

Calculating Center-Glass Performance Indices of Windows

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ABSTRACT

Building envelope performance is strongly influenced by solar gain and heat transfer through windows. The majority of this energy gain or loss passes through the center-glass area of the glazing system. Various methods have been devised to calculate the corresponding center-glass performance indices. Solar heat gain coefficient (SHGC) and U-factor are the quantities most frequently sought. Hand calculations have given way to computer-based techniques. Computer simulation offers the opportunity to employ more detailed models plus the ability to model the large number of glazing systems made possible by design options, such as low-emissivity or solar-control coatings, selective glass tints, substitute fill gases, and glazing layers, that partially transmit longwave radiation. A new, more accurate method is presented in this paper for manipulating spectral optical data while calculating the energy related optical properties of glazing layers and glazing systems. The use of the same technique to track visible and ultraviolet radiation is also demonstrated. In addition, more refined methods are documented for calculating SHGC and U-factor while accounting for the thermal resistance of individual glazings.

INTRODUCTION

Building envelope performance is strongly influenced by solar gain and heat transfer through windows. A one-dimensional simulation can accurately quantify the mechanisms of energy transfer in the center-glass area. The effect of the thermal short circuit caused by edge-seal and frame components must be dealt with using a two- or three-dimensional model. Considerable attention has been devoted to the implementation of edge-glass and frame calculations because of the higher level of complexity involved, but it should be remembered

that the center-glass is generally the single component area where the majority of energy gain or loss takes place.

The most important energy-related indices are solar heat gain coefficient (SHGC) and U-factor. Other indices of importance include the solar, visible, and ultraviolet (UV) transmittance values.

Various methods have been devised to calculate center-glass performance indices. Computer simulation offers the ability to conveniently solve the nonlinear problem involving coupled modes of convective and radiative heat transfer. This, in turn, allows for complete examination of the many possible combinations of low-emissivity or solar-control coatings, glass tints, pane spacings, tilts, and substitute fill gases. Computer simulation also provides the capability to properly quantify the impact of glazing layers that are selective in the solar wavelength band. Many of the newer coatings and tints are strongly selective, and spectral data are now available for almost all commercially available glazing layers.

Two center-glass analysis programs, VISION and WINDOW, are used widely in North America. VISION is specified for energy rating in Canada (CSA 1993), and the WINDOW program is used in the U.S. (NFRC 1991). Early versions of these programs were based on work by Wright (Wright 1980; Hollands and Wright 1980, 1983) and Rubin (1982), respectively. These heat transfer models surpassed previous methods because they introduced the capability of analyzing glazing layers that partially transmit longwave radiation (i.e., diathermanous glazing layers). This is an important feature and its value as a design tool has been demonstrated (Wright 1980, 1985; Wright and Sullivan 1987).

The capabilities of the programs mentioned above have been extended in more recent versions of the software (Wright and Sullivan 1995a; Arasteh et al. 1989; Finlayson et al. 1993). New features include an accounting for the thermal resistance

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of glazing layers and a spectral analysis of the glazing system in the solar wavelengths, allowing for the calculation of visible and UV transmittance plus a more accurate calculation of solar transmittance and SHGC. However, the model extensions incorporated in the two programs are not identical. A wealth of useful information about newer models can be found in consensus standards currently being developed (ASHRAE 1996; ISO 1996), but little detail has been recorded in the technical literature.

This paper provides a record of the most up-to-date models used for center-glass analysis. Information is provided on the way in which these models, or others, are used in the current software, and a discussion regarding the comparison of these models is included. In the particular case of spectral calculations, a new approach is developed offering increased accuracy.

APPROACH

Glazing system energy analysis takes advantage of the fact that there is no appreciable overlap between the band of solar wavelengths below ~ 3 μm and the band of longer wavelengths by which radiant heat transfer takes place. This naturally leads to a two-step process. First, an optical analysis determines how much of the incident solar radiation is absorbed at each of the glazing layers and how much is transmitted to the indoor space. Second, a heat transfer analysis is used to impose an energy balance on each glazing layer. The net heat transfer from any glazing layer must equal the amount of absorbed solar radiation.

The solar optical calculation can be completed without any information regarding glazing temperatures or heat transfer. The only information from the solar optical step needed in the heat transfer step is the amount of solar radiation absorbed at each glazing layer. These two steps yield the temperature profile and the rate of heat transfer at each glazing layer or interpane gap, which can then be used to determine the center-glass U-factor and SHGC.

Within the two-band framework, it is possible to use a multi-band (i.e., spectral) solar optical model whereby reflected, absorbed, and transmitted amounts of solar radiation are summed by wavelength. Similarly, a spectral calculation can be used to determine other optical properties such as the visible transmittance and ultraviolet transmittance of the glazing system.

SPECTRAL CALCULATIONS

Spectral Data

The glazing system is treated as an n -node array consisting of $n-2$ glazing layers together with the indoor ($i=1$) and outdoor ($i=n$) nodes. This arrangement is shown in Figure 1. It is assumed that each glazing layer is flat and specular.

Three solar optical properties are needed to describe the i^{th} glazing layer, and each should be specified as a function of the wavelength of the incident radiation, λ : (1) the front

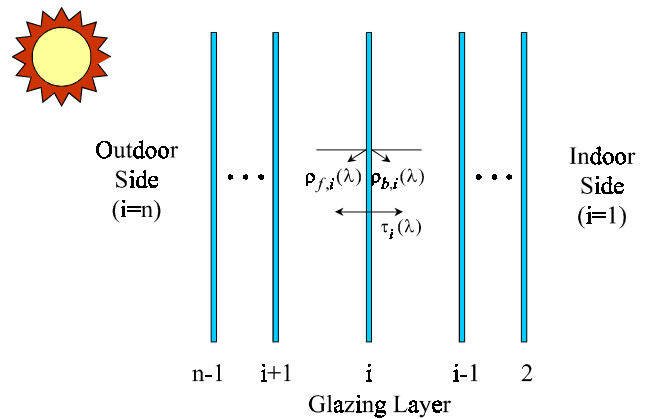


Figure 1 Numbering arrangement and solar optical properties of glazing layers.

(outdoor side) reflectance, $\rho_{f,i}(\lambda)$, (2) the back (indoor side) reflectance, $\rho_{b,i}(\lambda)$, and (3) the transmittance, $\tau_i(\lambda)$.

It is customary to use solar optical properties that pertain to normal incidence, but properties for any given angle of incidence can be used in this analysis to obtain the corresponding set of results. Increased accuracy can be obtained by accounting for the effect of polarization. It is convenient to assume that the incident solar radiation consists of two components (parallel and perpendicular) of equal strength. Measured optical properties pertaining to each of these components are not readily available but can be found on the basis of theory for uncoated glazing layers (e.g., Siegel and Howell 1972) and averaged. More approximate methods can be used for coated glazing layers (Ferguson and Wright 1983; Finlayson et al. 1993). The models presented in this paper could be extended to account individually for the components of polarization, but, in order to concentrate on the spectral calculations, that issue is not pursued.

Note that the spectral optical properties are measured and reported at discrete values of λ (λ_j). All three properties for a given glazing layer are reported at the same values of λ , but the set of λ_j used to describe one glazing layer may not exactly match the sets used for all of the other glazing layers. This is particularly true knowing that spectral data are available from a variety of sources such as measured research data, manufacturers data, and more comprehensive data libraries (e.g., NFRC 1997). Similarly, the solar spectral irradiance, $E(\lambda)$, the photopic response of the eye, $R(\lambda)$, and other functions of interest are tabulated for specific values of λ where, again, each set of λ_j is unique in order to capture the desired level of detail. For example, ASTM Standard E891-87 (air mass 1.5) lists $E(\lambda_j)$ at 121 values of λ_j from 0.305 μm to 4.045 μm , with 34 of those data falling in the visible band for which ASTM Standard E308-90 lists $R(\lambda_j)$ at 81 values of λ_j .

It is assumed that all of the solar spectral curves ($\rho_{f,i}[\lambda]$, $\rho_{b,i}[\lambda]$, $\tau_i[\lambda]$, $E[\lambda]$, etc.) are constructed such that intermediate values can be reliably evaluated by linear interpolation.

Calculating Total Optical Properties of a Glazing Layer

Equation 1 shows the way in which any total solar optical property, i.e., P_s , can be determined by averaging the corresponding spectral data, $P(\lambda)$, over the solar wavelength band using $E(\lambda)$ as a weighting function.

$$P_s = \frac{\int_{\lambda=0}^{\infty} P(\lambda)E(\lambda)d\lambda}{\int_{\lambda=0}^{\infty} E(\lambda)d\lambda} \quad (1)$$

P_s is called the total (or grey) optical property and is useful because a glazing layer having this property, independent of wavelength, will transmit, reflect, or absorb the same amount of incident solar energy. This can be seen in Equation 1 by recognizing that the integral in the denominator of the right hand side is the total incident flux of radiant energy, I_s .

Equation 1 can be evaluated numerically by replacing the integrals with summations applied over $m-1$ wavelength bands.

$$P_s = \frac{\sum_{j=1}^{m-1} P(\lambda_{j/j+1})E(\lambda_{j/j+1})\Delta\lambda_j}{\sum_{j=1}^{m-1} E(\lambda_{j/j+1})\Delta\lambda_j} \quad \Delta\lambda_j = \lambda_{j+1} - \lambda_j \quad (2)$$

where $P(\lambda_{j/j+1})$ and $E(\lambda_{j/j+1})$ are values of P and E , respectively, and are representative of the wavelength band from λ_j to λ_{j+1} .

Several schemes have been formulated for evaluating Equation 2. Each uses the set of λ_j for which $E(\lambda_j)$ data are tabulated. VISION4 subdivides each band from λ_j and λ_{j+1} into four equal subpanels and evaluates representative values of $E(\lambda)$ and $P(\lambda)$ individually by averaging the corresponding values at the endpoints of each subpanel. WINDOW 4.0 evaluates each of $E(\lambda)$ and the $P(\lambda)E(\lambda)$ product at λ_j and λ_{j+1} and uses the average. The ASHRAE draft standard SPC 142 (ASHRAE 1996) specifies that $E(\lambda_{j/j+1})$ and $P(\lambda_{j/j+1})$ be evaluated by calculating each of the corresponding quantities at λ_j and λ_{j+1} , averaging them and then multiplying the two averages to obtain a representative $P(\lambda_{j/j+1})E(\lambda_{j/j+1})$ product.

Each of these schemes entails some approximation. By using the set of λ_j at which the solar irradiance values, $E(\lambda_j)$, are tabulated, $E_i(\lambda_{j/j+1})$ is correctly determined. However, by averaging endpoint values, uncertainty arises in evaluating $P(\lambda_{j/j+1})$ because $P(\lambda)$ may contain discontinuities between λ_j and λ_{j+1} . Figure 2a shows a hypothetical situation that illustrates this point. Furthermore, the $P(\lambda_{j/j+1})E_i(\lambda_{j/j+1})$ product will not be linear between λ_j and λ_{j+1} and may also include discontinuities. Uncertainty in estimating values of $P(\lambda_{j/j+1})$ and the $P(\lambda_{j/j+1})E_i(\lambda_{j/j+1})$ product will be small in most cases, but the size of this error is a function of the particular glazing layers being used and cannot be predicted. It depends on the

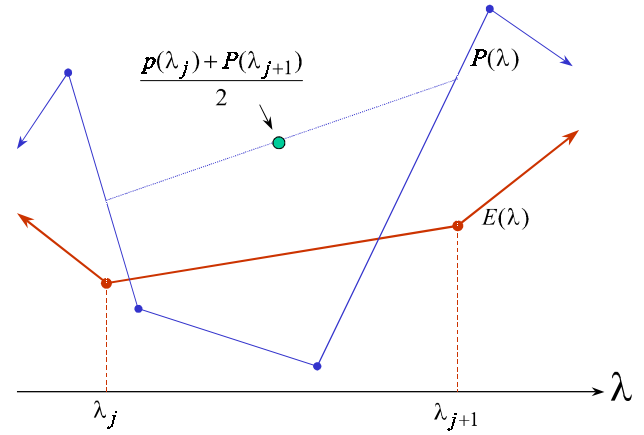


Figure 2a Discontinuity of optical property, $P(\lambda)$, within solar radiation band.

shapes of the various spectral data curves and the way in which the wavelengths chosen to characterize those curves interact with each other.

Fortunately, it is possible to formulate a computer algorithm that can be used to evaluate Equation 2 without the uncertainties described above. The critical component of this more accurate approach is the choice of wavelengths used to execute the summation. Instead of using a set of λ_j corresponding to $E(\lambda)$, a larger set of λ_j is assembled that includes all of the wavelengths used in tabulating $E(\lambda)$ and the optical properties of the glazing layer. The larger set of λ_j can readily be assembled using a simple sorting routine. This system of specifying a more detailed set of spectral bands by using the complete set of λ_j will be called the “complete wavelength set” (CWS) method. The CWS wavelengths corresponding to Figure 2a are shown in Figure 2b. The benefit of using the CWS approach is that not only $E(\lambda)$, but all of the optical properties can be taken as linear in order to evaluate each term in the summation. Therefore, within each CWS band:

$$E(\lambda) = a_E + b_E\lambda \quad (3)$$

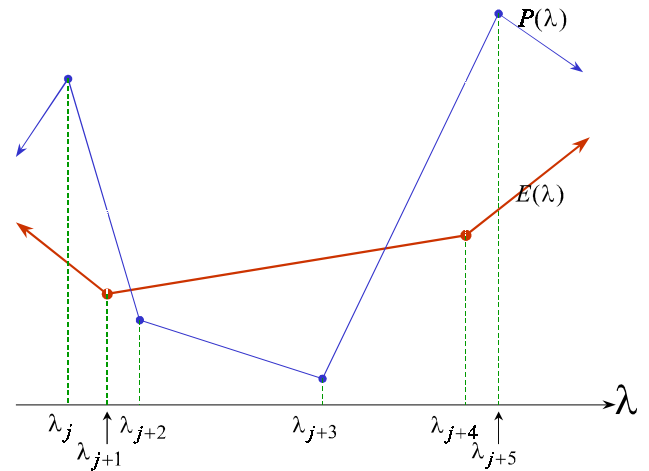


Figure 2b CWS wavelengths based on $E(\lambda)$ and $P(\lambda)$.

and
$$P(\lambda) = a_p + b_p \lambda \quad (4)$$

where

$$a_p = P(\lambda_j) - b_p \lambda_j \quad b_p = \frac{P(\lambda_{j+1}) - P(\lambda_j)}{\lambda_{j+1} - \lambda_j}$$

$$a_E = E(\lambda_j) - b_E \lambda_j \quad b_E = \frac{E(\lambda_{j+1}) - E(\lambda_j)}{\lambda_{j+1} - \lambda_j}$$

Now the integrals in Equation 1 can be evaluated exactly across each sub-band, and the results from each integration can be summed.

$$P_s = \frac{\sum_{j=1}^{m-1} \left\{ \int_{\lambda_j}^{\lambda_{j+1}} (a_p + b_p \lambda)(a_E + b_E \lambda) d\lambda \right\}}{\sum_{j=1}^{m-1} \left\{ \int_{\lambda_j}^{\lambda_{j+1}} (a_E + b_E \lambda) d\lambda \right\}} \quad (5)$$

Note that m now corresponds to the larger set of CWS wavelengths. Evaluating the integrals analytically:

$$P_s = \frac{\sum_{j=1}^{m-1} \left\{ a_p a_E (\lambda_{j+1} - \lambda_j) + \frac{1}{2} (a_p b_E + b_p a_E) (\lambda_{j+1}^2 - \lambda_j^2) + \frac{1}{3} b_p b_E (\lambda_{j+1}^3 - \lambda_j^3) \right\}}{\sum_{j=1}^{m-1} \left\{ a_E (\lambda_{j+1} - \lambda_j) + \frac{1}{2} b_E (\lambda_{j+1}^2 - \lambda_j^2) \right\}} \quad (6)$$

Multi-Layer Analysis by the Net Radiation Method

The rate at which radiation of a given wavelength is absorbed in the conditioned space and at each glazing layer can be found by considering the fluxes of radiant energy flowing between the i^{th} and $i+1^{\text{th}}$ glazings, $I_i^+(\lambda)$ and $I_i^-(\lambda)$. The + and - superscripts denote radiation flowing toward the outdoor and indoor sides, respectively. With reference to Figure 3, it can be seen that the following relations apply.

$$I_i^+(\lambda) = \tau_i(\lambda) I_{i-1}^+(\lambda) + \rho_{f,i}(\lambda) I_i^-(\lambda) \quad i = 1 \text{ to } n-1 \quad (7)$$

$$I_i^-(\lambda) = \tau_{i+1}(\lambda) I_{i+1}^-(\lambda) + \rho_{b,i+1}(\lambda) I_i^+(\lambda) \quad i = 2 \text{ to } n-2 \quad (8)$$

This formulation can be found in a variety of references (e.g., Edwards 1977; van Dijk 1996; Siegel 1973; Shurcliff 1974; Wijesundera 1975).

With the transmittance of the indoor surfaces set to zero ($\tau_1[\lambda] = 0$), this system of equations can be solved. If the window is considered to be a small surface in a large enclosure, the reflectance of the indoor surfaces can be (and usually is) set to zero ($\rho_{f,1}[\lambda] = 0$). Knowing all values of $I_i^+(\lambda)$ and $I(\lambda)$, the portion of incident radiation of wavelength λ , absorbed at the i^{th} glazing layer, $A_i(\lambda)$, is simply

$$A_i(\lambda) = \frac{I_i^-(\lambda) - I_i^+(\lambda) + I_{i-1}^+(\lambda) - I_{i-1}^-(\lambda)}{I_{n-1}^-(\lambda)} \quad (9)$$

The portion transmitted to the indoor space is

$$\tau_{gs}(\lambda) = \frac{I_1^-(\lambda)}{I_{n-1}^-(\lambda)} \quad (10)$$

and the portion reflected from the glazing system is

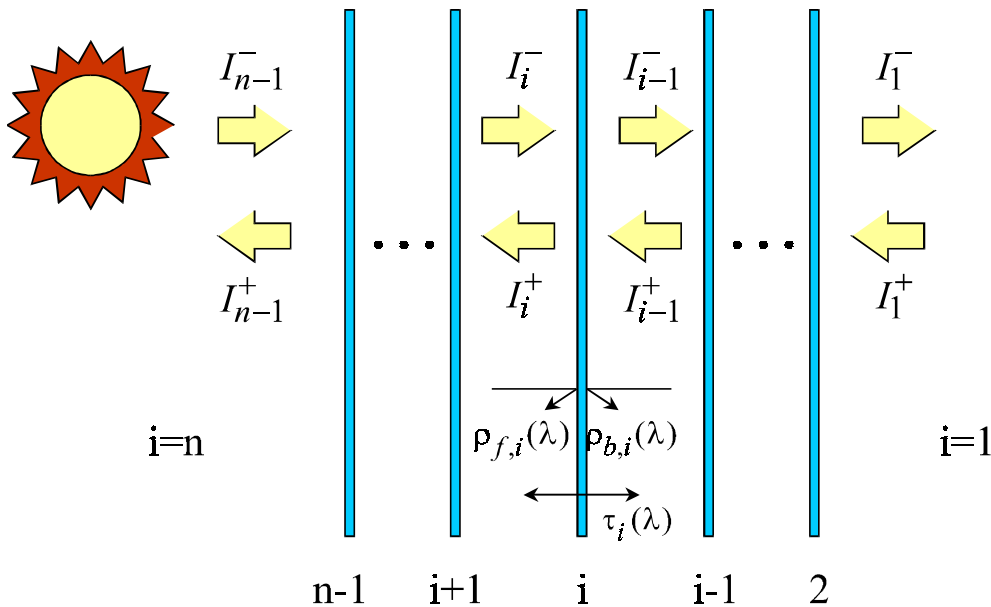


Figure 3 Solar flux within the glazing system.

$$\rho_{gs}(\lambda) = \frac{I_{n-1}^+(\lambda)}{I_{n-1}^-(\lambda)} \quad (11)$$

Note that $I_{n-1}^-(\lambda)$, the incident flux, can be set equal to any nonzero value for the purpose of solving Equations 7 through 11.

Solution of the Multi-Layer Model

A variety of methods are available for solving Equations 7 and 8. The most direct approach is matrix reduction, but recursive (e.g., Edwards 1977; Finlayson et al. 1993) solvers can also be used. All solvers must, by necessity, produce the same result. One method is presented in this paper because of its simplicity, ease of implementation, and because it can be easily developed and understood.

“Ray tracing” is used to develop two recursion relations¹ on the basis of the arrangement shown in Figure 4. Consider the effect of adding one more glazing layer, the i^{th} glazing layer, to the outdoor side of an existing glazing system. Assuming that the reflectance of the original glazing system (including multiple internal reflections), r_{i-1} , is known, we seek to determine the reflectance of the glazing system with the i^{th} glazing layer in place (i.e., r_i). Noting that the incident flux on the i^{th} glazing layer is $I(\lambda)$, the reflected flux can be found by summing the reflected rays. Omitting the spectral notation for convenience, the resulting relation is

$$r_i = \frac{I_i^+}{I_i^-} = \rho_{f,i} + \tau_i^2 r_{i-1} + \tau_i^2 r_{i-1}^2 \rho_{b,i} + \tau_i^2 r_{i-1}^3 \rho_{b,i} + \tau_i^2 r_{i-1}^4 \rho_{b,i} + \dots \quad (12)$$

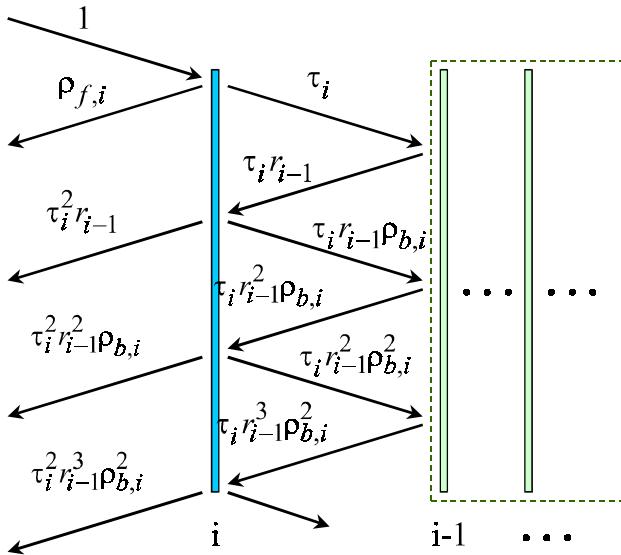


Figure 4 Ray tracing to develop solar flux analysis.

¹ The development of the same recursion relations using the net radiation method can be found in Edwards (1977).

$$\text{or } r_i = \rho_{f,i} + \tau_i^2 r_{i-1} \sum_{j=0}^{\infty} (r_{i-1} \rho_{b,i})^j \quad (13)$$

This can be recognized as a geometric series.

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x} \quad \text{for } |x| < 1 \quad \text{geometric series} \quad (14)$$

$$\text{so } r_i = \rho_{f,i} + \frac{\tau_i^2 r_{i-1}}{1 - \rho_{b,i} r_{i-1}} \quad (15)$$

Notice that without any further development, this recursion can be used to calculate all values of r_i by working from $r_1 = \rho_{f,1}$ to r_{n-1} .

Also consider $I_i^-(\lambda)$ and $I_{i-1}^-(\lambda)$, the fluxes of radiation on either side of the i^{th} glazing layer, flowing in the indoor direction. Again, ray tracing can be used to find the ratio between these two quantities, t_i .

$$t_i = \frac{I_{i-1}^-}{I_i^-} = \tau_i + \tau_i(\rho_{b,i} r_{i-1}) + \tau_i(\rho_{b,i} r_{i-1})^2 + \tau_i(\rho_{b,i} r_{i-1})^3 + \dots \quad (16)$$

$$\text{or } t_i = \frac{\tau_i}{1 - \rho_{b,i} r_{i-1}} \quad (17)$$

Now Equation 17 can be used to obtain t_i from $i=2$ to $i=n-1$. The rest of the solution follows by first calculating the flux reflected from the glazing system to the outdoor side $I_{n-1}^- = r_{n-1} I_{n-1}^+$ and then marching toward the indoor side from $i=n-2$ to $i=1$ calculating the remaining fluxes ($I_i^- = t_{i+1} I_{i+1}^-$ and $I_i^+ = r_i I_i^-$).

CWS Calculation of Solar and Visible Transmittance

The CWS method can be used to calculate the solar transmittance of the glazing system, τ_s . In this case, the CWS wavelength set consists of all the λ_j used to tabulate $E(\lambda)$ and the optical properties of all glazing layers. Assuming that $\tau_{gs}(\lambda)$ is linear over each CWS band, τ_s can be calculated with Equation 6 by setting $P(\lambda)$ equal to $\tau_{gs}(\lambda)$.

The visible transmittance of the glazing system, τ_v , can be calculated using a weighting function that represents the photopic response of the eye, $R(\lambda)$. ASTM Standard E308-90 (ASTM 1990) lists a suitable set of $R(\lambda)$, tabulated as $\bar{y}(\lambda)$, at 81 values of λ from 0.38 μm to 0.78 μm for the CIE 1931 standard observer. The visible transmittance is

$$\tau_v = \frac{\int_{\lambda=0}^{\infty} \tau_{gs}(\lambda) R(\lambda) E(\lambda) d\lambda}{\int_{\lambda=0}^{\infty} R(\lambda) E(\lambda) d\lambda} \quad (18)$$

Using the CWS approach, it is possible to eliminate errors associated with evaluating representative values of $R(\lambda)$ (i.e., errors caused by any mismatch between the wavelengths at which $R(\lambda)$, $E(\lambda)$, and the optical properties of glazing layers are reported). Replacing $R(\lambda)$ with

$$R(\lambda) = a_R + b_R \lambda \quad (19)$$

and assuming $\tau_{gs}(\lambda) = a_\tau + b_\tau \lambda$ (20)

Equation 18 becomes

$$\tau_v = \frac{\sum_{j=1}^{m-1} \left\{ \begin{aligned} & a_E a_\tau a_R (\lambda_{j+1} - \lambda_j)^+ \\ & \frac{1}{2} (a_E a_\tau b_R + a_E b_\tau a_R + b_E a_\tau a_R) (\lambda_{j+1}^2 - \lambda_j^2)^+ \\ & \frac{1}{3} (a_E b_\tau b_R + b_E a_\tau b_R + b_E b_\tau a_R) (\lambda_{j+1}^3 - \lambda_j^3)^+ + \frac{1}{4} b_E b_\tau b_R (\lambda_{j+1}^4 - \lambda_j^4)^+ \end{aligned} \right\}}{\sum_{j=1}^{m-1} \left\{ \begin{aligned} & a_R a_E (\lambda_{j+1} - \lambda_j)^+ \\ & \frac{1}{2} (a_R b_E + b_R a_E) (\lambda_{j+1}^2 - \lambda_j^2)^+ \\ & \frac{1}{3} b_R b_E (\lambda_{j+1}^3 - \lambda_j^3)^+ \end{aligned} \right\}} \quad (21)$$

In this case, m corresponds to a set of λ_j consisting of all the λ_j used to tabulate $R(\lambda)$, $E(\lambda)$, and the optical properties of all glazing layers. Note that the magnitude of m can be reduced substantially by accepting only the wavelengths where $R(\lambda)$ is nonzero (i.e., $0.38 \mu\text{m} \leq \lambda \leq 0.78 \mu\text{m}$) and by stripping out duplicate values of λ_j .

CWS Calculation of Ultraviolet Transmittance

The UV transmittance of the glazing system, τ_{uv} , can be calculated according to Equation 18 simply by replacing $R(\lambda)$ with a weighting function, $\Gamma(\lambda)$, that restricts the calculation to the UV wavelengths. For example, the UV band used in WINDOW 4.0 (Finlayson et al. 1993) is specified by

$$\begin{aligned} \Gamma(\lambda) &= 1 & 0.3\mu\text{m} \leq \lambda \leq 0.39\mu\text{m} \\ \Gamma(\lambda) &= 0 & \text{otherwise} \end{aligned} \quad (22)$$

Thus, the calculation of τ_{uv} can be carried out using Equation 21 with $R(\lambda)$ replaced by $\Gamma(\lambda)$. This is equivalent to applying Equation 6 (with $P[\lambda] = \tau_{gs}[\lambda]$) to the range of λ , where $\Gamma(\lambda)$ is nonzero. The set of m wavelengths used to evaluate τ_{uv} will be relatively small because values of λ_j falling outside the range where $\Gamma(\lambda)$ is nonzero can be excluded.

It is common to choose $\Gamma(\lambda)$ such that greater weight is given to the more damaging radiation at shorter wavelengths. The Krochman “UV relative damage factor” can be used for this calculation (Krochman 1978).

$$\begin{aligned} \Gamma(\lambda) &= e^{12.28 - 25.56\lambda} = 2153.46e^{-25.56\lambda} & 0.3\mu\text{m} \leq \lambda \leq 0.5\mu\text{m} \\ \Gamma(\lambda) &= 0 & \text{otherwise} \end{aligned} \quad (23)$$

If $\Gamma(\lambda)$ is of the form $\Gamma(\lambda) = \alpha e^{-\beta\lambda}$ and $\tau_{sg}(\lambda)$ is assumed to be linear in each CWS band, the damage weighted UV transmittance can be calculated as

$$\tau_{duv} = \frac{\sum_{j=1}^{m-1} \left\{ \frac{a_\tau a_E}{\beta} \left(e^{-\beta\lambda_{j+1}} - e^{-\beta\lambda_j} \right)^+ \right\}}{\sum_{j=1}^{m-1} \left\{ \frac{a_\tau a_E}{\beta} \left(e^{-\beta\lambda_{j+1}} - e^{-\beta\lambda_j} \right)^+ \right\}} \frac{a_\tau b_E + a_E b_\tau}{(\beta)^2} \left(e^{-\beta\lambda_{j+1}} (\beta\lambda_{j+1} + 1) - e^{-\beta\lambda_j} (\beta\lambda_j + 1) \right)^+ \quad (24)$$

Note that α does not appear in the final expression because it can be cancelled from numerator and denominator. Only β remains, which, for the Krochman damage function, is $\beta = 25.56 \mu\text{m}^{-1}$.

CWS Calculation of Absorbed Solar Flux

The absorbed solar flux at the i^{th} glazing layer appears in the heat transfer analysis and is denoted S_i . The amount of solar radiation of wavelength λ within an infinitesimal wavelength band, $d\lambda$, and absorbed at the i^{th} glazing layer, dS_i , is

$$dS_i = A_i(\lambda)E(\lambda)d\lambda \quad (25)$$

Summing over all wavelengths,

$$S_i = \int_{\lambda=0}^{\infty} A_i(\lambda)E(\lambda)d\lambda \quad (26)$$

Equation 26 must be evaluated numerically.

$$S_i = \sum_{j=1}^{m-1} A_i(\lambda_{j/j+1})E(\lambda_{j/j+1})\Delta\lambda_j \quad \Delta\lambda_j = \lambda_{j+1} - \lambda_j \quad (27)$$

where $A_i(\lambda_{j/j+1})$ and $E_i(\lambda_{j/j+1})$ are values of A_i and E_i , respectively, that are representative of the wavelength band from λ_j to λ_{j+1} .

Several schemes have been formulated for evaluating Equation 27, and each uses the set of λ_j for which $E(\lambda_j)$ data are tabulated. Each of these schemes entails some amount of error for the same reasons previously discussed.

Using CWS yields a result similar to the numerator of Equation 6. Specifically, assuming $A_i(\lambda)$ is linear in each CWS band,

$$A_i(\lambda) = a_A + b_A \lambda \quad (28)$$

then

$$S_i = \sum_{j=1}^{m-1} \left\{ a_A a_E (\lambda_{j+1} - \lambda_j) + \frac{1}{2} (a_A b_E + b_A a_E) (\lambda_{j+1}^2 - \lambda_j^2) + \frac{1}{3} b_A b_E (\lambda_{j+1}^3 - \lambda_j^3) \right\} \quad (29)$$

The portion of the total incident solar flux, I_s , absorbed at the i^{th} glazing layer is simply $A_i = S_i/I_s$.

showing that the approximate nature of Equation 41 disappears as convergence is reached.

ENERGY RELATED INDICES OF MERIT

Center-Glass U-Factor

The center-glass U-factor, U_c , can be determined for any environmental condition involving indoor/outdoor temperature difference and any level of incident solar radiation. It can be expressed as the reciprocal of the total center-glass thermal resistance, R_{tot} ,

$$U_c = \frac{1}{R_{tot}} \quad (43)$$

where R_{tot} is found by summing the individual thermal resistance values of the glazing cavities and glazing layers from the indoor side to the outdoor side.

$$R_{tot} = \sum_{i=1}^{n-1} R_{gap,i} + \sum_{i=2}^{n-1} R_{gl,i} \quad (44)$$

The thermal resistance of the i^{th} glazing cavity is

$$R_{gap,i} = \frac{T_{f,i} - T_{b,i+1}}{q_i} \quad (45)$$

and the thermal resistance of the i^{th} glazing layer is

$$R_{gl,i} = \frac{t_{gl,i}}{k_{gl,i}} \quad (46)$$

The exception to Equation 45 is the resistance on the outdoor side of the glazing system, $R_{gap,n-1}$, where, for conditions other than fully cloudy conditions, the extra cooling caused by radiant exchange with the clear portions of the sky must be considered. In this case, the effective ambient temperature, T_{ae} , must be used instead of the outdoor air temperature, $T_{b,n}$ (ASHRAE 1996; Wright and Sullivan 1992, 1995a).

Center-Glass Solar Heat Gain Coefficient

The center-glass solar heat gain coefficient, SHGC, can be found by summing the directly transmitted solar gain and the absorbed/redirected solar gain (Wright and Sullivan 1995b).

$$SHGC = \tau_s + \frac{1}{I_s} \sum_{i=2}^{n-1} N_i S_i \quad (47)$$

where I_s is the incident solar flux and N_i is the portion of S_i that is redirected by heat transfer to the indoor space (i.e., the inward flowing fraction).

Equation 47 suggests that SHGC cannot be calculated for a condition with no solar radiation ($I_s = 0$), but this is not true. Recognizing that $S_i = A_i I_s$, Equation 47 becomes

$$SHGC = \tau_s + \sum_{i=2}^{n-1} N_i A_i \quad (48)$$

Also, recognizing that $N_1 = 1$ and $A_1 = \tau_s$, Equation 48 can be written as

$$SHGC = \sum_{i=1}^{n-1} N_i A_i \quad (49)$$

The inward flowing fraction, N_i , is known to be equal to the ratio of the thermal resistance from the location of the absorbed solar radiation to the outdoor side and the total indoor/outdoor thermal resistance. For example, if most of the thermal resistance comprising R_{tot} exists between the i^{th} glazing and the outdoor side, then most of S_i will make its way to the indoor space.

If the thermal resistance of the glazing layers is neglected, N_i can be calculated by considering only the glazing cavity resistances:

$$N_i = \frac{\sum_{j=i}^{n-1} R_{gap,j}}{R_{tot}} \quad (50)$$

However, if the thermal resistance of the glazing layers is included, the calculation of N_i is more involved. Consider an infinitesimal layer within the i^{th} glazing layer of thickness, dx , and at distance, x , from the indoor side of the glazing layer. This arrangement is shown in Figure 8. If the thermal resistance between the outdoor environment and the outdoor side of the glazing layer is R_{out} and the flux of solar radiation absorbed within dx is dS_i , the inward flowing flux, dq_{in} , will be

$$dq_{in} = \frac{R_{out} + \frac{t_{gl,i} - x}{k_{gl,i}}}{R_{tot}} dS_i \quad (51)$$

Substituting

$$dS_i = \frac{S_i}{t_{gl,i}} dx \quad (52)$$

and summing through the thickness of the glazing layer,

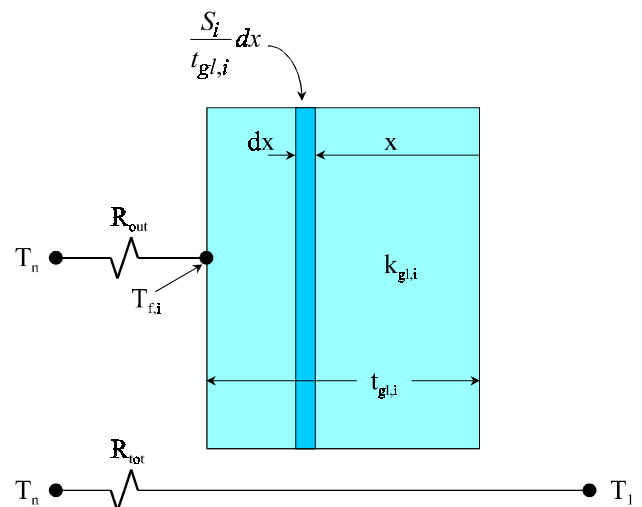


Figure 8 Calculating the inward flowing fraction of absorbed solar radiation from a glazing layer with thermal resistance.

$$q_{in} = \frac{S_i}{R_{tot} t_{gl,i}} \int_{x=0}^{t_{gl,i}} R_{out} + \frac{t_{gl,i}^{-x}}{k_{gl,i}} dx \quad (53)$$

Solving

$$\frac{q_{in}}{S_i} = N_i = \frac{R_{out} + \frac{1}{2} \frac{t_{gl,i}}{k_{gl,i}}}{R_{tot}} \quad (54)$$

Thus, given the assumption that solar radiation is absorbed uniformly through the glazing layer, N_i can be calculated as if the solar radiation were absorbed at the midplane of the glazing layer. Therefore, no discrepancy between VISION4 (based on uniform distribution) and WINDOW 4.0 (based on solar absorption at the midplane) exists because of the way in which N_i is calculated.

Finally, the full expression for N_i can be written as

$$N_i = \frac{\sum_{j=i}^{n-1} R_{gap,j} + \sum_{j=i+1}^{n-1} \frac{t_{gl,j}}{k_{gl,j}} + \frac{1}{2} \frac{t_{gl,i}}{k_{gl,i}}}{R_{tot}} \quad (55)$$

CONCLUSIONS

An up-to-date model has been presented for calculating the center-glass performance indices of windows. This procedure includes several important components.

1. A new solar spectral calculation, the complete wavelength set (CWS) method, has been formulated. The CWS approach is free of almost all of the uncertainty inherent in previous methods and is completely free of uncertainty caused by discontinuities and wavelength mismatches in data describing optical properties and spectral weighting functions.
2. A comprehensive heat transfer model has been specified in full detail. Step-by-step instructions have been provided for using linear matrix reduction methods to solve the nonlinear governing equations.
3. Details regarding the calculation of all indices of merit, optical and energy related, are presented and discussed. Singularities in calculating the U-factor when adjacent glazing layers are at the same temperature, or in calculating SHGC when no solar radiation exists, have been avoided.
4. All energy related calculations account for the thermal resistance of individual glazings. It has also been shown that the assumption of uniform vs. center-plane absorption of solar radiation within a glazing layer has no bearing on the calculation of solar gain.

Sufficient information has been provided so that the solution method is readily programmable by the potential user.

ACKNOWLEDGMENTS

This research was supported by NRCan (Natural Resources Canada) and the Department of Mechanical Engineering, University of Waterloo.

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