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Stability-based fire resistance duration of unbraced steel frames

Unbraced steel frames

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Abstract

Purpose – The collapse of a structure resulting from the instability of steel frames due to fire is the worst failure mode to consider in fire-structural engineering, and should be avoided. A new method is proposed for estimating the minimum possible duration of a fire event that could result in the instability of an unbraced steel frame.

Design/methodology/approach – The proposed method is in the form of a constrained minimization problem that determines the worst case fire scenario that can cause instability of a structure, and is solved using nonlinear constrained mathematical programming algorithms. The formulation is demonstrated via a numerical example.

Findings – For frames subjected to fire events modelled with monotonically increasing fire curves, the worst case fire causing instability of a frame is always one where all of the compartments catch fire at the same time. For frames subjected to fire events where fire curves decay, the minimization problem must be solved rigorously. The results are significantly affected by the fire curves and amount of insulation applied to each member.

Originality/value – The proposed method is an extension of a method previously established by Xu *et al.* (2018) to assess the stability of unbraced steel frames subjected to elevated member temperatures. The previous method does not consider fire duration and heat transfer mechanics, which are included in the proposed method. The proposed method is potentially useful for designers in conducting fire scenario analysis in the performance-based design of structures.

Keywords Fire, Minimization, Stability, Scenario, Steel frames, Unbraced

Paper type Research paper

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1. Introduction

Building fires are not uncommon, and often result in loss of life and property damage. At worst, building fires can result in structural collapse, such as the September 11, 2001, collapses of World Trade Centers 1, 2 and 7 (Kodur, 2003; Kodur, 2004). The instability of a structure subjected to fire is the least desirable failure mode against fire and should be avoided in design. To mitigate the risks of fire towards structural damage, building codes requirements have traditionally been based on the fire tests of individual members. However, it has been demonstrated through the Cardington Tests (Armer *et al.*, 1996) that buildings perform better as a whole than when considering their individual members. As such, there is a need for performance-based methods in determining the fire resistance of structural systems under the instability failure mode.

A new method is presented to estimate the minimum fire resistance of an unbraced steel frame based on its stability failure mode. The minimum fire resistance herein is defined as the shortest duration of fire, in minutes, that can result in instability of the frame, with all

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possible fire scenarios considered. Since the location of fire and spread mechanics are highly unpredictable, there are many different fire scenarios that can similarly result in instability, and the critical scenario is the scenario that most quickly causes instability. As such, identifying the critical scenario determines the minimum fire resistance. This is accomplished using the proposed non-linear constrained minimization problem, which minimizes the time of fire in the frame required to cause instability. For frames subjected to standard fire curves, the solution to the minimization problem is trivial, where all compartments catch fire at the same time. For frames subjected to decaying fires, the minimization problem needs to be solved rigorously. The proposed formulation is demonstrated via numerical example, whereby the worst case duration of fire causing instability was determined for a four bay frame subjected to parametric fires. The analysis also shows that varying the amounts of insulation applied in highly combustible compartments significantly impacts the worst case scenario.

2. Background

Storey-based stability refers to the overall performance of a structural system against destabilizing effects. Yura (1971) concluded that storey-based buckling of an unbraced frame must occur with all the columns in a storey buckling in a lateral sway mode simultaneously, and thus the contribution of each member towards the frame stability must be considered. LeMessurier (1977) and Aristizabal-Ochoa (1997) developed this idea further by proposing matrix methods of frame analysis. Xu (2001) later introduced the concept of variable loading by proposing a minimization problem for determining the worst case axial loading scenario causing the instability of an unbraced frame. Variable loading refers to the investigation of different loading scenarios where all of the loads are independent of each other, and involves identifying the worst case loading scenario causing failure. Recently, Xu *et al.* (2018) extended the method of variable loading in storey frames towards fire-structural analysis in determining the worst-case member temperature scenario causing instability. In this case, the variable load under consideration is temperature loading instead of axial loading. However, fire resistance is more conveniently and universally measured in units of time rather than temperature. This paper presents a new minimization problem for estimating the minimum duration of fire resulting in the instability of an unbraced steel storey frame. It may be viewed as an extension to the member-temperature based method presented by Xu *et al.* (2018).

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3. Proposed method

The primary difference between the proposed method and the one in Xu *et al.* (2018) is that the variables in Xu *et al.* (2018) are the member temperatures of the frame, whereas the variables in the proposed method are the fire durations in the bays of the frame. Moreover, the proposed method considers the heating of all members in the frame whereas the Xu *et al.* method (2018) only considers the heating of beams. Consider the 2D planar storey frame with n bays and $(n + 1)$ columns, shown in Figure (1). Note that the out-of-plane direction is assumed to be adequately restrained.

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The frame in Figure (1) is subjected to gravity loads, G_b , in addition to compartment fires. Each bay is assigned a fire curve and duration, t_j , over which the fire in that bay has transpired. Note that multiple bays may be part of the same compartment, in which case the fire curves in these bays may be identical. The indices i and j correspond to the numbering of the columns and beams, respectively. Similarly, the subscripts c and b correspond to columns and beams, respectively. The elastic modulus, moment of inertia and length of each member are E , I , and L , respectively. All connections are generalized rotational springs, and

the column lower and upper end fixity factors for each column are $r_{l,c,i}$ and $r_{u,c,i}$ respectively. The end fixity factors were originally defined by [Monforton and Wu \(1963\)](#) and are shown in equation (1). Unbraced steel frames

$$r_{u,c,i} = \frac{1}{1 + 3E_{c,i}I_{c,i}/R_{u,c,i}L_{c,i}}; \quad R_{u,c,i} = \sum_{j_u=1}^{m_u} R_{i,j_u} \quad (1a)$$

$$r_{l,c,i} = \frac{1}{1 + 3E_{c,i}I_{c,i}/R_{l,c,i}L_{c,i}}; \quad R_{u,c,i} = \sum_{j_l=1}^{m_l} R_{i,j_l} \quad (1b)$$

where $R_{u,c,i}$ and $R_{l,c,i}$ are the rotational stiffness of the upper and lower end connections, respectively. m_u and m_l are the number of beams connected to the upper and lower ends of column i , respectively. The rotational resistance provided by beam j to column i at the corresponding end, $R_{i,j}$ can be calculated using [equation \(2\)](#) as follows:

$$R_{i,j} = \frac{6E_{b,j}I_{b,j}r_{b,j,N}}{L_{b,j}} \left[\frac{2 + vr_{b,j,F}}{4 - r_{b,j,N}r_{b,j,F}} \right] \quad (2)$$

where $r_{b,j,N}$ and $r_{b,j,F}$ are the end-fixity factors for the near and far ends of beam j connected to column i . The near end of beam j is the end that is connected to column i , while the far end is the opposite end. Let v be the ratio of rotation of the near end of the beam, θ_N , to the rotation of the far end of the beam, θ_F . It was demonstrated by [Xu and Liu \(2002\)](#) that accurate estimations of results can be obtained by assuming the case of asymmetric buckling, $v = 1$, for unbraced partially restrained frames.

3.1 Assumptions

Based on the fire duration in the bay, t_j , the temperature of the fire in the bay, $T_{f,j}$, is calculated from an assumed fire curve. The temperature of each member can be calculated based on either traditional heat transfer principles and using an incremental method ([Pettersson et al., 1976](#)) or by using simplified methods such as the one in ([Dwaikat and Kodur, 2013](#)). More detailed analyses such as the use of finite elements may also be used, and it is emphasized that the proposed method can be used in conjunction with different mathematical models that specify the temperature of a member with respect to the duration of fire. The degradation of a member subjected to elevated temperatures is commonly modeled via a reduction to its elastic modulus and yield stress. Owing to its

Figure 1.
General unbraced
storey frame
subjected to fire
loading

popularity and comprehensiveness, the Eurocode 3 (BSI, 2005) model is adapted for this study. Its general stress-strain curve contains four segments, shown in Figure (2):

- (1) linear-elastic behavior;
- (2) non-linear behaviour modelled using a tangent modulus;
- (3) perfectly plastic behavior; and
- (4) failure.

In Figure (2), the proportional limit is denoted by the point (ε_p, f_p) , and the onset of yielding is defined by the point (ε_y, f_y) . E_a is the elastic modulus in the linear portion of the curve. The values of these quantities are given in Eurocode 3 for any temperature, T (BSI, 2005). For the proposed method, member failure is assumed to occur once f_y is reached. Based on this model, the elastic modulus of steel in a column is assumed to be a function of the axial load, P , and temperature, T , using the relation in equation (3).

$$E(P, T) = \lambda_T E_0;$$

$$\lambda_T = \frac{1}{E_0} \begin{cases} E_a(T) & \sigma \leq f_p(T) \\ (b/a)(\varepsilon_y(T) - \varepsilon) \left[a^2 - (\varepsilon_y(T) - \varepsilon)^2 \right]^{-0.5}, & f_p(T) < \sigma < f_y(T) \\ 0 & \sigma \geq f_y(T) \end{cases} \quad (3)$$

E_0 is the elastic modulus at ambient temperature equal to 200 GPa, and λ_T is the reduction factor. The strain ε and coefficients a and b are given in Eurocode 3 (BSI, 2005). The stress in a column for the proposed method may be taken as $P/A_{c,i}$, where the $A_{c,i}$ is the cross-sectional area of the column. The reduced elastic modulus monotonically decreases with increasing temperature, and is assumed to reach zero at 1,200°C. The temperatures throughout each member are assumed to be uniform for the reason of simplification. If a member is heated in such a way that would cause thermal gradients, the uniform temperature can be assumed to be the maximum temperature in the gradient. Particularly for the case of interior columns situated between two non-identical fires, the maximum of the column temperatures computed using either of the adjacent fire curves may be taken as the

Figure 2.
Eurocode 3 stress-strain model for steel at elevated temperature (BSI, 2005)

column temperature. Note that doing so also neglects the effects of thermal bowing. However, it has been shown by [Usmani *et al.* \(2001\)](#) that thermal bowing can be treated similar in a similar way to column initial imperfections, which only affects deformation analysis rather than stability analysis ([Ziemian, 2010](#)).

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To further simplify the analysis process, the fire curves are assumed to be independent of each other. In reality, heat transfer between adjacent compartments may accelerate or decelerate the development of the fires in either compartment. Although this effect is significant when walls collapse and allow radiation to occur between compartments, it can be assumed to be negligible if the transfer of flame and ignition between adjacent compartments is assumed to occur via small passages, eliminating the transfer of heat via radiation. Finally, the assumptions of the Euler-Bernoulli beam theory are assumed to apply.

3.2 Stability equation

The lateral stability of the frame in [Figure \(1\)](#) can be assessed by computing its lateral stiffness, which is a measure of the ability of the frame to resist lateral deformations and maintain the stability when subjected to applied loads. If the contribution to the lateral stiffness from each column is $S_{c,i}$, then the storey lateral stiffness of the frame, ΣS , subjected to elevated temperatures can be calculated using [equation \(4\)](#), which was presented in [Xu *et al.* \(2018\)](#). Note that the frame has $n + 1$ columns and n bays.

$$\Sigma S = \sum_{i=1}^{n+1} S_{c,i} = \sum_{i=1}^{n+1} \left[\frac{12E_{c,i}I_{c,i}}{L_{c,i}^3} \beta_{c,i}(\varphi_{c,i}, r_{l,c,i}, r_{u,c,i}) \right] \quad (4)$$

where $\beta_{c,i}$ is a modifier accounting for stiffness degradation due to second-order effects, and expressed in [equation \(5\)](#) ([Xu, 2001](#)).

$$\begin{aligned} \beta_{c,i}(\varphi_{c,i}, r_{l,c,i}, r_{u,c,i}) &= \frac{\varphi_{c,i}^3}{12} \frac{a_1 \varphi_{c,i} \cos \varphi_{c,i} + a_2 \sin \varphi_{c,i}}{18r_{l,c,i}r_{u,c,i} - a_3 \cos \varphi_{c,i} + (a_1 - a_2) \varphi_{c,i} \sin \varphi_{c,i}} \\ a_1 &= 3[r_{l,c,i}(1 - r_{u,c,i}) + r_{u,c,i}(1 - r_{l,c,i})] \\ a_2 &= 9r_{l,c,i}r_{u,c,i} - (1 - r_{l,c,i})(1 - r_{u,c,i})\varphi_{c,i}^2 \\ a_3 &= 18r_{l,c,i}r_{u,c,i} + a_1\varphi_{c,i}^2 \end{aligned} \quad (5)$$

where $\phi_{c,i}$ is equal to $\pi\sqrt{P_i/P_{ei}}$ in which $P_i = G_i + H_i$ is the total axial load in column i caused by the gravity load G_i and axial load induced by restraint against thermal strains, H_i . P_{ei} is the Euler buckling load given in [equation \(6\)](#).

$$P_{ei} = \frac{\pi^2 E_{c,i} I_{c,i}}{L_{c,i}^2} \quad (6)$$

H_i may be taken as zero where there are no physical restraints against thermal strains, or calculated using [equation \(7\)](#) ([Xu and Zhuang, 2014](#)). In using [equation \(7\)](#) it is assumed that the different axial shortenings among columns in the same storey of the frame are ignored ([Xu and Zhuang, 2014](#)).

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$$H_i = \frac{G_i + k_i L_{c,i} \int \alpha(T_{c,i}) dT_{c,i}}{1 + \frac{k_i L_{c,i} (E_0 - E_{c,i})}{A_{c,i} E_0 E_{c,i}}} - G_i \quad (7)$$

Where k_i is the equivalent spring stiffness of the column restraint against axial strains and can be computed in (Zhuang, 2013), α is the coefficient of thermal expansion given in equation (8) (Kodur and Hamarthy, 2002) and T_0 is the ambient temperature and may be taken as 20°C.

$$\alpha(T_{c,i}) = (0.004T_{c,i} + 12) \times 10^{-6} \text{ } ^\circ\text{C}^{-1} \quad (8)$$

It is important to note that since both H_i and P_{ei} depend on the elastic modulus, $\phi_{c,i}$ and $\beta_{c,i}$ are also both functions of the column temperature. Note that equation (4) is applicable for values of P_i greater than zero (compression) and less than the yielding load, P_{yi} , and rotational buckling limit, P_{bi} , of the column given in equation (9).

$$0 \leq P_i \leq P_{ui} = \min\{P_{yi}, P_{bi}\} \quad \forall i = 1, 2, \dots, n+1 \quad (9a)$$

$$P_{yi} = f_y A_{c,i} \quad (9b)$$

$$P_{bi} = \frac{\pi^2 E_{c,i} I_{c,i}}{(K_{c,i} L_{c,i})^2} \quad (9c)$$

The equivalent column length factor, $K_{c,i}$, may be approximated in equation (10) (Xu, 2003).

$$K_{c,i} = \sqrt{\frac{(\pi^2 + (6 - \pi^2)r_{u,c,i}) \times (\pi^2 + (6 - \pi^2)r_{l,c,i})}{(\pi^2 + (12 - \pi^2)r_{u,c,i}) \times (\pi^2 + (12 - \pi^2)r_{l,c,i})}} \quad (10)$$

By inspection of equation (4), $\beta_{c,i} < 0$ results in the column having a negative contribution to the lateral stiffness of the frame. When $\beta_{c,i} < 0$ the column relies on the lateral stiffness of other columns in the frame to maintain lateral stability (Xu, 2001). Based on the concept of storey-based buckling, the frame is laterally stable if and only if $\Sigma S > 0$. Thus, when $\Sigma S = 0$ the frame becomes laterally unstable because it cannot resist any lateral load. If $\Sigma S < 0$ then the frame has already become unstable before the given scenario is realized. Note that for the unbraced frames covered within the scope of this method, generally the governing stability failure mode is sway buckling corresponding to $\Sigma S = 0$, which occurs prior to or simultaneously with the rotational buckling mode of individual columns (Zhuang, 2013; Ziemian, 2010), with exception to the case where P_{yi} in equation (9) is reached first. In any case, the analysis of the sway buckling mode via the prescription of the upper bound load, P_{ui} , is sufficient to address the issue of instability most pertinent to unbraced storey frames.

3.3 Minimization problem

A minimization problem can be formulated to determine the worst case duration of fire resulting in lateral instability, presented in equation (11). The objective function in equation

(11a) minimizes the duration of fire in the frame, and is subject to two stability constraints in equations (11 b) and (11c), in addition to four logical constraints in equations (11d)-(11g). Unbraced steel frames

$$\text{minimize } F = \max_{j \in [1, n]} (t_j) \quad (11a)$$

subject to:

$$\Sigma S = \sum_{i=1}^n S_{c,i} = \sum_{i=1}^n \left[\frac{12E_{c,i}I_{c,i}}{L_{c,i}^3} \beta_i \right] = 0 \quad (11b)$$

$$P_i \leq P_{ui} \quad \forall i = 1, 2, \dots, n+1 \quad (11c)$$

$$t_j \geq 0; \quad \forall j = 1, 2, \dots, n \quad (11d)$$

$$t_j - \text{median}\{t_r, t_j, t_s\} \geq 0; \quad \forall 1 \leq r < j < s \leq n \quad \forall 2 \leq j \leq n-1 \quad (11e)$$

$$t_j = t_k \quad \forall \{j, k\} \in C_x | 1 \leq j, k \leq n; \quad 1 \leq x \leq n_x \quad (11f)$$

$$t_j t_{j+1} \left(\left| t_j - t_{j+1} \right| - RR_{j,j+1} \right) \geq 0; \quad 1 \leq j \leq n-1 \quad (11g)$$

In equation (11a), the frame fire duration, F , is the duration of a fire event in the n -bay frame. The bay fire duration, t_j , is the duration of fire within bay j during the fire event. The variables in the minimization problem are the bay fire durations. That is, the bay fire durations are independent on any other variables and any combination of the bay fire durations satisfying the constraints is valid for consideration in the objective function. As will be shown, from the results of the minimization problem, the possible development of a fire event leading to the worst case scenario can be postulated. This approach is similar to the philosophy of performance-based fire resistance design, which often aims to satisfy performance requirements regardless of how the fires develop due to their variable nature. Anyway, depending on the combinations of the bay fire durations, the originating bay of the fire can be identified via $F = t_j$. Since fire takes time to spread to other compartments, $F \geq t_j$ in all other bays. As there are many different fire scenarios that can similarly result in frame instability (i.e. $\Sigma S = 0$) and the duration associated with each of these fire scenarios can be different, the worst-case fire scenario can be defined as the combination of possible bay fire durations, t_j , yielding the lowest frame fire duration, F , and satisfying $\Sigma S = 0$. Thus, the instability of the frame is presented as the stability constraint, $\Sigma S = 0$, in equation (11 b). Since the lateral stiffness equation in equation (11 b) is only applicable with $P_i \leq P_{ui}$ from equation (9), the additional stability constraint in equation (11c) must be included. Additionally, the fire duration, t_j must be non-negative in equation (11d).

3.3.1 Single-Fire constraint. As fires usually spread via adjacent compartments, an optional “single-fire constraint” in equation (11e) similar to the one proposed in Xu *et al.* (2018) can be introduced. In applying this constraint, the possible scenarios to consider are

limited to fires initiated at a single origin. It is normally unlikely for multiple fires to start in the same building at the same time.

As mentioned previously, the fire duration in the originating bay of the fire is the highest in the frame. Also, the durations of fire in downstream bays should be equal or less than the durations of fire in their relative upstream bays from the originating location of the fire. To render all other fire scenarios inadmissible, the domain of feasible solutions in the problem space must be restricted in a way that satisfies [equation \(12\)](#) for an n -bay frame.

$$t_1 \leq \dots \leq t_{k-1} \leq t_k = F \geq t_{k+1} \geq \dots \geq t_n \quad (12)$$

where k is the compartment of fire origin. [Equation \(12\)](#) is difficult to formulate as a constraint that can be incorporated in a solver algorithm, because the location of k must be determined for each scenario. However, [equation \(12\)](#) can be equivalently satisfied by requiring that the duration of fire in any interior bay, or row of adjacent interior bays, be not less than both the durations of fire in the two bays immediately exterior to the bays being considered. An equivalent version of [equation \(12\)](#) is presented in [equation \(11e\)](#), which can be facilitated into solver algorithms since there is no need to determine the fire originating bay for each scenario during the solving process. In [equation \(11e\)](#), every combination of the bay indices r , j , and s in which bay j is interior to bays r and s must satisfy the given expression. If in any case the right-hand side of [equation \(11e\)](#) is negative, then the trial fire scenario violates [equation \(12\)](#) and should be rejected before conducting any further analysis on it. Of course, the use of the single-fire constraint is optional, as it should only be used if fires are not likely to bypass intermediate compartments.

3.3.2 Multi-bay compartment constraint. [Equation \(11f\)](#) is called the multi-compartment constraint. Where a compartment spans multiple bays of the frame without fire separation between the bays, the fire durations for all of the bays in the compartment may be constrained to be equal in a process commonly referred to as variable linking. This constraint can be facilitated in a solver algorithm according to [equation \(11f\)](#), where C_x is the set of indices of bays belonging to compartment x , and n_x is the number of compartments that span multiple bays. For example, let there be a frame containing two compartments that each span multiple bays without fire separation between the bays. The first compartment spans Bays 1, 2 and 3, while the second compartment spans Bays 4 and 5. Then $n_x = 2$, $C_1 = \{1, 2, 3\}$ and $C_2 = \{4, 5\}$. The resulting set of constraints given by [equation \(11f\)](#) is $t_1 = t_2 = t_3$ and $t_4 = t_5$. This constraint can be programmed into solver algorithms either directly as a constraint on the vector of all variables, t_1 through t_m , or by simply replacing all of the variables contained in each compartment with a single variable corresponding to the fire duration in the entire compartment. The latter method is recommended since it reduces the dimensionality of the minimization problem.

3.3.3 Fire-separator constraint. [Equation \(11 g\)](#), called the fire-separator constraint, is an optional constraint that accounts for the presence of fire separations between compartments. The constraint eliminates the cases where fires in the models spread too quickly between compartments where separations are installed. $RR_{i,i+1}$ is the minimum expected fire resistance of the separator, in minutes, between bays i and $i + 1$. Simply put, if fires exist in both of the adjacent bays ($t_i > 0$ and $t_{i+1} > 0$) then the difference between the fire durations in the bays must at least $RR_{i,i+1}$. However, if one of the bays is at ambient temperature ($t_i = 0$ or $t_j = 0$), then the constraint is automatically satisfied.

3.3.4 Solving the minimization problem. Let F^* be the optimal value of F from solving [equation \(11\)](#). F^* is the minimum fire resistance of the frame. [Equation \(11\)](#) may be solved using mathematical programming algorithms for minimization problems with nonlinear

constraints. The Generalized Reduced Gradient (GRG) Nonlinear (Lasdon *et al.*, 1973) algorithm, when used in combination with multi-start (György and Kocsis, 2011), appears to perform well for this problem. Note that the prescription of complicated fire curves and heat transfer analysis methods increases computation time.

3.4 Solutions based on monotonic fire curves

The solution to the minimization problem in equation (11) for any given set of applied column loads, G_i ($i = 1, 2, \dots, n + 1$) is trivial if the time-temperature curves for all of the fires are monotonically increasing functions, such as in the case of the standard ASTM E119 (ASTM, 2016) and ISO 834 (ISO, 1999) fire curves. In the absence of separator resistance constraints, the obtained solution from equation (11) consists of uniform fire duration in all bays (i.e. $t_j = F^*$ for all j). This is proven in the following:

- Given that the fire compartment temperature only increases with time, it is obvious that the temperatures of the members, and thus, the amount of degradation on the lateral stiffness of the columns experiencing the fire, can only increase with time.
- If there exists a scenario represented by the set $\{t_1, t_2, \dots, t_n\}$ where $F = \max(t_j) = F_0$ and $\Sigma S < 0$, then $F_0 > F^*$. The reason for this is that some of the values of t_j can be reduced to achieve $\Sigma S = 0$. In other words, a scenario with negative ΣS would have already become unstable at an earlier time.
- The uniform fire duration scenario ($t_j = F_0$ for all j) always results in the lowest possible value of ΣS out of all the scenarios satisfying $F = F_0$. The reason for this is that as discussed in (a), the lateral stiffness contribution of a column can only decrease with the increase of time, so by increasing the fire duration in every bay as much as possible while satisfying $F = F_0$, the lateral stiffness is reduced to its minimum possible value for $F = F_0$. In other words, given a specified duration of the fire event in the frame, the case where the duration of fire in every bay is equal to the duration of the fire event results in the most degradation in the frame.
- Given both (b) and (c), if the uniform fire scenario $t_j = F_0$ for all j results in $\Sigma S < 0$, then $F_0 > F^*$. Conversely, if the uniform scenario results in $\Sigma S > 0$, then there exists no possible scenario given $F = F_0$ where $\Sigma S = 0$, which means $F_0 < F^*$. Therefore, a unique solution exists for F^* where $t_j = F^*$ for all j . In other words, the unique solution to the minimization problem in a monotonically increasing fire is the uniform fire scenario causing in instability.

Clearly, the solution to equation (11) is trivial when the fire curves increase monotonically with time. However, realistic building fires will decay after some time, and if a member is cooling, its elastic modulus will be restored over time. For decaying fire curves, the solution to equation (11) is not trivial and the minimization problem should be solved rigorously. This is demonstrated via a numerical example in the following section. It should also be noted that if no feasible solution to equation (11) can be found and $\Sigma S > 0$ at ambient conditions, then the assumed fire curves are not severe enough for frame instability to become a possibility.

4. Numerical example

A numerical example is provided to demonstrate the use of the proposed method. This example illustrates a case where non-uniform fire durations govern the worst case fire scenario in a frame. Upon investigation it was discovered that for typical frame configurations and compartment properties, uniform or near-uniform fire durations often

governed the worst case scenarios despite the use of non-monotonically increasing fire curves. Consider the single-storey unbraced steel frame shown in [Figure \(3\)](#), adapted from [Xu et al. \(2018\)](#).

From left to right, the bays and beams are numbered from 1 through 4, and the columns numbered from 1 through 5. The first two columns are fixed to the ground ($r_f = 1$) and the other columns are pinned to the ground ($r_f = 0$). The beam-to-column connections are rigid connections ($r_u = 1$). The columns are subjected to the prescribed axial loading case shown. For all columns, $L_{c,i} = 4.877$ m and $E_0 = 200$ GPa. The moments of inertia of the W310X60 and W200X36 columns are $129 \times 10^6 \text{ mm}^4$ and $34.1 \times 10^6 \text{ mm}^4$, respectively. For all beams $L_{b,j} = 7.315$ m, $E_0 = 200$ GPa and $I_{b,j} = 245 \times 10^6 \text{ mm}^4$. Assume $\nu = 1$ for all beams and $H_i = 0$ for all columns, as they are assumed not restrained from axial deformations for the reason of simplicity. The parametric curve and step-by-step incremental methods in [Pettersson et al. \(1976\)](#) were used to establish fire temperature-time relationships in each bay, and compute member temperatures, respectively. The [Pettersson et al. \(1976\)](#) method is based on the energy balance equation shown in [equation \(13\)](#).

$$I_C = I_L + I_W + I_R + I_B \quad (13)$$

Where I_C is the heat released during combustion of fuel, I_L is the heat removed due to the replacement of hot gases by cold air through openings, I_W is the heat dissipated to and through the boundary wall of the compartment, I_R is the heat dissipated by radiation through windows and/or openings, and I_B is the quantity of heat stored in the gas volume within the fire compartment ([Pettersson et al., 1976](#)). After the fire temperature, T_f , is determined as a function of the duration of fire via [equation \(13\)](#), the temperature of a steel member, T_s , can be estimated by [equation \(14\)](#) over sequential time steps Δt (Buchanan, 2001).

$$T_s(t + \Delta t) = T_s(t) + \frac{\lambda_I (T_f(t + \Delta t) - T_s(t))}{c_s l_I (W/D) + c_I \rho_I l_I / 2} \quad (14)$$

Where the subscripts I and s refer to the material properties of insulation and steel, respectively. λ , c and ρ correspond to the thermal conductivity, specific heat capacity, and density, respectively. l_I is the thickness of insulation, W is the unit weight of the steel section, and D is the heated perimeter of the steel section. For this example, $\lambda_I = 0.12 \text{ W/mK}$, $c_s = 600 \text{ J/kgK}$, $c_I = 1500 \text{ J/kgK}$, $\rho_s = 7850 \text{ kg/m}^3$, and $\rho_I = 400 \text{ kg/m}^3$, the unit weight, W , of the sections are 60 kg/m for $W310 \times 60$, 36 kg/m for $W200 \times 36$, and 67 kg/m for $W410 \times 67$. The heated

Figure 3.
Four-bay unbraced
storey frame for
numerical example

Unbraced steel
frames

T1

perimeters of the sections, D , are 1.40 m for $W310 \times 60$, 1.05 m for $W200 \times 36$, and 1.52 m for $W410 \times 67$. Bay 1 is poorly ventilated and contains a high fuel load, represented by Fire Curve #1. The rest of the bays form a single compartment ($t_2 = t_3 = t_4$), and are relatively well-aerated and contain lower fuel loads, represented by Fire Curve #2. However, Fire Curve #1 also contains a lower ventilation factor, which means that its fuel will burn more slowly, resulting in a longer fire. The compartment properties are given in Table I.

The dashed lines in Compartment 2 indicate the free passage of flame between the bays of the compartment ($RR = 0$), while the solid line between the compartments indicates a minimum separation resistance of $RR_{1,2} = 75$ min. Note that the properties of Bays 2 through 4 are lumped into a single compartment, although dividing the properties equally among the three bays and considering them separately would result in the same fire curve. The fire curves are also affected by the amount of insulation applied on the members in the respective compartments. The thickness of insulation applied on each member was calculated according to the prescriptive formula in Lie (1992) shown in equation (15). The Lie (1992) formula estimates the amount of insulation required to provide an individual steel member with the desired fire resistance rating.

$$l_f = \frac{25.4R}{1.03^{W/D+42}} \quad (15)$$

where l_f is the thickness of insulation (mm) required to provide the desired fire resistance rating, R (min), for a member with weight W (kg/m), and heated perimeter, D (m). The desired fire rating for all members of the frame was 90 min. Based on equation (15), the insulation thicknesses for the columns ranged between 27 and 30 mm, and the insulation thickness applied to the beams was 26 mm. In terms of material properties, structural steel was assumed to have a density of $7,850 \text{ kg/m}^3$ and a heat capacity of 600 J/kgK . The insulation was assumed to have a density of 400 kg/m^3 , a heat capacity of $1,500 \text{ J/kgK}$, and a thermal conductivity of 0.12 W/mK . The fire curves are plotted in Figure (4) using the heat balance method and assumptions given in Pettersson *et al.* (1976).

F4
AQ: 3

	Bays	Opening area, A_{open}	Total area, A_{total}	Opening height, h_{open}	Fuel load, ε_f
Fire Curve #1	1	5.0 m^2	282.8 m^2	2.0 m	$1,200 \text{ MJ/m}^2$
Fire Curve #2	2, 3, 4	45.0 m^2	848.4 m^2	2.0 m	300 MJ/m^2

Table I.
Parametric curve
compartment
properties for
numerical example

Figure 4.
Time-temperature
plot of parametric fire
curves for numerical
example

JSFE

Since Fire Curve #1 contains a large fuel load and a low ventilation factor the fire curve grows slowly over many hours. In contrast, Fire Curve #2 reaches higher temperatures but begins to decay after only 35 min due to having a lower fuel load. It can be observed from [Figure \(4\)](#) that a fire originating in Bay 1 and spreading to Bays 2 through 4 later on would have much worse effect on the structure than if the fire started simultaneously in all four bays at once, since, in the first case, such a scenario would cause all of the bays to have high temperature fires at the same time. In fact, when the minimization problem in equation (11) was solved, this was the worst case scenario causing instability for this example. The results of solving the minimization problem, are shown in [Table II](#). Note that the damage contribution factor, $D_{c,i}$ for each column, introduced in [Xu et al. \(2018\)](#), was also evaluated. $D_{c,i}$ is the portion of the total reduction in lateral stiffness of the frame experienced by the individual column, shown in [equation \(16\)](#).

T2

$$D_{c,i} = \left(\Delta S_{c,i} / \sum_{i=1}^n \Delta S_{c,i} \right) * 100\% \tag{16}$$

$$\Delta S_{c,i} = S_{o,c,i} - S_{T,c,i}$$

where $S_{o,c,i}$ is the lateral stiffness of column i under ambient temperature conditions, $S_{T,c,i}$ is the lateral stiffness of column i under elevated temperature conditions, and $\Delta S_{c,i}$ is the change in lateral stiffness of column i resulting from elevated temperatures. In other words, $D_{c,i}$ reports the relative damage experienced by an individual column.

Based on the results, the worst case scenario represents a situation where a fire starts in Bay 1 and spreads to the multi-bay compartment after a delay of 75 min, equal to the fire resistance of the separator. The analysis shows that the frame could become unstable in 153 min, but not less. This is a significant improvement over the fire resistance ratings of the individual members, which were 90 min. It was also found that the temperatures of the beams in Bays 2, 3 and 4 would have reached their maximum temperatures at 91 min, while the columns in these bays would have reached their maximum temperatures at 90 min. Past these durations, the member temperatures in these bays would decrease, causing gradual restoration of lateral stiffness to the frame. Due to the separator constraint, however, the fire duration in Compartment 2 was constrained to be at most 78 min, given the duration of fire in Compartment 1. If this constraint were lifted, a lower fire duration causing instability would have governed, with these members closer to their peak temperatures.

4.1 Modified example

In terms of damage contribution, it can be seen from [Table II](#) that the degradation of the first two columns accounts for as much as 91.5 per cent of the total loss of stiffness in the frame,

Table II.
Details of worst case fire scenario causing instability in numerical example

Minimum fire resistance, F^*		153 min			
Bay #	1	2	3	4	
Bay fire durations, t_j	153 min	78 min	78 min	78 min	
Beam temperatures, $T_{b,j}$	582°C	385°C	385°C	385°C	
Column #	1	2	3	4	5
Column temperatures, $T_{c,i}$	583°C	583°C	392°C	392°C	386°C
Column damage contribution, $D_{c,i}$	41.1 %	50.4 %	2.0 %	2.0 %	4.5 %

T3

since the fire is most severe there. Subsequently, the effect of increasing the insulation on the members in Bay 1 (Beam 1, Column 1 and Column 2) to provide an additional hour (total 150 min) of fire resistance rating was investigated. The results of the worst-case scenario analysis with modified insulation thicknesses are given in [Table III](#).

From increasing the amount of insulation on the members in Bay 1, the minimum fire resistance rating is increased to 272 min (+78 per cent) since it takes longer for these members to heat up. In the worst case scenario, all of the other bays have the same fire duration of 90 min, resulting in the maximum possible heating of the members in Compartment 2. Similar to the original example, Columns 1 and 2 collectively contribute to 91.6 per cent of the total loss in lateral stiffness of the frame during the worst case scenario, since the fire is most severe in Bay 1. Note that if a uniform fire had occurred in the frame then it would have taken 278 min for the frame to collapse. In such a scenario, the members in the multi-bay compartment would have partially cooled down (ranging between 164°C and 166°C) while the members in Bay 1 continued to degrade, as a result of the long-burning fire in Bay 1, until instability of the frame occurred at 278 min. Thus, it has been demonstrated that the worst-case fire scenario causing instability of a given frame may not always be a uniform fire. Fire scenarios that simultaneously cause severe heating to as many of the members of the frame as possible are likely candidates for the worst case scenario. Also, the thickness of insulation applied on the members appears to have a significant effect on the fire resistance of the frame. As demonstrated by the modified example, the fire resistance of the frame can be improved when the insulation thicknesses in each bay are increased in proportion to the severity of fire that can be expected at that bay. In terms of design, the proposed minimization problem can be repeated by the designer for various scenarios related to the insulation on the members to determine the best arrangement.

4.2 Comparison to original method

The results of the proposed method are further compared to the results of the original variable fire loading analysis method proposed in [Xu et al. \(2018\)](#) on the same example to better highlight the advantages of the proposed method. The [Xu et al. \(2018\)](#) method determines the worst case scenario defined as instability caused by the minimum average temperature of beams in the frame while assuming that the columns temperatures are proportional to the beam temperatures via the factor k . That is, the temperature of a column can simply be assumed to be k times the maximum temperature of the connecting beams. By examining the results in [Table II](#), the value of $k = 1$ was selected to be most appropriate for this frame and is thus adopted for the analysis. To be clear, the proposed method computes the temperatures of the members in relation to the duration of fire, whereas the [Xu et al. \(2018\)](#) method assigns member temperatures directly without regard to the temperature-time

Unbraced steel frames

Minimum fire resistance, F^*		272 min			
Bay #	1	2	3	4	
Bay fire durations, t_j	272 min	90 min	90 min	90 min	
Beam temperatures, $T_{b,j}$	586°C	391°C	391°C	391°C	
Column #	1	2	3	4	5
Column temperatures, $T_{c,i}$	586°C	586°C	398°C	398°C	398°C
Column damage contribution, $D_{c,i}$	40.3%	51.3%	2.0%	2.0%	4.5%

Table III.
Details of worst case fire scenario causing instability in modified numerical example with increased insulation

JSFE

relationships. The minimization problem proposed in [Xu et al. \(2018\)](#) is similar to equations (11) but with some differences, shown in equation (17).

$$\text{minimize } \frac{1}{n} \sum_{j=1}^n T_{b,j}$$

(17a)

subject to:

$$\Sigma S = \sum_{i=1}^n S_{c,i} = \sum_{i=1}^n \left[\frac{12E_{c,i}I_{c,i}}{L_{c,i}^3} \beta_i \right] = 0$$

(17b)

$$P_i \leq P_{ui} \quad \forall i = 1, 2, \dots, n + 1$$

(17c)

$$20^\circ\text{C} \leq T_{b,j} \leq 1,200^\circ\text{C}; \quad \forall j = 1, 2, \dots, n$$

(17d)

$$T_{b,j} - \text{median}\{T_{b,r}, T_{b,j}, T_{b,s}\} \geq 0; \quad \forall 1 \leq r < j < s \leq n \quad \forall 2 \leq j \leq n - 1$$

(17e)

where $T_{b,j}$ is the temperature of the beam in Bay j . Solving the minimization problem in equation (17) yields the results in [Table IV](#).

From the results of [Table IV](#) it can be seen that the worst case fire based on the average temperatures of the beams in the frame corresponds to a localized fire in Bay 1 leading to instability. When considering the time-temperature relationships via the proposed method, a similar configuration of member temperatures can be produced with a fire burning solely in Bay 1 for 158 min prior to modifying the insulation thickness of members in Bay 1, or 280 min following the modification. The result of the proposed method ($F^* = 153$ min) is therefore similar to the one obtained by the [Xu et al. \(2018\)](#) method, but is not identical and has a slightly lower fire resistance due to the differing assumptions. Particularly, the assumption of $k = 1$ is overly simplistic and does not consider the time-temperature relationships of fire which are more realistically addressed in the proposed method. Overall, although the [Xu et al. \(2018\)](#) method is simpler it is unable to capture the details of fire spread and member heating effects of time, and only reports the critical temperatures of the members rather than the critical duration of fire leading to such scenarios.

5. Conclusion

Presented in this paper is a new formulation for determining the minimum fire resistance duration of an unbraced steel storey frame based on the storey-buckling failure mode. The

Table IV.
Worst case fire scenario causing instability based on member temperatures obtained via [Xu et al. \(2018\)](#)

Average beam temperature		164°C				
Bay #		1	2	3	4	
Beam temperatures, $T_{b,j}$		596°C	20°C	20°C	20°C	
Column #		1	2	3	4	5
Column temperatures, $T_{c,i}$		596°C	596°C	20°C	20°C	20°C
Column damage contribution, $D_{c,i}$		42.3%	57.7%	0.0%	0.0%	0.0%

procedure involves assuming fire curves in each bay, and determining the fire scenario that results in the fastest possible instability of the frame via a nonlinear constrained minimization problem. For frames with only monotonically increasing fire curves, the solution to the minimization problem is uniform fire duration across all bays when fire separations are not considered. However, for frames with decaying fires, the minimization problem should be solved rigorously. A numerical example is presented to demonstrate the use of the proposed formulation on an example frame using parametric fire curves. The details of fire spread in the worst case scenario can be inferred from the results of the proposed method. In frames with bays containing fire curves that decay quickly, the worst-case scenario fire can include fire durations in bays corresponding to the peak member temperatures. The worst case scenario is also influenced by the amount of insulation applied on the members, and increasing the amount of insulation in highly-combustible compartments helps to improve the worst case scenario. The results of the proposed method are also compared with those obtained via the original variable fire loading method proposed in [Xu et al. \(2018\)](#), and it is shown that the proposed method can more realistically represent the developing nature of fires over time. Overall, the proposed formulation is a stepping stone for performance-based fire-structural design as it shows that considering the entire structure as a whole most often yields higher fire resistance than the prescribed fire resistance ratings of individual members.

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