Framework for the Strategic Management of Dimensional Variability of Structures in Modular Construction

by

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Author’s Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.
Abstract

Challenges in construction related to dimensional variability exist because producing components and assemblies that have perfect compliance to dimensions and geometry specified in a design is simply not feasible. The construction industry has traditionally adopted tolerances as a way of mitigating these challenges. But what happens when tolerances are not appropriate for managing dimensional variability? In applications requiring very precise dimensional coordination, such as in modular construction, the use of conventional tolerances is frequently insufficient for managing the impacts of dimensional variability. This is evident from the literature and numerous industry examples. Often, there is a lack of properly understanding the rationale behind tolerances and about how to derive case specific allowances. Literature surrounding the use of tolerances in construction indicates that dimensional variability is often approached in a trial and error manner, waiting for conflicts and challenges to first arise, before developing appropriate solutions. While this is time consuming, non-risk averse, prone to extensive rework and very costly in conventional construction, these issues only intensify in modular construction due to the accumulation of dimensional variability, the geometric complexity of modules, and discrepancy between module production precision and project site dimensional precision. This all points to a need for a systematic and strategic approach for managing dimensional variability in modular construction.

This thesis explores dimensional variability management from a holistic construction lifecycle viewpoint, examining key project stages (manufacture, fabrication, aggregation, handling, transportation and erection) to identify critical variability sources and proposing adequate strategies to control dimensional variability. The scope of this work relates primarily to the structural system of commercial building modules, based on the assumption that the sequence of production and dimensional variability of building subsystems (mechanical, electrical, plumbing, architectural) hinge upon the dimensional variability of the structure. A novel method for quantifying dimensional variability is developed, which uses 3D imaging by way of laser scanning and building information models to compute deviations between the intent of a geometric design and the actual as-built construction. Novel strategies for managing dimensional variability are also developed, and include adaptation of manufacturing-based principles and practices for use in construction systems. The inspiration and foundation of these new strategies is derived from the original research of Dr. Colin Milberg, who explored how to apply tolerance theory used in manufacturing into civil construction systems. The new techniques developed in this thesis, along with other previous research, demonstrate that there is a clear correlation between manufacturing industries such as aerospace and automotive assembly production, and that of modular construction assembly production. In light of this, there is an opportunity to improve modular construction processes if these manufacturing-based methods can be appropriately implemented. This is the basis for the proposed methodology presented in this thesis.

Application of the proposed methodology using case study examples demonstrates that dimensional variability in modular construction should be approached from a holistic viewpoint. Furthermore, it needs to incorporate much more consideration into the key factors and critical sources of variability rather than pursuing the traditional construction approach of developing inefficient trial and error solutions.
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1. Introduction

The terms "modular" and "prefab" have become popular catchwords within the construction industry, especially in recent years due to the growing use and popularity of modularization and prefabrication as construction delivery methods. Previous trends of modularization and prefabrication (e.g., the US manufactured housing industry during the 1960’s and 1970’s) were often driven by strict demands for extremely short build times and lower project costs. Following WWII, there were large demands placed on construction industries throughout Europe and North America to rapidly fill the deficit of housing and commercial buildings, which had been destroyed during the war, and to provide affordable housing for veterans returning home. Although prefabrication and offsite fabrication methods were used as a means of providing mass temporary housing, these large scale movements ultimately did not achieve their targets in terms of quantity and quality (Phillipson 2001). As the result of these (and other) ‘top-down’ attempts at forcing the implementing of prefabrication (Smith 2011), several errors were made, which lead to their steady decline and poor perceptions of the terms "prefab" and "modular." Some of the most significant errors that lead to previous declines in prefabrication and modularization were (1) their significantly poor quality, (2) a lack of attention to technical detail and (3) life cycle performance failures associated with maintenance and durability (Phillipson 2001). Examples that point to both poor design and workmanship are inadequate thermal performance as well as air and water penetration at joints (Phillipson 2001). What is clear from previous attempts at introducing modularization and prefabrication on a large scale is that a great deal of additional design effort and corresponding high quality workmanship is required in order to make them successful construction delivery techniques.

1.1. Current State of Modular Construction

In recent years, there has been a resurgence in modularization and prefabrication trends within the construction industry from its prior days of providing mass produced manufactured homes (Figure 1). There are numerous modern examples of high rise buildings, which have utilized modular construction as their delivery method: Victoria Hall in Wolverhampton UK, Leadenhall Building in London UK, B2 BKLYN in New York USA, SOHO Tower in Darwin Australia, T30 Hotel in Changsha China, Hilton Palacio Del Rio Hotel in San Antonio Texas and the Paragon Building in London UK. Due to recent advancements in technology and high precision offsite manufacturing techniques, many sectors of construction are shifting away from conventional ‘stick-built’ construction practices and towards the use of prefabrication and modularization. As site safety issues continue to increase, and urban centres continue to become more and more congested, sectors of the construction industry are turning to modular construction. These sectors are finding that modular construction can yield numerous advantages including shorter project schedules, lower costs, increased safety and improved quality control (Burke and Miller 1998, Gibb and Isack 2003, Jaillon et al. 2009, Nadim and Goulding 2010, Sacks et al. 2010). In a study by Jaillon and Pool 2009, the choice of modular construction over traditional methods resulted in a 20% reduction in project duration, 56% reduction in construction waste, 9.5% reduction in labour requirements, improved safety, as well as less dust and noise on the construction site. A recent survey of more than 800 contractors, engineers and architects indicated that the primary reason for current usage of prefabrication and modularization is its ability to increase productivity (McGraw Hill Construction 2011). Data from the same survey indicated that 84% of contractors, 90% of engineers, and 76% of architects in the construction industry were using some degree of prefabrication or modularization on their projects.
Clearly, construction companies are discovering that modularization and prefabrication can result in substantial benefits if done correctly.

Figure 1: Example of 1960’s prefabricated construction: single-wide mobile manufactured house (left). Example of a modern modular construction project: modular apartment building (right). (Catpal 2013, Champion 2013)

Despite the numerous advantages of modular construction, there are several challenges that still exist. For instance, project pre-planning, project coordination, preliminary design, transportation and site fit-up are very challenging aspects of modular construction (Goodier and Gibb 2005, Haas et al. 2000, Lu 2009b). This is evident through limited initial design options, complex interfacing, long lead-in times, delayed planning processes, and design inflexibility (Haas et al. 2000, Lu 2009b, Pan et al. 2007, Taylor et al. 2009). A study conducted in the UK found that two of the biggest challenges in modular construction projects is the inability to freeze design specifications early into the project, and key decisions being made early in the project, which constrain the successful implementation of the project (Blismas et al. 2005). A similar study in the US found that the most prominent challenges for modular construction is the inability to make changes onsite, transportation restraints, and that general contractors do not have the sufficient expertise to properly assemble modules onsite (Lu 2009a). The challenges currently faced in modular construction leads to the formation of risk, which can have a profound impact on the benefits of modularization. Of the 24 risks identified in a recent study, the five most prominent risks were found to be (1) poor cooperation between numerous interfaces, (2) inappropriate design standards (using stick-built practices in the context of modular construction is not satisfactory since there are much stricter dimensional and geometric demands in modular construction), (3) lack of proper management and experience, (4) enormous difficulty obtaining a return on initial investment and (5) a lack of a sufficient quality monitoring mechanisms during production processes (Li-zi Luo et al. 2015). Modularization also introduces risk related to module damage and interfacing problems, resulting in rework and project delays (Taylor et al. 2009).

The current state of modular construction has made significant advancements and improvements since the 1960’s and 1970’s state of mass produced manufactured homes. However, there are still a large number of risks that can emerge, which requires a concerted effort by both industry and research in order to expose the full potential of modular construction. As such, modular construction “…is a technology that commands respect [when considering what it is capable of on a given project] but is also one that the
construction industry is only just beginning to come to terms with, at least on a large scale” (Lawson et al. 2014). This is the context upon which this thesis is based.

1.2. Problem Statement

One major challenge in the execution of a modular construction project is the management of dimensional and geometric variability (Lawson et al. 2014). Due to the increased quality control capabilities and use of advanced manufacturing technologies in modular construction, dimensional and geometric variability is typically approached through the use of precise methods of production. However, problems often persist due to discrepancies between precise production tolerances and larger site tolerances, as well as geometric variations and damage occurring from transportation and handling loads. As such, site fitting can be problematic since there is less forgiveness in module geometry once on site to accommodate varying site conditions (Smith 2011). A study of two modular prefabricated high rise buildings found that the geometric inflexibility of modular units once on site was a major design limitation to selection of modular construction techniques (Jaillon and Poon 2014). Failure to make design considerations about geometric changes to modules can be “a questionable decision as the modules move during transport and assembly, resulting in costly adjustments on site” (Johnsson and Meiling 2009). The management of dimensions, geometry and build tolerances are critical factors in modular construction. The industry currently lacks a proper understanding of how to properly manage dimensional variability, which previous studies have clearly demonstrated (Acharjee 2007, Johnsson and Meiling 2009, Milberg and Tommelein 2003a, Smith 2011).

The following example from a recent modular construction project demonstrates the current lack of understanding towards dimensional and geometric variability and how it can profoundly impact a modular construction project. In a recent project, a construction company was responsible for designing and fabricating more than 900 modules in a multi-story apartment building. During the erection of modules on site, delays on the order of 2 to 6 times the planned story erection time pushed the project far behind schedule. Extensive rework was required to ensure proper site-fitting during erection. This occurred due to poor planning and understanding of how dimensional and geometric variability can interact to cause misalignments between modules. The original design was based on the assumption that stiff and rigid modules would ensure no deformation during transportation and handling, which would improve the ease of erection. Unfortunately, once on site, the geometry of the modules was not conformant to the original design, which led to extensive rework to ensure proper fit up. Even so, the gaps between modules were not tight enough, leading to water damage. As a result of these challenges, the project was completely halted halfway through the project timeline due a multi-million dollar lawsuit, with lots of finger pointing as to which party was to blame for the dimensional and geometric conflicts. This example demonstrates a lack of proper understanding of the management of dimensional and geometric variability. When dimensional variability is not properly managed, the consequences experienced can far outweigh the potential benefits of modular construction.

This thesis provides a systematic and holistic framework for properly managing dimensional variability in modular construction. Simply put, dimensional variability relates to the deviation of dimensions or geometric properties from nominal values or specifications. Dimensional variability arises due to process capabilities (e.g., the accuracy and precision of certain construction processes for achieving specified dimensions and assembly geometry), design tolerances (the selected allowable variation from nominal
parameter values) and the interaction and accumulation of the final geometric properties of components and assemblies (which is a function of both allowable tolerances and the actual deviations).

1.3. Research Objectives

The primary objective of this thesis is to develop a strategic framework for managing dimensional variability throughout the lifecycle (i.e., fabrication, aggregation, transportation, and erection stages) of a modular construction project.

In addition to this primary objective, some secondary objectives are:

- to determine the major factors that impact dimensional variability on a modular construction project;
- to develop a procedure for quantifying the discrepancies between as-built and as-designed assembly states;
- to quantify dimensional variability at distinct project stages and to study the accumulation of variability throughout the lifecycle of a modular construction project;
- to identify current methods for managing dimensional variability in modular construction;
- to determine practical analogies and tools used in the manufacturing industry to adopt and apply to modular construction systems for appropriately managing dimensional variability.

1.4. Research Approach

This thesis explores the impact of dimensional variability in modular construction and proposes a strategic framework for properly managing it. The manufacturing industry is used analogously during the production of modules for ensuring the impact of dimensional variability on component aggregation is properly managed, since it was found to be the industry benchmark in this regard. In cases where analogies to the manufacturing industry are not suitable, other analyses and methodologies are adapted or developed and then subsequently implemented. One example of this is the use of kinematics-chain based dimensional variability analysis, where robotics theories are drawn upon in order to derive an efficient method for module connection design.

For capturing data associated with dimensional variability, this thesis utilizes discrete 3D data capture (e.g., total station) as well as continuous 3D data capture (e.g., laser scanning). 3D imaging concepts are employed in order to analyse data related to dimensional variability. The reason for the use of 3D imaging as a means of data capture and analysis is due to the ability to obtain rapid and accurate feedback regarding dimensions and geometry of construction components and assemblies. One of the most common means of quantifying dimensional variability is by way of a developed method of comparing as-built data (i.e., 3D point clouds) with as-designed data (i.e., BIM), which is herein referred to as Deviation Analysis. A comprehensive overview on the use of deviation analysis is provided in this thesis.

1.5. Scope

The scope of the management of dimensional variability in this thesis primarily focuses on the structural aggregation of a module and its erection on site. Furthermore, dimensional variability is explored explicitly in the context of modular construction. While the methodology derived in this thesis can be implemented in other types of offsite construction applications, the scope is focused on modular
construction, due to the unique challenges involved with the management of dimensions and geometry of often complex three-dimensional volumetric modules.

The reason for focusing primarily on the structural aggregation of a module is based on the order of typical fabrication processes employed on most modular construction projects:

- the structure is aggregated first;
- the partitions, walls, enclosures, architectural systems (etc.) are fabricated or aggregated next; and
- the building services, mechanical and or other systems are fabricated or aggregated last.

As a result of the typical fabrication process progression for the main systems in a modular construction project, the geometry of the structure will have an impact on the dimensional fit-up and geometry of succeeding fabrication processes. The progression of fabrication processes is such that datums, otherwise known as reference points of the as-built state are often used for the fit-up and positioning of succeeding construction components. This effectively means that the position, orientation and form of the module structure influences the dimensional variability of other systems (e.g., architectural, service, mechanical, etc.). Furthermore, if the structure of a module is fabricated first, when rework of the structure is required, the succeeding fabricated systems are ‘pulled back’, or exposed, in order to provide access to the structure, which can be very time consuming and costly. This means that corrections to the structure can sometimes come at larger cost and schedule impact than changes to other systems. As such, the scope of this thesis focuses on managing the dimensional variability of the structure of a module.

In addition to the management of dimensional variability of the structure, focus is placed on critical interfaces between the components, modules and project site. The reason being that the management of dimensional variability has certain impact categories. The ability to properly aggregate components and modules is one of the most significant impacts of dimensional variability. As such, focus is placed on dimensional and geometric compliance between components and modules at interfaces. While the aggregation of components in stick-built construction offers the ability to incrementally adjust and correct geometry at interfaces during construction, in modular construction the aggregation of components must be designed and well-executed before erection on site takes place.

### 1.6. Terminology

Some of the key terminology used in this thesis is provided in Table 1.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition in this Thesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variability</td>
<td>how a particular dimension varies from a mean parameter value (e.g., nominal dimension as specified in a design)</td>
</tr>
<tr>
<td>Variation</td>
<td>the continuous range of variability</td>
</tr>
<tr>
<td>Deviation</td>
<td>discrete value of variation</td>
</tr>
<tr>
<td>Dimensional variability</td>
<td>variability associated with all dimensions (e.g., linear dimensions, angular dimensions, geometric dimensions)</td>
</tr>
<tr>
<td>Linear dimension</td>
<td>a two-point measurement</td>
</tr>
<tr>
<td>Angular dimension</td>
<td>angle defined between two lines or two planes</td>
</tr>
<tr>
<td>Geometric properties/dimensions</td>
<td>combination of both linear and angular dimensions to describe the geometry of a component/assembly</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Tolerance</td>
<td>the amount of permissible variation from a mean parameter value (e.g., +/- 2 mm for a particular dimension of interest)</td>
</tr>
<tr>
<td>Dimensional tolerance</td>
<td>a tolerance placed on any kind of dimension (linear, angular or a geometric property)</td>
</tr>
<tr>
<td>Geometric tolerance</td>
<td>a tolerance placed on a specific geometric property (e.g., allowable amount of out-of-plane bending in a beam)</td>
</tr>
<tr>
<td>Component-feature</td>
<td>a geometric element of a component, such as a line, plane, or mathematically defined curve or surface</td>
</tr>
<tr>
<td>Assembly</td>
<td>the physical arrangement of components</td>
</tr>
<tr>
<td>Aggregation</td>
<td>the process of assembling components together</td>
</tr>
<tr>
<td>Assembly plan</td>
<td>the order or manner in which components are aggregated into an assembly</td>
</tr>
<tr>
<td>Envelope</td>
<td>External portion of a component or assembly described in terms of a series of lines, planes or surfaces</td>
</tr>
<tr>
<td>Interface</td>
<td>the point, line, plane, and surface of components along with the clearance zone or gap between two components being aggregated together</td>
</tr>
<tr>
<td>Tie-in point</td>
<td>critical interface coordinate on the component being aggregated into an assembly</td>
</tr>
<tr>
<td>Control point</td>
<td>the corresponding (matching) critical interface coordinate on an assembly for a component being aggregated</td>
</tr>
</tbody>
</table>

1.7. **Thesis Organization**

This thesis is organized into eight chapters:

- Chapter 1 provides the introduction to the problem statement, objectives and scope of the research contained in this thesis.
- Chapter 2 provides a comprehensive overview of the literature and background information necessary for the technical topics explored in this research. Examples from industry are provided, and the current approach for management of dimensional variability in construction is summarized. Relevant information surrounding the manufacturing industry is also provided since it is used analogously to solve some of the proposed research objectives.
- Chapter 3 presents the proposed methodology. Development of the proposed methodology is broken down into the key steps involved, and the scope, objectives and constraints are also addressed.
- Chapter 4 presents the development of a deviation analysis method, which is the approach used to quantify dimensional variability in this research.
- Chapter 5 provides a detailed case study of a modular construction project, where key sources of dimensional variability are identified, quantified, and subsequently analysed.
- Chapter 6 presents the development of and examples on novel design-based strategies for managing dimensional variability.
- Chapter 7 presents the development of and examples of novel production-based strategies for managing dimensional variability.
- Chapter 8 summarizes all of the developments of this research into key conclusions. Future work is also discussed in order to strengthen and supplement the proposed methodology and research undertaken in this thesis.
2. Background

Throughout this thesis, dimensional variability is used to describe variations in both dimensions (commonly used in construction as linear two-point measurements), and geometry (which is often expressed in a series of two-point measurements as well as other properties such as angles, volume, area, levelness, perpendicularity, etc.) of construction components and assemblies. A tolerance is a permissible variation from a specified requirement, and in the context of construction can be applied to parameters such as dimensions (e.g., clearance between components, member lengths, thicknesses, etc.), quantity, or alignment and position in three dimensional space. The need for tolerances for the production of parts (i.e., during manufacturing, fabrication and assembly) arises because variability is an inevitable reality. Regardless of the amount of effort placed into controlling the dimensional and geometric varying outcomes of production processes, some degree of variability cannot be avoided. However, in terms of the accumulating effects of variability, there are certain levels (or ranges) of variability which have larger impacts than others on overall goals of production. Therefore, these specified limits of variability known as tolerances are often used in order to target critical sources of variability and to control certain dimensional and geometric attributes of parts so that production goals can be met in way that balances cost, quality and customer satisfaction (Creveling 1997). Production variability is an issue that emerges in many industries, including manufacturing and construction.

This chapter provides background information related to the way that both construction and manufacturing industries manage dimensional variability. One of the most common approaches for this is done through the specification of tolerances. Accordingly, theory related to tolerance specification is a key focus of this chapter. The relationship between the construction industry and the manufacturing industry is first explored in order to demonstrate how modular construction can be viewed as a type of manufacturing process. Then the state of dimensional variability management in construction is summarized. The next part of this chapter explores how the manufacturing industry manages dimensional variability. Finally, literature related to recent technological advancements in 3D imaging and BIM (building information modelling) and how these methods can be used in the analysis and quantification of dimensional variability in construction is presented.

2.1. Modular Construction as a Manufacturing Process

The process of aggregating components in construction can be defined by two stages: (1) mating, which consists of bringing components into alignment with each other and (2) joining, which consists of fixing or fastening components together. In offsite construction, there are several categories to describe the extent of which the manufacturing and aggregation of components occurs within an offsite facility. A popular structure for describing this is known as PPMOF, which stands for Prefabrication, Preassembly, Modularization and Offsite Fabrication (Haas and Fagerlund 2002, Josephson and Hammarlund 1999). A useful structure for demonstrating the types of offsite construction along with examples and type of aggregation is shown in Table 2. The type of aggregation involved in offsite construction can be described in terms of being either ‘volumetric’, where fabricated components enclose a usable space, or ‘non-volumetric’, where components do not enclose a usable space (Gibb 1999). This breakdown of volumetric versus non-volumetric becomes very important when analyzing the impacts of dimensional variability in 3-dimensions, since volumetric assemblies often incorporate a higher degree of complexity than 1-dimensional or 2-dimensional (i.e., non-volumetric) assemblies.
Due to economies of scale involved in offsite construction, it shares many similarities with other more mainstream manufacturing applications (e.g., automotive, aerospace industries). All forms of offsite construction utilize some aspect of a manufacturing process in terms of the production and aggregation of components and assemblies. However, to say that offsite construction simply shares some similarities with mainstream manufacturing sectors is not descriptive enough nor is it a meaningful comparison to make due to the wide range of manufacturing models in existence. This is why research has explored specific manufacturing models in which to classify forms of offsite construction. Some common manufacturing models include mass production, lean production, complex systems production and component shop production (Winch 2003). These manufacturing models are based on the type of materials flow (i.e., low volume versus high volume) and the type of product information flow (i.e., concept/tender or new product development). Stick-built construction sees most of its work being done directly at a job site and has the benefit of allowing onsite construction adjust for dimensional variability challenges in an ad-hoc fashion. On the other hand, a high volume production model requires that components be able to aggregate properly in order to minimize the amount of rework related to dimensional variability. It is not possible to make a blanket statement about which manufacturing model best describes all modular construction, since each project can vary drastically from the next. For instance, modular construction could be used to mass produce volumetric bathroom pods, which would best resemble a mass production manufacturing model. However, modular construction could also be used to produce large complex pipe spool assemblies, in which case the best manufacturing model would be a complex systems production. Although selection of a specific manufacturing model for modular construction varies from project to project, there is certainty that modular construction can be viewed as a manufacturing process. This conclusion is important in this thesis, as theories and concepts used in the manufacturing industry are used analogously for the development of the proposed framework for managing dimensional variability.

While there are many similarities between modular construction and certain manufacturing models, key differences include certain unavoidable stick-built practices such as construction of foundations, service tie-ins, and module erection (Gann 1996, Gibb 2001). This is because despite utilizing a manufacturing model for the production and aggregation of components, the construction assembly is ultimately erected on a project site, which is dominated by certain stick-built practices. The offsite manufacture of components and assemblies allows for the use of high precision tools and processes, which means that construction assemblies can be fabricated with a high degree of precision. However, most site interfaces

Table 2: Breakdown of offsite construction types, with examples and predominant form of aggregation

<table>
<thead>
<tr>
<th>Offsite Construction Categories</th>
<th>Sub-categories and Examples</th>
<th>Aggregation Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prefabrication/Offsite Fabrication:</td>
<td>Components (i.e. bricks, tiles, structural steel, etc.)</td>
<td>Non-volumetric</td>
</tr>
<tr>
<td>manufacture and sub-assembly within a factory or on another site from project site</td>
<td>Sub-assemblies (i.e. doors, windows, etc.)</td>
<td>Non-volumetric</td>
</tr>
<tr>
<td>Preassembly: preassembled units which create usable spaces and usually installed into independent structures</td>
<td>Non-volumetric (skeletal, planar, or complex)</td>
<td>Non-volumetric</td>
</tr>
<tr>
<td></td>
<td>Volumetric (installed either into or onto another building)</td>
<td>Volumetric</td>
</tr>
<tr>
<td>Modular Building: preassembled volumetric units which form the actual structure of a building</td>
<td>Single story (horizontal aggregation)</td>
<td>Volumetric</td>
</tr>
<tr>
<td></td>
<td>Multi-story (horizontal and vertical aggregation)</td>
<td>Volumetric</td>
</tr>
</tbody>
</table>
such as foundations are still constructed in a traditional stick-built manner. As a result, modular construction is unique from any other manufacturing process in that it deals with levels of precision for its manufactured assemblies on the order of millimeters up to levels of precision on the order of several centimeters for field installed components (Ballast 2007).

2.2. Dimensional Variability Management in Modular Construction

While modular construction can utilize high levels of precision for the manufacture of modules, it is often constrained to the variability associated with certain unavoidable stick-built construction practices (Figure 2). This is the case for projects where modules are being installed into an existing building with stick-built levels of precision, or in projects where the level of precision for the construction of foundations does not match that of the modules. It should be noted that the production of modules has the potential of achieving certain manufacturing levels of precision, however there are many instances where stick-built levels of precision are also brought into the offsite manufacturing facility, in which case dimensional variability can become very challenging for the aggregation of assemblies. So how important is it to have compatibility between production and erection levels of precision in modular construction? And what are the impacts of not properly managing dimensional variability throughout the lifecycle (that is, during fabrication, assembly, transportation and erection stages) of a modular construction project? In order to address these questions, some examples of poor dimensional variability management are highlighted along with their consequences in terms of project performance and risk. Strategies currently employed for dealing with dimensional variability in construction are also presented in order to understand the knowledge gap underlying the proposed methodology.

![Figure 2: Predominant levels of precision involved in modular construction (Rausch et al. 2016).](image)

2.2.1. Consequences of Dimensional Variability in Modular Construction

The following examples of poor dimensional variability come from industry examples and from cited research work. The first set of examples come from the B2 BKLYN modular high rise project in New York. When complete, this project is set to be the tallest modular high rise building in the world. With an accomplishment like this, there has been a lot of challenges faced during the fabrication, aggregation and erection of modules on site. Two notable challenges which relate to dimensional variability are misalignments of the facade panels, and misalignments of the modules at joints.

Critical dimensions of doors between side panels on the facade were out-of-tolerance during the erection of certain panels which meant they were not able to close properly. This out-of-tolerance is problematic not only from an aggregation standpoint (i.e., building components do not fit properly), but having large
gaps or missing parts of the cladding increases the risk exposure for water leaks. In addition to some of the doors not being able to fit, there were several facade panels that were misaligned during erection, which required rework in order to make the panels fit correctly (Figure 3).

![Misalignment at joints](image)

Figure 3: Misalignments of facade panels on B2 BKLYN building (photo used by permission of Norman Oder, Atlantic Yards/Pacific Park Report)

Another example of a challenge related to dimensional variability on this project was misaligned modules at joints (Figure 4). This challenge was eventually resolved through use of brute force to manipulate the geometry of modules in order to bring modules into correct alignment with each other (Oder 2015).

![Panel not able to fit properly](image)

Figure 4: Misalignment of modules at joints on B2 BKLYN building (photo used by permission of Norman Oder, Atlantic Yards/Pacific Park Report)

While both of these challenges related to dimensional variability in this project were addressed (either before or after they occurred), what is significant is that the building developer stated that large parts of their cost overruns (which could be as high as $100 million once the project is complete) are attributed to the challenge of bringing modules into alignment (Oder 2015). While the exact impact of the management
of dimensional variability in this project is not known, it is clear that the management of dimensions and geometry of modules at joints can have serious challenges and consequences in modular construction.

The next set of examples come from a modular construction project which is the focus of a case study in Chapter 5. In this project, significant dimensional variability arose in numerous cases, and created challenges for aggregation of components, and erection of modules on site. During the fabrication stage, the aggregation of subsystems between modules was conducted based on the geometry of modules while supported on temporary cribbing. Since the geometry of modules were slightly different when on the site foundations, the fit-up of subsystems was challenging due to several misalignments. Another challenging aspect faced in this project was the variation in floor height between adjacent concrete precast panels. In terms of final misalignments on site, the interfaces between modules had large gaps at the location of tie-in plates as well as at column faces. These dimensional variability challenges are summarized in Figure 5.

![Dimensional variability challenges](image)

*Figure 5: Dimensional variability challenges faced in a modular construction project.*

eventually resolved before the building was put into service, however there was impacts in the form of rework, cost overruns and project delays. In addition to these examples of dimensional variability challenges gathered from experience with different projects, the following examples highlight other
researchers’ work in dealing with dimensional variability. Since there has not been a comprehensive study
of the management of dimensional variability in modular construction, the following examples come from
offsite construction and other Architecture, Engineering, Construction (AEC) projects. The challenges
faced in the following examples and the resulting consequences are not uncommon to modular
construction (and in some cases would even be even more challenging in the context of modular
construction).

A case study by Milberg et al. (2002) and later again published by Milberg and Tommelein (2009)
analyzed dimensional challenges for the installation of soldier piles. The piles being installed were not
oriented correctly in some cases, requiring trimming and welding of the clip angles used to attach roof
girders between two sets of piles. Of the 640 connections between clip angles and roof girders, 10 were
problematic from a geometric standpoint, which lead to rework and created delays. In this case study it
was noted that there were no general guidelines found in literature for tolerance specifications of soldier
piles in walls, which is partly why the dimensional variability challenges emerged (Milberg and
Tommelein 2009). Furthermore, this study found that the progressive accumulation of dimensional
variability (referred to as tolerance stackup) was also a significant factor in the dimensional variability of
soldier piles.

In a separate case study conducted by Milberg (2006) the interface between a prefabricated window and a
cast in place concrete building frame was examined. This case study analyzed a tolerance problem where
some windows would not fit properly into the building (65 of the 560 total windows in the building did
not fit properly). In terms of the gap tolerance between the outer envelope of the prefabricated window
and the concrete frame, two tolerances caused conflicts for the installation. The caulking joint around
the window was limited to a minimum of 3/8” (10 mm) for caulking performance (which left a +/- 10 mm
variation). However the concrete tolerances were allowed to vary as much as 1” (25 mm), resulting in
some windows not being able to fit properly in cases where the concrete variation was at a maximum. The
design assumed the tolerance of window frame, sill, head and jambs was 0 mm. Furthermore, the design
did not properly account for the concrete tolerances. Once the concrete was cast, the concrete tolerances
(as per ACI standards) were too large for the caulking gap tolerances (Figure 6).

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Max Allowed</th>
<th>Min Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Caulking Gap</td>
<td>10 mm + 0 mm</td>
<td>10 mm + 0 mm</td>
</tr>
<tr>
<td>Concrete Variation</td>
<td>Nominal + 25 mm</td>
<td>Nominal – 6 mm</td>
</tr>
</tbody>
</table>

**Dimensional Variability Conflict:** If the concrete reaches the + 25 mm maximum allowable tolerance value as permitted by ACI, this would exceed the 10 mm gap required between the window edge and concrete (for caulking). This leads to a dimensional variability conflict where deviations of a certain tolerance do not allow aggregation to take place.

**Figure 6:** Depiction of caulking gap and concrete variation (with explanation of tolerance values)

In a study by Acharjee (2007), dimensional variability associated with the installation of bleachers at a
tennis court was examined. Inadequate considerations were made with regards to process capabilities
which led to conflicts in the interaction between design, fabrication and installation tolerances. Some of the problems encountered in this project include steel framework and decking re-adjustments, variations in floor levels at base points, distorted and misaligned handrails and other misalignments between the bleacher modules and foundations. The impacts of these dimensional variability challenges resulted in increased wait time, increased costs and compromised quality (Acharjee 2007).

The examples presented above demonstrate some of the challenges of dimensional variability faced during the fabrication, aggregation and erection of construction assemblies. In addition to encountering challenges with respect to dimensional variability in these project stages, dimensional variability can also be a challenge during the transportation and handling of modules. One example of this comes from a study conducted by Johnsson and Meiling (2009) which examined the cause of defects encountered in prefabricated timber housing modules. In this study, there were notable effects of dimensional variability associated with transportation processes, where either doors or windows needed to be adjusted or walls were cracked due to the movement of the structure. In a modular high-rise project, it was found that the design of modules experienced their worst-case structural loads during transportation. Even after providing adequate strength based on an assumed transportation load, some damage was still experienced, revealing the fact that transportation can create changes in the geometry of modules, leading to potential dimensional variability (in the form of geometric changes), or even damage (Gardiner 2015).

Comparison of risk between offsite and onsite construction reveals that some of the largest risk exposure in offsite construction relates to tolerances (discrepancies between onsite and offsite components), handling (cumbersome large scale unit installation), fit (offsite components do not fit properly), and overall quality (Smith 2011). Each of the examples shown above demonstrate that dimensional variability can be challenging and problematic in modular construction. The poor management of dimensional variability whether in the form of gaps between components, misalignments at joints, poor tolerance specification, or geometric changes during transportation, has an impact in terms of project performance, cost and schedule. All of these factors can be viewed in terms of risk (Shahtaheri 2014):

\[
\text{Risk of a certain event due to poor dimensional variability management} = (\text{Probability of event occurring}) \times (\text{Impact in terms of cost, schedule or performance}) \quad \text{[Eqn. 1]}
\]

Risk related to poor dimensional variability management often requires some form of rework in order to achieve adequate aggregation or compliance with certain performance or quality requirements. In some cases, rework can contribute to over 50% of a project’s cost-growth and over 25% of the variance in cost-growth (Love 2002a). The direct costs of rework include tangible quantities such as materials, man-hours, equipment, and spatial occupancy (Tommelein et al. 2007). Indirect costs relate to delays, loss of schedule, decreased productivity, litigation, claims, and low operational efficiency. Since indirect costs cannot be easily quantified, they are often ignored when addressing the total impact of rework (Simpeh et al. 2012). Indirect rework can have a significant contribution of total rework in a project, comprising between 75% and 600% of direct rework costs (Love 2002b, Simpeh et al. 2012). Rework related to dimensional variability defects can be a very significant portion of the total rework involved in projects employing prefabrication due to the complexities of merging manufactured products with stick-built components. Dimensional defects have also been the focus of recent studies, in order to determine and evaluate causation. In a study by Jingmond and Ågren (2015), the cause of construction tolerance defects (which has direct links to dimensional variability) was cognitively mapped to unclear tolerance management, a lack of a holistic approach, technical deficiencies, complex behavior of materials and a
lack of knowledge. Clearly, rework and defects related to dimensional variability can be very problematic. While there has been some effort to quantify causes of rework and defects related to dimensional variability, it is clear that numerous impacts and consequences can emerge without a strategy for properly managing dimensional variability.

2.2.2. Classification of Tolerances in Construction

Through search of various construction literature, no comprehensive works to classify dimensional variability or its impact as a whole throughout the lifecycle of the construction process (i.e., from manufacture, to fabrication, to aggregation, to final erection on site) were discovered. However, numerous codes and standards for specific materials and applications (e.g., precast concrete construction, steel bridge construction, timber residential construction etc.) are available and provide guidance for how to specify critical tolerances. In addition, several guidelines exist for specifying the values of these critical tolerances. As a whole, these resources present distinct categories for grouping tolerances and the reasons for why they are needed. This section aims to summarize tolerance classification for common modular construction building materials (e.g., concrete, steel, and timber) as seen in Table 3 and then provides an amalgamated categorization structure which is used in the proposed methodology for classifying the impacts of dimensional variability in modular construction.

<table>
<thead>
<tr>
<th>Resource</th>
<th>Categorization of Tolerances (and additional notes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handbook of Construction Tolerances (Ballast 2007).</td>
<td>This resource provides recommended tolerance values for a comprehensive list of materials and applications within construction. This list contains information about building layout, concrete, steel unit masonry, stone, structural lumber, finish carpentry and architectural woodwork, curtain walls, finishes, glazing, and doors and windows. Recommended tolerance values are grouped together based on a categorization structure of manufacture, fabrication and installation.</td>
</tr>
<tr>
<td>Steel Construction Institute</td>
<td>This resource provides a range of publications dealing with various topics within structural engineering and draws upon British Standards and Eurocodes for construction. Within their National Structural Steelwork Specification for Building Construction publication, the tolerance categorization structure found is materials, fabrication (“buildability”), erection and final construction quality. In addition to this publication, the Steel Construction Institute has an interesting breakdown of dimensional imperfections in a separate publication (SCI P185) Guidance Notes on Best Practice in Steel Bridge Construction where dimensions are expressed as being either random errors (which have a general tendency to be self-compensating when accounting for their statistical probabilities), or systematic errors (will accumulate, and not be self-compensating).</td>
</tr>
<tr>
<td>BS EN 1090-2: Execution of steel structures and aluminum structures.</td>
<td>This standard has tolerances grouped into two types based on whether they are “essential tolerances” (associated with strength and stability) or “functional tolerances” (associated with fit-up). In addition, there are several tolerance classes: Class 1 is used for normal structures, whereas Class 2 is</td>
</tr>
<tr>
<td>Technical requirements for steel structures</td>
<td>used when tighter tolerances are required (as would be the case for most interfaces in modular construction).</td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Institute of Civil Engineers</td>
<td>This resource provides a publication Manual of Structural Design which has a chapter dedicated to the topic of building movements and tolerances (Silva 2012). The tolerance categorization structure found in this publication is: standard tolerances (required for all buildings, and based on codes and standards), particular tolerances (tighter than standard tolerances, and are required for fit-up or other requirements), and special tolerances (tighter than standard tolerances, applied to the entire structure, and are required for special serviceability, aesthetic or aggregation requirements).</td>
</tr>
<tr>
<td>Design in Modular Construction (Lawson et al. 2014)</td>
<td>This resource refers to BS EN 1090-2 for classification of required tolerances (note that this standard being referred to is for steel and aluminum structures). In addition to referring to this standard, this resource provides recommended tolerance categories specific to modular construction. For instance, it presents a separate tolerance category for the structural effect of ‘out-of-verticality’ in modular high rise applications. This tolerance is to ensure the placement of modules on each level are within a certain threshold to minimize eccentricity of loads and to ensure that modules can be erected on site properly. For steel-framed modules, two special tolerance categories are presented: ‘gross-dimension’ manufacturing tolerance (how the width, length or height of a module needs to be controlled), and positioning tolerance (how accurate the placement of a given module needs to be in 3 dimensional space).</td>
</tr>
<tr>
<td>Prefab Architecture – A Guide to Modular Design and Construction (Smith 2011)</td>
<td>This resources categorizes tolerances for prefabricated construction into two classes: part or subassembly tolerance (related to components, panels or module), and assembly tolerance (related to the assembly itself as well as the process of placing the subassemblies on site). Through this definition, there is a strong focus on the effect of accumulation of tolerances between subassemblies and components.</td>
</tr>
<tr>
<td>American Institute of Steel Construction (AISC)</td>
<td>This resource has two standards in particular which outline required tolerances for steel construction: AISC 360-10 Specification for Structural Steel Buildings and AISC 303-10 Code of Standard Practice for Steel Buildings and Bridges (AISC 2010a, AISC 2010b). In both of these codes, there are three main categories for tolerances: materials (primarily mill dimensional tolerances as per ASTM A6/A6M), fabrication and erection (erection tolerances were developed through long-standing usage and observations). Distinction is made here that a tolerance not falling into one of these classes does not mean it has a value of zero. While this might seem to be somewhat intuitive, these codes imply that the tolerance categories proposed do not constitute all tolerances required in a project.</td>
</tr>
<tr>
<td>American Concrete Institute</td>
<td>This resource has a publication ACI 117-10 Specification for Tolerances for Concrete Construction and Materials (with Commentary) which provides an extensive overview of tolerances in concrete construction. An interesting definition for tolerances is presented in the introduction of this publication,</td>
</tr>
</tbody>
</table>
where they are referred to as a means of establishing permissible variation in dimension and location. The important part of this definition is the emphasis on both dimension (relating to the component itself) as well as the location (relating to how that component is placed within an assembly). This is one of the only resources which explicitly classifies the location aspect of specified tolerances. This resource has another publication ACI 117.1R-14 Guide for Tolerance Compatibility in Concrete Construction which is one of the most comprehensive works for specifying tolerances, ensuring compatibility between assemblies and suggesting methods for accommodating required tolerances on a project. Tolerances in this publication are expressed in terms of material, product, erection and a separate category is given for measurement.

<table>
<thead>
<tr>
<th>Resource</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCI Design Handbook (PCI Industry Handbook Committee 2004)</td>
<td>In this resource, tolerances are categorized into two main groups: product tolerances and erection tolerances. Product tolerances relate to the manufacture and fabrication of precast concrete elements, and sub-categories for tolerances relate to overall dimensions, sweep or horizontal misalignment, position of strands, camber, weld plate, haunches of columns and wall panels, warping and bowing, and smoothness. For erection tolerances, sub-categories include provisions based on equipment required, type of building components they are installed into and connections and bearing type.</td>
</tr>
<tr>
<td>Victoria Building Institute Guide to Standards and Tolerances (Victorian Building Commission 2007)</td>
<td>This resource provides an overview of tolerances required for building construction. Within this resource, there is special attention given to tolerances required for timber. Tolerances are grouped based on the surface (horizontal or vertical), measurement method, door spacing, and roofs.</td>
</tr>
<tr>
<td>Canadian Hardwood Plywood and Veneer Association</td>
<td>This resource provides an online manual which provides an overview of categories for dimensional variability (Pierre Walsh 2010). The interesting thing about this particular resource is that while most wood design manuals make no mention about tolerances, this resource extends past the use of tolerances right to the discussion about dimensional variability. Categories include raw material (anisotropy of the raw material), expected finish tolerances, and environmental conditions.</td>
</tr>
</tbody>
</table>

In summary, there are many resources which recognize the need for tolerances in the design of construction components. Tolerance categories for all materials explored (concrete, steel and timber) can be broadly categorized into the following structure: material tolerances, manufacturing tolerances, fabrication tolerances, aggregation tolerances, measurement tolerances, erection tolerances, final assembly quality tolerances, functional tolerances and finally performance tolerances. Among the most common building materials in modular construction, resources related to concrete were found to have the most extensive approach for specification and classification of tolerances. This is primarily due to the fact that in concrete construction, there are many prefabricated components (e.g., precast) in which the dimensions and geometry of components need to be controlled within acceptable limits in order to ensure
proper fit on site. Of the materials explored (concrete, steel and timber), timber had the least comprehensive approach for specification of tolerances. This is perhaps due to the fact that timber can be easily adjusted in order to control dimensions and geometry. In the context of construction, timber construction is primarily used in the construction of homes, bathroom pods, temporary buildings, and small to medium sized commercial buildings.

2.2.3. Rationale for Tolerance Specification and Dimensional Variability Management

In construction, there is a wide belief that dimensional variability can be properly managed if only the correct tolerances are specified (Milberg and Tommelein 2009). This belief overlooks the fact that certain construction processes have intrinsic degrees of variability. By specifying strict tolerances, the underlying process capabilities do not necessarily change. Rather, process capabilities need to be altered in way which affects their dimensional variabilities in order to achieve certain tolerance values. However, this often comes at an increased cost (Creveling 1997). In order to understand why tolerances are specified, this section provides a condensed overview of the rationale behind tolerance specification as outlined in construction literature. This rationale can be directly used for management of dimensional variability, since tolerances are traditionally used in construction for controlling the adverse effects of dimensional variability.

Tolerance categories differ from tolerance rationale in that categories are used in more of a checklist format, ensuring designers provide adequate information for those involved with various construction processes (i.e., manufacture, fabrication, aggregation, erection). However tolerance rationale is why tolerances are needed. Tolerance rationale is used to determine tolerance values. Although in practice, most tolerance values are based on a priori knowledge, in some cases special tolerances for a project need to be derived from a first principles approach. In this case, tolerance rationale is extremely important to understand. This section presents tolerance rationale for the key building materials explored in this thesis: steel, concrete and timber.

2.2.3.1. Steel Tolerance Rationale

For steel construction, material tolerances (note that often ‘material’ is used to describe standard structural shapes and not the steel material itself) are required in order to control the geometry and dimensions of components against manufacturing related processes such as roll wear, thermal distortion, forming rolls, and differential cooling distortion. Another important aspect of material tolerances deal with structural properties, which includes variations in yield strength, tensile strength, unit weight, etc. (as outlined in codes such as ASTM A6/A6M). While the variations in material properties are not directly related to dimensional variations, they can have an effect when subjected to various loads, which is why tolerances on material properties are also required. It is important to note that material tolerances do not need to be so strict as to have a perfect geometric cross section for steel products since perfect geometry is not necessary for structural or even architectural (i.e., aesthetic) reasons (AISC 2010a). Similar to material tolerances, the tolerances associated with fabrication processes are used to ensure components can be aggregated properly, while at the same time ensuring proper downstream requirements are met (such as structural performance, aesthetics and serviceability). The National Structural Steelwork Specification for Building Construction outlines the rationale for fabrication tolerances as being in terms of essential tolerances (those affecting the mechanical resistance and stability of structures), functional tolerances (limits used for fit-up and aesthetics) and special tolerances (those used for other project
specific requirements). Often, fabrication tolerances are mainly specified to ensure proper coordination and aggregation of components (Silva 2012). Erection tolerances are required for ensuring that components can be properly aggregated on site (especially with respect to the interfaces between substructures and foundations), for ensuring compliance with final performance requirements (which could be either building-related, or related to the building services or equipment), for ensuring proper aesthetics of the final assembly, and for maintaining certain site boundary limitations (AISC 2010a, Silva 2012, Steel Construction Institute 2016). In addition to these reasons for tolerances in steel construction, there has been a concerted focus on the structural significance of tolerances (i.e., the safety of a structure). The effects related to structural safety has explored misalignments at joints, multi-story out-of-verticality, and in some cases has even explored the second order effects of installation tolerances in modular high rise buildings (Lawson and Richards 2010, Mann and Morris 1984, Steel Construction Institute 2016).

2.2.3.2. Concrete Tolerance Rationale

For concrete construction, material tolerances are specified for similar reasons as for steel, with special attention to the heterogenous properties of concrete versus steel or even timber (Smith 2011). The purpose of specifying material tolerances (note that ‘material’ is often used to describe the mixture of water, cement and air) for concrete construction is to control structural properties such as compressive strength, density, flexural strength, tensile strength, shear strength, modulus of elasticity, etc. Material tolerances which impact the structural performance include air content, moisture content, and aggregate size (American Concrete Institute 2004). Dimensional tolerances are also very important in terms of the structural performance of concrete. These tolerances include overall cross sectional dimensions, positions of reinforcing/anchors, and plumbness (PCI Industry Handbook Committee 2004). In terms of the overall rational behind concrete tolerances, numerous codes and guidelines outline the following reasons: structural safety (ensuring design is not sensitive to critical variations), feasibility (i.e., constructability by making sure components can fit together properly), aesthetics of final assembly, economy (i.e., ensuring ease and speed of production and erection), functionality (i.e., making sure the final assembly achieves certain performance requirements, both for the structure and non-structural elements), and legal or contractual (i.e., ensuring properly lines are not encroached upon) (ACI 2002, American Concrete Institute 2004, Malisch and Suprenant 2005, PCI Industry Handbook Committee 2004).

2.2.3.3. Timber Tolerance Rationale

Unlike steel and concrete design codes, standards and guidelines, which are explicit in the rationale for tolerances, timber does not have similar coverage. Through examination of both Canadian and American timber design guides, there is little-to-no explanations about tolerances (with the exception of tolerances for certain steel fasteners). However, certain timber specialty trades (such as hardwood veneer) have outlined rationale for controlling dimensional variability. In one online resource, reasons for controlling dimensional variability included controlling geometry against the effects of warping, shrinkage, thickness variability, flatness variability, which can all stem from both raw material properties (e.g., wood grain, knots, etc.) manufacturing processes (e.g., sawn, machining, cutting) and environmental conditions (e.g., moisture content, temperature) (Pierre Walsh 2010). In this resource, the emphasis on controlling dimensional variability is to ensure proper structural performance and functionality of the construction assembly.
In summary, there are many reasons why tolerances are needed in construction for controlling dimensional variability. Of the materials explored in this literature review (steel, concrete and timber), the main reasons for specifying tolerances include safety of structure, constructability, aesthetics, performance, and quality.

2.2.4. Current Approaches for Managing Dimensional Variability in Construction

The management of dimensional variability in construction is often approached by specifying standardized tolerances, or use of various ad-hoc strategies, which often includes adjusting dimensions and geometry on site by custom fitting components at the job site (Figure 7).

Numerous sources outline standard tolerances and ad-hoc dimensional variability management strategies. First of all, one of the most common approaches for controlling dimensional variability is through the specification and control of very tight (small) tolerances (Ballast 2007). While this approach might be effective in some cases, it is expensive to modify construction process capabilities to match the desired tolerance values (which is justified if process modifications save more than their upfront costs). For this reason, ad-hoc strategies are used, which move away from the use of tight tolerances, and towards (often) more cost-effective solutions. These solutions require an understanding of intricate design details and how to effectively accommodate dimensional variability. As such, the use of ad-hoc strategies are effective, yet can be very time consuming to implement correctly, since it requires a priori knowledge, past experience or requires continuous improvement in order to yield effective solutions. Some examples of ad-hoc strategies as outlined in literature can be found in Table 4.
<table>
<thead>
<tr>
<th>Resource</th>
<th>Ad-hoc strategies for managing dimensional variability in construction</th>
</tr>
</thead>
</table>
| **Handbook of Construction Tolerances (Ballast 2007)** | Custom joint design: (1) selection of type of joint (butt, reveal, covered, sliding, offset or sealant joint), (2) anticipate accumulation of tolerances and dimensional variability and (3) designing joints to manage dimensional variability properly based on a set of predefined objectives. This approach is used for many architectural connections on projects. Comprehensive list of other ad-hoc dimensional variability management strategies for the following materials and applications:  
  - cast-in place or precast concrete frames,  
  - cast-in place concrete joints,  
  - amalgamated precast and cast-in-place concrete joints,  
  - brick/stone and cast-in-place joints,  
  - curtain walls on concrete frames,  
  - doors/windows in precast or cast-in-place concrete,  
  - precast to precast joints,  
  - precast and steel joints,  
  - masonry and precast systems,  
  - accumulated tolerances in columns and steel frames,  
  - brick/stone and steel joints,  
  - curtain walls on steel,  
  - brick/stone and masonry joints,  
  - doors/windows in masonry,  
  - timber frames,  
  - prefabricated structural timber, and  
  - panelling on stick-built substrates |
| **Institute of Civil Engineers (Silva 2012)** | Onsite adjustment: this resource states that the probability of accumulation of tolerances is low for manufacture, fabrication and erection tolerances, and that simple onsite adjustments can be used to avoid the accumulative effect of deviations. Other ad-hoc strategies also included:  
  - For steel: packing pieces, slotted holes and threaded rods;  
  - For concrete: packing pieces at non-structural connections, whilst slabs can either be ground or coated with liquid latex to smooth out imperfections;  
  - For timber: cut members to size or level with packers to smooth out imperfections, store timber in conditions similar to the intended final condition in order to allow its moisture content to equalised prior to fabrication and erection;  
  - For structural aluminium: top-hung facade panels, packing pieces, slotted holes, threaded rods. Since fit tolerances tend to govern, ensuring an accurate pre-erection survey is done is paramount for aluminium. |
| **Prefab Architecture – A Guide to Modular Design and Construction (Smith 2011)** | Mateline stitching between panels and modules. Fitting mechanisms: sliding fit (components overlapped), adjustable fit (oversized or slotted hole connections), and reveal (purposely put gap between components which can have varying dimension). |
| **American Institute of Steel Construction** | Onsite adjustment: when member straightness is non-compliant during erection, it can often be corrected through adjustment of the geometry (i.e., members are often flexible enough to warp/rack into place). Ad-hoc geometric design strategies: |
- oversized holes, and
- specification of a plumb tolerances for components or an overall envelope tolerance.

**Construction strategies:**
- position, secure and align components in groups through the use of a jig,
- use two baseplates in cases where tolerances for bolts are too tight, and
- use advanced layout techniques.

There have been very few attempts to introduce a systematic approach for managing dimensional variability in construction. Previous researchers have demonstrated that there is often discontinuity between process capabilities and tolerances used in specifications (Milberg and Tommelein 2003a). As such, research has focused on analytical methods for improving the management of dimensional variability through compatibility between process capabilities and desired tolerances. One such method is referred to as **tolerance mapping** which utilizes a manufacturing tolerance notation called **Geometric Dimensioning and Tolerancing, GD&T** (Milberg and Tommelein 2005). Furthermore, it utilizes graph theory in order to ‘map out’ connected components in an assembly. The general procedure for creating a tolerance map can be described in three steps: (1) create an assembly network to define how all parts are geometrically related to each other in an assembly, (2) create component diagrams to define how all component-features are geometrically related to each other in each component (note: component features represent the geometrical properties of a component such as lines, angles, and surfaces), and (3) amalgamate all component diagrams in the assembly network to create the overall tolerance map.

Tolerance mapping has been shown to be an effective way of managing dimensional variability (Milberg 2006), however it only addresses the specification of tolerances. Furthermore, it can be challenging to use without proper knowledge about GD&T notation, or proper consideration of only critical sources of variability (otherwise, tolerance maps can become excessively large and too tedious to use practically).

### 2.2.5. Current Approach for Managing Dimensional Variability Risk in Modular Construction

There are many types of risk related to the management of dimensional variability as was demonstrated in previous sections. Similar to the ways that the construction industry as a whole manages dimensional variability, modular construction also employs standardized tolerances and ad-hoc strategies. Despite the categorization of tolerances and the widespread belief that “the inherent probability of all unfavourable extreme deviations occurring together is small, and simple means of on-site adjustment can be incorporated to avoid the cumulative accumulation of deviations” (Silva 2012), what happens when the accumulation of deviations does cause problems? This raises the question about how to effectively manage risk related to dimensional variability. Often, strategies for managing dimensional variability are developed without consideration of critical sources of variability, or about the financial impacts. For instance, one such risk management strategy could be the implementation of very precise methods of production. In some projects, this management strategy is successful, however in other cases component geometry can change from its nominal design state as the result of flexing and warping due to handling, transporting and installation (Neelamkavil 2009). Another risk management strategy is the use of strict fabrication tolerances. Despite utilizing strict fabrication tolerances, erection is still often problematic since there is discontinuity between offsite fabrication precision and the onsite construction precision. Both of these examples often do not consider the financial cost associated with them, nor do they consider impacts throughout the entire construction lifecycle that can negatively affect the management of dimensional variability. For this reason, the current approach for risk management of dimensional variability in modular construction is often not effective.
variability in modular construction encompasses a wide range of trial and error strategies (Shahtaheri et al., 2015). Trial and error strategies can be summarized into two distinct approaches (Table 5) depending upon whether they are reactive (responding to issues only once they have occurred) or proactive (anticipating potential issues and implementing certain avoidance measures).

**Table 5: Examples of Risks Related to Dimensional Variability & Corresponding Risk Responses in Construction**

<table>
<thead>
<tr>
<th>Risk Related to Dimensional Variability</th>
<th>Examples of Risk Response</th>
<th>Reactive Strategies</th>
<th>Proactive Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component(s) too large</td>
<td>Trim parts, exchange with another component, ream holes, warping &amp; racking</td>
<td>Splicing &amp; lapping at joints, pipe cut-lengths, strict production tolerances, produce smaller component and fill-in gaps</td>
<td></td>
</tr>
<tr>
<td>Component(s) too small</td>
<td>‘Fill in the gap’: spacers, shim plates, grouting, caulking</td>
<td>Splicing &amp; lapping at joints, pipe cut-lengths, strict production tolerances</td>
<td></td>
</tr>
<tr>
<td>Component(s) not level</td>
<td>Grouting, shim plates, spacers</td>
<td>Strict production tolerances, self-levelling technologies, flexible and adjustable connections</td>
<td></td>
</tr>
<tr>
<td>Excessive geometry changes &amp; misalignments</td>
<td>Warping &amp; racking, discard component &amp; replace</td>
<td>Flexible or adjustable connections, increasing relative stiffness to withstand larger loads, rigging strategies, temporary bracing strategies</td>
<td></td>
</tr>
<tr>
<td>On-site fit-up &amp; aggregation conflicts</td>
<td>Warping &amp; racking, force-fit into place</td>
<td>Flexible or adjustable connections</td>
<td></td>
</tr>
</tbody>
</table>

Both proactive and reactive trial and error strategies present challenges for dealing with risk posed by dimensional variability in construction. The challenge of using proactive strategies is that not all dimensional management issues are identified, which means that reactive strategies are still often required. The challenge of using reactive strategies is that the cost to fix a problem can often be very expensive and time consuming in comparison to proactive strategies. While the use of reactive trial and error strategies have traditionally worked well in stick-built construction, they do not function as well for modular construction since module production occurs offsite before being interfaced with the site. This therefore encourages the use of proactive strategies for managing risk related to dimensional variability since module conditions and interface properties need to be anticipated and designed-for long before erection on site occurs.

2.3. Dimensional Variability Management in the Manufacturing Industry

The manufacturing industry has long been using proactive strategies for effectively managing dimensional variability. Since modular construction encompasses many similarities with the manufacturing industry, it can be used analogously in many applications for adopting effective dimensional variability management strategies. This section provides a very brief overview of tolerancing theory used for managing dimensional variability in manufacturing applications such as aerospace and automotive assembly production. A much more comprehensive overview of the tolerancing theory used in manufacturing for managing dimensional variability is found in Appendix A.

The manufacturing industry can be regarded as the birthplace of comprehensive tolerance design due to the importance of ensuring part interchangeability and functionality in mass produced assemblies. The
The overall purpose behind tolerancing in manufacturing is to appropriately target dimensional variability, and to either (1) anticipate and design a system that can function with it or (2) control it through a more intricate product and process design. Tolerancing is carried out at several levels, starting with component-features, then components and finally for the overall assembly. Informed decision making in tolerance design requires an understanding of how variability affects both production and overall product functionality. Tolerances are specified in order to control deviations associated with size (linear and angular dimensions), form (lines and surfaces), orientation (lines and surfaces), location and runout (Henzold 2006, Meadows 2009). A useful classification structure provided in Appendix A which outlines the types of dimensional variability and tolerances used in manufacturing is shown in Figure 8.

**Figure 8: Classification of dimensional variability and corresponding tolerances in manufacturing**

In order to control the negative impact of dimensional variability in manufacturing, three approaches are often used, which stem from key design principles: (1) design for manufacture (DfM), (2) design for assembly (DfA) and (3) design for manufacture and assembly (DfMA). These approaches are discussed in the following section in terms of addressing the ability to control geometric variability.

In design for manufacture (DfM), components are designed to be economically manufactured, usually with less focus on the economical aspect of aggregation. Therefore, in order to control dimensional variability in DfM, it must be carried out during the assembly process. In interchangeable component manufacturing, components are often produced and aggregated at random (Asha et al. 2008). Random aggregation is time consuming and prone to rework, which is why the concept of selective assembly is applied. Selective assembly is a concept used in manufacturing to improve the aggregation of interchangeable components in a high volume assemblies. Rather than relying on adherence to tight tolerances for every component, components are manufactured with slightly more relaxed (or loose) tolerances and then grouped into categories which are then selected for assembly on the basis of interchangeability. Even as early as the 1950’s, this idea proved to be an economical approach for manufacturing high volume assemblies (Mansoor 1961). Selective assembly has become a popular way of enabling high-precision aggregation using low-precision manufacturing techniques (Pugh 1986).

Research has since focused on methods for optimizing the number of bins and the selection process for

In design for assembly (DfA), components are designed to be economically aggregated, usually with less focus on the economical aspect of manufacturing. Therefore, in order to control geometric variability in DfA, components must be extensively toleranced, usually at the expense of more costly and time consuming manufacturing techniques. In tolerance design, a trade-off is usually made between product quality and the cost to achieve certain tolerances (Creveling 1997). A rigorous tolerance design can be very intensive, both from a design and manufacturing standpoint. The design process employs use of concepts such as tolerance analysis, tolerance synthesis, and tolerance transfer, which can be very time consuming to properly implement. The design process also requires proper tolerance communication (i.e., Geometric Dimensioning & Tolerancing) which can be very time consuming to properly use. The success of a tolerance design also relies on controls implemented during the manufacturing and aggregation process. Depending on specific geometrical tolerance requirements, high precision equipment and processes must be used, in addition to detailed inspection measures. Therefore, it can be expensive and time consuming to manufacture components with very low geometric variability due to the various design, communication and control measures required. As such, a more balanced approach (i.e. design for manufacture and assembly) is often preferred because it is the most economical way to control the negative effects of geometric variability.

Design for manufacture and assembly (DfMA) is a common design approach used in manufacturing for minimizing overall production costs by addressing the tradeoff between manufacturing and aggregation process capabilities (Kamrani and Sa'ed 2002). DfMA provides great flexibility with regards to where and how geometric variability can be controlled. Rather than placing strict controls on either manufacturing or aggregation process, DfMA can be used to target the most prominent negative aspects of geometric variability. As such, this design approach is commonly preferred over DfM or DfA for its flexibility and ability to optimize overall production costs.

In addition to the design approach used for the specification of tolerance, there are numerous concepts and tools used to help predict and analyze both variability and adequacy of tolerance values. Among these tools are variation noise diagrams, dimensional variation analysis models, dimensional tolerance chain models, 3-D tolerance propagation models, tolerance assistance models, variation simulation tolerance analysis, torsor vector models, computer aided tolerancing, tolerance charting and process charting (Creveling 1997, Drake 1999, Hong and Chang 2002, Hong-Chao Zhang 1997, Liggett 1993, Sleath and Leaney 2013b). All of these tools have different applications and provide the designer with information and analysis guidance on how to properly specify tolerances.

In summary, tolerancing has evolved with time to become a foundation in manufacturing design, with the core emphasis on optimizing the overall cost trade-off between product tolerances and process capabilities. As the construction industry continues to adopt methods related to modular construction which have strong comparisons to manufacturing, tolerance theory using in manufacturing can be adopted for utilization in construction.
2.4. 3D Imaging as a Means of Quantifying Dimensional Variability

Various methods of measurement can be used to quantify dimensions and geometric properties of construction assemblies. Among the most commonly used include metal measuring tapes, laser rangefinders, carpenter’s levels, digital inclinometers, transits and construction lasers, electronic instruments and laser scanners (Ballast 2007). Each of these devices provide different information and have different levels of accuracy. Among these devices, the use of 3D imaging (e.g., laser scanners) is becoming used much more frequently on projects due to its ability to capture rapid geometric data in huge quantities (millions of data points), and has very good accuracy (Table 6).

Table 6: Comparative summary of dimensional data capture tools in construction

<table>
<thead>
<tr>
<th>Dimensional Data Capture Tool</th>
<th>Data Type/Quantity</th>
<th>Estimated Accuracy (Ballast 2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal tape measure</td>
<td>Single linear measurements</td>
<td>Varies between +/- 0.3 mm for 2 meter length to 20.3 mm for 100 meter length (depends primarily on length and manufacturing class) as per ISO 9001</td>
</tr>
<tr>
<td>Laser rangefinder</td>
<td>Single linear measurements</td>
<td>+/-1.5 mm at 200 m distance to +/- 3 mm at 100 m (note that the accuracy decreases over longer distances)</td>
</tr>
<tr>
<td>Carpenter’s level (requires use of tape measure)</td>
<td>Single angular measurements</td>
<td>Subject to accuracy of tape measure, but are typically equal to +/- 1°</td>
</tr>
<tr>
<td>Digital inclinometer</td>
<td>Single angular measurements</td>
<td>Can be as accurate as +/- 0.1°</td>
</tr>
<tr>
<td>Transits and construction lasers</td>
<td>Single linear measurements</td>
<td>1.6 mm at 30 m</td>
</tr>
<tr>
<td>Electronic instruments</td>
<td>Floor flatness (F-number)</td>
<td>NA</td>
</tr>
<tr>
<td>Laser scanners</td>
<td>Continuous surface data (point clouds)</td>
<td>Can be as accurate as 0.025 mm (however does decrease at larger distances)</td>
</tr>
</tbody>
</table>

Measuring geometric deviations and component alignment on project sites is a challenging task that needs to be performed in order to monitor compliance with design specification as well as to control excessive accumulation of dimensional variability. Traditional methods for geometric measurements can be prone to error, are time consuming and lack a sufficient level of automation. As such, 3D imaging in construction has emerged as a powerful tool for geometric quality monitoring and discrepancy quantification (Nahangi et al. 2014, Nahangi and Haas 2014). For measuring dimensional data, 3D image (point cloud) registration is a solution for comparing the as-built state with the as-designed state. While the as-designed state of components can be obtained through a computer-aided design (CAD) model and integrated with the building information model (BIM), the as-built state can be obtained through the use of laser scanning or structure-from-motion systems (also known as stereo vision). Semantic information about the as-built state from the acquired point clouds is generally extracted using two different approaches: (1) extracting objects from the point clouds and then comparing to the as-designed state integrated in the Building Information Model (BIM). This method is also called scan-to-BIM (Bosché et al. 2015), and (2) comparing the as-built and as-designed dimensions directly and detecting deviations for tracking the quality and progress of the project. This is also called scan-vs-BIM. Deviation analysis (which compares the as-built and as-designed states) quantify discrepancies, and is used in a range of applications in construction, including quality defect detection (Akinci et al. 2006), dimensional compliance control (Bosché 2010), floor flatness evaluation (Tang et al. 2010) assessing quality of as-built BIM (Anil et al. 2013, Tang et al. 2011) and tolerance analyses (Kalasapudi et al. 2015).
3. Proposed Methodology

While the ad-hoc dimensional variability management solutions presented in the background section may be adapted and applied for use in various modular construction applications, a more systematic approach for managing dimensional variability is often required. This is because the accumulation of dimensional variability can be very complex for modular assemblies, which can significantly impact the expected project performance. The systematic management of dimensional variability not only helps to manage overall project risk, but targets critical sources of variability in order to develop an optimal management approach. This chapter presents a framework that applies to the entire construction lifecycle of a modular construction project for managing dimensional variability. The manufacturing industry is used analogously to assist in the aggregation of components and assemblies.

The proposed framework is shown in Figure 9. The premise behind the proposed framework is to target critical sources of dimensional variability, then to develop an appropriate management strategy (which can be comprised of design-based, production-based, handling-based or onsite-based considerations). Then after implementing the developed strategy, a method for monitoring the performance of the strategy is used so that adjustments during the project can be made if necessary. Novel developments made in this thesis are highlighted in yellow, which relate to identification of critical sources of dimensional variability, design-based strategies, production-based strategies, and finally the method for monitoring the performance of the overall dimensional variability management strategy.

![Figure 9: Proposed framework for managing dimensional variability throughout the construction lifecycle (manufacture, fabrication, aggregation, transportation, handling and erection) of a modular construction project. New developments made in this thesis are highlighted in yellow.](image-url)
This chapter presents the proposed framework in terms of each step in Figure 9, followed by the scope, objectives, and constraints that govern the framework.

Following this chapter, research developments made in this thesis are outlined as follows:

- Development of the deviation analysis method used to monitor the strategy (Chapter 4)
- Case study for analyzing critical dimensional variability in a modular building (Chapter 5)
- Development of novel design-based strategies and case study demonstration (Chapter 6)
- Development of novel production-based strategies and case study demonstration (Chapter 7)

3.1. Determine Factors Influencing Variability

The first part of the proposed strategy focuses on determining factors that influence variability. This can be done by considering a predefined classification used in the manufacturing industry for sources of variability (Table 7). This table is structured according to the classification structure of materials, machines, manufacturing methods, manpower, measurement and environment which comes from lean six sigma literature (Keller 2011). This structure can be directly adapted for modular construction due to the similarities it shares with manufacturing. The actual sources of variability themselves do however vary from manufacturing when applied to the context of modular construction.

Table 7: Sources of variability for manufacturing and examples for modular construction (Rausch et al. 2016)

<table>
<thead>
<tr>
<th>Variability Categories</th>
<th>Variability Sources in Manufacturing (Henzold 2006)</th>
<th>Variability Sources in Modular Construction</th>
</tr>
</thead>
</table>
| Materials              | • rigidity of the part (shape)  
                        | • material properties (chemical composition, homogeneity, hardness, etc.)  
                        | • embedded material stresses  
                        | • intrinsic material properties of concrete, steel, timber, polymers, composites, etc.  
                        | • expansion and contraction of steel due to heat of sun.  |
| Machines (i.e. tools & equipment) | • equipment capabilities (precision & accuracy)  
                        | • static and dynamic stability during operation  
                        | • thermal properties  
                        | • time dependent tool and fixture wear  
                        | • cutting, welding, milling, and other fabrication tools and machines  
                        | • pre- and post-heat treatment equipment  
                        | • quality of jigs, fixtures and fixturing tables  |
| Methods of manufacturing (i.e. processes) | • clamping & fit-up methods  
                        | • processing parameters (temperature, speed, cycle time, depth of cut, pressure, etc.)  
                        | • steel mill production & fabrication processes (cutting, welding, bending, cambering, bolting, etc.)  
                        | • concrete production processes (curing, pre-stressing, post-tensioning, vibration, etc.)  
                        | • component fit-up and alignment capabilities  |
| Manpower               | • skills of workers (education, previous experience, etc.)  
                        | • precision of clamping & fit-up  
                        | • skill and experience of fit-up, fabrication and production craft  
                        | • multifaceted union labor force vs. specialized contracted labor force  
                        | • manual vs. automated processing  |
| Measurement            | • precision and accuracy of measuring processes (i.e. inspection capabilities)  
                        | • quantity and quality of measurement techniques  
                        | • quality control/assurance procedures  |
| Environment            | • loadings (thermal, static, dynamic)  
                        | • moisture level  
                        | • temperature changes  
                        | • setup conditions and duty cycle (end use)  
                        | • material handling loads  
                        | • transportation and crane loads  
                        | • alignment and erection loads  
                        | • transient and permanent construction loads  |
3.2. Determine Impact of Variability Sources

The next part of the strategy involves determining the specific impacts of variability sources. This is required in order to determine which factors are most significant in the context of the holistic construction lifecycle so that the proper management strategy can account for these critical sources of variability. The impact classes of dimensional variability are derived from both construction and manufacturing literature and can be described in terms of primary and secondary importance levels as follows:

1. Primary impact level:
   a. **Structural safety**: this class relates to dimensional variations which change the load resistance or stability of the structure in such a way that can create safety issues. This class is ranked as the most important category since the safety of the building must be upheld before examining any cost or performance tradeoffs of other impact classes. Examples include column eccentricity, story-drift in a high rise building, location of rebar, cross sectional dimensions, second-order-effects, etc.

2. Secondary impact level:
   a. **Constructability**: this class relates to the ability of the construction crew to fabricate and aggregate the construction assembly. For this class, two conditions generally govern the constructability of most aggregation processes: maximum material condition (MMC), and least material condition (LMC) (these conditions are heavily covered in manufacturing literature). MMC refers to the case where components cannot be physically aggregated due to dimensional clashes (e.g., bolt is too large to fit into bolt-hole). LMC refers to the case where components can be physically aggregated together, but do not achieve certain performance requirements (e.g., bolt is very small in contrast with bolt-hole and has too much movement, losing its intended performance). Beside these two conditions, the critical factor for successful component aggregation lies with the quality of the “mating parts” (Asha et al. 2008), which represent the physical features of components (or sub-assemblies) being interfaced together. Technically, this means that regardless of where dimensional variability stems from, component aggregation is impacted the greatest when the geometric properties of the mating parts are not compliant or compatible. Finally, this impact class can be described in terms of internal aggregation (the ability to fabrication and aggregate an assembly or module), and external aggregation (how well that assembly or module can aggregate with adjacent assemblies or modules).
   b. **Aesthetics**: this class relates to the overall perceived quality of the final constructed project. Of importance for ensuring adequate aesthetics is managing dimensional variability at joints so that the joints appear to align properly. This category is heavily covered in detail in many architectural design guides. Examples include alignment of facade, column splice alignment, floor levelness and smoothness, or any other noticeable deviations or misalignments that would point to poor perceived quality.
   c. **Performance & Functionality**: this class is related to both the serviceability of a building, as well as the performance of subsystems. Examples of dimensional variability which affect serviceability include whether doors or windows can open properly, floor or roof deflections, or any other form of dimensional variability which would make occupants feel unsafe to use the building (note that the perception of safety is not as important as the structural safety of the building). Examples related to the performance of subsystems include the dimensional requirements for proper functionality of subsystems such as equipment (e.g., motors needing to
operate on a level surface), utility/service systems (e.g., pipes requiring certain slopes for material flow), etc.

Although this list covers the vast majority of impact categories for dimensional variability management, there might be cases where other impact classes might be required in special cases. For example, the author of this thesis spoke with a dimensional control firm, whose services were retained to resolve a contractual conflict related to dimensional variability. On this project there were several different contractors, where one party was responsible for the erection of a modular steel frame structure, while another party was responsible for the installation of exterior cladding. From the perspective of the contractor installing the steel frame, it would be important to be within certain tolerance limits to avoid legal or contractual complications with the exterior cladding contractor who had a contractual clause stating the building frame must have certain tolerances before commencing their work. This example demonstrates that in some cases the management of dimensional variability might have an impact on the legal or contractual obligations within a project. The purpose of providing a list of impact classes is not to encapsulate all possible scenarios for needing to manage dimensional variability, but rather to function as a bare minimum baseline.

3.3. Identify Critical Sources of Dimensional Variability

The next step in the proposed framework is isolating the sources of dimensional variability which are the most critical for the overall project. In many cases, there will be numerous impacts for a given dimensional variation. For instance, the eccentricity between stacked columns of two modules can create impacts in terms of structural safety, constructability, aesthetics, performance & functionality, as well as legal or contractual. However, each of these impacts requires a distinct deviation before dimensional variability becomes significant. As such, selection of the critical sources of dimensional variability can be very challenging to address due to the large amount of tradeoffs that exist. Fortunately, in most cases where an impact can relate to structural safety, and constructability and or aesthetics, typically the deviation limits required for aesthetics govern. However this may not always be the case, and as such additional methods are required to determine the most critical sources of variation to strategically manage. Some of the available methods include previous experience, a priori knowledge, case study database information, or prototyping. Previous experience and a-priori knowledge are the most common approaches for management of dimensional variability in the construction industry as a whole. This is why numerous codes, standards and design guides will specify recommended or ‘standardized’ tolerances since these are the experiential limits of deviations to minimize the risk of a certain impact. However this cannot always be used in modular construction applications. For modular construction, the use of case studies and prototyping can provide the designer with more realistic and case-specific guidance on critical sources of variability.

To provide a useful case study on the critical sources of dimensional variability in a modular steel framed building, a detailed analysis is provided in Chapter 5.

3.4. Develop the Variability Management Approach

For this step, a series of difference approaches can be used. The most proactive approach is to utilize design-based strategies in order to fully anticipate the effect of variability and develop the best geometric design which manages dimensional variability. This thesis includes two design-based strategies: (1) application of tolerance mapping framework to modular construction, and (2) kinematics chain based
dimensional variation analysis modelling. These two strategies are developed and demonstrated using case studies as shown in Chapter 6.

Production-based strategies manage dimensional variability during the fabrication and aggregation stages of the project rather than during the geometric design. The idea behind production-based strategies is to determine the best management approach as dimensional variability unfolds during the production (rather than upfront in the geometric design). In this way, it is not necessary to predict and account for every single source of variability in the geometric design, but rather the fabrication and aggregation processes can be adapted on-the-fly in order to ensure critical sources of variability do not create problems. In many ways, production-based management of variability is a significant aspect of certain trades’ and craft-workers’ technique behind their skillsets. For instance, welders control dimensional variability through a method of selectively welding joints in a way which reduces the effects of welding distortion. There are many other seemingly ad-hoc methods for managing dimensional variability during production which are conducted as the dimensional variability unfolds. In many modular construction scenarios it may be difficult to predict or quantify expected variability before it occurs, and for this reason, production-based management strategies are effective to use. The challenge with current production-based strategies or approaches is that they are not systematic, but are rather experimental. This creates problems for projects which have distinct configurations, and have not been replicated before. As such, this thesis has explored several systematic approaches employed in the manufacturing industry to demonstrate that systematic management of dimensional variability can also be achieved during production. The methods adapted from manufacturing for use in modular construction include selective assembly and optimal assembly and involve minimizing the effect of variability of as-built components. Development and demonstration of these methods using case studies are shown in Chapter 7.

The purpose of handling-based strategies is to control the geometric response of the module against structural actions associated with handling and transportation. Transportation loading places much different structural actions on a module than those encountered during in-situ service loads. In some cases, the transportation loading of a structural design can be the limiting factor (Gardiner 2015). As such, handling-based strategies to manage dimensional variability are typically in the form of temporary bracing or lifting frame systems to ensure that plastic and elastic distortions of the module are controlled within certain limits. Often, these strategies are based on back-of-envelop calculations and are not very systematic.

Finally, onsite strategies for managing dimensional variability involve correcting, adapting or adjusting the geometry of the module in order to properly manage dimensional variability. Common applications currently employed involve the use of adjustable connections in 3 degrees of freedom so that any previously incurred accumulation of dimensional variability can be accommodated onsite.

3.5. Implementing and Monitoring the Dimensional Variability Strategy

Once the strategy has been developed it can be implemented. After being implemented, the strategy should be monitored in order to gage its effectiveness. To do this, it will be necessary to obtain as-built information regarding the resulting accumulation of dimensional variability in the construction assembly. For this purpose, this thesis has developed a deviation analysis method which can analyze the as-built state of a construction assembly in order to assess dimensional quality (ensuring dimensional compliance and to monitor accumulation of variability). The proposed deviation analysis method is developed in
detail in Chapter 4 and uses data in the form of laser scans (3D point clouds) and a building information model (BIM). These two data sources can be compared in different ways in order to assess dimensional quality. Monitoring the dimensional variability management strategy is intended to be done until the project is complete. That way if additional sources of variability are identified which were not originally considered, or if the developed strategy is not functioning properly, then it can be adapted on-the-fly during the project.

3.6. Scope, Objectives and Constraints of the Proposed Methodology

The scope of the proposed methodology relates to certain project parameters which, by altering, will change the overall management of dimensional variability. The goal of the proposed methodology is to optimally specify, alter or control these parameters so that dimensional variability can be properly managed. These project parameters are shown in Figure 10. Project parameters are grouped in terms of geometric design parameters (explicitly related to specification of dimensions and geometry), and parameters which govern the geometric response to structural actions. Each project parameter can be specified with a corresponding tolerance in order to define an allowable range of variation from mean parameter values. The breakdown of project parameters in Figure 10 is not exhaustive and may encompass other parameters which must be determined on a project-specific basis.

Geometric design parameters are based on dimensions and interface geometry. The classification of dimensions comes a manufacturing standard related to tolerance specification (ISO 8015:1985) which classifies dimensions in terms of being linear, angular or a combination thereof in the form of geometric properties (Figure 11). Parameters which govern the geometric response of a module to structural actions are based on the strength, stiffness, stability, and support conditions of a module, as these factors can influence the way that the module geometry responds to the structural actions caused by certain handling, transportation, and erection loads. Project stages are summarized into production (manufacture,
fabrication and aggregation), transportation & handling and finally erection. Variability can be located in components, modules or the site. Distinct categories for components are made based on whether they are ‘outsourced’ or ‘insourced’ since there is generally less control on parameters related to dimensional variability of outsourced components or assemblies.

Design-based strategies involve specifying project parameters and corresponding tolerances upfront before the production, transportation, handling and erection stages occur. This requires information in order to optimally specify these parameters, which can be challenging. Production-based strategies involve altering parameters related to explicit dimensions and geometric properties and those which govern the geometric response to structural actions involved during production. Handling-based strategies involve altering parameters related to the geometric response to structural actions during transportation and handling (e.g., crane load, transportation load, etc.). Onsite-based strategies involve altering parameters related to explicit dimensions and geometric properties as well as parameters which govern the geometric response to structural actions involved during erection on site.

Figure 11: Scope of project parameters which relate to design-based, production-based, handling-based or onsite-based dimensional variability management strategies
The goals of the proposed methodology are to either directly specify project parameters and tolerances or to indirectly adjust parameters in order to manage dimensional variability. For each of the key project stages (manufacture, fabrication, aggregation, handling, transportation, erection, in-service/operation stages) there are constraints and key goals for managing dimensional variability, which are summarized in Figure 13.

It is important to note that the proposed methodology (Figure 9), scope (Figure 10) and project stage specific goals and constraints (Figure 13) are not validated in this thesis due to a lack of being able to implement the proposed methodology on modular construction projects. These developments are rather a synthesis of analyses made on various modular construction projects, findings explored in literature and iterative developments made in conjunction with support from the industry sponsor of this research.
4. Development of Deviation Analysis Method for Quantifying Dimensional Variability

This chapter comes primarily from the following publication:


The primary author of this publication prepared the literature review, proposed methodology, assisted with data collection, analyzed results, and prepared the conclusions. The second and fifth author assisted with data collection and analysis, and the third and fourth readers provided guidance and reviewed the final paper.

Comparisons between as-built and as-designed states are very useful for evaluating the dimensional variability of fabrication and aggregation in construction. Deviation analysis employs the use of as-built data, which can be readily collected through the use of 3D images and point clouds (e.g., laser scans). While the deviation analysis method using as-built and as-designed data is not new, its application in the context of modular construction for quantifying dimensional variability is new. The two proposed types of analyses are structured into (1) direct comparison and (2) indirect comparison analyses. These analyses employ different data sets and comparisons, and have distinct applications and accuracies (Figure 14).

<table>
<thead>
<tr>
<th>Direct Comparison Analysis</th>
<th>Indirect Comparison Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Datasets</strong></td>
<td></td>
</tr>
<tr>
<td>Data sets are first registered together, and then measurements made on registered data set.</td>
<td>Measurements performed on each data set then compared.</td>
</tr>
<tr>
<td><strong>Comparisons</strong></td>
<td></td>
</tr>
<tr>
<td>Scan-vs-BIM(^1)</td>
<td>Scan-vs-Scan(^3)</td>
</tr>
<tr>
<td>A) Point-to-feature(^{2,3})</td>
<td>B) Feature-to-feature(^{4,5})</td>
</tr>
<tr>
<td>C) Point-to-point(^{2,3})</td>
<td>D) Feature-to-feature(^{4,5})</td>
</tr>
<tr>
<td><strong>Applications</strong></td>
<td></td>
</tr>
<tr>
<td>A) Detecting protruding elements in assembly that could create conflicts for assembly interfacing or module fitup</td>
<td></td>
</tr>
<tr>
<td>B) Fabrication compliance with BIM for entire assembly (i.e. assessing floor levels, welding distortion detection)</td>
<td></td>
</tr>
<tr>
<td>C) Detecting overall change of object between fabrication, aggregation, handling &amp; transportation processes</td>
<td></td>
</tr>
<tr>
<td>D) Detecting multiple feature changes of object between fabrication, aggregation, handling &amp; transportation processes</td>
<td></td>
</tr>
<tr>
<td>E) Fabrication compliance with BIM for discrete dimension on assembly (i.e. length or angular changes of an element)</td>
<td></td>
</tr>
<tr>
<td>F) Detecting single feature changes of object between fabrication, aggregation, handling &amp; transportation processes</td>
<td></td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td></td>
</tr>
<tr>
<td>(^1)Error associated with scan (3D point cloud) due to accuracy of scan acquisition equipment</td>
<td></td>
</tr>
<tr>
<td>(^2)Error associated with accuracy of registration (i.e. method of registering point cloud(s) and or with BIM)</td>
<td></td>
</tr>
<tr>
<td>(^3)Error associated with fitting substitute feature to point cloud data (no error with BIM feature)</td>
<td></td>
</tr>
<tr>
<td>(^4)Error associated with fitting substitute feature to each point cloud (i.e. twice the error as in (^3))</td>
<td></td>
</tr>
</tbody>
</table>

Figure 14: Direct and indirect comparison analysis in terms of data sets, comparisons, applications and accuracy

4.1. Requirements & Assumptions

The acquisition of 3D as-built data is a prerequisite for conducting deviation analyses. For conducting direct comparison deviation analyses, software that can register as-built point clouds and BIM is required. Among the most commonly used commercial software for this step is PolyWorks\(^\circ\), which is commercial
metrology software. For point cloud registration, the iterative closest points (ICP) technique is employed. This step is preceded by a rough alignment step, employing principal component analysis (PCA) for instance, in order to improve the match quality and robustness (Nahangi and Haas 2014). In analyses where BIM is used, it is assumed that the as-built state and the as-designed state (BIM) are geometrically compatible within a certain tolerance (Anil et al. 2013) since this influences the accuracy of the quantification process. Data sets for as-built data need to be in the form of point clouds, while the BIM should be kept in a surface or solid model format to reduce accrued errors (the exception being for commercial software which only registers point clouds, for which a solid model can be substituted for a high density point cloud).

4.2. Types of Comparisons and Applications

Data sets can either be compared directly or indirectly. Direct comparison (i.e., scan-vs-BIM and scan-vs-scan) requires registration, where the data sets are superimposed and iteratively matched. The BIM can also be converted to a point cloud for computational simplicity in scan-vs-BIM. For a complex assembly or module configuration, registration is typically constrained at one or more specified datum points based on the design. Indirect comparison (i.e., scan-to-BIM and scan-to-scan) does not require registration; rather data sets are kept isolated, and measurements for dimensional assessment are performed in each data set separately. The choice between direct or indirect comparison is dependent on numerous factors including geometric complexity, discrete vs. continuous assessment, quality of point cloud (degree of noise, occlusions, density, full coverage, etc.), and geometric compatibility between as-built state and BIM. Ultimately, the choice between direct or indirect comparison comes down to accuracy of data set registration and the amount and type of measurements being made.

When a single object is being analyzed, it is preferable to conduct a scan-vs-BIM analysis since the geometry is generally simple and there is compatibility between the two data sets, meaning that registration will have fewer potential errors and discrepancies (refer to the first case study example shown below for the dimensional assessment of a structural beam). However in cases where the geometry is more complex, or in cases where a partially fabricated element is being compared to a final BIM assembly, it may be more favorable to conduct a scan-to-BIM analysis (since there may be large discrepancies with registration). In cases where dimensional quality of a production process of a geometrically complex assembly is being assessed, it is preferable to conduct a scan-vs-scan analysis. However if there are discrepancies between these data sets (e.g., if there were occlusions, noise or partial coverage in the point clouds), then it may be more preferable to conduct a scan-to-scan analysis, since registration errors are omitted. While these examples demonstrate how each of the four analyses can be used in specific cases based on the type of data being compared, there are also specific applications based on the type of measurements being made. The three general types of measurements in deviation analyses are (1) point-to-point, (2) point-to-feature and (3) feature-to-feature (Figure 15).
Measurements which involve points from the as-built data (point cloud) can be very useful for measuring how far an object protrudes out of an assembly, as this can indicate if any alignment problems may exist. Point-to-point measurements are typically used for comparing as-built data sets which are registered together. When using BIM in a deviation analysis, it is best to use features, since this represents an ideal characteristic rather than converting the BIM into a point cloud. For instance, assessing floor levelness can be done by comparing points in the as-built data set with the floor feature in the BIM. Point-to-feature measurements are generally for comparing as-built and BIM data, however it can also be utilized for assessing a single as-built data set. For instance, throughout this thesis one of the most common uses of point-to-feature comparison is in the form of plane deviation analysis. In this type of comparison, a group of points are selected in an as-built data set (point cloud) and a best-fit plane is created (numerous software can provide this feature, however PolyWorks® was used in this thesis for creating best-fit planes). After creating the best-fit plane, points in the as-built data set are ‘mapped’ to the plane in terms of their Euclidean distances. This ‘mapping’ step associates a certain colour to each point based on how far it lies from the plane. Using plane deviation analyses are very useful for assessing the form variation of a component (how an as-built feature varies from a nominal form). The key steps involved in creating a plane deviation analysis are shown in Figure 16. Feature-to-feature measurements require fitting a feature to points in the as-built data, which can either incur measurement error or reduce point cloud error. Noise in a point cloud can be reduced by fitting a geometrically ideal substitute feature (refer to Appendix A for explanation on different feature types) to these points, however the fitting of a substitute feature also removes non-geometrically ideal distortions that may be present in the as-built state. For instance, if tracking longitudinal distortion in a beam, fitting geometrically ideal features to as-built data points will eliminate the form variation (e.g., longitudinal waviness), thereby resolving the dimensional analysis to purely angular, linear and size variations. Therefore, fitting geometrically ideal features should not be used in cases where the form variation is being tracked.
4.3. Accuracy

Accuracy is an important aspect to consider when conducting deviation analyses. There are three general types of errors that can occur that may affect the accuracy of the results: (1) errors in as-built data, (2) registration errors (for scan-vs-BIM and scan-vs-scan), and (3) measurement errors. Influences on as-built data accuracy include sensor measurement error, noise, and occlusions. Akinci et al. (2006) provide insight into other factors, which include area-based measurements, surface orientation and surface material type. Area-based measurements can reduce error since noise in the data can be filtered. Surface orientations affect the accuracy of point clouds since resolution is decreased for obliquely viewed angles (Akinci et al. 2006). Finally, surface material can affect sensor measurement (e.g., dark materials that absorb a lot of light, or highly reflective surfaces which can create noise). Registration error stems from geometric discrepancies between two data sets. While this can be reduced through advancements in registration methods, it is limited by the degree of geometric compatibility between the two data sets (i.e., how close the as-built state matches the BIM at a given production stage). Measurement errors also influence overall accuracy of deviation analyses. Measurement error in this case refers to errors between the two elements under comparison. For instance, for feature-to-feature comparisons, there is some error in the feature fitting process for the as-built data; this is unavoidable. The calculation of dimensional discrepancy between two data sets is done using minimum Euclidean distance, which can introduce errors (especially where registration errors are present).

4.4. Case Study Example 1: Deviation Analysis for Quantifying Compliance with Tolerances

This case study demonstrates how geometric distortion of a steel beam can be quantified using deviation analyses. Two S75x8 beams with different as-built geometries were compared to a BIM element (Figure 17). The first analysis demonstrates how compliance to mill tolerances can be quantified, and the second
analysis demonstrates how geometric distortions can be quantified during fabrication. As seen in Figure 17b, the top flange of the beam is distorted in comparison to the cross section shown in Figure 17a. Laser scans of each geometric state were acquired using a close range laser scanner (FARO® Edge Arm), which has an accuracy of 0.024 mm for the working length that was used (FaroARM 2014). The deviation analysis type used in this case is scan-vs-BIM, where each scan is compared to the BIM element using industrial 3D metrology software (PolyWorks®). The first deviation analysis (beam 1), shown in Figure 18a, reveals that the overall cross section of the scanned beam is compliant with the BIM element (all deviations are less than 1 mm). Furthermore, qualitative analysis of the colour pattern (created by the range of deviations) indicate that the section shape along the length is consistent, showing no indication of warping along the length of the beam. The second analysis (beam 2), shown in Figure 18b reveals that there is a substantial amount of geometric discrepancy between the as-built status and the BIM element (deviations exceed 4 mm in numerous locations as noted by the color scale). Qualitative analysis reveals that the most severe discrepancy stems from the distorted upper flange (as is expected).
A feature-to-feature comparison of the top flange reveals the dimensional quality of the beam. The feature being compared in this case is a line of best fit on the top of the flange. Feature to feature comparison reveals the angular discrepancy is equal to 6° (Table 8). The maximum allowable angular discrepancy in accordance with ASTM A6/A6M is 2°, meaning that corrective fabrication rework would be required for compliance (i.e., 4° correction of the top flange).

Table 8: Quantified Dimensional Quality of Top Flange using Feature-to-Feature Comparison for Beam 2

<table>
<thead>
<tr>
<th>Feature Properties</th>
<th>As-built Feature</th>
<th>Feature Properties</th>
<th>Nominal Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>X,Y,Z (mm)</td>
<td>2233.7, 1573.5, -1.3</td>
<td>X,Y,Z (mm)</td>
<td>2237.8, 1571.7, 0.00</td>
</tr>
<tr>
<td>Angle i,j,k (rad)</td>
<td>-0.994, -0.107, 0.012</td>
<td>Angle i,j,k (rad)</td>
<td>-1.000, 0.001, 0.000</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>55.4</td>
<td>Length (mm)</td>
<td>57.4</td>
</tr>
<tr>
<td>Angle with X (°)</td>
<td>173.8</td>
<td>Angle with X (°)</td>
<td>179.9</td>
</tr>
<tr>
<td>Angle with Y (°)</td>
<td>96.1</td>
<td>Angle with Y (°)</td>
<td>89.9</td>
</tr>
<tr>
<td>Angle with Z (°)</td>
<td>89.3</td>
<td>Angle with Z (°)</td>
<td>90.0</td>
</tr>
<tr>
<td>Angle between axes in X direction (°)</td>
<td>6.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.5. Case Study Example 2: Deviation Analysis for Quantifying Dimensional Quality

This case study demonstrates how dimensional quality of a building module can be quantified during fabrication and aggregation. This particular modular construction project experienced difficulties during module fit-up on site for two reasons: (1) the outer width of modules exceeded the original design allowance, and (2) gross misalignments of components occurred between modules. As-built data of the steel structure was collected using a FARO® LS 840HE laser scanner, which has an accuracy of 2 mm at a distance of 25 m (FARO 2007). The deviation analyses were conducted in MATLAB® using a high-density point cloud created from the BIM and laser scans of the as-built states.

In order to evaluate the dimensional quality of fabrication (for detection of dimensional quality defects in fabrication and aggregation of the steel structure), a deviation analysis was used to compare the as-built outer width of the structure to that of the design. This analysis utilized a feature-to-feature comparison between a laser scan of the as-built state of the raw steel structure and the BIM (Figure 19). Although a scan-vs-BIM analysis was not strictly required since only the width of the module was being quantified, it was useful to conduct this type of registration in order to assess overall dimensional compliance of the structure. The result of this deviation analysis revealed that the outer width of the steel structure was actually well within the specified design width (by 10 mm). As such, the fabrication of the steel structure had adequate dimensional quality, and actually offset the observed growing module width.

The second deviation analysis conducted in this case study compares two separate as-built states to determine the dimensional quality of the module while subjected to two different temporary support conditions during fabrication and aggregation. Since components between modules were frequently misaligned on site, it was determined that module geometry during initial component fit-up may be a probable cause. A deviation analysis was conducted using scan-vs-scan, point-to-point comparison to reveal that while components were being fit-up (modules were placed on shop floor), the modules deflected up to 30 mm (Figure 20). Although this deflection was elastic, this type of geometric variation could lead to alignment errors on site since the initial fit-up of components between modules was based on a deflected geometry, while the modules once on site did not have the same deflected geometry.
4.6. Case Study Example 3: Deviation Analysis for Detecting Module Alignment Conflicts

This case study demonstrates how dimensional quality can be quantified for alignment of an industrial pipe spool module. In the case of industrial pipe modules, being able to accurately quantify dimensional quality is very important for proper module alignment on site, since discrepancies can result in extensive rework and cost overruns (Nahangi and Haas 2014). For this case study, as-built data was collected (using a FARO® LS 840 HE laser scanner) and a BIM was obtained for a reconfigurable pipe spool module (Figure 21). To simulate a dimensional quality defect with regards to alignment of a module interface, an isolated joint in the pipe spool was rotated by $60^\circ$ (Figure 21a). Then using a scan-vs-BIM deviation analysis, a tolerance threshold of 5 mm was established, such that deviations in exceedance would be plotted in red, and data points within tolerance would be plotted as green. The deviation analysis for this case study was performed in PolyWorks®.
As seen in Figure 21c, the largest discrepancy (in terms of density of red data points) is near the rotated joint (as is expected). Furthermore, the angle of this discrepancy (in this case 60°) can be quantified by comparing a line feature to the as-built joint to the same BIM feature (Table 9). The ability to accurately quantify the angular discrepancy is extremely useful for being able to apply corrective adjustments before the module leaves the fabrication facility in order to avoid expensive site fit-up.

Table 9: Quantified Angular Discrepancy of Rotated Joint using Feature-to-Feature Comparison

<table>
<thead>
<tr>
<th>Feature Properties</th>
<th>As-built Feature</th>
<th>Feature Properties</th>
<th>Nominal Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>X,Y,Z (mm)</td>
<td>-1088.4, 20.6, 2687.7</td>
<td>X,Y,Z (mm)</td>
<td>-1052.4, 59.9, 2689.0</td>
</tr>
<tr>
<td>Angle i,j,k (rad)</td>
<td>-0.857, 0.505, 0.107</td>
<td>Angle i,j,k (rad)</td>
<td>0.008, 1.000, 0.024</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>96.0</td>
<td>Length (mm)</td>
<td>95.8</td>
</tr>
<tr>
<td>Angle with X (°)</td>
<td>148.9</td>
<td>Angle with X (°)</td>
<td>89.5</td>
</tr>
<tr>
<td>Angle with Y (°)</td>
<td>59.7</td>
<td>Angle with Y (°)</td>
<td>1.5</td>
</tr>
<tr>
<td>Angle with Z (°)</td>
<td>83.8</td>
<td>Angle with Z (°)</td>
<td>88.6</td>
</tr>
<tr>
<td>Angle between axes (°)</td>
<td></td>
<td></td>
<td>60.0</td>
</tr>
</tbody>
</table>

4.7. Discussion

The framework presented for deviation analysis method shows how as-built and as-designed data can be systematically compared in order to quantify dimensional quality. Three case studies are presented to demonstrate how deviation analysis can be used in modular construction applications. The results of the case studies demonstrate how the following types of dimensional compliance can be quantified: mill tolerances, steel beam flange distortion, outer module structure width, geometric distortions from varying temporary support conditions, and overall module alignment. While these case studies represent only a
small subset of the full use of deviation analysis in modular construction, it is clear that deviation analysis is a very powerful tool for quantifying dimensional quality.

Important limitations of the proposed use of deviation analysis include (1) the accuracy of results and (2) discretionary timeliness and frequency of analyses. First of all, while methods to obtain as-built data can be very accurate (e.g., sub-millimeter level of accuracy), registration of and comparisons between data sets can introduce error. For very critical dimensional assessments, this error can be reduced through sampling by conducting numerous analyses, however the practitioner must be aware that error will still exist. Secondly, deviation analyses need to be conducted in a very systematic manner in order to appropriately track dimensional quality and alignment at critical production stages. Prolonging an analysis for too long runs the risk of increasing potential rework costs. However, conducting too frequent deviation analyses can be very timely and may be unnecessary. Since the purpose of using deviation analyses is to reliably quantify dimensional quality, they should only be conducted at critical project stages.
5. Case Study to Quantify Critical Dimensional Variability in Modular Project

To demonstrate how dimensional variability can accumulate throughout a modular construction assembly during the project lifecycle (i.e., from part manufacture to final erection on site) a sample case study is shown. A construction company was contracted to construct two 805 m² data center projects comprised of 16 prefabricated building modules each (Figure 22). During the fabrication, aggregation, transportation and site erection stages, 3D as-built geometric data was collected and analyzed in order to quantify how dimensional variability accumulates.

For this case study, 3D as-built geometric data was primarily collected using a terrestrial laser scanner; however a robotic total station was also employed for secondary data collection during the transportation

Figure 22: Case study to demonstrate how dimensional variability accumulates throughout an offsite construction assembly during the life cycle.

For this case study, 3D as-built geometric data was primarily collected using a terrestrial laser scanner; however a robotic total station was also employed for secondary data collection during the transportation
and site-erection stages. The decision to use a laser scanner for geometric data collection comes from its ability to yield dense (e.g., millions of data points for a single scan of medium resolution) and accurate (e.g., 2 mm error at a distance of 25 m for the laser scanner employed in this research) data (FARO 2007). Secondary data in the form of robotic total station readings was also employed in this case study since flexible vinyl weatherproofing on the modules during transportation made the capture of surface data of the modules via laser scanning very challenging and not intuitively useful. A brief overview of the differences between the two data collection devices is summarized in Figure 23. For a technical comparison in terms of the dimensional compatibility between these two devices used, please refer to the calibration study found in Appendix B.

3D Laser Scanner
- **Full 3D data** in form of a “point cloud”
- Point cloud compared to design model to quantify geometric variations
- Large post-processing

Robotic Total Station
- Collects **coordinates** of single points of interest
- Requires resurveying original targets to quantify variations
- Medium post-processing

**Figure 23:** Comparative overview of 3D laser scanning versus robotic total station for as-built data collection (FARO 2007, Sokkia 2015)

The project lifecycle is broken down into seven stages to track distinct changes to the module geometry and accumulation of dimensional variability: (1) fabrication of precast concrete panels, (2) fabrication of floor frames, (3) fabrication of roof frames, (4) aggregation of the structural system (floor frame, roof frame, concrete panels and columns), (5) the aggregated module under temporary support conditions, (6) the aggregated module during transportation and handling, and (7) the aggregated module during erection loading. The core assumptions made in this case study with respect to tracking dimensional variability is that the aggregation of the structural system was the focus during fabrication, and that critical points on the module structure were the focus during transportation, handling and erection on site. When analyzing building systems, dimensional variability has an impact on more systems than simply just the structure. However the assumption made in this case study is that the geometry and dimensional variability of non-structural building systems (e.g., building service systems, enclosure, finishes) are either directly or indirectly influenced by the geometry and dimensional variability of the structural system. This is why the geometry of the structural system was directly analyzed.

From all of the main fabrication processes outlined by the construction company involved in this case study, only a select number were investigated directly in terms of their dimensional variability impact (Table 10). A hierarchy was created in order to understand which of the fabrication processes played the largest role in contributing to dimensional variability. In addition to fabrication processes, handling,
transportation and on-site erection were also evaluated in terms of their contribution to dimensional variability and ranked accordingly.

### Table 10: Main fabrication processes involved in case study (as outlined by Construction Company)

<table>
<thead>
<tr>
<th>Fabrication Process</th>
<th>Investigated?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formwork table fit-up (i.e., setting up framing table into proper position)</td>
<td>Y</td>
</tr>
<tr>
<td>Rebar cutting</td>
<td>N</td>
</tr>
<tr>
<td>Concrete pour</td>
<td>Y</td>
</tr>
<tr>
<td>Welding columns to base</td>
<td>Y</td>
</tr>
<tr>
<td>Welding (miscellaneous – everything other than columns, HSS, clips, lugs and pans)</td>
<td>Y</td>
</tr>
<tr>
<td>Welding HSS</td>
<td>Y</td>
</tr>
<tr>
<td>Welding precast concrete panel frames</td>
<td>Y</td>
</tr>
<tr>
<td>Material handling (steel frame delivery)</td>
<td></td>
</tr>
<tr>
<td>Fit-up of roof frame to base frame</td>
<td></td>
</tr>
<tr>
<td>Welding roof frame to base frame</td>
<td></td>
</tr>
<tr>
<td>Structural steel cutting (all columns, slices, angles, HSS)</td>
<td>Y</td>
</tr>
<tr>
<td>Install base plates with nelson studs</td>
<td>N</td>
</tr>
<tr>
<td>Install tie-in plates – cutting, fit-up, punching, and welding</td>
<td>Y</td>
</tr>
<tr>
<td>Install lifting lugs – cutting, moving, and fit-up</td>
<td>N</td>
</tr>
<tr>
<td>Install metal decking – cutting, moving, fit-up and shooting (steel studs used for install)</td>
<td>N</td>
</tr>
<tr>
<td>Rough carpentry</td>
<td>N</td>
</tr>
<tr>
<td>Parapet framing</td>
<td>N</td>
</tr>
<tr>
<td>Install FRP</td>
<td>N</td>
</tr>
<tr>
<td>Sealants and fire stop</td>
<td>N</td>
</tr>
<tr>
<td>Exterior metals panels</td>
<td>N</td>
</tr>
<tr>
<td>Wall framing</td>
<td>N</td>
</tr>
<tr>
<td>Wire mesh install</td>
<td>N</td>
</tr>
<tr>
<td>Install wall board</td>
<td>N</td>
</tr>
<tr>
<td>Tape, mud and sand</td>
<td>N</td>
</tr>
<tr>
<td>Column frame/board/mud/tape</td>
<td>N</td>
</tr>
<tr>
<td>Install ACT</td>
<td>N</td>
</tr>
<tr>
<td>Floor preparation for VCT</td>
<td>N</td>
</tr>
<tr>
<td>Install VCT &amp; rubber base</td>
<td>N</td>
</tr>
<tr>
<td>Painting of structural steel</td>
<td>N</td>
</tr>
<tr>
<td>Paint ceiling (decking)</td>
<td>N</td>
</tr>
<tr>
<td>Paint interior</td>
<td>N</td>
</tr>
<tr>
<td>Paint doors/frames</td>
<td>N</td>
</tr>
<tr>
<td>Floor protection</td>
<td>N</td>
</tr>
<tr>
<td>Install washroom accessories</td>
<td>N</td>
</tr>
<tr>
<td>Install pre-cast</td>
<td>Y</td>
</tr>
<tr>
<td>Fly roof frame over base frame</td>
<td>Y</td>
</tr>
<tr>
<td>Shipping preparation</td>
<td>N</td>
</tr>
<tr>
<td>Install HVAC</td>
<td>N</td>
</tr>
<tr>
<td>Install mechanical</td>
<td>N</td>
</tr>
<tr>
<td>Install electrical</td>
<td>N</td>
</tr>
</tbody>
</table>

The fabrication of the main structural system for each module can be expressed in a series of key processes based on those which impact the dimensional variability examined in this case study. The only other fabrication processes not included in Figure 24 are the manufacture and aggregation of precast concrete panels which make up the floor system.
Figure 24: Schematic representation of the key fabrication processes involved with construction of module structural system (with additional processes outlined for fit-up and fastening of tie-in plates).

5.1. Stage 1: Fabrication of Precast Concrete Panels

To provide a brief background on this production stage, the basic workflow of processes included: (a) light-gage steel frame fabrication using a framing table for layout and dimensional control, (b) formwork placement on the shop floor, (c) fit-up of a rebar mesh, (d) concrete pouring and leveling, (e) concrete finishing and screeding (Figure 25). After these basic processes, the concrete panels were stacked until they were aggregated into the floor frames.

For each floor frame, the precast concrete panels have the same length and height (2559 mm and 102 mm respectively), however the width of each panel varies, as outlined in Figure 26. Furthermore, the design allowed for a gap tolerance between the concrete panels and floor frame of + 3 mm (i.e., the width or length of any panel can only exceed the specified dimension by 3 mm).

**Figure 26: Width of each precast concrete panel, with a 3 mm tolerance for spacing between panels**

For the production of these concrete panels, guidance on the types of dimensional variability to consider and analyze is taken from the Precast/Prestressed Concrete Institute Design Handbook. This handbook outlines critical tolerances on precast concrete panels as being (1) overall dimensions (i.e., length, width, and height), (2) sweep or horizontal misalignment, (3) position of strands/rebar, (4) camber and differential camber, (5) weld plate tolerances, (6) corbel or haunch positions and wall panels, (7) warping and bowing, and (8) smoothness (PCI Industry Handbook Committee 2004). Of these eight critical dimensional variability types, the following dimensional analyses were conducted:

1. The overall dimensions of the concrete panels (i.e., length, width and height)
2. Warping, bowing and smoothness of the concrete surface

5.1.1. **Analysis 1: Overall Panel Dimensions**

The analysis considered point cloud data which was collected for the stack of concrete panels (Figure 25). After obtaining the point cloud for overall stack of panels, each panel was manually extracted from the overall point cloud. As a result, a point cloud can be obtained for each panel, however due to the limited line-of-sight of the laser scanner, the concrete surfaces of each panel were not consistent, with large occlusions in in the center (Figure 27). The approach taken to quantify the overall dimensions of concrete panels was to manually extract linear dimensions from the point cloud data for the outer edges, along with an average height. Although the use of an automated dimensional extraction process could have been utilized for this step, due to some noise in the point cloud data, it was deemed more efficient to manually extract dimensions since an automated process would require further point cloud cleaning (Figure 27). As seen in Table 11, the majority of panel dimensions are less than the corresponding nominal dimensions as specified in the design.
Table 11: As-built and nominal dimensions for length, width and height of concrete panels

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>X1 (mm)</td>
<td>1367</td>
<td>1362</td>
<td>1497</td>
<td>1494</td>
<td>1482</td>
<td>1367</td>
<td>1356</td>
<td>1367</td>
<td>1361</td>
<td>1500</td>
<td>1518</td>
</tr>
<tr>
<td>X2 (mm)</td>
<td>1367</td>
<td>1362</td>
<td>1497</td>
<td>1486</td>
<td>1486</td>
<td>1367</td>
<td>1359</td>
<td>1367</td>
<td>1365</td>
<td>1500</td>
<td>1476</td>
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<tr>
<td>Y1 (mm)</td>
<td>2559</td>
<td>2554</td>
<td>2559</td>
<td>2552</td>
<td>2565</td>
<td>2559</td>
<td>2554</td>
<td>2559</td>
<td>2553</td>
<td>2559</td>
<td>2554</td>
</tr>
<tr>
<td>Y2 (mm)</td>
<td>2559</td>
<td>2555</td>
<td>2559</td>
<td>2554</td>
<td>2565</td>
<td>2559</td>
<td>2552</td>
<td>2559</td>
<td>2553</td>
<td>2559</td>
<td>2561</td>
</tr>
<tr>
<td>Z_{AVG} (mm)</td>
<td>102</td>
<td>102</td>
<td>100</td>
<td>102</td>
<td>101</td>
<td>102</td>
<td>99</td>
<td>102</td>
<td>98</td>
<td>102</td>
<td>95</td>
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</table>

<table>
<thead>
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<td>X1 (mm)</td>
<td>1497</td>
<td>1484</td>
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<tr>
<td>X2 (mm)</td>
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<td>1474</td>
<td>1367</td>
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<td>Y1 (mm)</td>
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<td>2559</td>
<td>2560</td>
<td>2559</td>
<td>2556</td>
<td>2559</td>
</tr>
<tr>
<td>Y2 (mm)</td>
<td>2559</td>
<td>2542</td>
<td>2559</td>
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<tr>
<td>Z_{AVG} (mm)</td>
<td>102</td>
<td>93</td>
<td>102</td>
<td>96</td>
<td>102</td>
<td>96</td>
<td>102</td>
<td>97</td>
<td>102</td>
</tr>
</tbody>
</table>

By comparing the as-built dimensions with the nominal dimensions, histograms can be created in order to visualize the distribution of dimensional deviations (Figure 28).
Figure 28: Histogram plots for dimensional variability of concrete panel as-built dimensions.

From observing the overall distributions for width (X1 and X2), length (Y1 and Y2), and height (Z), it was found that the deviations of as-built dimensions are normally distributed. In most manufacturing and construction applications, the assumption of normally distributed process capabilities is very common (Drake 1999; Milberg and Tommelein 2009). In determining the probability distribution function to match each data set, chi-square goodness of fit tests were conducted through use of the formula

$$\chi^2 = \sum_k \left( \frac{(O_i - E_i)^2}{E_i} \right)$$ \hspace{1cm} [Eqn. 2]

where $O_i$ is the observed frequency for bin $I$ and $E_i$ is the expected frequency for bin $i$. which is found by

$$E_i = N \left( F(Y_u) - F(Y_l) \right)$$ \hspace{1cm} [Eqn. 3]

where $F$ is the cumulative distribution function, $Y_u$ is the upper limit for class $i$, $Y_l$ is the lower limit for class $i$, and $N$ is the sample size for a given dimension. While the significance level (i.e., $\alpha$) for the $x$, $y$ and $z$ dimensions yielded very large values (> 25% significance of the null hypothesis), the significance level for the sample of all dimensions yielded a very small value (0.18%) indicating that the panel dimensions are normally distributed (note: a low significance level means the null hypothesis which states that a given sample is drawn from the distribution in question must be rejected, and therefore the sample is in fact consistent with the distribution in question). The second best fit (applying distributions recommended in (Drake 1999) and similar sources) was a logistic function which had a significance level of 4.1% (worse than for the normal distribution fitting). For all panel dimension variations, the fitting parameters of the normal distribution fit are:

- Mean ($\mu$) = -5.0 mm
- Standard Deviation ($\sigma$) = 7.1 mm
5.1.2. Analysis 2: Warping, Bowing and Smoothness

In addition to assessing the overall panel dimensions, the top surface dimensional variability can also be quantified in order to detect any instances of excessive warping, bowing or lack of surface smoothness. A plane deviation analysis (which assesses deviation of the surface with respect to a best fit plane on the concrete surface) is very versatile since it can be used to assess all three of the dimensional variabilities of interest. The plane deviation analysis was conducted on a concrete panel of interest using PolyWorks® (Figure 29).

![Figure 29: Plane deviation analysis output using PolyWorks® of the top surface of a typical concrete panel (deviations shown are in mm).](image)

Observation of this plane deviation analysis can be somewhat non-conclusive for inferring relationships about the dimensional variabilities of interest upon first glance since there is a large gap in the center of the point cloud. Again, this is due to large occlusions in the panel surfaces since they were stacked on top of each other in the plant. However, upon observing the bottom edge, there is a gradual incline of approximately 5 mm (note that along the top edge that there is only a variation of only 2 mm). Since the bottom right corner has a deviation which is 5 mm below all other corners, it can be deduced that this particular panel has a warp deviation of 5 mm (note: PCI defines warping as a deviation from a plane in which the corners of the panel do not fall within the same plane). Furthermore, along the left edge there is a bowing pattern of the panel (note: PCI defines bowing in the case where two opposite edges of a panel may fall along a plane but the portion between the two edge falls out of plane). The degree of bowing is such that there is a deviation of 5 mm of the center with respect to the top and bottom edges. When observing the overall deviation color pattern across the entire surface, the largest deviations are equal to about 6 mm (from maximum deviation to minimum deviation), which represents the degree of
smoothness. In summary, analysis of a given concrete panel is found to have deviations of warping, bowing and smoothness of 5 mm, 5 mm, and 6 mm respectively. What is interesting is that in comparison to the recommended tolerances outlined by PCI, the allowable deviations for warping, bowing and smoothness of a panel which is 2559 mm in length is 13.3 mm, 7.1 mm, and 6.4 mm respectively. While none of these deviations were exceeded, the smoothness of the panels had the greatest deviation analyzed, and yet has the strictest tolerance of these three dimensional variations.

In summary, as a result of the approach taken in this dimensional variability analysis, panel bowing or wracking (in plan view) was not considered. Furthermore, the dimensional variability of structural related impacts (e.g., variability of the mix, location of rebar) were not explored. However several in-depth analyses were conducted in order to address the ease of aggregation for panels. The overall panel dimensions were analyzed and correlated to a probability distribution function in order to understand the properties of the dimensional variability. Through this analysis, it was found that as a whole, the panel dimensional variability is normally distribution with a mean of -5 mm, and a standard deviation of 7 mm. The large standard deviation value might be of concern based on six sigma theory as positive deviations from the mean value can create problems for aggregation of panels into steel floor frame (explored in Chapter 5.4.1). In addition to assessing the variability associated with overall panel dimensions, a plane deviation analysis was conducted on the top surface to assess whether there is any warping, bowing or flatness deviations. Through this analysis it was determined that the deviations associated with warping, bowing and smoothness were equal to 5 mm, 5 mm, and 6 mm respectively.

5.2. Stage 2: Fabrication of Floor Frames

The second production stage of interest is the fabrication of floor frames, in which the precast concrete panels are placed in. The floor frames were outsourced by the construction company, and arrived to the plant in a series of standardized frames. Since each module has different load requirements (due to various mechanical and electrical equipment stored within the building), each standardized frame was fit with custom steel bracing. These custom braces were fit-up and welded into each frame by utilizing a framing table (Figure 30). Information about the method in which the outsourced standardized frames were fabricated was not made available. Overall, the fabrication processes involved with the production of the floor frames that have an impact on dimensional variability include cutting, welding, measuring, fit-up and grinding.

![Figure 30: Left: Standardized frame from outsourced supplier. Right: Standardized frame fit with custom bracing (note: custom bracing installed on framing table)](image)
For the production of these frames, guidance on the types of dimensional variability to consider and analyze is taken from the *AISC Code of Standard Practice for Steel Buildings and Bridges* which outlines some of the critical fabrication tolerance criteria as: (1) straightness, (2) variation in length, (3) camber, and (4) mill tolerances (AISC 2010a). Of these categories, the types of dimensional variability which have the largest impact for this specific frame is overall assembly straightness, variations in member lengths, and the fit-up dimensional variability between the concrete panel support beams. To analyze these dimensional variability sources, the following analysis are conducted:

1. Quantification of the slot dimensions for which each precast concrete panel is placed in (assumed to use the same dimensions considered for the panels). This type of dimensional variability has an impact on internal aggregation (i.e., how well the panels fit into the frames).
2. Quantification of the overall geometry of the frame, and specifically assessing if there is any warping or bowing. The overall geometry has an impact on external aggregation (i.e., module-to-module aggregation).

### 5.2.1. Analysis 1: Slot Dimensions

This analysis considered point cloud data which was collected for a series of floor frames. The dimensional variability of the precast concrete panels and the corresponding frame slots in which they are assembled into are assessment in a proceeding fabrication stage. The approach taken for quantifying the slot dimensions followed the same method described in Chapter 5.2.1 through manual extraction of dimensions for the widths, lengths and an average height (Figure 31). An automated approach could have also been employed for extracting the critical dimensions, however due to the amount of post-processing involved with cleaning the point clouds, it was deemed impractical.

![Figure 31: Demonstrating critical dimensions from precast concrete panel (left) and the corresponding slot on the floor frame (right)](image)

After manually extracting the slot dimensions for a single floor frame, nominal dimensions and the values for extracted as-built dimensions were summarized in a chart for further analysis (Table 12).
Table 12: As-built and nominal dimensions for length, width and height of frame slots

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<td>X1 (mm)</td>
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<td>1384</td>
<td>1500</td>
<td>1495</td>
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<td>X2 (mm)</td>
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<td>Y1 (mm)</td>
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<td>Z_AVG (mm)</td>
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By comparing the as-built dimensions with the nominal dimensions, histograms can be created in order to visualize the distribution of dimensional deviations Figure 32.

![Histogram plots for the dimensional variability of frame as-built dimensions.](image)

From observing the overall distributions for width (X1 and X2), length (Y1 and Y2), and height (Z), it can be shown that the deviations of as-built dimensions are consistent with the normally distribution at an α equal to 11.4%. In determining the probability distribution function to match each data set, chi-square goodness of fit tests were conducted. While the significance level (i.e., α) for the x, y and z dimensions yielded very large values (> 25% significance of the null hypothesis), the significance level for the sample of all dimensions yielded a smaller value (11.4%). Typically for chi-square goodness of fit tests, an acceptable significance level of 5% is required in order to correlate a probability distribution of interest. However, through testing numerous probability distributions, it was found that the lowest significance
level was correlated to a normal distribution (which is consistent with assumptions made in most manufacturing and construction process capability sources). The second best fit was a logistic distribution, which had a significance level of 25.8%. For all panel dimension variations, the fitting parameters of the normal distribution fit are:

- Mean (μ) = 2.17 mm
- Standard Deviation (σ) = 8.31 mm

5.2.2. Analysis 2: Overall Geometry

For the second critical dimensional variability being analyzed, plane analyses were utilized, where out-of-plane bowing or warping could be captured in the form of distinct deviation patterns. Plane analyses were conducted along the length and width of a floor frame. The output of these analyses are shown in Figure 33 and Figure 34.

Figure 33: Plane analysis output using PolyWorks® on sample width of floor frame (deviations in in mm)

Figure 34: Plane analysis output using PolyWorks® for both lengths of the floor frame of interest
From these plane deviation analysis, it was found that there was no distinct deviated pattern (e.g., bowing or warping) for the width of the frame (note that deviations for the width were consistently between 2 and 3 mm). However there was a distinct deviation pattern in the form of bowing for the lengths of the floor frame. The pattern of bowing is illustrated in Figure 35, where the dimensional variability is quantified as being as much as 13 mm.

![Figure 35: Deviation pattern in the form of bowing along the lengths of a floor frame (not to scale)](image)

In summary, it was found that the fabrication of roof frames yielded slot dimensions which were consistently larger than the nominal specifications. By fitting the overall slot dimensions with a probability distribution, the best fit was found to be a normal distribution with a mean value of 2.17 mm and a standard deviation of 8.31 mm. The large standard deviation indicates that the ability to comply with nominal slot dimensions can result in dimensions which are lower than the nominal values, which can create problems for aggregation (which is discussed in Chapter 5.4.1). Analysis of the frame for distinct deviation patterns resulted in the conclusion that the combined effects of cutting, measuring, fit-up, welding, grinding, etc. yielded pronounced bowing distortion with dimensional variability of as much as 13 mm.

5.3. Stage 3: Fabrication of Roof Frames

The third production stage of interest is the fabrication of roof frames. The roof frames are positioned directly over the floor frame and are connected by a series of columns. Each frame was outsourced by the construction company and arrived to the plant in a series of standardized frames. The roof frames did not require additional structural bracing, however of particular importance for critical sources of dimensional variability is the installation of tie-in plates which are used to connect adjacent modules together. The construction company installed the tie-in plates as well as corrugated metal decking on the top of the floor frame (which serves as the roof enclosure). The fit-up and installation of the metal decking and tie-in plates was done on a framing table (Figure 36). Overall the fabrication processes that have an impact on critical dimensional variability is cutting, fit-up, measuring, and welding. For this production stage, there are two main types of dimensional variability being considered:

- The overall geometry of the frames (which will have an impact on how well the modules can be interfaced together)
- The positioning of the tie-in plates (which also has an impact on how well the modules can be interfaced together).
A laser scan of the roof frame was obtained, and converted into a point cloud in order to analyze the dimensional variability of interest.

Figure 36: Left: Standardized roof frame on framing table. Right: Installed corrugated metal decking and tie-in plates.

### 5.3.1. Analysis 1: Overall Geometry

The first analysis considers the overall geometry of the roof frame, by quantifying dimensional variability associated with length, width and deviation patterns (e.g., bowing or warping) along the length of the frame. Plane analyses were conducted along the width and length of a roof frame in order to determine deviation patterns in the form of bowing or warping. The results of these plane analyses are shown in Figure 45. From these plane deviation analyses, it was found that there was a slight deviation pattern (warping) for the width of the frame (however these deviations were only on the order of 2 to 3 mm). There was a distinct deviation pattern in the form of bowing for the lengths of the roof frame. The pattern of bowing is illustrated in Figure 46, where the dimensional variability is quantified as being as much as 17 mm.

Figure 37: Plane analyses outputs using PolyWorks® for length and width of roof frame (deviations in mm)
In addition to assessing the form of the frame (e.g., warping and bowing), the overall length and width dimensional variations were quantified. This was done by comparing extracted dimensions from the obtained point cloud of the roof frame to the nominal dimensions specified in the design drawings. Comparison of the length and width dimensions to the nominal dimensions resulted in variations of -4 mm and -1 mm respectively (Figure 39).

5.3.2. **Analysis 2: Position of Tie-in Plates**

The second analysis conducted for the roof frame examined the dimensional variability associated with the position of tie-in plates along the length and width of the frame. To quantify this variability, dimensions were extracted from the obtained point cloud and then compared with the nominal dimensions from the design drawings. Results of this comparison are shown in Figure 39. By analyzing the deviations, it was determined that the positional deviations were on the order of 2 to 5 mm.

Figure 38: Deviation pattern in the form of bowing along the lengths of the roof frame (not to scale)

Figure 39: Comparison of nominal dimensions for overall length, width and location of tie-in plates to extracted as-built dimensions from the point cloud for the roof frame (note: as-built dimensions are in red, and deviations with respect to the nominal dimensions are displayed in blue with a plus/minus value, all dimensions in mm).
In summary, it was found that there was a distinct deviation pattern in the form of bowing along the length of the frame, where deviations were as much as 17 mm. Deviations along the width of the frame was much smaller than along the length, where deviations in the form of warping were on the order of 2 to 3 mm. Finally, positional deviations of the tie-in plates was quantified as being equal to as much as 5 mm. It should be noted that this analysis did not consider the vertical deviations of the frame. This was due to the fact that the frame was positioned on a framing table, which had a very controlled surface. Furthermore, the frames were found to experience a small amount of elastic movement under its own self weight depending on the handling loads involved with moving the frames within the fabrication facility.

5.4. **Stage 4: Aggregation of the Structural System**

The fourth production stage of interest examines the aggregation of the entire structural system for each module. This stage involves installing the precast concrete panels into the floor frame, installing columns on the floor frame and finally installing the roof frame on to the columns (Figure 40). All of the aggregation processes for this stage was completed on a framing table (i.e., this is how the construction company went about controlling dimensional variability). In terms of the critical sources of dimensional variability, the following fabrication processes were considered: fit-up, measuring, and welding (concrete panel outer frames to floor frame, columns to floor frame and roof to columns).

![Figure 40: Overall aggregation of the structural system. Top Left: floor frame. Top Right: concrete panels and columns installed on floor frame. Bottom Left: roof frame fit-up in place and installed on to columns. Bottom Right: completed assembly on shop floor.](image-url)
For this production stage, there are three main types of dimensional variability being considered:

- Comparison of panel dimensions versus frame dimensions (to assess degree of dimensional compliance for aggregation)
- Overall geometry of the modules (which has an impact on how well modules can be aggregated together)
- Position and orientation of tie-in plates (which has an impact on how well modules can be aggregated together)

5.4.1. Analysis 1: Aggregation of Panels into Frame

For the aggregation of the concrete panels into the floor frame, there are dimensional variability limits (tolerances) governing upper bound (referred to as maximum material condition, MMC in GD&T) and lower bound (referred to as least material condition, LMC in GD&T) conditions. For the MMC condition, the panel dimensions must be less than the slot dimensions on the frame so that the panels can physically fit into the frame. For the LMC condition, the panel dimensions cannot be less than 15 mm from the corresponding slot dimensions, otherwise the gap is too large between the panel and the frame and aggregation cannot proceed without rework.

In the analysis of the panel and frame dimensions, several tables of dimensions were created (Table 11 and Table 12) as well as histograms to graphically visualize and compare the dimensional distributions of the deviations between nominal and as-built dimensions (Figure 28 and Figure 32). By comparing these dimensions and distributions, some key conclusions can be made about the dimensional compliance of the panel dimensions and the frame slot dimensions. In the initial analysis, it was determined that the population of panel dimensions and frame slot dimensions could be modelled by normal distributions. In order to properly aggregate panels into the frame slots, the panel dimensions need to be less than the frame slot dimensions minus the allowable gap. In the initial design, there was a gap of 3 mm around the perimeter of the panel and the frame slot. There are three ways in which the dimensional variability of panels and frame slots are assessed for aggregation compliance: (1) one-to-one comparison of dimensions to obtain a percentage of dimensional compliance, (2) binary panel-to-slot aggregation test and (3) probabilistic distribution comparison to assess aggregation compliance.

Analysis 1 which examines the one-to-one comparison of panel and frame slot dimensions finds that four dimensions of the total 55 are non-compliant (i.e., panel dimension exceeds the corresponding frame slot dimension) for a percentage of non-compliance of 7%. Analysis 2 examines whether a given panel has complete dimensional compliance with its corresponding frame slot (if any panel dimension exceeds its corresponding frame slot dimension, that panel of interest fails the binary pass-fall test). Analysis 2 reveals that four of the eleven panels require some sort of realignment or rework to make them fit into their corresponding frame slot, for a percentage of non-compliance of 36%. Finally analysis 3 compares the probability distributions of the panel dimensions (as a whole) with that of the frame slots (as a whole). Recall that the probability distributions for the panel dimensions and frame slots had the following properties:

- Panel dimensions: Mean ($\mu$) = -5.0 mm, Standard Deviation ($\sigma$) = 7.1 mm (normal distribution)
- Frame slot dimensions: Mean ($\mu$) = 2.17 mm, Standard Deviation ($\sigma$) = 8.31 mm (normal distribution)
By comparing the intersection of these probability distributions to assess non-compliance in terms of the probability that any random panel dimension exceeds any random frame slot dimension, it is found that the percentage of non-compliance is equal to 26%. This was found by computing the reliability index of the two dimensional variability (normal) distributions using the following formula:

$$\beta = \frac{\mu_{frame} - \mu_{panel}}{\sqrt{\sigma_{frame}^2 + \sigma_{panel}^2}} \text{ and } P_{\text{non-compliance}} = \Phi(-\beta) \quad \text{[Eqn. 4]}$$

This method of calculating the probability of non-compliance between the panels and frame is analogous to calculating probability of failure in structural reliability theory (Thoft-Cristensen & Baker, 2012). In structural reliability theory, probability of failure is calculated by integrating the cumulative distribution function (CDF) of the structure resistance with the probability density function (PDF) of the load distribution over a specified domain (i.e., $-\infty$ to $+\infty$). This calculation can also be expressed through a closed form solution using the reliability index ($\beta$) as shown in [Eqn. 4]. After obtaining the reliability index, probability of failure (or non-compliance in the case of assessing compliance of panel and frame dimensions) is calculated using the cumulative distribution function for the standard normal distribution function (denoted as $\Phi$ in [Eqn. 4]). The integration of the PDF for panel dimensional variability with the CDF for frame dimensional variability is shown in the graph on the right in Figure 41. Using either numerical integration (Figure 41) or the closed form solution [Eqn. 4], the value for the reliability index is found to be equal to 0.66, which equates to a probability of non-compliance of 26%.

![Figure 41: Comparison of probability distributions for panel and frame dimensions. Probability of non-compliance equal to 26% based on reliability index calculation. Dimensions in mm.](image_url)
While each of the preceding methods to quantify aggregation compliance of panels and frames result in different probabilities (7%, 36%, and 26%), they are consistent with the actual experience of the contractor for this project. Although the actual percentage of panels which did not fit into frames could not be obtained, the contractor noted that “several” panels (of the 176 in total) did not fit properly into the floor frames. Furthermore, the contractor noted that some sort of rework was “often” required to make each panel fit properly indicating that the non-compliance of panel and frame dimensions created challenges.

5.4.2. Analysis 2: Overall Module Geometry

The second analysis examines the overall geometry of the module assembly in order to assess variability associated with form (e.g., warping or bowing of the entire assembly), as well as the height and fit-up of columns. For assessing overall form deviations, a scan-to-BIM deviation analysis was conducted using PolyWorks®. Comparison of a laser scan of the overall module to the BIM model enables comparisons to be made about the overall fabrication and aggregation compliance of the assembly. In order to conduct this deviation analysis by way of scan-to-BIM, some point cloud cleaning was done, where the concrete panels in the point cloud were removed. This was done since the BIM model used for comparison was not available with the installed concrete panels. As a result, there were numerous errors during data registration between the point cloud and the BIM model, since the method used for registration (Iterative Closest Point method), relies on comparison of points in the point cloud to closest points in the BIM model. Since there were large gaps in the BIM model where the concrete panels were installed, registration was very challenging and a successful result was not obtained. This is why the panels were removed from the point cloud prior to registration. Results of the deviation analysis (Figure 42), reveal that the overall deviations between the as-built state and the nominal state are as much as 18 mm. It should be noted that this deviation magnitude closely matches the maximum deviation found in the roof frame fabrication analysis. It is very challenging to assess the form deviations in terms of a distinct deviation pattern. However, the deviation analysis results show that the largest deviations are experienced on the roof frame near the ends of the module, which is of particular importance when assessing the ability for this particular module to aggregate with adjacent modules.

Figure 42: Deviation analysis output using PolyWorks® for the overall module assembly (note: concrete panels are not shown or assessed in this analysis, all deviations in mm)
For assessing the dimensional variability associated with the height of the columns, comparisons between the obtained point cloud and the nominal dimensions were conducted for a sample column on either end of the module. The results of this comparison (Figure 51) reveal deviations are on the order of 4 to 5 mm.

Figure 43: Comparison of nominal dimensions for length of columns to extracted as-built dimensions from the point cloud (note: as-built dimensions are in red, and deviations with respect to the nominal dimensions are displayed in blue with a plus/minus value). Dimensions in mm.

5.4.3. Analysis 3: Position and Orientation of Tie-in Plates

For this analysis, the position and orientation of the tie-in plates were once again assessed. The reason for reassessing these was to compute deviations with respect to a datum on the lower floor frame (which was specified as the datum in the design drawings). For aggregation of this module with adjacent modules, it is important to quantify the dimensional variability of the tie-in plates with respect to the floor frame as this determines the overall protrusion of plates out-of-plane. For this analysis, a plane deviation analysis was conducted using PolyWorks®, where a plane of best-fit was created with respect to the floor frame. Then deviations of the tie-in plates were quantified (Figure 44). From the output of this analysis, it can be seen that the deviation of the tie-in plates with respect to their nominal position (based on the datum of the floor frame) is between 6 and 17 mm (protruding away from the length of the module).
In summary, the dimensional variability associated with the aggregation of the structural system may have larger deviations than in previous fabrication stages based on comparison of the maximum deviation values identified. The effects of accumulation of variability yield much more complex form variations for bowing and warping of the steel structure. It was found that the overall assessment of the structure resulted in deviations up to 18 mm. Furthermore, the maximum deviations occurred on the roof frame portion of this module. This indicates that as the assembly is aggregated from planar 2D systems to a volumetric 3D system, the effects of dimensional variability compound significantly. In terms of addressing the ability to aggregate this module with adjacent modules, the dimensional variability associated with the location of the tie-in plates with respect to the datum (floor frame) revealed deviations as large as 17 mm.

5.5. **Stage 5: Temporary Support Conditions**

This stage examines the dimensional variability associated with different temporary support conditions (Figure 37). Three support conditions were examined in this project: continuous supports of a BLUCO® framing table (used for fit-up and aggregation of the module), shop floor cribbing (only at 4 corners of the module), and shop floor cribbing (with the same number of supports as the onsite foundations). Temporary support conditions have a pronounced impact on the elastic changes to the module geometry, especially considering the self-weight of the modules was very large in this project. As such, several deviation analyses were conducted in order to understand the effect that temporary supports have on the geometry of the structure. Laser scans were taken of the BLUCO® fixturing system (on which the framing table sits on top of), the shop floor, the shop floor cribbing (with only 4 supports at the corners of the module), and the shop floor cribbing (matching the number of supports as on site).
5.5.1. Analysis 1: BLUCO® Fixturing System

During the aggregation of the structural system, a framing table was used for the fit-up and positioning of sub-assemblies. This table is positioned on a BLUCO® Modular Fixturing system (Figure 38), and has very tight flatness tolerances (+/- 0.0046 mm per 300 mm length) and precise modular bores (tolerance of +/- 0.025 mm) which can be used for specialized table set ups (BLUCO Corporation 2016). During the fabrication of this project, it was brought into question how accurate the levelness of the framing table was with respect to a ‘perfect’ plane (angular deviations of 0°). Although the framing ‘rails’ have very tight dimensional tolerances, the installation of this system requires accurate surveying to ensure the rails are level. In order to assess the levelness of this fixturing system, a plane deviation analysis was conducted (Figure 47).
The plane deviation analysis reveals that the main fixturing rails (note in Figure 47 that there are six rails that run the length of the floor), have more dimensional variability than the accuracy of the table system. Overall, the variability between rails (along the width of the floor) is very low, where variations are +/− 2 mm. Along the 17 meter length however, deviations on each rail vary by as much as 9 mm. The general deviation pattern of each rail is a bowing shape, where the center is lower than the left edge by about 7 mm, and lower than the right edge by 1 to 6 mm. This dimensional variability reveals that as a whole, the framing table system has a levelness deviation that is not insignificant, and should not be considered to have the same level of precision as the tables which fit into the floor fixturing system.
5.5.2. Analysis 2: Shop Floor Cribbing

Shop floor cribbing in this project was comprised of placing stacks of steel plates (smallest thickness used was 3 mm and the largest was 50 mm). By stacking different thicknesses of steel plates, it was possible to achieve certain vertical elevations (through the use of surveying) for temporary supports. Two different shop floor cribbing configurations were used: four supports at the module corners, and six supports. Six supports was introduced during the project because the module was noted to experience a substantial amount of elastic distortion from only four supports at the corners. To demonstrate the amount of dimensional variability associated with the shop floor, a plan deviation analysis was conducted using PolyWorks® (Figure 48). This plane analysis reveals that the shop floor under the shop floor cribbing has a large amount of dimensional variability, where the flatness changes by as much as 30 mm across the 18 m by 5 m area of floor examined.

Figure 48: Plane deviation analysis output using PolyWorks® to assess the dimensional variability associated with the shop floor.

A separate plane deviation analysis was conducted to assess the dimensional variability associated with the surfaces of the floor cribbing. In this analysis (Figure 49), the range of deviations were found to be equal to only 8 mm, which is significantly less than that of the shop floor. This result clearly shows how dimensional variability can be reduced through the use of shop floor cribbing. However, in terms of understanding the effect that the dimensional variability of temporary supports has on the overall goals of managing dimensional variability, it is important to analyze the elastic changes to the module geometry in different temporary loading conditions.
In order to quantify the effect that temporary supports has on the module geometry, a scan-to-scan analysis was conducted to compare the geometric state of the module while sitting on the BLUCO® framing table, and on four supports (Figure 50). The reason for analyzing the module in these two states is that this was found to be the two extreme cases (versus comparison of the module on four and six supports).

Figure 49: Shop floor cribbing conditions: first condition comprised only the outer four supports, however during the project two additional cribbing supports were added in the center. This figure also shows the output of a plane deviation analysis using PolyWorks® of the dimensional variability associated with the vertical elevation of the cribbing (deviations are in mm).

Figure 50: Demonstration of the module elastic deflection while sitting on only four cribbing supports at the corners.
The scan-to-scan analysis was conducted by using MATLAB® and point clouds of the two geometric states (Figure 51). While a scan-to-scan analysis could have alternatively been conducted through use of an open-source software called Cloud Compare®, the purpose of conducting an analysis in MATLAB® was to have more control over the analysis process. In this case, the midspan deflection was of interest, and so in order to have the ends of the point clouds set to a deviation value of 0 mm (matching at ends), an algorithm was prepared in MATLAB®. Refer to Appendix C for the development of the MATLAB® code used. This deviation analysis shows that the midspan (elastic) deflection of the module between the two temporary support conditions is equal to about 30 mm. This is a very significant deflection value, and is caused by the large self-weight of the module and the fact that the shop floor cribbing sits off the ground by as much as 40 mm in some locations. In order to validate this deflection value with regard to the structural response due to self-weight, a simplified deflection analysis was done (Appendix C), revealing that deflection due to self-weight of the steel frame is equal to 39 mm.

In summary, this stage examined the dimensional variability associated with the geometric response of the module in different temporary support conditions. Analysis of the first temporary support system (BLUCO® framing table), resulted in variations in levelness by as much as 9 mm along the 17 m length of the system. Comparison of variations of the shop floor to the profile of steel plate cribbing shows that a substantial amount of variability can be controlled through the use of various thicknesses of steel plates and surveying. However, the most interesting results of this stage was from the geometric response of the module itself. It was shown that by comparing the geometric state of the module while sitting on the framing table to 4 shop floor cribbing supports that the module can deflect by as much as 30 mm along its 16 m length. While this deflection was assumed to be completely elastic (that is, recoverable), it plays a very significant role for the fit-up and installation of subsystems between modules. For instance, if a series of pipes were to be installed between two modules along the length, while the modules were only supported by 4 corner supports, the vertical location of certain pipes would change substantially on site.
since modules are essentially continuously supported and experience much less deflection than the 30 mm observed.

5.6. Stage 6: Aggregated Module during Transportation & Handling (Geometric Changes)

This stage examines the dimensional variability associated with the geometric response of the module when subjected to different external loading scenarios. The two loading scenarios examined are when the module is being lifted by a crane in the shop and loads from transportation and handling required to get the module to and from the shop, temporary storage and finally to the project site (Figure 52).

The analysis of the loads from the crane considered only elastic deflection of the structure. This was due to the fact that analyses before and after crane loading resulted in less than 1 mm of plastic deflections (which is less than the accuracy of the laser scanner used). For analysis of the crane loads, a laser scan of the bare steel frame structure was taken before and during the lift. While suspended in the air by the crane, the module was supported at the ends to ensure it would not swing in the air (and thus yield inaccurate deviation values). A scan-to-scan deviation analysis was conducted using the open source software called Cloud Compare®, since this type of analysis was not easily done using PolyWorks®. As seen in this analysis output (Figure 53), the deviations between the two geometric states are between 1
and 3 mm. Furthermore the largest deviations occur at the ends of the module. This is likely because the crane loads put the top frame into compression, while the bottom frame is in tension (this would cause a general bowing shape where the two ends are slightly more deviated than in the center). Again, it should be noted that these deviations shown are elastic, meaning they only exist during the crane load. A separate analysis was conducted to detect plastic deflections as the result of the crane loads, however this analysis revealed deviations less than 1 mm and thus non conclusive in light of the accuracy of the laser scanner.

![Figure 53: Scan-to-scan deviation analysis using Cloud Compare® to quantify dimensional variability associated with geometric response of crane load (deviations shown are in m).](image-url)

In order to analyze the dimensional variability associated with the geometric response of the module to transportation loading, a separate type of data collection and analysis method was required. This is because collection and analysis of surface data (e.g., point clouds from laser scans) was not practical since the modules were wrapped in weatherproofing on the exterior. The weatherproofing is made of a plastic material which can move very easily. As such, the weatherproofing does not give an accurate representation of the plastic deflections of the structure. Furthermore it was not feasible to take laser scans on the interior of the structure since the module was fit-up with drywall and other interior finishes. In light of these challenges, a total station was used to quantify the deflection of the structure due to transportation loading. Since a different data collection and analysis method was utilized (total station versus laser scanning), a calibration study between these two approaches was conducted in order to validate that the results of both methods yield similar results. This calibration study is found in Appendix B.

24 permanent targets were placed on exposed steel on the interior of the module along the top and bottom frames along the length. These targets were surveyed before and after transportation of modules to a temporary storage area. By surveying these targets, their relative positions were obtained through analysis
in MATLAB® (the code used for this analysis is available in Appendix C). A wireframe model was created in order to visualize the deviations of target locations before and after transportation (Figure 54).

![Figure 54: Deviation analysis using MATLAB® code and surveying of 24 key locations on the module in order to quantify dimensional variability associated with transportation loading.](image)

As seen in this analysis output, the permanent deflection of the structure along the length as the result of transportation loading is on the order of 3 mm. In light of the accuracy of the total station used for this analysis (2 mm), these results show a very low amount of plastic distortion associated with transportation loading.

In summary, the effect of geometric responses due to transportation and handling on dimensional variability of the steel frame structure itself is quite low. Only elastic deflections were substantial sources of dimensional variability for the crane handling, and the likelihood of causing permanent changes to the geometry from crane handling in this project is quite low. One reason for this could be due to the lifting frame used to hoist the module. The purpose of a lifting frame ensures that the crane loads are distributed along the module, rather than being concentrated at a few locations. Furthermore the effect of transportation loading on the module geometry was also quite low. The transportation loading caused small plastic distortion to the module frame, however the magnitude of these distortions was low.

5.7. **Stage 7: Aggregated Module during Erection Loading (Geometric Changes)**

The final construction stage in this case study examines the dimensional variability associated with the geometric response of the module to erection loads (Figure 55). For this stage, it was not feasible to obtain a laser scan of the module before or after erection due to site constraints and schedule logistics (the project was slightly delayed, and there was no allowance for any further delays that would be caused by bringing a laser scanner on site to conduct scans). Furthermore a laser scanner would not be practical to
use since the inside of the module was fit-up with drywall and interior finishes which would not yield accurate information regarding geometric changes to the module structure. As such, a total station was used to obtain locations of key targets placed on the interior of the module on the structure.

Similar to the analysis of the geometric response of the module to transportation loads, the relative positions of permanent targets placed inside the module were analyzed. Due to challenges with placement and clear line-of-sight to targets within the module, only eight targets were placed at the ends of the module. However eight targets is still a sufficient quantify for analyzing the geometric response of the module due to erection loads (e.g., the loads required to bring modules into alignment with foundation and adjacent module connection points). The targets were surveyed before transportation of modules to the project site, and after the final erection on site was complete. By surveying these targets, their relative positions were obtained through analysis in MATLAB® (the code used for this analysis is available in Appendix C). A wireframe model was created in order to visualize the deviations of target locations before and after transportation (Figure 56). Since surveying of targets was done before and after transportation to site, the deviations shown in Figure 56 reflect the geometric response of the module to

Figure 55: Examining the dimensional variability associated with erection loads. Top left: project site, where modules are first brought into rough alignment, and then subsequently connected together at the tie-in plates. Bottom left: project site after completion of all construction activities. Right: demonstration of the amount of large gaps between modules at tie-in plate locations as the result of the accumulation of dimensional variability.
both transportation and erection loading (roughly 7 mm of deviations). Taking the deviations observed from transportation loading alone in the previous stage (roughly equal to 3 mm), then it can be expected that the erection loading can contribute to dimensional variability of approximately 4 mm. This is a very reasonable estimate of the dimensional variability associated with the geometric response to erection loads since the module is subject to elastic deflections under its own self weight by as much as 30 mm.

Figure 56: Deviation analysis using MATLAB® code and surveying of eight key locations on the module in order to quantify dimensional variability associated with transportation loading.

5.8. Conclusions on Dimensional Variability Case Study

In conclusion, this case study examined the dimensional variability of a steel framed module structure during seven key construction lifecycle stages. Surface data in the form of a laser scanner was the primary data collection used which was then analyzed using three software: PolyWorks®, MATLAB® and Cloud Compare®. In light of challenges to effectively analyze the geometric response of the module to applied loading, surveying through use of a total station was also used. While this case study may not have captured every source of dimensional variability that has an impact on the goals of dimensional variability management, it quantified key sources based on a list of key construction processes.

Due to spatial limitations on site, it was difficult to quantify the final accumulation of dimensional variability. However, some visual observations made on site indicate the accumulation of dimensional variability was quite significant. For instance, the tie-in plates used to connect modules on site at the roof frame level had very large gaps (refer to the image on the right in Figure 55). In some cases the gaps between module tie-in plates were as large as 50 mm. While gaps of this magnitude may not be directly the result of the accumulation of variability, it was noted in discussion with the site erection crew that the modules consistently had geometric discrepancies with foundations and adjacent modules. The final management strategy employed on this project for dealing with the challenges created by dimensional variability was to space the modules further apart from each other at the matelines than originally specified in the design. This practice is referred to as ‘the growing building phenomenon’, which can have serious implications for some modular construction projects (especially in the case of modular high rise
construction or in cases where the foundations cannot accommodate a ‘growing’ building). To summarize the critical sources of dimensional variability in this project, a table was created, which includes the type of variability, its magnitude and impact on specific aspects of the project (Table 13).

Table 13: Summary of the key sources of dimensional variability examined in this case study, along with estimated magnitudes and impact on the overall project.

<table>
<thead>
<tr>
<th>Construction Stage</th>
<th>Critical Dimensional Variability &amp; Estimated Value</th>
<th>Source of Variability</th>
<th>Impact of Dimensional Variability on Specific Aspects of Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabrication of precast concrete panels</td>
<td>Panel dimensions modelled by: • Normal distribution • Mean = -5 mm • Standard deviation = 7 mm</td>
<td>Fabrication processes: measuring, cutting, welding of steel pans, and concrete mixing, pouring, curing and finishing</td>
<td>Aggregation is the critical impact, however if gap between panels and frame is too large, aesthetics are also impacted.</td>
</tr>
<tr>
<td></td>
<td>Warping deviation = 5 mm</td>
<td></td>
<td>Warping, bowing and smoothness deviations have an impact on aggregation (potential mismatch between geometry of frames), aesthetics and serviceability as final surface of floor is not flat or smooth.</td>
</tr>
<tr>
<td></td>
<td>Bowing deviation = 5 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Smoothness deviation = 6 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fabrication of floor frames</td>
<td>Frame dimensions modelled by: • Normal distribution • Mean = +2 mm • Standard deviation = 8 mm</td>
<td>Fabrication processes: measuring, cutting, fit-up and welding</td>
<td>Aggregation with the concrete panels is the critical impact. If gap between panel and frame slots is too large, aesthetics also emerge as an impact.</td>
</tr>
<tr>
<td></td>
<td>Bowing deviations = 13 mm</td>
<td>Welding distortion</td>
<td>Aggregation with adjacent modules is the critical impact, as the frame has out-of-plane protrusions.</td>
</tr>
<tr>
<td>Fabrication of roof frames</td>
<td>Bowing deviation = 17 mm</td>
<td>Welding distortion</td>
<td>Aggregation with adjacent modules is the critical impact, as the frame has out-of-plane protrusions.</td>
</tr>
<tr>
<td></td>
<td>Tie-in plate positional deviation = 2 to 5 mm</td>
<td>Measuring, fit-up, fastening method</td>
<td>Aggregation is the critical impact, since the location of bolt holes on tie-in plates need to match up between adjacent modules for proper aggregation.</td>
</tr>
<tr>
<td></td>
<td>Frame dimension deviations = -1 mm to -4 mm</td>
<td>Measuring and cutting</td>
<td>Aggregation is the critical impact, since the overall length of the frame governs aggregation with adjacent modules.</td>
</tr>
<tr>
<td>Aggregation of the structural system</td>
<td>Aggregation of panels into steel frame: • Non-compliance based on comparison of corresponding dimensions = 7% • Non-compliance based on binary test (i.e., all dimensions for panel and slot are compliant) = 36% • Non-compliance based on comparison of process</td>
<td>• Measuring, cutting, welding of steel pans, and concrete mixing, pouring, curing and finishing. • Measuring, cutting, fit-up and welding of frame</td>
<td>Aggregation is the critical impact when conducting non-compliance tests for dimensions between concrete panels and steel frame. However if gaps between panels and frame are large enough, aesthetics are also impacted.</td>
</tr>
</tbody>
</table>
### Among all of the sources of dimensional variability examined, the most significant include the large form deviations of steel frames. The floor and roof frames in this project were comprised of large steel members, which required an extensive amount of welding. The result of welding distortion was very significant, and contributed to the majority of the final geometric deviations in the module structural system. The welding distortion changed the geometry of the frames such that the final position of tie-in plates had significant deviations (by as much as 17 mm). Furthermore, the elastic response of the module due to temporary support conditions was very significant (where the module deflected by 30 mm at midspan).
6. Development of New Design-Based Strategies and Demonstration through Case Studies

This chapter presents design-based strategies which can be used to manage dimensional variability. The methods presented are: (1) tolerance mapping based on work by Milberg et al. (2005), and (2) kinematics-chain based dimensional variation analysis. This chapter comes primarily from the following publication:


The primary author of this publication prepared the literature review, conducted the data collection, assisted with analysis of results, and prepared the conclusions. The primary author also conducted the example case study for tolerance mapping. The second author developed the details of the kinematics chain model, and conducted the example and full case study for kinematics chain dimensional variation analysis in this publication. The third author provided overall research guidance and reviewed the paper.

6.1. Tolerance Mapping Framework for Modular Construction

This section applies a generic tolerance mapping framework to case studies in modular construction for the purpose of analyzing the dimensional variability between as-built construction assemblies and the as-designed (tolerance-based) state. This tolerance mapping framework was originally developed by Dr. Colin Milberg (Milberg et al. 2002, Milberg and Tommelein 2003a, Milberg and Tommelein 2003b, Milberg and Tommelein 2004, Milberg and Tommelein 2005, Milberg 2007, Milberg 2006). As introduced in the background chapter of this thesis, tolerance maps are used as a tool to specify adequate tolerances, or to analyze if there are any over-constrained tolerance chains. Over-constrained tolerance chains arise when a tolerance for a series of connected components cannot be achieved based on the tolerances required for individual components within that chain.

6.1.1. Tolerance Mapping Case Study 1: Modular Steel Bridge Component

To demonstrate how tolerance mapping can be used to properly manage dimensional variability, a simple example is shown below which outlines the dimensional relationships of all component-features in a modular steel bridge component (Figure 57). This steel bridge is approximately 6 metres long, and has been designed in a modular manner, with 40 separate assemblies in 2 trusses. In this example, a tolerance map is created along with a separate map for the actual fabrication-related deviations. The purpose of creating this second ‘deviation map’ is to evaluate the compliance with tolerance specifications in order to revise the tolerance values for future applications.
Three variation categories are employed in the tolerance map to define relationships between component-features and components within assembly. Recall that component-features are the geometrical properties of a component such as lines, angles, and surfaces. These categories are based on GD&T notation: (1) orientation and location variations, which define a component-feature’s spatial state, (2) form variation, which define how straight, flat, or round a component-feature is, and (3) size variations which defines two-point measurements of a component-feature. For the assembly diagram, typically only orientation and location tolerances are used, since the assembly diagram defines how the sub-components or parts are spatially related (Figure 58). The creation of component diagrams and the overall tolerance map (Figure 59) follows the same approach taken for the assembly diagram (i.e., component-features are geometrically related using GD&T notation).
Figure 59: Example of steps involved for creating a tolerance map: (1) create the assembly diagram, (2) create diagrams for each component and (3) amalgamate all component diagrams into the assembly diagram to obtain the overall tolerance map.

The tolerances in this design are summarized in Table 14. Derivation of these initial tolerance values were based on a list of suggested tolerances by fabricators. In addition, size and form tolerances for the steel components used in this bridge were based on mill tolerances provided by ASTM A6/A6M-14.

Table 14: Equipment and assumptions about tolerance type and value in tolerance mapping case study 1

<table>
<thead>
<tr>
<th>ID (for map)</th>
<th>Equipment/Process</th>
<th>Tolerance Type(s)</th>
<th>Tolerance Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Tape measure</td>
<td>Size, location</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>b</td>
<td>Caliper</td>
<td>Size, location</td>
<td>0.025 mm</td>
</tr>
<tr>
<td>c</td>
<td>Manual taping + marking</td>
<td>Size, location, orientation</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Cutting</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Upright bandsaw</td>
<td>Size, form, orientation</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>e</td>
<td>Horizontal bandsaw</td>
<td>Size, form, orientation</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>f</td>
<td>Chopsaw</td>
<td>Size, form, orientation</td>
<td>2.0 mm</td>
</tr>
<tr>
<td>g</td>
<td>Drill Press</td>
<td>Size, form</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Trimming</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>Belt sander (assumed planar)</td>
<td>Size</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>i</td>
<td>Hand grinder</td>
<td>Size, form</td>
<td>2.0 mm</td>
</tr>
<tr>
<td>Positioning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>j</td>
<td>Fit-up (drilling)</td>
<td>Location</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>k</td>
<td>Fit-up (welding)</td>
<td>Location</td>
<td>2.0 mm</td>
</tr>
<tr>
<td>Fixturing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>Clamps + fixture (welding)</td>
<td>Location, orientation</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>m</td>
<td>Clamps + jigs (drilling)</td>
<td>Form, location</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Welding</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Movement (uneven cooling)</td>
<td>Form, orientation, location</td>
<td>3.0 mm</td>
</tr>
<tr>
<td>Mill Production</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>o</td>
<td>Mill – size tolerance (HSS)</td>
<td>Size</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>p</td>
<td>Mill – form tolerance (HSS + plates)</td>
<td>Form</td>
<td>0.4 mm</td>
</tr>
</tbody>
</table>
Development of the part diagrams, tolerance map and deviation map used in this analysis are included in Appendix D. Comparison of the values in the tolerance map and deviation map yield some key conclusions for improving the tolerance specifications in future projects which employ the fabrication processes used in this project. For instance, certain observed process capabilities (i.e., deviations) were consistently lower than specified product tolerances. This was true for the combined effect of tape measuring and marking, which had a deviation value of 0.5 mm on average compared to the tolerance of 1.5 mm. In other cases however, the specified tolerances were much smaller than the observed process capabilities. This was true for the tolerance associated with the horizontal bandsaw, which had deviations on average of 5 mm compared to the allowable tolerance of 1 mm. By creating a tolerance map, deviations of fabrication processes can be tracked in order to better manage dimensional variability.

The key limitation in this case study is that the use of a tolerance map is very tedious. There are currently no automated approaches for creating tolerance maps. As such, tolerance maps are only practical to use for very critical components (e.g., components in a nuclear construction assembly), unless assumptions can be made in order to increase the practicality of using tolerance maps in modular construction assemblies (which is demonstrated in the second case study).

6.1.2. Tolerance Mapping Case Study 2: Tie-in Plates for a Modular Building

In light of the challenges with using a tolerance map as demonstrated in the first case study, this section shows how tolerance mapping can be used in a much more simplified manner for managing critical dimensional variability associated with the aggregation of modules in a modular building. This case study comes from the same project shown in Chapter 5, where the dimensional variability of tie-in plates of a module were mapped with respect to the global datum as shown in Figure 60.

![Figure 60: 3D view of the module structure used in tolerance mapping case study 2 for outlining the position of tie-in plates along the length of the module with respect to the global datum (referred to as the Assembly Reference Frame).](image-url)
In this case study, a tolerance map and deviation map were created in order to evaluate the quality of tolerance specifications with respect to fabrication process capabilities. Furthermore, since tolerances for modular construction are typically much tighter (smaller) than for stick-built construction, two separate tolerance maps were created to determine where this particular as-built state falls with respect to recommended tolerances for modular construction and stick-built construction. For this analysis, it was assumed that the tie-in plates located on the upper beam in the steel module functioned as the critical parts for interfacing between modules, and as such the deviations were analyzed in the negative direction of the Y-axis as shown in Figure 60.

Rather than outlining every component-feature as was shown in the first case study for tolerance mapping, this case study focuses on key features and components in the overall structural assembly which are assumed to have large contributions to the position of the tie-in plates along the length of the module. The key components considered are the floor beam, roof beam, combined effect of columns and the tie-in plates. After creating part diagrams for each of these components, the most critical component-features were amalgamated into the overall tolerance map. Figures for part diagrams, assembly diagram, tolerance maps and deviation map are included in Appendix D. Two resources were used for deriving tolerance values: (1) ‘stick-built’ construction tolerances from the AISC Code of Standard Practice (COSP) for fabrication and erection tolerances in lieu of tolerances specified in the design (AISC 2010a), and (2) recommended ‘modular’ tolerances from Design in Modular Construction (DIMC) for the overall allowable envelope tolerances of a single module (Lawson et al. 2012).

By using tolerance values from these two distinct resources, a tolerance map can be created for stick-built tolerance values and modular tolerance values in order to assess tolerances in the modular tolerance map which are much tighter (smaller) than the stick-built tolerance map. Finally, a deviation map was created in order to assess the actual project deviations with respect to both tolerance maps. Assessment of the project deviations (Figure 61) are presented in terms of whether they are: (a) above stick-built tolerances, (b) between stick-built and modular tolerances, or (c) below modular tolerances. This assessment is valuable for indicating which processes need to be revised in order to meet desired deviation values.
Figure 61: Comparison of deviations with respect to both tolerance maps employed in the second case study for assessing process capabilities.
Based on the creation of the tolerance maps, deviation map and the comparison map in this case study, the following observations can be made:

- Module External Beam Bow Tolerance: the Code of Standard Practice for Steel Construction (COSP) has very loose tolerances in comparison to similar tolerance values as recommended in Design in Modular Construction (DIMC). An example of this is the form tolerance where COSP had a tolerance of 32.0 mm as compared to DIMC, which had the same tolerance of 4.0 mm. This leads to the conclusion that stick-built tolerances can often be an order of magnitude larger than equivalent recommended modular construction tolerances.

- An interesting observation about using tolerances recommended in COSP is that the “accumulation of mill tolerances and fabrication tolerances cannot cause the erection tolerances to be exceeded.” (AISC 2010a). In many cases, the erection tolerances specified in COSP are the same value as the fabrication tolerances. This means that fabrication processes cannot be mutually exclusive from mill processes. Therefore an overall tolerance should be allocated for the combined effect of mill plus fabrication rather than deterministic values for both.

- Of the 35 specified tolerances in this case study, 11 were between COSP and DIMC tolerance values, 18 were above COSP tolerances and 6 were below DIMC tolerance values. These results are subject to the setup of the analysis, meaning that analyzing the tolerances and deviations as ‘pass-fail’ criteria is not representative of this project’s ability to meet certain tolerances.

This case study was not as detailed of an analysis as case study 1, and furthermore involved analyzing an entire modular construction assembly (whereas case study 1 only looked at a single component). This case study demonstrates that a rigorous tolerance analysis is not always needed in order to evaluate and determine critical factors that lead to potential out-of-tolerance issues in a modular construction assembly. Finally, this case study demonstrates that more consideration must be made in modular construction with respect to the specification of tolerances. If fabrication and aggregation solely relies on the recommended tolerances found in stick-built construction, then the overall accumulation of tolerances can be much higher than if tighter modular tolerances are specified. The use of an overall tolerance envelope (as found in Design for Modular Construction) would be extremely useful for specification of tolerances in modular construction assemblies, rather than specifying tolerances for each component (Lawson et al. 2012).

### 6.1.3. Tolerance Mapping Conclusions from Case Studies

The use of tolerance mapping as a design-based strategy for managing dimensional variability was demonstrated using two case studies. The goals of tolerance mapping are to assist in the specification of adequate tolerances and to identify over-constrained tolerance chains. The second goal (identifying over-constrained tolerance chains) was not explored in these case studies, since there were no additional tolerances specified for a chain of components or features. However, the use of tolerance mapping did successfully help in identifying certain process capabilities (i.e., variations) which did not meet the required tolerances. Overall, tolerance mapping can be quite difficult to use as a design-based strategy since it requires understanding of GD&T notation, and is best used for identifying over-constrained tolerance chains (which do not always occur in modular construction projects). Furthermore, in order to aid with tolerance specification, deviation mapping is created after the design is complete, unless the use of prototyping is employed.
### 6.2. Development of Kinematics Chain Based Dimensional Variation Analysis

In aerospace and automotive manufacturing, dimensional variation analysis (DVA) is used to ensure that the effect of dimensional variability on parts and assemblies is properly controlled. Common mathematical models used in a DVA assembly include worst case, statistical or sampled mathematical models (Table 15) and can be modelled in 1D, 2D or 3D (Chase and Parkinson 1991, Hong and Chang 2002, Scholz 1995).

**Table 15: Mathematical models used in dimensional variation analysis (DVA)**

<table>
<thead>
<tr>
<th>Mathematical Model</th>
<th>Formula</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst Case</td>
<td>$T_{\text{accum}} = \sum aT_i$</td>
<td>$T_{\text{accum}} =$ tolerance accumulation of a chain of tolerances</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T_i =$ single tolerance in a chain</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a = +/- 1$ (outer/inner bound)</td>
</tr>
<tr>
<td>Root Mean Square</td>
<td>$T_{\text{accum}} = \sqrt{\sum ((1 - n_i) c_i a_i T_i)^2}$</td>
<td>$c_i =$ inflation factor (accounts for sensitivity between tolerances in a chain)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n_i =$ mean shift ratio (applicable for processes which have a tendency to shift the mean tolerance value)</td>
</tr>
<tr>
<td>Six Sigma</td>
<td>$T_{\text{accum}} = \sum \left( \frac{T_i}{3 C_p (1 - n_i)} \right)^2$</td>
<td>$C_p =$ process capability ratio (ratio of specified tolerance range to the process capability)</td>
</tr>
<tr>
<td>Sampled Data (i.e., Monte Carlo Simulation)</td>
<td>$T_{\text{accum}} = \text{AVG}(T_j)$</td>
<td>$T_j =$ tolerance accumulation value from simulation $j$</td>
</tr>
</tbody>
</table>

There are many challenges for properly using one of these mathematical models. Worst case or root mean square models do not account for aggregation sequences, six sigma and Monte Carlo simulation is not practical when process capability information is not available (Yang et al. 2013). One DVA which can overcome many of these challenges however is a vector loop based model. A key part of creating a vector loop based DVA is the use of kinematic constraints (sometimes referred to as chains) between components which accounts for the aggregation sequence. A kinematics chain based DVA is a robust solution since it can be used on simple or complex assemblies, and reduces the number of errors associated with mathematical parameters (Sleath and Leaney 2013). In the proposed design-based strategy, the dimensional relationships between components are modelled using kinematics chains, which comes from robotics theory. Construction assemblies are assumed to be very similar to robot arms with mutual degrees of freedom. Dimensional variations are modelled parametrically and assigned to certain design variables, so that variations of critical component-features can be controlled systematically. The analogy of construction assemblies with robot arms was first used by Nahangi et al. (2014), in order to quantify incurred discrepancies in construction assemblies. It was then used to calculate the required changes for realigning defective assemblies (Nahangi et al. 2015) by solving the inverse kinematics problem.

_The derivation of the proposed kinematics chain model (shown below) was developed by Dr. Mohammad Nahangi (the co-author of the original publication for this method). Using his permission, the following derivation and proposed algorithms are included in this thesis in order to provide the necessary background information for how this method was developed. The contribution made by the author of this thesis comes in the application of this method (kinematics chain based DVA) to a modular construction project for the management of dimensional variability, as shown in Chapter 5._
An overview of the proposed methodology for kinematics chain based DVA is shown in Figure 62. Critical information integrated in the building information model (BIM) is required to develop the kinematics chain for analyzing dimensional variations. As shown in Figure 62, critical interfaces and an assembly diagram are required for identifying the critical chains for the DVA. Figure 63 illustrates the identification of the aggregation sequence and critical kinematics chains required.

Figure 62: Overview of the proposed method for kinematics chain-based dimensional variation analysis.

Figure 63: A typical structural frame and a hypothetical assembly diagram (highlighted path). The joints with variable parameters are identified in the assembly path. The position of the critical feature is therefore modelled as a function of the joint parameters and variables.

For developing the kinematics chain, a similar approach to (Nahangi et al. 2014) is employed. Transformations are then derived using the Denavit-Hartenberg (D-H) convention (Denavit 1955). While it is possible to use any consistent convention for the derivation of the transformations, the D-H
convention is a systematic method that can be programmed and integrated with other components of the proposed framework. D-H parameters represent any homogeneous transformation as a combination of four transformations, as illustrated in Figure 64.

![Figure 64: Illustration of D-H parameters for a typical connection. D-H parameters are used for developing the kinematics chain to relate the geometric relationships of an assembly.](image)

Of these four transformations (illustrated in Figure 64), two are rotational and two are translational transformations as:

\[
T_i = (\text{Rot}_{x,\theta_i})(\text{Trans}_{z,d_i})(\text{Trans}_{x,a_i})(\text{Rot}_{x,\alpha_i})
\]

\[
= \begin{bmatrix}
    c\theta_i & -s\theta_i & 0 & 0 & 1 & 0 & 0 & a_i & 1 & 0 & 0 & 0 \\
    s\theta_i & c\theta_i & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & d_i & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & c\alpha_i & s\alpha_i & a_i c\theta_i & 0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 1 & -c\theta_i s\alpha_i & a_i s\theta_i & -s\theta_i c\alpha_i & 0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

in which, $\theta_i$, $d_i$, $a_i$, and $\alpha_i$ are parameters associated with link $i$ and joint $i$ (Figure 65). $c\theta$ and $s\theta$ denote $\cos \theta$ and $\sin \theta$, respectively. The four parameters $\theta_i$, $d_i$, $a_i$, and $\alpha_i$ are also known as “link length”, “link twist”, “link offset”, and “joint angle”, respectively.

Generally, two types of joints are required for defining the characteristics of a connection between two components in an assembly (Figure 65):

1. Rotational joints: dimensional variation occurs in the form of a rotation. These joints are sometimes referred to as revolute joints.
2. Translational joints: dimensional variation occurs in the form of a translation. These joints are sometimes referred to as prismatic joints.
In order to model the variation of connections and joints, \( \theta_i \) is assumed to be the design variable used for rotational joints, and \( d_i \) is assumed to be the design variable used for translational joints. For modelling the geometric relationship between components, the appropriate joint type is identified and incorporated in the kinematics chain. Often, an amalgamated joint comprised of a combination of rotational and translational joints need to be modelled at one node in the kinematics chain (Figure 65 demonstrates this). The position of the critical interface is then represented as a mathematical function with potential variations modelled as design variables. Once the assembly and potential dimensional variations are modelled mathematically by developing the kinematics chain, the DVA of a critical feature becomes systematic and algorithmic. The assembly diagram which is integrated with the building information model identifies how various components are aggregated. The potential variation and allowable tolerances are identified and the kinematics chain can be developed. The kinematics chain identifies the position of the critical feature or connection as a function of the potential variations incorporated into the chain. The variation of the critical feature can then be modelled and analyzed for design and further considerations. A hypothetical example is shown in Figure 66.

\[
P = f(\delta, \theta)
\]

As seen in Figure 66, the position \( P \) of the hypothetical critical feature is identified as a function of the potential design variables \( \theta \) and \( \delta \): \( P = f(\theta, \delta) \). A perfectly fabricated state is associated with \( \theta = 0 \) and
δ = 0. By changing the design variables within the allowable tolerance limits for each component and comparing the resulting positions with the perfectly fabricated state, the variation of the critical feature can be analyzed for design considerations. The deviation from the perfectly fabricated state is calculated as $Dev = f(\theta, \delta) - f(\theta = 0, \delta = 0)$.

Since the dimensional variation is modelled mathematically, a wide range of analyses become possible for design of modular construction components. For instance, the rate of accumulating dimensional variability in the critical region can be calculated by differentiating the kinematics chain with respect to the design variables. Components with large contributions to the dimensional variation of a critical feature can be identified systematically, and required actions for variation control can then be planned automatically.

6.2.1. Kinematics Chains Based Dimensional Variation Analysis Case Study

The case study shown in this section relates to the same project described in detail in Chapter 5. The focus of this case study is to develop a kinematics-chain based dimensional variation analysis on the critical aggregation features of this construction assembly: the tie-in plates between modules. The DVA is carried out in order to analyze the variations of tie-in plates in 3D (a sample deviation from the actual project in one direction is shown in Figure 67).

![Figure 67: Depiction of misalignment of tie-in plates (left), and adequate alignment (right). Red arrows indicate deviation associated with misalignment, while blue arrows indicate direction of interest.](image)

6.2.1.1. Implementation of the kinematics-chain based dimensional variation analysis

Using the assembly diagram (Figure 24), kinematics chains are developed for analyzing the dimensional variation of critical features (i.e., tie-in plates). The transformation required for analyzing the tie-in plates is represented as a chain of transformations between various local coordinate systems. These local coordinate systems are located where either a deviation might occur or where a tolerance has been specified. The kinematics chain for analyzing each tie-in plate in this case study is shown in Figure 68.
Figure 68: Kinematics chain and transformations for representing critical features in the local \( \{l_i\} \) and global \( \{G\} \) coordinate systems.

As seen in Figure 68, each tie-in plate is identified with a corresponding transformation \( [T_i] = [\mathbf{0}^T][\mathbf{1}] \) consisting of a chain of transformations that relate the local coordinate system \( [l_i] \) of the critical feature \( i \) to the global coordinate system \( [G] \). The local and global coordinate systems are then related to each other as:

\[
\{P_i\} = [T_i]\{p_i\}
\]  \hspace{1cm} \text{[Eqn. 6]}

Where, \( \{P_i\} \) and \( \{p_i\} \) are the positions of critical features in the global and local coordinate systems respectively. The kinematics chain (transformation \( [T_i] \)) is the link for relating the global and local coordinate systems together. Figure 69 shows how the kinematics chain is used to relate the local and global coordinate systems.

Figure 69: Relationship between positions in the local \( (p_i) \) and global \( (P_i) \) coordinate systems. Coordinate systems are defined based on D-H convention.
As such, the dimensional variation can be analyzed from two perspectives:

1. From local to global coordinate systems: when the coordinates of critical points in a critical feature are known and the kinematics chain is used to calculate the coordinates of the critical points in the global coordinate system.
2. From global to local coordinate systems: when the coordinates of critical points in a critical feature is measured in the global coordinate system, and the local coordinates are then identified for analyzing the variabilities and comparing to the acceptable tolerance ranges.

In addition, two types of dimensional variation analyses can be performed: (1) as-designed (model-based) dimensional variation analysis, and (2) as-built (laser-based) dimensional variation analysis. Analyses are explained in the following sections.

6.2.1.2. Model Based DVA

As-designed (model-based) analysis is performed when acceptable tolerances and variations are investigated based on information provided in the building information model. In other words, this analysis identifies how acceptable tolerances propagate through fabrication processes. Typical analyses on the case study used in this thesis are shown and discussed in this section.

For investigating the case study (see Figure 68), two stages of tolerance propagation can identify the variation of the tie-in plates:

1. How the tie-in plate is installed and assembled with respect to the roof frame, and
2. How the roof frame is installed with respect to the floor frame (global coordinate system)

The variation of the tie-in plate and the impact of tolerance propagation can then be modelled by developing the kinematics chain relating the global to the local coordinate system, as shown in Figure 68. The allowable tolerance impact can then be measured in the global coordinate system in order to investigate the propagation of the tolerances.

Based on the explanation provided for the proposed methodology, the D-H parameters for the system of coordinates of the case study (Figure 69) can be defined as shown in Table 16.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$a$</th>
<th>$d$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$l_1$</td>
<td>$90 + \theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>$l_2$</td>
<td>$\theta_2$</td>
</tr>
</tbody>
</table>

In Table 16, the values of $l_1$ and $l_2$ are constant and are extracted as the as-designed dimensions from the 3D CAD drawings integrated with the building information models. $\theta_1$ and $\theta_2$ are the design variables to be analyzed for dimensional variability of the tie-in plates. As discussed earlier, allowable tolerances on each part can be modelled with one rotational and one translation joint. For simplifying the illustration of the results (Figure 70 and Figure 71), we assume that the roof frame can only rotate about the perpendicular axis to the frame plane (i.e., an axis parallel to the columns direction). In other words, the translational DOF's and rotational DOF's about other axes are ignored. Although all DOF's can be considered and modelled using the kinematics analogy explained here, this simplification is made to
better illustrate the results. Considering more DOF’s will result in highly multi-variate functions as the design variable functions, which are difficult to illustrate.

Figure 70: Typical results for model-based DVA. Deviation surfaces and contour lines for the bolt holes (BH) are illustrated. The results are shown for the tie-in plate 2 (TP2) illustrated in Figure 68.

Figure 71: Propagation of dimensional variation along the roof frame for different tie-in plates. As seen, tie-in plates further from the datum have larger impacts. Using kinematics chains, thorough analyses are possible using mathematical relationships between different components.
The analysis results show that as the tie-in plates are spaced further away from the datum, the potential deviations are prone to increase. This is due to the fact any rotations of the beam with respect to the global datum will cause linearly increasing absolute deviations in the tie-in plates (this is with respect to the global datum). The input for the model-based design includes allowable tolerances which were either provided by the contractor in the design drawings or were taken from provisions listed in AISC Code of Standard Practice (AISC 2010a). The tolerances used in the model-based DVA are for orientation and location deviations. As such, form tolerances (which are used to control the profile of a line or surface and are typically referred to in terms of straightness of an edge or flatness of a surface) are not considered in a kinematics chain based DVA.

The results of the model-based DVA show that for the critical features in this assembly (i.e., tie-in plates 1, 2, 3, 4, and 5) that the absolute deviations in 3D range between 1.6 mm and 53 mm. Interestingly, there were no specification of tolerances to control the dimensional variation of tie-in plates in this project. Conducting a model-based DVA before the geometric design and allowable tolerances were finalized would have revealed that the deviations in the tie-in plates would have created large challenges for fit-up and erection of modules on site.

6.2.1.3. Laser Based DVA

As-built (laser based) analysis is performed when the built status of a construction assembly has been acquired. Feeding the built status information (via 3D point cloud from a laser scan) into the dimensional variation analysis framework developed here provides accurate information that can be used for as-built modelling, updating the BIM and for understanding contributions of out-of-tolerances. In this type of DVA, the actual constructed dimensions are extracted from point cloud models of the construction components and the kinematics chain is then populated. The variation and deviations are therefore analyzed using the actual constructed dimensions. Typical analyses for laser based DVA in the case study are shown and discussed in this section. Using the kinematics chains that were developed for the model-based DVA, input of the actual as-built dimensions yields results which show 2D deviation surfaces for the two design variables used (Figure 72) and the propagation of dimensional variation for all tie-in plates along the length of the module (Figure 73).

![Figure 72: Results for laser-based DVA of the case study. Deviation surfaces and contour lines for the bolt holes (BH) are illustrated. The results are shown for the tie-in plate 2 (TP2) illustrated in Figure 68.](image)
Figure 73: Propagation of dimensional variation along the roof frame of the case study in different tie-in plates. As seen, tie-in plates further from the datum have higher impacts. Using kinematics chains and input from point cloud data yields output in the form of absolute deviations.

The results of the laser-based DVA are extremely close to that of the model-based DVA. In this case, input into the DVA was provided in the form of extracted dimensions from a 3D point cloud. The laser scanner used in this case study was a FARO® LS 840HE which has an accuracy of ±2 mm for the distance used (FARO 2007). PolyWorks® was used to extract the as-built dimensions through use of a simple feature which computes the point-to-point Euclidean distance between two user-selected points (Figure 74). Since this approach was performed in a semi-automated fashion, extraction of all critical dimensions was somewhat time consuming and required careful selection of points. Furthermore, minor point cloud cleaning was employed in order to reduce slight noise around edges of components in the model. The results of the laser-based DVA show that for the critical features in this assembly (i.e., tie-in plates 1, 2, 3, 4, and 5) that the absolute deviations range between 1.7 mm and 53 mm.

Figure 74: Sample extracted dimensions (in mm) from 3D point cloud model using PolyWorks®
6.2.1.4. Comparison between Model-Based and Laser-Based DVA

Comparison of deviations obtained from the model-based and laser-based methods reveal remarkably similar final deviation results for the tie-in plates (Table 17).

Table 17: Comparison of maximum deviation values for model-based and laser-based DVA

<table>
<thead>
<tr>
<th>Tie-in Plate</th>
<th>Model Based DVA: Maximum Deviation (mm)</th>
<th>Laser Based DVA: Maximum Deviation (mm)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP1</td>
<td>1.64</td>
<td>1.65</td>
<td>-1.0%</td>
</tr>
<tr>
<td>TP2</td>
<td>13.86</td>
<td>13.85</td>
<td>0.7%</td>
</tr>
<tr>
<td>TP3</td>
<td>26.96</td>
<td>26.95</td>
<td>0.4%</td>
</tr>
<tr>
<td>TP4</td>
<td>40.05</td>
<td>40.05</td>
<td>0.0%</td>
</tr>
<tr>
<td>TP5</td>
<td>53.20</td>
<td>53.22</td>
<td>-0.4%</td>
</tr>
</tbody>
</table>

The fact that the model-based DVA results closely match that of the laser-based DVA indicates the method of using kinematics chains for analyzing dimensional variability is a reliable method (since the as-built and as-designed analyses closely match each other). The low percent differences between the two DVA methods shows the original tolerances specified were adequate, since the final as-built data yielded in the same overall deviation value. In contrast with tolerance mapping, the main difference in this method lies in the fact that form variations are not modelled in kinematics chains. This is because kinematics chains assume rigid body deformation, and do not account for distorted geometries. As such, it needs to be clearly stated that a kinematics chains DVA cannot be used when excessive distortions in geometry exist. An example of this in the context of construction would be if the effects of welding distortion in a large steel structure were significantly larger than the variations associated with fit-up (i.e., position and orientation deviations).

While comparison of the two DVA approaches seem to have only slight differences, it should be noted that there were offsetting effects involved in the as-built assembly which did not match the as-design state, yet resulted in nearly identical overall deviations. For instance, the as-built column placement deviation was +5 mm from the nominal specification, while the placement of the roof frame had a deviation of -4 mm. These two deviations offset each other such that the effective out-of-plane deviation is only +1 mm. If the deviations in this case did not offset each other and in fact accumulated, then the results of the model-based and laser-based DVA would be very different. This leads into a discussion about rework minimization and adaptive fabrication process control. By modelling the kinematic systems of components in a construction assembly, it is possible to analyze each source of variability as it occurs during the progression of fabrication activities. Then using as-built data (laser scans), it is possible to quantify and determine how to optimally correct or adapt fabrication and aggregation approaches to offset deviations such that the critical features are within tolerance.

6.2.2. Kinematics Chain Based Dimensional Variation Analysis Conclusions from Case Study

The proposed method for DVA assumes rigid body deformation, where deviations are in the form of rotational and translational degrees of freedom. In comparison with other analytical DVA methods, the proposed method does not account for form variation. This case study demonstrates how to derive assembly equations in order to model the accumulation of dimensional variability. Two approaches are used in the case study in order to validate the proposed methodology. The first approach is an as-built
DVA which utilizes tolerances and the assembly configuration contained in a BIM model (thus, it is referred to as a model-based DVA). The second approach is an as-built DVA, which uses data pertaining to the actual constructed assembly in the form of point clouds from laser scans (thus, it is referred to as a laser-based DVA). The results of both DVA approaches yielded remarkably similar deviation values, with percent differences less than 1%. The limitations of the proposed methodology need to be clearly understood. The proposed method does not function adequately in cases where the rigid body deformation assumption is invalid; in other words, where there are large form deviations (e.g., bending, bowing, warping of component geometry). If large form deviations do exist, the proposed method can be adapted by modelling the local coordinate system of a component to adequately account for rigid body deformation.
7. Development of New Production-Based Strategies and Case Study Demonstration

This Chapter presents existing production-based strategies which can be adapted for the management of dimensional variability in modular construction. The two methods presented herein are: (1) selective assembly and (2) optimal assembly. The selective assembly method comes from the manufacturing industry and has been modified for use in modular construction. The optimal assembly method is a form of selective assembly which utilizes an optimization framework in order to minimize rework and or accumulation of dimensional variability. This chapter comes primarily from the following publications:


The primary author of this publication conducted the literature review, developed the detailed methodology, assisted with data collection, assisted with data analysis, and analyzed the results. The second author assisted with the analysis of results, and provided valuable feedback on the developed method. The third author assisted with data collection and analysis. The fourth and fifth authors reviewed the paper and provided valuable feedback and overall research guidance.


The primary author of this publication conducted the literature review, assisted with development of the detailed methodology, conducted data collection, and analyzed the results of the case studies. The second reader developed the detailed methodology and programmed the methods for use in case studies. The third reader assisted with data collection and analysis. The fourth and fifth readers reviewed the paper and gave valuable feedback and overall research guidance.

7.1. Selective Assembly Framework for Modular Construction

Production variability is an issue that emerges in many industries, including manufacturing. While the use of expensive equipment and precise production methods is a common approach for controlling critical variability in manufacturing, selective assembly is a valuable quality improvement tool which sidesteps the use of expensive equipment and precise production methods. Select assembly is a dimensional quality improvement tool used to determine optimal pairs of mating parts from stockpiles which are nominally identical (Asha et al. 2008, Pugh 1986, Tan and Wu 2012).

One of the simplest examples used to demonstrate selective assembly is the aggregation of shaft and sleeve parts. In this example, the critical dimensions for aggregation are isolated (e.g., the radius of each part). Assuming the critical aggregation dimension of a given part can be modelled by a normal distribution with a certain tolerance threshold (Figure 75-a), then sample distributions will vary within an allowable variability region (Figure 75-b). Then, as the stockpile for each part is populated, critical dimensions can be quantified and parts organized into bins which enable optimal aggregation. As seen in Figure 75-c, if four randomly selected sleeve and shaft parts are selected from their respective stockpiles, it is clear that based on their respective dimensional distributions, that an optimal set of best-fit pairs does exist.
Selective assembly has been used in the manufacturing industry for years. Typical applications include the aggregation of ball bearings and joints (Mansoor 1961), production of pistons and cylinders (Pugh 1986), production of scroll compressor shells (Thesen and Jantayavichit 1999), and is still used in the aggregation of engines, transmissions, and compressors (MARPOSS 2016). The use of statistical selective assembly was introduced as a way of optimizing aggregation based on the dimensional distribution of mating components, which for a number of years focused on parts with similar dimensional attributes that follow the Normal distribution (Pugh 1986). In order to generalize statistical selective assembly, researchers began introducing novel grouping methods to reduce the dependency on the constraint that parts must have similar distributions (Chan and Linn 1998, Pugh 1992). The efforts to generalize statistical selective assembly opened a new area of research surrounding optimal binning strategies for a range of applications. Binning strategies can either be designed before production, during production, or post-production (note that in the case of post-production, parts have been manufactured, but mating parts still await aggregation). Since selective assembly encompasses measuring part dimensions after they have been produced, it is often preferable to design the binning strategy after the design stage, in a prototypical manner. Selective assembly is typically better suited for batch production rather than mass production, since the extra steps to utilize selective assembly in mass production can create congestion and bottlenecks, which is less likely to occur in batch production due to the lower production rate (Tan and Wu 2012).

While selective assembly has been traditionally used in manufacturing applications, reasons for why it can also be used in modular construction include (1) the ability to achieve tight tolerance requirements without the use of a rigorous tolerance design, and (2) the use of production techniques currently employed in stick-built construction rather than adoption of highly precise equipment which can be very
expensive. Modular construction also often resembles batch production manufacturing more so than mass production, making it very favorable for application of selective assembly.

Since selective assembly relies on sorting pairs of components based on the dimensional distribution of features that make up the physical interface between components, it is easier to develop an optimal strategy using as-built data rather than theoretical predictions. As such, the proposed use of selective assembly is applied after production has finished (i.e., parts are manufactured, but await aggregation). The proposed framework (Figure 76), is comprised of three key steps: (1) identify critical interfaces between components and tolerances, (2) calculate the minimum number of bins required, and (3) organize parts into bins based on a binning strategy.

![Figure 76: Framework of proposed use of selective assembly in modular construction](image)

### 7.1.1. Isolating the Critical Interfaces and Determining Aggregation Tolerances

The critical interfaces between components are the physical regions or points on components in direct contact with each other. The dimensional variation of features that make up the critical interfaces on the as-built components must be determined. Laser scanning is proposed for this purpose based on its ability to yield rich and accurate data (Bosché 2010). One of the most challenging aspects of applying selective assembly lies with determining tolerances that govern adequate component aggregation. For this purpose,
three approaches can be taken: (1) measure variation on prototyped pairs (2) use previous experience or benchmark tolerances, or (3) conduct a systematic tolerance analysis.

### 7.1.2. Calculating the Minimum Number of Bins Required for the Binning Strategy

Before calculating the minimum number of bins required in a binning strategy, it is important to ensure that all components are dimensionally compatible based on the allowable aggregation tolerances. For checking dimensional compatibility, each isolated component must have at least one possible mating component, otherwise it will not be able to aggregate properly. In the event that dimensional incompatibility exists (i.e., the geometry variability of a particular component is such that it cannot connect with any other component), major rework or component scrapping is required (depending on which option is least expensive or time consuming). Finally, the minimum number of required bins ($B_{\text{min}}$) is calculated using:

$$B_{\text{min}} = \text{ceiling}(\frac{\text{Var}_{\text{max}}}{T})$$  \hspace{1cm} [Eqn. 7]

where $\text{Var}_{\text{max}}$ is the maximum measured interface variation between all possible pairs, $T$ is the allowable aggregation tolerance and $\text{ceiling}$ is a function that rounds up to the nearest whole integer. Although the minimum required bins is calculated here, a larger number may be used, depending on desired accuracy. In general, increasing the number of bins decreases assembly variations, but also increases the likelihood of having surplus parts, disproportional bin populations, and decreased overall effectiveness (Thesen and Jantayavichit 1999). Selective assembly in manufacturing usually aims to achieve low assembly deviations since most mating parts are moving (e.g., pistons in automotive engines). However for modular construction, the level of assembly deviations only needs to ensure adequate component aggregation, and parts are typically not designed to experience movement after aggregation. As such, it is preferable to minimize the number of bins for selective assembly in modular construction.

### 7.1.3. Developing the Binning Strategy

The next step for applying selective assembly is determining the binning strategy, which outlines how components are organized into bins and how bins are matched together. Two common ways to partition the dimensional attributes of bins are (1) equal dimensional width or (2) equal probability. Equal width partitioning divides the total interface variability equally between bins, while equal probability partitioning ensures that each bin has equal populations of components (Mease et al. 2004). Matching criteria defines how components are matched between bins. Traditional methods include one-to-one matching (each bin has exactly one other matching bin), or one-to-three matching (each bin has one matching bin but can pull from adjacent bins to the matching bin if need be) (Thesen and Jantayavichit 1999). The methodology for developing the a binning strategy is outside of the scope of this thesis, however based on the results of the case study, the authors recommend using equal probability bin partitioning and one-to-one matching due to its simplicity. Since modular construction typically has a lower number of mating pairs of components than in manufacturing, and since part inventories are not common, everything should be matched on each project. These factors lend themselves to have equal probability bins (avoiding surplus parts), and one-to-one matching (to ensure that every part is matched). After the binning strategy has been determined, components between matched bins can be randomly aggregated.
7.2. **Selective Assembly in Modular Construction Case Study**

A small scale bridge was originally designed and built by an undergraduate team at the University of Waterloo for a steel bridge competition held by the AISC (American Institute of Steel Construction) and ASCE (American Society of Civil Engineering). In accordance with the competition rules, the bridge is approximately $6 \text{ m}$ long and is comprised of assemblies which are $305 \text{ mm}$ by $152 \text{ mm}$ by $102 \text{ mm}$. The bridge is comprised of hollow steel section members, and has five types of assemblies or modules that are bolted together (Figure 77). As part of a modularization strategy, 24 assemblies were designed as interchangeable top and bottom pairs (A2 and A3 in Figure 77c). During construction, selective assembly was utilized as an approach for mitigating the impact of fabrication error (accumulating effects of cutting, milling, fit-up, measurement, welding distortion, and inspection). Selective assembly was applied for the aggregation of the top and bottom pairs.

![Figure 77](image)

**Figure 77**: (a) Fully assembled steel bridge, (b) deconstructed single truss, (c) assembly diagram of single truss showing the five main assembly types.

7.2.1. **Critical Aggregation Interfaces**

For each top and bottom assembly pair (A2 and A3), there are three direct contact points that make up the aggregation interfaces. Of these interfaces, two critical dimensions are extracted: (1) an angular dimension, $\Theta$, and (2) a linear dimension, $X$, as illustrated in Figure 79. Aggregation between top and bottom pairs is assumed to rely on the compatibility of these critical dimensions.

![Figure 78](image)

**Figure 78**: Repetitive assembly pair in case study, with critical interface dimensions as $\Theta$ (yellow) and $X$ (red)
The as-built data of the critical interface dimensions were obtained by conducting coordinate probing using a laser scanner (FARO® Edge Arm). This device has an accuracy of 0.024 mm for the working length employed (FaroARM 2014). Coordinate probing was used since each part can be reduced from its as-designed model into a centerline model, and then to a series of critical coordinates at the interface points (Figure 79).

![Diagram](image)

**Figure 79:** Process of extracting critical coordinates from original as-designed model

Selective assembly in this case study depended solely on the centerline alignment of the physical interfaces for top and bottom pairs. Rework during aggregation is assumed to be constrained to shimming (i.e., extending the length of a member at an interface) and grinding (i.e., reducing the length of a member at an interface). This type of rework is much less intensive than having to cut, realign and weld a member into proper position (which does not necessarily eliminate the need for grinding and shimming at interfaces). After critical interface coordinates were obtained, populations of top and bottom components were sorted based on the distribution of each critical dimension (Table 18).

**Table 18: Sorted populations of top and bottom components based on distribution of critical aggregation dimensions**

<table>
<thead>
<tr>
<th>Dim Θ (degrees)</th>
<th>Dim X (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>A2 (top)</td>
</tr>
<tr>
<td>-----</td>
<td>---------</td>
</tr>
<tr>
<td>10</td>
<td>30.67</td>
</tr>
<tr>
<td>6</td>
<td>31.62</td>
</tr>
<tr>
<td>9</td>
<td>31.67</td>
</tr>
<tr>
<td>2</td>
<td>31.90</td>
</tr>
<tr>
<td>5</td>
<td>31.94</td>
</tr>
<tr>
<td>7</td>
<td>32.29</td>
</tr>
<tr>
<td>4</td>
<td>32.32</td>
</tr>
<tr>
<td>12</td>
<td>32.51</td>
</tr>
<tr>
<td>11</td>
<td>32.97</td>
</tr>
<tr>
<td>8</td>
<td>33.07</td>
</tr>
<tr>
<td>3</td>
<td>33.51</td>
</tr>
<tr>
<td>1</td>
<td>33.60</td>
</tr>
</tbody>
</table>

Nominal Dim – 32.9 Nominal Dim – 396.88
7.2.2. Binning for Critical Angular Dimension

Using the sorted populations of top and bottom components, the first iteration of binning was carried out for the critical angular dimension, $\Theta$ (Figure 78). Since the allowable tolerance for this dimension was not specified in the design, it was determined through prototyping. The maximum angular discrepancy between $\Theta$ values of top and bottom components for a successfully aggregated pair was measured as 2.72° from testing all possible component pairs. Since all possible top and bottom pair combinations result in angular discrepancies equal to or less than 2.72°, a single bin can be used for each top and bottom population for aggregation based on $\Theta$. As such, a binning strategy was not required for the critical angular dimension, since random aggregation can proceed between all top and bottom pairs.

7.2.3. Binning for Critical Linear Dimension

Using the sorted populations of top and bottom components, the second iteration of binning was carried out for the critical linear dimension, $X$ (Figure 78). The allowable tolerance for this dimension was specified in the design as +/- 1/16” (1.588 mm), since the bolt hole diameters are 1/16” (1.588 mm) larger than the bolt diameter used. As such, the tolerance range is equal to 1/8” (3.175 mm) to account for the case where an upper bound deviation is matched with a lower bound deviation. One final check was performed before determining the minimum number of bins to ensure that no components would be rejected based on dimensional incompatibility: for every bottom component there is at least one top component that does not exceed the allowable tolerance, and for every top component there is at least one bottom component that does not exceed the allowable tolerance. This check yielded no dimensionally incompatible components. Using the largest deviation between critical linear dimensions for all possible pairs (6.41 mm) and the tolerance (3.175 mm), the minimum number of bins was calculated using [Eqn. 7]: $Bins_{min} = \text{ceiling} \left( \frac{6.41 \text{ mm}}{3.175 \text{ mm}} \right) = 3$. Since the minimum number of bins is greater than 1, random aggregation cannot proceed between all top and bottom pairs, and a binning strategy is required. The binning strategy uses a one-to-one strategy, where every bin for top components has exactly one matching bin for bottom components. Furthermore, equal probability partitioning is employed to avoid having surplus components. Since the combined width of all bins (3* tolerance = 9.525 mm) is larger than the largest deviation between all possible pairs (6.41 mm), several bin arrangements are possible. A script was compiled in MATLAB® to find that there 8 possible bin arrangements (Figure 80).

![Figure 80: Bin populations for all possible bin arrangements based on the critical linear dimension](image-url)
7.2.4. Selection of Optimal Bin Arrangement

While each of the 8 possible bin arrangements yield dimensionally compatible component pairs, there exists a specific arrangement with a statistically optimal amount of rework avoidance. There are two approaches for selecting the optimal bin arrangement in this case study: (1) absolute rework avoidance or (2) least expected rework. Absolute rework avoidance finds the pairs of components in each bin with the largest possible deviation. Then, these values are summed together and used for comparison to find the bin arrangement with the least overall deviation (Table 19). Least expected rework associates a probability of selecting a random pair with its deviation value. As such, this approach finds the most probable deviation to be expected between pairs in each bin. The expected deviations for each bin are summed together and compared to find the bin arrangement with the least expected overall deviation (Table 20). As seen from both approaches, bin arrangement 1 is optimal since it has the least absolute rework and least expected rework. As such, it was selected for as the bin arrangement for this case study.

Table 19: Maximum deviations for critical linear aggregation interface for all possible pairings in each bin arrangement (all deviations in mm)

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bin 1</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>3.11</td>
<td>3.11</td>
<td>3.11</td>
<td>3.11</td>
</tr>
<tr>
<td>Bin 2</td>
<td>2.04</td>
<td>1.83</td>
<td>1.51</td>
<td>1.48</td>
<td>1.91</td>
<td>1.70</td>
<td>1.38</td>
<td>1.35</td>
</tr>
<tr>
<td>Bin 3</td>
<td>1.41</td>
<td>2.58</td>
<td>2.79</td>
<td>3.11</td>
<td>1.41</td>
<td>2.58</td>
<td>2.79</td>
<td>3.11</td>
</tr>
<tr>
<td>Sum</td>
<td>3.93</td>
<td>4.89</td>
<td>4.79</td>
<td>5.07</td>
<td>6.43</td>
<td>7.39</td>
<td>7.29</td>
<td>7.57</td>
</tr>
<tr>
<td>Average</td>
<td>1.31</td>
<td>1.63</td>
<td>1.60</td>
<td>1.69</td>
<td>2.14</td>
<td>2.46</td>
<td>2.43</td>
<td>2.52</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.64</td>
<td>0.87</td>
<td>0.95</td>
<td>1.08</td>
<td>0.71</td>
<td>0.58</td>
<td>0.75</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Table 20: Expected deviations for critical linear aggregation interface for all bin arrangements (all deviations in mm)

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bin 1</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>1.56</td>
<td>1.56</td>
<td>1.56</td>
<td>1.56</td>
</tr>
<tr>
<td>Bin 2</td>
<td>0.79</td>
<td>0.72</td>
<td>0.65</td>
<td>0.66</td>
<td>0.67</td>
<td>0.59</td>
<td>0.51</td>
<td>0.52</td>
</tr>
<tr>
<td>Bin 3</td>
<td>0.85</td>
<td>1.01</td>
<td>1.08</td>
<td>1.14</td>
<td>0.85</td>
<td>1.01</td>
<td>1.08</td>
<td>1.14</td>
</tr>
<tr>
<td>Sum</td>
<td>2.11</td>
<td>2.20</td>
<td>2.21</td>
<td>2.28</td>
<td>3.07</td>
<td>3.15</td>
<td>3.14</td>
<td>3.21</td>
</tr>
<tr>
<td>Average</td>
<td>0.70</td>
<td>0.73</td>
<td>0.74</td>
<td>0.76</td>
<td>1.02</td>
<td>1.05</td>
<td>1.05</td>
<td>1.07</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.16</td>
<td>0.22</td>
<td>0.25</td>
<td>0.28</td>
<td>0.38</td>
<td>0.40</td>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>

7.2.5. Discussion about Results

The binning strategy in this case study was conducted by isolating repetitive pairs of component in a modular assembly, classifying them by distinct dimensional attributes and sorting them into bins that yielded component pairs that could be correctly aggregated. Laser scanning was employed for as-built data collection primarily for its ease of capturing rapid and accurate data. Currently, the method of determining critical aggregation interfaces, and tolerances is manual and requires proper user judgement. Although the critical angular dimension was not directly used in this case study, it should be noted that a binning strategy needs to incorporate all critical dimensions. For instance, the ID numbers shown in Table 18 do not match up for a given component between each dimension. The distribution for the angular dimension does not match with the distribution for the linear dimension. This means that as the number of
critical dimensions increases, it becomes increasingly more difficult to decrease the minimum number of possible bins.

As a result of the binning strategy shown in this case study, the component aggregation of mating pairs in the modular steel bridge proceeded with no major rework or wasted components. Before applying selective assembly, the fabrication team of this bridge attempted to apply random aggregation of top and bottom components without a binning strategy. Due to compounding effects of fabrication error, there were two instances of extensive rework, where members had to be cut, realigned and welded into proper alignment. While the exact quantitative impact of this rework is unknown, the team found the results of binning components to yield component pairs that could be successfully aggregated.

The primary limitation of this case study lies in the assumption that component aggregation is solely based on the three direct contact points between top and bottom components (Figure 78). In this regard, the binning strategy finds optimal pairs locally, but does not consider the impact that a given assembly has on its adjacent neighbors (i.e., assemblies on either side).

7.3. Optimal Assembly Framework for Modular Construction

This section presents a framework which builds off of selective assembly by determining the optimal arrangement of as-built components being aggregated into an assembly. The optimal assembly framework presented herein is an automated method which uses as-built data collected by a laser scanner in the form of a point cloud. When interchangeable modular segments are being installed and erected on construction sites, there are multiple ways to aggregate the components. Finding an assembly plan (i.e., the way in which components are arranged together in an assembly) with minimum dimensional deviation from the as-designed status is key to minimizing rework related to dimensional variability. In this way, the dimensional deviations are systematically controlled and therefore the rework associated with such deviations are minimized, which is the key contribution of this framework.

The derivation of the proposed optimal assembly method (shown below) was developed by Dr. Mohammad Nahangi (the co-author of the original publication for this method). Using his permission, the following derivation and proposed algorithms (Chapter 7.3.2, Chapter 7.3.3 and Chapter 7.3.4) are included in this thesis in order to provide the necessary background information for how this method was developed. The contribution made in this thesis is through the application of this method (optimal assembly framework) to two modular construction projects for the management of dimensional variability, as shown in Chapter 7.3.5 and Chapter 7.3.6.

7.3.1. Terminology Used in Proposed Framework

A tie-in point is a term that is typically used in the context of pipe network assemblies (e.g., oil and gas industry) to describe the location and specification of a prefabricated pipe network or module that connects into another pipe network, vessel or other equipment. The terms “tie-in point” and “control point” are used to describe corresponding critical points that govern the aggregation (mating and joining) of modular components: \( \{\text{Comp} 1, \text{Comp} 2, \ldots, \text{Comp} N\} \) for serial-parallel assemblies or volumetric assemblies (Figure 81). For a given modular component (e.g., \( \text{Comp} N \)), the tie-in points refer to critical interface points defined by the component and the control points refer to the corresponding interface points on the assembly (where the component will be directly mated to).
Figure 81: Tie-in points and control points for: (a) serial/parallel assemblies, and (b) volumetric assemblies.

7.3.2. Methodology

Laser scanning is used for data acquisition in the form of point clouds which are then imported as an input to the processing framework for calculating the optimum assembly plan. As seen in Figure 82, the proposed framework has three primary steps: (1) analyzing modular segments locally, (2) matching the segments globally, and (3) optimizing the assembly plan by minimizing the resulting dimensional deviations. By minimizing dimensional deviations, aggregation and erection costs are saved and schedule delays are minimized. Each step is discussed extensively in the following sections.

The global control points are first initialized from the design model. Then, critical interface points (defined in Chapter 1.6) are extracted from the as-built status (point clouds) manually from automatically acquired point clouds using sophisticated point cloud analysis software packages. As previously explained, critical points change based on the type of module being investigated. In the proposed framework, the extraction of critical interface points is the only manual process. However, the key objective of this research is to automatically plan the optimum aggregation of modular components because this is difficult or impossible to do manually. For the purpose of the critical points extraction, an auxiliary software for point cloud manipulation is employed. After capturing the point cloud, the assemblies are imported into an auxiliary software and then manipulated to get the coordinates of critical interface points in the local coordinate system in which they were scanned. These points are stored in an array for further manipulation and required calculations.

Once tie-in and control points for each segment are identified, the required transformation from local to global coordinate system must be calculated. A similar approach suggested by (Kim et al. 2013, Nahangi
and Haas 2014) is used here for calculating this transformation. This transformation from local to global coordinate system is denoted by $[T_{lG}]$, as shown in Figure 83. $[T_{lG}]$ is then applied as follows:

$$\{P\}_G := [T_{lG}] \{P\}_l$$

[Eqn. 8]

In which, $\{P\}_l$ is the point set that represents the tie-in points in the local coordinate system, and $\{P\}_G$ is the point set in the global coordinate system that matches $\{P\}_l$.

Figure 83: Parameters for transformation of modular components from local to global coordinate system.

As a homogeneous transformation, $[T_{lG}]$ consists of a rotational ($R$) and a translational ($T$) part. For calculating the rotation and translation matrix Principal Component Analysis (PCA) is used. PCA aligns the points by aligning the principal axes of the point sets. For the PCA alignment, the first step is to calculate the covariance matrix as follows:

$$K = \text{cov}(p) = \frac{1}{n} \sum_{i=1}^{n} (p_i - \bar{p})(p_i - \bar{p})^T$$

[Eqn. 9]

Where, $\bar{p}$ is the centroid of the point set $p$, and $n$ is the number of points in the point set. Once the covariance is calculated, its eigenvector is calculated using single value decomposition (SVD) as follows:

$$K = \mathbf{U} \Sigma \mathbf{U}^T$$

[Eqn. 10]

Where, $\Sigma$ is a diagonal matrix, and $\mathbf{U}$ is the eigenvector.

The eigenvector for both point sets in the local and global coordinate systems is calculated using [Eqn. 9] and [Eqn. 10]. $\mathbf{U}_l$ and $\mathbf{U}_G$ are the eigenvectors for the point sets in the local and global coordinate systems, respectively. The rotation matrix $R$ is therefore calculated as follows:

$$R = \mathbf{U}_G \times \mathbf{U}_l^{-1}$$

[Eqn. 11]

and the required translation $T$ is calculated as follows:

$$T = \bar{p}_G - R \times \bar{p}_l$$

[Eqn. 12]

In which $\bar{p}_G$ and $\bar{p}_l$ are the centroids of the tie-in points in the global and local coordinate systems, respectively. The procedure for finding the transformation required for matching the tie-in points from local to the global coordinate system is summarized in Algorithm 1. In the following section, the optimization strategies for assembly planning are explained.
Algorithm 1: Transformation for matching modular segments from local to global coordinate system

**Input(s):** Tie-in points in local and global coordinate systems: \( \{P_i\}_l \) and \( \{P_i\}_G \) (Figure 83)

**Output(s):** The transformation from local to global coordinate system: \( [T]_lG \)

For each pair \( \{P_i\}_l, \{P_i\}_G \)

1. Calculate \( K_l = \text{cov}(\{P_i\}_l) \) and \( K_G = \text{cov}(\{P_i\}_G) \)  [Eqn. 9]
2. Calculate \( U_l \) and \( U_G \) using SVD  [Eqn. 10]

Calculate \( R \)  [Eqn. 9]

Calculate \( T \)  [Eqn. 10]

End for

Return \( [T]_lG = \begin{bmatrix} R_{3 \times 3} & T_{3 \times 1} \\ 0 & 1 \end{bmatrix} \)

The next step for optimum assembly planning is to find the best order for aggregating and erecting modular components. For this purpose two strategies are proposed: (1) minimizing rework of the final assembly by finding the sequence of components for each slot that minimizes the dimensional deviation at the end point of assembly, and (2) avoiding rework that finds the best component for each segment that minimizes the dimensional deviation of critical points for each slot. For a serial-parallel assembly, both optimization strategies are applicable depending on the critical metric measured for minimizing the dimensional deviation and variability. However, for a volumetric assembly, approach I is not applicable. The reason is that in a volumetric assembly, the variability of components in various slots are independent; while the dependency of the mating parts (i.e., adjacent assemblies) is the key factor that relates the slots in the sequence of the assembly plan. These strategies are briefly illustrated in Figure 84 and extensively described in the following sections.

**Figure 84:** Overview of the optimization strategies for assembly planning. Approach I: minimizing final rework, this approach finds all potential assembly plans and picks the plan with least resulting deviation. Approach II: avoiding rework occurrence, this approach minimizes the resulting deviation for each segment. Optimization steps for each framework are highlighted.
7.3.3. Optimization Approach 1: Minimizing Overall Dimensional Variation

The first strategy takes the critical points for the assembly from the designed drawings, existing in the building information model (BIM), and returns the best combination that minimizes the dimensional deviation at the end of the assembly. As shown in Figure 85, all possible combinations of assembly plans are evaluated. For each assembly plan, the components are matched from the start assembly point using the algorithm explained in the previous section. The corresponding assembly for each slot is transformed to the global coordinate system by the same transformation calculated previously. The tie-in points for the next slot are updated as the critical points from the previous component matched to the previous slot. This procedure is continuously performed for all of the slots until the assembly is completed. Finally, the dimensional deviation is identified by calculating the deviation between the as-built and as-designed statuses. This dimensional deviation is stored in the same array that the assembly is stored. Once the dimensional deviation is calculated for all possible assembly plans, the assembly plan associated with the minimum dimensional deviation is extracted as the optimum assembly plan. This way, the rework associated with aligning the final critical region is minimized. Therefore, the related labor and equipment costs are minimized assuming the rework and dimensional deviation are directly proportional.

![Diagram](image)

Figure 85: Hypothetical example for optimizing the assembly plan using approach I (minimizing final rework). The assembly plan associated with the minimum $\delta$ is picked as the plan that minimizes the dimensional deviation. $\{l\}$ and $\{G\}$ are local and global coordinate systems respectively.

The explained strategy for optimum assembly planning is summarized as a pseudo-code in Algorithm 2. Related parameters and metrics are illustrated in Figure 85.
Algorithm 2: Final rework minimizing strategy for optimum assembly planning

**Input(s):** \{\(P_i\)_\(l\) and \{\(P_i\)_\(G\)\}\}
- \{\(P_i\)_\(l\)\}_\(\text{start}\): tie-in points at the start point of the assembly
- \{\(P_i\)_\(l\)\}_\(\text{end}\): critical points the end point of the assembly

**Output(s):** The optimum assembly plan \(AP_{opt}\) that minimizes the rework incurred

- Build up all possible assembly plans → \([AP]\)
- For each Slot
  - Match \{\(P_i\)_\(l\)\} and \{\(P_i\)_\(G\)\} for the current \(Comp\) as follows:
    - \([T]\) ← Algorithm1 \{(\(P_i\)_\(l\) and \(P_i\)_\(G\))\}
    - \{\(P_i\)_\(G\)\} = \([T]\){\(P_i\)_\(l\)}
    - \{\(P_{i+1}\)_\(G\)\} = \{\(P_i\)_\(G\)\}
- **Next Slot** in the assembly plan
  - \(\delta = \|\{P_i\}_\text{end} - \{P_i\}_\text{end}\|\)
- End for

**Return** \(\delta_{min}\) and the associated \(AP\) as \(AP_{opt}\)

### 7.3.4. Optimization Approach 2: Local Rework Avoidance

This strategy finds the most suitable components for each slot by minimizing the incurred dimensional deviation. First, transformations for matching the components to the as-designed state is calculated for the slot being investigated using Algorithm 1. As shown in Figure 86, the most suitable component (closest to the as-designed status) is assigned to the current slot. The as-built status is then updated by calculating and updating the tie-in points for the next slot. This procedure is performed until the assembly is completed. Rather than comparing the final critical component to the as-designed status and minimizing the incurred error for that region (explained as Approach I and summarized in Algorithm 2), this strategy avoids error accumulation as the components are aggregated. In the approach explained, allowable geometric tolerances can also be controlled at each critical location (i.e. the tie-in points), and if any components are identified as being “out-of-tolerance” it will be marked for realignment or replacement. Required realignment actions can be calculated using the approach presented by (Nahangi et al. 2015a).

![Figure 86: Hypothetical example for optimizing the assembly plan using approach II (rework avoidance). For each segment, the component which is closest to the as-designed status is picked from the remaining components until the assembly plan is complete (\(\delta_i\) is the minimum deviation for the remaining components). \{\(l\)\} and \{\(G\)\} are local and global coordinate systems respectively.](image-url)
The rework avoidance strategy for optimum assembly planning is summarized as a pseudo-code in Algorithm 3. Required parameters and metrics are illustrated in Figure 86. The subscript \( l \) and \( G \) refer to the local and global coordinate systems, respectively.

**Algorithm 3: Rework avoidance strategy for optimum assembly planning**

| Input(s): | \( \{P_i\}_l \) and \( \{P_j\}_l \) for all components \( \{\text{Comp}\} \) \( \{P_i\}_G \) and \( \{P_j\}_G \) for all Slots |
| Output(s): | The optimum assembly plan \([AP]\) that minimizes deviation occurrence for each Slot |
| \( AP \leftarrow \left[ \right] \) |
| For each Slot |
| For the remaining \( \text{Comp} \)'s for assembly |
| Match \( \{P_i\}_l \) and \( \{P_i\}_G \) as: \( [T] \leftarrow \text{Algorithm1} \ \{(P_i)_l \text{ and } (P_i)_G\} \) \( \{P'_j\}_G = [T]\{P_j\}_l \) \( \delta = \|[P'_j]_G - \{P_j\}_G\| \) |
| End for |
| Assign the \( \text{Comp}^* \) associated with \( \delta_{\text{min}} \rightarrow \text{Current Slot} \) |
| Update the assembly plan: \([AP] \leftarrow [AP] \ \text{Comp}^*\) |
| Remove \( \text{Comp}^* \) from \( \{\text{Comp}\} \) |
| Update the tie-in points in the assembly: \( \{P_{i+1}\}_G = \{P'_j\}_G \) |
| Next Slot |
| End for |
| Return \([AP]\) |

The methodology explained, along with the required metrics and functions, is programmed and implemented in MATLAB®. The processing time as the key verification metric is benchmarked on a computer with a 3.7 GHz×12 processing unit and a 32 GB RAM. For the purpose of validation, two case studies are performed: a small scale modular bridge (case study I), and a full scale concrete panel assembly (case study II). Case study I is an example of a serial-parallel assembly, and Case study II is an example of a volumetric assembly. Two laser scanners are employed for data acquisition of the as-built assembly status. Both case studies and the results of the assembly planning are fully explained in the following sections.

### 7.3.5. Optimal Assembly Case Study 1: Modular Steel Bridge

This case study demonstrates how optimal assembly can be used to aggregate the same bridge examined using the selective assembly approach. This bridge was designed in three types of modules (nine modules in total) that are bolted together into a parallel system. The bridge contains six Type I modules (Figure 87c) and one Type II (Figure 87d). The third module type (legs at the ends) was not considered in this case study for simplicity.
Figure 87: Case Study I. (a) actual image, (b) 3D model the assembled bridge, dimensions of modules type I (c) and II (d). Dimensions are in mm.

The bridge was aggregated into Type I and Type II modules as shown in Figure 88. Although the six Type I modules should be theoretically interchangeable, fabrication process capabilities introduced geometric variability, impacting the degree of interchangeability. As a result, depending on the specific module assembly plan, gaps can be introduced between interfaces, causing the bridge to have different overall lengths.

Figure 88: Assembly planning for Case I. (a) 6 interchangeable modules of type I are scanned and critical points are extracted (see Figure 89). (b) Critical points in the 3D models are extracted at each slot. (c) Module type II is installed in the Slot 4, which makes a constraint in the assembly planning and the optimization problem involved.
To collect accurate as-built data (Figure 88), a laser scanner (FARO® Edge Arm) and PolyWorks® were used (FAROARM 2014, PolyWorks 2015). The laser scanner used in this case study can probe an object with an accuracy between 0.024 mm to 0.064 mm depending on the given working length (summarized in Table 21). In addition, a laser line probe with an accuracy of 0.025 mm was utilized to create point clouds quicker than probing individual points. However it should be noted that probing the center of each interface would have sufficed for the data collection in this case study. Point clouds of the entire assemblies are acquired only for clarification purposes. On the other hand, the coordinates of the critical interface points from the as-designed status are also extracted for calculating the dimensional deviation from the as-built status and planning for the optimum assembly. Figure 89 shows typical examples for the extraction of critical points for assembly Type I in this case study.

**Table 21: FARO® Arm performance comparison**

<table>
<thead>
<tr>
<th>Working Length (m)</th>
<th>1.8</th>
<th>2.7</th>
<th>3.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>FARO® Edge (mm)</td>
<td>0.024</td>
<td>0.029</td>
<td>0.064</td>
</tr>
</tbody>
</table>

![Reference frame for assembly (a), local coordinate system (b)](image)

**Figure 89:** Critical points extraction for the 3D model (a), and laser scanned point cloud representing the as-built status (b).

Once the critical point are extracted, the implemented optimization approaches I and II (explained in the methodology) are applied on the modular bridge components. Both approaches are applicable on the modular bridge as the dimensional deviation at each slot or the final segment may be critical. The results of the optimization approach I are shown in Figure 90. 720 possible assembly plans are considered and the resultant deviation at the final segment is measured by comparing the resulting critical points to the design drawings. As seen in Figure 90, the dimensional deviation changes from 12.9 mm to 68.7 mm for various assembly plans.
Figure 90: The dimensional deviation for all feasible assembly plans for Case Study I. Number of possible assembly plans equals the permutation of 6 interchangeable modules (6! = 720). The threshold value for acceptable deviation is considered 15 mm and all 51 assembly plans are identified as red points. The best five assembly plans are reported in Table 22.

A threshold value of 15 mm is considered as the acceptable variation limit (tolerance). As such only 51 assembly plans are deemed acceptable based on the variation limit. Of these 51 assembly plans, the 5 best plans are reported in Table 22, where the most suitable assembly plan can be chosen based on existing constraints in the fabrication plant which were not considered in the optimization step. The most suitable assembly plan using approach II for the optimization step is also reported in Table 22.

Table 22: Summary of the result for assembly planning applied on Case I.

<table>
<thead>
<tr>
<th>Optimization strategy</th>
<th>Assembly plan</th>
<th>Deviation (mm)</th>
<th>Processing time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach I</td>
<td>AP1: {6,1,2,4,7,5,3}</td>
<td>12.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AP2: {5,6,2,4,1,3,7}</td>
<td>12.95</td>
<td>80.54</td>
</tr>
<tr>
<td></td>
<td>AP3: {6,5,2,4,7,3,1}</td>
<td>12.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AP4: {6,3,2,4,1,5,7}</td>
<td>12.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AP5: {6,2,1,4,7,5,3}</td>
<td>13.15</td>
<td></td>
</tr>
<tr>
<td>Approach II</td>
<td>AP: {6,2,1,4,7,5,3}</td>
<td>13.15</td>
<td>0.51</td>
</tr>
</tbody>
</table>

One notable observation is the time effectiveness of the optimization approach II. As seen in Table 22, the processing time for the optimization approach II is 0.51 seconds which is significantly faster than the total processing time for the approach I (80.54 s). If it is preferable to reduce the overall dimensional deviation of the bridge by choosing an assembly plan which has the least amount of gaps between components, and an overall bridge length which matches the design length best, then approach I should be employed. If however, the overall dimensional deviation of the bridge is less important as minimizing the overall amount of rework associated with aggregation, then approach II should be employed. Time-related aspects of the case study I are summarized in Table 23.
**Table 23: Summary of the processing time at each step for the case study I**

| Data collection | ~ 15-20 minutes to scan each module  
|                 | ~ 1-2 minutes to probe the critical points of each module |
| Processing time | ~ 80 seconds for approach I  
|                 | < 1 second for approach II |
| Total           | ~ 16 minutes (approach I)* |

*Probed points are used for critical interface points*

The benefit of using either optimization approach I or II can be expressed in terms of rework minimization or minimizing overall dimensional variation. Assuming that the overall deviation associated with aggregating the components in the bridge is required to be less than 15 mm without incurring rework, then the probability of rework is equal to 92% (since 669 of the total 720 assembly plans result in an overall deviation greater than 15 mm. Furthermore, assuming the average amount of rework to be 4 hours for cutting, grinding and re-welding of components to ensure adequate assembly geometry, then the time savings is approximately equal to 3.4 hours, or 86% of the total time required for rework.

### 7.3.6. Optimal Assembly Case Study 2: Precast Concrete Panel Aggregation

This case study covers the installation of precast concrete panels in steel frames for the case study shown in Chapter 5. The precast concrete panels were cast into sixteen light-gage steel frames (Figure 91b) and then aggregated into a steel floor frame (Figure 91c). Five different sizes of concrete panels were used; each 102 mm thick, 2559 mm long and vary in width. The dimensions of the concrete panels used in this case study are summarized in Table 24.

**Table 24: Dimensional properties of the concrete panel types used for Case II**

<table>
<thead>
<tr>
<th>Concrete panel type</th>
<th>Dimensions (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type a</td>
<td>1367×2559</td>
</tr>
<tr>
<td>Type b</td>
<td>1497×2559</td>
</tr>
<tr>
<td>Type c</td>
<td>1393×2559</td>
</tr>
<tr>
<td>Type d</td>
<td>1058×2559</td>
</tr>
<tr>
<td>Type e</td>
<td>1162×2559</td>
</tr>
</tbody>
</table>

The floor frame contains eleven slots for concrete panels with cross bracing to support the sides of each panel. The panel types must be placed in the order specified in Figure 91a, for the panels to be properly supported on all sides. The designers accounted for anticipated geometric deviations by specifying a 3 mm gap between the frame (Figure 91b) and all the panels (Figure 91c). However, as the result of fabrication process capabilities, geometric deviations occasionally resulted in panels being too large or too small to easily fit into the steel frame. In the case of panel misfit, additional work was required to correct geometry. One correction strategy could have been to substitute interchangeable panels with each other in order to optimally match panel tie-in points with the frame control points. For each steel frame, there are 288 distinct assembly plans which could have been explored, thereby increasing the aggregation flexibility for ensuring adequate panel fit.
To quantify and optimize the assembly plan, laser scans of the floor frame (Figure 91d) and the concrete panels (Figure 91e) were taken with a laser scanner (FARO® LS 840HE) for accurate, as-built information. The laser scanner employed has an error of 2 mm at 25 m according to the instructions manual (FARO 2007). The coordinates of the bottom corners of the panels were selected as tie-in points and corresponding interface points on the floor frame were selected as control points.

![Laser scan of floor frame and concrete panels](image)

**Figure 91**: Case II: full scale concrete panels. (a) Assembly plan and the panel types for each slot in the floor frame. Images of the floor frame (b), and the stack of the concrete panels (c). Laser scan of the floor frame (d), and the stack of concrete panels (e).

As mentioned earlier, volumetric assemblies utilize optimization approach II only. Each assembly type is investigated for the associated slots in the design. The geometric deviation of the resulting plan from the as-designed state is minimized for each assembly type, with the constraint that each assembly has an allowable variation limit (tolerance) of 3 mm. In case that any assembly plan results in greater values than the allowable threshold, the assembly plan is ignored. The explained procedure is implemented and modified for the concrete panels and the floor frame investigated in this case study. Key results are reported in Table 25.
Table 25: Summary of the results for assembly planning of the concrete panels used in case II

<table>
<thead>
<tr>
<th>Assembly Type</th>
<th>Assembly plan</th>
<th>Processing time</th>
<th>Average deviation at each critical point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type a</td>
<td>a-1, a-3, a-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type b</td>
<td>b-3, b-2, b-4, b-1</td>
<td>2.23 s</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>Type c</td>
<td>c-1’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type d</td>
<td>d-2, d-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type e</td>
<td>e-1’</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As seen in Table 25, the procedure for assembly planning is time effective and the average deviation at each critical point is less than the acceptable threshold (3 mm). The processing time including data collection, manual manipulation, and assembly planning optimization for this case study is summarized in Table 26.

Table 26: Summary of the processing time at each step for the case study II

<table>
<thead>
<tr>
<th>Step</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data collection</td>
<td>~ 28 minutes for the floor frame (3 scans)</td>
</tr>
<tr>
<td></td>
<td>~ 16 minutes for the panels (2 scans)</td>
</tr>
<tr>
<td>Preprocessing (critical interface point extraction)</td>
<td>~ 20 minutes for both the panels and the frames</td>
</tr>
<tr>
<td>Processing</td>
<td>~ 2-3 seconds</td>
</tr>
<tr>
<td>Total</td>
<td>~ 1 hour</td>
</tr>
</tbody>
</table>

The benefit of using optimization approach II in this case study can be expressed in terms of a minimization of localized rework (i.e., rework associated with aggregation of each panel). While the time required to optimally plan the assembly is on the order of 1 hour for an entire frame of 11 concrete panels, the time associated with rework is challenging to calculate precisely. However, from observing the fabrication crew, it can be reasonably estimated that the impact of rework is low to medium with a schedule impact on the order of 5 man-hours per concrete panel. If only a single panel requires rework, the time savings is equal to approximately 4 hours or 80% of the total time required for rework. Due to the high quantity of floor frames in this building (16 frames and 176 concrete panels), utilization of the proposed optimum assembly planning framework can be extremely beneficial for reducing rework associated with geometric variability.

7.3.7. Conclusions from Optimal Assembly Case Studies

Through the two case studies, it was shown how the proposed optimal assembly algorithm can be used to manage dimensional variability during the aggregation of modular components. Rather than placing strict controls on the geometry of components, there is an optimal way to match mating components in order to minimize dimensional variability. The key assumption in this approach is that component dimensions have dimensional variation distributions that are approximately normal. In the case where the variation has a discrete bias in its mean and low degree of variability, then optimal assembly would be difficult to employ.
Despite demonstrating the applicability of the proposed optimal assembly planning algorithm, some limitations can be identified. Firstly, the extraction of critical interfacing points for component aggregation is currently done manually, which requires judgement and can be time consuming. Furthermore, the success of the optimal assembly plan relies heavily on the specification of aggregation type (serial-parallel or volumetric) and the quantity and accuracy of tie-in points and control points used. In addition, the process of extracting critical points from as-built status data (3D point clouds) assumes that there are no occlusions or incomplete data in the laser scans. In order to ensure a reliable analysis procedure, the as-built status must adequately capture critical data at the interface points.

7.4. **Comparison of Selective Assembly and Optimal Assembly Methods**

Selective assembly involves grouping dimensionally similar components into groups and then proceeding with a strategy between groups or “bins” for aggregation between two types of components. Optimal assembly is similar except that aggregation is performed on a component by component basis. The optimal assembly algorithm finds the best matches between components using as-built data in the form of 3D point clouds. Essentially, optimal assembly is a form of selective assembly in which the number of bins used is equal to the number of components being aggregated together.

For choosing between selective assembly or optimal assembly in practice, the key factors lie with the number of interchangeable components (i.e., high volume vs. low volume manufacturing) and with the complexity of interfaces (i.e., single critical dimension for aggregation vs. numerous critical dimensions for aggregation). The choice between either approach is summarized in Table 27.

<table>
<thead>
<tr>
<th>Manufacture Quantity</th>
<th>Number of Critical Dimensions for Aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Volume</td>
<td>Optimal or Selective Assembly</td>
</tr>
<tr>
<td>High Volume</td>
<td>Selective Assembly</td>
</tr>
<tr>
<td></td>
<td>Optimal Assembly</td>
</tr>
</tbody>
</table>

Table 27: Factors influencing the choice between Selective Assembly or Optimal Assembly methods
8. Summary and Recommendations

The management of dimensional variability in construction has traditionally been non-strategic. This is a result of numerous factors, which have led to the emergence of trial and error or ad-hoc solutions for managing dimensional variability. While this approach may be satisfactory for most construction projects, modular construction demands more dimensional coordination to yield benefits over conventional construction methods. This thesis approaches dimensional variability from a holistic construction lifecycle perspective (i.e., throughout all key stages from manufacture to erection) by identifying critical sources of variability and developing appropriate variability management strategies. This thesis presented one primary objective and several sub-objectives. Each of these objectives have been successfully addressed, as summarized in the following sections.

8.1. Summary of Proposed Methodology

The primary objective of this thesis was to develop a strategic framework for managing dimensional variability throughout the lifecycle of a modular construction project. For this purpose a flowchart was developed, which outlines the steps required to target key sources of variability, and depending on the nature of the project, to develop an overall strategy, which can include design-based methods, production-based methods, handling-based methods or onsite-based methods.

Design-based methods are the most proactive variability management strategies, since each of the critical sources of variability are targeted up front before the completion of the geometric design, which is then followed by selection of adequate parameter values and corresponding tolerances. Since all variability sources must be identified up front, design-based strategies are often the most difficult, which is why downstream approaches are favourable. Production-based methods manage dimensional variability during the manufacturing, fabrication and aggregation processes for constructing modules. Rather than conducting detailed analyses to determine adequate parameter values and corresponding tolerances, key variability management goals are emphasized, which for production is typically the goal of ensuring proper aggregation while respecting downstream goals for the final in-situ building. Handling-based methods aim to control the geometric response of modules during various structural actions (e.g., crane loads, transportation loads, temporary support gravity loads, etc.). Finally, onsite-based strategies aim to fix or remedy any conflicts related to dimensional variability during the final installation process. This approach is the most reactive form of dimensional variability management and is the default approach unless other more proactive strategies are employed.

In addition to the generic flowchart for creating a variability management strategy, a method was developed for quantifying dimensional variability using 3D imaging. This approach, referred to as Deviation Analysis, compares as-built data using laser scans (i.e., 3D point clouds) and building information models (BIM) in order to quantify the deviation between as-built and as-designed states. Deviation analysis is effective for quantifying process capabilities such as fit-up, cutting, welding or for the geometric response of a module to various structural actions such as crane loading, temporary support conditions, or transportation.

8.2. Summary of Novel Research Developments

The novel research developments made in this thesis relate to dimensional variability management strategies from design-based and production-based standpoints.
The novel design-based methods function as tools for designers to either analyse the effect that a particular tolerance specification has on an assembly in terms of its accumulating effects, or to design critical connections. A framework originally developed by Dr. Colin Milberg called tolerance mapping was adapted and applied to two modular construction case studies. The result from these case studies was that tolerance mapping (which is essentially a combination of graph theory and Geometric Dimensioning and Tolerancing, GD&T notation) can be an effective design-based method for managing variability, but requires an extensive knowledge of the mapping method and GD&T. To develop a more familiar approach for those working within the context of construction systems, a kinematics chain based dimensional variation analysis method was adapted to function in a similar manner as tolerance mapping. In this approach, a construction assembly is modelled in the form of a series of connected links which experience transformations in the form of rotations and or translations at joints. The method was applied to a construction assembly in order to determine the amount of dimensional variability accumulated through an assembly. The results of this case study show that a kinematics chain based approach is capable of modelling dimensional variability in an assembly by use of rigid body transformations. However, in cases where the rigid body deformation assumption cannot be used (e.g., large welding distortion in a frame, or for very flexible components), then a kinematics chain based design strategy is not feasible, and another design method for proper dimensional specification is required.

Novel developments were also made with respect to production-based methods. The first of these methods is the application of selective assembly to modular construction. Selective assembly, which selectively matches interchangeable components together based on their unique geometric deviations, was demonstrated in a simple case study of a modular steel bridge. It was determined through this case study that the overall principle of selective assembly has the potential to be used as an effective dimensional management strategy, but is not recommended in cases where the quantity of interchangeable parts is low, or in cases where the aggregation between two components is very complex (e.g., many connection points). The second production-based method was optimal assembly, where components being aggregated into an assembly are optimally placed based on their geometric deviations. Optimal assembly is based on the same principle as selective assembly, but is much more practical for most modular construction projects since it can be used for any number of interchangeable components, and any degree of complexity. The optimal assembly method also contains an optimization engine, which can produce the assembly sequence that minimizes localized rework or total assembly dimensional variability.

8.3. Limitations of Proposed Methodology

While this research developed several novel aspects of dimensional variability management, these come with some limitations, which need to be addressed.

The background chapter was primarily based upon an extensive literature review focused on construction management and structural engineering fields. As such, the literature review does not present a great amount of information coming from architectural practices for the management of dimensional variability. This limitation is not critical for the work presented however, since the scope of this thesis primarily focuses on managing the dimensional variability associated with the structure of a module, rather than all subsystems.

The scope of the proposed methodology includes several project parameters, which have an impact on various aspects of dimensional variability. The breakdown of these parameters are by no means exclusive.
nor exhaustive and may require identification of other project parameters on a case specific basis. Furthermore, the goals of dimensional variability management are based on the developed classification of impacts (structural safety, constructability, aesthetics, performance and functionality). These impacts are also not exhaustive and may include other categories (e.g., legal or contractual impact classes) on a case specific basis. The proposed methodology, including the steps involved for developing a variability management strategy on a given project, is not validated in this thesis. This is primarily due to a lack of being involved on numerous modular construction projects from the beginning stages to implement and refine the proposed methodology. Recommended future work is to implement and refine the proposed methodology. The methodology presented in this thesis is a synthesis of observations made on various modular construction projects. The range of projects include two modular data centres, one temporary commercial modular building, a modular home building production line, and a preliminary modular bathroom pod production. Smaller scale modular projects include two steel bridge design competitions, in which the author was involved with the actual fabrication and aggregation of both bridges. In addition to these first hand experiences, the author also gained insight into numerous industry examples (stemming from literature), which range from high rise modular construction to industrial pipe spool modular construction.

The limitations of the proposed methodology also need to be addressed. The deviation analysis method is subject to the accuracy of the data collection tools. Furthermore, when applying deviation analysis, the user must often perform manipulations on point clouds and BIM models to extract meaningful information. These processes must be currently performed manually, which is time consuming and can introduce certain errors. The kinematics chain based DVA method cannot be used in cases where the rigid body deformation assumption is not valid. The selective assembly method is not practical for instances where low volume production and complex interfacing between components occurs. Finally, the optimal assembly method is only practical to use for interchangeable components in an assembly.

8.4. Future Work and Recommendations

The recommended future area of work related to this research is the implementation and refinement of the proposed methodology. This is required for validating and improving this research. In addition, several areas of recommended research are proposed. The development of more strategies for managing dimensional variability should be explored. This includes design-based, production-based, handling-based and onsite-based methods. Since the proposed methodology relates to the management of dimensional variability from a high-level project viewpoint (and not the exhaustive development of optimal strategies), it was not feasible to explore all facets of dimensional variability strategies. Future work should explore a range of dimensional variability strategies for each management approach (e.g., design-based strategies) in order to determine or provide recommendations for the best strategy on a case specific basis. Finally, future work should investigate how to incorporate all subsystems of a modular project and the project-site itself into the proposed methodology, rather than simply focusing primarily on the structure, and only on the ‘module’ side of a project.
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Appendix A: Review of Tolerance Theory from Manufacturing

The manufacturing industry can be regarded as the birthplace of comprehensive tolerancing design due to the importance of ensuring part interchangeability and functionality in mass produced assemblies. The overall purpose behind tolerancing in manufacturing is to appropriately target dimensional and geometric variability, and to either (1) anticipate and design a system that can function with it or (2) control it through a more intricate product and process design. Tolerancing is carried out at several levels, starting with features, then parts and finally for the overall assembly. Informed decision making in tolerance design requires an understanding of how variability affects both production and overall product functionality. Numerous concepts and tools are used to help predict and analyze both variability and adequacy of tolerance values. Tolerancing has evolved with time to become a staple in manufacturing design, with the core emphasis on optimizing the overall cost trade-off between product tolerances and process capabilities.

Introduction

One of the earliest uses of tolerancing started in the manufacture of guns in the late 1700’s to ensure adequate part interchangeability (Curtis 2002) and has since continued in development to be a core design principle used in many manufacturing applications. The development and use of tolerance theory within manufacturing is much more comprehensive and in-depth than when compared with similar applications in construction. Often, manufacturing entails a high volume production of an intricate arrangement of tightly aggregated parts, where even small deviations and geometric distortions (on the sub-millimeter level) can have a profound impact on overall production and post-production requirements. The same cannot be said for construction as a whole. The control of dimensional and geometric variability is much different in construction since variations (and tolerances) for the in situ project site are larger than the variations (and tolerances) of offsite construction components. As such construction is unique for having such a large range in tolerance values. While offsite construction components can have tolerances on the order of millimeters, the in situ project site often has tolerances on the order of several centimeters (Ballast 2007). As a result, the traditional approach for controlling dimensional and geometric variability in construction has been to specify standardized tolerances for components and processes based on codes, standards and experience. Construction codes often categorize tolerances into distinct classes (material, fabrication and erection tolerances) in order to attempt to control adverse effects of variability (ACI 2002, AISC 2010a, AISC 2010b). However, even with standardized tolerances, there is a general lack of understanding of how variability and tolerances accumulate due to industry fragmentation, lack of process capability data, and proper knowledge about tolerancing theory (Milberg et al. 2002). The framework used for presenting this literature review of tolerance theory is shown in Figure A1.

![Figure A1: Sequential flow of topics within the body of knowledge for tolerance theory](image-url)
This structure helps to address three core questions in tolerance theory:

1. **Why** should certain tolerances be specified?
2. **How** do you ensure they these tolerances are designed and communicated properly?
3. **What** tools and notation are used in tolerancing?

These three questions follow each other sequentially, and cover various topics within the body of knowledge including decision making approaches (design philosophies), decision making processes (science), methods, models, charts, and other design tools, communication techniques (design processes) and notation. When describing the various topics within tolerancing, it is important to distinguish between engineering processes (deriving tolerance values) and subsequent design processes (communicating tolerances). This distinction can be very effective for fixing tolerance problems in mass production applications, since engineering-process related tolerance problems are often far more problematic (i.e. improper tolerance value) than design-process related tolerance problems (i.e. miscommunicated tolerance value). Correcting a miscommunicated tolerance value can be as simple as revising a drawing, however correcting an improper tolerance value can be much more tedious and time consuming (Creveling 1997).

**Variability**

Broadly defined, variability within the context of manufacturing refers to the change in shape or size of features, parts and assemblies (Liggett 1993). Sources of variability are often broken down into the following distinct categories:

- Materials
- Machines (i.e. tools and equipment)
- Methods of manufacturing (i.e. processes)
- Manpower
- Measurement (i.e. inspection capabilities)
- Environment

These sources of variation are covered extensively in lean six sigma manufacturing literature, and are often referred to as the “5M’s and E” (Keller 2011). Some of the influences for these sources of variability are summarized in Table A1.

<table>
<thead>
<tr>
<th><strong>Source of Variability</strong></th>
<th><strong>Influences</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>Rigidity of the part (shape)</td>
</tr>
<tr>
<td></td>
<td>Material properties (chemical composition, homogeneity, hardness, etc.)</td>
</tr>
<tr>
<td></td>
<td>Stress in material (from various loads)</td>
</tr>
<tr>
<td>Machines (i.e. tools and equipment)</td>
<td>Equipment capabilities (precision and accuracy)</td>
</tr>
<tr>
<td></td>
<td>Static and dynamic stability during operation</td>
</tr>
<tr>
<td></td>
<td>Thermal properties</td>
</tr>
<tr>
<td></td>
<td>Time dependent tool and fixture wear</td>
</tr>
<tr>
<td>Methods of manufacturing (i.e. processes)</td>
<td>Clamping and fit-up methods</td>
</tr>
<tr>
<td></td>
<td>Processing parameters (temperature, speed, cycle time, depth of cut, pressure, etc.)</td>
</tr>
</tbody>
</table>

**Table A1: Influences and sources of variability in manufacturing, adapted from (Henzold 2006)**
Types of Tolerances

Under the classification of dimensional tolerances, there are three distinct categories: (1) linear tolerances, (2) angular tolerances and (3) geometrical tolerances. This categorization structure (Figure 12 in thesis main body) is used for describing acceptable variations between points (linear tolerances), lines (angular and geometrical tolerances) and surfaces (geometrical tolerances). The breakdown of dimensional tolerances enables the communication of tolerance decisions (i.e. *Geometric Dimensioning and Tolerancing* or *GD&T*, which is the most common tolerance language used in practice).

Using the breakdown of dimensional tolerances, dimensional variability can be divided into two distinct categories for different applications: (1) those on the “individual feature level” which describe how an actual feature (i.e. the profile of a line) varies from a substitute feature (i.e. an averaged or best-fit line through the actual feature) and (2) those on the “referenced feature level” which describe how a feature varies from another feature or datum. These two applications (individual features and referenced features) stems from industry standards and common tolerance practice (Henzold 2006, Standard 2009). A brief discussion about different feature types (Figure A2) is important for understanding the two different applications of dimensional tolerances:

- **Actual feature**: This is a feature defined by the actual physical profile of a line or surface.
- **Substitute feature**: This is a geometrically ideal feature defined by a method of best fitting or averaging an ideal feature to the actual feature.
- **Nominal feature**: This is the geometrically ideal feature as defined by the design/drawing.

<table>
<thead>
<tr>
<th>Manpower</th>
<th>Skills of workers (education, previous experience, etc.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Precision of clamping and fit-up</td>
</tr>
<tr>
<td>Measurement (i.e. inspection</td>
<td>Precision and accuracy of measuring processes</td>
</tr>
<tr>
<td>capabilities)</td>
<td></td>
</tr>
<tr>
<td>Environment</td>
<td>Loadings (thermal, static, dynamic), moisture level,</td>
</tr>
<tr>
<td></td>
<td>temperature changes, duty cycle (end use) and setup</td>
</tr>
<tr>
<td></td>
<td>conditions.</td>
</tr>
</tbody>
</table>

| Comparison of Actual Feature    | Actual Feature (line)                                    |
| & Substitute Feature            |                                                          |
| Comparison of Actual Feature,   | Substitute Feature (line)                                 |
| Substitute Feature & Nominal    |                                                           |
| Feature                         | Nominal Feature (line)                                    |

**Figure A2: Comparison of actual feature, substitute feature and nominal feature**

Therefore, tolerances are specified to control different types of dimensional variability which can be grouped into linear, angular and geometrical classes (Figure 8 in thesis main body). The following section
provides a brief explanation on linear and angular tolerances (size, orientation), and geometrical tolerances (form, orientation, location, and run-out).

**Size Tolerance**

This tolerance is used to control the difference between a nominal size and the actual size of a feature. *Size* can be used in a linear context (2 point measurement of a feature) or in an angular context (angular measurement of averaged lines) (Henzold 2006, ISO 1985).

**Angularity Tolerance**

Sometimes a second tolerance is used for describing angular variations. Although angularity is often considered to be within the category of orientation tolerances (Henzold 2006), it can be used to describe the general orientation of nominal feature lines from the actual feature lines (ISO 1985).

**Geometrical Tolerances**

*Geometrical Tolerances* are the most widespread and in-depth category of dimensional tolerances, and describes the form, orientation or location of a particular feature (ISO 1985). Geometrical tolerances can describe surface variations in 1D (line), 2D (planes) or 3D (surfaces). Various researchers, practitioners, or standards organizations structure the breakdown of geometrical tolerances differently. The breakdown of geometrical tolerances is not as important as ensuring that the critical geometric variations are controlled. The breakdown structure chosen for this document is adapted from Georg Henzold (2006) and ASME Y14.5 and is shown in Table A2. The following sections provide descriptions about form, orientation, location and run-out tolerances.

<table>
<thead>
<tr>
<th>Application</th>
<th>Type of Geometrical Tolerance</th>
<th>Description</th>
<th>Specific Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Part Feature (how the actual feature varies from substitute feature)</td>
<td>Form</td>
<td>Profile of line</td>
<td>Straightness</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Roundness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Profile of surface</td>
<td>Flatness</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cylindricity</td>
</tr>
<tr>
<td>Related Part Feature (how a feature (actual or substitute) varies from another feature or specific datum description)</td>
<td>Orientation</td>
<td>Angularity</td>
<td>Parallelism</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Perpendicularity</td>
</tr>
<tr>
<td></td>
<td>Location</td>
<td>Position</td>
<td>Coaxiality</td>
</tr>
<tr>
<td></td>
<td>Run-out</td>
<td>Circular</td>
<td>Symmetry</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Radial</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Axial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>Radial</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Axial</td>
</tr>
</tbody>
</table>

**Form**

A form tolerance describes the allowable variation of an actual feature from its substitute feature. As such, form tolerances are the maximum permitted value of the form deviation – actual form from its nominal form (Henzold 2006). Ultimately, form tolerances are used to control variation of surfaces,
however depending on how the surface is defined (1D, 2D, or 3D) form tolerances are applied to lines or surfaces (2D or 3D). For the same of simplifying the specification of tolerances, surface tolerances are usually described in 2D (as per design drawings), however with the use of more advanced computing and modelling capabilities, 3D surface tolerances can also be utilized.

When describing the profile of a line, the two general variation types are roundness and straightness (Henzold 2006, ISO 2012). The selection of a specific variation control (tolerance) depends on the form of the nominal feature (i.e. if the nominal feature is supposed to be round, then a roundness tolerance is specified, but if the nominal feature is supposed to be straight, then a straightness tolerance is specified). A feature with a straightness tolerance is required to be contained between two parallel lines that are separate by the straightness tolerance value (Drake 1999, Henzold 2006). Similarly, a feature with a roundness tolerance is required to be contained between two equidistant lines which are separated by the roundness tolerance value. Figure A3 summarizes the two types of form tolerances for describing the form of a line.

![Figure A3: Depiction of form tolerances of a line (straightness and roundness)](image)

When describing the profile of a surface, the two general variation types are flatness and cylindricity. Similar to the justification of specifying straightness and roundness tolerances for lines, flatness and cylindricity are extensions for describing acceptable variations of a surface. A feature with a flatness tolerance is required to be contained between two parallel planes that are separated by a flatness tolerance value (similar to the straightness tolerance value as seen in Figure A3). Similarly, a feature with a cylindricity tolerance is required to be contained between two equidistantly spaced surfaces which have the same form as the nominal feature (similar to the roundness tolerance value as seen in Figure A3).

**Orientation**

Orientation tolerances describe the allowable variation of a substitute feature’s orientation from the nominal feature’s orientation. Orientation tolerances require the specification of a datum. Within the category of orientation tolerances there are generally three distinctions based on the way the datum is specified: (1) perpendicularity which requires the substitute feature to be perpendicular to the nominal datum, (2) parallelism which requires the substitute feature to be parallel to the nominal datum or (3) angularity which requires the substitute feature to be at a specific angle to the nominal datum (Henzold 2006, Standard 2009). While some practices apply hard and fast rules to perpendicularity, parallelism or
angularity (i.e. the spacing of containment planes and the orientation value of the substitute feature to the nominal datum), the specification of the required tolerance criteria is ultimately decided during the tolerance design process.

**Location**

Location tolerances describe the allowable variation in the position of a substitute feature to the nominal feature. Location tolerances require the specification of a datum. Sometimes, a tolerance zone is specified which defines a volumetric space in which the finally positioned part must be contained in. Within the category of location tolerances, there are generally three distinctions based on the way the datum is specified: (1) positional tolerance where the centre axis (or distinct coordinate) of a part is located some distance (in x, y, z) with respect to the nominal datum, (2) coaxiality tolerance where the centre axis of a part is located directly on a nominal datum, and (3) symmetry tolerance where the centre axis of a part is contained between two nominally defined planes (Henzold 2006, Standard 2009). Regardless of how the position of a part is located with respect to a datum, the purpose of specifying location tolerances is to control the placement of a part in an assembly. In addition, location tolerances also place some level of control over orientation and form tolerances of part (Henzold 2006).

**Run-out**

Run-out tolerances are specifically used to control variability associated with the inspection of cylindrical parts (especially those which rotate) (Coban Engineering 2015). Run-out tolerances are specified with respect to a datum defined at the centre-axis of a cylindrical part. Within the category of run-out tolerances, there are two distinctions: (1) circular radial run-out tolerances which ensure that every cross section of a cylindrical part are within a confined circular tolerance zone, and (2) total run-out tolerances which ensure that the entire cylindrical surface of a part is within a confined tolerance volumetric zone (Henzold 2006). Run-out tolerances are distinct from other geometric tolerances because they implicitly control form tolerances (i.e. roundness), orientation tolerances (i.e. angularity) and location tolerances (i.e. coaxiality).

**Accumulation of Variability and Tolerances**

While it is inevitable that variability exists in any manufacturing process, the geometric errors associated with variability will propagate, resulting in an accumulation (or stack-up) of error. In addition to geometric errors of parts and part-features, functional gaps between connected parts play a role in the accumulation and management of geometric variability. Tolerances which serve the purpose of describing allowable geometric error (for parts and part-features), tolerances are also used for describing the functional gaps between components. However tolerances merely act as boundaries, and depending on the resulting variability of a process, there will be some degree of dimensional discrepancy with respect to that specified tolerance. The accumulating effects of variability and tolerances can have a profound impact on the ability to achieve certain goals in the manufacture and assembly of components. While variability primarily stems from materials and processes, it is manifested as geometric variability (product variability), on the level of features, parts, and assemblies (Figure A4).
The accumulation (or stack up) of product aggregation variability starts on the level of *features* (features represent geometrical elements of a part, such as lines, planes, circles, cylinders, cones, spheres, helixes, tori, and mathematically defined curves and surfaces) and experience variation in terms of their size, form, orientation, location, waviness, roughness, surface discontinuities, edge deviations (Henzold, 2006). The next level of variability is the overall change in size or shape of a part, which has an envelope defined by all of its geometric features. Finally, variability escalates to the assembly level, which accounts for part variability and kinematic adjustments resulting from the interaction of mated parts (Chase and Parkinson 1991). The accumulating effects of variability, tolerances and functional gaps is demonstrated in a simple example of two parts being separated by a functional gap in an assembly (Figure A5).
Using the notation defined in Figure A5 the following two equations can be used to describe the nominal assembly envelope dimension ($X_{\text{nominal}}$) and the actual assembly envelope dimension ($X_{\text{actual}}$):

$$
X_{\text{nominal}} = (\mu_1 \pm 2t_1) + (\mu_G \pm 2t_G) + (\mu_2 \pm 2t_2) \quad [A1]
$$

$$
X_{\text{actual}} = V_1 + V_G + V_2 \quad [A2]
$$

An adequate design needs to ensure that $X_{\text{actual}}$ achieves the same function as $X_{\text{nominal}}$. This requires knowledge about adequate tolerance specification.

**Tolerancing Specification**

Tolerance specification is often regarded as the engineering process concerned with how to systematically select tolerance types and values in order to find an appropriate balance between cost, quality and customer satisfaction (Creveling 1997, Hong and Chang 2002). Tolerance types are determined through...
use of tolerancing schemes, which represent different approaches for making decisions with respect to managing variability. Selecting suitable tolerance values is equally as important as determining (the critical) tolerance types. Ultimately, tolerance specification is concerned with answering the following questions:

- What types of tolerances are the most critical?
- What is the rationale behind specification of a given tolerance?
- What is the tolerance value?
- What control metrics are required to ensure tolerance values are met?

To answer these questions, decision makers require an understanding of two fundamental entities: process capabilities (and/or limitations) and corresponding required product tolerances. Process capabilities need to be fully understood in order to achieve desired product tolerances. Once the process capabilities and related design requirements or limitations are identified, appropriate tolerance values can be specified. The choice of tolerance values not only requires knowledge about localized process capabilities (i.e. each manufacturing process), but also knowledge about how variability and tolerances accumulate through an assembly and through the overall production process.

In general, the major processes which influence the dimensional and geometric properties of parts include: manufacturing, assembly, inspection, functional use, operations, and or maintenance. Once the effect of each process on the product is known, tolerances can be specified while accounting for the additional accumulation (or stack up) effects. The final step of tolerance-related decisions involves the specification of a control metric known as datums. Datums can have a profound impact on the communication, adherence, or control and inspection of tolerance related criteria. The sequence of specifying tolerances is summarized in Figure A6.

![Figure A6: Sequential information flow for making tolerance-related decisions in manufacturing](image)

In order to account for the major processes that have an impact on dimensional and geometric variability, it is important to understand the capabilities and limitations of production processes (manufacturing, assembly and inspection), as well as requirements based on end-use functionality and customer needs. Once the required knowledge about production processes and end-use requirements is acquired, a suitable tolerance scheme can be selected, which finally leads to the specification and control of tolerances.
Process Capabilities, Limitations, and End-Use Requirements

A production process is classified as a sequence of tasks which change the geometry and or material properties of a part or assembly, and can be further categorized in terms of processes which reduce, maintain or increase the mass of a part or assembly (Hong-Chao Zhang 1997). The major production processes considered in this section include manufacturing, assembly and inspection. In addition to production processes, this section discusses end-use requirements in the form of product functionality and any other customer requirements. Finally, the role of overall tolerance compatibility is presented, which describes how specific individual process tolerances govern while others can be ‘flexed’ or ‘relaxed’ in order to optimize overall production costs.

Manufacturing Capabilities and Limitations

The goal of specifying tolerances for manufacturing is to ensure that parts can be produced economically. Specification of adequate manufacturing tolerances is essential in the implementation of Design for Manufacturability (DfM) which attempts to provide compatibility between process capabilities and resulting product tolerances. General manufacturing tolerances should be equal to or larger than the normal workshop accuracy (Henzold 2006). This requirement is based on cost effectiveness since tolerances required to be tighter (or smaller) than the normal workshop accuracy come at a certain cost and labor increase. As such, larger tolerances make parts easier and less expensive to manufacture (Chase and Parkinson 1991).

Some of the factors influencing the selection of manufacturing-related tolerances include:

- Production cost
- Process selection
- Production cycle time
- Equipment choice
- Machine precision and accuracy
- Production cost
- Process selection
- Production cycle time
- Equipment choice
- Machine precision and accuracy

In order to understand the variability or workshop accuracy for various manufacturing processes, manufacturers often utilize application-specific deviation charts. When application-specific information is not available, manufacturers often use standard tolerance tables (available through standards organizations such as the International Organization for Standardization), which outline the expected normal variation and corresponding general tolerances for various processes such as welding, casting, cutting, milling, etc.

Variability in manufacturing is broadly classified into two groups: random unpredictable variations, and time-dependent controllable variations (Hong-Chao Zhang 1997). The specification of manufacturing tolerances can benefit from the application of statistical modelling in order to mathematically derive deviations and thus, corresponding tolerances. This is possible in mass production applications where access to statistical data is feasible and economical to obtain. The discussion about manufacturing related tolerances up to this point has focused exclusively on understanding the variability associated with a predetermined set of manufacturing processes. However, when a certain product tolerance requirement is stricter than a predetermined manufacturing processes allow for, a specialized manufacturing processes (with tighter tolerances) is required. This happens when downstream processes place tighter tolerance constraints on a product than those associated with the normal workshop accuracy, as is often the case for assembly-related tolerances.
Aggregation Requirements

The goal of specifying tolerances for the aggregation of parts is to ensure that (1) they can be physically aggregated and (2) in a way which minimizes costs and reduces cycle time. The first aspect of ensuring parts can be aggregated in an assembly is that they can meet two material conditions: (1) maximum material condition (MMC) and (2) least material condition (LMC). These two conditions act as boundaries to ensure that parts can be physically aggregated or mated, and that they can fulfil certain functional requirements once aggregated or mated:

- **The maximum material condition (MMC):** represents the absolute minimum interface gap between two parts being aggregated together. MMC is typically used for features with an axis, are cylindrical or has two opposite parallel planes (Henzold 2006). The classic example used in tolerancing literature to demonstrate the maximum material concept is a bolt and a bolt hole, where the assembly requirement is the largest bolt size that fits in the smallest hole. In this example, the bolt has its maximum material condition at its largest diameter, and the bolt hole has its maximum material condition at its smallest diameter.

- **Least material condition (LMC):** used to maintain the position of a part being mated to another component (Liggett 1993) and represents the absolute maximum interface gap between the two components (before the mating becomes too loose, and functionality is sacrificed). An example used to describe the least material condition is the minimum wall thickness of a slot in order to prevent breakout (i.e. due to pressure in a tube).

In addition to these two material condition, a third assembly requirement is also considered: envelope requirement. The envelope requirement specifies that at the maximum material condition, a part cannot exceed the geometrically nominal envelope (Henzold 2006, ISO 1985). Once tolerance boundaries are determined to ensure parts can physically or functionally be aggregated in an assembly, assembly tolerance values can be optimized in order to minimize cost and aggregation cycle time. This process involves determining the assembly process capabilities and associated tolerance costs (similar to the tradeoff for manufacturing tolerances).

**Inspection Capabilities**

High precision inspection (metrology) is required throughout manufacturing and assembly to ensure that variability is controlled within the limits specified by certain tolerances. The equipment used to measure deviations cannot do so with perfect accuracy, which means that there is a certain measurement uncertainty associated with inspection equipment. Typically, less precise inspection methods are not expensive but can be time consuming to use, whereas more precise inspection methods are expensive yet less time consuming to use (Henzold 2006). The selection of an inspection method or equipment depends on the desired level of accuracy and precision. Regardless of the selected inspection method or equipment, it is important to understand that there will be some degree of error with the quantification of variability, which means that tolerances need to be specified to account for inspection error.

The process of quantifying variability is usually part of a larger dimensional inspection plan, which aims to identify only critical geometric characteristics (those which have the most significant impact on manufacturability, assemblability, and desired end-use functionality) to inspect (Meadows 1995). The dimensional inspection plan requires knowledge about the accuracy of the inspect method, as well as how
the accuracy may change over time (precision between measurements). Inspection equipment includes gages which capture coordinates of set points of a feature (coordinate measuring machines, micrometers, calipers, etc.), soft gages (mathematical-based computing) which compares sensed data to nominal data, or more advanced metrology techniques which can capture large amounts of geometric data (Henzold 2006, Liggett 1993, Meadows 1995).

When describing the process of inspecting the dimensional and geometric properties of a part, it is important to understand the accrued error involved with capturing the actual geometry of a part in terms of substitute features (often used for inspection):

- Error between actual surface and sensed surface (measurement error)
- Error between the sensed surface and the substitute surface (fitting a substitute surface to a sensed surface)

The accumulation of error between the actual feature of a part and the substitute feature represents the required tolerance of a given inspection method (Figure A7). The process of deriving a substitute surface from a sensed surface can be done in different ways, but two of the most common approaches are a Gauss Approach (sum of square deviations of the sensed surface and the substitute surface) and a Chebyshev Approach (maximum deviation of the sensed surface and the substitute surface).

![Figure A7: Actual surface, sensed surface and substitute surface, adapted from (Miller et al. 1962)](image)

**Product End-Use Requirements**

A manufactured part or assembly has requirements and or constraints based on end-use functionality and specific customer needs. These functions include basic pass-fail criteria such as fit (i.e. a part either fits into an assembly or it does not), quantitative performance criteria (i.e. strength or water-tightness) or time-based reliability criteria (i.e. performance over a specified lifetime). Ultimately, there are certain functional requirements placed on the tolerancing of a part in order to ensure that the customer is satisfied, and that certain product end-use requirements are met. Customer tolerances are often used to describe the critical product tolerances that have a direct impact on potential displeasure or economic loss due to off-target performance (Creveling 1997). Establishing the end-use functional and customer tolerances requires answering the following questions:

- What are the targets in terms of desired functionality and customer satisfaction?
• How far off-target can the product be…
  o …before incurring intolerable economic loss (Creveling 1997)?
  o …before the functionality of the product is sacrificed?
• What is the relationship between a certain target criteria and associated dimensional variations?

**Datums**

Datums are theoretically exact references (points, axes, or planes) used to establish the location and orientation of *tolerance zones*—geometrically nominal features used in the tolerancing of parts (Henzold 2006, Meadows 1995). A single datum is typically part of a larger datum system, depending on the importance and number of features being tolerated. A single datum can be used for tolerancing a complete part while multiple number of datums can also be used for tolerancing a single feature. Since datums can be used for features, parts or assemblies, there is often a network of three types of datums used in a datum system: (1) datum reference frame, which is a datum specified on the feature level, (2) part reference frame, which is a datum used at the part level, and finally (3) assembly reference frame which is a datum used on the overall assembly level (Milberg 2006). Datums are specified for at least three different conditions: (1) functionality, (2) convenience, or (3) ensure special relationships are met (Liggett 1993). Datums are chosen from physical features of individual or related parts or fixtures. Since datums themselves are theoretical entities, they do not physically exist on a part, but rather require manufacturing and inspection equipment to model their virtual existence. The selection of a datum feature is based on part functionality and its relationship to the part or feature being tolerated (Meadows 1995). There is a form tolerance between a nominal datum and the physical feature being used to derive the datum. As such it is important to account for this additional tolerance during tolerance measurement and part positioning (Figure A8).

![Figure A8: Depiction of form tolerance for substitute datum features](image)

Another important aspect of using datum features to understand is that since the physical feature has a form tolerance, multiple measurements from the same feature can vary. This is especially the case for rough surfaces or flexible parts (Liggett 1993). What is often used is a datum target system which aims to
control the six degrees of freedom of a part with respect to specific point-targets used as datums (Henzold 2006, Liggett 1993, Meadows 1995). To aid in restraining these 6 degrees of freedom (i.e. translations and rotations about each Cartesian axis), priorities are placed on datum features in terms of a series of planes. Each datum plane (primary, secondary and or tertiary) requires a certain amount of contact points or targets in order to establish each plane (Figure A9). The selection of a datum priority (i.e. how many degrees of freedom a particular feature needs) is a very important aspect in the design of a datum system.

![Figure A9: Depiction of primary, secondary and tertiary datum planes and required targets, adapted from (Hong-Chao Zhang 1997)](image)

The design of a datum system also plays a large role in the accumulation of tolerances. If a sequence of features all have their own datums, there will be a greater accumulation of location and orientation tolerances than if a single datum were used for numerous features since each new datum has its own location and orientation tolerances. As such, the design of a datum system is a critical aspect of tolerancing.

**Overall Tolerance Compatibility**

Tolerances for a specific feature can be specified based on manufacturability, assemblability, inspectability, or end-use functionality. However, one of these specific requirements typically governs and requires the strictest tolerance. Since deviations incurred throughout production (manufacturing, assembly and inspection) are accumulative, tolerances based on customer requirements must be controlled during manufacturing and assembly. Tolerance specification is hierarchical, where the “high-level” assembly tolerances are determined by customer specifications and end-use functionality and “low-level” part tolerances are determined by manufacturing and aggregation requirements (Creveling 1997). When determining which factors govern in the overall accumulation of variability and tolerances, it is important to identify the critical geometric characteristics (sometimes referred to as the key characteristics). Critical characteristics represent the variation with the most significant impact on the
overall performance and are often determined using a pareto-based approach (i.e. 80% of overall output variation is influenced by about 20% of the input critical characteristics) (Liggett 1993). The critical geometric characteristics are typically chosen by selecting features which are directly related to interfacing between parts in an assembly. In addition, it is preferable to design a datum system which allows for maximized functionality and ease of measurement for the critical geometric characteristics (Drake 1999). Low-level part tolerances often involve performing cost tradeoffs between manufacturability and assemblability. Design for Manufacture and Assembly (DfMA) is a common design approach used in manufacturing for minimizing overall production costs by addressing the tradeoff between manufacturing and aggregation process capabilities (Kamrani and Sa'ed 2002). DfMA provides great flexibility with regards to where and how geometric variability can be controlled. Rather than placing strict controls on either manufacturing or aggregation process, DfMA can be used to target the critical geometric variability characterizes. As such, this design approach is commonly preferred over Design for Manufacture (DfM) or Design for Assembly (DfA) for its flexibility and ability to optimize overall production costs.

Tolerance Analysis and Tolerance Synthesis

There are two common approaches used to aid with tolerance specification: (1) tolerance analysis, and (2) tolerance synthesis (Figure A10). Tolerance analysis involves analyzing component tolerances and determining acceptable overall assembly tolerances, while tolerance allocation involves taking constrained assembly tolerances and allocating allowable component tolerances (Chase and Parkinson 1991, Hong and Chang 2002). Between these two approaches, if end-use functionality and customer requirements are very specialized or strict, tolerance allocation is the preferred method; often at an increased production cost. However if it is more favorable to minimize production costs at the expense of looser overall assembly tolerances, then tolerance analysis is the preferred method. While tolerance analysis and tolerance synthesis can be regarded as mutually exclusive tolerance specification approaches, they are also used in conjunction with each other, since tolerance specification is an iterative process (Hong-Chao Zhang 1997).

**Tolerance Analysis**

![Features → Parts → Assembly](image)

**Loose Tolerance**

Acceptable Variation Region

Acceptable Variation Threshold (Tolerance)

Unacceptable Variation Region

Distribution of Variation

**Tolerance Synthesis**

Assembly → Parts → Features

**Strict Tolerance**

Figure A10: Sequence of specification for tolerance analysis and tolerance synthesis
One of the goals for using tolerance analysis in design is to determine the overall accumulation of tolerances. By developing a model for tolerance accumulation, the effects of geometric variability of localized production processes on the overall finished product can be known. Specific goals for analysing tolerance accumulation include ensuring interchangeability between nominally identical components, ensuring adequate clearance between components, ensuring adequate aesthetic performance, ensuring proper product functionality and ensuring that processes can adequately match required product tolerances (Liggett 1993). Common approaches for predicting tolerance accumulation in an assembly include worst case, statistical or sampled mathematical models and can be modelled in 1D, 2D or 3D (Chase and Parkinson 1991, Hong and Chang 2002). A summary of the most common mathematical models used in tolerance accumulation analyses along with generic formulas and notation are included in Table 15.

Of the common mathematical models used in tolerance accumulation analyses, the Worst Case approach is the most expensive to implement (from a tolerance accumulation standpoint) and represents only the extreme probable limits of tolerance accumulation. The Root Mean Square approach is slightly less expensive to implement than the Worst Case approach, and provides a reasonable estimate of probable tolerance accumulation. The Six Sigma approach represents the long term probability of tolerance accumulation (i.e. a change in the mean tolerance value is expected over time) and also accounts for process capability ratios. Finally, the Sampled Data approach, usually in the form of Monte Carlo Simulations is the most computationally heavy method, however can analyze numerous “what-if” scenarios in order to determine the resiliency of selected tolerance values (Geng 2004).

As previously described, the purpose behind tolerance synthesis is to balance cost and quality through the allocation of a required overall assembly tolerance into part and feature tolerances (Figure A10). Tolerance synthesis is carried out in a direction opposite to tolerance analysis, starting with the tolerance of the function of interest (i.e. assembly tolerance) and working its way back to individual tolerances (Hong and Chang 2002). Tolerance synthesis uses optimization and heuristic (solving) techniques to determine appropriate tolerances, and most literature focuses on cost-tolerance optimization (Hong and Chang 2002). Often, the same mathematical models used in tolerance accumulation analyses are also used for tolerance synthesis, with the exception that individual part and feature tolerances are solved for rather than the overall tolerance accumulation (Roy et al. 1991). Tolerance synthesis can involve numerous iterations in order to ensure adequacy of the overall tolerance accumulation.

### Tolerancing Schemes

The discussion about tolerance specification thus far has focused on the philosophy and required considerations for tolerance-related decision making. This section provides a brief overview of different tolerancing schemes that can be utilized.

A tolerance scheme represents a specific design approach, and selection of a particular scheme requires an understanding of how to best manage variability. Tolerancing schemes all have slightly different functions and there is no single approach that does it all (Creveling 1997). There are two major categories of tolerancing schemes: (1) product tolerance design and (2) process tolerance design. A summary of some of the most common tolerance schemes and categories are shown in Table A3.
### Table A3: Most common tolerancing schemes and categories

<table>
<thead>
<tr>
<th>Tolerance Scheme</th>
<th>Categories</th>
<th>Notes/Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product Design</strong></td>
<td><strong>Parametric tolerancing:</strong> identification of parameters and limits. (i.e. dimensions are one form of control parameters) (Hong and Chang 2002, Hong-Chao Zhang 1997)</td>
<td><strong>Conventional (plus/minus) tolerancing:</strong> this represents the worst-case scenario, where only upper and lower bounds are specified. This method often requires the least amount of design time, but results in the most amount of tolerance accumulation.</td>
</tr>
<tr>
<td><strong>Statistical tolerancing:</strong> an extension of conventional tolerancing, but uses statistical modelling to evaluate tolerances (Hong and Chang 2002)</td>
<td>Refer to Root Mean Square, Six Sigma and Sampled Data models</td>
<td></td>
</tr>
<tr>
<td><strong>Vectorial tolerancing:</strong> an entire workpiece is defined by a set of substitute features. Each feature is given a location and orientation vector (Hong and Chang 2002)</td>
<td>Can be very time consuming to use, but is beneficial for conducting tolerance accumulation analyses.</td>
<td></td>
</tr>
<tr>
<td><strong>Geometric tolerancing:</strong> applies tolerances directly to geometric features of a part</td>
<td><strong>Various models can be used:</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Geometric Models</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>2-D</strong></td>
<td><strong>3-D</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Wireframe</strong></td>
<td><strong>Wireframe</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Surface Model</strong></td>
<td><strong>Solid Model</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Figure adapted from (Kamrani and Sa'ed 2002)</strong></td>
<td></td>
</tr>
</tbody>
</table>


### Tolerance Representation

Tolerance representation is the act of communicating tolerance decisions in practice. Tolerance representation is often carried out through communicated information on drawings, in order to convey all necessary design intentions to the manufacturing team (Creveling 1997, Meadows 1995). Drawings which do not have properly communicated tolerances can result in gross deviations and out-of-tolerances throughout production that result in rework, defects, damage, and a failure to meet certain functional goals (Henzold 2006). However, when drawings contain too much information (i.e. if every single part feature has a tolerance), then those involved with production can be overloaded with information, reducing productivity. As such, tolerance representation is often carried out through use of standards which provide recommendations for communicating tolerances effectively. *Geometric Dimensioning and Tolerancing* is the most common type of tolerance representation, and the *International Organization of*
Standardization (ISO) has numerous standards (ISO 286, ISO 1101, ISO 5459, and ISO 10579) which outline how to appropriately communicate tolerances on drawings (Henzold 2006).

To summarize the key aspects of effectively communicating tolerances, several ‘rules’ are outlined in various literature (Curtis 2002, Henzold 2006, Meadows 1995):

- Every dimension must have a tolerance (this does not mean that these tolerances need to appear on every drawing, but where appropriate, reference to the normal workpiece accuracy needs to be clear)
- Datum should be minimized and have no redundancy
- Dimensions should be prescribed in accordance with datum that are functional (reduction of tolerance accumulation)
- Drawings should not prescribe manufacturing methods (as this falls in the realm of the manufacturing engineer). The actual method of achieving tolerances should not be placed on drawings, but should still account for certain non-mandatory processes (i.e. finishes)
- Dimensions should be displayed in several views in order to communicate sufficient information
- Design tolerances need to account for a standard unit of temperature (i.e. manufacturing facility temperature of 20 degrees Celsius). Thermal effects also play a role in the effect of achieving tolerances.

Tolerance Tools and Techniques used in Manufacturing

Tolerancing tools can be used to help supplement decision making during design, or to properly communicate tolerance considerations during production. The core concepts used in design tools include tolerance analysis, tolerance synthesis and tolerance transfer. For communication tools in production, the core concept used is a system of ‘checks and balances’ that can be used to ensure that certain tolerances are being met, or to act as an early warning system to bring attention to potential dimensional or geometric issues that might arise. A complete list of tolerancing tools is not feasible to attain since almost every application within manufacturing requires a unique tool in order to carry out a particular function. As such, a list of some general tools used within manufacturing has been included in Table A4.

<table>
<thead>
<tr>
<th>Tolerancing Tool</th>
<th>Description and Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variation (noise) diagrams</td>
<td>A system map that outlines all sources of variation that can affect the overall goals of tolerancing (Creveling 1997).</td>
</tr>
<tr>
<td>Dimensional variation analysis</td>
<td>Analogous to a dimensional tolerance analysis model (except for variations rather than tolerances). Also used to predict how minor variations in a part can propagate through an entire assembly (Sleath and Leaney 2013a). A separate type of model called “Variation Solid Model” can be used to model non-polygonal solids.</td>
</tr>
<tr>
<td>Dimensional tolerance chain</td>
<td>Mathematical models used to predict tolerance accumulation in a tolerance chain. Most common application is through use of a linear (i.e. 1-D) tolerance accumulation (Hong and Chang 2002).</td>
</tr>
</tbody>
</table>

Table A4: List of commonly used tolerancing tools in manufacturing
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D tolerance propagation models</td>
<td>Similar to a dimensional tolerance chain, except that it accounts for all accumulation effects in 3-D. Requires creation of tolerance zones and a mechanism in which to add effects of zones together (Hong and Chang 2002).</td>
</tr>
<tr>
<td>Tolerance assistance models</td>
<td>Graphical tool for representing a manufacturing sequence and for checking that tolerance accumulation meets required specification (Hong-Chao Zhang 1997)</td>
</tr>
<tr>
<td>Variation simulation tolerance analysis</td>
<td>A form of commercial simulation software which models the effects of variation and how specified tolerances will perform for a simulated production and use of a product (Drake 1999)</td>
</tr>
<tr>
<td>Torsor vector models</td>
<td>Torsors act as the displacement of a rigid body and are used in model the effect of distortions on overall assembly. Used during design as a method of checking resiliency of overall product. (Hong and Chang 2002)</td>
</tr>
<tr>
<td>Computer aided tolerancing</td>
<td>Encompasses the use of computers to automate the execution of the following: statistical tolerancing, empirical tolerancing and cost-based optimal tolerancing. Used when there is a high degree of computation is required, or if there is a large amount of empirical data to process.</td>
</tr>
<tr>
<td>Tolerance charting</td>
<td>Graphical method for communicating the dimensions of a part for production purposes (Hong-Chao Zhang 1997). Acts as a form of dimensional quality control.</td>
</tr>
<tr>
<td>Process charting</td>
<td>A tool which outlines all processes of a part feature, and the variation and accumulating effect of each subsequent process so that those involved with production can have a form of checks and balances to ensure that the product is conforming to required tolerances (Liggett 1993).</td>
</tr>
</tbody>
</table>
Appendix B: Calibration of 3D Geometric Capture Devices for Deviation Analysis

The purpose of this section is to highlight and compare the differences between as-built state dimensional analyses using four 3D geometric capture tools which were used during the developments and analyses contained in this thesis:

- Trimble® GX 3D Laser Scanner
- FARO® Laser Scanner 880 HE
- Sokkia® 30R Robotic Total Station
- Bosch® GLR825 Laser Meter

The laser scanners capture 3D geometric data in the form of large dense 3D point clouds of the surface of an object of interest, the total station captures single 3D points on the surface of an object of interest and the laser meter captures data in the form of a series of linear (2-point) measurements on the surface of an object of interest. The scope of this calibration experiment consisted of analyzing two distinct as-built states of a small test frame (1.8 m x 1.9 m x 2.9 m) configuration. The two distinct geometric states of this frame were configured such that the first state would resemble a dimensional tolerance compliant state (geometry would match the as-designed state as best possible), and the second state would resemble a dimensional tolerance non-compliant state (frame configured with clear misalignments as joints) as shown in Figure B1 and Figure B2.

Figure B1: Dimensional tolerance compliant state of the test frame used for calibration
For the distance from each device to the frame, the claimed accuracies are as follows:

- Trimble® GX 3D Laser Scanner: +/- 2 mm
- FARO® Laser Scanner 880 HE: +/- 3 mm
- Sokkia® 30R Robotic Total Station: +/- 1 mm
- Bosch® GLR825 Laser Meter: +/- 1 mm

The two distinct geometric configurations of the frame were captured using all devices within a short time interval (80 minutes) for each configuration in order to minimize minute movements during the data collection of each device. To compare the accuracy and form of data between devices, deviation analyses were conducted using an open source software called Cloud Compare®, where data sets for each as-built state were directly overlapped and deviations plotted. Cloud Compare® is a unique software in that it enables the output of deviations to be plotted in the form of a histogram, which can be used for calibration and comparison purposes. Since the data obtained from the total station and laser meter are in the form of discrete point measurements rather than dense continuous surface point clouds (as is the case for the laser scanners), the discrete data was converted into continuous data using a line plotting feature in AutoCAD 3D. For this purpose the discrete point sets were converted into line plots, and subsequently into uniform point clouds, in order to have continuous data sets for all devices (Figure B3). It should be noted that the continuous data sets for both the total station and laser meter devices are only truly accurate at the critical data points obtained (i.e., at the joints between members), and the continuous data is only used to visualize the linear progression of deviations between these critical points.
After obtaining continuous data sets for all devices, deviation analyses were created in Cloud Compare®. The deviation analysis outputs for all devices is shown in Figures B4, B5, B6 and B7. Each deviation analysis displays distances between 0 mm and 25 mm, which correspond to the discrepancies between the two geometric states imposed on the frame. Comparisons between the continuous data capture devices and discrete data capture devices are first made, and then comparisons between all devices are made, in order to understand similarities and differences between similar data capture devices and between discrete and continuous types.

Figure B3: Comparison of discrete data capture (i.e., total station and laser meter) versus continuous data capture (i.e., laser scanner)
Figure B4: Deviation analysis output for Trimble® GX 3D Laser Scanner

Figure B5: Deviation analysis output for FARO® Laser Scanner 880 HE
In Figure B8, the main difference between continuous data sets are outlined in white boxes. Difference 1 shows the Trimble analysis revealing deviations which are approximately 10 mm larger on average than the FARO analysis. Differences 2, 3, and 4 reveal that deviations in the FARO analysis are approximately
10 mm larger on average than the Trimble analysis. While the exact reason as to the 4 localized differences between the two laser scan analyses is not known, the overall analysis results are very similar between the Trimble and FARO laser scanners. As a whole, deviations are consistent in terms of absolute distances as well as the histogram distribution (Figure B9).

In Figure B8 there are also 4 key differences between the two discrete device data sets. Difference 1, 2 and 4 show that the total station deviation is approximately 8 mm larger on average than the laser meter. Difference 3 shows the opposite, where the laser meter deviations are approximately 10 mm larger on average than the total station analysis. Reasons as to why these differences most likely stem from the intrinsic errors involved with obtaining critical deviations from each device (even though both devices have the same claimed accuracy for each setup). Furthermore, the differences observed between the total station and laser meter analyses are not directly from the critical deviations, that is to say the actual 16 critical coordinates (i.e., joints) obtained to create the continuous data sets are very consistent. By observing only the critical coordinates, it will be observed that the two data capture devices are essentially identical. As a whole, both discrete data capture devices have similar deviation analysis results, however notable differences emerge when observing the histogram distribution of deviations (Figure B9).
After drawing comparisons between both continuous and discrete data capture devices, some conclusions can be made by examining the histogram distributions of deviations for all devices. Metrics for comparison which are explored in this calibration study include mode and mean deviation values. Figure B10 displays the approximate mode and mean deviation values between continuous and discrete data capture devices, which was obtained from analysis in Cloud Compare® software.

As seen in Figure B10 the two continuous data capture devices, which collect data in the form of point clouds from surfaces, are very similar. Both data sets have the exact same mean, with only a slight difference coming from the mode value. The difference in mode values is approximately equal to 1 mm, which in comparison to the claimed device accuracies is well within an acceptable limit. However, upon examining the mean and mode values for the discrete data devices, some notable differences are apparent. Mode and mean approximate valued vary by 5 mm and 2 mm respectively. It should be noted however that mode and mean value comparisons from the continuous data sets is not an accurate method for comparison, since line fitting on the 16-pt data sets was created. However using Cloud Compare®, it was not possible to create a deviation analysis by only using the initial 16 critical deviation values from the total station or laser meter (this is a further reason why these 16 point cloud sets were converted into continuous data sets).

In addition to comparison of discrete and continuous data capture devices, a separate comparison between the FARO® Laser Scanner 880 HE and Sokkia® 30R Robotic Total Station is necessary for the case
study for quantifying dimensional variability in modular construction. For this comparison between a continuous and discrete data capture device, only the critical coordinates from the Sokkia® 30R Robotic Total Station analysis are compared to the corresponding coordinates in the FARO® Laser Scanner 880 HE analysis. From comparison of these two data capture devices, there are 3 main differences between the deviation analyses (Figure B11). Difference 1 and 3 shows that the total station deviations exceeds the FARO deviation at the same joints by approximately 10 mm. However this discrepancy arises since the targets used to measure the deviation with the total station was placed 20 mm away from the actual joints (in order to be in view of the crosshairs of the total station). In light of this, the deviations at the location of the targets (for difference 1 and 3) in the total station analysis are only larger than the FARO analysis by approximately 3 mm. Difference 2 shows that the total station deviation at the joint is about 15 mm smaller than that of the FARO analysis. However, again due to the placement of the target (being about 20 mm away from the actual joint), the actual discrepancy is closer to 5 mm.

Figure B11: Key differences between the critical joint coordinate data for discrete and continuous data capture devices

Overall each of the 4 data capture devices used in this calibration experiment have similar results in terms of their deviation analyses output. Between the two laser scanners, although there are some notable differences (where discrepancies were as large as 10 mm), comparison of the histogram distributions for these continuous data capture devices were nearly identical. Similarly for the discrete data capture devices, overall their deviation analysis were very comparable. Again, in localized areas, discrepancies were found to be as large as 10 mm. Comparison of the histogram distributions of deviations for the discrete data capture devices is not practical due to the fact that these distributions were based on comparison of a ‘converted’ continuous data set. From the results of this calibration study, it was found that use of the two laser scans can be interchangeable on a given project since they have very similar claimed accuracies (2 mm for the Trimble® GX 3D Laser Scanner and 3 mm for the FARO® Laser Scanner 880 HE), and differences between deviation analyses revealed very similar overall results. For use of discrete data capture devices, again these devices can be interchangeable based on their claimed accuracies. Finally, comparison of continuous data capture (i.e., laser scanner) versus discrete data capture (i.e., total station coordinates), reveals that either device is comparable for capturing geometric data about two distinct geometric states of a steel frame.
Appendix C: Additional Information for Case Study in Chapter 5

MATLAB Code: Area of overlap between two probability distributions

```matlab
%%read excel file and assign vectors for all dimensions of interest
[dev]=xlsread('ref.xlsx');
x_panel=dev(:,1); xbins=-20:5:20;
y_panel=dev(:,2); ybins=-13:2:5;
z_panel=dev(:,3); zbins=-11:2:3;
a_panel=dev(:,4); allbins=-25:2:25;
x_frame=dev(:,5)
y_frame=dev(:,6)
z_frame=dev(:,7)
a_frame=dev(:,8)

%%Chi-square goodness of fit
pd_panels = fitdist(a_panel, 'Normal')
h1 = chi2gof(a_panel, 'CDF', pd_panels, 'Alpha', 0.0018);

%% if h=0, reject null, probability significant to value of alpha, h=1, no correlation

calc_overlap_twonormal(pd_panels.std,pd_frame.std,(pd_panels.mu-5),pd_frame.mu,
35,35,0.01)

%% numerical integral of the overlapping area of two normal distributions:
% s1,s2...sigma of the normal distributions 1 and 2
% mu1,mu2...center of the normal distributions 1 and 2
% xstart,xend,xinterval...defines start, end and interval width
% example: [overlap] = calc_overlap_twonormal(2,2,0,1,-10,10,0.01)
function [overlap2] = calc_overlap_twonormal(s1,s2,mu1,mu2,xstart,xend,xinterval)
clf
x_range=xstart:xinterval:xend;
plot(x_range,[normpdf(x_range,mu1,s1)' normpdf(x_range,mu2,s2)']);
hold on
area(x_range,min([normpdf(x_range,mu1,s1)' normpdf(x_range,mu2,s2)']));
overlap=cumtrapz(x_range,min([normpdf(x_range,mu1,s1)' normpdf(x_range,mu2,s2)']));
overlap2 = overlap(end);
legend([num2str(overlap2)]);
```

MATLAB Code: scan-to-scan deviation analysis for module on two temporary support conditions

```matlab
clear all
M=textread('revit.txt');
M=M(:,1:3);
D=textread('scan.txt');
D=D(:,1:3);
index=min([length(M); length(D); 50000]);
for i=1:index
    DDS(:,i)=D(ceil(rand()*length(D)),:);
    %creates a random sampling for the scanned points to be the same with number of available points in M matrix
    MDS(:,i)=M(ceil(rand()*length(M)),:);
end
D=D*rot_mat(pi,pi,0);
DDS=transpose(DDS'*rot_mat(pi,pi,0));
plot3(DDS(1,:),DDS(2,:),DDS(3,:),'b',MDS(1,:),MDS(2,:),MDS(3,:),'r');
xlabel('X'); ylabel('Y'); zlabel('Z');
```
\[ \text{Ricp} = \text{icp(MDS, DDS, 30)}; \]
\[ \text{Dicp} = \text{Ricp} \times D' + \text{repmat(Ticp, 1, length(D))}; \]
\[ \text{DDScicp} = \text{Ricp} \times \text{DDS} + \text{repmat(Ticp, 1, length(DDS))}; \]
\[ [f, d] = \text{match\_bruteForce(MDS, DDScicp)}; \]
\[ \text{plot3(DDScicp(1,:), DDScicp(2,:), DDScicp(3,:), '.b', MDS(1,:), MDS(2,:), MDS(3,:), '.r');} \]
\%n = Number of colors
\n\n\n\n\n\n\[ n = 11; \]
\[ \text{colorth(MDS, DDScicp, n);} \]
\[ \text{colorbar('location', 'EastOutside', 'YTickLabel', ...} \]
\[ \{'0 mm', '2 mm', '4 mm', '6 mm', '8 mm', ... \]
\[ \{'10 mm', '12 mm', '14 mm', '16 mm', '18 mm', '20 mm'})); \]
\[ \text{axis equal;} \]

**Additional Programmed Functions:**

```matlab
function m = rot_mat(omiga, phi, kappa)
    % omiga= input('rotation about x-axis=');
    % phi= input('rotation about x-axis=');
    % kappa= input('rotation about x-axis=');
    m_omiga = [1 0 0; 0 cos(omiga) sin(omiga); 0 -sin(omiga) cos(omiga)];
    m_phi = [cos(phi) 0 -sin(phi); 0 1 0; sin(phi) 0 cos(phi)];
    m_kappa = [cos(kappa) sin(kappa) 0; -sin(kappa) cos(kappa) 0; 0 0 1];
    m = m_kappa * m_phi * m_omiga;
end
```

```matlab
function [vis, d] = color(q, p, nc)
    % p is the transformed point cloud and nc is the number of colors for
    % visualization
    [f, d] = match_bruteForce(q, p);
    c = d;
    c = round((nc-1)*c/.2+1);
    figure;
    vis = scatter3(p(1,:), p(2,:), p(3,:), 3, c, 'filled');
end
```

```matlab
function [match, mindist] = match_bruteForce(q, p)
    m = size(p, 2);
    n = size(q, 2);
    match = zeros(1, m);
    mindist = zeros(1, m);
    for ki = 1:m
        d = zeros(1, n);
        for ti = 1:3
            d = d + (q(ti,:) - p(ti, ki)).^2;
        end
        [mindist(ki), match(ki)] = min(d);
    end
    mindist = sqrt(mindist);
end
```
Structural calculations for the elastic deflection due to self-weight of the structure on 4 supports at the corner of the module

Assumptions:
- neglect weight of ‘light’ bracing (shown in red):

- neglect weight of corrugated roofing
- assume weight of concrete = 24 kN/m$^3$
- column height -> use average centreline-centreline height (since slight slope across length of module).

General Information:
- concrete thickness = 100 mm
- concrete width (between longitudinal beams) = 2560 mm
- all columns = W200x59
- all longitudinal beams = W310x74

Calculations:
Column Height: Nominal column centreline-centreline height = (3350 + 3660)/2 = 3505 mm. Subtract depth of beam above/below column to get just height of column. Depth of beam (W310x74) = 310mm Therefore height of all columns = 3505 - 310 = 3195 mm.
### General Information

**Beam/Support Properties**

<table>
<thead>
<tr>
<th>Section</th>
<th>Unit Weight (kN/m)</th>
<th>Section</th>
<th>$I \times 10^6$ mm$^4$</th>
<th>$E$ (GPa)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W310x74</td>
<td>0.726</td>
<td>W310x74</td>
<td>164</td>
<td>200</td>
<td>15.48</td>
</tr>
<tr>
<td>W250x89</td>
<td>0.879</td>
<td>Outer-outer Length of Beam (m)</td>
<td>15.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W150x37.1</td>
<td>0.364</td>
<td>Support-support Length (subtract W250x89 flange width)</td>
<td>15.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W200x59</td>
<td>0.582</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Deflection Due to Framing

<table>
<thead>
<tr>
<th>Section</th>
<th>Length (m)</th>
<th>Weight (kN)</th>
<th>Point Load (kN)</th>
<th>&quot;x&quot; (m)</th>
<th>Largest Distance from (either) Support, &quot;a&quot; (m)</th>
<th>M.S. Deflection (m)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>W310x74</td>
<td>3.195</td>
<td>2.32</td>
<td>2.32</td>
<td>7.74</td>
<td>9.625</td>
<td>0.0050</td>
<td>second column from left</td>
</tr>
<tr>
<td>W310x74</td>
<td>3.195</td>
<td>2.32</td>
<td>2.32</td>
<td>7.74</td>
<td>11.985</td>
<td>0.0035</td>
<td>third column from left</td>
</tr>
<tr>
<td>3*W150x37 + W310x74</td>
<td>1.275</td>
<td>2.318</td>
<td>2.318</td>
<td>7.74</td>
<td>9.625</td>
<td>0.0050</td>
<td>upper frame load path to second column</td>
</tr>
<tr>
<td>W310x74</td>
<td>5.855</td>
<td>4.251</td>
<td>4.251</td>
<td>7.74</td>
<td>9.625</td>
<td>0.0092</td>
<td>upper frame load path to second column</td>
</tr>
<tr>
<td>2*W150x37 + W200x59</td>
<td>1.275</td>
<td>1.670</td>
<td>1.670</td>
<td>7.74</td>
<td>11.985</td>
<td>0.0025</td>
<td>upper frame load path to third column</td>
</tr>
<tr>
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<td>4.675</td>
<td>3.3941</td>
<td>3.3941</td>
<td>7.74</td>
<td>11.985</td>
<td>0.0050</td>
<td>upper frame load path to third column</td>
</tr>
<tr>
<td>W150x57</td>
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<td>0.4641</td>
<td>0.464</td>
<td>7.74</td>
<td>14.125</td>
<td>0.0003</td>
<td>1st cross brace</td>
</tr>
<tr>
<td>W150x37</td>
<td>1.275</td>
<td>0.4641</td>
<td>0.464</td>
<td>7.74</td>
<td>12.625</td>
<td>0.0006</td>
<td>2nd cross brace</td>
</tr>
<tr>
<td>W150x37</td>
<td>1.275</td>
<td>0.4641</td>
<td>0.464</td>
<td>7.74</td>
<td>11.125</td>
<td>0.0008</td>
<td>3rd cross brace</td>
</tr>
<tr>
<td>W200x89</td>
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<td>1.1207</td>
<td>1.1207</td>
<td>7.74</td>
<td>9.625</td>
<td>0.0024</td>
<td>4th cross brace</td>
</tr>
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<td>0.464</td>
<td>7.74</td>
<td>8.27</td>
<td>0.0011</td>
<td>5th cross brace</td>
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<td>0.464</td>
<td>7.74</td>
<td>8.985</td>
<td>0.0011</td>
<td>6th cross brace</td>
</tr>
<tr>
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<td>1.275</td>
<td>0.4641</td>
<td>0.464</td>
<td>7.74</td>
<td>10.485</td>
<td>0.0009</td>
<td>7th cross brace</td>
</tr>
<tr>
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<td>0.742</td>
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<tr>
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<td>0.464</td>
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</tr>
<tr>
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<td>0.464</td>
<td>7.74</td>
<td>14.315</td>
<td>0.0002</td>
<td>10th cross brace</td>
</tr>
</tbody>
</table>

**Notes**

- For columns - use average centreline height
- SUM 0.0392
- "x" in formula is half beam length = 15.48m/2

### Deflection Due to Concrete

<table>
<thead>
<tr>
<th>Unit Weight (kN/m3)</th>
<th>Thickness (m)</th>
<th>Half Unit Width (m)</th>
<th>UDL (kN/m)</th>
<th>M.S. Deflection (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>0.1</td>
<td>1.28</td>
<td>2.944</td>
<td>0.06711</td>
</tr>
</tbody>
</table>
Concrete width = 2560mm
Concrete thickness = 100mm
Weight ~ assume reinforced =

A = 1355
B = 1500mm
C = 1165mm

All columns = W200x59
All longitudinal beams = W310x74

Inside-inside = 2550mm

All columns = W200x59
All longitudinal beams = W310x74
Appendix D: Additional Information for New Design-Based Strategies

Case Study 1: Assembly in Modular Steel Bridge

Figure D1: Scale Drawing of Bridge Truss

Figure D2: Simplified Relationship of Assemblies in a Single Bridge Truss
Figure D3: Shop Drawing of Assembly 7
Figure D4: Tolerance Relationship between Parts in Assembly 7

Creation of Part Diagrams

Figure D5: Shop Drawing for Part 25
Figure D6: Part Diagram for Part 25

Figure D7: Shop Drawing for Part 8
Figure D8: Part Diagram for Part 8
Figure D9: Shop Drawing for Part 9

Figure D10: Part Diagram for Part 9
Figure D11: Shop Drawing for Part 11

Figure D12: Part Diagram for Part 11
Figure D13: Shop Drawing of Part 15

Figure D14: Part Diagram of Part 15
Figure D15: Assembly Network for Case Study 1
Tolerance Map and Deviation Map

Rather than displaying the Tolerance Map and Deviation Map similar to the Assembly, this section contains segmented Tolerance Maps and Deviation Maps for each part, as seen in the following figures. For process ID’s in the Tolerance Maps, refer to the fabrication tolerances in the thesis main body.

Figure D16: Tolerance Map (above) and Deviation Map (below) for Part 25
Figure D17: Tolerance Map (above) and Deviation Map (below) for Part 8
Figure D18: Tolerance Map (above) and Deviation Map (below) for Part 9 (left)
Figure D19: Tolerance Map (above) and Deviation Map (below) for Part 9 (right)
Part 15 Diagram (DRF @ 90 degrees from Part 9 DRF)

Part 15 (Left) Diagram (DRF @ 90 degrees from Part 9 DRF)

Figure D20: Tolerance Map (above) and Deviation Map (below) for Part 15 (left)
Part 15 Diagram (DRF @ 90 degrees from Part 9 DRF)

Part 15 (Right) Diagram (DRF @ 90 degrees from Part 9 DRF)

Figure D21: Tolerance Map (above) and Deviation Map (below) for Part 15 (right)
Part 11 Diagram (inclined DRF from Part 9 DRF)

Part 11 (left) Diagram (inclined DRF from Part 9 DRF)

Figure D22: Tolerance Map (above) and Deviation Map (below) for Part 11 (left)
Part 11 Diagram (inclined DRF from Part 9 DRF)

Part 11 (right) Diagram (inclined DRF from Part 9 DRF)

Figure D23: Tolerance Map (above) and Deviation Map (below) for Part 11 (right)
Case Study 2: Assembly in Modular Steel Bridge

Analysis Procedure

The part diagram for floor beam 1 includes all features on the top surface of the beam. Figure D25 shows information from the shop drawings used to create the part diagram. The part diagram shown in Figure D26 includes all features and datums in 2D (X and Y axes), however all relationships not required for the overall Y-dimensional aspect of the tie-in plate analysis have been greyed out.

Figure D24: Assembly Diagram for Case Study 2 (Tie-in Plate Deviations)

Figure D25: Relevant Information from Shop Drawings for Creating Part Diagram for Floor Beam 1
Figure D26: Part Diagram for Floor Beam 1

Part Diagram for Floor Beam 2 (Top Surface of Flange Only)

The part diagram for floor beam 2 includes all features on the top surface of the beam. Figure D27 shows information from the shop drawings used to create the part diagram. The part diagram shown in Figure D28 includes all features and datums in 2D (X and Y axes), however all relationships not required for the overall Y-dimensional aspect of the tie-in plate analysis have been greyed out.

Figure D27: Relevant Information from Shop Drawings for Creating Part Diagram for Floor Beam 2
The part diagram for the tie-in plates includes all features on the top surfaces of the angle (only features on the XY and XZ planes). Figure D29 shows information from the shop drawings used to create the part diagram. The part diagram shown in Figure D30 includes all features and datums in 3D (X, Y, and Z), however all relationships not required for the overall Y-dimension aspect of the tie-in plate analysis are greyed out.
Figure D30: Part Diagram for Tie-in Plates
Figure D31: Assembly Network for Case Study 2 (Tie-in Plate Deviation Analysis)
Table D1: Legend Used for the Assembly Network

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Tolerance Terminology</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Zo" /> <img src="image" alt="XL" /> <img src="image" alt="YL" /></td>
<td>Orientation + Location Tolerance</td>
<td>Tolerances to account for: (1) column sizes, (2) difference in column sizes, (3) column alignment, and (4) welding distortion</td>
</tr>
<tr>
<td><img src="image" alt="Zo" /> <img src="image" alt="XL" /> <img src="image" alt="YL" /></td>
<td>Orientation + Location Tolerance</td>
<td>Tolerances to account for: (1) cross bracing size, (2) difference in sizes between cross braces, and (3) welding distortion</td>
</tr>
<tr>
<td><img src="image" alt="Zo" /> <img src="image" alt="XL" /> <img src="image" alt="YL" /></td>
<td>Orientation + Location Tolerance</td>
<td>Tolerances to account for: (1) positional fit-up, and (2) welding distortion</td>
</tr>
<tr>
<td><img src="image" alt="Beam bottom edge substitute boundary ARF X axis" /></td>
<td>Datum Feature</td>
<td>Reference axis (out-of-plane tolerances of tie-in plates measured with respect to this axis)</td>
</tr>
<tr>
<td><img src="image" alt="XZ Plane Actual Surface Feature" /></td>
<td>Actual Feature</td>
<td>Out-of-plane tolerances of tie-in plates</td>
</tr>
</tbody>
</table>

Populating the Tolerance Map

Sources for deriving tolerance values include:

- ASTM A6/A6M-14 for manufacturing (mill) tolerances
- Tolerance indications on PCL-PMC shop drawings
- Erection tolerances from Design in Modular Construction where applicable
- AISC Code of Standard Practice (COSP) for general fabrication + erection tolerances in lieu of indications on shop drawings or suggested values in Design for Modular Construction

The following sources and equations are used for derivation of form and size tolerances in the Tolerance Map:

- Variability in material (mill): values taken from ASTM A6/A6M-14
  - Sweep = 1/8” x (length in feet)/10
- Fabrication: values taken from COSP
  - For non-compression members, sweep = ASTM stipulated value
- Design in Modular Construction – beam “bowing” tolerance < h/1000
- Tie-in plate form tolerance = 3/128 x (number of inches of flange)

The following sources and equations are used for derivation of orientation and location tolerances in the Tolerance Map:

- Out-of-verticality = h/500 from COSP and Design for Modular Construction
- Out-of-horizontality = h/500 from COSP and Design for Modular Construction
- Orientation between substitute x-axes of beam 1 and beam 2 = maximum envelope for column working points (assumption) from COSP = 1 ½ in = 38 mm
- Position tolerance of tie-in plates (in Y) = assume 3 mm
- Orientation tolerance of tie-in plates (in theta Z) = assume 1 mm
- Position (in Y-direction) of substitute X-axes for beam 1 and beam 2 = -1 /+3 mm (as per shop drawings).

An example of the derivation of orientation tolerances in the Z-direction between Beam 1 and Beam 2 (both on the floor level and at the roof level) is shown in Figure D32.

**Figure D32: Derivation of Orientation Tolerances in Z direction between Beams in Case Study 2**

An example of the derivation of location tolerances in the Y-dimension between Beam 1 and Beam 2 (both on the floor level and on the roof level) is shown in Figure D33.
Populating the Deviation Map

The following procedure was used to derive values in the Deviation Map:

- Collecting as-built data by way of laser scanning (Faro LS 840HE)
- Extraction of a 3D point cloud
- Manipulation of the 3D point cloud in a commercial software (Polyworks)
- Isolation of points of interest in the commercial software
- Manual extraction of coordinates of points of interest
- Calculation using extracted coordinates

Tolerance Maps, Deviation Map and Discrepancies

The following pages contain two Tolerance Maps (Figure D34 and Figure D35), a Deviation Map (Figure D36) and a final comparison map (Figure D37) that outlines discrepancies between all three maps.
Figure D34: Tolerance Map 1 for Case Study 2 (Tie-in Plate Deviation Analysis)
Figure D35: Tolerance Map 2 for Case Study 2 (Tie-in Plate Deviation Analysis)
Figure D36: Deviation Map for Case Study 2 (Tie-in Plate Deviation Analysis)
Figure D37: Comparison Map for Case Study 2 (Tie-in Plate Deviation Analysis)