FORCED CONVECTION FROM A PAIR OF SPHERES:
A THREE-TEMPERATURE PROBLEM

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ABSTRACT
Interaction of spherical particles in a fluid flow is important in combustion, chemical processes and air pollution. In this study, a recently developed technique for characterizing multi-temperature convective heat transfer is used to investigate convection from a pair of spheres with different surface temperatures. The technique entails numerical solutions of the full set of governing equations and subsequent solutions of the energy equation with perturbed boundary conditions. Steady-state heat transfer in intermediate Reynolds number flows over spheres in side-by-side and tandem arrangements in a water tunnel is studied. The results are expressed in terms of local and total Nusselt numbers. The variation of sphere-sphere and sphere-fluid Nusselt numbers with flow conditions is examined.

INTRODUCTION
Interaction of spherical particles in a fluid flow is important in the analysis of particulate and multiphase flow systems in such areas as combustion, chemical processes and air pollution. In many cases, the single sphere data does not provide an accurate prediction of the system behaviour [1] and interaction of spheres moving in close proximity must be examined. This interaction has been studied by various authors. Kim et al. [2], for instance, numerically studied three-dimensional flow over two identical spheres arranged side by side in a uniform stream at moderate Reynolds numbers. Tal et al. [3,4] studied fluid flow and heat transfer in assemblies of spherical particles. Patnaik [5] and Raju and Sirignano [6] studied the interactions between two moving vaporizing droplets in tandem arrangement for Re<200. Yoon and Yang [7] studied flow-induced forces on two identical nearby spheres in various arrangements at Re=300. Prahl et al. [8] further examined the interaction between two spheres in tandem at Re=300 using both steady and pulsating inflow conditions. Liang et al. [9] studied the drag force in various multi-particle arrangements for 30<Re<106. The interaction between groups of droplets in low to moderate Reynolds numbers is of special importance in combustion. A comprehensive review of the theory of droplet array combustion has been recently published by Sirignano [10]. Kleinreuer et al. [11] computed the transient velocity and temperature fields around interacting vaporizing fuel droplets for an initial Reynolds number of 100. Zhu and Dunn-Rankin [12] used a spectroscopy technique to perform temperature measurements in a stream of combusting droplets.

NOMENCLATURE
A surface area
C_d drag coefficient
D diameter
h heat transfer coefficient
k thermal conductivity
L center-to-center spacing
Nu Nusselt number
Pr Prandtl number
Q heat transfer rate
Re Reynolds number
T temperature
δ denotes small change
Subscripts
i at source i
ij from source i to source j
Juncu [1] has studied unsteady convective heat and mass transfer from two spheres in tandem. In most of the published studies, e.g. references [3,4], the array of spheres has been assumed to be isothermal, i.e. no temperature difference between individual particles. Based on this assumption, the problem of convective heat transfer becomes a two-temperature problem. Nevertheless, if the particles are at different surface temperatures, a multi-temperature convection problem occurs. The set of heat transfer coefficients needed for fully characterizing a multi-temperature convection problem cannot be obtained using conventional methods, e.g. CFD.

In the present work, the three-temperature problem of forced convection from a pair of spheres, with different temperatures, is examined using a recently developed technique, dQdT. The flow of water, Pr=7, in a 1.2m×1.2m×2.4m water tunnel is considered in a range of intermediate Reynolds numbers, 0<Re<250. The spheres are placed in side-by-side and tandem arrangements, with the center-to-center spacing between the spheres equal to twice their diameter, L/D=2. The present study is conducted as part of a feasibility study for the experimental implementation of the dQdT technique in a water tunnel.

METHODOLOGY
The dQdT technique: A technique for characterizing three-temperature convection problems called dQdT is used in this study. This technique entails a numerical solution of the full set of governing equations and consequent solutions of the energy equation with perturbed boundary conditions. A paired heat transfer coefficient, \( h_{ij} \), characterizing the influence of the temperature of one heat source, \( T_i \), on heat transfer from another source, \( Q_j \), is obtained as shown in Equation 1. The term paired is used here to designate heat transfer between one source and another designated heat source, as opposed to overall, which is used to designate heat transfer between one source and all the other sources.

\[
\begin{align*}
  h_{ij} &= -\frac{1}{A_j} \left. \frac{\partial Q_j}{\partial T_i} \right|_{h_{ij}=\text{const}} = -\frac{1}{A_j} \frac{\delta Q_j}{\delta T_i} \quad (1)
\end{align*}
\]

In Equation 1, \( A_j \) is the surface area of source j, \( \delta Q_j \) denotes the observed changed in the rate of heat transfer at source j as a result of a change, \( \delta T_i \), in the temperature of source i. The condition of \( h_{ij}=\text{const} \) is satisfied by solving the energy equation while the flow field and the fluid properties are fixed, i.e. prevented from changing after a perturbation is introduced in the temperature boundary conditions.

A heat transfer coefficient, \( h_{ij} \), calculated using Equation 1, is converted to dimensionless form, i.e. the paired Nusselt number \( Nu_{ij} \), using Equation 2.

\[
Nu_{ij} = \frac{h_{ij}D}{k} \quad (2)
\]

The paired heat transfer coefficients calculated by dQdT can also be used to characterize the thermal resistor model of three-temperature heat transfer problems. Characterizing thermal resistor networks in terms of a set of resistances (heat transfer coefficients) is significantly beneficial in the analysis of thermal systems and in decreasing the computational expense of modeling such systems, e.g. in building energy simulation.

Numerical solutions: The commercial CFD code ANSYS FLUENT is used to solve the equations of continuity, momentum and energy in steady state. Constant thermophysical properties are assumed. The SIMPLE scheme is used for pressure-velocity coupling, along with 2nd order discretization schemes for all the governing equations. A schematic of the side view of the computational domain is shown in Figure 1. Uniform velocity and temperature profiles are set at the inlet of the domain, while zero gauge pressure is set as the boundary condition at the outlet. The domain sides, representing the four walls of the water tunnel, are modeled as adiabatic impermeable no-slip walls. The boundary conditions at the spheres are zero velocity and constant temperature.

![Figure 1](2.4 m)

Schematic of the computational domain (side view)

For every Reynolds number, the full set of governing equations is first solved to obtain the temperature and velocity fields. Heat transfer rates at the spheres are then computed by integrating the...
temperature field. To obtain paired heat transfer coefficients defined by Equation 1, the temperature boundary condition at one of the isothermal heat sources, say one of the spheres, is perturbed. Next, the energy equation is solved and the new heat transfer rate at the unperturbed source, the other sphere, is calculated.

**Verification:** The present numerical solutions are verified by comparison to previously published results. Table 1 provides a comparison of the drag coefficient of side-by-side spheres as predicted by the current solutions and the numerical results of Kim et al. [2]. Note that these results are for a free stream, *i.e.* with the boundary conditions at the sides of the domain set as symmetric (zero normal gradient).

<table>
<thead>
<tr>
<th>Re</th>
<th>Present study</th>
<th>Kim et al. [1]</th>
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<tbody>
<tr>
<td>50</td>
<td>1.67</td>
<td>1.70</td>
</tr>
<tr>
<td>100</td>
<td>1.16</td>
<td>1.18</td>
</tr>
<tr>
<td>200</td>
<td>0.95</td>
<td>0.96</td>
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**RESULTS & DISCUSSION**

**Side by side arrangement:** When two spheres of equal diameter are placed side by side, they “see the flow” identically. In other words, they are placed symmetrically with respect to the flow. Therefore, assuming the fluid properties to vary negligibly with temperature, heat transfer coefficients of the two spheres will be equal, regardless of their surface temperatures, T_L and T_R. Nevertheless, each sphere is in thermal communication not only with the fluid, but also with the other sphere. Thermal interaction between the spheres is function of the driving temperature difference, distance between the spheres and the thickness of the thermal boundary layer developed around each sphere. The latter is itself function of flow conditions and fluid properties. If the spacing between spheres is small enough, roughly smaller than the combined thickness of the two thermal boundary layers, the two boundary layers interfere. In this situation, a change in the surface temperature of one sphere will not alter the heat transfer at the other sphere. The sphere-sphere Nusselt number, N_u_L_R, characterizes this interaction. In Figure 2, N_u_L_R is plotted versus Re. As Re increases, the boundary layer thickness decreases and thus, at a fixed spacing, the thermal interaction between the two spheres becomes weaker. Heat transfer between spheres and the fluid, on the other hand, is enhanced as Re increases. This is well known from the theory of external forced convection. This trend is captured by the paired sphere-fluid Nusselt number, N_u_L∞, also shown in Figure 2.

Figure 3 shows the distribution of the overall Nusselt number of a sphere in side-by-side arrangement as well the local distribution of the paired sphere-fluid Nusselt number at Re=100. At Re=100, sphere-sphere heat transfer is zero. Therefore, heat transfer at a sphere is chiefly between the sphere and the fluid. It is therefore expected that paired sphere-fluid and overall sphere Nusselt numbers, obtained from dQdT and the baseline CFD solution respectively, be equal (N_u_L∞=N_u_L). Maximum and minimum heat transfer coefficients are observed at θ=0 and θ=140°, stagnation and separation points respectively.

This three-temperature problem can be represented by a thermal resistor network such as the one shown in Figure 4. A set of three paired Nusselt numbers, N_u_L, characterizes this thermal network for any given combination of geometry, fluid, flow, and temperature boundary conditions. As mentioned previously, with constant fluid properties, the two sphere-fluid Nusselt numbers are equal and thus the...
thermal network of Figure 4 is characterized by two Nusselt numbers. It is noteworthy that prior to the development of the dQdT technique, there were no tools, numerical or experimental, available for determining the three Nusselt numbers (resistors) of this network.

**Tandem arrangement:** When arranged in tandem, the spheres will see the flow differently, *i.e.* they are placed asymmetrically with respect to the flow. Due to the blocking effect of the upstream sphere, $Q_u$ is smaller than $Q_d$. This difference increases with the Reynolds number. Moreover, due to their asymmetric arrangement, the spheres are expected to have asymmetric effects on each other. This asymmetry in thermal communication between two heat sources is referred to as *preferential advection*. To demonstrate this effect further, sphere-sphere Nusselt numbers are plotted against Re in Figure 5. In this figure, $N_{ud}$ corresponds to the heat transfer coefficient obtained by perturbing the surface temperature of the downstream sphere, $T_d$, and computing the resulting change in the heat transfer rate at the upstream sphere, $Q_u$, according to Equation 1. Hence, $N_{ud}$ is a measure of the influence of $T_d$ on $Q_u$. Similarly, $N_{du}$ characterizes the influence of $T_u$ on $Q_d$. At Re=0, the pure conduction limit, $N_{ud}=N_{du}$. However, for Re>0, the influence of $T_u$ on $Q_d$ is much larger than that of $T_d$ on $Q_u$ which is zero for Re<50. For 50<Re<200, via recirculation between the spheres, $T_d$ can affect $Q_u$ and hence the non-zero values of $N_{du}$. Maximum $N_{ud}$ also occurs in this interval, indicating the enhancement of the influence of the $T_u$ on $Q_d$ by the recirculating zone.

In Figure 6, sphere-fluid Nusselt numbers are plotted versus the Reynolds number. Corresponding to each sphere, two distinct Nusselt numbers are obtained: one by perturbing the sphere surface temperature and observing the resulting change in the total heat transfer (from both spheres) to the fluid, and the other by perturbing the temperature of the approaching free stream and computing the resulting change in the heat transfer rate at the sphere of interest. See Equation 1. Preferential advection is observed between spheres and the fluid too. Note that in Figure 6, the sphere-fluid curve of Figure 2 is reproduced for comparison. It is seen that with $N_{ud}$ substantially zero, $N_{∞u}$ is very close to $N_{∞L}$, which is another indication of the small thermal communication from the downstream sphere to the upstream sphere. Then difference between $N_{ud}$ and $N_{∞L}$, on the other hand, is considerable due to significant thermal communication from the upstream sphere to the downstream sphere, as reflected by $N_{ud}$ values of Figure 5.

The observed difference between $N_{u_i}$ and $N_{u_j}$ indicates that the tandem arrangement cannot be
represented by the thermal network of Figure 4. In fact, the paired heat transfer coefficients calculated through dQdT can be used to determine whether a multi-temperature system can be properly modeled as a thermal resistor network.

Local paired heat transfer coefficients on a plane parallel to the flow are shown in Figure 7. Note that the angular positions on the periphery of the two spheres are measured in opposite directions. As expected, Nud_u, characterizing the influence of T_u on Q_u, is maximum at θ_u=0, smoothly decaying to zero at θ_u≈60°. The local effect of T_d on heat transfer at the downstream sphere, on the other hand, is more complicated. Starting from Nud_d~1 at θ_d=0, Nud_d reaches its maximum at θ_d≈60°, the counterpart of [θ_u≈60°; Nud_u=0]. Note that θ_d=60° is also an inflection point, beyond which Nud_d decreases to its minimum at θ_d≈150°, the second inflection point, where flow separation occurs.

It is noteworthy that following the convention in heat transfer, a temperature ratio must be introduced to characterize the temperature arrangement of this three-temperature problem. The overall heat transfer coefficients, obtained from measurement or a numerical solution, will then be a function of this temperature ratio. Using dQdT, however, the obtained paired heat transfer coefficients are independent of the temperature ratio. In other words, the paired Nusselt numbers presented in this work can be used for any temperature arrangement. This is an important advantage of dQdT in characterizing multi-temperature forced convection.

Remarks on experimental dQdT: This study was conducted to assess the feasibility of implementing dQdT in a water tunnel study of flow over nearby spheres. To avoid complexities of unsteady flow and transition to turbulence, it is preferable to perform an experimental dQdT study in the steady laminar regime. Moreover, when performing measurements, the heat transfer response to a temperature perturbation must be at least an order of magnitude larger than the associated errors to provide meaningful data for calculating paired heat transfer
coefficients. Note that dQdT is in essence a sensitivity analysis. The presented results indicate little thermal communication between the spheres in the steady laminar regime. For $0 < \text{Re} < 250$, $\text{Pr}=7$ and $L/D=2$, sphere-sphere Nusselt numbers are at least one order of magnitude smaller than sphere-fluid Nusselt numbers. In side-by-side arrangement, $\text{Nu}_{LR}$ is essentially zero for $\text{Re}>1$. In tandem arrangement, spacing and Reynolds number have variable effects on sphere-sphere interaction depending on the flow structure, especially the size and strength of the recirculating zone between the spheres. Nevertheless, for $0 < \text{Re} < 250$, the obtained sphere-sphere Nusselt numbers are very small. Based on the presented results, successful implementation of experimental dQdT in intermediate Reynolds numbers does not seem feasible.

**CONCLUSION**

The three-temperature problem of convection from two spheres with the same size but different surface temperatures was studied using the dQdT technique. Intermediate Reynolds number flow of water over spheres in side-by-side and tandem arrangements was considered. It has been shown that due to the symmetry of the side-by-side arrangement with respect to the flow, Nusselt numbers corresponding to sphere-fluid heat transfer are the same at the two spheres. In tandem arrangement, however, the blocking effect of the upstream sphere leads to a large difference between heat transfer rates at the two spheres. Moreover, considerable difference was observed between the Nusselt numbers associated with any two of the isothermal sources. This difference is due to preferential advection, caused by the asymmetric position of the spheres with respect to the flow. Sphere-sphere Nusselt numbers are significantly smaller than sphere-fluid Nusselt numbers. This difference can impose a practical barrier to the experimental implementation of dQdT in intermediate Reynolds numbers.

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