current are evaluated based on their convergence.

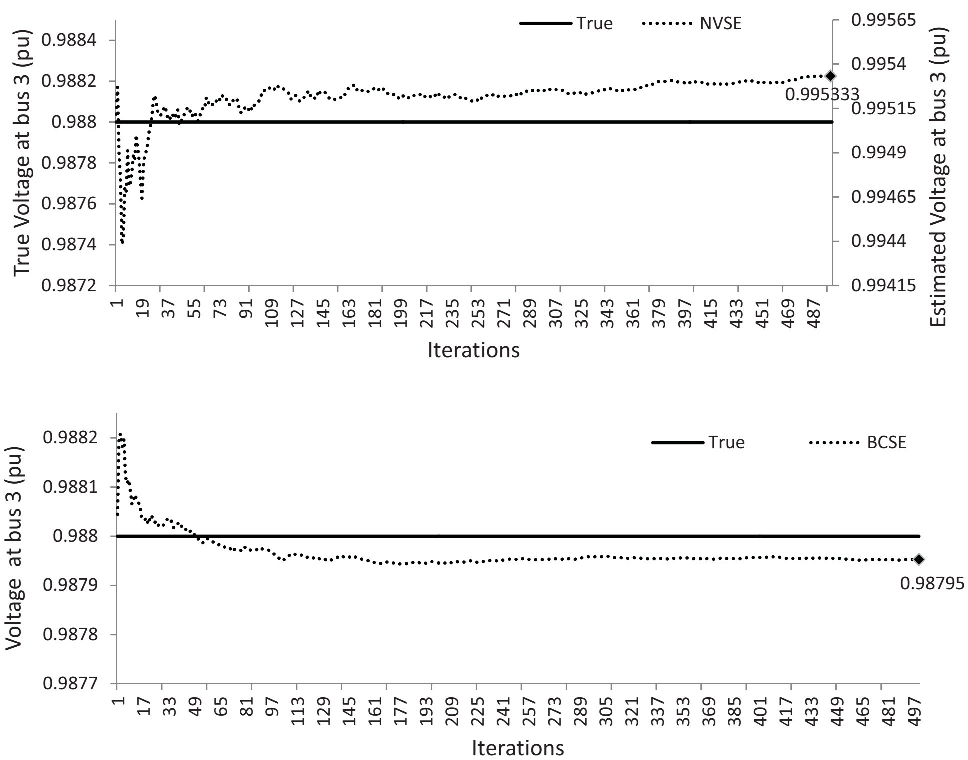


Figure 3.3: Convergence plot of bus 3 voltage estimate (a) NVSE method (b) BCSE method

The convergence plot for bus 3 voltage estimate for NVSE and BCSE method is given in Figure 3.3. Comparing with the true bus 3 voltage, the BCSE estimation shows closer convergence. Also the convergence speed is comparatively higher for BCSE than NVSE method. A similar result is observed for the convergence plot of bus 32 voltage angle for NVSE and BCSE methods. As shown in Figure 3.4, the convergence of BCSE is faster and closer the estimation to its true voltage angle.

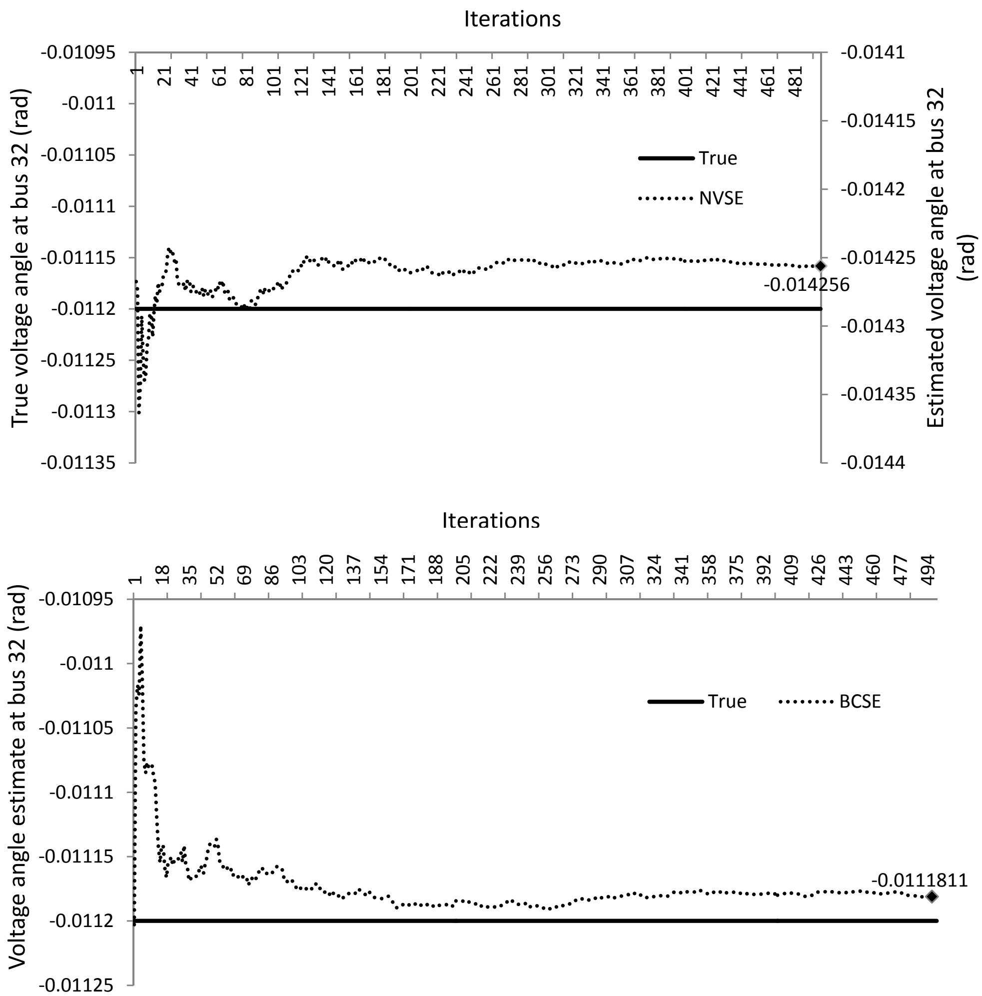


Figure 3.4: Convergence plot of bus 32 voltage angle estimate (a) NVSE method (b) BCSE method

The branch current magnitude convergence plot for BCSE method is as given in Figure 3.5 which provides an estimation near similar to its true value. Thus it is noted that the accuracy of DSSE is more in BCSE compared to NVSE as the various system states getting settled to the best possible value in less time.

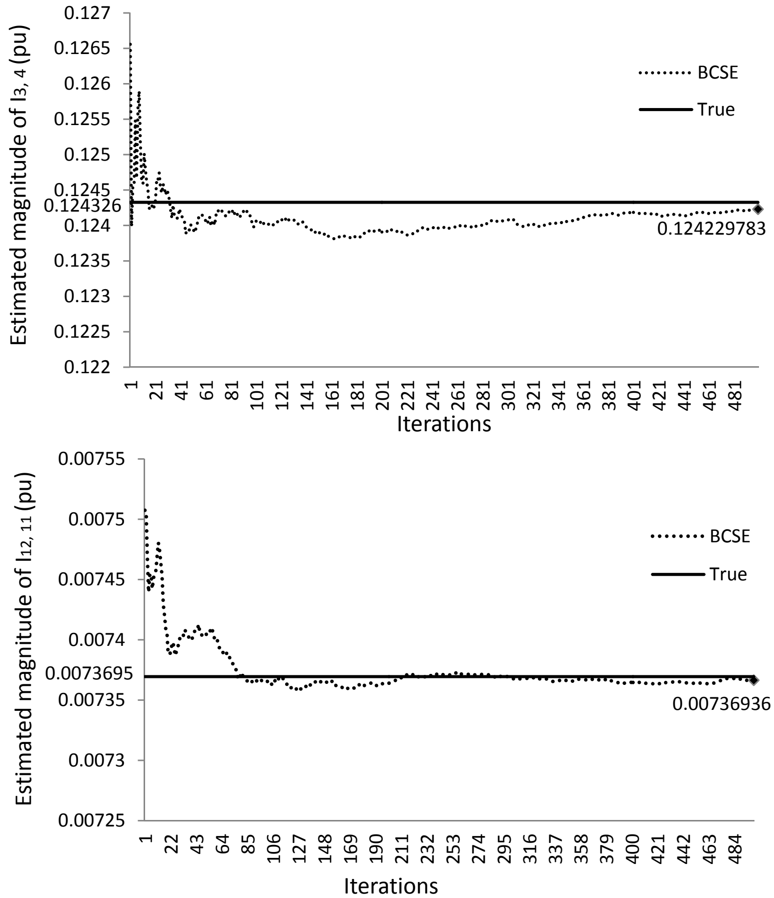


Figure 3.5: Branch current magnitude convergence plots in the BCSE Method (a) Branch current magnitude at 3-4 (b) Branch current magnitude at 12-11

The ARMSE plot for voltage magnitudes and voltage angles obtained by varying the number of PMUs used for measuring the $V^m$ and $\delta^m$ is shown in Figure 3.6. It is noted that there is a clear dependence of the DSSE error on the total number of installed PMUs $\mathcal{M}$. The plot of ARMSE are non-increasing functions of ARMSE values with respect to the number of PMUs $\mathcal{M}$. From these plots it can be concluded that as the number of PMUs installed in the system increases, the accuracy of the DSSE increases. In order to obtain the best DSSE, more PMUs should be installed in the system, which is however not a cost effective method.

It is to be noted that the research advancements in the design of low cost PMUs such

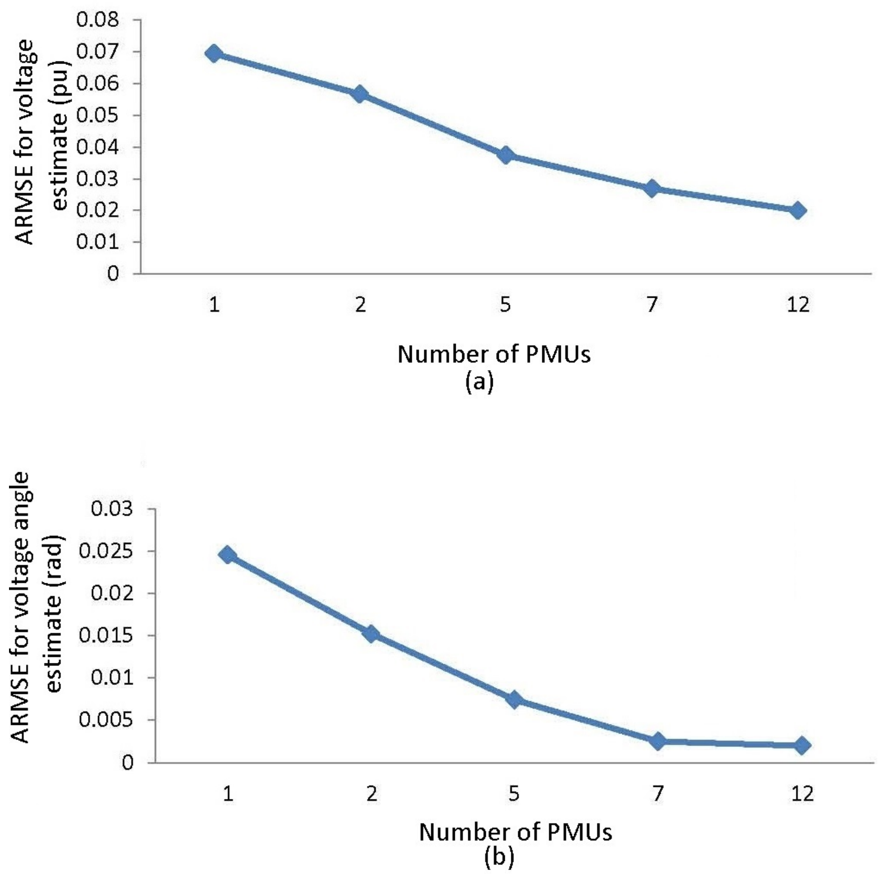


Figure 3.6: ARMSE as function of number of PMUs ($\mathcal{M}$) for NVSE Method (a) ARMSE of V (b) ARMSE of $\delta$

as microPMUs [39] can be deployed in the distribution network which serves the purpose of conventional PMUs in order to achieve increased accuracy in DSSE at much reduced cost.

### 3.5.2 Performance Comparison

Along with faster settling of the estimated state, the accuracy with which the DSSE is performed should be also examined. The performance of NVSE and BCSE is compared based on the squared error estimates of bus voltages and angles, as shown in Figure 3.7.

From this plot it can be concluded that BCSE is the most accurate method, between the two, as the error value is significantly low as compared to that of NVSE.

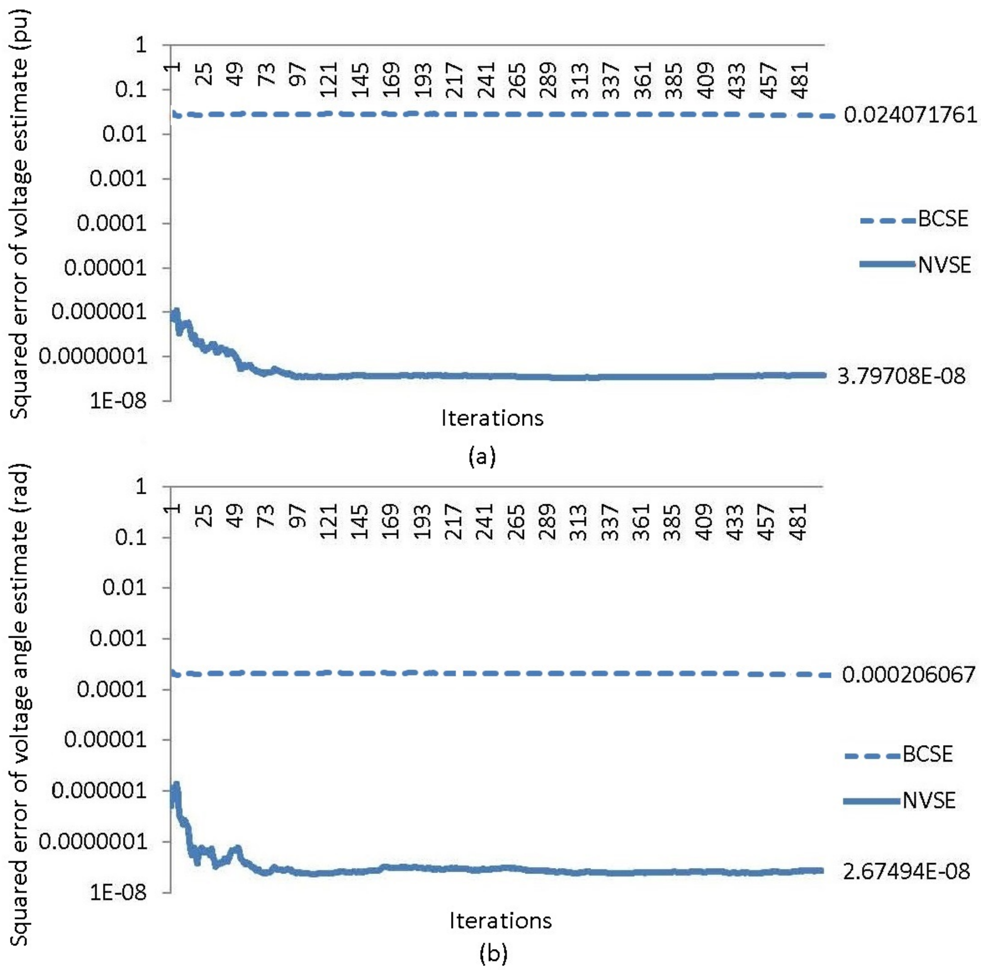


Figure 3.7: Comparison of NVSE and BCSE methods (a) Squared error of voltage estimates (b) Squared error of voltage angle estimates

The two methods can be compared on the basis of ARMSE for voltage magnitudes $ARMSE\_V$ (in pu), voltage angles $ARMSE\_delta$ (in rad) and branch current magnitudes $ARMSE\_I$ (in pu) by calculating the estimated values of theb deviation from the corresponding true state values, and are shown in Figure 3.8. It is observed that the $ARMSE\_V$ value is lower for BCSE than NVSE, while for $ARMSE\_delta$ the BCSE and NVSE values

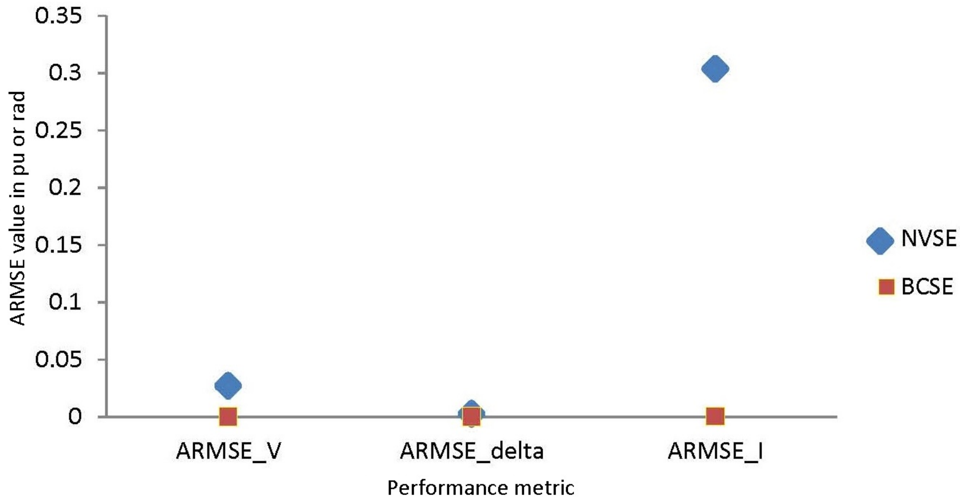


Figure 3.8: NVSE and BCSE method comparison based on ARMSE values

are very close. As branch current magnitude is not considered as a parameter in NVSE, $ARMSE\_I$ is high, *i.e.,* least accurate compared to that in BCSE.

## 3.6 Conclusions

The chapter presented two DSSE algorithms, the conventional NVSE and BCSE methods, which are formulated as optimization problems using the WLS estimator. To study the impact of the number of PMU measurements used for state estimation, the optimization problem was solved by varying the number of PMU units installed for network observability and the ARMSE value for voltage and angle estimates showed improved performance as the number of PMUs increases. Also, the results obtained showed a faster convergence of voltage and angle estimates using the BCSE method as compared to the NVSE method. The two methods were also compared in terms of ARMSE values of various system states and the squared error of system state values; the BCSE method performed better for DSSE than the NVSE method. The computational complexity accompanies the BCSE method as the branch current components allocates a lot of memory.

# Chapter 4

# Optimization Approaches to DSSE for Optimal Meter Placement

In this chapter, optimization based approaches are proposed to address the optimal meter placement problem considering different objectives such as minimization of cost, WLS residual estimate, and a multi-objective function comprising cost and WLS, and the ARMSE of the estimated state vector. The various optimization models are tested on the 33-bus distribution feeder. In Section 4.1 the PMU and PQ meter placement is formulated as optimization problems based on different objective functions as discussed above. In Section 4.2, the performance of the different methods are compared based on their meter placement cost and ARMSE of voltage estimates. In Section 4.3, the major findings are listed and the best performing among the five optimization methods for meter placement is reported.

## 4.1 PMU and PQ Meter Placement as an Optimization problem

State estimation is widely used in power transmission systems for obtaining a real time network model where measurements of bus voltages and line power flows are available. On the other hand, in distribution systems, with limited availability of measurements, and additional measurements being expensive, careful selection of location for the placement of meters becomes important. The measurement meters typically considered are PMUs and PQ meters.

Real-time monitoring of the operating conditions is a requirement for the secure operation of a power system. This can be achieved with synchrophasor technology, which uses the PMUs to measure electrical signals at the different locations across the power grid and synchronizes them using a common GPS satellite time signal. A PMU placed at a particular bus can measure the voltage phasor of that bus and the current phasors of all the branches radiating from the bus. The optimal PMU placement problem is formulated as minimizing the number of PMU installations, subject to full network observability.

In order to decide whether meters are to be placed at a bus or not, a binary decision variable can be used in the minimization problem. In this work various Mixed Integer Non-Linear Programming (MINLP) optimization models are formulated and compared to determine the optimal meter placement for a distribution system.

The cost considerations usually limit the number of PMUs that can be placed on distribution feeders, usually below the minimum needed for state estimation. To overcome this observability problem, traditionally, forecast load data is included as pseudo-measurements. In this work, instead of considering pseudo-measurements, PQ meters that measures real and reactive power injections are considered. The optimal meter placement problem considered in this chapter, is formulated in the form of the following minimization problem.

$$min\ J = \sum_{i=1}^{N}[W_{PMU}(i) + W_{PQ}(i)]f(x) \tag{4.1}$$

In (4.1), the PMU placement decision variables are:

$$W_{PMU}(i) = \begin{cases} 1 & PMU\ placed\ at\ bus\ i \\ 0 & otherwise \end{cases}$$

The PQ placement decision variables are:

$$W_{PQ}(i) = \begin{cases} 1 & PQ\ meter\ placed\ at\ bus\ i \\ 0 & otherwise \end{cases}$$

Subject to the following *Performance Requirement Constraints*:

- Observability Constraint: This constraint ensures that each bus in the network is observed at least once by PMUs.

$$\sum_{j} A_{i,j} W_{PMU}(i) \geq\ b \qquad \forall\ i\ and\ j \in N \tag{4.2}$$

The connectivity matrix $A_{i,j}$ ($\forall\ i,\ j\ \in\ N$) is defined as:

$$A_{i,j} = \begin{cases} 1 & if\ i\ =\ j \\ 1 & if\ i\ and\ j\ are\ connected \\ 0 & otherwise \end{cases}$$

b is a unity column matrix of dimension Nx1.

- Meter Placement Constraint: This constraint ensures that one meter is placed on every bus.

$$W_{PMU}(i) + W_{PQ}(i) = 1 \qquad \forall\ i \in N \tag{4.3}$$

- Measurement Constraints: These constraints correspond to the state variable measurements according to the corresponding binary decision variable. The PMUs will return the voltage and angle measurements, while the active and reactive power injection measurements will be provided by the PQ meters.

$$\begin{aligned} V_i^m &= W_{PMU}(i) \cdot (V_i^* + \epsilon_{V_i}) \quad && \forall\ PMUs \\ \delta_i^m &= W_{PMU}(i) \cdot (\delta_i^* + \epsilon_{\delta_i}) && \forall\ PMUs \\ P_i^m &= W_{PQ}(i) \cdot (P_i^* + \epsilon_{P_i}) && \forall\ PQs \\ Q_i^m &= W_{PQ}(i) \cdot (Q_i^* + \epsilon_{Q_i}) && \forall\ PQs \end{aligned} \tag{4.4}$$

- Measurement Function Constraint: These constraints are defined by non-linear equations $h(x)$ that relates the predicted value of measurement at bus $i$ to system states $x$, as given below:

$$h(x) = \begin{cases} \hat{V}_i & \forall\ i \\ \hat{\delta}_i & \forall\ i \\ \sum_j \hat{V}_i \hat{V}_j Y_{ij} cos(\theta_{ij} + \hat{\delta}_j - \hat{\delta}_i) & \forall\ i \\ -\sum_j \hat{V}_i \hat{V}_j Y_{ij} sin(\theta_{ij} + \hat{\delta}_j - \hat{\delta}_i) & \forall\ i \end{cases} \tag{4.5}$$

Different optimization problems can be formulate by appropriate choice of the function $f(x)$ in (4.1); for example $fx)$ can be a meter placement cost function, WLS estimate, a combined function of the cost and WLS estimate, or the ARMSE of voltage magnitudes. These are presented in the following sub-section.

### 4.1.1 Cost Based Optimal Meter Placement

In this method [30], the optimal placement of PMUs and PQ meters is formulated as a cost minimization problem, *i.e.*, minimizing the meter placement cost subject to various performance constraints as dedfined below.

$$min\ J_{Cost} = \sum_{i=1}^{N} C_{PMU} W_{PMU}(i) + \sum_{i=1}^{N} C_{PQ} W_{PQ}(i) \tag{4.6}$$

$$\text{Subject to: Performance Requirement Constraints}$$

where, $C_{PMU}$ is the cost of PMU installation and $C_{PQ}$ is the cost of PQ installation.

### 4.1.2 WLS Based Optimal Meter Placement

This formulation is used when considering a static state estimation; the objective is to minimize the sum of squares of the weighted residuals of the estimated measurements from the actual measurements, and hence estimate the system states. The formulation of this optimization model is as follows.

$$\begin{aligned} minimize\ J_{WLS} = &\sum_{i=1}^{N} \Big(\frac{V_i^m - \hat{V}_i}{\sigma_{V_i}}\Big)^2 \cdot W_{PMU}(i) + \sum_{i=1}^{N} \Big(\frac{\delta_i^m - \hat{\delta}_i}{\sigma_{\delta_i}}\Big)^2 \cdot W_{PMU} \\ &+ \sum_{i=1}^{N} \Big(\frac{P_i^m - \hat{P}_i}{\sigma_{P_i}}\Big)^2 \cdot W_{PQ}(i) + \sum_{i=1}^{N} \Big(\frac{Q_i^m - \hat{Q}_i}{\sigma_{Q_i}}\Big)^2 \cdot W_{PQ}(i) \end{aligned} \tag{4.7}$$

$$\text{Subject to: Performance Requirement Constraints}$$

### 4.1.3 Mixed Form Objectives for Optimal Meter Placement

Multi-objective optimization is an area of multiple criteria decision making that deals with simultaneous optimization of more than one objective function. In this work the aim is to minimize the cost of meter placement along with minimizing the WLS residual estimate. Two types of optimization approaches are proposed.

1. Goal Programming

2. Pareto Optimization

### Goal Programming for Optimal Meter Placement

Goal programming [40] is a well established multi-objective optimization method to handle multiple or conflicting objective measures. Each of these measures is assigned a goal or target value to be achieved, the achievement function minimizes the deviations from this set of target values. In the proposed goal programming problem the objective function seeks to minimize the weighted sum of two quantities: the normalized values of WLS estimate and meter placement costs, based on their minimum and maximum values as follows.

$$\begin{gathered} minimize\ J_{Goal} = W\widetilde{J}_{WLS} + (1-W)\widetilde{J}_{Cost} \\ \text{Subject to: Performance Requirement Constraints} \end{gathered} \tag{4.8}$$

where, W is the weight factor that ranges from 0 to 1. The normalized value of $\widetilde{J}_{WLS}$ and $\widetilde{J}_{Cost}$ are calculated as follows:

$$\begin{gathered} \widetilde{J}_{WLS} = \Big(\frac{J_{WLS} - \underline{J}_{WLS}}{\overline{J}_{WLS} - \underline{J}_{WLS}}\Big) \\ \widetilde{J}_{Cost} = \Big(\frac{J_{Cost} - \underline{J}_{Cost}}{\overline{J}_{Cost} - \underline{J}_{Cost}}\Big) \end{gathered} \tag{4.9}$$

$\underline{J}_{WLS}$ and $\overline{J}_{WLS}$ are the lowest and highest possible values of the WLS estimate, while $\underline{J}_{Cost}$ and $\overline{J}_{Cost}$ are the lowest and highest possible values of the cost estimate.

### Pareto Optimization for Optimal Meter Placement

In the Pareto optimal solution [40] approach, instead of generating a single optimal solution, many solutions are generated using different objectives, that satisfy the Pareto optimality criterion. According to this criteria, a solution point is accepted only if there are no solutions better than that with respect to the other objectives. For example let $P$ be a solution point. Even if $P$ is worse compared to another solution $P_1$ with respect to one objective, $P$ is still accepted if it is better than $P_1$ in at least one of the objectives. In this work, the objective function is formulated considering the WLS estimate and meter placement costs as a normalized function to obtain the Pareto optimal solution between the two objectives, as follows:

$$\begin{gathered} minimize\ J_{Pareto} = \sqrt{(\tilde{J}_{WLS})^2 + (\tilde{J}_{Cost})^2} \\ \text{Subject to: Performance Requirement Constraints} \end{gathered} \tag{4.10}$$

### 4.1.4 ARMSE Based Optimal Meter Placement

The ARMSE is a measure of the difference between the values predicted by an estimator and the actual observed values. ARMSE is explained in [41] as an effective performance metric that can be well utilized for DSSE efficiency analysis. ARMSE is defined as the root mean square error of the estimated system states from their true values, averaged over the number of buses (N). This is also referred to as the average prediction error of estimating the states $(V, \delta)$, of which, most relevant ones are the voltage magnitudes. Therefore the optimization problem is formulated in terms of true voltage at bus $i$ of the $l^{th}$ run $(V_i^l)^*$ and the estimated voltage at bus $i$ for the $l^{th}$ run $(\hat{V_i^l})$ as follows:

$$\begin{gathered} minimize\ ARMSE = \sqrt{\frac{1}{N}\frac{1}{L}\sum_{l=1}^{L}\sum_{i=1}^{N}[(V_i^l)^* - (\hat{V_i^l})]^2} \\ \text{Subject to: Performance Requirement Constraints} \end{gathered} \tag{4.11}$$

## 4.2 Results and Discussions

For the 33-bus radial distribution system, the OPF problem is solved to obtain the true states $V_i^*$, $\delta_i^*$, $P_i^*$, $Q_i^*$. For simulation purposes, a normally distributed white noise of zero mean and error variance of 1% of the OPF solution of $V$, $\delta$ are considered; for active and reactive power injections, a normally distributed white noise of zero mean and error variance of 3% of the OPF solution are considered. The state estimates are initialized at 1 p.u. for voltage estimates and 0 rad for angle estimates. The optimization models are formulated as MINLP problems and solved using the MINOS solver in GAMS [38].

The various optimization methods for optimal PMU and PQ meter placement discussed earlier are compared in terms of the number of buses where PQ and PMU meters are placed,

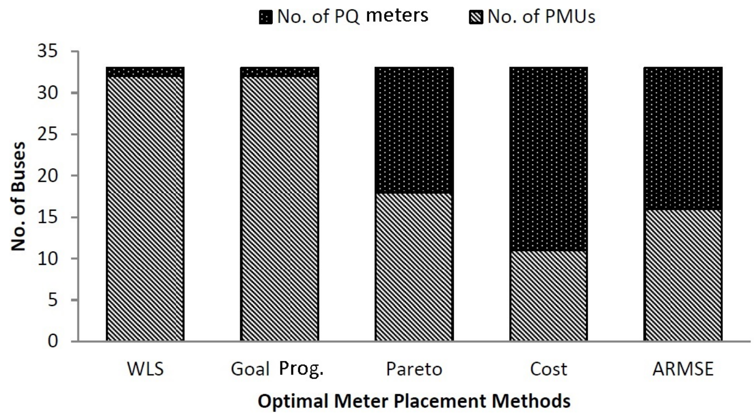


Figure 4.1: Optimal meter placements for various optimization objectives

total cost of meter placement and ARMSE of voltage magnitudes. It is considered that the cost of PMU placement is $40,000 [42] per unit. Also, it is assumed, without any loss of generality, that cost of PMUs are 40 times the cost of PQ meters, and accordingly, PQ meter placement cost is $1000 per unit.

Figure 4.1 shows the number of PMUs and PQ meters installed in the various optimization methods. It is noted that the cost based optimal meter placement method selects the fewest number of PMUs; the Pareto optimal solution and ARMSE based meter placement also selects mostly PQs and only a few PMUs. On the other hand, the WLS based and goal programming results in high number of PMUs being installed.

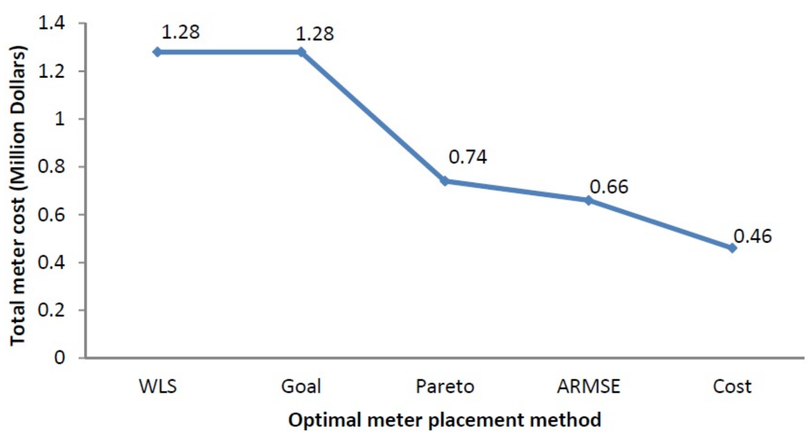


Figure 4.2: Cost of meter placement for various optimization objectives

Meter placement cost is the next performance metric used for comparing the various optimization methods and is presented in Figure 4.2. It is noted that the cost based and ARMSE based optimal meter placement methods yields the lowest cost solution while the WLS and goal programming based methods are the most expensive placement methods as they mostly select the expensive PMUs.

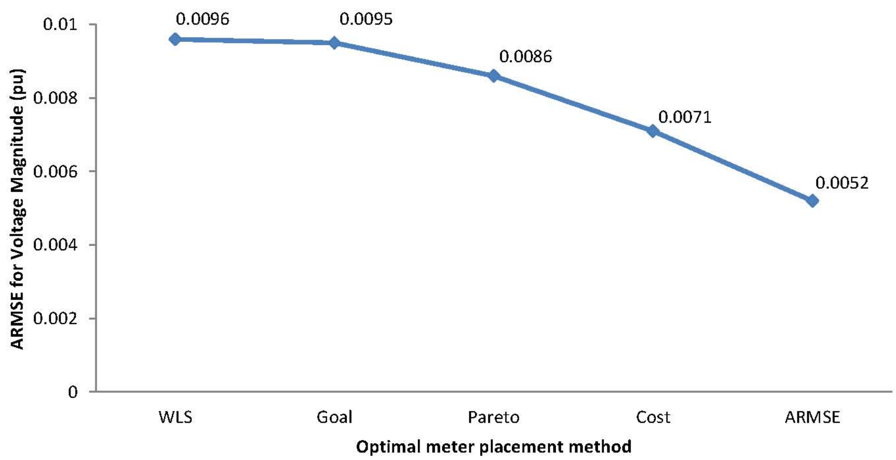


Figure 4.3: Comparison of ARMSE for various optimization methods

Figure 4.3 shows the performance comparison of these optimization methods based on the ARMSE of voltage, evaluated over L= 5 runs. It is noted that the best among the five optimization formulations is the ARMSE based optimal meter placement method. Cost minimization and Pareto optimality based methods also have comparatively low ARMSE values, while the WLS and goal programming based methods yield high ARMSE values, *i.e.,* these methods have the highest state estimation errors compared to other methods.

Figure 4.4 shows the true and estimated voltage plots for various optimal meter placement methods. Analyzing the plots it is noted that the ARMSE based method results in voltage estimates much closer to the true voltages. The cost based optimization approach and the Pareto optimal solution results in good estimations as well, where the estimated voltages are close to the true system states, as compared to the WLS based and goal programming based methods where the estimated voltages are more deviated from their true values.

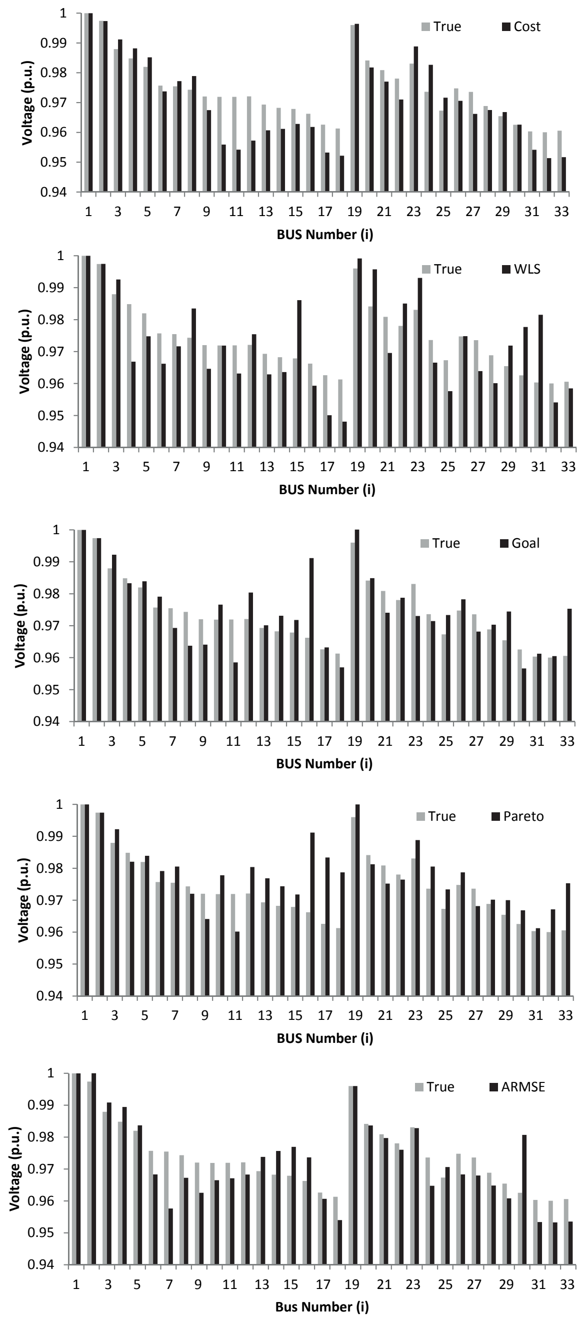


Figure 4.4: True and estimated voltages for various meter placement methods

The optimal locations obtained from various optimization methods are shown in Table 4.1, where '✓' shows the placement of PQ meter and 'X' shows the placement of PMU at bus *i*. It is noted that the cost based and ARMSE based optimal meter placement methods choses mostly PQ meters rather than PMUs. The WLS based and goal programming based methods selects mostly PMUs than PQ meters.

Table 4.1: Locations selected by different optimization objectives

| Bus | Method | | | | |
|---|---|---|---|---|---|
| | Cost | WLS | Goal | Pareto | ARMSE |
| 1 | ✓ | ✓ | ✓ | ✓ | X |
| 2 | X | X | X | X | ✓ |
| 3 | ✓ | X | X | X | X |
| 4 | ✓ | X | X | X | ✓ |
| 5 | X | X | X | X | ✓ |
| 6 | ✓ | X | X | X | X |
| 7 | ✓ | X | X | ✓ | X |
| 8 | X | X | X | ✓ | X |
| 9 | ✓ | X | X | X | X |
| 10 | ✓ | X | X | X | ✓ |
| 11 | X | X | X | X | ✓ |
| 12 | ✓ | X | X | X | X |
| 13 | ✓ | X | X | ✓ | ✓ |
| 14 | X | X | X | ✓ | ✓ |
| 15 | ✓ | X | X | X | X |
| 16 | ✓ | X | X | X | ✓ |
| 17 | X | X | X | ✓ | ✓ |
| 18 | ✓ | X | X | ✓ | X |
| 19 | ✓ | X | X | X | ✓ |
| 20 | ✓ | X | X | ✓ | X |
| 21 | X | X | X | X | ✓ |
| 22 | ✓ | X | X | ✓ | ✓ |
| 23 | ✓ | X | X | ✓ | ✓ |
| 24 | X | X | X | ✓ | X |
| 25 | ✓ | X | X | X | X |
| 26 | ✓ | X | X | X | X |
| 27 | X | X | X | X | ✓ |
| 28 | ✓ | X | X | ✓ | ✓ |
| 29 | ✓ | X | X | ✓ | X |
| 30 | X | X | X | ✓ | X |
| 31 | ✓ | X | X | X | X |
| 32 | X | X | X | ✓ | ✓ |
| 33 | ✓ | X | X | X | ✓ |

## 4.3 Conclusions

In this chapter, various optimization approaches for DSSE to provide the best possible system state estimate, while determining the best meter placement plan were presented. The results showed that the ARMSE based optimal meter placement method was the best as it selected the least number of PMUs and the ARMSE value of voltage estimates was the lowest. Cost based optimal meter placement and Pareto based optimal meter placement are also sufficiently good placement approaches, according to the performance metrics and the limited number of PMUs selected. The formulations based on WLS algorithm and goal programming can be used in placement problems where large number of PMUs are needed to provide a real time tagged high accurate voltage measurements of the network and accuracy is much more critical compared to cost considerations.

# Chapter 5

# Iterative Extended Kalman Filtering for DSSE

In this chapter, the detailed mathematical formulation of DSSE using IEKF method and the analysis of DSSE performance are discussed in detail. In Section 5.1, the detailed steps involved in IEKF for DSSE are explained. In Section 5.2, the results of DSSE based on the IEKF approach are discussed, and the impact of the length of apriori information and PMU measurement error on DSSE performance are examined. Section 5.3 summarizes the work presented in this chapter.

## 5.1 Iterative Extended Kalman Filtering

One of the newest Kalman filtering method is the Iterative Kalman filtering which is characterized as an iterative application of the Kalman filter for the case of non-linear processes. The IEKF [29] is an iterative application of EKF, discussed in Chapter 2, wherein along with the non-linearity of the system model, the observation model is also a non-linear differentiable function because of the presence of non-linear measurements. The non-linear form of (2.5) can thus be expressed as:

$$Z_t = h(X_t^*) + \nu_t \tag{5.1}$$

Where, $h(X_t^*)$ is a non-linear function to compute the predicted measurements from the actual system states. Thus in the update step, of the EKF method (Sub-section 2.3.2), given by (2.18), the measurement residual vector at time $t$ based on non-linear measurement row vector $Z_t$ and the non-linear measurement or observation model is given by $\nu_t = Z_t - h(X_t)$. The function $h(X_t)$ cannot be directly applied hence the Jacobian $H_t$ of the measurement function is used instead, as follows:

$$H_t = \frac{\partial h(X_t^*)}{\partial X_t^*} \tag{5.2}$$

Where, the function $h(X_t^*)$ comprises measurement estimates, considered as true states in this work, and is given by:

$$h(X_t^*) = \begin{pmatrix} V_i^* \\ \delta_i^* \\ P_i^* \\ Q_i^* \end{pmatrix} \tag{5.3}$$

The steps involved in the IEKF algorithm are as follows:

1. Obtain the distribution system topology information $Y_{ij}\angle\theta_{ij}$ and use this in the OPF to obtain the true states of the system. The true voltages $V_i^*$, true angles $\delta_i^*$, the true real and reactive power injections $P_i^*$,$Q_i^*$ are thus obtained.

2. The demand profile $u_{k,t}$, for different time steps $t$ can be obtained by multiplying $[P_i^{*T} Q_i^{*T}]^T$ with the hourly load variation ratio to obtain $u_{k,t} = [P_{i,t}^T\ Q_{i,t}^T]^T$, $\forall\ i \in \{2, ...33\}$. A typical 24 hour demand curve of the Ontario power system is considered for the purpose. An error is associated with this generated load profile, accordingly, the error covariance matrix is represented as follows:

$$S_t = \begin{bmatrix} \sigma_{P_{i,t}}^2 & 0 \\ 0 & \sigma_{Q_{i,t}}^2 \end{bmatrix} \tag{5.4}$$

   Where, $\sigma_{P_{i,t}}^2$ and $\sigma_{Q_{i,t}}^2$ are the standard deviations of the error associated with the real and reactive power injection measurements, respectively.

3. Using all available information on the state estimate at time *t-1*, calculate $J_{t-1}$ in (2.10), which is the Jacobian of the power flow equations as explained earlier.

4. The system state matrix $X_{k,t} = [V_{i,t}^T\ \delta_{i,t}^T]^T$, $\forall\ i \in \{2, .., N\}$ is determined using the true bus voltages and angles in the state propagation model, *i.e.,* prediction equations,

given as follows:

$$X_{k,t} = X_{k,t-1} + J_{t-1}^{-1}[u_{k,t} - u_{k,t-1}] \tag{5.5}$$

5. Now find the Jacobian of the measurement function $h(X_t^*)$ represented as $H_t$.

$$H_t = \begin{bmatrix} \frac{\partial V_{i,t}^*}{\partial \delta_i^*} & \frac{\partial V_{i,t}^*}{\partial |V_i^*|} \\ \frac{\partial \delta_{i,t}^*}{\partial \delta_i^*} & \frac{\partial \delta_{i,t}^*}{\partial |V_i^*|} \\ \frac{\partial P_{i,t}^*}{\partial \delta_i^*} & \frac{\partial P_{i,t}^*}{\partial |V_i^*|} \\ \frac{\partial Q_{i,t}^*}{\partial \delta_i^*} & \frac{\partial Q_{i,t}^*}{\partial |V_i^*|} \end{bmatrix} \tag{5.6}$$

Where,

$$\begin{aligned} P_{i,t}^* &= \sum_{j=1}^{N} V_{i,t}^* V_{j,t}^* Y_{ij} cos(\theta_{ij} + \delta_{j,t}^* - \delta_{i,t}^*) \qquad \forall\ i,\ i \neq slack \\ Q_{i,t}^* &= -\sum_{j=1}^{N} V_{i,t}^* V_{j,t}^* Y_{ij} sin(\theta_{ij} + \delta_{j,t}^* - \delta_{i,t}^*) \quad \forall\ i,\ i \neq slack, \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i \neq NPV \end{aligned} \tag{5.7}$$

The partial differentials $\frac{\partial V_{i,t}^*}{\partial \delta_i^*}$ and $\frac{\partial \delta_{i,t}^*}{\partial |V_i^*|}$ will return (N-1)x(N-1) matrices whereas the partial differentials $\frac{\partial V_{i,t}^*}{\partial |V_i^*|}$ and $\frac{\partial \delta_{i,t}^*}{\partial \delta_i^*}$ will be (N-1)x(N-1) identity matrices. The other partial differential terms are calculated in the same way as in (2.10).

6. An assumption of 1% error for voltages and angles ($X_{k,t}$) referred to as 'PMU errors', and 3% error for the real and reactive power injections ($u_{k,t}$) referred to as 'PQ meter errors' is made. These assumptions are used as measurement error ($\nu_t$), with error covariance matrix $R_t$, $\forall\ i \in \{1,..,N\},\ i \neq$ slack.

$$R_t = \begin{bmatrix} \sigma_{V_{i,t}}^2 & 0 & ... & 0 \\ 0 & \sigma_{\delta_{i,t}}^2 & ... & 0 \\ 0 & ... & \sigma_{P_{i,t}}^2 & 0 \\ 0 & 0 & ... & \sigma_{Q_{i,t}}^2 \end{bmatrix} \tag{5.8}$$

7. The noisy measurements are determined using $Z_t = H_t X_{k,t} + \nu_t$ where $\nu_t = normrnd(0, R_t)$ which is a normally distributed noise of zero mean and $R_t$ as error covariance matrix.

8. Initialize the state vector estimate $\widehat{X}_{k,1}$ and the associated state error covariance matrix as $\widehat{C}_1$ which is a diagonal matrix with diagonal terms calculated as follows:

$$\widehat{C}_1 = E[\widehat{X}_{k,1} - X_{k,1}][\widehat{X}_{k,1} - X_{k,1}]^T \tag{5.9}$$

   The estimation is not performed at bus 1 as it is the slack bus and the voltage magnitude and angle at bus 1 are fixed to 1 pu and 0 rad respectively.

9. Execute the prediction process to find the predicted current states from the previous time estimates, the Jacobian matrix $J_t$ and the load profile variations.

$$\bar{X}_{k,t+1} = \hat{X}_{k,t} + J_t^{-1}[u_{k,t+1} - u_{k,t}] \qquad \forall\ k \neq \text{slack} \tag{5.10}$$

   The prediction of error covariance matrix of the predicted states for all buses except the slack bus is given as,

$$\bar{C}_{t+1} = \widehat{C}_t + S_t \tag{5.11}$$

10. Now use the error covariance matrix of the predicted states and the Jacobian of the measurement function $H_t$ to calculate the Kalman gain and hence adjust the estimated state to make it close enough to the true value, as follows: *i.e.,*

$$K_{t+1} = \bar{C}_{t+1} H_{t+1}^T (H_{t+1} \bar{C}_{t+1} H_{t+1}^T + R_{t+1})^{-1} \tag{5.12}$$

11. Using $K_{t+1}$ from (5.12), update the state estimates and its covariance matrix using the following equations:

$$\hat{X}_{k,t+1} = \bar{X}_{k,t+1} + K_{t+1}[Z_{t+1} - H_{t+1}\hat{X}_{k,t+1}] \qquad \forall\ k \neq \text{slack} \tag{5.13}$$

$$\widehat{C}_{t+1} = \bar{C}_{t+1} + K_{t+1} H_{t+1} \bar{C}_{t+1} \tag{5.14}$$

In order to start the IEKF method, the length of apriori information of estimation (T) is selected, which is the length of estimated state vectors that is available prior to estimating a state at time $t = T + 1$, where $t$ is the time frame of estimation. The IEKF algorithm based on the steps explained earlier, is presented in Figure 5.1.

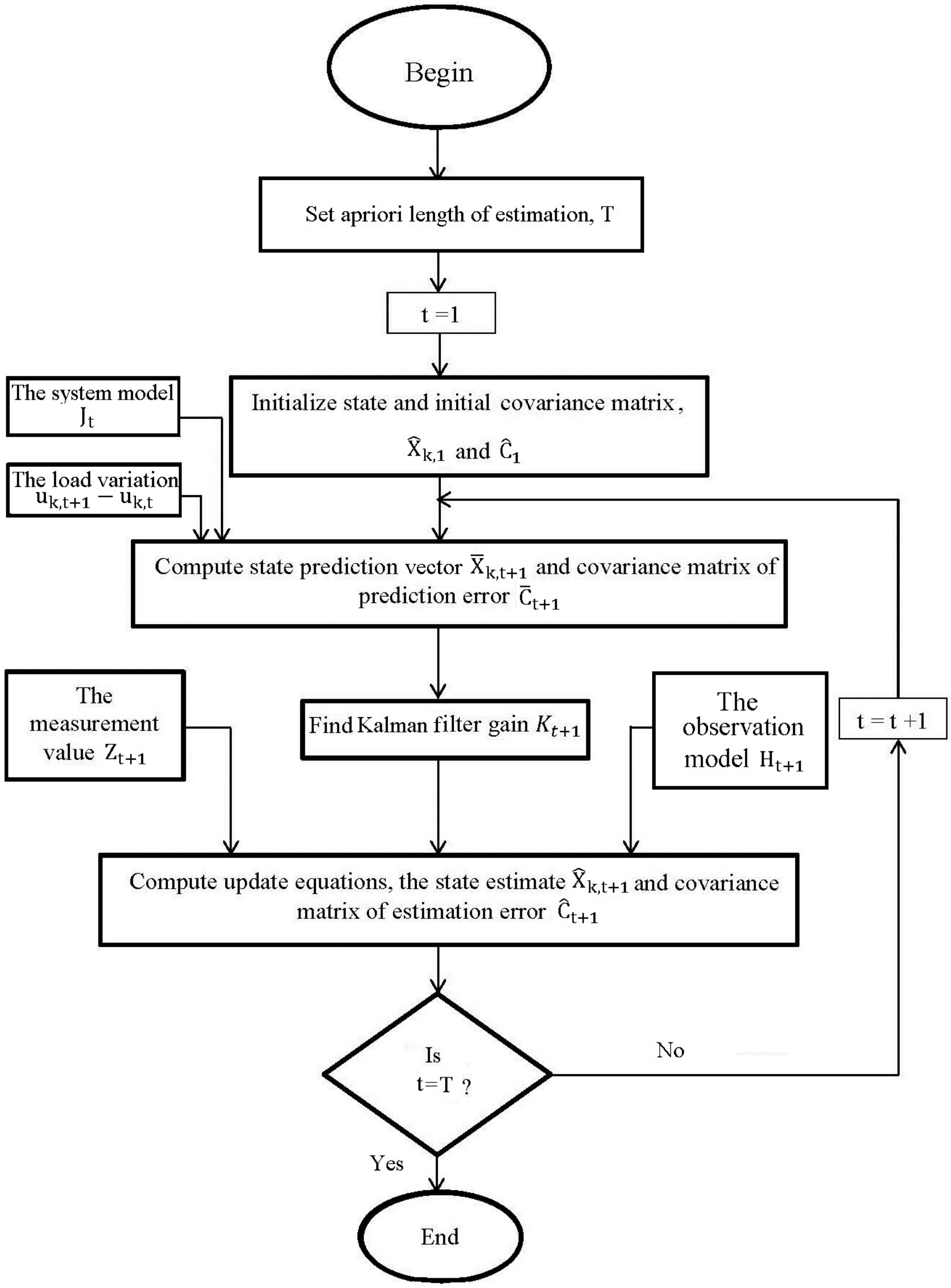


Figure 5.1: The flowchart of the IEKF method

## 5.2 Results and Discussions

For simulation purpose the balanced 33-bus distribution system used in [43] is considered where the bus-1 is the slack bus with no PV buses. For calculation of the system model, the Jacobian of the load flow equation is determined by (2.10). In the square matrix $J_t$, each individual matrices will be a 32x32 matrix, whereas the Jacobian $H_t$ of measurement function $h_t$ is a non-square matrix with each individual matrices of size 32x32. The initial true state of the system is the OPF solution and the state propogation equation (5.5) is applied on this initial true value to find out the true states corresponding to each time step. The initial voltage estimate at time $t$=1 is assumed as 1 pu and angle estimate as 0 rad which is the starting point of the state estimation process. Assume the error associated with PMU measurements are 1% of bus voltage and voltage angles and PQ meter measurements are 3% of real and reactive power injections.

In transmission systems, the time frame for monitoring and estimating the system states varies between 5 to 15 minutes [44], whereas the choice of an appropriate timescale for DSSE problem is an open question. In this work, the main model analysis is performed considering the length of apriori information (T) as 21 minutes, *i.e.,* the state estimation starts from time $t$=1 min and ends at $t$=22 min progressing in a time step of 1 minute. The convergence criteria is based on the two threshold values $\epsilon_v$ and $\epsilon_\delta$, which are chosen as 4x10$^{-5}$ and 7x10$^{-5}$ respectively. The proposed algorithm is simulated in MATLAB.

### 5.2.1 Effect of Length of Apriori Information (T) on DSSE

For DSSE using IEKF an important aspect is the selection of the value of T for the estimation process. For effective estimation with minimum error, this is really important as the system states converges over the range of T=1 to $t-1$, thereby reaching the best possible value at time $t$. In this work, a new state estimate is computed for every time step $\Delta t$.

For computational purposes, T is varied from 1 to 21 minutes, in time step of 1 minute, to obtain the state estimates while load variation $u_{k,t} - u_{k,t-1}$ is not significant. Thus, for T=21 min, state estimation starts with a rough intitialization from $t$=1 min, utilizing the real time measurements and the previous time state estimate, progressing in time steps of 1 minute. The convergence criteria used in this work is based on the ARMSE values of system states, and are defined as follows:

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}|\hat{V}_{i,t}-V_{i,t}^{*}|^2} \leq \epsilon_v \tag{5.15}$$

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}|\hat{\delta}_{i,t}-\delta_{i,t}^{*}|^2} \leq \epsilon_\delta \tag{5.16}$$

where $\epsilon_v$, $\epsilon_\delta$ represents the preset threshold to be achieved, and are chosen prior to the estimation. The value of T chosen for the state estimation process has a direct dependence on the rate of convergence of system states.

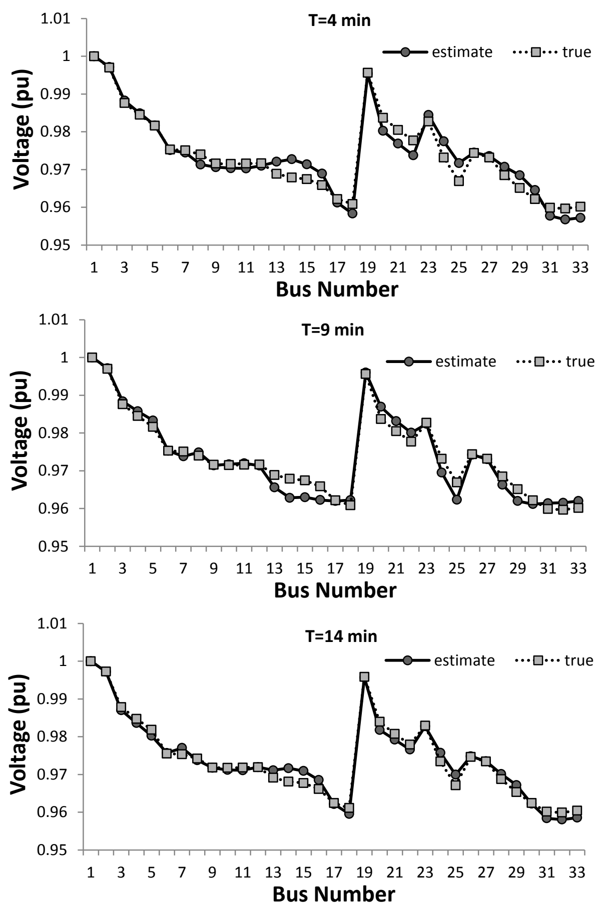


Figure 5.2: Convergence of system voltage estimates with variation in T

Figure 5.2 and 5.3 shows the convergence of the bus voltages and voltage angles to its true states with variation in T.

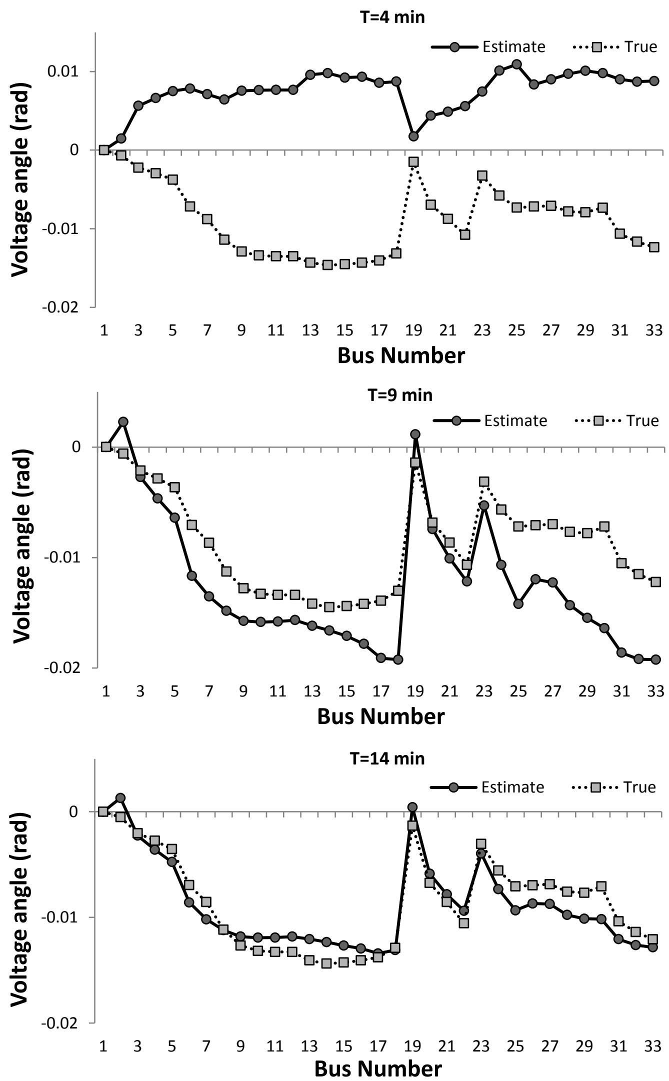


Figure 5.3: Convergence of voltage angle estimates with variation in T

From the above plots, it is observed that the convergence improves as T increases. It is noted that the voltage angle estimates are inferior in convergence as compared to bus voltages, for the same value of T. For T=21 min and beyond, the quality of connvergence for the bus voltage angles attains acceptable level of accuracy.

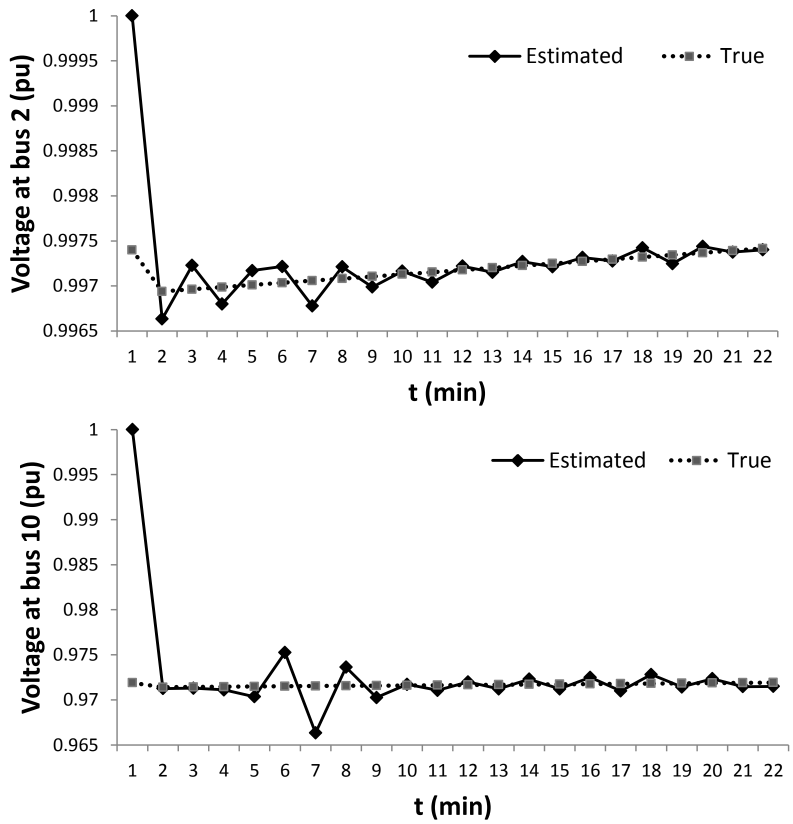


Figure 5.4: Process of convergence of bus voltage estimates over the range from T=1 to 21 min

Figure 5.4 and 5.5 shows the convergence of bus voltages and angles at two buses. These figures explain how well the IEKF based DSSE works over the range of T=1 to 21 min. Starting from an initialization of voltage magnitudes and angles to 1 pu and 0 rad respectively, at $t$=1 minute, settling to a particular value over the time horizon t=22 min. In Figure 5.4, the bus voltages converge to their true values by $t$=22 min, *i.e.,* T=21 min. The bus voltage angle estimates acquire the best possible value at $t$=21 min, *i.e.,* T=20 min, as shown in Figure 5.5.

In IEKF, the resultant state estimates from the prediction step in (5.10) is the states estimated without the knowledge of real time measurements termed as apriori estimates. On the other hand the resultant estimates obtained from the update step (5.13) are termed as aposteriori estimates which are updations of apriori estimates using Kalman gain and current

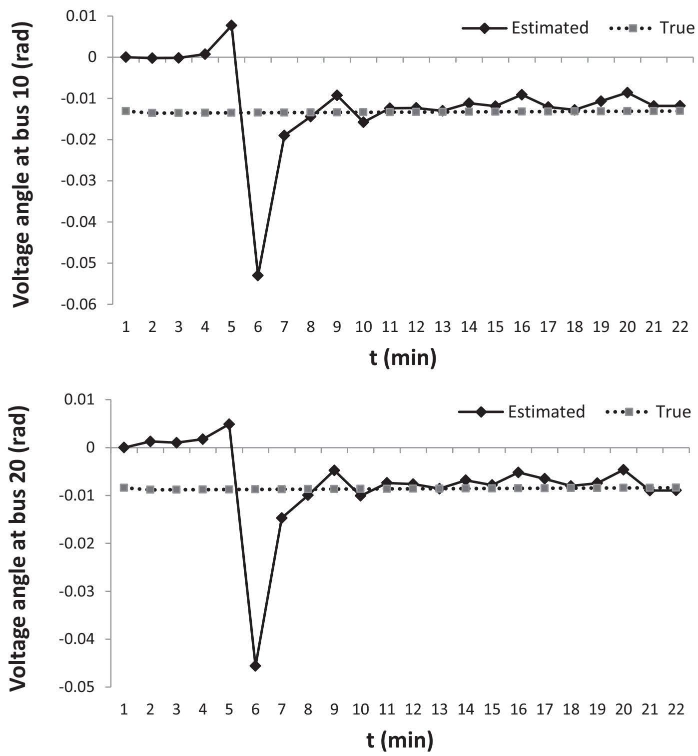


Figure 5.5: Process of convergence of bus voltage angle estimates over the range from T=1 to 21 min

time measurements $Z_t$. The Kalman gain is an mxn matrix where m corresponds to number of state variables and n corresponds to the real time observations. Kalman gain is computed using the estimated covariance matrix of the state forecasts and all the information up to period $t-1$. Kalman gain matrix weights are in such a way that it will adjust the prediction of a time instant so as to make the current state estimate closer to the actual true state of the system there by minimizing $\hat{X}_{k,t} - X^*_{k,t}$. This is explained in Section 5.1 using (5.12).

The estimation starts with a flat start of aposteriori voltage and angle estimates at time $t$=1 min, of 1 pu and 0 rad respectively. The apriori state estimation calculation is carried out for time step $t$=2 min and these estimates are adjusted iteratively at each time step using the Kalman gain to reduce the deviations of final estimated results from their actual true states. This results in the aposteriori estimates at each time step. At $t$=22 min the Kalman gain settles to a value of 0, and the aposteriori estimates converges to the apriori estimates *i.e.*, $\hat{X}_{k,t+1} = \bar{X}_{k,t+1}$, as shown in Figure 5.6.

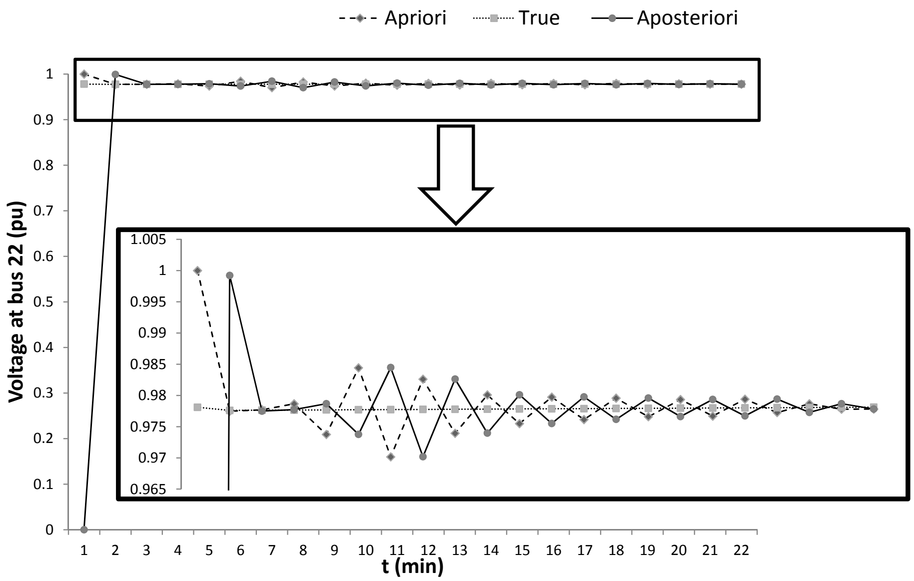


Figure 5.6: Apriori and Aposteriori estimates at T=21 min

Table 5.1: ARMSE for various T and PMU measurement errors

| Apriori Length (T) | PMU Errors | | |
|---|---|---|---|
| | 1% | 5% | 10% |
| 14 | 5.59E-05 | 1.5E-04 | 2.58E-03 |
| 17 | 4.63E-05 | 1E-04 | 3.1E-04 |
| 21 | 4.07E-05 | 4.98E-05 | 5.23E-05 |

The Table 5.1 shows the ARMSE values of voltage estimates for different values of T and three different caases of PMU errors (1%, 5% and 10%). As mentioned at the beginning of this section, the considered convergence criteria are $\epsilon_v$= 4x10$^{-5}$ and $\epsilon_\delta$=7x10$^{-5}$. It is noted that only the case of T=21 min and PMU error=1%, satisfies this criteria. From the Table, it is noted that the ARMSE decreases as the value of T increases and as the PMU error decreases. For 5% and 10% PMU errors, a value of T greater than 21 is needed, in order to meet the convergence criteria in (5.15). In short, for higher values of PMU errors, the estimation accuracy can be increased only by providing more apriori estimates, which will increase the overall time horizon of the estimation and the computational complexity.

### 5.2.2 Effect of PMU Measurement Error on DSSE

As discussed earlier, state estimation uses real time measurements and previous time state estimates to calculate the current state of the system. Real-time measurements are obtained using PMUs and PQ meters. To reduce the computational complexity and obtaining the best possble state estimate, while keeping the cost factor out, all buses except the slack bus, are monitored using a PMU and a PQ meter. Therefore, for all the buses, other than slack bus, there will be a set of voltage and voltage angle measurements provided by PMUs and real and reactive power measurements provided by PQ meters.

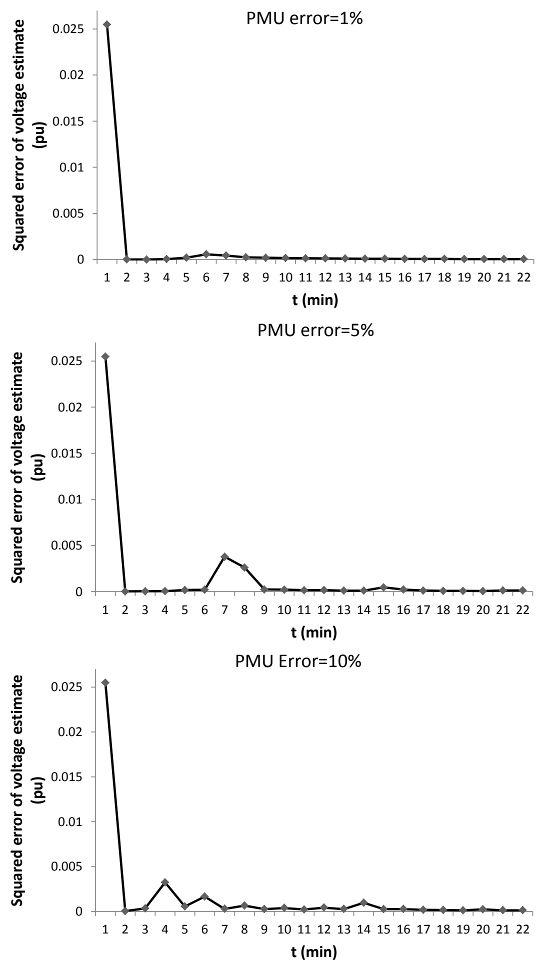


Figure 5.7: Squared error of voltage estimates for different PMU measurement errors

Since the distribution system states are represented in terms of the bus voltages and voltage angles, the PMU measurement accuracy will directly influence the DSSE performance. For this case study, three PMU measurement errors, *i.e.,* 1%, 5%, 10% are considered. The squared error of bus voltages and voltage angles based on the convergence criteria in (5.15), are as shown in Figure 5.7 and 5.8 respectively.

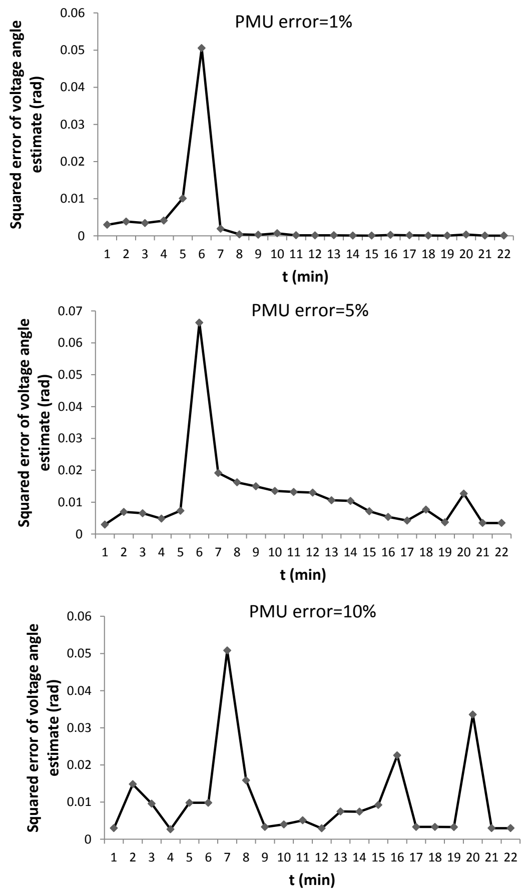


Figure 5.8: Squared error of voltage angle estimates for different PMU measurement errors

It can be observed that as the PMU error increases, the convergence time increases. This is particularly evident for convergence of voltage angle estimates. It is noted that for 1% PMU error, convergence is attained at t=15 min, and for 5% and 10% PMU errors, within t=20 to 22 min. When the error value is between 10% to 20%, then such measurements are refered to as pseudo measurements. In such cases, the convergence is not smooth, which indicates the a poor state estimation result compared to the other two cases.

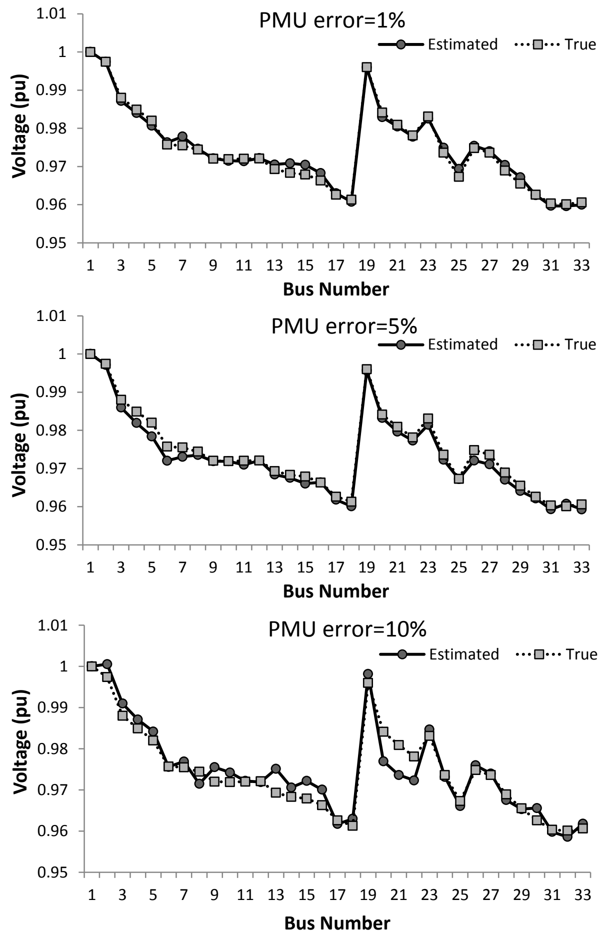


Figure 5.9: Bus votage estimates for different PMU measurement errors for T=21 min

It is also important to examine the impact of PMU measurement errors on IEKF based DSSE results. The plot of bus voltages and voltage angles for various cases of PMU errrors (1%, 5% and 10%) are shown in Figures 5.9 and 5.10 respectively. The accuracy achieved by voltage estimates are nearly similar for 1% and 5% PMU errors, but is significantly deteriorted for 10% PMU error, which is not acceptable, as the distribution system is sensitive to small variations in voltage magnitudes. The voltage angle estimation is accurate for PMU error of 1% but in the presence of pseudo measurements (10% error), the IEKF based DSSE results are significantly inferior.

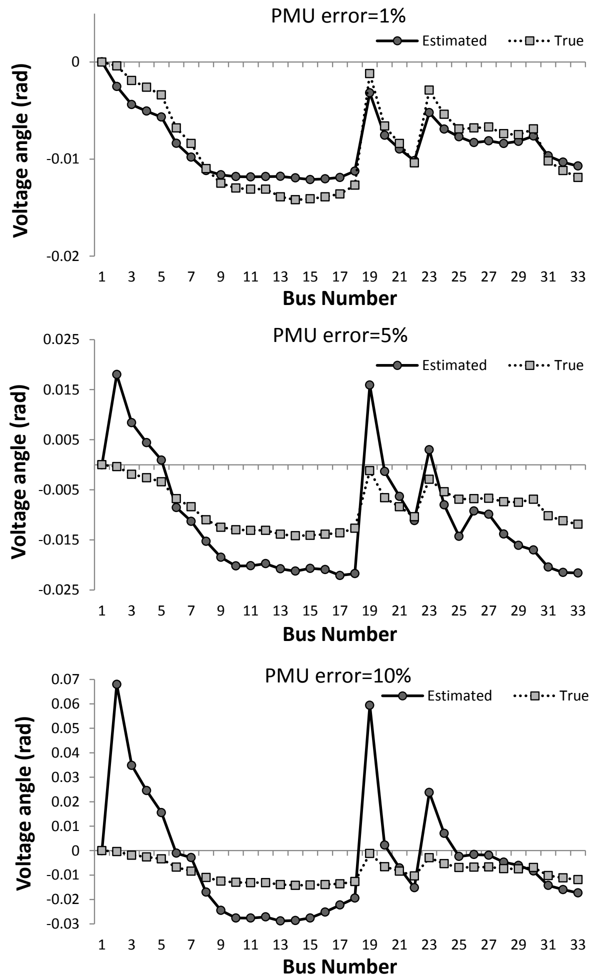


Figure 5.10: Bus voltage angle estimates for different PMU measurement errors for T=21 min

In a Kalman filter based state estimation, one of the most important factors that play a key role in between the update step and the prediction step is the Kalman gain. The Kalman gain is an $m$x$n$ matrix where $m$ corresponds to number of state variables and $n$ corresponds to the real time observations. Kalman gain is computed using the estimated covariance matrix of the predicted state, the observation model and its error covariance up to period $t-1$. Kalman gain matrix weights are in such a way that it will adjust the prediction of a time instant so as to make the current state estimate closer to the actual true

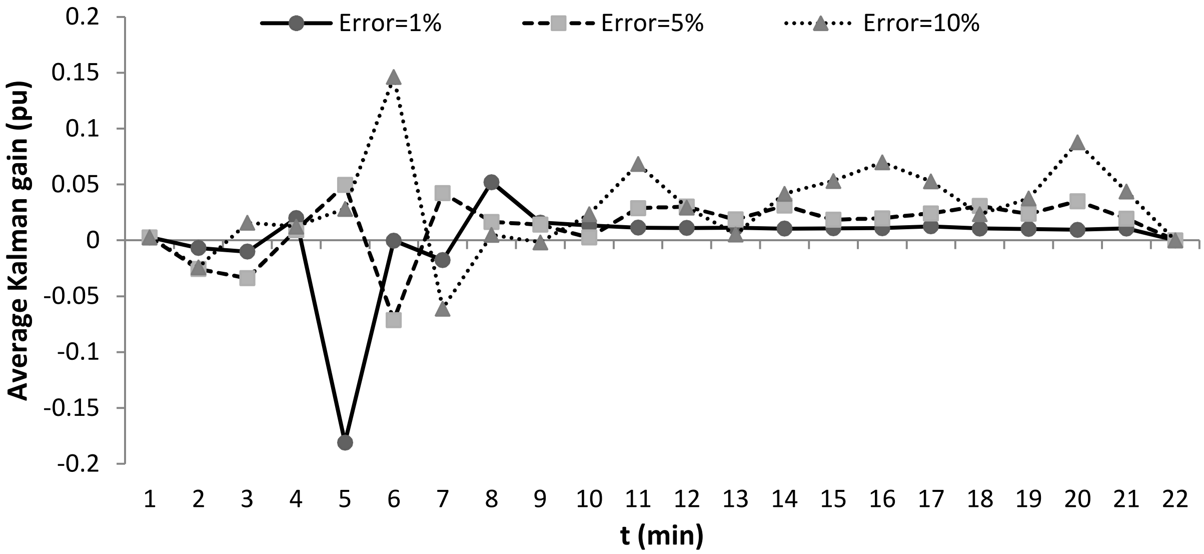


Figure 5.11: Average Kalman gain over time for various PMU measurement errors

state of the system there by minimizing $\hat{X}_{i,t} - X^*_{i,t}$. This is explained in Section 5.1 using (5.12).

The Kalman gain adjust the apriori estimates as given in (5.13), resulting in the aposteriori estimates at each time step. By the end of the estimation time *i.e.,* t=22 min, Kalman gain is tuned to a value of zero, thereby ensuring the equality of predicted state and estimated state at the correction step. The Kalman gain will be also impacted by the PMUs error values. The average Kalman gain is plotted over various time steps for different error values in Figure 5.11. The figure depicts that the average Kalman gain weights for 1% PMU error takes relatively lower values whereas for higher PMU errors, the average Kalman gain weights becomes comparitively higher to ensure efficiency at the correction step, resulting in increased computational complexity.

## 5.3 Conclusions

The chapter presented the dynamic DSSE using an IEKF, which is an iterative application of the EKF. The detailed step-by-step procedure and the mathematical modelling for IEKF was proposed. Furthermore, a simplified flowchart for IEKF based DSSE was presented. The voltage and angle estimates obtained using the IEKF algorithm are observed to satisfy the required convergence criteria. The estimation results for different values of T were compared in terms of the ARMSE value of voltages for different PMU error scenerios, and it was also noted that the DSSE accuracy was highly influenced by the selection of T. The effect of PMU measurement error on DSSE performance was also analyzed by comparing the squared errors of bus voltages and voltage angle estimates. The best DSSE performance was obtained for T=21 min and a PMU error of 1%. It was also observed that the DSSE performance was worst, corresponding to pseudo measurements of 10% PMU error. This impact could be mitigated by using higher values of T at the cost of increased estimation time $t$ and computational complexity.

# Chapter 6

# Conclusions and Future Work

## 6.1 Summary and Conclusions

State estimators plays an important role in ensuring the safe operation of distribution networks, hence detailed studies of various DSSE algorithms and the factors affecting their performance quality are needed. As real time measurements are used for DSSE, there is a need for cost effective placement of measurement devices without compromising the estimation quality. This thesis presents detailed studies on various static and dynamic DSSE algorithm and optimal meter placement in distribution systems. The main contents of this thesis are summarized briefly, as follows:

- The motivations of this research, a brief review of the literature, and the research objectives were presented in Chapter 1.
- Chapter 2 presented a brief overview of the background topics such as WLS based static DSSE method, cost based optimal meter placement technique and the concept of linear Kalman filters and EKF for dynamic DSSE.
- Chapter 3 presented the two DSSE algorithms, namely, the conventional NVSE and BCSE methods, formulated as optimization problems and the performance of the estimates were compared. It was noted that the BCSE method performed better state estimation than the NVSE method. Studies were conducted on the NVSE method to demonstrate the direct dependency of the number of PMU measurements used and the quality of DSSE.
- Chapter 4 discussed various optimization models for the optimal meter placement problem for DSSE, considering minimization of cost, WLS residual, an objective comprising cost and WLS residual, and the ARMSE value of voltages. These methods were compared based on the criteria of total meter cost and DSSE accuracy achieved.

Studies reveal that the ARMSE based optimal meter placement method was the best out of all the proposed methods.

- Chapter 5 presented the concept of dynamic DSSE using an IEKF, which was an extension of the EKF model presented in Chapter 2. The impacts of the selection of the T parameter, and PMU measurement errors, on DSSE accuracy were also discussed by conducting different case studies. Investigation revealed that to improve the DSSE accuracy in the presence of larger PMU errors, higher values of T were needed, which however increased the estimation time.

## 6.2 Contributions

The main contributions of the research presented in this thesis are as follows:

- The two well known static DSSE algorithms, namely, the NVSE and BCSE based on WLS estimation, are formulated as optimization problems which helps to arrive at the optimal state estimates faster and with high accuracy. The performance of the two algorithms are compared for accuracy and speed of convergence.
- Different optimization models are proposed, considering minimization of meter cost, WLS estimate, a combination of cost and WLS estimate, and ARMSE values, to solve the optimal meter placement problem for static DSSE.
- The thesis presented the studies on IEKF for dynamic DSSE and discussed the impact of various parameters such as the selection of apriori information length (T) and PMU measurement error, on the accuracy of estimated results.

## 6.3 Future Work

In the context of dynamic DSSE, one major assumption made was that the real time measurements, such as voltages and power injection values, were available from all the buses. But since PMUs are very expensive units, an effective meter placement strategy should be adopted so that the entire system is observable, while at the same time, the quality of estimation is not compromised. If PMUs are not installed at a particular bus, the power injections obtained from PQ meters can be used to determine the voltage measurements, and which can be considered to be pseudo voltage measurements, that are available at buses without PMUs, so as to carry out DSSE. Thus an extension of this work will be to develop an IEKF for DSSE with optimal meter placement consideration.

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