Constrained Rationality

Formal Value-Driven Enterprise Knowledge Management
Modelling and Analysis Framework
for Strategic Business, Technology and Public Policy
Decision Making & Conflict Resolution

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Majed Al-Shawa

Abstract

The complexity of the strategic decision making environments, in which businesses and governments live in, makes such decisions more and more difficult to make. People and organizations with access to the best known decision support modelling and analysis tools and methods cannot seem to benefit from such resources. We argue that the reason behind the failure of most current decision and game theoretic methods is that these methods are made to deal with operational and tactical decisions, not strategic decisions. While operational and tactical decisions are clear and concise with limited scope and short-term implications, allowing them to be easily formalized and reasoned about, strategic decisions tend to be more general, ill-structured, complex, with broader scope and long-term implications. This research work starts with a review of the current dominant modelling and analysis approaches, their strengths and shortcomings, and a look at how pioneers in the field criticize these approaches as restrictive and unpractical. Then, the work goes on to propose a new paradigm shift in how strategic decisions and conflicts should be modelled and analyzed.

Constrained Rationality is a formal qualitative framework, with a robust methodological approach, to model and analyze ill-structured strategic single and multi-agent decision making situations and conflicts. The framework brings back the strategic decision making problem to its roots, from being an optimization/efficiency problem about evaluating predetermined alternatives to satisfy predetermined preferences or utility functions, as most current decision and game theoretic approaches treats it, to being an effectiveness problem of: 1) identifying and modelling explicitly the strategic and conflicting goals of the involved agents (also called players and decision makers in our work), and the decision making context (the external and internal constraints including the agents priorities, emotions and attitudes); 2) finding, uncovering and/or creating the right set of alternatives to consider; and then 3) reasoning about the ability of each of these alternatives to satisfy the stated strategic goals the agents have, given their constraints. Instead of assuming that the agents' alternatives and preferences are well-known, as most current decision and game theoretic approaches do, the Constrained Rationality framework start by capturing and modelling clearly the context of the strategic decision making situation, and then use this contextual knowledge to guide the process of finding the agents' alternatives, analyzing them, and choosing the most effective one.

The Constrained Rationality framework, at its heart, provides a novel set of

modelling facilities to capture the contextual knowledge of the decision making situations. These modelling facilities are based on the Viewpoint-based Value-Driven - Enterprise Knowledge Management (ViVD-EKM) conceptual modelling framework proposed by Al-Shawa (2006b), and include facilities: to capture and model the goals and constraints of the different involved agents, in the decision making situation, in complex graphs within viewpoint models; and to model the complex cause-effect interrelationships among theses goals and constraints. The framework provides a set of robust, extensible and formal Goal-to-Goal and Constraint-to Goal relationships, through which qualitative linguistic value labels about the goals' operationalization, achievement and prevention propagate these relationships until they are finalized to reflect the state of the goals' achievement at any single point of time during the situation.

The framework provides also sufficient, but extensible, representation facilities to model the agents' priorities, emotional valences and attitudes as value properties with qualitative linguistic value labels. All of these goals and constraints, and the value labels of their respective value properties (operationalization, achievement, prevention, importance, emotional valence, etc.) are used to evaluate the different alternatives (options, plans, products, product/design features, etc.) agents have, and generate cardinal and ordinal preferences for the agents over their respective alternatives. For analysts, and decision makers alike, these preferences can easily be verified, validates and traced back to how much each of these alternatives contribute to each agent's strategic goals, given his constraints, priorities, emotions and attitudes.

The Constrained Rationality framework offers a detailed process to model and analyze decision making situations, with special paths and steps to satisfy the specific needs of: 1) single-agent decision making situations, or multi-agent situations in which agents act in an individualistic manner with no regard to others' current or future options and decisions; 2) collaborative multi-agent decision making situations, where agents disclose their goals and constraints, and choose from a set of shared alternatives one that best satisfy the collective goals of the group; and 3) adversarial competitive multi-agent decision making situations (called Games, in gamete theory literature, or Conflicts, in the broader management science literature).

The framework's modelling and analysis process covers also three types of conflicts/games: a) non-cooperative games, where agents can take unilateral moves among the game's states; b) cooperative games, with no coalitions allowed, where agents still act individually (not as groups/coalitions) taking both unilateral moves and cooperative single-step moves when it benefit them; and c) cooperative games, with coalitions allowed, where the games include, in addition to individual agents, agents who are grouped in formal alliances/coalitions, giving themselves the ability to take multi-step group moves to advance their collective position in the game.

Special modelling and analysis concepts and methods are offered within the framework to deal with the specific needs of multi-agent adversarial competitive decision making situations (conflicts or games). The framework defines formally: the conflict's states from the agents' alternatives; the conflict's structure (iterations, stages, or phases) from the conflict's states; the calculation of the agents' preferences over the conflict's states; the calculation of the preferences' strengths, for each of the agents, for each of the states; the different types of moves agents have within the three types of conflicts (noncooperative, cooperative without coalitions, and cooperative with coalitions); the stability and equilibrium solution concepts within these types of conflicts (and the interrelationship among these solution concepts and their respective strength sets); and the qualitative strength measures for stabilities/equilibriums, for each state, for each agent. In addition, the framework offers algorithms: to decide on the existence of a stability/equilibrium, for each state of the game, under each of the solution concepts; and to calculate the strength of this stability/equilibrium.

The research work used illustrative and exploratory case studies, and application examples, to demonstrate the effectiveness and benefits of using the Constrained Rationality approach in comparison to what the current dominant approaches provide. The cases cover a wide range of real-life strategic decision making situations and conflicts: the Cuban Missile Crisis (a non-cooperative historical political conflict); the Elmira Groundwater Contamination Conflict (a cooperative, without coalitions, historical governmental and environmental policy conflict); the showdown between RIM and NTP over intellectual property rights (a cooperative multi-phase, with coalitions, historical strategic business conflict); a multi-stakeholder requirements engineering collaborative decision making case (a hypothetical case study, based on a real-life industrial case simplified to fit the scope of our research context); a car manufacturer's strategic business decision to accept governmental bailout or file for bankruptcy (a hypothetical one-agent case study, based on real-life case simplified to fit the scope of our research context); Howard's personal dilemma (a hypothetical case, with high degree of emotionality, from the literature); in addition to some of the classical paradoxes of rationality games (the prisoner's dilemma, the iterative prisoner's dilemma, Tit for Tat, and the game of Chicken).

In all these case studies, and where there are models and analysis for them in the literature using other frameworks, we showed how the Constrained Rationality framework: performed better in addressing many of the limitations of current frameworks; provided better modelling and analysis facilities; captured more contextual knowledge about the case and the decision maker's motives and constraints; and provided more and better insight, learning and predictions. And, where there are no formal models and analysis exist in the decision and game theoretic literature for a conflict, such as the case of RIM v. NTP –an important business conflict with far reaching implications on patent laws and business practices–, we showed how the Constrained Rationality framework not only provided models and analysis for this multi-stage multi-game complex real-life strategic conflict, but also produced accurate predictions of how the confict would evolve.

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and

My Wife, Farihan

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My Son, Abdul Rahman

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Chapter 1

Introduction

Efficiency is doing things right; effectiveness is doing the right things.

Peter Drucker

Why do good, well managed, companies fail? Why did Digital Equipment Co. go down? Why did Nortel struggle to survive, and in its way to disappear? Why did IBM almost collapse? And how did it managed to recover? Why did Microsoft, with its less than perfect products, manage to survive, while Apple almost disappeared? Why did Sony fail to see the changes happening in its own market, while Apple, a complete outsider, recognized them? And why did Research In Motion (RIM), with its revolutionary product been heavily used and relied on by the important security and political establishment of the post 9/11 US, almost fail to maintain its presence in the US, its biggest market? All these companies, and many more, were well managed companies with exceptional sales and marketing arms that listened and measured customer needs and demands; all possessed exceptional funding, outstanding R&D, impressive intellectual capital portfolios; and most importantly, all had unparalleled access to all decision support techniques and tools.

The complexity of the strategic decision making environment in which businesses live in, especially these days, makes strategic decision making more and more difficult. In fact, Christensen (1997) argued that it is precisely because these firms, and the like, listened to their customers, invested aggressively in new technologies that provided their customers more and better products of the sort they wanted, and because they carefully studied market trends and systematically allocated investment capital to innovations that promised the best returns, they lost their positions of leadership, or worse failed to survive.

But, is this dilemma limited to the business domain? If so, how do we explain the US, with all its might and resources, failed to foresee the failures they faced in their war against Iraq? And how do we explain Britain's failure in the 1956 Suez War, the Tripartite aggression against Egypt? Did the US envision that Japan and Germany, two countries that lost the war, would become so powerful economically? Did the US envision China to become the largest creditor to the US government? And did the US envision the ousting of some of its most trusted and heavily supported allies in the Middle East, such as the Mubarak regime of Egypt and the Ben-Ali regime of Tunisia, by popular uprising? It is clear that the problematic strategic-decision-making dilemma is not limited to the business world, or even the political world. The problem lies in the nature of the decisions to be made, not on who make them.

So, what makes such decisions so hard that people and organizations with access to the best known decision support tools and methods cannot seem to benefit from such resources? Let us break the question to two parts: 1) what makes strategic decisions hard to make? and 2) why do the decision support tools and methods fail to help in the strategic decision making cases listed above? The answer lies in the characteristics of strategic decisions, and how they differ from other tactical and operational decisions; and, therefore, whether the decision tools and methods, used by decision makers, are designed to support strategic decision making.

1.1 Strategic vs. Operational Decisions

The USA Department of Defense (2008) defines three levels of military decisions: strategic, operational and tactical. The strategic level is concerned with the nation (or multi-nation) strategic security and military objectives and guidance, deciding on long-term global action plans, defining limits and assessing risks for use of military and other instruments of national power. The operational level is where campaigns and operations are planned, conducted, and sustained to achieve strategic objectives within theatres or other operational areas. The operational decisions link tactics and strategy by establishing operational objectives needed to achieve the strategic objectives. At the tactical level, battles and engagements are planned and executed to achieve military objectives assigned to tactical units or task forces. Decisions at this level focus on the immediate ordered arrangement and maneuver

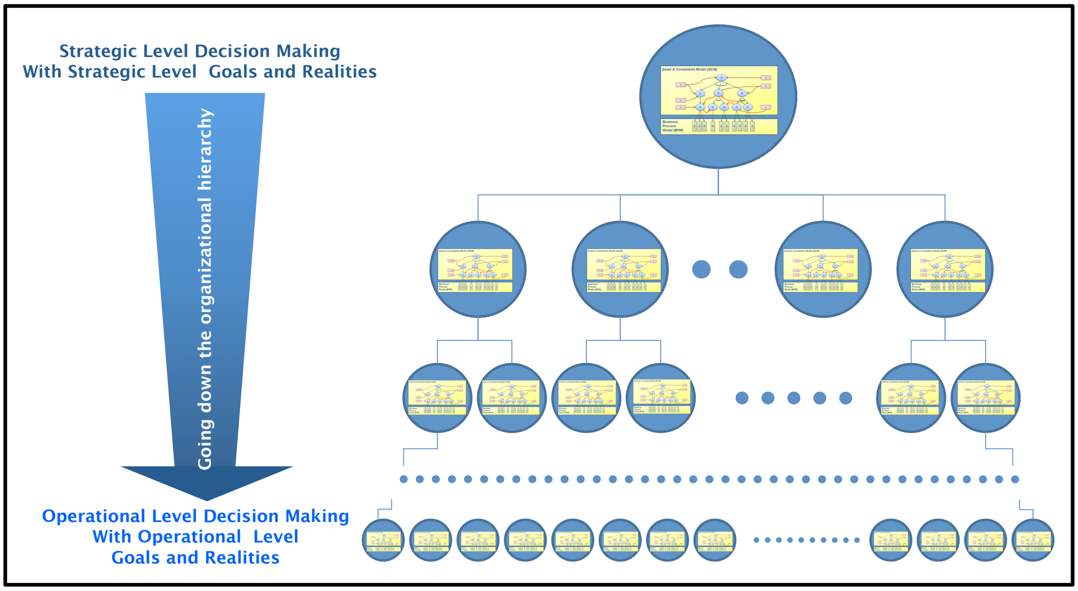


Figure 1.1: Goals and Decisions at the Strategic Level of the Organization versus at the Operational Level

of combat elements in relation to each other and to the enemy to achieve combat objectives.

And while the strategic goals and decisions are dealt with at the highest level of the US government and the military ranks, the operational and tactical ones are dealt with at lower ranks. Similarly, any other organization, whether governmental, business, or social, will have some form of these levels (strategic, operational and tactical). The nature (content and context) of the decisions at all these levels will differ from one organization to another, but the main characteristics of the decisions at these levels will mostly stay the same.

Figure 1.1 shows the hierarchal breakdown of goals and decision making environments within the organization. Strategic Goals are at the top of the organizational hierarchy, and are different from the Operational or Tactical ones at the middle and bottom of the hierarchy. While operational and tactical goals are clear and concise with limited scope and short-term implications, allowing them to be easily formalized and reasoned about, strategic goals tend to be more general, less clear, complex, with broader scope and long-term implications. As we go down the hierarchy shown in the figure, and as per the definitions provided by USA Department of Defense (2008), we see the following:

- Goals become clearer;
- Goals become less survival and prosperity type goals;

- Goals' scopes decrease;
- Level of control/influence over the environment increases;
- Goals increasingly become inter-related forming a complex network of dynamic affect and get-affected relations;
- Constraints increasingly shift from being external to being internal;
- Problems increasingly shift to be optimization problems (how to best use and allocate resources);
- Options/Alternatives increasingly become easier to identify and rationalize about; and
- Making the wrong decisions increasingly become less costly.

Strategic Decisions to address the strategic goals are enterprise-wide decisions that deal with the survival and prosperity of the origination, its directions, its partners, products to have, technologies to invest in, markets it wants to be in, lobbying it must do, etc. Conflicting goals, conflicting realities, internal constraints, outside regulations, and so on, are the norm of a strategic decision making environment. At the same time, departments and lower rank managers consider operational and tactical decisions that are limited scope decisions with the environment mostly well defined and under their control.

Even at the individual (human) agents level, strategic goals, such as balancing life and career goals or deciding on a career/educational path, seem difficult to deal with and very hard to mobilize or decided on. This is while mobilizing/deciding on tactical matters such as "what movie to watch tonight", or "which route to take to reach work today" are all easy decisions.

Dealing with the complexity of strategic decisions, and how to bring about the strategic goals the organization/individual have, is very challenging. In most cases, it is still considered more an art than a science. Many high ranking officers/directors, and people in general when facing strategic decisions, still rely on what they consider intuition, or gut feeling.

So, why not use sound mathematical quantitative and precise decision aids and tools, such as Decision Theory or Game Theory. Why do such theories fail in every day life to help decision makers, especially at the strategic level, to predict failures or see creative outcomes. What makes the strategic decision right? Is it maximization of expected utility? Is it finding the right Nash stable state? or something else?

1.2 Strategic Decision Support: Being Effective vs. Being Efficient

Answering the questions about why current decision analysis theories, such as decision theory and game theory, fail in modelling, and therefore aiding in analyzing, strategic decision making situations/conflicts, we look at whether these theories are designed to deal with decision making at the strategic level in the first place. We look at four major problems which any methodological framework to model and analyze strategic decision making situations must deal with.

1) The Incomplete-Knowledge Problem: Unknown or Incomplete Options, Utilities and Preferences

At the tactical level, options can be identified clearly, preferences can be elicited with no vagueness, and strict assumptions can be accommodated. But at the strategic decisions level, options are not complete/clear and preferences are harder to establish. Therefore, decision and game theoretic approaches can be used effectively at the tactical decision making situations (ignoring the criticism, the failures and the lack of use or interest by practice, which has been reported in the literature as we will see in Chapter 2). But, these theories, which require complete identification or determination of all possible options, as well as the utilities and/or preferences over them, could not be applied to situations where options are not complete, preferences are hard to establish, and utilities are very subjective and difficult to define, as it is the case when strategic decisions are considered. The strict assumptions imposed by these theories are unrealistic at best, as we will see in Chapter 2.

2) The Effectiveness vs. Efficiency Problem: Quantitative Analysis of the highly Qualitative Humanistic Systems

In his insightful early paper, Zadeh (1973) stated: "the conventional quantitative techniques of system analysis are intrinsically unsuited for dealing with humanistic systems or, for that matter, any system whose complexity is comparable to that of humanistic systems. The basis for this contention rests on what might be called the *principle of incompatibility*. Stated informally, the essence of this principal is that as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristic. It is in this sense that the precise quantitative analyses of the behaviour of humanistic systems are not likely to have much relevance to the real world societal, political, economic, and other types of problems which involve humans either as individuals or in groups".

The majority of the decision and game theoretic approaches, proposed to address the decision making problems, are limited in their use, not only because of their restrictive assumptions and pre-requisites, but also and most importantly because the majority of them deal with decision making, whether strategic or tactical, as quantitative-tactical-effectiveness-optimization problems. Framing the strategic decision making problem as such brings with it, to the problem definition, many troubling assumptions and restrictions. For example, if the strategic decision making problem is an optimization-effectiveness problem, then all the options/alternatives are known, and the problem is all about searching the options space to find an option that will maximize a benefit (or expected subjective utility). If this is true, how do we explain, or deal with, a creative decision maker "creates" a completely new set of options that were not part of the options space. The "out of the blue" phrase used casually to describe this dilemma could not be explained/modelled by using optimization decision or game theoretic approaches.

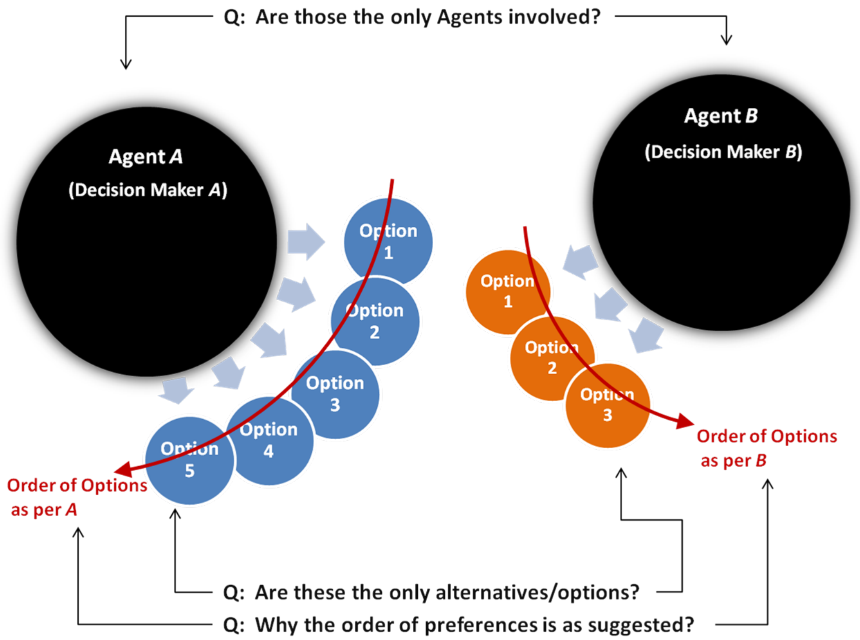
When the decision making problem is about increasing efficiency (or *doing the thing right*, as Drucker defined it in the quote presented at the beginning of this chapter), quantitative analysis methods and tools could be easily used. This happens usually at the operational/tactical decision making level, where the agents have all the data (all the options, all the resources needed, etc.), have a clear definition of the problem and how to solve it. For example, the decision making problems of an assembly line manager are about scheduling materials and people to achieve the optimal (maximized) production of the assembly line. All options are known, preferences well established, and utilities can easily be defined or found. This is similar to the case of a lower rank commander in the army. The objectives, plans and resources are all given to him. His job is to efficiently finish the mission utilizing the given resources and plan. The ultimate goal at this level looks more like the fine tuning of a mechanical machine with the purpose of maximizing the output.

In contrast to this, take the example of a CEO, or a General in the army, in one of the strategic decision making situations she might have. The theme of the strategic decision making situation will not be to increase efficiency (this is usually left to mid-to-lower rank managers/officers), but rather deciding on the best direction in which to take the whole organization/army. The question is more about effectiveness, rather than efficiency. Drucker, in the quote cited earlier, and discussed in depth in his landmark business strategy book (Drucker, 1967), calls this problem an effectiveness-problem, or a problem about *getting the right things done*, and considers it to be "the" type of problems/decisions that an executive level manager should be concerned with. The rest (the increasing-efficiency problems/decisions) should be delegated to lower rank managers.

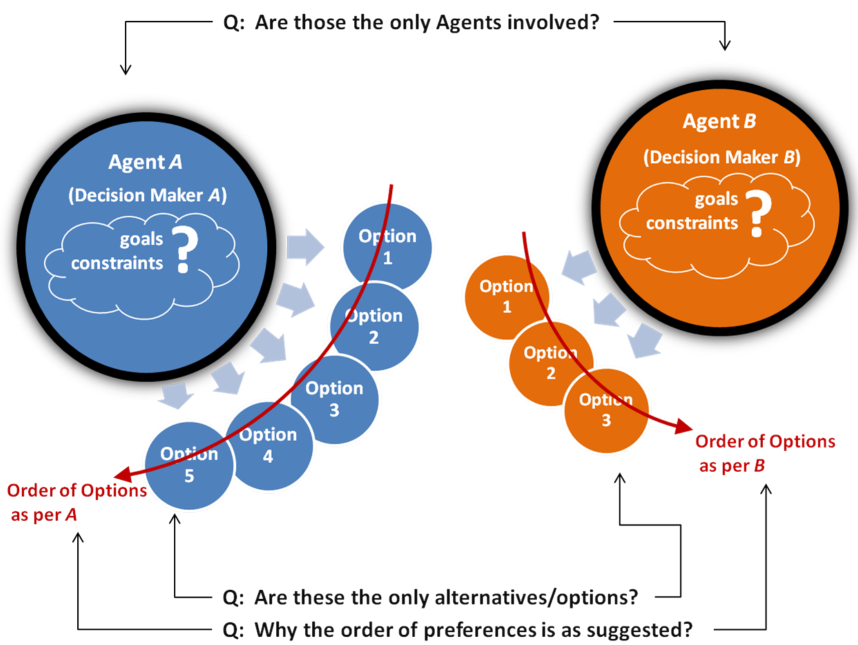
The effectiveness-problems are strategic problems that could make or break the organization. Decisions about such problems are ill-defined, with many factors involved, players are not all fully known, options are not all known, preferences are hard to decide on, and so on. In this type of decision scenarios, it is not about maximizing profit or minimizing costs (such as in the case of efficiency/optimization operational level decisions), but rather about leveraging and trade-offs, ensuring that all goals are achieved to some satisfactory levels for the time being, and all constraints are considered. Hardly anything could be quantitatively defined here. Things look as if they are defined in ranges and very subjectively. This is the qualitative nature of the strategic humanistic decision making systems (Zadeh, 1973). Compare this to the quantitative nature of the tactical/operational decision making.

3) The Black-Box-Agents Problem: Before there are Options to Analyze, there are Goals to Achieve and Satisfy

In reality, the lack-of-support exhibited by the current decision and game theoretic approaches for the needs of strategic decision making at its heart is a problem-framing problem. The decision making problem in general is not about choosing among options for the sake of choosing an option, but rather choosing among options to satisfy the strategic goals the agents (decision makers or game players –as these terms are used interchangeably in our research to mean the same thing–) have. Both decision theory and game theory, and by extension almost all related theories, deal with the agents' goals and motivation by establishing numeric strict proxies such as preference ordering and utility functions, instead of mapping how the options will help, support or hinder the achievement of the agents' multiple strategic goals.



(a) Decision Theory and Game Theory model the Decision Makers as Black-Box Agents.



(b) In reality Agents have goals that motivates their decision making experience and how they identify their respective options and evaluate them

Figure 1.2: Most of the current Rational Decision Making and Conflict Analysis theories deal with Decision Makers (Agents) as Black-Boxes, reasoning only about their pre-determined options and preferences. Goals and Constraints, that the involved agents have, are completely ignored, or dealt with through numeric strict proxies such as preference ordering and utility functions.

As shown in Figure 1.2, dealing with agents as black-boxes can be very dangerous. It changes the decision making problem. At the beginning, as Figure 1.2b shows, it was about: how the different options that the agent has, or could create, will contribute to her inter-related and complex set of needs and wants (goals), given all the internal and external realities/constraints of the environment and the situation. Decision theory and game theory changes the problem definition from one that is shown in Figure 1.2b to a new problem, same as the one shown in Figure 1.2a. The new problem is a about choosing one option/plan from a set of predetermined fully-known ones that each of the players (agents) has (no assumption of agents' creativity is allowed). These options are all ordered clearly and fully based on predetermined preferences (cannot change over time or have a gap among them at certain times).

In the black-box agents model, shown in Figure 1.2a, the purpose is to maximize a strictly defined subjective utility function/s, or find an equilibria based on the agents' preferences order. What had happened to the satisfaction the options supposed to bring to the inter-related and conflicting strategic goals the agents have? What will happen now if the agents' goals change over time and/or the agents' constraints change over time? Are these all the options that agents could have? How does an agent know if there is a need to become creative, or even ask for help to find another option in order to satisfy his ultimate strategic goals? How will the agent be able to tell which options have the best positive effect on his strategic goals, or be able to tell what constraints actually affect his ability to achieve his strategic goals? How will one be able to verify the correctness and completeness of the preferences given? and so on.

In reality, the modeller, analyst or consultant, usually meets with the decision maker/s to know his/their goals and constraints. Based on this acquired information (goals, constraints, and other information about who the players are, what are their options, and so on), he then defines the players' options and preferences. Then, suddenly all the original information disappear and only a set of options, preferences and utilities stay as part of the decision/conflict model. The problem now is not about goals to be achieved, but rather about these options and preferences. Now, if any of this information changes over time, it does not exist any longer as part of the decision/game model to validate/verify the changes against it, and therefore update the model accordingly. Clearly, there is a need to explicitly capture and include all the agents contextual information, namely the agents' goals and constraints, within the decision/conflict models the analyst builds.

Keeney (1992) acknowledged the need to go back to the root of the decision making problem: achieving the decision maker's strategic goals (and not just the mere choosing of one alternative among a pre-selected set of alternatives and using a pre-set cardinal preferences over such alternatives). In fact, Keeney went on to say: “”You do not control decision situations that you approach through alternative-focused thinking [in a reference to how decision theory and game theory model the decision making problems]. This standard mode of thinking is backward, because it puts the cart of identifying alternatives before the horse of articulating values [referring to the decision makers' goals and constraints]. It is values that are fundamentally important in any decision situation. Alternatives are relevant only because they are means to achieve your values. ...Many methodologies and techniques to aid decisionmaking have been developed over the past forty years. So why bother with yet another approach? Invariably, existing methodologies are applied to decision problems once they are structured, meaning after the alternatives and objectives are specified. Such methodologies are not very helpful for the ill-defined decision problems where one is in a major quandary about what to do or even what can possibly be done. Certainly if the alternatives are not known, one cannot characterize the decision problem by the alternatives. In addition, most decision methodologies try to find the best alternatives from a prespecified list. But where does this list come from?”

4) The Agent's Irrational-Behaviour Problem: Creativity, Emotions, Limitations and Biases

Furthermore, these current theories and approaches failed to explain what is been referred to as “irrational” behaviour of decision makers. The term “irrational” suggested by decision and game theory literature to describe some decisions made by decision makers, not because those agents show crazy insane foolish decision making behaviour, but rather because the decisions made by those agents do not fit the rational behaviour as defined by the theories. One has to wonder then, if Apple's decision to create the iPod, and go after a market that is not Apple's market, a market which is “the” market for giants like Sony, Panasonic, and Samsung, was an “irrational” decision, what good the “rational” decisions are or should be?

In business, the commonly used phrase “thinking outside the box”, which describes Apple's decision and many others, is in direct conflict with what decision and game theories suggest and propose. Not only because what Zadeh (1973) calls the *principle of incompatibility*, since these theories propose precise and quantitative methods to describe a completely complex and mostly qualitative humanistic world and systems, but also because these theories ignore the creative abilities of humans, and humanistic systems, to come up with new alternatives that did not exist before and never been thought of. Sometimes causing a complete shift in the paradigm of thinking.

In addition, many studies, as presented later in Chapter 2, discuss the effect of emotions, attitudes, cognitive limitations and biases on the decision making processes. These factors fail to be explained or account for using the current theories. Some of these factors are essential to analyze the agents' behaviour in conflict situations. For example, in political conflicts, players such as the US mostly have aggressive policies and actions, while others have soft reactions. Such attitudes could not be considered part of the preferences, because it could change suddenly in the middle of the game when circumstances and emotions change. There is a need to capture these factors within the model, and use them to guide the reasoning and analysis about the situation/conflict.

Constrained Rationality: Addressing the need for a Systematic and Methodological Qualitative Formal Modelling and Reasoning Framework for Strategic Decision Making

From above, it is clear that there is a need for a formal reasoning framework to address the challenges of dealing with strategic goals and decisions about them. This thesis document presents a new theory of rational strategic decision making. *Constrained Rationality* is a formal qualitative goals and constraints reasoning framework for the modelling and analysis of single and multi agents strategic decision and conflict. *The theory suggests bringing back the strategic decision making problem to its roots: reasoning about options, and alternatives, to satisfy the strategic and conflicting goals an agent (whether an individual, an organization, or a robot) has, given the internal and external complex and conflicting realities the agent has.* No linear equations are assumed. No single or multi variable utility functions are required. No probability distributions satisfying restrictive statistical assumptions are demanded. No criterion setting or pre-determination of options/alternatives is mandated. And, most importantly, no suppression of conflicts among goals and other knowledge concepts, in favour of compromises and consistent well-behaved agents/systems, is assumed.

Constrained Rationality is concerned with the effectiveness problem, i.e. finding the right strategy to adopt, or finding the right thing to do as Drucker put it, more than being concerned with the tactical efficiency problem. Constrained Rationality is a theory to explain how people, and humanistic systems/agents, can reason, create (be creative), realize limitations as well as opportunities, and deal with internal and external conflicts. And even though, it proposes a new way to look at the strategic decision making problem, it also tries to make use of the well established methods and techniques within decision and game theory. It will help these methods to capture formally what they missed, as well as validate their assumptions.

1.3 Research Objective

The objective of this research work: to propose a qualitative formal goals and constraints/context conceptual modelling and reasoning framework, for decision-making agent/s (also called players or decision makers) within single and multi-agent systems and environments, to use in order to effectively help the agent(s) systematically analyze his(their) strategic decisions and conflict situations.

The problem and application domains include: business strategic decision making, such as decisions about product development, R&D, intellectual property, market positioning; analysis of societal and political conflicts; and analysis of multiple stakeholders' requirements for optimal system design and development. The proposed framework should be able to handle the reasoning and analysis of decision making and conflict situations whether current or historic, and whether the changes happening to the conflict are tracked in real-time or as off-line simulated single or series of what-ifs.

The central argument (proposition) of the research is that:

By *structuring and conceptually modelling the strategic decision making problem (within single or multi agent decision making environment and whether the decision-maker/agent is an individual, an organization, a robot, or a coalition) by bringing the problem back to its roots: reasoning about options, and alternatives, to satisfy the strategic and conflicting goals each agent has, given the internal and external complex and conflicting realities (limitations and opportunities) each has.*

Then *more realistic modelling, insight and analysis of the strategic decision making situations, or the real-life conflicts, could be reached, allowing for:*

- *better understanding of the full extent of the players' options (current and potential alternatives)*
- *better decision making, and sensitivity analysis*
- *better stability analysis and outcome prediction*
- *better dynamic modelling and simulation of evolving and changing decision-making/conflicts*
- *better representation and testing of players' different patterns of behaviour (based on priorities, needs, wants and emotional states)*

Compared to *what alternative-focused (as called by Keeney (1992)) decision theory, and game theory, can provide. Not to mention that the proposed methodology, and the models it produces, can be used in fact to provide and/or verify as well as explain the proxy cardinal and/or ordinal preferences these theories use and need to order decision makers' options.*

Especially when applied to *strategic decision making, conflicts and problems where: options of players are not clear or unknown, preferences could not be defined or unclear, utility functions are hard to establish, and so on.*

1.4 Outline of the Thesis

The thesis research work argues that there is a need for a new reasoning framework for agent (and multi-agent) strategic decision making, that is based on agents' goals and constraints. Therefore, the thesis document starts, in Chapter 2, with a background and an analytical review of the current modelling and reasoning strategic decision making frameworks.

Chapters 3 and 4 discuss the main components of the Constrained Rationality Modelling and Reasoning Framework. Chapter 3 introduces the basic Constrained Rationality formal qualitative goals and constraints modelling and reasoning framework for strategic decision making situations. The chapter shows how goals and constraints of the different agents are modelled as viewpoint models, how the complex relationships these goals and constraints are modelled, and how value properties of the agents' goals, such as their achievement levels, propagate through these relationships until they are finalized. Chapter 4 shows how the framework proposes

Figure 1.3: Organization of the Thesis Document

to deal with and model the agents' priorities, emotions and attitudes towards acting rationally or emotionally. In addition, it will show how the agents preferences over their respective alternatives are elicited from the model.

As Figure 1.3 shows, Chapter 3 and 4, and because they introduce the core concepts and methods of the Constrained Rationality framework, present illustrative examples that focus on single agent decision making situations, or multi-agent situations where the agents act in an individualistic manner with no regard to others' current or future options and decisions. The real-life like multi-agent decision making situations, and how the Constrained Rationality framework aids in modelling and analyzing them, are discussed in the Chapters 5 - 9, as the figure shows.

Chapter 5 defines the two modes/types of multi-agent decision making situations: the collaborative situations and the adversarial competitive situations (these situations are also referred to as conflicts or games, as been called in the game theory literature). Then, it discusses the process of modelling and analyzing these two types of decision making situations, as it modifies and extends the Constrained Rationality concepts and methods introduced in Chapter 3 and 4 for the simple one-agent decision making situations. Chapter 5, also, provides an illustrative case study of a collaborative multi-agent system requirements engineering decision making situation. But, because of the complexity of multi-agent adversarial situations, or conflicts, the topic of discussing how Constrained Rationality proposes to analyze conflicts is not covered by Chapter 5. It is covered through out the following four chapters after that.

Chapter 6 defines the players' moves and the stability solution concepts (and their strengths) for non-cooperative strategic conflicts; and provides a thorough modelling and analysis of the Cuban Missile Crisis as a case study for such conflicts. Chapter 7 looks at the different characteristics the stability solution concepts introduced in Chapter 6; the theoretical interrelationships these solution concepts have among them; and how knowing about these characteristics and interrelationships brings more insight into the results of the conflict analysis process.

Chapter 8 defines the players' moves and the stability solution concepts (and their strengths) for cooperative strategic conflicts, where coalitions are not allowed/considered; and provides a thorough modelling and analysis of the Elmira Groundwater Contamination Conflict, a 1989 environmental policy conflict between the provincial government of Ontario and Uniroyal Chemical Ltd., as a case study for such conflicts. Chapter 9 defines the players' moves and the stability solution concepts (and their strengths) for the broadest type of multi-agent strategic conflicts: the cooperative conflicts where coalitions are allowed and considered. Chapter 9, then, provides a thorough modelling and analysis of the 2001-2006 showdown between Research and Motion (RIM) and NTP, over intellectual property rights, as a case study for cooperative conflicts where coalitions are involved.

Figure 1.4 provides a more detailed look at the Constrained Rationality framework as a process that branches at one point to three paths based on the type of decision making situation modelled and analyzed:

- single-agent decision making situations: covered mainly by the basic Constrained Rationality framework's concepts and methods introduced in Chapter 3 and 4. Recall, as we said above, that multi-agent decision making

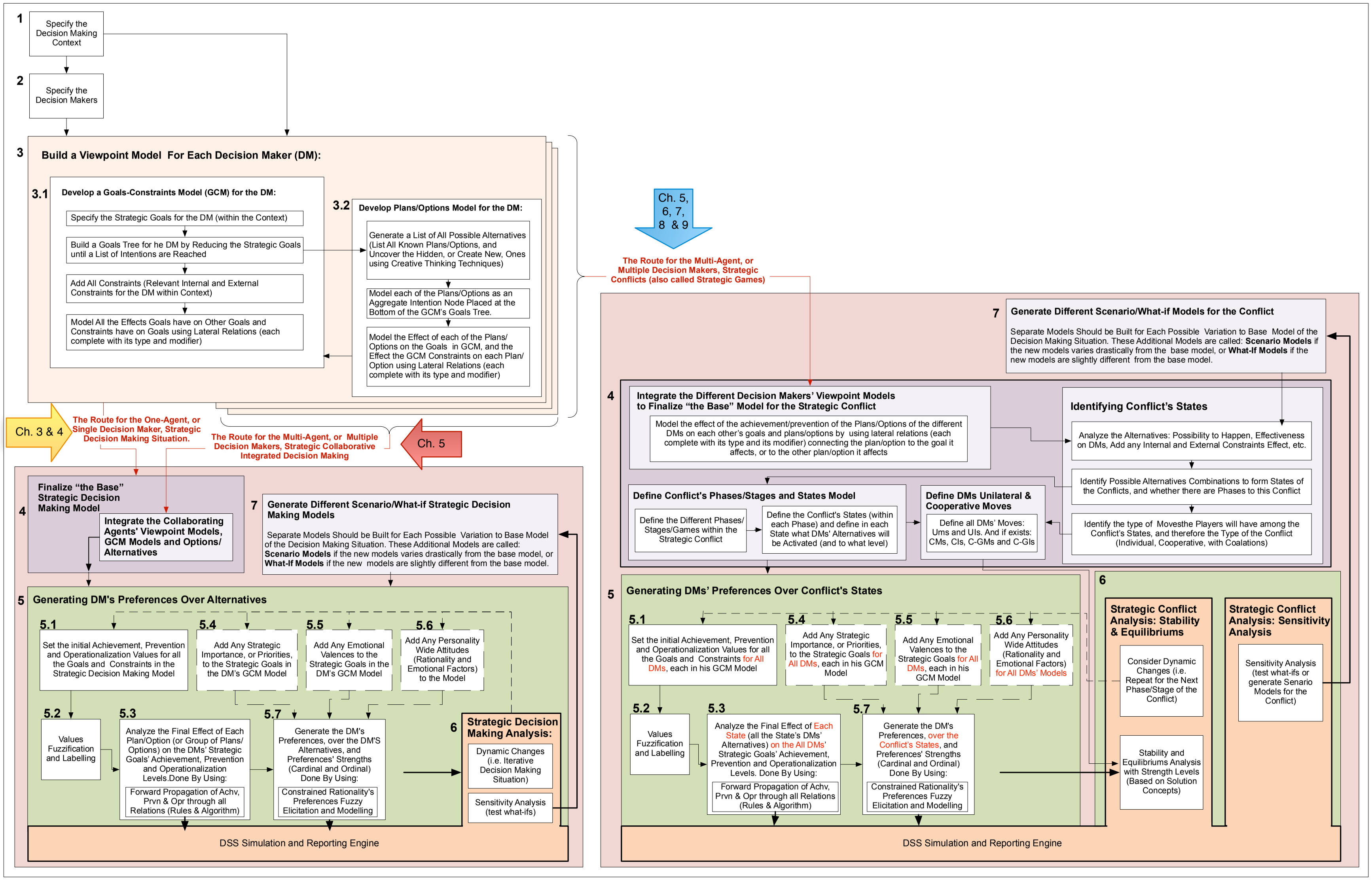


Figure 1.4: The Process of Modelling and Analyzing Single and Multi-Agent Decision Making Situations & Conflicts

situations where the agents act in an individualist manner, with no regard to others' current and future decisions and options, use the same concepts and methods the basic Constrained Rationality framework offer for modelling and analyzing single-agent decision making situations.

- collaborative multi-agent decision making situations: covered mainly in Chapter 5.
- adversarial competitive multi-agent decision making situations (also referred to as conflicts or games): an overview is provided in Chapter 5, with details given in Chapters 6, 7, 8, and 9 based on the type of the conflict (non-cooperative, cooperative without coalitions, or cooperative with coalitions).

Finally, the thesis finishes with Chapter 10 providing a statement about the contributions of the thesis work, and a list of suggested future work. The appendices provided additional material for Chapter 3. Namely, Appendix A provides the axiomatization of the Goal-Goal and Constraint-Goal Relationships introduced in Chapter 3; and Appendix B provides proof of soundness and completeness of the value-labels forward propagation algorithm defined in Chapter 3.

Chapter 2

Background and Literature Review

2.1 introduction

In this chapter, we present a review of how others addressed the research problem: how an agent can reason about her own goals and reality (constraints and context), and others' goals and reality, and therefore be able to strategize her next moves. Of course, we will be looking at models (representation) and methods for reasoning and planning that are computable, i.e. formal or could be formalized, and that could be integrated in a simulation tool and/or in a near-real-time Decision Support System (DSS). The review presented will make clear the fact that the modelling methodology usually goes hand in hand with the reasoning methodology used, because of the dependance and reliance of the reasoning method on how the agents, and their domain, or problems/games are been presented. In fact, as we will see later in this thesis, the goals and realities reasoning problem at its heart is a representation problem. The actual reasoning mechanisms are completely constrained and limited by the representation mechanisms offered.

Figure 2.1 shows a high level view of the research done regarding strategic decision making as led by different disciplines. We begin, in Section 2.1, by reviewing the the strategic decision making process and its challenges. We will end the section reporting on the gap between literature and practice regarding decision support models/aids. Section 2.2 reviews the typical mathematical and economical decision making and game/conflict analysis related work, which form the dominant formal

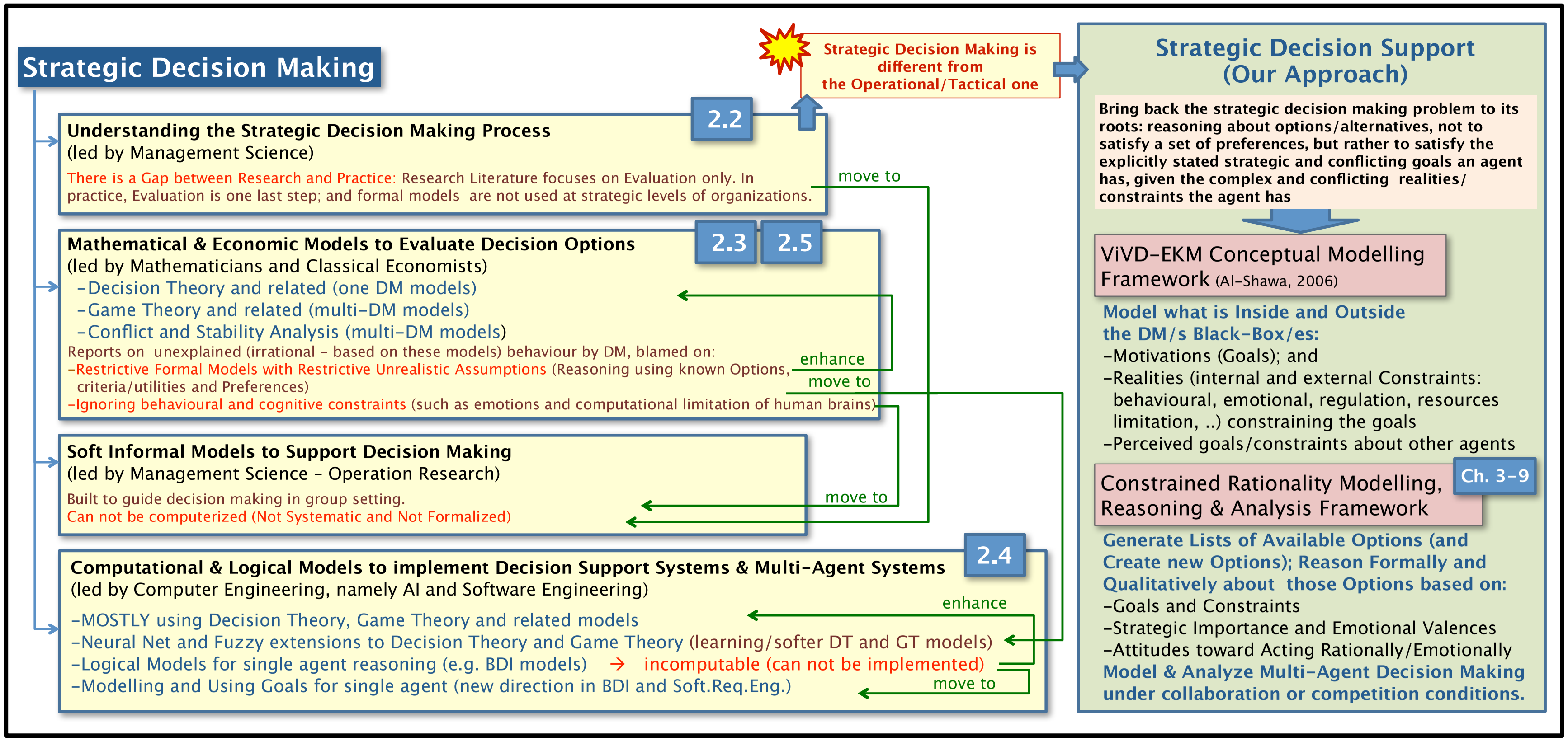


Figure 2.1: Research in Strategic Decision Making grouped in the left, as covered in this chapter. Green arrows show the moves by researchers from one area to another, in an effort to address the challenges discovered.

models available. Section 2.3 reviews additional models offered by Artificial intelligence (AI), especially within the Multi-Agents Systems (MAS) domain, to help agents reason about their goals and options. While most of MAS domain's research in this area uses and focuses on the typical mathematical and economical models, the domain shows increased interest to move beyond these limited models especially in the direction of modelling and reasoning about goals and beliefs. In Figure 2.1, we show how research flowed (shown as green arrows) to other areas in an effort to address the challenges discovered (shown in red text). One area of research shown in the figure, but we will not cover the Soft Informal Systems Models (proposed by Checkland (1999)) because these models are highly informal and mainly used to guide groups through problem solving/modelling sessions. In Section 2.4, we will look at the question of rationality and biases, and end the chapter with a summary and conclusion.

2.2 Strategic Decision Making Process

Before we start analyzing the strategic decision making process, let us clarify some of the terms that will be used in this proposal. We will use here the definitions used by Mintzberg et al. (1976).

First, we define a *decision* as a specific commitment to action or plan (usually this plan involves some form of commitment to allocate or use resources - structured/physical or intellectual resources). Second, we define a *strategic decision process* as a set of actions and dynamic factors that begins with the identification of a stimulus for action and ends with the specific commitment to action. We will assume that the decision processes we will refer to here are strategic decision processes that have not been encountered in quite the same form and for which no predetermined and explicit set of ordered responses exists in the organization. This assumption is consistent with the fact that most strategic decisions are not momentary, but take time and require some form of a process or stages/phases to come about. And finally we define *strategic* to simply mean highly important, long-term and enterprise-wide, in terms of the actions taken, the resources committed, or the precedents set.

We can categorize the strategic decision making processes proposed in the literature to two categories: *sequential* and *non-sequential*. The sequential type is the oldest and predominant , while the non-sequential is the most recent and realistic process. We will briefly look at each category and some of the process models proposed in it. Then we will comment on them.

2.2.1 Sequential Strategic Decision Making Process Models

We can trace back the first general theory to define stages for the decision making process to the French philosopher and mathematician Condorcet (1743-1794) in his motivation for the French constitution of 1793. Condorcet identified three stages for the decision making process. The first stage is concerned with "discuss(ing) the principles that will serves as the basis for the decision in a general issue",and "examine(ing) the the various aspects of this issue and the consequences of different ways to make the decision". At this stage the opinions formed are personal, with no attempts made to form a majority. The second stage is a discussion stage where "the question is clarified, opinions approach and combine with each other to a small number of more general opinions". The second stage discussion produces a manageable set of alternatives, which the third stage effort is concentrated on choosing among them [Hansson (2007) reporting on Condorcet (1793)].

The insightful distinction that Condoncet has between two the first two stages of the decision making process is virtually forgotten and not been referenced or reflected on in modern decision theory (Hansson, 2007). Nevertheless, this distinction is important. Why? Because it separates between what Condoncet considers a personal discussion of the issue at hand leading to the formation of one's opinion/s about it, as part of the first stage of the decision making process, and the analysis of the issue in its full social context (or a *game*, as has been called in modern decision making and game theories literature). This second stage, thinking about the issue as a game, will lead to the formation of the set of options, or alternatives, that one should consider to choose among them.

No account for serious discussion about the decision making process was conducted, as far as we know, until Dewey (1910). According to Dewey, each reflective thought, and problem solving, instance has five logically distinct steps: (1) a felt difficulty; (2) defining the difficulty; (3) suggestion of possible solutions; (4) development by reasoning of the bearings of the suggestion; and (5) further observation and experiment leading to the acceptance or rejection of the suggestion.

Herbert Simon took Dewey's five-steps problem-solving process and modified it to fit the context of decision making within organizations. According to Simon

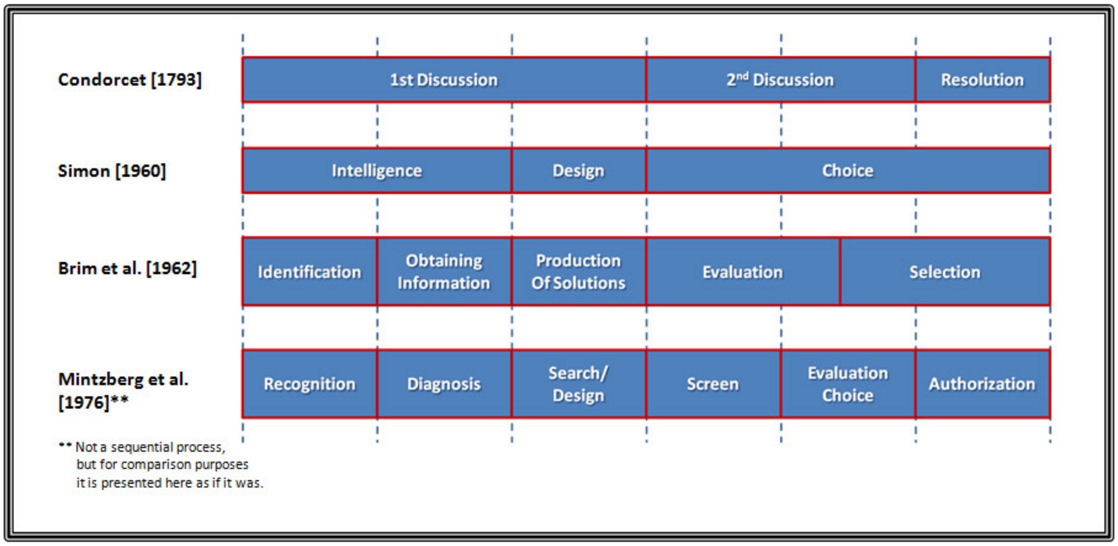


Figure 2.2: Comparing the Decision Making Processes' Stages as proposed by Condorcet (1793), Simon (1960), Brim et al. (1962), and Mintzberg et al. (1976)

(1960), the decision making process has mainly three distinct phases: (1) the *intelligence* phase: finding occasion for making a decision; (2) the *design* phase: finding possible courses of action; and (3) the *choice* phase: choosing among courses of action.

Later, Brim et al. (1962) proposed their decision process which included six phases: (1) identification of the problem; (2) obtaining necessary information; (3) production of possible solutions; (4) evaluation of possible solutions; (5) selection of a strategy for performance; and finally (6) implementation of the decision.

The decision making process as been proposed by Condorcet (1793), Dewey (1910), Simon (1960) and Brim et al. (1962) is a *sequential process* with the decision maker going through the process one phase/stage at a time. Once the stage at hand is completed, the decision maker then moves to the next stage in the process. The process under all decision making contexts will always have the same stages, and those stages come in the same order, as can be seen in Figure 2.2.

Many had disagreed with this notion of a sequential decision making process and criticized it as unrealistic and unpractical. For example, Witte (1972) reports on empirical studies done by him when he observed that these decision making stages are done actually in parallel rather than in sequence; and that "we believe that human beings cannot gather information without in some way simultaneously developing alternatives. They cannot avoid evaluating these alternatives immediately, and in doing this they are forced to a decision. This is a package of operations and the succession of these packages over time constitutes the total decision making process" (Witte, 1972).

A more realistic decision making process model in the view of Witte (1972) and others is a model which does not define the order of the stages in a rigid predefined fashion, but rather demonstrates flexibility by allowing different formations of these stages as demanded by the decision making instance's context and circumstances.

2.2.2 Non-Sequential Strategic Decision Making Process Models

One of the most well known non-sequential model for strategic decision making is the one proposed by Mintzberg et al. (1976). The model, which is shown in Figure 2.3, consists of three phases: (1) identification; (2) development; and (3) selection phase. These three phases are equivalent to the ones proposed by Simon (1960), shown in Figure 2.2 above, and could be easily mapped to Simon's intelligence, Design, and Choice phases. But the three phases that Mintzberg et al. (1976) proposed in their model have no simple sequential relationships connecting them.

1. The ***identification*** phase consists of two routines:

   (a) ***decision recognition*** routine: in which "opportunities, problems and crisis" are recognized in "the streams of ambiguous, largely verbal data that decision makers receive"

   (b) ***diagnosis*** routine: in which the decision maker "seeks to comprehend the evoking stimuli and determine cause-effect relationships for the decision situation", this is usually done by "tapping of existing information channels and the opening of new ones to clarify and define the issues"

2. The ***development*** phase is the heart of the decision making process and has two routines:

   (a) ***search*** routine: is evoked to find obvious alternatives and "ready-made solutions"

   (b) ***design*** routine: is evoked to find new alternatives, and to "develop custom-made solutions or modify ready-made ones" usually using complex iterative procedures

3. The ***selection*** phase consists of three routines:

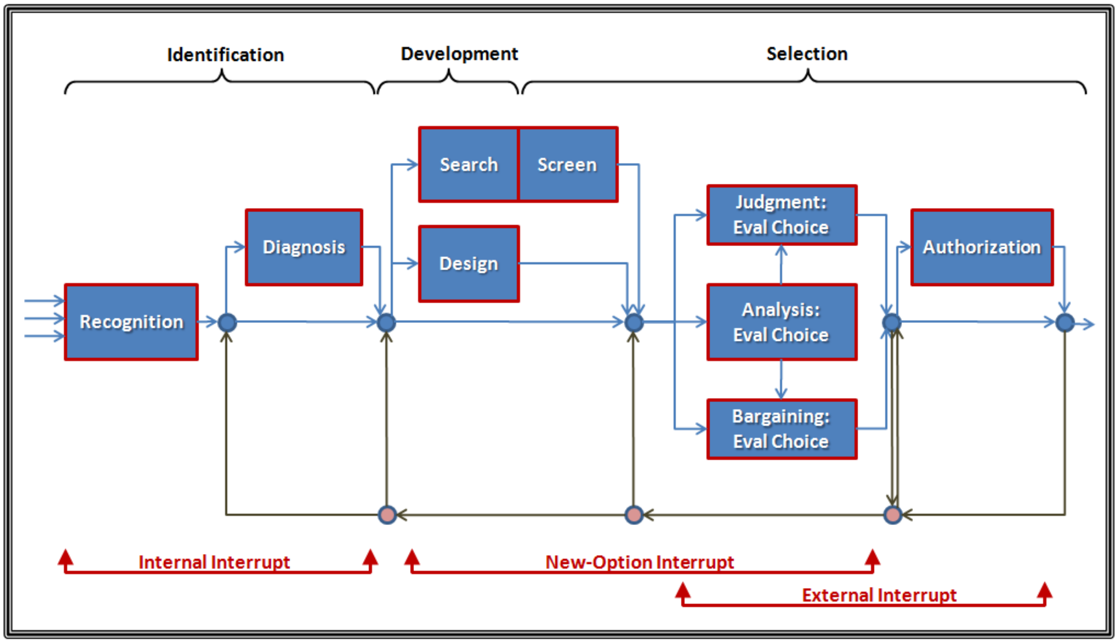


Figure 2.3: Comparing the Decision Making Processes' Stages as proposed by Mintzberg et al. (1976)

(a) ***screen*** routine: is evoked when the search routine above "generates more ready-made alternatives than can be intensively evaluated"; it is a "superficial routine more concerned with eliminating what is infeasible than with determining what is appropriate"

(b) ***evaluation-choice*** routine: during which the actual choice between alternatives happen, and may be considered to use one or more of three modes: (1) judgment, in which choice is made "with procedures that one cannot explain", i.e. intuitively; (2) bargaining, in which choice is made by "a group of decision makers with conflicting goal systems, each experiencing judgement"; and (3) analysis, where "factual evaluation is carried out, generally by technocrats, followed by managerial choice by judgement or bargaining"

(c) ***authorization*** routine: in which the decision follow "a tiered route of approval up the hierarchy and perhaps also out to parties in the environment that have the power to block it"

In their final analysis of their model and empirically testing it across many strategic decision making scenarios, Mintzberg et al. (1976) found that "the delineation of steps in almost any strategic decision process shows that there is not a steady, undisturbed progression from one routine to another; rather, the process is dynamic, operating in an open system where it is subjected to interferences, feedback loops, dead ends, and other factors. These dynamic factors are perhaps the most characteristic and distinguishing features of decision processes that are strategic. . . . We find in our study that dynamic factors influence the strategic decision process in a number of ways. They delay it, stop it, and restart it. They cause it to speed up, to branch to a new phase, to cycle within one or between two phases, and to recycle back to an earlier point in the process".

According to Mintzberg et al., the relation between their model's phases and routines is circular rather than sequential or linear, as been illustrated in Figure 2.3. The decision maker "may cycle within identification to recognize the issue during design, he may cycle through a maze of nested design and search activities to develop a solution during evaluation, he may cycle between development and investigation to understand the problem he is solving . . . he may cycle between selection and development to reconcile goals with alternatives, ends with means".

2.2.3 Strategic Decision Making Process: in Practice vs. in Literature

While the literature on strategic decision making has focused solely, for the largest part of it, on the evaluation-choice routine of the process's selection phase, the reality of the mater is decision makers spend the majority of their time in activities related to the first two phases of the process. First, Simon reported that executives spend a large fraction of their time in intelligence activities, an even larger fraction in design activities, and only a small fraction in choice related activities (Simon, 1960). Then, Mintzberg et al. (1976) confirmed this with the findings of their empirical study.

According to Mintzberg et al., out of the 25 strategic decision making processes they studied, 21 processes has the development phase activity appeared to dominate the other two phases' activity. This was "rather curious" to them, since "by far the largest part of the literature on strategic decision process has focused on the evaluation-choice routine" and that "this routine seems to be far less significant in many of the decision processes we studied". Mintzberg et al. also noted that:

> "The normative literature emphasizes the analytic mode, clearly distinguishing fact and value in the selection phase. It postulates that

> alternatives are carefully and objectively evaluated, their factual consequences explicitly determined along various goal, or value, dimensions and then combined according to some predetermined utility function a choice finally made to maximize utility. A more pragmatic rendition of this view sees the analyst presenting his factual analysis of the consequences of various alternatives to the manager who determines the value trade-offs in his head and thereby makes a choice.
>
> Our study reveals very little use of such an analytic approach, a surprising finding given the importance of the decision processes studied..."

In fact they went on to cite other empirical studies which also provide little evidence to support the prevailing normative views of decision making. For example, they cite the work of Soelberg (1967) and Carter (1971a,b) who have addressed the use of utility functions within the decision making process, and found no evidence to support their existence. In addition to Soelberg and Carter, Cyert et al. (1956) also "note rather that the criteria used in decision processes are multiple and noncomparable. No study finds that even weightings on individual goal dimensions are established in advance of making choices; rather the weights are determined implicitly, in the context of making choices". In fact, Soelberg (1967) goes a step further describing a confirmation period, the decision maker has before announcing his decision, during which he rationalizes to himself his implicit choices as well as the goals it represents. This means that the determination of criteria in effect follows the making of choice.

These observations lead us to the discussion of rationality within the context of strategic decision making. But let us first, present a review for some of most important and relevant rationality models for strategic decisions making and conflict analysis.

2.3 Mathematical and Economical Models of Rational Strategic Decision Making and Conflict Resolution

2.3.1 Decision Theory

Decision theory is a mathematical theory of rational decision making. Decision theory defines a rational agent as one that maximizes expected utility. It is a prescriptive approach to help an individual make a choice among a set of pre-specified alternatives/outcomes. The generation of alternatives is not generally addressed within decision theory, except where particular analysis of options can lead to the suggestion of new ones. The usual analysis has two components: an uncertainty analysis and a utility (preferences) analysis (Keeney and Raiffa, 1976).

Decisions can be analyzed in the following way. The set of alternative plans/actions is given and pre-specified $\{P_1, \ldots, P_m\}$. There is also a pre-specified set of criteria of concern $\{C_1, \ldots, C_n\}$. Each of the actions then can be evaluated for each criteria, yielding a vector of values for each action, and forming what is usually referred to as a decision matrix. Comparison between two actions involves the comparison of the two vectors, and weights can be used to give emphasis to criteria (objectives). The most common way to represent the values in the action-to-criteria vectors is to assign utilities to them. And, the most common way to represent the criteria weights is to assign probabilities to them.

Mainstream decision theory is almost exclusively devoted to problems that can be expressed in matrices of this type, utility matrices. With Expected Utility, or probability-weighted utility, playing the dominating approach or the major paradigm in decision making since the Second World War (Schoemaker, 1982). As per this paradigm, each alternative is assigned a weighted average of its utility values under different criteria (or sometimes states-of-nature), and the probabilities of these criteria/states are used as weights (Russell and Norvig, 2003). Mainstream decision theory is almost exclusively devoted to problems that can be represented in matrices of this type, expected utility matrices. As a result, most modern decision theoretic methods require numerical information about these expected utilities.

There are, of course, a number of complications to this decision theoretic representation of decision making models. First, the criteria items are most likely to be of incommensurable units, making direct comparisons difficult. Intangibles, especially psychological aspects, wile need to be taken into account, they are usually difficult to measure and scale. Time also has an effect, as consequences will affect the next decisions to be taken. Finally the uncertainties involved require consideration of multi-variate probability distributions (Bell et al., 1977). The interdependence among the criteria items is also usually ignored.

In addition, in many practical decision problems, especially the strategic decision problems, we have much less precise value information (perhaps best expressed by an qualitative information rather than quantitative numerical ones). This is at the heart of what Zadeh (1973) refers to as the *principle of incompatibility* ("the conventional quantitative techniques of system analysis are intrinsically unsuited for dealing with humanistic systems or, for that matter, any system whose complexity is comparable to that of humanistic systems"). However, the position that most decision theoretic methods took, with few excepts which we will refer to below, is that it is much more difficult to construct methods that can deal effectively with non-numerical information. Hence, the domination of numerical information in decision-theoretic methods' representation of decision problems.

As characterized by decision theory, decisions involve individual agents. The agent will be making a choice among a number of pre-determined pre-specified options (not deduced) based on some evaluation of the options. The criteria of evaluation used is also pre-specified, and not deduced. This may be contrasted with a game/conflict, in which there is more than one agent, each with a different perspective, a different set of strategic goals, and different set of constraints and realities/context.

Decision theory has a set of very restrictive assumptions. So, if, in order to deal with multiple agents conflicts, a new list of options is generated in which the options of the individual agents are combined to represent different outcomes for the decision, the usual assumptions of decision theory do not apply (Galbraith, 1977). While decision theory does allow for alternative evaluations to be taken into account, this is to allow for uncertainty, and then probability theory to be used to analyze the alternatives. If the differences arise not from uncertainty but from different perspectives, no probabilistic analysis can be used. This does not mean that decision theory has no role in conflict resolution. Agents can still use decision theory individually, to decide whether to accept a particular option/plan, to justify such decisions, to decide what action to take next in the process, and to persuade the other agents that a solution is satisfactory.

Multi-agent decision making needs are ignored within the classical decision theory. Although decision analysis attempts to prescribe approaches for each individual agent, in which the actions of other agents can be regarded as uncertainties, there is no means to handle multi-agent decision making. Keeney and Raiffa (1976) use this observation to suggest uses of decision analysis for agent's personal conviction, advocacy and reconciliation. This is not to say that the theory is not useful, the theory can still in conflict resolution situations provide, to each of the involved agents, supporting evidence for the various options the agent has. This is assuming that the options are all completely known to the agent before the analysis starts.

The evolution of decision theoretic methods over the years focused into dealing with the challenges and strict restrictions come to exist because of the the dominant expected utility approach to model decision making problems. Some focused on dealing the probabilistic knowledge about the decision making situation, to the point that Luce and Raiffa (1957) categorized decision making based on how complete/reliable this probabilistic knowledge to: decisions under certainty, under risk, and under uncertainty. Alexander (1975) added the category of decisions under ignorance (Luce and Raiffa (1957) referees to this category as extreme uncertainty).

But the literature show many views on what complete, and what reliable, probabilities means. For some only objective probabilities (actual frequencies of things happening in real life) should be considered . Others allow for subjective estimates of (objective) probabilities to be considered. Yet, others considered probabilities to be a purely mental phenomenon represented by a degree of belief (Bayesian probabilistic beliefs, or subjective (personalistic) probability). leading to what seems a dominant acceptable. But, as we know objective probabilities are hard, or impossible, to establish in strategic decision making situations, where every decision is different and does not repeat as often. Therefore, the most used probabilities are the subjective estimates, which are often unreliable (Tversky and Kahneman, 1986; Lichtenstein et al., 1982), and the Bayesian probabilistic beliefs, which are also criticized by many as being more popular among statisticians and philosophers than among more practically oriented decision scientists. An important reason for this detachment of Bayesianism from real practical use is that it is much less operative than most other forms of expected utility/probabilities. Theories based on objective (or subjective estimates of objective) utilities and/or probabilities more often give rise to predictions that can be tested. It is much more difficult to ascertain whether or not Bayesianism, with its strict assumptions, is violated (Weirich, 1986; Tversky and Kahneman, 1981, 1986).

The literature in decision theoretic methods show also a focus on dealing with the concept of utilities as a proxy measure of the decision makers objectives, needs, wants, and well being. Some offered variations to the expected utility approach. For example, Loomes and Sugden (1982), Bell (1982) and Sugden (1985) offered the use of a two-attribute utility function that incorporates two measures of satisfaction: utility of outcomes, as in expected utilities; and a quantity of regret (defined as the painful sensation of recognizing that “what is” compares unfavourably with “what might have been"). Sowden (1984) offered to add numerical values to represent attitudes towards risk and certainty, forming what is referred to as process utilities or a generalized expected utility (criticized by Weirich (1986); Luce and Raiffa (1957) and others for including double counting of attitudes). Kahneman and Tversky (1979), and Tversky and Kahneman (1981), offered what they refer to as prospect theory, a descriptive theory with no normative claims that is developed to explain the results of their experiments with decision problems that were stated in terms of monetary outcomes and objective probabilities. One of the unique features is that it distinguishes between two stages in the decision process: 1) editing phase "to organize and reformulate the options so as to simplify subsequent evaluation and choice", identifying the gains and losses for the different options; and 2) evaluation phase of these options using two scales, one replaces the monetary outcomes given in the decision problem and the other replaces the objective probabilities given in the decision problem.

The literature suggests many other forms of utility functions to deal with specific decision making situations under deferent levels of completeness of the situations' probabilistic knowledge and type of this knowledge. But, some went on to eliminate the need for utility cardinal functions altogether replacing them with simple ordering/scoring methods, especially when the decision situation has many options to be ordered/scored based on multiple criteria/attributes, and when utility functions are hard to establish (as it is the case in most strategic decision making situations). For example, Hwang and Yoon (1981) described a Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method. The basic principle of it is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution; Roy and Vanderpooten (1996) tacked about the ELECTRE method, proposed originally on 1966, that offers an outranking relations to compare pairs of actions/options; and Saaty (1980) proposed the Analytic Hierarchy Process (AHP), one of the most known ranking multi criteria decision making analysis tools.

But, unfortunately many of these methods, including TOPSIS, ELECTRE and AHP, suffer from many weaknesses that made them the target for many criticism by researchers and practitioners alike. One of these is the most severe criticism related to what came to be known as the rank reversal problem in which these methods tend to show contradictory results (ranking of options) for the same/similar problems. In particular,when a new decision alternative is added to a decision problem, and while the assessments concerning the original decision alternatives remain unchanged, the new alternative may cause rank reversals between the ranks or the utility estimates of the original decision alternatives (Leskinen and Kangas, 2005). For example, Belton and Gear (1983) and Triantaphyllou and Mann (1989) explain types of the rank reversal problem for the AHP method.

Group Decision Making is an area that is related to decision theory. It is a normative study of how individual preferences can be combined into a group decision. Arrow (1967) started much of the work in this area. Arrow defined the problem of group decision making as that of finding a fair welfare function for combining individual preference rankings into a social preference. Arrow proposed conditions for what could be considered as a good welfare function. But, unfortunately Arrow proved his conditions are inconsistent whenever there are three or more choices, so that no welfare function can satisfy them all. He tried, in later work, to examine relaxing his set of conditions, but with little success. Luce and Raiffa (1957) discussed the use of the majority rule as one of the most popular means of determining social preference, but the biggest problems with majority rule is that it can lead to intransitive social preferences.

Zeleny (1982) questions insistence on independence from irrelevant criteria attributes, as suggested by Arrow and others, citing empirical studies which show that the inclusion of unavailable options can reveal more about the strength of individual agent's preferences, as well as altering their perceptions of what is desirable. Instead, Zeleny proposes his theory of the displaced ideal. In his theory, each agent rates the options, and the infeasible combination of each agent's highest rated options, is used as a goal to guide the search for a feasible combination. The option which comes closest to this ideal goal will be chosen.

Nevertheless, the work done on group decision making extending decision theory to deal with multi-agent decision making situations, group decision making still suffers from the strict assumptions used in game theory, which will be discussed shortly. One of which is that all the options, for all the agents, must be known before the modelling and analysis phase starts. No to mention, the disconnect between the alternatives/option, and the proxy utilities/ranking and the agents' goals, which the alternatives are supposed to help in achieving them.

2.3.2 Game Theory

Game Theory is a close relative of decision theory, and dates back to the seminal work of von Neumann and Morgenstern (1953). It studies the interactions among

Table 2.1: Prisoner's Dilemma in a Normal Form Model

		Player P_b: *Don't Confess*	Player P_b: *Confess*
Player P_a	*Don't Confess*	1 year each ($2^{*}, 2^{**}$)	10 years for P_a, and 3 moths for P_b ($4, 1$)
	Confess	3 moths for P_a, and 10 years for P_b ($1, 4$)	8 years each ($3, 3$)

* Ordinal Preference for Player P_a (1-4: highest to lowest)
** Ordinal Preference for Player P_b (1-4: highest to lowest)

agents (called players or decision makers) where each agent is accorded a utility function (Binmore, 1992). Rapoport (1974a) defines Game Theory to be a theory of rational decision in conflict situations. Game theory examines the pre-specified options used by the agents in the process of trying to achieve particular outcomes. It is usually assumed that the set of outcomes is known, though not necessarily finite; and the outcomes are states formed by combining the available pre-specified options for the agents. A calculable payoff, for each agent, is associated with each state. All agents are assumed to be rational in that each agent's preference among the game's states is determined solely by the payoffs for that agent. In addition, agents assume that all other agents are rational in the same way they are. Games are classified according to whether they are two-player or n-player ($n > 2$); whether the players' choices of their strategies are independent (non-cooperative games) or coordinated (cooperative games); and whether the sum of the payoffs for the players is constant (zero-sum game such as chess) or not (non-zero sum games).

The payoffs for a game are usually shown a tabular form (a representation that is usually referred to in the literature as the *normal form*). To illustrate, Table 2.1 gives the payoff table for the prisoner's dilemma, a game which has received a lot of attention in game theory literature (Rapoport, 1974b). Each agent (a prisoner) must choose whether to confess, or not. The choices will affect the sentence given to the agents: confession implicates the other prisoner. Whatever the other player does, the payoff for confessing is a smaller sentence, and so a rational strategy is always to confess. However, both players could improve their payoff by agreeing not to confess (cooperate among them), but they need to be able to trust each other not to break the agreement, as the payoffs tempt them to do so. The prisoner's dilemma shows that a rational strategy might not always be the best one. The prisoner's dilemma has been studied in tournaments of repeated games (Axelrod, 1984). In this case agents need to persuade each other, through their play, that they can be trusted. The repetitions allow agents to use moves as punishments and rewards for previous actions.

Game theory has a number of important limitations, which make the results less useful than they might otherwise seem. The biggest limitation is the restricted set of actions available in a game. For example, in the prisoner's dilemma, each player can choose only a co-operative or a non-cooperative action each move. Sophisticated games could introduce a larger set of actions, the rules still to specify a bounded set of actions, and therefore states/outcomes. This is rarely the case in real-life conflicts and situations, where agents usually get creative in finding new ways (options) to achieve their goals. Also, game theory deals with payoffs which are usually assumed to be known and defined, for each outcome for each agent, with certainty. It assumes that all players have access to the same information about such payoffs. As it is rarely the case that payoffs are known exactly in real-life conflicts and situations, this restricts the applicability of the results of game theory. Agents usually have different perceptions of the game and the payoffs for other agents involved in the game.

Furthermore, game theory assumes that agents are rational, and selfishly motivated. This means that the agents for example must be induced into co-operation strategies. It also assumes that the preferences of the agents' options are known with certainty, let alone shared among the agents, which is rarely the case in real-life conflicts and situations. Not to mention, that the players' goals/motivations, and behavioural historical patterns, are not explicitly included, alluded or referred to in game theory approaches. If at all considered, then it will be part of the pre-preparation of the preferences data, and will be assumed but not explicitly modelled or referred to in preferences' order.

In addition, many formal techniques that have been developed to analyze decision situations by game theory are founded upon various assumptions, which was set by the pioneers who started Game theory (and its mathematical frameworks and tools): von Neumann and Morgenstern (1953) ; and Luce and Raiffa (1957). New formal framework and tools, which are related to game theory, but differ in some of the modelling assumptions or analysis principles, were proposed over the years to deal with some of these limitations in modelling and analyzing interactive games and decision problems. Some of these frameworks include: Metagame Theory (Howard, 1971), Conflict Analysis (Fraser and Hipel, 1984), Hypergame Analysis (Bennett, 1977, 1980; Wang et al., 1988), the Graph Model for Conflict Resolution (Fang et al., 1993), and Drama Theory (Bennett and Howard, 1996; Bryant, 2003; Howard, 1999; Howard et al., 1993).

Game theory rely almost completely on restrictive mathematical models and tools, that encompass a structural reduction limitations. The games models produced do not account for the players' goals, player's realities and constraints, interpersonal relations, environment's dynamics. The limitations of game theory's, and game theory related mathematical models and tools, which are formal, systematic, and abstract, are acknowledged by Luce and Raiffa (1957): "Game theory does not, and probably no mathematical theory could, encompass all the diverse problems which are included in our brief characterization of conflict of interest". Lately, Raiffa et al. (2002) acknowledged the limitations of these frameworks and their models: "for a long time I found the assumptions made in standard game theory too restrictive for it to have wide applicability".

2.3.3 Graph Model for Conflict Resolution (GMCR) and Stability Analysis

The Graph Model for Conflict Resolution (GMCR), proposed by Kilgour et al. (1987) and Fang et al. (1993), is a methodological approach for modelling an interactive decision situation/conflict in a format to which stability analysis can be applied. It serves as a prospective or retrospective strategic assessment tool for disputes. The GMCR is also supported by an automated decision support system, GMCR II (Hipel et al., 1997; Fang et al., 2003a,b). GMCR proposed a *graph form* representation to games/conflicts, as a better alternative to the widely used *normal form* (proposed by von Neumann and Morgenstern (1953), and builds on the binary-representation-based *option form* which was proposed by Howard (1971) and later used by Fraser and Hipel (1984).

Figure 2.4 shows that GMCR is part of a movement within the research domain of game analysis taking a direction to relieve game analysis from of the strict assumptions that classical game theory imposes. Namely, this new representation direction moves away from the utilities-based cardinal preferences required by game theory, in favour of a more realistic ordinal/relative preferences that could be elicited easily from the decision makers in a strategic decision making situation.

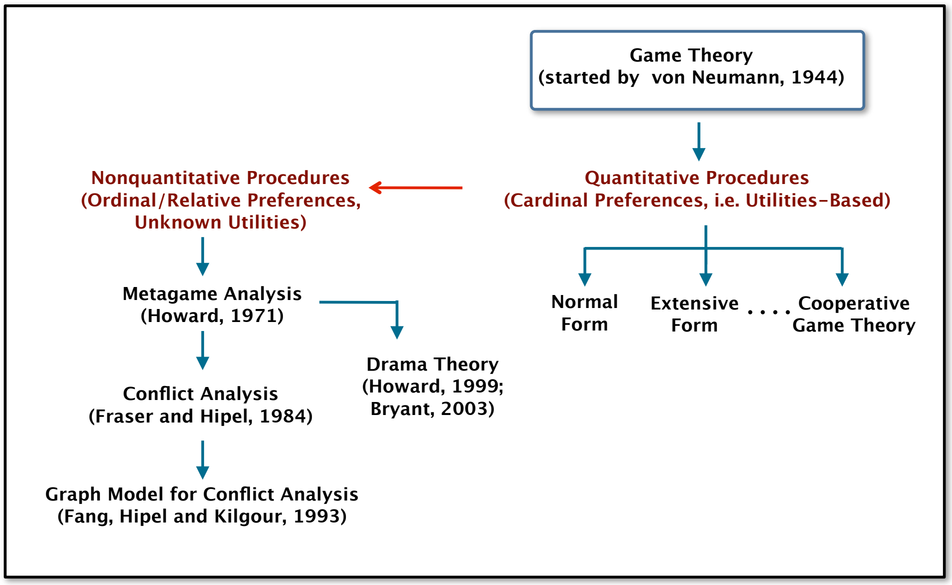


Figure 2.4: Game Theory and Related Theories. A Diagram adopted from Hipel and Obeidi (2005) (with modifications) showing how Conflict Analysis (Fraser and Hipel, 1984) and Graph Model for Conflict Resolution (Fang et al., 1993) relates to Game Theory (von Neumann and Morgenstern, 1953)

The movement started with (Howard, 1971) offering a new representation (option form) and a set of analysis tools (meta-rationality analysis), taking on the challenge of addressing the shortcomings of the representation proposed by (von Neumann and Morgenstern, 1953) and the Nash stability and equilibrium analysis proposed by Nash (1950). Shortcomings which are responsible, as per Howard, for the inability to deal with dilemmas such as the prisoner's dilemma (collectively referred to as Paradoxes of Rationality).

After the work of Howard (1971), Howard and others went on to propose Drama Theory (Howard, 1999; Bryant, 2003), while (Fraser and Hipel, 1984) took a more progressive systematic systems-engineering approach, building on Howard's work, offering a set of conflict analysis tools and methods. GMCR, which originated in conflict analysis (Fraser and Hipel, 1984) and in metagame theory (Howard, 1971) as Figure 2.4 shows, is the latest major advancement in that branch of research, employing definitions and terminology from graph theory, set theory, and logic to model and analyze conflicts.

As per GMCR, players' possible moves from state to state are depicted using a directed graph in which nodes represent states and arcs indicate state transitions controlled by the player. Each state represents an outcome, a player option or

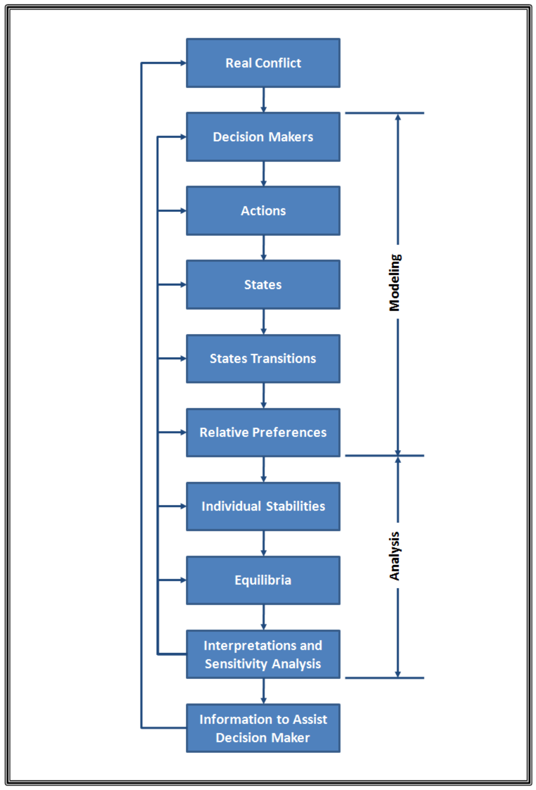


Figure 2.5: Applying the Graph Model for Conflict Resolution

combination of players' options (i.e. a scenario of how the game will progress). The model is considered to always be at some state, and changes of states are controlled by the players. The Graph Model specifies an initial state, or status quo.

Applying GMCR follows, as shown in Figure 2.5, two main stages: (1) Modelling Stage, in which the game/conflict is structured by determining the key ingredients: the players (or Decision Makers(DM), as called by GMCR), the states, the possible state transitions controlled by each DM, and each DM's relative preferences over the states; and (2) Analysis Stage, in which stability of each state from each DM's viewpoint is determined. Stable States represents a resolution to the the conflict.

The stability of states, for players, is defined by various solution concepts, or stability definitions. These are mathematical descriptions of patterns of players' social behaviour, in decision making situations. Some of the most commonly used solution concepts used for assessing stability of states and equilibria includes: Nash Stability (Nash, 1950, 1951) reflects a player who thinks only one step ahead. In General MetaRationality (GMR) (Howard, 1971), and Sequential Stability (SEQ) (Fraser and Hipel, 1984), a player considers two steps ahead. A player, in Symmetric MetaRationality (SMR) (Howard, 1971), contemplates three steps by assessing available moves by which he can escape from any sanctions that may be imposed by other players. On the other hand, in the Limited Move Stability (LhS) concept (Fang et al., 1993) a player has a foresight of horizon h moves, and in Non-Myopic Stability (Brams and Wittman, 1981a) the player's foresight is unlimited moves ahead. But, behavioural game theorists, such as Johnson et al. (2002), have challenged the notion of unlimited step-ahead strategizing.

Despite the fact that the GMCR framework is related to game theory, it is considered more advanced and feasible than other game theory approaches. Some of the restrictions are removed or relaxed. For example, GMCR require relative preferences for the states, for each of the players. No need for restrictive utility-based preferences' order. In addition, GMCR makes use of the very useful reliance on the solid mathematical and practical Graph Theory. This allowed for some aspects of the game/conflict dynamics to be captured and modelled formally. Namely, the moves from state to state by players.

Over the years, many researchers proposed enhancements to the GMCR framework, adding to it or dealing with some of its limitations. But these extensions to GMCR, for the most part, offered limited enhancements to GMCR. For example, Hamouda et al. (2004, 2006) proposed an extension to the preference relationship, of the original GMCR of Fang et al. (1993), to represent a realistic but limited weak-or-strong strength of preference; Obeidi et al. (2005) offered a method to deal with include or not include decision makers' options based on positive or negative emotions the have decision makers towers their options (the method did not model emotions or emotions effect beyond the include or not-include options); Li et al. (2004b,a) offered a systematic method to analyze the conflicts' states starting from the status quo of the conflict (the method employ strict move rules such as a player cannot take two consecutive moves); Li et al. (2004, 2005) added dealing with uncertainties in preferences (the method while mathematically sound, it failed to ground the uncertainties to why they exist in the first place); Inohara et al. (2007) added dealing with decision makers emotional feelings/attitudes toward each other; and Xu (2009) as well as Xu et al. (2009) offered a matrix representation to accompany/replace GMCR in addition to a very unrealistic and unverifiable representation to preferences' strength that assumes the decision makers will pre-specify strengths of their preferences in a scale/rank, much like AHP (the preferences' strength representation is not connected to the player's goals or constraints, and produce in what seems a break in the stability concept strengths structure). We intend to show the weaknesses and disadvantages of some of these extensions in the near future (as a future research work), in comparison to the our framework and the advancement (and extensions) it provides to GMCR.

In addition, the GMCR framework, and many of these extensions, still suffer from many of game theory limitations. For instance, GMCR demands a set of pre-specified options for each player, with a pre-specified preferences order for the states (defined from the player's options). Not to mention, here too, that the players goals/motivations, and behavioural historical patterns, are not explicitly included, alluded or referred to. This means that most of the human-intensive pre-preparation work which the framework user completes before he start building the GMCR model and analysis, is all not modelled or captured explicitly in a formal form.

In our thesis research, we build on the strengths of the conflict work of Fraser and Hipel (1984) and the GMCR work of Fang et al. (1993) (and the extensions offered to these works), and deal with their weaknesses. We show how we used, redefined, and/or advanced-on concepts and methods of these works, especially when we show how our proposed Constrained Rationality framework deals with the analysis of multi-agents conflicts.

2.3.4 Value Focused Thinking, as a Response to the Challenges Facing Decision & Game Theories

Keeney (1992) introduced an alternative thinking framework, for the problem of decision making, to the dominant alternative-focused thinking frameworks (i.e. decision theory and game theory, including multi-criteria decision making theories). Keeney, in his book, talked about the importance of values (goals, ethics, characteristics, and so on) on the decision making process:

> "Focusing on alternatives is a limited way to think through decision situations. It is reactive, not proactive. If you wish to be the master of your decisionmaking, it makes sense to have more control over the decision situations you face. You do not control decision situations that you approach through alternative-focused thinking. This standard mode of thinking is backward, because it puts the cart of identifying alternatives before the horse of articulating values. It is values that are fundamentally important in any decision situation. Alternatives are relevant only because they are means to achieve your values. Thus your thinking should focus first on values and later on alternatives that

might achieve them... Such thinking, which I refer to as *value-focused thinking*, can significantly improve decision making because the values guide not only the creation of better alternatives but the identification of better decision situations. These better decision situations, which you create for yourself, should be thought of as *decision opportunities* rather than as decision problems...

> Many methodologies and techniques to aid decisionmaking have been developed over the past forty years. So why bother with yet another approach? Invariably, existing methodologies are applied to decision problems once they are structured, meaning after the alternatives and objectives are specified. Such methodologies are not very helpful for the ill-defined decision problems where one is in a major quandary about what to do or even what can possibly be done. Certainly if the alternatives are not known, one cannot characterize the decision problem by the alternatives. In addition, most decision methodologies try to find the best alternatives from a prespecified list. But where does this list come from?" (Keeney, 1992)

Keeney, also went on, in his book, advocating of spending the time and effort to go through the process of modelling the decision maker values (which we call goals and constraints). It is definitely obvious, that we share the same concern and motivation with Keeney's (beyond alternative-focused thinking and methodologies, and the importance of what he calls values [which have in them what we call in Constrained Rationality goals and constraints]). But beyond this shared concern and motivation, we go about addressing the challenge in two different ways:

1. Keeney uses the goals (he calls them objectives) to generate a list of alternatives. The objectives, to him, have identifiable attributes (criteria) to judge their achievement through what he calls value functions, which are nothing more that utility functions. The problem lies in these utility functions, which he himself considers subjective, but still are at the heart of his analysis of the alternatives.

2. Keeney splits the decision maker's goals-reduction-tree to two layers. The top layer starts with the strategic goals of the decision maker reduced "nicely", i.e. forming an objectives-hierarchy while ignoring any interrelations or interdependencies among the goals (interrelations which we call in our Constrained

> Rationality framework lateral-relations). The second lower layer has objectives network, forming a structure similar to influence diagrams. While this objectives-network captures some interrelations among the objectives in it, it does not consider any affect of them or through them on the achievement of related/linked objectives. These interrelations are informal, and merely visual links, that are not considered when establishing the utility functions. That's because the purpose of the second layer is to help in generating alternatives only. There is no effect of these interrelations, or means-ends links, on strategic goals achievement, not even to the level offered by influence diagrams.

We have to remember that Keeney is one of the established researchers within the decision theory school of thinking. So, while he acknowledged the importance to go beyond the classical views of the alternative-focused domain of thinking, and therefore include some aspects of goals-oriented analysis and reasoning (mostly done to acknowledge the importance of creativity in the decision making process –hence, the name of his book), he showed extreme loyalty to the classical views of the dominant alternative-focused-thinking school by using utility functions (he calls them value functions in his book) and cardinal preferences as "the only means" to achieve judgement about alternatives. In his book, Keeney shows many examples where he went all the way to generate quantitative utility-based representation, sometimes crossing the objectivity line. He appeared to acknowledge this himself, after a lengthy chapter on Quantifying Objectives with a Value Model (i.e. setting utility functions as means to measure objectives', or the achievability of goals, as we call them):

> "Are Value Models Scientific or Objective? The final issue concerns the charge that value models are not scientific or objective. With that, I certainly agree in the narrow sense. Indeed, values are subjective, but they undeniably are a part of decision situations. Not modelling them does not make them go away. It is simply a question of whether these values get included implicitly and perhaps unknowingly in a decision process or whether there is an attempt to make them explicit and consistent and logical. In a broader sense, the systematic development of a model of values is definitely scientific and objective. It lays out the assumptions on which the model is based, the logic supporting these assumptions, and the basis for data (that is, specific value judgments). This makes it possible to appraise the implications of different value

judgments. All of this is very much in the spirit of scientific analysis. It certainly seems more reasonable-even more scientific-to approach important decisions with the relevant values explicit and clarified rather than implicit and vague." (Keeney, 1992)

In summary, we see Keeney's Value-Focused Thinking framework, and his acknowledgement about the need to go back to the root of the decision making problem, achieving the decision maker's strategic goals (and not just the mere choosing of one alternative among a pre-selected set of alternatives and using a pre-set cardinal preferences over such alternatives) as an additional support to our research motivation and also to its approach. After that, our research and Keeney's approach go about representing the goals of the decision makers, and formally using such goals and their inter-relations and inter-dependencies to reason about the known (and generated) alternatives, in two different ways.

But, this should not, in any way, lessen the importance of the support Keeney provides to our research's motivation and approach, by his realization of the need to bring back the strategic goals to the decision making models (in order to generate alternatives and to establish better judgement about the worth of such alternatives). Especially, when we know that these acknowledgements and realizations are coming from a researcher of Keeney's caliber and status within the communities of decision theory research and practice. In fact, Keeney's acknowledgements about the need to adopt a goal-oriented thinking when addressing decision analysis, and his criticism of the dominant alternative-focused theories, provided the motivation for our Al-Shawa (2006b) work, and provides support to our Constrained Rationality research direction and thinking.

2.4 AI Multi-Agents Rationality Models

2.4.1 Distributed Artificial Intelligence, and Multi-Agent Systems

Agent and Multi-Agents started in AI as studies of Distributed Artificial Intelligence (DAI). DAI studied how intelligence can be modelled in the cooperation of a set of agents (Huhns, 1987), questioning the usual AI assumption that a single self-consistent entity (such as a conventional knowledge base) can demonstrate intelligence. Problem solving activities can be divided up among agents, according to their specialist knowledge. As the system runs, the agents communicate partial solutions, and possibly control information (e.g. re-allocation of tasks), amongst themselves, until they converge on a final agreed solution. The premise is that intelligence is an emergent feature of co-operative behaviour.

The DAI domain of studies evolved to be called Multi-Agent Systems (MAS), in which the scope increased to cover cooperative and non-cooperative distributed intelligent agents. The MAS paradigm has a natural ability to handle conflicting knowledge without the usual logical contortions arising from inconsistent conclusions. It allows agents to develop and maintain alternative hypotheses. Different agents will contain different knowledge, which may compliment or conflict with knowledge contained in other agents.

This is demonstrated in the blackboard system (Nii, 1986a,b), which is typically used for recognition problems. Separate knowledge sources communicate partial hypotheses using a shared blackboard. The knowledge sources modify and extend existing hypotheses on the blackboard, until one of them adequately explains the phenomena being observed. In the blackboard model, the separate knowledge sources have no knowledge of each other (Erman and Lesser, 1975). Removing the global blackboard requires that agents communicate directly. In the BEINGS system, agents simply broadcast messages, usually in the form of requests for help, and hope that some other agent will reply (Lenat, 1975). Such systems still assume problems can be partitioned into totally independent sub-problems, and so the co-operation is reduced to that of trading tasks and sharing results. However, this is an unrealistic assumption, and is essentially conflict avoidance. There is a realization that most problems cannot be partitioned in this way (Ginsberg, 1987).

Various techniques have been proposed to allow belief-based epistemic reasoning, especially in multi-agent planning systems. Agents must be able to reason about what other agents know and are capable of in order to make full use of their existence, and to co-operate effectively (Konolige and Nilsson, 1980). Similarly, to communicate properly, agents must be sure their communications contain enough context information to be understood probably, and that they are useful to the recipient agent (Appelt, 1980).

Also, most MAS systems assume benevolent agents working towards the same goal. Rosenschein (1985) notes that in real world situations, perfect co-operation never happens, as the goals of any two agents will never coincide exactly. Rosenschein examines payoffs from game theory as a way of comparing goals, and discusses various situations in which conflict of goals can occur, and how they can be resolved (Rosenschein, 1985).

While MAS has contributed a variety of computational models of agent interaction, it has not progressed much beyond the game theoretical studies of conflict resolution (Sycara, 1988; Parsons et al., 2002; Wooldridge, 2000). We have discussed the limitations of these models, and the reasons behind these limitations, in Al-Shawa (2006b).

2.4.2 Goals Modelling and Reasoning, as a Response to the Challenges Facing MAS

Because of the limitation of the decision theory and game theory approaches to design and implement multi-agent systems (Wooldridge, 2000; Braubach et al., 2004), a new direction started emerging and becoming of interest to researchers in the area: Modelling Goals and Reasoning about them.

The popularity of the Belief-Desire-Intention (BDI) framework (Bratman, 1987; Busetta and Ramamohanarao, 1998; Mora et al., 1998; Wooldridge and Parsons, 1998) within the MAS research community, and the the Knowledge Acquisition in autOmated Specification (KAOS) framework (Dardenne et al., 1993; van Lamsweerde et al., 1995) within the agent-oriented software requirements analysis research community, since the mid-1990's, was instrumental in enriching the agent modelling and development research and extended it to be well suited for describing an agent's mental state. Both frameworks used the desires (goals) concept of an agent to represent its motivational stance and are the main source for the agent's actions.

Nevertheless, currently available BDI agent platforms mostly have abstract form of goals and do not represent them explicitly. The same could be said about agent-oriented requirements analysis and development frameworks other than BDI, including KAOS and i* (Yu, 1995; Yu and Mylopoulos, 1994). So even though that Goals are an integral part of a large body of theoretical work within these bodies of research, there is only a limited amount of work that provides an explicit structure for goals, and reasoning about goals, that can be directly implemented in a BDI-like agent system. Some of the relevant existing work is described below.

The support for any form of goal related reasoning is limited and varied in current agent development systems. PRS (Ingrand et al., 1992) and dMARS

(D'Inverno et al., 2004) represent goals as events with no internal structure. Other systems like JACK (Busetta et al., 1999) allow various information to be stored within the goal structure, but the goals are transient in that if they cannot be pursued as they occur they are ignored. On the other hand, Huber's JAM (Huber, 1999) keeps track of goals that cannot be immediately pursued for a later attempt. JAM also uses the goal structure to reason about when goals are satisfied. This is while others, such as Thangarajah et al. (2003b,a) and Shaw and Bordini (2007), uses hierarchical task networks embedding goals within plan trees to direct plan commitment to be more in line with goals adopted.

Irrespective of how these frameworks and systems represent goals, none of them support the management of interactions between concurrent goals. Most, in fact, consider goals that are tactical or procedural, and/or goals that are embedded within plan structures, i.e. none deals with strategic goals for agents. This mainly due to the nature of the bodies leading such research direction: the agent software development community. Agent implement-ability, and management of agent/system resources, are at the top of those researchers' research objectives.

Additionally, one reason for this shortcoming is that most of these frameworks, old and new (such as JACK (Howden et al., 2001), JAM (Huber, 1999), or Jason (Bordini et al., 2007)) are natural successors of the first generation BDI systems, such as PRS (Ingrand et al., 1996) and (Georgeff and Lansky, 1987) , which had to concentrate on performance issues and do without computationally expensive deliberation processes due to scarce computational resources. Furthermore, these frameworks are mostly based on formal agent languages like AgentSpeak(L) (Rao, 1996) which focus on the procedural aspects of goals and treat them in an event based fashion.

In the theory of agents and MAS, there has been some work on the formal properties of goals (Doyle et al., 1991; van Linder et al., 1995; Bell and Huang, 1997; Hindriks et al., 2000; van Riemsdijk et al., 2003) and an agent's commitment towards achieving its goals (Rao and Georgeff, 1992; Cohen and Levesque, 1990) . At the same time, there has been a significant amount of work on conflict management within MAS (Tessier et al., 2000) and interactions between agents (Cohen et al., 1997) . However, most of this work is presented within a logical framework that cannot be easily mapped into a practical agent, or agents, goal reasoning based system suitable for large scale conflict analysis real-time/simulation applications.

The goal-directed requirement engineering community has, also, contributed to the use of goals modelling and goals reasoning within MAS. The leading work of the KAOS team started a wave of interest in modelling goals formally and using it to direct the implementation of software systems (Dardenne et al., 1993; van Lamsweerde et al., 1995). i* (Yu, 1995; Yu and Mylopoulos, 1994), proposed informal reasoning about goals at early stage of software implementation, while Tropos (Mylopoulos and Castro, 2000; Giunchiglia et al., 2002) proposed a goal reasoning framework that formally reason about system implementation goals including non-functional goals (Mylopoulos and Castro, 2000). However, the frameworks proposed by the requirements engineering community focused on software implementation. Goals reasoning in KAOS for example, will lead to an object-oriented software model; and in i* will lead to a set of informal diagrams that help guide the software designers take decisions. Tropos's Goals reasoning, on the other hand, are done for: one agent systems; in isolation of the effect of both the environment and other agents; and no consideration is given to MAS conflict analysis or game playing (Mylopoulos and Castro, 2000; Giorgini et al., 2004, 2005).

Nevertheless, the need for explicit goal representation is expressed in several recent publications (Winikoff, Harland, and Padgham, 2002; Winikoff, Padgham, Harland, and Thangarajah, 2002), and is additionally supported by the classic BDI theory, which treats desires (possibly conflicting goals) as one core concept (Bratman, 1987). The importance of explicit and declarative goal representation in the modelling area is underlined by BDI agent methodologies like Prometheus (Padgham and Winikoff, 2002), Tropos (Mylopoulos and Castro, 2000; Giunchiglia et al., 2002) and requirements engineering techniques like KAOS (Dardenne et al., 1993; van Lamsweerde et al., 1995; Letier and van Lamsweerde, 2002). If the the declarative aspect of goals is omitted by the modelling framework, then the the ability to reason about goals is lost (Winikoff et al., 2002). This means that the representation of goals is a necessary precondition when one wants reasoning about goals to become possible. But the current frameworks lacks the representation mechanisms to support modelling goals, and therefore reason about it. Al-Shawa (2006b) talked about the short comings of the current frameworks and the need to extend them at different levels to be able to make the agent-oriented framework well suited to build multi-agent knowledge-based systems.

Recently, we proposed in Al-Shawa (2006b) a new conceptual knowledge modelling and management framework to address these short comings. especially with regards to dealing with strategic knowledge management and decision making in MAS. The Viewpoints-based Value Driven - Enterprise Knowledge Management

(ViVD-EKM) framework offers a new and practical way to model agents and multi-agents systems, the agents' perspectives of the world, and reason about it. An overview will be provided later on how an agent's viewpoints, goals and constraints are modelled within ViVD-EKM.

2.5 Rational Reasoning and Decision Making: Limitations and Biases

Are strategic decision makers rational agents? This question was the focus of many studies across many scientific disciplines and for a long time. The modern discussion started with the *Rational Action Theory* that grew out from the principles set forward by Von Neumann and Morgenstern's *expected utility theory* which they discussed in von Neumann and Morgenstern (1953). The expected utility theory suggests that rational decisions focuses primarily on maximizing the individual's economic condition/wealth.

Simon (1990) had critically studied rational theory and subjective expected utility theory. These theories depict the rational human decision maker as one who "contemplates, in one comprehensive view, everything that lies before him ... understands the range of alternative choices open to him, not only at the moment but over the whole panorama of the future ... understands the consequences of each of the available choice strategies, at least up to the point of being able to assign a joint probability distribution to future states of the world. (And) ... has reconciled or balanced all his conflicting partial values and synthesized them into a single utility function that orders, by his preference for them, all these future states of the world" (Simon, 1995). Simon concluded, "when these assumptions are stated explicitly, it becomes obvious that subjective expected utility theory has never been applied, and never can be applied" (Simon, 1990).

Dawes (1988) considered the influences on decision-making that lead to irrational outcomes and identified among many things: habit, tradition, religious beliefs, others' choices, past choices, etc. The investigation of rational action theory can lead to the conclusion that making a rational choice is beyond the capability of an average individual. Habit, tradition, religious beliefs, social relationships, interests, goals, experiences, perceptions, and biases, all influence human behaviour resulting in a complex decision-making environment.

Therefore, a more realistic view of rationality is needed. In response, Simon (1990) introduced Bounded Rationality. The theory of bounded rationality recognizes that human thought processes are bounded or limited. Although we live in a world with "millions of variables that in principle could affect each other", humans will only detect or acknowledge a reasonable number of them when facing a problem, which requires action (Simon, 1990). Although the theory of bounded rationality loosens the demands of "rationality", as described in game theory and decisions theory, on agent's decision-making, there still exists a gap between rationality and the observed so called "irrationality".

For an agent to exercise rational thinking, though bounded , the agent must be capable of searching for and generating alternative solutions, though not necessarily an exhaustive search. Once the alternatives have been identified, facts about the environment must be gathered to allow the decision-maker to draw inferences from these facts, relate them to each of the alternative solutions, reasonably project the consequences that may result from each alternative. Lastly, the agent should be able to compare each alternative to the desired goal and make a rational choice. Bounded rationality considers the limitations of human computational ability and the environmental constraints that exist in the real world. But, not all limitations are due to computational abilities.

The Gap Between Goals and Actions:

Simon's thesis is that "human behaviour is generally rational, and that it cannot be understood without finding the connections between its actions and its goals" (Simon, 1995). On the other hand, Mintzberg had stated that manager's work with "verbal information and intuitive (non explicit) processes" (Mintzberg, 1973). This is not to say that intuition is necessarily irrational, but when comparing the concept of intuition to the theory of rational action, it is inconclusive as to whether agents are completely rational in their actions.

The complexity in this picture quickly emerges because a political or business decision, for example, is not constructed in isolation between an agent and the economic alternatives that lie before him. The complexity arises from many factors such as the environment, culture, context, individual interests of the agents involved, communication limitations, the expectations and biases that are extremely influential on the agent's decision-making abilities and therefore the agent's decisions. The value of information and its effective use in rational action is dependent not only on the environment and the social factors but also on the individual agent. Tversky and Kahneman (1974, 1981); Kahneman and Tversky (1973, 1979, 1984, 1996); Bazerman (1986, 1990); Rachlin (1980) investigated the biases that may influence the decision-making behaviour of agents.

Abercrombie (1960) illustrates the intricate process behind the development of judgment and the mental schemata, which is the scheme in the human mind that allows information to be organized. The human mind scans all current information in order to find matching schemata. The mind may distort the new information in order to make it fit into current assumptions. Information that does not fit may be rejected. Humans may not remember information as it was originally presented; instead it may be remembered incorrectly, forming the individual perspective. This *individual perspective* (or *viewpoint*) is one reason for the limits of rational action or decision-making.

Cognitive heuristics (or biases) develop to perform this task. As a result, judgments, which lead to decisions, are strongly influenced. These judgmental biases or heuristics may be perceived as a negative influence, but in reality they allow agents to make decisions on real-time, allowing the agents to live normal life. For example, Kahneman and Tversky (1979, 1984); Tversky and Kahneman (1981) discovered that the way a situation is presented to a decision maker can influence the resulting decision. Their research revealed that different decisions are made simply based on whether a problem/conflict outcome results in a gain or a loss (even when the statistical probability of the two problems were exactly the same). An individual agent will avoid risk for a gain, yet, will seek risk to avoid a loss. In addition, Bazerman (1986) found that decision makers have a tendency to seek alternatives that confirm a prior decision. As a result they may ignore alternatives that may suggest a more optimal solution. Not to mention that many had discussed the effect/influence emotions have on decision making and how they shape and strengthen our beliefs (Frijda and Manstead, 2000; Elster, 1999) and therefore rearrange our priorities and revise our goal hierarchies (Simon, 1982).

These agents' biases and realities affect their ability to take decisions, or more accurately what decision making criteria to adopt, and therefore what decisions to take. Unfortunately, the mathematical and economical models, typically used to help decision makers analyze their options, ignores these factors. This is mainly because of the limitations and restrictive assumptions of these models and tools. The models do not even directly provide facilities to capture the agents' goals and constraining realities/biases, and reason about them.

2.6 Decision Support Methods and Models: Evaluation, Verification and Validation

The question of how best to evaluate and validate conceptual modelling methodologies and its products, is by itself a research topic. The issue is at the heart of what constitutes a scientific research, and what a valid methodology is, with very polarized conflicting views and camps dominate the scientific research landscape. Many research, arguments and counter-arguments could be cited here on what is the best way to evaluate and validate a methodology. Because this topic is not within the scope of our research, we will answer the question based on how evaluation and validation will be addressed in our thesis research.

2.6.1 Methodology Evaluation: the Essential Role of Case Studies and Applications

Similar theses research work used case studies to demonstrate the usage of the proposed methodologies in real-life examples, and to illustrate how more insight will be gained by applying the proposed methodologies over the usage of other existing methodologies. The more-insight, in each of these works, is explained and shown by providing an interpretation of the studied conflict model/s generated by using the newly proposed methodology. This more-insight is provided, in most cases, by informally explaining the differences of the old compared to the new models, and how the new models have better modelled the conflict studied. No controlled experiments, experimental data, nor statistical or logical (in mathematical terms) inferences are provided. Some examples of the very fine scholarly theses research work, old and recent, which proposed decision-making modelling and analysis methodologies include: Fang (1989), Obeidi (2006), AL-Mutairi (2007), Kassab (2007), and Sheikhmohammady (2009).

But this is not limited to theses works, or conflict modelling and analysis as an area of research. Consider for example the published literature on proposed methodologies on conceptual modelling, such as the work published on the benefits of the famous Object-Oriented methodology (Jacobson and G., 1992; Booch, 1994), and its more elaborate and expanded version, the UML methodology and conceptual framework (Jacobson and Rumbaugh, 1999); or take the research work in requirements engineering methodologies (for example Yu (1995)); or even work of systems and decision modelling within Operational Research such as the work of Checkland (1999) on Soft Systems Methodology (SSM) and Eden (1989) on Strategic Options Development and Analysis (SODA); and so on.

None of these research work, which proposed conceptual modelling and analysis methodologies and frameworks, used empirical experiments with numerical statistical data analysis to show how their newly proposed methodology (at the time) performs in comparison to the current existing methodologies (at the time), in order to qualify any of these proposed methodologies as scientific and valid. The lack of demands, by the receiving scientific community on the researchers and their newly proposed methodologies, to provide "statistical" or "data-oriented" proofs could be explained by the scholarly views cited and briefly discussed below.

In Checkland (1999) words: "Methodologies and their models are simply logical machines for carrying out a purposeful transformation process expressed in a root definition. Measuring the performance of a logical machine can be expressed through an instrumental logic which focuses on three issues: checking that the output is produced; checking whether minimum resources are used to obtain it; and checking, at a higher level, that this transformation is worth doing because it makes a contribution to some higher level or longer-term aim. This gives definition of the '3Es' which will be relevant for every model: the criteria efficacy (E_1), efficiency (E_2) and effectiveness (E_3) (Forbes and Checkland, 1987; Checkland and Scholes, 1990)." Checkland, the well-known in Operation Research and Systems Engineering communities, also argues that a methodology should be taken as a process of social and scientific inquiry which aims to bring about improvement in areas of concern by articulating a learning cycle (based on systems concepts) which can lead to action (Checkland, 1999).

In addition, Checkland and Holwell (1998a,b) and Checkland (1999) agreeing with Kurt Lewin's views, developed in the 1940s, that real social conflicts and events could not be studied in a laboratory, argued for some mid ground between the strong criterion of repeatability (of the happenings) insisted on by some in the scientific research community and the weak criterion of plausibility suggested by others in social sciences. They argue that: "action research, part of systems engineering, should be conducted in such a way that the whole process is subsequently *recoverable* by anyone interested in critically scrutinizing the research. This means declaring explicitly, at the start of the research, the intellectual frameworks and the process of using them which will be used to define what counts as knowledge in this piece of research. By declaring the epistemology of their research process in this way, the researchers make it possible for outsiders to follow the research and see whether they agree or disagree with the findings. If they disagree, well-informed discussion and debate can follow. Also, the learning gained in a piece of organization-based action research may concern any or all of: the area focused on in the research; the methodology used; or the framework of ideas embodied in the methodology."

Eden (1995) describes the methodologies concerned with decision analysis and support to be economically viable when used to support ill-structured, complex and probably strategic decision making. Therefore, as he puts it while discussing decision making in group settings: "Ashby's Law of Requisite Variety (Ashby, 1954) suggests that a complex system will be needed to support a complex situation. The criteria of success for such systems will inevitably be equally complex. . . . If the system [methodology] is designed specifically to address real groups (with a history and a future) working on complex issues then it is no use taking out those very characteristics that: make it complex in order to control experiments. Research with students using structured problems will say absolutely nothing about the performance of a GDSS [Group Decision Support Systems] in relation to its designed aims. As Checkland put it: "Methodology can be tested only in conjunction with a problem to which it is applied" (Checkland, 1981, p. 242), and the problem will always be complex."(text within square brackets is added to the original Eden text for clarification purposes). Meaning that case studies on real-life problems, not control experiments, should be considered the best way to test and evaluate methodologies, especially the ones that aim to address complex issues and systems such as ours (modelling and analysis of real-life strategic decision making and conflict).

It is also useful here to mention some research on what evaluation criteria should be considered to best evaluate software engineering methodologies and tools. For example, Kitchenham et al. (1997) identified the following evaluation criteria, grouped them in three successive levels, and defined them as follows: 1) Basic: Is the component description complete, understandable, usable, internally consistent etc.? Would potential users have confidence that they could use it for real or carry out a trial?; 2) Use: Is the component helpful? That is, when it was used, did it achieve its objective, produce the specified results, produce usable and relevant results, behave as expected, and not require expert assistance?; and 3) Gain: Is it better than what was available previously (the control situation), i.e. lead to better decision.

This is just one example of research on evaluation criteria, but it clearly confirms with the views of researchers in systems design and operations research that evaluation of methodologies best done within the context of case studies and the criteria of evaluation are best described by Checkland's "3Es" listed earlier (Checkland, 1999; Eden, 1995). One also has to remember that case studies usually are conducted to gain insight on whether the research arguments and propositions are satisfied or not and why, and this should constitute the evaluation criterion (Yin, 2003) (and as best shown by the best-selling scholarly case study of Allison and Zelikow (1999), again as per Yin (2003)).

Therefore, when it comes to evaluating our Constrained Rationality, with its conceptual framework and methodology, we will use case studies and illustrative application examples as a mechanism. At the time of using a case study or an illustrative application example, we will state: why the case or example was chosen; how it is used; what is the process used in conducting the modelling and analysis parts of it; and how the resultant models and analysis compare to what other methodologies provided (if any), i.e. what is the insight gained by using the Constrained Rationality framework versus what others provided (if any).

2.6.2 Verification and Validation

Now, discussing how best to judge the validity of a methodology and its models is an equally contentious research topic. While we were not able to find any discussion on validation of the methodologies provided in comparable research in decision making modelling and analysis (theses, or otherwise, such as the ones listed above), we discuss the topic of methodologies and the validation of their models as defined by the Requirements Engineering research and practice circles. Requirements Engineering research share many of the concerns that providers and users of decision making models, or any models describing humanistic system (as called by Zadeh in his principle of incompatibility - in Zadeh (1973)) or phenomenon, especially: how to judge the validity of models built to describe the world?

In requirements engineering, validation is the process of establishing that the requirements and models elicited from the stakeholders provide an accurate account of their requirements. Explicitly describing the requirements is a necessary precondition not only for validating requirements, but also for resolving conflicts between stakeholders. But rarely, in the requirements engineering literature and standards, validation is discussed separately. It is always accompanied by discussion on Verification. Take for example how the *IEEE Standards for Software Engineering* set a complete *Verification and Validation* standards document (IEEE, 1998), in which a definition of *Validation* is given to be: the confirmation that the "particular requirements for a specific intended use" are fulfilled; and *Verification* is defined to be: the confirmation that "specified requirements" have been fulfilled.

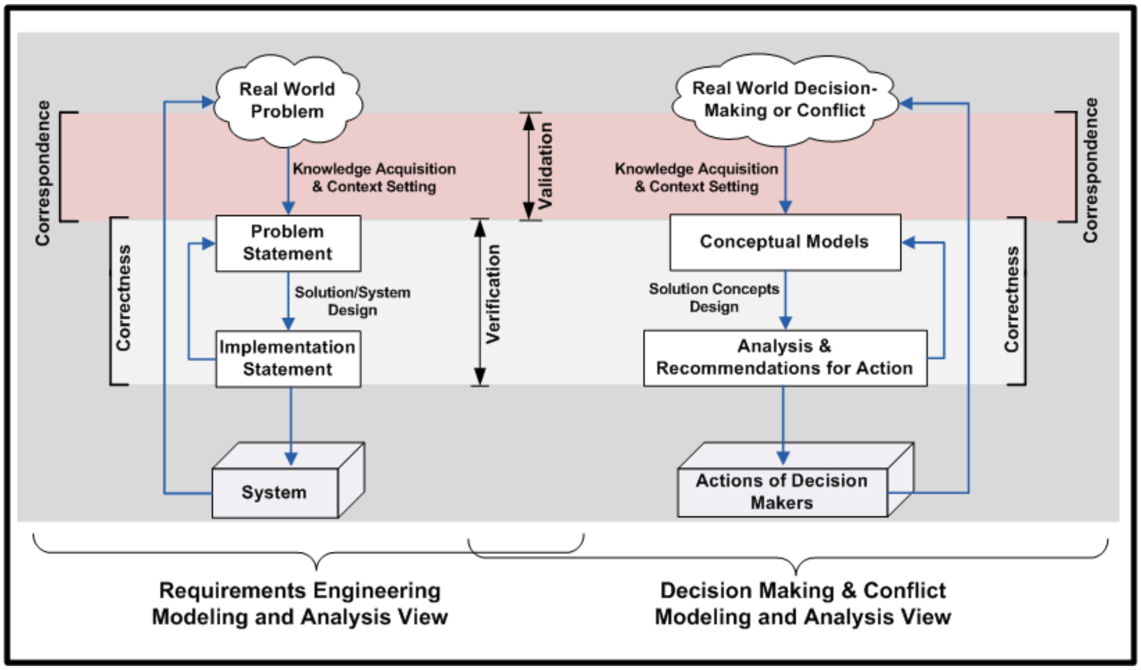


Figure 2.6: Verification vs. Validation

Figure 2.6 shows the difference between verification and validation as discussed in the system requirements engineering literature (Boman et al., 1997; Loucopoulos and Karakostas, 1995; Lauesen, 2002; Blum, 1992). The figure also shows the equivalent and similar steps that systems requirement engineering and decision/conflicts modelling and analysis have. In both, the conceptual models describing the real-world problem come as a product of a thorough knowledge acquisition process conducted by an analyst/modeller eliciting the stakeholders/players' view of the problem.

Many consider validating the produced models, and the methodology used to produce them, to be very difficult; and the same is not true when it comes to verification. Nuseibeh and Easterbrook (2000) recalled that the question of verification is easier to address, than validation, especially if techniques such as formal representation and analysis is used by the methodology and the models it produces, because such techniques tend to concentrate on the coherence of the models and descriptions of the world: are they consistent, and are they structurally complete? These are the type of questions that verification tries to answer. In contrast, validation is geared towards testing a correspondence with the real world problem. Therefore, the techniques used usually softer and mostly concerned with checking if all aspects of the problem that the stakeholders regard as important have been covered. Some of the aids used in testing and ensuring validity include prototyping, animation, and the use of scenarios. The use of models and replaying the elicited clear description of the problem are considered very helpful aids to communicate to the stakeholders/player what the analyst managed to capture from their knowledge, and to confirm whether this captured view is accurate.

Nuseibeh and Easterbrook (2000) stated that "requirements validation is difficult for two reasons. The first reason is philosophical in nature, and concerns the question of truth and what is knowable. The second reason is social, and concerns the difficulty of reaching agreement among different stakeholders with conflicting goals. . . We can compare the problem of validating requirements with the problem of validating scientific knowledge. . . Requirements engineers (who) adopt a logical positivist approach - essentially the belief that there is an objective world that can be modelled by building a consistent body of knowledge grounded in empirical observation. In RE, this view says that the requirements describe some objective problem that exists in the world, and that validation is the task of making sufficient empirical observations to check that this problem has been captured correctly." Then Nuseibeh and Easterbrook went on to cite the observations of Popper (1963) on the limitations of empirical observation: scientific theories can never be proved correct through observation, they can only be refuted , and this applies to requirements modelling.

We also have to remember that logical positivism was severely criticized in the latter part of the twentieth century (Blum, 1996). Kuhn (1962) observed that science tends to move through paradigm shifts, where the dominant paradigm determines the nature of current scientific theories. This leads to the realization that observation is not value-free, rather it is theory-driven, and is biased by the current paradigm, hinging on its view of the world. For requirements engineers, the methods and tools they use dominate the way that they see and describe problems. Nuseibeh and Easterbrook (2000) described an extreme case of this, when the problem of validating requirements models shifts to a problem of convincing stakeholders that the chosen representation for requirements models is appropriate. Jackson (1995) captures this perspective through his identification of problem frames. If stakeholders do not agree with the choice of problem frame, it is unlikely that they will ever agree with any statement of the requirements. Can this be reasonably accepted? At the other extreme, ethnomethodologists attempt to avoid the problem altogether, by refusing to impose modelling constructs on the stakeholders (Goguen and Linde, 1993). By discarding traditional problem modelling and analysis tools, ethnomethodologists seek to apply value-free observations of stakeholder activities, and therefore circumvent the requirements validation issue altogether.

The decision making and conflict analysis case studies and literature show quite striking resemblance and similarities. This could be confirmed by the case study results of Allison and Zelikow (1999), the much celebrated case study about the Cuban Missile Crisis. At the beginning, Allison and Zelikow stated the argument of their study, formulated in three propositions:

1. Professional analysts of foreign affairs and policymakers (as well as ordinary citizens) think about problems of foreign and military policy in terms of largely implicit conceptual models that have significant consequences for the content of their thought;
2. Most analysts explain (and predict) behaviour of national governments in terms of one basic conceptual model, here entitled Rational Actor Model [This is the dominant conceptual modelling and reasoning framework, within the research community in decision making and conflict analysis, and centres around the views of decision theory and game theory];
3. Two alternative conceptual models, labeled an Organizational Behaviour Model, and Governmental Politics Model, provide a base for improved explanations and predictions.

Then they conclude the study with the following observations:

1. The source of differences is the conceptual models each employed;
2. Differences in Interpretations based on taking alternative conceptual angles, looking at the conflict, presenting a number of significant differences in emphasis and interpretations;
3. Different Answers or Different Questions: Competing interpretations reflect each model's tendency to produce different answers to the same question. Observing the models at work, what was equally striking are the differences in the ways the analysts conceive of problems, shape puzzles, unpack summary questions and dig into the evidence in search for an answer

So, if different models captured different aspects of the world problem, who can judge which model is valid, invalid, more valid, or less valid than the others. And most importantly who can say that one methodology or conceptual framework is not valid because it captured some aspect of the problem but not all.

The other essential difficulty in requirements, and models in general, centres on how to validate the models of the world-problem if there is disagreement among the stakeholders/players. In Constrained Rationality, we explicitly model players' goal hierarchies and goal inter-dependencies making the problem clear: stakeholders/players have goals that conflict with one another. Avoiding the problem of resolving such conflicts, as current decision and conflict methodologies suggest, does not mean that the problem does not exist, nor means that it has been resolved. You only need to see the effect of suppressing the stakeholders' conflicting interests and goals on system implementation projects causing many failures and challenges, as many industry reports shows.

So, how should we address the issue of validity in our methodology and its produced models, knowing that methodologies and models validity is a research topic on its own right, as we know by now? Do we just concern ourselves with the verification problem, which is easier to address in our case because we employ a logical and formal representation/modelling and reasoning approach, and ignore validation. We do believe that this will be impractical, especially because we hope that the methodology will be used and employed by others to analyze their strategic decision making and real-life conflicts.

We see the verification problem of any methodology to be the most essential for the methodology to be a methodology in the first place. Why? Because it is concerned with the coherence, consistency and correctness of the models and methods employed within the framework structure of the methodology. This is especially true if the methodology assumes formality (mathematical and logical concepts and constructs). On the question of validity, we are with the view that validation testing and enhancing is what makes the methodology usable and useful for interested users. The methodology itself could not be invalidated except on the grounds of failing to be logically grounded, sound and verifiable, and here too we may have many of the scholars we cited their work above protest. On the other hand, models built (by the methodologies) to describe a problem of the world could be invalidated on the ground that it did not correspond to the real-world problem and events. But, the problem here is what constitutes the "true" definition of the problem, or the "truth" about any of the world affairs, especially if the stakeholders/players state the problem as they believe it, and the methodology is just a tool to capture their belief and then analyze it.

If a decision maker made an incorrect statement about his options, preferences and utilities, is it true then that the incorrect decision theory models (produced by an analyst based on this stated description of the decision making problem) will invalidate the decision theory and its applied methodology? Nobody can say that it is Decision Theory's fault, and therefore it is an "invalid" theory. Take another example from requirements modelling: if the stakeholders, of a systems to be built to solve a real-life problem, have agreed that the requirements' conceptual models for the system are absolutely correct, then, few months down the road, it is been found that what the stakeholders stated and agreed on as "the" model of the world and the system they want happened to be incorrect and incomplete (because they did not know better at the time). Who is at fault here? The methodology itself, and therefore it has to be declared invalid because it produced these incorrect and incomplete models, or because it employed some specific methodological and systematic-way to capture the world and model it, but then the process failed to produce the right models? is it the analyst's fault? or is it the stakeholders' fault? Suggesting an invalidation of methodologies because they provide models based on errors in knowledge-elicitation or failures of players/stakeholders to recognize and state the "truth" about their world and their problem is not logical at best.

The validation problem is not a methodology problem, except in one aspect: failing to help and aid the stakeholders to specify clear and accurate definition of their domain world or problem. In short, it is a knowledge elicitation problem. Yes, it is an important problem to address, especially when there is investment in time, money and resources at stake, such as the case when building a multi-million dollar business system. And that's why requirements engineering literature talks about validation and how to test it and solve its dilemmas, but only when it comes to models validation, not methodology validation.

In our research, we were and still concerned with model correctness, completeness, quality and reliability and that's why we discussed them in Al-Shawa (2006b), and highlighted there aspects we considered important to address, even informally, and aspects that we considered for future research. Within this thesis research, our methodology calls for context setting of the problem definition, as well as helps and challenges the analyst and the decision-makers/players to describe not only the decision making, or conflict, environment but also disclose their own interests, goals, and realities; and then captures all of this in formal models and structure. The opposite is practiced and encouraged by the current dominant frameworks and methodologies, where many aspects of the decision-making, or conflict, are reduced to a set of proxies such as preferences and/or utility functions (understood, and therefore verifiable, by specialists only, not the end users).

Our research, beside guiding the decision makers through the process of elicitation, it also allows for further validation of the models to be produced by providing a systematic methodological decision support system, tools and methods, that aids in the process of modelling, reasoning, simulation, stability analysis, sensitivity analysis, and what-if testing of one and multi-agent decision making situations.

2.7 Summary

In this chapter, we have looked at the strategic decision making process as presented in the literature; and how the literature focused by far on one small part, the evaluation of alternatives part, of the process. We then discussed the two main theories dominated the strategic decision making research: decision theory and game theory; and how these theories, and related theories, are inherently limited.

While the Multi-Agent Systems research community mostly used the dominant decision and game theories to address the decision making problem, recent work in the field (especially after the BDI framework appeared in the mid 1990s), as we discussed, start looking at reasoning-about goals and beliefs as a better way to frame the problem and therefore the solution. We concluded with some discussion on decision making theories and the concept of "Rationality", and how many of these theories could not explain the deviation observed in real life case studies from what these theories suggest as "the" rational behaviour; and with discussion on evaluation, verification and validation of conceptual modelling frameworks and how we define these concepts and intend to apply them in our research.

The following chapter will present the Constrained Rationality, a formal goal and constraints reasoning framework for strategic decision making and conflict analysis for single and multi-agent systems and environments. The framework suggests, as discussed in Chapter 1, *bringing back the strategic decision making problem to its roots: reasoning about options/alternatives, not to satisfy a set of preferences, but rather to satisfy the explicitly stated strategic and conflicting goals an agent* *has, given the internal and external complex and conflicting realities/constraints the agent has.*

Chapter 3

Constrained Rationality: A Formal Qualitative Goals-and-Constraints Modelling and Reasoning Framework

3.1 Introduction

This chapter presents an important part of the Constrained Rationality framework. At the heart of the framework is the modelling mechanisms the framework provides to model the goals and realities/constraints that the agents (or the involved decision makers in the decision making situation) have; how these goals and constraints interact and affect each other; how these goals, constraints and the relationships among them are used to reason about the goals and the different alternatives (options to choose from) the agents have; and how this reasoning will guide the agents to choose the best alternatives that provides the most achievement of their respective goals given their respective constraints.

The main purpose of the framework is to allow the agent to represent and reason about the goals that she believes in and the plans she is about to adopt in order to achieve these goals. By reasoning about goals, we mean: be able to judge the degree of operationalization that the agent is committing to these goals, and therefore the degree of achievement possible for such goals; as well as, be able to judge what plans to commit to, and how to order such plans based on the degree

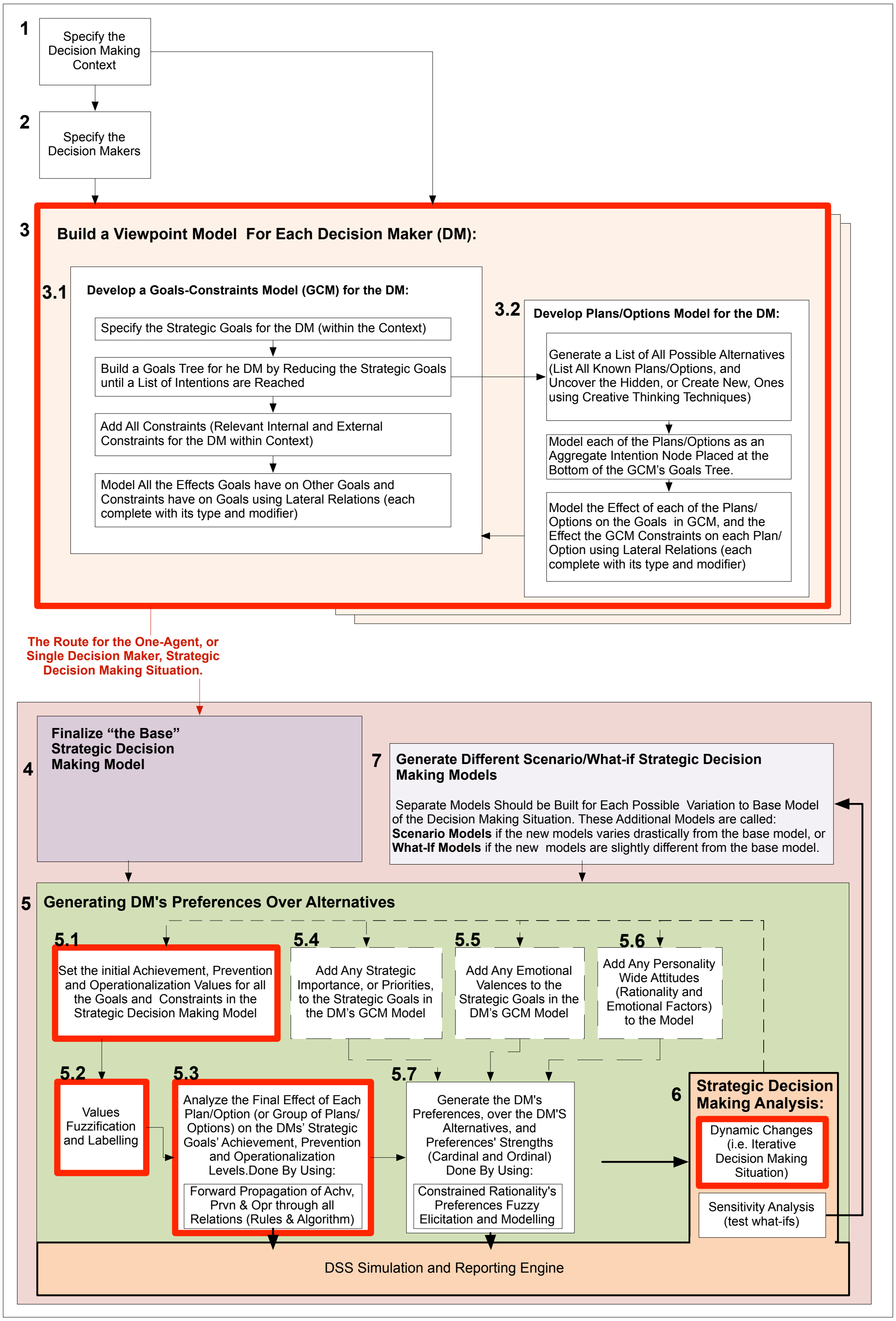


Figure 3.1: The Process of Modelling and Analyzing Single (and Multi-Agent Decision Making & Conflicts, where agents act in an individualistic manner with no consideration to others' current or future choices and decisions). Modelling agents' Viewpoints, Goals-and-Constraints-Models (GCMs), as well as modelling and finalizing the reasoning Value Properties are shown in the Highlighted Boxes and will be covered in this chapter.

of operationalization and achievement those plans will provide to the agent's goals, if the agents decide to commit to them.

Al-Shawa (2006b,a) discussed how ViVD-EKM represent agent's goals and constraints conceptually, and that goals and constraints are mainly captured as part of the Goals & Constraints Model (GCM), a sub-model of an agent's Viewpoint model. Figure 3.2 shows a simple one goal tree GCM model. We need now to find a satisfactory formal goals reasoning framework. The research objective is looking for a formal qualitative reasoning framework which an agent can use formally and systematically in order to rationalize about different aspects of its goals: its operationalization level, its achievement level, any inconsistencies that exit, any conflicting goals, and so on.

To tackle the problem of finding the right satisfactory formal reasoning framework, a step by step approach is followed. We started by tackling the simplest, and the limited in its scope, problem definition where the agent has only one simple goals-reduction-tree and where there are no conflicting or hindering relations among such goals. We, then, slowly increased the scope of the problem to include other items or more details such as goals-goals relation and goals-constraints relations. Once we reach a satisfactory and sufficiently formal reasoning techniques at each level/step of the problem, we move to the next level/step where we introduce additional concepts and relations from the GCM model. The process continued until we have a complete sufficient and effective formal reasoning framework that will satisfy our research objective. Following the steps of the research's plan of attack, we will dedicate one section in this chapter, to each step of the plan. This means that our formal reasoning framework, the Constrained Rationality framework, will be presented, as it was built, gradually. At the end, we will conclude with a summary.

Figure 3.1 shows the Constrained Rationality framework's process of modelling the decision making situation. The figure covers the process of modelling single-agent situations, or multi-agent decision making situations where agents act in an individualistic manner with no consideration to others' current or future choices and decisions). The figure also highlights the parts of the process that will be covered in this chapter. As the figure shows, this chapter will show how the goals and constraints of the different agents in the situation are modelled as viewpoints models and how the value properties, which the agents need for their reasoning abilities, are modelled and finalized. The following chapter will cover the aspects of the framework responsible for modelling the agents' priorities and emotions; and generating the agents' preferences over their alternatives. The chapter after that will look at how the framework's process and components presented in this chapter and the following one are extended and modified in order to model and analyze multi-agent decision making situations.

The chapter starts with Section 3.2 introducing the Goal and Constraints Model (GCM) and its constructs, and Section 3.3 discussing the concept of qualitative labelling of goals' value properties, such as the goals' achievement, operationalization and prevention. The following two sections, Sections 3.4 and 3.5, introduce a formalization of the relations that exist among goals and with constraints, and then discuss how the goals' value labels get propagated across these relations. All the pieces will be put together in Section 3.6. Then, the section will introduce a formal forward qualitative value propagation algorithm to analyze plans/options based on expressed initial set of value properties of the agent's goals and constraints.

Sections 3.7 and 3.8 discuss how the framework will deal with complexities such as the existence of multiple strategic goals, therefore multiple goal trees, and dynamic changes happen to the goals and constraints over time. Then, Section 3.9 will demonstrate the extensibility of the framework by showing some possible extensions such as new types of relations among goals that could be added when the applications/users need such. The chapter then ends with presenting some preliminary experiment, and a summary.

3.2 Agent's Goals & Constraints Model (GCM)

Al-Shawa (2006b,a) proposed the Viewpoints-based Value-Driven Enterprise Knowledge Management (ViVD-EKM) framework, a conceptual modelling framework to model Multi-Agent Systems in the context of knowledge management and analysis. As per ViVD-EKM, the agent has *Viewpoint* models about the world he perceives. These inter-related Viewpoints are structured in a way that each could represent the agent's own knowledge about a subject-matter, a decision making situation of his, or ones of another specific agent/player in his world.

At the heart of each Viewpoint model is the *Goals & Constraints Model* (GCM), a sub-model of the agent's Viewpoint model. GCM captures the agent's goals and constraints with regard to the specific situation/conflict which his viewpoint model is concerned with. The goals within GCM are operationalized by a set of plans (or

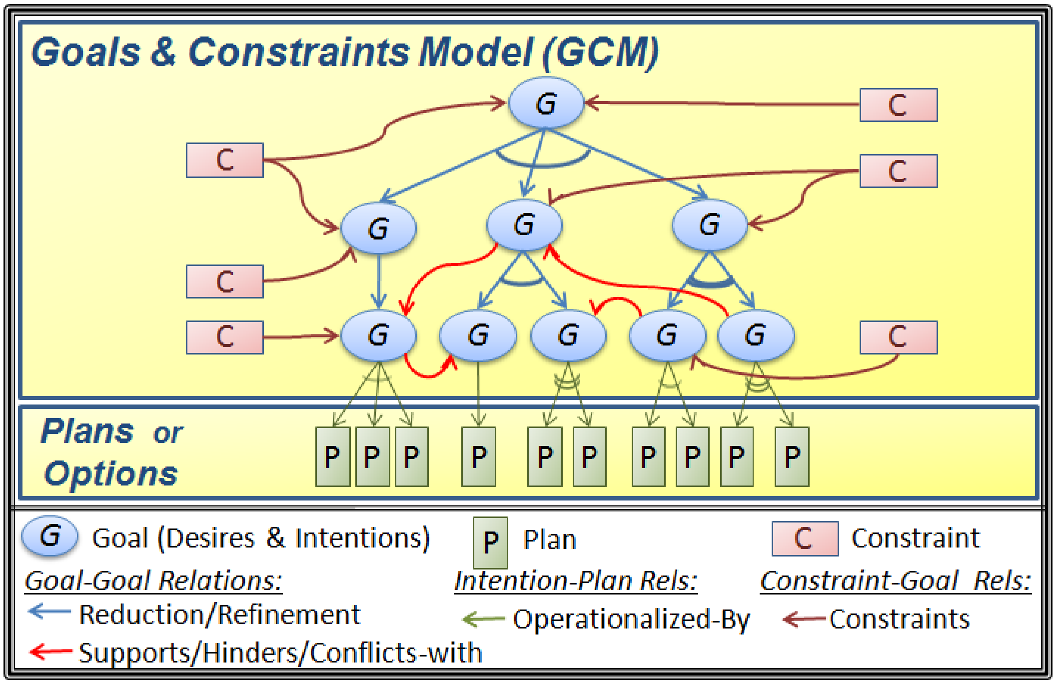


Figure 3.2: Goals & Constraints Model (GCM), with simple one goal-tree

processes), a set of assets (physical or intellectual), products, features of products, etc.

The detailed ontology of GCM, Viewpoint, and the full ViVD-EKM conceptual modelling framework are given in Al-Shawa (2006b). In this thesis document, we will use the GCM part of the viewpoint model because it has all the components to reason about: the goals and constraints. It has all what the judgment about the options (plans, processes, intellectual property or physical assets, products, features, etc.) is based on. But, for simplification and space constraints, we assume that the goals within GCM get operationalized by a set of options/plans through intentional primitive goals to adopt such plans, i.e. there will be no discussion on the details of the options/plans.

Figure 3.2 shows an illustration of a simple one goals-tree GCM model, with goals within the tree affecting each other, and a set of constraints constraining these goals. The lower refined goals at the bottom of goal-tree are operationalized by a set of plans (each is modelled as aggregate plan concept, i.e. with no details about the actions and rules within such plans).

The goal nodes in GCM represent the motivation the agent has within the topic, situation, or conflict that the viewpoint model is concerned with; or they could be represent all the agent's goals, needs and wants, in life in general. Goal nodes are modelled by first inserting the ultimate strategic goals the agent has. Then, these big goals, called *Desires* in ViVD-EKM, go through a refinement/reduction process, using reduction relations, refining them to a set of smaller Desires, and so on until a set of primitive very-refined goals, called *Intentions* in ViVD-EKM, are produced. Intentions are goals that could be operationalized by means of Plans, whilst Desires are goals that could be operationalized by other Desires or Intentions.

The end result of the goals reduction process is a goals tree, or a set of goals trees, where ultimate strategic Desires form the roots of these trees, and with Intentions present at the bottom of each goal tree. We will discuss the reduction relation that connects the upper goals to the immediate lower ones within the trees later in this thesis document.

Goals could have among them different type of relations, other than reduction relations. We call these relations *Goal-to-Goal (G-G) Lateral Relations*, to differentiate them from the top-down G-G reduction relations . These relations represent the supporting, hindering or conflicting effect which some goals have on other goals. For example, in the CM's GCM shown in Figure 3.11, the achievement of "Outsourcing Customer Services" goal (G_{23}) will have a negative impact (hinders or prevents to some degree) the achievement of "Improve Customer Services" goal (G_{13}). We will discuss later the different types/effects of G-G Lateral Relations.

An important component of the GCM model, is the set of *Constraint* nodes it has. As per the ViVD-EKM framework, these nodes could represent internal constraints within the agent, such as resources or capabilities constraints, or external constraints, such as governmental regulation or industrial known constraints. A detailed discussion about Constraints, within the ViVD-EKM framework, is given by Al-Shawa (2006b).

In this thesis document, we treat all constraints similarly as Constraint nodes without differentiating between internal and external (to the agent). An important fact about constraints is that they represent not only limitations on goals' achievement, i.e. affecting goals negatively, but also they could represent opportunities. For example, In the CM example, the "Government Bailing out Auto Industry" constraint (C_6) will provide a positive opportunity for CM to achieve its "Get a bailout/help from the Gov." goal (G_{26}). The effect Constraint nodes have over goal nodes is represented through the use of Constraint-to-Goal (C-G) Lateral Relations, a set of relations similar to the G-G Lateral relation but slightly different. We will discuss them later in this chapter.

Note that the constraint will not guarantee achievement to the goal, it only provides an opportunity for the goal to be achieved to an extent set by the constraint (decided by the bailout amount/percentage decided by the government). The actual achievement of CM's G_{26} intention (goal) will happen through some plan that CM will commit itself to. An intention, which through this plan operationalizing it, will most-likely have a negative impact on other goals CM has, such as the outsourcing goal indicated earlier (because the government will not allow outsourcing as part of the bailout conditions - for example).

Each modelling construct/concept, within the ViVD-EKM conceptual framework, has multiple *Attributes* and *Values* (*Value Properties*). Attributes for concept nodes in general stay not changed through out the life of the instances of the concept node, or at least tend to not change frequently as it is the case of Value Properties attached to the nodes. Values are more dynamic in nature and tend to change with time as the agent realizes changes in the instance value of the Value Property, as the environment around him changes, or his understanding of the world evolves and therefore his interests change.

Attributes are narrowly defined by ViVD-EKM; and a suggested practice of the framework is to use them to represent only conceptual modelling features of the concept, such as: the formal definition of the goal, informal definition of it, its name, its type (desire/intention, or strategic/operational,..), etc. On the other hand, we can have as many Value Properties attached to the concept, based on our needs and the purpose of the knowledge representation exercise. ViVD-EKM recognizes that Value as a concept has many interpretations. This is because it represents an opinion about any of the features or benefits of something.

Generally speaking, *Value* is a concept which describes a measure of the utility of a world object (thing) from the perspective of the person/organization looking at this object. Value is a relative concept that does not exist in the abstract and must be addressed in the context of time, place, evaluator, potential owner, and potential users.

For the purpose of introducing a formal reasoning framework, in this thesis document, we will not use attributes (beyond Goal/Constraints' formal names and numbers) and we will only use Value Properties since they are used intensively in the reasoning abilities of the agents Three important Value Properties are attached to each goal, and two to each constraint:

Goal Achievement is a value property that provides a measure of the achievement level of the goal. Goals' achievement levels propagates up the goals reduction tree from the intentions at the bottom (based on results from the plans attached to these intentions) and up the goals tree until a value is assigned to the achievement level of the goal, or through the G-G lateral relations.

Goal Prevention is a property that describes the hindering (negative) effect that another goal's achievement has on the goal. For example, if an agent has a goal to "increase Sale Price for Product A" and another goal to "increase Sales Numbers for A by 50% this year", then we know for sure, from experience, that increasing the sale price of A will impact negatively the goal to increase A sales, preventing it from happening at least in the short term. The question is by how much? The Goal Prevention value aims to answer this question. The Prevention property is especially important to track conflicting/hindering effect that may be hidden otherwise (if we have only achievement level indicators for goals).

Goal Operationalization is a value property that describes the operationalization level of the goal node. This property will state whether the agent has committed itself to a set of plans that will ensure a degree of operationalization for the goal, or not. Higher goals in the trees have operationalization levels that reflect the degree of operationalization that is provided to each by the lower level goals, mainly the Intentions (who get their operationalization levels through their direct connection to the operationalizing plans).

It is important to track Operationalization, separate from Achievement, because the maximum level of achievability possible for any goal depends on the level of operationalization the agent commits to it. For examples, if a strategic goal has been reduced to a set of goals but none is an intention (committed-to by attaching it to a plan), then the operationalization level of this strategic goal is zero, and therefore its achievement level will be zero (at best), modelling a pure wishful thinking by the agent.

Constraint Achievement is a value property attached to constraints to reflect the true reality/strength of the constraint as imposed by the enforcer, or as believed to be enforced/exist.

Constraint Prevention is a value property attached to constraints, to reflect the prevention the constraints suffer from, stopping them fully or partially from having their effect on the goals they are attached to.

In the following section, we will discuss the process of fuzzy-labelling these value properties with meaningful linguistic qualitative labels, then we will discuss how the value labels of these value properties get propagated through the relations connecting goals and constraints.

3.3 Fuzzy Labeling of Goals' and Constraints' Value Properties

As a matter of notation, the *GCM* model is a graph like structure $\langle\mathcal{G},\mathcal{C},\mathcal{R}\rangle$ where $\mathcal{G}$ is a set of goals, $\mathcal{C}$ is a set of constraints, and $\mathcal{R}$ is a set of relations over $\mathcal{G}$ and $\mathcal{C}$. Let the value properties of Operationalization, Achievement and Prevention for each goal $G_i \in \mathcal{G}$ be represented as variables $Opr(G_i)$, $Achv(G_i)$, and $Prvn(G_i)$ respectively; and the Achievement and Prevention value properties for each constraint $C_j \in \mathcal{C}$ be represented as variables $Achv(C_j)$, and $Prvn(C_j)$ respectively. In addition, let the set of variables, for each goal and constraint, tracks the different level-of-satisfaction for the value property it represents for the goal/constraint. In general, the level of satisfaction for each value property could be expressed numerically as a percentage number (0-100%).

For the purpose of the Constraint Rationality's qualitative reasoning framework, let us consider a limited number of satisfaction levels (instead of considering all the levels between 0-100%) for these value properties' variables. And let these limited set of levels be defined as fuzzy sets, each is given a name which represent a meaningful linguistic label such as *Full*, *Little*, *Some*, *Big*, etc. Each of these fuzzy sets is to be defined by a fuzzy membership function mapping the actual satisfaction level of the property (within the fuzzy domain of the property satisfaction level: 0-100%) to a set membership degree $[0, 1]$.

While the fuzzy domain of any value property's satisfaction-levels can be divided into any number of fuzzy sets, as deemed sufficient and beneficial to the framework user, caution should be exercised to maintain usability (i.e. this should not be taken as a restriction). For this thesis document, we introduce a simple but sufficient (and extendable if needed) scheme to divide the fuzzy satisfaction level domain of each value property to seven sets: *Full*, *Big*, *Much*, *Moderate*, *Some*, *Little*, and *None.*

These fuzzy sets will cover all the value properties (Operationalization, Achievement or Prevention) for goals/constraints, as shown in Figure 3.3. The figure shows

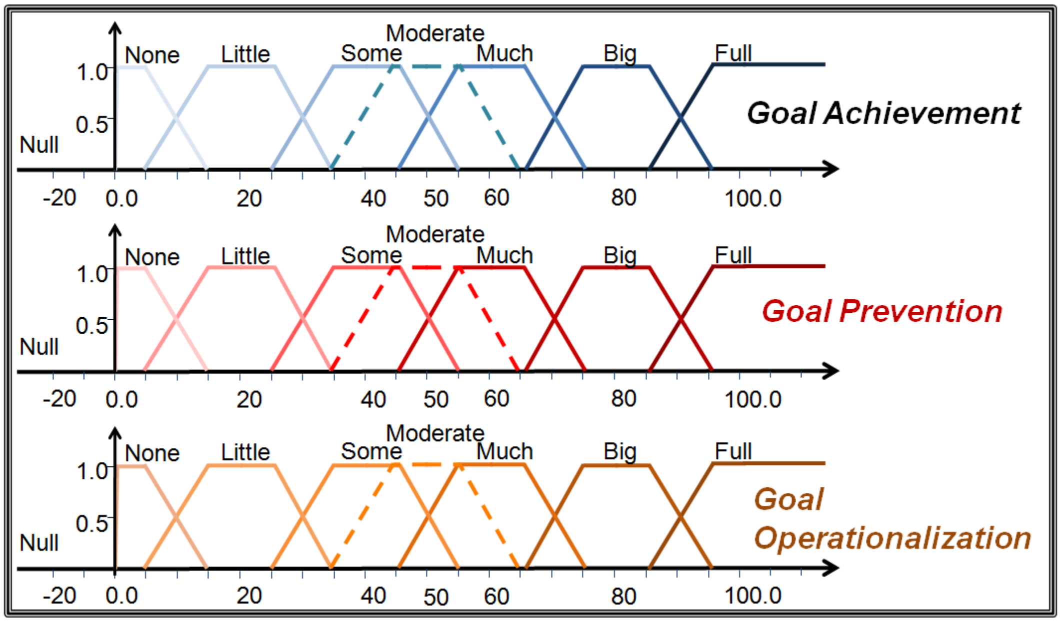


Figure 3.3: Fuzzy Sets dividing the satisfaction levels domain of the different Goals' Value Properties (operationalization, achievement, and prevention)

the membership functions for each set to be trapezoidal in shape, for simplicity only (not as a restriction). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements. Some users will consider a goal *Fully* Achieved if it reached an achievement level of 80-100%, whilst others will consider the goals to be *Fully* Achieved only if their satisfaction levels are 95-100%.

Now, let us introduce $\mathcal{L}$ as a set of labels. The elements of $\mathcal{L}$ matches in number and names the fuzzy sets chosen to divide the satisfaction levels domain of the operationalization, achievement, and prevention value properties. In our case, $\mathcal{L} = \{Full, Big, Much, Moderate, Some, Little, None\} = \{F, B, M, Mo, S, L, N\}$. And let $Full > Big > Much > Moderate > Some > Little > None$, or $F > B > M > Mo > S > L > N$, matching the order of the fuzzy sets coverage over the satisfaction levels domain, with the meaning that the *Full* label represents a higher satisfaction level than *Big*, and so on.

Let the *Achievement* value property of a goal G_i is represented as $Achv(G_i) = L_{achv}$, where $L_{achv} \in \mathcal{L}$, and L_{achv} is a label that matches the name of the fuzzy set which the achievement level of G_i has the highest membership of. For example: if x_i represent the achievement level of the goal G_i and $x_i = 94$, and the achievement level of G_i has memberships of $\mu_{Full}(x_i) = 0.9$, $\mu_{Big}(x_i) = 0.1$ and $\mu_{Much}(x_i) = \cdots = \mu_{None}(x_i) = 0$. This makes $Achv(G_i) = Full$.

Similarly, let the *Operationalization* value property of a goal G_i is represented as $Opr(G_i) = L_{opr}$, where $L_{opr} \in \mathcal{L}$, and where L_{opr} is a label that matches the name of the fuzzy set which the operationalization level of G_i has the highest membership of; and let the *Prevention* value property of a goal G_i is represented as $Prvn(G_i)$=L_{prvn}, where L_{prvn}∈$\mathcal{L}$, and where L_{prvn} is a label that matches the name of the fuzzy set which the prevention level of G_i has the highest membership of. Same to be said about constraints' Achievement and Prevention value properties.

We also use the proposition *Null* to represent the *Null* trivially true statement that the status of the satisfaction level of the value property for a goal/constraint is unknown or negative. Meaning that if x_i represents the achievement level of the goal G_i and x_i is unknown or a negative number, then the achievement level of G_i has memberships of $\mu_{Full}(x_i) = \mu_{Big}(x_i) = \cdots = \mu_{None}(x_i) = 0$, and therefore $Achv(G_i) = Null$. We also add the *Null* label to the set of labels $\mathcal{L}$, introduced earlier, to make $\mathcal{L} = \{Full, Big, Much, Moderate, Some, Little, None, Null\} = \{F, B, M, Mo, S, L, N, Null\}$, and *Full*>*Big*>*Much*>*Moderate*>*Some*>*Little*>*None*>*Null*, or $F > B > M > Mo > S > L > N > Null$.

In the following sections we will introduce the relations that could exist among goals and constraints within the GCM model, and how the value labels of the goal and constraint nodes will propagate through these relations. It is important to mention here, before we discuss the relations, that we are providing a complete set of ground relation axioms for all the goal-to-goal and goal-to-constraint relations, discussed later in this chapter, as an appendix. It should be understood that the propagation rules presented are generated by aggregating/generalizing the extensive ground axioms we started with. The rules and axioms are tested for soundness and completeness. Proofs of such are included in the appendix.

3.4 Goal-to-Goal Relations

There are two types of relations that could exist among goal nodes within GCM: *Reduction Relations*, and *Goal-to-Goal (G-G) Lateral Relations*.

3.4.1 Goal Reduction/Refinement Relations

The main objective of the goal reduction process is to produce eventually a set of primitive goals whose operationalization, through known plans (processes, products, features, or otherwise) by the agent, are obvious. To be able to reduce strate-

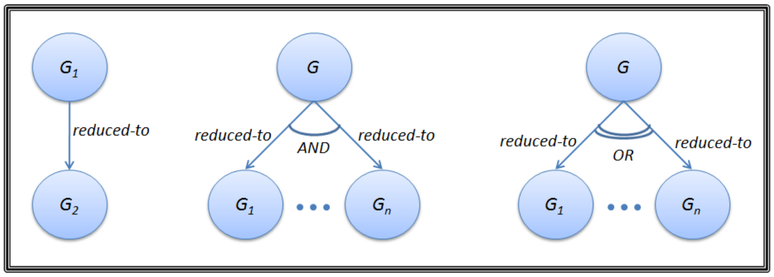


Figure 3.4: Goal Reduction/Refinement Relations

gic big goals (Desires) of the agent to a set of such smaller can-be-operationalized goals (Intentions), we make use of what is called *Goal Reduction/Refienment Relationships*. These are relations that can model a strategic goal as a conjunction or disjunction of a number smaller operational goals, decomposing the higher bigger goal to a lower (level) more detailed ones.

Goal reduction relations, especially *AND* and *OR* ones, are the easiest type of relations that could exist among goals, and the most widely used since the early days of conceptual modelling and AI (e.g. Nilsson (1971)). The goal reduction/refinement process, as said earlier, is responsible for generating the tree like structures found in goal-tree/s. Because of the popularity of this type of relations, we will not expand on it except to add few clarifications.

In this thesis document, for simplicity, only binary *AND* and *OR* goal reduction relations are considered (shown in Figure 3.4. This should not be taken as a restriction, in fact any n-ary operator could be used. The operators we use and consider in our framework, such as $\wedge$, $\vee$, min, max, etc., are all associative, therefore can be used as n-ary operators. And because strategic goals at the top of the goals-tree are reduced using n-ary operators, we can use such decomposition to propagate meaningful properties (or value properties' labels) of these goals across such relations.

The Propagation Rules of Value Labels using Goal Reduction Relations:

$$(G_1, G_2) \xrightarrow{and} G: \quad Opr(G) = \min\{Opr(G_1), Opr(G_2)\} \tag{3.1}$$
$$Achv(G) = \min\{Achv(G_1), Achv(G_2)\} \tag{3.2}$$
$$Prvn(G) = \max\{Prvn(G_1), Prvn(G_2)\} \tag{3.3}$$

$$(G_1, G_2) \xrightarrow{or} G: \quad Opr(G) = \max\{Opr(G_1), Opr(G_2)\} \tag{3.4}$$
$$Achv(G) = \max\{Achv(G_1), Achv(G_2)\} \tag{3.5}$$
$$Prvn(G) = \min\{Prvn(G_1), Prvn(G_2)\} \tag{3.6}$$

3.4.2 Goal-Goal Lateral Relations

Goal-to-Goal (G-G) Lateral Relationships are relations that represent the effect of goals other than the immediate successor and follower goals, part of the goal own reduction tree, on the goal itself. It is commonly known, in real life, that working to achieve a specific goal will make achieving another goal a bit easier or harder, in some cases even conflict completely with it. For example, if a person has a goal of "becoming rich", achieving this goal will make achieving another goal of hers, not part of the becoming-rich goal's reduction tree, such as "buying a new house" to be more easier, while making the goal of "spending more time with family" to be more harder to achieve than ever (since she had to spend more time now on accumulating wealth).

In this section, we introduce a formalization of the G-G lateral relations introduced informally in Al-Shawa (2006a), and briefly mentioned earlier: *Supports*, *Hinders* and *Conflicts-with*. Figure 3.5 lists 12 possible combinations of cause-effect relations which we could have, assuming that the cause comes in the form of reaching either a full or a partial satisfaction level of one of the three value properties (operationalization, achievement or prevention) of the *start goal*, or the goal G_1 which is in left side of the lateral relation $G_1 \xrightarrow{lr} G$, and the effect comes in the form of reaching either a full or a partial satisfaction level of one the three properties for the goal G on the right side of the relationship, the *end goal*.

These lateral relations are named based on whether the cause/effect is positive (achievement or operationalization) or negative (prevention) on the goal at that end of the relation. For example, if G_1 is achieved fully and this will cause G to be fully achieved as well, then we call the relation: a "$_{++}$" relation; and if having G_1 fully achieved will cause G to be fully prevented, then the relation is called: a "$_{+-}$" relation. And, to differentiate between fullness and partiality of effect, we put round brackets around the sign which represent the effect, when the effect is partial. For example, if achieving G_1 fully will cause G to be partially prevented, then the lateral relation between them will be called: a "$_{+(-)}$" relation, and we represent this relation as: $G_1 \xrightarrow{+(-)} G$. The degrees of partiality of the "effect", such as the effect is small or moderate, will be addressed by adding what we call a *Modifier* to the effect side of the relation's definition. We will discuss later in this section the Modifier concept, but for the time being let us assume that the partiality in the effect is dealt with in the aggregate as "Some" and represented with the round brackets around the effect's positive or negative sign, as shown above.

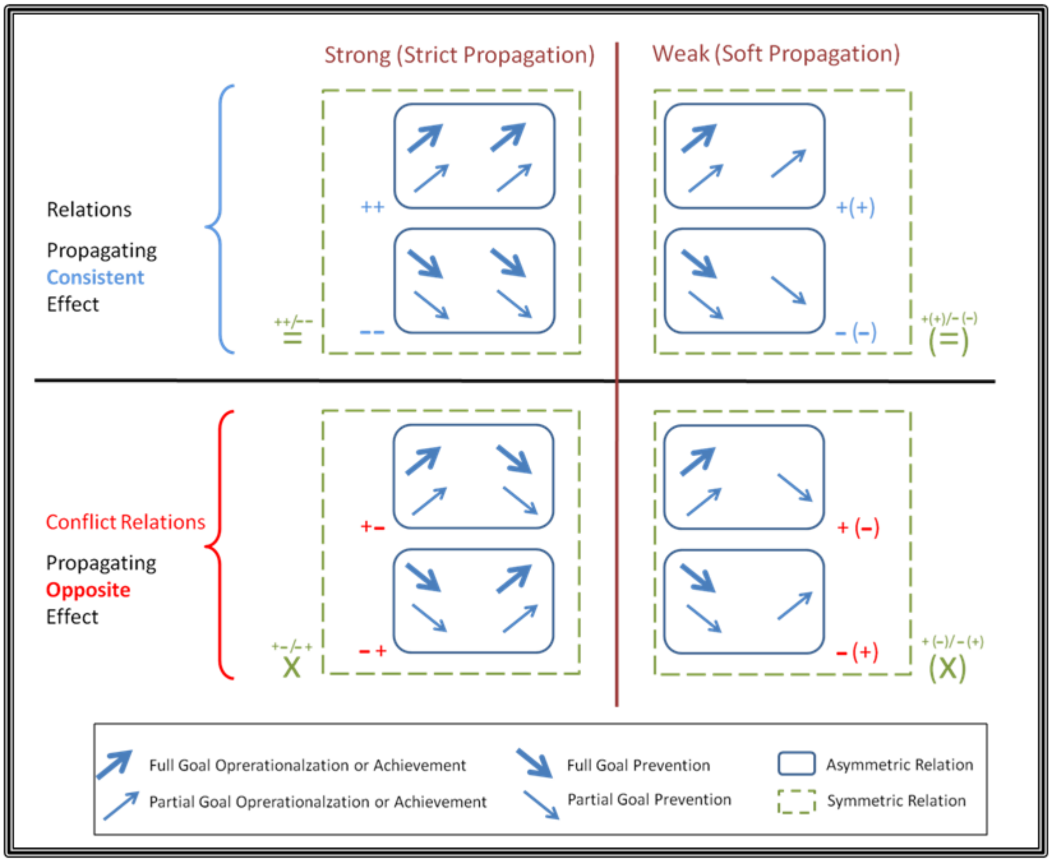


Figure 3.5: Goal-to-Goal (G-G) Lateral Relations

Figure 3.5 shows four lateral relations categorized as *Strong/Strict Lateral Relations*. These relations have the strength level of the cause (operationalization, achievement or prevention in the start goal) match the strength level of the effect on the end goal of the relation. The Strong Lateral Relations include the following relations: “++”, “−−”, “+−”, and “−+”. On the other hand, the *Weak/Soft Lateral Relations* are relations which propagate a *partial* effect on the end goal of the relation without consideration to the strength level of the cause. In this category of relations, the names of the relations will always have the effect sign between round brackets to show that these relation do not cause severe/full operationalization, achievement or prevention to the goal on the end side of the lateral relation, hence the name. Relations belong to this category are: “+(+)”, “−(−)”, “+(−)”, and “−(+)”. It is worth mentioning that we did not find the need to add lateral relations that have a partial level cause to produce a full level effect on the end goal of the relation, for two reasons: 1) it is not a usual relation to happen in real life; and 2) it will not be hard to add these relations when proved needed or beneficial, as we will show later in Section 3.9.

The above mentioned eight lateral relations could be categorized differently. In Figure 3.5, the lateral relations are split horizontally into two distinct groups. The top group includes relations in which the causes propagate consistently to the relations' effects. In other words, the sign of the cause matches the sign of the effect. This groups has the "$_{++}$", "$_{--}$", "$_{+(+)}$", and "$_{-(-)}$" relations. The group at the bottom includes relations in which the cause sign is always different from the effect sign, or the opposite of it. Each of these relations represents a conflict among its two goals. If the start goal is achieved, the end goal will be prevented, and vice versa. Relations that are part of this group are: "$_{+-}$", "$_{-+}$", "$_{+(-)}$", and "$_{-(+)}$".

A fair question to ask here is: why we called the second group *Conflict Relations* and not *inconsistency* relations? ViVD-EKM differentiate between an inconsistency and a conflict. If there are two relations starting from the same start goal, say G_1, and pointing at the same end goal, say G, one described as "++" relation and the other as "+−", then this is a clear instance of an *inconsistency*, that hint at a problem in the knowledge-base, or to be more specific a confusion about what is the actual effect of the source goal on the target goal. A problem that must be resolved. On the other hand, if we have the two relations connecting G_1 to G to be "+−" and "−+", then this represent a conflict-with relation between G_1 and G (with the meaning that if $G1$ is been achieved, then G will be prevented; and if $G1$ is been prevented, then G will be achieved). The "conflict-with" relationship is allowed to exist, because it represents a clear and valid relation among goals in real life; and does not represent a confusion, or a problem in the knowledge base.

Inconsistencies are not allowed in ViVD-EKM, but conflicts are permitted. Not only conflicts are allowed to be captured/modelled as fully supported modelling constructs, but also they are allowed within the reasoning process. This is quite different from what other knowledge modelling frameworks do. We felt the need to allow for representing conflicts because conflicts in the real world are the source of opportunities as we explained and discussed thoroughly in Al-Shawa (2006b). Therefore, Conflict Relations are supported in Constrained Rationality while inconsistencies are highlighted to be brought to the modeller's attention, in order to be resolved. Conflict Relations such as the ones discussed above, namely conflicts among goal nodes within the GCM model, represent one example of how conflicts are represented within the ViVD-EKM framework.

It is worth mentioning here that the eight lateral relations discussed above are all *Asymmetric Lateral Relations*. If an asymmetric relation states that an achievement in the start goal will result on an achievement of the end goal, then it is *not true* that the statement also supports the argument that a prevention of the start goal will result on a prevention of the end goal. Now, let us introduce four *Symmetric Lateral Relations*: "$_=$", "$_{(=)}$", "$_\times$" and "$_{(\times)}$". Each of these relations is the equivalent of combining two of the Asymmetric Relations listed above: ("$_{++}$" ∪ "$_{--}$"); ("$_{+(+)}$" ∪ "$_{-(-)}$"); ("$_{+-}$" ∪ "$_{-+}$"); and ("$_{+(-)}$" ∪ "$_{-(+)}$") respectively. Out of the four Symmetric Lateral Relations, both "$_=$" and "$_{(=)}$" relations produce consistent cause-effect, whilst "$_\times$" and "$_{(\times)}$" relations represent conflict relations. At the same time, "$_=$" and "$_\times$" are strong lateral relations, while the "$_{(=)}$" and "$_{(\times)}$" are weak lateral relations.

The set of lateral relations introduced above will be able to represent sufficiently any support, hinders or conflicts-with relations that could exist between any two goals in the agent's GCM model. But before listing the propagation rules for the G-G lateral relations, let us first add the concept of a *Modifier* to the lateral relation. So far, we used a lateral relation of "$_{+(+)}$" to represent a relation in which a full or partial achievement of the start node will make the achievement level of the targeted end node be partial, or "*Some*". In fact, it is like stating that the relation is "$_{+(Some+)}$". The *Some* part of the relation's definition is what we call the relation's *Modifier*. The relation's *Modifier* M is a label that belongs to the same set of labels $\mathcal{L}$ used for value properties, i.e. $M \in \mathcal{L}$, where $\mathcal{L} = \{Full, Big, Much, Moderate, Some, Little, None, Null\}$. *Modifier* M defines the at-most achievement/prevention (based on whether the end part of the lateral relation is "$_+$" or "$_-$" respectively) level of the end-goal targeted by the relation. For example, if a lateral relation is defined as $G_1 \stackrel{+(Little-)}{\longrightarrow} G$, then this means a "*Full*" achievement of G_1 will cause at most a "*Little*" prevention to G. Also, it is worth noting here that an assignment of *Null* as a label to a relation's *Modifier* makes the relation has no effect on the targeted end node, i.e. as if the relation does not exist.

The Propagation Rules of Value Labels using G-G Lateral Relations:

1) for the Symmetric Consistent G-G Lateral Relations:

$$G_1 \stackrel{=}{\longrightarrow} G: \quad Opr(G) = Opr(G_1) \tag{3.7}$$

$$Achv(G) = Achv(G_1) \tag{3.8}$$

$$Prvn(G) = Prev(G_1) \tag{3.9}$$

$$G_1 \stackrel{(M=)}{\longrightarrow} G: \quad Opr(G) = \min\{Opr(G_1), M\} \tag{3.10}$$

$$Achv(G) = \min\{Achv(G_1), M\} \tag{3.11}$$

$$Prvn(G) = \min\{Prvn(G_1), M\} \tag{3.12}$$

2) for the Symmetric Conflict G-G Lateral Relations:

$$G_1 \xrightarrow{\times} G : \quad Achv(G) = Prev(G_1) \tag{3.13}$$

$$Prvn(G) = Achv(G_1) \tag{3.14}$$

$$G_1 \xrightarrow{(M\times)} G : \quad Achv(G) = \min\{Prvn(G_1), M\} \tag{3.15}$$

$$Prvn(G) = \min\{Achv(G_1), M\} \tag{3.16}$$

3) for the Asymmetric Consistent G-G Lateral Relations:

$$G_1 \xrightarrow{++} G : \quad Opr(G) = Opr(G_1) \tag{3.17}$$

$$Achv(G) = Achv(G_1) \tag{3.18}$$

$$G_1 \xrightarrow{+(M+)} G : \quad Opr(G) = \min\{Opr(G_1), M\} \tag{3.19}$$

$$Achv(G) = \min\{Achv(G_1), M\} \tag{3.20}$$

$$G_1 \xrightarrow{--} G : \quad Prvn(G) = Prev(G_1) \tag{3.21}$$

$$G_1 \xrightarrow{-(M-)} G : \quad Prvn(G) = \min\{Prvn(G_1), M\} \tag{3.22}$$

4) for the Asymmetric Conflict G-G Lateral Relations:

$$G_1 \xrightarrow{+-} G : \quad Prvn(G) = Achv(G_1) \tag{3.23}$$

$$G_1 \xrightarrow{+(M-)} G : \quad Prvn(G) = \min\{Achv(G_1), M\} \tag{3.24}$$

$$G_1 \xrightarrow{-+} G : \quad Achv(G) = Prev(G_1) \tag{3.25}$$

$$G_1 \xrightarrow{-(M+)} G : \quad Achv(G) = \min\{Prvn(G_1), M\} \tag{3.26}$$

3.5 Constraint-Goal Relations

We have studied the effect of goals on goals through reduction and lateral goal-to-goal relations, but the effect on goals could be caused by more than just goals. Constraints also could affect goals, as discussed earlier. Constraints represent realities about the internal affairs of the agent, limiting the agent's ability to achieve some of its goals. It could also represent external and/or industrial limiting realities about the the domain.

Constraints are connected to goal nodes through Constraint-to-Goal (C-G) Lateral Relations, which are similar to the G-G Lateral ones. Figure 3.6 shows the effect a constraint will have on a goal through the C-G Lateral Relation. The constraint will not "add" achievability to the goal's achievement value property, but rather it will set an upper limit to the level of achievement the value property could

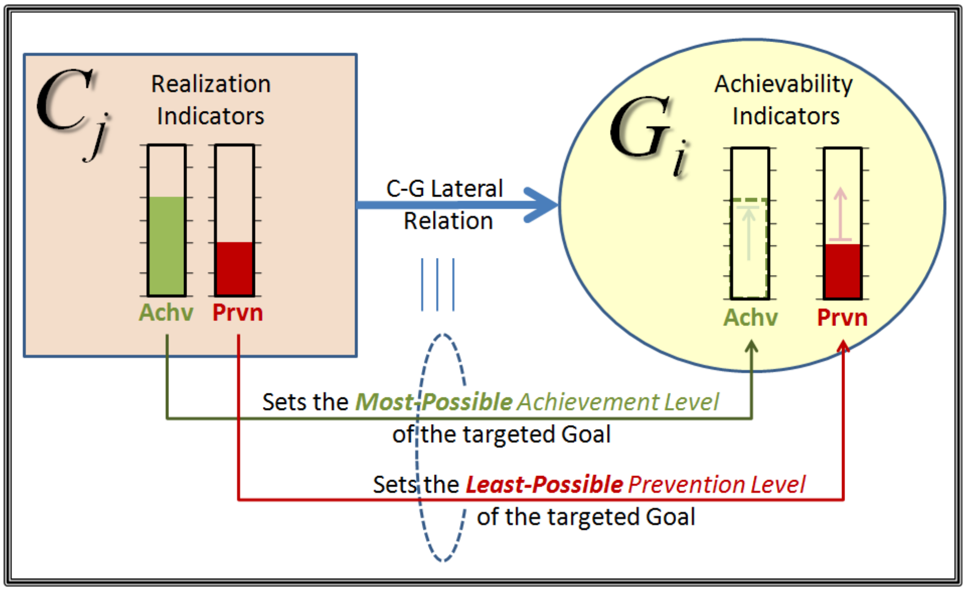


Figure 3.6: The effect of a Constraint-to-Goal (C-G) Lateral Relation on the Achievability and Prevention Value Properties of the targeted Goal Node

have. With regards to the targeted goal's prevention value property, the constraint through the C-G Lateral Relation will set the minimum prevention level the value property could have. This means that the constraint, and based on the type of the C-G lateral relation which connects the constraint to the targeted end goal node, will set the potential (the upper limit) of achievability which the end goal could have, and/or will set the minimum level (the lower-starting-level) of prevention the end goal must have.

The Constraint-to-Goal (C-G) Lateral Relations syntactically are similar to the G-G Lateral ones, i.e. show the same possible propagation flows as the ones shown in Figure 3.5 for the G-G lateral relations. Therefore, the propagation rules of value properties' labels using C-G Lateral Relations are similar to the G-G ones introduced above, with one exception: constraints do not have Operationalization values and do not affect the Operationalization values of their target end goal nodes (connected to them using the C-G lateral relation).

One also has to remember that, as discussed above, the constraint's effect on a targeted end goal by a lateral relation is different from the one a goal has on an end goal, even though the lateral relations' type used look similar. For example, whilst a "++" G-G lateral relation ensures that the targeted end goal gets an achievability level similar to the one of the start goal (of the lateral relation), the similar-in-type "++" C-G lateral relation will only set *the upper limit* of achievability that the targeted end goal node *could have* to a level similar to achievability level of the start constraint.

Therefore, if the C-G lateral relation has a positive effect on the targeted end goal, the propagation rules of the relation will set the upper-limit $Achv_{up-lim}(G)$, rather than setting the goal's achievability level itself, or $Achv(G)$, which similar (in type) G-G lateral relation's propagation rules will do. Likewise, if the C-G lateral relation describes that the start constraint (of the relation) has a negative effect on the targeted end goal, then the propagation rules of the relation will set the least possible prevention level that the targeted goal must have, or the lower-limit $Prvn_{lo-lim}(G)$, rather than setting the prevention level of the end goal, or $Prvn(G)$, which similar (in type) G-G lateral relation's propagation rules will do.

The Propagation Rules of Value Labels using C-G Lateral Relations:

1) for the Symmetric Consistent C-G Lateral Relations:

$$C \xrightarrow{=} G: \qquad Achv_{up-lim}(G) = Achv(C) \tag{3.27}$$

$$Prvn_{lo-lim}(G) = Prev(C) \tag{3.28}$$

$$C \xrightarrow{(M=)} G: \qquad Achv_{up-lim}(G) = \min\{Achv(C), M\} \tag{3.29}$$

$$Prvn_{lo-lim}(G) = \min\{Prvn(C), M\} \tag{3.30}$$

2) for the Symmetric Conflict C-G Lateral Relations:

$$C \xrightarrow{\times} G: \qquad Achv_{up-lim}(G) = Prev(C) \tag{3.31}$$

$$Prvn_{lo-lim}(G) = Achv(C) \tag{3.32}$$

$$C \xrightarrow{(M\times)} G: \qquad Achv_{up-lim}(G) = \min\{Prvn(C), M\} \tag{3.33}$$

$$Prvn_{lo-lim}(G) = \min\{Achv(C), M\} \tag{3.34}$$

3) for the Asymmetric Consistent C-G Lateral Relations:

$$C \xrightarrow{++} G: \qquad Achv_{up-lim}(G) = Achv(C) \tag{3.35}$$

$$C \xrightarrow{+(M+)} G: \qquad Achv_{up-lim}(G) = \min\{Achv(C), M\} \tag{3.36}$$

$$C \xrightarrow{--} G: \qquad Prvn_{lo-lim}(G) = Prev(C) \tag{3.37}$$

$$C \xrightarrow{-(M-)} G: \qquad Prvn_{lo-lim}(G) = \min\{Prvn(C), M\} \tag{3.38}$$

4) for the Asymmetric Conflict C-G Lateral Relations:

$$C \xrightarrow{+-} G: \qquad Prvn_{lo-lim}(G) = Achv(C) \tag{3.39}$$

$$C \xrightarrow{+(M-)} G: \qquad Prvn_{lo-lim}(G) = \min\{Achv(C), M\} \tag{3.40}$$

$$C \xrightarrow{-+} G: \qquad Achv_{up-lim}(G) = Prev(C) \tag{3.41}$$

$$C \xrightarrow{-(M+)} G: \qquad Achv_{up-lim}(G) = \min\{Prvn(G_1), M\} \tag{3.42}$$

3.6 Constrained Rationality's Qualitative Forward Reasoning Framework

We talked about how the value labels of goals propagate or get affected along individual relations, whether these relations are G-G or C-G relations, but still we did not discuss the overall effect that all these relations sinking into the goal node, or fanning out from it, will have on its value properties' labels. In this section, we will present first how the final value labels of each of the goals' value properties will be calculated. Then, we will present a forward propagation algorithm that will update all the value properties of all the goal nodes in the GCM model with their final value labels.

Let $\mathcal{R}_{G-G} \subseteq \mathcal{R}$, where $\mathcal{R}_{G-G}$ is the set of relations in $\mathcal{R}$ which includes all goal reduction and goal-to-goal lateral relations exist in $\mathcal{R}$; and $\mathcal{R}_{C-G} \subseteq \mathcal{R}$, where $\mathcal{R}_{C-G}$ is the set of relations in $\mathcal{R}$ which includes all constraint-to-goal lateral relations in $\mathcal{R}$. And, let $(\mathcal{R}_{G-G} \cap \mathcal{R}_{C-G}) = \varnothing$ whilst $(\mathcal{R}_{G-G} \cup \mathcal{R}_{C-G}) = \mathcal{R}$. For each goal $G_i \in \mathcal{G}$, let: the set of G-G relations (reduction and lateral) that targets/ends-with G_i is the set $\mathcal{R}_{G-G_i} \subseteq \mathcal{R}_{G-G}$; the set of C-G lateral relations that targets/ends-with G_i is the set $\mathcal{R}_{C-G_i} \subseteq \mathcal{R}_{C-G}$; and $Achv_r(G_i)$, $Opr_r(G_i)$, and $Prvn_r(G_i)$ are the value properties of the goal G_i as a result of the relation r. When the relation $r \in \mathcal{R}_{C-G_i}$, i.e. when r is a constraint-to-goal lateral relation targeting goal G_i, then the $Achv_r(G_i)$ and $Prvn_r(G_i)$ results from r are the $Achv_{up-lim}(G_i)$ and $Prvn_{lo-lim}(G_i)$, respectively, that r produces by applying the propagation rules for C-G lateral relations listed in the previous section (rules: 3.27- 3.42).

Then, *the final value labels of G_i's value properties, at any time t, are concluded by the following propagation rules:*

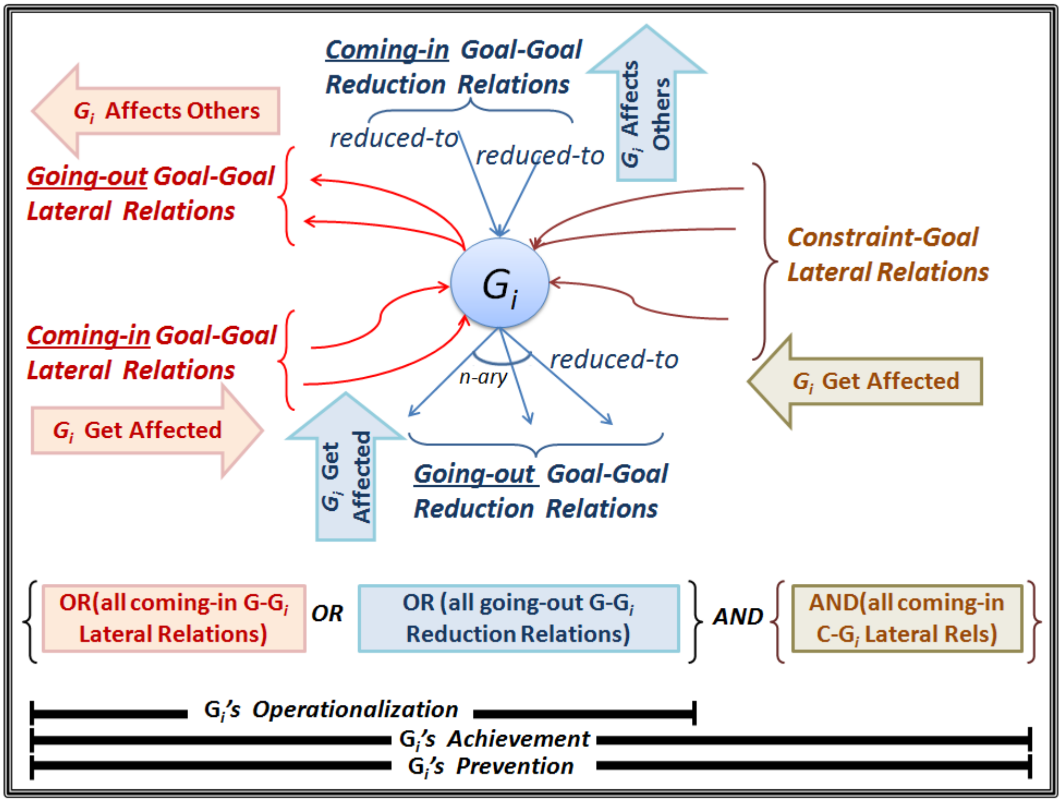


Figure 3.7: Dealing with multiple Goal-to-Goal and Constraint-to-Goal Relations coming-in to a Goal Node, or going-out from it, and the final effect of such relations on the node's value properties

$$Opr(G_i) = \left(Opr(G_i) \vee \bigvee_{r_j \in \mathcal{R}_{G-G_i}} Opr_{r_j}(G_i) \right) \tag{3.43}$$

$$Achv(G_i) = \left(Achv(G_i) \vee \bigvee_{r_j \in \mathcal{R}_{G-G_i}} Achv_{r_j}(G_i) \right) \wedge \left(\bigwedge_{r_k \in \mathcal{R}_{C-G_i}} Achv_{up\text{-}lim_{r_k}}(G_i) \right) \tag{3.44}$$

$$Prvn(G_i) = \left(Prvn(G_i) \wedge \bigwedge_{r_j \in \mathcal{R}_{G-G_i}} Prvn_{r_j}(G_i) \right) \vee \left(\bigvee_{r_k \in \mathcal{R}_{C-G_i}} Prvn_{lo\text{-}lim_{r_k}}(G_i) \right) \tag{3.45}$$

Figure 3.7, and the propagation rules above, show that the effect on any goal's achievement (and operationalization), based on all G-G relations (whether reduction or lateral) targeting the goal is maximized, whilst the goal's prevention is minimized. This follows the same rules of the *OR* reduction relation introduced earlier. In other words, each goal-to-goal relation, whether reduction or lateral, is considered to represent a different mean to achieve, operationalize and/or prevent the targeted end goal, and therefore all these relations act as if connected to the end goal by an *OR* relation. At the same time, the figure and the rules show that each C-G lateral relation plays a role of limiting (minimizing) the end goal's achievement to an upper limit set by the relation if it has positive effect on the goal, or increasing the end goal's prevention to a level match one set by the constraint if the constraint has a negative effect on the goal. Otherwise stated, a constraint effect on a goal follows the same rules of *AND*ing two goals' values (the *AND* relation rules are introduced earlier), but here with two values of the same goal: the first is the original value before the constraint's effect and the second is the value after the constraint's effect. Overall, the effect of all the C-G lateral relations on a specific targeted end goal are *AND*ed together and then *AND*ed with the goal's value properties after the effect of all the G-G relations targeting it had been applied.

Goals' Value Labels Forward Propagation Algorithm:

Let there be four arrays: *Initial_C* is an array that holds the value-labels of $\langle Achv(C_i), Prvn(C_i)\rangle$ for each $C_i \in \mathcal{C}$ part of the GCM model graph $\langle \mathcal{G}, \mathcal{C}, \mathcal{R}\rangle$; *Initial_G* is an array that holds the value-labels of $\langle Opr(G_j), Achv(G_j), Prvn(G_j)\rangle$ for each $G_j \in \mathcal{G}$ part of the GCM model graph $\langle \mathcal{G}, \mathcal{C}, \mathcal{R}\rangle$; *Previous_G* holds the previous value-labels for each G_i as per the last run of the propagation algorithm loop; and *Current_G* holds the current value-labels for each G_i as per the current run of the algorithm. The number of elements in *Initial_C* is $|\mathcal{C}|$; and in each of *Initial_G*, *Previous_G* and *Current_G* is $|\mathcal{G}|$. Now, the value-labels forward propagation algorithm is given as Algorithm 3.1

Termination and Complexity: A note worth mentioning, before we present our theorem about the algorithm termination and complexity, is that the formulas in our framework are all propositional Horn Clauses (one is headless clause, and the rest are all headed). This means that deciding if a ground assertion holds not only decidable, but also decidable in polynomial time (Horn, 1951; Clocksin and Mellish, 1984). Nevertheless, the following theorem and its proof show the at-most number of loops that the algorithm needs to go through for it to terminate.

Theorem 3.6.1: *The Label_GCM_Goals algorithm will terminate after at most* $(3|\mathcal{L}||\mathcal{G}|) + 1$ *loops.*

Proof. From lines 15, 16 and 17 of the *Label_GCM_ Goals* algorithm, each of the value property variables, for each goal G_i, is either monotonically increasing

Algorithm 3.1 Goals' Value-Labels Forward Propagation Algorithm

```
value-label-array Label_GCM_Goals(GCM_Graph⟨G,C,R⟩, c-value-label-array Initial_C, value-label-array Initial_G)
// Start with the Goals value-labels given in Initial_G
Curren_G=Initial_G

repeat
   Previous_G=Current_G
   // For every Goal, apply all Relations feeding into it
   for all G_i ∈ G do
      //OR all Goal-to-Goal Relations affecting G_i
      for all R_j ∈ R_{G−G} such that end_goal(R_j) == G_i do
         Opr = Apply_G_to_G_Opr_Rules(G_i, R_j, Previous_G)
         Achv = Apply_G_to_G_Achv_Rules(G_i, R_j, Previous_G)
         Prvn = Apply_G_to_G_Prvn_Rules(G_i, R_j, Previous_G)
         //OR with the effect of all previous G-G Rel affected G_i so far
         Current_G[i].Opr = max(Opr, Previous_G[i].Opr)
         Current_G[i].Achv = max(Achv, Previous_G[i].Achv)
         Current_G[i].Prvn = min(Prvn, Previous_G[i].Prvn)
      end for

      //AND all Constraint-to-Goal Relations affecting G_i
      for all R_k ∈ R_{C−G} such that end_goal(R_k) == G_i do
         Achv = Apply_C_to_G_Achv_Rules(G_i, R_k, Initial_C)
         Prvn = Apply_C_to_G_Prvn_Rules(G_i, R_k, Initial_C)
         //AND with the effect of all previous G-G/C-G Rel affected G_i so far
         Current_G[i].Achv = min(Achv, Previous_G[i].Achv)
         Current_G[i].Prvn = max(Prvn, Previous_G[i].Prvn)
      end for
   end for
until (Current_G==Previous_G)

return Current_G
```

or monotonically decreasing, but not both. Whilst both *Current_G[i].Opr* and *Current_G[i].Achv* value property variables, tracking G_i's operationalization and achievement levels respectively, monotonically increase over the life of the repeat-until loop cycles, the *Current_G[i].Prvn* value property variable, tracking the prevention level of goal G_i, monotonically decreases during the algorithm's repeat-until loop life. The labels that could be assigned to each value property variable are element of $\mathcal{L}$, in our case $\mathcal{L}=\{Full, Big, Much, Moderate, Some, Little, None, Null\}$, where $Full>Big>Much>Moderate>Some>Little>None>Null$, and $|\mathcal{L}|=8$.

In order for the algorithm's repeat-until loop not to terminate, one of the monotonically increasing value properties must change to a higher level (e.g. from *Null* to *None*, *None* to *Little*, or *Some* to *Full*) and/or the monotonically decreasing value property to change to a lower level (e.g. from *Full* to *Big*, *Much* to *Some*, or *Little* to *None*). And since we know that each of the $3|\mathcal{G}|$ value property variables (*Opr*(G_i), *Achv*(G_i) and *Prvn*(G_i)) takes one of $|\mathcal{L}|$ possible value labels, or eight possible labels for our specific $\mathcal{L}$, and that in each none final run (of the repeat-until loop) at least one of these $3|\mathcal{G}|$ value property variables will either increase (if the variable is one of the monotonically increasing value property variables, i.e one of *Opr*(G_i) and *Achv*(G_i)), or decrease (if the value property variable is *Prvn*(G_i)), then we can conclude that all the $3|\mathcal{G}|$ value properties will settle and reach to their final values in at most $|\mathcal{L}| * 3|\mathcal{G}|$ runs of the repeat-until loop of the algorithm. But the loop must continue for an additional one more run to satisfy the until-condition. This makes the algorithm to terminate after at most $3|\mathcal{L}||\mathcal{G}| + 1$ (or $24|\mathcal{G}| + 1$ using our specific case of $\mathcal{L}$) runs of its repeat-until loop structure. □

One could ask whether lines 25 and 26 of the algorithm could make the values of the value property variables zigzagging up and down, contradicting what is been indicated in the proof above that lines 16 and 17 ensure a monotonically increasing *Achv*(G_i) value property variable and a monotonically decreasing *Prvn*(G_i) value property variable. Answering this, we have to remember that "*AND*ing all constraint-to-goal relations affecting G_i" part of the algorithm (lines 20-27) has only a limiting effect on the value properties, making *Achv*(G_i) to not exceed a level of achievement set by the constraint and the lateral connection type, and/or making the *Prvn*(G_i) to not go below a level of prevention set by the constraint and the lateral connection type. This limiting effect, which the constraint-to-goal lateral relations set to the achievement level or prevention level of any goal, is consistent through all the repeat-until loop runs ensuring that the limit is satisfied for the specific goal at hand, G_i in this case. This limit will not affect other goals, but rather maintain G_i's own achievement/prevention to the satisfactory levels (set in the previous run) if it has been changed in the current run to a level that is not confirming with the collective upper/lower limits (respectively) established by all C-G relations affecting G_i . Thus, lines 25 and 26 of the algorithm will not change the fact that *Achv*(G_i) monotonically increases, and that *Prvn*(G_i) monotonically decreases, but rather will stop the increase, or decrease, at a level confirming with all the constraints affecting the goal G_i.

It is worth noticing that the algorithm will terminate much much sooner than the at-most $(3|\mathcal{L}||\mathcal{G}|) + 1$ loops ceiling indicated in the theorem because many updates to the value properties variables will happen in parallel; and because many of the value updates will not follow a step-wise increase/decrease (e.g. from $None$ to *Little*, then to *Small* and finally to $Moderate$) but we most likely see jumps, such as from $None$ to $Moderate$ in one iteration of the repeat-until loop. In fact, we have noticed from the experiments which we have conducted that the algorithm will not reach this pessimistic at-most running time, but rather usually terminates after just few runs of its repeat-until loop.

Finally, it is clear that the framework and this algorithm will not try to consolidate the value properties of each goal node within the GCM model. Meaning that the framework purposely keeps the achievement, operationalization and prevention values of the goal node all separate from each other. The idea is to highlight the achievement and operationalization which the goal node managed to gain, and highlight the prevention value it managed to receive. Therefore, the model user will be able to track what caused this achievement, operationalization, and/or prevention for each goal node. Understandably, the final achievement that a goal receives ideally is the result of subtracting its final prevention value from its final achievement one. But, if the real achievement value of a goal, tracked in real-time or in-simulation, is different than the one suggested by the model, or if in reality the goal managed to accumulate achievement value more than the operationalization level that the model suggests it could have, then the modeller/user must resolve this inconsistency between reality and the model by ensuring that all the relations targeting the goal are accurately captured and represented.

Soundness and Completeness: In Appendix B, we also provide Theorem B.0.2 and its proof proving the correctness and completeness for the goals' value-label forward propagation algorithm, *Label_GCM_Goals*, provided above as Algorithm 3.1. Theorem B.0.2 shows that the value label assignments for all the goals' value properties (achievement, operationalization and prevention) which are returned by the Label_GCM_Goals (GCM_Graph $\langle \mathcal{G}, \mathcal{C}, \mathcal{R} \rangle$, c-value-label-array Initial_C, value-label-array Initial_G) algorithm, Algorithm 3.1, are the correct, complete and final value label assignment statements that could be deduced from $Initial\_G$ and $Initial\_C$.

3.7 Multiple Strategic Goals, and Goal Trees

An agent who represents a human or an intelligent life/system will usually have multiple strategic goals it aspires to achieve at any single point of time. This is understandable, knowing that there are usually a number of *basic needs*, or *survival* *goals*, that the agent must achieve to stay alive and survive. For example, for an organization to survive, it must be able to pay its employees' salaries and pay the office rent; and for a robot, some of it needs could be to ensure at minimum a battery charge of 10%, and to be at no more than 100m away from the charging station. These basic needs are survival goals that has nothing to do with why the agent existed in the first place. On the other hand, any agent tends to have a set of *wants*, or *strategic value-creation goals*, that the agent is aspiring to achieve whether to add meaning to its life/operation or in order to exchange the value created by operationalizing and achieving such goals with things that satisfy its survival needs and continuous operation.

These multiple strategic goals the agent has, whether represent needs or wants, the agent usually must reduce them to a set of more primitive goals in order to be able to operationalize and achieve them, as we discussed before. This means that the agent will have multiple goal trees represented within its ultimate aggregate GCM model. Even if the agent decided to decompose these goals and model them as part of separate GCM models, each model to represent goals on a specific area/time/priority/etc., still at an aggregate level the agent must integrate all goals and see how these different goals will affect each other. This will lead to the situation where the agent has multiple goal-trees in its GCM model, at least at an aggregate level.

So far in this chapter, and for simplicity, we treated the GCM model as a one-goals-tree model. The question that this section will answer is: if the agent has multiple goal-trees, what will happen to our goals' value-label axioms and propagation rules that we have introduced earlier? Will they need to be changed or adjusted in order to accommodate the multiple goal-trees case and its complexities? We will examine whether the previously introduced value labels propagation rules and algorithm need to be changed, or adjusted, in order to deal with multiple goal trees within the GCM model, such as the one shown in Figure 3.8 instead of the one goals tree we dealt with so far (shown in Figure 3.2).

In this section, and for simplicity, we will assume that the agent is completely invariant about all its strategic goals $\mathcal{G}_{strategic}$ (root nodes of the goal-trees). We will discuss representing different priorities and orders for the agent's goals in the following chapter, but for now let us assume that all goals within $\mathcal{G}_{strategic}$ are equally important to the agent.

What we have here is an easy straight forward goals representation exercise.

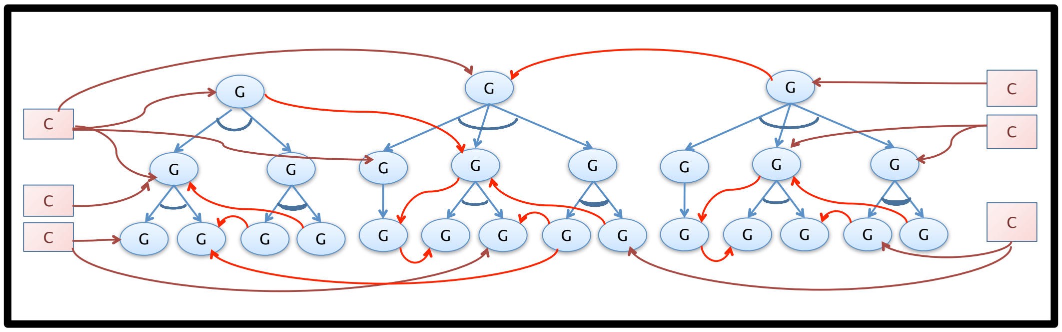


Figure 3.8: An agent's GCM with multiple goal-trees, where the root goals represent strategic goals for the agent and the goals in different goal-trees support, hinder or conflict each other through G-G lateral relationships

Whether we have all the goals in $\mathcal{G}_{strategic}$ (underneath each is its goal reduction tree attached to it) connected to a new higher overall strategic goal (e.g. 'success in life" goal) allowing for all the goals in $\mathcal{G}_{strategic}$ to form a one goal reduction tree for the agent, or we have them as isolated root nodes, each with its own reduction tree, this should not make a difference in terms of the reasoning mechanics of the formal framework presented in this chapter. The propagation rules and algorithm need not be changed.

For example, in the Car Manufacturer (CM) example, used later in this chapter, Figure 3.11 shows the CM's GCM with both of the company's short-term (G_2) and long-term (G_1) strategic goals connected to the ultimate overall goal "CM to Survive & Prosper" (G_0) using an AND redaction operator. Later in the chapter, we will show the results of running the algorithm to produce a final set of value labels for the value properties of all CM's goals including the three strategic goals CM has (G_0, G_1, and G_2).

CM could choose not to connect the two strategic goal is has, G_1 and G_2, to a higher overall goal(G_0). This will produce a two strategic goals GCM model (i.e. forming a multiple goal-trees case) with two goal-trees affecting each other through lateral relations running across them. For this case too, we will show the results of running the algorithm to produce a final set of value labels for all value properties of all CM's goals including the two strategic goals CM has (G_1, and G_2).

In addition, we will show an extreme case of a GCM with multi-goal-trees: the case where the agent's goals are all set as individual goal nodes with no reduction relations among them. The Howard's Dilemma example, shows goals connected to each other using a complex network of lateral goal-to-goal relations, forming no reduction trees. Again here, the algorithm managed to deduce the final value labels for all goals' value properties.

Within *each* of the multiple goal trees, all the relations and rules that were introduced earlier to explain and propagate value labels of goals' value properties (within a single reduction tree) will work. The only implication of having multiple goal reduction trees, is that goal nodes, which belong to different trees, will affect each other. This affect could come in the form of support, hinder or conflict-with relations. This means that lateral goal-to-goal relations will run between goals that belong to different reduction goal trees.

At the goal node level, the final value label of each value property still need to be concluded by ORing the results of all the goal-goal (reduction or lateral) relations the goal node is at the receiving end of, and ANDing the results of all the constraint-goal relations the node is constrained by, same as shown in Figure 3.7, and in the rules (3.43) - (3.45). Goal nodes across different goal trees, affecting each other, will be treated exactly the same way goal nodes at far ends of a heavenly branched single goal tree will be treated. Both have no direct goal reduction relations connecting them, and only lateral goal-goal relation affecting them.

One could also ask about the effect of constraints on goals of different trees. A constraint can be connected to many goals with different constraint-goal lateral relations. This should not make a different on how the goals properties' values are calculated, it will be the same way it was calculated before. Recall that value-labels are derived at the goal-node level by ORing and ANDing the results of all the value-label propagation relations that the node receives. This holds whether the goal node represents a strategic goal the agent has, or a very primitive goal (intention) at one of the goal reduction trees the agent has.

The only implication having-multiple-strategic-goals have on any agent is not how the goals value-labels propagate, but what goals to work on operationalizing and achieving first. In other words, the agent should be concerned now with what strategic goals are more important to work on achieving, and therefore what order/priority the agent should give to the different plans he uses to operationalize each of these goals. This will be the focus of the next chapter.

3.8 Reasoning as a Dynamic Process: Dealing with Dynamic Changes

The framework so far offered formal rules and techniques to identify and calculate the final value labels for GCM goals' value properties based on the effect goals

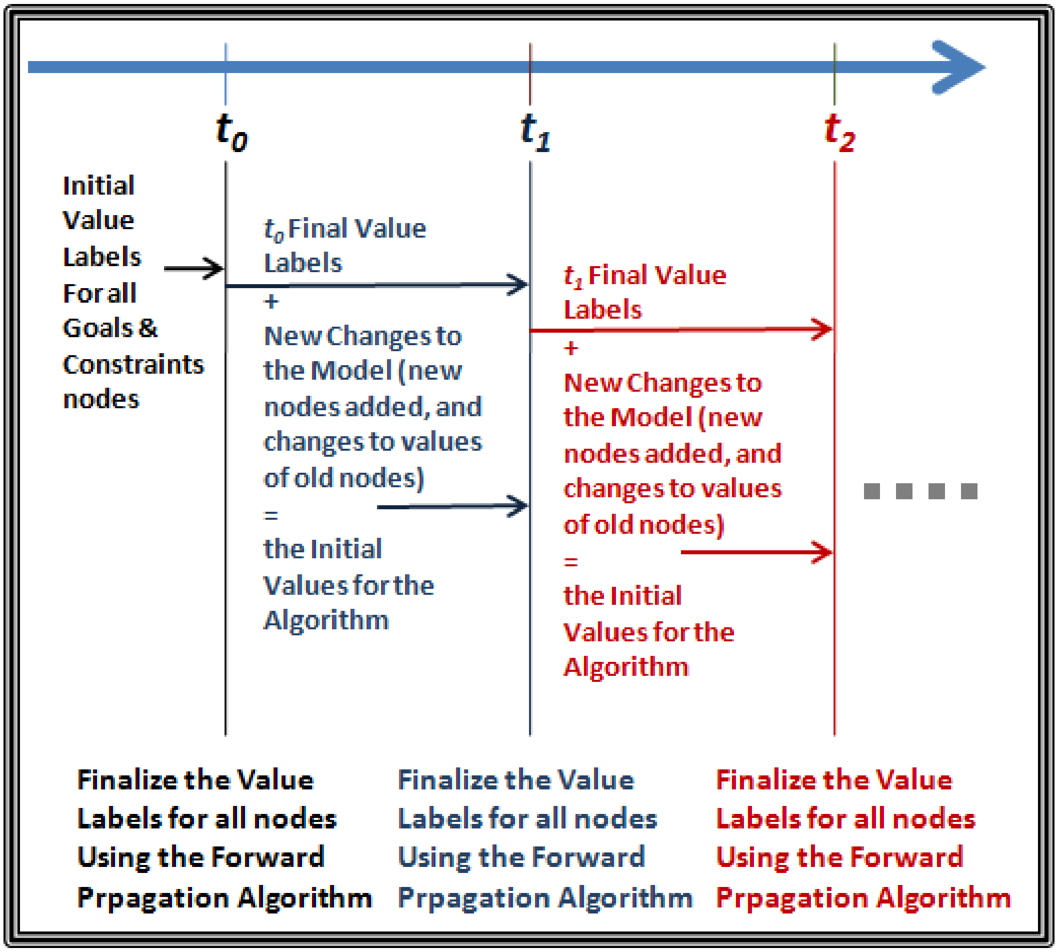


Figure 3.9: Dealing with the dynamic Changes happening to the GCM Model and its nodes over time, and how the framework will determine the nodes' final value labels at each point of time

and constraints have on each other, but what about changes that happen to the GCM model over time? In real life, agents tend to abandon goals or adopt new one; and their goals' achievement levels will increase over time, as plans feed to the lower intention nodes new satisfaction levels. Not to mention the fact that realities, internal and external constraints, of the agent change over time. This means new nodes are added, some old nodes are removed, and value labels assigned to nodes' value properties change. So, how the forward value labels propagation algorithm (Algorithm 3.1) will be used to decide on the nodes' values as changes happen to them over time?

Figure 3.9 shows how these dynamic changes will be dealt with using the Constrained Rationality's reasoning framework. No modification to the framework's rules and algorithms are needed. The agent needs only to do at each point of time the following steps:

1. Update the $Initial\_G$ and $Initial\_C$ arrays with the values that are coming from the previous point of time (or the values fed initially to the arrays - if this is the first time running the algorithm);

2. Update $GCM\_Graph\langle\mathcal{G},\mathcal{C},\mathcal{R}\rangle$ with any changes happened to the GCM model, including any new goals, constraints or relations added (new knowledge added), and/or any changes happened to the current goals, constraints or relations;

3. Update the $Initial\_G$ and $Initial\_C$ arrays with values for the new goals and constraints added, and/or with changes in values happened to the old goals and constraints just before this point of time; and then

4. Run the value-label forward propagation algorithm to have all previous values, as well as the new value changes, take effect and propagate through out the GCM's graph. *By the end of this step, all goal nodes' value properties will have the final value labels, for this point of time, assigned to them.*

For the next point in time, the same steps should be followed, as shown in Figure 3.9. In fact, the Constrained Rationality modelling and reasoning tool is designed and implemented to use these steps in its simulation engine.

3.9 Extending the Constraint Rationality Modelling and Reasoning Framework

The Constrained Rationality reasoning framework presented in this chapter is flexible enough to allow for many extensions. We present here two possible extensions, and leave many as future work.

3.9.1 Increasing the Number of Qualitative Fuzzy Linguistic Value Labels

One possible extension is to increase the number of linguistic labels ($\mathcal{L}$) that could be assigned to the value properties of goals (and constraints) beyond the simple scale adopted so far ($\mathcal{L}$={*Full*, *Big*, *Much*, *MOderate*, *Some*, *Little*, *None*, *Null*}). We said that one could increase, or decrease, the number of these value labels, as per his needs or his application's need. For example, in Appendix A, we used a smaller set of fuzzy sets and labels (shown in Figure A.1) in order to make the process of presenting the axioms of the framework's propagation rules simpler and the axioms more manageable and presentable. The analyst can use such smaller set of $\mathcal{L}$ or make it bigger as he sees fit.

3.9.2 Adding New Types of G-G and C-G Relationships

Another possible extension is to add new types of goal-to goal (or constraint-to-goal) lateral relationships to satisfy special reasoning/application needs. For example, Figure 3.10 shows a number of possible new lateral relationships, compared to some of the ones we introduced earlier. In the figure, the previously introduced strong G-G lateral relation $G_i \stackrel{++}{\rightarrow} G_j$ is shown in Figure 3.10.(1), and the weak relation $G_i{}^{+(MO\underset{\longrightarrow}{derate}+)} G_j$ is shown in Figure 3.10.(2). Both subfigures provide visual representations of the relations' value label propagation.

If an application of the framework requires a new weak relation such as the one given in Figure 3.10.(2) but with different value label propagation rules, then one could define a new relation such as the one given in Figure 3.10.(3) or the one shown as Figure 3.10.(4). The only requirement for any lateral relation is to maintain monotonicity of the value propagation.

One additional new type of G-G lateral relations shown in the figure is one where there is now a *Modifier* label at the start goal side (the cause) of the relation, not just at the end (targeted) goal of it, as in the previously introduced lateral relations. For example, relations shown in Figure 3.10.(5) and Figure 3.10.(6) are new relations with *Modifier* Labels present at both sides of the lateral relation: the start (cause) and the end (effect) sides. Relations in Figure 3.10.(7) and Figure 3.10.(8) have *Modifier* Labels at only the cause side of the lateral relations, whilst a lateral relation such as the one in Figure 3.10.(2) has the *Modifier* Label at only the effect side of the relation.

Many other relations could be added to make the modelling, and therefore the reasoning, of the GCM model more expressive and reflective. For example, new reduction relations could be used, as we mentioned earlier when we introduced goal reduction; and/or new value properties for both goals and constraints could be added whenever needed by the framework's application/user. We will show in

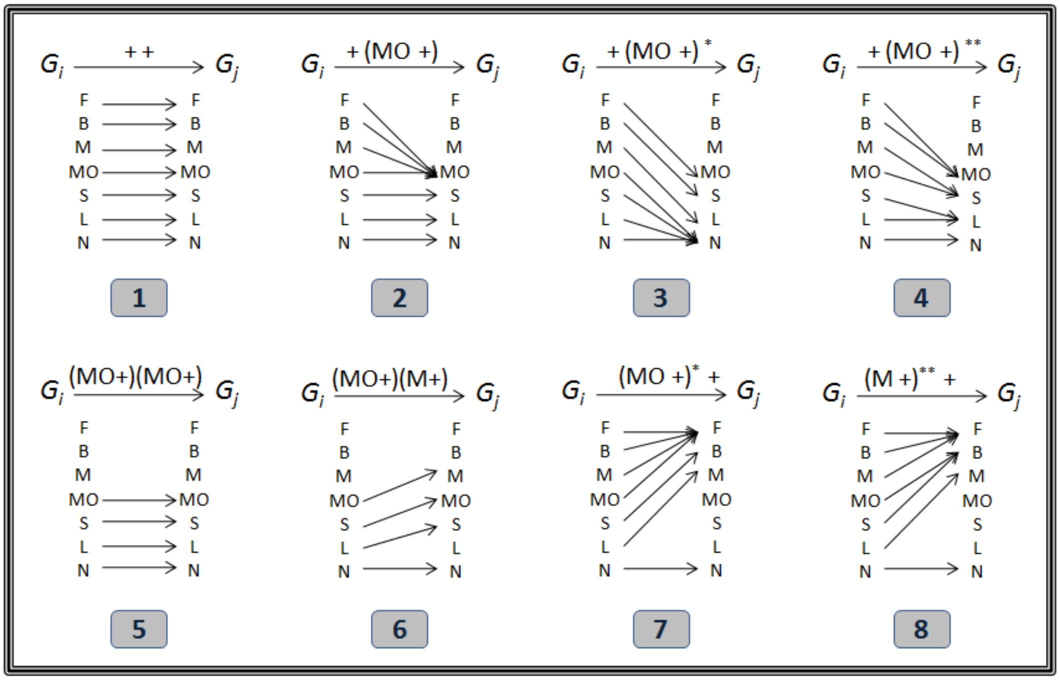


Figure 3.10: Some new possible types of Lateral Relations, that could be used in the framework in addition to the ones introduced earlier in Figure 3.5

the following chapter how we extended the framework to include modelling and reasoning-with emotions and attitudes, as well as priorities and ordering. And we will report on how we used these extensions to add more sensitivity analysis capabilities to the framework when used to model and analyze real industrial and political strategic decision making cases.

3.10 Examples and Experimental Results

3.10.1 A Car Manufacturer's Bailout vs. Bankruptcy Strategic Decision

Al-Shawa and Basir (2009) used the example of a leading Car Manufacturer (CM) facing the the economical crisis of 2008-2009. We will use this example as a preliminary experiment, and report on its results.

Figure 3.11 shows a business strategy decision making scenario in which CM modelled its strategic goals to survive the hardships it is facing, and to achieve prosperity in the future. The company used the Constrained Rationality framework to prepare a complex model of its goals and constraints (a simplified smaller

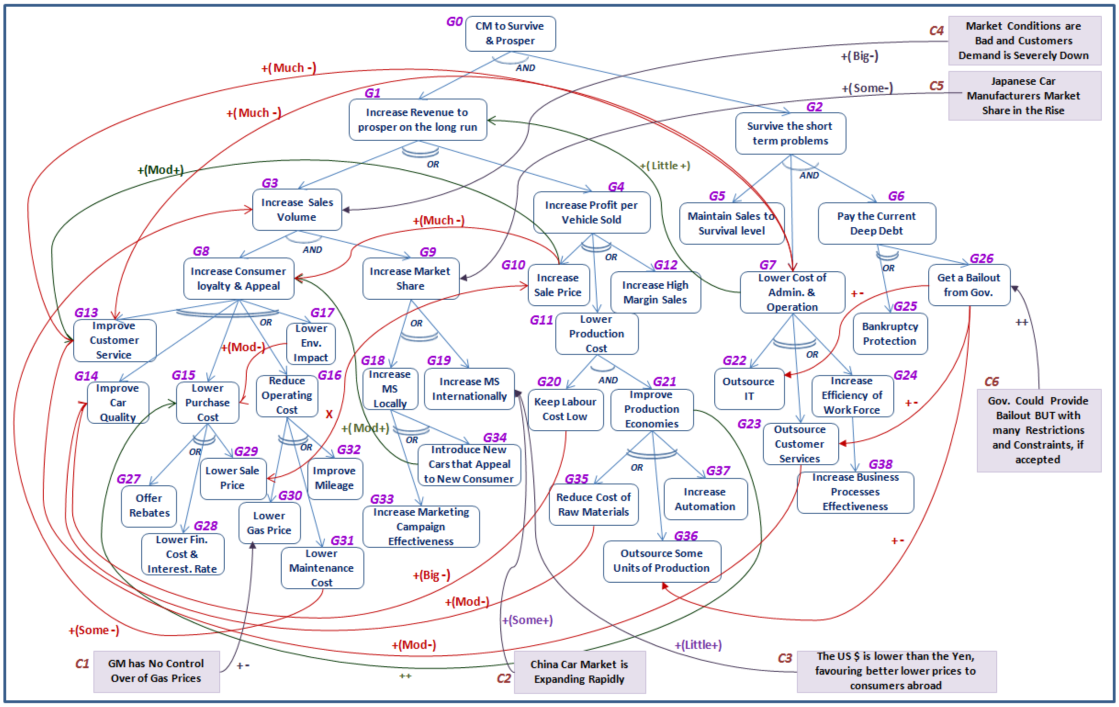


Figure 3.11: Goals & Constraints Model (GCM) for a Car Manufacturer (CM) Bailout-vs-Bankruptcy Strategic Decision Making Case [simplified for this illustrative example]

version of this model is used in this example and shown in the figure) with goals affecting/conflicting with each other.

Using the reasoning framework, CM wants to test two plans, to see which one satisfy its immediate short-term survival needs while keeping in mind the long-term prosperity goals: 1) plan to accept a bailout from the government (represented in Figure 3.11 as an intention node $G26$); and 2) reject the bailout and instead declare bankruptcy, based on chapter 11 of the US Bankruptcy Code (represented in Figure 3.11 as an intention node $G25$). CM's GCM shows that accepting a government bailout comes with conditions limiting it from, among other things, outsourcing to cut operational cost on the long run.

By using the Constrained Rationality framework's modelling and reasoning methods, presented in this chapter, we found that CM is better off rejecting the government's bailout and instead operationalize the bankruptcy intention by declaring bankruptcy (under Chapter 11 of the United States Bankruptcy Code). This is assuming bankruptcy will provide full achievement of goal $G6$ by allowing restructuring of CM's current debt. When fed with the initial values for the goal and constraint nodes, and by selecting to activate one of the intentions representing

CM's options ($G25$ or $G26$), Algorithm 3.1 will produce the final achievement and prevention value labels for the three strategic goals CM has: the CM to Survive and Prosper $G0$ goal; the Increase Revenue to Prosper in the Long Run $G1$ goal; and the Survive the Short Term Problems $G2$ goal. Note that the strategic goal $G0$ is an "AND" of both $G1$ and $G2$ (i.e. $G1,G2 \overset{AND}{\Longrightarrow} G0$). or as it could be put differently: $G1$ and $G2$ are the result of an AND reduction relationship.

The algorithm's repeat-until loop runs for both scenarios, using initial values representing each scenario (and for simplicity assuming operationalizability equals achievability for all goals), are shown in Figure 3.12. As shown, whilst both options, accepting the bailout or declaring bankruptcy, provide moderate achievement to the strategic goals CM has, accepting the bailout will have a prevention effect on the long-term prosperity goal ($G1$), and therefore the ultimate survive and prosper goal ($G0$). A negative effect that the bankruptcy will not have on these goals.

The experiment, even though simplified to fit this chapter's need, demonstrates the complexity of strategic decision making, knowing the interdependency and interrelations goals and constraints have among them, and how the reasoning framework we proposed can help identify the effect of adopting certain plans/goals on the ultimate (conflicting) strategic goals the agent has. It is worth mentioning here how the algorithm deduced the final value labels of all goals' value properties in just 5 runs of its loop, i.e. much less than the at-most ceiling of $3|\mathcal{L}||\mathcal{G}| + 1$, or $3(8)(39) + 1$, given by Theorem 3.6.1.

In addition, we could test the effect of CM having two separate strategic goals $G1$ and $G2$, without "ANDing" them to produce the one joint ultimate strategic goal $G0$. This could be done by removing the strategic goal $G0$ from CM's GCM model given in Figure 3.11, and removing the AND reduction relationship that connects $G1$ and $G2$ to the late $G0$ node. In effect, this will make CM has two goal-trees. One with the root strategic goal $G1$ on top, and another with the root strategic goal $G2$ on top. Individual goals within each tree are still affecting goals within the other tree, as shown in Figure 3.11.

Now, if we feed Algorithm 3.1 with the same initial values for the goal and constraint nodes, and by selecting to activate one of the intentions representing CM's options ($G25$ or $G26$), The algorithm will produce the same final achievement and prevention value labels for all CM's goals, including its two strategic goals $G1$ and $G2$. The results will be the same as the ones shown in Figure 3.12, but with no $G0$ node or final values for it .The fact that each of the two goals are separate and

CM Mobilize a Plan to Accept Government Bailout (G26)

Node	Initial Achv	Initial Prvn	run 1 Achv	run 1 Prvn	run 2 Achv	run 2 Prvn	run 3 Achv	run 3 Prvn	run 4 Achv	run 4 Prvn	run 5 Achv	run 5 Prvn
G0							L		O	M	O	M
G1					L		O		O		O	
G2					O		O	M	O	M	O	M
G3				B		B	L	B	L	B	L	B
G4					O		O		O		O	
G5	M		M		M		M		M		M	
G6	F		F		F		F		F		F	
G7			O		O	M	O	M	O	M	O	M
G8					O		O		O		O	
G9			L	S	L	S	L	S	L	S	L	S
G10												
G11			O		O		O		O		O	
G12												
G13				O		O		O		O		O
G14				O		O		O		O		O
G15			O		O		O		O		O	
G16-18												
G19	L		L		L		L		L		L	
G20	O		O		O		O		O		O	
G21			L		L		L		L		L	
G22				F		F		F		F		F
G23	O		O	F	O	F	O	F	O	F	O	F
G24			L	M	L	M	L	M	L	M	L	M
G25												
G26	F		F		F		F		F		F	
G27	S		S		S		S		S		S	
G28	O		O		O		O		O		O	
G29												
G30				F		F		F		F		F
G31-35												
G36	L		L	F	L	F	L	F	L	F	L	F
G37												
G38	L	M	L	M	L	M	L	M	L	M	L	M

C1-C6	F		Constraints' Initial Values

CM Mobilize a Plan to Reject Bailout & Declare Bankruptcy (G25)

Node	Initial Achv	Initial Prvn	run 1 Achv	run 1 Prvn	run 2 Achv	run 2 Prvn	run 3 Achv	run 3 Prvn	run 4 Achv	run 4 Prvn	run 5 Achv	run 5 Prvn
G0							L		O		O	
G1					L		O		O		O	
G2					O		O		O		O	
G3				B		B	L	B	L	B	L	B
G4					O		O		O		O	
G5	M		M		M		M		M		M	
G6			F		F		F		F		F	
G7			O		O		O		O		O	
G8					O		O		O		O	
G9			L	S	L	S	L	S	L	S	L	S
G10												
G11			O		O		O		O		O	
G12												
G13				O		O		O		O		O
G14				O		O		O		O		O
G15			O		O		O		O		O	
G16-18												
G19	L		L		L		L		L		L	
G20	O		O		O		O		O		O	
G21			L		L		L		L		L	
G22												
G23	O		O		O		O		O		O	
G24			L	M	L	M	L	M	L	M	L	M
G25	F		F		F		F		F		F	
G26												
G27	S		S		S		S		S		S	
G28	O		O		O		O		O		O	
G29												
G30				F		F		F		F		F
G31-35												
G36	L		L		L		L		L		L	
G37												
G38	L	M	L	M	L	M	L	M	L	M	L	M

C1-C6	F		Constraints' Initial Values didn't change (Same Realities)

Labels:	Full "F"	Big "B"	Much "M"	Moderate "O"	Some "S"	Little "L"	None "N"	Null " "

Figure 3.12: Algorithm Runs for the CM Example. G25 and G26 Represent the two intentions to adopt Accept Gov. Bailout and Declare Bankruptcy, respectively. And, CM's strategic goals: G0, G1 and G2.

has its own reduction goal-tree, with goals in both trees affecting each other, did not change how Algorithm 3.1 finalizes the value labels for the goals. Not only this, if we add loops and multiple interacting loops, the algorithm will still terminate and finalize the value labels for all its nodes, as we indicated earlier when we discussed the algorithm's termination and complexity.

3.10.2 Howard's Personal Dilemma

An interesting real-life "personal dilemma" was presented by Thagard and Millgram (1995). As per the dilemma, Howard, a professor in a good university, receives an offer to move to Harvard. Howard has many personal goals that range from research-related to family-related goals. The goals conflict with each other. Satisfaction of some will lead to dissatisfaction of others. Typically, a situation such as the Howard Dilemma will be modelled using decision theory models; more accurately using multiple-criteria decision models. But these models usually hide all the details about how and why such criteria came to be "the criteria" in the first place. These models also tend to hide all the conflicting relations among the chosen criteria. This is mostly done to show independence of events (an assumption that is

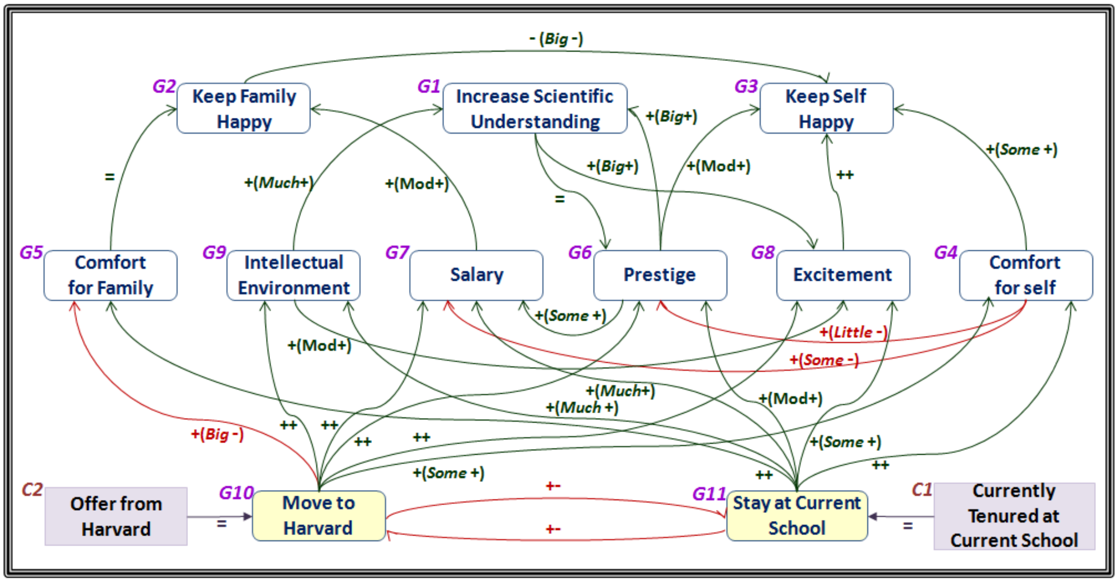


Figure 3.13: Goal & Constraints Model of the Howard's Dilemma example

not realistic in most cases) in order to satisfy the use of probability and statistical analysis, which is heavily relied on within decision theory.

Figure 3.13 shows Howard Dilemma modelled using the Constrained Rationality framework. We used the goals stated by Thagard and Millgram (1995), but added some additional relations and conflicts among the goals to increase the complexity of the model. It is worth mentioning that the model presented by Thagard and Millgram (1995) does not show any goals' reduction.The goals presented as islands (separate nodes) with no reduction, but with links which are numerically weighted (done by the authors mainly to model the situation as a neural-like-network). We believe that the example is better served if the dilemma's model used goals' reduction as well as inter-relations, but we decided to use the same structure suggested by the authors (mainly to show the usefulness of our model in an extreme multi strategic goals situation). Therefore, in our model of the dilemma, we use the goals as suggested, but the links among them are all labeled G-G lateral relations. The value labels are assigned to the goals' value properties through fuzzy membership functions, and the label propagation is done by following the Constrained Rationality framework's rules and algorithm.

Algorithm 3.1 repeat-until loop runs for this GCM model are shown in Figure 3.14, for the two scenarios: 1) Howard continues to work at his current school ($G11$) with no job offer received from Harvard ($C1$); and 2) Howard decides to activate the "move to Harvard" intention goal ($G10$) after the "Offer from Harvard"

Howard at his Current School, and has no offer from Harvard

	Initial		run 1		run 2		run 3		run 4		run 5	
	Achv	Prvn	Achv	Prvn	Achv	Prvn	Achv	Prvn	Achv	Prvn	Achv	Prvn
G1					M		M		M		M	
G2					F		F		F		F	
G3					O		O		M		M	
G4			F		F		F		F		F	
G5			F		F		F		F		F	
G6			O		O	L	M	L	M	L	M	L
G7			M		M	S	M	S	M	S	M	S
G8			S		O		M		M		M	
G9			M		M		M		M		M	
G10				F		F		F		F		F
G11	F		F		F		F		F		F	
C1	F											
C2		F										

Howard, if he accepts Harvard's offer

	Initial		run 1		run 2		run 3		run 4	
	Achv	Prvn	Achv	Prvn	Achv	Prvn	Achv	Prvn	Achv	Prvn
G1					B		B		B	
G2					O	B	O	B	O	B
G3					F		F	B	F	B
G4			S		S		S		S	
G5				B		B		B		B
G6			F		F	L	F	L	F	L
G7			F		F	S	F	S	F	S
G8			F		F		F		F	
G9			F		F		F		F	
G10	F		F		F		F		F	
G11				F		F		F		F
C1		F								
C2	F									

Labels: Full "F" | Big "B" | Much "M" | Moderate "O" | Some "S" | Little "L" | None "N" | Null " "

Figure 3.14: Algorithm Runs for the Howard Dilemma Example.

constraint ($C2$) changed from full prevention (no offer) to full achievement (offer received). For the first scenario, the figure shows that while Howard's goal of keeping his family happy ($G2$) is fully achieved, Howard's goals of "increase scientific understanding"($G1$) and "keep self happy" ($G3$) are moderately achieved.

For the second scenario, the figure shows the effect of Howard's decision to move to Harvard on all his goals. Here, while he managed to satisfy fully $G1$, his and his family's happiness goals received prevention. Simple tweaking to the Constrained Rationality model, by changing some of the lateral relation types, or their modifier labels, could provide more insight on how Howard thinks, and what will happen to his goals as his thinking changes. For example, the relation between G_2 and G_3 reflects how Howard believes that his own happiness is really connected to his family's. Is it true that this relation is only from G_2 to G_3, and not the other way around as well? And, is it true that this relation is of "$-(Big-)$" type (propagate prevention of G_2 to G_3) or is it more of a "$=$" type (propagate both achievement as well as prevention) that goes both ways? What about the relation between G_5 and G_2 which reflects how Howard believes that his family's happiness is really achieved? The current "$=$" relation between them shows that he believes that their happiness is achieved "only" by making them comfortable. Is this true? Any change in the model to "better" capture Howard's actual state-of-mind, or to test what-if scenarios by him, will show the effect on all his goals.

3.11 Summary

In this chapter, we presented a formal qualitative goals and constraints modelling reasoning framework for strategic decision making. We defined formally many types of relationships that could exist among goal and constraint nodes; multiple value properties that a goal node has to track the achievement, operationalization, and prevention effect it receives from other nodes; how the values assigned to these value properties are fuzzified and converted to meaningful linguistic qualitative value labels; and how value labels of the goals' value properties propagate through the different goal-to-goal and constraint-to-goal relations among the GCM model nodes.

The chapter then proposed a value label forward propagation algorithm which can conclude the final value labels for each value property of each goal node within the GCM model. The value labels propagation rules which form the basis for the algorithm were given, and shown to be a translation of a complete set of ground axioms (provided in Appendix A and the proof of sound and completion is provided in Appendix B). The chapter also provided proof that the algorithm terminates and produces a correct and complete set of final value label assignment statements deduced from a set of initial assignment statements. At the end, two illustrative strategic decision making examples, one for an enterprise business and another for an individual agent, were modelled and analyzed to illustrate the framework, its modelling and reasoning abilities.

This chapter showed how the goals and constraints of the different agents in the one agents decision making situation (and multi-agent decision making situations where agents act in an individualistic manner with no consideration to others' current or future choices and decisions) are modelled as viewpoints models and how the value properties, which the agents need for their reasoning abilities, are modelled and finalized. The following chapter will cover the aspects of the framework responsible for modelling the agents' priorities and emotions; and generating the agents' preferences over their alternatives. The chapter after that will look at how the framework's process and components presented in this chapter and the following one are extended and modified in order to model and analyze multi-agent decision making situations.

Chapter 4

Modelling Decision Makers' Priorities, Emotions and Preferences in Strategic Decision Making Situations

4.1 Introduction

In Chapter 3, when discussed multiple goal-trees, we said that an agent, especially if the agent represents a human or an intelligent life/system, will usually have multiple goals which it aspires to achieve at any single point of time. Some of these goals represent *basic needs*, or *survival goals*, that the agent must achieve to stay a live and survive. Others could represent *wants*, or *strategic value-creation goals*, that the agent is aspiring to achieve whether to add meaning to its life/operation or to exchange the value created by operationalizing and achieving such goals with things that satisfy its survival needs and continuous operation.

For this reason, goals which any agent has should not be treated equally. It is clear that achieving needs has priority over achieving wants. Prioritization of goals is a natural process embedded within an agent firmware, or purpose of creation.Therefore, we will look, in this chapter, into the process of how an agent will be able to assign different priorities/orders to strategic goal nodes within its GCM model.

We assumed, in Chapter 3, that the agent is indifferent about all its strategic

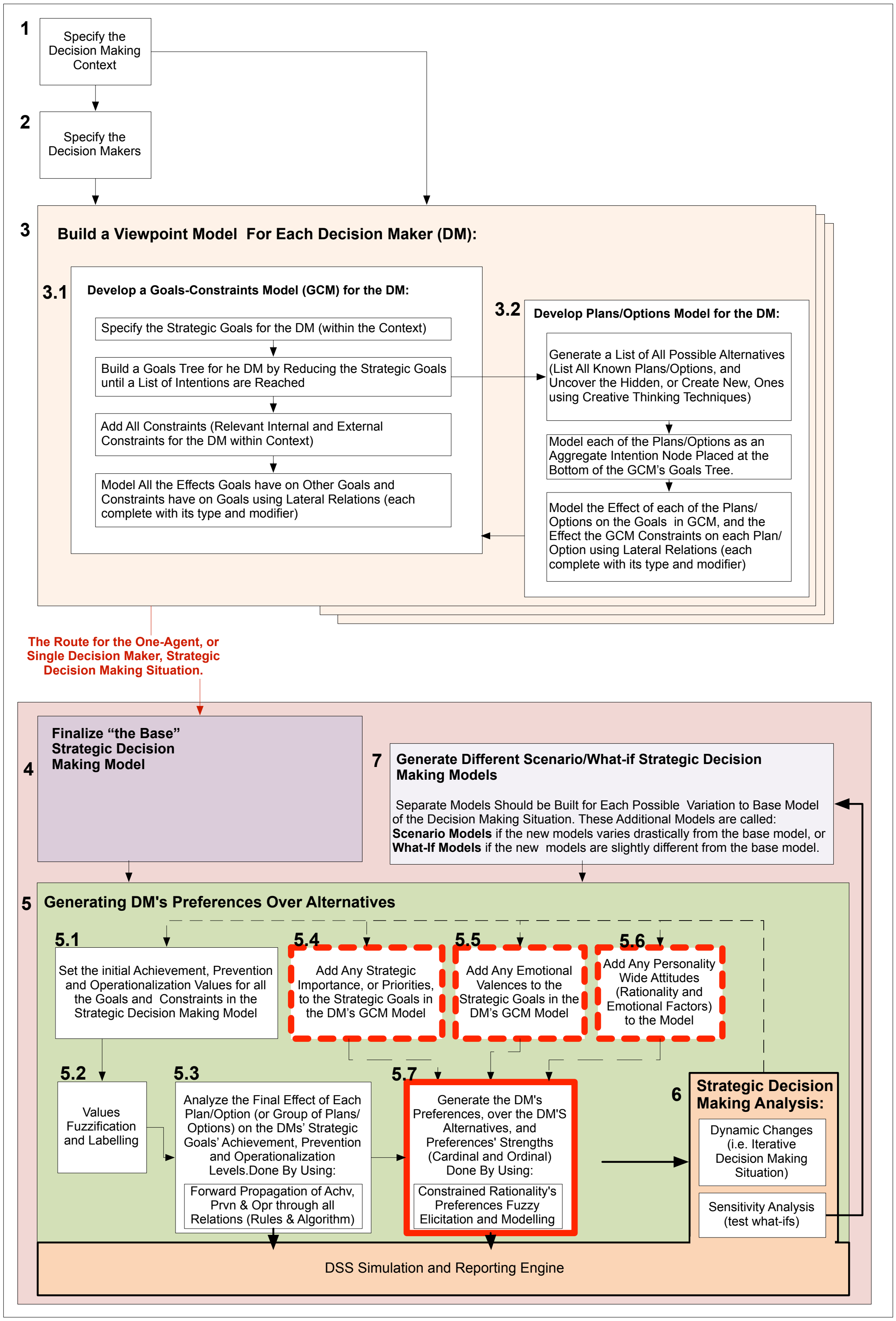


Figure 4.1: The Process of Modelling and Analyzing Single (and Multi-Agent Decision Making & Conflicts, where agents act in an individualistic manner with no consideration to others' current or future choices and decisions). Modelling agents' Priorities and Emotions, and generating agents' Preferences are shown in the Highlighted Boxes and will be covered in this chapter.

goals. This was to focus the discussion, then, on having multiple goal-trees and the implication such multiple trees has on our Constrained Rationality formal framework of goals' value labels axioms, rules and algorithm. In this chapter, we will remove the strictness of the indifference condition, and allow the agent, as it is usually the case, to have different priorities, order, or emotional feelings over the different strategic goals it has. Figure 4.1 shows the Constrained Rationality modelling and analysis process. Some of the process parts were discussed in the previous chapter. In this chapter, we will discuss the highlighted boxes in the process shown in the figure.

The first section of this chapter will discuss how the different strategic goals and alternatives the agent has are identified and modelled. The second section will present how the final achievement levels of agents' strategic goals are elicited for each of the alternatives the agents' have; how the strategic importance and the emotional valences for each of the agents over their own strategic goals are captured and modelled; as well as how the personality wide attitudes of the agents' toward acting rational and/or emotional are modelled.

The two sections to follow will show: how all these values will be used to determine the cardinal and ordinal preferences of the agents over their own alternatives; and how the strength of the agents' preferences are determined and represented. The chapter will close with an illustrative example, showing how the concepts and methods presented in the chapters are applied in a close-to-real-life one-agent decision making situation.

This chapter presents the foundational concepts and methods to model the importance and feelings of agents' over their strategic goals, and how to use these values (in addition to the achievement, prevention and operationalization values captured and modelled for the genets' goals in the previous chapter) to determine the agents' preferences over their alternatives. But, because this chapter is concerned more about presenting these foundational concepts and methods, it focuses on the simplest form of decision making situations: single-agent situations.

The methods will also apply to multi-agent decision making situations, but only to the ones where the agents act and decide in an individualistic manner with no regard to others' choices and decisions, or how these choices might affect their own objectives and choices. The following chapter will take the concepts and methods of this chapter further. It will discuss how to model collaborative and competitive multi-agent decision making situations; and how the concepts and methods presented here will be applied to determine the preferences of all agents in these situations.

4.2 Agent's Strategic Goals and Alternatives

In this section, we will first show how some of the agent's goals are singled out to be the "strategic goals" of the agent in the decision making situation at hand. And, how alternatives/options an agent have are modelled as intention nodes (each to represent an intention to adopt the alternative) within the agent's GCM model. In addition, we will set the stage for some of the notation to be used from now on.

4.2.1 Identifying Agent's Strategic Goals

Let, each decision maker $DM_i \in \mathcal{DM}$, at time t of the decision making situation, has a set of strategic goals $\mathcal{SG}_{DM_i,t}$, and $\mathcal{SG}_{DM_i,t} \subseteq \mathcal{G}_{DM_i,t}$, where $\mathcal{G}_{DM_i,t}$ is the set of all goals part of DM_i's GCM model $GCM\_Graph_{_{DM_i,t}} \langle \mathcal{G}_{DM_i,t}\,, \mathcal{C}_{DM_i,t}\,, \mathcal{R}_{DM_i,t} \rangle$. What differentiate the goals in $\mathcal{SG}_{DM_i,t}$ from the rest of goals in $\mathcal{G}_{DM_i,t}$ is that the strategic goals are the aims, or the ends, DM_i is ultimately looking to achieve while the rest of the goals form the means.

Also, the strategic goals in $\mathcal{SG}_{DM_i,t}$ are usually the top goals or root goals of the goal-trees part of the $GCM\_Graph_{_{DM_i,t}} \langle \mathcal{G}_{DM_i,t}\,, \mathcal{C}_{DM_i,t}\,, \mathcal{R}_{DM_i,t} \rangle$. But, this could not be the case all the time. While strategic goals within $\mathcal{SG}_{DM_i,t}$ which represent *Needs* are, or should be, considered part of DM_i's $\mathcal{SG}_{DM_i,t}$, strategic goals that represent *Wants* to DM_i could be part of $\mathcal{SG}_{DM_i,t}$ or not, depending on DM_i's state of mind at that moment of time or the current situation/conflict that DM_i is involved in. In reality, both strategic Needs and Wants are part of DM_i's set of strategic goals $\mathcal{SG}_{DM_i,t}$, but the strategic importance of these goals change from time to time. This is why we must consider the strategic importance, of the strategic goals within each agent's $\mathcal{SG}_{DM_i,t}$ set, and how DM_i feels about working for and achieving each of these strategic goals, as value properties attached to these goals as we will see in the next section.

4.2.2 Identifying Agent's Alternatives

The current alternatives for an agent are the current plans, moves, products, markets, or options which the agent is considering to take/adopt in the decision making situation, game or conflict. These alternatives could either be modelled as intention nodes at the bottom of the GCM's goal trees, or as Plans/Processes, Products, Features, etc. part of one of the many focused-sub-viewpoint models that the agent's viewpoint model could have (such as Business Processes Model, Business Offering Model, and so on (Al-Shawa, 2006b)). If the latter way is adopted to model the alternatives (as it is recommended by the ViVD-EKM framework), then these alternatives will be connected to intentions and goals in the GCM model by means of lateral relations.

In this research work, we will simplify the representation by modelling the alternatives/options that agent has in a decision making situation as individual intention nodes at the bottom of his GCM's goal-trees. With the meaning that each intention node represent an intention by the agent to adopt the specific alternative it represents without considering how the agent will actually do adopt the alternative. For example, a company's alternative to "Accept a Government Control Order As-Is", as we will see in a case study in a later chapter, is represented as an intention node in the company's GCM model, with the understanding that the company has many ways to operationalize this intention: send a letter to government indicating this, do this in public, drop the appeal process they started, and so on. These ways are considered detail plans to operationalize the intention. None is shown in the case study's GCM models, and none are considered in the analysis either, because these are considered not important compared to the actual real alternative "Accept a Government Control Order As-Is" adopted by the company.

Let $\mathcal{A}_{DM_i,t}$ be the set of alternatives (plans, options, moves) that the decision maker $DM_i \in \mathcal{DM}$ is considering at time t in his decision making situation. Each alternative $A \in \mathcal{A}_{DM_i,t}$ produce certain level of operationalization, achievement, prevention to each strategic goal $SG \in \mathcal{SG}_{DM_i,t}$, by propagating value labels for such value properties through the goal-trees and up to the strategic goals.

In addition, and as a matter of notation, let $\mathcal{DM}$ represent the set of all decision makers considered by the agent in his perceived decision making situation, and modelled in the situatuation's overall Viewpoint model; the the set of all alternatives considered by all agents involved in the decision making situation is $\mathcal{A}$; and the set of all strategic goals for all agents involved in the decision making situation is $\mathcal{SG}$, where:

$$\mathcal{A} = \bigcup_{DM_i \in \mathcal{DM}} \mathcal{A}_{DM_i,t} \qquad \textit{and} \qquad \mathcal{SG} = \bigcup_{DM_i \in \mathcal{DM}} \mathcal{SG}_{DM_i,t} \tag{4.1}$$

4.3 Modelling Agents Strategic Priorities, Emotional States and Attitudes

In this section we will present the different mechanisms by which an agent will be able to prioritize/order the strategic goals he has. We will show how strategic importance of deferent goals will be modelled, and how emotional valences the agent holds about these goals will be captured in the decision maiming model. In addition, we will present a robust and flexible modelling mechanisms to capture the personality-wide attitudes the agent has toward acting rationally and/or emotionally. But before we do so, we will show how the achievement values for strategic goals are consolidated in order to be used as the basis for judging the effectiveness of each of the alternatives which the agent has. The modelling mechanisms which will be presented in this section will be used in the following section to generate/validate the preferences each agent has over his own alternatives

4.3.1 Modelling the Final Achievement Levels of the Agent's Strategic Goals

We have stated in Chapter 3 that the Constrained Rationality framework's reasoning algorithm (Algorithm 3.1) purposely keeps track of the achievement, operationalization and prevention values of the goal nodes, within the agent's GCM model, all separate from each other. The value-propagation reasoning algorithm will not try to consolidate the value properties for each goal node to a single achievement value for the node. In fact, the decision support system we built for the framework displays graphically each of the value properties' values for each goal and constraint node in the GCM model.

The idea is to highlight the achievement and operationalization which each individual goal node manages to gain, and highlight the prevention value it manages to receive. Therefore, the DM, or the analyst modelling the decision making situation on his behalf, will be able to track what caused this achievement, operationalization, and/or prevention for each goal node in the model. And as a result, she will be able to reflect on the current state of affairs, her goals, constraints and her current alternatives to operationalize her goals; and help focus her critical as well as creative thinking efforts in changing the current available alternatives or generating new ones.

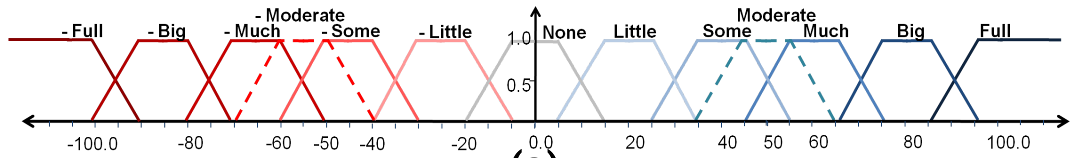
But for the purpose of evaluating each alternative (move, option or plan) the agent has in the strategic decision making situation at hand, a consolidated achievement level value property is as a logical choice to adopt. This new value property will track how much achievement that an alternative manages to give to a goal, especially a strategic important goal. This value property could be adopted as a measure of how good the alternative is in helping the agent getting closer to his ultimate goals. In this section, we propose a method and a set of mechanisms to evaluate alternatives based on their effect on the agent's strategic goals.

We introduce here a consolidated value property called Final Achievement. For a strategic goal SG, its Final Achievement value property will be denoted as $FAchv(SG)$. Understandably, $FAchv(SG)$ should receive a value that represent the result of subtracting its final prevention value ($Prvn(SG)$) from its final achievement one ($Achv(SG)$), taking in consideration the achievement upper limit that is set by both the constraints targeting SG and the level of operationalization SG managed to gain from the alternative the agent adopts. If in fact the real achievement value of SG, tracked in real-time or in-simulation, is different from the value suggested by the model, or if in reality SG managed to accumulate an achievement value more than the operationalization level that the model suggests it could have, then the analyst must resolve this inconsistency between reality and the model by ensuring that all the relations targeting SG are accurately captured, accounted for and represented.

At time t of the decision making situation and for a strategic goal $SG \in \mathcal{SG}_{DM_i,t}$, let the three final value labels that Algorithm 3.1 produces for SG's operationalization, achievement and prevention are denoted as $Opr(SG)$, $Achv(SG)$ and $Prvn(SG)$, respectively. The *Final Achievement* value of SG, denoted as $FAchv(SG)$, is defined as follows:

$$FAchv(SG) = \begin{cases} Achv(SG) \ominus Prvn(SG) & \text{if } Achv(SG) \leq Opr(SG) \\ Opr(SG) \ominus Prvn(SG) & \text{if } Achv(SG) > Opr(SG). \end{cases} \tag{4.2}$$

Let the fuzzy linguistic value label given to $FAchv(SG)$ based on the definition above is denoted as L_{FAchv} and is assigned by applying the "$\ominus$" operator's table shown in Figure 4.2b (implemented as reasoning rules). In other words, $FAchv(SG) = L_{FAchv}$, where $L_{FAchv} \in \mathcal{L} = \{$*Full*, *Big*, *Much*, *Moderate*, *Some*, *Little*, *None*, *-Little*, *-Some*, *-Moderate*, *-Much*, *-Big*, *-Full*, *Null*$\} = \{F, B, M, Mo, S, L, N, -L, -S, -Mo, -M, -B, -F, Null\}$. And, with the complete order of $F > B > M > Mo > L > N >$



(a)

⊖	N	L	S	Mo	M	B	F	Null
N	N	−L	−S	−Mo	−M	−B	−F	Null
L	L	N	−L	−S	−S	−M	−B	Null
S	S	L	N	−L	−L	−S	−M	Null
Mo	Mo	S	L	N	−L	−S	−Mo	Null
M	M	S	L	L	N	−L	−S	Null
B	B	M	S	S	L	N	−L	Null
F	F	B	M	Mo	S	L	N	Null
Null	Null	Null	Null	Null	Null	Null	Null	Null

(b)

Figure 4.2: (a) the membership functions of the extended Fuzzy Sets dividing the satisfaction levels domain of the Final Achievement value property $FAchv(G)$; and (b) the definition of the "$\ominus$" operator given as a table showing the resultant linguistic value label produced from the operation $V_1 \ominus V_2$, where V_1 and V_2 are value linguistic labels belong to the set $\mathcal{L} = \{F, B, M, Mo, S, L, N, -L, -S, -Mo, -M, -B, -F, Null\}$

$-L > -S > -Mo > -M > -B > -F > Null$, where the labels range from representing *Full* goal achievement to *Full* goal prevention, covering the Final Achievement satisfaction level of 100% to -100% or -1 to 1, and that the *Null* label represents an unknown achievement/prevention of the goal.

The fuzzy membership functions defining these linguistic value labels are given in Figure 4.2a. The figure shows the membership functions for each label's fuzzy set to be trapezoidal in shape, for simplicity only (not as a restriction) as indicated in Chapter 3. In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements. Some users will consider a goal *Fully* achieved if it reached an achievement level of 80-100%, whilst others will consider the goals to be *Fully* achieved only if their satisfaction levels are 95-100%.

4.3.2 Modelling the Strategic Importance of Agent's Goals

To model the priorities a DM might give to his strategic goals, we introduce here the *Strategic Importance* value property. A value property that will be attached to each strategic goal node the DM has, and given a fuzzy linguistic value label that will represent qualitatively the importance/priority the DM gives to this strategic goal.

For a decision maker $DM_i \in \mathcal{DM}$, and at time t of the decision making situation, let the *Strategic Importance* value property for a strategic goal $SG \in \mathcal{SG}_{DM_i,t}$ be denoted as $SImprt(SG)$. And, let $SImprt(SG) = L_{SImprt}$, where L_{SImprt} is a fuzzy linguistic value label that represents the name of the fuzzy set which the strategic importance of SG, as assigned to it by DM_i, has the highest membership of. And, $L_{SImprt} \in \mathcal{L} = \{Full, Big, Much, Moderate, \ Some, Little, None, NULL\} = \{F, B, M, Mo, S, L, N, Null\}$, with the complete order of $F > B > M > Mo > L > N > Null$. $\mathcal{L}$, here, is the same value labels set used before (for the goal's achievement, prevention and operationalization value properties) with the same fuzzy membership functions. But now, it is defined over the strategic importance levels ranging from 0% to 100%, or from 0 to 1, where 0 is represented by the importance label *None* and 1 is represented by the importance label *Full*, and where the label *Null* represents negative or unknown importance level.

The suggested membership functions for the fuzzy linguistic value labels in $\mathcal{L}$ are given in Figure 4.3a. The figure shows the membership functions to be trapezoidal in shape, for simplicity only (not as a restriction). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements, as we indicated in Chapter 3 for the basic value properties we introduced there.

This representation of the strategic importance of goals allows the agent the freedom to set his strategic goals to be *all* with *Full* importance, or set some goals to be of *Full* importance, set some to be of *Moderate* importance and others to be of *Little* importance. In other words, the total of all the strategic importance values given to the agent's strategic goals do not have to add up to be 1, or 100%, as many other game/decision-analysis representation frameworks demand (for example, the priorities in the famous Analytic Hierarchy Process must add up to 1 (Saaty, 1980), same as all probability-based or Bayesian-belief- network-based frameworks). In our representation, the agent has the freedom to choose the importance value label that reflect his own belief about the importance of each of his strategic goals considered individually, i.e. without consideration or comparison to the importance of the other strategic goals he has.

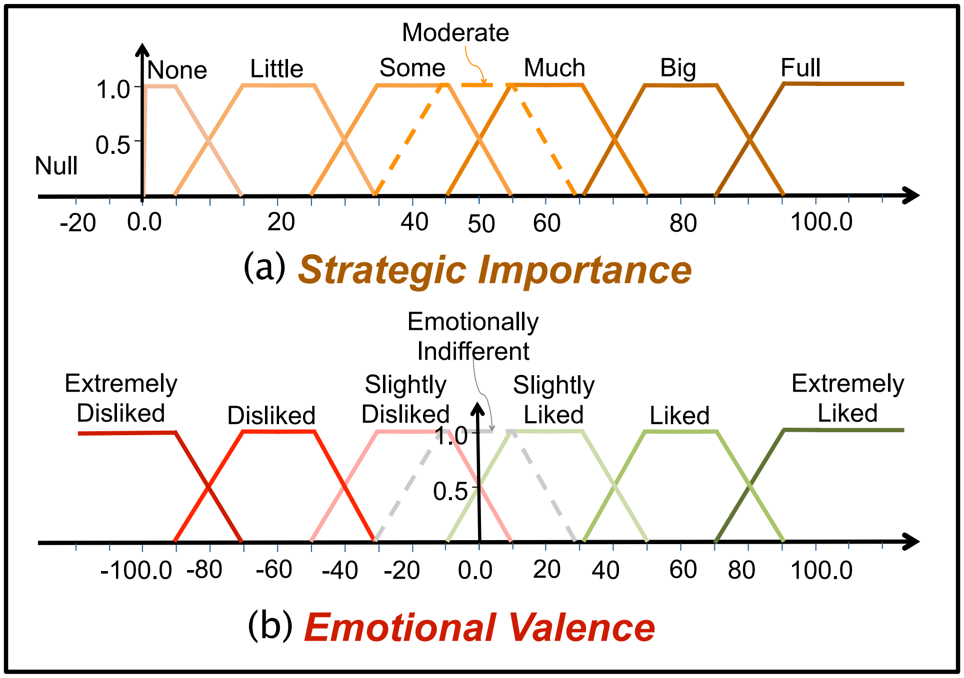


Figure 4.3: (a) the membership functions of the Fuzzy Sets dividing the importance levels domain of the Strategic Importance value property $SImprt(G)$; and (b) the membership functions of the Fuzzy Sets dividing the emotionally likeness and dislike levels domain of the Emotional Valence value property $EVlnc(G)$

4.3.3 Modelling Agent's Emotional Likes and Dislikes

Following the steps of many researchers who have proposed to introduce emotions into cognitive models by adding valences or affective tags (such as Thagard (2000); Bower (1981, 1991); Fiske and Pavelchak (1986); Lodge and Stroh (1993); Ortony et al. (1988); Sears et al. (1986)), we propose here to represent the effect of emotions on concepts in our models by adding emotional valence values. For example, Thagard (2000, 2006) proposed for his emotional coherence theory to include an emotional valence that can take positive or negative numbers for elements/concepts. These numerical valences can indicate likability, desirability, or other positive or negative attitudes towards the concept by the agent. Thagard (2006) cites the work by Kahneman (1999) in which Kahneman reviewed experimental evidence that evaluation on the good/bad (positive/negative) dimension is a ubiquitous component of human thinking.

There are many ways we can model emotional valences for the different concepts within the Constrained Rationality models (goals, constraints, and plans): (1) model valences as *Internal Constraints*; or (2) model valences as *Value Prop-*

erties. Modelling Valences as Internal Constraints, while easier since it could be added to the framework with no additional concepts, modelling mechanisms or reasoning mechanisms, it is limiting. Not to mention that every thing we added as a constraint in our framework, we demanded at minimum that the agent "knows" it, i.e. believes that it is true with justification (based on ViVD-EKM definition of *knowledge* given in Al-Shawa (2006b)). The problem with emotions and its effect is that it is tied to feelings and attitudes, where justification for truth of beliefs is not clear or possible. We use Constraints as a mechanisms to model some aspects of emotions, only when emotions play the role of constraining a goal(s), or a plan(s), and when there is true a justification for believing this constraining effect.

On the other hand, modelling emotional valence as a value property of concepts, such as goals, plans, constraints and relations, will provide a more powerful and flexible way to deal with emotions. In this thesis work, we propose to attach an *Emotional Valence* value property to each strategic goal the agent has. This value property will capture a different, but real –as suggested by many researchers such as the ones cited above–, importance level than the importance level given by the Strategic-Importance value property discussed above. Surely, a highly desired/liked strategic goal to achieve should not be treated in a similar manner as a strategic goal that the agent must/forced-to accomplish but does not like.

The *Emotional Valence* value property can hold non-fuzzy numeric value that is within the range [-1,1] or from -100% to +100%. But, in reality assigning precise numeric value for emotional valence is not practical, or even real. Therefore, we use the same mechanism to capture values for this property as we did for all value properties we used in our qualitative reasoning part of the Constrained Rationality framework. We suggest here that the emotional valence value properties be assigned a qualitative linguistic value label based on fuzzy membership functions.

For a decision maker $DM_i \in \mathcal{DM}$, and at time t of the decision making situation, let the *Emotional Valence* value property for a strategic goal $SG \in \mathcal{SG}_{DM_i,t}$ be denoted as $EVlnc(SG)$. And, let $EVlnc(SG) = L_{EVlnc}$, where L_{EVlnc} is a fuzzy linguistic value label that represents the name of the fuzzy set which the degree of like or dislike, that DM_i feels toward working and achieving SG, has the highest membership of. And, let $L_{EVlnc} \in \mathcal{L_{EV}} = \{$*ExtremelyLiked*, *Liked*, *SlightlyLiked*, *EmotionallyIndifferent*, *SlightlyDisliked*, *Disliked*, *ExtremelyDisliked*, $NULL\} = \{EL, L, SL, EI, SD, D, ED, Null\}$, with the complete order of $EL > L > SL > EI > SD > D > ED > Null$.

The suggested membership functions for the fuzzy linguistic value labels in $\mathcal{L}_{\mathcal{EV}}$ are given in Figure 4.3b. Again here, the figure shows the membership functions to be trapezoidal in shape, for simplicity only (not as a restriction). In practice, the number of fuzzy sets and their membership functions should be defined, shortened or extended based on the user needs and requirements, as we indicated earlier.

This representation of the emotional valence gives the agent the freedom to set his strategic goals to be all *ExtremelyLiked* , or some goals to be *ExtremelyLiked*, some to be just *Liked* and others to be *Disliked*. In other words, the total of all the emotional valence values given to the agent's strategic goals do not have to add up to be 1, or 100%. The agent has the freedom to choose the emotional valence value labels that reflects his own belief about each of his strategic goals considered individually.

4.3.4 Dealing with Agent's Overall Rationality and Emotionality Attitudes

To account for situations in which a decision maker decides to act completely rational even when having strong emotional likes/dislikes (i.e. act emotionless or in an extremely disciplined manner) or act completely emotional (i.e. give no regard to the strategic importance of goals), we offer two weighting factors: the *Rationality Factor* and *Emotionality Factor*. The two factors are intended to show the overall agent's attitude toward acting rationally and/or emotionally in general, and at times when there are conflicts among the strategic order versus the emotional order of goals.

For a decision maker $DM_i \in \mathcal{DM}$, and at time t of the decision making situation, let the *Rationality Factor* of DM_i be denoted as RF_{DM_i}, and let his *Emotionality Factor* be denoted as EF_{DM_i}. And, let $RF_{DM_i} = L_{RF}$ and $EF_{DM_i} = L_{EF}$, where L_{RF} and L_{EF} are fuzzy linguistic value labels that represent the name of the fuzzy sets which the rationality weighting factor and the emotionality weighting factor, respectively (that DM_i holds or expected to hold at time t of the decision making situation) have the highest memberships of. These two factors are intended to describe the overall attitude DM_i exhibits, or expected to exhibit, toward acting rationally or acting emotionally at that point of time.

Let $L_{RF} \in \mathcal{L}$ and $L_{EF} \in \mathcal{L}$, where $\mathcal{L} = \{Full, Big, Much, Moderate, Some, Little, None, NULL\} = \{F, B, M, Mo, S, L, N, Null\}$, with the complete order of $F >$

$B > M > Mo > L > N > Null$. $\mathcal{L}$, here too, is the same value labels set used before (for the goal's achievement, prevention and operationalization value properties) with the same fuzzy membership functions. But now, it is defined over the rationality/emotionality levels ranging from 0% to 100%, or from 0 to 1, where 1 is represented by the label *Full* to indicate *Fully Rational/Emotional* (depending on whether the label *Full* is assigned to L_{RF} or to L_{EF}, respectively) and where 0 is represented by the label *None* to indicate *Not Rational/Emotional* (depending on whether the label *Full* is assigned to L_{RF} or to L_{EF}, respectively).

The suggested membership functions for each of the fuzzy linguistic value labels in $\mathcal{L}$ are the same as the ones given for the strategic importance's $\mathcal{L}$ in Figure 4.3a. The figure shows the membership functions to be trapezoidal in shape, for simplicity only (not as a restriction). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements, as we indicated in Chapter 3 for the basic value properties we introduced there.

We will show in the next subsection, how these factors will be used. We will also show the flexibility this representation scheme provide to the modelling of decision makers' preferences. The last section of this chapter, will show the usage of these factors in modelling a single-agent decision making situation. Chapter 8 will show an example of using these factors to test different what-if models of players' preferences in a real-life multi-agent decision making conflict.

4.4 Eliciting Agents' Preferences over Alternatives

Now, for the purpose of calculating the effectiveness of adopting an alternative over another, for DM_i, we need to calculate how much each alternative contributes to the final achievement of the strategic goals in $\mathcal{SG}_{DM_i}$. The Final Achievement value property, $FAchv(SG)$ $\forall SG \in \mathcal{SG}_{DM_i}$, which was introduced above and represents the final achievement value that a goal SG managed to accumulate. But, $FAchv(SG)$ does not take into consideration how important is SG to DM_i and how DM_i emotionally feels about working to achieve SG. It is clear that the Final Achievement value property is not enough. We need to be able to differentiate between alternatives that provide means to achieve extremely important strategic goals from the ones that provide means to achieve less important strategic goals.

To provide such differentiation, we introduce the concept of a *Weighted Final Achievement* value property. A property that will be attached to each of strategic goals in $\mathcal{SG}_{DM_i}$, and provide a weight-adjusted Final Achievement value. The adjustment weight is based on the importance that DM_i gives to the strategic goal, the emotional valence given to the goal by DM_i, in addition to DM_i's overall attitudes towards rationality and emotionality. This new value property will be represented as a numerical value, not a qualitative fuzzy linguistic value label such as the ones we give to the Final Achievement value property, $FAchv(SG)$.

This means that the new *Weighted Final Achievement* value property will have a numerical value capturing a relationship between number of value properties ($FAchv(SG)$, $SImprt(SG)$, $EVlnc(SG)$, RF_{DM_i} and EF_{DM_i}), all of which are represented by fuzzy linguistic value labels (indicating a fuzzy membership functions). Therefore, we need to adopt a defuzzification scheme to reduce the fuzzy value labels to single scalars representing numerical values. While many defuzzification methods available in the literature, we will use here the simple but effective Centroid method (also known as Centre of Area or Centre of Gravity). Despite of its simplicity, this method is the most prevalent, physically appealing computationally efficient of all the defuzzification methods (Sugeno, 1985; Lee, 1990).

The centroid defuzzification method is appealing specially for calculating weighted averages when the memberships functions are symmetrical, something that we have here in our application. As per the method, the numerical representative value for the symmetrical fuzzy membership function (or its name: the fuzzy linguistic value label) is the centroid, or the mean, for its respective shape. For example, let the defuzzified value of $FArch(SG)$ be denoted as $FArch^*(SG)$, and as per the fuzzy memberships given in Figure 4.2a: if "*Big*" is assigned to $FArch(SG)$, then $FArch(SG)$ will have a representative centroid value of 0.8, and therefore $FArch^*(SG) = 0.8$; "*None*" will be represented by $FArch^*(SG) = 0.0$; "*-Some*" will be represented by $FArch^*(SG) = -0.4$; and "*Full*" will be represented by $FArch^*(SG) = 1.0$ (rounded). We will use the same method for the defuzzification of the Strategic Importance $SImprt(GS)$, the Emotional Valence $EVlnc(SG)$, the Rationality Factor RF_{DM_i} and the Emotionality Factor EF_{DM_i} value properties. Let the defuzzified values for them be denoted as $SImprt^*(SG)$, $EVlnc^*(SG)$, $RF^*_{DM_i}$ and $EF^*_{DM_i}$, respectively.

For a decision maker DM_i, at time t, let the Weighted Final Achievement of a strategic goal $SG \in \mathcal{SG}_{DM_i}$, as a result of having alternative $A \in \mathcal{A}$ been adopted, to be denoted as $WFAchv(SG, DM_i, A, t)$, and calculated algebraically as follows:

$$WFAchv(SG, DM_i, A, t) = \begin{cases} W(SG, DM_i, t) \cdot FAchv(SG, A, t) & \text{if } W(SG, DM_i, t) \geq 0 \\ 0 & \text{if } W(SG, DM_i, t) < 0. \end{cases} \quad (4.3)$$

where:

$$W(SG, DM_i, t) = (RF^*_{DM_i} \cdot SImprt^*(SG)) + (EF^*_{DM_i} \cdot EVlnc^*(SG)) \quad (4.4)$$

where $SImprt^*(SG)$, $EVlnc^*(SG)$, $RF^*_{DM_i}$ and $EF^*_{DM_i}$ represent the defuzzified values of their respective fuzzy values, and where none of the original fuzzy values is $Null$, and all reflect the state of mind and beliefs of DM_i at time t

$$FAchv(SG, A, t) = [FAchv^*(SG)]_{\text{if } A \text{ was fully applied to the GCM model at time } t-1} \quad (4.5)$$

where $FAchv^*(SG)$ represents the defuzzified values of $FAchv(SG)$, and $FAchv(SG) \neq Null$; and where "A was fully applied at time $t-1$" means that the intention to apply A was fully achieved, i.e. $Achv(A) = F$, at $t-1$

As per Equation 4.3, the Weighted Final Achievement of SG for DM_i at time t after alternative A is been adopted is given as the algebraic product of two values: the defuzzified value of SG's Final Achievement after A is applied ($FAchv(SG, A, t)$) and the weighting factor $W(SG, DM_i, t)$. $W(SG, DM_i, t)$ is calculated in Equation 4.4 as the sum of: 1) the defuzzified value of the strategic importance that DM_i assigns to SG ($SImprt^*(SG)$) weighted by the defuzzified value of the overall attitude DM_i has over acting rationality with no regards to emotions ($RF^*_{DM_i}$); and 2) the defuzzified value of the emotional valence that DM_i assigns to SG ($EVlnc^*(SG)$) weighted by the defuzzified value of the overall attitude DM_i has over acting emotionally with no regards to rationality ($EF^*_{DM_i}$).

The way the weighting factor $W(SG, DM_i, t)$ is calculated suggests that if DM_i has a $RF_{DM_i} = F$ and $EF_{DM_i} = N$ (i.e. DM_i will act completely rational with no regard to his emotions), then what matters as a weighting factor is the strategic importance that DM_i assigns to the strategic goal SG. On the other hand, if DM_i has a $RF_{DM_i} = N$ and $EF_{DM_i} = F$ (i.e. DM_i will act completely emotional with no regard to the strategic importance of his goals), then what matters as a weighting factor is the emotional valence that DM_i assigns to the strategic goal SG. But, if DM_i has none of the two factors (the rationality one and the emotionality one) set to *None*, then the strategic importance of SG as well as its emotional valence both come to play as a weighting factor but after each get adjusted by how much rationality/emotionality DM_i is exhibiting at the time. Some special cases could follow from this. For example, DM_i could be fully rational and fully emotional (i.e. $RF_{DM_i} = EF_{DM_i} = F$, and as a result his very important goals which are extremely liked too will get an additional weight in comparison to other very important goals but are disliked. As another extreme special case, DM_i could be completely careless and emotionless, or completely indifferent (i.e. $RF_{DM_i} = EF_{DM_i} = N$, and as a result none of his goals will matter to him whether achieved or not achieved. This shows the powerfulness and the flexibility of this weighting mechanism.

From Equation 4.4, the weighting factor $W(SG, DM_i, t)$ will have a final value that is in the range of $[-1, 2]$. This is because the defuzzified value ranges for RF_{DM_i}, $SImprt(SG)$, EF_{DM_i}, and $EVlnc(SG)$ are $[0, 1]$, $[0, 1]$, $[0, 1]$ and $[-1, 1]$, respectively. Note that Equation 4.4 requires that none of the original fuzzy value labels for these value properties to be *Null*, i.e. all these value properties must have *known* values (recall that *Null* label means that the value is unknown or out of range). This requirement/assumption is very reasonable, considering that the weighting factor is used in calculating the effect of applying an alternative A on achieving one of DM_i's strategic goals, or $WFAchv(SG, DM_i, A, t)$. If there is no known value for how important the strategic goal to DM_i, how DM_i feels about the goal, or how rational and/or emotional DM_i is expected to act, then it will not be logical to assume that $WFAchv(SG, DM_i, A, t)$ could give a valid assessment of the effect/contribution of A on DM_i's strategic goal SG.

In addition, note that Equation 4.5, which shows how to calculate $FAchv(SG, A, t)$, requires that the final achievement of the goal SG after A is been applied, or $FAchv(SG)$, to have a *known* value label. In other words, $FAchv(SG)$ should not have a fuzzy value label of *Null* assigned to it. This requirement/assumption is a logical one, for the same reasons explained above.

Knowing that the weighting factor $W(SG, DM_i, t)$ will have a final value in the range of $[-1, 2]$, and the defuzzified value of $FAchv(SG, A, t)$ is in the range $[-1, 1]$, then from Equation 4.3, we can be assured that the final value of $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-2, 2]$. It is worth noting here, that $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-2, 2]$, if and only if the modeller considered emotionality in calculating the weighting factor $W(SG, DM_i, t)$, i.e. when $(EF_{DM_i} \neq N) \vee (EVlnc(SG) \neq EI)$. On the other hand, if the modeller did not include emotionality in $W(SG, DM_i, t)$, whether by setting $(EF_{DM_i} = N) \vee (EVlnc(SG) = EI)$, then the value of $W(SG, DM_i, t)$ will be in the range $[0, 1]$, and therefore $WFAchv(SG, DM_i, A, t)$ will be in the range $[-1, 1]$.

Furthermore, because an alternative that is been taken by a decision maker can affect the the achievability of the strategic goals of another agent, we not only need to track the effect of the alternative on the decision maker who adopted the alternative but also the effect of it on all other decision makers. For this reason, we defined $\mathit{WFAchv}(SG, DM_i, A, t)$ in Equation 4.3 with the alternative $A \in \mathcal{A}$. A could be any alternative that is part of the total alternatives available for all decision makers in the decision making situation, and we did not limit A to be $A \in \mathcal{A}_{DM_i}$.

But, the weighted final achievement value, $\mathit{WFAchv}(SG, DM_i, A, t)$ calculated in Equation 4.3, shows only the affect of adopting an alternative A to one specific strategic goal SG that DM_i has. We need a measure that reflects the total effect of A, once adopted, on all DM_i's strategic goals in $\mathcal{SG}_{DM_i}$. This measure is what the *Total Weighted Final Achievement* value property tracks.

For a decision maker DM_i, at time t, let the effect of the full application of alternative $A \in \mathcal{A}$ into DM_i's strategic goals in the none-empty $\mathcal{SG}_{DM_i}$ collectively be represented by a Total Weighted Final Achievement value property; and let this value property be denoted as $\mathit{TWFAchv}(DM_i, A, t)$, and calculated algebraically as follows:

$$\mathit{TWFAchv}(DM_i, A, t) = \frac{1}{|\mathcal{SG}_{DM_i}|} \sum_{SG \in \mathcal{SG}_{DM_i}} \mathit{WFAchv}(SG, DM_i, A, t) \tag{4.6}$$

For the *Total Weighted Final Achievement* $\mathit{TWFAchv}(DM_i, A, t)$ value to reflect the effect of alternative A, and A only, on DM_i, then the GCM with all its constructs and value properties' value must stay the same and only A is applied fully, i.e. the achievement value of the intention representing the intention to implement/apply alternative A changes from $\mathit{Achv}(A) = N$ to $\mathit{Achv}(A) = F$. All other alternatives' intentions' achievement values must stay the same unchanged, preferably unselected and stay at the *None* level, i.e. $(\forall A_k \in \mathcal{A} \ : \ A_k \neq A) \quad \mathit{Achv}(A_k) = N$. Then after the values forward propagation algorithm, Algorithm 3.1, finalized the value labels for all goals for time t, we calculate $\mathit{TWFAchv}(DM_i, A, t)$. The value of $\mathit{TWFAchv}(DM_i, A, t)$, now, reflect the effect of applying alternative A, and only A, on DM_i's strategic goals.

All the requirements/assumptions given for calculating $W(SG, DM_i, t)$ and $\mathit{FAchv}(SG, A, t)$ applies to calculating $\mathit{TWFAchv}(DM_i, A, t)$ too. The values of the overall rationality and emotionality factors, RF_{DM_i} and EF_{DM_i}, must not have a fuzzy vale label of *Null* assigned to them . As well, none of the fuzzy value labels assigned to $SImprt(SG)$, $EVlnc(SG)$ and $FAchv(SG)$ for each $SG \in \mathcal{SG}_{DM_i}$, should be *Null*. These assumptions are logical for the same reasons we indicated earlier. In addition, Equation 4.6 requires that the set of strategic goals that decision maker DM_i have must not be empty, i.e. it requires that $|\mathcal{SG}_{DM_i}| \neq 0$. This is also a logical assumption. If DM_i has no strategic goals that he cares about, then there is no basis to judge how effective an alternative versus another. Recall that we use the contribution that the different alternatives have on the achievement of the decision maker's strategic goals as a measure of effectiveness for these alternatives.

We said above that $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-2, 2]$, if emotionality is captured within the model (i.e. $\exists DM_i \in \mathcal{DM} \;:\; ((EF_{DM_i} \neq N) \wedge (\exists SG \in \mathcal{SG}_{DM_i} \;\; EVlnc(SG) \neq N))$). This makes the value of $TWFAchv(DM_i, A, t)$ to be in the same range of $[-2, 2]$, under the same condition of having emotionality been captured as part of the model. On the other hand, if the modeller did not consider modelling emotionality as part of modelling the decision making situation (i.e. $\forall DM_i \in \mathcal{DM} \;:\; ((EF_{DM_i} = N) \vee (\forall SG \in \mathcal{SG}_{DM_i} \;\; EVlnc(SG) = N))$), then $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-1, 1]$, and therefore the value of $TWFAchv(DM_i, A, t)$ will be in the range of $[-1, 1]$.

In the case where the modeller included emotionality in the model, for at least one of the decision makers in the decision making situation modelled, then we need to normalize the value of $TWFAchv(DM_i, A, t)$ calculated for every and each decision maker involved in the decision making situation (and included in the model) to ensure consistency across the model. The new normalized value of $TWFAchv(DM_i, A, t)$ is calculated as follows:

$$TWFAchv(DM_i, A, t)\Big|_{\text{normalized}} = \frac{1}{2|\mathcal{SG}_{DM_i}|} \sum_{SG \in \mathcal{SG}_{DM_i}} WFAchv(SG, DM_i, A, t) \tag{4.7}$$

But, how $TWFAchv(DM_i, A, t)$ maps to the concepts of cardinal preference and ordinal preference used extensively in the literature of both game theory and decision theory? We should first differentiate between preferences over a set of alternatives a decision maker have and preferences over a set of states a multi-agent decision making situation has. Preferences over a set of alternatives, that a decision maker have in a single-agent decision making situation, is what we will discuss in this section, and what this chapter is concerned with. In the next chapter, we will modify our method of calculating preferences over alternatives (given in Equations 4.3 - 4.6) to account for preferences over states and shared-alternatives of multi-agent decision making situations.

For a single decision maker DM_i, in a single-agent decision making situation, at time t, let the *Cardinal Preference* that DM_i has over alternative $A \in \mathcal{A}_{DM_i}$ be represented as a *Weighted Payoff* value property attached to A, and be denoted as $WP(A, DM_i, t)$. Let $WP(A, DM_i, t)$ have a numerical value in the range of $[-1, 1]$ and calculated as follows:

$$WP(A, DM_i, t) = TWFAchv(DM_i, A, t)\Big|_{\text{normalized}} \tag{4.8}$$

$$= \frac{1}{2|\mathcal{SG}_{DM_i}|} \sum_{SG \in \mathcal{SG}_{DM_i}} WFAchv(SG, DM_i, A, t) \tag{4.9}$$

Recall that $TWFAchv(DM_i, A, t)$ does not need any normalization, if the modeller did not consider modelling emotionality as part of modelling the decision making situation (i.e. $\forall DM_i \in \mathcal{DM} \ : \ ((EF_{DM_i} = N) \vee (\forall SG \in \mathcal{SG}_{DM_i} \ \ EVlnc(SG) = N))$). Because in such case, $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-1, 1]$, and therefore the value of $TWFAchv(DM_i, A, t)$ will be in the range of $[-1, 1]$. But, if emotionality is captured within the model (i.e. $\exists DM_i \in \mathcal{DM} \ : \ ((EF_{DM_i} \neq N) \wedge (\exists SG \in \mathcal{SG}_{DM_i} \ \ EVlnc(SG) \neq N))$), then the value of $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-2, 2]$, and therefore the value of $TWFAchv(DM_i, A, t)$ will be in the range of $[-2, 2]$. In this case, normalization is needed to maintain the values of $TWFAchv(DM_i, A, t)$ and $WP(A, DM_i, t)$ in the range of $[-1, 1]$. Hence, the devision by 2 shown in 4.9.

Based on the cardinal preferences, or weighted payoffs, calculated for DM_i over each of his alternatives $A \in \mathcal{A}_{DM_i}$, there will be a *Preference Vector*, denoted as $Pref(DM_i, \mathcal{A}_{DM_i})$, showing the order of the alternatives in $\mathcal{A}_{DM_i}$ from the *most preferred* to the *least preferred*. It is assumed here that $Pref(DM_i, \mathcal{A}_{DM_i})$ represents only ordinal ranking of the alternatives in $\mathcal{A}_{DM_i}$ based on how much each of these alternatives contributes to the achievement of DM_i's strategic goals, given the importance weight and emotional valence that DM_i assigned to each of his strategic goals, and given a specific rationality factor RF_{DM_i} and emotionality factor EF_{DM_i} describing the attitudes DM_i is exhibiting towards acting rationally or emotionally at that point of time.

The preference order of a specific alternative $A \in \mathcal{A}_{DM_i}$, to DM_i at time t, is given as an *Ordinal Preference* value property attached to A, and is denoted by $OP(A, DM_i, t)$. Let $OP(A, DM_i, t)$ be given an integer number that reflects

A's position in DM_i's preference vector $Pref(DM_i, \mathcal{A}_{DM_i})$ at that point of time. The smaller the integer number assigned to $OP(A, DM_i, t)$ the more preferred the alternative A is, to DM_i at time t. The alternative that has $OP(A, DM_i, t) = 1$ is the most preferred alternative and the one with $OP(A, DM_i, t) = |\mathcal{A}_{DM_i}| - 1$ is the least preferred alternative. This is because the alternatives in the $Pref(DM_i, \mathcal{A}_{DM_i})$ vector are ordered from the alternative that has the highest weighted payoff value to the the one that has the lowest weighted payoff.

$$OP(A, DM_i, t) = n + 1 \quad \text{where } 0 \leq n \leq |\mathcal{A}_{DM_i}| - 1 \quad (4.10)$$
$$\text{and } n \text{ reflects } A\text{'s position in } Pref(DM_i, \mathcal{A}_{DM_i})$$

In the next chapter, we will modify our method of calculating the cardinal and ordinal preferences over alternatives (given in Equations 4.3 - 4.10) to account for preferences over states and shared-alternatives of multi-agent decision making situations. In the following section, we will present how we will elicit and model the strength of preferences over alternatives that we have calculated above.

4.5 Modelling Preferences' Strength

To capture the strength of alternative A's preference over other alternatives, for a specific decision maker DM_i, we use a distance measure between the cardinal preference of A and the cardinal preference of each other alternative DM_i has in his set of alternatives $\mathcal{A}_{DM_i}$ for the decision making situation at hand. Based on this distance measure a binary relation among each pair of these alternatives is assigned.

For decision maker DM_i, and at time t, let the distance measure among the two preferences which DM_i has over the two alternatives A_a and A_b, both in $\mathcal{A}_{DM_i}$, be denoted as $d(A_a, A_b, DM_i, t)$. And, let its value be given as a real number calculated as follows:

$$d(A_a, A_b, DM_i, t) = [WP(A_a, DM_i, t) - WP(A_b, DM_i, t)] \quad (4.11)$$

Because each of $WP(A_a, DM_i, t)$ and $WP(A_b, DM_i, t)$ is in the range of $[-1, 1]$, whether normalized as per Equation 4.9 or not normalized because the modeller did not consider modelling emotionality as part of the decision making situation's model (as discussed earlier), then the distance value will be in the range of $[-2, 2]$.

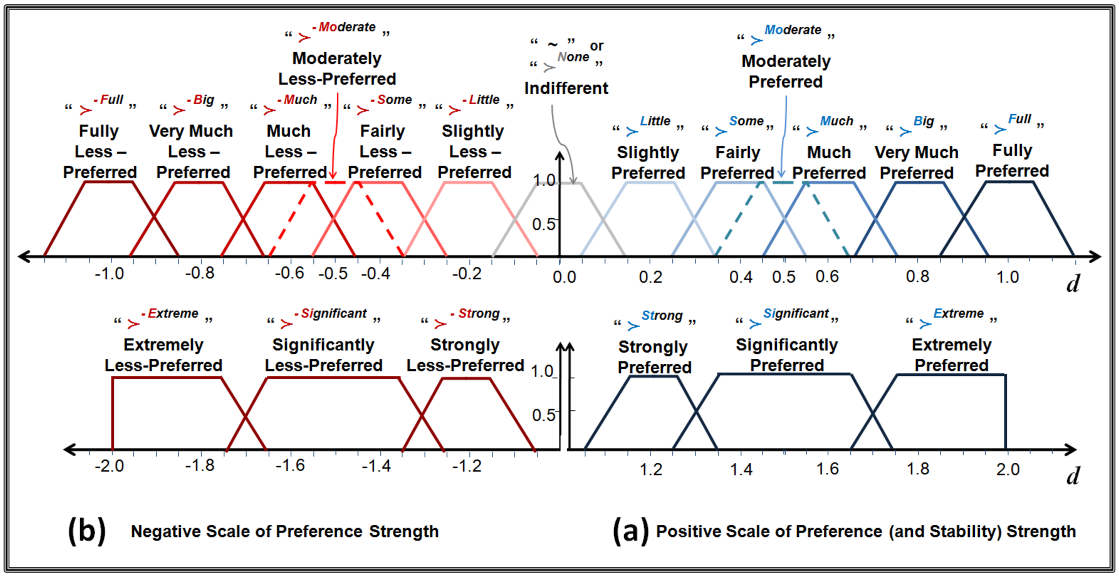


Figure 4.4: The membership functions of the Fuzzy Sets dividing the satisfaction levels domain of the Preference Strength value property $PrefStrngth(A_a, A_b, DM_i, t)$: (a) shows the fuzzy sets covering the defuzzified real values > 0; and (b) shows the fuzzy sets covering the defuzzified real values < 0

The sign of $d(A_a, A_b, DM_i, t)$ shows which alternative, of the two, that decision maker DM_i prefers. If $d(A_a, A_b, DM_i, t) > 0$, then state A_a is preferred by DM_i over A_b; if $d(A_a, A_b, DM_i, t) = 0$, then DM_i is indifferent; otherwise, if $d(A_a, A_b, DM_i, t) < 0$, then DM_i prefers A_b over A_a. The actual value of $d(A_a, A_b, DM_i, t)$ represents the strength of the preference that DM_i has for alternative A_a over A_b.

Let *Preference Strength* be a value property, denoted as $PrefStrngth(A_a, A_b, DM_i, t)$, and be given the fuzzified value of the distance value property. In other words, let $PrefStrngth(A_a, A_b, DM_i, t) = \underset{\sim}{d}(A_a, A_b, DM_i, t)$. $PrefStrngth(A_a, A_b, DM_i, t)$ will be assigned a fuzzy linguistic value label L_{PS} based on the fuzzy memberships functions given in Figure 4.4. The strength expressed by $PrefStrngth(A_a, A_b, DM_i, t) = L_{PS}$ is meant to represent the distance between the weighted preference values for the two alternatives considered here, A_a and A_b. And, where L_{PS} is a fuzzy linguistic value label that represents the name of the fuzzy set which the preference strength value has the highest membership of. And, $L_{PS} \in \mathcal{L_{PS}} =$ {*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*, *Little*, *None*, *-Little*, *-Some*, *-Moderate*, *-Much*, *-Big*, *-Full*, *-Strong*, *-Significant*, *-Extreme*, *Null*} $= \{Ex, Si, St, F, B, M, Mo, S, L, N, -L, -S, -Mo, -M, -B, -F, -St, -Si, -Ex, Null\}$. And, with the complete order of $Ex > Si > St > F > B > M > Mo > L > N > -L > -S > -Mo > -M > -B >$

$-F > -St > -Si > -Ex > Null$, where the labels range from representing *Extreme* preference of A_a over A_b to *Extreme* less-preference of A_a over A_b (or the *Extreme* preference of A_b over A_a).

The L_{PS} fuzzy label assigned to $PrefStrngth(A_a, A_b, DM_i, t)$ will cover the preference's strength levels in the range $[-2, 2]$, with the understanding that the *Null* label represents an unknown preference strength. The fuzzy membership functions defining these preference strength's linguistic value labels are given in Figure 4.4. The figure shows the membership functions for each label's fuzzy set to be trapezoidal in shape, for simplicity only (not as a restriction) as indicated in Chapter 3. In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements.

Now, let the preference strength of alternative A_a over alternative A_b, for decision maker DM_i at time t, given in $PrefStrngth(A_a, A_b, DM_i, t) = L_{PS}$, be represented by a binary relationship between the two alternatives. Let this binary relation be denoted as $A_a \succ^{L_{PS}}_{DM_i,t} A_b$. The notation of this relation is shown in Figure 4.4 for each possible preference strength fuzzy linguistic label L_{PS} (shown in the figure above each label's membership function).

In the next chapter, we will extend this method of calculating and representing the strength of the decision maker's preferences over alternatives he has, to cover the strength of preferences over states and shared-alternatives in multi-agent decision making situations. But, for now, we will show an example of calculating preferences and preferences' strengths for a single-agent decision making situation.

4.6 Example: Howard's Personal Dilemma

In Chapter 3, we presented as an example the Constrained Rationality GCM model for an interesting "personal dilemma" that represents a real-life conflict: Howard's Personal Dilemma (based on the personal conflict originally presented in Thagard and Millgram (1995)). As per the example, Howard, a professor in a good university, receives an offer to move to Harvard. Howard has many personal goals that range from research-related to family-related goals. The goals conflict with each other. Satisfaction of some will lead to dissatisfaction of others. We showed there, in Figure 3.13, the Howard Dilemma modelled using the Constrained Rationality framework; and we showed how Howard's two alternatives (stay at his current school or move to Harvard) contribute differently to Howard goals' achievement and prevention value properties.

We ran the framework's forward labels propagation algorithm (Algorithm 3.1) to generate the final reasoning about the two scenarios: 1) Howard continues to work at his current school (G_{11}) with no job offer received from Harvard (C_1); and 2) Howard decides to activate the "move to Harvard" goal (G_{10}) after the "Offer from Harvard" constraint (C_2) changed from full prevention (no offer) to full achievement (offer received). The analysis presented in Section 3.10 showed that for the first scenario: Howard's goal of keeping his family happy (G_2) will be fully achieved, but Howard's goals of "increase scientific understanding"(G_1) and "keep self happy" (G_3) will be moderately achieved. For the second scenario, the analysis showed the effect of Howard's decision to move to Harvard on all his goals: He will manage to satisfy fully G_1, but his and his family's happiness goals will receive prevention. We also showed that with simple tweaking to the dilemma's Constrained Rationality model (by changing some of the lateral relation types or their modifier labels, for example), we will able to provide more insight on how Howard thinks, and what will happen to his goals as his thinking changes. In addition, we said there that any change in the GCM model to "better" capture Howard's actual state-of-mind, or to test what-if scenarios by him, will show the effect on all his goals.

In this section, we will use the same dilemma, with Howard having the same two alternatives:

A_{H0} : stay at current school; and

A_{H1} : move to Harvard.

But, let the GCM model, presented in Figure 3.13 be modified to generate for each of these two alternatives a set of different final achievement and prevention value labels for Howard's ultimate three strategic goals:

SG_{H1} : increase scientific understanding;

SG_{H2} : keep family happy; and

SG_{H3} : keep self happy.

And, let the final achievement and prevention labels of these three strategic goals be the ones given in the table shown in Figure 4.5a. This table shows also the Final Achievement value for each of these goals, given in both its forms: as fuzzy linguistic value label, and as a defuzzified real number.

(a)

Achv & Prvn values for Goals within the decision making situation

Strategic Goals of DMs	SGs:	Howard		
		SG_{Howard}		
		SG_{H1}	SG_{H2}	SG_{H3}
Alternative A_{H0} Status Quo (Howard Stays at Current School) { Achv(A_{H0})=F }	Achv(SG_k)	S	M	Mo
	Prvn(SG_k)	N	N	N
	FAchv(SG_k)	S	M	Mo
	FAchv*(SG_k)	0.40	0.60	0.50
Alternative A_{H1} Howard Moves to Harvard { Achv(A_{H1})=F }	Achv(SG_k)	B	M	F
	Prvn(SG_k)	N	L	B
	FAchv(SG_k)	B	S	L
	FAchv*(SG_k)	0.80	0.40	0.20

(b)

	Strategic Goals of DMs	SGs:	decision making situation # 1 (Howard)			decision making situation # 2 (Howard)		
			SG_{Howard}			SG_{Howard}		
			SG_{H1}	SG_{H2}	SG_{H3}	SG_{H1}	SG_{H2}	SG_{H3}
	Strategic Importance	SImprt(SG_k)	F	B	Mo	F	B	Mo
	Emotional Valence	EVlnc(SG_k)	L	EL	SL	El	EL	SL
Extremely Rational (Rationality Factor = 1.0, Emotionality Factor = 0.0)	**Alternative A_{H0}** Status Quo (Howard Stays at Current School) { Achv(A_{H0})=F }	FAchv(SG_k, A_{H0},t)	0.40	0.60	0.50	0.40	0.60	0.50
		W(SGk,H,t)	1.00	0.80	0.50	1.00	0.80	0.50
		WFAchv(SGk,H,A_{H0},t)	0.40	0.48	0.25	0.40	0.48	0.25
		WP(A_{H0}, H, t)	0.38			0.38		
		OP(A_{H0}, H, t)	2 (Worst)			2 (Worst)		
	Alternative A_{H1} Howard Moves to Harvard { Achv(A_{H1})=F }	FAchv(SG_k, A_{H1},t)	0.80	0.40	0.20	0.80	0.40	0.20
		W(SGk,H,t)	1.00	0.80	0.50	1.00	0.80	0.50
		WFAchv(SGk,H,A_{H1},t)	0.80	0.32	0.10	0.80	0.32	0.10
		WP(A_{H1}, H, t)	0.41			0.41		
		OP(A_{H1}, H, t)	1 (Best)			1 (Best)		
Extremely Emotional (Rationality Factor = 0.0, Emotionality Factor = 1.0)	**Alternative A_{H0}** Status Quo (Howard Stays at Current School) { Achv(A_{H0})=F }	FAchv(SG_k, A_{H0},t)	0.40	0.60	0.50	0.40	0.60	0.50
		W(SGk,H,t)	0.60	1.00	0.20	0.00	1.00	0.20
		WFAchv(SGk,H,A_{H0},t)	0.24	0.60	0.10	0.00	0.60	0.10
		WP(A_{H0}, H, t)	0.31			0.23		
		OP(A_{H0}, H, t)	Same			1 (Best)		
	Alternative A_{H1} Howard Moves to Harvard { Achv(A_{H1})=F }	FAchv(SG_k, A_{H1},t)	0.80	0.40	0.20	0.80	0.40	0.20
		W(SGk,H,t)	0.60	1.00	0.20	0.00	1.00	0.20
		WFAchv(SGk,H,A_{H1},t)	0.48	0.40	0.04	0.00	0.40	0.04
		WP(A_{H1}, H, t)	0.31			0.15		
		OP(A_{H1}, H, t)	Same			2 (Worst)		
Fully Rational & Emotions are Considered Fully (Rationality Factor = 1.0, Emotionality Factor = 1.0)	**Alternative A_{H0}** Status Quo (Howard Stays at Current School) { Achv(A_{H0})=F }	FAchv(SG_k, A_{H0},t)	0.40	0.60	0.50	0.40	0.60	0.50
		W(SGk,H,t)	1.60	1.80	0.70	1.00	1.80	0.70
		WFAchv(SGk,H,A_{H0},t)	0.64	1.08	0.35	0.40	1.08	0.35
		WP(A_{H0}, H, t)	0.69			0.61		
		OP(A_{H0}, H, t)	2 (Worst)			1 (Best)		
	Alternative A_{H1} Howard Moves to Harvard { Achv(A_{H1})=F }	FAchv(SG_k, A_{H1},t)	0.80	0.40	0.20	0.80	0.40	0.20
		W(SGk,H,t)	1.60	1.80	0.70	1.00	1.80	0.70
		WFAchv(SGk,H,A_{H1},t)	1.28	0.72	0.14	0.80	0.72	0.14
		WP(A_{H1}, H, t)	0.71			0.55		
		OP(A_{H1}, H, t)	1 (Best)			2 (Worst)		
Mostly Rational & Slightly Emotional (Rationality Factor = 0.8, Emotionality Factor = 0.2)	**Alternative A_{H0}** Status Quo (Howard Stays at Current School) { Achv(A_{H0})=F }	FAchv(SG_k, A_{H0},t)	0.40	0.60	0.50	0.40	0.60	0.50
		W(SGk,H,t)	0.92	0.84	0.44	0.80	0.84	0.44
		WFAchv(SGk,H,A_{H0},t)	0.37	0.50	0.22	0.32	0.50	0.22
		WP(A_{H0}, H, t)	0.36			0.35		
		OP(A_{H0}, H, t)	2 (Worst)			Same		
	Alternative A_{H1} Howard Moves to Harvard { Achv(A_{H1})=F }	FAchv(SG_k, A_{H1},t)	0.80	0.40	0.20	0.80	0.40	0.20
		W(SGk,H,t)	0.92	0.84	0.44	0.80	0.84	0.44
		WFAchv(SGk,H,A_{H1},t)	0.74	0.34	0.09	0.64	0.34	0.09
		WP(A_{H1}, H, t)	0.39			0.35		
		OP(A_{H1}, H, t)	1 (Best)			Same		

Figure 4.5: Howard's Weighted and Ordinal Preferences over his Alternatives: (a) Howard's Strategic Goals and their individual Achievement, Prevention and Final Achievement value labels (in addition to the Final Achievement defuzzified value) for each of his alternatives; and (b) Calculating Howard's Preferences using different Rationality and Emotionality Factors, and with different Emotional Valences attached to the strategic goals.

We analyze Howard's dilemma to generate Howard's cardinal and ordinal preferences over his two alternatives. But, we assume different scenarios where different strategic importance, emotional valences, rationality factor and emotionality factor values are employed. This is done mainly to test how the preferences change as these values change. The results of the analysis are presented in a set of tables, shown in Figure 4.5b.

First, all the decision making situations which we analyzed for the Howard's Dilemma will have the following two sets for Howard: Howard's strategic goals set $\mathcal{SG}_H = \{SG_{H1}, SG_{H2}, SG_{H3}\}$; and Howard's alternatives set $\mathcal{A}_H = \{A_{H0}, A_{H1}\}$. Second, the two main situations which we analyzed with different Rationality and Emotionality factors are:

decision making situation # 1

Howard's three strategic goals SG_{H1}, SG_{H2} and SG_{H3} have the following strategic importance values: $SImprt(SG_{H1}) = F$, $SImprt(SG_{H2}) = B$ and $SImprt(SG_{H3}) = Mo$, respectively, with the meaning that Howard gives his "increase scientific knowledge" full importance while giving his "keep family happy" and "keep self happy" goals less importance levels (big and moderate importance, respectively). The three goals, also, have the following emotional valences: $EVlanc(SG_{H1}) = L$, $EVlanc(SG_{H2}) = L$ and $EVlanc(SG_{H3}) = L$, respectively, with the meaning that: Howard gives "keep family happy" the extremely-liked emotional valence (understandably) while keeping the achievement of his "increase scientific knowledge" and "keep self happy" goals emotionally in the second and third places by giving them the liked and slightly-liked emotional valence labels, respectively.

decision making situation # 2

In this situation, Howard keeps the strategic importance of his three goals SG_{H1}, SG_{H2} and SG_{H3} the same (as in decision making situation # 1). He, also, keeps the same emotional valences attached to the goals, with the exception of his "increase scientific knowledge" (SG_{H1}). In this situation, he elevated this goal from being just liked to extremely-liked, i.e. from $EVlanc(SG_{H1}) = L$, as in the first situation, to $EVlanc(SG_{H1}) = EL$ here. This makes SG_{H1} extremely-liked as it is the case for his "keep family happy" SG_{H2} goal. But, both goals still differ in their strategic importance levels with $SImprt(SG_{H1}) = F$ and $SImprt(SG_{H2}) = B$. The idea behind this situation is to test if a slight change in the emotional "likeness" of SG_{H1} will make a difference in the preferences of Howard over his two alternatives.

Figure 4.5b presents these two decision making situation and their respective calculated values in two separate columns. The figure shows the calculations of both the cardinal weighted preference and ordinal preference, for each of Howard'a two alternatives, for the two situations, but analyzed under four different Rationality and Emotionality Factors for Howard:

Howard is Extremely Rational

In this case, Howard is fully rational ($RF_H = F$) and considers no emotionality at all ($EF_H = N$). As this case's table in Figure 4.5b shows, both alternatives A_{H0} and A_{H1} maintained the same weighted and ordinal preference values, in both decision making situations #1 and #2: A_{H1} is slightly preferred to A_{H0}. This means that, for Howard, and if he is completely rational, "moving to Harvard" is slightly better than "staying at his current school". This stays the same whether he likes his "increase scientific knowledge" as much as he likes to "keep family happy", or less than 'keep(ing) family happy" (situation #1 and #2, respectively).

The strength of the preference of A_{H1} over A_{H0} is given as $PrefStrngth(A_{H1}, A_{H0}, H, t) = N$, and represented as $A_{H1} \succ^{N}_{H,t} A_{H}0$. Recall that the *None* fuzzy value label given here to describe the strength of the preference does not mean that there is no preference, but rather there is a very minimal preference. Under the fuzzy sets/labels scheme we adopted to describe the preferences' strengths, discussed above and shown in Figure 4.4, the label *None* is assigned to the preference's strength here because the real number (defuzzified) value of it (0.03) falls in the range covered by the *None* fuzzy-set/value-label.

Howard is Extremely Emotional

In this case, Howard is fully emotional ($EF_H = F$) and considers no rationality at all ($RF_H = N$). This case's table in Figure 4.5b shows that the weighted and ordinal preference values for A_{H0} and A_{H1} depends on how much Howard likes his SG_{H1} compared to SG_{H2}. For this case and under decision making situation #1, where $EVlanc(SG_{H1}) < EVlanc(SG_{H2})$, both alternatives have the same preference levels (with strength level of 0.0 and therefore assigned the *None* strength value label). But, under decision making situation #2, where $EVlanc(SG_{H1}) = EVlanc(SG_{H2})$, alternative A_{H0} is more preferred to A_{H1}. In other words, for Howard, the preference of staying at his school is slightly more than moving to Harvard. The preference strength of A_{H0} to A_{H1} for this scenario is still minimal, though slightly better, at the numeric value of 0.07. Therefore, $A_{H0} \succ^{N}_{H,t} A_{H}1$.

Howard is Fully Rational, but also Considers his Emotions Fully

In this case, Howard is fully rational ($RF_H = F$), but he also considers his emotions fully ($EF_H = f$). Therefore, the strategic importances will be added to the emotional valences to form the weighting factor for each of the goals' final achievement harnessed by each alternative. This case's table in Figure 4.5b shows that the weighted and ordinal preference values for A_{H0} and A_{H1}, here too, depends on how much Howard likes his SG_{H1} compared to SG_{H2}.

For this case, and under decision making situation #1, where $EVlanc(SG_{H1}) < EVlanc(SG_{H2})$, A_{H1} is shown to be slightly more preferred to A_{H0}. The strength level is minimal at 0.02. Therefore, $A_{H1} \succ^{N}_{H,t} A_{H}0$. But, under decision making situation #2, where $EVlanc(SG_{H1}) = EVlanc(SG_{H2})$, the reverse is now true. A_{H0} is shown to be slightly more preferred to A_{H1}. The strength level is minimal at 0.06. Therefore, $A_{H0} \succ^{N}_{H,t} A_{H}1$.

Howard is Mostly Rational and Slightly Emotional

In this case, Howard is more rational (with rationality at about the 80% level, i.e. $RF_H = B$) and slightly emotional (with emotionality at about the 20% level, i.e. $EF_H = L$). In other words, the weighting factor for each of the goals' final achievement, harnessed by each alternative, will be made for the most part(80%) from the strategic importance given to the individual goals. And, with the emotional valence forms a little part (20%) of the weighting factor. This case's table in Figure 4.5b shows that the weighted and ordinal preference values for A_{H0} and A_{H1}, here too, depends on how much Howard likes his SG_{H1} compared to SG_{H2}.

For this case, and under decision making situation #1, where $EVlanc(SG_{H1}) < EVlanc(SG_{H2})$, A_{H1} is shown to be slightly more preferred to A_{H0}. The strength level is minimal at 0.03. Therefore, $A_{H1} \succ^{N}_{H,t} A_{H}0$. But, under decision making situation #2, where $EVlanc(SG_{H1}) = EVlanc(SG_{H2})$, both alternatives have the same preference levels (with strength level of 0.0 and therefore assigned the *None* strength value label).

In this example, we illustrated how the different importance and emotional-likeness of specific strategic goals, as well as the personality wide attitudes towards acting rationally or emotionally, can affect the preferences (and their strengths) over alternatives for a specific agent. The Weighted Preference WP and the Ordinal Preference OP for Howard's alternatives are calculated for each of the four cases, and for each of the two decision making situations, #1 and #2 (where the emotional valences attached to the goals slightly changes to reflect two possible state-of-mind that Howard could be in). Figure 4.5 shows these preferences change as Howard's personality wide attitude mode change, i.e. as Howard Rationality Factor RF_H and Emotionality Factors EF_H change.

But in this example, we also experienced the importance of adopting the right fuzzy sets to describe a value property such as the strength of a preference. If the sets, and their respective fuzzy linguistic value labels cover wider satisfaction real-number/defuzzified values, then the labels will be less expressive, or descriptive. In our case, because the preference strength/distance is minimal, the fuzzy value label assigned to the value property is *None*. This could be satisfactory in this example, because Howard should know that the preference an alternative have over the other is minimal, and this is important for him to take in consideration when deciding on the matter of moving to Harvard or not. This scale is better than adopting a more detailed/expressive scale that will in effect inflate the strength level of the preference, or call it with a value label that is more than what it deserves to be described with.

4.7 Summary

We started this chapter by discussing how the different strategic goals and alternatives, the agent has, are identified and modelled. Then the chapter presented how the final achievement levels of agents' strategic goals are elicited for each of the alternatives the agents' have; how the strategic importance and the emotional valences for each of the agents over their own strategic goals are captured and modelled; as well as how the personality wide attitudes of the agents' toward acting rational and/or emotional are modelled.

The chapter, then, showed: how all these values are used to determine the cardinal and ordinal preferences of the agents over their own alternatives; and how the strength of the agents' preferences are determined and represented. At the end, the chapter presented an illustrative example that shows how the concepts and methods presented in the chapters are applied in a close-to-real-life one-agent decision making situation.

The following chapter will take the concepts and methods of this chapter further. It will discuss how to model collaborative and competitive multi-agent decision making situations; and how the concepts and methods presented here will be applied to determine the preferences of all agents in such situations.

Chapter 5

Modelling and Analyzing Multi-Agent Strategic Decision Making

5.1 Introduction

In the previous two chapters, we introduced the foundational concepts and methods of the Constrained Rationality framework. We showed how the agents' goals and realities are modelled; how they affect each other through reduction and lateral relationships; how qualitative linguistic value-labels of the agents' goals and constraints propagate through these relationships; and how value properties of the goals, such as their operationalization, achievement and prevention levels, are finalized and updated to reflect the changes happening over time to the agents and their decision making situation. In addition, we showed how the priorities and feelings of the agents' over their strategic goals are modelled, and how these values are used to determine the agents' preferences over their alternatives.

But, the previous two chapters focused more on single-agent decision making situations, and multi-agent decision making situations where the agents act and decide in an individualistic manner with no regard to others' choices and decisions, and to how these choices might affect their own objectives and choices. This chapter discusses how to model collaborative and competitive multi-agent decision making situations; and takes further the concepts and methods, introduced in the last two chapters, showing how they will be applied to determine the preferences of agents in these collaborative and competitive multi-agent decision making situations.

This chapter starts with discussing the two main modes/types of multi-agent decision making situations, the collaborative decision making situations and the competitive conflicts modes. The chapter, then, introduces the process to model and analyze both types. First, the process of modelling is discussed, including important topics such as: how the framework suggests the different agents' viewpoint models be modelled and integrated ; how the agents' alternatives will be modelled, in both collaborative and competitive decision making situations; how the situation's structure and states are defined, for the competitive conflicts; and how the base-model of the decision making situation is to be validated and finalized.

Then, the topic of modelling priorities, capturing emotions, and eliciting agents' preferences is discussed thoroughly for both types of multi-agent decision making situations. The chapter, then closes the modelling-process section with a discussion on how the framework models the dynamics of multi-agent situations, specifically the different moves that agents who are engaged in competitive conflicts make or can make; and how the different types of moves, agents have in conflicts, are used to identify three main types of multi-agent competitive decision making conflicts: non-cooperative, cooperative with no coalitions/alliances (i.e. agents cooperate but still act individually and do not sacrifice their individual positions for the good of the group), and finally cooperative conflicts where agents are allowed to form coalitions, act as groups and take multi-step moves including intermediate temporarily scarifies along the way for the good of all members of the group they belong to.

We will use these three types of multi-agent conflicts to structure and organize our discussion about modelling and analyzing multi-agent conflicts (or games, as they are also referred to in games theory literature), and what concepts and tools the Constrained Rationality framework offers to deal with the specific needs of modelling and analyzing these conflicts. We will provide a separate and dedicated chapter, after this one, to discuss each conflict type, including the concepts and methods to be used to model and analyze that type. But, in this chapter, we provide a section to give an overview on the topic of analyzing multi-agent decision making situation, covering not only the competitive conflict ones but also the collaborative ones. This section will discuss: the process of analyzing multi-agent decision making situations; how the agents' preferences over alternatives and states are calculated; how the stability of the different states in these conflicts is determined (for each agent) and how equilibriums are identified; how sensitivity and what-if analysis is conducted; and finally, why the analyst should compare the produced analysis and predictions to what actually happen/happened in real-life.

The chapter closes by providing an illustrative example. The example shows how the process, concepts and methods introduced in this chapter are used to model and analyze a collaborative multi-agent decision making situation: a system requirements engineering situation. The chapter does not provide any example for competitive conflict situations, because the next few chapters will provide detailed case studies that cover all the three types of conflicts: non-cooperative conflicts, cooperative conflicts without coalitions, and cooperative conflicts with coalitions.

5.2 Modes of Multi-Agents Decision Making

The domain of multi-agent decision making situations can be divided into two groups. The first group is the one that includes the *Collaborative Decision Making Situations*, where all agents collaboratively and jointly try to choose among many alternatives they have. The alternatives are known to all agents, and form a shared alternatives space. The goals and constraints are also shared, for the most part. Examples of such situations include Product Selection, Product Development, Systems Development, Requirements Engineering, Scientific Research, Policy Making, and so on.

The agents who are involved in collaborative decision making situations could all be internal agents, part of a whole entity, such as internal sub-organizations of the enterprise (as in Product Development initiatives). They also could be completely separate agents who only come together to solve a problem, build a system, or decide on the best policy. The fact that agents could be part of a whole entity does not mean that these agents have the same goals, or even aligned goals. Agents, even internal ones, tend to have conflicting goals, which could hinder or negatively affect each others' goals, and their constraints also could challenge each other. But in collaborative decision making situations, they all come together, and disclose their goals (needs and wants) for the purpose of choosing the option that "best fit" their goals collectively.

The second group of multi-agent decision making situations is the one that includes the *Adversarial/Competitive Decision Making Situations*. In these situations, all involved agents are competing with each other in some form or another, and to some degree or another. These situations are commonly referred to in game theory literature as *Games*, and in the wider management and behavioural science literature as *Conflicts*. In our research work, we use the two terms, game and conflict, interchangeably to mean the same thing: multi-agent adversarial decision making situation, where the agents involved have different conflicting gaols, and each has a separate set of alternatives/options/moves to choose from.

The agents involved in multi-agent conflicts also could be internal agents that belong to one whole bigger agent or completely separate independent agents. But what differentiates agents involved in games/conflicts than the ones involved in collaborative multi-agent decision making situations is the fact that the former have their own alternatives while the later have a shared set of alternatives to choose from. Therefore, collaborative situations are less dynamic, as the agents move together from one collective decision to another. There is no relying on individual agents choices/moves as the case in multi-agent conflicts.

Conflicts, on the other hand, are dynamic and could be represented as state machines where the choices/moves agents take at any pint of time (future state) depend on the choices they took at the previous point of time (current state). And, while collaborative situations' collective decisions could be represented as states, each of these sates is in fact defined by one shared alternative/option selected by all the agents. On the other hand, each of the states in a game/conflict situation is defined by a set of alternatives/options that the agents select, from their own set of alternatives, at the time they enter the state.

Splitting the domain of multi-agent decision making situations to the two groups, the collaborative decision making situations and the adversarial competitive ones, is a generalization that could help us categorize the situations in order to study and analyze them. This splitting is not theoretical. It is rooted in organizational behavioural studies, such as the work of Thomas (1976). But in reality, the two groups if defined to exclude competing collaborators or cooperating competitors form two extreme points of the domain space, as shown by Figure 5.1, an illustrative digram put together by Thomas (1976).

In reality, collaborating agents tend also to have competitiveness among them (such as scientific researchers in the same filed collaborating to solve key problems and at the same time competing for funding sources), and competing agents tend to co-operate with each other (for example, industrial corporations forming standards alliances even though they compete with each other in the same market space). In other words, the two "general groups" we identified above, are not the extreme

Figure 5.1: Behavioural Modes of tackling a Conflict (an illustration originally put by Thomas (1976)), with an overlay showing the two general groups, we are considering in our research, divide the multi-agent decision making situations domain.

opposite points of the domain, but rather they have within each of them decision making situations that embody degrees of shades of the characteristics defines the other group. Figure 5.1 shows how those two general groups maps to the extreme points of behaviour exhibited by decision makers in multi-agent decision making situations; and shows how the two groups generally divide the domain space of multi-agent decision making situations space with all its shades of behaviour modes.

A multi-agent decision making situation could have all the major characteristics of the collaborative-multi-agent-decision-making group, but have some agents who do not disclose all their relevant goals, because of reasons such as hidden competition. The situation should still be modelled and analyzed as a collaborative decision making situation, despite this competing-agents problem. The problem should be addressed (for example, by building separate what-if models to analyze the effect such hidden goals have on the ultimate decision the group as a whole will take), but should not affect how the situation be modelled (at least, in its base-model).

Similarly, in a situation where the agents are locked-in a competing adversarial conflict, but some agents are showing signs of cooperation or forming an

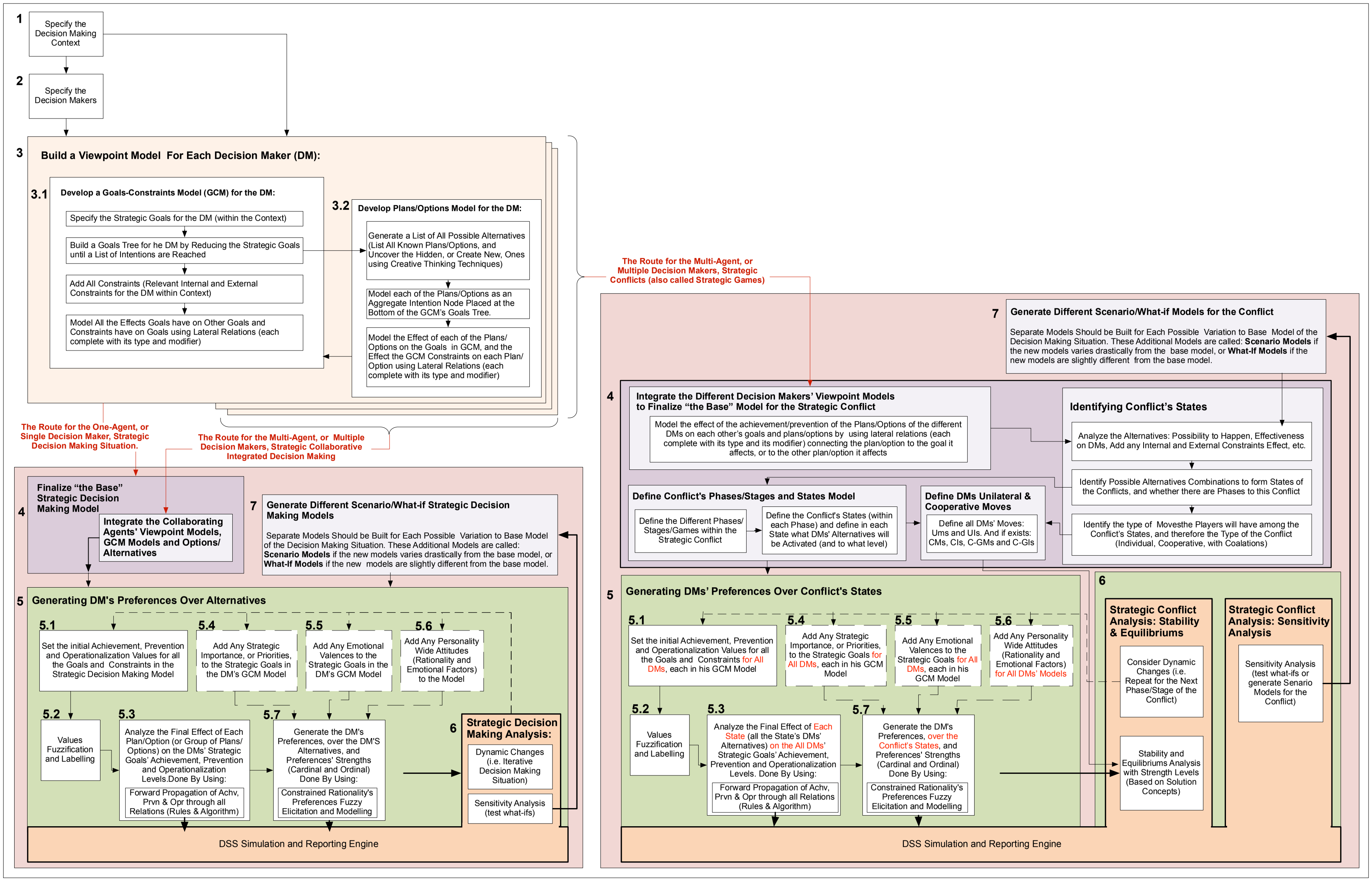


Figure 5.2: The Process of Modelling and Analyzing Single and Multi-Agent Decision Making & Conflicts

alliance, concerned agents/analyst should build what-if models of the conflict to test/validate the motives behind such suggested or expected cooperation. But, the situation itself should still be modelled as a conflict because it embodies all the major characteristics of an adversarial multi-agent decision making situation.

These shades/degrees of competition in collaborative decision making situations, or degrees of collaboration in conflict situations, comes from the perceptions and/or deceptions that the situations' agents have or participate in. It is a knowledge problem: How much we know? Does what we know truly reflect the real world as it is? How much of what we know, believe and perceive is truly justified?

The ViVD-EKM framework's answer to this problem is to compartmentalize knowledge and model perceived or hypothesized knowledge, as well as aggregated or specialized knowledge, as Viewpoint Models. The framework offers knowledge modelling mechanisms to support operations such as aggregation, decomposition, generalization and specialization. We will not dive deep into how these mechanisms work within the ViVD-EKM framework since this is an out of scope topic (Al-Shawa (2006b,a) offers details). But, we will show how viewpoint models can help model collaborative multi-agent decision making situations as well as multi-agent games.

5.3 Modelling Multi-Agent Decision Making Situations

In this section, we will introduce the process to follow when modelling a multi-agent decision making situation, whether it is a collaborative integrated situation or a game/conflict situation. Figure 5.2 provides an illustration of the process. The modelling and analysis process as a whole, and as shown in the figure, has two parts to it: the modelling process part, and the analysis process part.

The section we start by listing first the modelling process a step wise fashion. Then, it will discuss some important modelling concepts and methods used in these steps: the agents' perceived viewpoints and GCM models, and how these models will be integrated to fit the specific needs of the decision making situation and its type (collaborative or competitive); how the agents' alternatives and the conflict's states are defined and modelled; and how the base-model of the situation is validated and finalized. Modelling agents' priorities, emotions and preferences, for both collaborative situations and conflicts, will be discussed separately in the following section.

5.3.1 The Process of Modelling Multi-Agent Decision Making Situations

The process of modelling a multi-agent decision making situation is part of the total process shown in Figure 5.2, and includes the following steps:

1. Specify the context of the decision making situation. This context will serve as guidelines on: what to include and not include in the decision making model; what to consider as relevant and what should be declared as irrelevant; who to include as decision makers; and so on.

2. Identify all the "real" decision makers involved in this decision making exercise.

3. Build a Viewpoint Model for each decision maker. If this decision making situation is a one-agent one, then one viewpoint model should be built with the agent's CGM. And, if this is a multi-agent decision making situation, a collaborative or a conflict one, then multiple viewpoints should be built. One viewpoint model for each of the agents. Each agent's viewpoint must include the agent's GCM, and the agent's alternatives (modelled as intention nodes at the bottom of the agent GCM's goals-tree/s).

4. Validating and finalizing the Base Model. At this step, different decision making situational modes take different routes:

   (a) *For a Single-Agent Decision Making Situation*, finalizing the base-model means verifying that all the agent's relevant goals and constraints are captured, the alternatives are all elicited, and the reduction and lateral relations are all accurately represented. Any uncertainties or variations that need to be tested should all be dealt with at the sensitivity and what-if analysis stage.

   (b) *For a Collaborative Multi-Agent Decision Making Situation*, finalizing the base-model means completing the following steps: 1) verifying that all the agents relevant goals and constraints are captured within their respective viewpoints' GCMs, and that all reduction and lateral relations are accurately represented; 2) integrating all agents' viewpoint models and capture the effect of each others' goals and constraints on the rest using lateral relations; 3) ensuring that all relevant known alternatives (products, architectures, designs, specifications, etc.) are captured, and add any new ones the collaborating decision makers (find through creative thinking, brain storming or any other thorough knowledge acquisition process); then 4) connecting the effect of the agents' constraints on the alternatives and the effect of choosing each of the alternatives on the agents' goals using lateral relations. Any uncertainties or variations that need to be tested should all be dealt with at the sensitivity and what-if analysis stage.

(c) *For a Competitive Multi-Agent Decision Making Conflict*, there are multiple steps to be taken: 1) verify that all the agents' relevant goals and constraints are captured within their respective viewpoints' GCMs, and that all reduction and lateral relations are accurately represented; 2) integrate all agents' viewpoint models and capture the effect of each others' goals and constraints on the rest using lateral relations; 3) capture all the relevant known alternatives/options/moves that the involved agents have in their respective viewpoint models, and find any other ones (through a creative thinking process or otherwise) that should be added to the base-model of the conflict; 4) add the effect of each of the agent's alternatives, when and if it is get selected by the agent, on the agent's goals, and add the effect of the agents' alternatives on other agents' goals; 5) define the conflict states (by identifying the realistically possible combinations of the agents alternatives happening at the same point of time during the conflict or happening at all), then identify any phases/iterations the conflict has and finalize the structure of the conflict with the states that exist in each phase/iteration; and 6) define all unilateral and cooperative moves the agents could make between the conflict's states.

There are two more steps that could be part of the modelling steps as well as the analysis steps. Both steps are shown in Figure 5.2, and are as follows: adding the prioritizes, emotional valences, personality wide attitudes toward rationality and emotionality (these values will be used later to generate the agents' preferences over the alternatives, for single-agent or multi-agent collaborative decision making situations, or over the conflict's states, for the multi-agent adversarial conflicts); and modelling the dynamics of conflicts, namely the agents moves and countermoves. We will discuss some aspects of each of these steps below. These steps are shown in Figure 5.2 as part of the boxes numbered (5) and (6), respectively.

5.3.2 Agents' Perceived Viewpoints and GCM Models

We said above that one of the best benefits of the ViVD-EKM conceptual modelling framework is the ability not only to aggregate and generalize knowledge but also to segregate/divide the agent's body of knowledge by subject areas, by involved players, type of knowledge (who, how, what, etc.), and so on. A feature we will use to model how an agent will build a model of a multi-agent decision making environment. Whether this environment is a collaborative environment, where all agents are selecting the best option for all, or an adversarial competitive environment, where each agent has his own options and plans his own moves. We said that the later environment is also referred to as a *Game*, mainly in the Games Theory literature, or as a *Conflict* in the broader management science literature. In our research, we will use the terms Game and Conflict interchangeably to mean the same thing: multi-agent decision making in an adversarial competitive environment.

Let us consider a situation where agent A is currently involved in a political conflict $\mathbb{C}$. Agent A wants to analyze all his options, in light of all what he knows about his ultimate strategic goals and options, as well as what he knows about the other players' strategic goals and options. Agent A will use the Constrained Rationality framework to model conflict $\mathbb{C}$, and analyze all his options in order to come up with the best option to take.

Following the process set in Chapter 3, agent A sat down and modelled his own GCM model. A's GCM model included all his strategic goals, as well as the internal and external constraints which are relevant to conflict $\mathbb{C}$, and what he is hoping to achieve from it. In addition, agent A did a great job reducing the strategic goals, and capture the lateral effect of goals on goals and constraints on goals. He even managed to model the Intentions and Options that he can have, or take. For A, it was not an easy job, but because he "knows" himself really well, he managed to end up with a true complete GCM model (a viewpoint model of himself that contains his Goals and Constraints Model only).

The next step for A, is to identify all the other players (agents or decision makers –recall that both terms are used interchangeably in our research) involved in conflict $\mathbb{C}$. Agent A "knows" that agents B, C and D are the only other players involved in the $\mathbb{C}$ (as far as he "knows"). He started to capture what he believes about each of these players: their goals, constraints and options. In other words, agent A built a separate viewpoint model for each of B, C and D, as he believes that they each see conflict $\mathbb{C}$. Each of these B, C or D viewpoint models will

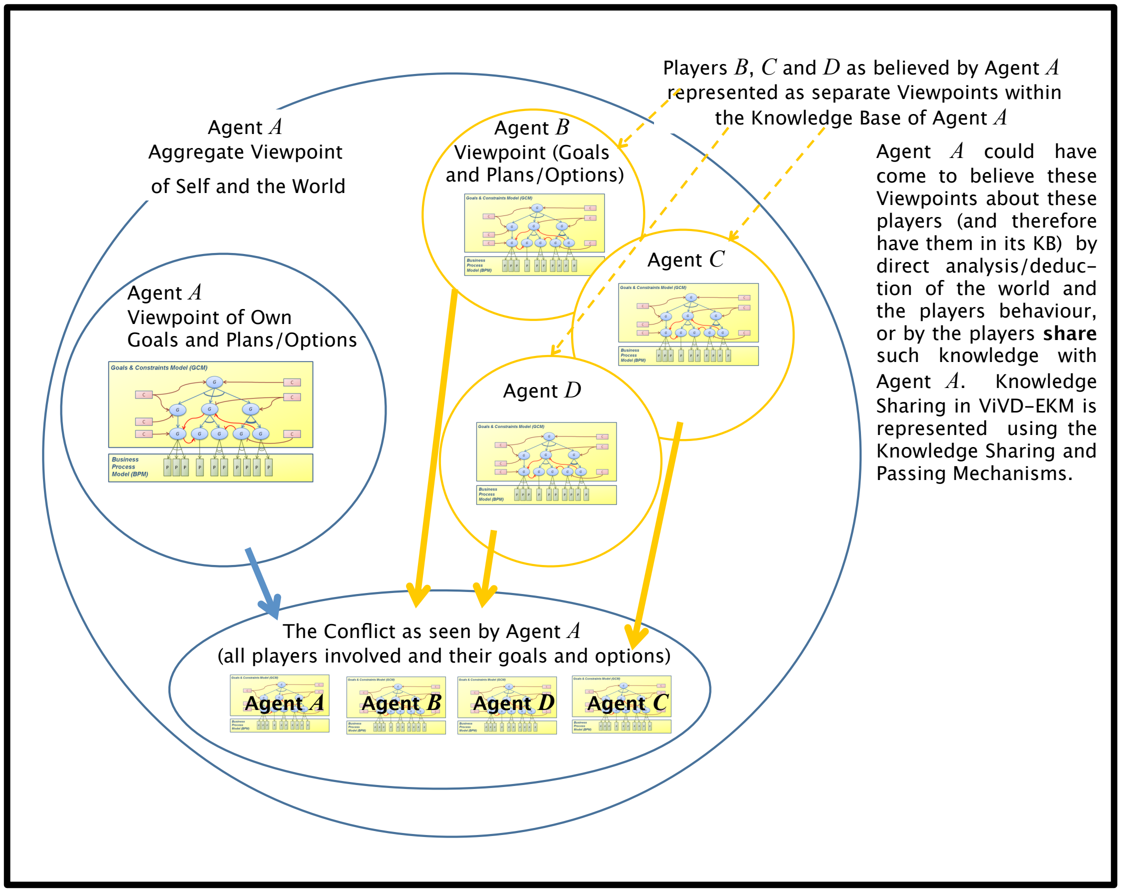


Figure 5.3: An Agent forming an Integrated View of a Game (Decision Making or Conflict Resolution) Environment, by integrating believed Viewpoints Models about all players involved, including itself

have a GCM model for the respective agent and his options. Figure 5.3 shows an illustration of A's work so far. To Agent A, the model he built to represent conflict $\mathbb{C}$ is an integrated viewpoint model that describes $\mathbb{C}$ as he perceives it. Conflict $\mathbb{C}$, to A, has four players (including himself), and each of the four players has his own goals, constraints and options captured in a separate GCM model.

A question may be asked whether Agent B, for example –if she builds a model for conflict $\mathbb{C}$–, will have the same model that Agent A have for $\mathbb{C}$. The answer is: maybe. In reality, it all depends on what agent B believes about conflict $\mathbb{C}$ and its players. If B is not aware, for example, that agent D is a player within $\mathbb{C}$, then B will have a model of $\mathbb{C}$ that does not have a GCM model for D. In her mind, B will have only three GCM models in the conflict $\mathbb{C}$ as she sees it: one for herself, one for A and another for C, as shown in Figure 5.4. The figure shows A has a full picture of $\mathbb{C}$ with all the four players been modelled, while B has a

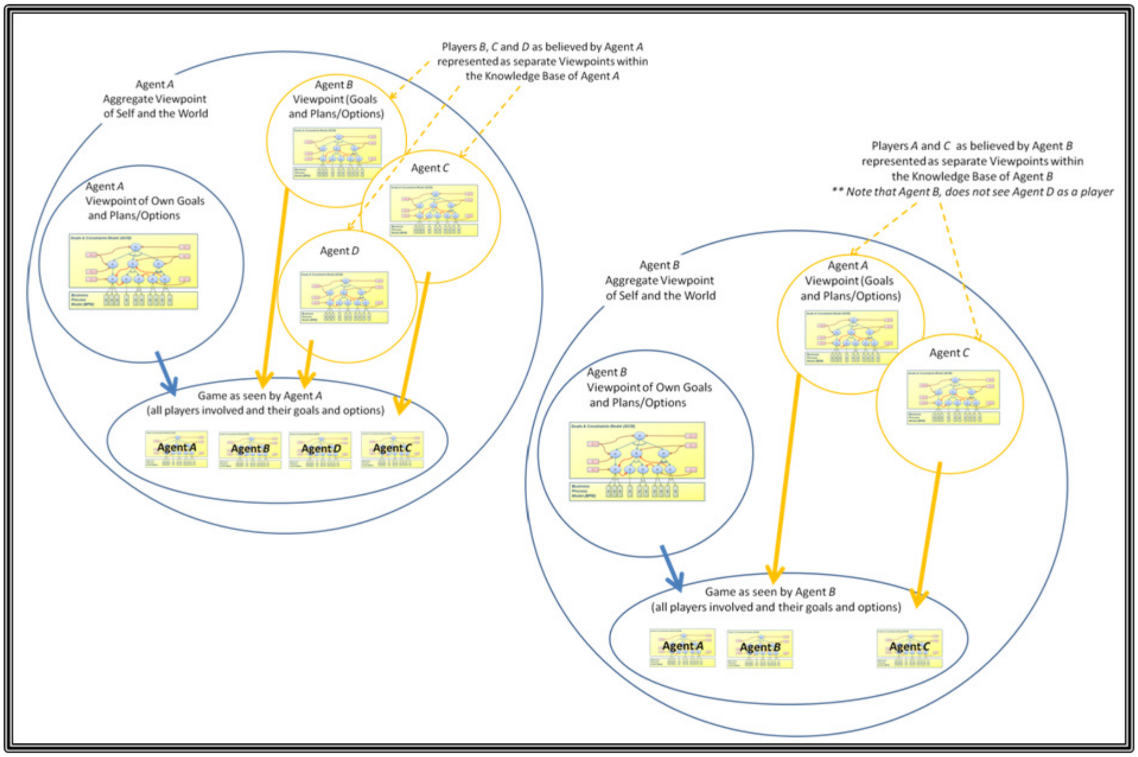


Figure 5.4: A Game as been by Two Different Agents, A and B, where Agent A sees more Game Players than Agent B, and therefore Agent A forms/perceives more Viewpoints about more Players, their Goals and Options than Agent B

less-than-complete model of $\mathbb{C}$. It is also possible that agent B's model for conflict $\mathbb{C}$ is the accurate model, whilst A's model is including a player who is not really part of the conflict. But the fact is still: A believes truly, with justification, that his $\mathbb{C}$'s model is accurate. At the same time, B believes that her model of $\mathbb{C}$ is an accurate model as she "knows" it (i.e. believes truly, with justification). A will play the game as he perceives it, and models it; and B will play the game as she perceives it, and models it.

The number of players involved in conflict $\mathbb{C}$ may not be the only difference between how A models $\mathbb{C}$, and how B models it. B could also have a different "knowledge" about A, and models A's goals and options differently, completely or partially. She may, or may not, know this fact. Similarly, A may, or may not, know this fact. A may have deceived B, leading her to have wrong beliefs about his goals and/or options. Alternatively, it could be that B held these wrong beliefs based on misreading of A's actions, without any deception from A. What matters at the end is that players play the game as they see it, perceive it, know it and model it in their minds.

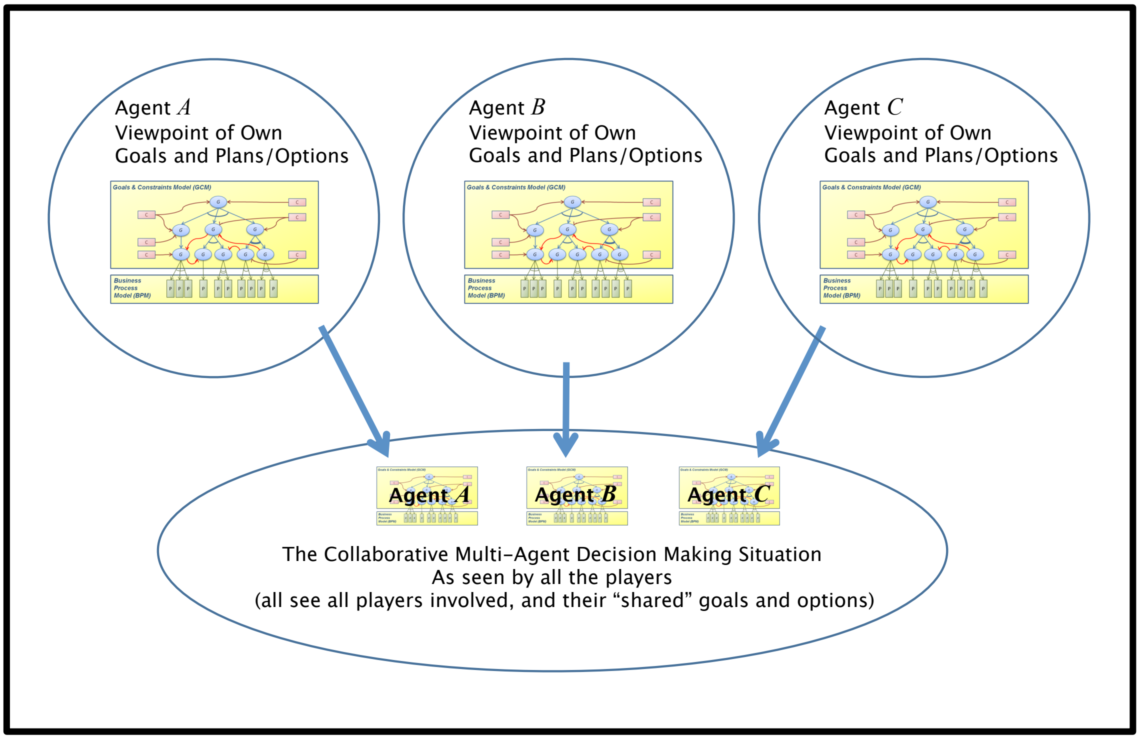


Figure 5.5: A Collaborative Multi-Agent Decision Making Situation, where all agents share their goals and constraints in an effort to reach the best compromise by selecting the most fit option/plan/product/.. to all involved

The literature refers to games, in which one or more of the players are not fully aware of the nature of the conflict situation and therefore players model the same game differently, as *Hypergames* (Bennett, 1977; Wang et al., 1988). In this research work, while we will not discuss hypergames specifically –we are leaving them as a future research topic–, we show how to model hypergames as different game models (each from the perception of one of the participating agents as shown in Figure 5.4) and analyze these models separately. We will also provide illustrative case studies, in the next few chapters, to show how the framework model and analyze these different models and scenarios. Beyond the simple modelling and analysis of hypergames provided in this thesis document, we intend to provide more mathematical comparative analysis models of hypergames in the future.

At the other end of the multi-agent decision making situations is the situation where all the agents collaborate with each other to choose one option (a product, a business process, a set of requirements specifications, etc.) out of many presented to them. They will choose the option that fit the most of what they collectively value and aim for (their goals). In other words, they try to reach a compromise. In this decision making situation, all involved agents will disclose and share their goals and their constraints. A GCM model for each agent is built to capture the agent's goals and constrains. All agents' GCMs are shared among the agents. Figure 5.5 illustrates how the GCMs for the involved agents are shared across. This is unlike a conflict/game multi-agent situation, where the agents are not likely to share their motives and constraints, and therefore build the conflict model with what they each believe to be theirs and others GCMs.

5.3.3 Agents' Alternatives and Conflict's States

After the agents are identified, a viewpoint model for each is built, and the agents' goals and constraints are added to their respective viewpoint models, comes the step of capturing the agents' alternatives. If there is one agent in the decision making situation, then her alternatives will be captured as part of her viewpoint model. The alternatives, which they will be modelled as intention nodes as we said earlier, could be reduced from the refined desire-type goals at the agent's goals-tree/s, or elicited directly from the agent herself.

The alternatives are then connected to the goals and constraints in the agent's GCM. Constraints are connected to any of the alternatives that they affect using C-G lateral relations, and alternatives will be connected to the upper goals that they affect positively or negatively through reduction relations (if the alternatives are reduced from the upper goals) or G-G lateral relations (if the alternatives are added to the viewpoint model by the agent and did not come through reduction from upper goals). This is the same process we used in the previous chapters, for one-agent decision making situations.

In the case of a multi-agent decision making situation, a collaborative or a competitive one, a different modelling process is used. After the real agents involved in the decision making situation are identified, a viewpoint model for each is built, and the GCMs models are added to the respective agents' viewpoint models, then comes the step to integrate all the agents' viewpoint models. This is done by capturing the the positive and negative effects that the different agents' goals has on each other's goals. The integration process is the same for collaborative as well as competitive multi-agent decision making situations, and intended mainly to test and highlight the effect of each agent's GCM on the others' GCMs. Figure 5.6 provides an illustration of how an end product of this integration process looks like.

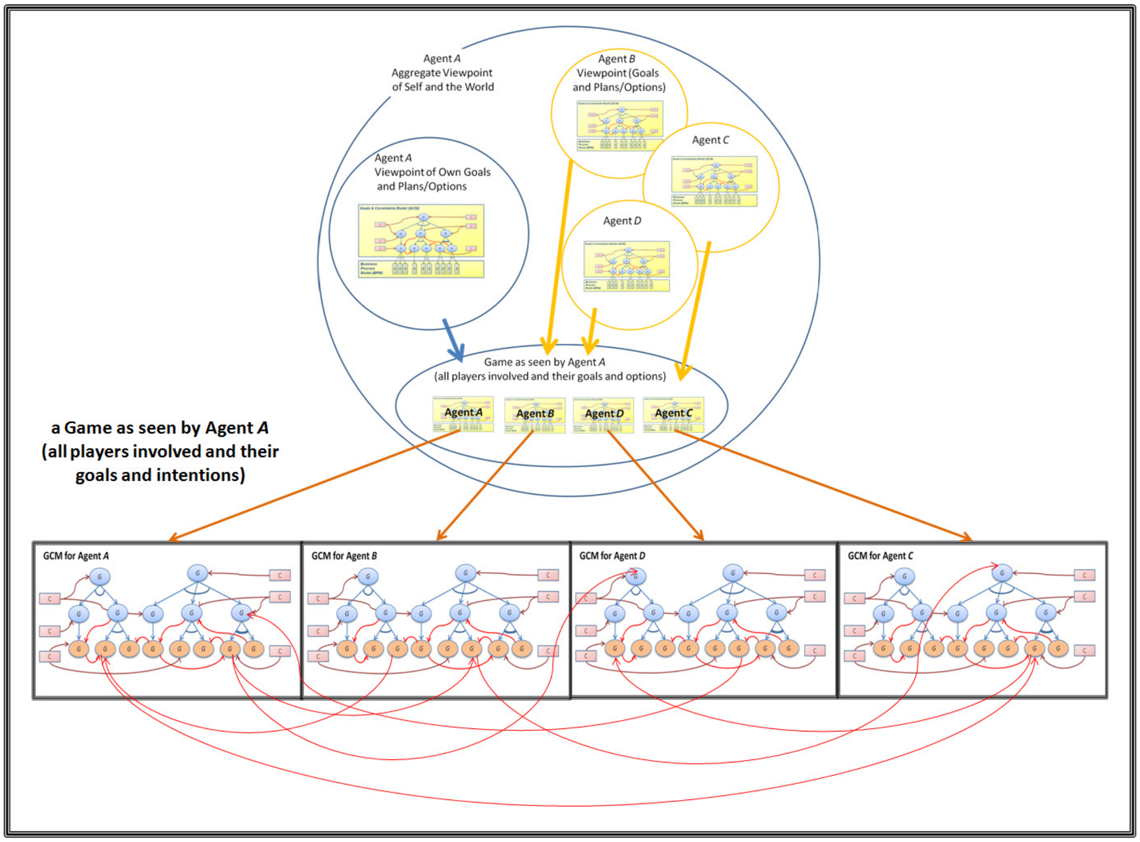


Figure 5.6: Goals (Desires and Intentions) of the different Players' GCM models Interact and affect each other using G-G Lateral Relations

Figure 5.6 shows agent A's model of the decision making situation, after he integrated the different viewpoints of all involved agents in the decision making situation. Agent A modelled the effect of each of the player's intentions on the the other agents' desires and intentions. He modelled such effects using the goal-to-goal (G-G) lateral relations. In the figure, the intentions were modelled to represent the plans/options the agents have, i.e. the model uses the earlier recommendation to further divide/decompose the intentions to form a one-to-one intention-to-plan, making the model cleaner and easier.

In this model, the analyst (or the focal agent, agent A) does not need to worry about the plans. All what he needs to test is how the satisfaction of different intentions (each represent a goal to implement a plan or adopt an option) will affect the satisfaction levels of the ultimate strategic goals of the agents. At the end of this chapter, we will show an example of a collaborative decision making case study, and in the next few chapters we will show the viewpoint integration in a number of competitive decision making situations/conflicts case studies.

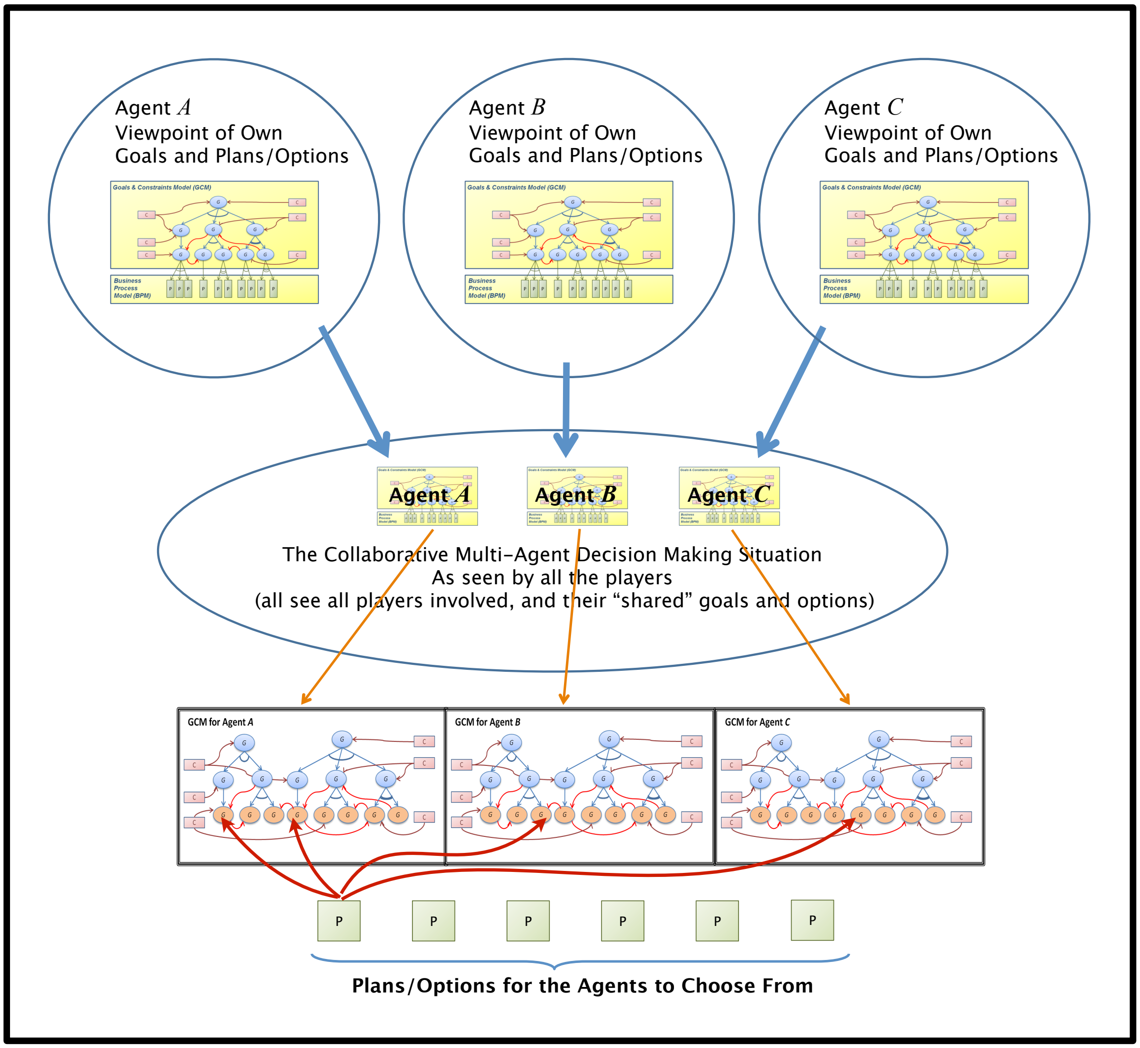


Figure 5.7: Common Alternatives, Options or Plans for all the involved agents, in a Multi-Agent Collaborative Decision Making Situation, to choose from. Alternatives contribute positively or negatively to the agents' goals.

5.3.3.1 Agents' Shared Alternatives in Collaborative Situations

As shown in Figure 5.2, the modelling process divert after the viewpoints integration step based on whether the multi-agent decision making situation is a collaborative one or a conflict (competitive) one. In collaborative situations, the agents, or the analyst helping them, should capture all the "shared" alternatives. Recall that the agents in these situations are supposed to choose from one of many alternatives that they all collectively have. The agents here do not have to decide separately on some alternatives that they need to choose from, away from the other agents (if this is the case, then they each have to consider their decision making situation as a single-agent situation, or alternatively they can model this as a collaborative multi-agent situation but must package the alternatives as sets as we will discuss below). Again here, and as before, alternatives could come: through reduction from lower goals in the agents' goals-trees; by directly eliciting these alternatives from the agents; and/or by discovering/uncovering them through the use of a creative thinking process.

After capturing the shared set of alternatives, as collective intention nodes shared among all the agents' viewpoints (part of the decision making aggregate base viewpoint model), the alternatives are connected to the agents' GCM models. The effect of each alternative on the agents' goal nodes is modelled using G-G lateral relations, and the effect of agents' constraints on the different alternative nodes is modelled using C-G lateral relations. Figure 5.7 provides an illustration of a model of a collaborative decision making situation, where the involved agents have a shared set of alternatives. Alternatives are connected to the goals of the different involved agents showing the effect on these gaols if the alternative get selected by the group (in the figure we show only one alternative connected to the agents' goals for clarity and presentation purposes only, we also eliminated the inter-viewpoint effects for the same reasons).

The shared alternatives modelling concept works perfectly in situations where the agents are required to decide on one of many options. Examples of such situations include: new product screening initiatives, where agents are required to decide on one of the products to invest in (design, produce, market, ..); product/system selection initiatives, where decision makers are required to choose the best fit for their organization from many offered by different vendors; and so on. In such situations, each of the products or the systems is represented as a shared alternative node (a shared intention node in the integrated viewpoint model for the decision making situations).

The shared alternatives modelling concept, also, works perfectly for situations where the collaborative agents are required to choose one of many alternatives, but these alternatives are more complicated in their structure than a simple product or a system. Consider, for example, a requirements engineering or a product development initiative. The decision makers here are required to chose one of many designs/specifications proposed. Each of these designs is a collection of features or specification statements. Will such alternatives need to be modelled differently? The answer is: No. Each of these "collective" designs, architectures, or specifications are represented as an alternative node (modelled as an intuition to select this option –as discussed before–). In other words, for each design, we package all its features, sub-systems, etc., and model it as an alternative. Then, we connect the alternative to the agents' goals using G-G lateral relations, showing the effect of choosing this alternative on the agents' wants and needs.

What if the alternatives are a set of different plans of actions, that the agents individually must commit to, will that make any difference? Again, not at all. Each of the alternatives, in such collaborative multi-agent situations (such as collaborative scientific research or industrial R&D initiatives), is modelled as one node. The node represents a complete coherent plan of actions. The items within coherent plan represent actionable items that individual agents have agreed to be responsible for. Variations of such plans will be modelled as completely separate new alternatives. The effect of choosing each of these alternative/plans is modelled using G-G lateral relationships, where the type of each relationship and its fuzzy label define the type of the effect (positive or negative) and the degree of the effect which choosing the plan has on the agents' wants and needs.

5.3.3.2 Defining the States of Conflicts

Now, back to the point of divergence which the modelling process has just after the viewpoints integration step. If the the multi-agent decision making situation is a competitive adversarial one (a conflict/game), then the analyst should capture the alternatives that each of the agents has. The alternatives in such situations are individual choices, options, stands or moves. Again here, for each agent, these alternatives could be the product of a reduction process (reduced from the lower goals in the agent GCMs' goals-tree/s), or elicited directly from the agent through a standard or creative knowledge acquisition process.

We said earlier that games, or conflicts as we also call them, are more dynamic than the collaborative multi-agent decision making situations. In the collaborative situations, the agents move together. They all choose one alternative at the same time. If they need to choose some other alternative later, then they will all move to choose another alternative after analyzing all their options. And, if their previous choices affect what they can choose later, then the agents need to consider modelling their alternatives as plans of actions as indicated earlier, or a series of point collaborative multi-decision making (as we will discuss later in Section 5.5 when the topic of modelling the dynamics in conflicts is discussed). Collaborative situations' dynamics can be simply dealt with, as they do not come close to the complexity

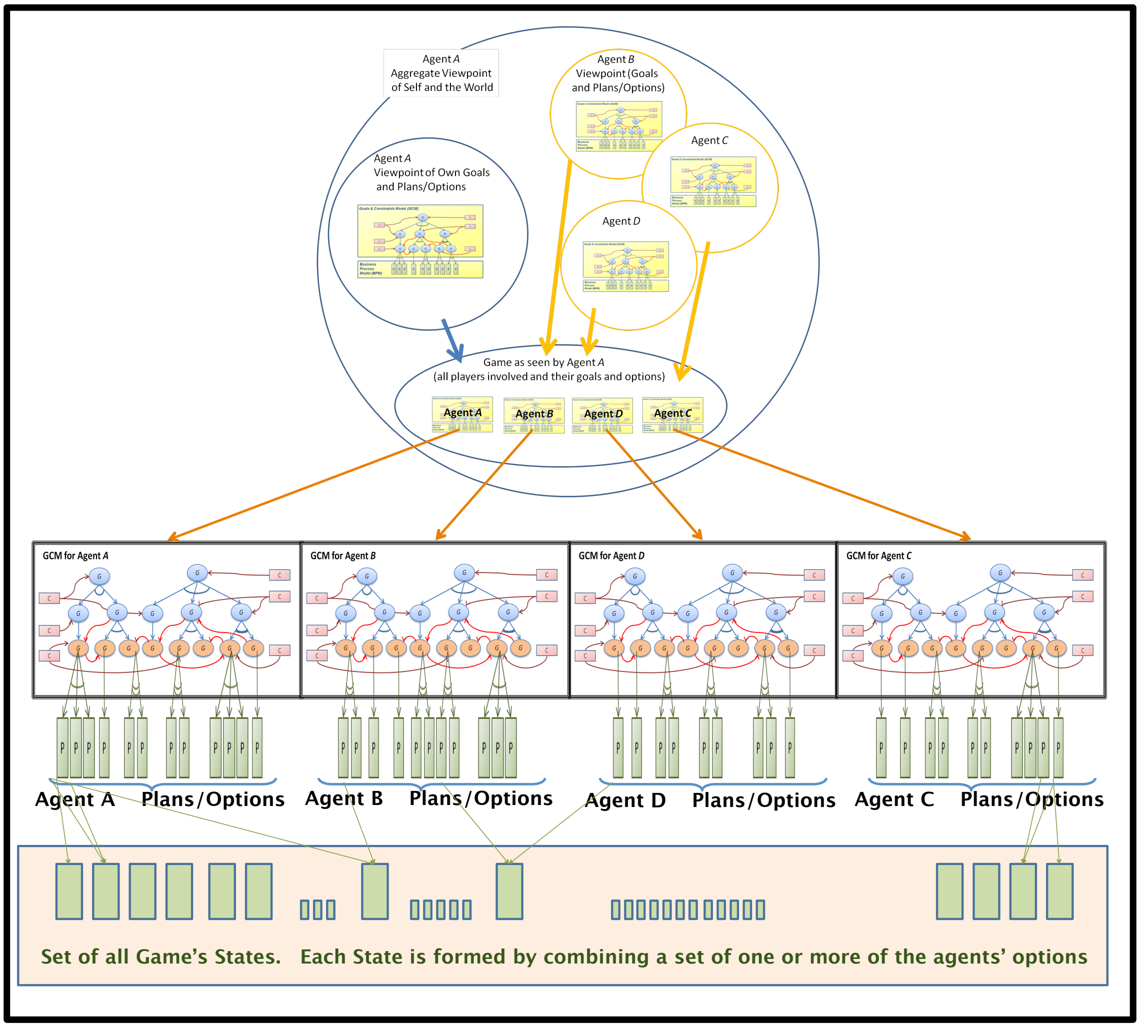


Figure 5.8: Players' Plans/Options, as extracted from how the different Players will be able to operationalize their Goals/Intentions

of completive conflicts' dynamics. This is because the agents' decisions on what their next moves should be depend completely on: their current positions in the conflict is; the current positions of their opponents; and the moves their opponents can make.

We said earlier that dynamics of conflicts could be best served if modelled as state machine, where each state represents a set of alternatives that the agents are expected to already have selected/applied, before the conflict can be declared to be in this state. The transitions/arcs represents moves or decisions by one or more agents to select/apply other alternatives (i.e. moving to another states).

In game theory literature, the players in a game/conflict are considered to be individual decision makers, or agents, who have various available actions (we call them alternatives) which jointly determine the outcomes (or as we call them here states) of the conflict. These possible actions/alternatives, and their relationship to the outcomes/states, are modelled differently by different game forms: the normal form (the most commonly used representation since von Neumann and Morgenstern (1953) proposed it), the option/binary form (proposed by Howard (1971)) and the graph form (first proposed by Fang (1989); Fang et al. (1993)).

The most advanced representation form for multi-agent conflicts is the graph form, as it captures the dynamics of conflict and how players move from one state to another in it. But despite their differences, these represenation forms share two important concepts, and consider them essential to any game representation: the individual actions (alternatives, options,..) of the decision makers involved in the game; and the outcomes/states of the game (each is represented as a set of the individual and joint actions that could be taken by the decision makers at a single time in the game during its evolution or when it ends).

We have seen how the different actions (planes, options or alternatives) are represented and integrated in the GCM models of agents. In Chapter 4, we showed how to: formally represent alternatives as intention goal nodes; connect them to the different goals and constraints in the agent's GCM model; and calculate the agent's cardinal and ordinal preferences over them based on how much each of these alternatives contribute to the strategic goals of the agent (given how important each goal to the agent and how does the agent feel about each goal). But, we did not yet formally define the concept of a state for a game/conflict. We will define it here.

For a set of decision makers $\mathcal{DM}$, where each $DM_i \in \mathcal{DM}$ is an involved player in a competitive decision making situation (such situation is also called in this research as a game or a conflict), the state s of the game is a vector that captures information about what alternatives/options have been adopted by the game players at a certain time t in the game, and what alternatives have not been adopted by them at time t. Let s be represented as a vector of $Achv(A)$ values for all the alternatives available in the game, and where the different $Achv(A)$ values show the degree of adoption/activation/implementation of each alternative at the time the sate is been reached in the game.

In other words, for a game with n alternatives, a state in the game is defined as: $[Achv(A_1), Achv(A_2), \ldots, Achv(A_n)]$ where $Achv(A_i)$ represents the level of implementation/achievement the alternative A_i gets at the time the state is reached in the game. For example, $s_1 = [F, N, N, N]$ represents a state of the game where there is a *Full* adoption of alternative A_1 (by the decision maker/s who can adopt A_1) only out of all the four alternatives available for all players in the game; and $s_2 = [F, N, F, N]$ represents a state of a game where two alternatives A_1 and A_3 are *Fully* adopted by the game's players who are responsible for (can adopt) these alternatives, while the other two alternatives A_2 and A_4 are not selected/adopted by the players who are responsible for (can adopt) them.

This representation, while modified to fit the concepts and needs of the Constrained Rationality framework and its modelling and reasoning tools, it is equivalent to the binary number/form representation suggested for game's states by many authors in the game theory and conflict analysis literature (such as Howard (1971), Fraser and Hipel (1984) and Fang et al. (1993)). But, whilst the binary representation of each of the game's states is mathematically attractive and sound, it provides a less-effective and more-costly representation to the game's states, especially for a game that have many alternatives. In addition, it suggests a very large set of states for the game, with states covering all possible combinations of the alternatives in the game. So, if a game has 10 alternatives, then the game could possibly have 2^{10} states (if we only consider two values of *None* and *Full*, or 0 and 1, for each alternative's achievement/adoption value $Achv(A)$). Definitely, there will be much more possible states if each of these states is represented as a vector of 10 fuzzy labels, or defuzzified numbers.

In our research, we adopt an equivalent representation, which is not only simpler to present and understand, but also more powerful and more computationally efficient to use. For a game with a total set of alternatives $\mathcal{A}$ that includes all the alternatives available to all agents involved in the game., we represent each state s as a set of $Achv(A_i) = L_{achv}$ statements for each $A_i \in \mathcal{A} : Achv(A_i) > N$ at the time state s is reached in the game. For example, state s_1 above will be represented as $\{Achv(A_1) = F\}$, and state s_2 as $\{Achv(A_1) = F, Achv(A_3) = F\}$. And, while this representation for game's states allows for an alternative A to be included in the state's set definition with $Achv(A) = L_{achv}$ and $N < L_{achv} \leq F$, we will use, in this thesis document and the examples presented in it, only states with alternatives that has their $L_{achv} = F$, i.e. fully implemented. We plan in the future to present examples that make use of the full spectrum of implementation/achievement levels that alternatives can have in a state's definition (such as allow states defined with alternatives half implemented). This state representation provides a flexible and expandable form to represent conflict's states.

Figure 5.8 provides an illustration of how the individual agents' alternatives are the basis of defining the total absolute set of states that the conflict can have. But in most conflicts, only a handful of these states are logical or possible to have. The rest are theoretical, but not logical or possible. We will show, in the next few chapters, how real-life conflicts we used in our case studies, despite the fact that they could have hundreds of states theoretically, only few states are possible and/or logical to have in the conflicts.

It is the analyst responsibility to remove all illogical and/or impossible states, or states that are not likely to occur within the context of the conflict (maybe add those as part of what-if models during the sensitivity analysis). The remaining states from the all-possible-combinations-of-alternatives set of states form the actual set of the conflict's states. In most case, very small number of states is relevant, possible or logical to include in the set of states a conflict has. The rest should be ignored by the analyst. In fact, our experience, and the case studies we studied and presented in the next chapters, show that only few states realistically make it to the set of states for for each real-life conflict studied.

5.3.4 Validation and Finalization of the Base Model

So far, we have the agents' viewpoints modelled then integrated. In addition, if the decision making situation is a collaborative one, the shared alternatives and their individual effects on the agents' goals are captured. And, if it is a competitive situation, a conflict, we have the conflict's states identified. At this time, it is important to validate what has been modelled so far, ensuring that it is as accurate as possible. Recall that in the ViVD-EKM framework, which the Constrained Rationality framework builds on, knowledge is defined as justified true beliefs. This means that the analyst not only need to capture the beliefs acquired from the decision makers, or the agent whom the conflict model is built based on his perceived reality, but also need to capture the justification for such beliefs. This will ensure that the agent, and the analyst, can at any time refer to who provided this information and what was the justification for believing its truthfulness at the time it was captured (added to the knowledge base).

In the validation step, the analyst should go through all the elements captured so far and ensure its accuracy. By the end of this step, the analyst is expected to finalize a base-model for the decision making situation, and generate a list of uncertainties and/or what-ifs to test and analyze. This list will be the basis for the sensitivity analysis stage of the process, and any what-if models the analyst should analyze at it.

5.4 Priorities, Emotions and Agents' Preferences over Alternatives or Conflict's States

In Chapter 4, we showed how an agent's priorities are captured as strategic importances fuzzy labels attached to the strategic goals the agent's has. We also showed how emotions, more specifically the degrees of like or dislike, the agent have towards working at and achieving the various strategic goals he has, are also attached to such goals as emotional valences fuzzy labels. In addition, we showed how the analyst can capture the overall attitudes the agent shows, specifically his general overall rationality and emotionality levels. We showed how the ordinal and weighted preferences are calculated for each of the agents over their respective individual alternatives. Preferences over alternatives are sufficient if the modelled and analyzed decision making situation is a single-agent decision making situation, or a multi-agent situation but the agents in it act individually with no regard to others' moves and decisions.

Preferences over individuals' alternatives could be used in multi-agent decision making situations, only to test the effectiveness of the individual alternatives on the agent himself, but not to guide the agent for the best plan of action in what is usually a dynamic situation. In this subsection, we will introduce how the collective preferences of decision makers, in a collaborative multi-agent decision making situation, over their shared alternatives are calculated. In addition, we will discuss how decision makers' preferences over the states of a multi-agent competitive decision making situations (games or conflicts, as we also call them) will be calculated.

But, before we show how the decision makers' preferences will determines in multi-agent decision making situations, we would like to point out here the importance of the care that the analyst must exhibit at this step of the modelling process. It is usually the case that justifications of beliefs about naturally fuzzy values (such as rationality, emotionality, importance, emotional valence) are very difficult to validate. Even if these values are elicited from the involved agents themselves, through a direct knowledge acquisition process, the analyst should ensure adequate sensitivity analysis been done. In addition, the analyst should check for any variations to these values based on what phase/iteration the game is in, if the game has multiple phases/iterations. Such variations, if exist, will surely affect the players' preferences, the stability of the conflict's states to the players, and ultimately affect which state forms an equilibrium to the conflict.

The analyst should test different variations of these values, and/or build complete what-if models (variations to the base-model) in order to highlight any implication on the agents' preferences and the stabilities of the conflict's states as a result of such variations. The extra care in conducting sensitivity analysis, especially for studying the implications of such variations, will serve two purposes: 1) keep the analyst on the alert, looking for any signs of changes in these variables, when and if such changes occur; and 2) point to the involved agents the importance to disclose the real beliefs and justifications for any of these values, and report any changes should some occur. The final analysis, stabilities, equilibriums and results, could depend on these values. They are important factors that impact how the agents' preferences are formed and calculated.

5.4.1 Agents' Preferences in Multi-Agent Collaborative Decision Making Situations

As indicated earlier, the decision makers involved in a multi-agent collaborative decision making tend to take decisions collectively, choosing the best one alternative that meets most of the decision makers goals. This one alternative will be chosen from a set of shared alternatives that the decision makers collectively have. So, how do the decision makers are supposed to rank their alternatives, or generate preferences over these shared alternatives? Answering this question, we will follow here the same steps we followed in Chapter 4, when we discussed generating the preferences one agent has over his own alternatives.

5.4.1.1 Identifying the Agents' Strategic Goals and Shared Alternatives

For each decision maker $DM_i \in \mathcal{DM}$, where $\mathcal{DM}$ is the set of all decision makers who are involved in the collaborative decision making situation, at time t, let their be a set of strategic goals $\mathcal{SG}_{DM_i}$ (chosen by DM_i). Let, the set of all strategic goals by all decision makers in $\mathcal{DM}$ is called the strategic goals of the collaborative decision making situation, and is denoted as $\mathcal{SG}$, where $\mathcal{SG} = \bigcup_{DM_i \in \mathcal{DM}} \mathcal{SG}_{DM_i}$. And, let the decision makers in $\mathcal{DM}$ collectively decided on a set of shared alternatives $\mathcal{A}$ to choose one from, based on how much each alternative contributes to the achievement of all strategic goals in $\mathcal{SG}$.

For each collaborative decision making situation, there are two important sets to be set up-front: 1) $\mathcal{SG}$: the set of all strategic goals, of all involved DMs in the situation; and 2) $\mathcal{A}$: the set of all shared alternatives that the DMs will choose one from. It is important for the DMs to disclose their strategic goals, if they want their goals be accounted for in the evaluation of the different alternatives. Similarly, all alternatives that the DMs believe collectively, or individually, to be serious alternatives to choose from must be included in $\mathcal{A}$.

5.4.1.2 Calculating the Final Achievement Values of Strategic Goals

The contribution of the alternatives in the shared-alternatives set $\mathcal{A}$ to the strategic goals' achievement will be measured for the each of these goals when each of the alternatives is applied/implemented. In other words, for each alternative $A_k \in \mathcal{A}$, we set the $Achv(A_k) = F$ then we run the value labels forward propagation algorithm (Algorithm 3.1), and then calculate the Final Achievement value for each strategic goal $SG \in \mathcal{SG}$:

$$FAchv(SG) = \begin{cases} Achv(SG) \ominus Prvn(SG) & \text{if } Achv(SG) \leq Opr(SG) \\ Opr(SG) \ominus Prvn(SG) & \text{if } Achv(SG) > Opr(SG). \end{cases} \tag{5.1}$$

Two notes about this equation. First, the "$\ominus$" operation used here is the same one defined in Section 4.3.1 in the previous chapter, and the fuzzy linguistic value labels assigned to $FAchv(SG)$ are the same value labels assigned in that section to the $FAchv(SG)$ calculated there (for one agent situations) with the same fuzzy sets defining these labels. Second, this final achievement value calculation is similar to the one we used in Chapter 4 for one-agent decision making situations. But here, the strategic goals do not belong to a single DM. They belong to one of many DMs involved in a multi-agent decision making situation. Also, the calculation of the final achievement values is done not after a DM chooses one of his own alternatives, but rather after all the DMs collectively choose one of the shared-alternatives they have.

5.4.1.3 Modelling the Strategic Importance, Emotional Valences, Rationality and Emotionality Factors

In collaborative decision making situations, DMs are not only allowed to set the strategic importance of their strategic goals, but are encouraged to do so. This is to ensure that important strategic goals of the different DMs are not treated as the less-important or the not-important-at-the-moment strategic goals. And, while most multi-agent decision making situations, especially the collaborative ones, tend to have the DMs act rationally with no consideration to emotions, or at least try hard not to consider emotions, research show that emotions in group decisions are part of the environment and their levels are as high or higher than their levels in single agent decision making situations –because of the interactions and the egos– (Barsade and Gibson, 1998; Bartel and Saavedra, 2000; Totterdell et al., 1998; Kelly and Barsade, 2001; Goleman, 1995; Mayer et al., 1999; De Bono, 1985, 1992).

Therefor, the Constrained Rationality framework allows for the individual DMs to attach emotional valences to their respective strategic goals; and allows for their personality wide attitudes (towards acting rationally and/or emotionally) to be captured as well. To do so, we follow the same steps and use the same value properties used in Sections 4.3.2, 4.3.3 and 4.3.4 to add the strategic importance $SImprt(SG)\ \ \forall SG \in \mathcal{SG}$, add the emotional valence $EVlnc(SG)\ \ \forall SG \in \mathcal{SG}$, and add both factors, the Rationality Factor RF_{DM_i} and the Emotionality Factor EF_{DM_i}, for each $DM_i \in \mathcal{DM}$. These values will have the same fuzzification scheme, sets and linguistic value labels used before for the same value properties in single-agent decision making situations.

When these values are used in the models, they must be used consistently. The same fuzzification scheme and value labels scheme, for each value property, must be used across all the agents. No two agents should be allowed to have different schemes to label their respective strategic goals. And, when one agent sets emotional valences and the others do not, for example, the modeller must encourage the agents who did not add their emotional valences to their goals to either: add their emotional valences; or, at least, apply the Emotionally Indifferent (EI) value label for their strategic goals.

In some of the industrial cases we have worked on, we have experienced a decision making situation in which the situation's model integrated two viewpoints: one viewpoint model for the targeted market segment (the product's potential customers), and the other viewpoint model is a collective/aggregate one for the company's different departments. While the company decided to ignore emotions and emotionality in their viewpoint, they decided to model them for the prospective customers. In such case, the modeller must set the company's Rationality Factor to be *Full*, and each of their strategic goals to have an emotional valence of Emotionally Indifferent. Consistency across the model must be maintained at all times in order to ensure reliable results. This is should not be seen as a strict requirement, but rather as a common sense requirement to ensure validity of the models and the results.

5.4.1.4 Eliciting Agents' Collective Preferences over Shared Alternatives

Each of the individual decision makers involved in the collaborative decision making situation can still use the same method and equations used in Section 4.4, in the previous chapter, to determine his own preferences over the shared alternatives, considering only how much each of these shared alternatives contribute to his, and only his, strategic goals. But here, we will modify that method and equations to determine the agents' collective preferences over the shared alternatives they have, considering how much each of these shared alternatives contributes to all the strategic goals that all DMs have in the situation.

For each decision maker $DM_i \in \mathcal{DM}$, at time t, let the *Weighted Final Achievement* of a strategic goal $SG \in \mathcal{SG}_{DM_i}$, where $\mathcal{SG}_{DM_i} \subseteq \mathcal{SG}$, as a result of having alternative $A \in \mathcal{A}$ been adopted collectively by $\mathcal{DM}$, to be denoted as $WFAchv(SG, DM_i, A, t)$, and calculated algebraically as follows:

$$WFAchv(SG, DM_i, A, t) = \begin{cases} W(SG, DM_i, t) \cdot FAchv(SG, A, t) & \text{if } W(SG, DM_i, t) \geq 0 \\ 0 & \text{if } W(SG, DM_i, t) < 0. \end{cases} \quad (5.2)$$

where:

$$W(SG, DM_i, t) = (RF^*_{DM_i} \cdot SImprt^*(SG)) + (EF^*_{DM_i} \cdot EVlnc^*(SG)) \quad (5.3)$$

where $SImprt^*(SG)$, $EVlnc^*(SG)$, $RF^*_{DM_i}$ and $EF^*_{DM_i}$ represent the defuzzified values of their respective fuzzy values, and where none of the original fuzzy values is $Null$, and all reflect the state of mind and beliefs of DM_i at time t

$$FAchv(SG, A, t) = [FAchv^*(SG)]_{\text{ if } A \text{ was fully applied to the GCM model at time } t-1} \quad (5.4)$$

where $FAchv^*(SG)$ represents the defuzzified values of $FAchv(SG)$, and $FAchv(SG) \neq Null$; and where "A was fully applied at time $t-1$" means that the intention to apply A was fully achieved, i.e. $Achv(A) = F$, at $t-1$

For all the decision maker in $\mathcal{DM}$ collectively, at time t of the situation, let the effect of the full joint application of alternative $A \in \mathcal{A}$ on all strategic goals in the none-empty $\mathcal{SG}$ is represented by a *Total Weighted Final Achievement* value property; and let this property value be denoted as $TWFAchv(\mathcal{DM}, A, t)$, and calculated algebraically as follows:

$$TWFAchv(\mathcal{DM}, A, t) = \frac{1}{|\mathcal{SG}|} \sum_{\substack{SG \in \mathcal{SG}_{DM_i} \\ DM_i \in \mathcal{DM}}} WFAchv(SG, DM_i, A, t) \tag{5.5}$$

For the *Total Weighted Final Achievement* $TWFAchv(\mathcal{DM}, A, t)$ value to reflect the effect of alternative A, and only A, on all DMs in $\mathcal{DM}$, then the situation's integrated viewpoint with all its constructs and value properties' values must stay the same, and only A is applied fully. The achievement value of the intention node representing the intention to implement/apply alternative A changes from $Achv(A) = N$ to $Achv(A) = F$. All other alternatives have their respective intentions' achievement values stay the same unchanged, preferably unselected and stay at the *None* level, i.e. $(\forall A_k \in \mathcal{A} : A_k \neq A) \;\; Achv(A_k) = N$. Then, after the values forward propagation algorithm, Algorithm 3.1, finalized the value labels for all goals for time t, we calculate $TWFAchv(\mathcal{DM}, A, t)$. The value of $TWFAchv(\mathcal{DM}, A, t)$, now, reflects the effect of applying alternative A, and only A, on all collaborating decision makers in $\mathcal{DM}$.

We said in Section 4.4 that $WFAchv(SG, DM_i, A, t)$, for a single agent, will be in the range of $[-2, 2]$, if emotionality is captured within the model. Here too in collaboration multi-agent decision making situations, if $(\exists DM_i \in \mathcal{DM} : EF_{DM_i} \neq N) \vee (\exists SG \in \mathcal{SG} : EVlnc(SG) \neq EI)$, then the value of $TWFAchv$ $(\mathcal{DM}, A, t)$ will be in the same range of $[-2, 2]$. On the other hand, if the modeller did not consider modelling emotionality as part of modelling the decision making situation (i.e. $(\forall DM_i \in \mathcal{DM} EF_{DM_i} = N) \wedge (\forall SG \in \mathcal{SG} EVlnc(SG) = EI)$), then $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-1, 1]$, and therefore the value of $TWFAchv(\mathcal{DM}, A, t)$ will be in the range of $[-1, 1]$.

In the case where the modeller includes emotionality in the model, for at least one of the DMs in the modelled decision making situation, then we need to normalize the value of $TWFAchv(\mathcal{DM}, A, t)$ calculated. The new normalized value of $TWFAchv(\mathcal{DM}, A, t)$ is calculated as follows:

$$TWFAchv(\mathcal{DM}, A, t)\Big|_{\text{normalized}} = \frac{1}{2|\mathcal{SG}|} \sum_{\substack{SG \in \mathcal{SG}_{DM_i} \\ DM_i \in \mathcal{DM}}} WFAchv(SG, DM_i, A, t) \tag{5.6}$$

Now, for all DMs in $\mathcal{DM}$, collectively, in a multi-agent collaborative decision making situation, at time t, let the *Cardinal Preference* that $\mathcal{DM}$ has over alternative $A \in \mathcal{A}$ be represented as a *Weighted Payoff* value property attached to A, and be denoted as $WP(A, \mathcal{DM}, t)$. Let $WP(A, \mathcal{DM}, t)$ have a numerical value in the range of $[-1, 1]$ and calculated as follows:

$$WP(A, \mathcal{DM}, t) = TWFAchv(\mathcal{DM}, A, t)\Big|_{\text{normalized}} \tag{5.7}$$

$$= \frac{1}{2|\mathcal{SG}|} \sum_{\substack{SG \in \mathcal{SG}_{DM_i} \\ DM_i \in \mathcal{DM}}} WFAchv(SG, DM_i, A, t) \tag{5.8}$$

Recall that $TWFAchv(\mathcal{DM}, A, t)$ does not need any normalization, if the modeller did not consider emotionality as part of modelling the decision making situation (i.e. $(\forall DM_i \in \mathcal{DM} EF_{DM_i} = N) \wedge (\forall SG \in \mathcal{SG} EVlnc(SG) = EI)$). This is because $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-1, 1]$, and therefore the value of $TWFAchv(\mathcal{DM}, A, t)$ will be in the range of $[-1, 1]$. But, if emotionality is captured within the model (i.e. $(\exists DM_i \in \mathcal{DM} : EF_{DM_i} \neq N) \vee (\exists SG \in \mathcal{SG} : EVlnc(SG) \neq EI)$), then the value of $WFAchv(SG, DM_i, A, t)$ will be in the range of $[-2, 2]$, and therefore the value of $TWFAchv(\mathcal{DM}, A, t)$ will be also in the range of $[-2, 2]$. In this case, normalization is needed to maintain the values of $TWFAchv(\mathcal{DM}, A, t)$ and $WP(A, \mathcal{DM}, t)$ within the range $[-1, 1]$. Hence, the devision by 2 shown in 5.8.

Based on the cardinal preferences, or weighted payoffs calculated for $\mathcal{DM}$ over each of their shared alternatives in $\mathcal{A}$, $\mathcal{DM}$ will have a *Preference Vector* $Pref(\mathcal{DM}, \mathcal{A})$ showing the order of the alternatives in $\mathcal{A}$ from the *most preferred* to the *least preferred*. It is assumed here that $Pref(\mathcal{DM}, \mathcal{A})$ represents only ordinal ranking of the alternatives in $\mathcal{A}$ based on how much each of these alternatives contributes to the achievement of all $\mathcal{DM}$'s strategic goals, given the importance weight and emotional valence that each DM_i assigned to each of his strategic goals, and given a specific rationality factor RF_{DM_i} and emotionality factor EF_{DM_i} describing the attitudes each DM_i exhibits towards acting rationally or emotionally at that point of time.

The preference order of a specific shared alternative $A \in \mathcal{A}$, collectively, to $\mathcal{DM}$ at time t, is given as an *Ordinal Preference* value property attached to A, and is denoted by $OP(A, \mathcal{DM}, t)$. Let $OP(A, \mathcal{DM}, t)$ be given an integer number that reflects A's position in $\mathcal{DM}$'s *Preference Vector* $Pref(\mathcal{DM}, \mathcal{A})$ at that point of time. The smaller the integer number assigned to $OP(A, \mathcal{DM}, t)$, the more preferred the shared alternative A is, to $\mathcal{DM}$ at time t. The alternative that has $OP(A, \mathcal{DM}, t) = 1$ is considered the most preferred shared alternative, and the one with $OP(A, \mathcal{DM}, t) = |\mathcal{A}| - 1$ is the least preferred shared alternative. This is because the alternatives in the $Pref(\mathcal{DM}, \mathcal{A})$ vector are ordered from the alternative with the highest weighted payoff value for $\mathcal{DM}$, to the one with the lowest weighted payoff.

$$OP(A, \mathcal{DM}, t) = n + 1 \quad \text{where } 0 \leq n \leq |\mathcal{A}| - 1 \quad \text{and } n \text{ reflects } A\text{'s position in } Pref(\mathcal{DM}, \mathcal{A}) \tag{5.9}$$

5.4.1.5 Modelling the Strength of DMs' Collective Preferences over their Shared Alternatives

To capture the strength of alternative A's collective preference over all other shared alternatives, for all DMs in $\mathcal{DM}$, we use the same distance measure we used in Section 4.5 for the strength of an individual DM's preferences. This distance measure captures the difference between the cardinal preference of A and the cardinal preference of another alternative in the set of shared alternatives $\mathcal{A}$ for the collaborative decision making situation at hand. Based on this distance measure a binary relation among each pair of alternatives in $\mathcal{A}$ is assigned.

For the set of all DMs in $\mathcal{DM}$, and at time t of the collaborative situation, let the distance measure among the two preferences which $\mathcal{DM}$ collectively has over the two alternatives A_a and A_b, both in $\mathcal{A}$, be denoted as $d(A_a, A_b, \mathcal{DM}, t)$. And, let its value be given as a real number calculated as follows:

$$d(A_a, A_b, \mathcal{DM}, t) = [WP(A_a, \mathcal{DM}, t) - WP(A_b, \mathcal{DM}, t)] \tag{5.10}$$

Because each of $WP(A_a, \mathcal{DM}, t)$ and $WP(A_b, \mathcal{DM}, t)$ has a value in the range $[-1, 1]$, whether normalized as per Equation 5.8 or not normalized (because the modeller did not consider modelling emotionality as part of the decision making situation's model), the distance value will be in the range of $[-2, 2]$. The sign of $d(A_a, A_b, \mathcal{DM}, t)$ shows which shared alternative of the two, that all DMs in $\mathcal{DM}$ collectively prefer.

Let *Preference Strength* be a value property, denoted as $PrefStrngth(A_a, A_b, \mathcal{DM}, t)$, and be given the fuzzified value of the distance value property. In other words, let $PrefStrngth(A_a, A_b, \mathcal{DM}, t) = \underset{\sim}{d}(A_a, A_b, \mathcal{DM}, t)$. $PrefStrngth(A_a, A_b,$

$\mathcal{DM}, t)$ will be assigned a fuzzy linguistic value label L_{PS} based on the fuzzy memberships functions given in Figure 4.4, the same fuzzy sets and labels that are used for the preferences strength of an individual DM over his own alternatives in Section 4.5. The strength expressed by the L_{PS} fuzzy label is meant to represent the distance between the weighted preference values for the two shared alternatives considered here, A_a and A_b.

Also here, as it was the case in Section 4.5 for individual's preferences, let the strength of the collective preference of alternative A_a over alternative A_b, for all DMs in $\mathcal{DM}$ at time t, given in $PrefStrngth(A_a, A_b, \mathcal{DM}, t) = L_{PS}$, be represented by a binary relationship between the two alternatives. Let this binary relation be denoted as $A_a \succ^{L_{PS}}_{\mathcal{DM},t} A_b$. The notation of this relation is the same as the ones shown in Figure 4.4 in Section 4.5, for each possible preference's strength fuzzy linguistic label L_{PS} (shown in Figure 4.4 above each label's membership function).

5.4.2 Agents' Preferences in Multi-Agent Conflicts

The decision makers involved in a competitive multi-agent collaborative decision making situation, a conflict, tend to take their decisions individually. Each DM will choose the best alternative/move that gives the most achievement levels to his strategic goals, given the state of the conflict at the time and what other already chose as their alternatives/moves. An involved DM, in such situation, can define his preferences over his own alternatives/moves, without consideration to others' choices and moves. He can do so by following the steps and equations given in the previous chapter (Chapter 4 for single-agent decision making situations.

But, if the DM does so, he will in effect treat a conflict, a competitive decision making situation, as a situation where the alternatives/moves that other DMs adopt as if they do not matter to him. This is not true, and defies the concept of a "competitive" multi-agent decision making situation (a game or conflict as mostly called in the literature). That's why we said earlier that what matters in conflict situations is not alternatives but rather states. Recall we defined a state as a collection of alternatives that have been adopted/implemented/taken by a number of agents at a single point of time in the conflict. Therefore, what matters for each DM in a conflict is not his preferences over his own alternatives, but rather his preferences over the conflicts states. We will show in this subsection how a DM can generate his own preferences over the conflict's states.

5.4.2.1 Identifying the Agents' Strategic Goals and Conflict's States

In conflicts, as in multi-agent collaborative decision making situations, individual agents have their own strategic goals, though their goals in games are more conflicting and competing. But, their alternatives will not be shared, for the agents to collectively choose one. Each agent will have his own alternatives to choose from. In conflicts,the agents, or the analysts modelling and analyzing them, will individually decide on their strategic goals and their alternatives and model them as part of their own GCM model as explained earlier.

For each decision maker $DM_i \in \mathcal{DM}$,, at time t, let there be a set of strategic goals $\mathcal{SG}_{DM_i}$, and a set of alternatives $\mathcal{A}_{DM_i}$ to chose from. Let the set of all alternatives in the conflict be denoted as $\mathcal{A}$, where $\mathcal{A} = \bigcup_{DM_i \in \mathcal{DM}} \mathcal{A}_{DM_i}$. And, let DM_i decision to choose/implement alternative $A \in \mathcal{A}$ at time t of the conflict depends on how much the state, that includes the implementation of this alternative, contributes the most to the final achievement levels for DM_i's strategic goals, given that others choose to implement their respective alternatives as per the state's definition.

In addition, let the set of all the conflict's states be given as $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where each state $s \in \mathcal{S}$ is defined as discussed above, as a set of $Achv(A_j) = L_{achv}$ value statements (where $N < L_{achv} \leq F$) for each $A_j \in \mathcal{A} : \; Achv(A_j) > N$ at the time state s is reached in the conflict (i.e. the list of all alternatives mentioned in the definition of a state are implemented/taken at the time of s by the agents who are responsible/can-take these alternatives). We said that what matters is the preferences that each decision maker has over the conflict's states, not just his own alternatives. This is because states will also account for the selection/implementation of other DMs in the conflict, at the same time the state is reached, and how much their selected alternatives affect the achievement of DM_i's strategic goals. Furthermore, using preferences over states, rather than individuals' alternatives, allow us to account for the ability of DMs to move between the conflict's states. This gives us the ability to analyze such dynamics, as we will show in the next few chapters.

5.4.2.2 Calculating the Final Achievement Values of Strategic Goals

The contribution of state $s \in \mathcal{S}$ (or more accurately stated: the contribution of all the alternatives that are part of s's definition) to the achievement of each of the strategic goals, for a specific DM in the conflict, will be measured for each of these goals when: all the alternatives part of state s's definition are implemented to the degree set by s's definition (recall that s's definition shows alternative's A_j implementation as: $Achv(A_i) = L_{achv}$, where $N < L_{achv} \leq F$; and all other alternatives, which are not part of s's definition, not implemented (such alternatives will have their achievement values set to *None*, or $Achv(A_k) = N$. After giving the alternatives that are part of s's definition their respective implementation/achievement levels, and the rest *None* implementation levels, we run the value labels forward propagation algorithm (Algorithm 3.1). We, then, calculate the *Final Achievement* value for each strategic goal $SG \in \mathcal{SG}_{DM_i}$, for each $DM_i \in \mathcal{DM}$:

$$FAchv(SG) = \begin{cases} Achv(SG) \ominus Prvn(SG) & \text{if } Achv(SG) \leq Opr(SG) \\ Opr(SG) \ominus Prvn(SG) & \text{if } Achv(SG) > Opr(SG). \end{cases} \quad (5.11)$$

Here too, the "$\ominus$" operation used is the same one defined in Section 4.3.1 in the previous chapter; and the fuzzy linguistic value labels assigned to $FAchv(SG)$ are the same value labels assigned in that section to the $FAchv(SG)$ calculated there (for one agent situations) with the same fuzzy sets defining these labels. Second, this final achievement value calculation is similar to the one we used in Chapter 4 for one-agent decision making situations. But here, the calculation of the final achievement value is done not after a DM chooses one of his own alternatives, but rather after all the alternatives that make up a state s are implemented.

5.4.2.3 Modelling the Strategic Importance, Emotional Valences, Rationality and Emotionality Factors

As in the different decision making situations we discussed before, in competitive decision making situations (conflicts/games) too, DMs (or the analysts modelling and analyzing them) are not only allowed to set the strategic importance of their respective strategic goals, but are encouraged to do so. This is to ensure that important strategic goals of the different DMs are not treated as the less-important or not-important-at-the-moment strategic goals for them.

And, while most competitive multi-agent decision making situations, especially the ones within the business or professional domains, tend to have the DMs act rationally with no consideration to emotions, or at least try hard not to consider emotions, many research show that emotions in group decisions are part of the environment and their levels are as high or higher than their levels in single agent decision making situations –because of the interactions and the egos– (Barsade and Gibson, 1998; Bartel and Saavedra, 2000; Totterdell et al., 1998; Kelly and Barsade, 2001; Goleman, 1995; Mayer et al., 1999; De Bono, 1985, 1992). The interaction and competition characterizing conflict situations, with the highly conflicting strategic goals the agents have, make emotions high and hard to ignore even in business/professional environments.

Therefor, the Constrained Rationality framework allows in conflict decision making situations too the individual DMs to attach emotional valences to their respective strategic goals; and allows for the personality wide attitudes towards acting rationally and emotionally to be captured as well. To do so, we follow the same steps and use the same value properties used in Sections 4.3.2, 4.3.3 and 4.3.4 to add the strategic importance $SImprt(SG)\ \forall SG \in \mathcal{SG}$, add the emotional valence $EVlnc(SG)\ \forall SG \in \mathcal{SG}$, and add both factors, the Rationality Factor RF_{DM_i} and the Emotionality Factor EF_{DM_i}, for each $DM_i \in \mathcal{DM}$. These values will have the same fuzzification scheme, sets and linguistic value labels used before for the same value properties in single-agent decision making situations.

Here too, and as in the previously discussed decision making situations, when these values are used in the models, they must be used consistently. The same fuzzification scheme and value labels scheme, for each value property, must be used across all the agents. No two agents should be allowed to have different schemes to label their respective strategic goals. And, when one agent sets emotional valences for his goals while the others do not, for example, the modeller must encourage the agents who did not add their emotional valences to their goals to either: add their emotional valences; or, at least, apply the Emotionally Indifferent (EI) value label for their strategic goals.

5.4.2.4 Eliciting Agents' Preferences over Conflict's States

We will modify here the method and equations used in Section 4.4, and the ones used in Ssection 5.4.1.4, to determine the agents preferences over their alternatives in single-agent decision making situations, and shared alternatives in collaborative multi-agent ones, respectively. The method and equations to follow will allow us to determine the DMs' preferences over the conflict's states, in a competitive decision making situation, rather than their own alternatives.

For each decision maker $DM_i \in \mathcal{DM}$, at state s of the conflict and at time t of it, let the *Weighted Final Achievement* of a strategic goal $SG \in \mathcal{SG}_{DM_i}$ as a result of being at s (and having all the alternatives in s's definition been adopted/implemented at the levels indicated by their respective *Achv* value label given in s's definition), to be denoted as $WFAchv(SG,\ DM_i, s, t)$, and calculated algebraically as follows:

$$WFAchv(SG, DM_i, s, t) = \begin{cases} W(SG, DM_i, t) \cdot FAchv(SG, s, t) & \text{if } W(SG, DM_i, t) \geq 0 \\ 0 & \text{if } W(SG, DM_i, t) < 0. \end{cases} \quad (5.12)$$

where:

$$W(SG, DM_i, t) = (RF^*_{DM_i} \cdot SImprt^*(SG)) + (EF^*_{DM_i} \cdot EVlnc^*(SG)) \quad (5.13)$$

where $SImprt^*(SG)$, $EVlnc^*(SG)$, $RF^*_{DM_i}$ and $EF^*_{DM_i}$ represent the defuzzified values of their respective fuzzy values, and where none of the original fuzzy values is $Null$, and all reflect the state of mind and beliefs of DM_i at time t

and

$$FAchv(SG, s, t) = [FAchv^*(SG)]_{\text{if } s \text{ was fully applied to the GCM model at time } t-1} \quad (5.14)$$

where $FAchv^*(SG)$ represents the defuzzified values of $FAchv(SG)$, and $FAchv(SG) \neq Null$; and where "s was fully applied at time $t-1$" means that the intentions to apply all the alternatives in s's definition was achieved at the levels indicated by their respective *Achv* values in s's definition, at $t-1$

For each $DM_i \in \mathcal{DM}$, at state s of the conflict and at time t of it, let the effect of the full application/implementation of s's alternatives (to the levels indicated by their respective *Achv* value labels given in s's definition) into all DM_i's strategic goals (part of his none-empty $\mathcal{SG}_{DM_i}$) is represented by a *Total Weighted Final Achievement* value property; and let this property value be denoted as $TWFAchv(DM_i, s, t)$, and calculated algebraically as follows:

$$TWFAchv(DM_i, s, t) = \frac{1}{|\mathcal{SG}_{DM_i}|} \sum_{SG \in \mathcal{SG}_{DM_i}} WFAchv(SG, DM_i, s, t) \quad (5.15)$$

For the *Total Weighted Final Achievement* $TWFAchv(DM_i, A, t)$ value to reflect the effect of state s, and only s, on decision maker DM_i, then the conflict's integrated viewpoint with all its constructs and value properties' values must stay the same and only the alternatives in s's definition are applied to the levels indicated by their respective *Achv* value labels given in s's definition. In other words, the achievement value of the intentions representing the intentions to implement/apply the alternatives included in s's definition must change from the *None* labels they used to have before s is reached to value labels that represent the achievement levels they are given in s's definition. All other intentions, representing alternatives that are not included in s's definition, must have their achievement values stay the same unchanged, preferably unselected and stay at the *None* level. Then, after the values forward propagation algorithm, Algorithm 3.1, finalize the value labels for all strategic goals for time t, we calculate $TWFAchv(DM_i, s, t)$. The value of $TWFAchv(DM_i, s, t)$, now, reflects the effect of being at state s of the conflict, and only s, on all DM_i's strategic goals.

As indicated in other decision making situations, the value of $WFAchv(SG, DM_i, s, t)$ will be in the range of $[-2, 2]$, if emotionality is captured within the model. So, if $\exists DM_i \in \mathcal{DM} : ((EF_{DM_i} \neq N) \vee (\exists SG \in \mathcal{SG}_{DM_i} : EVlnc(SG) \neq EI))$, then the value of $TWFAchv\ (DM_i, s, t)$, for each DM_i, be in the same range of $[-2, 2]$. On the other hand, if the modeller did not consider modelling emotionality as part of modelling the conflict situation (i.e. $\forall DM_i \in \mathcal{DM}((EF_{DM_i} = N) \wedge (\forall SG \in \mathcal{SG}_{DM_i}\ \ EVlnc(SG) = EI)))$, then $WFAchv(SG, DM_i, s, t)$ will be in the range of $[-1, 1]$, and therefore the value of $TWFAchv(DM_i, s, t)$ will be in the range of $[-1, 1]$.

In the case where the modeller includes emotionality in the model, for at least one of the DMs involved in the modelled conflict, then we need to normalize the value of $TWFAchv(\mathcal{DM}, s, t)$ calculated. The new normalized value of $TWFAchv(DM_i, s, t)$ is calculated as follows:

$$TWFAchv(DM_i, s, t)\Big|_{\text{normalized}} = \frac{1}{2|\mathcal{SG}|} \sum_{SG \in \mathcal{SG}_{DM_i}} WFAchv(SG, DM_i, s, t) \tag{5.16}$$

Now, for each state $s \in \mathcal{S}$ of the conflict, at time t of it, each player $DM_i \in \mathcal{DM}$ in the conflict has a *Cardinal Preference* of s over other states in $\mathcal{S}$. Let this cardinal preference which DM_i has over s be represented as a *Weighted Payoff* value property attached to s, and be denoted as $WP(s, DM_i, t)$. Let $WP(s, DM_i, t)$ have a numerical value in the range of $[-1, 1]$ and calculated as follows:

$$WP(s, DM_i, t) = TWFAchv(DM_i, s, t)\Big|_{\text{normalized}} \tag{5.17}$$

$$= \frac{1}{2|\mathcal{SG}_{DM_i}|} \sum_{SG \in \mathcal{SG}_{DM_i}} WFAchv(SG, DM_i, s, t) \tag{5.18}$$

Recall that $\mathit{TWFAchv}(DM_i, s, t)$ does not need any normalization, if the modeller did not consider emotionality as part of modelling the decision making situation (i.e. $\forall DM_i \in \mathcal{DM}((EF_{DM_i} = N) \wedge (\forall SG \in \mathcal{SG}_{DM_i}\ EVlnc(SG) = EI)))$. In such case, $\mathit{WFAchv}(SG, DM_i, s, t)$ will be in the range of $[-1, 1]$, and therefore the value of $\mathit{TWFAchv}(DM_i, s, t)$ will be in the range of $[-1, 1]$ too. But, if emotionality is captured within the model (i.e. $\exists DM_i \in \mathcal{DM} : ((EF_{DM_i} \neq N) \vee (\exists SG \in \mathcal{SG}_{DM_i} : EVlnc(SG) \neq EI)))$, then the value of $\mathit{WFAchv}(SG, DM_i, s, t)$ will be in the range of $[-2, 2]$, and therefore the value of $\mathit{TWFAchv}(DM_i, s, t)$ will be in the range of $[-2, 2]$. In this case, normalization is needed to maintain the values of $\mathit{TWFAchv}(DM_i, s, t)$ and $WP(s, DM_i, t)$ within the range $[-1, 1]$. Hence, the devision by 2 shown in 5.18.

Based on the cardinal preferences, or weighted payoffs calculated for DM_i over each of the conflict's states in $\mathcal{S}$, DM_i will have a *Preference Vector* $Pref(DM_i, \mathcal{S})$ showing the order of the states in $\mathcal{S}$ from the *most preferred* for DM_i to the *least preferred*. It is assumed here that $Pref(DM_i, \mathcal{S})$ represents only ordinal ranking of the states in $\mathcal{S}$ based on how much each of these states contributes to the achievement of all DM_i's strategic goals (in $\mathcal{SG}_{DM_i}$) given the importance weight and emotional valence that DM_i assigned to each of his strategic goals, and given the specific rationality factor RF_{DM_i} and emotionality factor EF_{DM_i} describing the attitudes DM_i exhibits towards acting rationally or emotionally at that point of time.

The preference order of a specific state $s \in \mathcal{S}$, for a specific $DM_i \in \mathcal{DM}$ at time t of the conflict, is given as an *Ordinal Preference* value property attached to s, and is denoted by $OP(s, DM_i, t)$. Let $OP(s, DM_i, t)$ be given an integer number that reflects s's position in DM_i's *Preference Vector* $Pref(DM_i, \mathcal{S})$ at that point of time. The smaller the integer number assigned to $OP(s, DM_i, t)$, the more preferred s is, to DM_i at time t. The state that has $OP(s, DM_i, t) = 1$ is considered the most preferred state for DM_i, and the one with $OP(s, DM_i, t) = |\mathcal{S}| - 1$ is the least preferred state for him. This is because the states in the $Pref(DM_i, \mathcal{S})$ vector are ordered from the state with the highest weighted payoff value for DM_i to the one with the lowest weighted payoff.

$$OP(s, DM_i, t) = n + 1 \quad \text{where } 0 \leq n \leq |\mathcal{S}| - 1 \quad \text{and } n \text{ reflects } s\text{'s position in } Pref(DM_i, \mathcal{S}) \tag{5.19}$$

5.4.2.5 Modelling the Strength of DMs' Preferences over Conflict's States

To capture the strength of DM_i's preference over a specific state $s \in \mathcal{S}$, at time t of the conflict, we use the same distance measure we used in Section 4.5 for the strength of an individual DM's preferences over their alternatives, but here we will have the distance between preferences over states instead. This distance measure captures the difference between the cardinal preference of a state s and the cardinal preference of another state in $\mathcal{S}$, for the specific decision maker DM_i. The preferences are take from his perspective and based how much he will gain from each state. Based on this distance measure a binary relation among each pair of states in $\mathcal{S}$ is assigned.

For $DM_i \in \mathcal{DM}$, and at time t of the conflict, let the distance measure between two preferences which DM_i has over the two state s_a and s_b, both in $\mathcal{S}$, be denoted as $d(s_a, s_b, DM_i, t)$. And, let its value be given as a real number calculated as follows:

$$d(s_a, s_b, DM_i, t) = [\,WP(s_a, DM_i, t) - WP(s_b, DM_i, t)] \tag{5.20}$$

Because each of $WP(s_a, DM_i, t)$ and $WP(s_b, DM_i, t)$ has a value in the range $[-1, 1]$, whether normalized as per Equation 5.18 or not normalized (because the modeller did not consider modelling emotionality as part of the conflict's model), then the distance value will be in the range of $[-2, 2]$. The sign of $d(s_a, s_b, DM_i, t)$ shows which state of the two DM_i prefers.

Let *Preference Strength* be a value property, denoted as $PrefStrngth(s_a, s_b, DM_i, t)$, and be given the fuzzified value of the distance value property. In other words, let $PrefStrngth(s_a, s_b, DM_i, t) = \underset{\sim}{d}(s_a, s_b, DM_i, t)$. $PrefStrngth(s_a, s_b, DM_i, t)$ will be assigned a fuzzy linguistic value label L_{PS} based on the fuzzy memberships functions given in Figure 4.4, the same fuzzy sets and labels that are used for the preferences strength of an individual DM over his own alternatives in Section 4.5. The strength expressed by the L_{PS} fuzzy label is meant to represent the distance between the weighted preference values for the two states considered here, s_a and s_b, from DM_i's perspective.

Also here, as it was the case in Section 4.5 for individual's preferences over their own alternatives, let the strength of DM_i's preference of state s_a over state s_b, at time t of the conflict, given in $PrefStrngth(s_a, s_b, DM_i, t) = L_{PS}$, be represented by a binary relationship between the two state. Let this binary relation be denoted as $s_a \succ^{L_{PS}}_{DM_i,t} s_b$. The notation of this relation is the same as the ones shown in Figure 4.4 in Section 4.5, for each possible preference's strength fuzzy linguistic label L_{PS} (shown in Figure 4.4 above each label's membership function).

5.5 Modelling The Dynamics of Conflicts

The Constrained Rationality framework takes care of the dynamics in multi-agent decision making situations that result from changes over time (happens to the agents' goals as well as the agents' internal and external realities/constraints – as we showed earlier in Section 3.8). The framework also provides a platform to analyze the dynamics produced by the nature of moves and countermoves agents have in conflicts model specifically. In addition, the framework allows the agents to have different moves for each phase/iteration of the game, based on the structure of the game and the players' preferences in each of these phases/iterations.

In the following chapters, we will study in details the different three conflict types: non-cooperative games, cooperative games where coalitions are not allowed, and cooperative games where coalitions exist and/or allowed. We will also present case studies in which we model and analyze real-life conflicts based on the concepts and tools that the Constrained Rationality framework offers. But in this section, we will provide an overview of the move types which agents can have, and an overview of the resultant different conflict types. In addition, we will provide an overview of the different patterns or structures conflicts in real-life take.

5.5.1 Agents' Unilateral, Cooperative and Coalition Moves

We said that multi-agent competitive decision making situations, or conflicts, are highly dynamic. The agents move between the conflict's states, and these moves depend on many factors. One factor is the ability for an agent to actually move from one specific state to another. The agent could in fact be able to move unilaterally between two specific states, both ways or in one direction only. But the agent could also be not able to move at all between those two states, or could only move between them if he cooperates with other agents (such as the case if the agent is moving from no-agreement state to a signed-agreement state). There is also the possibility of sanction moves that other agents could take to block any improvements the agent might have hoped when he took his original move. And, what about countermoves the agent might have to regain some of the lost improvement. In fact, the agents' moves and countermoves, in addition to their preferences over the conflict's states, are the basis of the stability and equilibrium analysis methods discussed in the game theory and conflict analysis literature.

We will not divert from the solid definitions and established methods that the game theory and conflict analysis literature provides. But, we will take steps to unify what seems fragmented definitions and analysis tools, and define (or more accurately re-define) them to offer a full comprehensive, consistent and coherent set of definitions and tools that not only be able to help model and analyze many types/variations of multi-agents conflicts, but also anchor these definitions and tools to verifiable and validate-able agents' preferences. Preferences which in turn are based on the captured and modelled agents' goals, constraints, priorities, emotions, and so on.

We will define, in the following chapters, a number of move types, agents in conflicts can have and take. Following the steps of Fraser and Hipel (1984) and Fang et al. (1993), we will define two types of unilateral moves for individual agents, for each state of the conflict: the set of Unilateral Moves (UMs) that an agent can take on his own from one state to another; and the set of Unilateral Improvement moves (UIs), a subset of the UMs the agent has from the state, but by the end of these moves, the agent will be at a better state (more preferred than the original state he started from). We also add additional moves to the ones originally offered by Fraser and Hipel (1984) and Fang et al. (1993). All these move types will be redefined or defined in the following chapters, within the context of using Constrained Rationality's conceptual modelling and analysis framework in different conflict types.

The UM and UI move types form the foundation of non-cooperative games, which we will study in the next chapter, where involved agents are not allowed or cannot make cooperative moves. We define Cooperative Moves (CMs) as moves that the individual agents cannot make on their own, and must cooperate together to jointly make the move (an example of such moves is a move from a state where two agents are engaged in a legal battle over intellectual property rights to a state where the agents both reach a full settlement agreement). These moves are not allowed within the context of non-cooperative conflicts, but are allowed in cooperative games. A subset of the CMs, agents have, are CMs in which engaged agents make the move from a less preferred state to a more preferred state, i.e. all engaged agents benefit from these moves. These moves are called Cooperative Improvement moves (CIs).

The concept of cooperative games is well known in the game theory literature, the literature for the most part insist on making the cooperative moves binding to all parties involved. Luce and Raiffa (1957) defined a cooperative game as a game in which the players have complete freedom of pre-play communication to make joint binding agreements. And, Hipel and Meister (1994) defined a different type of cooperation that lasts throughout the duration of the conflict.

While we followed the steps of Fraser and Hipel (1984) and Fang et al. (1993) in their definitions of non-cooperative UMs and UIs (but redefined them within the context, terminology and notation of the Constrained Rationality framework), we defined cooperative moves by the conflict's players differently. Fang et al. (1993) defined the Graph Model for Conflict Resolution (GMCR) techniques to fall under the category of non-cooperative game theory. But later modifications to GMCR, namely by Kilgour et al. (2001) and then by Inohara and Hipel (2008b,a), added cooperative moves by a coalition of players in the conflict. In this thesis work, specifically in Chapter 8 and 9, we define different types of cooperative moves.

In the Constrained Rationality's cooperative games, the agents can make a single-step joint cooperative move (CM), that none can make on their own, and later defect. This is to ensure realistic representation of real-life scenarios, where agents reach agreements, sign them, but later act as if the agreement does not exist. This deceiving behaviour should be modelled within the conflict models to ensure that the stability analysis uncover any potential benefits agents will have in going back on their commitments, or even backing down from the negotiating process at an earlier stage of it. In other words, our definition of cooperative games, and cooperative moves, is broader than the ones found generally in game theory literature.

In addition, we differentiate between two classes of cooperative moves: one-step cooperative moves (CMs) and group's cooperative moves (G-CMs). One-step CMs are committed by two or more agents, all moving jointly from one state of the game to another in one step. The agents make this joint move because none of them can make the move on his own. They all must cooperate to make the move. But they are all still concerned only about their individual gains and losses, and do not share the same goals or values that the other agents have. They all act as

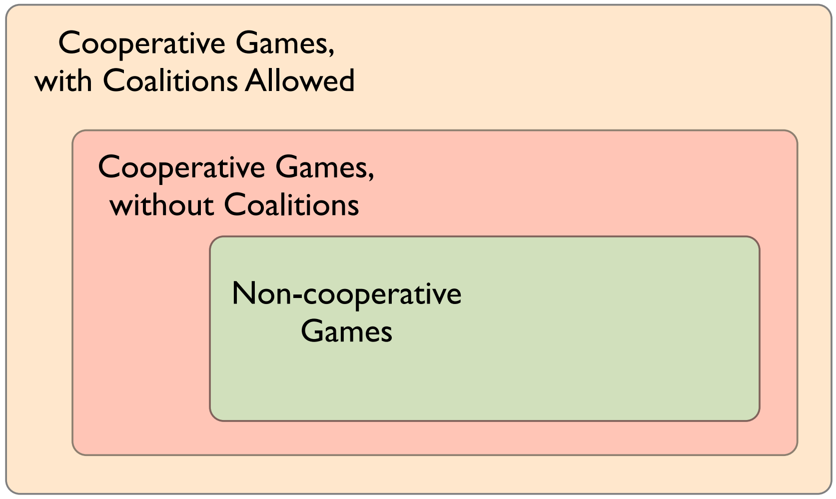


Figure 5.9: Venn Diagram showing the relationship between the three classes of multi-agent conflicts

separate entities, and their cooperation to make the CM move is merely because they all can benefit from this one-time one-step cooperative move.

On the other hand, we find in real-life conflicts, that agents form coalitions or alliances and move together. The differences between the individualistic CMs and the coalition group moves (C-GMs) is that G-CMs can be multi-step moves (a combination of UMs and CMs), where the coalition harness the power of grouping their abilities. And, while individual agents, for the most part, make a CM because it is in fact a CI move for all involved agents, the coalition can afford have some scarifies in the process, as long as the end result for the combined multi-step move is a better more-preferred state for all members of the coalition, most of the coalition members, or the big/strong members in the coalition (based on how the coalition is structured and organized). These moves are called in the Constrained Rationality framework: C-GIs or Coalition Group Improvement moves. We will study in later chapters the two classes of cooperative games: the cooperative games, with no coalitions; and the cooperative games, with coalitions.

The relationship between the three classes of conflicts is best described by the Venn diagram shown in Figure 5.9. As the figure shows, the broadest class of multi-agent competitive adversarial decision making situations (conflicts) is the class of cooperative conflicts with coalitions. These games not only allow coalitions and their moves to exist, but also allow agents to unilaterally or cooperatively move. Cooperative games without coalitions allow agents to have UMs and UIs, and allow them to come together and move jointly to states that they will not be able to reach individually. But cooperative games without coalitions allow only single-step cooperative moves by individual agents. Cooperative games with coalitions have both types of agents: individuals who can move unilaterally or cooperatively in one step UMs, UIs, CMs and/or CIs; and coalitions who can take multi-step moves (which surely include one-step UM/UI/CM/CI moves and more-than-one-step moves consisting of consecutive one-step UM/UI/CM/CI moves by the coalition members). On the other hand, non-cooperative games can have individual agents only, each with his own UM and UI moves. Therefore, and as Figure 5.9 shows, non-cooperative games are a subset of cooperative games without coalitions, which in turn is a subset of cooperative games where coalitions are allowed to exist and participate as whole entities.

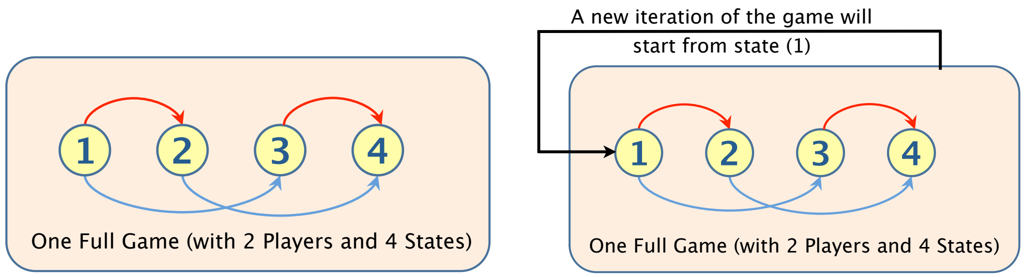
5.5.2 Game Structures, Phases and Iterations

The analyst should also check if the conflict's state have different contribution levels to the agents' goals based on what phase or iteration the conflict is in, or if there are different states available to the individual agents at different phases in the conflict. For example, in a legal conflict going through the courts, the players could have different options based on the court level (low, high/appeals, or supreme court) the conflict's case is at, and/or what the court's decision is (for or against a specific agent). In Chapter 9, we will study a legal intellectual property conflict that has multiple phases based on the level of the court and based on the nature of the courts' decisions.

The structure, or the grouping of a conflict's states, could be one of many patterns. Figure 5.10 shows three general patterns of conflicts' structures. Sub-Figure 5.10a shows a conflict with one stage/phase, i.e. a one time game. The states of the game are all part of this one phase. Once the game ends, the players have no other options or moves. An example of such games is the classical Prisoner's Dilemma, which will be studied extensively in Chapter 7.

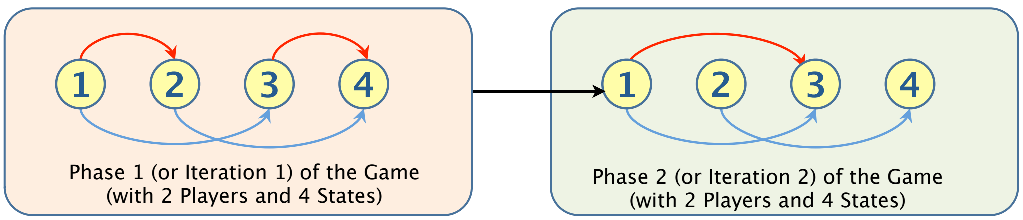
Sub-Figure 5.10b shows a game with the game's one-phase repeats infinitely, or for a finite number of time. The states in each iteration stay the same, as well as the players' preferences and moves. An example of such game is the Tit for Tat Iterative Prisoner's Dilemma, which also will be studied extensively in Chapter 7.

The third general structure is similar to the iterative game structure, but the iterations, or better called phases/stages, could have different states, different play-



(a) A Conflict that has one iteration/phase.

(b) An Iterative Conflict, with each iteration has identical states, players preferences and moves. Each iteration starts with state no. 1.



(c) A Conflict that has two phases/stages. The two phases has the same states, but the "red" player's preferences change in the second phase and therefore his UI moves changes as a result.

Figure 5.10: Three different structures of a conflict between two players. Each of the structures shows the conflict's phases/iterations, if any, the states within the phase/iteration, the two players UI moves between the states (shown in red for the "red" player and in blue for the "blue" player).

ers' preferences and/or players' moves. Sub-Figure 5.10c shows a simple two phases game. The two phases have similar states, but in the second phase one of the players has a change in his preferences leading to a change in the UIs (Unilateral Improvements moves –will be defined formally in the next few chapters for the three different conflicts' types) available to him at this phase. The legal intellectual property conflict referred to above, and will be studied extensively in Chapter 9, is an example of a complicated multi-phase conflict.

The analyst, most likely, will try a number of configurations of how the conflict states are grouped and structured, before he will settle on a structure that best represent the base-model of the conflict to study and analyze. He, also, could have some of these possible structures for the conflict as additional what-if models to analyze during the sensitivity analysis stage of the conflict analysis process.

Care should also be given to validate how the states are grouped in the conflict's phases, if the conflict has multiple phases, and whether these states and the players' alternatives that define them show different contributions (achievement, prevention and operationalization levels) to the different players' goals or not. Because such changes could result on changes in the players' preferences over these states, and by extension to the players' move, the stabilities of the different conflict' states to the different players, and finally to the conflict's equilibrium states.

5.6 Analyzing Multi-Agent Decision Making and Conflicts

We showed above how to model formally, systematically and methodologically multi-agent decision making situations, both collaborative and adversarial. In this section, we will discuss how these situations will be analyzed. Ultimately, the analysis steps are intended to provide the involved decision makers with enough information to guide them to make better decisions in these situations. The analysis steps differ for collaborative situations from the ones for games. This is due mainly to the highly dynamic competitive nature of games in comparison to the joint collaborative decision making situations. The additional complexity of game/conflict situations, is the reason behind the additional analysis steps that are required for these situations, as we will see, in comparison to the steps needed for the collaborative ones.

The analysis steps are shown in Figure 5.2 to be part of the overall modelling and analysis process presented. They include the following steps (numbered as shown in the figure):

5. Generating the agents' preferences over the alternatives (for single-agent or multi-agent collaborative decision making situations) or over the conflict's states (for the multi-agent adversarial conflicts).

6. Decision Making and Conflict Analysis:

    (a) Conducting a stability analysis study of all conflict's states, and identifying the states that form equilibriums, or possible resolution points for the conflict.

(b) Conducting a thorough search for all variations to the base-model that need to go through sensitivity and what-if analysis.

(c) Considering the changes that will happen over time to values and constructs of the base modes, as well as the sensitivity what-if models.

7. Generate the different sensitivity/what-if/scenario models for the conflict, and re-start the modelling and analysis steps for each of them.

For multi-agent adversarial conflicts, specifically, the three chapters to follow will provide more details and discussion on each of these steps for the three different conflict classes (non-cooperative games, cooperative games without coalitions, and cooperative game with coalitions); and will provide examples to illustrate how the analyst should go through each step. But, we will give in this section a general idea about them, and discuss some aspects of each of these steps.

5.6.1 Calculating Agents' Preferences over Alternatives or Conflict's States

In Chapter 4, we showed how the ordinal and weighted preferences are calculated for individual agents over their respective alternatives, in single-agent decision making situations or multi-agent situations where agents act in an individualistic manner with no regard to other agents past, current or future choices and moves. For collaborative multi-agent situations, the preferences generated for the agents over the stated "shared" alternatives represent the evaluation results for each of these alternatives (products, product designs, hardware specifications, software requirements, . . .). Section 5.4.1 showed how the agents preferences are calculated over shared alternatives in collaborative decision making situations. And, Section 5.4.2 showed how the agents preferences are calculated over games' states in competitive conflict situations, respectively.

Recall that these preferences map directly to the agents' goals and realities/constraints. Not to mention that they are calculated with the agents' priorities, emotions and personality-wide attitudes toward rationality and emotionality are in mind. Making the preferences generated, or the evaluations generated, more contextually rich than any evaluation results provided by the traditional methods used in decision theory, specifically the multi-criteria decision analysis.

The evaluation of the options, which decision makers have, represents the main decision making analysis step for the collaborative type multi-agent decision making situations. It provides the involved agents a clear preference ordinal ordering of their alternatives, as well as a strength of each of these preferences (weighted cardinal preferences) that maps directly to their objectives and constraints. Fo collaborative situations, this step usually followed by identifying all variations (deviations from the base-model) that should be studied. For the adversarial multi-agent conflicts, the evaluation step of the agents' individual options is just a step in the analysis process followed by establishing which of the agents' moves constitute improvement moves (UIs, CIs, or C-GIs) based on the agents' preferences. Both steps must be completed before the stability analysis is conducted for each of the conflict's states.

5.6.2 Conflict's Stability Analysis and Equilibriums

The stability analysis of a multi-agent adversarial game/conflict is a process by which the analyst will determine the stability of each conflict's feasible states (outcomes) for every agent involved in the conflict. If a state is stable for a given agent, it will not benefit this agent to move away from this state to any other state. A state that is stable for all agents involved in the game forms an equilibrium for the conflict, and constitutes a possible resolution to the conflict. Because the stability analysis step, part of the overall conflict analysis process, is used to predict the possible equilibriums for the conflict, it is commonly referred to as the prediction or forecasting step.

Clearly, the stability of a state to an individual agent depends on: what moves the agents have out from this state (i.e. the UM or CM moves he has out of this state); whether any of these UMs and/or CMs is in fact a move that constitute a UI or a CI move by which the agent will be able to enhance his position from the current state to a more preferred state; the strength of the gained preference, if any; the ability of other agents to sanction such UI or CI moves; and the ability of the agent to regain some of the lost preference by such sanction (i.e. the ability for the agent to have a countermove to the sanction allowing it to regain all or some of the advances made by his initial UI/CI).

This makes the conflict's stability analysis process and concepts requires separate and thorough discussion, more than what this chapter can provide. The four chapters to follow will focus on providing formal definitions to agents moves types, stability and equilibrium types, for each of the three different classes of multi-agent conflict, with one chapter dedicated to study the relationships among the different stability types (referred to as stability solution definitions) which we will use in our conflict stability analysis. The four chapters will also provide full illustrative case studies that cover different types of real-life conflicts including a non-cooperative cold-war political conflict, a cooperative-without-coalitions environmental/policy conflict, a cooperative-with-coalitions industrial intellectual property conflict, in addition to classical conflicts such as the prisoners dilemma and the game of chicken.

5.6.3 Sensitivity Analysis, What-if Analysis, and Analysis of Changes Over Time

The purpose of conducting sensitivity analysis for a multi-agent decision making situation, whether a collaborative one or an adversarial conflict one, is to be able to assess the validity of the results produced so far by the analysis steps. The idea is to model any variations, the analyst deems important, as what-if models and test them in order to identify any changes in the preferences structure and any implications that the DMs should be aware of. When doing so, few questions are important to keep in mind: what could change the preferences, and how bad/good the effect of each of these changes on the decision, or in comparison to the decision, if it is been taken based on the base-model analysis. One also need to include changes that could happen over time, not as a variation to the base-model, but happen to the base-model's context (goals and constraints, as well as the number and identity of the agents involved in the decision making situation).

Most of the variations that the analyst will identify as candidates for sensitivity testing will be –most likely– small variations that relate to the fuzzy values the base-model include (such as rationality, emotionality, importance, and emotional valence attached to the different agents' goals). This is because of the fuzzy nature of these values. Certainty and justification for the beliefs about these naturally fuzzy values are very difficult to elicit and validate. Even if these values are elicited from the involved agents themselves, through direct knowledge acquisition process, the analyst should ensure adequate sensitivity analysis has been conducted. The analyst should test different variations of these values, build complete what-if models (variations to the base-model), if needed, and highlight any implications on the agents' preferences and the stabilities of the conflict's states that could happen as a result.

But, the analyst should look at other variations too, especially the ones related to whether the agents., or some of them, have different goals, hidden goals or additional goals than the ones captured in the base-model of the decision making situation. In addition, he should verify that all the alternatives are captured within the base-model, including alternatives that the agents do not have now but will/may have in the future. Testing all these variations and possibilities is an important task, in order to ensure that the analysis produced and the recommendation of action/decision given to the focal decision makers are extensive and thoroughly considered all possible scenarios.

Some of the most important variations that could exist, but mostly ignored by analysts, are variations based on changes that could happen to the decision making situation over time. Assuming the base-model of a situation, including all its values and constructs, is accurate, there is always a possibility for changes to happen over time. These Changes could impact the situation and drastically affect its model, and therefore the analysis and predictions produced for it. For example, consider changes such as: new agents joined the conflict; one agent acquire another agent, consolidating some of the players or ending the conflict; some agents, due to certain market conditions, legal ruling, or new governmental regulations/policies, have drastic changes to their goals and/or constraints affecting their preferences over their alternatives or the conflict states; new phases/iterations are added to the conflict, and the analyst was not aware of them; and so on.

These variations should be tested as conflicts with different phases/stages, where the conflict moves from one phase at a certain point of time to another phase at a later stage. The way we deal with modelling such changes is similar to the way we discussed modelling the dynamics of single-agent decision making situations over time in Chapter 3 but here the new model is a new phase/stage of the conflict. This results of a new what-if model which has multiple phases, each with different agents, goals, alternatives, or states (similar to the model shown in Sub-Figure 5.10c.

The extra care the analyst puts in conducting sensitivity analysis, especially for the implications of any variations or deviation from the base-model, will serve two purposes: 1) keep the analyst on the alert, looking for signs of changes in these variables, when and if such changes occur; and 2) point to the involved agents the importance of disclosing the real beliefs and justifications for these values, and any changes happen to them should some occur. The final analysis, stabilities, equilibriums and results, could depend on such values. They form an important factor on how the agents' preferences are formed and calculated.

5.6.4 Comparing Analysis to Reality

If the modelled and analyzed multi-agent decision making situation, whether it is a collaborative one or a conflict one, is a historical situation happened in the past, then the analyst must compare the models as well as the analysis produced to what actually happened in reality. This is also true, if the modelled situation is unfolding before the analyst and the involved agents. The analyst must monitor how the situation unfolds in reality, compare it with the base-model analysis and prediction, understand the reasons behind any deviation from the base-model, re-adjust the base-model and conduct a thorough analysis for the new model (and for sure inform the focal decision makers).

Comparing the base-model analysis and predictions with how the situation evolved, or evolves, over time will benefit the analyst in many ways. First, this comparison serves as a mechanism to test if the base-model is/was accurate and valid. Second, it will help the analyst understand the reasons behind any deviation from the base-model, and take measures to re-tune the base-model, analyze it, conduct sensitivity testing on it, and provide better predictions and decision support on the next actions to be taken (especially if the conflict is unfolding in real-time before the analyst). Third, it will serve as a mechanism to learn about the reliability of: the knowledge used in building the base-model; the sources which the knowledge is elicited from; and finally, the elicitation and modelling process used. Forth, it will uncover any deception or misinformation, the focal agent and/or the analyst was a target of (i.e. it will uncover the existence of a hyper-game).

In addition, comparing the results of the analysis with the reality in the ground could serve as a mechanism to uncover certain aspects about the conflict that could not be discovered otherwise. In Chapter 6, for example, we modelled and analyzed the Cuban Missiles Crises. Certain questions about this important hysterical conflict were the focus of many studies, and still (Allison and Zelikow, 1999). For example, why the USSR did not escalate the conflict instead of withdrawing the missiles? Why the USSR did not insist on the US to withdraw the missiles it has in Turkey, before it removes the missiles from Cuba? Why the US chose the blockade on Cuba as a strategy to deal with the conflict, and did not employ a strategy that involved a show of force such as an air strike? and so on. By modelling the conflict and analyze it using our framework, we managed to get answers to these questions, and show that the hysterical outcome of the conflict, as well as the hysterical flow of events, are similar to what the analysis predicts. The Constrained Rationality's model for the conflict and its analysis provided answers to these questions that confirm with the information uncovered after the cold war had ended.

Therefore, the analysis vs. reality comparison step is an important step not only to ensure valid and reliable support for the focal decision makers as they consider taking their next moves, but also as tracking and learning tool. It helps the analyst uncovers any deviation from the base mode, and take the right measures to understand why the deviation happened, modify the base-model and produce new analysis and predictions based on the new model. It will also help the analyst track knowledge reliability problems, uncover them and take the proper measures to correct them for the conflict at hand, and all other future conflicts to be modelled and analyzed.

5.7 Example: System Requirements Engineering, Collaborative Multi-Agent Decision Making

The next few chapters will include many case studies showing how the Constrained Rationality framework is used to model and analyze multi-agent adversarial decision making situations (conflicts or games as also called in our research, and as they are called in game theory literature). We will cover conflicts that vary in their types (non-cooperative, cooperative without coalitions, and cooperative with coalitions), in their structure (single phase, iterative and multiple-different-phases) and in the nature of the conflict (whether they represent real industrial, environmental, policy, or intellectual property conflicts, or represent hypothetical classical paradoxes or rationality).

In this section, we will cover an example of a collaborative decision making situation (the other type of multi-agent decision making situations –other than conflicts–). We said above that collaborative situations are easier to model and analyze than conflicts, mainly due to the less dynamic nature of these situations in comparison to conflicts. Collaborative situations have shared alternatives that all the agents choose from. In conflicts, agents have their own set of alternatives to chose from. In conflicts, agents have moves, and have states been defined as stable or unstable for them. In collaborative situations, all agents move together. There is no competition. The agents still have conflicting goals, but they are not trying to win over each other such as the case in conflicts. In essence, collaborative decision making situations is aiming for a cooperative and collaborative win-win decision on a choice that will best satisfy the goals and constraints of all agents.

We used real-life hysterical conflicts or classical paradoxes of rationality games in our case studies to illustrate how the Constrained Rationality framework will be used to model and analyze different multi-agent conflict types. This also served our illustrative case studies well. Because these conflicts either happened in the past, or suggested and studied extensively in the game theory literature. This made the process of comparing our analysis and analysis's predictions to the hysterical events, and/or to others' analyses, much easier. Unfortunately, it proved to be harder to find hysterical collaborative multi-agent decision making cases suitable to be used as illustrative case studies. Cases that could be used to study and compare its historical outcomes to our models and analysis's predictions. At the heart of the difficulty in finding such cases is to have access to the case's players and their contextual knowledge for the situations (goals, constraints, etc.), or previous studies that report on such knowledge.

In an effort to validate the Constrained Rationality framework and its suitability for the collaborative type of multi-agent decision making situations, we have used the framework to study, model and analyze two complex industrial collaborative multi-agent decision making situations: a strategic product development initiative and a strategic software requirements engineering initiative. The two initiatives were very successful, but contractually confidential (because of the sensitivity of the information each covered) to be able to use them here. And, although the two initiatives were easier in modelling and analyzing than the hysterical and classical conflicts, we used in our conflicts case studies, in some aspects (namely because of the apparent alignment in the agents goals and their motivation to reach agreement), we faced some very challenging aspects in modelling them. Most of the challenges seems to stem from the size of the models, making model management and documentation a challenging task even with the help of some tools that we developed specifically to automate some parts of the task. Nevertheless, we intend to report on the two initiatives, and the learning from them, after removing/altering the confidential details with the help of the involved industrial partners, in separate research studies.

In this section, we will use a simpler and heavily scaled-downed version of a software requirements engineering example to show how the Constrained Rationality framework can be used to model and analyze a collaborative type multi-agent decision making situations. This simple example will be sufficient to illustrate the use of the framework at the level required for this thesis document and this chapter. To model this collaborative system requirements decision making situation, we follow the same modelling steps shown in Figure 5.2:

1. Define the Context: The purpose of this collaborative decision making initiative is to decide on the best architecture for a software system. There are three architectures to be reviewed for best fit. Best fit is defined as the most accommodating to the needs of all the stakeholders involved.

2. Relevant Decision Makers: The software architecture for such small system will be decided on by four agents. The involved agents include: two agents represent the business users of the system; one system architect/designer (also represents the rest of the technical team, namely the developers and testers); and one project manager (responsible for the contractual obligations, delivery timing and reporting requirements).

3. Build a Viewpoint Model for each Decision Maker: The analyst will acquire the knowledge needed to build a viewpoint model for each of the stakeholders, meeting individually with them and/or meeting with them as a group (there are benefits for doing both). For each of the decision makers, she needs to build a GCM model to capture all the goals (needs and wants) and all the constraints that the decision maker has for the to-be-built system. For this type of decision making situations, the collaborative ones, the agents' individual viewpoint models will include mainly the GCM models of the respective agents. Recall that the alternatives are "shared" for the group to choose from, and are not individuals' alternatives. Deciding on the alternatives, the different architectures to use for the system, can be done in many ways: collect all different alternative architectures that the agents suggest, evaluate them all against the agents' goals, or choose a subset of all alternatives and evaluate those. But, before the evaluation step, the following step must be completed.

4. Integrate All Viewpoint Models and Finalize the Base Mode: The analyst will take all the individual agents' viewpoints (GCM models) and integrate them, adding cause-effect qualified fuzzy-labeled lateral relationships between the different goal and constraint nodes reside in different GCM models.The result will be similar to Figure 5.11. Please note that the figure shows an integration of very simple GCM models, for a very simple system requirements example.

Note that each of the agents has a GCM model with a two-layers goal-tree, and that the goals in both GCMs conflict with each other. In real-life requirements engineering initiatives, there are usually many more stakeholders, each with a complex GCM model that has multiple goal-trees with many goals conflict with each other and conflict with other goals across the individual GCM boundaries. Also, Figure 5.11 shows the three alternative architectures the agents should choose from at the bottom of the integrated viewpoints model, with each alternative been connected to the agents' goals through lateral relations to show the positive or negative achievement/contribution that implementation of these alternative has if implemented on the goals. In real-life requirements engineering initiatives though, there are many more alternatives (and each alternative has many shades/variations/configurations). Please remember that the example we use here is intended to show a simple case for illustration of use purposes only.

5. Add Priorities, Emotional Valences and Rationality Factors, then Generate Agents' Preferences over Alternatives: In this step, the analyst will follow the same process we used in Section 5.4.1 in modelling the strategic importance and emotional valences the agents attach to their respective strategic goals. Recall that the rationality and emotionality factors are as important in collaborative multi-agent decision making situations as they are in one-agent situations and the multi-agent conflicts. But, before adding these value properties, the analyst should first calculate the Final Achievement levels that each of the alternative architectures, once adopted to be implemented for the system, will contribute to the agents' strategic goals. This is done by calculating the achievement and prevention levels these goals will receive by activating each of these alternatives individually in the integrated model of the decision making situation (and followed by running the same forward label propagation discussed in Chapter 3). Figures 5.11, 5.12 and 5.13 show the effect (the achievement and prevention values) of implementing Architecture 1, 2 and 3, respectively, on the agents' goals (needs and wants). After calculating the final achievement values for each strategic goal, for each of the alternative architectures, then the analyst can add the importance values and emotional valences for each of the agents' strategic goals, and generate the group's preferences over these alternatives (both the ordinal and the cardinal preferences) as shown in Section 5.4.1.

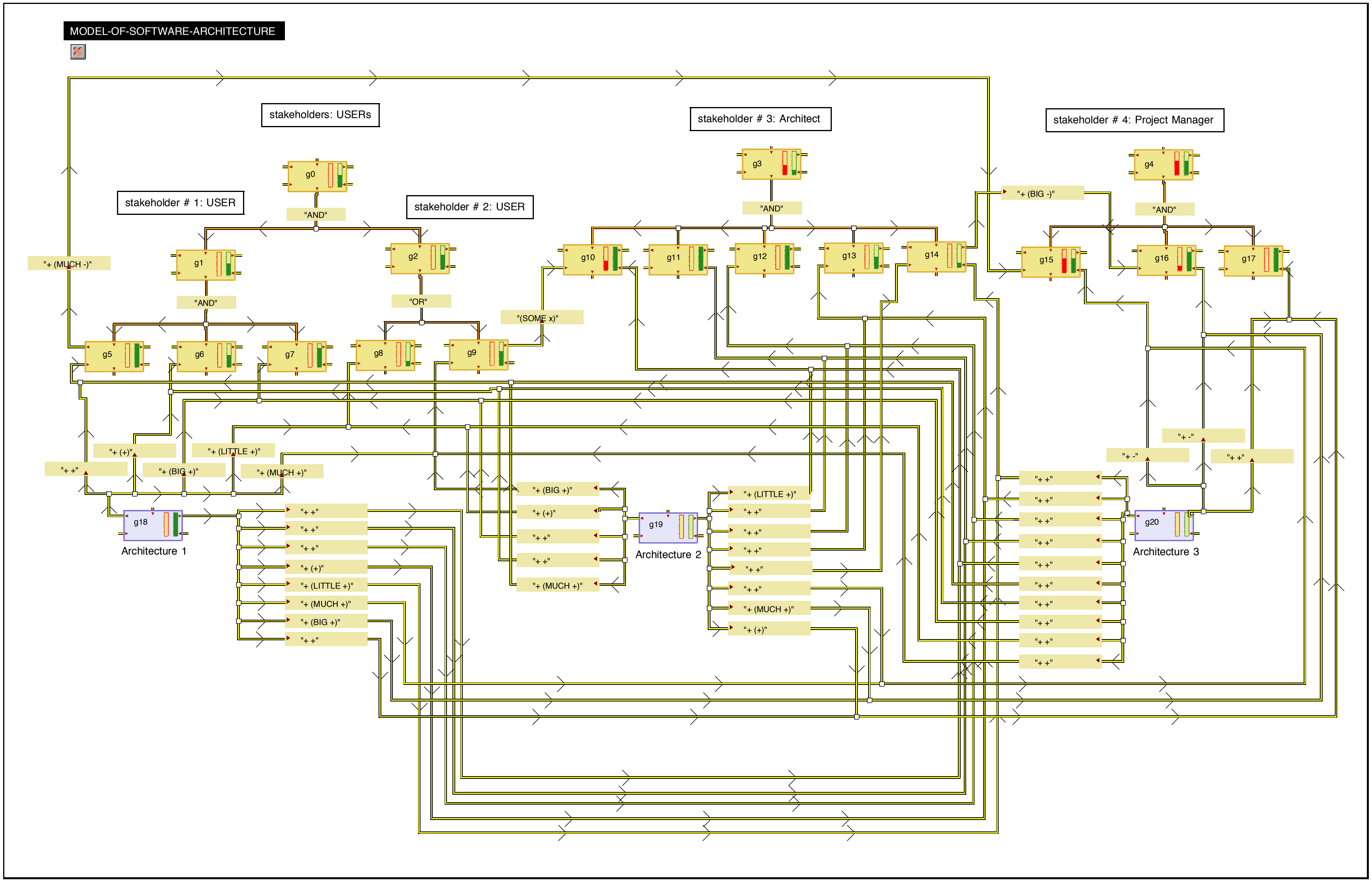


Figure 5.11: A Model of a Collaborative Multi-Stakeholder System/Software Requirements Decision Making Case. Mobilizing an intention to implement a specific design, Architecture 1 in this figure, will how much achievement/satisfaction each of the stakeholders have for their (conflicting) goals.

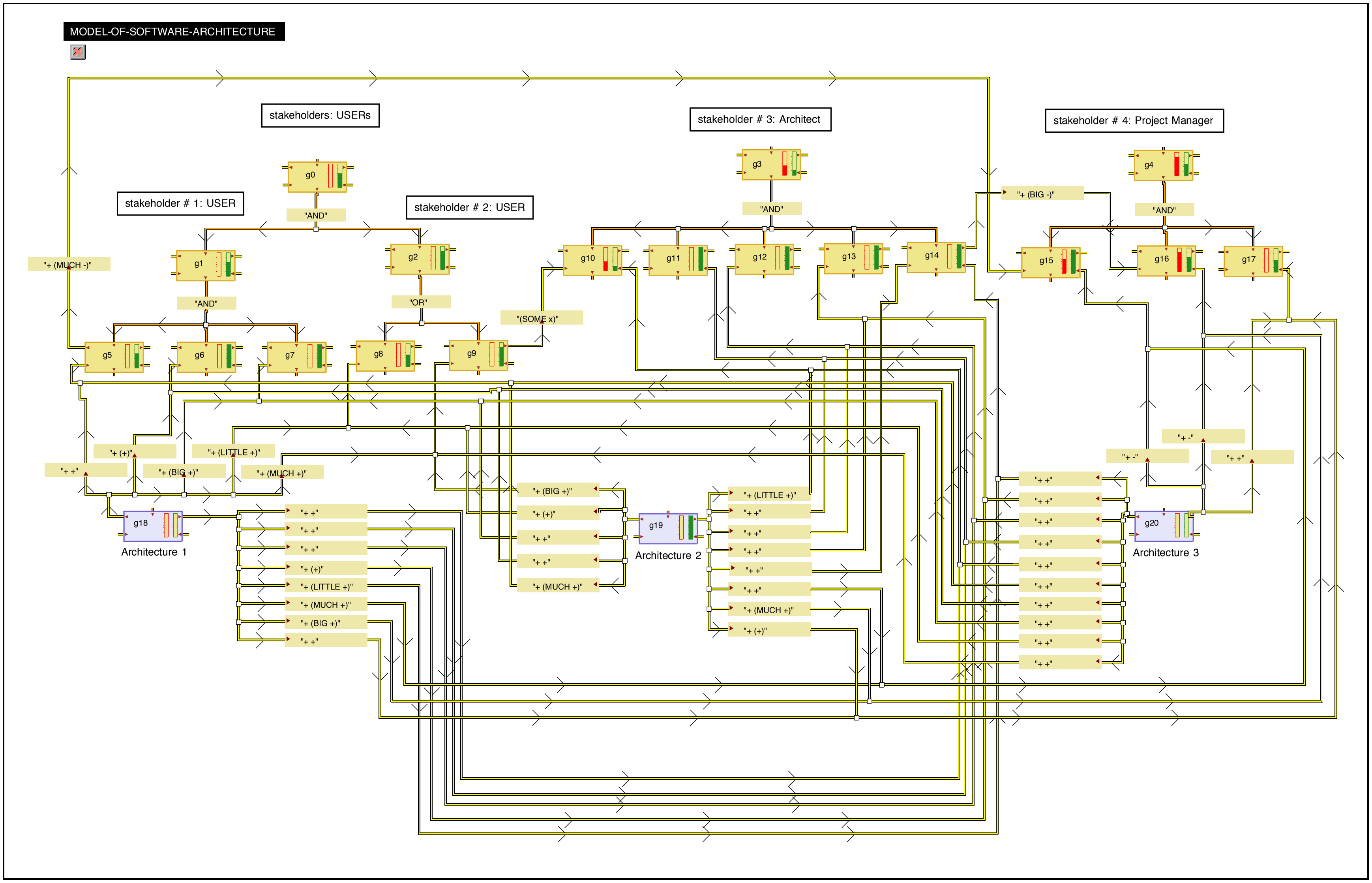


Figure 5.12: A Model of a Collaborative Multi-Stakeholder System/Software Requirements Decision Making Case. Mobilizing an intention to implement a specific design, Architecture 2 in this figure, will how much achievement/satisfaction each of the stakeholders have for their (conflicting) goals.

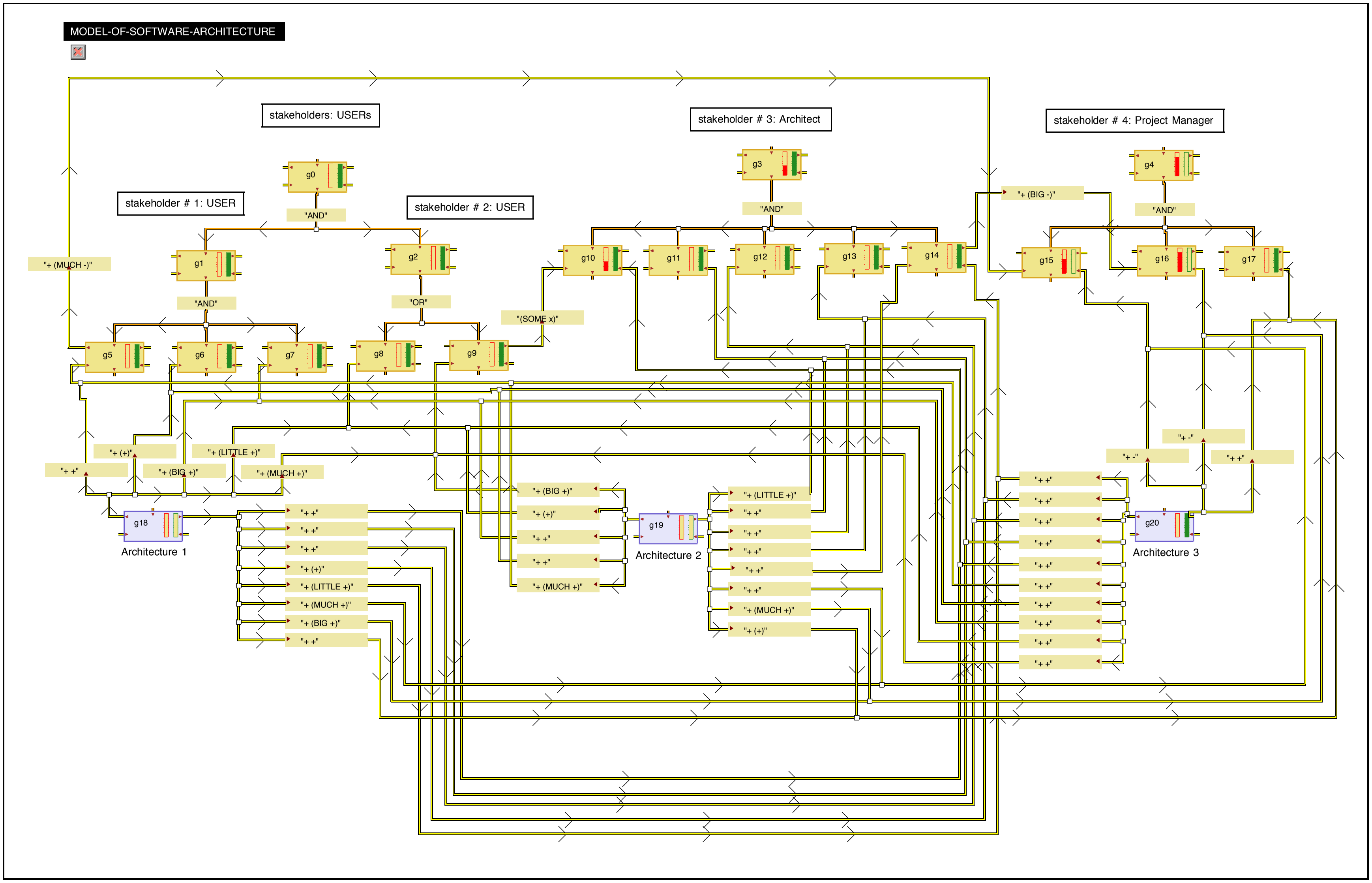


Figure 5.13: A Model of a Collaborative Multi-Stakeholder System/Software Requirements Decision Making Case. Mobilizing an intention to implement a specific design, Architecture 3 in this figure, will how much achievement/satisfaction each of the stakeholders have for their (conflicting) goals.

6. Sensitivity and What-If Analysis: The analyst at this step will test variations to the value properties or constructs of the decision makers' individual GCMs, or the integrated decision making situation as a whole. In this simple example, one cannotice that the three alternatives that the base-model have all show some serious weakness. For example, Alternative 3, the best overall performing alternative out of the three ones the base-model includes, is shown in Figure 5.13 to satisfy most of the goals of the system users and the technical architect of the project, but fail to meet any of the goals that the project manager value (most likely goals such as "meeting delivery timeline", "keep implementation cost within the approved budget", or "have all development skills/resources needed to implement and deliver the system"). In such case, which is not far from what real-life requirements projects face, the group has to rethink their alternatives, come up with other creative alternatives, break the system/project to smaller systems/projects, ask for a bigger budget and/or more resources, etc. The group, with the help of the analyst, generate a what-if variation model, for each of the other alternatives and/or creative solutions they come up with. They should, then, test each what-if scenario model looking for the one that offer the best fit architecture they all agree on.

This collaborative decision making situation shows that even in these situations the collaborative agents, such as the stakeholders in the example have (users, architect, and project manager), have different and conflicting goals. Despite the fact that we intentionally simplified the example and its model for clear illustration purposes, the example and its model (presented in Figures 5.11, 5.11 and 5.11) show the impact that different architectures/products have on satisfying each of the stakeholders' goals (requirements), in addition to the affect that different goals, different stakeholders have, have on each others' achievement or prevention.

But, in our experience, the best benefit these models provide is that they show in a consolidated visual form the effect of the different needs the agents have on each other. These models, then, serve as tools to guide the collaborating decision makers to negotiate and/or rethink their alternatives. They serve as tools to highlight the challenges some areas in the situation have, and where creative thinking must be applied to overcome such challenges. In the many experiments we had, using the Constrained Rationality framework to model and analyze collaborative decision making cases, we have seen that the sensitivity and what-if analysis step, and the following remodelling or retuning needed after that, become the area where most of the involved agents realized the strength and fruits of the framework and its methods and tools.

5.8 Summary

In this chapter, we discussed how to model collaborative and competitive multi-agent decision making situations; and took further the concepts and methods, introduced in the previous two chapters, showing how they are applied to determine the preferences of agents in collaborative and competitive multi-agent decision making situations. The chapter started with discussing the two main modes/types of multi-agent decision making situations, the collaborative decision making situations and the competitive conflicts modes. Then, it introduced the process to model and analyze both types.

It showed how the different agents' viewpoint models be modelled and integrated ; how the agents' alternatives will be modelled, in both collaborative and competitive situations; how the situation's states are defined, for the competitive conflicts; and how the base-model of the decision making situation is to be validated and finalized. The chapter then moved to discuss modelling priorities, capturing emotions, and eliciting agents' preferences within both types of multi-agent decision making situations; and how the framework models the dynamics of multi-agent situations, specifically the different moves that agents have in competitive conflicts.

The chapter discussed how these different types of moves are used to identify three main types of multi-agent competitive decision making conflicts: 1) non-cooperative; 2) cooperative with no coalitions/alliances (i.e. agents cooperate but still act individually and do not sacrifice their individual positions for the good of the group); and 3) cooperative conflicts where agents are allowed to form coalitions, act as groups and take multi-step moves including intermediate temporarily scarifies along the way for the good of all members of the group they belong to. We said that these three types of conflicts will be used to structure and organize our discussion about modelling and analyzing multi-agent conflicts, and what concepts and tools the Constrained Rationality framework offers to deal with the specific needs of modelling and analyzing these conflicts.Each of the following four chapters form a separate and dedicated chapter to discuss one of these conflict type, including the concepts and methods to be used to model and analyze that type.

Next, the chapter discussed the process of analyzing multi-agent decision making situations; how the agents' preferences over alternatives and states are calculated; how the stabilities of the different states in these conflicts are determined (for each agent) and how equilibriums are identified; how sensitivity and what-if analysis is conducted; and finally, why the analyst should compare the produced analysis and predictions to what actually happen/happened in real-life. At the end, the chapter provided an illustrative example that showed how the process, concepts and methods introduced in this chapter are used to model and analyze a collaborative multi-agent decision making situation: a system requirements engineering situation. The chapter did not provide any example for competitive conflict situations, because the next few chapters will provide detailed case studies that cover all three types of conflicts: non-cooperative conflicts, cooperative conflicts without coalitions, and cooperative conflicts with coalitions.

Chapter 6

Non-Cooperative Strategic Conflicts: Analysis and Stability Solution Concepts

6.1 Introduction

This chapter discusses the analysis of non-cooperative games, as per the Constrained rationality framework. The decision makers of this type of games are able to make move types that are unilateral non-cooperative moves. In other words, there will be no consideration of any type of cooperative moves. Such moves will be covered in cooperative games, with and without coalitions, in the following chapters.

We will start by looking at the type of moves the players of these games are allowed to make, and are important to the stability analysis concepts. Then, we will define four different stability and equilibrium solution concepts. These concepts will guide the stability analysis of each of the games' states, for each of the games' players. Next, we will define the strength of the stability under such solution concepts, and propose a set of algorithms to help identify the strength level of each of these stabilities.

We will finish the chapter with a case study in which we apply the concepts proposed in this chapter. In this case study, we analyze thoroughly the Cuban Missile Crises. We start by giving a brief background on this important political conflict. We then model the players goals, constraints and alternatives; analyze their GCMs; identify the conflict's states; elicit the players' cardinal and ordinal preferences over these states; and then identify the players unilateral moves among these states. The stabilities of the conflict's states will be analyzed under the four stability solution concepts, and the strength of these stabilities will be identified. Then, we will look at the over all equilibrium states for the conflict; and how the conflict could have progressed from the time the US discovered the missile bases in Cuba. We conclude the case study by showing how our analysis results compares to what historically happened in the conflict, and to what others offered as models and analysis to the conflict, after the fact.

A reminder on the notation which will be used in this chapter. Let the set of all the game states be given as $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where $|\mathcal{S}| = m$ the total number of states in the game, and states are defined as discussed earlier. And let $\mathcal{S}^{a,b,\ldots}_{D,t} \subseteq \mathcal{S}$ where $\mathcal{S}^{a,b,\ldots}_{D,t}$ represents a subset of $\mathcal{S}$'s states which has common characteristics described in the subset's notation as $a, b, \ldots$ as been perceived by decision maker D and at time t. The set of decision makers in the game is given as $\mathcal{DM} = \{DM_1, DM_2, \ldots, DM_n\}$, where $|\mathcal{DM}| = n$ the total number of decision makers, the involved agents/players, in the game.

As an additional reminder, and in relation to the notation used in this chapter and the following ones, we use the terms game and conflict interchangeably to mean the same thing: a multi-agent strategic conflict. Also, the terms agent, player and decision maker will be used interchangeably to mean the same thing: an autonomous independent agent, in the strategic conflict, who is capable of perceiving the world around, holding beliefs, justifying beliefs, holding knowledge, representing knowledge, extracting new knowledge, reasoning about held knowledge, and acting independently.

6.2 Types of Decision Makers' Moves

Decision makers in non-cooperative games are only allowed to have individual unilateral moves. First, we will define these unilateral moves. Then, we will define the type of sanction moves that players can do to block certain other players from benefiting from any unilateral improvement moves they have. Understanding these types of players' moves is essential to define the stability solution concepts which will be used to analyze the stability of the games' states for the games' players.

6.2.1 Types of Non-Cooperative Moves by Individual DMs

There are two types of movements that an individual decision maker, alone and non-cooperatively, can make in a non-cooperative game: Unilateral Moves (UMs) and Unilateral Improvements (UIs). We said in the previous chapter, in Section 5.5.1, that we follow the steps of Fraser and Hipel (1984) and Fang et al. (1993) in their definition of both UI and UM moves for individual agents. But, we define here both type of moves to be within the context, terminology and notation of the Constrained Rationality framework. We also provide a game knowledge structure and an algorithm to generate the UIs of all agents in the conflict from a given list of all their UMs and calculated preferences over the conflict's states.

Definition 6.2.1 (**Unilateral Move (UM)**)**:** *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and state $s \in \mathcal{S}$ of the game: a move to state $s_{um} \in \mathcal{S}$ is considered a Unilateral Move (UM) for DM_i at time t from state s, denoted as $s_{um} \in \mathcal{S}^{UM}_{DM_i,\,t}(s)$, iff DM_i can move unilaterally from state s to state s_{um} in one move, reaching s_{um} at time $t{+}1$.*

Definition 6.2.2 (**Unilateral Improvement (UI)**)**:** *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and state $s \in \mathcal{S}$ of the game: a move to state $s_{ui} \in \mathcal{S}$ is considered a Unilateral Improvement (UI) for DM_i at time t from state s, denoted as $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$, iff 1) $s_{ui} \in \mathcal{S}^{UM}_{DM_i,t}(s)$; and 2) $s_{ui} \succ^{L_{PS}}_{DM_i t} s : L_{PS} > N$, i.e. when $PrefStrength(s_{ui}, s, DM_i, t) > None$.*

Evidently, $\mathcal{S}^{UI}_{DM_i,t}(s) = \mathcal{S}^{UM,>N}_{DM_i,t}(s)$; and $\mathcal{S}^{UI}_{DM_i,t}(s) \subseteq \mathcal{S}^{UM}_{DM_i,t}(s) \subseteq \mathcal{S}$.

Now, let each game has a *Game Configuration Structure*, referred hereafter to it as *Game-Structure*. This data structure provides essential initial information about the game and its players, all organized and in a computerized DSS system is written in a file structure. The Game-Structure for an Non-Cooperative Game must have the following information:

- $\mathcal{S}$: the set of all states in the game (as perceived, known -believed true with justification- and elicited from the known players' alternatives by the focal Decision Maker whom the modeller is capturing/representing his knowledge about the game);
- $\mathcal{DM}$: the set of all DMs in the game;

- $\mathcal{S}^{UM}_{DM_i,t}(s)$ for every $DM_i \in \mathcal{DM}$ and for every $s \in \mathcal{S}$: given as a set of graphs describing the UMs that DMs have from each state the game has (one graph per DM in the game, with the game's states represented as the graph's nodes and the UMs are represented as its arcs); and

- $WP(s, DM_i, t)$ for every $DM_i \in \mathcal{DM}$ and for every $s \in \mathcal{S}$: the weighted (cardinal) preference of each DM over each of game's states, as calculated using the Constrained Rationality Framework's agents preferences elicitation and modelling method discussed in Chapter 5.

A Game-Structure will describe the game at a specific point of time t, as perceived and known by the focal decision maker whom the game is modelled based on his knowledge of it. Any updates or changes to what is known about the game by the focal decision maker should initiate a generation of a new Game-Structure to reflect the changes; and a new analysis of the updated game, treating the structure as a new game.

One important step of analyzing a game is to generate the the UIs that DMs will have from each state of the game. Given the Game-Structure for an Non-Cooperative Game, Algorithm 6.1 is proposed to generate the UM and UI sets for all DMs in the game.

6.2.2 Types of Sanction Moves

Sanction moves are moves that are made to block certain decision maker from benefiting from a UI move that he made. Other players in the game could decide to sanction this UI, by moving the game to a state which is less or equally proffered than the original state the decision maker made his UI from. In effect, the sanction move by the other player renders the UI made useless, or in the worst case scenario harmful because it could make the game at a state that is worse than the original state that the decision maker took a UI move from. The concept of a sanction move was initially proposed by Howard (1971), and is essential to some of the stability solution concepts games could have, as we will discuss later. Again here, we follow the steps of Howard (1971), Fraser and Hipel (1984) and Fang et al. (1993) in their definition of sanction moves for individual agents. But, we define them here to be within the context, terminology and notation of the Constrained Rationality framework.

Algorithm 6.1 Generating the UM and UI Sets for all DMs in a Game

void Generate_DMs_UM_and_UI_Sets (Game-Structure)

// Game-Structure file starts with empty UM and UI sets for DMs.
// Only, $\mathcal{S}$, $\mathcal{DM}$, and the UM graph for each DM are given.
// A UM graph for a DM, has $\mathcal{S}$'s states as vertices/nodes of the graph,
// while the directed arcs of the graph represent DM's UMs in the game.
//

for all $DM_i \in \mathcal{DM}$ **do**
  // Generate DM_i's UM Sets (one for each of the game's states), and
  // for each state find the DM_i's UI Set. All these UM and UI sets
  // will be initially empty. If not, empty them [not included here].
  // And, by the end, some of these UM/UI sets will be empty sets.

  for all $s \in \mathcal{S}$ **do**
    for all s_{um} destination state of each arc DM_i has out of s **do**
      $\mathcal{S}^{UM}_{DM_i,t}(s) = \mathcal{S}^{UM}_{DM_i,t}(s) \cup \{s_{um}\}$
      // moving to s_{um} from s is also considered a UI for DM_i
      // if and only if $s_{um} \succ^{L_{PS}}_{DM_i t} s : L_{PS} > N$
      if $PrefStrngth(s_{um}, s, DM_i, t) > None$ **then**
        $\mathcal{S}^{UI}_{DM_i,t}(s) = \mathcal{S}^{UI}_{DM_i,t}(s) \cup \{s_{um}\}$
      end if
    end for
  end for
end for
Add all generated UM and UI sets for each DM, for each state, to the Game- Structure file.
return

Definition 6.2.3 (Sanction Move (SM)): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, a UI by DM_i to state $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$ is said to have against it a Sanction Move (SM) at time $t+1$ to state $s_{sm} \in \mathcal{S}$ iff* $\exists\, ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{ui}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$.

As per the definition, a move to s_{ui} by DM_i is said to be *sanctioned/able* by DM_j's move to s_{sm}; and DM_j's move to s_{sm} is called a Sanction Move (SM) against DM_i's UI to s_{ui}. In addition, it is important to notice that the definition of an SM does not assume that the SM by $DM_j \in \{\mathcal{DM} - DM_i\}$ to state s_{sm} from state s_{ui} be a UI move by DM_j. In other words, DM_j's move to s_{sm} will be considered a Sanction Move even if $s_{sm} \notin \mathcal{S}^{UI}_{DM_j,t+1}(s_{ui})$. The definition assumes that DM_j's motive for the SM is to hurt DM_i and sanction DM_i's UI move to s_{ui}, even if this SM will put DM_j himself at a less preferred state.

To differentiate between an SM by DM_j which is not a UI for DM_j, as per the

SM definition above, and an SM by DM_j which is also a UI move for DM_j, we will call the second SM type: SMI move (read as Sanction Move and Improvement) by DM_j. As we will see later in the chapter, this *stricter* type of SMs, or SMI as we decided to call it, is required for some stability solution concepts, such as Sequential Stability (SEQ).

Definition 6.2.4 (**Inescapable Sanction Move (ISM)**): *For Decision Maker* $DM_i \in \mathcal{DM}$ *at time* t *and at state* $s \in \mathcal{S}$ *of the game, a UI by* DM_i *to state* $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$ *is said to have against it an Inescapable Sanction Move (ISM) at time* $t+1$ *to state* $s_{ism} \in \mathcal{S}$ *iff* $\exists\ ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_{ism} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{ui})))$: $[(s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \wedge (\forall s_{esc} \in \mathcal{S}^{UM}_{DM_i,t+2}(s_{ism}) \quad s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s))]$.

As per the definition, DM_i's UI move to s_{ui}, from s, is said to have against it *an inescapable sanction move* (ISM) by DM_j's move to s_{ism}, because DM_i has no move away from s_{ism} by which he will be able to mitigate, or lessen, the negative effect which DM_j's sanction to s_{ism} left DM_i in.

6.3 Stability Solution Concepts and Equilibriums for Non-Cooperative Conflicts

Two classes of solution concepts will be discussed in this chapter: 1) solution concepts that are extremely individualistic and shortsighted in their definitions, in a way that they do not consider other players countermoves; and 2) solution concepts that tries to include other players' countermoves, therefore these concepts show more foresight.

6.3.1 Solution Concepts with No Consideration to Others' Moves

One solution concept that falls under the class of solution concepts with no consideration to other players' countermoves will be covered in this subsection. This solution concept is Nash Stability, and is defined as follows:

Definition 6.3.1 (**Nash Stability (NASH)**): *For Decision Maker* $DM_i \in \mathcal{DM}$ *at time* t*, state* $s \in \mathcal{S}$ *is considered a Nash Stable (NASH) state, denoted as* $s \in \mathcal{S}^{NASH}_{DM_i,t}$*, iff* $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$.

As per the definition, state s is considered NASH stable for a decision maker DM_i at time t if and only if s is the best that DM_i can achieve at time t, given the total states of the game $\mathcal{S}$. Therefore, states that are not NASH stable are unstable states since DM_i can unilaterally improve his position from any one of them.

Despite the considerable appeal in the notion that a state is a NASH state for a certain decision maker, it is widely accepted that Nash stability represents a very shortsighted understanding of game playing in real-life conflicts. This is because it does not take into account the possible responses of other decision makers to the unilateral improvement made by the player. Howard (1971) argued that not only such reactions would be important to decision makers, but that they would take such considerations into account in advance of choosing their moves/alternatives in the game. His metagame analysis formalized these considerations into two additional stability solution concepts: General MetaRationality (GMR) and Symmetric MetaRationality (SMR). Both solution concepts imply that a decision maker can find certain states not to be NASH and yet find them to be stable nonetheless. Howard in his new two stability concepts used the idea of a *Sanction Move* (Howard (1971)), which we defined in the previous section.

6.3.2 Solution Concepts with Consideration to Others' Moves

We will discuss here three stability solution concepts consider in their definitions other players' moves and countermoves. These solution concepts are: General MetaRationality, Symmetric MetaRationality and Sequentially Stability. We follow the steps of Howard (1971), Fraser and Hipel (1984) and Fang et al. (1993) in their definition of these stability solution concepts, but we redefine them here to be within the context, terminology and notation of the Constrained Rationality framework, as well as using the definition of the agents' unilateral moves and sanction moves provided earlier.

Definition 6.3.2 (General MetaRational (GMR) Stability): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a General MetaRational (GMR) Stable state, denoted as $s \in \mathcal{S}^{GMR}_{DM_i,t}$, iff $\forall s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)$ $[\exists\, ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,]$.*

As per the definition, a state s is called GMR Stable for DM_i at time t if and only if there exists an SM against every UI that DM_i has out of s. In other words,

DM_i cannot in any way reach a more preferred position/state, to the current state s he is in, by using any of his UIs out of s, because other players will sanction his UIs. The GMR stability solution concept assumes that decision maker DM_i believes that other players *surely* would apply a sanction against any of his UIs out of s. Therefore, he will not move away from s, and s is a GMR stable state for him.

Definition 6.3.3 (**Symmetric MetaRational (SMR) Stability**): *For Decision Maker* $DM_i \in \mathcal{DM}$ *at time* t*, a state* $s \in \mathcal{S}$ *is considered a Symmetric MetaRational (SMR) Stable state, denoted as* $s \in \mathcal{S}^{SMR}_{DM_i,t}$*, iff* $\forall s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s) \;\; [\exists((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1))) : (s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))]$.

As per the definition, a state s is called SMR Stable for DM_i at time t if and only if there exists an inescapable sanction ISM against every UI that DM_i has out of s. In other words, DM_i cannot in any way reach a more preferred position/state, to the current state s he is in, by using any of his UIs out of s; and DM_i will not be able to mitigate the sanction moves. DM_i will not be able to reach a preferred state (to s) by moving away unilaterally from the less-preferred states result from the sanctions that other players can impose on his original UI move away from s.

SMR stability solution concept assumes that decision maker DM_i believes that other players *surely* would apply a sanction against any of his UIs out of s, and this sanction is an *inescapable* one: DM_i will not be able to benefit from any move away from the state produced by the sanction move. Therefore, DM_i will not move away from s, and s is SMR stable for him. SMR takes into consideration one further move in the game (the possible countermove -but not helpful one- by DM_i after the sanction) than what GMR considers. GMR does not consider whether DM_i have countermoves after the sanction to mitigate the sanction's effect.

The metarationality GMR and SMR solution concepts, as proposed by Howard (1971) and defined here, do not require the sanction moves imposed on the decision maker's UI away from the "GMR/SMR stable" state s be a UI for any of the players committing a sanction (or inescapable sanction) move. This is interesting, because Howard's proposed definitions for the metarationality concepts assume a completely rational DM_i who only moves if and only if he has a UI and surely will end up in a better position/state, but DM_i's opponents could be "irrational" players who might move to less-proffered states, for them, just to hurt DM_i and sanction his UIs. Fraser and Hipel (1984) proposed a stability concept that assumes all decision makers act consistently in a rational way even when sanctioning each other moves. In other words, sanctions must be UI moves by the imposing parties. The following is the definition of this stability solution concept.

Definition 6.3.4 (**Sequentially Stability (SEQ)**): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a Sequentially (SEQ) Stable state, denoted as $s \in \mathcal{S}^{SEQ}_{DM_i,t}$ iff $\forall s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)$ $[\exists((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)]$.*

As per the definition, a state s is SEQ stable for DM_i at time t if and only if there exists a UI by another decision maker sanctions every UI that DM_i makes out of s, i.e. render DM_i's UIs to be not beneficial for DM_i. DM_i cannot in any way reach a position/state more preferred to the current state s he is in, by using any of his UIs out of s, because other players will be able to sanction his UIs with SMI sanction moves, putting DM_i at the end into a state which is either less preferred, or equally preferred, to the original state s he started from.

SEQ is similar to GMR, but has one important difference. SEQ requires the sanction moves by other players to be SMI moves, not just merely SMs (as the GMR definition requires). SEQ stability solution concept assumes that decision maker DM_i believes that other players *surely* would apply a UI move they have to sanction against any of his UI moves out of s. Therefore, he will not move away from s, and s is a SEQ stable state for him.

6.3.3 Equilibrium States in Non-Cooperative Games

The concept of an Equilibrium states is an important concept in game theory and conflict analysis. Equilibrium is tied to the concept of Stability Solution Concepts, and explains ultimate stability states in the game. We use here the commonly used definition for an Equilibrium state, but we will add to it in the next section strength levels to describe the level of strength the overall stability the Equilibrium has. For now, here is the basic definition of an Equilibrium.

Definition 6.3.5 (**Equilibrium (EQ.)**): *A state $s \in \mathcal{S}$ is considered an Equilibrium for a non-cooperative game, at time t, under a specific Solution Concept SC definition, denoted as $s \in \mathcal{S}^{SC\ EQ.}_{\mathcal{DM},t}$, if and only if $\forall\, DM_i \in \mathcal{DM}$ $s \in \mathcal{S}^{SC}_{DM_i,t}$.*

As per the definition, a state s is stable for the game as a whole, i.e. an equilibrium, under a specific solution concept, such as NASH or GMR, if and only if the state s is stable under this solution concept for each and every decision maker in the game.

In any specific game/conflict, there may be a number of equilibrium states under one or more stability solution concepts. Equilibrium states represent the most likely outcomes for the game, and constitutes possible resolutions to the game. Once one of these states arise, this state is likely persist. But, the strength of this persistence depends on which solution concept the equilibrium state is under, and what is the strength of the state's stability under this solution concept for each of the DMs in the game.

6.4 Stability Strength of Solution Concepts and Equilibriums for Non-Cooperative Conflicts

In the first subsection, we will define the concept of a measure of "strength" for solution concepts. Then, we will discuss the mechanisms by which one can identify the strength of the stability for any given state for any given DM under specific solution concept. In the second subsection, we will discuss the strength of an equilibrium under a specific solution concept for a state in a non-cooperative conflict.

6.4.1 Stability Strength of Solution Concepts

Let the *Stability Concept Strength* a value property, denoted as StabilityStrength($StabilityConcept, s, DM_i, t$), be given a fuzzy linguistic value label L_{SS} based on the fuzzy memberships functions given in Figure 6.1. The strength expressed by the L_{SS} fuzzy label is meant to represent the strength of the stability concept type *StabilityConcept* (where $StabilityConcept \in \{NASH, GMR, SMR, SEQ\}$) for state s, for decision maker DM_i at time t.

As we will see from the definitions for the stability strength of the different solution concepts, the StabilityStrength value property, for a specific state for a specific DM, is tied to the strength of the preference of the state the player ends with in comparison to the state the state he started from. And as we discussed in Section 5.4.2 of the previous chapter, preferences' strengths when not normalized before fuzzification have numeric values in the range $[-2, 2]$, and the fuzzy label equivalent will be between *Extremely Preferred* and *Extremely Less-Preferred.* If the preferences' strength are normalized before fuzzification, then its numeric value will be in the range $[-1, 1]$, and the fuzzy label equivalent will be between *Fully Preferred* and *Fully Less-Preferred.* The stability strength, as we will see later in this section covers only the positive numeric value scale $[0, 2]$, before fuzzification and without normalization; and the scale $[0, 1]$ before fuzzification and with normalization.

When the StabilityStrength value property before fuzzification is not normalized, i.e. its numeric value is in $[0, 2]$, then its fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*,*Little*, *None*, *Null*$\} = \{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. And, with the complete order of $Ex > Si > St > F > B > M >$!$Mo > L > N > Null$, where the labels range from representing *Extreme*ly strong stability of s (based on the definition of the solution concept given in *StabilityConcept*) to *None* strength level for s (meaning very weak stability strength and close to non-existing strength or close to indifferent). When StabilityStrength, before fuzzification, is normalized, i.e. its numeric value is in $[0, 1]$, then its fuzzy labels will include the same labels as above with the exception of the three higher labels covering the range from $[1, 2]$ and those are: *Extreme*,*Significant* and *Strong*.

The L_{SS} fuzzy label assigned to $\mathit{StabilityStrength}(\mathit{StabilityConcept}, s, DM_i, t)$ will cover the stability strength satisfaction levels, with the understanding that the *Null* label represents an unknown stability strength or totaly-non-existing-stability. The fuzzy membership functions defining these stability strength's linguistic value labels are given in Figure 6.1. The figure shows the membership functions for each label's fuzzy set to be trapezoidal in shape, for simplicity only (not as a restriction) as indicated in Al-Shawa and Basir (2010). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements, as we indicated in Al-Shawa and Basir (2009, 2010).

It is worth mentioning here that the following set of definitions for strength of stability, under each of the four stability solution concepts defined above, are different from what others provide. For example, traditional stability solution concepts definitions provided for GMCR by Fang et al. (1993) decide only whether a conflict's state is stable or not-stable under a certain stability concept, i.e. no strength of stability is provided. These GMCR definitions were later extended by Hamouda et al. (2004), adding two strength levels for a state's stability under a certain solution concept, and by Xu et al. (2009), subsequently generalizing it to two or more levels. But all these GMCR stability-strength extended-definitions are based on given pre-determined agents' preferences that are set upfront, i.e. unlike the Constrained Rationality preferences, these preferences are not calculated as done here, and could not be verified nor validated. On the other hand, the stabilities' strengths that Constrained Rationality defined here all can be easily calculated, verified, val-

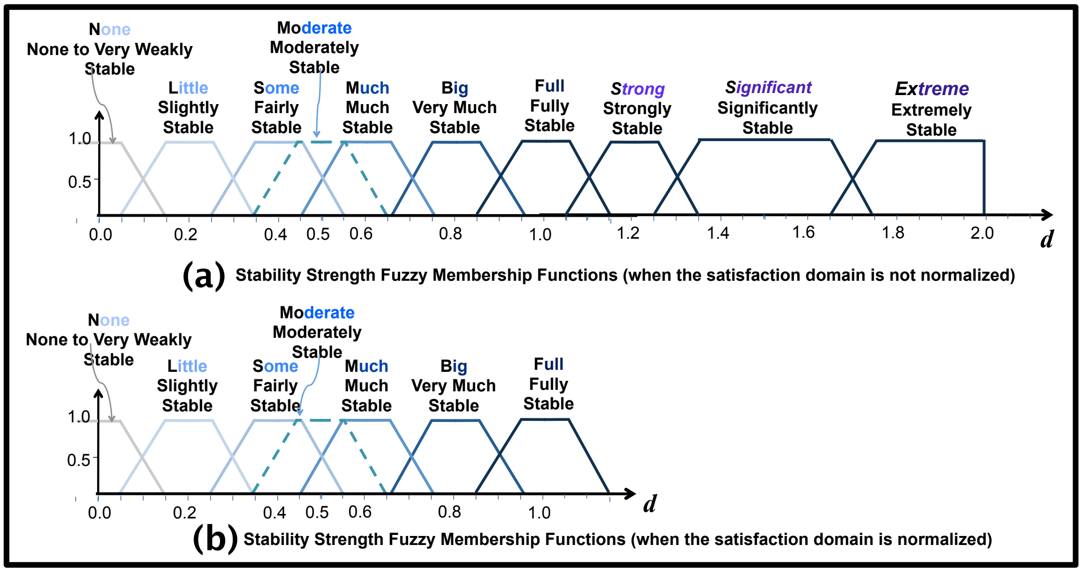


Figure 6.1: The membership functions of the Fuzzy Sets dividing the satisfaction levels domain of the *StabilityStrength* value property: (a) when the value property is not normalized; and (b) when it is normalized

idated, and tied to the players' motives and constraints. Changes in any of the contextual information about the conflict and its players, will be reflected on the stability strength of the conflict's states for the agents.

Now, we define the stability strength, for non-cooperative games, for each of the solution concepts we introduced in the previous section.

Definition 6.4.1 (Strength of NASH Stability): *For decision maker DM_i at time t, and for a NASH stable state $s \in \mathcal{S}^{NASH}_{DM_i,t}$, the strength of s's NASH stability, to DM_i at time t, i.e. $StabilityStrength(NASH, s, DM_i, t)$, is calculated as follows:*

$$(\forall s_{bfr} : \quad s \in \mathcal{S}^{UM,\geq N}_{DM_i,t}(s_{bfr}))$$
$$(StabilityStrength(NASH, s, DM_i, t) = |\max_{s_{bfr}}\{PrefStrength(s_{bfr}, s, DM_i, t), -Extreme\}|)$$

As per the definition, the strength of s's NASH stability strength is the positive strength equivalent of the negative preference of the state that the worst UI move executed/could-be-executed by DM_i at time $< t$ in order to move to s. Let the NASH's stability strength of a state s for DM_i at time t be denoted as $NASH(L_{SS})$, where $StabilityStrength(NASH, s, DM_i, t){=}L_{SS}$. Algorithm 6.2 uses Definition 6.4.1 to calculate the NASH's stability strength and assign the strength's fuzzy linguistic label.

Algorithm 6.2 Calculating a State's NASH Stability Strength for a DM in a Non-Cooperative Game

***strength-value-label Strength_of_NASH_Stability** (s, DM_i, Game-Structure)*
// start with the assumption that s is Not NASH stable
NASH_Strength = Null
// check if DM_i has any UIs from s at time t
if $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ **then**
    //s is NASH Stable State for DM_i at t. Now, find the NASH stability's strength
    NASH_Strength = Strength_of_Nash(s, DM_i, *Game-Structure*)
end if
return *NASH_Strength*

strength-value-label Strength_of_Nash(s, DM_i, Game-Structure)
// this routine will return the strength of the weakest UI, by DM_i, that yields to reaching s
// set Nash strength initially to "Extremely Strong" (this will be the case if s has no UIs
// that leads to it).
$Strength = -Extreme$
// find s's NASH strength
for all $s_{bfr} : s \in \mathcal{S}^{UM,\geq N}_{DM_i,t}(s_{bfr})$ **do**
    $Strength = max\{Strength, PrefStrength(s_{bfr}, s, DM_i, t)\}$
end for
// return the equivalent positive strength label, if *Strength<N*
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return *Strength*

Definition 6.4.2 (Strength of GMR Stability): *For DM_i at time t, and for a GMR stable state $s \in \mathcal{S}^{GMR}_{DM_i,t}$, the strength of s's GMR stability, to DM_i at time t, , i.e. StabilityStrength*(GMR, s, DM_i, t)*, is calculated as follows:*

$$(\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \quad \rightarrow \quad (\forall s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s))$$
$$[\forall (DM_j \in \{\mathcal{DM} - DM_i\}) \forall (s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{ui}) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$$
$$(\mathit{StabilityStrength}(GMR, s, DM_i, t) =$$
$$|\max_{s_{ui}} \{\min_{s_{sm}} \{PrefStrength(s_{sm}, s, DM_i, t), None\}, -Extreme\} |)]$$

And,

$$(\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \quad \rightarrow \quad \mathit{StabilityStrength}(GMR, s, DM_i, t) = None$$

As per the definition, the strength of the GMR stability of s is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs from s, given the fact that DM_i will choose the UI that will yield the best less-preferred end state.

Let GMR's stability strength of a state s for DM_i at time t be denoted as $GMR(L_{SS})$, where StabilityStrength(GMR, s, DM_i, t)=L_{SS}. Algorithm 6.3 uses Definition 6.4.2 to calculate the GMR's stability strength and assign the strength's fuzzy linguistic label.

Algorithm 6.3 Calculating a State's GMR Stability Strength for a DM in a Non-Cooperative Game

---

strength-value-label Strength_of_GMR_Stability **(**s**,** DM_i**,** *Game-Structure***)**
// start with the assumption that s is not GMR stable
$GMR\_Strength = NULL$
// check if DM_i has any UIs from s at time t
if $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ **then**
  //s is NASH Stable State for DM_i at t
  $GMR\_Strength = None$
else if $(\forall s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s))$ [$\exists$ an SM sanction] **then**
  // sanction exists against each of DM_i's UIs $\Rightarrow$ s is GMR stable; find GMR's strength.
  $GMR\_Strength = Strength\_of\_Sanctions(s,\ DM_i,\ Game\text{-}Structure)$
end if
return $GMR\_Strength$

strength-value-label Strength_of_Sanctions*(**s**,** DM_i**,** *Game-Structure)**
// this routine will return the strength of the sanction that yields the worst result for DM_i,
// given the fact that DM_i will choose the UI that will minimize his loss.
// set sanction's end state strength initially to "Extremely Less Preferred"
$Strength = -Extreme$
// find s's GMR strength
for all $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$ **do**
  $SancStrength = None$
  for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
    for all $s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{ui}) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
      $SancStrength = min\{SancStrength, PrefStrength(s_{sm}, s, DM_i, t)\}$
    end for
  end for
  $Strength = max\{Strength, SancStrength\}$
end for
// return the equivalent positive strength label, if $Strength{<}N$
if $Strength < None$ **then**
  $Strength = |Strength|$
end if
return $Strength$

Definition 6.4.3 (**Strength of SMR Stability**): *For DM_i at time t, and for a SMR stable state $s \in \mathcal{S}^{SMR}_{DM_i,t}$, the strength of s's SMR stability, to DM_i at time t, , i.e. StabilityStrength(SMR, s, DM_i, t), is calculated as follows:*

$$
\begin{aligned}
(\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \quad \rightarrow \quad & (\forall s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)) \\
& [\forall(DM_j \in \{\mathcal{DM} - DM_i\})\forall(s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{ui}) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \\
& [(\forall s_{cm} \in \mathcal{S}^{UM}_{DM_i,t+2}(s))(StabilityStrength(SMR, s, DM_i, t) = \\
& \qquad |\max_{s_{ui}}\{\min_{s_{sm}}\{\max_{s_{cm}}\{PrefStrength(s_{sm}, s, DM_i, t), \\
& \qquad PrefStrength(s_{cm}, s, DM_i, t)\}, None\}, -Extreme\}|)]]
\end{aligned}
$$

And,

$$(\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \quad \rightarrow \quad StabilityStrength(SMR, s, DM_i, t) = None$$

As per the definition above, the strength of the SMR stability of s is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs from s, given the fact that DM_i will choose the UI that will yield the best less-preferred end state after the counter move that he has to mitigate the sanctions.

Let SMR's stability strength of a state s for DM_i at time t be denoted as $SMR(L_{SS})$, where StabilityStrength(SMR, s, DM_i, t)=L_{SS}. Algorithm 6.4 uses Definition 6.4.3 to calculate the SMR's stability strength and assign the strength's fuzzy linguistic label.

Definition 6.4.4 (**Strength of SEQ Stability**): *For DM_i at time t, and for a SEQ stable state $s \in \mathcal{S}^{SEQ}_{DM_i,t}$, the strength of s's SEQ stability, to DM_i at time t, i.e. StabilityStrength(SEQ, s, DM_i, t), is calculated as follows:*

$$
\begin{aligned}
(\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \quad \rightarrow \quad & (\forall s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)) \\
& [\forall(DM_j \in \{\mathcal{DM} - DM_i\})\forall(s_{sm} \in \mathcal{S}^{UI}_{DM_j,t+1}(s_{ui}) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \\
& (StabilityStrength(SEQ, s, DM_i, t) = \\
& \quad |\max_{s_{ui}}\{\min_{s_{sm}}\{PrefStrength(s_{sm}, s, DM_i, t), None\}, -Extreme\}|)]
\end{aligned}
$$

And,

$$(\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \quad \rightarrow \quad StabilityStrength(SEQ, s, DM_i, t) = None$$

As per the definition above, the strength of the SEQ stability of s is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs from s, given the fact that DM_i will choose the UI that will yield the best less-preferred end state. But recall here that as per Definition 6.3.4, for SEQ stability to be established the sanctions

Algorithm 6.4 Calculating a State's SMR Stability Strength for a DM in a Non-Cooperative Game

strength-value-label Strength_of_SMR_Stability $(s,\ DM_i,\ \textit{Game-Structure})$
// start with the assumption that s is not SMR stable
$\textit{SMR\_Strength} = \textit{NULL}$
// check if DM_i has any UIs from s at time t
if $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ **then**
    //s is NASH Stable State for DM_i at t
    $\textit{SMR\_Strength} = \textit{None}$
else if $(\forall s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s))$ [$\exists$ an inescapable sanction ISM] **then**
    // inescapable sanction exists against each of DM_i's UIs $\Rightarrow$ s is SMR stable; strength?
    $\textit{SMR\_Strength} = \textit{Strength\_of\_Inescapable\_Sanctions}(s,\ DM_i, \textit{Game-Structure})$
end if

return $\textit{SMR\_Strength}$
strength-value-label Strength_of_Inescapable_Sanctions $(s,\ DM_i,\ \textit{Game-Structure})$
// this routine will return the strength of the ISM that yields the worst result for DM_i,
// given the fact that DM_i will choose a UI and a counter move that will minimize his loss.
// Set sanction's strength (after DM_i's counter move) initially to "Extremely Less Preferred"
$\textit{Strength} = -\textit{Extreme}$
for all $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$ **do**
    $\textit{ISancStrength} = \textit{None}$
    for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{ui}) : ((s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \wedge (\forall s_{cm} \in \mathcal{S}^{UM}_{DM_i,t+2}(s_{sm})\ s_{cm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$ **do**
            $\textit{ISancStrength} = \textit{min}\{\textit{ISancStrength}, \textit{PrefStrength}(s_{sm}, s, DM_i, t)\}$
            $\textit{CntrStrength} = -\textit{Extreme}$
            for all $s_{cm} \in \mathcal{S}^{UM}_{DM_i,t+2}(s_{sm})$ **do**
                $\textit{CntrStrength} = \textit{max}\{\textit{CntrStrength}, \textit{PrefStrength}(s_{cm}, s, DM_i, t)\}$
            end for
            if $\textit{ISancStrength} < \textit{CntrStrength}$ **then**
                $\textit{ISancStrength} = \textit{CntrStrength}$
            end if
        end for
    end for
    $\textit{Strength} = \textit{max}\{\textit{Strength}, \textit{ISancStrength}\}$
end for
if $\textit{Strength} < \textit{None}$ **then**
    $\textit{Strength} = |\textit{Strength}|$
end if
return $\textit{Strength}$

imposed by other players on DM_i's UIs out of s must be UI moves by those other-players. In other word, they must act "rationally". They will not hurt themselves in order to sanction DM_i's UIs. And this is at the heart of the difference between GMR stability and SEQ stability.

Let SEQ's stability strength of a state s for DM_i at time t be denoted as $SEQ(L_{SS})$, where StabilityStrength$(SEQ, s, DM_i, t){=}L_{SS}$. Algorithm 6.5 uses Definition 6.4.4 to calculate the SEQ's stability strength and assign the strength's fuzzy linguistic label.

Algorithm 6.5 Calculating a State's SEQ Stability Strength for a DM in a Non-Cooperative Game

strength-value-label Strength_of_SEQ_Stability *(s, DM_i, Game-Structure)*
// start with the assumption that s is not SEQ stable
$SEQ\_Strength = NULL$
// check if DM_i has any UIs from s at time t
if $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ **then**
    //s is NASH Stable State for DM_i at t
    $SEQ\_Strength = None$
else if $(\forall s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s))$ [$\exists$ an SMI sanction] **then**
    // SMI sanction exists against each of DM_i's UIs $\Rightarrow$ s is SEQ stable; find SEQ's strength
    $SEQ\_Strength = Strength\_of\_UISanctions(s,\ DM_i,\ Game\text{-}Structure)$
end if
return $SEQ\_Strength$

strength-value-label Strength_of_UISanctions *(s, DM_i, Game-Structure)*
// this routine will return the strength of the sanction that yields the worst result for DM_i,
// given the facts that: 1) the sanction move must be a UI (for the provider); and
// 2) DM_i will choose the UI that will minimize his loss.
// set sanction's end state strength initially to "Extremely Less Preferred"
$Strength = -Extreme$
// find s's SEQ strength
for all $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$ **do**
    $UISancStrength = None$
    for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_{sm} \in \mathcal{S}^{UI}_{DM_j,t+1}(s_{ui}) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
            $UISancStrength = min\{UISancStrength, PrefStrength(s_{sm}, s, DM_i, t)\}$
        end for
    end for
    $Strength = max\{Strength, UISancStrength\}$
end for
// return the equivalent positive strength label, if $Strength{<}N$
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return $Strength$

6.4.2 Equilibrium Strength

Let the *Equilibrium Strength* under a Solution Concept *StabilityConcept* is a value property attached to a state s of the game at time t, denoted as *Equilibrium-Strength*(*StabilityConcept*, s, t), be given a fuzzy linguistic value label L_{SS} based on the same fuzzy memberships functions setup for the *StabilityStrength* value property discussed in the previous subsection. The strength expressed by the L_{SS} fuzzy label is meant to represent the strength of the equilibrium under the specific stability concept type *StabilityConcept* (where *StabilityConcept* $\in \{NASH, GMR, SMR, SEQ\}$) for state s of the game at time t.

As indicated in the equilibrium definition given earlier (Definition 6.3.5), the equilibrium concept must be defined under a specific stability solution concept. An equilibrium state under a specific stability solution concept is a state that is stable for all the decision makers in the game under the same stability solution concept. For example, if a state is an equilibrium under GMR, then this means that the state is GMR stable from every player in the game. As a result, the strength of the equilibrium for a specific state s under a specific solution concept SC is tightly coupled with the strength of the SC stabilities of s for each player in the game.

We said at the beginning of the previous subsection that the *StabilityStrength*value property, for any state, before fuzzification and without normalization has a numeric value in the range $[0, 2]$, therefore it could have a fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*,*Little*, *None*, *Null*$\}$=$\{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. With the understanding that the complete order of these labels is $Ex > Si > St > F > B > M > Mo > L > N > Null$. We also said that the *StabilityStrength* before fuzzification and with normalization has a numeric value that is in the range $[0, 1]$, therefore it could have the same fuzzy labels listed above with the exception of the three higher labels covering the range from $[1, 2]$ and those are: *Extreme*,*Significant* and *Strong*.

As for the *StabilityStrength* value property, and because of the dependancy, the *Equilibrium Strength* fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*,*Little*, *None*, *Null*$\}$=$\{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. And, with the complete order of $Ex > Si > St > F > B > M > Mo > L > N > Null$, where the labels range from representing *Extreme*ly strong equilibrium stability (based on the definition of the solution concept given in *StabilityConcept*) of s to *None* strength level (meaning very weak equilibrium strength and close to non-existing strength) for s.

The L_{SS} fuzzy label assigned to *EquilibriumStrength*(*StabilityConcept*, s, t) will cover the equilibrium stability strength satisfaction levels, with the understanding that the *Null* label represents an unknown equilibrium stability strength or totaly-non-existing-equilibrium. The fuzzy membership functions defining these stability/equilibrium strength's linguistic value labels are given in Figure 6.1. The figure shows the membership functions for each label's fuzzy set to be trapezoidal in shape, for simplicity only (not as a restriction) as indicated in Al-Shawa and Basir (2010). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements, as we indicated in Al-Shawa and Basir (2009, 2010).

Now, we define the equilibrium strength under any specific solution concepts, for non-cooperative games.

Definition 6.4.5 (**Strength of an Equilibrium**)**:** *For $\mathcal{DM}$, all decision makers in a specific non-cooperative game, at time t, and for state s which is an Equilibrium for the game under a specific Solution Concept SC, i.e. $s \in \mathcal{S}^{SC\ EQ.}_{\mathcal{DM},t}$, the strength of s's Equilibrium stability, to $\mathcal{DM}$ at time t, is calculated as follows:*

$$(\exists\, DM_i \in \mathcal{DM}\ \ s \notin \mathcal{S}^{SC\ EQ.}_{DM_i,t}) \quad \rightarrow \quad EquilibriumStrength(SC,s,t) = NULL$$

And,

$$(\forall\, DM_i \in \mathcal{DM}\ \ s \in \mathcal{S}^{SC\ EQ.}_{DM_i,t}) \quad \rightarrow \quad EquilibriumStrength(SC,s,t) = \min_{DM_i}\{StabilityStrength(SC,s,DM_i,t)\}$$

As per the definition, the strength of the Equilibrium at s under the Solution Concept SC is the minimum of the strength of s's stability under SC for each decision maker in the game. This means that s must be stable under SC for each player in order for it to be an equilibrium for the game under SC (Definition 6.3.5), then the minimum of all DMs SC stabilities' strengths is considered to be the strength level of this equilibrium at s.

Algorithm 6.6 Calculating a State's Equilibrium Strength, under a specific Solution Concept SC, in a Non-Cooperative Game

```
strength-value-label Strength_of_Equilibrium (s, SC, Game-Structure)
// start with the assumption that s is not an Equilibrium under SC
SC_EQ_Strength = NULL
// check if s is stable for all DMs in the game under Solution Concept SC
if (∀ DM_i ∈ DM  s ∈ S^{SC EQ.}_{DM_i,t}) then
    // s is an Equilibrium for the game under Solution Concept SC; find EQ's strength
    // set equilibrium's strength initially to "Extremely Strong"
    SC_EQ_Strength = Extreme
    // find s's equilibrium strength
    for all DM_i ∈ DM do
        SC_EQ_Strength = min{SC_EQ_Strength, StabilityStrength(SC,s,DM_i,t)}
    end for
end if
return SC_EQ_Strength
```

Let the Equilibrium's stability strength of state s of the game at time t be denoted as $SC\ EQ(L_{ss})$, where EquilibriumStrength(SC, s, t)=L_{ss}. Algorithm 6.6 uses Definition 6.4.5 to calculate the equilibrium's strength and assign the strength's fuzzy linguistic label.

6.5 Case Study: The Cuban Missile Crisis

6.5.1 Background

The Cuban Missile Crisis stands as an one of the most important events in the history of mankind. History offers no parallel to the thirteen days (16-28) of October 1962, when the two rivalry post-second-world-war superpowers, the United States (US) and the Soviet Union (Union of Soviet Socialist Republics - USSR), were at the verge of starting the first nuclear war in history, and the most destructive war ever. Had war come, previous wars and natural disasters of history would have faded into insignificance. Given the high probability, which President Kennedy estimated as "between one out of three and even", of starting the disastrous nuclear war, the fact that humanity escaped such fate seems a historic awesome event. This made the Cuban Missile Crisis symbolizes what it meant to live in the nuclear age; and made it to be one of the the most studied events in modem history (Allison and Zelikow (1999)).

For our research, the Cuban Missile Crisis of 1962 gives us an interesting multi-agent strategic decision making conflict that is worth analyzing using our framework, because of: 1) its significance as one of a kind strategic political conflict; and 2) the many studies that analyzed and modelled the conflict using other commonly used rationality frameworks. The crisis has been studied and analyzed by many (Abel, 1969; Allison, 1971; Fraser and Hipel, 1984). This makes the Cuban Missile Crisis form a good example to show case the use of the framework, and its stability analysis tools, which we discussed in this chapter, in modelling and analyzing a political policy making strategic conflict; and to compare the new model and analysis to the existing ones available in the literature.

6.5.2 Players' Strategic Goals

Now, we will follow the steps given in the previous chapter (for modelling multi-agent conflicts) to model the Cuban Missile Crisis. In modelling the crisis, we assume that the crisis is modelled from the perspective of the US executive branch based on what the US "knows" about itself and the USSR. The porches, model and analysis we will convey in this section, is the same which a Constrained Rationality modeller will use to help the US executive branch understand the conflict in order to guide the US decision makers in the decision they will take.

We start by identifying the strategic goals of the players, using the iterative two step process discussed in the previous chapter. First, we define the immediate goals, the most related goals to the conflict context and why the conflict is considered a conflict in the first place. In this case, the two obvious goals that the players have were: the US wants the USSR to withdraw the missile bases it had setup in Cuba (Goal $G_{US\,4}$ in Table 6.1), and the USSR wants the US to remove the missile bases it had established in Turkey (Goal $G_{USSR\,5}$ in Table 6.1).

Second, we identify the higher strategic goals of the players by asking the "why" question: Why the US wants to achieve $G_{US\,4}$ of having the missiles withdrawn from Cuba? Why the US had put missile bases In Turkey? why the USSR had shipped the missiles to Cuba to be installed there? Why the USSR wants to achieve $G_{USSR\,5}$ of having the US remove the missile bases In Turkey? Answering these question will lead to uncovering the ultimate strategic goals (or higher goals in the players' goal tree/s) behind why the players considered the obvious goals, elicited in the first step, to be "the" goals for them in the conflict in the first place. This process will identify a number of strategic goals for both players, the US and the USSR. Table 6.1 provides a list of the most notable and relevant ones. During this steps the modeller will link these goals by lateral and reduction goal-to-goal relationships, and qualitatively labelling them as per the Constrained Rationality GCM modelling discussed in Chapter 3.

Ideally, after going through the first two steps few times, building and refining the goal tree/s for each of the players in the conflict, one should have a fully developed multi-level goal tree structures that form the bases for the players' GCM models. But because our focus in this case is to illustrate the use of the conflict analysis concepts presented in this chapter, the goals listed in Table 6.1 are sufficiently complete. These goals are the same goals/whys, for the US and USSR in this conflict, that are indicated by the literature (Abel, 1969; Allison, 1971; Allison and Zelikow, 1999; Fraser and Hipel, 1984); and in total explains sufficiently: why the the missiles in Cuba and Turkey are important; why the the USSR put the missiles in Cuba; and why the US wants them removed.

Table 6.1: Cuban Missile Crisis: Strategic Goals for the US and USSR

US Strategic Goals:	
$G_{US\,1}$	Bring All South American Countries Under US Influence (a long standing strategic goal of the US)
$G_{US\,2}$	Do not enter a war with the USSR (any war will eventually be a nuclear war)
$G_{US\,3}$	Contain the USSR's power (hard and soft powers)
$G_{US\,4}$	Control Cuba and/or Contain it
$G_{US\,5}$	Remove the USSR missiles from Cuba
$G_{US\,6}$	Maintain the US missile bases In Turkey
USSR Strategic Goals:	
$G_{USSR\,1}$	Attract new third world nations opposing to the US power/influnece to joint the Eastern Bloc and offer protection to them (a long standing strategic goal of the USSR)
$G_{USSR\,2}$	Do not enter a war with the US (any war will eventually be a nuclear war)
$G_{USSR\,3}$	Protect and defend the USSR from any potential threat close to its Border
$G_{USSR\,4}$	Protect the Communist Government in Cuba and Support it militarily and politically
$G_{USSR\,5}$	Remove the US missiles installed in Turkey

6.5.3 Players' Alternative Actions

In this step, the alternative course of actions for both players are identified. Because of the nature of the conflict as a strategic military about keep/remove missile bases, the players' alternatives are obvious. They range from peaceful means (such as do-nothing, apply diplomatic pressure, or negotiate) to full scale war, all in between. A list of the US and the USSR alternatives in this conflict is given in Table 6.2.

These alternatives are, for the most part, similar to the ones both the US and the USSR considered at the time of the conflict (Abel, 1969; Allison, 1971; Allison and Zelikow, 1999). Some alternatives, such as do-nothing and negotiation were not part of the list of actions considered by the Executive Committee formed by President Kennedy, even though the USSR was hoping the US would include them as alternatives. Instead for the US opted for a show of force approach, for many reasons that range from pure-attitudinal reason (naturally aggressive, trying to save face after being embarrassed publicly by the failure of the Bay of Pigs invasion, ..) to more rational reasons (behavioural political maneuvering, the missiles form an at most risk to the US national security and well-being, ..). Nevertheless, the decision to include these alternatives in our model is made for two reasons: 1) the USSR considered them as options for the US; and 2) a full scale analysis of the conflict should include these options to show wether the US was better off using one of them.

Table 6.2: Cuban Missile Crisis: US and USSR Alternatives/Options

The Set of Alternatives available to US (A_{US}) :	
$A_{US\,0}$	*Threaten* Cuba and USSR asking them to remove the missiles. This option includes diplomatic pressure through the United Nations or other non-aggressive measures.
$A_{US\,1}$	Impose a *Blockade*. The US will enforce an embargo on military shipments to Cuba.
$A_{US\,2}$	Carry out a surgical *Air Strike*, destroying the missile bases in Cuba using conventional but limited air attach.
$A_{US\,3}$	Conduct a *Full Attack* against Cuba. This option is most likely be as a follow-up invasion of Cuba if other less dramatic measures fail to remove the missile bases.
$A_{US\,4}$	Negotiate a *Deal* with USSR, with the most likely outcome of this deal is the US removes the missile bases in Turkey wile the USSR removes the Cuban missile bases.
$A_{US\,5}$	*War*. Ultimate escalation of the conflict, this could come as a response to the USSR escalating the conflict to a point where the US security, well being and reputation are at stake such as when the USSR launches a ballistic missile attack on the US.
The Set of Alternatives available to USSR (A_{USSR}) :	
$A_{USSR\,0}$	*Do Not Withdraw* the missiles from Cuba.
$A_{USSR\,1}$	*Withdraw* the missiles from Cuba, without this being part of a negotiated deal with the US. This option also represents *Missiles Destroyed & USSR Does Nothing* option (the US destroys the missiles, and the USSR decides to "do nothing").
$A_{USSR\,2}$	*Escalate* the conflict, by carrying on actions that tops the aggressive action the US took against Cuba. This escalation could be done by assaulting US Naval ships, bombing southeastern American targets from Cuba, invading West Berlin, etc.
$A_{USSR\,3}$	Negotiate a *Deal* with US, in which the US removes the missile bases in Turkey, as a condition to have the USSR removes the Cuban missile bases.
$A_{USSR\,4}$	*War*. Ultimate escalation of the conflict, this could come by commencing ballistic missile attack on the US.

6.5.4 Analysis of the Players' GCMs and Alternatives

In the previous steps, we built a rough GCM model for both players in this in conflict, the US and the USSR. At this stage, we should complete the GCM models (goal-trees, constraints, alternatives, and interrelationships among them) adding any missing component/relationship. The main purpose of this stage is to validate the players' alternatives, uncover any additional ones, then complete and validate the effect of these alternatives on both players' goals. The alternatives' effect is modelled by using lateral goal-to-goal relationships. At the end, this stage will provide the necessarily information for the next two stages in the modelling and analysis process: identifying the conflicts' states, and modelling the players' preferences over them.

Figure 6.2 shows the players' GCMs. The models could be enhanced and extended, but we believe the shown ones are sufficient for the analysis of this conflict, and the purpose of the case study (illustrate the conflict analysis concepts discussed in this chapter). The figure shows the result of the first test we do, after

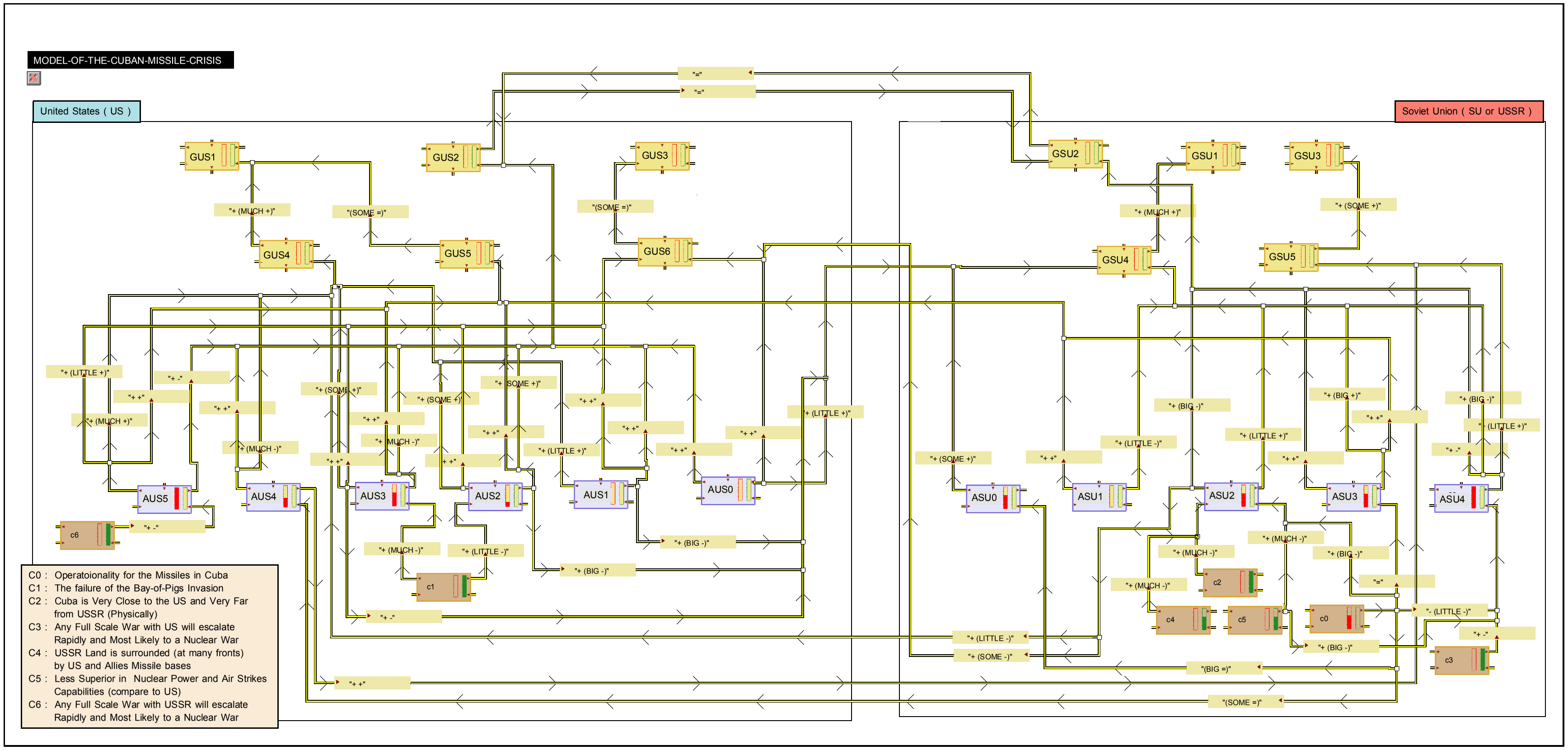


Figure 6.2: Model of the Cuban Missile Crisis: The conflict's known constraints effect on players' options shows how much potential influence both of the US and the USSR have on the game.

completing the GCM models and identify all known and potential alternatives for the players. This test checks the validity of the players' alternatives, knowing the players' relaities/constraints.

The conflict's model is presented in Figure 6.2 as it stands after setting the value labels for the conflict's constraints and running the Constrained Rationality forward propagation reasoning algorithm (Algorithm 3.1 discussed in Chapter 3). The figure shows the known constraints affects negatively the ability of the players to choose many of the alternatives they have. For example, the fact that, at the time the US spy U2 plans uncovered the Cuban missile base and the conflict started, the the missile base in Cuba was not operational, or could be made operational yet (Constraint C_0 in the model) prevents partially the ability for the USSR to use the Cuban missiles as a bargaining chip in order to induce the US to remove its missile bases in Turkey. This therefore affects negatively the ability for the USSR to choose the Negotiation Alternative. For the US, the failure of the Bay of Pigs Invasion (Constraint C_1 in the model) affects negatively the chances of succeeding in making another full scale attack or invasion on Cuba.

The first test, as we can see from the figure, reveals clearly that none of the US and USSR's alternatives will be eliminated as an option by just applying the effect of the conflict's constraints on them. Therefore, we should keep all the alternatives in the model, despite the fact that some are "less-likely" to be chosen by the players as the test demonstrates, and proceed to the next test. But it is important here to note that this first test model provides us insight on how much influence the players have on the game based on the known constraints/realities. For example, if the US's U2 spy plane discovered the Cuban missiles at a later stage of their installation, and when they are operational, then the conflict will be different. Constraint C_0 will propagate positive weight to negotiation between the US and USSR, and will affect negatively any aggressive alternative the US have.

The second test is about the relevance of the players' goals to the analysis of the game. As Figure 6.2 shows, some of the higher goals, in the players' GCM goal trees, are not affected directly by the actions selected by the players, but indirectly through the lower goals. For example, while goals $G_{US\,1}$ and $G_{US\,3}$ must stay in the model to remind us why the US wants to "control Cuba" and "maintain the existence of its missile bases in Turkey", respectively, these goals can be sufficiently dealt with by the lower US goals in the model that feeds into them. Therefore, we can see at a later stage in our analysis that the set of strategic goals for the US used to calculate the US preferences will include only goals $G_{US\,2}$, $G_{US\,4}$, $G_{US\,5}$ and $G_{US\,6}$ explained above in Table 6.1. Similarly, the USSR strategic goals set used for calculating the USSR preferences later include only goals $G_{USSR\,2}$, $G_{USSR\,4}$ and $G_{USSR\,5}$ (all explained in the same table above).

In summary, the second test helps identify for each player: what goals are in the model just to capture the "why"; and what subset of strategic goals are representative and more relevant to consider when it is time to calculate the player preferences. The "why" goals are mostly higher goals in the player's GCM goal-tree/s. They are important to stay in the model in order to keep the analyst reminded about the strategic importance of the player's lower more relevant goals. The very lower goals, which are at the bottom of the player's GCM goal tree/s, are usually not strategic enough to be considered in order to be used for calculating the player's preferences, but our Cuban Missile Crisis model shown in Figure 6.2 is a special case. This is because the GCM models we have for the players are flat and have two-level goal trees. So the distinction between what is very strategic high level goal and what is very tactical lower level goal, within the goal trees, is not as obvious or important as it would be if the goal trees were deeper multi-level trees. In our case, we consider all of the US and the USSR goals are strategic for them to consider as they analyze the conflict. But some of these goal are redundant, and the effect on them is already been captured through other goals, for them to be used for calculating the players' preferences. Including them will constitute a double counting problem.

The third test is to check for all combination of players' alternatives that could form valid conflicts states. Defining conflict states is not a straight forward binary combinations of the two players' alternatives. It is true some of the many binary combinations are valid states, but not all. For example, the US to impose a blockade on Cuba ($A_{US\,1}$ discussed in Table 6.2) and the USSR not withdrawing the missiles from Cuba ($A_{USSR\,0}$ discussed in the same table) for a very valid state (state s_2 shown in Table 6.3 discussed in the following section). However, a state based on combining the actions of the US conducting one of its aggressive options, such as Air Strike or Full Attack (shown in the table above as $A_{US\,2}$ and ($A_{US\,3}$ respectively), and the USSR Negotiate ($A_{USSR\,3}$ is impossible, because negotiation of a deal needs both parties to choose to negotiate as their action.

At the same time, the question about whether a state is a valid state or not is not black and white. Some states are not clear wether they form valid states or not. For example, a state based on the US chooses threatening as its action plan ($A_{US\,0}$) while the USSR chooses to escalate ($A_{USSR\,2}$) is not an impossible state to exist, but it is logically a very not likely state to exist. If at any point the analyst is in doubt, then he should try to add the state at the sensitivity testing stage of the conflict analysis process. The analyst there can test different what-if scenario variations of the base model.

But, this step of the process is not only about establishing a set of all valid states for the conflict to analyze. It is also about adding additional alternatives because they make the states more clearly defined; or removing alternatives because they proved redundant. For example, and based on the model that we built for the conflict, the USSR's $A_{USSR\,1}$ "Withdrawing the Missiles from Cuba" alternative could be also used as the USSR's "Do Nothing when the US destroy the missile base in Cuba" alternative. Both alternatives will have the same effect on the USSR's goals. So if we originally added an alternative for the USSR representing the Do Nothing action that it may take, we should remove it and combine the Do Nothing part to the $A_{USSR\,1}$, as we did in Table 6.2. Nevertheless, if for some reason the modeller felt that there is an important difference, then by all means he should add an alternative node to the USSR's GCM model, in the base model for the conflict, to represent the "Do Nothing" option and wire it to the USSR's goals showing the effect it has on them once adopted as "the" alternative to act on. Alternatively, the modeller could have a variation model to the base one and conduct the analysis on it at the sensitivity testing stage of the overall conflict analysis process.

By now, we have enough information about the conflict and we are ready to define the conflict states and the players' preferences over them.

6.5.5 The Conflict's States

Table 6.3 shows a complete list of the valid states for the Cuban Missile Crisis conflict, based on the analysis done above to the conflict model (the US and USSR's GCM model interacting with each other). The table defines each state in terms of the alternatives the players selects. The table rows correspond to the state definition notation we are using. For example, the first state's row in the table should mean the same as $s_0 = \{A_{US\,0} = F, A_{USSR\,0} = F\}$.

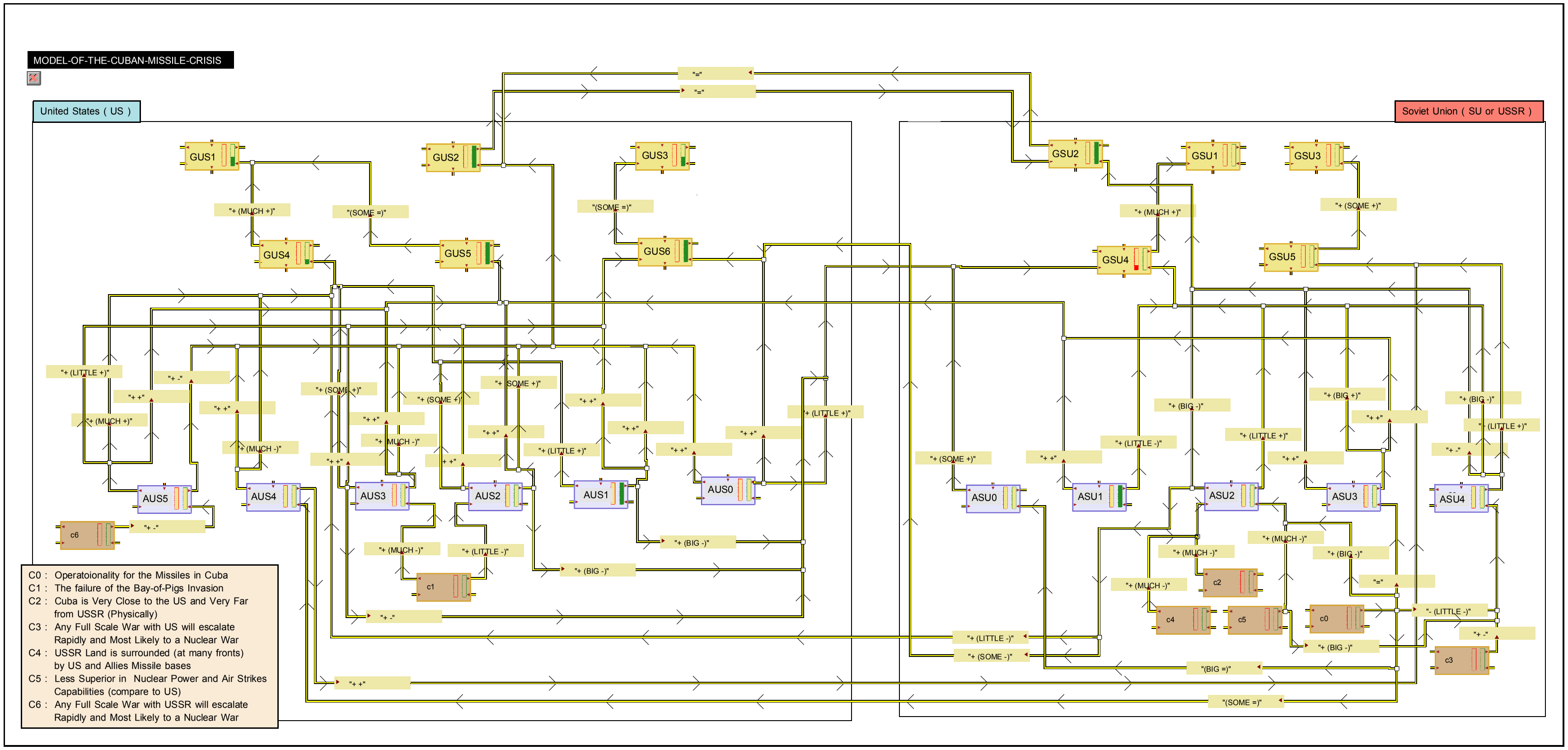


Figure 6.3: Model of the Cuban Missile Crisis showing the achievement levels that the US and USSR can gain from being in state s_3 (the US imposes a blockade on Cuba, and the USSR withdraws the missiles)

Table 6.3: Cuban Missile Crisis: Defining the Game States

The Set of All States $\mathcal{S}$ for the US x USSR Cuban Missile Crisis:		
State	**US Options**	**USSR Options**
s_0 Status Quo (Oct. 14, 1962)	Threaten $A_{US\,0}$	Do Not Withdraw $A_{USSR\,0}$
s_1	Threaten $A_{US\,0}$	Withdraw $A_{USSR\,1}$
s_2	Blockade $A_{US\,1}$	Do Not Withdraw $A_{USSR\,0}$
s_3	Blockade $A_{US\,1}$	Withdraw $A_{USSR\,1}$
s_4	Air Strike $A_{US\,2}$	Withdraw (Destroyed) $A_{USSR\,1}$
s_5	Air Strike $A_{US\,2}$	Escalate $A_{USSR\,2}$
s_6	Full Attack $A_{US\,3}$	Withdraw (Destroyed) $A_{USSR\,1}$
s_7	Full Attack $A_{US\,3}$	Escalate $A_{USSR\,2}$
s_8	Deal $A_{US\,4}$	Deal $A_{USSR\,3}$
s_9	War $A_{US\,5}$	War $A_{USSR\,4}$

6.5.6 Player's Preferences over the States of the Conflict

To calculate the preferences of the players over the conflict's states, we calculate how much each state (with all its players' alternatives selected) contribute to the achievement of the players' strategic goals. The sets of strategic goals for the players were defined earlier to be: $\mathcal{SG}_{US} = \{G_{US\,2}, G_{US\,4}, G_{US\,5}, G_{US\,6}\}$ and $\mathcal{SG}_{USSR} = \{G_{USSR\,2}, G_{USSR\,4}, G_{USSR\,5}\}$. Each of the individual goals is explained above in Table 6.1. The set of the conflict's states is defined earlier to be: $\mathcal{S} = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9\}$, each of which is defined in Table 6.3.

The players' preferences, and the strength of their preferences, depend on the level of achievement each state contribute to the players' strategic goals, when the state is in place (i.e. when the players select the alternatives as defined by the state). For example, the contribution of state s_3 (the US impose a *Blockade* on Cuba and the USSR *Withdraw* the missiles from Cuba) to both players' goals is calculated by first setting the US's alternative $A_{US\,1} = F$ and the USSR's alternative $A_{USSR\,1} = F$ in the conflicts' model (the interacting players' GCM models presented in Section 6.5.4). Then, run the forward label propagation reasoning algorithm (Algorithm 3.1 discussed in Chapter 3). The state's contribution to the players' strategic goals can be seen from the conflict's model shown in Figure 6.3. Note that we removed the effect of the constraints on the alternatives in the model shown, this is to conduct the stability analysis for the conflict under the assumption that all players' alternatives are equally valid and possible. This will allow us to elicit the states' stabilities without limiting their validity with whether the players could afford to commit to the alternatives defining these states. This produces a broader sense of the states' stabilities.

Finally, we calculate the *Weighted Payoff* value for each player using the method presented in Section 5.4.2 of of the previous chapter. The players' weighted payoffs for each state of the conflict are shown in Figure 6.4. The figure shows the level of achievement the state contributes to the players' strategic goals. It also shows the strategic importance and the emotional valence each of the players has for their respective strategic goals (all assumed to be fully important in our base model); in addition to the Rationality and Emotionality Factors for both players (assumed in the base model to be set to Full and None, or 1.0 and 0.0, respectively, in other words the players are assumed fully rational and not emotional as one could expect from governments with institutional collective rationality). The analyst can test different rationality and emotional values, and the implication of such values, by setting up what-if scenario models as variations to the base model as part of the sensitivity analysis stage.

Now, from the calculate weighted payoffs of each state to each of the players, we calculate the Ordinal Preferences for the players over the conflict's states. Figure 6.4 shows both the weighted payoffs and the ordinal preferences for each state of the conflict, for each player.

The strengths of the players' preferences of each state over each of the other states are also calculated from the calculate weighted payoffs using the method discussed in Section 4.5. Table 6.4 shows the US and USSR's preferences vectors, and the preferences strengths of each state over each other state in the conflict.

Table 6.4: Cuban Missile Crisis: Players' Preferences

US's Preferences (*Most* to *Least* Preferred)										
Pref(***US***)	s_3	s_1	s_4	s_2	s_0	s_6	s_5	s_8	s_7	s_9
WP	0.900	0.875	0.850	0.800	0.750	0.738	0.700	0.688	0.675	0.600
US Preferences' Strengths										
$\succ^{L_{PS}}_{US,t}$	s_3	s_1	s_4	s_2	s_0	s_6	s_5	s_8	s_7	s_9
s_3	N	N	N	L	L	L	L	L	L	S
s_1	N	N	N	N	L	L	L	L	L	L
s_4	N	N	N	N	L	L	L	L	L	L
s_2	-L	N	N	N	N	N	L	L	L	L
s_0	-L	-L	-L	N	N	N	N	N	N	L
s_6	-L	-L	-L	N	N	N	N	N	N	L
s_5	-L	-L	-L	-L	N	N	N	N	N	L
s_8	-L	-L	-L	-L	N	N	N	N	N	N
s_7	-L	-L	-L	-L	N	N	N	N	N	N
s_9	-S	-L	-L	-L	-L	-L	-L	N	N	N

USSR's Preferences (*Most* to *Least* Preferred)										
Pref(***USSR***)	s_8	s_0	s_1	s_3	s_2	s_4	s_6	s_5	s_7	s_9
WP	0.967	0.733	0.667	0.633	0.600	0.533	0.367	0.333	0.267	0.233
USSR Preferences' Strengths										
$\succ^{L_{PS}}_{USSR,t}$	s_8	s_0	s_1	s_3	s_2	s_4	s_6	s_5	s_7	s_9
s_8	N	L	S	S	S	S	M	M	B	B
s_0	-L	N	N	L	L	L	S	S	Mo	Mo
s_1	-S	N	N	N	N	L	S	S	S	S
s_3	-S	-L	N	N	N	L	L	S	S	S
s_2	-S	-L	N	N	N	N	L	L	S	S
s_4	-S	-L	-L	-L	N	N	L	L	L	S
s_6	-M	-S	-S	-L	-L	-L	N	N	L	L
s_5	-M	-S	-S	-S	-L	-L	N	N	N	L
s_7	-B	-Mo	-S	-S	-S	-L	-L	N	N	N
s_9	-B	-Mo	-S	-S	-S	-S	-L	-L	N	N

Rationality Factor = 1.0 (for US & USSR)
Emotionality Factor = 0.0 (for US & USSR)

		Cuban Missile Crisis						
		US				USSR		
DM's Strategic Goals	SGs:	SG_{US}				SG_{USSR}		
		$SG_{US\,2}$	$SG_{US\,4}$	$SG_{US\,5}$	$SG_{US\,6}$	$SG_{USSR\,2}$	$SG_{USSR\,4}$	$SG_{USSR\,5}$
Strategic Importance	SImprt(SG_k)	F	F	F	F	F	F	F
State S_0	Achv(SG_k)	F	N	N	F	F	S	N
	Prvn(SG_k)	N	N	N	N	N	N	N
Status Quo (Oct. 14, 1962)	FAchv(SG_k)	F	N	N	F	F	S	N
US: Threaten USSR: Keep	TWFAchv(SG_k,DM)	1.00	0.00	0.00	1.00	1.00	0.40	0.00
{ Achv($A_{US\,0}$)=F,	WP(S_0, DM)	0.750				0.733		
Achv($A_{USSR\,0}$)=F }	OP(S_0, DM)	5				2		
State S_1	Achv(SG_k)	F	N	F	F	F	L	N
	Prvn(SG_k)	N	N	N	N	N	L	N
US: Threaten	FAchv(SG_k)	F	N	F	F	F	N	N
USSR: Withdraw	TWFAchv(SG_k,DM)	1.00	0.00	1.00	1.00	1.00	0.00	0.00
{ Achv($A_{US\,0}$)=F,	WP(S_1, DM)	0.875				0.667		
Achv($A_{USSR\,1}$)=F }	OP(S_1, DM)	2				3		
State S_2	Achv(SG_k)	F	S	N	F	F	S	N
	Prvn(SG_k)	N	N	N	N	N	B	N
US: Blockade	FAchv(SG_k)	F	S	N	F	F	-S	N
USSR: Keep	TWFAchv(SG_k,DM)	1.00	0.40	0.00	1.00	1.00	-0.40	0.00
{ Achv($A_{US\,1}$)=F,	WP(S_2, DM)	0.800				0.600		
Achv($A_{USSR\,0}$)=F }	OP(S_2, DM)	4				5		
State S_3	Achv(SG_k)	F	S	F	F	F	N	N
	Prvn(SG_k)	N	N	N	N	N	L	N
US: Blockade	FAchv(SG_k)	F	S	F	F	F	-L	N
USSR: Withdraw	TWFAchv(SG_k,DM)	1.00	0.40	1.00	1.00	1.00	-0.20	0.00
{ Achv($A_{US\,1}$)=F,	WP(S_3, DM)	0.925				0.633		
Achv($A_{USSR\,1}$)=F }	OP(S_3, DM)	1 (Best)				4		
State S_4	Achv(SG_k)	S	S	F	F	S	N	N
	Prvn(SG_k)	N	N	N	N	N	L	N
US: Air Strike	FAchv(SG_k)	S	S	F	F	S	-L	N
USSR: Do Nothing	TWFAchv(SG_k,DM)	0.40	0.40	1.00	1.00	0.40	-0.20	0.00
{ Achv($A_{US\,2}$)=F,	WP(S_4, DM)	0.850				0.533		
Achv($A_{USSR\,1}$)=F }	OP(S_4, DM)	3				6		
State S_5	Achv(SG_k)	S	S	F	F	S	L	N
	Prvn(SG_k)	M	L	N	S	B	B	N
US: Air Strike	FAchv(SG_k)	-L	L	F	M	-S	-M	N
USSR: Escalate	TWFAchv(SG_k,DM)	-0.20	0.20	1.00	0.60	-0.40	-0.60	0.00
{ Achv($A_{US\,2}$)=F,	WP(S_5, DM)	0.700				0.333		
Achv($A_{USSR\,2}$)=F }	OP(S_5, DM)	7				8		
State S_6	Achv(SG_k)	N	M	F	F	N	N	N
	Prvn(SG_k)	M	N	N	N	M	L	N
US: Full Attack	FAchv(SG_k)	-M	M	F	F	-M	-L	N
USSR: Do Nothing	TWFAchv(SG_k,DM)	-0.60	0.60	1.00	1.00	-0.60	-0.20	0.00
{ Achv($A_{US\,3}$)=F,	WP(S_6, DM)	0.750				0.367		
Achv($A_{USSR\,1}$)=F }	OP(S_6, DM)	6				7		
State S_7	Achv(SG_k)	N	M	F	F	N	L	N
	Prvn(SG_k)	M	L	N	S	M	F	N
US: Full Attack	FAchv(SG_k)	-M	S	F	M	-M	-B	N
USSR: Escalate	TWFAchv(SG_k,DM)	-0.60	0.40	1.00	0.60	-0.60	-0.80	0.00
{ Achv($A_{US\,3}$)=F,	WP(S_7, DM)	0.675				0.267		
Achv($A_{USSR\,2}$)=F }	OP(S_7, DM)	9				9		
State S_8	Achv(SG_k)	F	N	F	N	F	B	F
	Prvn(SG_k)	N	Mo	N	N	N	N	N
US & USSR	FAchv(SG_k)	F	-Mo	F	N	F	B	F
Negotiate a Deal	TWFAchv(SG_k,DM)	1.00	-0.50	1.00	0.00	1.00	0.80	1.00
{ Achv($A_{US\,4}$)=F,	WP(S_8, DM)	0.688				0.967		
Achv($A_{USSR\,3}$)=F }	OP(S_8, DM)	8				1 (Best)		
State S_9	Achv(SG_k)	N	M	F	L	N	N	L
	Prvn(SG_k)	F	N	N	N	F	B	N
Full Scale War	FAchv(SG_k)	-F	M	F	L	-F	-B	L
	TWFAchv(SG_k,DM)	-1.00	0.60	1.00	0.20	-1.00	-0.80	0.20
{ Achv($A_{US\,5}$)=F,	WP(S_9, DM)	0.600				0.233		
Achv($A_{USSR\,4}$)=F }	OP(S_9, DM)	10 (Worst)				10 (Worst)		

Figure 6.4: Cuban Missile Crisis: Players' Ordinal and Normalized Weighted Preferences

6.5.7 Player's Moves over the States of the Conflict

At this step, we define the unilateral moves for each of the US and the USSR. These moves are shown in Figure 6.5. We should note that the states in the figure are shown with their corresponding numbers. For example state s_5 is shown as a circle with the number 5 at its centre. It is also worth noting that the conflict's states includes a Negotiation state, or s_8, which none of the players can move to it unilaterally. This state is considered a cooperative move, a type of players' moves that we will define later when we discuss cooperative games. But because of the nature of the conflict and the players' preferences structures, this state will not materialize as an improvement move (unilateral or cooperative). As a result, it will not affect the analysis of the conflict by demanding the conflict to be analyzed as a cooperative game.

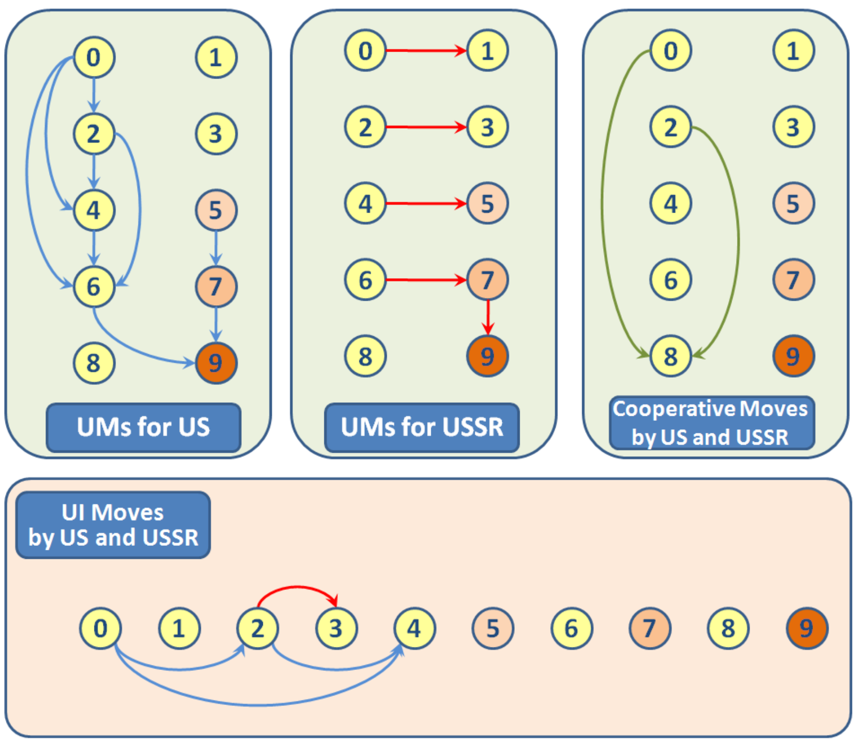


Figure 6.5: The Cuban Missile Crisis: The Unilateral, Cooperative and Unilateral Improvement Moves by the players, the US and the USSR

By feeding the players' UMs and preference structures to Algorithm 6.1, the Player's UI moves are defined. Figure 6.5 shows that the US has two UI moves from the status quo state s_0 to states s_2 and s_4 (the states in which the US imposes a *Blockade* on Cuba, and the US commits a surgical *Air Strike* to remove the missiles, respectively); and another UI from state s_2 to state s_4. On the other hand, the USSR has only one UI move, and that is from state s_2, the US imposing a *Blockade* state, to state s_3 in which the USSR *Withdraw* the missiles from Cuba after the US's Blockade.

6.5.8 Stability Analysis of the Cuban Missile Crises States

The stability of each of the Cuban Missile Crisis's states for both players, the US and the USSR, is defined in Table 6.5. The table shows whether each state is stable under the stability solution concept of NASH, GMR, SMR, and SEQ (defined in Definitions 6.3.1, 6.3.2, 6.3.3, and 6.3.4 respectively); and what is the strength of this stability (as per Definitions 6.4.1, 6.4.2, 6.4.3, and 6.4.4 respectively, and their implementations given in Algorithms 6.2, 6.3, 6.4 and 6.5 respectively).

First, Table 6.5 shows the stability analysis of the conflict's states for the US. At the top of the US's table, the states are listed in the order of the US's preferences vector deduced in Section 6.5.6, and shown in Table 6.4. Beneath some of the states in the preferences vector are states that the US can move unilaterally to, improving its position (preference order and weighted payoff). The result of the UI move is a state appears to the left of the column of the current state the UI occurs from, reflecting the fact that the UI move will result with a more preferred state. For example, the US has two UIs from the status quo just-threaten state s_0. One to the impose-blockade state s_2, and the other to the perform-surgical-air-strike state s_4. Both of s_2 and s_4 are preferred to the US than the status quo s_0. Recall that in all of the three states, the USSR maintains having the same alternative as its strategy to deal with the conflict, and that is Not Withdraw the Missiles from Cuba (A_0). The US, on the other hand, chooses a different alternative in each of these states as a strategy to deal with the conflict. This is why the US is considered moving unilaterally among them. In a similar manner all the USSR's Uls are listed under the appropriate state in the USSR's preference vector in the stability analysis Table 6.5.

Second, the table shows beneath each state in the player's preference vector the strength of the state's stability for the player under each of the four stability solution concepts (NASH, GMR, SMR and SEQ), if and only if the state is stable for the player under the speechified solution concept. If the state is not stable for the player under the specified solution concept, the table will show nothing.

For example, consider state s_0 for the US. The US has two UIs out of this state, making it unstable under any of the solution concepts as per the mathematical properties given for each in Definitions 6.3.1 - 6.3.4. Therefore, Table 6.5 shows nothing in the US tabular under s_0. On the other hand, state s_0 is stable for the USSR under NASH stability Definition 6.3.1 (with Extreme stability strength based on NASH Stability Strength given in Definition 6.4.1); and therefore stable for the USSR under GMR, SMR and SEQ stability solution concepts as per their respective definitions indicated earlier (with None stability strength based on the stability strength respective definitions and algorithms indicated earlier). These stability strengthes are shown under state s_0 in the USSR tabular within Table 6.5.

Table 6.5: Cuban Missile Crisis: Stability Analysis

US

	s_3	s_1	s_4	s_2	s_0	s_6	s_5	s_8	s_7	s_9
UIs				s_4 (UI)	s_2 (UI) s_4 (UI)					
NASH	Ex	Ex	N			Ex	Ex	Ex	Ex	Ex
GMR	N	N	N	L		N	N	N	N	N
SMR	N	N	N	L		N	N	N	N	N
SEQ	N	N	N			N	N	N	N	N

USSR

	s_8	s_0	s_1	s_3	s_2	s_4	s_6	s_5	s_7	s_9
UIs					s_3 (UI)					
NASH	Ex	Ex	Ex	N		Ex	Ex	Ex	Ex	Ex
GMR	N	N	N	N		N	N	N	N	N
SMR	N	N	N	N		N	N	N	N	N
SEQ	N	N	N	N		N	N	N	N	N

Table 6.6: Cuban Missile Crisis: Equilibrium States

	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
NASH EQ.		Ex		N	N	Ex	Ex	Ex	Ex	Ex
GMR EQ.		N		N	N	N	N	N	N	N
SMR EQ.		N		N	N	N	N	N	N	N
SEQ EQ.		N		N	N	N	N	N	N	N

While Table 6.5 shows the stability for each of the conflict's states being analyzed from a particular player's point of view, Table 6.6 shows the equilibrium states of the conflict. Under each state on the top row of Table 6.6, the table lists the strength of the equilibrium under each of the stability solution concepts if the state constitutes an equilibrium under the solution concepts. If the state is not an equilibrium state for the conflict under the specified solution concept, then the table will show nothing under the state. Recall that the state is considered an equilibrium under a specific solution concept if it is stable under the solution concept for both players (Definition 6.3.5); and the strength of the equilibrium is determined by applying Definition 6.4.5 and its implementation procedure given in Algorithm 6.6.

6.5.9 Results of the Cuban Missile Crisis Analysis

The results of the above analysis provide us with significant insights into the Cuban Missile Crisis. We will first discuss the insights gained, focusing on two aspects: analyzing the equilibrium states of the conflict found by the previous step of our analysis; and analyzing how the conflict is expected to evolve over time starting from the status quo state s_0. Then, we will compare our findings with what actually happened in this important historical conflict. Finally, we will compare what we learned from our analysis with the analysis of other methods found in the conflict analysis literature which addresses the Cuban Missile Crisis conflict. For our comparison analysis we will specifically compare our work to the work of Fraser and Hipel (1984), and the very famous work of Allison and Zelikow (1999).

1) Analysis of the Cuban Missile Crisis's Equilibrium States:

Table 6.5 shows both the US and the USSR have states s_1, s_3, s_4, s_5, s_6, s_7, s_8, and s_9 all as NASH stable states. This reflects the fact that both players have no UI moves out of any of these states. As a consequence, all these states are shown in the equilibrium analysis, Table 6.6, to be equilibrium states for the conflict under NASH stability solution concept. Two questions immediately arise: 1) how could the conflict have as many equilibrium states? Normally, conflicts have few equilibrium states; and 2) why Table 6.6 shows the strength of the NASH equilibrium stability for states s_3 and s_4 to be at the *None* level while the rest of the states have their NASH stability shown to be at the *Extreme* strength level?

First, most conflicts will have few equilibrium state, as our case studies in this thesis work show and as suggested by many scholars (e.g. Fraser and Hipel (1984)).

This fact is in general a reflection of the complexity of the conflicts analyzed. If a conflict has many of its states found to be stable for all players, particularly under NASH solution concept, and therefore form equilibriums (NASH EQs) suggesting that the conflict could end at any of them, then this represents a situation where the players have less maneuverability in the conflict. This is exactly what we find in the Cuban Missile Crisis. None of the players wants to escalate the conflict to a point where a nuclear war becomes possible. Figure 6.5 shows this clearly. States that have the US, the USSR or both adopt a very aggressive alternative, such as "full attack" or "escalate", the players have no UI moves to them (leading to them) and from them (taking them to a more aggressive state escalating the conflict further). In fact, this proves that none of the players was willing to take the risk of leading the conflict in that direction.

Another reason, for this too-many-equilibrium-states situation, is the possibility that some or many of these equilibrium states should not be considered as legitimate states, in the first place, in our stability analysis. This might be true, for example, in the case of the Negotiation state (s_3). Indeed our analysis of the players' GCMs and alternatives, conducted earlier in Section 6.5.4, showed that the US will not take this alternative as an option, and the USSR should cut any hopes of having this alternative as a valid strategy that the US will agree to. This is especially true because of the fact that the US's U2 spy planes had uncovered the USSR's Cuban missiles scheme before the missiles were operational or close to be operational. This removed the element of surprise which the USSR was counting on; and of course took out with it the possibility for the USSR to use this Cuban missiles base as a counter measure to the US's Turkey missile bases, or even as a bargaining chip to negotiate the removal of the US's Turkey bases in exchange of the removal of the USSR's Cuban base. This not-operational-missile-bases fact and its effect on the players' alternatives, and by propagation on their strategic goals, were captured in the conflict model we showed then. This should have suggested that including the negotiation s_3 state was extra and not necessary. We said then that we are including it to enrich the analysis and to see if this state will have any chance to form an end to the conflict (recall that to do so we removed the not-operational constraining effect when eliciting the players preferences). Despite this, the state came at the end of the US's preference vector, while came first, as expected, at the USSR's vector. And, with no UIs for the players to reach to it from the status quo state s_3, as we will see later in this section, it ended to be a mere hypothetical equilibrium that could not materialize as a resolution state to the conflict.

Second, a state's equilibrium strength under a specific solution concept is a function of its stability strength, under the same solution concept, for each of the players, as noted in Definition 6.4.5. For states s_1, s_5, s_6, s_7, s_8, and s_9, both the US and the USSR have them stable under NASH stability concept. Because none of these states have UIs leading to them, by the players individually, these states are assigned *Extreme* strength for their NASH stability for each player (as per Definition 6.4.1 and Algorithm 6.2). As a result, these states all form *Extreme* Equilibriums for the conflict under NASH stability concept.

On the other hand, while both players have no UIs away from states s_3 and s_4, and therefore have these two states stable under NASH, but the stability strength for them are showed to be in Table 6.6 at the *None* strength level (i.e. with very little/weak strength). This is due to the fact that the players can reach these two states as a result of using UIs from other states which are less preferred to them. For the US, s_4 can be reached through its UIs from s_0 or s_2, and we know from Table 6.4 that $s_2 \succ^{N}_{US,t} s_4$ and that $s_0 \succ^{-L}_{US,t} s_4$. By applying Definition 6.4.1 and its accompanying Algorithm 6.2, the NASH stability of state s_4 to the US is determined to be at the *None* strength level. For the USSR, state s_3 can be reached by using a UI from s_2, and Table 6.4 shows that $s_2 \succ^{N}_{USSR,t} s_3$. By applying the same definition and algorithm, the strength of s_3's NASH stability, to the USSR, is determined to be at the *None* strength level too. Applying Definition 6.4.5, the strength of the NASH equilibriums at s_3 and s_4 will be the minimum of the NASH stability strength for the two players of the conflict, making these two states at the *None* NASH equilibrium strength level.

It is important to remember here that the fuzzy strength label for a stability or an equilibrium set to be *None* does not mean that there is no stability or equilibrium here. It just means that the strength is at the "very little/weak" level, or at 10% or less level based on the fuzzy membership function we used to define the label *None*. Also, recall that if a state has no UIs out of it for a player, i.e. NASH stable for him, it will automatically be GMR, SMR and SEQ stable for the player, as per Definitions 6.3.2, 6.3.3 and 6.3.4, respectively. The strength of these GMR, SMR and SEQ stabilities will be set to None level as per Definition 6.4.2, 6.4.3 and 6.4.4, respectively (and by applying their accompanying Algorithms 6.3, 6.4 and 6.5, respectively).

We so far discussed how and why our analysis found too many equilibrium states for the Cuban Missile Crisis, and how valid these states as resolution end points for the conflict. We will now further validate our findings that many of these equilibrium points are hypothetical and do not form real resolution stable end points for the conflict. We will do so by conducting an analysis on the possible pathes that the conflict could take, or evolve to, starting for the status quo state of the conflict (October 14, 1962). As we will see below, many of these states are unreachable by the players, if both players will act "rationality" in this conflict, i.e. act consist with their respective preferences order. Despite the fact that the "rationality" assumption is a very valid and realistic assumption, considering the collective rationality of the players' institutions and their understanding of the consequences for acting against it, one should not rule out the possibility that one of the players could decide, by mistake or by design, escalate the conflict putting it at a path that could lead to the most destructive war humanity could face: a nuclear war. We will look also at how this unfortunate evolution could happen.

2) Evolution of The Cuban Missile Crisis Starting from the Status Quo:

Table 6.7 shows the possible scenarios of how the Cuban Missile Crisis could evolve starting from the status quo state s_0, where the US threatens (and does nothing else or uses only diplomatic pressure) and the USSR decides to keep the missile base in Cuba (working on completing the installation of it or hoping to do so in the near future). Giving the status quo's current time the symbol t, we will look at the player's option at time t and moving forward:

At time t: Let us consider at this point whether the US and the USSR both will be compelled to stay at the status quo s_0 state, and as a result the conflict would never progress beyond this point (Scenario 1 in Table 6.7). This scenario is unlikely to persist because s_0 is unstable to the US (see Table 6.5) even though it is a NASH stable state for the USSR. The US has a UI out of this state to either state s_2 or state s_4, and will rationally activate one of them leading the conflict to adopt Scenario 2 or Scenario 6, respectively.

At time $\mathbf{t+1}$: With the exception of Scenario 1, all the scenarios presented in Table 6.7 expect the conflict to extend beyond the status quo state. And, knowing that Scenario 1 is unlikely to persist, as discussed above, then most likely the conflict will move one step further reaching an end at or after $t+1$. Looking at the scenarios extending the conflict beyond t, we find that all show the US taking one of the two UIs it has out of the status quo state s_0.

Table 6.7: Cuban Missile Crisis: Evolution Scenarios (starting from the status quo s_0)

Scenario No.	No.of Steps	Conflict Evolution (0 1 2 3)	End State: EQ	End State: Stability for US	End State: Stability for USSR
1	0	0		Unstable	NASH (*Ex*)
2	1	0 →(US, UI) 2		GMR (*L*) & SMR (*L*)	Unstable
3	2	0 →(US, UI) 2 →(USSR, UI) 3	NASH (*N*)	NASH (*Ex*)	NASH (*N*)
4	2	0 →(US, UI) 2 →(US, UI) 4	NASH (*N*)	NASH (*N*)	NASH (*Ex*)
5	3	0 →(US, UI) 2 →(US, UI) 4 →(USSR, UM) 5	NASH (*Ex*)	NASH (*Ex*)	NASH (*Ex*)**
6	1	0 →(US, UI) 4	NASH (*N*)	NASH (*N*)	NASH (*Ex*)
7	2	0 →(US, UI) 4 →(USSR, UM) 5	NASH (*Ex*)	NASH (*Ex*)	NASH (*Ex*)**

** irrational last move by the USSR (made just to hurt the US)

Scenarios 2, 3, 4 and 5 show the US takes a UI to s_2, while Scenarios 6 and 7 show the US takes a UI to s_4.

One of the two maneuvers the US could do, after threatening the USSR and the USSR ignoring its threat (state s_0), is to activate a UI move it has out of s_0 to state s_2 where it imposes a blockade on Cuba. The purpose of this blockade is mainly to not allow the USSR to complete the missiles installation in Cuba, and also to inflict strict economical sanctions on Cuba. The second maneuver is to activate the UI it has from s_0 to state s_4 where it carries out a surgical air strike against the missile base in Cuba destroying it and eliminating its threat. This option is direct to the point but bear some serious risk. So what is the most rational move the US could have out of s_4? Let us consult the the stability analysis presented above.

s_2 is GMR and SMR stable for the US, with a higher degree of stability strength than the NASH (with its by product de facto *None* weak GMR and SMR) stability to be gained if the US moved to s_4 instead. Recall that GMR and SMR stability concepts consider other players' counter moves, while NASH does not. s_4 is mainly NASH stable for the US. The GMR and SMR stability for the US for this state is because the US has no UIs out of it and not because we considered the USSR's counter moves. s_2, on the other hand, is not NASH stable for the US, but GMR and SMR stable, this is due to the possibility of the USSR moving to s_5 if the US decided to use a UI move it has to s_4 from s_2, putting itself and the US at a less preferred position where the conflict could escalate further by both players acting irrationally to hurt each other.

This makes the US moving to state s_2 from the status quo state s_0, reaching it at time $t+1$, to be a more likely scenario (Scenario 2 in Table 6.7) than the one in which the US moves to state s_4 from s_0 (Scenario 6 in the same table), reaching it at time $t+1$. In addition, moving to state s_2 gives the US more maneuverability since it can escalate and take the conflict to state s_4 (Scenario 4) if the USSR does not take its UI to state s_3 removing the missiles from Cuba (Scenario 3). On the other hand, if the US moves the conflict to state s_4 then the USSR could act just to hurt the US by taking a non-UI move it has to state s_5 putting itself and the US at a less preferred state (Scenario 5). In other words, for the US, while choosing to adopt Scenario 2 gives the USSR a chance to take a UI leading the conflict to evolve to Scenario 3, choosing to adopt Scenario 6 leaves the USSR with one of two: living with a less preferred state (Scenario 6); or choosing to hurt/punish the US even if this means the USSR itself will end at an even further less preferred state (Scenario 7).

One question we should answer here: Will the conflict end at one of the two states s_2 and s_4 reached at the $t+1$ point of time (Scenarios 2 and 6, respectively), or will it continue beyond this point? State s_2 is unstable for the USSR since it has a UI out of it to state s_3 which the US cannot disimprove upon. But, state s_2 is GMR and SMR stable for the US, and this will make the US try to hold on to it and not take its UI to state s_4 out of it, in fear of the USSR retaliating by moving to state s_5 in an effort to hurt the US and save its face as a superpower in the world, even if this means hurting itself too. This makes state s_2 not an equilibrium for the conflict and a less likely candidate to mark the end of it. But, it is definitely a more rational state to be in, for the US, than moving directly to s_4 from the status quo s_0.

Ending the conflict by having the US move to s_4 is possible, since the state in NASH stable for both players. They both have no UIs out of this state. So if the US and the USSR both acted rationally, state s_4 is a NASH equilibrium and could be a possible end to the conflict. But, if the USSR decided to act irrationally, i.e. against its justifiable preferences order seen in Table 6.4, in order to just hurt the US as we said above, then the USSR will take a step move to s_5 escalating the conflict (this could be done by invading/attacking West Berlin for example - something the US does not want to happen). From state s_5, the conflict will likely escalate further by both players, each acting against their respective preferences order, just to hurt the other player and save its superpower face in the world. For this reason, and rationally speaking, the US is not likely to move to s_4 reaching at time $t+1$ in the first place, but try instead to maintain an s_2 position at $t+1$ giving the USSR a chance to act rationally and utilize a UI move it has to s_3 removing the missiles from Cuba and putting an end to the conflict.

At time $\mathbf{t+2}$: There are four scenarios from the ones presented in Table 6.7 that could extend the conflict beyond the $t+1$ point of time. Three of these scenarios, Scenarios 3, 4 and 5, extend the conflict if the US chooses to move to state s_2 at time t reaching it at time $t+1$, and the conflict did not end at s_2 (i.e. did not end to be a Scenario 2 situation). The fourth scenario is Scenario 7 which extends the conflict if the US chooses to move to state s_4 at time t reaching it at time $t+1$, and the conflict did not end at s_4 (i.e. did not end to be a Scenario 6 situation).

The most rational evolution of the conflict will be consistent with Scenario 3. As said before, the US took this route to give the USSR the chance to act rationally and utilize its UI out of s_2 reaching state s_3 at time $t+2$ by dismantling the Cuban missiles and as a result ending the conflict. But, if the USSR delayed taking its UI to s_3 from s_2, then the US will interpret this as a sign that the USSR decide to complete the installation of the missile base in Cuba. A hint of such action by the USSR, will immediately make the US take the conflict to state s_4, acting on a UI it has from s_2. This is a rational move by the US and will definitely put the conflict at a NASH equilibrium. The US has the missiles destroyed by the carried out surgical air strike and has no further rational intention, or UI, to escalate the conflict beyond this point ($t+2$). Similarly, the USSR has no UI out of s_4, and rationally should not escalate the conflict beyond this point. In other words, the conflict could end rationally at either s_3 or s_4 (Scenario 3 or 4, respectively). Both states are NASH equilibrium states, but state s_3 is a more likely ending of the conflict (Scenario 3) because it constitutes ending the conflict at a more preferred state. Recall, from Table 6.4, that $s_3 \succ^{N}_{US,t} s_4$ and $s_3 \succ^{L}_{USSR,t} s_4$ making state s_3 more preferred than s_4 for both players. And therefore, Scenario 3 constitutes, for both players, a more preferred path for the conflict to evolve to than Scenario 4.

On the other hand, the US could move to state s_4 because it ran out of patience for having the USSR take a UI to s_3 after it made its move to state s_2 from the status quo state. It also, out of fear from allowing the USSR to finish installing the missiles, could moved to state s_4 directly from the status quo state. In any of these two scenarios takes place, and as a response the USSR acts irrationally escalating the conflict in retaliation, moving it to state s_5, then conflict could end up been taken the Scenario 5 route, or the the Scenario 7 route, based on whether the US moved to s_4 from s_2 or directly from s_0, respectively. In both cases, the USSR acts irrationally against its own preferences order, motivated only by revenge and the fact that this will hurt the US. This makes the conflict ends at a less preferred state for both the US and the USSR. These are unlikely evolution paths for the conflict, because both assume that the USSR will act irrationally hurting itself. But, they are very possible scenarios considering that the cold war politics at the time did not mature enough. In such scenarios, it is possible that both parties will come to their senses, act rationally not escalate the conflict further, and end the conflict at state s_5, a NASH equilibrium. But we have to remember that at the time both parties were in their early stages of their coalitions building and they both were topping each other in their military capabilities. Going to state s_5, for the USSR means showing the world, and possible coalition partners, that it is a superpower and equal to the US. One could expect, as a result, the US to have a change of its preferences vector (the one shown in Table 6.4) making a retaliation and further escalation a possible scenario. Indeed, a very dangerous and destructive evolution of the conflict. For this reason, both players are not likely to take the conflict these two scenarios' ways.

At time $\mathbf{t+3}$: if the conflict extended beyond $t+2$ and was not resolved by then, then there is only one scenario the conflict is going: Scenario 5. According to this scenario, the US had moved to s_2 from the status quo state, then waited for the USSR to remove the missiles from Cuba taking a UI to state s_3 but the USSR did not. The US, then, carried out a surgical air strike destroying the missiles and putting the conflict in state s_4 at time $t+2$. The conflict did not end by the USSR acting rationally according to its preferences order (Scenario 4). Instead, the USSR retaliated by escalating the game and putting it in state s_5 at time $t+3$ (Scenario 5). As we said above, if the conflict actually evolved to be following this dangerous route, then this could lead to either: both parties act rationally and end the conflict at state s_5, a NASH equilibrium; or act irrationally and take further steps to escalate the conflict beyond s_5, maybe all the way to the nuclear war (state s_9).

We can conclude from the analysis above that while Scenarios 3, 4, 5, 6 and 7 shown in Table 6.7 form possible evolution paths for the Cuban Missile Crisis, Scenario 3 proves to be the most logical evolution path for the conflict. Second to it is Scenario 4. This is true if and only if both the US and the USSR act rationally (according to preferences order). If, on the other hand, one or both of them act in an irrational manner, for some reason or another, the possibility of having the conflict reach the escalating point of s_5 (Scenario 5 or 7), or go beyond (dragging the conflict further to s_7 or s_9), becomes very real.

3) Comparing our Analysis with What Really Happened:

Let us compare how the conflict actually evolved, historically, to how our analysis predicts it would. Table 6.8 presents the events of the Cuban Missile Crisis in their historical order, as given by Allison and Zelikow (1999), starting from the time the US put the conflict at the status quo state, with information about the state that the conflict is at during the event, who made the move to the state (in brackets we show whether the move is a UI move for the mover), the step number (or as it could be read: time $t+$ no. of steps/moves by the players so far), in addition to comments added to the event's description.

As Table 6.8 shows, the conflict in reality followed Scenario 3, which we described in Table 6.7 and discussed above. One should take note of important observations from how the conflict historically followed Scenario 3, and how much the analysis produced by our framework resembles what had had happened on the ground at the time. First, it is interesting to see that the US at the same time (October 22, 1962) it threatened the USSR demanding a removal of the Cuban missiles, putting the conflict at the status quo state s_0, it actually moved the conflict to state s_2 immediately after that, by announcing the plan to impose the blockade on Cuba. In effect, this shows that the US was certain that the USSR will not move from state s_0 to s_1 removing the missiles. This is because the USSR has no UI move

Table 6.8: Cuban Missile Crisis: Analysis vs.Reality

Step	Date	Actual Conflict Evolution	Move by	to/at State
0	Oct. 22, 1962	Kennedy initiates a public confrontation by announcing the Soviet action to the world, demanding Soviet removal of their missiles, putting U.S. strategic forces on alert, and warning the Soviet Union that any missile launched from Cuba would be regarded as a Soviet missile and met with a full retaliatory response.	US	s_0 Status Quo
1	Oct. 22, 1962	Kennedy as part of his public confrontation ordered a U.S. quarantine of Soviet weapon shipments to Cuba.	US (*UI*)	s_2
	Oct. 23, 1962	Khrushchev orders Soviet strategic forces to alert and threatens to sink U.S. ships if they interfere with Soviet ships en route to Cuba.		s_2
	Oct. 24, 1962	Soviet ships stop short of the U.S. quarantine line.		s_2
	Oct. 26, 1962	Khrushchev private letter, to Kennedy, says the necessity for the Soviet deployment would disappear if the U.S. will pledge not to invade Cuba. *Khrushchev tries to suggest for both parties to negotiate a deal, i.e. go to state s_8.*		s_2
	Oct. 27, 1962	Khrushchev second, but public, letter demanding US withdrawal of Turkish missiles for Soviet withdrawal of Cuban missiles. *Khrushchev tries, again, to suggest for both parties to negotiate a deal, ie.e go to state s_8, citing here the real concern of the USSR: the US missiles in Turkey.*		s_2
	Oct. 27, 1962	US responds affirmatively to first Khrushchev letter but says that, first, missiles now in Cuba must be rendered inoperable and urges quick agreement. Robert Kennedy adds privately that missiles in Turkey will eventually be withdrawn but that the missiles in Cuba must be removed immediately and a commitment to that effect must be received the next day, otherwise military action will follow. *In effect, the US is hinting here a move to state s_4, if the USSR does not move to state s_3, and is telling the USSR that state s_8's negotiation is not an option at this time.*		s_2
2	Oct. 28, 1962	Khrushchev publicly announces that the USSR will withdraw its missiles in Cuba	USSR (*UI*)	s_3
				Scenario 3

from s_0 to s_1, so the US decided to move directly to state s_2 and save precious time could have been wasted otherwise.

Second, the US chose to move to s_2, from the status quo state s_0, and not move to state s_4 instead. The analysis provided above sheds light on why the US chose the s_2 path. It is less risky. It gives the USSR a chance to utilize a UI it has from s_2 to s_3 removing the missiles from Cuba and ending the conflict. That is if the USSR acts rationally. It, also, allows the US to test the USSR's preferences order, and how rational the USSR will act in the conflict. Judging from the USSR response and reaction, whether the USSR will respect the blockade or try to escalate, how far the USSR managed to get the missiles operational. If indeed the the missiles were operational or very close to be, then the USSR will act aggressively and decisively, otherwise it will pay lips service but respect the blockade. In addition, the move to state s_2 does not eliminate the option of going to state s_4 (carry out air strikes to destroy the missiles). Going to s_4 after being in s_2 is still a valid option. In fact the US has a UI to do so, if the USSR fails to take its own UI from s_2 to s_3 in a reasonable time (a long period beyond the week time frame means that the USSR is working on making the missiles operational). If, instead, the US moves to state s_4 carrying out air strikes, then this does not give the US the option to use blockade if the air strikes fail to destroy the missiles (a complete impressment to the US considering its earlier failure of the Bay of Pigs Invasion). The US also does not to invoke an irrational a move by the USSR escalating the conflict beyond Cuba. By moving to state s_2 it ensures the conflict stays about Cuba and not get escalated to include Europe (West Berlin specifically).

Third, once the conflict reached state s_2, the USSR while initially paid lip service to not caving in to the US demands and blockade threats (event of October 23, 1962), it actually did respect the blockade. This demonstrated by the event of October 24, 1962, when the Soviet ships stopped short of the US quarantine line. This shows that the USSR had no intention to move the conflict to the escalation path. This is confirmed by our analysis showing that the USSR has no UI out of state s_2 to a state where it escalates the conflict, such as states s_5, s_7 or s_9.

Forth, one could not but notice the attempt by the USSR to suggest going the negotiation path. Khrushchev letters of October 26 and 27, 1962 shows two things. It shows the true strategic goals the USSR have in mind, and why it tried to deploy the missiles in Cuba. These goals were modelled within the GCM we have for the USSR. Recall that we have tested each alternative the USSR have, and by extension each state of the conflict, against these strategic goals. These alternatives/states contribution to the achievement of these goals guided the process of eliciting the true preferences order for the USSR. In addition, these letters confirm the preferences order we elicited for the USSR. They show how the USSR prefers going to the negotiation state s_8 more than anything else because it provides the best achievement to its strategic goals.

Fifth, the move by the US to ignore USSR's suggestion to go to the negotiating rout (go to state s_8 from state s_2) is fully explained by our conflict model and analysis, presented in Section 6.5.4. The effect of the realities on the ground, the constraints modelled as part of the plays GCM models, show that the fact the lack of operationally of the missiles at the status quo time rendered the negotiation alternative for the US to be not needed or desired. It also show that the USSR should not keep it hopes up for the US choosing this option. The USSR, by sending these letters, tried to confirm if the US is aware of the actual stage of completion and operationally of the Cuban missiles at the time, or they are ignorant about it. The US responded on October 27 by ignoring USSR's suggestion to go to state s_8 moving together to negotiate a deal to end the conflict, and instead threatened to go state s_4 if the USSR does not go to s_3 removing the missiles. This response by the US confirmed to the USSR thatthe US is aware of the fact that they had discovered the missiles before they are operational or close to be operational.

Last, the path the conflict took, or Scenario 3, confirms that the analysis was right in predicting rational behavior by both the US and the USSR. None of the players have any incitive in escalating the conflict or expanding it. In fact, for most of the observers and analysts (Allison and Zelikow (1999) for example), the moves by the US to impose a blockade (s_2) instead of carrying out air strikes (s_4), and by the USSR to state s_3 removing the missiles instead of escalating the conflict, were puzzling and needed much lengthy discussion on how and why, with no mathematical or logical models to provide the answers to these question in a concise manner. The models and analysis we provided provides a clear cut answer to why each step the players took in reality, during the conflict, is the most rational step to take; and why the sequence of events observed in reality is the most rational sequence of events for the conflict to take.

4) Comparing our Model and Analysis with Others' Work:

We will look at how our model and analysis of the Cuban Missile Crisis compare to the models and analysis of two important works in the field: The first is the work of Fraser and Hipel (1984), which presents a fairly normative and mathematical modelling and analysis of the Cuban Missile Crisis; and the second is the work of Allison (1971), the very famous and the most elaborate work on the crisis (updated to include many information revealed from the archives of the US and the USSR after the cold war ended in the early 90's and published as Allison and Zelikow (1999)), which provides mostly a descriptive analysis of the conflict, with no mathematical models to backup the analysis.

The work of Fraser and Hipel (1984) provides a normative conflicts modelling and analysis framework similar to the ones typically presented in the game theory literature, but differ in an important aspect: it is based on an ordinal representation of the players' preferences over the states of the conflict, not a cardinal one. The states are given binary numbers reflecting the selections players made from a menu of options/actions they can take. The work is based on solid mathematical grounds and provides many analytical tools, building on the work of Howard (1971), but suffers from some of the many weaknesses typical game theory methodologies suffers from (see the discussion in Chapter 2).

First, the work provides no means to model the mapping between the preferences and their ordering and the strategic objectives the players have, within the context of the conflict or in general. For example, the model of Cuban Missile Crisis presented does not show or explain why the the players' preferences are the way given in the model, and what should change, in terms of players' goals/priorities or realities on the ground, to make the preferences have different order. Comparatively, in our models we show the direct link between the players goals and conflict realities to how the players' preferences are modelled, or elicited as should be noted. In fact, any change to the players' goals, priorities, internal or external constraints/realities will be reflected on how these preferences get ordered. The analyst can immediately test why a state is more/less preferred than another for a certain player, and what should be done to reverse this order or change it.

Second, the ordinal representation of the players' preferences does not show any degrees of preference's strength beside the simple binary relations the ordering is based on: indifferent $\sim$ relation and the preferred-to $\succ$ relation. The work of Hamouda et al. (2004) tried to rectify this weakness by proposing degrees of preference to mainly the the preferred-to $\succ$ relation, offering relations such as $\succ\succ$ to show much-preferred status. Still, all these relations do not show "why" the degree of preference existed in the first place, and how it will affect the satisfaction of the player's strategic goals. The relations presented in Fraser and Hipel (1984) and the modified ones in Hamouda et al. (2004) are all based on strict logic: yes or no, applies or does not apply. On the other hand, our models provide a one complete preferred-to relation that: 1) has fuzzy qualifier to show the degree of strength for the preference, in addition to the preference order; and 2) offers traceability of the preference order and degree of strength back to the player's strategic goals and realities, answering why and how the preference came to be the one elicited. In our models, the analysis does not take the player word, or more accurately, his guess as a justification for the preferences order and strength. The justification, and validation, process is embedded within the way the preferences are elicited and represented.

Take for example how Fraser and Hipel (1984) show that the US prefers imposing a blockade while the USSR keeps the Cuban missiles (what we call state s_2) over conducting a surgical air strike to destroy the missiles (state s_4 in our model). Why? Is this a true preference order that reflects the benefits the US gets from both states? In reality, the US benefits more from the air strikes. As shown from the contribution that the states have to the strategic goals of the US in its GCM model: s_4 will ensure not giving the USSR time to complete installing the missiles; it will make sure the missiles are destroyed; it will provide some face saving at the world stage after the damage happened because of the failure of the Bay of Pigs invasion; and it will help the US in its long established strategy of bringing south American countries to its fold by any means (including fear and force). And, if its true that the US prefers s_2 over s_4, then the US has no improvement out of imposing the blockade state s_2 to the air strike state s_4 if and when the USSR do not remove the missiles because of the blockade. In reality, and as reflected in our model, the US prefers s_4 over s_2 (as also indicated by the discussion of the US's Joint Chiefs of Staff at the time, Allison and Zelikow (1999)). The only reason the US did not go to s_4, and instead went the s_2 way is the fear that the USSR will escalate the conflict out of retaliation (this fact is presented in our analysis of Table 6.5 as s_2 has GMR and SMR stability reflecting the US's fear in taking the UI it has from s_2 to s_4).

Third, because of the way Fraser and Hipel (1984) defines the State concept and the UI move concept, some of the states and UIs presented in their model of the Cuban Missiles Crisis are not realistic or feasible. For example, the model presents and includes in the stability analysis the following two states: A state in which the US chooses to conduct a surgical air strikes to destroy the Cuban Missiles, while the missiles neither destroyed or withdrawn; and another where the US conducts the air strikes with the USSR withdrawn the missiles. Conducting a surgical air strikes, in principle, will not lead to the missiles withdrawal but rather to the missiles destruction. The failure in carrying the air strike and not destroying the missiles is not a reasonable assumption, considering the proximity of Cuba, the US's powerful means and capabilities in doing so, and the fact that this is not an invasion where failure is a possibility. As a second example, their model presents states where the US implement both a blockade on Cuba and Air strikes. This contradicts the logical sequence of possible events. There is no need for a blockade, if air strikes are carried out; and blockades usually are used to help control the environment and weaken the enemy before carrying aggressive military actions. In our models, the analyst can rely in logical reasoning to eliminate redundant states, such as the ones included by Fraser and Hipel (1984) for only mathematical reasons (the states are presented as binary numbers where players' options are given a one bit of true or false (1 or 0) selection).

We also believe that UI representation in Fraser and Hipel (1984) models is an additional problematic area. Consider, for example, how the US is shown to have a UI move from a state where the US conducts an air strike and simultaneously impose a blockade to a state where it just carries out air strikes alone. In effect, the provided model suggests that the US will be able to eliminate the fact that it carried out a blockade after this has been done. This is not a realistic, or even valid, assumption to have. Once things happen in real-life, they cannot be reversed and deemed undone. Such problem is shown in the model because the conceptual modelling foundation of the Fraser and Hipel (1984) is putting mathematical representation in the driving seat. A UI move, to it, happens when all the bits, of the states' binary numbers, for one player is changing from less preferred state to a preferred state, while the bits of the other player are not changed. The representation is less concerned with the validity of such move, or the ability of the player to take it in real-life.

A problem that we do not have in our conceptual modelling framework. The analyst in our case is in the driving seat. He will, and should, examine the validity of each move. In our models, and consistent with a realistic view of how things happen in real-life, a UI move is a real unilateral move the player can take to enhance his position. The automated DSS system implementing our framework, as a knowledge modelling and management framework, demands justification of the beliefs held by the analyst of the ability players to make moves, UM or UIs. It cannot sacrifice validity in favor of ease of automating and reliance on binary value flipping as a justification. In fact the later works of the research group behind Fraser and Hipel (1984) rectified some of these problems, but not completely, by enhancing the representation scheme of the moves specifically to be based on directed graph presentation instead, at least at the visual level, while still relaying in checking for binary bits flipping as a mechanism to trigger the identification of players' moves.

In summary, when we compare the work of Fraser and Hipel (1984) to our work presented in this chapter, we used how Fraser and Hipel modelled the Cuban Missile Crisis as an example to illustrate the differences. While both works rely on mathematical and logical modelling and analysis schemes, they differ in many areas. The work of Fraser and Hipel, as most of the game theory methodologies in the literature, does not model the decision makers' goals and realities, and therefore do not provide direct mapping between how the preferences over the conflict's states happen to be modelled and how these states satisfy the strategic needs and wants of the players, or get affected by the realities on the ground. This important feature is at the heart of what we believe should be the starting point of modelling strategic conflicts. Dealing with strategic goals satisfaction through proxies such as stated preferences order or deceiving subjective utilities, while makes the mathematical presentation looks simpler and nicer, violates the principal of rationality employed by the normative decision analysis literature: choosing the best option that satisfies/satisfices the goals of the decision maker. We also differ in the way how states, players moves, as well as preferences relations and structures, defined and presented. We discussed how the models and analysis we provided in this chapter resulted on more insights about conflict's dynamics and better analysis of the players' options.

At a different front, comparing the rich and extensive study of the Cuban Missile Crisis provided by Allison (1971) to our model and analysis is quite an undertaking. So while we intend to do this in a separate extended analytical comparative study (a future work), in order to give justice to it, we will focus our discussion here on some few important differences.

Allison (1971) is a descriptive analysis of the Cuban Missile Crisis, whilst our work, presented in this chapter, is a normative one in its nature. Game theory literature, and the conflict analysis literature in general, differentiate between the two. Normative analysis prescribes (says what should be done) or evaluates (says what's good/bad); and a descriptive one describes (says how the world in fact is/was/will be). Beside the motivation difference, the methods employed also differs, dramatically. Normative decision making and conflict analysis studies rely on mathematical and logical models to shed light on conflicts, suggest optimal courses of actions and/or predict the flow of events. At the same time, descriptive studies uses experimental methods and/or fact findings analytical techniques. This leads to a list of important distinctions and observations:

First, Allison's model is an important model because it provided an account of the conflict based on what had happened, factually and based on hard evidence. But this account will only be possible after the fact. It will not help, or guide, the decision makers on what actions they have or should take at/during the conflict. The only possible way that the players can use such modelling technique is to rely on setting up a set of hypothesis, hopefully the right ones, and test them through discussion and textual analysis. Much the same way Allison did in his Rational Actor model for the Cuban Crisis. The lack of visuals, mathematical and logical tools, stability solution concepts, and so on, in such approach make the process one of a kind each time, each and every time it takes place. This one-of-a-kind modelling and analysis process has many problems: 1) it is more troublesome to conduct and follow for the non-specialist (most decision makers fall under this category of people); 2) not easy to verify or validate the produced models and analysis; and 3) transfer of lessons-learned (and the knowledge gained in general) from one conflict to another, or one iteration of a conflict to another becomes very hard to specialists, not to mention novices, because of the nature of the specific on-of-a-kind descriptive models/analysis. Consider the fact that a specialist such as Allison had to *rewrite*, not just update, the book once new documents/facts came to light. If this is so for Allison, then what should a bureaucrat/professional, who is expert in his field but not grounded on the since of decision making and conflict analysis, should do.

On the other hand our modelling technique provided a systematic and methodological process to follow each and every time a conflict, or a new evolution/what-if-variation of the same conflict, is to be analyzed. This makes the analysis of each conflict, or variation of the conflict, to be easily compared to other conflicts/variations and easily verifiable ensuring: all the right questions are asked, all the steps are followed, all the options are explored and all scenarios are studied.

Second, we see the two types of modelling techniques, the normative and the descriptive, complementary. Each of the two needs the other. The normative model needs verification, to ensure reflecting reality; and the descriptive model needs validation, to ensure proof of the hypotheses it is examining, and to ensure broadening the analysis beyond such hypotheses and without relying on narrow selective/emotional fact finding.

A normative model such as ours needs to be validated through comparison with what had happened in reality to: uncover any deficiencies and why the models missed to address them; and learn from this to enhance the process, add safe guards to ensure adequate attention is given to address such deficiencies in the next iteration of the conflict or in any new conflict we analyze.

Third, when one look at what Allison (1971) calls the Rational Actor model for the Cuban Missile Crisis, one will immediately notice the difference between that model and our model, or even Fraser and Hipel's (1984) model which is a normative model too. Allison (1971)'s model is a textual descriptive account of how the events went and why based on governmental documents. In fact, Allison found himself compelled to publish an update, a second edition (Allison and Zelikow, 1999), *rewriting* the original book published in 1971, once many of the cold war era governmental archives for both the US and the Soviets had been declassified in the early 90's. Allison did not present in his Rational Actor model any mathematical or logical presentation/analysis of the conflict.

Fourth, the one of kind (changes every time) process that Allison (1971) adopts, while rich and detailed, it will be hard to automate in the form of a Decision Support System (DSS). The maximum one could expect for automating this process is by developing a check list to ensure the analyst cover all aspects of the model/analysis. On the other hand, our modelling and analysis framework is easy to automate in the form of a DSS system, as we did and will show at the end of this thesis work. This is because it is a systematic, methodological, mathematical and logical approach to modelling and analysis. The analysis, expert or novice, can use it and benefit from it. Surely, the expert the models and produced analysis will get more insight and add more value, compared to the novice. Nevertheless, the system will benefit the novice by providing good start to conflict modelling and analysis, and help/guide his training to become an expert.

Fifth, Allison (1971), and its rewrite Allison and Zelikow (1999), provide three models to the Cuban Missile Crisis: 1) the Rational Actor model; 2) Organizational Behaviour model; and 3) Governmental Politics model. The author/s claim is that the second and third models are needed to cover for the unexplained events of the conflict which the first model could not explain. For example, why the Soviets did a poor job hiding the missiles in Cuba? the imposition of the blockade by the US? the withdrawal of the missiles from Cuba? Clearly, our model of the conflict answer many of the questions that Allison and Zelikow states as unanswered by (traditional) rational modelling and analysis techniques. For instance, our model and analysis of it show why the US chose to impose the blockade in stead of invade or carry out air strikes, and why USSR decided to withdraw the missiles instead of escalating the conflict. We show also the logical analytical proof of why these moves were been chosen by the players. Nevertheless, some of what Allison and Zelikow's put in their second and third model is important. Namely, the inter-organizational dynamics in each of the US and the USSR governmental institutes, and especially the misalignment in the goals and practices. Such dynamics, and misalignments, which is mostly uncovered by the declassification of the US and USSR's archives of the cold war time, are usually and in fact almost always ignored by rational models of game theory, decision analysis, and conflict analysis literature. So, does our framework ignore such important aspects of real-life conflicts? The answer is: No it does not.

We focused our discussion on this thesis work on implementing *better* rational models that tie the objectives and realities of the decision makers to the actions and option they have, elicits from the player's objectives their preferences, and provide analytical methods, tools, and concepts that will guide strategists in their strategic decision making situations. But, one has to remember that this thesis work is an extension to our previous research work: the Viewpoint-based Value Driven Enterprise Knowledge Management (ViVD-EKM) framework. The main objective of the ViVD-EKM framework and its conceptual modelling and analysis tools is to offer the analysts more than what traditional game theory, decision analysis, requirements engineering methods is offering: a Knowledge modelling and Management tools capable of dealing with the complexity of the real world conflicts, problems, and systems/software modelling needs. For example, the ViVD-EKM framework models agents and sub-agents (such as internal organizations/entities within the bigger political entity), each with their goals and realities, exploring misalignment problems in the goals and realities that could happen. Not only this, the ViVD-EKM models of agents/entities model the knowledge transfer and movement among the agents, exploring the possibility of miscommunication or broken-links for example. Something that Allison and Zelikow's second model attributes the failure of the USSR to hide their missiles deployment efforts in Cuba to.

So, while we intentionally focused our modelling and analysis of the Cuban Missile Crisis on the rationality aspect of the conflict, we intended to show how the ViVD-EKM framework, as a whole, will be used to model the organizational behaviour and governmental politics of the conflict and produce results comparable to the results of Allison and Zelikow's second and third models. This extended and thorough modelling and analytical study is beyond the scope of this thesis work, and will be done separately as a future work.

6.6 Summary

In this chapter, we introduced many concepts that are important for to analyze non-cooperative games. The chapter started by defining the types of moves the players in these games are allowed to take: unilateral moves (UMs), unilateral improvement moves (UIs), sanction moves (SMs), and inescapable sanction moves (ISMs). The definitions provided for such moves are then used in defining the four different solution stability and equilibrium concepts: Nash (NASH), General MetaRational (GMR) , Symmetric MetaRational (SMR), and Sequential (SEQ) stabilities. The chapter then defined the strength of the stability under such solution concepts, and proposed a set of algorithms to help identify the strength level of each of these stabilities.

The chapter finished with a case study in which we applied the concepts and methods proposed in it. In the case study, we analyzed thoroughly the Cuban Missile Crises. We started by giving a brief background on this important political conflict. We, then, modelled the players goals, constraints and alternatives; analyzed their GCMs; identified the conflict's states; elicited the players' cardinal and ordinal preferences over these states; and then identified the players unilateral moves among these states. The stabilities of the conflict's states were analyzed under the four stability solution concepts, and the strength of these stabilities were identified for each state, for each player. We looked at the equilibrium states for the conflict; and how the conflict could have progressed from the time the US discovered the missile bases in Cuba. We concluded the case study by showing how our analysis results compares to what historically happened in the conflict, and to what others offered as models and analysis to the conflict, after the fact.

The next chapter will study further the four solution concepts we defined in this chapter, and will be used in later chapters. It will look at the characteristics of each, how they relate to each other, and how understanding such characteristics and relations enrich the process of modelling and analyzing multi-agent conflicts. The two chapters after that will take the same concepts and methods (moves, stability solution concepts, strength of stability under these concepts, etc.), redefine and extend them to fit with the characteristics of cooperative games, without and with coalitions and their analysis needs.

Chapter 7

Stability Solution Concepts for Multi-Agent Conflicts: Characteristics and Interrelationships

7.1 Introduction

In this chapter, we discuss the differences and similarities among the stability solution concepts, beyond the mathematical definitions introduced in the previous chapter. These solutions concepts, which will continue to be used in the following chapters, are at the heart of our conflict analysis framework for non-cooperative conflicts and cooperative conflicts, with and without coalitions analysis. The chapter intends to answer: how these concepts differ? What are the main characteristics that differentiates them? And, are there any interrelationships among them? If so, what are the interrelationships among the strength sets of them?

First, the chapter compares the four solution concepts introduced in the previous chapter, NASH, GMR, SMR and SEQ, based on common and important practical characteristics, or properties, of these concepts. Second, the chapter presents interesting theoretical relationships among the solution concepts, and among the strength sets of these solution concepts. The chapter, then, looks at how the knowledge about how the solution concepts differ, and the interrelationships among them, can be very informative when analyzing conflicts and how it helps shed additional insight on the conflicts and their analysis. This will be done by case studies, in which additional non-cooperative conflicts are modelled and analyzed. Namely, the chapter will analyze two of the few games collectively called Paradoxes of Rationality. Both, the Prisoner's Dilemma and game of Chicken will be modelled and analyzed at the end of this chapter, with special emphasis given to how understanding the differences and interrelationships among the stability solution concepts bring additional insight into conflict analysis. Finally, the chapter will close with a summery of what has been discussed in it.

7.2 Characteristics of the Different Stability Solution Concepts

We start by looking at the characteristics that the four stability solution concepts, we discussed in the last chapter, have. Figure 7.1 presents a comparison table of these concepts based on some important criteria shown in the header of the table's columns. An early version of this table was given by Fang et al. (1993). We have expanded, clarified and added additional comparison criteria to that of Fang et al. (1993), discussing the four stability solution concepts within the context, terminology and notation of the Constrained Rationality framework. The first two columns of the table, in Figure 7.1, give respectively the names of the solution concepts accompanied by the acronym we use to reefer to the concept, and a brief description about it.

The third column presents an important aspect to differentiate the stability solution concepts based on: *Foresight*. Foresight, here, reefers to the number of lookahead steps in the future that the decision maker, whom the stability solution concept is defined from his perspective, uses when adopting the solution concept as criterion to decide wether the current state is stable or unstable. For example, NASH stability considers no moves ahead. All, what the decision maker needs to know is if he/she has any UIs out of the current state at this point of time. Compare this to the one step ahead the decision maker needs to identify whether his current state is stable based on GMR or SEQ stability concepts. If he takes a UI out the current state at this point of time, will one of his opponents have a sanction move against it in the next step of the conflict? The SMR stability solution concept has the highest foresight, comparatively speaking. The decision maker has to know if one of his opponents has a sanction move against any of the UIs he can take at this

<table>
<tr><th rowspan="3">Solution Concepts</th><th rowspan="3">Stability Descriptions</th><th colspan="7">Characteristics</th></tr>
<tr><th rowspan="2">Foresight (# of steps ahead)</th><th colspan="2">Disimprovement from current state</th><th colspan="3">Knowledge of Preferences</th><th rowspan="2">Attitude towards Risk</th></tr>
<tr><th>By Others</th><th>Self Recovery</th><th>Own</th><th>Others</th><th>Could Act Irrationally</th></tr>
<tr><td>Nash Stability (NASH)</td><td>A focal DM cannot unilaterally move to a more preferred state.</td><td>0</td><td>Possible</td><td>Possible</td><td>Yes</td><td>No</td><td>Possible</td><td>Ignores Risk</td></tr>
<tr><td rowspan="2">General Meta-Rationality (GMR)</td><td rowspan="2">All of the focal DM's unilateral improvements are sanctioned by subsequent unilateral moves by others.</td><td rowspan="2">1</td><td rowspan="2">Yes</td><td rowspan="2">Possible</td><td rowspan="2">Yes</td><td>No</td><td>Possible</td><td rowspan="2">Avoids Risk</td></tr>
<tr><td>Yes</td><td>Yes</td></tr>
<tr><td rowspan="2">Symmetric Meta-Rationality (SMR)</td><td rowspan="2">All focal DM's unilateral improvements are still sanctioned even after possible responses by the focal DM</td><td rowspan="2">2</td><td rowspan="2">Yes</td><td rowspan="2">No</td><td rowspan="2">Yes</td><td>No</td><td>Possible</td><td rowspan="2">Avoids Risk</td></tr>
<tr><td>Yes</td><td>Yes</td></tr>
<tr><td>Sequential Stability (SEQ)</td><td>All of the focal DM's unilateral improvements are sanctioned by subsequent unilateral improvements by others.</td><td>1</td><td>Yes</td><td>Possible</td><td>Yes</td><td>Yes</td><td>No</td><td>Takes Some Risk (assumes Rational Opponents)</td></tr>
</table>

Figure 7.1: Comparison of the Stability Solution Concepts based on some of their important characteristics and assumptions

point of time. Then, he has to consider, 2 steps later, whether he has a counter move to mitigate this sanction or no. Of course, there are other solution concepts in the literature that considers higher degree of foresight than SMR, such as nonmyopic stability (Brams and Wittman, 1981b). But these stability concepts are mostly impractical considering the dynamics of strategic conflicts and the creativity of decision makers to come up with new alternatives if given the time. This makes the 0-3 lookahead steps range much practical to consider in stability solution concepts definition for real-life conflicts.

The *Disimprovement* column in the table shown in Figure 7.1 describes the disimprovement others can do to the focal decision maker's position at the current state of the conflict. it also shows whether the the decision maker to recover back or mitigate the disimprovement is considered within the stability solution concept's definition. For example, all the stability concepts consider the disimprovement by others to a decision maker's UI from the current state when it is time for them to move. Only NASH does not consider disimprovement moves by others. This makes it possible for others to disimrpove a position/state claimed by NASH to be stable for a decision maker. The table also shows that SMR is the only solution concept that considered a countermove by the decision maker to recover/mitigate the disimprovement caused by others' sanctions to his UI from current state. But, SMR stability, by definition, considered all possible countermoves to each and every sanction move and declared them to be insufficient to put back the decision maker to the same level of preference he advanced to by his UI move. Other solution concepts do not consider the decision maker's countermoves, and therefore recovery from others' disimprovement moves is possible but not guaranteed.

The third criterion used in Figure 7.1's table to compare solution concepts is the focal decision maker's *Knowledge of Preferences*. As shown in the table, all solution concepts, by definition, assume that the focal decision maker has full knowledge of his own preferences (over the conflict's states). SEQ is the only concept that assumes full knowledge by the decision maker of his opponents' preferences, whilst NASH is the only solution concept that assumes that the decision maker has no knowledge of others' preferences.

The final criterion is the *Attitude towards Risk Taking* that the focal decision maker is showing, as per the definition of the stability solution concept if and when he uses this concept to define whether the current state is stable for him or not. This attitude is in some way connected to the assumption by the solution concept of whether the focal decision maker's opponents could act irrationally, i.e. against their preferences order. The last sub-column of the Knowledge of Preferences column in Figure 7.1's table shows if the concept considers the opponents fully rational people/actors or not. For example, SEQ considers all players to be rational actors, therefore a decision maker who uses SEQ to define the stability of his current position/state is, in fact, decided to take some risk. The risk here is the possibility for some of the opponents to act irrationally, or against the order of their preferences vector. NASH, on the other hand, because it neither demands knowing the opponents' preferences nor assumes all opponents to be fully rational actors, is a concept that advocates ignoring risks. Finally, both GMR and SMR concepts assumes that opponents could act irrationally against their preferences, if it happened that the focal decision maker happen to know these preferences. Therefore, both suggests that the decision maker should avoid risking his current position at the current state of the conflict. He should not use any of his UIs out from the current state to a better state, out of fear that his opponents might sanction his UI moves even if the sanction move is irrational to them (against their preferences order). Both concepts, in effect, assumes the opponents will be motivated to take such sanction moves in order to hurt the focal decision maker, even if they get hurt too.

We can conclude this section with the observation that the stability solution concepts, we introduced in the previous chapter and will continue to use in the following chapters, satisfy different characteristics and assumptions about the focal player and his opponents in the conflict. These assumptions and characteristics makes each solution concept a vital tool for analysts to analyze the conflict and its players' behaviour. In the next section, we move to study the fundamental interrelationships among the solution concepts.

7.3 Interrelationships among Stability Solution Concepts

The following interrelationships among NASH, GMR, SMR and SEQ have been proven by Fraser and Hipel (1984) and Fang et al. (1993) for their respective conflict stability analysis frameworks. We prove them here too for our Constrained Rationality conflict modelling and analysis framework.

Theorem 7.3.1 (Interrelationships among NASH, SMR and GMR): *For decision maker DM_i at time t:* $(s \in \mathcal{S}^{NASH}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{SMR}_{DM_i,t})$*; and* $(s \in \mathcal{S}^{SMR}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{GMR}_{DM_i,t})$.
In other words: $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{GMR}_{DM_i,t}$

Proof. If a state s is NASH Stable for decision maker DM_i at time t, i.e. $s \in \mathcal{S}^{NASH}_{DM_i,t}$, then $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ as per Definition 6.3.1. And, as indicated earlier, having $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ is a special case that trivially satisfies Definition 6.3.3 of SMR stability. Therefore, s is an SMR stable state, or $s \in \mathcal{S}^{SMR}_{DM_i,t}$.

Now, if s is an SMR stable state for DM_i at time t, then any sanction move, others have against any of DM_i's UIs from s, that satisfies the the requirements of Definition 6.3.3 will definitely satisfy the weaker requirements of the sanction moves given in Definition 6.3.2 for s to be identified as a GMR stable state, or $s \in \mathcal{S}^{GMR}_{DM_i,t}$.

Therefore, $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{GMR}_{DM_i,t}$ □

The relationship between NASH, GMR and SMR given above in Theorem 7.3.1 is shown graphically in the Venn diagram presented in Figure 7.2. The Venn diagram reminds also that all these sets are subsets of set $\mathcal{S}$ which the set of all the conflict's states.

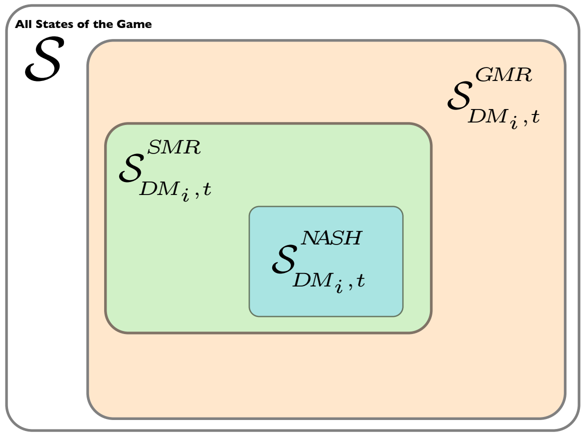


Figure 7.2: Interrelationships among NASH, GMR and SMR Solution Concepts

Theorem 7.3.2 (Interrelationships among NASH, SEQ and GMR): *For decision maker DM_i at time t: $(s \in \mathcal{S}^{NASH}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{SEQ}_{DM_i,t})$; and $(s \in \mathcal{S}^{SEQ}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{GMR}_{DM_i,t})$.*
In other words: $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{GMR}_{DM_i,t}$

Proof. If a state s is NASH Stable for decision maker DM_i at time t, i.e $s \in \mathcal{S}^{NASH}_{DM_i,t}$, then $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ as per Definition 6.3.1. And as indicated earlier, having $\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset$ is a special case that trivially satisfies Definition 6.3.4 of SEQ stability. Therefore, s is an SEQ stable state, or $s \in \mathcal{S}^{SEQ}_{DM_i,t}$.

If s is an SEQ stable state for DM_i at time t, then any sanction move, others have against DM_i's UIs from s, that satisfys the the requirements of Definition 6.3.4 of having the sanction move be also a UI move, for the other player, will definitely satisfy the weaker requirements of the sanction moves given in the GMR stability Definition 6.3.2, which demands the sanction move to be only a UM move for the other player. This means, if s is an SEQ stables state, or $s \in \mathcal{S}^{SEQ}_{DM_i,t}$, then it must be also a GMR stable state, or $s \in \mathcal{S}^{GMR}_{DM_i,t}$.

Therefore, $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{GMR}_{DM_i,t}$ □

The relationship between NASH, GMR and SEQ given above in Theorem 7.3.2 is shown graphically in the Venn diagram presented in Figure 7.3. The Venn diagram reminds again that all these sets are subsets of set $\mathcal{S}$ which the set of all the conflict's states.

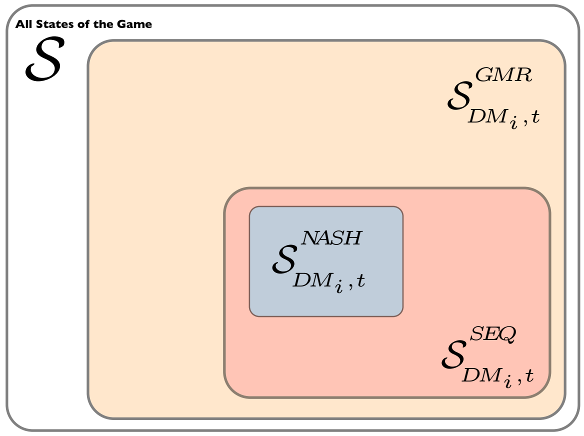


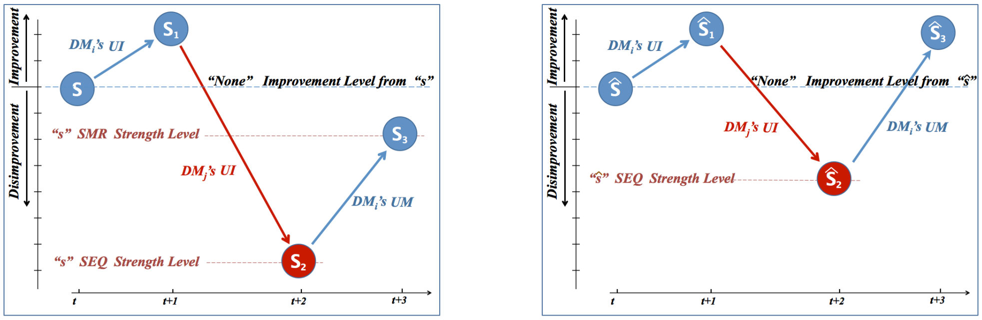
Figure 7.3: Interrelationships among NASH, GMR and SEQ Solution Concepts

Observation 7.3.3 (**Interrelationships among SEQ and SMR**): *For decision maker DM_i at time t: It is not necessarily always true that* $\mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t}$, *nor that* $\mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t}$

Proof. This observation can be proved by looking at two cases one to prove that the relationship between SMR and SEQ to be $\mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t}$, and another to show the relationship to be $\mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t}$. We use Figures 7.4 and 7.5 to illustrate the two cases.

Case 1: As suggested by Figure 7.4a, let decision maker DM_i, at time t of the game, have only one UI move from state s to state s_1. Let us also assume that an opponent of DM_i, another player of the game who we call DM_j in the figure, has a sanction move (to DM_i's UI). This sanction move is a UI move to DM_j; and will put DM_i at state s_2 which is a worse place for DM_i than the original state s, the state he exercised his UI out of it. This sanction will qualify s to be an SEQ stable state for DM_i, as per Definition 6.3.4. This makes $s \in \mathcal{S}^{SEQ}_{DM_i,t}$. Similarly, let DM_i have another state $\hat{s}$ which is also an SEQ stable state, as illustrated in Figure 7.4b. As a result, $\mathcal{S}^{SEQ}_{DM_i,t} = \{s, \hat{s}\}$.

Now, let state s also be an SMR stable state for DM_i, because DM_i has a countermove to the sanction (from state s_1 to state s_2) imposed by DM_j. As shown in Figure 7.4a, this countermove will make DM_i's position better (at state S_3), but still worse than his original position at s before he executed his UI move to s_1. For state $\hat{s}$, let DM_i has a UM countermove to the sanction move that could be imposed by DM_j on DM_i's UI (out from $\hat{s}$ to $\hat{s_1}$). But, let this countermove puts DM_i at a position better than his original position at

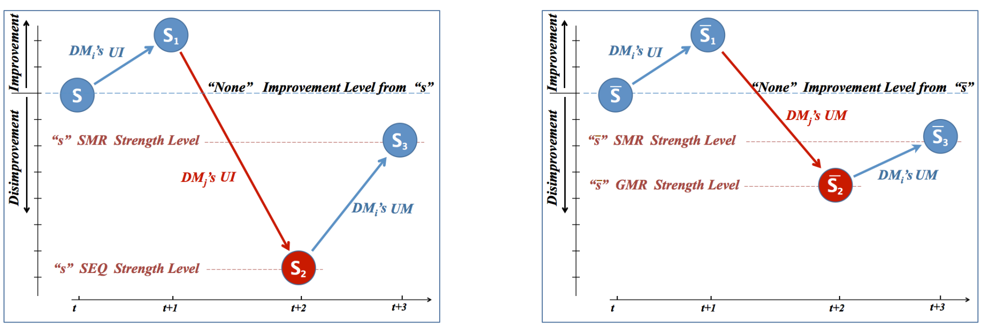


(a) State s is SEQ and SMR stable for DM_i at time t

(b) State $\hat{s}$ is only SEQ stable for DM_i at time t

Figure 7.4: Case 1 of Observation 7.3.3 Proof : Showing $\mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t}$ to be true

$\hat{s}$, as illustrated in Figure 7.4b. In other words, while state s is SMR stable for DM_i at time t, state $\hat{s}$ is *not* SMR stable for him. As a result, $\mathcal{S}^{SMR}_{DM_i,t} = \{s\}$. Therefore, in this case ,and for DM_i at time t of the game, it is true that $\mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t}$.



(a) State s is SEQ and SMR stable for DM_i at time t

(b) State $\overline{s}$ is only SMR stable for DM_i at time t

Figure 7.5: Case 2 of Observation 7.3.3 Proof: Showing $\mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t}$ to be true

Case 2: Let us change Case 1 slightly. Let us assume that there is no other SEQ stable state to DM_i at time t other than state s, making $\mathcal{S}^{SEQ}_{DM_i,t} = \{s\}$. And, as in Case 1, let state s be SMR stable for DM_i at time t, as shown in Figure 7.5a. But here in Case 2, let us assume that we have state $\overline{s}$ which is an SMR stable state to DM_i at time t but not an SEQ stable state to him, because, as illustrated in Figure 7.5b, the sanction that the other player have to DM_i's UI move (away from $\overline{s}$ to $\overline{s_1}$)is a UM but not a UI move for the other player. This makes $\mathcal{S}^{SMR}_{DM_i,t} = \{s, \check{s}\}$. And as a result, in this case, and for DM_i at time t of the game, it is true that $\mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t}$.

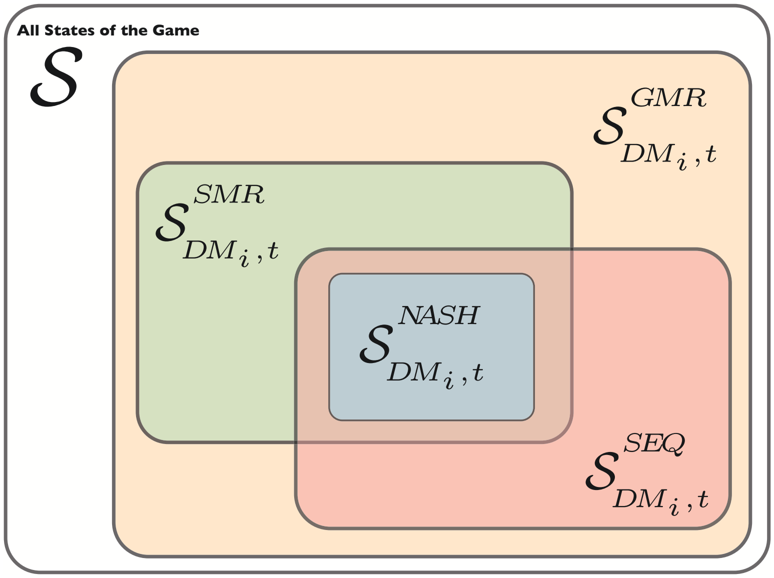


Figure 7.6: Interrelationships among NASH, GMR, SMR and SEQ Stability Solution Concepts

By showing that each of the two statements $\mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t}$ and $\mathcal{S}^{SEQ}_{DM_i,t} \subset \mathcal{S}^{SMR}_{DM_i,t}$ could be true, for different situations, the Observation is proven. □

As a consequence to Observation 7.3.3, and the relationships established by Theorem 7.3.1 and Theorem 7.3.2 above, is shown in Figure 7.6. This figure combines the separate Venn diagrams shown in Figures 7.2and 7.2 in one overall Venn diagram that represents the interrelationships among all of the solution concepts: NASH, GMR, SMR and SEQ.

7.4 Interrelationships among Strength Sets of the Stability Solution Concepts

In this section, we present, and prove, a new set of interrelationships that exist between the Stability Solution Concepts, and their respective strength sets. This new set is unique to our Constrained Rationality framework and the way it defines stability strength for each of the solution concepts. We believe our framework's definitions and the interrelationships that follow from them are more representative of the dynamics and characteristics of real-life conflicts, as we will see later from the case studies provided later in the chapter.

Observation 7.4.1 (**Strength Subsets of NASH, SEQ, SMR and GMR**): *For decision maker DM_i at time t:*
$\forall\, L_{SS} \in \{\mathcal{L} - \{Null\}\}$

$$\mathcal{S}^{NASH}_{DM_i,t} = \bigcup_{L_{SS}} \mathcal{S}^{NASH(L_{SS})}_{DM_i,t} \qquad \mathcal{S}^{GMR}_{DM_i,t} = \bigcup_{L_{SS}} \mathcal{S}^{GMR(L_{SS})}_{DM_i,t} \qquad \mathcal{S}^{SMR}_{DM_i,t} = \bigcup_{L_{SS}} \mathcal{S}^{SMR(L_{SS})}_{DM_i,t} \qquad \mathcal{S}^{SEQ}_{DM_i,t} = \bigcup_{L_{SS}} \mathcal{S}^{SEQ(L_{SS})}_{DM_i,t}$$

Proof. The above observed relations can be easily proved using the corresponding definitions for the stability concepts (Definitions 6.3.1, 6.3.2, 6.3.3 and 6.3.4) and their respective strength definitions (Definitions 6.4.1, 6.4.2,, 6.4.3, and 6.4.4). It is worth mentioning here why the label "Null" is excluded from being a strength label. Because, when a stability concept for a state s has a "Null" assigned to it, this means that s is not stable at all, for the decision maker at the time, under this specific stability concept definition. □

Theorem 7.4.2 (**Strength of NASH in relation to Strengths of SEQ, SMR and GMR**): *For decision maker DM_i at time t:*
$(s \in \mathcal{S}^{NASH}_{DM_i,t}) \rightarrow [(s \in \mathcal{S}^{GMR(N)}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SMR(N)}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ(N)}_{DM_i,t})]$.

Proof. The theorem states that: If a state s is NASH stable, then no matter what the strength of s's NASH stability is, it will always be SEQ, SMR and GMR stable and the stability strength for each will be at the *None* level.

If a state s is NASH stable, then no matter what the NASH stability strength for s is, to DM_i at time t, it will be always true, as per Definition 6.3.1, that $\mathcal{S}^{UI}_{DM_i,t} = \emptyset$. It thus follows from this fact, as per the GMR strength's Definition 6.4.2, that the StabilityStrength$(GMR, s, DM_i, t) = None$, so as to make the statement $(s \in \mathcal{S}^{GMR(N)}_{DM_i,t})$ be always true.

Similarly, following the same logic and by applying the SMR strength's Definition 6.4.3 and the SEQ strength's Definition 6.4.4, respectively, we can prove that if a state s is NASH stable, and no matter what the NASH stability strength of s is, to DM_i at time t, the following statements will be always true: $(s \in \mathcal{S}^{SMR(N)}_{DM_i,t})$ and $(s \in \mathcal{S}^{SEQ(N)}_{DM_i,t})$.

Therefore, $(s \in \mathcal{S}^{NASH}_{DM_i,t}) \rightarrow [(s \in \mathcal{S}^{GMR(N)}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SMR(N)}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ(N)}_{DM_i,t})]$. □

In effect, Theorem 7.4.2 states that: $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SEQ(N)}_{DM_i,t}$; $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SMR(N)}_{DM_i,t}$; and $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{GMR(N)}_{DM_i,t}$.

Corollary 7.4.3 (NASH's Strength Subsets in relation to GMR and SMR's Strength Subsets): *For decision maker DM_i at time t:* $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SMR(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(N)}_{DM_i,t}$.

Proof. The set of all states that are NASH stable for DM_i at time t, regardless of the strength of each state's NASH stability, is shown in Observation 7.4.1 to be $\mathcal{S}^{NASH}_{DM_i,t}$. From Theorem 7.3.1, we know that $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SMR}_{DM_i,t} \subseteq \mathcal{S}^{GMR}_{DM_i,t}$. But, Theorem 7.4.2 states that if a state s is NASH stable for a specific decsion maker at a specific time, then s will belong to none other than the *None* strength level subsets for both SMR and GMR, or $\mathcal{S}^{SMR(N)}_{DM_i,t}$ and $\mathcal{S}^{GMR(N)}_{DM_i,t}$, respectively. Therefore, $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SMR(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(N)}_{DM_i,t}$. □

Corollary 7.4.4 (NASH's Strength Subsets in relation to GMR and SEQ's Strength Subsets): *For decision maker DM_i at time t:* $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SEQ(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(N)}_{DM_i,t}$.

Proof. The set of all states that are NASH stable for DM_i at time t, regardless of the strength of each state's NASH stability, is shown in Observation 7.4.1 to be $\mathcal{S}^{NASH}_{DM_i,t}$. From Theorem 7.3.2, we know that $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SEQ}_{DM_i,t} \subseteq \mathcal{S}^{GMR}_{DM_i,t}$. But, Theorem 7.4.2 states that if a state s is NASH stable for a specific decision maker at a specific time, then s will belong to none other than the *None* strength level subsets for both SEQ and GMR, or $\mathcal{S}^{SEQ(N)}_{DM_i,t}$ and $\mathcal{S}^{GMR(N)}_{DM_i,t}$, respectively. Therefore, $\mathcal{S}^{NASH}_{DM_i,t} \subseteq \mathcal{S}^{SEQ(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(N)}_{DM_i,t}$. □

Theorem 7.4.5 (Strength of SMR in relation to Strength of GMR): *For decision maker DM_i at time t:*

$$\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SMR}_{DM_i,t}))$$
$$[StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SMR, s, DM_i, t) \geq N].$$

Proof. From Definition 6.4.2, it is clear that StabilityStrength$(GMR, s, DM_i, t) \geq$ None; and from Definition 6.4.3, it is clear that StabilityStrength$(SMR, s, DM_i, t) \geq$ None. This is because if it happened that a GMR or an SMR Stability for a state is given the label "Null", then that state is not at all stable under GMR definition, nor stable under SMR definition.

From the definitions of both GMR and SMR (Definitions 6.3.2 and 6.3.3, respectively), we know that for the same game, at the same state s and time t, the same set of sanction moves (SMs) by other players (other than DM_i) will be used/considered to check if the state s is GMR stable and to check if it is SMR stable. And, from Theorem 7.3.1, we know that any state s which is SMR stable, then it will also be GMR stable. This is because SMR stability for a state requires that it must have an inescapable sanction (ISM), by one or more of the other players, for each DM_i's UI out of s (the state tested for stability), that will put DM_i at a lower or equally preferred state to state s. In other words, s must be GMR stable (GMR's Definition 6.3.2 asks for a sanction only) before it could be tested for the stricter and additional requirements of an inescapable sanction needed for s to be SMR stable (as per Definition 6.3.3).

From Definition 6.4.2, the strength of GMR stability is defined to be the strength of the worst of the sanction moves imposed by other players, sanctioning DM_i's UIs out of s. This is measured by how much the end state of the worst sanction move is less-preferred compared to s. The definition takes in consideration that DM_i when considering how bad the idea is to take any UI out of s, the essence of the GMR stability definition, he will most likely look at each of his UIs out of s and how bad the sanction will be for each. Then, the UI that will lead to the least less-preferred end state by the sanctions of others, or the best out of all bad scenarios, will be the one UI to use in quantifying the strength of s's GMR stability for DM_i.

The same set of UIs (by DM_i out of s), and the same sanction moves (by others) will be used to test the state s for SMR stability. But, now one should consider any countermoves DM_i has to mitigate/lessen the effect of the sanctions. As per Definition 6.4.3, if DM_i has only countermoves that will lead to even worse state, i.e. less-preferred state than the one the sanction move end up with, then s is SMR stable with strength that matches the less-preference strength of the sanction's end state. Otherwise, if DM_i has countermoves that will lessen the bad effect of the sanction move, i.e. the countermove will end up with a state that is still less preferred that s (needed for the state to be called SMR stable as per Definition 6.3.3) but more preferred than the end state of the sanction move, then s is SMR stable with strength that matches the less-preference strength of the countermove's end state. In other words, SMR strength will either has the same strength as the sanction, or less than that. And since we know that the strength of GMR stability of a state is measured by the strength of the sanctions, then SMR's stability strength will always be equal to the GMR's stability strength or less than that. But, we also proved earlier that both GMR and SMR stability's strengths for a state s are always bigger or equal than the *None* strength level.

Therefore, for a state s that is both GMR and SMR stable, for DM_i at time t, $StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SMR, s, DM_i, t) \geq N$. □

Corollary 7.4.6 (SMR's Strength Subsets in relation to GMR's Strength Subsets): *For decision maker DM_i at time t:*
$(\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SMR}_{DM_i,t}))(\forall L_{SS} \in \{\mathcal{L} - \{Null\}\})) \ [\mathcal{S}^{SMR(L_{SS})}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq L_{SS})}_{DM_i,t}]$.

Proof. To prove the Corollary, we will give the proof for only two values of $L_{SS} \in \{\mathcal{L} - \{Null\}\}$. For the rest of L_{SS} values , the theorem can be proved easily following the same logic.

For $L_{SS} = N$, we need to show that $\mathcal{S}^{SMR(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq N)}_{DM_i,t}$. For DM_i at time t, for all states that are SMR stable with stability strength at level *None*, i.e. $\forall s \in \mathcal{S}^{SMR(N)}_{DM_i,t}$, $StabilityStrength(SMR, s, DM_i, t) = N$. Theorem 7.4.5 states that $StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SMR, s, DM_i, t) \geq N$ for each state s that is both GMR and SMR stable for the given decision maker at the specified time. This makes $StabilityStrength(GMR, s, DM_i, t) \geq N$, and therefore $s \in \mathcal{S}^{GMR(\geq N)}_{DM_i,t}$. In other words, $\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SMR}_{DM_i,t})) \ [(s \in \mathcal{S}^{SMR(N)}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{GMR(\geq N)}_{DM_i,t})]$. This proves that $\mathcal{S}^{SMR(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq N)}_{DM_i,t}$.

For $L_{SS} = Ex$, we need to show that $\mathcal{S}^{SMR(Ex)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t}$. For DM_i at time t, for all states that are SMR stable with stability strength at level "Extreme", i.e. $\forall s \in \mathcal{S}^{SMR(EX)}_{DM_i,t}$, $StabilityStrength(SMR, s, DM_i, t) = Ex$. Theorem 7.4.5 states that $StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SMR, s, DM_i, t) \geq N$ for each state s that is both GMR and SMR stable for the given decision maker at the specified time. This makes $StabilityStrength(GMR, s, DM_i, t) \geq Ex$, and therefore $s \in \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t}$. In other words, $\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SMR}_{DM_i,t})) \ [(s \in \mathcal{S}^{SMR(Ex)}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t})]$. This proves that $\mathcal{S}^{SMR(Ex)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t}$. Interestingly enough, this also shows that SMR stable states with "extremely strong" strength level will only be part of the set of "extremely strong" GMR stable states, and will not be part of any other set of less-strong GMR stable states. □

The relationships between the strength sets of each of NASH, GMR and SMR, given above in Corollary 7.4.6 and Corollary 7.4.3, are illustrated graphically in the Venn diagram presented in Figure 7.7.

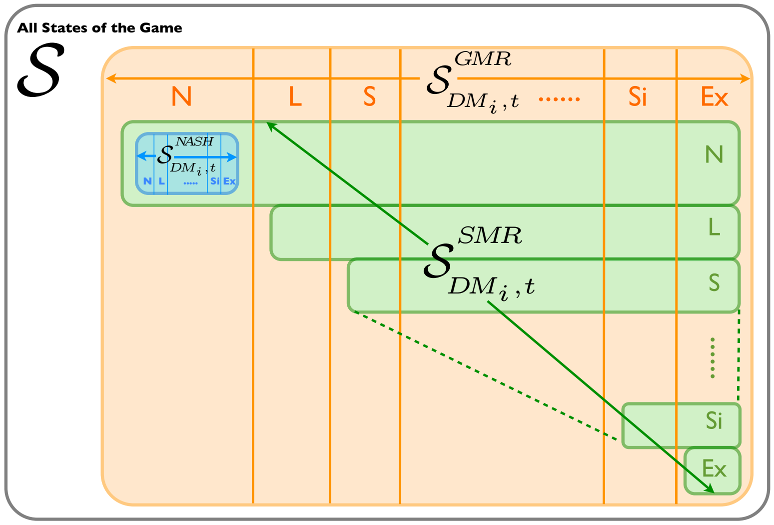


Figure 7.7: Venn Diagram showing the Interrelationships among the Strength Sets of NASH, GMR and SMR Solution Concepts

Theorem 7.4.7 (Strength of SEQ in relation to Strength of GMR): *For decision maker* DM_i *at time* t*:*

$$\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ}_{DM_i,t}))$$
$$[StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SEQ, s, DM_i, t) \geq N].$$

Proof. From Definition 6.4.2, it is clear that $StabilityStrength(GMR, s, DM_i, t) \geq$ None; and from Definition 6.4.4, it is clear that $StabilityStrength(SEQ, s, DM_i, t) \geq$ None. This is because if it happened that a GMR or an SEQ Stability for a state is given the label "Null", then that state is not at all stable under GMR definition, nor stable under SEQ definition.

From Theorem 7.3.2, we know that any state s which is SEQ stable, then it will also be GMR stable. And, from the definitions of both GMR and SEQ (Definitions 6.3.2 and 6.3.4, respectively), we know that for the same game, at the same state s and time t, the set of sanction moves (SMs) by other players (other than DM_i) which will be used/considered to check whether the state s is SEQ stable, or not, is only a subset of the sanction moves to be used for considering whether s is GMR stable, or not. This is because SEQ stability for a state requires that it must have a sanction by another player (other than DM_i) that is also a UI for the player performing the sanction move, not just a merely UM for him. In other words, sanction moves demanded for SEQ stability must be SMUIs (as per Definition 6.3.4), not just SMs as GMR stability requires (as per Definition 6.3.2). This means that s must be GMR stable before it could be tested for the stricter requirements of SEQ stability.

From Definition 6.4.2, the strength of GMR stability is defined to be the strength of the worst sanction moves imposed by other players, sanctioning DM_i's UIs out of s. This is measured by how much the end state of the worst sanction move is less-preferred compared to s. The definition takes in consideration that DM_i when considering how bad the idea is to take any UI out of s, the essence of the GMR stability definition, he will most likely look at each of his UIs out of s and how bad the sanction will be for each. Then, the UI that will lead to the least less-preferred end state by the sanctions of others, or the best out of all bad scenarios, will be the one UI to use in quantifying the strength of s's GMR stability.

The same set of UIs (by DM_i out of s), but only a subset of the sanction moves (must be also UIs for the players imposing the sanctions) will be used to test the state s for SEQ stability, as per Definition 6.4.4. If the worst sanction move that is used to define GMR stability of s happens to be a SMUI move as well, not just an SM move, then the strength of s's GMR stability will be the same as the strength level of its SEQ stability. Otherwise, if the worst sanction move (SM) is not an SMUI move, then this means that the worst sanction move will be used to define s's GMR stability's strength and the not-as-worse sanction (the SMUI) will used to define s's SEQ stability's strength. In other words, SEQ strength will either has the same strength as the sanction used to identify s's GMR strength, or less than that. Therefore, SEQ's stability strength will always be equal to the GMR's stability strength or less than that. But, we also proved earlier that both GMR and SEQ stability's strengths for a state s are always bigger or equal than the *None* strength level.

Therefore, for a state s that is both GMR and SEQ stable, for DM_i at time t, $StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SEQ, s, DM_i, t) \geq \text{None}$. □

Corollary 7.4.8 (SEQ's Strength Subsets in relation to GMR's Strength Subsets): *For decision maker DM_i at time t:*
$(\forall s: ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ}_{DM_i,t}))(\forall L_{SS} \in \{\mathcal{L} - \{Null\}\}))\ [\mathcal{S}^{SEQ(L_{SS})}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq L_{SS})}_{DM_i,t}]$

Proof. To prove the Corollary, we will give the proof for only two values of $L_{SS} \in \{\mathcal{L} - \{Null\}\}$. For the rest of L_{SS} values, the theorem can be proved easily following the same logic.

For $L_{SS} = N$, we need to show that $\mathcal{S}^{SEQ(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq N)}_{DM_i,t}$. For DM_i at time t, for all states that are SEQ stable with stability strength at level *None* , i.e.

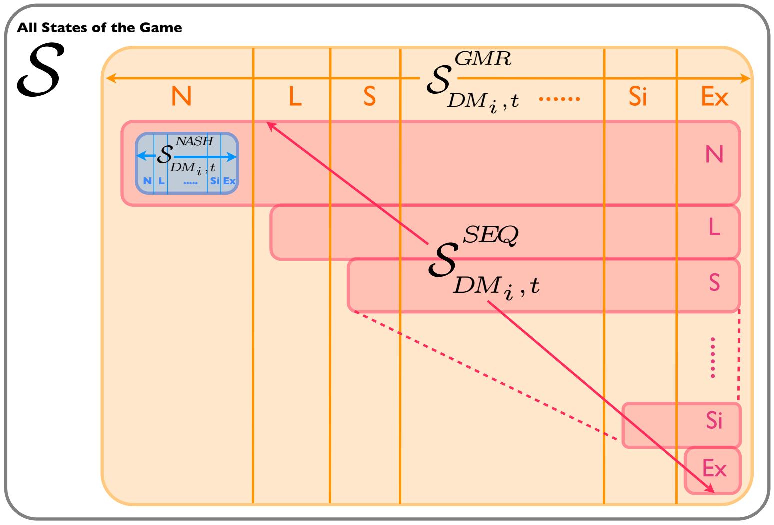


Figure 7.8: Venn Diagram showing the Interrelationships among the Strength Sets of NASH, GMR and SEQ Solution Concepts

$\forall s \in \mathcal{S}^{SEQ(N)}_{DM_i,t}$, $StabilityStrength(SEQ, s, DM_i, t) = N$. Theorem 7.4.7 states that $StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SEQ, s, DM_i, t) \geq N$ for each state s that is both GMR and SEQ stable for the given decision maker at the specified time. This makes $StabilityStrength(GMR, s, DM_i, t) \geq N$, and therefore $s \in \mathcal{S}^{GMR(\geq N)}_{DM_i,t}$. In other words, $\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ}_{DM_i,t})) \ \ [(s \in \mathcal{S}^{SEQ(N)}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{GMR(\geq N)}_{DM_i,t})]$. This proves that $\mathcal{S}^{SEQ(N)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq N)}_{DM_i,t}$.

For $L_{SS} = Ex$, we need to show that $\mathcal{S}^{SEQ(Ex)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t}$. For DM_i at time t, for all states that are SEQ stable with stability strength at level "Extreme", i.e. $\forall s \in \mathcal{S}^{SEQ(EX)}_{DM_i,t}$, $StabilityStrength(SEQ, s, DM_i, t) = Ex$. Theorem 7.4.7 states that $StabilityStrength(GMR, s, DM_i, t) \geq StabilityStrength(SEQ, s, DM_i, t) \geq N$ for each state s that is both GMR and SEQ stable for the given decision maker at the specified time. This makes $StabilityStrength(GMR, s, DM_i, t) \geq Ex$, and therefore $s \in \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t}$. In other words, $\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ}_{DM_i,t})) \ \ [(s \in \mathcal{S}^{SEQ(Ex)}_{DM_i,t}) \rightarrow (s \in \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t})]$. This proves that $\mathcal{S}^{SEQ(Ex)}_{DM_i,t} \subseteq \mathcal{S}^{GMR(\geq Ex)}_{DM_i,t}$. Interestingly enough, this also shows that SEQ stable states with "extremely strong" strength level will only be part of the set of "extremely strong" GMR stable states, and will not be part of any other set of less-strong GMR stable states. □

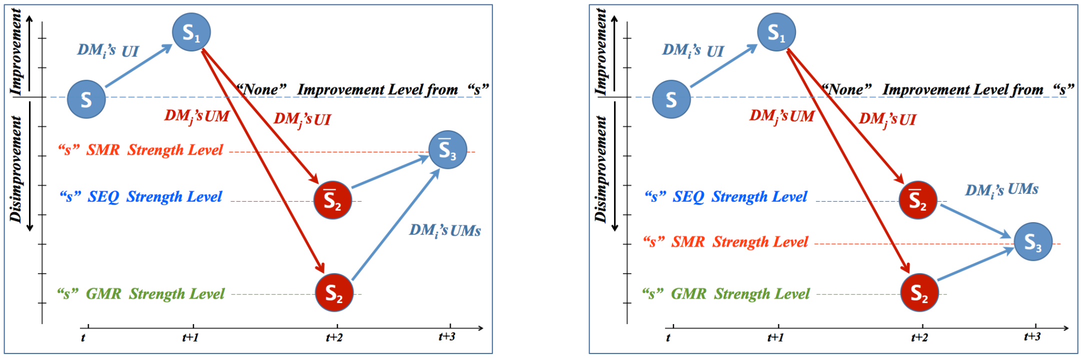
The relationships between the strength sets of each of NASH, GMR and SEQ, given above in Corollary 7.4.8 and Corollary 7.4.4, are illustrated graphically in the Venn diagram presented in Figure 7.8.

Observation 7.4.9 (The Strength of SEQ in Relation to Strength of SMR): *For decision maker DM_i at time t:* $\forall s: \ ((s \in \mathcal{S}^{GMR}_{DM_i,t}) \wedge (s \in \mathcal{S}^{SEQ}_{DM_i,t}))$
It is not necessarily always true that
$[\, StabilityStrength(SMR, s, DM_i, t) \geq StabilityStrength(SEQ, s, DM_i, t) \geq N\,]$, *nor that* $[\, StabilityStrength(SEQ, s, DM_i, t) \geq StabilityStrength(SMR, s, DM_i, t) \geq N\,]$.

Proof. First, the observation states that for a state s which is stable under both SMR and SEQ stability solution concepts, SMR and SEQ's stabilities strengths for s are greater than or equal *None* strength level. This part of the observation is easy to prove. From Definition 6.4.3, it is clear that $StabilityStrength(SMR, s, DM_i, t) \geq$ None; and from Definition 6.4.4, it is clear that $StabilityStrength(SEQ, s, DM_i, t) \geq$ None. This is because if it happened that an SMR or an SEQ Stability for a state is given the label "Null", then that state is not at all stable under SMR definition, nor stable under SEQ definition.

We now prove the second part of the observation which states that for a state s which is stable under both SMR and SEQ stability solution concepts there will be times where StabilityStrength$(SMR, s, DM_i, t) \geq$StabilityStrength(SEQ, s, DM_i, t) is true, and times where the opposite is true. This part of the observation can be proved by looking at two cases one to prove that StabilityStrength$(SMR, s, DM_i, t) \geq$ StabilityStrength(SEQ, s, DM_i, t) is true is true, and another case to show that StabilityStrength$(SEQ, s, DM_i, t) \geq$StabilityStrength(SMR, s, DM_i, t) can sometimes be true too. We use Figure 7.9 to illustrate the two cases.

Case 1: We use Figure 7.9a to illustrate this case. Let decision maker DM_i, at time t of the game, have only one UI move from state s to state s_1. Let us also assume that an opponent of DM_i, another player of the game who we call DM_j in the figure, has two sanction moves (to DM_i's UI). One sanction move is a UM move from s_1 to state s_2 putting DM_i at an extremely less preferred state than the state he was originally in, state s. We will call this move by the acronym we decided to use for such moves: SM (for Sanction Move). The other sanction move, DM_j has, will take DM_i from state s_1 to state $\overline{s}_2$, a moderately less-preferred state to state s. This second sanction move by DM_j, happen to be also a UI for DM_j, making it a special kind of a sanction move which we used the acronym SMUI for. The presented case so far tells us that state s is stable under both GMR and SEQ solution concepts. The strength of the s's GMR stability is calculated using Algorithm 6.3 using



(a) Case 1: The SEQ stability's strength for state s is greater than its SMR stability's strength

(b) Case 2: The SMR stability's strength for state s is greater than its SEQ stability's strength

Figure 7.9: Proving Observation 7.4.9 by showing that, for a specific state that is both SMR and SEQ stable, the strength of its SMR stability could be sometimes greater than, and some times less than, the strength of its SEQ stability

the sanction move, out of the two, that will yield the biggest disimprovement level to DM_i. This makes GMR uses the absolute value of the disimprovement level reached by SM. For SEQ's stability strength, only SMUI can be used, as per SEQ's Definition 6.3.4. Using Algorithm 6.5, SEQ's stability strength will be the absolute value of the disimprovement level reached by SMUI.

This case's illustration, given in Figure 7.9a, also shows that DM_i have two countermoves. One out of s_2 to state $\overline{s}_3$, mitigating the effect of DM_j's SM; and the other out of $\overline{s}_2$ to state $\overline{s}_3$, mitigating the effect of DM_j's SMUI. Note that both countermoves put DM_i at the same end state of $\overline{s}_3$. A state which better where the sanctions left DM_i in, but still less-preferred than his original state s. This makes s to be an SMR stable state for DM_i. The strength of this stability will be calculated using Algorithm 6.4 to be the absolute value of the disimprovement level reached after DM_i makes his countermoves. In this case, the same disimprovement level is reached by both countermoves, the one DM_i makes as a response to SM and the other he takes as a response to SMUI. This makes s's SMR stability *less than* its SEQ stability (which *less-than* its GMR stability). In other words, this case proved that StabilityStrength(SEQ, s, DM_i, t)$\geq$StabilityStrength(SMR, s, DM_i, t).

Case 2: We use Figure 7.9b to illustrate this case. Notice that this case is similar to case 1 discussed above, in terms of DM_i's UI move out from state s to state s_1, and also in terms of the SM and SMUI sanction moves DM_j has out of s_1

to states s_2 and $\overline{s}_2$, respectively. Such similarity, makes the strength of both s's GMR and SEQ stability to be at the same strength levels calculated for case 1, as shown in Figure 7.9b when compared to case 1's Figure 7.9a.

The difference in the two cases is in the countermoves DM_i have to the sanction moves SM and SMUI. Figure 7.9b shows that DM_i has a countermove out of each. The first countermove taks him out of $\overline{s}_2$ to state s_3, putting him at an even worse position than the one which SMUI left him at. This makes Algorithm 6.4, which is used for calculating the strength of SMR stabilities, assumes that DM_i, being a rational and intelligent decision maker, will not take the countermove he has out of $\overline{s}_2$ to state s_3. The Algorithm will calculate the strength of s's SMR stability, if done only based on the SMUI sanction move with no consideration to the SM move, to be at the same strength level of s's SEQ stability.

The second of DM_i's countermoves is shown in Figure 7.9b to take him from state s_2 to state s_3. This enhances DM_i's position a bit, since s_3 is a little more preferred to him than s_2. But, still this countermove leaves him at a less preferred state than the original state s, making the situation satisfy the requirement of SMR stability. Recall that Algorithm 6.4, and the definition of SMR Strength (Definition 6.4.3), assumes that DM_i's opponents in the game knows his moves, countermoves and preferences, therefore they will try to put DM_i at the worst disimprovement level possible. In this case, and as per the illustration given in its figure, DM_i's opponents will prefer to take the SM sanction, and not the SMUI one (SMUI type of sanctions are not required by SMR Definition 6.3.3). As a result, the Algorithm 6.4will calculate the strength of s's SMR stabilities to be the absolute value of the disimprovement level which DM_i end up with after taking the s_2-to- s_3 countermove, i.e. the disimprovement level at state s_3.

As shown Figure 7.9b, this makes s's SMR stability *greater than* its SEQ stability (notice that both stabilities have their strengths to be still bound by the upper/higher stability strength of GMR). In other words, this case proved that StabilityStrength$(SMR, s, DM_i, t) \geq$StabilityStrength(SEQ, s, DM_i, t).

Neither StabilityStrength$(SEQ, s, DM_i, t) \geq$ StabilityStrength(SMR, s, DM_i, t) nor StabilityStrength$(SMR, s, DM_i, t) \geq$ StabilityStrength(SEQ, s, DM_i, t)) can be the case all the time. By showing that one can be true in some cases, while the other is true in some other cases, this Observation is proven. □

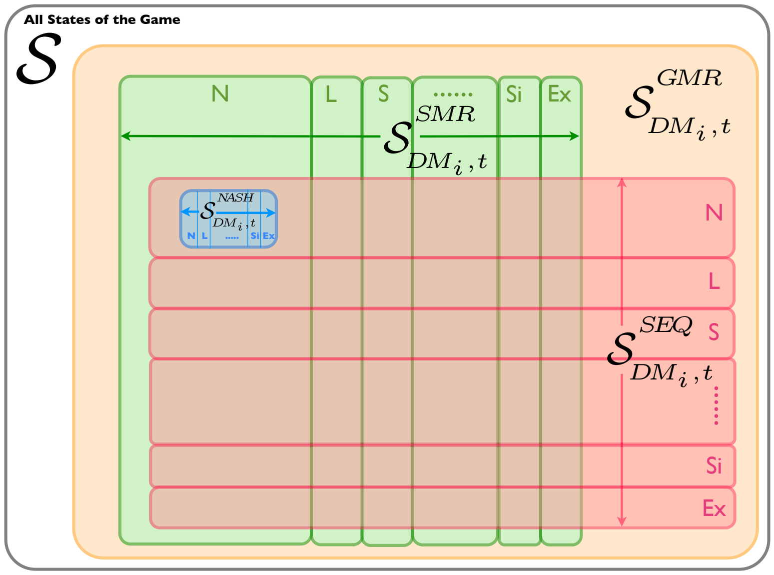


Figure 7.10: Venn Diagram showing the Interrelationships among the Strength Sets of NASH, SMR and SEQ Solution Concepts, and their relationship with the generic set of the GMR Solution Concept

The above observation leaves us with no established formal generic relationships that could link the strength sets of SMR stability with the strength sets of SEQ stability, such as the ones we proved to be among GMR and SMR strength sets (Corollary 7.4.6), or the ones we proved to be among GMR and SEQ strength sets (Corollary 7.4.8). This is not surprising, given the fact that we observed earlier this lack of established generic and consistent formal relationship among the generic-SMR, the set of all SMR strength subsets, and the generic-SEQ, the set of SEQ strength subsets, when we discussed and proved Observation 7.3.3.

As a consequence to Observation 7.4.9 above, and the relationships established before by Corollaries 7.4.3 and 7.4.4, is shown in Figure 7.10. This figure is similar to Figure 7.6 presented in the previous section of this chapter, but in this figure the strength subsets of NASH, SMR and SEQ are shown. Notice that this new figure does not show the strength subsets of GMR. This is done to not complicate the Venn diagram. Recall that a Venn diagram showing the relationships between the strength sets of NASH, GMR and SMR is shown above in Figure 7.7; and another one showing the relationships between the strength sets of NASH, GMR and SEQ is shown in Figure 7.8. The two earlier Venn diagrams illustrate, separately and respectively, how GMR's strength sets relate to SMR's strength sets and how GMR's strength sets relate to SEQ's strength sets. All of NASH's strength sets fall within the *None* strength set of GMR, SMR and SEQ, as given by Theorem 7.4.2

and illustrated by the three diagrams shown in Figures 7.7, 7.8 and 7.10.

7.5 Case Studies: Paradoxes of Rationality

7.5.1 The Prisoner's Dilemma

7.5.1.1 Background

In this section we will model and analyze the *Prisoner's Dilemma* (PD) game using the Constrained Rationality framework. We will not discuss here the history of the game, and the importance of it. It is one of the most discussed and analyzed games, not only within the game theory literature, but also across many economical and social sciences. This is mainly due to the cited usefulness of the game, as it represents a simplification of many real-life conflict situations, and also because of the trouble the game causes to anyone tries to solve it.

Poundstone (1992) provides a good review of the game, and its history. Poundstonealso summarizes the challenge PD poses: "How one should act in a prisoners's dilemma? In the main this is still an unanswered and probably unanswerable question". And, ends with few words on what the game theorists Luce and Raiffa, who gave the poisoner's dilemma great emphasis in Luce and Raiffa (1957), wrote: "The hopelessness that one feels in such a game as this cannot be overcome by a play on the words 'rational' or 'irrational'; it is inherent in the situation".

In short, the PD game is a 2x2 zero-sum game, and while the description of the game took many variations over the years, the hypothetical story behind the game and its name is still the same. The story goes as follows: two criminals have been arrested under the suspicion of having committed a crime together. The police do not have enough evidence for prosecuting and convicting them. So, the police try to cut a deal with at least one of them. While keeping them separated in different rooms, the police offers each of them the same deal.

The prisoner who confesses, i.e. defects his partner, will get a lesser sentence. If they both cooperate with each other, by rejecting the police's offer and both refuse to confess to the crime, each of them will get a short jail time sentence (for what the police has as evidence on them and be able to get a conviction for). If, on the other hand, they both defects and confesses on the crime they committed, then each of

Table 7.1: The Prisoner's Dilemma in a Normal Form Model

		Player P_b: *Don't Confess*	Player P_b: *Confess*
Player P_a	*Don't Confess*	1 year each $(2^{*}, 2^{**})$	10 years for P_a, and 3 moths for P_b $(4, 1)$
	Confess	3 moths for P_a, and 10 years for P_b $(1, 4)$	8 years each $(3, 3)$

* Ordinal Preference for Player P_a (1-4: highest to lowest)
** Ordinal Preference for Player P_b (1-4: highest to lowest)

them will get a longer, but the same for both, prison time. However, if only one defects his partner and confesses, while the other does not confess, then the person who defects will get a very minimal jail time and the other will get a very long prison time. Let the penalties for cooperating with each other (or not confessing) and defecting (or confessing) be the times set in the table shown in Table 7.1.

In fact, Table 7.1 represents the PD game using the mostly used game modelling technique in the game theory literature: the *Normal Form* modelling. A game's Normal Form model is a tabular with rows and columns. The rows represent one player's' different options/strategies, whilst the columns represent the other's different options/strategies. Each of the tabular cells represents an outcome, or what we refer to in our framework as a game's state. Each of the cells will also show the ordinal/cardinal preference for that outcome/state for each of the players. For example, in the PD's Normal Form shown in Table 7.1, the red ordinal preference numbers represent the ordinal preferences for each of the game's states for the "column" player, P_a, while the blue ordinal preferences shown represent the preferences for the "row" player, P_b.

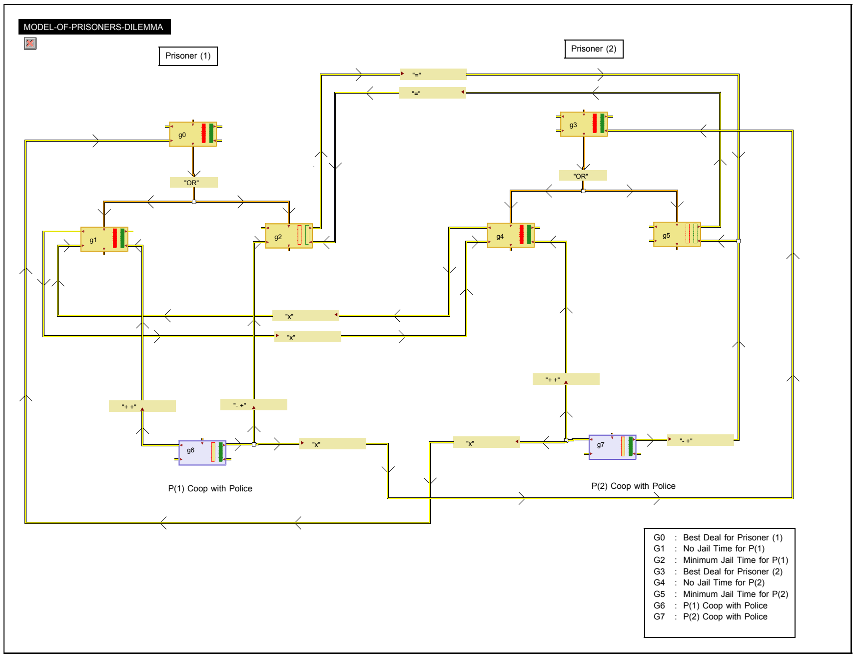
The dilemma arises from each prisoner need to make a wise decision which is not possible without knowing the other's choice. A restriction that is embedded in the structure of the game. None of the players knows what the others' will select as their option/strategy moving forward in the game. This is a common restriction in game theory literature. Table 7.1 shows why PD is considered a dilemma, in fact one of the most known talked about paradoxes of rationality games. Let us take for example the prisoner represented in the rows of the game's Normal Form shown in the figure. This prisoner could decide to defect, act selfishly and confess to police, because this is the only way for him to get the least punishment of 3 months jail time. He surely could get this least punishment, but only if the other prisoner did not confess. Since both prisoners kept separated while held in the police's custody, our "row" prisoner, or Player P_a as called in the figure, will not be able to know what the "column" prisoner, or Player P_b decides or is leaning towards. If it happened that prisoner P_b is also acting selfishly, defecting and confessing, then our first prisoner, Player P_a, will sure get hit by a very long prison time of 8 years.

Rationally talking, at least according to the very famous John Nash and his Nash Stability definition, each of the prisoners is better off acting selfishly by selecting the defection/confession path. This game is called dilemma, or more accurately a paradox as some refer to it, because the prisoners are actually collectively better off if they cooperate with each other and not confess to the police. Doing so, each of them will only get one year jail time, and more importantly not risk getting 8 or even 10 yeas prison time. Many scholarly attempts were made to build tools and solution solution concepts that better help analyze the PD game, especially to show that cooperation is better than defection as a rational choice for the prisoners/players. The most famous of these attempts, is the successful attempt by Howard (1971) proposing the meta-rationality concepts (GMR and SMR, discussed in the previous chapter and defined with strength levels within the context of our own framework). But the defection choice, or the NASH stable choice, is still the mostly used strategy to show that the player is rational, otherwise if a player choses cooperation then many in the game-theory community, specially the economists among them, will definitely not be shy to call this player irrational, or exhibiting irrational behaviour.

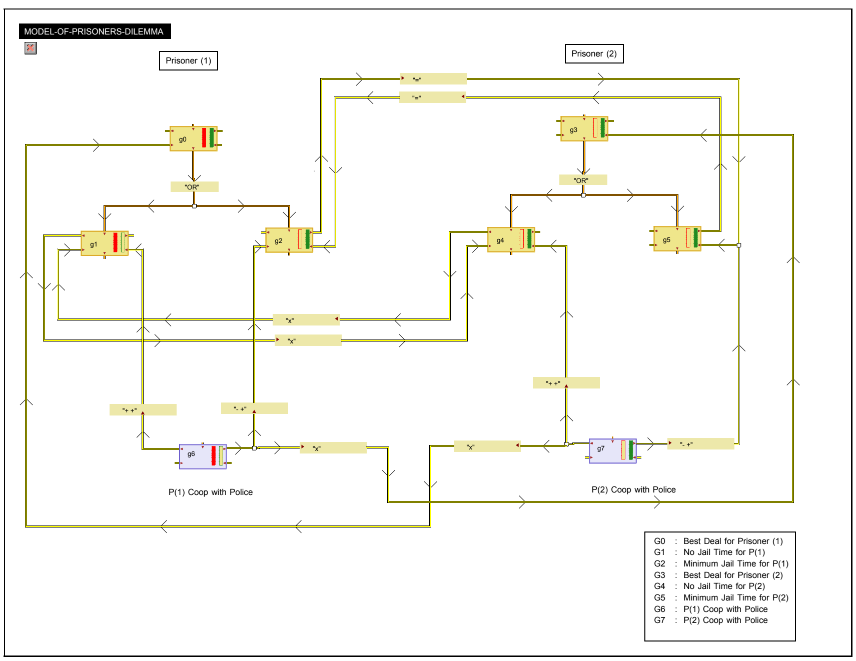
We now model the PD game using the Constrained Rationality modelling and analysis tools discussed in the previous chapter for non-cooperative games. We will focus our attention, though, on illustrating the additional insight to the multi-agent games gained by using the learnings from the concepts, theorems and observations discussed earlier in this chapter to analyze these games, especially the PD game.

7.5.1.2 Players' Strategic Goals and Alternatives

By design, the PD game, in its structure, does not present the players/prisoners with more than one goal for each: achieve "Best Deal". The best-deal is the one minimizes the prisoner's jail/prison time, knowing after-the-fact what the other prisoner chose as a strategy. The game has many assumptions inherent in its

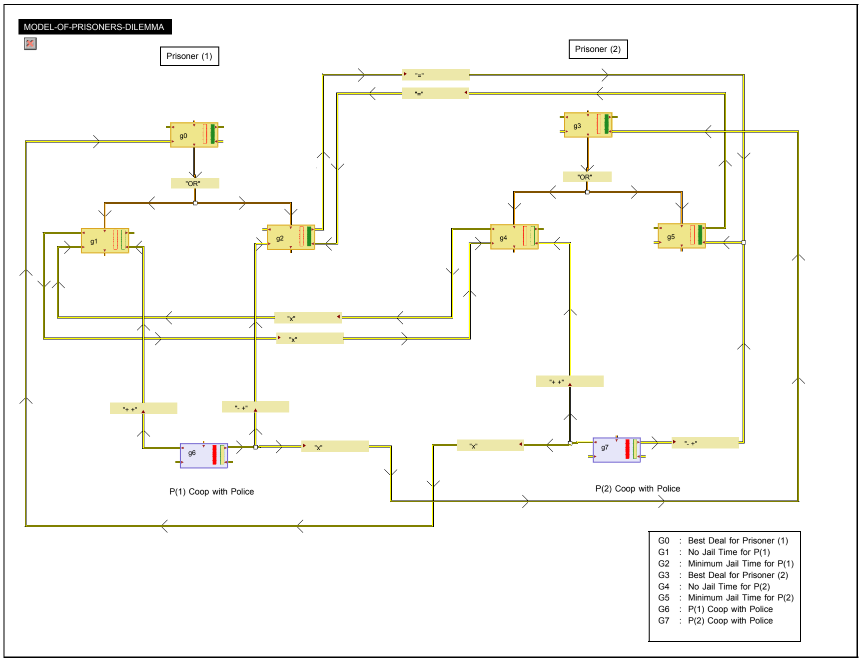


(a) Prisoner's Dilemma with both Prisoners Defecting (Cooperating with the Police)



(b) Prisoner's Dilemma with Prisoner (1) Not Defecting (i.e. Not Co-operating with the Police) and Prisoner (2) Defecting

Figure 7.11: Prisoner's Dilemma: GCM models for the players, and the how the players' decisions to cooperate or defect affect their respective ultimate strategic goal in the game



(c) Prisoner's Dilemma with both Prisoners Cooperating with each other (i.e. both are Not Cooperating with the Police

Figure 7.11: Prisoner's Dilemma: GCM models for the players, and the how the players' decsions to cooperate or defect affect their respective ultimate strategic goal in the game

structure. In summary, the game is not a game of many-goals to reason about, but rather a game of a challenging structure.

Table 7.2: Prisoner's Dilemma: Players' Alternatives/Options

Set of Alternatives available to Player P_a (A_{P_a}) :	
$A_{P_a\,0}$	*Do Not Defect* by not confessing and cooperating with Police.
$A_{P_a\,1}$	*Defect* by confessing and cooperating with Police, in other words turn in Prisoner b in exchange for getting lesser jail time.
Set of Alternatives available to Player P_b (A_{P_b}) :	
$A_{P_b\,0}$	*Do Not Defect* by not confessing and cooperating with Police.
$A_{P_b\,1}$	*Defect* by confessing and cooperating with Police, in other words turn in Prisoner a in exchange for getting lesser jail time.

The alternatives that the prisoners have in the game are also limited to two for each: Not to Defect (not confessing to police), and Defect (confess to police). Let the names of the prisoners in the game be: Player P_a and Player P_b; and let the players' alternatives named as shown in Table 7.2.

Even though that the game is not a game of many-goals to reason about, but rather a game of a challenging structure, still it is worth looking at the players'

GCM models and how they interact especially when different players' alternatives are activated. Figures 7.11a - Figure 7.11c show the GCMs model of all scenarios possible, as provided by the definition of the PD game. The figures show that the win-win scenario happens only when both players cooperate with each other and not confess to the police (Figure 7.11c). All other scenarios will produce less than full satisfactory achievement to both prisoners.

The GCMs model, also, offer a better understanding of the game's structure and challenges, because the models capture formally the game's assumptions and structure beyond what game theory, and related, forms (whether tabular or graphical) offer. Game theory, and related, forms only provide a description of the players' preferences in the game, such as what the famous Normal Form shown inTable 7.1 provides. No explanation is given to why the prisoners' preferences are as the way they are presented. Furthermore, such models do not show what will happen to the game's structure and player's ultimate goals when some tweaks happen to the game's assumptions (testing what-ifs). In the GCMs model of the PD game, everything is captured within the model, even though the game is not a typical game of many-conflicting-goals as most real-life business and political games are.

Knowing that the GCMs model reveals the effect of the players' choices (the strategies/alternatives) they adopt on their respective strategic goals, make the model a great tool to elicit the players' preferences for the game. No need to assume the players' preferences, or even take their word. The analyst, now, can verify the players' preferences. But, let us first identify the game's states.

Table 7.3: Prisoner's Dilemma: Defining the Game's States

The Set of All States $\mathcal{S}$ for the Prisoner's Dilemma:		
State	**Player P_a Options**	**Player P_b Options**
S_1	Do Not Defect $A_{P_a 0}$	Do Not Defect $A_{P_b 0}$
S_2	Do Not Defect $A_{P_a 0}$	Defect $A_{P_b 1}$
S_3	Defect $A_{P_a 1}$	Do Not Defect $A_{P_b 0}$
S_4	Defect $A_{P_a 1}$	Defect $A_{P_b 1}$

7.5.1.3 Game's States

By design, the game has two alternatives only for each player. This is the reason behind calling it 2x2 game, giving it four outcomes, or states, only. These states are shown in Table 7.3 with the players' selected alternatives for each.

7.5.1.4 Players' Preferences over States of the Game

The jail/prison time the prisoners will be given for each of the states (their selected alternatives) is shown in the Normal Form of the game, Table 7.1. We use these jail/prison times as measurements of how much punishment, or prevention of freedom. We assign these values, after fuzzification, to each of the prisoner's Prevention $Prvn$ value property of their strategic goal of best deal (or least jail/prison time). For example, Player P_a's $Prvn(SG)$ will get the following values: *Little* for state s_1 (for the 3 months jail time he will get if this state materialized); *Full* for state s_2 (for the 10 years prison time he will get if this state materialized); *None* for state s_3 (for the no jail time he will get if this state materialized); and *Big* for state s_4 (for the 8 years time he will get if this state materialized). We then continue the process of eliciting both the ordinal (numerical order) and the cardinal (weighted value) preferences for each of the prisoners over the game's states. The final results are summarized in Figure 7.12.

Figure 7.12 show the prisoners each with one strategic gaol, as per the game design as we discussed earlier. It also shows that the prisoners give their respective one-strategic-goal, understandably, *Full* strategic importance (i.e. $SImprt(SG) = F$) and *INdifferent* emotional likeness (i.e. $EVlnc(SG = IN)$. The figure shows that both prisoners have *Full* label for their respective rationality factor ($RF = F$) and *Full* label for their respective emotionality factor ($EF = N$). These values ensure that the game is modelled as designed and used in the literature: full rationality with no emotionality.

From Figure 7.12, the preferences vector for each of the prisoners/players is defined and shown in Table 7.4. The table also shows the strength preferences, for each of the players, over all the game's states.

Rationality Factor = 1.0 (for Pa & Pb)

Emotionality Factor = 0.0 (for Pa & Pb)

		Prisoner's Dilemma	
		Pa	Pb
DMs' Strategic Goals	SGs:	SG_{Pa}	SG_{Pb}
		$SG_{Pa\ 0}$	$SG_{Pb\ 0}$
Strategic Importance	$SImprt(SG_k)$	F	F
Emotional Likeness	$ELike(SG_k)$	IN	IN
State S_1	$Achv(SG_k)$	N	N
	$Prvn(SG_k)$	L	L
	$FAchv(SG_k)$	-L	-L
	$TWFAchv(SG_k,DM)$	-0.20	-0.20
{ $Achv(A_{Pa\ 0})$=F,	$WP(S_1, DM)$	-0.200	-0.200
$Achv(A_{Pb\ 0})$=F }	$OP(S_1, DM)$	2	2
State S_2	$Achv(SG_k)$	N	N
	$Prvn(SG_k)$	F	N
	$FAchv(SG_k)$	-F	N
	$TWFAchv(SG_k,DM)$	-1.00	0.00
{ $Achv(A_{Pa\ 0})$=F,	$WP(S_2, DM)$	-1.000	0.000
$Achv(A_{Pb\ 1})$=F }	$OP(S_2, DM)$	4 (Worst)	1 (Best)
State S_3	$Achv(SG_k)$	N	N
	$Prvn(SG_k)$	N	F
	$FAchv(SG_k)$	N	F
	$TWFAchv(SG_k,DM)$	0.00	-1.00
{ $Achv(A_{Pa\ 1})$=F,	$WP(S_3, DM)$	0.000	-1.000
$Achv(A_{Pb\ 0})$=F }	$OP(S_3, DM)$	1 (Best)	4 (Worst)
State S_4	$Achv(SG_k)$	N	N
	$Prvn(SG_k)$	B	B
	$FAchv(SG_k)$	B	B
	$TWFAchv(SG_k,DM)$	-0.80	-0.80
{ $Achv(A_{Pa\ 1})$=F,	$WP(S_4, DM)$	-0.800	-0.800
$Achv(A_{Pb\ 1})$=F }	$OP(S_4, DM)$	3	3

Figure 7.12: Prisoner's Dilemma: Players' Ordinal and Weighted Preferences

Table 7.4: Prisoner's Dilemma: Players' Preferences

P_a **Preferences** (*Most* to *Least* Preferred)				
Pref(P_a)	S_3	S_1	S_4	S_2
WP	0.00	-0.20	-0.80	-1.00
P_a **Preferences' Strengths**				
$\succ^{LPS}_{P_a,t}$	S_3	S_1	S_4	S_2
S_3	N	L	B	F
S_1	-L	N	M	B
S_4	-B	-M	N	L
S_2	-F	-B	-L	N

P_b **Preferences** (*Most* to *Least* Preferred)				
Pref(P_b)	S_2	S_1	S_4	S_3
WP	0.00	-0.20	-0.80	-1.00
P_b **Preferences' Strengths**				
$\succ^{LPS}_{P_b,t}$	S_2	S_1	S_4	S_3
S_2	N	L	B	F
S_1	-L	N	M	B
S_4	-B	-M	N	L
S_3	-F	-B	-L	N

7.5.1.5 Players' Moves over States of the Game

As per the classical PD game, the players have no moves between states. This is because the game is a one-shot, or one-time, game. Each of the players is expected to select one alternative: confess or do not confess. The players are not allowed to change their selection, and the game's state is defined after the fact based on their selected alternatives. No player is allowed to have a second move. Consider for example the situation where prisoner P_a chooses to not-defect, i.e. alternative $A_{P_a\,0}$, and P_b chooses to defect, i.e. alternative $A_{P_b\,1}$. Recall that, as per the game rules, they select their choices in separation and none of them know what the other's selection will be. The game, based on the prisoners' selections, is now at state s_2. What is next? Nothing. The game is complete, and both prisoners get the expected punishments as set by state s_2: P_a gets 10 years prison time, and P_b gets 3 months jail time. Can P_a change this situation, by deciding to defect now for example? No.

For a one-shot game, where players are not allowed to sanction each other or respond with countermoves, modelling the UMs and UIs of the players become useless. In fact, this restriction, among others the PD game has, form the basis of the criticism the game receives from many scholars and practitioners alike for its limited use. Sure, the classical PD represents a real-life game that had happened and continues to happen in police stations around the world , but it is hard to imagine that it represents economical, trade or business conflicts, as some argue. To make the standard PD more useful, a slightly modified version of it, was proposed allowing the player/prisoner to punish the other player if the other player defected when he didi not This new modified game is referred to in the literature as the *Iterated Prisoner's Dilemma* (IPD).

The first reported experiment that involved IPD was the 1950 experiment done by the two RAND scientists, Merrill Flood and Melvin Dresher, who invented the PD game. Flood and Dresher were concerned that John Nash's NASH equilibrium point solution could be unsatisfactory; and there can be cases where the NASH equilibrium state is not a good outcome. Their experiment, in which they recruited two RAND employees to be the players in it, involves playing what came to be known as the Prisoner's Dilemma (PD) game repeatedly 100 times. The experiment and its results were published in Flood (1952). But by far, the most important result this experiment has in the filed of game theory, and conflict analysis in general, is introducing: 1) the Prisoner's Dilemma (PD) game; and 2) the concept of repeated or iterative games, especially the Iterative Prisoner's Dilemma (IPD) game.

After the Flood (1952) report, the interest in IPD game grew steadily. One reason for this is IPD seemed more realistic with more practical applications than the one-shot PD. There are many flavours of IPD in the literature, some employ sophisticated strategies and some employ simple ones in order to allow the player to gain the best. The interest on IPD and IPD's strategies had sparked by the effort of by Robert Axelrod, and his IPD tournaments, and his book The Evolution of Cooperation (1984).

We will start our analysis of the players moves in the PD game by looking at the moves the players can have in the two flavours of PD we talked about so far: 1) the standard classical one-shot PD; and 2) the standard IPD. The standard IPD is simply a repeated PD game with no pre-defined strategies the players follow. Later in this chapter, we will analyze IPD games with players using one of the most known strategies, and by far the most successful: the Tit for Tat (TFT) strategy. But for now, we will consider only the standard, no pre-defined player' strategies PD and IPD game.

1) Players' Moves over States of the Classical One-Shot Prisoner's Dilemma:

The classical one-shot PD gives the prisoners/players no moves or second chances after their one and only one selection of their alternatives. But, we can assume fairly that the game starts for each prisoner/player at the No-Confess , or the No-Defection, option, because the prisoners did actually confess at the beginning before the game starts. Now, hypothetically speaking, the prisoners mentally, but not physically, can have some moves starting from that option. Consider for example the case of prisoner P_a. He starts the game knowing that he did not confess/defect yet.

This makes his version of the Prisoner's Dilemma game actually starts at either state s_1 or state s_2, based on whether he thinks/believes P_b will select to Not Defect or Defect, respectively. P_a can envision mentally changing his selected alternative from Not Defect/Confess to Defect/Confess, before he actually change his selection to Defect/Confess. This makes him has two mental UM moves. One from s_1 to s_3, and a second from s_2 to s_4. Both UMs are, in fact, UI moves for P_a based on his preferences. These mental P_a moves, and the equivalent moves for prisoner P_b are shown in Figure 7.13.

The figure does not show reverse UM moves for P_a from s_3 to s_1 or from s_4 to s_2, because P_a can not change his position/selection from Confessing to Not

Confessing. Even if we modelled this as a mental UM move for him, this move will never be able to materialize. Once a confession is doe, it is done. There is go back. This is especially true knowing that the prisoner in this one-shot game can not even change his strategy/selection in a following game, or iteration of a game. Similarly, there are no reverse UM moves for P_b from s_2 to s_1 or from s_4 to s_3, as Figure 7.13 correctly show.

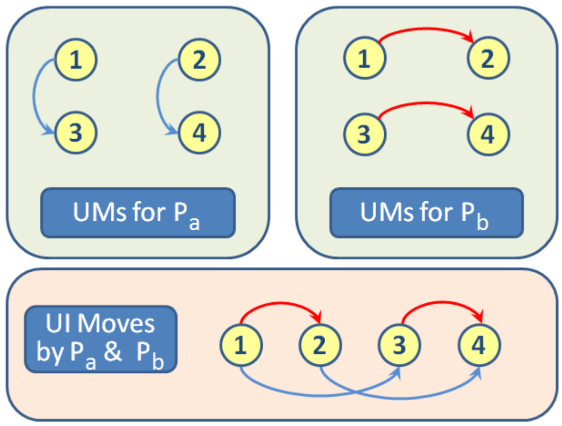


Figure 7.13: The Prisoner's Dilemma - One-Shot Standard Game: The Unilateral Moves and the Unilateral Improvement Moves by the players

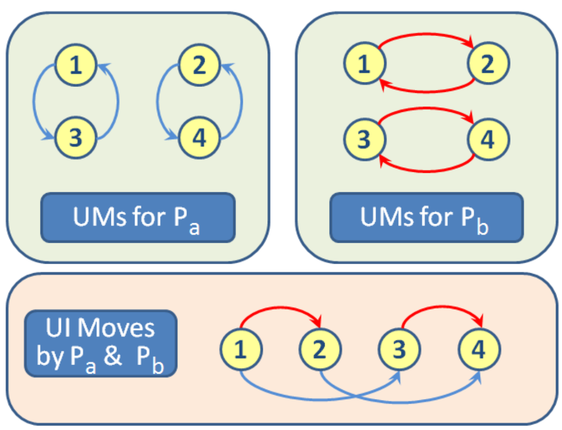


Figure 7.14: Prisoner's Dilemma - Iterated Standard Game: Players' Unilateral Moves and Unilateral Improvement Moves

2) Players' Moves over States of the Iterated Standard Prisoner's Dilemma:

For the IPD, and unlike the one-shot classical PD, the prisoners actually can change selection from confessing to confessing in the iteration that followed. It is true that each iteration is an instant of the one-shot PD, but each of the prisoners will have the chance to change his position, or retaliate, in the following instance of the game. Therefore, the physical restriction that the one-shot game has of not allowing player to not confess if he already confessed, is no longer a restriction of the players moves in the iterated version of the game. This is illustrated in Figure 7.14 by showing the players have the reverse moves prohibited to have in the classical one-shot PD.

7.5.1.6 Stability Analysis and Analysis Results of the Prisoner's Dilemma

In this section we analyze both versions of the PD game, the classical one shot and the iterated one. For each version of the game, we follow the same process we use in our framework for stability analysis and results discussion. We start by conducting stability analysis, followed by equilibrium states analysis, and closing by some results and comparisons.

1) Analysis of the Classical One-Shot Prisoner's Dilemma:

The classical one-shot PD has no moves by the focal decision maker, nor sanctioning moves by others after the one and only first move is completed. This makes the concept of sanction moves (SMs), defined in defined in Definition 6.2.3, not applicable to this type of game. Recall that GMR, SMR and SEQ stability solution concepts require in their definitions (Definition 6.3.2, 6.3.3 and 6.3.4, respectively) some form of SMs to exist. Therefore, these solutions concepts are not applicable to this game by design. Only NASH does not require, in its Definition 6.3.1, SMs by others to declare a state stable after the focal decision maker takes a UI move out of it.

One also can immediately notice the implication of this, and the complication this could cause to finding an equilibrium to the classical one-shot PD game. The first implication is that by allowing only one move per player, i.e. no retaliation is allowed or even considered, the game become with foresight of zero steps. As we discussed in Section 7.2, one of the main characteristics of NASH is it has a foresight of zero lookahead steps. The players who uses NASH solution concept to define wether a state is stable for him or not does not need to know anything but his UI moves. He is not even required to know the preferences of others or their moves, if they are different from his. This leads us to the second implication: the absence of knowledge about others' preferences and moves, makes the focal decision maker vulnerable to others' moves to disimrpove his positions, as shown in Figure 7.1.

The third implication is a result of the first two ones. The believers in NASH, as a stability solution concept, exhibit an attitude toward risk taking that is correctly could be describes as: ignoring risks. Ignoring risks does not mean the risks are not there.

Table 7.5: Prisoner's Dilemma - One-Shot Standard Game: Stability Analysis

	$\mathbf{P_a}$				$\mathbf{P_b}$			
	S_3	S_1	S_4	S_2	S_2	S_1	S_4	S_3
UIs		S_3 (UI)		S_4 (UI)		S_2 (UI)		S_4 (UI)
NASH	L		L		L		L	

Table 7.6: Prisoner's Dilemma - One-Shot Standard Game: Equilibrium States

	S_1	S_2	S_3	S_4
NASH EQ.				L

The stability analysis of the classical one-shot PD is provided in Table 7.5. Notice that states are only tested for NASH stability for each of the players, as per our discussion above; and that each has two states that are NASH stable with strength calculated using Algorithm 6.2. But, only one state is NASH stable for both players. It is state s_4, the state that both player defect/confess. This makes s_4 the only equilibrium state for the game, as shown in Table 7.6. A bad state for both players to be in, based in their respective preferences and the long prison time both will get out of it.

The complication that arises from the implications listed above is demonstrated by the stability analysis results shown in Tables 7.5 and 7.6. This complication is at the heart of why the PD is considered a dilemma or a paradox of rationality. Attempts to solve, or suggest solution concepts, to a dilemma that represents a conflict with a multi-agent social context to it by just ignoring all other agents/players and concentrate on own gains and moves, will definitely fail to recognize the dangers as well as the opportunities lie ahead. So, it is understandable that others questioned the logic behind the self imposed restrictions of the one-shot classical PD, by its own design, and proposed instead to look at it as a repeated/iterated game to give it the depth, the foresight, and dynamics it deserves as a social conflict.

2) Analysis of the Iterated Standard Prisoner's Dilemma:

In the IPD, players are allowed to retaliate, punish, and reward other players position/moves of the previous iteration/instance of the game. Sanction moves here are considered, and as a result all solution concepts are allowed to compete in their findings. Table 7.7 shows the stability analysis of the game based on all four solution concepts.

Beside the NASH stable states that the stability analysis for the classical one-shot PD, shown in Table 7.5, there is now a new state emerging to be stable for under GMR, SMR and SEQ solution concepts. State is s_1, where both players select to cooperate with each other and not defect/confess, is now stable stable for both players, and therefore form a new equilibrium state for the game, as shown in Table 7.8.

Table 7.7: Prisoner's Dilemma - Iterated Standard Game: Stability Analysis

	$\mathbf{P_a}$				$\mathbf{P_b}$			
	S_3	S_1	S_4	S_2	S_2	S_1	S_4	S_3
UIs		S_3 (UI)		S_4 (UI)		S_2 (UI)		S_4 (UI)
NASH	L		L		L		L	
GMR	N	M	N		N	M	N	
SMR	N	M	N		N	M	N	
SEQ	N	M	N		N	M	N	

Table 7.8: Prisoner's Dilemma - Iterated Standard Game: Equilibrium States

	S_1	S_2	S_3	S_4
NASH EQ.				L
GMR EQ.	M			N
SMR EQ.	M			N
SEQ EQ.	M			N

The stability analysis results of the IPD (the standard game) given in Tables 7.7 and 7.8 provide us with much insight, especially when compared with the results of the classical one-shot game given in Table 7.5 and 7.6. The following is a list of some of the most insightful observations:

- Foresight: By allowing the players the ability to react, sanction or have countermoves in general, the IPD gives the players the ability to strategize and think ahead of the consequences of their initial moves. The one-shot classical PD deprives the players from this strategy-playing, and instead the game come across as take-a-chance game (this could explain why people went on to add probabilities to the mix).

- Risk taking: while the classical one-shot game forced all players to adopt an attitude of ignoring risk, the iterated version of the game allowed each of the players to chose among many attitudes to adopt: 1) ignore risks, therefore use NASH stability analysis to guide his decisions on what he should do; 2) avoid risks, for this he can use GMR stability analysis to know if the other player can sanction his UI, even if this sanction is made solely to hurt him, or go even further using SMR to check if he will be able to mitigate any sanction others can impose on him; or 3) strategize by taking acceptable risks, risks with the assumption that the other player is rational player and will not sanction him just for the sack of hurting him, for this he can use the SEQ stability analysis.

- Stability of selfishness: The states in which the players act selfishly to maximize their gain (minimize their jail/prison time), or states s_2 and s_3, are not equilibria states, i.e. they are not stable for all players at the same time. Once of a state of them is reached in an instance of the game, in the second instance a retaliation will happen brining the game to the all-must-loose s_4 state. Making any short maximization of gains by one of the players to be short lived. The player who acted selfishly will find himself in state s_4, loosing in the second round much more what he gained in the first round.

- How much gain is there in defection? The gain one should expect by moving from not-defecting, or not-confessing, states to defecting, or confessing, states is indeed *Little*. Table 7.9show the amount of gain a prisoner should expect from defecting/confessing. It is clear from the table that gain is little. This fact is also can be elicited from looking at the preference strength between no-defection states and defection states. The preferences' strength as shown to be in Table 7.4 at the L, or Little, level for both the one-shot classical game and the iterated one. This fact is also reflected more clearly in the NASH stability and equilibrium's strength for defection states, for both types of games, as shown in Tables 7.5 - 7.8.

Table 7.9: Prisoner's Dilemma: Is Defection worth it?

Player	Defection Move (UI)		Gain from Defection	Preference	
	From	To		Relation	Strength
P_a	s_1 (1 year jail)	s_3 (3 moths jail)	9 months less of jail time	$s_3 \succ^{L}_{P_a,t} s_1$	L
	s_2 (10 year prison)	s_4 (8 years prison)	2 year less of prison time	$s_4 \succ^{L}_{P_a,t} s_2$	L
P_b	s_1 (1 year jail)	s_2 (3 moths jail)	9 months less of jail time	$s_2 \succ^{L}_{P_b,t} s_2$	L
	s_3 (10 year prison)	s_4 (8 years prison)	2 year less of prison time	$s_4 \succ^{L}_{P_b,t} s_3$	L

- Cooperation's stability in comparison to defection: For the players individually, the defection states (shown in the "To" column in Table 7.9) are all NASH stable with *Little* strength. This true for both types of PD games, the classical one-shot and the iterated one, as can be seen from Table 7.5 and Table 7.7, respectively. Recall that NASH stability, no matter what strength it has, is still a weak stability when it is compared to stability under all other solution concepts we studied (Theorem 7.4.2). Therefore, all these defection states have weak stabilities for the individual players, and when one is stable for both players then it forms a *weak* NASH equilibrium. Compare this with the cooperation state. When both players cooperate with each other in the game they move to state s_1 which is stable for both players under GMR, SMR and SEQ solution concepts, with strength level of *Much*. In other words, the cooperation state has a higher stability type, and stability strength, once the players start thinking strategically and start looking ahead to see what others have to sanction/disimprove their moves.

7.5.2 Iterative Prisoner's Dilemma : The Tit for Tat Way

7.5.2.1 Background

Tit for Tat (TFT) is a highly effective well known strategy for the IPD. It was first proposed by Anatol Rapoport in Robert Axelrod's two tournaments, held in the late 1970's. It was introduced in Axelrod (1980a,b) to be the most effective strategy for the IPD game, as per the tournaments results, in spit of the fact that it was the simplest and the least sophisticated (the TFT submission to the tournament was a BASIC program with four lines of code). A player adopting a TFT strategy in playing the IPD will start the first iteration/instance of the game by cooperative action and then behaves the same as the other in future steps. So after the first iteration, the player will follow what the other player did in the previous iteration. If the other player defected in the previous iteration, he will defect in this one; and if the other player cooperated, then he, likewise, will cooperate in this one too.

We will model and analyze the IPD with two players using the TFT strategy against each other, and with one player playing using TFT and the other uses the standard IPD game that we discussed above in this section. Because the two versions of the IPD TFT games we are concerned with in this section assume that the players have the same alternatives the players of the standard PD (single-shot and iterated) have,. These alternatives are discussed above in Section 7.5.1.2, so we will not re-discuss the players' alternatives again. And because the players' alternatives do not change in the TFT versions of the game, The TFT games will still have the same four states that the standard games have. So, we will also not re-discuss the TFT games' states, because they are discussed already in Section 7.5.1.3. But, we will start the modelling and analysis process of the TFT games from the players' preferences elicitation step, where change is start to happen.

7.5.2.2 Players' Preferences over States of the Game

Traditionally, when people model the TFT IPD, they will model the actions and the mechanical moves the players who adopt TFT do in the game. But, Axelrod (1980a) when introduced Rapoport's TFT submission to his IPD tournaments, he actually did an important analysis to find out what common characteristics TFT and the other runner up submissions have. His findings were descriptive, but insightful in a sense that they shed light on the motivation/goals behind such strategies.

Axelrod (1980a,b) described these characteristics as properties of successful rules (of game playing), and they are, in order: niceness, effectiveness with the kingmakers (strategies which exhibit not-niceness and do not do for themselves but help establish the ranking of the top contending strategies in the tournament), forgiveness, provocability (immediately defects after an uncalled-for defection from the other, but goes to forgiveness once the other goes back to cooperation). There are many ways to model these "attitudes" using our Constrained Rationality framework. One way to model these here is by adding another strategic goal to the TFT strategy adopter. Recall that each player of the PD is given in our model, so far, one strategic goal "Get the Best Deal", denoted as $SG_{P_a 0}$ or $SG_{P_b 0}$ depends if the player is P_a or P_b, respectively. Now, the players who adopt TFT will have an additional strategic goal "Win By Cooperating with Cooperators". This goal could alternatively be called "Have a Win-Win Deal". Let this strategic goal be denoted $SG_{P_a 1}$ or $SG_{P_b 1}$ depends if the player is P_a or P_b, respectively.

It is also very apparent form Axelrod's list of attitudes exhibited by TFT, and a like strategies, adopter that these attitudes/qualities have emotional dimension to them. Considering that the very first attitude in the list, and the most important was niceness, one can not just capture the importance of this attitude with just a strategic goal, even if we add to this goal a full rational strategic importance. Niceness, forgiveness and provocability are emotional attitudes, and can be captured effectively in the model by addition an An *Extremely Liked* emotional valance label to this second strategic goal we just added. Then, for those with the new strategic goals and emotional valences, we re-elicit their preferences based on the new GCM model they have. Because TFT player's goals and the strategic/emotional importance of these goals are different than the ones that players of standard PD games have, then the TFT players' preferences will be different. Therefore, we will start by looking at the changes happening to the players' preferences structure and strength over the game's states.

1) Players' Preferences over States of the Tit-For-Tat-Both-Ways Prisoner's Dilemma:

In the IPD game where both players playing Tit for Tat, both players have two rational strategic goals, with the second goal for each is also emotionally extremely liked. Figure 7.20 shows the elicitation of the ordinal and weighted/cardinal preferences of both players over the four states of the games. Table 7.10 shows the preferences vector and preferences' strengths for both players.

Rationality Factor = 1.0 (for Pa & Pb) Emotionality Factor = 1.0 (for Pa & Pb)		Prisoner's Dilemma (Both Play Tit For Tat)			
		Pa		Pb	
DMs' Strategic Goals	SGs:	SG_{Pa}		SG_{Pb}	
		$SG_{Pa\,0}$	$SG_{Pa\,1}$	$SG_{Pb\,0}$	$SG_{Pb\,1}$
Strategic Importance	$SImprt(SG_k)$	F	F	F	F
Emotional Likeness	$ELike(SG_k)$	IN	EL	IN	EL
State S_1	$Achv(SG_k)$	N	F	N	F
	$Prvn(SG_k)$	L	N	L	N
	$FAchv(SG_k)$	-L	F	-L	F
	$TWFAchv(SG_k,DM)$	-0.20	2.00	-0.20	2.00
{ $Achv(A_{Pa\,0})$=F,	$WP(S_1, DM)$	0.725		0.725	
$Achv(A_{Pb\,0})$=F }	$OP(S_1, DM)$	1 (Best)		1 (Best)	
State S_2	$Achv(SG_k)$	N	N	N	Mo
	$Prvn(SG_k)$	F	F	N	N
	$FAchv(SG_k)$	-F	-F	N	Mo
	$TWFAchv(SG_k,DM)$	-1.00	-2.00	0.00	1.00
{ $Achv(A_{Pa\,0})$=F,	$WP(S_2, DM)$	0.125		0.625	
$Achv(A_{Pb\,1})$=F }	$OP(S_2, DM)$	4 (Worst)		2	
State S_3	$Achv(SG_k)$	N	Mo	N	N
	$Prvn(SG_k)$	N	N	F	F
	$FAchv(SG_k)$	N	Mo	-F	-F
	$TWFAchv(SG_k,DM)$	0.00	1.00	-1.00	-2.00
{ $Achv(A_{Pa\,1})$=F,	$WP(S_3, DM)$	0.625		0.125	
$Achv(A_{Pb\,0})$=F }	$OP(S_3, DM)$	2		4 (Worst)	
State S_4	$Achv(SG_k)$	N	N	N	N
	$Prvn(SG_k)$	B	Mo	B	Mo
	$FAchv(SG_k)$	B	-Mo	B	-Mo
	$TWFAchv(SG_k,DM)$	-0.80	-1.00	-0.80	-1.00
{ $Achv(A_{Pa\,1})$=F,	$WP(S_4, DM)$	0.275		0.275	
$Achv(A_{Pb\,1})$=F }	$OP(S_4, DM)$	3		3	

Figure 7.15: Prisoner's Dilemma - Tit For Tat by Both Players: Players' Ordinal and Normalized Weighted Preferences

Table 7.10: Prisoner's Dilemma - Tit For Tat by Both Players: Players' Preferences

P_a **Preferences** (*Most* to *Least* Preferred)				
$Pref(P_a)$	S_1	S_3	S_4	S_2
WP	0.725	0.625	0.275	0.125
P_a **Preferences' Strengths**				
$\succ^{LPS}_{P_a,t}$	S_1	S_3	S_4	S_2
S_1	N	L	Mo	M
S_3	-L	N	S	Mo
S_4	-Mo	-S	N	L
S_2	-M	-Mo	-L	N

P_b **Preferences** (*Most* to *Least* Preferred)				
$Pref(P_b)$	S_1	S_2	S_4	S_3
WP	0.725	0.625	0.275	0.125
P_b **Preferences' Strengths**				
$\succ^{LPS}_{P_b,t}$	S_1	S_2	S_4	S_3
S_1	N	L	Mo	M
S_2	-L	N	S	Mo
S_4	-Mo	-S	N	L
S_3	-M	-Mo	-L	N

2) Players' Preferences over States of the Tit-For-Tat-One-Way Prisoner's Dilemma:

In the IPD game where only one of the players uses Tit for Tat as a strategy, then only this player will have two rational strategic goals, with the second goal for each is also emotionally extremely liked. The player who does not use TFT, but rather the standard IPD, then he will maintain having only one strategic goal as before. Figure 7.16 shows the elicitation of the ordinal and weighted/cardinal preferences of both players over the four states of the games.

Rationality Factor = 1.0 (for Pa & Pb)
Emotionality Factor = 1.0 (for Pa only, Pb is 0)

		Prisoner's Dilemma (Pa Plays Tit For Tat)		
		Pa		Pb
DMs' Strategic Goals	SGs:	SG_{Pa}		SG_{Pb}
		$SG_{Pa\,0}$	$SG_{Pa\,1}$	$SG_{Pb\,0}$
Strategic Importance	$SImprt(SG_k)$	F	F	F
Emotional Likeness	$ELike(SG_k)$	IN	EL	IN
State S_1	$Achv(SG_k)$	N	F	N
	$Prvn(SG_k)$	L	N	L
	$FAchv(SG_k)$	-L	F	-L
	$TWFAchv(SG_k,DM)$	-0.20	2.00	-0.20
$\{Achv(A_{Pa\,0})=F,$	$WP(S_1, DM)$	0.725		0.400
$Achv(A_{Pb\,0})=F\}$	$OP(S_1, DM)$	1 (Best)		2
State S_2	$Achv(SG_k)$	N	N	N
	$Prvn(SG_k)$	F	F	N
	$FAchv(SG_k)$	-F	-F	N
	$TWFAchv(SG_k,DM)$	-1.00	-2.00	0.00
$\{Achv(A_{Pa\,0})=F,$	$WP(S_2, DM)$	0.125		0.500
$Achv(A_{Pb\,1})=F\}$	$OP(S_2, DM)$	4 (Worst)		1 (Best)
State S_3	$Achv(SG_k)$	N	Mo	N
	$Prvn(SG_k)$	N	N	F
	$FAchv(SG_k)$	N	Mo	F
	$TWFAchv(SG_k,DM)$	0.00	1.00	-1.00
$\{Achv(A_{Pa\,1})=F,$	$WP(S_3, DM)$	0.625		0.000
$Achv(A_{Pb\,0})=F\}$	$OP(S_3, DM)$	2		4 (Worst)
State S_4	$Achv(SG_k)$	N	N	N
	$Prvn(SG_k)$	B	Mo	B
	$FAchv(SG_k)$	B	-Mo	B
	$TWFAchv(SG_k,DM)$	-0.80	-1.00	-0.80
$\{Achv(A_{Pa\,1})=F,$	$WP(S_4, DM)$	0.275		0.100
$Achv(A_{Pb\,1})=F\}$	$OP(S_4, DM)$	3		3

Figure 7.16: Prisoner's Dilemma - Tit For Tat by P_a and Standard by P_b: Players' Ordinal and Normalized Weighted Preferences

Table 7.11 shows the preferences vector and preferences' strengths for both players. Notice that the TFT player has his preferences changed, while the player who uses the standard strategy maintained the preferences order and strength as shown in Figure 7.12 and Table 7.4of the standard PD (both the one-shot and the iterative versions).

Table 7.11: Prisoner's Dilemma - Tit For Tat by P_a and Standard by P_b: Players' Preferences

P_a **Preferences** (*Most* to *Least* Preferred)				
Pref(P_a)	S_1	S_3	S_4	S_2
WP	0.725	0.625	0.275	0.125
P_a **Preferences' Strengths**				
$\succ^{LPS}_{P_a,t}$	S_1	S_3	S_4	S_2
S_1	N	L	Mo	M
S_3	-L	N	S	Mo
S_4	-Mo	-S	N	L
S_2	-M	-Mo	-L	N

P_b **Preferences** (*Most* to *Least* Preferred)				
Pref(P_b)	S_2	S_1	S_4	S_3
WP	0.500	0.400	0.100	0.000
P_b **Preferences' Strengths**				
$\succ^{LPS}_{P_b,t}$	S_2	S_1	S_4	S_3
S_2	N	L	S	Mo
S_1	-L	N	S	S
S_4	-S	-S	N	L
S_3	-Mo	-S	-L	N

7.5.2.3 Players' Moves over States of the Game

Because the players' preferences changed in the TFT IPD games, while the players' unilateral moves are the same, only the unilateral moves of the players actually change.

1) Players' Moves for the Tit-For-Tat-Both-Ways Prisoner's Dilemma:

Figure 7.17 shows the UMs and UIs for the two players playing an IPD while each adopting TFT as their strategy.

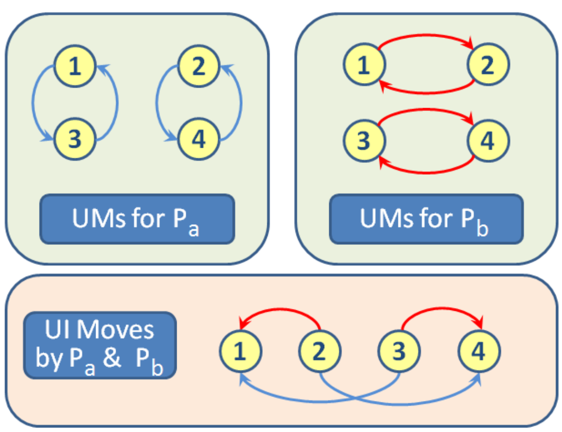


Figure 7.17: Prisoner's Dilemma - Tit For Tat by Both Players: Players' Unilateral Moves and Unilateral Improvement Moves

2) Players' Moves for the Tit-For-Tat-One-Way Prisoner's Dilemma:

Figure 7.18 shows the UMs and UIs for the two players playing an IPD with one adopting TFT as his strategy while the other playing the standard strategy.

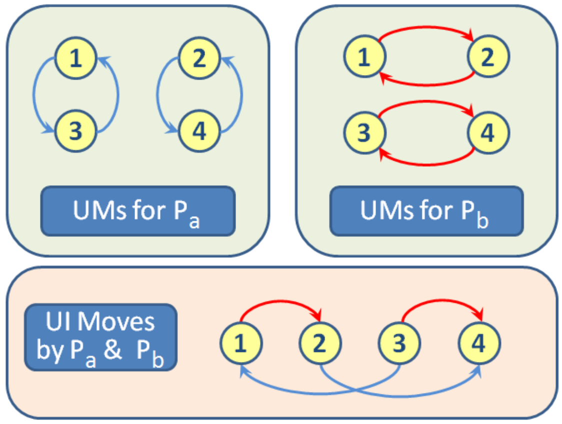


Figure 7.18: Prisoner's Dilemma - Tit For Tat by P_a and Standard by P_b: Players' Unilateral Moves and Unilateral Improvement Moves

7.5.2.4 Stability Analysis and Analysis Results of the Prisoner's Dilemma

Earlier, we said that the IPD players are allowed to retaliate, punish, and reward other players position/moves of the previous iteration/instance of the game. Sanction moves here are considered, and as a result all solution concepts are allowed to compete in their findings. But, the players have consistent similar preferences structures and strategies to direct their moves in the game. On the other hand, in the two special IPD games, which we are analyzing here, the players, or some of them, have special (other than the standard) strategies to direct their retaliation process and therefore the game flow of events and/or have different strategies to do so. These changes will effect the stability of the game's states for the players, and on the game as a whole.

1) Analysis of the Tit-For-Tat-Both-Ways Prisoner's Dilemma:

Table 7.12 shows the stability analysis based on all four solution concepts for the IPD in which both players player the TFT strategy. Table 7.13 shows the equilibrium states for the game under the same solution concepts. One will notice immediately some differences between these two tables and the stability/equilibrium analysis for the standard IPD shown in Tables 7.7 and 7.8.

Table 7.12: Prisoner's Dilemma - Tit For Tat by Both Players: Stability Analysis

	$\mathbf{P_a}$				$\mathbf{P_b}$			
	S_1	S_3	S_4	S_2	S_1	S_2	S_4	S_3
UIs		S_1 (UI)		S_4 (UI)		S_1 (UI)		S_4 (UI)
NASH	L		L		L		L	
GMR	N	Mo	N		N	Mo	N	
SMR	N	S	N		N	S	N	
SEQ	N		N		N		N	

First, It is true that in the standard IPD each player have two NASH stable state. But, as Table 7.7 only one is a shared NASH stable state, and that is state s_4 which is a very bad state, based on the players' preferences and the punishment received, reached by both players defect/confess. Now, in the IPD game where both players adopt the TFT strategy, and as shown in Table 7.12, there are two NASH stable states for each player and these states are the same for both players. Both players, now, have states s_1 and s_4 stable under NASH. This makes this game to have two states to be equilibrium states under NASH stability solution concept, as Table 7.13 shows.

Table 7.13: Prisoner's Dilemma - Tit For Tat by Both Players: Equilibrium States

	S_1	S_2	S_3	S_4
NASH EQ.	L			L
GMR EQ.	N			N
SMR EQ.	N			N
SEQ EQ.	N			N

Second, in this game we notice that the state which represents the players cooperating with each other and not defecting/confessing is stable. Not only this, but knowing that the first iteration/instance of the IPD game always starts from no-confession, or no-defection, by both players (state s_1), this makes none of the two players have any incentive (UI) to move from the cooperation s_1 state. Therefore, state s_1 is expected to persist from the time the game has its first iteration. This will only change if one of the players moved *irrationally* against their preferences and UIs.

Third, we can see from Table 7.12 that state s_3 is now a GMR and SMR stable to P_a. Similarly, s_2 is now a GMR and SMR stable to P_b. Recall that these two states are NASH stable for the standard IPD game, as shown by Table 7.7 above. These states are now GMR and SMR stable for their respective players, because the players should expect to get sanctioned if they decide to take their respective UIs out from them to state s_1. The question is why one of the player decides to sanction his opponent if that opponent is actually taking him to his at most preferred state s_1. As Table 7.10 shows, s_1 is the at most preferred state for both players of this type of game. The answer is simple. GMR and SMR do not assume the opponent to be a fully rational agent. In fact GMR and SMR assumes that the opponent could move in the game against his own preferences, motivated only by the desire to hurt the focal decision maker.

For example, in this case, s_3 is GMR stable for P_a because if P_a decides to take his UI to s_1 from s_3, P_b could sanction this UI by utilizing a normal UM he has from s_1 from s_2 which a less preferred state to P_a than state s_3 which P_a started from. s_3 is also SMR stable for P_a because P_a will not be able to recover from P_b's sanction. Importantly, both states s_3 and s_2 are not SEQ stable to P_a and P_b, respectively. Why? Because SEQ stability concept assumes full rational players that do not act against their preferences and UIs. In summary, s_3 and s_2 stability can only be considered seriously if the assumption that all players are rational players is called into question.

Finally, Table 7.13 is clear in it finding. The are two equilibrium states for this game. One is s_1, the both cooperating state; and the other is s_4, the both defecting state. Bust since we know that the game in its first iteration starts at state s_1, the expectation is that rational players in the game will not move out of this state, and cooperation will persist. If one, or both, players decided to act selfishly (and irrationally according to the their preferences structures), then it is expected that the game will progress reaching state s_4 within few rounds (two iterations -at least- to be specific). So, if one defect, or try defecting in an iteration, then the other surely will defect. The real problem for both players start her. Because, according to the TFT strategy, the players will be stuck at s_4. This especially true in an automated IPD game, where the players are automated agents that are programmed to act based on their preferences and moves. The only solution for the players, or automated agents, is to test the willingness of the other player to go back cooperating with each other. If the other player is willing, or acting rationally according to his preferences, then surely it will take a UI to state s_1 where both can claim a Win-Win situation. Otherwise, if the other player failed the test, and continue defecting, then the focal player can bring the game back to s_4.

From above, one can see the opportunity and danger that adopting TFT has for both players. It is either all-cooperate or all-defect. In other words, both players are expected to gain almost the same. There will be no player to be called the real winner in the game, or the one who claim the most out of the game. One could argue if one of he players become unpredictable, then this player might harness more from the game than the player who strictly follow TFT. This is actually proven to be untrue. Many simple and sophisticated strategies, including selecting moves randomly, were tried against TFT over the years, but TFT continued to show superiority in effectiveness against any players' manipulation or selfishness behaviour (Axelrod, 1980b). Next, we show that this is consist of our analysis of a IPD with only of the players adopting TFT as a strategy while the other adopting a standard IPD strategy.

2) Analysis of the Tit-For-Tat-One-Way Prisoner's Dilemma:

We analyze here the IPD game where P_a plays according to the TFT strategy (TFT goals, preferences and moves) while P_b plays a standard IPD strategy (one goal, self behaviour, etc.). First, Table 7.14 shows the players have different stable states for them under the fours stability solution concepts we use in our study. For example, the TFT player P_a has s_1 and s_4 to be NASH stable; and the standard IPD player P_b has s_2 and s_4 to be NASH stable.

Table 7.14: Prisoner's Dilemma - Tit For Tat by P_a and Standard by P_b: Stability Analysis

	$\mathbf{P_a}$				$\mathbf{P_b}$			
	S_1	S_3	S_4	S_2	S_2	S_1	S_4	S_3
UIs		S_1 (UI)		S_4 (UI)		S_2 (UI)		S_4 (UI)
NASH	L		L		L		L	
GMR	N	Mo	N		N	S	N	
SMR	N	S	N		N	S	N	
SEQ	N	Mo	N		N	S	N	

In addition, notice that Table 7.14 shows s_3 to be GMR, SMR and SEQ stable (with stability strength that range from small to moderate) for P_a. This means that it does not matter if P_a believes P_b to be a rational or irrational player (acts according to his preferences or against them, respectively), P_a has s_3 stable for the long run. He should not move out of it by taking a UI to s_1 , the more preferred state to him. Why? because the GMR, SMR and especially the SEQ stability of s_3 to him means that P_b will try to take advantage of his UI move to s_1 by sanctioning it (this is an SM as well as SMUI move for P_b at the same time). In other words, player P_a is advised to demonstrate some selfishness if he reaches s_3 knowing that the other player does not cooperation as he does.

Similarly, Table 7.14 shows s_1 to be GMR, SMR and SEQ stable (with small stability strength) for P_b. This is because if P_b takes advantage of the cooperation P_a is demonstrating, by staying at s_1, and decide to active a selfish UI he has to s_2, P_a will definitely retaliate moving the game to the all-loose state s_4. In other words, P_b is well advised to show some restrain and not to act selfishly when the game reach the all-cooperate s_1.

Table 7.15: Prisoner's Dilemma - Tit For Tat by P_a and Standard by P_b: Equilibrium States

	S_1	S_2	S_3	S_4
NASH EQ.				L
GMR EQ.	N			N
SMR EQ.	N			N
SEQ EQ.	N			N

Table 7.15 shows that this game has one NASH equilibrium state, and that is state s_4. Not a good state for both players. It is clear is clear that the selfishness of one of the players will definitely doom the game to go that way, in spite of the other player's appreciation and well intentions to cooperate. The table also, shows a weak equilibrium, under GMR, SMR and SEQ solution concepts, at state s_1. This send also a clear message that cooperation is *stable* to both players if and only if both of them think strategically and have a longer foresight when analyzing the game (lookahead to see what the other player have as sanctions to your moves). But notice that s_1 is a very weak equilibrium under GMR, SMR and SEQ, with strength of *None* (0-10% in a numerical none-fuzzy scale). The weakness of this stability is because of the selfishness that one player demonstrates, and the cautiousness the other will show in response. Indeed, a very insightful experiment that is consistent with both Axelrod (1980a,b) findings and his tournaments results.

7.5.3 The Game of Chicken

7.5.3.1 Background

Same like the Prisoner's Dilemma, the *Game of Chicken* poses a paradox of rationality. The game of Chicken is a game that has been studied extensively in the game theory literature. Rapoport and Guyer (1966) in their survey of 2x2 games included Chicken as game number 66. The name "Chicken" has its origins in a daredevil duel in which two drivers drive towards each other on a collision course. One driver must swerve, or both may die in the crash. But, if one driver swerves and the other does not, the one who swerved will be called a "Chicken" (means a "coward").

The game of Chicken is initially used in the literature to represent generally, or symbolically, conflicts where two parties engage in a showdown where they have nothing to gain, and only pride stops them from backing down. Then, the game is developed in game theory literature to represent a mathematical phenomenon with the preference structure in Chicken is the main item of interest; the story of the car drivers driving towards each other merely adds colour and intuition to the analysis. But later on, the game of Chicken started to be taken more seriously to represent real-life problems, especially after Bertrand Russell famously used the game to model geopolitical conflicts, namely Russell (1959) saw in the Game of Chicken a metaphor for nuclear stalemate. The game now is used to represent, with simplification, a class of problems that involves the introduction of an element of uncontrollable risk. Even if all players act rationally in the face of risk, uncontrollable events can still trigger the catastrophic outcome.

Table 7.16: Game of Chicken: Normal Form Model

		Player P_b	
		Don't Swerve	*Swerve*
Player P_a	*Don't Swerve*	$4^{*}, 4^{**}$	$1, 3$
	Swerve	$3, 1$	$2, 2$

* Ordinal Preference for Player P_a
** Ordinal Preference for Player P_b
*,** 1-4 Preference: highest to lowest

Again here, we will not discuss further the history or applications of the game of chicken. We will present how the game which is typically modelled within the game theory literature in its normal formal, as shown in Table 7.16, can be modelled and analyzed using the Constrained Rationality framework. The new model and analysis of the game will provide more information and insight into the dynamics of the game and the stability of its states to its players.

7.5.3.2 Players' Strategic Goals and Alternatives

The Game of Chicken, as the Prisoner's Dilemma, in its structure does not present the players with more than one goal for each: achieve "Best Deal" for the player. The best-deal is the shows the player as the "brave" one, hopefully while still alive, knowing after-the-fact what the other player chose as a strategy. The game has many assumptions inherent in its structure; and ultimately it is not a game of many-goals to reason about, instead it is more a game of a challenging structure. Saying so, still here too as in the Prisoner's Dilemma, modelling the game using the Constrained Rationality framework will show that the win-win scenario will only happen with both player cooperating with each other, i.e. both Swerve. All other scenarios will produce less than full satisfactory achievement to both players.

We provide in this section a very rational model for the game of Chicken. This model will not involve an emotional valences or factors, even though the game by its nature as a daredevil automobile duel has a very visible emotional dimension to it. After all, the players are fighting for their pride, to show they are "men" and to boost their status among their peers. But, our choice to model the game as a complete rational game, with no emotional component to it, come because the game is mostly used to symbolize real-life similar conflict where the players involved are rational individuals, or entities with collective institutional rationality. The importance of the game does not come from its roots as a daredevil automobile duel, but rather from the fact that it shows, with simplification, a way to model geopolitical conflicts. For example, Russell (1959) saw in the game of Chicken a metaphors for nuclear stalemate.

In a game of Chicken among two completely rational players, we model the GCM for each with one strategic goal: Reach the Best Outcome. This "Best Outcome" goal is reduced, using reduction goal-to-goal relationships, to three goals that explain what it means to reach a best outcome for the player: 1) come out of the game Alive; 2) come out Not a Chicken; and 3) Do Not Swerve. The following figures, Figures 7.19a - 7.19c, show the players' GCM models and how they interact

Table 7.17: Game of Chicken: Players' Alternatives/Options

The Set of Alternatives available to Player P_a (A_{P_a}) :	
$A_{P_a 0}$	*Don't Swerve*
$A_{P_a 1}$	*Swerve*
The Set of Alternatives available to Player P_b (A_{P_b}) :	
$A_{P_b 0}$	*Don't Swerve*
$A_{P_b 1}$	*Swerve*

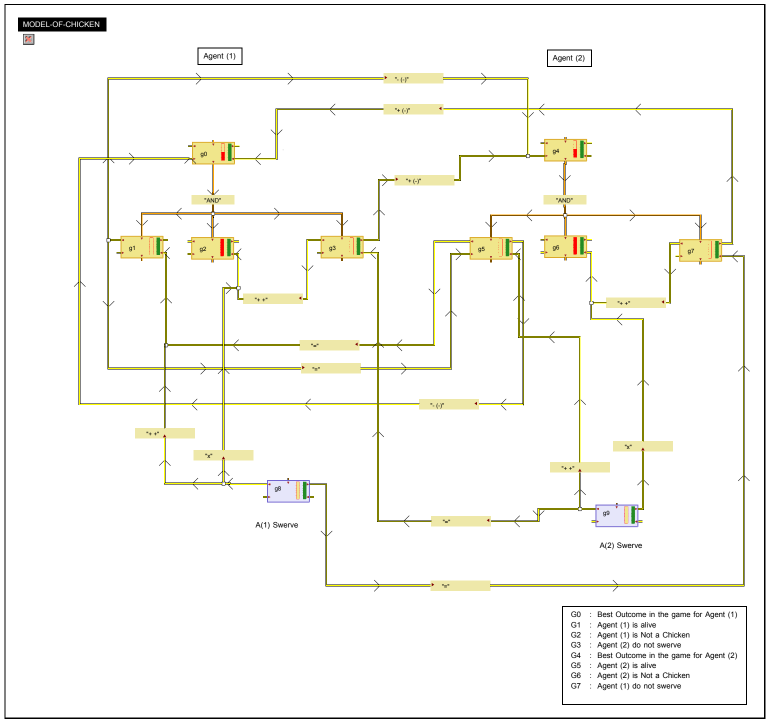
with each other for all scenarios possible, as provided by the definition of the Game of Chicken.

By design, the standard structure of the game of Chicken gives each of the player two options only: Swerve or Do Not Swerve. These options are shown in Table 7.17. Each one of these alternatives contributes differently to the player's achievement of his ultimate strategic goal of "reaching the Best Outcome". Notice that the options selected by each of the players will also contribute positively or negatively, depending on the selected option, to the other player's achievement of his ultimate strategic goal. Hence the lateral relationships across the players' GCMs shown in Figures 7.19a - 7.19c.

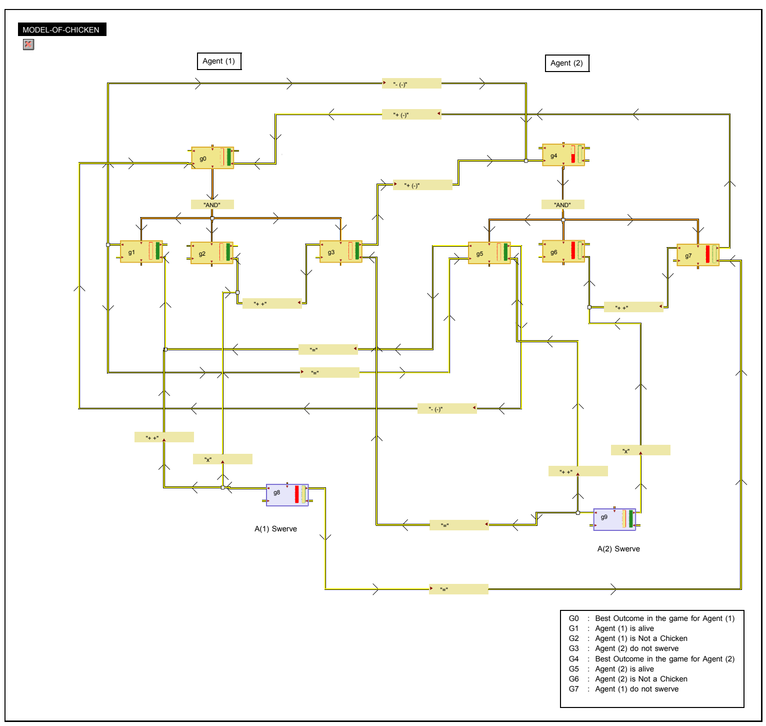
The Constrained Rationality model of the players' GCMs for the game of Chicken offer a better understanding of the game structure and challenges, because the models capture formally the game assumptions and structure beyond what the game theory, or related, forms whether tabular or graphical provide. All what the game theory, and related, forms provide is a game of preferences, such as the one shown in Table 7.16. No explanation is given to why the prisoners' preferences are as the way they are presented. Furthermore, such models do not show what will happen to the game structure and player's ultimate goals when some tweaks happen to the game's assumptions (testing what-ifs).

In the players' GCMs model everything is captured within the models, even though the game is not a typical game of many-conflicting-goals, such as the ones happen in most of real-life conflicts. Knowing that the GCMs model reveals the effect of the players' choices in the strategy/alternative to adopt on their strategic goals, make the model a great tool to elicit the players' preferences for the game. No need to assume the players' preferences, or even take their word. The analyst, now, can verify the players' preferences.

Figures 7.19a - 7.19c show that the win-win scenario happens only when both

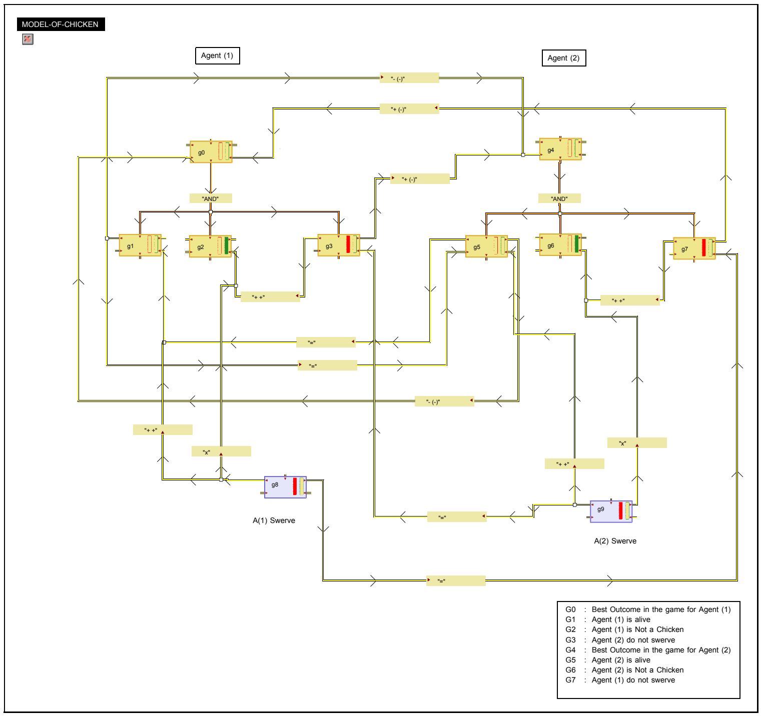


(a) Game of Chicken with both Players Swerving



(b) Game of Chicken with Player (1) Not Swerving while Player (2) Swerves

Figure 7.19: Game of Chicken: GCM models for the players, and the how the players' decisions to swerve or not-to-swerve affect their respective ultimate strategic goal in the game



(c) Chicken Game with Both Players Not Swerving

Figure 7.19: Game of Chicken: GCM models for the players, and the how the players' decisions to swerve or not-to-swerve affect their respective ultimate strategic goal in the game

players cooperate with each other, i.e. both Swerve. All other scenarios will produce less than full satisfactory achievement to both players. We will use these modes to elicit the players' preferences in the game, and use these preferences to analyze the stability of the game. But, let us first identify the game' s states.

Table 7.18: Game of Chicken: Defining the Game States

The Set of All States $\mathcal{S}$ for the Game of Chicken:		
State	**Player P_a Options**	**Player P_b Options**
S_1	Don't Swerve $A_{P_a 0}$	Don't Swerve $A_{P_b 0}$
S_2	Don't Swerve $A_{P_a 0}$	Swerve $A_{P_b 1}$
S_3	Swerve $A_{P_a 1}$	Don't Swerve $A_{P_b 0}$
S_4	Swerve $A_{P_a 1}$	Swerve $A_{P_b 1}$

7.5.3.3 Game's States

Each of the players is given two alternatives to choose one from: Swerve or Not-to-Swerve. The game in its structure, and as proposed by the literature, has 4 outcomes/states. These states are shown in Table 7.18.

7.5.3.4 Players' Preferences over States of the Game

Using the GCM models shown above to represent the players in the game of Chicken, the analyst will be able to elicit the cardinal/weighted and ordinal preferences for the players over the four states the game has. These preferences are shown in Figure 7.20. Then, the preferences' strengths are calculated and shown in Table 7.19.

Rationality Factor = 1.0 (for Pa & Pb)
Emotionality Factor = 0.0 (for Pa & Pb)

		Game of Chicken	
		Pa	Pb
DMs' Strategic Goals	SGs:	SG_{Pa}	SG_{Pb}
		$SG_{Pa\,0}$	$SG_{Pb\,0}$
Strategic Importance	SImprt(SG_k)	F	F
Emotional Valence	EVInc(SGk)	IN	IN
State S_1 Don't Swerve, Don't Swerve	Achv(SG_k)	N	N
	Prvn(SG_k)	F	F
	FAchv(SG_k)	F	F
	TWFAchv(SG_k,DM)	−1.00	−1.00
{ Achv($A_{Pa\,0}$)=F,	WP(S_1, DM)	0.000	0.000
Achv($A_{Pb\,0}$)=F }	OP(S_1, DM)	4 (Worst)	4 (Worst)
State S_2 Don't Swerve, Swerve	Achv(SG_k)	F	N
	Prvn(SG_k)	N	B
	FAchv(SG_k)	F	B
	TWFAchv(SG_k,DM)	1.00	−0.80
{ Achv($A_{Pa\,0}$)=F,	WP(S_2, DM)	1.000	0.100
Achv($A_{Pb\,1}$)=F }	OP(S_2, DM)	1 (Best)	3
State S_3 Swerve, Don't Swerve	Achv(SG_k)	N	F
	Prvn(SG_k)	B	N
	FAchv(SG_k)	−B	F
	TWFAchv(SG_k,DM)	−0.80	1.00
{ Achv($A_{Pa\,1}$)=F,	WP(S_3, DM)	0.100	1.000
Achv($A_{Pb\,0}$)=F }	OP(S_3, DM)	3	1 (Best)
State S_4 Swerve, Swerve	Achv(SG_k)	N	N
	Prvn(SG_k)	L	L
	FAchv(SG_k)	−L	−L
	TWFAchv(SG_k,DM)	−0.20	−0.20
{ Achv($A_{Pa\,1}$)=F,	WP(S_4, DM)	0.400	0.400
Achv($A_{Pb\,1}$)=F }	OP(S_4, DM)	2	2

Figure 7.20: Game of Chicken: Players' Ordinal and Normalized Weighted Preferences

Table 7.19: Game of Chicken: Players' Preferences

P_a **Preferences** (*Most* to *Least* Preferred)				
Pref(P_a)	S_2	S_4	S_3	S_1
WP	1.00	0.40	0.10	0.00
P_a **Preferences' Strengths**				
$\succ^{LPS}_{P_a,t}$	S_2	S_4	S_3	S_1
S_2	N	M	F	F
S_4	-M	N	S	S
S_3	-F	-S	N	L
S_1	-F	-S	-L	N

P_b **Preferences** (*Most* to *Least* Preferred)				
Pref(P_b)	S_3	S_4	S_2	S_1
WP	1.00	0.40	0.10	0.00
P_b **Preferences' Strengths**				
$\succ^{LPS}_{P_b,t}$	S_3	S_4	S_2	S_1
S_3	N	M	F	F
S_4	-M	N	S	S
S_2	-F	-S	N	L
S_1	-F	-S	-L	N

7.5.3.5 Players' Moves over States of the Game

We said above that we will model the game of Chicken as an iterative game, with the game's states represent the outcome of an iteration/instance of the game. Figure 7.21 presents the UMs of the players in the game. Notice the similarity between the players' UMs of the Chicken game and the ones shown earlier in Figure **??** for the players in the IPD game. In fact, this similarity is not limited to these two types of games, but shared across all 2x2 games. But, in this game, and as Figure 7.21 shows, the players will not have UMs our of state s_1. This state represent full destruction for both players, or the doom scenario. So, once this state is reached the players will not be able to get out of it.

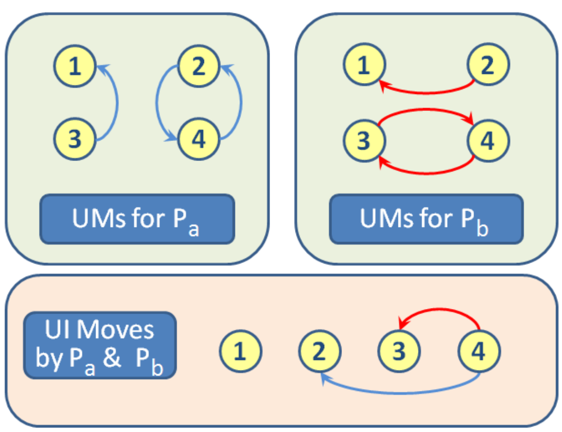


Figure 7.21: The Game of Chicken: The Unilateral Moves and the Unilateral Improvement Moves by the players

The game of Chicken has unique UIs for the players, not similar to the ones IPD has. Recall that UIs, by Definition 6.2.2, are UMs but the destination state is more preferred to the player than the start/source state. Knowing what UMs the players have in this game and their preferences structure as presented above, and using Algorithm 6.1 the players' UIs are identified. They are as shown in Figure 7.21.

7.5.3.6 Stability Analysis and Analysis Results of the Game of Chicken

Table 7.20 presents the stability analysis of the game of Chicken's four states, for both players, under NASh, GMR, SMR and SEQ solution concepts. Table 7.21 shows the equilibrium states for the game under the four solution concepts.

Table 7.20: Game of Chicken: Stability Analysis

	$\mathbf{P_a}$				$\mathbf{P_b}$			
	S_2	S_4	S_3	S_1	S_3	S_4	S_2	S_1
UIs		S_2 (UI)				S_3 (UI)		
NASH	M		Ex	Ex	M		Ex	Ex
GMR	N	S	N	N	N	S	N	N
SMR	N	S	N	N	N	S	N	N
SEQ	N		N	N	N		N	N

Table 7.21: Game of Chicken: Equilibrium States

	S_1	S_2	S_3	S_4
NASH EQ.	Ex	M	M	
GMR EQ.	N	N	N	S
SMR EQ.	N	N	N	S
SEQ EQ.	N	N	N	

The analysis shows that all the game of Chicken's states are stable for the players, individually and collectively, in one stability solution concept or another. But, the differences are in the strength of the stabilities, as well as the in the types of solution concept under which the stabilities are formed. The tables provide much insight on the game dynamics.

First, Table 7.20 shows that the doom-scenario state, the nobody-swerve state s_1, is NASH stable for both players with *Extreme* strength level. This state has no UIs for the players to move out of it, hence its is NASH stable. But for this state, there is no option. The destruction that happens to both players because of reaching this state, leaves the players unable to take another round/iteration of the game. Also, the fact that the strength of the NASH stability for the state is at the extreme level, and that it has no UIs leading to it by any of the players, shows that no rational player will go willingly to this state.

Second, if a previous iteration/instance of the game ended with both players swerving, i.e. ended at state s_4, then each player has an incentive to use his UI out of s_4. P_a can go to state s_2 by not swerving in this iteration, assuming that P_b will also "chicken" out this time too. Similarly, P_b can use his UI from s_4 to state s_2 by not swerving in this iteration, assuming that P_a will "chicken" out this time too. This makes s_4 not a NASH stable for both players. Notice that the states which the players have just moved in are "much" preferred that s_4 they left, and the players have no UIs out of them, making these states to be NASH stable for the players at "much" strength level.

Third, by the time state s_2 or state s_3 is reached in a previous iteration of the game, the players now have their positions revealed. Who is the "Chicken" is known now. If the previous iteration ended with state s_2, then P_b was the "Chicken" because he swerved. P_a does not have a UI move from this state, and would like to stay at s_2, his most preferred state according to his preference vector shown in Table 7.19. For P_b, if it happened that the previous iteration of the game ended with state s_3, then P_a was the "Chicken" and P_b has no incentive/UI to move out from his most preferred state, s_3.

But states s_2 and s_3 pose a real challenge to the other player who ended up the "Chicken" at them. For example, if the previous iteration of the game ended with s_2, then P_b was declared the "Chicken". The fact that P_a now knows that P_b swerved in the previous iteration enforces his feeling of stability at s_2. So, P_a will continue to not-swerve. This is a serious problem to P_b. If P_b decides to not-swerve this time, the game will end with state s_1 which is disastrous for both players. Recall that, for P_b, state s_1 is less preferred than state s_2. This means that P_b has no UI of s_2 and therefore it is NASH stable for him. Any move out of s_2 is not rational for P_b. In other words, once a player is found to be the "Chicken" in an iteration, this player can not change this status without the serious risk of ending up at the doom-scenario state, state s_1.

Finally, Table 7.21 show that state s_4, where both players end up swerving, is the only state with stability that is reached by having a higher foresight of looking ahead beyond the current state one is at. s_4 is found to be GMR and SMR stable for both players; and by avoiding risks. A player at this state will fear the sanction the other player will impose on any UI he has out of s_4. The SMR stability adds to this fear, the fact that recovery from this sanction will not be possible. So, the player is better off staying at s_4 and not move out. Notice that the sanctions which the players have to each other UIs out of s_4 are not UI moves themselves. So, if a player wants to avoid risks only if they are rational risks, i.e. if the other player is motivated by improving his respective position not by his desire to hurt others, then s_4 is not stable. Hence, s_4 is not SEQ stable for each of the players.

In summary, Table 7.21 shows that if the players believe they should avoid risks completely, they should stay at s_4 by not playing this game, or declare/find everybody to be a “Chicken”. Otherwise, if the players are of the type that ignore risks, believe that others will never act to hurt themselves just to hurt others, or have a foresight of zero steps ahead, then by all means they can rely on the NASH stability of the other states. But, they have to be cautious because they could end up with either the doom-scenario state s_1, or a state where one of them is found to be the “Chicken” and there is no recovery is possible.

7.6 Summary

The chapter started by comparing the four solution concepts introduced in the previous chapter, NASH, GMR, SMR and SEQ, based on common and important practical characteristics, or properties, of these concepts. The chapter, then, presented interesting theoretical relationships among the solution concepts, and among the strength sets of these solution concepts. Finally, the chapter looked at how the knowledge about how the solution concepts differ, and the interrelationships among them, can be very informative when analyzing conflicts and how it helps shed additional insight on the conflicts and their analysis. This was done by modelling and analyzing additional two non-cooperative conflicts: the Prisoner’s Dilemma and the game of Chicken. Both games were modelled and analyzed, with special emphasis given to how understanding the differences and interrelationships among the stability solution concepts brought additional insight into conflict analysis.

Chapter 8

Cooperative Strategic Conflicts: Analysis and Stability Solution Concepts

8.1 Introduction

This chapter discusses the analysis of cooperative multi-agent games, as per the Constrained rationality framework The decision makers of this type of games are able to make move types that are unilateral non-cooperative moves, as well as cooperative one-step moves. In other words, the decision makers in these games not only are able to take individually the traditional unilateral non-cooperative moves, discussed in Chapter 6, but also they are able to take cooperative moves as groups. These cooperative moves are unique to the cooperative games.

But this chapter will not deal with all cooperative games. Some of these games, namely the cooperative games with coalitions have wider range of cooperative moves for the decision makers to select from. Coalitions move as a group with collective objectives and power. They can take few hits, i.e. some of the players might get disadvantaged for short periods of time. All for the good of the coalition as a whole. Coalitions can stage multi-step moves that include individual players' moves and cooperative players' moves, but all players must be members of the coalition.

On the other hand, in the games discussed in this chapter, players are not part of coalitions. They do not have aligned or even partially aligned objectives and powers. They cooperate in simple one-step moves, such as decide to sign a binding agreement, or settle a law suit outside the court, etc. The players here do not have shared goals and common enemies. What motivates them to take a cooperative move is the common benefit that they will get for moving together in the same direction for one step ahead to a specific state of the game. Something they will not be able to do unilaterally. Therefore, no multi-step group moves will be considered in this chapter. Such moves will be covered in cooperative games with coalitions, which will be discussed in the following chapter.

We will start by looking at the type of moves the players of cooperative games, without coalitions, are allowed to make, and are important to the stability analysis concepts. Then, we will define for these games the same four different stability and equilibrium solution concepts which we defined for the non-cooperative games in Chapter 6 . These concepts will guide the stability analysis of each of the games' states, for each of the games' players. Next, we will define the strength of the stability under such solution concepts, and propose a set of algorithms to help identify the strength level of each of these stabilities.

We will finish with a case study in which we apply the concepts proposed in this chapter. In this case study, we analyze thoroughly the Elmira Groundwater Contamination Conflict, a 1989 environmental policy conflict between the provincial government of Ontario and Uniroyal Chemical Ltd. We start by giving a brief background on the conflict and the players. We model the players goals, constraints and alternatives; analyze their GCMs; identify the conflict's states; elicit the players' cardinal and ordinal preferences over these states; and then identify the players unilateral moves among these states. The stabilities of the conflict's states will be analyzed under the four stability solution concepts, and the strength of these stabilities will be identified. We will look at the over all equilibrium states for the conflict; and how the conflict could have evolved over time under different scenarios. We conclude the case study by showing how our analysis results compares to what historically happened in the conflict, and to what others offered as models and analysis to the conflict, after the fact.

In terms of the notation used in this chapter it will be the same as the one used in Chapter 6, included here as a reminder. Let the set of all the game states be given as $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where $|\mathcal{S}| = m$ the total number of states in the game, and states are defined as discussed earlier. And let $\mathcal{S}^{a,b,\ldots}_{D,t} \subseteq \mathcal{S}$ where $\mathcal{S}^{a,b,\ldots}_{D,t}$ represents a subset of $\mathcal{S}$'s states which has common characteristics described in the subset's notation as $a, b, \ldots$ as been perceived by decision maker D and at time t.

The set of decision makers in the game is given as $\mathcal{DM} = \{DM_1, DM_2, \ldots, DM_n\}$, where $|\mathcal{DM}| = n$ the total number of decision makers, the involved agents/players, in the game.

Also as a reminder, we use the terms game and conflict interchangeably to mean the same thing: a multi-agent strategic conflict. Also, the terms agent, player and decision maker will be used interchangeably to mean the same thing: an autonomous independent agent, in the strategic conflict, who is capable of perceiving the world around, holding beliefs, justifying beliefs, holding knowledge, representing knowledge, extracting new knowledge, reasoning about held knowledge, and acting independently.

8.2 Types of Decision Makers' Moves

Decision makers in cooperative games can have either individual unilateral moves, or cooperative one-step moves. First, we define the unilateral moves individual players can have. These moves are similar to the ones individual players use in non-cooperative games. Second, we define cooperative one-step moves which a group of decision makers can have in cooperative, without coalitions, games. Lastly, we define the type of sanction moves that players can do, as individuals or groups, to block certain other players from benefiting from any unilateral or cooperative improvement moves they have. Understanding these types of players' moves is essential to define the stability solution concepts which will be used to analyze the stability of cooperative games' states for the games' players.

As a reminder, we discussed in Chapter 5, Section 5.5.1, and in Chapter 6, Section 6.2.1, that we followed the steps of Fraser and Hipel (1984) and Fang et al. (1993) in their definitions for UI, UM and SM moves for individual agents. But, we defined all these type of moves to be within the context, terminology and notation of the Constrained Rationality framework. These individual agent's moves are the same as the ones we defined in Chapter 6 for non-cooperative games, included here for completeness and coherence of addressing the needs of modelling and analysis of cooperative games, without coalitions, in this chapter.

It is also worth including a reminder here that the cooperative moves, defined in this chapter for cooperative games without coalitions, are different from the cooperative moves defined by Kilgour et al. (2001) and Inohara and Hipel (2008b,a) for GMCR. We discussed in Chapter 5, Section 5.5.1, how these GMCR's cooperative moves, or coalition moves as been called in the cited work, are limited in their scope and in their applications to real-life conflicts. We mentioned there how the Constrained Rationality provide broader, more advanced and practical cooperative moves that reflect the needs of complex real-life multi-agent conflicts. In Constrained Rationality, the cooperation among agents within a conflict could happen between agents that are not part of a coalition. Some of the Constrained Rationality's cooperative moves will be discussed in this chapter, namely the one-step cooperative moves, while the multi-step coalition ones will be added in Chapter 9 when we discuss cooperative games with coalitions.

8.2.1 Types of Non-Cooperative Moves by Individual DMs

In this subsection, we define the following important types of movements that an individual decision maker, alone and non-cooperatively, can make in the game.

Definition 8.2.1 (Unilateral Move (UM)): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and state $s \in \mathcal{S}$ of the game: a move to state $s_{um} \in \mathcal{S}$ is considered a Unilateral Move (UM) for DM_i at time t from state s, denoted as $s_{um} \in \mathcal{S}^{UM}_{DM_i,t}(s)$, iff DM_i can move unilaterally from state s to state s_{um} in one move, reaching s_{um} at time $t+1$.*

Definition 8.2.2 (Unilateral Improvement (UI)): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and state $s \in \mathcal{S}$ of the game: a move to state $s_{ui} \in \mathcal{S}$ is considered a Unilateral Improvement (UI) for DM_i at time t from state s, denoted as $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$, iff 1) $s_{ui} \in \mathcal{S}^{UM}_{DM_i,t}(s)$; and 2) $s_{ui} \succ^{L_{PS}}_{DM_i t} s : L_{PS} > N$, i.e. when $PrefStrength(s_{ui}, s, DM_i, t) > None$.*

Evidently, $\mathcal{S}^{UI}_{DM_i,t}(s) = \mathcal{S}^{UM,>N}_{DM_i,t}(s)$; and $\mathcal{S}^{UI}_{DM_i,t}(s) \subseteq \mathcal{S}^{UM}_{DM_i,t}(s) \subseteq \mathcal{S}$.

One important step of analyzing a game is to generate the the UIs that DMs will have from each state of the game. Given a *Game-Structure* for a cooperative game, without coalitions, that resembles the one proposed for the non-cooperative games and discussed in Chapter 6, we use Algorithm 6.1 to generate the UM and UI sets for all DMs in the game. Notice that Algorithm 6.1 is the algorithm proposed previously in Chapter 6 to be used for generating the UM and UI sets for the players of non-cooperative games. The same algorithm is used for these two different type of games, non-cooperative and cooperative-without-coalitions, because UM and UI sets exist for individual players regardless of the players' abilities to have cooperative moves.

Now, let each game has a *Game Configuration Structure*, referred to it as a *Game-Structure* in Chapter 6. This data structure provides essential initial information about the game and its players, all organized and in a computerized DSS system is written in a file structure. As a reminder, a Game-Structure will describe the game at a specific point of time t, as perceived and known by the focal decision maker whom the game is modelled based on his knowledge of it. Any updates or changes to what is known about the game by the focal decision maker should initiate a generation of a new Game-Structure to reflect the changes; and a new analysis of the updated game, treating the structure as a new game.

The Game-Structure for a cooperative game must have the same information required for non-cooperative games, and listed in Section 6.2.1: the set of the game's states, $\mathcal{S}$; the set of the game's DMs, $\mathcal{DM}$; $\mathcal{S}^{UM}_{DM_i,t}(s)$ for each DM in the game; and $WP(s, DM_i, t)$ for every $DM_i \in \mathcal{DM}$ and for every $s \in \mathcal{S}$. *Game-Structures* for cooperative games, with no coalitions, differ from the one for non-cooperative games in one aspect. A Game-Structure for cooperative games, without coalitions, has additional information in its structure about the cooperative moves the players have in such game, as we will see next when we discuss cooperative moves for players. Something that non-cooperative games' Game-Structures do not have.

8.2.2 Types of Cooperative Moves

The are two types of one-step movements that a set of decision makers, cooperatively, can make in the game. We assume here no coalition or alliance exist among the decision makers of this set. They are only motivated to do such moves because they all have to agree and cooperate to make the move (as a condition to reach the destination state)Most likely, these cooperative moves happen because all cooperating decision makers benefit from the move.

It is important to note here that all types of moves we consider in this section are one-step moves, i.e. no moves consistent of consecutive series of moves are considered. We consider this type of moves as coalition moves because it demands strategic coordinating and grouping of the member decision makers' moves to establish the final effect expected. In this section, we assume all decision makers are acting individually, each for his own benefit, but might "in some circumstances" cooperate to reach a state that they will not individually be able to reach, such as reaching a formal agreement to end the conflict.

Definition 8.2.3 (**Cooperative Move (CM)**): *For a Group of Decision Makers $DM_g \subseteq \mathcal{DM}$, where $|DM_g| \geq 2$, at time t and at state $s \in \mathcal{S}$ of the game, the move by DM_g from s in one step to state $s_{cm} \in \mathcal{S}$ is considered a Cooperative Move (CM) for DM_g from s at time t, denoted as $s_{cm} \in \mathcal{S}^{CM}_{DM_g,t}(s)$ iff DM_g cannot make the move unless each and every member of DM_g agrees to the move and cooperates by doing what is necessary to reach the state s_{cm}, from the starting state s.*

As per the definition, a Cooperative Move is a movement from one state of the game to another that requires a group of decision makers (more than two) to make the move. No single decision maker can do the move on his own. Similarly no subset of a group of decision makers can do the move on their own, every single decision maker in the group is needed to participate in the move to make it happen. Additionally, it is evident that $\mathcal{S}^{CM}_{DM_g,t}(s) \subseteq \mathcal{S}$.

Definition 8.2.4 (**Cooperative Improvement (CI)**): *For a Group of Decision Makers $DM_g \subseteq \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, the move by DM_g from s in one step to state $s_{ci} \in \mathcal{S}$ is considered a Cooperative Improvement (CI) for DM_g from s at time t, denoted as $s_{ci} \in \mathcal{S}^{CI}_{DM_g,t}(s)$ iff 1) $s_{ci} \in \mathcal{S}^{CM}_{DM_g,t}(s)$; and 2) $\forall\ DM_i \in DM_g\ \ s_{ci} \succ^{L_{PS}}_{DM_i,t} s\ :\ L_{PS} > N$, i.e. when $PrefStrength(s_{ci}, s, DM_i, t) > None$ for every $DM_i \in DM_g$.*

From the definition, it is evident that $\mathcal{S}^{CI}_{DM_g,t}(s) = \mathcal{S}^{CM,>N}_{DM_g,t}(s) \subseteq \mathcal{S}^{CM}_{DM_g,t}(s) \subseteq \mathcal{S}$.

One important step of analyzing a cooperative game, with no coalitions, is to generate the CIs that DMs will have from each state of the game. The Game-Structure for a cooperative game, without coalition (and even with coalitions as we will see in the next chapter when we discuss coalition analysis), must include information about the CMs that all the DMs have in the game from each of the game's states. It must include $\mathcal{S}^{CM}_{DM_i,t}(s)$, for every $DM_i \in \mathcal{DM}$ and for every $s \in \mathcal{S}$, given as a set of graphs describing the CMs that DMs have from each state the game has (one graph per DM in the game, with the game's states represented as the graph's nodes and the CMs are represented as its arcs (each of these arcs annotated with the names of the other cooperating players). Given a Game-Structure, for a cooperative game, without coalitions, that has all this required information, we use

Algorithm 8.1 Generating the CM and CI Sets for Cooperating DMs in a Game

void Generate_DMs_CM_and_CI_Sets (Game-Structure)

// Game-Structure file starts with empty CM and CI sets for DMs.
// Only, $\mathcal{S}$, $\mathcal{DM}$, and the CM graph for DMs are given.
// A CM graph for DMs, has $\mathcal{S}$'s states as vertices/nodes of the graph,
// while the directed arcs of the graph represent DMs' CMs in the game.
// In addition, each directed arc is annotated with a list of the names
// of the DMs that must cooperate to make the move to the destination state.

// make a list of the groups of Decision Makers who combined have CM moves.
// This is done by scanning all the CM arcs in the CM graph generating a
// a group for each new set of DMs that have any CM.

for all $DM_g \subseteq \mathcal{DM}$ **do**
    // Generate DM_g's CM Sets (one for each of the game's states), and
    // for each state find DM_g's CI Set. All these CM and CI sets
    // will be initially empty. If not, empty them [not included here].
    // And, by the end, some of these CM/CI sets will be empty sets.

    for all $s \in \mathcal{S}$ **do**
        for all s_{cm} destination state of each arc DM_g has out of s **do**
            $\mathcal{S}^{CM}_{DM_g,t}(s) = \mathcal{S}^{CM}_{DM_g,t}(s) \cup \{s_{cm}\}$
            // moving to s_{cm} from s is also considered a CI for DM_g
            // if and only if $\forall DM_i \in DM_g \;\; s_{cm} \succ^{L_{PS}}_{DM_i,t} s : L_{PS} > N$
            if $\forall DM_i \in DM_g \;\; PrefStrngth(s_{cm}, s, DM_i, t) > None$ **then**
                $\mathcal{S}^{CI}_{DM_g,t}(s) = \mathcal{S}^{CI}_{DM_g,t}(s) \cup \{s_{cm}\}$
            end if
        end for
    end for
end for
Add all generated CM and CI sets for each group, for each state, to the Game- Structure file.
return

Algorithm 8.1 to generate the CM and CI sets for all DMs in the game.

8.2.3 Types of Sanction Moves

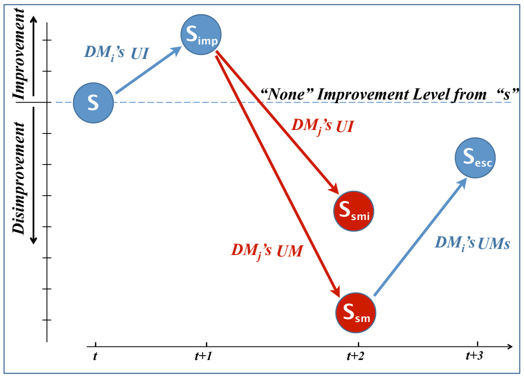
We will expand here the definitions for a sanction move (SM) and an inescapable sanction move (ISM), to include sanction moves that are committed not only by individual decision makers, but also by a group of decision makers acting cooperatively. Additionally, we will include in the new definitions sanction moves that are intended to sanction a group of decision makers's CI moves, not just individual's UI moves.

Definition 8.2.5 (Sanction Move (SM)): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, a UI by DM_i to state $s_{imp} \in \mathcal{S}^{UI}_{DM_i,t}(s)$, or a CI by a group $DM_g : DM_i \in DM_g$ to state $s_{imp} \in \mathcal{S}^{CI}_{DM_g,t}(s)$, is said to have against it a Sanction Move (SM) at time t+1 to state $s_{sm} \in \mathcal{S}$ iff $[\,\exists\,((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{imp}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \vee [\,\exists\,((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_{sm} \in \mathcal{S}^{CM}_{DM_p,t+1}(s_{imp}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,]$.*

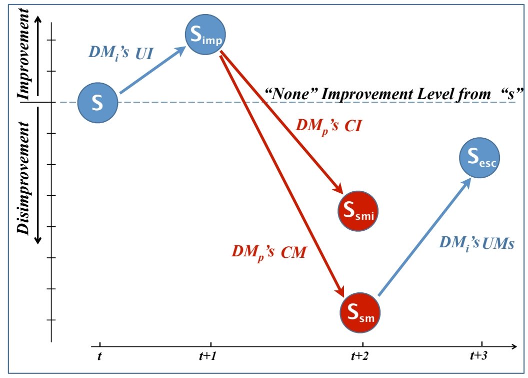
As per the definition, a move to s_{ui} by DM_i is said to be *sanctioned/able* by DM_j's move to s_{sm}, and DM_j's move to s_{sm} is called a Sanction Move (SM) against DM_i's UI to s_{ui}. This definition, adds to the previous definition given in Chapter 6 for SM , the possibility that the SM could be a CM move by a group of DMs instead of being only a UM move by an individual DM.

Figure 8.1 shows the four possible sanction move types available to players of cooperative games, without coalitions, based on the definition above. Figure 8.1a shows a UI by a DM faced with an SM that is a UM by another DM, whilst Figure 8.1b shows the same UI faced with an SM that is a CM by a group of DMs acting cooperatively. On the other hand, Figure 8.1c shows a CI by a cooperative DMs faced with an SM that is a UM by another DM, whilst Figure 8.1d shows the same CI faced with an SM that is a CM by another group of DMs acting cooperatively.

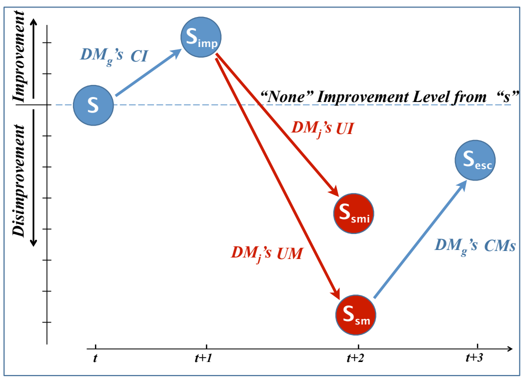
In addition, it is important to notice that the definition of a SM does not assume that the SM to be a UI or a CI move by the DM/s committing the sanction. The definition assumes that the motive of the DM/s committing the sanction for their SM is to hurt DM_i and sanction DM_i's UI move to s_{imp}, or to hurt DM_g and sanction DM_g's CI move to s_{imp}, even if this SM will put the DM/s committing the sanction himself/themselves at a less preferred state. To differentiate between an SM by DM/s which is not required to be a UI or CI, as per the SM definition above, and an SM which is also a UI or a CI move for the committing parties, we will call the second SM type as SMI move (read as Sanction Move and Improvement). These SMIs are shown in the Figures 8.1a - 8.1d as moves from state s_{imp} to state s_{smi} (instead of having the destination be state s_{sm} which cannot be reached by an SM that is also a UI or a CI). As we will see later, this *stricter* type of SM, or SMI - as we decided to call it, is required for some stability solution concepts, such as Sequential Stability (SEQ).



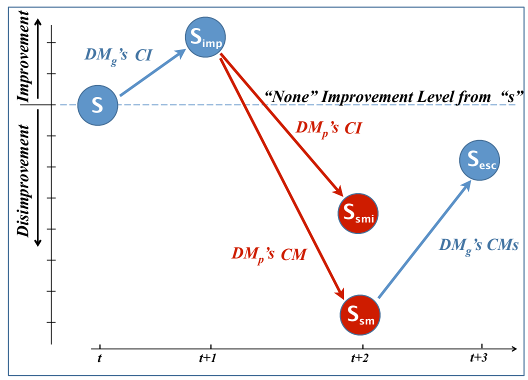
(a) A UI by a DM is faced with an SM/ISM by another individual DM



(b) A UI by a DM is faced with an SM/ISM by a cooperative group of DMs



(c) A CI by a cooperative group of DMs is faced with an SM/ISM by an individual DM



(d) A CI by a cooperative group of DMs is faced with an SM/ISM by another cooperative group of DMs

Figure 8.1: Type of Sanction Moves available to players of cooperative games, without coalitions

Definition 8.2.6 (Inescapable Sanction Move (ISM)): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, a UI by DM_i to state $s_{imp} \in \mathcal{S}^{UI}_{DM_i,t}(s)$, or a CI by a group $DM_g : DM_i \in DM_g$ to state $s_{imp} \in \mathcal{S}^{CI}_{DM_g,t}(s)$, is said to have against it an Inescapable Sanction Move (ISM) at time $t+1$ to state $s_{ism} \in \mathcal{S}$ iff* $[\exists ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_{ism} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{imp}))) : (s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_{esc} \in \mathcal{S}^{UM}_{DM_i,t+2}(s_{ism}) \quad s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))] \vee [\ \exists ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_{ism} \in \mathcal{S}^{CM}_{DM_p,t+1}(s_{imp})$ *and where* DM_p *reaches* s_{ism} *at time* $t+1+k)) : (s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_{esc} \in \mathcal{S}^{UM}_{DM_i,t+1+k}(s_{ism}) \quad s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))]$.

As per the definition, the DM_i's UI move to s_{ui}, from s, is said to have against it *an inescapable sanction move* (ISM) by DM_j's move to s_{ism}, because DM_i has no move away from s_{ism} by which he will be able to mitigate, or lessen, the negative effect DM_j's sanction to s_{ism} lift DM_i in. This definition, adds to the previous definition given in Chapter 6 for ISM , the possibility that the ISM could be a CM move by a group of DMs instead of being only a UM move by an individual DM. These additional cases as shown in Figure 8.1.

8.3 Stability Solution Concepts and Equilibriums for Cooperative Games without Coalitions

The same four stability solution concepts, we discussed in Chapter 6 for non-cooperative games, will be discussed in this chapter for cooperative games, without coalitions. But we will expand these solution concepts to accommodate the new co-operative moves that decision makers in cooperative games, without coalitions, can have. We will group these solution concepts into two classes, as we did in Chapter 6 for non-cooperative games: 1) solution concepts that are extremely individualistic and shortsighted in their definitions, in a way that they do not consider other players countermoves; and 2) solution concepts that tries to include other players' countermoves, therefore these concepts show more foresight.

As a reminder, we said in Chapter 6, Section 6.3, that we followed the steps of Fraser and Hipel (1984) and Fang et al. (1993) in their definitions for the four stability solution concepts for non-cooperative games. But, we defined all these solution concepts, for non-cooperative games, to be within the context, terminology and notation of the Constrained Rationality framework; and using the definitions of the agents' non-cooperative unilateral moves and sanction moves introduced in Chapter 6. In this section, we will extend the definitions of the four stability solution concepts, presented in Chapter 6, to deal with the Constrained Rationality's new additional cooperative moves introduced above for cooperative games without coalitions; and to deal with the changes happened accordingly to the definitions of sanction moves in such games (presented above in this chapter).

It is also worth including here a reminder that the definitions of the stability solution concepts for cooperative games without coalitions, which will follow, are different from the ones presented by Inohara and Hipel (2008b,a) for GMCR and called coalition stability solution concepts. This is due to the different moves that GMCR and Constrained Rationality employ. We discussed in Chapter 5, Section 5.5.1, how the GMCR's cooperative moves, or coalition moves as been called by Inohara and Hipel (2008b,a), are limited in their scope and in their applications to real-life conflicts; and how the Constrained Rationality provide broader and more practical collection of cooperative moves that reflect the needs of complex real-life multi-agent conflicts. We discussed there how the Constrained Rationality allow for cooperation among agents within a conflict could happen between agents that are not part of a coalition. And, because each framework employs different cooperation among agents and different definitions of cooperative moves, the frameworks' definitions of the stability solution concepts, which are based on the definitions of the cooperative moves employed, will definitely be different.

8.3.1 Solution Concepts with No Consideration to Others' Moves

NASH solution concept is the only stability solution concept that does not consider in its definition the moves and countermoves of other players. We expand its definition to include the cooperative moves that players within cooperative games, without coalitions, can have.

Definition 8.3.1 (**Nash Stability (NASH)**): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t, state $s \in \mathcal{S}$ is considered a Nash Stable (NASH) state, denoted as $s \in \mathcal{S}^{NASH}_{DM_i,t}$, iff $[\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset]$.*

As per the definition, state s is considered NASH stable for a decision maker DM_i at time t if and only if s is the best that DM_i, individually or cooperatively, can achieve at time t, given the total states of the game $\mathcal{S}$. Therefore, states that are not NASH stable are unstable states since DM_i can improve his position from any one of them: unilaterally by activating one of his UIs out of these states; or cooperatively by activating one of the CIs that he and other individual DMs have from these states.

8.3.2 Solution Concepts with Consideration to Others' Moves

We will discuss here the same three stability solution concepts we discussed in Chapter 6 for non-cooperative games, General MetaRationality, Symmetric Meta-Rationality and Sequentially Stability. But we will expand the definitions of these solution concepts to include the cooperative moves that players within cooperative games, without coalitions, can have.

Definition 8.3.2 (**General MetaRational (GMR) Stability**): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a General MetaRational (GMR) Stable state, denoted as $s \in \mathcal{S}^{GMR}_{DM_i,t}$, iff*

$$\forall s_1 : \quad ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)))$$
$$[\exists ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)] \vee$$
$$[\exists ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)]$$

The previous GMR stability definition given in Chapter 6 for non-cooperative games, is expanded in this definition to account for the possible CI moves DM_i has from state s at time t. It is also expanded to include not only sanctions that could be imposed by individual decision makers against DM_i's UIs/CIs from s, but also sanctions that could be imposed by groups of decision makers cooperating together to hurt DM_i and put him at a less preferred state than even the original state s from which his UIs/Cis start from.

The GMR stability solution concept assumes that decision maker DM_i believes that other players *surely* would apply, unilaterally or cooperatively, a sanction against any of his UIs/CIs out of s. Therefore, he will not move away from s, and s is a GMR stable state for him.

Definition 8.3.3 (**Symmetric MetaRational (SMR) Stability**): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a SMR Stable state, denoted as $s \in \mathcal{S}^{SMR}_{DM_i,t}$, iff*

$$\forall s_1 : \quad ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)))$$
$$[\exists ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1))) :$$
$$(s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))] \vee$$
$$[\exists ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1)) \textit{ and } s_2 \textit{ is reached at time } t+k+1) :$$
$$(s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+k+1}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))]$$

The previous SMR stability definition given in Chapter 6 for non-cooperative games, is expanded in this definition to account for the possible CI moves DM_i has from state s at time t. It is also expanded to include not only inescapable sanctions that could be imposed by individual decision makers against DM_i's UIs/CIs from s, but also inescapable sanctions that could be imposed by groups of decision makers cooperating together to hurt DM_i and put him at a less preferred state than even the original state s from which his UIs/Cis start from.

The SMR stability solution concept assumes that decision maker DM_i believes that other players *surely* would apply, unilaterally or cooperatively, a sanction against any of his UIs out of s, and this sanction is an *inescapable* one (i.e. an ISM). In other words, DM_i will not be able to benefit from any move away from the state produced by the inescapable sanction move. Therefore, DM_i will not move away from s, and s is SMR stable for him. Notice that SMR takes into consideration one further move in the game (the possible countermove -but not helpful one- by DM_i after the sanction) than what GMR considers. GMR does not consider whether DM_i have countermoves after the sanction to mitigate the sanction's effect.

Definition 8.3.4 (**Sequentially Stability (SEQ)**)**:** *For Decision Maker $DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a Sequentially (SEQ) Stable state, denoted as $s \in \mathcal{S}^{SEQ}_{DM_i,t}$, iff*

$$\begin{aligned}
\forall s_1 : \quad & ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s))) \\
& [\, \exists\, ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \vee \\
& [\, \exists\, ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{CI}_{DM_p,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,]
\end{aligned}$$

The previous SEQ stability definition given in Chapter 6 for non-cooperative games, is expanded in this definition to account for the possible CI moves DM_i has from state s at time t. It is also expanded to include not only the SMI sanctions that could be imposed by individual decision makers against DM_i's UIs/CIs from s, but also SMI sanctions that could be imposed by groups of decision makers cooperating together to hurt DM_i and put him at a less preferred state than even the original state s from which his UIs/Cis start from.

Recall that the SEQ solution concept requires that the imposed sanction against DM_i's UIs/CIs from s to be also a UI, a CI or a G-CI by the party committing the sanction. In other words, SEQ assumes that all decision makers in the game to be rational. Unlike the GMR solution concept, SEQ assumes that no player will move in the game, whether individually or cooperatively, for the sake of hurting others and in the process hurt himself/themselves. All players in the game will only commit themselves to moves that will benefit them. The SEQ stability solution concept also assumes that decision maker DM_i believes that other players *surely* would apply, unilaterally or cooperatively, a sanction (must be an SMI sanction) against any of his UIs/CIs out of s. Therefore, he will not move away from s, and s is a SEQ stable state for him.

8.3.3 Equilibrium States in Cooperative Games without Coalitions

The concept of an Equilibrium states is an important concept in game theory and conflict analysis. Equilibrium is tied to the concept of Stability Solution Concepts, and explains ultimate stability states in the game. The same definition for an Equilibrium state which we provided in Chapter 6 for non-cooperative games is the same as the one we provide here for cooperative games, without coalitions. This is because both types of games consider the games' players as individual DMs who are concerned only with their own well-being, regardless of the fact that they are able to cooperate in certain one-step moves within the context of cooperative games, without coalitions. Both game types assume that he players will not surrender their concern for their own well-being to any other DM or group of DMs. Therefore, the concept of overall stability, or equilibrium, for the games at a certain state must account for the stability of this state to each and every player in the game.

Definition 8.3.5 (Equilibrium (EQ.)): *A state $s \in \mathcal{S}$ is considered an Equilibrium for a cooperative game without coalitions, at time t, under a specific Solution Concept SC definition, denoted as $s \in \mathcal{S}^{SC\ EQ.}_{\mathcal{DM},t}$, iff $\forall\, DM_i \in \mathcal{DM}\ \ s \in \mathcal{S}^{SC}_{DM_i,t}$.*

As per the definition, a state s is stable for a cooperative game, without coalitions, as a whole, i.e. an equilibrium for the game, under a specific solution concept, such as NASH or SMR, if and only if the state s is stable under this solution concept for each and every decision maker in the game.

In any specific cooperative game/conflict, without coalitions, there may be a number of equilibrium states under one or more stability solution concepts. Equilibrium states represent the most likely outcomes for the game, and constitutes possible resolutions to the game. Once one of these states arise, this state is likely persist. But, the strength of this persistence depends on which solution concept the equilibrium state is under, and what is the strength of the state's stability under this solution concept for each of the DMs in the game.

8.4 Stability Strength of Solution Concepts and Equilibriums for Cooperative Conflicts without Coalitions

In this section, we will discuss the mechanisms by which one can identify the strength of the stability, under the four stability solution concept, for any given state in a cooperative game without coalitions, for any given DM in the game. We expand here the stability strength definitions and algorithms provided in Chapter 6 for non-cooperative games to accommodate the new cooperative moves that are possible within the context of cooperative games, and not possible in non-cooperative games. Then, we will discuss the strength of an equilibrium under a specific solution concept for a state in a cooperative conflict, without coalitions.

8.4.1 Stability Strength of Solution Concepts

The same *Stability Concept Strength* value property, denoted as StabilityStrength($StabilityConcept, s, DM_i, t$), discussed in Chapter 6 for non-cooperative games, will continue to be used here for cooperative games, without coalitions. Also, the same fuzzy linguistic value label L_{SS} used for for *StabilityStrength* will continue to be used here with the same fuzzy memberships functions given in Figure 4.4-(a). And, the strength expressed by the L_{SS} fuzzy label is meant to represent the strength of the stability concept type *StabilityConcept* (where *StabilityConcept* $\in \{NASH, GMR, SMR, SEQ\}$) for state s, for decision maker DM_i at time t.

As a reminder, we said that the StabilityStrength value property before fuzzification and without normalized has numeric value is in the range $[0, 2]$, therefore it will have a fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*, *Little*, *None*, *Null*$\}$=$\{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. And, with the understanding that the complete order of these labels is: $Ex>Si>St>F>B>M>Mo>L>N>Null$. And, when the StabilityStrength, before fuzzification, is normalized, i.e. its numeric value is in $[0, 1]$, then its fuzzy labels will include the same labels as above with the exception of the three higher labels covering the range from $[1, 2]$ and those are: *Extreme*,*Significant* and *Strong*.

The L_{SS} fuzzy label assigned to $\mathit{StabilityStrength}(\mathit{StabilityConcept}, s, \mathit{DM}_i, t)$ will cover the stability strength satisfaction levels. Where the labels range from representing *Extreme*ly strong stability of s (based on the definition of the solution concept given in *StabilityConcept*) to *None* strength level for s (meaning very weak stability strength and close to non-existing strength or close to indifferent). And, with the understanding that the *Null* label represents an unknown stability strength or totaly-non-existing-stability.

We define, now, the stability strength for cooperative games, without coalitions, for each of the solution concepts we introduced in the previous section. Recall that within the context of cooperative games, without coalitions, players are still looked at and dealt with as individual players with individualistic aims and objectives in the game, despite the fact that will be able occasionally to cooperate with other player for specific one-step moves.

Definition 8.4.1 (**Strength of NASH Stability**): *For decision maker DM_i at time t, and for a NASH stable state $s \in \mathcal{S}^{NASH}_{DM_i,t}$, the strength of s's NASH stability, to DM_i at time t, i.e. $\mathit{StabilityStrength}(NASH, s, \mathit{DM}_i, t)$, is calculated as follows:*

$$(\forall s_{bfr} : \quad ((s \in \mathcal{S}^{UM,\geq N}_{DM_i,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CM,\geq N}_{DM_g,t}(s_{bfr}))))$$
$$(\mathit{StabilityStrength}(NASH, s, \mathit{DM}_i, t) = |\max_{s_{bfr}}\{\mathit{PrefStrength}(s_{bfr}, s, \mathit{DM}_i, t), -Extreme\}|)$$

As per the definition, the strength of s's NASH stability strength is the positive strength equivalent of the negative preference of the state that the worst UI/CI move executed/could-be-executed by DM_i, individually or cooperatively, at time $< t$ in order to move to s.

Let the NASH's stability strength of a state s for DM_i at time t be denoted as $NASH(L_{SS})$, where $\mathit{StabilityStrength}(NASH, s, \mathit{DM}_i, t){=}L_{SS}$. Algorithm 8.2 uses Definition 8.4.1 to calculate the NASH's stability strength and assign the strength's fuzzy linguistic label.

Algorithm 8.2 Calculating a State's NASH Stability Strength for a DM in a Cooperative Game, without Coalitions

strength-value-label Strength_of_NASH_Stability (s, DM_i, *Game-Structure*)
// start with the assumption that s is not NASH stable
NASH_Strength = *Null*
// check if DM_i has any UIs/CIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset]$ **then**
    //s is NASH Stable State for DM_i at t; find NASH stability's strength
    NASH_Strength = *Strength_of_Nash*(s, DM_i, *Game-Structure*)
end if
return *NASH_Strength*

strength-value-label Strength_of_Nash(s, DM_i, *Game-Structure*)
// this routine will return the strength of the weakest UI by DM_i, or CI by a cooperating
// group DM_g he belongs to, that yields to reaching s. First, set Nash strength initially
// to "Extremely Strong" (this will be the case if s has no UIs/CIs that leads to it).
$Strength = -Extreme$
// find s's NASH strength
for all $s_{bfr} : ((s \in \mathcal{S}^{UM,\geq N}_{DM_i,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CM,\geq N}_{DM_g,t}(s_{bfr})))$ **do**
    $Strength = max\{Strength, PrefStrength(s_{bfr}, s, DM_i, t)\}$
end for
// return the equivalent positive strength label, if $Strength<N$
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return *Strength*

Definition 8.4.2 (Strength of GMR Stability): *For decision maker DM_i at time t, and for a GMR stable state $s \in \mathcal{S}^{GMR}_{DM_i,t}$, the strength of s's GMR stability, to DM_i at time t, i.e. StabilityStrength*(GMR, s, DM_i, t)*, is calculated as follows:*

$$((\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \vee ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) \neq \emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s))))$$

$$[\,[\,(\forall (DM_j \in \{\mathcal{DM} - DM_i\})\, \forall (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))) \;\wedge$$

$$(\forall (DM_p \subseteq \{\mathcal{DM} - DM_i\})\, \forall (s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\;\,]$$

$$(StabilityStrength\,(GMR, s, DM_i, t) =$$

$$|\max_{s_1} \{\min_{s_2}\{PrefStrength(s_2, s, DM_i, t), None\}, -Extreme\}\,|\,)\,]$$

And,

$$((\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \wedge ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset))$$

$$\Rightarrow \quad (StabilityStrength\,(GMR, s, DM_i, t) = None)$$

As per the definition, the strength of the GMR stability of s is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs/CIs from s, individually or cooperatively, given the fact that DM_i will choose the UI/CI that will yield the best less-preferred end state.

Let GMR's stability strength of a state s for DM_i at time t be denoted as $GMR(L_{SS})$, where StabilityStrength(GMR, s, DM_i, t)=L_{SS}. Algorithm 8.3 uses Definition 8.4.2 to calculate the GMR's stability strength and assign the strength's fuzzy linguistic label.

Algorithm 8.3 Calculating a State's GMR Stability Strength for a DM in a Cooperative Game, without Coalitions

```
strength-value-label Strength_of_GMR_Stability (s, DM_i, Game-Structure)
// start with the assumption that s is not GMR stable
GMR_Strength = NULL
// check if DM_i has any UIs/CIs from s at time t
if [S^{UI}_{DM_i,t}(s)=∅] ∧ [(∀DM_g ⊆ DM : DM_i ∈ DM_g)  S^{CI}_{DM_g,t}(s)=∅] then
   //s is NASH Stable State for DM_i at t
   GMR_Strength = None
else if (∀s_1 : ((s_1 ∈ S^{UI}_{DM_i,t}(s)) ∨ (s_1 ∈ ⋃_{DM_g⊆DM: DM_i∈DM_g} S^{CI}_{DM_g,t}(s)))) [∃ an SM sanction] then
   // sanction exists against each of DM_i's UIs/CIs ⇒ s is GMR stable; find GMR's strength
   GMR_Strength = Strength_of_Sanctions(s, DM_i, Game-Structure)
end if
return GMR_Strength

strength-value-label Strength_of_Sanctions(s, DM_i, Game-Structure)
// this routine will return the strength of the sanction that yields the worst result for DM_i,
// given the fact that DM_i will choose the UI/CI that will minimize his loss
// set sanction's strength initially to "Extremely Less Preferred"
Strength = −Extreme
for all s_1 : ((s_1 ∈ S^{UI}_{DM_i,t}(s)) ∨ (s_1 ∈ ⋃_{DM_g⊆DM: DM_i∈DM_g} S^{CI}_{DM_g,t}(s))) do
   SancStrength = None
   for all DM_j ∈ {DM − DM_i} do
      for all s_2 ∈ S^{UM}_{DM_j,t+1}(s_1) : s_2 ∈ S^{≤N}_{DM_i,t}(s) do
         SancStrength = min{SancStrength, PrefStrength(s_2, s, DM_i, t)}
      end for
   end for
   for all DM_p ⊆ {DM − DM_i} do
      for all s_2 ∈ S^{CM}_{DM_p,t+1}(s_1) : s_2 ∈ S^{≤N}_{DM_i,t}(s) do
         SancStrength = min{SancStrength, PrefStrength(s_2, s, DM_i, t)}
      end for
   end for
   Strength = max{Strength, SancStrength}
end for
if Strength < None then
   Strength = |Strength|
end if
return Strength
```

Definition 8.4.3 (Strength of SMR Stability): *For decision maker DM_i at time t, and for an SMR stable state $s \in \mathcal{S}^{SMR}_{DM_i,t}$, the strength of s's SMR stability, to DM_i at time t, i.e. StabilityStrength(SMR, s, DM_i, t), is calculated as follows:*

$$
\begin{aligned}
&((\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \vee ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) \neq \emptyset)) \\
&\qquad \Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)))) \\
&\qquad\qquad [\,[(\forall (DM_j \in \{\mathcal{DM} - DM_i\})\forall (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))\forall (s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s))) \\
&\qquad\qquad \wedge (\forall (DM_p \subseteq \{\mathcal{DM} - DM_i\})\forall ((s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))\forall (s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s)))] \\
&\qquad\qquad (\mathit{StabilityStrength}(SMR, s, DM_i, t) = \\
&\qquad\qquad\qquad |\max_{s_1}\{\min_{s_2}\{\max_{s_3}\{PrefStrength(s_2, s, DM_i, t), \\
&\qquad\qquad\qquad\quad PrefStrength(s_3, s, DM_i, t)\}, None\}, -Extreme\}\,|\,)\,]
\end{aligned}
$$

And,

$$
\begin{aligned}
&((\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \wedge ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset)) \\
&\qquad \Rightarrow \quad (\mathit{StabilityStrength}(SMR, s, DM_i, t) = None)
\end{aligned}
$$

As per the definition, the strength of the SMR stability of s is the positive strength equivalent of the negative strength of the worst ISM sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs/CIs from s, individually or cooperatively, given the fact that DM_i will choose the UI/CI that will yield the best less-preferred end state.

Let SMR's stability strength of a state s for DM_i at time t be denoted as $SMR(L_{SS})$, where StabilityStrength$(SMR, s, DM_i, t) = L_{SS}$. Algorithm 8.4, and its additional routine listed as Algorithm 8.5, use Definition 8.4.3 to calculate the SMR's stability strength and assign the strength's fuzzy linguistic label.

Algorithm 8.4 Calculating a State's SMR Stability Strength for a DM in a Cooperative Game, without Coalitions

strength-value-label Strength_of_SMR_Stability *(s, DM_i, Game-Structure)*
// start with the assumption that s is not SMR stable
SMR_Strength = NULL
// check if DM_i has any UIs/CIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset]$ **then**
  //s is NASH Stable State for DM_i at t
  SMR_Strength = None
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s))))$ [∃ an ISM sanction] **then**
  // ISM exists against each of DM_i's UIs/CIs ⇒ s is SMR stable; find SMR's strength
  SMR_Strength = Strength_of_Inescapable_Sanctions(s, DM_i, Game-Structure)
end if
return *SMR_Strength*

Algorithm 8.5 The "Strength_of_Inescapable_Sanctions" used in Algorithm 8.4

strength-value-label Strength_of_Inescapable_Sanctions(s***,*** DM_i***,*** *Game-Structure****)***
// this routine will return the strength of the ISM that yields the worst result for DM_i, given
// the fact that DM_i will choose a UI/CI and a counter move that will minimize his loss.
// Set sanction's strength (after DM_i's counter move) initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)))$ **do**
  $ISancStrength = None$
  for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
    for all $s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : ((s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$ **do**
      $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
      $CntrStrength = -Extreme$
      for all $s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)$ **do**
        $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_i, t)\}$
      end for
      if $ISancStrength < CntrStrength$ **then**
        $ISancStrength = CntrStrength$
      end if
    end for
  end for
  for all $DM_p \subseteq \{\mathcal{DM} - DM_i\}$ **do**
    for all $s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : ((s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$ **do**
      $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
      $CntrStrength = -Extreme$
      for all $s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)$ **do**
        $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_i, t)\}$
      end for
      if $ISancStrength < CntrStrength$ **then**
        $ISancStrength = CntrStrength$
      end if
    end for
  end for
  $Strength = max\{Strength, ISancStrength\}$
end for
if $Strength < None$ **then**
  $Strength = |Strength|$
end if
return $Strength$

Definition 8.4.4 (Strength of SEQ Stability): *For decision maker DM_i at time t, and for a SEQ stable state $s \in \mathcal{S}^{SEQ}_{DM_i,t}$, the strength of s's SEQ stability, to DM_i at time t, i.e. StabilityStrength(SEQ, s, DM_i, t), is calculated as follows:*

$$
\begin{aligned}
&((\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \vee ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) \neq \emptyset)) \\
&\qquad \Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)))) \\
&\qquad\qquad [\,[\,(\forall (DM_j \in \{\mathcal{DM} - DM_i\}) \forall (s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))) \;\wedge \\
&\qquad\qquad\quad (\forall (DM_p \subseteq \{\mathcal{DM} - DM_i\}) \forall (s_2 \in \mathcal{S}^{CI}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\;] \\
&\qquad\qquad (\mathit{StabilityStrength}(SEQ, s, DM_i, t) = \\
&\qquad\qquad\qquad |\max_{s_1}\{\min_{s_2}\{PrefStrength(s_2, s, DM_i, t), None\}, -Extreme\}|\,)\,]
\end{aligned}
$$

And,

$$
\begin{aligned}
&((\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \wedge ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset)) \\
&\qquad \Rightarrow \quad (\mathit{StabilityStrength}(SEQ, s, DM_i, t) = None)
\end{aligned}
$$

As per the definition, the strength of the SEQ stability of s is the positive strength equivalent of the negative strength of the worst SMI sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs/CIs from s, individually or cooperatively, given the fact that DM_i will choose the UI/CI that will yield the best less-preferred end state.

But recall here that as per Definition 8.3.4, for SEQ stability to be established the sanctions imposed by other players on DM_i's UIs/CIs out of s must be UI/CI moves by those other-players, i.e. their moves must be SMI sanctions. In other word, they must act "rationally". They will not hurt themselves in order to sanction DM_i's UIs/CIs. This is at the heart of the difference between GMR stability and SEQ stability.

Let SEQ's stability strength of a state s for DM_i at time t be denoted as $SEQ(L_{SS})$, where StabilityStrength$(SEQ, s, DM_i, t) = L_{SS}$. Algorithm 8.6 uses Definition 8.4.4 to calculate the SEQ's stability strength and assign the strength's fuzzy linguistic label.

8.4.2 Equilibrium Strength

The same *Equilibrium Strength* value property attached to a state s of the game at time t, denoted as *EquilibriumStrength*(*StabilityConcept*, s, t), discussed in Chapter

Algorithm 8.6 Calculating a State's SEQ Stability Strength for a DM in a Cooperative Game, without Coalitions

strength-value-label Strength_of_SEQ_Stability *(s, DM_i, Game-Structure)*
// start with the assumption that s is not SEQ stable
$SEQ\_Strength = NULL$
// check if DM_i has any UIs/CIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset]$ **then**
    //s is NASH Stable State for DM_i at t
    $GMR\_Strength = None$
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{DM_g \subseteq \mathcal{DM}: DM_i \in DM_g} \mathcal{S}^{CI}_{DM_g,t}(s))))$ [$\exists$ an SMI sanction] **then**
    // an SMI exists against each of DM_i's UIs/CIs $\Rightarrow$ s is SEQ stable; find SEQ's strength
    $SEQ\_Strength = Strength\_of\_UISanctions(s,\ DM_i,\ Game\text{-}Structure)$
end if
return $SEQ\_Strength$

strength-value-label Strength_of_UISanctions(*s, DM_i, Game-Structure)*
// this routine will return the strength of the sanction that yields the worst result for DM_i,
// given the facts that: 1) the sanction move must be a UI/CI (for the provider); and
// 2) DM_i will choose the UI/CI that will minimize his loss.
// set sanction's strength initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{DM_g \subseteq \mathcal{DM}: DM_i \in DM_g} \mathcal{S}^{CI}_{DM_g,t}(s)))$ **do**
    $UISancStrength = None$
    for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
            $UISancStrength = min\{UISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
        end for
    end for
    for all $DM_p \subseteq \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{CI}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
            $UISancStrength = min\{UISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
        end for
    end for
    $Strength = max\{Strength, UISancStrength\}$
end for
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return $Strength$

6 for non-cooperative games, will continue to be used here for cooperative games, without coalitions. Also, the same fuzzy linguistic value label L_{SS} used for for *EquilibriumStrength* will continue to be used here with the same fuzzy memberships functions. And, the strength expressed by the L_{SS} fuzzy label is meant to represent the strength of the equilibrium under the specific stability concept type *StabilityConcept* (where $StabilityConcept \in \{NASH, GMR, SMR, SEQ\}$) for state s,

for decision maker DM_i at time t.

As indicated in the equilibrium definition given earlier (Definition 8.3.5), the equilibrium concept must be defined under a specific stability solution concept. An equilibrium state under a specific stability solution concept is a state that is stable for all the decision makers in the game under the same stability solution concept. For example, if a state is an equilibrium under GMR, then this means that the state is GMR stable from every player in the game. As a result, the strength of the equilibrium for a specific state s under a specific solution concept SC is tightly coupled with the strength of the SC stabilities of s for each player in the game.

As a reminder, we said that the StabilityStrength value property before fuzzification and without normalized has numeric value is in the range $[0, 2]$, therefore it will have a fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*,*Little*, *None*, *Null*$\}$=$\{Ex, Si, St,\ F, B, M, Mo, S, L, N, Null\}$. And, with the understanding that the complete order of these labels is: *Ex*>*Si*>*St*>*F*> $B{>}M{>}Mo{>}L{>}N{>}Null$. And, when the StabilityStrength, before fuzzification, is normalized, i.e. its numeric value is in $[0, 1]$, then its fuzzy labels will include the same labels as above with the exception of the three higher labels covering the range from $[1, 2]$ and those are: *Extreme*,*Significant* and *Strong*.

As for the *StabilityStrength* value property, and because of the dependancy, the *Equilibrium Strength* fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*,*Little*, *None*, *Null*$\}$=$\{Ex, Si, St,\ F, B, M, Mo, S, L, N, Null\}$. And, with the complete order of $Ex{>}Si{>}St{>}F{>}B{>}M{>}Mo{>}L{>}N{>}Null$, where the labels range from representing *Extreme*ly strong equilibrium stability (based on the definition of the solution concept given in *StabilityConcept*) of s to *None* strength level (meaning very weak equilibrium strength and close to non-existing strength) for s.

The L_{SS} fuzzy label assigned to $EquilibriumStrength(StabilityConcept, s, t)$ will cover the equilibrium stability strength satisfaction levels, with the understanding that the *Null* label represents an unknown equilibrium stability strength or totaly-non-existing-equilibrium. The fuzzy membership functions defining these stability/equilibrium strength's linguistic value labels are given in Figure 6.1. The figure shows the membership functions for each label's fuzzy set to be trapezoidal in shape, for simplicity only (not as a restriction) as indicated in Al-Shawa and Basir (2010). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements, as we indicated in

Al-Shawa and Basir (2009, 2010).

Now, we define the equilibrium strength for cooperative games, without coalitions, for each of the solution concepts we introduced in the previous section. Recall that within the context of cooperative games, without coalitions, players are still looked at and dealt with as individual players with individualistic aims and objectives in the game, despite the fact that will be able occasionally to cooperate with other player for specific one-step moves.

Definition 8.4.5 (Strength of an Equilibrium): *For $\mathcal{DM}$, all decision makers in a specific cooperative game, without coalitions, at time t, and for state s which is an Equilibrium for the game under a specific Solution Concept SC, i.e. $s \in \mathcal{S}^{SC\ EQ.}_{\mathcal{DM},t}$, the strength of s's Equilibrium stability, to $\mathcal{DM}$ at time t, is calculated as follows:*

$$(\exists\, DM_i \in \mathcal{DM}\ \ s \notin \mathcal{S}^{SC\ EQ.}_{DM_i,t}) \quad \rightarrow \quad EquilibriumStrength(SC,s,t) = NULL$$

And,

$$(\forall\, DM_i \in \mathcal{DM}\ \ s \in \mathcal{S}^{SC\ EQ.}_{DM_i,t}) \quad \rightarrow \quad EquilibriumStrength(SC,s,t) = \min_{DM_i}\{StabilityStrength(SC,s,DM_i,t)\}$$

As per the definition above, the strength of the Equilibrium at s under the Solution Concept SC is the minimum of the strength of s's stability under SC for each decision maker in the game. This means that s must be stable under SC for each player in order for it to be an equilibrium for the game under SC (Definition 8.3.5, then the minimum of all DMs SC stabilities' strengths is considered to be the strength level of this equilibrium at s.

Let the Equilibrium's stability strength of state s of the game at time t be denoted as $SC\ EQ(L_{SS})$, where EquilibriumStrength(SC, s, t)=L_{SS}. Algorithm 8.7 uses Definition 8.4.5 to calculate the equilibrium's strength and assign the strength's fuzzy linguistic label.

We said earlier that the same definition for an equilibrium state for cooperative games without coalitions (Definition 8.3.5) is the same definition provided in Chapter 6 for an equilibrium state in non-cooperative games. This is because both types of games consider the games' players as individual DMs who are concerned only with their own well-being, regardless of the fact that they are able to cooperate in certain one-step moves within the context of cooperative games, without coalitions. Both game types assume that he players will not surrender their concern for their own well-being to any other DM or group of DMs. Therefore, the concept of overall stability, or equilibrium, for the games at a certain state must account for the stability of this state to each and every player in the game. Therefore, the definition for equilibrium strength for cooperative games without coalitions (Definition 8.4.5) is the same as the definition for equilibrium strength for non-cooperative games (Definition 6.4.5); and as a result Algorithm 8.7 used here is similar to Algorithm 6.6used for eliciting equilibrium strength in non-cooperative games.

Algorithm 8.7 Calculating a State's Equilibrium Strength, under a specific Solution Concept SC, in a Cooperative Game without Coalitions

strength-value-label Strength_of_Equilibrium **(***s***,** *SC***,** *Game-Structure***)**
// start with the assumption that s is not an Equilibrium under SC
$SC\_EQ\_Strength = NULL$
// check if s is stable for all DMs in the game under Solution Concept SC
if $(\forall\, DM_i \in \mathcal{DM}\;\; s \in \mathcal{S}^{SC\ EQ.}_{DM_i,t})$ **then**
  // s is an Equilibrium for the game under Solution Concept SC; find EQ's strength
  // set equilibrium's strength initially to "Extremely Strong"
  $SC\_EQ\_Strength = Extreme$
  // find s's equilibrium strength
  for all $DM_i \in \mathcal{DM}$ **do**
    $SC\_EQ\_Strength = min\{SC\_EQ\_Strength, \text{StabilityStrength}(SC,s,DM_i,t)\}$
  end for
end if
return $SC\_EQ\_Strength$

8.5 Case Study: The Elmira Groundwater Contamination Conflict

8.5.1 Background

The town of Elmira is a small but a prosperous town located in the rich agricultural land of Southern Ontario, Canada, about 15 kilometers north of the twin cities of Kitchener and Waterloo. An underground aquifer is the main source of water for the town's population of close 7,500 residents in the late 1989, at the time of the conflict.

In 1989, the Ontario Ministry of Environment (MoE) discovered that a carcinogen, N-nitroso demethylamine (NDMA) was contaminating the underground aquifer. Immediately, the pesticide and rubber products plant of Uniroyal Chemical Ltd (UR) was the main suspect. UR had a history of environmental problems and was known to use processes that could produce NDMA as a byproduct. As a result, a Control Order was issued by MoE, under the *Environmental Protection Act of Ontario*, requesting UR to implement a long term collection and treatment system. UR cooperation was ordered to help in the determination of the cause as well as the best way to cleanse the contaminated aquifer and to carry out the necessary cleaning actions under the supervision of MoE. But, UR quickly responded by appealing the Control Order, in what seemed as an effort by UR to lengthen the process hoping that the Control Order would be canceled or at least modified.

The main decision makers (DMs) involved in this strategic environmental conflict were mainly: MoE and UR. The local governmental bodies, the township of Woolwich and the Regional Municipality of Waterloo, have no actual say on the strategic decisions of either MoE or UR, but has a lot of influence as they represent the people who will mainly get affected by this environmental crisis. This explains the fact that both MoE and UR tried to use the local governmental bodies as well as local economical and environmental interest groups to put pressure on the other main player. But, clearly this is a game/conflict between MoE and UR, ultimately because environmental issues and control orders are handled by the provincial government. MoE has the responsibility over such issues not the local governments. The local governments and the interest groups were encouraged and used by both MoE and UR to influence the other party's decision, but as the conflict unfolded and the end results show: the only players in the game were really: MoE and UR.

We will model and analyze the conflict, based on the Constrained Rationality framework and the concepts we discussed in this chapter. To do so, we will assume that the modeller/analyst is called to perform this task just after MoE issued its Control Order in 1989 and the immediate response taken by UR to appeal the order, in effect employing delaying tactics. In other words, the starting point for the conflict, or as will be referred to hereafter "the status quo" state for the Elmira conflict, is the state of MoE has (or keeping its) Control Order against UR, and UR employing delaying tactics.

8.5.2 Players' Strategic Goals and Alternatives

In this illustrative case study analysis, we provide a simplified but sufficient and realistic modelling of what the conflict's decision makers' Goals & Constraints Models (GCMs) could look like at the time. These GCMs are shown in Figure 8.2. The

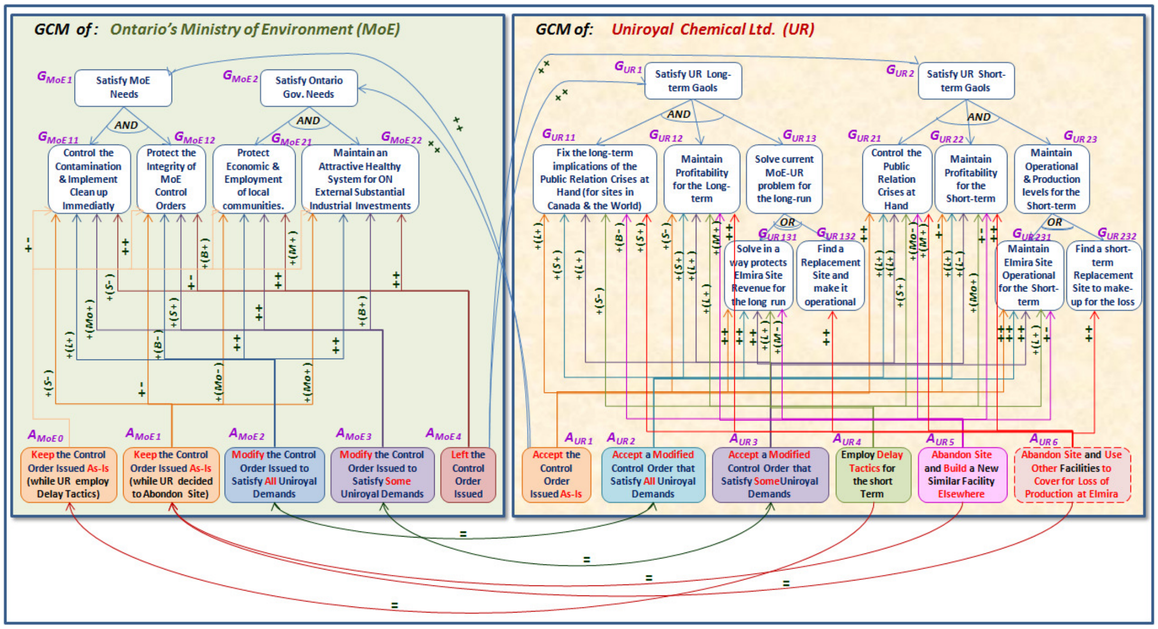


Figure 8.2: The Elmira Conflict: The GCM models showing Goals and Alternatives of both Decision Makers: Ontario Ministry of Environment (MoE) and Uniroyal Chemical Ltd. (UR)

figure shows that the main decision makers (DMs), MoE and UR, have different strategic objectives. Not only this, but as it is usually the case in strategic decision making and conflict situations, each of MoE and UR has within their cognitive thinking a set of conflicting goals to achieve.

As Figure 8.2 shows, MoE wants to carry out its environmental responsibilities in an effective and efficient way (shown in the figure as *Satisfy MoE Needs* goal labeled as $G_{MoE\,1}$). This strategic goal is reduced to two subgoals: starting immediately the process of cleaning up the contamination, for safety and health concerns; and protect the integrity of MoE's Control Orders. $G_{MoE\,1}$ goal reduction is shown in Figure 8.2 and details of it and its two subgoals, labeled as $G_{MoE\,11}$ and $G_{MoE\,12}$ respectively, are given in Table 8.1.

But unfortunately, MoE does not live in a vacuum. It is part of the provincial government which was at that time trying hard to survive the late 80's and early 90's recession and economical hardships. So, MoE, logically speaking, must have considered the implication of any decision it will take on the economical and employment situation at both the local township level and at the provincial economy and industrial base. Therefore, within the MoE's GCM model shown in Figure 8.2, we captured these provincial concerns of MoE under the strategic goal *Satisfy Ontario Government Needs* labeled as $G_{MoE\,2}$. This strategic goal is also reduced to

Table 8.1: The Elmira Conflict: Players' Strategic Goals

MoE Strategic Goals:		
Satisfy MoE Needs ($G_{MoE\,1}$)	$G_{MoE\,11}$	Starting immediately the process of cleaning up the contamination (for safety and health concerns)
	$G_{MoE\,12}$	Protect the integrity of its Control Orders in order for these orders to be taken seriously by the industry, and therefore prevent any future environmental miss-behaving by all operating industrial players in the province
Satisfy Ontario Gov. Needs ($G_{MoE\,2}$)	$G_{MoE\,21}$	Protect the economic and employment conditions of local communities
	$G_{MoE\,22}$	Maintain an attractive healthy economical system for Ontario to current and future external substantial industrial Investments
UR Strategic Goals:		
Satisfy UR Long-term Goals ($G_{UR\,1}$)	$G_{UR\,11}$	Fix the long-term implications of the public relation crises at hand (for all UR sites in Canada and in the World at large since now governments will take notice of UR actions and operations)
	$G_{UR\,12}$	Maintain UR operational profitability for the Long-term
	$G_{UR\,13}$	Solve current MoE-UR problem for the long-run. This goal is further reduced to two sub-goals: $G_{UR\,131}$, solve the current MoE Control Order issue in way that will maintain and protect the long term operation and revenue of the Elmira site; and $G_{UR\,132}$, find another replacement site elsewhere and make it operational, in order to substitute for the operation and therefore revenue stream which is currently supplied by the Elmira site.
Satisfy UR Short-term Goals ($G_{UR\,2}$)	$G_{UR\,21}$	Control the public relation crises at hand (to maintain short-term gains and operations)
	$G_{UR\,22}$	Maintain profitability at the current level for the short-term
	$G_{UR\,23}$	Maintain for the short-term an equivalent operational and production levels to Elmira site levels before the crisis. This goal is further reduced to two sub-goals: $G_{UR\,231}$, maintain the Elmira site operational for the short-term; and $G_{UR\,232}$, find a short-term replacement site or arrangements elsewhere to substitute for the loss of operation and production at the Elmira site.

two subgoals: protect the economic and employment conditions of local communities; and maintain an attractive healthy economical system for Ontario. $G_{MoE\,2}$ goal reduction is shown in Figure 8.2, and details of it and its two subgoals, labeled as $G_{MoE\,21}$ and $G_{MoE\,22}$ respectively, are given in Table 8.1.

On the other hand UR would like the Control Order, which had been issued by

MoE, to be lifted or modified. But, as we try to model UR goals to understand its alternatives and preferences, we surely can see that UR, as an international industrial business, considered the implication of any decision it may take on both its short-term goals, as well as its long-term ones. In Figure 8.2, we show a simplified but sufficient and realistic GCM model of UR that has two strategic goals, each with its reduction goal-tree. The first strategic goal, UR has, is *Satisfy UR Long-term Goals*, labeled in the figure as $G_{UR\,1}$. This strategic goal is reduced to a number of subgoals, details of which are given in Table 8.1. These subgoals include dealing with the long-term implications of the public relation crises at hand, maintaining UR operational profitability for the Long-term and having a long-term solution to MoE's control order and the Elmira site.

But UR is surely not only concerned with the long-term issues at hand, but also must deal with the short-term implications and issues. Therefore, the second strategic goal, UR has and shown in Figure 8.2 labeled as $G_{UR\,2}$, is *Satisfy UR Short-term Goals*. This strategic goal is reduced also to a number of subgoals, all of which are detailed in Table 8.1. These subgoals are similar to the ones $G_{UR\,1}$ is reduced to, but those are short-term goals.

If MoE has two strategic goals as we said above, then which one of them is more important to MoE? The analyst who is modelling the conflict from UR's perspective will not be sure whether the Ontario's MoE will see its own needs ($G_{MoE\,1}$) as important as the needs of the government of Ontario as a whole ($G_{MoE\,2}$), or less important. A strong case could be made either way. The safest thing to do is to analyze both scenarios. The first with $SImprt(G_{MoE\,1})$ =*Full* and $SImprt(G_{MoE\,2})$ =*Full*. And, the second with $SImprt(G_{MoE\,1})$ =*Little* and $SImprt(G_{MoE\,2})$ =*Full*.

Similarly, for an analyst who is modelling the conflict from MoE's perspective, which of UR's two strategic goals should she consider more important for UR? A strong case could be made that UR, strategically speaking, will consider its long-term strategic goal ($G_{UR\,1}$) as important as its short-term strategic goal ($G_{UR\,2}$). But, the fact that UR is a public company, and such companies most of the time act in favour of aggressively fixing short-term problems under the immediate pressure of the market and bad publicity. Therefore, a strong case could be made that UR will surely consider its short-term strategic goal more important than its long-term one. But, for the analyst who is modelling the conflict from MoE's perspective, the safest thing to do is to analyze the conflict under both scenarios. The first with $SImprt(G_{UR\,1})$ =*Full* and $SImprt(G_{UR\,2})$ =*Full*. And, the second with

$SImprt(G_{UR\,1})$ =*Little* and $SImprt(G_{UR\,2})$ =*Full*.

In this case, we will look at two versions of the four we have for Elmira conflict (four possible versions of the conflict based on the 2x2 scenarios). The first, we call *Game 1*, is the conflict with the assumption that both MoE and UR consider their respective two strategic goals equally and fully important. In other words, for MoE: $SImprt(G_{MoE\,1}) = SImprt(G_{MoE\,2})$ =*Full*; and for UR: $SImprt(G_{UR\,1})$ =*Full* and $SImprt(G_{UR\,2})$ =*Full*. The second game, we will call *Game 2*, is the conflict with the assumption that both MoE and UR consider only one of their respective two strategic goals fully important while the other with little importance. In other words, for MoE: $SImprt(G_{MoE\,1})$ =*Little* and $SImprt(G_{MoE\,2})$ =*Full*; and for UR: $SImprt(G_{UR\,1})$ =*Little* and $SImprt(G_{UR\,2})$ =*Full*.

Now, we move to the next step of modelling the conflict: finding the alternatives/options that the conflict players have. Both MoE and UR have many alternatives in dealing with the conflict and operationalize their respective goals. Both players are supposed to list all their known alternatives, and analyze their situation looking for any additional alternatives that they can come up with (whether through a generic process such as brainstorming or a methodological creative thinking process such as lateral thinking (De Bono, 1971). The process of generating players' alternatives can be done by each individual player in the conflict, or as in our case, and based on the discussion we have in Chapter 5, can be done by one of the players modelling the whole conflict and all its players's GCMs, including his.

Let us assume that conflict is modelled by MoE and UR separately and independently; and that both players came up with the same list of alternatives for themselves and the other player. The list of both decision makers alternatives is given in Table 8.2, and shown in Figure 8.2.

MoE, as shown in Table 8.2, can choose to: insist that UR implement the control order as-is; modify the issued control order to satisfy all UR demands (most likely this will render the control order useless); modify the control order to satisfy some of UR demands (a compromise solution); or lift the control order completely. UR, on the other hand, is shown in Table 8.2 to have the following options: accept MoE control order as it is; accept a modified control order by MoE that meets all what UR asks MoE to change; accepts a modified control order by MoE that meets only some of what UR asks MoE to change; employ delay tactics (use the legal system and governmental bureaucratic processes to their fullest extent in order to exhaust the government and keep the Emlira site operational for the longest possible time);

Table 8.2: The Elmira Conflict: Players' Alternatives/Options

The Set of Alternatives available to MoE (A_{MoE}) :	
$A_{MoE\,0}$	Insist on UR to Implement the issued Control Order *As-Is*, while UR is employing Delay tactics
$A_{MoE\,1}$	Insist on UR to Implement the issued Control Order *As-Is*, while UR decided to abandon site
$A_{MoE\,2}$	*Modify* the issued Control Order to satisfy *All* UR Demands
$A_{MoE\,3}$	*Modify* the issued Control Order to satisfy *Some* of UR Demands
$A_{MoE\,4}$	Lift completely the issued Control Order against UR.
The Set of Alternatives available to UR (A_{UR}) :	
$A_{UR\,1}$	*Accept* the issued Control Order *As-Is* and implement it
$A_{UR\,2}$	*Accept a Modified* Control Order that satisfy *All* UR demands
$A_{UR\,3}$	*Accept a Modified* Control Order that satisfy *Some* UR demands
$A_{UR\,4}$	*Employ* all available *Delay Tactics* for the short Term
$A_{UR\,5}$	*Abandon Elmira Site* and *Build a New Similar Facility Elsewhere* (though it will take time to build the site and for the site to be operational)
$A_{UR\,6}$	*Abandon Elmira Site* and *Use Other Facilities Elsewhere* to cover the loss of production at Elmira (no substantial loss of production will happen since UR will be able within short period of time to use other facilities to reach the same production levels of what is produced at the current Elmira site).

or abandon the Elmira site and stop the current, and any future, operational plans in Ontario.

But Table 8.2 seems to show that MoE has two alternatives, $A_{MoE\,0}$ and $A_{MoE\,1}$, that seem to be similar. Both alternatives seem to cover MoE's option to keep its control order as-is and insist that UR implement it. But this option has two different effect of MoE goals based on whether UR chooses to answer by employing delay tactics or decide to take the drastic measure of abandoning the Elmira site and any operational plans it has in Ontario. So, for the benefit of showing/studying these two different effects, this one option could have in two different contexts, we decided: it is better to model them as two alternatives in MoE's GCM model, as shown in Figure 8.2. This will be better, from modelling and analysis perspective, than having two sets of GCM models for MoE; or modify the lateral relations which connects what will be one alternative and MoE's goals based on what UR choses.

Table 8.2 also shows two alternatives for UR, $A_{UR\,5}$ and $A_{UR\,6}$, that seem to be similar. Both alternatives seem to convey that UR abandons the Elmira site. But upon examination, one should notice that $A_{UR\,5}$ covers the site abandoning option for UR, but with UR announcing clearly, or hinting indirectly, that it intends to build a new similar facility to the Elmira one –it is abandoning– somewhere else.

An option that will solve UR's long-term needs, but will not solve the immediate operational needs for UR, its sales forecast, market expectation, and/or contractual obligations. On the other hand, $A_{UR\,6}$ covers the same Elmira site abandoning option for UR, but with UR communicating clearly, or hinting indirectly, that it will use the extra production capacity that it has through its facilities elsewhere to cover the loss of production at Elmira. This option will satisfy UR immediate short-term needs while it is working on building its replacement site to the Elmira one.

Sure, $A_{UR\,6}$ could be a true option for UR, if it actually has the capacity to transfer the production supposed to be covered by the Elmira site to other facilities around the world with no substantial loss of business. This option also could be not true, and only intended to mislead MoE into believing that UR can abandon the Elmira operation and still be able to meet its short-term and long-term goals (i.e. a hypergame situation). Whether UR has actually the ability to implement alternative $A_{UR\,6}$, or is successful in misleading MoE and Ontario Government in believing that this alternative is a real option for UR, and UR has the ability to implement/adopt it, MoE should feel at a serious disadvantage and tremendous pressure given the economic situation at the time.

It is important for UR to analyze both alternatives $A_{UR\,5}$ and $A_{UR\,6}$. $A_{UR\,5}$ because it is a real option, and $A_{UR\,6}$ because it is a real option, or to test if it is worth investing in making MoE believe that it is a real option for UR. From MoE's perspective, it should analyze both alternatives for UR because it is not sure what UR is capable of doing. So, it is safe for MoE to analyze how the conflict can progress in both scenarios. Therefore, we, as any good analysts, will analyze the conflict with both scenarios: a) UR has $A_{UR\,5}$ as an option, in addition to its other five alternatives $A_{UR\,0}$ - $A_{UR\,4}$; and b) UR has $A_{UR\,6}$ as an option, instead of $A_{UR\,5}$ and in addition to its other five alternatives. But, recall that we said above that we will analyze the conflict under two goals-prioritization contexts in what we called Game 1 and Game 2. This makes the different configurations, contexts, or what-if versions of the Elmira conflict, which we will analyze, to be four:

Game 1:

- for MoE: $SImprt(G_{MoE\,1}) = SImprt(G_{MoE\,2}) = Full$;
- for UR: $SImprt(G_{UR\,1}) = SImprt(G_{UR\,2}) = Full$; and
- UR has alternative $A_{UR\,5}$ in addition to its alternatives $A_{UR\,0}$ - $A_{UR\,4}$.

Game 1 (with s_6 replacing s_5):

- for MoE: $SImprt(G_{MoE\,1}) = SImprt(G_{MoE\,2})$ =*Full*;
- for UR: $SImprt(G_{UR\,1}) = SImprt(G_{UR\,2})$ =*Full*; and
- UR has alternative $A_{UR\,6}$ in addition to its alternatives $A_{UR\,0}$ - $A_{UR\,4}$.

Game 2:

- for MoE: $SImprt(G_{MoE\,1})$ =*Little* and $SImprt(G_{MoE\,2})$ =*Full*;
- for UR: $SImprt(G_{UR\,1})$ =*Little* and $SImprt(G_{UR\,2})$ =*Full*; and
- UR has alternative $A_{UR\,5}$ in addition to its alternatives $A_{UR\,0}$ - $A_{UR\,4}$.

Game 2 (with s_6 replacing s_5):

- for MoE: $SImprt(G_{MoE\,1})$ =*Little* and $SImprt(G_{MoE\,2})$ =*Full*;
- for UR: $SImprt(G_{UR\,1})$ =*Little* and $SImprt(G_{UR\,2})$ =*Full*; and
- UR has alternative $A_{UR\,6}$ in addition to its alternatives $A_{UR\,0}$ - $A_{UR\,4}$.

Table 8.3 shows the differences in the definitions of these four what-if versions of the Elmira conflict.

Table 8.3: The Elmira Conflict: The Four What-if Versions/Games of the Conflict

		Strength of the Elmira-Abandonening Option for UR (Strength in terms of how realistic the option is for UR)	
		State s_5 UR could abandon the Elmira site, but it will get affected operationally at least in the short-to-medium-term	**State s_6** UR could seriously abandon the Elmira site and move its production elsewhere without suffering much operational loss
Strategic Importance of Players' Goals	**for MoE:** $SImprt(G_{MoE\,1})$ =*F* ; $SImprt(G_{MoE\,2})$ =*F* *needs of the Ontario Government are as important as MoE's needs* **for UR:** $SImprt(G_{UR\,1})$ =*F* ; $SImprt(G_{UR\,2})$ =*F* *UR short-term needs are as important as its long-term needs*	**Game 1**	**Game 1 (s_6 replacing s_5)**
	for MoE: $SImprt(G_{MoE\,1})$ =*L* ; $SImprt(G_{MoE\,2})$ =*F* *needs of the Ontario Government are more important than MoE's needs* **for UR:** $SImprt(G_{UR\,1})$ =*L* ; $SImprt(G_{UR\,2})$ =*F* *UR short-term needs are more important than its long-term needs*	**Game 2**	**Game 2 (s_6 replacing s_5)**

Now, we will define the conflict's states.

8.5.3 Conflict's States

Before defining the states of the the Elmira conflict, we need to review the integrated model of players' GCMs shown in Figure 8.2. This is to ensure that we have all the player' alternatives are included in the integrated GCMs model, and to understand all the possible realistic states the conflict could have. The analyst, first, will check the possible alternatives of the players, as we did in the previous subsection; and ensure that they are all captured within each player's GCM. Then, all de facto combinations of the players' alternatives, as well as all possible ones, will be identified. Each of these combinations form a state.

States that cannot be reached unless both players move to them jointly are de facto end-states of cooperative moves. For example, $A_{UR\,2}$ alternative of UR is shown in Figure 8.2 to be tightly coupled with MoE's $A_{MoE\,2}$ alternative by connecting each alternative's intention node to the other by a "=" lateral relation; and therefore a strong candidate for a state and a CM's end-state. The tight coupling of these two alternatives is to demonstrate the fact that MoE cannot choose $A_{MoE\,2}$ without UR choosing to select $A_{UR\,2}$ as its strategic move, and vice versa. This means that these alternatives/moves are cooperative moves that must be taken by both parties at the same time. It is not logical to assume that MoE will decide to modify the Control Order it issued to satisfy all UR demands (as stated by $A_{MoE\,2}$) without negotiating and agreeing with UR on what its demands are, how the control order will look like, and UR agreeing to accept the modified version of the control order (as stated by $A_{UR\,2}$). Nobody expects, at least not in a real-life like conflict, MoE to modify the order and wait to see if UR will accept. If there is a chance that MoE, for example, could do such move without cooperation (agreement between MoE and UR) then the alternatives should be de-coupled in the integrated GCMs conflict model. The same should be said about alternatives $A_{UR\,3}$ and $A_{MoE\,3}$ which are shown in Figure8.2 to be also tightly coupled forming an end-state for a cooperative move to happen simultaneously by both MoE and UR.

Table 8.4 lists all the states that we found valid and realistic in this conflict, each with the players' choices/alternatives that makes up the state. In two of the games/configurations, we are analyzing for the Elmira conflict, state s_5 will be replaced by state s_6, which is exactly similar to s_5 in the fact that it includes MoE chooses $A_{MoE\,1}$ as its option, but differ in the alternative that UR chooses. In s_6, UR chooses $A_{UR\,6}$ as its alternative instead of $A_{UR\,5}$ which it chooses in state s_5.

Table 8.4: The Elmira Conflict: Defining the Conflict's States

The Set of All States $\mathcal{S}$ for the MoE-UR Conflict:

State	s_0 Status Quo (1989)	s_1	s_2
MoE	Keeps Order As-Is $[A_{MoE\,0}]$	Keeps Order As-Is $[A_{MoE\,1}]$	Modifies Order Fully $[A_{MoE\,2}]$
UR	Delays $[A_{UR\,4}]$	Accepts $[A_{UR\,1}]$	Accepts New Order $[A_{UR\,2}]$

State	s_3	s_4	s_5
MoE	Modifies Order Partially $[A_{MoE\,3}]$	Lifts Control Order $[A_{MoE\,4}]$	Keeps Order As-Is $[A_{MoE\,1}]$
UR	Accepts New Order $[A_{UR\,3}]$		Abandons Site $[A_{UR\,5}]$

8.5.4 Players' Preferences over States of the Conflict

To calculate the preferences of the players over the conflict's states, we calculate how much each state (with all its players' alternatives selected) contribute to the achievement of the players' strategic goals. The sets of strategic goals for the players were defined earlier to be: $\mathcal{SG}_{MoE} = \{G_{MoE\,1}, G_{MoE\,2}\}$; and $\mathcal{SG}_{UR} = \{G_{UR\,1}, G_{UR\,2}\}$. Each of the individual strategic goals is explained above in Table 8.1. The set of the conflict's states is defined in Table 8.4. Two of the four games, or configurations, we are analyzing for the Elmira conflict, have $\mathcal{S} = \{s_0, s_1, s_2, s_3, s_4, s_5\}$, and those are *Game 1* and *Game 2*. The other two games have $\mathcal{S} = \{s_0, s_1, s_2, s_3, s_4, s_6\}$, and those are *Game 1 (with s_6 replacing s_5)* and *Game 2 (with s_6 replacing s_5)*.

For each of the four games, we calculate the *Weighted Payoff* value for each state for each player using the method presented in Section 5.4.2. The players' weighted payoffs for each state of the conflict, for all four games, are shown in Figure 8.3. The figure shows the level of achievement the state contributes to the players' strategic goals. It also shows the strategic importance, each of the players has for their respective strategic goals (differ based on the game/configuration); in addition to the Rationality and Emotionality Factors for both players (assumed in the games/configurations' base models to be set to *Full* and *None*, or 1.0 and 0.0, respectively, in other words the players are assumed fully rational and not

Rationality Factor = 1.0 (for UR & MoE) Emotionality Factor = 0.0 (for UR & MoE)		Game 1				Game 2			
		UR		MoE		UR		MoE	
Strategic Goals of DMs	SGs:	SG_{UR}		SG_{MoE}		SG_{UR}		SG_{MoE}	
		$SG_{UR\ 1}$	$SG_{UR\ 2}$	$SG_{MoE\ 1}$	$SG_{MoE\ 2}$	$SG_{UR\ 1}$	$SG_{UR\ 2}$	$SG_{MoE\ 1}$	$SG_{MoE\ 2}$
Strategic Importance	$SImprt(SG_k)$	F	F	F	F	L	F	L	F
State S_0 Status Quo (1989) MoE Keeps Order & UR Delay { $Achv(A_{MoE\ 0})$=F, $Achv(A_{UR\ 4})$=F }	$Achv(SG_k)$	N	L	N	M	N	L	N	M
	$Prvn(SG_k)$	S	N	F	N	S	N	F	N
	$FAchv(SG_k)$	−S	L	−F	M	−S	L	−F	M
	$TWFAchv(SG_k,DM)$	−0.40	0.20	−1.00	0.60	−0.08	0.20	−0.20	0.60
	$WP(S_0, DM)$	−0.10		−0.20		0.06		0.20	
	$OP(S_0, DM)$	4		6 (Worst)		3		5	
State S_1 MoE Keeps Order As-Is & UR Accepts { $Achv(A_{MoE\ 1})$=F, $Achv(A_{UR\ 1})$=F }	$Achv(SG_k)$	N	N	F	F	N	N	F	F
	$Prvn(SG_k)$	S	F	N	N	S	F	N	N
	$FAchv(SG_k)$	−S	−F	F	F	−S	−F	F	F
	$TWFAchv(SG_k,DM)$	−0.40	−1.00	1.00	1.00	−0.08	−1.00	0.20	1.00
	$WP(S_1, DM)$	−0.70		1.00		−0.54		0.60	
	$OP(S_1, DM)$	5		1 (Best)		5		1 (Best)	
State S_2 MoE Modify Fully & UR Accepts New Order { $Achv(A_{MoE\ 2})$=F, $Achv(A_{UR\ 2})$=F }	$Achv(SG_k)$	S	L	N	F	S	L	N	F
	$Prvn(SG_k)$	N	N	B	N	N	N	B	N
	$FAchv(SG_k)$	S	L	−B	F	S	L	−B	F
	$TWFAchv(SG_k,DM)$	0.40	0.20	−0.80	1.00	0.08	0.20	−0.16	1.00
	$WP(S_2, DM)$	0.30		0.10		0.14		0.42	
	$OP(S_2, DM)$	2		4		2		3	
State S_3 MoE Modify Some & UR Accepts New Order { $Achv(A_{MoE\ 3})$=F, $Achv(A_{UR\ 3})$=F }	$Achv(SG_k)$	L	N	S	B	L	N	S	B
	$Prvn(SG_k)$	N	L	N	N	N	L	N	N
	$FAchv(SG_k)$	L	−L	S	B	L	−L	S	B
	$TWFAchv(SG_k,DM)$	0.20	−0.20	0.40	0.80	0.04	−0.20	0.08	0.80
	$WP(S_3, DM)$	0.00		0.60		−0.08		0.44	
	$OP(S_3, DM)$	3		2		4		2	
State S_4 MoE Lefts Control Order { $Achv(A_{MoE\ 4})$=F }	$Achv(SG_k)$	F	F	N	F	F	F	N	F
	$Prvn(SG_k)$	N	N	F	N	N	N	F	N
	$FAchv(SG_k)$	F	F	−F	F	F	F	−F	F
	$TWFAchv(SG_k,DM)$	1.00	1.00	−1.00	1.00	0.20	1.00	−0.20	1.00
	$WP(S_4, DM)$	1.00		0.00		0.60		0.40	
	$OP(S_4, DM)$	1 (Best)		5		1 (Best)		4	
State S_5 MoE Keeps Order As-Is & UR Abandons Site { $Achv(A_{MoE\ 1})$=F, $Achv(A_{UR\ 5})$=F }	$Achv(SG_k)$	N	N	F	N	N	N	F	N
	$Prvn(SG_k)$	B	F	N	Mo	B	F	N	Mo
	$FAchv(SG_k)$	−B	−F	F	−Mo	−B	−F	F	−Mo
	$TWFAchv(SG_k,DM)$	−0.80	−1.00	1.00	−0.50	−0.16	−1.00	0.20	−0.50
	$WP(S_5, DM)$	−0.90		0.25		−0.58		−0.15	
	$OP(S_5, DM)$	6 (Worst)		3		6 (Worst)		6 (Worst)	

(a) for Game 1, with all players' Strategic Goals set as"Fully' Important; and for Game2, with the players prioritize their Strategic Goals to "Fully" and "Partially" Important

Figure 8.3: The Elmira Conflict: Preferences of both Players, Ontario Ministry of Environment (MoE) and Uniroyal Chemical Ltd. (UR), for all games/configurations of the conflict

emotional as one could expect from governments with institutional collective rationality). The analyst can test different rationality and emotional values, and the implication of such values, by setting up what-if scenario models as variations to the games/configurations' base models as part of the sensitivity analysis stage.

From the calculate weighted payoffs of each state, to each of the players, we calculate the Ordinal Preferences for the players over the conflict's states. Figure 8.3 shows both the weighted payoffs and the ordinal preferences for each state of the conflict's games/configurations, for each player. Figure 8.3 shows that both the cardinal/weighted preferences and the ordinal ones change for the states from one

Rationality Factor = 1.0 (for UR & MoE)
Emotionality Factor = 0.0 (for UR & MoE)

		Game 1 (with S_6 replacing S_5)				Game 2 (with S_6 replacing S_5)			
		UR		MoE		UR		MoE	
Strategic Goals of DMs	SGs:	SG_{UR}		SG_{MoE}		SG_{UR}		SG_{MoE}	
		$SG_{UR\ 1}$	$SG_{UR\ 2}$	$SG_{MoE\ 1}$	$SG_{MoE\ 2}$	$SG_{UR\ 1}$	$SG_{UR\ 2}$	$SG_{MoE\ 1}$	$SG_{MoE\ 2}$
Strategic Importance	SImprt(SG_k)	F	F	F	F	L	F	L	F
State S_0	Achv(SG_k)	N	L	N	M	N	L	N	M
	Prvn(SG_k)	S	N	F	N	S	N	F	N
Status Quo (1989) MoE Keeps Order & UR Delay	FAchv(SG_k)	–S	L	–F	M	–S	L	–F	M
	TWFAchv(SG_k,DM)	–0.40	0.20	–1.00	0.60	–0.08	0.20	–0.20	0.60
{ Achv($A_{MoE\ 0}$)=F,	WP(S_0, DM)	–0.10		–0.20		0.06		0.20	
Achv($A_{UR\ 4}$)=F }	OP(S_0, DM)	5		6 (Worst)		4		5	
State S_1	Achv(SG_k)	N	N	F	F	N	N	F	F
	Prvn(SG_k)	S	F	N	N	S	F	N	N
MoE Keeps Order As-Is & UR Accepts	FAchv(SG_k)	–S	–F	F	F	–S	–F	F	F
	TWFAchv(SG_k,DM)	–0.40	–1.00	1.00	1.00	–0.08	–1.00	0.20	1.00
{ Achv($A_{MoE\ 1}$)=F,	WP(S_1, DM)	–0.70		1.00		–0.54		0.60	
Achv($A_{UR\ 1}$)=F }	OP(S_1, DM)	6 (Worst)		1 (Best)		6 (Worst)		1 (Best)	
State S_2	Achv(SG_k)	S	L	N	F	S	L	N	F
	Prvn(SG_k)	N	N	B	N	N	N	B	N
MoE Modify Fully & UR Accepts New Order	FAchv(SG_k)	S	L	–B	F	S	L	–B	F
	TWFAchv(SG_k,DM)	0.40	0.20	–0.80	1.00	0.08	0.20	–0.16	1.00
{ Achv($A_{MoE\ 2}$)=F,	WP(S_2, DM)	0.30		0.10		0.14		0.42	
Achv($A_{UR\ 2}$)=F }	OP(S_2, DM)	3		4		3		3	
State S_3	Achv(SG_k)	L	N	S	B	L	N	S	B
	Prvn(SG_k)	N	L	N	N	N	L	N	N
MoE Modify Some & UR Accepts New Order	FAchv(SG_k)	L	–L	S	B	L	–L	S	B
	TWFAchv(SG_k,DM)	0.20	–0.20	0.40	0.80	0.04	–0.20	0.08	0.80
{ Achv($A_{MoE\ 3}$)=F,	WP(S_3, DM)	0.00		0.60		–0.08		0.44	
Achv($A_{UR\ 3}$)=F }	OP(S_3, DM)	4		2		5		2	
State S_4	Achv(SG_k)	F	F	N	F	F	F	N	F
	Prvn(SG_k)	N	N	F	N	N	N	F	N
MoE Lefts Control Order	FAchv(SG_k)	F	F	–F	F	F	F	–F	F
	TWFAchv(SG_k,DM)	1.00	1.00	–1.00	1.00	0.20	1.00	–0.20	1.00
{ Achv($A_{MoE\ 4}$)=F }	WP(S_4, DM)	1.00		0.00		0.60		0.40	
	OP(S_4, DM)	1 (Best)		5		1 (Best)		4	
State S_6	Achv(SG_k)	S	M	F	N	S	M	F	N
	Prvn(SG_k)	N	N	N	Mo	N	N	N	Mo
MoE Keeps Order As-Is & UR Abandons Site	FAchv(SG_k)	S	M	F	–Mo	S	M	F	–Mo
	TWFAchv(SG_k,DM)	0.40	0.60	1.00	–0.50	0.08	0.60	0.20	–0.50
{ Achv($A_{MoE\ 1}$)=F,	WP(S_6, DM)	0.50		0.25		0.34		–0.15	
Achv($A_{UR\ 5}$)=F }	OP(S_6, DM)	2		3		2		6 (Worst)	

(b) for Game 1 and Game 2, but with UR has a stronger more effective plan to abandon the Elmira Site with less damaging effect on the short-term (i.e. with s_6 replacing s_5)

Figure 8.3: The Elmira Conflict: Preferences of both Players, Ontario Ministry of Environment (MoE) and Uniroyal Chemical Ltd. (UR), for all games/configurations of the conflict

game/configuration to another. In other words, prioritizing strategic goals for the player, as Game 2 do, changes the preferences' order for both players in comparison to the all-goals-are-equally-important Game 1. Similarly, replacing s_5 with the more dramatic s_6 state in both games, Game 1 and 2, changes the preferences' order for the players in both games.

Now, we elicit the preferences' strengths for each of the players, MoE and UR, for each of the four games/configurations of the Elmira conflict.

1) Players' Preferences for Game 1

The strengths of the players' preferences of each state over each of the other states are also calculated from the calculate weighted payoffs using the method discussed in Section 5.4.2. Table 8.5 shows the MoE and UR's preferences vectors, and the preferences strengths of each state over other states in Game 1 configuration of the conflict.

Table 8.5: The Elmira Conflict - Game 1: Players' Preferences

UR Preferences (*Most* to *Least* Preferred)						
UR	s_4	s_2	s_3	s_0	s_1	s_5
	1.00	0.30	0.00	-0.10	-0.70	-0.90
UR Preferences' Strengths						
$\succ^{L_{PS}}_{UR,t}$	s_4	s_2	s_3	s_0	s_1	s_5
s_4	N	S	Mo	M	B	F
s_2	-S	N	L	L	Mo	M
s_3	-Mo	-L	N	N	S	Mo
s_0	-M	-L	N	N	S	S
s_1	-B	-Mo	-S	-S	N	L
s_5	-F	-M	-Mo	-S	-L	N

MoE Preferences (*Most* to *Least* Preferred)						
MoE	s_1	s_3	s_5	s_2	s_4	s_0
	1.00	0.60	0.25	0.10	0.00	-0.20
MoE Preferences' Strengths						
$\succ^{L_{PS}}_{MoE,t}$	s_1	s_3	s_5	s_2	s_4	s_0
s_1	N	L	S	Mo	Mo	M
s_3	-L	N	L	L	S	S
s_5	-S	-L	N	N	L	L
s_2	-Mo	-L	N	N	N	L
s_4	-Mo	-S	-L	N	N	L
s_0	-M	-S	-L	-L	-L	N

2) Players' Preferences for Game 1 (with s_6 replacing s_5)

Table 8.6 shows the MoE and UR's preferences vectors, and the preferences strengths of each state over other states in Game 1 (with s_6 replacing s_5) configuration of the conflict.

Table 8.6: The Elmira Conflict - Game 1 (with s_6 replacing s_5): Players' Preferences

UR Preferences (*Most* to *Least* Preferred)						
UR	s_4	s_6	s_2	s_3	s_0	s_1
	1.00	0.50	0.30	0.00	-0.10	-0.70
UR Preferences' Strengths						
$\succ^{L_{PS}}_{UR,t}$	s_4	s_6	s_2	s_3	s_0	s_1
s_4	N	L	S	Mo	M	B
s_6	-L	N	L	L	S	M
s_2	-S	-L	N	L	L	Mo
s_3	-Mo	-L	-L	N	N	S
s_0	-M	-S	-L	N	N	S
s_1	-B	-M	-Mo	-S	-S	N

MoE Preferences(*Most* to *Least* Preferred)						
MoE	s_1	s_3	s_6	s_2	s_4	s_0
	1.00	0.60	0.25	0.10	0.00	-0.20
MoE Preferences' Strengths						
$\succ^{L_{PS}}_{MoE,t}$	s_1	s_3	s_6	s_2	s_4	s_0
s_1	N	L	S	Mo	Mo	M
s_3	-L	N	L	L	S	S
s_6	-S	-L	N	N	L	L
s_2	-Mo	-L	N	N	N	L
s_4	-Mo	-S	-L	N	N	L
s_0	-M	-S	-L	-L	-L	N

3) Players' Preferences for Game 2

Table 8.6 shows the MoE and UR's preferences vectors, and the preferences strengths of each state over other states in Game 2 configuration of the conflict.

Table 8.7: The Elmira Conflict - Game 2: Players' Preferences

UR Preferences (*Most* to *Least* Preferred)						
UR	s_4	s_2	s_0	s_3	s_1	s_5
	0.60	0.14	0.06	-0.08	-0.54	-0.58
UR Preferences' Strengths						
$\succ^{L_{PS}}_{UR,t}$	s_4	s_2	s_0	s_3	s_1	s_5
s_4	N	L	L	S	M	M
s_2	-L	N	N	L	S	S
s_0	-L	N	N	N	S	S
s_3	-S	-L	N	N	L	L
s_1	-M	-S	-S	-L	N	N
s_5	-M	-S	-S	-L	N	N

MoE Preferences(*Most* to *Least* Preferred)						
MoE	s_1	s_3	s_2	s_4	s_0	s_5
	0.60	0.44	0.42	0.40	0.20	-0.15
MoE Preferences' Strengths						
$\succ^{L_{PS}}_{MoE,t}$	s_1	s_3	s_2	s_4	s_0	s_5
s_1	N	N	N	L	L	S
s_3	N	N	N	N	L	L
s_2	N	N	N	N	L	L
s_4	-L	N	N	N	L	L
s_0	-L	-L	-L	-L	N	L
s_5	-S	-L	-L	-L	-L	N

4) Players' Preferences for Game 2 (with s_6 replacing s_5)

Table 8.6 shows the MoE and UR's preferences vectors, and the preferences strengths of each state over other states in Game 2 (with s_6 replacing s_5) configuration of the conflict.

Table 8.8: The Elmira Conflict - Game 2 (with s_6 replacing s_5): Players' Preferences

UR Preferences (*Most* to *Least* Preferred)						
UR	s_4	s_6	s_2	s_0	s_3	s_1
	0.60	0.34	0.14	0.06	-0.08	-0.54
UR Preferences' Strengths						
$\succ^{L_{PS}}_{UR,t}$	s_4	s_6	s_2	s_0	s_3	s_1
s_4	N	L	L	L	S	M
s_6	-L	N	L	L	L	S
s_2	-L	-L	N	N	L	S
s_0	-L	-L	N	N	N	S
s_3	-S	-L	-L	N	N	L
s_1	-M	-S	-S	-S	-L	N

MoE Preferences(*Most* to *Least* Preferred)						
MoE	s_1	s_3	s_2	s_4	s_0	s_6
	0.60	0.44	0.42	0.40	0.20	-0.15
MoE Preferences' Strengths						
$\succ^{L_{PS}}_{MoE,t}$	s_1	s_3	s_2	s_4	s_0	s_6
s_1	N	N	N	L	L	S
s_3	N	N	N	N	L	L
s_2	N	N	N	N	L	L
s_4	-L	N	N	N	L	L
s_0	-L	-L	-L	-L	N	L
s_6	-S	-L	-L	-L	-L	N

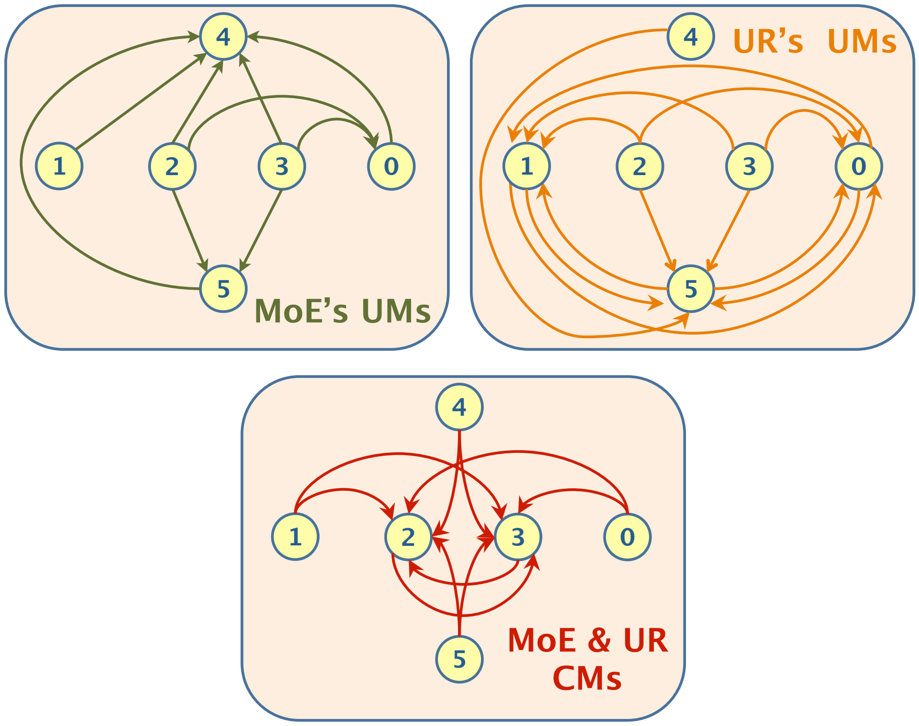


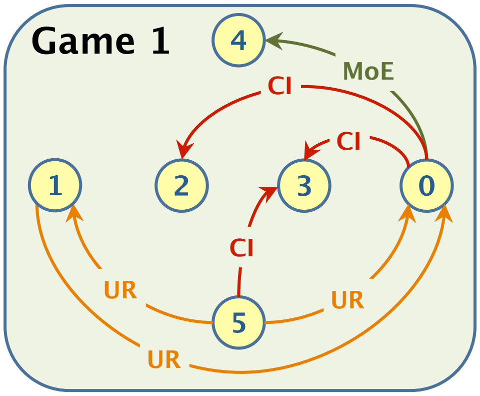
Figure 8.4: The Elmira Conflict: The players' UM and CM moves between the conflict's states

8.5.5 Players' Moves over States of the Conflict

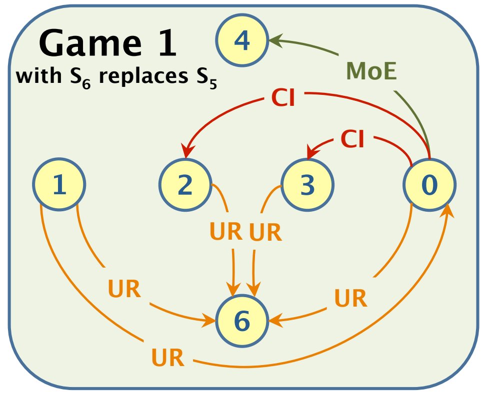
At this step, we define the UM moves for each of the players, MoE and UR. These moves are shown in Figure 8.4. We should note that the states in the figure are shown with their corresponding numbers. For example state s_2 is shown as a circle with the number 2 at its centre.

The figure also shows the CM moves that none of the players can take on their own, but they can cooperatively take together. States s_2 and s_3 cannot be reached except through cooperative moves, because they represent states in which the parties agree on some form of a modified control order. But, notice that each of the players, individually through a UM, can move away from any of these two states, representing a breakdown in the agreement or the negotiation for the agreement (this happens quite often in real-life conflicts).

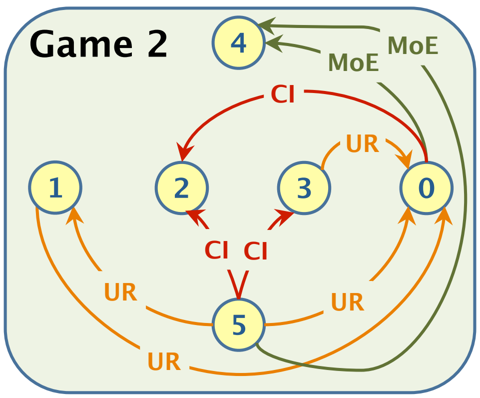
By feeding the players' UMs and preference structures to Algorithm 8.1, the Player's UI and CI moves are defined. Figure 8.5 shows both players individual UIs and their joint cooperative CIs, for each of the four games/configurations we are analyzing for the Elmira conflict.



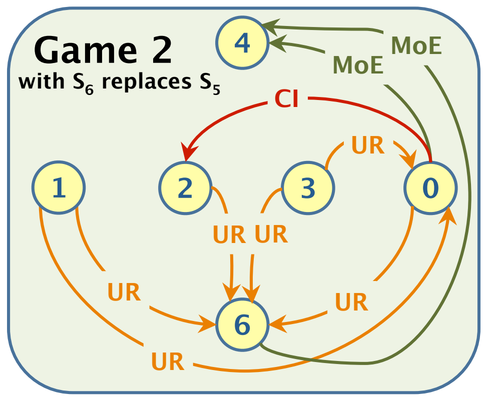
(a) Game 1: UIs and CIs by MoE and UR



(b) Game 1 with (s_6 replacing s_5): UIs and CIs by MoE and UR



(c) Game 2: UIs and CIs by MoE and UR



(d) Game 2 with (s_6 replacing s_5): UIs and CIs by MoE and UR

Figure 8.5: The Elmira Conflict: The players' UI and CI moves between the conflict's states, based on their Preferences

8.5.6 Stability Analysis of the Elmira Conflict

In this section, we will analyze the four versions of the Elmira conflict. For each, we start by conducting stability analysis, followed by equilibrium states analysis. We will discuss the insight that both analyses provide. We will look at the long-term equilibrium and resolution states for the conflict, based on each of the four configurations/versions. Also, we will discuss the short-terms strategies that the players can take, and how such strategies affect the conflict and its evolution over time.

Table 8.9: The Elmira Conflict - Game 1: Stability Analysis

	UR						***MoE***					
	s_4	s_2	s_3	s_0	s_1	s_5	s_1	s_3	s_5	s_2	s_4	s_0
UIs &				s_2 (CI)	s_0 (UI)	s_0 (UI)			s_3 (CI)			s_4 (UI)
CIs				s_3 (CI)		s_1 (UI)						s_2 (CI)
						s_3 (CI)						s_3 (CI)
NASH	Ex	Ex	Ex				Ex	Ex		Ex	L	
GMR	N	N	N	N			N	N	L	N	N	
SMR	N	N	N	N			N	N	L	N	N	
SEQ	N	N	N				N	N		N	N	

1) Stability Analysis for Game 1

Table 8.9 presents the stability analysis of the the six states of Game 1, for both players, under NASH, GMR, SMR and SEQ solution concepts. Table 8.10 shows the equilibrium states for the game under the four solution concepts.

Table 8.9 shows that all the NASH stable states for UR states that UR cannot reach on its own. For UR, reaching s_4, which is its most preferred state, is under the control of MoE. UR cannot decide to lift the control order issued by MoE. In addition, both s_2 and s_3 are states that can be reached only by both parties, MoE and UR, cooperate together taking their joint CMs (available only from certain states). But, if UR reaches s_2, s_3 or s_4, then this state will be NASH stable for it, because it does not have any UI it can take individually, or CI that it can take cooperatively, out from this state. The NASH stability of these states, to UR, is shown in the table to be at the *Extreme* strength level. This is because UR cannot reach these states on its own. In such case, both Definition 8.4.1 and Algorithm 8.2 put the strength of the states's NASH stability at the *Extreme* strength level to reflect the fact that the player, UR here, cannot reach the state (with a move that it can benefit from) and cannot leave the state (using a move that it will benefit from).

On the other hand, Table 8.9 shows that MoE have three states (s_2, s_3 and s_1) that are NASH stable and MoE cannot reach them on its own, and one (s_4) which is NASH stable but MoE can move unilaterally to it. As in the case of UR: both s_2 and s_3 are states that can be reached only by both parties, MoE and UR, cooperate together taking their joint CMs (available only from certain states); and the NASH stability of these states, to MoE, is shown in the table to be at the

Extreme strength level, because MoE cannot reach these states on its own. And similar to the case of UR and s_4, for MoE to reach s_1, which is its most preferred state, is under the control of UR. MoE cannot decide that UR accepts the control order as-is without all the appeal process (and other delay tactics UR is rightfully and skillfully applying) .

But in contrast with the *Extreme* NASH stability of s_2 and s_3 to MoE, s_4 is shown to be of NASH stability but at *Little* strength level. This is to reflect the fact that MoE can move to s_4 using a UI it has from the status quo state (s_0); and the gain it will achieve by the this UI, calculated as a difference in preference strength level, is *Little* (because $s_4 \succ^{L}_{MoE,t} s_0$). In other words, for MoE, moving from the status quo to s_4 by lifting the control order is of "little" benefit, but nevertheless it is better than staying at the status quo state (where UR employing delay tactics, the environmental problems with the effected people demand immediate cleanup action by the government, and potential investors take notice of the inflexibility of the government in dealing with the situation).

Table 8.9 provides additional insight on Game 1. It shows that each of the players has another state that constitutes a longer foresight stability. The status quo state, s_0, is a GMR and an SMR stable for UR. UR will not take any of the two joint cooperative CI moves, it has with MoE to reach s_2 and s_3, because it fears that MoE will unilaterally back down, sanction and therefore disimprove UR's CIs. Two important aspects of this SM by MoE. First, this SM is a UM move for MoE and not a UI, i.e. the motive by MoE to take it is just to hurt UR and not to benefit itself from it. In other words, MoE will act irrationally if it takes this sanction. This, in fact, why the SM by MoE is not considered an SMI move and why s_0 is not considered SEQ stable for UR. Second, this SM move by MoE puts the game back at s_0, the state where the CIs started from. This makes the strength of the GMR and SMR stability of s_0, for UR, to be at the *None* level, i.e. UR does not gain any benefit (or close to nothing) by taking the joint CIs out of s_0 to s_2 or s_3. Therefore, UR will stay at s_0 and considers it stable (based on both GMR and SMR solution concepts).

The table also shown that MoE has state s_5 as GMR and SMR stable. Even though MoE has a joint CI move, with UR, to state s_3 from s_5, MoE will not take the CI move out of fear that UR will unilaterally back down, sanction and therefore disimprove MoE's CI. This SM by UR has also two important aspects to it. First, this SM is a UM move by UR and not a UI, i.e. the motive by UR in

Table 8.10: The Elmira Conflict - Game 1: Equilibrium States

	s_0	s_1	s_2	s_3	s_4	s_5
NASH EQ.			Ex	Ex	L	
GMR EQ.			N	N	N	
SMR EQ.			N	N	N	
SEQ EQ.			N	N	N	

taking it is just to hurt MoE and not to benefit itself from it. In other words, UR will act irrationally if it takes this sanction. This, in fact, why the SM by UR is not considered an SMI move and why s_5 is not considered an SEQ stable for MoE. Second, this SM move by UR puts the game either back at s_5, the state where the CI started from, or at the less preferred status quo state s_0. This makes the strength of the GMR and SMR stability of s_5, for MoE, to be at the *Little* level, i.e. MoE will most likely end up at a *Little* less-preferred state if takes the joint CI out of s_5 to s_3. Therefore, MoE will stay at s_5 and considers it stable (based on both GMR and SMR solution concepts).

Now, moving to the equilibrium analysis of Game 1. Table 8.10 shows the overall equilibrium states for Game 1 configuration of the Elmira conflict, under the four solution concepts. As the table shows, Game 1 has three equilibrium states. State s_4 is NASH equilibrium of *Little* strength level. This is because it is of *Little* strength to MoE, even though UR has it as *Extremely* strong NASH stable state. Both states s_2 and s_3 are *Extremely* NASH stable for both players, and therefore they are *Extremely* NASH equilibrium for the game.

Table 8.10 is very telling about the conflict and the way it is expected to progress. All three equilibrium states for the conflict are not the best states that MoE could hope for. They all involve some form of concession by MoE, either by lifting the control order (s_4) or loosening it to an agreed-up terms with UR (s_2 or s_3). The best equilibrium state for MoE is the most preferred state (from those three states), and it is state s_3 where MoE *agrees with* UR on *some* modifications for the current control order. For sure, such modifications will be better for UR but not for MoE. On the other hand, the three equilibrium states shown in Table 8.10 are the best states that UR hopes for.

But the strengths of the three equilibrium states that Table 8.10 shows are also telling. Out of the three long term equilibrium states, which form possible resolutions to the conflict, two states (s_2 and s_3) have high stability strength level.

Those two states are more likely to form the end/resolution to the conflict, than the weaker equilibrium state of s_4. Therefore, UR should expect that MoE will have harder time going to s_4. MoE should expect to give concessions. In addition, MoE should expect UR to use/abuse the situation as much as possible, because it holds the power to agree on any modified control order. Otherwise, UR can go back to the delay tactics, or it can abandon the site.

The equilibrium states analysis provided in Table 8.10 gives us insight on the long-term resolution states for the conflict. But, what about the short term strategies that the players should take or consider. For this, we go back to Table 8.9. Two states are shown in the table to play some role on the interim. Those are states s_5 (UR abandons the Elimra site) and s_0 (the status quo state where UR employing all possible delay tactics). Each of which is shown in the table to be GMR and SMR stable for one player or the other.

The fact that MoE has state s_5, in which UR abandons the Elmira site, as GMR and SMR stable, tells us a lot about the conflict. If UR made the move to abandon the Elmira site, or declared its intention to do so, then MoE is set to gain more by not negotiating a lesser strict control order against UR. This puts pressure on UR to accept the control order as-is; and this will also has the added benefit of showing the business community in Ontario how serious the government in dealing with the environmental problems caused by industry. In addition, the fact that the GMR and SMR stabilities of s_5, for MoE, is at strength level *Little* shows that MoE will scarifies little, at the preference scale, if it decides to negotiate a deal with UR to loosen the control order. The worst UR can do is bring the game back to s_5, or to the status quo state s_0. Not to mention that the sanction UR will make in this case is not a UI for it. And we can be sure that UR, as a mature business entity, will act rationally and not just for the sake of hurting MoE or Ontario government. So, UR will most likely stay at s_3 which is an equilibrium state for the game as a whole, with an *Extreme* NASH strength level.

Additionally, the fact that UR has the status quo state s_0, in which UR employs delay tactics and does not accept the control order issued by MoE, as GMR and SMR stable, tells us a lot about the conflict from UR's perspective. For UR, staying at s_0 is the best strategy for many reasons. First, staying at s_0 keeps the pressure on MoE to either lift its order (s_4), or loosen its order and reach an agreement with UR (s_2 or s_3). MoE has no other option. This is what historically happen in the conflict. Second, if UR and MoE reach an agreement on a better-terms-for-UR control order, and later MoE backs down on this (because of the pressure that the city, the people , the media, etc., put on it, or for any other reason), then UR is set to lose nothing. UR will come back to s_0. Third, if UR runs out of delay tactics to employ, and it cannot bear the cost of the clean up as required by the control order, then it can abandon the Elmira site (s_5). And, if it can bear the cost, it can move to s_1 by accepting the control order.

Therefore, s_0 is the best state to be in, at least for the short to medium range, and this is why UR immediately moved to s_0 after MoE issued its control order; and why s_0 is stable for UR in the foreseeable future. This state's GMR and SMR stability, for UR, has higher foresight (one or two steps ahead, as per the discussion given in Section 7.2), and forms a great proposition to force MoE in either lifting the order or loosening it on terms that both parties agree on (the NASH stable states (s_4 or s_2/s_3, respectively).

2) Stability Analysis for Game 1 (with s_6 replacing s_5)

Table 8.11 presents the stability analysis of the the six states of Game 1 (with s_6 replacing s_5), for both players, under NASh, GMR, SMR and SEQ solution concepts. Table 8.12 shows the equilibrium states for the game under the four solution concepts.

Recall that Game 1 (with s_6 replacing s_5) is a what-if version of the Elmira conflict in which UR has a stronger case for abandoning the Elmira site. Hence, state s_6 replacing the weaker abandoning case represented in state s_5 of the conflict's base model given in Game 1. It does not matter whether UR truly has the capabilities to allow it shift all/most of the Elmira production capacity to one of its other inter-

Table 8.11: The Elmira Conflict - Game 1 (with s_6 replacing s_5): Stability Analysis

	UR						***MoE***					
	s_4	s_6	s_2	s_3	s_0	s_1	s_1	s_3	s_6	s_2	s_4	s_0
UIs &			s_6 (UI)	s_6 (UI)	s_6 (UI)	s_0 (UI)						s_4 (UI)
CIs					s_2 (CI)	s_6 (UI)						s_2 (CI)
					s_3 (CI)							s_3 (CI)
NASH	Ex	L					Ex	Ex	Ex	Ex	L	
GMR	N	N					N	N	N	N	N	
SMR	N	N					N	N	N	N	N	
SEQ	N	N					N	N	N	N	N	

national facilities or it managed somehow (e.g. through media reports) to convince MoE and Ontario that it has such capabilities (UR is playing a hypergame). The conflict now, under Game 1 (with s_6 replacing s_5) configuration and assumptions, is different than the conflict under Game 1 configuration and assumptions.

Figure 8.3 and Tables 8.5 - 8.6, presented earlier in Section 8.5.4, show clear differences in the cardinal, and as a result in the ordinal, preferences of UR over the states of the conflict between the base model in Game 1 and its variation Game 1 (with s_6 replacing s_5). The figure and tables show that MoE's preferences stayed the same. From MoE's perspective, abandoning the site by UR in s_6 is the same as abandoning it by UR in s_5. But, despite the no change to MoE's preferences, the changes in UR's preferences have significant impact on the stability of the states for the players and for the conflict as a whole.

The stability analysis of Game 1 (with s_6 replacing s_5) shown in Table 8.11 is different than the stability analysis of the base Game 1 shown in Table 8.9 in four aspects. First, whilst the agreement states s_2 and s_3 in Game 1 are NASH stable for both MoE and UR, these states in the new Game 1 (with s_6 replacing s_5) maintained their NASH stability for MoE but are not stable at all for UR. In the new modified Game 1, UR is showing itself to be not interested in changing the control order, fully (s_2) or partially(s_3). If UR is truly, or tactically shows itself, capable of abandoning the Elmira site without losing much in operational capacity, revenue generation and demand satisfaction, then UR will be in avery good position to get more concisions from MoE in the conflict.

Second, Table 8.11 shows that state s_6, which replaces in this new game state s_5 used in Game 1, is now NASH stable for both UR and MoE. For UR, this is the essence of showing that it is seriously considering the abandon-Elmira-site option. It is to show that UR has the capability to move on, and that the state which represent this option is stable for UR. The table shows the strength of s_6's NASH stability for UR to be at the *Little* strength level. On the other hand, MoE has no UM to move to s_6 and no UI/CI to move out from it. This makes s_6 to be NASH stable for MoE with *Extreme* strength level.

Third, The Game 1's stability analysis given in Table 8.9 shows the status quo state s_0 to be GMR and SMR stable for UR. This state in Game 1 (with s_6 replacing s_5) is not GMR or SMR stable to UR, as shown in Table 8.11. Because the status quo state, where UR employing delay tactics, becomes unstable for UR in this version of the game, UR ideally should not stay long in this state. UR should

Table 8.12: The Elmira Conflict - Game 1 (with s_6 replacing s_5): Equilibrium States

	s_0	s_1	s_2	s_3	s_4	s_6
NASH EQ.					L	L
GMR EQ.					N	N
SMR EQ.					N	N
SEQ EQ.					N	N

either take its UI to s_6 (by abandoning the Elmira site), or its joint CIs with MoE to one of the two agreement state (s_2 or s_3). This has a significant implication to how the game will progress. This puts more pressure on MoE to modify the control order and agree with UR to make the game go the s_2 or s_3 direction, and not the s_6 direction. MoE should move fast to loosen its position, otherwise UR can take its UI to s_6.

Fourth, state s_5 is shown in Table 8.9 to be GMR and SMR stable for MoE in Game 1, and unstable for UR. In Game 1 (with s_6 replacing s_5), s_6 is becoming NASH stable for both players as we mentioned above. Recall that s_6 is NASH stable for MoE because it has no UIs to reach the state, not to mention no UMs either to reach to the state. This makes the state's NASH stability *Extremely* strong for MoE. On the other hand, for UR, the state is NASH stable since UR has no UIs out of it. But because UR can reach this state from many other states, the strength of the state's NASH stability to UR reflect the lesser difference in preference between the state UR starts from and s_6 (the end state of the UI). This makes the strength of s_6's NASH stability to UR is set at the *Little* strength level. This *Little* strength shows that UR is not extremely excited to abandon the Elmira site (s_6). But, absent an agreement with MoE in the making, UR can go to s_6. This, while keeps the pressure on MoE to change the control order, makes the case for MoE not go all the way in agreeing to all UR demands.

Table 8.11 also shows that state s_4, as in the case of Game 1, is still NASH stable for both players for this version of the conflict. For UR, reaching s_4, which is its most preferred state, is under the control of MoE. UR cannot decide to lift the control order issued by MoE, making state s_4's NASH stability for UR to be at the *Extreme* strength level. For MoE, s_4 is Nash stable but at the *Little* strength level, similar to the situation in Game 1.

Now, moving to the equilibrium analysis of Game 1 (with s_6 replacing s_5). Table 8.12 shows the overall equilibrium states for this new configuration of the Elmira conflict, under the four solution concepts. As the table shows, this game has only two equilibrium states. Similar to Game 1, Game 1 (with s_6 replacing s_5) has state s_4 as a NASH equilibrium state of *Little* strength level. But unlike Game 1, the new game does not have s_2 and s_3 as equilibrium states. Instead it has state s_6 as a NASH equilibrium of *Little* strength level.

Table 8.12 is very telling about the conflict under the Game 1 (with s_6 replacing s_5) configuration and the way it is expected to progress. With two equilibrium states, one that has MoE lift the control order and a second that has UR abandons the Elmira site, MoE finds itself in a very intense situation. UR shows signs that it intends to leave the province, and therefore leaves MoE to bear all the fallout of the environmental disaster it caused. In this game, even compromise agreements, where MoE loosens its control order, are shown to be not so exciting to UR (as is the case in Game 1). MoE has to move fast and try to reach an agreement that will give UR enough incentive to sway it from abandoning the site.

For UR, this configuration of the conflict is much better than Game 1. It increases the pressure on MoE to lift the control order or drastically change it to meet UR demands. But, this added value comes at a price. The fact that the status quo state s_0, where UR is employing delay tactics, is unstable to UR in this game, UR cannot stay for long in this state. Staying at s_0 for a long time could suggest that UR does not actually have the capability to move production elsewhere, as suggested by s_6. UR could be seen as a company that is bluffing in an attempt to deceive MoE, and Ontario at large. This leads to a more damaging public relation for the company and its current and future business in canada and elsewhere.

3) Stability Analysis for Game 2

Table 8.13 presents the stability analysis of the the six states of Game 2, for both players, under NASh, GMR, SMR and SEQ solution concepts. Table 8.14 shows the equilibrium states for the game under the four solution concepts.

In Section 8.5.2, we mentioned that Game 2 what-if version of the Elmira conflict is similar to Game 1 in the goals and alternatives the players have. Game 2 differs only in the importance weights the players put into some of their goals. In Game 2, MoE has $SImprt(G_{MoE\,1})$ =*Little* and $SImprt(G_{MoE\,2})$ =*Full*, instead of having the *SImprt* for both goals to be *Full* as in Game 1. Similarly, UR in Game 2 has $SImprt(G_{UR\,1})$ =*Little* and $SImprt(G_{UR\,2})$ =*Full*, again instead of having the importance value property label for both goals to be *Full* as in Game 1.

Table 8.13: The Elmira Conflict - Game 2: Stability Analysis

	UR						***MoE***					
	s_4	s_2	s_0	s_3	s_1	s_5	s_1	s_3	s_2	s_4	s_0	s_5
UIs & CIs			s_2 (CI)	s_0 (UI)	s_0 (UI)	s_0 (UI) s_1 (UI) s_2 (CI) s_3 (CI)					s_4 (UI)	s_4 (UI) s_2 (CI) s_3 (CI)
NASH	Ex	Ex					Ex	Ex	Ex	L		
GMR	N	N	N				N	N	N	N	L	N
SMR	N	N	N				N	N	N	N		
SEQ	N	N					N	N	N	N		

As a result, the players' cardinal and ordinal preferences change. Figure 8.3a, Table 8.5 and Table 8.7, presented earlier in Section 8.5.4, show the differences in the players' preferences over the states of the conflict between the base model in Game 1 and the Game 2 variation. We will show here that these changes in MoE and UR's preferences have significant impact on the stability of the states for the players and as a result for the conflict as a whole.

The stability analysis of Game 2 shown in Table 8.13 is different than the stability analysis of the base Game 1 shown in Table 8.9 in three aspects. First, in Game 2, the agreement state s_3, in which MoE modifies the control order to fully satisfy UR requests and UR accepts to comply with the new order, is no longer a stable state for UR under any of the four solution concepts. In Game 1, s_3 is a NASH stable state for UR. And while s_3 is an *Extreme* NASH stable state for MoE in both, the change in the stability status of the state for UR has an impact on how the conflict could progress as we will discuss when we analyze the equilibrium states of the conflict under Game 2 versus Game 1.

Second, While Table 8.13 shows that MoE in Game 2 keeps all its NASH stable states from Game 1 (s_1, s_2, s_3 and s_4), with the same stability strength levels, there are changes to its non-NASH stable states. In Game 1, s_5 is both a GMR and SMR stable state for MoE with stabilities' strength at level *Little*. In Game 2, s_5 is only a GMR stable state for MoE and with strength level set at the very weak level *None*. s_5 is no longer SMR stable for MoE, because UR has no ISM for MoE's UI to s_4 from s_5. Also, the lowering of s_5's GMR stability, for MoE, from *Little* to *None* is due to the change in the available sanctions for MoE's UIs/CIs from s_5, and the change in MoE's preferences' strengths.

Table 8.14: The Elmira Conflict - Game 2: Equilibrium States

	s_0	s_1	s_2	s_3	s_4	s_5
NASH EQ.			Ex		L	
GMR EQ.	N		N		N	
SMR EQ.			N		N	
SEQ EQ.			N		N	

Third, MoE in Game 2 has a new GMR stable state that is unstable in Game 1. State s_0, the status quo, is GMR stable for MoE in Game 2. This is because UR can sanction MoE's UI to s_4 from s_0, in an irrational move to s_5 motivated by the need to hurt MoE (based on GMR Definition 8.3.2 but an unrealistic assumption to adopt). And because $s_5 \succ^{-L}_{MoE,t} s_0$, and by applying Definition 8.4.2 and Algorithm 8.3 , the strength of s_0's GMR stability for MoE is determined to be at the *Little*, i.e. at a higher strength than s_5's GMR stability for MoE. The fact that the status quo state s_0 is GMR stable for MoE, as it is for UR (as shown in Table 8.13), makes it a GMR equilibrium state for the conflict; and this will impact how the players are expected to act in the conflict, as we will discuss shortly.

Table 8.14 shows the overall equilibrium states for Game 2 configuration of the Elmira conflict, under the four solution concepts. As per the table, Game 2 has two equilibrium states. State s_4 is NASH equilibrium of *Little* strength level. This is because it is of *Little* strength to MoE, even though UR has it as *Extremely* strong NASH stable state. State s_2 is *Extremely* NASH stable for both players, and therefore an *Extremely* NASH equilibrium for Game2. Both s_4 and s_2 are also NASH equilibrium states under Game 1, the base configuration of the Elmira conflict, as shown in Table 8.10.

The comparison between the equilibrium states analysis for Game 2 (shown in Table 8.14) and the one for Game 1 (shown in Table 8.10) provides us with insights on the expected resolution for the Elmira conflict and the way the conflict is expected to progress. The first thing one should notice is that both games, Game 1 and Game 2, has states s_4 and s_2 as NASH equilibrium states for the conflict. Both states involve MoE giving concessions to UR, either by lifting the control order completely (s_4) or by *modifing* the control order to *satisfy all* UR requirements. For sure, such modifications will be better for UR, but not for MoE. In other words, both of these equilibrium states are not considered happy ending for the conflict, from MoE's perspective. On the other hand, the two equilibrium states are the best states that UR could hope for (most preferred as shown in Table 8.7).

Recall from the equilibrium analysis of Game 1 (shown in Table 8.10) that s_3, in which MoE *modify* the control order partially to fit with *some* of UR requirements, is a NASH equilibrium with *Extreme* strength level. In Game 2, s_3 is no longer an equilibrium state. This definitely adds pressure on MoE. All the foreseeable equilibrium states for the conflict, under Game 2 configuration, requires MoE to go further in satisfying UR demands for modifying the control order if MoE wants the conflict to end. In Game 1, MoE is in a comparatively better position, because Game 1 has an equilibrium state in which MoE and UR agree on partial modification to the control order that will satisfy some but not all UR requirements.

Furthermore, Table 8.14 shows that Game 2 has a new equilibrium state that is not considered an equilibrium under Game 1. State s_0, the status quo in which MoE insists that UR implement the control order as-is and UR employs all possible delay tactics, is a GMR equilibrium state in Game 2. This has significant impact on how the players are expected to act in the conflict, at least in the interim.

In Game 1, s_0 is unstable for MoE. This makes MoE must move from s_0 to another stable state. It cannot stay for long in s_0. At the same time, s_0 is GMR and SMR stable for UR, so UR is expected to be not in a rush to leave this stable state at least for a while (until UR runs out of delay tactics to employ). In Game 2, on the other hand, s_0 is a GMR stable state for both MoE and UR and SMR stable for UR alone. This makes both players relatively stable at s_0 and not in a rush to move out from this state. This is advantageous to MoE, in comparison to its situation in Game 1, giving it some leverage in the short term. UR will not be happy with this, because this will remove some of the pressure on MoE, in comparison to Game 1, to lift/relax the control order it issued against it.

In addition, the strengths of the three equilibrium states that Table 8.14 shows for Game 2 are also telling. Out of the two long term equilibrium states, which form possible resolutions to the conflict, one state (s_2) has high strength level. The other equilibrium state (s_4) has a weaker strength level. The stronger equilibrium at s_2 will more likely form the end/resolution for the conflict, more so than the weaker equilibrium state of s_4. Therefore, UR should expect that MoE will have harder time going to s_4. On the other hand, MoE should expect to give *serious* concessions. And, by *serious* concessions we mean: MoE will modify the control order to satisfy many if not all UR demands.

But unlike Game 1, in which MoE has no leverage what so ever , Game 2 gives

MoE some room to stay at the status quo state, the newly established GMR stable state for it. We said earlier, when we analyzed the equilibriums of Game 1, that MoE should expect UR to use/abuse the situation as much as possible, because it holds the power to agree on any modified control order. Otherwise, UR can go back to the delay tactics, or it can abandon the site. This is still true, to an extent, but MoE in Game 2 can at least wait at the status quo state s_0 for a while in an effort to put some pressure on UR to moderate its demands. This pressure will most likely come in the form of public relations pressures (locally and at the international stage) and/or operational pressure that could affect the short-term productivity of the Elmira site.

Saying so, one should notice that the equilibrium at state s_0 is: 1) an equilibrium under the GMR solution concept, i.e. MoE is expecting UR to act irrationally in an effort to hurt MoE even if this means that it will hurt itself, a very unlikely scenario in the business world with institutional rationality in place; and 2) the strength of this GMR equilibrium is set to be at the very weak *None* strength level. For these reasons, the status quo could not be expected to form a resolution to the conflict, not even a stable state for more than a short period of time (recall that UR has s_0 as GMR stable at the *None* strength level).

4) Stability Analysis for Game 2 (with s_6 replacing s_5)

Table 8.15 presents the stability analysis of the the six states of Game 2 (with s_6 replacing s_5), for both players, under NASh, GMR, SMR and SEQ solution concepts. Table 8.16 shows the equilibrium states for the game under the four solution concepts.

Recall that Game 2 (with s_6 replacing s_5) is a what-if version of the Elmira con-

Table 8.15: The Elmira Conflict - Game 2 (with s_6 replacing s_5): Stability Analysis

	UR						***MoE***					
	s_4	s_6	s_2	s_0	s_3	s_1	s_1	s_3	s_2	s_4	s_0	s_6
UIs & CIs			s_6 (UI)	s_6 (UI), s_2 (CI)	s_0 (UI), s_6 (UI)	s_0 (UI), s_6 (UI)					s_4 (UI), s_2 (CI)	s_4 (UI)
NASH	Ex	L					Ex	Ex	Ex	L		
GMR	N	N					N	N	N	N	L	N
SMR	N	N					N	N	N	N		
SEQ	N	N					N	N	N	N		

flict in which UR has a stronger case for abandoning the Elmira site, in comparison to the Game 2 configuration. Hence, state s_6 replacing the weaker abandoning case represented in state s_5 of the conflict's model given in Game 2. As in the case of Game 1 (with s_6 replacing s_5), here to it does not matter for Game 2 (with s_6 replacing s_5) whether UR truly has the capabilities to allow it shift all/most of the Elmira production capacity to one of its other international facilities or it managed somehow (e.g. through media reports) to convince MoE and Ontario that it has such capabilities (UR is playing a hypergame). The conflict now, under Game 2 (with s_6 replacing s_5) configuration and assumptions, is different than the conflict under Game 2 configuration and assumptions.

Figure 8.3 and Tables 8.7 - 8.8, presented earlier in Section 8.5.4, show clear differences in the cardinal, and as a result in the ordinal, preferences of UR over the states of the conflict between Game 2 and its variation Game 2 (with s_6 replacing s_5). The figure and tables show that MoE's preferences stayed the same. From MoE's perspective, abandoning the site by UR in s_6 is the same as abandoning it by UR in s_5. But, despite the no change to MoE's preferences , the changes in UR's preferences have significant impact on the stability of the states for the players and for the conflict as a whole.

Comparing the stability analysis of Game 2 (with s_6 replacing s_5), shown in Table 8.15, to the stability analysis done for Game 2, shown in Table 8.9, one will notice some differences. But these differences are only for the stability of some sates to UR. For MoE, the stabilities of the states stay exactly the same, even their strengths stay the same. This was not the case in the comparison we provided earlier between the states' stabilities for Game 1 and the ones for Game 1 (with s_6 replacing s_5). There, we have noticed differences for both players. So, what are these differences that UR have in states' stabilities between Game 2 and Game 2 (with s_6 replacing s_5)?

First, state s_2 in Game 2 is a NASH stable for both MoE and UR, but this state maintained its NASH stability for MoE only in the new Game 2 (with s_6 replacing s_5). State s_2 is not stable at all for UR. In the new modified Game 2, UR is showing itself to be not interested in changing the control order, fully (s_2) or partially(s_3). If UR is truly, or tactically shows itself, capable of abandoning the Elmira site without losing much in operational capacity, revenue generation and demand satisfaction, then UR will be in avery good position to get more concisions from MoE in the conflict. This is similar to Game 1 (with s_6 replacing s_5) where

Table 8.16: The Elmira Conflict - Game 2 (with s_6 replacing s_5): Equilibrium States

	s_0	s_1	s_2	s_3	s_4	s_6
NASH EQ.					L	
GMR EQ.					N	N
SMR EQ.					N	
SEQ EQ.					N	

MoE find itself in the same situation under the same pressure.

Second, state s_6, which replaces in this new game state s_5 used in Game 2, is shown in Table 8.15 to be NASH stable for UR and GMR stable for MoE. For UR, this is the essence of showing the it is seriously considering the abandon-Elmira-site option. It shows that UR has the capability to move on, and that the state which represent this option is stable for UR. The table shows the strength of s_6's NASH stability for UR to be at the *Little* strength level. For MoE, the UR-abandoning-Elmira state s_5 stays as a GMR stable even if this state intensifies in the form of s_6 with UR shows itself to be more willing to take the step to abandon the site. In Game 2 (with s_6 replacing s_5), the strength of s_6's GMR stability for MoE stays at the same very weak *None* strength level it has for s_5 in Game 2.

Table 8.16 shows the overall equilibrium states for Game 2 configuration of the Elmira conflict, under the four solution concepts. As per the table, Game 2 (with s_6 replacing s_5) has two equilibrium states. State s_4 is NASH equilibrium of *Little* strength level, as it is in Game 2. The strength level of *Little* is because s_4's NASH stability for MoE is at the *Little* strength level, despite the fact that UR has s_4 as *Extremely* strong NASH stable state. The second equilibrium state is state s_6 which is shown in the table to be an equilibrium state under the GMR solution concept with very weak strength set at the *None* strength level.

The comparison between the equilibrium states analysis for Game 2 (with s_6 replacing s_5), shown in Table 8.16, and the one for Game 2, shown in Table 8.14, provides us with insights on the expected resolution for the Elmira conflict and the way the conflict is expected to progress. The first thing one will notice is that both games, Game 2 and Game 2 (with s_6 replacing s_5), has state s_4 as NASH equilibrium states for the conflict. This state involves MoE giving that at most concession to UR: lift the control order completely. This equilibrium state is not considered a happy ending for the conflict, from MoE's perspective. On the other hand, this states is the best state that UR could hope for in both games (most preferred as shown in Table 8.7 and Table 8.8).

The second difference is that s_6 an equilibrium state in Game 2 (with s_6 replacing s_5) under the GMR solution concept. State s_6 is not an equilibrium state in Game 2. The fact that s_6 is an equilibrium in this new game shows the UR-abandons-Elmira state is seriously considered as a resolution for the conflict. But we should notice that the equilibrium at state s_6 is: 1) an equilibrium under the GMR solution concept, because MoE has this state stable only under GMR , i.e. MoE is expecting UR to act irrationally in an effort to hurt MoE even if this means that it will hurt itself, a very unlikely scenario in the business world with institutional rationality in place; and 2) the strength of this GMR equilibrium is set to be at the very weak *None* strength level. For these reasons, s_6 could not be expected to form a resolution to the conflict, except as a theoretical non-practical resolution.

The third difference is that unlike Game 2, Game 2 (with s_6 replacing s_5) does not have s_2 as a NASH equilibrium states for the conflict. This is very telling about the conflict. With two equilibrium states, one that has MoE lift the control order, and a second that has UR abandons the Elmira site, MoE finds itself in a very intense situation. UR shows signs that it intends to leave the province, and therefore leave MoE to bear all the fallout of the environmental disaster it caused. In this game, even compromise agreements, where MoE loosens its control order, are shown to be not so exciting to UR (as is the case in Game 2). MoE has to move fast and try to reach an agreement that will give UR enough incentive to sway it from abandoning the site.

The fact that s_0, the status quo state, is not an equilibrium state in Game 2 (with s_6 replacing s_5) as it is in Game 2 forms the forth difference one will notice when comparing between the stability analysis of both games, given in Table 8.16) and Table 8.14) respectively. For UR, the Game 2 (with s_6 replacing s_5) configuration of the conflict is much better than the Game 2 one. It increases the pressure on MoE to lift the control order or drastically change it to meet UR demands. But, this added value comes at a price. The fact that the status quo state s_0, where UR is employing delay tactics, is not an equilibrium state because it is unstable to UR in this game, UR cannot stay for long in this state. Staying at s_0 for a long time could suggest that UR does not actually have the capability to move production elsewhere, as suggested by s_6. UR could be seen as a company that is bluffing in attempt to deceive MoE. This leads to a more damaging public relation for the company and its current and future business in canada and elsewhere.

8.5.7 Results of the Elmira Conflict Analysis

The results of the stability analysis, presented in the previous subsection, provide us with a significant amount of insight into the Elmira conflict. In this subsection, we will compare the findings of the stability analysis for the Elmira conflict with what actually happened in this environmental historical conflict; and with the analysis and findings of others' analysis as provided in the literature. We will also compare the four what-if versions/configurations of the Elmira conflict in terms of how the conflict is expected to evolve over time based on each; discuss which one of these four versions better represent what happened historically; and discuss the benefits of analyzing different what-if models of the conflict.

1) Evolution of The Elmira Conflict Starting from the Status Quo:

Table 8.17 shows the possible scenarios, for each of the four versions/games of the Elmira conflict. Each of these scenarios shows an evolution path that the Elmira conflict could take starting from the status quo state s_0. Recall that s_0 is the sate where MoE insists UR implement the control order it issued as-is and the UR employs delay tactics. Also as a reminder, Table 8.3 shows the differences between the definitions of the four versions, configurations or what-if games which we studied individually in the previous subsections.

For each of these versions of the conflict, Table 8.17 shows how the conflict could progress over time. Starting from the status quo state s_0, and by having the players' take their individual UIs or cooperative CIs out of s_0 until the conflict reach one of its equilibriums (for the specific version of the conflict). For example, Scenario 2 of Game 1, presented in Table 8.18a, shows that the game will move from s_0 by MoE taking its UI to s_4. The game then stops, because it reaches an equilibrium state for the game (under the definition of NASH solution concept). Similarly, Scenario 3 of Game 1, also presented in Table 8.18a, shows that the game will move from s_0 by both players, MoE and UR acting cooperatively, taking their joint CI to s_2. The game then stops, because it reaches an equilibrium state for the game (also under the definition of NASH solution concept).

As indicated when analyzed earlier the four what-if versions/games of the Elmira conflict, the stability analysis for each shows MoE giving major concessions to UR. In fact, Table 8.17 shows repeatedly that the conflict, under all versions of it, could end by a move by MoE from s_0 to s_4 (MoE lifting the control order). If MoE

Table 8.17: The Elmira Conflict: Evolution Scenarios (starting from status quo s_0)

(a) Game 1: Evolution Scenarios

Scenario No.	No.of Steps	Conflict Evolution 0 1	End State		
			EQ	Stability for UR	Stability for MoE
1	0	0		GMR (N) & SMR (N)	Unstable
2	1	0 → 4 (MoE, UI)	NASH (L)	NASH (Ex)	NASH (L)
3	1	0 → 2 (UR & MoE, CI)	NASH (Ex)	NASH (Ex)	NASH (Ex)
4	1	0 → 3 (UR & MoE, CI)	NASH (Ex)	NASH (Ex)	NASH (Ex)

(b) Game 1 (s_6 replacing s_5): Evolution Scenarios

Scenario No.	No.of Steps	Conflict Evolution 0 1	End State		
			EQ	Stability for UR	Stability for MoE
1	0	0		Unstable	Unstable
2	1	0 → 4 (MoE, UI)	NASH (L)	NASH (Ex)	NASH (L)
3	1	0 → 6 (UR, UI)	NASH (L)	NASH (L)	NASH (Ex)

(c) Game 2: Evolution Scenarios

Scenario No.	No.of Steps	Conflict Evolution 0 1	End State		
			EQ	Stability for UR	Stability for MoE
1	0	0	GMR (N)	GMR (N) & SMR (N)	GMR (L)
2	1	0 → 4 (MoE, UI)	NASH (L)	NASH (Ex)	NASH (L)
3	1	0 → 2 (UR & MoE, CI)	NASH (Ex)	NASH (Ex)	NASH (Ex)

(d) Game 2 (s_6 replacing s_5): Evolution Scenarios

Scenario No.	No.of Steps	Conflict Evolution 0 1	End State		
			EQ	Stability for UR	Stability for MoE
1	0	0		Unstable	GMR (L)
2	1	0 → 4 (MoE, UI)	NASH (L)	NASH (Ex)	NASH (L)
3	1	0 → 6 (UR, UI)	GMR (N)	NASH (L)	GMR (N)

does not lift the control order, then it is either expected to: modify the order to fully satisfy UR requirements (end state s_2: scenario 3 in Game 1 and scenario 3 in Game 2); modify the order to partially satisfy UR requirements (end state s_3: scenario 3 in Game 1); or prepare itself to take full control of the clean up effort and full responsibility of all costs associated with this effort because UR abandoned the Elmira site (end state s_6: scenario 3 in Game 1 (s_6 replacing s_5) and scenario 3 in Game 2 (s_6 replacing s_5)).

The table also points to the fact that none of the expected evolution scenarios of the Elmira conflict, not in any of the what-if versions of the conflict, has as an end state UR agreeing to implement the control order as-is and without modifications that address its requirements (s_1). This, in by itself, is a very telling discovery by the analysis and should have made MoE think twice before issuing the control order and put its reputation and the integrity of its control orders at stake. All expected scenarios of how the conflict will progress over time show MoE backing down from its original demands, and/or taking full responsibility to clean up the site itself.

In addition, some scenarios, or evolution paths, shown in Table 8.17 make the case that: it is possible for the conflict to end with UR abandoning the Elmira site (s_6). But, one should notice that this possibility is only valid if and only if UR can abandon the site and move its production elsewhere, satisfying what the Elmira site used to cover in terms of quantity and quality of material produced. If UR cannot abandon the Elmira site without suffering great loss, i.e. cannot be in state s_6, then its only option is to consider s_5 (abandon but suffer on the short term operationally and financially until another site is built and made operational). It is important to note here that if s_5 is the sate considered in the game, then none of the possible evolution scenarios predict an equilibrium that ends with UR abandoning the Elmira site. Such ending is only possible if s_6 is used in the game, i.e. if UR can really abandon the site without much loss in the short-term. Recall that s_6 is used only in Game 1 (s_6 replacing s_5) and in Game 2 (s_6 replacing s_5)). Game 1 and Game 2 has the not-so-solid-case-to-abandon-Elmira state (s_5) instead.

From Table 8.17, one can also observe that the agreement states (s_2 in which MoE modifies the control order to fully satisfy UR demands, and s_3 in which MoE modifies the control order to partially satisfy UR demands) are not equilibrium states when the strong-abandoning-case-for-UR state s_6 is considered in the the conflict. In other words, the agreement states, s_2 and/or s_3, form possible resolution to the conflict only in Game 1 and Game 2 (Scenarios 3 and 4 in Game 1 and

Scenario 3 in Game 2). Neither s_2 nor s_3 can form an equilibrium end state to any scenario the Elmira conflict could follow over time, if the conflict is Game 1 (s_6 replacing s_5) or Game 2 (s_6 replacing s_5).

The status quo state s_0 itself is an important state to analyze its stability for the two players in the conflict. Table 8.17 shows s_0 to form an equilibrium only in one configuration of the conflict. If the conflict follows the what-if version of Game 2, s_0 will be an equilibrium under the GMR solution concept. But one should notice that the strength of this GMR equilibrium is at the very weak *None* strength level. In addition, the nature of this state does not allow it to be a long lasting resolution to the conflict. Recall that s_0 represent a state where MoE insist on UR implementing its control order as-is and UR employing delay tactics. Once UR delay tactics, mostly done through appeals and legal procedures, come an end, s_0 expires and UR must make a move, unilaterally or cooperatively, to another state.

For these reasons, the status quo could not be expected to form a resolution to the conflict, not even a stable state for more than a short-to-medium period of time (recall that UR has s_0 as GMR stable at the *None* strength level). What might drag the stability of s_0 beyond the short-term stability to more of a medium-range stability is the fact that MoE, under Game 2 configuration, has this state GMR stable with strength at the*Little* level (i.e. a bit more stable for MoE than for UR). So, MoE is expected to delay the move out from this state to any state under its control (a UI move by MoE or a CI move that MoE can stop by not cooperating). But, eventually as we can see from Table 8.17, MoE cannot hold on forever to s_0. If UR does not run out of delay tactics, MoE cannot stay too long without mobilizing the clean up efforts, i.e MoE has to move out of s_0. Unfortunately, MoE can only move unilaterally to s_4, lifting the control order and ending the conflict; or it can cooperate with UR reaching an agreement that satisfy *all* UR demands and ending the conflict at state s_2.

Now, which one of the scenarios presented in Table 8.17 reflect what happened historically in the Elmira conflict? And, how an analyst will use the stability analysis and conflict evolution analysis provided above to identify which of the what-if versions of the conflict accurately represent the conflict before the conflict reaches its end? By knowing which game the players are playing, the analyst can better guide and help the player he represents in the conflict. We will discuss all of this in what follows.

2) Comparing our Analysis with What Really Happened:

To compare how the conflict actually evolved, historically, to how our analysis predicts it would, we first need to know the historical time line of this conflict's evens. Table 8.18 shows the time line for the Elmira conflict's events in their historical order, starting from the time MoE issued the control order against UR on December 30, 1989. This event is shown in the table as the first event, and formed the pre-status-quo state. The table provides information about the state that the conflict is at during each of the conflict's events, who made the move to the state (in brackets we show whether the move is a UI/CI move for the mover/s), the step number (or as it could be read: time t+number of steps/moves by the players so far), in addition to comments added to the event's description.

As per Table 8.18, the conflicts enters its status quo state, s_0 (the state which we are analyzing the conflict from), when UR appealed the control order to the Ontario Environmental Appeal Board on January 12, 1990. By this move, UR declared that it will use all legal delay tactics it has available to it. The events that follow, and shown in the table , are based on the documents related to the conflict in the archive of the Ontario Environmental Appeal Board, and based on the Uniroyal Chemical / Crompton Company (Elmira, Ontario) Collection (Laurier Library Archives, Wilfrid Laurier University, 2009).

By looking at Table 8.18, one will notice two important features of the list of events that historically took place in the Elmira conflict. First, the players stayed at the status quo state s_0 an extended period of time (22 months), signalling a relative stability for both players at this state. Second, MoE and UR at the end entered an agreement that satisfies all UR demands. Cameron (1995) lists many concerns raised by locals and environmentalists alike about the agreement: giving UR close to 40 years to clean-up; not requiring UR to excavate buried wastes deep enough; MoE, and Ontario at large, taking big part of the responsibility to do the clean-up and pay a big share of its costs; and so on. In fact, Cameron (1995) cites sources stating that MoE appointed a well-known pro-Unirouyal person to supervise the clean-up efforts, satisfying a demand by UR in order to reach a deal. Clearly, this agreement, and the new MoE modified control order that resulted from it, satisfied all UR demands, and not just part of these demands. This signals the conflict reaching state s_2.

The only scenario, from all those listed in Table 8.17, that satisfies those two features is Scenario 3 of Game 2. It has the conflict staying at s_0 in a relative

Table 8.18: The Elmira Conflict: Analysis vs.Reality

Step	Date	Actual Conflict Evolution	Move by	to/at State
	Dec. 30, 1989	Ontario's Minsitry of Envieronment (MoE) issued a control order against Uniroyal Chemical Ltd (UR), after N-nitroso demethylamine (NDMA) was discovered to contaminate the underground aquifer. The control order came as an Emergency Direction from the Director, Hamilton Regional Office, Ministry of the Environment under s. 32(1) of the Ontario Water Resources Act as amended.	MoE	
0	Jan. 12, 1990	UR submitted its appeal application to the Ontario Environmental Appeal Board.	UR	s_0 Status Quo
	Jan. 18, 1990	MoE added to the control order a Director's Notice from the Director, Hamilton Regional Office, Ministry of the Environment under s. 8 of the Ontario Water Resources Act as amended.	MoE	
	Jan. 19, 1990	UR amended its appeal application submitted to Ontario Environmental Appeal Board in response to the Jan. 18 Director's Notice by MoE.	UR	
	Jan. 26, 1990	MoE added to the control order a Director's Notice from the Director, Hamilton Regional Office, Ministry of the Environment under the Ontario Water Resources Ac.	MoE	
	Feb. 2, 1990	MoE added to the control order a Director's Order from the Director, Hamilton Regional Office, Ministry of the Environment under the Ontario Water Resources Act.	MoE	
	Feb. 9, 1990	UR submitted a appeal application submitted to Ontario Environmental Appeal Board in response to the Jan. 26 and Feb. 2 Director's Notice and Order by MoE, respectively.	UR	
	Aug. 28, 1990	The Director, West Central Region, MoE issued a new control against UR replacing the old order dated Dec. 30, 1989. The new control order was more extensive, setting out procedures for monitoring, sampling and reporting results for DMNA in wastewater going to the Elmira sewage treatment plant.	MoE	
	Sep. 12, 1990	UR submitted a new appeal application to Ontario Environmental Appeal Board with respect to the entirety of MoE's new control order dated Aug. 28, 1990.	UR	
1	October , 1991	The Government of Ontario, as represented by the Minister of the Environment entered into an agreement with UR. In this agreement, MoE and UR agreed to share the work and the costs involved in the cleanup efforts. The agreement also provided for its implementation by the issuance of a new control order amending the order of August 28,1990. This amended control order was issued in November 4,1991. Later the same day, in consideration of the issuance of the amended order, UR withdrew all outstanding appeals. The withdrawals were followed later that day by the issuance of a Revocation Order by the Director, West Central Region, Ministry of the Environment. This new order revoked the orders of all previous MoE control orders, directives and notices against UR had issued before this date. The clean-up was expected to take decades, conclude in 2028, and costs would be shared between the two parties. The province would pay one-third of the capital cost and other one-time costs, along with half the annual operating costs of the treatment system, and UR would pay the balance.	MoE and UR (*CI*)	s_2
				Scenario 3 Game 2

stability (equilibrium under GMR(N): for UR it is GMR(N) and SMR(N) stable, and for MoE it is GMR(L) stable). These strengths of the s_0's stabilities to both players are very telling by themselves. UR has a very weak stability at s_0, while MoE has s_0 stable under a slightly more stability strength. MoE clearly demonstrated this stability in actions during its stay at s_0, as shown in Table 8.18. Also, Scenario 3 of Game 2 has the conflict reaching state s_2 by both players taking their cooperative improvement move out from s_0.

3) Sensitivity Analysis and What-if Models:

Sensitivity analysis within the context of conflict analysis is achieved by tweaking, or modifying, the established base model of the conflict, and checking if such tweaks/ modifications lead to different results than the ones produced by the base model analysis. In the Elmira conflict analysis provided in this chapter, we used what we called different versions, configurations or what-if games, with Game 1 representing the base model of the conflict and the rest (Game 1 (s_6 replacing s_5), Game 2, and Game 2 (s_6 replacing s_5)) representing various variations of the base model.

An analyst, whether working for MoE, UR or independently, will not be initially sure about certain aspects of the players: how they prioritize their goals, whether certain alternatives are considered real/serious options for their respective players, and so on. In our case, and as analysts modelling the Elmira conflict, we were not sure about: 1) whether the economic crisis of the early 90's will make MoE give more importance to the goals of the Ontario government dealing with the recession and the downturn that resulted from it, more so than its own goals, rules and integrity of its orders; 2) whether the short-terms operational and financial needs of UR will have more importance to it than its long-term goals; and 3) do UR has a real option to abandon the Elmira site after the environmental disaster was uncovered?

To deal with the uncertainties around (1) and (2), we decide to build Game 2 as a second what-if model for the conflict, as opposed to what we considered the base model of the conflict, Game 1. In Game 1, both UR and MoE treat all their strategic goals similarly, with all fully important. In Game 2, MoE treats Ontario government needs as more important than its own, at the same time that UR treats its short-term goals more important than its long-term ones. For an illustrative case study, as ours, we think these two variations are sufficient enough. In reality, an analyst will try at least to check more variations, such as while MoE differentiate between its needs and the needs of the government as a whole, UR has its short-term goals to be as important as the long-term ones.

In addition, to deal with the uncertainty around (3), whether UR has a real option to abandon the Elmira site, we added a new alternative A_{UR6} for UR to abandon the site and move its operation somewhere else with less impact on its bottom line and production needs in the short term. This new alternative A_{UR6} is similar to the old A_{UR5} except in A_{UR5} UR is expected to suffer in the short term because it cannot move the operation elsewhere. If it abandons Elmira, it needs to build a production facility somewhere, therefore it will suffer financially and operationally in the short term. Recovery is uncertain and will be very slow, as opposed to UR taking A_{UR6}, if and only if A_{UR6} actually exist as a real option for UR. So, absent of any indication of whether UR has A_{UR5} or A_{UR6} as its abandoning-Elmira-site option, as analysts we decided to consider variations for our both Game 1 and Game 2 models that will have A_{UR6} instead of A_{UR5} . Recall that states of the conflict are shaped by the alternatives the players have in the conflict. Therefore, the states that Game 1 and Game 2 have will be different than the ones that the new variations for them will include. Hence, the names of these two additional variations for the conflict: Game 1(s_6 replacing s_5) and Game 2 (s_6 replacing s_5).

After the analyst establishes his base model and all its what-if variations, then he will analyze all of these models as we did. But, he cannot stay still waiting for the conflict to end to know which model variation of the conflict that the conflict resembles the best in reality. A good practice, by a good analyst, is monitor the conflict closely as it unfolds for signs to see which model variation the conflict is going towards, before the conflict ends. At the same time, this monitoring activity might uncover new options, new players, new goals, new constraints, and so on. In such case, the analyst must move into action establishing new models, or what-if variations, to test the implication of the newly uncovered facts about the conflict by conducting thorough analysis of these new models.

In our case study, and if we were analysts modelling the conflict at the status quo state, i.e. during January, 1990, then we would analyze the four variation of the conflict we identified, exactly as we did in the previous subsection. But, as the conflict unfolds, we would look for the following:

- *Signs for A_{UR6} as a areal option to UR:* The existence of this option as a real and possible option that UR is seriously considering will shift the Elmira conflict from being either Game 1 or Game 2 conflict, to more of Game 1 (s_6 replacing s_5) or Game 2 (s_6 replacing s_5) conflict. It is important to note here that if UR signals through staged PR campaigns, or by any other means, that it is considering the A_{UR6} option seriously, but shown signs of relative stability at s_0 and not moving fast to adopt A_{UR6} then this puts the conflict back as Game 1 or Game 2 (Table 8.17 shows clearly that the existence of A_{UR6} through state s_6 is always accompanied by having state s_0 to be unstable for UR).

- *Signs of stability exhibited by the players at the status quo state s_0:* A stability that the players is showing at s_0 beyond the few months range, especially if accompanied by MoE not showing signs to back down (knowing that it should in order for the conflict to end), this shows a leaning toward Game 2 or Game 2 (s_6 replacing s_5) more so than Game 1 and Game 1 (s_6 replacing s_5), as shown clearly in Table 8.17.

As analysts modelled and analyzed the four versions of the Elmira conflict during january 1990, and looking for these signs, we would have notice the following:

- *No strong signs for A_{UR6} as a areal option to UR :* Although UR threatened to leave the site during its early communications with MoE official (Cameron, 1995), these threats never materialized. In fact, UR did not show that it is seriously considering this option. No media reports of UR moving, even partially, some of its production needs to a different plant somewhere else. Not to mention, the second test of UR behaviour that will lead us to determine if A_{UR6} really exists: how much stability UR is exhibiting towards s_0. Any observer monitoring the situation at the time will be amazed by the effort UR is showing to stay at s_0: employing all available legal means to delay implementing MoE's control order; questioning test procedures and the acceptable levels of DMNA; lobbying the locals at the city; lobbying the business bodies; questioning environmental reports; overloading the appeal procedures with discussions and requested related to details and procedures; and so on. At the same time, no serious effort to shift operation elsewhere. This makes the conflict, based on our analysis, to be a Game 1 or Game 2, eliminating the possibility of the conflict being a Game 1 with s_6 replacing s_5 or Game 2 with s_6 replacing s_5.

- *Signs of stability exhibited by the players at the status quo state s_0:* We talked above about the stability that UR showed in staying at state s_0. But, MoE too demonstrated stability staying at s_0. The hesitation of MoE to negotiate a deal with UR, and instead the demonstrated on-going effort by MoE at the time to be bogged down with what seemed endless requests to enhance and clarify its procedures and policies. Not to mention, the lobbying effort MoE was doing among the locals and environmental groups in an effort to distract the public from its lack of ability to reach a resolution with UR, start the clean-up, or take decisions. To be fair to MoE, it was in a very tough situation, any decision it will take will be heavily criticized. Environmental friendly policies and decisions are always criticized by economists and businesses, this is more so at tough economical periods, and vice versa. Nevertheless, MoE, by feeling stable at s_0 and not consider it an unstable state, defined the conflict as a Game 2 or its variant Game 2 with s_6 replacing s_5.

From above, an analyst, with few months passed in the conflict and way before MoE reached an agreement, could justifiably say that the conflict is going the Game 2 model way. If he is working for MoE, he might prepare his employer that the conflict has two absolute equilibrium states: s_4 and s_2. Both of which are not in favour of MoE. Therefore, MoE must work to achieve the better of both, i.e. s_2, and try to use the time in enhancing the the agreement terms and make the clean up effort start early. If the analyst is working for UR instead, then he might suggest that UR continue the delay tactics. This is if we put aside the ethical obligations that UR should feel regarding the environmental disaster they caused. Something we are personally and completely against, but for the sake of the argument we assume UR would do (focus rather on purely business side of things) –which in fact what UR unfortunately demonstrated in reality and historically speaking with their handling of the conflict–. By continuing the employment of all possible delay tactics, UR guarantees to put more pressure on MoE to come to an agreement that will satisfy all UR demands, or lift the control order completely. MoE cannot afford to delay the clean up effort indefinitely.

The Elmira conflict analysis, we provided in this chapter, clearly shows the lose-lose position governments, environmentalists and affected local citizens, are doomed to face when an environmental disaster occurs because of the ill-practices and irresponsibility committed by an international company operating within their territory. Absent any clear international laws to force them do the clean up immediately and be responsible themselves, their holding/owning companies, and their subsidiaries everywhere in the world for these such disasters, the world will continue to see case after case of polluters get away with what they cause, either by getting what they want (favourable terms subsidized by tax-payers money), or by them leaving the affected place and operating in a new place as if nothing had happened. Example after example show that this what happens in the absence of international laws and policies to deal with such matters. From the UR caused disaster at Elmira Ontario in 1989, to the Union Carbide caused disaster at Bhopal India in 1984, to the disasters caused by oil companies in Nigeria. Compare this to what happened recently to the environmental disaster caused by Shell in the Gulf of Mexico in which the US, motivated by the politics of the time –just before mid-election time– and by its economical influence around the world, forced Shell to immediately move into action committing itself to start its clean up efforts and try everything possible to solve the problem.

4) Comparing our Model and Analysis with Others' Work:

We will look at how our model and analysis of the Elmira conflict compares to the models and analysis of Kilgour et al. (2001). Kilgour et al. used the Graph Model for Conflict Resolution (GMCR), introduced in Fang (1989) and in Fang et al. (1993), to model and analyze the conflict. GMCR provides normative models that is based on an ordinal representation of the players' preferences over the states of the conflict, not a cardinal one. GMCR is based on solid mathematical grounds and provides many analytical tools, building on the work of Howard (1971) and the work of Fraser and Hipel (1984), adding a graphical representation to the players moves, but suffers from some of the many weaknesses typical game theory methodologies suffer from (see our discussion in Chapter 2).

First, the work provides no means to model the mapping between the preferences, and their ordering, and the strategic objectives the players have within the context of the conflict, or in general. For example, the model of Elmira conflict presented in Kilgour et al. (2001) does not show or explain why the the players' preferences are the way given in the model, and what should change, in terms of players' goals/priorities or realities on the ground, to make the preferences have different order. Comparatively, in our models we show the direct link between the players goals and the conflict realities to how the players' preferences are modelled, or elicited as should be noted. In fact, any change to the players' goals, priorities, internal or external constraints/realities will be reflected on how these preferences get ordered. The analyst can immediately test why a state is more/less preferred than another for a certain player, and what should be done to reverse this order or change it. In our models, we showed how changes in the strategic goals of UR and MoE changed the preferences of both, and leading to a complete set of what-if games to test.

Second, the ordinal representation of the players' preferences does not show any degrees of preference's strength beside the simple binary relations the ordering is based on: indifferent $\sim$ relation and the preferred-to $\succ$ relation. The work of Hamouda et al. (2004) tried to rectify this weakness by proposing degrees of preference added to mainly the the preferred-to $\succ$ relation, offering relations such as $\succ\succ$ to show much-preferred status. Still, all these relations do not show "why" the degree of preference existed in the first place, and how it will affect the satisfaction of the player's strategic goals. The relations presented in Kilgour et al. (2001), similar to the ones presented by Fraser and Hipel (1984) and the modified ones in Hamouda et al. (2004), are all based on strict logic: yes or no, applies or does not apply. On the other hand, our models provide a one complete preferred-to relation that: 1) has a fuzzy qualifier to show the degree of strength for the preference, in addition to the preference order; and 2) offers traceability of the preference order and degree of strength back to the player's strategic goals and realities, answering why and how the preference came to be the one elicited. In our models, the analysis does not take the player word or the analyst word, or more accurately their guesses as a justification for the preferences order and strength. The justification, and validation, process is embedded within the way the preferences are elicited and represented.

For example, in the Elmira conflict's model presented by Kilgour et al. (2001), UR is shown to prefer the state in which UR employes delay tactics (state 1 in their model, and state s_0 in our model) more than the MoE-Modify-Order-and-UR-Accept state (state 4 in their model, and state s_2 or s_3 in our model, based on whether the modifications made by MoE to the control oder satisfy UR demanded modification fully or partially, respectively). A very strange assumption to make. As typically done in the game theory literature, and similar theories literature, Kilgour et al. (2001) dos not mention any reasons for this preferences order, nor any justification for such assumption. In reality, delay tactics are employed for a reason, are temporarily in nature, and cannot be assumed to form an indefinite solution. So, can one assume the preference of the delay state over reaching-a-modified-control-order state to be a logical one? and if it is logical, under what context? The model does not answer any. As a second example, the model given by Kilgour et al. (2001) shows UR preferring the Abandon-Elmira-site state (state 9 in their model, and state s_5 or s_6 in our model, based on the version of the conflict and the ability of UR to find an alternative facility elsewhere to carry the burden of covering for the Elmira production) to the state in which UR accepts MoE original control order. Same questions apply here: is this logical for a business? under what context? why? for what reason?

On the other hand, one can immediately answer why UR has the modified-agreements states (both modified-fully and modified-partially states) to be preferred to the delay state; and why UR in some version what-if models of the conflict has the Abandon-Elmira-site state is preferred or less-preferred than accepting MoE's original order with no modification. The preferences elicitation tables (shown in Figure 8.3 for all versions/games of the conflict we analyzed) present how each state of the conflict contribute to the achievement of each of the players' strategic goals. Hence, the preferences are fully justified by: how much each state help the player achieve his goals. Not only this, our models go further to show the strength of such preferences, by calculating the weighted preferences, ordinal preferences, and preferences with linguistic fuzzy qualifiers (as given in Figure 8.3 and Tables 8.5 - 8.8 for all versions of the conflict we studied). So, even the strengths given to the preferences are justified. Now, the analyst, and the analysis-sponsoring decision makers, will know exactly why the models, and the analysis based on them, are the way they are presented. No hidden, unexplained, or unjustified assumptions exist. Once, the analysts notice any deviation on the behaviour of one player, or more, from what is given in the models, then he can just build a what-if or a variation model to test the implications of such deviation on the analysis end results.

Third, because of the way GMCR used by Kilgour et al. (2001) defines the State concept and the UI move concept, some of the states and UIs presented in their model of the Elmira conflict are not realistic or feasible. For example, the model presents and includes in its stability analysis two states in which the MoE modifies the control order, while UR does not accept the modifications but rather employs delay tactics. A highly unlikely state of such conflict. What these two states describe is an extreme form of failure in logic and rationality by MoE, if such states happen. MoE loses the credibility of its control order for no gains at all from the other side. In reality, opponents enter a negotiating step. By the end of this step, they either have a binding agreement, or they don't. MoE would not modify its order and then wait for UR to accept or not.

Forth, Kilgour et al. (2001) fails to recognize the concept of cooperative moves, which we explained earlier in this chapter. That's why their model and stability analysis failed to predict that the agreement between MoE and UR forms an end of the conflict. In their paper, they proposed an addition to the GMCR framework to correct this err in prediction. The paper proposed coalition analysis as a solution. The paper suggested that MoE and UR formed a coalition, allowing them to jump from one step to another in a multi-step sequence of moves to reach a favourable state by all parties. The problem with this approach is two folds:

1. the concept of a coalition or an alliance cannot describe in any form or shape a relationship between a governmental agency, who's responsibility is to enforce the laws, and a private enterprise company, who ignored the laws and created an environmental disaster. The idea that MoE and UR formed a coalition, against the local governments, will be troubling to rationalize for the players, the people of Ontario, or even general observers and analysts.

2. the concept of a multi-step moves by a coalition is offered to handle a shortcoming of the GMCR's framework: the inability to recognize players cooperating in a single move. Cooperative moves are moves that players take jointly to advance their position in the conflict to a more preferred state for all. A natural concept that is implemented all the time in real-life conflicts. For example, the MoE and UR move to negotiate an agreement and reach one is a joint cooperative move that none of the players can do on their own.

At the heart of this GMCR's shortcoming is the fact that the framework defines: states as binary numbers with 1's and 0s' representing players selecting or not selecting, respectively, strategies form the conflict's set of players' alternatives; and player's moves are individual moves between states with the player's alternatives-selection bits are changing from 1's to 0's or vice versa. GMCR, as defined by Fang (1989) and in Fang et al. (1993), still loyal to its origins as a framework build on the work of Fraser and Hipel (1984), which we mentioned when we compared their Cuban Crisis model and analysis with ours in Chapter 6. We said then that Fraser and Hipel (1984), and similarly GMCR, have a problematic UI moves representation in their models. Such problem is shown in their models because the conceptual modelling foundation of the Fraser and Hipel (1984), and GMCR by extension, is putting mathematical representation in the driving seat. A UI move, to them, happens when all the bits, of the states' binary numbers, for one player is changing from less preferred state to a preferred state, while the bits of the other player are not changed.

While GMCR tried to add the directed graph representation to solve some of the problems that the work of Fraser and Hipel (1984) suffered from. Still, the representation is less concerned with the validity of such move, or the ability of the player to take it in real-life. So, when it comes to cooperative moves, the framework was/is not capable of representing such moves unless through coalition's multi-step moves, even though the concept of coalition could not describe/explain the relationship among the involved parties, such as the case of the MoE and UR in the Elmira conflict. Not to mention, the fact that some of these expressed as multi-step moves by Kilgour et al. (2001), such as MoE and UR reaching an agreement, could not realistically be called multi-step moves.

In our conceptual modelling framework, we do not have such problems. The analyst in our case is in the driving seat. He will, and should, examine the validity of each move. In our models, and consistent with a realistic view of how things happen in real-life, a UI move is a real unilateral move the player can take to enhance his position, and a CI move is a joint cooperative move that a number of players take at the same time to enhance the position of each one of them in the conflict. A CI move cannot be reached by the individual players on their own. They must cooperate to do so. The automated DSS system implementing our framework, as a knowledge modelling and management framework, demands justification of the beliefs held by the analyst about the ability of the players to make all types of moves (UMs, UIs, CMs and CIs). It cannot sacrifice validity in favour of ease of automation and reliance on binary value flipping as a justification.

Fifth, in the Elmira conflict model provided by Kilgour et al. (2001), the model shows three players: MoE, UR and the Local Government (LG, a collective player representing the Township of Woolwich and the Regional Municipality of Waterloo). By the end of their analysis for the conflict, Kilgour et al. (2001) showed no effect at all to including LG as a player. In fact, the end result of the conflict historically showed that the conflict was between MoE and UR and ended with an agreement between MoE and UR.

This failure in recognizing the real players in the conflict could have been avoided by building a GCM model for each of these proposed players. We actually built GCM model for LG, and immediately realized that all LG can have as goals, or do as options are wishful thinking. LG's GCM model could not affect the GCM's of neither MoE nor UR. The only interaction that LG's GCM model has with other players' GCMs is to represent: the other players try to get LG's support for their positions, for public relations reasons, and nothing more. The conflict between MoE and UR is: at the provincial level; and it is a legal and policy conflict. So, we eliminated LG from the conflict model we presented, and our prediction matched the real historical flow of events happened on the ground. The local governments did not have any effect on the resolution of the conflict. The analysts could test the viability of including or excluding any player in a conflict model, through the use of what-if variant models to the base model. They will be able to identify a real player in the conflict from the players who have no influence in the conflict and its outcomes at the GCM modelling phase of the process.

Finally, in our opinion, the Elmira conflict's GMCR model and analysis provided by Kilgour et al. (2001) failed to provide any context of the conflict: the fact that the conflict happened at the late 80s and early 90s where recession and economical hardships described the situation for both the government of Ontario and Ontario's citizens at the time. The conflict's model of Kilgour et al. (2001) did not capture this context, nor it did show the effect of such reality on the players' preferences. In addition, when Kilgour et al. (2001) suggested that there was a coalition forming between MoE and UR as a justification to explain why the GMCR failed to predict the agreement between both players as an end state to the conflict (and as a result proposed a coalition analysis add-on), one should stop and wonder: *How can the government of Ontario ally itself with an Environmental-Disaster-Creator such as UR? And how the proposed conflict's model could rationalize the fact that the Ontario Government at the time, and the one who sighed the final agreement with UR, is an Ontario New Democratic Party (ONDP) Government, the most environmentally friendly political party that Ontario had at the time and the only ONDP government Ontario had, so far.*

While the models presented by Kilgour et al. (2001) fails to answer these two questions, our framework, and the Elmira conflict's models and analysis we provided, presented the answer: the economical hardship at the time forced the government to put saving jobs at the head of its agenda. Keeping businesses in Ontario, and showing that ONDP is also a business friendly party, was second. An agenda, in addition to many other reasons, may have contributed to the big failure the party faced in gaining the people's trust and votes in the following election.

In summary, when we compare the work of Kilgour et al. (2001) to our work presented in this chapter, we used how Kilgour et al. modelled the Elmira conflict as an example to illustrate the differences. While both works rely on mathematical and logical modelling and analysis schemes, they differ in many areas. The work of Kilgour et al. (2001), as most of the game theory methodologies in the literature, does not model the decision makers' goals and realities, and therefore do not provide direct mapping between how the preferences over the conflict's states happen to be modelled and how these states satisfy the strategic needs and wants of the players, or get affected by the realities on the ground. This important feature is at the heart of what we believe should be the starting point of modelling strategic conflicts. Dealing with strategic goals' satisfaction through proxies such as stated preferences order or deceiving subjective utilities, while makes the mathematical presentation looks simpler and nicer, violates the principal of rationality employed by the normative decision analysis literature: choosing the best option that satisfies/satisfices the goals of the decision maker.

We also differ in the way how states, players moves, as well as preferences relations and structures, defined and presented. And in relation to the type of games this chapter is concerned about, Cooperative Games without Coalitions, GMCR which Kilgour et al. (2001) uses and adds on, does not accommodate the concept of cooperative moves made jointly by players who cannot take the moves on their own without cooperation. A concept, we illustrated in the case study of the Elmira conflict the usefulness and practicality of it in modelling real-life conflicts. We also discussed how the models and analysis we provided in this chapter resulted on more insights about the conflict's dynamics and better analysis of the players' options. In addition, how these models represented not only the players, their goals, their options, but also the context of the conflict at the time whether through the use of the concept of constraints, or by including the "right" strategic goals the players have at the time, similar to what we did in the Elmira conflict's models.

8.6 Summary

The chapter discussed the analysis of cooperative multi-agent games, as per the Constrained rationality framework. It started by defining the type of moves the players of cooperative games, without coalitions, are allowed to make. Then, the chapter provided definitions for the four different stability and equilibrium solution concepts, which were defined for the non-cooperative games in Chapter 6, but here for cooperative games. These concepts guide the stability analysis of each of the games' states, for each of the players in these games. The chapter, then, defined the strength of the stability under such solution concepts, and proposed a set of algorithms to help identify the strength level of each of these stabilities.

The chapter finished with a case study in which the concepts and methods proposed in this chapter are applied. In the case study, we analyzed thoroughly the Elmira Groundwater Contamination Conflict, a 1989 environmental policy conflict between the provincial government of Ontario and Uniroyal Chemical Ltd. We started by giving a brief background on the conflict and the players. We, then, modelled the players goals, constraints and alternatives; analyzed their GCMs; identified the conflict's states; elicited the players' cardinal and ordinal preferences over these states; and then identified the players unilateral moves among these states. Next, the stabilities of the conflict's states were analyzed under the four stability solution concepts, and the strength of these stabilities were identified. We looked at the equilibrium states for the conflict; and how the conflict could have evolved over time under different scenarios. We concluded the case study by showing how our analysis results compares to what historically happened in the conflict, and to what others offered as models and analysis to the conflict, after the fact.

The following chapter takes the same concepts and methods (moves, stability solution concepts, strength of stability under these concepts, etc.) defined in this chapter for cooperative games with no coalitions, redefines these concepts and methods and extend them to fit with the characteristics of cooperative games with coalitions and their analysis needs.

Chapter 9

Coalitions Analysis and Stability Solution Concepts for Cooperative Strategic Conflicts

9.1 Introduction

This chapter discusses the analysis of cooperative multi-agent games with coalitions, i.e. cooperative games in their broadest configuration possible. The decision makers of this type of games are able to make move types that are not limited to the unilateral non-cooperative moves and the cooperative one-step moves, which players of normal cooperative games discussed in Chapter 8 can perform. Decision makers of cooperative games with coalitions are also allowed to have multi-step coalition moves. A coalition move can be: a one-step unilateral move by one member of the coalition, a one-step cooperative move by a subset of the coalition members, or a number of these unilateral and cooperative moves performed consecutively forming a multi-step cooperative move.

The decision makers in these games can stay independent players, or join any of the coalitions available in the game. Therefore, they can act individually and move unilaterally among the games' states; or cooperate with other individual and/or coalitions moving from one state to another, if this one-step move is beneficial for the cooperating decision makers. They also can join coalitions in which they will have much flexibility in their moves. Coalitions have aligned objectives and powers, and therefore can have sophisticated multi-step moves in an effort to advance forward the coalition members collectively toward a more preferred states, or to sanction UIs of others who are not part of the coalition. These multi-step coalition moves are unique to the games discussed in this chapter: the cooperative games with coalitions.

We will start by looking at the type of moves the players of the cooperative games, with coalitions, are allowed to make, and are important to the stability analysis concepts. Then, we will define, for these games, the same four different stability and equilibrium solution concepts which we defined for the non-cooperative games in Chapter 6 and for the cooperative-without-coalitions games in Chapter 8. These concepts will guide the stability analysis of each of the games' states, for each of the games' players. Next, we will define the strength of the stability under such solution concepts, and propose a set of algorithms to help identify the strength level of each of these stabilities.

We will finish the chapter with a case study in which we apply the concepts and methods proposed in this chapter. In the case study, we analyze thoroughly a strategic business conflict over intellectual property rights. The case will examine whether it is worth fighting a patent troll. We use the showdown between Research and Motion, the prosperous wireless email technology and product innovator, and NTP to help us answer this question. We start by giving a brief background on the conflict and the players. We, then, model the players goals, constraints and alternatives; analyze their GCMs; identify the conflict's states; elicit the players' cardinal and ordinal preferences over these states; and then identify the players unilateral moves among these states. We also examine the possible coalitions that could be formed in the game. The stabilities of the conflict's states will be analyzed under the four stability solution concepts, and the strength of these stabilities will be identified. We will look at the equilibrium states for the conflict; and how the conflict could have evolved over time under different scenarios. We will analyze the effect of the coalition formations on the conflict, its stabilities, and its equilibrium states. We conclude the case study by showing how our analysis results compares to what historically happened in the conflict.

In terms of the notation used in this chapter it will be the same as the one used in Chapter 6 and Chapter 8, included here as a reminder. Let the set of all the game states be given as $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where $|\mathcal{S}| = m$ the total number of states in the game, and states are defined as discussed earlier. And let $\mathcal{S}^{a,b,\ldots}_{D,t} \subseteq \mathcal{S}$ where $\mathcal{S}^{a,b,\ldots}_{D,t}$ represents a subset of $\mathcal{S}$'s states which has common characteristics described in the subset's notation as $a, b, \ldots$ as been perceived by decision maker D and at time t. The set of decision makers in the game is given as $\mathcal{DM} = \{DM_1, DM_2, \ldots, DM_n\}$, where $|\mathcal{DM}| = n$ the total number of decision makers, the involved agents/players, in the game.

Also as a reminder, we use the terms game and conflict interchangeably to mean the same thing: a multi-agent strategic conflict. Also, the terms agent, player and decision maker will be used interchangeably to mean the same thing: an autonomous independent agent, in the strategic conflict, who is capable of perceiving the world around, holding beliefs, justifying beliefs, holding knowledge, representing knowledge, extracting new knowledge, reasoning about held knowledge, and acting independently.

9.2 Coalition and Coalition's Preferences

As explained in Chapter 8, the decision makers within the context of cooperative games, without coalitions, are all considered individual players of the games. They pursue their own individual interests in the games, and look only after themselves. The only difference between a decision maker within a cooperative games, without coalitions, and a player of non-cooperative games (discussed in Chapter 6) is that the former has the ability to cooperate with other players in moving from one state of the game to another in one step. The motivation for this single-step cooperative moves could be the fact that the moves' destination state is more preferred to them than the start state; or that their desire to hurt some other player of the game is more important to them collectively to the point they are willing to hurt themselves (move to a less preferred state –for them or for some of them–) to accomplish this. Examples of such cooperative moves could include entering negotiation, signing an agreement, entering war together against an enemy, breaking a signed agreement, breaking an established partnership, signing an intellectual property licensing agreement or cancelling one.

In cooperative games, with coalitions, those individual players are still considered as players here too. They are allowed to take individual unilateral moves, as well as cooperative moves. Their motivations are also the same in these games as the ones they have in non-cooperative games and cooperative games, without coalitions. They could cooperate here too for the purpose of benefit themselves by advancing to a more preferred state together. Their purpose for cooperation in single-step moves could also be to hurt some other player regardless of whether these moves will benefit them or hurt them.

But, in cooperative games, with coalitions, a new type of decision makers is considered legitimate. In these games, players could joint together and setup a coalition or an alliance. This gives the coalition players collectively the power to coordinate a multi-step moves for the benefit of the coalition and its members, or to hurt other non-coalition members. We define here what a coalition is, and how the coalition preferences over the game's state will be calculated. Before doing so, and as a reminder, one should note that the Constrained Rationality's definition of a coalition and that of the coalition's preferences are different from that of the GMCR framework's ones (proposed by Kilgour et al. (2001) and Inohara and Hipel (2008b,a)).

9.2.1 What is a Coalition?

Definition 9.2.1 (**Coalition**): *A Group of Decision Makers* $DM_C \subseteq \mathcal{DM}$*, at time t of the game, is considered a Coalition iff* DM_C *is an official coalition/alliance, a known unofficial coalition/alliance among group of decision makers, or a subset of the game's decision makers* $\mathcal{DM}$ *who their intent to move from one state of the game to another or more (could be because this will be benefit all or the majority of* DM_C*'s members or for other reason may fit* DM_C *collectively) is made known/declared (by means of an agreement, voting, etc.) or believed to be true (with justification).*

This definition of a coalition considers permanent long time alliance a coalition, and as well it considers a temporarily established coalition of decision makers a coalition. It also considers a well known highly publicized alliance to be a coalition. And at the same time, it considers a believed-to-be-true but-never-been-declared coalition among some decision makers to be a coalition as long as their is a justification for this belief by the knowledge holder, the modeller or the focal decision maker whom the modeller is capturing his viewpoint of the world. This demand for a justification is to be consistent with how knowledge is defined within ViVD-EKM (Al-Shawa, 2006b) and to ensure that "why" the viewpoint holder believed this coalition existed is captured in the knowledge-base.

As we will see in the next section, a coalition of decision makers has the ability to commit a multi-step moves which none of the decision makers can do on his own, or cooperate with others to do if they are not part of a coalition.

9.2.2 Coalition's Preferences over Game's States

In Section 5.4.2, of Chapter 5, we discussed how an individual decision maker's ordinal and cardinal/weighted preferences over the game's states is elicited and modelled. So, how the preferences of a coalition made up of a group of decision makers, each with his/her own preferences, are calculated? The answer to this question is: it depends.

Ideally, a coalition is established among a group of decision makers that have aligned strategic goals in the game. As a result, the preferences should match. But, we all know that this type of a coalition does not exist in real-life. And even among close partners, strategic goals never found to be fully aligned or matches. Therefore, traditionally coalitions, alliances, parties, and similar groups setup mechanisms by which the "best-alternative-for-all", or the all-agreed-upon-ranking for all alternatives, is defined within its charters or articles of incorporation. Definitely, as in the case of individual decision makers, a coalition's preferences over the game's states is decided by how they rank their alternatives. As we saw earlier, a game's states are made from the alternatives all the game's players have.

Mechanisms such voting is very popular within such bodies to choose alternatives, or rank them. But as we know, voting is not by all means the one and only, or the at-most fair mechanism. The rules used in voting to declare the "best" (such as the majority rule), or treating different players within the coalition differently (because of their size, market share, the time they joined the coalition, their investment in the coalition, etc.) could cause unfairness in the process of choosing alternatives. For example, in standards alliances within the telecommunication technologies domain, big players have a bigger say on what the alliance decides to be "the standard". Smaller players, or startup companies, are at a disadvantage. So, the alliance choice is not necessarily "the best" choice for all.

But, regardless of the fairness of the mechanism which the coalition adopts to decide on what is the "best alternative" for the coalition, or to decide on the ranking for the alternative that the coalition have collectively, the most important thing for the coalition members is that: there is a mechanism for the coalition to decide, collectively, on their choices. And as for modellers and analysts of the coalition in a game, they now have a way to know how the alternatives contribute to the coalition objectives, or how the alternatives been ranked for the coalition. Therefore, the modellers and analysts have a method for this coalition, and almost every coalition, to elicit the coalition's preferences over the game's state.

In our notation, when we say that $s_1 \succ^{Big}_{DM_i,t} s$, we know that this means that $PrefStrength(s_1, s, DM_i, t) = Big$. Similarly, if $s_1 \in \mathcal{S}^{\geq N}_{DM_i,t}(s)$, then we know that $PrefStrength(s_1, s, DM_i, t) \geq None$. This is easy because DM_i is a single decision maker. So, for a coalition DM_C, what does it mean to say that $s_1 \succ^{Big}_{DM_C,t} s$ or that $s_1 \in \mathcal{S}^{\geq N}_{DM_C,t}(s)$, knowing that DM_C has many member decision makers part of it?

$s_1 \in \mathcal{S}^{\geq N}_{DM_C,t}(s)$ could mean that state s_1 is preferred over state s, with strength level greater than or equal to the *None* strength level, for every decision maker member of the coalition DM_C. In other words, $\forall DM_i \in DM_C \; s_1 \in \mathcal{S}^{\geq N}_{DM_i,t}(s)$. But, this is a very simplistic assumption, because we said coalitions tend to have sophisticated mechanisms by which they decide their preferences over alternatives and by extension over states.

Consider for example an alliance DM_A with two classes/groups of members, the first class DM_{A1} has the elite big-in-size founding members, and the second class DM_{A2} has all other smaller players who joined after the alliance was established. Let us assume that the elite members have the voting powers and that all must have the same preference, order and strength level, for any state the alliance is considering moving in or out from. Now, for alliance DM_A, if $\forall DM_i \in DM_{A1} \; s_1 \in \mathcal{S}^{\geq N}_{DM_i,t}(s)$ and $\forall DM_j \in DM_{A2} \; s_1 \notin \mathcal{S}^{\geq N}_{DM_i,t}(s)$ then we can clearly say that statement $s_1 \in \mathcal{S}^{\geq N}_{DM_A,t}(s)$ is true. What about the fact that DM_{A2} does not prefer s_1 over s? DM_{A2} has no say. It is only DM_{A1} who has the power to decide on the preferences, as per the alliance setup.

In the absence of having one universal way that coalitions have to decide on their preferences, we will assume in this chapter the ideal case: If $s_1 \in \mathcal{S}^{\geq N}_{DM_C,t}(s)$, then $\forall DM_i \in DM_C \; s_1 \in \mathcal{S}^{\geq N}_{DM_i,t}(s)$, and vice versa. And, if $s_1 \succ^{Big}_{DM_C,t} s$, then this means that $\forall DM_i \in DM_C \; PrefStrength(s_1, s, DM_i, t) = Big$. In other words, all members of the coalition must agree on the preference, its order, and its strength. This should not be considered in any way as a restriction on the framework, but rather a convenient simple way to refer to coalition's preferences. In reality, the modeller should take note of how the coalition actually decides on its preferences, capture it in an algorithm, and then use it in the stability definitions which are proposed in this chapter.

For the most part of this chapter, we will use the generic notation $\mathcal{S}^{a,b,\ldots}_{DM_C,\,t}$ to represents a subset of $\mathcal{S}$'s states which has common characteristics described in the subset's notation as $a, b, \ldots$ as been perceived "collectively" by the coalition DM_C and at time t. The modeller/analyst job is to ensure that the "collectively" part is qualified to be true by applying the algorithm/mechanism/method that the coalition actually uses to agree on these common characteristics and ensure that it reflects the coalition's true state of affairs .

9.3 Types of Decision Makers' Moves

Decision makers in cooperative games, with coalitions, can have either individual unilateral moves, cooperative one-step moves, or coalition multi-step complex moves made of a combination of unilateral and/or one-step cooperative moves. First, we define the unilateral moves individual players can have. These moves are similar to the ones individual players use in non-cooperative games. Second, we define cooperative one-step moves which a group of decision makers can have. These moves are similar to the ones that group of players can have in cooperative, without coalitions, games. Then, we define a special type of moves only coalitions can make: multi-step cooperative moves. Lastly, we define the type of sanction moves that players (as individuals, groups or coalitions) can make to block certain other players, or groups/coalitions of players, from benefiting from any unilateral or cooperative improvement moves they have. Understanding these types of players' moves is essential to define the stability solution concepts which will be used to analyze the stability of cooperative games' states for the games' individual players, as well as its coalitions.

As a reminder, we discussed in Chapter 5, Section 5.5.1, and in Chapter 6, Section 6.2.1, that we followed the steps of Fraser and Hipel (1984) and Fang et al. (1993) in their definitions for UI, UM and SM moves for individual agents. But, we defined all these type of moves to be within the context, terminology and notation of the Constrained Rationality framework. These individual agent's moves are the same as the ones we defined in Chapter 6 for non-cooperative games, and in Chapter 8 for cooperative games without coalitions, included here for completeness and coherence of addressing the needs of modelling and analysis of cooperative games, with coalitions, in this chapter.

It is also worth including a reminder here that the cooperative moves and coalition moves, defined in this chapter for cooperative games with coalitions, are different from the cooperative moves defined by Kilgour et al. (2001) and Inohara and Hipel (2008b,a) for GMCR. We discussed in Chapter 5, Section 5.5.1, how these GMCR's cooperative moves, or coalition moves as been called in the cited work, are limited in their scope and in their applications to real-life conflicts. We mentioned there how the Constrained Rationality provide broader, more advanced and practical cooperative moves that reflect the needs of complex real-life multi-agent conflicts. We also mentioned how the Constrained Rationality cooperative moves differ from those defined for the GMCR; and how, in Constrained rationality, the cooperation among agents within a conflict could happen between agents that are not part of a coalition. Some of the Constrained Rationality's cooperative moves were discussed in Chapter 8, namely the single-step cooperative moves, and will be mentioned in this chapter too. But, the Constrained Rationality's multi-step group cooperative moves are discussed in this chapter only. All types of the Constrained Rationality's cooperative moves are allowed within the context of cooperative games with coalitions.

9.3.1 Types of Non-Cooperative Moves by Individual DMs

In this subsection, we define the following important types of movements that an individual decision maker, alone and non-cooperatively, can make in the game.

Definition 9.3.1 (**Unilateral Move (UM)**): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and state $s \in \mathcal{S}$ of the game: a move to state $s_{um} \in \mathcal{S}$ is considered a Unilateral Move (UM) for DM_i at time t from state s, denoted as $s_{um} \in \mathcal{S}^{UM}_{DM_i,t}(s)$, iff DM_i can move unilaterally from state s to state s_{um} in one move, reaching s_{um} at time $t+1$.*

Definition 9.3.2 (**Unilateral Improvement (UI)**): *For Decision Maker $DM_i \in \mathcal{DM}$ at time t and state $s \in \mathcal{S}$ of the game: a move to state $s_{ui} \in \mathcal{S}$ is considered a Unilateral Improvement (UI) for DM_i at time t from state s, denoted as $s_{ui} \in \mathcal{S}^{UI}_{DM_i,t}(s)$, iff 1) $s_{ui} \in \mathcal{S}^{UM}_{DM_i,t}(s)$; and 2) $s_{ui} \succ^{L_{PS}}_{DM_i t} s : L_{PS} > N$, i.e. when $PrefStrength(s_{ui}, s, DM_i, t) > None$.*

Evidently, $\mathcal{S}^{UI}_{DM_i,t}(s) = \mathcal{S}^{UM,>N}_{DM_i,t}(s)$; and $\mathcal{S}^{UI}_{DM_i,t}(s) \subseteq \mathcal{S}^{UM}_{DM_i,t}(s) \subseteq \mathcal{S}$.

One important step of analyzing a game is to generate the the UIs that DMs will have from each state of the game. Given a *Game-Structure* for a cooperative game, with coalitions, that resembles the one proposed for the cooperative games, without coalitions, and discussed in Chapter 8, we use Algorithm 6.1 to generate the UM and UI sets for all DMs in the game. Notice that Algorithm 6.1 is the algorithm proposed previously in Chapter 6 to be used for generating the UM and UI sets for the players of non-cooperative games. The same algorithm is used for these two different type of games, non-cooperative and cooperative-with-coalitions, because UM and UI sets exist for individual players regardless of the players' abilities to have cooperative moves.

Now, let each game has a *Game Configuration Structure*, referred to it as a *Game-Structure* in Chapter 6 and in Chapter 8. This data structure provides essential initial information about the game and its players, all organized and in a computerized DSS system is written in a file structure. As a reminder, a Game-Structure will describe the game at a specific point of time t, as perceived and known by the focal decision maker whom the game is modelled based on his knowledge of it. Any updates or changes to what is known about the game by the focal decision maker should initiate a generation of a new Game-Structure to reflect the changes; and a new analysis of the updated game, treating the structure as a new game.

The Game-Structure for a cooperative game must have the same information required for non-cooperative games, and listed in Section 6.2.1: the set of the game's states, $\mathcal{S}$; the set of the game's DMs, $\mathcal{DM}$; $\mathcal{S}^{UM}_{DM_i,t}(s)$ for each DM in the game; and $WP(s, DM_i, t)$ for every $DM_i \in \mathcal{DM}$ and for every $s \in \mathcal{S}$. We said in Chapter 8 that the *Game-Structures* for cooperative games, with no coalitions, differ from the one for non-cooperative games in one aspect. A Game-Structure for cooperative games, with coalitions, as the Game-Structure for cooperative games, without coalitions, has additional information in its structure about the cooperative moves the players have in such game, as we will see next when we discuss cooperative moves for players. Something that non-cooperative games' Game-Structures do not have.

9.3.2 Types of Cooperative Moves by Decision Makers Groups

There are two groups of cooperative moves available for DM groups in a cooperative game, with coalitions. The first is the group that include all the simple one-step cooperative moves that DMs, even the ones that do not belong to the same coalition, can have. These cooperative moves are similar to the ones DMs of cooperative games, without coalitions, can have. The second is the group that include the multi-step cooperative moves that only coalitions can make, and only available within the context of cooperative games, with coalitions. Coalitions' cooperative moves could consist of a series of consecutive unilateral moves and/or one-step cooperative moves. We will discuss here both groups.

9.3.2.1 Types of One-Step Cooperative Moves

There are two types of one-step movements that a group/coalition of decision makers, cooperatively, can mae in the game.

Definition 9.3.3 (**Cooperative Move (CM)**): *For a Group of Decision Makers $DM_g \subseteq \mathcal{DM}$, where $|DM_g| \geq 2$, at time t and at state $s \in \mathcal{S}$ of the game, the move by DM_g from s in one step to state $s_{cm} \in \mathcal{S}$ is considered a Cooperative Move (CM) for DM_g from s at time t, denoted as $s_{cm} \in \mathcal{S}^{CM}_{DM_g,t}(s)$ iff DM_g cannot make the move unless each and every member of DM_g agrees to the move and cooperates by doing what is necessary to reach the state s_{cm}, from the starting state s.*

As per the definition, a Cooperative Move is a movement from one state of the game to another that requires a group of decision makers (more than two) to make the move. No single decision maker can do the move on his own. Similarly no subset of a group of decision makers can do the move on their own, every single decision maker in the group is needed to participate in the move to make it happen. What differentiates a CM move from the coalition's group move (C-GM), discussed shortly, is that a CM move is a one-step move done by all the group members at once from one state to another, and could not be made of a series of consecutive moves.

Definition 9.3.4 (**Cooperative Improvement (CI)**): *For a Group Decision Makers $DM_g \subseteq \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, the move by DM_g from s in one step to state $s_{ci} \in \mathcal{S}$ is considered a Cooperative Improvement (CI) for DM_g from s at time t, denoted as $s_{ci} \in \mathcal{S}^{CI}_{DM_g,t}(s)$ iff 1) $s_{ci} \in \mathcal{S}^{CM}_{DM_g,t}(s)$; and 2) $\forall\ DM_i \in DM_g\ \ s_{ci} \succ^{L_{PS}}_{DM_i,t} s\ :\ L_{PS} > N$, i.e. when $PrefStrength(s_{ci}, s, DM_i, t) > None$ for every $DM_i \in DM_g$.*

From the definition, it is evident that $\mathcal{S}^{CI}_{DM_g,t}(s) = \mathcal{S}^{CM,>N}_{DM_g,t}(s) \subseteq \mathcal{S}^{CM}_{DM_g,t}(s) \subseteq \mathcal{S}$.

One important step of analyzing a cooperative game, with coalitions, is to generate the CIs that DMs will have from each state of the game. The Game-Structure for a cooperative game, with coalition (same as cooperative games without coalitions), must include information about the CMs that all the DMs have in the game from each of the game's states. It must include $\mathcal{S}^{CM}_{DM_i,t}(s)$, for every $DM_i \in \mathcal{DM}$ and for every $s \in \mathcal{S}$, given as a set of graphs describing the CMs that DMs have from each state the game has (one graph per DM in the game, with the game's states represented as the graph's nodes and the CMs are represented as its arcs (each of these arcs annotated with the names of the other cooperating players). Given a Game-Structure, for a cooperative game, with coalitions, that has all this required information, we use Algorithm 8.1 to generate the CM and CI sets for all DMs in the game. This is the same algorithm we used in Chapter 8 for cooperative games, without coalitions, to generate the CIs for their players. This is because groups of decision makers in cooperative games, with or without coalitions, can have CM and CI moves regardless of the fact that these groups are part of coalitions or not.

9.3.2.2 Types of Multi-Step Coalition Cooperative Moves

There are two types of multi-step movements that a coalition, or an alliance, of decision makers cooperatively can make in the game.

Definition 9.3.5 (Coalition Group Movement (C-GM)): *For a Group of Decision Makers $DM_G \subseteq \mathcal{DM}$, where $|DM_G| \geq 2$, at time t and at state $s \in \mathcal{S}$ of the game, the move by DM_G from s in one step, or a series of consecutive moves, to state $s_{cgm} \in \mathcal{S}$ is considered a Coalition Group Movement (C-GM) for DM_G from s at time t, denoted as $s_{cgm} \in \mathcal{S}^{C\text{-}GM}_{DM_G,t}(s)$ iff DM_G is an official coalition, a known unofficial coalition among group of decision makers, or a subset of the game's decision makers $\mathcal{DM}$ who their intent to move from one state of the game to another (because this will be benefit all or the majority of DM_G's members) is made known/declared (by means of an agreement, voting, etc.) or believed to be true (with justification), and:*

$[(DM_G \text{ reaches } s_{cgm} \text{ at time } t+1) \rightarrow ((\exists\ DM_i \in DM_G : s_{cgm} \in \mathcal{S}^{UM}_{DM_i,t}(s)) \vee (\exists\ DM_g \subseteq DM_G : s_{cgm} \in \mathcal{S}^{CM}_{DM_g,t}(s)))] \quad \vee$

$[(DM_G \text{ reaches } s_{cgm} \text{ at time } t+h) \rightarrow (\forall \text{ stepwise move to state } s_k\ \forall\ k : ((0 < k \leq h) \wedge (s_0 = s) \wedge (s_h = s_{cgm}))[(\exists\ DM_i \in DM_G : s_k \in \mathcal{S}^{UM}_{DM_i,t+k-1}(s_{k-1})) \vee (\exists\ DM_g \subseteq DM_G : s_k \in \mathcal{S}^{CM}_{DM_g,t+k-1}(s_{k-1}))]\,)]$.

In other words, the move to s_{cgm} by DM_G is considered a C-GM for the group, if and only if the group is a legitimate group, or believed to be a legitimate group, of the game's total set of decision makers, and: 1) if DM_G reaches s_{cgm} in one step, then there exists at least one of the group members (or a subset of the group members) has the capacity to move the game unilaterally (or cooperatively, in the case of a subset) at time t to s_{cgm} from the current state s and in one step, reaching s_{cgm} at time $t+1$; or 2) if DM_G reaches s_{cgm} in h consecutive steps, then for each of these steps there exists at least one of the group members (or a subset of the group members) has the capacity to move the game unilaterally (or cooperatively, in the case of a subset) to the next step's state in one step and in one increment of the time, allowing the group to start from the current state s and ultimately, after h stepwise moves, reach s_{cgm} at time $t+h$. Additionally, it is evident that $\mathcal{S}^{C\text{–}GM}_{DM_G,t}(s) \subseteq \mathcal{S}$.

Definition 9.3.6 (**Coalition Group Improvement (C-GI)**): *For a Group Decision Makers $DM_G \subseteq \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, the move by DM_G from s in one step, or a series of consecutive moves, to state $s_{cgi} \in \mathcal{S}$ is considered a Coalition Group Improvement (C-GI) for DM_G from s at time t, denoted as $s_{cgi} \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)$ iff 1) $s_{cgi} \in \mathcal{S}^{C\text{–}GM}_{DM_G,t}(s)$; and 2) $s_{cgi} \succ^{L_{PS}}_{DM_C,t} s \ : \ L_{PS} > N$.*

The second condition in the definition requires the destination state s_{cgi} to be preferred to the start state s for coalition DM_G. Recall that we said judging the preference over states of the game by a coalition requires knowledge about how the coalition decides collectively on its preferences, and for simplicity and convenience reasons we are opting for the use of the ideal case where all the coalition members must have agreement on the preferences and their strength. In other words, the second condition of the definition, and by using the ideal case, should read: $\forall\ DM_k \in DM_G \ \ s_{cgi} \succ^{L_{SS}}_{DM_k,t} s \ : \ L_{SS} > N$, i.e. when $PrefStrength(s_{cgi}, s, DM_k, t) > None$ for every $DM_k \in DM_G$. Also, we said earlier that, in reality and when using such preference statements about a coalition, the modeller/analyst must qualify the truthfulness of such statements by applying the actual method/algorithm the coalition uses to decide on its collective preferences.

From the definition, it is evident that $\mathcal{S}^{C\text{–}GI}_{DM_G,t}(s) = \mathcal{S}^{C\text{–}GM,>N}_{DM_G,t}(s) \subseteq \mathcal{S}^{C\text{–}GM}_{DM_G,t}(s) \subseteq \mathcal{S}$. And, this is true for each coalition regardless of how the coalition decides on its preferences, as long as it is consistent in how it decides on these preferences.

One important step of analyzing a cooperative game, with coalitions, is to generate the the G-CIs that each coalition of DMs will have from each state of the game. Given a Game-Structure, for a cooperative game, with coalitions, that has all the required information listed earlier, we use Algorithm 9.1, and its additional routine listed in Algorithm 9.2, to generate the G-CM and G-CI sets for all coalitions of DMs in the game. The Game-Structure used by this algorithm does not require any additional information beside what is required for Algorithms 8.1 and 6.1 combined to generate the UMs, UIs, CMs and CIs for all the game's DMs. Notice that Algorithms 9.1 and 9.2 uses the ideal case in which the coalition members must have full agreement of the preferences over the game's states, and in the strengths of these preferences.

Algorithm 9.1 Generating the C-GM and C-GI Sets for each Coalition in a Game

void Generate_Coalition_C-GM_and_C-GI_Sets (Game-Structure)
// Game-Structure file starts with empty C-GM and C-GI sets for each DM_C Coalition.
// $\mathcal{S}$, $\mathcal{DM}$, the UM and UI sets for each $DM_i \in DM_C$, and the CM and CI sets for each
// $DM_g \subseteq DM_C$ are given as part of the Game-Structure.
for all $DM_C \subseteq \mathcal{DM}$ **do**
  // Generate DM_C's C-GM Sets (one for each of the game's states), and for each state find
  // DM_C's C-GI Set. All these C-GM and C-GI sets will be initially empty. If not, empty
  // them [not included here]. And, by the end, some of these C-GM/C-GI sets will be
  // empty sets. Setup C-GM, and C-GI, Paths sets consist of ordered sets of different
  // lengths, each in the form of $(s_a, s_b, \ldots, s_n)$ representing a set of consecutive moves: from
  // the start state to s_a, from $(s_a$ to $(s_b$, and so on.
  for all $s \in \mathcal{S}$ **do**
    // Initialize the G-CM Paths set.
    $\mathcal{P}^{C\text{–}GM}_{DM_C,t}(s) = \emptyset$
    path = ""
    call Add_C-GM_Paths(s, path, DM_C)
    // Now, we have all DM_C's C-GM paths, the simple one-step ones and the ones
    // that are consistent of a series of steps. Generate $\mathcal{S}^{C\text{–}GM}_{DM_C,t}(s)$, and $\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)$
    for all $Path \in \mathcal{P}^{C\text{–}GM}_{DM_C,t}(s)$ **do**
      s_{cgm} = the last state in $Path$
      $\mathcal{S}^{C\text{–}GM}_{DM_C,t}(s) = \mathcal{S}^{C\text{–}GM}_{DM_C,t}(s) \cup \{s_{cgm}\}$
      // moving to s_{cgm} from s is also considered a C-GI for DM_C
      // if and only if $\forall DM_j \in DM_C \;\; s_{cgm} \succ^{L_{PS}}_{DM_j,t} s : L_{PS} > N$
      if $\forall DM_j \in DM_C \;\; PrefStrngth(s_{cgm}, s, DM_j, t) > None$ **then**
        $\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s) = \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s) \cup \{s_{cgm}\}$
      end if
    end for
  end for
end for
return

Algorithm 9.2 The "Add_C-GM_Paths" Routine used in Algorithm 9.1

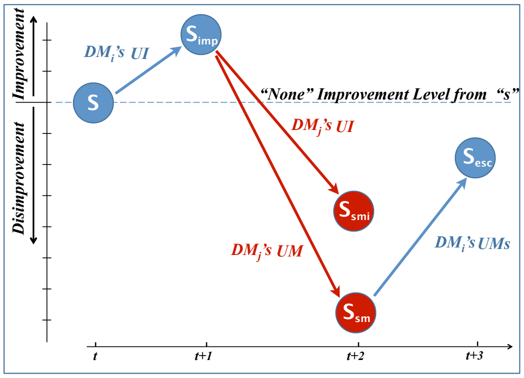
void Add_C-GM_Paths$(s$, ***List,*** DM_C, $\mathcal{DM}$, $\mathcal{S})$
for all $DM_i \in DM_C$ **do**
    for all $s_{um} \in \mathcal{S}^{UM}_{DM_i,t}(s)$ **do**
        $\mathcal{P}^{C-GM}_{DM_C,t}(s) = \mathcal{P}^{C-GM}_{DM_C,t}(s) \cup \{(List + \text{“}s_{um}, \text{''})\}$
        List1 = List + "s_{um},"
        call Add_C-GM_Paths(s_{um}, List1, DM_C)
    end for
end for
for all $DM_g \in DM_C$ **do**
    for all $s_{cm} \in \mathcal{S}^{CM}_{DM_i,t}(s)$ **do**
        $\mathcal{P}^{C-GM}_{DM_C,t}(s) = \mathcal{P}^{C-GM}_{DM_C,t}(s) \cup \{(List + \text{“}s_{cm}, \text{''})\}$
        List1 = List + "s_{cm},"
        call Add_C-GM_Paths(s_{cm}, List1, DM_C)
    end for
end for

return

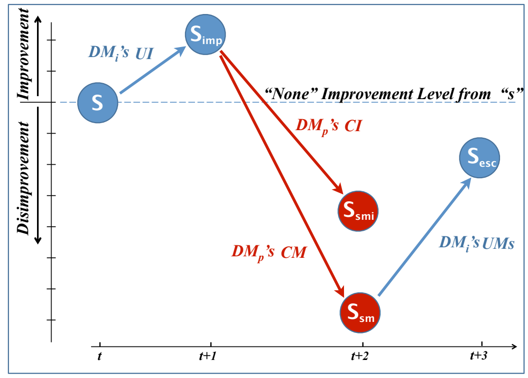
9.3.3 Types of Sanction Moves

We will expand here the definitions provided in Chapter 8, for cooperative games without coalitions, for a sanction move (SM) and an inescapable sanction move (ISM), made against a decision maker's UI, to include sanction moves that are committed not only by other individual decision makers but also by coalitions of decision makers. Additionally, we will include in the new definitions sanction moves that are intended to sanction a group/coalition of decision makers's CI/C-GI moves, not just individual's UI moves.

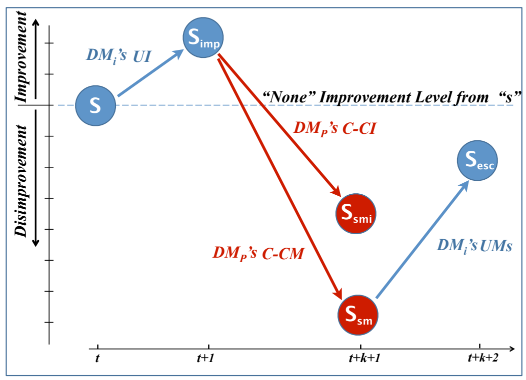
Definition 9.3.7 (**Sanction Move (SM)**): ***1) For an Individual Decision Maker*** $DM_i \in \mathcal{DM}$ *at time* t *and at state* $s \in \mathcal{S}$ *of the game, a UI by* DM_i *to state* $s_{imp} \in \mathcal{S}^{UI}_{DM_i,t}(s)$*, or a CI by a group* $DM_g : DM_i \in DM_g$ *to state* $s_{imp} \in \mathcal{S}^{CI}_{DM_g,t}(s)$*, is said to have against it a Sanction Move (SM) at time* $t+1$ *to state* $s_{sm} \in \mathcal{S}$ *iff* $[\ \exists\ ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{imp}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)] \vee [\ \exists\ ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_{sm} \in \mathcal{S}^{CM}_{DM_p,t+1}(s_{imp}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)] \vee [\ \exists\ ((DM_P \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_{sm} \in \mathcal{S}^{C-GM}_{DM_P,t+1}(s_{imp}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)]$*; and* ***2) For a Coalition of Decision Makers*** $DM_G \subseteq \mathcal{DM}$ *at time* t *and at state* $s \in \mathcal{S}$ *of the game, a C-GI by* DM_G *to state* $s_{cgi} \in \mathcal{S}^{C-GI}_{DM_G,t}(s)$*, reaching it at time* $t+h$*, is said to have against it a Sanction Move (SM) at time* $t+h$ *to state* $s_{sm} \in \mathcal{S}$ *iff* $[\ \exists\ ((DM_j \in \{\mathcal{DM} - DM_G\}) \wedge (s_{sm} \in \mathcal{S}^{UM}_{DM_j,t+h}(s_{cgi}))) : s_{sm} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)] \vee [\ \exists\ ((DM_p \subseteq \{\mathcal{DM} - DM_G\}) \wedge (s_{sm} \in \mathcal{S}^{CM}_{DM_p,t+h}(s_{cgi}))) :$



(a) A UI by a DM is faced with an SM/ISM by another individual DM



(b) A UI by a DM is faced with an SM/ISM by a cooperative group of DMs



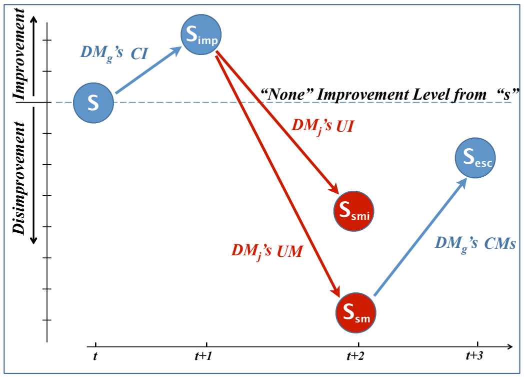
(c) A UI by a DM is faced with an SM/ISM by a coalition of DMs

Figure 9.1: Type of Sanction Moves available to players of cooperative games, with coalitions, in response to a UI by a DM

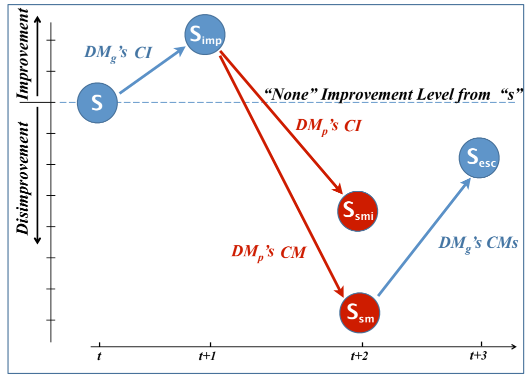
$s_{sm} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)] \vee [\ \exists\ ((DM_P \subseteq \{\mathcal{DM} - DM_G\}) \wedge (s_{sm} \in \mathcal{S}^{C\text{–}GM}_{DM_P,t+h}(s_{cgi})))\ :\ s_{sm} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)]$.

Cooperative games, with coalitions, can have two types of players: 1) individuals, not part of any coalition; and 2) coalitions. The SM definition above covers all types of SMs that both type of players could face. In total there are nine types of SMs within the context of cooperative games, with coalitions. All are shown in Figures 9.1 - 9.3. Recall that there are only four types of SMs within the context of cooperative games, without coalitions. So, adding coalitions to mix of players allowed within cooperative games, increased not only the types of cooperative moves the players can have (as indicated in the previous subsection) but also expanded the types of SMs the players can face in these games.

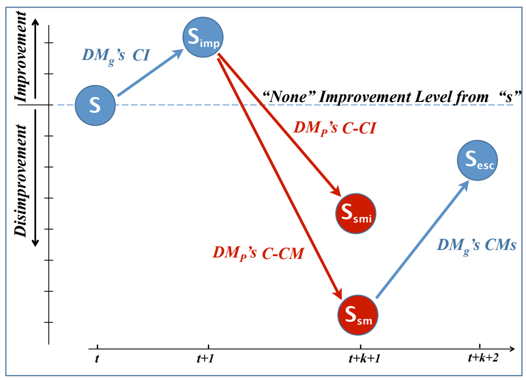
The subfigures within Figures 9.1 and 9.2 show what Definition 9.3.7 describes



(a) A CI by a cooperative group of DMs is faced with an SM/ISM by an individual DM



(b) A CI by a cooperative group of DMs is faced with an SM/ISM by another cooperative group of DMs

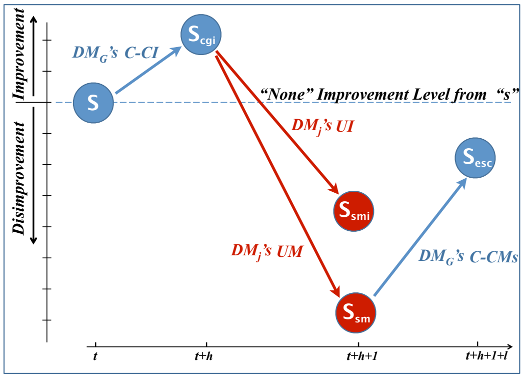


(c) A CI by a cooperative group of DMs is faced with an SM/ISM by a coalition of DMs

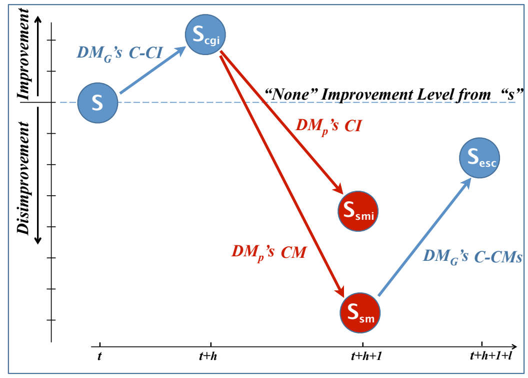
Figure 9.2: Type of Sanction Moves available to players of cooperative games, with coalitions, in response to a CI by a cooperative group of DMs

in its first part "for an Individual Decision Maker" as SMs against this individual's UI or cooperative CI move from state s to state s_{imp}. In Figure 9.1a, DM_i's UI move to s_{imp} is sanctioned by the move of another individual player in the game, DM_j, to state s_{sm}. In Figure 9.1b, the SM against DM_i's UI comes from a group of decision makers, DM_p, who are acting cooperatively but not as a coalition. The SM against DM_i's UI could also come, as shown in Figure 9.1c, from coalition DM_P in the form of a series of consecutive k moves progressing the game from s_{imp}, which is preferred by DM_i to state s, to state s_{sm} which is not only worse than s_{imp} but could be equally or less preferred to the original state s. Similarly, Figures 9.2a - 9.2c covers the same types of SMs but instead of sanctioning DM_i's UI move to s_{imp}, they sanction DM_g's CI move to s_{imp} and where $DM_i \in DM_g$.

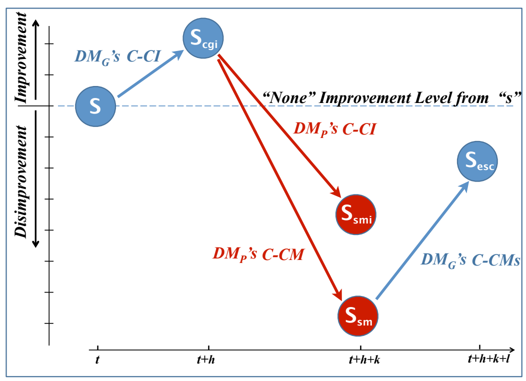
Figures 9.3a - 9.3c covers the same type of SMs but here the sanction moves



(a) A C-CI by a coalition of DMs is faced with an SM/ISM by an individual DM



(b) A C-CI by a coalition of DMs is faced with an SM/ISM by a cooperative group of DMs



(c) A C-CI by a coalition of DMs is faced with an SM/ISM by another coalition of DMs

Figure 9.3: Type of Sanction Moves available to players of cooperative games, with coalitions, in response to a C-CI by a coalition of DMs

are committed against a coalition DM_G's C-GI move from s to a more preferred state for the coalition, state s_{cgi}. These three SM types cover what Definition 9.3.7 describes as SMs against a coalition, not just an individual DM, in its second part "for a coalition of Decision Makers". Recall that a C-GI move by the coalition DM_G could be a single move or a series of consecutive moves.

In addition, it is important to notice here that the new expanded definition of a SM presented above does not assume that the SM to be a UI, a CI or a C-GI move by the DM/s committing the sanction. The definition assumes that the motive of the DM/s committing the sanction for their SM is to hurt the individual DM_i, or coalition DM_G, even if this SM will put the DM/s committing the sanction himself/themselves at a less preferred state. To differentiate between an SM by DM/s which is not required to be a UI, CI or C-GI, as per the SM definition above, and an SM which is also a UI, a CI or a C-GI move for the committing parties, we will call the second SM type as SMI move (read as Sanction Move and Improvement). As we will see later, this *stricter* type of SM, or SMI - as we decided to call it, is required for some stability solution concepts, such as Sequential Stability (SEQ).

The concept of an SM that is also a UI, CI or C-GI move by the committing party is illustrated in the subfigures of Figures 9.1 - 9.3, by showing in each the additional scenario of having an SMI move from state s_{imp} to state s_{smi} (instead of having the destination be state s_{sm} which cannot be reached by an SM that is also a UI, a CI or a C-GI). This means that we are expanding here, the concept of a SMI move to include not only sanction moves by committing parties wether individuals, groups or coalitions, but also to include sanction moves against the UI of individual DMs, the CI of cooperating groups of DMs, and the C-GIs by coalitions of DMs.

It is important here to also remember that wherever $s_{sm} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)$ is mentioned in the second part, the "for a Coalition of Decision Makers" part, of the SM definition above, the modeller/analyst should qualify what the preferences for the coalition DM_G are, and how they are calculated. We said that ideally: $(s_{sm} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)) \equiv (\forall DM_i \in DM_G \quad s_{sm} \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$. This is assuming that all the coalition members agree on the preferences and their strength. Otherwise, judging the preference over the states of the game by a coalition requires knowledge about how the coalition decides collectively on its preferences. The ideal case of coalition's preferences is adopted in this chapter only for simplicity and convenience reasons. In any case, and especially when modelling a real-life cooperative conflict, with coalitions, the modeller/analyst must qualify the truthfulness of any statement about a coalition's preferences by applying the actual method/algorithm the coalition uses to decide on its collective preferences.

Definition 9.3.8 (Inescapable Sanction Move (ISM)): ***1) For an Individual Decision Maker*** *$DM_i \in \mathcal{DM}$ at time t and at state $s \in \mathcal{S}$ of the game, a UI by DM_i to state $s_{imp} \in \mathcal{S}^{UI}_{DM_i,t}(s)$, or a CI by a group $DM_g : DM_i \in DM_g$ to state $s_{imp} \in \mathcal{S}^{CI}_{DM_g,t}(s)$, is said to have against it an Inescapable Sanction Move (ISM) at time $t+1$ to state $s_{ism} \in \mathcal{S}$ iff $[\exists\ ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_{ism} \in \mathcal{S}^{UM}_{DM_j,t+1}(s_{imp}))) : (s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_{esc} \in \mathcal{S}^{UM}_{DM_i,t+2}(s_{ism}) \quad s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))] \vee [\exists\ ((DM_p \in \{\mathcal{DM} - DM_i\}) \wedge (s_{ism} \in \mathcal{S}^{CM}_{DM_p,t+1}(s_{imp}))) : (s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_{esc} \in \mathcal{S}^{UM}_{DM_i,t+2}(s_{ism}) \quad s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))] \vee [\exists\ ((DM_P \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_{ism} \in \mathcal{S}^{C-GM}_{DM_P,t+1}(s_{imp})$ and where DM_P reaches s_{ism} at time $t+1+k)) : (s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_{esc} \in \mathcal{S}^{UM}_{DM_i,t+1+k}(s_{ism}) \quad s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))]$; and* ***2) For***

a Coalition of Decision Makers $DM_G \subseteq \mathcal{DM}$ *at time* t *and at state* $s \in \mathcal{S}$ *of the game, a C-GI by* DM_G *to state* $s_{cgi} \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)$*, reaching it at time* $t+h$ *is said to have against it an Inescapable Sanction Move (ISM) at time* $t+h$ *to state* $s_{ism} \in \mathcal{S}$ *iff* $[\ \exists\ ((DM_j \in \{\mathcal{DM} - DM_G\}) \wedge (s_{ism} \in \mathcal{S}^{UM}_{DM_j,t+h}(s_{cgi}))) : ((s_{ism} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)) \wedge (\forall s_{esc} \in \mathcal{S}^{C\text{–}GM}_{DM_G,t+h+1}(s_{ism}) \quad (s_{esc} \in \mathcal{S}^{\leq N}_{DM_G,t}(s))))] \vee [\ \exists\ ((DM_p \in \{\mathcal{DM} - DM_G\}) \wedge (s_{ism} \in \mathcal{S}^{CM}_{DM_p,t+h}(s_{cgi}))) : ((s_{ism} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)) \wedge (\forall s_{esc} \in \mathcal{S}^{C\text{–}GM}_{DM_G,t+h+1}(s_{ism}) \quad (s_{esc} \in \mathcal{S}^{\leq N}_{DM_G,t}(s))))] \vee$ $[\ \exists\ ((DM_P \subseteq \{\mathcal{DM} – DM_G\}) \wedge (s_{ism} \in \mathcal{S}^{C\text{–}GM}_{DM_P,t+h}(s_{cgii})$ *and where* DM_P *reaches* s_{ism} *at time* $t+h+k)) : ((s_{ism} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)) \wedge (\forall s_{esc} \in \mathcal{S}^{C\text{–}GM}_{DM_G,t+h+k}(s_{ism}) \quad (s_{esc} \in \mathcal{S}^{\leq N}_{DM_G,t}(s))))]$.

The concept of an inescapable sanction move is expanded by this definition to include not only inescapable sanction moves by committing parties wether individuals or groups, but also to include inescapable sanction moves against UI, CI or C-GI moves (UI by individuals, CI by cooperating groups of DMs and C-GIs by coalitions of DMs). Again here, illustrations of the different scenarios covered by the definition are provided as part of the subfigures of Figures 9.1 - 9.3. From the figures, it is clear that the concept of inescapable sanction move is building on the concept of a sanction move and narrowing it down to demand that that the decision maker/s, whose UI, CI or C-GI is targeted by the sanction move, will not be able to escape the negative effect of the sanction move (hence the name: inescapable sanction move (ISM)).

In addition, it is important here to remember that wherever $(s_{ism} \in \mathcal{S}^{\leq N}_{DM_G,t}(s))$ and $(s_{esc} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)))$ mentioned in the second part, the "for a Coalition of Decision Makers" part, of the ISM definition above, the modeller/analyst should qualify what the preferences for the coalition DM_C are, and how they are calculated. We said that ideally: $(s_{ism} \in \mathcal{S}^{\leq N}_{DM_G,t}(s)) \equiv (\forall DM_i \in DM_G \;\; s_{ism} \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$; and $(s_{esc} \in \mathcal{S}^{\leq N}_{DM_G,t}(s))) \equiv (\forall DM_i \in DM_G \;\; s_{esc} \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$. This is assuming that all the coalition members agree on the preferences and their strength. Otherwise, judging the preference over the states of the game by a coalition requires knowledge about how the coalition decides collectively on its preferences. The ideal case of coalition's preferences is adopted in this chapter only for simplicity and convenience reasons. In any case, and especially when modelling a real-life cooperative conflict, with coalitions, the modeller/analyst must qualify the truthfulness of any statement about a coalition's preferences by applying the actual method/algorithm the coalition uses to decide on its collective preferences.

9.4 Stability Solution Concepts and Equilibriums for Cooperative Games with Coalitions

The same four stability solution concepts, we discussed in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions, will be discussed in this chapter for cooperative games with coalitions. But we will expand these solution concepts to accommodate the new multi-step cooperative moves that coalitions of decision makers in cooperative games with coalitions can have.

We will group these solution concepts into two classes, as we did in Chapter 6 for non-cooperative games: 1) solution concepts that are extremely individualistic, or coalition-centric, and shortsighted in their definitions, in a way that they do not consider other players countermoves; and 2) solution concepts that tries to include other players' countermoves, therefore these concepts show more foresight.

As a reminder, we said in Chapter 6, Section 6.3, that we followed the steps of Fraser and Hipel (1984) and Fang et al. (1993) in their definitions for the four stability solution concepts for non-cooperative games. But, we defined all these solution concepts, for non-cooperative games, to be within the context, terminology and notation of the Constrained Rationality framework; and using the definitions of the agents' non-cooperative unilateral moves and sanction moves introduced in Chapter 6. In Chapter 8, we extended the definitions of the four stability solution concepts, presented in Chapter 6, to deal with the Constrained Rationality's single-step cooperative moves, added in Chapter 8 for cooperative games without coalitions; and to deal with the changes happened accordingly to the definitions of sanction moves in such games. In this section, we expand the definitions of the four stability solution concepts further. The expanded ones, which will follow, deal with the Constrained Rationality's single-step cooperative moves, as well as multi-step coalition cooperative moves added above; and to deal with the changes happened accordingly to the definitions of sanction moves in cooperative games with coalitions (as discussed earlier in this chapter).

It is also worth including here a reminder that the definitions of the stability solution concepts for cooperative games with coalitions, which will follow, are different from the ones presented by Inohara and Hipel (2008b,a) for GMCR and called coalition stability solution concepts. This is due to the different moves that GMCR and Constrained Rationality employ. We discussed in Chapter 5, Section 5.5.1, how the GMCR's cooperative moves, or coalition moves as been called by Inohara and Hipel (2008b,a), are limited in their scope and in their applications to real-life conflicts; and how the Constrained Rationality provide broader, more advanced and practical collection of cooperative moves that reflect the needs of complex real-life multi-agent conflicts. We discussed there how the Constrained Rationality's cooperative moves differ from those defined for the GMCR; and how, in Constrained rationality, the cooperation among agents within a conflict could happen between agents that are not part of a coalition. And, because each framework employs different cooperation among agents and different definitions of cooperative moves, the frameworks' definitions of the stability solution concepts, which are based on the definitions of the cooperative moves employed, will definitely be different.

9.4.1 Solution Concepts with No Consideration to Others' Moves

There is one stability solution concept which does not consider in its definition the moves and countermoves of other players. It is the NASH solution concept. We expand its definition to include not only NASH stability for individual players within cooperative games, with coalitions, but also we define NASH stability for a coalition of players.

Definition 9.4.1 (**Nash Stability (NASH)**):
1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, state $s \in \mathcal{S}$ is considered a Nash Stable (NASH) state, denoted as $s \in \mathcal{S}^{NASH}_{DM_i,t}$, iff*
$[\,\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset\,] \wedge [(\forall DM_g \subseteq \mathcal{DM} \,:\, DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset\,] \wedge$
$[(\forall DM_G \subseteq \mathcal{DM} \,:\, DM_i \in DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset\,]$*; and*
2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, state $s \in \mathcal{S}$ is considered a Nash Stable (NASH) state, denoted as $s \in \mathcal{S}^{NASH}_{DM_C,t}$, iff*
$[\,\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)=\emptyset\,] \wedge [(\forall DM_G \subseteq \mathcal{DM} \,:\, DM_C \subseteq DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset\,]$

The definition, in its first part, describes what it means for an individual DM in a cooperative game, with coalitions, to have a state s be a NASH stable state for him. In the second part, the definitions describe what it takes for a coalition of DMs to consider a state s a NASH stable state for the coalition.

First, as per the definition, state s is considered NASH stable for an individual decision maker DM_i at time t if and only if s is the best that DM_i, individually or cooperatively (with or without coalitions), can achieve at time t, given the total states of the game $\mathcal{S}$. Therefore, states that are not NASH stable are unstable states since DM_i can improve his position from any one of them: unilaterally by activating one of his UIs out of these states; cooperatively by activating one of the CIs that he and other individual DMs have from these states; or cooperatively through the coalition, if DM_i is part of one, by activating one of the C-GIs the coalition have from these states.

Second, as per the definition, state s is considered NASH stable for a coalition of decision makers DM_C at time t if and only if s is the best that DM_C collectively can achieve at time t, given the total states of the game $\mathcal{S}$. Therefore, states that are not NASH stable are unstable states since coalition DM_C can improve its position from any one of them by activating one of its C-GIs from these states, or the C-GIs of bigger coalitions that it is part of.

9.4.2 Solution Concepts with Consideration to Others' Moves

We will discuss here the same three stability solution concepts we discussed in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions. These solutions concepts are General MetaRationality, Symmetric MetaRationality and Sequentially Stability. But in this section we will expand the definitions of these solution concepts to include not only the stability under these solution concepts for individual players within cooperative games, with coalitions, but also we define the stability under these solution concepts for a coalition of players.

Definition 9.4.2 (General MetaRational (GMR) Stability):
1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a General MetaRational (GMR) Stable state, denoted as $s \in \mathcal{S}^{GMR}_{DM_i,t}$, iff*

$$\forall s_1 : \quad ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s)))$$
$$[\exists\, ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \vee$$
$$[\exists\, ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \vee$$
$$[\exists\, ((DM_P \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{C-GM}_{DM_P,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \qquad ; \text{ and}$$

2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, state $s \in \mathcal{S}$ is considered a General MetaRational (GMR) Stable state, denoted as $s \in \mathcal{S}^{GMR}_{DM_C,t}$, iff*

$$\forall s_1 : \quad \big(\,((s_1 \in \mathcal{S}^{C\text{--}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}:\\ DM_C \subseteq DM_G}} \mathcal{S}^{C\text{--}GI}_{DM_G,t}(s)))\ \textit{and } s_1 \textit{ is reached at time } t{+}h\,\big)$$

$$[\,\exists\,((DM_j \in \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+h}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)\,] \vee$$

$$[\,\exists\,((DM_p \subseteq \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{CM}_{DM_p,t+h}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)\,] \vee$$

$$[\,\exists\,((DM_P \subseteq \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{C\text{--}GM}_{DM_P,t+h}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)\,]$$

The previous GMR stability definitions given in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions, are expanded in this definition. This new definition explains what it means to have a state be GMR stable not only for an individual decision maker in a cooperative game, with coalitions, but also for a coalition of decision makers in such games.

The first part of new GMR definition, Definition 9.4.2, includes now all type of sanction moves that could be imposed, in a cooperative game with coalitions, against an individual decision maker DM_i's UIs or CIs from s. These types include: SMs by other individual decision makers; single-step cooperative SMs by groups of decision makers cooperating together; and multi-step cooperative SMs by coalitions of decision makers. These SMs are motivated mainly by the desire of the parties committing them to hurt DM_i and put him at a less preferred state than even the original state s from which DM_i's UIs/Cis start from. Hence, these SMs are not required by the definition to be UIs, CIs or C-GIs by the committing parties, but instead the definition states that they could be UMs, CMs, or C-GMs for them, respectively.

In addition, the second part of Definition 9.4.2 provides what it means for a state s, in a cooperative game with coalition, to be considered GMR stable for a coalition DM_C. The definition considers s to be GMR stable for the coalition if and only if any C-GI move that the coalition has out of s could be faced by one or more SMs. These SMs could be: single-step SMs by other individual decision makers; single-step cooperative SMs by groups of decision makers cooperating together; or multi-step cooperative SMs by other coalitions of decision makers. Again here, and as per the definition, these SMs are motivated mainly by the desire of the parties committing them to hurt the focal coalition DM_C and put it at a less preferred state than even the original state s from which DM_C's C-GIs start from. Hence, these SMs are not required by the definition to be UIs, CIs or C-GIs by the committing parties, but instead the definition states that they could be UMs, CMs, or C-GMs for them, respectively.

The GMR stability solution concept assumes that decision maker DM_i (or coalition DM_C) believes that other players *surely* would apply, unilaterally or cooperatively, a sanction against any of his UIs/CIs (the coalition's C-GIs) out of s. Therefore, DM_i (or DM_C) will not move away from s, and s is a GMR stable state for DM_i (DM_C) . In addition, in this chapter we continue to use the same notation to denote the set of all states which are GMR stable for DM_i at time t as $\mathcal{S}^{GMR}_{DM_i,t}$. But we add here that the set of all states which are GMR stable for coalition DM_C at time t will be denoted as $\mathcal{S}^{GMR}_{DM_C,t}$.

As a reminder, it is important here to mention that wherever $s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ is mentioned in the second part, the "for a Coalition of Decision Makers" part, of the GMR definition above, the modeller/analyst should qualify what the preferences for the coalition DM_C are, and how they are calculated. We said that ideally: $(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \;\equiv\; (\forall DM_i \in DM_C \quad s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$. This is assuming that all the coalition members agree on the preferences and their strength. Otherwise, judging the preference over the states of the game by a coalition requires knowledge about how the coalition decides collectively on its preferences. The ideal case of coalition's preferences is adopted in this chapter only for simplicity and convenience reasons, and should not be taken as a restriction. In any case, and especially when modelling a real-life cooperative conflict, with coalitions, the modeller/analyst must qualify the truthfulness of any statement about a coalition's preferences by applying the actual method/algorithm the coalition uses to decide on its collective preferences.

Definition 9.4.3 (Symmetric MetaRational (SMR) Stability):
1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a Symmetric MetaRational (SMR) Stable state, denoted as $s \in \mathcal{S}^{SMR}_{DM_i,t}$, iff*

$$\forall s_1 : \quad ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s)))$$

$$[\exists\, ((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1))) :$$
$$(s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\,] \vee$$
$$[\exists\, ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1))) :$$
$$(s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\,] \vee$$
$$[\exists\, ((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{C-GM}_{DM_p,t+1}(s_1)) \textit{ and } s_2 \textit{ is reached at time } t+k+1) :$$
$$(s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+k+1}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\,] \quad \textit{; and}$$

__2) For a Coalition of Decision Makers__ $DM_C \subseteq \mathcal{DM}$ at time t, state $s \in \mathcal{S}$ is considered a Symmetric MetaRational (SMR) Stable state, denoted as $s \in \mathcal{S}^{SMR}_{DM_C,t}$, iff

$$\forall s_1 : \quad \Big(\big((s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)) \big) \textit{ and } s_1 \textit{ is reached at time } t{+}h \Big)$$

$$[\, \exists \, ((DM_j \in \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{UM}_{DM_j,t+h}(s_1))) :$$

$$(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_C,t+h+1}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))\,] \vee$$

$$[\, \exists \, ((DM_p \subseteq \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{CM}_{DM_p,t+h}(s_1))) :$$

$$(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_C,t+h+1}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))\,] \vee$$

$$[\, \exists \, ((DM_P \subseteq \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{C\text{–}GM}_{DM_P,t+h}(s_1)) \textit{ and } s_2 \textit{ is reached at time } t{+}h{+}k) :$$

$$(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_C,t+h+k}(s_2) \quad s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))\,]$$

The previous SMR stability definitions given in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions, are expanded in this definition. This new definition explains what it means to have a state be SMR stable not only for an individual decision maker in a cooperative game, with coalitions, but also for a coalition of decision makers in such games.

The first part of new SMR definition, Definition 9.4.3, includes now all type of ISM moves that could be imposed, in a cooperative game with coalitions, against an individual decision maker DM_i's UIs or CIs from s. These types include: ISMs by other individual decision makers; single-step cooperative ISMs by groups of decision makers cooperating together; and multi-step cooperative ISMs by coalitions of decision makers. In all of which, DM_i cannot escape or mitigate the negative effect of them by moving away from the state produced by them. Again here, as in the case for GMR stability, these ISMs are motivated mainly by the desire of the parties committing them to hurt DM_i and put him at a less preferred state than even the original state s from which DM_i's UIs/Cis start from. Hence, these ISMs are not required by the definition to be UIs, CIs or C-GIs by the committing parties, but instead the definition states that they could be UMs, CMs, or C-GMs for them, respectively.

In addition, the second part of Definition 9.4.3 provides what it means for a state s, in a cooperative game with coalition, to be considered SMR stable for a coalition DM_C. The definition considers s to be SMR stable for the coalition if and only if any C-GI move that coalition has out of s could be faced by one or more ISMs. These ISMs could be: single-step ISMs by other individual decision makers; single-step cooperative ISMs by groups of decision makers cooperating together; or multi-step cooperative ISMs by other coalitions of decision makers. And for all of these ISM, the coalition DM_C cannot escape or mitigate the negative effect of them by moving away from the state produced by them. Also, as per the definition, these ISMs are motivated mainly by the desire of the parties committing them to hurt the focal coalition DM_C and put it at a less preferred state than even the original state s from which DM_C's C-GIs start from. Hence, these ISMs are not required by the definition to be UIs, CIs or C-GIs by the committing parties, but instead the definition states that they could be UMs, CMs, or C-GMs for them, respectively.

The SMR stability solution concept assumes that decision maker DM_i (or coalition DM_C) believes that other players *surely* would apply, unilaterally or cooperatively, a sanction against any of his UIs/CIs (the coalition's C-GIs) out of s. And, DM_i (or coalition DM_C) cannot escape the negative effect of sanction by moving away from the state results from the sanction to a more preferred state than s, or an equally preferred state to s. Therefore, DM_i (or DM_C) will not move away from s, and s is a SMR stable state for DM_i (DM_C) . In addition, in this chapter the set of all states which are SMR stable for coalition DM_C at time t will be denoted as $\mathcal{S}^{SMR}_{DM_C,t}$

Also here and as a reminder, it is important to mention that wherever $s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ is mentioned in the second part, the "for a Coalition of Decision Makers" part, of the GMR definition above, the modeller/analyst should qualify what the preferences for the coalition DM_C are, and how they are calculated. We said that ideally: $(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \equiv (\forall DM_i \in DM_C \quad s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$. This is assuming that all the coalition members agree on the preferences and their strength. Otherwise, judging the preference over the states of the game by a coalition requires knowledge about how the coalition decides collectively on its preferences.

The ideal case of coalition's preferences is adopted in this chapter only for simplicity and convenience reasons, and should not be taken as a restriction. In any case, and especially when modelling a real-life cooperative conflict, with coalitions, the modeller/analyst must qualify the truthfulness of any statement about a coalition's preferences by applying the actual method/algorithm the coalition uses to decide on its collective preferences.

Definition 9.4.4 (**Sequentially Stability (SEQ)**):
1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, a state $s \in \mathcal{S}$ is considered a Sequentially (SEQ) Stable state, denoted as $s \in \mathcal{S}^{SEQ}_{DM_i,t}$, iff*

$$\forall s_1 : \quad ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s)))$$
$$[\,\exists\,((DM_j \in \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \vee$$
$$[\,\exists\,((DM_p \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{CI}_{DM_p,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \vee$$
$$[\,\exists\,((DM_P \subseteq \{\mathcal{DM} - DM_i\}) \wedge (s_2 \in \mathcal{S}^{C-GI}_{DM_P,t+1}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)\,] \qquad ;\ and$$

2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, state $s \in \mathcal{S}$ is considered a Sequentially (SEQ) Stable state, denoted as $s \in \mathcal{S}^{SEQ}_{DM_C,t}$, iff*

$$\forall s_1 : \quad (((s_1 \in \mathcal{S}^{C-GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s)))\ \textit{and } s_1 \textit{ is reached at time } t+h\,)$$
$$[\,\exists\,((DM_j \in \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{UI}_{DM_j,t+h}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)\,] \vee$$
$$[\,\exists\,((DM_p \subseteq \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{CI}_{DM_p,t+h}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)\,] \vee$$
$$[\,\exists\,((DM_P \subseteq \{\mathcal{DM} - DM_C\}) \wedge (s_2 \in \mathcal{S}^{C-GI}_{DM_P,t+h}(s_1))) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)\,]$$

The previous SEQ stability definitions given in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions, are expanded in this definition. This new definition explains what it means to have a state be SEQ stable not only for an individual decision maker in a cooperative game, with coalitions, but also for a coalition of decision makers in such games.

The first part of new SEQ definition, Definition 9.4.4, includes now all type of SMI sanction moves that could be imposed, in a cooperative game with coalitions, against an individual decision maker DM_i's UIs or CIs from s. These types include: SMIs by other individual decision makers; single-step cooperative SMIs by groups of decision makers cooperating together; and multi-step cooperative SMIs by coalitions of decision makers.

In addition, the second part of Definition 9.4.4 provides what it means for a state s, in a cooperative game with coalition, to be considered SEQ stable for a coalition DM_C. The definition considers s to be SEQ stable for the coalition if and only if any C-GI move that coalition has out of s could be faced by one or more

SMIs. These SMIs could be: single-step SMIs by other individual decision makers; single-step cooperative SMIs by groups of decision makers cooperating together; or multi-step cooperative SMIs by other coalitions of decision makers.

Recall that the SEQ solution concept requires that the imposed sanction against DM_i's UIs/CIs/C-GIs from s to be also a UI, a CI or a G-CI by the party committing the sanction. In other words, SEQ assumes that all decision makers, individuals or coalitions, in the game to be rational players and act as such. Unlike the GMR solution concept, SEQ assumes that no single player or coalition of players will move in the game, whether individually or cooperatively, for the sake of hurting others and in the process hurt himself/itself. All players in the game will only commit themselves to moves that will benefit them.

The SEQ stability solution concept assumes that decision maker DM_i (or coalition DM_C) believes that other players *surely* would apply, unilaterally or cooperatively, a SMI sanction against any of his UIs/CIs/C-GIs (the coalition's C-GIs) out of s. Therefore, DM_i (or DM_C) will not move away from s, and s is a SEQ stable state for DM_i (DM_C) . In addition, in this chapter we continue to use the same notation to denote the set of all states which are SEQ stable for DM_i at time t as $\mathcal{S}^{SEQ}_{DM_i,t}$. But we add here that the set of all states which are SEQ stable for coalition DM_C at time t will be denoted as $\mathcal{S}^{SEQ}_{DM_C,t}$

As it was the case for the GMR stability definition above, and as a reminder, it is important here to mention that wherever $s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ is mentioned in the second part, the "for a Coalition of Decision Makers" part, of the SEQ definition above, the modeller/analyst should qualify what the preferences for the coalition DM_C are, and how they are calculated. We said that ideally: $(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \equiv (\forall DM_i \in DM_C \quad s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$. This is assuming that all the coalition members agree on the preferences and their strength. Otherwise, judging the preference over the states of the game by a coalition requires knowledge about how the coalition decides collectively on its preferences.

The ideal case of coalition's preferences is adopted in this chapter only for simplicity and convenience reasons, and should not be taken as a restriction. In any case, and especially when modelling a real-life cooperative conflict, with coalitions, the modeller/analyst must qualify the truthfulness of any statement about a coalition's preferences by applying the actual method/algorithm the coalition uses to decide on its collective preferences.

9.4.3 Equilibrium States in Cooperative Games with Coalitions

The concept of an Equilibrium states is an important concept in game theory and conflict analysis. Equilibrium is tied to the concept of Stability Solution Concepts, and explains ultimate stability states in the game. The previous definitions for an Equilibrium state, which is provided in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions, will be expanded here. This new definition will account for the two types of players that cooperative games with coalitions have: individual DMs, and coalitions.

Definition 9.4.5 (Equilibrium (EQ.)): *A state $s \in \mathcal{S}$ is considered an Equilibrium, for a cooperative game with coalitions, at time t, under a specific Solution Concept SC definition, denoted as $s \in \mathcal{S}^{SC\ EQ.}_{\mathcal{DM},t}$, iff*

$$[\,\forall\ Coalition\ DM_C \subseteq \mathcal{DM}\ \ s \in \mathcal{S}^{SC}_{DM_C,t}\,] \ \wedge\ [\,\forall\, DM_i \in \{\mathcal{DM} - \bigcup_{\substack{Coalition\ DM_G:\\ DM_G \subseteq \mathcal{DM}}} DM_G\}\ \ s \in \mathcal{S}^{SC}_{DM_i,t}\,]$$

As per the definition, a state s is stable for a cooperative game, with coalitions, as a whole, i.e. an equilibrium for the game, under a specific solution concept, such as NASH or GMR, if and only if the state s is stable under this solution concept for each and every coalition in the game and each and ever individual decision maker in the game who is not part of a coalition. Notice that this definition is a bit different from the equilibrium definition given in in Chapter 6 for non-cooperative games and in Chapter 8 for cooperative games without coalitions. We have to consider the fact that there are coalitions here; and the assumption that we are making here is that the decision makers who are part of coalitions have surrendered their stability needs for the sake of the stability needs of the coalitions that they belong to. For individual DMs who are not part of any coalition their stability needs must be accounted for separately. If it happened that individual DMs who are part of coalitions decided to part from their coalitions and act individually (not considering the coalition's stable states to be also stable for them), then the definition is still valid. Because those parted DMs no longer belong to any coalition in the game, and are considered therefore as individual DMs, and their stability needs must be accounted for separately.

As in the other types of games, in any specific cooperative game/conflict with coalitions there may be a number of equilibrium states under one or more stability solution concepts. Equilibrium states represent the most likely outcomes for the game, and constitutes possible resolutions to the game. Once one of these states arise, this state is likely persist. But, the strength of this persistence depends on which solution concept the equilibrium state is under, and what is the strength of the state's stability under this solution concept for each of the DMs in the game.

9.5 Stability Strength of Solution Concepts and Equilibriums for Cooperative Conflicts with Coalitions

In this section, we will discuss the mechanisms by which one can identify the strength of the stability, under the four stability solution concept, for any given state in a cooperative game with coalitions, for any given DM in the game. We expand here the stability strength definitions and algorithms provided in Chapter 6 for non-cooperative games, and in Chapter 8 for cooperative games without coalitions. The new definitions and algorithms will accommodate the new multi-step coalition cooperative moves that are possible only within the context of cooperative games with coalitions. Then, we will discuss the strength of an equilibrium under a specific solution concept for a state in a cooperative conflict, with coalitions. We will discuss stabilities strengths, and equilibriums' strengths, for both types of players in these conflicts: individual decision makers, and coalitions.

9.5.1 Stability Strength of Solution Concepts

The same *Stability Concept Strength* value property, denoted as StabilityStrength($StabilityConcept, s, DM_i, t$), discussed in Chapter 8 for cooperative games without coalitions, will continue to be used here for cooperative games with coalitions. Also, the same fuzzy linguistic value label L_{SS} used for for *StabilityStrength* will continue to be used here with the same fuzzy memberships functions given in Figure 4.4-(a). And, the strength expressed by the L_{SS} fuzzy label is meant to represent the strength of the stability concept type *StabilityConcept* (where *StabilityConcept* $\in \{NASH, GMR, SMR, SEQ\}$) for state s, for decision maker DM_i at time t. The only difference is that we will also be able to take the stability strength for a coalition DM_C at time t for a specific solution concept, as we do for the individual decision maker DM_i.

As a reminder, we said that the StabilityStrength value property before fuzzification and without normalized has numeric value is in the range $[0, 2]$, therefore it will have a fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*, *Little*, *None*, *Null*$\}$=$\{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. And, with the understanding that the complete order of these labels is: $Ex{>}Si{>}St{>}F{>}B{>}M{>}Mo{>}L{>}N{>}Null$. And, when the StabilityStrength, before fuzzification, is normalized, i.e. its numeric value is in $[0, 1]$, then its fuzzy labels will include the same labels as above with the exception of the three higher labels covering the range from $[1, 2]$ and those are: *Extreme*,*Significant* and *Strong*.

The L_{SS} fuzzy label assigned to $\mathit{StabilityStrength}(\mathit{StabilityConcept}, s, DM_i, t)$, or $\mathit{StabilityStrength}(\mathit{StabilityConcept}, s, DM_C, t)$ for a coalition, will cover the stability strength satisfaction levels. Where the labels range from representing *Extreme*ly strong stability of s (based on the definition of the solution concept given in *StabilityConcept*) to *None* strength level (to mean very weak stability strength and close to non-existing strength or close to indifferent). And, with the understanding that the *Null* label represents an unknown strength or totaly-non-existing-stability.

We define, now, the stability strength for cooperative games, with coalitions, for each of the solution concepts we introduced in the previous section. Recall that within the context of cooperative games, with coalitions, there are two types of players: 1) the individual players, with their individualistic aims and objectives in the games, despite the fact that are able occasionally to cooperate with other player for specific one-step moves; and 2) coalitions, with their collective aims and objectives in the game and their ability to perform multi-step coalition cooperative moves. We will define the stabilities' strengths for both types of players.

Definition 9.5.1 (Strength of NASH Stability):
1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, and for a NASH stable state $s \in \mathcal{S}^{NASH}_{DM_i,t}$, the strength of s's NASH stability, to DM_i at time t, i.e. $\mathit{StabilityStrength}(NASH, s, DM_i, t)$, is calculated as follows:*

$$(\forall s_{bfr}\colon\ ((s \in \mathcal{S}^{UM,\geq N}_{DM_i,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}:\\ DM_i \in DM_g}} \mathcal{S}^{CM,\geq N}_{DM_g,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}:\\ DM_i \in DM_G}} \mathcal{S}^{C-GM,\geq N}_{DM_G,t}(s_{bfr}))))$$

$$(\mathit{StabilityStrength}(NASH, s, DM_i, t) = |\max_{s_{bfr}}\{\mathit{PrefStrength}(s_{bfr}, s, DM_i, t), -Extreme\}|); and$$

2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, and for a NASH stable state $s \in \mathcal{S}^{NASH}_{DM_C,t}$, the strength of s's NASH stability, to DM_C at time t, i.e. $StabilityStrength(NASH, s, DM_C, t)$, is calculated as follows:*

$$(\forall s_{bfr}\colon\; ((s \in \mathcal{S}^{C\text{-}GM,\geq N}_{DM_C,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}:\\ DM_C \subseteq DM_G}} \mathcal{S}^{C\text{-}GM,\geq N}_{DM_G,t}(s_{bfr}))))$$
$$(StabilityStrength(NASH, s, DM_C, t) = |\max_{s_{bfr}}\{PrefStrength(s_{bfr}, s, DM_C, t), -Extreme\}|)$$

As per the definition, the strength of s's NASH stability strength for an individual DM_i, in a cooperative game with coalitions, is the positive strength equivalent of the negative preference of the state that the worst UI/CI/C-GI move executed/could-be-executed by DM_i, individually, cooperatively, or part of a coalition that he belongs to, at time $<t$ in order to move to s.

Let the NASH's stability strength of a state s for an individual decision maker DM_i at time t be denoted as $NASH(L_{SS})$, where $StabilityStrength(NASH, s, DM_i, t) = L_{SS}$. Algorithm 9.3 uses Definition 9.5.1 to calculate the NASH's stability strength for individual DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

Also as per the definition, for a coalition DM_C, the strength of s's NASH stability strength is the positive strength equivalent of the negative preference of the state that the worst C-GI move executed/could-be-executed by DM_C, collectively (including any UI that individual member of the coalition has or any CI that a group of coalition members -alone or with others outside the coalition- can make), at time $<t$ in order to move to s.

Let the collective NASH's stability strength of a state s for a coalition DM_C at time t be denoted as $NASH(L_{SS})$, where $StabilityStrength(NASH, s, DM_C, t) = L_{SS}$. Algorithm 9.4 uses Definition 9.5.1 to calculate the NASH's stability strength for a coalition of DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

It is important to note here that Algorithm 9.4, and Definition 9.5.1 which the algorithm is based on, refer to the collective preference of a coalition without specifying how this collective-preference will be calculated. Recall that we said in Section 9.2 that in the absence of having one universal way which coalitions use to decide on their preferences, we will assume in this chapter the ideal case: members of the coalition must all agree on the preference, its order and its strength. Therefore, for the algorithm:

Algorithm 9.3 Calculating a State's NASH Stability Strength for an Individual DM in a Cooperative Game with Coalitions

strength-value-label Strength_of_NASH_Stability **(**s**,** DM_i**,** *Game-Structure***)**
// start with the assumption that s is not NASH stable
NASH_Strength = *Null*
// check if DM_i has any UIs/CIs/C-GIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset]$ **then**
  // s is NASH Stable State for DM_i at t; find NASH stability's strength
  NASH_Strength = *Strength_of_Nash*(s, DM_i, *Game-Structure*)
end if
return *NASH_Strength*

strength-value-label Strength_of_Nash**(**s**,** DM_i**,** *Game-Structure***)**
// this routine will return the strength of the weakest UI by DM_i, or C-GI by a cooperating
// group/coalition DM_g he belongs to, that yields to reaching s. First, set Nash strength
// initially to "Extremely Strong" (the case if s has no UIs/CIs/C-GIs that leads to it).
$Strength = -Extreme$
// find s's NASH strength
for all $s_{bfr} : ((s \in \mathcal{S}^{UM,\geq N}_{DM_i,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CM,\geq N}_{DM_g,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C\text{–}GM,\geq N}_{DM_G,t}(s_{bfr})))$ **do**
  $Strength = max\{Strength, PrefStrength(s_{bfr}, s, DM_i, t)\}$
end for
// return the equivalent positive strength label, if $Strength<N$
if $Strength < None$ **then**
  $Strength = |Strength|$
end if
return *Strength*

$$(s \in \mathcal{S}^{C\text{–}GM,\geq N}_{DM_C,t}(s_{bfr})) \equiv [(s \in \mathcal{S}^{C\text{–}GM}_{DM_C,t}(s_{bfr})) \wedge (\forall DM_i \in DM_C \quad (s \in \mathcal{S}^{\geq N}_{DM_i,t}(s_{bfr})))]$$

And because all the members of the coalition must agree on the preference level, adopting the ideal case assumes that the safest way to calculate the strength for a coalition's preference is to take the minimum of the preference's strengths across all the individual members of the coalition. In other words, for Algorithm 9.4:

$$PrefStrength(s_{bfr}, s, DM_C, t) = \min_{DM_i \in DM_C} \{PrefStrength(s_{bfr}, s, DM_i, t)\}$$

Using the ideal case (that all members of the coalition must agree on the preference, its order, and its strength) should not be considered in any way as a restriction on the framework, but rather a convenient simple way to refer to coalition's preferences. In reality, the modeller should take note of how the coalition actually decides

Algorithm 9.4 Calculating a State's NASH Stability Strength for a Coalition of DMs in a Cooperative Game with Coalitions

***strength-value-label Strength_of_NASH_Stability** (s, DM_C, Game-Structure)*
// start with the assumption that s is not NASH stable
NASH_Strength = *Null*
// check if DM_C has any C-GIs from s at time t
if $[\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset]$ **then**
    //s is NASH Stable State for DM_C at t; find NASH stability's strength
    NASH_Strength = *Strength_of_Nash*(s, DM_C, *Game-Structure*)
end if
return *NASH_Strength*

***strength-value-label Strength_of_Nash(s,** DM_C**,** Game-Structure)*
// this routine will return the strength of the weakest C-GI by DM_C, or C-GI by a cooperating
// group/coalition DM_G it belongs to, that yields to reaching s. First, set Nash strength
// initially to "Extremely Strong" (the case if s has no C-GIs that leads to it).
$Strength = -Extreme$
// find s's NASH strength
for all $s_{bfr} : ((s \in \mathcal{S}^{C\text{–}GM,\geq N}_{DM_C,t}(s_{bfr})) \vee (s \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} \mathcal{S}^{C\text{–}GM,\geq N}_{DM_G,t}(s_{bfr})))$ **do**
    $Strength = max\{Strength, PrefStrength(s_{bfr}, s, DM_C, t)\}$
end for
// return the equivalent positive strength label, if $Strength < N$
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return *Strength*

on its preferences, capture it in an algorithm, and then modify Algorithm 9.4 to fit with the way the coalition takes decision on its preferences.

The need for the modeller/analyst, especially when modelling a real-life cooperative conflict with coalitions, to qualify the truthfulness of any statement about a coalition's preferences is very important. He must apply the actual method/algorithm the coalition uses to decide on its collective preferences, and should not assume that all the members of the coalition agree on the preferences, their order, and/or their strengths. It will be hard to find the ideal case reflects the current affairs of real-life coalitions. In reality, it is usually the case that the members in the coalition who who are the biggest in size or market share, the foundering members, the members who owns the key technologies, or the members with the most political/physical powers, have more say on the overall coalition's preferences over a conflict's states than smaller or less powerful members of the Coalition.

Definition 9.5.2 (**Strength of GMR Stability**):

1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, and for a GMR stable state $s \in \mathcal{S}^{GMR}_{DM_i,t}$, the strength of s's GMR stability, to DM_i at time t, i.e. StabilityStrength$(\mathrm{GMR}, s, DM_i, t)$, is calculated as follows:*

$$((\mathcal{S}^{UI}_{DM_i,t}(s)\neq\emptyset) \vee ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g)\mathcal{S}^{CI}_{DM_g,t}(s)\neq\emptyset) \vee ((\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G)\mathcal{S}^{C-GI}_{DM_G,t}(s)\neq\emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g\subseteq\mathcal{DM}:\\ DM_i\in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G\subseteq\mathcal{DM}:\\ DM_i\in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s))))$$

$$[[\,(\forall(DM_j \in \{\mathcal{DM}-DM_i\})\,\forall(s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))) \;\wedge$$

$$(\forall(DM_p \subseteq \{\mathcal{DM}-DM_i\})\,\forall(s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))) \;\wedge$$

$$(\forall(DM_P \subseteq \{\mathcal{DM}-DM_i\})\,\forall(s_2 \in \mathcal{S}^{C-GM}_{DM_P,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\;]$$

$$(StabilityStrength\,(GMR, s, DM_i, t) =$$

$$|\max_{s_1}\{\min_{s_2}\{PrefStrength(s_2, s, DM_i, t), None\}, -Extreme\}|\,)\,]$$

And,

$$((\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset) \wedge ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g)\mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset) \wedge ((\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G)\mathcal{S}^{C-GI}_{DM_G,t}(s)=\emptyset))$$

$$\Rightarrow \quad (StabilityStrength\,(GMR, s, DM_i, t) = None)$$

2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, and for a GMR stable state $s \in \mathcal{S}^{GMR}_{DM_C,t}$, the strength of s's GMR stability, to DM_C at time t, i.e. StabilityStrength$(\mathrm{GMR}, s, DM_C, t)$, is calculated as follows:*

$$((\mathcal{S}^{C-GI}_{DM_C,t}(s)\neq\emptyset) \vee ((\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C-GI}_{DM_G,t}(s)\neq\emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{C-GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G\subseteq\mathcal{DM}:\\ DM_C\in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s))) \quad and \quad s_1 \text{ is reached at time } t+h\,)$$

$$[[\,(\forall(DM_j \in \{\mathcal{DM}-DM_C\})\,\forall(s_2 \in \mathcal{S}^{UM}_{DM_j,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))) \;\wedge$$

$$(\forall(DM_p \subseteq \{\mathcal{DM}-DM_C\})\,\forall(s_2 \in \mathcal{S}^{CM}_{DM_p,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))) \;\wedge$$

$$(\forall(DM_P \subseteq \{\mathcal{DM}-DM_C\})\,\forall(s_2 \in \mathcal{S}^{C-GM}_{DM_P,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))\;]$$

$$(StabilityStrength\,(GMR, s, DM_C, t) =$$

$$|\max_{s_1}\{\min_{s_2}\{PrefStrength(s_2, s, DM_C, t), None\}, -Extreme\}|\,)\,]$$

And,

$$((\mathcal{S}^{C-GI}_{DM_C,t}(s)=\emptyset) \wedge ((\forall DM_G \subseteq \mathcal{DM} : DM_C \in DM_G) \quad \mathcal{S}^{C-GI}_{DM_G,t}(s)=\emptyset))$$

$$\Rightarrow \quad (StabilityStrength\,(GMR, s, DM_C, t) = None)$$

As per the definition, the strength of s's GMR stability strength for an individual DM_i, in a cooperative game with coalitions, is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs/CIs/C-GIs from s, individually or cooperatively, given the fact that DM_i will choose the UI/CI/C-GIs which will yield the best less-preferred end state.

Let the GMR's stability strength of a state s for an individual decision maker DM_i at time t be denoted as $GMR(L_{SS})$, where $StabilityStrength(GMR, s, DM_i, t)= L_{SS}$. Algorithm 9.5 uses Definition 9.5.2 to calculate the GMR's stability strength for individual DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

Also as per the definition, for a coalition DM_C, in a cooperative game with coalitions, the strength of s's GMR stability strength is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_C\}$, against DM_C's C-GIs from s, given the fact that DM_C will choose collectively the C-GIs which will yield the best less-preferred end state.

Let the collective GMR's stability strength of a state s for a coalition DM_C at time t be denoted as $GRM(L_{SS})$, where $StabilityStrength(GMR, s, DM_C, t) = L_{SS}$. Algorithm 9.6 uses Definition 9.5.2 to calculate the GMR's stability strength for a coalition of DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

It is also important to note here that Algorithm 9.6, and Definition 9.5.2 which the algorithm is based on, refer to the collective preference of a coalition without specifying how this collective-preference will be calculated. Recall that we said in Section 9.2 that in the absence of having one universal way which coalitions use to decide on their preferences, we will assume in this chapter the ideal case: members of the coalition must all agree on the preference, its order and its strength. Therefore, for the algorithm:

$$(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \equiv (\forall DM_i \in DM_C \quad (s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$$

And because all the members of the coalition must agree on the preference level, adopting the ideal case assumes that the safest way to calculate the strength for a coalition's preference is to take the minimum of the preference's strengths across all the individual members of the coalition. In other words, for Algorithm 9.6:

$$PrefStrength(s_2, s, DM_C, t) = \min_{DM_i \in DM_C} \{PrefStrength(s_2, s, DM_i, t)\}$$

As a reminder of what we said earlier, using the ideal case (that all members of the coalition must agree on the preference, its order, and its strength) should not be considered in any way as a restriction on the framework, but rather a convenient simple way to refer to coalition's preferences. In reality, the modeller

Algorithm 9.5 Calculating a State's GMR Stability Strength for an Individual DM in a Cooperative Game with Coalitions

strength-value-label Strength_of_GMR_Stability *(s, DM_i, Game-Structure)*
// start with the assumption that s is not GMR stable
$GMR\_Strength = NULL$
// check if DM_i has any UIs/CIs/C-GIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G) \quad \mathcal{S}^{C\text{-}GI}_{DM_G,t}(s)=\emptyset]$ **then**
  // s is NASH Stable State for DM_i at t
  $GMR\_Strength = None$
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}:\\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}:\\ DM_i \in DM_G}} \mathcal{S}^{C\text{-}GI}_{DM_G,t}(s))))$ $[\exists$ an SM$]$ **then**
  // sanction exists against each of DM_i's UIs/CIs/C-GIs $\Rightarrow$ s is GMR stable; find strength
  $GMR\_Strength = Strength\_of\_Sanctions(s,\ DM_i,\ Game\text{-}Structure)$
end if
return $GMR\_Strength$

strength-value-label Strength_of_Sanctions *(s, DM_i, Game-Structure)*
// this routine will return the strength of the sanction that yields the worst result for DM_i,
// given the fact that DM_i will choose the UI/CI/C-GI that will minimize his loss
// set sanction's strength initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}:\\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}:\\ DM_i \in DM_G}} \mathcal{S}^{C\text{-}GI}_{DM_G,t}(s)))$ **do**
  $SancStrength = None$
  for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
    for all $s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
      $SancStrength = min\{SancStrength, PrefStrength(s_2, s, DM_i, t)\}$
    end for
  end for
  for all $DM_p \subseteq \{\mathcal{DM} - DM_i\}$ **do**
    for all $s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
      $SancStrength = min\{SancStrength, PrefStrength(s_2, s, DM_i, t)\}$
    end for
  end for
  for all $DM_P \subseteq \{\mathcal{DM} - DM_i\}$ **do**
    for all $s_2 \in \mathcal{S}^{C\text{-}GM}_{DM_P,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
      $SancStrength = min\{SancStrength, PrefStrength(s_2, s, DM_i, t)\}$
    end for
  end for
  $Strength = max\{Strength, SancStrength\}$
end for
if $Strength < None$ **then**
  $Strength = |Strength|$
end if
return $Strength$

Algorithm 9.6 Calculating a State's GMR Stability Strength for a Coalition of DMs in a Cooperative Game with Coalitions

strength-value-label Strength_of_GMR_Stability *(s, DM_C, Game-Structure)*
// start with the assumption that s is not GMR stable
$GMR\_Strength = NULL$
// check if DM_C has any C-GIs from s at time t
if $[\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset]$ **then**
    $//s$ is NASH Stable State for DM_C at t
    $GMR\_Strength = None$
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s))))$ [$\exists$ an SM sanction] **then**
    // sanction exists against each of DM_C's C-GIs $\Rightarrow s$ is GMR stable; find GMR's strength
    $GMR\_Strength = Strength\_of\_Sanctions(s,\ DM_C,\ Game\text{-}Structure)$
end if
return $GMR\_Strength$

strength-value-label Strength_of_Sanctions(*s, DM_C, Game-Structure)*
// this routine will return the strength of the sanction that yields the worst result for DM_C,
// given the fact that DM_C will choose the C-GI that will minimize the coalition loss
// set sanction's strength initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \in DM_G}} s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)))$ **do**
    h = the number of steps which DM_C/DM_G needs to reach state s_1 starting from s
    $SancStrength = None$
    for all $DM_j \in \{\mathcal{DM} - DM_C\}$ **do**
        for all $s_2 \in \mathcal{S}^{UM}_{DM_j,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ **do**
            $SancStrength = min\{SancStrength, PrefStrength(s_2, s, DM_C, t)\}$
        end for
    end for
    for all $DM_p \subseteq \{\mathcal{DM} - DM_C\}$ **do**
        for all $s_2 \in \mathcal{S}^{CM}_{DM_p,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ **do**
            $SancStrength = min\{SancStrength, PrefStrength(s_2, s, DM_C, t)\}$
        end for
    end for
    for all $DM_P \subseteq \{\mathcal{DM} - DM_C\}$ **do**
        for all $s_2 \in \mathcal{S}^{C\text{–}GM}_{DM_P,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ **do**
            $SancStrength = min\{SancStrength, PrefStrength(s_2, s, DM_C, t)\}$
        end for
    end for
    $Strength = max\{Strength, SancStrength\}$
end for
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return $Strength$

should take note of how the coalition actually decides on its preferences, capture it in an algorithm, and then modify Algorithm 9.6 accordingly.

Definition 9.5.3 (**Strength of SMR Stability**):

1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, and for a SMR stable state $s \in \mathcal{S}^{SMR}_{DM_i,t}$, the strength of s's SMR stability, to DM_i at time t, i.e. StabilityStrength(SMR, s, DM_i, t), is calculated as follows:*

$$((\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \vee ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g)\mathcal{S}^{CI}_{DM_g,t}(s) \neq \emptyset) \vee ((\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G)\mathcal{S}^{C-GI}_{DM_G,t}(s) \neq \emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s))))$$

$$[\,[(\forall (DM_j \in \{\mathcal{DM} - DM_i\})\forall(s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))\forall(s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s)))$$

$$\wedge(\forall (DM_p \subseteq \{\mathcal{DM} - DM_i\})\forall((s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))\forall(s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s)))$$

$$\wedge(\forall (DM_P \subseteq \{\mathcal{DM} - DM_i\})\forall((s_2 \in \mathcal{S}^{C-GM}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \text{ and } s_2 \text{ is reached at time } t+k+1)\forall(s_3 \in \mathcal{S}^{UM}_{DM_i,t+k+1}(s)))]$$

$$(StabilityStrength(SMR, s, DM_i, t) = |\max_{s_1}\{\min_{s_2}\{\max_{s_3}\{PrefStrength(s_2, s, DM_i, t), PrefStrength(s_3, s, DM_i, t)\}, None\}, -Extreme\}|\,)\,]$$

And,

$$((\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \wedge ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g)\mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset) \wedge ((\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G)\mathcal{S}^{C-GI}_{DM_G,t}(s) = \emptyset))$$

$$\Rightarrow \quad (StabilityStrength(SMR, s, DM_i, t) = None)$$

2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, and for a SMR stable state $s \in \mathcal{S}^{SMR}_{DM_C,t}$, the strength of s's SMR stability, to DM_C at time t, i.e. StabilityStrength(SMR, s, DM_C, t), is calculated as follows:*

$$((\mathcal{S}^{C-GI}_{DM_C,t}(s) \neq \emptyset) \vee ((\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C-GI}_{DM_G,t}(s) \neq \emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{C-GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} s_1 \in \mathcal{S}^{C-GI}_{DM_G,t}(s))) \text{ and } s_1 \text{ is reached at time } t+h)$$

$$[[(\forall (DM_j \in \{\mathcal{DM} - DM_C\})\forall(s_2 \in \mathcal{S}^{UM}_{DM_j,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))\forall(s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+1}(s)))$$

$$\wedge(\forall (DM_p \subseteq \{\mathcal{DM} - DM_C\})\forall(s_2 \in \mathcal{S}^{CM}_{DM_p,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))\forall(s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+1}(s)))$$

$$\wedge(\forall (DM_P \subseteq \{\mathcal{DM} - DM_C\})\forall((s_2 \in \mathcal{S}^{C-GM}_{DM_P,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \text{ and } s_2 \text{ is reached at time } t+h+k)\forall(s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+k}(s)))]$$

$$(StabilityStrength(SMR, s, DM_C, t) = |\max_{s_1}\{\min_{s_2}\{\max_{s_3}\{PrefStrength(s_2, s, DM_C, t), PrefStrength(s_3, s, DM_C, t)\}, None\}, -Extreme\}|\,)\,]$$

And,

$$((\mathcal{S}^{C-GI}_{DM_C,t}(s) = \emptyset) \wedge ((\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C-GI}_{DM_G,t}(s) = \emptyset))$$

$$\Rightarrow \quad (StabilityStrength(SMR, s, DM_C, t) = None)$$

As per the definition, the strength of s's SMR stability strength for an individual DM_i, in a cooperative game with coalitions, is the positive strength equivalent of the negative strength of the worst ISM sanction, imposed by other decision makers

$\{\mathcal{DM} - DM_i\}$, against DM_i's UIs/CIs/C-GIs from s, individually or cooperatively, given the fact that DM_i will choose the UI/CI/C-GIs which will yield the best less-preferred end state.

Let the SMR's stability strength of a state s for an individual decision maker DM_i at time t be denoted as $SMR(L_{SS})$, where $StabilityStrength(SMR, s, DM_i, t)= L_{SS}$. Algorithm 9.7, and its additional routine listed as Algorithm 9.8, use Definition 9.5.3 to calculate the SMR's stability strength for individual DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

Algorithm 9.7 Calculating a State's SMR Stability Strength for an Individual DM in a Cooperative Game with Coalitions

strength-value-label Strength_of_SMR_Stability *(s, DM_i, Game-Structure)*
// start with the assumption that s is not SMR stable
SMR_Strength = NULL
// check if DM_i has any UIs/CIs/C-GIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \;\; \mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G) \;\; \mathcal{S}^{C-GI}_{DM_G,t}(s) = \emptyset]$ **then**
    //s is NASH Stable State for DM_i at t
    SMR_Strength = None
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{DM_g \subseteq \mathcal{DM}: DM_i \in DM_g} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{DM_G \subseteq \mathcal{DM}: DM_i \in DM_G} \mathcal{S}^{C-GI}_{DM_G,t}(s))))$ [$\exists$ an ISM] **then**
    // ISM exists against each of DM_i's UIs/CIs/C-GIs $\Rightarrow$ s is SMR stable; find strength
    SMR_Strength = Strength_of_Inescapable_Sanctions(s, DM_i, Game-Structure)
end if
return *SMR_Strength*

Also as per the definition, for a coalition DM_C, in a cooperative game with coalitions, the strength of s's SMR stability strength is the positive strength equivalent of the negative strength of the worst inescapable sanction (ISM), imposed by other decision makers $\{\mathcal{DM} - DM_C\}$, against DM_C's C-GIs from s, given the fact that DM_C will choose collectively the C-GIs which will yield the best less-preferred end state.

Let the collective SMR's stability strength of a state s for a coalition DM_C at time t be denoted as $SRM(L_{SS})$, where $StabilityStrength(SMR, s, DM_C, t)=L_{SS}$. Algorithm 9.9, and its additional routine listed as Algorithm 9.10, use Definition 9.5.3 to calculate the SMR's stability strength for a coalition of DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

As a reminders, it is also important to note here that Algorithm 9.9, and its additional routine listed as Algorithm 9.10, and Definition 9.5.3 which the algorithms are based on, refer to the collective preference of a coalition without specifying

Algorithm 9.8 The "Strength_of_Inescapable_Sanctions" used in Algorithm 9.7

strength-value-label Strength_of_Inescapable_Sanctions(*s****,*** DM_i***,*** *Game-Structure)*
// this routine will return the strength of the ISM that yields the worst result for DM_i, given
// the fact that DM_i will choose a UI/CI/C-GI and a countermove that will minimize his loss.
// Set sanction's strength (after DM_i's countermove) initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s)))$ **do**
    $ISancStrength = None$
    for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{UM}_{DM_j,t+1}(s_1) : ((s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$ **do**
            $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
            $CntrStrength = -Extreme$
            for all $s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)$ **do**
                $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_i, t)\}$
            end for
            if $ISancStrength < CntrStrength$ **then**
                $ISancStrength = CntrStrength$
            end if
        end for
    end for
    for all $DM_p \subseteq \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{CM}_{DM_p,t+1}(s_1) : ((s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)) \wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$ **do**
            $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
            $CntrStrength = -Extreme$
            for all $s_3 \in \mathcal{S}^{UM}_{DM_i,t+2}(s_2)$ **do**
                $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_i, t)\}$
            end for
            if $ISancStrength < CntrStrength$ **then**
                $ISancStrength = CntrStrength$
            end if
        end for
    end for
    for all $DM_P \subseteq \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{C-GM}_{DM_P,t+1}(s_1) : (((s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))$ and s_2 is reached at time $t+k+1)$
            $\wedge (\forall s_3 \in \mathcal{S}^{UM}_{DM_i,t+k+1}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$ **do**
            $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
            $CntrStrength = -Extreme$
            for all $s_3 \in \mathcal{S}^{UM}_{DM_i,t+k+1}(s_2)$ **do**
                $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_i, t)\}$
            end for
            if $ISancStrength < CntrStrength$ **then**
                $ISancStrength = CntrStrength$
            end if
        end for
    end for
    $Strength = max\{Strength, ISancStrength\}$
end for
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return $Strength$

Algorithm 9.9 Calculating a State's SMR Stability Strength for a Coalition of DMs in a Cooperative Game with Coalitions

strength-value-label Strength_of_SMR_Stability *(s, DM_C, Game-Structure)*
// start with the assumption that s is not SMR stable
SMR_Strength = NULL
// check if DM_C has any C-GIs from s at time t
if $[\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset]$ **then**
  //s is NASH Stable State for DM_C at t
  SMR_Strength = None
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{DM_G \subseteq \mathcal{DM}:\, DM_C \subseteq DM_G} s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s))))$ [$\exists$ an ISM sanction] **then**
  // ISM exists against each of DM_C's C-GIs $\Rightarrow$ s is SMR stable; find SMR's strength
  SMR_Strength = Strength_of_Inescapable_Sanctions(s, DM_i, *Game-Structure*)
end if
return *SMR_Strength*

how this collective-preference will be calculated. Recall that we said in Section 9.2 that in the absence of having one universal way which coalitions use to decide on their preferences, we will assume in this chapter the ideal case: members of the coalition must all agree on the preference, its order and its strength. Therefore, for the algorithm, and as an example:

$$(s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \equiv (\forall DM_i \in DM_C \quad (s_3 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$$

And because all the members of the coalition must agree on the preference level, adopting the ideal case assumes that the safest way to calculate the strength for a coalition's preference is to take the minimum of the preference's strengths across all the individual members of the coalition. In other words for Algorithm 9.6, as an example:

$$PrefStrength(s_3, s, DM_C, t) = \min_{DM_i \in DM_C} \{PrefStrength(s_3, s, DM_i, t)\}$$

As a reminder of what we said earlier, using the ideal case (that all members of the coalition must agree on the preference, its order, and its strength) should not be considered in any way as a restriction on the framework, but rather a convenient simple way to refer to coalition's preferences. In reality, the modeller should take note of how the coalition actually decides on its preferences, capture it in an algorithm, and then modify Algorithm 9.9, and its additional routine listed as Algorithm 9.10, to fit with the way the coalition takes decision on its preferences.

Algorithm 9.10 The "Strength_of_Inescapable_Sanctions" used in Algorithm 9.9

strength-value-label Strength_of_Inescapable_Sanctions(**s**,** DM_c**,** Game-Structure**)
// this routine will return the strength of the ISM that yields the worst result for DM_C, given
// the fact that DM_C will choose a C-GI and a countermove that will minimize DM_C's loss.
// Set sanction's strength (after DM_C's countermove) initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{C-GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} s_1 \in \mathcal{S}^{C-GI}_{DM_G,t}(s)))$ **do**
  h = the number of steps which DM_C/DM_G needs to reach state s_1 starting from s
  $ISancStrength = None$
  for all $DM_j \in \{\mathcal{DM} - DM_C\}$ **do**
    for all $s_2 \in \mathcal{S}^{UM}_{DM_j,t+h}(s_1) : ((s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \wedge (\forall s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+1}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))$ **do**
      $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_C, t)\}$
      $CntrStrength = -Extreme$
      for all $s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+1}(s_2)$ **do**
        $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_C, t)\}$
      end for
      if $ISancStrength < CntrStrength$ **then**
        $ISancStrength = CntrStrength$
      end if
    end for
  end for
  for all $DM_p \subseteq \{\mathcal{DM} - DM_C\}$ **do**
    for all $s_2 \in \mathcal{S}^{CM}_{DM_p,t+h}(s_1) : ((s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \wedge (\forall s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+1}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))$ **do**
      $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_C, t)\}$
      $CntrStrength = -Extreme$
      for all $s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+1}(s_2)$ **do**
        $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_C, t)\}$
      end for
      if $ISancStrength < CntrStrength$ **then**
        $ISancStrength = CntrStrength$
      end if
    end for
  end for
  for all $DM_P \subseteq \{\mathcal{DM} - DM_C\}$ **do**
    for all $s_2 \in \mathcal{S}^{C-GM}_{DM_P,t+h}(s_1) : (((s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))$ and s_2 is reached at time $t+h+k)$
      $\wedge (\forall s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+k}(s_2)\ s_3 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))$ **do**
      $ISancStrength = min\{ISancStrength, PrefStrength(s_2, s, DM_C, t)\}$
      $CntrStrength = -Extreme$
      for all $s_3 \in \mathcal{S}^{C-GM}_{DM_C,t+h+k}(s_2)$ **do**
        $CntrStrength = max\{CntrStrength, PrefStrength(s_3, s, DM_C, t)\}$
      end for
      if $ISancStrength < CntrStrength$ **then**
        $ISancStrength = CntrStrength$
      end if
    end for
  end for
  $Strength = max\{Strength, ISancStrength\}$
end for
if $Strength < None$ **then**
  $Strength = |Strength|$
end if
return $Strength$

Definition 9.5.4 (Strength of SEQ Stability):

1) For an Individual Decision Maker *$DM_i \in \mathcal{DM}$ at time t, and for a SEQ stable state $s \in \mathcal{S}^{SEQ}_{DM_i,t}$, the strength of s's SEQ stability, to DM_i at time t, i.e. StabilityStrength(SEQ, s, DM_i, t), is calculated as follows:*

$$((\mathcal{S}^{UI}_{DM_i,t}(s) \neq \emptyset) \vee ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g)\mathcal{S}^{CI}_{DM_g,t}(s) \neq \emptyset) \vee ((\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G)\mathcal{S}^{C-GI}_{DM_G,t}(s) \neq \emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_g \subseteq \mathcal{DM}: \\ DM_i \in DM_g}} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_i \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s))))$$

$$[\,[\,(\forall (DM_j \in \{\mathcal{DM} - DM_i\})\,\forall (s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))) \;\wedge$$

$$(\forall (DM_p \subseteq \{\mathcal{DM} - DM_i\})\,\forall (s_2 \in \mathcal{S}^{CI}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s))) \;\wedge$$

$$(\forall (DM_P \subseteq \{\mathcal{DM} - DM_i\})\,\forall (s_2 \in \mathcal{S}^{C-GI}_{DM_P,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))\;]$$

$$(StabilityStrength\,(SEQ, s, DM_i, t) =$$

$$|\max_{s_1}\{\min_{s_2}\{PrefStrength(s_2, s, DM_i, t), None\}, -Extreme\}\,|\,)\,]$$

And,

$$((\mathcal{S}^{UI}_{DM_i,t}(s) = \emptyset) \wedge ((\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g)\mathcal{S}^{CI}_{DM_g,t}(s) = \emptyset) \wedge ((\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G)\mathcal{S}^{C-GI}_{DM_G,t}(s) = \emptyset))$$

$$\Rightarrow \quad (StabilityStrength\,(SEQ, s, DM_i, t) = None)$$

2) For a Coalition of Decision Makers *$DM_C \subseteq \mathcal{DM}$ at time t, and for a SEQ stable state $s \in \mathcal{S}^{SEQ}_{DM_C,t}$, the strength of s's SEQ stability, to DM_C at time t, i.e. StabilityStrength(GMR, s, DM_C, t), is calculated as follows:*

$$((\mathcal{S}^{C-GI}_{DM_C,t}(s) \neq \emptyset) \vee ((\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C-GI}_{DM_G,t}(s) \neq \emptyset))$$

$$\Rightarrow \quad (\forall s_1 : ((s_1 \in \mathcal{S}^{C-GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \in DM_G}} \mathcal{S}^{C-GI}_{DM_G,t}(s))) \quad and$$

$$s_1 \text{ is reached at time } t+h\,)$$

$$[\,[\,(\forall (DM_j \in \{\mathcal{DM} - DM_C\})\,\forall (s_2 \in \mathcal{S}^{UI}_{DM_j,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))) \;\wedge$$

$$(\forall (DM_p \subseteq \{\mathcal{DM} - DM_C\})\,\forall (s_2 \in \mathcal{S}^{CI}_{DM_p,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s))) \;\wedge$$

$$(\forall (DM_P \subseteq \{\mathcal{DM} - DM_C\})\,\forall (s_2 \in \mathcal{S}^{C-GI}_{DM_P,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)))\;]$$

$$(StabilityStrength\,(SEQ, s, DM_C, t) =$$

$$|\max_{s_1}\{\min_{s_2}\{PrefStrength(s_2, s, DM_C, t), None\}, -Extreme\}\,|\,)\,]$$

And,

$$((\mathcal{S}^{C-GI}_{DM_C,t}(s) = \emptyset) \wedge ((\forall DM_G \subseteq \mathcal{DM} : DM_C \in DM_G) \quad \mathcal{S}^{C-GI}_{DM_G,t}(s) = \emptyset))$$

$$\Rightarrow \quad (StabilityStrength\,(SEQ, s, DM_C, t) = None)$$

As per the definition, the strength of s's SEQ stability strength for an individual DM_i, in a cooperative game with coalitions, is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_i\}$, against DM_i's UIs/CIs/C-GIs from s, individually or cooperatively, given the fact that DM_i will choose the UI/CI/C-GIs which will yield the best less-preferred end state. But recall here that as per Definition 9.4.4, for SEQ stability to be established the sanctions imposed by other players on DM_i's UIs/CIs/C-GIs out

Algorithm 9.11 Calculating a State's SEQ Stability Strength for an Individual DM in a Cooperative Game with Coalitions

strength-value-label Strength_of_SEQ_Stability **(**s**,** DM_i**,** *Game-Structure***)**
// start with the assumption that s is not SEQ stable
$SEQ\_Strength = NULL$
// check if DM_i has any UIs/CIs/C-GIs from s at time t
if $[\mathcal{S}^{UI}_{DM_i,t}(s)=\emptyset] \wedge [(\forall DM_g \subseteq \mathcal{DM} : DM_i \in DM_g) \quad \mathcal{S}^{CI}_{DM_g,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_i \in DM_G) \quad \mathcal{S}^{C\text{-}GI}_{DM_G,t}(s)=\emptyset]$ **then**
    //s is NASH Stable State for DM_i at t
    $SEQ\_Strength = None$
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{DM_g \subseteq \mathcal{DM}: DM_i \in DM_g} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{DM_G \subseteq \mathcal{DM}: DM_i \in DM_G} \mathcal{S}^{C\text{-}GI}_{DM_G,t}(s))))$ [$\exists$ an SMI] **then**
    // SMI exists against each of DM_i's UIs/CIs/C-GIs $\Rightarrow$ s is SEQ stable; find strength
    $SEQ\_Strength = Strength\_of\_SMISanctions(s,\ DM_i,\ Game\text{-}Structure)$
end if
return $SEQ\_Strength$

strength-value-label Strength_of_SMISanctions **(**s**,** DM_i**,** *Game-Structure***)**
// this routine will return the strength of the sanction that yields the worst result for DM_i,
// given the facts that: 1) the sanction move must be a UI/CI/C-GI (for the provider); and
// 2) DM_i will choose the UI/CI/C-GI that will minimize his loss.
// set sanction's strength initially to "Extremely Less Preferred"
$Strength = -Extreme$
for all $s_1 : ((s_1 \in \mathcal{S}^{UI}_{DM_i,t}(s)) \vee (s_1 \in \bigcup_{DM_g \subseteq \mathcal{DM}: DM_i \in DM_g} \mathcal{S}^{CI}_{DM_g,t}(s)) \vee (s_1 \in \bigcup_{DM_G \subseteq \mathcal{DM}: DM_i \in DM_G} \mathcal{S}^{C\text{-}GI}_{DM_G,t}(s)))$ **do**
    $SMISancStrength = None$
    for all $DM_j \in \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{UI}_{DM_j,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
            $SMISancStrength = min\{SMISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
        end for
    end for
    for all $DM_p \subseteq \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{CI}_{DM_p,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
            $SMISancStrength = min\{SMISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
        end for
    end for
    for all $DM_P \subseteq \{\mathcal{DM} - DM_i\}$ **do**
        for all $s_2 \in \mathcal{S}^{C\text{-}GI}_{DM_P,t+1}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)$ **do**
            $SMISancStrength = min\{SMISancStrength, PrefStrength(s_2, s, DM_i, t)\}$
        end for
    end for
    $Strength = max\{Strength, SMISancStrength\}$
end for
if $Strength < None$ **then**
    $Strength = |Strength|$
end if
return $Strength$

of s must be UI/CI/C-GI moves by those players who are committing the sanctions. In other word, they must act "rationally". They will not hurt themselves in order to sanction DM_i's UIs/Cis/C-GIs. And this is at the heart of the difference between GMR stability and SEQ stability.

Let the SEQ's stability strength of a state s for an individual decision maker DM_i at time t be denoted as $SEQ(L_{SS})$, where $StabilityStrength(SEQ, s, DM_i, t) = L_{SS}$. Algorithm 9.11 uses Definition 9.5.4 to calculate the SEQ's stability strength for individual DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

Also as per the definition, for a coalition DM_C, in a cooperative game with coalitions, the strength of s's SEQ stability strength is the positive strength equivalent of the negative strength of the worst sanction, imposed by other decision makers $\{\mathcal{DM} - DM_C\}$, against DM_C's C-GIs from s, given the fact that DM_C will choose collectively the C-GIs which will yield the best less-preferred end state. Again here, for SEQ stability to be established the sanctions imposed by other players on coalition DM_C's C-GIs out of s must be UI/CI/C-GI moves by those players who are committing the sanctions.

Let the collective SEQ's stability strength of a state s for a coalition DM_C at time t be denoted as $SEQ(L_{SS})$, where $StabilityStrength(SEQ, s, DM_C, t) = L_{SS}$. Algorithm 9.12 uses Definition 9.5.4 to calculate the SEQ's stability strength for a coalition of DMs in a cooperative game, with coalitions, and assign the strength's fuzzy linguistic label.

Again, it is important to note here that Algorithm 9.12, and Definition 9.5.4 which the algorithm is based on, refer to the collective preference of a coalition without specifying how this collective-preference will be calculated. Recall that we said in Section 9.2 that in the absence of having one universal way which coalitions use to decide on their preferences, we will assume in this chapter the ideal case: members of the coalition must all agree on the preference, its order and its strength. Therefore, for the algorithm:

$$(s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)) \equiv (\forall DM_i \in DM_C \quad (s_2 \in \mathcal{S}^{\leq N}_{DM_i,t}(s)))$$

And because all the members of the coalition must agree on the preference level, adopting the ideal case assumes that the safest way to calculate the strength for a coalition's preference is to take the minimum of the preference's strengths across all the individual members of the coalition. In other words, for Algorithm 9.12:

$$PrefStrength(s_2, s, DM_C, t) = \min_{DM_i \in DM_C} \{PrefStrength(s_2, s, DM_i, t)\}$$

Algorithm 9.12 Calculating a State's SEQ Stability Strength for a Coalition of DMs in a Cooperative Game with Coalitions

strength-value-label Strength_of_SEQ_Stability *(s, DM_C, Game-Structure)*
// start with the assumption that s is not SEQ stable
$\mathit{SEQ\_Strength} = \mathit{NULL}$
// check if DM_C has any C-GIs from s at time t
if $[\mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)=\emptyset] \wedge [(\forall DM_G \subseteq \mathcal{DM} : DM_C \subseteq DM_G) \quad \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)=\emptyset]$ **then**
    $//s$ is NASH Stable State for DM_C at t
    $\mathit{SEQ}_S\mathit{trength} = \mathit{None}$
else if $(\forall s_1 : ((s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \subseteq DM_G}} s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s))))$ [$\exists$ an SMI] **then**
    // SMI exists against each of DM_C's C-GIs $\Rightarrow$ s is SEQ stable; find SEQ's strength
    $\mathit{SEQ\_Strength} = \mathit{Strength\_of\_SMISanctions}(s,\ DM_C,\ \mathit{Game\text{-}Structure})$
end if
return $\mathit{SEQ\_Strength}$

strength-value-label Strength_of_SMISanctions *(s, DM_C, Game-Structure)*
// this routine will return the strength of the sanction that yields the worst result for DM_C,
// given the facts that: 1) the sanction move must be a UI/CI/C-GI (for the provider); and
// 2) DM_C will choose the C-GI that will minimize the coalition loss.
// set sanction's strength initially to "Extremely Less Preferred"
$\mathit{Strength} = -\mathit{Extreme}$
for all $s_1 : ((s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_C,t}(s)) \vee (s_1 \in \bigcup_{\substack{DM_G \subseteq \mathcal{DM}: \\ DM_C \in DM_G}} s_1 \in \mathcal{S}^{C\text{–}GI}_{DM_G,t}(s)))$ **do**
    h = the number of steps which DM_C/DM_G needs to reach state s_1 starting from s
    $\mathit{SMISancStrength} = \mathit{None}$
    for all $DM_j \in \{\mathcal{DM} - DM_C\}$ **do**
        for all $s_2 \in \mathcal{S}^{UI}_{DM_j,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ **do**
            $\mathit{SMISancStrength} = \mathit{min}\{\mathit{SMISancStrength}, \mathit{PrefStrength}(s_2, s, DM_C, t)\}$
        end for
    end for
    for all $DM_p \subseteq \{\mathcal{DM} - DM_C\}$ **do**
        for all $s_2 \in \mathcal{S}^{CI}_{DM_p,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ **do**
            $\mathit{SMISancStrength} = \mathit{min}\{\mathit{SMISancStrength}, \mathit{PrefStrength}(s_2, s, DM_C, t)\}$
        end for
    end for
    for all $DM_P \subseteq \{\mathcal{DM} - DM_C\}$ **do**
        for all $s_2 \in \mathcal{S}^{C\text{–}GI}_{DM_P,t+h}(s_1) : s_2 \in \mathcal{S}^{\leq N}_{DM_C,t}(s)$ **do**
            $\mathit{SMISancStrength} = \mathit{min}\{\mathit{SMISancStrength}, \mathit{PrefStrength}(s_2, s, DM_C, t)\}$
        end for
    end for
    $\mathit{Strength} = \mathit{max}\{\mathit{Strength}, \mathit{SMISancStrength}\}$
end for
if $\mathit{Strength} < \mathit{None}$ **then**
    $\mathit{Strength} = |\mathit{Strength}|$
end if
return $\mathit{Strength}$

As a reminder of what we said earlier, using the ideal case (that all members of the coalition must agree on the preference, its order, and its strength) should not be considered in any way as a restriction on the framework, but rather a convenient simple way to refer to coalition's preferences. In reality, the modeller should take note of how the coalition actually decides on its preferences, capture it in an algorithm, and then modify Algorithm 9.12 to fit with the way the coalition takes decision on its preferences.

9.5.2 Equilibrium Strength

The same *Equilibrium Strength* value property of a game at time t, denoted as $EquilibriumStrength(StabilityConcept, s, t)$, discussed in Chapter 8 for cooperative games without coalitions, will continue to be used here for cooperative games with coalitions. Also, the same fuzzy linguistic value label L_{SS} used for for *EquilibriumStrength* will continue to be used here with the same fuzzy memberships functions. And, the strength expressed by the L_{SS} fuzzy label is meant to represent the strength of the equilibrium under the specific stability concept type *StabilityConcept* (where $StabilityConcept \in \{NASH, GMR, SMR, SEQ\}$) for state s, for decision maker DM_i at time t. The only difference between the *Equilibrium Strength* value property for a cooperative game without coalition, and another for a cooperative game with coalition, is that the latter will consider two types of players, individual players and coalitions, while the former considers only individual players.

As indicated in the equilibrium definition given earlier (Definition 9.4.5), the equilibrium concept must be defined under a specific stability solution concept. An equilibrium state under a specific stability solution concept is a state that is stable for all the decision makers, whether coalitions or non-coalition individual players, in the game under the same stability solution concept. For example, if a state is an equilibrium under SEQ, then this means that the state is SEQ stable from every coalition and non-coalition player in the game. As a result, the strength of the equilibrium for a specific state s under a specific solution concept SC is tightly coupled with the strength of the SC stabilities of s for each coalition and non-coalition individual player player in the game.

As a reminder, we said that the StabilityStrength value property before fuzzification and without normalized has numeric value is in the range $[0, 2]$, therefore it will have a fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*, *Little*, *None*, *Null*$\}$=$\{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. And, with the understanding that the complete order of these labels is: $Ex{>}Si{>}St{>}F{>}B{>}M{>}Mo{>}L{>}N{>}Null$. And, when the StabilityStrength, before fuzzification, is normalized, i.e. its numeric value is in $[0, 1]$, then its fuzzy labels will include the same labels as above with the exception of the three higher labels covering the range from $[1, 2]$ and those are: *Extreme*,*Significant* and *Strong*.

As for the *StabilityStrength* value property, and because of the dependancy, the *Equilibrium Strength* fuzzy label $L_{SS} \in \{$*Extreme*, *Significant*, *Strong*, *Full*, *Big*, *Much*, *Moderate*, *Some*,*Little*, *None*, *Null*$\}=\{Ex, Si, St, F, B, M, Mo, S, L, N, Null\}$. And, with the complete order of $Ex{>}Si{>}St{>}F{>}B{>}M{>}Mo{>}L{>}N{>}Null$, where the labels range from representing *Extreme*ly strong equilibrium stability (based on the definition of the solution concept given in *StabilityConcept*) of s to *None* strength level (meaning very weak equilibrium strength and close to non-existing strength) for s.

The L_{SS} fuzzy label assigned to *EquilibriumStrength*(*StabilityConcept*, s, t) will cover the equilibrium stability strength satisfaction levels, with the understanding that the *Null* label represents an unknown equilibrium stability strength or totaly-non-existing-equilibrium. The fuzzy membership functions defining these stability/equilibrium strength's linguistic value labels are given in Figure 6.1. The figure shows the membership functions for each label's fuzzy set to be trapezoidal in shape, for simplicity only (not as a restriction) as indicated in Al-Shawa and Basir (2010). In practice, the number of fuzzy sets and their membership functions should be defined based on the user needs and requirements, as we indicated in Al-Shawa and Basir (2009, 2010).

We define, now, the equilibrium strength for cooperative games, with coalitions, under each of the solution concepts we introduced earlier in the chapter. Recall that within the context of cooperative games, with coalitions, there are two types of players: 1) the individual players, with their individualistic aims and objectives in the games, despite the fact that are able occasionally to cooperate with other player for specific one-step moves; and 2) coalitions, with their collective aims and objectives in the game and their ability to perform multi-step coalition cooperative moves. So, as per Definition 9.4.5, we have to consider the fact that there are coalitions here, and the assumption that we are making here is that the decision makers who are part of coalitions have surrendered their stability needs for sake of the stability of the coalitions that they belong to. For individual DMs who are not part of any coalition their stability needs must be accounted for separately. If it happened that individual DMs who are part of coalitions decided to part from their coalitions and act individually (not considering the coalition's stable states also stable to them), then the definition is still valid. Because those parted DMs no longer belong to any coalition in the game, and are considered therefore as individual DMs, and their stability needs must be accounted for separately.

Definition 9.5.5 (**Strength of an Equilibrium**)**:** *For $\mathcal{DM}$, all decision makers in a specific cooperative game, with coalitions, at time t, and for state s which is an Equilibrium for the game under a specific Solution Concept SC, i.e. $s \in \mathcal{S}^{SC\ EQ.}_{\mathcal{DM},t}$, the strength of s's Equilibrium stability, to $\mathcal{DM}$ at time t, is calculated as follows:*

$$[(\exists\,(Coalition\,DM_C \subseteq \mathcal{DM})\ \ s \notin \mathcal{S}^{SC\ EQ.}_{DM_C,t}) \vee (\exists\,(DM_i \in \{\mathcal{DM} - \bigcup_{\substack{Coalition\ DM_G:\\ DM_G \subseteq \mathcal{DM}}} DM_G\})\ \ s \notin \mathcal{S}^{SC\ EQ.}_{DM_i,t})]$$

$$\Rightarrow EquilibriumStrength(SC,s,t) = NULL$$

And,

$$[(\forall\,(Coalition\,DM_C \subseteq \mathcal{DM})\ \ s \in \mathcal{S}^{SC\ EQ.}_{DM_C,t}) \wedge (\forall\,(DM_i \in \{\mathcal{DM} - \bigcup_{\substack{Coalition\ DM_G:\\ DM_G \subseteq \mathcal{DM}}} DM_G\})\ \ s \in \mathcal{S}^{SC\ EQ.}_{DM_i,t})]$$

$$\Rightarrow EquilibriumStrength(SC,s,t) = \min\{\min_{DM_C}\{StabilityStrength(SC,s,DM_C,t)\}\,,\ \min_{DM_i}\{StabilityStrength(SC,s,DM_i,t)\}\,\}$$

Algorithm 9.13 Calculating a State's Equilibrium Strength, under a specific Solution Concept SC, in a Cooperative Game with Coalitions

---

```
strength-value-label Strength_of_Equilibrium (s, SC, Game-Structure)
// start with the assumption that s is not an Equilibrium under SC
SC_EQ_Strength = NULL
// check if s is stable for all DMs in the game under Solution Concept SC
if ((∀(Coalition DM_C ⊆ 𝒟ℳ) s ∈ 𝒮^{SC EQ.}_{DM_C,t}) ∧ (∀(DM_i ∈ {𝒟ℳ − ⋃_{Coalition DM_G: DM_G ⊆ 𝒟ℳ} DM_G}) s ∈ 𝒮^{SC EQ.}_{DM_i,t}))
  then
    // s is an Equilibrium for the game under Solution Concept SC; find EQ's strength
    // set equilibrium's strength initially to "Extremely Strong"
    SC_EQ_Strength = Extreme
    // find s's equilibrium strength
    for all Coalition DM_C ⊆ 𝒟ℳ do
        SC_EQ_Strength = min{SC_EQ_Strength, StabilityStrength(SC,s,DM_C,t)}
    end for
    for all DM_i ∈ {𝒟ℳ − ⋃_{Coalition DM_G: DM_G ⊆ 𝒟ℳ} DM_G} do
        SC_EQ_Strength = min{SC_EQ_Strength, StabilityStrength(SC,s,DM_i,t)}
    end for
end if
return SC_EQ_Strength
```

As per the definition above, if a state s is stable under a Solution Concept SC, then the state form an equilibrium for the game (Definition 9.4.5. The strength of the Equilibrium at s is the minimum strength level of all strengths of s's stability under SC for each coalition and non-coalition individual decision maker in the game.

Let the Equilibrium's stability strength of state s of the game at time t be denoted as $SC\ EQ(L_{SS})$, where EquilibriumStrength(SC, s, t)=L_{SS}. Algorithm 9.13 uses Definition 9.5.5 to calculate the equilibrium's strength and assign the strength's fuzzy linguistic label.

9.6 Case Study: Is it Worth Fighting a Patent Troll? The Showdown between RIM and NTP, as an example

9.6.1 Background

In late 2001, Research in Motion Ltd. (RIM) of Waterloo, Ontario, the company famous of its game-changer BlackBerry® wireless email device had found itself in the middle of what looks like a normal patent infringement lawsuit. NTP, Inc. (NTP) a Virginia-based patent holding company, founded in 1992 by inventor Thomas J. Campana Jr. and Lawyer Donald E. Stout, had filed a lawsuit in the US District Court for the Eastern District of Virginia, claiming that over forty system and method claims from its several patents-in-suit had been knowingly infringed upon by various configurations of the BlackBerry® system (Court of Appeals, 2005). This seemed to many observers at the time as a normal intellectual property legal case that is brought against RIM by a patent troll. Therefore, the case did not get any serious media attention at the time. But four and a half years later, RIM found itself mysteriously embroiled in a battle for its own survival.

The conflict between RIM and NTP holds many features of the classical legal cases/conflicts between patent trolls and real product innovators. Although, NTP is not a patent troll per se, it holds one of the key features of a patent troll: it does not own a product that it actively markets, only patents on papers. In the intellectual property law parlance, a "patent troll" is an individual or a company that owns vague or overly broad patents, but does not commercialize them, and waits for an operating company to infringe them (Melnitzer, 2006). NTP doesn't exactly fit this definition. It was co-founded by Thomas J. Campana Jr., an engineer who in 1990 created a system to send e-mails between computers and wireless devices. His wireless e-mail innovations were shown at the Comdex computer show in Las Vegas. Mr. Campana was working for his own company, but his primary customer, a wireless carrier called Telefind, was unraveling. To protect his work, he formed NTP along with a northern Virginia attorney, Don Stout (Associated Press, 2005; McKenna et al., 2006). In other words, Mr. Campana and his company, NTP, have products that used his patents, but they were at the early stage of development when the funding was cut. So when we use NTP as an example for a patent troll, by no means we say that NTP is actually a patent troll or in any form undermine the work, the intellectual abilities, and/or the ethical practices of Mr. Campana, an engineer who has more than 30 years in the wireless communication industry.

The fact that NTP does not have a wireless email device, such as the BlackBerry®, that it is actively marketing in the marketplace, it has only patents on papers, and that it went after the number one player in the wireless email space, RIM, for infringing on NTP's patents, made the case looks like a patent troll going after a real product company. The other key feature of patent troll cases that fit the RIM v. NTP case is: NTP has nothing to lose, except licensing fees, but RIM has much more than money to lose. Patent trolls, in general, understand this very well. In fact, if a patent troll does not find the product-company has the financial capacity to pay "good" licensing fees, he can go after the product-company's customers, if they are much bigger and wealthier. For example, see the typical patent troll case, reported by (Varchaver, 2001), of Jerome Lemelson, or "The Patent King", going after the customers who use barcode scanners instead of going after the manufacturer of the barcode scanners.

Most of the time, and once the patents found to be valid in courts of law, the patent trolls get their wishes and the defendants sign licensing agreements, pay and settle. What separates the RIM v. NTP case from almost all patent troll cases, and similar cases to the RIM v. NTP case came before and after it, is the fact that RIM did not pay and settle. RIM fought back hard to the point that experts, both within the legal domain and the business domain, called RIM's moves and actions as: gamble, risky, bold, arrogant, ill-advised, and so on (e.g. Green (2005); McKenna et al. (2006)).

So, was RIM right in fighting and not settling the case with NTP? and was RIM right in continuing its fight against NTP even after the courts, in all its levels, sided with NTP and against RIM's position? Looking back, the answer is easy. RIM paid more than ten times what it was supposed to pay if it settled at the time the lower federal court issued its decision favouring NTP's position. Also, if RIM had settled at that time, it would not find itself fighting for survival four and half years later, threatened to be shutdown from its biggest market, and losing current and potential market share to its rivals who allied themselves with NTP. We will look, at the end of this section, at how the conflict evolved over time, and compare the actual path the conflict took with the findings and predictions of our analysis.

For decision makers, looking back and judging whether a decision was a good or a bad one is easy. The difficulty arises when decision makers are required to choose one strategy versus another on real time. This is when conflict analysis and decision support methods are supposed to come to play, offering decision makers methodological methods to test scenarios, weigh options, predict opponents' next moves, and offer a look at possible equilibrium end points to the conflict in hand. In this case study, we look at the RIM v. NTP conflict, model it and analyze it using the concepts and methods introduced earlier in this chapter. What makes this conflict an interesting case study to end this chapter with is the fact that it is not just a conflict that fits the definition of a cooperative game but also it has coalitions (between NTP and RIM's rivals) that could be formed, and as a result affect the main conflict between RIM and NTP.

9.6.2 Structure of the the RIM v. NTP Conflict

Figure 9.4 shows the structure of the RIM v. NTP conflict. Following a conflict structure that is imposed by the legal system, the conflict moves from stage to another as it goes from one court to a higher level one through the appeal process. In the case of RIM v. NTP, and after NTP had filed a federal lawsuit against RIM accusing it of knowingly infringing upon NTP's patented technologies, the lower federal court (the U.S. District Court for the Eastern District of Virginia) can take a decision either: 1) against RIM, finding that RIM in fact did and still infringe upon NTP patents; or 2) against NTP, finding that RIM did not infringe on NTP patents and therefore the lawsuit is baseless. Once the lower federal court takes its decision, then the losing party can file an appeal to the higher court, the U.S. Court of Appeals for the Federal Circuit. This higher court also has to take a decision that is either against RIM or against NTP.

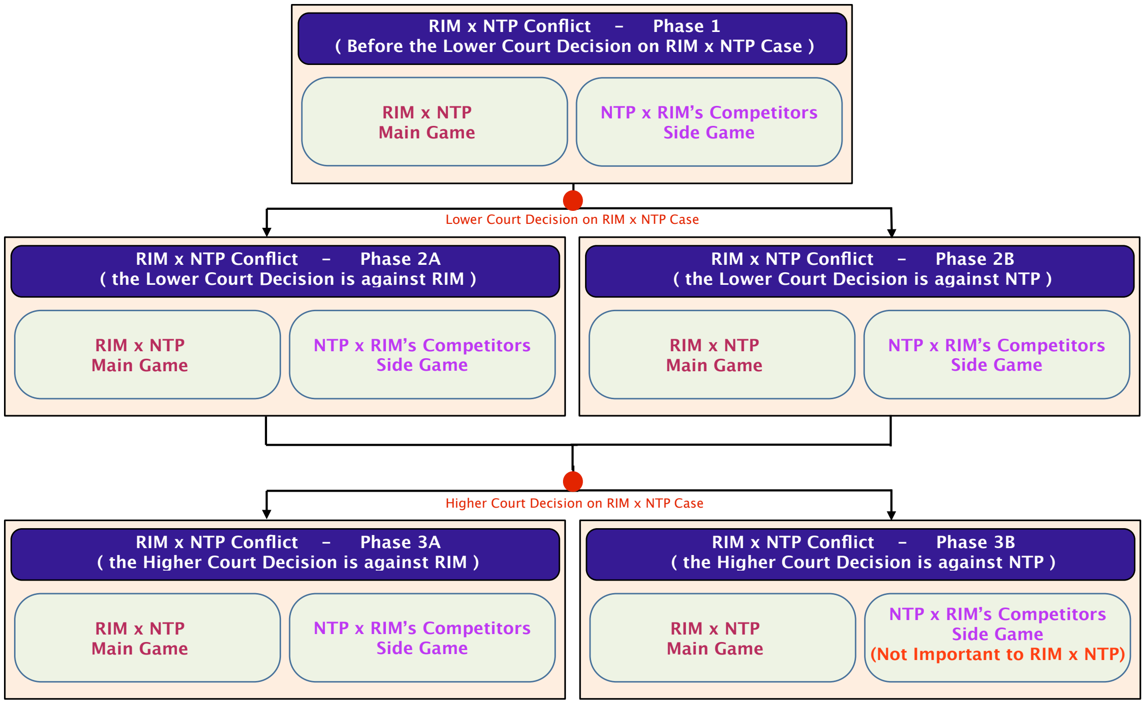


Figure 9.4: The RIM v. NTP Conflict: Phases/Iterations of the conflict, and in each phase there is a main game, between RIM and NTP, and a side game that NTP plays with RIM compatitors (we use Nokia as an example because it is the biggest in terms of market share).

Within each of these stages, we call them phases, that the lawsuit goes through, there are two games played: the main game between RIM and NTP; and a side game between NTP and RIM's rivals in the marketplace. In the side game, of these phases, we intend to test whether RIM's rivals will find it more stable to sign license agreements with NTP, or not. Essentially, we will test whether a coalition between NTP and RIM's rivals is likely to happen, or not. And, if this coalition is likely to happen, then will NTP be able to capitalize on this coalition to affect the main game between it and RIM, puting the main game at a more favourable state to NTP and a less favourable state to RIM.

Figure 9.4 shows the conflict goes from Phase 1, before the lower federal court issue its decision on the case, to either: Phase 2A, if the lower court decision comes in favour of NTP; or Phase 2B, if the lower court decision comes in favour of RIM. Then either Phase 2A or Phase 2B, the conflict will move to either: Phase 3A, if the higher court decision comes in favour of NTP; or Phase 3B, if the lower court decision comes in favour of RIM. In each of these phases, we will analyze both the main game and the side game, with the exception of Phase 3B where we will analyze only the phase's main game (we found no value to the side game at that stage –but one can follow the same analysis methods and add it if found some value for it–).

9.6.3 Players' Strategic Goals and Alternatives

In this case, we will use a simplified flat modelling of the players' GCMs. In other words, we will not model the players' full goals-tree/s, we will just list the main strategic goals each of the players have. We, then, let each of the alternatives/option that the players have contribute to the strategic goals that the players have through lateral goal (the start intention node representing the alternative/option) to goal (the end goal is the strategic goal of the player) relationships.

Table 9.1 shows the strategic goals that each of RIM and NTP has in this conflict. Recall that RIM and NTP are playing in the main game of each phase of the conflict. Both, RIM and NTP are expected to maintain these goals throughout all the phases. On the other hand, NTP and RIM's competitors (we are using Nokia –the biggest of them– as an example) are playing a side game within each of the conflict phases.

Table 9.2 shows the strategic goals that each of NTP and Nokia (and other competitors of RIM) have in these side games. We treat NTP strategic goals in the main game as different than its goals in the side game, because theoretically they can be different, even though in this case we assumed that NTP will maintain its strategic goals in both games, the main and the side game, in each of the conflict's

Table 9.1: RIM v. NTP Conflict: Strategic Goals for the Main Game Players, RIM and NTP

RIM Strategic Goals:	
$G_{RIM\ 1}$	Keep the Financial Cost as low as possible and the Finical Gain as high as possible
$G_{RIM\ 2}$	Protect RIM Technologies (especially the key mobile email ones)
$G_{RIM\ 3}$	Protect Current Customer Base and Market Share
$G_{RIM\ 4}$	Protect Potential Growth in Customer Base and Market Share
NTP Strategic Goals:	
$G_{NTP\ 1}$	Increase the (Potential) Financial Licensing Rewards from RIM as much as possible
$G_{NTP\ 2}$	Increase the (Potential) Financial Licensing Rewards from Other Manufacturers as much as possible
$G_{NTP\ 3}$	Protect the ownership of the key technologies to the mobile email devices

Table 9.2: RIM v. NTP Conflict: Strategic Goals for the Side Game Players, NTP and RIM's Competitors (Nokia –the biggest of them– as an example)

Nokia Strategic Goals : (these will be the same goals that other RIM's competitors will have)	
$G_{NOK\,1}$	Keep the Financial Cost as low as possible and the Finical Gain as high as possible
$G_{NOK\,2}$	Capitalize on RIM Problems and Attract its Customers/Markets
$G_{NOK\,3}$	Protect Current Customer Base and Market Share
$G_{NOK\,4}$	Protect Potential Growth in Customer Base and Market Share

NTP Strategic Goals:	
$G_{NTP\,1}$	Increase the (Potential) Financial Licensing Rewards from RIM as much as possible
$G_{NTP\,2}$	Increase the (Potential) Financial Licensing Rewards from Other Manufacturers as much as possible
$G_{NTP\,3}$	Protect the ownership of the key technologies to the mobile email devices

phases. This is done because we believe NTP demonstrated a consistent behaviour of a patent troll, despite the fact that NTP is not by any stretch of an imagination a typical patent troll as we said earlier. NTP showed that it is motivated by maximizing the financial rewards of using its patent portfolio and protecting these patents.

Table 9.3 shows the alternatives that each of RIM and NTP has in the conflict These are expected to be consistent across all the phases, but limited to the main games (between RIM and NTP) of these phases. On the other hand, Table 9.4 shows the alternatives that NTP and RIM's competitors (Nokia is used here as example because it is the biggest of them) have in the conflict. Again the alternatives that NTP and Nokia (and others) have in the conflict are expected to stay the same across the side games of the different phases of the conflict.

As per Table 9.3, RIM have the option to: continue to fight NTP demands legally ($A_{RIM\,0}$), by all means available to it (such as defending itself in courts, appealing to higher courts, filing to invalidate NTP patents, and so on); continue to fight NTP demands and at the same time find a technical workaround ($A_{RIM\,3}$), allowing RIM to run its BlackBerry® service and network without using NTP's technology; reach an agreement with NTP to either license NTP's technology for future usage of it and pay for previous usage ($A_{RIM\,1}$) or just pay for previous usage ($A_{RIM\,2}$); or finally, withdraw from the US market and stop any operations there, including stop providing the BlackBerry® service to current customers there ($A_{RIM\,4}$). Definitely, some of these alternatives are more preferred to RIM than others but such preferences depend also on the state that the conflict is in, in terms

Table 9.3: RIM v. NTP Conflict: RIM and NTP's Alternatives/Options

The Set of Alternatives available to RIM (A_{RIM}) :	
$A_{RIM\,0}$	*Fight*, in courts, NTP's demands to license their technology, and pay for the claimed RIM's usage of their Technology so far.
$A_{RIM\,1}$	Reach a *Full Agreement* satisfying all NTP's demands to licence the technology and pay for previous usage.
$A_{RIM\,2}$	Reach a *Part Agreement* satisfying some of NTP's demands, namely paying for RIM's previous usage of NTP technology.
$A_{RIM\,3}$	Fight all of NTP's demands, but Find a *Workaround* to what NTP claims to be their technology.
$A_{RIM\,4}$	*Stop US Sales* and Operations.
The Set of Alternatives available to NTP ($A_{NTP\text{-}\mathbf{main}}$) :	
$A_{NTP\,0}$	*Fight* RIM, in courts, asking it to pay for previous usage of NTP's technology, and have it sign a licensing agreement for future usage.
$A_{NTP\,1}$	Reach with RIM a *Full Agreement* satisfying all NTP's demands to licence the technology and pay for previous usage.
$A_{NTP\,2}$	Reach with RIM a *Part Agreement* satisfying some of NTP's demands, namely having RIM pay for previous usage of NTP's technology.
$A_{NTP\,3}$	*Ally* and sign Full Licence Agreements *with RIM's Competitors.*
$A_{NTP\,4}$	*Give Up* and stop the fight, i.e. no longer want to enforce NTP patents.

of what phase the conflict is in and what NTP chooses as a strategy.

NTP, on the other hand, has the following options within the context of the main game of the conflict, as shown in Table 9.3: continue to fight RIM ($A_{NTP\,0}$) by all legal means available to it in an effort to force RIM to license NTP's technology and pay for using it; continue to fight RIM and sign full licensing agreements with RIM's competitors ($A_{NTP\,3}$) in an effort to add more pressure on RIM and show the judge and jury that RIM's competitors are licensing NTP technology (validating NTP's case in front of the legal system); reach an agreement with RIM to either license NTP's technology for RIM's future usage of it and pay for previous usage ($A_{NTP\,1}$) or just pay for previous usage ($A_{NTP\,2}$); or finally, stop the fight with RIM over their usage of NTP's patented technology. It is obvious here too that some of these alternatives are more preferred to NTP than others but such preferences depend also on the state that the conflict is in, in terms of what phase the conflict is in and what RIM chooses as a strategy.

In the side game of the conflict, Table 9.4 shows that NTP and RIM's competitors (we continue to use Nokia –the biggest of them– as an example) have the following options to: NTP to wait for its fight with RIM to finish and settle ($A_{NTP\,a}$) before it goes after RIM's competitors; Nokia to wait for NTP to challenge it legally ($A_{NOK\,a}$) before it enters in any talks with NTP; NTP to fight Nokia over its usage

Table 9.4: RIM v. NTP Conflict: Alternatives of NTP and RIM's Competitors (Nokia —the biggest of them– as an example)

The Set of Alternatives available to NTP ($A_{NTP\text{-}\mathbf{side}}$) :	
$A_{NTP\ a}$	*Wait* for the legal fight with RIM to finish before challenging other companies (e.g. Nokia) to licence-and-pay or go-to-court.
$A_{NTP\ b}$	*Fight*, in courts, these companies (e.g. Nokia) asking them to licence NTP's technology and pay for previous usage.
$A_{NTP\ c}$	Ally and Reach with these companies (e.g. Nokia) *Full Agreements* satisfying all NTP's demands, to license the technology and pay for previous usage.
$A_{NTP\ d}$	Reach with these companies (e.g. Nokia) *Part Agreements* satisfying some of NTP's demands, namely paying for previous usage of NTP technology.

The Set of Alternatives available to Nokia (A_{NOK}) :	
$A_{NOK\ a}$	*Wait* for NTP to challenge/fight Nokia legally.
$A_{NOK\ b}$	*Fight*, in courts, NTP demands for licensing their technology and paying for previous usage.
$A_{NOK\ c}$	Ally with NTP and Reach a *Full Agreement* satisfying All NTP's demands to licence the technology and pay for previous usage.
$A_{NOK\ d}$	Reach a *Part Agreement* satisfying only some of NTP's demands, namely paying for previous usage of NTP technology.

of NTP' technology in courts and use all legal means to force them into entering in a full licensing agreement ($A_{NTP\ b}$); and Nokia to defend itself in courts and use all legal means available to it to fight back ($A_{NOK\ b}$).

Alternatively, Table 9.4 shows, both companies, NTP and Nokia (and possibly other competitors of RIM), can ally and join forces by entering into a full licensing agreement through which Nokia will use NTP's technology and NTP collect royalties (Nokia's $A_{NOK\ c}$ alternative and NTP's $A_{NTP\ c}$ alternative). This alternative will benefit both NTP and Nokia (and other RIM's rivals) by allowing NTP to add more pressure of RIM as well as validate its case in front of the legal system, and allowing Nokia (and others) to capitalize on RIM problems by targeting its current and future customers. Finally, NTP and Nokia (and others) have the equivalent of the last alternative but instead of reaching a full licensing agreement the parties reach a partial agreement that covers Nokia (and other RIM's competitors) paying for previous usage of NTP's technology but not licensing NTP's technology for future usage (Nokia's $A_{NOK\ d}$ alternative and NTP's $A_{NTP\ d}$ alternative).

We will see, when studying the players' preferences over the conflict's states, that both NTP and Nokia (and others) have different preferences over these alternatives that depend on the stage the conflict is in and on the strategies chosen by the opponent.

Table 9.5: RIM v. NTP Conflict - Phase 1: Defining the Conflict's States

The Set of All States $\mathcal{S}$ for the RIM v. NTP Conflict - Phase 1:					
States of RIM v. NTP Game			**States of NTP x Nokia/Others Game**		
State	**RIM Options**	**NTP Options**	**State**	**NTP Options**	**Nokia Options**
s_0 Status Quo	Fight $A_{RIM\ 0}$	Fight $A_{NTP\ 0}$	s_a Status Quo	Wait $A_{NTP\ a}$	Wait $A_{NOK\ a}$
s_1	Full Agrmnt $A_{RIM\ 1}$	Full Agrmnt $A_{NTP\ 1}$	s_b	Fight $A_{NTP\ b}$	Fight $A_{NOK\ b}$
s_2	Part Agrmnt $A_{RIM\ 2}$	Part Agrmnt $A_{NTP\ 2}$	s_c	Full Agrmnt $A_{NTP\ c}$	Full Agrmnt $A_{NOK\ c}$
s_3	Workaround $A_{RIM\ 3}$	Fight $A_{NTP\ 0}$	s_d	Part Agrmnt $A_{NTP\ d}$	Part Agrmnt $A_{NOK\ d}$
s_4	Fight $A_{RIM\ 0}$	Ally w Compt $A_{NTP\ 3}$			

9.6.4 Conflict's States

In this subsection, we will define the states for the RIM v. NTP conflict. This includes all the states for each phase of the conflict, and for both games of the phase, when applicable: the main between RIM and NTP, and the side game between NTP and RIM's competitors.

1) Conflict's States for Phase 1 of the RIM v. NTP conflict

In Phase 1 of the RIM v. NTP conflict, both players have all their alternatives available for them to choose from. But, it is unlikely that RIM will consider stopping its operations and BlackBerry® service at this stage ($A_{RIM\ 4}$) and before the courts forces RIM to do so. Similarly, NTP is not expected to stop the fight against RIM and enforce its patents ($A_{NTP\ 4}$). The final list of possible states at Phase 1 of the conflict, and within the main game between RIM and NTP, is shown in Table 9.5. Each state is defined in the table as a set of alternatives that will be chosen by the players in order to allow the conflict to enter the state. The table also shows the states for Phase 1's side game between NTP and RIM's competitors. In the side game, both NTP and Nokia (and other RIM's competitors) have all their alternatives available for them to choose from.

2) Conflict's States for Phase 2 (A and B) of the RIM v. NTP conflict

In Phase 2 of the conflict, and similar to Phase 1, both players have all their alternatives available for them to choose from. Again here, it is unlikely that RIM

Table 9.6: RIM v. NTP Conflict - Phase 2: Defining the Conflict's States

The Set of All States $\mathcal{S}$ for the RIM v. NTP Conflict - Phase 2: covering states for both Phase 2 cases (A and B, or NTP Wins and RIM Wins)					
States of RIM v. NTP Game			**States of NTP x Nokia/Others Game**		
State	**RIM Options**	**NTP Options**	**State**	**NTP Options**	**Nokia Options**
s_5, s_{10}	Fight $A_{RIM\,0}$	Fight $A_{NTP\,0}$	s_e, s_i	Wait $A_{NTP\,a}$	Wait $A_{NOK\,a}$
s_6, s_{11}	Full Agrmnt $A_{RIM\,1}$	Full Agrmnt $A_{NTP\,1}$	s_f, s_j	Fight $A_{NTP\,b}$	Fight $A_{NOK\,b}$
s_7, s_{12}	Part Agrmnt $A_{RIM\,2}$	Part Agrmnt $A_{NTP\,2}$	s_g, s_k	Full Agrmnt $A_{NTP\,c}$	Full Agrmnt $A_{NOK\,c}$
s_8, s_{13}	Workaround $A_{RIM\,3}$	Fight $A_{NTP\,0}$	s_h, s_l	Part Agrmnt $A_{NTP\,d}$	Part Agrmnt $A_{NOK\,d}$
s_9, s_{14}	Fight $A_{RIM\,0}$	Ally w Compt $A_{NTP\,3}$			

will consider stopping its operations and BlackBerry® service at this stage ($A_{RIM\,4}$) and before the courts forces RIM to do so. Similarly, NTP is not expected to stop the fight against RIM and enforce its patents ($A_{NTP\,4}$). And these exceptions will apply to both players regardless of what the lower court decision was, in favour of RIM or in favour of NTP, because both players are motivated to not lose at this stage while they both have options to escalate the fight, and possibly reverse the lower court's decision by the higher court .

The final list of possible states at Phase 2 of the conflict, and within the main game between RIM and NTP, is shown in Table 9.6. Each state is defined in the table as a set of alternatives that will be chosen by the players in order to allow the conflict to enter the state. The table shows the states for Phase 2 under: Phase 2A, which applies just after the lower court's decision was found to be in favour of NTP and against RIM; and Phase 2B, which applies just after the lower court's decision was found to be in favour of RIM and against NTP. States of Phase 2A are shown in the left side of the "States" column and states of Phase 2B are shown in the right side of the column. The table also shows the states for Phase 2's side game between NTP and RIM's competitors, again for both Phase 2A and Phase 2B. In the side game, both NTP and Nokia (and other RIM's competitors) have all their alternatives available for them to choose from.

3) Conflict's States for Phase 3 (A and B) of the RIM v. NTP conflict

In Phase 3 of the conflict, and unlike Phase 1 and 2, it is possible that RIM will consider stopping its operations and BlackBerry® service at this stage ($A_{RIM\,4}$), or be forced to do so, if the higher court orders it to seize any operation in the US until it satisfies its obligation towards NTP's rights. Similarly, NTP could decide to stop the fight against RIM and therefore stop enforcing its patents ($A_{NTP\,4}$), especially if the higher court rejects its claims towards RIM and found its case baseless. But two other alternatives have no effect on how the conflict will progress from now on or end.

Alternatives $A_{RIM\,3}$, in which chooses to have a work around NTP technology while it is continuing to fight NTP, is useless to RIM at this stage of the conflict because this will not change the fact that the case has been decided on by the higher court. If the higher court decides against RIM, RIM must stop its operation or satisfy NTP demands, as decided by the court, whether RIM finds a workaround to its usage of NTP's technology or not. And, if the court decision comes against NTP, RIM does not need to look for a workaround.

In addition, Alternative $A_{NTP\,3}$ for NTP has an effect on the conflict and how it will progress or end., only if the higher court decided against RIM. If this is the case, allying with RIM's rivals will give some assurance to the court, and current RIM customers, that the service will not be completely lost. Companies allied with NTP will ensure a valid alternative (at RIM's expense). This definitely will put additional pressure on RIM to settle, otherwise RIM will be forced to seize any operation in its biggest market, the US. On the other hand, if the higher court decides against NTP wishes, then the alliance with RIM's rivals, and any agreements NTP signs with them, will not benefit NTP at this stage (this is also the reason that the side game between NTP and RIM's rivals does not matter at Phase 3B). NTP can only take the fight higher to the supreme court or end the fight.

Beside these exceptions both players will have all their other alternatives available to them regardless of what the higher court decision come to be, in favour of RIM or in favour of NTP. Both NTP or RIM can take the fight to the supreme court if they are motivated to not lose at this stage, but RIM options in this regard seem much limited especially if the courts ordered it to seize any operations in the US until the supreme court makes its decision (a most likely outcome if both lower levels of the legal system decided against RIM). RIM can still fight through the United States Patent and Trademark Office (USPTO) to invalidate NTP's patents and/or claims in these patents, but RIM must comply with the higher court order if it gives/confirms a seize-operation ruling. On the other hand, NTP fighting options seem open, ranging from appeals to multiple cases to appeals through the

USPTO. Not to mention that NTP has nothing to lose and everything to win if it continues the fight, especially when RIM starts feeling the heat in the marketplace with customers leaving because of the uncertainty and its share price suffers.

The final list of possible states at Phase 3 of the conflict, and within the main game between RIM and NTP, is shown in Table 9.6. Each state is defined in the table as a set of alternatives that will be chosen by the players in order to allow the conflict to enter the state. The table shows the states for Phase 3 under: Phase 3A, which applies just after the higher court decision was found to be in favour of NTP and against RIM; and Phase 3B, which applies just after the higher court decision was found to be in favour of RIM and against NTP. States of Phase 3A are shown in the left side of the "States" column and states of Phase 3B are shown in the right side of the column. The table shows the states for a side game between NTP and RIM's competitors only within Phase 3A of the conflict. As we said above, the side game is not relevant to the conflict between RIM and NTP at Phase 3B and will not affect how the conflict progress or end.

9.6.5 Players' Preferences over States of the Conflict

To calculate the preferences of the players over the conflict's states, for each phase and for each game of the state, we calculate how much each state (with all its play-

Table 9.7: RIM v. NTP Conflict - Phase 3: Defining the Conflict's States

The Set of All States $\mathcal{S}$ for the RIM v. NTP Conflict - Phase 3: covering states for both Phase 3 cases (A and B, or NTP Wins and RIM Wins)					
States of RIM v. NTP Game			**States of NTP x Nokia/Others Game**		
State	**RIM Options**	**NTP Options**	**State**	**NTP Options**	**Nokia Options**
s_{15}, s_{20}	Fight $A_{RIM\ 0}$	Fight $A_{NTP\ 0}$	s_m	Wait $A_{NTP\ a}$	Wait $A_{NOK\ a}$
s_{16}, s_{21}	Full Agrmnt $A_{RIM\ 1}$	Full Agrmnt $A_{NTP\ 1}$	s_n	Fight $A_{NTP\ b}$	Fight $A_{NOK\ b}$
s_{17}, s_{22}	Part Agrmnt $A_{RIM\ 2}$	Part Agrmnt $A_{NTP\ 2}$	s_o	Full Agrmnt $A_{NTP\ c}$	Full Agrmnt $A_{NOK\ c}$
s_{18}	Stop US Sales $A_{RIM\ 4}$		s_p	Part Agrmnt $A_{NTP\ d}$	Part Agrmnt $A_{NOK\ d}$
s_{19}	Fight $A_{RIM\ 0}$	Ally w Compt $A_{NTP\ 3}$			
s_{23}		Give Up $A_{NTP\ 4}$			

ers' alternatives selected as shown in Tables 9.5 - 9.7) contributes to the achievement of the players' strategic goals. The sets of strategic goals for the players were defined earlier to be: $\mathcal{SG}_{RIM} = \{G_{RIM\,1}, G_{RIM\,2}, G_{RIM\,3}, G_{RIM\,4}\}$; $\mathcal{SG}_{NTP} = \{G_{NTP\,1}, G_{NTP\,2}, G_{NTP\,3}\}$; $\mathcal{SG}_{NOK} = \{G_{NOK\,1}, G_{NOK\,2}, G_{NOK\,3}, G_{NOK\,4}\}$. All the strategic goals are explained individually above in Tables 9.1 - 9.2. The sets of the conflict's states, for each of the phases and for each of the games within the phases, are defined in Tables 9.5 - 9.7.

For each game within each of the 3 phases the RIM v. NTP conflict has, we calculate the *Weighted Payoff* value for each state for each player using the method presented in Section 5.4.2. As we will see, the strategic importance of the different strategic goals, the players have, changes based on the phase the conflict is in and the game they are in. But, for all the games, in all the phases, we set the Rationality and Emotionality Factors for the players to Full and None, or 1.0 and 0.0, respectively. In other words, all players are assumed to be fully rational and not emotional, as one could expect from business entities with institutional collective rationality. Saying so, the analyst can still test different values for the rationality and emotional factors, and the implication of such values, by setting up what-if scenario models as variations to the games' base models during the sensitivity analysis stage. This might be important especially if the analyst suspects that some of the players start acting emotionally, or showing signs of such.

1) Players' Preferences for Phase 1 of the RIM v. NTP conflict (before the lower court's decision)

From the calculated Weighted Payoffs, for each state to each of the players, we calculate the Ordinal Preferences for the players over the conflict's states in Phase 1. Figure 9.5 shows the weighted payoffs and the ordinal preferences for each state of Phase 1's games, for each player. Figure 9.5a shows the preferences of RIM and NTP in the main game of the phase. And, Figure 9.5b shows the preferences of NTP and Nokia (as an example of RIM's competitors) in the side game of the phase.

In the main game of Phase 1, as we are showing in Figure 9.5 , NTP is considering all its strategic goals of *Full* importance. This is because both financial and technology-ownership related goals are all important to NTP at this stage of the conflict, and will continue to be so through out the conflict. RIM, on the other hand, has its market-share and technology-ownership related strategic goals to be of *Full* importance to it. But, because at the time when NTP started to fight RIM

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

RIM x NTP Conflict: Phase # 1								
		RIM				NTP		
Strategic Goals of DMs	SGs:	SG_{RIM}				SG_{NTP}		
		$SG_{RIM\,1}$	$SG_{RIM\,2}$	$SG_{RIM\,3}$	$SG_{URIM\,4}$	$SG_{NTP\,1}$	$SG_{NTP\,2}$	$SG_{NTP\,3}$
Strategic Importance	SImprt(SG_k)	M	F	F	F	F	F	F
State S_0	Achv(SG_k)	N	B	F	F	B	B	B
	Prvn(SG_k)	L	N	N	N	N	N	N
Status Quo (2003) NTP insists & RIM fights	FAchv(SG_k)	–L	B	F	F	B	B	B
	TWFAchv(SG_k,DM)	–0.12	0.80	1.00	1.00	0.80	0.80	0.80
{ Achv($A_{RIM\,0}$)=F,	WP(S_0, DM)	0.67				0.80		
Achv($A_{NTP\,0}$)=F }	OP(S_0, DM)	1 (Best)				3		
State S_1	Achv(SG_k)	N	L	B	M	F	F	F
	Prvn(SG_k)	B	N	N	N	F	N	N
RIM agrees to Pay for old use + signs Lic. future use	FAchv(SG_k)	–B	L	B	M	N	F	F
	TWFAchv(SG_k,DM)	–0.48	0.20	0.80	0.60	1.00	1.00	1.00
{ Achv($A_{RIM\,1}$)=F,	WP(S_1, DM)	0.28				1.00		
Achv($A_{NTP\,1}$)=F }	OP(S_1, DM)	4				1 (Best)		
State S_2	Achv(SG_k)	N	L	B	M	Mo	Mo	M
	Prvn(SG_k)	M	N	N	N	F	N	N
RIM agrees to Pay for old Only	FAchv(SG_k)	–M	L	B	M	Mo	Mo	M
	TWFAchv(SG_k,DM)	–0.36	0.20	0.80	0.60	0.50	0.50	0.60
{ Achv($A_{RIM\,2}$)=F,	WP(S_2, DM)	0.31				0.53		
Achv($A_{NTP\,2}$)=F }	OP(S_2, DM)	3				4		
State S_3	Achv(SG_k)	N	M	B	M	L	L	B
	Prvn(SG_k)	S	N	N	N	S	N	N
NTP insists & RIM fights +RIM develops Workaround	FAchv(SG_k)	–S	M	B	M	L	S	B
	TWFAchv(SG_k,DM)	–0.24	0.60	0.80	0.60	0.20	0.20	0.80
{ Achv($A_{RIM\,3}$)=F,	WP(S_3, DM)	0.44				0.40		
Achv($A_{NTP\,0}$)=F }	OP(S_3, DM)	2				5 (Worst)		
State S_4	Achv(SG_k)	N	L	M	S	F	F	B
	Prvn(SG_k)	L	N	N	N	F	N	N
NTP insists & RIM fights +NTP signs Lic Agr. w Others	FAchv(SG_k)	–L	L	M	S	N	F	B
	TWFAchv(SG_k,DM)	–0.12	0.20	0.60	0.40	1.00	1.00	0.80
{ Achv($A_{RIM\,0}$)=F,	WP(S_4, DM)	0.27				0.93		
Achv($A_{NTP\,3}$)=F }	OP(S_4, DM)	5 (Worst)				2		

(a) Preferences for RIM and NTP in Phase 1 of the conflict's main game.

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

Nokia x NTP Conflict: Phase # 1								
		Nokia				NTP		
Strategic Goals of DMs	SGs:	SG_{Nokia}				SG_{NTP}		
		$SG_{Nokia\,1}$	$SG_{Nokia\,2}$	$SG_{Nokia\,3}$	$SG_{UNokia\,4}$	$SG_{NTP\,1}$	$SG_{NTP\,2}$	$SG_{NTP\,3}$
Strategic Importance	SImprt(SG_k)	M	F	F	F	F	F	F
State S_a	Achv(SG_k)	N	L	F	F	B	B	B
	Prvn(SG_k)	N	N	N	N	N	N	N
Status Quo (2003) NTP Waits & Nokia Waits	FAchv(SG_k)	N	L	F	F	B	B	B
	TWFAchv(SG_k,DM)	0.00	0.20	1.00	1.00	0.80	0.80	0.80
{ Achv($A_{NTP\,a}$)=F,	WP(S_a, DM)	0.55				0.80		
Achv($A_{NOK\,a}$)=F }	OP(S_a, DM)	1 (Best)				2		
State S_b	Achv(SG_k)	N	N	F	F	M	B	B
	Prvn(SG_k)	L	N	N	N	N	N	N
NTP Fights & Nokia Fights	FAchv(SG_k)	–L	N	F	F	M	B	B
	TWFAchv(SG_k,DM)	–0.12	0.00	1.00	1.00	0.60	0.80	0.80
{ Achv($A_{NTP\,b}$)=F,	WP(S_b, DM)	0.47				0.73		
Achv($A_{NOK\,b}$)=F }	OP(S_b, DM)	2				3		
State S_c	Achv(SG_k)	N	S	B	Mo	F	F	F
	Prvn(SG_k)	B	N	N	N	N	N	N
Nokia agrees to Pay for old use + signs Lic. future use	FAchv(SG_k)	–B	S	B	Mo	F	F	F
	TWFAchv(SG_k,DM)	–0.48	0.40	0.80	0.60	1.00	1.00	1.00
{ Achv($A_{NTP\,c}$)=F,	WP(S_c, DM)	0.33				1.00		
Achv($A_{NOK\,c}$)=F }	OP(S_c, DM)	4 (Worst)				1 (Best)		
State S_d	Achv(SG_k)	N	S	B	M	L	L	B
	Prvn(SG_k)	M	N	N	N	N	N	N
Nokia agrees to Pay for old (Partial Agreement with NTP)	FAchv(SG_k)	–M	S	B	M	L	L	B
	TWFAchv(SG_k,DM)	–0.36	0.40	0.80	1.00	0.20	0.20	0.80
{ Achv($A_{NTP\,d}$)=F,	WP(S_d, DM)	0.46				0.40		
Achv($A_{NOK\,d}$)=F }	OP(S_d, DM)	3				4 (Worst)		

(b) Preferences for NTP and Nokia (the biggest competitor to RIM) in Phase 1 of the conflict's side game between NTP and RIM Competitors.

Figure 9.5: The RIM v. NTP Conflict - Phase 1 (before lower court order is issued): Preferences of RIM, NTP and RIM Competitors (Nokia as an example).

Table 9.8: RIM v. NTP Conflict - Phase 1: Players' Preferences in the main game (RIM v. NTP)

RIM Preferences (*Most* to *Least* Preferred)					
RIM	s_0	s_3	s_2	s_1	s_4
	0.67	0.44	0.31	0.28	0.27
RIM Preferences' Strengths					
$\succ^{L_{PS}}_{RIM,t}$	s_0	s_3	s_2	s_1	s_4
s_0	N	L	S	S	S
s_3	-L	N	L	L	L
s_2	-S	-L	N	N	N
s_1	-S	-L	N	N	N
s_4	-S	-L	N	N	N

NTP Preferences (*Most* to *Least* Preferred)					
NTP	s_1	s_4	s_0	s_2	s_3
	1.00	0.93	0.80	0.53	0.40
NTP Preferences' Strengths					
$\succ^{L_{PS}}_{NTP,t}$	s_1	s_4	s_0	s_2	s_3
s_1	N	N	L	Mo	M
s_4	N	N	L	S	Mo
s_0	-L	-L	N	L	S
s_2	-Mo	-S	-L	N	L
s_3	-M	-Mo	-S	-L	N

Table 9.9: RIM v. NTP Conflict - Phase 1: Players' Preferences in the side game (NTP x RIM's Competitors (Nokia as an example))

NTP Preferences (*Most* to *Least*)				
NTP	s_c	s_a	s_b	s_d
	1.00	0.80	0.73	0.40
NTP Preferences' Strengths				
$\succ^{L_{PS}}_{NTP,t}$	s_c	s_a	s_b	s_d
s_c	N	L	L	M
s_a	-L	N	N	S
s_b	-L	N	N	S
s_d	-M	-S	-S	N

Nokia Preferences (*Most* to *Least*)				
NOK	s_a	s_b	s_d	s_c
	0.55	0.47	0.46	0.33
NOK Preferences' Strengths				
$\succ^{L_{PS}}_{NOK,t}$	s_a	s_b	s_d	s_c
s_a	N	N	N	L
s_b	N	N	N	L
s_d	N	N	N	L
s_c	-L	-L	-L	N

for license rights over the technology used by RIM, as it claims, RIM was prospering and enjoying huge success in the market, RIM has its financial related strategic goal a bit less important than the other. The figure shows RIM has its financial goal, $G_{RIM\,1}$, set at the *Much* importance level, while all other technology-ownership and market related goals set at the *Full* importance level.

The strengths of each of the players' preferences, within each game of the phase, are then elicited from the weighted preferences shown in Figures 9.5a and 9.5b. Table 9.8 shows the preferences' strengths for NTP and RIM in the main game of Phase 1. And, Table 9.9 shows the preferences' strengths for NTP and Nokia (as an example of RIM's competitors) in the side game of Phase 1. Both tables also show the preferences vector for each player in each one of these two games.

2) Players' Preferences for Phase 2A of the RIM v. NTP conflict (after the lower court decides against RIM's position)

From the calculated Weighted Payoffs, for each state to each of the players, we calculate the Ordinal Preferences for the players over the conflict's states in Phase 2A. Figure 9.6 shows the weighted payoffs and the ordinal preferences for each state of Phase 2A's games, for each player. Figure 9.6a shows the preferences of RIM and NTP in the main game of the phase. And, Figure 9.6b shows the preferences of NTP and Nokia (as an example of RIM's rivals) in the side game of the phase.

In our analysis of the RIM v. NTP conflict, and what happened in it, historically speaking, we have noticed that RIM puts a lot more importance on its financial strategic goal, within the context of this conflict, than it should. We will elaborate later, when we discuss the stability analysis of the conflict and its results, on this observation and the implication of this on the conflict as it actually unfolded in real-life. In this section, and for the sake of following the same presentation style of modelling and analyzing conflicts as used so far in this document, we will show two preferences models for RIM at Phase 2A of the conflict. First, the *Should-Be* RIM's preferences model based on RIM placing "Little" importance level of its financial strategic goal, $G_{RIM\,1}$. This reflects the fact that RIM, once been handed a court decision against it at the lower federal court, should have seen the clock should start ticking and should feel that all its future now is at stake (its current customers and market in the US). Therefore, RIM places *Little* importance on how much resolving this conflict will cost it, but still keeps placing *Full* importance on all its market and technology-ownership goals. Recall that this is similar to the importance levels placed by RIM, as per the Phase 1 preference model for RIM, shown in Figure 9.5a, with the exception that $G_{RIM\,1}$ in that model is given a *Much* importance level compared to the *Little* importance level this goal is been given in the Should-Be RIM's preferences model for Phase 2A, as shown in Figure 9.6a.

The second preferences model for RIM at Phase 2A is the one which RIM actually demonstrated historically in the conflict as the conflict was unfolding in real-life. In this second model, which we will call hereafter as the *the Observed* RIM's preferences model for this phase, RIM keeps the same importance levels it placed to all its strategic goals in Phase 1 of the conflict (shown in Figure 9.5a). RIM keeps the financial goal $G_{RIM\,1}$ set at the *Much* importance level while all other goals are set to the *Full* importance level. In other words, RIM still thinks that settling the conflict with NTP is not worth paying for, and if it should, then the

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

RIM x NTP Conflict: Phase # 2A (Lower Court Order AGAINST RIM)								
		RIM (SHOULD BE)				NTP		
Strategic Goals of DMs	SGs:	SG_{RIM}				SG_{NTP}		
		$SG_{RIM\ 1}$	$SG_{RIM\ 2}$	$SG_{RIM\ 3}$	$SG_{URIM\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	SImprt(SG_k)	L	F	F	F	F	F	F
State S_5	Achv(SG_k)	N	S	Mo	S	B	B	B
	Prvn(SG_k)	L	N	N	N	N	N	N
	FAchv(SG_k)	–L	S	Mo	S	B	B	B
NTP insists & RIM fights	TWFAchv(SG_k,DM)	–0.04	0.40	0.50	0.40	0.80	0.80	0.80
{ Achv($A_{RIM\ 0}$)=F,	WP(S_5, DM)	0.32				0.80		
Achv($A_{NTP\ 0}$)=F }	OP(S_5, DM)	4				3		
State S_6	Achv(SG_k)	N	L	B	M	F	F	F
	Prvn(SG_k)	B	N	N	N	F	N	N
RIM agrees to Pay for old use + signs Lic. future use	FAchv(SG_k)	–B	L	B	M	N	F	F
	TWFAchv(SG_k,DM)	–0.16	0.20	0.80	0.60	1.00	1.00	1.00
{ Achv($A_{RIM\ 1}$)=F,	WP(S_6, DM)	0.36				1.00		
Achv($A_{NTP\ 1}$)=F }	OP(S_6, DM)	2				1 (Best)		
State S_7	Achv(SG_k)	N	L	B	M	Mo	Mo	M
	Prvn(SG_k)	M	N	N	N	F	N	N
RIM agrees to Pay for old Only	FAchv(SG_k)	–M	L	B	M	Mo	Mo	M
	TWFAchv(SG_k,DM)	–0.12	0.20	0.80	0.60	0.50	0.50	0.60
{ Achv($A_{RIM\ 2}$)=F,	WP(S_7, DM)	0.37				0.53		
Achv($A_{NTP\ 2}$)=F }	OP(S_7, DM)	1 (Best)				5 (Worst)		
State S_8	Achv(SG_k)	N	S	M	S	M	Mo	B
	Prvn(SG_k)	S	N	N	N	S	N	N
NTP insists & RIM fights +RIM develops Workaround	FAchv(SG_k)	–S	S	M	S	M	Mo	B
	TWFAchv(SG_k,DM)	–0.08	0.40	0.60	0.40	0.60	0.50	0.80
{ Achv($A_{RIM\ 3}$)=F,	WP(S_8, DM)	0.33				0.63		
Achv($A_{NTP\ 0}$)=F }	OP(S_8, DM)	3				4		
State S_9	Achv(SG_k)	N	L	Mo	L	F	F	B
	Prvn(SG_k)	L	N	N	N	F	N	N
NTP insists & RIM fights +NTP signs Lic Agr. w Others	FAchv(SG_k)	–L	L	Mo	L	N	F	B
	TWFAchv(SG_k,DM)	–0.04	0.20	0.50	0.20	1.00	1.00	0.80
{ Achv($A_{RIM\ 0}$)=F,	WP(S_9, DM)	0.22				0.93		
Achv($A_{NTP\ 3}$)=F }	OP(S_9, DM)	5 (Worst)				2		

(a) Preferences for RIM and NTP in Phase 2A of the conflict's main game. RIM's Preferences shown here are the ones that should-be RIM's preferences.

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

Nokia x NTP Conflict: Phase # 2A								
		Nokia				NTP		
Strategic Goals of DMs	SGs:	SG_{Nokia}				SG_{NTP}		
		$SG_{Nokia\ 1}$	$SG_{Nokia\ 2}$	$SG_{Nokia\ 3}$	$SG_{UNokia\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	SImprt(SG_k)	S	F	F	F	F	F	F
State S_e	Achv(SG_k)	N	L	B	Mo	B	B	B
	Prvn(SG_k)	N	N	N	N	N	N	N
	FAchv(SG_k)	N	L	B	Mo	B	B	B
NTP Waits & Nokia Waits	TWFAchv(SG_k,DM)	0.00	0.20	0.80	0.60	0.80	0.80	0.80
{ Achv($A_{NTP\ a}$)=F,	WP(S_e, DM)	0.40				0.80		
Achv($A_{NOK\ a}$)=F }	OP(S_e, DM)	3				2		
State S_f	Achv(SG_k)	N	N	M	Mo	M	B	B
	Prvn(SG_k)	M	N	N	N	N	N	N
NTP Fights & Nokia Fights	FAchv(SG_k)	–M	N	M	Mo	M	B	B
	TWFAchv(SG_k,DM)	–0.36	0.00	0.60	0.50	0.60	0.80	0.80
{ Achv($A_{NTP\ b}$)=F,	WP(S_f, DM)	0.19				0.73		
Achv($A_{NOK\ b}$)=F }	OP(S_f, DM)	4 (Worst)				3		
State S_g	Achv(SG_k)	N	B	B	B	F	F	F
	Prvn(SG_k)	Mo	N	N	N	N	N	N
Nokia agrees to Pay for old use + signs Lic. future use	FAchv(SG_k)	–Mo	B	B	B	F	F	F
	TWFAchv(SG_k,DM)	–0.30	0.80	0.80	0.80	1.00	1.00	1.00
{ Achv($A_{NTP\ c}$)=F,	WP(S_g, DM)	0.53				1.00		
Achv($A_{NOK\ c}$)=F }	OP(S_g, DM)	2				1 (Best)		
State S_h	Achv(SG_k)	N	B	B	B	L	L	B
	Prvn(SG_k)	S	N	N	N	N	N	N
Nokia agrees to Pay for old (Partial Agreement with NTP)	FAchv(SG_k)	–S	B	B	B	L	L	B
	TWFAchv(SG_k,DM)	–0.24	0.80	0.80	0.80	0.20	0.20	0.80
{ Achv($A_{NTP\ d}$)=F,	WP(S_h, DM)	0.54				0.40		
Achv($A_{NOK\ d}$)=F }	OP(S_h, DM)	1 (Best)				4 (Worst)		

(b) Preferences for NTP and Nokia (the biggest competitor to RIM) in Phase 2A of the conflict's side game between NTP and RIM Competitors.

Figure 9.6: The RIM v. NTP Conflict - Phase 2A (after the lower court order comes against RIM): Preferences of RIM, NTP and RIM Competitors (Nokia as an example).

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

		RIM x NTP Conflict: Phase # 2A (Lower Court Order AGAINST RIM)						
		RIM (OBSERVED)				NTP		
Strategic Goals of DMs	SGs:	SG_{RIM}				SG_{NTP}		
		$SG_{RIM\ 1}$	$SG_{RIM\ 2}$	$SG_{RIM\ 3}$	$SG_{URIM\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	$SImprt(SG_k)$	M	F	F	F	F	F	F
State S_5	$Achv(SG_k)$	N	S	Mo	S	B	B	B
	$Prvn(SG_k)$	L	N	N	N	N	N	N
	$FAchv(SG_k)$	−L	S	Mo	S	B	B	B
NTP insists & RIM fights	$TWFAchv(SG_k,DM)$	−0.12	0.40	0.50	0.40	0.80	0.80	0.80
{ $Achv(A_{RIM\ 0})$=F,	$WP(S_5, DM)$	0.30				0.80		
$Achv(A_{NTP\ 0})$=F }	$OP(S_5, DM)$	2				3		
State S_6	$Achv(SG_k)$	N	L	B	M	F	F	F
	$Prvn(SG_k)$	B	N	N	N	F	N	N
RIM agrees to Pay for old use + signs Lic. future use	$FAchv(SG_k)$	−B	L	B	M	N	F	F
	$TWFAchv(SG_k,DM)$	−0.48	0.20	0.80	0.60	1.00	1.00	1.00
{ $Achv(A_{RIM\ 1})$=F,	$WP(S_6, DM)$	0.28				1.00		
$Achv(A_{NTP\ 1})$=F }	$OP(S_6, DM)$	4				1 (Best)		
State S_7	$Achv(SG_k)$	N	L	B	M	Mo	Mo	M
	$Prvn(SG_k)$	M	N	N	N	F	N	N
RIM agrees to Pay for old Only	$FAchv(SG_k)$	−M	L	B	M	Mo	Mo	M
	$TWFAchv(SG_k,DM)$	−0.36	0.20	0.80	0.60	0.50	0.50	0.60
{ $Achv(A_{RIM\ 2})$=F,	$WP(S_7, DM)$	0.31				0.53		
$Achv(A_{NTP\ 2})$=F }	$OP(S_7, DM)$	1 (Best)				5 (Worst)		
State S_8	$Achv(SG_k)$	N	S	M	S	M	Mo	B
	$Prvn(SG_k)$	S	N	N	N	S	N	N
NTP insists & RIM fights +RIM develops Workaround	$FAchv(SG_k)$	−S	S	M	S	M	Mo	B
	$TWFAchv(SG_k,DM)$	−0.24	0.40	0.60	0.40	0.60	0.50	0.80
{ $Achv(A_{RIM\ 3})$=F,	$WP(S_8, DM)$	0.29				0.63		
$Achv(A_{NTP\ 0})$=F }	$OP(S_8, DM)$	3				4		
State S_9	$Achv(SG_k)$	N	L	Mo	S	F	F	B
	$Prvn(SG_k)$	L	N	N	N	F	N	N
NTP insists & RIM fights +NTP signs Lic Agr. w Others	$FAchv(SG_k)$	−L	L	Mo	S	N	F	B
	$TWFAchv(SG_k,DM)$	−0.12	0.20	0.50	0.40	1.00	1.00	0.80
{ $Achv(A_{RIM\ 0})$=F,	$WP(S_9, DM)$	0.25				0.93		
$Achv(A_{NTP\ 3})$=F }	$OP(S_9, DM)$	5 (Worst)				2		

(c) Preferences for RIM and NTP in Phase 2A of the conflict's main game. RIM's Preferences shown here are the ones demonstrated by RIM's behaviour at the time.

Figure 9.6: The RIM v. NTP Conflict - Phase 2A (after the lower court order comes against RIM): Preferences of RIM, NTP and RIM Competitors (Nokia as an example).

cost should not be that much.

Because of the fact that we are considering two models of RIM's preferences at Phase 2A, we have two figures showing the preferences of Phase 2A's main game. First, Figure 9.6a shows the preferences of RIM and NTP in the main game of Phase 2A, with RIM's preferences are shown as they *Should-Be* the preferences for RIM. Second, Figure 9.6c shows the preferences of RIM and NTP in the main game of Phase 2A, with RIM's preferences are shown as they were *Observed* historically and as demonstrated by RIM at the time. In both figure, NTP's preferences does not change. NTP still considers all its strategic goals *Fully* important. It is not under pressure to reconsider such thinking.

The strengths of each of the players' preferences, within each game of the phase, are then elicited from the weighted preferences shown in Figures 9.6a, 9.6c and 9.6b. Table 9.10 shows the preferences' strengths for NTP and RIM (based on RIM's Should-Be preferences as shown in Figure 9.6a) in the main game of Phase

Table 9.10: RIM v. NTP Conflict - Phase 2A: Players' Preferences in the main game (RIM v. NTP) - RIM's Preferences shown here are the Should-be ones (not observed in reality)

RIM Preferences (*Most* to *Least* Preferred)					
RIM	s_7	s_6	s_8	s_5	s_9
	0.37	0.36	0.33	0.32	0.22
RIM Preferences' Strengths					
$\succ^{L_{PS}}_{RIM,t}$	s_7	s_6	s_8	s_5	s_9
s_7	N	N	N	N	L
s_6	N	N	N	N	L
s_8	N	N	N	N	L
s_5	N	N	N	N	L
s_9	-L	-L	-L	-L	N

NTP Preferences (*Most* to *Least* Preferred)					
NTP	s_6	s_9	s_5	s_8	s_7
	1.00	0.93	0.80	0.63	0.53
NTP Preferences' Strengths					
$\succ^{L_{PS}}_{NTP,t}$	s_6	s_9	s_5	s_8	s_7
s_6	N	N	L	S	Mo
s_9	N	N	L	S	S
s_5	-L	-L	N	L	L
s_8	-S	-S	-L	N	L
s_7	-Mo	-S	-L	-L	N

Table 9.11: RIM v. NTP Conflict - Phase 2A: Players' Preferences in the main game (RIM v. NTP) - RIM's Preferences shown here are the ones actually-demonstrated by RIM at the time

RIM Preferences (*Most* to *Least* Preferred)					
RIM	s_7	s_5	s_8	s_6	s_9
	0.31	0.30	0.29	0.28	0.25
RIM Preferences' Strengths					
$\succ^{L_{PS}}_{RIM,t}$	s_7	s_5	s_8	s_6	s_9
s_7	N	N	N	N	N
s_5	N	N	N	N	N
s_8	N	N	N	N	N
s_6	N	N	N	N	N
s_9	N	N	N	N	N

NTP Preferences (*Most* to *Least* Preferred)					
NTP	s_6	s_9	s_5	s_8	s_7
	1.00	0.93	0.80	0.63	0.53
NTP Preferences' Strengths					
$\succ^{L_{PS}}_{NTP,t}$	s_6	s_9	s_5	s_8	s_7
s_6	N	N	L	S	Mo
s_9	N	N	L	S	S
s_5	-L	-L	N	L	L
s_8	-S	-S	-L	N	L
s_7	-Mo	-S	-L	-L	N

Table 9.12: RIM v. NTP Conflict - Phase 2A: Players' Preferences in the side game (NTP x RIM's Competitors (Nokia as an example))

NTP Preferences (*Most* to *Least*)				
NTP	s_g	s_e	s_f	s_h
	1.00	0.80	0.73	0.40
NTP Preferences' Strengths				
$\succ^{L_{PS}}_{NTP,t}$	s_g	s_e	s_f	s_h
s_g	N	L	L	M
s_e	-L	N	N	S
s_f	-L	N	N	S
s_h	-M	-S	-S	N

Nokia Preferences (*Most* to *Least*)				
NOK	s_h	s_g	s_e	s_f
	0.54	0.53	0.40	0.19
NOK Preferences' Strengths				
$\succ^{L_{PS}}_{NOK,t}$	s_h	s_g	s_e	s_f
s_h	N	N	L	S
s_g	N	N	L	S
s_e	-L	-L	N	L
s_f	-S	-S	-L	N

2A. Table 9.11 shows the preferences' strengths for NTP and RIM (based on RIM's Observed preferences as shown in Figure 9.6c) in the main game of Phase 2A. And,

Table 9.12 shows the preferences' strengths for NTP and Nokia (as an example of RIM's competitors) in the side game of Phase 2A.

3) Players' Preferences for Phase 2B of the RIM v. NTP conflict (after the lower court decides in favour of RIM's position)

From the calculated Weighted Payoffs, for each state to each of the players, we calculate the Ordinal Preferences for the players over the conflict's states in Phase 2B. Figure 9.7 shows the weighted payoffs and the ordinal preferences for each state of Phase 2B's games, for each player. Figure 9.7a shows the preferences of RIM and NTP in the main game of the phase. And, Figure 9.7b shows the preferences of NTP and Nokia (as an example of RIM's rivals) in the side game of the phase.

In the main game of Phase 2B, as we are showing in Figure 9.7, NTP is considering all its strategic goals of *Full* importance. This is because both financial and technology-ownership related goals are all important to NTP also at this stage of the conflict, and will continue to be so through out the conflict. For RIM, and unlike the situation imposed on it in Phase 2A (by the lower federal court been against it), this phase puts it in a very good position and provides it with an opportunity to maintain the importance level it assigned to its strategic goals at the same levels of Phase 1.

In this phase RIM's current US customers and potential US market are not threatened to be lost, if RIM decides to continue the fight and not settle the conflict with NTP. A situation, that RIM will find itself in if the court's decision is against it and in favour of NTP's position. In fact, in Phase 2B, the lower court just handed RIM a ruling that enforced its position against NTP. Therefore, RIM should not lower the importance of the not-paying-much strategic goal, $G_{RIM\,1}$, to end the conflict. Instead, RIM will keep $G_{RIM\,1}$ at the the *Much* importance level, and will keep its market-share and technology-ownership related strategic goals set at the *Full* importance levels, as shown in Figure 9.7.

The strengths of each of the players' preferences, within each game of the phase, are then elicited from the weighted preferences shown in Figures 9.7a and 9.7b. Table 9.13 shows the preferences' strengths for NTP and RIM in the main game of Phase 2B. And, Table 9.14 shows the preferences' strengths for NTP and Nokia (as an example of RIM's competitors) in the side game of Phase 2B. Both tables also show the preferences vector for each player in each one of these two games.

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

		RIM x NTP Conflict: Phase # 2B (Lower Court Order AGAINST NTP)						
		RIM				NTP		
Strategic Goals of DMs	SGs:	SG_{RIM}				SG_{NTP}		
		$SG_{RIM\ 1}$	$SG_{RIM\ 2}$	$SG_{RIM\ 3}$	$SG_{URIM\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	$SImprt(SG_k)$	M	F	F	F	F	F	F
State S_{10}	$Achv(SG_k)$	N	B	F	B	L	L	Mo
	$Prvn(SG_k)$	L	N	N	N	N	N	N
NTP insists & RIM fights	$FAchv(SG_k)$	–L	B	F	B	L	L	Mo
	$TWFAchv(SG_k,DM)$	−0.12	0.80	1.00	0.80	0.20	0.20	0.50
{ $Achv(A_{RIM\ 0})$=F,	$WP(S_{10}, DM)$	0.62				0.30		
$Achv(A_{NTP\ 0})$=F }	$OP(S_{10}, DM)$	2				2		
State S_{11}	$Achv(SG_k)$	N	L	F	L	L	L	L
	$Prvn(SG_k)$	B	N	N	N	N	N	N
RIM agrees to Pay for old use + signs Lic. future use	$FAchv(SG_k)$	–B	L	F	L	L	L	L
	$TWFAchv(SG_k,DM)$	−0.48	0.20	1.00	0.20	0.20	0.20	0.20
{ $Achv(A_{RIM\ 1})$=F,	$WP(S_{11}, DM)$	0.23				0.20		
$Achv(A_{NTP\ 1})$=F }	$OP(S_{11}, DM)$	5 (Worst)				4		
State S_{12}	$Achv(SG_k)$	N	L	F	L	N	N	L
	$Prvn(SG_k)$	M	N	N	N	N	N	N
RIM agrees to Pay for old Only	$FAchv(SG_k)$	–M	L	F	L	N	N	L
	$TWFAchv(SG_k,DM)$	−0.36	0.20	1.00	0.20	0.00	0.00	0.20
{ $Achv(A_{RIM\ 2})$=F,	$WP(S_{12}, DM)$	0.26				0.07		
$Achv(A_{NTP\ 2})$=F }	$OP(S_{12}, DM)$	4				5 (Worst)		
State S_{13}	$Achv(SG_k)$	N	B	F	F	L	L	S
	$Prvn(SG_k)$	S	N	N	N	N	N	N
NTP insists & RIM fights +RIM develops Workaround	$FAchv(SG_k)$	–S	B	F	F	L	L	S
	$TWFAchv(SG_k,DM)$	−0.24	0.80	1.00	1.00	0.20	0.20	0.40
{ $Achv(A_{RIM\ 3})$=F,	$WP(S_{13}, DM)$	0.64				0.27		
$Achv(A_{NTP\ 0})$=F }	$OP(S_{13}, DM)$	1 (Best)				3		
State S_{14}	$Achv(SG_k)$	N	M	F	B	L	S	Mo
	$Prvn(SG_k)$	L	N	N	N	N	N	N
NTP insists & RIM fights +NTP signs Lic Agr. w Others	$FAchv(SG_k)$	–L	M	F	B	L	S	Mo
	$TWFAchv(SG_k,DM)$	−0.12	0.60	1.00	0.80	0.20	0.40	0.50
{ $Achv(A_{RIM\ 0})$=F,	$WP(S_{14}, DM)$	0.57				0.37		
$Achv(A_{NTP\ 3})$=F }	$OP(S_{14}, DM)$	3				1 (Best)		

(a) Preferences for RIM and NTP in Phase 2B of the conflict's main game.

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

		Nokia x NTP Conflict: Phase # 2B						
		Nokia				NTP		
Strategic Goals of DMs	SGs:	SG_{Nokia}				SG_{NTP}		
		$SG_{Nokia\ 1}$	$SG_{Nokia\ 2}$	$SG_{Nokia\ 3}$	$SG_{UNokia\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	$SImprt(SG_k)$	M	F	F	F	F	F	F
State S_i	$Achv(SG_k)$	N	L	M	S	L	L	Mo
	$Prvn(SG_k)$	N	N	N	N	N	N	N
NTP Waits & Nokia Waits	$FAchv(SG_k)$	N	L	M	S	L	L	Mo
	$TWFAchv(SG_k,DM)$	0.00	0.20	0.60	0.40	0.20	0.20	0.50
{ $Achv(A_{NTP\ a})$=F,	$WP(S_i, DM)$	0.30				0.30		
$Achv(A_{NOK\ a})$=F }	$OP(S_i, DM)$	1 (Best)				2		
State S_j	$Achv(SG_k)$	N	N	B	S	L	L	S
	$Prvn(SG_k)$	L	N	N	N	N	N	N
NTP Fights & Nokia Fights	$FAchv(SG_k)$	–L	N	B	S	L	L	S
	$TWFAchv(SG_k,DM)$	−0.12	0.00	0.80	0.40	0.20	0.20	0.40
{ $Achv(A_{NTP\ b})$=F,	$WP(S_j, DM)$	0.27				0.27		
$Achv(A_{NOK\ b})$=F }	$OP(S_j, DM)$	2				3		
State S_k	$Achv(SG_k)$	N	L	B	S	L	S	Mo
	$Prvn(SG_k)$	B	N	N	N	N	N	N
Nokia agrees to Pay for old use + signs Lic. future use	$FAchv(SG_k)$	–B	L	B	S	L	S	Mo
	$TWFAchv(SG_k,DM)$	−0.48	0.20	0.80	0.40	0.20	0.40	0.50
{ $Achv(A_{NTP\ c})$=F,	$WP(S_k, DM)$	0.23				0.37		
$Achv(A_{NOK\ c})$=F }	$OP(S_k, DM)$	4 (Worst)				1 (Best)		
State S_l	$Achv(SG_k)$	N	L	B	S	N	N	L
	$Prvn(SG_k)$	M	N	N	N	N	N	N
Nokia agrees to Pay for old (Partial Agreement with NTP)	$FAchv(SG_k)$	–M	L	B	S	N	N	L
	$TWFAchv(SG_k,DM)$	−0.36	0.20	0.80	0.40	0.00	0.00	0.20
{ $Achv(A_{NTP\ d})$=F,	$WP(S_l, DM)$	0.26				0.07		
$Achv(A_{NOK\ d})$=F }	$OP(S_l, DM)$	3				4 (Worst)		

(b) Preferences for NTP and Nokia (the biggest competitor to RIM) in Phase 2B of the conflict's side game between NTP and RIM Competitors.

Figure 9.7: The RIM v. NTP Conflict - Phase 2B (after the lower court order comes against NTP): Preferences of RIM, NTP and RIM Competitors (Nokia as an example).

Table 9.13: RIM v. NTP Conflict - Phase 2B: Players' Preferences in the main game (RIM v. NTP)

RIM Preferences (*Most* to *Least* Preferred)					
RIM	s_{13}	s_{10}	s_{14}	s_{12}	s_{11}
	0.64	0.62	0.57	0.26	0.23
RIM Preferences' Strengths					
$\succ^{L_{PS}}_{RIM,t}$	s_{13}	s_{10}	s_{14}	s_{12}	s_{11}
s_{13}	N	N	N	S	S
s_{10}	N	N	N	S	S
s_{14}	N	N	N	S	S
s_{12}	-S	-S	-S	N	N
s_{11}	-S	-S	-S	N	N

NTP Preferences (*Most* to *Least* Preferred)					
NTP	s_{14}	s_{10}	s_{13}	s_{11}	s_{12}
	0.37	0.30	0.27	0.20	0.07
NTP Preferences' Strengths					
$\succ^{L_{PS}}_{NTP,t}$	s_{14}	s_{10}	s_{13}	s_{11}	s_{12}
s_{14}	N	N	L	L	S
s_{10}	N	N	N	L	L
s_{13}	-L	N	N	N	L
s_{11}	-L	-L	N	N	L
s_{12}	-S	-L	-L	-L	N

Table 9.14: RIM v. NTP Conflict - Phase 2B: Players' Preferences in the side game (NTP x RIM's Competitors (Nokia as an example))

NTP Preferences (*Most* to *Least*)				
NTP	s_k	s_i	s_j	s_l
	0.37	0.30	0.27	0.07
NTP Preferences' Strengths				
$\succ^{L_{PS}}_{NTP,t}$	s_k	s_i	s_j	s_l
s_k	N	N	L	S
s_i	N	N	N	L
s_j	-L	N	N	L
s_l	-S	-L	-L	N

Nokia Preferences (*Most* to *Least*)				
NOK	s_i	s_j	s_l	s_k
	0.30	0.27	0.26	0.23
NOK Preferences' Strengths				
$\succ^{L_{PS}}_{NOK,t}$	s_i	s_j	s_l	s_k
s_i	N	N	N	N
s_j	N	N	N	N
s_l	N	N	N	N
s_k	N	N	N	N

4) Players' Preferences for Phase 3A of the RIM v. NTP conflict (after the higher court decides against RIM's position)

The Ordinal Preferences for the players over the conflict's states in Phase 3A are calculated from the calculate Weighted Payoffs of each state to each of the players. Figure 9.8 shows the weighted payoffs and the ordinal preferences for each state of Phase 3A's game, for each player. Figure 9.8a shows the preferences of RIM and NTP in the main game. And, Figure 9.8b shows the preferences of NTP and Nokia (as an example of RIM's rivals) in the side game of this phase.

In Phase 3A, and as shown in Figure 9.8a, NTP continue to consider all its strategic goals of *Full* importance as before. RIM, on the other hand, has the situation worsened for it. If Phase 2A, in which the lower court agrees with NTP claims, is a bad phase for RIM, Phase 3A is even worse. In this phase, the higher federal court agrees with NTP demands, empowering NTP's position and anteing

Rationality Factor = 1.0 (for RIM & NTP) Emotionality Factor = 0.0 (for RIM & NTP)		RIM x NTP Conflict: Phase # 3A (Higher Court Order AGAINST RIM)						
		RIM				NTP		
Strategic Goals of DMs	SGs:	SG_{RIM}				SG_{NTP}		
		$SG_{RIM\ 1}$	$SG_{RIM\ 2}$	$SG_{RIM\ 3}$	$SG_{URIM\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	$SImprt(SG_k)$	L	S	F	F	F	F	F
State S_{15}	$Achv(SG_k)$	N	S	S	L	Mo	B	F
	$Prvn(SG_k)$	L	N	N	N	N	N	N
NTP insists & RIM fights	$FAchv(SG_k)$	−L	S	S	L	Mo	B	F
	$TWFAchv(SG_k,DM)$	−0.04	0.16	0.40	0.20	0.50	0.80	1.00
{ $Achv(A_{RIM\ 0})$=F,	$WP(S_{15}, DM)$	0.18				0.77		
$Achv(A_{NTP\ 0})$=F }	$OP(S_{15}, DM)$	3				3		
State S_{16}	$Achv(SG_k)$	N	L	M	M	F	F	F
	$Prvn(SG_k)$	B	N	N	N	N	N	N
RIM agrees to Pay for old use + signs Lic. future use	$FAchv(SG_k)$	−B	L	M	M	N	F	F
	$TWFAchv(SG_k,DM)$	−0.16	0.08	0.60	0.60	1.00	1.00	1.00
{ $Achv(A_{RIM\ 1})$=F,	$WP(S_{16}, DM)$	0.28				1.00		
$Achv(A_{NTP\ 1})$=F }	$OP(S_{16}, DM)$	2				1 (Best)		
State S_{17}	$Achv(SG_k)$	N	L	M	M	M	M	F
	$Prvn(SG_k)$	M	N	N	N	N	N	N
RIM agrees to Pay for old Only	$FAchv(SG_k)$	−M	L	M	M	M	M	F
	$TWFAchv(SG_k,DM)$	−0.12	0.08	0.60	0.60	0.60	0.60	1.00
{ $Achv(A_{RIM\ 2})$=F,	$WP(S_{17}, DM)$	0.29				0.73		
$Achv(A_{NTP\ 2})$=F }	$OP(S_{17}, DM)$	1 (Best)				4		
State S_{18}	$Achv(SG_k)$	N	L	N	N	N	F	F
	$Prvn(SG_k)$	F	N	N	N	N	N	N
RIM stops sales & operation in US	$FAchv(SG_k)$	−F	L	N	N	N	F	F
	$TWFAchv(SG_k,DM)$	−0.20	0.08	0.00	0.00	0.00	1.00	1.00
{ Achv(ARIM 4)=F }	$WP(S_{18}, DM)$	−0.03				0.67		
	$OP(S_{18}, DM)$	5 (Worst)				5 (Worst)		
State S_{19}	$Achv(SG_k)$	N	L	L	N	Mo	F	F
	$Prvn(SG_k)$	M	N	N	N	N	N	N
NTP insists & RIM fights +NTP signs Lic Agr. w Others	$FAchv(SG_k)$	−M	L	L	N	Mo	F	F
	$TWFAchv(SG_k,DM)$	−0.12	0.08	0.20	0.00	0.50	1.00	1.00
{ Achv(ARIM 4)=F }	$WP(S_{18}, DM)$	0.04				0.83		
	$OP(S_{18}, DM)$	4				2		

(a) Preferences for RIM and NTP in Phase 3A of the conflict's main game.

Rationality Factor = 1.0 (for RIM & NTP) Emotionality Factor = 0.0 (for RIM & NTP)		Nokia x NTP Conflict: Phase # 3A						
		Nokia				NTP		
Strategic Goals of DMs	SGs:	SG_{Nokia}				SG_{NTP}		
		$SG_{Nokia\ 1}$	$SG_{Nokia\ 2}$	$SG_{Nokia\ 3}$	$SG_{UNokia\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	$SImprt(SG_k)$	S	F	F	F	F	F	F
State S_m	$Achv(SG_k)$	N	L	B	Mo	B	B	B
	$Prvn(SG_k)$	N	N	N	N	N	N	N
NTP Waits & Nokia Waits	$FAchv(SG_k)$	N	L	B	Mo	B	B	B
	$TWFAchv(SG_k,DM)$	0.00	0.20	0.80	0.60	0.80	0.80	0.80
{ $Achv(A_{NTP\ a})$=F,	$WP(S_e, DM)$	0.40				0.80		
$Achv(A_{NOK\ a})$=F }	$OP(S_e, DM)$	3				2		
State S_n	$Achv(SG_k)$	N	N	M	Mo	M	B	B
	$Prvn(SG_k)$	M	N	N	N	N	N	N
NTP Fights & Nokia Fights	$FAchv(SG_k)$	−M	N	M	Mo	M	B	B
	$TWFAchv(SG_k,DM)$	−0.36	0.00	0.60	0.50	0.60	0.80	0.80
{ $Achv(A_{NTP\ b})$=F,	$WP(S_f, DM)$	0.19				0.73		
$Achv(A_{NOK\ b})$=F }	$OP(S_f, DM)$	4 (Worst)				3		
State S_o	$Achv(SG_k)$	N	B	B	B	F	F	F
	$Prvn(SG_k)$	Mo	N	N	N	N	N	N
Nokia agrees to Pay for old use + signs Lic. future use	$FAchv(SG_k)$	−Mo	B	B	B	F	F	F
	$TWFAchv(SG_k,DM)$	−0.30	0.80	0.80	0.80	1.00	1.00	1.00
{ $Achv(A_{NTP\ c})$=F,	$WP(S_g, DM)$	0.53				1.00		
$Achv(A_{NOK\ c})$=F }	$OP(S_g, DM)$	2				1 (Best)		
State S_p	$Achv(SG_k)$	N	B	B	B	L	L	B
	$Prvn(SG_k)$	S	N	N	N	N	N	N
Nokia agrees to Pay for old (Partial Agreement with NTP)	$FAchv(SG_k)$	−S	B	B	B	L	L	B
	$TWFAchv(SG_k,DM)$	−0.24	0.80	0.80	0.80	0.20	0.20	0.80
{ $Achv(A_{NTP\ d})$=F,	$WP(S_h, DM)$	0.54				0.40		
$Achv(A_{NOK\ d})$=F }	$OP(S_h, DM)$	1 (Best)				4 (Worst)		

(b) Preferences for NTP and Nokia (the biggest competitor to RIM) in Phase 3A of the conflict's side game between NTP and RIM Competitors.

Figure 9.8: The RIM v. NTP Conflict - Phase 3A (after the higher court order comes against RIM): Preferences of RIM, NTP and RIM's competitors (Nokia as an example).

up the price for RIM to settle the conflict with NTP. Therefore, RIM at this stage gives little importance to the financial cost of resolving the conflict, i.e. sets the importance level for its $G_{RIM\,1}$ to the *Little* level. As well, the technology-ownership goal now is taking a *Some* importance level, because it is clear that RIM is more and more losing this battle based on the courts' decision. RIM now shifts it focus to keep its customers-base from leaving to other more stable services, as it appears more and more to the base that RIM is losing control of the situation, and also protect as much as possible the future potential that it could harness in the US market, strategic goals $G_{RIM\,2}$ and $G_{RIM\,3}$, respectively, giving each of these two goals a *Full* importance level.

Table 9.15: RIM v. NTP Conflict - Phase 3A: Players' Preferences in the main game (RIM v. NTP)

RIM Preferences (*Most* to *Least* Preferred)					
RIM	s_{17}	s_{16}	s_{15}	s_{19}	s_{18}
	0.29	0.28	0.18	0.04	-0.03
RIM Preferences' Strengths					
$\succ^{L_{PS}}_{RIM,t}$	s_{17}	s_{16}	s_{15}	s_{19}	s_{18}
s_{17}	N	N	L	L	S
s_{16}	N	N	L	L	S
s_{15}	-L	-L	N	L	L
s_{19}	-L	-L	-L	N	N
s_{18}	-S	-S	-L	N	N

NTP Preferences (*Most* to *Least* Preferred)					
NTP	s_{16}	s_{19}	s_{15}	s_{17}	s_{18}
	1.00	0.83	0.77	0.73	0.67
NTP Preferences' Strengths					
$\succ^{L_{PS}}_{NTP,t}$	s_{16}	s_{19}	s_{15}	s_{17}	s_{18}
s_{16}	N	L	L	L	S
s_{19}	-L	N	N	L	L
s_{15}	-L	N	N	N	L
s_{17}	-L	-L	N	N	N
s_{18}	-S	-L	-L	N	N

Table 9.16: RIM v. NTP Conflict - Phase 3A: Players' Preferences in the side game (NTP x RIM's Competitors (Nokia as an example))

NTP Preferences (*Most* to *Least*)				
NTP	s_o	s_m	s_n	s_p
	1.00	0.80	0.73	0.40
NTP Preferences' Strengths				
$\succ^{L_{PS}}_{NTP,t}$	s_o	s_m	s_n	s_p
s_o	N	L	L	M
s_m	-L	N	N	S
s_n	-L	N	N	S
s_p	-M	-S	-S	N

Nokia Preferences (*Most* to *Least*)				
NOK	s_p	s_o	s_m	s_n
	0.54	0.53	0.40	0.19
NOK Preferences' Strengths				
$\succ^{L_{PS}}_{NOK,t}$	s_p	s_o	s_m	s_n
s_p	N	N	L	S
s_o	N	N	L	S
s_m	-L	-L	N	L
s_n	-S	-S	-L	N

The strengths of each of the players' preferences, within each game of the phase, are then elicited from the weighted preferences shown in Figures 9.8a and 9.8b.

Table 9.15 shows the preferences' strengths for NTP and RIM in the main game of Phase 3A. And, Table 9.16 shows the preferences' strengths for NTP and Nokia (as an example of RIM's rivals) in the side game of Phase 3A.

5) Players' Preferences for Phase 3B of the RIM v. NTP conflict (after the higher court decides in favour of RIM's position)

For Phase 3B, the Ordinal Preferences for the players over the conflict's states are calculated from the calculate Weighted Payoffs of each state to each of the players. Figure 9.9 shows the weighted payoffs and the ordinal preferences for each state of Phase 3B's game, for each player. Figure 9.8a shows the preferences of RIM and NTP in the main game and only game of the phase. Recall that we said earlier the side game between NTP and RIM's competitors is not important in Phase 3B. The side game could have an effect on the main conflict between RIM and NTP only before the higher court takes a decision on the case, or if the higher court takes a decision that is against RIM's position. After the higher court decision, and if the court's decision is against NTP's position, the side game, and any outcome of it, will not have an effect on the main game.

Figure 9.9 shows that NTP continues in Phase 3B to consider all its strategic

Rationality Factor = 1.0 (for RIM & NTP)
Emotionality Factor = 0.0 (for RIM & NTP)

RIM x NTP Conflict: Phase # 3B (Higher Court Order AGAINST NTP)		RIM				NTP		
Strategic Goals of DMs	SGs:	SG_{RIM}				SG_{NTP}		
		$SG_{RIM\ 1}$	$SG_{RIM\ 2}$	$SG_{RIM\ 3}$	$SG_{URIM\ 4}$	$SG_{NTP\ 1}$	$SG_{NTP\ 2}$	$SG_{NTP\ 3}$
Strategic Importance	$SImprt(SG_k)$	L	S	F	F	F	F	F
State S_{19}	$Achv(SG_k)$	N	S	M	S	L	L	S
NTP insists & RIM fights	$Prvn(SG_k)$	B	N	N	N	N	N	N
{ $Achv(A_{RIM\ 0})$=F,	$FAchv(SG_k)$	−B	S	M	S	L	L	S
$Achv(A_{NTP\ 0})$=F }	$TWFAchv(SG_k,DM)$	−0.04	0.40	0.60	0.40	0.20	0.20	0.40
	$WP(S_{19}, DM)$	0.34				0.27		
	$OP(S_{19}, DM)$	3				3		
State S_{20}	$Achv(SG_k)$	N	L	M	L	S	Mo	M
RIM agrees to Pay for old use + signs Lic. future use	$Prvn(SG_k)$	Mo	N	N	N	N	N	N
{ $Achv(A_{RIM\ 1})$=F,	$FAchv(SG_k)$	−Mo	L	M	L	S	Mo	M
$Achv(A_{NTP\ 1})$=F }	$TWFAchv(SG_k,DM)$	−0.10	0.08	0.60	0.20	0.40	0.50	0.60
	$WP(S_{20}, DM)$	0.20				0.50		
	$OP(S_{20}, DM)$	4 (Worst)				1 (Best)		
State S_{21}	$Achv(SG_k)$	N	S	M	Mo	L	S	Mo
RIM agrees to Pay for old Only	$Prvn(SG_k)$	S	N	N	N	N	N	N
{ $Achv(A_{RIM\ 2})$=F,	$FAchv(SG_k)$	−S	S	M	Mo	L	S	Mo
$Achv(A_{NTP\ 2})$=F }	$TWFAchv(SG_k,DM)$	−0.08	0.40	0.60	0.50	0.20	0.40	0.50
	$WP(S_{21}, DM)$	0.36				0.37		
	$OP(S_{21}, DM)$	2				2		
State S_{22}	$Achv(SG_k)$	B	F	B	B	N	N	N
NTP Stops the fight	$Prvn(SG_k)$	N	N	N	N	N	N	N
	$FAchv(SG_k)$	B	F	B	B	N	N	N
	$TWFAchv(SG_k,DM)$	0.16	0.40	0.80	0.80	0.00	0.00	0.00
	$WP(S_{22}, DM)$	0.54				0.00		
{ $Achv(A_{NTP\ 4})$=F }	$OP(S_{22}, DM)$	1 (Best)				4 (Worst)		

Figure 9.9: The RIM v. NTP Conflict - Phase 3B (after the higher court order comes against NTP): Preferences of RIM and NTP the conflict's main game. The side game between NTP and RIM's competitors does not matter at this stage of the conflict.

Table 9.17: RIM v. NTP Conflict - Phase 3B: Players' Preferences in the main game (RIM v. NTP)

RIM Preferences *(Most to Least)*				
RIM	s_{23}	s_{22}	s_{20}	s_{21}
	0.54	0.36	0.34	0.20
RIM Preferences' Strengths				
$\succ^{L_{PS}}_{RIM,t}$	s_{23}	s_{22}	s_{20}	s_{21}
s_{23}	N	L	L	S
s_{22}	-L	N	N	L
s_{20}	-L	N	N	L
s_{21}	-S	-L	-L	N

NTP Preferences *(Most to Least)*				
NTP	s_{21}	s_{22}	s_{20}	s_{23}
	0.50	0.37	0.27	0.00
NTP Preferences' Strengths				
$\succ^{L_{PS}}_{NTP,t}$	s_{21}	s_{22}	s_{20}	s_{23}
s_{21}	N	L	L	Mo
s_{22}	-L	N	L	S
s_{20}	-L	-L	N	L
s_{23}	-Mo	-S	-L	N

goals of *Full* importance, as before. RIM, on the other hand, has the situation much better than if it is in Phase 3A (where the higher court order is against RIM), but still the situation is not good. Despite the fact that the higher court here supports RIM against NTP demands, NTP can still go to the supreme court or restart all over with new law suits. This means that NTP will continue to drag RIM into continuos sources of distractions to its operations and market, making RIM's problems continue and grow.

Therefore, RIM at this stage too, much like its situation in Phase 3A, gives little importance to the financial cost of resolving the conflict, i.e. sets the importance level for its $G_{RIM\,1}$ to the *Little* level. As well, the technology-ownership goal now is taking a *Some* importance level, because it is clear that RIM is gaining some support based on the court's decision but this support is not enough to let the cloud hanging over RIM's operations and market go away. This is at the heart of the patent troll cases, the patent troll has nothing to lose, and their opponents have much more to lose. In fact the opponents' best case scenarios are the ones that cut short the loss and the distraction as early as possible.

But, as in Phase 3A, RIM puts more emphasis into the importance of keeping its customers-base from leaving to other more stable services, as it appears more and more to the base that RIM has a lot of problems to resolve, and also protect as much as possible the future potential that it could harness in the US market, strategic goals $G_{RIM\,2}$ and $G_{RIM\,3}$, respectively, giving each of these two goals a *Full* importance level.

The strengths of each of the players' preferences, within the game of this phase, are then elicited from the weighted preferences shown in Figure 9.9. Table 9.17 shows the preferences' strengths for NTP and RIM in the game of Phase 3B. The table also shows the preferences vector for each player in the game.

9.6.6 Players' Moves over States of the Conflict

At this step, we define the UM moves for each of the players, RIM, NTP and Nokia (as an example of RIM's competitors). These moves will be shown in this subsection for each phase of the conflict and for each game within the phases. We should note that the states in the figures shown in this subsection are all represented with circles surrounding their corresponding numbers. For example state s_2 will be shown as a circle with the number 2 at its centre. The figures will also show the CM moves that none of the players can take on their own, but they can cooperatively, but not as a coalition, take together.

From these UM and CM moves, and by using the players' preferences, we will be able to elicit the players' individual UIs, cooperative CIs and the coalitions' C-GIs. We will use for this purpose Algorithm 6.1 to generate each of the players' list of UIs, Algorithm 8.1 to generate the cooperative CI moves for the players, and Algorithm 9.1 to generate the coalitions' C-GIs.

In this section, we will also check if NTP and RIM's competitors can have a fruitful coalition in which they all will benefit from some C-GI moves the coalition have against their common enemy here, RIM. The list of states that the conflict phases have, in their main game between NTP and RIM, include states in which NTP reaches an agreement with RIM's competitors putting additional pressure on RIM (giving RIM's current and potential customers a safe exist from RIM's service to a stable will-not-be-shut-down licensed service) and giving NTP's claims additional legitimacy in the eyes of the courts, the jury, and the market.

But these states, cannot be reached by NTP alone (using some UMs it have), nor it can reach them cooperatively in one steps. NTP must use multiple-step group moves, or C-GMs to reach such states, and these moves require a trusted coalition relation among NTP and RIM's competitors. But such coalition will not form, or more accurately the involved parties will not have an incentive to form the coalition, unless these moves are not only C-GM moves but C-GI moves, by which the coalition will reach ultimately states that will benefit all, NTP and the involved RIM's competitors. Therefore, we will be looking for the existence of C-GIs by the possible coalition of NTP and RIM's competitors. Once these moves are identified, then the analysis should suggest the likelihood of having the coalition formed and take such C-GIs.

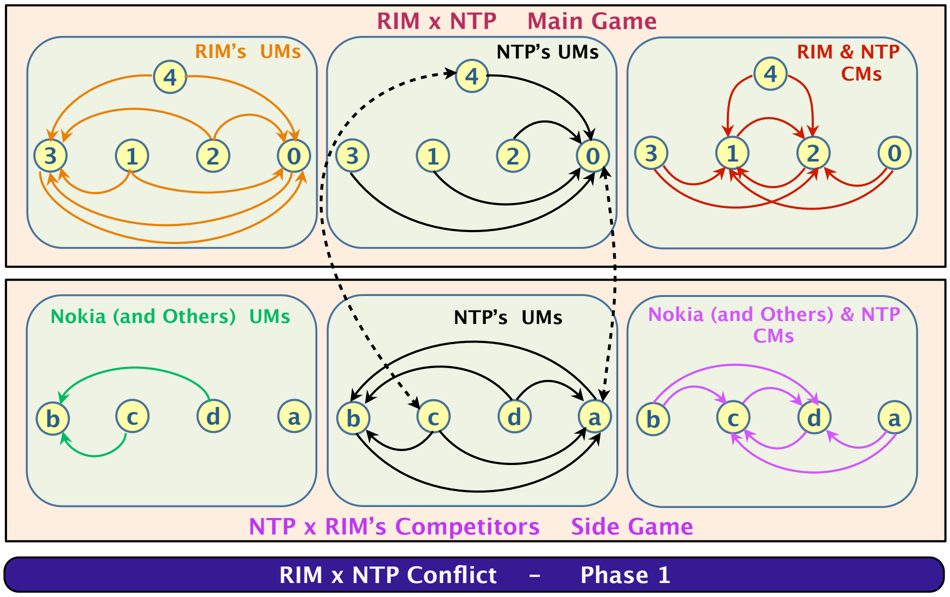
1) Players' Moves in Phase 1 of the RIM v. NTP conflict (before the lower court's decision)

Figure 9.10a shows the players' UMs in both the main game (between NTP and RIM) and the side game (between NTP and RIM's competitors) of the conflict at this phase (before the lower federal court rules in the RIM v. NTP case). The figure also shows the one-step CM moves that the players, in each game of the phase, have and can act upon jointly. Recall that these moves are identified as possible moves that the players can have, either individually in the case of UMs or cooperatively in the case of CMs, but may not lead to improvements in the players' positions in the conflict. Only a subset of these moves, the UI and CI moves, that can lead to such improvements.

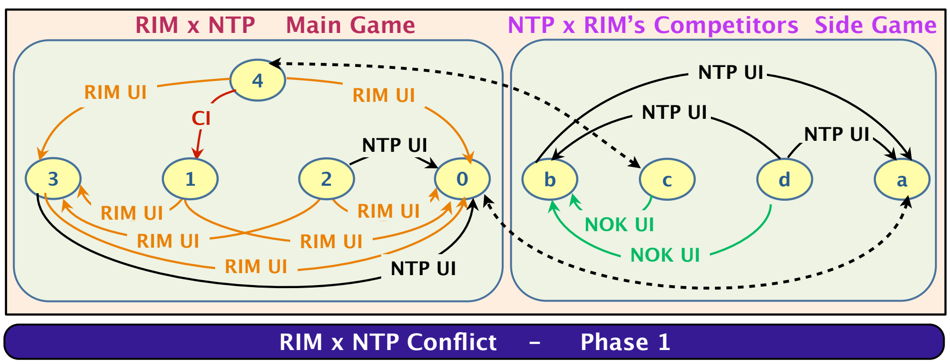
Figure 9.10a also shows moves in dotted lines signifying the existence of UM moves that NTP has between states in different games, the main game and the side game, of the conflict. As indicated earlier, we will analyze each game in each of the phases separately, but the fact that one of the players is participating in both games and may choose to use gains from one game to elevate his position in the other, makes it more likely that this player will move between the games at certain points/states. In our case, NTP is a player in the main game between RIM and NTP, and also a player in the side game between NTP and RIM's competitors. NTP will be looking, if it is in its benefit –and ultimately also in the benefit of RIM's competitors–, to sign licensing agreements with these companies, i.e. reach state s_4, allowing them to capitalize on RIM's uncertain future within the US market and at the same time add a layer of legitimacy to NTP claims for technology ownership.

But for NTP to reach s_4, it must: 1) move from the current status quo s_0 at the main game to the current status quo at the side game s_a (the dotted line between s_0 and s_a); 2) check for a coalition C-GI move that could allow it to reach s_4; and 3) if this C-GI move is available, then NTP will follow all the one-step UIs and CIs it has in the side game until it reaches the signing-license-deal-with-RIM-rivals state s_c, and then move from s_c to s_4 through a UM it has (the dotted line between s_c and s_4); or 4) if this C-GI does not exist, then NTP will take a UM back from the status quo state s_a of the side game to the status quo state s_0 of the main game, i.e. go back to the main game (the dotted line between s_0 and s_a). Such scenarios demand these UM moves, denoted graphically as dotted lines in Figure 9.10a.

By feeding the players' UMs and CMs and preference structures separately to Algorithm 6.1 and then to Algorithm 8.1, the players' UI and then CI moves will be



(a) Unilateral Moves (UMs) and Cooperative Moves (CMs) by RIM, NTP and RIM's Competitors (Nokia as an example) in both games, the main game and the side game, of the conflict at Phase 1



(b) Unilateral Improvements (UIs) and Cooperative Improvements (CIs) by RIM, NTP and RIM's Competitors (Nokia as an example) in both games of the conflict at Phase 1

Figure 9.10: The RIM v. NTP Conflict - Phase 1 (before the lower federal court decision is issued): Moves by RIM, and NTP in the conflict's main game, and moves by NTP and RIM's Competitors (Nokia as an example) in the conflict's side game.

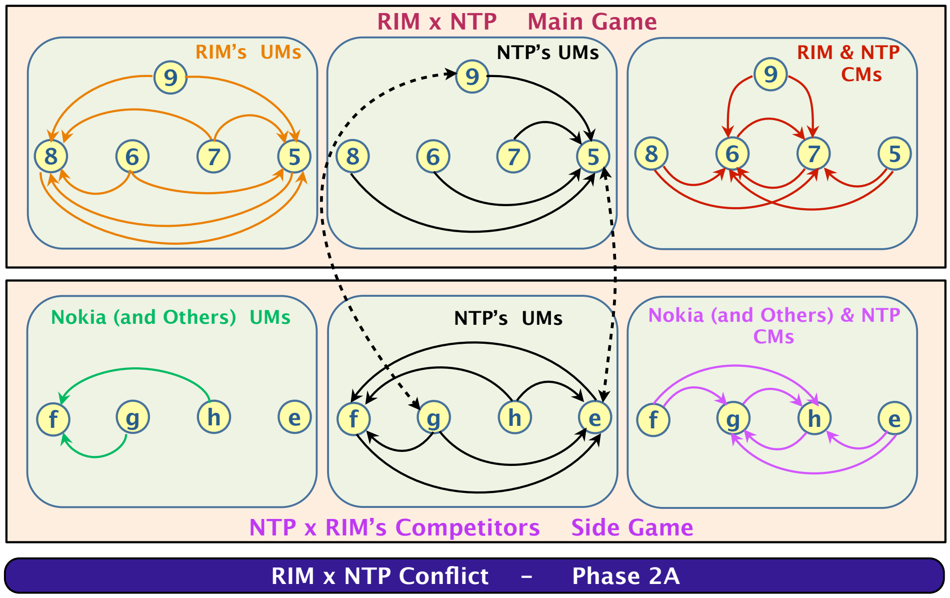
defined. Figure 9.10b shows the resultant set of UI and CI moves that the players of both games, the main game and the side game, have in the conflict at this phase. It is important to note here that by feeding the players' UMs and CMs and preference structures to Algorithm 9.1, assuming a coalition among NTP and Nokia (and other competitors of RIM) is formed, the coalition will not have C-GIs that could lead to s_4 in the main game, the state that the coalition's members can all benefit from. The coalition will be stuck at state s_a of the side game, because none of the coalition's members have a unilateral UI or cooperative CI out from it. In other words, the coalition between NTP and RIM's competitors is not likely to form. We will elaborate more on this, once the stabilities of the different states to the different players, individually and collectively, are analyzed in the next subsection. But, for now, there are no C-GIs of any value to the possible coalition to show.

2) Players' Moves in Phase 2A of the RIM v. NTP conflict (after the lower court decides against RIM's position)

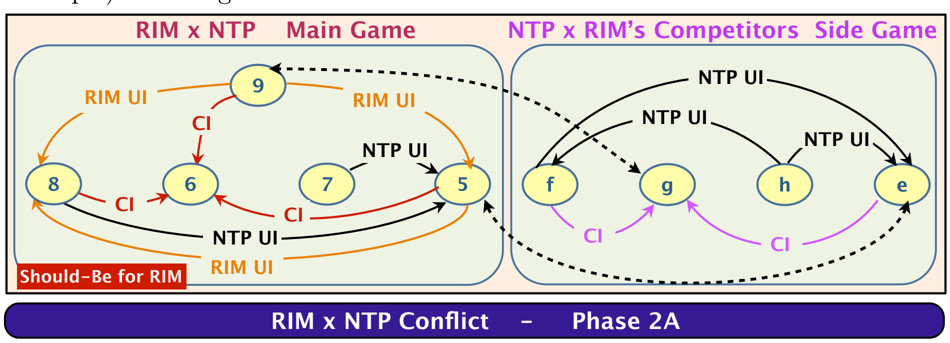
Figure 9.11a shows the players' UMs in both the main game (between NTP and RIM) and the side game (between NTP and RIM's competitors) of the conflict's Phase 2A (after the lower federal court rules against RIM and in favour of NTP in the RIM v. NTP case). The figure also shows the one-step CM moves that the players, in each game of the phase, have and can act upon jointly. These moves are similar to the ones that we identified above as possible moves that the players can have, either individually in the case of UMs or cooperatively in the case of CMs, in Phase 1 of the conflict. This is because the states that make up Phase 2A are similar, in their definitions, to the states make up Phase 1. They differ only on the timing of when the states occur (before or after the lower federal court ruling).

By feeding the players' UMs and CMs and preference structures separately to Algorithm 6.1 and then to Algorithm 8.1, the player's UI and then CI moves for Phase 2A will be defined. But because of the fact that we have studied two preferences structures for RIM, the should-have-been preferences and the preferences actually demonstrated by RIM at the time, then we sure will have two sets of UIs and CIs especially for the main game of Phase 2A in which RIM is a key player. Both Figure 9.11b and Figure 9.12a show the UI and CI moves that the players of both games, the main game and the side game, have in the conflict's Phase 2A. But, the former figure shows the UIs and CIs for the players, RIM and NTP, in the main game based on the should-be preferences for RIM, whilst the latter shows the UIs and CIs for RIM and NTP based on the preferences RIM demonstrated at the time.

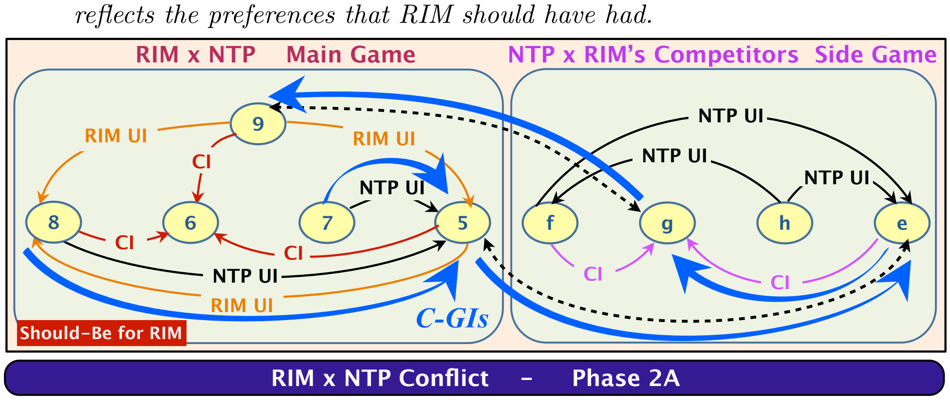
In both versions of the UIs and CIs for the conflict's players at Phase 2A, shown in Figure 9.11b and Figure 9.12a, the UIs and CIs for the players in the side game of the phase do not change. Looking at both figures, one will notice that the UI and CI moves for NTP and RIM's competitors (Nokia –the biggest of them– as an



(a) UMs and CMs by RIM, NTP and RIM's Competitors (Nokia as an example) in both games of the conflict's Phase 2A.

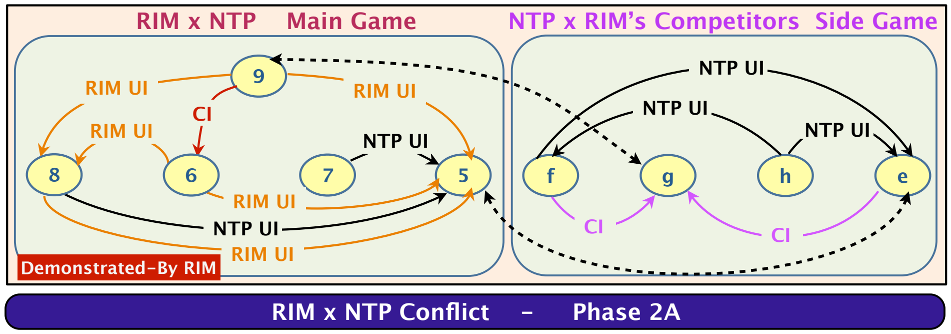


(b) UIs and CIs by RIM, NTP and RIM's Competitors (Nokia as an example) in both games of the conflict's Phase 2A. *RIM's UIs and CIs shown here reflects the preferences that RIM should have had.*

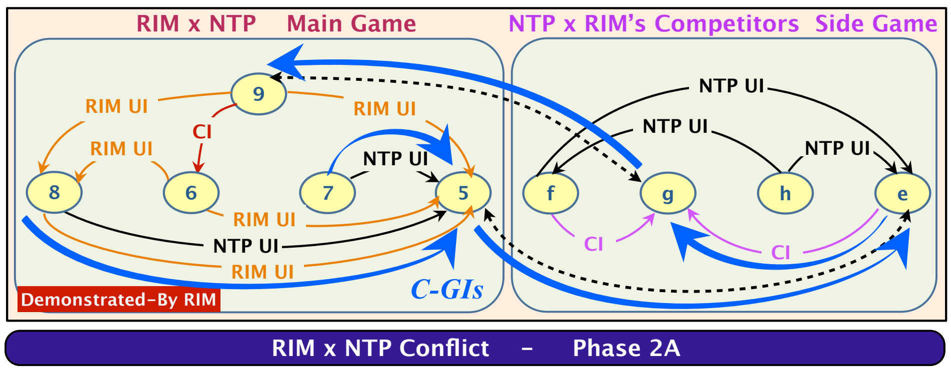


(c) Coalition Group Improvement (C-GI) multi-step moves by the NTP-Nokia-Others Coalition in both games of the conflict's Phase 2A.

Figure 9.11: The RIM v. NTP Conflict - Phase 2A (after the lower court decision comes against RIM): Moves by RIM, and NTP in the conflict's main game, and moves by NTP and RIM's Competitors (Nokia as an example) in the conflict's side game, including their possible coalition's multi-step C-GI moves. [all based on RIM's *Should-Be* Preferences]



(a) UIs and CIs by RIM, NTP and RIM's Competitors (Nokia as an example) in both games of the conflict's Phase 2A. *RIM's UIs and CIs shown here reflects the preferences that RIM actually demonstrated at the time.*



(b) Coalition Group Improvement (C-GI) multi-step moves by the NTP-Nokia-Others Coalition in both games of the conflict's Phase 2A.

Figure 9.12: The RIM v. NTP Conflict - Phase 2A (after the lower court decision comes against RIM): Improvement Moves by RIM, and NTP in the conflict's main game, and Improvement Moves by NTP and RIM's Competitors (Nokia as an example) in the conflict's side game, including their possible coalition's multi-step C-GI moves. [all based on RIM's *Observed* Preferences]

example) stay the same, and this is because these moves are not affected by RIM's preferences. Only NTP's cooperative CI moves with RIM in the main game of the phase are affected.

It is important to note here that by feeding the players' UMs and CMs and preference structures to Algorithm 9.1, assuming a coalition among NTP and Nokia (and other competitors of RIM) is formed, the coalition will have C-GIs that could lead to s_4 in the main game, the state that the coalition's members can all benefit from. Figure 9.11c shows the G-CIs for the coalition formed by NTP and Nokia (and other RIM's competitors who would like to join). The figure shows G-CIs, through which the coalition can, at different starting states in the main game or the side game, end up at state s_4 of the main game. These C-GI multi-step moves are made possible because the coalition members prefer the licensing-agreement state s_g more than the wait-wait state s_e.

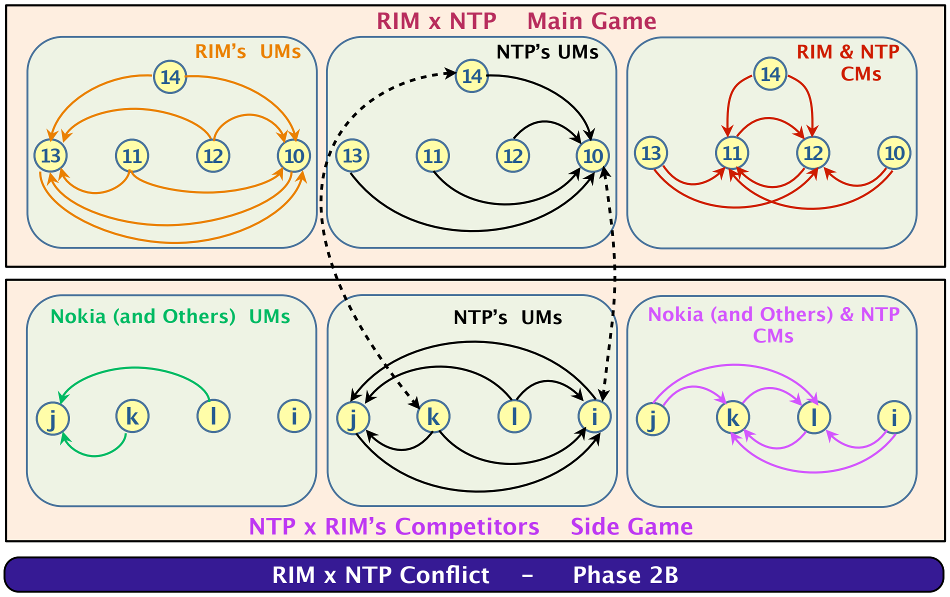
Figure 9.11c shows the NTP-Nokia-and-Others coalition's C-GIs on a graph that shows also the UIs and CIs of the players in Phase 2A, with RIM's preferences are the ones that should-have-been RIM's preferences. Figure 9.12b shows the same C-GIs for the coalition, but on a graph that also shows the UIs and CIs of the players in the phase, but with RIM's preferences are the ones that RIM actually demonstrated at the time. It is important to note here that the coalition's C-GIs in both figures, under the two contexts, are the same. The reason is quite obvious. As we have said above, the UIs and CIs of NTP and RIM's competitors in the side game do not depend on RIM's preferences. Therefore the coalition's multi-steps, which are made of a sequence of UM, UI, CM, and/or CI moves by the member players of the coalition, as a result will not be affected by RIM's preferences (and any UIs/CIs depend on these preferences).

3) Players' Moves in Phase 2B of the RIM v. NTP conflict (after the lower court decides in favour of RIM's position)

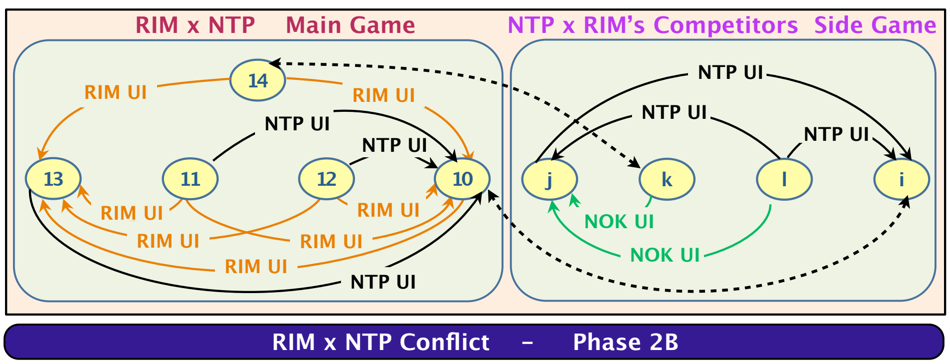
Figure 9.13a shows the players' UMs in both the main game (between NTP and RIM) and the side game (between NTP and RIM's competitors) of the conflict's Phase 2B (after the lower federal court rules in favour of RIM's position and against NTP). The figure also shows the one-step CM moves that the players, in each game of the phase, have and can act upon jointly.

These moves are similar to the ones that we identified above as possible moves that the players can have, either individually in the case of UMs or cooperatively in the case of CMs, in Phase 1 as well as in Phase 2A of the conflict. This is because the states that make up Phase 2B are similar, in their definitions, to the states make up Phase 1 and Phase 2A. They differ only in the timing of when the states occur (before or after the lower federal court's ruling), and or in the context (the lower court's ruling is in favour of RIM or NTP).

By feeding the players' UMs and CMs and preference structures separately to Algorithm 6.1 and then to Algorithm 8.1, the player's UI and then CI moves will be defined for this phase. Figure 9.13b shows the resultant set of UI and CI moves that the players of both games, the main game and the side game, have in the conflict at this phase. It is important to note here that by feeding the players' UMs and CMs



(a) Unilateral Moves (UMs) and Cooperative Moves (CMs) by RIM, NTP and RIM's Competitors (Nokia as an example) in the conflict's both games: the main game and the side game.



(b) Unilateral Improvements (UIs) and Cooperative Improvements (CIs) by RIM, NTP and RIM's Competitors (Nokia as an example) in the conflict's both games: the main game and the side game.

Figure 9.13: The RIM v. NTP Conflict - Phase 2B (after the lower court decision comes against NTP): Moves by RIM, and NTP in the conflict's main game, and moves by NTP and RIM's Competitors (Nokia as an example) in the conflict's side game.

and preference structures to Algorithm 9.1, and agin as before assuming a coalition among NTP and Nokia (and other competitors of RIM) is formed, the coalition will not have C-GIs that could lead to s_4 in the main game, the state that the coalition's members can all benefit from. The coalition will be stuck at state s_i of the side game, because none of the coalition's members have a unilateral UI or cooperative CI out from it. In other words, the coalition between NTP and RIM's competitors is not likely to form. We will elaborate more on this, once the stabilities of the different states to the different players, individually and collectively, are analyzed in the next subsection. But, for now, there are no C-GIs of any value to the possible coalition to show.

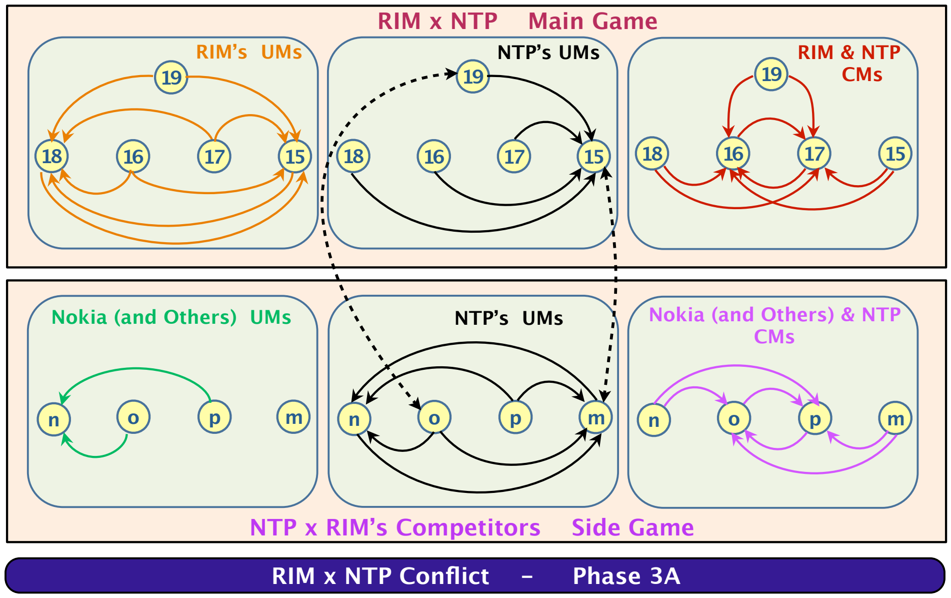
4) Players' Moves in Phase 3A of the RIM v. NTP conflict (after the higher court decides against RIM's position)

Figure 9.14a shows the players' UMs in both the main game (between NTP and RIM) and the side game (between NTP and RIM's competitors) of the conflict's Phase 3A (after the higher federal court rules against RIM's position and in favour of NTP). The figure also shows the one-step CM moves that the players, in each game of the phase, have and can act upon jointly.

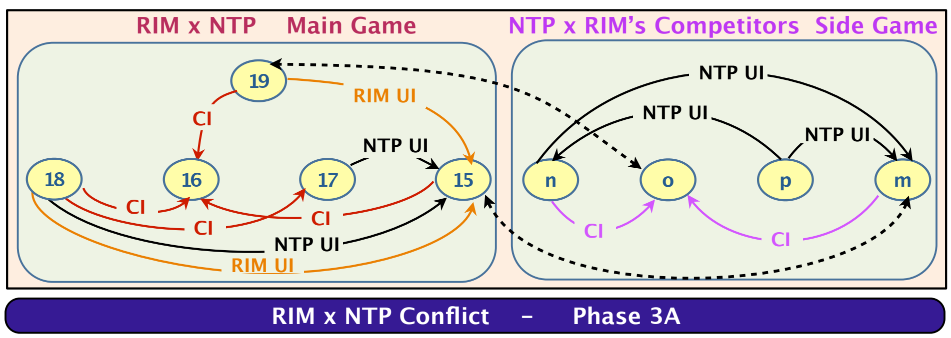
These moves are similar to the ones that we identified above as possible moves that the players can have, either individually in the case of UMs or cooperatively in the case of CMs, in Phase 1 as well as in Phase 2A and 2B of the conflict. But there are some differences. Most notably, in this phase, RIM has a state replaces the workaround state that it used to have before (states s_3, s_8 and s_{13} in Phase 1, 2A and 2B, respectively) with state s_{18}. s_{18} represents RIM leaving the US market and seizing any operations there. These changes in the states landscape caused some differences in the moves allowed for the players at this phase of the conflict.

By feeding the players' UMs and CMs and preference structures separately to Algorithm 6.1 and then to Algorithm 8.1, the player's UI and then CI moves will be defined for this phase. Figure 9.14b shows the resultant set of UIs and CIs that the players of the main game, and the side game, have in the conflict at this phase.

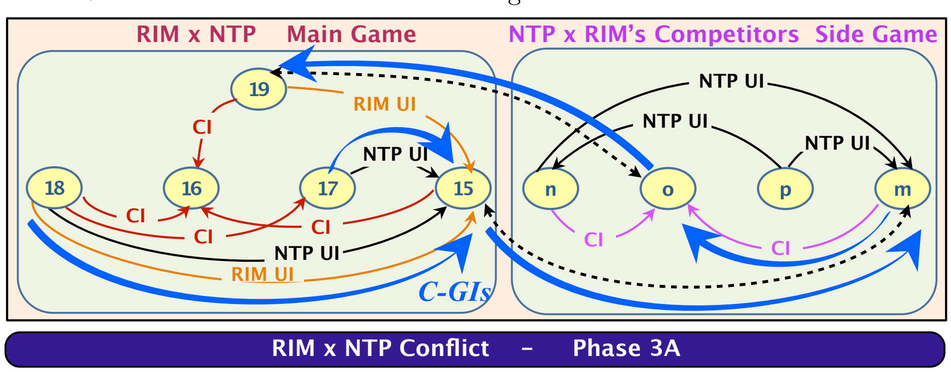
It is important to note here that by feeding the players' UMs and CMs and preference structures to Algorithm 9.1, assuming a coalition among NTP and Nokia (and other competitors of RIM) is formed, the coalition will have C-GIs that could lead to s_{19} in the main game, the state that the coalition's members can all benefit from. Figure 9.14c shows the G-CIs for the coalition formed by NTP and Nokia (and other RIM's rivals who would like to join). The figure shows G-CIs, through which the coalition can, by starting at some states in the main game or the side game, end up at state s_{19} of the main game. These C-GI multi-step moves are made possible because the coalition members prefer the licensing-agreement state s_o over the wait-wait state s_m.



(a) Unilateral Moves (UMs) and Cooperative Moves (CMs) by RIM and NTP in the conflict's main game.



(b) Unilateral Improvements (UIs) and Cooperative Improvements (CIs) by RIM and NTP in the conflict's main game.



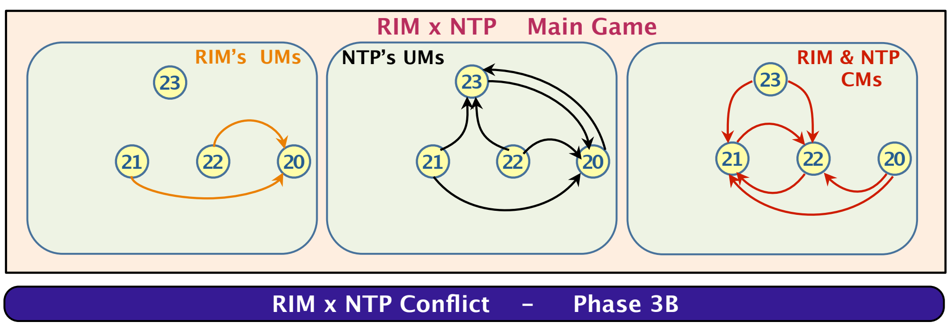
(c) Coalition Group Improvement (C-GI) multi-step moves by the NTP-Nokia-Others Coalition in both games of the conflict's Phase 3A.

Figure 9.14: The RIM v. NTP Conflict - Phase 3A (after the higher court decision comes against RIM): Moves by RIM and NTP in the conflict's main game, and moves by NTP and RIM's Competitors (Nokia as an example) in the conflict's side game, including their possible coalition's multi-step C-GI moves.

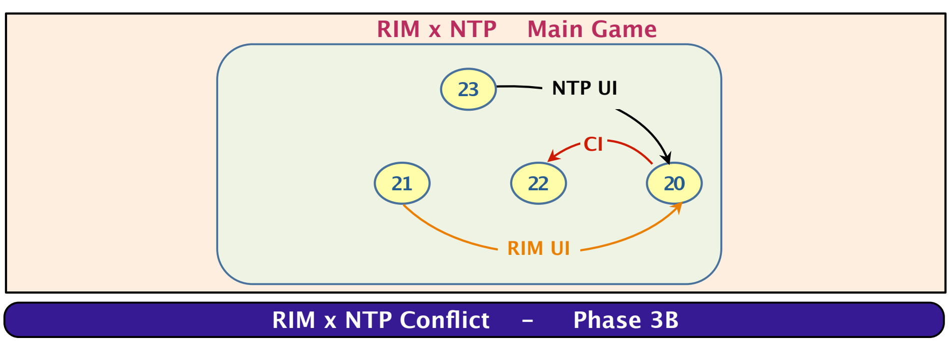
5) Players' Moves in Phase 3B of the RIM v. NTP conflict (after the higher court decides in favour of RIM's position)

Figure 9.15a shows the players' UMs in both the main game (between NTP and RIM) and the side game (between NTP and RIM's competitors) of the conflict's Phase 3B (after the higher federal court rules in favour of RIM's position and against NTP in the RIM v. NTP case). The figure also shows the one-step CM moves that the players have in the main game, and the only game that this phase has.

In this phase there is no state equivalent to the state represented, in the previous phases, NTP reaching a licensing agreement with RIM's competitors (states s_4, s_9 and s_{14} in Phase 1, 2A and 2B, respectively). This is because such an agreement, after the higher court takes a decision, does not help NTP (and that's why we do



(a) Unilateral Moves (UMs) and Cooperative Moves (CMs) by RIM and NTP in the conflict's main game.



(b) Unilateral Improvements (UIs) and Cooperative Improvements (CIs) by RIM and NTP in the conflict's main game.

Figure 9.15: The RIM v. NTP Conflict - Phase 3B (after the higher court decision comes against NTP): Moves by RIM and NTP in the conflict's main game. The conflict's side game between NTP and RIM's Competitors in not important to the main game in this phase.

not need to study the side game at this phase of the conflict). Also, RIM has no state representing the workaround state that it used to have before (states s_3, s_8 and s_{13} in Phase 1, 2A and 2B, respectively), or a state representing RIM leaving the US market and seizing any operations there (state s_{18} in Phase 3A). In addition, because of the higher federal court ruling against NTP in this phase, NTP has a new state that represents giving up the fight with RIM (state s_{23}). These changes in the states landscape caused some limitations to the moves the players can have at this phase of the conflict.

By feeding the players' UMs and CMs and preference structures separately to Algorithm 6.1 and then to Algorithm 8.1, the player's UI and then CI moves will be defined for this phase. Figure 9.15b shows the resultant set of UI and CI moves that the players of the main game, and the only game we have in the conflict at this phase. It is important to note here that, similar to the situation in Phase 3A of the conflict, there is no coalition's moves here since the side game is no longer important to the conflict between RIM and NTP. The conflict is now just between RIM and NTP, and is guided by whatever UIs they have individually or CIs they have cooperatively at this phase. Therefore, there is no coalition at this phase too, and there are no C-GIs to show as a result.

9.6.7 Stability Analysis of the RIM v. NTP Conflict

In this section, we will analyze the three phases of the RIM v. NTP conflict, each with the games it has. For each, we start by conducting stability analysis, followed by equilibrium states analysis. We will discuss the insight that both analyses provide.

1) Stability Analysis for Phase 1 of the RIM v. NTP conflict (before the lower court's decision)

Table 9.18 presents the stability analysis of the the five states of Phase 1's main game, for both RIM and NTP, under NASH, GMR, SMR and SEQ solution concepts. Table 9.19 shows the equilibrium states for the main game under the four solution concepts.

As Table 9.18 shows, RIM's only stable state, under any solution concept, is the status quo state s_0. This is because the worst that NTP can do to RIM in this game, at this phase, is to take RIM to state s_0. which is the most preferred state to RIM in the game. Therefore, NTP cannot do any disimprovement to RIM, if RIM takes a UI to s_0 from any of the game's states. As a result, none of the games states is stable to RIM with the exception of s_0.

And, because RIM has no UI/CI out of s_0 to any other state, then this state is a NASH stable state for RIM. Also, because RIM can reach this state through many UI moves it has from other states in the main game, the strength of the NASH stability of this state to RIM equals the least preference strength it has with these other states (NASH stability strength for individual player within a cooperative conflict with coalitions is given in Definition 9.5.1 and calculated by applying Algorithm 9.3). Referring back to RIM's preferences in the main game of Phase 1 shown in Table 9.8, we see that out of all the states that can lead to s_0 through a UI, coming from s_3 provides RIM with the least preference gain, of *Little* gain. This makes the strength of s_0's NASH stability to RIM set at the *Little* strength level.

NTP, on the other hand, has all states of Phase 1's main game stable to it under one or more of the solution concepts. NTP has a UI from each of s_3 and s_2 to the status quo state s_0. RIM can disimprove any of these two NTP's UIs by sanctioning them and moving the game to s_3, leaving NTP with GMR stability for s_3 and for s_2. The GMR stability strength for each of s_3 and s_2 matches the disimprovement that could happen to NTP because of RIM's sanctions. Notice that both s_3 and s_2 are not SMR stable for NTP because NTP can recover from such disimprovement by moving back to s_0. Also, both states are not SEQ stable for NTP, because these RIM's sanctions are not UI moves to RIM. This means that RIM, if it takes any of these two sanctions, it will be acting in an irrational way against its preferences structure, motivated by the urge to hurt NTP even if this means that it will hurt itself in the process.

In addition, NTP has state s_4 as a GMR, SMR and SEQ stable state for it in this game. This is because the cooperative CI move that NTP can have jointly with RIM might get sanctioned by RIM afterwords by RIM taking one of two UIs it has from s_4 to either state s_0 (a *Little* less preferred state for NTP than s_4) or to state s_3 (a *Moderately* less preferred state for NTP than s_4). This makes s_4 GMR as well as SEQ stale for NTP, with strength set at the *Moderate* level. And, because NTP has no countermove to the first sanction and a countermove to the sanctions that leaves the game at state s_0, state s_4 is also an SMR stable with strength set at the

Table 9.18: RIM v. NTP Conflict - Phase 1: Stability Analysis for the main game (RIM v. NTP)

	RIM					***NTP***				
	s_0	s_3	s_2	s_1	s_4	s_1	s_4	s_0	s_2	s_3
UIs, CIs & C-GIs		s_0 (UI)	s_0 (UI) s_3 (UI)	s_0 (UI) s_3 (UI)	s_0 (UI) s_3 (UI) s_1 (CI)		s_1 (CI)		s_0 (UI)	s_0 (UI)
NASH	L					Ex		L		
GMR	N					N	Mo	N	L	N
SMR	N					N	L	N		
SEQ	N					N	Mo	N		

Table 9.19: RIM v. NTP Conflict - Phase 1: Equilibrium States for the main game (RIM v. NTP)

	s_0	s_1	s_2	s_3	s_4
NASH EQ.	L				
GMR EQ.	N				
SMR EQ.	N				
SEQ EQ.	N				

Little level.

Furthermore, NTP has states s_1 and s_0 as NASH stable states for it at strengths *Extreme* and *Little* levels, respectively. This is because NTP has no UIs, CIs, or C-GIs out of these two states. It also cannot reach s_4 without a coalition that is unlikely to form because NTP and RIM's competitors has no incentive, or C-GI leading them to s_4, to form such coalition (see the stability analysis for the side game of this phase which will be presented shortly). Hence, the *Extreme* NASH stability of s_4 for NTP. At the same time, the status quo state s_0 can be reached, by way of an NTP's UIs, from states s_3 and s_2. By applying Algorithm 9.3, s_0's NASH stability for NTP is found to be at the *Little* strength level.

The overall stability of the states in Phase 1's main game is shown in Table 9.19. The only stable state for all players within this game, i.e. equilibrium state for the game, is the status quo state s_0. This means that all the other states in the game are unstable for at least one player. In this case, RIM has all other stables as unstable to it. And, for this reason the main game between RIM and NTP is likely to be stuck at the status quo s_0 state, where both fighting each other in the court system. This is because s_0 is the only equilibrium state for the conflict at

Table 9.20: RIM v. NTP Conflict - Phase 1: Stability Analysis for the side game (NTP x RIM's Competitors (Nokia as an example)

	NTP				***NOK***			
	s_c	s_a	s_b	s_d	s_a	s_b	s_d	s_c
UIs, CIs & C-GIs			s_a (UI)	s_a (UI) s_b (UI)			s_b (UI)	s_b (UI)
NASH	Ex	N			Ex	N		
GMR	N	N			N	N		
SMR	N	N			N	N		
SEQ	N	N			N	N		

Table 9.21: RIM v. NTP Conflict - Phase 1: Equilibrium States for the side game (NTP x RIM's Competitors (Nokia as an example)

	s_a	s_b	s_c	s_d
NASH EQ.	N			
GMR EQ.	N			
SMR EQ.	N			
SEQ EQ.	N			

this phase, before the lower federal court decides on the RIM v. NTP case, as per the analysis presented. Historically, this is exactly what happened.

We have said above that a coalition between NTP and RIM's competitors is not likely to form at Phase 1 of the RIM v. NTP conflict. The reason behind this will be more obvious, once we present the stability and equilibrium analysis of the side game of Phase 1. Table 9.20 presents the stability analysis of the the four states of Phase 1's side game, for both NTP and Nokia (the biggest of RIM's competitors used here as an example), under NASH, GMR, SMR and SEQ solution concepts. Table 9.21 shows the equilibrium states for the side game under the four solution concepts.

From able 9.20, we can see that Nokia (and other RIM's competitors) will not likely settle, or sign any agreement, with NTP (states s_c or s_d). Nokia will likely use the UIs it has from these states to the fight-NTP-legally state s_b. All other RIM's competitors will do the same thing, based on their preferences, and logically speaking since all want to see if NTP has a strong case against RIM. In addition, RIM is not yet suffering in the marketplace because of this case, and RIM's customers are not likely at this stage to look for alternatives because of fear of disruption or cut in the services. These things can happen only when RIM start losing legally to NTP and/or the cloud of uncertainty start thickening around its service and future in the US market. NTP cannot sanction the UIs that Nokia, and others, will take out of states s_c or s_d to state s_b, a NASH stable for them but a very weak one (at strength level set at the *None* level). If NTP does take the move it has from s_b to the status quo s_a, which is a UI move from NTP, then Nokia and other RIM's competitors will be at the best state for them, preference-wise. Therefore, s_a is a NASH stable for Nokia, and others, with strength set at the *Extreme* level, whilst s_c and s_d are not stable for them.

On the other hand, NTP in this side game has s_b and s_d as unstable states, because it has UIs out from them to state s_a or to states s_a and s_b (respectively), and Nokia/Others cannot sanction such UIs. But, as the table shows, s_c is a NASH stable state to NTP because NTP does not have a UI out of it, and with strength set at the *Extreme* level because it cannot reach this state on its own.

The only state in Phase 1's side game that is stable for all players, NTP and Nokia (and others), is the status quo state s_a, which represents all players wait for the results of the lower federal court ruling on RIM v. NTP case. s_a is NASH for all players, because none of the side game players has a UI, CI or C-GI out of s_a. And, the strength for s_a's NASH stability for Nokia (and others) is set at the *Extreme* level, because they cannot reach this state on their own, and their stay there is at the mercy of NTP's decision to challenge them legally for what it claims its rights. Also, the strength of s_a's NASH stability for NTP is calculated applying Algorithm 9.3 to be set at the very-weak *None* level. This is because NTP can reach state s_a by using UIs it has from state s_b (where NTP and Nokia–and others– are fighting in the courts) and from state s_c (where NTP and Nokia–and others– reach a partial agreement that does not include a licensing component). The least preference gain from these UIs is the one accomplished by moving from the s_b to s_a with NTP gaining almost nothing in such move. Hence, the very weak *None* strength of s_a's NASH stability for NTP.

This makes s_a a NASH equilibrium for Phase 1's side game with strength set at *None* level. In other words, the side game in Phase 1 of the conflict has the weakest strength level. NASH, by definition is a weak stability; and NASH at strength *None* brings down the stability of this state. This is understandable for many reasons. For all players in the side game, NTP and RIM's competitors, it is a wait and see game. NTP can challenge those players legally but will gain nothing in the process. If NTP waits until the courts agree with its case with RIM, it will be in a better position. It will be able to sign those companies for more licensing fees, and can use such agreements to add another layer of legitimacy to its claims. Not to mention, the additional pressure RIM will face if it decides not to settle. For RIM's competitors, it is also wait and see. They do not like the idea to license NTP's technology and pay for it, but this may change if RIM is dragged, willingly or forcefully, into a long battle with NTP. Then, there may be a chance for a coalition between NTP and all other against RIM (the market leader). This coalition, then, will benefit all of the players except RIM, their common enemy.

So far, and based on the analysis of Phase 1's side game, the coalition is not likely because none will benefit from it. A good indication that no member of this NTP-Nokia-Others coalition will benefit from such coalition is the fact that there is no C-GIs that could materialize for this coalition and affect the dynamics within the main game between NTP and RIM. The side game in this phase has no C-GIs that could change the main game from one state to another. Therefore, based on this analysis of the side game, the side game is more likely to stay put at its current status quo s_a because this state is the only equilibrium to the side game at this phase. And, even though this equilibrium is of weak strength, it is stable for all the players, nevertheless.

2) Stability Analysis for Phase 2A of the RIM v. NTP conflict (after the lower court decides against RIM's position)

Phase 2A represent the stage of the conflict where the lower federal court decides against RIM. Table 9.22 presents the stability analysis of the the five states of Phase 2A's main game, for both RIM and NTP, under NASH, GMR, SMR and SEQ solution concepts. Table 9.23 shows the equilibrium states for the main game under the four solution concepts. Recall that we have two versions of RIM's preferences structure for Phase 2A of the conflict: 1) the should-be RIM's preferences; and 2) the observed preferences that historically RIM demonstrated at the time. Table 9.22 and Table 9.23 show the stability analysis and the equilibrium analysis, respectively, of Phase 2A's main game using RIM's should-be preferences. The stability analysis and equilibrium analysis of Phase 2A's main game using the preferences that RIM actually demonstrated at the time are presented later, after the should-be analysis tables, in Table 9.24 and Table 9.25, respectively.

As the stability analysis of Phase 2A's main game based on the should-be RIM's preferences provided in Table 9.18 shows, RIM has all the phase's states stable under

Table 9.22: RIM v. NTP Conflict - Phase 2A: Stability Analysis for the main game (RIM v. NTP) - based on the Should-be RIM's Preferences not the ones observed in reality at the time

RIM

	s_7	s_6	s_8	s_5	s_9
UIs, CIs			s_6 (CI)	s_8 (UI)	s_5 (UI)
& C-GIs				s_6 (CI)	s_8 (UI)
					s_6 (CI)
NASH	Ex	Ex			
GMR	N	N	L	L	N
SMR	N	N	N		
SEQ	N	N			

NTP

	s_6	s_9	s_5	s_8	s_7
UIs, CIs		s_6 (CI)	s_6 (CI)	s_5 (UI)	s_5 (UI)
& C-GIs			s_9 (C-GI)	s_6 (CI)	s_9 (C-GI)
				s_9 (C-GI)	
NASH	Ex				
GMR	N	S	L	N	
SMR	N	N			
SEQ	N				

* All C-GIs here are C-GI moves by the Coalition of NTP and Nokia (and other RIM's competitors).

one or more of the solution concepts, and at different stabilities' strength levels. Both states s_7 and s_6, in which RIM reaches an agreement with NTP (partial agreement that covers old usage for state s_7 and full licensing agreement for state s_6), are NASH stable states for RIM because RIM has no UIs or CIs out of them, and with stabilities' strength set at the *Extreme* level for both because RIM cannot reach these states on its own through any of its UIs.

State s_8, in which RIM comes out with a workaround to NTP's wireless email technology, is GMR stable for RIM because if RIM decides to cooperate with NTP in reaching an agreement putting the conflict at state s_6, NTP might decide to cut the agreement talks short, sanction it and go back to fighting RIM. Of course, this is not a rational sanction by NTP, if NTP decides to take it. Hence, the fact that s_8 is not SEQ stable for RIM. But, because RIM has a countermove from this sanction to state s_5, lowering the bad effect of the sanction from *Little* (the GMR strength level) to *None*, s_8 is considered SMT stable for RIM at stability strength level of *None*.

In addition, the both-parties-fighting state s_5 is GMR stable for RIM because if RIM tries to enhance its position by going to state s_8, in which it will keep fighting but at the same time prepare a workaround NTP's technology for its service, or by going to state s_6, in which it cooperates with NTP to reach a full scale licensing agreement, NTP might just sanction any of these UI moves by RIM using a multi-step move NTP has with its coalition (a coalition formed by NTP and RIM's competitors). This coalition's sanction is a C-GI move if it starts from s_8, but just a C-GM move if it starts from s_6 (because of this, s_5 is not SEQ stable for RIM), and will put the game at state s_9. This sanction will put RIM at a worse place in the game because now NTP, partnering with RIM's competitors in the marketplace, manages to legitimize its claims for the technology in front of the court and in front current/potential customers of RIM. RIM will end up at state s_9, which is *Little* less-preferred than s_5, therefore the strength of s_5's GMR stability for RIM is set at the *Little* level.

Also, notice that s_5 is not SMR stable for RIM, because RIM can ignore the facts on the ground created by s_9 and go to sate s_8, which is more preferred to it than s_5 which it started from. But in reality, RIM cannot just ignore the facts on the ground created by NTP and its new coalition moving the game to state s_9. If RIM decides to take any of its individual UIs or cooperative CIs from s_9 (shown in Table 9.22 under RIM's s_9), the coalition will use a multi-step sanction move, it has from any of these UIs/CIs end states, bringing the game back to s_9. Yes, the coalition will gain nothing. Hence, the fact that s_9 is GMR stable for RIM at strength level of *None*. And, if the coalition sanction started from s_6, then this sanction is not rational for the coalition, namely NTP –its main player–. Hence, the fact that s_9 is not SEQ stable for RIM. In addition, RIM can still persist fighting, and, or even better, fight but find a workaround solution, i.e. RIM can enhance its position from s_9, as a countermove to the coalition sanction. This makes s_9 not an SMR stable state for RIM in this game.

For NTP, this phase of the conflict is a good phase. The lower federal court have just agreed with its claims, putting pressure on RIM to cave in to NTP demands. In terms of stability of the conflict's states for RIM, the picture is not bad either, as can be seen from Table 9.22. Understandably, State s_6 is NASH stable for NTP because NTP has no UI or CI out of it, and with strength set at the *Extreme* level because NTP cannot reach this state on its own, it must cooperate with RIM to do so. State s_9 is GMR stable for NTP because NTP fears RIM moving away from an initial cooperative CI joint move by NTP and RIM to reach a licensing agreement (s_6). This sanction by RIM is not a rational one for RIM, based on the should-be RIM's preferences structure. Hence, state s_9 is SEQ stable for NTP. And, NTP has a countermove for this sanction, bringing the game back to the facts in the ground created by its coalition's licensing-agreement move. This countermove makes the strength of s_6's SMR stability for NTP set at the *None* level, easing the state's GMR stability of *Small* strength level (equal to the loss at the preference scale that NTP will have with RIM's sanction been the one that takes the game to s_8).

Table 9.23: RIM v. NTP Conflict - Phase 2A: Equilibrium States for the main game (RIM v. NTP) - based on the Should-be RIM's Preferences not the ones observed in reality at the time

	s_5	s_6	s_7	s_8	s_9
NASH EQ.		Ex			
GMR EQ.	L	N		N	N
SMR EQ.		N			
SEQ EQ.		N			

Both states s_5 and s_8 are GMR stable for NTP, making NTP not wanting to take any of its UI, CI and or C-GIs of these two states. NTP will be afraid that RIM will sanction its improvement moves by either moving to state s_5 or s_8. This puts the strengths of s_5's GRM stability for NTP at *Little* level, and s_8 at the *None* GMR strength level. Definitely, NTP can fight back by putting the game at state s_9 which is much preferred to it, making in the process both s_5 and s_8 not SMR stable to it. And, because some of RIM's sanction moves, especially the ones in response to NTP's CI move to s_6, are not rational moves for RIM (under the should-be RIM's preferences model), both s_5 and s_8 cannot be SEQ stable state for NTP.

The overall stability of the states in Phase 2A's main game, with RIM's should-be preferences considered, is shown in Table 9.23. As the table shows, state s_6, in which RIM reaches a licensing agreement with NTP, forms a NASH equilibrium because neither RIM nor NTP has a UI out of this state. The strength of the state's NASH is set at the *Extreme* level because neither can reach this state on its own. They have to cooperate jointly to make it to s_6.

Table 9.23 shows another important fact about the equilibrium states, and potential resolution points for the RIM v. NTP conflict at Phase 2A of the conflict. It shows that s_6 is the only equilibrium for the conflict at this stage, assuming that both players, NTP and RIM, act rationally. The table shows that the conflict has three other equilibriums at this stage, at states: s_5, s_8 and s_9. But all these equilibrium states are equilibriums under the GMR solution concepts. In other words, each of the players at these states will fear the other player act for the purpose of hurting the opponent, even if it means that the opponent will hurt himself in the process. An unlikely scenario, given the collective rationality established at the institutional level within the business world. Therefore, s_6 is a more likely resolution to the conflict at Phase 2A, if and only if the conflict ends at this phase, given

Table 9.24: RIM v. NTP Conflict - Phase 2A: Stability Analysis for the main game (RIM v. NTP) - based on RIM's Preferences that were actually-demonstrated by RIM at the time

RIM

	s_7	s_5	s_8	s_6	s_9
UIs, CIs & C-GIs			s_5 (UI)	s_5 (UI) s_8 (UI)	s_5 (UI) s_8 (UI) s_6 (CI)
NASH	Ex	N			
GMR	N	N	N	N	N
SMR	N	N			
SEQ	N	N	N	N	

NTP

	s_6	s_9	s_5	s_8	s_7
UIs, CIs & C-GIs		s_6 (CI)	s_9 (C-GI)	s_5 (UI) s_9 (C-GI)	s_5 (UI) s_9 (C-GI)
NASH	Ex				
GMR	N	S	L	N	
SMR	N	N			
SEQ	N	S	L		

* All C-GIs here are C-GI moves by the Coalition of NTP and Nokia (and other RIM's competitors).

the rationality of both players in the conflict. Saying so, one should notice that s_5 is slightly more stable as a GMR equilibrium, strength-wise, than the other GMR equilibriums. Again, showing that if both players act rationally, they will fear the escalation of the fight. The escalator will soon see his opponent escalates the fight from his side too.

But, how will Phase 2A's stability and equilibrium analysis, provided above, change, if we consider the preferences structure that RIM actually demonstrated historically at the time, instead of using RIM's should-be preferences structure? The question could be easily answered by comparing the analysis tables for Phase 2A given RIM's should-be preferences, Tables 9.22 and 9.23, with the phase's stability and equilibriums analysis tables given RIM's preferences observed at the time, Tables 9.24 and 9.25.

First, by comparing RIM's should be preferences shown in Table 9.10 to the actual preferences demonstrated by RIM at the time shown in Table 9.11, one will notice: 1) the ordinal preferences for states s_5, s_6 and s_8 have changed from $s_6 \succ^{N}_{RIM,t} s_8 \succ^{N}_{RIM,t} s_5$, based on the should-be preferences for RIM, to $s_5 \succ^{N}_{RIM,t} s_8 \succ^{N}_{RIM,t} s_6$, based on the actually demonstrated ones; and 2) while RIM's weighted preferences show very weak preferences across all the states, in both cases –the should-be and the demonstrated preferences–, with each state is preferred or less-preferred to the other states by a strength degree of *None*, the should-be preferences structure show a stronger less-preference of state s_9 to all other states by a strength degree of *Little*.

At the heart of the reasons behind these changes is the fact that RIM still valued what it considered a fair finical value of NTP settlement more than the actual settlement of the case, and moving on to focus on its core business of stabilizing its position in the market and growing its customer base (see Figures 9.6a and 9.6c for the strategic importance RIM assigned to its goals based on both cases). Thus, RIM underestimated the impact of continue its fight with NTP and the fact that its competitors in the market will capitalize on its distraction, and what seemed RIM's obsession to win the legal case it has with NTP, to win over RIM's customer and gain market share.

The changes in RIM's preferences, between the should-be and the actual structures, resulted on a change in RIM's UIs and the cooperative CIs it has in the main game of Phase 2A, as can be seen by comparing Figure 9.11b with Figure 9.12a. This lead to the changes we see in the stability analysis of the different states based on the two different structures. Comparing Table 9.22 with Table 9.24, we can notice the following:

1. RIM maintained the NASH stability it has for state s_7, but replaced what should have been a very strong *Extreme* NASH stability at the full-licensing-settlement state s_6 (based on the should-be preferences model) by a very weak NASH stability at the fight-fight state s_5 with strength set at the *None* level;

2. while state s_8 is GMR stable for RIM based on both models (with a bit more strength in the model based on RIM's demonstrated preferences), s_8 is shown to be SEQ stable for RIM only based on the demonstrated preferences model, i.e. the sanction against RIM's UI out of s_8 committed by the NTP-Nokia-Others coalition is considered in this case a rational move by the coalition;

3. state s_8 lost its SMR stability for RIM, in the model which is based on demonstrated-preference, because RIM thinks now that its countermove to the coalition sanction bringing the game back to the fight-fight s_5 state, and ignoring the facts created on the ground by the coalition sanction, is a preferred state (regardless of the fact that it is a very superficial belief because in reality RIM cannot just ignore the facts on the ground created by the coalition sanction –at least, the market will take notice and will not ignore–);

4. state s_6 is a GMR and SEQ stable for RIM, based on the demonstrated-preferences model, with strength for both stabilities set at the very weak *None* level, for the same reasons that state s_8 is GMR and SEQ stable for RIM based on this model; and

Table 9.25: RIM v. NTP Conflict - Phase 2A: Equilibrium States for the main game (RIM v. NTP) - based on RIM's Preferences that were actually-demonstrated by RIM at the time

	s_5	s_6	s_7	s_8	s_9
NASH EQ.					
GMR EQ.	N	N		N	N
SMR EQ.					
SEQ EQ.	N	N			

5. The stabilities of the different states of the game for NTP, based on RIM's demonstrated preferences model, did not change from the ones NTP has based on the the should-be RIM's preferences model except in a minor way: both state s_9 and state s_5 are SEQ stable for NTP based on RIM's demonstrated preferences model because now RIM's sanctions are "rational" (for sure this is based on RIM's thinking and how it perceives its goals and therefore its preferences at this stage of the game –i.e. regardless whether the rationality of these beliefs is justified by the outside observers or not–).

The changes in Phase 2A's individual state's stabilities based on the two different preferences structures for RIM, the should-be and the actually-demonstrated-at-the-time, impact also the overall stability of these states for the conflict at this phase. This is illustrated by comparing the equilibrium states analysis of Phase 2A based on the should-be RIM's preferences, shown in Table 9.23, and the same analysis based on the demonstrated RIM's preferences, shown in Table 9.25. One will immediately notice the following:

1. there is no equilibrium states for Phase 2A, under the NASH stability solution concept, if and when RIM's demonstrated preferences at the time is considered in the analysis;

2. state s_6, in not a NASH equilibrium in the analysis based on RIM's demonstrated preferences, as it is the case if RIM's preferences were the should-be ones;

3. state s_6, nevertheless, maintained in both cases a GMR and SEQ stability for both players with strength set at the very weak *None* level;

4. the fight-fight s_5 state is not only GMR stable for both players, for the conflict at its Phase 2A based on RIM's demonstrated preferences as it is the

case based on the should-be RIM's preferences, but also SEQ stable for both players; and

5. states s_8 and s_9 maintained their very weak equilibrium status under the GMR solution concept.

This list gives us great insight on why the RIM v. NTP conflict's Phase 2A progressed historically in real-life the way it did, as we will elaborate later when we discuss the results of our overall stability analysis of the conflict. But, we can at least point at some interesting results elicited from the list above:

1. the only two states that could form an equilibrium for the conflict at Phase 2A, assuming the players of the conflict act rationally, i.e. according to their preferences structures, are: the full-licensing-agreement s_6 state and the fight-fight s_5 state;

2. if state s_5 persisted as an equilibrium to the conflict at the Phase 2A stage of it, then the conflcit will continue to the next phase of the conflict: Phase 3, and whether the conflict is going the Phase 3A way or Phase 3B way depends completely on what the higher federal court decides; making state s_6 to be the true possible rational resolution to the conflict at Phase 2A of it;

3. the partial-agreement state, s_7, is not a candidate as a resolution to the conflict at this state; and

4. each of the players will not be escalating the conflict individually, because the other player will respond by escalating the conflict to a less-preferred state (suggested by the GMR stability of both s_8 and s_9).

Now, we analyze the stability of each of Phase 2A side game's states, with the purpose of checking whether a coalition between NTP and RIM's competitors is possible at this stage of the conflict. And, to check whether the members of this possible coalition will all expect to benefit from some C-GI moves that could affect the main game of the phase and put their common "enemy" at a worse position. First, we introduce the stability tables of the side game of Phase 2A, and then we will discuss the contents of these tables. Table 9.26 presents the stability analysis of the the four states of Phase 2A's side game, for both NTP and Nokia (the biggest of RIM's competitors used here as an example), under NASH, GMR, SMR and

Table 9.26: RIM v. NTP Conflict - Phase 2A: Stability Analysis for the side game (NTP x RIM's Competitors (Nokia as an example)

	NTP				***NOK***			
	s_g	s_e	s_f	s_h	s_h	s_g	s_e	s_f
UIs, CIs		s_g (CI)	s_e (UI)	s_e (UI)			s_g (CI)	s_g (CI)
& C-GIs			s_g (CI)	s_f (UI)				
NASH	Ex				Ex	Ex		
GMR	N	N			N	N	L	N
SMR	N	N			N	N	L	N
SEQ	N				N	N		

Table 9.27: RIM v. NTP Conflict - Phase 2A: Equilibrium States for the side game (NTP x RIM's Competitors (Nokia as an example)

	s_e	s_f	s_g	s_h
NASH EQ.			Ex	
GMR EQ.	N		N	
SMR EQ.	N		N	
SEQ EQ.			N	

SEQ solution concepts. Table 9.27 shows the equilibrium states for the side game under the four solution concepts.

From Table 9.26, we can see that Nokia (and other RIM's competitors) will likely sign a full licensing agreement with NTP (states s_g). The full-agreement s_g state is NASH stable for NTP and Nokia (and others) with *Extreme* strength level because none can reach s_g on their own, they all must cooperate to be able to do so. On the other hand, the partial agreement s_h state, even though it is a NASH stable state for Nokia (and other RIM's competitors) with *Extreme* strength because Nokia cannot get to it without cooperating with NTP, it is unstable to NTP because NTP can take one of its UIs out of s_h to either s_e or s_f.

The table shows also that the fight-fight s_f state is unstable for NTP because it will always be able to use its UI to wait s_e or cooperate with RIM's competitors and go to state s_g. For Nokia, and others, the fight state is GMR and SMR stable because NTP can sanction them after they all start putting together the full agreement by bringing the side game back to the fight-fight state s_f. Hence, the stability strength of s_f's GMR and SMR for Nokia, and others, is set at the very weak *None* level. In addition, the wait-wait s_e state is GMR and SMR stable for both NTP and Nokia (and others) because while both can move cooperatively to reach s_g, each will fear a sanction move that the other has bringing the side game back to the fight-fight s_f state. The strength of s_e's GMR and SMT for each of NTP and Nokia (and others) differ based on how less-preferred s_f is to state s_e for each.

Table 9.26 shows also that neither s_e nor s_f form an SEQ stable state for any of the players, giving us an indication that this fear, that the other party will sanction the CI the players take cooperatively to reach to s_g from any of these two states, is not justified because none of the players will act irrationally against their preferences just to hurt the other. This is especially true in a business environment with institutional collective rationality.

The stability analysis results of Phase 2A's individual states are the input needed to definite the overall equilibrium states for the side game at this phase. Table 9.27 shows that this side game has two equilibrium states. As expected, the full-agreement s_g state is a NASH equilibrium, with strength set at the *Extreme* level because none of the players in this game will be able to reach this state on their own. They all have to cooperate to do so. The second equilibrium state is state s_e, the wait-wait state, which found to be an equilibrium under both the GMR and the SMR stability solution concepts. But, this equilibrium has a very weak strength set at the *None* level. Not to mention, that s_e is also a weak equilibrium, because it is based on fear the individual players has that the opponent/s will act irrationally –against their preferences– just to hurt them (an additional indication of this is the fact that this state is not SEQ stable to any of the players in the side game). An unjustified fear, knowing that within the context of the business environment all entities have collective institutional rationality, eliminating the possibility of emotional reactions and irrational behaviour.

The results of the side game's analysis suggest that the coalition between NTP and RIM's competitors is very likely because all will benefit from it. A good indication of this strong possibility is the formation of a number C-GI moves that the coalition can have to affect the main game between NTP and RIM, putting it at a state that is preferred to all members of the coalition. These multi-step C-GI moves, the coalition can have, are shown above in Figure 9.11c. It is also worth noting here that the stabilities, equilibriums and these C-GIs will not change whether RIM adopted the should-be preferences structure or the one that RIM actually demonstrated at the time. This is because these stabilities, equilibriums and C-GIs are based on the preferences of NTP and RIM's competitors (Nokia and others).

From Figure 9.11c, we can see that the resultant coalition's C-GIs will put the main game at state s_9. A state that is preferred to NTP in its fight with RIM, especially if the main game was stuck at one of the states that is less preferred to NTP: the fight-fight s_5 state, the RIM-fights-and-have-a-workaround-solution s_8 state, or the partial-agreement s_7 state. In addition, putting the main game between NTP and RIM at state s_9 is better for all RIM's competitors, despite the fact that they are not direct players in the main game. But, putting their common rival at a much worse state in the conflict, and as a result in the marketplace, will definitely help them capitalize on RIM's problems and uncertain future in order to take away current customer of RIM and increase their market share. Let us not forget that RIM was the biggest player in the wireless/mobile email devices market (later started to be known as smart phones market) with the biggest market share and the most advanced devices. So, slowing down RIM's growth and bogging it down with legal problems is considered a very positive outcome for RIM's competitors. Especially, if this also comes packaged with legitimizing these companies devices, offering RIM's customers a stable alternative away from the uncertainties produced by RIM's no-sign-of-settling strategy and the possibility that RIM will end up forced to stop operating in the US as a result.

3) Stability Analysis for Phase 2B of the RIM v. NTP conflict (after the lower court decides in favour of RIM's position)

Phase 2B represent the stage of the RIM v. NTP conflict where the lower federal court decides against NTP and in favour of RIM's position. Table 9.28 shows the stability analysis of the the five states of Phase 2B's main game, for both RIM and NTP, under NASH, GMR, SMR and SEQ solution concepts. Table 9.29 shows the equilibrium states for the main game under the four solution concepts.

Table 9.28 shows RIM having two states stable for this phase of the conflict. State s_{13} is now a NASH stable for RIM, because RIM has no UI out of it. And, the strength of s_{13}'s NASH stability for RIM is set at the *None* level, because RIM has many UIs leading to this state and the strength of its NASH stability is set based on the lows gain in preference that RIM will earn by taking any of these UIs to s_{13}. Recall that s_{13} is the state in which RIM develops a workaround to NTP's claimed-technology, despite the fact that the lower federal court has just ruled in favour of RIM's position, as it also continues the fight NTP in the courts. The

Table 9.28: RIM v. NTP Conflict - Phase 2B: Stability Analysis for the main game (RIM v. NTP)

	RIM					***NTP***				
	s_{13}	s_{10}	s_{14}	s_{12}	s_{11}	s_{14}	s_{10}	s_{13}	s_{11}	s_{12}
UIs, CIs		s_{13}(UI)	s_{13}(UI)	s_{13}(UI)	s_{13}(UI)			s_{10}(UI)	s_{10}(UI)	s_{10}(UI)
& C-GIs			s_{10}(UI)	s_{10}(UI)	s_{10}(UI)					
NASH	N					Ex	N			
GMR	N	N				N	N	N		
SMR	N					N	N			
SEQ	N	N				N	N	N		

Table 9.29: RIM v. NTP Conflict - Phase 2B: Equilibrium States for the main game (RIM v. NTP)

	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}
NASH EQ.					
GMR EQ.	N			N	
SMR EQ.					
SEQ EQ.	N			N	

move to s_{13} is a rational move to RIM, act as an insurance policy that strengths its position in the marketplaces and through out the fight with NTP (just in case the higher court rule against RIM and in favour of NTP).

In addition, RIM has the fight-fight s_{10} state as a GMR and SEQ stable state in this phase. This is because RIM will not activate its UI from s_{10} to s_{13} fearing that NTP will sanction this UI by bring the game back to state s_{10}. Three things we should notice here: 1) this sanction by NTP is a UI for it, hence the SEQ stability of this state to RIM; 2) the strength of s_{10}'s GMR and SEQ for RIM is set at the very weak *None* level, making RIM's loss to be minimal if in fact it ignored this stability and activated a UI to s_{13}; and 3) this state is not SMR stable to RIM, because RIM can bring the game to state s_{13} restating the facts on the ground that it has moved on to a new technology that does not depend on what NTP claims to be its own.

NTP, on the other hand, appears to be at a disadvantage in this phase, because the lower federal court ruled against its claims and demands. But, this appears to be the case only on the surface. In reality, NTP plays the time game too. Continue the fight with RIM (s_{10}) and/or escalate the market pressure on RIM by partnering with RIM's competitors (s_{14}) seem to be the best strategies for NTP going forward. Table 9.28 shows this clearly. State s_{14} is NASH stable for NTP because NTP has no UIs out of it. The strength of s_{14}'s NASH stability for NTP is set at the *Extreme* level because NTP cannot reach this state on its own, it must ally with RIM's competitors to do so. This coalition is not likely to materialize during this phase, as we will see later, making this state in effect unreachable to NTP. In addition, state s_{10} is NASH stable for NTP, with no UIs out from it too. The strength of this Nash stability is set at the very weak *None* level because NTP can reach this state by using any of its UIs from other states with at least one of these UIs providing *None* level preference gain.

NTP, also, has state s_{13} as a GMR and SEQ stable state in this phase. This is because NTP will not activate its UI from s_{13} to s_{10} fearing that RIM will sanction this UI by bringing the game back to state s_{13} reminding all with the facts on the ground (that RIM now is using a new technology that is not what NTP claims to be its own). Again here too, we should notice: 1) this sanction by RIM is a UI for it, hence the SEQ stability of this state to NTP; 2) the strength of s_{13}'s GMR and SEQ for NTP is set at the very weak *None* level, making NTP's loss to be minimal if in fact it ignored this stability and activated a UI to s_{10}; and 3) this state is not SMR stable to NTP, because NTP can bring the game to state s_{10} reminding RIM, RIM's customers and RIM's shareholders that it can hurt them by keep fighting till the end, generating in the process uncertainty around RIM services, operations and future.

Table 9.29 shows the overall stability of the states in Phase 2B's main game. Two states are shown in the table to form equilibrium states for the conflict at this phase. Both states s_{10} and s_{13} are found to form equilibrium states under the GMR and the SEQ stability solution concepts. And, because these two states are stable for the players not only under GMR but also under SEQ, the hesitation the players will experience, when trying to move out from these two states to more preferred states fearing sanction moves, is justified as these sanctions are also rational UI moves to the players committing them.

But, the table also shows that s_{10} and s_{13} stabilities have strength levels that are set at the very weak *None* level. With the meaning that these equilibriums are weak and if any of the players ignores them, taking a UI out of any, that player will only suffer minimally. Saying so, if one will check Table 9.28 again, he will find that RIM has one UI only out from s_{10} , a UI that will put the main game at s_{13}; and NTP has one UI only out of s_{13} , a UI that will put the main game back at s_{10}.

Table 9.30: RIM v. NTP Conflict - Phase 2B: Stability Analysis for the side game (NTP x RIM's Competitors (Nokia as an example))

	NTP				***NOK***			
	s_k	s_i	s_j	s_l	s_i	s_j	s_l	s_k
UIs, CIs & C-GIs			s_i (UI)	s_i (UI) s_j (UI)			s_j (UI)	s_j (UI)
NASH	Ex	N			Ex	N		
GMR	N	N			N	N		
SMR	N	N			N	N		
SEQ	N	N			N	N		

Table 9.31: RIM v. NTP Conflict - Phase 2B: Equilibrium States for the side game (NTP x RIM's Competitors (Nokia as an example))

	s_i	s_j	s_k	s_l
NASH EQ.	N			
GMR EQ.	N			
SMR EQ.	N			
SEQ EQ.	N			

This will ensure a loop that signifies one important fact of Phase 2B, if and only if Phase 2B materialize by the lower federal court ruling against NTP and in favour of RIM, and that is: the fight will continue and NTP will not declare defeat. This means bad news for RIM, RIM's customers and shareholders. If they are hoping for NTP to give up the fight after the court agree with RIM's position, they are sure heading for a big disappointment. What the analysis of Phase 2 is suggesting here is not only understandable but also logical. By NTP keeping the fight on, it cannot lose: 1) it might get the higher courts to agree on its claims and demands, reversing the lower court's decision; and 2) the pressure on RIM will increase over time, internally and externally. The distraction this continuous fight will cause to RIM's operations, and the uncertainty it will cause to RIM's products and future, will surely increase with time.

We have said above that a coalition between NTP and RIM's competitors is not likely to form at Phase 2B of the RIM v. NTP conflict. The reason behind this will be more obvious, once we present the stability and equilibrium analysis of the side game of Phase 2B. Table 9.30 presents the stability analysis of the the four states of Phase 2B's side game, for both NTP and Nokia (the biggest of RIM's competitors used here as an example), under NASH, GMR, SMR and SEQ solution concepts.

Table 9.31 shows the equilibrium states for the side game under the four solution concepts.

From able 9.30, we can see that Nokia (and other RIM's competitors) will not likely settle, or sign any agreement, with NTP (states s_k or s_l). Nokia will likely use the UIs it has from these states to the fight-NTP-legally state s_j. All other RIM's competitors will do the same thing, based on their preferences, and logically speaking since all want to see if NTP can prevail at the end in its case against RIM. In addition, RIM will not likely to suffer that much in the marketplace because the lower court has just handed RIM a rule in its favour and against NTP. RIM's customers will also not likely at this stage to look for alternatives because of fear of disruption or cut in the services. These things can happen only when RIM start losing legally to NTP and/or the cloud of uncertainty start thickening around its service and future in the US market. NTP cannot sanction the UIs that Nokia, and others, will take out of states s_k or s_l to state s_j, a NASH stable for them but a very weak one (at strength level set at the *None* level). If NTP does take the move it has from s_j to state s_i, which is a UI move by NTP, then Nokia and other RIM's competitors will be at the best state for them, preference-wise. Therefore, s_i is a NASH stable for Nokia, and others, with strength set at the *Extreme* level, whilst s_k and s_l are not stable for them.

On the other hand, NTP in this side game has s_j and s_l as unstable state, because it has UIs out from them to state s_i or to states s_i and s_j (respectively), and Nokia/Others cannot sanction such UIs. But, as the table shows, s_k is a NASH stable state to NTP because NTP does not have a UI out of it, and with strength set at the *Extreme* level because it cannot reach this state on its own.

The only state in Phase 2B's side game that is stable for all players, NTP and Nokia (and others), is state s_i, which represents all players wait for the results of the higher federal court ruling on RIM v. NTP case. s_i is NASH for all players, because none of the side game players have a UI, CI or C-GI out of s_i. And, the strength for s_i's NASH stability for Nokia (and others) is set at the *Extreme* level, because they cannot reach this state on their own, and their stay there is at the mercy of NTP's decision to challenge them legally for what it claims its rights. Also, the strength of s_i's NASH stability for NTP is calculated applying Algorithm 9.3 to be set at the very-weak *None* level. This is because NTP can reach state s_i by using UIs it has from state s_j (where NTP and Nokia–and others– are fighting in the courts) and from state s_l (where NTP and Nokia–and others– reach a partial agreement that does not include a licensing component). The least preference gain from these UIs is the one accomplished by moving from the s_l to s_i with NTP gaining almost nothing in such move. Hence, the very weak *None* strength of s_a's NASH stability for NTP.

This makes s_i a NASH equilibrium for Phase 2B's side game with strength set at *None* level. In other words, the side game in Phase 2B of the conflict has the weakest strength level. NASH, by definition is a weak stability; and NASH at strength *None* brings down the stability of this state. This is understandable for many reasons. For all players in the side game, NTP and RIM's competitors, it is a wait and see game. NTP can challenge those players legally but will gain nothing in the process. If NTP waits until the higher courts agree with its case with RIM, it will be in a better position. It will be able to sign those companies for more licensing fees. For RIM's competitors, it is also wait and see. They do not like the idea to license NTP's technology and pay for it, but this may change if NTP managed to gain the support of the higher federal court in its fight with RIM.

Based on the analysis of Phase 2A's side game, the coalition is not likely to form because none will benefit from it. A good indication that no member of this NTP-Nokia-Others coalition will benefit from such coalition is the fact that there is no C-GIs that could materialize for this coalition and affect the dynamics within the main game between NTP and RIM. The side game in this phase has no C-GIs that could change the main game from one state to another. Therefore, based on this analysis of the side game, the side game is more likely to stay put at its starting state, s_i, because this state is the only equilibrium to the side game at this phase. And, even though this equilibrium is of weak strength, it is stable for all the players, nevertheless.

4) Stability Analysis for Phase 3A of the RIM v. NTP conflict (after the higher court decides against RIM's position)

At Phase 3A of the conflict, the higher federal court decides against RIM and in favour of NTP's claims and demands. Table 9.32 shows the stability analysis of the the five states of Phase 3A's main game, for both RIM and NTP, under NASH, GMR, SMR and SEQ solution concepts. We also present here Table 9.33 which shows the equilibrium states for the main game under the four solution concepts.

Table 9.32 provides us with insight on the expected dynamics between RIM and NT at this phase of the conflict. At the first glance, the table shows state s_{16}, which

Table 9.32: RIM v. NTP Conflict - Phase 3A: Stability Analysis for the main game (RIM v. NTP)

RIM

	s_{17}	s_{16}	s_{15}	s_{19}	s_{18}
UIs, CIs & C-GIs			s_{16} (CI)	s_{15} (UI) s_{16} (CI)	s_{15} (UI) s_{16} (CI) s_{17} (CI)
NASH	Ex	Ex			
GMR	N	N	L		
SMR	N	N	N		
SEQ	N	N			

NTP

	s_{16}	s_{19}	s_{15}	s_{17}	s_{18}
UIs, CIs & C-GIs		s_{16} (CI)	s_{16} (CI) s_{19} (C-GI)	s_{15} (UI) s_{19} (C-GI)	s_{15} (UI) s_{16} (CI) s_{17} (CI) s_{19} (C-GI)
NASH	Ex				
GMR	N	L	L	N	N
SMR	N	N			
SEQ	N				

* All C-GIs here are C-GI moves by the Coalition of NTP and Nokia (and other RIM's competitors).

Table 9.33: RIM v. NTP Conflict - Phase 3A: Equilibrium States for the main game (RIM v. NTP)

	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}
NASH EQ.		Ex			
GMR EQ.	L	N	N		
SMR EQ.		N			
SEQ EQ.		N			

represents RIM and NTP reaching a full settlement agreement, is NASH stable for RIM and for NTP. None of the players has a UI out of s_{16}. In addition, the strength of s_{16}'s NASH stability, for both players, is as expected set at the *Extreme* level because none of the players can reach this state alone. They both have to cooperate to do so.

On the other hand, state s_{17}, which represents RIM and NTP reaching a partial settlement agreements, is NASH stable for RIM only, and GMR stable for NTP. RIM does not have a UI out of s_{17}, and cannot reach s_{17} on its own, as it must cooperate to do so. This makes s_{17} NASH stable for RIM, with strength set at the *Extreme* level. NTP, and because it has a UI (to s_{15}) and C-GI (to s_{19} as part of the moves that its coalition with RIM's rivals have), state s_{17} is not NASH stable for it at this phase of the conflict. But, NTP could decide not to take the UI or the C-GI it has from s_{17} out of fear that RIM will sanction such moves. RIM has non-UI SMs to state s_{18}, making s_{17} GMR stable for NTP at strength set at the *None* level. And, because NTP has a countermove to s_{19}, a much better state than

s_{17}, then s_{17} is not SMR stable for NTP. Also, because RIM sanctions are not UI moves by RIM, s_{17} is not SEQ stable to NTP.

Therefore, state s_{17}, or reaching a partial agreement with RIM, is a weak stable state for NTP (based on GMR stability –a weak stability by definition– and the strength of this GMR stability is set at the very weak *None* level). NTP can enhance on it by either add more pressure on RIM by partnering with its rivals in the marketplace, or keep fighting RIM especially after the higher court decision siding with NTP. NTP fears that RIM will sanction its unilateral UI move or coalition C-GI move is based on the weak assumption that RIM is ready to act against its own preferences structure just to hurt NTP. An unlikely assumption, given the fact that both parties are businesses with collective institutional rationality.

States s_{16} and s_{17} also form equilibrium states of the conflict at this phase, as shown in Table 9.33. But, as expected, State s_{16} form an *Extreme* NASH equilibrium state, because it is NASH stable for both RIM and NTP at the same level of strength. And, state s_{17} is an equilibrium to the conflict at this state under the GMR stability definition, with strength that matches the lowest strength of s_{17}'s GMR stability that the players have: *None* strength level.

Table 9.32 also shows the fight-fight s_{15} to be stable for both NTP and RIM under the GMR. This means that RIM will fear to take its CI out of s_{15}, to s_{16} entering agreement talks with NTP, because NTP might walk away from the agreement sanctioning the CI move by putting the game back at the fight-fight s_{15} state, or worse partner with RIM's rivals (s_{19}). And, NTP will fear to take its CI out of s_{15}, to s_{16} entering agreement talks with RIM, or its C-GI coalition multi-step move out of s_{15}, to s_{19} partnering with RIM's competitors, because RIM might sanction such moves by putting the game back at the fight-fight s_{15} state, or worse at s_{18} leaving its US operations all together (an irrational and unlikely move - hence the state is GMR stable not SEQ stable for NTP).

The GMR and SMR stability of s_{15} for RIM, especially when we see that the strengths of these stabilities are set at the weak levels(*Little* for GMR, and *None* for SMR), and the fact that s_{15} is not SEQ stable for RIM, suggest: 1) this fear is based on NTP taking an irrational sanction against NTP's own preferences, an unlikely move considering that the players are business entities (with collective institutional rationality) operating in a professional business environment; 2) RIM cannot recover from NTP's sanction, but the maximum harm that NTP can do to RIM is to bring the game back to the fight-fight s_{15} state (hence the *None* strength level for the SMR stability); 3) because this is a weak stability, if RIM decides to ignore it and therefore take its CI out of s_{15}, the expected loss for RIM is close to *None* even if NTP act irrationally and sanction the CI move, therefore it is better for RIM to try to get out of this state than act based on fear of an irrational NTP; and 4) because the weak stability of continuing the fight with NTP, if RIM is looking for a better strong stability position in this conflict, in order to get back to its core business, then RIM needs to reach an agreement with NTP (through a serious move to state s_{16} meeting NTP demands).

In addition, the GMR stability of state s_{15}, for NTP at this phase, with strength set at the *Little* level, and the fact that the table shows s_{15} not SEQ stable for NTP, suggest the following: 1) the fear NTP has, which this GMR stability is based on, is justified by RIM taking an irrational sanction to stop operating in the US or continue the fighting course, against RIM's own preferences, an unlikely move considering that the players are business entities (with collective institutional rationality) operating in a professional business environment; 2) NTP can recover from RIM's sanction by partnering with RIM's rivals in the marketplace, bringing the game to state s_{19} putting more pressure on RIM to settle and meet NTP demands and comply with the court's ruling (s_{19} is more preferred to NTP than s_{15}, hence s_{15} is not SMR stable for NTP in this phase); and 3) the weak stability of continuing the fight with RIM, and if NTP is looking for a better strong stability position in this conflict, in order for NTP to maximize its income from licensing of its technologies, then NTP need to reach an agreement with RIM (through a serious move to state s_{16} with reasonable demands which RIM can accommodate and meet).

As we can see, the GMR stability of s_{15} for both RIM and NTP at this phase could be a source of the contentious moving in and out of settlement talks between RIM and NTP at this phase of the conflict, as demonstrated by the players historically at the time. For the observers at the time, it seemed that neither RIM nor NTP would like to move on beyond this fight-fight s_{15} state, despite the fact that reaching an agreement even if it is an unfair one is better for both (especially for RIM because it has the most to lose with this ongoing uncertainty this is causing to its customers and market).

One could easily explain this phenomena, or predict it a head of the time, by just looking at Tables 9.32 and 9.33. Table 9.33 shows state s_{15} as an equilibrium state for the conflict at this phase, under the GMR solution concept. The table shows this equilibrium as: 1) weak, but not very weak, with strength set at the *Little* level, with the meaning that it is not a lasting equilibrium especially with the existence of a better rational NASH equilibrium state that both players can reach (at state s_{16}); and 2) based on fear of the other player acting irrationally just to hurt the other, a weak assumption knowing that both players are business entities acting in a highly rational (profit maximization and loss minimization) type of environment.

Tables 9.32 also shows that state s_{18}, which represent RIM leaving the US market and stop its operation there, is unstable for RIM because RIM has UI and CIs out of such state/thinking. The table also shows state s_{18} with very weak GMR stability for NTP with strength set at the *None* level. This weak stability is understandable given the fact that NTP will gain nothing by staying in it, and it is better for NTP to escalate the fight with RIM within the US and elsewhere, or even better cooperate with RIM in reaching a long lasting agreement. This stability is also based on a weak assumption of RIM acting irrationally, against its preferences, and decides to stop operating in the US. An unlikely course of events and unjustified assumption given the business environment the players working in, and the collective institutional rationality they have.

The analysis table shows also state s_{19}, which represent NTP partnering with RIM's competitors in the market space (such as Nokia, Good Technology, Visto, and any RIM rival who did not yet join NTP coalition–if formed in Phase 2A–), is unstable for RIM because RIM has a UI and a CI out of it. But, the table show state s_{19} with GMR stability for NTP with strength set at the *Little* level, and SMR stability for NTP with strength set at the *None* level. This stability though is based on a weak assumption of RIM acting irrationally, against its preferences, and decides to stop operating in the US or prefer to go back to the fight-fight situation. It is unlikely that RIM will act that way, given the business environment the players working in, and the collective institutional rationality RIM and NTP have.

Finally, because both s_{18} and s_{19} are unstable for one player or the another, both are not shown in Table 9.33 as equilibrium states, under any solution concept, for the main game at this phase of the conflict. This should indicate two things to the strategists in NTP's camp:

1. RIM is not likely to leave the US market or accept the rules order to stop its operations in the US, understandably, as this outcome is the least preferred

outcome to RIM and RIM has a UI to continue the fight with NTP, in addition to a CI move to the equilibrium state s_{16} reaching an agreement with NTP (no matter how unfair this agreement will be it will still better than leaving its biggest market, the US); and

2. Partnering with RIM's rivals should be considered as means to put more pressure on RIM, and cannot be used as means to end the conflict with RIM replacing RIM in the marketplace, because RIM is not likely to leave the US market.

Similarly, the fact that s_{18} and s_{19} are unstable for one player or the another, and therefore do not form equilibrium states, under any solution concept, should indicate to RIM's strategists that:

1. NTP is not likely to put up the fight even if RIM stops its operations in the US, but still work through third party partners or some other technical workarounds, because NTP has two strategies/moves to deal with this: a) partner with RIM's rivals in the marketplace forming a coalition (s_{19}) that in effect gives more stable alternative to RIM's BlackBerry® services and products (shortly we will see that the analysis show that this coalition formation is likely to happen, or expand if it started in Phase 2A); and b) if the courts do not shut RIM's operations completely in the US market, NTP can still go legally after the Telcos that RM is providing its services and products through them and/or all its big enterprise customers (see for example the classical US case of Jerome Lemelson, or "The Patent King" as he was nicknamed by the media, going after the barcode scanner customers not the smaller company sold the scanners to those customers, discussed in (Varchaver, 2001)).

2. fighting NTP is a temporarily strategy, and a bad one if RIM continue adopting it in the long run (we will elaborate more on this later when we discuss the overall results of the analysis), and RIM is better off moving to the long-lasting equilibrium state s_{16} with serious intention to meet NTP demands and end the conflict.

Before we end the stability analysis of Phase 3A, we would like to discuss the stability of the Phase 3A side game's states, with the purpose of checking whether a coalition between NTP and RIM's competitors is possible at this stage of the conflict. And, to check whether the members of this possible coalition will all

Table 9.34: RIM v. NTP Conflict - Phase3A: Stability Analysis for the side game (NTP x RIM's Competitors (Nokia as an example)

	NTP				***NOK***			
	s_o	s_m	s_n	s_p	s_p	s_o	s_m	s_n
UIs, CIs		s_o (CI)	s_m (UI)	s_m (UI)			s_o (CI)	s_o (CI)
& C-GIs			s_o (CI)	s_n (UI)				
NASH	Ex				Ex	Ex		
GMR	N	N			N	N	L	N
SMR	N	N			N	N	L	N
SEQ	N				N	N		

Table 9.35: RIM v. NTP Conflict - Phase 3A: Equilibrium States for the side game (NTP x RIM's Competitors (Nokia as an example)

	s_m	s_n	s_o	s_p
NASH EQ.			Ex	
GMR EQ.	N		N	
SMR EQ.	N		N	
SEQ EQ.			N	

expected to benefit from some C-GI moves that could affect the main game of the phase putting their common "enemy" at a worse position.

Table 9.34 presents the stability analysis of the the four states of Phase 3A's side game, for both NTP and Nokia (and/or any other rival of RIM who did not join NTP in its coalition at Phase 2A of the conflcit), under NASH, GMR, SMR and SEQ solution concepts. Table 9.35 shows the equilibrium states for the side game under the four solution concepts.

As one can see, Phase 3A's side game stability analysis Table 9.34 and equilibrium analysis Table 9.35 are exact copies of the respective tables for the side game of Phase 2A. Notice that the states used in the side games of both phases, 2A and 3A, are similar but with different names: states s_e,s_f, s_g and s_h in the side game of Phase 2A are replaced with s_m, s_n, s_o and s_p in the side game of Phase 3A, respectively. Keeping the name differences in mind, the discussion of Phase 2A side game's stability and equilibrium analysis tables will apply here for Phase 3A side game tables.

And as in the equilibrium states analysis of Phase 2A, Table 9.35 of Phase 3A's side game shows two equilibrium states. The full-agreement s_o state is a NASH equilibrium, with strength set at the *Extreme* level because none of the players in the side game will be able to reach this state on their own. They all have to cooperate to do so. The second equilibrium state is state s_m, the wait-wait state, which is found to be an equilibrium under both the GMR and the SMR stability solution concepts. But, this equilibrium has a very weak strength set at the *None* level. Not to mention, that s_m is also a weak equilibrium because it is based on fear the individual players have that the opponent/s will act irrationally –against their preferences– just to hurt them (an additional indication of this is the fact that this state is not SEQ stable to any of the players in the side game). An unjustified fear, knowing that within the context of the business environment where all entities have collective institutional rationality, eliminating the possibility of emotional reactions and irrational behaviour.

The results of the side game's analysis at this phase suggest that the coalition between NTP and RIM's competitors is very likely to happen, because all will benefit from it. A good indication of this strong possibility is the formation of a number C-GI moves the coalition can have to affect the main game between NTP and RIM putting it at a state that is preferred to all members of the coalition. These multi-step C-GI moves, the coalition can have, are shown above in Figure 9.14c. Recall that the results of analyzing the side game of Phase 2A indicated a similar likely hood for the coalition to form. So, if the coalition between NTP and RIM's rivals did not form in Phase 2A then it is more likely to form here at this stage; and if the coalition was formed during Phase 2A but some of RIM's rivals did not join at the time, then they are most likely to join at this phase. In fact, the historical events of the conflict show that this what had happened. We will discuss this further when we compare what had happened historically in the conflict to what our analysis of the conflict suggested and predicted.

From Figure 9.14c, we can see that the resultant coalition's C-GIs will put the main game at state s_{19}. A state that is preferred to NTP in its fight with RIM, especially if the main game was stuck at one of the states that is less preferred to NTP: the fight-fight s_{15} state, the RIM-stopes-its-operation-in-the-US s_{18} state, or the partial-agreement s_{17} state. In addition, putting the main game between NTP and RIM at state s_{19} is better for all RIM's competitors, despite the fact that they are not direct players in the main game. But, putting their common rival at a much worse state in the conflict, and as a result in the marketplace, will definitely help them capitalize on RIM's problems and uncertain future in order to take away current customer of RIM and increase their market share. Let us not forget that RIM was the biggest player in the wireless/mobile email devices market (later started to be known as smart phones market) with the biggest market share and the most advanced devices. So, slowing down RIM's growth and bogging it down with legal problems is considered a very positive outcome for RIM's competitors. Especially, if this also comes packaged with legitimizing these companies devices, offering RIM's customers a stable alternative away from the uncertainties produced by RIM's no-sign-of-settling strategy and the possibility that RIM will end up forced to stop operating in the US as a result. This situation is similar to the situation that the conflict has during Phase 2A of it, as we discussed above.

5) Stability Analysis for Phase 3B of the RIM v. NTP conflict (after the higher court decides in favour of RIM's position)

At Phase 3B of the conflict, the higher federal court decides against NTP and in favour of RIM's position. Table 9.36 shows the stability analysis of the the four states of Phase 3B's main game, for both RIM and NTP, under NASH, GMR, SMR and SEQ solution concepts. We have said earlier that, for Phase 3B of the conflict and similar to Phase 3A, we will not study the side game between NTP and RIM's competitors (Nokia and others) because the side game cannot influence/affect the main game and its states, moves, stabilities and so on, which is the main focus of our case study. Hence, a state which represents NTP signing licensing agreements with RIM's competitors is not part of Phase 3B's states, while similar states were part of each phase we studied before this phase. We also present here Table 9.37 which shows the equilibrium states for the main game under the four solution concepts.

Table 9.36: RIM v. NTP Conflict - Phase 3B: Stability Analysis for the main game (RIM v. NTP)

	RIM				***NTP***			
	s_{23}	s_{22}	s_{20}	s_{21}	s_{21}	s_{22}	s_{20}	s_{23}
UIs, CIs & C-GIs			s_{22} (CI)	s_{20} (UI)			s_{22} (CI)	s_{20} (UI)
NASH	Ex	Ex			Ex	Ex		
GMR	N	N	N		N	N	N	
SMR	N	N	N		N	N	N	
SEQ	N	N			N	N		

Table 9.37: RIM v. NTP Conflict - Phase 3B: Equilibrium States for the main game (RIM v. NTP)

	s_{20}	s_{21}	s_{22}	s_{23}
NASH EQ.			Ex	
GMR EQ.	N		N	
SMR EQ.	N		N	
SEQ EQ.			N	

Table 9.36 provides us with insight on the expected dynamics between RIM and NTP at this phase of the conflict. The table shows that NTP will not give up the fight with RIM (s_{23}), even after the higher court decided in favour of RIM. NTP will most likely escalate the case to the supreme court, and additionally use any other legal and political means it has. Having state s_{23} as unstable state for NTP in the game, will not help RIM and its NASH stability for this state (with strength set at the *Extreme* level because RIM cannot reach this state itself). Therefore, RIM should be aware that its problems will not end by the higher court agreeing with its position. The court's decision will weaken NTP's resolve, but will not break it. NTP, most likely, will keep dragging RIM into the distraction and uncertainties this continuos legal fight brings.

Similarly, NTP's hopes of RIM agreeing to sign a full licensing agreement with it (s_{21}), is no longer justified. Yes, NTP has this state NASH stable with strength level set at the *Extreme* level (because NTP cannot reach this state/agreement on its own). But, NTP must remember that state s_{21} is unstable for RIM. RIM will most likely take its UI to the fight-fight s_{20} state, armed with the higher federal court ruling supporting its position.

State s_{22}, which represent both RIM and NTP reach a partial settlement agreement, is shown in Table 9.36 to be NASH stable for RIM and for NTP. None of the players has a UI out of s_{22}. In addition, the strength of this state's NASH stability is, as expected, set at the *Extreme* level because none of the players can reach it alone. They both have to cooperate to do so. This makes s_{22} to be an *Extreme* NASH equilibrium state for the conflict at this phase, as Table 9.37 shows. As a result, s_{22} forms the most expected resolution to the conflict at this phase.

Table 9.36 also shows the fight-fight s_{20} to be stable for both NTP and RIM under the GMR and the SMR solution concepts. This means that RIM will fear to take one of its CIs out of s_{20}, in order to enter agreement talks with NTP, and then NTP walks away from the agreement sanctioning the CI move by putting the game back at the fight-fight s_{20} state. The GMR and SMR stability of s_{20} for RIM, especially when we see that the strengths of these stabilities are set at the very weak *None* level and s_{20} is not SEQ stable for RIM, suggest: 1) this fear is based on NTP taking an irrational sanction against NTP's own preferences, an unlikely move considering that the players are business entities (with collective institutional rationality) operating in a professional business environment; 2) RIM cannot recover from NTP's sanction, but the maximum harm that NTP can do to RIM is to bring the game back to the fight-fight s_{20} state (hence the *None* strength level for the stabilities); and 3) because of the weak stability of continuing the fight with NTP, if RIM is looking for a better strong stability position in this conflict, in orderer for RIM to get back to its core business, then RIM need to reach an agreement with NTP (through a serious move to state s_{22} meeting some of NTP demands).

In addition, the GMR and SMR stabilities of state s_{20}, for NTP at this phase, with strengths set at the *None* levels, and the fact that the table shows s_{20} not SEQ stable for NTP, suggest the following: 1) the fear NTP has, which these stabilities are based on, is justified by RIM taking an irrational sanction against RIM's own preferences, an unlikely move considering that the players are business entities (with collective institutional rationality) operating in a professional business environment; 2) NTP cannot recover from RIM's sanction to bring the game back to the fight-fight s_{20} state (hence the *None* strength level for the SMR stability); and 3) because of the weak stability of continuing the fight with RIM, if NTP is looking for a better strong stability position in this conflict, in order for NTP to start earning income from licensing/capitalization of its technologies, then NTP need to reach an agreement with RIM (through a serious move to state s_{20} with reasonable demands which RIM can accommodate and meet). NTP must remember that the court's ruling is not in its favour.

As we can see here too, the GMR and SMR stabilities of s_{20} for both RIM and NTP at this phase could be a source of the contentious moving in and out of settlement talks between RIM and NTP at this phase of the conflict. It will seem, for an observer, that neither RIM nor NTP would like to move on beyond the fight-fight s_{20} state, despite the fact that reaching an agreement even if it not a fair one is better for both (especially for RIM –here too– because it has the most to lose with this ongoing battle). One could easily explain this phenomena, or predict it a head of the time, by just looking at Tables 9.36 and 9.37. Table

9.37 shows state s_{20} as an equilibrium state for the conflict at this phase, under both GMR and SMR solution concepts. The table shows this equilibriums as: 1) very weak with strength set at the *None* level, under both solution concepts, with the meaning that it is not a lasting equilibrium especially with the existence of a better NASH equilibrium states that both players can reach; and 2) based on fear of the other player acting irrationally just to hurt the opponent, a weak assumption knowing that both players are business entities acting in a highly rational (profit maximization and loss minimization) type of environment.

9.6.8 Results of the RIM v. NTP Conflict Analysis

The stability analysis for the RIM v. NTP conflict, presented in the previous subsection, provides us with a significant insight into the conflict, the players motives, the options they can choose from, and the implications of such choices. In this subsection, we will compare the findings of the stability analysis for the RIM v. NTP conflict with what actually happened in this landmark historical intellectual property conflict. We will try to see why did the conflict go through the lengthy painful roller-coaster-style path it took? and whether there are some generalized conclusions that can be elicited from this case, namely, on how smiler cases will evolve over time and eventually end?

Also, we will shed some light on the coalition analysis aspect of the overall conflict stability analysis, especially the possibility of coalition formation and how the coalition could benefit the coalition members. Finally, we will discuss additional interesting aspects of modelling and analyzing the RIM v. NTP conflict, such as: the multi-stage nature of the conflict, the effect of side games on the main conflicts, the effect of the courts system (as an example of non-involved player/s in the conflict who certainly can control some parts of the conflict's flow and dynamics), and lastly the need for sensitivity/what-if analysis in conflicts such as this one.

1) Comparing our Analysis with What Really Happened:

In other conflicts, we discussed in earlier chapters, we presented for each an analysis of the different evolution paths the conflict will take, as predicted by the stability analysis conducted for the conflict, and then compared such evolution paths with what happened historically, or expected to happen logically, in the conflict. In the case of RIM v. NTP conflict, and because of the size of the case and the fact that we discussed before how such evolution analysis is done for any conflict, we will skip this step. We have enough insight from the stability and equilibrium analysis, we conducted for this conflict, to move directly to the comparison between the predicted path the conflict will take and the path the conflict actually took historically.

It is worth mentioning here that the task of putting together a table to show the major events of the conflict, in order to trace the evolution of the conflict as it happened, was not an easy task. The legal battle between NTP and RIM lasted for about 4.5 years with the majority of the events reported by the sensational media (in all its flavours: biased, unbiased, analytical, informative and dis-informative) and companies' press releases (mostly represented messages that the companies would like to get out through their public relation campaigns/propaganda). Despite the hardship of sorting facts from fiction, and pure reporting from passion wishful thinking, we managed to put together a table that show the real events took place during the conflict period. We compared any of the news reports, or articles, to court documents, or looked for multiple sources to ensure the events in the table, and the description given to each, represent the reality on the ground at the time, as much as possible.

The result is shown in Table 9.38. The table is broken to three parts, each reflect one of the phases that the conflict went through. Recall that we decided at the beginning of the modelling exercise of this conflict to break the conflict into a number of phases or frames, based on the level of the court taking the decision on the case between RIM and NTP, and whether the decision, the court makes, agrees with RIM's position or against it. The table also, for each of the events listed, indicates the player responsible for the event (took the move), the type of the payer's move (UI, CI, or C-GI), and the end state which this event/move keeps, or leaves, the game at.

Figure 9.16 shows visually the conflict's moves, explained in Table 9.38. The figure shows all the players' UI, CI and C-GI moves for all phases of the conflict (moves that are shown in Section 9.6.6 for each phase separately), with an overlay indicating the actual moves the players took actually at the time. These actual moves are shown in the figure as a consecutive series of arrows, with each arrow representing a step explained in detail in Table 9.38. Recall that we studied two versions of Phase 2A of the conflict (one using the should-be RIM's preferences and the other using the actual preferences RIM demonstrated at the time). Because of

Table 9.38: RIM v. NTP Conflict: Analysis vs.Reality

Phase 1				
Step	**Date**	**Actual Conflict Evolution**	**Move by**	**to/at State**
0	Nov. 13, 2001	NTP filed a lawsuit in the US District Court for the Eastern District of Virginia, claiming that over forty system and method claims from its several patents-in-suit had been infringed by various configurations of the BlackBerry® system (Court of Appeals, 2005).	NTP	s_0 Phase 1

Phase 2A				
Step	**Date**	**Actual Conflict Evolution**	**Move by**	**to/at State**
•	Nov. 21, 2002	After a 13 day jury trial commenced on Nov. 4, 2002, the jury found direct, induced, and contributory infringement by RIM on all 14 asserted claims of the patents that were submitted to the jury trial. The jury also found that the infringement was willful. It rejected every defence proposed by RIM and awarded to NTP damages of approximately $23M, representing a royalty rate of 5.7% of RIM's US BlackBerry® sales up to this point in time.Following the jury verdict, RIM filed with the District Court a petition for judgments as a matter of law or, in the alternative, for a new trial. (District Court, 2003b; Court of Appeals, 2005; Teska, 2006).	Lower Court	s_5 Phase 2A
	Feb. 28, 2003	The U.S. District Court for the Eastern District of Virginia orders RIM and NTP to begin mediation. As part of the ruling, RIM is ordered to account for sales and services through Feb. 28, 2003 for the purposes of determining potential damages (Peacock, 2005).		
	May 23, 2003	The U.S. District Court for the Eastern District of Virginia denied RIM's motion f for judgments as a matter of law or, in the alternative, for a new trial, which RIM filed with the District Court following the jury verdict on Nov. 21, 2002 (District Court, 2003b). The court also criticized RIM's "questionable litigation tactics throughout", specifically that "RIM attempted to confuse and mislead the jury by conducting a demonstration of the TekNow! system which RIM asserted as prior art, by using updated software that did not exist at the time the system was used" (District Court, 2003b; McKenna et al., 2006) .		
	Aug. 5, 2003	Justice James Spencer from the Virginia District Court entered final judgment, in favour of NTP, awarding it monetary damages totalling $53.7M. In his final order, he also issued a permanent injunction order against RIM, enjoining it from further manufacture, use, importation, and/or sale of all accused BlackBerry® systems, software, and handhelds. The injunction was stayed pending RIMs appeal (District Court, 2003a).		
0	Aug. 29, 2003	RIM appealed the District Courts decision (District Court, 2005).	RIM	
1	Jun. 14, 2004	Nokia Corporation entered into a patent licensing agreement with NTP that includes the five patents currently under litigation between RIM and NTP. Nokia took the step to go forward with US sales of a version of its 6820 phone that can connect to RIM's popular BlackBerry® e-mail server, as Nokia officials said. "We've gone ahead and licensed the NTP patents, and this gives us the opportunity to get these things out on the market," said Keith Nowak, a Nokia spokesman (Nobel, 2004). Months later, the other RIM-rivals, but smaller rivals, signed an agreement with NTP: Good Technology (Mar. 11, 2005) and Visto (Dec. 14, 2005) (Green, 2005)	Coalition of NTP-Nokia-Others *(C-GI)*	s_9

Table 9.38: RIM v. NTP Conflict: Analysis vs.Reality (Continued)

Phase 2A (Continued..)				
Step	**Date**	**Actual Conflict Evolution**	**Move by**	**to/at State**
	Mar. 11, 2005	Good Technology entered into a patent licensing agreement with NTP that includes the five patents currently under litigation between RIM and NTP (Green, 2005). Good Technology is a smaller rival to RIM, but nevertheless poses a serious threat to RIM because the company's markets its products and services to the corporate-enterprise market, RIM's main market.	Coalition of NTP-Nokia-Others *(C-GI)*	
2	Mar. 16, 2005	RIM announced that it agreed to pay $450M to settle the dispute, sending its stock soaring more than 17 per cent (Teska, 2006). The announced settlement–one of the largest settlements in history–to fully rest all claims to date against RIM, as well as provide for a perpetual, fully paid license for NTPs patented technologies going forward (RIM, 2005; Peacock, 2005). * most likely this was an attempt to reach s_6, not a serious one though, given the fact next event happened.	RIM & NTP *(CI*)*	s_6^*
3	Jun. 9, 2005	RIM and NTP fail to finalize a settlement, and RIM says it will ask a judge to enforce the terms of the March deal (Peacock, 2005). RIM claims that it is acting in good faith on the terms of the settlement while NTP is refusing to finalize the documents needed to complete the settlement. NTP, however, accuses RIM of stalling in the face of the U.S. Patent and Trademark Offices re-examination of the patents (Peacock, 2005). ** if RIM is right in its claim, then this is a sanction move by NTP to hurt RIM and put more pressure on RIM to settle for more [and the move follows the Should-be RIM's Preferences model]; but *** if RIM actually the ones delaying finalizing the deal, as NTP claims, a move which is more consistent with the facts on the ground where RIM seems to the observers more playing the delay game in hopes that it will get the USPTO to re-examine and invalidate NTP's patents before it run out of legal options, then this is a UI move by RIM [and the move follows the Demo nstrated-By RIM's Preferences model].	NTP *(SM**)* — or — RIM *(UI***)*	$s_5^{**,***}$
4	Jun. 16, 2005	RIM announces it has developed and tested workarounds that would eliminate its reliance on any of NTPs patented technology. It claims that the new workaround will work with all BlackBerry® models and on all networks. Unfortunately for RIM, the next day a widespread BlackBerry® outages occurred across the United States. Networks are down for periods anywhere between two minutes to two hours, depending on the network and location. An undisclosed hardware failure is blamed. Speculation abounds that RIM was secretly testing its new workaround, which apparently failed. And on Jun. 22, 2005, i.e. in less than a week, another BlackBerry® disruption occurred. Again, the disruption is blamed on a hardware failure, with RIM claiming it to be completely unrelated to the first. Again, speculation arises that the BlackBerry® workarounds were being tested publicly and once again failed. (Peacock, 2005). **** this is mostly perceived as just an attempt by RIM to reach, or appear to reach, state s_8 in an effort to enhance its position in the game, given the fact that these repeated BlackBerry® service outages happened within few days after the announcement, making the actual UI move by RIM is to state s_5 instead.	RIM *(UI)*	s_8^{****}

Table 9.38: RIM v. NTP Conflict: Analysis vs.Reality (Continued)

Phase 3A				
Step	**Date**	**Actual Conflict Evolution**	**Move by**	**to/at State**
•	Aug. 2, 2005	The U.S. Court of Appeals for the Federal Circuit issued its final decision.The court upholds most of the claims that RIM infringed on NTP's patents.The court also says part of the lower court had erred in some claims' infringement, specifically related to the lower courts misconstruction of the originating processor term, and orders the lower court to take another look at the case, and consider what effect, if any,thismight have had on the jurys assessment of damages and on the scope of the injunction. And, on the matter of RIM's claim that the United States Patent Act states that patent rights are restricted to the U.S.. the appeals court held that since the control and beneficial use of the infringing system was in the U.S., RIM could be subject to penalties and injunctions for the infringement (Court of Appeals, 2005).	Higher Court	s_{15}
0	Oct. 7, 2005	Responding to a motion by RIM, the U.S. Appeals Court refuses to reconsider its earlier ruling on the case. RIM announces it will appeal the case to the U.S. Supreme Court (Singer, 2005).	RIM	
	Oct. 26, 2005	U.S. Supreme Court Chief Justice John Roberts rejects RIM's motion to stay case while the high court decides whether to hear an appea l(Levine, 2005).		
	Nov. 30, 2005	The case returns to Richmond, where a judge refuses to force NTP to accept the proposed $450M settlement with RIM back on March 2005. The judge also decided not to delay the case, pending a review of NTP's patents by the U.S. Patent & Trademark Office, as per RM's request (District Court, 2005; Teska, 2006).		
1	Dec. 14, 2005	Visto entered into a patent licensing agreement with NTP that includes the five patents currently under litigation between RIM and NTP (Krazit, 2005). NTP said it took a stake in Visto to strengthen its legal position in asking for the injunction against RIM. Rather than simply being a patent holding company that doesn't sell any products or services, it now has stakes in two companies that are in the mobile e-mail business. "If there is an argument about whether an injunction is appropriate, we aren't in the same set of circumstances," says Don Stout, co-founder of NTP (Green, 2005). With uncertainty and concern rising about the outcome of the RIM-NTP suit, RIM's competitors have been fielding a growing number of calls from BlackBerry® customers, according to executives at Good Technology, Visto, and Microsoft (Green, 2005).	Coalition of NTP-Nokia-Others *(C-GI)*	s_{19}
2	Dec. 19, 2005	RIM submitted a petition to the Supreme Court for a writ of certiorari, requesting the court to issue a writ of certiorari to review the judgment of the United States Court of Appeals for the Federal Circuit issued on Aug. 2, 2005 (Fenster et al., 2005). On Jan. 23, 2006, the Supreme Court denied RIM's petition for certiorari to review the Appeals Court's decision issued on Aug. 2, 2005 (Supreme Court, 2006; Teska, 2006).	RIM *(UI)*	s_{15}
	Feb. 24, 2006	NTP asked for an immediate injunction that would have shut down BlackBerry® after 30 days, as well $126M in damages in addition to royalty payments. As a result, the district court judge James Spencer declined to issue an injunction that would have immediately shut down BlackBerry® service to US customers. In his ruling, the judge said it was clear that RIM had infringed on NTP patents, and he would decide on damages and whether to	NTP	

Table 9.38: RIM v. NTP Conflict: Analysis vs.Reality (Continued)

Phase 3A (Continued..)				
Step	**Date**	**Actual Conflict Evolution**	**Move by**	**to/at State**
		eventually issue an injunction as soon as reasonably possible. Observers reported that the judge appeared upset that the two parties had not resolved the dispute on their own. The judge is reported saying to RIM and NTP representatives: "I must say I am surprised, absolutely surprised, that you have left this incredibly important and significant decision to the court. I've always thought that this, in the end, was really a business decision. And yet you have left the decision in the legal arena, and that's what you're going to get, a legal decision" (Broache and Krazit, 2006; Krazit, 2006).		
	Mar. 3, 2006	RIM and NTP announced that they have signed a definitive licensing and settlement agreement. All terms of the agreement have been finalized and the litigation against RIM has been dismissed today by a court order. The agreement eliminates the need for any further court proceedings or decisions relating to damages or injunctive relief. As part of the agreement, RIM has paid NTP $612.5M in full for final settlement of all claims against RIM, as well as for a perpetual, fully-paid up license going forward (Krazit and Broache, 2006; **?**).	RIM & NTP *(CI)*	s_{16}

this, we have Figure 9.16 uses the should-be RIM's preferences in Phase 2A of it and Figure 9.17 uses the demonstrated preferences model.

So, how does the flow of the conflict as it happened at the time compare to the flow predicted by the stability analysis we provided for the conflict in the previous subsection? Let us take the conflict phase by phase:

Phase 1 (before the lower court decision on the RIM v. NTP case):

***Main Game - Analysis*:** As per the equilibrium states analysis for the main game of this phase, given in Table 9.19, there is only one equilibrium state: the fight-fight s_0 state. And, this equilibrium is under the NASH solution concept, with the meaning that neither RIM nor NTP has a UI or a CI out of this state. The stability analysis Table 9.18 shows that the strength of s_0's NASH stability to both parties is set at the *Little* level, showing the parties have states which are *Little* less-preferred to s_0. A margin of preference strength that, if ignored by any of the parties acting irrationally against his preference vector, might make that player choose a state that show some signs of a little bit of escalation (such as going to state s_3 for RIM acknowledging some truth to NTP claims by announcing a workaround, or state s_2 for NTP accepting a partial deal

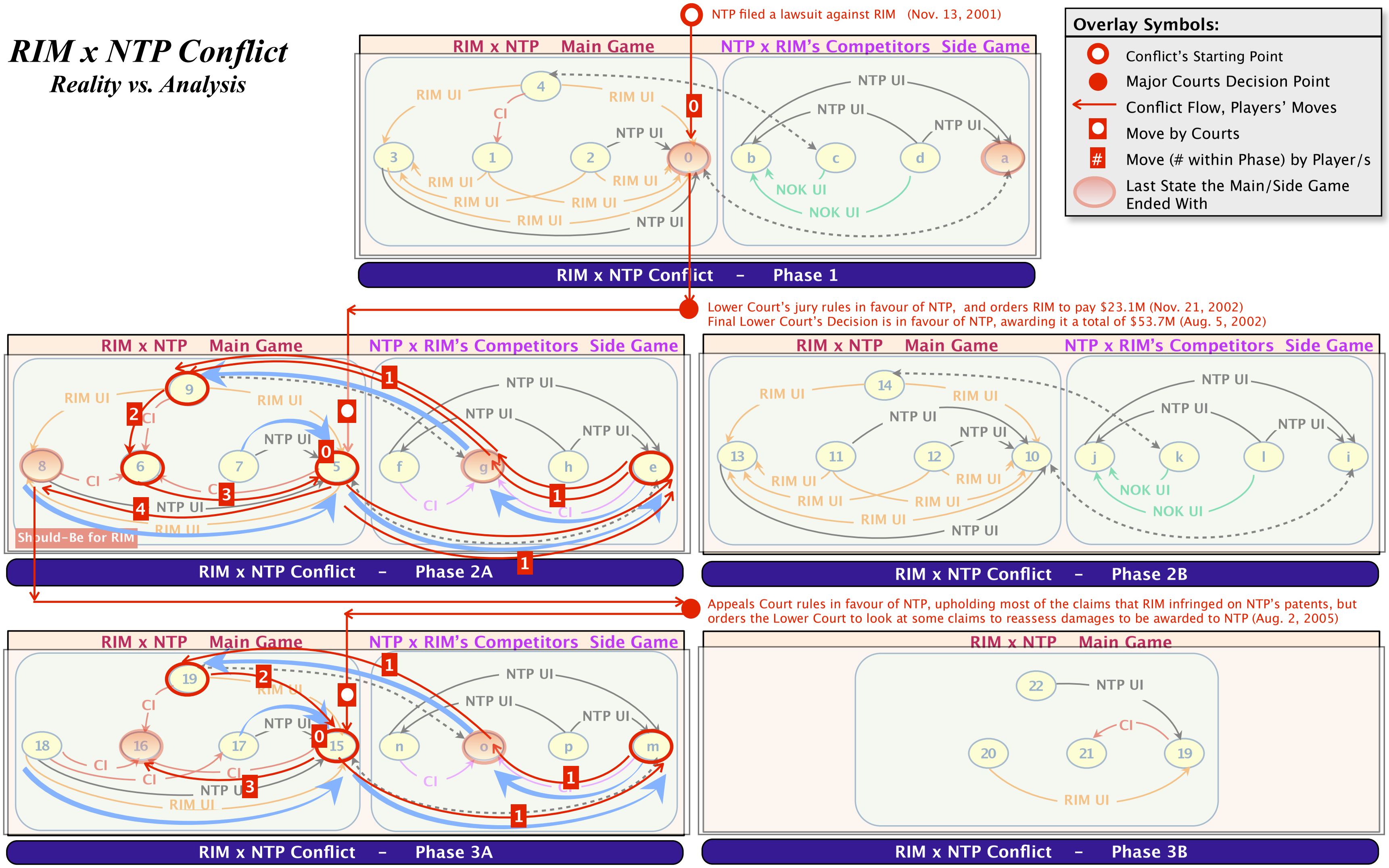


Figure 9.16: RIM v. NTP Conflict: Comparing our analysis with what really happened historically in all phases of the conflict [Note: Phase 2A model in this figure is based on the Should-Be RIM's Preferences not what RIM actually demonstrated at the time]

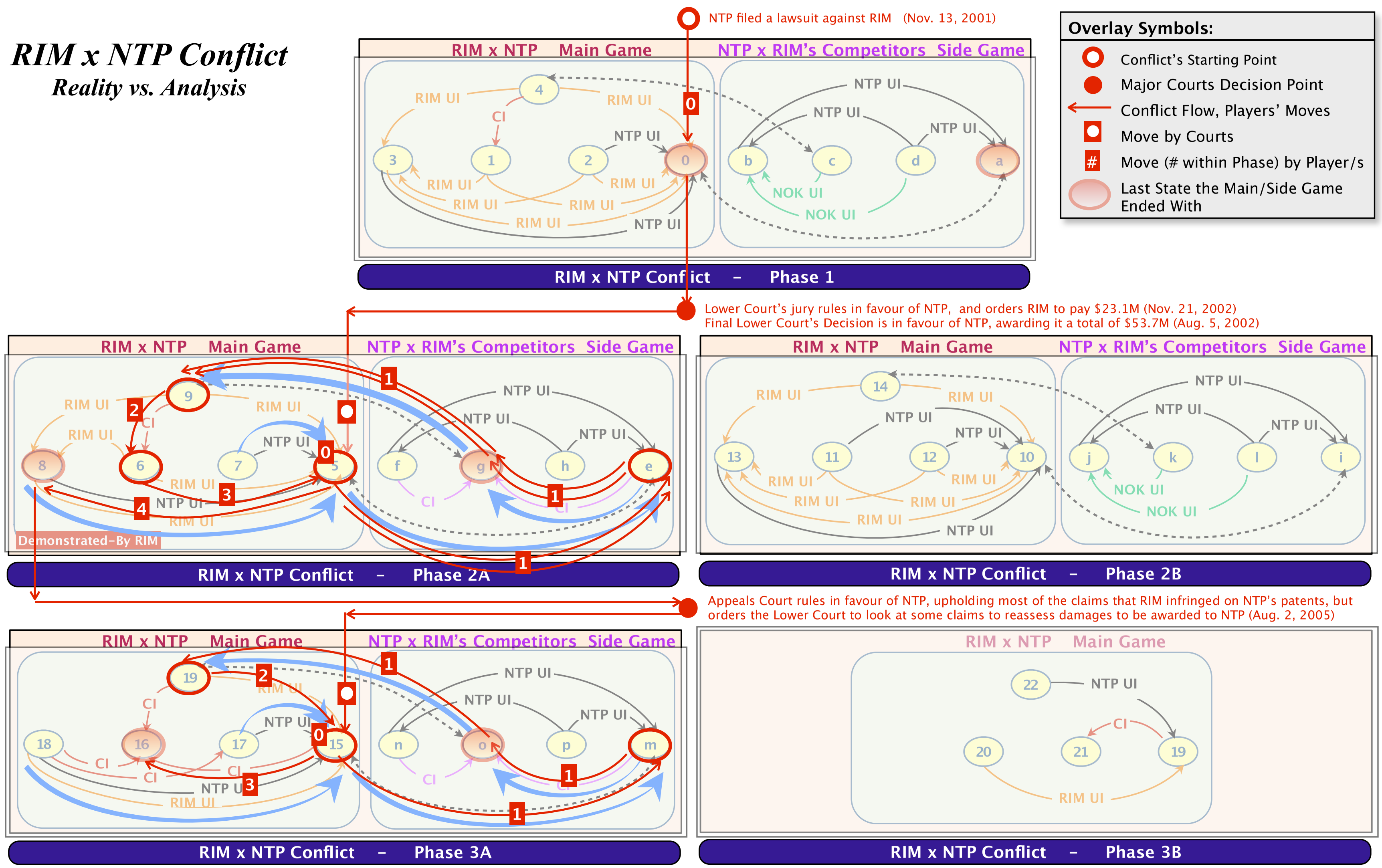


Figure 9.17: RIM v. NTP Conflict: Comparing our analysis with what really happened historically in all phases of the conflict [Note: Phase 2A model in this figure is based on the Preferences that RIM actually demonstrated at the time]

with RIM). But, if both players act rationally, as we expect them to do, they will stay put at the fight-fight s_0 state.

Main Game - Reality: After NTP filed its lawsuit against RIM on Nov. 13, 2001, both players were engaged in the legal battle to convey their respective positions to the court. No signs of talks between the parties to negotiate a settlement to end the dispute. In other words, **the conflict at this phase was deadlocked at the fight-fight state s_0, as predicted by the analysis**.

Side Game - Analysis: Table 9.21 shows that the wait-wait s_a state is the only equilibrium for the game between NTP and RIM's rivals at this phase of the conflict. This equilibrium is also under the NASH solution concept. In other words, neither NTP nor any of RIM's rivals has a UI or a CI out of this state. But, the very weak strength of this equilibrium (set at the *None* level) suggest that at least one of the players (NTP in this game) has a slightly less-preferred state to s_a that NTP have a UI from it to s_a. This state is the fight-fight s_b state. This means that the equilibrium at state s_a is pretty much under the control of NTP. If NTP decides to compromise on rationality a bit, though not expected, it could act against its preferences vector and go to the slightly less-preferred state of fighting RIM's rivals in courts (state s_b). In addition, the fact that both sides will prefer the wait-wait s_a is the reason that a coalition among them is not expected to form at this phase.

Side Game - Reality: During this phase, neither NTP nor any of RIM's competitors in the marketplace show signs of negotiating or reaching a licensing deal. **Both sides preferred the wait-wait s_a, as suggested by the analysis**. In addition, notably and as a result, **the coalition/alliance did not form at this stage of the game, as also predicted by the analysis**.

Phase 2A (after the lower court decision came against RIM's position): During this phase, observing analysts (as well as NTP analysts) have/should-have noticed that RIM was acting based on the expected Should-Be RIM's preferences model. When this starts to be obvious, the analysts should try to build what-if preferences model/s for RIM, to understand the behaviours it is demonstrating at the time, and the motives behind such preferences. In the stability analysis for Phase 2A, discussed in the previous sub-sections, we showed a preferences-model for RIM that explains its behaviour at the time, and the motives behind such behaviour (the strategic importance RIM placed at the time for its strategic goals), and we referred to this model as the Demonstrated-By or Observed RIM's Preferences model. We also discussed the implication of this what-if model on the players moves and states' stabilities for each player. Here, we will show the *Main-Game - Analysis* part for both models, and then compare the results with what really happened at the time.

***Main Game - Analysis*:**

***Based on the Should-Be RIM's Preferences*:**

Table 9.23 shows that the main game between NTP and RIM at this phase has one and only one rational equilibrium, and it is at state s_6 where both parties reach a full licensing agreement that will end the conflict. The equilibrium at s_6 is under NASH solution concept, and its strength is set at the *Extreme* level, showing that none of the players has a UI/CI out of it and none of the players can reach it on their own. The table also shows GMR based equilibriums at: the fight-fight s_5 state, the RIM-finds-a-workaround-but-continue-the-fight s_8 state, and the NTP-ally-with-RIM-rivals-and-sign-licensing-agreements-with-them s_9 state. But, these GMR based equilibriums are based on the players no improving their position in the game fearing that their opponents will act irrationally against their own preferences vectors hurting themselves and others in the process. An unreasonable assumption given the business environment that conflict is in, and the collective institutional rationality that each player embodies. Nevertheless, these GMR equilibriums suggest temporarily stable states at different points of time within this phase. The stability strength for each of these states, to the individual players, will suggest how the players will move and what state they prefer to stay put at. Table 9.22 shows that NTP has higher GMR and SMR stability at state s_9, while RIM has higher GMR and SMR stability at state s_8. But, because each has a UI out from the more GMR and SMR stable state of his opponent to the fight-fight state s_5 or its own more stable GMR and SMR state, we will expect to see movements back and forth among those states. We can expect movements by NTP to s_9 (if coalition with RIM's rivals is possible), followed by RIM taking the game back to s_5 or s_8; and movements by RIM to s_8 (if it can build a workaround to NTP's technology), followed by NTP taking the game back to s_5 or s_9. The fight-fight s_5 state has higher shared GMR equilibrium strength than s_8 and s_9, making the game more likely to stay at it than the other two state. This is if and only if the players do not move cooperatively to the only rational equilibrium of this phase, to state s_6, reaching a full lasting agreement and ending the conflict. Based on this analysis, it expected that rational players will try to move to s_6 with serious well-intentioned plans to end the conflict.

***Based on the Demonstrated-By RIM's Preferences*:**

The equilibrium states analysis, for this phase but based on the preferences RIM demonstrated at the time, given in Table 9.25, shows that the very weak GMR based equilibriums at states s_8 and s_9 stayed the same, with the same strength level, as the equilibriums shown at these states based on the should-be model. This suggests that the players will show signs of moving back and forth between the three states, s_8, s_9 and s_5, as suggested by the analysis based on the should-be RIM's preferences model. No change here. But, for states s_5 and s_6, we see a big difference. First, the fight-fight s_6 state does not form an equilibrium under NASH solution concept for this phase, showing that at least one of the players (RIM in this case) has a UI/CI out of this state to other more preferred states. Second, s_6 is now an equilibrium under GMR and SEQ, with the meaning that each of the two players will stay put at this state once reached and not take any UI/CI out of it fearing the other player will sanction (with a rational SMI sanction) his move. But the lack of SMR equilibrium suggests that at least one of the players can get out of the bad effect of this sanction. Third, s_5 is now, not only an equilibrium state under the GMR solution concept, but also under the SEQ solution concept. This makes the fear, the players have to get out of s_5, to be well grounded in rationality, i.e. a justified fear since the sanctions the players will face are SMI sanctions (SM + UI moves). These three differences are quite insightful and telling on how the conflict is expected to evolve over the period of this phase.

Here, state s_6 is no longer a state that no player has a UI out of it. So, if state s_6 is reached, or the players showing signs of attempting to reach the state by negotiating a full agreement to end the conflict, soon one of the players (RIM in this case) will break away from this state to a better state, in a completely justifiable rational move (at least in his mind). Therefore, any stay at state s_6 will be short lived. The same, to an extent, could be said about the equilibrium at the fight-fight s_5 state. State s_5 has stability that is backed by rational behaviour of all player involved, but the fact that it is a very weak equilibrium, that players can mitigate sanctions imposed on them, and that the players have moves to other GMR equilibriums (at s_8 and s_9), we should see more in and out movements to s_5 than to s_6. Recall that moves to s_6 demands cooperation, while moves in and out of s_5 can be done unilaterally. Based on this analysis, it is expected that the conflict will not reach its end by the players arriving at state s_6. But rather, we will see a cycle of moving in and out from the three states, s_5, s_8 and s_9, showing the players continuously fighting each other and putting pressure on each other. Any move to negotiate a final full settlement, will be short lived.

***Main Game - Reality*:** Immediately after the lower federal District Court showed signs that it was agreeing with NTP's position, that RIM infringed on the patents, especially after the jury found the infringement to be wilful, RIM entered into a tireless fight with the courts employing all the tactics it has legally to change the ruling, reverse or at least delay any injunction order against RIM, the court was expected to issue, shutting its sales and operations in the US. Table 9.38 lists a number of these actions that RIM took against NTP, including filing for a reexamination of NTP patents by the US Patents and Trademarks Office (USPTO) –not listed in the table but will be discussed later in this subsection–. Once the lower court issued its final decision, as anticipated, against RIM, RIM appealed the decision to the higher federal court. In effect, enforcing the game stay at state s_5 in this phase, which the court order put the conflict at. Interestingly, shortly after the court final decision, NTP entered into a licensing agreement with Nokia, RIM's biggest rival in the marketplace, and then with the second in-line competitor to RIM's mobile email technology, Good Technology. These agreements marked the establishment of a coalition, or a soft alliance, forming between NTP and RIM's rivals in the marketplace, moving the game to state s_9, where NTP puts pressure on RIM to settle, and RIM's rivals capitalize on RIM's problems and signs of uncertainty that start showing for the continuation of its service in the marketplace. In less than a week, after NTP signed a licensing agreement with Good Technology, RIM announced that it is negotiating a final licensing settlement with NTP. In other words, an attempt to reach state s_6. But as expected by the analysis (based on the preferences demonstrated by RIM at the time), the settlement talks broke down, and RIM took a UI out of this state to the fight-fight s_5 state. Shortly after that, in fact after exactly a week passed as Table 9.38 shows, RIM tried to enhance its position in the game by moving the game to state s_8 announcing a workaround NTP's technology that will ensure continuos service to its customers no matter what the courts decide. But as we stated in the table, the outage in RIM services that happened in the following day and days, made the move by RIM to be perceived by the market and the industry more as an attempt to move the game from s_5 to s_8, instead of an actual solid move to s_8. This left the game ending the phase at state s_5, or at state s_8 if one believes RIM and discredit the reposts that the outage was caused by RIM testing its workarounds. In other words, **the players behaved as predicted by the analysis. The conflict moved between the variations of continue-the-fight states, s_5, s_8 and s_9. And, when an attempt to reach s_6 was made, the attempt was short lived by RIM taking a UI out of s_6 to the fight-fight s_5 state**.

***Side Game - Analysis*:** Table 9.27 shows state s_g, in which NTP will reach full licensing agreements with RIM's rivals (all or some of them), is the only rational equilibrium for the side game at this phase of the conflict. This equilibrium is under the NASH solution concept, with strength set at the *Extreme* level, with the meaning that NTP and any of RIM's rivals decide to join in has a UI or a CI out of this state, and the joining partners cannot reach this state on their own. The table shows anther equilibrium at the wait-wait s_e state. But this equilibrium is under the GMR and SMR solution concepts, with the meaning that the players will decide to stay s_e for fear that the other player will retaliate by sanctioning any improvement move the player will try to make. But, because state s_e is not an equilibrium under the SEQ solution concept, then the sanction moves are not UI/CI for the players committing them, i.e. the equilibrium at s_e is based on irrational sanctioning players who are willing to hurt themselves (work against their preferences) just to hurt others. This is an unjustified assumption especially in the business environment where players embody institutional collective rationality. In addition, the GMR and SMR based equilibrium at s_e has very weak strength level of *None*. This makes the equilibrium at s_g the only rational equilibrium for completely rational players in the side game. In other words, the analysis suggest that NTP and RIM's rivals will join forces, establish a coalition or a soft alliance, and sign full licensing agreements, i.e. reach s_e in the side game, and allow NTP (through the coalition) to reach s_9 in the main game. This will benefit all parties. NTP will put more pressure at RIM to join in, settle the case, and sign a full licensing agreement. And, RIM's rivals (Nokia –the biggest–, Good Technologies –the second in wireless email technology–, and others) will benefit by offering an alternative, to RIM's services and devices, to frightened current customers of RIM (afraid to lose their services and investment in such devices) and potential customers (who are looking for a stable services with certainty).

***Side Game - Reality*:** On Jun. 14, 2004, while the RIM v. NTP case was in its mid way through the federal appeal court, NTP and Nokia announced a full licensing agreement that will allow Nokia to sell wireless email devices and service, comparable to RIM's devices and services, within the US. In a smiler move, in what seemed a move to consolidate the establishment of a soft alliance between NTP and RIM's rivals, NTP and Good Technology announced on Mar. 11, 2005 a full licensing agreement that will allow Good Technology to sell its wireless email services and devices in the US market. In other words, **a coalition between NTP and the two major rivals of RIM had formed as predicted by the analysis. This allowed RIM's rivals to sign full licensing agreements with NTP, i.e. reach state s_g in the side game, and allowed NTP to reach state s_9 in the main game of the conflict at this phase, also as predicted by the analysis**.

Phase 3A (after the higher court decision came against RIM's position):

***Main Game - Analysis*:** Table 9.33 shows three equilibriums states for the main game at this stage of the conflict. The first is an equilibrium state under the NASH solution concept, and it is at state s_{16}, in which NTP and RIM reach a full licensing agreement and settle the conflict. This is the only rational equilibrium (assumes that all players are rational player who will not disimprove their positions just to hurt others) that this game has. Its NASH stability to the players, and its *Extreme* strength level, suggest that the players do not have UI/CI/C-GIs out of it, and none of the players can reach this state on their own. They have all to cooperate to reach it. The other two equilibrium states, shown in Table 9.33 , are the the equilibriums at the fight-fight s_{15} state and at the reaching-a-partial-settlement-agreement s_{17} state. Both equilibriums are under the GMR solution concept, and not equilibriums under the SEQ solution concept This means that both are based on unjustified fear of opponents cting irrationally against their own preferences vector just to hurt other players. An assumption that is not likely to be true, especially within the context of a business environment where all/most players embody collective institutional rationality. In addition, both states are not equilibriums under the SMR solution concept, i.e. players can improve their positions out of the sanctions that the GMR stability is based on. The strengths of these two GMR-based equilibriums also show that the fight-fight s_{15} state is more stable to the players, than the partial-agreement state s_{16} state. The stability analysis for this phase, Table 9.32, shows that state s_{19} is GMR and SMR stable for NTP, but unstable for RIM. This means that NTP will keep its coalition with RIM's rivals (if this coalition was established at phase 2A of the conflict), setup one (if not established at phase 2A), and add new members to it. This move by NTP is expected, only if the side game could show that a coalition formation is possible, and is beneficial to all parties involved. But, NTP should know that RIM is expected to sanction any such move by bringing the main game back to the fight-fight s_{15} state. In summary, the analysis predicts that the end state of the conflict at this phase is state s_{16}, in which RIM and NTP reach a full agreement to settle the conflict. It is also expected that any stability at the fight-fight s_{15} state, or state s_{17} (where the parties seemed negotiating a partial agreement) will be a short lived stability that will not persist. In addition, NTP will try to move the game, if the analysis of the side game shows that a coalition is possible, to the NTP-Coalition-Licensing-Deals s_{19} state. NTP will try to consolidate, maintain, and add new members to its soft alliance against RIM, in an effort to amount additional pressure on RIM to settle the conflict by signing a full licensing agreement. But, it is expected that any such move by NTP will be faced by a swift sanction by RIM bringing the conflict to the fight-fight state, most likely with escalation of the fight in a form of a new motion or an announcement to show that it is determined to fight the case (in other words, declare a back-to-s_{15} move).

***Main Game - Reality*:** Table 9.38 shows the conflict moved to state s_{15} as the higher Appeals Court decided against RIM's position, with RIM shortly after that enforcing the games state to be s_{15} by filing motions asking the Appeals Court to reconsider its ruling. NTP, then, cut a deal with the third ranking rival to RIM in the wireless email space, Visto (an innovative company that owns many patents of its own in this space). The licensing deal with Visto, gave NTP also a share of the company. This bold move by NTP, enforced the view that NTP built a strong alliance against RIM in the marketplace, with NTP as not just a licensor to the technology but also as a company that has services and products of its own. This marked a move for the game from state s_{15} to state s_{19}, putting RIM under real pressure, with its current and potential customers having real more certain alternatives to its services and products. In just five days, RIM filed a petition to the Supreme Court, despite the fact that the supreme court turns down most patent cases, putting the game back at state s_{15}. But, by now RIM started to face the heat in the marketplace, from the stock market, from its customers and partners, and the courts alike. RIM and NTP announced, on Mar. 3, 2006, a final deal that gave RIM full license to NTP's technologies and compensated NTP for all legal costs, ending the main game of phase 3A and the conflict at state s_{16}. In summary, **the conflict ended, as predicted by the analysis, at the equilibrium state s_{16}. Before that, also as predicted, the coalition of NTP and RIM's rivals made a move to state s_{19}, with RIM responding shortly after that with a move bringing the conflict back to state s_{15}.**

***Side Game - Analysis*:** The side game of Phase 3A was shown in the analysis provided in the previous subsection to follow the same pattern of stabilities and equilibriums as the side game of Phase 2A. The equilibrium at state s_o, where the coalition of NTP and RIM's rivals (whomever did not join the coalition by now) reaching full licensing agreement, is shown to be the only rational equilibrium for completely rational players in the side game at this phase. In other words, the analysis suggest that NTP and RIM's rivals will join forces, establishing a coalition or a soft alliance, if not formed yet, or enforcing the alliance, if already existed by adding new members, and sign full licensing agreements, i.e. reach s_o in the side game, and allow NTP (through the coalition) to reach s_{19} in the main game.

***Side Game - Reality*:** On Dec. 14, 2005, NTP and Visto (the third, in market share size and importance, competitor to RIM) announced Visto getting a full licensing agreement and NTP getting a share in Visto. This move marked NTP strengthening its coalition with RIM's rivals, adding new ones, and taking part in the services and devices delivery, put the side game at state s_o. The type of arrangements announced by the deal show the benefits that both sides are getting. NTP bringing additional pressure on RIM to settle, and on the courts which agreed to not delay the case any longer (but hesitated to shut down a telecommunication service that may affect the work of governmental agencies who uses it). RIM's rivals, now having a field day with all reporting increase in sales especially to people who were with RIM or were thinking of RIM but opted for a more certain service offered by RIM's competitors. In other words, **a coalition between NTP and the third major RIM rival had formed as predicted by the analysis. This allowed RIM's rivals to sign full licensing agreements with NTP, i.e. reach state s_o in the side game, and allowed NTP to reach state s_{19} in the main game of the conflict at this phase, also as predicted by the analysis**.

2) Notes on the RIM v. NTP Conflict's Analysis vs. Reality:

So, the Constrained Rationality analysis we conducted for the RIM v. NTP conflict predicted the flow and the outcome of each of the conflict's phases. But, surely there are many issues and questions we intentionally left out addressing or answering, in order to not clutter the analysis with matters we thought has no implication on the conflict and how it will evolve over time. Some of these issues are at the heart of the conflict (such as: Who actually refused to finalize the $450M settlement announced early on March 2005? Is it RIM or NTP?), or add-on questions that we believe are not part of the conflict but nevertheless interesting to know because of its proximity to the conflict (such as the question: Was RIM justified on its belief that playing the delay game will be better off for it?). In this subsection, we will address these questions and issues which we thought are better addressed after the analysis is completed and the hysterical events of the conflict are discussed.

Who Refused to Finalize the $450M Settlement Deal on Jun 9, 2005?

The announcement RIM made on March 16, 2005 that it had agreed to pay NTP $450M to settle and end the conflict (RIM, 2005), was received very positively by the market and RIM's customers alike. In fact, the marker rewarded RIM with a jump in RIM's stock price. But, shortly after that, more specifically on Jun. 9, 2005, a breakdown on finalizing the agreement between RIM and NTP was announced (shown in Table 9.38 as Step No. 3 in Phase 2A), with the parties blaming each other for this breakdown. RIM claims that it is acting in good faith on the terms of the settlement while NTP is refusing to finalize the documents needed to complete the settlement. NTP, however, accuses RIM of stalling in the face of the U.S. Patent and Trademark Offices re-examination of the patents (Peacock, 2005).

So, who is really to be blamed? It is hard to tell, unless one have full knowledge of the negotiation happening at the time. But, we can take both sides argument and match them to the moves in the Constrained Rationality models we have for Phase 2A of the conflict (the stage of the conflict after the lower federal court decided against RIM in its ruling and before the Appeal Court decided on it). In Table 9.38, we listed the claims of NTP and RIM, and how such claims could be interpreted in terms of players moves from state s_6 to state s_5:

1. If RIM was right in its claim that there was a complete settlement agreement, i.e. both players were at state s_6, but NTP refused to finalize it and moved the game to the fight-fight s_5 state, then this could be read as one of two:
   (a) if both parties were actually and truly at state s_6, then NTP's move to s_5 can only be considered as a sanction move to hurt RIM, put more pressure on it, and hurt itself in the process. Let us not forget that the USPTO office started by then re-examining NTP's patents (based on a

request by RIM made on Jan. 14, 2003), and in fact on Apr. 6, 2005 rejected one of the patents (Peacock, 2005). So, NTP surly made a risky move by taking such sanction move that could hurt its chance of getting a full licensing agreement with RIM. And, with its patents getting struck down one at time by the USPTO then it might lose the court's support that it managed to harness so far. Could it be really that NTP is an irrational player, who decided to hurt itself and its chances in order to hurt RIM and punish it?

(b) alternatively, both parties were actually not at state s_6, and the negotiation was not serious and never materialized as a final agreement on the details of the settlement, as RIM announced in its press release at the time (RIM, 2005). In other words, the conflict was still at state s_9, the state where the NTP-Nokia-GoodTechnology coalition left the game at after the Mar. 11, 2005 announcement. There is some arguments to support this. The biggest and ultimately the most important of them came after RIM asked the Federal Circuit to stay the appeal and to have Judge Spencer, of the District Court, decide this latest dispute of finalizing a settlement claimed to be a done-deal. In a briefing on that issue, NTP asserted that RIM, once again, was simply attempting to avoid the consequences of infringement. The Federal Circuit refused RIM's request. Ultimately, on Nov. 30, 2005, Judge Spencer ruled there had been no settlement (District Court, 2005). Recall that RIM had a jump in its stock price, after its announcement of the settlement, and that RIM's announcement came just 5 days after NTP announced its alliance with Good Technology, the number two Competitor of RIM in the wireless email mart space. The fact the court rejected the claim that there was a done-deal between RIM and NTP, and the fact that RIM had benefited from such announcement, made some analysts question the "bold" move RIM made by this announcement (e.g. Teska (2006)).

2. If NTP was right in its claim that RIM stalled the talks regarding a complete final settle agreement in the face of the USPTO re-examining NTP's patents (responding to RIM's request submitted to the office on Jan. 14, 2003), and the office rejecting one of NTP's patents on Apr. 6, 2005 (Peacock, 2005), then the main game at Phase 2A of the conflict moved from s_6 to state s_5 by RIM deciding that the fight-fight state is better for it. After all the USPTO

could reject all NTP's patents, and therefore, there will be no basis for the NTP infringement case against RIM. This move by RIM could be read as one of two:

(a) if both parties were actually and truly at state s_6, then RIM's move to s_5 can only be considered as:

   i. an SM by RIM to hurt NTP and hurt itself, if we use the Should-be RIM's Preferences model. A move that surely RIM will be hurt in, even more than NTP, given the ruling that the lower court took against RIM's position (District Court, 2003a), and the direction the Appeal Courts was taking by its initial and non-final decision of Court of Appeals (2004) (agreeing mostly with the lower court's ruling of District Court (2003a)). This move does not even explain the harm that RIM could suffer from. So, it is an illogical explanation at best.

   ii. a UI by RIM to enhance its position from state s_6 to state s_5, based on the Demonstrated-By RIM's preferences model. A move, and an explanation, that has many arguments to support. Despite the fact that most observers and analysts questioned the rationality behind the move (e.g. Green (2005); McKenna et al. (2006)), because they all were thinking based on the Should-Be RIM's preferences model and that settling-the-conflict-at-any-price and ending the uncertainty is better for RIM and its future, RIM was completely rational (in its collective mind) to stall-the-deal and continue-the-fight based on the Demonstrated-By RIM performance model. In RIM's mind, it attempt to invalidate NTP's patents at the USPTO office, started to pay off. Three weeks after RIM rationally moved to cut a final settlement with NTP (on Mar. 16, 2005), the USPTO on Apr. 6, 2005 (Peacock, 2005) rejected the first of NTP's patents-in-suit. So, playing the delay game with NTP and the courts by bringing the conflict at this phase to the fight-fight s_5 state is quite rational and logical. This is only if one believes that RIM had at the time the Demonstrated-By RIM's preferences model. RIM was in effect believing that the cost of the settlement does not justify ending the conflict and ending the uncertainty it causes to RIM's future in the marketplace (Recall that RIM has the strategic importance lower for the finical cost in its Should-Be-RIM's preferences model that the Demonstrated-By-RIM one). A belief that is not shared by many in the industry, the market, and the courts at the time (we will discuss shortly the justification of this delay strategy of RIM).

(b) alternatively, both parties were actually not at state s_6 and the negotiation was not serious and never materialized as a final agreement on the details of the settlement, as RIM announced in its press release at the time (RIM, 2005). In other words, the conflict was still at state s_9, the state where the NTP-Nokia-GoodTechnology coalition left the game at, after the Mar. 11, 2005 announcement. This certainly could be the case, as stated above, given the District Court's decision on Nov. 30, 2005 that there had been no settlement (District Court, 2005).

So, who killed the most-hoped-to-be-be-finalized $450M settlement of March, 2005? Firstly, whatever scenario, out of all possible scenarios discussed above, that you think happened, and whether one believes RIM's story or NTP's one, this scenario is shown to be mapped to the Constrained Rationality model and stability analysis of the conflict. The analyst will not only understand how the players behaved and predict their next moves, but also understand the rationality behind any of the taken and/or to-be-taken moves. Secondly, looking at the scenarios above, and assuming that all players in this conflict are rational actors (a safe and logical assumption in a business environment where the players are not emotional individuals but institutions with collective rationality), then there are only two scenarios that can explain what happened at the time: the UI-by-RIM scenario (scenario No. 2.a.ii) based on the Demonstrated-By RIM Preferences model; and the alternative scenario that the game was never at s_6 state in the first place, i.e. there was no serious negotiation to end the conflict going on between the parties (at least one of the players was not serious, if not both, in negotiating the final settlement).

Was RIM justified in its belief that the delay game will be better for it?

This question could be put differently: Why RIM showed no rush to end the conflict? There are many reasons that could be given. Some will argue that RIM was outspoken about these reasons in words and deeds. Here is a list of the most cited reasons:

- **the US Patent and Trademark Office (USPTO) will invalidate all NTP patterns:** It is no secret that RIM tried hard to invalidate NTP's patents-in-suit, as a strategy to invalidate the bases for the infringement lawsuit that NTP filed against RIM. Two months after a federal jury determined that RIM knowingly infringed upon eight patents held by NTP (Peacock, 2005), more specifically on Jan. 14, 2003 RIM filed a request with the USPTO to re-examine five of the eight patents-in-suit owned by NTP. In Apr. 6, 2005, just three weeks after RIM announced that it will pay a $450M to settle the case with NTP, USPTO after re-examination rejected one of the five patents-in-suit. This as indicated above is/could-be one "the reason", or one of the reasons, that led to the breakdown of the settlement talks with NTP. In Jun. 22, 2005, two more patents are rejected by USPTO after been re-examined.

  Apparently, RIM find the strategy working and decided to file for many motions at different court levels to stay, i.e. delay, pending the USPTOs reexamination of several of the patents-in-suit. There is one serious problem with this strategy though. The re-examination process for any entities' patents is not a one-step process finished and completed at the USPTO. For most analysts and observers, who cared about RIM and its services and products, the strategy was troubling because it shows that RIM was indeed ill-advised, despite the fact that RIM at the time was already a big company with many resources and no longer the little small start-up company based on Waterloo-Ontario.

  For many experts familiar with the patents laws and procedures, striking down the patents is only the first step in what would be an extremely long legal process (Peacock, 2005; Teska, 2006). Surely, NTP will more than likely appeal the decision by the patent office, which would be sent to a patent board and then ultimately to the US. Circuit Court of Appeals, which is the same court that decided in favour of NTP in both its rulings (Court of Appeals, 2004, 2005). In his ruling, denying RIMs Motion for Stay of Proceedings Pending Reexamination of NTP (District Court, 2005), Judge Spencer explained the flaw in the line of thinking, that RIM will soon get off the hock if the USPTO struck down all the patents-in-suit under examination:

  > "Over the course of this litigation, at both the trial and appellate levels, RIM has moved on four separate occasions to stay the proceedings based at least in part on the ongoing reexamination of the patents-in-suit by the United States Patent and Trademark Office

> (the "PTO"). RIM's first three attempts were unsuccessful. ...
>
> The Court is not persuaded that the PTO will issue final actions in RIMs favor "within the next few months," as RIM asserts. RIM's Mem. Supp. 6. The PTO has not even finished issuing all of its first actions. Furthermore, NTP will have the opportunity and has already indicated its intention to respond to the first actions. The PTO, after considering NTP's responses, will then issue another office action which may or may not be "final." Even in the unlikely event that all final office actions were taken in the next few months, NTP, if not satisfied, could appeal the PTO's findings.
>
> Reality and past experience dictate that several years might very well pass from the time when a final office action is issued by the PTO to when the claims are finally and officially "confirmed" after appeals. See, e.g., In re Am. Acad. of Sci. Tech Ctr., 367 F.3d 1359 (Fed. Cir. 2004) (affirming the claim construction of the Board of Patent Appeals and Interferences in a case where, after numerous rehearing requests and appeals, the PTO's findings were not confirmed until ten years after a reexamination was first requested)." [source: District Court (2005), underlines are from the source]

Therefore, and because of the fact that a strategy of delaying the case, and any settlement of it until a re-examination of NTP's patents-in-suit is completed, is flawed, we did not see the need to include this strategy as an option/alternative for RIM in the conflict. We included such "delay" tactics within the fight-fight states that each of the conflict phases in the model has.

- **the US Government will not allow RIM services to be shut down in the US:** It is fair to assume that RIM would not settle the case, as it finally did on March 2006, if there had been no injunction against it that threatened to shut down RIM's services and operation in the US (court order of District Court (2003a), a stay on it until the Appeal Courts finalize its decision is removed after Court of Appeals (2005) and a new request by RIM to stay it further is denied by District Court (2005)). Therefore, immediately after the lower court decision of District Court (2003a), RIM hired political lobbyists and public relations specialists in Washington in order to: 1) to speed up the invalidation of NTP's patents-in-suit re-examination at the USPTO, a process which typically very slow, and picked up speed at the USPTO unfairly according to NTP (2006) and most observers (Teska, 2006; McKenna et al., 2006) (but still the legal process itself after the USPTO decision is still along one, as we said above); and 2) stop the court from implementing the injunction against RIM, at least until the USPTO finalized its decision.

RIM's heavy weight lobbyists managed to get the USPTO speed up its typically-slow internal re-examination process, not to mention undisclosed meetings with RIM (NTP, 2006), but could not change the pace of the legal process followed by the courts, or the nature of the decisions the courts took (McKenna et al., 2006). Saying so, RIM managed to get the US's Department of Justice (DOJ), on behalf of many US governmental departments and agencies, to file briefs on Nov. 2005 and later on Feb. 2006 asking the federal court to allow RIM to stay and continue because of the large number of BlackBerry® users in the US Federal Government (Noguchi, 2005; Wong, 2006).

But all these efforts succeeded to delay the courts and the process for a while, but it will not be likely that the court will wait for a long time. In addition, the move by NTP and its coalition (Nokia, Visto, Good Technology) to offer safe and equivalent alternatives, and NTP suggesting that only US governmental staff to be exempted from the shutdown of RIM's BlackBerry® services, the court has no other choice but to seriously threaten RIM that the court is running out of patience. This exactly what the court did on Feb. 24, 2006 (Broache and Krazit, 2006; Krazit, 2006).

Based on all of the above, it is safe to say that lobbing-the-govenment served, and in most cases can only serve, as a "delay tactic". And similar to the re-examination of NTP's patents-in-suit, we did not see the need to include this as an option/alternative for RIM in the conflict. We included such "delay" tactics within the fight-fight states that each of the conflict phases in the model has.

- **the customers will stay with RIM no matter what:** This was probably was RIM's worst assumption. And, even if RIM did not assume that its customers will stay with it and not move on, it sure acted as if it did hold such belief. The delay tactics employed by RIM put many of RIM's US valuable customers, the enterprise business, at risk of losing what become to be "the communication device" used by them and the investment in setting up their systems around it.

Some called RIM's delay strategy a "'Legal Gamble' (Green, 2005), because of its effect on RIM's current and potential customers base. Green (2005) explains why it is a gamble: "With uncertainty and concern rising about the outcome of the RIM-NTP suit, RIM's competitors have been fielding a growing number of calls from BlackBerry® customers, according to executives at Good Technology, Visto, and Microsoft. Licensing agreements with NTP give individuals and companies an added reason to check out these alternatives, analysts say. "It's a huge opportunity to try to steal some market share while RIM is in limbo," says Gene Signorini, an analyst at researcher Yankee Group". The consulting company Gartner advised its clients, on Dec. 2005, to stop or delay crucial BlackBerry® rollouts, pending the outcome of the case. "I would say that every 2 out of 10 companies are starting to investigate something else, but are still hoping it goes away before they have to do anything," said Ken Dulaney, a Gartner analyst (Green, 2005; Perez, 2005).

In other words, while RIM was busy delaying a settlement deal, its competitors in the marketplace (through the coalition of NTP, Nokia, Good Technology, Visto) moved aggressively not only to acquire new customers who are looking for a stable-sure-to-stay wireless email services and devices, but also start taking some of RIM's customers at the time. One do not need more proof than RIM itself admitting in the same day it announced the final settlement with NTP, Mar. 3, 2006, but in a separate press release (RIM, 2006) that the number of new subscribers to its BlackBerry® service will fall short of expectations by as much as 120,000 subscribers in the fourth quarter of that year. The uncertainty caused by the dispute with NTP was evidently causing many potential customers to either put off their BlackBerry® purchases or choose a competing wireless product (RIM, 2006).

Not to mention that while RIM was busy with its battle with NTP, other new comers to the market ware preparing new devices and services. Apple announced on Jan. 2007, few months after RIM settled the case, its game-changer smartphone, the iPhone®. Apple at the time said it was working on the phone for about two years (Honan, 2007). So, Apple was building the "next big thing" during the second half of the 4.5 years legal battle of RIM vs. NTP. This is the time in which RIM was not only tied down and distracted by the case, but also working hard employing delay tactics to extend the time for the legal battle further.

- **the financial cost of a final settlement is too high:** This seems to be the most logical reason that RIM can rationalize to itself and others. But, RIM forgot that the cost of letting the case continue for so long has also finical implications. Some were really harsh on RIM, but nevertheless stated the consequences of its delay tactics and actions rightly so, such as McKenna et al. (2006): "Through ... and bad advice, RIM's potential bill had shot up from a few million dollars before the trial to roughly $20-million when its case headed south at trial, to now hundreds of millions of dollars."

2) So, Is It Worth Fighting a Patent Troll?

From a purely technical point of view, we used the case of RIM v. NTP mainly to show how a complicated intellectual property business (and legal) multi-stage cooperative conflict, with coalitions involved, can be modelled and analyzed using our Constrained Rationality framework; and to illustrate the use of the conflict analysis concepts and methods, introduced earlier in this chapter, in a real-life conflict. But, surely one can stop and ask the question: Can we generalize this RIM v. NTP case to the point that we can state with confidence "that's why one should not fight a patent troll"? The answer, with confidence, is: No. Generalization, especially when it comes to real-life cases, is very dangerous. Context is very important in any real-life conflict, and it will be hard to find two cases with the same context.

Nevertheless, there are some general lessons to be taken from this case. After all, this case is one of the the few cases that a defendant challenged a patent troll plaintiff (or a plaintiff that holds the key characteristics of a patent troll –specifically that he does not have a product, only patents on paper, and the fact the he has nothing to lose and everything to gain from these legal cases–) in courts, at all levels, for as long as RIM challenged NTP. Not only RIM challenged NTP in courts for a long time, but also vigorously employed and tried to employ all delay tactics possible to extend the case as much as possible (hoping to invalidate the patent-troll's patents and claims). Most companies settle the cases with patent trolls, and do not let it go for long, as RIM did, threatening its business, its customers, and market. We will list here some of the general technical lessons, or patterns, elicited from the stability analysis we provided for the RIM v. NTP conflict, and most likely will show in the majority of patent troll cases:

Phase 1 - Before the Lower Court Decides on the Case:

- **No settlement in phase 1. Phase 1 will serve to validate the patent troll case and claims.** In analyzing the RIM v. NTP case, one could not but notice that the only equilibrium Phase 1 has was the fight-fight s_0 state (Table 9.19). This is understandable, given that the defendant in this case, and all other similar cases, would like to validate the patent troll case and claims in courts of law. Especially, when the defendant is not sure, or does not believe, that the patent-troll's patents-in-suit are valid or apply to his case. So, it is safe to wait for the court to decide on the validity. But, if the defendant knows that the patents-in-suit and claims are valid, waiting for the court decision will only serve to raise the cost of settling the case.
- **No coalition between the patent troll and the defendant's rivals in the marketplace is likely to form in phase 1.** The waiting game for the main parties involved in the infringement case between the plaintiff (the patent-troll) and the defendant, is likely to extend to the plaintiff and any other smaller companies (the defendant's rivals in the marketplace) who also infringe on the plaintiff's patents. As the stability analysis of the side game of Phase 1 of the RIM v. NTP conflict shows, in Table 9.21, that the wait-wait s_a state is the only equilibrium for this game at this stage of the conflict. This leaves the main defendant (usually the biggest player in the market –that's why the patent troll went after him first) safe to assume that a coalition is not likely to happen between the patent troll and its own rivals. Not before the courts validate, or in its way to validate, the patent troll claims, and not before the defendant's products and services start suffering from uncertainty in the marketplace.

Phase 2A - After the Court Decision Comes in Favour of the Plaintiff:

- **The only rational equilibrium at this stage is a full and final settlement.** This is expected to last, i.e. beyond Phase 2A to Phase 3A, as long as the courts at the higher levels support the lower court's decision in favour of the plaintiff (the patent troll). The equilibrium analysis tables for the main game of both Phases 2A and 3A, Tables 9.23 and 9.33, show that all other equilibriums are not rational (assumes some of the players will act irrationally against their preferences vectors just to hurt their opponents). In addition, the defendant should expect the total cost for a final settlement to increase as time passes and as the courts amount more support in favour of the plaintiff.

- **A coalition between the patent troll and the defendant's rivals in the marketplace is more likely to form in phase 2A, last and add new members in Phase 3A as long as the courts keep agreeing with the plaintiff's position.** As the equilibrium analysis of the side game for Phase 2A and 3A of the RIM v. NTP conflict shows, in Tables 9.27 and 9.35, that the rivals-sign-full-licensing-agreements state is the only equilibrium for this game at these stages of the conflict. In other words, the defendant should expect that his rivals will most likely form a coalition, or a soft alliance, with the plaintiff allowing both sides to benefit. The defendant will be increasingly under pressure as time passes to either settle or keep losing financially and market-share-wise.

Phase 2B - After the Lower Decision Comes in Favour of the Defendant:

- **The patent troll is expected to continue the fight.** This is expected to last, i.e. beyond Phase 2B to Phase 3B, as long as the courts at the higher levels support the lower court's decision in favour of the defendants. The defendant should not expect that the patent troll to stop the fight. After all, the patent troll has nothing to lose, and everything to gain, if he continues the fight. On the other hand, the defendant will continue to live with the noise and distraction caused by the plaintiff's case. The equilibrium analysis tables for the main game of both Phases 2B and 3B, Tables 9.29 and 9.37, show that all rational equilibrium states (under solution concepts that assumes all players to be rational players at all times) at these stages of the conflict have the patent troll chooses "fighting" as his strategy (alternative of choice).

- **No coalition between the patent troll and the defendant's rivals in the marketplace is likely to form in phase 2B, and as long as the courts decisions continue to be in favour of the defendant's position.** As the equilibrium analysis of the side game for Phase 2B of the RIM v. NTP conflict shows, in Table 9.31, that the wait-wait state is the only equilibrium for this game at this stage of the conflict.

In summary, as for the classical cases of a patent troll (who has no products but patents on paper) going after a prosperous company with products and services, the analysis shows that the patent troll has nothing to lose by fighting for the alleged rights, while the prosperous company has everything to lose from having this fight extend beyond the point the courts found that the patent troll claims are valid. Any delay in settling the case beyond this point, will only play into the patent troll game, and will result with more loss at many fronts: customers-base, market potential, stock market reaction, competitors eagerly seizing the opportunity, and so on. It was unfortunate to see RIM fall into this trap, upping the ante, the pressure on itself and the odds of it getting out of this legal battle without paying. RIM should have paid-and-settled as early as possible, once it was clear that the system decided the case in favour of NTP.

3) Final Notes on Coalition Analysis and Sensitivity Analysis:

Two final notes before we end this case study. One on coalition analysis and another on sensitivity analysis. First, it is worth reminding the reader that there are many coalition formations and types. Some are legally binding while others are voluntarily based. Some are strictly defined as coherent bodies, while others softly defined as groups of members with common objectives even though they are free agents. The coalition between NTP and RIM's rivals could be defined as individually-legally-binding between NTP and each of the members, but soft in its structure that all the members are free agents who share one objective: put RIM at a worse position and capitalize on this. Saying so, one could not but notice that close to the end of 2005, NTP started to be not only signing licensing agreements with these companies but taking shares in them, namely in Visto and Good Technology, moving the coalition from the soft structure it had to be more coherent and solid in nature.

Analysts, when analyzing coalitions in cooperative conflicts, must understand the nature of these coalitions, how the members benefit from it, and how decisions/moves are taken (all should benefit, the big original founders must benefit but the rest is not as important, and so on). As we discussed at the beginning of this chapter when we defined the concept of a coalition, the analyst must be able to answer these questions clearly, before starting the analysis. We said, for example, that defining C-GI moves and separating them from the mere C-GM moves depends on such answers.

The second note relates to conducting sensitivity analysis for complex conflicts such as the RIM v. NTP conflict we studied. We said earlier that, generally speaking, an analyst will build a base model, that fits most of the known facts, and safely assumed, about the conflict and its players. In the RIM v. NTP case study, the base model for the conflict is the one which includes the conflict's Phase 2A model which is based on the Should-Be RIM's Preferences. The analyst will then conducts sensitivity analysis by building variant models, called what-if or scenario models in this research work. Analyze these models separately and compare results. Sensitivity analysis, by building what-if models, can be done: up front when the conflict is in the horizon or at its initial stage, motivated by testing some uncertain aspects of the conflict or the players (such as alternatives that they may or may not have or can afford, or goals they may have or not have); while the conflict progressing over time, motivated by new discoveries about the conflict, new players surfaced, new players' options uncovered, or some players exhibiting unusual behaviour, deviating from the base model assumptions, such as the case of RIM's behaviour in Phase 2A of the RIM v. NTP conflict; or after the conflict had ended, to study lessons learned, opportunities missed, and so on.

Good practices in conflict analysis, based on our experience, shows that the care that the analyst puts in structuring the conflict, will eventually pay off. This is true in any problem solving situation, but it is more so and especially important when conducting sensitivity analysis as part of a bigger conflict analysis assignment. While theoretically speaking, the analyst can build complete new models each time he wants to test a small minor change to the base model he originally built, it is a matter of common sense to assume that re-building new models is not only a time consuming practice, but also wastes the analyst valuable time and resources. Better planning and model structuring, saves the analyst time and energy, and allows for the much needed comparative analysis based on a base model. For example, in the RIM v. NTP conflict, which we have analyzed above, and because we structured the conflict model based on the legal process phases that such conflicts follow, we were able to build a whole new what-if model for the conflict with only Phase 2A's main game model slightly changed to reflect what RIM demonstrated at the time as preferences. The rest of the conflict phases stayed as is, including Phase 2A's side game stayed unchanged. This allowed for comparative analysis of both scenarios.

9.7 Summary

This chapter discussed the analysis of cooperative multi-agent games with coalitions. It started by defining the type of moves the players of the cooperative games, with coalitions, are allowed to make. Then, the chapter provided definitions for the four different stability and equilibrium solution concepts which were defined for the non-cooperative games in Chapter 6 and for the cooperative-without-coalitions games in Chapter 8. These concepts guide the stability analysis of each of the games' states, for each of the games' players. The chapter, then, defined the strength of the stability under such solution concepts, and proposed a set of algorithms to help identify the strength level of each of these stabilities.

The chapter finished with a case study in which the concepts and methods proposed in this chapter were applied. In the case study, we analyzed thoroughly a strategic business conflict over intellectual property rights. The case examined whether it is worth fighting a patent troll, and used the showdown between Research and Motion and NTP to help us answer this question. We started by giving a brief background on the conflict and the players. We, then, modelled the players goals, constraints and alternatives; analyzed their GCMs; identified the conflict's states; elicited the players' cardinal and ordinal preferences over these states; and then identified the players unilateral moves among these states. We also examined the possible coalitions that could be formed in the game. Next, the stabilities of the conflict's states were analyzed under the four stability solution concepts, and the strength of these stabilities were identified. We looked at the equilibrium states for the conflict; and how the conflict could have evolved over time under different scenarios. We analyzed the effect of the coalition formations on the conflict, its stabilities, and its equilibrium states. We concluded the case study by showing how our analysis results compares to what historically happened in the conflict.

Chapter 10

Contributions and Future Work

10.1 Summary of Contributions

We started the thesis document, in Chapter 1, by stating our research objective: to propose a qualitative formal goals and constraints/context conceptual modelling and reasoning framework, for decision-making agent/s (also called players or decision makers) within single and multi-agent systems and environments, to use in order to effectively help the agent(s) systematically analyze his (their) strategic decisions and conflict situations. The novel formal Constrained Rationality framework, which deliver on this research objective is the main contribution of our research work. The framework forms a new paradigm for modelling and analyzing single and multi-agent decision making situations by bringing the decision support problem back to its roots: reasoning about options, and alternatives, to satisfy the strategic and conflicting goals each agent has, given the internal and external complex and conflicting realities (limitations and opportunities) each has. Each of its components and methods represent a contribution to the field of decision making modelling and analysis, both in one-agent and in multi-agent decision support environments. This section summarizes the contributions that this framework and its components provide.

The foundational concepts and methods of the Constrained Rationality conceptual modelling and reasoning framework were introduced in Chapters 3 and 4. In Chapter 3, we showed how the Constrained Rationality framework extends the conceptual modelling mechanisms of the Viewpoint-based Value Driven - Enterprise Knowledge Management framework (ViVD-EKM), proposed by Al-Shawa (2006b,a), to model the goals, realities/constraints and plans/alternatives which the agents (or the involved decision makers in the decision making situation) have and reason about. Specifically, Chapter 3 introduced:

- The Goals & Constraints Model (GCM) for each of the involved agents, a model that includes the agent's goals and constraints as interacting nodes, affecting each other.
- Goal-to-Goal (G-G) reduction relationships allowing the goals of the agent to form a hierarchal tree-like structures, where the goals at the top of the goal-trees are the agent's strategic goals (needs and wants), and the goals at the bottom represent the reduced-to operational goals.
- Goal-to-Goal (G-G) lateral relationships, a special set of relationships to model the support, hinder and conflict-with relations that exist among goals.
- Constraint-to-Goals (C-G) lateral relations, to model the affect that constraints have on the individual goal nodes within the GCM model.
- A set of value properties for goal nodes, given fuzzified qualitative linguistic value labels, based on fuzzy membership functions, to represent: the amount of operationalization and achievement goal nodes harness through the network of relationships connecting them to other goals and constraints, and the amount of prevention goal nodes receive through these relationships.
- A set of fully axiomatized propagation rules for these fuzzy linguistic value labels propagation through the different G-G and C-G relationships; and how the final value labels, for each value property, for each individual goal node in the GCM model, are calculated at any single point of time (axiomatization of the relationships is given in Appendix A).
- A goals' qualitative value labels forward propagation algorithm, in Section 3.6; and we proved in the section that the algorithm terminates in polynomial time, and proved the correctness and completeness of the algorithm in Appendix B. In addition, we showed in the chapter that the algorithm supports finalizing the value labels of goals part of multiple goal-trees within the agent's GCM.
- How the reasoning framework deals with the dynamic changes happening to the agents' GCM models over the time (such as the addition of new goals or constraints, the removal of some of these goals or constraints, and/or the changes that could happen to the current goals and constraints, including changes to the value labels of their value properties or to their interrelationships).

- The extendability and flexibility of the framework's modelling mechanisms, by showing possible extensions, such as: changing the qualitative fuzzy linguistic value labels or increasing/decreasing their number; and changing or adding new lateral relationships (G-G or C-G) to satisfy the specific needs of the decision making modelling initiative.

In Chapter 4, we showed how the Constrained Rationality framework captures the agents' priorities, emotions and attitudes within the agents' decision making models, and showed how the agents' preferences over their alternatives are generated. More specifically, the chapter introduced:

- How the agent's strategic goals are identified and separated from the rest of the goals he has; and how the agent's alternatives are represented within the agent's GCM as intention goal nodes (lower refined operational goal nodes), each node representing an intention by the agent to adopt a specific alternative.
- How the agent's priorities are modelled as strategic importance value properties attached to the agent's strategic goals, and how these properties are given fuzzy qualitative linguistic value labels based on fuzzy membership functions.
- How the agent's emotion/feeling towards working to achieve a specific strategic goal is modelled as an emotional-valence value property attached to that strategic goal, and how this property is given a fuzzy qualitative linguistic value label based on fuzzy membership functions (representing a range of emotions from extremely-like to extremely-dislike).
- How the framework allows for personality wide attitudes, such as attitudes towards acting rationally or emotionally (whether or not the importance and/or emotional valences exist), to be modelled; and how this modelling mechanism allows for a wide range of personalities to be captured, modelled and used in the reasoning mechanisms.
- How the agent's cardinal and ordinal preferences over his alternatives are calculated based on how much each alternative contribute to the achievement of the agent's strategic goals, given: the constraints affecting these goals, the importance the agent assigned to his strategic goals, the emotional valences assigned to these goals, and the attitudes the agent is demonstrating towards acting rationally or emotionally in this situation.
- How the strengths of the agent's preferences, over his alternatives, are calculated, and assigned a fuzzy qualitative linguistic value label based on fuzzy

membership functions. These value labels represent the preferences' strengths, with strength labels that range from Extremely-Less-Preferred to Extremely-Preferred, including the Indifferent preference strength label). Note that these preferences' strengths are calculated, i.e. not predetermined. In other words, one can trace back (verify and validate) how and why each preference's strength is at the level it is given, and not less or more.

- A novel, unique and complete calculated-and-verifiable preference relationship of $\succ^{L_{PS}}_{DM_i,t}$, where L_{PS} is the fuzzy linguistic value label representing the preference's strength, to represent any preference a decision maker DM_i, at time t, has over any two alternatives he has. For example, $A_a \succ^{N}_{DM_i,t} A_b$ represents the fact that DM_i is indifferent among both A_a and A_b; $A_a \succ^{-E}_{DM_i,t} A_b$ represents A_a to be extremely-less-preferred to DM_i than A_b; and $A_a \succ^{E}_{DM_i,t} A_b$ represents A_a to be extremely-preferred to DM_i than A_b. This modelling mechanism of preferences, and their strengths, is a major contribution and advancement over the way current methods provide. For example, the GMCR provides two relations, one for preference "$\succ$" and another for indifference "$\sim$" (Fang et al., 1993). Some of GMCR extensions provide additional relationships: Hamouda et al. (2004) added "$>>$" to represent a strong preface and "$>$" a weak preference; and Xu (2009) went further to the extent of offering "$>> \cdots >$" to represent a preference with strength that matches the number of "$>$" in the relation . One should remember that all these preferences are pre-determined set upfront, i.e. unlike the Constrained Rationality preferences, these preferences are not calculated, and could not be verified nor validated.

Chapter 3 and 4 focused on the modelling and analysis of one-agent decision making situations (and multi-agent decision making situations where the agents act in an individualistic manner with no regard to others' choices and decisions), for simplicity, and in order to maintain the main focus of the two chapters on the Constraints Rationality's foundational concepts and methods. Chapter 5, on the other hand, introduced the framework's modelling and analysis process for multi-agent decision making situations. Specifically, Chapter 5 introduced:

- The identification of two distinct types/modes of multi-agent decision making situations: collaborative situations, and adversarial competitive situations (the lateral called games in game theory literature or conflicts in the broader management science and conflict analysis literature).

- Two different processes for the modelling and analysis of the two different multi-agent decision making situations.
- How the different concepts and methods of the Constrained Rationality modelling and reasoning framework, introduced in Chapter 3 and 4, are modified and extended to deal with the modelling and analysis of multi-agent decision making situations.
- How agents' cardinal and ordinal preferences, and their preferences' strengths, are calculated and modelled in collaborative decision making situations, and in conflicts.
- How states of conflicts are defined, how complex conflicts structures (with iterations and multiple phases) are modelled, and how the different types of moves agents have among the conflict states help defining three types of conflicts: non-cooperative conflicts, cooperative conflicts with no coalitions, and cooperative conflicts with alliances and coalitions allowed and competing in the conflicts.
- The relationships among the three conflict types, more specifically how cooperative games with coalitions form the most general and broad type of multi-agent conflicts, where cooperative game without coalitions form a subset of the set of cooperative conflicts with coalitions, and non-cooperative conflicts form a subset of the set of cooperative conflicts without coalitions.

Each of Chapters 6, 8 and 9 provided a detailed look at the specific modelling and analysis needs of the three conflict types: non-cooperative, cooperative without coalitions, and cooperative with coalitions, respectively. These chapters defined formally the analysis process, moves, stability solution concepts, equilibrium concepts, for each of the three conflict types. More specifically, Chapters 6, 8 and 9 defined formally for each of non-cooperative conflicts, cooperative conflicts without coalitions, and cooperative conflicts with coalitions, respectively, the following:

- The different types of non-cooperative unilateral moves, and sanction moves, that individual agents are allowed to make in all conflict types.
- The different types of cooperative single-step moves, and sanction moves, that agents are allowed to make in cooperative games (with and without coalitions).
- The multi-step cooperative moves, and sanction moves, that groups/coalitions are allowed to make in cooperative games with coalitions.

- Four stability solution concepts, for all conflict types: Nash stability, General MetaRational (GMR) stability, Symmetric MetaRational (SMR) stability, and Sequentially Stability (SEQ).
- Equilibrium states under these stability solution concepts.
- A unique and novel definition of the strength of a stability/equilibrium, under each of the four solution concepts, with the strength of a stability/equilibrium is assigned a fuzzy qualitative linguistic value label based on fuzzy membership functions. This unique strength of a stability solution concept is based on the novel complete calculated-and-verifiable preference relationship indicated earlier. It provides a calculated and verifiable stability strength. For example, traditional stability solution concepts definitions provided for GMCR by (Fang et al., 1993) decides only whether a conflict's state is stable or not-stable under a certain stability concept, i.e. no strength of stability is provided. But, these GMCR definitions were later extended by Hamouda et al. (2004), and others such as Xu (2009), adding two/more level strength levels for a state's stability under a certain solution concept. But all these GMCR stability-strength extended definitions are based on given pre-determined agents' ordinal preferences that are set upfront, i.e. unlike the Constrained Rationality preferences, these preferences are not calculated, and could not be verified nor validated. Therefore, the stabilities' strengths that Constrained Rationality define can easily be calculated, verified, validated, and tied to the players' motives and constraints. Changes in any of the contextual information about the conflict and its players, will be reflected on the stability strength of the conflict's states for the agents.
- A set of formal robust algorithms to calculate the strength of stabilities and equilibriums under each of the four solution concepts, and assign the right strength value label for each state's stability and/or equilibrium.

In Chapter 7, we provided a detailed discussion about the characteristics of the four stability solution concepts (NASH, GMR, SMR and SEQ); the theoretical interrelationships among these solution concepts, generally; and the theoretical interrelationships among these solution concepts' strength sets. More specifically, Chapter 7 defined formally the following (within the context of non-cooperative games –for simplicity only–):

- a detail comparative look at the characteristics of the NASH, GMR, SMR and SEQ stabilities.

- a set of theorems and proofs describing formally the interrelationships among the general sets of (stable states under each of) NASH, GMR, SMR and SEQ.
- a novel and unique robust set of theorems and proofs describing formally the interrelationships among the sets (of states stable under) different stability strengths of NASH, GMR, SMR and SEQ.

The Constrained Rationality framework is build from the ground up as a set of modelling and analysis modules connected through a coherent and complete systematic and methodological process. This gives its users the flexibility to use it from start to finish as a complete framework and process to model and analyze their decision making situation/conflict, or use only some modules of it in conjunction with other frameworks. For example, a user of a game theoretic approach, or a user of a graph model for conflict resolution approach, whose structure uses or mandates the use of such an approach, or a tool that embodies the use of such a procedure, can continue using their approach for conflict modelling and analysis. At the same time, they can use the Constrained Rationality's goals and constrained modelling module, the players' priorities and emotions modelling module and the preferences generation module for the purpose of validating the players' preferences in the conflicts they are studying. Constrained Rationality, in such case, is used to test the validity of what is usually given as-is pre-determined (without justification) preferences, ensuring the alignment of the players' preferences with their respective objectives and realities within the context of the conflict. Saying so, we still recommend using Constrained Rationality as a complete framework and process, even for people who are familiar with, or mandated to use, other approaches. Using Constrained Rationality framework, in its totality with all its modelling and analysis modules and process, will give these users a validation not only to the players' preferences, but also to the conflict analysis results generated using other approaches.

Through out the thesis documents, we used illustrative and exploratory case studies (as defined by Yin (2003), and as used by other comparative thesis and research works – discussed in Section 2.6–) to show the effectiveness and benefits of using the Constrained Rationality approach, in modelling and analyzing strategic decision making situations and conflicts, in comparison to what current dominant approaches provide. In selecting the illustrative and exploratory cases studies, and application examples, to use, model and analyze in our thesis work, we tried to cover:

- Classical conflicts which are extensively used and modelled in the literature (using the traditional and new-comer approaches alike). This allowed us to compare our approach and models to what is provided in the literature.
- Historical conflicts which are extensively studied, modelled and analyzed in the literature (such as the case of the Cuban Missile Crises). Using such conflict cases allows for objective comparison of the models and analysis produced by our approach to what others provided.
- Cases and examples that spanned across the different types of problem and application domains (listed in Chapter 1 under research objective as our research's application domains): from one agent (decision-maker) to multi-agent cases; from political conflicts to business conflicts; from organizational (group) conflicts to personal dilemmas; from historical conflicts to conflicts of current interest; from strategic business/managerial conflicts to technology design conflicts; and so on.
- Cases and application examples that allowed us to illustrate the full extent of all the claimed benefits of our approach. For example, we used the Howard's Dilemma case (a hypothetical case used in the literature) to demonstrate the modelling and analysis of a conflict where high emotions and personal interests/attitudes exist.
- Cases that allowed us to illustrate the claimed "better than" benefits of our approach (over other approaches), listed in Chapter 1 under research arguments and propositions. For example, in order to show how our approach compares-to, enhances and complements other approaches, namely here the Conflict Analysis (Fraser and Hipel, 1984), and the Graph Model of Conflict Resolution (Fang, 1989; Fang et al., 1993) and the traditional Game-Theory's Rational Actor Model (Allison and Zelikow, 1999), we chose to model and analyze the Cuban Missile Crisis (a political historical conflict), a noncooperative conflict, and the Elmira Groundwater Contamination Conflict (a historical governmental and environmental policy conflict), a cooperative without coalitions conflict.

The following case studies, and application examples, were used in this thesis work. Each of these case studies/examples, its models and analysis using the Constrained Rationality framework (including the comparison with how others' modelled and analyzed the case) is a contribution to the research domain of decision making modelling and analysis. The case studies/ examples used in this research work include:

1. One-Agent Decision Making Cases:
   (a) Howard's Personal Dilemma (hypothetical case from literature, with a mix of high degrees of emotions and rationality) [Chapter 3 and 4]
   (b) Car Manufacture's Strategic Business Decision (hypothetical case study, but based on a current real-life case simplified to fit the scope of our research context) [Chapter 3]
2. Multi-Agent Decision Making Cases:
   (a) Collaborative Multi-Agent Decision Making Situations:
      i. System Requirements Engineering (hypothetical case study, but based on a real-life industrial case simplified to fit the scope of our research context) [Chapter 5]
   (b) Competitive Adversarial Multi-Agent Decision Making Situations (also called Conflicts or Games):
      i. The Cuban Missile Crisis (non-cooperative historical political conflict - from literature) [Chapter 6]
      ii. The Elmira Groundwater Contamination Conflict (a cooperative, without coalitions, historical governmental and environmental policy conflict, with possible hypergames - from literature) [Chapter 8]
      iii. Is it Worth Fighting a Patent Troll? The Showdown between RIM and NTP, as an example (a cooperative multi-phase, with coalitions, historical intellectual property strategic business conflict) [Chapter 9]
3. Paradoxes of Rationality Cases:
   (a) Classical Prisoner's Dilemma (non-cooperative conflict - from literature) [Chapter 7]
   (b) Iterative Prisoners Dilemma, Classical and Tit-For-Tat (non-cooperative conflict - from literature) [Chapter 7]
   (c) Game of Chicken (non-cooperative conflict - from literature) [Chapter 7]

In these case studies, we demonstrated that our research central argument, given in Chapter 1 under research objective, is true. In other words, using these case studies, and application examples, we proved that:

By *structuring and conceptually modelling the strategic decision making problem (within single or multi agent decision making environment and whether the the decision-maker/agent is an individual, an organization, a robot, or a coalition) by bringing the problem back to its roots: reasoning about options, and alternatives, to satisfy the strategic and conflicting goals each agent has, given the internal and external complex and conflicting realities (limitations and opportunities) each has.*

Then *more realistic modelling, insight and analysis of the strategic decision making situations, or the real-life conflicts, could be reached, allowing for:*

- *better understanding of the full extent of the players' options (current and potential alternatives)*
- *better decision making, and sensitivity analysis*
- *better stability analysis and outcome prediction*
- *better dynamic modelling and simulation of evolving and changing decision-making/conflicts*
- *better representation and testing of players' different patterns of behaviour (based on priorities, needs, wants and emotional states)*

Compared to *what alternative-focused (as called by Keeney (1992)) decision theory and game theory can provide. Not to mention that the proposed methodology, and the models it produces, can be used in fact to provide and/or verify as well as explain the proxy cardinal and/or ordinal preferences these theories use and need to order decision makers' options.*

Especially when applied to *strategic decision making and conflicts and problems where: options of players are not clear or unknown, preferences could not be defined or unclear, utility functions are hard to establish, and so on.*

In all these case studies, and where there are models and analysis for them in the literature using other frameworks, we showed how the Constrained Rationality framework: performed better in addressing many of the limitations of current frameworks (discussed in Chapter 2); provided better modelling and analysis facilities; captured more contextual knowledge about the cases and the decision maker's motives and constraints; and provided more and better insight, learning and predictions. And, where there are no formal models and analysis exist in the decision and game theoretic literature for a conflict, such as the case of RIM v. NTP – an important business conflict with far reaching implications on patent laws and business practices–, we showed how the Constrained Rationality framework not only provided models and analysis for this multi-stage multi-game complex real-life strategic conflict, but also produced accurate predictions of how the conflict would evolve.

10.2 Future Work

This research work proposed foundational concepts and methods, for the Constrained Rationality framework, that could be extended in many directions and applied to many application domains. We see the follow-up future work, based on this research, as a full scale research program with many research areas. The following is a list of these areas, with some of the work items we intend to deliver, within each area, in the near future:

Extensions to the Constrained Rationality Framework:

- Build risk analysis methods and tools for the framework.
- Add methods and tools to assist in modelling and reasoning strategic decision making under uncertainties or unknowns (we have started work on this item, based on our experience, using the framework in a product-development industrial study which we have just finished).
- Add a set of more systemic and formal methods and tools to help automate the process of status quo analysis.
- Propose additional stability solution concepts under the different conflict types/patterns, and redefine some of the useful existing ones (available in the literature beyond what we have included –NASH, GMR, SMR and SEQ–).
- Add goal prioritization (and emotional valences) for all goals, not just the strategic goals; and offer modelling and reasoning facilities to propagate and consolidate these priorities (and emotional valences) under different decision making situations and patterns.
- Offer modelling and reasoning facilities to handle additional attitudes, beside the overall-acting Rational and Emotional ones the framework has currently, such as: attitudes towards specific agents, types of agents, and

types of decision making situations. In addition, offer modelling and reasoning facilities to handle more attitudes types, such as: aggressiveness, easy-going, etc.

- Add Constraint-to-Constraint lateral relationships to capture the effect that constraints have on each other; and Goal-to-Constraint lateral relationships to capture the effect that goals' achievement have on some constraints.
- Expand on the use of Constraints and Alternatives within the framework to match the rich and detailed definitions of both concepts given in the ViVD-EKM conceptual modelling framework which Constrained Rationality is based on.
- Add modelling facilities within the framework to make use of sophisticated known creative thinking tools, in order to help the decision makers generate alternatives/options creatively.
- Identify patterns of conflicts, and provide additional customized modelling and analysis facilities for them.
- Offer additional formal modelling and analysis facilities specifically for multi-level hypergames.
- Provide modelling and analysis facilities to capture and reason-about different aspects of multi-agent conflicts, such as the organizational and behavioural aspects Allison and Zelikow (1999) discussed for the Cuban Missile Crisis.

Applications and Case Studies: Model and analyze real-life industrial decision making situations and conflicts in the following areas, and report on the findings and learnings:

- New Product Development
- Product Line Strategies
- Intellectual Property Strategies
- Business Acquisitions Strategies
- Research and Development Business Strategies
- Consumer Adoption and Marketing Strategies
- Systems Requirements Engineering

- User Centric Designs and Usability Analysis
- Governmental Environmental and Infrastructure Policies
- Governmental Research and Development Funding Strategies
- Enhancing the Learning Environment and the Students Learning Experience in Higher Education

Experimental Studies and Comparative Analysis:

- Test the effectiveness of Constrained Rationality, in comparison to traditional decision support methods, in controlled strategic decision making experiments
- Test the effectiveness of Constrained Rationality in capturing the effect of emotions and attitudes, in controlled strategic decision making experiments
- Conduct comparative analysis studies: the Constrained Rationality framework, its concepts and methods, versus other frameworks.
- Provide case studies to show how the Constrained Rationality framework compliment other decision and conflict modelling frameworks, such as game theory and the graph model for conflict resolution, and test the effectiveness of Constrained Rationality in doing so, especially when it comes to offering these framework mechanisms to validate the predetermined given-as-is options and preferences that they rely on, and to connect them to the decision makers goals and constraints.

Decision Support System (DSS) and Tools:

- Integrate many of the Constrained Rationality's DSS modules and tools, which we built during this research work, into one DSS.
- Build model management facilities, specifically to deal with usability and presentation issues due to the size of models (specially for real-life full scale industrial decision making models).
- Complete the simulation engine, and integrate it into the DSS system.
- Add animation and visualization capabilities to the DSS system, specially for dynamic changes under simulation.
- Add an enhanced reporting facility to the DSS.

- Add new pre-built modelling templates and components, for each specific application area and conflict pattern, to aid the analysts and decision makers through the modelling and analysis processes.
- Enhance the DSS and its components based on the learnings from the application case studies, and the feedback from users.

Appendices

Appendix A

Axiomatization of Goal-Goal and Constraint-Goal Relationships

In this appendix, we will provide a complete set of ground relation axioms for all the goal-to-goal and goal-to-constraint relationships, discussed in Chapter 3.

For this appendix, and for space and presentation reasons, we will assume that the fuzzy value labels set $\mathcal{L}$, which is introduced in Section 3.3 and has its elements match in number and names the fuzzy sets chosen to divide the satisfaction levels domain of the operationalization, achievement, and prevention value properties, to be $\mathcal{L} = \{Full, Some, None, Null\} = \{F, S, N, Null\}$, reflecting the fuzzy sets definition as shown in Figure A.1, and with the order $F > S > N > Null$. This reduced set used here replaces the full $\mathcal{L} = \{Full, Big, Much, Moderate, Some, Little, None, Null\}$ with the order $Full > Big > Much > Moderate > Some > Little > None > Null$, introduced in Section 3.3 and reflected the sets definition as shown in Figure 3.3; and with the understanding that this is done for space and presentation concerns only and because of the fact that the fuzzy set "*Some*" and its representative value label could be broken down, or extended using the same logic presented in this appendix, to form the full list of fuzzy sets/labels between *Full* and *None* to: *Big*, *Much*, *Moderate*, *Some*, and *Little*; or any other fuzzy sets/labels division of the value properties' domains that the framework's user/application see fit.

For each value property, we introduce a set of predicates over goals and constraints, where $F_{achv}(G_i)$ represents $Achv(G_i) = Full$, and $S_{achv}(G_i)$ represents $Achv(G_i) = Some$, $N_{achv}(G_i)$ represents $Achv(G_i) = None$ and $Null_{achv}(G_i)$ represents $Achv(G_i) = Null$. We then introduce a total order where $\forall G \in \mathcal{G} : F_{achv}(G) \geq$

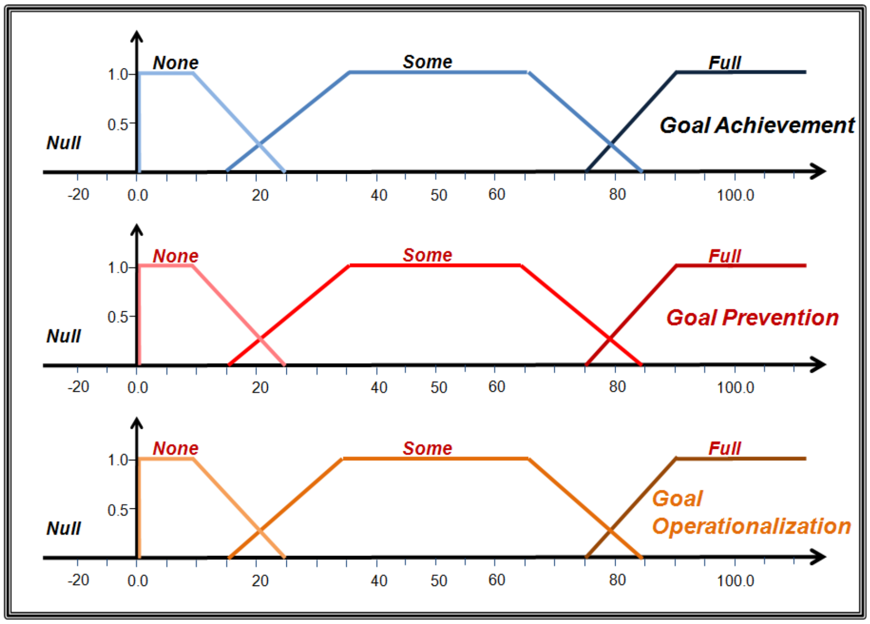


Figure A.1: Smaller set of the Fuzzy Sets introduced earlier to divide the satisfaction levels domain of the different Goals' Value Properties (operationalization, achievement, and prevention)

$S_{achv}(G) \geq N_{achv}(G) \geq Null_{achv}(G)$. The same order exists for Opr and $Prvn$ predicates over goals, and for $Achv$ and $Prvn$ predicates over constraints. Then, the following is the axiomatization of all the goal-to-goal relations (reduction and lateral relations introduced in Section 3.4 of the paper), and all the constraint-to-goal lateral relations (introduced in Section 3.5):

Axioms for the *AND* Reduction Relationship

$(G_1, G_2) \xrightarrow{and} G:$

$$(F_{opr}(G_1) \wedge F_{opr}(G_2)) \rightarrow F_{opr}(G) \tag{A.1}$$
$$(S_{opr}(G_1) \wedge S_{opr}(G_2)) \rightarrow S_{opr}(G) \tag{A.2}$$
$$(F_{opr}(G_1) \wedge S_{opr}(G_2)) \rightarrow S_{opr}(G) \tag{A.3}$$
$$(S_{opr}(G_1) \wedge F_{opr}(G_2)) \rightarrow S_{opr}(G) \tag{A.4}$$
$$(N_{opr}(G_1) \wedge N_{opr}(G_2)) \rightarrow N_{opr}(G) \tag{A.5}$$
$$(N_{opr}(G_1) \wedge (F_{opr}(G_2) \vee S_{opr}(G_2))) \rightarrow N_{opr}(G) \tag{A.6}$$
$$((F_{opr}(G_1) \vee S_{opr}(G_1)) \wedge N_{opr}(G_2)) \rightarrow N_{opr}(G) \tag{A.7}$$
$$(Null_{opr}(G_1) \vee Null_{opr}(G_2)) \rightarrow Null_{opr}(G) \tag{A.8}$$

$$(F_{achv}(G_1) \wedge F_{achv}(G_2)) \rightarrow F_{achv}(G) \tag{A.9}$$
$$(S_{achv}(G_1) \wedge S_{achv}(G_2)) \rightarrow S_{achv}(G) \tag{A.10}$$

$$(F_{achv}(G_1) \wedge S_{achv}(G_2)) \rightarrow S_{achv}(G) \quad \text{(A.11)}$$
$$(S_{achv}(G_1) \wedge F_{achv}(G_2)) \rightarrow S_{achv}(G) \quad \text{(A.12)}$$
$$(N_{achv}(G_1) \wedge N_{achv}(G_2)) \rightarrow N_{achv}(G) \quad \text{(A.13)}$$
$$(N_{achv}(G_1) \wedge (F_{arch}(G_2) \vee S_{arch}(G_2))) \rightarrow N_{achv}(G) \quad \text{(A.14)}$$
$$((F_{arch}(G_1) \vee S_{arch}(G_1)) \wedge N_{achv}(G_2)) \rightarrow N_{achv}(G) \quad \text{(A.15)}$$
$$(Null_{arch}(G_1) \vee Null_{arch}(G_2)) \rightarrow Null_{arch}(G) \quad \text{(A.16)}$$

$$(F_{prvn}(G_1) \vee F_{prvn}(G_2)) \rightarrow F_{prvn}(G) \quad \text{(A.17)}$$
$$(S_{prvn}(G_1) \wedge S_{prvn}(G_2)) \rightarrow S_{prvn}(G) \quad \text{(A.18)}$$
$$(S_{prvn}(G_1) \wedge (N_{prvn}(G_2) \vee Null_{prvn}(G_2))) \rightarrow S_{prvn}(G) \quad \text{(A.19)}$$
$$((N_{prvn}(G_1) \vee Null_{prvn}(G_1)) \wedge S_{prvn}(G_2)) \rightarrow S_{prvn}(G) \quad \text{(A.20)}$$
$$(N_{prvn}(G_1) \wedge N_{prvn}(G_2)) \rightarrow N_{prvn}(G) \quad \text{(A.21)}$$
$$(N_{prvn}(G_1) \wedge Null_{prvn}(G_2)) \rightarrow N_{prvn}(G) \quad \text{(A.22)}$$
$$(Null_{prvn}(G_1) \wedge N_{prvn}(G_2)) \rightarrow N_{prvn}(G) \quad \text{(A.23)}$$
$$(Null_{prvn}(G_1) \wedge Null_{prvn}(G_2)) \rightarrow Null_{prvn}(G) \quad \text{(A.24)}$$

Axioms for the *OR* Reduction Relationship

$(G_1, G_2) \xrightarrow{or} G$:

$$(F_{opr}(G_1) \vee F_{opr}(G_2)) \longrightarrow F_{opr}(G) \quad \text{(A.25)}$$
$$(S_{opr}(G_1) \wedge S_{opr}(G_2)) \longrightarrow S_{opr}(G) \quad \text{(A.26)}$$
$$(S_{opr}(G_1) \wedge (N_{opr}(G_2) \vee Null_{opr}(G_2))) \longrightarrow S_{opr}(G) \quad \text{(A.27)}$$
$$((N_{opr}(G_1) \vee Null_{opr}(G_1)) \wedge S_{opr}(G_2)) \longrightarrow S_{opr}(G) \quad \text{(A.28)}$$
$$(N_{opr}(G_1) \wedge N_{opr}(G_2)) \rightarrow N_{opr}(G) \quad \text{(A.29)}$$
$$(N_{opr}(G_1) \wedge Null_{opr}(G_2)) \rightarrow N_{opr}(G) \quad \text{(A.30)}$$
$$(Null_{opr}(G_1) \wedge N_{opr}(G_2)) \rightarrow N_{opr}(G) \quad \text{(A.31)}$$
$$(Null_{opr}(G_1) \wedge Null_{opr}(G_2)) \rightarrow Null_{opr}(G) \quad \text{(A.32)}$$

$$(F_{achv}(G_1) \vee F_{achv}(G_2)) \longrightarrow F_{achv}(G) \quad \text{(A.33)}$$
$$(S_{achv}(G_1) \wedge S_{achv}(G_2)) \longrightarrow S_{achv}(G) \quad \text{(A.34)}$$
$$(S_{achv}(G_1) \wedge (N_{achv}(G_2) \vee Null_{achv}(G_2))) \longrightarrow S_{achv}(G) \quad \text{(A.35)}$$
$$((N_{achv}(G_2) \vee Null_{achv}(G_2)) \wedge S_{achv}(G_2)) \longrightarrow S_{achv}(G) \quad \text{(A.36)}$$
$$(N_{achv}(G_1) \wedge N_{achv}(G_2)) \rightarrow N_{achv}(G) \quad \text{(A.37)}$$
$$(N_{achv}(G_1) \wedge Null_{achv}(G_2)) \rightarrow N_{achv}(G) \quad \text{(A.38)}$$
$$(Null_{achv}(G_1) \wedge N_{achv}(G_2)) \rightarrow N_{achv}(G) \quad \text{(A.39)}$$

$$(Null_{achv}(G_1) \wedge Null_{achv}(G_2)) \rightarrow Null_{achv}(G) \tag{A.40}$$

$$(F_{prvn}(G_1) \wedge F_{prvn}(G_2)) \longrightarrow F_{prvn}(G) \tag{A.41}$$

$$(S_{prvn}(G_1) \wedge S_{prvn}(G_2)) \longrightarrow S_{prvn}(G) \tag{A.42}$$

$$(F_{prvn}(G_1) \wedge S_{prvn}(G_2)) \longrightarrow S_{prvn}(G) \tag{A.43}$$

$$(S_{prvn}(G_1) \wedge F_{prvn}(G_2)) \longrightarrow S_{prvn}(G) \tag{A.44}$$

$$(N_{prvn}(G_1) \wedge N_{prvn}(G_2)) \rightarrow N_{prvn}(G) \tag{A.45}$$

$$(N_{prvn}(G_1) \wedge (F_{prvn}(G_2) \vee S_{prvn}(G_2))) \rightarrow N_{prvn}(G) \tag{A.46}$$

$$((F_{prvn}(G_1) \vee P_{prvn}(G_1)) \wedge N_{prvn}(G_2)) \rightarrow N_{prvn}(G) \tag{A.47}$$

$$(Null_{prvn}(G_1) \vee Null_{prvn}(G_2)) \rightarrow Null_{prvn}(G) \tag{A.48}$$

Axioms for the *Asymmetric Consistent* G-G Lateral Relationships

$G_1 \stackrel{++}{\longrightarrow} G:$

$$F_{opr}(G_1) \longrightarrow F_{opr}(G) \tag{A.49}$$

$$S_{opr}(G_1) \longrightarrow S_{opr}(G) \tag{A.50}$$

$$N_{opr}(G_1) \longrightarrow N_{opr}(G) \tag{A.51}$$

$$Null_{opr}(G_1) \longrightarrow Null_{opr}(G) \tag{A.52}$$

$$F_{achv}(G_1) \longrightarrow F_{achv}(G) \tag{A.53}$$

$$S_{achv}(G_1) \longrightarrow S_{achv}(G) \tag{A.54}$$

$$N_{achv}(G_1) \longrightarrow N_{achv}(G) \tag{A.55}$$

$$Null_{achv}(G_1) \longrightarrow Null_{achv}(G) \tag{A.56}$$

$G_1 \stackrel{+(M+)}{\longrightarrow} G:$

$$\text{if} \quad M = \textit{Full}, \text{ then} \quad [G_1 \stackrel{+(\textit{Full}+)}{\longrightarrow} G] \equiv [G_1 \stackrel{++}{\longrightarrow} G] \tag{A.57}$$

i.e. follow axioms: A.49 - A.56

if $M = \textit{Some}$, then $G_1 \stackrel{+(Some+)}{\longrightarrow} G:$

$$(F_{opr}(G_1) \vee S_{opr}(G_1)) \longrightarrow S_{opr}(G) \tag{A.58}$$

$$N_{opr}(G_1) \longrightarrow N_{opr}(G) \tag{A.59}$$

$$Null_{opr}(G_1) \longrightarrow Null_{opr}(G) \tag{A.60}$$

$$(F_{achv}(G_1) \vee S_{achv}(G_1)) \longrightarrow S_{achv}(G) \tag{A.61}$$

$$N_{achv}(G_1) \longrightarrow N_{achv}(G) \tag{A.62}$$

$$Null_{achv}(G_1) \longrightarrow Null_{achv}(G) \tag{A.63}$$

if $M = None$, then $G_1 \overset{+(None+)}{\longrightarrow} G:$

$$(F_{opr}(G_1) \vee S_{opr}(G_1) \vee N_{opr}(G_1)) \longrightarrow N_{opr}(G) \tag{A.64}$$

$$Null_{opr}(G_1) \longrightarrow Null_{opr}(G) \tag{A.65}$$

$$(F_{achv}(G_1) \vee S_{achv}(G_1) \vee N_{achv}(G_1)) \longrightarrow N_{achv}(G) \tag{A.66}$$

$$Null_{achv}(G_1) \longrightarrow Null_{achv}(G) \tag{A.67}$$

if $M = Null$, then $G_1 \overset{+(Null+)}{\longrightarrow} G:$

$$Null_{opr}(G) \tag{A.68}$$

$$Null_{achv}(G) \tag{A.69}$$

$G_1 \overset{--}{\longrightarrow} G:$

$$F_{prvn}(G_1) \longrightarrow F_{prvn}(G) \tag{A.70}$$

$$S_{prvn}(G_1) \longrightarrow S_{prvn}(G) \tag{A.71}$$

$$N_{prvn}(G_1) \longrightarrow N_{prvn}(G) \tag{A.72}$$

$$Null_{prvn}(G_1) \longrightarrow Null_{prvn}(G) \tag{A.73}$$

$G_1 \overset{-(M-)}{\longrightarrow} G:$

$$\text{if } M = Full, \text{ then } [G_1 \overset{-(Full-)}{\longrightarrow} G] \equiv [G_1 \overset{--}{\longrightarrow} G] \tag{A.74}$$

i.e. follow axioms: A.70 - A.73

if $M = Some$, then $G_1 \overset{-(Some-)}{\longrightarrow} G:$

$$(F_{prvn}(G_1) \vee S_{prvn}(G_1)) \longrightarrow S_{prvn}(G) \tag{A.75}$$

$$N_{prvn}(G_1) \longrightarrow N_{prvn}(G) \tag{A.76}$$

$$Null_{prvn}(G_1) \longrightarrow Null_{prvn}(G) \tag{A.77}$$

if $M = None$, then; $G_1 \overset{-(None-)}{\longrightarrow} G:$

$$(F_{prvn}(G_1) \vee S_{prvn}(G_1) \vee N_{prvn}(G_1)) \longrightarrow N_{prvn}(G) \tag{A.78}$$

$$Null_{prvn}(G_1) \longrightarrow Null_{prvn}(G) \tag{A.79}$$

if $M = Null$, then $G_1 \overset{-(Null-)}{\longrightarrow} G:$

$$Null_{prvn}(G) \tag{A.80}$$

Axioms for the *Asymmetric Conflict* G-G Lateral Relationships

$G_1 \overset{+-}{\longrightarrow} G:$

$$F_{achv}(G_1) \longrightarrow F_{prvn}(G) \tag{A.81}$$

$$S_{achv}(G_1) \longrightarrow S_{prvn}(G) \tag{A.82}$$

$$N_{arch}(G_1) \longrightarrow N_{prvn}(G) \tag{A.83}$$

$$Null_{arch}(G_1) \longrightarrow Null_{prvn}(G) \tag{A.84}$$

$G_1 \overset{+(M-)}{\longrightarrow} G$:

$$\text{if} \quad M = \mathit{Full}, \text{ then} \quad [G_1 \overset{+(\mathit{Full}-)}{\longrightarrow} G] \equiv [G_1 \overset{+-}{\longrightarrow} G] \tag{A.85}$$

i.e. follow axioms: A.81 - A.84

if $M = \mathit{Some}$, then $G_1 \overset{+(\mathit{Some}-)}{\longrightarrow} G$:

$$(F_{achv}(G_1) \vee S_{achv}(G_1)) \longrightarrow S_{prvn}(G) \tag{A.86}$$

$$N_{achv}(G_1) \longrightarrow N_{prvn}(G) \tag{A.87}$$

$$Null_{achv}(G_1) \longrightarrow Null_{prvn}(G) \tag{A.88}$$

if $M = \mathit{None}$, then $G_1 \overset{+(\mathit{None}-)}{\longrightarrow} G$:

$$(F_{achv}(G_1) \vee S_{achv}(G_1) \vee N_{achv}(G_1)) \longrightarrow N_{prvn}(G) \tag{A.89}$$

$$Null_{achv}(G_1) \longrightarrow Null_{prvn}(G) \tag{A.90}$$

if $M = \mathit{Null}$, then $G_1 \overset{+(\mathit{Null}-)}{\longrightarrow} G$:

$$Null_{prvn}(G) \tag{A.91}$$

$G_1 \overset{-+}{\longrightarrow} G$:

$$F_{prvn}(G_1) \longrightarrow F_{achv}(G) \tag{A.92}$$

$$S_{prvn}(G_1) \longrightarrow S_{achv}(G) \tag{A.93}$$

$$N_{prvn}(G_1) \longrightarrow N_{achv}(G) \tag{A.94}$$

$$Null_{prvn}(G_1) \longrightarrow Null_{achv}(G) \tag{A.95}$$

$G_1 \overset{-(M+)}{\longrightarrow} G$:

$$\text{if} \quad M = \mathit{Full}, \text{ then} \quad [G_1 \overset{-(\mathit{Full}+)}{\longrightarrow} G] \equiv [G_1 \overset{-+}{\longrightarrow} G] \tag{A.96}$$

i.e. follow axioms: A.92 - A.95

if $M = \mathit{Some}$, then $G_1 \overset{-(\mathit{Some}+)}{\longrightarrow} G$:

$$(F_{prvn}(G_1) \vee S_{prvn}(G_1)) \longrightarrow S_{achv}(G) \tag{A.97}$$

$$N_{prvn}(G_1) \longrightarrow N_{achv}(G) \tag{A.98}$$

$$Null_{prvn}(G_1) \longrightarrow Null_{achv}(G) \tag{A.99}$$

if $M = \mathit{None}$, then $G_1 \overset{-(\mathit{None}+)}{\longrightarrow} G$:

$$(F_{prvn}(G_1) \vee S_{prvn}(G_1) \vee N_{prvn}(G_1)) \longrightarrow N_{achv}(G) \tag{A.100}$$

$$Null_{prvn}(G_1) \longrightarrow Null_{achv}(G) \tag{A.101}$$

if $M = \mathit{Null}$, then $G_1 \overset{-(\mathit{Null}+)}{\longrightarrow} G$:

$$Null_{achv}(G) \tag{A.102}$$

Axioms for the *Symmetric Consistent* G-G Lateral Relationships

$$G_1 \xrightarrow{=} G \quad \equiv \quad [G_1 \xrightarrow{++} G] \cup [G_1 \xrightarrow{--} G] \tag{A.103}$$

i.e. follow axioms: A.49 - A.56 and A.70 - A.73

$$G_1 \overset{(M=)}{\longrightarrow} G \quad \equiv \quad [G_1 \overset{+(M+)}{\longrightarrow} G] \cup [G_1 \overset{-(M-)}{\longrightarrow} G] \tag{A.104}$$

i.e. follow axioms: A.57 - A.69 and A.74 - A.80

Axioms for the *Symmetric Conflict* G-G Lateral Relationships

$$G_1 \xrightarrow{\times} G \quad \equiv \quad [G_1 \xrightarrow{+-} G] \cup [G_1 \xrightarrow{-+} G] \tag{A.105}$$

i.e. follow axioms: A.81 - A.84 and A.92 - A.95

$$G_1 \overset{(M\times)}{\longrightarrow} G \quad \equiv \quad [G_1 \overset{+(M-)}{\longrightarrow} G] \cup [G_1 \overset{-(M+)}{\longrightarrow} G] \tag{A.106}$$

i.e. follow axioms: A.85 - A.91 and A.96 - A.102

Axioms for the *Asymmetric Consistent* C-G Lateral Relations

$C \xrightarrow{++} G$:

$$F_{achv}(C) \longrightarrow F_{achv_{up-lim}}(G) \tag{A.107}$$

$$S_{achv}(C) \longrightarrow S_{achv_{up-lim}}(G) \tag{A.108}$$

$$N_{achv}(C) \longrightarrow N_{achv_{up-lim}}(G) \tag{A.109}$$

$$Null_{achv}(C) \longrightarrow Null_{achv_{up-lim}}(G) \tag{A.110}$$

$C \overset{+(M+)}{\longrightarrow} G$:

$$\text{if} \quad M = \textit{Full}, \text{ then} \quad [C \overset{+(\textit{Full}+)}{\longrightarrow} G] \;\equiv\; [C \xrightarrow{++} G] \tag{A.111}$$

i.e. follow axioms: A.107 - A.110

if $M = \textit{Some}$, then $C \overset{+(\textit{Some}+)}{\longrightarrow} G$:

$$(F_{achv}(C) \vee S_{achv}(C)) \longrightarrow S_{achv_{up-lim}}(G) \tag{A.112}$$

$$N_{achv}(C) \longrightarrow N_{achv_{up-lim}}(G) \tag{A.113}$$

$$Null_{achv}(C) \longrightarrow Null_{achv_{up-lim}}(G) \tag{A.114}$$

if $M = \textit{None}$, then $C \overset{+(\textit{None}+)}{\longrightarrow} G$:

$$(F_{achv}(C) \vee S_{achv}(C) \vee N_{achv}(C)) \longrightarrow N_{achv_{up-lim}}(G) \tag{A.115}$$

$$Null_{achv}(C) \longrightarrow Null_{achv_{up-lim}}(G) \tag{A.116}$$

if $M = Null$, then $C \overset{+(Null+)}{\longrightarrow} G$:

$$Null_{achv_{up-lim}}(G) \tag{A.117}$$

$C \overset{--}{\longrightarrow} G$:

$$F_{prvn}(C) \longrightarrow F_{prvn_{lo-lim}}(G) \tag{A.118}$$

$$S_{prvn}(C) \longrightarrow S_{prvn_{lo-lim}}(G) \tag{A.119}$$

$$N_{prvn}(C) \longrightarrow N_{prvn_{lo-lim}}(G) \tag{A.120}$$

$$Null_{prvn}(C) \longrightarrow Null_{prvn_{lo-lim}}(G) \tag{A.121}$$

$C \overset{-(M-)}{\longrightarrow} G$:

$$\text{if} \quad M = Full, \text{ then} \quad [C \overset{-(Full-)}{\longrightarrow} G] \equiv [C \overset{--}{\longrightarrow} G] \tag{A.122}$$

i.e. follow axioms: A.118 - A.121

if $M = Some$, then $C \overset{-(Some-)}{\longrightarrow} G$:

$$(F_{prvn}(C) \vee S_{prvn}(C)) \longrightarrow S_{prvn_{lo-lim}}(G) \tag{A.123}$$

$$N_{prvn}(C) \longrightarrow N_{prvn_{lo-lim}}(G) \tag{A.124}$$

$$Null_{prvn}(C) \longrightarrow Null_{prvn_{lo-lim}}(G) \tag{A.125}$$

if $M = None$, then $C \overset{-(None-)}{\longrightarrow} G$:

$$(F_{prvn}(C) \vee S_{prvn}(C) \vee N_{prvn}(C)) \longrightarrow N_{prvn_{lo-lim}}(G) \tag{A.126}$$

$$Null_{prvn}(C) \longrightarrow Null_{prvn_{lo-lim}}(G) \tag{A.127}$$

if $M = Null$, then $C \overset{-(Null-)}{\longrightarrow} G$:

$$Null_{prvn_{lo-lim}}(G) \tag{A.128}$$

Axioms for the *Asymmetric Conflict* C-G Lateral Relations

$C \overset{+-}{\longrightarrow} G$:

$$F_{achv}(C) \longrightarrow F_{prvn_{lo-lim}}(G) \tag{A.129}$$

$$S_{achv}(C) \longrightarrow S_{prvn_{lo-lim}}(G) \tag{A.130}$$

$$N_{achv}(C) \longrightarrow N_{prvn_{lo-lim}}(G) \tag{A.131}$$

$$Null_{achv}(C) \longrightarrow Null_{prvn_{lo-lim}}(G) \tag{A.132}$$

$C \overset{+(M-)}{\longrightarrow} G$:

$$\text{if} \quad M = Full, \text{ then} \quad [C \overset{+(Full-)}{\longrightarrow} G] \equiv [C \overset{+-}{\longrightarrow} G] \tag{A.133}$$

i.e. follow axioms: A.129 - A.132

if $M = Some$, then $C \overset{+(Some-)}{\longrightarrow} G$:

$$(F_{achv}(C) \vee S_{achv}(C)) \longrightarrow S_{prvn_{lo-lim}}(G) \tag{A.134}$$

$$N_{achv}(C) \longrightarrow N_{prvn_{lo-lim}}(G) \tag{A.135}$$

$$Null_{achv}(C) \longrightarrow Null_{prvn_{lo-lim}}(G) \tag{A.136}$$

if $M = None$, then $C \overset{+(None-)}{\longrightarrow} G$:

$$(F_{achv}(C) \vee S_{achv}(C) \vee N_{achv}(C)) \longrightarrow N_{prvn_{lo-lim}}(G) \tag{A.137}$$

$$Null_{achv}(C) \longrightarrow Null_{prvn_{lo-lim}}(G) \tag{A.138}$$

if $M = Null$, then $C \overset{+(Null-)}{\longrightarrow} G$:

$$Null_{prvn_{lo-lim}}(G) \tag{A.139}$$

$C \overset{-+}{\longrightarrow} G$:

$$F_{prvn}(C) \longrightarrow F_{achv_{up-lim}}(G) \tag{A.140}$$

$$S_{prvn}(C) \longrightarrow S_{achv_{up-lim}}(G) \tag{A.141}$$

$$N_{prvn}(C) \longrightarrow N_{achv_{up-lim}}(G) \tag{A.142}$$

$$Null_{prvn}(C) \longrightarrow Null_{achv_{up-lim}}(G) \tag{A.143}$$

$C \overset{-(M+)}{\longrightarrow} G$:

$$\text{if } M = Full\text{, then } [C \overset{-(Full+)}{\longrightarrow} G] \equiv [C \overset{-+}{\longrightarrow} G] \tag{A.144}$$

i.e. follow axioms: A.140 - A.143

if $M = Some$, then $C \overset{-(Some+)}{\longrightarrow} G$:

$$(F_{prvn}(C) \vee S_{prvn}(C)) \longrightarrow S_{achv_{up-lim}}(G) \tag{A.145}$$

$$N_{prvn}(C) \longrightarrow N_{achv_{up-lim}}(G) \tag{A.146}$$

$$Null_{prvn}(C) \longrightarrow Null_{achv_{up-lim}}(G) \tag{A.147}$$

if $M = None$, then $C \overset{-(None+)}{\longrightarrow} G$:

$$(F_{prvn}(C) \vee S_{prvn}(C) \vee N_{prvn}(C)) \longrightarrow N_{achv_{up-lim}}(G) \tag{A.148}$$

$$Null_{prvn}(C) \longrightarrow Null_{achv_{up-lim}}(G) \tag{A.149}$$

if $M = Null$, then $C \overset{-(Null+)}{\longrightarrow} G$:

$$Null_{achv_{up-lim}}(G) \tag{A.150}$$

Axioms for the *Symmetric Consistent* C-G Lateral Relations

$$C \overset{=}{\longrightarrow} G \quad \equiv \quad [C \overset{++}{\longrightarrow} G] \cup [C \overset{--}{\longrightarrow} G] \tag{A.151}$$

i.e. follow axioms: A.107 - A.110 and A.118 - A.121

$$C \xrightarrow{(=)} G \quad \equiv \quad [C \xrightarrow{+(M+)} G] \cup [C \xrightarrow{-(M-)} G] \tag{A.152}$$

i.e. follow axioms: A.111 - A.117 and A.122 - A.128

Axioms for the *Symmetric Conflict* C-G Lateral Relations

$$C \xrightarrow{\times} G \quad \equiv \quad [C \xrightarrow{+-} G] \cup [C \xrightarrow{-+} G] \tag{A.153}$$

i.e. follow axioms: A.129 - A.132 and A.140 - A.143

$$C \xrightarrow{(\times)} G \quad \equiv \quad [C \xrightarrow{+(M-)} G] \cup [C \xrightarrow{-(M+)} G] \tag{A.154}$$

i.e. follow axioms: A.133 - A.139 and A.144 - A.150

We, now, introduce the axioms for concluding the effect of multiple goal-to-goal (reduction or lateral) and constraint-to-goal (lateral) relations on a single goal node G_i. But first, let the achievement value predicate over the goal G_i be called $L_{achv}(G_i)$ where $L_{achv}(G_i) \in \{F_{achv}(G_i), S_{achv}(G_i), N_{achv}(G_i), Null_{achv}(G_i)\}$. Similarly, let the operationalization value predicate over G_i be $L_{opr}(G_i) \in \{F_{opr}(G_i), S_{opr}(G_i), N_{opr}(G_i), Null_{opr}(G_i)\}$, and the prevention value predicate over G_i be $L_{prvn}(G_i) \in \{F_{prvn}(G_i), S_{prvn}(G_i), N_{prvn}(G_i), Null_{prvn}(G_i)\}$. For each relation r, and where $r \in \mathcal{R}_{G-G_i}$ if r is a reduction or a G-G lateral relation that targets/ends-with G_i, or $r \in \mathcal{R}_{C-G_i}$ if r is a C-G lateral relation that targets/ends-with G_i, let the achievement, operationalization and prevention value predicates over G_i which results from the relation r be represented as $L_{achv_r}(G_i)$, $L_{opr_r}(G_i)$ and $L_{prvn_r}(G_i)$, respectively. Finally, let the initial values' predicates over G before the effect of the relation is applied be noted as $L_{achv_0}(G_i)$, $L_{opr_0}(G_i)$ and $L_{prvn_0}(G_i)$; and the final values' predicates over G after the effect of the relation be noted as $L_{achv_f}(G_i)$, $L_{opr_f}(G_i)$ and $L_{prvn_f}(G_i)$.

Axioms for concluding the effect of multiple Relations on a goal G_i

If $\mathcal{R}_{G-G_i} = \mathcal{R}_{C-G_i} = \emptyset$

i.e. if there exist no such relation $r \in \mathcal{R}$ such that r targetes/ends-with G_i

$$L_{opr_0}(G_i) \xrightarrow{=} L_{opr_f}(G_i) \quad \text{[then as per axiom A.103]} \tag{A.155}$$

$$L_{achv_0}(G_i) \xrightarrow{=} L_{achv_f}(G_i) \quad \text{[then as per axiom A.103]} \tag{A.156}$$

$$L_{prvn_0}(G_i) \xrightarrow{=} L_{prvn_f}(G_i) \quad \text{[then as per axiom A.103]} \tag{A.157}$$

If $\mathcal{R}_{G-G_i} \neq \emptyset$, then for each $r \in \mathcal{R}_{G-G_i}$

$$(L_{opr_0}(G_i), L_{opr_r}(G_i)) \xrightarrow{or} L_{opr_f}(G_i) \quad \text{[then as per axioms A.25-A.32]} \tag{A.158}$$

$$(L_{achv_0}(G_i), L_{achv_r}(G_i)) \xrightarrow{or} L_{achv_f}(G_i) \quad \text{[then as per axioms A.33-A.40]} \tag{A.159}$$

$$(L_{prvn_0}(G_i), L_{prvn_r}(G_i)) \xrightarrow{or} L_{prvn_f}(G_i) \quad \text{[then as per axioms A.41-A.48]} \tag{A.160}$$

If $\mathcal{R}_{C-G_i} \neq \emptyset$, then for each $r \in \mathcal{R}_{C-G_i}$

$$(L_{achv_0}(G_i), L_{achv_{up-lim_r}}(G_i)) \xrightarrow{and} L_{achv_f}(G_i) \quad \text{[then as per axioms A.9-A.16]} \tag{A.161}$$

$$(L_{prvn_0}(G_i), L_{prvn_{lo-lim_r}}(G_i)) \xrightarrow{and} L_{prvn_f}(G_i) \quad \text{[then as per axioms A.17-A.24]} \tag{A.162}$$

Appendix B

Soundness and Completeness of the Goals' Value-Labels Forward Propagation Algorithm

In this appendix, we prove the correctness and completeness for the goals' value-label forward propagation algorithm, *Label_GCM_Goals*, provided in Chapter 3 as Algorithm 3.1.

Let each of goal G_i's *Achv(G_i)*, *Opr(G_i)* and *Prvn(G_i)* be represented generically as a value function $V(G_i)$. And similar to what has been said earlier in the paper, that a value label assignment statement to the achievement value property of a goal G_i, for example, represented as $Achv(G_i) = L_{achv}$ where $L_{achv} \in \mathcal{L}$, a generic value label assignment statement for $V(G_i)$ will be represented here as $V(G_i) = L$. $L \in \mathcal{L}$, and $\mathcal{L}$ is the universal set of actual value labels which could be assigned to any of the value properties/functions of any goal, and given earlier as $\mathcal{L} = \{Full, Some, None, Null\}$, where $Full > Sume > None > Null$. It is important to remember that the value label assignment statements for the different value properties of goal nodes are also represented as predicates over goals. In addition, the value label assignment statements of value properties of constraint nodes are represented as predicates over constraints. Each of the $V(G_i) = L$ and $V(C_i) = L$ statements has a predicate that is equivalent to it.

Let $V(G) = L$ is said to be *deduced* from $(V(G_1) = L_{g1})$, $(V(G_2) = L_{g2})$, $\ldots (V(G_n) = L_{gn})$ and $(V(C_1) = L_{c1})$, $(V(C_2) = L_{c2})$, $\ldots (V(C_m) = L_{cm})$ by means of a Relation Axiom, one of the axioms listed in this appendix (A.1 - A.162), that leads to this deduction. For example, ($Achv(G) = Full$) can be deduced from ($Achv(G_1) = Full$) and ($Achv(G_2) = Full$) by means of the Relation Axiom (A.9), if and when ($(G_1, G_2) \stackrel{and}{\longrightarrow} G$) relation exists among G, G_1 and G_2.

On the other hand, let $V(G) = L$ is said to be *derives* from ($V(G_1) = L_{g1}$), ($V(G_2) = L_{g2}$), $\ldots$ ($V(G_n) = L_{gn}$) and ($V(C_1) = L_{c1}$), ($V(C_2) = L_{c2}$), $\ldots$ ($V(C_{cm}) = L_m$) by means of a relation's Propagation Rule, one of the propagation rules listed earlier in the paper (3.1 - 3.45), that leads to this derivation. For example, ($Prvn(G)$=*Some*) derives from ($Achv(G_1)$=*Full*) by means of the propagation rule (3.24), if and when ($G_1 \stackrel{+(Some-)}{\longrightarrow} G$) relation exists among G and G_1.

The following Lemma states that the propagation rules listed in the paper are nothing but a translation, or more precisely an aggregation/generalization, of the axioms listed in this appendix.

Lemma B.0.1: $V(G) = L$ derives *from* ($V(G_1) = L_{g1}$), ($V(G_2) = L_{g2}$), $\ldots$ ($V(G_n) = L_{gn}$) *and/or* ($V(C_1) = L_{c1}$), ($V(C_2) = L_{c2}$), $\ldots$ ($V(C_m) = L_{cm}$) *by means of the Propagation Rules (3.1 - 3.45), if and only if* $V(G) = L$ *can be* deduced *from* ($V(G_1) = L_{g1}$), ($V(G_2) = L_{g2}$), $\ldots$ ($V(G_n) = L_{gn}$) *and/or* ($V(C_1) = L_{c1}$), ($V(C_2) = L_{c2}$), $\ldots$ ($V(C_m) = L_{cm}$) *by applying one of the Relation Axioms (A.1 - A.162).*

***Proof*.** This Lemma states that propagation relation rules (3.1 - 3.45) are straightforward translation of the the relation axioms (A.1 - A.162). To prove the Lemma, we will give the proof for only two of these propagation rules, and the rest can be proved easily following the same logic.

Firstly, for the Propagation Rule (3.2), which states that "$(G_1, G_2) \stackrel{and}{\longrightarrow} G : Achv(G) = \min\{Achv(G_1), Achv(G_2)\}$" :

From before, we said that any goal G_j will have, at any single point of time, an achievement value property statement of ($Achv(G_j) = L_{achv}$), where $L_{achv} \in \mathcal{L}$ and $\mathcal{L}=\{F, S, N, Null\}$. So, only one of these statements will be true for G_j at any single point of time: ($Achv(G_j)$=F), ($Achv(G_j)$=S), ($Achv(G_j)$=N), or ($Achv(G_j)$=$Null$).

if. With two achievement value statements for two different goals, G_1 and G_2, at the left side of the rule, and each has 4 possible labels assigned to them, then there are 2^4, or 16, possibilities for the combination of $Achv(G_1)$ and $Achv(G_2)$. For each combination, we shall show that a value for $Achv(G)$ derives by applying the rule, matches one-to-one the value for it that is deduced by applying one axiom (among the ones listed earlier: A.1 - A.162).

If ($Achv(G_1)$=F) and ($Achv(G_2)$=F) then ($Achv(G)$=F) is derived, using the propagation rule (3.2), since $\min\{F, F\}$=F. This matches one-to-one the fact that from ($Achv(G_1)$=F) and ($Achv(G_2)$=F) axiom (A.9) is applied so that ($Achv(G)$=F) is deduced. Similarly, if ($Achv(G_1)$=S) and ($Achv(G_2)$=S) then ($Achv(G)$=S) is derived, and this matches one-to-one the fact that from ($Achv(G_1)$=S) and ($Achv(G_2)$=S) axiom (A.10) is applied so that ($Achv(G) = S$) is deduced; and if ($Achv(G_1)$=N) and ($Achv(G_2)$=N) then ($Achv(G)$=N) is derived, then this matches one-to-one the fact that from ($Achv(G_1)$=N) and ($Achv(G_2)$=N) axiom (A.13) is applied so that ($Achv(G)$=N) is deduced.

But, if ($Achv(G_1)$=F) while ($Achv(G_2)$=S), or vice versa, then ($Achv(G)$=S) is derived since $\min\{F, S\}$=S. This matches one-to-one the fact that from ($Achv(G_1)$=F) and ($Achv(G_2)$=S) axiom (A.11), or axiom (A.12) if vice versa, is applied so that ($Achv(G)$=S) is deduced. Similarly, if ($Achv(G_1)$=N) while (($Achv(G_2)$=F) or ($Achv(G_2)$=S)), or the opposite (i.e.($Achv(G_2)$=N) while (($Achv(G_1)$=F) or ($Achv(G_1)$=S))), then ($Achv(G)$=N) is derived because $\min\{F, S, N\}$=N. This matches one-to-one the fact that from ($Achv(G_1)$=N) and (($Achv(G_2)$=F) or ($Achv(G_2)$=P)) axiom (A.14), or axiom (A.15) if the opposite, is applied so that ($Achv(G)$=N) is deduced.

Finally, we covered so far nine possibilities. The other remaining seven possibilities, which we still need to cover, are the ones where either ($Achv(G_1)$= $Null$) or ($Achv(G_2)$=$Null$), or both. In such cases, ($Achv(G)$=$Null$) is derived, because it represent the minimum in the order of all value labels exist in $\mathcal{L}$, and this matches one-to-one the fact that, in all these cases, axiom (A.16) is applied so that ($Achv(G)$=$Null$) is deduced.

only if. The value of $Achv(G)$ deduced by applying one of the axioms (A.1 - A.16) matches one-to-one the value derives by means of propagation rule (3.2).

Secondly, for the Propagation Rule (3.24), which states that
"$G_1 \xrightarrow{+(M-)} G : \quad Prvn(G) = \min\{Achv(G_1), M\}$" :

if. Since the relation's Modifier label $M \in \mathcal{L} = \{Full, Some, None, Null\} = \{F, S, N, Null\}$, then this rule covers four forms of the "$+(M-)$" relation. One form for each possible value of M. We also know from before that, at any single point of time, the prevention value property statement of any goal G_j is given as $(Prvn(G_j) = L_{prvn})$, where $L_{prvn} \in \mathcal{L}$ and $\mathcal{L} = \{F, S, N, Null\}$. So, only one of these statements will be true for G_j at any single point of time: $(Prvn(G_j) = F)$, $(Prvn(G_j) = S)$, $(Prvn(G_j) = N)$, or $(Prvn(G_j) = Null)$. Similarly, at any single point of time, the achievement value property statement of any goal G_j is given as $(Achv(G_j) = L_{achv})$, where $L_{achv} \in \mathcal{L}$ and $\mathcal{L}=\{F, S, N, Null\}$. Therefore, only one of these statements will be true for G_j at any single point of time: $(Achv(G_j)=F)$, $(Achv(G_j)=S)$, $(Achv(G_j)=N)$, or $(Achv(G_j)=Null)$. This means that for each of the four forms of the "$+(M-)$" relation, based on the M value, there are four possible values of $Achv(G_1)$ to cover, to deduce a value for $Prvn(G)$ by means of the rule.

First, when M=$Full$, then the relationship is $G_1 \stackrel{+(Full-)}{\longrightarrow} G$, or $G_1 \stackrel{+-}{\longrightarrow} G$ by the equivalence stated in axiom (A.85): Now, if $(Achv(G_1)=F)$, then $(Prvn(G)= F)$ is derived by means of rule (3.24), since $\min\{F, F\} = F$. This matches one-to-one the fact that from $(Achv(G_1) = F)$ axiom (A.81) is applied so that $(Prvn(G) = F)$ is deduced. If $(Achv(G_1) = S)$, then $(Prvn(G) = S)$ is derived, since $\min\{S, F\} = S$. This matches one-to-one the fact that from $(Achv(G_1)=S)$ axiom (A.82) is applied so that $(Prvn(G)=S)$ is deduced. And if $(Achv(G_1)=N)$ $[(Achv(G_1)=Null)]$, then $(Prvn(G)=N)$ $[(Prvn(G)=Null)]$ is derived, since $\min\{N, F\}=N$ $[\min\{Null, F\}=Null]$. This matches one-to-one the fact that from $(Achv(G_1)=N)$ $[(Achv(G_1)=Null)]$ axiom (A.83) [axiom (A.84)] is applied so that $(Prvn(G)=N)$ $[(Prvn(G)=Null)]$ is deduced.

Second, when M=$Some$, the relationship is "$G_1 \stackrel{+(Some-)}{\longrightarrow} G$: Here, if $(Achv(G_1)= F)$, or if $(Achv(G_1)=S)$, then $(Prvn(G)=S)$ is derived by means of rule (3.24), since $\min\{F, S\}=S$ and $\min\{S, S\}=S$. This matches one-to-one the fact that from $(Achv(G_1) = F)$, or from $(Achv(G_1) = S)$, axiom (A.86) is applied so that $(Prvn(G)=S)$ is deduced. And if $(Achv(G_1) = N)$ $[(Achv(G_1) = Null)]$, then $(Prvn(G) = N)$ $[(Prvn(G) = Null)]$ is derived, because $\min\{N, S\} = N$ $[\min\{Null, S\}=Null]$. This matches one-to-one the fact that from $(Achv(G_1)= N)$ $[(Achv(G_1)=Null)]$ axiom (A.87) [axiom (A.88)] is applied so that $(Prvn(G)= N)$ $[(Prvn(G)=Null)]$ is deduced.

Third, when $M = None$, the relationship is "$G_1 \stackrel{+(None-)}{\longrightarrow} G$: If $(Achv(G_1)=F)$, $(Achv(G_1)=S)$, or $(Achv(G_1)=N)$, then $(Prvn(G)=N)$ is derived by means of rule (3.24), since $\min\{F, N\} = \min\{S, N\} = \min\{N, N\} = N$. This matches one-to-one the fact that from $(Achv(G_1)=F)$, $(Achv(G_1)=S)$, or $(Achv(G_1)=$

N), axiom (A.89) is applied so that ($Prvn(G) = N$) is deduced. And if ($Achv(G_1)$=$Null$), then ($Prvn(G)$=$Null$) is derived, because $\min\{Null, N\}$= $Null$. This matches one-to-one the fact that from ($Achv(G_1)$=$Null$) axiom (A.90) is applied so that ($Prvn(G)$=$Null$) is deduced.

Finally, when $M = Null$, the relationship is "$G_1 \overset{+(Null-)}{\longrightarrow} G$: If ($Achv(G_1)$= F), ($Achv(G_1)$=S), ($Achv(G_1)$=N), or ($Achv(G_1)$=$Null$) then ($Prvn(G)$= $Null$) is derived by means of rule (3.24), since $\min\{F, Null\}$=$\min\{S, Null\}$ = $\min\{N, Null\}$=$\min\{Null, Null\}$=$Null$. This matches one-to-one the fact that from ($Achv(G_1)$=F), ($Achv(G_1)$=S), ($Achv(G_1)$=N), or ($Achv(G_1)$=$Null$), axiom (A.91) is applied so that ($Prvn(G)$=$Null$) is deduced.

only if. The value of $Prvn(G)$ deduced by applying one of the axioms (A.81 - A.91) matches one-to-one the value derives by means of propagation rule (3.24).

□

Now, we state the correctness and completeness theorem for the goals' value-label forward propagation algorithm, *Label_GCM_Goals*, provided in Chapter 3 as Algorithm 3.1.

Theorem B.0.2: *Let Final_G be the returned value-label-array by the algorithm Label_GCM_Goals (GCM_Graph $\langle \mathcal{G}, \mathcal{C}, \mathcal{R} \rangle$, c-value-label-array Initial_C, value-label-array Initial_G). The value statement ($V(G_i)$=L_i) is true in Final_G if and only if the statement ($V(G_i)$=L_i) can be deduced from Initial_G and Initial_C by applying the relation axioms (A.1 - A.162).*

Proof. Let the "deduced from $Initial\_G$ and $Initial\_C$ in k steps" be defined inductively: (1) a value-label assignment statement/assertion in $Initial\_G$ can be deduced from $Initial\_G$ in 0 steps; and (2) a value-label assignment statement/assertion can be deduced from $Initial\_G$ and $Initial\_C$ in up to $k+1$ steps if either it can be deduced from $Initial\_G$ and $Initial\_C$ in up to k steps or it can be deduced by applying a relation axiom to some assertions, all of which deduced from $Initial\_G$ and $Initial\_C$ in up to k steps.

Let $Current\_G_k$ be the value label assignments statements of $Current\_G$ after k repeat-until loops (of the algorithm). We show that the value label assignment statement ($V(G_i) = L_i$) is true in $Current\_G_k$ if and only if ($V(G_i) = L_i$) can be deduced from $Initial\_G$ and $Initial\_C$ in up to k steps.

The thesis follows from the fact that $Final\_G = Current\_G_k$ for some k. We then reason by induction on k: (1) the base case, in which $k = 0$, is obvious as $Current\_G_0 = Initial\_G$; and (2) by inductive hypothesis, we assume the thesis for k and we prove it for $k + 1$.

If. If the value assignment statement $(V(G_i)=L_i)$ is deduced from $Initial\_G$ and $Initial\_C$ in up to $k + 1$ steps, then $(V(G_i) = L_i)$ is deduced by applying one of the relation axioms (A.1 - A.162) to some value assignment assertions $(V(G_1) = L_{g1})$, $(V(G_2) = L_{g2})$, $\ldots(V(G_n) = L_{gn})$, and/or $(V(C_1) = L_{c1})$, $(V(C_2)=L_{c2})$, $\ldots(V(C_m)=L_{cm})$, all of which could be deduced from $Initial\_G$ and $Initial\_C$ in up to k steps. By inductive hypothesis, $(V(G_1)= L_{g1})$, $(V(G_2) = L_{g2})$, $\ldots(V(G_n) = L_n)$ and $(V(C_1) = L_{c1})$, $(V(C_2) = L_{c2})$, $\ldots(V(C_m)=L_{cm})$ occur in $Current\_G_k$. By Lemma B.0.1, $(V(G_i)=L_i)$ derives from $(V(G_1)=L_{g1})$, $(V(G_2)=L_{g2})$, $\ldots(V(G_n)=L_{gn})$, and/or $(V(C_1)= L_{c1})$, $(V(C_2)=L_{c2})$, $\ldots(V(C_m)=L_{cm})$, by means of the propagation rules (3.1 - 3.45). Thus, if $V(G_i)$ is one of $Opr(G_i)$, $Arch(G_i)$ or $Prvn(G_i)$, then L_i is a value label assigned to $V(G_i)$ in $Current\_G_{k+1}$ through one of the statements in lines 15, 16 or 17 [or lines 15, 25 or 26 if the goal is affected by constraints] of the Label_GCM_Goals algorithm, respectively. These algorithm lines, as directly stated in the algorithm, uses the propagation rules (3.1 - 3.45) to conclude such value assignment. Thus, the value label assignment statement $(V(G_i)=L_i)$ is true in $Current\_G_{k+1}$.

Only if. Let statement $(V(G_i)=L_i)$ be true in $Current\_G_{k+1}$. If $(V(G_i)=L_i)$ is also true in $Current\_G_k$, then, by inductive hypothesis, $(V(G_i)=L_i)$ can be deduced from $Initial\_G$ and $Initial\_C$ in up to k steps, and therefore in $k+1$ steps. Otherwise, if $(V(G_1)=L_{g1})$, $(V(G_2)=L_{g2})$, $\ldots(V(G_n)=L_{gn})$ and $(V(C_1)=L_{c1})$, $(V(C_2)=L_{c2})$, $\ldots(V(C_m)=L_{cm})$ value assignment statements are true in $Current\_G_k$, and from which the value statement $(V(G_i) = L_i)$ is derived by applying one of the propagation rules (3.1 - 3.45). Then, by inductive hypothesis, $(V(G_1) = L_{g1})$, $(V(G_2) = L_{g2})$, $\ldots(V(G_n) = L_{gn})$ and $(V(C_1) = L_{c1})$, $(V(C_2) = L_{c2})$, $\ldots(V(C_m) = L_{cm})$ can be deduced from $Initial\_G$ and $Initial\_C$ in up to k steps. And by Lemma B.0.1, $(V(G_i) \geq L_i)$ can be deduced from $(V(G_1) = L_{g1})$, $(V(G_2) = L_{g2})$, $\ldots(V(G_n) = L_{gn})$ and $(V(C_1)=L_{c1})$, $(V(C_2)=L_{c2})$, $\ldots(V(C_m)=L_{cm})$ by the application of one the relation axioms (A.1 - A.162). Thus, the value label assignment statement $(V(G_i) = L_i)$ can be deduced from $Initial\_G$ and $Initial\_C$ in up to $k + 1$

steps. □

By Theorem B.0.2, the value label assignments for all the goals' value properties (achievement, operationalization and prevention) which are returned by the Label_GCM_Goals (GCM_Graph $\langle \mathcal{G}, \mathcal{C}, \mathcal{R} \rangle$, c-value-label-array Initial_C, value-label-array Initial_G) algorithm, Algorithm 3.1, are the correct, complete and final value label assignment statements that could be deduced from $Initial\_G$ and $Initial\_C$.

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