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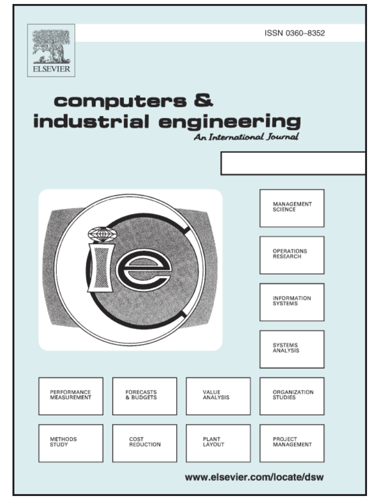

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Optimal Randomized Ordering Policies for a Capacitated Two-Echelon Distribution Inventory System

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Abstract

We propose a new formulation for controlling inventory in a two-echelon distribution system consisting of one warehouse and multiple non-identical retailers. In such a system, customer demand occurs based on a normal distribution at the retailers and propagates backward through the system. The warehouse and the retailers have a limited capacity for keeping inventory and if they are not able to fulfill the demand immediately, the demand will be lost. All the locations review their inventory periodically and replenish their inventory spontaneously based on a periodic Randomized Ordering (RO) policy. The RO policy determines order quantity of each location in each period by subtracting corresponding on-

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hand inventory at the beginning of that period from a deterministic decision variable. We propose a mathematical model to find the optimal RO policies such that an average systemwide cost consisting of ordering, holding, shortage, and surplus costs is minimized. We use the first and second moments of on-hand inventory as auxiliary variables. A remarkable advantage of this model is calculating the immediate fill rate of all locations without adding new variables and facing the curse of dimensionality. Using two numerical examples with stationary and non-stationary demand settings, we validate and evaluate the proposed model. For validation, we simulate the optimal RO policy and demonstrate that the optimal first and second moments of on-hand inventory from our model reasonably follow the corresponding moments obtained through simulation. Furthermore, we evaluate the RO policy by drawing a comparison between the optimal RO policy and the optimal well-known (R, s_n^*, S_n^*) policy. The results confirm that the RO policy could outperform (R, s, S) policy in terms of the average systemwide annual cost.

***Keywords*:** Multi-echelon inventory system; Periodic review; Randomized ordering policy; Stationary and non-stationary demand patterns; Simulation-based optimization

1. Introduction and Brief Literature Review

A supply chain is a network in which procurement of raw material, transformation of raw material to intermediate and finished products, and distribution of finished products to customers are performed (Lee and Billington (1993)). In different stages of such networks, inventory may be kept in the form of raw material, work-in-process, and finished product to confront the uncertainties. In many industries, inventory is the second largest cost after production costs (Ertogral and Rahim, 2005). Ganeshan (1999) states a fact that between 20 to 60 percent of the total assets in a company is assigned to inventory. Therefore, one of the main goals of industries might be to find the optimal policies to control inventories such that the respective costs are minimized. In other words, to stay competitive in today's fast changing business environment, companies should have an efficient control policy for managing their inventories.

Inventory management has been studied for more than half a century. After developing the well-known policy of Economic Order Quantity (EOQ) proposed by Harris (1913) for managing inventory in a single echelon inventory system, many researchers and practitioners have investigated this issue using different operating policies under various assumptions for single and multi-echelon inventory systems. The study of multi-echelon inventory management dates back to the 1960's, when Clark and Scarf (1960) investigate a two-echelon serial inventory system and presented the optimality conditions of $(S-1, S)$ policy.

Moreover, the study of multi-echelon distribution systems dates back to the 1960's, when Sherbrook (1968) investigated the $(S-1, S)$ policy, called METRIC, for a one warehouse multi-retailer (OWMR) distribution system. As an extension to Sherbrook's model, Graves (1985)

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proposed an exact expression for the expected value and the variance of unsatisfied orders at the retailers under $(S-1, S)$ policy. Afterwards, researchers have studied the OWMR distribution system under different settings. We summarize all the relevant work in the literature of the OWMR distribution system in Table 1. Although the literature on the OWMR distribution systems is rich, there are still some restrictions to be relaxed.

Figure 1. An OWMR distribution system

As illustrated in Table 1, the majority of researchers have considered unsatisfied demand as backordered demand, while in reality approximately 85% of unsatisfied demand is lost (Bijvank and Vis (2011)). Modeling complexity of the lost demand situations is one of the reasons that the majority of researchers have considered backordered demand situations which are less realistic compared to similar situations with lost demand. Moreover, in a real distribution system, retailers may have different characteristics such as demand size, capacity limitation, ordering policy, and etc. In this case, dealing with all the retailers in an OWMR system identically is a simplifying assumption that makes the problem less realistic. Table 1 shows that this simplification has been a common assumption in the literature. Furthermore, considering capacity limitation for all the locations (i.e., warehouse and retailers) is not a straightforward task.

Table 1. Literature review on the OWMR distribution system

In this study, we consider an OWMR distribution system consisting of one-warehouse and N non-identical retailers as illustrated in Figure 1. In such a system, all locations, i.e., the warehouse and all the retailers, monitor their inventory periodically. Customer demand happens just at the lowest echelon where the retailers are located. We would like to contribute to the literature of the OWMR distribution systems by providing a new mathematical model under the following assumptions:

1) Lost sale situation at all the locations,
2) Non-identical retailers,
3) Different capacity limitation for all the locations,
4) Stationary and non-stationary customer demand patterns, and
5) A new randomized ordering policy, which is straightforward to implement in reality.

We were inspired by a model called FP (authors' initials) model which proposed by Fletcher and Ponnambalam (1996) for reservoir management. The FP model is stablished based on the first and the second moments of storage as stochastic variables. Mahootchi et al. (2012) applied the FP model to a single location inventory system and showed that the extended FP model outperforms (s, S) policy in terms of the expected annual cost and service level. In this work, we extend the single location model, proposed by Mahootchi et al. (2012), to a two-echelon distribution system consisting of one warehouse and multiple non-identical retailers. Based on the common definition of the OWMR distribution systems, customer demand occurs just at the

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lowest echelon, where retailers are located. Thus, this makes it challenging to apply our model to an OWMR system. For instance, it is not clear that what is the best way to transfer demand from downstream echelon to the upstream echelon. Or how to estimate the first two moments of on-hand inventory at the warehouse. Clearly, we face joint probabilities at the upstream echelon and calculating cost terms, immediate fill rate, and the first two moments of on-hand inventory locations which are located at upstream is not a straightforward task.

This paper is organized as follows: Section 2 is dedicated to present the problem definition and the respective notations. This section also provides detailed information on finding the first and second moments of on-hand inventory, calculating the immediate fill rates, and extending a mathematical formulation for the problem. Section 3 presents validation and evaluation of the proposed model using two test problems with different demand patterns. Concluding remarks are presented in Section 4.

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2. Problem Formulation

2.1. Problem Description and Notations

This research was conducted based on a capacitated two-echelon distribution system consisting of one warehouse and N retailers, in which the warehouse, at the upper echelon, supplies all the retailers at the lower echelon, and in turn the retailers supply products to the end customers (see Figure 1 for more illustration). The central warehouse receives products from an outside supplier with ample stock. We use term *location* to refer to either the warehouse or a retailer. All the locations monitor their inventory, periodically. We use subscript i to indicate the location (0 is the warehouse and retailers are locations 1 through N) and superscript t to indicate the review period. In each period, the customer demand happens just at the retailers, which is independent and normally distributed with mean μ_i^t and standard deviation σ_i^t. Let D_i^t be the demand during period t. At each location, there is a capacity limit S_i^t that serves as an upper limit on the amount of on-hand inventory that can be kept in each period. Let I_i^t be on-hand inventory of location i at the end of period t. Keeping inventory imposes a holding cost h_i^t per item per period. All locations use a periodic randomized ordering policy to take place an order equal to Π_i^t. Replenishment decisions take effect instantaneously and there is no replenishment lead time. Although we can relax this constraint by assuming that the lead time, *l*, is a small predefined value which is sufficiently less than the respective period, *t*. It means that the optimal order obtained through solving the mathematical model should be performed in *t-l* to be replenished at the end of period *t*. Each order incurs a fixed cost A_i^t regardless of the size of the order. When the order arrives at location i and the location does not have enough space, then the additional products should be sold in a secondhand market with price q_i^t per item, which is less than purchase price. In addition, unsatisfied demand is lost and a penalty cost p_i^t per item and per time unit should be paid. The goal is to obtain the optimal ordering policies for all the locations in the system such that the long-run systemwide annual cost consisting of ordering, holding, shortage, and surplus costs is minimized. Without loss of generality, we assume that the planning horizon is finite and it is divided into T periods of equal length.

For more concision, define the following mathematical notations: $[x]^+ = max\,(x, 0)$, $[x]^- = max\,(-x, 0)$, and $x \vee y = max\,(x, y)$, $x \wedge y = min\,(x, y)$. Let $\mathbb{E}\{.\}$ be the expectation operation and $f_X(x)$ be the probability density function of random variable X.

2.2. Randomized Ordering Policy

In the remainder of this paper, we call a solution Π to our problem a policy. Any policy Π for the problem is represented by a $(N + 1) \times T$ matrix where each Π_i^t represents the order quantity of location i at the beginning of period t. Since the order quantity cannot be a negative number, then for all $i = 0,1,2,..,N;\ \ t = 1,2,..,T,$ we have $\Pi_i^t \geq 0$. The policy Π is a periodic ordering policy in which the order quantity of each location at the beginning of each period is determined by subtracting the on-hand inventory at the end of the previous period from a non-

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negative number. Then, $\Pi_i^t = \mathrm{k}_i^t - I_i^{t-1}$, where k_i^t is a non-negative decision variable. Because on-hand inventory at the end of each period I_i^{t-1} is a stochastic variable, then the ordering policy Π_i^t is stochastic, as well. Therefore, we call policy Π a *Randomized Ordering* (RO) policy. It is well-known that the structure of optimal ordering policies for most multi-echelon stochastic inventory systems are either unknown or extremely complex; consequently, it is challenging to implement them in practice (Chu and Shen 2010). However, the RO policy is easy to understand and to implement in practice.

2.3. Analysis

We break down the demand D_i^t into two parts, the constant part (the mean of demand) and the random part (white noise). Then, we can write D_i^t as

$$D_i^t = \mu_i^t - \Psi_i^t, \tag{1}$$

where Ψ_i^t is a zero-mean random normal variable $N(0, \sigma_i^t)$. Let $f_{\Psi_i^t}(\psi_i^t)$ be the probability density function of Ψ_i^t, then

$$f_{\Psi_i^t}(\psi_i^t) = \frac{1}{\sqrt{2\pi}\sigma_i^t}\ \mathrm{e}^{-\frac{1}{2}\left(\frac{\psi_i^t}{\sigma_i^t}\right)^2} \tag{2}$$

All replenishment lead times are assumed to be zero, i.e., ordered products are distributed and received instantaneously or lead times are less than considered period interval, *t*. There are four main events during a period, which may occur based on the following order:

I. At the beginning of each period, the warehouse monitors its on-hand inventory and then determines the order quantity from the outside supplier. The retailers, respectively, monitor their on-hand inventory and then decide on their order quantities from the warehouse.
II. The products from the outside supplier arrive at the warehouse and the warehouse then allocates them to the retailers.
III. The products from the warehouse arrive at the retailers and the retailers with received products, meet customer demands during the period.
IV. At the end of each period, all the locations should sell their surplus (i.e., more on-hand inventory than their capacity limit) in a second market.

Remark: Since all the locations place their order at the beginning of the period and they have information on current available space $S_i^t - I_i^{t-1}$, it is not sensible to order more than $S_i^t - I_i^{t-1}$; otherwise, they have to sell the surplus in the second market with a lower price than purchase price. This logic is consistent with the results that we get from optimization in the numerical analysis part. Hence, under optimal RO policy the surplus is zero. We are aware of the fact that surplus concept in inventory management is not realistic; however, we need this concept for the theoretical concept of capacity.

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2.3.1. The First Two Moments of On-hand Inventory at Downstream

We define the dynamics of the on-hand inventory at location i in terms of two consecutive periods. Let us introduce $\mathcal{X}_i^t$ which represents the potential on-hand inventory at the end of period t. Then we have

$$\mathcal{X}_i^t = I_i^{t-1} + \Pi_i^t - D_i^t \tag{3}$$

where D_i^t is demand during period t, $I_i^{t-1}$and Π_i^t are on-hand inventory and order quantity at the beginning of period t, respectively. $\mathcal{X}_i^t$ represents the potential on-hand inventory at the end of period t. Actually, Expression (3) represents the on-hand inventory at the end of period t for a situation without capacity constraint. We know $\Pi_i^t = k_i^t - I_i^{t-1}$ and $D_i^t = \mu_i^t - \Psi_i^t$. Then we can rewrite Expression (3) as follows.

$$\mathcal{X}_i^t = k_i^t - \mu_i^t + \Psi_i^t \tag{4}$$

After applying the capacity constraint, the on-hand inventory at the end of period t is

$$I_i^t = \left[S_i^t \wedge \mathcal{X}_i^t\right]^+ \tag{5}$$

Equivalently, we can write Expression (5) in terms of indicator functions as Expression (6).

$$I_i^t = \left(k_i^t - \mu_i^t + \Psi_i^t\right).1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right) + S_i^t.1_{(S_i^t,+\infty)}\left(\mathcal{X}_i^t\right) \tag{6}$$

The indicator function of the potential on-hand inventory $\mathcal{X}_i^t$ takes zero or one value according to the following conditions:

$$1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right) = \begin{cases} 1 & 0 \le \mathcal{X}_i^t \le S_i^t \\ 0 & otherwise, \end{cases} \tag{7}$$

$$1_{(-\infty,0)}\left(\mathcal{X}_i^t\right) = \begin{cases} 1 & \mathcal{X}_i^t < 0 \\ 0 & otherwise, \end{cases} \tag{8}$$

$$1_{(S_i^t,+\infty)}\left(\mathcal{X}_i^t\right) = \begin{cases} 1 & \mathcal{X}_i^t > S_i^t \\ 0 & otherwise, \end{cases} \tag{9}$$

At the end of each period, at most the value of one of these three indicator functions is one and the value of the other two is zero. It means that the potential on-hand inventory at the end of each period is either negative (shortage situation), or more than S_i^t (surplus situation), or bounded by zero and S_i^t.

Now we can take the expected value of both sides of Expression (6) and apply linearity of expectation:

$$\mathbb{E}\{I_i^t\} = \mathbb{E}\left\{\left(k_i^t - \mu_i^t + \Psi_i^t\right).1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\} + S_i^t.\mathbb{E}\left\{1_{(S_i^t,+\infty)}\left(\mathcal{X}_i^t\right)\right\}, \tag{10}$$

Since $1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right).1_{(S_i^t,+\infty)}\left(\mathcal{X}_i^t\right) = 0$, after raising Expression (6) to the power of two and expanding it, we can take the expected value of both sides and apply linearity of expectation:

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$$\mathbb{E}\{(I_i^t)^2\} = \mathbb{E}\left\{\{(k_i^t - \mu_i^t)^2 + 2\Psi_i^t(k_i^t - \mu_i^t) + (\Psi_i^t)^2\}.\,1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\} + \left(S_i^t\right)^2.\,\mathbb{E}\left\{1_{(S_i^t,+\infty)}\left(\mathcal{X}_i^t\right)\right\}. \quad (11)$$

As a result, Expressions (10) and (11) represent the first and second moments of on-hand inventory at retailer at the end of period t, respectively.

2.3.2. The First Two Moments of On-hand Inventory at Upstream

As mentioned before, the customer demand occurs only in the downstream echelon where the retailers are located. Hence, there is no direct customer demand for the upstream echelon where the warehouse is located. However, the customer demand propagates backwards through the distribution network. It means that we should estimate the demand at the warehouse using the orders placed by downstream. We investigate the following three alternative ways to estimate the demand at the warehouse.

$$\text{I.}\quad D_0^t = \sum_{i=1}^{N} D_i^t, \quad (12)$$

$$\text{II.}\quad D_0^t = \sum_{i=1}^{N} D_i^t - I_i^{t-1}, \quad (13)$$

$$\text{III.}\quad D_0^t = \sum_{i=1}^{N} D_i^t - I_i^{t-1} + I_i^t. \quad (14)$$

In the first alternative, we estimate demand at upstream warehouse using the customer demand at all downstream retailers. Since the current on-hand inventory of the retailers has not been taken into account, this estimation is not comprehensive enough. In the second alternative, we subtract the initial on-hand inventory of all the retailers from the total customer demand. Since, the second alternative avoids ordering from warehouse more than the total actual demand, the second alternative is more efficient than the first alternative. Knowing that at the end of period t, at each retailer the left on-hand inventory is equal to I_i^t, leads to make a more accurate estimation. Therefore, by adding the total left on-hand inventories at the end of the period to the second alternative, we will have alternative III.

We restrict our analysis to the third alternative, which is the most comprehensive estimation for demand at upstream echelon. Then, using the same logic, we can derive the potential on-hand inventory at the end of period t for the warehouse as fallows.

$$\mathcal{X}_0^t = k_0^t - \sum_{i=1}^{N} \mu_i^t - \Psi_i^t - I_i^{t-1} + I_i^t, \qquad \forall t \quad (15)$$

Similar to the retailers, we can write Expression (15) in terms of indicator functions. Then,

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$$I_0^t = \left(k_0^t - \sum_{i=1}^{N} \mu_i^t - \Psi_i^t - I_i^{t-1} + I_i^t \right) 1_{[0,S_0^t]}(\mathcal{X}_0^t) + S_0^t 1_{(S_0^t,+\infty)}(\mathcal{X}_0^t), \quad \forall t \tag{16}$$

Now, we can take the expected value of both sides of Expression (16) and apply linearity of expectation:

$$\mathbb{E}\{I_0^t\} = \mathbb{E}\left\{ \left(k_0^t - \sum_{i=1}^{N} \mu_i^t - \Psi_i^t - I_i^{t-1} + I_i^t \right) 1_{[0,S_0^t]}(\mathcal{X}_0^t) \right\} + S_0^t \mathbb{E}\left\{ 1_{(S_0^t,+\infty)}(\mathcal{X}_0^t) \right\}, \qquad \forall t \tag{17}$$

Also, Since $1_{[0,S_i^t]}(\mathcal{X}_0^t).1_{(S_i^t,+\infty)}(\mathcal{X}_0^t) = 0$, after raising Expression (16) to the power of two and expanding it, we can take the expected value of both sides and apply linearity of expectation:

$$\mathbb{E}\{(I_0^t)^2\} = \mathbb{E}\left\{ \left(\left(k_0^t - \sum_{i=1}^{N} \mu_i^t - I_i^{t-1} + I_i^t \right)^2 + 2\Psi_0^t \left(k_0^t - \sum_{i=1}^{N} \mu_i^t - I_i^{t-1} + I_i^t \right) + (\Psi_0^t)^2 \right) 1_{[0,S_0^t]}(\mathcal{X}_0^t) \right\} + (S_0^t)^2 \mathbb{E}\left\{ 1_{(S_0^t,+\infty)}(\mathcal{X}_0^t) \right\}, \qquad \forall t \tag{18}$$

As a result, Expressions (17) and (18) represent the first and second moments of on-hand inventory at the warehouse at the end of period t, respectively.

From Expression (14) we can derive the mean and variance of demand at warehouse as Expressions (19) and (20), respectively.

$$\mu_0^t = \sum_{i=1}^{N} \mu_i^t - \mathbb{E}\{I_i^{t-1}\} + \mathbb{E}\{I_i^t\}, \qquad t = 1,2,\dots,T \tag{19}$$

$$\mathbb{V}ar(\Psi_0^t) = \sum_{i=1}^{N} \left(\sigma_i^t\right)^2 + \mathbb{V}ar\{I_i^{t-1}\} + \mathbb{V}ar\{I_i^t\}, \qquad t = 1,2,\dots,T \tag{20}$$

Preposition 1. *The first and second moment of on-inventory of each location (i.e., the warehouse and the retailers) at the end of period* t *can be presented as the following closed-form expressions, respectively.*

1) $$\mathbb{E}\{I_i^t\} = \frac{(k_i^t - \mu_i^t)}{2} \left\{ \mathrm{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}} \right) - \mathrm{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}} \right) \right\} + \left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}} \right) \left\{ e^{-\frac{(\mu_i^t - k_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}} - e^{-\frac{(S_i^t - (k_i^t - \mu_i^t))^2}{2\mathbb{V}ar(\Psi_i^t)}} \right\} + \frac{S_i^t}{2} \left\{ 1 - \mathrm{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}} \right) \right\}, \qquad \forall t, i$$

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2) $$\mathbb{E}\left\{\left(I_i^t\right)^2\right\} = \frac{\left(\left(k_i^t - \mu_i^t\right)^2 + \mathbb{V}ar\left(\Psi_i^t\right)\right)}{2}\left\{\mathbb{erf}\left(\frac{S_i^t - \left(k_i^t - \mu_i^t\right)}{\sqrt{2\mathbb{V}ar\left(\Psi_i^t\right)}}\right) - \mathbb{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar\left(\Psi_i^t\right)}}\right)\right\} + 2\left(k_i^t - \mu_i^t\right)\left(\sqrt{\frac{\mathbb{V}ar\left(\Psi_i^t\right)}{2\pi}}\right)\left\{\mathbb{e}^{-\frac{\left(k_i^t-\mu_i^t\right)^2}{2\mathbb{V}ar\left(\Psi_i^t\right)}} - \mathbb{e}^{-\frac{\left(S_i^t-\left(k_i^t-\mu_i^t\right)\right)^2}{2\mathbb{V}ar\left(\Psi_i^t\right)}}\right\} - \left(\sqrt{\frac{\mathbb{V}ar\left(\Psi_i^t\right)}{2\pi}}\right)\left\{\left(k_i^t - \mu_i^t\right)\mathbb{e}^{-\frac{\left(k_i^t-\mu_i^t\right)^2}{2\mathbb{V}ar\left(\Psi_i^t\right)}} + \left(S_i^t - \left(k_i^t - \mu_i^t\right)\right)\mathbb{e}^{-\frac{\left(S_i^t-\left(k_i^t-\mu_i^t\right)\right)^2}{2\mathbb{V}ar\left(\Psi_i^t\right)}}\right\} + \frac{\left(S_i^t\right)^2}{2}\left\{1 - \mathbb{erf}\left(\frac{S_i^t - \left(k_i^t - \mu_i^t\right)}{\sqrt{2\mathbb{V}ar\left(\Psi_i^t\right)}}\right)\right\}, \quad \forall t, i$$

Proof of Preposition 1. For the proof of this proposition, we refer the reader to Appendixes A and B.

2.4. The Immediate Fill Rates

Here, the product availability at each location throughout the distribution network is calculated. We introduce service level as *immediate fill rate* or "off-the-shelf" concept. In other words, *immediate fill rate* is equivalent to a fraction of requirements met without delay (Lee and Billington (1993)).

Lemma 1: *Let $\mathfrak{f}_i^t$ be the immediate or "off-the-shelf" fill rate at location i in period t. Then, for all $i = 0,2, \dots, N$; and $t = 1,2, \dots, T$,*

$$\mathfrak{f}_i^t = 1 - \frac{1}{2}\left\{1 + \mathbb{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar\left(\Psi_i^t\right)}}\right)\right\}.$$

Proof of Lemma 1. The random variable $1_A(\mathcal{X})$ is the indicator random variable for event $\mathcal{X}$. When $\mathcal{X}$ occurs, then $1_A(\mathcal{X}) = 1$; otherwise, $1_A(\mathcal{X}) = 0$.

$$\mathbb{E}\{1_A(\mathcal{X})\} = 1.\,\mathbb{P}(1_A(\mathcal{X}) = 1) + 0.\,\mathbb{P}(1_A(\mathcal{X}) = 0) = 1.\,\mathbb{P}(1_A(\mathcal{X}) = 1) = \mathbb{P}(1_A(\mathcal{X}) = 1).$$

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We can use shortage event to calculate the immediate *fill rate*. Hence, $\mathscr{f}_i^t = 1 - \mathbb{P}\big(1_{(-\infty,0)}(\mathcal{X}_i^t) = 1\big)$. Based on Appendixes A and B, we can find $\mathbb{P}\left(1_{(-\infty,0)}(\mathcal{X}_i^t)\right)$ and $\mathbb{P}\left(1_{(-\infty,0)}(\mathcal{X}_0^t)\right)$, respectively. Then,

$$\mathscr{f}_i^t = 1 - \frac{1}{2}\left\{1 + \mathbb{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right\}.$$

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2.5. The Total Cost

The expected systemwide annual cost, $C(\mathbf{\Pi})$, can be expressed as

$$\min_{\Pi} C(\mathbf{\Pi}) = \mathbb{E}\{ORC\} + \mathbb{E}\{HOC\} + \mathbb{E}\{SHC\} + \mathbb{E}\{SUC\} \tag{21}$$

where $\mathbb{E}\{ORC\}$ is the expected systemwide annual ordering cost, $\mathbb{E}\{HOC\}$ the expected systemwide annual holding cost, $\mathbb{E}\{SHC\}$ the expected systemwide annual shortage cost, and $\mathbb{E}\{SUC\}$ the expected systemwide annual surplus cost. We can express each of these costs as follows;

The expected total annual ordering cost is

$$\mathbb{E}\{ORC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} A_i^t \left(1 \wedge \left(k_i^t - \mathbb{E}\{I_i^{t-1}\}\right)\right) \tag{22}$$

The expected total annual holding cost is

$$\mathbb{E}\{HOC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} h_i^t \left(\left(\frac{\mathbb{E}\{I_i^{t-1}\} + \mathbb{E}\{I_i^t\}}{2}\right)\right) \tag{23}$$

The expected total annual shortage cost is

$$\mathbb{E}\{SHC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} p_i^t \left(\left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right).\,\mathbb{e}^{-\frac{(k_i^t-\mu_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}} - \frac{(k_i^t - \mu_i^t)}{2}\left(1 + \mathbb{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)\right) \tag{24}$$

The expected total annual surplus cost is

$$\mathbb{E}\{SUC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} q_i^t \left(\left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right).\,\mathbb{e}^{-\frac{\left(S_i^t-(k_i^t-\mu_i^t)\right)^2}{2\mathbb{V}ar(\Psi_i^t)}} + \frac{(k_i^t - \mu_i^t - S_i^t)}{2}\left(1 - \mathbb{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)\right) \tag{25}$$

We have deferred all the corresponding proofs to Appendix C.

3. Numerical Experiments

In order to examine the validity and effectiveness of our formulation, we have performed a numerical study. In total, we consider two different test problems consisting of one warehouse and two non-identical retailers. One test problem involves stationary customer demand and the other considers non-stationary customer demand. In both test problems, we have an annual planning horizon with monthly review periods. Table 2 represents the parameter settings for the test problem with stationary demand. Also, Tables 3.1, and 3.2 represent the parameter settings for the test problem with non-stationary demand. Note that in the non-stationary test problem, the customer demands of the two retailers are different.

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Table 2. Parameter settings for test problem with stationary demand pattern

Table 3.1. Parameter settings for the test problem with non-stationary demand pattern

Table 3.2. Parameter settings for the test problem with non-stationary demand pattern

As it is obvious, the proposed optimization model is non-convex, and therefore it is sensitive to initial solutions. Thus, the respective optimization process is implemented multiple times using different initial solutions. The best solution in terms of minimal optimal cost is selected for the verification step. The optimal values of k_i^t for all locations and periods are demonstrated in Tables 4 and 5 for both test problems.

Table 4. The optimal values of k_i^t for each location in the test problem with non-stationary demand

Table 5. The optimal values of k_i^t for each location in the test problem with stationary demand

3.1. The Validation of the Optimal RO Policy

For each test problem, we find the best RO policy for each location based on our model. We also obtain the expected value and standard deviation of on-hand inventory at all location. The accuracy of our formulation is then evaluated by simulation. We run a simulation for 96000 periods (8000 years), in which the obtained RO policies from our model are used. The process is performed for both demand patterns and the respective results are illustrated in Figures 2 and 3.

Figure 2. The comparison of $\mathbb{E}\{I_i^t\}$ and $\sqrt{\mathbb{V}ar(I_i^t)}$ from the optimization and the simulation for stationary demand

In Figures 2 and 3, we draw a comparison between the proposed model and simulation for the test problems with stationary and non-stationary demand patterns, respectively. As it is clear in Figures 2 and 3, the proposed model is capable of estimating the various inventory statistics of all the locations in the OWMR distribution system for both the stationary and the non-stationary demand patterns, because the results of optimization and simulation are close to each other. As can be expected, the results are more accurate for expected values than standard deviations. These results also confirm that our approximations and estimations are quite effective leading to an efficient RO policy.

Figure 3. The comparison of $\mathbb{E}\{I_i^t\}$ and $\sqrt{\mathbb{V}ar(I_i^t)}$ from the optimization and the simulation for non-stationary demand

The model can be validated by the average annual cost as well. This measurement criterion can be obtained using the following equation (Seifbarghy and Jokar, 2006):

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$$\epsilon = \frac{\left|C^{Sim}(\mathbf{\Pi}^*) - C^{Opt}(\mathbf{\Pi}^*)\right|}{C^{Sim}(\mathbf{\Pi}^*)} \tag{26}$$

As it is demonstrated in Table 6, the errors for both stationary and non-stationary are quite reasonable which could be promising for managing other type of inventory system with more echelons and different configurations (e.g., parallel inventory structure and lateral shipment).

Table 6. Comparison of results obtained from optimization and simulation models

3.2. Evaluation of the RO policy

The performance of our proposed optimization model, which results in a periodic policy with the review period ($R = 1$), could be evaluated using some classical policies such as the (R, s, S) policy with $R = 1$ in which the system is reviewed periodically. If the inventory position is at or below s, an order should be placed to bring the inventory position up to level S. Otherwise, no orders should be placed.

Unfortunately, there is no exact method in the literature to find the optimal parameters of (R, s, S) where the unsatisfied demand is lost. A good approximate method to find the optimal parameters of (R, s, S) where the unsatisfied demand is backordered was presented by Schneider & Rink (1991), and Schneider et al. (1995). Here, an iterative approximation method is developed for the lost sale situation. The initial solution obtained from Schneider et al. (1995) algorithm is also used as an initial solution to implement the proposed iterative method for finding the parameters of the policy. This iterative method consists of two main phases: simulation and optimization. In the simulation phase, the current policy is evaluated by calculating the average annual cost, while in the optimization phase the current policy should be improved. We use the Matlab© optimization toolbox which is a derivative-based optimization algorithm, called Matlab "Fmincon" function, to generate new policies at each iteration. This process should be continued until a stop criterion is satisfied. The graphical view of the developed algorithm is demonstrated in Figure 4.

Figure 4. An iterative simulation-based optimization

In this method, the order quantity based on (R, s, S) policy is determined by min and max functions as Expression (27).

$$Q_i^t = \left\{1 - \left(1 \wedge \left(0 \vee \left(I_i^{t-1} - s_i^t\right)\right)\right)\right\}\left(S_i^t - I_i^{t-1}\right), \qquad \forall t, i \tag{27}$$

where Q_i^t is order quantity of location i in period t.

The results of the optimal RO and (R, s_n^*, S_n^*) policies for both test problems are provided in Table 7. We indicate the optimal RO policy by Π^* policy.

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Table 7. Comparison of the optimal (R,s_n^*,S_n^*) and Π^* policies

As demonstrated in table 7, the RO policy is superior for both test problems.

4. Conclusion

In this paper, we propose a new formulation for controlling inventory in a two-echelon distribution system consisting of one warehouse and multiple non-identical retailers called OWMR system. In such systems, customer demands occur based on a normal distribution at the lowest echelon, where the retailers are located, and propagates backward through the system. The warehouse and the retailers have a limited capacity for keeping inventory and if they are not able to fulfill the demand immediately, the demand will be lost. All the locations review their inventory periodically and replenish their inventory spontaneously or within a small interval based on a periodic Randomized Ordering (RO) policy. The RO policy determines order quantity of each location in each period by subtracting corresponding on-hand inventory at the beginning of that period from a deterministic decision variable. We propose a mathematical model to find the optimal RO policies such that the average systemwide cost consisting of ordering, holding, shortage, and surplus costs is minimized. We use the first and second moments of on-hand inventory as auxiliary variables. A remarkable advantage of our model is calculating the immediate fill rate of all locations without adding new variables and facing the curse of dimensionality.

Using two numerical examples with stationary and non-stationary demand settings, we validate and evaluate the proposed model. For the validation, we simulate the optimal RO policy and show that the optimal first and second moments of on-hand inventory from our model suitably follow the corresponding moments from simulation. Furthermore, we evaluate the RO policy by drawing a comparison between the optimal RO policy and the optimal well-known (R, s_n^*, S_n^*) policy. The results demonstrate that the RO policy outperforms (R, s, S) policy in terms of the average systemwide annual cost.

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Appendix

Appendix A. Downstream Echelon (Retailers)

Appendix A.1. Calculation of $\mathbb{E}\{1_{[.]}(\mathcal{X}_i^t)\}$, (downstream echelon)

$$\mathbb{E}\left\{1_{[0,S_i^t]}(\mathcal{X}_i^t)\right\} = \mathbb{P}(0 \le \mathcal{X}_i^t \le S_i^t) = \mathbb{P}(\mu_i^t - k_i^t \le \Psi_i^t \le S_i^t - (k_i^t - \mu_i^t)) = \int_{\mu_i^t - k_i^t}^{S_i^t - (k_i^t - \mu_i^t)} f_{\Psi_i^t}(\psi_i^t)\, d\psi_i^t$$

where $f_{\Psi_i^t}(\psi_i^t)$ is a normal probability density function of random variable Ψ_i^t with mean zero. Then,

$$\mathbb{E}\left\{1_{[0,S_i^t]}(\mathcal{X}_i^t)\right\} = \int_{\mu_i^t - k_i^t}^{S_i^t - (k_i^t - \mu_i^t)} \frac{1}{\sqrt{2\pi \mathbb{V}ar(\Psi_i^t)}} . \mathbb{e}^{-\left(\frac{\psi_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)^2} d\psi_i^t = \frac{1}{\sqrt{\pi}} \int_{\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}}^{\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}} \mathbb{e}^{-x^2} dx = \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}}^{\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}} \mathbb{e}^{-x^2} dx\right)$$

Knowing that the error function is defined as; $\mathbb{erf}(a) = \frac{2}{\sqrt{\pi}} \int_0^a \mathbb{e}^{-x^2} dx$, then

$$\mathbb{E}\left\{1_{[0,S_i^t]}(\mathcal{X}_i^t)\right\} = \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}}^{\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}} \mathbb{e}^{-x^2} dx\right) = \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_0^{\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}} \mathbb{e}^{-x^2} dx - \frac{2}{\sqrt{\pi}} \int_0^{\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}} \mathbb{e}^{-x^2} dx\right)$$

$$= \frac{1}{2}\left(\mathbb{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right) - \mathbb{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)$$

In a similar way, we can calculate the following terms.

$$\mathbb{E}\{1_{(-\infty,0)}(\mathcal{X}_i^t)\} = \mathbb{P}(\mathcal{X}_i^t < 0) = \mathbb{P}(\Psi_i^t < \mu_i^t - k_i^t) = \int_{-\infty}^{\mu_i^t - k_i^t} f_{\Psi_i^t}(\psi_i^t)\, d\psi_i^t = \frac{1}{2}\left(1 + \mathbb{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)$$

$$\mathbb{E}\left\{1_{(S_i^t,+\infty)}(\mathcal{X}_i^t)\right\} = \mathbb{P}(S_i^t < \mathcal{X}_i^t) = \mathbb{P}\{S_i^t - (k_i^t - \mu_i^t) < \Psi_i^t\} = \int_{S_i^t - (k_i^t - \mu_i^t)}^{+\infty} f_{\Psi_i^t}(\psi_i^t)\, d\psi_i^t = \frac{1}{2}\left(1 - \mathbb{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)$$

Appendix A.2. Calculation of $\mathbb{E}\left\{\Psi_0^t . 1_{[0,S_0^t]}(\mathcal{X}_0^t)\right\}$ and $\mathbb{E}\left\{(\Psi_0^t)^2 . 1_{[0,S_0^t]}(\mathcal{X}_0^t)\right\}$

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Similar to the calculation of $\mathbb{E}\left\{1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\}$ we can calculate $\mathbb{E}\left\{\Psi_i^t.1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\}$ and $\mathbb{E}\left\{(\Psi_i^t)^2.1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\}$ as follows;

$$\mathbb{E}\left\{\Psi_i^t.1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\}=\left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right)\left\{e^{-\frac{(k_i^t-\mu_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}}-e^{-\frac{\left(S_i^t-(k_i^t-\mu_i^t)\right)^2}{2\mathbb{V}ar(\Psi_i^t)}}\right\}$$

$$\begin{aligned}\mathbb{E}&\left\{(\Psi_i^t)^2.1_{[0,S_i^t]}\left(\mathcal{X}_i^t\right)\right\}\\ &=\left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right)\left\{-(k_i^t-\mu_i^t)e^{-\frac{(k_i^t-\mu_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}}-(S_i^t-(k_i^t-\mu_i^t))e^{-\frac{\left(S_i^t-(k_i^t-\mu_i^t)\right)^2}{2\mathbb{V}ar(\Psi_i^t)}}\right\}\\ &+\left(\frac{\mathbb{V}ar(\Psi_i^t)}{2}\right)\left(\mathrm{erf}\left(\frac{S_i^t-(k_i^t-\mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)-\mathrm{erf}\left(\frac{\mu_i^t-k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)\end{aligned}$$

Appendix B. Upstream Echelon (Warehouse)

Appendix B.1. Calculation of individual $\mathbb{E}\{1_{[.]}\left(\mathcal{X}_0^t\right)\}$, (upstream echelon)

$$\begin{aligned}\mathbb{E}\left\{1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}&=\mathbb{P}(0\le\mathcal{X}_0^t\le S_0^t)=\mathbb{P}\left(0\le k_0^t-\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t-\Psi_0^t\le S_0^t\right)\\ &=\int_{\vec{0}}^{\vec{s}}\int_{\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t-k_0^t}^{S_0^t-\{k_0^t-\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t\}}f_{(\vec{I},\Psi_0^t)}(\vec{\boldsymbol{I}},\psi_0^t)\,d\vec{\boldsymbol{I}}\,d\psi_0^t\end{aligned}$$

Where $\vec{S}=(S_1^t,S_2^t,\ldots S_N^t)$ and $f_{(\vec{I},\Psi_0^t)}(\vec{\boldsymbol{I}},\psi_0^t)$ is a joint probability density function of the total demand in downstream as well as on-hand inventory of all the retailers at the beginning and the end of period t. Clearly, the total demand in downstream is independent of total on-hand inventory in downstream at the beginning of the period. For simplification, we assume that the total demand in downstream is independent of total on-hand inventory in downstream at the end of the period. Therefore, based on the aforementioned independencies, we can write

$$\begin{aligned}\mathbb{E}\left\{1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}&=\int_{\vec{0}}^{\vec{s}}\left\{\int_{\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t-k_0^t}^{S_0^t-\{k_0^t-\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t\}}f_{\Psi_0^t}(\psi_0^t)\,d\psi_0^t\right\}f_{\vec{I}}(\vec{\boldsymbol{I}})d\vec{\boldsymbol{I}}\\ &=\mathbb{E}\left\{\mathbb{P}\left(\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t-k_0^t\le\Psi_0^t\le S_0^t-\left\{k_0^t-\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t\right\}\right)\right\}\qquad(*)\end{aligned}$$

Using Tylor's series first order approximation about $\mathbb{E}\{\vec{\boldsymbol{I}}\}$, we generate approximations of Equation $(*)$, as below.

$$\begin{aligned}\mathbb{E}\left\{1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}&=\mathbb{P}\left(\sum_{i=1}^{N}\mu_i^t-I_i^{t-1}+I_i^t-k_0^t\le\Psi_0^t-\sum_{i=1}^{N}I_i^t-I_i^{t-1}\le S_0^t-\left\{k_0^t-\sum_{i=1}^{N}\mu_i^t\right\}\right)\\ &=\mathbb{P}\left(\sum_{i=1}^{N}\mu_i^t-\mathbb{E}\left\{I_i^{t-1}\right\}+\mathbb{E}\{I_i^t\}-k_0^t\le\Psi_0^t\le S_0^t-\left\{k_0^t-\sum_{i=1}^{N}\mu_i^t-\mathbb{E}\left\{I_i^{t-1}\right\}+\mathbb{E}\{I_i^t\}\right\}\right)\end{aligned}$$

By putting

$$\mu_0^t=\sum_{i=1}^{N}\mu_i^t-\mathbb{E}\{I_i^{t-1}\}+\mathbb{E}\{I_i^t\},\quad\text{and}\quad\mathbb{V}ar\{\Psi_0^t\}=\sum_{i=1}^{N}\sigma_i^t+\mathbb{V}ar\{I_i^{t-1}\}+\mathbb{V}ar\{I_i^t\}$$

Then we can write down $\mathbb{E}\left\{1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}$ as the following closed-form expression.

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$$\mathbb{E}\left\{1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\} = \mathbb{P}\left(\mu_0^t - k_0^t \le \Psi_0^t \le S_0^t - \left(k_0^t - \mu_0^t\right)\right) = \frac{1}{2}\left(\mathrm{erf}\left(\frac{S_0^t - \left(k_0^t - \mu_0^t\right)}{\sqrt{2\mathbb{V}ar(\Psi_0^t)}}\right) - \mathrm{erf}\left(\frac{\mu_0^t - k_0^t}{\sqrt{2\mathbb{V}ar(\Psi_0^t)}}\right)\right)$$

In a similar way, we can calculate the following terms;

$$\mathbb{E}\left\{1_{(S_0^t,+\infty)}\left(\mathcal{X}_0^t\right)\right\} = \mathbb{P}(\mathcal{X}_0^t \ge S_0^t) = \frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{S_0^t - \left(k_0^t - \mu_0^t\right)}{\sqrt{2\mathbb{V}ar(\Psi_0^t)}}\right)\right)$$

$$\mathbb{E}\left\{1_{(-\infty,0)}\left(\mathcal{X}_0^t\right)\right\} = \mathbb{P}(\mathcal{X}_0^t \le 0) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\mu_0^t - k_0^t}{\sqrt{2\mathbb{V}ar(\Psi_0^t)}}\right)\right)$$

Appendix B.2. Calculation of $\mathbb{E}\left\{\Psi_0^t . 1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}$ and $\mathbb{E}\left\{(\Psi_0^t)^2 . 1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}$

Similar to the calculation of $\mathbb{E}\left\{1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}$ we can calculate $\mathbb{E}\left\{\Psi_0^t . 1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}$ and $\mathbb{E}\left\{(\Psi_0^t)^2 . 1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\}$ as follows;

$$\mathbb{E}\left\{\Psi_0^t . 1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\} = \left(\sqrt{\frac{\mathbb{V}ar(\Psi_0^t)}{2\pi}}\right)\left\{\mathrm{e}^{-\frac{(k_0^t-\mu_0^t)^2}{2\mathbb{V}ar(\Psi_0^t)}} - \mathrm{e}^{-\frac{\left(S_0^t-(k_0^t-\mu_0^t)\right)^2}{2\mathbb{V}ar(\Psi_0^t)}}\right\}$$

$$\begin{aligned}\mathbb{E}&\left\{(\Psi_0^t)^2 . 1_{[0,S_0^t]}\left(\mathcal{X}_0^t\right)\right\} \\ &= \left(\sqrt{\frac{\mathbb{V}ar(\Psi_0^t)}{2\pi}}\right)\left\{-(k_0^t - \mu_0^t)\mathrm{e}^{-\frac{(k_0^t-\mu_0^t)^2}{2\mathbb{V}ar(\Psi_0^t)}} - (S_0^t - (k_0^t - \mu_0^t))\mathrm{e}^{-\frac{\left(S_0^t-(k_0^t-\mu_0^t)\right)^2}{2\mathbb{V}ar(\Psi_0^t)}}\right\} \\ &+ \left(\frac{\mathbb{V}ar(\Psi_i^t)}{2}\right)\left(\mathrm{erf}\left(\frac{S_0^t - \left(k_0^t - \mu_0^t\right)}{\sqrt{2\mathbb{V}ar(\Psi_0^t)}}\right) - \mathrm{erf}\left(\frac{\mu_0^t - k_0^t}{\sqrt{2\mathbb{V}ar(\Psi_0^t)}}\right)\right)\end{aligned}$$

Appendix C. Calculation of the Expected Annual Cost

Appendix C.1. Expected Fixed Ordering Cost

We calculate the expected fixed ordering cost of location i in period t by the following expression

$$A_i^t(1 \wedge E\{\Pi_i^t\}) = A_i^t\left(1 \wedge E\left\{k_i^t - I_i^{t-1}\right\}\right) = A_i^t\left(1 \wedge \left(k_i^t - E\left\{I_i^{t-1}\right\}\right)\right), \quad \forall i, t$$

Then,

$$\mathbb{E}\{ORC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} A_i^t\left(1 \wedge \left(k_i^t - \mathbb{E}\left\{I_i^{t-1}\right\}\right)\right)$$

Appendix C.2. Expected Holding Cost

We calculate the expected holding cost of location i in period t by taking the arithmetic mean of the expected on-hand inventory at the beginning $(E\{I_i^{t-1}\})$ and at the end of the period $(E\{I_i^t\})$. Thus, we can write

$$\mathbb{E}\{HOC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} h_i^t\left(\left(\frac{\mathbb{E}\left\{I_i^{t-1}\right\} + \mathbb{E}\left\{I_i^t\right\}}{2}\right)\right)$$

Appendix C.3. Expected Shortage Cost

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Recall that $\mathcal{X}_i^t < 0$ represents shortage situation and $(0 - \mathcal{X}_i^t)$ represents the amount of the shortage. Then, the expected shortage cost of location i in period t is equal to $p_i^t . \mathbb{E}\{(0 - \mathcal{X}_i^t) . 1_{(-\infty,0)} (\mathcal{X}_i^t)\}$. By expanding $\mathbb{E}\{(0 - \mathcal{X}_i^t) . 1_{(-\infty,0)} (\mathcal{X}_i^t)\}$, we have

$$\mathbb{E}\{(0 - \mathcal{X}_i^t) . 1_{(-\infty,0)} (\mathcal{X}_i^t)\} = \mathbb{E}\left\{\left((\mu_i^t - k_i^t) - \Psi_i^t\right). 1_{(-\infty,0)} (\mathcal{X}_i^t)\right\} = \mathbb{E}\{(\mu_i^t - k_i^t). 1_{(-\infty,0)} (\mathcal{X}_i^t)\} - \mathbb{E}\{\Psi_i^t . 1_{(-\infty,0)} (\mathcal{X}_i^t)\}$$

$$= (\mu_i^t - k_i^t)\mathbb{E}\{ 1_{(-\infty,0)} (\mathcal{X}_i^t)\} - \mathbb{E}\{\Psi_i^t . 1_{(-\infty,0)} (\mathcal{X}_i^t)\}$$

$$= \frac{(\mu_i^t - k_i^t)}{2}\left(1 + \mathrm{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right) - \int_{-\infty}^{\mu_i^t - k_i^t} \psi_i^t f_{\Psi_i^t}(\psi_i^t)\, d\psi_i^t$$

$$= \frac{(\mu_i^t - k_i^t)}{2}\left(1 + \mathrm{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right) + \left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right). e^{-\frac{(k_i^t - \mu_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}}$$

$$= \left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right). e^{-\frac{(k_i^t - \mu_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}} - \frac{(k_i^t - \mu_i^t)}{2}\left(1 + \mathrm{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right).$$

Therefore,

$$\mathbb{E}\{SHC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} p_i^t \left(\left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right). e^{-\frac{(k_i^t - \mu_i^t)^2}{2\mathbb{V}ar(\Psi_i^t)}} - \frac{(k_i^t - \mu_i^t)}{2}\left(1 + \mathrm{erf}\left(\frac{\mu_i^t - k_i^t}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)\right)$$

Appendix C.3. Expected Surplus Cost

When $\mathcal{X}_i^t > S_i^t$, we are in surplus situation and the amount of the surplus is equal to $(\mathcal{X}_i^t - S_i^t)$. Thus, the expected surplus cost of location i in period t is equal to $q_i^t . \mathbb{E}\left\{(\mathcal{X}_i^t - S_i^t) . 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\}$. By expanding $\mathbb{E}\left\{(\mathcal{X}_i^t - S_i^t) . 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\}$, we have

$$\mathbb{E}\left\{(\mathcal{X}_i^t - S_i^t) . 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\} = \mathbb{E}\left\{\left((k_i^t - \mu_i^t - S_i^t) + \Psi_i^t\right). 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\}$$

$$= \mathbb{E}\left\{(k_i^t - \mu_i^t - S_i^t). 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\} + \mathbb{E}\left\{\Psi_i^t . 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\}$$

$$= (k_i^t - \mu_i^t - S_i^t)\mathbb{E}\left\{ 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\} - \mathbb{E}\left\{\Psi_i^t . 1_{(S_i^t,\infty)} (\mathcal{X}_i^t)\right\}$$

$$= \frac{(k_i^t - \mu_i^t - S_i^t)}{2}\left(1 - \mathrm{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right) + \left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right). e^{-\frac{\left(S_i^t - (k_i^t - \mu_i^t)\right)^2}{2\mathbb{V}ar(\Psi_i^t)}}$$

Therefore,

$$\mathbb{E}\{SUC\} = \sum_{t=1}^{T}\sum_{i=0}^{N} q_i^t \left(\left(\sqrt{\frac{\mathbb{V}ar(\Psi_i^t)}{2\pi}}\right). e^{-\frac{\left(S_i^t - (k_i^t - \mu_i^t)\right)^2}{2\mathbb{V}ar(\Psi_i^t)}} + \frac{(k_i^t - \mu_i^t) - S_i^t}{2}\left(1 - \mathrm{erf}\left(\frac{S_i^t - (k_i^t - \mu_i^t)}{\sqrt{2\mathbb{V}ar(\Psi_i^t)}}\right)\right)\right)$$

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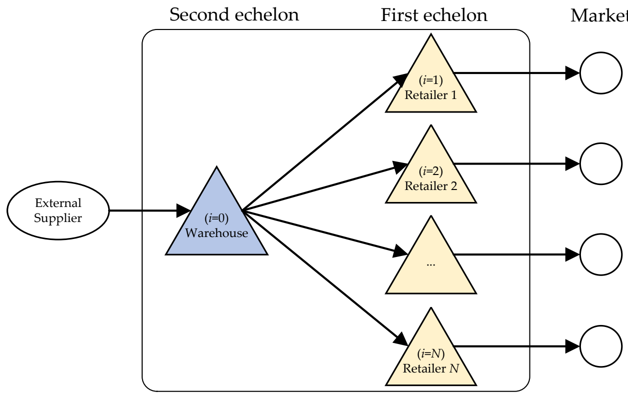


Figure 1. An OWMR distribution system

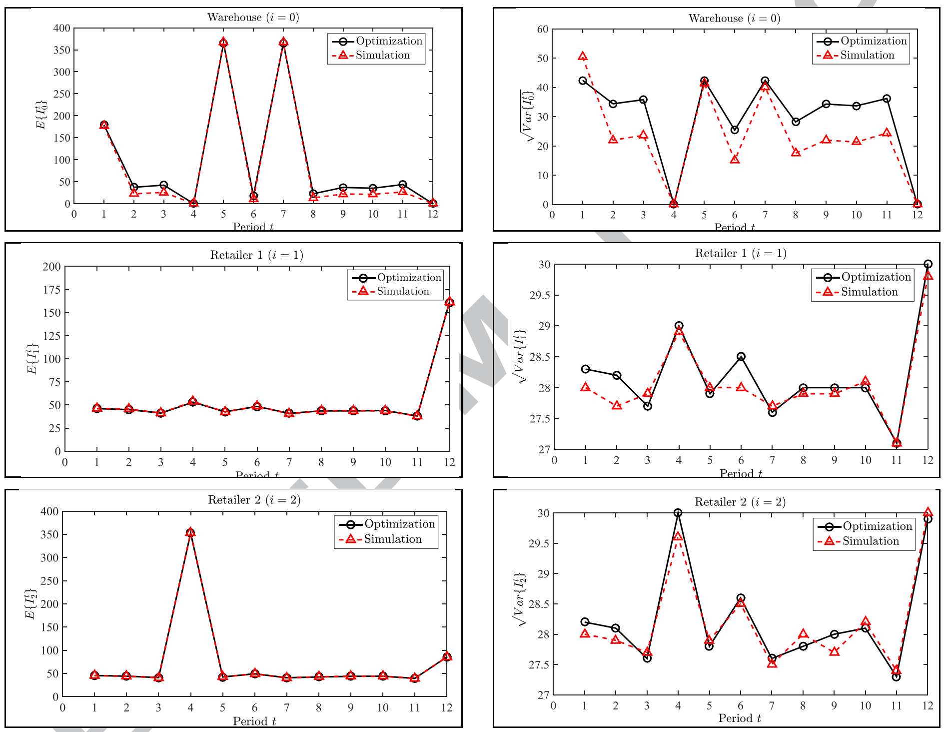
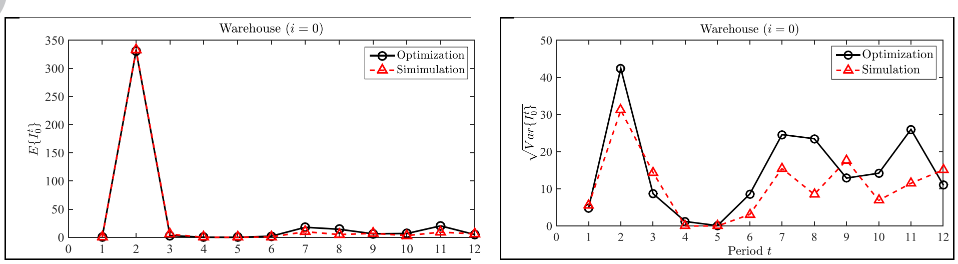


Figure 2. The comparison of $\mathbb{E}\{\mathrm{I}_i^t\}$ and $\sqrt{\mathbb{V}ar(\mathrm{I}_i^t)}$ from the optimization and the simulation for stationary demand



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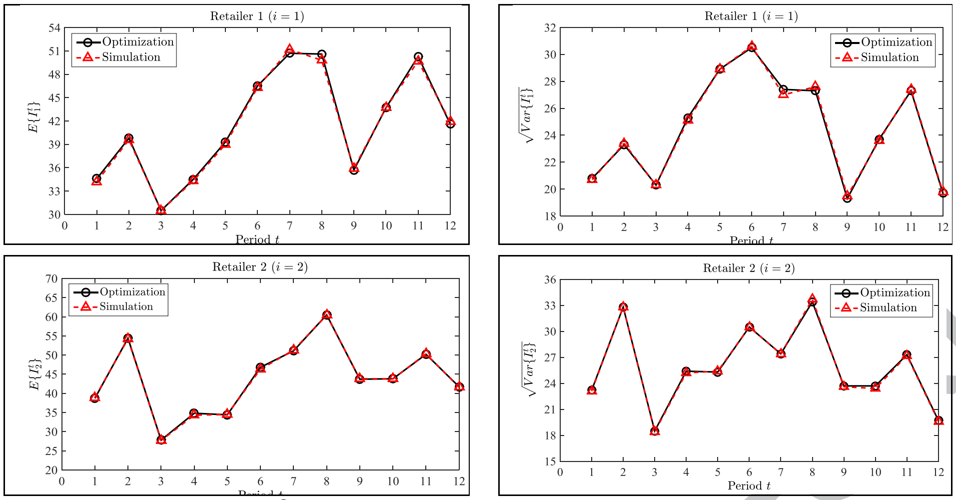

Figure 3. The comparison of $\mathbb{E}\{I_i^t\}$ and $\sqrt{\mathbb{V}ar(I_i^t)}$ from the optimization and the simulation for non-stationary demand

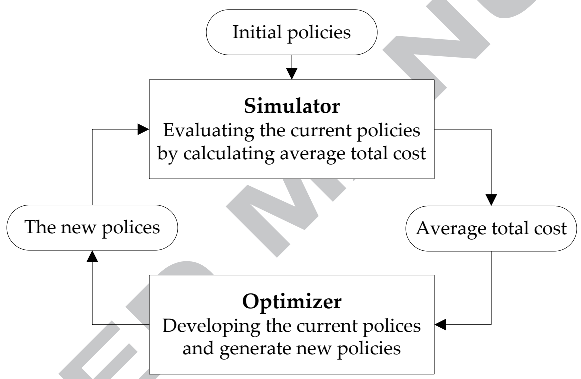

Figure 4. An Iterative simulation-based optimization

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Table 1. Literature review on the OWMR distribution system

Authors	Demand type	Ordering policy	Shortage	Review	Retailers
Sherbrook (1968)	Poisson	$(S-1,S)$	B	C	I
Graves (1985)	C. Poisson	$(S-1,S)$	B	C	I
Schwarz et al (1985)	Poisson	(r,Q)	B	C	I
Park and Kim (1989)	Normal	$(R,T)/(r,Q)$	B	C/P	I
Axsater (1990)	Poisson	$(S-1,S)$	B	C	I
Schneider and Rinks (1991)	Stochastic	(s,S)	B	P	I
Axsater (1993)	Poisson	(r,Q)	B	C	I
McGavin et al. (1993)	Poisson / Gamma	Two-interval	L	P	I
Schneider et al. (1995)	Stochastic	(s,S)	B	P	I
Graves (1996)	Poisson	$(S-1,S)$	B	C	I
Axsater (1998)	Poisson	(r,Q)	B	C	N
Ganeshan (1999)	Poisson	(r,Q)	B	C	I
Cachon and Fisher (2000)	Stochastic	(r,nQ)	B	C	I
Andersson and Melchiors (2001)	Poisson	$(S-1,S)$	B / L	C	I
Cachon (2001a)	Poisson	(r,Q)	B	C	N
Cachon (2001b)	Stochastic	(r,nQ)	B	C	I
Axater (2003)	C Poisson	(r,Q)	B	C	I
Akbari Jokar and Seifbarghy (2006)	Normal	(r,Q)	B / L	C	I
Seifbarghy and Akbari Jokar (2006)	Poisson	(r,Q)	B / L	C	I
van Houtum (2006)	Stochastic	$(S-1,S)$	B	P	I
Al-Rifai and Rossetti (2007)	Poisson	(r,Q)	B	C	I
Axsater et al. (2007)	C. Poisson	(r,Q)	B	C	I
Gallego et al. (2007)	Poisson / C. Poisson	$(S-1,S)$	B	C	I / N
Hill et al. (2007)	Poisson	$(nr,(n-1)Q)$	L	C	I / N
Monthatipkul and Yenradee (2008)	Stochastic	IDP	L	P	I
Haji et al. (2009)	Poisson	$(S-1,S)$	L	C	N
Chu and Shen (2010)	Stochastic	Power-of-two	B	P	N
Geng et al. (2010)	Stochastic	Up-to-level	L	P	I / N
Duc et al. (2010)	Stochastic	Up-to-level	B	P	I
Lee and Jeong (2010)	Deterministic	Power-of-two	None	P	I
Atan and Snyder (2012)	Deterministic	$(S-1,S)$	B	P	I / N
Yang et al. (2012)	Deterministic	Batch size	B	P	I
Basten and van Houtum (2013)	Poisson	$(S-1,S)$	B	C	I
Wang (2013)	Poisson	Up-to-level	B	P	I
Berling and Marklund (2014)	Normal / C. Poisson	(r,nQ)	B	C	I
Howard et al. (2015)	Poisson	$(S-1,S)$	B / L	C	I / N
Mateen et al (2015)	Normal	Up-to-level	B	P	I
Gayon et al. (2016)	Deterministic	JRP	B / L	P	I
Stenius et al. (2016)	C. Poisson	(r,nQ)	B	C	I
Feng et al. (2017)	Stochastic	Up-to-level	L	P	I
Turan et al. (2017)	Stochastic	Batch size	L	-	I
Verma and Chatterjee (2017)	Deterministic	Batch size	None	P	N

C. Poisson: compound Poisson; L: lost sale; B: backorder; C: continuous; P: periodic; I: identical; N: non-identical

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Table 2. Parameter settings for test problem with stationary demand pattern

i	0	1	2
D_i^t	-	N(200, 900)	N(200, 900)
S_i^t	1000	500	500
A_i^t	900	750	750
h_i^t	2	4	4
p_i^t	50	50	50
q_i^t	40	40	40
I_i^0	100	100	100

Table 3.1. Parameter settings for the test problem with non-stationary demand pattern

i	0	1	2
S_i^t	2000	1000	1000
A_i^t	500	400	400
p_i^t	80	50	50
I_i^0	1000	500	100

Table 3.2. Parameter settings for the test problem with non-stationary demand pattern

t	1	2	3	4	5	6	7	8	9	10	11	12
h_0^t	0.2	0.2	0.2	0.6	0.6	0.6	0.2	0.2	0.2	0.1	0.1	0.1
q_0^t	1	1	1	2	2	2	6	6	6	2	2	2
μ_1^t	120	150	120	200	260	270	200	200	100	150	200	100
$(\sigma_1^t)^2$	480	600	480	800	1040	1080	800	800	400	600	800	400
h_1^t	3	3	3	7	7	7	2	2	2	2	2	2
q_1^t	3	3	3	7	7	7	2	2	2	2	2	2
μ_2^t	150	300	100	200	200	270	200	300	150	150	200	100
$(\sigma_2^t)^2$	600	1200	400	800	800	10080	800	1200	600	600	800	400
h_2^t	3	3	3	7	7	7	2	2	2	2	2	2
q_2^t	3	3	3	7	7	7	2	2	2	2	2	2

Table 4. The optimal $\boldsymbol{k}_i^t$ for each location in test problem with non-stationary demand

t	1	2	3	4	5	6	7	8	9	10	11	12
$i = 1$	210.4	802.3	157.9	306.8	302.0	498.4	412.2	501.5	202.2	290.3	421.0	166.8
$i = 2$	154.0	189.2	149.6	232.7	297.3	315.3	250.3	250.1	135.3	193.3	249.8	141.5
$i = 3$	188.0	353.5	127.0	233.2	232.6	315.6	250.7	359.9	193.4	193.4	249.7	141.5

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Table 5. The optimal $\boldsymbol{k}_i^t$ for each location in test problem with stationary demand

t	1	2	3	4	5	6	7	8	9	10	11	12
$i = 1$	374.3	379.1	381.2	369.2	393.7	364.5	698.8	364.9	382.5	379.7	378.7	359.8
$i = 2$	245.2	244.0	240.0	252.5	241.4	247.7	239.6	242.6	242.6	242.9	236.3	361.0
$i = 3$	194.6	193.1	189.8	502.7	191.1	198.6	189.3	191.3	193.0	193.0	187.8	235.3

Table 6. Comparison of results obtained from optimization and simulation models

	Model type	test problem with stationary demand	test problem with non-stationary demand
$C(\boldsymbol{\Pi}^*)$	Optimization	36453.0	19464.2
	Simulation	36009.0	19401.0
ϵ		0.012	0.03

Table 7. Comparison of the optimal (R,s_n^*,S_n^*) and Π^* policies

	stationary demand		non-stationary demand	
Optimal ordering policy	(R,s_n^*,S_n^*)	Π^*	(R,s_n^*,S_n^*)	Π^*
Total average cost	39744.6	36453.0	26498.0	19464.2
Dominant policy	Π^*		Π^*	

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Highlights:

1- A new formulation is proposed for controlling inventory in a two-echelon distribution system consisting of one warehouse and multiple non-identical retailers.

2- A periodic randomized ordering (RO) policy is extended to manage the inventory in a one warehouse and multiple retailers (OWMR) configuration subject to the following assumptions:

- considering capacity limitation for all locations,
- considering lost-sale situation instead of backorder,
- considering stationary and non-stationary demand patterns,
- considering identical and non-identical retailers.

3- The immediate fill rate of all locations are obtained without adding new variables and facing the curse of dimensionality.

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