

Entropy-based Demand Splits in a Hospital-Warehouse Profit Center

by

Nidarsha Attanayake

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Management Sciences

Waterloo, Ontario, Canada, 2014

© Nidarsha Attanayake 2014

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Financial pressures on healthcare industry in the United States and elsewhere have forced the industry to address their supply costs, their fastest growing cost sector currently comprising over 40 % of their total spend. In the USA, the healthcare supplies market is dominated by a few large distributors and significant barriers to entry. Cost reducing measures to date have relied on Group Purchasing Organizations to leverage economies of scale in negotiating price reductions. Recently, the healthcare industry has been deemphasizing this practice. In doing so, healthcare organizations have merged to form large Integrated Delivery Networks, leveraging their collective purchasing capacity to negotiate price reductions. These organizations have essentially created their own internal Group Purchasing Organizations to compete with external suppliers. Although these ventures have been publicized to be “successful”, their overall success cannot be independently validated. Furthermore, the operational details of creating these ventures, financial analyses, and operations are not publically available.

Our ultimate objective is to model the creation of ventures in which healthcare organizations enter price competitions with their external vendors using the currently prevalent market parameters and practices. Specifically, the models would identify and quantitate the parameters that determine venture success, here referred to as Venture Success Metrics. Such models would comprise multiple external suppliers of different products that belong to different categories. This thesis is our first step towards that objective. It represents a simplified venture in which the hospital runs its own warehouse as a profit center that competes with one external vendor on a single supply item. The model is based on currently prevalent healthcare industry practices. In particular, it incorporates discount schedules that accurately account for the unique healthcare industry practice of offering year-2 volume-based discounts based on year-1 volumes, restricted only to the contract period. Modeling a simplified venture enabled us to identify and quantitate the parameters that determine venture success. These

parameters comprise the vendor and warehouse year-1 profit objectives as well as the bias of the hospital's purchases from its own warehouse.

Pursuing the models of the thesis induced the development of healthcare-relevant sigmoidal discount schedules. These functions accurately represent the tabular step-function discount schedules while averting the infinite and discontinuous derivatives of the latter. Their "continuous derivatives" advantage renders the sigmoidal discounts readily useable in computing price equilibria, a feat that was not easily achievable with the rigid step-function discounts.

The thesis also introduces novel demand split functions in which a customer's total demand can be *equitably* apportioned across all suppliers subject to diverse business objectives such as price constraints or biasing purchases in favor of one or more suppliers while *retaining equitability*. The ultimate economic goal for achieving equitability is to conserve supply source. The demand split methodology introduced in this thesis can be characterized as "achieving equitability under business constraints". A series of examples are provided to illustrate the methods developed in this research. Finally, the thesis concludes with a synopsis of the findings and future extensions.

Acknowledgements

I would like to express my high appreciation of and deep gratitude for Professor James Bookbinder for giving me the opportunity to develop, refine, and articulate the nebulous ideas that arose during my supply chain operations tenure in the healthcare industry. I am highly appreciative of his willingness to expand his research venues into healthcare supply chain. His insightful questions and comments invariably led to profound pedagogical and developmental outcomes. In addition to his inspirational influence, his ever readiness with the kind word and moral support on academic as well as life matters eased the challenge.

I am also deeply indebted for Professors Selcuk Onay and Benny Mantin for their kind willingness to read and critique my thesis.

I would also like to thank Dr. Tariq Andrea for explaining the concept of entropy, its applications in physics, and its possible utility in operations research.

Last but not least, I would like to thank my parents for encouraging and supporting me to reach to this stage and my daughter Atara who patiently kept herself busy while I was pursuing my research and course work.

Table of Contents

Author's Declaration.....	ii
Abstract	iii
Acknowledgements	v
Table of Contents	vi
List of Figures	viii
List of Tables	x
Chapter 1 Introduction.....	1
1.1 Problem statement	1
1.2 Supply chain management in healthcare.....	2
1.3 Motivation and objective	3
1.4 Models and approaches.....	5
1.5 Organization of the thesis	6
Chapter 2 Literature Review.....	8
2.1 Use of game theory in supply chain	8
2.2 Supply chain practices in healthcare.....	9
2.3 Exponential demand	10
2.4 Quantity discounts	11
2.5 Demand splitting	13
Chapter 3 Determination of Healthcare Discount Structure Based on Previous Year's Purchases	15
3.1 Single-tiered discount schedule.....	21
3.2 Single-tiered discounted pricing.....	23
3.3 Special implementation.....	24
3.4 Determining discount breakpoints V_0	25
3.5 Multi-tiered discount	26
3.6 Numerical example.....	28
Chapter 4 General Methodology of Computing Inter-Supplier Demand Splits.....	32
4.1 EOQ and Logit-based demand-splits.....	33
4.2 Entropy-based demand split across two suppliers	35
Chapter 5 Determining Venture Success Metrics	40
5.1 Problem setup.....	42
5.2 Demand-split functions	44

5.3 Discount and pricing schedules	47
5.4 Cost and profit functions.....	50
5.4.1 Hospital cost functions.....	50
5.4.2 Vendor profit functions	51
5.4.3 Warehouse profit functions	52
5.5 Derivative functions and computing equilibrium markups.....	53
5.6 Venture Success Metrics (VSM).....	55
5.6.1 Variables that determine VSM values	56
5.6.2 VSM computation sequence.....	59
5.7 Computations, results, and conclusions	61
5.7.1 Computing year-1 markups (σ_1, ζ_1) – the Bertrand Nash equilibrium	62
5.7.2 Variation of (σ_e, ζ_e) with the demand split exponent f	66
5.7.3 Computing Venture Success Metrics.....	69
Chapter 6 Conclusions and Future Research	82
6.1 Healthcare-relevant discount schedules based on previous year’s purchases	82
6.2 Entropy-based demand splitting methodology	83
6.3 Venture Success Metrics	85
References	87

List of Figures

1-1 A typical healthcare supply chain	3
1-2 Thesis organization	7
2-1 Price variation with the discount schedule of equation 2–1.....	12
3-1 Price variation with a single-tiered step-function	16
3-2 Price with a three-tiered step-function discount.....	16
3-3 Price variation with a linear discount schedule	17
3-4 Discount schedule (V) of equation 3-1 ($V_m = 0.4, V_0 = 0.6, \mu = 12$)	22
3-5 Discount schedule (V) of equation 3-1 ($\mu = 10, 20, 50; V_m = 0.4, V_0 = 0.6$)	22
3-6 Discount schedule (V) of equation 3-1 for four different discount schedules where $\{(V_m, V_0) =$ $(0.6, 0.2), (0.6, 0.4), (0.4, 0.6), (0.3, 0.8); \mu = 25\}$	23
3-7 Year-2 markups (σ_2 ; equation 3-2) under four different discount schedules where $\{(V_m, V_0) =$ $(0.6, 0.2), (0.6, 0.4), (0.4, 0.6), (0.3, 0.8); \mu = 25\}$	24
3-8 Discount schedule (V) of equation 3-4 for four different discount schedules where $\{\kappa = 0.2, .4,$ $0.6, 0.8; V_0 = 0.4; \mu = 25\}$	25
3-9 Year-2 markups (σ_2 ; equation 3.2) for four different discount schedules where $\{V_0 = 0.2, 0.4,$ $0.6, 0.8; \kappa = 0.4; \mu = 25\}$	25
3-10 The three components of the multi-tiered discount schedule (equation 3-8) $\{n = 3; (V_{m,1},$ $V_{m,2}, V_{m,3}) = (0.05, 0.07, 0.12); (V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)\}$	27
3-11 Three-tiered discount schedule (V, equation 3-8) $\{n = 3; (V_{m,1}, V_{m,2}, V_{m,3}) = (0.05, 0.07, 0.12);$ $(V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)\}$	27
3-12 Three-tiered year-2 markup using the three-tiered discount schedule (V, equation 3-8) $\{n = 3;$ $(V_{m,1}, V_{m,2}, V_{m,3}) = (0.05, 0.07, 0.12); (V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)\}$	28
3-13 Year-2 discount schedules for four values of π_V^O	30
3-14 Year-2 markups for four values of π_V^O	31
5-1 Pre- and post-venture situations, 1 and 2 respectively. L and S are long and short shipping distances. M, V, W, and H represented manufacturer, vendor, warehouse, and hospital, respectively.	43
5-2 Vendor fractions (v) as a function of σ for $f = 0, -0.5, \text{ and } -1$	45
5-3 Warehouse fraction ($1 - v$) as a function of σ for $f = 0, -0.5, \text{ and } -1$	45
5-4 Vendor discount schedule (V; equation 5 – 4; $V_m = 0.4, V_0 = 0.6, \mu = 12$).....	49

5-5 Vendor discount schedule (V; equation 5 – 4; $V_m = 0.4$, $V_0 = 0.6$) for $\mu = 10, 20$, and 50	49
5-6 Vendor discount schedule (V; equation 5 –6; $\mu = 25$) with $\{(V_0, \kappa) = (0.4, 0.2), (0.4, 0.4), (0.4, 0.6), (0.4, 0.8)\}$	50
5-7 Computation sequence.....	61
5-8 Vendor year-1 profit function equation 5 – 33.....	64
5-9 Warehouse year-1 profit function 5-30	64
5-10 Vendors (red) and warehouse (blue) lines of maxima for $f = 0.2, 0, -0.2, -0.4$. The (σ, ζ) values at the line intersections corresponds to equilibrium (σ_e, ζ_e) markups.....	65
5-11 Variation of the equilibrium markups (σ_e, ζ_e) with f value.....	66
5-12 Variation of π_{w1} (equation 5–51d, $\sigma_1 = 1.3$) for $f = 0.4, 0, -0.2, -0.4$	68
5-13 Variation of the warehouse fraction $(1 - v_1; v_1$ computed with 3 – 3, $\sigma_1 = 1.3$) for $f = 0.4, 0, -0.2, -0.4$	68
5-14 Variation of π_{v1} (equation 5–51g, $\zeta_1 = 1.3$) for $f = 0.4, 0, -0.2, -$ and 0.4	69
5-15 Variation of the vendor fraction $(v_1, \text{equation } 5-3, \zeta_1 = 1.3)$ for $f = 0.4, 0, -0.2, -$ and 0.4	69
5-16 Hierarchy of figures 5 – 17 to 5 – 23.....	71
5-17 Variation of the first braces term in equation 5 –52 with v_1	72
5-18 Variation of σ_2 and v_2^{en} with v_1	73
5-19 Variation of the $\sigma_2 v_2^{en}$ product, the second term in equation 5–52, with v_1	73
5-20 Variation of ζ_2 and $(1 - v_2^{en})$ terms with v_1	73
5-21 Variation of the $\zeta_2 (1 - v_2^{en})$ product, the third term in equation 5–52, with v_1	74
5-22 Variation of the sum of the second and thirds terms in equation 5–52 with v_1	74
5-23 Variation of δC (equation 5–52) with v_1	74

List of Tables

3-1 Product information.....	28
3-2 Profit objectives, V_0 , and V_m . (values in this table computed using $(\kappa, \mu) = (0.5, 50)$)	29
3-3 Combinations of κ , π_V^0 and μ that yield similar σ_2 at $\nu_1 = 0.5$	30
5-1 Variation of the equilibrium markups (σ_e, ζ_e) with f value	66
5-2 Variation of $1 - \nu_1^{\min}$ and with (π_V^0, π_W^0) at $f = 0$	77
5-3 Values of δC for three different $[f, \sigma_e, \zeta_e]$ sets over an entire (π_V^0, π_W^0) grid.....	78
5-4 Values of T_{wp} for three different $[f, \sigma_e, \zeta_e]$ sets over an entire (π_V^0, π_W^0) grid	79
5-5 Values of τ for three different $[f, \sigma_e, \zeta_e]$ sets over an entire (π_V^0, π_W^0) grid	80
5-6 Variation of average VSM values with f value.....	81

Chapter 1

Introduction

Due to current financial pressures, companies have targeted their supply chain to reduce costs and improve efficiencies. Healthcare providers, globally, represent one of the hardest hit sectors. In most healthcare organizations warehouse facilities are underutilized and inefficiently operated. This is a consequence of lack of competition that drives optimization. This thesis develops models that determine the viability of cost reducing ventures in which a hospital's warehouse enters price competitions with its vendors. We develop novel entropy-based demand-splitting methodology and healthcare-relevant discount schedules to compute Venture Success Metrics. These methodologies will be illustrated with numerical examples.

In this thesis we refer to four different entities: hospital, warehouse, vendor, and manufacturer. The manufacturer is the entity that manufactures the supply items used by the hospital. The manufacturer does not sell products directly to the hospital. Instead, the manufacturer sells products to the vendor and warehouse only. The latter two entities in turn, sell products directly to the hospital. Direct manufacturer-to-hospital sale does not occur.

The vendor and warehouse are commonly referred to as distributors or suppliers. They differ in the following regard. The warehouse is established by the hospital with the sole purpose of entering price competition with the vendor with the ultimate objective of enhancing the hospital's profits.

1.1 Problem statement

Healthcare systems in the United States and elsewhere are under significant pressure to cut cost. A Global Healthcare Exchange report indicates that supply chain spending in hospitals and healthcare organizations accounted for 40 – 45 % of their total operating expenses. This corresponds to over \$300 billion of the total \$800 billion spent on healthcare reported by National Center for

Health Statistics in 2010. Current growth in healthcare supply chain spending is outpacing all other spend categories. Not surprisingly, it remains a target of aggressive cost cutting.

Medical-surgical distribution in the United States is an oligopolistic market with few large players and significant barriers to entry. This character enables the use of game theoretic approaches for strategic decision making. The “game” is a price competition between an external supplier (vendor) and a central warehouse serving as an internal supplier whose payoffs are computed using their respective profit functions.

1.2 Supply chain management in healthcare

Healthcare supply chain is multi-echelon. Typically, a large central warehouse serves multiple hospitals. Figure 1-1 shows a typical operational model with three hospitals that belong to the same organization, each with its own warehouse connected to, and replenished by, a central warehouse. All external suppliers deliver products to the central warehouse, which serves as a make-or break-bulk facility and which also distributes these products to individual hospitals. The order is placed with the manufacturer or the distributor and will be received based on the lead times and contractual obligations.

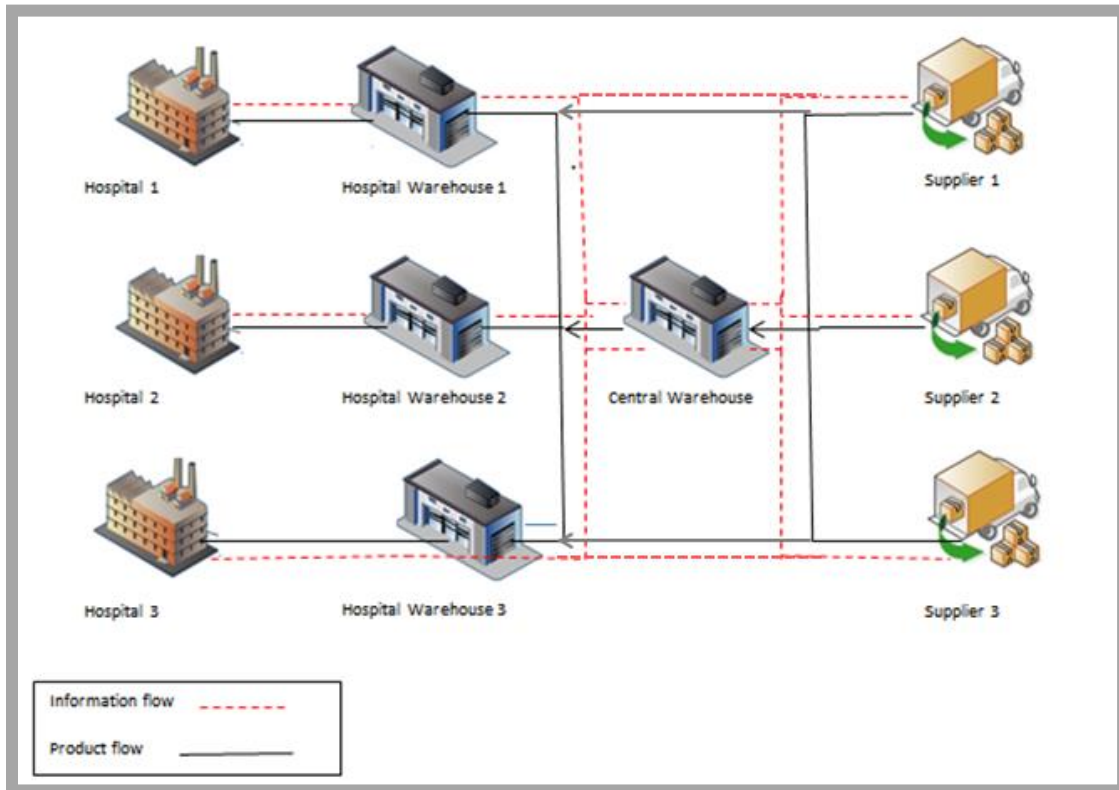


Figure 1-1: A typical healthcare supply chain

Hospitals typically renew their product contracts annually. Each supplier is sent a Request for Proposal which lists the items that the hospital expects to purchase in the coming year and the hospital-proposed preferred prices. Additionally, hospitals consider product quality and lead times. In the models of this thesis, lead times and product quality are assumed to be the same of the vendor and warehouse and that their competition is exclusively price based. Once a price is set and agreed to by the hospital, it will remain in effect throughout the contract period which is typically 12 months.

1.3 Motivation and objective

The Annual Pain in the healthcare supply chain survey conducted by UPS Solutions, 60% of the respondents indicated that supply chain costs are their biggest concern. Historically, healthcare

systems have engaged Group Purchasing Organizations (GPO's) to leverage economies of scale in negotiating price reductions. There has been a recent trend to move away from using GPO-negotiated contracts. This has led to mergers of large hospitals and healthcare systems, and the concomitant development of Integrated Delivery Networks (IDN's). These are exemplified by Novant Health (North Carolina), Greater New York Hospital Alliance and Yankee Alliance (New York). The preceding IDN's, and others, manage their own supply chain operations, in-house. However, healthcare organizations such as Sisters of Mercy hospital system have done just that. Mercy has formed its own supply chain organization, "ROI," (<http://www.roiscs.com/>), and has been able to compete with its vendors.

Our ultimate objective is to create models that identify successful ventures in which healthcare organizations enter price competition with their vendors. Such models would comprise multiple external suppliers of distinct products that belong to different categories. Achieving this objective requires thorough understanding of the discount and pricing paradigms of the healthcare industry. Having access to multiple suppliers for a homogenous product is of great importance to health systems. Advance knowledge of the stock status at Supplier 1 may enable an expedited order to Supplier 2 to avoid a stockout at the hospital. This would ensure continuous delivery of patient care. The model proposed here—

1. Allows multiple sources for a homogenous product. This eliminates the risk of depending on a single source of supply.
2. Relies on analytically robust discount schedules that are relevant to the healthcare industry.
3. Develops Venture Success Metrics that enable the hospital to determine the impact of its various decision variables on the success of the venture. (The entity discussed here is the hospital warehouse which is setup by the hospital as a separate business entity, a profit center that competes with the external vendor.)

This thesis is our first step towards meeting those three objectives. The specific objective of this thesis is to identify the parameters that lead to the success of a venture in which a hospital creates its own warehouse that enters into price competition with a single vendor over a single supply item using volume discount paradigms that are unique to the healthcare industry (year-2 discounts based on year-1 volumes). Implementing this model necessitated the development of novel demand split methodologies which, in retrospect had business applications beyond healthcare.

This one-vendor one warehouse model enables us to identify and quantify the variables that determine the viability of the concept of this venture: vendor and warehouse year-1 profit as well as the bias of the hospital's purchases from its own warehouse.

1.4 Models and approaches

In the models of this thesis, the competing entities use Bertrand-Nash equilibrium methodology to determine their equilibrium markups subject to the demand split parameters set by the hospital. According to Perloff (2012; p. 467), Bertrand-Nash equilibrium is defined as “a Nash equilibrium, where prices are defined as a set of prices such that no firm can gain higher profit by choosing a different price if the other firms continue to charge these prices”. Equilibrium markups are computed for the vendor and warehouse profit functions which comprise sales revenue, holding, shipping and ordering costs components. Bertrand Nash equilibrium identifies the best response and the dominant strategies. Perloff (2012; p. 488) defined the best response as “the strategy that maximizes a player's payoff given its beliefs about its rivals' strategies”.

To model the competition between the warehouse and the vendor, and to set the warehouse as a separate business venture, it is important to adhere to the current practices of the healthcare industry. The model will focus on a pair of exam gloves with deterministic demand. From a patient-care standpoint, the hospitals should always have access to several suppliers selling similar items to

avoid any out-of-stock or back-order situations. We develop an entropy-based demand splitting methodology to divide the demand equitably between the two players, subject to price constraints. Entropy-based split, first developed in statistical physics, has found its way into other application. This methodology allows the hospital to utilize both suppliers, thereby protecting supply sources.

Another business practice that is adapted in this thesis is the quantity discount paradigm that is unique to the healthcare industry. Volume-based price discounting is prevalent (Axsäter (2000); Choi, et. al (2005)). It is commonplace for healthcare suppliers to offer volume-based discounts to incentivize customers to purchase larger quantities, simultaneously boosting supplier profits while minimizing hospitals' costs. Typically, suppliers offer all-unit price discounts for order quantities that exceed a preset breakpoint. In a practice that is unique to healthcare industry discounts, the breakpoint is based on the previous years' sales. Such discounts can be one-, two-, or multi-tiered. Despite the prevalence of this practice in healthcare, it is absent from the academic literature where the all unit quantity discounts offered are based on purchases made in the current period. Such healthcare discount schedules are step-functions with discontinuous and infinite derivatives. In order to use such discount schedules in computing price equilibria, we replace them with Sigmoidal functions which closely approximate the step-functions with the additional advantage of having continuous and finite derivatives.

1.5 Organization of the thesis

Figure (1-2) shows the organization of the thesis. Chapter 1 defines the problem, gives a synopsis of healthcare supply chain challenges, and outlines the solutions proposed in this thesis. Chapter 2 reviews the literature on problems in solution of healthcare supply chain. Chapters 3, 4 and 5 will discuss and compute the demand-splitting methodology, healthcare-relevant quantity discounts, and the Venture Success Metrics, respectively. Chapter 5 gives a numerical example that details the

application of the methodologies developed in chapters 3, 4 and 5. Chapter 6 concludes with a discussion of the findings and proposes future extensions of the methodologies developed in this research.

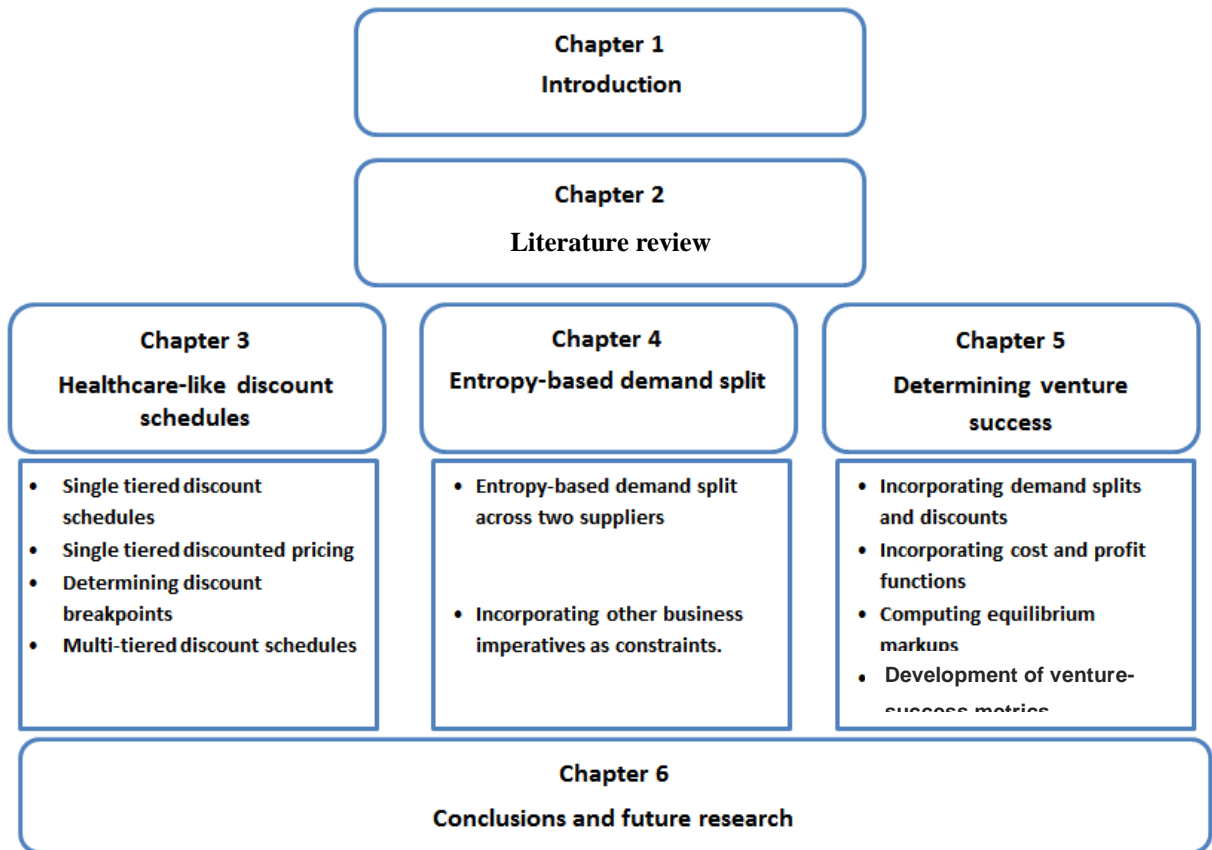


Figure 1-2: Thesis organization

Chapter 2

Literature Review

This chapter reviews the literature pertaining to the topics of this thesis. Section 1 reviews the literature on game-theoretic applications in supply chain. This is followed by literature on supply chain practices in healthcare, and use of the exponential distribution in modeling demand. Next, the chapter focuses on literature on quantity discounts and finally offers a comprehensive literature review on demand-splitting methodologies.

2.1 Use of game theory in supply chain

Game- theoretical applications in supply-chain practices are prevalent in literature, falling into two categories: cooperative and non-cooperative games. Nagarajan and Susic (2008) and Feistras-Janeiro et al. (2011) exemplify cooperative games. Feistras-Janeiro et al. (2011) conducted a comprehensive review on cooperative game theory applications in centralized inventory management practices and Nagarajan and Susic (2008) surveyed the applications of cooperative game theory in supply chain with a main focus on stability and profit allocation. Nagarajan and Susic (2008) also analyzed several bargaining models to allocate profit among the supply chain players. Chinchulum et al. (2008) solved single and multi-period supply chain problems using cooperative and non-cooperative game theory. A Seller-buyer supply chain models under non-cooperative and cooperative situations were researched by Esmaeilli et al. (2009). Stackelberg or leader-follower solution methodologies were used to model the non-cooperative applications and the seller and buyer alternated as leader and follower in this case. Cooperative situations were demonstrated using pareto-optimal scenarios. Osborne (2002) provided a basic introduction to game theory with a main focus on oligopolistic situations such as Cournot and Bertrand models. Shenoy (1980) presented a non-zero-sum game of the world oil market, where optimal prices of a barrel of oil for OPEC (Organization of

Petroleum Exporting Countries) and OPIC (Organization of Petroleum Importing countries) were analytically determined in a non-cooperative sense but the research was also extended to model the optimum price of a barrel of oil in a cooperative sense using the Von Neumann-Morgenstern solution.

An inventory allocation study in multi-channel distribution center using game theory was introduced by Geng and Malik (2007). The idea behind the latter paper is somewhat similar to this thesis where we split hospital demand between the vendor and warehouse in favor of the warehouse. Their model was biased toward the manufacturer, thereby allocating more capacity towards their own warehouses. In this thesis, a “warehouse-bias” is used to bias demand in favor of the warehouse with the objective of enhancing the odds of venture success. However, Geng and Malik (2007) used a Stackelberg game to model the scenarios, whereas the Bertrand-Nash model is employed here.

Literatures on game theoretic models in healthcare supply chain are sparse. In one such application, Hu et al. (2012) used game theory to model healthcare supply chain with and without GPO's (Group Purchasing Organizations). Current healthcare industry trend is to move away from GPO's.

Typically, Bertrand-Nash equilibria are used to model the “retail world” as competing suppliers that are highly sensitive to each other's pricing and marketing strategies. Sinha and Sarma (2010) analyzed a Bertrand-Nash model without channel coordination, akin to our models. Shamir (2011) and Zhang et al. (2012) used Bertrand-Nash games to model the competition among retailers, and to demonstrate the efficiencies in information sharing in a supply chain.

2.2 Supply chain practices in healthcare

The literature on supply chain practices for medical surgical products is very limited. Distinct product categories in the healthcare supply chain are treated differently. There is a large amount of research on inventory management of perishable products such as blood and pharmaceuticals (Bakker et al. 2001; Hammelmayer et al. 2010; Stanger et al. 2012; Kelle et al. 2012; Uthayakumar and Priyan

2013). This thesis focuses on commonly used medical-surgical items such as gloves which require no special handling.

Guerrero et al (2013) performed a simulation of a joint inventory system for a University Medical Center in France with a single warehouse, using stochastic demand, batching, and an order-up-to level ordering policy. Their simulation also included expedited deliveries, a practice that is commonplace in healthcare. The business objective of Guerrero et al. (2013) was to minimize on-hand inventory. Duan and Liao (2013) explored inventory replenishment policies for perishable items in healthcare. They used metaheuristic simulation-optimization methods to minimize expected system outdate rate under a fill-rate constraint. Bernstein et al. (2006) simulated vendor managed inventory in a hospital setting. Arts and Kiesmuller (2012) simulated a two-echelon inventory system with two supply nodes under periodic reviews. They proposed a dual-index policy with three base stock levels. Although, these literature sources showcase the intricacies of the healthcare inventory practices, they are vastly different from the work presented in this thesis. In Chapter 1 we indicated that the thesis focus is strategic. It is to identify cost cutting ventures in which in-house supply chain functions compete with external vendors. The tactical endpoints of literature reports are not useful for our objective.

2.3 Exponential demand

The use of exponential demand functions is prevalent in the economics literature. Lau & Lau (2003) indicate that linear demand functions are not sensible for realistic situations, and that demand more often is iso-elastic or exponential in price. They further stated that the shape of the demand curve can give drastically different analytical results for profit and revenue calculations, especially for multi-echelon systems. They also point out that exponential demand curves are commonly used in analytical modeling and assumed in the empirical estimation of demand curves. Linear demand

curves on the other hand are expressed as $(d = a - bp)$, where d is the quantity demanded, a is the intercept, b the slope, and p the price. In order for the seller to be profitable, the manufacturer's unit price has to be less than a/b . However, iso-elastic and exponential functions do not have such a requirement for the seller to be profitable. Furthermore, according to Huang et al. (2013), the iso-elastic curve $(d = Kp^{-\alpha})$ requires *constant* price elasticity that is not realistic with demand becoming infinite at zero price. Exponential demand curves do not have this drawback.

Exponential demand curves are also used by Xu et al. (2003) and Chongchao et al. (2006) in disruption management and by Uddin and Sano (2011) in their work on coordination and optimization in integrated supply chains. The latter authors cite the explicit price-elasticity term in the exponential demand function, and its simpler manipulations, as their main reasons for using it.

2.4 Quantity discounts

Various types of quantity discounts are in practice today and the healthcare industry is no stranger to such discounts. Quantity discounts can help both the buyer and the seller in optimizing their profit objectives. Our research introduces an all-unit quantity discount in the second year. Such a discount differs from what has already been introduced in the academic literature. In this thesis, we will stay close to the healthcare industry practice in which the quantity discount is based on the *previous years'* purchases by the hospital. We were unable to find academic or trade literature on this type of quantity discount. The literature on more commonplace quantity discount models is discussed below.

In order to circumvent computational obstacles, authors often resort to analytically convenient simplifications such as linear discounts that are weak representations of reality. In one report, Hu et al. (2012) address the impact of GPO's on prices of healthcare supplies using advanced game theoretic models in which quantity based discounted prices represent the game strategies. Their

implementation however reverts to linear discounts, whose use they justify based on prior publications.

Schotanus et al. (2009) highlight the conspicuous absence of analytic non-linear discount functions in the vast literature on discounts. They offer the following form as an alternative

$$p(q) = p_m + \frac{S}{q^\eta} \quad (2-1)$$

Where p , p_m , q , S and η are price, minimum price, quantity, discount scale factor, and discount exponent respectively. The shape of this function is shown in figure 2-1. Other discount schedules with simple non-linear forms have been deployed elsewhere (Marvel et al. 2008).

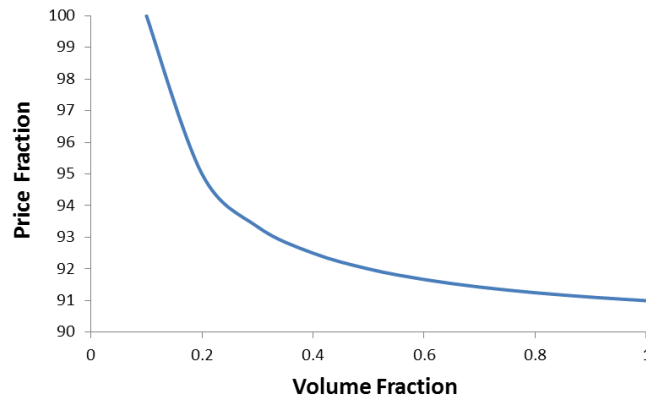


Figure 2 – 1: Price variation with the discount schedule of equation 2–1

Other discounting approaches have been pursued in the literature. Zhang (2010) studied a single-period news vendor problem with all-unit supplier quantity discounts and budget constraints using a nonlinear mixed integer programming model with a Lagrangian relaxation heuristic. Monahan (1984) developed an analytical model for supplier profits by anticipating buyers' response to discounts. In a more recent work, Ke and Bookbinder (2012) found optimal quantity discount from a supplier perspective using non-cooperative two-player Stackelberg-Nash equilibrium and joint decision making. These and related proposals suffer one or more of the troika of shortcomings:

discontinuous derivatives, lack of practical realism, and inability to accurately represent the industry's step-function discounts.

2.5 Demand splitting

Our models require splitting the hospital's demand for one specific product between the vendor and warehouse. Several mathematical models are used in the literature for splitting demand, purchase orders or making route choices. In this section we provide a brief synopsis of the literature of demand splits. Another thorough analysis on EOQ and Logit-based demand splits can be found in Chapter 4 of this thesis.

Literature on splitting replenishment orders among vendors or suppliers is ample. Hill (1996) and Chiang (2001) modeled order splitting under continuous and periodic inventory review policies. Both these authors applied cycle stock reduction policies in their research, while Chiang (2001) used a Lagrangian based optimization policy to divvy up the replenishment orders. Ramasesh et al. (1991) and Abinehchi et al (2013) developed mathematical models for dual and multiple sourcing respectively. Their model calculates order splitting among n suppliers at the time the inventory level reaches the re-order point. Thomas and Tyworth (2006) produced a comprehensive list of such literature specifically pertaining to reducing lead time risks, thereby reducing the safety stock levels. Although these analytical models vary in terms of their level of novelty, their applications are at a tactical level. This thesis on the other hand is focused on splitting the demand at the time of the contract award. That allows a strategic demand split.

The logit function is another widely used model to depict modal split decisions and consumer choice. Murthy and Ashtakala (1987) conducted a modal-split study using logit and log-linear models to determine the future commodity transportation choices in the province of Alberta. Ghareib (1996) evaluated the significance of logit and probit models in modal-split situations. Basu et al. (2007) found optimal prices under a Logit demand in their research. They modeled the consumers' utility

function for the competing brands using a Logit model and found the optimum prices using Nash equilibrium. Logit splits are popular in mode and consumer choices. However, its use here for demand split purposes is novel.

In this thesis, we use an *entropy-based* demand split function to apportion the hospital's demand between vendor and warehouse. Entropy maximization is widely used in statistical physics, it has found its way into econometrics and other areas. Wilson (1967, 1969) derived entropy-maximizing or probability-maximizing models to compute modal splits. His earlier paper (Wilson 1967) employs a conventional gravity model and adds constraints to limit equal distribution of the demand choice, whereas Wilson(1969) extends these findings to determine route choice. When compared with the logit models, entropy-maximizing models which guaranteed equitably splitting demand under price and other constraints do not require the ready availability of market demand data in deriving modal splits.

This summarizes the current literature pertaining to the different topics addressed in this thesis. The next Chapter details the healthcare-relevant discount schedules.

Chapter 3

Determination of Healthcare Discount Structure Based on Previous Year's Purchases

In most industries, the typical volume-based discount depends just upon the quantity purchased on the given requisition or during the current year. Suppliers to the healthcare industry, however, offer a discount percentage whose magnitude depends on the volume purchased in the *previous* year. Such a discount plan is fairly unique, making it difficult for procurement professional to compute economic equilibria that would aid the supplier in deciding its "optimal" discount to offer.

The purpose of this section is to develop analytic discount schedules that approximate the tabular ones found in the healthcare industry, and to derive the set of prices that are based upon those discounts. Numerical examples illustrate how a vendor could achieve a particular profit target by manipulating various parameters.

Historically, healthcare systems have engaged Group Purchasing Organizations (GPO's) to leverage economies of scale in negotiating price reductions. Although there are many types of product contracts, a typical such agreement negotiated by GPO's generally has multi-tiered discounts. Healthcare systems can qualify for these tiers based on their *previous year's* sales. There has been a recent trend to move away from the practice of using contracts negotiated by GPO's. Nevertheless, the product contracts and the determination of discount tiers have not changed.

Accurate knowledge of the industry's discounting and pricing paradigms is important in determining the success of venture described in the problem statement section of this thesis (Chapter 1) In particular, discount schedules that carefully follow the practices of the healthcare industry are required. For the purposes of research and analytical generality, however, those discount schedules need to be sufficiently mathematically robust for facile computation of price equilibria. We were surprised that our extensive literature searches revealed a paucity of work in this area. One of the few

relevant references is Hu et al. (2012). Those searches encouraged the present chapter, in which we conduct examples of such analyses below.

Discounts can be one-, two-, or multi-tiered as shown in figures 3-1 and 3-2. In a variation of this theme, the discounted price is offered only for units ordered in excess of the breakpoint. Contracts negotiated by GPO's as well as health systems themselves are based on multi-tiered volume-based discount schedules. These discount schedules are non-analytic (often tabular) step-functions. The non-differentiability at the points of discontinuity frustrates the use of multi-objective optimization of cost and profit.

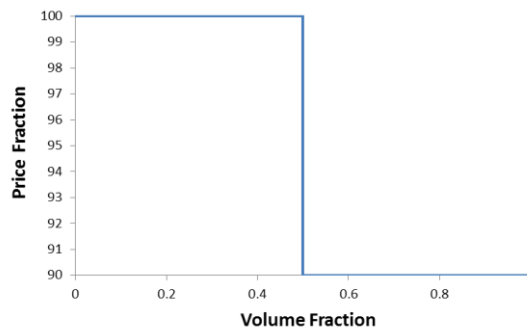


Figure 3-1: Price variation with a single-tiered step-function

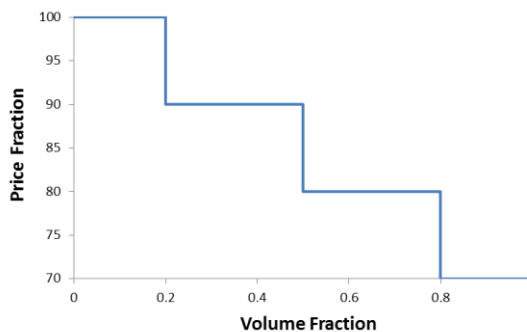


Figure 3-2: Price with a three-tiered step-function discount

To circumvent computational obstacles, authors often resort to analytically convenient simplifications such as linear discounts (figure 3-3) which can poorly approximate the practical situation. In one report, Hu et al. (2012) address the impact of GPO's on prices of healthcare supplies

using game theoretic models in which volume-based discounted prices represent the game strategies. Their implementation reverts to linear discounts, whose use they justify based on prior publications.

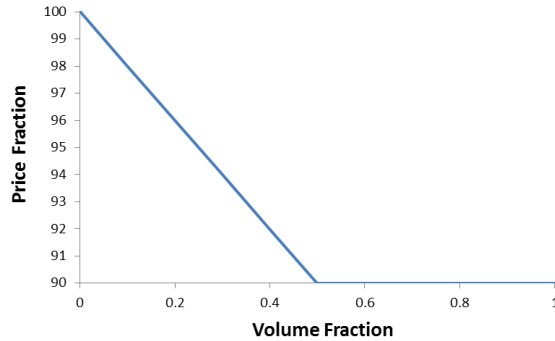


Figure 3-3: Price variation with a linear discount schedule

In summary, our extensive literature surveys were not able to find published realistic, analytically robust, and healthcare-relevant discount schedules. In this paper, we develop discount schedules that meet the following critical features;

1. Represent multi-tiered discounts
2. Analytic with continuous and finite derivatives
3. Easily adaptable for single-tiered and two-tiered discounting
4. Their limiting behavior as the “slope” parameter $\mu \rightarrow \infty$ (equations 3–1) is a step-function.

This paper also utilizes these discount schedules to develop pricing schedules that are consistent with the healthcare industry’s pricing practices in which GPO-related discounts which, absent prior volume commitment by the customer, are rarely offered on first year prices. Instead, second year prices are set by discounting their first year counterparts using all-unit discount schedules in which the magnitude of discount is based on first year volume.

This thesis uses the following variables, acronyms and parameter values.

1. Product Demand

D – Hospital’s annual demand (SKU/year)

υ – Fractional demand satisfied by vendor

υ_1^{en} – Entropy-based year-1 vendor fraction of total demand

υ_1 – Biased year-1 vendor fraction of total demand

υ_2^{en} – Entropy-based year-2 vendor fraction of total demand

υ_1^{min} – Vendor fraction of total demand that minimizes total hospital cost

β – Price sensitivity of demand ($\$^{-1}$)

2. Product Prices and markup

P – Manufacturer price to vendor and warehouse ($\$/\text{SKU}$)

(σ, ζ) – Vendor and warehouse markups

(σ_e, ζ_e) – Vendor and warehouse equilibrium markups

(σ_1, ζ_1) – Vendor and warehouse year-1 markups

(σ_2, ζ_2) – Vendor and warehouse year-2 markups

3. Ordering costs

K – Vendor from manufacturer ($\$/\text{order}$)

k – Warehouse from manufacturer ($\$/\text{order}$)

A – Hospital from vendor and warehouse ($\$/\text{order}$)

4. Lot sizes

N – Manufacturer to vendor (SKU)

n – Manufacturer to warehouse (SKU)

Q – Vendor to hospital (SKU)

q – Warehouse to hospital (SKU)

Q_1 – Vendor to hospital in situation-1 (SKU)

5. Shipping costs

η – Long shipment (\$/SKU) – Long shipments: manufacturer to vendor; manufacturer to warehouse; vendor to hospital

λ – Short shipment (\$/SKU) – Warehouse to hospital

6. Storage costs

ρ – Warehouse, vendor, hospital (\$/SKU)

7. Discount

V, W – Vendor and warehouse discount schedules

V_0, W_0 – Vendor and warehouse discount breakpoints

V_m, W_m – Vendor and warehouse maximal discounts

μ – Discount slope parameter

κ – Maximal discount-to-breakpoint ratio

8. Cost and profit functions

C_0, C_1, C_2 – Hospital's pre- and post-venture cost functions

π_{W1}, π_{W2} – Warehouse's post-venture profit functions

$\pi_{V0}, \pi_{V1}, \pi_{V2}$ – Vendor's pre- and post-venture profit functions

δC – Total (year one and year two) costs for the hospital

T_{wp} – Total (year one and year two) warehouse profit

$\tau = T_{wp} - \delta C$

9. Profit objectives

π_V^0, π_W^0 – Vendor and warehouse year-1 profit objectives

10. Other

$\xi, \Omega, \omega, \alpha, \gamma, Q_1, z$ – defined in equation (5-35)

Parameter values used in the computations –

1. Product Demand

$D = 10,000$ (SKU/year)

$\beta = 0.5$ ($\$^{-1}$)

2. Product Prices and markup

$P = 10$ ($\$/SKU$)

3. Ordering costs

$K = 18$ ($\$/order$)

$k = 18$ ($\$/order$)

$A = 18$ ($\$/order$)

4. Shipping costs

$\eta = 0.7$ ($\$/SKU$)

$\lambda = 0$ ($\$/SKU$)

5. Storage costs

$\rho = 1$ ($\$/SKU$)

6. Discount

$\mu = 50$

$\kappa = 0.4$

Acronyms

1. SOH – shipping, ordering, and holding
2. VSM – Venture Success Metrics ($\delta C, T_{wp}, \tau$)
3. GPO – Group Purchasing Organization
4. IDN – Integrated Delivery Network
5. EOQ – Economic Order Quantity
6. SKU – Stock Keeping Unit

3.1 Single-tiered discount schedule

The shortcomings of commonly used discount schedules outlined above are easily overcome by using discount schedules that have “sigmoidal” analytic forms as described below. Such functions are commonplace in electrical engineering (Park et al. 1991; Yu 2013; Zhang et al. 2013) and other fields as analytic representations of switching functions routinely used for approximating on-off step-functions. With that, a vendor’s discount schedule V (equation 3-1; figure 3.4) is a sigmoidal increasing function of the fraction of total demand (v_1) that is apportioned to that vendor.

$$V = \frac{V_m}{1 + e^{-\mu(v_1 - V_0)}} \quad (3-1)$$

v_1 is bounded by $0 \leq v_1 \leq 1$. V has three parameters: maximal discount (V_m), discount breakpoint (V_0), and the “slope” parameter (μ). V_0 is the value of v_1 that delineates the no-discount/discount volume. Discounts are only offered for $v_1 \geq V_0$. V_m is the maximal discount as v_1 approaches 1. The parameter μ controls the slope of V at midpoint. As μ approaches infinity V approaches a step-function (figure 3-5) depending upon whether v_1 is greater or less than V_0 . Although we do not propose specific methodology for determining μ , we do set its value high enough so that the resulting sigmoidal

discount schedule “sufficiently” emulates a step-function in practice. For market practitioners, this is intuitively understood.

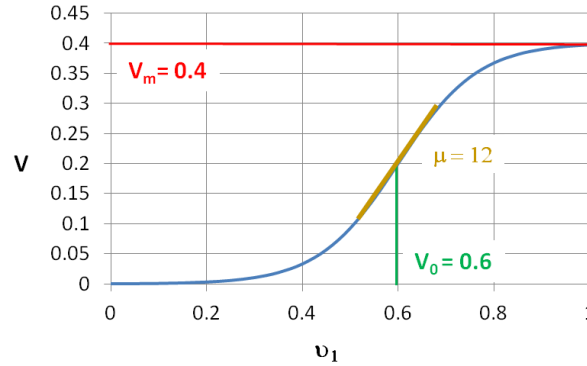


Figure 3-4: Discount schedule (V) of equation 3-1 ($V_m = 0.4$, $V_0 = 0.6$, $\mu = 12$)

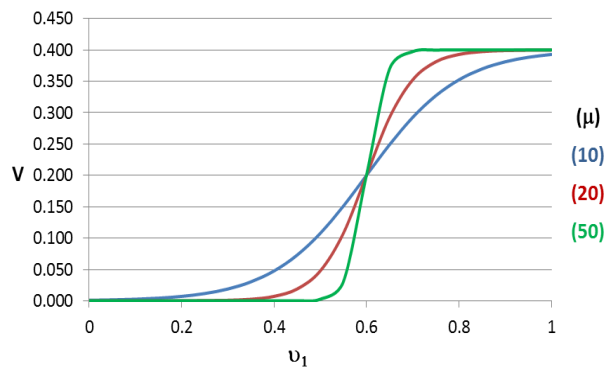


Figure 3-5: Discount schedule (V) of equation 3-1 ($\mu = 10, 20, 50$; $V_m = 0.4$, $V_0 = 0.6$)

The V discount schedule (equation 3–1; figure 3 – 4) has the following properties;

1. $0 \leq V \leq V_m$ for all v_1 .
2. Values of $v_1 \geq V_0$ trigger higher discounts, approaching V_m as v_1 approaches 1.
3. Values of $v_1 < V_0$ lead to lower discounts, approaching 0 as v_1 approaches 0.

Figure 3-6 shows the variation of V for different (V_m , V_0) combinations.

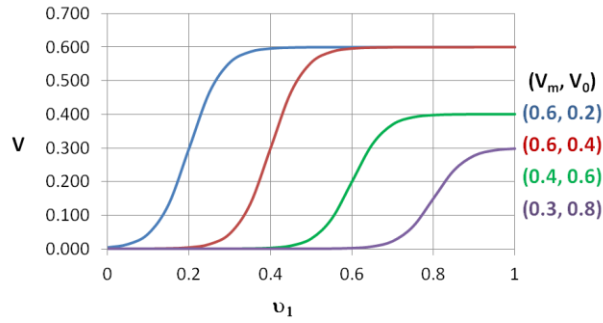


Figure 3-6: A typical discount schedule (V) of equation 3-1 for four different parameter values $\{(V_m, V_0) = (0.6, 0.2), (0.6, 0.4), (0.4, 0.6), (0.3, 0.8)\}; \mu = 25\}$

3.2 Single-tiered discounted pricing

As mentioned in the introduction, a commonplace practice in GPO related contract pricing is to determine second year price markups (σ_2) by discounting their first year counterparts (σ_1) using all-unit discount schedules in which the magnitude of discount is based on first year volume v_1 . The magnitude of the discount is determined by the vendor discount schedule V . In order to clarify the discounting rationale, we note that σ_1 is a markup over the unit price that the vendor pays for purchasing an item. For example, $\sigma_1 = 1.3$ comprises a cost of “1” and a vendor profit of 0.3 that corresponds to 30 % profit. σ_2 is computed by discounting the $(\sigma_1 - 1)$ profit part of σ_1 . That is

$$\sigma_2 = \sigma_1 - V\{\sigma_1 - 1\}$$

$$0 \leq V \leq 1$$
(3-2)

For $\sigma_1 = 1.3$ and $V = 0.2$ (20 % discount), $V(\sigma_1 - 1) = 0.2 * 0.3 = 0.06$, $\sigma_2 = 1.24$, corresponding to a 24 % profit in year-2. Figure 3-7 shows year-2 markups σ_2 as a function of v_1 for $\sigma_1 = 1.3$ and different (V_m, V_0) combinations.

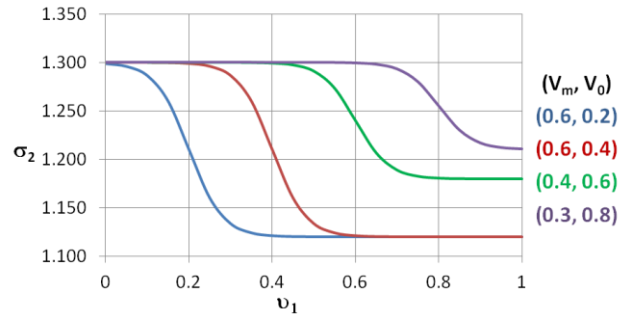


Figure 3-7: Year-2 markups σ_2 equation 3-2) under four different parameter values $\{(V_m, V_0) = (0.6, 0.2), (0.6, 0.4), (0.4, 0.6), (0.3, 0.8)\}; \mu = 25\}$

3.3 Special implementation

Different vendors in various industries utilize diverse protocols for setting their maximal discounts and discount breakpoints (V_m, V_0) in equation 3-1. One implementation of this discount schedule relies on an economic and business rationale in which V_m is proportional to V_0 with a slope κ .

$$V_m = \kappa V_0 \tag{3-3}$$

This, when substituted in equation 3-1, gives

$$V = \frac{\kappa V_0}{1 + e^{-\mu(v_1 - V_0)}} \tag{3-4}$$

Equation 3-4 is a discount schedule in which explicit dependence on V_m is replaced by κ . The value of the parameter κ is determined by the business imperatives of the vendor. Figure 3-8 is a graph of that equation for different values of κ . This figure shows that the maximal discount varies with the discount breakpoint. In practice, this means that the larger the volume fraction received by the vendor in year-1, the higher is the maximal discount offered in year-2. When the discount schedule of equation 3-4 is used in computing year-2 markups in equation 3-2, the results are shown in figure 3-9,

computed for different values of V_0 at constant κ . Figure 3-9 clearly shows that, as year-1 vendor fraction breakpoint V_0 increases, the year-2 markups become progressively lower.

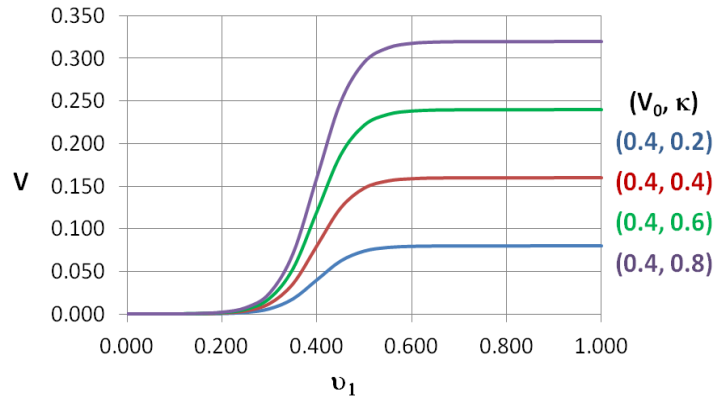


Figure 3-8: Discount schedule (V) of equation 3-4 for four different discount schedules where $\{\kappa = 0.2, 0.4, 0.6, 0.8; V_0 = 0.4; \mu = 25\}$

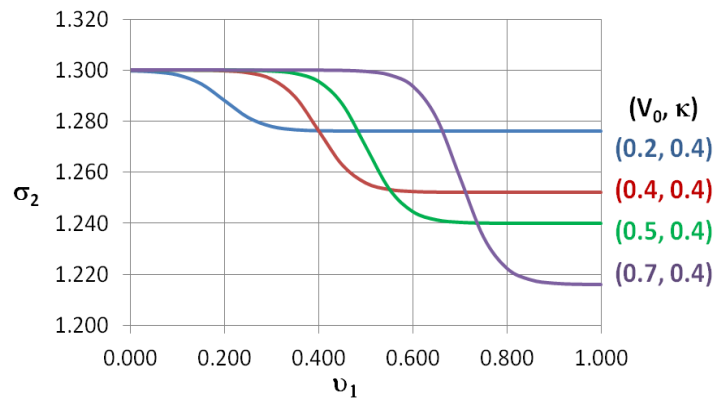


Figure 3-9: Year-2 markups (σ_2 ; equation 3.2) for four different discount schedules where $\{V_0 = 0.2, 0.4, 0.6, 0.8; \kappa = 0.4; \mu = 25\}$

3.4 Determining discount breakpoints V_0

To determine the discount breakpoint V_0 that sets the vendor fraction that triggers maximal discount, we use the following economic logic:

1. Set year-1 profit objective
2. The discount breakpoint V_0 is the vendor fraction that enables the vendor to meet its profit objective
3. If $v_1 \geq V_0$, the vendor meets its year-1 profit objective

4. Hence, year-1 volume equals to or in excess of v_1 leads to year-2 discounts

This logic is implemented here, using year-1 vendor's profit objectives and profit function, π_v^0 and π_{v_1} , respectively. For illustration purposes, we employ the following rudimentary vendor profit function

$$\pi_{v_1} = (\sigma_1 - 1)v_1 \quad (3-5)$$

More advanced and business-relevant profit functions, such as ones that incorporate shipping, ordering, and holding cost, would have a term corresponding to a product of profit with demand or demand fraction such as the one in equation 3-5. V_0 is the value of v_1 that triggers a year-2 discount. Thus, discount is contingent on its garnering sufficient year-1 volume to achieve year-1 profits (π_{v_1}) that meet or exceed vendor's year-1 targeted profit objective, π_v^0 . Thus, setting $\pi_{v_1} = \pi_v^0$ in equation 3-5,

$$(\sigma_1 - 1)v_1 = \pi_v^0 \quad (3-6)$$

gives

$$V_0 = v_1 = \frac{\pi_v^0}{(\sigma_1 - 1)} \quad (3-7)$$

3.5 Multi-tiered discount

The single-tiered discount schedule in equation 3-1 is easily generalized to multi-tiered discount schedules via

$$V = \sum_{i=1}^n \frac{V_{m,i}}{1 + e^{-\mu(v_1 - V_{0,i})}} \quad (3-8)$$

For $n = 3$, $\mu = 100$, and the values of the three maximal discounts $(V_{m,1}, V_{m,2}, V_{m,3}) = (0.05, 0.07, 0.12)$ and the corresponding discount breakpoints $(V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)$, the three individual discount functions in equation 3-8 are shown in figure 3-10. Their multi-tiered sum V is shown in figure 3-11. Employing V from equation 3-8 to compute the year-2 markups using equation 3-2 is shown in figure 3-12.

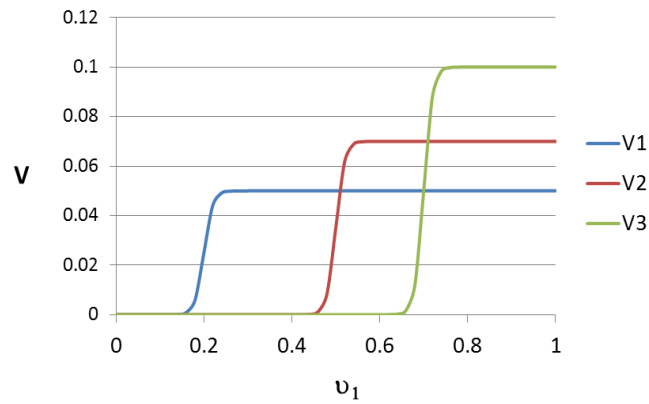


Figure 3-10: The three components of the multi-tiered discount schedule (equation 3-8) $\{n = 3; (V_{m,1}, V_{m,2}, V_{m,3}) = (0.05, 0.07, 0.12); (V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)\}$

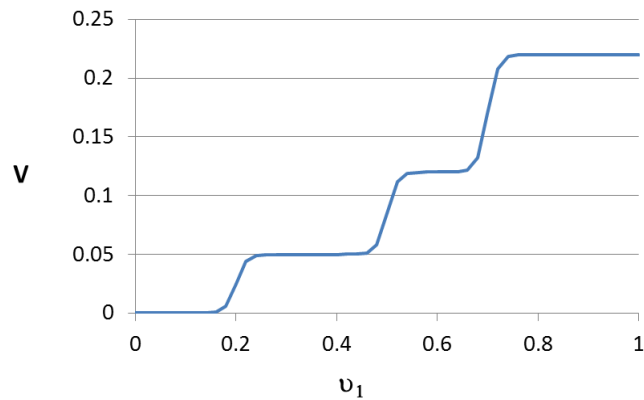


Figure 3-11: Three-tiered discount schedule (V , equation 3-8) $\{n = 3; (V_{m,1}, V_{m,2}, V_{m,3}) = (0.05, .07, .12); (V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)\}$

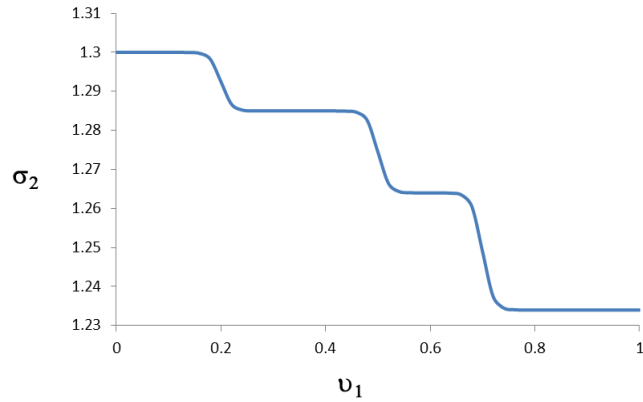


Figure 3-12: Three-tiered year-2 markup using the three-tiered discount schedule (V , equation 3-8) $\{n = 3; (V_{m,1}, V_{m,2}, V_{m,3}) = (0.05, 0.07, 0.12); (V_{0,1}, V_{0,2}, V_{0,3}) = (0.2, 0.5, 0.7)\}$

3.6 Numerical example

The following numerical example illustrates the discount schedules and pricing functions presented in this paper. We consider an item frequently used by hospitals to demonstrate the practical application of the discount paradigm detailed above.

The product and its demand, price, and markup profiles are shown in Table 3-1. The first year profit markup (σ_1) is arbitrarily set at 1.33, meaning a 33% profit. Vendor price is taken from a catalog of Baxter Inc; demand for this particular item is representative of that from a large healthcare system in the Southeastern United States.

Table 3-1: Product information	
Catalog #	5K7797P
Description	Latex free exam glove, non-sterile, powder free, small
Unit	1 Carton; 100 gloves each
Annual demand	10,000 Cartons
Manufacturer price	\$ 19.6
Vendor price	\$ 26.01
σ_1	1.33

The computations below show the year-2 volume-based discount schedules and corresponding markup functions for four values of vendor profit objectives $\pi_v^0 = 0.05, 0.10, 0.15, 0.20$. The demand breakpoints (V_0) and discount maxima (V_m) are computed using equations 3-7 and 3-3, respectively, and shown in Table 32. These parameters enable computation of the year-2 discount schedule (V) and markup (σ_2) as functions of first year vendor fraction (v_1). Results are shown in figures 3-13 and 3-14.

Table 3-2: Profit objectives, V_0, and V_m. (values in this table computed using $(\kappa, \mu) = (0.5, 50)$)				
π_v^0	0.05	0.10	0.15	0.20
V_0	0.15	0.31	0.46	0.61
V_m	0.08	0.15	0.23	0.31

Figure 3-13 shows the discounts a vendor can offer in year 2, based on the year-1 demand fractions. For example, according to this graph, if the first year demand fraction (v_1) is 0.5, the second year discount (σ_2) is approximately 0.225 with a 15% first year profit objective, $\kappa=0.5$, and $\mu=50$. Figure 3-14 shows the corresponding markups a vendor could charge, based on the discounts determined in Figure 3-13. Therefore, the corresponding second year markup is 1.256. This means that the vendor achieves a 25.6 % profitability in the second year. The vendor has the ability to change the second year discount schedule by altering κ , μ or π_v^0 . Table 3-3 shows the various combinations of κ , μ and π_v^0 that yield essentially the same second year markup σ_2 ($1.33 < 1.36$).

Table 3-3: Combinations of κ , π_V^0 and μ that yield similar σ_2 at $u_1 = 0.5$			
(κ, π_V^0)	$\mu = 20$	$\mu = 50$	$\mu = 80$
(0.2, 0.05)	1.33	1.33	1.33
(0.4, 0.05)	1.34	1.34	1.34
(0.6, 0.05)	1.34	1.34	1.34
(0.2, 0.1)	1.34	1.34	1.34
(0.4, 0.1)	1.34	1.34	1.34
(0.6, 0.1)	1.35	1.35	1.35
(0.2, 0.15)	1.34	1.34	1.34
(0.4, 0.15)	1.35	1.35	1.35
(0.6, 0.15)	1.36	1.36	1.36

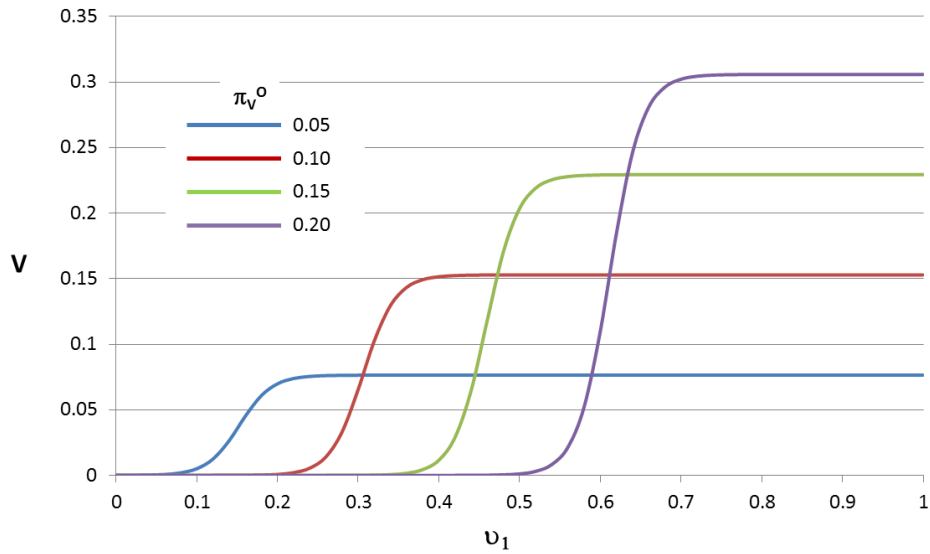


Figure 3 – 13: Year-2 discount schedule for four values of π_V^0

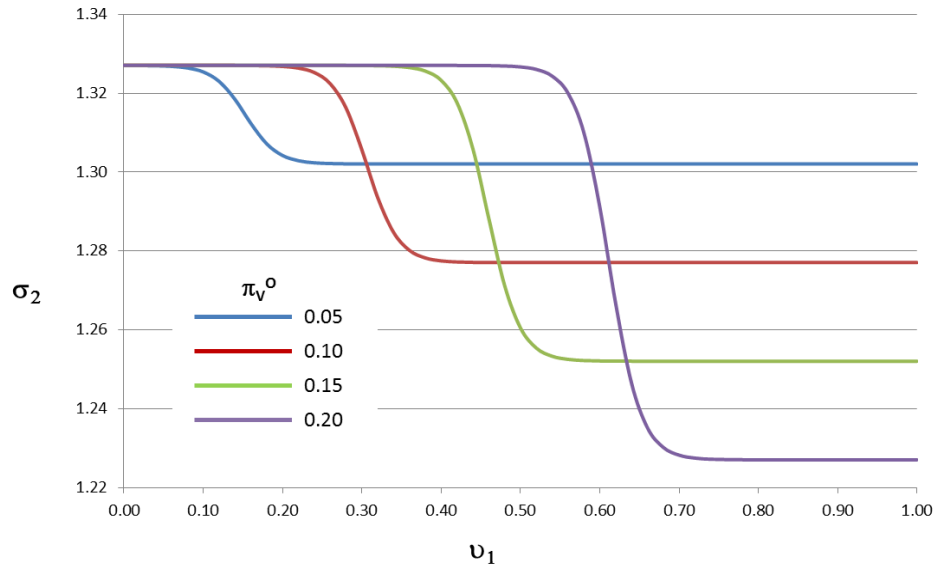


Figure 3 – 14: Year-2 markups for four values of π_v^0

In Summary, we developed analytically convenient healthcare-like discount functions that are based on previous year's purchases. We extended the single-tiered discount schedule developed in this chapter to include multi-tier discount schedules and presented a special implementation of the discount structures where the explicit dependence on the maximal discount was avoided. A series of numerical calculations was presented to showcase the application of the model. Next, we develop an entropy-based demand-splitting methodology to apportion the demand between two suppliers.

Chapter 4

General Methodology of Computing Inter-Supplier Demand Splits

Industrial organizations have a need for general demand-split methodologies and functions that are sufficiently robust to incorporate general business imperatives. By “demand-split,” we shall mean the allocation of the firm’s purchases of the given item between the two or more available suppliers. Our literature review reveals that routine methodologies are limited in scope, often with ad-hoc goals that limit their generality. Contemporary literature is permeated by the ethos of a perfect market in equilibrium. That notwithstanding, organizations that operate in imperfect markets still find it necessary to split their purchases across a spectrum of suppliers in a manner consistent with their business imperatives.

The parameters that determine the demand-split decision are the respective numbers of suppliers, brands, and customers, and the number of units purchased by a single customer. In the cell phone market, for instance, several suppliers offer a few brands to a large customer population in which each customer needs no more than one or two units. In the commodity medical supplies market, on the other hand, a few large suppliers sell several functionally equivalent brands to a moderate number of hospitals, each making extensive purchases of those product units. Other markets have different distributions of demand-split parameters.

Demand-split decisions are made from different perspectives. Vendors study demand-splits to try to determine their market shares. However, customers, such as hospitals and manufacturers that purchase in bulk, make demand-split decisions that best serve their business objectives. Customers that need no more than a few units of product available in several brands may not engage in extensive demand-split decisions. They would typically decide based on a combination of brand preference, specific attributes of each brand, and price.

The objective of this chapter is to develop a robust demand-split methodology that enables ready incorporation of general business objectives. Such a methodology should address the general case of a number of suppliers selling multiple brands to a set of customers, each of which requires a widely-ranging number of units subject to a diversity of business objectives such as non-linear discounts and skewed demand. In this thesis, we introduce the methodology for the special case of multiple suppliers selling the same brand at distinct prices to a single customer purchasing in bulk under different business imperatives. This paper will clearly show how to incorporate business imperatives in demand-split decisions. We reiterate that such functionality is not available with existing methods. Examples of such imperatives are volume-based discounts that rely on a plethora of non-linear discount schedules and rebates, as well as purchase decisions requiring the customer to skew demand in favor of one or more suppliers.

4.1 EOQ and Logit-based demand-splits

The literature on demand-split is large. Our reviews revealed two general commonly used approaches to determine demand-split and several ad hoc one-offs. We refer to the two prevalent approaches as EOQ-based and logit-based demand-splits. Below, we briefly summarize each followed by an outline of their limitations.

For simplicity, we restrict our discussions to a single customer purchasing a large number of units of an identical product from multiple suppliers that offer the product at different prices. Generalizing this discussion to include the effects of customer brand preference and brand attributes of diverse functionally-comparable brands is straightforward.

In EOQ-based demand-split methodology, exemplified in references Rosenblatt et al. (1998), Dai and Qi (2007) and Chang (2006), the customer cost function is expressed as a combination of product per unit price, ordering, and holding cost from different suppliers. Such functions are generally expressed as

$$C = \sum_{i=1}^n \left\{ p_i d_i + \frac{a_i}{d_i} + b_i d_i \right\} \quad (4-1)$$

$$D = \sum_{i=1}^n d_i \quad (4-2)$$

where n is the number of suppliers, D is total demand, and the p_i , a_i , and b_i are the constants representing price, ordering, and holding costs associated with supplier “ i ” and d_i is the average periodic quantity order from supplier “ i ”. The set of values $\{d_1, d_2, \dots, d_n\}$ represent the sought after inter-supplier demand-split. This split is determined, as usual, by setting the derivatives of C with respect to each d_i to 0, giving

$$d_i = \sqrt{\frac{a_i}{p_i b_i}} \quad (4-3)$$

Logit-based split functions on the other hand have the following forms (Basu et al. (2007); Aksoy-Pierson et al. (2013))

$$v_j = \frac{e^{-\beta u_j}}{\sum_{i=1}^n e^{-\beta u_j}} \quad (4-4)$$

Here v_j is the fraction of total demand satisfied by supplier j and n is the number of suppliers. u_j is the utility function which quantifies the utility of the product from vendor j , typically expressed as a linear combination of product attributes ($X_{1j}, X_{2j}, \dots, X_{mj}$) (with constant coefficients $a_1, a_2 \dots a_m$) of the product offered by vendor j and a price term (P_j) with price sensitivity β .

$$u_j = \sum_{k=1}^m a_k X_{kj} - \beta P_j \quad (4-5)$$

As mentioned above, the products procured from different suppliers are identical with respect to all attributes except price. Under that assumption, the sum term from the above equation is identical for all suppliers, and can be removed as a vendor-distinguishing factor without loss of generality. With this simplification, the demand-split function u_j becomes

$$u_j = \frac{e^{-\beta P_j}}{\sum_{i=1}^n e^{-\beta P_i}} \quad (4-6)$$

This is the Logit demand-split function. It is shown in a later section that, if the only two business imperatives were that the total demand is distributed across all suppliers and the total budget is fixed, the entropy-based methodology would lead to the same results as the price based logit demand-split shown in equation 4 – 4.

The goal of this chapter is to develop a demand-split methodology that enables ready incorporation of business imperatives under possibly imperfect market conditions.

4.2 Entropy-based demand split across two suppliers

The demand-split methodology developed in this paper draws on lines of reasoning first introduced in statistical physics, and later adapted in different disciplines. In one operations-research-related application, entropy-based approaches were reformulated by Wilson (1967 and 1969) for traffic modal split problems. The discussions in this section focus on one customer purchasing bulk quantities of an identical item from two different suppliers. Its generalization to multiple suppliers is shown in the next section. The demand-split challenge for this case can be stated as follows. A customer's total demand of N units can be split among two suppliers, 1 and 2, who provide n_1 and n_2 units respectively. The number of ways w to achieve this (n_1-n_2) split is given by w , where

$$w = \frac{N!}{n_1! n_2!} \quad (4-7)$$

For a rational customer, it is of vital interest to achieve an Effective Split (ES), defined as an equitable demand-split that strives to conserve and protect all supply sources by distributing demand as equitably as possible among them. In the above formulation, this is achieved with an (n_1-n_2) split in which $n_1 = n_2$ for even N and $(n_1-n_2) = 1$ for odd N ; the difference of one unit for large N (bulk purchasing) is trivial. Such (n_1-n_2) splits maximize w , and achieve ES. Thus, ES is achieved by maximizing w , as yet unencumbered by any business imperatives other than conserving and protecting supply sources.

Obtaining an ES that meets business imperatives is achieved with a constrained maximization of ES in which each constraint reflects a different imperative. We illustrate this with two business imperatives –

1. Imperative-1 – the combined units obtained from the two suppliers must meet the total demand
2. Imperative-2 – the total purchase cost must meet the customer’s budget (C) given the suppliers’ unit pricing of c_1 and c_2

These requirements are represented by constraints (4 – 10) and (4 – 11) respectively. Before proceeding, we note that (n_1-n_2) splits that maximize w also maximize its natural logarithm $\ln(w)$. This correspondence is routinely used in entropy-based treatments to simplify the derivations. The “entropy” designation comes from statistical physics where entropy (S) is defined as

$$S = R \ln(w) \quad (4-8)$$

R is a constant that has a specific physical interpretation with little relevance here, and can be set to 1 without loss of generality, giving

$$S = \ln(w) \quad (4-9)$$

The problem at hand becomes one of maximizing entropy

$$S = \ln(w) = \ln\left(\frac{N!}{n_1! n_2!}\right) \quad (4-10)$$

Subject to two constraints

$$N = n_1 + n_2 \quad (4-11)$$

$$C = c_1 n_1 + c_2 n_2 \quad (4-12)$$

Expanding the entropy function gives

$$S = \ln(N!) - \ln(n_1!) - \ln(n_2!) \quad (4-13)$$

Using Stirling's approximation which, for large values of x

$$\ln(x!) = x \ln(x) - x \quad (4-14)$$

gives

$$S = N \ln(N) - N - n_1 \ln(n_1) + n_1 - n_2 \ln(n_2) + n_2 \quad (4-15)$$

The constrained optimization of S is done using two Lagrange Multipliers, α and β . Recasting the constraints (4-11) and (4-12) as

$$g(n_1, n_2) = N - n_1 - n_2 = 0 \quad (4-16)$$

$$h(n_1, n_2) = N - c_1 n_1 - c_2 n_2 = 0 \quad (4-17)$$

The constrained optimization of S thus require that, α and β , satisfy

$$\nabla S(n_1, n_2) + \alpha \nabla g(n_1, n_2) + \beta \nabla h(n_1, n_2) = 0 \quad (4-18)$$

The symbol “ ∇ ” represents the 2-dimensional gradient vector

$$\nabla S(n_1, n_2) = \begin{bmatrix} \frac{\partial S}{\partial n_1} \\ \frac{\partial S}{\partial n_2} \end{bmatrix} = \begin{bmatrix} -\ln(n_1) \\ -\ln(n_2) \end{bmatrix}$$

(4 – 19)

$$\alpha \nabla g (n_1, n_2) = \alpha \begin{bmatrix} \frac{\partial g}{\partial n_1} \\ \frac{\partial g}{\partial n_2} \end{bmatrix} = \begin{bmatrix} -\alpha \\ -\alpha \end{bmatrix}$$

(4 – 20)

$$\beta \nabla h (n_1, n_2) = \beta \begin{bmatrix} \frac{\partial h}{\partial n_1} \\ \frac{\partial h}{\partial n_2} \end{bmatrix} = \begin{bmatrix} -\beta c_1 \\ -\beta c_2 \end{bmatrix}$$

(4 – 21)

Substituting (4 – 19) – (4 – 21) in (4 – 18) yields

$$\nabla S(n_1, n_2) + \alpha \nabla g (n_1, n_2) + \beta \nabla h (n_1, n_2) = \begin{bmatrix} -\ln(n_1) - \alpha - \beta c_1 \\ -\ln(n_2) - \alpha - \beta c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(4 – 22)

whose solution is given by

$$\ln(n_1) = -\alpha - \beta c_1$$

(4 – 23)

$$\ln(n_2) = -\alpha - \beta c_2$$

(4 – 24)

$$n_1 = e^{-\alpha - \beta c_1}$$

(4 – 25)

$$n_2 = e^{-\alpha} e^{-\beta c_2}$$

(4 – 26)

The values of α and β are determined by applying the constraints (4 – 11) and (4 – 12).

Substituting (4 – 25) and (4 – 26) in (4 – 11) results in

$$N = n_1 + n_2 = e^{-\alpha} \{e^{-\beta c_1} + e^{-\beta c_2}\}$$

(4 – 27)

$$e^{-\alpha} = \frac{N}{\{e^{-\beta c_1} + e^{-\beta c_2}\}}$$

(4 – 28)

Substituting (4 – 28) in (4 – 25) and (4 – 26) gives

$$\frac{n_1}{N} = \frac{e^{-\beta c_1}}{\{e^{-\beta c_1} + e^{-\beta c_2}\}}$$

(4 – 29)

$$\frac{n_2}{N} = \frac{e^{-\beta c_2}}{\{e^{-\beta c_1} + e^{-\beta c_2}\}}$$

(4 – 30)

Dividing constraint (4 – 12) through by N gives

$$c_1 \frac{n_1}{N} + c_2 \frac{n_2}{N} = \frac{C}{N} \quad (4 - 31)$$

Substituting (4 – 29) and (4 – 30) in constraint (4 – 31), and defining $\gamma = C / N$, we find

$$\frac{c_1 e^{-\beta c_1}}{\{e^{-\beta c_1} + e^{-\beta c_2}\}} + \frac{c_2 e^{-\beta c_2}}{\{e^{-\beta c_1} + e^{-\beta c_2}\}} = \gamma \quad (4 - 32)$$

The value of β can be determined by solving (4 – 32) numerically. This completes the derivation of the entropy-based demand-split for two suppliers.

In summary, businesses have a need for general demand-split methodologies and functions that are sufficiently robust to incorporate general business necessities. Commonly used methods cannot be easily adapted to meet these requirements. This Chapter developed demand-split functions in which a customer’s total demand is *equitably* apportioned across suppliers subject to diverse business requirements such as price constraints and demand bias while *retaining equitability*. The ultimate economic objective for achieving equitability is to conserve supply sources. The demand split methodology introduced in this thesis can be characterized as “achieving equitability under business constraints”. This demand-split methodology will be used in chapter 5 to compute price markups and the Venture Success Metrics.

Chapter 5

Determining Venture Success Metrics

Healthcare systems in the US and elsewhere are under significant pressure to cut cost. According to National Center for Health Statistics, supply chain and procurement spending in hospitals and healthcare organizations accounted for 40 – 45 % of total spend, or over \$300 billion of the total \$800 billion spent in 2010. Its current growth is outpacing all other spend sectors. Not surprisingly, it remains a target of aggressive cost cutting.

The medical-surgical supplies market in the United States is oligopolistic with few large players and many barriers to entry. It is dominated by a small number of large suppliers such as Owens & Minor and Cardinal Health. Healthcare systems have engaged Group Purchasing Organizations (GPO's) to leverage economies of scale in negotiating price reductions. Recently, however, to further reduce cost and to avert potential conflicts of interest inherent in the GPO's business model, there has been a movement away from the GPO practice. The trend has led to large mergers of hospitals and healthcare systems, and the concomitant development of Integrated Delivery Networks (IDN's). These are exemplified by Novant Health (North Carolina) and Greater New York Hospital Alliance and Yankee Alliance (New York) that manage their supply chain operations in-house.

One such venture has been created by the Sisters of Mercy Health System. They established the Resource Optimization and Innovation (ROi) division (<http://www.roiscs.com/>) to run as profit center that combines the functions of GPO and supplier. ROi was eventually spun out as a separate company that offers supply chain optimization services to the healthcare industry. The development and evolution of ROi's venture are outlined on their website and qualitatively described in a case study white paper (www.supplychainbrain.com). Although competitors are not explicitly addressed in

these write-ups, it is virtually certain that they were amply considered in creating the venture. Our extensive literature searches did not find published models of this venture, rendering it of little utility as a template for new ones.

The literature is replete with papers on inventory management of medical supplies including blood products ((Bakker et al. 2001; Hammelmayer et al. 2010; Stanger et al. 2012), pharmaceuticals (Uthayakumar & Priyan 2013; Kelle et al. 2012), and other generic items (Guerrero et al. 2013; Duan and Liao 2013; Chen and Federgruen 2006). This literature is exclusively focused on warehouse operational process improvements. No publications address profitability. Despite the vast operations research literature, we were not able to find healthcare relevant work that provides the basis for modeling such ventures.

A comprehensive venture model would feature a warehouse competing with multiple vendors for the hospital's demand. The model would also deal with a hospital's diverse item categories, each with a unique inventory profile. These categories are exemplified by lower cost non-perishable medical-surgical supplies like gloves and scalpels that are ordered and stocked in bulk, high value perishables such as blood supplies and protein therapeutics that require specialized shipping and storage, and expensive implants and prosthetics that are typically ordered as needed. Such a model would also feature stochastic demand and the time variation of its parameters.

In this chapter, we develop the methodologies that form the foundation for more comprehensive models. This objective is pursued by focusing on a simple case: a single vendor in price competition with an internal hospital warehouse, run as a profit center to procure the entire hospital demand (deterministic) of a single product with a commonplace inventory profile that requires no special shipping, storage, or handling. The chapter determines the venture's conceptual underpinnings and identifies the factors that are critical for success. It addresses vendor-warehouse competition and determines price equilibria using game-theoretic methodologies. Here, Venture

Success Metrics (VSM) are developed and deployed to assess the venture viability under different vendor and warehouse profit-objective scenarios.

The developments in this paper necessitated two additional breakthroughs. First is a methodology for splitting demand across multiple suppliers that are consistent with user-specified business imperatives. The second comprises analytic functions with continuous and finite derivatives that encode multi-tiered discount schedules that are specific to the healthcare industry and which are radically different from the commonly used linear and non-linear volume-based discount schedules.

5.1 Problem setup

The venture presented (figure 5 – 1) has three stakeholders: hospital, vendor, and warehouse (H, V, and W). In this venture, the hospital starts its own warehouse and runs it as a profit center, competing with the vendor. Both vendor and warehouse purchase their supplies from the same manufacturer (M). The venture is deemed to be successful if the post-venture (Scenario-2) hospital's costs are lower than its pre-venture (Scenario-1) costs, and the warehouse is profitable. Throughout this venture, the vendor is retained as a full-fledged player that competes with the warehouse for the hospital's business. Vendor retention serves two purposes. First, it represents a second source of supplies as a hedge against warehouse failure. In addition, the vendor provides a source of price-competition to the newly established warehouse. Hospital's costs and vendor and warehouse profits are determined by the respective price markups of vendor and warehouse and by the vendor-warehouse "demand-splits" chosen by the hospital (i.e. the choice by hospital of its allocation of total demand between vendor and warehouse) to minimize its costs.

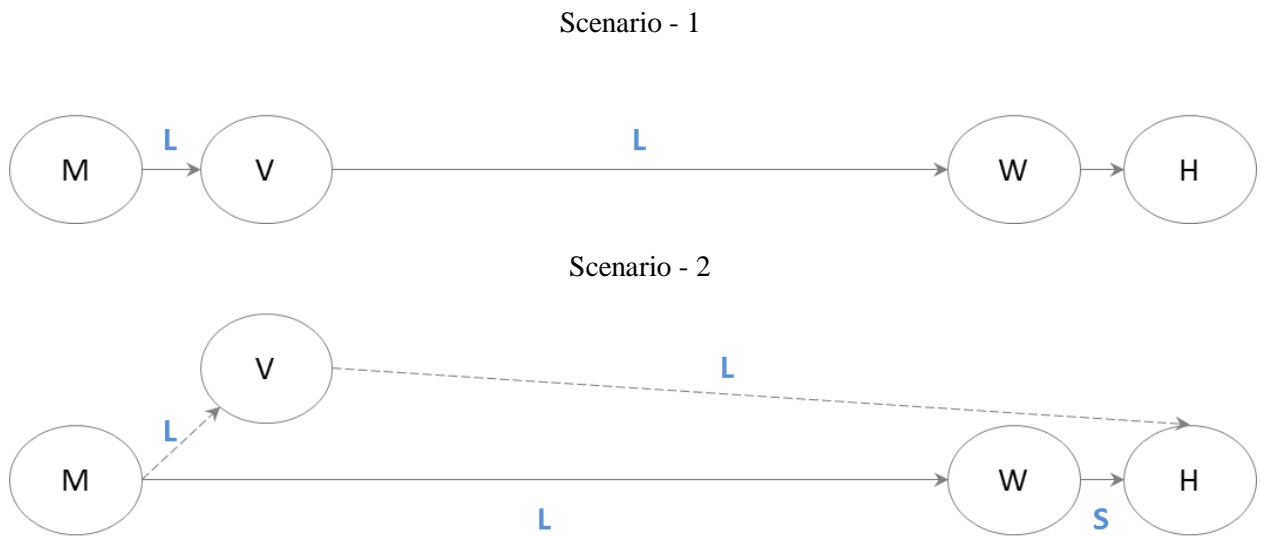


Figure 5-1: Pre- and post-venture situations, 1 and 2 respectively. L and S are long and short shipping distances. M, V, W, and H represented manufacturer, vendor, warehouse, and hospital, respectively.

In this model, the interplay and balance of several competing forces determines venture success. Higher vendor markups increase vendor profitability but lower the vendor's share of hospital purchases. It also increases hospital's costs, motivating the hospital to pursue the venture. Higher warehouse markups have similar impact on warehouse profitability and hospital's costs. Finally, while shipping, ordering, and holding costs are constant, the warehouse has a slight advantage in lower warehouse-to-hospital shipping costs and their overall impact is to drive up costs for all three parties in this venture.

In calculating the Venture Success Metrics, the three stakeholders control different variables that are relevant to venture success. Vendor and warehouse each control their respective markups and profit objectives. Year-1 markups are set by the vendor and warehouse based on hospital-imposed warehouse demand bias. The vendor and warehouse also set their year-1 profit objectives which, as

we show below, determine their respective year-1 discounts, as well as year-2 markups and splits. Finally, the hospital determines its year-1 vendor-warehouse demand split that minimizes its two year costs. While this profile does not have a simple controller-controlled structure, it does reflect business practices that can complicate setting business needs into academically well-defined paradigms.

5.2 Demand-split functions

Demand-split functions are used here to compute the vendor, warehouse, and hospital profit and cost functions derived in section 5.5. Utilizing the demand-split functions developed in chapter 4, the entropy-based vendor (v^{en}) and warehouse ($1 - v^{en}$) fractions of total demand are given by

$$v^{en} = \frac{e^{-\beta\sigma}}{\{e^{-\beta\sigma} + e^{-\beta\zeta}\}} \quad (5-1)$$

$$(1 - v^{en}) = \frac{e^{-\beta\zeta}}{\{e^{-\beta\sigma} + e^{-\beta\zeta}\}} \quad (5-2)$$

Where β is the price sensitivity parameter and (σ, ζ) are the vendor and warehouse's markups, respectively. The current venture allows the hospital to bias its year-1 purchases in favor of the warehouse. We refer to this as the "Venture Success bias." It "penalizes" the entropy-based vendor fraction with a power function ($\sigma^f; f < 0$) of the vendor markup (σ) of the form

$$v = \sigma^f v^{en} \quad (5-3)$$

The "Venture Success" bias was introduced for the following reasons;

1. It is an attempt to skew demand split towards venture success

- In the unlikely case that the vendor and warehouse have identical SOH costs, the unbiased v^{en} leads to a trivial Bertrand Nash equilibrium in which the year-1 vendor and warehouse markups are equal (section 8 of this chapter). Biasing v_1^{en} circumvents this trivial outcome.

Figures 5- 2 and 5-3 show the vendor and warehouse demand splits, v and $(1 - v)$ respectively, as functions of σ for three f values.

Note that for $f = 0$, $v = v^{en}$.

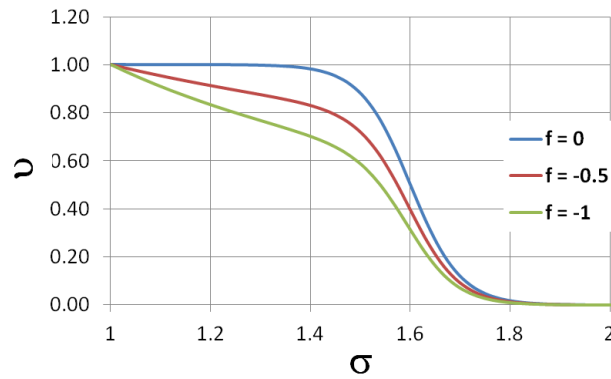


Figure 5-2: Vendor fractions (v) as a function of σ for $f = 0, -0.5,$ and -1

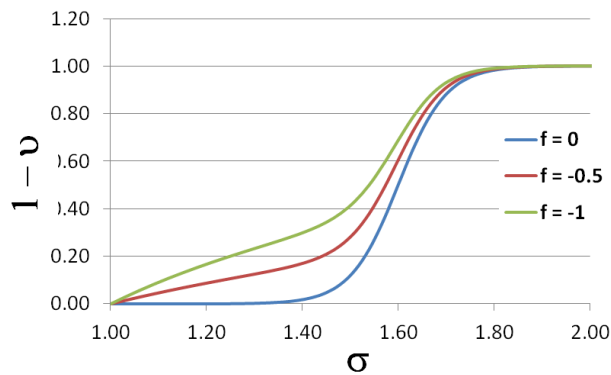


Figure 5-3: Warehouse fraction ($1 - v$) as a function of σ for $f = 0, -0.5,$ and -1

Figure 5-2 shows that, for fixed σ , the vendor fraction decrease as the demand split exponent f increases, satisfying the objective of Venture Success bias. Figure 5-3 on the other hand shows that increasing f increases the warehouse fraction, also satisfying the objective of Venture Success bias.

The demand split chapter (4) defines three different split functions, v_1^{en} , v_1 , and v_2^{en} . These variables are used to compute various cost and profit functions in subsequent sections of this chapter. Section 8 of this chapter introduces a fourth split variable: v_1^{min} . Here, we discuss the difference between these splits. The discussion below refers exclusively to “vendor fraction.” However, it is to be understood that in each such reference there is an implied corresponding “warehouse fraction” computed as $(1 - \text{vendor fraction})$.

The four split functions are defined to serve different purposes –

1. v_1^{en} is the entropy-based vendor fraction obtained from equation (5 – 1). It is a function of the vendor and warehouse markups, σ and ζ , without further bias.
2. v_1 is calculated using equation (5 – 3) to bias hospital purchases in favor of the warehouse. It is used in computing the vendor and warehouse year-1 profit functions and the ensuing Bertrand Nash equilibrium markups in section 8.
3. v_2^{en} is the year-2 entropy-based vendor fraction (equation 5 – 1) and employed in later sections to calculate the year-2 hospital costs.
4. v_1^{min} is the value of the year-1 vendor fraction that minimizes δC .

Why use the entropy-based vendor fraction (v_2^{en}) in year-2 instead of its biased counterpart?

The reason for biasing year-1 purchases was to find non-trivial equilibrium markups. As shown in chapter 3 on discounts and in the subsequent sections of this chapter. Year-2 markups, however, are not computed using equilibrium methods. Instead, they are determined by discounting the year-1 markups, thereby circumventing the need for biasing the purchases.

5.3 Discount and pricing schedules

Obtaining the hospital's cost functions in section 5.5 requires discount and price schedules that accurately reflect the practices of the industry and are sufficiently mathematically robust for computing price equilibria. Multi-tiered discounts are commonplace in healthcare, particularly in contracts negotiated by Group Purchasing Organizations (GPO's). Our extensive literature searches, detailed in Chapter 2, revealed that tiered discount schedules used in practice are non-analytic step-functions with infinite and discontinuous derivatives, shortcomings that frustrate their application in equilibrium multi-objective optimizations. Chapter 3 showed that infinite derivatives and the non-representativeness of commonly used discount schedules can be remedied by using sigmoidal analytic discount schedules of the form

$$V = \frac{V_m}{1 + e^{-\mu(v_1 - V_0)}} \quad (5 - 4)$$

V is the vendor's discount schedule and it has three parameters: maximal discount (V_m), discount breakpoint (V_0), and the "slope" parameter (μ). v_1 is the fraction of total demand satisfied by the vendor. v_1 and V are bounded by $[0, 1]$.

Figure 5-4 is a graph of function 5 - 4 . Discounts are offered only for $v_1 \geq V_0$. V_m is the maximal discount as v_1 approaches 1. The parameter μ controls the slope of V at midpoint. As μ approaches infinity V approaches a step-function (figure5-5). In that limit, V_0 is the breakpoint such that a discount is offered only for $v_1 \geq V_0$ and vice versa. In subsequent computations, μ is set "sufficiently" high so that the resulting sigmoidal discount schedule emulates a step-function in practice. Relying on economics logic, Chapter 3 developed an implementations of this discount schedule in which V_m increases linearly with V_0 with a slope κ .

$$V_m = \kappa V_0 \quad (5-5)$$

This leads to

$$V = \frac{\kappa V_0}{1 + e^{-\mu(v_1 - V_0)}} \quad (5-6)$$

This variation of V with κ is depicted in figure 5-6. As outlined in Chapter 3, V_0 can be computed

using the vendor's year-1 profit objective π_v^0 and the vendor's year-1 profit function π_{v1} (5-33).

Applying this procedure leads to equation (5-7)

$$\sqrt{V_0} = x = \frac{\Omega}{2(\sigma_1 - z)} \left\{ 1 \pm \sqrt{1 + \left[\frac{4\pi_v^0(\sigma_1 - z)}{\Omega^2} \right]} \right\} \quad (5-7)$$

In typical GPO-negotiated contracts, second year price markups (σ_2) are determined by discounting their first-year counterparts (σ_1) using all-unit discount schedules in which the magnitude of discount is based on first-year volume. In this thesis, magnitude of the discount is determined by the vendor's discount schedule (equation (5-6) and the following pricing function

$$\sigma_2 = \sigma_1 - V\{\sigma_1 - 1\} \quad (5-8)$$

Using similar logic, the corresponding warehouse discount schedules and pricings are given by

$$W = \frac{W_m}{1 + e^{\mu(v_1 - W_0)}} \quad (5-9)$$

$$W_m = \kappa W_0 \quad (5-10)$$

$$W = \frac{\kappa W_0}{1 + e^{\mu(v_1 - W_0)}} \quad (5-11)$$

$$\sqrt{1 - W_0} = x = \frac{\omega}{2(\zeta_1 - z)} \left\{ 1 \pm 1 \sqrt{1 + \left[\frac{4\pi_w^0(\zeta_1 - z)}{\omega^2} \right]} \right\} \quad (5 - 12)$$

$$\zeta_2 = \zeta_1 - W\{\zeta_1 - 1\} \quad (5 - 13)$$

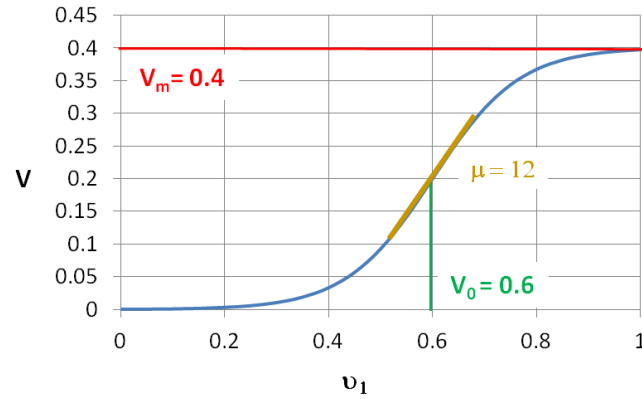


Figure 5-4: Vendor discount schedule (V ; equation 5 - 4, $V_m = 0.4$, $V_0 = 0.6$, $\mu = 12$)

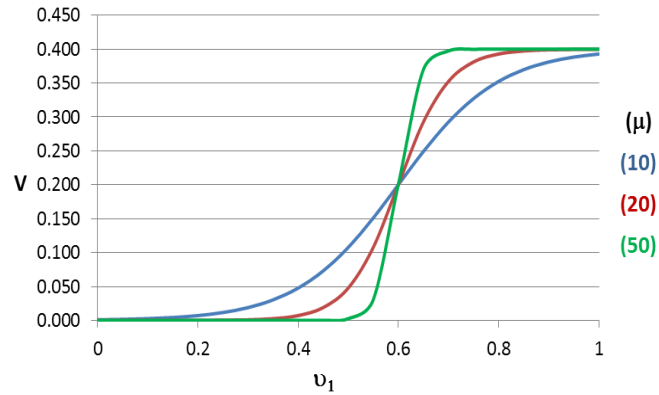


Figure 5-5: Vendor discount schedule (V ; equation 5 - 4; $V_m = 0.4$, $V_0 = 0.6$) for $\mu = 10, 20$, and 50

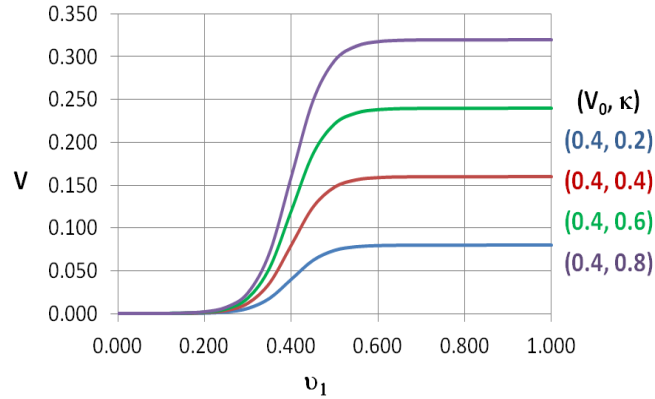


Figure 5-6: Vendor discount schedule (V ; equation 5 – 6; $\mu = 25$) with $\{(V_0, \kappa) = (0.4, 0.2), (0.4, 0.4), (0.4, 0.6), (0.4, 0.8)\}$

5.4 Cost and profit functions

Computing the VSM's in section 5.8 of this chapter requires knowledge of the profit functions of the pre- and post-venture healthcare organizations and of the vendor and warehouse functions developed in this section.

5.4.1 Hospital cost functions

The hospital has three different cost functions representing pre-venture costs (C_0), and post-venture year-1 and year-2 costs (C_1 and C_2). Each cost function has four components: total price expressed as the algebraic product of unit price and demand, shipping, ordering, and holding (SOH) costs.

In Scenario-1, the pre-venture cost C_0 is given by

$$C_0 = \sigma_0 PD + \eta D + A \frac{D}{Q_1} + \rho Q_1 \quad (5 - 14)$$

The four terms in this function represent total purchase price, shipping, ordering, and holding costs respectively. Scaling C_0 by the manufacturer's annual revenue (PD) gives

$$C_0^{scaled} = \frac{C_0}{PD} = \sigma_0 + \frac{\eta}{P} + \frac{A}{PQ_1} + \frac{\rho}{PD} Q_1$$

(5 – 15)

Eliminating the “scaled” superscript gives

$$C_0 = \sigma_0 + \frac{\eta}{P} + \frac{A}{PQ_1} + \frac{\rho}{PD}Q_1$$

(5 – 16)

In Scenario-2, the post-venture year-1 cost (C_1) is

$$C_1 = \{\sigma_1 v_1 + \zeta_1(1 - v_1)\}PD + \{\eta v_1 + \lambda(1 - v_1)\}D + AD \left\{ \frac{v_1}{Q} + \frac{1 - v_1}{q} \right\} + \rho\{Q + q\}$$

(5 – 17)

The first term represents the total purchase price paid to the vendor and warehouse based on the demand split v_1 . The last three terms correspond to shipping, ordering, and holding costs, respectively. The PD-scaled C_1 is

$$C_1 = \{\sigma_1 v_1 + \zeta_1(1 - v_1)\} + \frac{\{\eta v_1 + \lambda(1 - v_1)\}}{P} + \frac{A}{P} \left\{ \frac{v_1}{Q} + \frac{1 - v_1}{q} \right\} + \frac{\rho}{PD} \{Q + q\}$$

(5 – 18)

PD-scaled post-venture year-2 cost (C_2) in Scenario-2 is

$$C_2 = \{\sigma_2 v_2 + \zeta_2(1 - v_2)\} + \frac{\{\eta v_2 + \lambda(1 - v_2)\}}{P} + \frac{A}{P} \left\{ \frac{v_2}{Q} + \frac{1 - v_2}{q} \right\} + \frac{\rho}{PD} \{Q + q\}$$

(5 – 19)

5.4.2 Vendor profit functions

The vendor’s pre-venture PD-scaled profit function in Scenario-1 is

$$\pi_{V_0} = (\sigma_0 - 1) - \frac{1}{P}\eta - \frac{1}{PN}K - \frac{\rho}{PD}N,$$

(5 – 20)

and for year-1 and year-2 in Scenario-2

$$\pi_{V_1} = (\sigma_1 - 1)v_1 - \frac{1}{P}v_1 \left\{ \eta + \frac{K}{N} \right\} - \frac{\rho}{PD}N$$

(5 – 21)

$$\pi_{v_2} = (\sigma_2 - 1)v_2 - \frac{1}{P}v_2 \left\{ \eta + \frac{K}{N} \right\} - \frac{\rho}{PD}N \quad (5-22)$$

5.4.3 Warehouse profit functions

The corresponding warehouse profit functions are

$$\pi_{W_1} = (\zeta_1 - 1)(1 - v_1) - \frac{1}{P}(1 - v_1) \left\{ \eta + \frac{k}{n} \right\} - \frac{\rho}{PD}n \quad (5-23)$$

$$\pi_{W_2} = (\zeta_2 - 1)(1 - v_2) - \frac{1}{P}(1 - v_2) \left\{ \eta + \frac{k}{n} \right\} - \frac{\rho}{PD}n \quad (5-24)$$

Setting the first derivatives of these cost and profits functions with respect to their particular lot sizes to zero yields the following economic order quantities (EOQ's)

$$Q_1 = \sqrt{\frac{AD}{\rho}}; Q = \sqrt{\frac{v_1AD}{\rho}}; q = \sqrt{\frac{(1-v_1)AD}{\rho}}; N = \sqrt{\frac{v_1KD}{\rho}}; n = \sqrt{\frac{(1-v_1)kD}{\rho}} \quad (5-25)$$

These can be recast as functions of Q_1

$$Q_1 = \sqrt{\frac{AD}{\rho}}; Q = \sqrt{v_1}Q_1; q = \sqrt{(1-v_1)}Q_1; N = \sqrt{\frac{v_1K}{A}}Q_1; n = \sqrt{\frac{(1-v_1)k}{A}}Q_1 \quad (5-26)$$

Substituting these EOQ's (5-26) into their corresponding cost and profit functions gives

$$C_0 = \sigma_0 + \alpha\eta + \xi \quad (5-27)$$

$$C_1 = \{\sigma_1 + \alpha\eta\}v_1 + \{\zeta_1 + \alpha\lambda\}(1 - v_1) + \xi\{\sqrt{v_1} + \sqrt{1 - v_1}\} \quad (5-28)$$

$$C_2 = \{\sigma_2 + \alpha\eta\}v_2 + \{\zeta_2 + \alpha\lambda\}(1 - v_2) + \xi\{\sqrt{v_2} + \sqrt{1 - v_2}\} \quad (5-29)$$

$$\pi_{W_1} = \{\zeta_1 - z\}(1 - v_1) - \sqrt{(1 - v_1)}\omega \quad (5-30)$$

$$\pi_{W_2} = \{\zeta_2 - z\}(1 - v_2) - \sqrt{(1 - v_2)}\omega \quad (5 - 31)$$

$$\pi_{V_0} = \sigma_0 - z - \Omega \quad (5 - 32)$$

$$\pi_{V_1} = (\sigma_1 - z)v_1 - \Omega\sqrt{v_1} \quad (5 - 33)$$

$$\pi_{V_2} = (\sigma_2 - z)v_2 - \Omega\sqrt{v_2} \quad (5 - 34)$$

where

$$\begin{aligned} \xi &= \left[\frac{\alpha A}{Q_1} + \gamma Q_1 \right]; \Omega = \sqrt{K} \left\{ \frac{\alpha \sqrt{A}}{Q_1} + \frac{\gamma Q_1}{\sqrt{A}} \right\}; \omega = \sqrt{k} \left\{ \sqrt{A} \frac{\alpha}{Q_1} + \frac{\gamma Q_1}{\sqrt{A}} \right\}; \alpha = \frac{1}{P}; \gamma = \frac{\rho}{PD}; \\ Q_1 &= \sqrt{\frac{AD}{\rho}}; z = 1 + \alpha\eta \end{aligned} \quad (5 - 35)$$

Equations (5 – 27) to (5 – 35) summarize the cost and profit functions employed in computing the Bertrand Nash equilibria and VSM's in the subsequent sections.

5.5 Derivative functions and computing equilibrium markups

Calculation of the Bertrand Nash equilibrium requires various derivatives of the cost and profit functions.

The Bertrand Nash equilibrium year-1 pricing (σ_e, ζ_e) is the point in (σ, ζ) space for which the following two conditions are simultaneously satisfied

$$\frac{\partial \pi_{V_1}(\sigma_e, \zeta_e)}{\partial \sigma_1} = 0 \text{ and } \frac{\partial \pi_{W_1}(\sigma_e, \zeta_e)}{\partial \zeta_1} = 0 \quad (5 - 36)$$

Adapting equations (5 – 1) – (5 – 3) to the current scenario,

$$v_1^{en} = \frac{e^{-\beta\sigma_1}}{\{e^{-\beta\sigma_1} + e^{-\beta\zeta_1}\}} \quad (5-37)$$

$$(1 - v_1^{en}) = \frac{e^{-\beta\zeta_1}}{\{e^{-\beta\sigma_1} + e^{-\beta\zeta_1}\}} \quad (5-38)$$

$$v_1 = \sigma_1^f v_1^{en} \quad (5-39)$$

The derivatives of the functions π_{w_1} and π_{v_1} (5-30) and (5-33) with respect to σ_1 and ζ_1 requires computing derivatives of the biased and entropy-based split functions, equations (5-37) – (5-39) with-respect-to to the same variables. These derivatives are

$$\frac{\partial v_1^{en}}{\partial \sigma_1} = -\beta v_1^{en} (1 - v_1^{en}) \quad (5-40)$$

$$\frac{\partial v_1^{en}}{\partial \zeta_1} = \beta v_1^{en} (1 - v_1^{en}) \quad (5-41)$$

$$\frac{\partial(1 - v_1^{en})}{\partial \sigma_1} = \beta v_1^{en} (1 - v_1^{en}) \quad (5-42)$$

$$\frac{\partial(1 - v_1^{en})}{\partial \zeta_1} = -\beta v_1^{en} (1 - v_1^{en}) \quad (5-43)$$

Differentiating (5-39) with respect to σ_1 gives

$$\frac{\partial v_1}{\partial \sigma_1} = v_1 \left\{ \frac{f}{\sigma_1} - \beta(1 - v_1^{en}) \right\} \quad (5-44)$$

Differentiating (1 - v_1) with respect to ζ_1 gives

$$\frac{\partial(1 - v_1)}{\partial \zeta_1} = -\beta v_1 (1 - v_1^{en}) \quad (5-45)$$

Using equations (5 – 40) – (5 – 45), setting the derivatives of the vendor and warehouse profit functions (5 – 33) and (5 – 30) to zero gives

$$\frac{\partial \pi_{V_1}}{\partial \sigma_1} = v_1 \left[1 + \left\{ \frac{f}{\sigma_1} - \beta(1 - v_1^{en}) \right\} \left\{ \sigma_1 - z - \frac{\Omega}{2\sqrt{v_1}} \right\} \right] = 0 \quad (5 - 46)$$

$$\frac{\partial \pi_{W_1}}{\partial \zeta_1} = (1 - v_1) - \beta v_1 (1 - v_1^{en}) \left\{ \zeta_1 - z - \frac{\omega}{2\sqrt{(1 - v_1)}} \right\} = 0 \quad (5 - 47)$$

Equations (5 – 46) and (5 – 47) are solved simultaneously using numerical methods to compute (σ_e , ζ_e).

5.6 Venture Success Metrics (VSM)

The objective of this section is to identify the parameters that influence the success of the venture and obtain values that lead to venture success. In this section, we define the Venture Success Metrics, specify the variables that determine the values of these metrics, and outline the sequence for calculating VSM's. Venture success is computed as the difference in the pre and post-venture hospital costs and total warehouse profits over the initial two year period, using the following Venture Success Metrics (VSM's):

$$\delta C = C_1 + C_2 - 2C_0 \quad (5 - 48)$$

$$T_{wp} = \pi_{W_1} + \pi_{W_2} \quad (5 - 49)$$

$$\tau = T_{wp} - \delta C \quad (5 - 50)$$

δC is the difference between the two-year post-venture ($C_1 + C_2$) and pre-venture ($2 C_0$) hospital costs; T_{wp} measures the two-year combined warehouse profits (π_{w1}, π_{w2}); τ is the difference of T_{wp} and δC . Venture success requires: $\delta C < 0$, $T_{wp} > 0$, and $\tau > 0$. The three metrics measure success of different parts of the venture. $\delta C < 0$ indicates that the venture has been able to reduce hospital costs. On the other hand, $T_{wp} > 0$ shows that the venture has succeeded in created a profitable warehouse. Finally, $\tau > 0$, if and only if, the overall venture is a success.

The following three scenarios in which $\tau > 0$ can be envisioned;

1. The venture simultaneously reduces hospital costs and turns a warehouse profit $\{\delta C < 0$ and $T_{wp} > 0\}$
2. The hospital fails to reduce costs, but warehouse profitability is sufficiently high to compensate $\{\delta C \geq 0, T_{wp} > 0; T_{wp} > \delta C; T_{wp} - \delta C > 0\}$
3. The hospital achieves an adequate cost reduction to overcome warehouse losses $\{\delta C < 0, T_{wp} < 0; \text{but } |\delta C| > |T_{wp}|; T_{wp} - \delta C > 0\}$

It could be argued that, from the healthcare organization's perspective, τ is the most important VSM.

5.6.1 Variables that determine VSM values

The variables that determine VSM values, termed here VSM Determinants, are the set of variables required to obtain values of the Venture Success Metrics, hence the venture's success. The discussion presented below will demonstrate that VSM's are determined by the year-1 profit objectives (π_v^0, π_w^0) set by the vendor and warehouse, and the split bias exponent f of equation (5 – 3). Equations 5 – 48, 49, and 50 show that VSM's are functions of pre and post-venture year-1 and

year-2 hospital costs (C_0, C_1, C_2) and post-venture warehouse profits (π_{W_1}, π_{W_2}) (equations 5 – 27 to 5 – 31). These cost and profit functions have two types of terms;

1. Type-1 terms represent SOH costs
2. Type-2 terms are algebraic products of markups and demands that reflect SKU costs and profits.

The requisite markups and demand splits are computed in an intricate sequence of steps outlined below. While SOH costs contribute to the eventual numerical values of costs and profits, their actual numerical values are small relative to those of Type-2 terms. Furthermore, they have no impact in determining markups or demands. Thus, to help simplify the discussions below, we create a modified set of cost and profit functions in which the SOH costs are set to zero. Subsequent numerical VSM computations, however, rely on the original forms of these functions that include both Type-1 and 2 terms. Setting to zero, the values of $\eta, \lambda, A, K, k,$ and ρ in equations 5 – 27 to 5 – 31 to, leads to zero values for $\xi, \Omega, \omega, \gamma,$ and Q_1 and $z = 1$ in equations 5 – 35 and the following cost and profit functions

$$\begin{aligned}
 & \text{a. } C_0 = \sigma_0 \\
 & \text{b. } C_1 = \sigma_1 v_1 + \zeta_1 (1 - v_1) \\
 & \text{c. } C_2 = \sigma_2 v_2^{en} + \zeta_2 (1 - v_2^{en}) \\
 & \text{d. } \pi_{W_1} = \{\zeta_1 - 1\} (1 - v_1) \\
 & \text{e. } \pi_{W_2} = \{\zeta_2 - 1\} (1 - v_2^{en}) \\
 & \text{f. } \pi_{V_0} = \sigma_0 - 1 \\
 & \text{g. } \pi_{V_1} = (\sigma_1 - 1) v_1 \\
 & \text{h. } \pi_{V_2} = (\sigma_2 - 1) v_2^{en}
 \end{aligned}$$

(5 – 51)

VSM Determinants are now identified. Equations 5 – 48 to 5 – 51 show that (parenthetical inclusions are to be read “is a function of”);

1. $\delta C (C_0, C_1, C_2)$
 - a. $C_0 (\sigma_0)$
 - Equating the vendor’s profit function 5 – 51f of the vendor’s year-1 profit objective π_v^0 , gives $\sigma_0 (\pi_v^0)$
 - $C_0 (\pi_v^0)$
 - b. $C_1 (\sigma_1, \zeta_1, \upsilon_1)$
 - (σ_1, ζ_1) are calculated by the equilibrium of π_{v1} and π_{w1} (equations 5 – 51d and 51g) employing the warehouse-biased demand-split function equation (5 – 3) with exponent f. Therefore $\sigma_1 (f), \zeta_1 (f)$
 - $C_1 (f, \upsilon_1)$.
 - c. $C_2 (\sigma_2, \zeta_2, \upsilon_2^{en})$
 - Employing equations 5 – 4 to 5 – 8, $\sigma_2 (\sigma_1, V), V (V_0, \upsilon_1), V_0 (\pi_v^0)$; thus $\sigma_2 (f, \pi_v^0, \upsilon_1)$
 - Utilizing equations 5– 9 to 5 – 13, $\zeta_2 (\zeta_1, W), W (W_0, \upsilon_1), W_0 (\pi_w^0)$; thus $\zeta_2 (f, \pi_w^0, \upsilon_1)$
 - Using equations 5 – 1, $\upsilon_2^{en} (\sigma_2, \zeta_2)$; thus $\upsilon_2^{en} (f, \pi_v^0, \pi_w^0, \upsilon_1)$
 - $C_2 (f, \pi_v^0, \pi_w^0, \upsilon_1)$
 - d. $\delta C (f, \pi_v^0, \pi_w^0, \upsilon_1)$
2. υ_1^{\min}
 - a. $\delta C (f, \pi_v^0, \pi_w^0, \upsilon_1)$
 - b. Assuming a δC and υ_1 are in one-to-one correspondence, that is for every υ_1 there is one and only one value of δC and vice versa, solving for equation δC in step 3 for υ_1 gives $\upsilon_1 (\delta C, f, \pi_v^0, \pi_w^0)$.
 - c. υ_1 varies in the range $[0, 1]$; υ_1^{\min} is the value of \square_1 in that range that minimizes δC .
 - d. $\upsilon_1^{\min} (\delta C_{\min}, f, \pi_v^0, \pi_w^0)$
3. $T_{wp} (\pi_{w1}, \pi_{w2})$
 - a. $\pi_{w1} (\sigma_1, \zeta_1, \upsilon_1^{\min})$, thus $\pi_{w1} (f, \pi_v^0, \pi_w^0)$
 - b. $\pi_{w2} (\sigma_2, \zeta_2, \upsilon_2^{en})$, hence $\pi_{w2} ((f, \pi_v^0, \pi_w^0)$
 - c. $T_{wp} (f, \pi_v^0, \pi_w^0)$

4. $\tau (T_{wp}, \delta C)$; thus $\tau (f, \pi_v^o, \pi_w^o)$
5. Therefore, the determinants of venture success are (f, π_v^o, π_w^o)

5.6.2 VSM computation sequence

The sequence used to calculate the VSM's is given in the steps below and outlined in figure 5-7 (numbers correspond to the sequence of steps below);

- I. Set the SOH variables and function parameter values listed in chapter 3. These values remain constant throughout this computational sequence.
- II. Set the initial value of f
 - a. Create the two dimensional Profit Objective Grid (POG) as a range of (π_v^o, π_w^o) profit objectives values. For each profit objective pair;
 1. Obtain σ_0 by setting $\pi_{v0} = \pi_v^o$ in equation 5 – 32 and solving for σ_0
 2. Calculate C_0 using equation 5 – 27
 3. Create a one dimensional grid of υ_1 points over $[0, 1]$
 4. Compute Bertrand Nash equilibrium markups (σ_e, ζ_e) that corresponds to the equilibrium of year-1 vendor and warehouse profit functions π_{v1} and π_{w1} using the warehouse-biased demand split function (equation 5 – 3 with υ and υ^{en} replaced by υ_1 and υ_1^{en})
 5. Set year-1 vendor and warehouse markups $(\sigma_1, \zeta_1) = (\sigma_e, \zeta_e)$
 6. Determine C_1 (equation 5 – 28) as a function of (σ_1, ζ_1) for the entire υ_1 grid
 7. Calculate (V_0, W_0) (equations 5 – 7 and 5 – 12) for each profit objective pair

8. Compute V and W discount schedules (equations 5 – 6 and 5 – 11) over the entire υ_1 grid
 9. Obtain (σ_2, ζ_2) (equations 5 – 8 and 5 – 13) by discounting (σ_1, ζ_1) over the entire υ_1 grid
 10. Find υ_2^{en} (equation 5 – 1) as a function of (σ_2, ζ_2) over the entire υ_1 grid
 11. Calculate C_2 (equation 5 – 29) as a function of $(\sigma_2, \zeta_2, \upsilon_2^{\text{en}})$
 12. Obtain δC (equation 5 – 48)
 13. Determine the value of υ_1^{min} that minimizes δC . Note that υ_1^{min} is a unique value of υ_1 over the entire υ_1 grid.
 14. Identify $\delta C(\upsilon_1^{\text{min}})$
 15. Compute π_{w1} , π_{w2} , and T_{wp} (equations 5 – 30, 5 – 31, and 5 – 49) as functions of υ_1^{min} , $\sigma_1, \zeta_1, \sigma_2, \zeta_2$
 16. Calculate τ (equation 5 – 50)
- b. Return to step II.a; set the next POG grid point; repeat until the grid is exhausted
- III. Return to step 2; increment f; repeat until all values of f are exhausted

Note that the biased demand split used in obtaining the equilibrium markups in step 4 above is different from the year-1 demand split υ_1 grid created in step 3. It is critical to understand the difference. The latter is controlled and exercised by the hospital. The biased demand-split in 4 is employed by the vendor and warehouse for the explicit purpose of computing their respective year-1 profit functions and the consequent equilibrium markups (σ_1, ζ_1) .

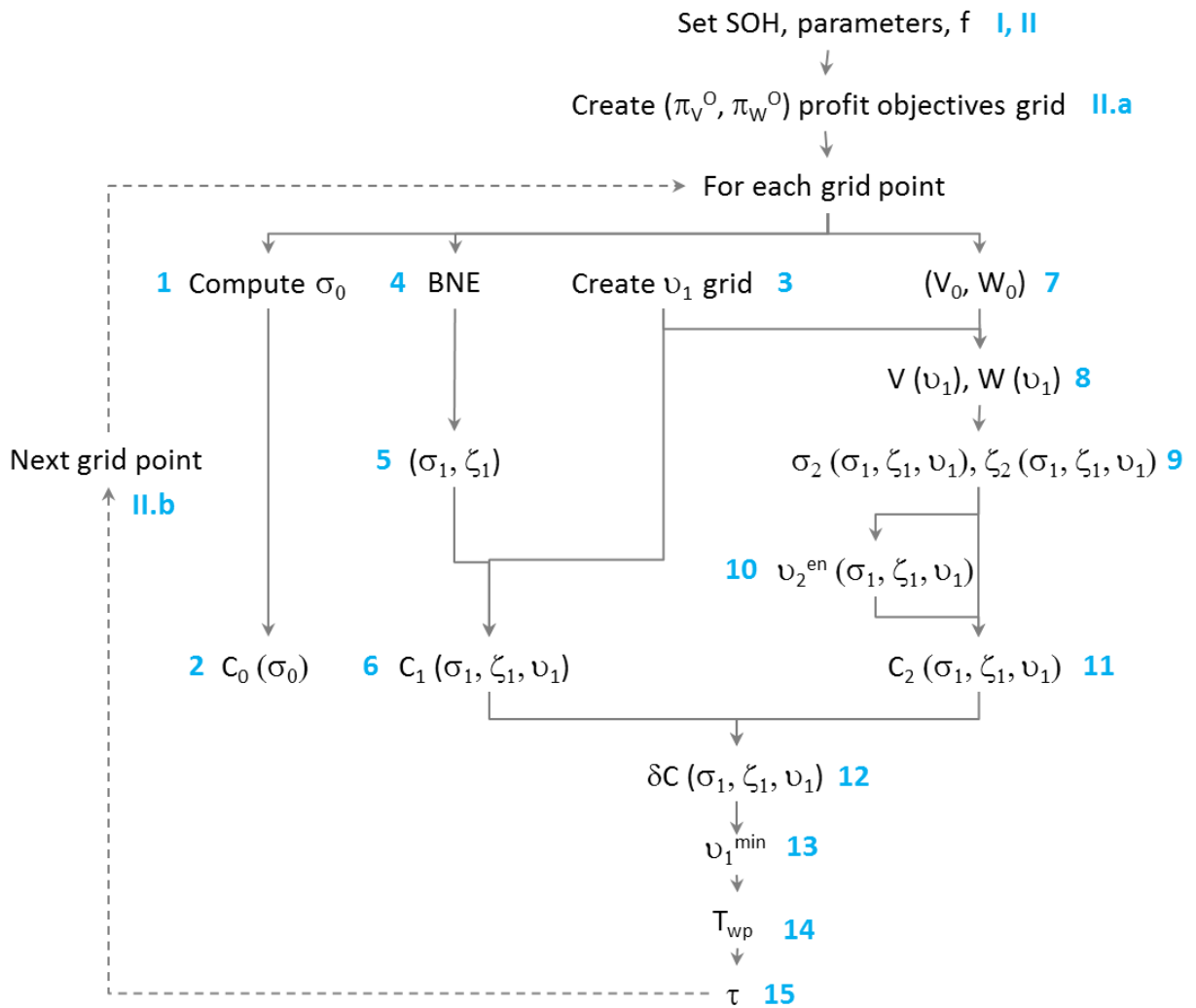


Figure 5-7 – Computation sequence

5.7 Computations, results, and conclusions

In this section, we will alternately refer to “increase in warehouse bias” or equivalently “decrease in f ” to indicate the decrease in f values in equation 5 – 3 from positive to negative and the resulting shift in year-1 demand function used to compute year-1 equilibrium markups in favor of the warehouse. This section will show that increasing warehouse bias leads to a decline in all VSM’s, compromising the venture. That is, biasing year-1 demand used in computing Bertrand Nash

equilibrium markups in favor of the warehouse with the intent of improving the venture's odds of success, leads to the opposite result of compromising the venture.

This result is explained below by showing that an increase in warehouse bias shifts year-1 markups (σ_e, ζ_e) to higher values, with ζ_e increasing more rapidly than σ_e as f decreases. This, despite discounts, leads to higher year-2 markups (σ_2, ζ_2). These elevated markups result in higher C_1 and C_2 , compromising δC , and eventually τ .

This section is organized as follows; first compute year-1 equilibrium markups, followed with a discussion of the variation of equilibrium markups with f . The section concludes by computing VSM's and discussing their variation with (π_v^0, π_w^0) and f .

5.7.1 Computing year-1 markups (σ_1, ζ_1) – the Bertrand Nash equilibrium

Oligopoly theory is premised on the assumption that, in setting price and production volumes, competing firms must seek equilibria that balance their respective advantages. Deviation from equilibrium by one or more competitors to increase own advantage is quelled by counter-moves from others (Osborne, 2004). The development of oligopoly equilibrium theory follows two paradigms: Bertrand and Cournot. Despite predating game theory and Nash equilibria by several decades, the Bertrand and Cournot paradigms have the appearance of being natural extensions of Nash theory and are invariably discussed within that context. Bertrand's model focuses on determining equilibrium prices and treats demand as a function of price (Bertrand, 1883). Cournot's on the other hand focuses on determining equilibrium demand and treats price as a function of demand (Cournot, 1927). Here, Bertrand Nash equilibrium methodology is employed to compute year-1 vendor and warehouse prices as the equilibrium markup (σ_e, ζ_e) for the corresponding profit functions. As outlined in section 6 of this chapter, Bertrand Nash equilibrium is computed by simultaneously solving equation 5 – 46 and 5

– 47 numerically for the equilibrium pricing (σ_e, ζ_e) . Year-1 markups are (σ_1, ζ_1) are equated to (σ_e, ζ_e) .

The vendor and warehouse profit functions whose equilibrium is sought are shown in figures 5 – 8 and 5 – 9) as contour diagrams of $\pi_{v_1}(\sigma, \zeta)$ and $\pi_{w_1}(\sigma, \zeta)$ functions 5 – 33 and 5 – 30 respectively. The main features of these functions are illustrated using the vendor’s profit surface figure 5 – 8. In this figure, as σ increases at fixed ζ , say at $\zeta = 1.1$, vendor profit increases, reaches a maximum at $\sigma \sim 1.3$ and decreases at higher values of σ . This “parabolic” behavior is a consequence of algebraic markup-demand product term in the vendor’s profit function (5 – 33). At low σ , $\upsilon \cong 1$ and this product increases with σ . Higher values of σ cause a rapid decrease in υ , and the product decreases. The maximum occurs at σ_{\max} when these two terms are equal. The value of σ_{\max} increases with ζ because of the effect of the latter on υ . Similarly, at a fixed σ , the vendor’s profit increases with ζ . This is exclusively due to the fact that increasing ζ lead to increasing υ , resulting in higher vendor profits. A similar synopsis can be developed for the warehouse profit surface in figure 5 – 9.

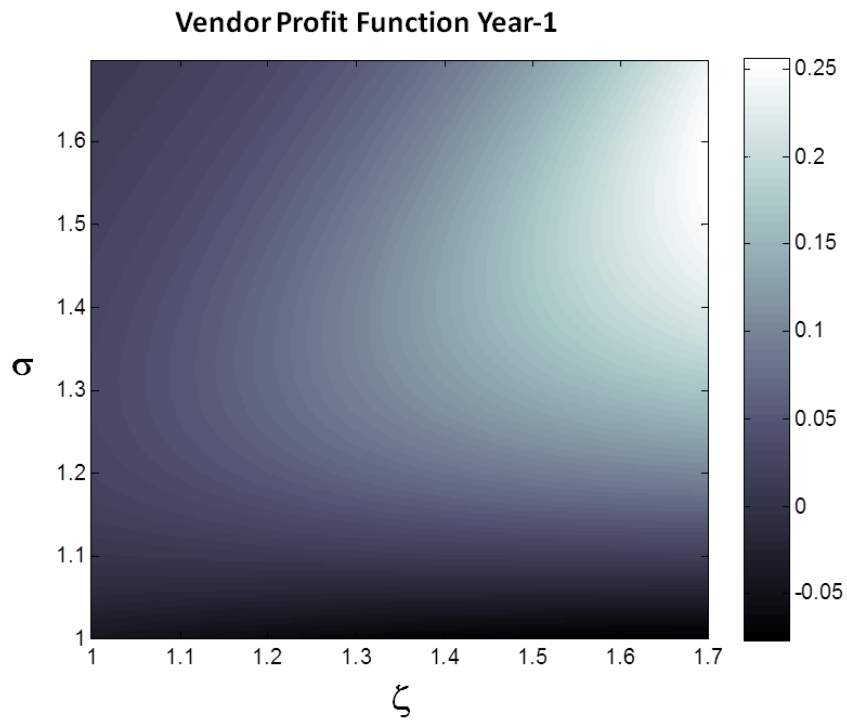


Figure 5-8: Vendor year-1 profit function equation 5 – 33

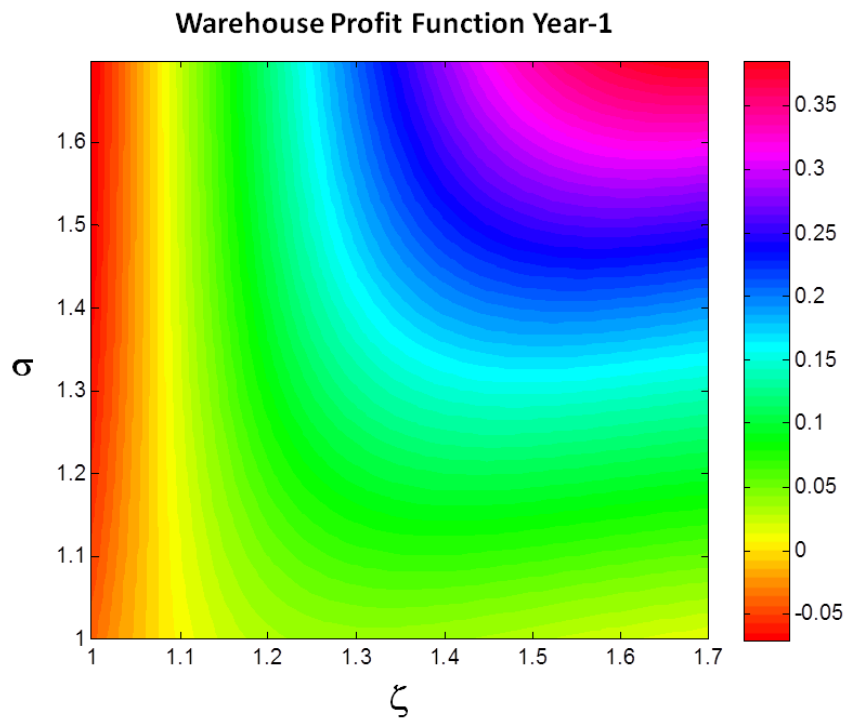


Figure 5-9: Warehouse year-1 profit function 5-30

Figure 5 – 10 shows the lines of maxima for the vendor (red) and warehouse (blue) profit functions for four different f values. This figure also shows the intersections of the red and blue lines that correspond to the Bertrand Nash equilibrium (σ_e, ζ_e) markups for each f value.

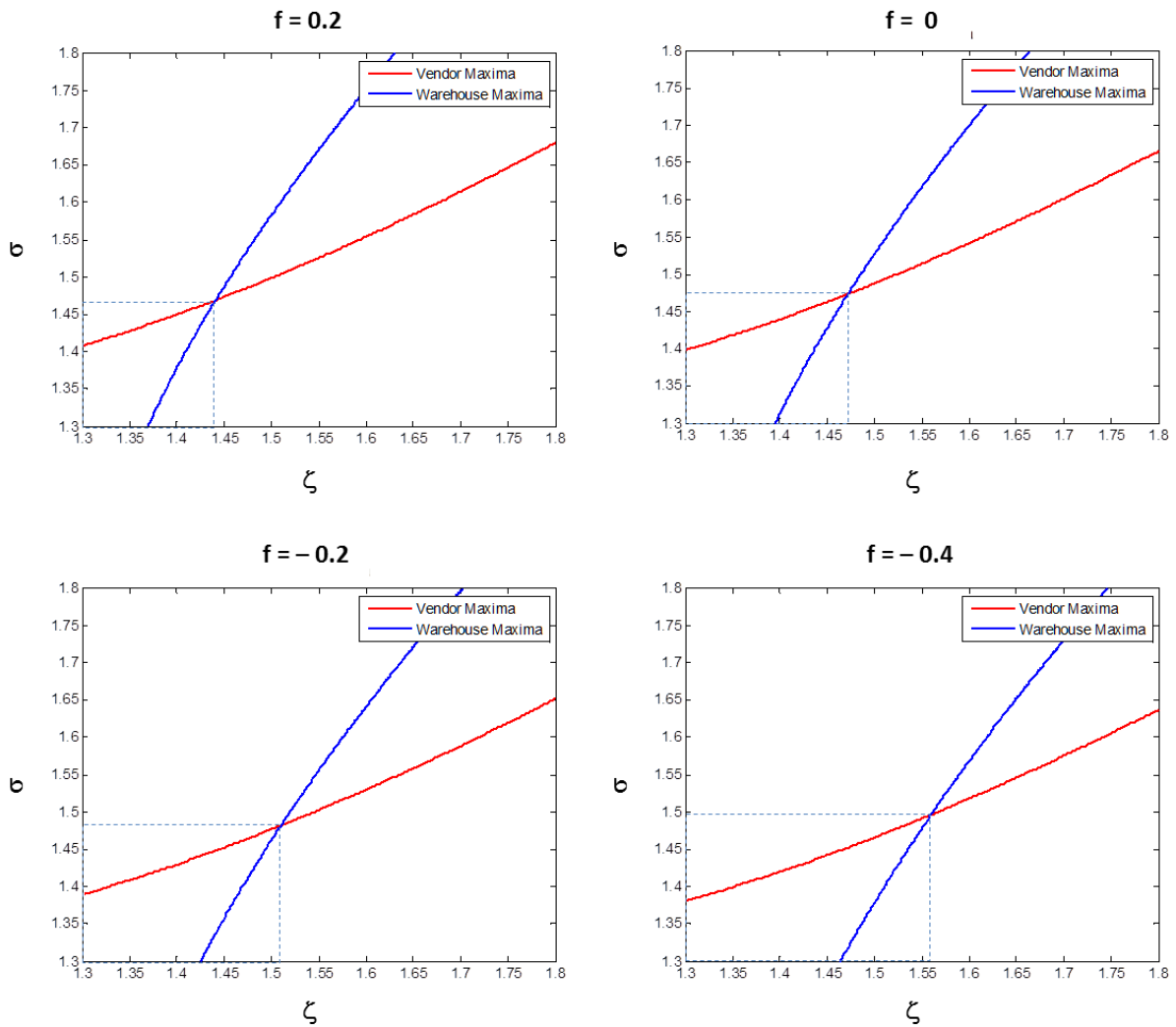


Figure 5-10: Vendors (red) and warehouse (blue) lines of maxima for $f = 0.2, 0, -0.2, -0.4$. The (σ, ζ) values at the line intersections corresponds to equilibrium (σ_e, ζ_e) markups.

5.7.2 Variation of (σ_e , ζ_e) with the demand split exponent f

The Bertrand Nash equilibrium diagrams in figure 5 – 10 clearly show that a decrease in f value leads to a rightward shift of the warehouse maxima (blue) and a slight downward shift in the vendor maxima (red). The figure also shows that σ_e and ζ_e increase as f decreases. Table 5 – 1 shows the values of σ_e and ζ_e for seven f values. Figure 5 – 11 is a graph of the data in table 5 – 1. This figure clearly shows that both σ_e and ζ_e increase as f decreases, with ζ_e increasing more rapidly than σ_e . The latter observation is reflected by “ ζ_e/σ_e ” ratios in table 5 –1.

Table 5-1: Variation of the equilibrium markups (σ_e, ζ_e) with f value				
f	σ_e	ζ_e	$\zeta_e - \sigma_e$	ζ_e/σ_e
-0.8	1.556	1.689	0.133	1.09
-0.6	1.525	1.630	0.105	1.07
-0.4	1.498	1.560	0.062	1.04
-0.2	1.481	1.510	0.029	1.02
0	1.472	1.471	-0.001	1.00
0.2	1.470	1.440	-0.030	0.98
0.4	1.470	1.410	-0.060	0.96

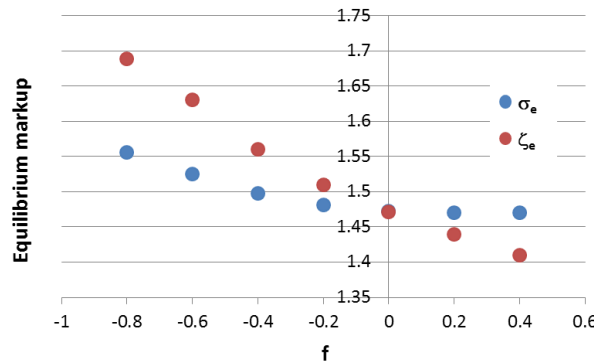


Figure 5-11: Variation of the equilibrium markups (σ_e , ζ_e) with f value

Understanding the VSM results discussed later in this section requires understanding the causes of the f -dependences of σ_e and ζ_e shown in table 5 – 1 and figure 5 – 11. As before, we simplify the discussion by noting that markup-demand term in π_{w_1} and π_{v_1} functions 5 – 30 and 5 – 33 dominate the SOH terms. With that, we use their equations 5 – 51e and 5 – 51g counterparts in which the SOH terms have been stripped.

$$\pi_{w_1} = (\zeta_1 - 1)(1 - v_1) \tag{5 – 51e}$$

$$\pi_{v_1} = (\sigma_1 - 1)v_1 \tag{5 – 51g}$$

Where

$$v_1 = \sigma_1^f v_1^{en}$$

Figure 5 – 12 depicts the shift of π_{w_1} (5 – 51d, $\sigma_1 = 1.3$) with f . It clearly shows that ζ_{\max} increases as f decreases and higher year-1 warehouse bias. This increase in ζ_{\max} is a consequence of the rightward shift of the $(1 - v_1)$ term in equation (5 – 51e) shown in figure 5 – 13.

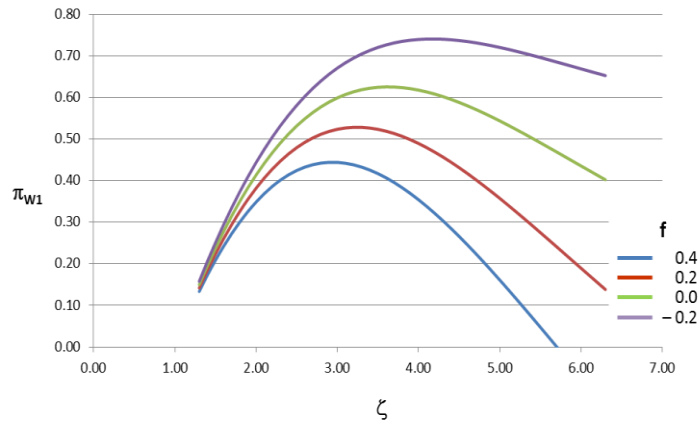


Figure 5-12: Variation of π_{w1} (equation 5 – 51d, $\sigma_1 = 1.3$) for $f = 0.4, 0, -0.2, -0.4$

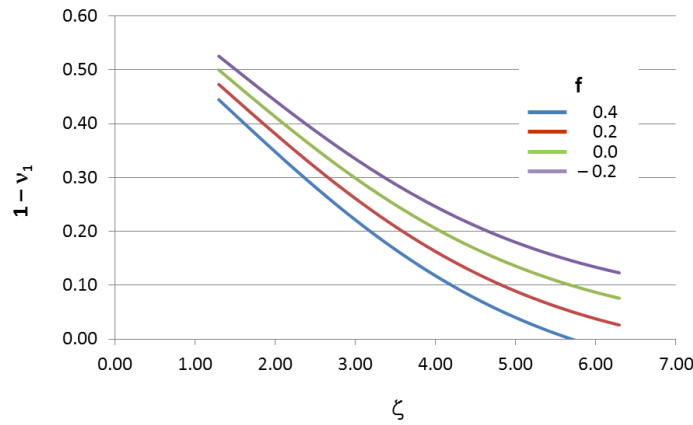


Figure 5-13: Variation of the warehouse fraction ($1 - v_1$; v_1 computed with 3 – 3, $\sigma_1 = 1.3$) for $f = 0.4, 0, -0.2, -0.4$.

Figure 5 – 14 demonstrates the variation π_{v1} (5 – 51g, $\zeta_1 = 1.3$) with f . It clearly shows that σ_{max} decreases as f decreases and year-1 warehouse bias increases. This decrease in σ_{max} is a consequence of a leftward shift of the v_1 term in (5 – 51g) with f , shown in figure 5 – 15.

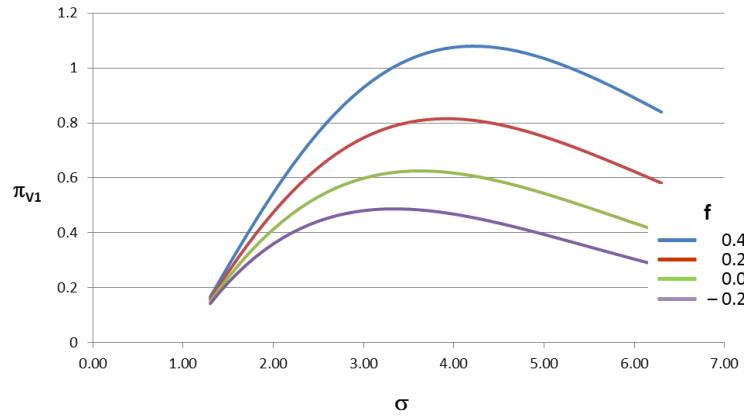


Figure 5-14: Variation of π_{v1} (equation 5 – 51g, $\zeta_1 = 1.3$) for $f = 0.4, 0, -0.2, -0.4$.

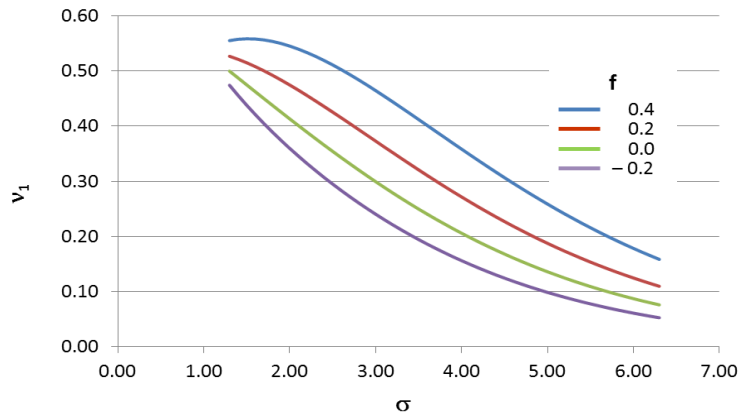


Figure 5-15: Variation of the vendor fraction (v_1 , equation 5 – 3, $\zeta_1 = 1.3$) for $f = 0.4, 0, -0.2, -0.4$.

5.7.3 Computing Venture Success Metrics

In Section 5.6 we showed that δC , T_{wp} , and τ are determined by three factors: the vendor and warehouse year-1 profit objectives (π_v^0 , π_w^0) and the demand biasing exponent f . That section also outlined the computational sequence for determining these VSM's, In this section VSM's are

computed for three different f ($-0.2, 0, 0.2$) over a profit objectives grid (π_v^0, π_w^0) with $0 \leq \pi_v^0 \leq 0.4$ and $0 \leq \pi_w^0 \leq 0.4$ with a grid spacing of 0.05 in both dimensions. After this initial setup, the following variables and functions are computed sequentially: σ_0, C_0 , Bertrand Nash equilibrium markups $(\sigma_e, \zeta_e), (\sigma_1, \zeta_1), V, W, (\sigma_2, \zeta_2), C_1, C_2$, and finally the VSM's: $\delta C, T_{wp}$, and τ .

At this point, it is important to clearly identify which venture parameters are controlled by which entities. Year-1 markups are set by the vendor and warehouse. However, these markups determined using a hospital-imposed warehouse demand bias. The degree of this bias that optimizes venture success is determined below. The vendor and warehouse also set their year-1 profit objectives (π_v^0, π_w^0) , hence their respective year-1 discounts, year-2 markups, and splits. In short, the vendor and warehouse control the values of $(\sigma_1, \zeta_1), (\sigma_2, \zeta_2)$, and υ_2^{en} .

Based on these values, the hospital computes δC as a function of υ_1 and determines υ_1^{\min} that minimizes δC . Thus, year-1 vendor/warehouse demand split is set by the hospital to a value that maximizes the hospital's advantage. $\upsilon_1^{\min}, (\sigma_1, \zeta_1), (\sigma_2, \zeta_2)$, and υ_2^{en} are used to determine the warehouse's profits and T_{wp} . In what follows, we report the computed values of the various VSM's as functions of the profit objectives (π_v^0, π_w^0) and f and a discussion of these results.

As shown below, the variation of VSM's with f is straightforward. Their variation with profit objectives however is more complicated. It is a direct consequence of the complex dependence of υ_1^{\min} on the profit objectives, as discussed below. In order to unravel this complexity, we use the simplified hospital cost functions (5 – 51a, b, and c) from which SOH cost are stripped. With that, δC is

$$\delta C = \{\sigma_1 v_1 + \zeta_1(1 - v_1) - 2\sigma_0\} + \sigma_2 v_2^{en} + \zeta_2(1 - v_2^{en}) \quad (5 - 52)$$

While the σ_0 , σ_1 , and ζ_1 variables are straightforward, σ_2 and ζ_2 are non-linear functions of (π_v^o, π_w^o) . Furthermore v_2^{en} is a non-linear function of σ_2 and ζ_2 . Thus, the dependence of δC , its minimum value, hence v_1^{\min} on (π_v^o, π_w^o) is highly non-linear and does not lend itself to a simple and intuitive interpretation. We will illustrate this complexity with an example.

Using function parameter values from table 1 – 1 and $(\sigma_1, \zeta_1) = (1.45, 1.4)$, $(\pi_v^o, \pi_w^o) = (0.25, 0.30)$, and $\sigma_0(\pi_v^o) = 0.25$, the various components of equation 5 – 52 are graphed in figures 5 – 17 through 5 – 23 as functions of v_1 . The hierarchy of these figures is shown in figure 5-16. The quantity of interest is v_1^{\min} depicted in figure 5 – 23 whose graph is a sum of the functions in figures 5 – 17 and 5 – 22. The latter is obtained by summing graphs 5 – 19 (a product of the two functions in figure 5 – 18) and 5 – 21 (a product of the two functions in figure 5 – 20)

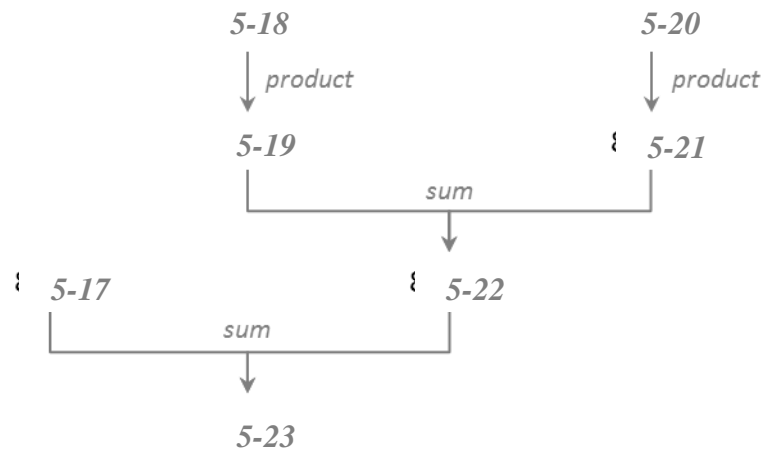


Figure 5-16: Hierarchy of figures 5 – 17 to 5 – 23

It is thus clear that v_1^{\min} in figure 5 – 23 is related to the minima in figures 5 – 19 and 5 – 21. The latter two minima are determined by corresponding products of functions shown in figures 5 – 18 and 5 – 20. Thus, ultimately, the shifts in v_1^{\min} with (π_v^0, π_w^0) are determined by the corresponding shifts of the four functions in figures 5 – 18 and 5 – 20 with (π_v^0, π_w^0) and, in turn, determine the value of the VSM's δC , T_{wp} , and τ . The non-linear dependence of v_1^{\min} and $(1 - v_1^{\min})$ on (π_v^0, π_w^0) is shown in table 5 – 2. The first part of this table shows that, for $\pi_w^0 = 0.3 - 0.4$, $(1 - v_1^{\min})$ has a minimum at $\pi_v^0 = 0.35$. The second part of this table shows a corresponding non-linearity in v_1^{\min} . As discussed above, the complex non-linear dependence of v_1^{\min} on (π_v^0, π_w^0) does not lend itself to simple and intuitive interpretations. It is the same for $\delta C (v_1^{\min})$, T_{wp} , and τ which are determined by v_1^{\min} .

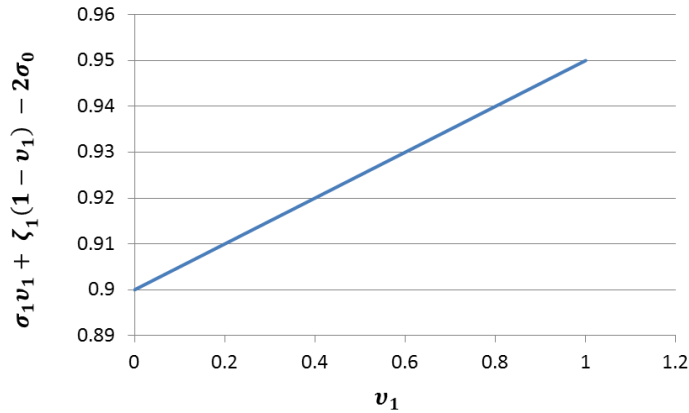


Figure 5-17: Variation of the first braces term in equation 5 – 52 with v_1

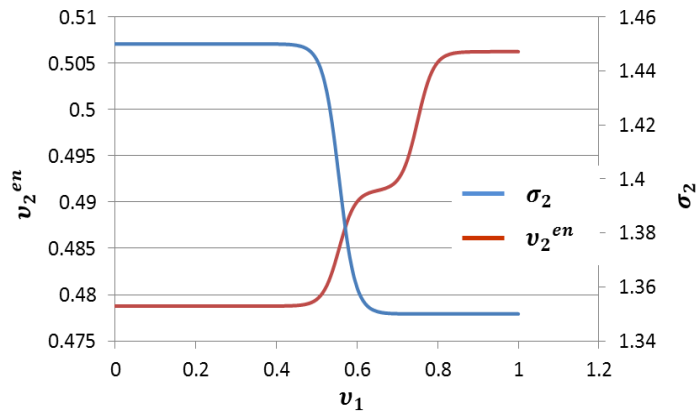


Figure 5-18: Variation of σ_2 and v_2^{en} with v_1

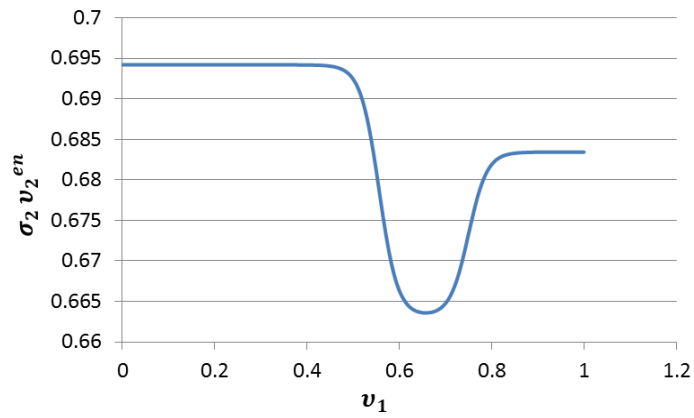


Figure 5-19: Variation of the $\sigma_2 v_2^{en}$ product, the second term in equation 5 – 52, with v_1

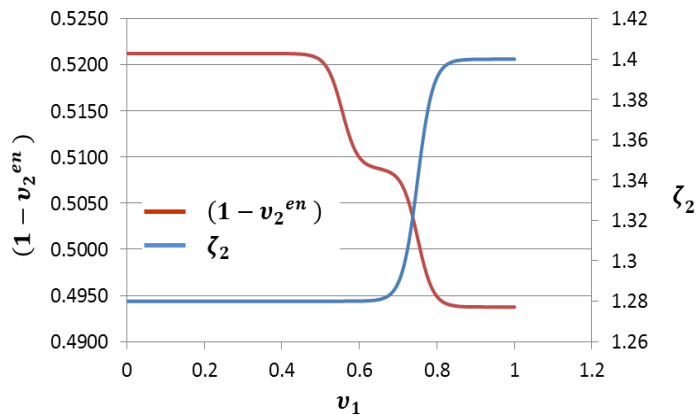


Figure 5-20: Variation of ζ_2 and $(1 - v_2^{en})$ terms with v_1

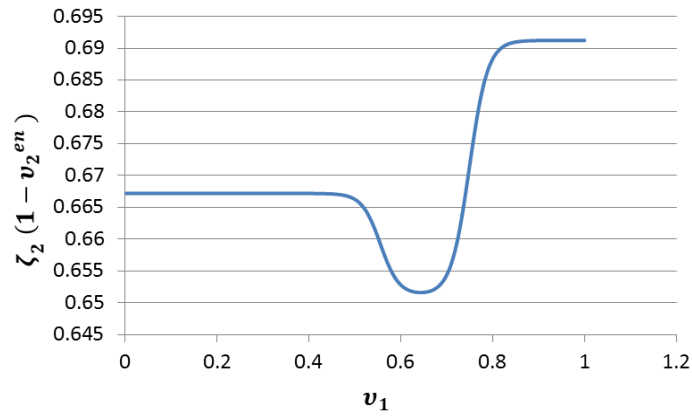


Figure 5-21: Variation of the $\zeta_2(1 - v_2^{en})$ product, the third term in equation 5 – 52, with v_1

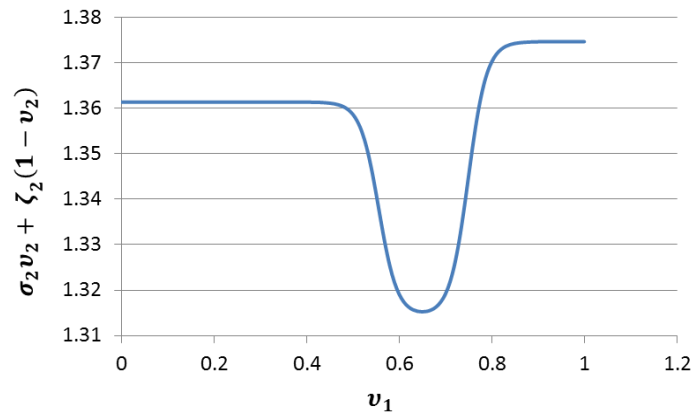


Figure 5-22: Variation of the sum of the second and third terms in equation 5 – 52 with v_1

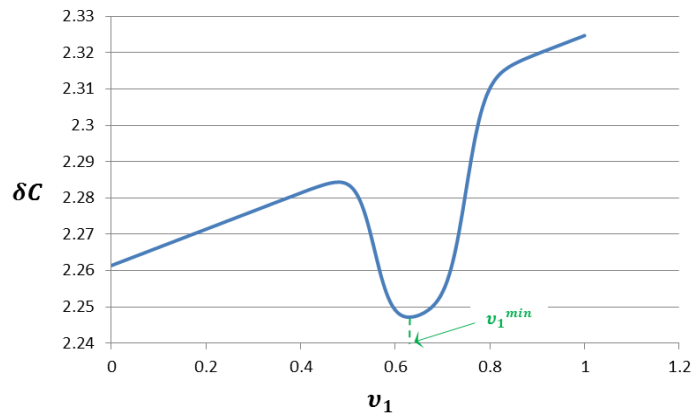


Figure 5-23: Variation of δC (equation 5 – 52) with v_1

Table 5 – 3 shows the variation of $\delta C (v_1^{\min})$ with (π_v^0, π_w^0) for the three f values. For $f = -0.2$ and fixed π_w^0 , increasing π_v^0 (moving to the right in each row) decreases δC , eventually attaining $\delta C < 0$ at the higher value of π_v^0 . Additionally, for fixed π_v^0 , increasing π_w^0 (moving down in each column) increases δC . These trends hold for the other two f values. Thus table 5 –3 shows that the hospital's advantage metric δC improves with higher π_v^0 and lower π_w^0 . Table 5 – 4 shows variation of T_{wp} with (π_v^0, π_w^0) . For $f = -0.2$ and $\pi_w^0 \geq 0.2$, T_{wp} has minima at $\pi_v^0 = 0.35$ for $\pi_w^0 = 0.2 - 0.4$. This non-linearity follows from T_{wp} 's dependence on v_1^{\min} . This trend holds for $f = 0$. From an organization's point of view, τ is the most significant VSM. Table 5 – 5 shows the variation of the τ metric with (π_v^0, π_w^0) . Interestingly, for this metric, for the range of (π_v^0, π_w^0) in table 5 – 5, non-linearity is observed for $f = -0.2$ and 0 but not for $f = 0.2$.

Now the variation of the VSM's with f is explored. Comparing δC values in table 5 – 3 that correspond to identical profit objective pairs for the three f values shows that δC decreases as f increases. This trend is also reflected in table 5 – 6 which shows the δC averages over the (π_v^0, π_w^0) grid for each f value. Comparing T_{wp} values in table 5 – 4 that correspond to identical profit objective pairs for the three f values shows different trends in T_{wp} dependence on f , decreasing for some profit objectives and increasing for others. Table 5 – 6 however, shows the T_{wp} averages over the (π_v^0, π_w^0) grid increase with f .

Finally, comparing τ values in table 5 – 5 that correspond to identical profit objective pairs for the three f values shows that τ increases with f . This trend is also reflected in table 5 – 6 which shows τ averages over the (π_v^0, π_w^0) grid for each f value.

Thus, biasing purchases in favor of the warehouse with more negative f compromises the venture. This counterintuitive result is due to the fact that increasing warehouse bias causes higher year-1 equilibrium markup (table 5 – 1 figure 5 – 11) which translates into higher year-2 markups, even after taking the discounts into account. The combined higher year-1 and year-2 markups with increasing warehouse bias, while they increase the warehouse profitability as reflected by T_{wp} , the higher prices undermines the hospital's cost reduction advantage as reflected by δC and a deteriorating combined decline in the venture success as measured by τ .

In summary, our ultimate objective is to develop models for measuring the success of ventures in which healthcare organizations enter price competitions with their external suppliers. This thesis is our first step towards that objective. It models a simplified venture in which the hospital runs its own warehouse as a profit center that competes with one external vendor on a single supply item based on currently prevalent healthcare industry practices. This model enabled us to identify and quantitate the following parameters as determinants of venture success: vendor and warehouse year-1 profit.

Table 5-2										
Variation of $1 - \upsilon_1^{\min}$ and with (π_v^o, π_w^o) at $f = 0$										
		$1 - \upsilon_1^{\min}$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.05	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.15	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.20	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.25	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.30	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0
	0.35	1.0	1.0	1.0	1.0	1.0	1.0	0.15	0.0	1.0
	0.40	1.0	1.0	1.0	1.0	0.41	0.28	0.15	0.0	1.0
		υ_1^{\min}								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
	0.35	0.0	0.0	0.0	0.0	0.0	0.0	0.85	1.0	0.0
	0.40	0.0	0.0	0.0	0.0	0.59	0.72	0.85	1.0	0.0

Table 5-3: Values of δC for three different $[f, \sigma_e, \zeta_e]$ sets over an entire (π_v^o, π_w^o) grid										
		$[f, \sigma_e, \zeta_e] = [-0.2, 1.48, 1.51]$								
		χ								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.05	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.10	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.15	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1
	0.20	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1
	0.25	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1
	0.30	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1
	0.35	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1
	0.40	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1
		$[f, \sigma_e, \zeta_e] = [0.0, 1.47, 1.47]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
	0.05	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.10	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.15	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.20	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.25	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.30	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	-0.1
	0.35	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	-0.1
	0.40	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	-0.1
		$[f, \sigma_e, \zeta_e] = [0.2, 1.47, 1.44]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
	0.05	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
	0.10	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
	0.15	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.20	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.25	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.30	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.35	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2
	0.40	0.6	0.5	0.4	0.3	0.2	0.1	0.0	-0.1	-0.2

Table 5-4										
Values of T_{wp} for three different $[f, \sigma_e, \zeta_e]$ sets over an entire (π_v^o, π_w^o) grid										
		$[f, \sigma_e, \zeta_e] = [-0.2, 1.48, 1.51]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.05	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.10	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.15	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.20	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.1	0.6
	0.25	0.6	0.6	0.6	0.6	0.6	0.6	0.2	0.1	0.6
	0.30	0.6	0.6	0.6	0.6	0.6	0.3	0.2	0.1	0.6
	0.35	0.6	0.6	0.6	0.6	0.3	0.3	0.2	0.1	0.1
	0.40	0.6	0.6	0.5	0.4	0.3	0.3	0.2	0.1	0.1
		$[f, \sigma_e, \zeta_e] = [0.0, 1.47, 1.47]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.05	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.10	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.15	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.20	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.25	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.30	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.1	0.6
	0.35	0.6	0.6	0.6	0.6	0.6	0.6	0.2	0.1	0.6
	0.40	0.6	0.6	0.6	0.6	0.3	0.3	0.2	0.1	0.6
		$[f, \sigma_e, \zeta_e] = [0.2, 1.47, 1.44]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.05	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.10	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.15	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.20	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.25	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.30	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.35	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.40	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

Table 5-5										
Values of τ for three different $[f, \sigma_e, \zeta_e]$ sets over an entire (π_v^o, π_w^o) grid										
		$[f, \sigma_e, \zeta_e] = [-0.2, 1.48, 1.51]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.01	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81
	0.05	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
	0.10	-0.02	0.08	0.18	0.28	0.38	0.48	0.58	0.68	0.78
	0.15	-0.03	0.07	0.17	0.27	0.37	0.47	0.57	0.67	0.77
	0.20	-0.04	0.06	0.16	0.26	0.36	0.46	0.56	0.14	0.76
	0.25	-0.06	0.04	0.14	0.24	0.34	0.44	0.10	0.14	0.74
	0.30	-0.07	0.03	0.13	0.23	0.33	0.06	0.10	0.14	0.73
	0.35	-0.08	0.02	0.12	0.22	0.01	0.06	0.10	0.14	0.22
	0.40	-0.10	0.00	-0.06	-0.03	0.01	0.06	0.10	0.14	0.22
		$[f, \sigma_e, \zeta_e] = [0.0, 1.47, 1.47]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.02	0.12	0.22	0.32	0.42	0.52	0.62	0.72	0.82
	0.05	0.01	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81
	0.10	-0.01	0.09	0.19	0.29	0.39	0.49	0.59	0.69	0.79
	0.15	-0.02	0.08	0.18	0.28	0.38	0.48	0.58	0.68	0.78
	0.20	-0.03	0.07	0.17	0.27	0.37	0.47	0.57	0.67	0.77
	0.25	-0.04	0.06	0.16	0.26	0.36	0.46	0.56	0.66	0.76
	0.30	-0.06	0.04	0.14	0.24	0.34	0.44	0.54	0.17	0.74
	0.35	-0.07	0.03	0.13	0.23	0.33	0.43	0.13	0.17	0.73
	0.40	-0.09	0.01	0.11	0.21	0.04	0.08	0.13	0.17	0.71
		$[f, \sigma_e, \zeta_e] = [0.2, 1.47, 1.44]$								
		π_v^o								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
π_w^o	0.00	0.03	0.13	0.23	0.33	0.43	0.53	0.63	0.73	0.83
	0.05	0.01	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81
	0.10	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
	0.15	-0.01	0.09	0.19	0.29	0.39	0.49	0.59	0.69	0.79
	0.20	-0.02	0.08	0.18	0.28	0.38	0.48	0.58	0.68	0.78
	0.25	-0.04	0.06	0.16	0.26	0.36	0.46	0.56	0.66	0.76
	0.30	-0.05	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75
	0.35	-0.06	0.04	0.14	0.24	0.34	0.44	0.54	0.64	0.74
	0.40	-0.07	0.03	0.13	0.23	0.33	0.43	0.53	0.63	0.73

Table 5-6			
Variation of average VSM values with f value *			
	f		
VSM	- 0.2	0	0.2
δC	0.277	0.222	0.179
T_{wp}	0.501	0.523	0.549
τ	0.225	0.301	0.369

* VSM's were averaged over the (π_v^0, π_w^0) grid for each f value

Chapter 6

Conclusions and Future Research

This chapter provides a synopsis of the findings of the research pursued in this thesis and the conclusions reached. It also outlines further expansions of the methods developed in the thesis. The main objective of this research is to explore novel ventures aimed at reducing costs of healthcare organizations by creating internal profit centers that compete directly with external vendors. The models comprise the hospital, its warehouse run as a profit center, and an external vendor. The hospital's warehouse and the vendor compete for the hospital's demand for a single product. The venture is assessed over a period of two years. A Bertrand-Nash game was developed to determine the year-1 price markups charged by the competing vendor and warehouse. A healthcare-relevant discount structure was enforced in the second year.

The research required creation of novel discount and pricing schedules that emulate discount practices that are unique to the healthcare industry. The models also relied on a somewhat innovative entropy-based demand split methodology. These methods were critical for computing the various cost and profit functions used in calculating the Venture-Success Metrics. These developments and their future expansions are discussed in the sections below.

6.1 Healthcare-relevant discount schedules based on previous year's purchases

Chapter 3 outlined the need for analytic discount schedules with continuous derivatives that reflect the commonplace practice of multi-tiered discounts. Our literature reviews of volume-based discounts revealed that typically-used discount schedules are either non-analytic with discontinuous and infinite derivatives, or analytic but lack the multi-tiered feature often required in discount schedules. We proposed sigmoid switching functions, routinely applied in engineering and other fields, to implement single-tiered and multi-tiered discount schedules.

The market relevance of the discount schedules represented by these functions coupled with the ready computability of all their derivatives, render them highly useful in the healthcare industry. These discount schedules provide one of the foundations for determining the viability of ventures in which healthcare organizations seize the reins of their procurement by evolving their warehouses to operate as mini distributors that compete with Owens & Minor, Cardinal Health, and the handful of other distributors that dominate the healthcare supplies market.

The discount schedules developed in this thesis can be extended and adapted to include;

1. Multiple items and item categories
2. Rebates and other discount paradigms (e.g., annual purchase commitments) heavily used in healthcare
3. Purchase history to compute discounts on *newly-introduced* products, based on prior year volumes of comparable older or discontinued products

6.2 Entropy-based demand splitting methodology

Chapter 4 introduced an entropy-based demand-split methodology to divide the hospital's demand among the vendor and warehouse. The EOQ-based demand-split methods proposed in the literature were not applied here due to their tactical nature and lack of generality. Logit-based demand splits, also proposed in the literature, have the same mathematical structure as the entropy-based demand split method for the limited case in which total cost is a constraint. Otherwise, logit-based methods cannot be extended to include more general business necessities such as enforcing purchases from one supplier as a function of purchases from another supplier. Such imperatives are frequently used to favor one supplier over another in order to meet contract requirements or other business objectives. Entropy-based demand-split methods are thus more general than prevalent state of the art methods and serve more strategic purposes. Entropy-based methods imply equitable splits among

suppliers, subject to constraints that enforce different business imperatives. The requirement of an “equitable-split” paradigm serves the business purpose of protecting supply sources, hence patient care, in a manner consistent with other business constraints. This technique was then incorporated in this thesis and applied in computing Bertrand-Nash equilibrium year-1 markups

The two business objectives introduced in this model as constraints were, the competing suppliers serve to satisfy the total demand, and the total cost is fixed. In practice, other business objectives (for example demand satisfied from supplier-1 must be twice the demand satisfied by supplier-2) can be enforced as constraints. The non-dependence of these split functions on market data to calculate effective demand-splits is of great value to new, as well as established business ventures when data are not readily available.

This model can be extended to;

1. Introduce stochastic demand. This is an example of how a simulation model of stochastic demand would be pursued –
 - a. Draw annual demand from a Poisson distribution
 - b. Compute VSM's as outlined in this thesis
 - c. Record the VSM's for that demand
 - d. Pick another demand from the same Poisson distribution and return to step b
 - e. Repeat for a statistically significant number of demands
 - f. Process the computed VSM's to determine their distribution parameters
2. Incorporate more items or item categories with various demand distributions
3. Integrate product complements along with the substitutes

6.3 Venture Success Metrics

Existing literature on healthcare inventory and warehouses exclusively deal with operational improvements of a warehouse without addressing profitability. Chapter 5 on the other hand defined the venture in which the hospital warehouse competes against an outside vendor with the objective of reducing costs. That chapter defined the Venture-Success Metrics used in assessing the venture's success or failure. The models required calculations of the various hospital, warehouse, and vendor's cost and profit functions which in turn rely on the discount schedules and demand split functions presented in the previous chapters.

Three Venture Success Metrics were used in Chapter 5:

- Cost reduction of the hospital
- Profitability of the warehouse
- Total profitability of the healthcare organization.

A demand-split bias was introduced to favor purchases from the warehouse. This relied on the intuitive notion that “if the hospital procures more of its supplies from the warehouse, the odds of venture success become higher”. The results showed that higher bias in favor of the warehouse leads to a less successful venture. This outcome is caused by higher bias which resulted in vendor and warehouse year-1 mark-ups when the bias is high. With that, it is necessary to find the optimum level of “bias” to make the venture a profit making center. The analytical models presented in this research allow the hospitals to adjust the “bias” according to their strategic objectives.

This work can be extended to include the following:

1. Larger number of competing suppliers. This would require updating the demand split functions to deal with multiple suppliers and their respective profit functions. It would also require updating the hospital's cost function to account for purchases from multiple suppliers.

2. Explore situations in which the manufacturer and vendor are the same entity. In this case the manufacturer-to-vendor shipping costs will be eliminated. Also, it will be interesting to find the warehouse's strategy as it will be competing against the manufacturer/vendor.
3. Expand the procured items to multiple products and multiple product categories. This expansion would require dealing with products that have different purchasing profiles such as large quantities of inexpensive items versus small quantities of expensive items. It would also include products with different inventory profiles such as perishables that have special shipping and storage requirements.
4. Integrate other constraints such as space and resource constraints, often very real in hospital warehouses.
5. In this research, we used Bertrand-Nash to determine the equilibrium price markups of the vendor and warehouse. However, an argument can be made to use Cournot-Nash to attain the equilibrium *quantity* demanded. Therefore a model can be designed to use Cournot-Nash in lieu of Bertrand-Nash to model the competition. The results of this exercise then can be evaluated using venture viability criteria developed in this thesis.

References

1. Abginehchi, S., Farahani, R. Z., & Rezapour, S. (2013). A mathematical model for order splitting in a multiple supplier single-item inventory system. *Journal of Manufacturing Systems*, 32(1), 55-67.
2. Aksoy-Pierson, M., Allon, G., & Federgruen, A. (2013). Price competition under mixed multinomial logit demand functions. *Management Science*, 59(8), 1817-1835.
3. Arts, J., and Kiesmüller, G. P. (2013). Analysis of a two-echelon inventory system with two supply modes. *European Journal of Operational Research*, 225(2), 263-272.
4. Bakker, M., Riezebos, J., and Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, 221(2), 275-284.
5. Basu, A., Mazumdar, T., and Raj, S. P. (2007). Components of optimal price under logit demand. *European journal of operational research*, 182(3), 1084-1106.
6. Bernstein, F., Chen, F., and Federgruen, A. (2006). Coordinating supply chains with simple pricing schemes: the role of vendor-managed inventories. *Management Science*, 52(10), 1483-1492.
7. Bertrand, J. (1988). Review of Walras's "Théorie Mathématique de la richesse sociale" and Cournot's "Recherches sur les principes mathématiques de la théorie des richesses", translated into English by James W. Friedman. *Cambridge University Press: Cambridge, UK*.
8. Chang, C. T. (2006). An acquisition policy for a single item multi-supplier system with real-world constraints. *Applied Mathematical Modelling*, 30(1), 1-9.
9. Chiang, C. (2001). Order splitting under periodic review inventory systems. *International Journal of Production Economics*, 70(1), 67-76.

10. Chinchuluun, A., Karakitsiou, A., and Mavrommati, A. (2008). Game theory models and their applications in inventory management and supply chain. In *Pareto Optimality, Game Theory And Equilibria* (pp. 833-865). Springer New York.
11. Chongchao, H., Gang, Y., Song, W., Xianjia, W. (2006), Disruption management for supply chain coordination with exponential demand function. *Acta Mathematica Scientia*, 26B(4), pp655-669.
12. Cournot, A. (1927: [1838]). “Researches into the Mathematical Principles of the Theory of Wealth,” translated by N. T. Bacon, 1897. New York: Macmillan.
13. Dai, T., and Qi, X. (2007). An acquisition policy for a multi-supplier system with a finite-time horizon. *Computers & operations research*, 34(9), 2758-2773.
14. Duan, Q., and Liao, T. W. (2013). A new age-based replenishment policy for supply chain inventory optimization of highly perishable products. *International Journal of Production Economics*, 145(2), 658-671. DOI: <http://dx.doi.org/10.1016/j.ijpe.2013.05.020>
15. Esmaeili, M., Aryanezhad, M. B., and Zeephongsekul, P. (2009). A game theory approach in seller–buyer supply chain. *European Journal of Operational Research*, 195(2), 442-448.
16. Feistra-Janeiro, M.G., Garcia-Jurado, I., Meca, A., and Mosquera, M.A., (2011), Cooperative game theory and inventory management, *European Journal of Operational Research*, 210, 459-466
17. Geng, Q., and Mallik, S. (2007). Inventory competition and allocation in a multi-channel distribution system. *European Journal of Operational Research*, 182(2), 704-729.
18. Ghareib, A. H. (1996). Evaluation of logit and probit models in mode-choice situation. *Journal of transportation engineering*, 122(4), 282-290.
19. Guerrero, W. J., Yeung, T. G., and Guéret, C. (2013). Joint-optimization of inventory policies on a multi-product multi-echelon pharmaceutical system with batching and ordering

- constraints. *European Journal of Operational Research*, 231(1), 98-108. DOI:
<http://dx.doi.org/10.1016/j.ejor.2013.05.030>
20. Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Savelsbergh, M. W. (2010). Vendor managed inventory for environments with stochastic product usage. *European Journal of Operational Research*, 202(3), 686-695.
 21. Hill, R. M. (1996). Order splitting in continuous review (Q, r) inventory models. *European Journal of Operational Research*, 95(1), 53-61.
 22. Hu, Q., Schwarz, L. B., and Uhan, N. A. (2012). The impact of group purchasing organizations on healthcare-product supply chains. *Manufacturing & Service Operations Management*, 14(1), 7-23.
 23. Huang, J., Leng, M. and Mahmut, P., (2013), Demand Functions in Decision Modeling: A comprehensive Survey and Research Directions. *Decision Sciences*, 44(3) pp557-609.
 24. Ke, G. Y., and Bookbinder, J. H. (2012). The optimal quantity discount that a supplier should offer. *Journal of the Operational Research Society*, 63(3), 354-367.
 25. Kelle, P., Woosley, J., and Schneider, H. (2012). Pharmaceutical supply chain specifics and inventory solutions for a hospital case. *Operations Research for Health Care*, 1(2), 54-63.
 26. Lau A.H. and Lau, H.S. (2003). Effects of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model. *European Journal of Operational Research*, 147, 530-548.
 27. Osborne, M. J. (2004). *An introduction to game theory* (Vol. 3, No. 3). New York: Oxford University Press.
 28. Marvel, H. P., & Yang, H. (2008). Group purchasing, nonlinear tariffs and oligopoly. *International Journal of Industrial Organization*, 26(5), 1090-1105.

29. Mercy ROi takes optimized supply chain all the way to patients' bedsides, <http://www.supplychainbrain.com/content/research-analysis/supply-chain-innovation-awards/single-article-page/article/mercy-roi-takes-optimized-supply-chain-all-the-way-to-patients-bedsides-1>, Retrieved on July 7th, 2014
30. Monahan, J. P. (1984). A quantity discount pricing model to increase vendor profits. *Management science*, 30(6), 720-726.
31. Murthy, A. N., and Ashtakala, B. (1987). Modal split analysis using logit models. *Journal of transportation engineering*, 113(5), 502-519.
32. Nagarajan, M., Sobic, G., (2008), Game-theoretic analysis of cooperation among supply chain agents: Review and Extensions. *European Journal of Operational Research*, 187, 719-745.
33. Osborne, M. J. (2004). *An introduction to game theory* (Vol. 3, No. 3, pp. 53-97). New York: Oxford University Press.
34. Perloff J. (2012), *Microeconomics* (6th ed.). Boston, MA: Addison Wesley.
35. Ramasesh, R. V., Ord, J. K., Hayya, J. C., and Pan, A. (1991). Sole versus dual sourcing in stochastic lead-time (s, Q) inventory models. *Management science*, 37(4), 428-443.
36. Rosenblatt, M. J., Herer, Y. T., and Hefter, I. (1998). Note. An acquisition policy for a single item multi-supplier system. *Management Science*, 44(11-part-2), S96-S100.
37. Schotanus, F., Telgen, J., and de Boer, L. (2009). Unraveling quantity discounts. *Omega*, 37(3), 510-521.
38. Shamir, N., (2012). Strategic information sharing between competing retailers in a supply chain with a endogenous wholesale price. *International Journal of Production Economics*, 136(2), 352-365.

39. Shenoy, P. P. (1980). A two-person non-zero-sum game model of the world oil market. *Applied Mathematical Modelling*, 4(4), 295-300.
40. Sinha, S., and Sarmah, S. P. (2010). Coordination and price competition in a duopoly common retailer supply chain. *Computers & Industrial Engineering*, 59(2), 280-295.
41. Stanger, S. H., Wilding, R., Yates, N., and Cotton, S. (2012). What drives perishable inventory management performance? Lessons learnt from the UK blood supply chain. *Supply Chain Management: An International Journal*, 17(2), 107-123.
42. Thomas, D. J., and Tyworth, J. E. (2006). Pooling lead-time risk by order splitting: a critical review. *Transportation Research Part E: Logistics and Transportation Review*, 42(4), 245-257.
43. Uddin, M. F. and Sano, K. (2012), Coordination and Optimization: The integrated supply chain analysis with non-linear price-sensitive demand. *An international journal of optimization and control: Theories and Applications* 2(1), 83-94.
44. UPS Solutions, Pain in the (supply) Chain Healthcare Survey
<http://www.ups.com/content/ca/en/bussol/browse/industries/pain-in-the-chain.html>,
Retrieved, June 29th, 2014
45. Uthayakumar, R., and Priyan, S. (2013). Pharmaceutical supply chain and inventory management strategies: Optimization for a pharmaceutical company and a hospital. *Operations Research for Health Care*, 2(3), 52-64.
46. Winston, W.L. Operations Research Applications and Algorithms, (4th Ed), Brook/Cole Cengage Learning, 2004
47. Wilson, A. G. (1967). A statistical theory of spatial distribution models. *Transportation research*, 1(3), 253-269.

48. Wilson, A. G. (1969). The use of entropy maximising models, in the theory of trip distribution, mode split and route split. *Journal of Transport Economics and Policy*, 108-126.
49. Xu, M., Qi, X., Yu G., Zhang, H., and Gao C (2003), The demand disruption management problem for a supply chain system with non-linear demand functions. *Journal of Systems Science and Systems Engineering*, 12(1), pp82-97.
50. Zhang, G. (2010). The multi-product newsboy problem with supplier quantity discounts and a budget constraint. *European Journal of Operational Research*, 206(2), 350-360.
51. Zhang, W., Fu, J., Li, H. and Xu, W., (2012). Coordination of supply chain with a revenue-sharing contract under demand disruptions when retailers compete. *International Journal of Production Economics*, 138(1), 68-75.