Location-Routing Problems with Economies of Scale

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The purpose of the location-routing problem is to select facility locations, assign customers to facilities, and design routes between facilities and customers. The most common objective is to achieve minimal cost. In general, for the standard location-routing problem, the total cost includes fixed costs to open Distribution Centers (DCs) plus the transportation costs from facilities to customers. In this thesis, we also consider the variable cost of facilities' operations. Two forms of the variable cost are proposed. One is a linear cost function with a constant variable (operating) cost per unit; the other employs a concave function of total throughput at any DC as its cost model. The latter is studied because economies of scale can be achieved for large facilities. By economies of scale, we mean that the variable cost per unit is a decreasing function of the number of units of throughput; that relationship is expressed by a concave function.

Two solution methods are developed, both based on a genetic algorithm. After some preliminary tests, one approach is employed for further testing. Computational experiments of the model without variable cost are performed on published data sets. Then extensive testing is done on modified data sets for cases with operating cost but without economies of scale, and for other cases when economies of scale are present. Analysis of the influence of economies of scale is provided. First, we briefly test how parameter values affect the economies of scale. Then we extensively analyze the tradeoffs between operating costs of facilities and transportation costs. Conclusions are drawn and further research is suggested.

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Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

The location-routing problem (LRP) is an important application in supply chain management. That problem combines the facility location problem and the execution of vehicle routes to customers from the chosen, open facilities. In more detail, the facility location problem begins with a pre-defined set of feasible facility locations; the model helps to decide which facilities to open. In the standard LRP, the goal is to minimize the total cost, which includes the fixed costs of the open facilities and the transportation costs of the vehicle routes between those facilities and the customers served. Throughout this thesis, the locations of customers are assumed to be known and fixed.

1.1 Facility Location Problem

As noted, the facility location problem begins with a pre-defined set of facility sites. At which locations should a facility be open? Each opened facility serves a group of customers. Various objectives are studied. The most common is minimization of total cost, i.e. the fixed costs of open facilities plus the variable costs of satisfying the demands of those customers allocated to each facility.

In some applications, e.g. Daskin (1982) and Gelvão (1996) and others, only the latter, i.e. the variable costs, are included. Sometimes, the number of facilities to open and operate is an input parameter of the model. The variable-cost objective may then be expressed as the minimization of the total travel time or of the total distances between facilities and customers; the minimization of the maximal travel time or cost; or the maximization of the number of customers that can be served within a given distance or time.

Klose and Drexl (2005) classify the types of facility location models. In particular, they review continuous location models and network location models. Various applications of facility location models are discussed. Melo et al. (2009) provide a survey on facility location problems associated with supply chain management. They discuss characteristics that support decision making in strategic supply chain planning. The book by Daskin (2013) is a treatise on the various types of facility location models, their applications, and the details of a number of solution methodologies.

1.2 Vehicle Routing Problem

The vehicle routing problem refers to the problem of designing routes to customers, from depots, where vehicles are stored. Usually, the objective is to minimize total cost. This always includes the costs (in time or distance) of the vehicle routes; sometimes there is an additional fixed cost for each vehicle utilized. Laporte (1992) discusses the main types of solution methods, including both exact algorithms and heuristic approaches. As stated by a number of publications, since the size of real-life problems is large, heuristics and meta-heuristics are often more suitable. Laporte (1992) also mentions that exact algorithms generally work only for small-size problems.

Eksioglu et al. (2009) classify more than 1000 papers on the vehicle routing problem and present a taxonomic review, providing a detailed classification of models studied by the researchers cited. They count those papers by year, by journal and by solution method, respectively. They point out certain shortcomings in the performance of various solution methods. Toth and Vigo (2014) summarize the definition and variants of vehicle routing problems, examine main approaches, and discuss a variety of applications.

1.3 Location-Routing Problem

The location-routing problem has a variety of applications in logistics, combining the decisions of locating one or more facilities and the optimization of routes. The standard, or single-echelon, location-routing problem involves routing at only one level (See Sec. 2.1). According to the literature, the two-echelon location-routing problem can be defined as a model including factory, DCs and customers, with consideration of routing at each echelon (between factory and DCs, and between DCs and customers). As mentioned by Prodhon and Prins (2014) and Drexl and Schneider (2015), researchers have studied the two-echelon location-routing problem for more than twenty years.

The majority of references consider routing from depots (warehouse or distribution centers) to customers. Only a few papers consider a factory or supply source for the distribution centers. Most publications thus do not worry about the flow of goods into the DCs. Rather, that supply of product is there, and routes are constructed from each DC to a number of customers (retailers). The routing is only at that level, hence this is a single-echelon location-routing problem. For instance, Perl and Daskin (1985) and Bookbinder and Reece (1988) present models with multiple supply sources for DCs; products are shipped directly from factories to each DC. Routes are then designed between DCs and customers.

1.4 Models Considering Economies of Scale

This thesis is concerned with economies of scale in the LRP. Such scale economies, i.e. concave costs, have been discussed in the content of other problems, e.g. in warehouse or plant operations. Manne (1964), Feldman et al. (1966), Soland (1974), and Kelly and Khumawala (1982) discuss the facility location problem with non-linear operating costs at the facilities. Cohen and Moon (1991) design a model to assign products to plants, where those plants exhibit economies of scale with increased levels of production. Dupont (2008) proposes a new type of model in which the objective function, including the fixed costs of operation, production and transportation, is expressed as a single concave function.

To the best of our knowledge, the following paper is the only one that considers economies of scale in dealing with the standard location-routing problem. Melechovsky et al. (2005) treat a single-echelon problem, i.e. a problem with routing at only one level. Those authors propose a non-linear cost function (but do not give its exact form) for the throughput of products handled in each facility (DC), incorporating economies of scale in the single-echelon location-routing problem. They emphasize solution methods. Further details are given in Chapter 2.

In practice, cost is of course associated with goods moving in or out of a facility. However, that facility-throughput cost is only sometimes studied in the LRP. And except in Melechovsky et al. (2005) and this thesis, that cost has not included economies of scale. Most often, that variable cost is a linear function of the throughput at the DC. However, for many facilities, economies of scale are present in the real-world situation. To represent such economies of scale, the operating cost, i.e. the variable cost of operation of each facility (usually a distribution center or DC), should be a concave function of facility throughput.

Thus, our research problem in this thesis focuses on the influence of economies of scale on a distribution center, when vehicle routes must also be considered. Introduction of economies of scale thus has some practical value. The proposed problem is an integration of a facility location problem and a vehicle routing problem, with the presence of concave operating costs at DCs. The latter costs are the variable costs of throughput at a distribution center. Throughout this thesis, the terms "variable cost" and "operating cost" shall be synonymous.

1.5 Remainder of this Thesis

As mentioned, this thesis considers a location-routing problem with economies of scale. Those economies of scale are expected to result in lower total cost. With respect to the algorithms for solving a location-routing problem, meta-heuristic approaches have been the most popular in the literature. With the inclusion of economies of scale, we will seek a meta-heuristic that can obtain quality solutions to our location-routing problem in a reasonable amount of computation time.

The remainder of this thesis is organized as follows: Chapter 2 presents a detailed literature review for the the location-routing problem. Next, we summarize published papers that treat concave costs, whether in the facility location problem or related problems. Solution methods for solving the location-routing problem are then discussed. Chapter 3 proposes a formulation of the location-routing problem when there are variable costs. We employ a concave power function to treat economies of scale.

Chapter 4 develops two hybrid meta-heuristic methods to solve this location-routing problem with variable cost. Both methods are based on a genetic algorithm. These are simulated annealing embedded within a genetic algorithm, and a genetic algorithm with ant colony optimization. Chapter 5 discusses results for the performance of our proposed methods, and presents arguments for the superiority of one of these algorithms. We employ our chosen method to analyze the influence of economies of scale. A conclusion is provided and future research directions are discussed in Chapter 6.

Chapter 2

Literature Review

The classical versions of the location-routing problem include research on capacitated depots and vehicles. Naturally, the earlier studies concerned problem with uncapacitated depots. (e.g. Tuzun and Burke,1999). Nagy and Salhi (2007) do a survey which examines the attributes of different location-routing models. Borges Lopes et al. (2013) consider each LRP model in the form of a taxonomical classification. Prodhon and Prins (2014) survey the research from years 2007 to 2013 on location-routing problems. They classify the literature into ten main types. According to the findings in the latter references, there are 72 papers in total which were published during those recent seven years, the majority concerned the *capacitated* location-routing problem.

2.1 Single-echelon Location-Routing Problem

Drexl and Schneider (2015), in the most recent survey on location-routing problems, define the standard LRP as a single-echelon problem. They do point out, however, that multi-echelon LRPs have also become attractive to researchers. We agree, that taking the supply echelon into consideration, that is, adding the factory-DC shipments to the standard model, has potential for industrial application.

Perl and Daskin (1985) present a mixed integer model for the LRP and propose a heuristic solution method. Their model concerns the locations of DCs, and routes between those DCs and customers. The supply sources for DCs are taken into consideration as well, with equality of inbound and outbound flows at each open DC. The problem is decomposed into three subproblems, for which corresponding methods are constructed. A case study is discussed, showing the ability to solve a practical problem based on their approach.

Bookbinder and Reece (1988) propose an LRP model with capacited warehouses. Those facilities also act as depots for vehicles in a distribution system. Warehouses are located, taking into account that routes naturally need to be designed from warehouse facilities to customers. Products are shipped from factories to customers via the facilities suggested by the model. Two versions of the algorithm are presented, based on Benders decomposition.

2.2 Two-echelon Location-Routing Problem

By definition, many of the two-echelon LRPs discussed in recent years develop routes at both levels. Sterle (2010) presents the two-echelon LRP model for a freight distribution system. A Tabu search method is proposed, based on a decomposition into four subproblems. A greedy randomized adaptive search procedure is proposed by Nguyen et al. (2012) for a two-echelon LRP with one main depot in the first (upper) level, and multiple capacitated satellites in the second layer between depot and customers. Routes are constructed from the satellites to customers. However, the depot is also allowed to serve customers directly.

2.3 Economies of Scale in Location or Production Problems

Although few publications have discussed economies of scale in the LRP, a number of researchers have studied the location problem with economies of scale. The following papers thus concern only location; there are no vehicle routes. Manne (1964) presents the first plant location model that considers both the manufacturing and transportation cost, concentrating upon economies of scale in manufacturing. The manufacturing cost contains two parts; one is a concave function of the amount produced at the given location, and the other is a fixed charge. The transportation cost is linear. The model is solved by a steepest-ascent, one-point move algorithm. The model has no capacity limitation.

Feldman et al. (1966) deal with the warehouse location problem, taking into consideration a nonlinear warehouse operating cost and a linear transportation cost. To solve the model, a heuristic with a drop-and-add routine was developed. The methodology was tested on problem with four factories, forty-nine warehouse sites, and two hundred customer locations that were laid out on a map of America. The proposed heuristic technique was said to generate near-optimal solutions. Because the paper was published in the 1960s, the method was tested only on two instances and not much data was shared.

Dupont (2008) presents a model of the uncapacited facility location problem. The total cost function is concave. Those total costs include the fixed cost of opening the facility, as well as production costs and the cost of delivery. The total cost function exhibits concavity with respect to the quantity delivered by the facility. The objective is to minimize the total cost, subject to the demand of each customer. He mentions that the marginal cost is decreasing because the overall cost is concave. The problem is formulated as a non-linear optimization model with integer variables. Some properties of the model are proved. A branch and bound approach is suggested, and a greedy algorithm is used to get the initial solution. The proposed method can solve problems with $20-25$ sites and more than 250 customers, given the running time allowed.

The work of Cohen and Moon (1991) emphasize production rather than location. However, their research is related to ours because of its treatment of economies of scale. Those authors address a plant loading problem with the goal of minimizing total annual costs including inbound (purchasing and transportation), production (fixed and variable), and outbound (transportation and handling) costs by selecting inbound raw material flows, the plant product mix, and outbound shipments of finished product. Cohen and Moon (1991) consider the existence of scale and complexity (scope) effects at the plant level, which result in the nonlinear production costs with respect to product-line volume. Specifically, if additional product lines are assigned to a plant, the plant must absorb additional fixed costs and will experience higher (but less than proportional) variable production costs. To solve this piecewise linear concave problem, they develop a variant of Benders decomposition which is implemented in GAMS. The proposed solution algorithm works successfully with 3 product groups, 4 plants, 10 DCs and 10 customers. That algorithm can be applied to problems with a cost function of any shape that can be expressed as a piecewise linear function.

2.4 Economies of Scale in Location-Routing Problems

To the best of our knowledge, only Melechovsky et al. (2005) discuss the LRP with warehouse economies of scale, i.e. non-linear variable costs, and develop a mixed integer non-linear model. They propose a meta-heuristic method which combines Variable Neighborhood Search with Tabu Search. Those authors show that these two approaches together are better than either one by itself. They focus on demonstrating the performance of the hybrid method. However, they do not discuss the non-linear cost function in detail. They do mention that those non-linear costs have an effect on the assignment of customers, in that a customer may be served by a facility which is not the closest.

2.5 Solution Methods in Solving Location-Routing Problems

The influence of *two* NP-hard problems, namely the location problem and the vehicle routing problem, makes it difficult to present an exact algorithm. There is thus only a small number of publications on exact solution methods for the LRP. As stated by Prodhon and Prins (2014), lower bounds have been obtained by only a few researchers, e.g. Belenguer et al. (2011) and Contardo et al. (2014), namely those implementing exact methodologies. Most location-routing problems therefore employ efficient meta-heuristics.

Wu et al. (2002) suggest such a method for solving the multi-depot location-routing problem with limited homogeneous fleets. Meta-heuristic methods are developed respectively for the location problem and the vehicle routing problem. Each subproblem is solved in a sequential and iterative manner by a simulated annealing algorithm with a tabu list to avoid cycling. The approach of Wu et al. is efficient for solving problems with up to 85 customers.

Prins et al. (2006a) design a greedy randomized adaptive search procedure for a general LRP with uncapacitated vehicles, based on the Clarke-Wright algorithm, combined with a learning process. Prins et al. (2007) solve location routing problems with a heuristic decomposition method. Their proposed method alternates between a facility-location phase, solved by a Lagrangian-relaxation approach, and a routing phase, handled by a granular Tabu search. Both formulations of Prins et al. are similar to that of Wu et al. (2002). However, either approach of Prins et al. (2006a, 2007) is able to solve problems with a greater number of customers than in Wu et al.

The model studied by Prins et al. (2007) is defined on a complete, weighted, and undirected network. The objective is to minimize the total cost, including fixed costs of depots plus total variable costs of the routes. Prins et al. (2007) compare their method with other approaches, and find that they can improve 80% of the values obtained by others on three data sets. They also analyze the factors that influence the cooperation and alternation between the location phase and the routing phase in their method, pointing out that their approach performs well on large scale instances.

Prins et al. (2007) tested their method on three data sets. Those are the publicly-available instances of Tuzun and Burke (1999), Barreto (2004), and Prins et al. (2006a). Prins et al. (2007) were able to solve instances with up to 200 customers. We will solve instances from these same data sets in our numerical experiments (Chapter 5) after first adapting or extending those instances to include (concave) operating costs of facilities.

We now turn to Chapter 3 to introduce our two mathematical models. Both include operating costs of facilities, but only the second (Sec. 3.4) has economies of scale.

Chapter 3

Mathematical Models

3.1 Problem Definition

We consider the location-routing problem with capacity on both depots and vehicles, and introduce variable cost of DCs to the formulation. The optimal locations of distribution centers are chosen from a set of feasible potential locations, taking into account that vehicle routes must be operated from the chosen sites. That is, customers should be assigned to the opened DCs and routes need to be constructed.

For the model without economies of scale, the preceding variable cost of a DC is expressed as a unit operating cost multiplied by the total demand. When economies of scale are present, a concave function of facility throughput is employed for calculating the variable cost of a DC. Both formulations have the objective of achieving minimal total cost.

Several assumptions pertain to our model:

- The vehicles are homogeneous, each with the same capacity.

- Each vehicle serves a route where total demands are within that capacity.
- There is a single product type.
- Facilities can have different capacities, but are the same in all other aspects. In other words, each facility has the same unit operating cost.

3.2 Notation

The notation below applies to both of our proposed models, i.e. with and without the economies of scale. The location routing problem with economies of scale can be characterized by the following parameters:

- I: set of indices of possible locations of DCs
- J: set of indices of customers
- L: $L = I \bigcup J$: set of nodes
- K: set containing vehicles in the fleet
- A_i: fixed cost of DC i, $\forall i \in I$
- U: unit operating cost of DCs for the case without economies of scale
- C_{ij}: transportation cost from node i to node j, $\quad \forall i, j \in L$
- W: vehicle capacity
- F: fixed cost per vehicle

- Q_i: maximum throughput (capacity) of DC i, $\forall i \in I$

- d_j: demand of customer j, $\forall j \in J$

Variables

- $x_{ijk} = \begin{cases} 1 & \text{if node } i \text{ is immediate predecessor of node } j \text{ on the vehicle } k \quad \forall i, j \in L, \forall k \in K \\ 0 & \text{otherwise} \end{cases}$

- $v_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is served by DC } i \\ 0 & \text{otherwise} \end{cases}$

- $y_i = \begin{cases} 1 & \text{if DC } i \text{ is open} \\ 0 & \text{otherwise} \end{cases}$

The objective of the model is to select locations, and at the same time, find best routes with the minimum total cost. That includes the fixed cost to locate a facility, nonlinear operating cost, and transportation cost. In addition, the following constraints should be satisfied:

- Each customer must be assigned to a single route.

- Each route starts and ends at a DC.

- The total load of every vehicle is within its capacity limit.

- The total demand of the customers served by any one DC cannot exceed the capacity of that DC.

The problem can be modeled based on the classical location-routing model and a location problem with economies of scale. The formulation presented below begins with the location-routing model given by Prins et al. (2006a). Our model differs from that of Prins et al. by the addition of the variable cost term in the objective function, but otherwise only in small change in notation. The constraints (3.2) through (3.11) in Sec. 3.3 are identical to those of Prins et al. (2006a). The objective function in the model $[LRPV]$ adds a variable-cost term, $\sum_{j \in J} U d_j$, which is not considered by Prins et al.

Note that when economies of scale are absent, the operating cost (variable cost) for that model simply represents a *constant* addition to the objective function of models of Prins et al. (2006a) and others that ignore operating cost. We include the variable-cost term so that we may compare the results without economies of scale to those when economies of scale are present. The latter model, $[LRPES]$, is presented in Sec. 3.4. The constraints are unchanged; the variable-cost term is replaced by Eq. (3.1) below.

That variable-cost term in the objective function of the model $[LRPES]$ thus ensures that the operating cost exhibits concavity. The concave variable cost function is f_i; its argument is the total throughput of facility i:

$$f_i(\sum_{j \in J} d_j v_{ij}) = \gamma(\sum_{j \in J} d_j v_{ij})^{\delta} \tag{3.1}$$

That total throughput is summed over each facility i in $[LRPES]$. The non-linear cost function is illustrated in Figure 3.1, in which we set $\gamma = 25$ and $\delta = 2/3$. With an increase in the quantity handled at a DC, the operating cost per unit (the slope of the curve) decreases. Attainment of such economies of scale requires that $0 < \delta < 1$ in Eq. (3.1).

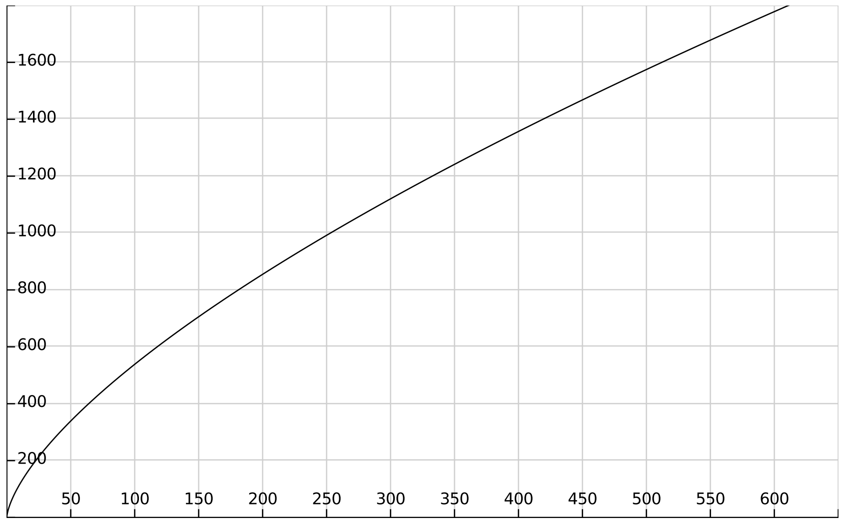


Figure 3.1: Variable cost function when $\gamma = 25$ and $\delta = 2/3$

3.3 Model without Economies of Scale

The location-routing problem with variable cost but no economies of scale is:

$$[LRPV] \qquad \min \quad z = \sum_{i \in I} A_i y_i + \sum_{j \in J} U d_j + \sum_{i \in L} \sum_{j \in L} \sum_{k \in K} C_{ij} x_{ijk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F x_{ijk}$$

subject to

$$\sum_{k \in K} \sum_{i \in L} x_{ijk} = 1 \quad \forall j \in J \tag{3.2}$$

$$\sum_{j \in J} \sum_{i \in L} d_j x_{ijk} \leq W \quad \forall k \in K \tag{3.3}$$

$$\sum_{j \in L} x_{ijk} - \sum_{j \in L} x_{jik} = 0 \quad \forall k \in K, \forall i \in L \tag{3.4}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \leq 1 \quad \forall k \in K \tag{3.5}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} \leq |S| - 1 \ \ \forall S \subseteq J, \ \ \forall k \in K \tag{3.6}$$

$$\sum_{u\in J} x_{iuk} + \sum_{u\in L\setminus\{j\}} x_{ujk} \leq 1 + v_{ij} \ \ \forall i \in I, \forall j \in J, \forall k \in K \tag{3.7}$$

$$\sum_{j\in J} d_j v_{ij} \leq Q_i y_i \qquad \forall i \in I \tag{3.8}$$

$$y_i \in \{0,1\} \quad \forall i \in I \tag{3.9}$$

$$v_{ij} \in \{0,1\} \quad \forall i \in I \ \ \forall j \in J \tag{3.10}$$

$$x_{ijk} \in \{0,1\} \quad \forall i \in L \ \ \forall j \in L \ \ \forall k \in K \tag{3.11}$$

The constraints will be discussed in the following section.

3.4 Model with Economies of Scale

$$[LRPES] \qquad \min \quad z = \sum_{i\in I} A_i y_i + \sum_{i\in I} f_i(\sum_{j\in J} d_j v_{ij}) + \sum_{i\in L}\sum_{j\in L}\sum_{k\in K} C_{ij} x_{ijk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} F x_{ijk}$$

subject to

(3.2)-(3.11)

These two models are very similar; differences lie in the objective functions. The objective function of each model is the sum of the operating cost including fixed cost, variable cost of operation, and transportation cost between DCs and customers. In Section 3.3, the variable cost is calculated by multiplying the total throughput by the unit operating cost. Since that unit cost is a constant, the total operating cost in $[LRPV]$ can be calculated directly by the total demand of all customers. However, the model of Section 3.4 employs the *power function* of Eq. (3.1); In $[LRPES]$, the marginal operating cost of an opened DC decreases with the number units of demand satisfied by that DC.

Otherwise, the two models satisfy the same constraints. Equality (3.2) guarantees that each customer will be connected to a unique predecessor by a single vehicle. Constraint (3.3) claims that the capacity of each vehicle will be respected. Relations (3.4) and (3.5) ensure the continuity of routes, and that all vehicles travel back to the origin, i.e. the DC. Sub-tours are eliminated by Constraint (3.6). Inequality (3.7) means that only those locations selected as DCs can become the origin of a route. Constraint (3.8) indicates that the sum of customers' demands satisfied by a given DC cannot exceed the capacity of that DC. Constraints (3.9) - (3.11) show the binary nature of decision variables.

The two models, [LRPV] and [LRPES] (Secs. 3.3 and 3.4), will undergo intense computational testing in Chapter 5. Before that, appropriate solution methods will need to be considered. Chapter 4 addresses that issue.

Chapter 4

Solution Methods

4.1 Overview

The combination of two NP-hard problems makes it difficult to present an exact algorithm; this has reduced the number of publications on exact solution methods. A report on the vehicle routing problem published by the School of Mathematics, University of Edinburgh (anon, n.d.) points out that exact methods have solved only "small" vehicle routing problem instances, and could take days to solve a particular instance consisting of less than one hundred customers. After experiments and comparison, that report suggests the use of meta-heuristics.

There are four survey papers on the LRP published since the year 2000: Nagy and Salhi (2007), Borges Lopes et al. (2013), Prodhon and Prins (2014), and Drexl and Schneider (2015). The first three indicate that approximate or meta-heuristic approaches are more suitable than exact algorithms. Prodhon and Prins (2014) emphasize the contrasting approaches for the location-routing problem. They present and compare various methods used to solve these problems. They point out that due to the complexity of the problem, very few papers can obtain lower bounds or propose an exact method, and that exact methods can solve only small instances.

The majority of papers thus develop meta-heuristics. According to the survey of Prodhon and Prins (2014), meta-heuristics give relatively better results than those of ordinary heuristics, or even those of an optimization model when the allowed number of iterations is limited. The survey also mentions the "hybridizing" of a heuristic approach with an exact method, but this class does not give a better performance than the pure meta-heuristics.

Prodhon and Prins (2014) compare several methods. Adaptive large neighborhood search, ant colony optimization, simulated annealing, and the greedy randomized adaptive search procedure are the top four performers based on three data sets: Tuzun and Burke (1999), Barreto (2004), and Prins et al. (2006a). (We remark that the first of these data sets treats only the case in which the facilities are uncapacited.) The last two are the data sets against which we test our algorithms in Chapter 5.

Meta-heuristics are thus efficiently employed in most location-routing problems. Some methods might offer a good quality of solution but are poor in term of running time. Ant colony optimization, however, gives a suitable compromise between them. Since the majority of papers cannot obtain a lower bound, authors often illustrate their solution quality by comparing their results with the best known results.

Melechovsky et al. (2005) use Tabu search, combined with a variable neighborhood search, for a problem with warehouse economies of scale. Because there is no best known result on this problem, they compare the solution obtained by their method with results returned by purely Tabu search and pure variable neighborhood search, respectively. The results show that the hybrid method gives a better performance.

4.2 Methods Considered

As mentioned above, we have chosen to employ meta-heuristics. We came up with two methods based on genetic algorithms to address our problem: simulated annealing embedded within a genetic algorithm, and a genetic algorithm with ant colony optimization. Details of the first approach are in Sec.4.3 (See Figure 4.1). The flow chart of the second method is given in Figure 4.8 and discussed in Sec. 4.4.

The foundation of either optimization algorithm is Darwin's evolutionary theory. Choice of location, customer aggregation, and vehicle selection are solved by our genetic system. Routing improvement is addressed by the addition of a 2-opt routine in a simulated annealing (SA) algorithm, or by ant colony optimization (ACO).

4.3 Simulated Annealing embedded within Genetic Algorithm

The flow chart for this method is presented in Figure 4.1.

4.3.1 Generation of Feasible Solutions

We construct a number of random feasible solutions as our initial population as follows. First, the algorithm develops a list of available DCs, and another list of customers that are not yet served. Next, it randomly assigns a vehicle to any DC (i.e. depot) on the list, and

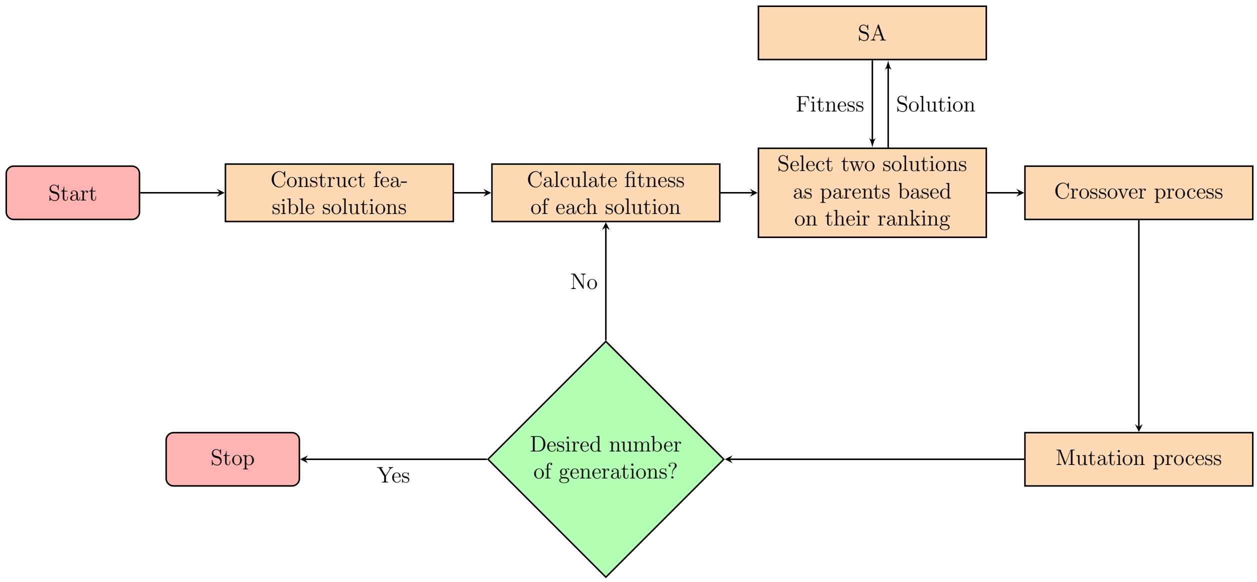


Figure 4.1: Simulated Annealing embedded within Genetic Algorithm

then assigns customers to the nearest DC. Whenever a vehicle's capacity is exceeded, a new route is started. If any DC's capacity becomes violated, that depot is eliminated from the list of those available depots. Any customer already served would also be deleted from the list of customers that are not yet served. The whole algorithm stops when that list no longer contains any customers.

4.3.2 Coding and Decoding

The initial population is thus found as in Sec. 4.3.1, yielding the feasible solutions. The algorithm then enters the next phase: coding each solution into a chromosome sequence. Each sequence indicates one route, beginning at a particular DC and indicating the order in which customers are served. Binary numbers are used to represent the DCs from which the vehicles depart, and the customers that each vehicle serves. Each chromosome corresponds to one vehicle.

To be specific, binary number 0 means a certain DC or customer is not selected on this vehicle; on the contrary, we use 1 to represent those selected. Figure 4.2 gives an example with three potential DCs and five customers. Here, DCs D1 and D2 are opened. Thus, the chromosomes shown in Figure 4.3, corresponding to Figure 4.2, are explained as vehicle 1 departing from depot 1 serving customers 2,3,5; and vehicle 2 departing from depot 2 serving customers 1 and 4.

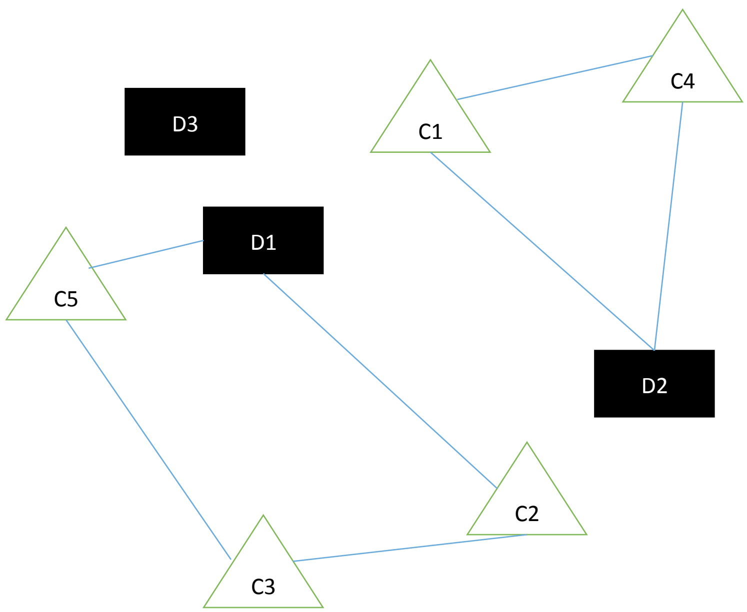


Figure 4.2: Sample customer assignments and routes

	Depot1	Depot2	Depot3	Customer1	Customer2	Customer3	Customer4	Customer5
Vehicle1	1	0	0	0	1	1	0	1
Vehicle2	0	1	0	1	0	0	1	0

Figure 4.3: Two sample chromosomes, corresponding to Figure 4.2

4.3.3 Fitness Calculation

$$[FC] = \sum_{i\in I} A_i y_i + \sum_{i\in I} f_i(\sum_{j\in J} d_j v_{ij}) + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} C_{ij} x_{ijk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} F x_{ijk}$$
$$+ \sum_{i\in I} P_1 \max(\sum_{j\in J} d_j v_{ij} - Q_i y_i, 0) + \sum_{k\in K} P_2 \max(\sum_{i\in J}\sum_{I\in V} d_j x_{ijk} - W, 0)$$

[FC] denotes the calculation of the fitness function. It consists of facility operating cost, transportation cost, the fixed cost of using a route, and two penalties. Each chromosome's fitness will be calculated by the function [FC]; smaller cost denotes a better level of fitness. Eq. (4.1), below, thus involves the calculated fitness FC_l of chromosome l.

The processes of crossover and mutation in our system may lead to infeasible solutions. We thus add penalties to the fitness function, so that an infeasible solution will have a lower fitness. P_1 is the penalty coefficient that is applied when the capacity of any depot is exceeded. P_2 is the corresponding coefficient for the per-unit violation of a vehicle's capacity. The penalty coefficients P_1, P_2 need to be "large" (consistent with the other parameters in the particular problem instance), so that the probability of an infeasible solution is small.

4.3.4 Selection and Route Optimization: 2-opt Exchange within SA

All chromosomes are then decoded for the simulated annealing process. Route optimization employs 2-opt exchange, which means that two particular customers interchange their positions.

A genetic algorithm proceeds from one generation to the next. However, before allowing it to do so, we have found that, within the genetic algorithm, it is helpful to employ simulated annealing, to enhance the probability of selection for the next generation of those chromosomes that have a better fitness level. This use of simulated annealing, in combination with a genetic algorithm, has been shown to be worthwhile in a previous publication about the vehicle routing problem (Tuzkaya et al. 2012).

The possibility of accepting a worse solution follows the principle of simulated annealing. Simulated annealing is a probabilistic meta-heuristic that imitates the annealing process in metallurgy. The goal is to escape a local optimum. The whole algorithm starts with an initial temperature and cooling-rate setting. The probability to accept a worse solution is calculated by the formula: $\exp(-\frac{\text{current cost}-\text{best cost}}{T})$. The gap between current route cost and best cost, and the temperature T, determines the chance to accept a worse solution. As we see from this formula, if the gap between the two solutions is larger, acceptance of this inferior solution would be less likely. Moreover, the temperature T becomes cooler (value becomes smaller) after a certain number of iterations. This guarantees that the probability of accepting a worse solution will gradually become smaller. The lowest cost of each chromosome explored by this algorithm is returned as the "fitness" of that chromosome.

4.3.5 Crossover

Next, each solution receives its ranking based on the fitness (smaller cost means better fitness and higher ranking). Then the system enters into our crossover process. Crossover refers to the replacement of part of the structure of the two parent individuals to generate new recombinations. Through crossover, the search capability of the genetic algorithm is improved. We select two chromosomes from the whole population (chromosomes with better ranking have larger probability to be selected). These two solutions are crossed over, to generate a totally new offspring. Specifically, two random numbers are initially drawn, then all genes between these two numbers are exchanged so that we get two new offspring chromosomes. For example, as shown in Figure 4.4, the genes in the positions between numbers 5 and 7 of two parent chromosomes are exchanged. The other parts of the parent chromosomes remain the same, and two offspring chromosomes are generated, as presented in Figure 4.5.

parent 1

1	0	0	0	1	1	0	1

parent 2

0	1	0	1	0	0	1	0

Figure 4.4: Parents-before crossover

offspring:

1	0	0	0	0	0	1	1

0	1	0	1	1	1	0	0

Figure 4.5: Offspring-after crossover

4.3.6 Mutation

Mutation, an important process to escape local optima, is forced to occur to preserve genetic diversity. A mutation causes a change in one or more genes of a chromosome. That is, a previous value of 0 becomes 1, or vice-versa (See Figure 4.6).

Most mutated chromosomes become bad solutions or even infeasible. Therefore, the mutation rate should be rather small. After defining our initial mutation rate, if a randomly-drawn number from $U(0,1)$ is less than that mutation rate, the chromosome will progress into mutation. There are two situations: customers on the original route will be switched to another route, or a mutated route's original depot will be randomly reassigned. As shown in Figure 4.6, in this mutation, customer 2 is removed from the original route (Figure 4.3) and served by vehicle 2 instead. Figure 4.7 depicts the second possibility. In this mutation, vehicle 1 is re-assigned from depot 1 (see Figure 4.3) to depot 3.

	Depot1	Depot2	Depot3	Customer1	Customer2	Customer3	Customer4	Customer5
Vehicle1	1	0	0	0	0	1	0	1
Vehicle2	0	1	0	1	1	0	1	0

Figure 4.6: Mutated chromosomes-reassignment of Customer 2

	Depot1	Depot2	Depot3	Customer1	Customer2	Customer3	Customer4	Customer5
Vehicle1	0	0	1	0	1	1	0	1
Vehicle2	0	1	0	1	0	0	1	0

Figure 4.7: Mutated chromosomes-vehicle reassigned to different depot

4.4 Genetic Algorithm with Ant Colony Optimization

The method that we tested for route improvement is to employ ant colony optimization within the genetic algorithm. Ant colony optimization has been used widely for solving routing problem, e.g. Yu et al. (2009) and Abdulkader et al. (2015). We previously suggested the incorporation of simulated annealing within the GA. Now, however, we decided for ant colony optimization within the genetic algorithm for our location-routing problem.

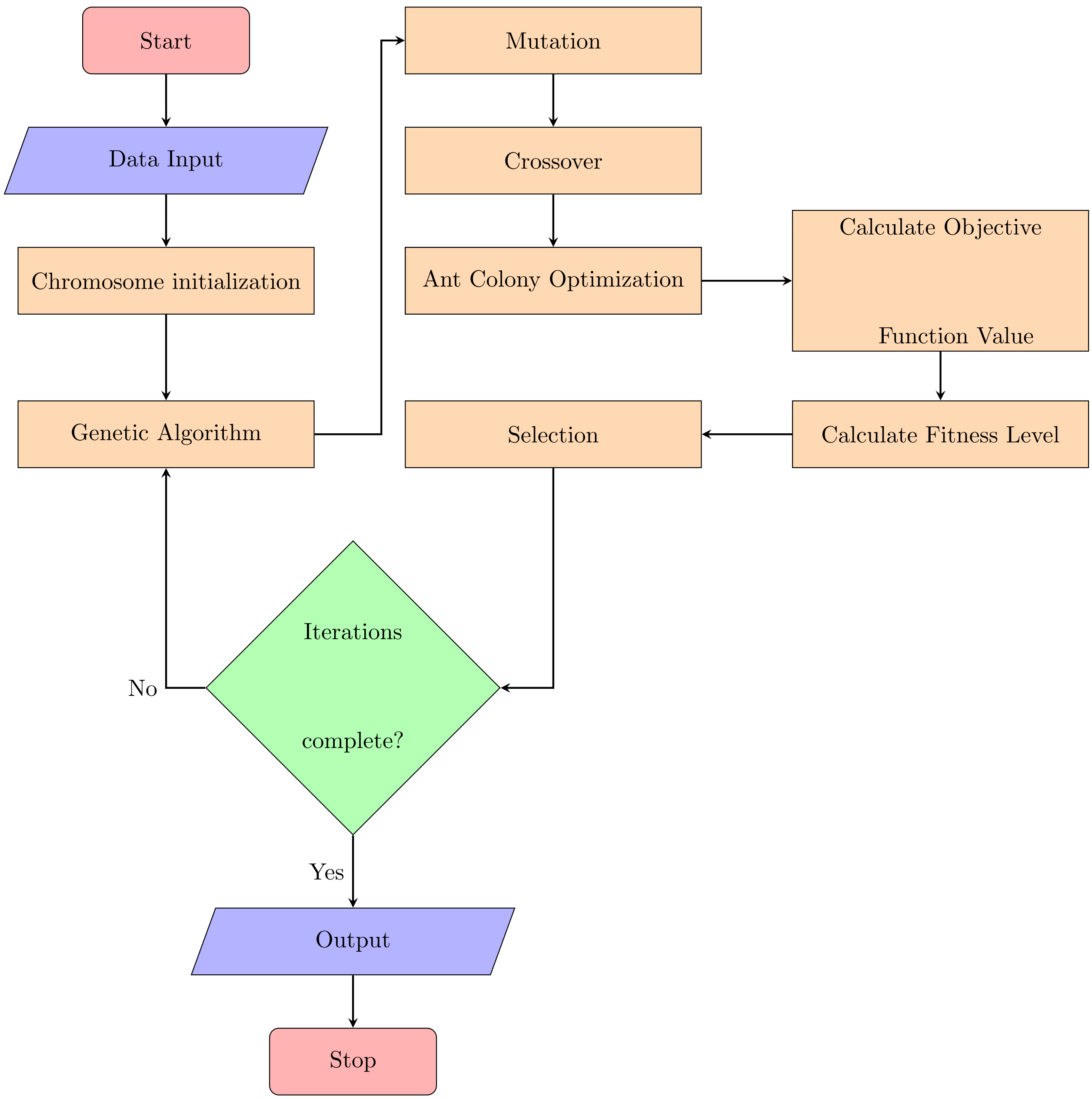


Figure 4.8: Flow chart of genetic algorithm with ant colony optimization

Ant colony optimization algorithm was first introduced by Dorigo (1992). It is based on the nature of ants' behavior in seeking food. As mentioned by Dorigo and Stützle (2009), ant colony optimization has since been successfully applied to various combinatorial optimization problems. Hence we employ ACO to improve the routes from the genetic algorithm. Some parts of this approach are the same for the case of simulated annealing within the GA (Fig. 4.1). Differences lie in chromosome selection and in route optimization. The flow chart of Figure 4.8 shows the entire algorithm.

4.4.1 Selection

We employ "roulette wheel" selection. The probability that a given chromosome will be selected is in direct proportion to its level of fitness. Let N denote the set of all chromosomes, and $n = |N|$ represent the cardinality. Suppose the size of the chromosome population is n, and chromosome l has the fitness level of FC_l (see Sec. 4.3.3). The probability that chromosome l will be selected is then given by formula (4.1):

$$p_l = \frac{FC_l}{\sum_{l \in N} FC_l}. \tag{4.1}$$

Figure 4.9 illustrates that every chromosome occupies one part of the pie chart; the size of the area is related to the chromosome's level of fitness. Let q_l be the cumulative probability of chromosome l ($l \in N$), which can be calculated by Eq. (4.2):

$$q_l = \sum_{l \in N} p_l \tag{4.2}$$

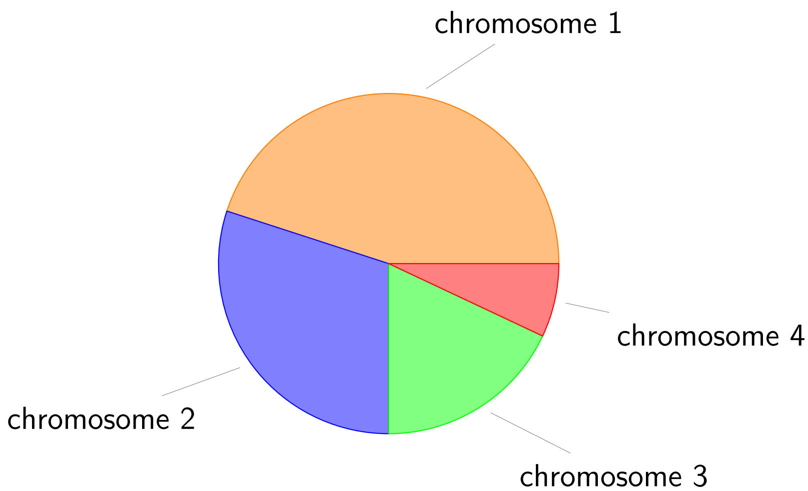


Figure 4.9: Roulette wheel selection

The following steps indicates how roulette wheel selection works:

- Generate a random number r from the uniform distribution on [0,1];
- Compare r with q_1. If $r \leq q_1$, chromosome 1 is selected;
- If $q_{l-1} < r \leq q_l (2 \leq l \leq n)$, chromosome l is selected.

4.4.2 Route Optimization: Ant Colony Optimization

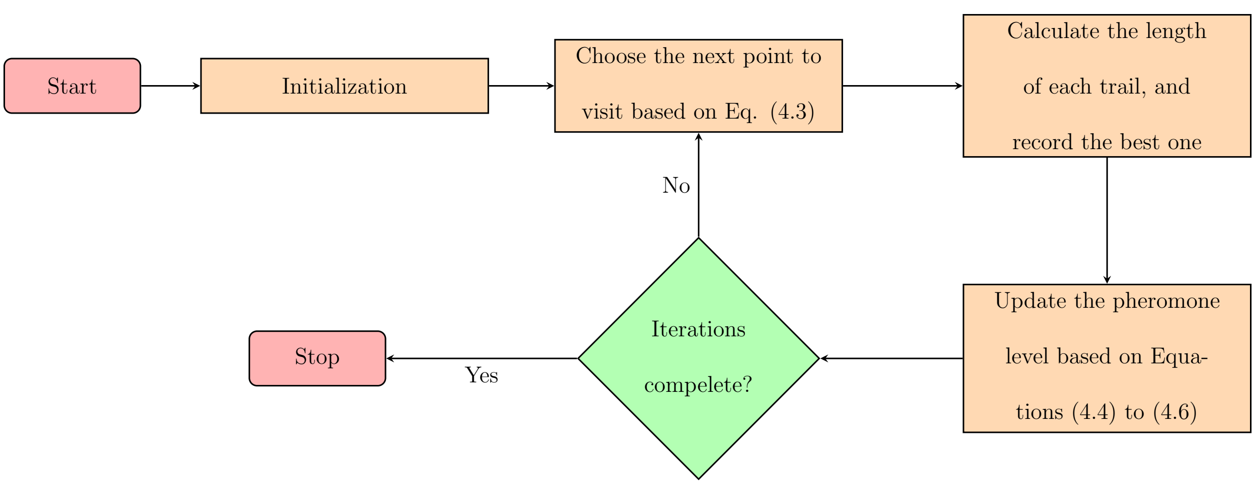


Figure 4.10: Ant colony optimization-detailed flow chart

In the solution obtained from previous steps (See Fig.4.8), ant colony optimization is used to improve the (randomly-chosen) initial routes. The flow chart is shown in Figure 4.10.

Each of the m ants selects the next point to visit based on the probability p_{ij}^k in Eq.(4.3). No ant is allowed to go to those demand points already visited by itself; this is controlled by a tabu list that records the visited points. An ant leaves *pheromone* on its trajectory. At the beginning, the amount of pheromone is the same on every route. During the ACO step in Figure 4.8, the quantity of pheromone (i.e. the pheromone level) decides the ant's movement. α, β, ρ are parameters chosen to control the relative importance of pheromone and to update the information obtained by the ACO heuristic. Specifically, α controls the influence of a trail and the effect of τ_{ij} (See Eq. 4.3). β indicates the importance of η_{ij} in that equation. ρ is the rate of evaporation of pheromone. The following parameter values are based on previous literature on ant colony optimization (Gaertner and Clark 2005, Çatay 2009): $\alpha = 1$, $\beta = 4$, $\rho = 0.1$, and the number of iterations is set to 50.

Let τ_{ij}^{α} represent the concentration of pheromone on edge (i, j), and p_{ij}^k denote the probability of ant k moving from node i to node j. That is calculated by the formula:

$$p_{ij}^k = \begin{cases} \frac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{s \in allowed_k} \tau_{is}^{\alpha} \cdot \eta_{is}^{\beta}}, & j \in allowed_k \\ 0, & \text{otherwise} \end{cases} . \tag{4.3}$$

η_{ij} means the "visibility" of node j, as seen from node i. We take $\eta_{ij}(t) = \frac{1}{d_{ij}}$, where d_{ij} is the distance between i and j. $allowed_k = 0, 1, ...n - 1 - tabu_k$, and represents the list of nodes that ant k can visit. After the ant finishes one iteration, the information on the route is updated by Equations (4.4) - (4.6):

$$\tau_{ij} \longleftarrow (1-\rho)\tau_{ij} + \Delta\tau_{ij} \tag{4.4}$$

$$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \tag{4.5}$$

$$\Delta\tau_{ij}^{k} = \begin{cases} \frac{Q}{L'_k}, & \text{if k-th ant traverses the edge } (i,j) \\ \\ 0, & \text{otherwise} \end{cases} . \tag{4.6}$$

In Equations (4.4) to (4.6), $(1-\rho)$ is less than 1, indicating that the concentration of pheromone left over will decrease with time. $\Delta\tau_{ij}^{k}$ represents the quantity of pheromone left by ant k on the edge between i and j, and L'_k is the total distance that ant k has moved in going from i to j. These equations are employed in each iteration of the ACO algorithm.

Sections 4.3 and 4.4 have thus summarized the methods that we consider. In Chapter 5, experiments are done for both approaches. We compare the results in terms of solution quality and running time. And the one that gives a better performance will be employed for the majority of tests.

Chapter 5

Computational Results

The performance of the proposed solution methods is now evaluated. The suggested algorithms are coded in Matlab 2015b, and tested on a computer with an i-5 Quad Core processor running at 2.6 GHz and 16 GB of memory.

In the first group of experiments (Sec. 5.1), the number of demand points n ranges from 10 to 100, with $m = 3$, 7, or 15 potential depots(DCs) opened. The coefficient of transportation cost C_{ij} is 2, which means that this cost is twice the Euclidean distance traveled on arc (i, j). Coordinates of DCs and demand points are generated from uniform distributions between [0, 50]. The capacity of each vehicle is 100. Opening a depot incurs a random cost in the range of 800 to 1000. And the capacity Q_i of each depot is 450. The demand of each customer is generated uniformly in the range [20,30]. For the experiments without economies of scale, we set the constant variable cost to be $8 per pallet. Here, we use 1 pallet of product as the unit of demand. When economies of scale are present, we employ the function f_i in Eq. (3.1), whose initial slope (see Figure 3.1) corresponds to $8 per pallet.

For all experiments, we first set the parameters of Eq. (3.1) after preliminary tests for the cases when economies of scale are present. Then, from the derivative of Eq. (3.1) $f'(x) = \delta\gamma x^{\delta-1}$, and, according to the average demand $\overline{x}$ in the particular data set, we set $f'(\overline{x})$ as the initial slope

5.1 Comparison of SA and ACO

		No economies of scale-ACO				No economies of scale-SA			
m	n	number of DC opened	fixed cost	total cost	CPU(s)	number of DC opened	fixed cost	total cost	CPU(s)
3	10	1	0.86	3.26	5.86	1	0.86	3.26	5.85
	20	2	1.66	6.85	10.65	2	1.66	6.85	11.67
	30	2	1.66	8.42	19.16	2	1.66	8.54	19.37
7	30	2	1.76	8.54	22.0	2	1.76	8.56	23.2
	30	2	1.66	8.78	22.8	2	1.66	8.80	23.7
	50	4	3.50	12.0	50.3	4	3.48	12.2	52.9
	50	4	3.52	12.1	51.7	4	3.52	12.6	54.1
	70	4	3.52	15.8	97.0	4	3.52	15.9	101.1
	70	4	3.52	15.6	96.1	4	3.52	15.6	100.3
15	100	6	5.28	18.9	307.7	6	5.30	19.2	309.9
	100	7	6.10	20.8	309.8	7	6.10	21.1	310.2

Table 5.1: Algorithm comparison: No economies of scale

All costs in \$000

		With economies of scale-ACO				With economies of scale-SA			
m	n	number of DC opened	fixed cost	total cost	CPU(s)	number of DC opened	fixed cost	total cost	CPU(s)
3	10	1	0.86	2.68	5.01	1	0.86	2.68	5.00
	20	2	1.66	5.43	12.28	2	1.66	5.43	13.23
	30	2	1.66	7.17	18.98	2	1.66	7.28	19.37
7	30	2	1.76	7.26	24.7	2	1.76	7.29	25.6
	30	2	1.66	7.24	25.2	2	1.66	7.28	26.6
	50	3	2.73	10.3	50.1	3	2.53	10.4	52.9
	50	3	2.53	9.98	49.2	3	2.53	10.4	52.0
	70	4	3.52	13.6	96.0	4	3.56	13.9	101.4
	70	4	3.52	13.5	96.8	4	3.52	13.7	102.7
15	100	5	4.38	16.0	308.2	5	4.40	16.4	311.7
	100	5	4.42	16.8	310.0	5	4.42	16.9	315.5

Table 5.2: Algorithm comparison: With economies of scale

All costs in \$000

Tables 5.1 and 5.2 compare the results returned by the genetic algorithm with ant colony optimization, to those from simulated annealing containing an embedded genetic algorithm. In those tables, we use $\gamma = 25$ and $\delta = 2/3$(See Eq. 3.1). The model in Table 5.1 has no economies of scale. Both tables show that for the small-size instance (3 DCs and 10 or 20 customers), the two algorithms give the same result. For example, when there are no economies of scale, a single DC is opened with a fixed cost of $860 and total cost of $3,260 for the instance including 3 DCs and 10 customers. Running times are almost the same for the two approaches.

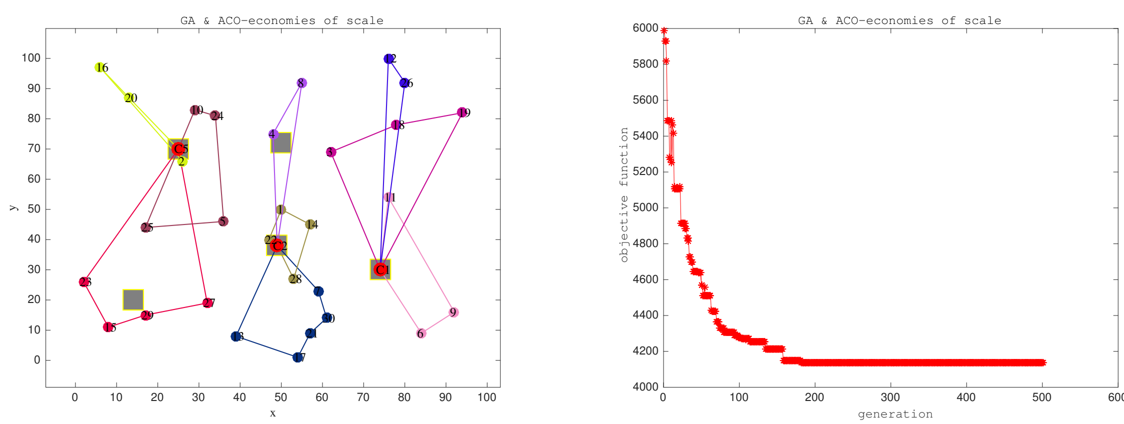
Deviations begin to appear as the size of the instance grows. From Table 5.1, consider the first row when $m = 7$ and $n = 50$. Both methods recommend four DCs, but since the resulting fixed costs differ, we know that the methods open distinct DCs. Although the ACO has a fixed cost of $3,500, $200 greater than that of SA, it has a lower total cost of $12,000. Besides, the running time is shorter for ACO. For the same instances but for the model with economies of scale, we find similar results in Table 5.2. In general, from the perspectives of cost and running time, the genetic algorithm with ant colony optimization performs better than simulated annealing embedded within a genetic algorithm in these experiments.

We suspect, but cannot prove, that the additional running time of SA over ACO may became greater as the size of problem instances increase. We thus use ant colony optimization with the genetic algorithm to test additional instances and analyze the influence of economies of scale.

5.2 Testing GA-ACO on Published Data Sets

In all groups of experiments, the ACO & GA algorithm is implemented with five runs for each instance. We set the probability of crossover equal to 0.7. In this section, we set $\delta = 2/3$ in Equation (3.1). In each run of the same instance, to obtain a better solution, we may change the size of the initial population, the number of generations, or the probability of mutation.

Figure 5.1 presents solutions and shows the trend in total cost over successive generations. This example has 5 DCs and 30 customers, and is solved by ant colony optimization with genetic algorithm. (In this example, all data are randomly generated. Each DC has a capacity ranging from 400 to 500, every customer's demand is uniform in the interval [30,50], vehicle capacity is 200, fixed cost of opening a DC ranges between 800 and 900, fixed cost to use a vehicle is 20, and the coefficient of transportation is $1/km.). In this experiment, as frequent occurs when economies of scale are present, we found that a given customer can be assigned to a DC with a higher total throughput, instead of to the nearest one [Fig. 5.1(a)].



(a) Solutions (b) Convergence of optimal objective function value

Figure 5.1: Example with 5 potential DCs and 30 customers

Note that often we cannot compare our algorithm with another paper dealing with variable cost. Those authors may use their own data, to which we have no access. However, the way we can illustrate the quality of our proposed method is by comparing results with papers published on the standard LRP, when we and they employ public data sets. Three data sets have been tested by a number of authors: instances introduced by Tuzun and Burke (1999), by Barreto (2004), and by Prins et al. (2006a). For the second and third ones, both the depots and vehicles are capacitated, which is the case in this thesis. Those are the data sets on which we will illustrate and test our method. Whether for Barreto or Prins et al, instances in our tables in this section follow the same sequence as in theirs.

5.2.1 Barreto's Instances

We will conduct two groups of experiments on instances generated by Barreto (2004). That data set, available at *http://prodhonc.free.fr/Instances/instances_us.htm*, contains 13 instances, between which there are great differences in parameters. Here, there is no fixed cost of using a vehicle. Depots and vehicles have capacities.

In the first group of experiments, we take away the variable cost, which makes the model become a standard LRP. We compare the results of our proposed algorithm with the best-known results (BKR) among GRASP (Prins et al., 2006a), a memetic algorithm with population management (MA|PM, Prins et al., 2006b), and Lagrangean relaxation-granular tabu search (LRGTS, Prins et al., 2007). The gap is calculated by $\frac{Total\ Cost - BKR}{BKR} \times 100\%$.

Results are given in Table 5.3. Although the figures in Tables 5.1 and 5.2 are expressed in \$000, results in the remainder of the thesis will be given in "dollars". This is in keeping

Instance	n	m	BKR	Total Cost	CPU(s)	Gap(%)
Christofides69	50	5	565.6	608.1	65.23	7.51
Christofides69	75	10	861.6	861.6	127.18	0.00
Christofides69	100	10	842.9	906.7	330.57	7.57
Daskin95	88	8	355.8	370.4	262.67	4.13
Daskin95	150	10	44011.7	45109.4	1523.53	2.49
Gaskell67	21	5	424.9	424.9	25.11	0.00
Gaskell67	22	5	585.1	605.1	28.08	3.41
Gaskell67	29	5	512.1	512.1	34.54	0.00
Gaskell67	32	5	562.2	562.2	33.19	0.00
Gaskell67	32	5	504.3	528.1	64.23	4.72
Gaskell67	36	5	460.4	473.1	67.23	2.69
Min92	27	5	3062.0	3062.0	58.92	0.00
Min92	134	8	5809.0	6136.2	560.79	5.63

Table 5.3: Results for the instances of Barreto (2004)-no variable cost

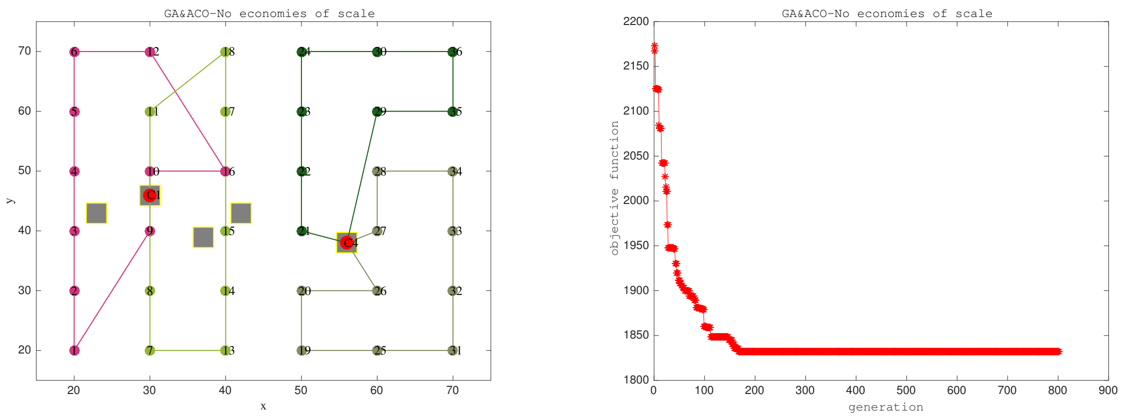
with common practice in the literature.

From Table 5.3, we find that our proposed algorithm obtained the same results as BKR on five of the thirteen instances. With the increase in the number of customers, a gap appears. Although three instances have a gap over 5%, the overall results (median gap = 2.69%) suggest that our proposed method may be acceptable.

The second group of experiments tests our proposed model containing DC operating cost. γ, the coefficient of the variable cost function in Eq. (3.1), is equal to 1 (for instances Gaskell67 when m=21, 22, 29 and 32), or 10 (remaining nine instances of Table 5.3). Of course, γ refers to the model with economies of scale. For the case without economies of scale, we set the operating cost per pallet equal to \$0.06 (for instances Gaskell67 when m=21, 22, 29 and 32), or \$1.50 (again, the nine other instances). Although \$0.06 per pallet seems very small, the demand is high and other costs are low. In such cases, we need to use a figure as small as six cents per pallet, to enable the DC operating cost to be of the same magnitude as the other costs in the model.

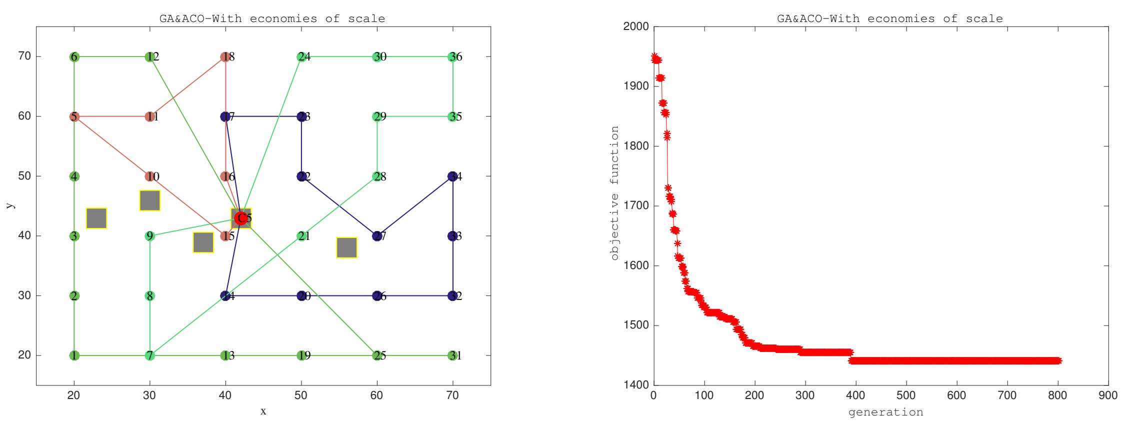
			No Economies of Scale				With economies of Scale			
Instance	**n**	**m**	**Number of DC opened**	**Operating Cost**	**Total Cost**	**CPU(s)**	**Number of DC opened**	**Operating Cost**	**Total Cost**	**CPU(s)**
Christofides69	50	5	4	1166	1790	75.24	2	1064	1650	78.81
Christofides69	75	10	4	2046	2926	130.52	2	1549	2391	133.28
Christofides69	100	10	4	2187	3079	342.67	2	1625	2479	19.99
Daskin95	88	8	-	-	-	-	-	-	-	-
Daskin95	150	10	-	-	-	-	-	-	-	-
Gaskell67	21	5	2	1350	1769	26.37	2	1004	1428	26.92
Gaskell67	22	5	3	611	1259	32.04	2	594	1200	33.48
Gaskell67	29	5	3	765	1318	35.68	2	687	1202	36.33
Gaskell67	32	5	2	1762	2332	33.37	2	1219	1797	33.60
Gaskell67	32	5	2	1762	2296	65.03	1	952	1460	67.42
Gaskell67	36	5	2	1350	1832	70.36	1	935	1440	69.85
Min92	27	5	2	12615	15675	60.90	2	5211	8297	62.98
Min92	134	8	4	11866	17825	565.21	3	5729	11550	569.02

Table 5.4: Results for the instances of Barreto (2004)-with variable cost



(a) Solutions (b) Convergence of optimal objective function value

Figure 5.2: Results of Gaskell67 (n=36, m=5), with variable cost but no economies of scale



(a) Solutions (b) Trends of optimal objective function value

Figure 5.3: Results of Gaskell67 with n=36 and m=5, with economies of scale

$\delta = 2/3$ in Eq. (3.1)

Table 5.4 provides the results, comparing the effects of economies of scale. For eight among the eleven instances we tested, more depots are opened when there are no economies of scale. In general, both the operating cost and total cost are lower when the DCs have economies of scale.

The reader will have noted that the two Daskin95 instances were not tested in Table 5.4. Table 5.3, in the absence of operating cost, show that our method of solution is sound, even for those two instances. However, with the inclusion of operating cost, not only allowed each term in the total-cost function to be of the same order of magnitude, but the parameters to achieve this should be "in proportion" to the respective parameters that reach attain this approximate equally in the other instances. We were unable to achieve this for the two Daskin95 instances.

Returning to the results of Table 5.4, Figures 5.2 and 5.3 compare solutions of the model, without and with the economies of scale, for the instance Gaskell67 (with n=5 and m=36) of Barreto (2004). (The specific data can be found in Table 5.4.) The total cost is lower when there are economies of scale; just one DC is opened (Figure 5.3). In the case of operating cost but no economies of scale, two DCs are used. There are four routes in both Figure 5.2 and Figure 5.3. Although the transportation cost is lower when two DCs are opened (Figure 5.2), the total cost is greater because there are no economies of scale.

5.2.2 Instances of Prins

First, as before, we leave off the variable cost, and test our algorithm on another set of standard benchmarks, $http://prodhonc.free.fr/Instances/instances\_us.htm$, designed by Prins et al. (2006a). This set comprises 30 instances with capacitated depots and vehicles. The number of depots is either 5 or 10, and the number of customers ranges from 20 to 200. The vehicle capacity W is equal to 70 or 150. Euclidean distance is used, and the transportation cost is 100 times the respective distance. The demand of each customer is uniformly distributed between 11 and 20. Results are given in Table 5.5. The best-known result is the terminal result obtained by GRASP or LRGTS for each respective instance. The BKR are available at the same website as above. The gap is calculated by $\frac{Total\ Cost - BKR}{BKR} \times 100\%$.

As shown in Table 5.5, our proposed algorithm obtains the same results on the first four instances. With an increase in the number of customers, a gap appears. For larger instances, the running time increases rapidly. The performance of our designed method is sometimes less acceptable; for three of 30 instances, we have a gap of over 10%. However, the median gap is below 3%. Moreover, works such as Prins et al. (2007) and Ting and Chen (2013) emphasize the performance of their algorithms on the *standard* LRP. The present research concentrates on the effects of economies of scale. Therefore, in that regard, results show that our method is acceptable to some extent.

Next, we experiment with those same instances of Prins et al. (2006a), but we now add the variable cost. For the model with economies of scale, after some preliminary tests, we set the coefficient of the power function to be $\gamma = 55$ in Eq. (3.1). For the one without economies of scale, the operating cost is a constant \$12 per pallet. Table 5.6 presents results of this group of experiments. In the majority of instances, the two models open the same number of DCs. Total cost and operating cost are both lower when there are economies of scale, for all instances. In seven of those thirty instances, the model without economies of scale uses one more DC than in the case of economies of scale. However, not all instances open more DCs when there are no economies of scale; in the data set of Prins et al., the fixed cost to open a facility is quite high. That fixed cost turns out to be much greater than

n	m	W	BKR	Total Cost	CPU(s)	Gap(%)
20	5	70	54,793	54,793	17.29	0.00
20	5	150	39,104	39,104	17.35	0.00
20	5	70	48,908	48,908	16.98	0.00
20	5	150	37,542	37,542	17.57	0.00
50	5	70	90,111	93,188	33.41	3.41
50	5	150	63,242	65,606	30.99	3.74
50	5	70	88,298	90,886	28.08	2.93
50	5	150	67,340	70,847	27.64	5.20
50	5	70	84,055	85,081	32.01	1.22
50	5	150	51,822	52,979	35.23	2.23
50	5	70	86,203	88,530	37.65	2.69
50	5	150	61,830	62,690	34.08	1.39
100	5	70	275,993	280,527	200.70	1.64
100	5	150	214,392	217,588	192.87	1.49
100	5	70	194,598	200,470	210.24	3.01
100	5	150	157,173	159,550	181.55	1.51
100	5	70	200,246	204,322	203.91	2.03
100	5	150	152,586	155,509	185.34	1.92
100	10	70	290,429	311,771	181.08	7.34
100	10	150	234,641	262,322	168.70	11.79
100	10	70	244,265	255,008	178.02	4.40
100	10	150	203,988	210,535	160.23	3.21
100	10	70	253,344	271,946	192.61	7.34
100	10	150	204,597	216,173	179.77	5.66
200	10	70	479,425	491,926	1237.09	2.61
200	10	150	378,773	416,753	1101.84	10.03
200	10	70	450,468	500,571	1198.05	11.12
200	10	150	374,435	378,970	998.76	1.21
200	10	70	472,898	500,004	1201.78	5.73
200	10	150	364,178	376,850	1022.23	3.48

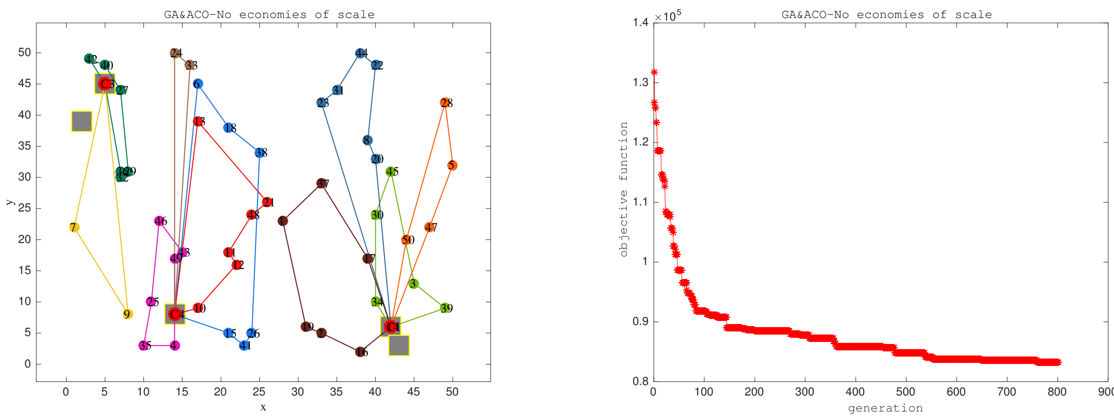
Table 5.5: Results for the instances of Prins et al. (2006a).
No DC operating cost, hence no economies of scale

			No Economies of Scale				With economies of Scale ($\delta = 2/3$)			
n	**m**	**W**	**DC opened**	**Operating Cost**	**Total Cost**	**CPU(s)**	**DC opened**	**Operating Cost**	**Total Cost**	**CPU(s)**
20	5	70	3	3,780	58,573	19.27	3	3,672	58,465	19.64
20	5	150	2	3,696	42,800	19.55	2	3,173	42,368	20.09
20	5	70	3	3,720	52,561	18.98	3	3,675	52,680	19.99
20	5	150	2	3,624	41,166	19.74	2	3,122	42,667	20.86
50	5	70	3	9,072	99,684	36.14	3	6,585	98,082	38.22
50	**5**	**150**	**3**	**9,072**	**79,572**	**33.99**	**2**	**5,756**	**72,275**	**36.13**
50	5	70	3	9,300	98,286	31.40	3	6,701	97,012	32.21
50	5	150	3	9,300	78,042	30.88	3	6,714	75,831	32.55
50	5	70	3	9,228	93,880	35.12	3	6,656	91,737	37.60
50	5	150	3	9,396	62,175	38.22	3	6,736	58,714	39.59
50	5	70	2	9,132	96,532	40.37	2	5,775	92,988	43.04
50	5	150	2	9,132	71,169	37.90	2	5,770	68,270	39.88
100	5	70	3	18,996	298,096	210.02	3	10,778	291,056	214.79
100	5	150	3	18,996	235,668	201.98	3	10,786	227,876	206.08
100	5	70	2	18,696	218,006	219.53	2	9,312	209,196	223.21
100	5	150	2	18,696	177,896	192.04	2	9,331	168,686	196.75
100	5	70	2	18,744	222,190	213.77	2	9,350	212,937	216.68
100	5	150	2	18,744	173,270	195.48	2	9,323	164,632	198.39
100	10	70	3	19,320	311,209	190.04	3	10,929	310,816	192.63
100	10	150	3	19,320	255,322	178.23	3	10,896	247,428	183.52
100	10	70	3	18,432	264,950	188.10	3	10,559	264,615	190.38
100	10	150	3	18,432	222,964	169.21	3	10,598	220,593	178.06
100	10	70	3	18,480	278,280	201.16	3	10,590	281,589	202.95
100	10	150	3	18,480	229,653	180.85	3	10,571	225,884	185.78
200	10	70	4	37,176	575,102	1254.23	3	16,869	508,779	1260.47
200	10	150	4	37,176	479,962	1118.43	3	16,848	432,631	1121.55
200	10	70	3	37,212	500,163	1215.40	3	16,845	508,816	1219.09
200	10	150	3	37,212	415,182	1014.76	3	16,872	395,152	1020.30
200	10	70	4	36,924	584,012	1219.65	3	16,785	516,815	1222.75
200	10	150	4	36,924	470,918	1040.01	3	16,770	392,745	1050.98

Table 5.6: Results for instances of Prins et al.(2006a), with addition of variable cost

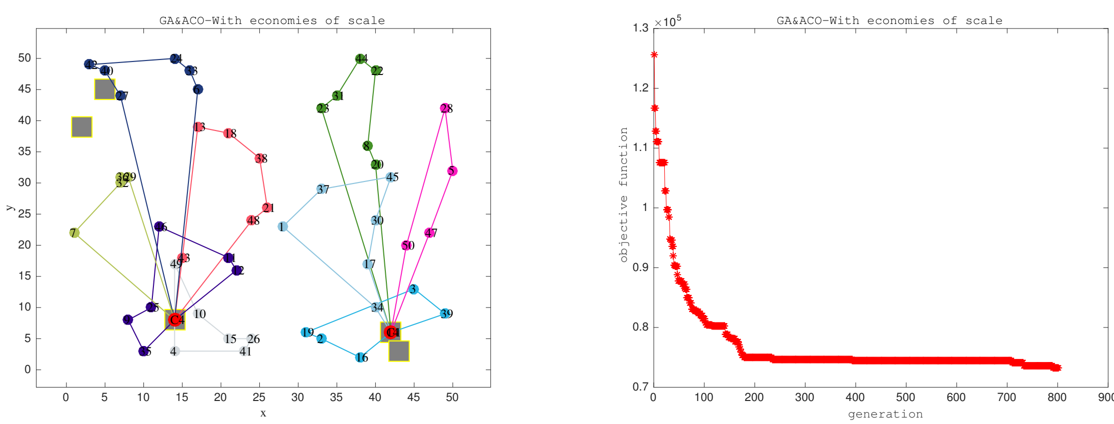
Bolded row corresponds to particular parameter values of Prins et al., for which the solutions are in Figures 5.4 and 5.5.

any savings in transportation cost. Figures 5.4 and 5.5 show the solutions and trends of in the objective function for the models without and with economies of scale, for an instance of Prins et al. (2006a) with n=50, m=5 and W=150. (See the bolded row in Table 5.6.)



(a) Solutions (b) Trends of optimal objective function value

Figure 5.4: Results for n=50, m=5, and W=150, with variable cost, but no economies of scale (See Table 5.6)



(a) Solutions (b) Convergence of optimal objective function value

Figure 5.5: Results for n=50, m=5, and W=150, with economies of scale (See Table 5.6)

5.3 Influence of Economies of Scale

In the previous experiments, when we compared the models with a constant variable cost per unit to those with economies of scale, we found that economies of scale have a positive impact on the operating costs and on total costs. Also, fewer DCs were generally used when economies of scale exist. To better understand the influence of economies of scale and the effect of changes in parameters, we now test some additional cases.

5.3.1 Impact of Exponent δ

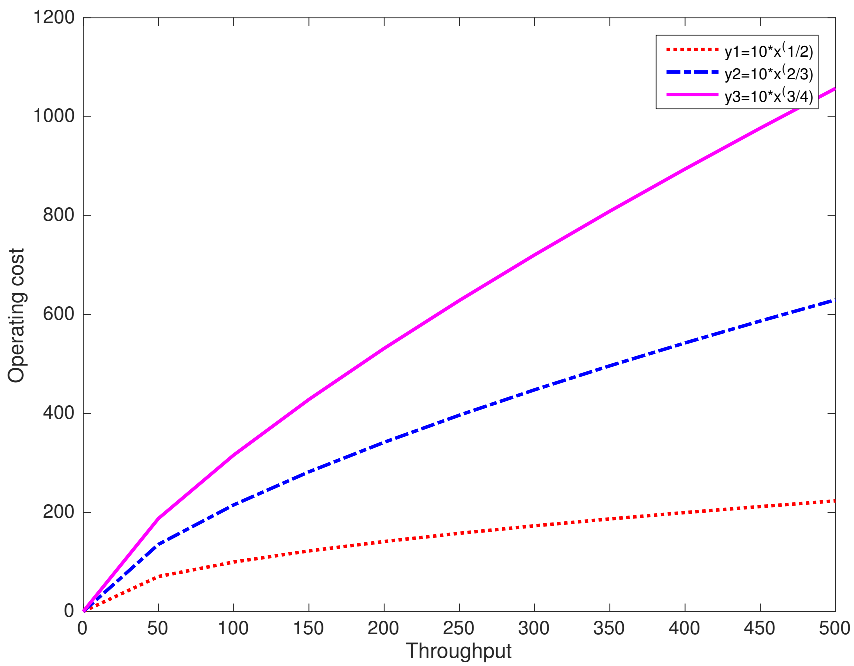


Figure 5.6: Three variable cost functions with differing economies of scale $\delta = 1/2$, $2/3$ or $3/4$ in Eq. (3.1)

To present the effect of the economy-of-scale function on the operating costs and total costs, we test six instances from the data sets of Barreto (2004) and of Prins et al. (2006a).

To better understand the influence of the change in the exponent δ, we choose similar-size instances from these two data sets and compare the results. In the previous section, we used $\delta = 2/3$ in Eq. (3.1) for all experiments with either data set (Tables 5.4 and 5.6). Here, we test three values for this exponent: 1/2, 2/3 and 3/4.

n	m	Instance	W	δ	Operating cost	Total cost	%
50	5	Barreto	160	1/2	395	990	39.9
				2/3	1,064	1,657	64.2
				3/4	1,750	2,340	74.8
50	5	Prins et al.	70	1/2	2,620	94,118	2.8
				2/3	6,585	98,082	6.7
				3/4	10,436	75,942	10.2
50	5	Prins et al.	150	1/2	2,189	68,660	3.2
				2/3	5,756	72,275	7.9
				3/4	9,430	68,660	12.4
100	10	Barreto	200	1/2	565	1550	36.2
				2/3	1,625	2,559	63.5
				3/4	2,810	3,713	75.7
100	10	Prins et al.	70	1/2	3,901	303,790	1.3
				2/3	10,929	310,816	3.5
				3/4	18,797	318,450	5.9
100	10	Prins et al.	150	1/2	3,837	240,421	1.6
				2/3	10,896	247,428	4.4
				3/4	18,102	254,651	7.1

Table 5.7: Impact of exponent δ on economies of scale

In Table 5.7, % in the last column indicates the percentage of total cost contributed by operating cost. We see that with an increase in δ, the operating cost and the percentage of operating cost in the total cost increase for all instances. As expected, the larger the δ, the less the effect of economies of scale, as shown in Figure 5.6. Note that for instance of the same size, operating cost represents a much larger percentage of the total cost in the data set of Barreto (2004). That is because the fixed cost of opening a depot is much lower there than in the instances of Prins et al. (2006a). Also, in the latter instances, the transportation cost is calculated as the distance multiplied by 100, plus the fixed cost of 1000 to use a vehicle. (Recall that in the instance of Barreto, there is no fixed cost to use a vehicle; the transportation cost is the distance multiply by 1.)

In conclusion, with other parameters remaining the same, the percentage of operating cost in the total cost is an increasing function of δ. The economies of scale thus contribute the greatest cost savings for a smaller value of δ. A reduced value of δ could be achieved, e.g. by an improvement of technology employed in the operation of a DC.

5.3.2 Tradeoff Between Operating Cost and Transportation Cost

In previous sections, we noticed that, when economies of scale are present, fewer DCs are often opened; customers may be assigned to a DC that is not the nearest, but which has lower unit operating cost. In other words, the total fixed cost plus DC variable cost is lowered, but the transportation cost is increased. Therefore, there is a tradeoff between the costs of a DC and the transportation cost. To better understand this situation, we run some additional experiments.

We again test the instances of Barreto (2004) and Prins et al. (2006a). In order to analyze the tradeoff, we make the total cost of certain instances equal (within 3% deviation). To achieve the approximate equality of total cost, particular cost parameters are changed. That is done in three ways for each data set, referred to as types a, b, and c for instances of Barreto (2004), and types d, e, and f for those of Prins et al. (2006a). Tables 5.9 and 5.10 give a summary of changes of those three types for each data set. "%" in all Tables

5.9, 5.10 and 5.12 indicates the percent difference in transportation cost (increase in the transportation costs between when there are economies of scale, and when those economies are absent), relative to the total costs without economies of scale.

Type	**Description**	**Results**
a	Add a fixed cost per vehicle to the original data set of Barreto (2004).	Table 5.9
b	Based on the original parameters, double the fixed cost to open a DC, and increase the unit transportation cost. The unit transportation cost is doubled for cases without economies of scale, and multiplied by a factor in the range [2.4, 3.6] when economies of scale are present.	Table 5.10
c	For each instance of Min92, the change in parameters is a combination of types a and b. Add a fixed cost of using a vehicle, and increase the unit transportation cost.	Tables 5.9 and 5.10

Table 5.8: Cost-parameter changes, Type a, b, c based on instances of Barreto (2004)

For type b, the multiplicative factor is chosen, for each instance, to make the respective total costs approximately equal.

Results for types a and b are respectively shown in Tables 5.9 and 5.10. (Note that type c in those tables strictly concerns the four instances of Min 92. Note also that Tables 5.9, 5.10 omit the two instances of Daskin95 that are contained in Table 5.4.)

For type a (Table 5.9), we first refer to the results of previous experiments presented in Table 5.4. There, we found that to achieve similar total cost, we could add a fixed cost per vehicle, or increase the unit transportation cost for the cases having economies of scale. However, if we only increase the coefficient of transportation cost, that could influence the

				No Economies of Scale				With Economies of Scale ($\delta = 2/3$)				
Instance	n	m	Types	FixedCost	Operating Cost	Transportation Cost	Total Cost	Fixed Cost	Operating Cost	Transportation Cost	Total Cost	%
Christodes69	50	5	a	160	1166	464	1790	80	1064	626	1770	9.05
Christodes69	75	10	a	160	2046	723	2929	80	1549	1284	2913	19.15
Christodes69	100	10	a	160	2187	732	3079	80	1625	1374	3079	20.85
Gaskell67	21	5	a	100	1350	319	1769	100	1004	660	1764	19.27
Gaskell67	22	5	a	150	611	498	1259	100	594	566	1260	5.40
Gaskell67	29	5	a	150	765	403	1318	100	687	527	1314	9.41
Gaskell67	32	5	a	100	1762	470	2332	100	1219	1003	2332	22.95
Gaskell67	32	5	a	100	1762	434	2296	50	952	1278	2280	36.76
Gaskell67	36	5	a	100	1350	382	1832	50	935	847	1832	25.38
Min92	27	5	c	544	12615	2516	15675	544	5211	9858	15613	46.84
Min92	134	8	c	1072	11866	4887	17825	804	5729	11272	17805	35.82

Table 5.9: Tradeoff between operating cost and transportation cost (Type a, c)

With variable cost (Barreto's instances)

				No Economies of Scale				With Economies of Scale ($\delta = 2/3$)				
Instance	n	m	Types	Fixed Cost	Operating Cost	Transportation Cost	Total Cost	Fixed Cost	Operating Cost	Transportation Cost	Total Cost	%
Christodes69	50	5	b	240	1166	862	2268	160	1067	1038	2265	7.77
Christodes69	75	10	b	320	2046	1589	3955	240	1776	1938	3954	8.83
Christodes69	100	10	b	320	2187	1821	4328	240	1878	2212	4330	9.03
Gaskell67	21	5	b	200	1350	542	2092	200	1006	866	2072	15.49
Gaskell67	22	5	b	300	611	927	1838	200	595	1050	1845	6.69
Gaskell67	29	5	b	400	765	591	1756	200	690	860	1750	15.32
Gaskell67	32	5	b	300	1762	775	2837	200	1221	1409	2830	22.35
Gaskell67	32	5	b	200	1762	793	2755	200	1200	1350	2750	20.22
Gaskell67	36	5	b	200	1350	781	2331	200	1174	903	2277	5.23
Min92	27	5	c	1088	12615	4350	18053	1088	5212	11228	17528	38.10
Min92	134	8	c	2144	11866	11266	25276	2144	6304	16822	25270	21.98

Table 5.10: Tradeoff between operating cost and transportation cost (Type b, c)

With variable cost (Barreto's instances) effect of economies of scale by opening more DCs than in the regular cases. We thus add a fixed cost of using vehicles for all instances. (For both instances of Min 92 in Table 5.9, type c, we also increase the unit transportation cost.) The preceding fixed costs are in the interval [20,1000]. Every instance uses a distinct coefficient because there are differences in all types of costs among each instance; sometimes those original costs differ greatly.

For each respective instance of type b (Table 5.10), we change some parameters to equalize the total costs of those instances in the given row. To be specific, we double the fixed cost of opening DCs. We also use \$2/km as the the coefficient of transportation for the cases without economies of scale, and between \$2.40/km and \$3.60/km as the transportation cost when economies of scale are present. Now consider the two instances of type c in Table 5.10. For the first instance of Min92 [(n,m) = (27,5)], we made the preceding changes plus one more, a fixed cost of \$1200 per vehicle. That fixed vehicle cost is included in the transportation cost for this instance, but not for the other [(n,m) = (134,8)]. It turns out the demand per customer is much higher in that first instance. The particular parameters used by Min et al. imply that equality of total cost cannot be achieved without the fixed cost per vehicle. However, equality of total cost *is* attainable without a fixed vehicle cost, when (n,m) = (134,8), the second instance of Min et al.

According to Tables 5.9 and 5.10, there is no distinct regularity in the change of percentage. The reason is that the parameters differ in each instance. And since the number of opened DCs changes, the total fixed cost of those DCs will also change, contributing to the difference in percentage in the final column. Nevertheless, in general, we find that with the increase in the number of customers, the difference in transportation cost goes up. For example, for the instance with 50 customers in Table 5.9, about 9% of the total cost (without economies of scale) can be saved with the presence of those economies of scale. That amount could be spent on additional transportation cost, to make up for the effect of opening fewer DCs. If the number of customers or the total demand of those customers is very large (instance Min 92 with m = 5, n = 27), the economies of scale save even more on the operating cost. As a result, the difference in total cost between the case without and with economies of scale is greater. And the percentage of the total cost that can be re-distributed to transportation becomes large, on average 38% in that instance of Min92.

Type	**Description**	**Results**
d	Same as the data in Table 5.6. Only add the variable cost to the original data set of Prins et al.	Table 5.12
e	Based on the original parameters in Table 5.6, increase the fixed cost of using a vehicle.	Table 5.12
f	For one instance when (n, m) = (50, 5), and four instances when (n, m) = (200, 10), increase the fixed cost of using a vehicle and raise the unit transportation cost.	Tables 5.12

Table 5.11: Description of Type d, e, f based on instances of Prins et al. (2006a)

For the instances of Prins et al. (2006a), we employ similar measures as in those of Barreto (2004). For type d, because the problem size is small ($n = 20$), we employ the same data as in Table 5.6. The only difference compared to the original data set of Prins et al. is that we add a variable cost. With respect to type e, we increase the fixed cost of using a vehicle. As for type f, we also raise the unit transportation cost in addition to increasing the

				No Economies of Scale				With Economies of Scale				
n	**m**	**W**	**Types**	**Fixed Cost**	**Operating Cost**	**Transportation Cost**	**Total Cost**	**Fixed Cost**	**Operating Cost**	**Transportation Cost**	**Total Cost**	**%**
20	5	70	d	25,549	3,780	29,244	58,573	25,549	3,672	29,244	58,465	0.00
20	5	150	d	15,497	3,696	23,607	42,800	15,497	3,173	23,698	42,368	0.21
20	5	70	d	24,196	3,720	24,712	52,628	24,196	3,675	24,809	52,680	0.18
20	5	150	d	13,911	3,624	23,631	41,166	13,911	3,122	23,644	40,677	0.03
50	5	70	e	25,442	9,072	65,170	99,684	25,442	6,585	67,690	99,717	2.53
50	5	150	f	24,830	9,072	45,670	79,572	15,385	5,756	58,320	79,461	15.90
50	5	70	e	29,319	9,300	59,667	98,286	29,319	6,701	62,352	98,372	2.73
50	5	150	e	29,319	9,300	39,423	78,042	29,319	6,714	42,003	78,036	3.31
50	5	70	e	19,785	9,228	64,867	93,880	19,785	6,656	67,387	93,828	2.68
50	5	150	e	18,763	9,396	34,016	62,175	18,763	6,736	36,596	62,095	4.15
50	5	70	e	18,961	9,132	68,439	96,532	18,961	5,775	71,799	96,535	3.48
50	5	150	e	18,961	9,132	43,076	71,169	18,961	5,770	46,432	71,163	4.72
100	5	70	e	132,890	18,996	146,210	298,096	132,890	10,778	154,370	298,038	2.74
100	5	150	e	132,890	18,996	83,782	235,668	132,890	10,786	91,940	235,616	3.46
100	5	70	e	102,246	18,696	97,064	218,006	102,246	9,312	106,424	217,982	4.29
100	5	150	e	102,246	18,696	56,954	177,896	102,246	9,331	66,314	177,891	5.26
100	5	70	e	88,287	18,744	115,159	222,190	88,287	9,350	124,519	222,156	4.21
100	5	150	e	88,287	18,744	66,239	173,270	88,287	9,323	75,599	173,209	5.40
100	10	70	e	154,942	19,320	136,947	311,209	154,942	10,929	145,267	311,138	2.67
100	10	150	e	154,942	19,320	81,060	255,322	154,942	10,896	89,460	255,298	3.29
100	10	70	e	145,956	18,432	100,562	263,950	149,586	10,559	104,882	265,027	1.63
100	10	150	e	145,956	18,432	58,576	221,964	149,586	10,598	62,756	222,940	1.87
100	10	70	e	139,411	18,480	120,389	278,280	144,699	10,590	122,889	278,178	0.90
100	10	150	e	139,411	18,480	71,762	229,653	144,699	10,571	74,364	229,634	1.13
200	10	70	f	317,619	37,176	220,307	575,102	285,784	16,869	272,125	574,778	9.01
200	10	150	f	317,619	37,176	125,167	479,962	285,784	16,848	175,350	477,982	10.46
200	10	70	e	280,370	37,212	182,581	500,163	280,370	16,845	202,671	499,886	4.02
200	10	150	e	280,370	37,212	97,600	415,182	280,370	16,872	117,840	415,082	4.87
200	10	70	f	322,384	36,924	224,704	584,012	272,528	16,785	293,520	582,833	11.78
200	10	150	f	322,384	36,924	111,610	470,918	272,528	16,770	178,072	467,370	14.11

Table 5.12: Tradeoff between operating cost and transportation cost, based on instances of Prins et al.(2006a)

With variable cost

fixed cost of using a vehicle. Thus, the type f changes include the type e changes, relative to the original data set of Prins et al.(2006a). Note that types d, e, f all add an operating cost to that original data set.

Table 5.12 presents the results for those types. Instances in our tables follow the same sequence as in Prins et al. Because we change nothing for type d and the numbers of opened DCs are the same for cases with and without economies of scale, the percent difference in transportation cost is very small, almost equal to 0. For type f, the five instances have the same situation: the cases having no economies of scale open one more DC than those when economies of scale are present. Therefore, to equalize the total cost, more money (savings of fixed cost of opening DCs) can be spent on transportation. And the type-f percentages are the largest among all instances, varying from 9.01 to 15.90. As for the instances of type e, the percentages are in between; on average, an additional 2.78% of total cost can be distributed to transportation when economies of scale are present, without increasing the total cost. For the instances of Prins et al., since the fixed cost of opening a DC is very high, the preceding tradeoffs are therefore not obvious. We would have to increase the other costs quite significantly, to cancel the influence of such a high fixed cost.

In conclusion, the presence of economies of scale will affect the company's decisions. On one hand, operating cost can be saved. On the other hand, even though we need to spend more on transportation, the total cost will not increase. That is, suppose greater economies of scale can be achieved in DC operation. Those additional savings will permit greater expenditure on transportation, without increasing the total cost.

Chapter 6

Conclusions and Future Work

In this thesis, a location-routing problem with variable cost of DCs is discussed. The problem minimizes the total cost, including fixed cost of opening DCs, operating cost which changes with the throughput, and transportation cost. We present two objective functions. Both consider operating cost of DCs, but one of them accounts for DC economies of scale. We represent those concave costs by employing a power function γX^{δ}, where X denotes the throughput of the facility.

Since the LRP combines two NP-hard problems (facility location problem and vehicle routing problem), the majority of papers use heuristics. We therefore consider two meta-heuristic methods: simulated annealing embedded within a genetic algorithm, and a genetic algorithm with ant colony optimization. Through preliminary tests, we decide to use the genetic algorithm with ant colony optimization for further experiments in this thesis. We first take away the variable cost part to compare the performance with other published methods. Employing publicly-available data sets, the results show that our proposed method is acceptable. The median gaps, relative to best known results, are less than 3% in Tables

5.3 and 5.5.

Then, with those data sets, we compare the cases with variable cost and no economies of scale, to those when economies of scale are present. We find that fewer DCs are opened and total costs are lower for the cases with economies of scale. Moreover, we analyze the influence of the exponent δ in the power function representing the economies of scale. We first test the how the value of power δ affects the total cost. We test three values of δ, and give mathematical proof, both showing that with an increase in δ, the operating cost increases, and there is less of an impact of economies of scale. Next, we analyze the tradeoff between operating cost and transportation cost. From previous experiments, we notice that when economies of scale are present, fewer DCs are opened, hence the distances traveled are greater. Scale of economies yield total costs that are lower, though the transportation costs are larger, and the required number of vehicles will not decrease.

By changing some parameters, we make the total cost of certain instances approximately equal to exhibit the tradeoff. We find that, without increasing the total cost, the savings achieved by economies of scale permit additional expenditure on transportation. That may enable the DCs to furnish enhanced service to retailers.

To the best of our knowledge, only one published paper on LRP (Melechovsky et al. 2005) considers concave cost at DCs or warehouses. And their focus is on demonstrating the performance of the hybrid algorithm they proposed. They do not discuss the influence of the concave costs. We are perhaps the first to carefully analyze the effects of economies of scale in the LRP. Also, the published data sets with which most authors experimented do not include the operating cost of DCs or warehouses. We add those variable cost parameters to the original data sets. Taking account the operating costs helps to achieve solutions closer to reality.

A number of research directions can be explored in the future. First, for ongoing research, variable cost can be considered as a part of the objective function for LRP. Even if the problem had no need to discuss economies of scale, adding the operating cost of DCs or warehouses has practical value.

Second, the model can be extended. Supply sources, i.e. plants, can be added to the current model. Even more, economies of scale can be taken into account for the two-echelon LRP, which considers the product from flow plants to DCs to customers, and routes are developed at both levels. That is, we can introduce variable cost into the two-echelon LRP to simulate real-world situations in the distribution system of a supply chain.

In addition, solution methods can be improved. Although results show that our proposed method may be acceptable, the gap compared to the best known result (BKR) is not small for some particular instances. Currently, the majority of research employs heuristic approaches, even on the standard LRP. Because of the lack of a lower bound, we can only compare with the BKR. Thus, new methods can be developed. Exact methods, based on Lagrangian relaxation or a branch-and-price algorithm, would be helpful.

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Appendix A

Impact of Economies of Scale on the Number of Opened DCs

Theorem. Opening more DCs will weaken the effect of economies of scale. In other words, the operating cost will be larger. The cost function we employ in Eq.(3.1) is: $f(x) = \gamma(x)^{\delta}(\gamma > 1,\ 0 < \delta < 1)$

Assumption. *Assuming the total demand is X units. The capacity of each DC will be respected.*

We start with a specific example: we compare the cost of opening two DCs with opening one DC. When open two DCs, one serves $A * X$ units, and the other one serves $(1 - A)X$ units $(0 < A < 1)$. That is, the operating cost when two DCs opened is $f_1(x) = \gamma(AX)^\delta + \gamma[(1-A)X]^\delta$, that of the one DC opened is $f_2(x) = \gamma(X)^\delta$.

$$\textbf{\textit{Proof}}.f_1(X) - f_2(X) = \gamma(AX)^\delta + \gamma[(1-A)X]^\delta - \gamma(X)^\delta = \gamma(X)^\delta[A^\delta + (1-A)^\delta - 1]$$

$$Define\ g(A) = f_1(X) - f_2(X)\ \ for\ fixed\ X.$$

$$Take\ the\ derivative\ with\ respect\ of\ A:\ \ g'(A) = \gamma(X)^\delta[\delta A^{\delta-1} - \delta(1-A)^{\delta-1}]$$

$$Setting\ g'(A) = 0,\ we\ obtain\ A^{\delta-1} = (1-A)^{\delta-1}$$

$$Since\ \delta < 1,\ we\ must\ have\ A = 1 - A \Longrightarrow A = 0.5.$$

$$Take\ the\ second\ derivative:\ \ g''(A) = g''(A) = \gamma(X)^\delta[\delta(\delta-1)][A^{\delta-2} + \delta(1-A)^{\delta-2}]$$

$$Since\ 0 < \delta < 1,\ 0 < 1 - \delta < 1,$$

$$\gamma(X)^\delta > 0,\ [\delta(\delta-1)] < 0,\ [A^{\delta-2} + \delta(1-A)^{\delta-2}] > 0$$

$$Thus,\ g''(A) < 0 for A = 0.5. And A = 0.5\ is\ a\ global\ maximum.$$

$$\Longrightarrow g(A)_{min} = g(1) = g(0) = 1$$

$$\Longrightarrow g(A) > 1 when 0 < A < 1.$$

Therefore, one DC will be opened for a lower operating cost.

Based on the proof of the specific case, we assume open n DCs cost more than open (n-1) DCs. We will prove open (n+1) DCs cost more than open n DCs.

That is, when open n DCs, the cost function is: $f_n(x) = \gamma(A_1X)^\delta + \gamma(A_2)X^\delta + ... + \gamma(A_nX)^\delta.(A_1 + A_2 + ... + A_n = 1)$. When open $n+1$ DCs, the cost is: $f_{n+1}(x) =$

$\gamma(B_1X)^\delta + \gamma(B_2)X^\delta + ... + \gamma(B_nX)^\delta + \gamma(B_{n+1}X)^\delta.(B_1 + B_2 + ... + B_n + B_{n+1} = 1).$

***Proof**. We want to prove* $f_{n+1}(x) \geq f_n(x)$

$$That\ is,\ we\ need\ show:\ B_1^\delta + B_2^\delta + ... + B_n^\delta + B_{n+1}^\delta \geq A_1^\delta + A_2^\delta + ... + A_n^\delta$$

$$Since\ 0 < \delta < 1,\ \sum_{n+1} B_i = 1,\ combine\ any\ two\ items,\ we\ must\ have\ B_1^\delta + B_i^\delta \geq (B_1 + B_i)^\delta$$

$$Thus,\ B_1^\delta + B_2^\delta + ... + B_n^\delta + B_{n+1}^\delta \geq (B_1 + B_i)^\delta + B_2^\delta + ... + B_{n+1}^\delta$$

$$\Longrightarrow\ B_1^\delta + B_2^\delta + ... + B_n^\delta + B_{n+1}^\delta \geq A_1^\delta + A_2^\delta + ... + A_n^\delta$$

Therefore, when economies of scale are present, fewer DCs will be opened to reach a lower operating cost.