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An Optimal Power Management Strategy for Power Split Plug-in Hybrid Electric Vehicles

A.Taghavipour, N. L. Azad, J. McPhee

Abstract

Model Predictive Control (MPC) can be an interesting concept for designing a power management strategy for hybrid electric vehicles according to its capability of online optimization by receiving current information from the powertrain and handling hard constraints on such problems. In this article a power management strategy for a power split plug-in hybrid electric vehicle is proposed using the concept of MPC to evaluate the effectiveness of this method on minimizing the fuel consumption of those vehicles. Also the results are compared with Dynamic Programming.

Keywords: Plug-in Hybrid Vehicles, Power Management Strategy, Model Predictive Control, Dynamic Programming, Fuel Consumption.

1. Introduction

Air pollution and rising fuel costs is an important concern for transportation industry. Hybrid electric vehicles have come to existence as a solution to this problem. Other sources of energy in hybrid vehicle powertrains have made the engines smaller and more efficient. Therefore these vehicles have less emissions and also better fuel economy. Hybrid electric vehicles (HEV) powertrains consist of an efficient engine and electric motor/generator in addition to a power storage device that is usually a battery. With the development of advanced battery technologies, the energy storage capacity of batteries has significantly improved. The plug-in hybrid electric drive train is designed to fully or partially use the energy of the energy storage to displace part of the primary energy source [1]. In plug-in hybrid vehicles (PHEVs) the battery is fully charged before starting off with the conventional home electric plugs. Therefore plug-in hybrid vehicles can go longer on pure electric mode. According to [2] about half of the daily driving distance is less than 64 km (40 miles). If a vehicle is designed to have 64 km (40 miles) of pure electric range, that vehicle will have half of its total driving distance from the pure electric vehicle (EV) mode [1]. In that case, there will be rarely need for starting the engine in most urban travels. This leads to a better fuel economy in plug-in hybrid vehicles with respect to conventional hybrids.

According to the Electric Power Research Institute (EPRI), more than 40% of the U.S. generating capacity operates at a reduced load overnight, and it is during these off-peak hours that most PHEVs could be recharged. Recent studies show that if PHEVs replace one-half of all vehicles on the road by 2050, only an 8% increase in electricity generation (4% increase in capacity) will be required [3]. At today’s electrical rates, the incremental cost of charging a PHEV fleet overnight will range from \$90 to \$140 per vehicle per year. This translates to an equivalent production cost of gasoline of about 60 cents to 90 cents per gallon [4]. So PHEVs are a very interesting option for the future of transportation.

Designing the power management is as important as choosing the architecture of a hybrid powertrain architecture. The best architecture can operate poorly in case of using an inappropriate control strategy. One way to design power management strategy for a PHEV is extending the strategies applied on conventional hybrid vehicles [5]. Of course design of a power management strategy needs some considerations, for instance the choice of the correct objective function to be minimized, forecasting the future load based on the information available at runtime and also the characteristics of the vehicle [6].

It should be mentioned that most of the strategies in commercial hybrid vehicles are rule-based [7].

Rule-based approaches put a constraint on the power split between different power sources on-board based on the current state of the powertrain (e.g., vehicle/engine speed, battery charge, power demand, etc.) through some map, or rule base [8]. Then some rules can be to ensure that the states of the system are as close as possible to the desired scheme. Those decisions can be conducted through some maps. These maps can be constructed from engineering expertise and insight, or using more formal methods such as optimization [9] or fuzzy logic [10]. Stochastic dynamic programming (SDP) method is quite appealing in this context because of its ability to optimize the system performance with respect to a probabilistic distribution of some different drive cycles [11]. However, this method has some computational complexity issues [12]. Moura et al. derived an optimal power management strategy for a plug-in hybrid vehicle (power split architecture) based on stochastic dynamic programming which rations battery charge by blending engine and battery power in a manner that improves engine efficiency and reduces total charge sustenance time [13].

Freyermuth et al. [14] simulated and compared four different control strategies for a power split PHEV with 16 km AER (All Electric Range) battery pack for a vehicle with similar performances with for couple of strategies.

In Electric Vehicle/ Charge Sustaining (EV/CS), the engine only turns on when the power demand is higher than available power of battery. Differential Engine Power strategy is similar to EV/CS but the engine-turn-on threshold is lower than the maximum power of electrical system. In Full Engine Power strategy, if the engine turns on it will supply all the power demand of the drive cycle and no power will drain from the battery. The aim of this strategy is to force the engine to operate in higher power demand and consequently in higher efficiency. Optimal Engine Power Strategy, similar to previous strategy, seeks to propel the engine more efficiently in higher power by restricting the engine operation close to peak efficiency.

Freyermuth et al. concluded EV/CS is equivalent to Differential Engine Power and Full Engine Power is the best of all and much better than optimal Engine Power [14].

Rule-based strategies are rigid and their performance is considerable for a known pattern of drive cycle (for taxi cabs or bus routes) but they’re not optimized. The same thing is expected for the offline optimization methods through which the strategy is designed according to a predefined drive cycle. So this strategy is not necessarily optimized for a deviated drive cycle. But more advanced control techniques are based on real-time optimization. Also referred to as causal systems, they rely on real-time feedback to optimize a cost function that is developed using past information [5]. This gap is covered in the approach we present in this article.

Trajectory power management algorithms require knowledge of future power demand. This approach uses this information to specify the future power contribution of different sources of energy on board. Such optimization can be performed offline for drive cycles known a priori using deterministic dynamic programming (DDP) [15], and can also be performed online using optimal model predictive control [16].

Gong et al. [17, 18] suggested that it is possible to improve the control strategy of PHEV if the trip information is determined a priori by means of recent advancements in intelligent transportation system (ITS) based on the use of global positioning system (GPS) and geographical information system (GIS).

In this paper a model-based strategy is proposed with the use of model predictive control (MPC) concept. MPC seems a proper method to exploit the potentials of modern concepts and to fulfil the automotive requirements since most of them can be stated in the form of a constrained multi-input multi-output optimal control problem and MPC provides an approximate solution of this class of problems [19]. In general, Model predictive control is the only advanced control technology that has made a substantial impact on industrial control problems: its success is largely due to its almost unique ability to handle, simply and effectively, hard constraints on control and states [20].

The popularity of MPC stems from the fact that the resulting operating strategy respects all the system and problem details, including interactions and constraints, which would be very hard to accomplish in any other way. Indeed, often MPC is used for the regulatory control of large multivariable linear systems with constraints, where the objective function is not related to an economical objective, but is simply chosen in a mathematically convenient way, namely quadratic in the states and inputs, to yield a “good” closed-loop response. Again, there is no other controller design method available today for such systems that provides constraint satisfaction and stability guarantees [21].

The only serious drawback of this method is the volume of calculations for any time step of control. This is the reason that MPC was mostly used for controlling chemical processes that are considered as slow systems. But by having faster processors nowadays, there is an obvious motivation for using this model-based control method for rather fast systems especially for the automotive systems.

Application of MPC to hybrid vehicles has been investigated before. Wang et al. [22] integrated the MPC controller and proposed a real time control system. The system can be used for all kinds of hybrid architectures based on engine and electric motor. They used a number of different performance indices that can be applied to the control system. By changing the operational weights in the cost function, the power control system can achieve different goals. Borhan et al. [23] applied MPC to a power split hybrid electric vehicle, whereas they ignored the dynamics of powertrain against other faster dynamics for the model inside the controller. They proposed that the fuel economies achieved with MPC are better than those reported by the rule-based PSAT simulation software.

In fact, this method has not been applied to design a power management strategy for a plug-in power split hybrid electric vehicle; the goal we seek in this article. It should be mentioned that plug-in powertrain is different from conventional hybrid vehicles in terms of initial conditions and constraints. In a PHEV, the battery capacity is larger and it can be charged from another source out of powertrain; the battery can be fully charged before vehicle is started, whereas it is an impossible option for a HEV. In a HEV powertrain, the battery state of charge (SOC) should be maintained inside a definite range (for instance between 0.60 to 0.65 in [23]) and final SOC value at the end of simulation time should be the same as initial SOC [1]. But generally in PHEVs, the battery is discharged from a high level and when SOC drops to a reference value, the control strategy tries to keep it as close as possible to that level. This reference value is lower than what it is in a HEV. The strategies that are applied on HEVs can be implemented on PHEVs, but should be modified for the best performance. Therefore, these are originally two different problems with different constraints.

To compare MPC results, we solved Dynamic Programming (DP) for this problem with the same dynamics and constraints as well. DP has been extensively used in literature in order to find a global solution for HEVs control strategies. DP cannot be implemented online. Therefore it's just a benchmark for developing heuristic strategies.

Dynamic programming yields results that are close to being global optimal. In the context of optimal control, DP and PMP are two different approaches to obtain optimal trajectories for deterministic optimal control problems. In the minimum fuel consumption problem of HEVs, the DP method guarantees a global optimal solution by detecting all possible control options [24].

Moreover we will compare the MPC performance with the result, that has been proposed in [24] according to Pontryagin's Minimum Principle (PMP) for a PHEV.

According to [25] for urban driving conditions, the power split showed best fuel economy in comparison with series and parallel configurations. In highway driving condition, power split and parallel architectures showed similar and better efficiency in comparison to series architecture.

At first the theory of MPC will be explained in brief. After designing the power management strategy, it will be implemented on the model and results of simulation will be compared to Dynamic Programming. The discussion and the conclusion sections come afterwards.

2. Theory

The general design objective of MPC is to compute a trajectory of a future input to optimize the future behaviour of the plant output. The optimization is performed within a limited time window based on the information of the plant at the start of the time window.

Moving horizon window is the time interval in which the optimization is applied. The length of this window is called prediction horizon (N_p). It determines how far we wish to predict the future. The objective of solving an MPC problem is to find a vector that contains the variation of inputs in order to reach the desired trajectory of outputs. The length of this vector is called control horizon (N_c).

In the planning process, we need the information about state variables at time t_i in order to predict the future. This information is denoted as $x(t_i)$ which is a vector containing many relevant factors, and is either directly measured or estimated. A good dynamic model will give a consistent and accurate prediction of the future [26].

Meanwhile an integrator is naturally embedded into the design, leading to the predictive control system tracking constant references and rejecting constant disturbances without steady-state errors.

For linear MPC, the model inside the controller is an augmented one which contains an integrator for each output.

For an augmented discrete system like:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{1}$$

Where x, u and y are state variable, input and output of the linear system.

Note that *D=0* due to the principle of receding horizon control, where a current information of the plant is required for prediction and control.

The relation between the predicted output of the system inside the prediction window (Y), time step k_i , measured states at t_i and the designed variation of the inputs will be (prediction equation):

$$Y = Fx(k_i) + \Phi\Delta U \tag{2}$$

Where

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \ldots \\ CA^{N_P} \end{bmatrix};\ \Phi = \begin{bmatrix} CB & 0 & 0 & \ldots & 0 \\ CAB & CB & 0 & \ldots & 0 \\ CA^2B & CAB & CB & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ CA^{N_P-1}B & CA^{N_P-2}B & CA^{N_P-3}B & \ldots & CA^{N_P-N_C}B \end{bmatrix} \tag{3}$$

$$\begin{aligned} Y &= [y(k_i+1\,|\,k_i)\ \ y(k_i+2\,|\,k_i)\ \ \ldots\ \ y(k_i+Np\,|\,k_i)]^T \\ \Delta U &= [\Delta u(k_i)\ \ \Delta u(k_i+1)\ \ \ldots \Delta u(k_i+N_c-1)]^T \end{aligned} \tag{4}$$

Where $y(k_i+2|k_i)$ means the predicted output on k_i+2 step based on the measurement on k_i step [26].

The performance of a control system can deteriorate significantly when the control signals from the original design meet with operational constraints. But with a small modification, the degree of performance deterioration can be reduced if the constraints are incorporated in the implementation, leading to the idea of constrained control. For modifying the controller, all the constraints must be changed in the form of variation in input signal. For the constraints on the amplitude of the input, variation of the inputs and the outputs:

$$\begin{gathered} U^{\min} \le (C_1 u(k_i-1) + C_2\Delta U) \le U^{\max} \\ \Delta U^{\min} \le \Delta U \le \Delta U^{\max} \\ Y^{\min} \le Fx(k_i) + \Phi\Delta U \le Y^{\max} \end{gathered} \tag{5}$$

Where

$$C_1 = \begin{bmatrix} I_{(N_c)\times(N_c)} \\ I_{(N_c)\times(N_c)} \\ \ldots \\ I_{(N_c)\times(N_c)} \end{bmatrix},\ C_2 = \begin{bmatrix} I_{(N_c)\times(N_c)} & o_{(N_c)\times(N_c)} & \cdots & o_{(N_c)\times(N_c)} \\ I_{(N_c)\times(N_c)} & I_{(N_c)\times(N_c)} & \cdots & o_{(N_c)\times(N_c)} \\ \ldots & \ldots & \ldots & \ldots \\ I_{(N_c)\times(N_c)} & I_{(N_c)\times(N_c)} & \cdots & I_{(N_c)\times(N_c)} \end{bmatrix} \tag{6}$$

If a quadratic objective function is used for the optimization, this is a quadratic programming problem.

3. Model Description

Among the different architectures for a hybrid electric vehicle, power split configuration seems to be the most efficient one for a limited capacity of battery. In a power split configuration, the engine, the electric motor and the generator are connected to each other by means of a planetary gear set. The start off the vehicle is pure electric. It means that the engine is shut off. This status continues for driving with low speed until the battery state of charge drops to a predefined level or the velocity increases. Hereby the engine is started and delivers power to the drivetrain and simultaneously charges the battery with the help of the generator. For full performance, the battery empowers the electric motor to propel the vehicle with the help of the engine. With regenerative braking, a part of dissipated energy returns to the battery by the electric motor.

For deriving the dynamics of the system it is assumed the mass of the pinion gears is small, there is no friction, no tire slip or efficiency loss in powertrain. By considering the vehicle longitudinal dynamics, the equation of the system will be according to (7), (8) and (9) which is derived by J. Liu et al [27].

$$\left(\frac{I_v^{'}(R+S)^2}{RI_e^{'}K}+\frac{I_v^{'}S^2}{RI_g^{'}K}+R\right)\dot{\omega}_r=\left(\frac{(R+S)^2}{RI_e^{'}}+\frac{S^2}{RI_g^{'}}\right)T_m+\frac{(R+S)}{I_e^{'}}T_e+\frac{S}{I_g^{'}}T_g-\left(\frac{(R+S)^2}{RI_e^{'}K}+\frac{S^2}{RI_g^{'}K}\right)C \tag{7}$$

$$\left(\frac{I_e^{'}R^2K}{(R+S)I_v^{'}}+\frac{I_e^{'}S^2}{(R+S)I_g^{'}}+(R+S)\right)\dot{\omega}_e=\left(\frac{R^2K}{(R+S)I_v^{'}}+\frac{S^2}{(R+S)I_g^{'}}\right)T_e+\frac{KR}{I_v^{'}}T_m-\frac{S}{I_g^{'}}T_g-\frac{R}{I_v^{'}}C \tag{8}$$

$$\dot{SOC}=-\frac{V_{oc}-\sqrt{V_{oc}^2-4\left(T_m\omega_r\eta_m^{-k}-T_g\omega_g\eta_g^{k}\right)R_{batt}}}{2R_{batt}Q_{batt}} \tag{9}$$

In this relation:

$$\begin{gathered}
I_v^{'}=m\frac{R_{tire}^2}{K}+I_mK+I_rK\\
I_g^{'}=I_g+I_s\\
I_e^{'}=I_e+I_c\\
C=mgf_rR_{tire}+0.5\rho Ac_d\left(\frac{\omega_r}{K}\right)^2R_{tire}^3\\
\omega_rR+\omega_gS=\omega_e(R+S)
\end{gathered} \tag{10}$$

Where the variables and parameters are defined in the Appendix.

If the macroscopic behavior of the battery is to be represented within a more complex system, as is typically the case in vehicle modeling, the battery is often represented by an equivalent circuit. We can use a simple circuit for modeling the battery. The external resistance represents the effect of chemical reactions. Since only one resistance is considered, the complex nonlinear effects such as diffusion and battery surface capacitance are not directly considered [28]. Therefore a simple internal resistance model for the battery is considered.

In PHEVs, the battery pack is discharged from a fully charged status to a reference SOC, where the vehicle is then operated as a regular hybrid [28]. The value we have chosen here as the reference SOC is 30% regarding to the life of the battery pack.

These idealized assumptions will result in a more optimistic fuel economy prediction. In this system there are 3 states: ring speed (ω_r) which is proportional to the vehicle velocity, engine speed (ω_e), and battery state of charge (SOC). Also there are 3 inputs: Engine (T_e), Motor (T_m) and Generator Torque (T_g). η_m and η_g represent motor drive and generator drive efficiency respectively (Including DC/DC convertor and DC/AC inverter) [29]. The readers are referred to [30] for an empirical estimation of the power electronics efficiency. When the battery is discharged k=1. But k=-1 for battery charging.

4. Problem Statement

The goal of this research is to design a control strategy for a plug-in hybrid vehicle with power split architecture. The battery in a plug-in hybrid vehicle is fully charged before vehicle start off. We assume that the vehicle goes on the pure electric mode until the charge of the battery drops to a reference state of charge, then the strategy enters a loop governed by MPC. This controller tries to keep the state of charge around the reference and simultaneously minimize the fuel consumption. In this problem there are 3 inputs that give flexibility to the control problem.

In each prediction window we need a cost function to be minimized that results in maximum fuel economy and tracking a predefined level of battery charge while following a predefined drive cycle. The cost function is:

$$J(k)=\sum_{i=1}^{N_p}\{\underbrace{\gamma_1\left(SOC_{ref}(k+i)-SOC(k+i)\right)^2}_{\text{Control of SOC}}+\underbrace{\gamma_2\left(\dot{m}_f(k+i)\right)^2}_{\text{Minimizing Fuel Consumption}}\}$$
$$+\sum_{i=1}^{N_c}\{\underbrace{\gamma_3\left(\Delta T_g(k+i)\right)^2}_{\text{Minimizing Generator Torque Variation}}+\underbrace{\gamma_4\left(\Delta T_m(k+i)\right)^2}_{\text{Minimizing Motor Torque Variation}}\} \tag{11}$$

Where $\Delta T(k+i)=T(k+i+1)-T(k+i)$.

The first term is related to keep the state of charge around reference. The second term is for minimizing the fuel consumption. The third and fourth terms try to minimize the input variations inside the prediction horizon. $\gamma_1,\gamma_2,\gamma_3,\gamma_4$ are weighting parameters that are chosen according to the predicted maximum value of the weighted variables. In most optimization problems except in rather rare cases (e.g., [31] and [32]), only minimizing fuel consumption is the objective, and pollution limitation is considered as a constraint of the process; as long as pollution is within predefined limits, it does not influence the optimization process [6].

We chose the FTP75 drive cycle to estimate fuel consumption. Also there are some constraints on this problem that are defined as following:

$$
\begin{aligned}
T_{\min-e} &< T_e < T_{\max-e} \\
T_{\min-m} &< T_m < T_{\max-m} \\
T_{\min-g} &< T_g < T_{\max-g} \\
\Delta T_{\min-m} &< \Delta T_m < \Delta T_{\max-m} \\
\Delta T_{\min-g} &< \Delta T_g < \Delta T_{\max-g} \\
\omega_{\min-e} &< \omega_e < \omega_{\max-e} \\
\omega_{\min-r} &< \omega_r < \omega_{\max-r} \\
\omega_{\min-g} &< \omega_g < \omega_{\max-g} \\
SOC_{\min} &< SOC < SOC_{\max}
\end{aligned}
\tag{12}
$$

For finding a simpler form of the controller and also using the linear MPC, the equations of the system were linearized for each time step around the operating point. Moreover, we use receding horizon control principle where the actual control input to the plant only takes the first sample of the control signal, while neglecting the rest of the trajectory. Also the fuel consumption map of the engine was estimated as:

$$\dot{m} = \alpha\omega_e^2 + \beta T_e \omega_e \tag{13}$$

Where α, β are constant.

As mentioned before, the current optimization problem can be converted to a quadratic form. Assume that the cost function is written in the form of

$$
\begin{aligned}
J(k) &= \frac{1}{2}\Delta U^T H \Delta U + \Delta U^T E \\
& M\Delta U \le N
\end{aligned}
\tag{14}
$$

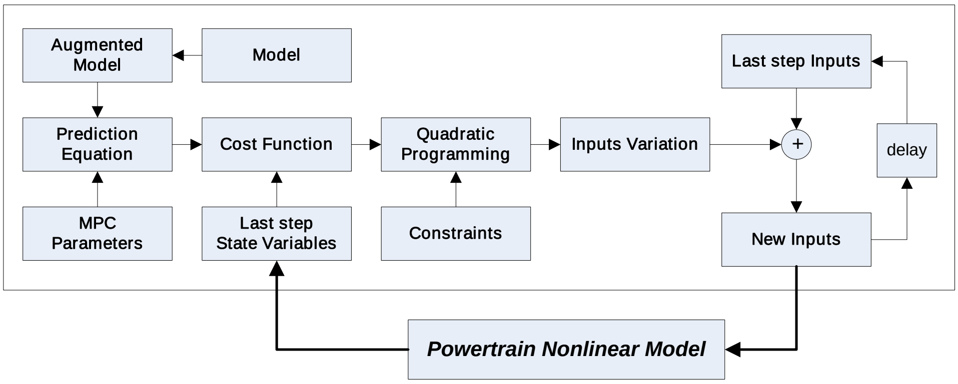
Where M and N are specified by the constraints of (12). Note that input for MPC problem is the inputs variation, with length of control horizon. Typical solution to this problem using Lagrangian multipliers can be found [33]:

$$\Delta U = -H^{-1}E - H^{-1}M^T\lambda \tag{15}$$

where $\lambda = -(MH^{-1}M^T)^{-1}\left(N + MH^{-1}E\right)$.

Since this problem must be solved in every time step we need a fast approach. Identifying active constraints in each time step would be helpful to accelerate the calculation procedure. In this paper we used Hildreth's quadratic programming procedure that suggests an iterative approach to identify the active constraints in order to solve the problem and find the second term in equation (15). Figure 1 summarizes the algorithm of MPC.

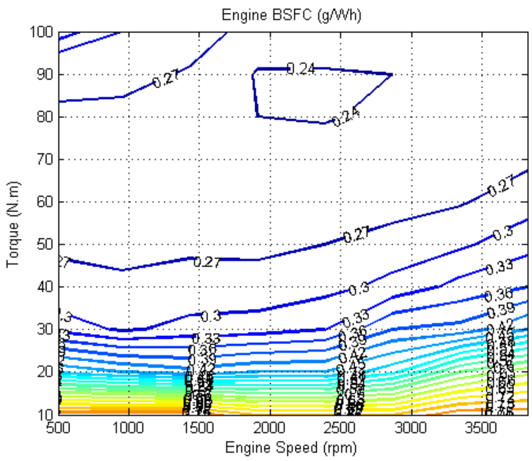
Figure 1. MPC structure



For Dynamic Programming, we only considered the fuel consumption inside the cost function. The quadratic term in (11) for controlling SOC was replaced with a hard constraint on SOC in charge sustaining mode. Other constraints and dynamics have remained unchanged.

Fuel consumption map of the engine is shown in figure 2.

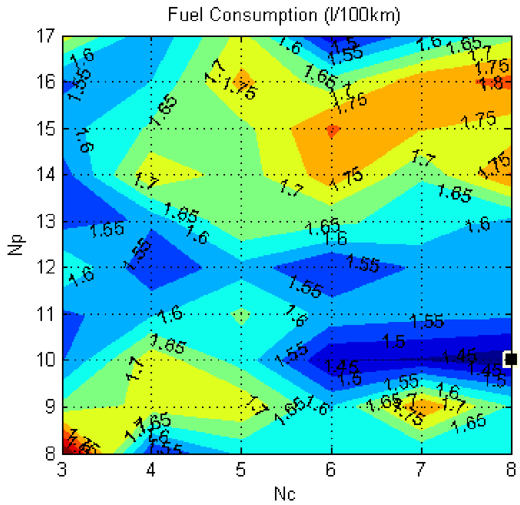
Figure 2. Engine fuel consumption map [29]



5. Results of Simulation

The simulation was done in the MATLAB environment. The requested torque was calculated based on 2 sequential FTP drive cycles. This torque is one of the inputs to the controller. Power management strategy uses this input and a linearized model of the powertrain to predict the future contribution of each power source on board. Outputs of the controller are applied to the nonlinear model of the powertrain (equations 7 to 9) so that we can find out critical state variables like battery state of charge, vehicle velocity and especially fuel consumption. Figure 3 shows the fuel consumption for different values of MPC parameters. Control horizon (*Nc*) is less than the prediction horizon(*Np*). The input horizon should be as large as the expected transient behavior. In practice, a value of $Nc \geq 3$ often seems to give performance close to the '*global optimal*'. To achieve closed-loop behavior close to open-loop behavior, $Nc=1$ will often be sufficient [34].

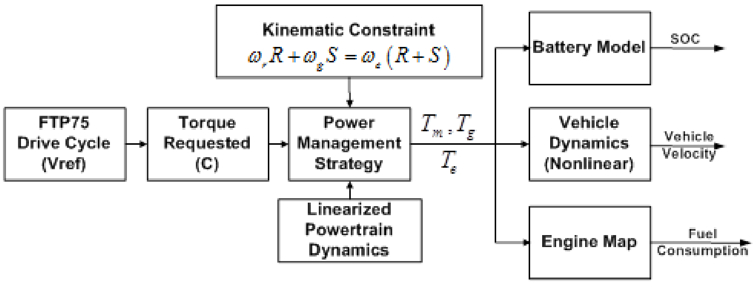
Figure 3. Fuel consumption vs. control and prediction horizon



It can be seen that the least fuel consumption can be found with $Nc=8$ and $Np=10$. All simulations have been done with respect to these values. Therefore fuel economy will not necessarily be improved by increasing prediction horizon.

Figure 4 shows the simulation procedure that has been followed in this paper.

Figure 4. Simulation Procedure



The battery is fully charged at the beginning of the drive cycle and the vehicle goes on a pure electric mode until the state of charge drops to 0.3, the reference state of charge. Fuel consumption in this period is zero.

Figure 5 and table I show the electric range of the PHEV for different reference states of charge.

Figure 5. Electric range of PHEV for different reference SOCs

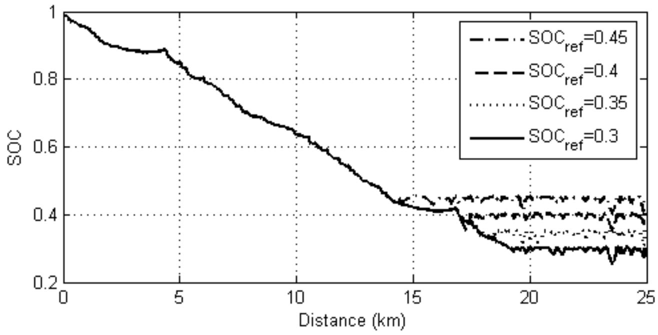


Table I . All Electric Range/ Fuel Consumption for different SOCs

Reference SOC	AER (km)	Fuel Consumption (l/100km)
0.3	19.14	1.41
0.35	17.80	2.06
0.4	17.26	2.33
0.45	14.01	3.37

Figure 6 shows the torque, speed and efficiency of the electric motor. As shown in figure 7 the generator torque on the pure electric mode is equal to zero since there is no need to charge the battery (SOC is higher than the reference).Figure 8 shows the cumulative fuel consumption which is around 1.41liter per 100 km. Engine is shut off and on many times to maximize the fuel economy.

Figure 6. Motor Torque, Speed, Efficiency

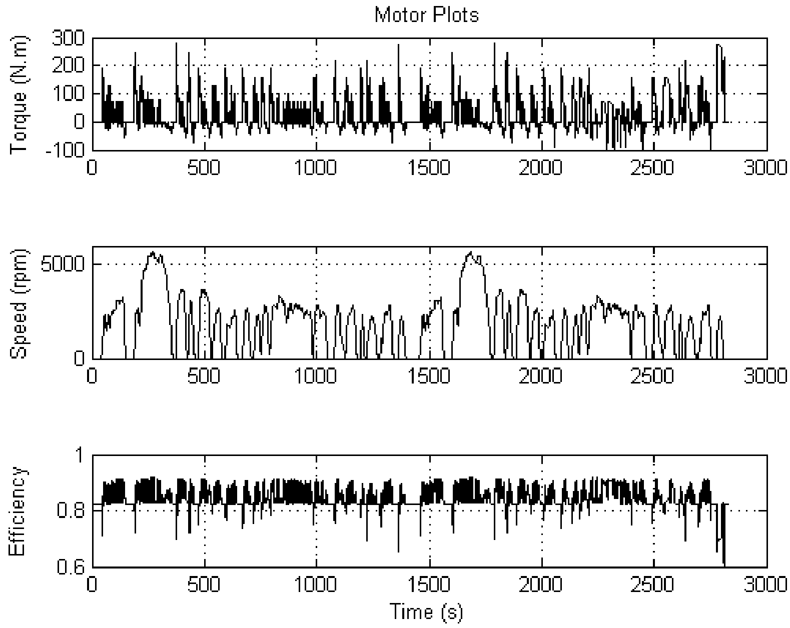


Figure 7. Generator Torque, Speed, Efficiency

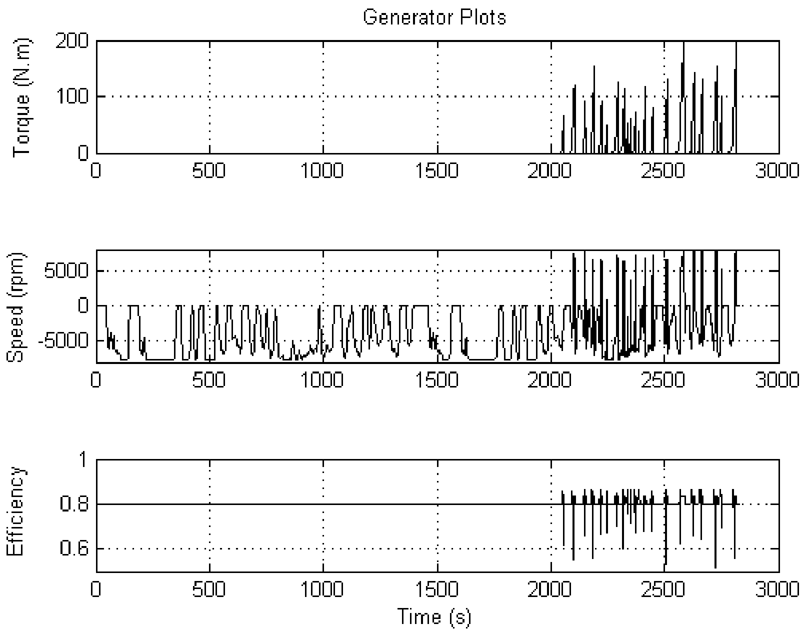


Figure 8. Engine Torque, Speed, Fuel Consumption

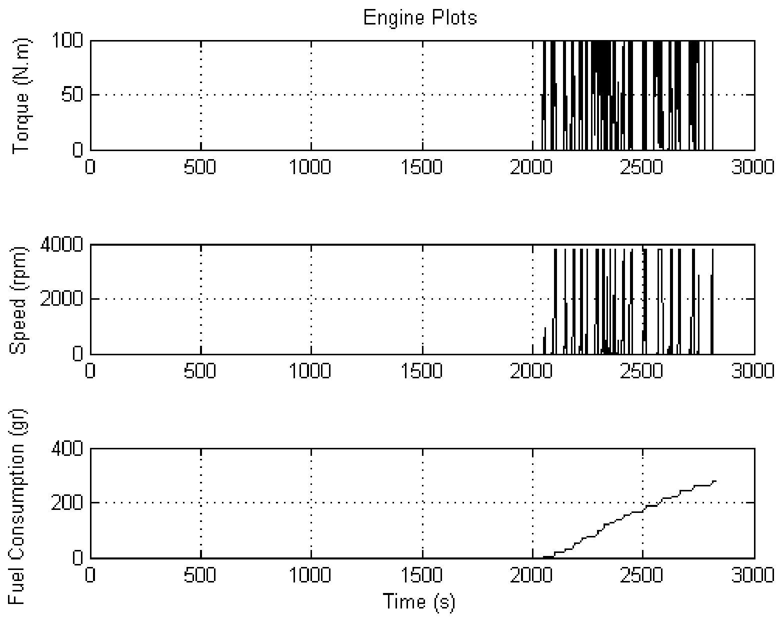


Figure 9 shows the cost function value along the drive cycle. For non-pure electric mode portion there is a tiny deviation from zero for the most time steps and for some points the cost function is not minimized according to the driving condition.

Figure 9. Cost Function Value

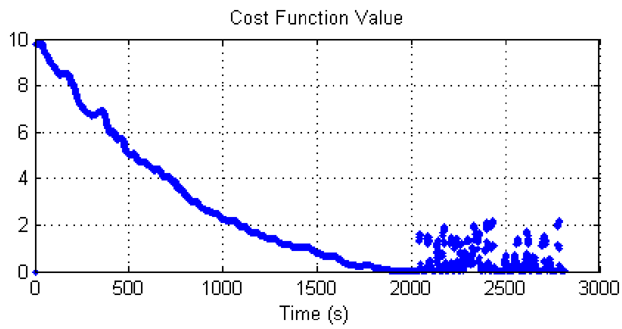


Figure 10 shows fuel consumption increase by considering the input variation inside the cost function.

Figure 10. Fuel consumption vs. reference SOC

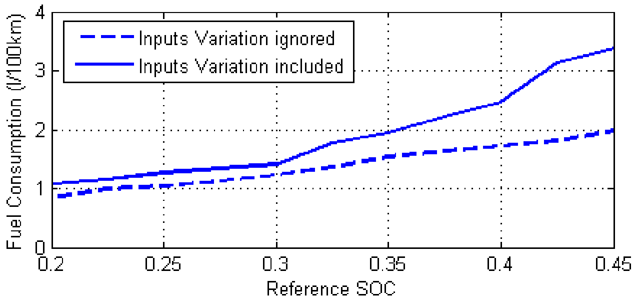


Figure 11 to 14 show the result of Dynamic Programming which was applied for charge sustaining mode.

Figure 11. Dynamic Programming result: Motor Torque, Speed, Efficiency

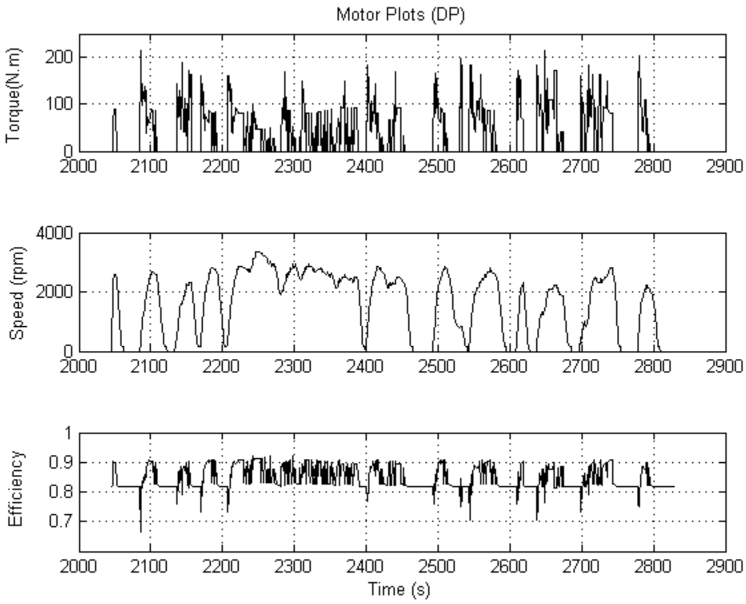


Figure 12. Dynamic Programming result: Generator Torque, Speed, Efficiency

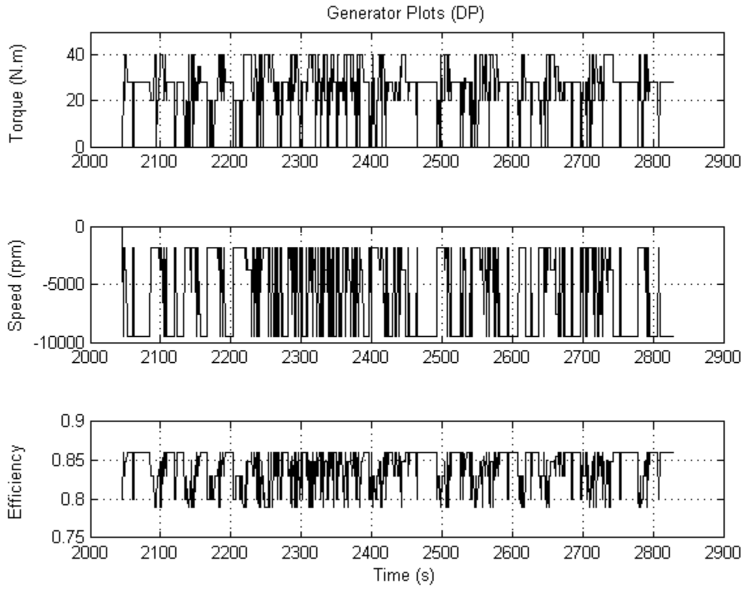


Figure 13. Dynamic Programming result: Engine torque, speed, fuel consumption

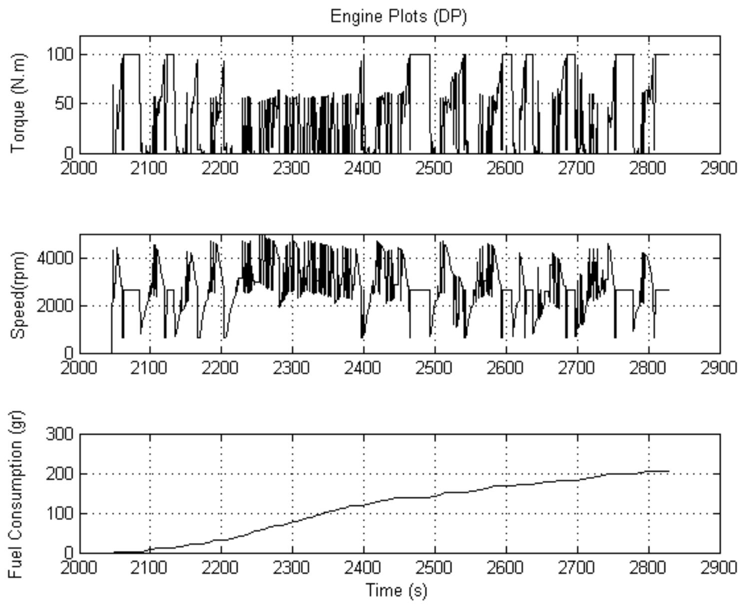
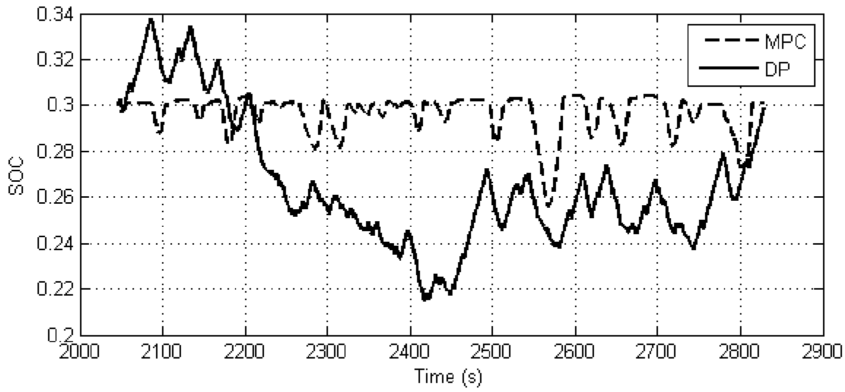


Figure 14. SOC comparison for Dynamic Programming and MPC in charge sustaining mode



6. Discussion

It’s evident that by reducing the reference state of charge we can reduce fuel consumption (Table I) and also increase the electric range of the PHEV. Reducing the state of charge reference is restricted by health parameters and life time of the battery. When SOC drops to a predefined level, the controller switches to charge sustaining (CS) mode and tries to maintain the state of charge as close as possible to the reference (0.3).

According to figure 6 the motor torque will increase upon accelerating especially in the full electric range. Also MG2 can capture a part of braking torque as shown by the negative torques. In figure 7 the generator speed is negative because of the power split device. Since the carrier part is stationary to keep the engine off, the sun gear which is connected to the generator rotates in the reverse direction of the ring gear which is connected to the motor. In charge sustaining mode, depending on the engine speed, the direction of sun gear rotation will change as shown in the speed plot figure 8. The job of the generator is to recharge the battery and also restart the engine (an effect that is ignored in the present work). The generator never stops rotating while the vehicle is moving so it potentially can produce electricity. But it needs a share of engine power which is sometime sufficient for both contributing to the power needed for vehicle propulsion and also recharging the battery. So the power management strategy can connect or disconnect the generator to or from the battery when required to keep the SOC as close as possible to the reference. Generator efficiency is not as high as that of the motor.

Figure 8 must be closely investigated with figure 9 since fuel consumption is one of the terms inside the cost function. In the electric range of travel, cost function value is proportional to the squared difference of SOC with reference SOC. Therefore by getting closer to the charge sustaining mode, the cost function value decreases and finally reaches to zero upon start of CS mode. From now on the main part of cost function value relates to fuel consumption and input variation constraints. Because of some constraints on engine, motor and generator torque, this value cannot be matched to its global minimum. It should be noted that the cost function is the summation of squared predicted input variations and fuel rate along the prediction window at any time step. One of our important concerns is sustaining the battery SOC closely to the reference. Changing the corresponding weight parameter (γ_1) inside the cost function in simulation has revealed that the least fuel consumption is obtained when γ_1 equals the inverse of lower and upper bounds average of SOC.

It's evident that by increasing the reference SOC, fuel consumption will decrease as shown in figure 10. In plug-in hybrid the reference SOC is set to a lower value if it is possible. This value is closely related to the battery life time. By removing the terms related to input variations from the cost function, we obtain better fuel economy as expected. Considering constraints in order to make a more realistic decision leads to fuel consumption increase. Another important issue is the trend of curves, which was predictable. The general solution to (15) is proportional to the *E* matrix. It should be mentioned that the reference SOC can be factorized from the *E* matrix. Therefore the solution of the quadratic programming problem has a proportional relation to the reference SOC. According to (13), we can justify the linear behavior shown in figure 10. By adding the input variation to the cost function, fuel consumption behavior has been changed to a piecewise linear curve.

The results of DP in charge sustaining mode are illustrated in figure 11 to 13. According to figure 12, there is no need for the generator to capture higher torque values to make a sudden increase in SOC (this issue was important for MPC) because of the constraint on SOC in charge sustaining mode. Also, the generator speed goes on higher levels in DP in comparison to figure 7, since the average speed of the engine is more than what it is in MPC. This makes operating points get closer to the engine sweet spot. Therefore, the resultant fuel consumption is 204.3 gr, although the engine never stops operating. The fuel consumption for MPC without considering input variation inside MPC cost function while SOC_{ref} =0.3 is 233.7 gr. This shows 14.4% increase in fuel consumption regarding DP result.

According to figure 14, SOC for DP is free to fluctuate in specific range around reference SOC and this makes fuel consumption less than MPC where the controller was enforced to maintain SOC around the reference.

Kim et al. [24] applied Pontryagin's minimum principle (PMP) to a power split PHEV based on FTP 72 (UDDS) drive cycle. They predicted 1.53 l/100km as the fuel consumption. By sticking to the same controller proposed in this paper and just changing the driving schedule it was revealed that MPC suggests even better fuel economy (1.29 l/100km). Moreover, MPC can also be implemented online.

By adding the variation of inputs inside the cost function and choosing appropriate MPC parameters, we can obtain fuel consumption equal to 1.41 l/100km according to the FTP 75 drive cycle.

Moreover, it took 35.4 second in real time for 2828 seconds simulation (for two successive FTP 75 drive cycles) to be completed. The simulation is conducted in the MATLAB environment and on a machine which is powered by a 3.16 GHz dual core CPU and a 4 GB memory. It would be even faster if the controller was implemented as a C-code. It means that MPC is capable of being implemented online.

In summary, a model-based controller was proposed to effectively handle the hard constraints on power management strategy design problem of a PHEV. In this work the effect of MPC parameters was investigated without ignoring the powertrain dynamics. MPC problem was solved by considering input variation inside the cost function to not even consider them as a hard constraint but to make the system operate as smoothly as possible.

7. Conclusion

In this article, a power management strategy for a plug-in hybrid vehicle was designed according to the discrete MPC concept with appropriate parameters and compared to dynamic programming. The model inside the controller was linearized and discretized. Simulation was done along 2 successive FTP 75 drive cycles to get an insight into the electric range of the PHEV. It was revealed that fuel economy will not necessarily be improved by increasing the prediction horizon. Also for making the analysis more realistic, the input variations were considered inside the cost function that should be minimized in each time step.

Simulation results showed a promising fuel consumption of 1.41 l/100km by following the FTP 75 drive cycle.

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Appendix

Table II reviews the model parameters and variables.

TABLE II. VARIABLES & PARAMETERS DESCRIPTION

Symbol	Unit	Value	Description
α	$\left(kg/h\right)\left(rad/s\right)^{-2}$	0.02	Engine Speed Coefficient
β	$\left(kg/h\right)(W)^{-1}$	1.86	Engine Power Coefficient
$\dot{m}$	kg/h	-	Fuel Rate
I_g	$kg.m^2$	0.1	Generator Equivalent Inertia
I_s	$kg.m^2$	0.1	Sun Equivalent Inertia
I_e	$kg.m^2$	0.5	Engine Equivalent Inertia
I_c	$kg.m^2$	0.1	Carrier Equivalent Inertia
I_m	$kg.m^2$	0.1	Motor Equivalent Inertia
I_r	$kg.m^2$	0.1	Ring Equivalent Inertia
R_{tire}	m	0.3	Tire Radius
K	-	6.75	Gear Ratio
m	kg	1380	Vehicle Mass
g	m/s^2	9.81	Gravity Acceleration
f_r		0.02	Friction Coefficient
ρ	kg/m^3	1.2	Air Density
A	m^2	2.5	Vehicle Frontal Area
c_d	-	0.2	Drag Coefficient
R	-	78	Ring Teeth No.
S	-	30	Sun Teeth No.
V_{oc}	V	345.6	Battery Open Circuit Voltage
R_{batt}	Ω	0.85	Battery Open Circuit Resistance
Q_{batt}	$A.s$	54167	Battery Capacity
SOC_{ref}	-	0.3	Reference SOC