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Calculating Center-Glass Performance Indices of Glazing Systems with Shading Devices

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ABSTRACT

Building energy consumption and loads are strongly influenced by solar gain and heat transfer through the centre-glass area of windows. Methods have been devised to calculate the corresponding energy performance indices (e.g., SHGC and U-factor). Simulation offers the opportunity to examine design options such as low-emissivity or solar-control coatings, glass tints, substitute fill gases and diathermanous glazing layers. Current models use a radiosity-based approach to quantify longwave radiant exchange. A new method is presented for the thermal analysis of multilayer systems. This method, by using a resistor network to quantify both convective and radiant exchange, offers exceptional generality. "Jump" resistors allow for airflow between layers, or diathermanous layers, or any combination of the two. The air and mean radiant temperatures can differ on both the indoor and outdoor sides. In addition a more general method has been devised for calculating indices of merit without restricting the generality of the simulation model – for any set of environmental temperature and insolation conditions. These methods are especially useful for the analysis of glazing systems used in combination with shading layers such as venetian blinds, curtains, roller blinds and insect screens. These methods also offer new possibilities, speed and convenience when used in conjunction with whole building performance simulations.

INTRODUCTION

The design of low-energy buildings, even net-zero energy buildings, has become a topic of heightened activity. The driving force behind this interest in energy-efficiency and sustainability has shifted from simple economic payback to a growing list of concerns that most directly impact future generations. These concerns include the consumption of non-renewable resources, pollution in general, smog, ozone depletion, greenhouse gas production and climate change. The possibility of downsizing equipment while providing increased comfort is an added benefit of good thermal design.

Conservation is the first step in a shift to sustainability and the use of alternate sources of energy. In building envelope design this is most readily seen in substantially increased levels of insulation. The walls become thermally benign; they allow little heat transfer and very little solar gain. In this context the window design, already important in conventional building design, becomes even more important. The use of components such as low-emissivity (low-e) coatings, substitute fill gas and additional glazing layers can substantially increase the thermal resistance of a window. However, designers are recognizing the fact that even moderate levels of solar gain or internal heat gain will cause serious overheating of a well insulated building. Solar gain is especially troublesome because it is generally the largest and most variable heat gain a building will experience. As a result window attachments that can be used for solar control are drawing attention and a renewed effort is being made to develop models for devices such as venetian blinds, drapes, roller blinds and insect screens (e.g., Rosenfeld 1996, Rosenfeld et al 2000, Pfrommer et al 1996, Collins and Harrison 2004, ISO 2000, Chantrasrisalai and Fisher 2004, Naylor and Collins 2004, van Dijk and Goulding 1996, Yahoda and Wright 2004, 2005, Kuhn et al 2006, Laouadi and Parekh 2007).

The energy saving potential of switchable glazing is widely understood. Solar gain can be admitted when and where heating is required, and rejected otherwise. Similarly, window shading attachments offer the benefit of being oper-

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able and many devices such as venetian blinds and roller blinds can be automated. Computer simulation offers a means to evaluate the energy saving performance of these devices, their potential to reduce peak cooling loads and the effectiveness of various control strategies.

Computer algorithms can readily solve the non-linear coupling of convective and radiative heat transfer found in glazing systems. Several computer programs have been written to model centre-glass energy performance of multi-layer glazing systems (e.g., Finlayson et al. 1993, Wright and Sullivan 1995, van Dijk and Goulding 1996). Several publications describe the basis of the associated analysis algorithms (e.g., Wright 1980, Hollands and Wright 1980, 1983, Rubin 1982). These algorithms are noteworthy because they allow for the possibility of layers that transmit longwave radiation (i.e., diathermanous layers). This capability is available because the radiant exchange is quantified using a "net radiation" approach. Balances are applied to determine the flux of radiant energy leaving any given surface - the radiosity. The net radiant flux between layers is obtained by taking the difference between radiosities at the bounding surfaces. The freedom to consider diathermanous layers has been demonstrated in a small number of studies (Wright 1980, 1985, Hollands and Wright 1983, Wright and Sullivan 1987a, 1987b, Wright and Hollands 1989) but is seldom needed in the analysis of conventional glazing systems because glass is opaque with respect to longwave radiation. In contrast shading layers are almost always diathermanous - by virtue of their openness. Therefore, the full capability of the net radiation analysis is much more valuable when shading devices are considered.

It is important to note that a diathermanous layer allows for heat transfer directly between two layers that are not adjacent to each other. Viewing the multilayer system as an electrical circuit, a series of temperature nodes connected by thermal resistors, the diathermanous layer gives rise to a resistor that bypasses, or jumps over, one of the nodes. The jump resistor causes complication in the calculation of indices of merit (U-factor and Solar Heat Gain Coefficient (SHGC)). Methods previously documented for calculating U-factor and SHGC of glazing systems (Wright 1998, Hollands et al. 2001) can be in error when a jump resistor is present, even though all temperatures and fluxes are determined correctly. Collins and Wright (2006) provided a closed-form solution to overcome this error in situations where a single diathermanous layer is located between two opaque layers. Derivation of solutions for systems with two or more consecutive diathermanous layers is a great deal more cumbersome and the resulting formulas, if available, are very complicated and difficult to apply. A simpler approach is needed.

The following sections provide details regarding a new method for calculating the indices of merit of multi-layer systems. The one-dimensional framework is retained. This new method is sufficiently general to handle any combination of diathermanous and opaque layers comprising a glazing/shading system. The system can be exposed to any combination of indoor/outdoor temperature difference and any level of incident solar radiation. It is also possible for the air temperature to differ from the mean radiant temperature on the indoor and/or outdoor side. The longwave radiant exchange is modeled using thermal resistors instead of the customary net radiation approach so that the jump resistors can be individually identified and assigned values. The effect of convective jump resistors (e.g., air flow between a drape and a window) can also be included.

The new method can be used to generate indices of merit for performance comparison. Perhaps of greater interest, the indices of merit can be used for on-the-fly calculation of heat gain data as an integral component of building energy simulation software. The latter possibility is especially valuable because the convective-radiant split, vital for estimating peak cooling load, can be retained. Very little CPU time is required because the detailed solar optical and heat transfer balances need to be executed only once per building simulation time step.

STRUCTURE OF THE PROBLEM

The problem of interest is shown in Figure 1. The glazing/shading system is comprised of layers numbered from $i = 1$ to $i = n$. In this study the numbering convention was chosen such that $i = 1$ designates the layer nearest the outdoor side.

The longwave hemispheric properties of the i-th layer include front-surface emissivity, $\varepsilon_{f,i}$, back-surface emissivity, $\varepsilon_{b,i}$, and transmittance, τ_i, as shown. Even though a layer may be spectrally selective it is safe to assume all temperatures in the system span a small enough range that Kirchoff's Law applies and so total emissivity and total absorptivity can be equated in the longwave band; $\alpha_{f,i} = \varepsilon_{f,i}$ and $\alpha_{b,i} = \varepsilon_{b,i}$. It is also possible to ascertain the front-surface and back-surface reflectivities, $\rho_{f,i}$ and $\rho_{b,i}$, by noting that $\rho_{f,i} + \alpha_{f,i} + \tau_i = 1$ and $\rho_{b,i} + \alpha_{b,i} + \tau_i = 1$.

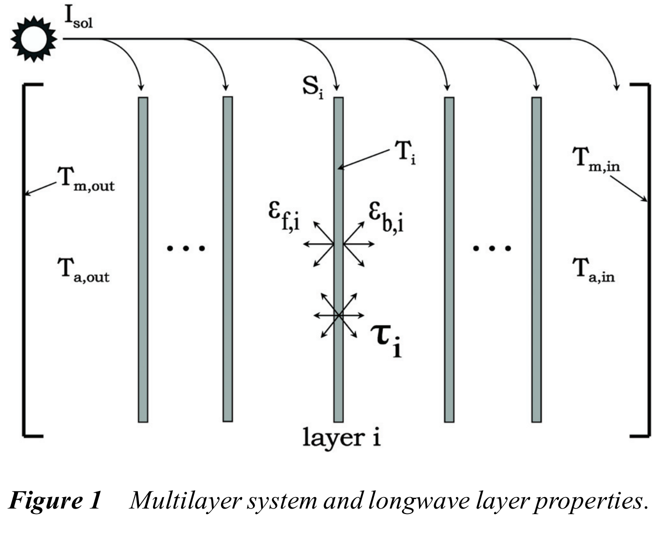


Figure 1 *Multilayer system and longwave layer properties.*

Values of $\varepsilon_{f,i}$, $\varepsilon_{b,i}$, and τ_i are readily available for many coated and uncoated commercially available glazing layers (NFRC 1997). Generally, $\varepsilon_{f,i} = \varepsilon_{b,i} = 0.84$ applies to uncoated glass and $\tau_i = 0$ applies to a glass layer whether coated or uncoated. The effective (i.e., spatially averaged) longwave properties of shading layers can readily be estimated (e.g., ISO 2000, Yahoda and Wright 2004).

It is also assumed that the absorbed flux of solar radiation at each layer, S_i, and the solar transmittance of the system, τ_{sol}, are known. The necessary solar properties of commercially available glazing layers are also readily available (NFRC 1997) and the effective solar properties of shading layers can be estimated (e.g., ISO 2000, Yahoda and Wright 2005). A variety of methods are available to calculate S_i and τ_{sol} (e.g., Wright and Kotey [2006]) as functions of the solar optical properties of the individual layers and the directional nature of the incident solar flux, I_{sol}.

The indoor environment is characterized by two temperatures; the air temperature, $T_{a,in}$, and the mean radiant temperature of the participating indoor surfaces, $T_{m,in}$. For the purpose of the radiant exchange model the indoor surfaces can be treated as an additional parallel surface in the multi-layer system, as shown in Figure 1. This indoor surface is opaque and because the window is usually small in relation to the size of the room the emissivity of the indoor surface must be high and is often treated as black by setting its emissivity equal to unity.

The outdoor environment is also characterized by two temperatures; the air temperature, $T_{a,out}$, and the mean radiant temperature of the participating outdoor surfaces, $T_{m,out}$. The set of outdoor surfaces can be treated as a single parallel surface. This outdoor surface is opaque and because the window is always small in relation to the outdoor environment the outdoor surface is assumed to be black, but this is not a restriction imposed by the nature of the analysis.

RESISTOR-BASED ENERGY ANALYSIS

The Resistance Network

Figure 2 shows a general resistance network that applies to the i-th layer, a glazing or shading layer, in a multi-layer system. The label attached to the each resistor is not resistance, R, but instead the corresponding heat transfer coefficient, shown as h with various subscripts. The conversion is

$$R = \frac{1}{h \cdot A_{cg}}, \tag{1}$$

where A_{cg} is the centre-glass area of the system.

Radiant exchange between the i-th layer and any other glazing or shading layer, layer j, is quantified by the heat transfer coefficient, $h_{r,i,j}$, and the corresponding convective heat transfer coefficient is $h_{c,i,j}$. Radiant exchange with the indoor and outdoor environments, at temperatures $T_{m,in}$ and $T_{m,out}$, is through $h_{r,i,in}$ and $h_{r,i,out}$, respectively. Similarly, convective heat transfer between layer i and the indoor and outdoor environments is through $h_{c,i,in}$ and $h_{c,i,out}$, respectively. In the unlikely event that heat transfer is possible directly between the indoor and outdoor environments $h_{r,in,out}$ and $h_{c,in,out}$ are also considered.

Formulation and Solution of the Energy Balance

If all of the heat transfer coefficients are known the set of layer temperatures can be obtained by applying an energy balance at each layer. At steady-state the flux of absorbed solar radiation at layer i must be balanced by the flux of heat transfer from the i-th layer to all possible destinations.

$$S_i = \sum_{\substack{i=1 \\ i \neq j}}^{n} (h_{r,i,j} + h_{c,i,j})(T_i - T_j) + h_{r,i,in}(T_i - T_{m,in}) + h_{c,i,in}(T_i - T_{a,in}) + h_{r,i,out}(T_i - T_{m,out}) + h_{c,i,out}(T_i - T_{a,out}) \tag{2}$$

Equation 2 constitutes a set of n linear equations that can be directly solved for the layer temperatures, $T_1, T_2, T_3, \ldots$. The solution can be obtained efficiently by means of matrix inversion. As an alternative Equation 2 can be rearranged to obtain the following:

$$T_i = \frac{S_i + \sum_{\substack{i=1 \\ i \neq j}}^{n} (h_{r,i,j} + h_{c,i,j})T_j + h_{r,i,in}T_{m,in} + h_{c,i,in}T_{a,in} + h_{r,i,out}T_{m,out} + h_{c,i,out}T_{a,out}}{\sum_{\substack{i=1 \\ i \neq j}}^{n} (h_{r,i,j} + h_{c,i,j}) + h_{r,i,in} + h_{c,i,in} + h_{r,i,out} + h_{c,i,out}} \tag{3}$$

which makes it apparent that a simple iterative solver such as a Jacobi or a Gauss-Seidel scheme could be used. It is interesting to observe, from Equation 3, that T_i increases with S_i, more strongly if the heat transfer coefficients are small, and

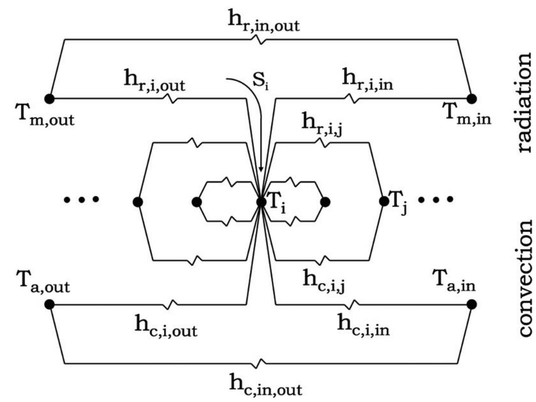


Figure 2 *Multilayer thermal resistance network.*

that T_i is otherwise a weighted average of all of the other system component temperatures where the weighting factors are given by the respective heat transfer coefficients.

The more realistic situation is that the heat transfer coefficients are not known in advance. Each heat transfer coefficient will be influenced by temperature and/or temperature difference. A thorough discussion can be found in (Hollands et al. 2001). In this situation a solution can be generated by following these steps: (1) make an initial estimate of the layer temperatures, T_i, (2) use the current set of T_i to calculate a set of heat transfer coefficients, $h_{r,i,j}$, $h_{c,i,j}$, etc. (3) solve the layer energy balances for a new set of layer temperatures, say T_i^{new}, (4) compare the set of T_i against T_i^{new} and if convergence has not been achieved return to step 2 with T_i replaced by T_i^{new}. Convergence is usually reached in several iterations.

Calculation of Heat Gain

Having solved Equation 2 for the layer temperatures the full set of heat transfer coefficients (i.e., the full set of resistors) will also be available. Any heat flux or heat flux component can then be obtained. The heat flux from the glazing/shading system to the indoor space, q_{in}, is of particular interest. It can be calculated by summing radiant and convective contributions from each of the system nodes as shown in Equation 4.

$$\begin{aligned} q_{in} = & \sum_{i=1}^{n} h_{r,i,in}(T_i - T_{m,in}) + \sum_{i=1}^{n} h_{c,i,in}(T_i - T_{a,in}) \\ & + h_{r,in,out}(T_{m,out} - T_{m,in}) + h_{c,in,out}(T_{m,out} - T_{a,in}) \end{aligned} \tag{4}$$

The first and third terms of Equation 4 include all long-wave radiant gain to the indoor space. The second and last terms include all convective gain. The second line of Equation 4, representing direct indoor/outdoor heat transfer, will very rarely be non-zero.

Calculation of Heat Transfer Coefficients—Longwave Radiation

In order to apply Equations 2, 3 and/or 4 it is necessary to obtain values for the radiative heat transfer coefficients between each pair of temperature nodes. The procedure presented here is similar to Gebhart's analysis of diffuse, grey enclosures (Gebhart 1957, 1959, 1961), a "unified method for radiation exchange", with an extension to specifically account for diathermanous layers.

Consider the system of layers shown in Figure 3. This system includes the outdoor surface ($i = 0$), all of the glazing/shading layers ($i = 1$ to n) and the indoor surface ($i = n + 1$). The radiosity of the front surface of the i-th layer, $J_{f,i}$, is comprised of emitted, reflected and transmitted components.

$$J_{f,i} = E_{front,i} + \rho_{f,i} J_{b,i-1} + \tau_i J_{f,i+1} \tag{5}$$

Accordingly, the radiosity of a back surface of i-th layer, $J_{b,i}$, is given by

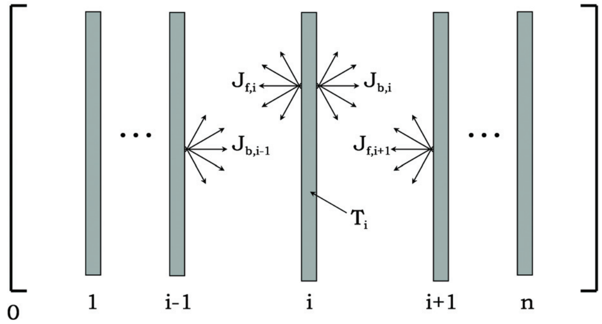


Figure 3 *Front-surface and back-surface radiosity at i-th layer.*

$$J_{b,i} = E_{back,i} + \tau_i J_{b,i-1} + \rho_{b,i} J_{f,i+1}\,. \tag{6}$$

The variable $E_{front,i}$ represents a source of radiant flux at the front surface of layer i and $E_{back,i}$ is a similar source at the back surface of layer i. The system is isolated because $\tau_0 = 0$ and $\tau_{n+1} = 0$.

Equations 5 and 6, applied layer-by-layer, represent a system of $(2n+2)$ equations that can be solved through matrix inversion to obtain $(2n+2)$ radiosity values. The matrix representation is given by $A \cdot x = B$ where

$$A = \begin{bmatrix} 1 & -\rho_{b,0} & 0 & 0 & & & & & \\ -\rho_{f,1} & 1 & 0 & -\tau_1 & & & & & \\ & & & & & & & & \\ & & -\rho_{f,i} & 1 & 0 & -\tau_i & & & \\ & & -\tau_i & 0 & 1 & -\rho_{b,i} & & & \\ & & & & & & & & \\ & & & & & -\tau_n & 0 & 1 & -\rho_{b,n} \\ & & & & & 0 & 0 & -\rho_{f,n+1} & 1 \end{bmatrix},$$

(entries not shown are zeros) (7)

x is a column vector whose transpose is

$$x^t = [J_{b,0}, J_{f,1}, J_{b,1}, \ldots, J_{f,i}, J_{b,i}, \ldots, J_{f,n}, J_{b,n}, J_{f,n+1}], \tag{8}$$

and B is a column vector whose transpose is

$$B^t = \begin{bmatrix} E_{back,0}, E_{front,1}, E_{back,1}, \ldots, E_{front,i}, E_{back,i}, \ldots, \\ E_{front,n}, E_{back,n}, E_{front,n+1} \end{bmatrix}. \tag{9}$$

If all values of $E_{front,i}$ and $E_{back,i}$ are set equal to the corresponding emissive powers ($E_{front,i} = \varepsilon_{f,i}\sigma T_i^4$ and $E_{back,i} = \varepsilon_{b,i}\sigma T_i^4$) this system yields radiosities that can be used to calculate the net radiant flux between any two adjacent layers and the entire radiant balance will have been obtained for a given set of layer temperatures. However, more detail is needed regarding heat transfer coefficients.

Instead $E_{front,i}$ and $E_{back,i}$ are used to create unit sources of flux, one surface at a time and including all surfaces, so that the flux absorbed at any layer, layer j, as a result of emission from layer i can be resolved. For example, consider the front surface of layer 3. A unit source of radiant flux is created at this surface by setting $E_{front,3} = 1$ while $E_{front,i} = 0$ for all other values of i, and $E_{back,i} = 0$ for all values of i. Now $B^t = [0, 0, 0, 0, 0, 1, 0, 0, \ldots, 0]$. The A-matrix remains unchanged. The system is solved to determine a set of radiosities which are now given superscripts to denote the surface of origin for all radiation in the multi-layer system. The radiosities at the front and back surfaces of layer j, "caused" by the front surface of layer 3, are $J_{f,j}^{(f,3)}$ and $J_{b,j}^{(f,3)}$, respectively. The set of $(2n+2)^2$ radiosities obtained in this exercise completes the set of information needed to calculate all of the radiant heat transfer coefficients.

The coefficient $h_{r,in,out}$ is the simplest to calculate. This heat transfer coefficient applies between the back surface of node 0 and the front surface of node $n + 1$. The rate of radiant heat flux from the outdoor surface to the indoor surface is the product of the emitted flux at surface (b,0), $E_{back,0} = \varepsilon_{b,0}\sigma T_0^4$, the portion of that flux that reaches surface(f, $n + 1$), $J_{b,n}^{(b,0)}$, and the longwave absorptivity of surface (f, $n + 1$), $\alpha_{f,n+1}$. The corresponding flux in the reverse direction is the product of $E_{front,n+1}$, $J_{f,1}^{(f,n+1)}$, and $\alpha_{b,0}$. The net indoor-to-outdoor heat flux, $q_{in,out}$, is the difference between the two products.

$$q_{in,out} = \varepsilon_{f,n+1}\sigma T_{n+1}^4 J_{f,1}^{(f,n+1)}\alpha_{b,0} - \varepsilon_{b,0}\sigma T_0^4 J_{b,n}^{(b,0)}\alpha_{f,n+1} \quad (10)$$

In interpreting Equation 10 it is helpful to recognize that the radiosity of the front surface of layer 1, $J_{f,1}^{(f,n+1)}$, is the irradiance of the back surface of surface 0 and the radiosity of the back surface of layer n, $J_{b,n}^{(b,0)}$, is the irradiance of the front surface of surface $n + 1$. Several substitutions and simplifications can be made: $\varepsilon_{f,n+1} = \alpha_{f,n+1}$, $\varepsilon_{b,0} = \alpha_{b,0}$, $T_0 = T_{m,out}$, $T_{n+1} = T_{m,in}$ and because the areas of the two surfaces are equal a reciprocity relationship proven by Gebhart (1957) gives $J_{f,1}^{(f,n+1)} = J_{b,n}^{(b,0)}$. Equation 10 becomes:

$$q_{in,out} = \varepsilon_{f,n+1}\varepsilon_{b,0}J_{f,1}^{(f,n+1)}\sigma(T_{m,in}^4 - T_{m,out}^4) \quad (11)$$

As expected, the radiant heat transfer between the two surfaces is driven by black emissive power and this heat transfer will be zero if either surface emissivity is zero. It is important to note that, in this context, each radiosity is really just a ratio and is therefore dimensionless. In this case $J_{f,1}^{(f,n+1)}$ is the ratio between the irradiance at surface b,0 and the emitted flux from surface f,n+1 that caused the irradiance at surface b,0. If any one of the intermediate layers is opaque $J_{f,1}^{(f,n+1)}$ and $q''_{in,out}$ will both be zero.

The indoor-outdoor heat transfer coefficient can now be calculated.

$$h_{r,in,out} = \frac{q_{in,out}}{T_{m,in} - T_{m,out}} = \varepsilon_{f,n+1}\varepsilon_{b,0}J_{f,1}^{(f,n+1)}\sigma\frac{T_{m,in}^4 - T_{m,out}^4}{T_{m,in} - T_{m,out}} \quad (12)$$

Applying a little more algebra (difference of squares), Equation 12 becomes

$$h_{r,in,out} = \varepsilon_{f,n+1}\varepsilon_{b,0}J_{f,1}^{(f,n+1)}\sigma(T_{m,in}^2 + T_{m,out}^2)(T_{m,in} + T_{m,out}) \quad (13)$$

This is the form most conveniently used in computer code because $h_{r,in,out}$ can be determined even when $T_{m,in} = T_{m,out}$.

Similar logic can be followed to obtain expressions for the remaining radiant heat transfer coefficients. The only difference is that more terms arise because two surfaces comprise each of the glazing/shading layers. The resulting equations are as follows.

$$h_{r,i,in} = \varepsilon_{f,n+1}(\varepsilon_{f,i}J_{b,n}^{(f,i)} + \varepsilon_{b,i}J_{b,n}^{(b,i)})\sigma(T_i^2 + T_{m,in}^2)(T_i + T_{m,in}) \quad (14)$$

$$h_{r,i,out} = \varepsilon_{b,0}(\varepsilon_{f,i}J_{f,1}^{(f,i)} + \varepsilon_{b,i}J_{f,1}^{(b,i)})\sigma(T_i^2 + T_{m,out}^2)(T_i + T_{m,out}) \quad (15)$$

$$h_{r,i,j} = \varepsilon_{f,j}(\varepsilon_{f,i}J_{b,j-1}^{(f,i)} + \varepsilon_{b,i}J_{b,j-1}^{(b,i)}) + \varepsilon_{b,j}(\varepsilon_{f,i}J_{f,j+1}^{(f,i)} + \varepsilon_{b,i}J_{b,j+1}^{(b,i)}) \times \sigma(T_i^2 + T_j^2)(T_i + T_j) \quad (16)$$

Calculation of Heat Transfer Coefficients—Convection

In contrast to the treatment of radiant exchange the quantification of convective heat transfer coefficients relies heavily on the use of empirical information. One of several correlations can be used to calculate $h_{c,i,i+1}$ for a glazing cavity (e.g., Elsherbiny et al 1982, Wright 1996, Shewen et al 1996). The three correlations referenced give similar results. These correlations are expected to also work well for the cavity between a window and an indoor shading layer as long as the shading device is airtight. This arrangement is approached by close-fitting roller blinds or drapes. A method is also available to determine convective heat transfer coefficients for a glazing cavity that contains a venetian blind (Huang et al 2006, Wright et al 2008).

Less information is known about convective heat transfer where a shading device is attached adjacent to the indoor surface of a window and air is able to flow between the two. This situation is important because (a) indoor retrofit attachments are common and (b) the combination of window surface and attachment roughly triples the amount of surface area available for convective heat gain to the room - heat gain that represents an immediate cooling load. This penalty may be greater for indoor venetian blinds because the convective heat transfer coefficients are expected to be especially high in instances where there is appreciable air movement through the blind.

Figure 4a shows the resistance network for convection that applies when the gap between a double-glazed window and the shading layer is not sealed. Recognizing that both the front and back surfaces of the shading layer are exposed to the room air, at $T_{a,in}$, the two unlabelled resistors can be combined in parallel as $h_{c,n,in}$. The equivalent indoor-portion of the resistance network is shown in Figure 4b. A method has been devised by Roeleveld et al (2007) to estimate the three heat transfer coefficients shown in Figure 4b. If the shading layer is far enough from the window the boundary layers associated with each surface will not interfere with each other. In this case it is reasonable to consider each surface on an individual basis with $h_{c,n-1,n} \approx 0$ and $h_{c,n-1,n} \approx \frac{1}{2} h_{c,n,in} \approx 4 \frac{\mathrm{W}}{\mathrm{m^2K}}$ for natural convection. If forced convection is considered the heat transfer coefficients will be larger and may need to be assigned on an individual basis depending on the location of the air stream.

INDICES OF MERIT

Functional Dependence

Consider a multi-layer system in which all layers are opaque with respect to longwave radiation. No jump resistors are present. This system is exposed to indoor and outdoor environments where the air temperature is equal to the mean radiant temperature, say $T_{m,in} = T_{a,in} = T_{in}$ and $T_{m,out} = T_{a,out} = T_{out}$. The incident flux of solar radiation is I_{sol}. Figure 5a shows the corresponding resistance network for a system with four layers. The two parallel resistors between each node correspond to the radiant and convective heat transfer components. Each pair of resistors can be combined to form a single resistor. The resulting network is a set of resistors in series as shown in Figure 5b. This network applies to all of the glazing systems for which U and SHGC are tabulated in the ASHRAE Handbook of Fundamentals (e.g., ASHRAE 2005). It can be shown (e.g., Wright and McGowan 1999) that q_{in} is given by the following set of equations.

$$q_{in} = U \cdot (T_{out} - T_{in}) + \mathrm{SHGC} \cdot I_{sol} \tag{17}$$

The energy-related indices of merit, U-factor and SHGC, are as follows:

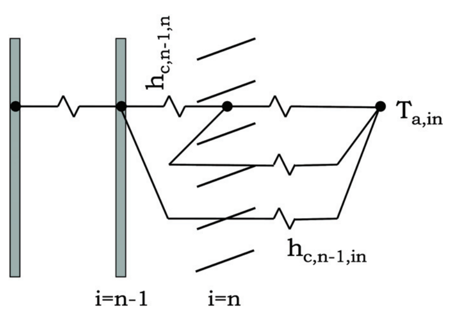
$$U = (R_{tot})^{-1} = \left(\sum_{i=0}^{n} R_i\right)^{-1} \tag{18}$$

$$\mathrm{SHGC} = \tau_{sol} + \frac{\sum_{i=1}^{n} N_i S_i}{I_{sol}} \tag{19}$$

and the inward flowing fraction of S_i is

$$N_i = \frac{\sum_{j=0}^{i=1} R_j}{R_{tot}} \tag{20}$$

Equations 18 through 20 contain a key piece of information. The indices of merit depend only upon the values of the various resistors, R_i. The consequence is that, knowing only



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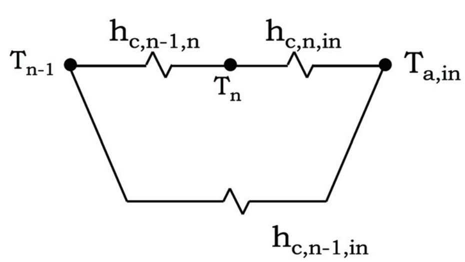
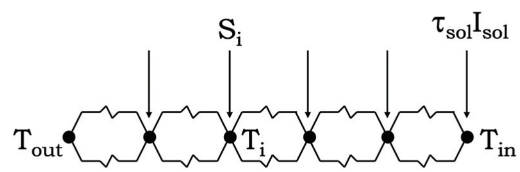


b

Figure 4 *(a) Double-glazed window with indoor shading layer and (b) equivalent resistance network—indoor side.*



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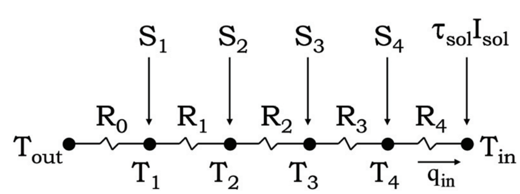

b

Figure 5 *(a) Thermal resistance network: opaque layers, n = 4 and (b) equivalent resistance network, n = 4.*

the values of R_i, (and S_i and τ_{sol} from a previous calculation), it is possible to calculate U and SHGC without knowing any of the temperatures. Going one step further, computational experiments can be used to determine the properties of the resistance network by applying well chosen stimuli - similar to the way one might measure the overall resistance of an intricate electrical network by applying an arbitrary voltage and observing the resulting current. The critical requirement is that the resistors (i.e., the $h_{c,i,j}$, $h_{r,i,j}$, $h_{c,i,in}$, etc.) must be held constant during these experiments and this is easily done in a calculation.

It is important to note that such computational experiments are equally valid if jump resistors are introduced. In fact, this is the situation where the new approach is most useful. The presence of jump resistors will make Equation 20 invalid. Equations 17 through 19 remain unchanged. Whether the replacement for Equation 20 can be derived is of little importance; the important point is that the applicable version of Equation 20 will invariably be a function of only the values of the resistors.

The Computational Experiments

In order to simplify the presentation the assumption that $T_{m,in} = T_{a,in} = T_{in}$ and $T_{m,out} = T_{a,out} = T_{out}$ will be retained for the moment. This restriction applies to both the ASHRAE winter and summer design conditions. A method for avoiding this restriction is presented subsequently. The freedom to specify unequal mean-radiant and air temperatures is of greater importance in the context of building energy simulation.

The first step in the calculation of energy-related indices of merit is to solve the multi-layer energy balance (Equation 2), subject to the environmental conditions of interest, those being T_{in}, T_{out}, and I_{sol}. Values of T_i ($i = 1$ to n) and all heat transfer coefficients, $h_{x,i,y}$ ($x = c$ or r, $i = 1$ to n, $y = in,out$ or j, $j = 1$ to n), are obtained simultaneously.

Now computational experiments can be undertaken. The heat transfer coefficients are held constant for all remaining calculations and it is because of this that the calculated indices of merit apply to the condition of interest.

The U-factor can be obtained by applying a temperature difference to the resistance network, with no solar radiation present, and observing the value of q_{in} that results. This is done by re-solving Equation 2 while (a) assigning arbitrary, but unequal, values to T_{out} and T_{in}, say T_{out}^{exp} and T_{in}^{exp}, and (b) setting the absorbed amounts of solar radiation to zero, $I_{sol}^{exp} = S_i^{exp} = 0$ ($i = 1$ to n). This will generate a new set of node temperatures, T_i^{exp} ($i = 1$ to n). Now use the sets $h_{x,i,y}$ and T_i^{exp} to obtain q_{in}^{exp} using Equation 4. Finally, rearranging Equation 17 and using the results of the experiment the U-factor is obtained.

$$U = \frac{q_{in}^{exp}}{T_{out}^{exp} - T_{in}^{exp}} \tag{21}$$

The foresight of choosing $T_{out}^{exp} = 1$ and $T_{in}^{exp} = 0$ will eliminate the need to execute Equation 21 - the subroutine used to evaluate Equation 4 will return the numerical value of U.

Evaluate SHGC by conducting a second experiment with zero temperature difference but with solar radiation present: $T_{out}^{exp} = T_{in}^{exp}$ and $S_i^{exp} = S_i$ ($i = 1$ to n). Again Equation 2 is re-solved with $h_{x,i,y}$ unchanged to obtain a new set of T_i^{exp}. Use $h_{x,i,y}$, T_i^{exp} and Equation 4 to obtain q_{in}^{exp}. In this case the inward flowing portion of all solar radiation absorbed in the glazing/shading layer system is $q_{in}^{exp} = \sum_{i=1}^{n} N_i S_i$ and Equations 17 and 19 give the following:

$$\mathrm{SHGC} = \tau_{sol} + \frac{q_{in}^{exp}}{I_{sol}} \tag{22}$$

In the event that no solar radiation is present ($I_{sol} = 0$) it is still possible to calculate SHGC. Defining A_i as the portion of S_i absorbed at the i-th layer:

$$A_i = \frac{S_i}{I_{sol}} \tag{23}$$

we can rewrite Equation 19 as:

$$\mathrm{SHGC} = \tau_{sol} + \sum_{i=1}^{n} N_i A_i \tag{24}$$

The values of A_i can be determined knowing only the solar optical properties of the layers that comprise the system and so can be determined even if $I_{sol} = 0$ (Wright 1998, Wright and Kotey 2006). Additional experiments can be conducted to determine the set of inward flowing fractions, N_i, of the absorbed solar quantities by introducing absorbed solar radiation one layer at a time. To do this set $T_{out}^{exp} = T_{in}^{exp}$, $S_j^{exp} = 0$ ($j = 1$ to n, $j \neq i$) and $S_i^{exp} = 1$. The inward flowing fraction is

$$N_i = \frac{q_{in}^{exp}}{S_i}, \tag{25}$$

which is equal to the numerical value of q_{in}^{exp} returned by Equation 4 because the source of absorbed solar flux introduced at the i-th layer, S_i^{exp}, was chosen to be unity.

Unequal Air/Mean Radiant Temperatures

The method presented above can readily be extended to account for unequal air and mean radiant temperatures. It is instructive to start by examining the approach that is routinely applied to systems in which the outdoor layer ($i = 1$) is opaque. Consider the resistance network between an opaque outdoor layer and environment. The resistors, and corresponding heat transfer coefficients, are shown in Figure 6. The environment temperatures $T_{a,out}$ and $T_{m,out}$ are replaced by an ambient-effective outdoor temperature, $T_{ae,out}$, which is evaluated by imposing the requirement that the heat flux remain unchanged between the outdoor layer and the environment.

$$h_{c,1,out}(T_1 - T_{a,out}) + h_{r,1,out}(T_1 - T_{m,out}) \\ = (h_{c,1,out} + h_{r,1,out})(T_1 - T_{ae,out}) \tag{26}$$

Rearranging, $T_{ae,out}$ is found to be a weighted average of $T_{m,out}$ and $T_{a,out}$

$$T_{ae,out} = \frac{h_{r,1,out}}{h_{c,1,out} + h_{r,1,out}} T_{m,out} + \frac{h_{c,1,out}}{h_{c,1,out} + h_{r,1,out}} T_{a,out}, \tag{27}$$

where the weighting factors are given by the values of the respective heat transfer coefficients. If we define a factor, say $f_{r,out}$, that represents the relative strength of the radiant mode of heat transfer on the outdoor side,

$$f_{r,out} = \frac{h_{r,1,out}}{h_{c,1,out} + h_{r,1,out}}. \tag{28}$$

Equation 27 becomes

$$T_{ae,out} = f_{r,out} T_{m,out} + (1 - f_{r,out}) T_{a,out}. \tag{29}$$

If $T_{m,out} = T_{a,out}$ then $T_{ae,out} = T_{m,out} = T_{a,out}$ for any value of $f_{r,out}$.

A similar development can be applied to the indoor side giving rise to the indoor ambient-effective temperature, $T_{ae,in}$, and the corresponding weighting factor, $f_{r,in}$.

$$T_{ae,in} = f_{r,in} T_{m,in} + (1 - f_{r,in}) T_{a,in} \tag{30}$$

Equation 17 is now rewritten in a more general fashion by replacing T_{out} and T_{in} with $T_{ae,out}$ and $T_{ae,in}$.

$$q_{in} = U \cdot (f_{r,out} T_{m,out} + (1 - f_{r,out}) T_{a,out}) \\ + (f_{r,in} T_{m,in} + (1 - f_{r,in}) T_{a,in}) + \mathrm{SHGC} \cdot I_{sol} \tag{31}$$

At the same time we wish to develop a method that is general in the sense that any layer may be diathermanous. See, for example, Figure 7. Therefore, we forfeit Equations 26 and 27. Instead, $f_{r,in}$ and $f_{r,out}$ are evaluated by conducting two more computational experiments. In taking this step it is important to examine Equation 27 and note that $f_{r,out}$ is a function of resistances only. This will also be the case if jump resistors are present. Therefore, it is legitimate to evaluate $f_{r,out}$ and $f_{r,in}$ by means of computational experiments. It is also noteworthy that $f_{r,out}$ and $f_{r,in}$ can be added to the set of energy related indices of merit.

Recall that the procedure starts by solving Equation 2 subject to the environmental conditions of interest. These conditions now consist of $T_{a,in}$, $T_{m,in}$, $T_{a,out}$, $T_{m,out}$ and I_{sol}. The sets of T_i and $h_{x,i,y}$ are obtained.

The procedure to find $f_{r,out}$ is devised by first rearranging Equation 31.

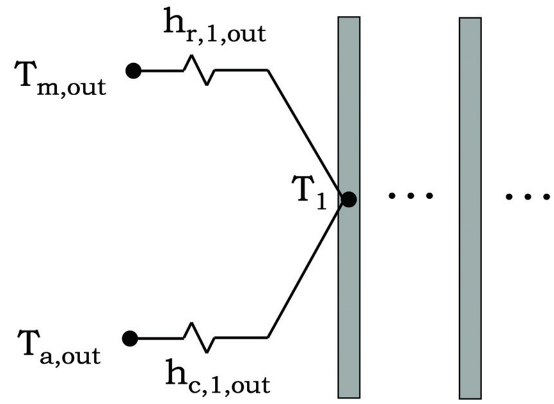

Figure 6 *Heat transfer between opaque surface and outdoor environment—unequal air and mean radiant temperatures.*

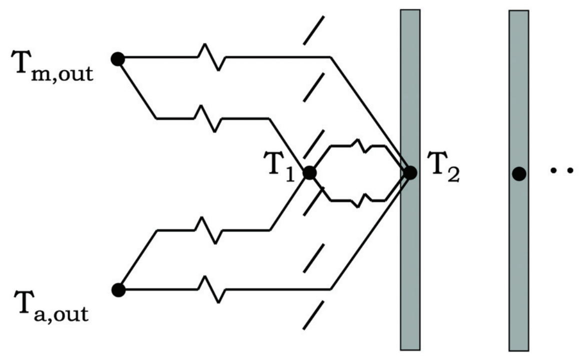

Figure 7 *Resistance network for diathermanous layer facing outdoor environment—unequal air and mean radiant temperatures.*

$$f_{r,out} = \frac{q_{in} - U \cdot (T_{a,out} - (f_{r,in} T_{m,in} + (1 - f_{r,in}) T_{a,in})) \\ -(\mathrm{SHGC} \cdot I_{sol})}{U \cdot (T_{m,out} - T_{a,out})} \tag{32}$$

Applying Equation 32 to a computational experiment while imposing the restrictions that $I_{sol}^{exp} = 0$ and $T_{a,in}^{exp} = T_{m,in}^{exp} = T_{in}^{exp}$ we obtain the following:

$$f_{r,out} = \frac{q_{in}^{exp} - U \cdot (T_{a,out}^{exp} - T_{in}^{exp})}{U \cdot (T_{in,out}^{exp} - T_{a,out}^{exp})} \tag{33}$$

If the experiment is conducted with the foresight of setting $T_{a,in}^{exp} = T_{m,in}^{exp} = T_{in}^{exp} = 0$, $T_{a,out}^{exp} = 1$, and $T_{m,out}^{exp} = 1$, $f_{r,out}$ can be evaluated simply by dividing the value of q_{in}^{exp} returned by Equation 4 by the U-factor.

Similarly, $f_{r,in}$ is found using $T_{a,out}^{exp} = T_{m,out}^{exp} = T_{out}^{exp} = T_{a,in}^{exp} = 1$, $T_{m,in}^{exp} = 0$, and $I_{sol}^{exp} = S_i^{exp} = 0$. Divide the newly calculated value of by U to obtain $f_{r,in}$.

APPLICATION IN COMPUTER CODE

Clearly the analysis of multilayer systems is best undertaken using a computer-based calculation. In general, application of the energy balance to obtain layer temperatures and heat transfer coefficients will be confined to one specific subroutine. Input parameters will describe the properties of the multilayer system and its components as well as boundary conditions associated with the indoor and outdoor environments. Output parameters will include information such as temperatures and heat flux values, as desired, and q_{in} should be included in this set. If so, it is convenient to recognize that same subroutine can be used to conduct the computational experiments described above. It is only necessary to add an input parameter that can be used to switch on/off the calculation of updated heat transfer coefficients during the solution of Equation 2. Heat transfer coefficients should be updated during the initial solution but they should be held constant, simply by *not* updating them, during the subsequent experiments. Care should be taken to retain the values of the heat transfer coefficients between subroutine calls either by passing the corresponding arrays back and forth between the subroutine and the calling routine or by declaring them as "static" variables at the subroutine level.

APPLICATION IN BUILDING ENERGY ANALYSIS ALGORITHMS

The methods described for calculating indices of merit have a worthwhile application in the field of computational building energy analysis. In this context any procedure must use very little CPU time because of the large number of building components under consideration, the large number of time steps and the highly iterative nature of the solver used to satisfy the energy balances imposed. This requirement can be satisfied by executing the following two steps *only once at each time step*.

1. Complete the solar optical analysis. Calculate values of τ_{sol}, A_i and S_i for the glazing/shading layer array exposed to the known level (and direction) of insolation (e.g., Wright and Kotey 2006).
2. Complete the thermal analysis. Calculate values of U, SHGC, $f_{r,in}$ and $f_{r,out}$ for the known outdoor conditions and approximate indoor temperatures - perhaps from the previous time step. Also determine the radiant/convective split of q_{in} (examine Equation 4).

As the building energy analysis progresses for the same time step, possibly through thousands of iterations, and indoor temperatures are updated the value of q_{in} can be updated by applying Equation 31. It is assumed that the indices of merit can be held constant during this procedure and this is expected to be a very safe assumption because of the small changes expected with respect to indoor temperatures and the exceptionally small changes in thermal resistance values that result. Similarly, the convective/radiant split of q_{in} is assumed to remain constant for the time step and this is also expected to be a very safe practice. It is common in current practice to hold the convective/radiant split constant for the duration of the entire simulation.

Note that significant benefits arise by updating the various parameters at each time step. For example, it will be possible to account for (a) the day/night variation of U-factor, (b) the hour-by-hour variation of τ_{sol} and A_i caused by the directional solar optical properties of glazing layers and shading layers (consider a venetian blind), and (c) changes in the convective/radiant split of q_{in} through the day and even as the configuration of a shading device is altered.

Case studies, along with more basic sample calculations, are planned for a subsequent calculation to learn more about the performance of shaded glazing systems and to confirm the validity of the assumptions listed above.

Future work will also be aimed at the development of similar theory to be used in conjunction with building simulation programs that account for radiant exchange between the window and multiple indoor surfaces instead of a single node at the mean radiant temperature of the indoor environment.

CONCLUSIONS

New methods have been devised for the thermal analysis of multilayer glazing/shading systems and the companion calculation of energy-related indices of merit. Several significant advances can be highlighted.

1. A thermal resistance network has been formulated in such a way that heat transfer, convective and/or radiant, can take place between any two layers in the system.
2. Theory has been presented for quantifying all of the longwave radiant resistors for any combination of glazing and/or shading layers, any of which can be diathermanous.
3. Procedures for conducting computational experiments have been devised such that any indices of merit related to the thermal resistance network can be evaluated.

This new technique offers remarkable generality. "Jump" resistors allow for airflow between layers and/or diathermanous layers. The air and mean radiant temperatures can differ on both the indoor and outdoor sides. The indices of merit can be obtained for any set of environmental temperature and insolation conditions - even if the indoor-outdoor temperature difference is zero and even if no solar radiation is present.

The new methods are especially useful for the analysis of glazing systems used in combination with shading layers such as venetian blinds, curtains, roller blinds and insect screens because these devices are generally diathermanous by virtue of their openness. These methods also offer new possibilities, speed and convenience when used in conjunction with whole building performance simulations.

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