A data-driven optimization approach for mixed-case palletization and three-dimensional bin packing

by

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Abstract

Palletization is the most standard method of packaging and transportation in the retail industry. Their building involves the solution of a three-dimensional packing problem with side practical constraints such as item support and pallet stability, leading to what is known as the mixed-case palletization problem. Motivated by the fact that solving industry-size instances is still very challenging for current methods, we propose a new solution methodology that combines data analysis at the instance level and optimization to build pallets. Item heights are first analyzed to identify possible layers and to derive relationships between item positions. Items are stacked in pairs and trios to create super items, which are then arranged to create layers of even height. The resulting layers are finally stacked to create pallets. The layers are constructed using a reduced-size mixed integer program as well as a two-dimension placement heuristic. Computational tests on industry data show that the solution approach is extremely efficient in producing high-quality solutions in fast computational times.

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My father, mother, sisters for being the cornerstone of my life and giving me everything so I could be here today.

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Dedication

I dedicate this to my love Julia, my family and friends, and all the ones like me that have always been curious and asked so many questions.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Transportation and warehousing play a fundamental role in every country's economy. For example, they represented 4.3% of Canada's Global Domestic Product (GDP) in 2015. In addition, world freight volume is expected to increase by a factor of 4.3 by 2050 (Transport Canada, 2016). Considering this growth, improving efficiency and providing cost effectiveness are essential to ensure financial sustainability and competitiveness in this industrial segment.

In in a survey of logistics service providers and companies, 98% of logistics suppliers stated that data-driven decision making is essential to the success of supply chains. Moreover, 85% of them believe that effective manipulation of big data will become a core competency in this sector (Capgemini and Langley, 2017). Furthermore, data analytics is the featured topic of the 2017 edition of the survey. When asked about the most important initiatives regarding data analytics in logistics and supply chain management, 71% of participants identified "Improving process quality and performance", followed by "Improving logistics optimization" with 70%.

In this work, we focus on one of the basic and most common forms of goods transportation, which is pallet transportation. The efficiency of pallet transportation depends on the palletization process, which is the process by which an order, typically very large, is packed in different pallets. This problem, known as the mixed-case palletization problem, is a generalization of the three-dimensional bin packing problem (3DBPP).

One and two-dimensional versions of bin packing problems have been extensively studied as opposed to the more challenging three-dimensional version (Zhao et al., 2016). Three-dimensional bin packing problems consist of packing items inside a container (bin) where items are placed side-by-side, parallel to the edges of the container. The problems are often categorized by the heterogeneity of items, the characteristics of the bin, and the objective of the packing (Bortfeldt and Wäscher, 2013).

Aside from basic geometric constraints, Bortfeldt and Wäscher (2013) define five additional categories. Container-related constraints include weight limit and distribution, while loading priority, orientation, and stacking are within the item-related category. Cargo-related constraints include shipment integrity and item-specific allocation. Positioning constraints impose separation of items, for example, food from cleaning products. Load-related constraints involve cargo stability. Most of the current work on 3DBPP does not consider these practical constraints as indicated by Bortfeldt and Wäscher (2013) and Zhao et al. (2016).

The traditional 3DBPP is already difficult to solve, and the inclusion of practical constraints makes the task even more challenging. As exact methods face limitations when dealing with large instances, approximation methods and heuristics are more suitable to tackle practical problems. Zhao et al. (2016), while comparing different solution methods, separate heuristics into placement and improvement. Arranging items in modular blocks such as walls or layers is the basis of most placement and construction heuristics.

Motivated by the need for more efficient solution methodologies, we propose a data-driven optimization approach for the mixed-case palletization and three-dimensional bin packing problems. We combine data analysis and optimization in an efficient solution methodology where analysis of the item dimensions at the instance level is used to enhance the optimization by fixing variable and reducing problem size, while also accounting for practical constraints such as support and stability.

The solution methodology is composed of two phases. The first phase is devoted to, analyzing the instance under study, identify groups of items that when stacked achieve a certain target height. A common height can include single as well as special two and three-item arrangements. The second phase is item placement, and for each common height a layer is constructed where only pallet depth is enforced. This long layer will later be divided into multiple layers. By construction, the layers are dense to maximize volume utilization and provide adequate support. The formation of layers is done using a reduced-size three-dimensional bin packing mixed-integer program (MIP) formulation as well as a two-dimensional placement heuristic proposed by Jylänki (2010).

The main contribution of this work is a data-driven solution methodology based on instance analysis and optimization. The data analysis at the instance level defines the main blocks of the optimization model and drives the heuristic methodology. It leads to solutions of large problems that are otherwise out of reach of exact methods and are very challenging for current heuristics.

The remainder of this thesis is organized as follows. In Chapter 2, a literature review is performed. Chapter 3 defines the problem and the solution methodology. Chapter 4 presents the numerical tests and results. Conclusions and future research are given in Chapter 5.

Chapter 2

Literature review

Being the core problem in mixed-case palletization, we start by reviewing the literature on 3DBPP. The review paper by Bortfeldt and Wäscher (2013) provides a definition and a classification scheme depending on problem characteristics.

Wäscher et al. (2007) classify container loading problem into seven types: Single Stock-Size Cutting Stock, Multiple Stock-Size Cutting Stock, Residual Cutting Stock, Single Bin-Size Bin Packing, Multiple Bin-Size Bin Packing, Residual Bin Packing, and Open Dimension Problems. More details on the specific definition of each category are given by Wäscher et al. (2007).

Bortfeldt and Wäscher (2013) extend Wäscher et al. (2007) by analyzing 163 papers, published between 1980 and 2011, focusing on the types of constraints considered. Weight limits are accounted for in 23 papers, while weight distributions are in 19. Meanwhile, loading priorities are only in 3 papers, orientation in 15, and stacking in 25 out of the 163 papers reviewed. In addition, 19 papers consider complete-shipment, 13 allocation, and 28 account for positioning constraints. Finally, load-related stability and complexity constraints are considered in 61 and 15 papers respectively. Although some constraint types are widely adopted, the authors observe that practicality (by addressing several constraints simultaneously) is still scarce. As more than 75% of the papers consider at most 2 types of constraints. This is most likely due to the solution methodologies not being ready to accommodate such complexity. The current work is a step in this direction.

The recent survey by Zhao et al. (2016) analyzes 113 papers, focusing mainly on solution approaches: heuristics (placement or improvement) or exact methods. Improvement heuristics create rules and mechanisms to place items inside the container in order to generate an initial solution and/or iterate between different solutions. by exploring the neighbourhood of the solution. Placement heuristics tackle the problem by building walls, layers, or blocks of items, and by creating a sequence for the placement. This search can be associated with a placement heuristic, to construct an initial solution. The authors note that the number of exact methods is relatively small compared to heuristics, and conclude that only a small portion of the papers addresses real-world constraints. They also emphasize that more challenging and realistic benchmark data-sets need to be proposed.

George and Robinson (1980) present a two-phase construction heuristic based on wall building. The first phase identifies an empty space to be filled and the second fill it. The walls are based on the dimensions of the first box placed, which is selected according to a ranking criteria. Bischoff and Marriott (1990) propose a compound technique based on two-dimensional packing. They propose a hybrid approach that considers 14 different heuristics, to select the best solution. The 14 different heuristic base methods come from combinations of 3 methods and 6 ranking rules, applied to instances ranging from 1 type of boxes to 20 different types. Based on the results for all methods, the authors propose a compound heuristic, that apply different methods in a sequence.

To maximize the number of items in a single bin, Mohanty et al. (1994) propose a heuristic based on the multi-dimensional knapsack problem. Bischoff and Ratcliff (1995) address the multi-drop container loading problem and present a heuristic that combines packing and a sequence based on destination. They are able to solve larger problems than Bischoff and Marriott (1990). Ratcliff and Bischoff (1998) incorporate load bearing constraints.

Faina (2000) proposes a geometric model to reduce the problem to a finite enumeration, presenting good results for problems with less than 128 items. Pimpawat and Chaiyaratana (2001) utilize a co-operative co-evolutionary genetic algorithm. Their results indicate better performance in terms of containers required, compared to standard genetic algorithm. Pisinger (2002) aggregates tree search algorithm to ranking criteria, achieving considerable improvements in the space filling.

Lodi et al. (2002) introduce a two-phase Tabu search heuristic. In the first phase items are grouped by height, and then sorted by area. The heuristic is then used as a base to generate solutions for the Tabu search. Farøe et al. (2003) propose a guided local search. It initiates with an upper bound generated by a greedy heuristic, then the algorithm iteratively reduces the number of bins searching for a feasible solution. Moura and Oliveira (2005) include modifications based on GRASP (greedy randomized adaptive search procedure) to the framework of George and Robinson (1980), increasing volume use and cargo stability.

Bischoff (2006) proposes a random search method that accounts for load bearing. It creates a scoring framework, accounting for suitability of items and impact on future placements. Crainic et al. (2008) present an extreme point placement heuristic. Extreme points are identified and items are placed based on a best-fit decreasing heuristic. This work is developed further in Crainic et al. (2009) where a two-level Tabu search (TS2pack) is introduced. The first level reduces the number of bins and the second level optimizes the packing.

Zhu et al. (2012) propose a set covering formulation with a prototype-based column generation approach. The goal is to minimize the cost of containers used while considering support constraints. The efficiency is improved by the generation of prototypes, which are good approximation of columns.

Gonçalves and Resende (2013) combine a biased random-key genetic algorithm to set the order in which the items are packed and a constructive heuristic combined with maximal-space representation to determine where each box is placed.

Hifi et al. (2014) use an integer linear programming heuristic that breaks the problem into a series of single knapsack problems. It is composed of two phases, selection and positioning. The first phase defines a subset of items susceptible to an active container, and the second packs the already selected items. A greedy heuristic is added to deal with larger instances. Elhedhli et al. (2017) propose a slicing heuristic based on the relative positioning mixed integer formulation that forces dense layers at the bottom of the bins. The results show good performance on item support.

Toffolo et al. (2017) propose a decomposition method with multi-phase heuristics. The problem is decomposed into two types of subproblems, stacks and bins. The method, based on a MIP model, dynamically build layers and pack them into containers. Local search is also applied to enhance solution performance. Mahvash et al. (2017) study a combined 3DBPP and vehicle routing problem. They decompose the problem and solve it with a column generation based technique. Wu et al. (2017) propose a three-stage heuristic with phases defined as: fastener packing, carton packing, and crate packing. The first is used to determine a packing strategy depending on the item shape, selecting predetermined packing patterns to maximize the filling rate. The second aims to increase utilization of the crates while considering sequence and allocation constraints, and the third is a heuristic based on multi-layer search used to load the blocks created in the previous stages into the containers.

Formulations based on MIP form an important part of the 3DBPP literature. Chen et al. (1995) developed a mixed integer formulation based on relative positioning that considers non-uniform sized items and containers.

Martello et al. (2000) propose a branch-and-bound method to the 3DBPP with orthogonal packing. This work proposes lower bounds on the number of bins. The work is extended in Martello et al. (2007), where algorithm 864 and a benchmark instance were proposed.

Fasano (2004) considers orthogonal placement with balancing conditions and proposes a MIP formulation heuristic. At each iteration the method increases the number of items picked, maintaining the relative position of the previous solutions. This work is extended in Fasano (2008) with a recursive procedure based on non-blind local search. Wu et al. (2010) extend Chen et al. (1995) to take box orientation into account.

Allen et al. (2012) introduce a space indexed formulation to overcome limitation of the relative positioning model. The method is claimed to be strong but faces limitations due to the number of variables generated. Paquay et al. (2016) use MIP to minimize unused space and incorporate some real-world constraints as weight distribution and stability. Paquay et al. (2017) introduce a heuristic to generate a good initial solution, based on decomposition and relax-and-fix, insert-and-fix and fractional relax-and-fix methods.

Junqueira et al. (2012b) incorporate vehicle routing into the 3DBPP also accounting for vertical stability, as an extension of Junqueira et al. (2012a).

Yildiz et al. (submitted in 2017) use a column generation approach where the pricing is a two-dimensional layer generation problem. They are able to solve large problems in fast computational time.

Chapter 3

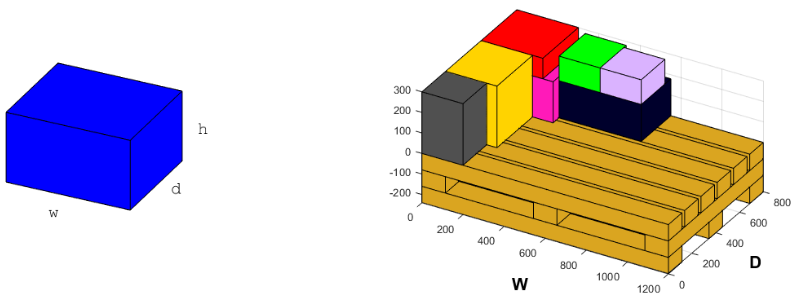
Problem Definition and Solution Methodology

In this chapter, we first formulate the 3DBPP as a mixed integer program and then give the details of the two-phase solution methodology we propose.

3.1 Mathematical Formulation

Consider a set of items $i \in \{1, 2, ..., n\}$ of rectangular shape, with width (w_i), depth (d_i) and height (h_i) that have to be packed in a pallet of width (W), depth (D), and height (H). The items have to be placed parallel to the edges of the pallet with no physical overlap. An illustration is given in figure 3.1.

Figure 3.1: Representation of an Item and a Pallet



Given a Cartesian co-ordinate system with origin at the lower left corner of the bin, the 3DBPP can be modeled as a MIP with decision variables:

c_i^s : the front, lower, left coordinate of item $i = 1, 2, ..., n$ along direction $s = x, y, z$

$$p_{ij}^s = \begin{cases} 1 & \text{if item } i = 1, 2, ..., n \text{ precedes item } j = i+1, 2, ..., n \text{ along direction } s = x, y, z, \\ 0 & \text{otherwise} \end{cases}$$

h : the maximum height of all packed items.

For now, let us assume that the height of the bin, H, is large enough to accommodate all items, and formulate what is known as the Open Bin Packing problem:

$$\begin{array}{llll}
\text{[BPP]:} & \min & h & \\
& \text{s.t.} & \sum_{s=x,y,z} (p_{ij}^s + p_{ji}^s) \geq 1 & i = 1, ..., n; j = i+1, ..., n \quad (3.1) \\
& & p_{ij}^s + p_{ji}^s \leq 1 & i = 1, ..., n; j = i+1, ..., n; s = x, y, z \quad (3.2) \\
& & c_i^s + l_i^s \leq c_j^s + L^s(1 - p_{ij}^s) & i = 1, ..., n; j = i+1, ..., n; s = x, y, z \quad (3.3) \\
& & 0 \leq c_i^s \leq L^s - l_i^s & i = 1, ..., n; s = x, y, z \quad (3.4) \\
& & c_i^z + l_i^z \leq h & i = 1, ..., n \quad (3.5) \\
& & p_{ij}^s \in \{0, 1\} & i = 1, ..., n; j = i+1, ..., n; s = x, y, z \quad (3.6) \\
& & h, c_i^s \geq 0 & i = 1, ..., n; s = x, y, z \quad (3.7)
\end{array}$$

where, $L^x = W, \quad L^y = D, \quad L^z = H, \quad l_i^x = w_i, \quad l_i^y = d_i, \quad$ and $\quad l_i^z = h_i; i = 1, ..., n.$

The first constraints (3.1) make sure that any two items differ in at least one direction, which means having at least one relative positioning variable greater than zero. Constraints (3.2) ensure that an item can not both proceed and precede an other. Constraints (3.3) prohibit overlap by setting the minimum distance between items along the direction of non-overlap. Constraints (3.4) place items inside the pallet limits, and constraints (3.5) define the bin height as the maximum of all item heights. The objective function minimizes the height of the pallet.

3.2 Solution Methodology

Due to the complexity of solving mixed-case palletization problems, especially when practical constraints such as support and stability are incorporated, we propose a two-phase solution approach that combines data analysis and optimization in an attempt to use information about item placement to reduce the complexity of the optimization. The approach has two main phases.

3.2.1 Phase 1 - Item Grouping

This step aims at identifying subsets of items that can be combined and treated as a single item. As we implicitly account for item support and pallet stability, we consider two and tree-item arrangement that are well supported. We call them pairs and trios, respectively. Figure 3.2 provides an illustration. In addition, we target common heights to be able to build long layers of even height. The grouping algorithm clusters items and defines the height of layers so that a stable and well-supported pallet can be built.

Figure 3.2: Representation of a Pair and a Trio

The objective is to generate subsets with defined item combinations to form a homogeneous layer (in terms of height), arranged in a way to ensure support. Stacked layers will form one or multiple pallets. The method starts with a set of items S, and at each iteration k, generates a list of trios T_k and pairs P_k of items. This information is then passed on to the placement/optimization phase. The following section provide the details.

Trio generation

A trio T_k is defined as a set of three items where two items, a_1 and a_2, are placed side by side over the third item. The base item (b) is selected to have an area that fully support the two items on top (a_1 and a_2). The heights of the top items are chosen to be as close as possible to each other and total height $b + a_1$ (or $b + a_2$) being within the target height interval.

The first target height is the tallest item's height. This is because it is a layer setting item, i.e., a layer should be at least as tall as this tallest item. For subsequent iterations, the function *combination* from library *itertools* in *python* is used. This function generates all combination of a predetermined number of elements (2 items - 1 base and 1 top). Then, the heights are summed for every combination and the value with the highest occurrence is taken.

After setting the target height r, two additional parameters are defined: height tolerance t_h and trio tolerance t_w. The height tolerance t_h is the maximum deviation allowed on an accepted height combination. It defines the allowable range as $r \pm t_h$. The trio tolerance t_w is analogous to t_h, but for the width dimension. The trio generation algorithm is presented next:

Algorithm 1 Trio Generation

```
 1: Input:
    set of items $i = 1, ..., n_1$ each with dimensions $(w_i, d_i, h_i)$
 2: Initialize:
 3: Layer height $\leftarrow r$
 4: Height tolerance $\leftarrow t_h$
 5: Trio width tolerance $\leftarrow t_w$
 6: $T_{temp}$: set of temporary Trios $\leftarrow \emptyset$
 7: $T$: set of Trios $\leftarrow \emptyset$
 8: $P_{temp}$: set of temporary Pairs $\leftarrow \emptyset$
 9: for item $i$ do
10:     for item $j$ do
11:         if $h_i + h_j \in [r - t_h, r + t_h]$ then
12:             include $i$ and $j$ as possible pair $\leftarrow (i, j) \in P_{temp}$
13:         end if
14:     end for
15: end for
16: for $(i_1, j) \in P_{temp}$ do Find $(i_2, j) \in P_{temp}$
17:     if $w_{i_1} + w_{i_1} \in [w_j - t_w, w_j + t_h]$ then
18:         include $i_1, i_2, j$ as possible trio $\leftarrow (i_1, i_2, j) \in T_{temp}$
19:     end if
20: end for
21: Calculate trios fitness
22: while $T_{temp}$ not $\emptyset$ do
23:     select trio with highest fitness
24:     include trio in final list
25:     remove selected items from temporary list
26: end while
27: return List of Trios for iteration $k$
```

An important parameter in the trio generation is the *fitness*. It is defined as: $|1 - \frac{\text{area of } a_1 + \text{area of } a_2}{\text{area of } b}|$. The trios are selected based on the best fitness (lowest value). Ties are decided for the item listed first. In addition, we also generate pairs of items that are stacked on top of each other, to achieve a predetermined height.

Pair generation

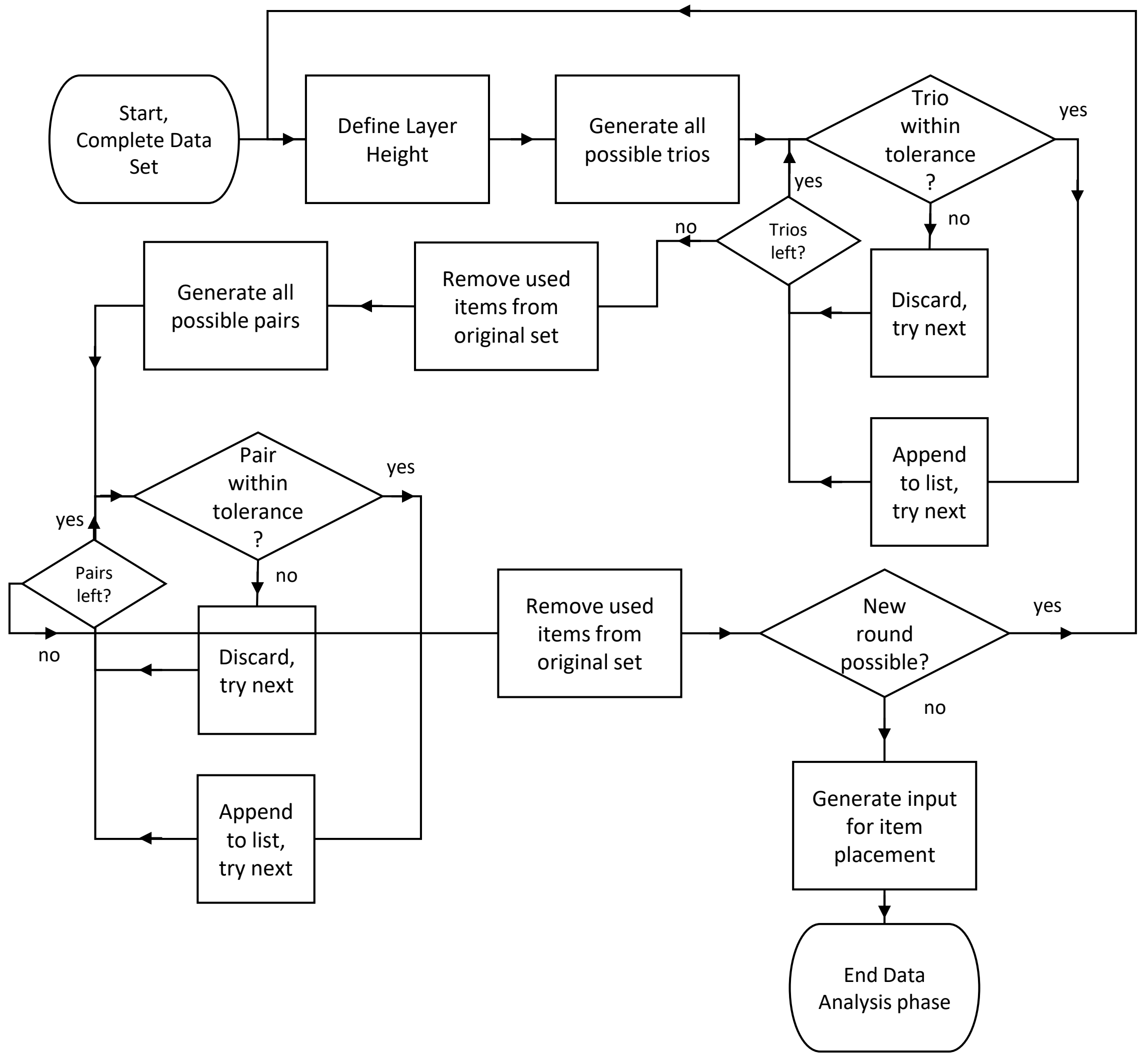
Similar to the trios, the remaining items are searched to generate a list of pairs of similar height. The algorithm to achieve this is described below:

Algorithm 2 Pair Generation

```
 1: Input:
    set of items $i = 1, ..., n_2$ each with dimensions $(w_i, d_i, h_i)$
 2: Initialize:
 3: Layer height $\leftarrow r$
 4: Height tolerance $\leftarrow t_h$
 5: $P_{temp}$: set of temporary Pairs $\leftarrow \emptyset$
 6: $P$: set of Pairs $\leftarrow \emptyset$
 7: for item $i$ do
 8:     for item $j$ do
 9:         if $h_i + h_j \in [r - t_h, r + t_h]$ then
10:             include $i$ and $j$ as possible pair $\leftarrow (i, j) \in P_{temp}$
11:         end if
12:     end for
13: end for
14: Calculate pair fitness
15: while $P_{temp}$ not $\emptyset$ do
16:     select pair with highest fitness
17:     include pair in final list
18:     remove selected items from temporary list
19: end while
20: return List of Pairs for iteration $k$
```

For pairs, the *fitness* is defined as: $|1 - \frac{\text{area of } a}{\text{area of } b}|$. The pairs are selected based on it. Once this complete procedure is done, three sets of items, pairs P, trios T, and singletons R are available to form layers. The pairs and trio generation steps are executed repeatedly until a stopping criteria is met, which can be a predetermined number of iterations or the size of the remaining-items set. The complete procedure is summarized in Figure 3.3.

Figure 3.3: Flowchart for phase 1



The procedure starts with the complete set of items. A target layer height is defined, and all possible trios are found. The items used for the trios are removed from the set. With the remaining items, pairs are generated. Again, the covered items are removed from the set. A check is performed to evaluate if another round is necessary, based on a minimum size of the remaining group or a predetermined number of rounds to be executed. If another round is necessary, the procedure is re-started, if not, it is terminated.

In the following section, we will detail how the output of phase 1 is incorporated into the placement methods - the mathematical model and the two-dimensional heuristic.

3.2.2 Phase 2 - Item Placement

3.2.2.1 Modified mathematical model

After the item grouping phase, the mathematical formulation [BPP] is modified to incorporate the information generated in phase 1. This will reduce the problem complexity.

Problem Decomposition

The first impact from the data analysis is that the placement will be executed on reduced and selected groups. Within each group, a reduced-size problem is solved using a modified version of [BPP] where H is fixed to the desired height and W is set to be sufficiently large to accommodate all items in a long strip. Complete layers will be generated within these groups. This model is solved repeatedly for different groups, generating multiple layers.

Relative Positioning

The grouping algorithm defines which items are stacked on top of each other given the predetermined layer height. This enforces $p^z_{ab} = 1$ where a is the bottom item and b the top item in pair (a, b). The same is applied to the trios (a_1, a_2, b), accounting for the relations between top items a_1 and a_2, and the base item b. The corresponding constraints are $p^z_{a_1b} = 1$ and $p^z_{a_2b} = 1$

The paired items should also be aligned, to make sure they sit on top of each other and not in different places in the x, y plane. This is achieved by the following constraints: $c^x_a - c^x_b = 0$ and $c^y_a - c^y_b = 0$. For the trios the constraints are: $c^x_{a_1} - c^x_b = 0$ and $c^y_{a_1} - c^y_b = 0$ for top item $a1$, and $c^x_{a_2} - c^x_{a_1} = l^x_{a_1}$ and $c^y_{a_2} - c^y_b = 0$ for item a_2.

Long layer approach

The resulting layer for each group has a homogeneous height, a fixed depth, and a long width ('long layer'). To be able to cut it into multiple layers having the depth and width of the pallet, we introduce artificial items, called separators. These items, have very small width, depth D and can be thought of as being sheets of cardboard. Their positioning is defined by: $c^x_{separator_k} = k \times W, \quad k = \{1, 2, ..., K\}, \qquad c^y_{separator_k} = c^z_{separator_k} = 0.$

Objective function

With the long layer approach, the objective function has to be adapted to minimize the total length and create dense layers. To achieve that, we maximize the area of items inside the pallet. In other words, we maximize the area of all items placed in the first section of the layer, which coincide with the pallet. We define a new binary variable x_i for $i = 1, ..., n$, which takes a value of 1 if the item i is in the first section of the layer.

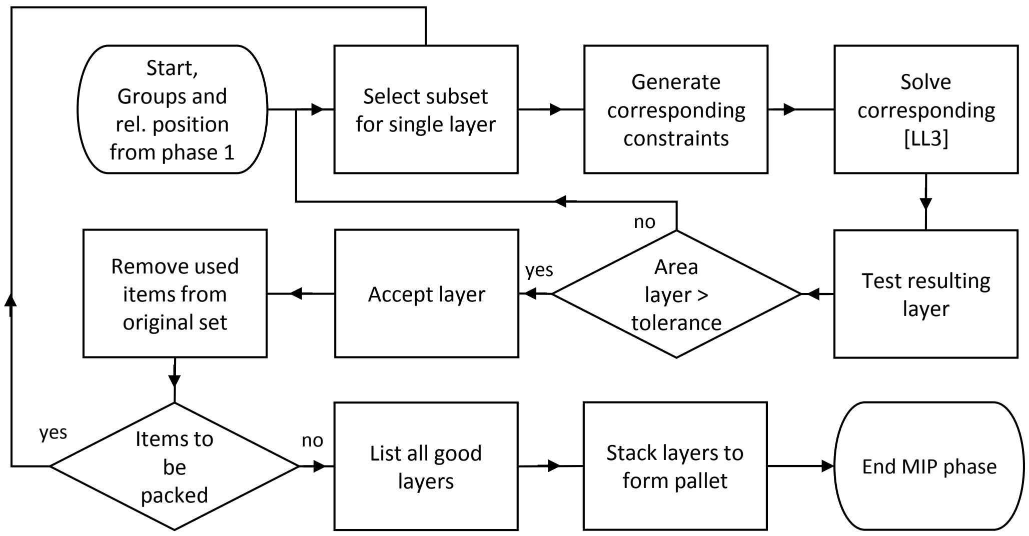
Modified model

Given trios T, pairs P, and separators n, the new mathematical model is:

$$
\begin{array}{lll}
\text{[LL3]:} & \max & \sum_i (l_i^x l_i^y) x_i \\
& \text{s.t.} & (3.1) - (3.6) \\
& & c_i^s + l_i^s \le r \qquad i = 1, ..., n \quad (3.7) \\
& & p_{ab}^z = 1 \qquad (a,b) \in P \quad (3.8) \\
& & p_{a_1 b}^z = 1, \qquad p_{a_2 b}^z = 1 \qquad (a_1, a_2, b) \in T \quad (3.9) \\
& & c_a^x - c_b^x = 0 \\
& & c_a^y - c_b^y = 0 \\
& & c_{a_1}^x - c_b^x = 0 \\
& & c_{a_1}^y - c_b^y = 0 \\
& & c_{a_2}^y - c_b^y = 0 \\
& & c_{a_2}^x - c_{a_1}^x = l_{a_1}^x \qquad (a_1, a_2, b) \in T \quad (3.10) \\
& & c_{separator_k}^x = k \times W \qquad k = \{1, ..., K\} \quad (3.11) \\
& & c_{separator_k}^y = c_{separator_k}^z = 0 \qquad k = \{1, ..., K\} \quad (3.12) \\
& & c_i^x + M x_i \le W + M \qquad i = 1, ..., n \quad (3.13) \\
& & c_i^x + W x_i \ge W \qquad i = 1, ..., n \quad (3.14) \\
& & x_i \in \{0, 1\} \qquad i = 1, ..., n \quad (3.15) \\
& & c_i^s \ge 0 \qquad i = 1, ..., n; s = x, y, z \quad (3.16)
\end{array}
$$

The model is used iteratively for a subset of items until the complete group is covered. The goal is to form 1 or 2 acceptable layers in each iteration. Constraints (3.7) define the maximum height as the target height r set in phase 1. Constraints (3.8) define the relative positioning (on the z coordinate) of items paired in the grouping phase. Constraints (3.9) do it for the items in a trio. Constraint (3.10) separates the top items in a trio by its width. Constraints (3.11) and (3.12) set the position of the artificial items (the separators). Constraints (3.13) and (3.14) ensure that the variable x_i has a value of 1 for the items in the first part of the layer, where we want to maximize the area coverage. The complete procedure is summarized in Figure 3.4.

Figure 3.4: Flowchart for the MIP phase



Start, Groups and rel. position from phase 1
Select subset for single layer
Generate corresponding constraints
Solve corresponding [LL3]
Test resulting layer
Area layer > tolerance
no
yes
Accept layer
Remove used items from original set
Items to be packed
yes
no
List all good layers
Stack layers to form pallet
End MIP phase

The procedure starts with the output of phase 1. A reduced subset of items is selected, as only one or two layers are expected to be built at each iteration. Then, the corresponding constraints are generated and [LL3] is solved. The resulting layers are accepted only if they exceed a minimum area coverage of 70%. The covered items are then removed from the list and new items selected for the next iteration. When all items are placed, a list with all accepted layers is created. The list is sorted by increasing area coverage and the layers are stacked to form the pallets.

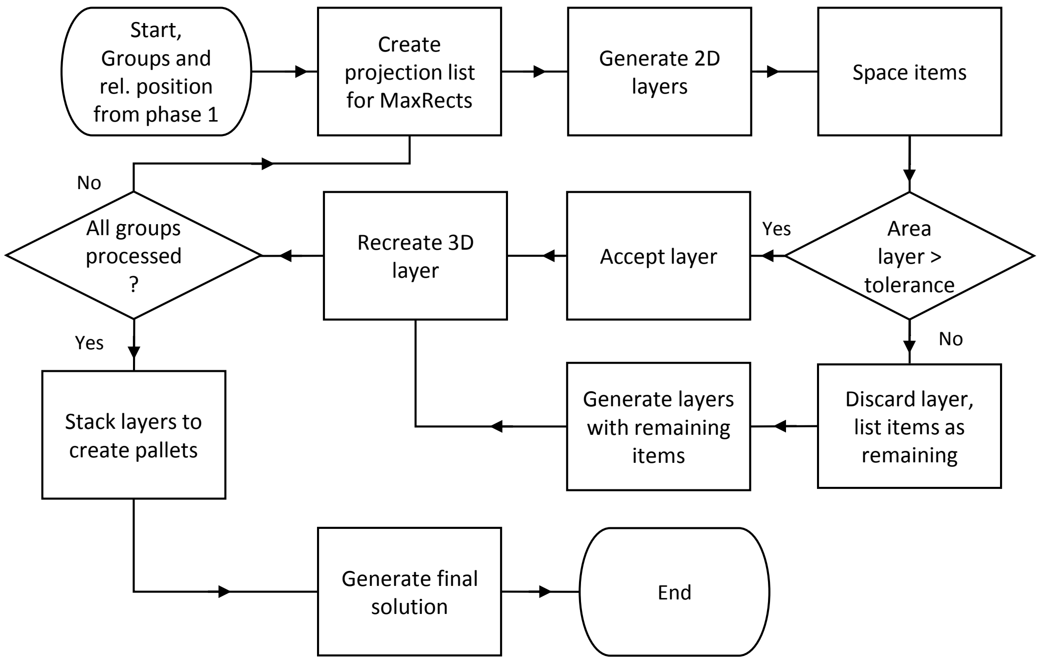
3.2.2.2 A placement heuristic

An alternative to solving [LL3] is to use a two-dimensional heuristic. The MaxRects placement heuristic is a two-dimensional bin packing algorithm that places items based on squares of free space (Jylänki, 2010). In order to adapt it to the two-phase methodology, tree-dimensional pair or trio are represented as a two-dimensional projection. For a pair, composed items a and b, the projection v_i is defined by the expression: $v_i = (\max\{w_a, w_b\}, \max\{d_a, d_b\})$; for a trio composed by items a_1, a_2, and b the expression is then $v_i = (\max\{(w_{a_1} + w_{a_2}), w_b\}, \max\{d_{a_1}, d_{a_2}, d_b\})$. For each group, a list of rectangles (projections) is the input for MaxRects. The algorithm returns the layers with the coordinates of each element. The top items are then added based on the information from the item-grouping phase. Several layers are created covering all items in that group.

The procedure is performed for each group, discarding the layers that do not satisfy the minimum coverage criteria (70%). These rejected pairs and trios are put together with the original remaining items and MaxRects is executed to create uneven layers (to be used as the pallet's top layer). After all items are covered, the three-dimensional layer are recreated from the MaxRects results, which are then stacked and the final pallet generated.

The flowchart for the two-phase method, using the MaxRects heuristic, is presented in Figure 3.5.

Figure 3.5: Flowchart for the MaxRects phase


Start, Groups and rel. position from phase 1
Create projection list for MaxRects
Generate 2D layers
Space items
No
All groups processed ?
Recreate 3D layer
Accept layer
Yes
Area layer > tolerance
No
Yes
Stack layers to create pallets
Generate layers with remaining items
Discard layer, list items as remaining
Generate final solution
End

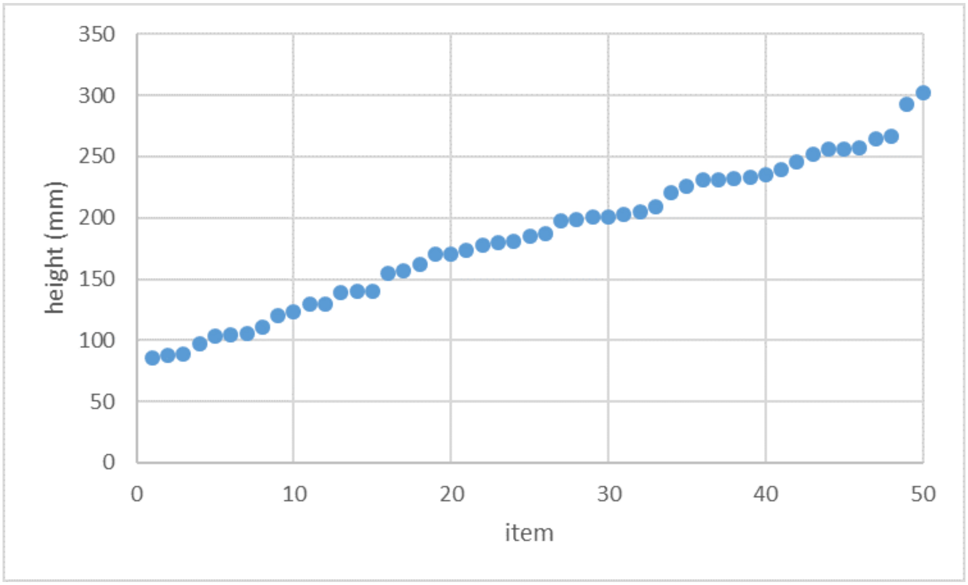
3.2.3 Support Evaluation

Support is an important requirement in mixed-case palletization. An item is said to have partial support if 3 out of 4 corners are supported by an item below it. An item has full support if all its bottom corners are in contact with items below it, or 70% of its area rests on items below it.

3.3 An Illustrative Example

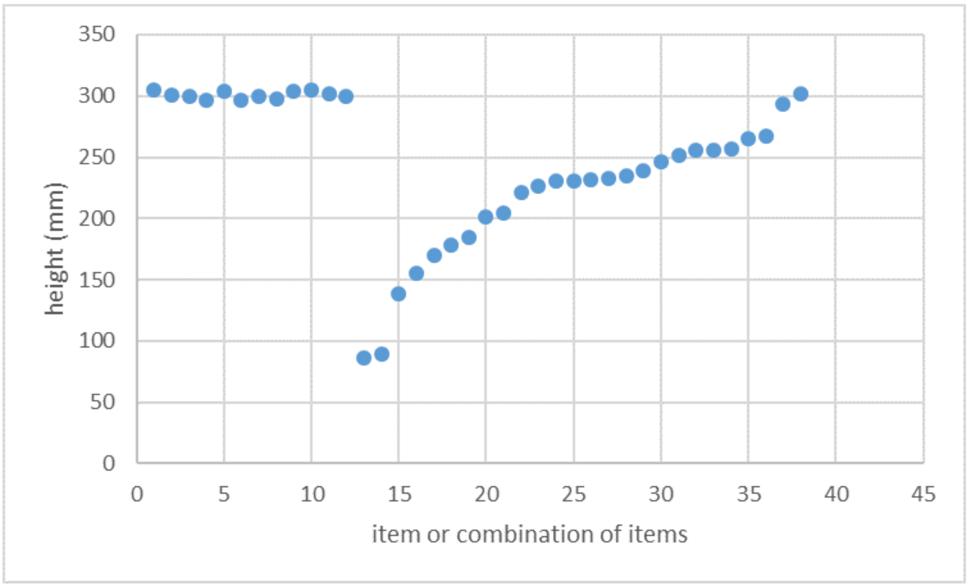
We illustrate the proposed methodology on a practical 50-item instance from an industrial partner. The heights, graphed in Figure 3.6, range from 86 to 302 mm.

Figure 3.6: Sample height profile



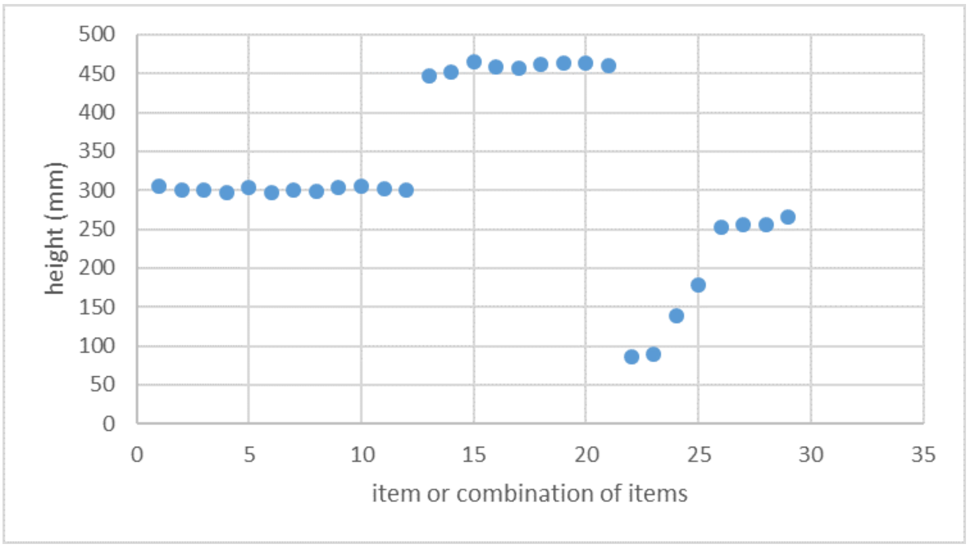
We start with a target height of 302 ± 3 mm, forming 12 pairs (no trios). With these pairings, the height distribution is as depicted in Figure 3.7.

Figure 3.7: Height profile after the first grouping step



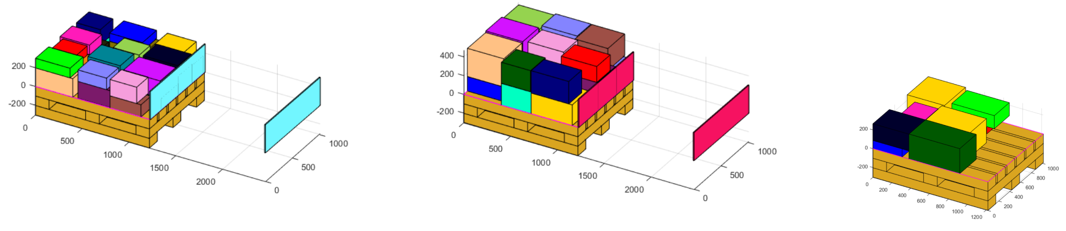
Performing a second pairing, 9 new pairs are formed within the height of 455 mm. The new height distribution is depicted in Figure 3.8.

Figure 3.8: Height profile after the second grouping step



After two rounds, the original set of 50 items is reduced to three groups, of 24, 18, and a remaining subset of 8 items.

As seen, the data analysis phase successfully modifies the height distribution from almost a straight line in Figure 3.6 to a step function with well-defined target heights. The first group has a target height of 302 mm and the second 455 mm. With this information, the optimization step focuses on the three groups of items defined. Model [LL3] is solved with a running time of 15 seconds for each group. The solutions are given in Figures 3.9a, 3.9b and 3.9c, respectively.



(a) Solution of the first group (b) Solution of the second group (c) Solution of the remaining items

Figure 3.9: Optimization results for the three groups

Figures 3.10 and 3.11 display the solution as a single layer and as a pallet, respectively. It is clear that the pallet has a good volume usage, is composed by layers with good items support, and does not have towers. Note that from an industry perspective, layers are preferred to towers as they increase volume usage and lead to stable pallets.

Figure 3.10: Optimization result as a long layer

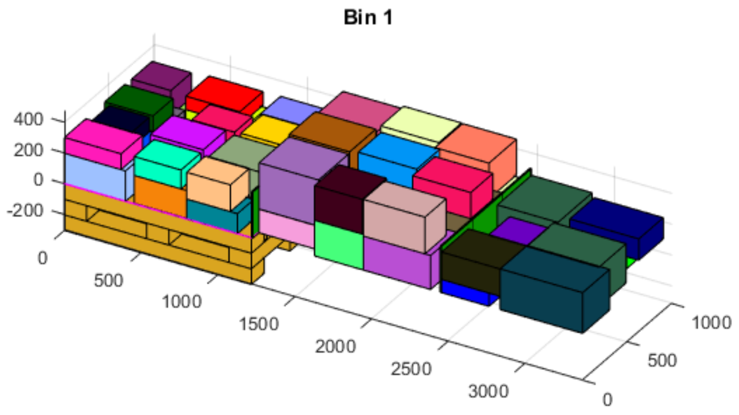
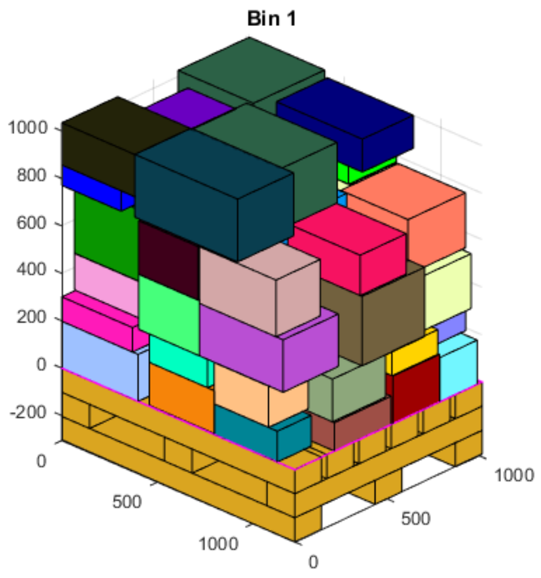


Figure 3.11: Final Pallet arrangement



Chapter 4

Numerical Testing

The proposed solution methodology is tested on industrial instances supplied by a global logistics company. All tests were performed using an Intel(R) Core(TM) i7-3517U CPU @ 1.90GHz 2.40GHz computer with 6.00 GB of Installed memory (RAM). Python 3.6 was used for data analysis and placement heuristics and MATLAB R2015b with CPLEX 12.6.2 to solve the mixed-integer program.

We start by testing on an industrial instance composed of 750 items. We use a North American standard pallet of size $W = 1219, D = 1016,$ and $H = 2200mm$.

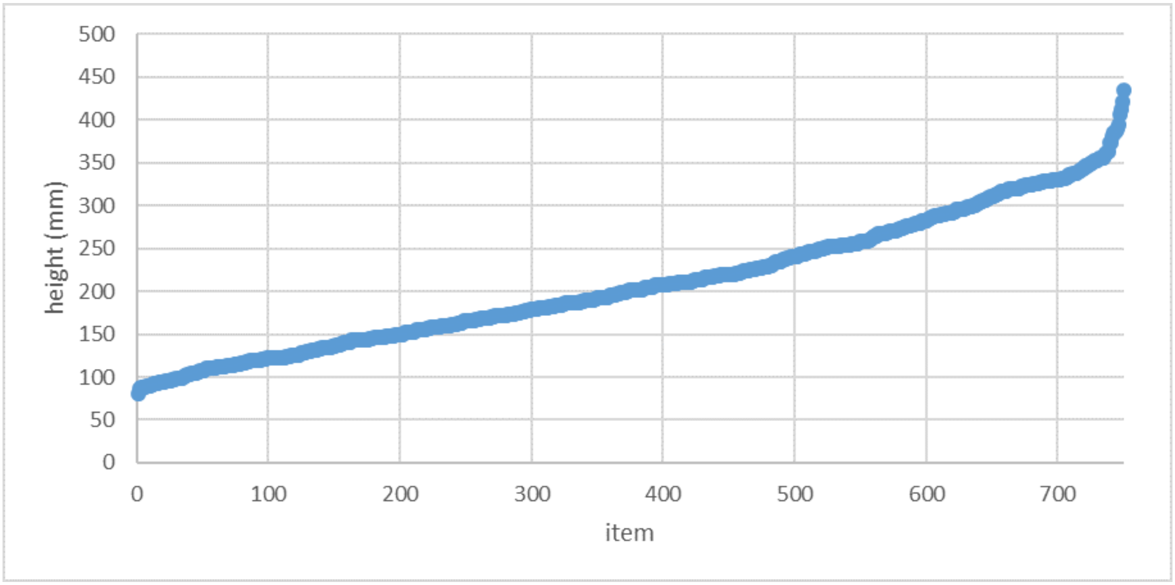
4.1 Tests on the 750-item instance

The 750-item instance is based on real orders. The width, height and depth distribution is summarized in table 4.1 and the height distribution is graphed in Figure 4.1

Table 4.1: Characteristics of the 750-item instance

	Width	Depth	Height
mean	358.1	246.9	209.2
st. dev.	76.2	53.1	75.8
min	182	164	81
max	602	403	435

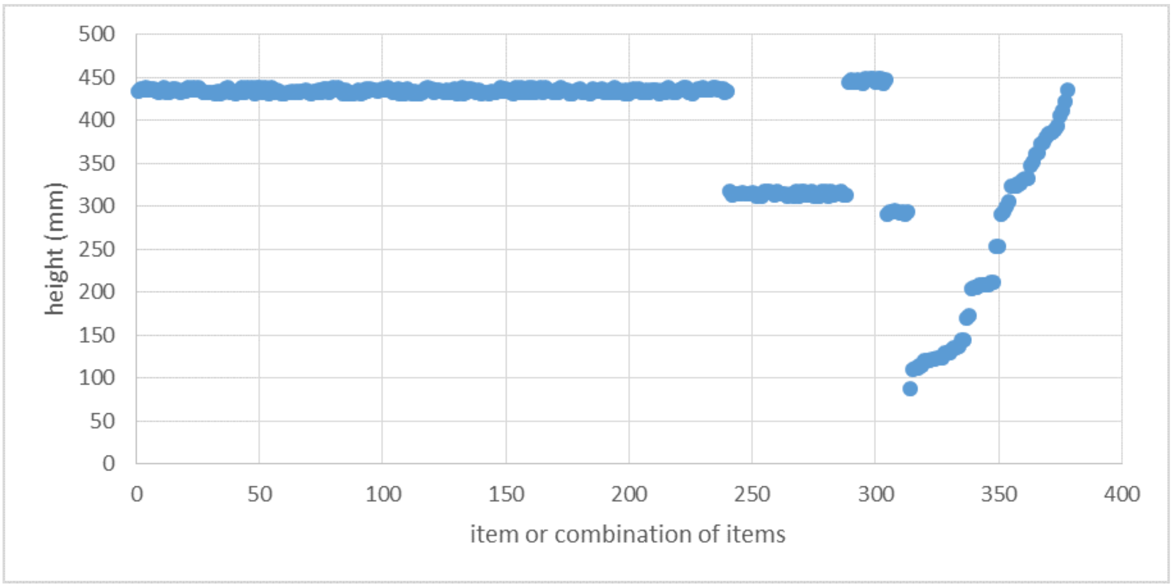
Figure 4.1: 750-item instance height profile



4.1.1 Phase 1 - Item grouping

Applying the grouping method leads to 4 groups of items and a remaining set. The first has a height around 435 mm, the height of the tallest item. It is composed of 53 trios and 187 pairs, covering 533 items. The second group has a height around 315 mm, with 3 trios and 45 pairs covering an additional 99 items. The third, contains 14 pairs and 2 trios among 34 items grouped around a height of 446 mm. The fourth and last has 19 items paired over 292 mm, with 1 trio and 9 pairs. By that, 685 items out of 750 are grouped, allowing multiple layers to be built within these subsets. The remaining 65 items are not grouped further, as the new groups would not constitute a layer. The corresponding height profile is graphed in Figure 4.2.

Figure 4.2: 750-item instance height profile after phase 1



Equipped with information about relative positioning and coordinate of items within the defined groups, optimization based on [LL3] and placement based on MaxRects will be used to construct layers.

4.1.2 Phase 2 - Item placement

MIP solution based on [LL3]

Layer generation based on [LL3] can be done for each group or for the entire set. Using the full group may provide the best results in terms of layer density, as all possible item combination are considered, it may not be practical depending on the size of the group. For instance, group 1 from the 750-item data set, has 533 items. The number of variables and constraints for a problem of this magnitude may lead to excessive computational times, way beyond the industrial standard of 2 minutes. To overcome this, 15 combinations of pairs or trios are selected at each iteration and fed to [LL3], allowing a running time of only 15 seconds. The solution is then checked if satisfies a minimum area coverage. If this resulting layer has more than 70% of area coverage, the layer is accepted and the corresponding items are removed from the total list. If not, new items are randomly selected and the optimization executed. This procedure is repeated until no more items are left in that group.

Processing of Groups 1- 4

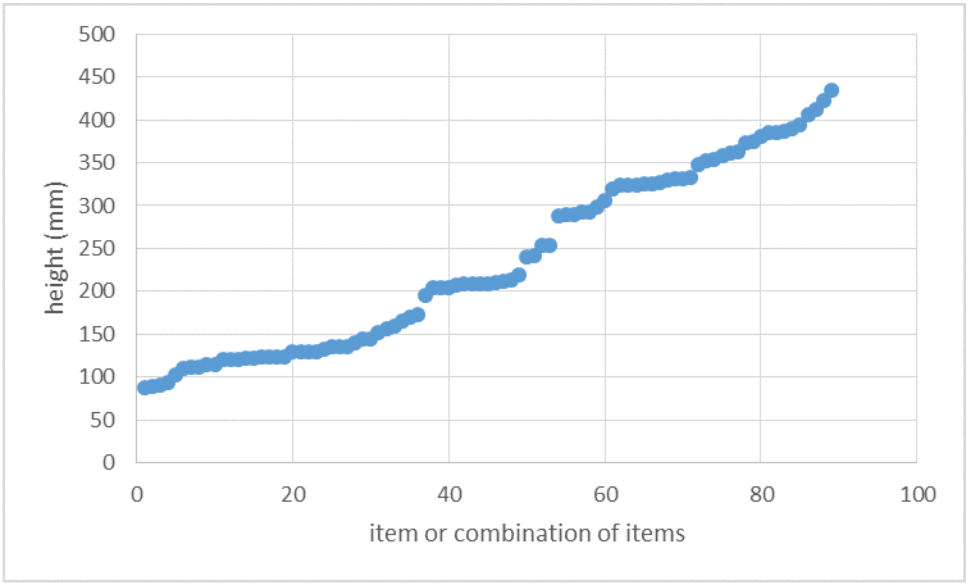
For item Group 1, it took 25 iterations to process the 533 items, leading to 25 accepted layers and one rejected (with 9 items). Group 2 had 5 iterations with 5 accepted layers covering 97 out of 99 items. For group 3, one layer with 21 items was selected and 13 items remained. Group 4 had all its 19 items placed in one accepted layer. The final result after processing these four groups is presented in table 4.2.

Table 4.2: Initial results based on [LL3]

Group	**Accepted layers**	**Covered items**	**Items remaining**	**Avg layer area %**	**min layer area %**	**max layer area %**
Group 1	25	524	9	80%	76%	86%
Group 2	5	97	2	82%	79%	84%
Group 3	1	21	13	77%	77%	77%
Group 4	1	19	0	68%	68%	68%

After processing all the groups, it was possible to generate 31 layers covering 661 items. The 24 uncovered items and the original 65 remaining set were reprocessed in phase 1 in an attempt to generate more layers. Figure 4.3 shows the distribution of the remaining items.

Figure 4.3: Height distribution of the remaining items



After applying the grouping algorithm, another 2 groups are formed, one with 24, and one with 25 items as seen in Figure 4.4:

Figure 4.4: Height distribution of the remaining items after grouping

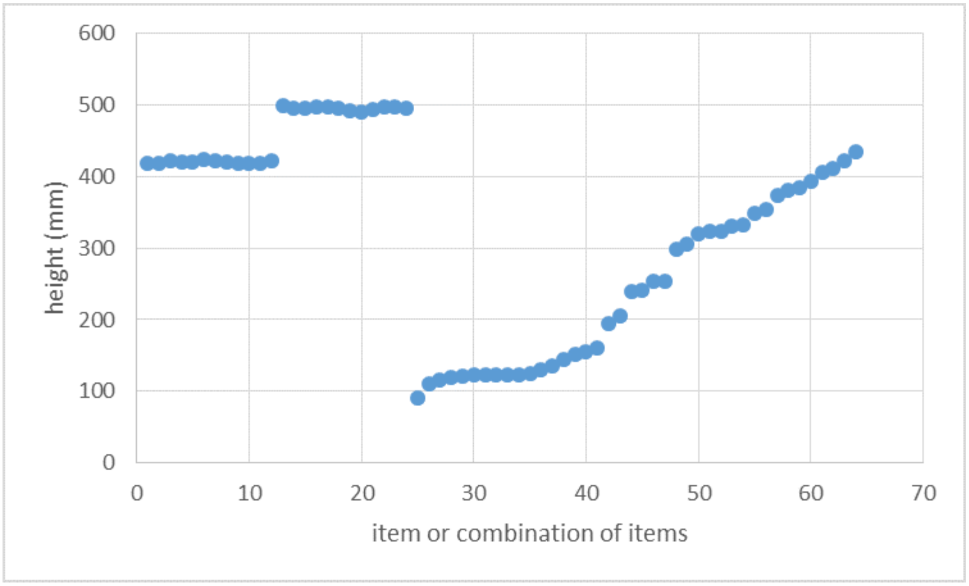


Table 4.3: Final results based on [LL3]

Group	**Accepted layers**	**Covered items**	**Avg layer area %**	**Item support %**
Group 1	25	524	80%	100%
Group 2	4	97	82%	100%
Group 3	1	21	77%	100%
Group 4	1	19	68%	100%
Group 5	1	24	73%	100%
Group 6	1	25	76%	100%
Remaining	N/A	40	N/A	N/A

With the remaining items, 5 new layer were formed by just placing items until the layer is full. This is performed using [BPP]. The items were sorted in descending height.

In total 39 layers were identified to form pallets. Then, they were stacked on top of each other based on area coverage (density). The pallets are displayed in Figures 4.5 and 4.6.

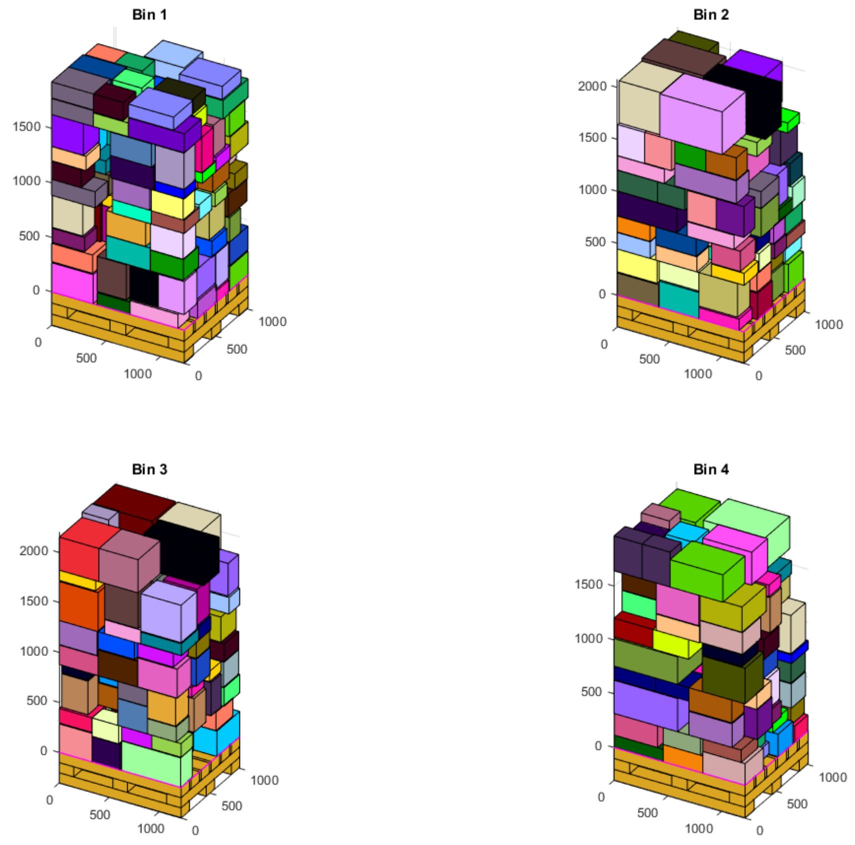


Figure 4.5: The final pallets using [LL3] - 1 to 4

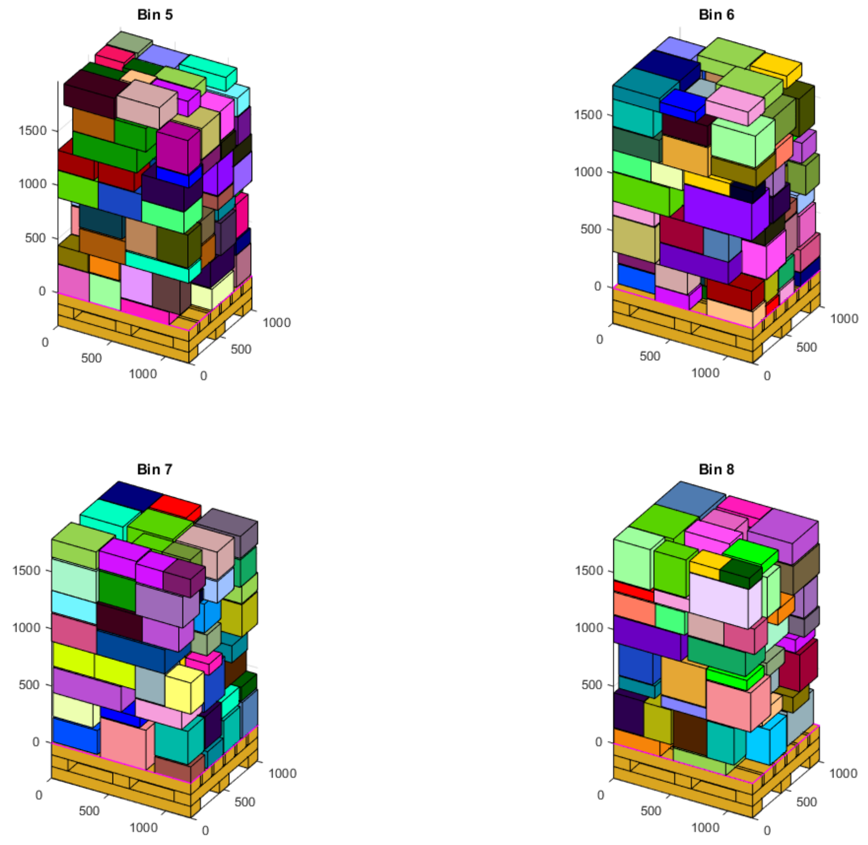


Figure 4.6: The final pallets using [LL3] - 5 to 8

We mentioned earlier that one of the benefits of using layer is item support, which is one of the features that distinguishes mixed-case palletization from 3DBPP. To assess to what extent item support is achieved, we evaluate the support of each item based on the condition that 3 out of 4 corners are supported by an item underneath (partial support). The results are displayed in table 4.4.

Table 4.4: 3-corner item support per bin - [LL3]

	Bin 1	**Bin 2**	**Bin 3**	**Bin 4**	**Bin 5**	**Bin 6**	**Bin 7**	**Bin 8**
Nb of items	111	94	99	90	101	97	80	78
Item Support (%)	86.5	83.0	80.8	80.0	80.2	81.4	82.5	87.2

The results for a more strict support criteria (where an item is fully supported if it has at least 70% of its area over other items beneath) are presented in table 4.5.

Table 4.5: Full support evaluation - [LL3]

	Average % area support	**Min % area support**	**Nb of items with support > 70%**
MIP	76.3	13.51	471 (62.8%)

With more than 62.8% of items fully supported, it is clear that the methodology does enforce support implicitly. The performance could be increased by modifying the pallet formation step.

Placement heuristic based on MaxRects

The same instance with the 750 items is processed now using the 2D heuristic MaxRects from rectpack[1] library. As MaxRects only determines the position of the base, post processing is necessary to place the top items and recreate the tree-dimensional layer.

Table 4.6 presents the groups and number of layers generated.

Table 4.6: Final results iterative method per group - MaxRects

Group	**Generated layers**	**Covered items**	**Avg layer area %**	**Item support %**
Group 1	26	533	86.5%	100%
Group 2	5	99	71.7%	100%
Group 3	3	52	61.3%	100%
Group 4	2	20	41.5%	100%
Remaining	N/A	46	N/A	N/A

The generation of layers using MaxRects is much faster that MIP, giving the flexibility to change the groups and test different arrangements to improve support. From first group, 26 layers were generated covering 533 of the 750 items. The layer with highest area coverage has 93.4% and the lowest 69.7%. Figure 4.7 displays the three most and least dense layers.

[1]https://github.com/secnot/rectpack

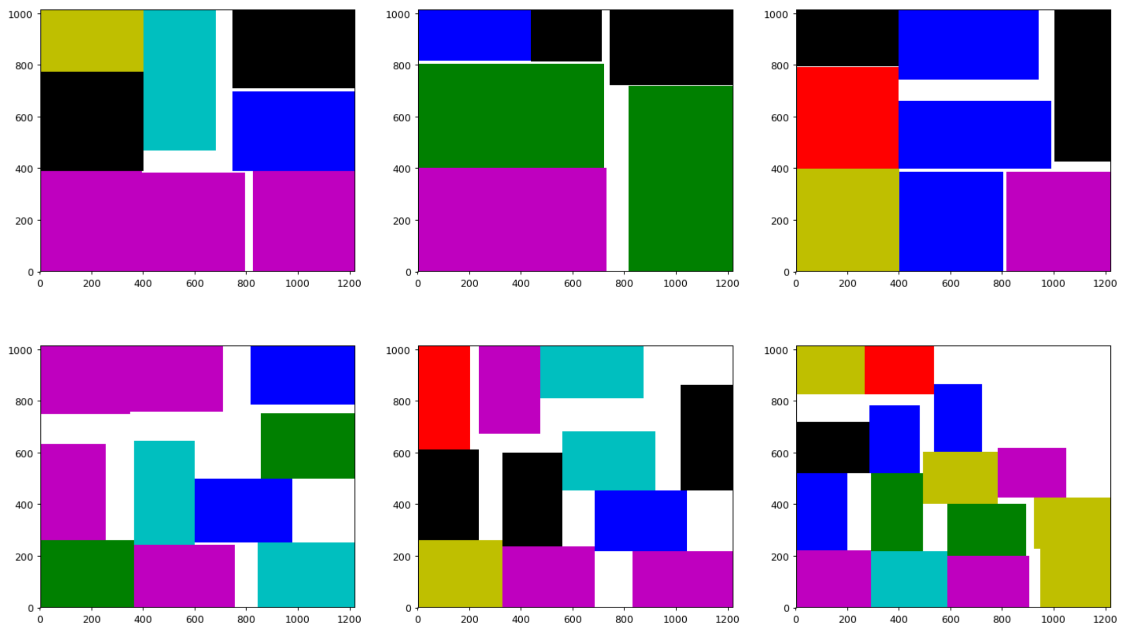


Figure 4.7: The 3 best and worst layers in terms of area coverage using MaxRects

From group 2, 5 layers were generated covering all 99 items with an average area coverage of 71.7%, a maximum of 91.7%, and a minimum of 31.8%. The layer projections are displayed in Figure 4.8.

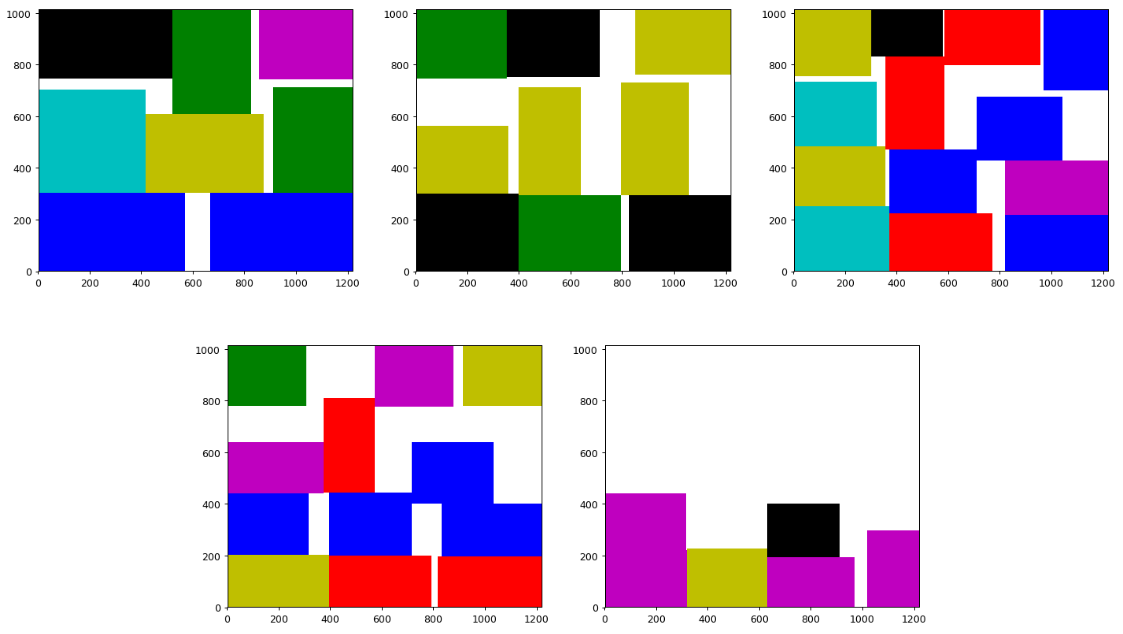


Figure 4.8: Layer coverage for group 2

Based on the 52 items in group 3, three layers were created. They have area coverage of 92.2%, 80.4%, and 16.8%.

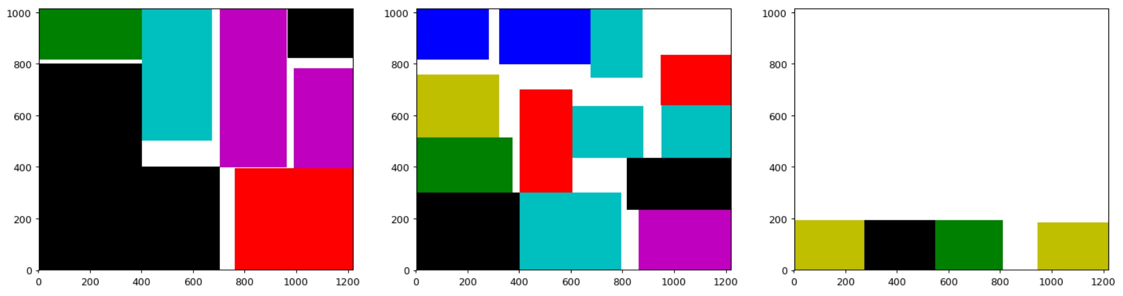


Figure 4.9: Layer coverage for group 3

Group 4 produces only two layers with its 20 items. One layer has 77.3% of area coverage and the other, with just two items, has 5.8%. Figure 4.10 presents the layers.

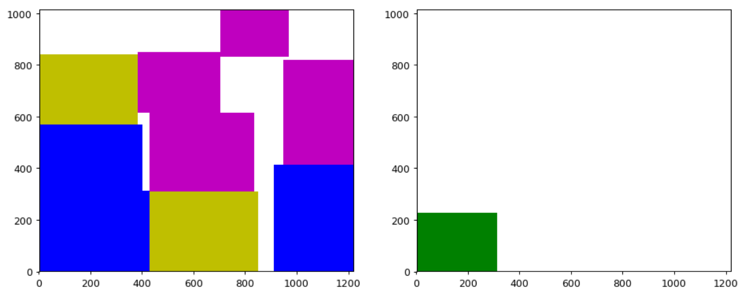


Figure 4.10: Layer coverage for group 4

After processing the four groups, 46 items remain unpaired. Out of the 35 resulting layers, 3 were discarded due to low area coverage (less than 65%). These items were put together with the unpaired items, for a total of 98, and the MaxRects heuristic was applied. This results in 8 additional layers. Next, the layers are arranged and the pallets are formed.

The final result has 40 layers covering the full 750 items. These layers were organized according to their density and are used to form 8 pallets. Figures 4.11 and 4.12 display the final pallets. Note that pairs and trios are represented as single items.

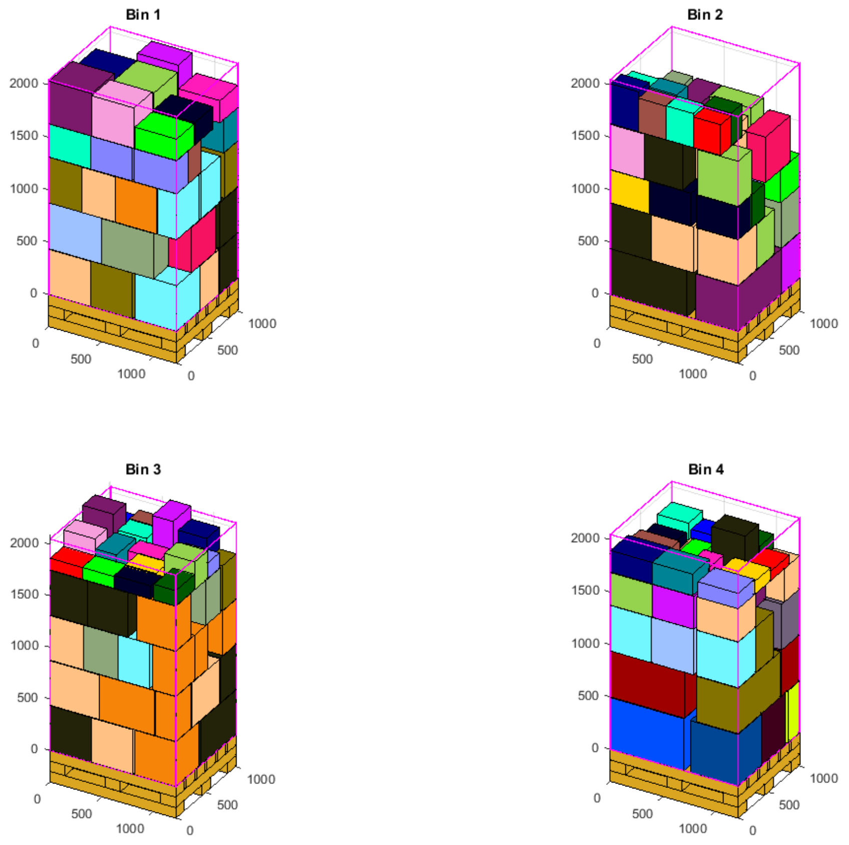


Figure 4.11: The final pallets using MaxRects - 1 to 4

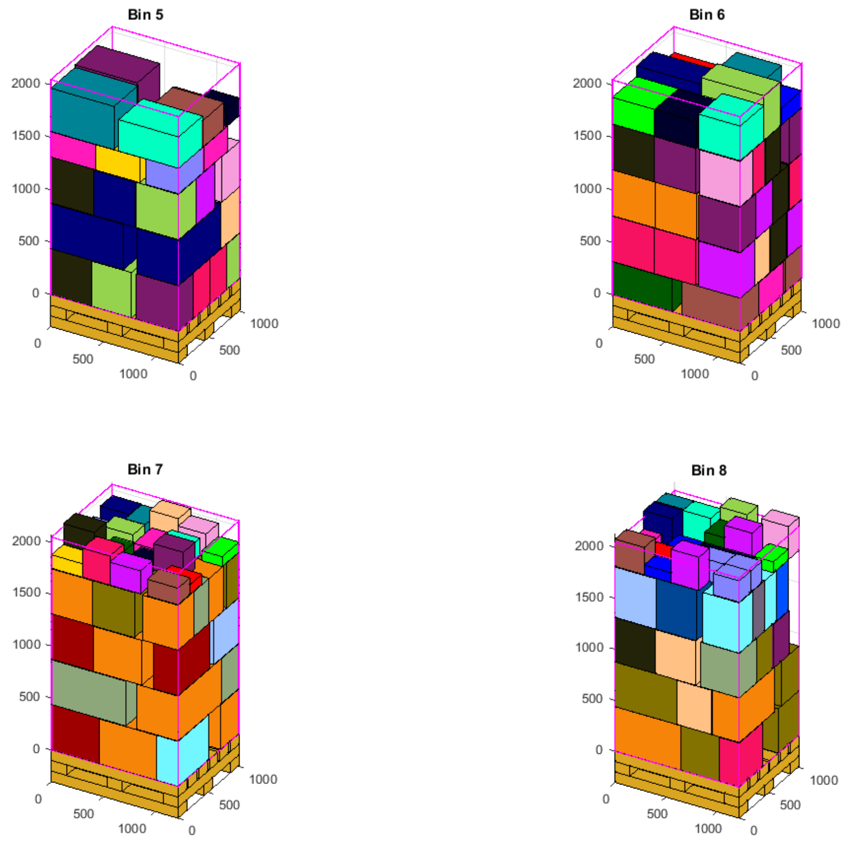


Figure 4.12: The final pallets using MaxRects - 5 to 8

Tables 4.7 and 4.8 display support results. According to the tables, around 90% of the items have a three-corner item support and 72.2% of items have full support.

Table 4.7: 3-corner item support per bin - MaxRects

	Bin 1	**Bin 2**	**Bin 3**	**Bin 4**	**Bin 5**	**Bin 6**	**Bin 7**	**Bin 8**
Nb of items	99	90	108	82	85	95	92	99
Item Support (%)	96.0	88.9	93.5	92.7	91.8	88.4	94.6	92.9

Table 4.8: Full support evaluation - MaxRects

	Average % area support	**Min % area support**	**Nb of items with support > 70%**
MaxRects	85.15	0.84	542 (72.2%)

The MaxRects solution has a higher average support than that based on [LL3]. 542 items are fully supported compared to 471; and the average area supported is 85% compared to 76%.

Chapter 5

Conclusion

In this thesis, we proposed a new data-driven solution approach for the mixed-case palletization problem that combines data analysis and optimization. Depending on the instance, we analyze the height distribution of items, combine them into pairs an trios to create distinct sets with uniform height. Items of the same height are then used to form layers, which are in turn stacked to form a three-dimensional packing. The layer formation are optimized using a mixed-integer programming model as well as fast placement heuristic.

Layers are a highly desired feature in palletization for several reasons. They tend to maximize volume usage, provide adequate support for items, lead to stable pallets, and are easy to build in automated warehouses. From a modeling perspective, building a three-dimensional packing based on layers reduces the problem to a two-dimensional bin packing problem.

Tests on a large industrial dataset composed of 750 items reveal that the approach is successful in reducing the set of items into 4 groups, each with a common height, as a well a fifth small group of remaining items. The uniform height groups are used to form layers by passing the relative positioning and the coordinate information of the grouped items into a three-dimensional mixed integer programming formulation. A fast placement 2D heuristic was also used. The resulting solution leads to eight well-built pallets where more than 85% of the items have at least 3 corner support.

Possible future research based on the two-phase method can focus on the incorporation of other practical constraints, such as guaranteed item support and load bearing. These can be handled at the layer formation or pallet building phases, respectively.

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