Scheduling in omnidirectional relay wireless networks

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The capacity of multiuser wireless network, unclear for many years, has always been a hot research topic. Many different operation schemes and coding techniques have been proposed to enlarge the achievable rate region. And omnidirectional relay scheme is one of them.

This thesis mainly works on the achievable region of the all-source all-cast network with omnidirectional relay scheme. In order to better understand this problem, we first describe the half-duplex model on the one-dimensional and two-dimensional regular networks. And we present an optimal operation scheme for them to have the maximum achievable rate. For the one-dimensional general network, we proposed an achievable region that indicates valued improvement compared to the previous results. In the full-duplex model of the one-dimensional general network, the maximum achievable rate is presented with a simpler proof in comparison with the previous results. In this thesis, we also show some discussions on more general networks.

Acknowledgements

Firstly, I would like to express the deepest appreciation to my supervisor Prof. Liang-Liang Xie, who guided me into the world of academics, a world full of fun. His continuous support and encouragement motivated me to make further progress. I think I will always remember and appreciate the precious two-year study here.

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Thirdly, I would like to express my sincere thanks to my dear friends. I really appreciate your accompany and care that bring me happiness and help me a lot.

And thanks also goes to my family for your love and support.

Dedication

To my parents.

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Chapter 1

Introduction

C. E. Shannon's landmark paper "A mathematical theory of communication" [23] has laid a solid theoretical foundation for information theory. In this paper, he introduced the basic mathematical concepts and essential models for communication systems. Among them, the introduction of the concept "capacity" was one of the most significant accomplishments. It showed that the capacity was the maximum limit of the transmission rates when messages are being transmitted through a communication channel.

For the single user channel, i.e. point-to-point channel, the capacity was determined, $C = \max_{p(x)} I(X;Y)$, where X denotes the input symbol of the channel and Y denotes the output symbol of the channel[23]. For the multi-user channels, the capacity region, i.e. the optimal set of rates at which the nodes can communicate reliably with each other, is still not completely answered. Actually, the nature of the multi-user channel is composed of multiple access channel[14, 1], broadcast channel [4, 15, 6, 5], relay channel[2, 7, 21] and many different coding schemes and techniques. Different from the single user channel, interference and cooperation occur in the multi-user channel and it is also them that motivates many coding schemes.

[9] developed node cooperation and studied two basic relay schemes: decode-forward scheme and compress-forward scheme. In the decode-forward scheme, as it is named, the relay helps by decoding the source's messages and then forwards to the destination. In terms of decoding, there are several main decoding techniques: irregular encoding, successive decoding[2], regular encoding, sliding-window decoding[12], regular encoding, backward decoding[12]. In the compress-forward scheme, the relay helps by compressing the source's messages and then forwards them to the destination. The compress-forward scheme is similar to the source coding with side information, thus Slepian-Wolf coding [25]

and Wyner-Ziv[29] coding can be applied. Just for the relay channel, with so many coding schemes mentioned and unmentioned, its capacity has not been completely determined.

The once hot research topic network coding inspired by computer network applications was first introduced in [18]. It has great contribution in enlarging the achievable rate when extended to multiuser channel coding schemes. [24] showed that by linear network coding, the optimal individual max-flow bound can be achieved at each receiving node. In [13], an algebraic framework was further presented for the capacity solutions of network problems. [27, 26] proposed noisy networking coding which combines network coding and compress-forward for the noisy relay channel.

Two-way communication channel is a simple and classical element of the multi-user channel. It was first studied in [22] in which the lower and upper bound of its capacity was found. [10] further shown the general coding scheme with which independent encoders can achieve the inner bound of the capacity region. Adding one relay in the two-way channel, [19] made a study on the two-way relay channel. Adding multiple relays in the channel, [35, 34, 36, 28] made analyses on the two-way multi-relay channel from different perspectives.

Following the idea of network coding, the technique of random binning[25] was investigated in [30] to show its advantage in the certain multiuser channels. Random binning scheme, which will be shown in later chapter, is widely used in encoding of correlated sources. In this thesis, we focus on the application of random binning technique in the omnidirectional relay networks.

In the omnidirectional relay networks, each node relays messages in many different directions. Here, a combination of random binning and decode-forward relay is utilized in each node. This combining scheme was first introduced in [30] and then had a full development in [31, 32] for more general networks. In the omnidirectional relay networks, when each node is an independent source to be transmitted to all the other nodes, then here comes the all-source all-cast problem[31, 32]. The combining scheme has much benefit in the all-source all-cast problem because it can cancel out all the interferences eventually.

1.1 Problem and Motivation

Consider the following wireless network as shown in figure 1.1, each round dot denoting a node that can transmit, relay and receive signals. In figure 1.1, each node is an independent source to be transmitted to all the other nodes with the omnidirectional relay scheme. What is the achievable rate? This is all-source all-cast problem. [8] shows the capacity of

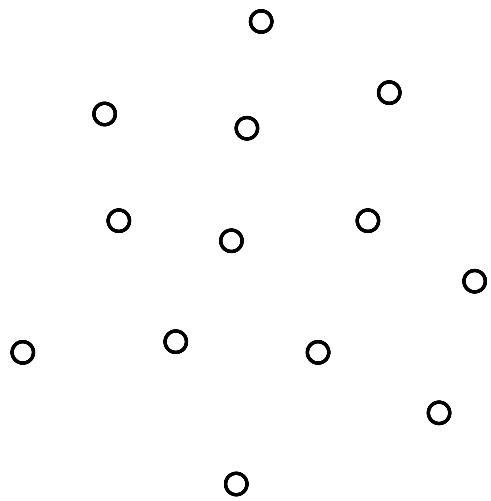

Figure 1.1: general wireless network

the graphical multi-source, multi-cast network when the sets of destination nodes are the same for every source. In the all-source all-cast problem, the sets of destination nodes are not the same for each source.

As the study of [16] and [33], we find that the achievable rate region in [16] outperforms that in [33] when the model in [33] is reduced the same model in [16]. This means that the greedy omnidirectional relay scheme proposed in [33] is not optimal. Then what kind of scheduling of the omnidirectional relay can achieve better rates? With this question, we explored from the regular networks in half-duplex mode to discover a scheduling rule for the optimal achievable rates.

1.2 Thesis Outline

The content of this thesis is organized as follows:

In Chapter 2, we will introduce some fundamental background information in information theory used throughout this thesis. First, some basic concepts on the channel capacity are given. Then, some relay networks and the core problem "all-source all-cast problem" are presented. Also, the random binning technique with the idea of network coding is well explained within the presentation of some relay networks. At the end of Chapter 2, we will review some previous results on the all-source all-cast problem with the omnidirectional relay scheme. And the primary motivation of taking on this research project is stated.

In Chapter 3, we start from the 1-dimensional and 2-dimensional regular network in the half-duplex mode. Then we propose the achievable regions for some general networks followed by the proofs for them.

Some Matlab simulations are given in Chapter 4 to clearly present the advantage of the omnidirectional relay scheme.

Chapter 5 concludes this thesis and states about the future work.

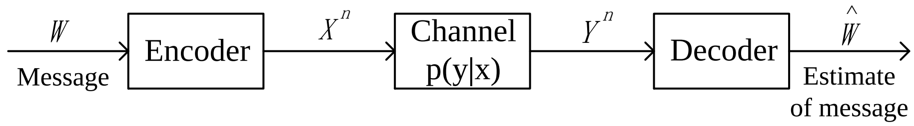

Figure 2.1: A communication channel

Chapter 2

Preliminaries

In this chapter, we will introduce some basic concepts and fundamental theorems in information theory. Some of the formal concepts and theorems following are referred from [3].

2.1 Channel Capacity

We now introduce some related concepts formally in a communication channel shown in figure 2.1.

Definition 1 *A discrete channel, denoted by* $(\mathcal{X}, p(y|x), \mathcal{Y})$*, consists of two finite sets* $\mathcal{X}$ *and* $\mathcal{Y}$ *and a collection of probability mass functions* $p(y|x)$*, one for each* $x \in \mathcal{X}$*, such that for every* x *and* y*,* $p(y|x) \geq 0$*, and for every* x*,* $\sum_y p(y|x) = 1$*, with the interpretation that* X *is the input and* Y *is the output of the channel. The channel is said to be memoryless if the probability distribution of the output depends only on the input at that time and is conditionally independent of previous channel inputs or outputs.*

Definition 2 *The* n*th extension of the discrete memoryless channel (DMC) is the channel* $(\mathcal{X}^n, p(y^n|x^n), \mathcal{Y}^n)$*, where*

$$p(y_k|x^k, y^{k-1}) = p(y_k|x_k), k = 1, 2, \ldots, n.$$

Definition 3 *An* (M, n) *code for the channel* $(\mathcal{X}, p(y|x), \mathcal{Y})$ *consists of the following:*

1. *An index set* $\{1, 2, \ldots, M\}$*.*

2. *An encoding function* $X^n : \{1, 2, \ldots, M\} \to \mathcal{X}^n$*, yielding codewords* $X^n(1), X^n(2), \ldots, X^n(M)$*. The set of codewords is called the codebook.*

3. *A decoding function*

$$g : \mathcal{Y}^n \to \{1, 2, \ldots, M\},$$

which is a deterministic rule which assigns a guess to each possible received vector.

Definition 4 *Probability of error: Let*

$$\lambda_i = Pr(g(Y^n) \neq i | X^n = X^n(i)) = \sum_{y^n} p(y^n|x^n(i)) I(g(y^n) \neq i)$$

be the conditional probability of error given that index i *was sent, where* $I(\cdot)$ *is the indicator function.*

Definition 5 *The maximal probability of error* $\lambda^{(n)}$ *for an* (M, n) *code is defined as*

$$\lambda^{(n)} = \max_{i \in \{1,2,\ldots,M\}} \lambda_i.$$

Definition 6 *The rate* R *of an* (M, n) *code is*

$$R = \frac{\log M}{n} \text{ bits per transmission.}$$

Definition 7 *A rate* R *is said to be achievable if there exists a sequence of* $(\lceil 2^{nR} \rceil, n)$ *codes such that the maximal probability of error* $\lambda^{(n)}$ *tends to 0 as* $n \to \infty$*. We will write* $(2^{nR}, n)$ *codes to mean* $(\lceil 2^{nR} \rceil, n)$ *codes.*

Definition 8 *The capacity of a discrete memoryless channel is the supremum of all achievable rates.*

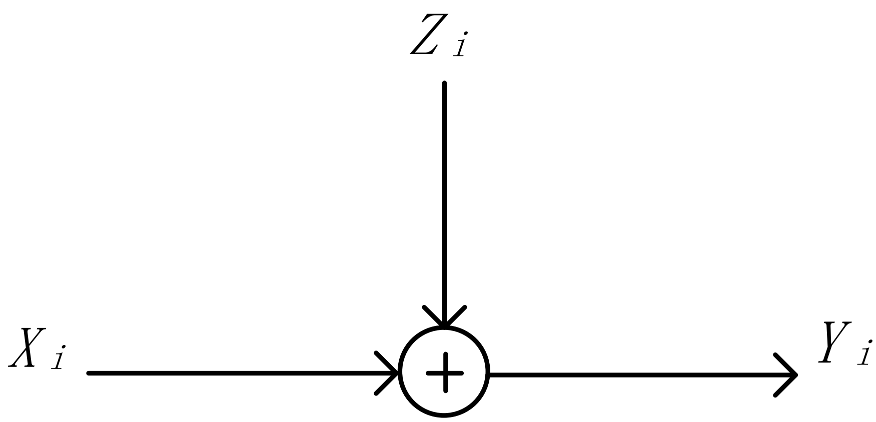

Figure 2.2: The Gaussian Channel

Theorem 1 *The channel coding theorem: All rates below capacity C are achievable. Specifically, for every rate $R < C$, there exists a sequence of $(2^{nR}, n)$ codes with maximum probability of error $\lambda^{(n)} \to 0$.*

Conversely, any sequence of $(2^{nR}, n)$ codes with $\lambda^{(n)} \to 0$ must have $R \leq C$.

Definition 9 *We define the "information" channel capacity of a discrete memoryless channel as*

$$C = \max_{p(x)} I(X;Y), \tag{2.1}$$

where the maximum is taken over all possible input distributions p(x).

2.2 The Gaussian Channel

The Gaussian channel depicted in Figure 2.2 is the most important continuous alphabet channel. It is a time discrete channel with output Y_i at time i, where Y_i is the sum of the input X_i and the noise Z_i. The noise Z_i is drawn i.i.d. from a Gaussian distribution with variance N. Thus

$$Y_i = X_i + Z_i, \qquad Z_i \sim \mathcal{N}(0, N) \tag{2.2}$$

The noise Z_i is assumed to be independent of the signal X_i. We assume an average power constraint on the input X. For any codeword $(x_1, x_2, \ldots, x_n)$ transmitted over the channel, we require

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 \leq P$$

Definition 10 *(Capacity of Gaussian channel)*

The information capacity of the Gaussian channel with power constraint P is

$$C = \max_{p(x):EX^2 \leq P} I(X;Y). \tag{2.3}$$

We can calculate the information capacity as follows: Expanding $I(X;Y)$, we have

$$\begin{aligned} I(X;Y) &= h(Y) - h(Y|X) \\ &= h(Y) - h(X+Z|X) \\ &= h(Y) - h(Z|X) \\ &= h(Y) - h(Z), \end{aligned} \tag{2.4}$$

Since Z is independent of X. Now, $h(Z) = \frac{1}{2}\log(2\pi e N)$. Also,

$$\begin{aligned} EY^2 &= E(X+Z)^2 \\ &= EX^2 + 2EXEZ + EZ^2 \\ &= P + N, \end{aligned} \tag{2.5}$$

since X and Z are independent and $EZ = 0$. Given $EY^2 = P + N$, the entropy of Y is bounded by $\frac{1}{2}\log 2\pi e(P+N)$.

So we obtain

$$\begin{aligned} I(X;Y) &= h(Y) - h(Z) \\ &\leq \frac{1}{2}\log 2\pi e(P+N) - \frac{1}{2}\log 2\pi e N \\ &= \frac{1}{2}\log(1 + \frac{P}{N}), \end{aligned} \tag{2.6}$$

Hence the information capacity of the Gaussian channel is

$$C = \max_{EX^2 \leq P} I(X;Y) = \frac{1}{2}\log(1 + \frac{P}{N}),$$

and the maximum is attained when $X \sim \mathcal{N}(0, P)$.

Definition 11 *A (M, n) code for the Gaussian channel with power constraint P consists of the following:*

1. An index set $\{1, 2, \ldots, M\}$.

2. An encoding function $x : \{1, 2, \ldots, M\} \to \mathcal{X}^n$, yielding codewords $x^n(1), x^n(2), \ldots, x^n(M)$, satisfying the power constraint P, i.e., for every codeword

$$\sum_{i=1}^{n} x_i^2(w) \leq nP, \qquad w = 1, 2, \ldots, M. \tag{2.7}$$

3. A decoding function

$$g : \mathcal{Y}^n \to \{1, 2, \ldots, M\},$$

Definition 12 *A rate R is said to be achievable for a Gaussian channel with a power constraint P if there exists a sequence of $(2^{nR}, n)$ codes with codewords satisfying the power constraint such that the maximal probability of error λ^n tends to zero. The capacity of the channel is the supremum of the achievable rates.*

Theorem 2 *The capacity of a Gaussian channel with power constraint P and noise variance N is*

$$C = \frac{1}{2}\log(1 + \frac{P}{N}) \text{ bits per transmission} \tag{2.8}$$

2.3 Omnidirectional Relay Networks

In the omnidirectional relay networks where there are multiple sources, a combination of random binning and the decode-and-forward relay strategy is utilized at each node. Before investigating the omnidirectional relay networks, some simple relay networks are to be presented.

2.3.1 The Relay Channel

The relay channel[7], introduced by Van Der Meulen in 1971, is a communication model between a sender and a receiver with the help of one or more relay nodes. As shown in figure 2.3, the relay channel combines a broadcast channel (X to Y and Y_1) and a multiple access channel (X and X_1 to Y).

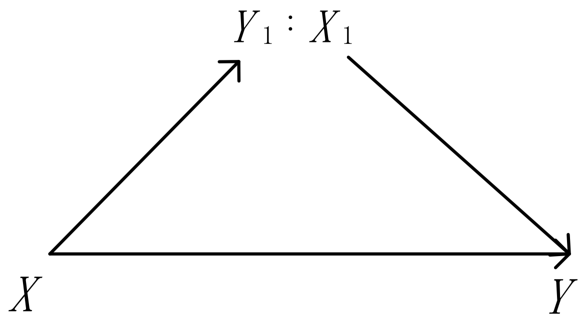

Figure 2.3: the relay channel

X	$X(W_1)$	$X(W_2)$	$X(W_3)$	$\ldots$	$X(W_B)$
X_1	$X_1(1)$	$X(\widehat{W_1})$	$X(\widehat{W_2})$	$\ldots$	$X(\widehat{W_B})$
Y	$Y(1)$	$Y(2)$	$Y(3)$	$\ldots$	$Y(B)$

Table 2.1: Block Markov Coding

Definition 13 *A* $(2^{nR}, n)$ *code for a relay channel consists of a set of integers* $\mathcal{W} = \{1, 2, \ldots, 2^{nR}\}$*,an encoding function*

$$X : \{1, 2, \ldots, 2^{nR}\} \to \mathcal{X}^n,$$

a set of relay functions $\{f_i\}_{i=1}^n$ *such that* $x_{1i} = f_i(Y_{11}, Y_{12}, \ldots, Y_{1i-1})$*,* $1 \leq i \leq n$*, and a decoding function,*

$$g : \mathcal{Y}^n \to \{1, 2, \ldots, 2^{nR}\}$$

.

- Decode-Forward Relay

  In the decode-forward relaying scheme[9], the relay decodes the source message in one block and transmits the re-encoded message in the following block. Block Markov Coding, one of the coding strategies that can be applied in the decode-forward relaying schemes, is shown in the table 2.1 to better demonstrate decode-forward relay scheme.

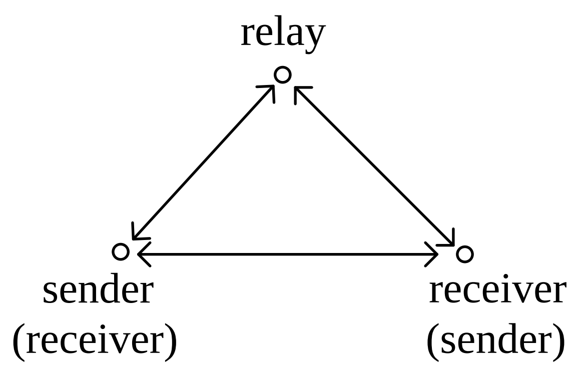

Figure 2.4: two-way relay channel

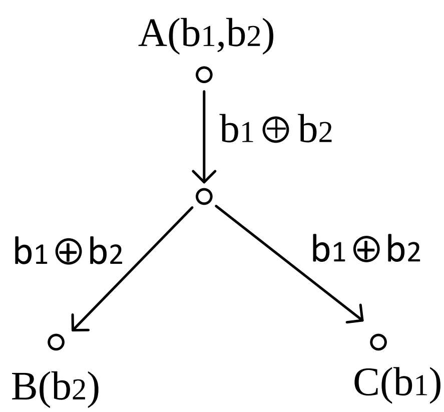

Figure 2.5: The idea of network coding

2.3.2 Two-Way Relay Channel

As shown in figure 2.4, when the receiver is also a sender and the sender is also a receiver, then the one-way relay channel becomes the two-way relay channel[19]. The relay needs to help to transmit the messages of two nodes at the same time. Binning Technique with the idea of network coding is applied in the coding scheme to achieve higher rates.

Network Coding and Random Binning for Multi-User Channels

- The idea of Network Coding

  The idea of network coding, first introduced in [18], can be generalized into the simple

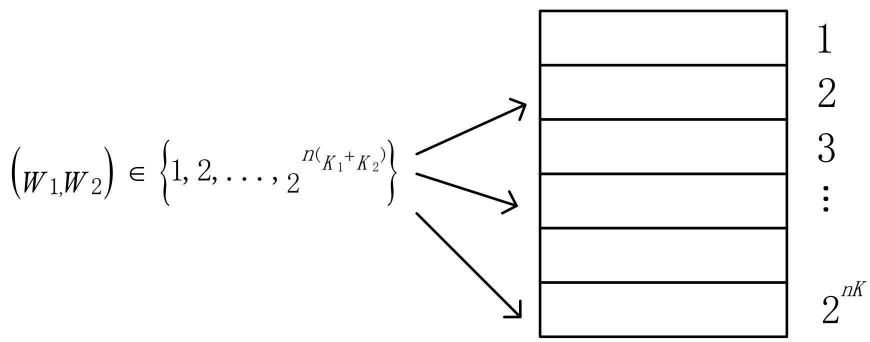

Figure 2.6: Random Binning Technique

network[30] in figure 2.5. In figure 2.5, node A wants to send two bits of information b_1 and b_2 to node B and node C respectively. When node B knows b_2 and node C knows b_1, node A only needs to send one bit of information $b_1 \bigoplus b_2$ to node B and node C instead of sending two bits of information. Node B can get b_1 by calculating $(b_1 \bigoplus b_2) \bigoplus b_2 = b_1$, and node C can get b_2 by calculating $(b_1 \bigoplus b_2) \bigoplus b_1 = b_2$. Obviously, there is an advantage of one bit less by using network coding.

- Random Binning Technique

  Following the idea of network coding, the random binning technique[25] achieves less bits transmission with the help of side information at the receivers. For example,suppose node A wants to send two messages w_1 and w_2 to node B and node C respectively, where w_1 takes K_1 different values and w_2 takes K_2 different values as shown in figure 2.6. By randomly throwing $K_1 \times K_2$ different vectors of (w_1, w_2) into $K \geq \max\{K_1, K_2\}$ bins instead of $K_1 \times K_2$ bins, node A only needs to send out messages of K different values. As long as $K \geq \max\{K_1, K_2\}$, the probability for two vectors containing the same w_1 or w_2 to be at the same bin is arbitrarily small as the transmission goes on. Here, the random binning technique realizes the idea of network coding.

2.3.3 Three-Way Relay Channel

When the relay in the two-way relay channel has its own message to send, it becomes the three-way relay channel. In the three-way relay channel, as shown in the figure 2.7, each node sends its messages to the other two nodes and acts as relay and receiver simultaneously. Decode-forward relay and random binning technique are applied at each node. That

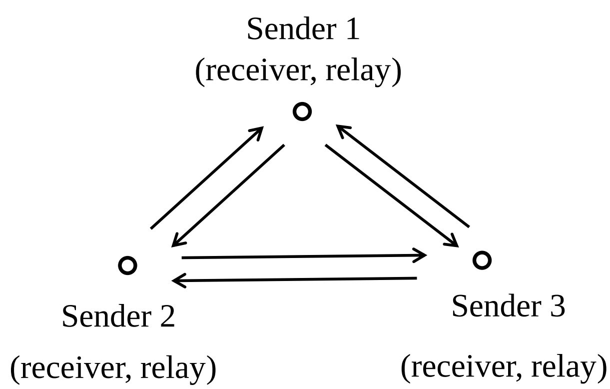

Figure 2.7: Three-Way Relay Channel

means, in this channel, each node needs to randomly throw vectors (w_1, w_2, w_3) containing the messages of three nodes into the bin.

2.3.4 Omnidirectional Relay Networks

Following the three-way relay channel, when there are more nodes located in different directions and each node needs to relay messages for all the other nodes, as shown in figure 1.1, then it becomes the omnidirectional relay networks. Furthermore, in the omnidirectional relay network, when each node is an independent source to be transmitted to all the other nodes, then here comes the all-source all-cast problem.

Omnidirectional relay shows great coding advantages in the all-source all-cast problem. Since each node can decode all the other nodes in the network, the all interferences received by each node will be cancelled out eventually and all signals being transmitted will be useful.

2.3.5 Previous Results

Consider a wireless network of n nodes $\mathcal{N} = \{1, 2, ..., n\}$.

We use the standard AWGN multiple access wireless channel model as the following:

$$Y_j(t) = \sum_{i \in \mathcal{N}, i \neq j} g_{i,j} X_i(t) + Z_j(t), \forall j \in \mathcal{N}, t = 1, 2, ... \tag{2.9}$$

where, $X_i(t) \in \mathbb{C}^1$ and $Y_i(t) \in \mathbb{C}^1$ respectively denote the signals transmitted and received by node $i \in \mathcal{N}$ at time t; $g_{i,j} \in \mathbb{C}^1 : i \neq j$ denotes the signal attenuation gains; and $Z_i(t)$ is zero-mean complex Gaussian noise with variance N.

The Key Technical Lemma[31, 32]

For the multiple access channel (2.9), with each source $i \in \mathcal{N}$ sending a message w_i at rate R_i with power P_i, there always exists some nonempty subset of $\{w_1, w_2, \ldots, w_n\}$ that can be decoded, as long as the following inequality holds:

$$\sum_{i \in \mathcal{N}} R_i < \log(1 + \frac{\sum_{i \in \mathcal{N}} |g_{i,j}|^2 P_i}{N}) \tag{2.10}$$

A Greedy Operation of Omnidirectional Relay[33]

Every node decodes as many messages as possible, and in the next block, relays all of them, with the restriction of adding at most one new message for each source.

All-Source All-Cast Problem

Consider a network of n nodes $\mathcal{N} = \{1, 2, ..., n\}$, with the channel modeled by

$$(\mathcal{X}_1 \times \cdots \times \mathcal{X}_n, p(y_1, \ldots, y_n | x_1, \ldots, x_n), \mathcal{Y}_1 \times \cdots \times \mathcal{Y}_n).$$

At each time $t = 1, 2, \ldots$, every node $i \in \mathcal{N}$ sends an input $X_i(t) \in \mathcal{X}_i$, and receives an output $Y_i(t) \in \mathcal{Y}_i$, and they are related via $p(Y_1(t), \ldots, Y_n(t) | X_1(t), \ldots, X_n(t))$.

Theorem 3 *With the greedy omnidirectional relay scheme, a rate vector* $(R_1, R_2, \ldots, R_n)$ *is achievable if for any nonempty subset* $\mathcal{S} \subset \mathcal{N}$*, there is a node* $i_0 \in \mathcal{S}$*, such that*

$$\sum_{j \in \mathcal{S}^c} R_j < I(X_{\mathcal{S}^c}; Y_{i_0} | X_{\mathcal{S}}) \tag{2.11}$$

for some $p(x_1)p(x_2)\cdots p(x_n)$*, where* $X_{\mathcal{S}^c} = \{X_j : j \in \mathcal{S}^c\}$*, and* $X_{\mathcal{S}} = \{X_i : i \in \mathcal{S}\}$*.*

Theorem 3 in [33] means that in figure 2.8, for any cut in the network, there is a node i_0 on the part of $\mathcal{S}$ that can decode at least one node on the part of $\mathcal{S}^c$.

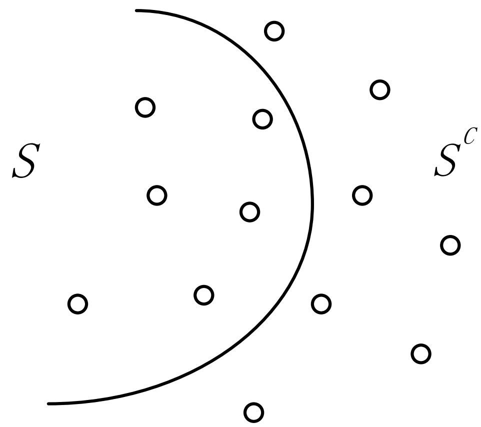

Figure 2.8: All-Source All-Cast Problem

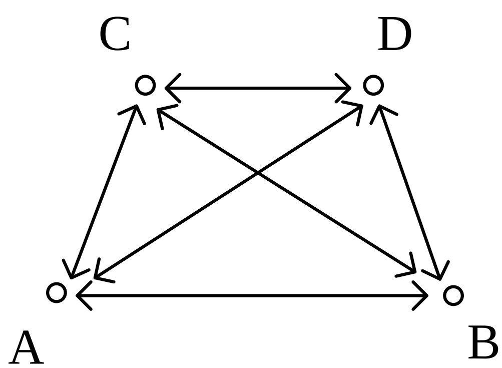

Figure 2.9: Two-way Two-relay channel

Two-way Two-relay channel

[16] proposed an achievable rate region for the two-way two-relay channel as shown in figure 2.9. In this channel, node A and node B work as nodes that have their messages to send, and node C and node D work as relays that help to transmit messages. The two-way two-relay channel can be seen as the four-source all-source all-cast problem with the rate for source C and source D equals 0.

The achievable rate region proposed in [16] shows improvement compared to that achieved by theorem 3, which indicates that greedy omnidirectional relay scheme is not optimal in the all-source all-cast problem. Therefore, we take on the research on the achievable rate region of all-source all-cast problem to discover more improvement.

Chapter 3

Scheduling in Omnidirectional Relay Networks

3.1 Scheduled Half-Duplex Networks

In the half-duplex networks, for simplicity, we consider a special model where the total time is divided into blocks of equal length and in each block, only one node is allowed to transmit. This may be a reasonable choice in practice when power is the most precious resource in the network, such as some sensor networks, so that bits per unit power rather than the rate itself (bits per second) becomes the key performance measure.

Consider a wireless network of n nodes $\mathcal{N} = \{1, 2, ..., n\}$.

We use the standard AWGN wireless channel model as the following:

$$Y_j(t) = \sum_{i \in \mathcal{N}, i \neq j} g_{i,j} X_i(t) + Z_j(t), \forall j \in \mathcal{N}, t = 1, 2, ... \tag{3.1}$$

where, $X_i(t) \in \mathbb{C}^1$ and $Y_i(t) \in \mathbb{C}^1$ respectively denote the signals transmitted and received by node $i \in \mathcal{N}$ at time t; $g_{i,j} \in \mathbb{C}^1 : i \neq j$ denote the signal attenuation gains; and $Z_i(t)$ is zero-mean complex Gaussian noise with variance N. Note that we are considering the special model where only one node is transmitting in any block. So for every t, there can only be one $i \in \mathcal{N}$ such that $X_i(t)$ is not zero.

We make a general assumption on the signal attenuation following [31, 32]. We assume that there is a non-increasing function to relate the magnitude of the gains in (3.1) to the distance:

$$|g_{i,j}| = g(d_{i,j}), \tag{3.2}$$

where $d_{i,j}$ is the distance between node i and node j, and $g(\cdot)$ is some non-increasing function. We consider the all-source all-cast problem, in which each node transmits its messages to all the other nodes. The following definition is from [31, 32].

Definition 14 *For each node i, define a set of nodes in its neighborhood as its 1-hop neighbors, and denote the set as $\mathcal{N}_{i(1)}$. If node j is a 1-hop neighbor of node i, it is said that j can reach i in one hop. If furthermore, i is a 1-hop neighbor of node l, then it is said that j can reach l in two hops. Similarly, it can be said that a node can reach another node in k hops, for any positive integer k. Now, for each node i, its k-hop neighbors is defined as the set of nodes that can reach it in k hops, but not in any less hops, and denote this set as $\mathcal{N}_{i(k)}$. Mathematically, $\mathcal{N}_{i(k)}$ can be sequentially defined as*

$$\begin{aligned}\mathcal{N}_{i(k)} =& \{j : j \in \mathcal{N}_{l(1)} \textit{ for some } l \in \mathcal{N}_{i(k-1)},\\ & \textit{and } j \notin i \cup \mathcal{N}_{i(1)} \cup \ldots \cup \mathcal{N}_{i(k-1)}\}\end{aligned}$$

It is clear that for any network of a finite number of nodes, there is a finite number L_i for each $i \in \mathcal{N}$, such that $\mathcal{N}_{i(k)} = \emptyset$ for $k > L_i$.

We operate the network in terms of rounds of blocks. As shown in figure 3.1, every round consists of n blocks, so that each node can use one block to transmit in every round. In the first round of n blocks, each node i transmits its own message $w_i(1)$, for any $i \in \mathcal{N}$. In the second round of n blocks, each node i transmits its own message $w_i(2)$, plus the first-block message of its one-hop neighbor $w_{\mathcal{N}_{i(1)}}(1)$, where $w_{\mathcal{N}_{i(1)}}(1)$ stands for $\{w_j(1) : j \in \mathcal{N}_{i(1)}\}$ for simplicity. So in general, in the r-th round, each node i transmits $\{w_i(r), w_{\mathcal{N}_{i(1)}}(r-1), \ldots, w_{\mathcal{N}_{i(r-1)}}(1)\}$. In the above operation scheme, the order of transmission by the n nodes in each round can be arbitrary. This flexibility of ordering the transmissions will not affect the achievable rate as we will show in the proof.

3.1.1 The 1-dimensional regular network

In this section, we consider the 1-dimensional regular network where all nodes are evenly spaced with distance d_0, as shown in figure 1. For this network, we define the 1-hop neighbors for each node i as $\mathcal{N}_{i(1)} = \{i-1, i+1\}$, for $2 \le i \le n-1$; and $\mathcal{N}_{1(1)} = \{2\}, \mathcal{N}_{n(1)} = \{n-1\}$. In general, obviously, the k-hop neighbors for each node i is defined

1st round	$w_1(1)$	$w_2(1)$	$w_3(1)$	$\cdots$	$w_n(1)$
2nd round	$w_1(2), w_{\mathrm{N}_{1(1)}}(1)$	$w_2(2), w_{\mathrm{N}_{2(1)}}(1)$	$w_3(2), w_{\mathrm{N}_{3(1)}}(1)$	$\cdots$	$w_n(2), w_{\mathrm{N}_{n(1)}}(1)$
rth round	$w_1(r)$ $w_{\mathrm{N}_{1(1)}}(r-1)$ $\cdots$ $w_{\mathrm{N}_{1(r-1)}}(1)$	$w_2(r)$ $w_{\mathrm{N}_{2(1)}}(r-1)$ $\cdots$ $w_{\mathrm{N}_{2(r-1)}}(1)$	$w_3(r)$ $w_{\mathrm{N}_{3(1)}}(r-1)$ $\cdots$ $w_{\mathrm{N}_{3(r-1)}}(1)$	$\cdots$	$w_n(r)$ $w_{\mathrm{N}_{n(1)}}(r-1)$ $\cdots$ $w_{\mathrm{N}_{n(r-1)}}(1)$

Figure 3.1: 1-dimensional regular network

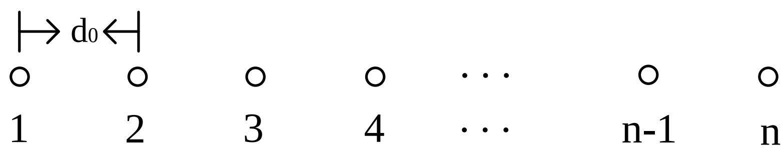

Figure 3.2: 1-dimensional regular network

as $\mathcal{N}_{i(k)} = \{i-k, i+k\}$, as long as $i-k \geq 1$ and $i+k \leq n$. For simplicity, the same transmit power constraint P is assumed for all the nodes. Thus, $|g_{i,j}|^2P$ is the corresponding received power at another node j when a node i is transmitting at its full power. Assume all nodes are transmitting at a common rate R. Therefore, when a node is transmitting at its full power P, the corresponding received power at its k-hop neighbors is $|g(kd_0)|^2P$, denoted as P_k.

Theorem 4 *For the half-duplex 1-dimensional regular network, with the operation scheme stated as above, the following average rate is achievable for all the sources.*

$$R < \frac{1}{n} \cdot \frac{1}{n-1} \sum_{i=1}^{n-1} \log(1 + \frac{P_i}{N}) \tag{3.3}$$

Obviously, the rate (3.3) is the maximum common rate based on the total power received by node 1 or node n.

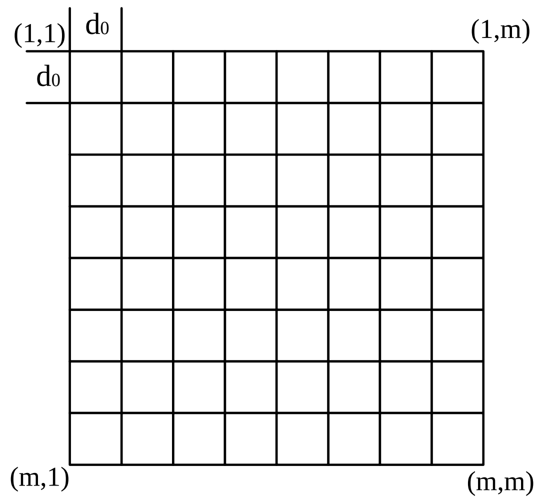

Figure 3.3: 2-dimensional regular network

3.1.2 The 2-dimensional regular network

In this section, we consider the 2-dimensional regular network composed of $m \times m$ nodes where all nodes are evenly space with distance d_0, as shown in figure 3.3. Each node can be represented by its coordinates (i, j) for any $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, m$. For this network, similarly, we define the 1-hop neighbors for each node (i, j) as $\mathcal{N}_{(i,j)(1)} = \{(i-1, j), (i+1, j), (i, j-1), (i, j+1)\}$ or equivalently, $\mathcal{N}_{(i,j)(1)} = \{(i_1, j_1) : |i_1 - i| + |j_1 - j| = 1\}$; and therefore, the k-hop neighbors for node (i, j) is $\mathcal{N}_{(i,j)(k)} = \{(i_1, j_1) : |i_1 - i| + |j_1 - j| = k\}$, as long as these coordinates are valid, i.e., representing any nodes. For simplicity, the same transmit power constraint P is assumed for all the nodes. Thus, $|g_{i,j}|^2 P$ is the corresponding received power at another node j when a node i is transmitting at its full power. Assume all nodes are transmitting at a common rate R. When a node (i, j), is transmitting at its full power P, the corresponding received power at node $(i \pm k, j \pm l)$, is $|g(\sqrt{k^2 + l^2} d_0)|^2 P$, denoted as $P_{k,l}$.

Theorem 5 *For the half-duplex 2-dimensional regular network, with the operation scheme as stated above, the following rate is achievable for all the sources.*

$$R < \frac{1}{m^2} \cdot \frac{1}{m^2 - 1} \sum_{i=0}^{m-1} \sum_{\substack{j=0 \\ i+j \neq 0}}^{m-1} \log(1 + \frac{P_{i,j}}{N}) \tag{3.4}$$

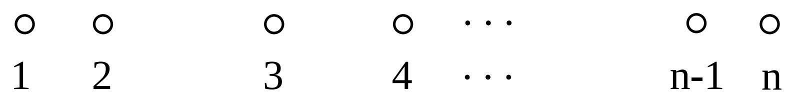

Figure 3.4: 1-dimensional general network

Obviously, the rate (3.4) is the maximum common rate based on the total power received by node (1,1) or node (m, m).

3.1.3 The 1-dimensional general network with the same transmission rate and power

In this section, we consider the 1-dimensional general network where all nodes are located on a straight line, with equal transmission rate R and transmission power P, as shown in figure 3.4. Following the notations in the regular network, we denote $P_{i,j} = |g_{i,j}|^2 P_i$, which is the corresponding received power at another node j when a node i is transmitting at its full power.

Theorem 6 *For the half-duplex 1-dimensional general network with equal transmission rate R and transmission power P, the following rate is achievable for all the sources.*

$$
\begin{aligned}
R < \frac{1}{n} \min\{ & \min_{1 \le i \le n-1} \frac{1}{n-i} \sum_{j=i+1}^{n} \log(1 + \frac{P_{j,i}}{N}), \\
& \min_{2 \le i \le n} \frac{1}{i-1} \sum_{j=1}^{i-1} \log(1 + \frac{P_{j,i}}{N})\}
\end{aligned} \tag{3.5}
$$

3.1.4 The 1-dimensional general network

In this section, we consider the 1-dimensional general network where all nodes are located on a straight line, each with transmission rate R_i and transmission power P_i, as shown in figure 3.4. Following the notations in the regular network, we denote $P_{i,j} = |g_{i,j}|^2 P$, which is the corresponding received power at another node j when a node i is transmitting at its full power.

Theorem 7 *For the half-duplex 1-dimensional general network, with the regulated greedy relay scheme, the following rate is achievable for all the sources. For any set S containing consecutive nodes, there is a node j outside of set S that satisfies*

$$\sum_{i \in S} R_i < \sum_{i \in S} \log(1 + \frac{P_{i,j}}{N}) \tag{3.6}$$

Or the set S can be divided into two subsets S_1 and S_2 containing consecutive nodes, $S_1 \bigcup S_2 = S$, there are two corresponding nodes j_1 and j_2 outside of set S that satisfies

$$\sum_{i \in S_1} R_i < \sum_{i \in S_1} \log(1 + \frac{P_{i,j_1}}{N}) \tag{3.7}$$

and

$$\sum_{i \in S_2} R_i < \sum_{i \in S_2} \log(1 + \frac{P_{i,j_2}}{N}) \tag{3.8}$$

Regulated greedy relay scheme: In every round, each node i will help to relay the nodes it can decode from the nearest to the farthest on every side respectively. For the m-th message of node i that is h-hop away from node j, node j transmits it at the $r_{h,m}$ round if node j decodes it earlier. $r_{h,m} = 1 + (m-1)h + \frac{h(h+1)}{2}$. Node j does not need to wait any rounds any more until it knows from its decoding that all other nodes between node i and node j have already decoded and relayed the message of node i. This makes sure that the round order of the messages it relays decreases according to the distance from it and each node will be relayed earlier by the node that is nearer to it on every side.

A detailed table 3.1 will explain the worst operation of the regulated greedy relay scheme. Here, the worst operation means the longest waiting time.

3.2 Half-Duplex Networks under greedy omnidirectional relay scheme

Following the same network model and definition as in 3.1 Scheduled Half-Duplex Networks, we operate the network in a different way. Same with the operation in the scheduled half-duplex networks, the network is also operated in terms of rounds of blocks: every round consists of n blocks, and each node can use one block to transmit in every round. What

round	i	a	b	c	$h-4$ nodes	j
1	i_1	a_1	b_1	c_1		j_1
2	i_2	$a_2 i_1$	b_2	c_2		j_2
3		$a_3 i_2$	$b_3 a_1$	c_3		j_3
4			$b_4 i_1$	c_4		j_4
5			$b_5 a_2$	$c_5 b_1$		j_5
6			$b_6 i_2$	$c_6 a_1$		j_6
7				$c_7 i_1$		j_7
$\vdots$				$c_8 b_2$		
$r(h,1)$				$c_9 a_2$		$j_{r(h,1)} i_1$
				$c_{10} i_2$		
$\vdots$						
$r(h,2)$						$j_{r(h,2)} i_2$
$\vdots$						
$r(h,m)$						$j_{r(h,m)} i_m$

Table 3.1: Regulated greedy relay scheme

differs from the scheduled half-duplex network is that in every round, each node transmits all it can decode in the last round, according to the greedy omnidirectional relay scheme. The order of transmission by the n nodes in every round can also be arbitrary.

Theorem 8 *For the half-duplex 1-dimensional regular network, 2-dimensional regular network and 1-dimensional general network, (3.3), (3.4) and (3.5) is achievable respectively for all the sources under the greedy relay scheme.*

3.3 Full-Duplex 1-dimensional regular Networks

We consider the full-duplex model, where each node can transmit and receive at the same time.

Theorem 9 *For the full-duplex 1-dimensional regular network, the following rate is achievable for all the sources.*

$$R < \frac{1}{n-1} \log(1 + \frac{\sum_{i=1}^{n-1} P_i}{N}) \tag{3.9}$$

(3.9) was shown in [32] to be achievable by a complicated mathematical analysis. Here, we prove (3.9) by a simpler argument.

3.4 Proofs

3.4.1 Proof of Theorem 4

We first prove a lemma for a half-duplex multiple-access network similar to Lemma 4.1 in [32]. Consider a general network where there are m senders and 1 receiver. Consider m time blocks of equal length where each node $i \in \mathcal{M}$ uses one block to transmit its message w_i at rate R_i. Assume that the corresponding received power is P_i. Then we have the following result similar to Lemma 4.1 in [32].

Lemma 1 *In the above network, there always exists some nonempty subset of* $\{w_1, w_2, ..., w_m\}$ *that can be decoded at the end of the m blocks, as long as the following inequality holds:*

$$\sum_{i\in\mathcal{M}} R_i < \sum_{i\in\mathcal{M}} \log(1+\frac{P_i}{N}) \tag{3.10}$$

Lemma 2 *In the case that nodes are helping each other relay previous messages, similar to Lemma 4.2 in [32], there always exists some nonempty subset of nodes whose transmissions in the current block can be decoded at the end of every round, as long as (3.10) holds:*

Proofs of Lemma 1 and Lemma 2 are analogous to the proofs of Lemma 4.1 and Lemma 4.2 in [32] and are stated briefly here.

Proof of Lemma 1: We use a contradiction argument. Suppose (3.10) does not hold for some $\mathcal{A} \subset \mathcal{M}$, i.e.,

$$\sum_{i\in\mathcal{A}} R_i \geq \sum_{i\in\mathcal{A}} \log(1+\frac{P_i}{N}) \tag{3.11}$$

Then by taking the difference between (3.10) and (3.11), we have

$$\sum_{i\in\mathcal{A}^c} R_i < \sum_{i\in\mathcal{A}^c} \log(1+\frac{P_i}{N}) \tag{3.12}$$

where, $\mathcal{A}^c = \mathcal{M}\backslash\mathcal{A}$. Now, by comparing (3.12) with (3.10), we get the same situation as (3.10) with $\mathcal{M}$ replaced by $\mathcal{A}^c$. Similar with the above process, if the inequality

$$\sum_{i\in\mathcal{S}^c} R_i < \sum_{i\in\mathcal{S}^c} \log(1+\frac{P_i}{N}) \tag{3.13}$$

holds for all nonempty $\mathcal{S}^c \subseteq \mathcal{A}^c$, then the subset $\{w_i : i \in \mathcal{A}^c\}$ can be decoded. If (3.13) does not hold for all nonempty $\mathcal{S}^c \subseteq \mathcal{A}^c$, then it arrives at the same situation with our previous supposition that (3.10) does not hold for some $\mathcal{A} \subset \mathcal{M}$. We can continue decreasing $\mathcal{A}^c$ like decreasing $\mathcal{M}$. There must be at least one nonempty subset, all of whose subsets hold for the inequalities of the type (3.13), and thus the messages of this nonempty subset can be decoded. Therefore, we proved that if (3.10) holds, there must be a nonempty subset $\mathcal{M}_1 \subseteq \mathcal{M}$ such that $\{w_i : i \in \mathcal{M}_1\}$ can be decoded, while the messages $\{w_i : i \in \mathcal{M}_2\}$ of $\mathcal{M}_2 = \mathcal{M}\backslash\mathcal{M}_1$ can not.

Proof of Lemma 2: In the case that nodes are helping each other relay previous messages, let us consider a two-block decoding situation. In the first block $\{w_i(1) : i \in \mathcal{M}_2\}$ are decoded while $\{w_i(1) : i \in \mathcal{M}_1\}$ are not and in the second block each node $\{i \in \mathcal{M}_2\}$ helps to transmit the messages $\{w_i(1) : i \in \mathcal{M}_1\}$ with its own messages of the second block $\{w_i(2) : i \in \mathcal{M}_2\}$. At the end of the second block, it is $\{w_i(2) : i \in \mathcal{M}_2\} \bigcup \{w_i(1) : i \in \mathcal{M}_1\}$ that needs to be decoded. In order to simplify the notation, we denote $w_{\mathcal{M}_2}(2) = \{w_i(2) : i \in \mathcal{M}_2\}$, $w_{\mathcal{M}_1}(1) = \{w_i(1) : i \in \mathcal{M}_1\}$, and $\{w_{\mathcal{M}_2}(2), w_{\mathcal{M}_1}(1)\} = \{w_i(2) : i \in \mathcal{M}_2\} \bigcup \{w_i(1) : i \in \mathcal{M}_1\}$, following the notations in [32]. Also, we denote $\Gamma_i \subset \mathcal{M}$ as the set of nodes that node i helps in the second block. Node i sends a codeword $X_i(w_i(2), w_{\Gamma_i}(1))$ by binning these vectors in the second block. Reversely, denote $\Lambda_i \subset \mathcal{M}$ as the set of nodes that will help node i to relay $w_i(1)$ in the second block.

For any subset $\mathcal{S} \subseteq \mathcal{M}$, let $\mathcal{S}_1 = \mathcal{S} \bigcap \mathcal{M}_1$, and let $\mathcal{S}_2 = (\mathcal{S} \bigcap \mathcal{M}_2) \bigcup (\bigcup_{i\in\mathcal{S}_1} \Lambda_i \bigcap \mathcal{M}_2)$. It means that $\mathcal{S}_2$ also consists of nodes from $\mathcal{M}_2$ that may not be in $\mathcal{S}$, but are helping transmitting $w_{S_1}(1)$.

The corresponding inequality of (3.10) can be written as

$$\sum_{i\in\mathcal{M}} R_i < \sum_{i\in\mathcal{M}_1} \log(1+\frac{P_i}{N}) + \sum_{i\in\mathcal{M}_2} \log(1+\frac{P_i}{N}) \tag{3.14}$$

We also use a contradiction argument. Suppose (3.14) does not hold for some $\mathcal{A} \subset \mathcal{M}$, i.e.,

$$\sum_{i\in\mathcal{A}} R_i \geq \sum_{i\in\mathcal{A}_1} \log(1+\frac{P_i}{N}) + \sum_{i\in\mathcal{A}_2} \log(1+\frac{P_i}{N}) \tag{3.15}$$

Then taking the difference between (3.14) and (3.15), we have

$$\sum_{i\in\mathcal{A}^c} R_i < \sum_{i\in\mathcal{A}_1^c} \log(1+\frac{P_i}{N}) + \sum_{i\in\mathcal{A}_2^c} \log(1+\frac{P_i}{N}) \tag{3.16}$$

where $\mathcal{A}^c = \mathcal{M}\backslash\mathcal{A}$, $\mathcal{A}_1^c = \mathcal{M}_1\backslash\mathcal{A}_1$, $\mathcal{A}_2^c = \mathcal{M}_2\backslash\mathcal{A}_2$.

Since $\mathcal{A} \subseteq \mathcal{A}_1 \bigcup \mathcal{A}_2$, then $\mathcal{A}^c \supseteq \mathcal{A}_1^c \bigcup \mathcal{A}_2^c$. Thus,

$$\sum_{i\in\mathcal{A}_1^c\bigcup\mathcal{A}_2^c} R_i \leq \sum_{i\in\mathcal{A}^c} R_i < \sum_{i\in\mathcal{A}_1^c} \log(1+\frac{P_i}{N}) + \sum_{i\in\mathcal{A}_2^c} \log(1+\frac{P_i}{N}) \tag{3.17}$$

$$\sum_{i\in\mathcal{A}_1^c\bigcup\mathcal{A}_2^c} R_i < \sum_{i\in\mathcal{A}_1^c} \log(1+\frac{P_i}{N}) + \sum_{i\in\mathcal{A}_2^c} \log(1+\frac{P_i}{N}) \tag{3.18}$$

This is the same situation as (3.14) with $\mathcal{M}$ replaced by $\mathcal{A}_1^c \bigcup \mathcal{A}_2^c$, $\mathcal{M}_1$ replaced by $\mathcal{A}_1^c$, $\mathcal{M}_2$ replaced by $\mathcal{A}_2^c$. As in the case of one-block multiple-access discussed earlier, we can continue decreasing $\mathcal{A}_1^c \bigcup \mathcal{A}_2^c$ until we find a nonempty subset of $\{w_{\mathcal{M}_2}(2), w_{\mathcal{M}_1}(1)\}$ that can be decoded.

Therefore, the inequality (3.14) ensures that there always exists a nonempty subset of $\{w_{\mathcal{M}_2}(2), w_{\mathcal{M}_1}(1)\}$ that can be decoded. When combining the two terms on the right side of (3.14), (3.14) becomes (3.10). It means that the inequality (3.10) makes sure that there always exist some messages that can be decoded, no matter whether it is one-block multiple-access, or two-block multiple-access with relays. Then generally, we get the same conclusion for K-block multiple-access with relays, which is lemma 2.

Now, we prove Theorem 4. In the half-duplex one-dimensional regular networks which has n nodes, each node has the same distance with its neighbors as shown in figure 1.

Since we have (3.3), where R, the average transmission rate equals $\frac{R_i}{n}$, the instantaneous transmission rate in Lemma 1 divided by n.

$$(n-1)R < \frac{1}{n}\sum_{i=1}^{n-1} \log(1+\frac{P_i}{N}) \tag{3.19}$$

then for $1 \leq k \leq n-1$,

$$kR < \frac{1}{n}\sum_{i=1}^{k} \log(1+\frac{P_i}{N}) \tag{3.20}$$

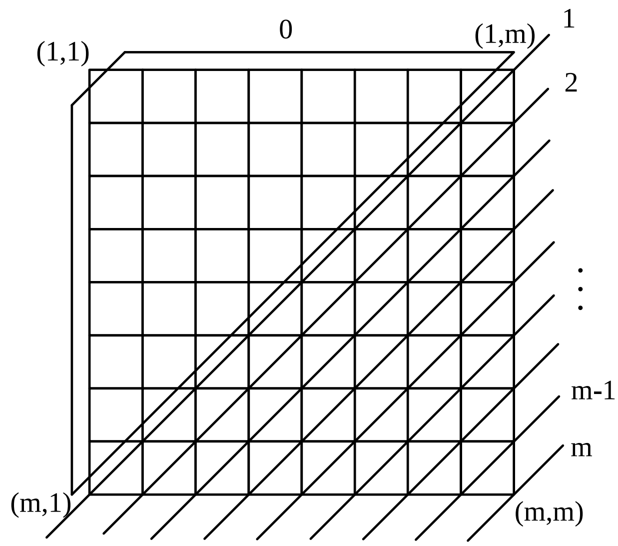

Figure 3.5: 2-dimensional regular network in lemma 3

Since in each block time of every round, only one node is transmitting, then we can consider any set of nodes that have relay relationship without any interference from other nodes. Considering the nodes on the left of node i, by applying (3.20) with $k = i - 1$ into Lemma 1 and Lemma 2, node i can decode node $i - 1$ since it is the nearest one. Considering the nodes on the right of node i, by applying (3.20) with $k = n - i$ into Lemma 1 and Lemma 2, node i can decode node $i+1$ since it is the nearest one. Thus each node i can decode its one-hop neighbors on both the left side and the right side. Therefore, each node can decode all the other nodes under (3.3) with the scheduled omnidirectional relay scheme.

3.4.2 Proof of Theorem 5

Lemma 3 *In the two-dimensional $m \times m$ regular networks,*

$$\frac{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}{(m-1)^2-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{m^2-1} \tag{3.21}$$

Proof of Lemma 3: As shown in figure 3.5, we denote the set of nodes in the upper left square $(m-1) \times (m-1)$ except node (1,1) as set A and the set of nodes on line

$[(m,1),(m,m)]$, and line $[(1,m),(m,m)]$ as set B. Thus, the average power received from set A by node i is $\frac{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}{(m-1)^2-1}$, denoted as P_A and the the average power received from set B by node i is $\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}-\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}{2m-1}$, denoted as P_B. The number of nodes in set A is denoted as n_A and the number of nodes in set B is denoted as n_B.

In figure 3.5, the nodes in set A can be divided into several parts, nodes in triangle 0 denoted as set a_0, nodes on line 1 in A denoted as set a_1, nodes on line 2 in A denoted as set a_2,..., nodes on line $m-2$ in A denoted as set a_{m-2}. Also, the nodes in set B can be divided into several parts, nodes on line 2 in B denoted as set b_2, nodes on line 3 in B denoted as set b_3,..., nodes on line m in B denoted as set b_m. The average power and number of the nodes in each set is denoted as P_{s_l} and n_{s_l} respectively, where $s \in \{a,b\}$ and $l \in \{0,1,2,...,m\}$.

Then, we have

$$\begin{aligned}
P_A =&\frac{n_{a_0}}{n_A}P_{a_0} + \frac{n_{a_1}}{n_A}P_{a_1} + \ldots + \frac{n_{a_{m-2}}}{n_A}P_{a_{m-2}} \\
P_B =&\frac{n_{b_1}}{n_B}P_{b_1} + \frac{n_{b_2}}{n_B}P_{b_2} + ... + \frac{n_{b_m}}{n_B}P_{b_m} \\
<&\frac{n_{b_1}}{n_B}P_{b_1} + \frac{n_{b_2}}{n_B}P_{b_2} + ... + \frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B}P_{b_{m-2}} \\
=&\frac{n_{a_1}}{n_A}P_{b_1} + (\frac{n_{b_1}}{n_B}P_{b_1} - \frac{n_{a_1}}{n_A}P_{b_1}) \\
&+ \frac{n_{a_2}}{n_A}P_{b_2} + (\frac{n_{b_2}}{n_B}P_{b_2} - \frac{n_{a_2}}{n_A}P_{b_2}) + ... \\
&+ \frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}} + (\frac{n_{b_{m-3}}}{n_B}P_{b_{m-3}} - \frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}}) \\
&+ \frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}} + (\frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B}P_{b_{m-2}} - \frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}}) \\
=&\frac{n_{a_1}}{n_A}P_{b_1} + \frac{n_{a_2}}{n_A}P_{b_2} + ... + \frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}} + \frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}} \\
&+ (\frac{n_{b_1}}{n_B}P_{b_1} - \frac{n_{a_1}}{n_A}P_{b_1}) + (\frac{n_{b_2}}{n_B}P_{b_2} - \frac{n_{a_2}}{n_A}P_{b_2}) + ... \\
&+ (\frac{n_{b_{m-3}}}{n_B}P_{b_{m-3}} - \frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}}) \\
&+ (\frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B}P_{b_{m-2}} - \frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}}) \\
=&\frac{n_{a_1}}{n_A}P_{b_1} + \frac{n_{a_2}}{n_A}P_{b_2} + ... + \frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}} + \frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}}
\end{aligned} \tag{3.22}$$

$$
\begin{aligned}
&+(\frac{n_{b_1}}{n_B}-\frac{n_{a_1}}{n_A})P_{b_1}+(\frac{n_{b_2}}{n_B}-\frac{n_{a_2}}{n_A})P_{b_2}+...\\
&+(\frac{n_{b_{m-3}}}{n_B}-\frac{n_{a_{m-3}}}{n_A})P_{b_{m-3}}\\
&+(\frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B}-\frac{n_{a_{m-2}}}{n_A})P_{b_{m-2}}\\
<&\frac{n_{a_1}}{n_A}P_{b_1}+\frac{n_{a_2}}{n_A}P_{b_2}+...+\frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}}+\frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}} \qquad (3.23)\\
&+(\frac{n_{b_1}}{n_B}-\frac{n_{a_1}}{n_A})P_{b_1}+(\frac{n_{b_2}}{n_B}-\frac{n_{a_2}}{n_A})P_{b_1}+...\\
&+(\frac{n_{b_{m-3}}}{n_B}-\frac{n_{a_{m-3}}}{n_A})P_{b_1}\\
&+(\frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B}-\frac{n_{a_{m-2}}}{n_A})P_{b_1}\\
=&\frac{n_{a_1}}{n_A}P_{b_1}+\frac{n_{a_2}}{n_A}P_{b_2}+...+\frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}}+\frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}}\\
&+[(\frac{n_{b_1}}{n_B}-\frac{n_{a_1}}{n_A})+(\frac{n_{b_2}}{n_B}-\frac{n_{a_2}}{n_A})+...\\
&+(\frac{n_{b_{m-3}}}{n_B}-\frac{n_{a_{m-3}}}{n_A})\\
&+(\frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B}-\frac{n_{a_{m-2}}}{n_A})]P_{b_1}\\
=&\frac{n_{a_1}}{n_A}P_{b_1}+\frac{n_{a_2}}{n_A}P_{b_2}+...+\frac{n_{a_{m-3}}}{n_A}P_{b_{m-3}}+\frac{n_{a_{m-2}}}{n_A}P_{b_{m-2}}+\frac{n_{a_0}}{n_A}P_{b_1}\\
<&\frac{n_{a_0}}{n_A}P_{a_0}+\frac{n_{a_1}}{n_A}P_{a_1}+\frac{n_{a_2}}{n_A}P_{a_2}+...+\frac{n_{a_{m-3}}}{n_A}P_{a_{m-3}}+\frac{n_{a_{m-2}}}{n_A}P_{a_{m-2}} \qquad (3.24)\\
=&P_A
\end{aligned}
$$

where (3.22) follows from the fact that $P_{b_m} < P_{b_{m-1}} < P_{b_{m-2}}$;(3.23) follows from the fact that $\frac{n_{b_{m-2}}+n_{b_{m-1}}+n_{b_m}}{n_B} > \frac{n_{b_{m-3}}}{n_B} = ... = \frac{n_{b_2}}{n_B} = \frac{n_{b_1}}{n_B} = \frac{2}{2m-1} > \frac{m-2}{(m-1)(m-1)-1} = \frac{n_{a_1}}{n_A} > \frac{n_{a_2}}{n_A} > ... > \frac{n_{a_{m-2}}}{n_A}$ and $P_{b_{m-2}} < P_{b_{m-1}} < \ldots < P_{b_2} < P_{b_1}$;(3.24) follows from the fact that $P_{b_1} < P_{a_0}, P_{b_1} < P_{a_1}, P_{b_2} < P_{a_2}, \ldots, P_{b_{m-2}} < P_{a_{m-2}}$. Then

$$\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}-\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2}P_{i,j}}{2m-1} < \frac{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2}P_{i,j}}{(m-1)^2-1}$$

$$\frac{2m-1}{(m-1)^2-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}-\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2}P_{i,j}}{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2}P_{i,j}}$$

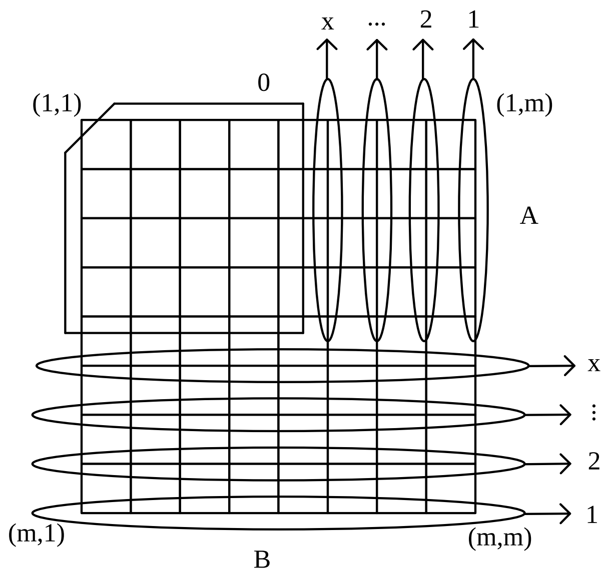

Figure 3.6: 2-dimensional regular network in lemma 4

$$1+\frac{2m-1}{(m-1)^2-1} > 1+\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j} - \sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}$$

$$\frac{m^2-1}{(m-1)^2-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}$$

$$\frac{\sum_{i=0}^{m-2}\sum_{j=0,i+j\neq 0}^{m-2} P_{i,j}}{(m-1)^2-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{m^2-1}$$

Lemma 4 *In the two-dimensional $m \times m$ regular networks,*

$$\frac{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{(m-x)m-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{m^2-1} \tag{3.25}$$

where $x < m$.

Proof of Lemma 4:

As shown in figure 3.6, we denote the set of nodes in the upper rectangular $(m-x)\times m$ except node (1,1) as set A and the set of nodes in the lower rectangular $m \times x$ as set B.

Thus, the average power received from set A is $\frac{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{(m-x)m-1}$, denoted as P_A and the the average power received from set B is $\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}-\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{mx}$, denoted as P_B. The number of nodes in set A is denoted as n_A and the number of nodes in set B is denoted as n_B.

In figure 3.6, the nodes in set A can be divided into several parts, nodes in square 0 denoted as set a_0, nodes on line 1 in A denoted as set a_1, nodes on line 2 in A denoted as set a_2,..., nodes on line x in A denoted as set a_x. Also, the nodes in set B can be divided into several parts, nodes on line 1 in B denoted as set b_1, nodes on line 2 in B denoted as set b_2,..., nodes on line x in B denoted as set b_x. The average power and number of the nodes in each set is denoted as P_{s_l} and n_{s_l} respectively, where $s \in \{a, b\}$ and $l \in \{0, 1, 2, ..., x\}$.

Then, we have

$$
\begin{aligned}
P_A =& \frac{n_{a_0}}{n_A}P_{a_0} + \frac{n_{a_1}}{n_A}P_{a_1} + \ldots + \frac{n_{a_x}}{n_A}P_{a_x} \\
P_B =& \frac{n_{b_1}}{n_B}P_{b_1} + \frac{n_{b_2}}{n_B}P_{b_2} + \ldots + \frac{n_{b_x}}{n_B}P_{b_x} \\
=& \frac{n_{a_1}}{n_A}P_{b_1} + (\frac{n_{b_1}}{n_B}P_{b_1} - \frac{n_{a_1}}{n_A}P_{b_1}) \\
& + \frac{n_{a_2}}{n_A}P_{b_2} + (\frac{n_{b_2}}{n_B}P_{b_2} - \frac{n_{a_2}}{n_A}P_{b_2}) + \ldots \\
& + \frac{n_{a_x}}{n_A}P_{b_x} + (\frac{n_{b_x}}{n_B}P_{b_x} - \frac{n_{a_x}}{n_A}P_{b_x}) \\
=& \frac{n_{a_1}}{n_A}P_{b_1} + \frac{n_{a_2}}{n_A}P_{b_2} + \ldots + \frac{n_{a_x}}{n_A}P_{b_x} \\
& + (\frac{n_{b_1}}{n_B}P_{b_1} - \frac{n_{a_1}}{n_A}P_{b_1}) + (\frac{n_{b_2}}{n_B}P_{b_2} - \frac{n_{a_2}}{n_A}P_{b_2}) \\
& + \ldots + (\frac{n_{b_x}}{n_B}P_{b_x} - \frac{n_{a_x}}{n_A}P_{b_x}) \\
=& \frac{n_{a_1}}{n_A}P_{b_1} + \frac{n_{a_2}}{n_A}P_{b_2} + \ldots + \frac{n_{a_x}}{n_A}P_{b_x} \\
& + (\frac{n_{b_1}}{n_B} - \frac{n_{a_1}}{n_A})P_{b_1} + (\frac{n_{b_2}}{n_B} - \frac{n_{a_2}}{n_A})P_{b_2} \\
& + \ldots + (\frac{n_{b_x}}{n_B} - \frac{n_{a_x}}{n_A})P_{b_x} \\
<& \frac{n_{a_1}}{n_A}P_{b_1} + \frac{n_{a_2}}{n_A}P_{b_2} + \ldots + \frac{n_{a_x}}{n_A}P_{b_x} \qquad (3.26) \\
& + P_{b_x}[(\frac{n_{b_1}}{n_B} - \frac{n_{a_1}}{n_A}) + (\frac{n_{b_2}}{n_B} - \frac{n_{a_2}}{n_A})
\end{aligned}
$$

$$
\begin{aligned}
&+\ldots+(\frac{n_{b_x}}{n_B}-\frac{n_{a_x}}{n_A})] \\
=&\frac{n_{a_1}}{n_A}P_{b_1}+\frac{n_{a_2}}{n_A}P_{b_2}+\ldots+\frac{n_{a_x}}{n_A}P_{b_x} \\
&+P_{b_x}[(\frac{n_{b_1}}{n_B}+\frac{n_{b_2}}{n_B}+\ldots+\frac{n_{b_x}}{n_B}) \\
&-(\frac{n_{a_1}}{n_A}+\frac{n_{a_2}}{n_A}+\ldots+\frac{n_{a_x}}{n_A})] \\
=&\frac{n_{a_1}}{n_A}P_{b_1}+\frac{n_{a_2}}{n_A}P_{b_2}+\ldots+\frac{n_{a_x}}{n_A}P_{b_x} \\
&+P_{b_x}[1-(1-\frac{n_{a_0}}{n_A})] \\
=&\frac{n_{a_0}}{n_A}P_{b_x}+\frac{n_{a_1}}{n_A}P_{b_1}+\frac{n_{a_2}}{n_A}P_{b_2}+\ldots+\frac{n_{a_x}}{n_A}P_{b_x} \\
<&\frac{n_{a_0}}{n_A}P_{a_0}+\frac{n_{a_1}}{n_A}P_{a_1}+\frac{n_{a_2}}{n_A}P_{a_2}+\ldots+\frac{n_{a_x}}{n_A}P_{a_x} \qquad (3.27)
\end{aligned}
$$

where (3.26) follows from the fact that $\frac{n_{b_1}}{n_B}=\frac{n_{b_2}}{n_B}=\ldots=\frac{n_{b_x}}{n_B}=\frac{1}{x}$, $\frac{n_{a_1}}{n_A}=\frac{n_{a_2}}{n_A}=\ldots=\frac{n_{a_x}}{n_A}=\frac{m-x}{m(m-x)-1}\leq\frac{1}{x}$, $\frac{n_{b_1}}{n_B}-\frac{n_{a_1}}{n_A}\geq 0, \frac{n_{b_2}}{n_B}-\frac{n_{a_2}}{n_A}\geq 0,\ldots,\frac{n_{b_x}}{n_B}-\frac{n_{a_x}}{n_A}\geq 0$, and $P_{b_1}<P_{b_2}<\ldots<P_{b_x}$; (3.27) follows from the fact that $P_{b_x}<P_{a_0}$ (by lemma 3), $P_{b_1}<P_{a_1}$, $P_{b_2}<P_{a_2},\ldots,$ $P_{b_x}<P_{a_x}$, then we have $P_B<P_A$

$$\frac{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{(m-x)m-1}>\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}-\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{mx}$$

$$\frac{mx}{(m-x)m-1}>\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}-\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}$$

$$1+\frac{mx}{(m-x)m-1}>1+\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}-\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}$$

$$\frac{m^2-1}{(m-x)m-1}>\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}$$

$$\frac{\sum_{i=0}^{m-x-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{(m-x)m-1}>\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{m^2-1}$$

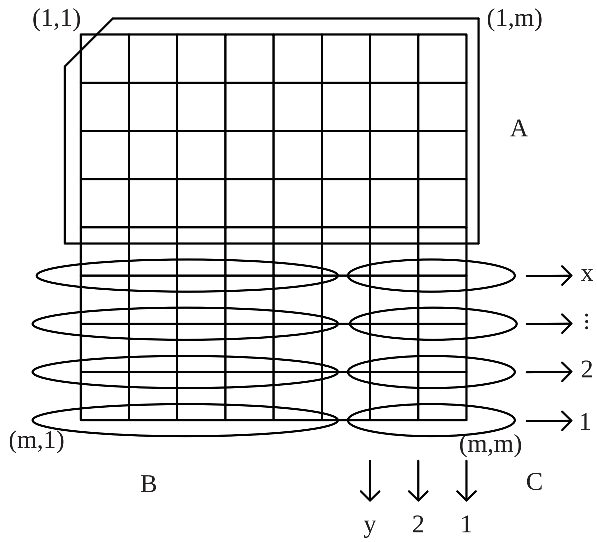

Figure 3.7: 2-dimensional regular network in lemma 5

Lemma 5 *In the two-dimensional $m \times m$ regular networks,*

$$\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j} - \sum_{i=m-x}^{m-1}\sum_{j=m-y,(m-y)(m-x)\neq 0}^{m-1} P_{i,j}}{m^2 - xy - 1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{m^2 - 1} \tag{3.28}$$

Proof of Lemma 5:

By lemma 4, it can be seen that in figure 3.7, the average power of nodes in A is larger than the average power of nodes in $B \bigcup C$, where A denotes the nodes on the up $m - x$ lines except node (1,1), B denotes the first $m - y$ nodes on the bottom x lines and C denotes the later y nodes on the bottom x lines.

Focusing on the nodes in $B \bigcup C$, we denote the average power of nodes in X on line i, where $X \in \{B, C\}$, $i \in \{1, 2, \ldots, x\}$, as P_{X_i}. Since obviously, on each bottom line 1,2,..., x,

$$P_{B_i} > P_{C_i}$$

then the average power of nodes in B is larger than that in C, we denote this by

$$P_B > P_C.$$

then we have

$$P_B > P_{B\bigcup C} > P_C$$

Also since

$$P_A > P_{B\bigcup C} > P_C$$

then

$$P_{A\bigcup B} > P_C$$

then we have equation (3.28).

When $(m-x)(m-y) = 0, (m-x)+(m-y) \neq 0$, Lemma 5 becomes Lemma 4

Proof of Theorem 5: By lemma 3 and lemma 4, we can get

$$\frac{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}}{(m-a)(m-b)-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{m^2-1} \tag{3.29}$$

where $a < m, b < m$.

$$\frac{m^2-1}{(m-a)(m-b)-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}} \tag{3.30}$$

Since when $x_2 > x_1 > 0$, we have

$$\frac{x_2}{x_1} > \frac{\log(1+x_2)}{\log(1+x_1)}$$

then for any $P_{i,j}$, we have

$$\frac{P_{i,j}}{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}} > \frac{\log(1+\frac{P_{i,j}}{N})}{\log(1+\frac{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}}{N})}$$

then, by adding all inequalities for each $P_{i,j}$, (3.30) continues to have the following inequality.

$$\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} P_{i,j}}{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1} \log(1+\frac{P_{i,j}}{N})}{\log(1+\frac{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}}{N})} \tag{3.31}$$

By the concavity of the logarithm function,

$$\log(1+\frac{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} P_{i,j}}{N}) < \sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1} \log(1+\frac{P_{i,j}}{N})$$

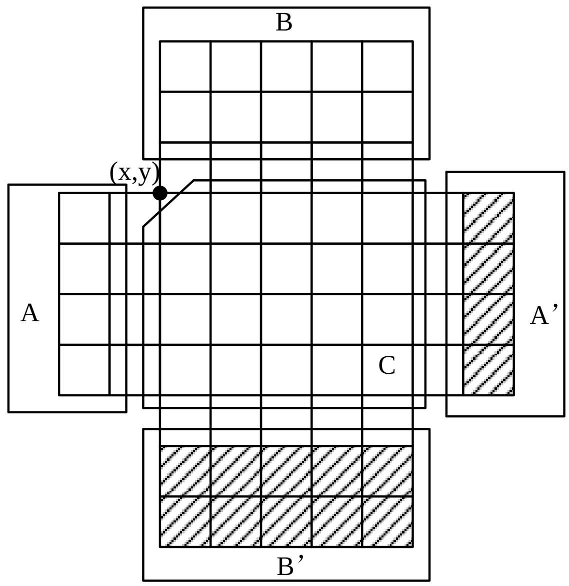

Figure 3.8: 2-dimensional regular network in theorem 5

Then, (3.31) continues to have the following inequality:

$$\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}\log(1+\frac{P_{i,j}}{N})}{\log(1+\frac{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1}P_{i,j}}{N})} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}\log(1+\frac{P_{i,j}}{N})}{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1}\log(1+\frac{P_{i,j}}{N})}$$

Then, (3.30) continues to have the following inequality:

$$\frac{m^2-1}{(m-a)(m-b)-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}\log(1+\frac{P_{i,j}}{N})}{\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1}\log(1+\frac{P_{i,j}}{N})}$$

By (3.4), we have

$$((m-a)(m-b)-1)R < \frac{1}{m^2}\sum_{i=0}^{m-a-1}\sum_{j=0,i+j\neq 0}^{m-b-1}\log(1+\frac{P_{i,j}}{N}) \tag{3.32}$$

By Lemma 5, we have equation (3.28). In other words, as shown in figure 3.8, the average power of nodes in $A'\bigcup B'\bigcup C$ received by the black focused node (x,y) is larger

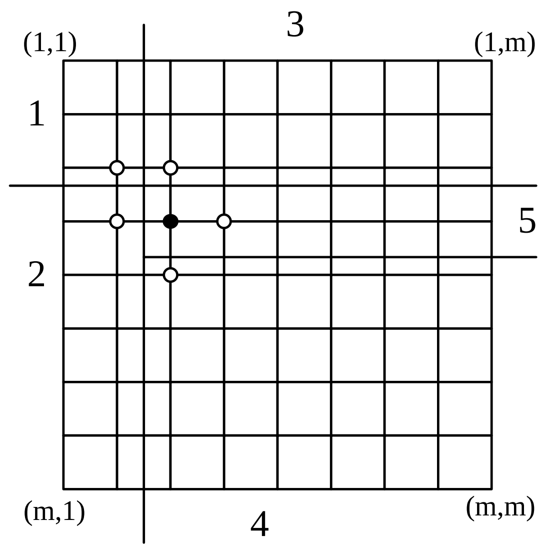

Figure 3.9: 2-dimensional regular network in theorem 5

than $\frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{m^2-1}$, where A, A', B, B', C denotes the set of nodes in its respective rectangular shown in figure 3.8. When we move the nodes in A' to A, and the nodes in B' to B, the power received by node (x, y) becomes larger. So we have for node (x, y)

$$\frac{P - \sum_{i=1}^{x-1}\sum_{j=1}^{y-1}P_{i,j}}{m^2-(x-1)(y-1)-1} > \frac{\sum_{i=0}^{m-1}\sum_{j=0,i+j\neq 0}^{m-1}P_{i,j}}{m^2-1} \tag{3.33}$$

where P denotes the total power received by node (x, y).

By similar analysis of (3.29)∼(3.32), we have for node (x, y)

$$[m^2-(x-1)(y-1)-1]R < \frac{1}{m^2}\sum_{i=1}^{x-1}\sum_{j=1}^{y-1}\log(1+\frac{P-P_{i,j}}{N}) \tag{3.34}$$

Following the idea of the half-duplex 1-dimensional network, we prove that in the half-duplex 2-dimensional network, each node can decode all its four 1-hop neighbors. We take a node (x, y) inside the network as an example, where $1 \le y \le x \le \frac{m}{2}$. As shown in figure 3.9 with the black node (x, y) focused, by applying Lemma 1 to (3.4), we can see that node (x, y) can decode at least one node in region 1, 2, 3, 4, 5. If node (x, y) can decode the hollow node in region 1, then all other nodes in 1 can either be taken away because they can be relayed by the hollow node in region 1 or not considered. By applying Lemma 1,2 to (3.34), we can see that node (x, y) can decode at least one more node in region 2, 3, 4, 5. In each region, the hollow node has the advantage of being decoded because it is the nearest one and the nodes it relays in each block are nearer, compared with the other nodes in its region. Obviously, its one-hop neighbor on its shortest side, the hollow node in region 2, has the advantage of being decoded by node (x, y) among all the hollow nodes in region 2, 3, 4, 5. Then all the nodes in region 2 can be taken out, since they have been relayed or not considered.

By (3.25) and similar analysis of (3.29)∼(3.32), we have for node (x, y)

$$[m^2 - m(y-1) - 1]R < \frac{1}{m^2}[\sum_{i=0}^{m-x} \sum_{j=0, i+j\neq 0}^{m-y} \log(1 + \frac{P_{i,j}}{N}) + \sum_{i=1}^{x-1} \sum_{j=0}^{m-y} \log(1 + \frac{P_{i,j}}{N})] \tag{3.35}$$

By applying Lemma 1,2 to (3.35), we can see that node (x, y) can decode at least one more node in region 3, 4, 5. Obviously, node (x, y) can decode its one-hop neighbor on its second shortest side, the hollow node in region 3. Then with all the nodes in region 3 taken out, since they have been relayed or not considered, by applying Lemma 1,2 to (3.32) with $a = x-1, b = y-1$, we can see that node (x, y) can decode at least one more node in region 4, 5. Obviously, node (x, y) can decode its one-hop neighbor on its third shortest side, the hollow node in region 4. Then with all the nodes in region 4 taken out, since they have been relayed or not considered, by applying Lemma 1,2 to (3.32) with $a = y-1, b = m-1$, we can see that node (x, y) can decode at least one more node in region 5. Then its last one-hop neighbor can be decoded. Therefore, each node in the half-duplex 2-dimensional regular network can decode the current block message of all its 1-hop neighbors. Similarly, other cases can be proved. Due to the page limit, the details are omitted here. Since the network is finite, then each node can decode all the other nodes under the scheduled omnidirectional relay scheme.

3.4.3 Proof of Theorem 6

The first part of (3.5) guarantees that for each node except the last one, it can decode its one-hop neighbor on its right side, by similar analysis with that in the half duplex 1-dimensional regular network. Also, the second part of (3.5) guarantees that for each node except the first one, it can decode its one-hop neighbor on its left side. Then in total, each node can decode its one-hop neighbors in both sides. Therefore, each node can decode all the other nodes by the scheduled omnidirectional relay scheme.

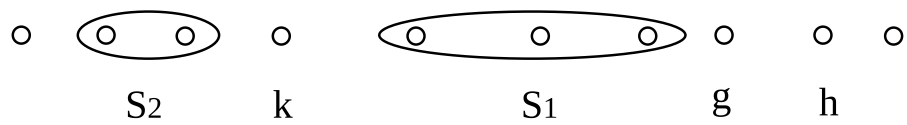

Figure 3.10: 1-dimensional general network in theorem 7

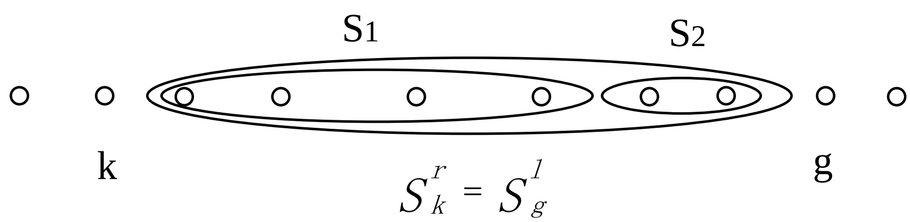

Figure 3.11: 1-dimensional general network in theorem 7

3.4.4 Proof of Theorem 7

In the 1-dimensional general network as shown in figure 3.10, firstly we make a supposition: node h cannot decode all the nodes on its left and suppose the nodes that it cannot decode are sets S_1 and S_2. Since node h can decode node g, then if node g can decode some node in set S_1, node h can also decode them by the regulated greedy relay scheme. Therefore, the supposition can be revised focusing on node g:

Supposition 1 *Node g cannot decode all the nodes on its left and suppose the nodes that it cannot decode are sets S_1 and S_2.*

For any node α, we denote the first consecutive set of nodes on its left that it cannot decode as S_α^l, the first consecutive set of nodes on its right that it cannot decode as S_α^r.

Lemma 6 *In the 1-dimensional general network as shown in figure 3.11, if $S_k^r = S_g^l$ and it satisfies the condition in theorem 7, then at least one node in the set can be decoded by node k or node g. Therefore, S_k^r or S_g^l does not exist.*

Proof of Lemma 6: In figure 3.11, $S_k^r = S_g^l$, if it satisfies (3.6), which means there is a node j outside of set S_k that satisfies

$$\sum_{i \in S_k^r} R_i < \sum_{i \in S_k^r} \log(1 + \frac{P_{i,j}}{N}) \tag{3.36}$$

then by applying Lemma 1,2 into (3.36), node k or node j can at least decode one node in this set. Therefore it cannot be S_k^r and S_g^l at the same time.

In figure 3.11, $S_k^r = S_g^l$, if it satisfies (3.7) and (3.8), which means there exist node k and node g outside of set S_1 and S_2 that satisfy

$$\sum_{i \in S_1} R_i < \sum_{i \in S_1} \log(1 + \frac{P_{i,k}}{N}) \tag{3.37}$$

$$\sum_{i \in S_2} R_i < \sum_{i \in S_2} \log(1 + \frac{P_{i,g}}{N}) \tag{3.38}$$

By the regulated greedy relay scheme, in the first several rounds when there is no mutual relay relationship between sets S_1 and S_2, node k and node g can decode some nodes in sets S_1 and S_2 respectively. In round r, the mutual relay relationship between sets S_1 and S_2 occurs. As shown in figure 3.12, we denote the node in set S_1 that node k can decode in round $r-1$ as node A. It needs to relay w_1 of some node denoted as a in S_2. Also, we denote the node in set S_2 that node g can decode in round $r-1$ as node B. It needs to relay w_1 of some node denoted as b in S_1.

As shown in figure 3.12, if node $a =$ node B, since node g decoded node B at the end of round $r-1$, then node g will relay node B at a later round by the regulated greedy relay scheme. Node k will at least know w_1 of node a. So this situation is the same with that when node A does not relay node a. Node k can still decode node A.

As shown in figure 3.13, if node a is at the right side of node B, since node B is nearer to node a than node A, then node B will relay node a at some previous round. Since node g decoded node B at the end of round $r-1$, it also decoded node a, then node g will relay node a at a later round by the regulated greedy relay scheme. Then node k will at least know w_1 of node a. So this situation is the same with that when node A does not relay node a. Node k can still decode node A.

Therefore, the last possibility is that node a is on the left side of node B. Similarly, it is only possible for node b to be on the right side of node A, as shown in figure 3.14, if. In this situation, the only reason that node A relays w_1 of node a instead of w_1 of node b is that node A can not decode node b. Similarly, the only reason that node B relays w_1 of node b instead of w_1 of node a is that node B can not decode node a. And the reason for node a to be able to be decoded by node A instead of node B is that node A is nearer to it than node B. Similarly, for node b, node B is nearer to it than node A. However, from figure 3.14, this makes contradiction. This situation also does not exist.

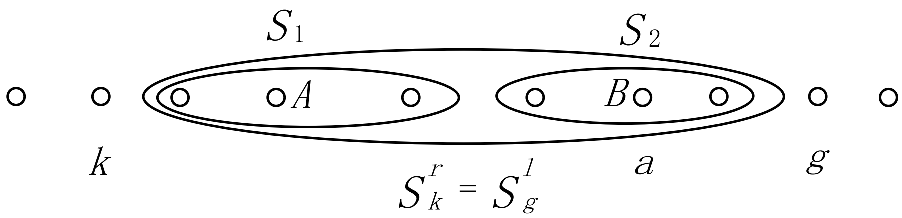

Figure 3.12: 1-dimensional general network in theorem 7

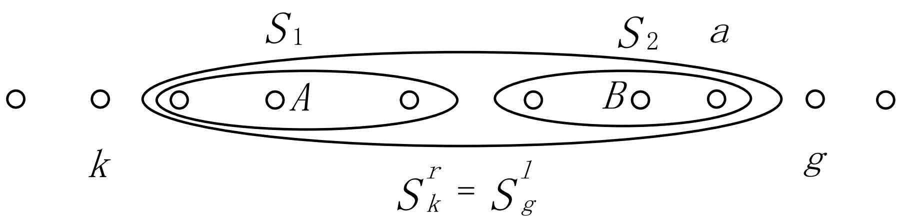

Figure 3.13: 1-dimensional general network in theorem 7

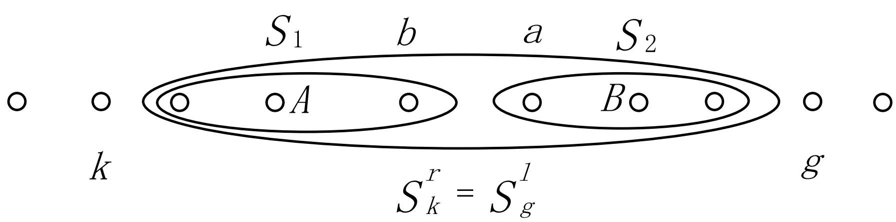

Figure 3.14: 1-dimensional general network in theorem 7

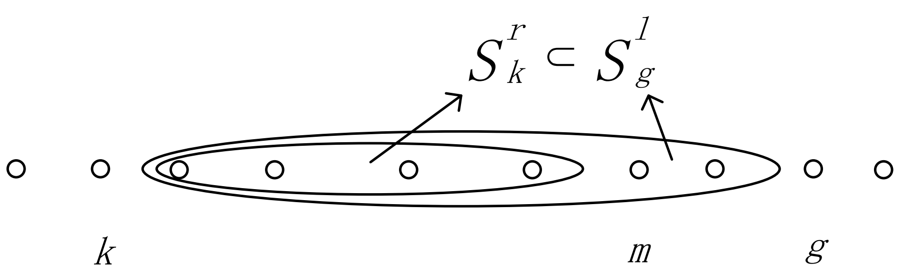

Figure 3.15: 1-dimensional general network in theorem 7

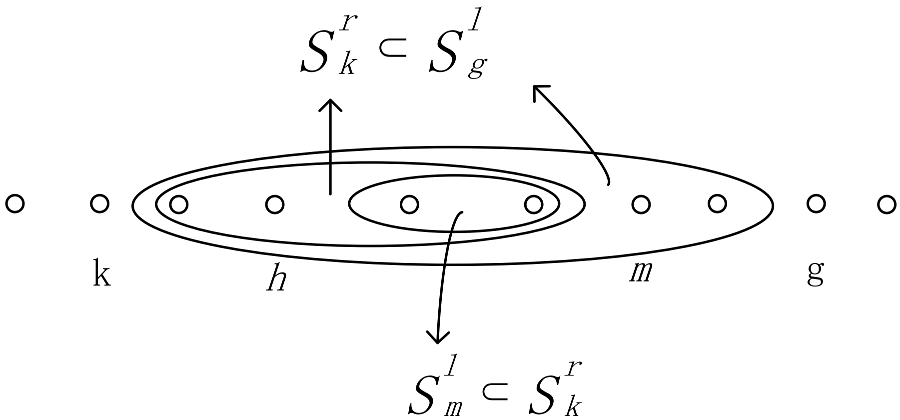

Figure 3.16: 1-dimensional general network in theorem 7

Lemma 7 *In the 1-dimensional general network as shown in figure 3.15, if $S_k^r \subset S_g^l$ and it satisfies the condition in theorem 7, then S_k^r and S_g^l do not exist.*

Proof of Lemma 7:

In figure 3.15, since node k can be decoded by node g, it can also be decoded by node m. Then $S_m^l \subseteq S_k^r$. Because node k and node m can decode each other. It is the same with the situation in figure 3.11 for node k and node g. By Lemma 6, S_m^l cannot be equal to S_k^r, thus $S_m^l \subset S_k^r$, as shown in figure 3.16.

In figure 3.15, by Lemma 7, if $S_k^r \subset S_g^l$ and it satisfies the condition in theorem 7, then $S_m^l \subset S_k^r$. Similarly, in figure 3.17, since $S_m^l \subset S_k^r$, then $S_h^r \subset S_m^l$. Subsequently, $S_q^l \subset S_h^r$ as shown in figure 3.17, and S_q^l does not exist. S_h^r can be decoded by node q, then by node h. This situation does not hold.

Proof of Theorem 7

As shown in figure 3.18, by Lemma 6 and Lemma 7, S_k^r needs to be larger than S_g^l, which is $S_g^l \subset S_k^r$. Subsequently, $S_k^r \subset S_h^l$. By the conditions of theorem 7, node h can decode at least one node in S_h^l which contradicts the definition of S_h^l. Therefore S_g^l does not exist and Supposition 1 does not hold.

The conditions in theorem 7 guarantee that each node can decode all the nodes on its left. With similar analysis, each node can decode all the nodes on its right. Above

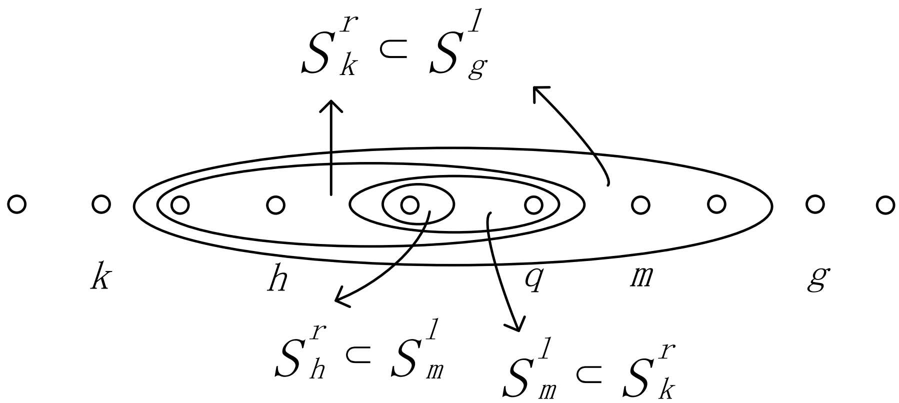

Figure 3.17: 1-dimensional general network in theorem 7

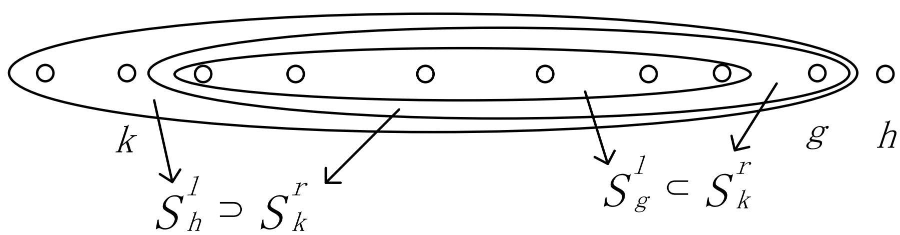

Figure 3.18: 1-dimensional general network in theorem 7

all, under the conditions in theorem 7, each node can decode all the other nodes by the regulated greedy relay scheme..

3.4.5 Proof of Theorem 8

Although all nodes are transmitting under the greedy relay scheme, the one-hop neighbors of every node still have the advantages of being decoded earlier. Thus under the greedy relay scheme, in the three networks we studied in 3.1.1, 3.1.2, 3.1.3, every node can still decode the current-block messages of all its one-hop neighbors. With faster pace, all other nodes can be covered under this greedy omnidirectional relay scheme.

3.4.6 Proof of Theorem 9

Case 1 : $i \le \frac{n}{2}$

In block b when $b \le i-1$, node i can decode $(w_{i-1}(b), w_{i-2}(b-1), \ldots, w_{i-b}(1))$ and $(w_{i+1}(b), w_{i+2}(b-1), \ldots, w_{i+b}(1))$ that node $i-1$ and node $i+1$ transmit respectively by (3.9) because they are symmetric, which means they have the same rate, the same transmit power, the same distance with node i and the same number of messages to transmit.

In block b when $b \ge i$, focusing on node i, node $i-1$ transmits the signal $(w_{i-1}(b), w_{i-2}(b-1), \ldots, w_1(b-i+2))$ which has $i-1$ messages, node $i+1$ transmits the signal $(w_{i+1}(b), w_{i+2}(b-1), \ldots, w_{\min\{i+b,n\}}(b-\min\{b, n-i\}+1))$ which has $\min\{b, n-i\}$ messages. Applying the Lemma 4.1 in [32] into (3.9), node i can decode node $i-1$ or node $i+1$ at the end of block b. Compared with node $i+1$, node $i-1$ transmits less messages, so node i can certainly decode node $i-1$ at the end of block b.

Since we have (3.9),

$$(n-1)R < \log(1 + \frac{\sum_{j=1}^{n-1} P_j}{N}) \tag{3.39}$$

then according to the concavity of the logarithmic function,for $1 \le k \le n-1$

$$kR < \log(1 + \frac{\sum_{j=1}^{k} P_j}{N}) \tag{3.40}$$

For any $l = 2, 3, 4, ..., i$, when $k = n - l$, we have

$$(n-l)R < \log(1 + \frac{\sum_{j=1}^{n-l} P_j}{N}) \tag{3.41}$$

Because

$$\begin{aligned} &\log(1 + \frac{\sum_{j=1}^{n-l} P_j}{N}) \\ =&\log(1 + \frac{\sum_{j=1}^{n-i} P_j + \sum_{j=n-i+1}^{n-l} P_j}{N}) \end{aligned} \tag{3.42}$$

and

$$l \le i < n-i+1$$

$$P_l > P_{n-i+1}$$

$$\sum_{j=l}^{i-1} P_j > \sum_{j=n-i+1}^{n-l} P_j$$

we get

$$(n-l)R < \log(1 + \frac{\sum_{j=1}^{n-i} P_j + \sum_{j=l}^{i-1} P_j}{N}) \tag{3.43}$$

By applying Lemma 4.1 in [32] into (3.43) with $l = 2$, it can be seen that node i can decode node $i-2$ or node $i+1$ with the knowledge of node $i-1$ at the end of block b. If it is node $i+1$ that can be decoded by node i, then we have come to the point that node i can decode node $i+1$. Otherwise, if it is node $i-2$ that can be decoded by node i, then (3.43) with $l = 3$ is needed to show that node i can decode either node $i-3$ or node $i+1$. Similarly, if it is not node $i+1$ that can be decoded by node i, then (3.43) with $l = 4$ is consecutively needed to show that node i can decode either node $i-4$ or node $i+1$. Until $l = i$, with the knowledge of all the signals sent from nodes on its left side, node i can decode block b messages of node $i+1$ at the end of block b. Thus in this case when $i \leq \frac{n}{2}$, node i can decode at least the current-block messages of its left and right one-hop neighbors.

Case II : $i \geq \frac{n}{2} + 1$

Node i can also decode the current block messages of both its left and right neighbors by (3.9). The proof is symmetric with that in case I.

Case III : $i = \frac{n+1}{2}$

In this case, node i is in the exact middle of the one-dimensional line. Because node $i-1$ and node $i+1$ are totally symmetric to node i, node i can decode the current-block messages of node $i-1$ and node $i+1$ under the condition (3.9) applied with the key technical lemma.

From the above discussion, it can be seen that no matter where node i is on the one-dimensional line, it can always decode the current-block messages of its two neighbors. Then under the neighboring omnidirectional relay scheme, node i can cover all the other nodes.

Chapter 4

Matlab Simulation

4.1 half-duplex 1-dimensional regular network

By comparing the matlab simulations of the achievable rates of the omnidirectional relay and the multi-hop relay in the half-duplex 1-dimensional regular network, the advantage of the omnidirectional relay is clearly stated.

By Theorem 4, the achievable rate for the half-duplex 1-dimensional regular network with the omnidirectional relay scheme is

$$R < \frac{1}{n} \cdot \frac{1}{n-1} \sum_{i=1}^{n-1} \log(1 + \frac{P_i}{N}) \tag{4.1}$$

The achievable rate for the half-duplex 1-dimensional regular network with the multi-hop relay scheme is

$$R < \frac{1}{n} \cdot \frac{1}{n-1} \log(1 + \frac{P_1}{N}) \tag{4.2}$$

Figure A.1 and figure A.2 show rates comparison in half-duplex one-dimensional regular network. In figure A.1, the dotted line represents the achievable rates with the omnidirectional relay scheme and the line represents the achievable rates with the multi-hop relay scheme. In figure A.2, when the signal attenuation becomes larger, the dotted line and the line drop more quickly.

4.2 half-duplex 2-dimensional regular network

By comparing the matlab simulations of the achievable rates of the omnidirectional relay and the multi-hop relay in the half-duplex 2-dimensional regular network, the advantage of the omnidirectional relay is clearly stated.

By Theorem 5, the achievable rate for the half-duplex 2-dimensional regular network with the omnidirectional relay scheme is

$$R < \frac{1}{m^2} \cdot \frac{1}{m^2 - 1} \sum_{i=0}^{m-1} \sum_{\substack{j=0 \\ i+j \neq 0}}^{m-1} \log(1 + \frac{P_{i,j}}{N}) \tag{4.3}$$

The achievable rate for the half-duplex 2-dimensional regular network with the multi-hop relay scheme is

$$R < \frac{1}{m^2} \cdot \frac{1}{m(m-1)} \cdot 2 \cdot \log(1 + \frac{P_1}{N}) \tag{4.4}$$

Figure A.3 and figure A.4 show rates comparison in half-duplex two-dimensional regular network. In figure A.3, the dotted line represents the achievable rates with the omnidirectional relay scheme and the line represents the achievable rates with the multi-hop relay scheme. In figure A.4, when the signal attenuation becomes larger, the dotted line and the line drop more quickly.

Matlab simulations clearly shows the advantage of the omnidirectional relay scheme. The advantage in the 1-dimensional network is more significant compared to the 2-dimensional one. In the 1-dimensional network, the node on the boundary only decodes the signal from its one-hop neighbor while in the 2-dimensional network the node on the corner decodes the signal from its two one-hop neighbors. As the distance goes further, the accumulated power decreases greatly. In the 2-dimensional one, the accumulated power received by the corner node from all the other nodes does not differ much from the added power received from its two 1-hop neighbours. So the advantage in the 1-dimensional network is more significant. This reason can be verified from the figure that as the attenuation factor becomes larger, the improvement becomes less obvious in the 2-dimensional network.

Chapter 5

Conclusion and Future Work

5.1 Conclusions

Based on the previous study[31, 32] of the achievable rate region of the all-source all-cast problem, the idea behind was questioned that all the subsets of all the nodes in this network are necessary. We start from the 1-dimensional and 2-dimensional half-duplex regular network to better understand the general network. Their optimal achievable rates are derived and proved in this thesis. Then for the general 1-dimensional half-duplex network, we proposed an achievable rate region with a new regulated greedy relay scheme, which shows that some consecutive sets may not need to be considered. This also indicates that greedy relay scheme is not optimal in the all-source all-cast problem. The key reason that the greedy scheme is not optimal is that offset encoding can help avoid some deadlock, which is similar as that stated in [20].

5.2 Future Work

Although the achievable rate region of the general 1-dimensional half-duplex network indicates that the greedy relay scheme is not optimal in the all-source all-cast problem, for the all-source all-cast network, it is still very hard to find the scheduling rule that brings improvement compared to the greedy relay scheme due to the difficulty of controlling of many flexible parameters.

The reason that we operate the network in the scheduled half-duplex mode depicted in 3.1 and figure 3.1 is just to eliminate the interference. This made the wireless network kind of like the wireline network, which is one shortcoming of this thesis. The regulated greedy omnidirectional relay scheme presented in theorem 7 and table 3.1 shows serious inefficiency before the operation goes to the steady state. This is basically due to the independence of each node. If there is cooperation in the network, which means if each node can know the decoding state of other nodes, the network can go into the steady state much faster. In the communication system, cooperation can always bring efficiency and rate benefits.

Further study will be done to explore and discover a better scheduling rule for the general network.

APPENDICES

Appendix A

Matlab Plots and Codes

A.1 half-duplex 1-dimensional regular network

In equation (4.1) and (4.2), in order to simplify the results, we make $N = 1$, $P_i = |g(id_0)|^2 P_0$, $P_0 = 1$, $d_0 = 1$, $g(x) = \frac{1}{x^\alpha}$.

When $\alpha = 2$, figure A.1 is the comparison figure.

When $\alpha = 4$, figure A.2 is the comparison figure.

A.1.1 Matlab codes

```
N=2:1:20;
K=N-1;
P=1./(K.^4);
F=log2(1+P);
R1=(1./N).*(1./K).*cumsum(F);
R2=(1./N).*(1./K).*F;
plot(N,R1,':ok',N,R2);
xlabel('number of nodes'), ylabel('rates');
hold on;
title('rates comparison in half-duplex one-dimensional regular network');
legend('the achievable rates with the omnidirectional relay scheme',
'the achievable rates with the multi-hop relay scheme');
```

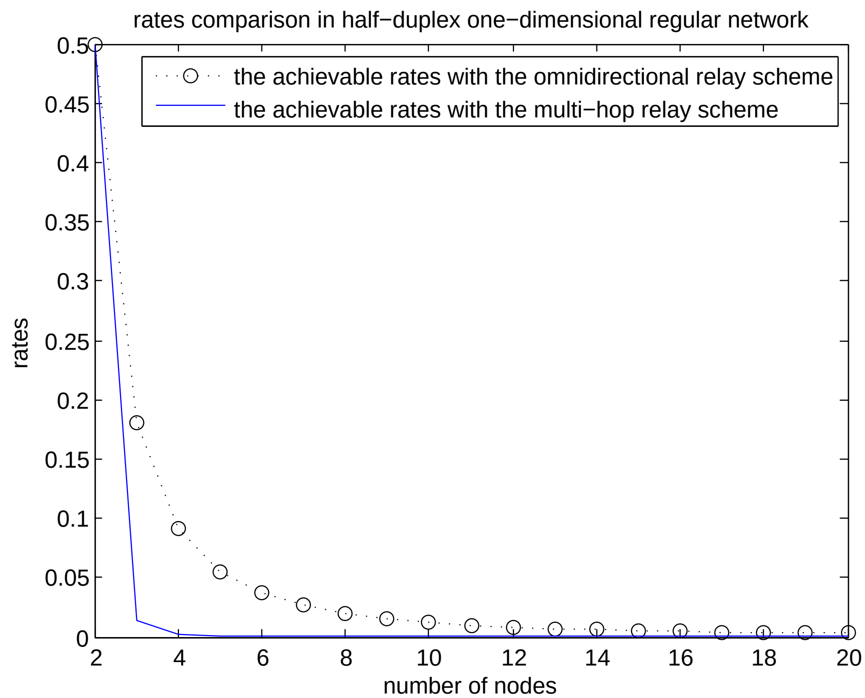

Figure A.1: rates comparison in half-duplex one-dimensional regular network

A.2 half-duplex 2-dimensional regular network

In equation (4.3) and (4.4), in order to simplify the results, we make $N = 1$, $P_{i,j} = |g(d_{i,j})|^2 P_0 = |\sqrt{i^2 + j^2} g(d_0)|^2 P_0$, $P_0 = 1$, $d_0 = 1$, $g(x) = \frac{1}{x^\alpha}$.

When $\alpha = 2$, figure A.3 is the comparison figure.

When $\alpha = 4$, figure A.4 is the comparison figure.

A.2.1 Matlab codes

```
D=15;a=2;
X=0:1:D;
Y=0:1:D;
K=X+1;
S=1:1:D;
```

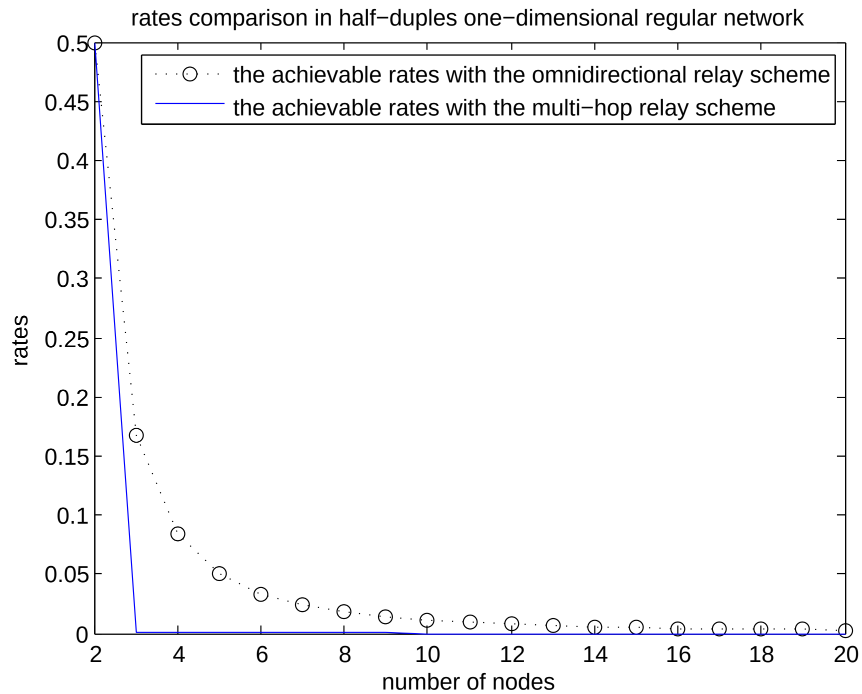

Figure A.2: rates comparison in half-duplex one-dimensional regular network

```
S(1:D)=K(2:D+1);
N=S.^2;
[new_X,new_Y]=meshgrid(X,Y);
P1=new_X.^2+new_Y.^2;
P1(1)=1;
P2=(1./P1).^a;
P3=log2(1+P2);
P3(1)=0;
F=1:1:D;
for T=1:D
F(T)=sum(sum(P3(1:T+1,1:T+1)));
end
R1=1./N./(N-1).*F;

R2=(1./N).*(1./(N-1)).*P3(1,2)*2;
```

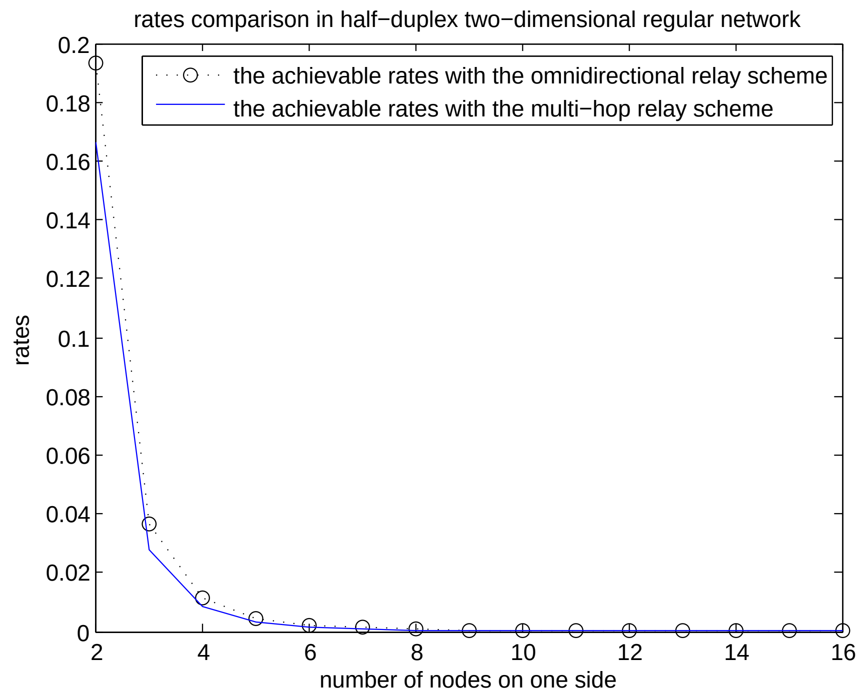

Figure A.3: rates comparison in half-duplex two-dimensional regular network

```
plot(S,R1,':ok',S,R2);
xlabel('number of nodes on one side'), ylabel('rates');
hold on;
title('rates comparison in half-duplex two-dimensional regular network');
legend('the achievable rates with the omnidirectional relay scheme',
'the achievable rates with the multi-hop relay scheme');
```

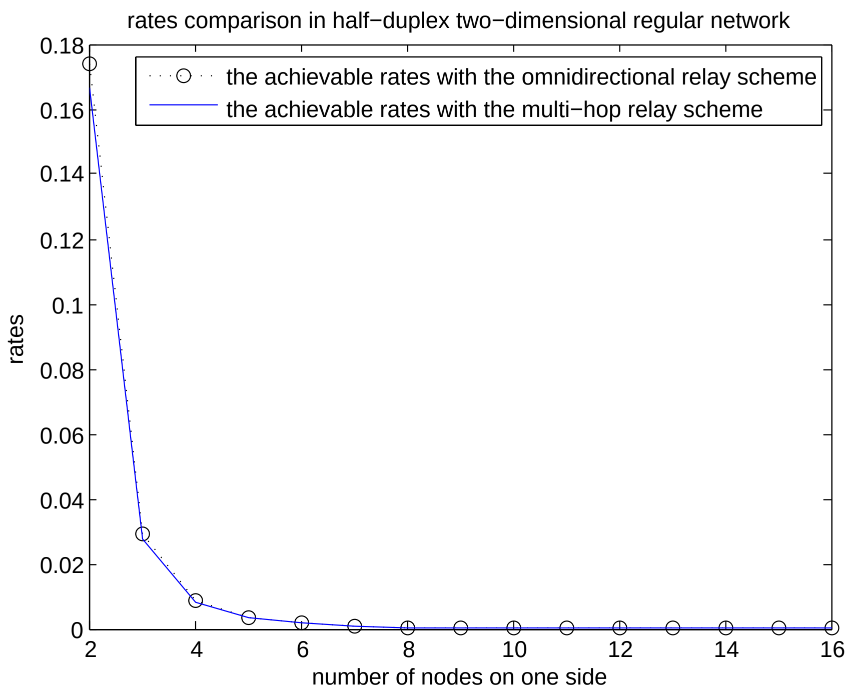

Figure A.4: rates comparison in half-duplex two-dimensional regular network

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