Performance Comparison Between Variable and Fixed Signature Codes in DS-CDMA Systems

by

Amir Tasbihi

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In a CDMA or SCMA system, users can use a fixed signature for the whole transmission interval. But there is the possibility for two users to have highly correlated signatures. Hence, high interference exists in the system, and it will degrade the performance. On the other hand, if the users use variable signatures for spreading each modulation symbol, then the interference between the users will not be fixed for all the transmissions.In this way, we are avoiding clustering error symbols resulted from the high interference. As we show in this thesis, this prediction about better performance of variable spreading is not always true. We have discussed several scenarios and shown the performance for both the fixed and variable signature codes.

Acknowledgements

Special thanks to Prof. Khandani, for supervising me during the past 2 years; undoubtedly his patience to tolerate a student like me is remarkable. Also I must appreciate my friend, Milad Johnny, who was so patient on discussing the problems, and in addition, taught me how to make perfect diagrams in LaTeX. Finally I would like to express my appreciation to the Huawei's Ottawa Research and Development Center for their financial support of this project.

To my beloved ones:

my parents,

my brother,

and my Uncle Mohammad

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Multiple access

In today's communications, we are interested to share the resources with several users. We can share time or frequency to have time division multiple access (TDMA) or frequency division multiple access (FDMA). These two methods, waste resources when the number of potential users is much larger than the number of simultaneous active users [6]. In another approach, we can discriminate users by dedicating a specific code to each user, which we call code division multiple access (CDMA). As the performance of the CDMA degrades gradually by increasing the number of users, it is often claimed that CDMA is superior to FDMA and TDMA. But it is shown in [7] that in an overloaded AWGN channel, their performances are similar.

In CDMA, each user spreads their signal via the signature dedicated to them; hence, in addition to multiple accessing, we can benefit from the properties of spread spectrum communication as well.

If the signatures of all the users are orthogonal, then there is no interference, but as the users are not synchronous, there is always some amount of interference in the system. The magnitude of the interference between two users, increases as their signatures' inner product increases.

To have a better estimation of the transmitted data, we must employ interference cancellation. Indeed interference plays the role of an additional noise source, which degrades the system's performance and it must be resolved.

1.2 Multiuser detection

Interference cancellation, plays a critical role in multiuser communication systems, and there have been a tremendous work on improving its performance, both in theoretical and practical aspects.

Multiuser detection schemes can be generally divided into two categories. Those, which employ a linear mapping on the received signals after despreading are called linear detectors. Decorrelating detector and MMSE detector fall in this category. The main drawback for these detectors is the difficulty in matrix inversion process. Indeed, in their process, we need to invert a very large matrix, which is not efficient. Combating this issue has led to several papers on circumventing the matrix inversion. For example, Moshavi in [23] has presented the polynomial expansion detectors, which approximate the inverse of a matrix just by the polynomial combination of the matrix. An efficient method for polynomial expansion detectors is presented in [15]. An iterative algorithm to calculate the inverse of the matrix in linear detectors in frequency domain is proposed in [11]. Although the proposed method in that paper, is equivalent to the desired linear multiuser detection for an infinite number of iterations, however, it can be approximated in a finite number of iterations which results the low complexity, low memory consumption and small detection delay.

Interference cancelers are another type of multiuser detection methods. The basic idea of this type of detectors, is to reduce the interference effect of some users, and continue to detect the remaining users in the lower interfered scenario. They can be divided into two subcategories: parallel interference cancelers (PIC) and successive interference cancelers (SIC). The PIC usually operates well in situations, where the users have relatively equal power, where as the SIC is better suited for users with a range of powers [3]. Due to the good performance and the simple implementation (there is no need for matrix inversion), interference cancelers have been focused more in research works. The main drawback of these detectors, is the sensitivity to the initial estimations. Indeed, if the estimations at the initial iterations are weak, then the error will propagate through the remaining iterations and worsen the performance. In another words, if an incorrect tentative decision is made, then the interference from that user will be enhanced rather than canceled [10]. In [35], a hybrid receiver is presented, which uses a linear detection at the first stage and several PIC detectors for the remaining stages. It will make the first decision more reliable. In [3] a hybrid method, combined of both SIC and PIC, is proposed. Although it is more complex than a simple SIC or PIC, but it yields a better performance for each user. The performance of interference cancelers in presence of CSI[1] error is discussed in [46].

1.3 Problem Discussion

We can use fixed spreading for each user during the whole transmission. It means, after dedicating the signatures to the users, each one uses their signature to spread all of their modulation symbols. The positive point of this method is the simplicity in detecting the signature of each user. Indeed, we can use the pre-detected signature of each user, to despread the users' signal, but it has a bad side-effect. In the case of high correlation between the signatures of two transmitters, the interference is significant for all the transmission time. An alternative to remedy this problem is using the variable signature spreading. In that case, although there is the possibility of high correlated signatures between users, but it lasts just for a modulation symbol. In the next modulation symbol, that we use another signature for users, the interference will change. So we have spread the interference-corrupted symbols in the whole transmission interval and by using a channel code, we can obtain a better performance. Does the variable spreading always outperform the fixed spreading? We discuss about this question in this thesis.

1.4 SCMA systems

There is a scenario which users selects their resources among a lot of available resources. If the number of chosen resources is so small regarding to the available resources, the system has a sparsity which is beneficial in decoding. SCMA is a category of multiple accessing which has such a sparsity. Should the users use a variable pattern for transmitting their data or it is better to use a fixed pattern? We will see we have a similar problem as the CDMA case.

1.5 Thesis outline

The thesis can be divided in two parts. First, we will cover the basic concepts needed to understand the problem, and finally, the problem is discussed.

[1] channel state information

In chapter 2, we present the important concept of code division multiple access. The spreading and despreading have been introduced and some methods for generating signature are presented. Its capability to combat multipath fading phenomena is discussed, and the rake receiver is introduced.

Chapter 3 deals with the concept of multiuser detection. Practically, there is interference between users and we must mitigate it, as it degrades the performance. The detectors are divided into two categories, linear and interference cancelers and each of them is discussed separately. Successive and parallel interference cancelers and their pros and cones are discussed at the end of this chapter.

Chapter 4 deals with three methods that can be employed, to enhance the performance. Although the detection methods discussed in chapter 3 mitigate the interference, however the performance is improved more by these methods.

In chapter 5, the main problem for CDMA systems is discussed. We discuss the effect of randomizing the signature during the transmission interval. We see in contrast to our prediction, variable spreading outperforms the fixed spreading only in some cases.Indeed there are occasions that fixed is better. The effect of spreading factor and the number of users on the performance of the system is discussed.

In chapter 6, we have introduced the SCMA systems. Then we have compared the fixed and variable SCMA in FER and maximum achievable rate aspects. The thesis finishes in chapter 7 by some conclusions and suggestions for future works. The proofs for some theorems are included in the appendices.

Chapter 2

Code Division Multiple Access

2.1 Multiple Access

The advent of *multiple access* goes back to 1873, when Thomas A. Edison transmitted two telegraphic messages in one direction through a wire simultaneously[38]. Since that, there was an increasing need to exploit the channel to transmit several signals from different users. For example, in a cellular network, there are several users connecting to a base station by transmitting radio waves in free space. As another example, several ground stations communicating with a satellite.

The main problem of this aim, is to recognize the users' signals. Indeed, as several users are transmitting to a receiver, we receive the superposition of all the transmitted signals and we must extract the signal transmitted by each user. There have been several methods to resolve this issue:

- TDMA: *Time division multiple access* is a method that we use each time slot to transmit the signal of a single user. Indeed, if there are k transmitters in the system, the first time interval is dedicated to the user 1, the second one to the second user $\ldots$, the k^{th} time interval to the k^{th} user and then, we start from the first user again. The non-ideal effects of the channel or the receiver could result the insertion of *guard time* to avoid *cochannel interference* [38]. In the case of 3-user transmission, TDMA procedure is depicted in figure 2.1. Blocks follow each other in time domain, the blue blocks represent users' signals and the gray blocks represent guard time intervals.

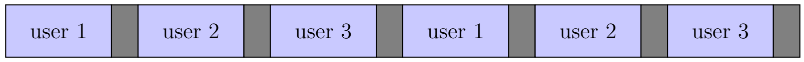


Figure 2.1: TDMA for 3-user system

The main drawback of this method is the common clock that must exist between the users. As it is apparent from Figure 2.1, all the transmitters must be synchronous in a way.

- FDMA: *Frequency division multiple access* is another method, which exploits the frequency band to transmit data from different users. Indeed, instead of time splitting in TDMA, we have frequency band splitting in this method. Different users are allowed to transmit in different bands, so we can discriminate users by filtering the received signal. in cellular networks, users within a cell have different bands, but as interference between users is attenuated severely from a cell to a sufficient far cell, the same band can be reused at far cells which inter-cell interference can be neglected [36]. The same as TDMA, to avoid interference between users, we must have a *guard band* between bands of users. Figure 2.2 depicts a 4-user FDMA system.

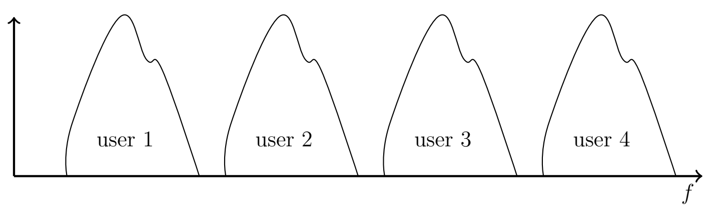


Figure 2.2: FDMA

- CDMA: In *code division multiple access*, the users are separated in code domain, so they can share *resource elements*. By resource element we mean a time-frequency slot which is shown in Figure 2.3. Apparently, in TDMA and FDMA, each resource element is dedicated to only one user, but in CDMA, it can be used by several users since they are separated in code domain. If different users use orthogonal codes for transmission, we can discriminate each other perfectly. For example, if we are interested in k^{th} user's signal, then we can multiply the received signal by the complex conjugate of the k^{th} user's code and integrate it over the code interval. As users' codes are orthogonal, then we have the signal from the k^{th} user at the output. However this is not the real case as it will be discussed in the section 2.2.

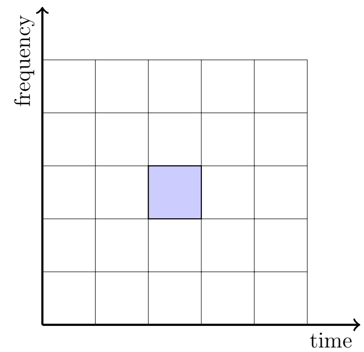


Figure 2.3: resource element definition

- SDMA: *Space division multiple access* is a technique which signals are transmitted from directed antennas. This method allows the frequency band reuse in different areas[41]

There are some other methods for multiple accessing, such as *random multiple access*[38] and *polarization division*[41] as well.

The words *multiplex* and multiple access are often used interchangeably, however, more technically, multiple access is referred to the case that message sources are not collocated and operate autonomously[38].

2.2 CDMA

2.2.1 inner product and orthogonality

As stated in section 2.1, one of the main problems of multiuser transmission is to reduce the interference between different users, and by this, we mean signals on one channel should not significantly increase the probability of error on another channel. For example, if the signals used by users are mutually orthogonal, then the interference among users is totally avoided as [6]:

$$\int_{-\infty}^{+\infty} s_i(t)s_j^*(t) = E_i\delta(i-j) \tag{2.1}$$

which s_i and s_j are the transmitted signals by the users i and j respectively, "*" sign denotes complex conjugate operation, δ is the Dirac delta function and E_i is the signal's energy of the i^{th} user. The left hand side of equation 2.1 is called *inner product* of s_i and s_j and is represented by $\langle s_i, s_j\rangle$, so

$$\langle s_i, s_j\rangle = \int_{-\infty}^{+\infty} s_i(t)s_j^*(t). \tag{2.2}$$

The inner product of two signals can be considered as a measurement of interference among them. In the case that all the transmission signals have the same energy, the inner product is maximized when the signals are the same and this is the worst possible case as we cannot discriminate signals. As it is apparent from equation 2.1, the inner product of orthogonal signals is zero and in this case there is no interference.

To clarify this, consider the system in Figure 2.4 which two users are transmitting antipodal signals to the base station, and their signals are orthogonal to each other. By antipodal we mean users transmit s_1 and s_2 for 1, $-s_1$ and $-s_2$ for 0, and in addition $\langle s_1, s_2\rangle = 0$ which is the definition of orthogonality. At the receiver we have a superposition of the transmitted signals and the additive noise which is considered to be added at the front end of the receiver. Hence:

$$y = b_1s_1 + b_2s_2 + n \tag{2.3}$$

which b_i is -1 for 0 and 1 for 1. The receiver wants to know b_1 and b_2. Thus it multiplies the received signal by s_i and the result is as follows:

$$\int_{-\infty}^{\infty} y(t)s_i^*(t) = b_iE_i + \int_{-\infty}^{\infty} n(t)s_i^*(t)dt = b_iE_i + n'(t) \tag{2.4}$$

As it is apparent from 2.4, there is no interference between users. However, if the users' signals are not orthogonal, there will be an interference term in 2.4.

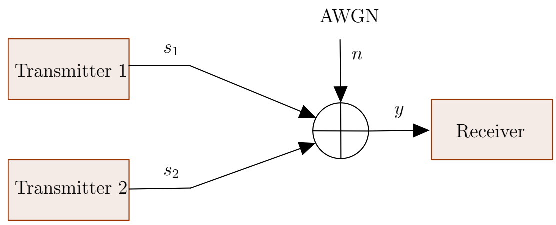


Figure 2.4: an uplink system with two users

2.2.2 spreading

Consider an uplink scenario which users want to transmit their data by a given modulation scheme. The ideal is to extract users' data with the lowest interference. So instead of transmitting modulation symbol m_i for the i^{th} user, we multiply it by a vector $\mathbf{s}_i$ and then we transmit the resulted vector $m_i\mathbf{s}_i$. We do this as we can choose these vectors for different users to be orthogonal. Hence we are able to discriminate users. In the analog domain, this is equivalent to multiplying the modulation signal by a signature signal which is comprised of several *chips*[1]. Figure 2.5 depicts the procedure.

The *spreading ratio* is the number of chips which the signature is comprised of and is denoted by G. From Figure 2.5, it is clear that

$$T_s = G \times T_c \tag{2.5}$$

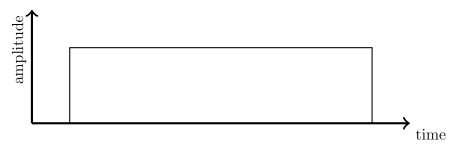
By the proposed explanation, and by the assumption of synchronous users, the output of the spreader is:

$$x(t) = \sum_{k=1}^{K} m_k(t)s_k(t) \qquad 0 < t < T_s \tag{2.6}$$

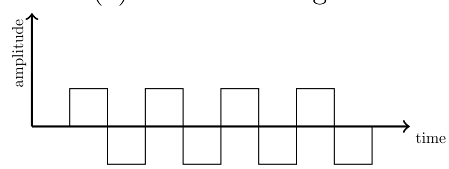
which

- K is the number of users
- $m_k(t)$ is k^{th} user's modulation symbol at the time t
- $s_k(t)$ is k^{th} user's spreading sequence at the time t

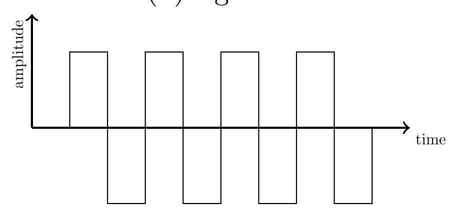
[1]Do not confuse with chirp which is a frequency varying sinusoidal signal



(a) modulation signal



(b) signature



(c) spread signal

Figure 2.5: spreading process

This method of multiple accessing is called code division multiple access, or in abridged form, CDMA.[1]. In the case of AWGN channel, the received signal is:

$$y(t) = x(t) + n(t) = \sum_{k=1}^{K} m_k(t)s_k(t) + n(t) \qquad 0 < t < T_s \tag{2.7}$$

which $n(t)$ is the white Gaussian noise at the time t with variance $\frac{N_o}{2}$ per dimension. If the users are not synchronous, then

$$x(t) = \sum_{k=1}^{K} m_k(t-\tau_k)s_k(t-\tau_k) \qquad 0 < t < T_s \tag{2.8}$$

and τ_k is the delay of the k^{th} user and is considered to be less than the modulation symbol's period, T_s. In this thesis, we will consider synchronous users. The transmission process

[1]It is called Direct Sequence CDMA (DS-CDMA) in some texts. DS-CDMA is used in spread spectrum (SS) communication. There are three methods for implementing SS systems as direct sequence spread spectrum(DSSS), frequency hopping spread spectrum(FHSS), time-hopped spread spectrum (THSS)[12]. Our considered system belongs to DSSS

is shown in Figure 2.6. The energy of the spreading signatures is set to be 1, to have the same amount of energy after despreading at the receiver side. so

$$\int_0^{T_s} |s(t)|^2 dt = \int_0^{GT_c} |s(t)|^2 dt = 1 \tag{2.9}$$

or in the vector notation we have

$$\mathbf{s}.\mathbf{s}^* = 1 \tag{2.10}$$

which dot represents inner product operation.

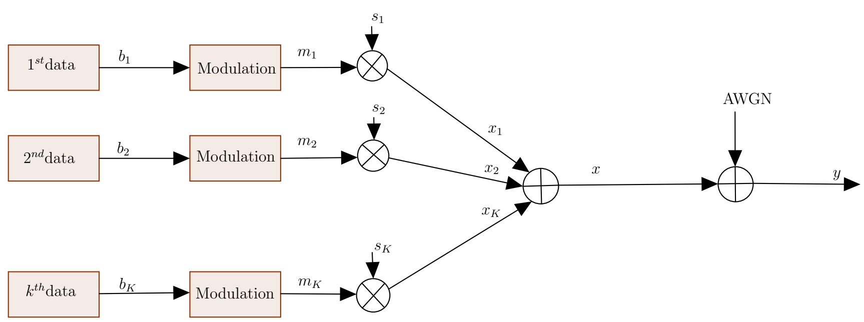


Figure 2.6: spreading and transmission

2.2.3 Despreading

As discussed by the example in Figure 2.4, in the optimal case that the users' signals are mutually orthogonal, there is no interference; the same thing happens when the spreading signatures of users are mutually orthogonal. By multiplying the received signal of the equation 2.7 by the complex conjugate of the i^{th} user's spreading signature, we have

$$\begin{aligned}
&r_i = \int_0^{T_s} y(t)s_i^*(t)dt = \int_0^{T_s} \Big(\sum_{k=1}^{K} m_k(t)s_k(t) + n(t)\Big)s_i^*(t)dt = \\
&\sum_{k=1}^{K} \int_0^{T_s} m_k(t)s_k(t)s_i^*(t)dt + \int_0^{Ts} n(t)s_i^*(t)dt = \\
&m_i + \sum_{\substack{k=1\\k\neq i}}^{K} \int_0^{T_s} m_k(t)s_k(t)s_i^*(t)dt + \int_0^{T_s} n(t)s_i^*(t)dt = \\
&m_i + \sum_{\substack{k=1\\k\neq i}}^{K} \Big(m_k \int_0^{T_s} s_k(t)s_i^*(t)dt\Big) + \int_0^{T_s} n(t)s_i^*(t)dt = \\
&m_i + \sum_{\substack{k=1\\k\neq i}}^{K} \Big(m_k\langle s_k, s_i\rangle\Big) + n'
\end{aligned} \tag{2.11}$$

and $m_k(t)$ came out from the integration since it is constant over a modulation symbol's duration. Figure 2.7 depicts the despreading operation.

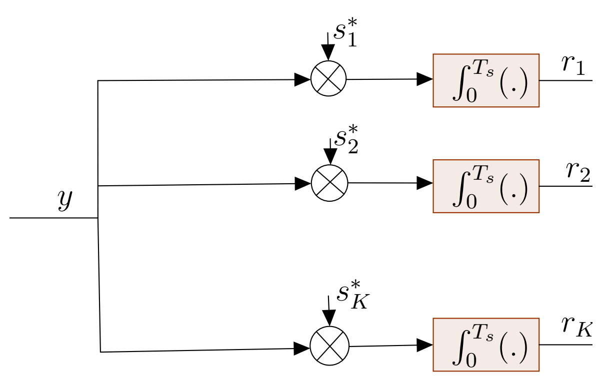


Figure 2.7: Despreading

In Equation 2.11, when the spreading signatures are orthogonal, $\langle s_k, s_i\rangle = 0$ for $k \neq i$ and r_i depends only on the data transmitted from the i^{th} user. In practice we have not orthogonality of signatures and there is an interference, which degrades the performance of the system. There are several methods to mitigate the effect of interference and are discussed thoroughly in chapter 3.

2.3 Signature sequences

Practically, the spreading factor and chip waveform are common among all the users, however it is the assigning of the codes to the users that determines the quality of the multiple access [38]. Indeed we are interested in signatures that will yield the lowest possible interference. An immediate but incorrect suggestion is to use orthogonal signatures. Orthogonal signatures are perfect and have no interference, but this is true only in the case that all the users are synchronous. If $\mathbf{s}_i$ and $\mathbf{s}_j$ are orthogonal, it is not guaranteed that their delayed versions are orthogonal again. So it cannot be a good solution for a more practical scenario where the users are asynchronous. Another approach is to select signatures, which result the cross-correlation between users to be minimized.

Consider we have K users, each with a signature of length G. The n^{th} chip of the k^{th} user is $s_k(n)$. To include the effect of asynchronous transmission, we consider the signatures are periodic with period G[1]. Then the periodic cross correlation, $CC_{jk}(\tau)$, between the sequences of the j^{th} and the k^{th} users is

$$CC_{jk}(\tau) = \sum_{n=1}^{G} s_j(n+\tau)s_k^*(n) \tag{2.12}$$

and the maximum absolute value of the cross-correlations among all the user pairs and delays is denoted as C_c. So

$$C_c = \max_{j \neq k} \max_{0 \leq \tau < G} |CC_{jk}(\tau)|. \tag{2.13}$$

In a similar way, the auto-correlation of j^{th} user is

$$AC_j(\tau) = \sum_{n=1}^{G} s_j(n+\tau)s_j^*(n) \tag{2.14}$$

and its maximum among all users and delays is C_a, hence

$$C_a = \max_{j} \max_{0 \leq \tau < G} |AC_j(\tau)|. \tag{2.15}$$

[1] or we can consider the cyclic shifts of the signatures

If

$$C_{\max} = \max(C_c, C_a) \tag{2.16}$$

then a good approach is to try to minimize $C_{\max}$ [31].

Theorem 2.3.1 *Welch's bound [44]: If there are J normalized complex vectors of length G, then the maximum absolute value of their inner product is bounded by:*

$$\max_{i \neq j} |\langle \mathbf{s}_i, \mathbf{s}_j \rangle|^2 \geq \frac{J - G}{(J-1)G} \tag{2.17}$$

Proof: See appendix A ■

Welch's bound only considers the actual vectors' inner product and does not include their delayed versions. So to apply it in our scenario, we consider KG signatures which are resulted by the original K signatures and all of their delayed versions(each signature has $G - 1$ delayed version as we have assumed to be periodic with period G). So we have KG signature vectors. If we let $J = KG$ in 2.17, then

$$C_{\max}^2 \geq \frac{K-1}{KG-1}. \tag{2.18}$$

Clearly from 2.18, the lower bound decreases if we increase the spreading factor or decrease the number of users. There have been several efforts to design *pseudonoise*(PN) sequences to meet the Welch's bound.

2.3.1 Pseudonoise sequences

In practice, a real random sequence needs arbitrary large storage at both the transmitter and the receiver. So we are interested in a pseudorandom sequence which resembles a random sequence relatively. To satisfy our desire of randomness, the pseudorandom sequence must satisfy the following conditions [40, 12]

- Relative frequencies of 0 and 1 are $\frac{1}{2}$
- For zeros and ones, half of all run lengths are of length 1, one quarter are of length 2, one eighth are of length 3 and so on

- If the random sequence is shifted by any nonzero number of elements, the resulted sequence must have an equal number of agreements and disagreements with the original sequence

The key randomness property of a random sequence, such as a Bernoulli one, can be successfully mimicked by a long deterministic periodic sequence, that can be generated by a simple linear operation of binary parameters [40]. For example, they can be generated by the linear feedback shift registers, as shown in Figure 2.8. In this example, we have 4 shift

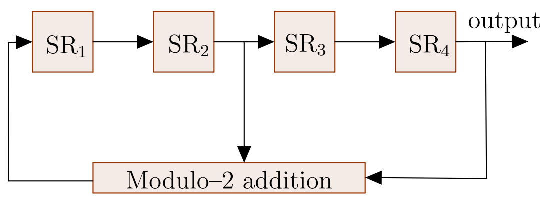


Figure 2.8: PN sequence generator

registers. When we have t shift registers, then there are 2^t states, so in this case we have 16 different states. We can have a *trellis diagram* that shows the transmission between different states. A trellis is a diagram that depicts the state transition as a function of time. For example if there are 4 different states, then the Figure 2.9 can be a trellis for those states, that shows we have transferred from the state 0 to the state 2, then to the state 1, after that to the state 3 and again to the state 0.

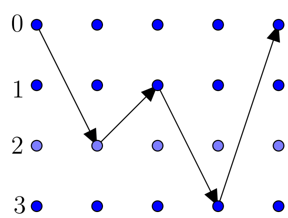


Figure 2.9: 4-states trellis

In a usual trellis (such as a convolutional code's trellis), there can be several transitions from a given state, indeed given the current state, the next state depends on the input. However, the state transitions of the pseudonoise generator, depends only on the current state as the input of the generator is fully determined by the current state. Thus, the transitions must repeat from some point, otherwise we must have infinite states. So the output of the generator is periodic. it is clear that the maximum period will be $2^r - 1$. Sequences that achieve this maximum period length are called *maximal-length* [38]. The output of the generator at Figure 2.8 is 000101 and has period 6, so it is not maximal-length.

There are several methods on designing a good pseudonoise sequence to meet Welch's bound. Kasami [17] and bent [27] sequences are two methods that meet Welch's bound. There is another type of signatures called Gold sequences, that they do not meet Welch's bound, but achieve another bound called Sidelnikov [31].

2.4 Rake receiver

Due to the obstacles, there can be several paths between a transmitter and a receiver. One of the main features of CDMA systems, is their capability to remedy multipath channel's effect. CDMA systems, can distinguish the received signals from different paths and combine them in a way, to exploit multipath diversity. Indeed, in a multipath scenario we have several replicas of the transmitted signal and if we know the channel properties perfectly, we can use them to have a better estimation. The receiver that performs this action is called *rake receiver* [20]. Consider there is only one user, transmitting towards a receiver, but the signal will travel through different paths with different gains and delays. Assume that the delays of paths are an integer multiple of the chip duration and there are M different paths. Then the received signal is

$$y(t) = \sum_{i=1}^{M} \alpha_i x(t - \tau_i) + n(t) \tag{2.19}$$

where α_i and τ_i are the gain and the delay of the i^{th} path respectively and are assumed to be the same for all the chips of a modulation symbol. In addition, assume that the maximum delay is less than a modulation symbol's period. So we can claim the received signal is something like Figure 2.10. Indeed at each time instant, we have the superposition of all delayed and weighted versions of the transmitted signal $x(t)$.

From equation 2.6 we know

$$x(t) = m(t)s(t) \qquad 0 < t < T_s \tag{2.20}$$

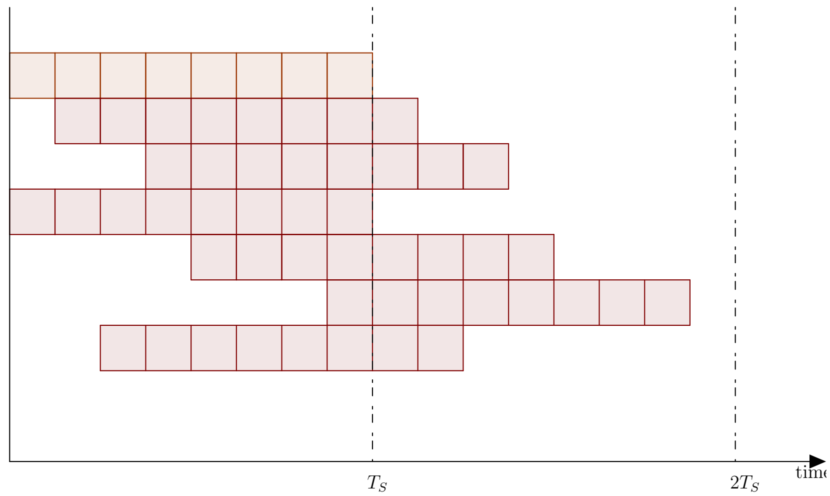


Figure 2.10: multipath transmission

so

$$\begin{aligned} y(t) &= \sum_{i=1}^{M} \alpha_i x(t-\tau_i) + n(t) \\ &= \sum_{i=1}^{M} \alpha_i m(t-\tau_i) s(t-\tau_i) + n(t). \end{aligned} \tag{2.21}$$

If we multiply both sides of equation 2.21 by $s^*(t-\tau_j)$ and then integrate over $(\tau_j, T_s+\tau_j)$, we have

$$\begin{aligned} f_j &= \int_{\tau_j}^{T_s+\tau_j} y(t) s^*(t-\tau_j) dt \\ &= \int_{\tau_j}^{T_s+\tau_j} \Big(\sum_{i=1}^{M} \alpha_i m(t-\tau_i) s(t-\tau_i) + n(t) \Big) s^*(t-\tau_j) dt \\ &= \sum_{i=1}^{M} \alpha_i \int_{\tau_j}^{T_s+\tau_j} m(t-\tau_i) s(t-\tau_i) s^*(t-\tau_j) dt + n'_j. \end{aligned} \tag{2.22}$$

As the modulation symbol is fixed over its interval then

$$
\begin{aligned}
f_j &= \sum_{i=1}^{M} m\alpha_i \int_{\tau_j}^{T_s+\tau_j} s(t-\tau_i)s^*(t-\tau_j)dt + n'_j \\
&= \sum_{i=1}^{M} m\alpha_i AC(\tau_j-\tau_i) + n'_j \\
&= m\beta_j + n'_j
\end{aligned}
\tag{2.23}
$$

If the auto-correlation of the signature, $AC(\tau)$, is relatively small for $\tau \neq 0$, then the main term of the equation 2.23 summation is the j^{th} path. Thus we have relatively distinguished signals from different paths. We call f_j, the j^{th} finger, and the whole process as fingering.

Till now, we have M versions of the transmitted signal. We can combine them in a manner to have a better estimation of the transmitted symbol. After weighting each finger and combining, we have

$$
f = \sum_{j=1}^{M} \lambda_j f_j. \tag{2.24}
$$

There are several combining methods:

- **Selection combining:** In this method, we choose the finger with the maximum SNR. If we assume for all fingers the noise terms are i.i.d. with zero mean and variance σ^2, then

$$
f = m\beta_i + n'_i \qquad \text{for a specific } i \tag{2.25}
$$

  and the output SNR for this method is

$$
SNR = \frac{|m|^2|\beta_{\max}|^2}{\sigma^2} \tag{2.26}
$$

- **Equal gain combining (EGC):** As it is apparent from its name, in this method we combine fingers with equal weights. So we have

$$
f = \lambda m \sum_{j=1}^{M} \beta_j + \lambda \sum_{j=1}^{M} n'_j \tag{2.27}
$$

and the SNR is

$$
\begin{aligned}
SNR &= \frac{|\lambda|^2|m|^2\left|\sum_{j=1}^{M}\beta_j\right|^2}{|\lambda|^2 M\sigma^2} \\
&= \frac{|m|^2\left|\sum_{j=1}^{M}\beta_j\right|^2}{M\sigma^2}
\end{aligned}
\tag{2.28}
$$

- **Maximum ratio combining (MRC):** In this method we choose those weights, which result in maximization of equivalent SNR. By equation 2.24, the combined signal is

$$
f = m\sum_{j=1}^{M}\lambda_j\beta_j + \sum_{j=1}^{M}\lambda_j n'_j \tag{2.29}
$$

and the SNR is

$$
SNR = \frac{|m|^2\left|\sum_{j=1}^{M}\lambda_j\beta_j\right|^2}{\sigma^2\sum_{j=1}^{M}|\lambda_j|^2}. \tag{2.30}
$$

By the Cauchy-Schwarz inequality

$$
\left|\sum_{j=1}^{M}\lambda_j\beta_j\right|^2 \le \Big(\sum_{j=1}^{M}|\lambda_j|^2\Big)\Big(\sum_{j=1}^{M}|\beta_j|^2\Big) \tag{2.31}
$$

and the equality holds when λ_j is a linear function of β_j^*. Back to equation 2.30, we have

$$
\begin{aligned}
SNR &= \frac{|m|^2\left|\sum_{j=1}^{M}\lambda_j\beta_j\right|^2}{\sigma^2\sum_{j=1}^{M}|\lambda_j|^2} \\
&\le \frac{|m|^2\Big(\sum_{j=1}^{M}|\lambda_j|^2\Big)\Big(\sum_{j=1}^{M}|\beta_j|^2\Big)}{\sigma^2\sum_{j=1}^{M}|\lambda_j|^2} \\
&\le \frac{|m|^2\Big(\sum_{j=1}^{M}|\beta_j|^2\Big)}{\sigma^2}
\end{aligned}
\tag{2.32}
$$

and the equality holds if and only if λ_j is a linear function of β_j^*.

For coherent detection with independent branch fading, MRC is the optimal combining method, but for non-coherent detection EGC is used commonly[20]. In above derivations we have assumed that only one modulation symbol has been transmitted, however we transmit a series of modulation symbols and hence in that case we face to inter-symbol interference, ISI (in this case we cannot extract $m(t)$ from the integral of equation 2.22). Beside ISI, we have interference of other users as well. In an attempt to handle both ISI and interference, the *generalized rake receiver* (G-rake receiver) was proposed by Bottomly et al. [43, 2]. In that method, the fingers placement and the combining method is different than the conventional rake receiver discussed above. In [19] there are several methods for reducing the complexity of the algorithm of G-rake receiver. The general structure of the rake receiver has been depicted in Figure 2.11.

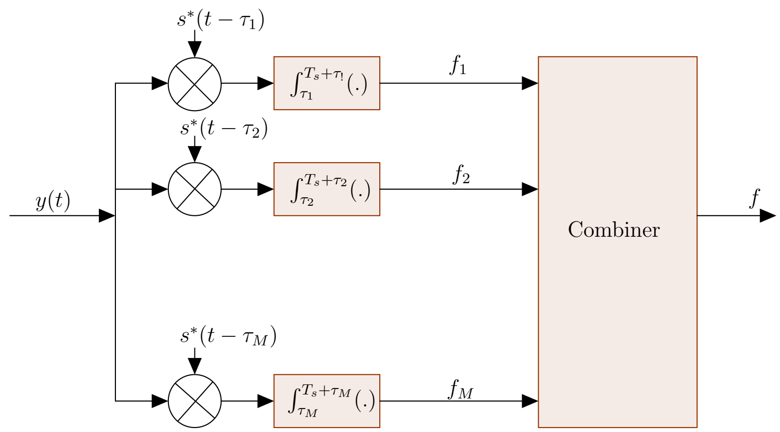


Figure 2.11: Rake receiver for just one user

2.5 Spreading applications

The initial applications of spreading were not in multiple accessing, but in military affairs[1]. As it is evident in Figure 2.5, the spread signal has a larger bandwidth relative to the original modulation signal, and for this reason we call this operation "spreading", as the bandwidth of the signal is spread.

in equations 2.9 and 2.10, we saw that the spreading signature has a unit energy, thus the energy of a modulation symbol before and after spreading must be the same. But as the bandwidth of the spread signal is larger than the bandwidth of the modulation signal, to have the same integral, it's power spectral density (PSD) must have a lower amplitude

[1]Unfortunately, many of the progresses in communication theory are the result of military demands

relative to the modulation signal's one. The larger the spreading ratio is, the larger the bandwidth widening is and hence the lower the amplitude of the PSD will be. It has an application in the case that we want the signal to be undetectable by unintended receivers. If the spreading ratio is large enough, then the PSD level of the spread signal will be less than the ambient noise's PSD level. Thus only the receiver that has the transmitter's spreading signature is able to detect and recover the transmitted data [12]. You must notice that it is totally different from encrypting the signal. In encryption, we change the signal in a way that unintended receivers would not be able to extract the data from their received signal(they detect the signal, but cannot extract information from it). However by spreading the signal, we can make the signal to be undetectable. It is better visualized by looking at Figure 2.12.

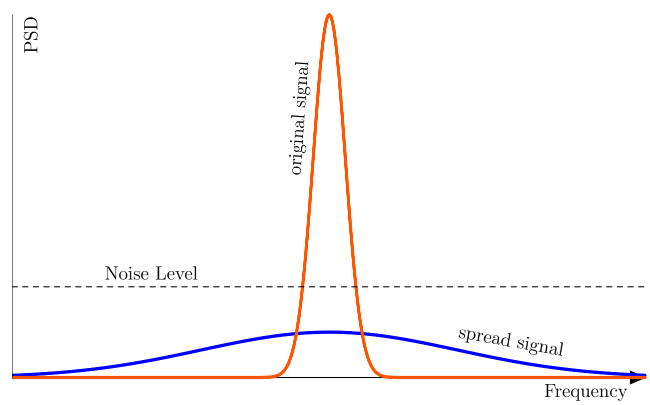


Figure 2.12: a spread signal which is undetectable

There are several other applications for spreading. For example it can be used to mitigate jamming [25] and by changing the structure of the spreading we can have a better performance in presence of jammers as well[5]. It is also used in spread-spectrum radars, to estimate the position and velocity of the objects [31].

Chapter 3

Multiuser Detection

3.1 What is the problem?

The CDMA is so attractive for cellular and personal communications, and it is perhaps for its potential capacity increasing, and some other factors such as anti-multipath fading capabilities [28]. However, systems employing CDMA have shown user capacity limit in the sense that there exists a maximum number of users, that can communicate simultaneously in the channel. This limitation is due to the interference among users that exists beside the channel noise [10]. The optimum detector for CDMA systems, is maximum likelihood sequence estimator. However its complexity arises exponentially with the number of users and therefore, it is impractical. Matched filtering has a low complexity; however, its performance is optimum only in the case that there is no interference, and it is very poor in the case of interfering users. So we must seek a sub-optimum detector with a lower complexity than the maximum likelihood detector and a better performance than the matched filtering [9]. As discussed in chapter 2, due to the time offset of users, it is impossible to have orthogonal signatures, and hence we have interference which we call *multiple access interference*(MAI). In equation 2.11, we saw the output of the despreader is:

$$r_i = m_i + \sum_{\substack{k=1 \\ k \neq i}}^{K} \Big(m_k \langle s_k, s_i \rangle \Big) + n' \tag{3.1}$$

If we write it in the vector-matrix notation, then

$$\mathbf{r} = \mathbf{m} + \Psi + \mathbf{n} \tag{3.2}$$

which $\mathbf{r}$, $\mathbf{m}$, Ψ and $\mathbf{n}$ are the despreader's output vector, the transmitted vector, the interference vector and the noise vector respectively and all of them are of size $K \times 1$. From equation 3.1, we can write the interference vector as:

$$\Psi = \mathbf{P} \times \mathbf{m} \tag{3.3}$$

which $\mathbf{P}$ is the square matrix of signatures' inner products of size K^2 and

$$(p_{i,j}) = \begin{cases} \mathbf{s}_j \;.\; \mathbf{s}_i^* & i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

The decoder that makes a decision on the soft output of equation 3.2 is called *conventional detector*. While the MAI caused by any user is generally small, as the number of interferes or their power increases, the MAI becomes substantial [23]. Another issue is the near-far problem. Such a situation arises, when the transmitters have different geographical locations relative to the receiver. In this case the signal of the closer transmitter undergoes less amplitude attenuation than the signals of the users that are further away[23, 13]. One way to remedy this problem is to look at the MAI as another source of noise, and hence consider the received signal as the output of the despreader in Figure 2.7. However, it is not optimal as there are some information from other users in the interference term, which can be used in detection. In addition, if we mitigate the MAI, its power does not contribute to the noise term, and can improve the performance of the system. So there have been several efforts to invent methods, to identify the interference term and reduce its effect.

The joint detection, measures the distance of the received vector to all of the possible vectors, and selects the most probable one. The problem is that its complexity is exponential with the number of users. Hence several other methods have been investigated, which can be categorized as *linear multiuser detectors* and *interference cancellation-based detectors*[13]. These methods are discussed in sections 3.2.1 and 3.2.2 respectively.

3.2 Several types of detectors

As mentioned in section 3.1, there have been several efforts in recent years on the methods for multiuser detection. However, most of the proposed methods can be placed into two categories: linear detectors and interference cancelers(IC).

In linear detectors, the output of the despreader in equation 3.2, is multiplied by a matrix, **L**, to mitigate the MAI [23]. So the problem is to find the **L**, which mitigates the interference relatively. There are several works that deal with this kind of detectors in different scenarios such as [16, 18, 23]. We will discuss these detectors in section 3.2.1. The general structure of a linear detector is depicted in Figure 3.1.

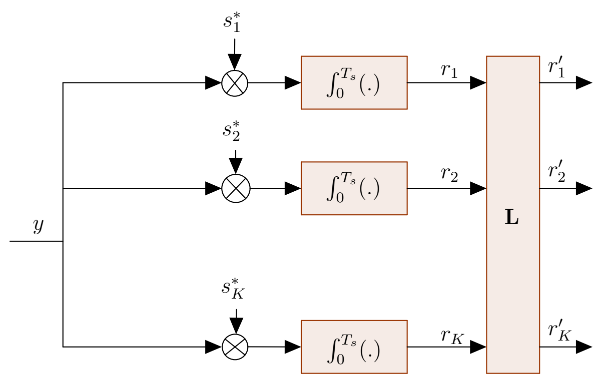


Figure 3.1: linear detector

The linear detectors does not decide on the interference term, but it contains information from other users' transmitted signal. So if make a goof estimation on the interference, then we can have a better estimation. In this method, after making the decision on the interference term, we reduce its effect. There are several methods on interference decision making and the whole process of interference cancellation. Some of these methods can be found in [37, 28, 35, 29, 22] . There is a thorough discussion on these IC detectors in section 3.2.2.

3.2.1 Linear multiuser detectors

By equation 3.2:

$$\begin{aligned}\mathbf{r} &= \mathbf{m} + \Psi + \mathbf{n} \\ &= \mathbf{m} + (\mathbf{P}\mathbf{m}) + \mathbf{n} \\ &= (\mathbf{I}_K + \mathbf{P})\mathbf{m} + \mathbf{n} \\ &= \mathbf{R}\mathbf{m} + \mathbf{n}\end{aligned} \tag{3.5}$$

which $\mathbf{I}_K$ is the identity matrix of size K^2. At the receiver side, we have $\mathbf{r}$ and we want to find $\mathbf{m}$ which is the transmitted vector and is unknown at the receiver side. If there was not any MAI, then $\mathbf{R} = \mathbf{I}_K$ and the problem decreases to a single user transmission over an AWGN channel. However due to the interference, $\mathbf{P}$ is not the null matrix. Linear multiuser detectors, applies a linear transformation on $\mathbf{r}$, to mitigate the effect of $\mathbf{P}$.

Decorrelating detector:

Consider the case that we have only 2 users, and each of them is transmitting 3 modulation symbols. Figure 3.2 depicts the asynchronous signal transmission for this scenario.

We can do the same thing as we did previously, and to detect the 1^{st} symbol of user 1,

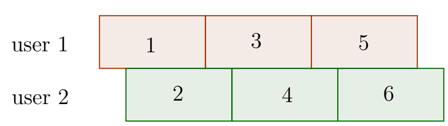


Figure 3.2: asynchronous users are transmitting 3 modulation symbols

only use the interference caused by the 1^{st} symbol of user 2 and so one. However, if we look more precisely, we see that to decode the symbol 1 we need to know the symbol 2, and to detect the symbol 2, we must detect the symbol 3 (as it has time overlap with the symbol 2), and to detect the symbol 3, we need to know the symbol 4 and so on. Hence, instead of making decision on a single symbol, it seems to be better if we make a decision on a block of symbols. As the length of the block increases, we have a more accurate decision on the symbols. But how can we deal with a block? The scenario in Figure 3.2 is equivalent to a system, which has 6 users, but only certain users have interference on a specific user. Indeed in that case, the user 1 has only interference with the user 2 and the user 2 has interference with the users 1 and 3. So we can use equation 3.5, but now, its interference matrix, $\mathbf{R}$, is

$$R = \begin{bmatrix} 1 & \rho_{2,1} & 0 & 0 & 0 & 0 \\ \rho_{1,2} & 1 & \rho_{3,2} & 0 & 0 & 0 \\ 0 & \rho_{2,4} & 1 & \rho_{4,3} & 0 & 0 \\ 0 & 0 & \rho_{3,4} & 1 & \rho_{5,4} & 0 \\ 0 & 0 & 0 & \rho_{4,5} & 1 & \rho_{6,5} \\ 0 & 0 & 0 & 0 & \rho_{5,6} & 1 \end{bmatrix} \tag{3.6}$$

which $\rho_{i,j}$ is the inner product of the i^{th} user's signature with the j^{th} user's one, and is equivalent to the interference of the user i on the user j. Now if $\mathbf{R}$ is nonsingular, then we can multiply both sides of equation 3.5 by $\mathbf{R}^{-1}$ to have

$$\mathbf{R}^{-1}\mathbf{r} = \mathbf{m} + \mathbf{R}^{-1}\mathbf{n}. \tag{3.7}$$

Indeed, we have assumed $\mathbf{L} = \mathbf{R}^{-1}$. As it is apparent from equation 3.7, the output depends only on the transmitted vector, and there is no interference, but the power of the noise has been changed. Indeed, the power of the noise in (3.7) is always greater than or equal to the noise power in (3.5). In addition to the noise power enhancement, the output noise is not white, and we must apply whitening filter to turn it back to a white noise. The main disadvantage of this method, is the matrix inversion process. Indeed if we are considering a block of size N, then the size of $\mathbf{R}$ is $(KN)^2$ and for large blocks or for the case that there are a lot of users, the inversion process is so time consuming[1] [23].

Minimum mean-squared error (MMSE) detector

As we saw, the decorrelating detector does not take into account the noise term, and deals only with the interference term. So, it is trivial to expect a method that considers both terms, interference and noise.

From its name, it is apparent that MMSE seeks a matrix $\mathbf{L}$ to minimize $\mathbb{E}\left[|\mathbf{m} - \mathbf{L}\mathbf{r}|^2\right]$ which $\mathbb{E}[.]$ is the expectation operation. For example, in the case of BPSK signal transmission, the expectation value is minimized when we choose $\mathbf{L}$ as

[1] In practice we try to avoid inverting matrices as long as it is possible, especially when the size of the matrix is so large. The logarithm of the determinant of a random matrix with Bernoulli distributed elements is about $\frac{n\log(n)}{2}$ when n is large enough [24]. Hence, for example in the case of $n = 1000$, the value of the determinant is of order of 10^{1500}.

$$\mathbf{L} = \left[\mathbf{R} + \frac{N_o}{2}\mathbf{A}^{-2}\right]^{-1} \tag{3.8}$$

which $\mathbf{A}$ is the square diagonal matrix, and $(a_{i,i})$ is the received amplitude of the BPSK for i^{th} user [23]. So, in the case that all the users are transmitting 1 or -1 over an AWGN channel, we have [45]

$$\mathbf{L} = \left[\mathbf{R} + \frac{N_o}{2}\mathbf{I}_K\right]^{-1} \tag{3.9}$$

As it is clear from equation 3.8, MMSE method considers both the interference and the noise terms and this is a superiority over the decorrelating detector. However, we still have to deal with the matrix inversion, which is impractical. If we want to implement MMSE (or decorrelating detector), we can solve the linear equation

$$\left(\mathbf{R} + \frac{N_o}{2}\mathbf{A}^{-2}\right)\hat{\mathbf{m}} = \mathbf{r} \tag{3.10}$$

by several efficient methods such as LU decomposition, forward elimination and back substitution [45]. In (3.8), if we let $N_o = 0$, then MMSE will reduce to the decorrelating detector. So although due to the noise term, MMSE detector outperforms decorrelating detector, but they show a similar behavior at the high SNRs.

3.2.2 Interference cancellation-based(IC) detectors

Interference cancellation detectors' philosophy, is that after detecting one or some of the users, we can cancel their effect from the received signal, to have a less interfered signal for detecting other users. In another way, it will feedback the decision, made on a user or some of the users, and makes a new decision based on that; so, they are sometimes referred as *decision-feedback detectors* [23]. For making decision at different steps, we can use several methods including decorrelating and MMSE decoders. There are several methods that are placed in this category.

One of the main drawbacks of this category of detectors, is their sensitivity to the first step decision. Indeed, if we make a bad decision at the first step, the error will propagate through the steps, and will worsen the detection. So, we must know that no cancellation is better than a bad cancellation [23]. Two main methods that are in this category are *successive interference cancellation* (SIC) and *parallel interference cancellation* (PIC).

Successive interference cancellation (SIC)

In this method, we detect the user among all the users that is more probable to be decoded correctly. Then reduce its equivalent effect from the received signal, and keep going with the same procedure for the remaining users. Another approach is to choose a user randomly and then see if we can decode it correctly or not. If we can, then decode it and (after re-spreading and impinging by the channel) reduce its effect, otherwise, decode another user and again return to this user later. If we assume y_i is the input signal of the i^{th} stage, then the diagram of SIC is as Figure 3.3. The flow chart of SIC can be found in [39].

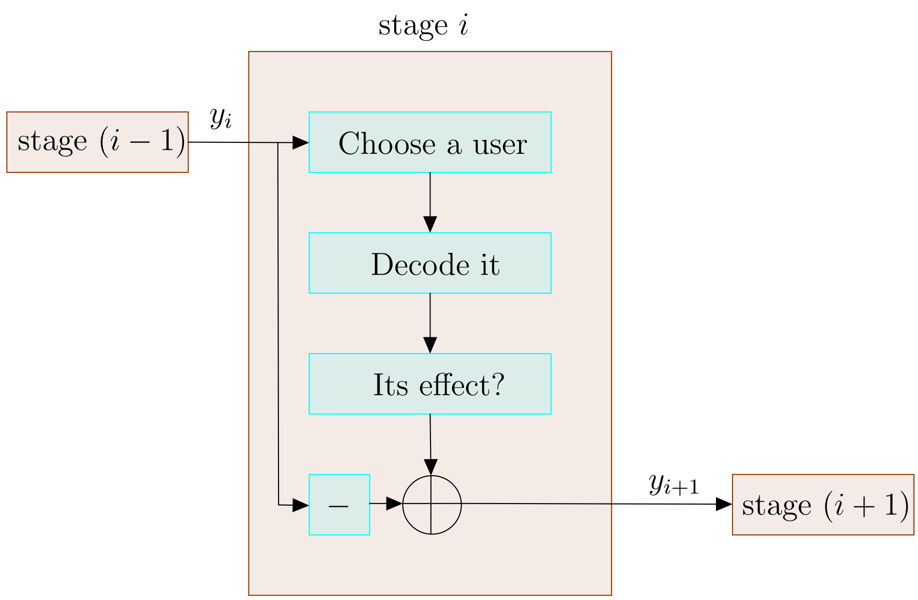


Figure 3.3: i^{th} stage of SIC

Parallel interference cancellation (PIC)

In contrast to SIC that at each stage we decode only one user, in PIC we decode all the users simultaneously. Then for all the users, we subtract the effect of others signals from the received signal, and keep going to the next stage but with taking decision on the new signals. So after each stage we have a set of K estimations for the K users' data. It has the same philosophy as Jacobi method for solving a system of linear equations. Given a system of linear equations, in Jacobi method, we have an initial guess for all the unknowns, and based on that guess, we continue to make new guesses for all the unknowns. If the received signal at the antenna's front end is y, and the signal for detecting the k^{th} user at the i^{th} stage is $y_{i,k}$, then the i^{th} stage is as Figure 3.4 [46]. You must notice that we have subtracted the other users' effect from the original received signal, not from the signal estimated at the previous stage. In addition, at the first stage $y_{1,k} = y$ for $1 \leq k \leq K$.

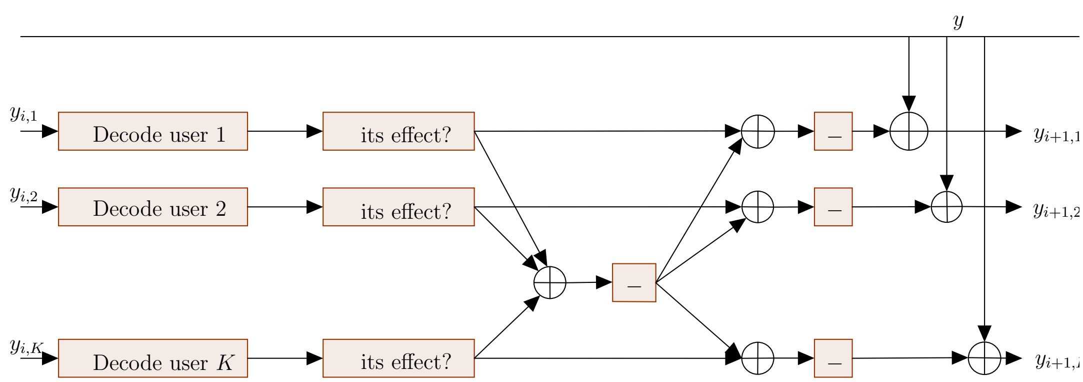


Figure 3.4: i^{th} stage of PIC

As it was mentioned at the beginning of this section, we must have a good detection at the first stage to prevent error propagation to the next stages. So, at the first stage, we can take advantage of several detection methods, which some of them have been discussed so far. However, we must be careful about it. For example, we cannot use MMSE detection for all the stages and we can implement it for the first stage only. This is true since for the next stages, the effect of interference is not the same as the matrix Ψ, and it has another format[1]. In addition, we can use soft decisions in middle stages. By this we mean, instead of approximating users' signals roughly, we can take the expected value of the users' transmitted signals given the received signals. For example, consider the case that there is a point r, which we want to take decision on. If there are 2 possibilities for transmitted signal (such as the case in BPSK), as m and m', then, instead of estimating r as one of m or m' based on its distance to these points, we calculate the probabilities that r is mapped to those points. Then use the expected value point as the approximation for r.

[1]Remember that we have canceled some interference in previous stages, and Ψ represents the interference at the front end of antenna

There are several papers, dealing with performance comparison of multiuser detection methods. Each detector performs better than the other one in some specific scenarios. PIC has this superiority that there is not users' ordering and all the users benefit from interference cancellation. But in SIC, for one of the users we cancel the interference of all other users, and for another one we decide without any interference cancellation [23]; but it needs less hardware than PIC (lower implementation complexity) [29]. In addition, PIC's delay can be smaller than SIC; in SIC we need at least K detection and estimation time intervals, but as we do this in parallel in PIC, it can take fewer time slots [10]. The PIC usually operates well in situations, where the users have relatively equal powers, whereas the SIC is better suited for users with a range of powers (near-far effect) [3].

We can use from the benefits of SIC and PIC simultaneously. In [14], there is a system, which the users are categorized as high rate and low rate. First, we decode one group and use PIC for canceling intra-group interference. Then, its effect is canceled (as same as SIC) for the second group and again we use PIC for the second group. [21] has proposed a method to mitigate the error propagation through the stages. It has used multi-branch multi-feedback detection with shadow area constraint[1].

[1]By shadow area constraint we mean the border between decision regions is not a line but an area

Chapter 4

Some methods to improve the performance

In chapter 3, we discussed how to resolve the multiuser interference to improve the performance of the system. In addition to those methods, we can do some other things to improve the performance further more. These methods are not limited to the CDMA case, and can be used for other scenarios as well. We will discuss these methods in large scale, and for further details, the reader is directed to some references.

4.1 Space diversity

If the channel parameters can be estimated at the receiver, and if the path gains between different antenna pairs behave independently, the use of multiple antennas will considerably increase the achievable rates on the fading channels[34]. The main idea, is the same as two ears are better than one. If we receive two replicas of a signal in a fading channel, then if one of the antennas is under a deep fading, there is the possibility to retrieve the signal via the other one. We can combine the output of the receivers by several methods as the same as combining fingers of the rake receiver. For example, by MRC, we give a greater weight to the antenna with a greater channel's gain.

Example 4.1. There are 2 users, transmitting toward a base station using QPSK modulation, and no channel coding. The multiple access method is CDMA, and the signature length is 8. The channels for the users are slow fading Rayleigh distributed, and have only one tap. There are two different scenarios. First, there is one receive antenna at the base station, and in the second scenario there are two antennas with independent paths. By independent, we mean paths h_1, h_2, h_3 and h_4 in Figure 4.1 are independent complex Gaussian random variables with unit variances and zero means (which results their magnitude to be Rayleigh distributed). The receivers are combined by MRC method. The BER and SER for the two scenarios are shown in Figure 4.2. $\triangle$

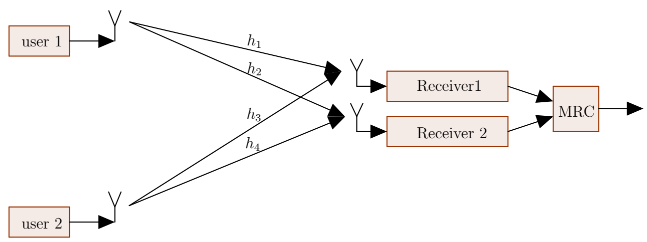


Figure 4.1: Example 4.1 scenario: two receivers

Although the example 4.1 describes the uplink system, however we can have a similar idea for the downlink system, but the problem is the space and the cost limitations. Indeed we can have several antennas at the base stations, but in the downlink scenario, which users are receiving the signal via a small device (such as a cell phone), we cannot have several receiver antennas. This problem was resolved in the seminal paper of Alamouti [1], which he showed we can have several transmitters instead of receivers and transmit by a simple technique and have the same performance as the multi-receiver scenario[1].

[1]Before Alamouti, Tarokh et al. proposed the same idea but they used trellis codes, shortly after their seminal paper [33], Alamouti proposed his method which he used a simple block code in his scheme. For further and deeper study, look at [42].

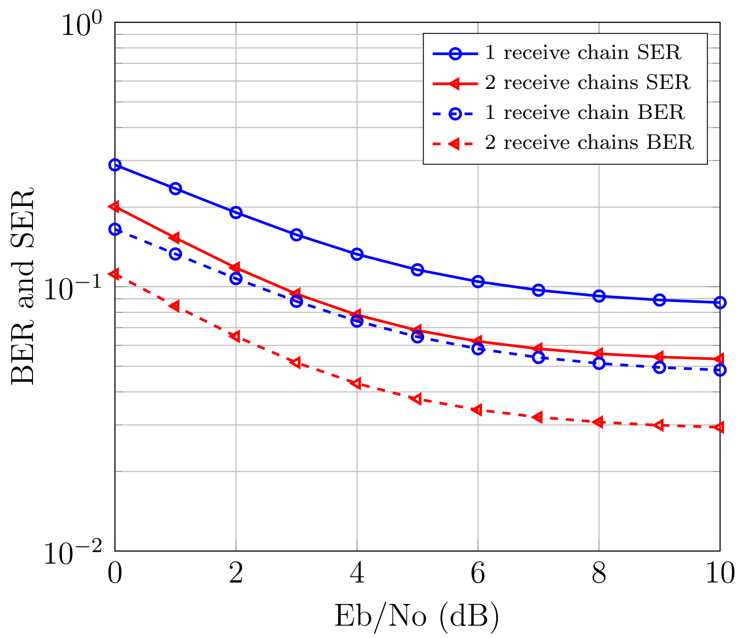


Figure 4.2: SER and BER performance for example 4.1

4.2 Random phase rotation

In a slow fading channel, the rate of the channel changing is low relative to the application[1]. So if we transmit several blocks of channel-encoded data over this channel, there is the possibility that all the transmissions, experience deep fading. It will result a burst error at the receiver side which can be beyond of the capability of the channel decoder to resolve. In a fast fading channel, the rate of the channel changing is high, and the symbols which experience deep fading, are distributed through the whole transmission. Hence the channel decoder may have the ability to correct them, although the averages of deep faded symbols are the same for both scenarios. The trade off, is the difficulty in training the channel's coefficients. In Figure 4.3 the significant better performance in fast fading channel is clear. It is for 2 receive chains, QPSK modulation and a $(127, 50, 13)$ BCH encoder.

[1]A channel can be regarded to be fast fading for a particular application, while is considered slow fading for another application. So we can not say generally that a channel is fast fading or flat fading[36]. Through the thesis when we say a channel is fast/slow fading, we mean relative to the application we have that case, but we do not mention that explicitly, unless it may result an ambiguity.

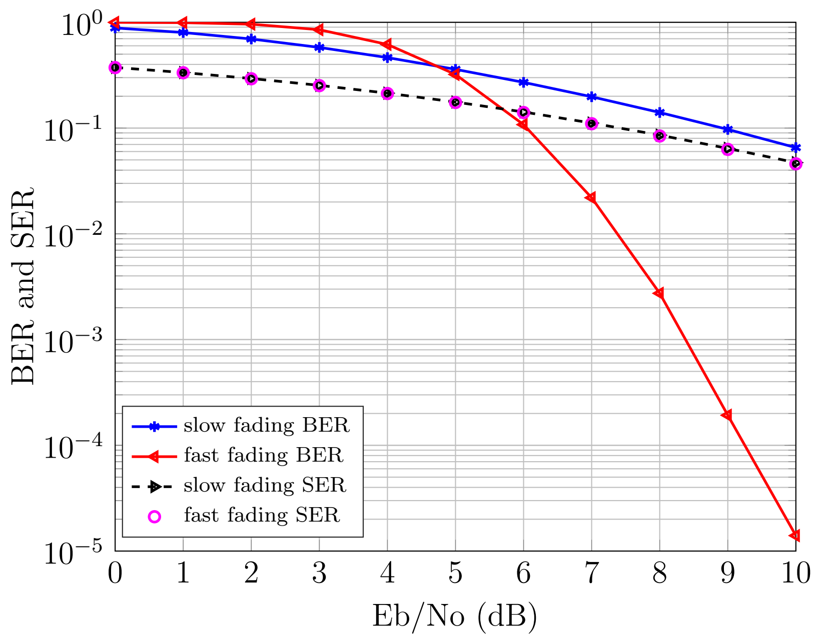


Figure 4.3: fast and slow fading channels' performance

Now consider the case that the channel is slow fading. Then, if we make it to show itself as a fast fading channel, then we can benefit from its better performance. The idea is to use several antennas and rotate their output randomly and then combine them. Due to the randomness of the rotations, their combination shows a fast fading behavior. The more the number of antennas is, the more resemblance to a fast fading Rayleigh channel is. Although we are using several antennas, however its complexity is not the same as the concept discussed in section 4.1. Indeed the main cost of decoding is due to the functions performed in the base-band, but this operation is at RF. So we can have one receive chain, which has more than one antenna. The scheme is shown in Figure 4.4.

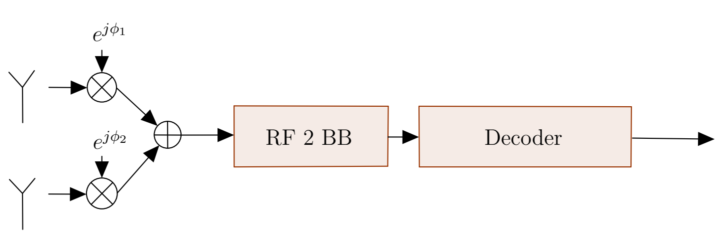


Figure 4.4: Random Phase Rotation

Note that we can get benefit from the section 4.1 in presence of random phase rotation as well. Indeed we can have several receive chains, each of them containing several RF antennas.

4.3 Symbols packing

By using a modulation with more than two symbols, we can transmit several bits in a single symbol transmission. The idea of symbols packing, is assigning bits to these symbols in a manner that yields different error probabilities for different bit positions. For example, consider the 4-ASK modulation as shown in Figure 4.5. As we have 4 symbols, each symbol represent 2 bits. Now what happens when we assign bits to symbols as in Figure 4.5? Clearly for the first bit, we have a minimum distance that results an error probability for that position. But after decoding the first bit, we have to decide on the second bit, given the first bit. So the minimum distance has been increased, and the error probability for this bit position is less than that for the first one. The correct decision's probability for the first and the second bit positions in an AWGN channel is equal to (See appendix B)

$$\begin{aligned} Pr(c_{b1}) &= 0.5\Big((1-A)+G+E+F\Big) \\ Pr(c_{b2}) &= 0.5\Big((1-A)+(1-E)\Big) \end{aligned} \tag{4.1}$$

which, its parameters are integrals of noise's Gaussian density function over some specific intervals shown in Figure 4.6. We have assumed the minimum distance between symbols in Figure 4.5 is $2d$. Note that $(1-A)$ is common in both terms, but

$$\begin{aligned} &G < C \\ &\rightarrow (G+E+F) < \underbrace{(C+E+F)}_{=(1-A)} \\ &\rightarrow (G+E+F) < (1-A) \end{aligned} \tag{4.2}$$

so we conclude

$$Pr(c_{b1}) < Pr(c_{b2}). \tag{4.3}$$

The main property of this method, is treating even and odd bits fairer. Indeed, instead of using a channel code for the whole input data stream, we can use two channel codes with different rates for the input data stream. Indeed, the channel code for odd output bits, has a lower rate as its error probability is higher than the even bits. So we split the input data stream in blocks of length $k_1 + k_2$ and encode the first k_1 bits with a channel code of rate r_1, and the remaining k_2 bits with a channel code of rate r_2 such that

$$r_1 = \frac{k_1}{n} \; , \; r_2 = \frac{k_2}{n} \tag{4.4}$$

note that both encoders maps the input blocks into outputs of the same length n. Then we combine the two encoder's output bit streams such that one of them (with lower encoder's rate) forms the odd bits and the other (higher rate) one, forms the even bits. Thus the total rate is

$$t_{\text{total}} = \frac{k_1 + k_2}{2n} \tag{4.5}$$

The process is depicted in Figure 4.7.

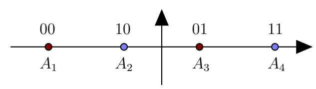


Figure 4.5: A proposed labeling

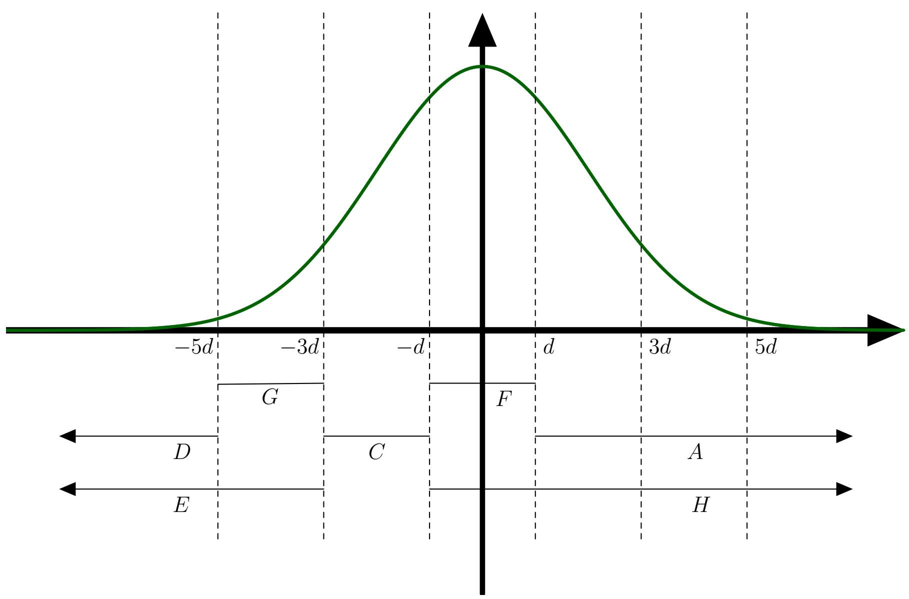


Figure 4.6: Integral intervals for equation 4.1

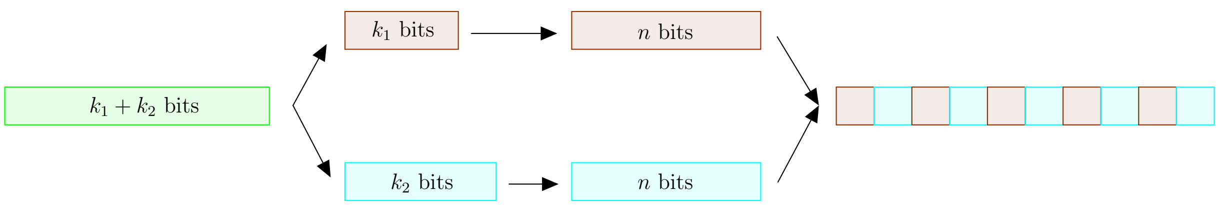


Figure 4.7: data encoding for a symbols packed scenario

Chapter 5

Variable and fixed signatures

5.1 Why variable?

In chapter 2, we saw that interference is a function of inner product of users' signatures. The greater the absolute value of the inner product is, the greater the interference will be. Consider the scenario where all the users are transmitting data toward a receiver. After choosing a signature of a given length randomly, they spread their modulation symbols via that signature through all of the transmissions. In other words, users will not change the signatures that they have chosen randomly. Since the selection is random, the signatures generally will not be orthogonal to each other. The problem that arises in this scenario is the possibility for two users to choose signatures which are highly correlated. In that case, they have a significant interference, which is for the whole transmission and diminishes the performance. This is also the problem when users spread their symbols by a specific signature for a large number of consecutive modulation symbols. To remedy this issue, we can use variable spreading code. This means each user chooses a different signature for each modulation symbol. Then the inner product and hence the interference vary from symbol to symbol, and there is no burst error due to the high multiuser interference. The following example clarifies this problem.

Example 5.1. There are two users, transmitting toward a base station. Both of them use BPSK modulation and a $(127, 50, 13)$ BCH code, for which the error correction ability of the codeis 13. Each user wants to transmit 1016 bits or equivalently 1016 modulation symbols via an AWGN channel. Consider two different scenarios as follows:

a) Both users transmit by fixed signature spreading code, for 100 consecutive symbols. This means that if they select a signature, then they will spread 100 consecutive modulation symbols by that signature. Therefor, we have 11 different inner products. If at least one of the inner products has a magnitude near unity, then the interference is high and we have errors in 100 consecutive symbols or 100 bits. It is a burst error beyond the capability of the channel code to correct.

b) Both users transmit by variable signature spreading code. In this case the interference between users shows a stochastic behavior. In a symbol, due to the great inner product, we have high interference and in another one, relatively low interference. So, if the ratio of symbols that are in deep interference is the same as the previous part, then about 100 symbols are in error. However, the difference is that these erroneous symbols are not clustered; they are distributed through the whole transmission interval, creating the probability for the channel encoder to correct the erroneous bits.

$\triangle$

It is clear that both scenarios have the same **average symbol** error rate. However as discussed in the example, the distributions of erroneous symbols are different; and by using a channel encoder, we can have a better performance in the variable scenario. In another words, the **average frame** error rate is improved by using a variable signature; as a distributed error is more tolerable than a burst error. In Figure 5.1, the symbol error rate and the frame error rate of the example 5.1 are shown. We can see that the average symbol error rate, as expected, is the same for both cases; however, as we are using a BCH channel code, it can correct some of the errors, and will result performance improvement in the variable signature case. As seen in Figure 5.1, this special case has an error floor for fixed spreading. We have used the conventional detector for this example.

5.2 Simulation and results

First, consider we are transmitting through an AWGN channel, and the parameters of the system are as Table 5.1. We assume the users are synchronous.

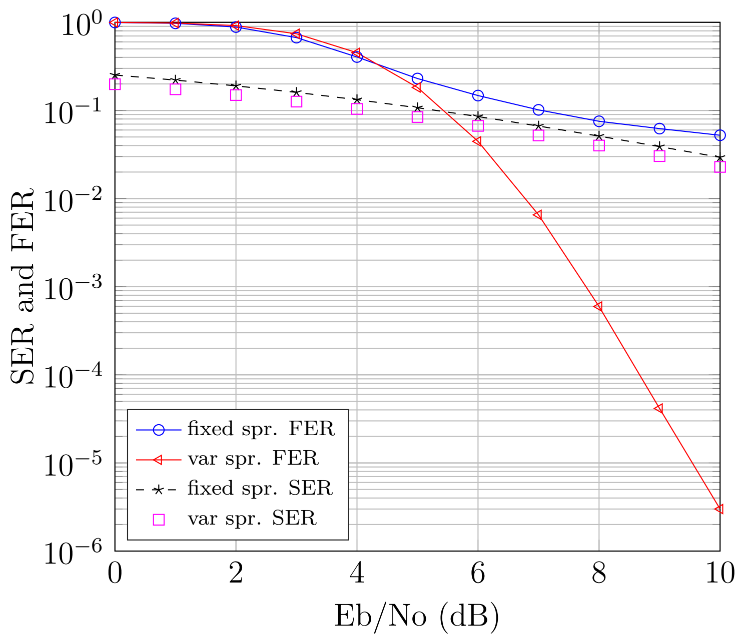


Figure 5.1: FER and SER for example 5.1

Table 5.1: AWGN channel parameters

no. of users	3
spreading ratio	8 , 16
signature type	normalized energy–complex
modulation	16-QAM
Average symbol's power	1
no. of receive chains	2
combining method	MRC

The interference of the users is related to the inner product of their signatures. Particularly, the interference of the j^{th} user over the i^{th} user in a conventional detection scheme will be

$$\sigma_{j,i} = \langle \mathbf{s}_j, \mathbf{s}_i \rangle d_j \tag{5.1}$$

We have assumed that the receive antennas have the same noise level with 2D power spectral density N_o. Then, as the noises of the antennas are independent, and we are combining the two antennas with MRC method (as the channels are AWGN, the coefficient for both antennas is 0.5 which is similar to EGC), the equivalent noise's power spectral density will be $\frac{N_o}{2}$. Since, the users are independent, and we have 2 interferes for each transmitter; so the average of interference power is

$$\mathbb{E}[I^2] = 2\,\mathbb{E}[\sigma^2] \tag{5.2}$$

where $\mathbb{E}[\sigma^2]$ is the variance of the inner product between two signatures. For different spreading ratios, $\mathbb{E}[\sigma^2]$ is represented in Table 5.2.

Table 5.2: interference variance for different spreading ratios

G	$\mathbb{E}[\sigma^2]$	E_b/N_o limit (dB)
4	0.2507	-3.0224
8	0.1259	-0.0312
16	0.0635	2.9414
32	0.0323	5.8819
64	0.0166	8.7680
128	0.0088	11.5268

So we see by doubling the spreading ratio, the average of interference power will be halved. Then the SINR is equivalent to

$$\text{SINR} = \frac{1}{2\,\mathbb{E}[\sigma^2] + \frac{N_o}{2}} \tag{5.3}$$

and when the noise level is so small, the equivalent $\frac{E_b}{No}$ is

$$\frac{E_b}{N_o} = \lim_{N_o \to 0} \frac{\frac{1}{4}}{2\,\mathbb{E}[\sigma^2] + \frac{N_o}{2}} = \frac{1}{8\,\mathbb{E}[\sigma^2]} \tag{5.4}$$

Its value for different values of spreading ratio is shown in Table 5.2 as well. The uncoded BER for spreading ratios of 8 and 16, and for the case where there is only one user transmitting over an AWGN channel, is shown in Figure 5.2. As can be seen, we have error floors at high SNRs, which is exactly equivalent to the BER value of single user at $\frac{E_b}{N_o}$ specified in Table 5.2.

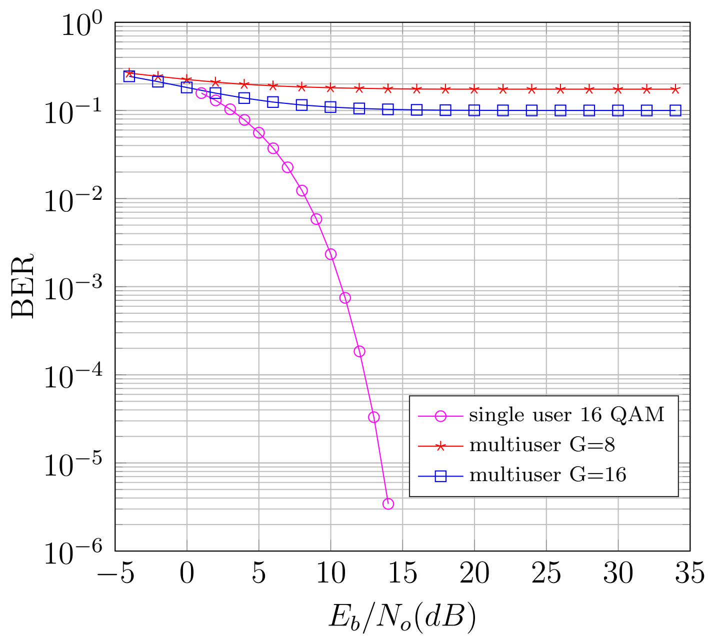


Figure 5.2: uncoded BER for AWGN channel, single user and multiuser

So our designed system is correct. In addition, in a Rayleigh fading scenario, the performance becomes worse. So we cannot have a good performance by using a typical block code, like BCH. In Figure 5.3, the uncoded BER performance of fixed and variable spreading for $G = 8$ is shown.

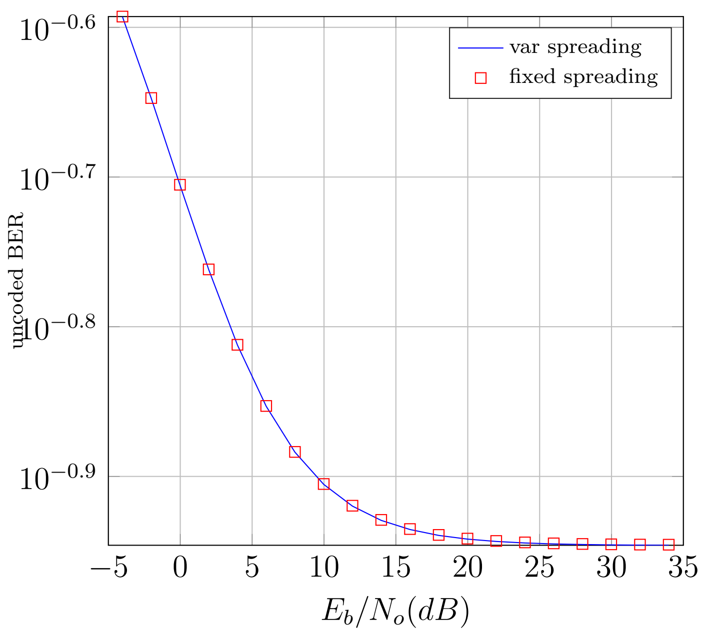


Figure 5.3: Uncoded BER performance of fixed and variable spreading in fading channel

As it is apparent from this figure, the average uncoded BERs for both scenarios are the same. However as variable spreading will distribute high interference errors among all the symbols (in contrast to fixed spreading, where we have a cluster of symbols in deep interference), we achieve performance improvement by using an error correction code. By using a very low rate BCH $(511, 10, 121)$ code, the frame error rate performance for the spreading ratios of 8, 16, 32, 64 and 128 is as seen in Figures 5.4 to 5.8 respectively. As is apparent from these figures, the variable signature scenario has a significant improvement as it has a lower error floor than the fixed signature scenario. Figure 5.9, depicts the uncoded BER performance of the system over a single tap Rayleigh fading channel. All the cases in that figure, have two receive chains. In the fading channel case, the calculation of the average interference power is not as easy as the AWGN channel. For example, in this specific case, the interference term of users 2 and 3 over first user is as follows:

$$i_1 = \sigma_{2,1}\frac{h_2}{h_1} + \sigma_{3,1}\frac{h_3}{h_1} \tag{5.5}$$

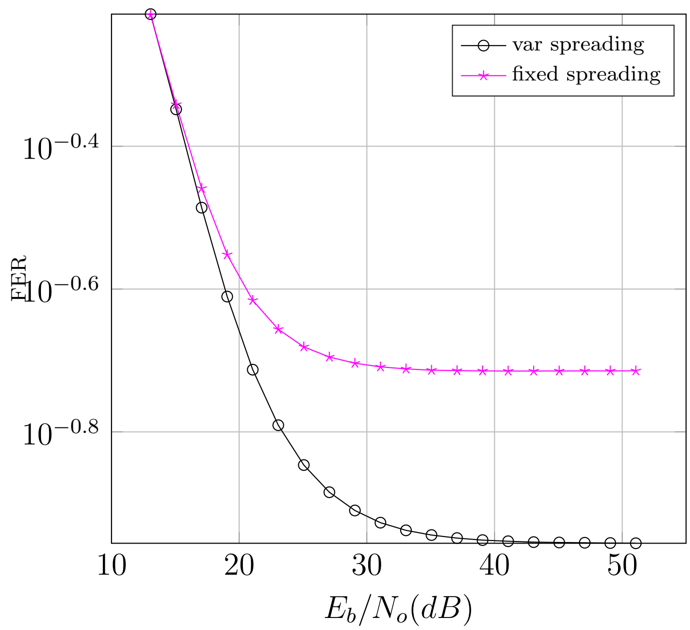


Figure 5.4: FER performance of BCH code with rate of $\frac{10}{511}$ for $G = 8$

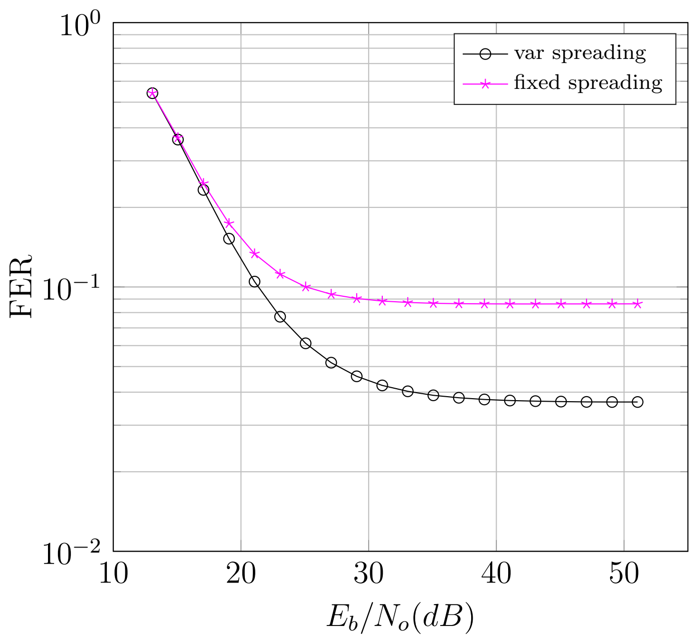


Figure 5.5: FER performance of BCH code with rate of $\frac{10}{511}$ for $G = 16$

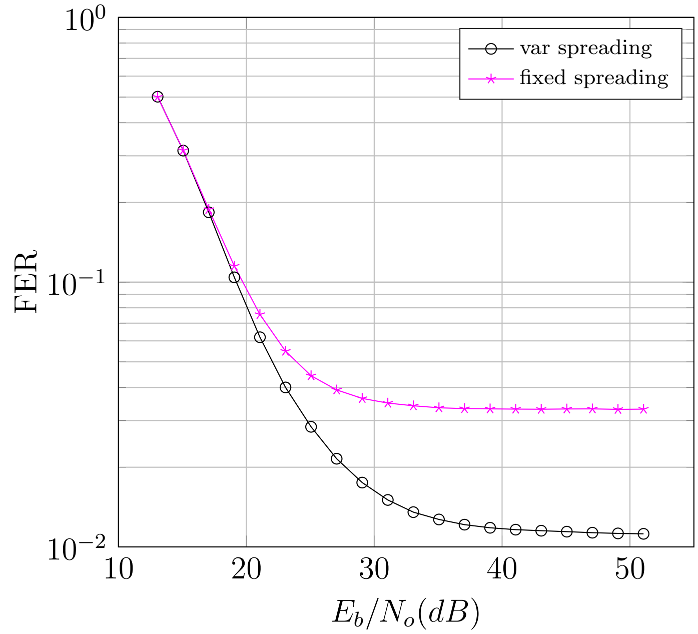


Figure 5.6: FER performance of BCH code with rate of $\frac{10}{511}$ for $G = 32$

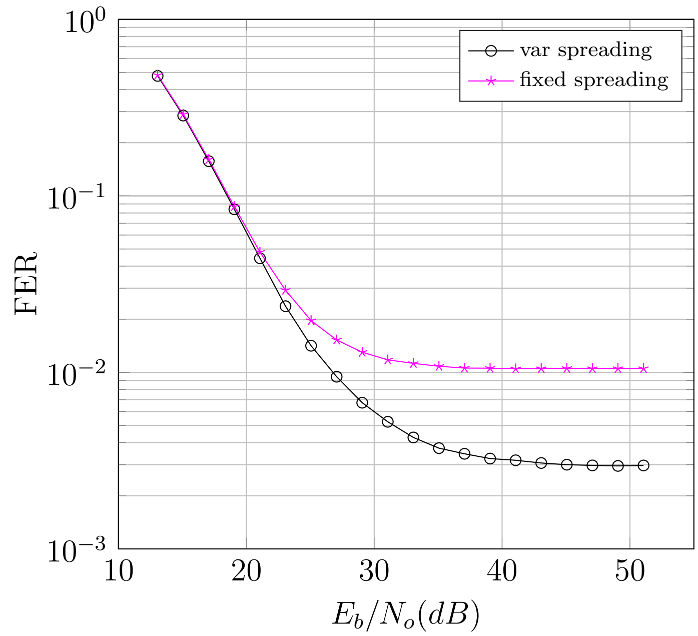


Figure 5.7: FER performance of BCH code with rate of $\frac{10}{511}$ for $G = 64$

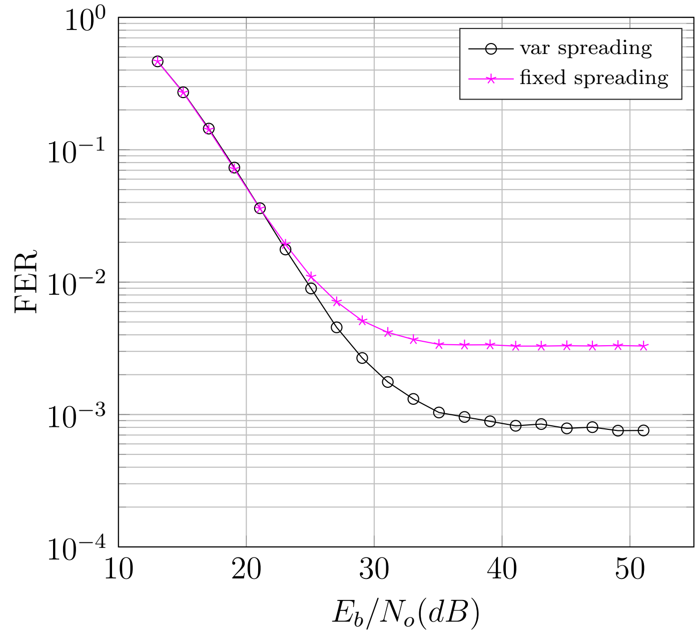


Figure 5.8: FER performance of BCH code with rate of $\frac{10}{511}$ for $G = 128$

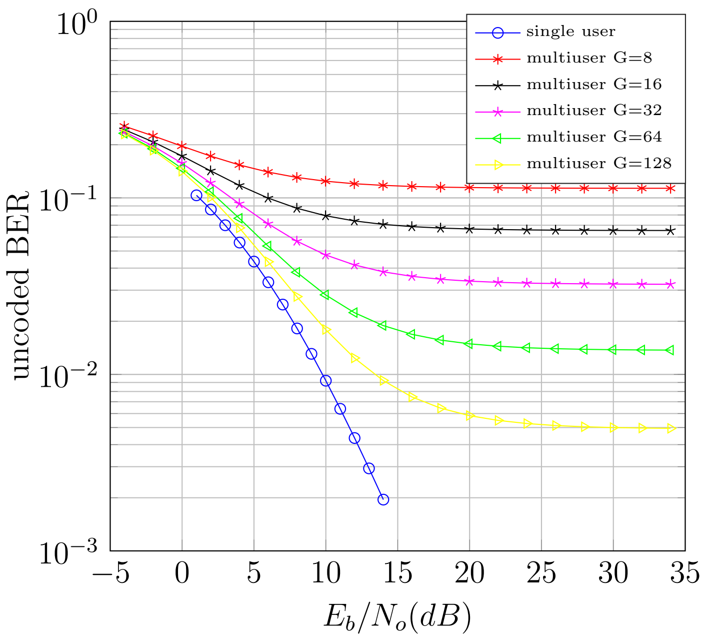


Figure 5.9: Uncoded BER for Rayleigh fading channel and 2 receive chains

Let's consider the performance improvement by using a Turbo code. As implementing this code is time consuming, we have assumed that the system will result an acceptable frame error rate at 1.5 dB greater than the Shannon's SNR, which is reasonable due to [4]. So, we have the following figures for outage probabilities. The graphs have been obtained by comparing the SINR for each user right before the channel decoder's input. If it is greater than a threshold, we can decode it correctly and if it is less than that threshold, we have an error. The only factor that makes difference in the decoder's input SINR of both the variable and the fixed scenarios, is the inner product of signatures. Indeed, in fixed spreading, this SINR is a function of inner product of signatures and the channel coefficients, but in variable spreading, as we are changing the signature from a modulation symbol to another, it is not a function of inner products and just a function of channel coefficients[1]. It is clear that, by increasing the spectral efficiency, the frame error rate of a given system deteriorates, and hence at a given received SNR, the outage probability will decrease. Figures 5.10 to 5.14 are for decoder's input SINRs of 1.5, 2, 2.5, 3 and 3.5 dB respectively.

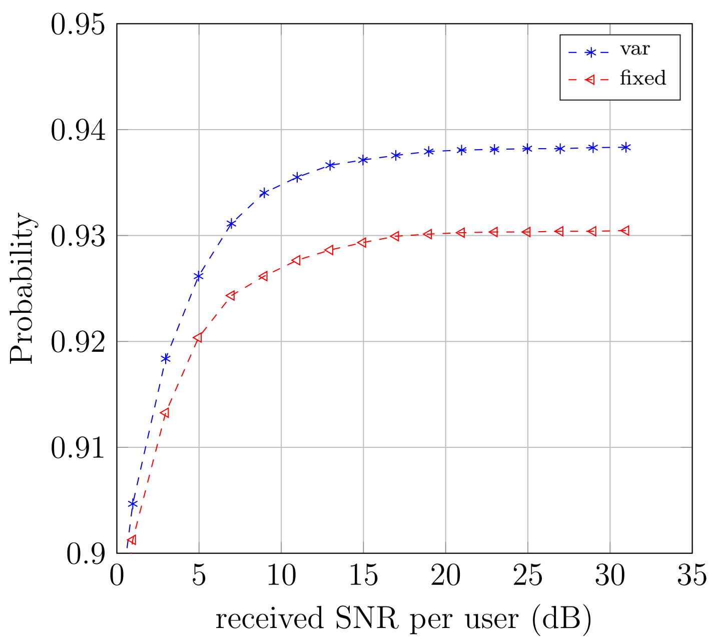


Figure 5.10: Outage vs received SNR per user in SE=$\frac{1}{8}$ bit/sec/user

[1]Indeed in its equation we have the average of inner products

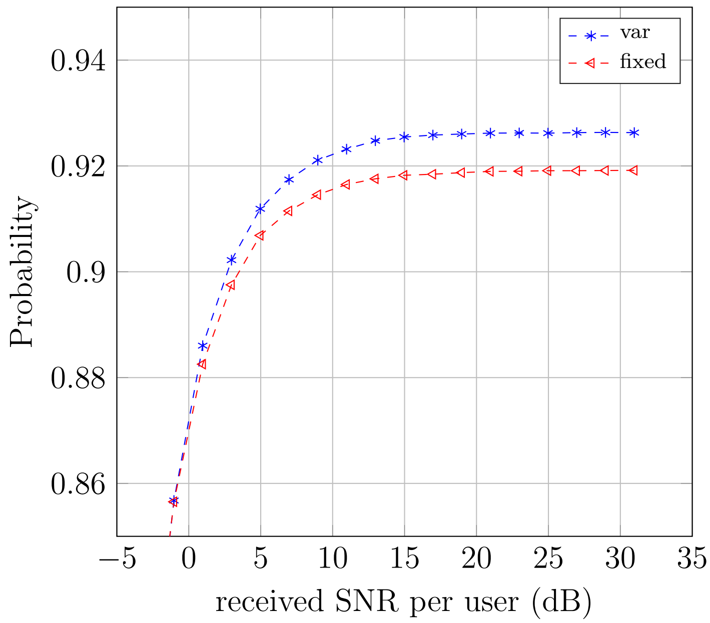


Figure 5.11: Outage vs received SNR per user in SE=$\frac{1.1}{8}$ bit/sec/user

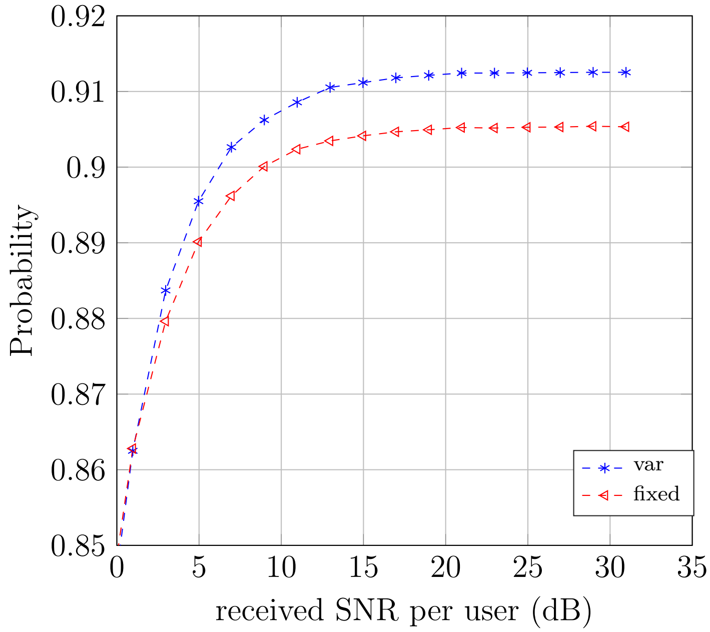


Figure 5.12: Outage vs received SNR per user in SE=$\frac{1.2}{8}$ bit/sec/user

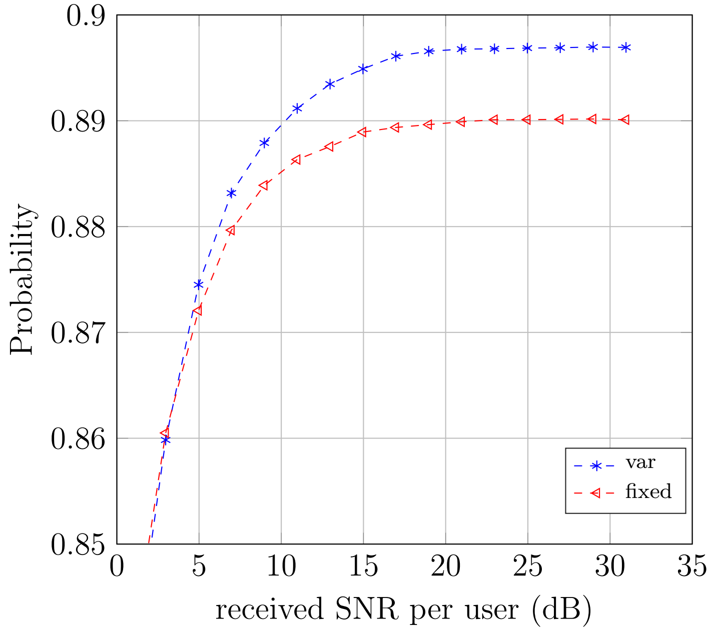


Figure 5.13: Outage vs received SNR per user in SE=$\frac{1.3}{8}$ bit/sec/user

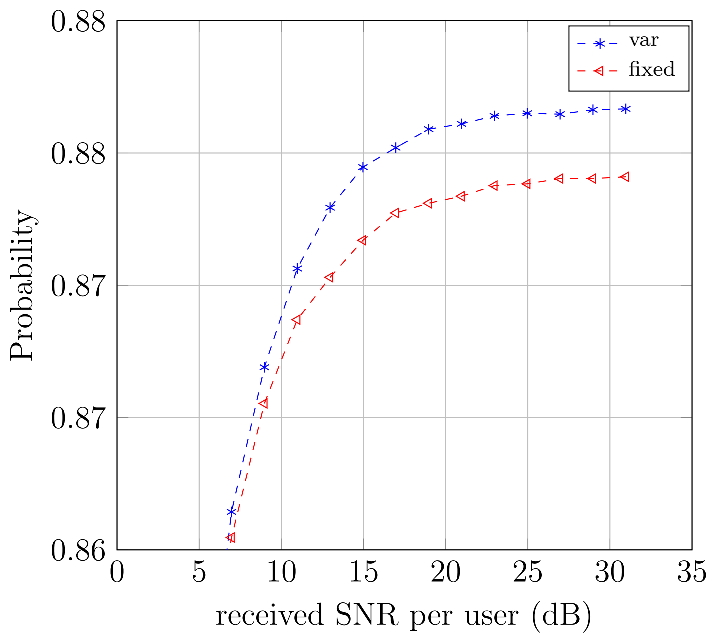


Figure 5.14: Outage vs received SNR per user in SE=$\frac{1.4}{8}$ bit/sec/user

5.3 Does variable spreading always outperform the fixed spreading?

By looking at Figures 5.10 to 5.14, we could conclude that variable signature code scenario always outperforms the fixed signature one; however, this is not true. Indeed, if we sketch the outage figures for a larger range of probabilities, we have performances described in Figures 5.15 to 5.17. As we see, there is the possibility for fixed spreading to be better in low SNRs. Even with high rate channel codes, the fixed spreading can act better for the whole range of SNR interval. But what is the reason for this behavior? Let's look back to the procedure of outage calculating. For a given received SNR, we see what will be the SINR right before the channel decoder. The SINR is a random variable, that depends on the channel coefficients and the inner product of users' signatures and noise power. In variable spreading the SINR is a function of average inner product of users, but in fixed spreading, it is a function of instantaneous inner products. More specifically, the summation of noise and interference for the fixed and variable spreading is

$$z_j^{var} = z_j^{fix} = i_j + n_j = \sum_{k \neq j} \langle \mathbf{s}_k, \mathbf{s}_j \rangle d_k \frac{h_k}{h_j} + \langle \mathbf{n}, \mathbf{s}_j \rangle \frac{1}{h_j} \tag{5.6}$$

where, $\langle \mathbf{s}_k, \mathbf{s}_j \rangle$ is a random variable for variable spreading in contrast to fixed spreading, which is a fixed number, as each user selects their signature. So its power for a single path slow fading channel is

$$\mathbb{E}[Z^2] = \frac{1}{|h_j|^2} \times \left(\sum_{k \neq j} \mathbb{E}[\sigma_{k,j}^2] |h_k|^2 + \frac{N_o}{2} \right) \tag{5.7}$$

for variable spreading and

$$\mathbb{E}[Z^2] = \frac{1}{|h_j|^2} \times \left(\sum_{k \neq j} \sigma_{k,j}^2 |h_k|^2 + \frac{N_o}{2} \right) \tag{5.8}$$

for fixed spreading. The channel coefficients come outside the expected value operation, as we have assumed for each running, the channel coefficients are fixed through the all the transmissions[1]. The SINR at the input of the channel decoder is

$$\text{SINR} = \frac{1}{\mathbb{E}[Z^2]}. \tag{5.9}$$

[1]but you should notice the channel coefficients for different users are different, and they vary from one MATLAB running to another

The SINR at 5.9 denoted by Λ is a random variable. This is due to the randomness of the channel and the inner product of the signatures.

Figure 5.18 illustrates the mean and the variance of fixed and variable spreading SINR for different received SNRs. From equations 5.7 and 5.8, we expect their means to be equal to each other. It can be seen in Figure 5.18; however, the variance of SINR for the fixed spreading is considerably larger than the variable case.

We are interested that the SINR at the input of channel decoder be greater than a threshold, denoted by η. In mathematical representation, we want to find $Pr(\Lambda \geq \eta)$.

Figure 5.19 is the probability density function of Λ, for received SNR= 31 dB. As it is apparent, it behaves nearly as a Gaussian distributed random variable. Let μ and ξ^2 represents the mean and the variance of Λ, hence

$$\begin{aligned} Pr(\Lambda \geq \eta) &= Pr\Big(\frac{\Lambda - \mu}{\xi} \geq \frac{\eta - \mu}{\xi}\Big) \\ &= \begin{cases} Q(\frac{\eta-\mu}{\xi}) & \eta \geq \mu \\ 1 - Q(\frac{\mu-\eta}{\xi}) & \eta < \mu \end{cases} \end{aligned} \tag{5.10}$$

For a chosen η, if it is less than the mean of SINR for a specific received SNR, then the probability of the ability to decode the received signal is $1 - Q(\frac{\mu-\eta}{\xi})$, which becomes larger as ξ decreases, hence in this case, the variable spreading is better than the fixed spreading. But if the chosen η is larger than μ, then the probability of the decoding ability is $Q(\frac{\eta-\mu}{\xi})$, which becomes larger as ξ increases; hence, in this case fixed spreading is better than variable spreading.

We saw variable spreading does not outperform the fixed scenario in all circumstances, and it depends on the threshold SNR of our channel code. If the number of users becomes larger, then the mean of SINR decreases, and for a specific channel code, there is the possibility that fixed spreading play a better role than variable spreading. It is shown in Figure 5.26. In another hand, for a given number of users and hence a given μ, by decreasing the channel code's rate, we are decreasing its threshold SNR, and hence we can see a better performance by employing variable spreading. Note that the PDF in Figure 5.19 is not Gaussian and we have assumed it to be Gaussian; hence, we see the outage probability, in the case that fixed spreading is better, is not bounded by 0.5 (the maximum of Q function), but we can say generally that the two outage probabilities will intersect at the point, where the cumulative distribution functions(CDF) of Λ_{fix} and Λ_{var} are equal.

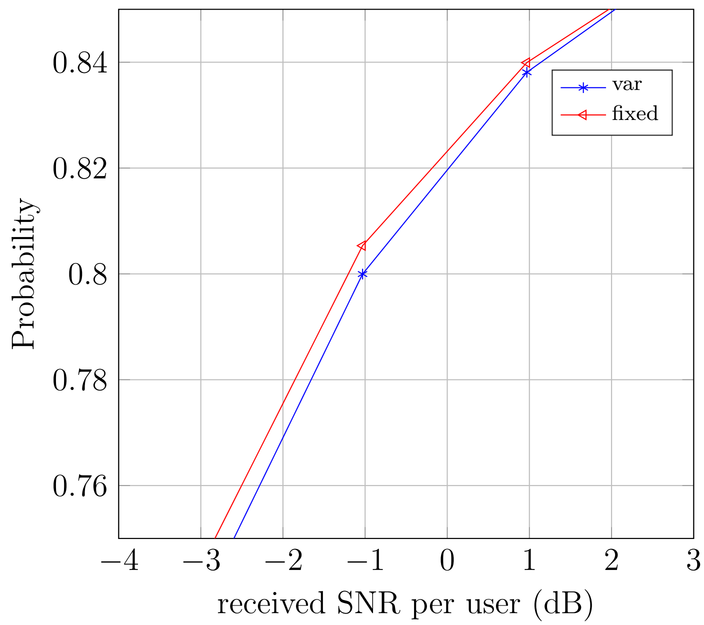


Figure 5.15: Outage vs received SNR per user in SE= $\frac{1.3}{8}$ bit/sec/user

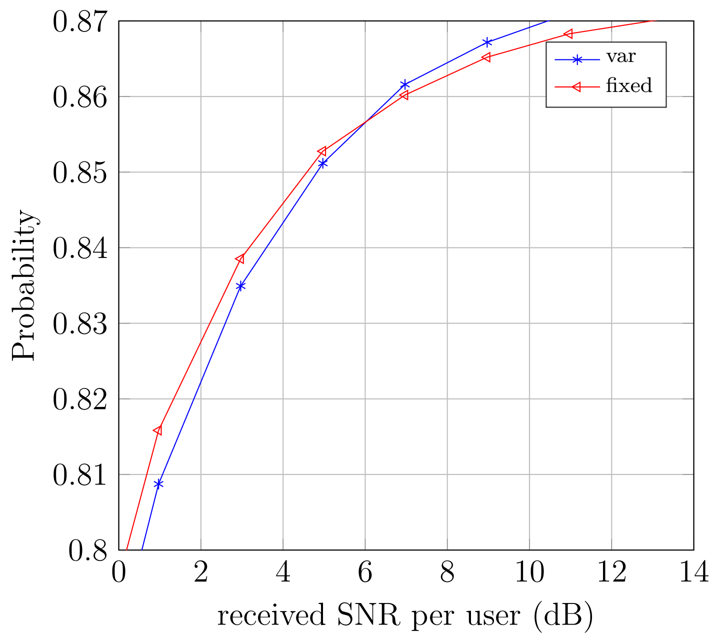


Figure 5.16: Outage vs received SNR per user in SE= $\frac{1.4}{8}$ bit/sec/user

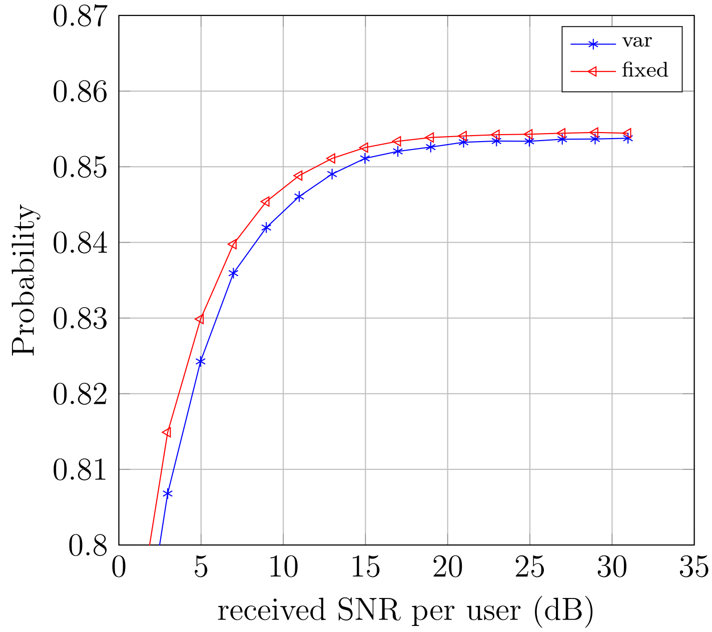


Figure 5.17: Outage vs received SNR per user in SE= $\frac{1.5}{8}$ bit/sec/user

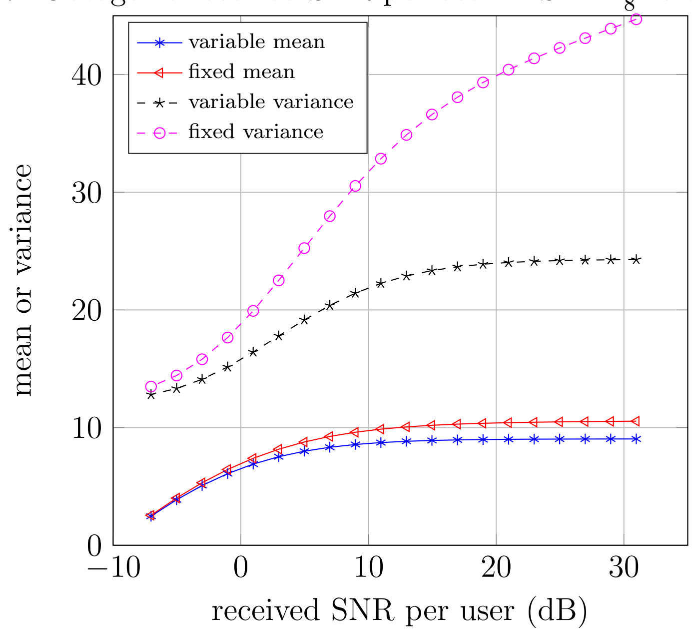


Figure 5.18: Mean and Variance of SINR of channel decoder's input

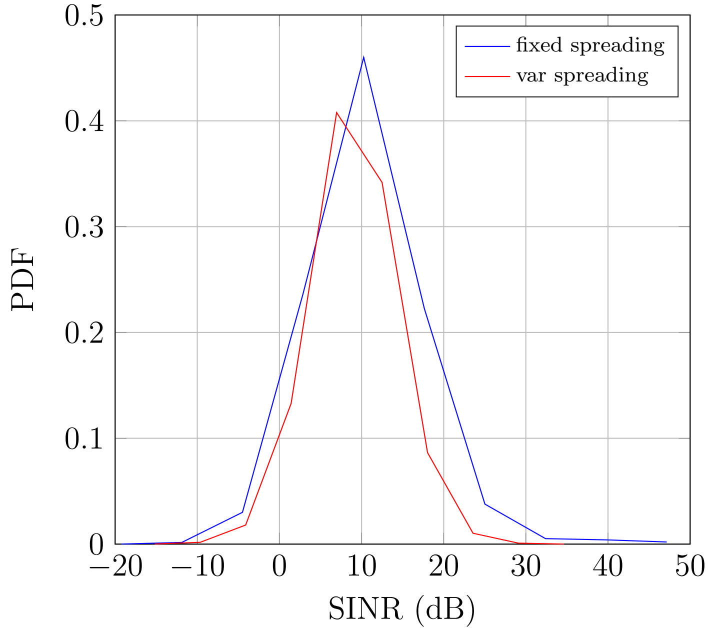


Figure 5.19: PDF of SINR for received SNR= 31 dB

5.4 The effect of the number of users and the spreading factor

Till now, we considered the case that there are 3 users in the system, and we changed the spreading ratio. Now let us change the number of users and the spreading ratio simultaneously, and see different behaviors. Indeed, we are interested in seeing the performance of the system in accordance with $\frac{k}{G}$, which is the ratio of the number of users to the spreading factor. For example, what happens if we increase the users and spreading factor simultaneously, such that their ratio remains the same. As it is apparent from Figures 5.20 to 5.23, for a given $\frac{k}{G}$, by increasing the number of users (or equivalently the spreading factor), the outage probability will deteriorate. For the AWGN channel, the mathematical description is easy. As we saw in Table 5.2, by increasing the spreading ratio by a factor of m, the inner product's mean square will be divided by m and the total interference power will be

$$I_t = (mk-1)\frac{\overline{\sigma^2}}{m} = k\overline{\sigma^2} - \frac{\overline{\sigma^2}}{m} \tag{5.11}$$

and by increasing the spreading factor, the total interference will increase; thus, the performance of the system will deteriorate.

It is clear, for a given number of users, by increasing the spreading factor, the performance will improve; but the trade off is in losing the spectral efficiency. As we can see in Figure 5.24, by increasing the spreading ratio for a given number of users, the performance of fixed and variable signature spreading will merge into each other. It is reasonable, as by increasing the spreading factor, the inner product of users' signatures will be smaller with higher probability. So there will be no benefit in using variable signature spreading in the case that the spreading factor is large enough. For the case of $\frac{k}{G} = \frac{3}{64}$, the outage probability has been magnified in Figure 5.25.

Until now we considered two different cases: first, we fixed $\frac{k}{G}$ and increased the number of users and the spreading factor with the same ratio, and we saw its effect. Then we fixed the number of users and increased the spreading factor and saw its effect as well. Now what if we fix the spreading factor and increase the number of users. The answer is evident, by increasing the number of users for a given spreading factor, we are increasing interference on each user, so the performance will deteriorate. Figure 5.26 shows the performance for the case that $G = 16$. As we see in that figure, by increasing the number of users, there is the possibility that fixed spreading outperforms variable spreading. In Figure 5.26, for 3 and 6 users, the variable is better, for 9 users they do not differ significantly and for 12 users, fixed spreading is better.

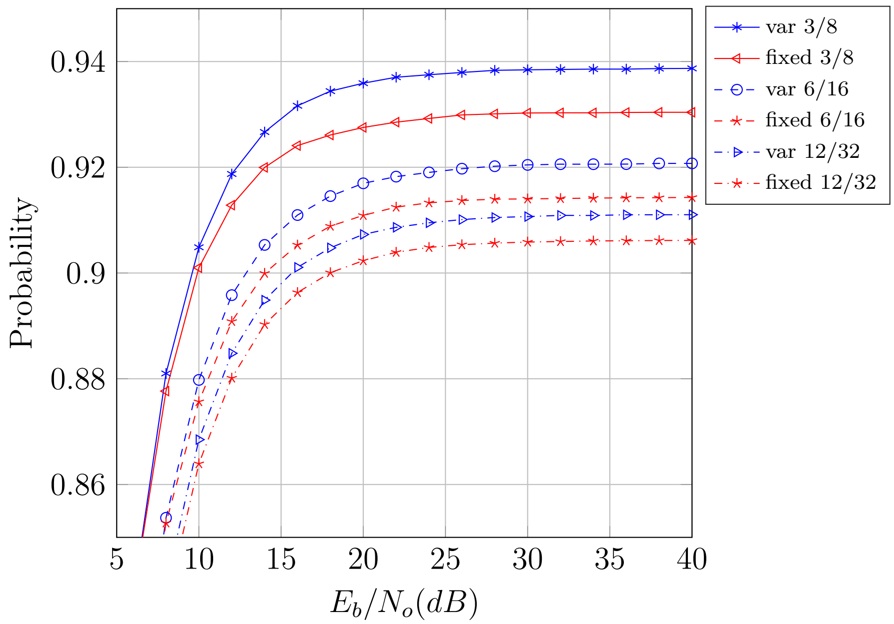


Figure 5.20: Comparison in case of Turbo code of rate $\frac{1}{4}$ for $\frac{k}{G} = \frac{3}{8}$

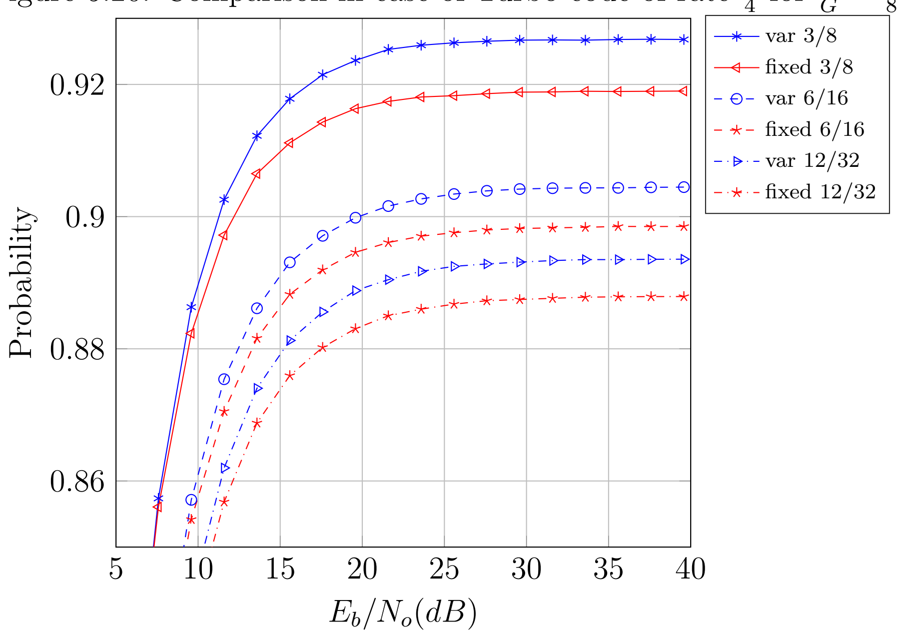


Figure 5.21: Comparison in the case of Turbo code of rate $\frac{1.1}{4}$ for $\frac{k}{G} = \frac{3}{8}$

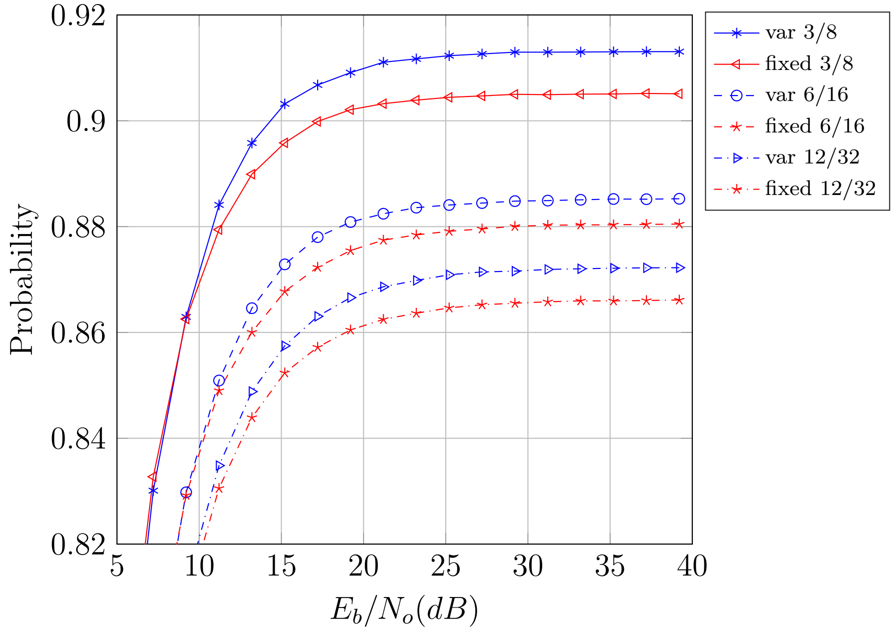


Figure 5.22: Comparison in the case of Turbo code of rate $\frac{1.2}{4}$ for $\frac{k}{G} = \frac{3}{8}$

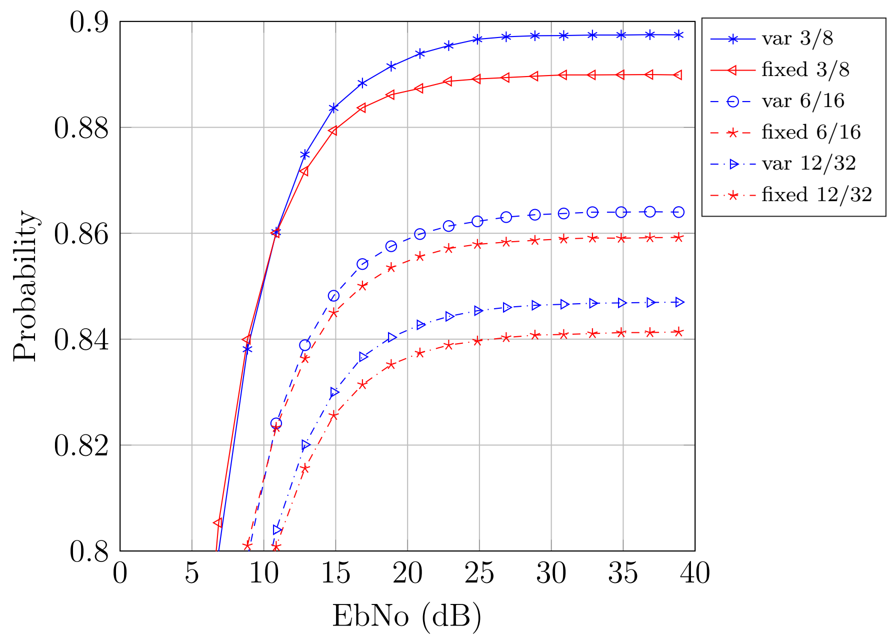


Figure 5.23: Comparison in the case of Turbo code of rate $\frac{1.3}{4}$ for $\frac{k}{G} = \frac{3}{8}$

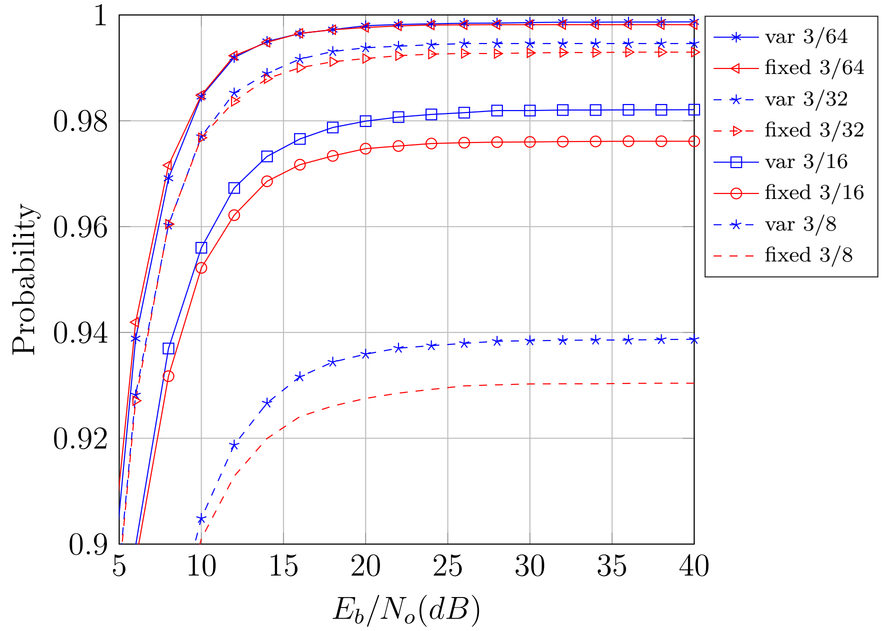


Figure 5.24: Comparison in the case of Turbo code of rate $\frac{1}{4}$ for 3 users

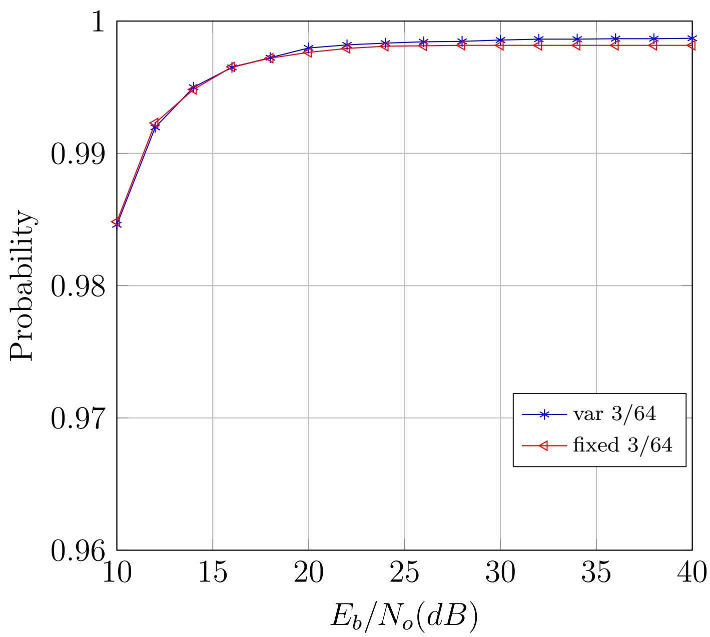


Figure 5.25: Outage comparison by using a Turbo code of rate $\frac{1}{4}$ and $G = 64$

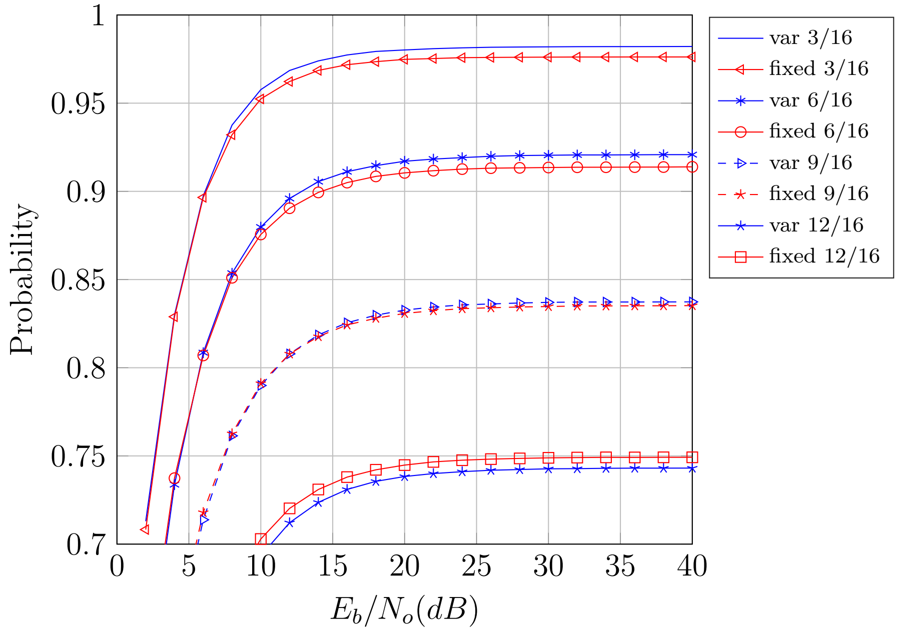


Figure 5.26: Comparison in case of Turbo code of rate $\frac{1}{4}$ for fixed G

Chapter 6

Fixed and variable SCMA

6.1 What is SCMA?

There is a type of spreading, which each user has N available resources to transmit each modulation symbol, but they will use just K of those N resources, and the remaining resources are set to be zero. In the case that $K << N$, we have low density spreading (LDS) [8]. The main property of this scheme is in multiuser detection. There are several methods with low complexity that can be accomplished for the case of sparsity of the signatures, such as *factor graphs*. If we define the number of users to the number of available resources as the *loading factor* and denote it by λ, then the LDS system has nearly a single user performance for $\lambda = 2$ [30].

There is another approach in spreading, which we combine the modulation and the spreading together and the bits are directly mapped to a multidimensional codeword. This scheme, known as *sparse code multiple access (SCMA)* benefits from the shaping gain of the code structure. In addition, due to the sparsity of the codes, it can have a low-complex receiver as well [26].

Suppose there are J users in the system, each one chooses K resources from N available resources in each transmission. Suppose each user has M codewords, and hence in each mapping, they will map $\log_2 M$ bits into a sequence of M complex numbers. The procedure is as follows [32].

- let $\mathbf{b}$ be a length $\log(M)$ bit stream

- $g(\mathbf{b})$ maps $\mathbf{b}$ into a complex string of length K
- $\mathbf{V} \times g(\mathbf{b})$ inserts $N - K$ zeros into the above string

There are J users, and each one transmits m packages of SCMA. Suppose we are using a channel code of length $N_c = mK$ and it can correct upto t errors in its block. The resource which is in common with at least two users is considered to be in error. Let summarize the parameters in a table.

Table 6.1: Parameters and their descriptions

parameter	description
N	total resources per SCMA package
K	used resources per SCMA package per user
J	number of users
m	number of transmitted SCMA package per user
M	cardinality of the range of the function g
N_c	the channel code's block length equal to mK
t	the channel code can correct upto t errors in its block
Ψ_e	number of collided resources in a SCMA package
Ψ_c	number of correct resources in a SCMA package
Ω	number of collided resources in m SCMA blocks

Now we want to compare the performance of the variable and the fixed SCMA from FER and maximum achievable rate aspects.

6.2 variable SCMA

In this case we are changing the $\mathbf{V}$ matrix for each SCMA package. In another words, we are changing the position of selected resources from package transmission to transmission. Let us denote the number of collided and correct resources for a transmitted package for a specific user by Ψ_e and Ψ_c respectively.

We want to find $Pr(\Psi_e = l) = Pr(\Psi_c = K - l)$. which $l = 0, 1, \ldots K$. For this aim, consider the simple case of $l = 0$. Then $Pr(\Psi_c = K) = \frac{\binom{N-K}{K}^{J-1}}{\binom{N}{K}^{J-1}}$ as the remaining users cannot choose those K resources which the first user has selected. Now let us calculate $Pr(\Psi_c = K-1)$. It means there is one resource in collision and the remaining resources are received without any collision. Hence there are $\binom{K}{1}$ choices for the collided resource. As the remaining $K-1$ resources are error free, then the other users must choose their resources from the remaining $N-(K-1)$ resources and there are $\binom{N-K+1}{K}^{J-1}$ combinations. But among the selections are those which yield no collision with that chosen erroneous resource. Hence we must reduce them from the possible selections. $\binom{N-K}{K}^{J-1}$ different choices exits for such a selection. So there are $\binom{N-K+1}{K}^{J-1} - \binom{N-K}{K}^{J-1}$ correct selections and thus

$$Pr(\Psi_c = K-1) = \binom{K}{1} \frac{\binom{N-K+1}{K}^{J-1} - \binom{N-K}{K}^{J-1}}{\binom{N}{K}^{J-1}} \tag{6.1}$$

What does the solution look like when there are $K-2$ correct resources. In that case we can choose which two resources are erroneous and thus we have $\binom{K}{2}$ difference cases. After selection of the erroneous resources, the remaining users must choose between $N-(K-2)$ resources which are totally $\binom{N-K+2}{K}^{J-1}$ different selections. But among them are those which either one of the erroneous resources are error free and hence we must reduce their effect. They are equal to $\binom{2}{1}\binom{N-K+1}{K}^{J-1}$ different choices. But we should notice that we have reduced the case that there is no erroneous resource twice. Hence we should add it up. So there are totally $\binom{2}{0}\binom{N-K+2}{K}^{J-1} - \binom{2}{1}\binom{N-K+1}{K}^{J-1} + \binom{2}{2}\binom{N-K}{K}^{J-1}$ valid selection for the remaining users and hence

$$Pr(\Psi_c = K-2) = \binom{K}{2} \frac{\binom{2}{0}\binom{N-K+2}{K}^{J-1} - \binom{2}{1}\binom{N-K+1}{K}^{J-1} + \binom{2}{2}\binom{N-K}{K}^{J-1}}{\binom{N}{K}^{J-1}}. \tag{6.2}$$

By the same procedure and using the *inclusion-exclusion principle*, we can say

$$Pr(\Psi_c = \eta) = \binom{K}{\eta} \frac{\sum_{i=0}^{K-\eta} (-1)^i \binom{K-\eta}{i}\binom{N-\eta-i}{K}^{J-1}}{\binom{N}{K}^{J-1}} \tag{6.3}$$

and if we let $\eta = K - l$ then

$$Pr(\Psi_e = l) = \binom{K}{l} \frac{\sum_{i=0}^{l} (-1)^i \binom{l}{i}\binom{N-K+l-i}{K}^{J-1}}{\binom{N}{K}^{J-1}} \tag{6.4}$$

Equation 6.4 is for the error in a single SCMA package, but users are transmitting m consecutive packages which make the channel code's block. What will be the error distribution for the channel code's block length? Let X_i denote the number of erroneous resources in the i^{th} SCMA block. Then we want to find the probability that $\sum_{i=1}^{m} X_i = \zeta$ subject to this condition that $X_i \leq K$ for $i = 1, 2, \ldots, m$. How many combinations do exit that satisfy the above conditions? To answer this question let's look at a simpler problem.

Suppose there are m different boxes and ζ same apples. How many combinations exist? We can denote the m boxes with $m-1$ vertical lines like $|$ and the ζ apples by ζstars like $\star$. Then each combination of apples and boxes is equivalent to a specific combination of those lines and stars. For example if there are three different boxes and five same apples, then the combination which there is one apple in the first box, two apples in the second and the third boxes can be represented as

$$\star | \star\star | \star\star$$

and if there are two apples in the first box and three apples in the third one, this is equivalent to

$$\star\star || \star\star\star$$

So as there are $\binom{\zeta+m-1}{\zeta}$ combinations for stars and lines, we conclude there are $\binom{\zeta+m-1}{\zeta}$ different solution for $\sum_{i=1}^{m} X_i = \zeta$, subject to this condition that all the variables are non-negative.

Let us return to our question. We want to find the number of solutions for $\sum_{i=1}^{m} X_i = \zeta$ given that the variables are non-negative and less than or equal to K. Let $\chi_i = K - X_i$, then any solution to $\sum_{i=1}^{m} X_i = \zeta$ by the mentioned conditions is equivalent to a solution to the equation $\sum_{i=1}^{m} \chi_i = mK - \zeta$ subject to the non-negative variables and vice versa. Hence there are totally $\binom{m(K+1)-\zeta-1}{m-1}$ different combinations for the errors in the m consecutive SCMA package witch their errors add up to ζ. If we denote the number of erroneous resources in m consecutive SCMA blocks by Ω, by using equation 6.4, we have

$$Pr(\Omega = \zeta) =$$
$$\sum_{\substack{l_1, l_2, \ldots, l_m \\ l_1 + l_2 + \ldots + l_m = \zeta}} \left(\prod_{i=1}^{m} \binom{K}{l_i} \frac{\sum_{i_m=0}^{l_m} \sum_{i_{m-1}=0}^{l_{m-1}} \cdots \sum_{i_1=0}^{l_1} (-1)^{i_1+\ldots+i_m} \prod_{j=1}^{m} \binom{l_j}{i_j} \binom{N-K+l_j-i_j}{K}^{J-1}}{\binom{N}{K}^{m(J-1)}} \right) \tag{6.5}$$

for $\zeta = 0, \ldots, mK$ and 0 elsewhere. Let us see the validity of 6.5 in some simple examples.

Example 6.1. If $\Omega = 0$, it means there is no error in the block and hence for all the m packages, the other $J-1$ users have selected their K resources from the remaining $N-K$ resources. Hence the probability is

$$Pr(\Omega = 0) = \frac{\binom{N-K}{K}^{m(J-1)}}{\binom{N}{K}^{m(J-1)}}. \tag{6.6}$$

Now look at the equation 6.5. There is only one combination for $(l_1, l_2, \ldots, l_m)$ that results $l_1 + l_2 + \ldots + l_m = 0$ and that is $(l_1, l_2, \ldots, l_m) = (0, 0, \ldots, 0)$. So $\binom{K}{l_i} = 0$ and for the summations in the numerator of the equation 6.5, $i_j = 0$. So we have

$$\begin{aligned} Pr(\Omega = 0) &= \frac{\prod_{j=0}^{m} \binom{l_j}{i_j} \binom{N-K+l_j-i_j}{K}^{J-1}}{\binom{N}{K}^{m(J-1)}} \\ &= \frac{\binom{N-K}{K}^{m(J-1)}}{\binom{N}{K}^{m(J-1)}}. \end{aligned} \tag{6.7}$$

Note that $0! = 1$. △

Example 6.2. Consider the case that $\Omega = 1$, so there is only one error in the whole block. How many combinations do exist with just one error? We are transmitting m packages and the error is in one of them. There are $\binom{m}{1}$ choices for the collided package. In addition just one resource is collided within that package. There are $\binom{K}{1}$ choices for the erroneous resource. Till now we have $\binom{m}{1}\binom{K}{1}$ different combinations. For the packages other than the selected collided one, the other users must choose their resources among $N-K$ available resources and hence $\binom{N-K}{K}^{(m-1)(J-1)}$ different combinations exist. For the collided package, at least one user must use the selected resource, and there are $\binom{N-K+1}{K}^{J-1} - \binom{N-K}{K}^{J-1}$ possible event. In the first term, $\binom{N-K+1}{K}^{J-1}$, users are allowed to choose among $N-K+1$ resources which the added resource is the collided one. Then we have subtracted all the choices which the other users do not choose that erroneous resource at all. So the total number of possibilities is

$$\binom{m}{1}\binom{K}{1}\binom{N-K}{K}^{(m-1)(J-1)}\left(\binom{N-K+1}{K}^{J-1} - \binom{N-K}{K}^{J-1}\right). \tag{6.8}$$

Now let's calculate the numerator of 6.5. There are m different solutions for $l_1 + l_2 + \ldots + l_m = 1$ and they are of the form $l_r = 1$ and $l_{r'} = 0$ which r' are packages other than r^{th} package. Due to the symmetry we consider just $l_1 = 1$ and then multiply the solution by m. Then the numerator is

$$m\binom{K}{1}\left(\binom{N-K+1}{K}^{J-1}\binom{N-K}{K}^{(m-1)(J-1)} - \binom{N-K}{K}^{m(J-1)}\right) \tag{6.9}$$

which is exactly equal to the term calculated directly. △

6.3 fixed SCMA

If we use a fixed $\mathbf{V}$ for transmitting all the m SCMA packages, then the collisions for the rest of the packages are the same as the first one. The probability distribution function for the erroneous resources in the first block is the same as equation 6.4, so in this case we have

$$Pr(\Omega = \zeta) = \begin{cases} \binom{K}{r}\frac{\sum_{i=0}^{r}(-1)^i\binom{r}{i}\binom{N-K+r-i}{K}^{J-1}}{\binom{N}{K}^{J-1}} & \zeta = rm, r = 0, \ldots, K \\ 0 & \text{otherwise} \end{cases} \tag{6.10}$$

6.4 Comparison

As the distribution of the erroneous resources for the first block of fixed and variable SCMA is the same, we can say the average of collided resources are the same for both scenarios. But as the distributions of the errors are different, we expect there is a difference in the performance when we use a channel code.

The channel decoder's error probability is equivalent to the probability that there are more than t collided resources in the m consecutive SCMA blocks. So generally we can say

$$Pr(\text{error}) = Pr(\Omega \geq t). \tag{6.11}$$

It is difficult to calculate the error probability from 6.11 for the variable case. However if we assume the length of the codewords is long enough, due to the law of large numbers, we can say with high probability we have $m\overline{\Psi_e}$ erroneous resources in the whole block, which $\overline{\Psi_e}$ denotes the expected value of resources in error in a single SCMA package. Indeed

$$\lim_{m\to\infty} Pr(|\frac{\Omega}{m} - \overline{\Psi_e}| > \epsilon) = 0 \tag{6.12}$$

For fixed SCMA, the probability of error is

$$\begin{aligned} Pr(\Omega \geq t) &= \sum_{\zeta=t+1}^{mK} Pr(\Omega = \zeta) \\ &= \sum_{\zeta=\lceil \frac{t}{m} \rceil}^{mK} Pr(\Omega = \zeta) \\ &= \sum_{r=\lceil \frac{t}{m} \rceil}^{K} \binom{K}{r} \frac{\sum_{i=0}^{r} (-1)^i \binom{r}{i} \binom{N-K+r-i}{K}^{J-1}}{\binom{N}{K}^{J-1}} \end{aligned} \tag{6.13}$$

This term is difficult for simplification as well. Let P_e denote the error probability for the fixed SCMA, calculated in 6.13.

For enough large value of m, there are about $m\overline{\Psi_e}$ erroneous resources with any desired high probability. So if $m\overline{\Psi_e} \leq t$, we can correct it by the channel decoder, otherwise we have an error. So if $m\overline{\Psi_e} \leq t$, we have no error for variable SCMA, but it is the case with probability $1 - Pe$ for the fixed case. Hence in this scenario, the variable one outperforms the fixed SCMA. In contrast, when $m\overline{\Psi_e} \geq t$, we have error with high probability for the variable SCMA, but it is the case for the fixed SCMA with probability P_e. Indeed in this scenario, the fixed SCMA outperforms the variable one.

Note that in equation 6.4, $\binom{N-K+l-i}{K} \leq \binom{N}{K}$, and they are equal to each other only when $l = K$ and $i = 0$. So by increasing the number of users, we are increasing the probability of $l = K$ (all the resources are in error) and hence the $\overline{\Psi_e}$ increases (tends to K). It is expectable as by increasing the number of users, we are increasing collision and hence we expect the average number of collided resources to be increased. So for a given channel code (fixed t), by increasing the number of users there is the possibility that $m\overline{\Psi_e} \geq t$. In that case we have error for variable SCMA with high probability (which goes toward 1 by increasing m). But for fixed spreading we can correct it by the probability $1 - P_e$. So we see the fixed spreading is better in this case.

6.5 Simulation

Now let's see some examples to verify the conclusions obtained in section 6.4. The CDF and the PDF plots of both scenarios for $m = 1000$ are shown in Figures 6.1 and 6.2. As we see, the variable SCMA has a smooth PDF(although it is discrete) and the fixed SCMA has an step behavior. These steps are due to the ceiling function in 6.13. The amplitude of the PDF of the variable SCMA is much lower than the fixed scenario's one. It has been depicted separately both in a linear and logarithmic y axis in Figures 6.3 and 6.4.

We saw for a fixed number of users, by decreasing the channel encoder's rate (which is equivalent to increasing the error correction ability), then it is more probable that variable SCMA outperforms the fixed SCMA. This can be seen in Figures 6.5 to 6.7. Note that in these figures, the variable SCMA behaves impulsively. This is due to this reason that the number of erroneous resources in the variable SCMA in a sufficient large number of consecutive SCMA packages is a fix number. So there is a specific rate, which we can correct the whole block with high probability (ideally 1) for encoders of lower rates, and we have error for encoders of higher rates. In addition as it is apparent in these figures, the fixed SCMA behaves as several steps. This can be justified by equation 6.13. As you can see, the lower limit of the summation has a ceiling function. Indeed the error probabilities for all values of t which are between two consecutive multiples of m are the same. In this example as the block length is $mK = 3000$ and $m = 1000$, we have three steps in each figure.

In figures 6.8 to 6.12, the effect of number of users is shown. As it is apparent, by increasing the channel code's correction ability, the variable SCMA can outperform the fixed one. This is compatible with the conclusion in section 6.4.

Now let us focus more on the lower FERs as we want it to be as small as possible. Tables 6.2 and 6.3 describe the FER for $m = 1000$ for different number of users, J, and different error correction abilities, t. As it is apparent from the tables, fixed SCMA shows a step-like behavior and it is not important for the fixed scenario that the error correction ability is 1 or 999. In addition, the impulsive behavior of the variable spreading is clear as well. As we see for a fixed number of users, before a threshold, the FER is nearly 1 and after that it falls suddenly to 0. It is clear that by decreasing the number of transmitted packages, m, the variable behavior becomes smoother.

As we see in Tables 6.2 and 6.3, for high rate codes that the error correction ability of the code is low, the fixed SCMA is better, but for low rate channel codes, the variable outperforms the fixed scenario. It is compatible with the theoritical derivations discussed so far. It is notable that these tables have been obtained by one million runnings, so each 0 entry[1] of them is equivalent to a FER of less than 10^{-6}.

Table 6.2: FER for fixed SCMA for $m = 1000$, $K = 3$ and $N = 64$

	$J = 2$	$J = 3$	$J = 4$	$J = 5$
$t = 100$	0.1322	0.2476	0.3459	0.4325
$t = 200$	0.1322	0.2476	0.3459	0.4325
$t = 300$	0.1322	0.2476	0.3459	0.4325
$t = 400$	0.1322	0.2476	0.3459	0.4325
$t = 500$	0.1322	0.2476	0.3459	0.4325
$t = 600$	0.1322	0.2476	0.3459	0.4325
$t = 700$	0.1322	0.2476	0.3459	0.4325
$t = 800$	0.1322	0.2476	0.3459	0.4325
$t = 900$	0.1322	0.2476	0.3459	0.4325
$t = 1000$	0.0062	0.0231	0.0479	0.0792
$t = 1100$	0.0062	0.0231	0.0479	0.0792
$t = 1200$	0.0062	0.0231	0.0479	0.0792
$t = 1300$	0.0062	0.0231	0.0479	0.0792
$t = 1400$	0.0062	0.0231	0.0479	0.0792
$t = 1500$	0.0062	0.0231	0.0479	0.0792
$t = 1600$	0.0062	0.0231	0.0479	0.0792
$t = 1700$	0.0062	0.0231	0.0479	0.0792
$t = 1800$	0.0062	0.0231	0.0479	0.0792
$t = 1900$	0.0062	0.0231	0.0479	0.0792
$t = 2000$	0.0001	0.0007	0.0023	0.0053

[1] note that we said 0, not 0.0000

Table 6.3: FER for variable SCMA for $m = 1000$, $K = 3$ and $N = 64$

	$J = 2$	$J = 3$	$J = 4$	$J = 5$
$t = 100$	0.9997	1.0000	1.0000	1.0000
$t = 200$	0	1.0000	1.0000	1.0000
$t = 300$	0	0.0294	1.0000	1.00000
$t = 400$	0	0	0.4109	1.0000
$t = 500$	0	0	0	0.7810
$t = 600$	0	0	0	0.0000
$t = 700$	0	0	0	0
$t = 800$	0	0	0	0
$t = 900$	0	0	0	0
$t = 1000$	0	0	0	0
$t = 1100$	0	0	0	0
$t = 1200$	0	0	0	0
$t = 1300$	0	0	0	0
$t = 1400$	0	0	0	0
$t = 1500$	0	0	0	0
$t = 1600$	0	0	0	0
$t = 1700$	0	0	0	0
$t = 1800$	0	0	0	0
$t = 1900$	0	0	0	0
$t = 2000$	0	0	0	0

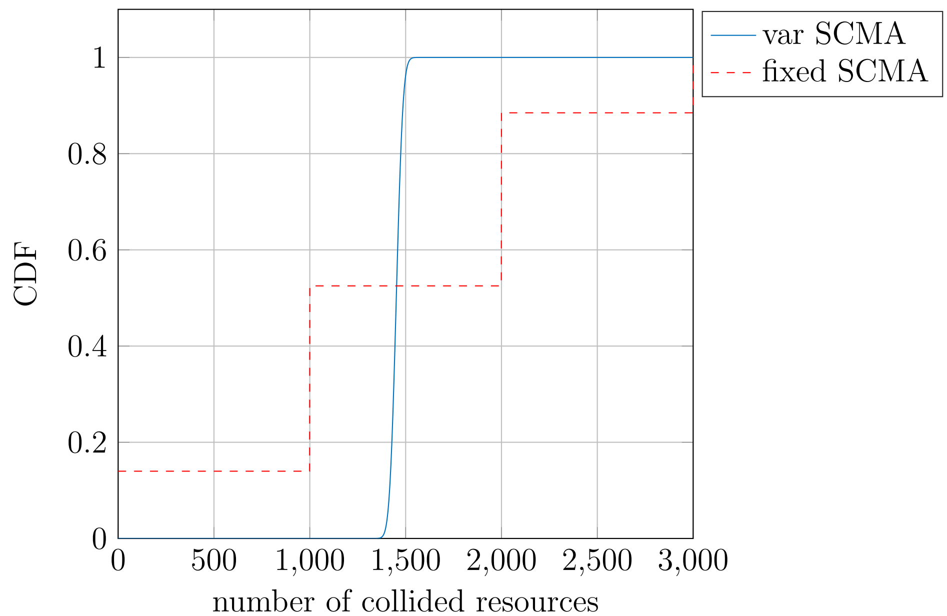


Figure 6.1: CDF for $N = 64$, $K = 3$ and $J = 15$

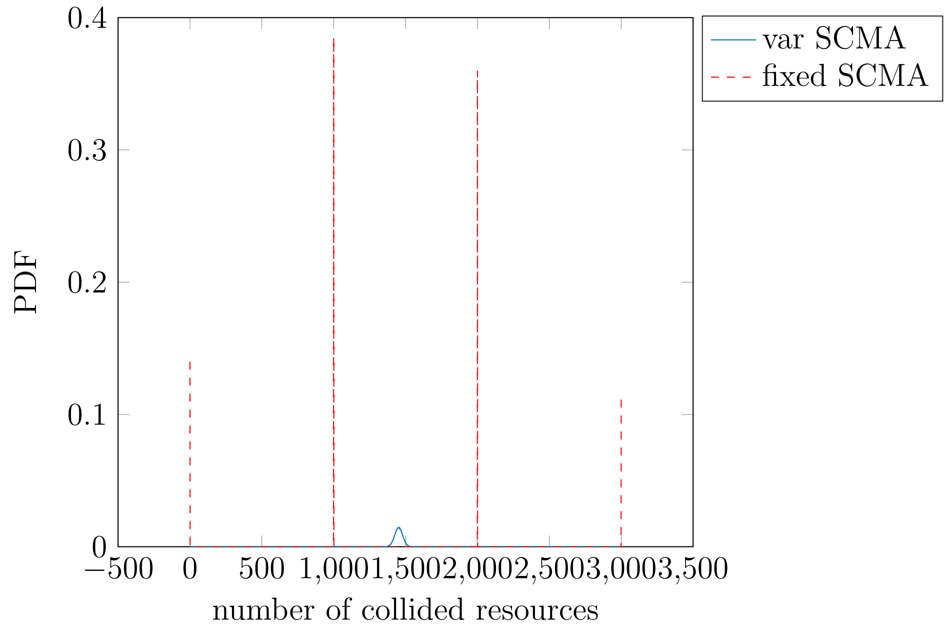


Figure 6.2: PDF for $N = 64$, $K = 3$ and $J = 15$

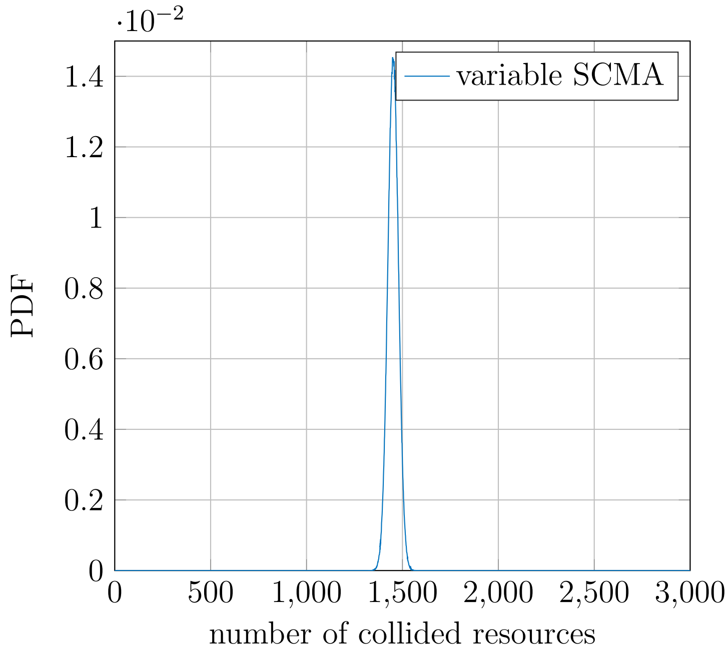


Figure 6.3: magnified PDF for variable SCMA for $N = 64$, $K = 3$ and $J = 15$

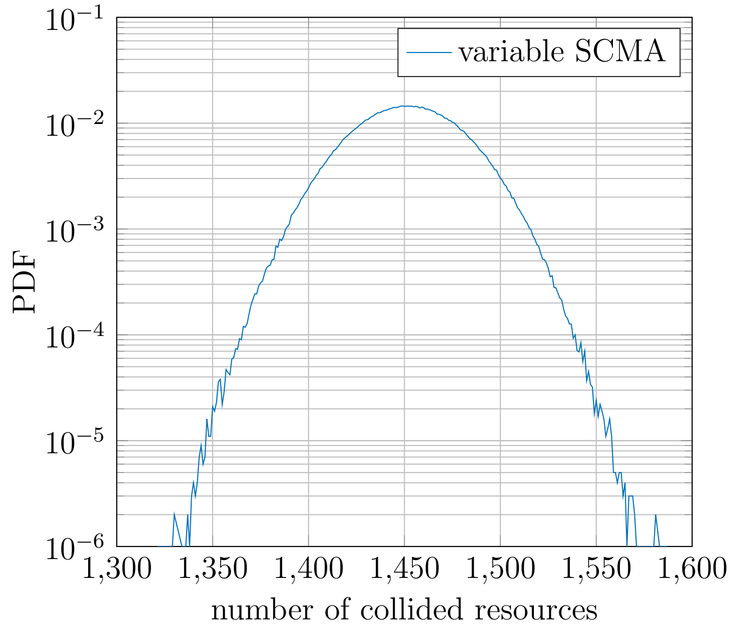


Figure 6.4: PDF for variable SCMA for $N = 64$, $K = 3$ and $J = 15$

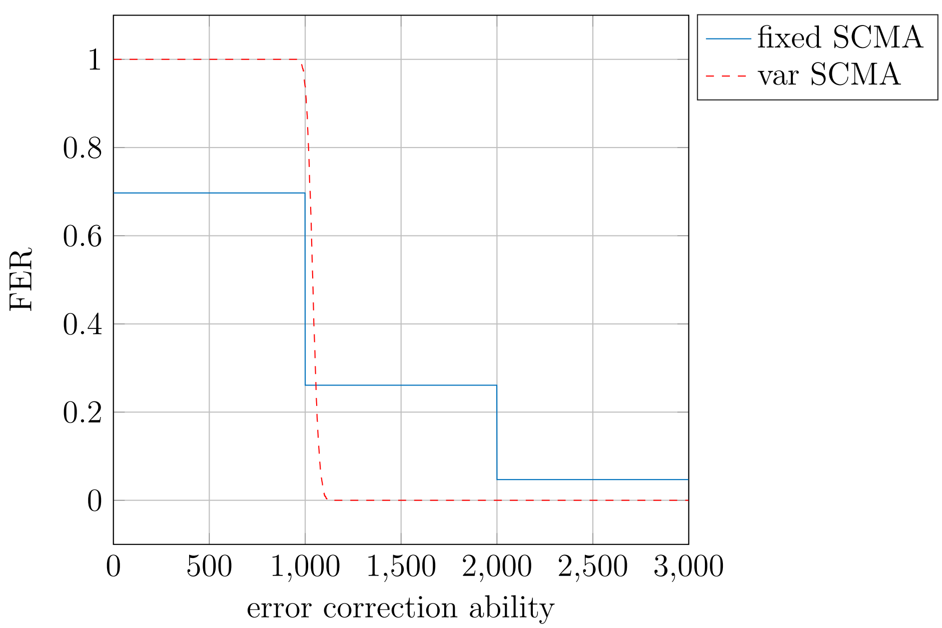


Figure 6.5: FER for $N = 64$, $K = 3$ and $J = 10$

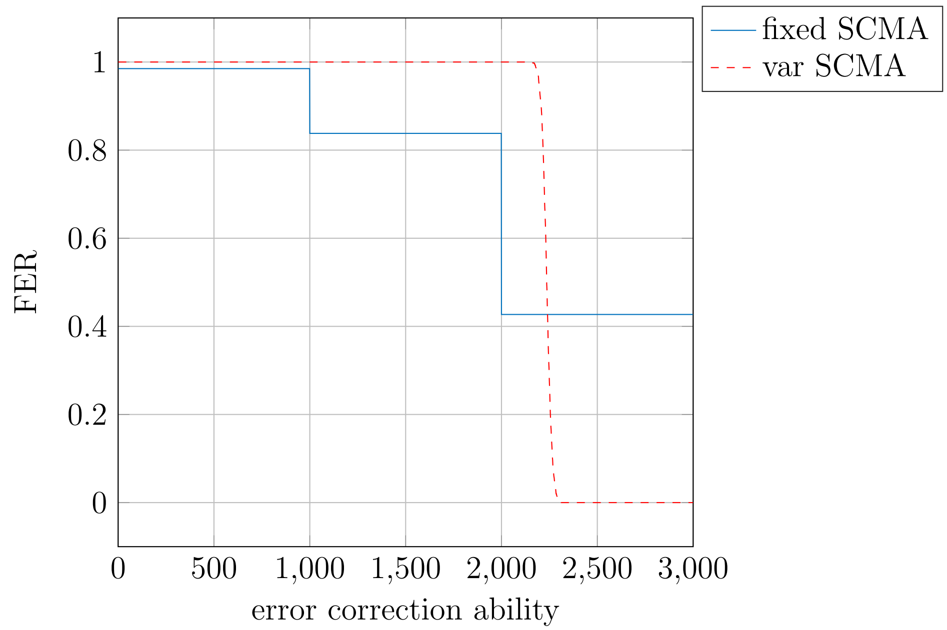


Figure 6.6: FER for $N = 64$, $K = 3$ and $J = 30$

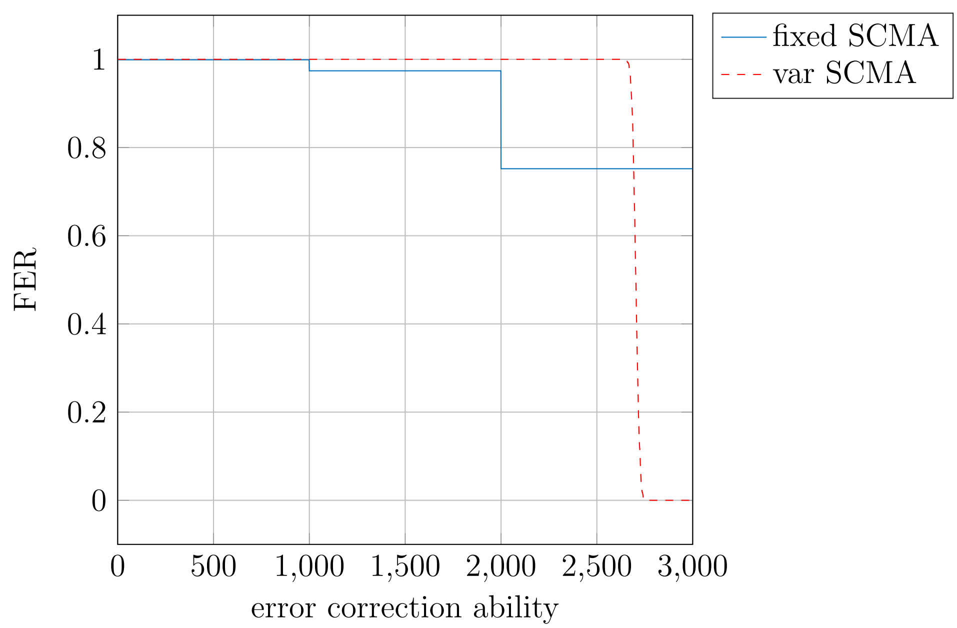


Figure 6.7: FER for $N = 64$, $K = 3$ and $J = 50$

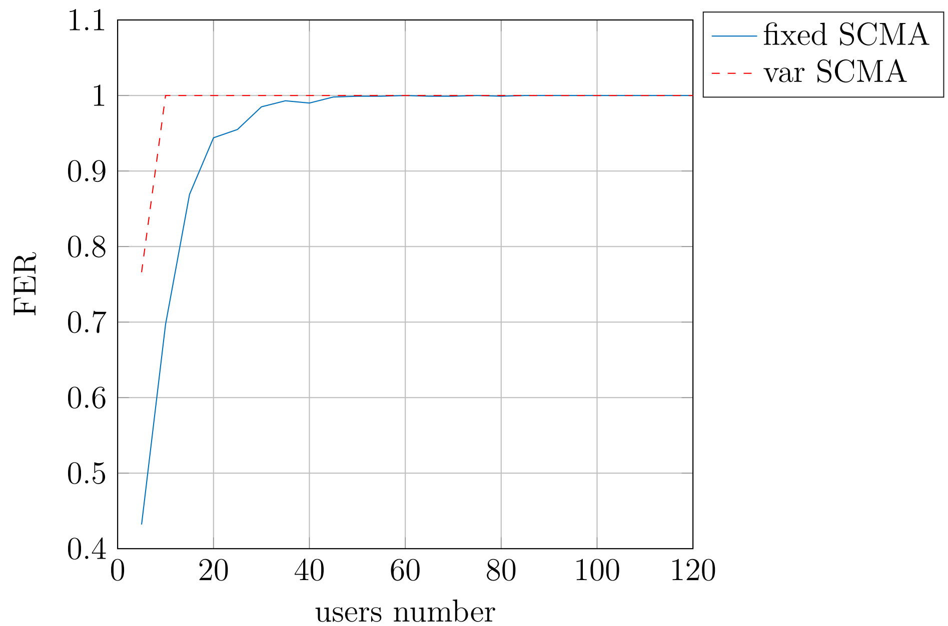


Figure 6.8: FER for $N = 64$, $K = 3$ and $t = 500$

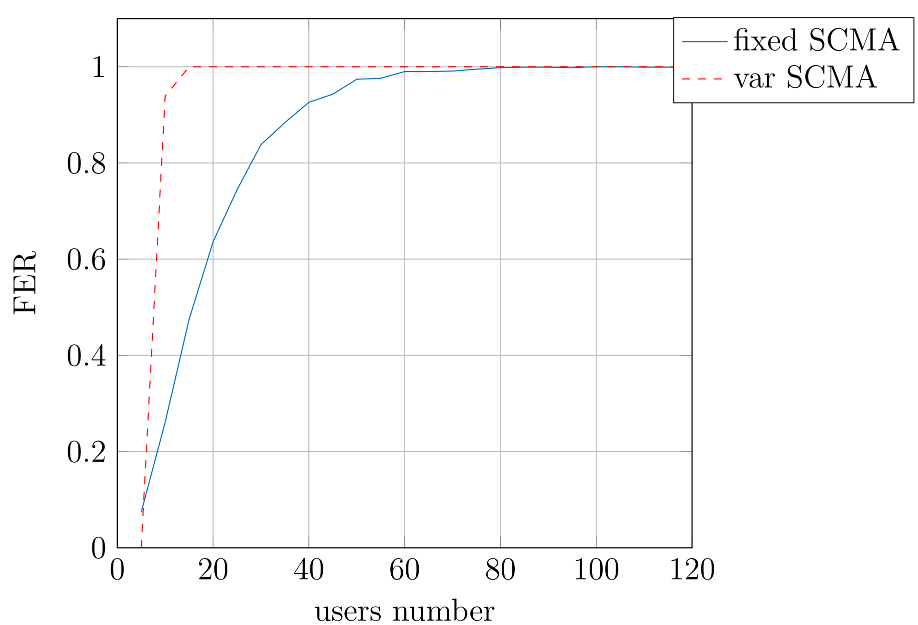


Figure 6.9: FER for $N = 64$, $K = 3$ and $t = 1000$

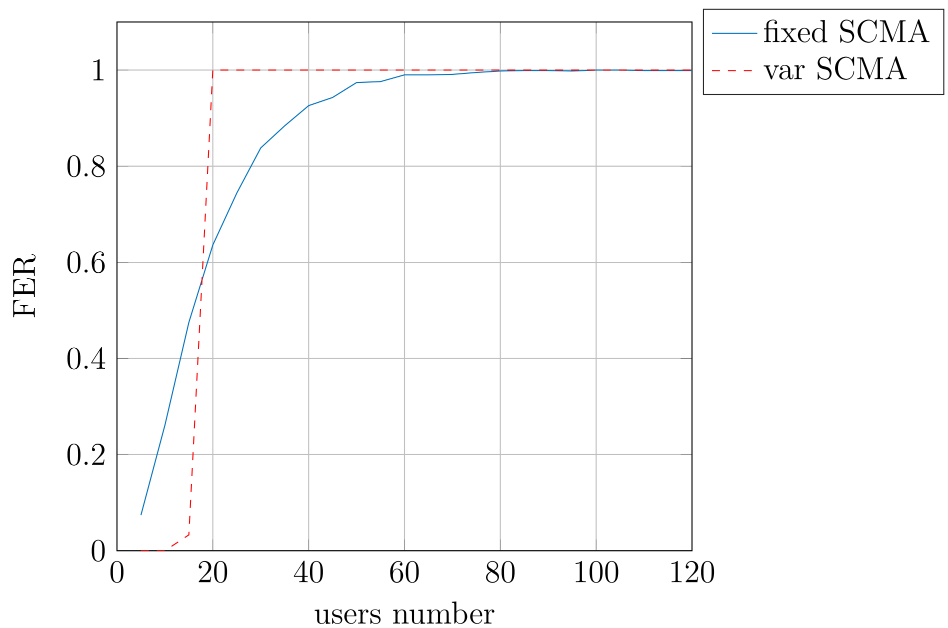


Figure 6.10: FER for $N = 64$, $K = 3$ and $t = 1500$

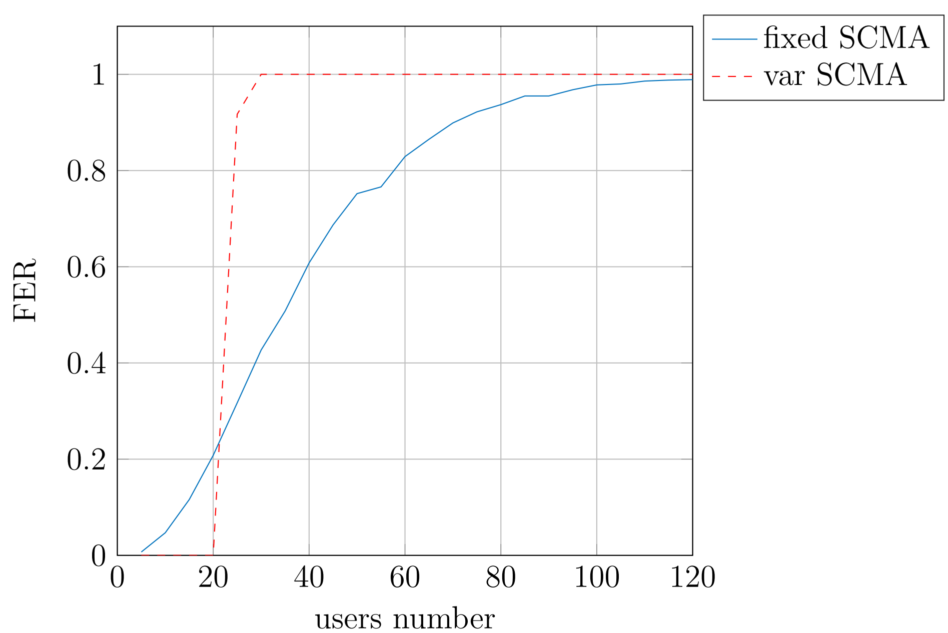


Figure 6.11: FER for $N = 64$, $K = 3$ and $t = 2000$

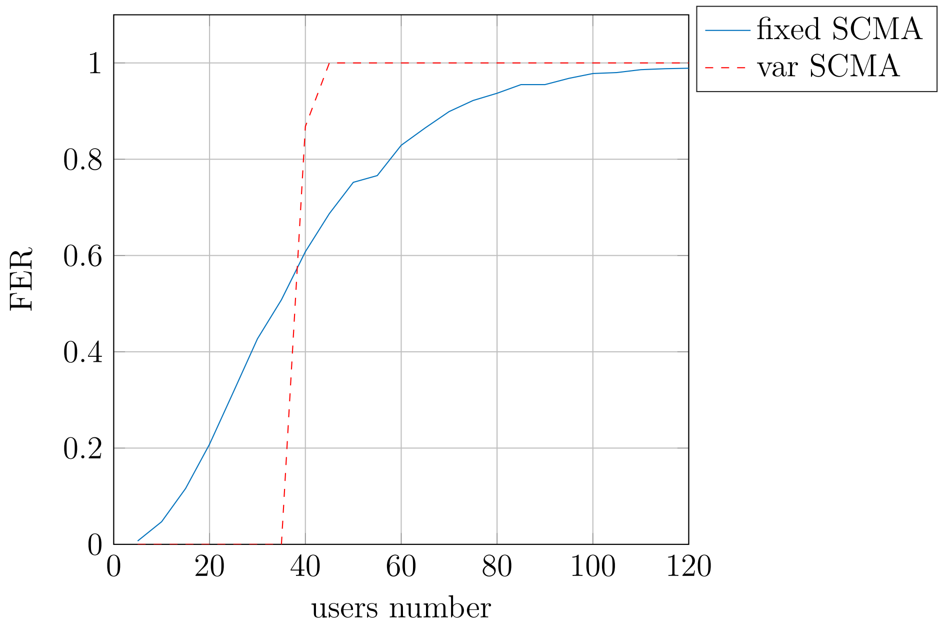


Figure 6.12: FER for $N = 64$, $K = 3$ and $t = 2500$

6.6 Rate comparison

In this section we want to compare the maximum achievable rates in fixed and variable scenarios.

6.6.1 Variable SCMA

By increasing the number of blocks in the variable SCMA scenario, we can find the achievable rate for each user. Indeed we have assumed the number of users and the available resources are fixed to specific J and N, and we want to see what is the best choice for K. If there is no channel encoder, then in a single user scenario, we can send $\log\{M\binom{N}{K}\}$ bits via a single SCMA package, in which M is the cardinality of the range of the function g. It is true as in the variable case we can have a specific g for each matrix V and hence we can achieve such a rate. It is maximum for $\lceil \frac{N}{2} \rceil$. But as there are other users, transmitting toward a base station, collision is unavoidable. Hence we must use a channel encoder. In that case the effective rate is $r \log\{M\binom{N}{K}\}$ bits per SCMA package, which r is the channel code's rate.

By using a channel code, we cannot say the maximum rate is for $K = \lceil \frac{N}{2} \rceil$, as by increasing K, the collision increases and hence the channel code's rate decreases. So we must find an optimum K.

In variable case, by increasing the number of SCMA packages, m, there are $m\overline{\Psi_e}$ erroneous resources with high probability[1]. Let us denote the channel code's error correction ability for a code of length mK by t_m. So to correct the errors, $t_m \geq m\overline{\Psi_e}$. We choose the minimum t_m as it will increase the channel code's rate. So let $t_m = m\overline{\Psi_e}$. As the RS codes are MDS, for a specific code's length and error correction ability, the maximum rate is achieved by them. Hence

$$t_m = N_c - K_c + 1 = mK - K_c + 1 \Rightarrow m\overline{\Psi_e} = mK - K_c + 1. \tag{6.14}$$

By dividing both sides by the block length, we have

$$\frac{\overline{\Psi_e}}{K} = 1 - r + \frac{1}{mK}, \tag{6.15}$$

so

$$r = \frac{m(K - \overline{\Psi_e}) + 1}{mK} \tag{6.16}$$

and by taking the limit, when m is so large we have

$$r = \frac{K - \overline{\Psi_e}}{K}. \tag{6.17}$$

Till now we know the effective rate for a user is

$$R = r \log\left\{M\binom{N}{K}\right\} = \frac{K - \overline{\Psi_e}}{K} \log\left\{M\binom{N}{K}\right\}. \tag{6.18}$$

Let us find $\overline{\Psi_e}$ to find an equation explicitly as a function of K. By using 6.4, it is easily seen that

$$\frac{\overline{\Psi_e}}{K} = \sum_{l=1}^{K} \binom{K-1}{l-1} \frac{\sum_{i=0}^{l} (-1)^i \binom{l}{i} \binom{N-K+l-i}{K}^{J-1}}{\binom{N}{K}^{J-1}} \tag{6.19}$$

[1] which goes toward 1 by increasing m

and hence the effective rate is

$$R = \left\{ 1 - \sum_{l=1}^{K} \binom{K-1}{l-1} \frac{\sum\limits_{i=0}^{l} (-1)^i \binom{l}{i} \binom{N-K+l-i}{K}^{J-1}}{\binom{N}{K}^{J-1}} \right\} \log \left\{ M \binom{N}{K} \right\}. \tag{6.20}$$

Now the problem turns into a combinatorial optimization which is difficult to solve generally. For $N = 64$, $M = 16$ and different number of users, the rates for different K are depicted in Figures 6.13 to 6.19. As it is apparent, for $J = 1$, the rate for each K is equal to $\log \left\{ M \binom{N}{K} \right\}$ as there is no collision.

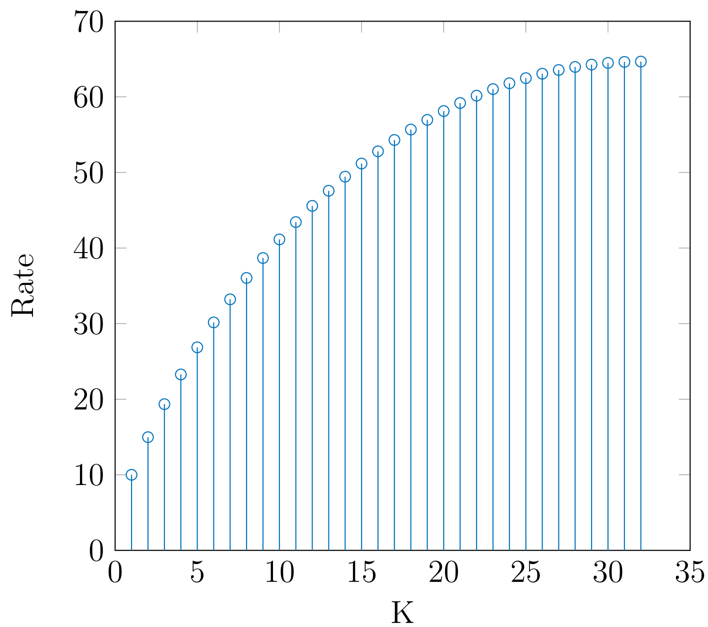


Figure 6.13: Rate for $J = 1$, $M = 16$, $N = 64$ and variable SCMA per transmitter

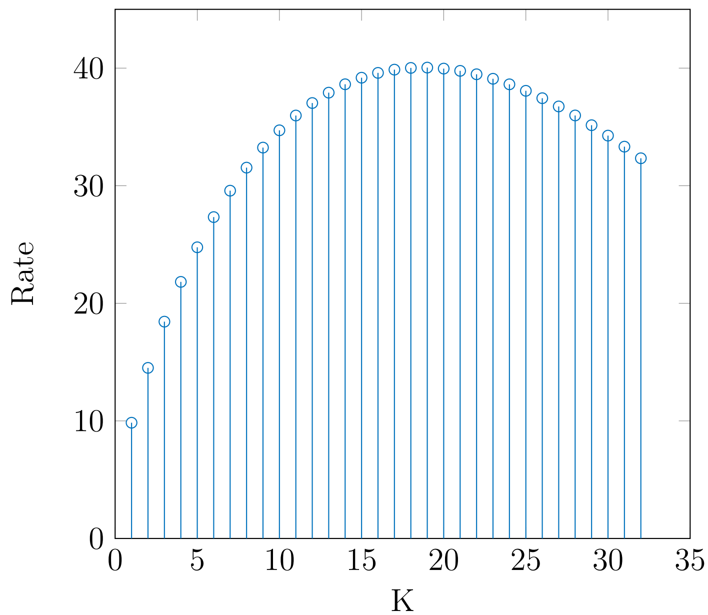


Figure 6.14: Rate for $J = 2$, $M = 16$, $N = 64$ and variable SCMA per transmitter

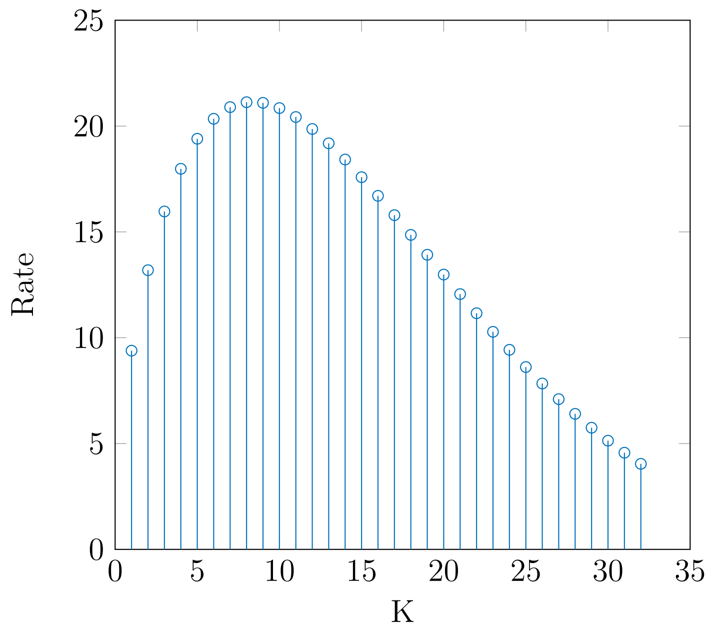


Figure 6.15: Rate for $J = 5$, $M = 16$, $N = 64$ and variable SCMA per transmitter

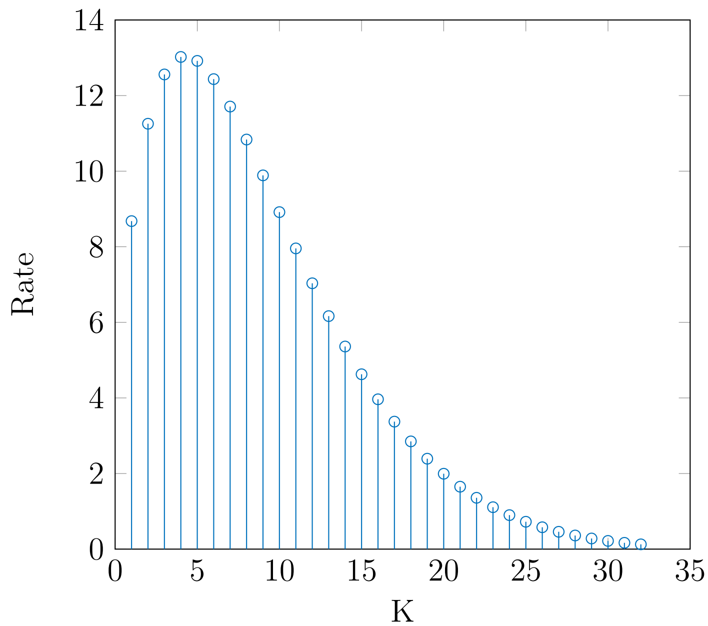


Figure 6.16: Rate of $J = 10$, $M = 16$, $N = 64$ and variable SCMA per transmitter

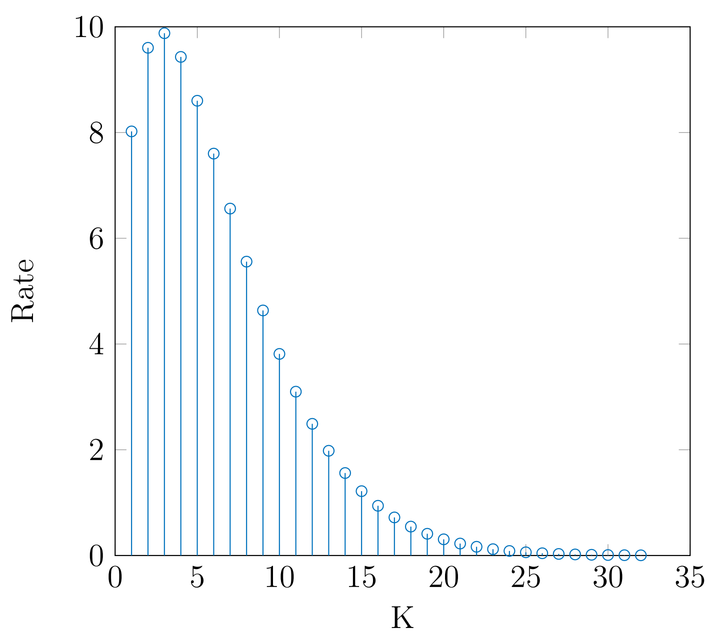


Figure 6.17: Rate of $J = 15$, $M = 16$, $N = 64$ and variable SCMA per transmitter

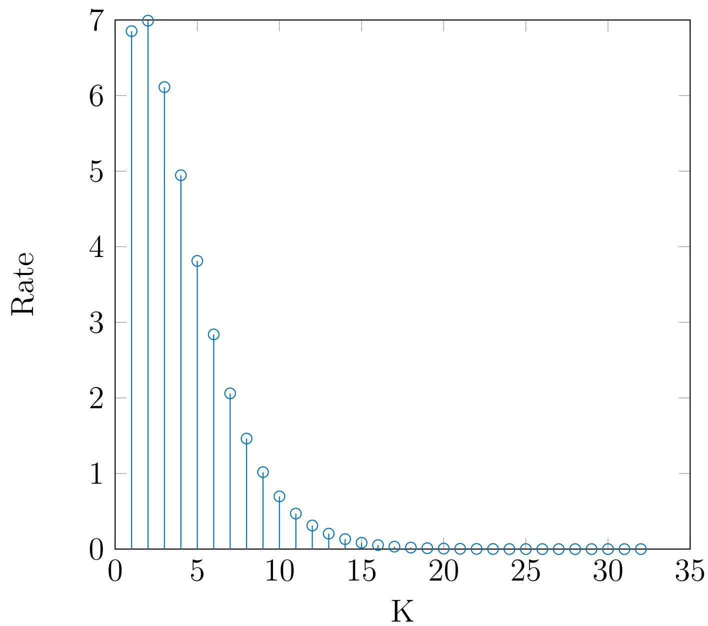


Figure 6.18: Rate of $J = 25$, $M = 16$, $N = 64$ and variable SCMA per transmitter

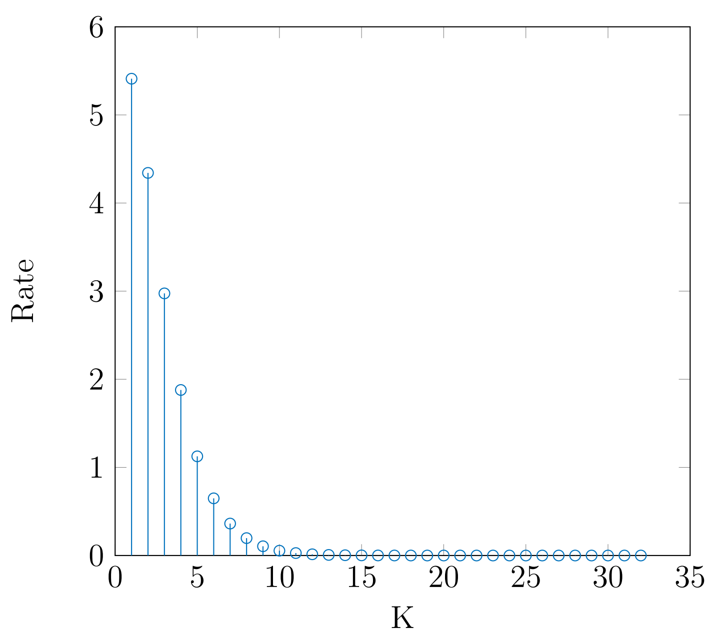


Figure 6.19: Rate of $J = 40$, $M = 16$, $N = 64$ and variable SCMA per transmitter

6.6.2 Fixed SCMA

The PDF of the number of erroneous resources in fixed SCMA, is discrete with $K+1$ points and the probabilities are independent from m. In other words, for two different m_1 and m_2, the probabilities of rm_1 collisions in the first case, and rm_2 collisions in the second case are the same. So even for large values of m, with a nonzero probability[1], all the resources are in error. So we cannot talk about rate[2]. In this case we talk about outage rate, which means we want to find the rate which yields a maximum frame error rate of α. So we are interested in $Pr(\Omega \geq t_m) \leq \alpha$. If we denote the CDF function of the number of erroneous resources in just a single SCMA package by F, then

$$\begin{aligned} &Pr(\frac{\Omega}{m} \geq \frac{t_m}{m}) = 1 - F(\frac{t_m}{m}) \leq \alpha \Rightarrow \\ &F(\frac{t_m}{m}) \geq 1 - \alpha. \end{aligned} \tag{6.21}$$

Let us denote the minimum t_m that satisfies 6.21 by $t^\star = r^\star m$. By using an MDS code like RS, the rate of the channel code is

$$r = 1 - \frac{t^\star - 1}{mK} = 1 - \frac{r^\star m - 1}{mK}. \tag{6.22}$$

The main difference in the fixed scenario with the variable one is that we have not $\log\left\{M\binom{N}{K}\right\}$ bits for each transmission. Indeed for the first transmission we have $M\binom{N}{K}$ choices, but as we aredealing with the fixed scenario, for the remaining $m-1$ transmissions, we have just M different symbols. So in the whole m packages we are transmitting $\log\left\{M\binom{N}{K}\right\} + (m-1)\log M$ bits. So the rate is

$$R = \left(1 - \frac{r^\star m - 1}{mK}\right) \times \frac{\log\left\{M\binom{N}{K}\right\} + (m-1)\log M}{m} \tag{6.23}$$

bits per SCMA package. For large values of m, the rate is

$$R = \left(1 - \frac{r^\star}{K}\right) \log M. \tag{6.24}$$

Optimizing 6.24 is more difficulte than the variable case as we must find the suitable t_m that satisfies the error condition. In Figures 6.20 to 6.23, the rates for different values of K

[1] which is independent of m

[2] as the error probability is not zero for all the rates

are depicted. The fixed SCMA does not show a smooth behavior as the variable case. The points which the rate is equal to zero are those which the probability of having the whole resources in collision is greater than $\alpha = 0.01$. As it is apparent from 6.24, the maximum rate for the fixed scenario is $\log M$ bits per SCMA package. So in this case that $M = 16$, the maximum rate is 4.

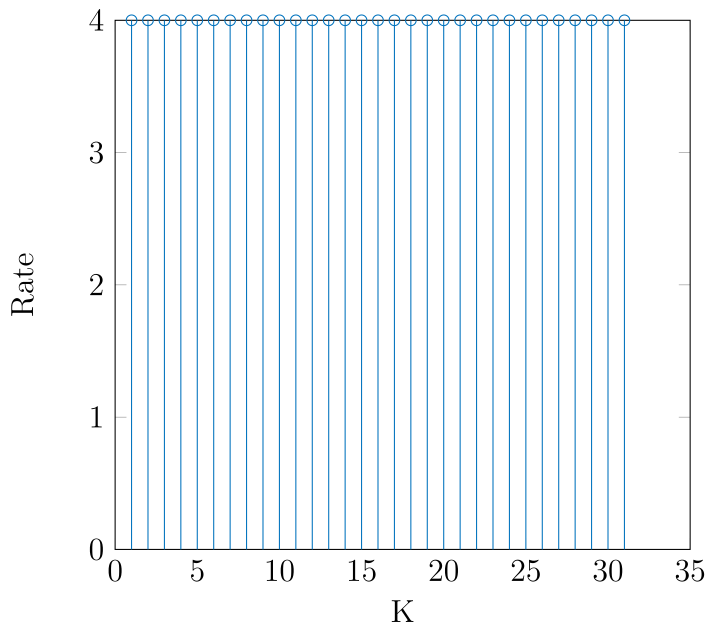


Figure 6.20: Rate of $(J, M, N, \alpha) = (1, 16, 64, 0.01)$ for fixed SCMA per user

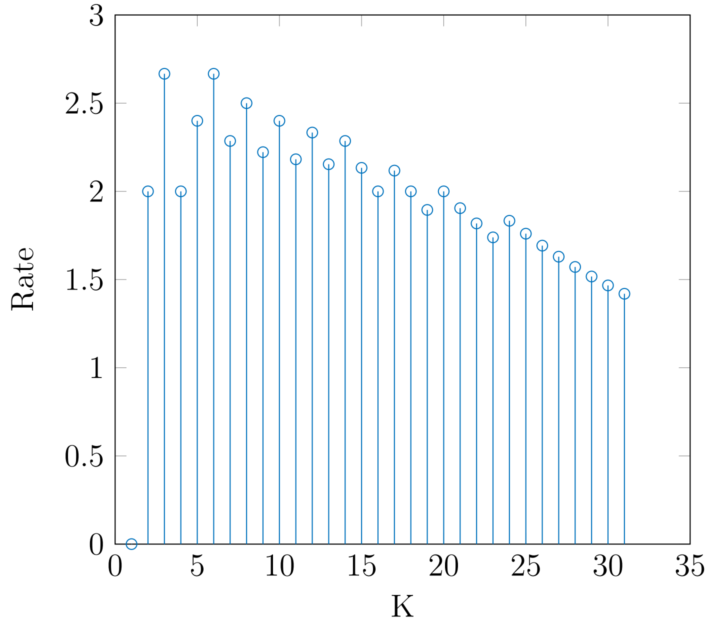


Figure 6.21: Rate of $(J, M, N, \alpha) = (2, 16, 64, 0.01)$ for fixed SCMA per user

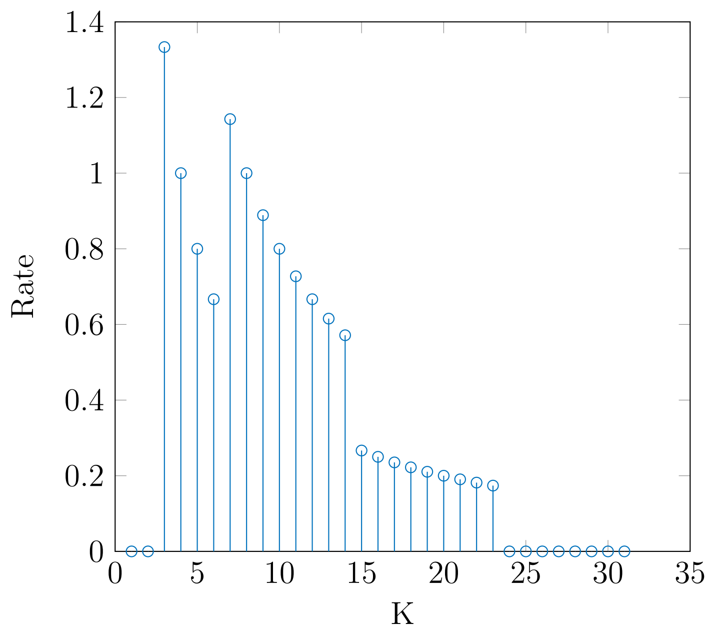


Figure 6.22: Rate of $(J, M, N, \alpha) = (5, 16, 64, 0.01)$ for fixed SCMA per user

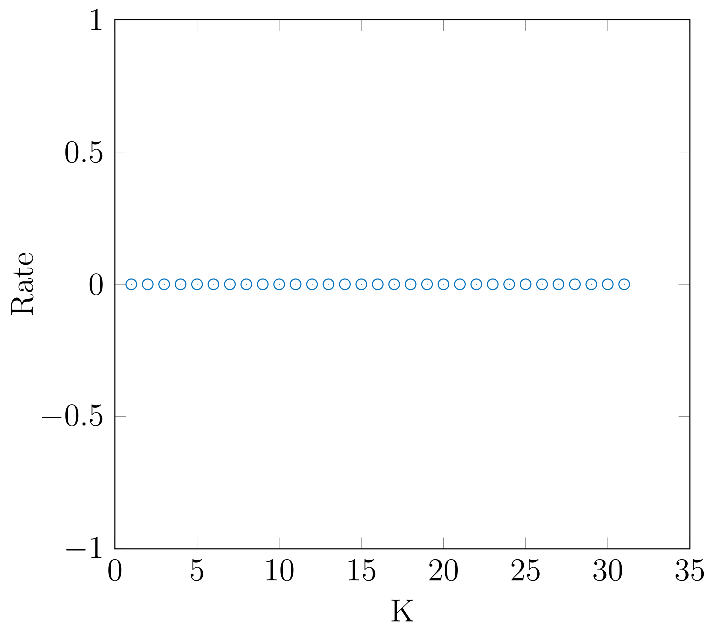


Figure 6.23: Rate of $(J, M, N, \alpha) = (10, 16, 64, 0.01)$ for fixed SCMA per user

6.6.3 Rates using limited number of SCMA packages

In the previous section we calculated the rate for the variable and the fixed SCMA, when the number of SCMA packages, m, is so large. In this section we obtain some practical results, using finite m. Furthermore, to have a fare comparison, we obtain the maximum rate to have the frame error rate of α[1].

Figures 6.24 and 6.25 depicts the rate for the case of $N = 64$, $M = 16$, $J = 20$, $m = 100$ and $\alpha = 0.01$. For each K, the maximum rate which the FER is less than α is depicted. We have used from the equation 6.25 in calculating the rate.

$$r = 1 - \frac{t-1}{N_c} = 1 - \frac{t-1}{mK} \tag{6.25}$$

As you see Figures 6.26 and 6.27 are compatiable with Figures 6.16 and 6.23 respectively. Figures 6.28 to 6.31 depict the rate for the previous parameters but with five and two users, indeed they are for $J = 5$ and $J = 2$.

[1] In the previous part, for the variable SCMA, we said we can achieve the rate which the FER is zero, but for the fixed case, we considered a maximum FER and denoted it by α. In this section we calculate the rate for both scenarios to have the FER of α.

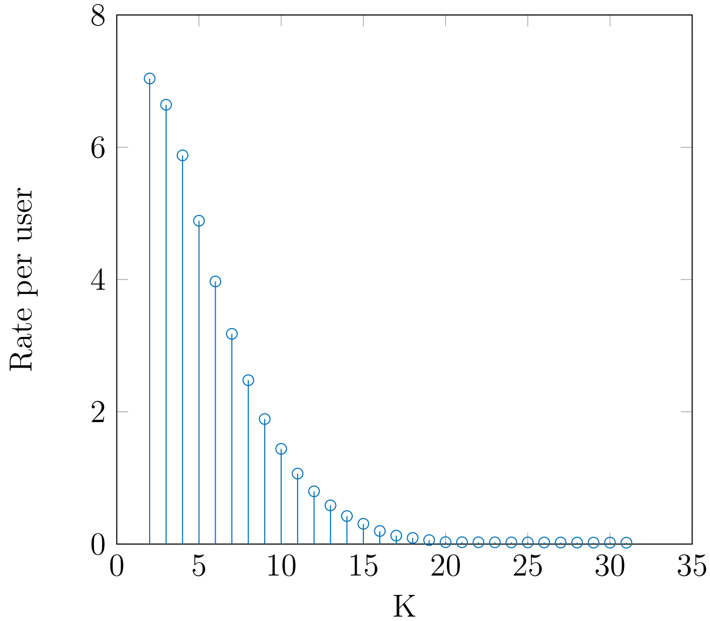


Figure 6.24: Rate for variable SCMA for $(J, N, M, m, \alpha) = (20, 64, 16, 100, 0.01)$

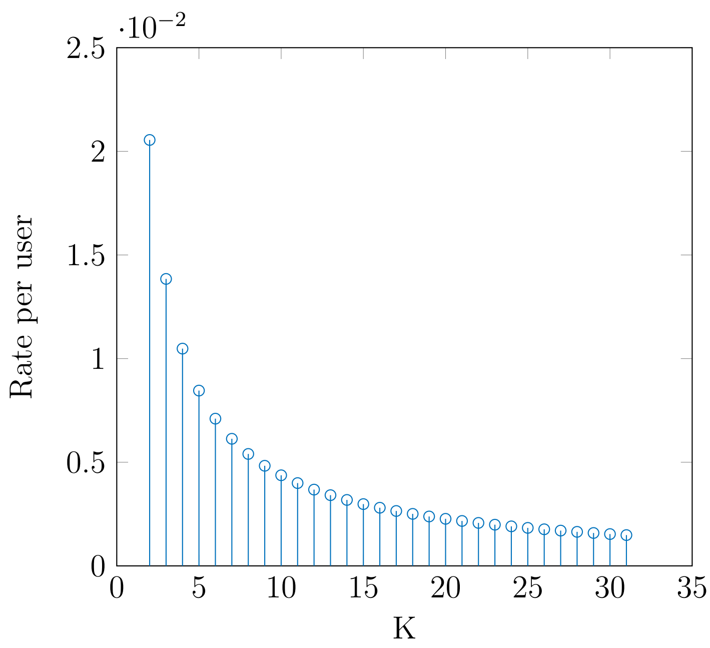


Figure 6.25: Rate for fixed SCMA for $(J, N, M, m, \alpha) = (20, 64, 16, 100, 0.01)$

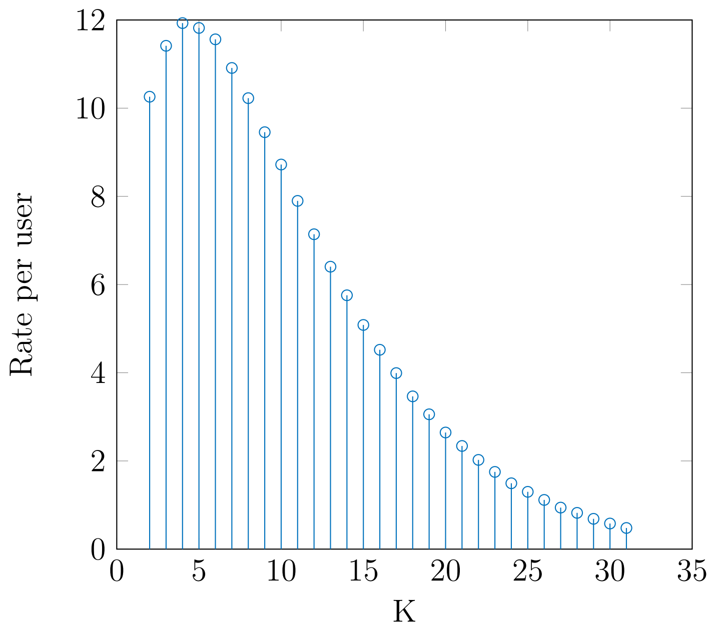


Figure 6.26: Rate for variable SCMA for $(J, N, M, m, \alpha) = (10, 64, 16, 100, 0.01)$

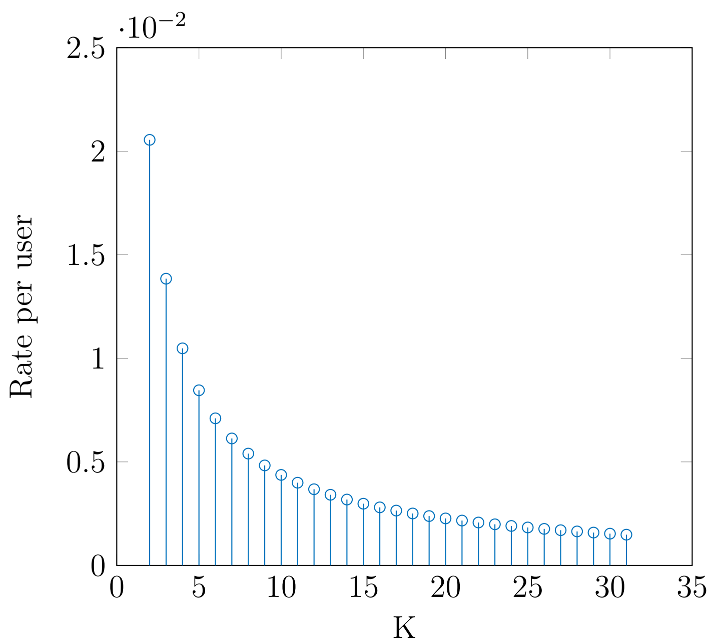


Figure 6.27: Rate for fixed SCMA for $(J, N, M, m, \alpha) = (10, 64, 16, 100, 0.01)$

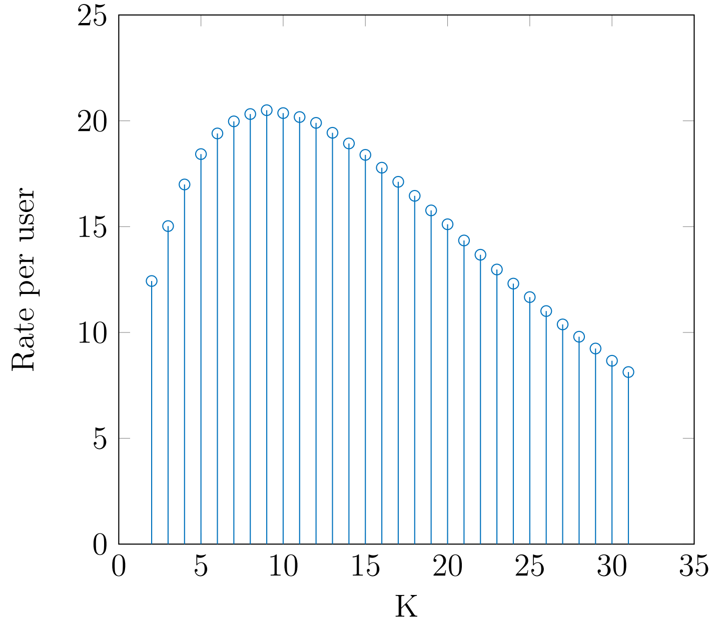


Figure 6.28: Rate for variable SCMA for $(J, N, M, m, \alpha) = (5, 64, 16, 100, 0.01)$

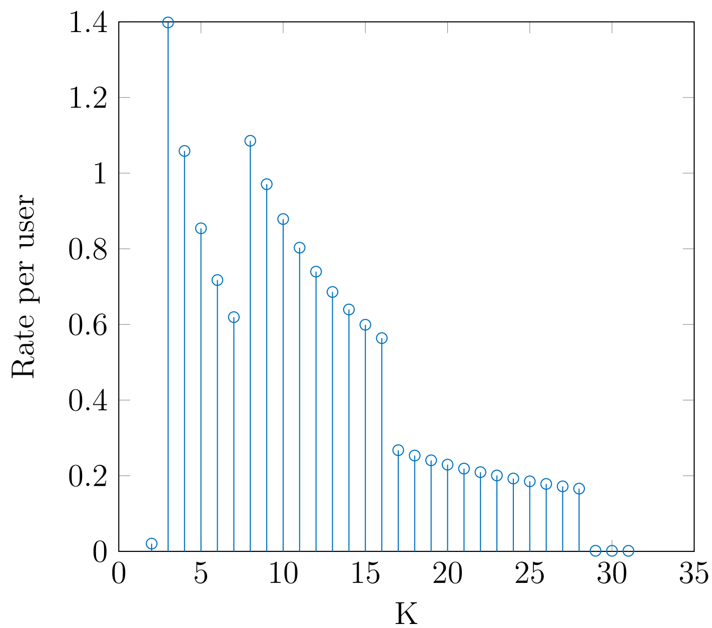


Figure 6.29: Rate for fixed SCMA for $(J, N, M, m, \alpha) = (5, 64, 16, 100, 0.01)$

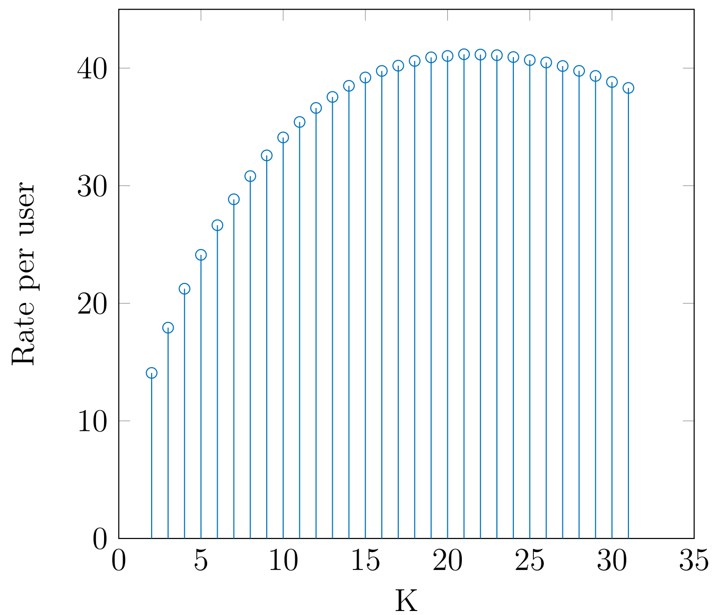


Figure 6.30: Rate for variable SCMA for $(J, N, M, m, \alpha) = (2, 64, 16, 100, 0.01)$

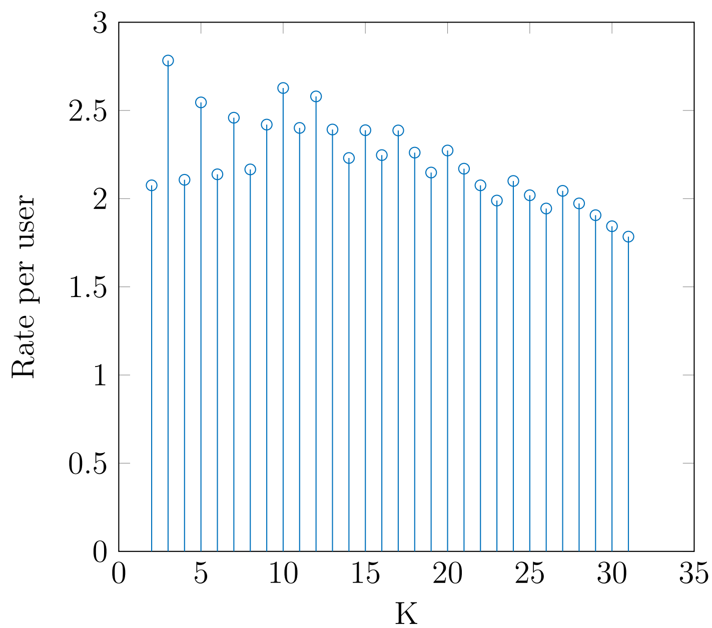


Figure 6.31: Rate for fixed SCMA for $(J, N, M, m, \alpha) = (2, 64, 16, 100, 0.01)$

Chapter 7

Conclusion and future works

We showed by using a variable signature spreading, when the number of users is not so large, and the effective rate of the system is small, the performance will improve. But if there are a lot of user in the system, the fixed spreading can be better than variable one.

We obtained the PDF for the number of erroneous resources in a single SCMA package.Then we calculated the PDF for m consecutive packages' erroneous resources. We compared the FER performance between the fixed and the variable code scenario and we showed the FER depends on several factors and there is the possibility for either of them to be better. However the scenarios which leads the fixed SCMA to outperform the variable one are usually those which the performance is not desirable. For example we showed by increasing the user's number, the fixed one can get better, however the overal performance of the system for a large number of users is not so good.

We discussed the maximum achievable rate for fixed and variable SCMA as well. We obtained a formula which needs combinatorial optimization to find the variable parameters. But we fixed some parameters on reasonable and applicable values and found the maximum rate by the simulation. We saw that in all of them the variable has significantly higher rates than the frixed SCMA.

In section 5.3, we assumed the PDF of SINR at the input of the channel's decoder is Gaussian. We said that it is a poor assumption as the Q function has a maximum of 0.5, but we see in outage figures that the intersection of variable and fixed graphs occurs in probabilities other than 0.5, so it is good to find the exact PDF of SINR and find the intersection point exactly. For large number of users, the interference can be modeled as Gaussian distributed and the SINR will be the logarithm of a Gaussian random variable. However, in the case that the number of users is not large enough, to approximate the interference's distribution, the PDF will take a non-simple form, which can be discussed separately.

For a block code like BCH, as it can correct a specified number of errors in the system, we know the performance without implementing the channel code. However, for a complicated channel code like Turbo codes, it is not valid. we have made a guess on the performance of the system by using a Turbo code, but we have not implemented it. The exact and more reliable FER for this type of channel code obtains by implementing the Turbo code in the system. So it is nice if we implement the Turbo code in the system for further researches. To make the system more realistic, we can consider the channel to be multipath and then use a rake receiver to exploit the multipath diversity.

In the SCMA section, we calculated the rates for both scenarios. But as their optimization was difficult, we fixed some of the parameters on reasonable values. It is better to solve them generally to obtain the optimized values.

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APPENDICES

Appendix A

Welch's Bound

Theorem: Welch's bound [44]: If there are J normalized complex vectors of length G, then the maximum absolute value of their inner product is bounded by:

$$\max_{i \neq j} |\langle \mathbf{s}_i, \mathbf{s}_j \rangle|^2 \geq \frac{J - G}{(J-1)G} \tag{A.1}$$

Proof: The proof is from [31]. Let $R_{\max}$ denotes $\max_{i \neq j} |\langle \mathbf{s}_i, \mathbf{s}_j \rangle|$. As the vectors are normalized, then we can say:

$$\begin{aligned}
&J(J-1)R_{\max}^2 + J \geq \sum_{i=1}^{J} \sum_{j=1}^{J} |\langle \mathbf{s}_i, \mathbf{s}_j \rangle|^2 = \\
&\sum_{i=1}^{J} \sum_{j=1}^{J} \sum_{n=1}^{G} \sum_{m=1}^{G} s_i(n) s_j^*(n) s_i^*(m) s_j(m) = \\
&\sum_{n=1}^{G} \sum_{m=1}^{G} \sum_{i=1}^{J} \sum_{j=1}^{J} s_i(n) s_i^*(m) s_j^*(n) s_j(m) = \\
&\sum_{n=1}^{G} \sum_{m=1}^{G} \left(\Big(\sum_{i=1}^{J} s_i(n) s_i^*(m) \Big) \Big(\sum_{j=1}^{J} s_j^*(n) s_j(m) \Big) \right) = \\
&\sum_{n=1}^{G} \sum_{m=1}^{G} \left(\Big| \sum_{i=1}^{J} s_i(n) s_i^*(m) \Big|^2 \right)
\end{aligned} \tag{A.2}$$

Now if we neglect those terms which $m \neq n$, then

$$\begin{aligned}
&\sum_{n=1}^{G}\sum_{m=1}^{G}\left(\left|\sum_{i=1}^{J} s_i(n)s_i^*(m)\right|^2\right) \geq \\
&\sum_{n=1}^{G}\left(\left|\sum_{i=1}^{J} |s_i(n)|^2\right|^2\right) \overset{(1)}{\geq} \\
&\frac{1}{L}\left|\sum_{n=1}^{G}\sum_{i=1}^{J} |s_i(n)|^2\right|^2 \overset{(2)}{=} \frac{J^2}{L}
\end{aligned} \tag{A.3}$$

in which (1), is a result of Cauchy's inequality for sum of squares and (2), is a result of considering normalized vectors. So

$$(J-1)R_{\max}^2 + 1 \geq \frac{J}{L} \Rightarrow R_{\max}^2 \geq \frac{J-L}{(J-1)L} \tag{A.4}$$

∎

Appendix B

Symbols Packing's bits error probability

We claimed in the special case of 4-ASK, the correct decision's probability for first and second positions are as

$$\begin{aligned} Pr(c_{b1}) &= 0.5\Big((1-A)+G+E+F\Big) \\ Pr(c_{b2}) &= 0.5\Big((1-E)+(1-A)\Big) \end{aligned} \tag{B.1}$$

Now we want to show it. The minimum distance between symbols is considered to be $2d$. We use minimum distance criterion for assigning a symbol to the receiver symbol r. r_1 and r_2 are the first and the second bits of the received symbol. The probability that the first bit is detected correctly is equal to

$$\begin{aligned} Pr(b_1 : \text{correct}) &= \sum_{i=1,3} Pr(r_1 = 0|A_i)Pr(A_i) + \sum_{i=2,4} Pr(r_1 = 1|A_i)Pr(A_i) \\ &= 0.25\Big(\sum_{i=1,3} Pr(r_1 = 0|A_i) + \sum_{i=2,4} Pr(r_1 = 1|A_i)\Big) \\ &= 0.5\Big(\sum_{i=1,3} Pr(r_1 = 0|A_i)\Big) \end{aligned} \tag{B.2}$$

which, we have assumed a priori probability is the same for all the modulation symbols. The first bit is detected as 0 if the received point is in $r < -2d$ or $0 < r < 2d$. So if n

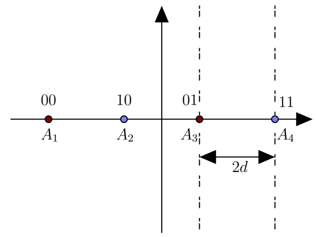

Figure B.1: The proposed labeling

represents the noise term, we can write

$$\begin{aligned} Pr(r_1 = 0|A_1) &= Pr(n < d) + Pr(3d < n < 5d) \\ &= (1 - A) + G \end{aligned} \tag{B.3}$$

and

$$\begin{aligned} Pr(r_1 = 0|A_3) &= Pr(n < -3d) + Pr(-d < n < d) \\ &= E + F \end{aligned} \tag{B.4}$$

in which, we have used from the Figure 4.6 to represent the probability of noise in a certain interval and noticing that $Pr(3d < n < 5d) = Pr(-5d < n < -3d)$. So

$$Pr(b_1 : \text{correct}) = 0.5\Big((1 - A) + G + E + F\Big). \tag{B.5}$$

In a similar manner we can write

$$Pr(b_2 : \text{correct}) = 0.5\Big(\sum_{i=1,2} Pr(r_2 = 0|A_i)\Big) \tag{B.6}$$

but

$$Pr(r_2 = 0|A_1) = Pr(n < 3d) = 1 - E \tag{B.7}$$

and

$$Pr(r_2 = 0|A_2) = Pr(n < d) = 1 - A \tag{B.8}$$

so

$$Pr(b_2 : \text{correct}) = 0.5\Big((1 - E) + (1 - A)\Big). \tag{B.9}$$